WIND WAVES AND SWELL

PRINCIPLES IN FORECASTING

Prepared for the

Hydrographic Office, U. S. Navy

by

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INTRODUCTION

Study of the problem of forecasting sea and swell was started at the request of the Army Air Forces and is being continued under the direction of the Hydrographic Office, U. S. Navy.

Four problems of forecasting are involved: (1) forecasting the length and height of the swell in the open sea, (2) forecasting the swell reaching exposed or partially exposed anchorages, (3) forecasting the height of breakers and the amount of surf on any given beach, and (4) forecasting the state of the sea in any given ocean area. The first problem involves two steps: (a) determination of height and period of the waves which emerge from any given wind area and which may arrive as swell on a distant coast, (b) determination of the travel time and the decrease of height of the waves as they proceed from the wind area. For the second and third problems an additional factor is involved, namely, the determination of the transformation of the waves as they enter into shallow water and wash the beach. The fourth problem involves two steps: (a) determination of the highest waves found under given wind conditions and (b) establishment of the relation of these waves to the state of the sea as described by a scale such as the Douglas Sea Scale.

This manual deals with the generation of waves by wind and with the travel of waves in deep water after they have left the regions of strong winds. Methods are described for determining the characteristics of wind waves by means of data from adequate, consecutive synoptic weather maps and for forecasting swell off coasts.

Relationships between waves and the three important variables, wind at the sea surface, fetch (the stretch of water over which the wind blows), and duration (the length of time the wind has blown) are discussed. Verifications and interpretations of the empirical laws developed by various observers of waves are given, together with graphs for use in forecasting wind waves and swell.

In order to use the graphs most effectively their physical significance and limitations must be clearly understood. Forecasts should therefore not be attempted until the forecaster has studied the first part of the paper which describes the processes leading to the growth and decay of waves.

Tests of the method made to date indicate that swell forecasts can be made with about the same certainty as that of most meteorological forecasts. Prognostic charts are not important for the forecasting of swell because considerable time elapses between the generation of waves in distant storm areas and their arrival at the coast. Thus, after experience has been gained, it is possible to forecast swell several days in advance. Forecasts of the state of the sea, on the other hand, must be based in part on prognostic weather maps and cannot be prepared for periods longer than those for which these maps can be considered valid.

It is contemplated that a more comprehensive edition of this manual will be issued in the near future. This will contain methods for determining the transformation of waves in shallow water and for forecasting surf from synoptic weather data or from observations of waves offshore.

SURFACE WAVES IN WATER

General Discussion

A wave is described by its length, L, i.e. the horizontal distance from crest to crest or trough to trough (see fig. 1A), and by its height, H, i.e. the vertical distance from trough to crest. A wave is furthermore characterized by its period, T, i.e. the time interval between the appearance of two consecutive crests at a given position.

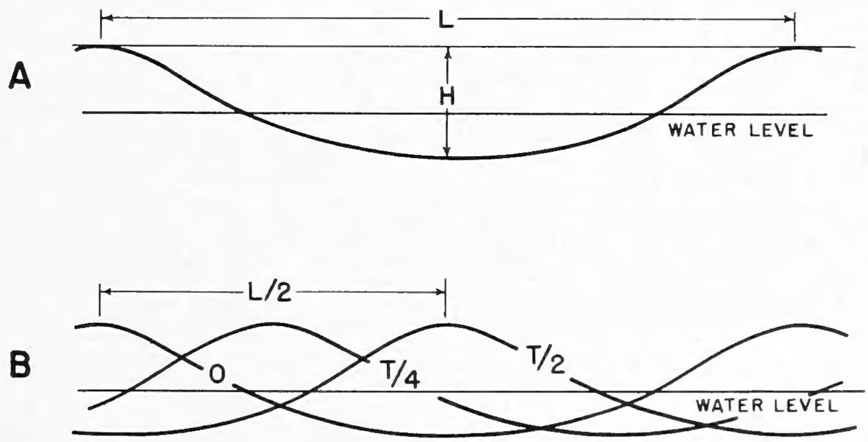


Figure 1. Surface waves. A. Profile of wave. B. Advance of wave, showing the wave profile at the times t = 0, t = T/4, and t = T/2. In the time T/2 the wave has advanced one half wave length, L/2.

A wave may be standing or progressive, but this discussion deals with progressive waves only. In a progressive wave, if the length and energy are constant, the wave height is the same at all localities and the wave crest appears to advance with a certain velocity (fig. 1B). During one wave period, T, the wave crest advances one wave length, L, and the velocity of the wave, C, is therefore defined as

$$C = \frac{L}{T}$$

The motion of the water particles depends on the wave length and the depth of the water. In general, it can be stated that the advance of the wave form is caused by convergences and divergences of the horizontal motion. In front of the crest the motion is converging and the surface is rising, but behind the crest the motion is diverging and the surface is sinking.

By energy of the wave is always understood the average energy over one wave length. The energy is in part potential, E_p, associated with the displacement of the water particles above or below the level of equilibrium, and in part it is kinetic, E_k, associated with the motion of the particles. In surface waves half the energy is present as kinetic and half as potential. The total average energy per square foot is $E = 1/8\ g\rho H^2$, where g is the acceleration of gravity and ρ is the density of the water. For a 10-foot high wave the total average energy is 800 foot-pounds per square foot. Since g and ρ can be considered constant the energy per unit area in a wave is proportional only to the square of the wave height.

For the total energy per unit width along a wave length it is necessary to multiply the energy per unit area by the wave length.

Waves of Very Small Height

By waves of very small height are understood waves for which the ratio of height to length is 1/100 or less. The simplest wave theory deals with such waves, the form of which can be represented by a sine curve (see fig. 3). In water of constant depth, d, such waves travel with the velocity

$$C = \sqrt{g \frac{L}{2\pi} \tanh 2\pi \frac{d}{L}}$$

where g is the acceleration due to gravity.

If d/L is large, that is, if the wave length is small compared to the depth, tanh $2\pi d/L$ approaches unity and one obtains

$$C = \sqrt{g \frac{L}{2\pi}}$$

These waves are called deep-water waves.

If d/L is small, that is, if the wave length is large compared to the depth, tanh $2\pi d/L$ approaches $2\pi d/L$ and one obtains

$$C = \sqrt{gd}$$

These waves are called shallow-water waves.

In general, waves have the character of deep-water waves when the depth to the bottom is greater than one half the wave length ($d > L/2$). However, for shallow-water waves the depth must be less than one twenty-fifth of the wave length ($d < L/25$).

In a low deep-water wave the water particles move in circles. At any depth, z, below the surface the radius of the circular path followed by a particle is

$$r = \frac{H}{2} e^{-2\pi \frac{z}{L}}$$

In this circle the velocity is

$$v = \frac{2\pi r}{T} = \frac{H}{T} e^{-2\pi \frac{z}{L}}$$

because the particles complete one revolution in the time T (see fig. 2).

A water particle at the sea surface remains at the surface throughout its orbit. A water particle at a given average depth below the sea surface is farthest from the surface when it moves in the direction of wave progress.

In a low shallow-water wave the vertical motion of the particles is negligible and the horizontal motion is independent of depth. The particles move back and forth, following nearly straight lines.

In a deep-water wave only half of the energy advances with wave velocity, whereas in a shallow-water wave all the energy advances with wave velocity. The reason for this difference is that in a deep-water wave only the potential energy varies periodically and advances with the wave form, but in a shallow-water wave both potential and kinetic energy vary periodically and both advance with the wave form. These laws can also be stated by saying that the energy advances at a rate which, in a deep-water wave, equals half the product of energy and wave velocity, whereas in a shallow-water wave it equals the product of energy and wave velocity.

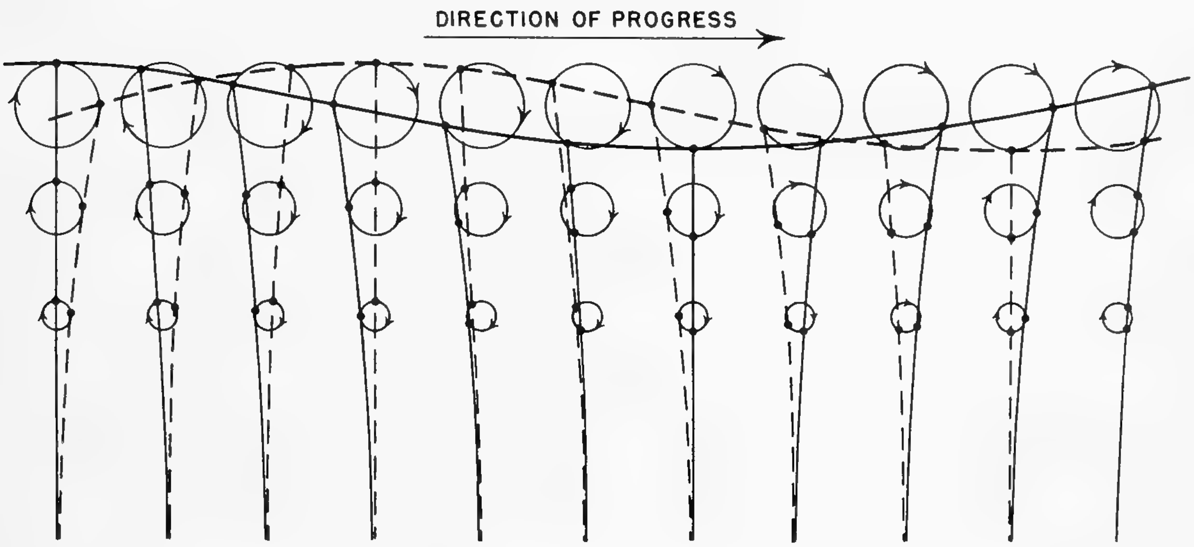


Figure 2. Movement of water particles in a deep-water wave of very small height. The circles show the paths in which the water particles move. The wave profiles and the positions of a series of water particles are shown at two instants which are one quarter of a period apart. The full-drawn, nearly vertical lines indicate the relative positions of water particles which lie exactly on vertical lines when the crest or the trough of the wave pass and the dashed lines show the relative positions of the same particles one quarter of a period later.

Deep-water Waves of Moderate and Great Height

By waves of moderate and great height are understood waves for which the ratio of height to length (H/L) is from 1/100 to 1/25 and from 1/25 to 1/7, respectively. The form of these waves can not be represented by a sine curve. For waves of moderate height the form closely approaches the trochoid, that is, the curve which is described by a point on a disc which rolls below a flat surface (fig. 3). Waves of great height deviate from the trochoid; the troughs are wider and flatter and the crests narrower and steeper. The wave form becomes unstable when the ratio H/L equals 1/7.

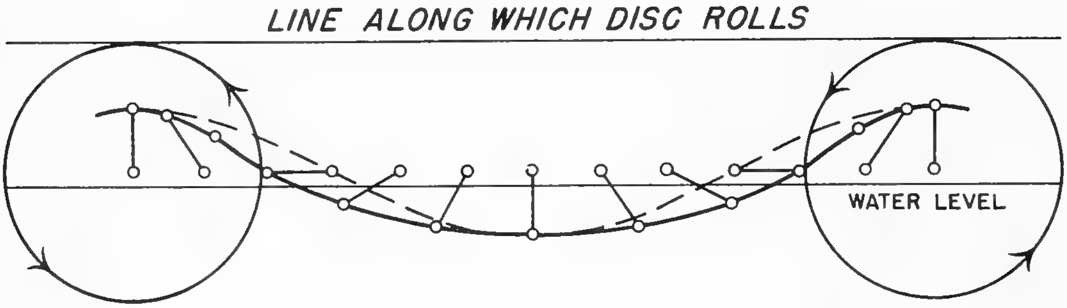


Figure 3. Profile of a trochoidal wave (full-drawn lines) and of a sine wave (dashed lines).

The wave velocity increases with increasing steepness (increasing values of H/L), but the increase of velocity never exceeds 12 per cent.

The water particles move approximately in circles, the radii of which decrease rapidly with depth. The particle velocity is not uniform but is greatest when the particles are near the top of their orbit (moving in the direction of wave progress), with the result that the particles upon completion of each nearly circular motion have advanced a short distance in the direction of progress of the wave (fig. 4). Consequently, there is a mass transport in the direction of progress of the wave. The <u>mass transport velocity</u> (<u>u'</u>) at the sea surface is expressed by the formula,

$$u' = (\pi \frac{H}{L})^2 C$$

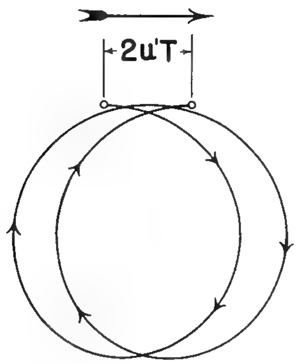


Figure 4. Orbital motion during two wave periods of a water particle in a deep-water wave of moderate or great height. In two wave periods the forward displacement equals 2u'T.

The velocity is appreciable for high, steep waves but is very small for low waves of long period. Mass transport in waves has received little attention in previous work because in most practical applications it is sufficient to consider the water particles as moving in circles regardless of the wave height. In order to understand the growth of waves through wind action, however, it is necessary to take the mass transport velocity into account.

Interference of Waves; Short-crested Waves; White Caps

When waves of different heights and lengths are present simultaneously the appearance of the free surface becomes very complicated. At some points the waves are opposite in phase and therefore tend to eliminate each other, whereas at other points they coincide in phase and reinforce each other.

As a simple case, consider two trains of waves which have the same height and nearly the same velocity of progress. Owing to interference, groups of waves are formed with wave heights roughly twice those in the component wave trains, and between the wave groups are regions in which the waves nearly disappear (fig. 5A). Analysis shows that these groups advance with a velocity which is nearly equal to one half of the average velocity of the two trains.

As another example, consider the simultaneous presence of long, low swell and short but high wind waves. The resultant pattern is illustrated in Figure 5B from which it is evident that the short, high waves dominate to such an extent that the presence of the swell is obscured.

So far, the discussion has dealt only with long-crested waves, that is, waves with very long straight crests and troughs. Waves can, however, also have short, irregular crests and troughs. In the presence of such short-crested waves the free surface shows a series of alternating "highs" and "lows", as indicated in Figure 6. This figure illustrates the topography of the sea surface, "highs" being shown with full-drawn lines and "lows" with dashed lines.

