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This report describes the CEL Mooring Dynamics Seminar, which was held on 10-11 January 1980. Nine experts, selected to represent the major disciplines relevant to mooring analysis, were invited to attend and informally discuss the field of mooring simulation. These discussions resulted in identification of the present state-of-the-art and promising research topics in mooring simulation. Suggestions were also made towards advancing the state-of-the-art in nonlinear systems identification techniques. This report summarizes

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INTRODUCTION

The Civil Engineering Laboratory (CEL) started its research in mooring simulation in FY 78 under the sponsorship of the Naval Facilities Engineering Command (NAVFAC). The goal of this development effort, called the Mooring Systems Prediction Project, is to develop and demonstrate a validated mooring analysis capability; the effort is being supported under the Ocean Facilities Engineering Exploratory Development Program (YF59.556).

NAVFAC provided CEL with two mooring analysis computer models for continued development. The first model, DESMOOR (for DESign MOORings), is an inexpensive, simplified model which gives approximate answers. The second model, DSSM (Deep Sea Ship Moor), is an advanced finite element mooring model for use in the final design stage. Neither model was verified by experimental or field measurements.

After the first year's effort into the problem of mooring simulations, it was realized that the behavior of moored ships involved complex mechanisms that were deeply interrelated. It was clear that an understanding of each of the fields related to the mooring phenomenon was necessary before rational decisions could be made regarding the development and use of a general mooring analysis capability.

SEMINAR

Background

CEL sponsored a workshop seminar at the beginning of 1980 for the purpose of reviewing the NAVFAC/CEL mooring analysis development effort. Specifically, the objectives of the seminar were as follows:

1. To define the present state of mooring analysis and simulation. Included here would be an evaluation of the framework of NAVFAC's mooring analysis capability, namely, DESMOOR and DSSM.
2. To identify the problem areas and uncertainties in the present state-of-the-art.
3. To develop specific guidelines for the further development of the Navy's mooring analysis capability.
4. To identify promising research topics for advancing the state-of-the-art of mooring analysis.

Participants

CEL invited nine prominent experts to attend. These participants were not necessarily experienced in the mooring dynamics area but were recognized for their expertise in subjects integral to the mooring phenomenon. A list of the attendees and their affiliations follows (in alphabetical order):

Dr. J. S. Bendat	--	J. S. Bendat Company
Dr. R. Bhattacharyya	--	U.S. Naval Academy
Dr. S. Calisal	--	U. S. Naval Academy
Dr. C. J. Garrison	--	C. J. Garrison and Associates
Dr. W. McCreight	--	David Taylor Naval Ship Research and Development Center
Dr. B. J. Muga	--	Duke University
Dr. M. K. Ochi	--	University of Florida
Dr. J. R. Paulling	--	University of California, Berkeley
Dr. R. L. Webster	--	Thiokol Corporation

The participants displayed a spirit of cooperation that led to the success of the meeting.

Format

The two-day seminar was organized into three sessions. Session I covered the first day and consisted of short presentations by each attendee summarizing the problems associated with his particular area of expertise. Guidelines as to subject matter for these presentations were outlined in advance by CEL to insure complete coverage of the disciplines associated with mooring analyses. A short discussion/question period followed each presentation.

Session II was held on the second morning and was the most important part of the seminar. Session II was an informal discussion which allowed the participants to build on the presentations of the first day. The intent of Session I was to brief each participant on the problems and limitations within each discipline (vessel motion, cable dynamics, etc.), while Session II was an opportunity for a free exchange of questions and ideas aimed at evaluating and extending the state-of-the-art in mooring analysis.

Session III was held at the end of the second day and started with a description of the Navy's mooring needs as seen by CEL. The intent of Session III was to get specific recommendations from the attendees regarding the development of a mooring analysis capability. This discussion was purposely placed last to avoid any bias in the Session II discussions on mooring models in general.

Session I

The seminar attendees were carefully chosen to represent all the "building blocks" necessary to evaluate and assemble mooring models. The presentations on the first day were divided into two groups: mooring system behavior and mooring system excitation and analysis. The following presentations were made during Session I:

Mooring System Behavior

- Mooring Dynamics Models . . . . Dr. B. J. Muga
- Mooring Cable Dynamics . . . . Dr. R. L. Webster
- Diffraction Theory, Including Mean Drift Forces . . . . Dr. C. J. Garrison
- Vessel Equations of Motion . . . . Dr. R. Bhattacharyya

Mooring System Excitation and Analysis

- Second-Order Drift Forces . . . . Dr. W. McCreight
- Random Wave Characteristics . . . . Dr. M. K. Ochi
- Spectral Analysis . . . . Dr. J. S. Bendat
- Mooring System Analysis . . . . Dr. J. R. Paulling
- Mooring Dynamics Model Development Example . . . . Dr. S. Calisal

Each attendee delivered a short paper concerning his particular subject; these are included in a separate section in this report. Each of these papers gives a concise overview of the fields important to mooring simulation by emphasizing assumptions, limitations, and applications. These papers are subjective in nature, and as such they are easily read and understood.

Session II

The discussions on the second day were intended to give CEL better insight into existing state-of-the-art mooring models and to allow the participants to delve into new ideas and approaches to mooring analysis problems. Both of these goals were achieved.

Much of the discussion centered on the calculation and importance of the slowly varying drift force. Although this force is small, it can become very important because it exists at frequencies close to the natural frequency of many moored systems. When this dynamic force is negligible, a linearized frequency-domain dynamic analysis can be used at a very small computation cost. However, if the force has a significant effect on the mooring, a nonlinear time-domain model is required. This model requires a great deal of computer time because the statistics of the system behavior must be calculated indirectly from several brute-force simulations. Thus, the magnitude of the slowly varying drift force determines whether an inexpensive frequency-domain or an expensive time-domain dynamic computer model is required. Other topics of discussion are included in the CONCLUSIONS FROM SEMINAR DISCUSSIONS section.

The discussions throughout the second day did result in progress toward extending the state-of-the-art in mooring analysis. As Dr. Muga points out in his paper, a nonlinear stochastic model would be the ideal analysis tool for moorings. This imaginary model would give statistical information directly and would eliminate the expensive intermediate results necessary with current time-domain nonlinear dynamic models. Dr. Bendat stated that he felt the time was right to extend linear time series analysis techniques and modify existing nonlinear techniques to obtain the necessary mathematics to describe nonlinear system behavior.

Session III

The final session began with an explanation by Dr. Webster on the DSSM computer model and a short summary by CEL on Navy mooring applications. This was followed by discussions on the strong and weak points of the DSSM model and suggestions on how to improve it.

A collection of conclusions from each participant, which addresses both general and specific conclusions from the seminar, is included in this report. Many of the items listed below are developed in these summary reports.

CONCLUSIONS FROM SEMINAR DISCUSSIONS

Th CEL Mooring Dynamics Seminar fully satisfied its objectives (i.e., to define the present state-of-the-art in mooring analysis and simulation, to identify the problem areas and uncertainties associated with available mooring models, and to recommend guidelines for the development of mooring models to suit Navy needs). The Seminar discussions also initiated a development effort that may lead to significant advances in the analyses of nonlinear dynamic systems. Some of the major contributions within each objective are outlined below.

Present Mooring Analysis Capabilities

As illustrated in Figure 1, there are several models available for mooring analysis. Each analysis technique is useful because of trade-offs in accuracy versus computational costs, which allow the mooring analyzer to choose the model that best suits his particular needs. For example, the fully nonlinear time-domain model, although the most accurate, is certainly not necessary for all applications. Alternatively, applying a large factor of safety and omitting the dynamic analysis, although it is very inexpensive, is likewise not appropriate in all cases.

The majority of available mooring analysis models known to CEL assume a ship-dominated system, with the mooring lines treated as massless springs. System response is determined in either the time or frequency domain. Mooring line tensions are determined in a subsequent quasi-static analysis with the ship displacement imposed on the cable. The most accurate mooring models have no major restrictions or assumptions, and are based on a time-domain representation of vessel and cable response.

It was generally agreed that DSSM is a very cost-effective mooring model. As demonstrated in Figure 1, DSSM uses a fully nonlinear static analysis model (finite element) and a fully coupled, but linearized, frequency-domain dynamic model. The only major improvement possible would be to add a nonlinear time-domain model, that would add at least an order of magnitude to the computation costs. Since the degree of nonlinearity (i.e., effect of the slowly varying drift force) for moorings involving Navy (intermediate-sized) ships is unknown, the need for a nonlinear dynamic solution is unknown. It was recognized that the accuracy of the linearized dynamic solution in DSSM might be adequate for Navy applications, and that major model improvements might not be necessary. Further details regarding the evaluation of DSSM are included after the next section.

