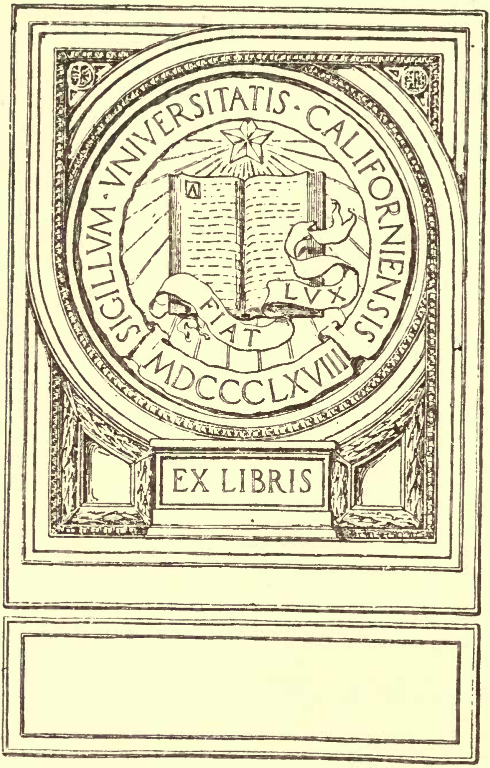
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ELEMENTARY TREATISE

ON THE

WAVE-THEORY OF LIGHT.

BY

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TO THE SECOND EDITION.

The former edition of this little work was given to the public in the shape of Lectures, as delivered, in compliance with the regulations of the Chair which the author then occupied, and without any expectation that its use would extend much beyond the circle of his immediate hearers. It has, however, found its way elsewhere; and the author has been urged, by some of his fellow-labourers in other Universities, to reprint it.

With this request he could not but comply; and he trusts that the delay in acceding to it may be excused to those who made it, by the desire of the author to render the work more deserving of their favourable estimation.

In the present edition some account is given of the more important discoveries in Physical Optics, which have been made since the publication of the former. In preparing these additions, the author has derived much aid from the *Rêpertoire d'Optique Moderne* of the Abbè Moigno,—a work which contains a full analysis, and critical discussion, of most of the recent researches in Optics. He has also to acknowledge his obligations to M. Moigno, for the favourable introduction of the former edition of the present work, in the pages of the "Repertoire," to the notice of Continental readers.

The form of Lectures has been abandoned; but the author fears that the style still retains more of the traces of the lecture-room than is consistent with a formal scientific treatise. His only aim has been to present, to those who were conversant with the elements of Mathematics, a clear and connected view of his attractive subject; and he has been compelled, by this limitation, to confine himself in many cases (as in all that relates to the Dynamics of Light) to a general account of methods, and of their results. Those who desire a more exact acquaintance with the science will, of course, study it in Sir John Herschel's *Essay on Light*, and in Mr. Airy's Tract on the *Undulatory Theory of Optics*.

Trinity College, Dublin,
March 18*th*, 1857.

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The important experiment, the principle of which is described in Art. (37), was first performed by M. Foucault. M. Fizeau was occupied with the problem at the same time, although independently; and the researches of the two experimenters were communicated to the French Academy on the same day.

ELEMENTS

OF

THE WAVE-THEORY OF LIGHT.

CHAPTER I.

PROPAGATION OF LIGHT.

(1) NATURAL bodies may be divided into two classes in relation to Light. Some possess, *in themselves*, the power of exciting the sense of vision, and of producing the sensation of light; while others are devoid of that property. Bodies of the former class are said to be *luminous;* those of the latter, *non-luminous*. The Sun and the fixed stars are all luminous bodies; terrestrial bodies are luminous, in the states of *incandescence*, *combustion*, or *phosphorescence*.

Non-luminous bodies acquire the power of exciting the sensation of light in the presence of a luminous body. Thus, a lamp or candle illuminates all the objects in a room, and renders them visible; and the light of the Sun illuminates the Earth and the planets. This property of bodies is due to their capacity of reflecting light, and belongs to them in different degrees.

(2) The foregoing distinction of bodies, obvious as it seems, was not fully comprehended by the ancients. According to

them, vision was performed by something which emanated *from the eye to the object;* and the sense of Sight was explained by the analogy of that of Touch. In this view, then, the sensation was represented as independent of the nature of the body seen; and all objects should be visible, whether in the presence of a luminous body or not. This strange hypothesis held its ground for many centuries. The Arabian astronomer, Alhazen, who lived in the latter part of the eleventh century, seems to have been the first to refute it, and to prove that the rays which constituted vision came from the object to the eye.

(3) The light of a luminous body emanates from it *in all directions.* Thus, the light of a lamp or candle is seen in all parts of a room, if nothing intervenes to intercept it; and the light of the Sun illuminates the Earth, the Planets, and their satellites, in whatever position they may be placed respecting it.

Each physical point of a luminous body is an independent source of light, and is called a *luminous point.*

(4) Non-luminous bodies are distinguished into two classes, according as they allow the light which falls upon them to pass freely through their substance, or intercept it. Bodies of the former kind are said to be *transparent;* those of the latter, *opaque.*

There are no bodies in nature actually corresponding to these extremes. The most transparent bodies, as *air* and *water*, intercept a sensible quantity of light, when of sufficient thickness; and, on the other hand, the most opaque bodies, such as the *metals*, allow a portion of light to pass through their substance, when reduced to laminæ of exceeding tenuity.

(5) In the same homogeneous medium, light is propagated *in right lines*, whether it emanates directly from luminous bodies, or is reflected from such as are non-luminous.

This is proved by the fact that when an opaque body is interposed in the right line connecting the eye and the luminous source, the light of the latter is intercepted, and it ceases to be visible. The same thing is proved also by the *shadows* of bodies, which, when received upon plane surfaces perpendicular to the path of the light, are observed to be similar to the section of the body which produces them.

This property of light was recognised by the ancients; and by means of it the few optical laws which were known to them became capable of mathematical expression and reasoning. Any one of these lines, proceeding from a luminous point, is called in optics *a ray.*

(6) In a perfectly transparent medium, the intensity of the light proceeding from a luminous point varies *inversely as the square of the distance.*

This is easily proved, if light be supposed to be a material emanation of any kind. For the intensity of the light, received upon any spherical surface whose centre is the luminous point, is as the quantity of the light directly, and inversely as the space over which it is diffused. But none of the light being lost, the quantity of light received upon any spherical surface is the same as that emitted, and is therefore constant; and the space of diffusion, or the area of the spherical surface, is as the square of its radius. Hence the intensity of the light is inversely as the square of the radius, i. e. inversely as the square of the distance.

Let the light be supposed to emanate from the points of an uniformly luminous surface, which we shall suppose to be a small portion of a sphere. Then the quantity of light emitted is proportional to the quantity emitted by a single point, and the number of points (or area) conjointly. Hence if a denote the area of the luminous surface, and i the quantity emitted from a single point, which is a measure of the *absolute brightness*, the intensity of the illumination, at any distance d, is

$$\frac{ai}{d^2}.$$

(7) A plane surface, whose dimensions are small in comparison with the distance, and which is perpendicular to the incident light, may, without sensible error, be considered as a portion of a spherical surface concentric with the luminary. The intensity of the illumination, therefore, or the quantity of light received upon a given portion of such a plane, is expressed by the formula of the preceding Article.

When the surface is *inclined* to the incident light, the quantity of the light received by any given portion is diminished in the ratio of unity to the sine of the angle of inclination. The intensity of the illumination is, therefore, diminished in the same proportion, and is expressed by the formula

$$\frac{ai \sin \theta}{d^2},$$

θ being the inclination of the surface to the incident light.

(8) Experience proves that the eye is incapable of comparing directly two lights, so as to determine their relative intensity. But, although unable to estimate *degrees*, the eye can detect *differences* of intensity with much precision; and with this power it is enabled (by the help of the principles just established) to compare the intensities of two lights *indirectly*.

Let two portions of the same paper (or any similar reflecting surface) be so disposed, that one of them shall be illuminated by one of the lights to be compared, and the other by the other, the light being incident upon each at the same angle. Then let the distance of one of the lights be altered, until there is no longer any appreciable difference in the intensities of the illuminated portions. The illuminating powers of the two lights will then be as the squares of their respective distances; and their absolute brightnesses as the illuminating powers directly, and as their luminous surfaces inversely. For, if i and i' denote the absolute brightnesses of the two lights, a and a' the areas of the luminous surfaces, and d and d' their distances from the paper, the intensities of illumination are $\frac{ai \sin \theta}{d^2}$ and $\frac{a'i' \sin \theta}{d'^2}$, respectively; and these being rendered equal in the experiment, we have

$$\frac{ai}{a'i'} = \frac{d^2}{d'^2}.$$

The following simple and convenient mode of practising this method was suggested by Count Rumford. A small opaque cylinder is interposed between the lights to be compared and a screen; in this case it is obvious that each of the lights will cast a shadow, which is illuminated by the other light, while the remainder of the screen is illuminated by both lights conjointly. If, then, one of the lights be moved, until the shadows appear of equal intensity, their illuminations are equal, and, therefore, the illuminating powers of the two lights are to one another as the squares of their distances from the screen.

(9) Light is propagated with a finite velocity.

This important discovery was made in the year 1676, by the Danish astronomer, Olaus Roemer. Roemer observed that when Jupiter was in opposition, and therefore nearest to the Earth, the eclipses happened *earlier* than they should according to the astronomical tables; while, when Jupiter was in conjunction, and therefore farthest, they happened *later*. He thence inferred that light was propagated with a finite velocity, and that the difference between the computed and observed times was due to the change of distance. This difference is found to be $8^m 13^s$; and accordingly the velocity of light is such, that it traverses 192,500 miles in a second of time.

(10) The velocity of light, combined with that of the Earth in its orbit, was afterwards applied by Bradley to explain the phenomenon of the *aberration of the fixed stars*. From the theory of aberration so explained, it followed that the velocity of the light of the fixed stars is to the velocity of the Earth in its orbit, as radius to the sine of the maximum aberration. This latter quantity—the *constant* of aberration, as it is called —is now found to be $20''\cdot36$; and the Earth's velocity being known, the velocity of the light of the fixed stars is deduced. The value so obtained is 191,500 miles in a second, which differs from that inferred from the eclipses of Jupiter's satellites, by only the $\frac{1}{200}$th part of the whole.

From this it follows, that the direct light of the fixed stars, and the reflected light of the satellites, travel with the same velocity.

(11) The velocity of light, emanating from a terrestrial source, has been recently measured by M. Fizeau, by direct experiment. The first idea of this experiment was communicated to M. Arago, by the Abbè Laborde, a few years before; its principle will be understood from the following description.

Let the light of a lamp be reflected nearly perpendicularly by a mirror placed at a considerable distance; let a toothed wheel, the breadth of whose teeth is equal to that of the interval between them, be interposed near the luminous source; and let the mirror be so adjusted that the light passing through one of these intervals is reflected to that diametrically opposite. If the eye be placed behind the latter interval, the wheel being at rest, it will perceive the reflected ray, which has traversed a space equal to double the distance of the mirror from the wheel. But if, on the other hand, the wheel be made to revolve rapidly, its velocity may be such that the light transmitted through the opening at one extremity of the diameter shall not pass through the opposite aperture on its return, but be intercepted by the adjacent tooth; and it will be continually invisible to the eye, so long as the wheel revolves with the same velocity. If the velocity of the wheel be doubled, the light will be transmitted, on its return, through the succeeding opening, and will reappear to the eye. If the velocity be trebled, the light will be intercepted by the next tooth, and there will be a second eclipse; and so on.

It is plain that if the velocity of the wheel, corresponding to the 1st, 2nd, 3rd, or m^{th} eclipse, be known, the velocity of the light may be calculated. Thus, if the wheel makes n revolutions in a second, and has p teeth, the time of passage of one tooth across the same point of space $= \frac{1}{np}$ of a second. Consequently, the first eclipse will correspond to $\frac{1}{2np}$ of a second. But in the same time the light has twice traversed the distance between the wheel and the mirror. If, therefore, that distance be denoted by a, the velocity of light will be

$$V = 2a \times 2np.$$

If n be the number of revolutions in a second corresponding to the m^{th} eclipse, the velocity of light will be given by the formula,

$$V = 2a \times \frac{2np}{2m-1}.$$

The apparatus devised by M. Fizeau for this experiment is ingenious and effective. It consists of two telescopes, directed towards each other, and so adjusted that an image of the object-glass of each is formed in the focus of the other. The light from the source is introduced laterally into the first telescope, through an aperture near the eye-piece. It is then received on a transparent plate, placed between the focus and the eye-glass, and inclined at an angle of 45° to the axis of the instrument. It is thus reflected along the axis of the first telescope, having passed through one of the apertures in the revolving wheel, and is received perpendicularly on the mirror in the focus of the second. It then returns by the same route, and is received by the eye at the eye-glass of the first telescope. The distance of the two telescopes in M. Fizeau's experiments was 9440 yards. The revolving disc had 720 teeth, and was connected with a counting apparatus which measured its velocity of rotation. The first eclipse took place when the wheel made 12·6 revolutions in a second. With double the velocity, the light was again visible; with treble the velocity, there was a second eclipse, and so on. The mean result of the experiments gave 196,000 miles, nearly, for the velocity of light.

(12) Let us now proceed to the physical explanation of the foregoing facts.

We have seen that light travels from one point of space to another *in time*, and with a prodigious velocity. Now, there are two distinct and intelligible ways of conceiving such a propagated movement. Either it is the *same individual body* which is found in different times in distant parts of space;—or there are a *multitude* of moving bodies, occupying the entire interval, each of which *vibrates* continually within certain limits, while the vibratory motion itself is communicated in succession from one to another, and so advances uniformly. These two modes of propagated movement may be distinguished by the names of the motion of *translation* and the motion of *vibration*. The former is more familiar to our thoughts, and is that which we observe, when with the eye we follow the path of a projectile in the air; or about which we reason, when we determine the course of a planet in its orbit. Motions of the latter kind, too, are everywhere taking place around us. When the surface of stagnant water is agitated by any external cause, the particles of the fluid next the origin of the disturbance are set vibrating up and down, and this vibratory motion is communicated to the adjacent particles, and from them onwards, to the boundaries of the fluid surface. All the particles which are elevated at the same instant constitute what is called a *wave;* and that this wave does not consist of the *same* particles in two successive instants may be seen in the movements of any floating body, which will be observed to rise and fall as it is reached and passed by the wave, but not to advance, as it must necessarily do if the particles of the fluid on which it rested had a progressive motion. The phenomena of sound afford another well-known instance of the motion of vibration. The vibratory motion is communicated from the sounding body to the ear, through all the intervening particles of the air, though each of the aerial particles moves back and forwards through a very narrow space.

Each of these modes of propagated motion has been applied to explain the phenomena of light; and hence the two rival theories—the *theory of emission* and the *wave-theory.* In the former the luminous body is supposed to send forth, or *emit*, continually, material particles of extreme minuteness, in all directions. In the latter, the same body is supposed to *excite the vibrations* of an *elastic ether*, which are communicated from particle to particle, to its remotest bounds. This ethereal medium is supposed to pervade all space, and to be of such extreme tenuity as to afford no appreciable resistance to the motions of the planets.

Such are the two systems, some traces of which may be found even in the recorded opinions of the ancients. It is only within a period comparatively recent, however, that either of them has been stated formally, or supported by any show of reasoning. Descartes put forward, very distinctly, the hypothesis that light consisted of small particles emitted by the luminous body, and he even endeavoured to explain the laws of reflexion and refraction on that supposition. But as Newton was the first to deduce the mathematical consequences of the theory of emission, he has been usually regarded as its author. The wave-theory was propounded by Hooke, in the year 1664; and was developed into several of its consequences, a few years later, by Huygens. Let us examine each of these theories by the only test to which a physical theory can be subjected,—namely, the accordance of its consequences with phenomena.

(13) The fundamental assumption of the theory of emission—the hypothesis that light consists of *bodies* moving with an immense velocity—would appear to be easily submitted to the test of experiment. If the weight of a molecule of light amounted to but *one grain*, its momentum would equal that of a cannon-ball, 150 pounds in weight, moving with the velocity of 1000 feet in a second. The weight of a single molecule may be assumed to be many millions of times less than what has been here supposed; but, on the other hand, many millions of such molecules may be made to act together, by concentrating them in the foci of lenses or mirrors, and the effects of their impulse might be expected in this manner to be rendered evident.

This apparently easy test of the materiality of light was appealed to by many experimental philosophers of the last century, and with various results. The effects observed have been traced, with much probability, to extraneous causes (such as aerial currents produced by unequal temperature); and it is now universally conceded that no sensible effect of the *impulse* of light has been ever perceived. The experiments of Mr. Bennet seem to be decisive on this point. In these experiments a slender straw was suspended horizontally by means of a single fibre of the spider's thread. To one end of this delicately suspended lever was attached a small piece of white paper, and the whole was inclosed in a glass vessel, from which the air was withdrawn by the air-pump. The sun's rays were then concentrated by means of a large lens, and suffered to fall upon the paper, but without any perceptible effect.

(14) But the actual velocity of light is not the only difficulty which the theory of emission has to encounter at the very outset. It has been further proved that this velocity is *one and the same*, whether the light is directly emitted from the sun or a fixed star, or reflected from a planet or its satellite; that it is, in short, independent of the luminous source, as well as of the subsequent modifications which it undergoes in the celestial spaces. It is not easy to account for these facts in the theory of emission. The emissive force, required to produce the known velocity, is calculated to be more than a million of million times greater than the force of gravity at the earth's surface; and it can hardly be supposed that this prodigious force is *the same* for all the various and independent bodies of the universe, and that it acts *equally* on all the particles of light, so as to generate in them the same velocity. Yet even this assumption will not avail. Laplace has shown, that if the diameter of a fixed star were 250 times as great as that of our sun, its density being the same, its attraction would be sufficient to destroy the whole momentum of the emitted molecules, and the star would be invisible at great distances. With a smaller mass there will be a proportionate retardation, so that the final velocities will be different, whatever be the initial ones. The suggestion of M. Arago seems to offer the only way of escaping the force of this objection. It may be supposed that the molecules of light are originally projected with different velocities, but that among these velocities there is but one which is adapted to our organs of vision, and which produces the sensation of light.

The uniform velocity of light is, on the other hand, an immediate consequence of the principles of the wave-theory. It follows from these principles, that the velocity with which vibratory movement is propagated in an elastic medium depends in no degree on the exciting cause, but varies solely with the *elasticity* of the medium and its *density*. If these then be supposed to be *uniform* in the vast spaces which intervene between the material bodies of the universe, the velocity will be the same, whatever be the luminous origin.

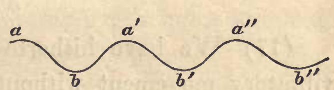
(15) The *rectilinear motion* of light has long been urged in favour of the theory of emission, and against the wave-theory. If light consists in the undulations of an ethereal medium (it has been said), as sound consists in the undulations of the air, it should be propagated in all directions from every new centre, and so bend round interposed obstacles. Thus luminous objects should be visible, even when an obstacle is between them and the eye, just as sounding bodies are heard, though a dense body may be interposed between them and the ear, and *shadows* could not exist.

To this objection, which was that chiefly urged by Newton himself, it might be enough to reply, that though vibratory motion in an elastic medium is propagated in *all* directions from every new centre, yet there is no reason to conclude that it is propagated with the *same intensity* in every direction, however inclined to that of the original wave. In fact, analogy furnishes grounds for an opposite conclusion; for there are a multitude of facts which prove that sound is *not* propagated with the same intensity in all directions, however inclined to the direction of the original motion. Now, if there be *any* difference between the intensity of the direct and lateral propagation, this difference may be ever so great; i. e. the ethereal medium may be so constituted that the intensity of the laterally-propagated vibration shall be insensible.

But the solution of the difficulty rests upon more solid grounds than analogy. A more minute examination of the nature and laws of vibratory motion has, in fact, shown this to be the case, as respects the luminiferous waves. It has been proved, that whatever be the intensity of the *partial* waves of the ether, which are propagated laterally round any interposed obstacle, the *total light* resulting from their joint action must *degrade* rapidly; and the luminous fringes which have been observed within the shadows of bodies do, in fact, represent the intensities resulting from these lateral waves, when submitted to the most rigid mathematical calculation.

(16) Let us now proceed to consider, somewhat more minutely, the nature of a wave and its mode of propagation.

Let us conceive, then, a cord stretched in a horizontal position, one end being attached to a fixed point, and the other held in the hand. If the latter extremity be agitated, by the motion of the hand up and down, a series of waves will be propagated along the cord, each of which will advance uniformly. Here it is evident that each particle of the cord has merely a vibratory motion in a vertical direction. But as this vibratory motion is communicated from each particle to the next, along the whole length of the cord,—it will follow that some of the particles reach their highest position, when others are in the lowest; while other particles, intermediate to these, are neither in their highest nor their lowest position, but in some intermediate state of their vibration. Thus, while each particle moves only to and fro vertically, an undulation or wave is propagated horizontally along the string; and there will be a succession of similar undulations as long as the original disturbance continues. The particles a, a', a'', or the particles b, b', b'', &c., are said to be in similar *phases* of vibration.



The *wave*, or undulation, consists of all the particles between two which are in similar phases,—as between a and a', or between b and b'; and the *length of a wave* is the distance between them, estimated in the direction in which the motion is propagated. It is evident from this description that a wave, or undulation, comprises particles in every phase of their vibration.

Now, instead of a single string, let us suppose an infinite number, all diverging from the same centre; and let us suppose that they are each made to undulate by a disturbing action at that centre, acting in a similar manner, and in the same degree, on all. It is obvious, then, that an undulation will be propagated along all the strings; and that these undulations will be equal in magnitude, and will be propagated with the same velocity, provided the strings be equal in tension, elasticity, and other respects. In this case, then, similar waves will be propagated to points equally distant from the origin of disturbance in the same time; and all the points which are in a similar phase of vibration will be situated on the surface of a *sphere*, of which that origin is the centre.

In the place of the actual strings we have been considering, let us imagine rows of ethereal particles connected by their mutual actions, and all that has been said will apply to the propagation of light, the luminous body being the source of disturbance. The length of the wave is the distance, estimated in any direction from the centre, of two particles which are in similar phases of vibration; and it is therefore the space through which the vibratory movement is propagated in the time of a single vibration. Accordingly, if λ denote the length of the wave, τ the time of vibration, and v the velocity of wave-propagation,

$$\lambda = \mathrm{v}\tau.$$

(17) We have hitherto considered the propagation of vibratory movement without reference to any diversity in its nature. It is obvious, however, that vibrations may differ from one another in two particulars,—namely, in the *space of vibration*, and in the *time*. In the aerial pulses the amplitude of the vibration determines the *loudness* of the sound; and the frequency of the pulses, or the time of vibration, determines its *note*. In like manner, the amplitude of the *ethereal* vibrations determines the *intensity of the light;* and their frequency, or the period of vibration, determines the *colour*.

Thus, two lights may differ from one another in intensity and colour, the former depending (according to the wave-theory) on the space of vibration, and the latter on the time.

But though the intensity of the light is obviously dependent on the amplitude of the vibration, yet it does not appear, *à priori*, by what power of the amplitude it is to be represented. In fact, we must *define* what we mean by a *double*, *triple*, &c. quantity of light, before we can know how that quantity is to be mathematically measured. If then we say that a *double light* is the *sum* of the lights produced by *two* luminous origins of equal intensity, placed close together, it is easy to prove that the quantity of light, in general, is measured by the *square* of the amplitude of the vibration. From this it follows that the intensity of the light diverging from any luminous origin must decrease inversely as the square of the distance; for, from the laws of wave propagation it appears that the space of vibration diminishes in the inverse simple ratio of the distance. Thus the known law of the variation of the intensity of light is deduced from the principles of undulatory propagation.

(18) The *colour* of the light (it has been said) depends on the number of impulses which the nerves of the eye receive, in a given time, from the vibrating particles of the ether,—the sensation of *violet* being produced by the most frequent vibrations, and that of *red* by the least frequent. But the number of vibrations performed in a given time varies inversely as the time of a single vibration; the colour of the light, therefore, varies with the time of vibration, or with the *length of the wave* in a given medium. By experiments, which will be described hereafter, it has been found that the length of a wave, in air, corresponding to the *extreme red* of the spectrum, is 266 ten-millionths of an inch, and that corresponding to the *extreme violet* 167 ten-millionths. The length of the wave

corresponding to the ray of mean refrangibility is nearly 200 ten-millionths, or $\frac{1}{50000}$th of an inch.

It appears, then, that the sensibility of the eye is confined within much narrower limits than that of the ear; the ratio of the times of the extreme vibrations which affect the eye being only that of 1·58 to 1, which is less than the ratio of the times of vibration of a fundamental note and its octave. There is no reason for supposing, however, that the vibrations themselves are confined within these limits. In fact, we know that there are *invisible* rays *beyond* the two extremities of the spectrum, whose periods of vibration (and lengths of wave) must fall without the limits now stated to belong to the visible rays.

(19) The aberration of light, it has been said, results from the movement of the Earth in its orbit, combined with the movement of light. Nothing can be simpler than its explanation in the theory of emission. In fact, we have only to combine the two coexisting motions according to the known mechanical law, and the apparent direction of the star is that of their resultant. The angle between this direction, and that of the principal component, is called the *aberration.*

In order to explain this phenomenon, in accordance with the principles of the wave-theory, it seemed necessary to suppose that the ether which encompasses the Earth does *not* participate in its motion, so that the ethereal current produced by their relative motion pervades the solid mass of the Earth "as freely," to use the words of Young, "as the wind passes through a grove of trees." Fresnel has developed this hypothesis, and has shown that it suffices to explain other phenomena also, in which the Earth's motion is concerned. Professor Stokes has lately shown that the same results may be deduced from a more plausible hypothesis relative to the mutual dependence of the ether and the Earth.

CHAPTER II.

REFLEXION AND REFRACTION.

(20) When light meets the surface of a new medium, a portion of it is always turned back, or *reflected*.

The reflexion of light is twofold. Thus, when a beam of solar light is admitted into a darkened chamber through an aperture in the window, and is allowed to fall upon a metallic mirror, a reflected beam is seen pursuing a determinate direction after leaving the mirror; and if the eye be placed in this direction, it will perceive a brilliant image of the sun. This beam is said to be *regularly reflected*, and its intensity *increases* with the polish of the mirror. But it is observed also, that in whatever part of the room the eye is placed, it can always distinguish the portion of the mirror which reflects the light; some of the rays, consequently, are reflected in all directions. This portion of the light is said to be *irregularly reflected*, and its intensity *decreases* with the polish of the mirror.

Irregular reflexion is due, mainly, to the inequalities of the reflecting surface, which is composed of an indefinite number of reflecting surfaces in various positions, and which therefore reflect the light in various directions.

(21) The *angles of incidence* and *reflexion* (or the angles which the incident and reflected rays make with the perpendicular to the reflecting surface at the point of incidence) *are in the same plane, and are equal.* This law is universally true, whatever be the nature of the light itself, or that of the body which reflects it.

(22) The *intensity* of the reflected light, on the other hand,

is found to vary greatly with the medium. The following leading facts have been established experimentally.

I. The quantity of light regularly reflected increases with the angle of incidence, the increase being very slow at moderate incidences, and becoming very rapid at great ones. Thus, water at a perpendicular incidence, according to the experiments of Bouguer, reflects only 18 rays out of 1000; at an incidence of 40° it reflects 22 rays; at 60°, 65 rays; at 80°, 333 rays; and at $89\frac{1}{2}$°, 721 rays.

II. The quantity of light reflected at the same incidence varies both with the medium upon which the light falls, and with that from which it is incident. Thus, at a perpendicular incidence, the number of rays reflected by water, glass, and mercury, are 18, 25, and 666, respectively, the number of incident rays being 1000. The dependence of the quantity of the reflected light upon the medium *from* which it is incident is easily shown by immersing a plate of glass in water or oil.

III. The differences in the reflective powers of different substances are much more marked at small, than at great incidences. Thus, water and mercury—the first of which reflects but the one-fiftieth part of the incident light at a perpendicular incidence, while the latter reflects two-thirds —are equally reflective at an incidence of $89\frac{1}{2}$°, the number of rays reflected at this angle being, in both cases, 721 out of 1000.

(23) When light is incident upon the surface of a transparent medium, a portion enters the medium, pursuing there an altered direction. This portion is said to be *refracted.*

When the ray passes from a rarer into a denser medium, the angle of incidence is, in general, greater than the angle of refraction, and the deviation takes place *towards the perpendicular* to the bounding surface. On the contrary, when the ray passes from a denser into a rarer medium, the angle of incidence is less than the angle of refraction, and the deviation is *from the perpendicular*.

(24) *The angles of incidence and refraction are in the same plane; and their sines are in an invariable ratio.*

In order to verify this law experimentally, it is only necessary to measure several angles of incidence at the surface of the same medium, and the corresponding angles of refraction. This was done by Ptolemy in the second century, and subsequently by Vitello in the thirteenth; but both of these observers failed in discovering the connecting law. The law of refraction, just stated, was discovered by Willebrord Snell, about the year 1621.

If ϕ and ψ be employed to denote the angles which the portions of the ray in the rarer and denser medium, respectively, make with the perpendicular to the common surface, the second part of the law of refraction is expressed by the equation,

$$\sin \phi = \mu \sin \psi,$$

μ being a constant quantity. This constant is termed the *index of refraction;* and since $\phi > \psi$, it is always greater than unity.

When a ray of light passes into any medium from a vacuum, the index of refraction is in that case termed the *absolute index* of the medium. For air, and the gases, it exceeds unity by a very small fraction; for water, $\mu = 1{\cdot}336$; for crown glass, $\mu = 1{\cdot}535$; for diamond, $\mu = 2{\cdot}487$; and, for chromate of lead, $\mu = 3$.

(25) When light traverses a prism,—that is, a medium bounded by two inclined plane surfaces,—the total deviation of the refracted ray is the sum of the deviations at incidence and emergence. Let ϕ and ϕ' denote the angles which the incident and emergent rays make with the perpendiculars to the faces at the points of incidence and emergence, ψ and ψ' the

angles which the portion of the ray within the prism forms with the same, then the deviations at incidence and emergence are, respectively, $\phi - \psi$, and $\phi' - \psi'$; and the total deviation $\delta = \phi + \phi' - (\psi + \psi')$. Now, it is easily shown that the algebraic sum of the angles, which the portion of the ray within the prism makes with the two perpendiculars, is equal to the vertical angle of the prism; or, denoting this angle by a, that

$$\psi + \psi' = a;$$

wherefore

$$\delta = \phi + \phi' - a.$$

(26) When a ray of light is incident nearly perpendicularly upon a thin prism, the total deviation is constant, and bears an invariable ratio to the angle of the prism.

For in this case the angles of incidence and refraction, being small, are proportional to their sines, so that

$$\phi = \mu\psi, \quad \phi' = \mu\psi'; \quad \text{and } \phi + \phi' = \mu(\psi + \psi') = \mu a.$$

Hence

$$\delta = (\mu - 1)\,a.$$

(27) The deviation produced by a prism is easily determined when the angles of incidence and emergence are equal.

For we have seen that, generally,

$$\phi + \phi' = a + \delta, \qquad \psi + \psi' = a.$$

But since, in this case, $\phi = \phi'$, there is also $\psi = \psi'$; and consequently

$$\phi = \tfrac{1}{2}(a + \delta), \qquad \psi = \tfrac{1}{2}a.$$

Hence we have

$$\sin \tfrac{1}{2}(a + \delta) = \mu \sin \tfrac{1}{2}a;$$

from which $a + \delta$, and therefore δ, is determined.

It may be shown that the angle of deviation, in this case, is the *least possible;* and accordingly, if the prism be turned slowly round its axis, the inclination of the emergent to the incident ray will first decrease, and afterwards increase, appearing for a moment to be stationary between the opposite changes. By this principle it is easy to place a prism, experimentally, in the position in which the refractions are equal at both sides.

(28) We are now enabled to determine the *refractive index* of a transparent solid experimentally.

The first step of this process is to polish two plane faces, inclined to one another at a sufficient angle, and to measure that angle by a goniometer. This being done, the prism is to be placed, with its refracting edge vertical, before the object-glass of the telescope of a theodolite, so as to refract to the cross wires in its focus the rays proceeding from a distant mark. The prism is then to be turned slowly round its axis, and the telescope moved, until the deviation is a minimum. The horizontal circle being read, and the prism removed, the telescope is to be turned directly to the distant mark, and the reading repeated; the difference of the two readings is the deviation. The angle of the prism and the deviation being obtained, the refractive index is given by the formula,

$$\mu = \frac{\sin \frac{1}{2}(a + \delta)}{\sin \frac{1}{2}a}.$$

To determine the refractive index of a fluid, we have only to inclose it in a hollow prism, whose sides are formed of glass plates with parallel surfaces. For the course of the ray will be the same as if it had been incident directly from the air into the fluid, and had emerged similarly, without passing through the glass.

(29) Let us now proceed to the physical explanation of the phenomena.

To account for the phenomena of reflexion and refraction, it is supposed, in the theory of emission, that the particles of bodies and those of light exert a *mutual action*;—that, when they are nearly in contact, this action is *attractive*;—that, at a distance a little greater, the attractive force is changed into a *repulsive* one;—and that these attractive and repulsive forces succeed one another for many alternations. Nothing can be more reasonable than this hypothesis, granting that light is material; for the succession of attractive and repulsive forces, here assumed, is altogether similar to that to which the known phenomena of molecular action are ascribed.

On these suppositions Newton has rigorously deduced the laws of reflexion and refraction. In the case of reflexion, it is shown that the whole perpendicular velocity of the molecule is restored to it in an opposite direction, by the operation of the supposed repulsive force; and, therefore, that the angles which its path makes with the perpendicular to the surface, before and after reflexion, are equal. In the case of refraction, it is proved that the effect of the attractive force is to increase the square of the perpendicular velocity of the molecule, by an amount which is constant for the same medium; from which it follows, that the sines of the angles which its course makes with the perpendicular to the surface, before and after refraction, are in the inverse ratio of the velocities in the two media. This problem was the first in which the effects of molecular forces were submitted to calculation; and its solution is justly regarded as forming an era in the history of science.

(30) But although the theory of emission is successful in explaining the laws of reflexion and refraction, considered as distinct phenomena, it is by no means equally so in accounting for their connexion and mutual dependence. When a beam of light is incident on the surface of any transparent medium, part is in all cases transmitted, and part reflected; the intensity of the reflexion being less, the less the difference of the refractive indices of the two media, and the reflexion ceasing altogether when this difference vanishes. How is it, then, that some of the molecules obey the influence of the repulsive force, and are reflected, while others yield to the

attractive force, and are refracted? To account for this, Newton was obliged to have recourse to a new hypothesis. The molecules of light, in their progress through space, are supposed to pass continually into two alternate states, or fits, which recur periodically and at equal intervals. While in one of these states, called the *fit of easy reflexion*, they are disposed to obey the repulsive or reflective forces of any body which they meet; and, on the other hand, they yield more readily to the attractive or refractive forces, when in the alternate state, or *fit of easy transmission*. Now, the molecules composing a beam of common light are supposed to be in every possible stage of these fits, when they reach the surface;—some in a fit of reflexion, and others in a fit of transmission. Some of them, consequently, will be reflected, and others refracted, and the proportion of the former to the latter will increase with the incidence.

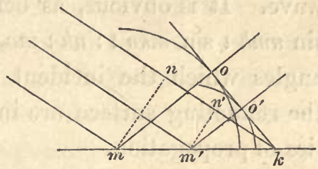
To account for the fits themselves, Newton assumed the existence of an ethereal medium, analogous to that of Huygens, although he did not assign to it the same office. The molecules of light were supposed to excite the vibrations of this ether, just as a stone flung into water raises waves on its surface. This vibratory motion was supposed to be propagated with a velocity greater than that of the molecules; so as to overtake them, and impress upon them the disposition in question, by conspiring with or opposing their progressive motion. In one of his queries Newton has even calculated the elastic force of this ether, as compared with that of air, in order that the velocity of propagation should exceed that of light.

(31) The hypothesis of the fits has lost much of its credit, since the phenomena of the colours of thin plates (phenomena which first suggested it to the mind of Newton) have been shown to be irreconcilable with it. The explanation which it yields of the facts now under consideration is alike un-

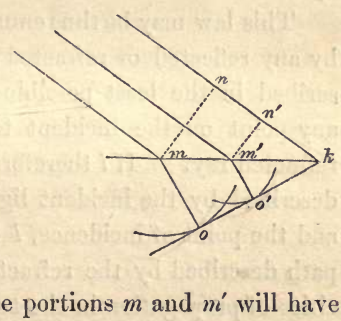
satisfactory. In fact, the molecules which are transmitted are not all in the *maximum* of the fit of transmission; but are supposed to reach the surface in *every possible phase* of this, which may be called the *positive* fit. But as a change of the fit from positive to negative is, in general, sufficient to overcome altogether the effect of the attractive force, and subject the molecule to the repulsive one, it is obvious that the phase of the fit must modify the effects of these forces in every intermediate degree; and that the molecules which do obey the attractive force must have their velocities augmented in *different degrees*, depending on their phase. Hence, as the direction of the refracted ray depends on its velocity, the transmitted beam should consist of rays refracted in widely different angles, and should be *scattered* and *irregular*.

(32) Let us now turn to the account which the other theory gives of the same phenomena, and of their laws.

The velocity of propagation, in the wave-theory of Light, depends on the elasticity of the vibrating medium as compared with its density. In the same homogeneous medium the velocity will be therefore *constant*, and the wave propagated from any centre of disturbance *spherical*. But when a wave reaches the surface of a new medium whose elasticity is different, it will give rise to two waves, one in each medium, and both differing in position from the original wave. For it is obvious that, in general, the several portions of the incident wave will reach the bounding surface at different moments of time. Each of these portions will be the centre of two new waves, one of which will be propagated in the first medium with the original velocity, while the other will be propagated in the new medium, and with the velocity which belongs to it; so that there will be an infinite number of *partial waves* in both media, diverging from the several points of the bounding surface. But, by the principle of the *coexistence of small motions*, the agitation of any particle of either medium is the sum of the agitations sent there at the same instant from these several centres of disturbance. The surfaces on which these are accumulated will be the *reflected* and *refracted waves*, and they are obviously those which *touch* all the small spherical waves at any instant.