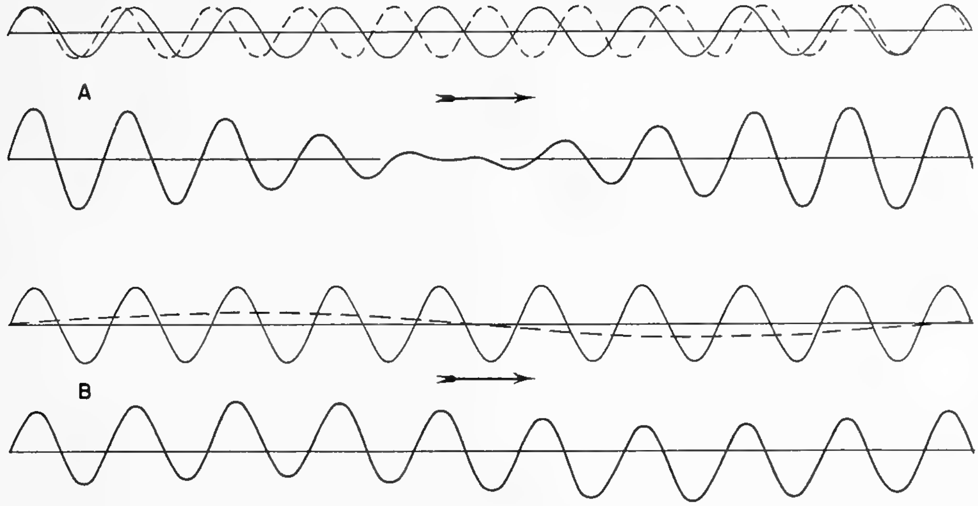


Figure 5. Wave patterns resulting from interference. A. Interference of two waves of equal height and nearly equal length, forming wave groups. B. Interference between short wind waves and long swell.

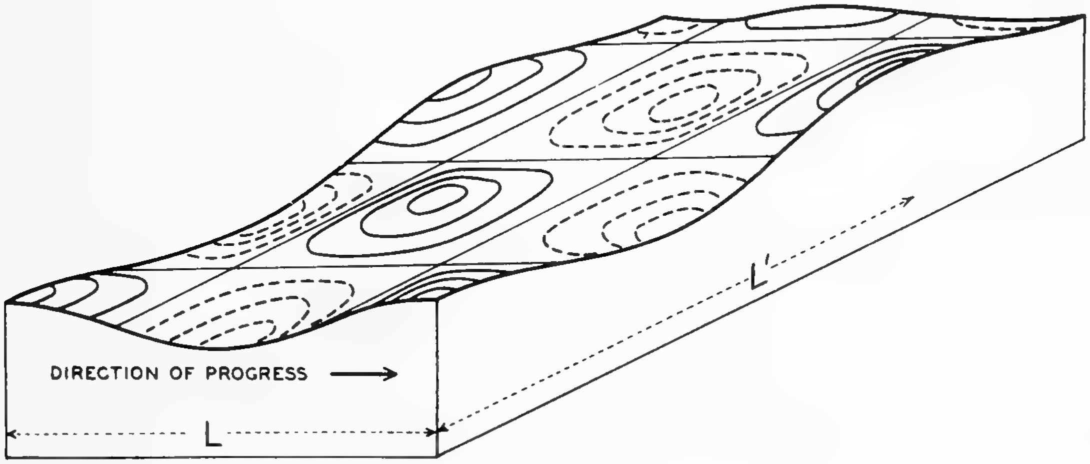


Figure 6. Short-crested waves. L = wave length, L' = crest length.

White caps are formed by the breaking of relatively short waves which often appear as "riders" on longer waves (fig. 5B). Such short waves may grow so rapidly that their steepness reaches the critical value H/L = 1/7 and they break. If interference occurs long waves may attain this steepness and break.

EMPIRICAL KNOWLEDGE OF WIND WAVES AND SWELL

Measurements of Waves and Swell

Wind waves are defined as waves which are growing in height under the influence of the wind.

Swell consists of wind-generated waves which have advanced into regions of weaker winds or calms and are decreasing in height.

So far, the discussion of surface waves has dealt mainly with waves which appear as rhythmic and regular deformations of the surface. Because of interference, the formation of short-crested waves, and the breaking of waves there is, however, little regularity in the appearance of the sea surface, particularly when a strong wind blows. Although individual waves can be recognized and their heights, periods, lengths, and velocities measured, such measurements are extremely difficult and comparatively inaccurate. The lengths of most waves and the heights of low waves are likely to be underestimated, while the heights of large waves are generally overestimated. Wave heights above 55 feet are extremely rare, yet the literature contains many reports of waves exceeding 80 feet in height. Such errors are probably due to the complexity of the sea surface and the movement of the ships from which measurements are made.

Reliable measurements of wave height, H, are so difficult that, in general, the reported values represent crude estimates. The height of a large wave is estimated as the eye height of the observer above the water line when the ship is on even keel in the trough of the wave, provided that the observer sees the crest of the wave coincide with the horizon. The height of a small wave is estimated directly, using the dimensions of the ship for comparison. On board a small ship the height of waves which are more than twice as long as the ship can be recorded by a microbarograph.

The wave period, T, can be measured by recording the time interval between successive appearances (on a wave crest) of a well-defined patch of foam at a considerable distance from the ship. In order to obtain a reliable value, observations should be made for several minutes and averaged.

The wave length, L, can be estimated by comparing the ship's length with the distance between two successive crests. This procedure leads to uncertain results, however, because it is often difficult to locate both crests relative to the ship and because of disturbance caused by the movement of the ship.

The velocity of the wave, C, can be found by recording the time needed for the wave to run a measured distance along the side of the ship and by applying a correction for the ship's speed.

Comparison of Measured and Computed Values

Theory indicated that velocity, length, and period for deep-water waves are interrelated by the formulae

$$C = \frac{L}{T} = \sqrt{g\,\frac{L}{2\pi}} = g\,\frac{T}{2\pi}; \quad L = \frac{2\pi}{g}\,C^2 = \frac{g}{2\pi}\,T^2; \quad T = \sqrt{\frac{2\pi}{g}\,L} = \frac{2\pi}{g}\,C$$

With C in knots, L in feet, and T in seconds:

$$C = 1.34\sqrt{L} = 3.03\ T$$

$$L = 0.555\ C^2 = 5.12\ T^2$$

$$T = 0.442\sqrt{L} = 0.33\ C$$

Thus, if one characteristic is measured the other two can be computed, and if two or three are measured the correctness of the theory as applied to ocean waves can be checked. Comparisons of measured and computed values have given satisfactory results, indicating that wind waves and swell in deep water do have the characteristics described above. In general, the conclusion that the ratio H/L always remains less than 1/7 is also confirmed by observations, as waves of this or greater steepness are very rarely reported.

Empirical Relationships between Wind and Waves

Observations of waves have not led to clear-cut conclusions about the empirical relationships between the wind and waves. The following nine approximate relationships have been proposed by various workers:

1. Maximum wave height and fetch. For a given wind velocity the wave height becomes greater the longer the stretch of water (fetch) over which the wind has blown. Even with a very strong wind the wave height for a given fetch does not exceed a certain maximum value. For fetches larger than 10 nautical miles it has been observed that

$$H_{max} = 1.5\sqrt{F}$$

where H_{max} represents the maximum probable wave height in feet with very strong winds and F is the fetch in nautical miles.

2. Wave velocity and fetch. At a given wind velocity the wave velocity increases with increasing fetch.

3. Wave height and wind velocity. The height in feet of the greatest waves with high wind velocities has been observed to be about 0.8 of the wind velocity in knots. If the entire range of wind velocities is considered, the observed data conform to

$$H = 0.026\ U^2$$

where U represents the wind velocity in knots.

4. Wave velocity and wind velocity. Although the ratio of wave velocity to wind velocity has been observed to vary from less than 0.1 to nearly 2.0, the average maximum wave velocity apparently slightly exceeds the wind velocity when the latter is less than about 25 knots, and is somewhat less than the wind velocity at higher wind speeds.

5. Wave height and duration of wind. The time required to develop waves of maximum height corresponding to a given wind

increases with increasing wind velocity. Observations show that with strong winds high waves will develop in less than 12 hours.

6. <u>Wave velocity and duration of wind</u>. Although observational data are inadequate, it is known that for a given fetch and wind velocity, the wave velocity increases rapidly with time.

7. <u>Wave steepness</u>. No well established relationship exists between wind velocity and wave steepness, that is, the ratio of wave height to length. This is probably due to the fact that wave steepness is not directly related to the wind velocity, but depends upon the stage of development of the wave. The stage of development, or <u>age of the wave</u>, can be conveniently expressed by the ratio of wave velocity to wind velocity (C/U), because during the early stages of their formation the waves are short and travel with a velocity much less than that of the wind, while at later stages the wave velocity may exceed the wind velocity. In order to establish the probable relation between wave steepness and wave age all wave observations were examined which appeared to be consistent with certain basic requirements and for which values of <u>H</u>, <u>L</u> (or <u>C</u> or <u>T</u>), and <u>U</u> were recorded. The corresponding values of H/L and C/U were plotted in a diagram (fig. 7). The scattering of the values is no greater than would be expected, considering the great errors of measurements. There appears to be a definite relationship between the steepness and the age of the wave. This relationship, shown by the curve in Figure 7, plays an important part in the theoretical discussion.

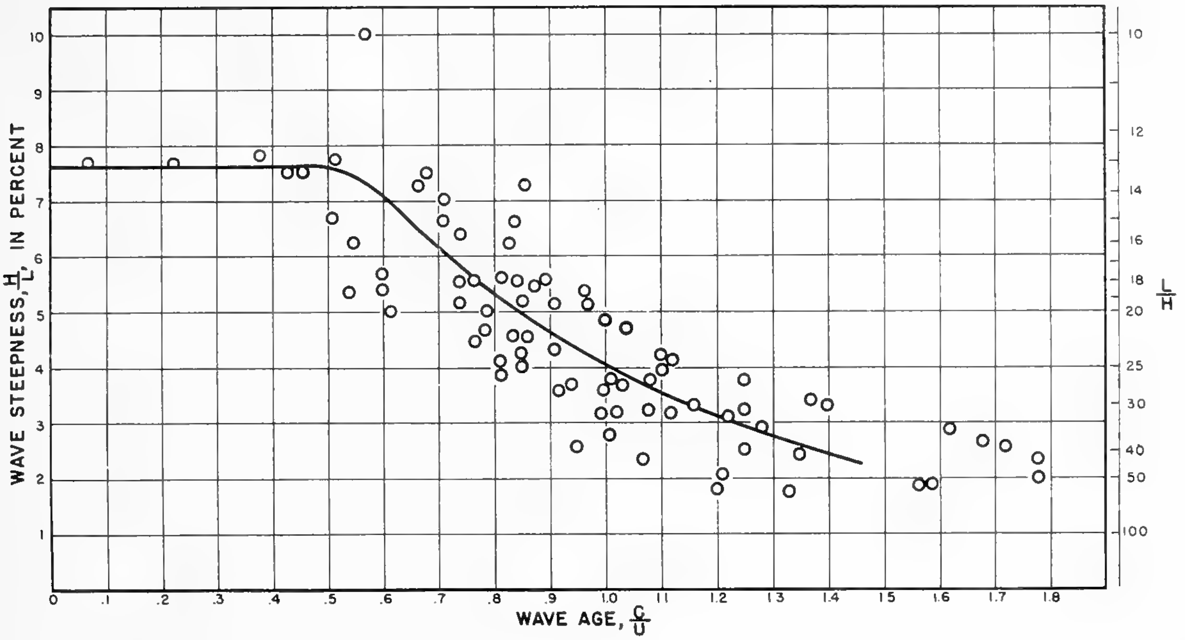


Figure 7. Relation between wave steepness as expressed by the ratio wave height to wave length, H/L, and wave age as expressed by the ratio wave velocity to wind velocity, C/U. Observed values shown by circles.

8. Decrease of height of swell. The height of swell decreases as the swell advances. Roughly, the waves lose one-third of their height each time they travel a distance in miles equal to their length in feet.

9. Increase of period of swell. Some authors claim that the period of the swell remains unaltered when the swell advances from the generating area, whereas others claim that the period increases. The greater amount of evidence at the present time indicates that the period of the swell increases as the swell advances.

GROWTH OF WIND WAVES

A knowledge of the height, velocity, and direction of progress of wind waves is necessary if their arrival as swell at a distant coast is to be predicted. Direct observations of these wind waves are rarely available, but their height and period can be determined from consecutive synoptic weather maps if the relationship between wind and waves is known.

In the area of wave formation the highest waves present at any time depend upon the wind velocity, the stretch of water over which the wind has blown (the fetch), the length of time the wind has been blowing over the fetch (the duration of the wind), and the waves which were present when the wind started blowing (the state of the sea). These four factors can all be determined if a sequence of weather maps is available showing the meteorological conditions over the oceans at intervals of, say 12 or 24 hours. These maps must be based on a sufficient number of ships' observations to make possible the plotting of fairly accurate isobars from which winds may be determined. In the tropics wind observations must be available from ships or exposed stations on islands. In middle and higher latitudes direct wind observations on ships will serve as checks on wind estimates from the isobars.

Thus, with adequate weather maps at one's disposal, an estimate of the wind waves can be made if accurate relationships between wave height and wind velocity, fetch, and duration are known. Such accurate relationships have not been developed in the past because of the inadequacy of observational data on waves, but they can be determined theoretically from a consideration of the wind energy available for wave formation if the fundamental assumption is made that the velocity (period) of a wave always increases with time.

The area in which waves are formed is called the generating area. In such an area waves receive energy from the wind by two processes, by the push of the wind against the wave crests and by the pull or drag of the wind on the water.

The energy transfer by push depends upon the difference between wind velocity and wave velocity. If the waves advance with a speed much less than that of the wind the push is great, but if the two velocities are equal no energy is transferred. If the waves travel faster than the wind they receive no energy by push but on the contrary they meet an air resistance comparable to the air resistance against a traveling automobile. The effect of the push of the wind or of the air resistance against the wave depends on the wave form. There enters, therefore, a fundamental coefficient which is related to the degree to which the wave is streamlined and which is called the "sheltering coefficient." The determination of this coefficient is necessary for an exact evaluation of energy transfer by push.

The pulling force of the wind always acts in the direction of the wind. It is the same at the wave crest and the wave trough but the effect differs. Energy is transferred from the air to the water (the movement of the surface layer is speeded up) if the surface water moves in the direction of the wind, but energy is given off from the water to the air (the movement of the surface water is slowed down) if the surface water moves against the wind. If wind and waves move in the same direction the water particles move in the direction of the wind drag while at the crest, but against the drag when in the trough (see fig. 2). In the absence of a mass transport velocity the particle velocities at the crest and the trough are equal but in opposite directions, so that the effect of the pulling force of the wind at the wave crest is exactly balanced by the effect at the wave trough. <u>In the presence of a mass transport velocity</u>, however, the forward motion at the crest is greater than the backward motion in the trough (fig. 4) and a net amount of energy is transferred to the water. No satisfactory explanation of the growth of waves can be given without assuming a transfer of energy due to the wind pulling at the water particles; and this fact is the best argument for the presence of a mass transport velocity in ocean waves.

Since the pulling force of the wind over the ocean is known, the energy transfer from the air to the water by wind drag can be computed with considerable accuracy from the theoretical values for mass transport velocity given on page 9. Even when the wave velocity exceeds the wind velocity, the effect of the wind drag remains nearly the same because it depends upon the <u>difference</u> between wind velocity and particle velocity in the water, and in general the water particles move much more slowly than the wind even when the wave form moves much faster. If the wind can not transfer energy to the water by pulling at the water particles, no satisfactory explanation can be given of the fact that waves frequently have a higher velocity than the wind which produces them.