Problem Areas and Uncertainties

Identification of the problems associated with state-of-the-art mooring simulation will be discussed without reference to any particular applications or computational cost limitations. Evaluation of these items is left to the reader. Some of the most significant problems are discussed below:

1. The most accurate and complete time-domain models have only one restriction — that the buoyancy of the vessel be linear with immersion. This does not result in any significant errors for moderate vessel motions. However, this restriction does introduce errors for severe vessel motion when bow/stern submergence occurs. This restriction is a consequence of the mathematics required to transform frequency-domain vessel motions to the time domain. Since no available vessel motion models can handle extreme vessel motions, this restriction is unimportant. However, recent efforts in the OTEC project towards the development of an extreme vessel motion model may spur research aimed at the development of a corresponding mooring model.

2. At the present time, there is no accepted technique for predicting and simulating the slowly varying drift forces on a floating vessel. These forces can be significant in comparison to the other environmental loads. Approximate techniques of unknown accuracy are available for estimating this load.

3. Another limitation in reducing the errors associated with mooring simulation comes from the uncertainty in defining the environmental loads, particularly the wind and wind-driven surface waves. Errors associated with the use of a wind wave model (Bretschneider, Pierson-Moskowitz, etc.) have been shown to approach 100% for spectral components as compared to actual measurements. These errors can seriously affect the simulation due to the frequency sensitivities of the vessel response and the use of the wave spectrum in determining the mean and slowly varying drift forces.

The development of spectral families for wind wave models by Dr. Ochi is an important development reported at the seminar. By identifying the error bounds (admittedly a statistically averaged value) in these wind wave spectral models, much of the uncertainty in the final results can be reduced. For many mooring models, the error introduced by using a single spectral model was significant compared to the error due to approximations in the mooring model itself.

4. It was also pointed out that developing and using a very accurate mooring model may not be cost effective if the criteria by which the results are evaluated are not well-defined. This is illustrated in Figure 2, which shows the uncertainty (also probability) in the simulated results, p(s), and the uncertainty in the criteria, p(c); the bandwidth of either curve is analogous to the standard deviation of the error. The area of overlap gives an indication of the probability of system failure. For example, in long-term applications, p(c) for failure loads may be large due to uncertainty in the corrosion, wear, etc. of system components. This has important implications because the mooring designer could simulate such a system with an inexpensive, simplified model and save computation costs from a more refined model. A more detailed illustration of model errors versus evaluation criteria is shown in Figure 3, using CEL's mooring models as an example. Definition of p(c) is dependent on each application, so generalizations would be difficult. Recognizing that the evaluation criteria play a role in the choice of analysis models is the first step.

Guidelines for Navy Mooring Research

The conclusions listed in the previous section are universal and are somewhat independent of specific needs. The conclusions in this section are applicable to the development of a Navy mooring analysis capability. An overview of this development within the CEL Mooring Systems Prediction Project can be found in CEL Technical Memorandum M-44-80-9.*

Some of the specific recommendations made during and after the seminar are listed below:

- A few minor additions could be made to DSSM to improve its generality. Examples are:

(1) Build in additional wind wave spectral models and allow for shoaling.

(2) Allow for wave orbital velocities in the dynamic analysis. In shallow water, these velocities would approach the magnitude of the dynamic motions and should be accounted for.

(3) Allow for unsteadiness in the wind loading by introducing a wind spectrum.

(4) Allow for cylindrical surface buoys to complement the spherical buoy dynamic characteristics already in DSSM. Results from an extensive investigation into the dynamic characteristics of floating cylinders will be reported soon from the U.S. Naval Academy.

- The relative effect of many of the idealizations used in the DSSM model can be determined through parametric studies. Examples of this include:

(5) Errors caused by the use of spheres to represent all surface buoys. The dynamics of buoys were considered of secondary importance compared to the ship when this section of the model was formulated.

(6) Errors associated with the linearization of the mooring cable response for the frequency-domain analysis. The moored ship response is first calculated in the frequency domain. A second time-domain analysis with fully nonlinear cable response is then performed separately, using the linear ship response as excitation to the top of the cables. Relative comparisons of the cable responses would help determine the significance of this linearization.

(7) Importance of the inclusion or exclusion of the dynamics of surface buoys in the dynamic analysis. The wave-induced motions of the buoys certainly contribute to the loads in

*Civil Engineering Laboratory. Technical Memorandum M-44-80-9: A review of the CEL mooring systems prediction product area, FY 79 and FY 80, by P. A. Palo. Port Hueneme, Calif., Sep 1980.

the hawsers and mooring lines, but the relative size of this contribution relative to the ship-induced dynamic loads has never been determined. This is important, since surface buoys add additional degrees-of-freedom to the solution and increase computation costs.

(8) Errors associated with the use of current and wind loads versus relative heading in the static analysis. Determining the sensitivity of typical mooring systems to changes in the static load coefficients would be extremely valuable, since the available data exhibit a large scatter.

(9) Determining the sensitivity of the model to errors in any input variable would be valuable and practical, particularly for actual studies where many values can only be estimated.

State-of-the-Art Advances

Discussions which evolved from the use of bispectra for ship resistance measurements indicated that an advance in the state-of-the-art may be possible in the analysis of nonlinear dynamic systems. Development of nonlinear systems identification techniques, as discussed under Session II, would be a major breakthrough not only for mooring analysis, but for all nonlinear dynamic systems. Efforts in this area have been initiated.

SUMMARY

The two-day Mooring Dynamics Seminar satisfied all of its objectives. Recommendations for development of a mooring analysis capability were made, and a potential contribution towards advancing the state-of-the-art in nonlinear dynamic analysis was initiated. The presentations and summary reports included in this report form a unique primer on the mooring analysis problem and state-of-the-art analysis techniques.

ACKNOWLEDGMENT

CEL gratefully acknowledges the cooperation and enthusiasm shown by the participants, and hopes that they, too, benefited from the discussions.

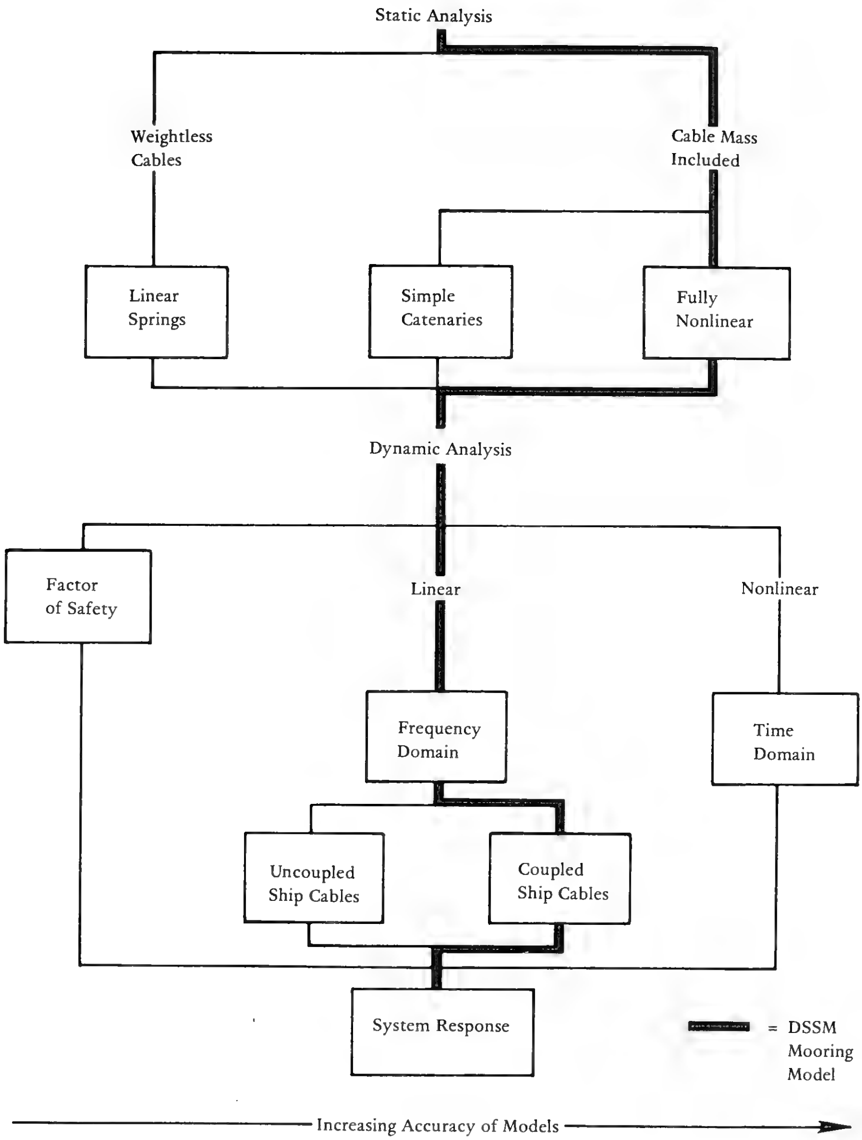

Figure 1. Hierarchy of deterministic mooring models.