Thus, let *mn* be the front of a *plane* wave, meeting the reflecting surface at *m*. Each portion of this wave, as it reaches the surface, becomes the centre of a diverging spherical wave in the first medium, which will be propagated with the velocity of the original wave. Accordingly, when the portion *n* reaches the surface at *k*, the portion *m* will have diverged into the spherical wave, whose radius, *mo*, is equal to *nk*. And, in like manner, if *m′n′* be drawn parallel to *mn*, the wave diverging from *m′* will in the same time have reached the spherical surface whose radius, *m′o′*, is equal to *n′k*. The surface which touches all these spheres at any instant is that of the reflected wave. But, as *mo* and *m′o′* are proportioned to *mk* and *m′k*, it is obvious that this tangent surface is plane; and since *mo* = *nk*, and the angles at *n* and *o* are right, it follows that the angles *nmk* and *okm* are equal,—or that the incident and reflected waves are equally inclined to the reflecting surface.



(33) The proof of the law of refraction is in all respects analogous to the preceding. Let *mn* be the position of the incident plane wave at any moment. In an interval of time proportional to *nk*, the portion *n* of this wave will have reached the surface at *k*, and the portions *m* and *m′* will have

become the centres of diverging spherical waves in the second medium,—the radii of these spheres, *mo* and *m'o'*, being to the intercepts, *nk* and *n'k*, in the constant ratio of the velocities of propagation in the two media. The surface which touches these spheres is that of the refracted wave. It is obvious, as before, that it is plane; and, since sin *nmk* : sin *mko* : : *nk* : *mo*, we learn that the sines of the angles which the incident and refracted waves make with the refracting surface are in the constant ratio of the velocities of propagation.

(34) Such is the demonstration of the laws of reflexion and refraction given by Huygens. The composition of the *grand* or primary wave, by the union of the several secondary or *partial* waves, in this demonstration, has been denominated the *principle of Huygens;* and it is obviously a case of the more general principle of the co-existence of small motions. It easily follows from this mode of composition, that the surface of the primary wave marks the extreme limits to which the vibratory movement is propagated in any given time; so that light is propagated from any one point to another in the *least possible time.* This is the well-known law of Fermat,—the law of swiftest propagation; and it will appear from what has been stated, that it holds, whatever be the modifications which the course of the light may undergo by reflexion or refraction.

This law may be thus enunciated:—"The course pursued by any reflected or refracted ray is that which would be described in the least possible time, by a body moving from any point on the incident to any point on the reflected or refracted ray." If l therefore denote the length of the path described by the incident light, between any assumed point and the point of incidence, l' the corresponding length of the path described by the refracted light, and v and v' the velocities of propagation in the two media, the sum of the times,

$\frac{l}{v} + \frac{l'}{v'}$, is a minimum; or, multiplying by v, and denoting the ratio $\frac{v}{v'}$ by μ,

$$l + \mu l' = minimum.$$

The constant factor, μ, is the refractive index of the medium.

In the case of reflexion, $\mu = 1$, and $l + l'$ is a minimum. The course pursued by a reflected ray is therefore such, that the sum of the paths described between any two points and the reflecting surface is the least possible.

(35) The *intensity* of the light, in the reflected and refracted waves, will depend on the relative densities of the ether in the two media. For we may compare the contiguous strata of ether in these media to two elastic bodies of different masses, one of which moves the other by impact; and it is easy to deduce, on this principle, the intensities of the reflected and refracted lights in the case of perpendicular incidence.

(36) On reviewing what has been said, we cannot but be struck by the remarkable fact, that theories so widely opposed as the theory of emission, and that of waves, should lead mathematically to the same result. According to both, we have seen, the ratio of the sines of incidence and refraction is equal to the ratio of the velocities of light in the two media, and is therefore constant. But there is this important difference between them: in the wave-theory, the sines of these angles are in the *direct* ratio of the velocities, while, according to the theory of emission, they are in the *inverse*. In other words, the velocity of light in the denser medium is *less* according to the former theory; while, according to the latter, it is *greater*. Here, then, the two theories are directly at issue upon a point of fact, and we have only to ascertain how this fact stands, in order to be able to decide between

them. The important experiment by which this was first accomplished was made by Arago; and the result, as will be shown hereafter, is conclusive in favour of the wave-theory.

(37) The conclusion deduced from the experiment here referred to presupposes the laws of Interference of Light—laws which, in themselves, are intimately connected with the principles of the wave-theory. It was desirable, therefore, to deduce the same conclusion, if possible, by *direct* means. The experiment by which this is effected has been recently made by M. Fizeau, upon a method devised by Arago; its principle will be understood from the following description.

Let a ray of light, reflected by a heliostat, be admitted into a darkened chamber in a horizontal direction, and fall upon a mirror which revolves about a vertical axis situated in its own plane. It is manifest that, as the mirror revolves, the reflected ray will move, in the horizontal plane passing through the point of incidence, with an angular velocity double of that of the mirror itself. Now, in this plane let a second mirror be placed, perpendicular to the right line joining the centres of the two mirrors. Then, when the ray reflected by the revolving mirror meets the fixed mirror, in the course of its angular movement, it will be turned back on its course, and, after a second reflexion by the revolving mirror, return towards the aperture.

It is plain that if the revolving mirror were for a moment to rest in this position, the ray, after a second reflexion by it, would return *precisely* by the path by which it came. But, owing to the progressive movement of light, the mirror describes a certain small angle round its axis, in the interval between the two appulses of the ray; and the ray, after the second reflexion, will *deviate* from its first position, by an angle which is double of that described by the mirror in the interval. Hence, if this angle can be observed, the velocity of light is known.

For, if t be the time taken by the light to traverse the interval of the two mirrors, forwards and backwards, the angle described by the mirror in that time will be $= \omega t$, ω denoting the angle described by the mirror in the unit of time. Hence, the angle described by the reflected ray in the time t, or the deviation, $= 2\omega t$. Let this angle be denoted by a, and there is

$$t = \frac{a}{2\omega}.$$

But the corresponding space is double the distance between the two mirrors, or $2a$. Consequently, the velocity of the light is

$$V = 4a \times \frac{\omega}{a}.$$

M. Fizeau has been enabled to observe an appreciable deviation of the reflected ray, when the distance of the two mirrors was 4 metres, and the revolving mirror made only 25 turns in a second. And as such a mirror has been made to revolve 1000 times in a second, it was obvious that the time taken by light to traverse even this short distance was capable of being measured with precision. It only remained to interpose a column of water between the mirrors, to observe the deviation, and to calculate the velocity. By these means M. Fizeau has established the fact, that the velocity of light is *less in water than in air*, in the inverse proportion of the refractive indices. The result is, therefore, decisive in favour of the wave-theory.

(38) The refractive index being equal to the ratio of the velocities of light in the two media (direct or inverse) it follows, whichsoever theory we adopt, that any change in the velocity of the incident ray must cause a variation in the amount of refraction, unless the velocity of the refracted ray be altered proportionally. Now the relative velocity of the light of a star is altered by the Earth's motion; and the amount of the change is obviously the resolved part of the Earth's velocity in

the direction of the star. It was, therefore, a matter of much interest to determine how, and in what degree, this change affected the refraction. The experiment was undertaken by Arago, at the request of Laplace. An achromatic prism was attached in front of the object-glass of the telescope of a repeating circle, so as to cover only a portion of the lens. The star being then observed, directly through the uncovered part of the lens, and afterwards in the direction in which its light was deviated by the prism, the difference of the angles read off gave the deviation. The stars selected for observation were those in the ecliptic, which passed the meridian nearly at 6 A.M. and 6 P.M., the velocity of the Earth being added to that of the star in the former case, and subtracted from it in the latter. No difference whatever was observed in the deviations.

This remarkable and unexpected result can be reconciled to the theory of emission, as Arago has observed, only by the hypothesis already adverted to,*—namely, that the molecules are emitted from the luminous body with various velocities; but that among these velocities there is but one which is adapted to our organs of vision, and which produces the sensation of light. It is explained, in accordance with the principles of the wave-theory, on the same hypotheses which have been already made to explain the aberration of light;† and it is shown, on these suppositions, that both the laws, and the amount of refraction, are independent of the Earth's motion.

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* Art. (14).

† Art. (19).

CHAPTER III.

DISPERSION.

(39) WE have hitherto supposed light to be *simple* or *homogeneous*. The light of the Sun, however, and most of the lights, natural or artificial, with which we are acquainted, are *compound*, each ray consisting of an infinite number of rays differing in *colour* and *refrangibility*. This important discovery we owe to Newton. We shall briefly describe the principal experiments by which it is established.

(40) When a beam of solar light is admitted into a darkened room through a small circular aperture, and received on a screen at a distance, a circular image of the Sun will be depicted there, whose diameter will correspond to that of the hole. If now the light be intercepted by a prism, having its refracting edge horizontal and perpendicular to the incident beam, the image of the Sun will be thrown upwards by the refraction of the prism, and will be no longer white and circular, but *coloured* and *oblong* ; the sides which are perpendicular to the axis of the prism being rectilinear and parallel, and the extremities semicircular. The breadth of this image, or *spectrum* (as it is called), is equal to the diameter of the unrefracted image of the Sun, but its length is much greater.

Now if the solar beam consisted of rays having all the same refrangibility, the refracted image should be circular, and of the same dimensions as the unrefracted image, from which it should differ only in position. For the rays composing the beam, being parallel at their incidence on the prism, must (on this supposition) be equally refracted by it, and therefore continue parallel after refraction. This not being the case, we conclude that the rays composing the incident beam are of *different degrees of refrangibility*, the more refrangible rays going to form the upper part of the spectrum, and the less refrangible the lower; and that the elongation of the solar image, and the variety of its colouring, arise from the separation of these rays in their refraction through the prism.

It further appears that the rays, which differ in *refrangibility*, likewise differ in *colour;* the spectrum being red at its lowest or least refracted extremity, violet at its most refracted extremity, and yellow, green, and blue, in the intermediate spaces, these colours passing into one another by imperceptible gradations. Sir Isaac Newton distinguished the spectrum, or coloured image of the Sun, into *seven* principal colours, and measured the spaces occupied by each. These colours, arranged in the order of their refrangibility, are *red*, *orange*, *yellow*, *green*, *blue*, *indigo*, *violet*;* of which the yellow and orange are the most luminous, the red and green next in order, and the indigo and violet weakest.

Any one of these rays may be separated from the rest by transmitting it through a small aperture in a screen which intercepts the remainder of the light. The ray thus separated may be examined apart from the rest, and will be found to undergo no dilatation, or change of colour, by any subsequent refractions or reflexions. We are, therefore, warranted in concluding that the solar light is *compound*, and consists of an infinite number of simple rays, which are permanent in their own nature, but differ from one another both in their *colour* and *refrangibility*.

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* The imperfection of Newton's classification of colours has been pointed out by Professor Forbes and others. The *indigo* ought not to have been distinguished from the *blue*, the difference to the eye being much less, in kind, than between any other two adjacent colours of the scale. We may, therefore, better distribute the colours of the spectrum into *six*, viz., *red*, *orange*, *yellow*, *green*, *blue*, and *violet*,—of which the red, yellow, and blue, may be regarded as *primary* colours, and the orange, green, and violet, as *secondary*.

(41) The following experiment may be considered as removing all doubt on this subject. Close behind the prism is placed a board, perforated with a small aperture, through which the refracted light is permitted to pass. This light is then received on a second board, similarly perforated, at a considerable distance from the first; so that a small portion of the light of the spectrum is suffered to pass through the aperture in the second board, the rest being intercepted. Close behind this aperture a second prism is fixed, having its axis parallel to that of the first. The first prism being then turned slowly round its axis, the light of the spectrum will move up and down on the second board, and the differently-coloured rays will be successively transmitted through the second aperture, and be refracted by the prism behind it. If then the places of these twice-refracted rays on the screen be noted, the red will be found to be lowest, the violet highest, and the intermediate colours in the same order as they are in the spectrum. Here, on account of the unchanged position of the two apertures, all the rays are necessarily incident upon the second prism at the same angle; and yet some of them are more refracted, and others less, in the same proportion as by the first prism.

From the foregoing we conclude, then, that the peculiar colour and refrangibility belonging to each kind of homogeneous light, are permanent* and original affections, and are not generated by the changes which that light undergoes in refractions or reflexions.

(42) In the experiments hitherto described, the *analysis*

* Professor Stokes has recently discovered that the refrangibility of light does undergo alteration in certain cases, some bodies possessing the property of *lowering the refrangibility* of the incident light—that is, of emitting rays of a lower refrangibility, when excited by those of a higher. This property belongs to the solution of sulphate of quinine, and to certain coloured glasses. Professor Stokes has denominated it *fluorescence*.

of solar light, or its resolution into its simple components, is far from being complete, inasmuch as there is a considerable mixture of the different species of simple light in the spectrum. This will be evident, if we consider that, as the several pencils of *homogeneous* light suffer no dilatation by the prism, each will depict on the screen a circular image, equal in magnitude to the unrefracted image of the Sun at the same distance; so that the spectrum consists of innumerable circles of homogeneous light, whose centres are disposed along the same right line, and whose common diameter is that of the Sun's unrefracted image. Wherefore the number of such circles mixed together in the spectrum, is to the corresponding number in the unrefracted image of the Sun, as the interval between the centres of two *contingent* circles (or the *breadth* of the spectrum), to the interval between the centres of the *extreme* circles, which is the *length* of the rectilinear sides. The mixture in the spectrum, therefore, varies as the breadth of the spectrum divided by its length; and if the breadth can be diminished, the length remaining the same, the mixture will be diminished in proportion.

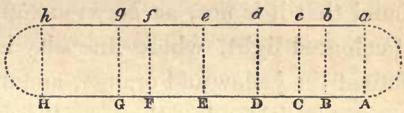
There are various ways of diminishing the breadth of the spectrum, or the diameter of the Sun's unrefracted image, amongst which that of Newton seems as convenient in practice as any. The solar beam, admitted through a small circular aperture, is received upon a lens of long focus, at the distance of double its focal length from the aperture; and at the same distance beyond the lens will be formed a distinct image of the hole, equal to it in magnitude. A prism being then placed immediately behind the lens, this image will be dilated in length, its breadth remaining unaltered, and thus a spectrum will be formed whose breadth is the diameter of the hole; whereas, without this contrivance, the breadth would be equal to that diameter, together with a line which (at the distance of the screen from the hole) subtends an angle equal to the apparent diameter of the Sun. Thus, by diminishing the diameter of the aperture, the breadth of the spectrum, and therefore the mixture, may be reduced at pleasure.

If the diameter of the aperture be very small, the spectrum is reduced to a narrow line, and is unfit for examination. To remedy this, Newton employed a narrow rectangular aperture, whose length, parallel to the axis of the prism, may be as great as we please, while its breadth is very small. In this manner we obtain a spectrum as broad as we wish, and whose light is as simple as before.

(43) In order to determine the laws of *dispersion*, it is necessary to find experimentally the indices of refraction of the several species of simple light, of which solar light is composed.

Newton's method was to determine the refractive indices of the extreme red and violet rays directly by means of the formula of (28), and to deduce those of the other rays by a simple proportion.

When the refracting prism was of crown-glass, the indices of refraction of the extreme rays were found to be $\frac{77}{50}$, $\frac{78}{50}$, respectively. To determine the refractive indices of the intermediate rays, it was necessary to measure the spaces which they occupied in the spectrum. For this purpose Newton delineated on paper the spectrum AH*ha*, and distinguished it by the cross lines A*a*, B*b*, C*c*, &c., drawn at the confines of the several colours; so that the



space AB*ba* is that occupied by the red light, BC*cb* that by the orange, CD*dc* the yellow, DE*ed* the green, EF*fe* the blue, FG*gf* the indigo, and GH*hg* the violet. He then found that, if the whole length of the rectilinear side, AH, be taken as unit, the distances to the confines of the several colours, AB, AC, AD, &c., will be denoted by the numbers $\frac{1}{8}$, $\frac{1}{5}$,

$\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{7}{9}$. Now the intervals AB, BC, CD, &c., occupied by the several colours in the spectrum, or the differences of the deviations which they subtend, are to one another as the corresponding variations of the index of refraction. If, therefore, the whole variation of μ, or $\frac{1}{50}$, be divided as the line AH is in the points B, C, D, &c., the refractive indices of the rays at the confines of the several colours will be as follow:—

$$\frac{77}{50},\quad \frac{77\frac{1}{8}}{50},\quad \frac{77\frac{1}{5}}{50},\quad \frac{77\frac{1}{3}}{50},\quad \frac{77\frac{1}{2}}{50},\quad \frac{77\frac{2}{3}}{50},\quad \frac{77\frac{7}{9}}{50},\quad \frac{78}{50}.$$

The *mean* refractive index is $\frac{77\frac{1}{2}}{50}$, or 1·55; and it appears from the preceding that it belongs to the rays at the confines of the green and blue.

(44) The *intensity* of the light is very different in the different parts of the spectrum. According to the experiments of Fraunhofer, the following numbers represent the intensities of the light in each of the coloured spaces, the maximum intensity (which occurs at the confines of the yellow and orange) being represented by 1000; viz., red, 94; orange, 640; green, 480; blue, 168; indigo, 31; violet, 6.

(45) On a minute examination of the solar spectrum, when every care has been taken in making the experiment, it is found that it is not, as Newton supposed, a continuous band of coloured light, whose intensity is greatest about the confines of the yellow and orange, and diminishes regularly to the two extremities; but that, on the contrary, there are at certain points *abrupt deficiencies of light*, total or partial, indicated by the existence of numerous dark lines or bands, crossing the spectrum in the direction of its breadth; while in the intermediate spaces the intensity of the light does not increase or decrease continually, but varies irregularly, or according to some very complex law. Solar light, then, does not consist

(as has been hitherto supposed) of rays of every possible refrangibility, within certain limits, for it is found that many rays corresponding to certain degrees of refrangibility are wanting in the spectrum.

Some of these lines are wholly black; others dark, of various degrees of illumination. Again, some of them are well defined and single; others are clustered together, so as to present the appearance of dark bands. They are irregularly disposed throughout the length of the spectrum. They are not, however, the result of any accidental cause; for, when solar light is used, and the refracting substance is the same, it is found that they preserve the *same relative position*, both with respect to one another and to the colours of the spectrum. On the other hand, when the refracting substance is varied, their relative positions with respect *to one another* are altered: but their positions as referred *to the colours* of the spectrum, as also their relative breadth and intensity, remain unchanged.

(46) If other kinds of light—as that of the *fixed stars*, *flames*, *the electric spark*—are examined in the same way, similar bands are discovered, but differing in each species of light in their position, &c.; so that each species of flame, and the light of each fixed star, has its own system of bands, which remains unalterable under all circumstances, and which, therefore, is a distinct physical characteristic of the species of light to which it belongs. Thus the light of the *electric spark* has bright bands, instead of dark ones. The flames of *oil*, *hydrogen*, and *alcohol*, have each a brilliant line between the red and the yellow. The red flames coloured by *nitrate of strontian* exhibit a brilliant blue line, which is detached from the rest of the spectrum; and the *salts of potash* give rise to a remarkable red ray, beyond the limits of the ordinary red of the spectrum, and separated from it by a dark interval. On the other hand, the spectrum of the flame of *cyanogen* exhibits great re-

gularity, as well in the distribution of the dark bands, as in the intensity of the intervening luminous spaces.

These bands depend on the rapidity of the combustion. Thus sulphur, when burning slowly, exhibits blue and green bands in the spectrum; in rapid combustion, its light is nearly homogeneous.

(47) These *fixed lines*, as they are called, were first noticed by Wollaston, in the year 1802. They have since been much more fully examined by Fraunhofer, who distinguished 590 in the solar spectrum, of which he has delineated 354. Of these he has selected seven principal ones, to serve as standards of comparison, and has designated them by the letters B, C, D, E, F, G and H. Of these, B and C are single lines in the red portion of the spectrum, the former near to its extremity; D is a double line, at the confines of the orange and yellow; E is a group of fine lines in the green; F is a strongly marked black line in the blue; G is a group of fine lines in the indigo; and H is a similar group in the violet, clustered round one much stronger line. They are of the utmost importance in optical investigations. On account of the accuracy of their delineation, their position may be observed with an accuracy equal to that of astronomical measurements, and the refractive indices of the rays, to which they correspond, thus determined with the utmost exactness.

(48) The dispersion of a ray which passes nearly perpendicularly through a thin prism is easily expressed.

If δ_1 and δ_2 denote the deviations of the red and violet rays, μ_1 and μ_2 the refractive indices of the prism for those rays, and a its refracting angle, we have

$$\delta_1 = (\mu_1 - 1)\,a, \qquad \delta_2 = (\mu_2 - 1)\,a;$$

whence

$$\delta_2 - \delta_1 = (\mu_2 - \mu_1)\,a.$$

Accordingly the dispersion, in this case, is equal to the angle of the prism multiplied by the difference of the refractive indices.

(49) The *dispersive power* of a substance is measured,—not by the absolute dispersion, which varies in general with the angle of refraction,—but by the ratio which that quantity bears to the total deviation, or by $\frac{\delta_2 - \delta_1}{\delta_1}$. But, in the case of a ray which passes nearly perpendicularly through a thin prism, this ratio is constant; for, dividing the third of the equations of the preceding article by the first,

$$\frac{\delta_2 - \delta_1}{\delta_1} = \frac{\mu_2 - \mu_1}{\mu_1 - 1}.$$

The dispersive power, therefore, is measured by the difference of the refractive indices of the red and violet rays, divided by the refractive index of the former *minus* unity.

(50) Newton supposed that the dispersive powers of all substances were the same. He was led to this erroneous conclusion, by observing that when a prism of glass was inclosed in a prism of water with a variable angle, their refracting angles being turned in opposite directions, the emergent ray was *coloured* when it was *inclined* to its original direction; while, on the other hand, it was *colourless* whenever, by varying the angle of the water prism, the refractions of the two prisms were made to compensate each other, or the ray to emerge *parallel* to the incident ray. Hence he concluded that the dispersion was always proportional to the total deviation; and that refraction could never take place without a separation of the refracted ray into its coloured elements.

When Newton's experiment with the two prisms was repeated a long time after, by Dollond, he found that the results were exactly the opposite to those stated by Newton;—that, in fact, the emergent ray was *coloured*, when the deviation was nothing, or the ray *parallel* to its original direction; and that, on the other hand, when the dispersions of the two prisms were made to correct each other by varying the angle, so that the ray emerged *colourless*, their refractions were no longer equal, and the ray was *inclined* to its original direction. This important discovery led to the construction of the achromatic telescope.

(51) It is easy to determine the condition of achromatism, when a ray of light passes nearly perpendicularly through two prisms, whose refracting angles are small.

The dispersions produced by the two prisms are $(\mu_2 - \mu_1)\, a$, and $(\mu_2' - \mu_1')\, a'$, respectively (48); and, therefore, when the total dispersion is nothing, we must have

$$(\mu_2 - \mu_1)\, a + (\mu_2' - \mu_1')\, a' = 0, \quad \text{or } \frac{a'}{a} = -\frac{\mu_2 - \mu_1}{\mu_2' - \mu_1'}.$$

The negative sign, in the second member, indicates that the angles of the two prisms must be turned in opposite ways.

(52) In order to ascertain the relative dispersive powers of different substances, they must be separately compared with some standard substance, such, *e.g.*, as water. For this purpose a vessel must be constructed, whose opposite sides, formed of parallel glass, are moveable on hinges, and may be inclined to one another at any angle. It is closed on the other two sides by metallic cheeks, to which the moveable sides are accurately fitted. The vessel being filled with water, it is evident that the transmitted ray will be refracted in the same manner as by the inclosed water prism, the parallel plates of glass producing no change in the direction of the refracted ray. The substance whose dispersive power is sought being formed into a thin prism, a beam of light is to be transmitted nearly perpendicularly through the two prisms, with their refracting angles turned in opposite directions; and the angle

of the water prism is to be varied, until the beam emerges colourless. The angle of the water prism being then measured, the ratio of the differences of the refractive indices (and thence that of the dispersive powers) will be given by the formula of the preceding article.

(53) We now proceed to the physical explanation of the foregoing phenomena.

To account for dispersion, the modern advocates of the theory of emission have been forced to assume that the molecules of light are *heterogeneous*, and that the attractions exerted on them by bodies vary with their nature, being in this respect analogous to chemical affinities. This supposition, as Young has justly observed, is but veiling our inability to assign a *mechanical* cause for the phenomenon.

According to the principles of the wave-theory, the *colour* of light is determined by the frequency of the ethereal vibrations, or by the length of the wave;—the longest waves producing the sensation of *red*, and the shortest that of *violet*. Now observation proves that the refractive index (or the ratio of the velocities of propagation in the two media) is different for the light of different colours. The velocity of propagation in a refracting medium, therefore, *varies with the length of the wave*. Here, then, we encounter a difficulty in this theory, which was long regarded as the most formidable obstacle to its reception. Analysis seemed to indicate that the velocity of wave-propagation depended solely on the elasticity of the medium as compared with its density, and should therefore be the same for light of all colours, as it is for sound of all notes; so that all rays should be equally refracted. It will be necessary to enter, in some detail, into the consideration of this difficulty.

(54) The conclusion of analysis to which we have just ad-

verted,—namely, that the velocity of wave-propagation is constant in the same homogeneous medium,—is deduced on the particular supposition, that the sphere of action of the molecules of a vibrating medium is indefinitely small compared with the length of a wave. If this restriction be removed, we have no longer any ground for concluding that the waves of different lengths will be propagated with the same velocity; and the conclusion hitherto acquiesced in must be regarded but as an approximate result. It was in this point of view that the question presented itself to M. Cauchy. Resuming the problem of wave-propagation with the more general equations, he has proved that there exists, generally, a relation between the *velocity of propagation* (or the refractive index *in vacuo*) and the *length of the wave;* and, therefore, that the rays of different colours will be differently refracted.

(55) Let us make, for abridgment,

$$k = \frac{2\pi}{\lambda}, \quad s = \frac{2\pi}{\tau},$$

in which λ and τ denote, as before, the wave-length and time of vibration. M. Cauchy has proved that k and s are connected by an equation of the form

$$s^2 = a_1 k^2 + a_2 k^4 + a_3 k^6 + \&c.,$$

in which the coefficients a_1, a_2, a_3, &c., vary with the medium. Now the velocity of wave-propagation is

$$V = \frac{\lambda}{\tau} = \frac{s}{k};$$

consequently,

$$V^2 = a_1 + a_2 k^2 + a_3 k^4 + \&c.$$

Accordingly, the velocity of propagation is a *function of the wave-length*, and varies with the colour.

(56) In a vacuum, and in media (such as atmospheric air)

which do not disperse the light, the coefficients a_2, a_3, &c., are insensible, and we have

$$V^2 = a_1 ;$$

that is, the velocity of propagation is independent of the wave-length, and the same for light of all colours.

In other media we may, as a first approximation, neglect the third and following terms of the series, and we have

$$V^2 = a_1 + a_2 k^2.$$

Hence, if V_1, V_2 denote the velocities of propagation for two definite rays of the spectrum, and k_1, k_2, the corresponding values of k,

$$V_1^2 - V_2^2 = a_2 (k_1^2 - k_2^2).$$

The truth of this formula has been verified by M. Cauchy, by introducing in it the values of the refractive indices and wave-lengths, as determined by Fraunhofer for the seven definite rays in certain media.

(57) The general formula, above given, is unsuited to a comparison with observation in its present form, inasmuch as the variable k $\left(= \frac{2\pi}{\lambda}\right)$ is not independent of V. This difficulty is overcome by M. Cauchy by *inverting* the first series. The result is of the form

$$k^2 = A_1 s^2 + A_2 s^4 + A_3 s^6 + \&c.$$

M. Cauchy has shown that this series, as well as the former, is convergent, and that all the terms after the third may be neglected. Hence, since

$$k = \frac{s}{V} = \mu s,$$

the velocity *in vacuo* being unity, we have

$$\mu^2 = A_1 + A_2 s^2 + A_3 s^4 ;$$

an equation expressing the refractive index in terms of the time of vibration, or of the wave-length *in vacuo*.

(58) The constants in this formula, A_1, A_2, A_3, will be determined, when we know three values of μ, with the corresponding values of s, or of the wave-length *in vacuo* ; and the formula may be then applied to calculate the values of μ corresponding to any other values of s, which may be thus compared with the results of observation. The comparison has been made by Professor Powell, and by M. Cauchy himself, by means of the observations of Fraunhofer on the refractive indices of water and several kinds of glass, and the agreement of the calculated and observed results is within the limits of the errors of observation.

But the truth of a formula, expressing the relation between the refractive index and the wave-length *in vacuo*, can only be satisfactorily tested in the case of highly-dispersive media; and for such media no observations of sufficient accuracy hitherto existed. To supply this want, Professor Powell undertook the laborious task of determining the refractive indices corresponding to the seven definite rays of Fraunhofer, for a great number of media, including those of a highly dispersive power, and of comparing them with the theory of M. Cauchy. The result of the comparison is, on the whole, satisfactory.

(59) It is an interesting consequence of the preceding formula, pointed out by Professor Powell, that as s diminishes, or the wave-length *in vacuo* increases, the value of μ approximates to a *fixed limit*, given by the equation

$$\mu^2 = A_1,$$

which, therefore, defines the limit of the spectrum on the side of the less refrangible rays. This *limiting index* corresponds to a point not greatly below the red extremity of the visible spectrum.

CHAPTER IV.

DOUBLE REFRACTION.

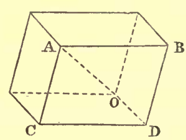
(60) It has hitherto been assumed that, when a ray of light is incident upon the surface of a transparent medium, the intromitted portion pursues, in all cases, a single determinate direction. This is, however, very far from the fact. In many,—indeed in most cases,—the refracted ray is divided into *two distinct pencils*, each of which pursues a separate course, determined by a distinct law.

This property is called *double refraction*. It was first discovered by Erasmus Bartholinus, in the well-known mineral called Iceland spar. After a long series of observations, he found that one of the rays within the crystal followed the known law of refraction, while the other was bent according to a new and extraordinary law not hitherto noticed. An account of these experiments was published at Copenhagen, in the year 1669, under the title "*Experimenta Crystalli Islandici dis-diaclastici, quibus mira et insolita refractio detegitur.*"

A few years after the date of this publication, the subject was taken up by Huygens. This distinguished philosopher had already unfolded the theory which supposes light to consist in the undulations of an ethereal fluid; and from that theory had derived, in the most lucid and elegant manner, the laws of *ordinary refraction* (33). He was, therefore, naturally anxious to examine whether the new properties of light, discovered by Bartholinus, could be reconciled to the same theory; and, in his desire to assimilate the two classes of phenomena, he was happily led to assign the true law of *extraordinary refraction*. The important researches of Huygens on this subject are contained in the fifth chapter of his "*Traitè de la Lumiere.*"

(61) The property of double refraction is possessed by all crystallized minerals, excepting those belonging to the *tessular system,* i. e. those whose fundamental form is the cube. It belongs likewise to all animal and vegetable substances, in which there is a regular arrangement of parts; and, in fine, to all bodies whatever, whose parts are in a state of unequal compression or dilatation. The separation of the two refracted pencils is in some cases considerable, and the course of each easily ascertained by observation; but it is generally too minute to be directly observed, and its existence is only proved by the appearance of certain phenomena, which are known to arise from the mutual action of two pencils. In Iceland spar, the substance in which the property was first discovered, the separation of the pencils is very striking: and, as this mineral is found in considerable masses, and in a state of great purity and transparency, it is well fitted for the exhibition of the phenomena.

(62) Carbonate of lime, of which Iceland spar is a variety, crystallizes in more than 300 different forms, all of which may be reduced by cleavage to the *rhombohedron,* which is accordingly the primitive form. The angles of the bounding parallelograms, CAB and ABD, in the rhombohedron of Iceland spar, are 101° 55′ and 78° 5′. Two of the solid angles, at A and O, are contained by three obtuse angles; while the remaining four are bounded by one obtuse and two acute angles. The line AO, joining the summits of the obtuse solid angles, is called the axis of the rhombohedron, and is equally inclined to the three faces which meet there. The angles at which the faces themselves are mutually inclined are 105° 5′ and 74 55′.



(63) If a transparent piece of this substance be laid upon a sheet of white paper, on which a small black spot is marked with ink, we see two images of the spot instead of one, on looking through the crystal; and if the eye be held perpendicularly above the surface, and the crystal turned round in its plane, one of these images will appear to describe a circle round the other, which is immoveable, the line connecting them being in the direction of the shorter diagonal of the rhombic face. We may vary this experiment, by substituting for the dark spot on the paper a luminous point on a dark ground,—as, for example, the light of the sky seen through a small aperture; but the most direct mode of performing the experiment is to transmit a ray of the Sun's light through the crystal, and receive the emergent pencils on a screen.

If now we examine the course of the two rays within the crystal, we shall find that, at a perpendicular incidence, the deviation of one of them is nothing; that, at any other incidence, the ray is bent towards the perpendicular, the sines of the angles of incidence and refraction being in the constant ratio of 1·654 to 1; and that these angles are always in the same plane. This ray, therefore, is refracted according to the known law, and is called the *ordinary ray*. On examining the other ray, however, we find that, at a perpendicular incidence, the deviation, instead of vanishing, is $6° 12'$; that, at other incidences, the refracted ray does *not* follow the law of the sines; and that, moreover, the angles of incidence and refraction are in *different planes*. This ray, therefore, is refracted according to a new and different law, and is called the *extraordinary ray*.

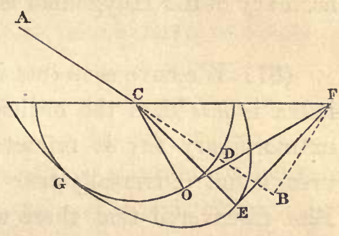
(64) In proceeding to the consideration of this law, we must observe, in the first place, that there is a certain direction in every double-refracting crystal, along which if a ray be transmitted, it is no longer divided. This line is called the *optic axis*, and all the phenomena of double refraction are

related to it. There are, properly speaking, an infinite number of such lines within the crystal, all parallel to one another; so that the optic axis is fixed, not in position, but in *direction* only. It has been already mentioned that the line connecting the obtuse solid angles of the rhombohedron of Iceland spar is the axis of the crystal. Now if we conceive a crystallized mass of this substance to be subdivided into its elementary molecules, which are of this form, the axis of each of these molecules will be an optic axis. The optic axis of the crystallized mass, therefore, is a direction in space parallel to the axes of the elementary molecules, or equally inclined to the three faces containing the obtuse solid angle.

(65) All the phenomena of double refraction are symmetrical round this line. To see this, we have only to polish an artificial face on the crystal, perpendicular to the optic axis, and to mark the course of the refracted rays. We shall then observe, that when the ray is incident perpendicularly on this face, or in the direction of the axis, it undergoes no deviation by refraction, and the ordinary and extraordinary rays coincide; that for every other incidence the ray is divided, the refracted rays being both in the plane of incidence, and the deviation of the extraordinary ray being less than that of the ordinary. This deviation of the extraordinary ray (and therefore the ratio of the sines) is the same for all rays equally inclined to the axis, whatever be the azimuth of the plane of incidence. But it is found, that the ratio of the sines of incidence and refraction of the extraordinary ray is not constant, but *diminishes* as the inclination of the incident ray to the optic axis *increases;* being least of all when the ray is perpendicular to the axis. This least value of the ratio is called the *extraordinary index;* in Iceland spar it is 1·483.

In the preceding cases, the plane of incidence *contains the optic axis*, and the extraordinary ray continues in that plane. This is generally true under the same circumstances, whatever be the refracting surface. To see it, we have only to look obliquely through a rhomb of Iceland spar at a point on a sheet of paper: the extraordinary image will be seen to revolve round the other, as the rhomb is turned, and will *twice* arrive in the plane of incidence,—namely, when that plane contains the optic axis. The same coincidence of the two planes occurs also when the plane of incidence is *perpendicular* to the optic axis; but in this case, the ratio of the sines of incidence and refraction of the extraordinary ray is *constant*, so that this ray then satisfies *both* the laws of ordinary refraction. This constant ratio is the extraordinary index already referred to; it is best determined by means of a prism of the crystal, having its refracting edge parallel to the optic axis.

(66) The directions of the two refracted rays are given by the following construction.



Let AC be the incident ray, and CF the section of the surface of the crystal made by the plane of incidence. Let the incident ray be produced anywhere to B, and let BF be drawn perpendicular to it, meeting the surface in F. Let CD : CB : : sine of refraction : sine of incidence of the ordinary ray; and from the centre C, and with the radius CD, let the sphere DOG be described. Let the spheroid of revolution GE be described with the same centre, its axis of revolution being in the direction of the optic axis of the crystal, and equal to the diameter of the sphere, while the other axis is greater in the ratio of the ordinary to the extraordinary index. Now, if through F a line be drawn perpendicular to the plane of the diagram, and through that line there be drawn tangent planes, FO and FE, to the sphere and spheroid, the lines CO and CE, drawn from the centre to the points of contact, will be the directions of the ordinary and extraordinary rays. This elegant construction was given by Huygens.

For this construction Newton substituted another, without stating the theoretical grounds on which he formed it, or even advancing a single experiment in its confirmation. In this unsatisfactory position the problem of double refraction was suffered to rest for nearly a century; and it was not until the period of the revival of physical optics, in the hands of Young, that any new light was thrown upon the question. Young was led by the theory of waves to assume the truth of the law of Huygens; and, at his instigation, Wollaston undertook the experimental examination, which recalled to it the attention of the scientific world, and ended in its universal admission. The French Institute soon after proposed the question of double refraction as the subject of their prize essay, and the successful memoir of Malus left no doubt remaining as to the accuracy of the Huygenian law.

(67) We have seen that in Iceland spar the extraordinary index is *less* than the ordinary, and that consequently the extraordinary ray is refracted *from the axis*. This, however, is not universally true of all double-refracting crystals. Biot discovered that there were many crystals in which the extraordinary index was *greater* than the ordinary, and in which, therefore, the extraordinary ray is refracted *towards the axis*. Crystals of this kind he called *attractive*, while those of the former were denominated *repulsive*. Among the attractive, or (as they are sometimes called) positive crystals, are *quartz*, *ice*, *zircon;* the repulsive or negative class is far more numerous, and includes, among others, *Iceland spar*, *sapphire*, *ruby*, *emerald*, *beryl*, and *tourmaline*.

The Huygenian law applies to attractive as well as to repulsive crystals, it being observed, that in the former case the axis of revolution of the ellipsoid must be the *greater* axis of the generating ellipse; or, in other words, that the spheroid is *prolate* instead of oblate.