Energy is dissipated by viscosity but the viscosity of the water is so slight that this process can be neglected. There is no evidence that energy is dissipated by turbulent motion in the wave. The chief processes which can alter the wave height or the wave velocity in deep water are therefore the push of the wind, which becomes an air resistance if the wave travels faster than the wind, and the drag or pull of the wind on the sea surface.

Knowing the rate of energy transfer from the wind and the rate at which the wave energy advances (page 6) it is possible to establish a differential equation from which the relationships between the waves and wind velocity, fetch, and duration are obtained as special solutions. The equation contains three numerical constants (including the "sheltering coefficient") which have to be determined in such a manner that all the nine empirical relationships are satisfied. This can be accomplished, and at the same time discrepancies between existing empirical relationships can be accounted for.

The growth of waves as determined in this manner is illustrated in Figures 8 and 9 which are constructed on the assumption that a wind of a constant velocity of 30 knots started to blow over an undisturbed water surface extending for 600 or more nautical miles from a coast line. Figure 8 shows the height and period of the waves as functions of the distance from the coast

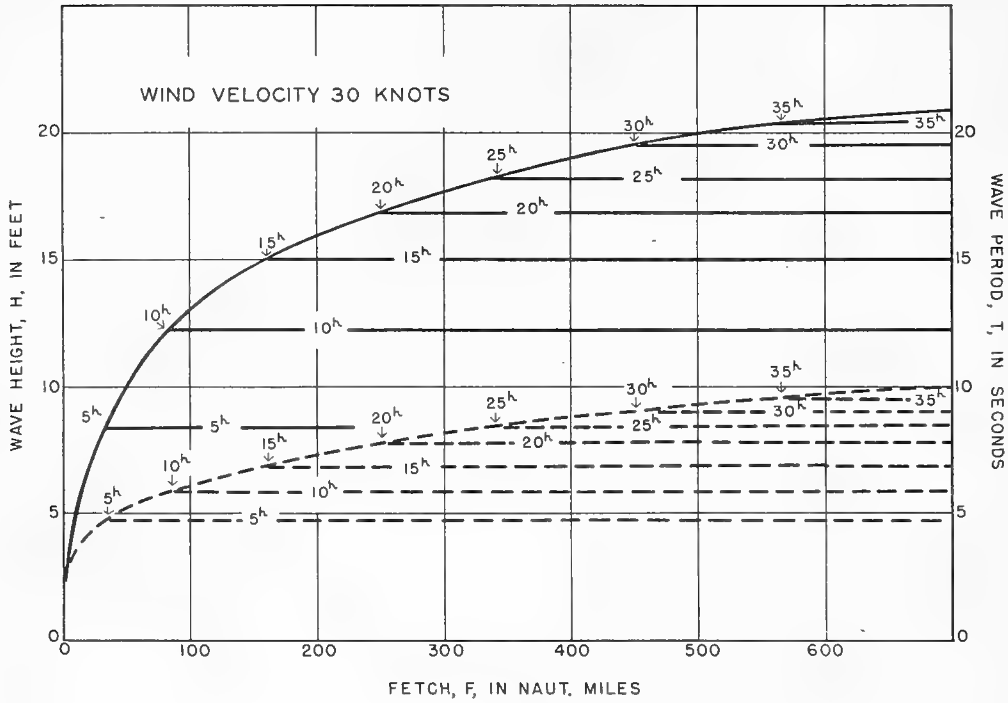


Figure 8. Wave height and wave period as functions of distance from coast line at 5^h to 35^h after a wind of 30 knots started to blow over an undisturbed water surface.

for every fifth hour after the wind started. First, small waves are formed, probably by eddies striking the sea surface. At the coast the waves remain low, but off the coast they travel with the wind and grow as they receive energy by push and pull. When the wind has blown for 5 hours one finds that with increasing distance from the coast the waves increase rapidly in height and period out to a distance of 35 miles. There the waves are 8.4 feet high with a period of 4.7 seconds. Beyond 35 miles similar waves are present but there exists a striking difference between conditions inside and beyond the 35-mile point. Inside of 35 miles a steady state has been reached, that is at any given point the waves do not change, no matter how long the wind lasts, but beyond 35 miles the waves continue to grow for a length of time which depends upon the distance from the coast. After 10 hours a steady state has been established to a distance of 85 miles, after 15 hours to a distance of 160 miles, and so on. In Figure 8 the full-drawn and dashed curves show the steady state. Parts of the curves and the horizontal lines represent wave height and period as functions of the distance from the coast at 5 to 35 hours after the constant wind of 30 knots started to blow.

The fetch shown in Figure 8 can be limited either by the presence of a coast line or by the characteristics of a wind system over the open ocean. It may be seen from the figure that for a given wind velocity the time needed to establish a steady state depends only upon the length of the fetch. For a given fetch this time depends, however, on the wind velocity and is longer for weak winds than for strong winds. This time is called the minimum duration and is measured in hours. Plate I shows the minimum duration as function of wind velocity and fetch.

Plates II and III show wave heights and periods as functions of fetch and wind velocity when the duration is longer than the minimum.

If the time is shorter than the minimum duration, the waves at the end of the fetch depend on the wind velocity and the duration in a manner similar to that shown for a 30 knot wind in Figure 8. For practical use Plates IV and V show wave heights and periods as functions of wind velocity and duration.

When using Plates II to V it should be borne in mind that the curves are constructed on the assumption that a constant wind suddenly starts to blow over an undisturbed water surface. If the wind velocity changes gradually, an average velocity has to be introduced according to rules which are discussed when dealing with the practical applications. Also, allowances must be made for waves that are present when the wind starts blowing.

Some other characteristics of the growing waves are shown in Figure 9. In the upper curve the wave steepness as expressed by the ratio H/L is plotted against the fetch for a wind of 30 knots. The curve shows the steady state and the horizontal lines show the stage of development after 10, 20, and 30 hours. Before a steady state has been reached, that is, when the duration is shorter than the minimum duration, the steepness decreases with time, and when a steady state has been established it decreases with fetch.

In the lower curve of Figure 9 the wave age as expressed by the ratio, wave velocity to wind velocity, C/U, is plotted against fetch. The wave age increases with duration before the minimum value is reached and with fetch after the establishment of a steady state.

If the corresponding values of H/L and C/U are plotted in a graph with wave steepness, H/L, and wave age, C/U, as coordinates they fall exactly on the curve in Figure 7, which represents the empirical data. Actually, this curve has been used for determining the constants needed for carrying out all computations. By means of the curves in Plates II to V it can be ascertained that the empirical relationships 1 to 6 are satisfied.

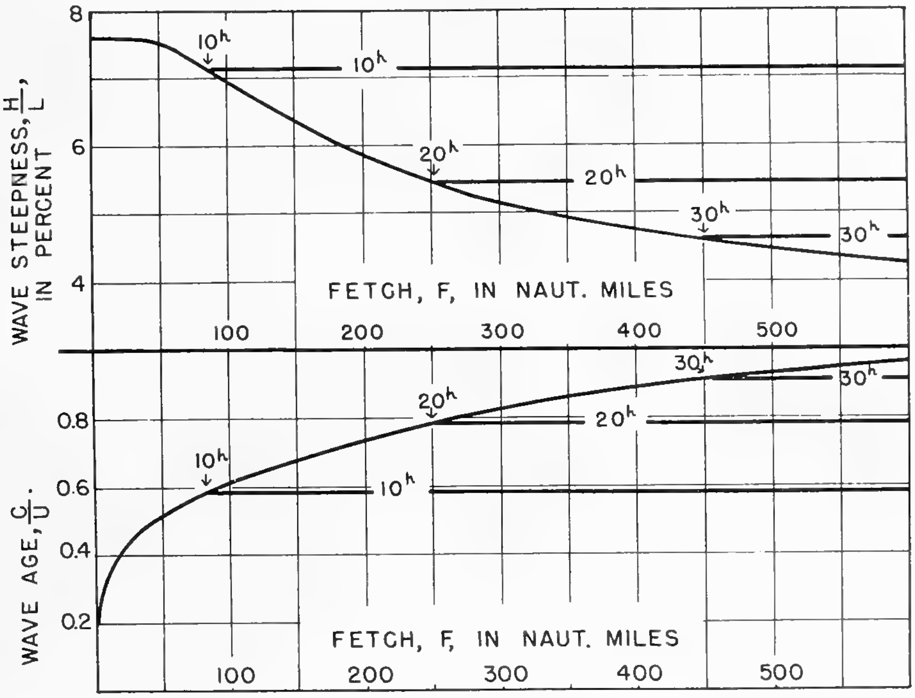


Figure 9. Wave steepness (upper graph), expressed by the ratio H/L, as function of distance from coast line at 10^h, 20^h, and 30^h after a wind of 30 knots started to blow over an undisturbed water surface, and corresponding representation of wave age (lower graph) expressed by the ratio of wave velocity to wind velocity, C/U.

According to Figure 9, with a 30-knot wind the wave velocity remains lower than the wind velocity at fetches of 600 miles or shorter. With increasing fetch the wave velocity would, however, exceed the wind velocity and the waves would continue to grow in height but decrease in steepness.

If the wave velocity exceeds the wind velocity the waves can no longer receive energy by push but will lose energy because of the air resistance they meet. They will however continue to receive energy by the pulling force of the wind and will grow in height until this gain is compensated by the loss due to air resistance, which occurs when the ratio C/U equals 1.45. The fetch and duration needed for reaching this stage increase rapidly with increasing wind velocity, as shown by the values in Table I. If the fetch and the duration are longer than those listed in the table the highest possible waves will be present regardless of how much longer the wind blows.

Table I

Highest Possible Waves Produced by Different Wind Velocities, and Corresponding Fetches and Durations.

(Ratio wave velocity to wind velocity equals 1.45, ratio wave height to wave length equals 1/45)

Wind velocity (knots)	Highest waves Height (feet)	Highest waves Period (seconds)	Fetch (naut. m.)	Duration (hours)
10	2.6	4.8	260	25
20	10.6	9.6	1040	50
30	23.7	14.4	2340	75
40	42.5	19.2	4150	100
50	66.2	24.0	6500	125

Waves of the character shown in Table I may be present in the trade wind regions and may be approached in the westerlies of the southern oceans. In the middle and higher latitudes of the Northern Hemisphere the fetches are so short that with strong winds the wave velocity always remains less than the wind velocity.

Plates II to V show only the highest waves present. These waves have traveled the entire distance from the beginning of the fetch. However, the wind can raise new waves anywhere in the fetch, and some of these may grow slowly and reach heights corresponding to the distances they travel, while others may grow rapidly and break. These contribute to the broken appearance of the sea surface which is described as the "state of the sea." The relationship between the wind and the state of the sea is discussed later.

DECAY OF WAVES

Waves Advancing into Regions of Calm

When waves spread out from a generating area into a region of calm only half of the energy of the wave advances with wave velocity. The consequence of this characteristic can be recognized by examining a simple example. Assume that a series of waves is formed by rhythmical strokes of a wave machine which at each stroke adds the energy E/2 in a given locality. The first stroke creates a wave of energy E/2. In the time interval between the first and the second stroke one half of this energy, E/4, advances one wave length and one half, E/4, is left behind. The second stroke adds E/2 to the part of the energy which was left behind. On completion of the second stroke two waves are present, one close to the wave machine with an energy 3E/4 and one which has advanced one wave length with energy E/4. By repeating this reasoning, Table II has been prepared, showing the distribution of energy in the waves after each of the first five strokes. As shown in the last line of the table a definite pattern has already developed after five strokes; the waves which have traveled the greatest distance have very little energy, the wave which has traveled half way has an energy E/2, and each of the waves closest to the machine has an energy which approaches the full amount E. When a large number of strokes have been completed these gradations are much clearer and the distribution of energy can be represented schematically by the curve in Figure 10, which shows that the energy advances with a definite "front." At the front the wave height increases from nearly zero to nearly its full value in a distance corresponding to a small number of wave lengths, and this front advances with half the wave velocity.

Table II

Advance of Waves from a Wave Machine into Still Water

Number of strokes	Relative energy of advancing waves				
1	1/2				
2	3/4	1/4			
3	7/8	4/8	1/8		
4	15/16	11/16	5/16	1/16	
5	31/32	26/32	16/32	6/32	1/32

When applying the above reasoning to the behavior of wind waves which advance into regions of calm it is necessary to consider also the following facts: (1) the wave loses energy because of the air resistance against the wave form, (2) the wave velocity (period) increases continuously.

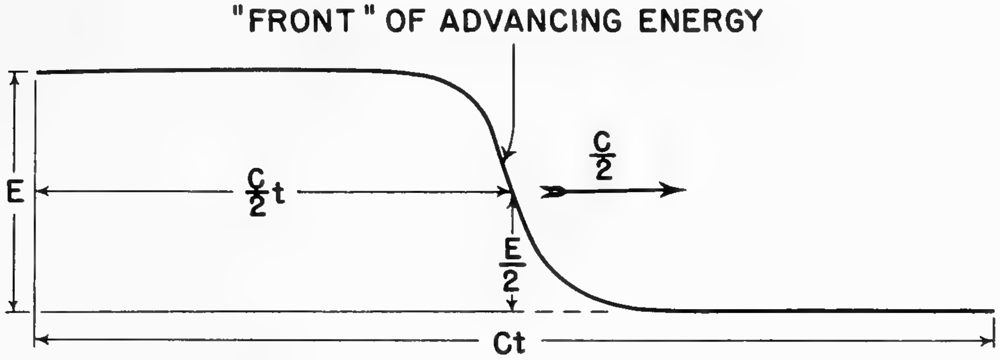


Figure 10. Advance of wave energy in time t from a source into still water. A very small amount of the energy has advanced the distance Ct. The region of rapid increase, "the front," has advanced the distance $\frac{C}{2}$ t.

When the problem is treated analytically it is not necessary to introduce any new constants. The travel time of the waves and the decrease in wave height can be obtained as special solutions of the fundamental equation which was discussed in the section on the growth of waves.

The results of this analysis are presented in Plate VI. The coordinates are the wave period at the end of the fetch, T_F, and the distance of decay, D, that is, the distance which the waves travel through areas of calm. The main part of the graph contains two sets of curves. One set gives the factor by which the wave height at the end of the fetch, H_F, must be multiplied in order to find the height of the swell at the end of the distance of decay, H_D. The other set gives the travel time, t_D, (in hours) for the distance, D. Inset I gives the length and velocity of a deep-water wave for which the period is known. Inset II gives the factor by which the period at the end of the fetch, T_F, must be multiplied in order to find the period at the end of the distance of decay, T_D. This factor depends only upon the reduction factor for the wave height, H_D/H_F. The use of the diagrams will be described when discussing the forecasting of swell.

Effect of Following or Opposing Winds

The effect of a following or an opposing wind on the decrease of the height of the swell is also found from a special solution of the fundamental equation of the "energy budget" of the wave. It is assumed that the increase in wave velocity over the distance of decay is not influenced by following or opposing winds. Although this assumption has little basis in either theory or observation it probably leads to approximately correct results. In the case of a following wind the computed wave heights may be somewhat too high and the wave periods somewhat too low, whereas in the case of an opposing wind the heights may be too low and the periods too high. Consistent differences between values computed on this basis and observed values may later be used to improve the theoretical approach.