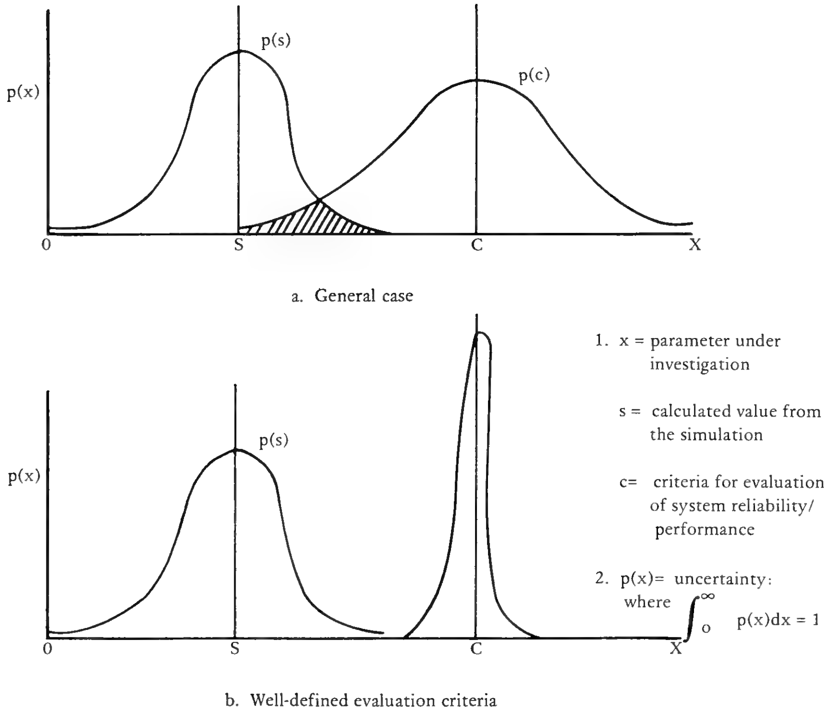

Figure 2. Role of uncertainty in system evaluation.

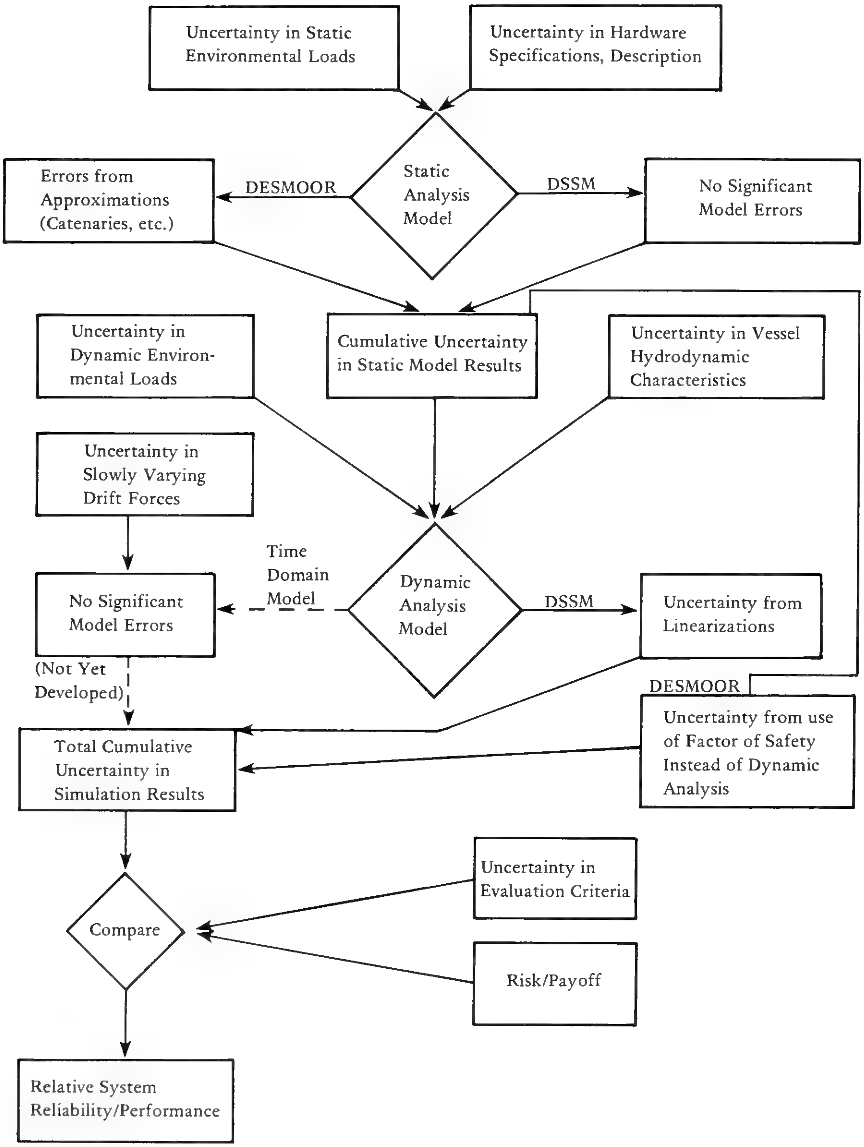

Figure 3. Mooring system evaluation using CEL computer models.

SEMINAR PRESENTATIONS

1980 CEL Mooring
Dynamics Seminar

MOORING DYNAMIC MODELS

By

Dr. B. J. Muga

INTRODUCTION

It is particularly appropriate that this seminar is being sponsored by the Civil Engineering Laboratory. As far as is known, this Laboratory was the first (at least in modern times) to begin the systematic study of moored ship behavior. The earliest group of studies consisted of (1) an aircraft carrier moored alongside a conventional pier at Bremmerton, Washington, (2) another aircraft carrier anchored off San Nicholas Island in what has come to be known as a single point mooring, and (3) an LST moored off a drilling platform in the Gulf of Mexico in what is known as a multipoint mooring or spread mooring or what in the industry is referred to as a sea berth mooring. In addition, a study of the motions of the CUSS I vessel as a part of Phase I of Project Mohole was carried out here at this Laboratory.

All of these studies were, for the most part, data collection efforts having rather specific objectives which were satisfied by what would be regarded as crude data analysis. In brief, they were field studies of prototypes. It should be remembered that when these studies were initiated over 20 years ago, analyses techniques were not as highly developed and even those there were were not well-known. Computer capability, in terms of what we have available today, was in a stage of infancy. Therefore, it is no wonder that the course of field studies was taken, even though some model studies were also carried out by outside contract. (As a matter of fact, even though field studies are very, very expensive, still they were not as expensive, relatively speaking, as they are today.)

The important role of these early studies — aside from satisfying the immediate objectives — was in drawing attention to the various deficiencies in understanding moored ship behavior.

An effort was made to remove some of these deficiencies by sponsoring one analysis study which is often cited in the literature and is well-known and somewhat of a milestone. This study entitled, <u>The Motions of a Moored Construction Type Barge and Their Influence on Construction Operations</u>, by Kaplan and Putz, was completed in 1962 but interest in the problem seems to have waned shortly thereafter. So, now it is refreshing to note that the Navy has recognized the need for renewing interest in this problem.

It has been observed that the dynamics of moored ship systems is a more general problem of the response of a vessel in a seaway or what has been referred to in the literature as the ship motion problem. This problem has occupied the attention of some of the best researchers and high-level mathematicians for the better part of this century. Therefore, nearly all of the analysis techniques that have been developed in connection with ship motion problems can be applied to the moored vessel problem. However, we have three differences. First, we are usually interested only in the zero forward speed case. Second, we have the added complication of the mooring system, and, third, we are usually interested in barges or other vessels which may not have a traditional ship plan form. This last difference is more than just a superficial interest since many of the simplifications which are employed for ship motion studies are no longer appropriate for a moored vessel.

MODEL CHARACTERISTICS

One approach toward understanding mooring dynamic models is to list all of the various models citing all the references, the underlying assumptions, the various limitations, and the validation, if any. Unfortunately, such an approach is time consuming, laborious and not particularly rewarding.