(68) It has been hitherto assumed that there is but one optic axis in every crystal, or one direction only along which a ray will pass without division. It was reserved for Sir David Brewster to discover that the greater number of crystals possessed *two optic axes*. Among the most remarkable of the crystals with two axes may be mentioned *arragonite*, *mica*, *sulphate of barytes*, *sulphate of lime*, *topaz*, and *felspar*. The angles range in magnitude through the entire quadrant; and they accordingly afford a new and important criterion for the distinction of mineral substances.

(69) It appears from the foregoing, that crystalline bodies may be divided into three classes, with respect to their action upon light, namely—

I. *Single-refracting crystals.*

II. *Uniaxal crystals*, or those which have one axis of double refraction.

III. *Biaxal crystals*, or those which have two such axes.

Sir David Brewster has established a connexion between these diversities of optical character and the varieties of crystalline form. He has shown that all the crystals of the first class, i. e. all *single-refracting* crystals, belong to the *tessular* system of Mohs; that all *uniaxal* crystals belong either to the *rhombohedral* or to the *pyramidal* system; and that crystals of the third class, or *biaxal* crystals, belong to one or other of the *prismatic* systems.

These important relations bear, in a very close and definite manner, upon the proximate cause of double refraction. It has been just mentioned, that the only crystals which do not possess the property of double refraction are those belonging to the *tessular* system, i. e. those whose fundamental form is the *cube*. Now in this, and its derived forms, we can assign three lines at right angles to one another, round which the whole figure is symmetrical; and we may, therefore, reasonably conclude that the density and elasticity of the crystal is the same in each of these directions, and consequently the same throughout. Again, crystals with one axis of double refraction belong either to the *rhombohedral*, or to the *pyramidal* system,—systems whose fundamental forms are the *rhombohedron* and the *straight pyramid*. In each of these forms there is *one axis of figure*, or one line round which the whole is symmetrical: and we may, therefore, assume that the density of the crystal is either greater or less in this direction than in others, while it is equal in all directions at right angles to it. The axis of form is, in this case, the axis of double refraction. Finally, in the *oblique pyramid*, which is the fundamental form of the *prismatic* systems, there is no one line, or axis of figure, round which the whole is symmetrical; and it is therefore probable that the density of the crystal is unequal in all the three directions. Such crystals are found to have *two optic axes*.

It has been stated, that in uniaxal crystals the optic axis is also the *axis of form*. In biaxal crystals, it did not at first appear that the optic axes were in any manner related to the lines which bound the elementary crystal. Sir David Brewster, however, ascertained that if two lines be taken, one bisecting the acute, and the other the obtuse angle contained by the optic axes, these (together with a third line at right angles to both) are closely connected with the primitive form.

These relations between the optical properties of crystals and their external forms are so close and intimate, that any change (however produced) in one of them, is found to be accompanied by a corresponding change in the other. Thus, if the form of a crystal be altered by mechanical compression, or change of temperature, its refracting properties undergo a corresponding change.

(70) It was long supposed that one of the refracted rays, in every crystal, followed the ordinary law of the sines, while the other was refracted according to the Huygenian law. But Fresnel has proved, both from theory and by experiment, that this is not the case, and that in biaxal crystals, *both rays* are refracted in an extraordinary manner, and according to a *new law*. It is, in fact, a consequence of his beautiful theory of double refraction, that the form of the wave, which is propagated in the interior of such a crystal, is neither a sphere nor spheroid, as in uniaxal crystals, but a curved surface of the fourth order. This surface is composed of two sheets; and if tangent planes be drawn to these, after the same manner as to the sphere and spheroid in the Huygenian law, the points of contact determine the directions of the two refracted rays. These more general laws of double refraction will be more fully considered hereafter.

(71) We may now proceed to illustrate some of the more remarkable effects of double refraction.

If a rhomboid of Iceland spar, or any other double-refracting crystal, be placed close to a small object,—as, for example, a black spot on a sheet of paper,—it will be observed that one of the images is sensibly *nearer* than the other; and that the difference of their apparent distances changes with the thickness of the crystal, and with the obliquity of the ray.

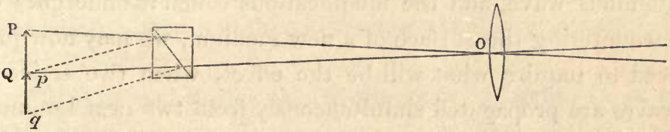
This effect is easily accounted for. It is a well-known principle of optics, that when an object is viewed through a denser medium bounded by parallel planes,—as, for example, a cube of glass,—the image is nearer to the surface than the object; the difference of their distances being to the thickness of the medium, as the difference of the sines of incidence and refraction to the sine of incidence. This interval, through which the image is made to approach, increases therefore with the refractive power of the medium; thus in water it is one-fourth of the thickness, in glass one-third, and so for other media. Now as double-refracting crystals have two refractive indices, of different magnitudes, there will be two images, at different distances from the surface. In Iceland spar, the ordinary index is greater than the extraordinary, and therefore the ordinary image is nearer than the other. The reverse is the case in positive crystals, such as quartz, in which the extraordinary index is the greater.

(72) The refractions being equal at the two parallel surfaces of the rhomb, whether the refraction be ordinary or extraordinary, the two rays will emerge parallel to the incident ray, and therefore parallel to one another; and the distance between them will be proportional to the thickness of the crystal. But if the surfaces be inclined, so as to form a prism, the deviation of the two rays will be different, and they will emerge inclined to one another; consequently the separation will increase with the distance.

Such a separation is of use in many experiments. In order to render the divergence of the emergent pencils greatest, the prism should be cut with its edge *parallel to the optic axis ;* so that the refraction may take place in a plane perpendicular to the axis. In this case the ordinary and extraordinary refractions differ by the greatest amount, and therefore the difference of the deviations of the two pencils is greatest. A double-refracting prism, so cut, is usually achromatized by a prism of glass, with its refracting angle turned in the opposite way.

A better arrangement has been suggested by Wollaston. Two prisms of the *same* substance, and of equal refracting angles, are cut in such a manner, that in one the refracting edge is *parallel* to the optic axis, and in the other *perpendicular* to it. They are then united, with their refracting angles turned in opposite directions, so as to form a parallelopiped ; and the effect of this arrangement is to double the separation of the images produced by either singly. By this duplication the weak double refraction of rock crystal is rendered very sensible.

(73) An achromatic prism of this kind is employed in the *double image micrometer*, an ingenious and valuable instrument invented by Rochon. It consists of a telescope, in which a prism, such as we have described, is introduced between the object-glass and its principal focus; and thus two images are



formed in the principal focus, whose interval is greater or less, according to the distance of the prism from that point. When the instrument is used, the prism is moved until the two images appear in contact, and its distance from the focus is then read on a graduated scale. The two angles in this case having the same subtense, the visual angle of the object is to the deviation produced by the prism, as the distance of the prism from the focus is to the focal length. Now the divergence of the two rays is constant for a given prism, and may be determined either by calculation or experiment; consequently, the visual angle is deduced from the preceding proportion. By this instrument Arago has determined the apparent diameters of the planets with great precision.

The same instrument has been also employed in war, to determine the distance of an inapproachable object. Thus, if it be required to ascertain the distance of the walls of a besieged town, in order to know whether they are within the range of shot, it is only necessary to measure by this instrument the angle subtended by a man, or any other object whose height is known approximately. The height of the object, divided by the tangent of the angle, is the distance required.

CHAPTER V.

INTERFERENCE OF LIGHT.

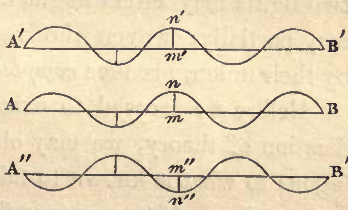
(74) HAVING considered the mode of propagation of a luminous wave, and the modifications which it undergoes on encountering the surface of a new medium, we may now proceed to inquire what will be the effect, when two series of waves are propagated simultaneously from two near luminous origins.

It is obvious that when two waves—one proceeding from each source—arrive at any instant at the same point of space, the particle of ether there will be thrown into vibration by both; and we are to consider what will be the result of this compound vibration. Now, it is demonstrated by analysis, that when *two small* vibrations are communicated at the same time to a material point, each of them will subsist independently of the other; and the motion of the point will, in consequence, be the *resultant* of the motions due to each vibration considered separately. This principle is denominated the *superposition of small motions.* Its nature may be made clear by a simple instance.

Let a pendulous body receive an impulse in any plane passing through the point of suspension: it will then, of course, vibrate in that plane. Now, at the lowest point of the arc of vibration, let a second impulse be given to the moving body, in a direction perpendicular to the plane in which it already vibrates. This impulse, if communicated to the body at rest, would cause it to vibrate in a plane at right angles to the former, and through an arc depending on the magnitude of the impulse. Now it will be found, on trial, that the distance of the body from the vertical, measured in either of these planes, is the same at any instant as if the other vibration did not exist; so that each vibration subsists *independently* of the other, and the result will be a compound elliptical vibration. We have here supposed the coexisting vibrations to take place in separate planes, in order that their independence may be more distinctly recognised. When the two vibrations are in the *same plane*, it is obvious that the resulting vibration will be also in that plane; and that its amplitude will be the *sum* of the amplitudes of the component vibrations when their directions conspire, and their *difference* when they are opposed.

(7[illegible]) Let us transfer this to the case of Light:—Let us suppose that two sets of waves start at the same time from two near luminous origins (which, for simplicity, we shall assume to be of equal intensity), and that a distant particle of ether is thrown into vibration by both at the same time. Then, supposing that these two vibrations are performed in the same plane, it follows from what has been said, that, when their directions conspire, they will be added together, and the resulting space of vibration will be *double* of either; and that, on the contrary, they will counteract one another, and the resulting vibration will be reduced to *nothing*, when their directions are opposed.

It is evident, further, that the *directions* of the vibrations will *conspire*, and therefore the space of vibration be doubled, when the two waves arrive in the *same phase;* and that, on the contrary, their directions will be opposed, and the resulting vibration reduced to nothing, when they arrive in *opposite phases.* Let the waving lines AB and A′B′, or AB and A″B″, represent the two undulations, the distance of any particle from its state of rest being represented by the ordinate, or perpendicu-



lar, at the corresponding point of the horizontal or *mean* line. Then, if the undulation A′B′ be superposed upon AB, the corresponding points of each being in the *same phase*, it is evident that the distances by which the particle at any point is removed from its state of rest by each, *mn* and *m′n′*, will be added together, and the space of vibration *doubled*. Whereas, if the undulations A″B″ and AB, whose corresponding points are in *opposite phases*, be superposed, the distances from the position of rest, *mn* and *m″n″*, lie on opposite sides of the mean line, and when added together *destroy one another*. Thus the space of vibration is doubled, when the waves arrive at the same point in the same phase: it is annihilated, when they arrive in opposite phases. Now the *intensity of the light* is as the square of the amplitude of vibration; the intensity, therefore, is *quadrupled* in the former case, and *destroyed* in the latter.

We have here taken, for the sake of illustration, two of the most important cases,—those, namely, in which the co-existing undulations are in complete *accordance*, or complete *discordance*. When this is not the case, and the waves meet in some intermediate stage of the vibratory movement, the *position* of the maximum will be altered, as well as its *magnitude*; and the rules for the composition of the coexisting vibrations bear a close analogy to the well-known rule for the composition of forces.

(76) We learn, then, as a result of the wave-theory, that two lights may either augment each other's effects; or they may partially, or even wholly, destroy one another, and thus, by their union, produce *complete darkness*.

Before we proceed to examine more particularly this indication of theory, we may observe that it is altogether analogous to what is known to take place in other cases of vibratory motion. If two waves of water arrive at the same point at the same instant, in such a manner that the crest of one wave coincides with that of the other, their effects will be added together, and the water at that point will be raised into a wave, whose height is the sum of the heights of the conspiring waves. If, on the other hand, the crest of one wave coincides with the *sinus*, or depression of the other, the height of the resultant wave will be the difference of the components; and, when these are equal, the resultant wave will entirely disappear.

We have a magnificent example of these effects in the well-known phenomena of the *spring* and *neap* tides; the tidal wave in the former case being the sum of the waves caused by the action of the Sun and Moon, and in the latter, their difference.

The peculiarity of the tides in the port of Batsha furnishes a still more striking instance of the principle of interference. The tidal wave reaches this port by two distinct channels, which are so unequal in length, that the time of arrival by one passage is exactly six hours longer than by the other. It follows from this that when the crest of the tidal wave, or the *high water*, reaches the port by one channel, it is met by the *low water* coming through the other; and when these opposite effects are also equal, they completely neutralize each other. At particular seasons, therefore, when the morning and evening tides are equal, there is *no tide* whatever in the port of Batsha; while at other seasons there is but *one tide in the day*, whose height is the difference of the heights of the ordinary morning and evening tides.

Analogous phenomena take place in sound, and produce the coincidences or *beats* in music. These effects occur when the condensed part of the aërial pulse, arising from one origin of sound, coincides with the rarified part of that proceeding from the other. They are often heard during the playing of a large organ, and give rise to the swelling and falling sounds which are heard, especially among the lower notes of the instrument.

(77) The interference of the aërial pulses may be exhibited to the eye. Let a compound tube be taken, consisting of two equal and similar branches terminating in a common trunk. It is evident, then, that if the air be thrown into the *same* state of vibration at the extremities of the two branches,—the particles going and returning simultaneously in both,—a *double* vibration will be propagated to the extremity of the main trunk, and may be rendered sensible by the agitation of the particles of sand on a stretched membrane. If, on the other hand, the air be in *opposite* states of vibration at the extremities of the branches, these will neutralize one another in the trunk, and the membrane, and the sand, will be quiescent. The conditions here described are attained, by bringing the ends of the branches over the parts of a vibrating plate which are in similar, or in opposite states of vibration. When the length of the tube is such that it is *in unison* with the vibrating plate, it will utter a distinct sound in the one case, while in the other it will be silent.

The alternate augmentation and intermission of sound observed by Young, when a tuning-fork is turned round its axis at a short distance from the ear, are easily referred to the same principles.

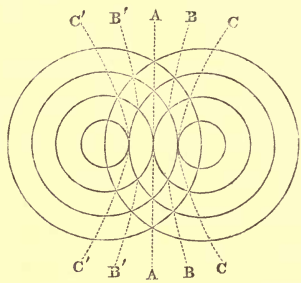
(78) That *two lights*, then, should *produce darkness*, is a result of the same kind as that two sounds should cause *silence*, or that two waves should make a dead level. But we are not left to analogy alone for the proof of this remarkable consequence of the wave-theory of light. The phenomenon itself has been established by the most direct and convincing experiments; and we shall soon see that it is observed in a multitude of cases where its existence was at first little suspected.

This important law—now known under the name of the *interference of light*—was for the first time distinctly stated and established by Young, although some facts connected with it were known to Grimaldi. The latter writer had even explicitly asserted that "an illuminated body may be rendered darker by the *addition of light*," and adduced a simple experiment in proof of it. Grimaldi's experiment was as follows. Let the Sun's light be admitted into a darkened chamber through two small and equal apertures of a circular form. Two diverging cones of light will be thus produced; and each of these cones will be surrounded by a penumbra in which the illumination is only partial. Now let these two beams be received on a screen at some distance, where the penumbras of the two cones overlap. It will be then observed, that although the greater part of this doubly illuminated space is brighter than the penumbra of one cone alone, yet the boundaries of the overlapping portions are much darker than the other parts of the penumbras which do not overlap; and if one of the beams be intercepted by an obstacle, this dark part will recover the brightness of the rest. Thus darkness may be produced by *adding light;* and, on the other hand, by withdrawing a portion of the light we may augment the illumination.

(79) This interesting experiment assumed a more distinct and decisive character in the hands of Young. If the two apertures be reduced to a very small size, and brought close together, and if the original light be homogeneous, we shall observe a series of alternate *bright* and *dark* bands, formed at those points where the waves proceeding from the two origins conspire, or are opposed. That these alternations of light and darkness are caused by the mutual action of the two beams, is proved by the fact, that if one of the beams be intercepted, the whole system of bands will disappear, and the light which remains become of uniform intensity. By withdrawing one of the lights, then, the dark intervals recover their brightness; so that darkness, in this case, must have been produced by the action of one light on the other.

(80) We shall best understand the circumstances of this phenomenon, by considering what takes place in another more familiar case of interference. If two stones be flung at the same instant into a pool of stagnant water, a series of circular waves will be propagated from each of the two centres of disturbance; and where these two sets of waves *cross*, they will produce effects similar to those we have been describing in the case of light. Where the *crest* of one wave falls upon the crest of another, they will be added together, and form a higher crest, or *ridge*, on the surface. And, on the contrary, where the crest of one wave meets the hollow, or *sinus*, of another, they will counteract one another's effects, and the water will stand at that point at its original level, as if undisturbed.

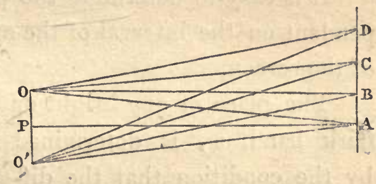
It is obvious that there will be several sets of consecutive points of each class, or several lines of *double disturbance* and *no disturbance*. One line of double disturbance, AA, will be produced by the meeting of waves *equidistant from the two centres;* —as the first of one with the first of the other, the second of one with the second of the other, &c. This line is necessarily *straight*. On either side of this there will be a line, BB, B′B′, consisting of those points where the first wave from one origin encounters the second from the other, the second from one the third from the other,—of all those points, in short, whose distances from the two centres *differ* by the *length of a single wave*. The next pair of lines, CC, C′C′, consist of those points whose distances from the two centres differ by the *length of two waves;* and so on. All these lines are *hyperbolas*, and on all of them the disturbance is doubled, and an elevated ridge raised on the surface. But there are likewise intermediate lines, composed of those points whose distances from the two centres differ by *half a wave*, by a *wave*



and half, by *two waves and half*, &c. On all these lines, the *crest* of the wave from one origin meets the *sinus* of a wave from the other; and these, therefore, are the lines of *no disturbance*. They are evidently hyperbolas like the former.

All that has been now said applies strictly to the phenomena of light, in the aspect under which they are presented by the wave-theory. In the same medium the waves of any given length are propagated with a constant velocity. When therefore two series of waves of equal length diverge at the same time from two centres, they will arrive at the same point in the *same phase*, provided that the lengths of the paths traversed are *equal*, or differ by any *whole number of undulations*. They meet in *opposite phases*, on the other hand, when the lengths of their paths differ by *half a wave*, or by *any odd multiple* of half a wave. The central bright band, then, is formed at those points where the distances traversed are *equal*. The next bright band on either side is produced where the distances traversed differ by the length of *one entire wave;* the succeeding pair where the distances differ by *two* whole waves; and so on. In the same manner, the first *dark* band is produced on either side of the central bright one, and at points for which the distances traversed differ by the length of *half a wave;* the second pair of dark bands where these distances differ by *one wave and half;* and so on.

(81) In Young's experiment, if the light diverging from the two apertures O and O′, be received on a screen, AD, it is found that the central bright band is formed at the point A, where the screen is intersected by the line PA,

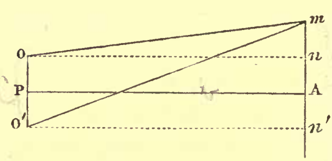


which bisects the line OO′ and is perpendicular to it. The *central* band, therefore, is formed where the paths traversed by the two pencils are *equal*. There will be a dark band on either side of the central bright one, and, beyond these, a pair of bright bands. If we measure the distances of one of these from the two apertures, we shall find that their difference, BO′ – BO, is a *constant* quantity, whatever be the position of the screen; this difference is the *length of a wave.* Beyond these is a second pair of bright bands, the difference of whose distances from the two centres, CO′ – CO, is double of the preceding, or equal to *two* whole waves; and in like manner, the difference of the lengths of the paths, at the place of each succeeding bright band, is found to be some exact multiple of the first difference, or of the length of a wave.

Performing the same measurements for the intermediate dark bands, we find that the difference in the lengths of the paths, where the first pair is formed, is half the difference, BO′ – BO, or *half the length of a wave.* The differences of the paths, at the place of each succeeding pair of dark bands, are found in like manner to be intermediate to the corresponding differences for the bright bands on either side, or to be *odd* multiples of half a wave.

The difference of the distances from the two apertures being constant for the successive points of the same band, it follows that these points must form an hyperbola, whose foci coincide with the two apertures. It will be easily seen that the curvature of these hyperbolic lines is very small, except close to their vertices; and that we may, without sensible error, consider them as coincident with their asymptots.

It is easy to determine the positions of the bands, as dependent on the interval of the apertures, and on the distance of the screen.



The place of any bright or dark band, m, is determined by the condition that the difference of its distances from the two apertures, $mO' - mO$, is an integer multiple of the length of half a wave. Now, drawing the lines On, O$'n'$, parallel to PA, and denoting the distance AP by b, the interval of the apertures OO$'$ by c, and the distance Am by x, the right-angled triangles Omn, O$'mn'$, give

$$\mathrm{O}m = \sqrt{b^2 + (x - \tfrac{1}{2}c)^2} = b + \frac{(x - \frac{1}{2}c)^2}{2b},$$

$$\mathrm{O}'m = \sqrt{b^2 + (x + \tfrac{1}{2}c)^2} = b + \frac{(x + \frac{1}{2}c)^2}{2b};$$

the distance b being very great in comparison with x and c. Hence

$$\mathrm{O}'m - \mathrm{O}m = \frac{(x + \frac{1}{2}c)^2 - (x - \frac{1}{2}c)^2}{2b} = \frac{cx}{b}.$$

But this difference is equal to $n\frac{\lambda}{2}$, λ being the length of a wave; we have, therefore, $2cx = nb\lambda$

$$x = \frac{nb\lambda}{2c};$$

$\lambda = \frac{2cx}{nb}$

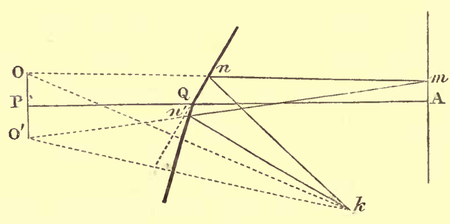
in which the even values of n correspond to the places of the bright bands, and the odd values to those of the dark ones.

The preceding formula enables us to compute the length of a wave of light, when the distances b, c, and x have been determined by accurate measurement. It has been found in this manner that the length of a wave is ·0000266 of an inch for the extreme red rays; ·0000167 for the extreme violet; and ·0000203, or about the $\frac{1}{50000}$ of an inch, for the mean rays of the spectrum.

(82) But though the principle of Interference seemed to be established by the experiments and reasonings of Young, it was not freed from all question. It might be supposed that the light passing by the edges of the apertures, in the experiment last described, underwent modifications of some kind or other which produced the observed effects. It was, therefore, of importance to show that these effects were

wholly independent of *apertures or edges;* and that *any two rays* proceeding from the same luminous origin, and meeting under a small obliquity, will *interfere* in the manner already described, whatever be the attending circumstances. This has been done by Fresnel; and the experiment, which he devised for the purpose, has been justly ranked among the most important and instructive in the whole range of Physical Optics.

Two plane mirrors are placed so as to meet at a very obtuse angle. A beam of light diverging from the focus of a lens is suffered to fall upon them; and there will be therefore two reflected beams, whose directions are inclined at a very small angle. Here, then, are two beams diverging from the same luminous origin, separated simply by reflexion at plane surfaces, without the intervention of edges, or of anything accidental which can be regarded as contributing to the result. These beams, however, still interfere, and produce a succession of alternate bright and dark bands, in the manner already explained. In order to satisfy ourselves that these bands are in fact produced by the mutual action of the two beams, we have only to intercept one of them, by covering one of the mirrors, and the whole system instantly vanishes.



Let Qn and Qn' represent the sections of the two mirrors, which we shall suppose to be perpendicular to the plane of the diagram; and let k be the luminous origin, or the focus of the lens in which the Sun's rays are concentered. Then taking the points O and O′ at equal distances on opposite sides of the mirrors, these points will be the foci of the two reflected pencils, or the points of divergence of the two beams. Now it is found, in the first place, that the bands are parallel to the line of intersection of the two mirrors; secondly, that they are symmetrically placed on either side of a plane passing through this intersection, and through the point P, which bisects the interval between the two foci O and O′; and thirdly, that in proceeding from the mirrors, they are propagated in hyperbolas, whose foci are O and O′, and whose common centre is P.

(83) These results are in exact accordance with theory. In fact, since $On = nk$, and $O'n' = n'k$, the difference of the paths traversed by the reflected rays, knm and $kn'm$, when they meet at m, is the same as if they had reached that point diverging directly from the points O and O′. All, then, that has been said respecting the interference of the pencils diverging from two near luminous origins, will apply to this case. Since $kQ = OQ = O'Q$, the line QP, which bisects the line OO′, is also perpendicular to it, and any point of it, as A, is equidistant from O and O′. The bands, therefore, are symmetrically situated with respect to this line; and the distance, Am, of the band of any order from the central band, is equal to $\frac{n\lambda \mathrm{AP}}{\mathrm{OO'}}$.

This distance is easily expressed in terms of given quantities. For $PQ = OQ \times \cos OQP = kQ \times \cos OQP$; and $OO' = 2OP = 2kQ \times \sin OQP$. But since the angles kQO, kQO', are bisected by the lines Qn and Qn', it is easy to see that the angle OQP (or the half of the angle OQO′) is equal to the inclination of the mirrors. If then this inclination be denoted by ε, and the distances kQ and QA by a and b, we have

$$OO' = 2a \sin \varepsilon, \qquad AP = a \cos \varepsilon + b = a + b;\ q.p.;$$

and therefore the distance of the band of the n^{th} order from the centre is expressed by the formula

$$\frac{(a + b)\, n\lambda}{2a \sin \varepsilon}.$$

(84) The phenomenon of interference is displayed in a striking manner by the mutual action of *direct* and *reflected* light; and the experiment in this form is more manageable than that of Fresnel. We have only to take a piece of plate glass, or a metallic reflector, and to place it in such a position that the rays diverging from the luminous origin shall be reflected at an angle of nearly 90°. A screen placed on the other side of the mirror will receive both the direct and reflected pencils; and as they meet under a small angle, and have traversed paths differing by a small amount, they are in a condition to interfere. It will be readily seen that the system of bands, formed in this manner, is but half of that produced in Fresnel's experiment.

(85) There is yet another mode of studying the fundamental phenomenon of interference, which is in some respects more convenient than any of the former. It is obvious that the original beam may be separated by *refraction*, as well as reflexion; and if the inclination of the two refracted pencils be small, similar results will take place. For this purpose it is only necessary to procure a prism with a very obtuse angle, and to allow the beam of light to fall perpendicularly on the opposite face. It is evident that this beam will be differently refracted, at emergence, by the two faces which contain the obtuse angle; and that it will be thus divided into two beams, which will be slightly inclined. These beams then proceed from one common origin, and meet under a small obliquity, and therefore fulfil all the conditions necessary for their interference. It is found, accordingly, that a series of alternate bright and dark bands is formed parallel to the edge of the prism.

(86) It will be evident, from what has been said, that the central fringe produced by the interference of two pencils is the locus of those points at which they arrive in the *same time;*

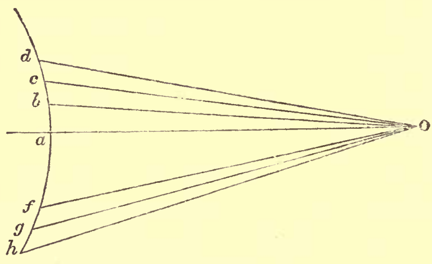
and, accordingly, when neither of the pencils has experienced any interruption in its progress, the points of that fringe will be equally distant from the two luminous origins. The case is altered, however, if we interpose a thin plate of a denser medium in the path of one of the interfering rays. If the light is *retarded* in the denser medium, it is obvious that the points reached in the same time will no longer be equally distant from the two centres, but will be *nearer* to that whose light has undergone the retardation. The reverse will be the case if the light is *accelerated* in the interposed plate; so that the central fringe, and the whole system, will be shifted *towards* the side of the interposed plate in the former case, and *from* it in the latter. Here then we have a complete *experimentum crucis*, by which to decide between the theory of emission and that of waves; and its result, as we have already stated, is conclusive in favour of the wave-theory.

The amount of the displacement of the fringes, in this important experiment, depends on the thickness of the interposed plate, and on its refractive index; so that any one of these quantities will be determined when the other two are known. Accordingly, by observing the displacement of the fringes produced by a plate of known thickness, the refractive index of the plate is found. Arago and Fresnel have employed this method to determine the refractive powers of the gases. The method is susceptible of very great precision. By observing the position of the fringes formed by two rays, one of which has passed through air, and the other through a vacuum, Arago has shown that the minutest changes in the refractive power of the air may be observed—such, for example, as would arise from a variation of temperature amounting to $\frac{1}{20}$th of a degree centigrade. By the same method it was ascertained that dry air was more refractive than air saturated with moisture, the difference amounting, very nearly, to the millionth of the refractive index.

In connexion with these results, Arago has shown, that the *scintillation* of the stars is a phenomenon of interference, due to changes in the refractive powers of portions of the atmosphere, through which different portions of light reach the eye.

(87) The principle of interference furnishes the complete answer to the difficulty suggested by Newton, and shows in what manner the *rectilinear propagation* of light is reconciled to the wave-theory. It had been objected, that if light consisted in the undulations of an elastic fluid, it should diverge in every direction from each new centre, and so bend round interposed obstacles, and obliterate all shadow. To this we reply, that light *does* diverge in every direction from each new centre,—that it *does* bend round interposed obstacles; but that *shadows* notwithstanding *exist*, because the several portions of this laterally-diverging light *destroy one another* by interference, and *no effect* is produced, except by those parts of the wave which are in the right line joining the luminous origin and the eye.

To see this, let *abcd* represent a portion of a spherical wave; and let O be the place of the eye, and O*a* the line drawn from it to the luminous centre. Commencing from the point *a*, let portions *ab*, *bc*, *cd*, &c., be taken, such that the differences of the distances of their extremities from the point O shall be the same for all, and equal to *half a wave*. Now we may suppose all these portions of the grand wave to be so many centres of disturbance; and it is obvious that the *secondary* waves, sent



by each pair of consecutive portions to the eye, are in complete discordance, and should wholly destroy one another if their intensities were equal. It is easy to see that this is the case with respect to portions, as *fg*, *gh*, which are remote from the point *a*. For the magnitudes of the waves sent by the several portions to any point depend—first, on the *magnitudes* of these portions themselves, and secondly, on the *angles* which the line drawn from them to that point makes with the front of the wave. As respects the former, it is evident that *ab* is greater than *bc*, *bc* than *cd*, and so on; but these differences continually diminish, and the magnitudes of the consecutive portions approach indefinitely to equality, as they recede from the point *a*. The same is true of the *obliquities*. Hence, the portions of the wave, *fg*, *gh*, remote from the point *a*, destroy one another's effects, and the effect on the eye, or on a screen at O, will be entirely due to those parts of the grand wave which are in the neighbourhood of the line connecting that point with the luminous origin.

Of these parts *ab* produces the greatest effect being both the largest and the least oblique. The effect of the neighbouring portions is, however, very sensible, and we shall have occasion hereafter to study some important phenomena to which they give rise. In the meantime, one remarkable consequence of this explanation is obvious—namely, that if the alternate portions *bc*, *de*, &c., whose effects are *negative*, be stopped, the total effect will be *augmented*, and the resulting light increased by intercepting a portion of the wave. We shall see hereafter that this startling conclusion is confirmed by experiment.

CHAPTER VI.

DIFFRACTION.

(88) It has been shown to be a result of the wave-theory, that the intensity of the light which encounters an obstacle must diminish rapidly within the edge of the geometric shadow. It now remains to consider the other phenomena which arise under these circumstances; and it will be found that the same theory affords the most complete account, not only of their general characters, but even of their most minute details.

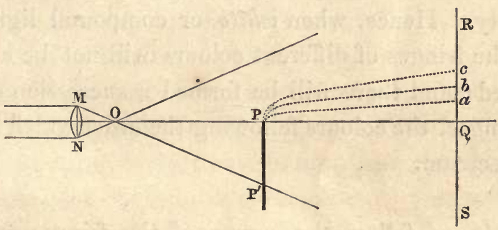
In order to understand the theory of *shadows*, it is necessary to investigate their laws in the simple case in which the magnitude of the luminous body is reduced to a point. The effects thus presented were first observed, and in some degree explained, by Grimaldi; and they have been since studied, as a separate branch of Optical Science, under the title of *diffraction* or *inflexion.*

Grimaldi found, that when a small opaque body was placed in the cone of light, admitted into a dark chamber through a very small aperture, its shadow was much larger than its geometric projection; so that the light suffered some deviation from the rectilinear course in passing by the edge. On observing these shadows more attentively, he found that they were bordered with three iris-coloured fringes, which decreased in breadth and intensity in the order of their distances from the shadow, and which preserved the same distance from the edge throughout its entire extent, unless where the body terminated in a sharp angle. Similar fringes were observed, under favourable circumstances, *within* the shadows of narrow bodies.

The phenomena of diffraction were subsequently examined by Hooke and by Newton; and, lastly, in the hands of Young and Fresnel, they have been forced to furnish evidence in favour of the wave-theory, which few who impartially examine it can continue to withstand. We shall first describe the most important of these phenomena, and afterwards examine them in their bearing upon the two theories.

(89) The most obvious of these phenomena are the modifications which light undergoes in passing by the edge of an obstacle of any kind.

Let a beam of homogeneous light, entering a dark chamber, fall on a lens of short focal length, MN, by which it is



brought to a focus at O, and thence diverges. Let an obstacle, PP′, be placed in the diverging beam, and let the shadow which it casts be received upon a sheet of white paper at Q, or on a piece of roughened glass. We shall then observe the following phenomena:

I. The line OPQ, which is the boundary of the *geometric shadow*, is not the actual boundary of light and shade.

II. The space below this line, QS, is not absolutely dark, but is enlightened by a faint light, which extends to a sensible distance within the geometric shadow, and gradually fades away as it recedes from the edge of this shadow at Q.

III. On the other side of the boundary of the geometric shadow, at QR, the paper is not uniformly illuminated by the diverging beam, but is observed to be covered with a series of alternate bright and dark bands, which are parallel to the edge of the shadow. The distances of these fringes *inter se*, and from the edge of the shadow, vary with the position of the screen, and diminish indefinitely as the screen is brought near the obstacle. These fringes succeed one another for many alternations, becoming, however, less marked as the distance from the edge of the geometric shadow increases, until at length they are wholly obliterated and lost. They preserve the same distances from the shadow in all parts, except only where the edge of the body forms a sharp angle.*

IV. The dimensions of the fringes vary with the colour of the light; being broadest in *red* light, narrowest in *violet* light, and of intermediate magnitude in the light of mean refrangibility. Hence, when *white* or compound light is employed, the fringes of different colours will not be accurately superposed; and there will be formed a succession of *iris-coloured* fringes, the colours following the order which they have in the spectrum.

(90) If we follow the course of the fringes from their origin, we shall observe that they are propagated in lines sensibly curved, whose concave side is turned towards the shadow. In order to obtain accurate measures of the distances of the fringes from the edge of the shadow, at different distances from the obstacle, Fresnel viewed them directly with an eye-piece, furnished with a micrometer. He thus ascertained that the curved path of each fringe was an *hyperbola*, whose summit coincided with the edge of the obstacle, and whose centre was the middle point of the line connecting that edge with the luminous origin.

* If this angle be *salient*, the fringes, instead of forming a similar angle, are observed to curve round the shadow. When the angle is *re-entrant*, they cross, and enter on the shadow at each side, without interfering with one another.

If we consider these hyperbolas as coincident with their asymptots (which may be done without sensible error, unless near the edge of the obstacle), and if we then determine the angles which they make with one another, and with the edge of the geometric shadow, we shall find that these angles increase rapidly as the distance of the obstacle from the luminous point diminishes. When this distance is about 40 inches, the fringes are very close together, the fringes of the first and second order making an angle with one another of less than 2′ in red light. At the distance of 4 inches this angle is increased to more than 5′; and at $\frac{4}{10}$ of an inch it exceeds 16′. Thus the fringes dilate, as the edge of the obstacle approaches the luminous origin.

(91) In this experiment the incident light is supposed to diverge from a luminous point. If the dimensions of the luminous origin had been considerable, it will be easily understood that each line in it, parallel to the edge of the obstacle, would give rise to a different system of fringes; and, as the dark bands of some of these systems must coincide with the bright bands of others, every trace of the phenomenon would be obliterated.

(92) The preceding experiments exhibit the effect of a *single* edge. When the light diverging from the luminous point is suffered to pass by *two near edges*, the phenomena will be varied in a very interesting manner.

Let a *fine wire* be placed in the pencil of light diverging from a luminous point, and let its shadow be received on a screen, or plate of roughened glass, as before. We then observe, outside the geometric shadow, a set of parallel bands, or fringes, analogous to those produced by the single edge in the former experiment. These are the *exterior fringes*. But we observe further that the whole space of the geometric shadow itself is also occupied by parallel stripes, alternately bright and dark. These are the *interior fringes ;* and they are in general closer, and more finely marked than the exterior. When the breadth of the obstacle is considerable, the interior fringes disappear, and the phenomena fall under the class already examined.

The interior fringes are propagated, like the exterior, in hyperbolic curves; but their curvature is less considerable, and the deviation from a right-lined course is scarcely perceptible within the limits at which they are commonly observed. They are also, like the exterior fringes, broader in red than in violet light, and of intermediate breadths in the light of intermediate refrangibility. Accordingly, in compound or white light, the fringes of different dimensions are superposed; and the bands are no longer alternately bright and black, but coloured with different tints, in the order of the colours of the spectrum.

(93) It still remains to examine the effects produced by two edges turned *inwards*, so as to form an *aperture* of any dimensions.

For this purpose Fresnel employed an instrument consisting of two metallic plates, one of which is fixed in the frame of the apparatus, while the other is moveable by means of a fine screw. The edges of these plates are right-lined and parallel, so that they form always a rectangular aperture; and, by means of the adjusting screw, the magnitude of this aperture may be varied at pleasure.

When a narrow rectangular aperture, thus formed, is substituted for the wire in the last experiment, the resulting phenomena are very remarkable. In the first place, the luminous beam diverges considerably after passing the aperture, so that the space which it occupies on the screen, or roughened glass, is much wider than the geometric projection of the aperture. Secondly, the entire of this space is covered with parallel bands, or fringes, alternately bright and dark, distributed symmetrically on either side of the line passing through the luminous point and the centre of the aperture.