The following or opposing wind may blow over only a part of the distance of decay. The problem is to determine how much more or how much less the wave height decreases in any given distance as compared to its decrease in the absence of any wind. This problem can be solved by means of Plate VII, the use of which will be explained when discussing the practical procedure.

Distance from which Observed Swell Comes; Travel Time; Velocity of Wind which Produced the Swell

If the height and the period of the swell are observed it is possible to find approximate values of the distance to the end of the generating area from which the swell came, of the travel time of the swell, and of the wind velocity in the generating area. In Plate VIII the coordinates are the height of the swell (in feet) and the period of the swell (in seconds). The plate contains three families of curves: full-drawn curves giving the distance to the generating area in nautical miles, light dashed curves giving the travel time from the generating area in hours, and heavy dashed curves giving the wind velocity in the generating area in knots.

The values which can be derived from the plate are only approximate because the height and period of the swell depend also upon the ratio between wave velocity and wind velocity (C/U) at the end of the fetch. The graph is constructed for C/U = 0.8, corresponding to average conditions, and gives too high values if C/U is smaller and too low values if C/U is larger. However, variations in C/U between 0.7 and 0.9 will not introduce errors exceeding 10 per cent, but errors will also arise from inaccuracies in the observations of height and period of the swell and from lack of knowledge as to changes caused by following or opposing winds. The values read off from the graphs may therefore be 25 per cent in error.

THE STATE OF THE SEA

The preceding discussion has dealt only with long-crested waves, that is, waves with very long crests and troughs. Waves may also have wave-shaped crests and troughs. In the presence of such short-crested waves the free water surface shows a series of alternating "highs" and "lows" (page 10). Furthermore, attention has been paid only to the waves which accumulate the largest amount of energy and attain the greatest heights and longest periods. In addition to these waves, and superimposed upon them, a large variety of shorter and lower waves will also be present. At wind velocities exceeding Beaufort 3 many of these shorter waves increase so rapidly in height that they break, forming white caps. It appears that at low wind velocities a great amount of energy goes into the formation of regular long-crested waves, while at high wind velocities a large part is used in the generation of small and short-crested waves.

After the waves leave the generating area, the small waves and the short-crested waves die out quickly because they contain little energy, and the long-crested waves of maximum height, which have been dealt with in the preceding analysis, are responsible for the emerging swell. In the generating area, however, the broken appearance of the sea surface is chiefly determined by the presence of the small, short-crested waves and is described by the term "state of the sea."

For the state of the sea there have been proposed several scales of which the Douglas Sea Scale is the most widely used.

There exist, however, discrepancies between definitions of the term "state of the sea" and disagreements as to the wave heights to be assigned to the descriptive terms. The following discussion appears in "Instructions to Marine Meteorological Observers." (U. S. Weather Bureau. Circular M, 6th ed., 1938). pages 53-55:

> "Ordinary waves which are moving with the wind constitute the 'sea' while a relatively low, undulating sea surface, with motion in a direction different from the local wind, is the 'swell.'
>
> "These definitions are not entirely satisfactory. Usually, the ocean surface is disturbed by both forms of wave motion, with the swell from distant winds crossing the local sea. The combined effect is the 'sea,' while the well-defined ridges of waves moving in a different direction from the local wind are the 'swells.'
>
> "The ... scale" /Table III, columns 1, 2, and 3/ "should be used in classifying the character of the sea disturbance. In recording observations in accordance with this scale, 'sea' may be considered to be composed of swells, combined with waves produced by the winds at the place of observation.
>
> "The scale of sea disturbance is approximate, based roughly on the observer's judgment as to the height of waves."

On the other hand, the "Admiralty Weather Manual," 1938, pages 50-51, states:

> "The state of the sea should be reported according to the Douglas Sea Scale (Code XIII), which is here reproduced with a table of heights of waves corresponding to the code figures" /Table III, columns 1, 2, and 4/ "... Careful distinction should be made between sea and swell, sea being the waves caused by the wind at the place and time of observation, while swell is wave motion due to past wind or wind at a distance. The direction from which the swell comes should be noted to nearest compass point."

On the *Meteor* Expedition wave heights were measured from stereophotogrammetric pictures and these wave heights were compared to simultaneous estimates of the state of the sea made by the ships' officers. A comparison of the two sets of observations led to the assignment of the wave heights which are given in Table III, column 5.

In view of the discrepancies between different systems for describing sea state, only a tentative assignment of wave heights (in feet) to the different terms of the Douglas Sea Scale is given in Table III, column 6. It should be noted that this assignment intends to relate the wave heights as obtained from Plates II and V to the terms of the Sea Scale. Considerable weight has been given to the *Meteor* data and to the fact that for low waves the observed wave heights are in general too low. If the latter feature is taken into account there is no great discrepancy between the values of the Admiralty Weather Manual (Table III, column 4) and the values introduced here (column 6). The validity of the tentative assignment can be tested by comparing reports of the state of the sea to values derived from wind fetches and durations as determined by means of weather maps.

In columns 7 and 8 are stated the corresponding wind velocities based on fetches of 500 miles and durations of 24 hours.

The frequency and direction of different states of sea in certain parts of the oceans, as well as the frequency and direction of swell, are given on H. O. charts Nos. 10; 712A, C, and E.

Table III

State of the Sea and Corresponding Wave Heights and Wind Velocities

Douglas Sea Scale		Wave Heights (feet)				Wind Velocity	Wind Force	Wind Force
No. (1)	Term (2)	a (3)	b (4)	c (5)	d (6)	(knots) (7)	Beaufort, e (8)	Beaufort, f (9)
0	Calm	0	0		0			
1	Smooth	1	0-0.5		0-1			
2	Slight	1-3	0.5-2	2.5-4	1-3			
3	Moderate	3-5	2-5	4-6.5	3-6	11-16	4	4
4	Rough	5-8	5-9	6.5-12	6-12	15-24	5,6	5,6
5	Very Rough	8-12	9-15	12-20	12-18	22-30	7	6,7
6	High	12-20	15-24		18-25	28-37	8	7,8
7	Very High	20-40	24-36		25-35	35-46	9,10	8,9
8	Precipitous	> 40	> 36		> 35	> 46	> 10	> 9
9	Confused	-	-		-		-	

a. Instructions to Marine Meteorological Observers. (U. S. Weather Bureau. Circular M, 6th ed., 1938).
b. Admiralty Weather Manual, 1938, pages 50-51.
c. Results of Meteor observations.
d. Computed wave heights assigned to the terms of the Douglas Sea Scale.
e. According to scale adopted by International Meteorological Committee.
f. According to Bowditch, H. O. no. 9, 1939, page 52.

FORECASTING OF WAVES FOR SHORT FETCHES

In areas such as the Mediterranean and other partially enclosed bodies of water, it is often necessary to forecast waves generated over short fetches which are determined entirely by coast lines and wind direction. In this case the problem of forecasting becomes primarily a meteorological problem of forecasting the direction and velocity of the wind. If this can be done, the wave height and wave period are found from Plate III if the fetch is shorter than 100 nautical miles, and from Plate II if it is longer. The duration is rarely limiting if the fetch is less than 200 nautical miles, but should such be the case Plate IV or V must be used.

Example

A strait running north-south has a width of 30 miles. At 12ØØ it is forecast that a northwest wind will reach the strait, will attain a velocity of 30 knots at 2ØØØ and will continue to blow with that velocity for 12 hours. What waves can be expected off the eastern shore of the strait at Ø6ØØ the next morning?

The pertinent values are:

Fetch	43 naut. m.
Wind velocity	30 knots
Duration, 2ØØØ to Ø6ØØ	10 hours
Minimum duration	6 hours (Plate I)

Since the duration is longer than the minimum duration Plate II is used, from which

$$H = 9.5 \text{ feet}, \quad T = 5.0 \text{ seconds}$$

The next noon the wind velocity decreases and it is forecast that from 18ØØ on it will be nearly calm. What waves can be expected to reach the eastern shore on the following morning at Ø6ØØ?

Assuming that the wind suddenly died at 15ØØ the following wave heights and periods are determined from Plate III:

Distance from lee shore (naut. m.)	Distance to windward shore (naut. m.)	Wave height (feet)	Wave period (seconds)
5	38	3.7	3.0
15	28	6.0	4.0
30	13	8.0	4.6

The time interval under consideration is 15 hours. From Plate VI it is evident that the waves listed above would travel 50 nautical miles or more in 15 hours. Therefore, the waves have died out before Ø6ØØ on the following morning.

The procedure indicated in this example can be modified according to the nature of the problem. The forecaster should attempt to gain local experience and modify his use of the graphs accordingly.

FORECASTING OF SWELL

Forecasts of swell can be made with considerable accuracy if adequate consecutive weather maps are available from which (1) wind direction, (2) wind velocity, (3) fetch, and (4) duration can be determined for the wave-generating areas. The details of the procedure will depend upon the character of the weather maps but some general principles can be outlined. In Tables IV, V, and VI are listed the fundamental and the auxiliary quantities which are used when preparing a forecast. The quantities summarized in Table IV always have to be computed, but when some experience has been gained the computations indicated in Table VI need not be carried out but an estimate of the final values can be made directly. In the following discussion the numbers in parentheses refer to corresponding terms in Tables IV and VI.

Determination of Wind, Fetch, and Duration

1. Wind direction. Outside of the tropics the wind direction over the ocean is obtained from the course of the isobars, applying the rule that the wind deviates 80 degrees to the right of the pressure gradient in the Northern Hemisphere and 80 degrees to the left in the Southern Hemisphere. Where the isobars are nearly straight (fig. 11, A and B) the winds to be considered in forecasting swell are those with directions within 30° of a line joining the generating area and the locality for which forecasts are to be made. Where the isobars are curved (fig. 11C) the winds to be considered are those with directions within 45° of a line joining the generating area and the locality for which forecasts are to be made. The generating areas to be considered are limited by these restrictions. The reasons for these rules are that the course of the isobars is not exactly known and that the swell probably spreads out somewhat when entering areas of calm. The spreading out will be greater from a region with curved isobars.

In the tropics the wind direction must be obtained from observations on board ships or at exposed island stations.

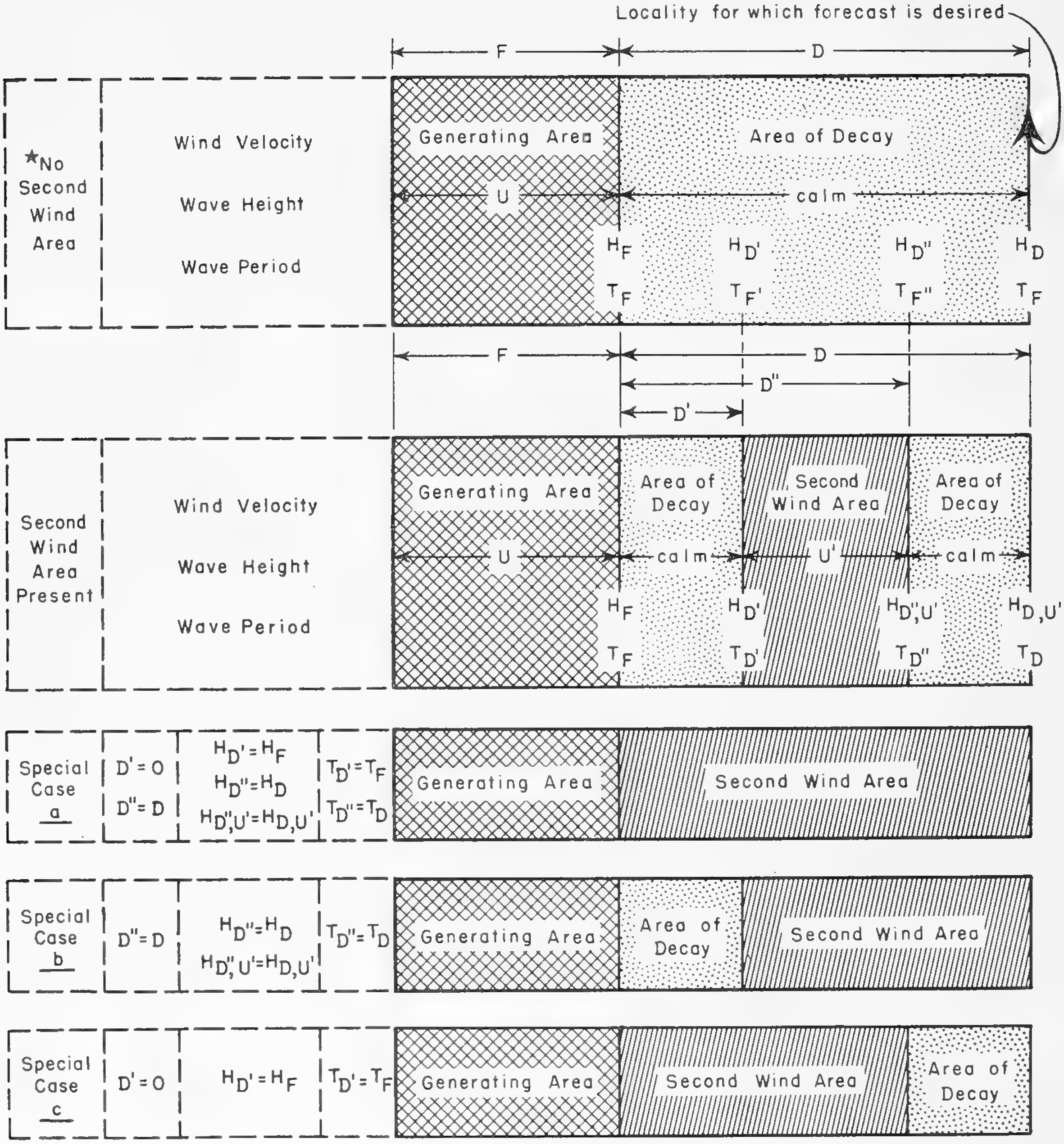
Table IV

Summary of Quantities to be Determined when Forecasting Swell in the Absence of Following or Opposing Winds

No.	Term	Symbol	Units	Source
1	Generating area, mean distance between isobars drawn at intervals of 5 mb	G	Degrees of latitude	Synoptic chart
2	Mean latitude	ϕ	Degrees	Synoptic chart
3	Curvature of isobars	-	-	Synoptic chart
4	Geostrophic wind	U_G	Knots	Tables or graph
5	Wind velocity at sea surface	U_S	Knots	From (4), considering curvature of isobars. Multiply by: 0.60 Great cycl. curv. 0.63 Small cycl. curv. 0.65 Straight isobars 0.67 Small anticycl. curv. 0.70 Great anticycl. curv.
6	Average wind velocity	U	Knots	Current and preceding synoptic charts
7	Observed wind at sea surface	(U)	Beaufort	Synoptic chart
8	Fetch	F	Naut. m.	Synoptic chart
9	Duration of wind	t_d	Hours	Current and preceding synoptic charts
10	Minimum duration	t_{min}	Hours	Plate I
11	Wave height at end of fetch	H_F	Feet	Plate II or III, using (6) and (8) if $t_d > t_{min}$; Plate IV or V, using (6) and (9) if $t_d < t_{min}$
12	Wave period at end of fetch	T_F	Seconds	(same as 11)
13	Distance of decay	D	Naut. m.	Synoptic charts
14	Reduction factor for wave height	H_D/H_F		Plate VI, using (12) and (13)
15	Wave height near coast	H_D	Feet	(11) times (14)
16	Factor of period increase	T_D/T_F		Plate VI, Inset II, using (14)
17	Period near coast	T_D	Seconds	(12) times (16)
18	Travel time	t_d	Hours	Plate VI, using (12) and (13)
19	Wave length	L	Feet	Plate VI, Inset I, using (17)
20	Wave velocity	C	Knots	Plate VI, Inset I, using (17)

Table V

Scheme of Nomenclature Used for Forecasting in the Presence of a Second Wind System



<u>Definitions</u>:

D	Distance from end of generating area to locality for which forecast is desired.
D'	Distance from end of generating area to beginning of second wind area.
D''	Distance from end of generating area to end of second wind area.
$H_{D'}$	Wave height at the beginning of second wind area.
$H_{D'',U'}$	Wave height at the end of second wind area
$H_{D''}$	Wave height at the end of second wind area, if secondary wind U' were zero.