Another approach, as followed herein, has for its basis the categorization of the various model characteristics. This approach is systematic and therefore, more instructive and illuminating. However, it suffers from the disadvantage of requiring some knowledge of the general nature of the problem and of the pertinent ship-motion literature. But, this is a meaningful requirement for a seminar such as this. Therefore, it is appropriate to categorize the various model characteristics in terms of a hierarchy, as shown in Figure 1, and to discuss the representative mathematical models in terms of these characteristics.

Deterministic versus Stochastic Models

It seems natural to consider that the most important division of mooring dynamic model characteristics is the division between deterministic and stochastic models, as shown in Figure 1. A few comments are in order. We know that if the input is deterministic, then the output is deterministic. We also know that if the input is stochastic, then the output should be stochastic. But, the problem that emerges is - How is it (i.e., the output) computed? Is it computed directly or indirectly? How many simulation runs are needed? How long should each run be? What will it cost? and so on.

An example of a direct computation stochastic model is one for which the non-linear terms appearing in the governing system of equations have undergone "equivalent statistical linearization." That is to say, that the process of linearizing the non-linear terms is one in which the errors (associated with the linearized terms) are minimized.

This is in contrast to equivalent linearization in the deterministic sense wherein the energy (or some other variable) is averaged over one "cycle".

Of course, another example of a direct stochastic model is one wherein the system is linear. In this case, the output statistics are obtained directly. Since there are all kinds of

Deterministic versus Stochastic

Linear versus Non-Linear

Time versus Frequency Domain

Quasi-Static versus Dynamic
(for Cable Stresses)

Analytical versus Experimental

Approximate versus Exact

Figure 1 - Hierarchy Levels of Model Characteristics.

non-linearities, we have to be careful to explain precisely what is meant by this distinction (linear versus non-linear) which is illustrated by the second level in the hierarchy of model characteristics (Figure 1).

In summary, if the system is strictly linear, the output statistics can be obtained directly. If the system in non-linear, an exact analytical derivation (of the output statistics) is generally impossible, except in very specialized cases (Caughey, 1963). Thus, two alternatives are possible: an approximate analytical solution, or an experimental determination of the response statistics by means of analog or digital simulations.

Approximate Analytical Techniques. If the system inputs are small, the usual approach, as cited above, is to linearize the non-linear equations and derive the output statistics from the resultant approximate linear model. This has been applied to problems of vibrating strings (Caughey, 1963) and ship rolling (Haddara, 1973) for the purpose of deriving response moments of first and second order. If the system non-linearities are small, then first and second order response moments for large inputs may often be obtained by using a classical perturbation approach (Crandall, 1963). For the moored ship, the first and second moments are important but they are not the only important statistics — estimates of, for example, peak distributions and the probabilities of exceeding maximum allowable values of stresses or displacements, especially under heavy sea conditions, are also needed.

Simulation Techniques. In order to obtain satisfactory estimates of system responses by simulation, long duration runs are required. This means that simulation algorithms used should be efficient as well as accurate, with the sampling rate close to the Nyquist rate for the bandlimited signals. It also means that algorithms for generating synthetic test-input sequences on the computer must be capable of being adjusted to give spectral characteristics and amplitude distributions similar to those obtained by the sampling of actual (i.e., measured) forcing function records. Thus, these simulation studies require:

(1) Algorithms employing random number generators and digital filtering techniques for the generation of synthetic correlated input sequences having characteristics which can be adjusted to closely match those of sequences which can be expected from the sampling of actual wave data, and

(2) Efficient and "statistically accurate" algorithms for the numerical solution of the non-linear state equations of the system for random inputs as described.

In regard to the second requirement, there are numerous approaches to the problem of numerically integrating both linear and non-linear differential state equations, and several comparisons of various methods have been made (e.g., Martens, 1969) for homogeneous equations or equations with simple specified (deterministic) forcing functions. A comparison of simulation algorithms (based on mean-square error criterion) for systems with random inputs has recently been completed (Kim, 1978). In this study, the system models included simple first and second order linear systems, and two highly simplified non-linear models for an ocean platform and a moored ship, respectively. The input signals used consisted of synthetically generated independent Gaussian sequences and sequences derived by sampling actual ocean wave-force records obtained during hurricane conditions. The simulation algorithms were derived using state-transition, z-transform and Runge-Kutta methods. Through the use of Shannon sampling expansion for bandlimited functions, a new type of state-transition algorithm and a modification of the classical fourth-order Runge-Kutta method were derived and shown to result in increased accuracy over the methods currently in common use. This is particularly noticeable for low sampling rates.

Linear versus Non-linear

In order to clarify this characteristic in the model hierarchy, it is illuminating to introduce a mooring dynamic model.

Consider, for example, a general deterministic 'linear' model as developed and contributed to by Cummins (1962), Ogilvie (1964) and others which has been found useful in analysing moored-ship behavior. The system of equations appears as

$$\sum_{j=1}^{6} \left\{ (M_{kj} + m_{kj}) \ddot{x}_j + \int_{-\infty}^{t} K_{kj}(t-\tau)\dot{x}_j(\tau)d\tau + C_{kj}\, x_j \right\} = X_k(t), \quad k = 1, 2, 3 \ldots 6 \tag{1}$$

M_{kj}	is the inertia matrix
C_{kj}	is the matrix of hydrostatic restoring force coefficients
K_{kj}	is an impulse response or retardation function
m_{kj}	is a constant inertia (frequency independent) coefficient matrix
$X_k(t)$	is a time-varying exciting force due to winds wave, currents and restraining (mooring) forces.

This system of equations is linear in the sense that the integral involves a superposition or summation. Also, although the above formulation is written in the time domain, there is an equivalent frequency domain description. This can be shown by letting the moored vessel perform simple harmonic motion in response to harmonic excitation.

By using the following relations, as provided by Ogilvie (1964), the frequency domain relations are obtained.

$$K_{kj}(t) = \frac{2}{\pi} \int_{0}^{\infty} b_{kj}(\omega) \cos \omega t d\omega,$$

$$m_{kj} = a_{kj}(\omega') + \frac{1}{\omega'} \int_{0}^{\infty} K_{kj}(\omega) \sin \omega' t dt,$$

where

a_{kj} = added mass coefficient (frequency dependent)

ω' = one particular but artibrary value of ω

b_{kj} = damping coefficient (frequency dependent).

Thus, the third category in model hierarchy (Figure 1) which distinguishes between frequency domain and time domain model descriptions is shown to be a case in which one is a subset of the other.

At this point, we no longer talk about the system of equations in a general way but instead we focus our attention on various terms of the equations and examine each in the light of the linear-non-linear description. We have already examined the impulse response term.

Consider the various restoration sub-models. The restoration forces consist essentially of two components. There are those that are due to bouyancy effects in roll, pitch and heave and those that are due to the mooring lines. For small motion amplitudes, these restoration forces are often approximated by linear functions. But the roll moment is well-known to be non-linear with respect to roll displacement, and the components due to the mooring lines are often non-linear with respect to geometry and/or material properties or both. For example, the mooring restoration forces in the case of a ship at a conventional pier where a combination of mooring lines and fenders result is a highly non-linear restoration function.

Thus, the restoration forces can be linear or non-linear, but, the important point is that given the instantaneous displacements, these forces can be determined. Also, for ship-dominated systems, a quasi-static treatment has been customary on the assumption that the system inertia is very large relative to the restoration forces. For cable-dominated systems, this may be no longer true and in these cases, it becomes necessary to consider the cable dynamics. It develops that this division constitutes one of model hierarchy characteristics (e.g., static or dynamic with respect to cable forces).

The time-varying excitation term can be linear or non-linear in the sense that they are non-linear with respect to amplitude or surface elevation. The non-linearities are attributable to two different sources but for the same reason. For example, they can result from monochromatic higher-order waves, or from frequency interactions between components of wave spectra. Solution of the free surface boundary value problem carried to second order in either case will disclose these non-linear occurrences.

In summary, the mooring dynamic model described by Equation (1), although 'linear' in one sense (i.e., the integral term), is suitable for handling non-linearities which occur in the other terms. It should be emphasized that this is a deterministic model and therefore will not yield the response statistics directly. Also, although the model is consistent for first order solutions, there is somewhat of an inconsistency in utilizing the model for higher order solutions. This will be illustrated in the next section.

As a final remark, all other time domain deterministic models can be shown to be subsets and/or special cases of the model described above. This includes any model having less than six (6) degrees of freedom, any model in which the modes are uncoupled, any model in which the mooring forces are linearized or any model in which the excitations are approximated by any simplifying assumption (e.g., slender body assumption leading to strip theory).