If we trace these fringes, from their origin at the aperture to any distance, we shall find that they are propagated in hyperbolas, like the former. The curvature of these hyperbolic branches, and their inclination to one another, depend on the breadth of the aperture, and on its distance from the luminous point. Fraunhofer, who observed this class of phenomena with great attention and care, found that the angular distances of the successive bands of any given colour from the central line formed an *arithmetical progression*, whose common difference was equal to its first term; and that, when different apertures were used, the distances of one and the same band from the central line were *inversely as the breadths of the apertures*. These fringes are broadest and most widely separated in red light; they are narrowest and closest in violet light, and of intermediate magnitudes in the intermediate rays of the spectrum. In white light, therefore, they present the succession of colours observed in other cases.

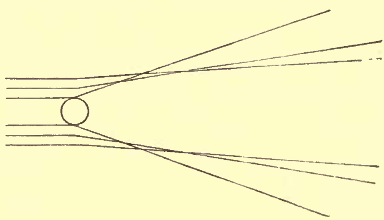
When the aperture is formed by two straight edges *slightly inclined*, Newton observed that the fringes were not accurately parallel to the edges, but became broader as they approached; and that they finally crossed, and formed two hyperbolic branches, one of whose asymptots is perpendicular to the line bisecting the angle of the edges, while the others are parallel to the edges themselves.

(94) It is scarcely necessary to observe that the phenomena of diffraction may be endlessly varied, by varying the form of the diffracting edge. The preceding cases have been selected as the most elementary. They are abundantly sufficient, when pursued into numerical details, to test the truth of any theory which may be applied to this class of phenomena; and such a theory being once established, the laws of

the more complex appearances are best sought for in its deductions. We shall proceed, therefore, to consider the preceding phenomena in their relation to the two theories of light.

(95) Newton conceived the rays of light to be *inflected* in passing by the edges of bodies, by the operation of the attractive and repulsive forces, which the molecules of bodies were supposed to exert upon those of light before they arrived in actual contact. By the operation of such forces, Newton was enabled to explain the laws of reflexion and refraction; and it was reasonable to suppose that the same forces played an important part in the phenomena now under consideration.

Thus, the rays passing by the edges of a narrow opaque body, such as a hair or fine wire, are supposed to be turned aside by its repulsion; and, as this force decreases rapidly as the distance increases, it follows that the rays which pass at a distance from the body will be less deflected than those which pass close to it, as is shown in the annexed diagram. The *caustic* formed by the intersection of these deflected rays will be concave inwards; and as none of the rays pass within it, it will form the boundary of the *visible shadow*.



Thus this supposition explains satisfactorily the *curvilinear* termination of the visible shadow, and its excess above the geometric one.

To account for the *fringes* which are parallel to the edge of this shadow, Newton appears to have supposed the attractive and repulsive forces to succeed one another for some alternations; and the molecules composing each ray, in their passage by the body, to be bent to and fro by these forces, in a serpentine course, and to be finally thrown off at one or other of the points of contrary flexure. The intersection of the rays thus thrown off at different points of the same serpentine course will form a caustic or fringe; so that each succeeding fringe will be produced by the rays which pass at a given distance from the edge of the body.

Finally, the separation of white light into its elements is explained, by supposing that the rays which differ in refrangibility differ also in *inflexibility*,—the body acting alike upon the less refrangible rays at a greater distance, and upon the more refrangible rays at a less distance.

It is needless to comment upon the vagueness of these explanations. Newton himself was dissatisfied with them, and the subject fell from his hands unfinished. Still, however, the mere guesses of such a mind as that of Newton must claim a deep interest; and it was natural that among his followers more weight should be attached to these conjectures, than he himself ever assigned to them. It seems necessary, therefore, to advert to some of the circumstances of the phenomena, which are not only unexplained by this theory, but which seem moreover entirely at variance with it.

(96) If the phenomena of inflexion be the effects of attractive and repulsive forces emanating from the interposed body,—and if these forces are the same, or even analogous to those to which the reflexion and refraction of light are ascribed in the theory of emission,—it will follow that they must exist in different bodies in *very different degrees;* so that the amount of bending of the rays, and therefore the breadth of the diffracted fringes, should vary with the *mass*, the *nature*, and the *form* of the inflecting body. Now it is clearly ascertained, on the contrary, that *all bodies*, whatever be their nature or the form of their edge, produce under the same circumstances fringes *identically the same;* and, in fact, the partial interruption of light, caused by the interposition of an obstacle of any kind, appears to be the only condition essential to the phenomenon.

Gravesende seems to have first observed that the *nature* or *density* of the body had no effect upon the magnitude of the diffracted images; and the fact has since been confirmed in the fullest manner by almost every inquirer in this branch of experimental science. It is now admitted that the inflecting forces, if such exist, must be independent of the chemical nature of the inflecting body, and altogether different from those to which, in the theory of emission, the phenomena of reflexion and refraction are ascribed.

To ascertain whether the *form of the edge* had any effect upon the fringes, Fresnel took two plates of steel, the edge of each of which was rounded in one half of its length, and sharp in the remaining half,—and placed the rounded portion of each edge opposite the angular part of the other. If then the position of the fringes depended on the form of the edge, the effect would thus be doubled, and the fringes appear broken in the midst. They were found, on the contrary, to be perfectly straight throughout their entire length.

Again, the inflecting forces (though they must be supposed to vary in intensity with the form and mass of the body, and with the distance of the luminous molecule from the edge) cannot be conceived to depend in any way upon the *distance previously traversed* by the molecule, before it arrives in the neighbourhood of that edge; so that the magnitude and position of the fringes, in this hypothesis, cannot vary with the distance of the inflecting edge from the luminous point. But this conclusion is the reverse of fact. The fringes *dilate*, and their mutual inclination is increased, as the obstacle approaches the luminous origin.

The phenomena of diffraction, therefore, do not arise from the operation of attractive and repulsive forces, exerted by the molecules of bodies upon those of light.

(97) The same objections apply to the hypothesis of Mairan and Du Tour, which ascribes these effects to the *re-*

fraction of small atmospheres encompassing the bodies, and of a different refractive power from the surrounding medium. For, if such an atmosphere be retained by the attraction of the body which it encompasses,—and this seems to be the only intelligible mode of accounting for its presence,—its density, and its form, must vary with those of the body itself; and, consequently, its effects upon the rays of light must vary also.

We are forced, then, to conclude, that the phenomena of diffraction are inexplicable in the system of emission; and we proceed to examine in what manner, and with what success, the principles of the wave-theory have been applied to their explanation.

(98) This important step in Physical Optics was made by Young, and all the complicated phenomena of diffraction are now reduced to the simple *principle of Interference.*

The *exterior* fringes, formed without the shadows of bodies, were ascribed by Young to the interference of two portions of light, one of which passed by the body, and was more or less deviated, while the other was obliquely reflected from its edge. The fringes formed by narrow apertures were, in like manner, supposed to arise from the interference of the two pencils reflected from the opposite edges; while the *interior* fringes, within the shadows of narrow bodies, were accounted for by the interference of the pencils which passed on either side of the body, and were bent into the shadow. The observed facts closely correspond with the calculated results of this theory; and in the case last mentioned, Young proved that the phenomenon admitted no other explanation. Placing a small opaque screen on either side of the diffracting body, so as to intercept the portion of light which passed by one of its edges, the whole system of bands immediately disappeared, although the light passing by the other edge was unmodified.

The *general* laws of the fringes—the dependence of their magnitudes upon the length of the wave, and upon the distances of the luminous origin and of the screen—are fully explained on these hypotheses. It is easy to infer from them that, as the position of the screen is varied, the successive points of the same fringe are not in a right line, but form an hyperbola; and that, when the distance of the luminous origin is lessened, the inclination of these hyperbolic branches (considered as coincident with their asymptots) augments, and the fringes dilate.

The theory of Young, however, did not bear a closer comparison with facts. If the exterior fringes arose from the interference of the direct light with that obliquely reflected from the edge of the obstacle, it would follow that the intensity of the light in them should depend on the extent and curvature of the edge. Fresnel found, on the contrary, that the fringes were wholly independent of the form of the diffracting edge; the fringes formed by the back and by the edge of a razor, for example, being precisely alike in every respect. In the other cases of diffraction also, he perceived that the rays grazing the edge of the body were not the only rays concerned in the production of the fringes; but that the light which passed by these edges at sensible distances was also deviated, and concurred in their formation. Fresnel was thus forced to seek a broader foundation for his theory.

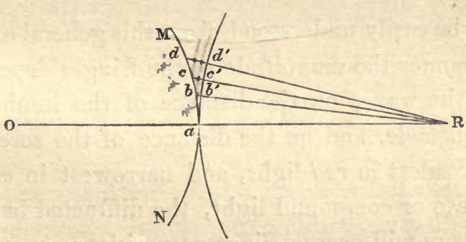
(99) In this theory the phenomena of diffraction are ascribed to the interference of the *partial*, or *secondary* waves, which are separated from the grand wave by the interposition of the obstacle. In applying this principle, Fresnel supposes the surface of the wave, when it reaches the obstacle, to be subdivided into an indefinite number of equal portions. Each of these portions may, by the principle of Huygens, be considered as the centre of a system of partial waves; and the mathematical laws of interference enabled him to compute the resultant of all these systems at any given point. This resultant vibration, Fresnel has shown, is in general expressed

by means of two integrals, which are to be taken within limits determined by the particular nature of the problem. Its square is the measure of the intensity of the light; and it is found that its value has several maxima and minima, which correspond to the intensities of the light in the bright and dark bands.

The problem of diffraction was thus completely solved; and its laws derived from the two principles to which the laws of reflexion and refraction are themselves referred,—the *principle of Interference* and the *principle of Huygens*. It only remained to apply the solution to the principal cases, and to compare the results with those of observation. The cases of diffraction selected by Fresnel are those whose laws have been already explained; viz. the phenomena produced—1, by a single straight edge; 2, by an aperture terminated by parallel straight edges; and 3, by a narrow opaque body of the same form. The agreement of observation and theory is so complete, that the computed places of the several bands seldom differ from those observed by more than the 100th part of a millimetre.

(100) The general circumstances of these phenomena may be deduced by very simple considerations from the principles already laid down; although the complete development of these principles demands the aid of a complicated analysis.

Thus, in the case of the fringes produced by a *single*



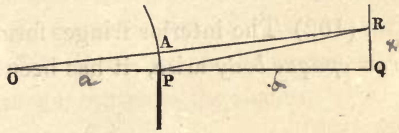
edge, let O be the luminous origin, MaN a diverging wave, and R any point at which the illumination is sought. From this point, as centre, let a circle be described, touching the circle MaN in a, and let the lines R$b'b$, R$c'c$, &c., be drawn in such a manner that the intercepts bb', cc', dd', &c., are equal respectively to one, two, three, &c. semi-undulations. The effect produced at the point R is then, by the principle of Huygens, the sum of the effects produced by each of the portions ab, bc, cd, &c., separately. But, the distances of these consecutive portions from the point R differing by half a wave, their effects will be opposed at that point; so that, if m denote the intensity of the light sent from the portion ab, m' that from bc, &c.—the light sent from the indefinite wave, aM or aN, being taken as unity—the actual light which reaches the point R will be 1, $1 + m$, $1 + m - m'$, $1 + m - m' + m''$, &c., according as the obstacle is placed at the point a, b, c, d, &c. And the intensity of the light when the obstacle is altogether withdrawn is

$$1 + m - m' + m'' - m''' + \&c. = 2.$$

Now, as the terms of this series are continually decreasing, and are affected alternately with opposite signs, it is manifest that if we stop at any term, the sign of the remainder will be the same as that of its first term, and therefore alternately positive and negative. Accordingly the intensities, $1 + m$, $1 + m - m'$, $1 + m - m' + m''$, &c., are alternately greater and less than 2; and the intensity of the light sent to the point R is alternately greater and less than when no obstacle is interposed.

It will be easily understood, from this general explanation, in what manner the magnitude of the fringes depends on the length of the wave, on the distance of the luminous origin from the obstacle, and on the distance of the screen. They must be broadest in *red* light, and narrowest in *violet* light; and in white or compound light, the diffracted bands of different colours will occupy different positions, so as to form a succession of *iris-coloured* bands having the violet or blue inside, and the red without. After a few successions these bands wholly disappear, owing to the superposition of bands of different colours.

(101) It is easy to compute the relative places of the same fringe, for different positions of the luminous point, and of the screen.



Let P be the edge of the obstacle, PA a portion of the wave, diverging from O, which has just reached that edge; and let QR be the screen, and R the place of a fringe of any given order. Then, in order that this point should belong to the same fringe, for every distance of the luminous origin and of the screen, it is only necessary that the interval of retardation, RP – RA, of the central and marginal parts of the wave should be constant. For in this case the whole wave, AP, may be divided into a given number of parts, such that the difference of the distances of the successive points of division from the point R shall be constant; and therefore the effective wave consists of the same number of elementary portions in the same relative state as to interference.

Now, denoting OP by a, PQ by b, and QR by x, we have

$$RP = \sqrt{b^2 + x^2} = b + \frac{x^2}{2b},$$

q. p., since x is very small in comparison with b. Similarly,

$$RO = \sqrt{(a+b)^2 + x^2} = a + b + \frac{x^2}{2(a+b)};$$

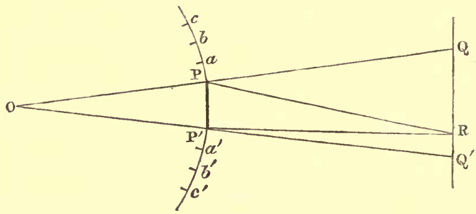
so that $RA = b + \dfrac{x^2}{2(a+b)}$, and $RP - RA = \frac{1}{2} x^2 \left(\dfrac{1}{b} - \dfrac{1}{a+b}\right)$.

But, by the condition of the question, this difference is a constant quantity; and denoting this constant by δ, we have

$$\tfrac{1}{2} x^2 \left(\frac{1}{b} - \frac{1}{a+b}\right) = \delta, \qquad x = \sqrt{\frac{2\delta\,(a+b)\,b}{a}}.$$

When b varies, a remaining unaltered,—i. e. when the position of the screen is varied,—the value of x is the ordinate of an hyperbola whose abscissa is b; so that the successive points of the same fringe belong to an hyperbola, whose summit is the edge of the obstacle.

(102) The interior fringes formed in the shadow of a *narrow opaque body* arise, it has been said, from the interference



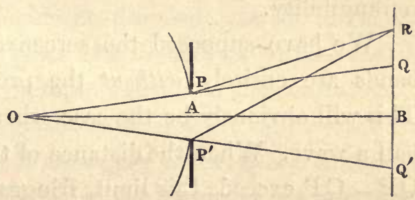
of the two portions of the wave which pass by the edges on either side. Let PP′ be the section of the opaque body, Pc and P′c' the two portions of the diverging wave which has just reached its edges, and R any point of the shadow. Then, if these portions be divided in the points a, b, c, &c., a', b', c', &c., in such a manner, that the difference of the distances of any two consecutive points from the point R is equal to half an undulation, the elementary wave sent from each portion will be in complete discordance with those sent from the two adjacent portions; so that, if the several portions be equal, they will neutralize one another's effects at the point R, with the exception of the extreme portions, Pa, P′a', the halves of which next the edges remain uncompensated.

Now the arcs Pa, ab, bc, &c., are very nearly equal, when the lines drawn from their extremities to the point R are sufficiently inclined to the normal,—or, in other words, when this point is sufficiently removed from the edge of the geometric shadow. In this case, then, the only efficacious parts of the wave are the halves of the extreme portions, Pa and P′a'; and the intensity of the light at the point R will be determined by the difference of their distances from that point, or (which comes to the same thing) by the difference of the lengths of the lines connecting it with the edges of the obstacle. The phenomena of interference are therefore the same as in the case of light emanating from two near origins, already considered; and we may transfer to the present case the conclusions arrived at in (81). Accordingly, if c denote the breadth of the obstacle, and b its distance from the screen, the distance, x, of any band from the centre of the shadow is

$$x = \frac{nb\lambda}{2c}.$$

(103) The positions of the fringes formed by a *narrow rectangular aperture* are determined by a similar formula.

Let PP′ be the section of the aperture, PAP′ the portion of the wave which has just reached it, diverging from the luminous origin at O; and let QQ′ be the projection of the aperture on the screen. Then, if we take the point R on this screen in such a manner, that the difference of its distances from the edges of the aperture, RP′ − RP, shall be equal to a whole number of semi-undulations, that point will be the centre of a *dark* or *bright* band, according as the assumed number is *even* or *odd*. For, in the former case, the wave PAP′ may be divided into an even number of parts, such that the distances of every two consecutive points of division from the point R differ by half an undulation; the waves sent by every two consecutive portions to the point R will therefore be in complete discordance, and the total effect at that point will be null. On the other hand, when the difference RP′ − RP is equal to an odd number of semi-undulations, the number of opposing portions of the wave will be odd, and as the alternate portions compensate



each other's effects at the point R, there will remain one portion producing there its full effect.

The successive bands being formed at the points for which $RP' - RP = n\lambda$, it is obvious that their distances, RB, from the centre of the projection of the aperture, will be given by the same formula as in the case last considered, c being now the breadth of the aperture,—with this difference, however, that the dark bands correspond to the even values of n, and the bright bands to the odd values, which is the reverse of what takes place in the bands formed within the shadow of an opaque obstacle. We learn then, 1st, that the distances of the successive fringes of any colour form an arithmetical progression whose common difference is equal to its first term; 2ndly, that they vary directly as the distance of the screen, and inversely as the breadth of the aperture; and 3rdly, that they are proportional to the length of the wave; and therefore greatest for the extreme red rays, least for the extreme violet, and of intermediate magnitude for the rays of intermediate refrangibility.

We have supposed the screen to be so remote that the bands are entirely *without* the projection of the aperture. This will obviously be the case when $QP' - QP$ is less than half a wave. When the distance of the screen is so small that $QP' - QP$ exceeds this limit, fringes will be visible also *within* the projection of the aperture. In this case the portions into which the wave is divided are sensibly different in magnitude, as well as obliquity. The reasoning above employed is therefore no longer applicable; and the points of maximum and minimum brightness can only be obtained by a complete calculation of the intensity of the light.

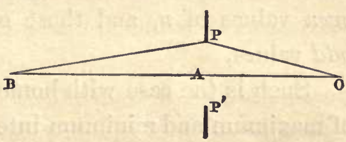
(104) The phenomena of diffraction hitherto considered are of the simplest class: but as such phenomena arise in every instance in which light is in part intercepted, it is obvious that they admit of endless modifications, varying with

the form of the interposed body. Some of these are too remarkable to pass unnoticed.

Among the most striking of these effects are those produced by light diverging from a luminous origin, and transmitted through a small *circular aperture*;—as, for example, that formed by a pin in a sheet of lead. When the transmitted light is viewed through a lens, the image of the aperture appears as a brilliant spot, surrounded by coloured rings of great vividness; and these vary in the most beautiful manner, as the distance of the aperture from the luminous origin, or from the eye, is altered. When the latter distance is considerable, the central spot is white, and the coloured rings follow the order observed in thin plates. As the eye approaches the aperture, the central white spot contracts to a point, and then vanishes. The rings then close in on it in order; and the centre assumes in succession the most vivid and beautiful hues, altogether similar to those of the reflected rings of thin plates.

This remarkable coincidence has been shown to be an exact result of theory. It has been demonstrated that the intensity of the light of any simple colour, at the central spot, —and the compound tint in the case of white light,—will be the same as that reflected from a plate of air, whose thickness bears a certain simple relation to the radius of the aperture, and its distances from the luminous origin and from the eye.

The points of maximum and minimum intensity are easily determined.



Let O be the luminous point, and OAB the line drawn from it through the centre of the aperture PP′; then the interval of retardation, δ, of the ray which reaches any point B on this line, coming from the edge of the aperture, is OP + PB − OB. Let OA = a, AB $= b$, and AP $= r$; then, since r is very small in comparison with a and b, it is easy to see that

$$OP = a + \frac{r^2}{2a}, \qquad BP = b + \frac{r^2}{2b}, \qquad \delta = \tfrac{1}{2} r^2 \left(\frac{1}{a} + \frac{1}{b} \right).$$

Now when this interval is equal to a whole number, n, of semi-undulations, the aperture may be divided by concentric circles, such that the rays which reach the point B, coming from any two successive circumferences, shall differ by the interval of half a wave. It follows from the preceding formula that the squares of the radii, and therefore the superficies of the successive circles thus formed, are as the numbers of the natural series; so that the annuli comprised between every two succeeding circumferences are equal. But the elementary waves proceeding from each annulus are in complete discordance with those from the two adjacent. The successive annuli will therefore destroy one another's effects, and the total intensity of the light at the point B will be null, or equal to that of the last, according as the number of annuli (the central circle included) is even or odd. Hence, for a given aperture, there will be a succession of points on the axis, at which the intensity of the light is alternately *nothing* and a *maximum;* and it is obvious from the preceding that the distances of these points will be the values of b given by the formula

$$r^2 \left(\frac{1}{a} + \frac{1}{b} \right) = n\lambda \, ;$$

in which the points of complete darkness correspond to the *even* values of n, and those of maximum brightness to the *odd* values.

Such is the case with homogeneous light. As the points of maximum and minimum intensity are different for the rays of different colours, there will be no point of complete darkness in *compound* light, but a succession of points, at which the centre of the aperture is richly coloured.

(105) The theory of Fresnel is not only in exact accordance with facts already known: it has also led to many new and unexpected conclusions, and predicted consequences which have been afterwards verified on trial. One of the most remarkable of these is the phenomenon of diffraction by an *opaque circular disc.* Poisson applied Fresnel's integrals to this case; and he was led to the startling result, that the illumination of the centre of the shadow was precisely the same as if the disc had been altogether removed. The principles already laid down will enable the reader to satisfy himself of the theoretical truth of this conclusion. Arago was the first to show that it was in accordance with fact, and his experiment may be repeated without much difficulty.

(106) We have seen that when light diverging from a luminous point passes by the edges of a fine hair or wire, a succession of coloured bands will be formed parallel to the edge of the shadow; and the distances of these bands from the shadow, and from one another, will be greater, the less the diameter of the wire. If many such wires be exposed to the diverging beam, and if, instead of being parallel, they are crossed and interlaced in every possible direction, it is easy to conceive that the coloured bands will be disposed in concentric circles, whose centre is the luminous point. These circles resemble the halos visible round the Sun and Moon in hazy weather. Their diameters vary in the inverse ratio of the thickness of the wires or fibres.

This law was applied by Young, in a very ingenious manner, to the comparison of the diameters of fibres, or small particles of any kind.

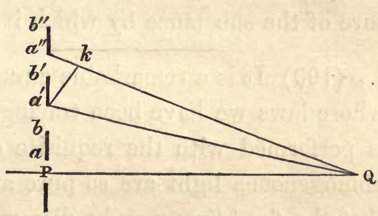
A plate of metal is perforated with a small round hole, about the $\frac{1}{50}$th of an inch in diameter, around which, at the distance of about $\frac{1}{2}$ or $\frac{1}{3}$ of an inch, is a circle of smaller holes. The flame of a lamp is then placed immediately behind the aperture, and the luminous point viewed through the substance to be examined. A ring or halo will be seen surrounding the aperture; and by moving the substance backwards and forwards on a graduated ruler, this ring may be brought to coincide with the circle of small holes pierced in the plate. The distance from the aperture is then read off on the ruler, and varies obviously in the inverse ratio of the angular diameter of the spectrum; but the diameters of the particles vary also in the same inverse ratio, so that the distance on the ruler at once becomes a measure of these diameters. In this manner Young compared the diameters of a great number of very minute substances,—such as the fibres of the finest wools, the globules of the blood, &c. The instrument itself he called the *Eriometer*.

(107) In the case last mentioned, we have supposed the intervals of the fibres, or fine wires, to be much greater than their thickness; in which case the phenomenon depends mainly on the diameter of the opaque fibre. When the intervening apertures are very small, the effect is influenced by *their* magnitude, and assumes a different character. Thus, if a *grating* be formed, by stretching a wire between two fine screws of equal thread, and if this grating be held in the beam diverging from a luminous point, we shall observe, on either side of the direct image, a series of spectral images richly coloured with all the prismatic tints; the spectra increasing in breadth, and diminishing in intensity, as they recede from the centre.

These phenomena are seen to most advantage by means of a telescope adjusted to the luminous origin. The grating being held before the object-glass of the telescope, the spectra are formed at its focus, and are there viewed, with all the advantages of distinctness and amplification, by means of the eye-glass. Fraunhofer, who observed these phenomena with much attention and care, traced no fewer than thirteen spectra on either side of the central image; the first pair being separated from the central image, and from the second pair, by intervals absolutely black. By a very accurate mode of measurement he ascertained that the deviations of any one colour from the central image, in the successive spectra, formed an arithmetical progression; and that the absolute amount of these deviations varied inversely as the intervals of the axes of the wires.

(108) These results flow readily from the principle of interference,—the *first* pair of spectra, on either side of the central image, being produced by the interference of those rays whose paths differ by *one* undulation; the *second* pair, by those whose paths differ by *two* undulations; and so on.

Let the light proceeding from a very remote origin fall on the grating, whose opaque parts are represented by ab, $a'b'$, $a''b''$, &c.; and let Q be the place of the eye. Then, if we take a portion of the grating, $a'a''$, composed of one opaque and one transparent portion, in such a manner that the difference of the distances of its extremities from the point Q, $Qa'' - Qa'$, shall be equal to the length of a wave, it is manifest that the corresponding portion of the incident wave, $a'a''$, may be divided into two parts very nearly equal, the waves sent from which to the point Q shall be in complete discordance. Without the grating, therefore, the effect of that portion of the incident wave would be null at the point Q, and no light from it would reach the eye. The effect of the grating, however, is to intercept the whole or part of one of the two interfering portions, and thus to render the other visible: and this effect is greatest when the opaque and transparent parts of the grating are equal. A bright band will therefore be visible in the direction Qa''. The same thing will happen for all the similar divisions of the grating, the distances of whose extremities from the point Q differ by



two, three, or any whole number of undulations; and thus there will be a succession of bright bands, visible at different angular distances from the direct ray PQ.

These angles are easily computed. Let $a'k$ be the arch of a circle described with the centre Q; then $a''k = a'a'' \cos a'a''k = a'a'' \sin PQa''$. But the interval of retardation, $a''k$, is equal to the length of a wave; so that, if the angle PQa'' be denoted by θ, and the interval composed of an opaque and transparent part of the grating, $a'a''$, by ε, we have

$$\sin \theta = \frac{\lambda}{\varepsilon}.$$

This is the angular distance of the *first* bright band from the central one; and it is obvious that the corresponding angle, for the band of the n^{th} order is given by the formula

$$\sin \theta_n = \frac{n\lambda}{\varepsilon}.$$

The position of each ray, in these spectra, therefore depends solely on the *length of the wave*, and is independent of the nature of the substance by which it is produced.

(109) It is a remarkable circumstance of the phenomenon whose laws we have been tracing, that when the experiment is performed with the requisite care, the several species of homogeneous light are so pure and unmixed in the spectra, that the *fixed lines* may be discerned. These lines, then, are wholly independent of refraction, and exist in the parts of the solar beam before they are separated by the prism. The phenomenon, when thus exhibited, is however distinguished by a remarkable peculiarity. The distances of the fixed lines in the *diffracted spectrum* are always *proportional*, whatever be the diffracting substance; while the ratios of the intervals of the fixed lines (or of the breadths of the coloured spaces), in the spectra formed by refraction, vary with the dispersive powers of the prisms. In fact, the angle θ being small, we may make $\sin \theta = \theta \sin 1'$; so that

$$\theta = \frac{n\lambda}{\varepsilon \sin 1'}.$$

Hence if, θ_1, θ_2, θ_3, denote the deviations corresponding to any three definite points in the spectrum, and λ_1, λ_2, λ_3, the corresponding wave-lengths, it follows that

$$\frac{\theta_2 - \theta_1}{\theta_3 - \theta_1} = \frac{\lambda_2 - \lambda_1}{\lambda_3 - \lambda_1};$$

or the intervals of the fixed lines of the spectrum are as the differences of the corresponding wave-lengths, and are therefore in an invariable ratio. The difference in the disposition of the fixed lines, in the spectra formed by diffraction and by refraction, will be seen in the diagrams of Art. (111), in which the points B, C, D, &c., of the horizontal line BH, represent the relative positions of the principal fixed lines, in the spectrum formed by a prism of flint-glass, and in the diffracted spectrum, respectively.

(110) The formula of Art. (108) suggests a very simple method of determining the *length of the wave* corresponding to any given ray of the spectrum. The value of ε, or the interval of the axes of the wires, may be ascertained with the greatest ease and precision; and we have, therefore, only to measure the angular deviation, θ, of the ray of any simple colour from the axis, in order to deduce the value of λ. Fraunhofer computed in this manner the lengths of the waves corresponding to the seven principal fixed lines of the spectrum; and the resulting values are perhaps the most exact optical *constants* we possess.

The wave-lengths, corresponding to the principal fixed lines, B, C, D, E, F, G, H, expressed in millionths of a millimetre, were thus found to be*

688, 656, 589, 526, 484, 429, 393.

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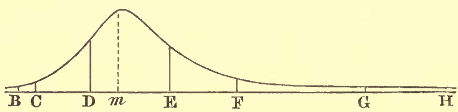
* M. Mossotti has pointed out a curious relation between these numbers and the lengths of the chords which produce the notes of the diatonic scale.

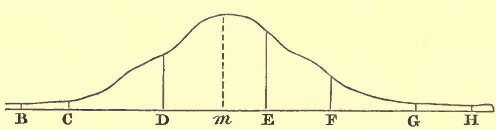
The wave-length corresponding to the *middle point* of the diffracted spectrum is 553·5 millionths of a millimetre. The wave-lengths corresponding to the *extreme visible points* are 738 and 369 millionths, respectively, the former of which is exactly double of the latter.

(111) But the diffracted spectrum is further distinguished by the simplicity of the law which governs the *intensity* of the light in its several parts. The intensity of the light in the ordinary spectrum (formed by a prism of flint-glass) was determined by Fraunhofer, for the points corresponding to the principal fixed lines. These intensities are represented by the ordinates of the curve in the annexed diagram. The ordinate



at the point m (situated between the fixed lines D and E, at a distance $Dm = \frac{7}{24}$ DE) corresponds to the maximum intensity, and divides the whole light of the spectrum into two equal parts, the areas of the two portions of the curve being equal.

The law of the intensity in the diffracted spectrum was deduced by Mossotti from the foregoing: it is represented by the ordinates of the curve in the following diagram. We see that—



I. The ordinate which divides the light into two equal portions corresponds to the *middle point* of the spectrum.

II. This ordinate is a *maximum* ; and the curve is *symmetrical* with respect to it as an axis.

Accordingly, the intensity of the light of the latter spectrum is a *maximum* at the *middle of its length*, and decreases thence *symmetrically* on either side. It is evanescent, when the wave-length increases, or decreases, by about one-third of the value corresponding to the maximum intensity.

Hence while, in the spectra formed by refraction, the ratios of the spaces occupied by the several colours, and the intensities of the light at the several points, *vary with the refracting substance*, they are, on the other hand, invariable in the diffracted spectrum. The latter spectrum, accordingly, must be regarded as the *normal* one, to which all others are to be referred.

(112) Gratings producing these effects may be formed in several ways—as, for example, by tracing a number of parallel lines on glass with a fine diamond point. Fraunhofer succeeded by such means in forming ruled surfaces in which the striæ were actually invisible under the most powerful microscopes, the interval of the grooves being only the $\frac{1}{30000}$ of an inch.

Analogous phenomena may be produced by *reflexion*. If a great number of parallel lines be engraved at very small and equal intervals upon a polished surface, the light reflected from the intervals of the grooves will interfere in a manner precisely analogous to that admitted through the apertures of the gratings; and will, by their interference, produce the most brilliant spectra. In some of the grooved metallic surfaces constructed by Mr. Barton, there are 10,000 lines to the inch. With surfaces so minutely divided, the spectra produced are as perfect as those formed by the finest prisms; and the colours which they display are little inferior to those of the diamond.

Similar appearances may be observed on metallic surfaces which have been polished with a coarse powder, the powder leaving minute striæ which produce the effects we have been describing. They may also be very simply produced by passing the finger over the surface of a piece of glass moistened by the breath. The striæ thus formed in the coating of vapour display very brilliant colours, which vary with the position of the eye.

(113) The beautiful colours of *mother of pearl* are natural instances of the same phenomena. This substance is composed of a vast number of very thin layers, which are gradually and successively deposited within the shell of the oyster, each layer taking the form of the preceding. When it is wrought, therefore, the natural joints are cut through in a great number of sinuous lines; and the resulting surface, however highly polished, is covered by an immense number of undulating ridges, formed by the edges of the layers. These striæ may be observed by the aid of a powerful microscope, although they are sometimes so close that 5000 of them occupy an inch. That they are the causes of the brilliant colours displayed by this substance has been placed beyond doubt by an experiment of Sir David Brewster. This experiment consisted simply in taking the impression of the surface of the pearl on wax, or any other substance fitted to receive it: it was found that the impressed surface displayed all the colours of the original body. In fact, the colours of striated surfaces indicate their structure, perhaps more unerringly than any other means: Sir David Brewster has made a very ingenious use of their laws, in investigating the curious and complicated structure of the crystalline lens in fishes and other animals.

(114) There remains another class of phenomena produced by diffraction, which it is important to notice.

We have already seen the effects produced, when light diverging from a luminous point is transmitted through a narrow aperture, and received on a screen. But if we vary the experiment, by placing a lens of considerable focal length (as the object-glass of a telescope) immediately behind the aperture, and receive the image on a screen at the conjugate focus, the appearances displayed are altered in a remarkable manner, and differ more widely from those produced in the former case, as the aperture is greater. In fact, the phenomena of diffraction are thus produced with apertures of considerable dimensions, and were observed by Sir William Herschel with the undiminished object-specula of his great telescopes: they are rendered more distinct, however, when the aperture of the telescope is limited by a diaphragm of moderate size. When a star is viewed through a telescope of high power, having its object-lens thus limited, its image is encompassed with a system of diffracted rings slightly coloured, succeeding one another at equal intervals;—the diameters of the rings varying inversely as those of the apertures. The phenomena vary in a very curious manner, when the form of the aperture is changed. Thus, when a *triangular* diaphragm is substituted for the circular one, the disc of the star appears surrounded by a black ring, from which diverge six rays at equal intervals.

These phenomena have been examined in detail by Sir John Herschel and by M. Arago. Their mathematical explanation has been given by Mr. Airy, in his valuable tract on the Undulatory Theory; and the deductions of theory are found to be in complete accordance with the observed facts.

CHAPTER VII.

COLOURS OF THIN PLATES.

(115) The *colours of thin plates* were first noticed by Boyle and Hooke. They are displayed in every instance in which transparent bodies are reduced to films of great tenuity. Boyle succeeded in blowing *glass* so thin as to exhibit the phenomena: they are more readily developed in *mica*, and some other transparent minerals, which possess a lamellar structure; but the most familiar instance of their exhibition is in the *froth of liquids*,—the fluid envelopes of the bubbles which compose it being in general of extreme thinness.

These colours vary with the thickness of the film, and disappear altogether when it passes certain limits. When the film exceeds a certain thickness, all the colours are equally reflected, and the reflected light is therefore *white*. On the other hand, when the thickness falls below a certain limit, no light whatever reaches the eye, and the surface of the film appears absolutely *black*.

(116) The foregoing facts may be observed in the common soap-bubble, when properly defended from the disturbing influence of currents of air. If the mouth of a wine-glass be dipped in water, which has been rendered somewhat viscid by the mixture of soap, the aqueous film which remains in contact with it after emersion will display the whole succession of these phenomena. When held in a vertical plane, it will at first appear uniformly white over its entire surface; but, as it grows thinner by the descent of the fluid particles, colours begin to be exhibited at the top, where it is thinnest. These colours arrange themselves in horizontal bands, and

become more and more brilliant as the thickness diminishes; —until finally, when the thickness is reduced to a certain limit, the upper part of the film becomes completely black. When the bubble has arrived at this stage of tenuity, cohesion is no longer able to resist the other forces which are acting on its particles, and it bursts.

Similar phenomena may be observed when a drop of *oil* is let fall on water. As the oil spreads rapidly over the surface, it is soon reduced to a very thin film, which displays the spectral colours.

Every one has noticed the fact that *steel* and *other metals*, when polished, acquire various shades of colour by exposure to the air. These colours are produced by a thin coating of metallic oxide, which is gradually formed on the surface. The formation of this oxide is greatly accelerated by an augmentation of temperature; and the colour thus formed is so invariably connected with the thickness of the film, and this latter with the degree of heat, that artists are in the habit of measuring the temperature by the colour developed. Thus steel, in the process of tempering, is said to have received a *yellow heat*, a *blue heat*, &c.

The same appearances are displayed in a still more striking manner by *air* itself, or even by a *vacuum*. If two plates of glass be pressed together by the fingers, we shall observe, round the point of nearest approach, a succession of coloured bands of great brilliancy, which dilate as the pressure is increased, and the inclosed plate reduced in thickness.

(117) In order to observe these phenomena, in such a manner as to be enabled to trace their laws, we must follow Newton. Newton's experiment consisted simply in laying a convex lens of glass upon a plane surface of the same material. The thickness of the inclosed plate of air increases as the square of the distance from the point of contact, and is, therefore, the same at all equal distances from that point; and, as

the reflected colour depends on the thickness, the bands of the same colour will be arranged in concentric circles, of which that point is the centre. The same succession of colours is produced when any other transparent fluid is inclosed between the glasses. The colours, however, are more vivid, the more the refractive power of the thin plate differs from that of the substances within which it is inclosed.

When we look attentively at these rings, the light being reflected always at the same angle, we observe that the central one is not a mere annulus, but a complete circle of nearly uniform colour. If then we diminish the thickness of the plate of air, by pressing the two glasses more closely together, this central circle is observed to dilate, and a new circle of a different colour to spring up in its centre. This will dilate in turn, driving the former before it, and another circle appear within it;—until at length a *black spot* shows itself in the centre of the system, after which no further diminution of thickness will alter the succession. When the black spot makes its appearance, we have obtained a plate of air so thin as no longer to reflect any colours, and the phenomenon is complete. Newton traced seven coloured rings round this spot, the colours of which are said to be of the *first*, *second*, *third*, &c., *order*, according to the order of the ring to which they belong. Thus, the red of the third order is the red in the third ring from the central black, &c. The whole succession of colours is called *Newton's scale*.