(The same system of notations applies to wave periods, except that $T_{D'',U'}$ always equals $T_{D''}$)

★ <u>Note</u>: $H_{D'}$, $H_{D''}$, $T_{D'}$, $T_{D''}$, are wave heights and periods in area of decay at distances D' and D'' from generating area. (No second wind area).

Table VI

Summary of Quantities to be Determined when Forecasting Swell in the Presence of Following or Opposing Winds

No.	Term	Symbol	Units	Source
21	Distance to beginning of second wind area	D'	Naut. m.	Current and prognostic synoptic charts
22	Distance to end of second wind area	D''	Naut. m.	Current and prognostic synoptic charts
23	Wind velocity in second wind area $(D'' - D')$	U'	Knots	Estimated from current and prognostic synoptic charts
24	Sign to be applied to U'	$\pm$		Following or opposing wind
25	Reduction factor for wave height for distance D' $(U' = 0)$	$H_{D'}/H_F$		Plate VI, using (12) from Table IV and (21)
26	Wave height at distance D' $(U' = 0)$	$H_{D'}$	Feet	(11), Table IV, times (25)
27	Factor of period increase for distance D'	$T_{D'}/T_F$		Plate VI, Inset II, using (25)
28	Period at distance D'	$T_{D'}$	Seconds	(12), Table IV, times (27)
29	Reduction factor for wave height for distance D'' $(U' = 0)$	$H_{D''}/H_F$		Plate VI, using (12), Table IV, and (22)
30	Wave height at distance D'' $(U' = 0)$	$H_{D''}$		(11), Table IV, times (29)
31	Factor of period increase for distance D''	$T_{D''}/T_F$		Plate VI, Inset II, using (29)
32	Period at distance D''	$T_{D''}$	Seconds	(12), Table IV, times (31)
33	Average period for distance $(D'' - D')$	$\overline{T}$	Seconds	Average of (28) and (32)
34	Ratio wind velocity to wave period in second wind area	$U'/\overline{T}$	Knots/sec	(23), considering (24), divided by (33)
35	Ratio of wave heights in second wind area $(U' = 0)$	$H_{D''}/H_{D'}$		(30) divided by (26)
36	Correction factor to (35)	$H_{D'',U'}/H_{D''}$		Plate VIII, using (34) and (35)
37	Wave height at end of second wind area	$H_{D'',U'}$	Feet	(30) times (36)
38	Reduction factor of wave height for distance $(D - D'')$	$H_{D,U'}/H_{D'',U'}$		Plate VI, using (32) and $(D - D'')$
39	Wave height near coast	$H_{D,U'}$	Feet	(37) times (38)

2. Wind velocity (5). Outside the tropics the wind velocity over the generating area is obtained from the pressure distribution. Instead of computing the gradient wind it is sufficient to compute the geostrophic wind (4) and to multiply the value so obtained by a reduction factor which takes into account the curvature of the isobars.

The following factors appear to be sufficiently accurate to dispose of the somewhat uncertain computation of the gradient wind:

Great cyclonic curvature of isobars	0.60
Small cyclonic curvature of isobars	0.63
Straight isobars	0.65
Small anticyclonic curvature of isobars	0.67
Great anticyclonic curvature of isobars	0.70

The computations may have to be carried out for different parts of the fetch in order to obtain the average wind velocity in the generating area. Ships' observations should be used as a check on the computed value. A difference of not more than one on the Beaufort Scale between computed and observed velocity is a satisfactory check.

The wind velocity obtained in this manner applies to the current weather map and may differ from the wind velocity over the same area according to the preceding map. A constant wind velocity was assumed in the preparation of Plates II to V which are used to determine the wave height, and it is therefore necessary to introduce an average wind velocity (6) which can be considered applicable to the entire time interval between the two maps. Although the manner in which the velocity has changed is not known, the fact that strong winds raise waves more rapidly permits the application of the following crude procedure:

Find the component of the wind which on the preceding map blew in the direction of the wind on the current map. Subtract one-fourth of the difference between these two velocities from the greater velocity. The result is considered the average velocity during the time interval between the maps.

If the wind is decreasing this rule should be applied only if the velocity remains above 15 knots. If the velocity drops below 15 knots the effect of a following wind should be examined.

This procedure may have to be modified according to the experience of the forecaster.

In the tropics the wind velocities have to be obtained from observations on board ships or at exposed stations on islands.

3. Fetch (8). The fetch is the length of the generating area in the direction of the wind, that is, the stretch between the rear and the front boundaries of this area. In general, the boundaries are determined by coast lines or by one of the following: (a) fanning out of isobars, (b) meteorological fronts, or (c) curvature of isobars, as shown schematically in Figure 11. When the boundaries have been decided upon the fetch is measured on the map. When the isobars have a great curvature two fetches should be measured, as shown in Figure 11C. Computations of wave height and wave period at the end of both fetches should be carried out, since inspection alone will not indicate which fetch should be used. In making the forecast consideration should be given the higher values.

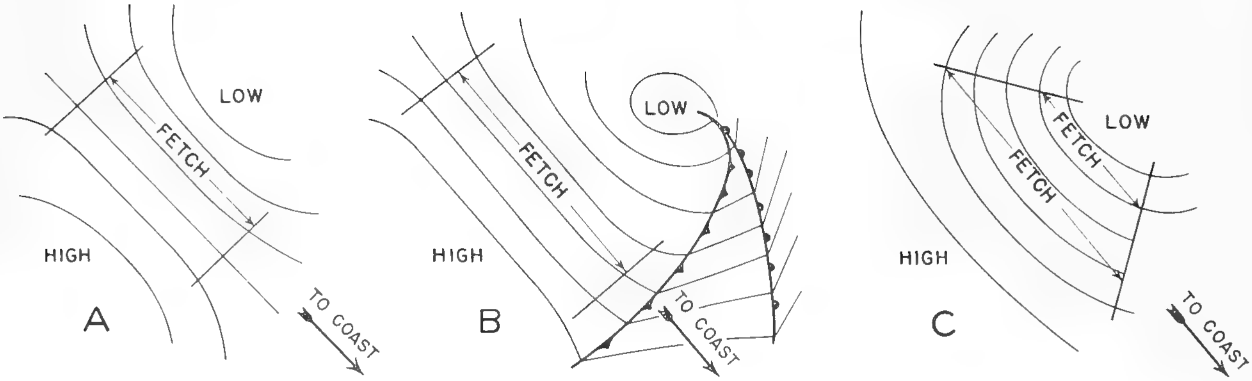


Figure 11. Boundaries of the fetch for different types of isobars.

4. Duration of wind (9). The duration of the wind is determined from a comparison of current and preceding weather maps. The duration of the average wind velocity equals the time interval between the last two maps plus a correction determined from the height of the waves present at the beginning of that time interval. These waves should be known from the examination of the preceding map. Only the waves which travel at an angle less than 45° from the average wind direction should be examined. The correction is found from Plate IV by the following procedure:

Enter the graph with the average wind velocity and follow a horizontal line to the curve which gives the wave height on the preceding day. The corresponding duration, as read off from the top or bottom scale, represents the correction to be added to the time interval between the maps.

Example

a.	Wind velocity from current map	32 knots
b.	Wind velocity 24 hours earlier	20 knots
c.	Average wind velocity for last 12 hours $(a - \frac{a-b}{4})$	29 knots
d.	Maximum wave height 24 hours earlier (from preceding map).	10 feet
e.	Time needed by 29-knot wind to raise 10-foot waves (Plate IV)	$7\frac{1}{2}$ hours
f.	Duration of 29-knot wind (24 hours + e). .	32 hours

Determination of Highest Wind Waves (11,12)

When wind velocity, fetch, and duration have been determined the minimum duration (10) is read off from Plate I. If the duration is longer than the minimum duration the wave height and wave period at the end of the fetch are obtained from Plates II or III, if it is shorter, from Plates IV or V.

Example

Wind velocity 29 knots, fetch 800 nautical miles, duration 32 hours.

From Plate I: minimum duration, 43 hours.
From Plate IV: wave height, 18.0 feet,
wave period, 9.0 seconds.

Determination of the Swell (13-20)

1. Waves advancing through regions of calm. The distance of decay (the distance from the end of the fetch to the locality for which the forecast is made) is measured on the map. Entering Plate VI with the distance of decay, D, and the period at the end of the

fetch, $\underline{T_F}$, the reduction factor to be applied to the wave height at the end of the fetch and the travel time (in hours) are read off.

Example

At end of fetch:		
Wave height	18	feet
Period	9	seconds
Distance of decay	600	naut. m.
Reduction factor to be applied to wave height. .	0.47	
Travel time	40	hours
Height of swell.	8.5	feet

From Inset II in Plate VI the factor is found by which the wave period at the end of the fetch, $\underline{T_F}$, must be multiplied in order to find the period of the swell at the end of the distance of decay.

Example

Wave period at end of fetch . .	9.0 seconds
Reduction factor to be applied to wave height	0.47
Factor of period increase (from Plate VI, Inset II)	1.27
Period of swell	11.4 seconds

From Inset I in Plate VI are found velocity and length corresponding to any given wave period. Exact values are obtained by using the formulas: C (in knots) = 3.03 T, L (in feet) = 5.2 T^2.

Example

	Period (seconds)	Velocity (knots)	Length (feet)
Waves at end of fetch	9.0	27.5	415
Swell at end of distance of decay	11.6	35.0	690

2. Following or opposing winds. In general, swell should be forecast on the basis of preceding and current weather maps, assuming that it travels through regions of calm. However, if a prognosis of the weather situation or if a subsequent weather map shows that the waves travel through regions where the wind has a component parallel to the direction of progress, the forecast should be modified by taking into account the effect of a following or an opposing wind.

The region of a following or an opposing wind has to be considered as a second wind area, the boundaries of which have to be selected as the boundaries of the region in which the component, U', of the wind parallel to the direction of progress of the swell exceeds 6 knots. The reasons for this limitation are that cross winds are not considered to influence the swell and that the effect of very weak winds is negligible. The wind velocity in the second wind area is obtained by estimate if a prognosis is made or in the manner described above if a subsequent weather map is used.

3. Effect of following or opposing winds. The effect of the following or opposing wind on the wave height only has to be determined, because it is assumed that the wave period is not influenced by these winds and that, consequently, the travel time remains unaltered. Travel time and wave period at the end of the distance of decay are therefore found by means of Plate VI in the manner described above.

In order to determine the wave height at the end of the distance of decay, $H_{D,U'}$, the auxiliary quantities listed in Table V have to be found. The values of $H_{D'}$ and $H_{D''}$ are obtained from Plate VI by entering the graph with the period at the end of the fetch, T_F, and the partial distances of decay, $\underline{D}'$, and $\underline{D}''$ (see Table V). The corresponding periods, $T_{D'}$ and $T_{D''}$, are obtained from Inset II to Plate VI in the manner described above. Having determined these quantities, $H_{D'',U'}$ is obtained from Plate VII in the following manner:

The average value of $T_{D'}$ and $T_{D''}$ is computed and called $\overline{T}$. The ratio between the wind velocity in the second wind area, U', and the average period, $\overline{T}$, is found and is taken as positive for a following wind and negative for an opposing wind. From Plate VII which is entered with the ratio U'/T and the ratio $H_{D''}/H_{D'}$, a correction factor, $H_{D'',U'}/H_{D''}$, is read off. Multiplying this factor by $H_{D''}$ the value of $H_{D'',U'}$ is found.

Finally, $H_{D,U'}$ is obtained from Plate VI by entering this graph with the period $T_{D''}$ and the distance (D - D").

If the second wind area extends over the entire distance of decay or if there is only one region of calm (see Table V) the procedure is shortened, as evident from the following examples.

Example 1 (Table V, special case a)

Wave height at end of fetch, H_F . . .	18	feet
Wave period at end of fetch, T_F . .	9.0	seconds
Distance of decay, D	600	naut. m.

It is estimated that a following wind of 10 knots will blow over the entire distance of decay. The computation of the wave height at the end of the distance of decay, $\underline{H_{D,U'}}$, is carried out as follows, using the symbols in Table VI:

Number	Symbol	Numerical Value	
21	D'	0	
22	D''	600	naut. m. ($D'' = D$)
22,23	U'	10	knots
29	H_D/H_F	0.47	($H_{D''} = H_D$)
30	H_D	8.5	feet
31	T_D/T_F	1.27	($T_{D''} = T_D$)
32	T_D	11.4	seconds
33	$\overline{T}$	10.2	seconds
34	$U'/\overline{T}$	0.98	
36	$H_{D,U'}/H_D$	1.43	[using (29) and (34)]
39	$H_{D,U'}$	12.2	feet

Thus, the corrected height of the swell is 12.2 feet. The period is 11.4 seconds, as in the preceding examples, and the travel time, 40 hours. It is probable that the method gives wave heights that are somewhat too great and periods that are too short.

Example 2 (Table V, special case b)

Wave height at end of fetch, H_F . .	18	feet
Wave period at end of fetch, T_F . .	9.0	seconds
Distance of decay, D	600	naut. m.

On the basis of the subsequent weather map it is estimated that the swell will meet an opposing wind of 30 knots over the last 200 nautical miles of the distance of decay. Again using the symbols in Table VI:

Number	Symbol	Numerical value	
21	D'	400	naut. m.
22	D"	600	naut. m. (D" = D)
23,24	U'	-30	knots
25	$H_{D'}/H_F$	0.58	
26	$H_{D'}$	10.4	feet
27	$T_{D'}/T_F$	1.2	
28	$T_{D'}$	10.8	seconds
29	H_D/H_F	0.47	$(H_{D''} = H_D)$
30	H_D	8.5	feet
31	T_D/T_F	1.27	$(T_{D''} = T_D)$
32	T_D	11.4	seconds
33	$\overline{T}$	11.1	seconds
34	$U'/\overline{T}$	-2.7	
35	$H_D/H_{D'}$	0.82	
36	$H_{D,U'}/H_D$	0.62	
39	$H_{D,U'}$	5.3	feet

Thus, the corrected wave height is 5.3 feet, but period and travel time remain unchanged.