STATE-OF-THE-ART PROCEDURE

Application of the foregoing mooring dynamic model (as formulated in the time domain) to the analysis of inertia-dominated systems as exemplified by a tanker is approached by the following stepwise procedure:

1. For each frequency corresponding to the partitioned components of the input wave spectrum, the following quantities are computed:
   a. First order forces corresponding to the sum of the incident and scattered wave potentials (that is, the first order forces corresponding to the diffraction potential). These forces are those that would exist if the vessel were fully restrained and the theory employed is that outlined by Wehausen and Laitone (1960). There are no simplifying assumptions in this procedure except that for an ideal irrotational fluid.
   b. First order unit amplitude radiation potentials and corresponding unit amplitude transfer functions. The real and imaginary components of the latter are directly related to the added mass and damping coefficients which are frequency dependent. Implicit in this process is the assumption that the vessel responds at the same frequency as the input exciting wave frequency.
   c. Using a frequency domain description of the dynamical equations of motion, the first order motions (i.e., amplitudes) are computed.

2. The first order motions obtained in paragraph 1 (c) above are used to scale up the unit amplitude radiation potentials to the radiation potentials. This is done for each frequency as is the next step.
3. These radiation potentials are combined with the diffracted potentials obtained from 1 (a) to yield, after some manipulation, the mean drift forces. The main assumption underlying this procedure is the far-field approximation (Newman, 1967).
4. The mean drift forces corresponding to the various frequency components are then combined using the procedure outlined by Newman (1974) to yield the time-varying drift (or second order) forces. This procedure is a recognition of the weak contribution from large frequency-difference combinations components. In other words, the off-diagonal contributions are neglected.
5. The first order force time histories are generated by superposition (either random phase or artibrarily specified phase) and combined with the second order force time histories to provide a set of synthetically generated excitation time histories corresponding to the input wave spectrum.
6. Using the relations derived by Ogilvie (1964) the constant inertia coefficients and the impulse response functions are obtained from the added mass and damping coefficients.
7. From the above, the time-domain system of equations is solved to yield the resulting motions and line loads in the form of time histories. The pertinent statistics of interest are computed. To provide additional confidence in the statistical results, steps 5 and 7 are repeated for different phase angles between the components of the wave spectrum and different phase relations between the first and second order time histories. Each

of the latter constitutes a numerical simulation. These are the so-called "Monte Carlo" simulations.

The main shortcoming of the above procedure is that it is laborious and does not yield the output statistics directly. Whereas the input is a stochastic variable, the mechanical model is deterministic. What is needed, then, is a mechanical model which is stochastic, or, at the least, capable of handling stochastic output directly. Such a model would be consistent with the nature of the phenomena.

It is believed that the procedure could easily be expanded to include the cable-dynamics effects for restoration-force dominated systems. At the same time, some simplifications might be possible.

CONCLUSIONS

It is believed that the mooring dynamic model described herein could be utilized for the analysis of moored-barge systems of interest to the Navy. This may require some modification and/or linkage with an existing cable-dynamics model. It is also suggested that a stochastic model might be developed which would be more consistent with the nature of the problem and potentially very rewarding.

REFERENCES

1. Caughey, T.K., Derivation and Applications of the Fokker-Planck Equation to Discrete Non-Linear Dynamic Systems Subjected to White Random Excitation, Journal Acoustical Society of America, Vol. 35, No. 11, Nov. 1963.

2. Crandall, S.H., Perturbation Techniques for Random Vibration of Nonlinear Systems, Journal Acoustical Society of America, Vol. 35, No. 11, Nov. 1963.

3. Cummins, W.E., "The Impulse Response Function and Ship Motions,"DTMB Report 1661, Washington, D.C., 1962.

4. Haddara, M.R., "On Nonlinear Rolling of Ships in Random Seas," International Shipbuilding Progress, Vol. 20, Oct. 1973.

5. Kaplan, P., and R.R. Putz, "The Motions of a Moored Construction-Type Barge in Irregular Waves and Their Influence on Construction Operation," N By-32206, Marine Advisers, Inc., La Jolla, 1962.

6. Kim, Y., Digital Simulation of Bandlimited and Narrow-Band Random Systems, (Ph.D. Dissertation), Department of Electrical Engineering, Duke University, Durham, N.C., 1978.

7. Martens, H.R., A Comparative Study of Digital Integration Methods, Simulation, Vol. 12, No. 2, Feb. 1969.

8. Newman, J.N., "The Drift Force and Moment on Ships in Waves," Journal of Ship Research, Vol. 11, No. 1, Dec. 1967.

9. Newman, J.N., "Second-Order, Slowly Varying Forces on Vessels in Irregular Waves," Symposium on Dynamics of Marine Vehicles and Structures in Waves, London, Apr. 1974.

10. Ogilvie, T.F., "Recent Progress Toward the Understanding and Prediction of Ship Motions," 5th Symposium on Naval Hydrodynamics, Bergen, 1964.

11. Wehausen, J.V., and E.V. Laitone, "Handbuch der Physik," Vol. 9, Springer Verlag, Berlin 1960.

MOORING CABLE DYNAMICS

by R. L. Webster
10 January 1980

The dynamic responses of the mooring lines themselves become an important consideration in deep water moors. It is the purpose of this note to outline some of the characteristics of mooring lines, to describe an approach to modeling them, and to focus on some of the problems encountered.

Important Characteristics of Mooring Lines

Special behavior features which may appear in significant combinations in long mooring lines are:

Geometric Nonlinearity - This is a large displacement effect which produces a stiffness dependent on the deflections and preloads in the line. As an example, consider a simple straight line segment with negligible bending stiffness. The stiffness of the segment along its axis is

$$K_{axial} = \frac{AE}{L} + \frac{P}{L} \tag{1}$$

where
- A = cross-section area
- E = Young's modulus
- L = segment length
- P = preload in the segment

Usually P is small compared to AE and is neglected; however, the stiffness transverse to the line is not zero when P is not zero. It is

$$K_{lateral} = \frac{P}{L} \tag{2}$$

When the displacements are not small, the changes in position of the line and the changes in magnitude of P must be accounted for in the equations of motion.

Material Nonlinearity - Most mooring lines exhibit a nonlinear relationship between load and elongation (i.e., Hooke's Law does not hold). Even when the effect is linearized, one must contend with differing behavior in tension and compression. The slenderness of mooring lines usually results in such low compressive load resistance that it is assumed to be zero. In most mooring lines (particularly with synthetic materials), the material damping or hysteresis is also an important effect.

Non-conservative/Nonlinear Loading - The dominant environmental loads on mooring lines are of the distributed pressure type (e.g., drag loading). This is typically dependent on the square of the relative velocity between the line and the fluid. It is also highly sensitive to the orientation of the velocity vector relative to the line. Dynamic effects produce additional non-conservative loads proportional to the relative acceleration (re Morrison's equation). It is common practice to neglect change of acceleration effects and to treat the acceleration terms with an added mass. The added mass is ususally assumed to be zero for acceleration parallel to the line.

Position Dependent Constraints - Limits must be imposed on the mooring components to prevent movement out of the water or below the bottom. The bottom interaction of catenary type lines is an important effect in moors of moderate to shallow depths.

Nonlinear Dynamics - The dynamic response of long lines is very complicated and may produce significantly different effects from those implied by the static responses. There may be significant coupling between mooring, buoy and ship responses. One should expect various forms of harmonic coupling. Thus, excitation at a single frequency may produce responses at other frequencies. Subharmonic responses are common and super-harmonic responses are possible. Damping and large amplitude/low frequency forces (drift forces and swells) are also important factors.

It should be noted that most moored ship analyses assume the lines are non-dynamic components modeled by a simple spring which is usually assumed to be linear. In some cases, a material nonlinearity has been introduced, but most other nonlinearities of the lines are ignored. Typically these analyses are conducted with 6 degrees of freedom (the rigid body components of the ship) and some limit the model to 3 degrees of freedom.

Finite Element Modeling of Mooring Lines

The finite element method offers an approach to representing the dynamic equations of the lines. It is a discrete approach which uses a finite set of nodal degrees of freedom to approximate the line behavior. The finite element method can be viewed as a special form of the Rayleigh-Ritz method where the assumed response functions are defined in finite sub-regions rather than on the entire body. Admissibility is readily assured when the assumed functions are displacement fields. The finite element approach using assumed displacement fields is known as the stiffness method.