(118) The principal laws of these phenomena are included in the following propositions:

I. In homogeneous light, the rings are alternately *bright* and *black;* the thicknesses corresponding to the bright rings of succeeding orders being as the *odd* numbers of the natural series, and those corresponding to the black rings as the intermediate *even* numbers.

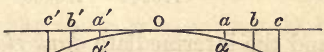
II. The thickness corresponding to the ring of any given order varies with the *colour* of the light,—being greatest in red light, least in violet, and of intermediate magnitude in light of intermediate refrangibility. In white or compound light, therefore, each ring will be composed of rings of different colours, succeeding one another in the order of their refrangibility.

III. The thickness corresponding to any given ring varies with the *obliquity* of the incident light, being very nearly proportional to the secant of the angle of incidence.

IV. The thickness varies with the *substance* of the reflecting plate, and in the inverse ratio of its refractive index.

(119) In order to establish the first of these laws, it is necessary to employ homogeneous light. This may be obtained by means of the prism: or we may adopt the method suggested by Mr. Talbot, and illuminate the glasses with a spirit lamp having a salted wick. The light of such a lamp being a yellow of almost perfect homogeneity, the rings will be alternately *black* and *yellow*; and their number is so great as to baffle any attempt to determine it.

The law of the thicknesses corresponding to the successive rings is easily established. Let O be the point of contact of the plane and spherical surfaces, and *aa'*, *bb'*, *cc'*, &c. the diameters of the successive rings formed round that point as a centre.



It is evident that the thicknesses of the plate of air at the points where these rings are formed, $a\alpha$, $b\beta$, $c\gamma$, &c., are as the squares of the distances O*a*, O*b*, O*c*, &c., or as the squares of the diameters of the rings: to determine the law of the thicknesses, therefore, we have only to measure these diameters. This was done by Newton with great accuracy, and it was found that the squares of the diameters were in *arithmetical progression*; consequently, the thicknesses corresponding to the successive rings formed a similar progression.

(120) But Newton did not stop here: he ascertained further the *absolute thickness* of the plate of air at which each ring was formed. It is manifest that if the thickness of the plate be determined for any one ring, that corresponding to the others will be given by the law just stated. Newton, accordingly, proceeded to ascertain this thickness for the dark ring of the *fifth* order. This was done by measuring its diameter accurately, and determining the radius of the spherical surface from the focal length of the lens and its refractive index. The thickness is thence immediately deduced; for it is equal to the square of the radius of the ring divided by the diameter of the spherical surface. The value thus deduced being suitably corrected, it was found that the thickness of the plate of air was the $\frac{1}{17800}$ of an inch, at the dark ring of the fifth order; and this thickness being decuple of that corresponding to the first bright ring, it followed that the thickness of the plate of air, at the place of the *first bright ring*, was the $\frac{1}{178000}$ of an inch. Thus the *bright rings* of the successive orders are formed at the thicknesses

$$\frac{1}{178000},\quad \frac{3}{178000},\quad \frac{5}{178000},\quad \frac{7}{178000},\quad \&c.$$

and the intermediate *dark rings* at the thicknesses

$$\frac{2}{178000},\quad \frac{4}{178000},\quad \frac{6}{178000},\quad \frac{8}{178000},\quad \&c.$$

These determinations belong to the most luminous rays of the spectrum, or those at the confines of the orange and yellow.

(121) The variation of the diameters of the rings (or of the thicknesses of the plate of air at which they are exhibited) with the *colour* of the light, may be observed by illuminating the glasses with different portions of the spectrum in succession,—or, yet more simply, by looking at the rings through coloured glasses; and it is found that the magnitude of the rings is greater, the less the refrangibility of the light. This

being understood, it is easy to comprehend the cause of the succession of colours in each ring, when *white* or *compound light* is used. For the rings, in this case, are the aggregate of the rings of different colours; and these being of different magnitudes, the compound ring will be variously coloured, the more refrangible rays occupying the interior, and the less refrangible the exterior parts of the ring. It is easy to see also, that all phenomena of colour must disappear after a few successions, the rings of different colours, belonging to different orders, being at length superposed.

The variation of the rings (and therefore of the thicknesses) with the *obliquity* of the incident light may be observed by depressing the eye. The rings are then seen to dilate rapidly with the obliquity of the reflected pencil; the thicknesses of the plate of air at which they are exhibited being nearly as the secants of the angles of incidence or reflexion.

The fourth and last law, which expresses the dependence of the thickness, at which any ring is formed, upon the *refractive power* of the plate, is easily verified by introducing a drop of water between the glasses. The rings are then observed to contract; and if we compare their diameters in air and in water, it will be found that the corresponding thicknesses of the plate are as four to three, or in the inverse ratio of the refractive indices.

(122) We have hitherto spoken only of the reflected rings. There is another system of rings formed by *transmission*, but much fainter than the former. The transmitted rings are found to observe the same laws as the reflected rings,—with this remarkable exception, that the colour transmitted at any particular thickness of the plate is always *complementary* to that reflected at the same thickness; so that, in homogeneous light, the bright transmitted ring is always at the same distance from the centre as the corresponding dark one of the reflected system.

(123) The phenomena of thin plates are exhibited, under a modified form, in the following experiment :

A little fine soap is spread upon a plate of black glass, and is distributed uniformly by rubbing the surface lightly with a piece of soft leather. If then we blow on the surface, thus prepared, through a short tube,—taking care to direct the tube always to the same part of the plate,—the vapour of the breath will be deposited in a thin film, whose thickness diminishes regularly from the point to which the tube is directed. This film will accordingly display a series of coloured rings analogous to those formed by the plate of air between two object-glasses,—with this difference, however, that the order of the rings is *reversed*, the outermost ring corresponding to the centre of Newton's scale. This little apparatus, contrived by Mr. Read, is denominated by him an *iroscope*.

(124) It is now time that we should enter on the physical account of these phenomena.

For their explanation, it has been already stated, Newton framed the hypothesis of the *fits of easy reflexion and transmission* already referred to: its application to the phenomena of thin plates is obvious. The molecule of light is in a *fit of easy transmission* in its passage through the *first* surface; this is succeeded by a fit of easy reflexion,—and so on alternately, the spaces traversed during the continuance of the fits being all equal. On arriving at the *second* surface, therefore, the molecule will be in a fit of easy transmission, or easy reflexion, according as the interval of the surfaces is an even or an odd multiple of the length of the fit. Thus the alternate succession of bright and dark rings, and the arithmetical progression of the thicknesses at which they are exhibited, are explained.

To explain the second law, it is necessary to suppose that the *length* of the fits *varies with the colour of the light*, being greatest in red light, least in violet, and of intermediate magnitude in light of intermediate refrangibility. Newton determined the absolute lengths of these fits for the rays of each simple colour, and found that they bore a remarkable numerical relation to the lengths of the chords sounding the octave.

To account for the two remaining laws, Newton was constrained to make new suppositions, and to attribute properties to the fits, which are inconsistent with every physical account which has been given of them. Thus, to explain the dilatation of the rings with the obliquity, he assumed that the *length* of the fits *augmented with the incidence*, and nearlyi n the ratio of the square of the secant of the angle of incidence. This assumption is at entire variance with the physical theory. If the fits are produced by the vibrations of the ether which are propagated faster than the luminous molecules, and which alternately conspire with and oppose their progressive motion, their lengths should continue the same in the same medium, whatever be the incidence.

The fourth law appears to be also irreconcileable with the theory. The thicknesses of the plates of different media, at which the same tint is exhibited, being in the inverse ratio of the refractive indices, it was necessary to suppose that the *lengths* of the fits varied in the same proportion; and since, in the Newtonian theory, the refractive indices are directly as the velocities of propagation, it would follow that, as the velocities augmented, the spaces traversed by the ray in the interval of its periodical states must *diminish*, and in the same proportion.

(125) Newton seems to have regarded this hypothesis as the mere expression of a physical fact, and in this light it was long considered. It cannot be denied that, as the thickness of the plate increases, the light appears by reflexion and transmission *alternately* ; and it is of no moment, it may be said, by what name these alternate states are called. But if we look more narrowly into the theory, we shall find that it *assumes*

the alternate appearance of the light, in the reflected and transmitted pencils, to be the effect of an alternate reflexion and transmission at a *single* surface, that surface being the *second* surface of the plate. Now it can be shown that this supposition is untrue; that light is reflected from *both* surfaces of the plate; and that the concurrence of these *two reflected pencils* is an essential condition of the phenomenon.

To show this, let us employ (instead of common light) light which is *polarized in a plane perpendicular to the plane of incidence;* and let it fall upon a plate of air inclosed between two transparent surfaces of different refractive powers. Under these circumstances it is found that the intensity of the light in the rings varies with the incidence; and that the whole system *disappears* in two cases, namely, when the incidence corresponds to the polarizing angle of either of the media.

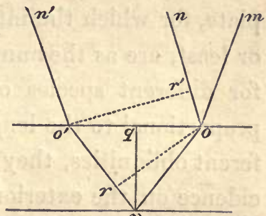
To understand the conclusion to which this leads, we must assume a property of light which will be hereafter established —namely, that when light, thus polarized, is incident upon a transparent surface at what is called the polarizing angle, it is *wholly transmitted*, and no portion of it whatever reflected. We see then, from the experiment, that the rings disappear when the light reflected from *either* of the two surfaces of the plate vanishes; and we are therefore warranted in concluding, that the light reflected from *both* surfaces of the plate is essential to their production.

(126) The preceding experiment, and the conclusion drawn from it, lead us to the very threshold of the true theory.

In fact, the light incident on the first surface of the plate is in part reflected, and in part also transmitted. The transmitted portion undergoes a similar subdivision at the second surface; and part of the light reflected at that surface will emerge through the first, and reach the eye along with that reflected there. Thus the reflected light consists of two portions, one reflected at the upper, and the other at the lower surface of the plate; and these two portions will *interfere*, and reinforce or weaken each other's effects, according as they reach the eye in the same or in opposite phases.

(127) This mode of explaining the phenomena of thin plates was pointed out by Hooke, in a remarkable passage in his *Micrographia*, some years before the subject was taken up by Newton. In this passage he very clearly describes the manner in which the rings of successive orders depend on the interval of retardation of the second "pulse," or wave, with respect to the first, and therefore on the thickness of the plate. But he does not seem to have had any distinct idea of the principle of Interference itself; and his conception of the mode in which the colours resulted from this "duplicated pulse" is entirely erroneous. Euler was the next who attempted to connect the phenomena of thin plates with the wave-theory of light; but the attempt, like all the physical speculations of this great mathematician, was signally unsuccessful, and the subject remained in this unsettled state, until the principle of Interference was discovered by Young. When this principle was combined with the suggestion of Hooke, the whole mystery vanished. The application was made by Young himself, and all the principal laws of the phenomena were readily and simply explained.

(128) Let *mon* be the course of a ray reflected at the first surface of a plate; *mopo'n'* that of the ray reflected at the second surface, and twice transmitted through the first. From the point *o'* let fall the perpendicular *o'r'* upon the reflected ray *on*; it will be also perpendicular to *o'n'*, and will therefore be parallel to the front of the two reflected waves.



Now let us conceive a wave reflected at the first surface, in the position $o'r'$, to meet at the same place an anterior wave reflected at the second surface, and let us calculate the original interval between them. From the time that they reached the first surface at o, one has travelled over the space or', and the other over the space $op + po'$. But, if we let fall the perpendicular or upon po', it is evident from the law of refraction that the spaces or' and $o'r$ are traversed in the same time in the two media; and, consequently, that the interval of retardation is the time of describing $op + pr$. Now $pr = op \cos 2opq$, and therefore $op + pr = op\,(1 + \cos 2opq) = 2op \cos^2 opq$. But $op \cos opq = pq$; and, consequently, the interval is $2pq \cos opq$. Or, if we denote that interval by δ; the thickness of the plate, pq, by t; and the angle opq by θ,—

$$\delta = 2t \cos \theta.$$

The two waves are in complete accordance or discordance, when the interval of retardation is an exact multiple of the length of half a wave: i. e. when

$$\delta = n\frac{\lambda}{2},$$

n being any number of the natural series. Equating these values of δ, therefore, we have, for the values of the thickness of the plate which will produce a complete accordance or discordance of the two waves,

$$t = \tfrac{1}{4} n\lambda \sec \theta.$$

We learn then, 1st, that the successive thicknesses of the plate, for which the intensity of the reflected light is greatest or least, are as the numbers of the natural series; 2ndly, that, for different species of simple light, these thicknesses are proportional to the lengths of the waves; 3rdly, that, for different obliquities, they vary as the secant of the angle of incidence on the exterior medium; and, 4thly, that, for plates of different substances, they are proportional to λ, and therefore in the direct ratio of the velocity of propagation, or in the inverse ratio of the refractive index of the substance of which the plate is composed.

(129) There is one part of the preceding explanation which demands a little further consideration. The two waves being in complete *accordance* when the interval of retardation is an *even* multiple of the length of half a wave, and in complete *discordance* when that interval is an *odd* multiple of the same quantity, it would seem, from the foregoing account, that the bright rings should be formed at all those points for which n is an even number in the formula above given, (or the thickness an even multiple of $\frac{1}{4}\lambda \sec \theta$), and the dark rings at those points for which it is odd. If this were true, the point of contact should be a point of accordance, and the rings should commence from a *bright centre*, instead of a *dark* one.

This apparent discrepancy is explained by the fact, that the two reflexions take place under opposite circumstances, one of the rays being reflected at the surface of a *denser*, and the other at that of a *rarer* medium.

The effect of this difference will be best understood by a simple illustration. When one elastic ball strikes another at rest, it communicates motion to it in all cases; but its own condition after the shock will depend on the relative masses of the two balls. If the balls be equal, the first will remain at rest after the shock. If they be unequal, it will move; and its motion will be *in the direction* of its former motion, when its mass *exceeds* that of the second ball,—it will be in the *opposite direction* when it is *less*. This will help us to understand what passes when a wave reaches the surface separating two media. The particles of ether next the bounding surface communicate motion to the adjacent particles of the second medium, and thus give rise to the *refracted wave*. But the former particles will not remain at rest afterwards, unless the density and elasticity of the ether be the same in the two media. When this is not the case, the particles of the first medium will move, after communicating motion to those of the second, and, in moving, give rise to the *reflected wave*. Thus *refraction* is always accompanied by *reflexion;* and this reflexion is greater, the greater the difference of the densities of the ether in the two media. It appears also, from what has been said, that the *direction* of the motions of the particles of the first medium, after they communicate motion to those of the second, will be different, according as the ether is *denser* or *rarer* in the first medium. In the former case the vibration of the particles is in the *same direction* that it was before; in the latter it is in the *opposite direction.* Thus there will be a reflected wave in both cases; but in one case this reflected wave is caused by a vibration in the *same* direction as that of the incident wave; in the other, by a vibration in an *opposite* direction.

The result of this difference is obviously the same as if one of the systems of waves were to gain or lose half an undulation on the other; so that when the two waves, reflected from the two surfaces of the plate, should be in complete accordance,—as far as depended on the difference of the lengths of their paths,—they will actually be in complete discordance, and *vice versâ.* Thus the *dark* rings will be formed where the thickness of the plate is any *even* multiple of $\frac{1}{4}\lambda \sec \theta$, and the *bright* ones where that thickness is an *odd* multiple of the same quantity; and the facts and the theory are reconciled.

(130) The principle which we have been illustrating has been experimentally established by M. Babinet, by an independent method. A pencil of rays diverging from a narrow aperture is separated into two, slightly inclined to one another, by means of the obtuse prism (85). These are allowed to fall on a thick plate of parallel glass, whose second surface is quicksilvered in one-half of its extent; and in such

a manner as to be both reflected by the *transparent* portion of that surface, or both by the *opaque* portion, or one by the former and the other by the latter. These two portions will interfere, and produce fringes after reflexion; and it is found that, in the two former cases, the central band is *white*, the two waves being in complete accordance: in the third case—i. e. when one of the pencils is reflected from the rarer, and the other from the denser medium—the central band is a *black* one; the two waves are, therefore, in complete discordance, and their phases differ by half an undulation.

It follows from the preceding, that in the system of rings formed between two object-glasses, the central spot will be *white*, if the thin plate is of a density intermediate to those of the two glasses; for it is evident that the reflexion takes place under the same conditions at the two surfaces—i. e. in both cases at the surface of a rarer, or in both at that of a denser medium. This anticipation of theory was verified by Young, by inclosing oil of sassafras between two object-glasses, one of which was of flint-glass, and the other of crown-glass.

(131) We have spoken of another set of rings visible by transmission. These are produced by the interference of the rays *directly* transmitted through the plate with those which penetrate it after *two interior reflexions*. It follows from the preceding considerations that they should be *complementary* to those seen by reflexion; and this is observed to be the case. The extreme paleness of the transmitted rings arises from the great difference in the intensities of the interfering pencils.

(132) The theory of thin plates, as it came from the hands of Young, laboured under an imperfection, which was, however, soon removed. It is obvious that the intensities of the two portions of light, reflected from the upper and under surfaces of the plate, can never be the same,—the light incident

on the second surface being already weakened by partial reflexion at the first. These two portions, therefore, can never wholly destroy one another by interference, and the intensity of the light in the dark rings can never entirely vanish, as it appears to do when homogeneous light is employed.

Poisson was the first to point out, and to remedy, this defect in the theory. It is evident, in fact, that there must be an infinite number of partial reflexions within the plate, at each of which a portion is transmitted; and that it is the *sum of all* these portions, and not the two first terms of the series only, which is to be considered in the calculation of the effect. When the problem is taken up in this more general form, it is found that, where the effective thickness of the plate is an exact multiple of the length of half a wave, the intensities of the reflected and transmitted lights will be the same as if it were removed altogether, and the bounding media placed in absolute contact. Hence, when these media are of the same refractive power, the reflected light must vanish altogether, and the transmitted light be equal to the incident.

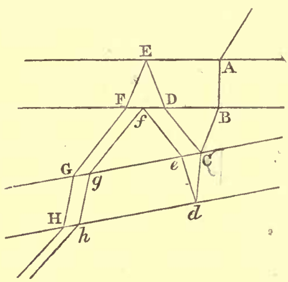
Here then we have reached a point, with respect to which the two theories are completely opposed. According to both, a certain portion of light is reflected from the first surface of the plate. This portion, in the Newtonian theory, is left in all cases to produce its full effect, and there should therefore be a considerable quantity of light in the dark rings; while, in the wave-theory, it is, at certain intervals, wholly destroyed by the interference of the other portions, and the dark rings should be absolutely *black* in homogeneous light.

The latter of these conclusions seems to accord with phenomena, while the former is obviously at variance with them. This is clearly shown by an experiment of Fresnel. A prism was laid upon a lens having its lower surface blackened, a portion of the base of the prism being suffered to extend beyond the lens. The light reflected from this portion, according to the Newtonian theory, should not surpass that of the dark rings in intensity. The roughest trial is sufficient to show that the intensities of the light in the two cases are widely different, and thus to prove that the dark rings cannot arise (as they are supposed to do in the theory of the fits) from the suppression of the second reflexion.

(133) When a pencil of light falls upon *two plates* in succession, some of the many portions into which it is divided by partial reflexion at the bounding surfaces, are frequently in a condition to interfere, and to give rise to the phenomena of colour.

Thus, when light is transmitted through *two parallel plates*, slightly differing in thickness, the colour is the same as that produced by transmission through a single plate, whose thickness is the *difference* of their thicknesses, and is found to be independent of the interval of the plates. This phenomenon was observed by Nicholson; and it has been shown by Young to arise from the interference of two pencils, one of which is twice reflected within the first plate, and the other twice reflected in the second. It is obvious, in fact, that if t be the thickness of the first plate, and t' that of the second, the first pencil will have traversed the thickness $3t + t'$ in glass, and the second the thickness $3t' + t$, the spaces traversed in air being the same; so that the interval of retardation is the time of describing the space $2(t - t')$ in glass. Sir David Brewster observed a similar case of interference, produced by two plates of *equal* thickness *slightly inclined;* the thickness traversed in the two plates being altered by their inclination.

In the foregoing cases, the interfering pencils are mixed up with, and overpowered by, the light directly transmitted, and some contrivance is necesssary to make the colours visible. The phenomena are much more obvious in the light *reflected* from both plates, which, on account of their inclination, is thus separated from the direct light. It is obvious, in fact, that the direct image of a luminous object, seen through two glasses slightly inclined, will be accompanied by several lateral images, formed by 2, 4, 6, &c. reflexions. These images Sir David Brewster observed to be richly coloured; the bands of colour being parallel to the line of junction of the two glasses, and their breadth being greater, the less the inclination of the plates. The colours in the first lateral image are produced by the interference of the two pencils ABCDEFGH, ABC*defgh*, into which the ray is divided at the first surface of the second plate; one of these portions being reflected externally by the second plate, and internally by the first, —while the other is reflected internally by the second, and externally by the first. The routes of these portions are obviously equal when the plates are parallel, and differ in length only by reason of their inclination.



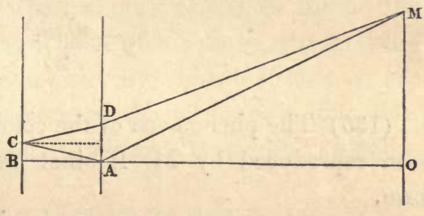
(134) The two preceding cases of interference may be produced with plates of any thickness. What are termed the *colours of thick plates*, however, are phenomena of another kind, and arise in circumstances wholly different. These phenomena were first observed by Newton.

In Newton's experiment a beam of light is admitted through a small aperture, and received on a concavo-convex mirror with parallel surfaces, the hinder of which is silvered. A screen of white paper being then held at the centre of the mirror, having a hole in the middle to let the beam pass and repass, a set of broad coloured rings will be depicted on it, similar to the transmitted rings of thin plates. The diameters of these rings vary inversely as the square roots of the thicknesses of the mirrors.

When the mirror is inclined a little, so as to throw the reflected image a little to one side of the aperture, the rings are formed as before; but their centre is in the middle of the line joining the aperture and its image. At this centre is a spot, which changes its appearance in a remarkable manner as the image recedes from the aperture, being alternately dark and bright in homogeneous light, and in white light assuming every gradation of tint in rapid succession.

(135) These phenomena have been shown to arise from the interference of the two portions of light, which are *irregularly scattered* in the passing and repassing of the ray through the refracting surface.

Thus, let O be the aperture through which the beam is admitted, and let it fall perpendicularly on a reflecting plate at A and B. A portion of the incident light will be irregularly scattered, in the passage of the ray OAB through



the first surface of the plate; and this portion will diverge from the point A in all directions. Let AC be one of the rays which compose this scattered portion: this is reflected at the second surface of the plate in the direction CD, and emerges in the direction DM. But the direct ray, AB, which is reflected back in BA, will again be partially scattered in repassing through the first surface. Let AM be one of the rays of this second pencil, meeting the ray DM of the first at the point M; and let us calculate their interval of retardation. The latter has traversed the space AB + BA in glass, and AM in air; while the former has described the space

AC + CD in glass, and DM in air. The interval of retardation is therefore the time of describing AM – DM in air, *plus* the time of describing 2 (AB – AC) in glass; and it is easy to prove that the corresponding space in air is equal to

$$\frac{by^2}{a(2b+\mu a)},$$

in which a denotes the distance, OA, of the screen from the plate; b the thickness, AB; and y the distance, OM, of any point on the screen from the aperture. Equating this to $n\frac{\lambda}{2}$, it follows that the successive bright and dark rings will be formed where

$$y^2 = n\lambda\frac{a(2b+\mu a)}{2b}.$$

When a is very great in comparison with b, as is usually the case, we have simply

$$y^2 = \frac{n\lambda\mu a^2}{2b}.$$

(136) The phenomena of the colours of thick plates have been reproduced by M. Babinet under a more instructive form.

The rays proceeding from a luminous point are refracted by a lens, and are then received upon a transparent plate with parallel surfaces, interposed between the lens and its focus. If now this plate be slightly tarnished, or covered with powder, a series of concentric rings will be formed around the focal image. The innermost of these is *white;* and this is followed by a series of *coloured rings* in the order of Newton's scale. Their diameters increase with the distance of the plate from the focus, and diminish as the thickness of the plate increases. They are formed, as before, by the interference of the two pencils, which are scattered in passing through the two surfaces of the plate.

The phenomenon is reduced to its simplest conditions, by receiving the light diverging from a narrow rectilinear aperture upon *two polished wires*, stretched parallel to the slit, and nearly in the same plane with it. If then the eye, fortified by a lens, be placed so that the sum of the distances of the aperture and of the eye from each wire is the same, a series of coloured fringes will be seen, formed by the interference of the pencils irregularly reflected by the two wires.

(137) When the interval between two glasses is filled with different substances,—such as water and air, or water and oil,—in a finely subdivided state, the portions of light which have traversed them are in a condition to interfere, the interval of retardation depending on the difference of the velocities of light in the two media. Accordingly, coloured rings will be seen, when a luminous object is viewed through the glasses, the rings being similar to those usually seen by transmission, but much larger. But when a dark object is behind the glasses, and the incident light somewhat oblique, the rings immediately change their character, and resemble those of the ordinary reflected system; one of the portions, in this case, being reflected, and therefore suffering a loss of half an undulation.

These phenomena were observed and explained by Young, and were denominated by him the *colours of mixed plates.* Young observed some similar effects in an unconfined medium. Thus, when the dust of the *lycoperdon* is mixed with water, the mixture exhibits a green tint by direct light, and a purple tint when the light is indirect; and the colours rise in the series, when the difference of the refractive densities is lessened by adding salt to the water. The interval of retardation, in this case, depends on the magnitude of the transparent particle.

(138) In concluding this review of the two theories, in

their application to the laws of unpolarized light, it should be observed, that any well-imagined hypothesis may be accommodated to phenomena, and seem to explain them, if only we increase the number of its assumed principles, so as to embrace each new class of phenomena as it arises. In a certain sense such an explanation may be said to be *true*, so long as it is thus made to represent all known facts; but it is no longer a *theory*, whose very essence it is to ascend in simplicity, at the same time that it rises in generality:—it is "a mob" of hypothetical laws, without connexion, order, or dependence.

These remarks apply to the explanation of the phenomena of thin plates adopted in the theory of emission. These phenomena, Newton saw, could not be accounted for on the bare hypothesis of molecules shot from the luminous body, and subjected to the attractive and repulsive forces of the bodies which they met in their progress. He was compelled to add a new property to light,—to endow the molecules with dispositions which seemed wholly alien to their other properties, and which could only be connected with them by assuming a material mechanism much more complicated than was at first proposed. But this was not all. *Each* of the laws of thin plates was found to require a *new* property in the fits to which they were referred; and *none* of these properties were in any way related to the rest, or to the mechanism on which they were supposed to depend.

These imperfections of the emission-theory are still more glaring when we pass from one class of phenomena to another. The phenomena of *diffraction*, for example, are referred to principles altogether different from those which seemed to be required in explaining the colours of *thin plates;* and the two classes of phenomena, in this way of accounting for them, bore no relation of any kind to each other.

All this is otherwise in the wave-theory. Here the several classes of phenomena are deduced from a common principle, and are, therefore, mutually related. The principle of interference is a *necessary* consequence of the nature of a vibration ; and this *one principle*, as we have seen, explains in the most complete manner the laws of the other phenomena.

But it is not merely in their reference to a common origin that these phenomena are thus related : they are even bound by the chain of number. The simple laws of *interference*,—the laws of *diffraction*,—and those of *thin plates*,—are all dependent upon a *single constant* for each kind of light,—the length of a wave in each medium ; and this constant being inferred from any one experiment, in any one class of phenomena, we can compute *numerically* the details of all the rest, and compare them with the results of measurement. The agreement is found to be complete.

CHAPTER VIII.

POLARIZATION OF LIGHT.

(139) In the various phenomena which take place when a ray of light encounters the surface of a new medium, it has been supposed that the direction and intensity of the several portions into which it is subdivided will continue the same, on whatever side of the ray the surface is presented, provided that the angle and the plane of incidence continue unchanged. In other words, it was taken for granted that a ray of light had *no relation to space*, with the exception of that dependent on its *direction;* that, around that direction, its properties were *on all sides alike;* and that, if the ray be made to revolve round that line as an axis, the resulting phenomena would be unaltered.

Huygens was the first to observe that this was not always the case. In the course of his researches on the law of double refraction, he found that when a ray of solar light is received upon a rhomb of Iceland crystal, in any but one direction, it is *always* subdivided into *two of equal intensity*. But, on transmitting these rays through a *second* rhomb, he was surprised to observe that the two portions, into which each of them was subdivided, were *no longer equally intense;* that their relative brightness depended on the *position* of the second rhomb with regard to the first; and that there were two such positions in which one of the rays *vanished altogether*.

On analyzing the phenomenon, it is found that these two positions are those in which the *principal sections* of the two crystals are *parallel* or *perpendicular*. When these sections are parallel, the ray which has undergone ordinary refraction by the first crystal will be also refracted ordinarily by the second;

and the ray which has been extraordinarily refracted by the first will be also extraordinarily refracted by the second. On the contrary, when the principal sections of the two crystals are perpendicular, the ray which has suffered ordinary refraction by the first crystal will undergo extraordinary refraction by the second; and the extraordinary ray of the first will be refracted according to the ordinary law in the second. In the intermediate positions of the two principal sections, each of the rays refracted by the first crystal will be divided into *two* by the second; and these two pencils are in general *different in intensity*, their intensities being measured by the square of the cosine of the distance from the position of greatest intensity.

(140) From this "wonderful phenomenon," as Huygens justly called it, it appears that each of the rays refracted by the first rhomb has acquired properties which distinguish it altogether from solar light. It has, in fact, acquired *sides*; and it is evident that the phenomena of refraction depend, in some unknown manner, on the relation of these sides to certain planes within the crystal. Such was the conclusion of Newton. "This argues," says he, "a virtue or disposition in those sides of the rays, which answers to, and sympathizes with, that virtue or disposition of the crystal, as the poles of two magnets answer to one another."

Although the phenomenon discovered by Huygens was one of such importance, in the mind of Newton, as to force him to admit the existence of properties in the rays of light which, until then, had never been imagined, yet the result remained, for more than 100 years, a unique fact in science; and the kindred phenomena—the properties which light acquires in a greater or less degree in almost every modification which it undergoes—remained unnoticed until the beginning of the present century.

(141) In the year 1808, while Malus was engaged in the

experimental researches by which he established the Huygenian law of double refraction, he happened to turn a double-refracting prism towards the windows of the Luxembourg palace, which then reflected the rays of the setting sun. On turning round the prism, he was astonished to find that the ordinary image of the window nearly disappeared in two opposite positions of the prism; while in two other positions, at right angles to the former, the extraordinary image nearly vanished. Struck with the analogy of this phenomenon to that which is observed when light is transmitted through two rhomboids of Iceland spar, Malus at first ascribed it to some property which the light had acquired in its passage through the atmosphere: but he soon abandoned this idea, and found that this new property was impressed upon the light by reflexion at the surface of the glass.

Pursuing the subject, he was led to the important discovery, that when a ray of light is reflected from the surface of glass, or water, or any other transparent medium, at certain angles, the reflected ray acquires all the characters which belong to the light which has undergone double refraction. When received upon a rhomb of Iceland spar, or a double-refracting prism, one of the two pencils into which it is divided vanishes in two positions of the rhomb,—namely, when the principal section of the crystal is parallel or perpendicular to the plane of reflexion; while, in intermediate positions, these pencils vary in intensity through every possible gradation.

A ray, then, may acquire *sides* or *poles*—may, in short, be *polarized*—by reflexion at the surface of a transparent medium, as well as by double refraction. The *plane of polarization* is the plane of reflexion at which the effect is produced; and it is experimentally known by its relation to the principal section of a double-refracting crystal,—the ray undergoing ordinary refraction only, when the principal section is parallel to the plane of polarization.

(142) But a polarized ray possesses other characters. When a ray of light, proceeding directly from a self-luminous body, is received upon a reflecting surface at a given angle, the intensity of the reflected beam will be unaltered, whether the surface be above or below, on the right or on the left of the incident beam. The case, however, is different, if, instead of the direct light of a self-luminous body, we submit to the same trial light which has been already polarized. It is then no longer indifferent on what side of the ray the new surface is presented. The *inclination* of the reflected or transmitted ray will, indeed, remain unaltered, on *whatever side* the surface be presented, but its *intensity* will be very different; and a ray which is *reflected* most intensely when the new surface is presented at one side, under a certain angle, will be *wholly transmitted* when it is offered to another, all other circumstances being identical.

It is evident then, that the ray which has suffered reflexion at the first surface, in this experiment, has in consequence acquired properties wholly distinct from the original light. The latter is equally reflected in every azimuth of the plane of reflexion; while, on the other hand, the intensity of the twice-reflected ray diminishes, as the angle between the reflecting planes increases; and it vanishes altogether, and the ray is wholly transmitted, when the plane of reflexion at the second surface is perpendicular to that at the first. These sides, or poles, once acquired, are retained by the ray in all its future course, provided it undergoes no further modification by reflexion or refraction; for, whether the plates be an inch or a mile asunder, the phenomena are the same.

(143) A polarized ray, then, is distinguished by the following characters:—

I. It is *not divided* into two pencils by a double-refracting crystal, in two positions of the principal section with respect to the ray; being refracted *ordinarily*, when the plane of polarization coincides with the principal section, and *extraordinarily* when it is perpendicular to it. In other cases it gives rise to two pencils, which vary in intensity according to the position of the principal section.

II. It suffers *no reflexion* at the polished surface of a transparent medium, when this surface is presented to it at a certain angle, and in a plane of incidence perpendicular to the plane of polarization; while it is *partially reflected*, when the reflecting surface is presented in other planes of incidence, or under different angles.

The apparatus best fitted for the exhibition of these phenomena is that devised by M. Biot. It consists of a tube, furnished at its extremities with two graduated rings, which are capable of revolving in a plane perpendicular to its axis. Each of these rings carries a plate of glass set in a frame, and held by two projecting arms. These plates are capable of revolving round a transverse axis, so that their inclination to the axis of the tube may be varied at pleasure; and a small graduated circle, attached to one of the arms, measures the inclination. The whole apparatus is connected with a vertical pillar, by a moveable joint, so that the tube may be inclined to the horizon at any angle. In this form of the apparatus, it is arranged to exhibit the properties of polarized light dependent on reflexion: in order to show the other properties, one of the glass plates may be removed, and a double-refracting prism, or a plate of tourmaline, substituted in its place.

(144) The angles of incidence at which light is polarized are called the *angles of polarization*. They are in general different for different substances: thus the angle of polarization of *glass* is 57°, and that of *water*, 53°.

The connexion between the polarizing angles and the other properties of substances with regard to light was discovered by Sir David Brewster, In the year 1811 he commenced an extensive series of experiments, with the view of determining the angles of polarization of different media, and of connecting them by a law. The result of these investigations was the simple and remarkable principle, that "*the index of refraction of the substance is the tangent of the angle of polarization.*" Hence, when the refractive index is known, the angle of polarization is inferred, and *vice versâ;* and we learn from the law that this angle ranges in different substances from 45° upwards, increasing always with the refractive power.

The refractive index being different, for the differently coloured rays which compose solar light, it follows that all the rays of the spectrum are not polarized at the same angle; so that if a beam of solar light be reflected successively by two glass plates, whose planes of reflexion are at right angles, the reflected beam will never be wholly extinguished, but will be coloured *red* or *blue*, according as the angle of incidence is the polarizing angle of the more, or of the less refrangible rays. When the angle of incidence is the angle of polarization corresponding to the *most luminous* portion of the spectrum, the reflected light is of a *purplish* tint, formed by the union of the remaining rays in different proportions. These effects are, of course, most observable in highly dispersive substances.

The law of Brewster may be presented in another form. We may say that the angle of polarization is such, that the *reflected and refracted rays form a right angle.* In fact, if this angle be denoted by π, and the corresponding angle of refraction by ρ, we have

$$\tan \pi = \mu, \quad \text{or} \quad \frac{\sin \pi}{\cos \pi} = \frac{\sin \pi}{\sin \rho};$$

$$\text{therefore,} \cos \pi = \sin \rho, \quad \text{and} \quad \pi + \rho = 90°.$$

Now the angle of reflexion is equal to the angle of incidence, π; consequently the angles of reflexion and refraction are complementary, and the reflected and refracted rays are perpendicular.

(145) The law of Brewster applies to the case of light reflected from the surface of a *rarer*, as well as that of a denser, medium; and it follows from it, that the *two angles of polarization*, at the bounding surface of the same two media, are *complementary*. For the index of refraction, from the denser into the rarer medium, is the reciprocal of the index when the light proceeds in the contrary direction; consequently, the tangents of the angles of polarization are reciprocals, and the angles themselves complementary.

It follows from this, that when a beam of light falls upon a medium bounded by parallel planes, and at the polarizing angle of the first surface, the portion which enters the medium will meet the second surface also at its polarizing angle, and be completely polarized by reflexion there. For the ray being incident upon the first surface at the polarizing angle, the angle of refraction will be the complement of the angle of incidence, and will be therefore equal to the angle of polarization at the second surface. But the surfaces being parallel, the angle of refraction at the first surface is equal to the angle of incidence at the second; the ray will therefore fall upon the second surface at its polarizing angle.

From the same principles it follows, that if several plates of glass, or of any transparent substance, be arranged parallel to one another, and a ray of light be incident upon the first surface at the polarizing angle, the several portions which reach the succeeding surfaces will meet them also at their polarizing angles, and the portions reflected at each will be completely polarized. Such a *pile of plates* is highly useful as a *polarizer*; for the reflected beam is necessarily far more intense than that produced by a single surface.

(146) It has been shown, that when a beam of light is

polarized by reflexion, is suffered to fall upon a second reflecting surface at the polarizing angle, the intensity of the twice-reflected beam will vary with the inclination of the planes of reflexion, being greatest when these planes are coincident, and vanishing when they are perpendicular. In all cases, *the intensity varies as the square of the cosine of the angle formed by the two planes of reflexion.* This law was at first conjecturally assumed by Malus; its truth has since been verified by the experiments of Arago.

It follows, as a consequence of this law, that a ray of common light may be conceived to be composed of two polarized rays of equal intensity, whose planes of polarization are perpendicular.* For if light, thus composed, is incident on a reflecting surface, and if a, and $90° - a$, denote the angles which the plane of reflexion makes with the planes of polarization of the two pencils, the intensity of the reflected light in one of these rays will be $I \cos^2 a$, and in the other $I \sin^2 a$, I denoting the intensity of each of the incident pencils; and the sum of these, or the total intensity of the reflected light, is

$$I\,(\cos^2 a + \sin^2 a) = I.$$

The intensity is therefore constant, and independent of the position of the plane of reflexion with respect to the ray; and this, we have seen, is the distinctive character of common, or unpolarized light.