Remarks on Forecasts

Estimates of the probable decrease and increase of the swell have to be based in part upon a prognosis of weather conditions. Usually the forecaster need not construct a prognostic map but can base his estimate on the conditions he anticipates from his examination of the weather maps. The following or the opposing winds can be estimated in a similar manner.

In order to arrive at an estimate of the rapidity with which swell may die out it is advisable to split the fetch into several parts and compute the swell from each.

In middle latitudes a sequence of low-pressure systems, that is, a sequence of generating areas, often travels across the oceans. It is recommended that the swell which is forecast from each generating area be plotted on graph paper, using height of swell and time of arrival as coordinates. Observed values should be entered on the same graph in order to test the accuracy of the forecasts.

In carrying out the forecasting it may be found that several wave trains arrive at approximately the same time; in this case the resulting swell will be complicated because of interference. The greatest wave heights may equal the sum of the heights in the individual wave trains but the average height will be that characteristic of the train having the highest waves. It appears probable that with experience the complexity of the expected swell can be forecast.

The general procedure which has been outlined should be modified according to the type of weather maps which are available and according to the experience of the forecaster. However, it should be emphasized that the continuity of the processes must be borne in mind.

Example

Forecast of swell for Casablanca and vicinity,
Northwest coast of Africa, November 7, 1931.

The forecast is based on the weather map for the North Atlantic of November 7 at 13ØØ, G.M.T. (fig. 12) and on preceding maps. The weather map of November 6 showed an elongated low-pressure area to the south of Greenland from which a cold front extended south in longitude 32° W, bending toward SW in latitude 40° N. Behind the cold front the wind was WNW with an average speed of about 30 knots. To the east of the front, toward the coast of Spain, the wind was nearly W and the average speed about 20 knots.

On November 7 the low-pressure area and the cold front had advanced toward SSE and a well-defined generating area was present to the northeast of the Azores (fig. 12). The isobars, drawn at intervals of 5 mb, were nearly straight and in 40° N they were 1.6 degrees of latitude apart. The corresponding geostrophic wind was 50 knots and, with à reduction factor of 0.65, the wind at the sea surface was 32.5 knots. Ships reported wind velocities of 8 Beaufort (30-35 knots according to the scale adopted by the International Meteorological Committee). The average wind velocity during the past 24 hours is found to be 29 knots, according to the rule given when the determination of the wind velocity was discussed.

In selecting the boundaries of the generating area the front boundary was placed somewhat behind the cold front because of the curving of the isobars, and the rear boundary was placed where the isobars fanned out. This selection gave a <u>fetch of 800 nautical miles</u>.

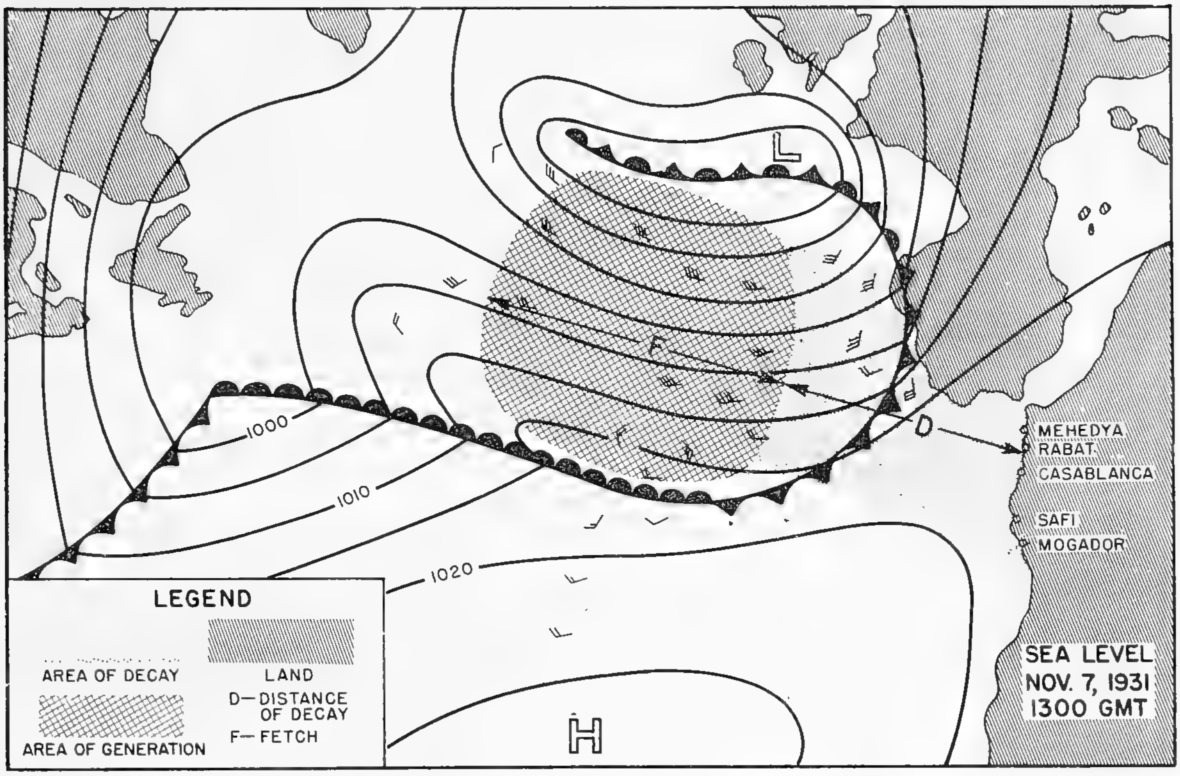


Figure 12. Isobars over the North Atlantic on Nov. 7, 1931, at 1300 G.M.T. taken from the meteorological charts of the Northern Hemisphere. Original observations are omitted, except a number of ships' observations of wind. A generating area and the distance of decay for swell traveling toward northwest Africa are indicated.

The duration was determined in the following manner: On November 6 a wind of 20 knots had blown over the generating area and had been preceded by stronger winds. The waves present on November 6, therefore, were the highest possible at that wind velocity, that is, according to Plates II or IV they were 10 feet high. A wind velocity of 29 knots would need 7 to 8 hours to raise these waves (Plate IV) and the duration of the wind was therefore 32 hours.

With these values one obtains from Plate I:

Minimum duration, $t_{min} = 43$ hours.

Since the duration is shorter than 43 hours, Plate IV is used, from which one obtains:

H_F - 18.0 feet, T_F = 9.0 seconds.

The distance of decay was 600 miles. Entering Plate VI with a period of 9.0 seconds and a distance of decay of 600 miles, a travel time of 40 hours and a reduction factor of 0.47 are read off. Consequently, the swell should arrive at the northwest coast of Morocco in 40 hours, that is, on November 9 at Ø5ØØ, G.M.T., with a height of 8.5 feet. From Inset II in Plate VI one finds a factor of 1.27 for the period increase, that is, the swell should arrive with a period of about 11.4 seconds.

The calculations can be tabulated as follows, using the symbols in Table IV:

Number	Symbols	Numerical value
1	G	1.6 degrees of latitude
2	ϕ	40° N.
3	-	straight
4	U_G	50 knots
5	U_S	32.5 knots (factor 0.65)
6	U	29 knots (adopted average)
7	(U)	8 Beaufort
8	F	800 naut. m.
9	t_d	32 hours

Number	Symbols	Numerical value
10	t_{min}	43 hours
11	H_F	18.0 feet
12	T_F	9.0 seconds
13	D	600 naut. m.
14	H_D/H_F	0.47
15	H_D	8.5 feet
16	T_D/T_F	1.27
17	T_D	11.4 seconds
18	t_D	40 hours
19	L_D	690 feet
20	C_D	35 knots

When preparing a forecast on the basis of this analysis it must be considered that the winds over the distance of decay can be expected to continue to blow in the direction of progress of the swell so that the decrease in height will be less than that obtained from Plate VI. From this prognosis of the weather conditions it is estimated that a following wind of 10 knots will be present over the entire distance of decay. According to the procedure outlined above (example 1) the swell should then arrive with a height of 12.2 feet and the period of the swell should remain unchanged, but this wave height may be somewhat high and the period may be too short. Furthermore, the wind system causing the swell will probably continue to advance towards the east so that the height of the swell can be expected to increase for some time as the distance of decay shortens. The following forecast should therefore be issued:

Casablanca and vicinity: On November 9 between 24ØØ and Ø8ØØ: Swell from NW, height 8 to 12 feet, period 11.4 to 14 seconds. Swell increasing during the day.

This forecast did not need modification on November 8 because the weather map of the 8th showed the estimate of the following wind to be nearly correct.

The following values were observed on the morning of November 9:

Locality	Approx. height (feet)	Period (seconds)	Swell from
Mehedya	7	15	NW
Rabat	9	15	NW
Casablanca	12	15	NW
Safi	6	12	W
Mogador	7	12	NW

The observations at Safi give consistently lower values than those at neighboring stations, possibly because the locality at which observations were made is less exposed. For the other stations the forecast height of the swell was nearly correct, but at the northern stations the forecast period was too short.

FORECASTING OF THE STATE OF THE SEA

A forecast of the state of the sea must be based on the conclusions as to the state of the sea drawn from preceding and current weather maps and upon a prognostic weather map. The procedure in using the prognostic weather map is exactly the same as that which applies to the current map. When winds, fetches, and durations have been estimated the wave heights and periods in the generating areas are found in the manner described when discussing the forecasting of swell. If desired, the state of the sea may be described by a term on the Douglas Scale, according to Table III.

Although the method has not yet been tested extensively, it is believed that the accuracy of the forecast will correspond to the accuracy of the prognostic map. It must again be emphasized that success can be expected only if the continuity of the processes are borne in mind.

Appendix

WAVES ENTERING SHALLOW WATER: BREAKERS AND SURF.

A manual on forecasting breakers and surf is in preparation. For temporary guidance the transformations of waves that enter shallow water are briefly discussed here.

Consider a wave which approaches a straight coast off which the depth to the bottom increases regularly and slowly, and assume that in deep water the wave crest is parallel to the coast line. At a distance from a coast at which the depth to the bottom, $\underline{d}$, is about 1/2 the wave length transformation from a deep-water wave to a shallow-water wave begins to be perceptible. The velocity of progress decreases but the period remains unaltered so that the decrease in velocity appears as a decrease in wave length. If the wave lengths in deep water, $\underline{L_o}$, and in shallow water, $\underline{L_s}$, are known, the depth to the bottom is obtained from the equation:

$$\tanh 2\pi \frac{d}{L_s} = \frac{L_s}{L_o}$$

Where the depth is less than 1/25 of L_o the equation is reduced to

$$d = \frac{1}{2\pi} \frac{L_s^2}{L_o}$$

These equations have been used to determine the bottom topography from aerial photographs of waves.

The wave height remains constant until a depth is reached which equals about 1/25 of the wave length in deep water. This is explained by the fact that if the effect of friction is disregarded changes in wave height depend upon changes in the rate at which energy advances. In deep water the amount of energy which advances through a cross section of the wave is 1/2 C_oE_o, where $C_o = \sqrt{\frac{g}{2\pi} L_o}$ and E_o is the mean energy of the wave per unit surface area. In shallow water the corresponding amount is C_sE_s where $C_s = \sqrt{gd}$. If no energy is lost by bottom friction as the wave advances toward shore, $1/2\ C_oE_o = C_sE_s$. Where $E_o = E_s$, one has $1/2\ C_o = C_s$ or $1/2\ L_o = L_s$. The corresponding depth is $d = \frac{L_o}{8\pi} \approx \frac{L_o}{25} \approx \frac{L_s}{12.5}$.

Therefore, the wave height, which is proportional to the square root of the wave energy, is the same in deep water and in shallow water where the depth is approximately $L_o/25$. The wave height however does appear higher. The steepness of the wave has been doubled because the wave length has decreased one half.

As the depth becomes less than $L_o/25$ the wave height increases rapidly and the wave length continues to decrease. When long and low swell approaches a gently sloping beach, narrow, steep crests, separated by long, flat troughs, appear to rise a short distance from the beach, and these crests soon become so steep that they break. It is the narrowness, however, and not the height of the crests which makes them plainly visible. The breaker height, H_b, and the depth of breaking, d_b, depend upon a number of factors: the steepness and direction of the waves in deep water, the slope and regularity of the bottom, the strength and direction of local winds, and the number of wave trains present. As yet no general rules can be given, but the ratio H_b/H_o appears to lie between one and two with the smaller value referring to steep waves on gently sloping beaches. The ratio d_b/H_b varies between one and three, the smaller value referring to a gently sloping beach.

Where a wave train approaches the coast at an angle the direction of progress changes as the waves enter shallow water. Since the velocity is less in shallow water, the part of the wave which first reaches shallow water progresses at a slower rate than the part which is still in deep water and consequently the wave front turns gradually until it becomes parallel to the beach. The height of the waves will be less than that of waves which advance directly against the coast as the energy must be distributed over a greater length of beach.

As a simple example consider a straight coast off which the depth contour lines are parallel. Call the energy of the waves in deep water E_o, let a_o be the angle which the wave crest in deep water forms with the coast line, and let a_s be the angle with the coast line where $d = L_o/25$. Where $d = L_o/25$ the energy of the wave equals $E_o \cos(a_o - a_s)$, and the wave height is

$$H = H_o \sqrt{\cos(a_o - a_s)}$$

because the wave height is proportional to the square root of the energy. Thus, the reduction in height is small because even for $(a_o - a_s) = 45°$

$$\sqrt{\cos(a_o - a_s)} = 0.84$$

If the bottom topography is not too complicated and a good chart is available the bending of the waves can be computed, but such computations should be checked by measurements or aerial photographs. Methods for computations will be dealt with in the forthcoming manual of forecasting breakers and surf.