Application of the stiffness method to the modeling of mooring lines requires the assumption of the geometric form as well as the displacement field for a typical sub-region or element. The simplest form is a 2-node system with a straight line connecting the nodes. The simplest displacement field is also linear in form. This field assumes that the displacement of any point on the line segment is obtained by simple linear interpolation form the displacements of the nodes. This is known as the 1-D simplex element. It also assumes there is no bending or torsional resistance so that only the spatial positions of the nodes are used. Thus the element has 3 degrees of freedom per node in a 3-D problem. The equations of motion for this element are obtained by an application of kinematic, material constitutive and energy relations. Since the assumed functions are defined only on the single element, the entire structure and its dynamic equations are obtained by summing the individual element contributions under the obvious assumption that the elements are joined at the nodes.

Element forms of higher order that the 1-D simplex can be used. The most readily defined forms are based on polynomials, and the next logical element form is one which uses a parabola for the geometry and the displacement fields. Such an element uses one more node along the line between the two end nodes, and the functional form is obtained using Lagrangian interpolation on the three nodes. The process could be extended to cubics and higher orders, but there appears to be no justification for doing so. Even though the higher order fields are more capable of modeling complicated geometries and responses with fewer elements, the number of nodes required does not change much while the complexity of the equations and the cost of their calculation increase considerably.

The very regular and orderly form of the stiffness method makes it very attractive for coding on a digital computer. The method also makes it very easy to introduce discrete bodies and special constraints. A very attractive feature is that the coding is insensitive to the geometric complexity. To be sure, more nodes and elements means more computation cost, but the method is insensitive to the degree of interconnection, multiplicity of materials and the irregularity of the geometry and boundary conditions.

A very strong feature of the stiffness method is the ability to develop the governing discrete equations directly for a variety of solution forms. Since it works with the governing equations from mechanics, it is as easy to get the incremental equation form as it is to get the total response equations. Frequency domain or time domain forms may be chosen. This is of particular value in large displacement solutions where dynamic effects occur relative to some static preloaded state. This allows the static and dynamic analyses to be done using consistent models. This is of very specific value in mooring analysis.

Two very real problems with the use of the finite element method deserve mention. First is the fact that this approach (like most discrete methods) tends to produce large order simultaneous coupled equations, and solution of these equations can be expensive. Often the novice code user will tend to "shot gun" the problem with many nodes, many design perturbations and many debug runs. Cost may not be a dominant factor if there is no other way and the answer must be had; however, one tends to vault from very crude models to excessively complex models with little thought about what is in between. Once the ability to analyze is given, one may tend to over-analyze or expect far too much from the analysis. The second problem is related to the first. When a very complicated problem is solved on the computer, the input generation is a major task and the output is voluminous. One tends to be overwhelmed by it all and jumps to the very convenient conclusion that since a successful run is finally obtained and it was done using an "all powerful black box" using a computer, then the answers obtained must be correct. These are two aspects of the "black box syndrome" and they tend to reduce the amount of intelligence put into the formulation of an analysis .... Without some careful control, they may lead to a very costly pile of garbage!

Typical Solution Forms

The most common approach to analyzing mooring dynamics begins with a static analysis to establish a stable initial or reference state of the cable system. The dominant nonlinearities are present in the static equations and all effects mentioned earlier must be accounted for except the dynamics. Because of the strong geometric nonlinearity present in most deep water moors, this may be a very difficult step in the analysis. Often the static reference state is not well defined by a simple connection of the unstretched elements. Usually the unpreloaded system represents a mechanism which is unable to support loads in one or more directions and/or it violates boundary conditions and compatibility constraints. Load resistance (stiffness) is only developed as the elements rotate and stretch.

Once the static reference state is obtained, various dynamic solution options are available. Some of them are:

1. Linearize equations and solve for small displacement perturbations about the static reference in the time domain.

$$[\bar{M}]\{\Delta\ddot{q}\} + [\bar{C}]\{\Delta\dot{q}\} + [\bar{K}]\{\Delta q\} = \{\Delta\bar{f}(t)\} \tag{3}$$

2. Linearize equations as above, transform to frequency domain for quasi-linear solution where equations are frequency dependent.

$$\{\Delta\bar{f}\} = \mathrm{Re}\ (\{F\}\ e^{i\omega t})$$

$$\{\Delta\bar{q}\} = \mathrm{Re}\ (\{Q\}\ e^{i\omega t}) \tag{4}$$

$$(-\omega^2 [\bar{M}] + i\omega\ [\bar{C}] + [\bar{K}])\{Q\} = \{F\}$$

3. Direct numerical integration of nonlinear time domain equations.

$$[M]\ \{\ddot{q}\} = \{f\} - \{g\} \tag{5}$$

where $[M]$ is the position dependent virtual mass matrix.

$\{f\}$ represents the time and position dependent external forces (dragloads, point loads, etc.)

$\{g\}$ represents the internal loads from the elements, reflecting the material and geometric nonlinearities and material damping.

The finite element method allows direct calculation of any of these terms for the mooring lines given the material properties (EA, mass, etc.), the nodal positions and the unstretched element lengths. The effects of lumped bodies; such as, buoys, platforms and ships, can be readily included if they are described in functional or tabular form appropriate to the solution form. Special rigid link multi-point constraints are used to tie the bodies into the mooring model.

Some Demonstration Solutions

Single Degree of Freedom System With Geometric Nonlinearity

Figure 1 shows a single degree of freedom system composed of two linear springs attached to a single mass point. It also shows the nonlinearity of its static response. When the mass point is released from a deformed state, it will oscillate about the reference state. Without damping, the oscillation will continue indefinitely with a period which is dependent on the magnitude of the initial displacement. The natural period for this system is 0.2639 seconds for an initial displacement of 20 cm. If the mass point is forced at some other frequency with a force magnitude equal to that required to produce the initial static defection, some interesting things occur. Two examples are shown in Figures 2 and 3. Excitation below the natural frequency induces a response similar to the linear case where the impressed frequency is dominant and the response amplitude approximates the static response. The little ripple in the response is at the natural frequency and would be expected to die out in the presence of appropriate damping. Excitation above the natural frequency introduces a new phenomenon. As before, there are two frequencies present: one at the imposed frequency and the other at a varying frequency. The variable frequency response appears as a damped transient because of the numerical damping that was included in the integrator. The varying frequency is a direct result of the geometric nonlinearity which causes the natural frequency to be a function of the amplitude of the response. An important aspect of this behavior is that the decay of the transient is long compared to the period of the excitation.

Figure 4 shows the results of an attempt to force the system at the apparent natural frequency. In this solution there is no damping in the model nor is there any intentionally in the numerical integrator. Although the plot is a crude one which attempts only to show the peaks and valleys, it shows behavior not found in the linear problem. The response is not unbounded and is quite complicated in form. There is not a single amplitude, and for the most part the frequency of the response is higher than the frequency of the input. It does appear that two amplitudes are in the response: one at about 35 cm and the other at about 15 cm. In addition, it appears that the oscillation pattern repeats on a period of about ten times the apparent natural period (coincidence?).

Taken together, these three figures are indicative of the typical response of a nonlinear oscillator as represented in Figure 5. They also show some of the difficulties in using numerical calculations of transient responses to correlate with theoretical steady state responses.

Static Excursion of a Moored Ship

A diagram of the DD692 Destroyer in a four point moor is shown in Figure 6. The lines are essentially catenaries in the quiescent state, and substantial lengths of line lie on the bottom. Figure 7 shows the combined effects of a 2 kt. surface current and a 30 kt. wind versus the heading relative to the ship. Figure 8 shows two calculations of the excursion the c.g. of the ship takes as the heading of the wind and current is varied through 180° relative to the original quiescent position of the ship. The effect of neglecting the bottom interaction with the lines is clearly shown. The moor appears much stiffer without the bottom interaction. The differences in stiffness as well as the change in ship position could have significant influence on dynamic response calculations. See Reference 1 for more details.

Frequency Domain Dynamic Response Calculations for Moored Ships

Following the approach represented by Equations (4) for some basic mooring configurations offers some insights. Reference 2 gives more detail and presents the figures which will be commented on briefly here.

The paper notes three important results:

First - Contrary to the assumption made in many mooring analyses, the mooring has significant influence on the dynamics of the moored vessel. This can be seen in the moored and free frequency response curves (Figures 3-6, 9, 10, OTC). For example, there is a significant reduction in peak response and a shift in the frequency where this occurs. This means it is not appropriate to analyze mooring legs by simply imposing the free ship motions at the upper end of the line.

Second - Contrary to the assumption made by some to get from 6 to 3 degrees of freedom, the use of mooring buoys and hawsers does not effectively isolate the moor from the heave/pitch/roll motions of the vessel. See OTC Table 4 and the discussion. This is due to the geometric stiffening effect of the hawser preload.