* This is not to be understood as describing the actual physical character of ordinary, or unpolarized light. This may be more correctly represented as polarized light, whose plane of polarization is incessantly changing; so that, in a given time, there are as many polarized rays in any one plane as in any other at right angles to it. This agreement has been verified experimentally by Professor Dove, by impressing mechanically a rapid motion of rotation upon the plane of polarization of the light; the phenomena presented by the resulting light agreeing in all respects with ordinary or unpolarized light. When analyzed by a double refracting prism, the two images were of equal intensities in all azimuths, so as to have similar properties in all planes passing through the ray.

(147) We now proceed to consider the effects which take place, when the light is incident upon the reflecting surface at an angle different from the polarizing angle.

Malus observed, that when the angle of incidence was either greater or less than the polarizing angle, the properties already described were only *in part* developed in the reflected light; that neither of the two pencils into which it was divided by a rhomb of Iceland spar ever wholly vanished; but that they varied in intensity between certain limits, these limits being closer the more remote the incidence from the polarizing angle. From this he naturally concluded that, in these circumstances, a *portion only* of the reflected light had received the modification to which he had given the name of polarization, that portion increasing as the incidence approached the polarizing angle; and that the remaining portion was unmodified, or in the state of common light. *Partially polarized* light, then, according to Malus, is composed of two portions, one of which is wholly polarized, while the other is in the state of ordinary or unpolarized light. In this supposition Malus has been followed by most subsequent philosophers.

If this partially polarized light be reflected at a second surface in the same plane, and at the same angle, the reflected pencil is found to contain a greater portion of polarized light; and by increasing the number of successive reflexions, the light may become, as to sense at least, wholly polarized. This fact was first observed by Sir David Brewster; and it was found that light may be polarized at any incidence, by a sufficient number of reflexions, the number of reflexions necessary to produce this result increasing as the incidence is more removed from the polarizing angle.

(148) It remains to describe the modification which light undergoes in *refraction*.

When common light is suffered to fall upon a plate of glass, a portion of it in all cases enters the plate, and is refracted; and this refracted portion is found to be *partially polarized.* The quantity of polarized light in the refracted light increases with the incidence, being nothing at a perpendicular incidence, and greatest when the incidence is most oblique. The plane of polarization is not (as in the case of reflected light) coincident with the plane of incidence, but perpendicular to it.

The connexion between the light thus polarized, and that polarized by reflexion, is very intimate, the two portions being always of equal intensity. This remarkable law was discovered by Arago, and may be thus enunciated—"*When an unpolarized ray is partly reflected at, and partly transmitted through, a transparent surface, the reflected and transmitted portions contain equal quantities of polarized light; and their planes of polarization are at right angles to each other.*"

(149) If the transmitted light be received upon a second plate parallel to the first, the portion of common light which it contains undergoes a new subdivision; and so continually, whatever be the number of plates. Hence, when that number is sufficiently great, the transmitted light will be, as to sense, *completely polarized*, the plane of polarization being perpendicular to the plane of incidence. These facts were discovered by Malus. The laws of the phenomena have been since investigated, in much detail, by Sir David Brewster; and he has drawn the conclusion, that when a ray of light is transmitted through any number of plates, in the same plane of incidence, the polarization will be complete, when the sum of the tangents of the angles of incidence is equal to a certain constant. Hence, when the plates are parallel, and the incidence therefore the same on all, the tangent of the angle of complete polarization is inversely as their number.

It is a remarkable consequence of these principles, that when a ray is incident upon a pile of parallel plates at the polarizing angle, after passing a certain number the intensity of the transmitted light will undergo no further diminution. For, when the transmitted light becomes wholly polarized, no portion of it whatever will be reflected by any of the succeeding plates, its plane of polarization being perpendicular to the plane of incidence; it is therefore transmitted without diminution through them, whatever be their number. The case is different, however, when the light is incident on the pile at any other than the polarizing angle; and it follows therefore that the intensity of the light transmitted through a thick pile is greatest, when it is incident at the polarizing angle.

(150) There are certain crystals which, like the pile of transparent plates, possess the property of polarizing the transmitted light. This property depends upon a peculiarity in the absorbing powers of double-refracting crystals,—namely, that the *absorption of a polarized ray varies with the position of its plane of polarization* with respect to the crystal. Thus, tourmaline absorbs a polarized ray more rapidly when the plane of polarization is parallel to the axis, than when it is perpendicular to it. But a ray of *common light* falling upon this crystal may be divided into two, one polarized in a plane passing through the axis, and the other in a plane perpendicular to it; and as the former of these is absorbed more rapidly than the latter, the transmitted light will be *partially polarized in the plane perpendicular to the axis of the crystal.* When the plate is sufficiently thick, the latter portion alone will be sensible, and the ray emerges *wholly polarized in the perpendicular plane.*

The tourmaline, accordingly, is of much use in experiments on polarized light, not only in affording a ready test of polarization, but also in producing a polarized beam. It has the disadvantages, however, that the polarization of the emergent light is never perfect, and that its intensity is much weakened by absorption — both the rays being absorbed in their passage through the crystal, though with unequal energies.

The polarization produced by double refraction is the most complete of any; while the intensity of the polarized pencils is greater than in any other case, being very nearly *one-half* of the intensity of the original light. The intensity of the light reflected from a plate of glass, at the polarizing angle, is not more than the $\frac{1}{12}$th part of that of the incident light.

(151) M. Haidinger has observed a remarkable phenomenon of polarized light, by which it may be recognised by the naked eye, and its plane of polarization ascertained. This phenomenon consists in the appearance of *two brushes*, of a pale orange-yellow colour, the axis of which coincides always with the trace of the plane of polarization; these are accompanied, on either side, by two patches of light, of a complementary or violet tint. In order to see them, the plane of polarization of the light must be turned quickly from one position to another, so as to shift the position of the brushes. Thus, they may be observed by looking for a few moments at one of the images of a circular aperture, formed by a rhomb of Iceland spar, and then at the other, and so alternately. They gradually disappear when the eye continues directed to them in the same position; but they may be made to reappear by shifting that position, or the plane of polarization on which it depends.

The most probable explanation of this phenomenon seems to be that given by M. Jamin, in which it is ascribed to the refracting coats of the eye. When polarized light falls upon a pile of parallel plates, the proportion of the refracted to the incident light varies with the plane of polarization, being a minimum when that plane coincides with the plane of incidence, and a maximum when it is perpendicular to it. These variations are nothing at a perpendicular incidence: they are greatest when the angle of incidence is equal to the angle of polarization. Accordingly, when the polarized light is incident obliquely on the plates, the refracted light should exhibit *two dark brushes*, enlarging from the centre to the circumference, in the plane of polarization; and *two bright brushes* in the perpendicular plane.

In the preceding explanation, the incident light is supposed to be *homogeneous*. When *white* light is used, the intensities of its several components, in the refracted pencil, will vary with the refractive indices, and consequently the brushes will be coloured. M. Jamin has shown that the effect of a single refracting surface will be to produce two yellow brushes, whose axis is in the plane of polarization.

CHAPTER IX.

TRANSVERSAL VIBRATIONS—THEORY OF REFLEXION AND REFRACTION OF POLARIZED LIGHT.

(152) Having in the preceding chapter stated the principal facts of polarization, we may proceed to consider their connexion with the physical theory.

It is strange that the department of optics, in which the wave-theory now stands unrivalled, should be the very one which Newton selected as affording the most decisive evidence against it:—"Are not," says he, "all hypotheses erroneous, in which light is supposed to consist in pressure, or motion, propagated through a fluid medium? . . . . for pressures or motions, propagated from a shining body through an uniform medium, must be on all sides alike; whereas it appears that the rays of light have different properties in their different sides." In this objection Newton seems to have had his thoughts fixed upon that species of undulatory propagation, whose laws he himself had so sagaciously unfolded. When *sound* is propagated through *air*, the vibrations of the particles of the air are performed in the same direction in which the wave advances; and if the vibrations of the ether which constitute light had been of the same kind, the objection would be insuperable. For, if the particles of the ether vibrated in the direction of the ray itself, the ray could not bear a different relation to the different parts of the surrounding space.

But the case is altered, if the vibrations of the ethereal particles be performed in a transverse direction. Let us suppose the direction of the vibrations to be *perpendicular* to that of the ray: then it is obvious that if that direction be *vertical*, for example, while the ray advances *horizontally*, the ray will bear a relation to the parts of space *above* and *below*, different from that which it bears to those parts which are on the *right hand* and on the *left*. Such is, in fact, the mode of vibration which is now assumed to belong to the ether, in the wave-theory, the ethereal molecules being supposed to vibrate *in the plane of the wave;* and we shall find that, with the help of this assumption, all the complicated phenomena of polarization and double refraction are explained in the fullest and most complete manner.

The principle of *transversal vibrations*, as it is called, seems to have first occurred to Hooke, and was announced, in 1672, in his *Micrographia*. Young and Fresnel arrived at the same principle independently; and the latter has reared upon its basis the noblest fabric which has ever adorned the domain of physical science,—Newton's system of the universe alone excepted.

(153) In order to conceive the manner in which an undulation may be propagated by transversal vibrations, let us imagine a cord stretched in a horizontal position, one end being attached to a fixed point, and the other held in the hand. If the latter extremity be made to vibrate, by moving the hand up and down, each particle of the cord will, in succession, be thrown into a similar state of vibration; and a series of waves will be propagated along it with a uniform velocity. The vibrations of each succeeding particle of the cord, being similar to that of the first, will all be performed in the same plane, and the whole will represent the state of the ethereal particles in a *polarized* ray.

Now if, after a certain number of vibrations in the vertical plane, the extremity of the cord be made to vibrate in another plane, and then in another,—and so on, in rapid succession,—each particle of the cord will, after a time proportional to its distance from the extremity, assume in succession all these varied vibrations; and the whole cord, instead of taking the form of a curve lying all in *one plane* (as in the last case), will be thrown into a species of *helical curve*, depending on the nature of the original disturbance. Such is the condition of the ethereal particles in a ray of common, or *unpolarized light*.

When, therefore, we admit a connexion to subsist among the particles of the ether, such as that which holds among the particles of the cord, there is no difficulty in conceiving how a vibration may be propagated in a direction perpendicular to that in which it is executed. It is true, the particles of the ether are not chained together by cohesive forces, like those of the cord; but the attractive forces which subsist among them are of the same kind, and may be shown to produce a similar effect. In fact, let us conceive the ether to be composed of separate molecules, which act on one another according to some law varying with the distance. When any row or line of such molecules is similarly displaced, through a space which is small compared with the separating intervals, the molecules of the succeeding row will be moved in the same direction by the forces developed with the change of distance; so that the vibrations of the particles composing the first row will be communicated to those of the second, and thus the vibratory motion will be propagated in a direction perpendicular to that in which it takes place. The rapidity of the propagation will depend on the magnitude of the force developed by the displacement.

To account for the fact, that there are no sensible vibrations in a direction normal to the wave, we have only to suppose the repulsive force between the molecules to be very great, or the resistance to compression very considerable. For, in this case, the force which resists the *approach of two strata* of the fluid is much greater than that which opposes their *sliding* on one another.

(154) But the existence of transversal vibrations—and of transversal vibrations only—is a necessary consequence of the laws of interference of polarized light, if the theory of waves be admitted at all. It has been *experimentally* proved, by Fresnel and Arago, that two rays *oppositely polarized* compound a single ray whose intensity is *constant*, whatever be the phases of vibration in which they meet. But *theory* shows, that the intensity of the light resulting from the union of two rays oppositely polarized will be constant, and independent of the phase, *only when the vibrations normal to the wave are evanescent.*

This conclusion is easily extended to the case of common, or unpolarized light. In unpolarized light, therefore, as in polarized, the vibrations are only in the plane of the wave: but in the latter, these vibrations are all *parallel to a fixed line;* while in the former they take place in every possible direction in the plane of the wave. The phenomenon of polarization consists simply in the resolution of these vibrations into two sets, in two rectangular directions, and the subsequent separation of the two systems of waves thus produced. When the resolved vibrations are not separated, but one of them is diminished in any ratio, the light is said to be partially polarized.

(155) We have stated that the vibrations of the molecules of the ether, in a polarized ray, are all parallel to a fixed direction in the plane of the wave: this fixed direction may be either *parallel* or *perpendicular* to the plane of polarization; and there was nothing in the phenomena, hitherto discovered, to determine the choice between these two positions. Hence, contrary suppositions have been made respecting it. In the theories of Fresnel and of Cauchy, the vibrations are assumed to be *perpendicular to the plane of polarization,*—in those of Mac Cullagh and Neumann, to be *parallel to it;* and

this difference in one of the postulates of the different theories has necessarily led to others, especially as respects the relative densities of the ether in different media.

Professor Stokes has recently arrived at a result, in the dynamical theory of diffraction, which seems to afford the means of deciding between these hypotheses. When a polarized ray is diffracted, the plane of vibration of the diffracted ray should differ from that of the incident, the positions of the two planes being connected by a very simple relation. This relation may be deduced in the following elementary manner.

When a polarized ray is incident perpendicularly upon a fine grating, the direction of its vibrations is (by the principle of transversal vibrations) in the plane of the grating, when the wave reaches it. Let a denote the angle formed by that direction with the lines of the grating: then, if the amplitude of the incident vibration be taken equal to unity, it may be resolved into two,—namely, $\cos a$, parallel to the lines of the grating, which will be unaltered by diffraction; and $\sin a$, perpendicular to them. The second component is to be resolved again, in the direction of the diffracted ray, and perpendicular to it, respectively; and of these the latter portion alone is propagated as light. Its value is $\sin a \cos \theta$, θ being the angle which the diffracted ray makes with the incident. Hence the two components of the diffracted ray are

$$\cos a, \quad \text{and} \quad \sin a \cos \theta;$$

and their ratio is equal to the tangent of the angle which the direction of the vibration in the diffracted ray makes with the lines of the grating. Denoting this angle by a', we have therefore,

$$\tan a' = \tan a \cos \theta.$$

Accordingly, the angle which the direction of the vibration makes with the lines of the grating is less in the diffracted than in the incident ray.

It would appear, therefore, that we had only to measure the angles which the *planes of polarization* of the incident and diffracted rays make, respectively, with the lines of the grating. If the latter is *less* than the former, the vibrations are *parallel* to the plane of polarization; if it be *greater*, they are *perpendicular* to it. The experiment has been made by Professor Stokes himself; and he has drawn the conclusion that the latter is the fact, and, therefore, that the original hypothesis of Fresnel is the true one. An opposite result has been since obtained by M. Holtzmann, on repeating the same experiment under somewhat different circumstances; and the question must therefore be regarded as still undetermined.*

(156) We now proceed to consider the application of the principle of transversal vibrations to the problem of reflexion and refraction.

The *direction* of the light reflected and refracted at the surface of a uniform medium, is a simple consequence of the theory of waves; and we have already explained Huygens' demonstration of the laws which govern this direction—a demonstration which holds good, whatever be the magnitude and direction of the propagated vibration, or, in other words, whatever be the *intensity* and *plane of polarization* of the light. The problem which we have now to consider is that which proposes to determine the latter quantities, or to deduce the intensities and planes of polarization of the reflected and refracted pencils, those of the incident pencil being given.

This important problem was first solved by Fresnel. In the attempt to generalize his theory, and to apply it to reflexion and refraction at the surfaces of crystallized media, Professor MacCullagh and Mr. Neumann were led to modify the

* In retaining, therefore, the demonstration of the laws of reflexion and refraction of polarized light, which was adopted in the former edition of this work, the author does not wish to be considered as giving a preference to the principles upon which it depends.

principles of Fresnel. The principles, so modified, are the following:—

I. The vibrations of polarized light are *parallel* to the plane of polarization.

II. The density of the ether is the same in all bodies as *in vacuo*.

III. The *vis viva* is preserved; from which it follows that the masses of ether put in motion, multiplied by the squares of the amplitudes of vibration, are the same before and after reflexion.

IV. The resultant of the vibrations is the same in the two media; and, therefore, in singly-refracting media, the refracted vibration is the resultant of the incident and reflected vibrations.

When the light is polarized *in the plane of incidence*, the fourth principle, alone, is sufficient to determine the magnitudes of the reflected and refracted vibrations. For, the directions of the vibrations, being in the plane of incidence and perpendicular to the rays, are necessarily inclined to one another at the same angles as the rays themselves; these angles therefore are

$$2\theta, \quad \theta - \theta', \quad \theta + \theta',$$

θ and θ' denoting the angles of incidence and refraction. Hence, if v and v' denote the amplitudes of the reflected and refracted vibrations, that of the incident vibration being taken as unity, we have

$$v = \frac{\sin(\theta - \theta')}{\sin(\theta + \theta')}, \qquad v' = \frac{\sin 2\theta}{\sin(\theta + \theta')}.$$

When the light is polarized *perpendicularly to the plane of incidence*, the vibrations in the incident, reflected, and refracted pencils are all parallel. The law of equivalent vibrations therefore gives, in this case, the following simple relation among them,

$$1 + w = w';$$

and another relation is required, in order to deduce the values of w and w'. This second relation is furnished by the principle of the *vis viva*, and is

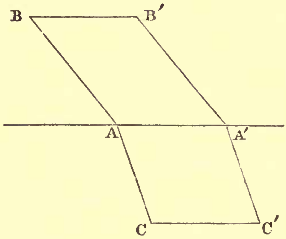
$$m(1-w^2) = m'w'^2,$$

m and m' denoting the masses of the ether in motion in the two media. Eliminating between these equations, we find

$$w = \frac{m-m'}{m+m'}, \quad w' = \frac{2m}{m+m'};$$

expressions which are remarkable as being identical with those for the velocities of two elastic balls after impact.

Let BA, AC represent the velocities and directions of the incident and refracted rays; AA′ the separating surface of the two media; and BB′, CC′, lines parallel to that surface. Then the masses of ether in motion in the two media are to one another as the parallelograms A′B, A′C; that is,



$$m : m' :: \text{AB} \sin \text{A}'\text{AB} : \text{AC} \sin \text{A}'\text{AC}$$
$$:: \sin\theta \cos\theta : \sin\theta' \cos\theta' :: \sin 2\theta : \sin 2\theta'.$$

Subtituting this ratio, therefore, in the expressions for w and w', given above,

$$w = \frac{\tan(\theta-\theta')}{\tan(\theta+\theta')}, \quad w' = \frac{\sin 2\theta}{\sin(\theta+\theta')\cos(\theta-\theta')}.$$

The *intensity* of the light is measured by the *vis viva*, or by the mass of the ether put in motion, multiplied by the square of the amplitude of the vibration. Hence, for light polarized in the plane of incidence, the intensities of the incident, reflected, and refracted rays will be m, mv^2, and $m'v'^2$, respectively;—or, if we take the intensity of the incident light as unity, 1, v^2, and $1-v^2$; since, by the law of the *vis viva*, $m(1-v^2) = m'v'^2$. Similarly, for light polarized in the perpendicular plane, the intensities in the three pencils are 1, w^2, and $1 - v^2$.

(157) Confining our attention for the present to the reflected vibration, it will be seen that its amplitude, and consequently the intensity of the light, increases with the incidence, whether the light be polarized in the plane of incidence, or in the perpendicular plane,—being least when the light is incident perpendicularly, and greatest when it is most oblique.

In the former case, i. e. when $\theta = 0$, the values of v and w are each equal to $\frac{\mu - 1}{\mu + 1}$, μ being the refractive index; and the intensity of the light reflected perpendicularly is

$$\left(\frac{\mu - 1}{\mu + 1}\right)^2.$$

This remarkable expression was first given by Young.

On the other hand, when $\theta = 90°$, or when the ray grazes the surface, the intensity of the reflected light is equal to that of the incident; or the whole of the light is reflected, whatever be the reflecting substance.

(158) We have seen that a ray of *common* light is equivalent to two polarized rays of equal intensity, whose planes of polarization are at right angles. Now, let such a ray, whose intensity = 1, be incident upon the surface of a transparent medium; and let it be resolved into two, each equal to $\frac{1}{2}$, polarized respectively in the plane of incidence, and in the perpendicular plane. Each of these polarized rays will give rise to a reflected and refracted ray; so that the actual reflected and refracted rays will consist of two portions, one polarized in the plane of incidence, and the other in the perpendicular plane. If these portions were of equal intensity, as they are in the incident light, the reflected and refracted rays would be *unpolarized:* but this, in general, is not the case.

In the case of the *reflected* beam,—the intensities of the two portions are

$$\tfrac{1}{2}v^2, \quad \tfrac{1}{2}w^2;$$

and the whole intensity of the reflected light is their sum. Now the first thing to be observed is, that these two quantities are *unequal;* or, that the two portions of which the reflected light consists, and which are polarized in opposite planes, are *different in intensity.* Hence the reflected light will not be of the nature of common, or unpolarized light; but will have an *excess* of light *polarized in the plane of incidence*, the former expression being always greater than the latter. This is otherwise expressed by saying, that the light is *partially polarized in the plane of incidence.* The quantity of polarized light is measured by the difference of the two portions, or by

$$\tfrac{1}{2}(v^2 - w^2).$$

Again, the intensities of the two *refracted* portions are

$$\tfrac{1}{2}(1 - v^2), \quad \tfrac{1}{2}(1 - w^2).$$

As the latter of these quantities is greater than the former, the refracted beam always contains an excess of light polarized *perpendicularly to the plane of incidence.* Their difference, $\frac{1}{2}(v^2 - w^2)$ is the same as in the former case; and accordingly, the reflected and refracted pencils contain *equal quantities of oppositely polarized light.* Thus, the experimental law of Arago is a necessary consequence of theory.

(159) The reflected light will be *completely polarized*, when one of the portions of which it consists vanishes; for, in this case, the whole of the reflected light will be polarized in a single plane. It is easy to see that the first portion, which is polarized in the plane of incidence, can never vanish. The second part vanishes, when $\theta + \theta' = 90°$, the denominator of the fraction becoming infinite; the reflected light then contains only the other portion, and is therefore *completely polarized in*

the plane of incidence. Since, in this case, $\theta + \theta' = 90°$, we have

$$\cos\theta = \sin\theta' = \frac{\sin\theta}{\mu}, \quad \text{and} \quad \tan\theta = \mu\,;$$

i. e. *the tangent of the angle of polarization is equal to the refractive index.* Thus the beautiful law, which Brewster had inferred from observation, is deduced as an easy consequence of Fresnel's theory.

When $\theta + \theta'$ is greater than 90°,—i. e. when the angle of incidence *exceeds* the polarizing angle,—the expression for the amplitude of the reflected, w, vibration *changes sign*, the light being polarized perpendicularly to the plane of incidence. This change of sign is equivalent to an alteration of the *phase* of the reflected vibration by 180°, as the incidence passes the polarizing angle; and the circumstance explains the remarkable fact noticed by Arago,—namely, that when Newton's rings are formed between a lens of glass and a metallic reflector (the incident light being polarized perpendicularly to the plane of reflexion), the rings change their character as the incidence passes the polarizing angle of the glass, the black centre being transformed into a *white* one, and the whole system of colours becoming *complementary* to what it was before. Mr. Airy was led to anticipate this result, from a consideration of the formula; and to show that a similar change must take place in the rings formed between two transparent substances of different refractive powers, as the incidence passes the polarizing angle of either substance.

(160) When a polarized ray undergoes reflexion, the reflected light is still polarized, but its *plane of polarization is changed*, the amount of the change depending on the incidence. When the angle of incidence is nothing, or the ray perpendicular to the reflecting surface, the new plane of polarization is inclined to the plane of incidence by the *same angle* as the old, but on the *opposite side.* As the angle of in-

cidence increases, the plane of polarization of the reflected ray approaches the plane of incidence, and finally *coincides* with it, when the incidence reaches the *polarizing angle*. As the angle of incidence still further increases, the plane of polarization of the reflected ray crosses the plane of incidence, and therefore lies on the *same side* of it with the original plane; and the two planes of polarization *finally coincide*, when the angle of incidence is 90°.

The azimuth of the plane of polarization of the reflected ray may be deduced from the theory we have been considering. For, let the vibration of the incident ray, a, be resolved into two, one in the plane of incidence, and the other in the perpendicular plane. If α denote the angle which it makes with the plane of incidence, these resolved portions are $a \cos \alpha$, and $a \sin \alpha$. After reflexion they become,* respectively,

$$-a \cos \alpha \frac{\sin(\theta - \theta')}{\sin(\theta + \theta')}, \quad a \sin \alpha \frac{\tan(\theta - \theta')}{\tan(\theta + \theta')};$$

and they compound a single vibration, inclined to the plane of incidence at an angle whose tangent is the ratio of the component vibrations. If, then, this angle be denoted by α', we have

$$\tan \alpha' = -\tan \alpha \frac{\cos(\theta + \theta')}{\cos(\theta - \theta')}.$$

The truth of this formula has been verified by the observations of Fresnel himself, and more fully since by those of Arago and Brewster.

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* In order to explain the facts above mentioned, the values of v and w (156) must be affected with *opposite* signs, at all incidences below the polarizing angle; and there are other phenomena which indicate that the former quantity is *negative*, and the latter *positive* (see Professor Powell's paper "On the Demonstration of Fresnel's Formulas," *Phil. Mag.*, Aug. 1856). This is equivalent to saying, that one of the waves *gains, or loses, half an undulation* in the act of reflexion. We shall see hereafter that the complete theory of reflexion includes a *progressive* change of phase; and that the conclusions of Art. (156) are only approximate.

When $a = 0$, $a' = 0$; and when $a = 90°$, $a' = 90°$. Accordingly, when the plane of polarization of the incident ray coincides with, or is perpendicular to, the plane of incidence, it is unchanged by reflexion. When $\theta + \theta' = 90°$, $a' = 0$, and the plane of polarization of the reflected ray *coincides* with the plane of incidence, whatever be the azimuth of the incident ray.

(161) The plane of polarization of a polarized ray is changed by *refraction*, as well as reflexion, but in an *opposite* direction, the plane being removed farther from the plane of incidence, instead of approaching it. This movement of the plane of polarization increases with the incidence; being nothing when the ray falls perpendicularly upon the refracting surface, and greatest when the incidence is most oblique. The law of the change is expressed by the simple formula,

$$\operatorname{cotan} a' = \operatorname{cotan} a \cos(\theta - \theta');$$

in which a and a' denote (as before) the angles which the planes of polarization form with the plane of incidence, before and after refraction. This law was discovered experimentally by Sir David Brewster: it is a necessary consequence of the theory already given, and is deduced by a process exactly similar to that of the preceding article.

CHAPTER X.

ELLIPTIC POLARIZATION.

(162) WHEN an ethereal molecule is displaced from its position of equilibrium, the forces of the neighbouring molecules are no longer balanced, and their resultant tends to drive the particle back to its position of rest.* The displacement being supposed to be very small, in comparison with the intervals between the molecules, the force thus excited will be proportional to the displacement; and from this it follows, according to known mechanical principles, that the trajectory described by the molecule will be an ellipse, whose centre coincides with the position of equilibrium. Hence the vibration of the ethereal molecules is, in general, *elliptic*, and the nature of the light depends on the *direction* and *relative magnitude of the axes.* By the principle of transversal vibrations, these elliptic vibrations are all *in the plane of the wave;* their axes, however, may either preserve constantly the *same direction* in that plane, or they may be continually shifting. In the former case, the light is said to be *polarized;* in the latter, it is unpolarized, or common light.

The relative magnitude of the axes of the ellipse determines the nature of the polarization. When the axes are *equal*, the ellipse becomes a *circle*, and the light is said to be *circularly polarized*. On the other hand, when the lesser axis vanishes, the ellipse becomes a *right line*, and the light is *plane-polarized*—the vibrations being in this case confined to a single plane passing through the direction of the ray.

* This is not strictly true, except in homogeneous or uncrystallized media.

In intermediate cases, the polarization is called *elliptical;* and its character may vary indefinitely between the two extremes of plane polarization and circular polarization.

(163) An *elliptic* vibration may be regarded as the resultant of *two rectilinear* vibrations, at right angles to one another, which differ in phase.

For, let x and y denote the distances of the molecule of the ether from its position of rest, in the two rectangular directions; a and b the amplitudes of the component vibrations; and t the time. Then

$$x = a \sin (vt - \alpha), \quad y = b \sin (vt - \beta);$$

whence

$$\alpha - \beta = \text{arc}\left(\sin = \frac{y}{b}\right) - \text{arc}\left(\sin = \frac{x}{a}\right).$$

Taking the cosines of both sides, and clearing the result of radicals, we obtain

$$\frac{y^2}{b^2} + \frac{x^2}{a^2} - 2\cos(\alpha - \beta)\frac{xy}{ab} = \sin^2(\alpha - \beta).$$

This is the equation of an *elliqse* referred to its centre.

When the component vibrations are *equal* in amplitude, and *differ* 90° *in phase*,

$$a = b, \quad \text{and} \quad \alpha - \beta = 90°;$$

and the preceding equation becomes

$$y^2 + x^2 = a^2.$$

The path described by the molecule is then a *circle.*

(164) The nature of the elliptic polarization is completely defined, when we know the *direction of the axes* of the ellipse, and the *ratio of their lengths.*

These may be determined experimentally. In fact, when the elliptically-polarized ray is transmitted through a double-refracting prism, whose principal section is parallel to one of the axes of the ellipse, it is resolved into two plane-polarized rays, one of which has the *greatest* possible intensity, and the other the *least*. Accordingly, the direction of the principal section, for which the two pencils are most unequal, is the direction of one of the axes; and the square roots of the intensities are in the ratio of their lengths.

The direction of the axes of the ellipse may be more conveniently determined by turning the prism until the two pencils are of *equal intensity:* the principal section is then inclined at an angle of 45° to each of the axes.

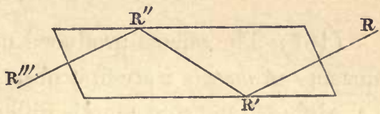
(165) When a plane-polarized ray undergoes reflexion, the reflected light is, generally, *elliptically-polarized*. For a plane-polarized ray may be resolved into two, polarized respectively in the plane of incidence, and in the perpendicular plane; and we shall presently see that the effect of reflexion is, in general, to alter the phases of these two portions, and by a different amount. Hence the reflected light is compounded of two plane-polarized rays, whose vibrations are at right angles, and whose phases are no longer coincident; it is therefore elliptically polarized (163).

The first case in which this effect was observed was that of *total reflexion.*

When the angle of incidence exceeds the angle of total reflexion (the light passing from the denser into the rarer medium), the expressions for the intensity of the reflected light, given in (156), become *imaginary*. But it is obvious that, in this case, the intensity of the reflected light is simply equal to that of the incident, there being no refracted pencil. How, then, are the imaginary expressions to be interpreted? They signify, according to Fresnel, that the *periods of vibration* of the incident and reflected waves, which had been assumed to coincide at the reflecting surface, *no longer coincide* there when the reflexion is total; or, in other words, that the ray undergoes a *change of phase* at the moment of reflexion. The amount of this change has been deduced by Fresnel, by a most ingenious train of reasoning, based upon the interpretation of imaginary formulæ. It varies with the incidence; and is different for light polarized in the plane of incidence, and in the perpendicular plane.

In the case of light polarized in any azimuth, we have only to conceive the incident vibration resolved into two, one in the plane of incidence, and the other in the perpendicular plane. The phases of these vibrations being differently altered by reflexion, the reflected vibration will be the resultant of two vibrations at right angles to one another, and differing in phase,—the amount of the difference depending upon the angle of incidence: this vibration, consequently, will be *elliptic*, and the reflected light *elliptically polarized.* When the azimuth of the plane of polarization of the incident ray is 45°, the amplitudes of the resolved vibrations will be *equal;* and if, moreover, their difference of phase is a *quarter of an undulation*, the ellipse will become a circle, and the light will be *circularly polarized.*

(166) Reducing his formulæ to numbers, in the case of St. Gobain's glass, Fresnel found that the difference of phase of the two portions of the reflected light amounted to *one-eighth** of an undulation, when the angle of incidence was 54° 37′. Polishing, therefore, a parallelopiped of this glass, whose faces of incidence and emergence were inclined to the other faces at these angles, it followed that a ray RR′R″R‴, incident perpendicularly on one of these sides, and once reflected at each of



the others, at R′ and R″, would emerge perpendicularly at the remaining side, the difference of phase in the two portions of

* In order to produce a difference of phase of a quarter of an undulation by a *single* reflexion, the refractive index should be = 4·142.

the twice-reflected ray amounting to a quarter of an undulation. If, then, the incident ray be polarized in a plane inclined at an angle of 45° to the plane of reflexion, the emergent light will be *circularly polarized.* This was found to be the case on trial, and the theory thereby verified. The parallelopiped described is well known under the name of *Fresnel's rhomb;* and is of essential service in all experiments relating to circular and elliptic polarization.

If the circularly polarized ray be made to undergo two more total reflexions, in the same plane and at the same angle, by transmitting it through a second rhomb placed parallel to the first, it will emerge *plane-polarized;* and its plane of polarization will be inclined 45° on the other side of the plane of reflexion. In fact, the two additional reflexions increase the difference of phase of the two portions, into which the light was originally resolved, from 90° to 180°; and we know that two equal vibrations, whose phases differ by 180°, compound a single right-lined vibration, whose direction bisects the supplement of the angle formed by their directions.

This property enables us to distinguish a circularly polarized ray from a ray of common light. On the other hand, it is at once distinguished from plane-polarized light, by the circumstance that it is divided into two rays of equal intensity by a double-refracting crystal, whatever be the position of the plane of the principal section.

(167) The effects produced upon light by reflexion at the surfaces of *metals* were first observed by Malus.

Malus found that metals differed from transparent bodies, in their action upon light, in this, that common light was never completely polarized by reflexion at their surfaces. The phenomenon of polarization was, however, *partially* produced; and the effect increased to a *maximum* at a certain angle of incidence.

When the incident light was polarized in a plane inclined at an angle of 45° to the plane of incidence, Malus observed that the reflected light was completely *depolarized*, the pencil being divided into two by a double-refracting prism in every position of its principal section.

(168) The subject of metallic reflexion was next studied by Sir David Brewster, and the laws of the phenomena investigated in much detail. These laws may be reduced to the following:—

I. When a ray of common light is incident upon a metallic reflector, the reflected light is *partially polarized*, the amount of polarized light in the reflected pencil increasing up to a certain incidence, which is thence called the angle of *maximum polarization.*

II. When the light is reflected several times in succession, in the same plane and at the same angle, the proportion of polarized light in the reflected pencil is increased; and by a sufficient number of reflexions the light becomes, as to sense, *wholly polarized* in the plane of incidence.

III. When a ray polarized in the plane of incidence, or in the perpendicular plane, falls upon a metallic reflector, it is polarized *in the same plane* after reflexion.

IV. A ray polarized in any other plane is, in general, *partly depolarized* by reflexion, the effect produced being greatest at the angle of maximum polarization.

V. When light, so depolarized, undergoes a second reflexion in the same plane, and at the same angle, its *polarization is restored*. The new plane of polarization lies on the opposite side of the plane of incidence from the original plane, and its azimuth is changed.

(169) From the last of the foregoing laws it is evident, that the light produced by the reflexion of a polarized ray is not common light. Neither is it plane-polarized light, since it does not vanish in any position of the analyzing rhomb.

It is *elliptically polarized;* and all the phenomena are explicable on the hypothesis, that the two oppositely polarized rays, into which the incident ray is resolved, *differ in phase* after reflexion, the difference amounting to 90° at the angle of maximum polarization. For it is plain that the effect of a second reflexion, in the same plane and under the same angle, will be to double the difference of phase, which thus becomes 180°; and the resulting light will be *plane-polarized*, the plane of polarization lying at the *opposite* side of the plane of incidence.

It is easy to see, from the foregoing, that the problem of metallic reflexion is reduced to the determination of the *intensity*, and the *phase*, of the reflected vibrations, in the case of light polarized in the two principal planes. For any polarized ray may be resolved into two, polarized respectively in the plane of incidence and in the perpendicular plane; and these planes, by the third of the preceding laws, are unaltered by reflexion. The two components, however, undergo changes both of intensity and phase; and when these are known, the character of the reflected pencil is completely determined.

This problem has been solved experimentally, by M. Jamin, in the most complete manner.

(170) The *intensity* of the light reflected by a metal at different incidences is determined by M. Jamin by comparison with the intensity of light reflected from glass under the same angle, which latter is known by Fresnel's formulæ (156). A plate of metal, and one of glass, are placed in the same plane, and in contact, and the light is allowed to fall partly upon each. When the incident light is *polarized in the plane of incidence,* the light reflected from the metal, as well as from the glass, continues polarized in that plane. If, therefore, the two reflected pencils be received on a double-refracting prism, whose principal section coincides with the plane of incidence, each of them will furnish a *single* refracted pencil.

But if the principal section of the prism be inclined to the plane of incidence at any angle, a, each of the reflected pencils will furnish *two* refracted pencils, whose intensities will vary with the azimuth of the principal section according to the known law of Malus.

Let I be the intensity of the light reflected from the metal, and I′ that of the light reflected from the glass. The intensities of the *ordinary* and *extraordinary* pencils, into which the former is subdivided by the prism, are respectively

$$I \cos^2 a, \qquad I \sin^2 a\,;$$

and those of the corresponding pencils, derived from the latter, are

$$I' \cos^2 a, \qquad I' \sin^2 a.$$

Hence, if the prism be turned, until the *ordinary* image of the light reflected from the *metal* is equal, in intensity, to the *extraordinary* image of the light reflected from the *glass*, $I \cos^2 a = I' \sin^2 a$, and

$$I = I' \tan^2 a.$$

The azimuth of the principal section, a, is measured by means of a graduated circle attached to the prism; and the value of I′ for each incidence is given by Fresnel's formulæ.

A second measure is obtained, by turning the prism until the *extraordinary* image of the light reflected from the *metal* is equal to the *ordinary* image of the light reflected from the *glass;* and similar processes are followed in the case of light polarized in the perpendicular plane.

The results of these observations prove, that when light polarized *in the plane of incidence* is reflected by a metal, the intensity of the reflected light increases continually, as the incidence increases from 0° to 90°,—the total variation, however, being very small. In the case of light polarized *perpendicularly to the plane of incidence*, on the other hand, the intensity of the reflected light diminishes from a perpendicular incidence, up to the angle of maximum polarization, and afterwards increases. The values found by experiment accord satisfactorily with the results of M. Cauchy's dynamical theory. The intensities of the reflected light, in the two cases, are equal at the extreme incidences: at all other incidences the intensity of the reflected light is less in the case of light polarized perpendicularly to the plane of incidence; and the inequality is greatest at the angle of maximum polarization.