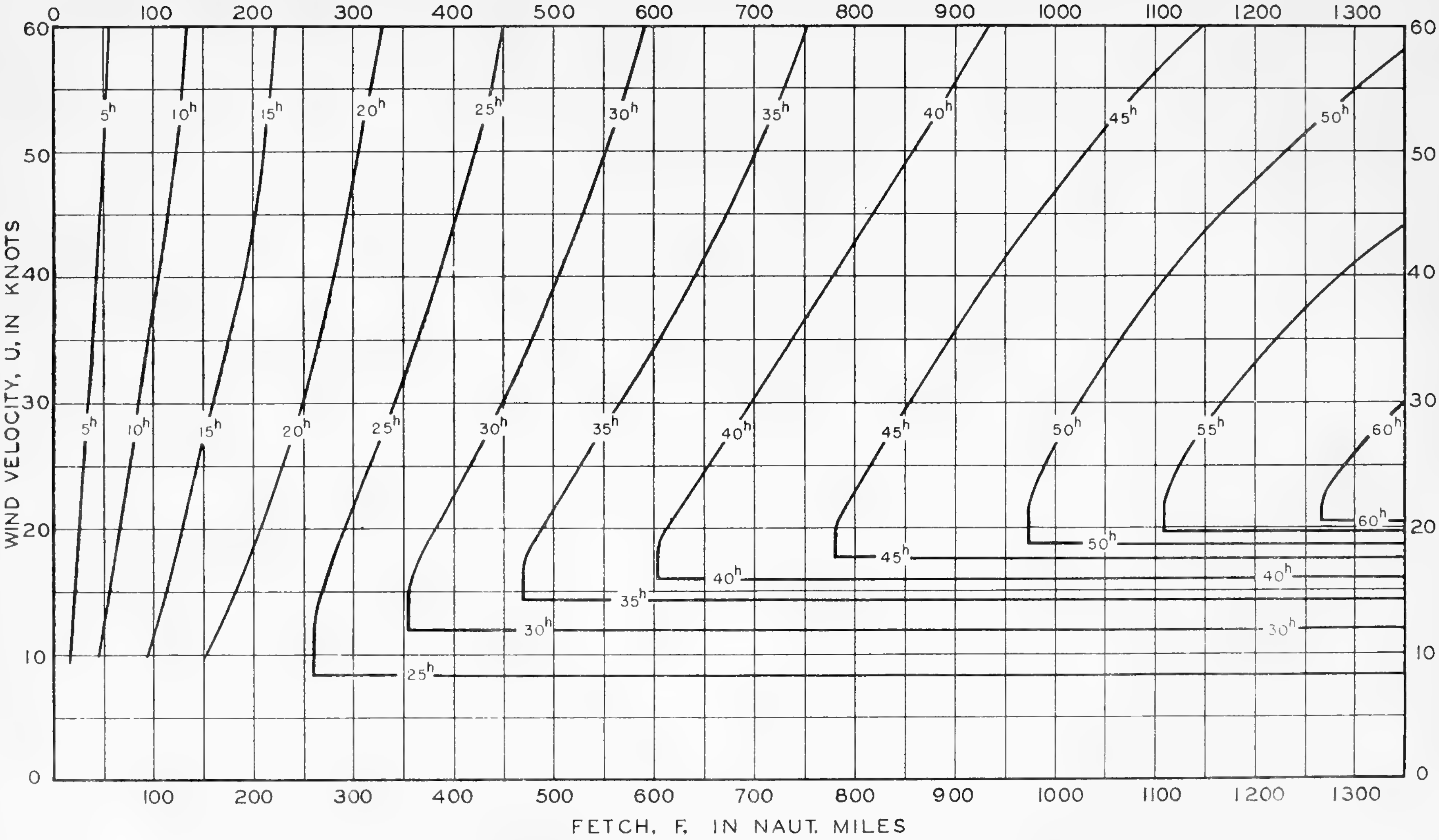


Plate I. Growth of wind waves. Time in hours needed by a wind of given velocity to raise the highest possible waves at the end of a given fetch.

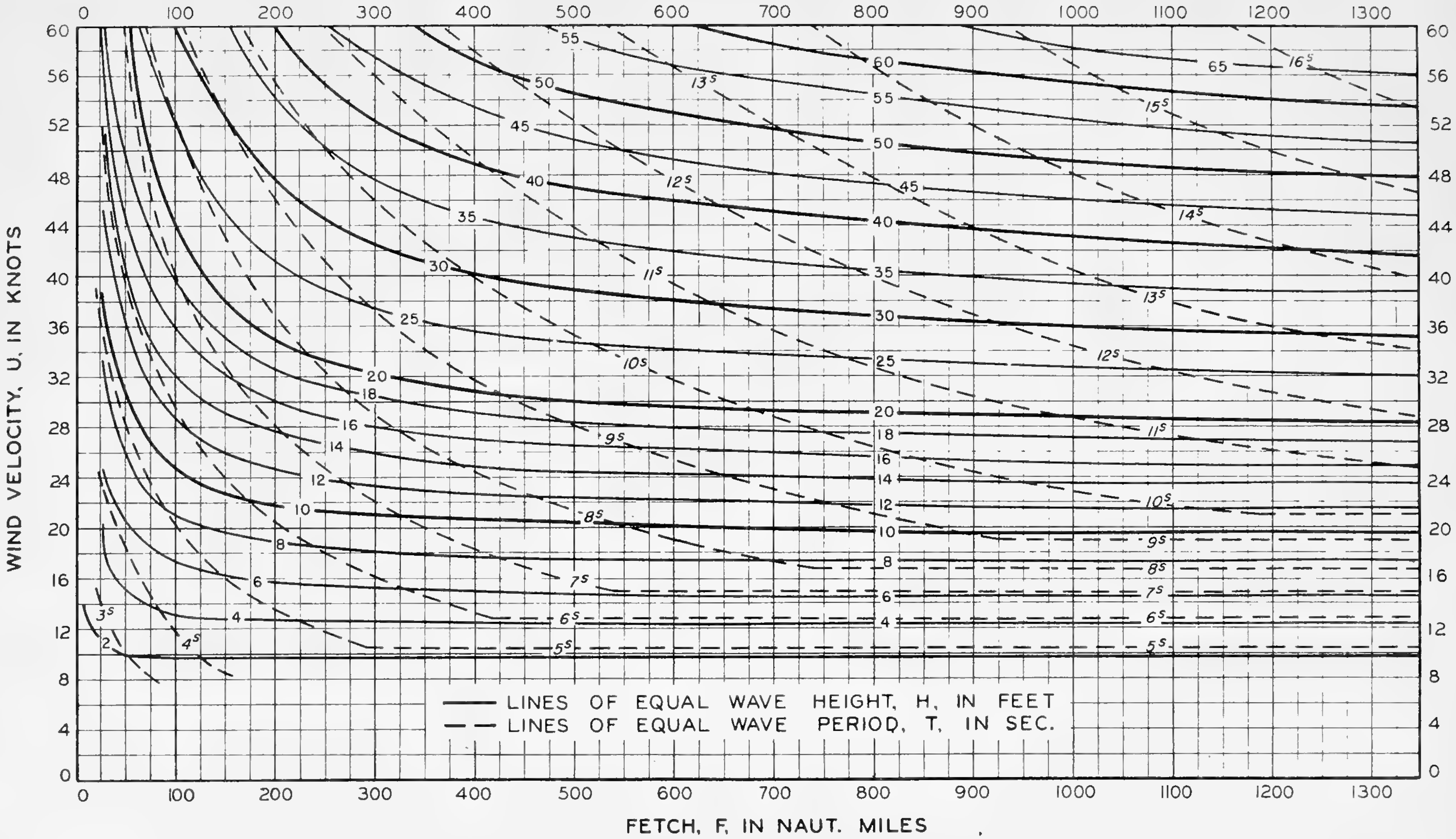


Plate II. Growth of wind waves. Wave height and wave period as functions of wind velocity and fetch.

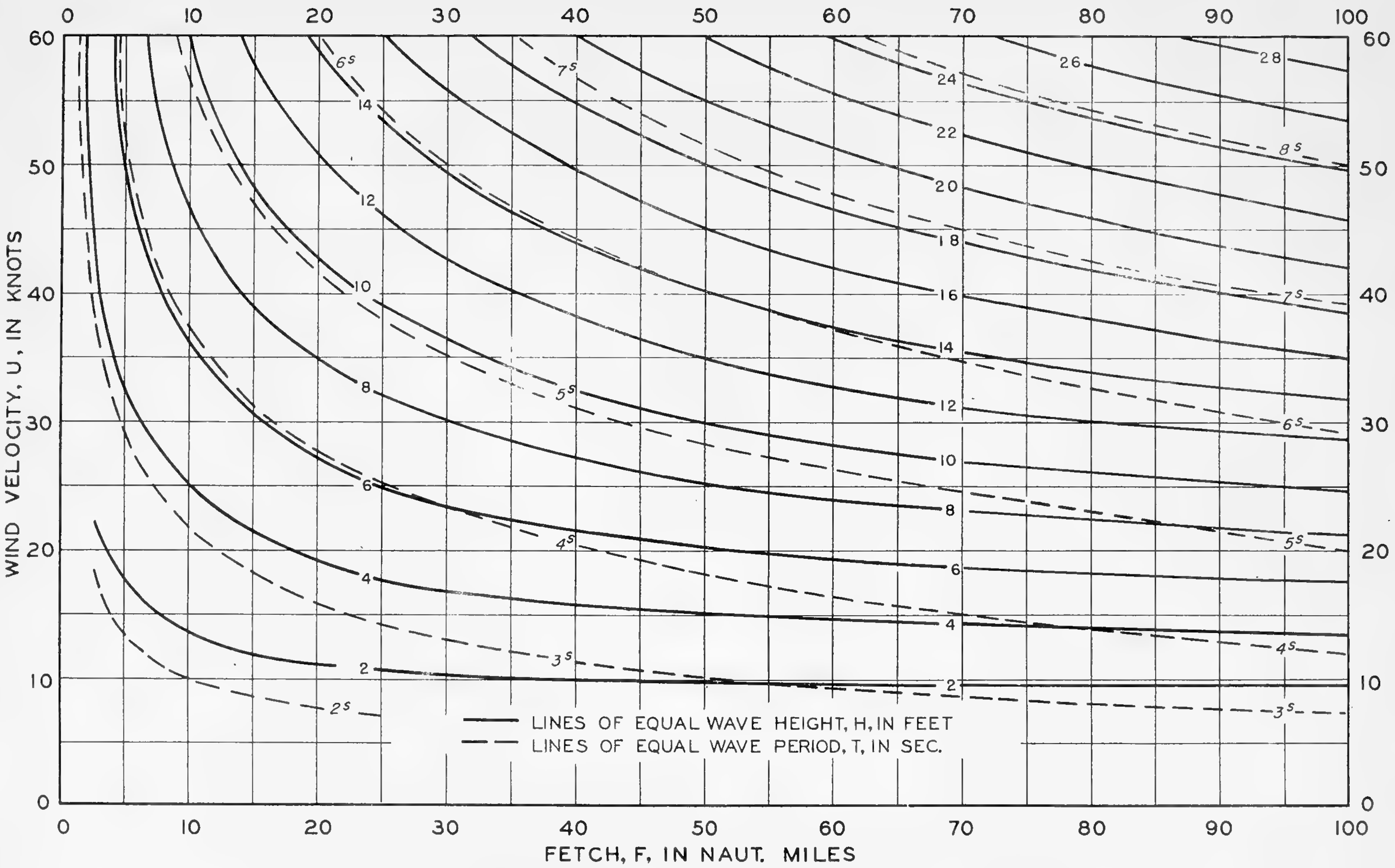


Plate III. Growth of wind waves. Wave height and wave period as functions of wind velocity and short fetch.

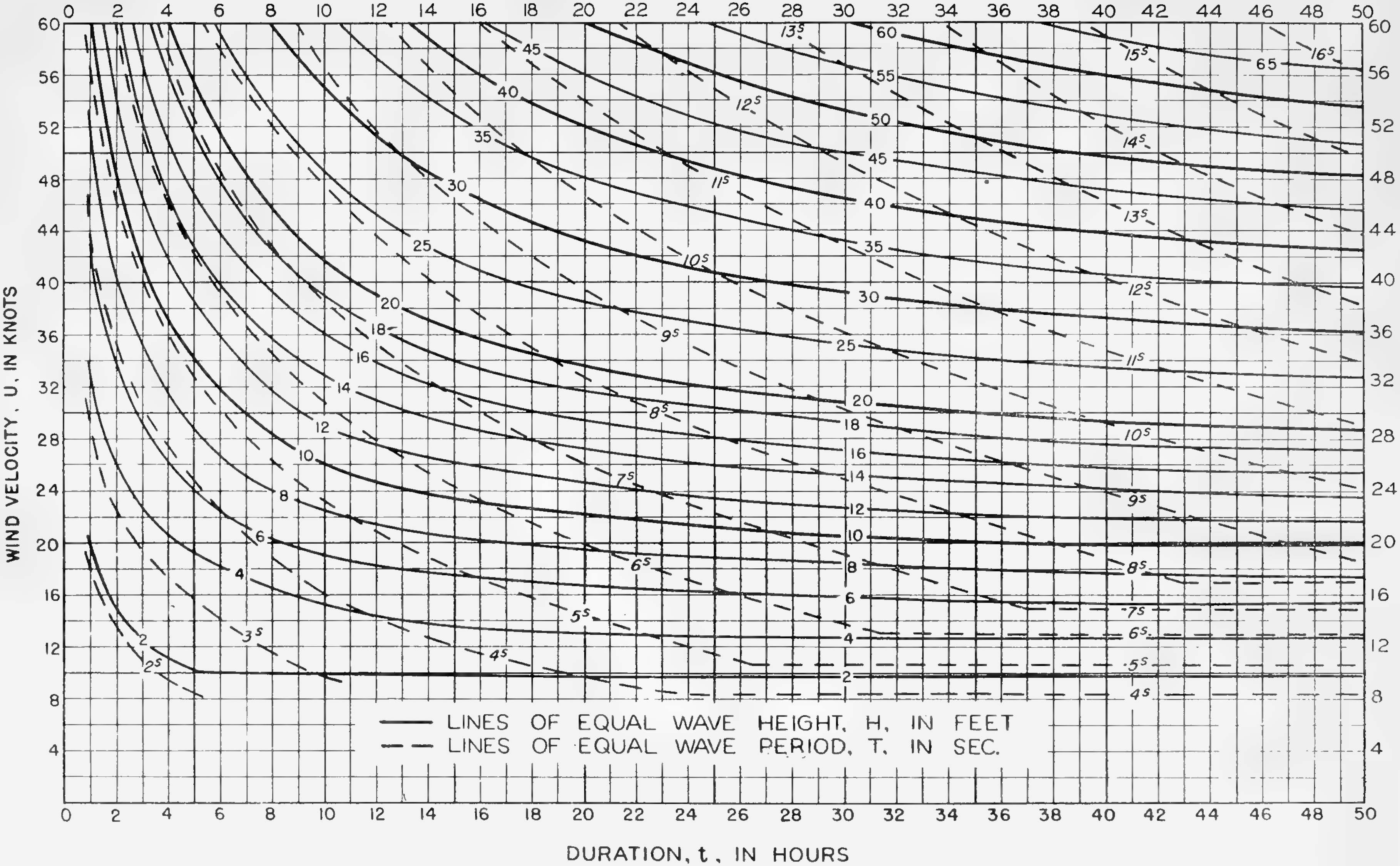


Plate IV. Growth of wind waves. Wave height and wave period as functions of wind velocity and duration of wind.