Third - At some frequencies, the mooring legs act in a nearly linear fashion while at others the dynamic behavior is decidedly nonlinear (see Figures 13 and 14, OTC). This particular phenomenon is quite difficult to predict and probably involves multi-frequency responses and resonances along with other large displacement effects. This calls into question the validity of the entire frequency domain solution procedure. The OTC paper suggests there is general qualitative agreement with the results obtained with the frequency domain solution and established mooring design procedures (DM-26), but the calculation of specific responses may be erroneous or difficult to interpret.

CONCLUSIONS

This brief discussion and the examples presented suggest that proper modeling of the deep sea mooring problem (and perhaps shallow water problems as well) requires very careful consideration of the nonlinear dynamics of the mooring lines. At present, this means that time domain models are preferred above frequency domain models. This further means there is a need to develop appropriate time domain models of the ships, platforms, buoys and other bodies used and to develop the appropriate description of the environment. It may also mean that there is a need to develop new solution techniques. Although not dealt with in this discussion, low frequency effects such as wave induced drift forces and swells acting in combination with wind and waves may require that large displacement responses of the mooring be considered even in the situations where the frequency domain solution may be an adequate model of the first order wave responses.

REFERENCES

1. Webster, R. L., "Finite Element Analysis of Deep Sea Moors and Cable Systems," Preprint 3033, ASCE Fall Convention and Exhibit, San Francisco, California, Oct. 17-21, 1977.

2. Webster, R. L., McCreight, W. R., "Analysis of Deep Sea Moor and Cable Structures," Paper OTC 3623 Presented at the 11th Annual Offshore Technology Conference, Houston, Texas, 30 April-3 May 1979.