(171) It remained to determine the *difference of phase* of the two component pencils corresponding to any incidence.

For this purpose two mirrors of the same metal were placed parallel to one another, with their reflecting surfaces opposed; and their distance was adjusted by means of a screw. A ray of light, incident upon one of the mirrors, will, after reflexion, fall upon the other in the same plane, and under the same angle. It will then return to the first, its plane and angle of incidence being unaltered; and will thus undergo a series of similar reflexions between the mirrors, the number of which depends on their distance, and on the angle of incidence.

Now the incident ray, polarized in any plane, may be resolved into two, polarized respectively in the plane of incidence, and in the perpendicular plane. The *planes of polarization* of these two components are unchanged by reflexion: but their *phases* are altered, and that unequally; and the reflected light, composed of them, is therefore *elliptically polarized*.

When there are *several* reflexions in the same plane, and under the same angle, the two components undergo the same modification of phase at each successive reflexion, and the difference of phase produced is equal to that produced by a single reflexion, multiplied by the number of reflexions. But the resulting light will be *plane-polarized*, when the difference of phase becomes a multiple of π: we have, therefore, only to increase the number of reflexions at the same incidence* until the light is plane-polarized, and the difference of phase produced by a single reflexion will be known. For, if ε denote the difference of phase sought, $n\varepsilon$ will be that produced by n reflexions. And, when the resulting light is plane-polarized, $n\varepsilon = m\pi$, m being any integer number; consequently

$$\varepsilon = \frac{m\pi}{n}.$$

It follows from these researches, that the phase of the ray polarized perpendicularly to the plane of incidence is always *retarded*, relatively to the other. The difference of phase increases regularly with the incidence, being equal to π at a perpendicular incidence, and to 2π at an incidence of 90°. At the angle of *maximum polarization*, $\varepsilon = \frac{3}{2}\pi$. This angle is, of course, different for different metals: it is, however, not far in any from 75°.

(172) It follows from the preceding, that there are $n - 1$ incidences between 0° and 90°, for which the ray is *restored* to the condition of *plane polarization* by successive reflexions. For the ray becomes plane-polarized, as often as the difference of phase of the two components is a multiple of π. But, with a single reflexion, the difference of phase increases by π between 0° and 90° of incidence. Consequently, with n reflexions, the difference increases by $n\pi$; and between these limits of incidence there are $n - 1$ multiples of π, and therefore $n - 1$ angles of incidence at which the polarization is restored.

The plane of polarization of the restored ray will be on the *same* side of the plane of incidence, as the plane of polarization of the incident ray, or on the *opposite*, according as the difference of phase is an *even* or *odd* multiple of π.

(173) It would appear from the foregoing, that *metals*

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* In practice it is more convenient to produce this effect by *increasing the incidence*, the number of reflexions remaining unchanged.

differ from *transparent* bodies, in their action upon light, in two particulars—namely, 1st, that they do not polarize common light completely at any incidence; and 2nd, that they alter plane-polarized light by reflexion into light elliptically polarized. It will be seen, presently, that these differences are only differences in degree.

It was long since observed by M. Biot, that *diamond* and *sulphur* did not polarize the light completely at any angle; and the property was extended, by Sir John Herschel, to all transparent bodies possessing an *adamantine lustre*. Mr. Airy has proved, that plane-polarized light becomes elliptically polarized, by reflexion from diamond. And, finally, Mr. Dale and Professor Powell have shown that these two properties, supposed peculiar to metals, belonged to all transparent bodies having a *high refractive power*.

In this state of the question, the problem of reflexion by transparent bodies was taken up by M. Jamin, and received, at his hands, its complete experimental solution. The conclusions deduced by M. Jamin from his observations may be summed up as follows:—

I. *All* transparent bodies polarize the light *incompletely* by reflexion—the polarization of the reflected light becoming a *maximum* at a certain angle of incidence.

II. They transform plane-polarized light into light *elliptically polarized*.

III. The *difference of phase* which they impress upon light, polarized in the two principal planes, undergoes the same variations as in metallic reflexion, within certain limits of incidence.

(174) It is necessary to enter a little more minutely into the consideration of this third law, which (it is obvious from the preceding) virtually includes the two others.

According to Fresnel's theory, when a ray polarized in any plane falls upon a transparent body, the reflected light continues polarized. But its plane of polarization is changed; and lies at the opposite side of the plane of incidence, when the incidence is less than the polarizing angle, and at the same side when it is greater (160). It follows from this, that the two components of the reflected ray, polarized respectively in the plane of incidence and in the perpendicular plane, *agree in phase* at all incidences above the angle of polarization, while they *differ* 180° *in phase* at all incidences below it. According to this theory, therefore, the difference of phase changes *abruptly*, from π to 2π, at that critical incidence. On the other hand, in reflexion from *metals*, the difference of phase of the two components increases *continuously* from π to 2π, as the incidence increases from 0° to 90°.

Now M. Jamin has shown that the latter is generally true for all bodies, whether *opaque* or *transparent;* and that the distinction of these bodies, as to their effects upon reflected light, consists only in this, that in transparent bodies the variation of phase is insensible, except in the neighbourhood of the angle of maximum polarization.

In transparent substances, accordingly, the difference of phase is *nearly constant*, at *low* and at *high* incidences; and passes from π to 2π, (not abruptly, as we are required to suppose in Fresnel's theory, but) between two incidences, one lower and the other higher than the angle of maximum polarization. The *elliptic polarization* of the reflected light will be sensible only within the same limits of incidence; and beyond them the light is (as to sense) plane-polarized. In substances of low refractive power, these limiting incidences differ from one another, and from the angle of maximum polarization, by a small amount; and for these, therefore, the change of phase (although not instantaneous) is very rapid, and Fresnel's laws are *approximately* true.

When the difference of phase $= \frac{3}{2}\pi$, the ellipticity of the reflected ray is greatest. The angle of incidence at which this occurs is the angle of *maximum polarization* in the case of common light, and is called the *principal incidence*. It is theoretically different from the angle given by Brewster's law; but the difference is in all cases small.

(175) M. Jamin has shown, further, that transparent bodies may be distinguished into *two classes*, with respect to their action upon reflected light. In some of them, as in *opal*, the phase of the component in the plane of incidence is *accelerated*, relatively to the other component; in others, as *hyalite*, it is *retarded*. The bodies of these classes are denominated, by M. Jamin, substances of *positive* and of *negative reflexion*, respectively. Intermediate to these two classes we should expect to find a *third*, characterized by the property that the phase is *unaltered by reflexion*, and for which, therefore, Fresnel's laws are *accurately* true. This class is very small; the only bodies observed to belong to it being *menilite* and *alum*.

These distinctions appear to be connected with the refractive power. Thus all bodies, whose refractive index is greater than 1·46, *accelerate* the phase of vibration in the plane of incidence; those whose refractive index is less than 1·46, *retard* it; while those bodies, for which $\mu = 1{\cdot}46$, reflect according to Fresnel's laws.

(176) The elliptical vibration of the reflected light will be completely known, when we know the *difference of phase* of the two principal components, and the *ratio of their intensities*. The difference of phase is determined experimentally by M. Jamin, by the process which restores the light to the condition of plane polarization; while the azimuth of the plane of polarization of the restored ray gives the ratio of the intensities of the two components. The results obtained have been compared with the formulæ given by M. Cauchy for the case of diamond; and the agreement has been found to be satis-

factory. These formulæ involve two constants,—the *refractive index*, and the *coefficient of ellipticity*; and these are determined, when we know the principal incidence, and the ratio of the amplitudes of the two vibrations at that incidence.

The fundamental difference between this theory, and that of Fresnel, consists (we have seen) in including a change of phase of the reflected vibration, varying with the incidence. This change of phase is due, according to M. Cauchy and Mr. Green, to the normal vibration, which—though evanescent at a short distance from the surface—modifies the phase.

(177) Professor Haughton has followed up the researches so ably commenced by M. Jamin, and has obtained some new and interesting results. The more important of these are comprised in the following laws:—

I. If plane-polarized light be incident on a transparent reflecting body, and the incidence be gradually increased from 0° to 90°, the ratio of the axes of the reflected elliptically polarized light diminishes from infinity, at 0°, to a *minimum*, at the *principal incidence*; and increases again to infinity, at 90°.

II. The *minimum* ratio of the axes varies with the plane of polarization of the incident light, and diminishes as the azimuth of that plane increases, until the latter reaches a certain value; after which the ratio again increases.

III. When the azimuth of the plane of polarization of the incident light reaches this value, the ratio of the axes becomes equal to *unity*, and the reflected light is *circularly polarized*.

This last conclusion is one which might have been anticipated. M. Jamin had shown, that the *difference of phase* of the two principal components of the reflected light was equal to 270°, at the principal incidence; so that the light reflected at this incidence must be *circularly polarized*, when the amplitudes of the two components are equal. This equalization of the reflected components can always be effected by varying the azimuth of the plane of polarization of the incident ray. α denoting this azimuth, the amplitudes of the two components are $\cos\alpha$ and $\sin\alpha$, that of the original vibration being unity; so that if v and w denote (as before) the ratios of the amplitudes of the reflected and incident vibrations in the two principal planes, the amplitudes of the two components in the reflected ray will be $v\cos\alpha$, and $w\sin\alpha$. These will be equal, and therefore the reflected light circularly polarized, when

$$\tan\alpha = \frac{v}{w} = \frac{\cos(\theta - \theta')}{\cos(\theta + \theta')}.$$

If the *principal incidence* were the same as the angle given by Brewster's law, $\cos(\theta + \theta') = 0$, and $\alpha = 90°$. But this not being the case, $\cos(\theta + \theta')$ is not actually evanescent; and the azimuth, α, at which the light is circularly polarized, is a few degrees less than $90°$.

CHAPTER XI.

FRESNEL'S THEORY OF DOUBLE REFRACTION.

(178) It has been stated (60, 66), that soon after the discovery of double refraction in Iceland crystal, Huygens succeeded in embracing its laws in the theory of waves, by a bold and happy assumption. He had already shown that the form of the wave which gives rise to the ordinary refracted ray, in glass and other uncrystallized substances, was the *sphere ;* or, in other words, that the velocity of undulatory propagation was the *same in all directions.* One of the rays in Iceland crystal, too, was found to obey the same law; and, judging that the law which governed the other, though not so simple, was yet *next in simplicity*, he assumed the form of its wave to be the *spheroid ;* that is, he supposed the velocity of propagation to be *different in different directions*, in accordance with the following construction :—" Let an ellipsoid of revolution be described round the optic axis, having its centre at the point of incidence; and let the greater axis of the generating ellipse be to the less in the ratio of the greatest to the least index of refraction: then the velocity of any ray will be represented by the radius vector of the ellipsoid which coincides with it in direction." We have already seen that the construction for the direction of the rays, derived from this assumption, was verified by experience; and we have here another instance, to which the history of science affords many parallels, of the value of analogical principles in directing scientific research.

(179) The law of Huygens was found to hold in many crystals besides that to which it was originally applied; and in

all of these there was *one optic axis*, or one line along which a ray of light passed without division. But when the researches of Brewster made known a class of crystalline bodies, having *two optic axes*, or two lines of no separation, Huygens's law was found not to be general; and it was ascertained that one of the rays, at least, in *biaxal* crystals, followed some new and unknown law.

In this state of the question, the problem of double refraction was taken up by Fresnel; and by the aid of a natural and simple hypothesis, combined with the principle of transversal vibrations, he has been conducted to its complete solution,—a solution which not only embraces all the known phenomena, but has even outstripped observation, and predicted consequences which were afterwards verified by experiment.

(180) Fresnel sets out from the supposition, that the elastic force of the vibrating medium, in every crystal, is different in different directions. This is, in fact, the most general supposition that can be made; and whether we suppose that the vibrating medium is the ether within the crystal, or that the molecules of the body itself partake of the vibratory movement, there will be obviously such a connexion and mutual dependence of the parts of the solid and those of the medium in question, that we cannot hesitate to admit for the one, what has been already established on the clearest evidence for the other.

It is easy to see, generally, that the phenomenon of double refraction is a necessary consequence of this hypothesis, and of the principle of transversal vibrations.

Let us take, for example, the simple case of a ray of light proceeding from an infinitely distant point, and falling *perpendicularly* on the surface of a uniaxal crystal, cut *parallel to the axis*. The incident wave being plane, and parallel to the surface of the crystal, the vibrations are also parallel to the same surface; and we may conceive them to be composed of vibrations *parallel* and *perpendicular to the axis* of the crystal. Now, the elasticity brought into play by these two sets of vibrations being different, they will be propagated with different velocities; and there will be *two waves* within the crystal, in which the vibrations are parallel to two fixed directions at right angles to one another,—or two rays *oppositely polarized.* If the second face of the crystal be *parallel* to the first, the two rays will emerge perpendicularly; and the only effect produced will be, that one will be *retarded* more than the other, in its progress through the crystal. But if the second face be *oblique* to the direction of the rays, they will be both *refracted* at emergence, and *differently*; and they will therefore diverge from one another.

(181) To return to the general theory. Let us suppose a disturbance to be produced in a medium such as we have been considering, and any particle of the medium to be displaced from its position of rest. The resultant of all the elastic forces which resist the displacement will not, in general, act in the direction of the displacement (as would be the case in a medium *uniformly* elastic), and therefore will not drive the displaced particle directly back to its position of equilibrium. Fresnel has shown, however, that there are *three* directions at right angles to each other, in every elastic medium, in each of which the elastic forces do act *in the direction of the displacement*, whatever be the nature or laws of the molecular action. He assumes that these three directions are *parallel* throughout the crystal. In fact, the first principles of crystallization oblige us to admit, that the arrangement of the molecules of the crystalline body is similar in all parallel lines throughout the crystal; and the same property must belong to the ether within it, if (as we have reason to presume) its elasticity be dependent on that of the crystal itself. These three directions Fresnel denominates *axes of elasticity*; and

he concludes that they are also axes of symmetry, with respect to the crystalline form.

If, on each of these axes, and on every line diverging from the same origin, portions be taken, which are as the square roots of the elastic forces in their directions, the locus of the extremities of these portions will be a surface, which Fresnel denominates the *surface of elasticity*. Its equation is

$$r^2 = a^2 \cos^2 \alpha + b^2 \cos^2 \beta + c^2 \cos^2 \gamma :$$

a^2, b^2, c^2, being the elasticities in the directions of the three axes; r the radius vector of the surface; and α, β, γ, the angles which it makes with the axes.

This surface determines the *velocity of propagation of the wave*, when the direction of its vibrations is given. For, the ethereal molecule vibrating in the direction of any radius vector, r, of this surface, the elastic force which governs its vibration will be proportional to r^2; and, since the velocity of wave-propagation is as the square root of the elastic force, it must, in this case, be represented by the radius vector of the surface of elasticity in the direction of the vibrations. Hence, if we conceive the vibration in the incident wave to be resolved into two within the crystal, performed in two determinate directions, these will be propagated with different velocities; and, as a difference of velocity gives rise to a difference of refraction, it follows that the incident ray will be divided into two within the crystal, which will in general pursue different paths. Thus, the bifurcation of a ray, on entering a crystal, presents no difficulty, provided we can explain in what manner the vibration comes to be resolved.

(182) To understand in what manner this takes place, let us conceive a plane wave advancing within the crystal. By the principle of transversal vibrations, the movements of the ethereal molecules are all parallel to the wave. But the motion of each molecule, when thus removed from its position of

equilibrium, is resisted by the elastic force of the medium; and that force is, in general, oblique to the direction of the displacement. If the plane containing the direction of the force and that of the displacement were *normal to the plane of the wave*, the force would be resolvable into two,—one perpendicular to the plane of the wave, which (by the principle of transversal vibrations) can produce no effect; and the other in the direction of the displacement itself, which will thus be communicated from particle to particle without change. But this, in general, is not the case. Fresnel has shown, however, that the displacement may be resolved in two directions in the plane of the wave, at right angles to one another, such that the elastic force called into action by each component will be in the plane passing through the component, and normal to the wave; and thus each component will give rise to a wave, in which the direction of the vibrations is preserved, and which therefore will be propagated with a constant velocity.

The two directions, above alluded to, are those of the *greatest and least diameters of the section of the surface of elasticity* made by the plane of the wave; so that if the original displacement be resolved into two, parallel to these directions, each component will give rise to a plane wave, in which the vibrations preserve constantly the same direction. The velocity of propagation being represented by the radius vector of the surface of elasticity in the direction of the displacement, the velocities of the two parts of the wave will be proportional to the *greatest* and *least diameters* of the section of the surface of elasticity, to which the vibrations are parallel. Thus it appears that an incident plane wave, in which the vibrations are in any direction, will be resolved into *two* within the crystal; and these will be propagated with *different velocities*, and consequently assume *different directions*.

(183) The vibrations in these waves being *parallel to two*

fixed lines,—namely, the greatest and least diameters of the section of the surface of elasticity,—it follows that the two refracted rays are *polarized*, and that their planes of polarization are at *right angles*, being the planes passing through the direction of the ray and these two lines. Hence it follows, that the plane of polarization of one of the rays bisects the dihedral angle made by the two planes, which pass through the *normal to the wave* and the *normals to the two circular sections* of the surface of elasticity; and that the plane of polarization of the other is perpendicular. This coincides, very nearly, with the rule previously given by M. Biot, namely, that *the plane of polarization of one of the pencils bisects the dihedral angle formed by planes drawn through the ray and the two optic axes; while that of the other is perpendicular, or bisects the supplemental dihedral angle.*

Thus the two fundamental facts of crystalline refraction—namely, the bifurcation of the ray, and the opposite polarization of the two pencils—are completely accounted for.

Further, the amplitudes of the resolved vibrations are represented by the cosines of the angles which the direction of the original vibration contains with the two fixed rectangular directions; and, as the squares of these amplitudes measure the intensities of the two pencils, the law of Malus respecting these intensities is a necessary consequence.

(184) The velocity of propagation of a *plane wave* in any direction being known, the *form of the wave*, diverging from any point within the crystal, may be found. For, if we conceive an indefinite number of plane waves, which, at the commencement of the time, all pass through the point which is considered as the origin of the disturbance, the *wave surface* will be that touched by all these planes at any instant. Fresnel has given the following elegant construction for its determination:—"Let an ellipsoid be conceived, whose semiaxes are a, b, c (the same as those of the surface of elasticity), and let

it be cut by any diametral plane. At the centre of this section let a perpendicular be raised; and on this line let two portions be taken, whose lengths (measured from the centre) are equal to the greatest and least radii of the section. The extremities of these perpendiculars will be the loci of the double wave."

The equation of the wave surface is of the fourth order; it has been thrown into the following symmetric form by Sir William Hamilton,

$$\frac{a^2 x^2}{x^2+y^2+z^2-a^2}+\frac{b^2 y^2}{x^2+y^2+z^2-b^2}+\frac{c^2 z^2}{x^2+y^2+z^2-c^2}=0.$$

(185) The form of the wave surface being known, the directions of the two refracted rays are determined by tangent planes drawn to the two sheets of the surface, according to the construction of Huygens. Conceive three surfaces, having their common centre at the point of incidence, and representing, respectively, the simultaneous positions of three waves diverging from that point,—the first in air, which is a *sphere;* and the other two within the crystal, which are the *two sheets* of the surface we have been considering. Let the incident ray be produced to meet the sphere, and at the point of intersection let a tangent plane be drawn. Through the line of intersection of this plane with the refracting surface, let two planes be drawn touching the two sheets of the refracted wave; the lines connecting the centre with the points of contact are the directions of the two refracted rays.*

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* If, in place of the ellipsoid mentioned above, we take that whose semi-axes are $\frac{1}{a}$, $\frac{1}{b}$, $\frac{1}{c}$, the three principal refractive indices of the medium, the surface derived from it by the same construction will represent the *normal slowness of the waves*, and is connected with the *wave surface* by a remarkable relation of reciprocity. The properties of this surface lead to the following elegant construction for the directions of the refracted rays, a construction which is, in many cases, more convenient than that given above:—"With the point of incidence, as a common centre, construct the surfaces of *wave-slowness* belonging to air and to the crystal, respectively. Let the

It may be shown that the direction of the vibratory movement, at any point of the surface of the wave, coincides with the projection of the radius vector upon the plane which touches the surface at that point. Hence, if perpendiculars be let fall from the centre, on the tangent planes to the two sheets of the wave surface, the lines connecting their feet with the points of contact are the directions of the vibrations in the two rays; and therefore determine their planes of polarization. The perpendiculars themselves measure the velocities of propagation of the *waves*, while the radii vectores represent those of the *rays*.

(186) From the construction of the wave surface, above given, it follows that there are two directions,—namely, the normals to the two circular sections of the ellipsoid,—in which the two sheets of the wave surface have a common radius vector, and therefore the two rays a common velocity. If ω and ω' denote the angles which any line drawn from the centre of the wave makes with these lines, v and v' the radii vectores in its direction terminating in the two sheets of the wave surface, the equation above given may be reduced to the following remarkable polar forms:—

$$v^{-2} = a^{-2}\sin^2 \tfrac{1}{2}(\omega + \omega') + c^{-2}\cos^2 \tfrac{1}{2}(\omega + \omega'),$$
$$v'^{-2} = a^{-2}\sin^2 \tfrac{1}{2}(\omega - \omega') + c^{-2}\cos^2 \tfrac{1}{2}(\omega - \omega').$$

Since the radius vector of the wave surface measures the

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incident ray be produced to meet the sphere, which represents the normal slowness of the wave in air; and from the point of intersection let a perpendicular be drawn to the refracting surface. This will cut the surface of slowness of the refracted waves, in general, in two points. The lines connecting these points with the centre will represent the direction and normal slowness of the *waves;* while the perpendiculars from the centre on the tangent planes at the same points will represent the direction and slowness of the *rays*." This construction was given by Sir William Hamilton and Professor Mac Cullagh.

velocity of the ray in its direction, the velocities of the two rays are given by the preceding formulæ. If we subtract the latter from the former, we find (after a simple trigonometrical reduction),

$$v^{-2} - v'^{-2} = (a^{-2} - c^{-2}) \sin\omega \sin\omega'.$$

Hence the difference of the squares of the reciprocal velocities, in the two rays, is proportional to the product of the sines of the angles made by their common direction with the lines in which the two rays have a common velocity. In all known crytals, these lines deviate very little from the *optic axes*,—or the lines in which the two parts of the *wave* have a common velocity; and thus the remarkable law, to the discovery of which M. Biot was led by analogy, and which has been also shown to flow from the constructions for the velocity given by Sir David Brewster, is a necessary consequence of Fresnel's theory.

The two sets of lines above alluded to—the lines of *single ray-velocity*, and *single wave-velocity*—are situated in the plane of the axes of greatest and least elasticity, the lines of each pair making equal angles with the axis of greatest elasticity on either side. The tangents of these angles are respectively,

$$\frac{a\sqrt{b^2 - c^2}}{c\sqrt{a^2 - b^2}}; \qquad \frac{\sqrt{b^2 - c^2}}{\sqrt{a^2 - b^2}}.$$

Hence, when $b^2 = c^2$, or $b^2 = a^2$, these angles become 0, or 90°; and the two optic axes *unite*,—coinciding in the former case with the axis of greatest elasticity, and in the latter with that of the least.

In each of these cases, then, $\omega = \omega'$, and the preceding equations become

$$v^{-2} = a^{-2} \sin^2\omega + c^{-2} \cos^2\omega, \quad v' = c;$$

the former of which is the equation of the *ellipsoid* of revolution, and the latter that of the *sphere*. Accordingly, the wave surface resolves itself into the sphere and spheroid of the Huygenian law; and the form of the wave in *uniaxal* crystals, which was *assumed* by Huygens, is deduced as a simple corollary from the general theory of Fresnel.

Finally, when the *three* elasticities are all equal, it will appear at once from the preceding equations that the spheroid becomes a sphere. The velocity is accordingly the same in all directions, and the law of refraction is reduced to the known law of Snell.

(187) It has been stated (70) that, as soon as a class of double-refracting substances was discovered, possessing two optic axes, the construction of Huygens was found not to be general. It was still thought, however, that the velocity of *one* of the rays in every crystal was constant; or, in other words, that one of the rays was refracted according to the ordinary law of the sines. According to Fresnel's theory, however, the velocity of neither of the rays in biaxal crystals was constant, and the refraction of both was performed according to a new law. It was, therefore, a matter of much interest to decide this question by accurate experiment. This experimental problem was solved by Fresnel himself, and the result was decisive in favour of his theory.

It has been already shown (81) that when light, diverging from a luminous origin, passes through two near apertures in a screen, the two pencils into which it is thus divided will interfere, and produce fringes,—the central fringe being the locus of those points at which the two rays have traversed equal paths. Now if two plates of *glass*, cut from the same plate, and of exactly the same thickness, be placed perpendicularly, one in the path of each ray, the two rays will be *equally retarded*, and the central fringe will remain undisplaced. But if, instead of glass plates, we employ plates cut *in different directions* from the *same biaxal crystal*,—the plates being of exactly the same thickness,—the fringes produced by the interference of the two ordinary* rays will remain still undisplaced, if the velocity of these rays is the same in the two plates; while, on the other hand, if the velocities be different, the fringes will be shifted from their original position. On trial, the result was found to be as Fresnel had anticipated: the fringes *were displaced;* and the amount of that displacement agreed with the calculated difference of velocity, which had been previously deduced from theory.

In a second experiment, two prisms were cut in different directions from the same crystal of topaz, cemented together, and ground to the same angle; and the compound prism thus formed was achromatized by a prism of glass. On looking through the combination at a line of light, Fresnel found that the ordinary image of the line was *broken* at the junction of the two prisms,—thus showing that the ray was unequally refracted in different directions.

(188) There are two remarkable cases of Fresnel's theory, which have since furnished a very striking confirmation of its truth.

If we make $y = 0$, in the equation of the wave surface, so as to obtain its intersection with the plane of xz, the resulting equation is reducible to the form

$$(x^2 + z^2 - b^2)\ [a^2 x^2 + c^2 z^2 - a^2 c^2] = 0.$$

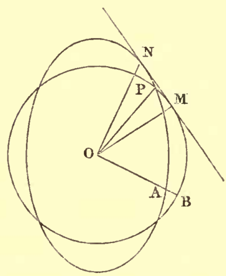
This equation is manifestly resolvable into the two following:

$$x^2 + z^2 = b^2, \quad a^2 x^2 + c^2 z^2 = a^2 c^2;$$

so that the surface intersects the plane of xz in a *circle*

* The ray whose velocity varies the least, in biaxal crystals, is sometimes, though improperly, called for distinction the *ordinary* ray.

and *ellipse.* As these two curves have a common centre, and as the radius of the circle, *b*, is of intermediate magnitude to the semiaxes of the ellipse, it follows that they must intersect in four points, as is represented in the annexed diagram.



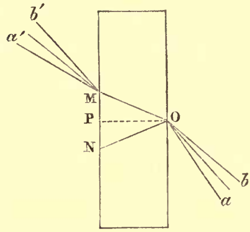
Now, when two rays pass within the crystal in any common direction, as OAB, their velocities are represented by the radii vectores of the two parts of the wave, OA and OB; and their directions, at emergence, are determined by the positions of the tangent planes at the points A and B. But in the case of the ray OP, whose direction is that of the line joining the centre with one of the four cusps, or intersections just mentioned, the two radii vectores *unite*, and the two rays have the *same velocity.* There are still, however, two tangents to the plane section at the point P; so that it might be supposed that the rays proceeding with this common velocity within the crystal would still be divided at emergence into two,—and two only,—whose directions are determined by the tangent planes. This seems to have been Fresnel's view of the case. Sir William Hamilton has shown, however, that there is a *cusp* at each of the four points just mentioned, not only in this particular section, but in every section of the wave-surface passing through the line OP; or, more properly, that there is a *conoidal cusp* on that surface at the four points of intersection of the circle and ellipse, and consequently *an infinite number of tangent planes*, which form a tangent cone of the second degree. Hence, a *single ray*, such as OP, proceeding within the crystal in one of these directions, should be divided into an *infinite number of rays* at emergence, whose directions and planes of polarization are determined by the tangent planes.

Again, it is evident that the circle and ellipse have *four* *common tangents*, such as MN ; and the planes passing through these tangents, and perpendicular to the plane of the section, are perpendicular to the optic axes of the crystal. Fresnel seems to have thought that these planes touched the wave surface in the two points just mentioned, and in these only; and, consequently, that a single ray, incident upon a biaxal crystal in such a manner that one of the refracted rays should coincide with an optic axis, OM, will be divided into two within the crystal, OM and ON, determined by the points of contact. But Sir William Hamilton has shown that the four planes of which we have spoken touch the wave surface,—not in two points only,—but in an *infinite number of points*, constituting each a small *circle of contact;* and, consequently, that a single ray of common light, incident externally in the above-mentioned direction, should be divided into an *infinite number of refracted rays* within the crystal.

(189) Here, then, are two singular and unexpected consequences of Fresnel's theory, not only unsupported by any facts hitherto observed, but even opposed to all the analogies derived from experience;—here are two remote conclusions of that theory, deduced by the aid of a refined analysis, and in themselves so strange, that we are inclined at first to reject the principles of which they are the necessary consequences. They accordingly furnish a test of the truth of that theory of the most trying nature that can be imagined.

Being naturally anxious to submit the wave-theory to this test, and to establish or disprove its new results, Sir William Hamilton requested the author to examine the subject experimentally. The result of this examination has been to prove the existence of both species of *conical refraction.*

The first case of conical refraction is that called by Sir William Hamilton *external conical refraction,* and was expected to take place, as we have seen, when a single ray passes within the crystal in the direction of either of the lines of *single ray-velocity*. These lines coincide nearly, but not exactly, with the optic axes of the crystal; and, in the case of arragonite (the crystal submitted to experiment), contain an angle of nearly 20°. The plate of arragonite employed had its faces perpendicular to the line bisecting the optic axes; consequently, the lines above mentioned were inclined to the perpendicular at an angle of about 10° on either side. Let these lines be represented by OM and ON, equally inclined to the perpendicular OP. A ray of common light traversing the crystal in the direction OM or MO, should emerge in a cone of rays, as represented in the figure; the angle of this cone depending on the relative magnitude of the three elasticities of the crystal, a^2, b^2, c^2. In the case of arragonite this angle is considerable, and amounts to 3° very nearly.



A thin metallic plate, perforated with a very minute aperture, was placed on each face of the crystal; and these plates were so adjusted, that the line connecting the two apertures should coincide with the line MO, or any parallel line within the crystal. The flame of a lamp was then brought near one of the apertures, and in such a position that the central part of the beam converging from its several points to the aperture should have an incidence of 15° or 16°. When the adjustment was completed, a brilliant annulus of light appeared, on looking through the aperture in the second surface. When the aperture in the second plate was ever so slightly shifted, so that the line connecting the two apertures no longer coincided with the line MO, the phenomenon rapidly changed, and the annulus resolved itself into two separate pencils.

The incident converging cone was also formed by a lens of short focus, placed at the distance of its own focal length from the surface; and in this case, the lamp was removed to a distance, and the plate on the first surface dispensed with. The same experiments were repeated with the sun's light; and the emergent rays were even thrown on a screen, and thus the section of the cone observed at various distances from its summit.

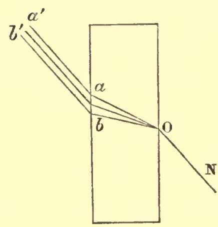
In the first experiments there was a considerable discrepancy between the results of observation and theory, both as to the magnitude of the cone, and some other circumstances of its appearance. These discrepancies were found to arise from the *sensible magnitude* of the little aperture on the second surface of the crystal, which suffered rays to pass which were inclined to the line OM at small angles. Accordingly, the magnitude of the observed cone required a correction before it could be compared with the results of theory: when this correction was applied, the agreement of the observed and theoretical angles was found to be complete.

The rays which compose the emergent cone are all polarized in different planes. It was discovered by observation that these planes are connected by the following law,—namely, "*the angle between the planes of polarization of any two rays of the cone is half the angle between the planes containing the rays themselves and the axis.*" This law was found to be in accordance with theory.

(190) A remarkable variation of the phenomenon took place, on substituting a narrow *linear aperture* for the small circular one, in the plate next the lamp, in the first-mentioned mode of performing the experiment,—the line being so adjusted, that the plane passing through it and the aperture on the second surface should coincide with the plane of the optic axes. In this case, according to the hitherto received views, all the rays transmitted through the second aperture should be

refracted doubly *in the plane of the optic axes*, so that no part of the line should appear enlarged in breadth, on looking through this aperture; while, according to Sir William Hamilton, the ray which proceeds in the direction OM should be refracted in *every plane*. The latter was found to be the case: in the neighbourhood of each of the optic axes, the luminous line was bent, on either side of the plane of the axes, into an oval curve. This curve, it is easy to show, is the *conchoid of Nicomedes*, whose asymptot is the line on the first surface.

(191) The other case of conical refraction—called *internal conical refraction* by Sir William Hamilton—was expected to take place when a single ray has been incident externally upon a biaxal crystal, in such a manner that one of the refracted rays may coincide with an optic axis. The incident ray in this case should be divided into a cone of rays within the crystal, the angle of which, in the case of arragonite, is equal to 1° 55′. The rays composing this cone will be refracted at the second surface of the crystal, in directions parallel to the ray incident on the first, so as to form a small *cylinder of rays in air*, whose base is the section of the cone made by the surface of emergence. This is represented in the annexed diagram, in which NO is the incident ray, *aOb* the cone of refracted rays within the crystal, and *aa′b′b* the emergent cylinder.



The minuteness of this phenomenon, and the perfect accuracy required in the incidence, rendered it much more difficult to observe than the former. A thin pencil of light, proceeding from a distant lamp, was suffered to fall upon the crystal, and the position of the latter was altered with extreme slowness, so as to change the incidence very gradually. When the required position was attained, the two rays suddenly spread out into a continuous circle, whose diameter was apparently equal to their former interval. The same experiment was repeated with the sun's light, and the emergent cylinder was received on a small screen of silver paper, at various distances from the crystal; and no sensible enlargement of the section was observable on increasing the distance. The angle of this minute cone within the crystal was found to agree, within very narrow limits, with that deduced from theory,—the observed angle being 1° 50′, and the theoretical angle 1° 55′.

The rays composing the internal cone are all polarized in different planes; and the law connecting these planes is the same as in the case of external conical refraction.

(192) We have seen that the problem to find the direction and magnitude of the reflected and refracted vibrations, when those of the incident vibration are given, was solved by Fresnel in the case of *ordinary* media. In the year 1831, M. Seebeck generalized, to a certain extent, the solution of Fresnel, and extended it to the case of reflexion by *uniaxal crystals* in the principal plane. The general solution of the problem of reflexion and refraction by crystalline media was obtained, a few years later, by Professor Mac Cullagh and M. Neumann upon other principles (156); and the memoir of the former is distinguished for the beauty and elegance of its geometrical laws. This solution, like that of Fresnel for ordinary media, does not include the *change of phase*, which is now proved to take place in reflexion at the bounding surfaces of all media (174). Its results, accordingly, are only approximately true, the degree of approximation being probably the same as in the case of Fresnel's laws themselves.

CHAPTER XII.

INTERFERENCE OF POLARIZED LIGHT.

(193) HAVING considered the theory and laws of double refraction, we are prepared to study the phenomena of interference which take place when polarized light is transmitted through crystalline substances. The effects displayed in such cases are probably the most splendid in optics; and when it is considered that, through them, an insight is afforded into the very laboratory of Nature itself, and that we are thus enabled almost to view the interior structure and molecular arrangement of bodies, the subject will hardly be thought inferior in importance to any other in the science.

The first discoveries in this attractive region were made by Arago, who presented a memoir to the Institute, in the year 1811, on the colours of crystalline plates when exposed to polarized light. The subject has since been prosecuted with unremitting ardour by the first physical philosophers of Europe, and among the foremost by Biot, Brewster, and Fresnel.

(194) It has been already shown (142), that when a beam of light, polarized by reflexion, is received upon a second reflecting plate at the polarizing angle, the twice-reflected light will vanish, when the plane of the second reflexion is perpendicular to that of the first. The first reflector, in any apparatus of this kind, is called the *polarizing plate*, and the second (for reasons which will presently appear), the *analyzing plate*. Now, if between the two reflectors we interpose a plate of any double-refracting substance, the capability of reflexion at the analyzing plate is suddenly restored, and a por-

tion of the light is reflected, whose quantity depends on the position of the interposed crystal. The light is thus said (though improperly) to be *depolarized* by the crystal; and it was by this property that the double-refracting structure was detected by Malus in a vast variety of substances, in which the separation of the two rays was too small to be directly perceived.

In order to analyze this phenomenon, let the crystalline plate be placed so as to receive the polarized beam perpendicularly, and let it be turned round in its own plane. We shall then observe that there are two positions of the plate in which the light totally disappears, and the reflected ray vanishes, just as if no crystal had been interposed. These two positions are at right angles to one another; and they are those in which the *principal section* of the crystal *coincides with the plane of the first reflexion*, or is *perpendicular* to it. When the plate is turned round from either of these positions, the light gradually increases; and it becomes a *maximum*, when the principal section is inclined at an angle of 45° to the plane of the first reflexion.

(195) In these experiments the reflected light is *white*. But if the interposed crystalline plate be very thin, the most gorgeous colours appear, which vary with every change of inclination of the plate to the polarized beam.

Mica and *sulphate of lime* are very fit for the exhibition of these beautiful phenomena, being readily divided by cleavage into laminæ of extreme thinness. If a thin plate of either of these substances be placed so as to receive the polarized beam perpendicularly, and be then turned round in its own plane, the tint does not change, but varies only in *intensity;* the colour *vanishing* altogether when the principal section of the crystal coincides with the plane of primitive polarization, or is perpendicular to it,—and, reaching a *maximum*, when it is inclined to the plane of primitive polarization at an angle of 45°.

If, on the other hand, the crystal be fixed, and the analyzing plate turned,—so as to vary the inclination of the plane of the second reflexion to that of the first,—the colour will be observed to pass, through every grade of tint, into the complementary colour; it being always found that the light reflected in any one position of the analyzing plate is *complementary*, both in colour and intensity, to that which it reflects in a position 90° from the former. This curious relation will appear more evidently, if we substitute a double-refracting prism for the second reflector; for the two pencils refracted by the prism have their planes of polarization—one coincident with the principal section of the prism, and the other at right angles to it, and are therefore in the same condition as the light reflected by the analyzing plate, with its plane of reflexion successively in these two positions. In this manner the complementary lights are seen together, and may be easily compared. But the accuracy of the relation is completely established by making the two pencils partially overlap; for, whatever be their separate tints, it will be found that the part in which they are superposed is absolutely *white*.