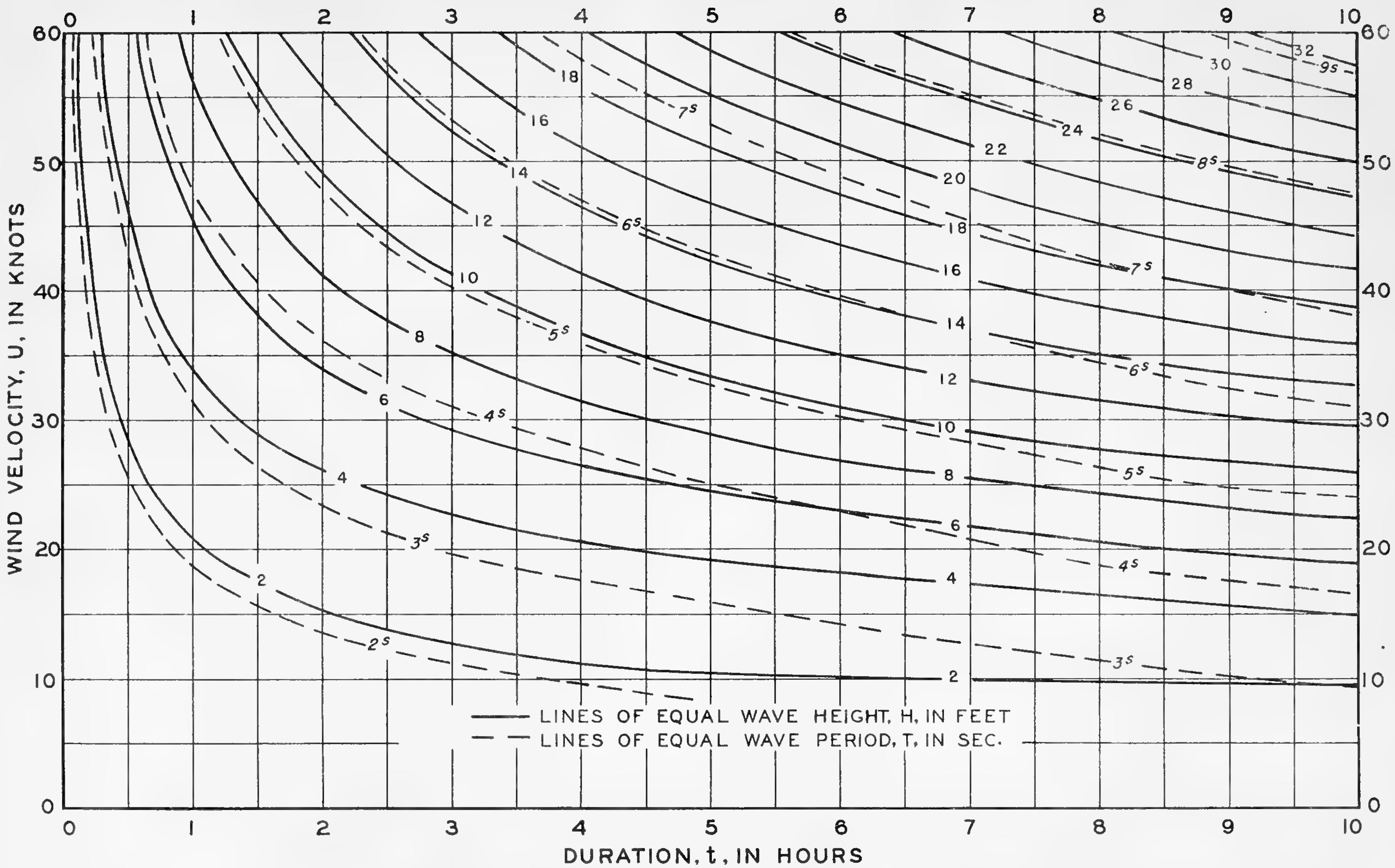


Plate V. Growth of wind waves. Wave height and wave period as functions of wind velocity and short duration of wind.

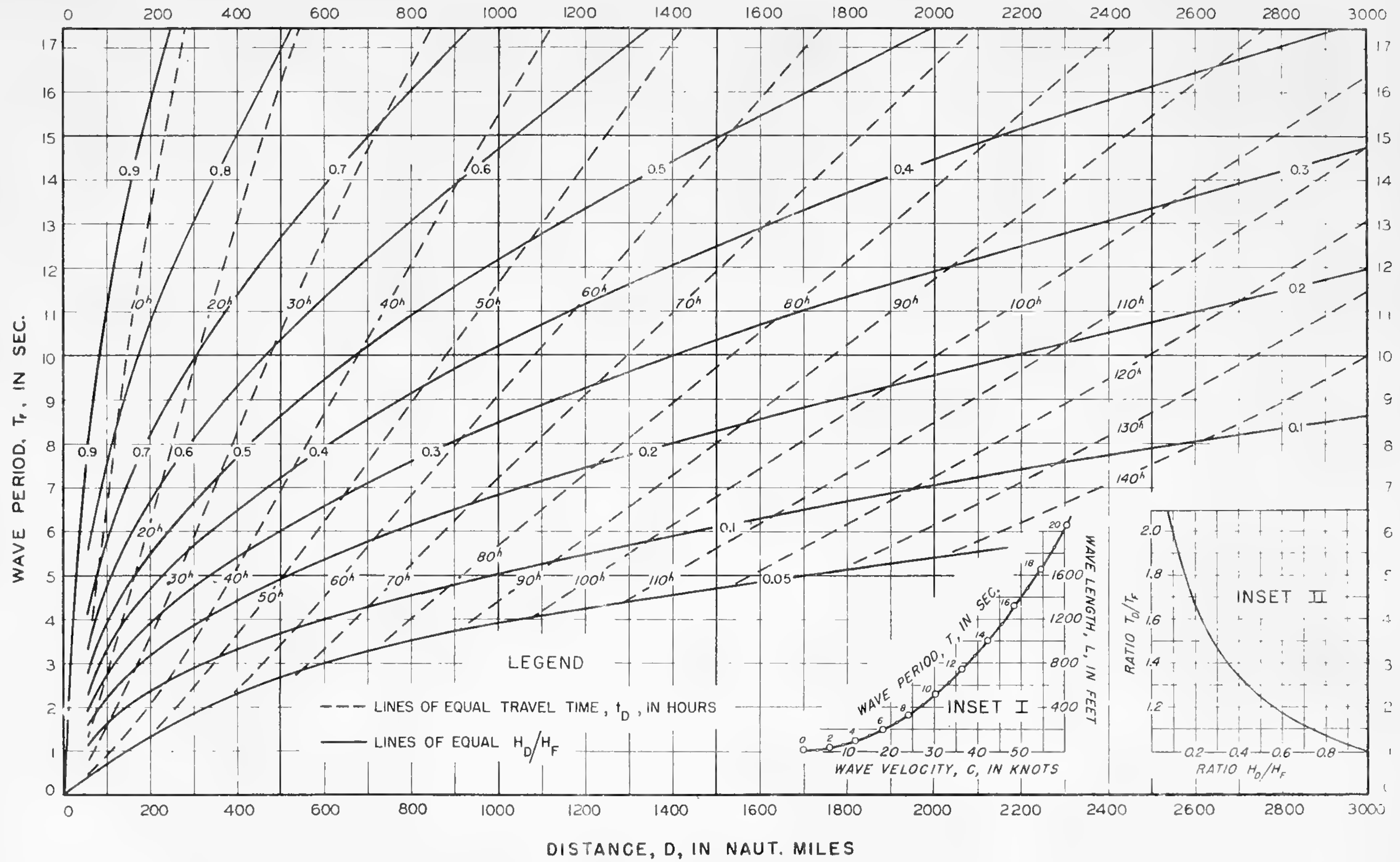


Plate VI. Decay of waves. Ratio between wave height at end of distance of decay, H_D, and at end of fetch, H_F, as functions of wave period at end of fetch and distance of decay.

Inset I. Wave velocity and length for different periods.

Inset II. Ratio between period at end of distance of decay, T_D, and at end of fetch, T_F, as function of ratio H_D/H_F.

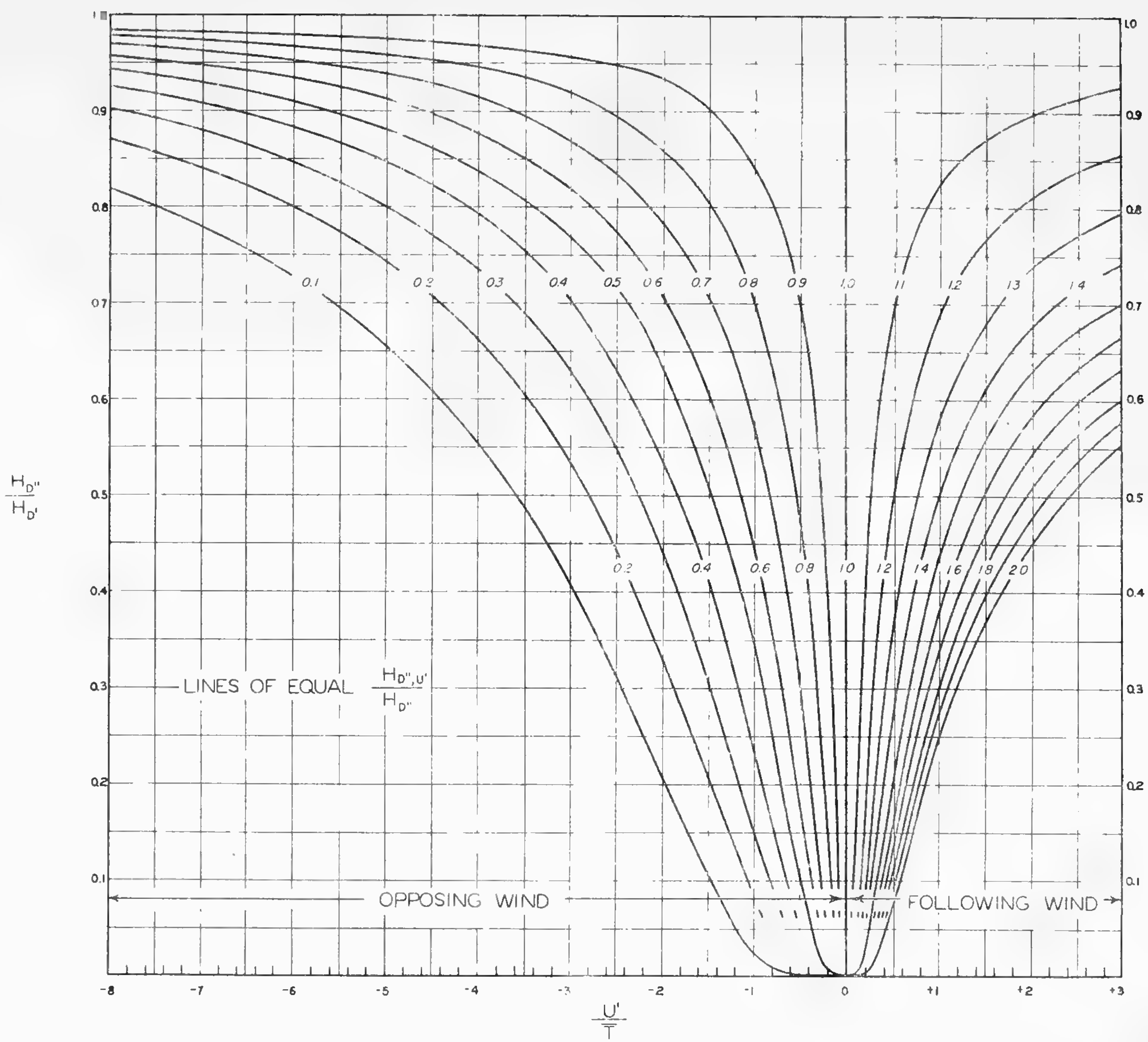


Plate VII. Change in height of swell due to following or opposing winds.

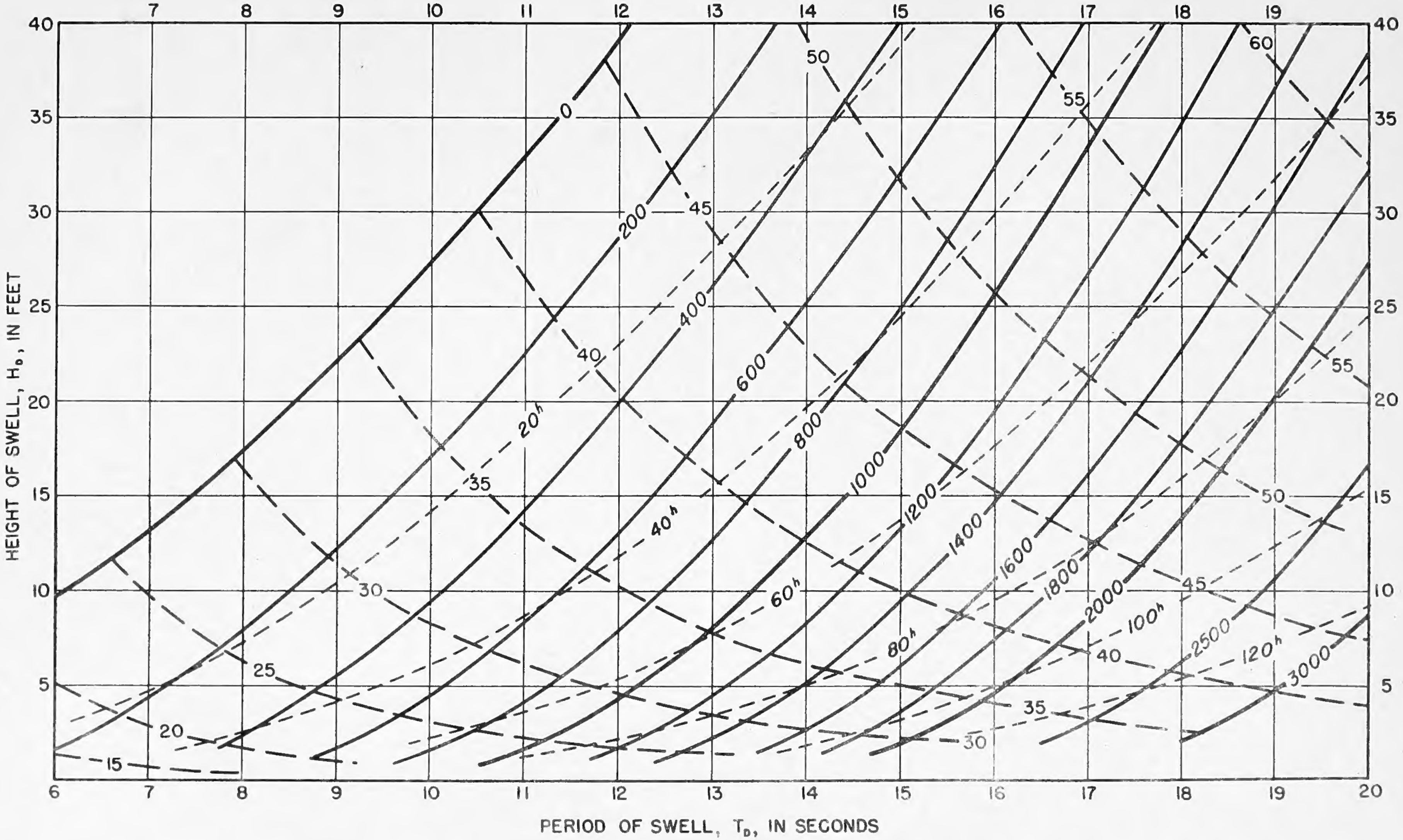


Plate VIII. Distance from which swell comes (in naut. m.), travel time (in hours) and wind velocity in generating area (in knots) as functions of observed height and period of swell.