(196) When laminæ of *different* thicknesses are interposed between the polarizing and analyzing plates, so as to receive the polarized beam perpendicularly, the tints are found to vary with the thickness. The colours produced by plates of the same crystal, of different thicknesses, follow the same law as the colours reflected from *thin plates* of air, the tints *rising in the scale* as the thickness is diminished; until finally, when this thickness is reduced below a certain limit, the colours disappear altogether, and the central space appears *black*, as when the crystal is removed. The thickness producing corresponding tints is, however, much greater in crystalline plates exposed to polarized light, than in thin plates of air, or any other uniform medium. The *black of the first order* appears in a plate of sulphate of lime, when the thickness is the

$\frac{1}{2000}$th of an inch. Between $\frac{1}{2000}$th and $\frac{1}{50}$th of an inch, we have the whole succession of colours of Newton's scale; and when the thickness exceeds the latter limit, the transmitted light is always *white*. The tint produced by a plate of mica, in polarized light, is the same as that reflected from a plate of air of only the $\frac{1}{400}$th part of the thickness.

Pursuing the examination of the same subject for *oblique* incidences, M. Biot found that, in uniaxal crystals, the tint developed—or rather the number of periods and parts of a period belonging to a ray of given refrangibility—was determined by the *length of the path* traversed by the light within the crystal, and by the *square of the sine* of the angle which its direction made with the optic axis, jointly. In biaxal crystals we must substitute, for the square of the sine, the *product of the sines* of the angles which the ray makes with the two axes.

(197) Let us now turn to the physical theory, and see in what manner it explains the appearances.

We have seen that the wave incident upon a crystal is resolved into two sets of waves within it, which traverse it in different directions, and with different velocities. One of these waves, therefore, will lag behind the other, and they will be in *different phases* of vibration at emergence. When the plate is thin, this *retardation* of one wave upon the other will amount only to a few undulations and parts of an undulation; and it would therefore appear that we have here all the conditions necessary for their *interference*, and the consequent production of colour. Such was the sagacious conjecture of Young.

But here we are met by a difficulty. So far as this explanation goes, the phenomena of interference and of colour should be produced by the crystalline plate alone, and in common light, without either polarizing plate or analyzing plate. Such, however, is not the fact, and the real difficulty in this case is,

—not so much to explain how the phenomena *are* produced, as to show why they are *not always* produced.

In seeking for a solution of this difficulty, we perceive that the two rays, whose interference is supposed to produce the observed results, are not precisely in the condition of those whose interference we have hitherto examined. They are *polarized*, and polarized in *opposite planes*. We are led then to inquire, whether there is anything peculiar to the interference of polarized rays which may influence these results; and the answer to this inquiry will be found to complete the solution of the problem.

(198) The subject of the *interference of polarized light* was examined, with reference to this question, by Fresnel and Arago, and its laws experimentally developed. It was found that two rays of light, *polarized in the same plane*, interfere and produce fringes, under the same circumstances as two rays of common light;—that when the planes of polarization of the two rays are *inclined* to each other, the interference is diminished, and the fringes decrease in intensity;—and that, finally, when the angle between these planes is a *right angle*, no fringes whatever are produced, and the rays no longer interfere at all. These facts may be established by taking a plate of tourmaline which has been carefully worked to a uniform thickness, cutting it in two, and placing one-half in the path of each of the interfering rays. It will be then found that the intensity of the fringes depends on the relative position of the axes of the two tourmalines. When these axes are *parallel*, and consequently the two rays polarized in the same plane, the fringes are best defined; they decrease in intensity, when the axes of the tourmalines are *inclined* to one another; and, finally, they vanish altogether when the axes form a *right angle*.

In this law we find the account of the fact which hitherto perplexed us,—namely, that no phenomena of interference or colour are produced, under ordinary circumstances, by the two rays which emerge from a crystalline plate, and which are polarized in opposite planes; and we learn that, to produce these phenomena in perfection, the *planes of polarization of the two rays must be brought to coincidence by the analyzer.*

The non-interference of rays, polarized in planes at right angles to one another, is a necessary result of the mechanical theory of transversal vibrations. In fact, it is a mathematical consequence of that theory, that the intensity of the resultant light in that case is *constant*, and equal to the *sum* of the intensities of the two component lights, whatever be the phases of vibration in which they meet.

But though the intensity of the light does not vary with the phase of the component vibrations, the character of the resulting vibration will. It appears from theory, that two rectilinear and rectangular vibrations compound a single vibration, which will be also *rectilinear* when the phases of the component vibrations differ by an exact number of semi-undulations; that, in all other cases, the resulting vibration will be *elliptic;* and that the ellipse will become a *circle*, when the component vibrations have equal amplitudes, and the difference of their phases is an odd multiple of a quarter of a wave. These results of theory have been completely confirmed by experiment.

(199) Fresnel and Arago discovered, further, that two oppositely polarized rays will not interfere, even when their planes of polarization are made to coincide, unless they belong to a pencil, the whole of which was originally *polarized in one plane;* and that, in the interference of rays which had undergone double refraction, *half an undulation* must be supposed to be *lost* or *gained*, in passing from the ordinary to the extraordinary system,—just as in the transition from the reflected to the transmitted system, in the colours formed by thin plates.

The principle of the allowance of half an undulation is a simple consequence of the theory of transversal vibrations. In fact, the vibration of the wave incident on the crystal is resolved into two within it, at right angles to one another, one in the plane of the principal section, and the other in the perpendicular plane. Each of these must be again resolved, in two fixed directions which are also perpendicular ; and it will easily appear from the process of resolution, that, of the four components into which the original vibration is thus resolved, the pair in one of the final directions must *conspire*, while in the other they are *opposed*. Accordingly, if the vibrations of the one pair are *coincident*, those of the other *differ by half an undulation*. Hence, when the plane of reflexion of the analyzing plate coincides successively with these two positions, the colours (which result from the interference of the portions in the plane of reflexion) will be *complementary*.

The former of the two laws explains the office of the *polarizing plate* in the phenomena. To account mechanically for the non-interference of the two pencils, when the light incident upon the crystal is unpolarized, we may regard a ray of common light as composed of two rays of equal intensity, oppositely polarized,* and whose vibrations are therefore perpendicular. Each of these vibrations, when resolved into two within the crystal, and these two again resolved in the

* More properly, a ray of common light must be regarded as composed of an indefinite number of rays polarized in all azimuths ; so that if any two planes be assumed at right angles, there will be an equal quantity of light actually polarized in each. Ordinary light, in fact, consists of a series of systems of waves, in each of which the vibrations are different ; the different systems succeeding one another so rapidly, that, in a moderate time, as many vibrations take place in any one plane, as in another at right angles to it. But the phenomena of interference, exhibited by common light, compel us also to admit (as Mr. Airy has observed) that the vibrations *do not change continually ;* and that in each system of waves there are, probably, several hundred vibrations which are all similar,—although the vibrations constituting one system bear no relation to those of another, and the different systems succeed one another with such rapidity as to obliterate all trace of polarization.

plane of reflexion of the analyzing plate, will exhibit the phenomena of interference. But the interval of retardation will differ by half a wave in the two cases; the tints produced will therefore be complementary, and the light resulting from their union will be of a uniform whiteness.*

(200) These laws of interference being kept in mind, the reason of all the phenomena is apparent. The ray is originally polarized in a single plane, by means of the polarizing plate. It is then divided into two within the crystal, which are polarized in opposite planes; and these are finally reduced to the same plane by means of the analyzing plate. The two pencils will therefore interfere; and the resulting tint will depend on the *retardation* of one of the rays behind the other, produced by the difference of the velocities with which they traverse the crystal.

It has been shown, that the difference between the reciprocals of the squares of the velocities, with which the two rays traverse the crystal, is proportional to the product of the sines of the angles which their direction makes with the optic axes; or, that if v and v' denote the velocities of the two rays, ω and ω' the angles which their direction makes with the two axes,

$$v^{-2} - v'^{-2} = c \sin\omega \sin\omega'.$$

But if t and t' denote the times occupied by the two rays in traversing the crystal, and θ the thickness actually traversed,

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* We have here supposed the resulting light to be simply the *sum* of the lights derived from each of the portions into which the original light was supposed to be resolved, without reference to their phase. The justice of this assumption depends upon the fact adverted to in the preceding note,—namely, that the two oppositely polarized portions, into which we have supposed common light to be resolved, differ in phase,—that difference continually varying. The same thing is true, therefore, of the final components; so that these must be regarded as lights proceeding from *different* sources, and compound a light equal in intensity to the sum of the components.

—or the thickness of the plate multiplied by the secant of the angle of refraction,

$$v^{-2} - v'^{-2} = \frac{t^2 - t'^2}{\theta^2} = \frac{t + t'}{\theta} \times \frac{t - t'}{\theta}.$$

Now the first factor of this product is very nearly constant; we have, therefore,

$$t - t' = \text{const} \times \theta \sin \omega \sin \omega';$$

or, the interval of *retardation* is proportional to the *product of the sines* of the angles which the direction of the rays makes with the two axes, and to the *thickness* of the crystal traversed, jointly. When the two axes coalesce, or the crystal becomes *uniaxal*, the retardation is proportional to the *square of the sine* of the angle which the direction makes with the axis. But the *tint* developed is measured by the interval of retardation; accordingly the laws of the tints, discovered experimentally by M. Biot, flow immediately from the theory.

(201) It is plain that the light issuing from the crystal is, in general, *elliptically polarized*, inasmuch as it is the resultant of two waves, in which the vibrations are at right angles, and differ in phase. Hence, when homogeneous light is used, and the emergent beam is analyzed with a double-refracting prism, the two pencils into which it is divided vary in intensity as the prism is turned,—neither, in general, ever vanishing.

When the thickness of the crystal is such, that the difference of phase of the two rays is an *exact number of semi-undulations*, they will compound a *plane-polarized* ray at emergence,—the plane of polarization coinciding with the plane of primitive polarization, or making an equal angle with the principal section of the crystal on the other side, according as the difference of phase is an even or odd multiple of half a wave. Accordingly, one of the pencils into which the light is divided by the analyzing prism will vanish in two positions of its principal section; and it is manifest that the successive thicknesses of the crystalline plate at which this takes place form a series in arithmetical progression.

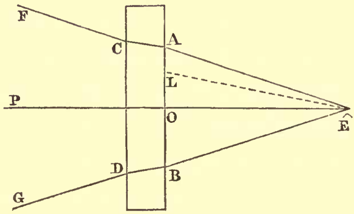
On the other hand, when the difference of phase is a *quarter of a wave-length*, or an odd multiple of that quantity, —and when, at the same time, the principal section of the crystal is inclined at an angle of 45° to the plane of primitive polarization—the emergent light will be *circularly polarized*. This is one of the simplest means of obtaining a circularly-polarized beam; but it has the disadvantage, that the required interval of phase is only exact for waves of one particular length, and that, therefore, the circular polarization is perfect only for one colour.

(202) It has been stated (195) that the phenomena of *colour* are only produced when the crystalline plate is thin. In thick plates, where the difference of phase of the two pencils contains a great many wave-lengths, the tints of different orders come to be superposed (as in the case of Newton's rings, where the thickness of the plate of air is considerable), and the resulting light is *white*. The phenomena of colour may still, however, be produced in thick plates, by superposing two of them in such a manner, that the ray which has the greater velocity in the first shall have the less in the second. We have only to place the plates with their principal sections *perpendicular* or *parallel*, according as the crystals to which they belong are of the *same*, or of *opposite* denominations. Thus, if the crystals be uniaxal, and both positive, or both negative, they are to be placed with their principal sections perpendicular; and, on the other hand, these sections should be parallel, when one of the crystals is positive and the other negative. The reason of this is evident.

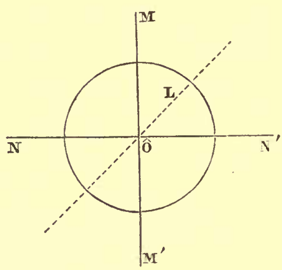
(203) Let us now consider the effects produced when a converging or diverging pencil of rays traverses a uniaxal crystal, in various directions inclined to the axis at small

angles; and let us suppose, for simplicity, that the crystalline plate is cut in a direction perpendicular to the axis.

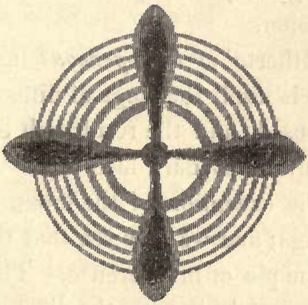
Let ABCD be the plate, and E the place of the eye. The visible portion of the emergent beam will form a cone, AEB, whose summit coincides with the place of the eye, and axis, EO, with the axis of the crystal. The ray which traverses the crystal in the direction of the axis, POE, will undergo no change whatever; and consequently will be reflected, or not, from the analyzing plate, according as the plane of reflexion there coincides with, or is perpendicular to, the plane of the first reflexion. But the other rays composing the cone will be modified in their passage through the crystal; and the changes which they undergo will depend on their inclination to the optic axis, and on the position of the principal section with respect to the plane of primitive polarization.

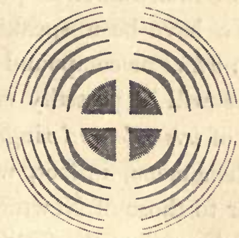


Let the circle represent the section of the emergent cone of rays made by the second surface of the crystal; and let MM′ and NN′ be two lines drawn through its centre at right angles, being the intersections of the plane of primitive polarization, and of the perpendicular plane, respectively, with the surface. Now the rays which emerge at any point of these lines will not be divided into two within the crystal, nor will their planes of polarization be altered; because the principal section of the crystal, for these rays, in the one case coincides with the plane of primitive polarization, and in the other is perpendicular to it.



These rays therefore will be reflected, or not, from the analyzing plate, according as the plane of reflexion there coincides with, or is perpendicular to, the plane of the first reflexion. In the latter case, therefore, a *black cross* will be displayed on the field, and in the former a *white one*,—as is represented in the annexed diagrams.





But the case is different with the rays which emerge at any other point, such as L. The principal section of the crystal for this ray, OL, neither coincides with, nor is perpendicular to, the plane of primitive polarization; and consequently the incident polarized ray will be divided into two within the crystal, whose planes of polarization are parallel and perpendicular to OL, respectively. The vibrations in these two rays are reduced to the same plane by means of the analyzing plate: they will therefore interfere, and the extent of that interference will depend on their difference of phase.

Now the difference of phase of the two rays depends on the interval of retardation. When this interval is an odd multiple of half an undulation, the two rays are in complete discordance; and, on the other hand, they are in complete accordance when it is an even multiple of the same quantity. We have seen (200) that, for a given plate, the interval of retardation is proportional to the square of the sine of the angle which the ray makes with the optic axis within the crystal. It may be easily shown that the sine of this angle is very nearly proportional to the sine of the angle LEO (see first fig. p. 190), which the emergent ray makes with the axis; and this latter to LO, the distance of the point of emergence from the centre. The retardation therefore varies as the square of the distance LO; and consequently the successive dark and bright lines will be arranged in circles, (as represented in the preceding diagrams) the squares of whose radii are in arithmetical progression.

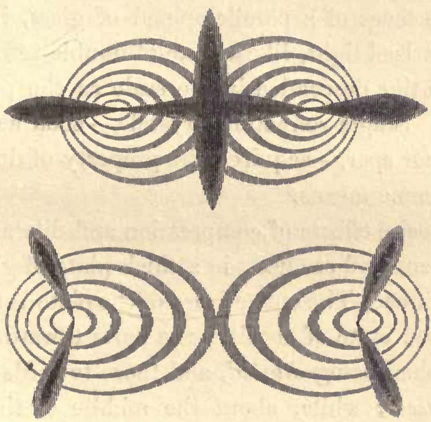
We have been speaking hitherto of *homogeneous* light. When *white* or compound light is used, the rings of different colours will be partially superposed, and the result will be a series of *iris-coloured* rings separated by dark intervals. All the phenomena, in fact, with the exception of the cross, are similar to those of Newton's rings; and we now see that they are both cases of the fertile principle of interference. These rings are exhibited even in *thick* crystals, because the difference of the velocities of the two pencils is very small for rays slightly inclined to the optic axis.

(204) Let us now consider briefly the case of *biaxal* crystals.

Let a plate of such a crystal be cut perpendicularly to the line bisecting the optic axes, and let it be interposed, as before, between the polarizing and analyzing plates. In this case the bright and dark bands will no longer be disposed in circles, as in the former, but will form curves which are symmetric with respect to the lines drawn from the eye in the direction of the two axes; the points of the same band being those for which the *interval of retardation* of the two rays, $t - t'$, is the same. Now this interval is proportional to the product of the sines of the angles which the direction of the rays makes with the optic axes (200); and these sines are, very nearly, as the distances of the points of emergence (measured on the face of the crystal) from the projections of the optic axes. Hence the product of these distances will be constant for all the points of the same curve. The curve formed by each band is therefore the *lemniscata* of James Bernouilli,—the fundamental property of which is, that the product of the radii vectores, drawn from any point to two fixed poles, is a constant quantity.

The exactness of this law has been verified, in the most complete manner, by the measurements of Sir John Herschel. The constant varies from one curve to another,—being proportional to the interval of retardation, and increasing therefore as the numbers of the natural series for the successive dark bands. For different plates of the same substance, the constant is inversely as the thickness.

The annexed diagrams represent the systems of rings in a biaxal crystal whose axes form a small angle with one another, in two positions of the crystalline plate, the planes of polarization of the polarizing and analyzing plates being at right angles.



The form of the *dark brushes*, which cross the entire system of rings, is determined by the law which governs the planes of polarization of the emergent rays. There is no difficulty in showing, on the principles of Fresnel's theory, that two such dark curves, in general, pass through each pole;

and that they are *rectangular hyperbolas*, whose common centre is the middle point of the line which connects the projections of the two axes.

(205) The phenomena of depolarization and of colour, impressed by double-refracting substances upon the transmitted light, are, we have seen, the necessary results of the interference of the two pencils into which the light is divided within them. These properties, therefore, become distinctive characters of the double-refracting structure; and thus enable us to discover the existence, and to trace the laws, of that structure, even in substances in which the separation of the two pencils is too minute to be directly observed. By such means it has been discovered that a double refracting structure may be communicated to bodies which do not possess it naturally, by *mechanical compression* or *dilatation*. Thus Sir David Brewster observed, that when pressure was applied to the opposite faces of a parallelopiped of glass, it developed a tint in polarized light, like a plate of double-refracting crystal; and the tint descended in the scale as the pressure was augmented. Single-refracting crystals,—such as muriate of soda, and fluor spar,—acquired the property of double refraction by the same means.

The opposite effects of compression and dilatation may be very well seen, and studied, in a thick plate of glass bent by an external force. The entire mass of the plate is thus thrown into an altered state of density, the parts towards the *convex* side of the plate being *dilated*, and those towards the *concave* side *compressed;* while, about the middle of the thickness, there is a surface in which the particles are in their natural state. Accordingly, when this body is interposed between the polarizing and analyzing plates, so as to form an angle of 45° with the plane of primitive polarization, two sets of coloured bands are seen, separated by a neutral line; and these vanish altogether when the compressing force is withdrawn.

The parts towards the *convex*, or dilated side of the neutral line, are found to have acquired a *positive* double-refracting structure, and those on the *concave*, or compressed side, a *negative* one.

In these cases of induced double refraction, the phenomena are related to the form of the entire mass; and the axes of double refraction are single lines within the substance, fixed in *position*, as well as *direction*. In this respect the phenomena are essentially different from those produced by *regular crystals*, in which the laws of the double refraction depend solely on the *direction*, and are the same in all parts of the substance.

(206) The phenomena described in the preceding article are in perfect accordance with the wave-theory. Owing to the connexion of the vibrating medium with the solid in which it is contained, its elasticity is rendered unequal in different directions by the effects of compression, the maximum and minimum of elasticity corresponding to the directions of *greatest* and *least pressure*. Accordingly the vibrations of the ray, on entering the substance, are resolved into two in these directions, and these are propagated with unequal velocities. The incident wave will therefore be separated into two within the medium, one of which will be in advance of the other, and these will be in different phases of vibration at emergence. The resolved parts of the vibrations, in the plane of reflexion of the analyzing plate, will accordingly interfere, and the tint developed will be determined by the interval of retardation.

These results of theory were experimentally confirmed by Fresnel; and he found that the velocity with which a ray traversed the glass was *greater* or *less*, according as its plane of polarization *coincided with*, or was *perpendicular to*, the line in which the pressure was exerted. The double refraction of the ray is a necessary consequence of this difference of velocities: but this was also established by Fresnel by direct experiment. A series of glass prisms were placed together, with their refracting angles alternately in opposite directions, and the ends of the alternate prisms were powerfully pressed by screws. A ray transmitted through the combination was found to be divided into *two oppositely polarized*. The compressed prisms, in this arrangement, acquired a double-refracting structure, the axis of pressure being also the axis of double refraction; and their refracting angles being all turned in the same direction, the divergence of the two rays was increased in proportion to their number, and thus rendered sensible. The intermediate prisms served to correct the deviation, and to render the combination achromatic.

(207) The effects of unequal density and elasticity may be much more regularly produced by the application of *heat*. These effects may be studied by applying a bar of hot iron to the edge of a rectangular plate of glass, and placing it in the polarizing apparatus, so that the heated edge may form an angle of 45° with the plane of primitive polarization. At the end of some time, the whole surface of the plate will be observed to be covered with coloured bands, the parts near the opposite edges having acquired a positive double-refracting structure, and those near the centre a negative one. The effects are reversed when a plate of glass, uniformly heated, is rapidly cooled at one of its edges; and all the appearances vanish when the glass acquires the same temperature throughout.

If we transmit heat from the surface to the axis of a glass cylinder, by immersing it in heated oil, it will display a system of rings similar to those of a *negative* crystal with one axis, the axis of the cylinder being also the axis of double refraction. When the heat reaches the axis, the double refraction begins to weaken; and the colours disappear altogether when the glass is uniformly heated. Again, if the cylinder, when in this state, be made to cool rapidly by surrounding it with a good conductor of heat, it will transiently assume the opposite character of a *positive* double-refracting crystal; and when it is restored to a uniform temperature throughout, all traces of double refraction again disappear. If we employ an *elliptic* cylinder, instead of a circular one, in the experiment just described, it will exhibit the coloured curves formed by a *biaxal* crystal: and the phenomena may be endlessly varied by varying the form of the glass to which the heat is applied.

If now, by any means, the glass be arrested in one of these transient states, it will acquire a *permanent* double-refracting structure. This has been accomplished by raising it to a red heat, and then cooling it rapidly at the edges. For, as the outer parts, which are thus more condensed, assume a fixed form in cooling, the interior parts must accommodate themselves to that form, and therefore retain a state of unequal density. The law of density, and therefore the double-refracting structure, will depend on the external form; and it is accordingly found that the coloured bands and patches, which such bodies display in polarized light, assume a regular arrangement varying with the shape of the mass.

(208) As the double-refracting structure is communicated to bodies which do not possess it naturally, by mechanical compression, or unequal temperature,—so, by the same means, that structure may be *altered* in the bodies in which it already resides. Thus Sir David Brewster and M. Biot found that the double refraction of *regular crystals* may be altered, and the tints they display made to rise or descend in the scale, by *simple pressure*.

But the changes induced by *heat* are more remarkable. Professor Mitscherlich discovered the important fact, that heat dilates crystals *differently in different directions*, and so alters their form; and their double-refracting properties are found to undergo corresponding changes. Thus, Iceland spar is dilated by heat in the direction of its axis; while it actually *contracts*, by a small amount, in directions perpendicular to it. The angles of the primitive form thus vary, the rhomboid becoming less obtuse, and approaching the form of the *cube*,—in crystals of which form there is no double refraction (69). Professor Mitscherlich accordingly conjectured, that the double-refracting energy of the crystal must, in these circumstances, be *diminished;* and the conjecture was verified by experiment. In fact, the extraordinary index in Iceland spar is found to increase considerably with the temperature, while the ordinary index undergoes little or no change.

We have seen (186) that the inclination of the optic axes, in biaxal crystals, is a simple function of the three principal elasticities of the vibrating medium, and that the plane of the axes is that of the greatest and least elasticities. If, then, these elasticities be altered by heat in *different proportions*, the inclination of the axes will likewise vary; and it may even happen that the plane of the axes will shift to a position at right angles to that which it formerly occupied. All these variations have been actually observed. Professor Mitscherlich found that, in *sulphate of lime*, the angle between the axes (which is about 60° at the ordinary temperature) *diminishes* on the application of heat; that, as the temperature increases, these axes approach until they *unite*; and that, on a still further augmentation of heat, they again separate, and *open out in a perpendicular plane.* Heat is found to dilate this crystal more in one direction than in another perpendicular to it.

CHAPTER XIII.

ROTATORY POLARIZATION.

(209) In the phenomena hitherto considered, the changes in the plane of polarization, which a polarized ray undergoes in reflexion or refraction, are determinate in amount, and are wholly independent of the distances traversed by the ray in either medium. There are certain cases, however, in which the change of the plane of polarization increases with the thickness of the medium traversed; and the plane is made to revolve, sometimes from left to right (like the hands on the dial-plate of a clock), and sometimes in the opposite direction. This remarkable phenomenon was first observed by Arago.

When a polarized ray, of any simple colour, traverses a plate of Iceland spar, beryl, or any other uniaxal crystal, *in the direction of its axis*, it suffers no change of any kind. But when the ray traverses in the same manner a plate of *rock-crystal*, its plane of polarization is found to be altered at emergence; and the change increases with the thickness of the plate. In some crystals of this substance, the plane of polarization is turned from *left to right*, while in others it is turned in an *opposite* direction; and the crystals themselves are called *right-handed* or *left-handed*, according as they produce one or other of these effects.

(210) The phenomena of *rotatory polarization* in rock-crystal were analyzed with great diligence and success by M. Biot, and were reduced by him to the following general laws.

I. In different plates of the same crystal, the rotation of the plane of polarization is always proportional to the *thickness* of the plate. The same thing holds, very nearly, in plates of different crystals.

II. When two plates are superposed, the effect produced is, very nearly, the same as that which would be produced by a single plate, whose thickness is the *sum* or *difference* of the thicknesses of the two plates, according as they are of the same or of opposite denominations.

III. The rotation of the plane of polarization is different for the different rays of the spectrum, and increases with their refrangibility. For a given plate, the angle of rotation is *inversely as the square of the length of the wave*. Thus, the angle of rotation, produced by a plate of rock-crystal whose thickness is a millimetre, is $17\frac{1}{2}°$ for the extreme red of the spectrum, 30° for the rays of mean refrangibility, and 44° for the extreme violet.

Since the rays of different colours emerge polarized in different planes, it follows that if a beam of *white* light be let fall upon the crystal, and be received after emergence upon an analyzing plate, this will reflect a portion of the light in every position of the plane of reflexion; and this light will be beautifully coloured, the colour varying with the thickness of the crystal, and the position of the analyzing plate. For the analyzing plate will reflect the rays of different colours in different proportions, depending on the positions of their planes of polarization with respect to the plane of reflexion; and the resulting colour will be a compound tint, which can be easily estimated.

(211) The curious distinction, which was found to subsist between different specimens of rock-crystal, has been connected by Sir John Herschel with a difference of crystalline form. The ordinary form of the crystal of quartz is the six-sided prism terminated by the six-sided pyramid. The solid angles, formed at the junction of the pyramid and prism, are sometimes *replaced* by small secondary planes, which are

oblique with reference to the original planes of the crystal; and the form of the crystal is then called *plagiedral.* In the same crystal these planes lean all in the same direction; and it is found that, when that direction is *to the right* (the apex of the pyramid being uppermost), the crystal is *right-handed;* and that, on the contrary, it is *left-handed,* when the planes lean in the opposite way.

Sir David Brewster subsequently discovered that the *amethyst,* or violet quartz, is made up of *alternate layers* of right-handed and left-handed quartz. This remarkable structure may be traced in the fracture of the mineral; for the edges of the layers *crop out,* and give to the fracture the undulating appearance which is peculiar to this mineral. But the structure in question is displayed in the most beautiful manner, when we expose a plate of this substance to polarized light.

The colours exhibited in polarized light likewise reveal the existence of crystals of quartz penetrating others in various directions, when no striæ, or other external appearances, indicate their presence.

(212) The connexion between the rotation of the plane of polarization and the crystalline form, discovered by Sir John Herschel in quartz, has since been observed in other substances. M. Pasteur has recently found that *tartaric acid,* and the *tartrates,* which are all plagiedral in the same direction, likewise deviate the plane of polarization to the same side. On the other hand, *para-tartaric acid,* and the *para-tartrates,* which have the same general form, are for the most part not plagiedral; while, in those salts of this class which are so, the facettes of the crystals are inclined sometimes to the right, and sometimes to the left,—and this difference is found to exist even in crystals belonging to the same specimen. M. Pasteur has found, accordingly, that the salts of the former class have *no effect* upon the plane of polarization; while those of the latter deviate the plane of polarization in the *same direction* as the facettes of the crystal.

This remarkable distinction among the para-tartrates has been traced by the same observer to their chemical composition. He has discovered that para-tartaric acid is composed of two distinct acids, which have the same general crystalline form; but which differ in this, that in one of them the facettes of the crystals are inclined to the right, and in the other to the left. These acids (one of which is the ordinary tartaric acid) accordingly deviate the plane of polarization—the former *to the right*, and the latter *to the left*, and by the same amount; and the difference in the optical properties of different specimens of the compound acid, and its salts, arises from the predominance of one or other of the component elements.

(213) The phenomena of rotatory polarization in rock-crystal, have been accounted for by the interference of *two circularly polarized* pencils, which are propagated along the axis with unequal velocities, one revolving from left to right, and the other in the opposite direction.

For a *plane polarized* ray is equivalent to two *circularly polarized* rays of half the intensity, in which the vibrations are in opposite directions. When a plane polarized ray, therefore, is incident perpendicularly upon a plate of rock-crystal, cut perpendicularly to the axis, it may be resolved into two such circularly polarized rays; and as these are supposed to be transmitted with different velocities, one of them will be in advance of the other when they assume a common velocity at emergence. They then compound a single ray, polarized in a single plane; and this plane, it can be shown, is removed from the plane of primitive polarization by an angle proportional to the interval of retardation, and therefore to the thickness of the crystal.

Thus the laws of rotatory polarization are completely explained; and it only remains to prove the truth of the assumption, that two circularly polarized pencils, whose vibrations are in opposite directions, are actually transmitted along the axis of quartz with different velocities. This supposition is easily put to the test of experiment; since such a difference of velocity must produce a difference of refraction, when the surface of emergence is oblique to the direction of the ray. According to this hypothesis, therefore, a polarized ray transmitted through a prism of rock-crystal, in the direction of the optic axis, should undergo *double refraction* at emergence; and the two pencils into which it is divided should be *circularly polarized.* This has been completely verified by Fresnel, by means of an achromatic combination of right-handed and left-handed prisms, arranged so as to double the separation; and he has shown that the two pencils are neither common nor plane-polarized light, but possess all the physical characters of light circularly polarized.

(214) The relation between the rotation and double refraction of rock-crystal, in the direction of its axis, has been very simply deduced by M. Babinet.

Let v and v' denote the velocities of the ordinary and extraordinary waves in the direction of the axis of the crystal; μ and μ' the corresponding refractive indices; then

$$\frac{\mu'}{\mu} = \frac{v}{v'}.$$

But, if θ be the thickness of the crystal, and δ the interval of retardation of the two waves after traversing it, the second member of the preceding equation is obviously equal to $\frac{\theta}{\theta - \delta}$, or to $1 + \frac{\delta}{\theta}$, δ being very small in comparison to θ. We have therefore

$$\mu' = \mu\left(1 + \frac{\delta}{\theta}\right); \qquad \mu' - \mu = \frac{\mu\delta}{\theta}.$$

Now the angle of rotation is proportional to the interval of retardation of the two circularly polarized pencils; and when that interval is equal to the length of a wave *in vacuo*, the angle of rotation is 180°. Hence the interval of retardation of the emergent rays, corresponding to any angle of rotation, ρ, will be $\lambda \frac{\rho}{180°}$, λ denoting the length of the wave; and the corresponding interval *within the crystal* is equal to this, multiplied by the velocity of propagation, or divided by the refractive index. Hence, if ρ be the rotation corresponding to the thickness of the crystal, θ, we have

$$\delta = \frac{\lambda}{\mu} \frac{\rho}{180°};$$

and substituting in the preceding formula,

$$\mu' - \mu = \frac{\lambda}{\theta} \frac{\rho}{180°}.$$

This difference is extremely small. When $\theta = 1$ millimetre, the angle of rotation, ρ, corresponding to the rays of mean refrangibility, $= 30°$. But for these rays, $\lambda = \cdot 0005$ of a millimetre; and therefore $\mu' - \mu = \cdot 00008$.

(215) The phenomena hitherto described take place only in the direction of the axis of the crystal. Mr. Airy discovered that when a plane polarized ray is transmitted through rock-crystal in any direction *inclined to the axis*, it is divided into two pencils which are *elliptically polarized;* the elliptical vibrations in the two rays being in opposite directions, and the greater axes of the ellipses coinciding respectively with the principal plane, and with the perpendicular plane. The ratio of the axes, in these ellipses, varies with the inclination of the ray to the optic axis,—being a ratio of equality when the direction of the ray coincides with the axis, and increasing indefinitely with its inclination to that line. With respect to the *course* of the refracted rays, Mr. Airy found that it was still determined by the Huygenian law; but that the sphere and spheroid, which determine the velocities and directions of the

two rays, *do not touch*, as in all other known uniaxal crystals, —the latter surface being contained *entirely within* the former. This is a necessary consequence of the fact, that the interval of retardation of the two pencils does not vanish, with the inclination of the ray to the optic axis.

Mr. Airy has given an elaborate calculation, founded on these hypotheses, of the forms of the rings, &c., displayed by rock crystal in plane polarized and circularly polarized light; and he has found the most striking agreement between the results of calculation and experiment. Among the most remarkable of the phenomena whose laws are thus developed, is that produced by the superposition of two plates of rock-crystal, of the same thickness, one of them being right-handed, and the other left-handed.

In order to complete the experimental investigation of this subject, it remained to determine the velocities of the two elliptically polarized rays, and the ratio of the axes of the ellipses, as dependent on the inclination of the rays to the axis of the crystal. This has been effected by M. Jamin, by measuring the amplitudes, and the differences of phase of the two component pencils, when the incident light is polarized in the plane of a principal section. From these data the quantities sought are deduced by calculation.

(216) All these complicated facts have been linked together, and their laws deduced, by Professor Mac Cullagh. In this remarkable investigation the author sets out by assuming the form of the differential equations of vibratory motion in rock-crystal; and from this assumed form he has deduced the elliptical polarization of the two pencils,—the law of the ellipticity as depending on the inclination of the ray to the axis, —the interval of retardation in the direction of the axis,—and the peculiar form of the wave-surface.

The ratio of the axes of the two ellipses is found to be equal to *unity* in the direction of the axis of the crystal. In all other directions it is given by a quadratic equation whose constant term is equal to unity; so that this ratio has two values, one of which is the *reciprocal* of the other. Hence the ratio of the axes is the same in both ellipses; and the greater axis of one coincides with the smaller axis of the other.

When the ray traverses the axis of the crystal, the rotation of the plane of polarization is given by the formula

$$\rho = \frac{C\theta}{\lambda^2};$$

which comprises all the experimental laws of M. Biot (210). The *sign* of the constant factor, C, determines the *direction* of the rotation.

It is a striking peculiarity of this theory, that it contains (in addition to the two refractive indices) but one constant, —and that this constant having been determined, from the known angles of rotation when the ray traverses the axis of the crystal, the ratio of the axes of the ellipses may be calculated, when the ray is inclined by any angle to the axis. The author has applied this calculation to the observations of Mr. Airy, and has found the calculated and observed results to agree.

(217) MM. Biot and Seebeck discovered that some of the *liquids*, and even of the *vapours*, possess the same property as quartz in the direction of its axis, and impress a rotation on the plane of polarization of the intromitted ray, which is proportional to the thickness of the substance traversed. The fact is easily observed by transmitting a polarized ray through a long tube filled with the liquid, and closed at each end by parallel plates of glass; and analyzing the emergent ray by a double-refracting prism. Among the liquids possessing this property are *oil of turpentine*, *oil of lemon*, *solution of sugar* in water, *solution of camphor* in alcohol, &c. The first-mentioned of these liquids is right-handed, and the others left-handed. They all possess the property in a much feebler degree than

quartz ; so that the ray must traverse a much greater thickness of the substance, in order to have its plane of polarization altered by the same amount. Thus a plate of rock-crystal, whose thickness is one millimetre, rotates the plane of polarization of the red ray through an arc of about 18°; a plate of oil of turpentine, of the same thickness, turns the plane of polarization only through a quarter of a degree.

The rotatory liquids do not lose their peculiar power (except in degree) by dilution with other liquids not possessing the property; and they retain it, even in the state of vapour. From these and other facts, M. Biot concludes that this property, in liquids, is inherent in their ultimate particles. In this respect the rotatory liquids are essentially distinguished from rock-crystal, which is found to lose the property when it loses its crystalline arrangement. Thus, Sir John Herschel observed, that quartz held in solution by potash (liquor of flints) did not possess the rotatory power; and the same thing has been remarked by Sir David Brewster with respect to fused quartz.

(218) When two or more liquids possessing this property are mixed together, the rotation produced by the mixture is always the sum, or the difference, of the rotations produced by the ingredients, in thicknesses proportional to the volumes in which they enter the mixture, according as the liquids are of the same or of contrary denominations. The same law holds good in many cases in which the liquids are chemically united.

M. Biot has made an important application of this principle to the analysis of compounds, containing a substance possessing the rotatory power combined with others which are *neutral*,—the quantity of which in the compound may (by the principle just stated) be determined, by observing the optical effects of the mixture. This application has been found of much industrial value, in the case of the *sac-*

charine solutions; and a very ingenious apparatus, called the *saccharimeter*, has been devised by M. Soleil for the purpose. This instrument is founded on the principle, that the rotatory solutions follow the same laws as rock-crystal, in their action upon the light of different colours; so that it is possible to compensate the effect of the solution by a plate of rock-crystal of a suitable thickness, and of the opposite action.