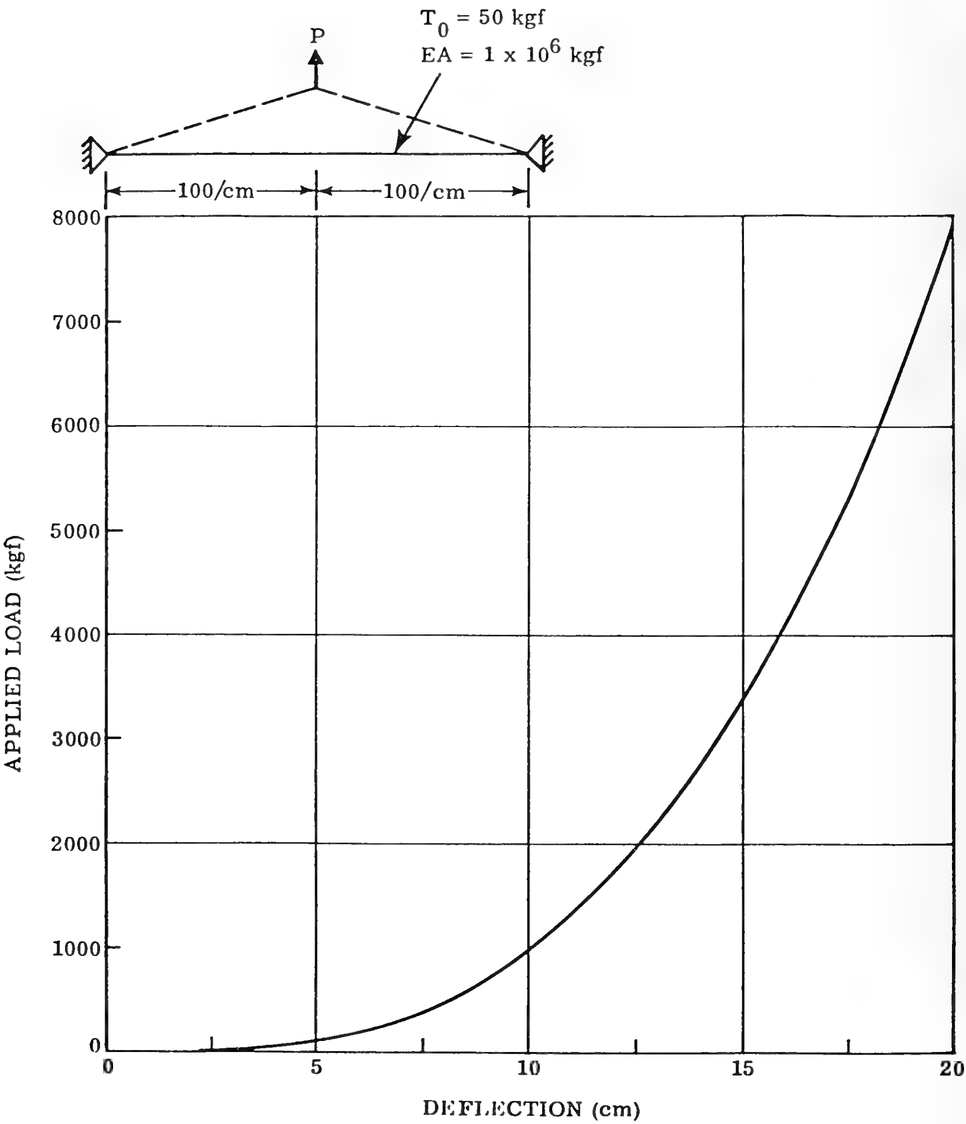

Figure 1. Stretched String with Point Load

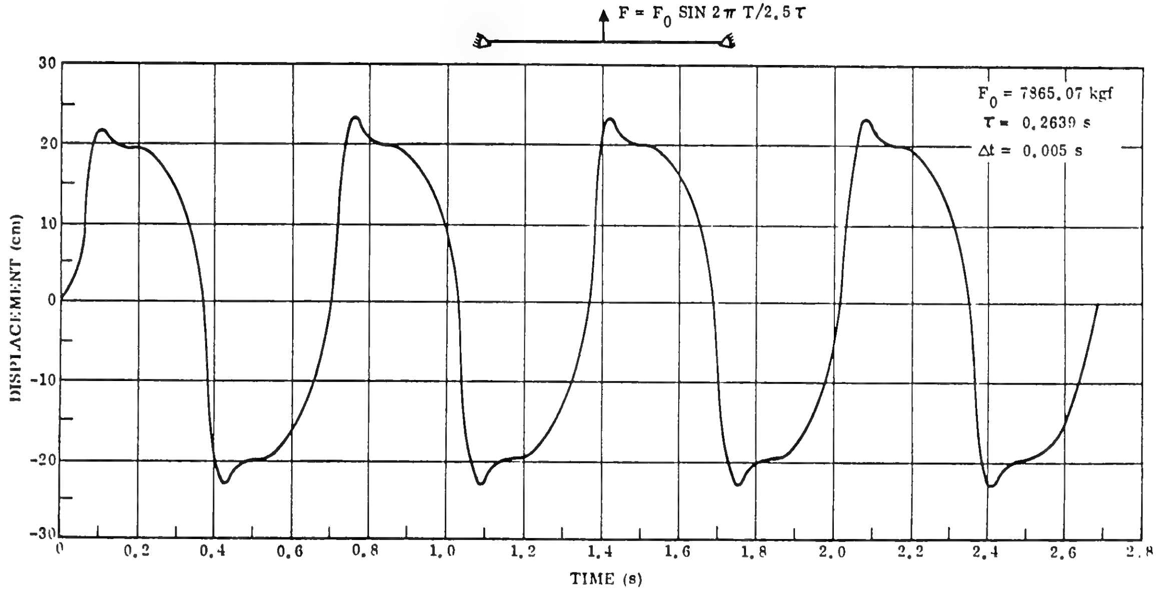

Figure 2. Superharmonic Response of Stretched String

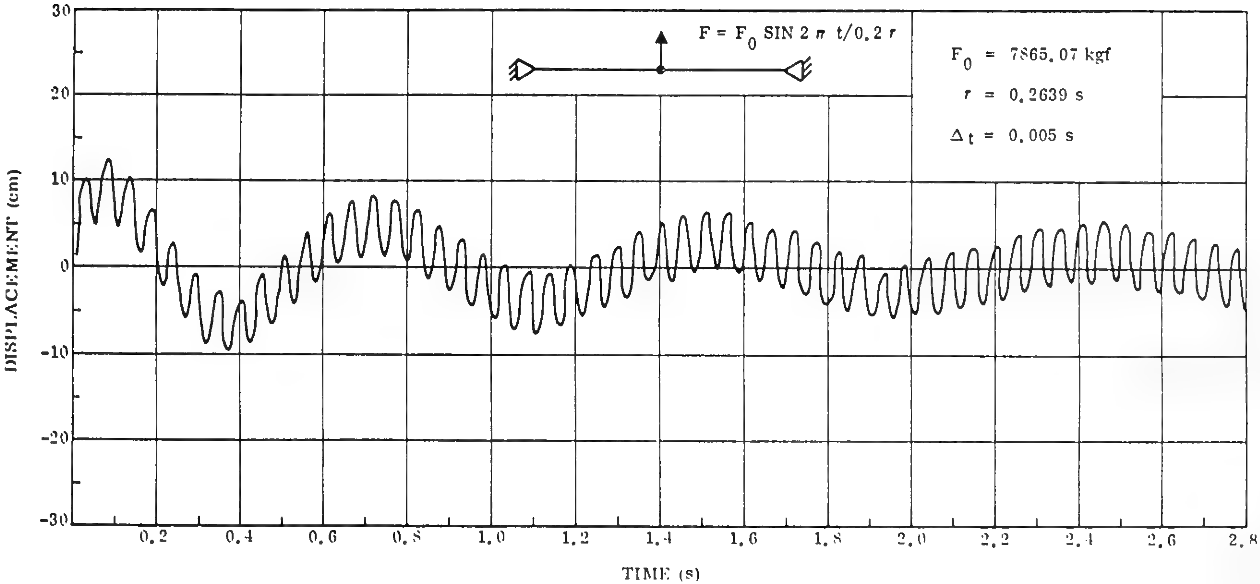

Figure 3. Subharmonic Response of Stretched String

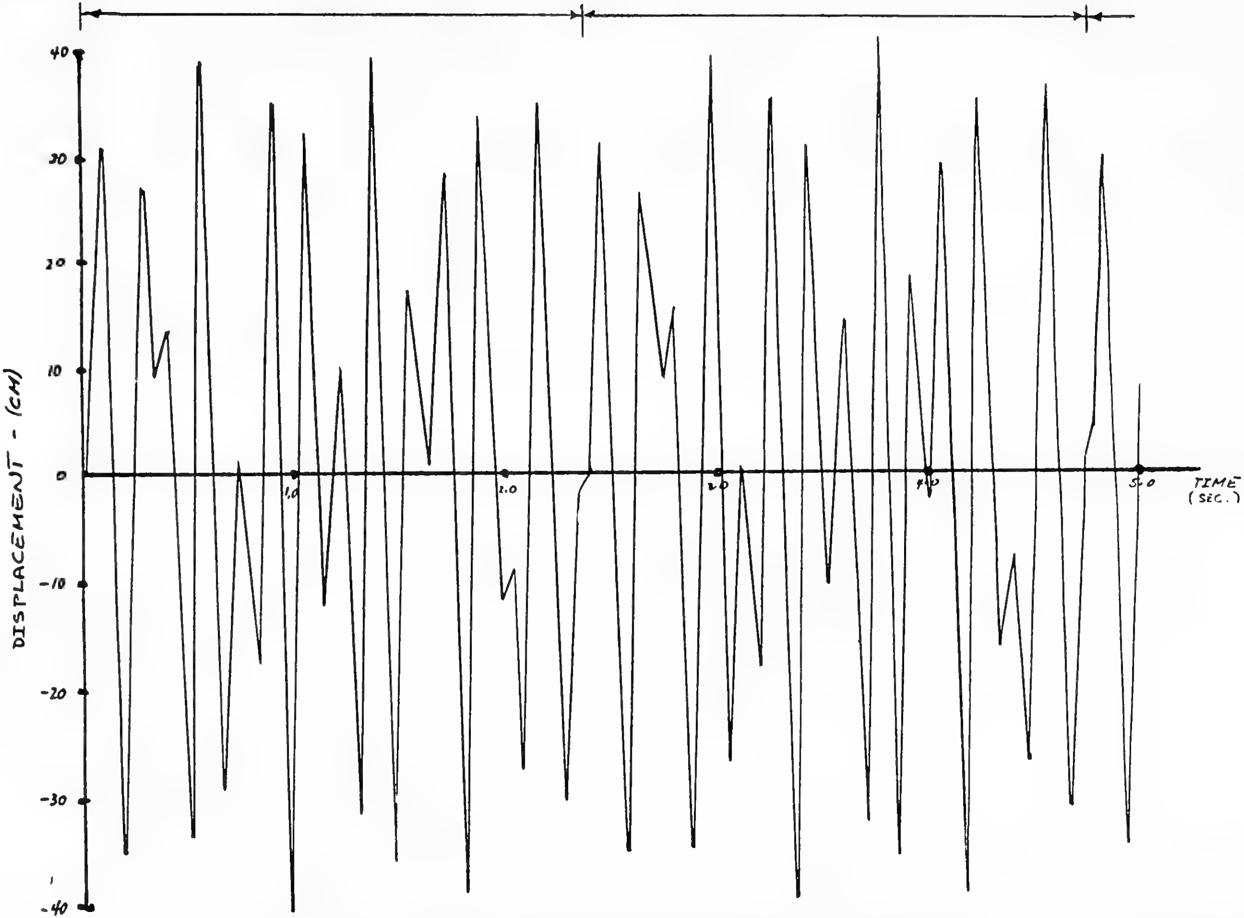

Figure 4. Stretched String Response When Forced at Natural Frequency

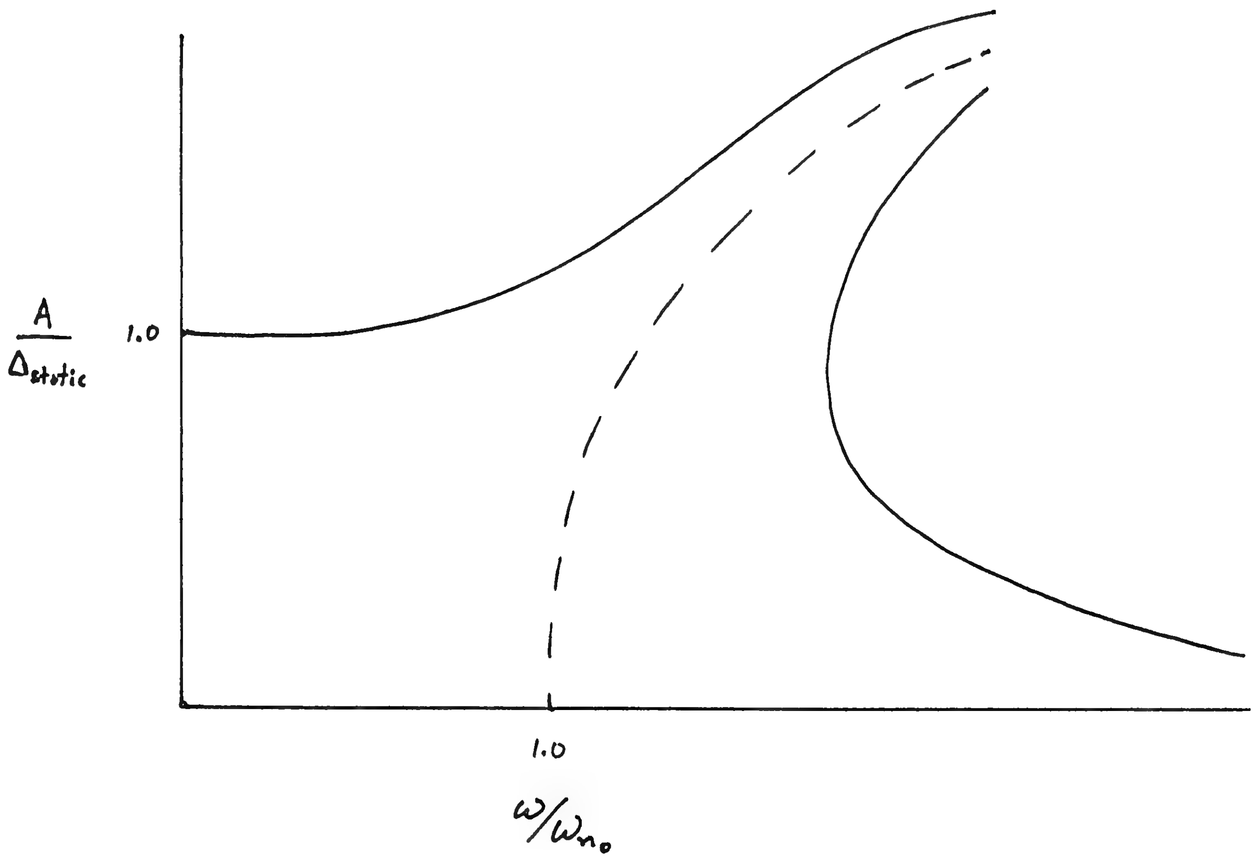

Figure 5. Typical Nonlinear Frequency Response

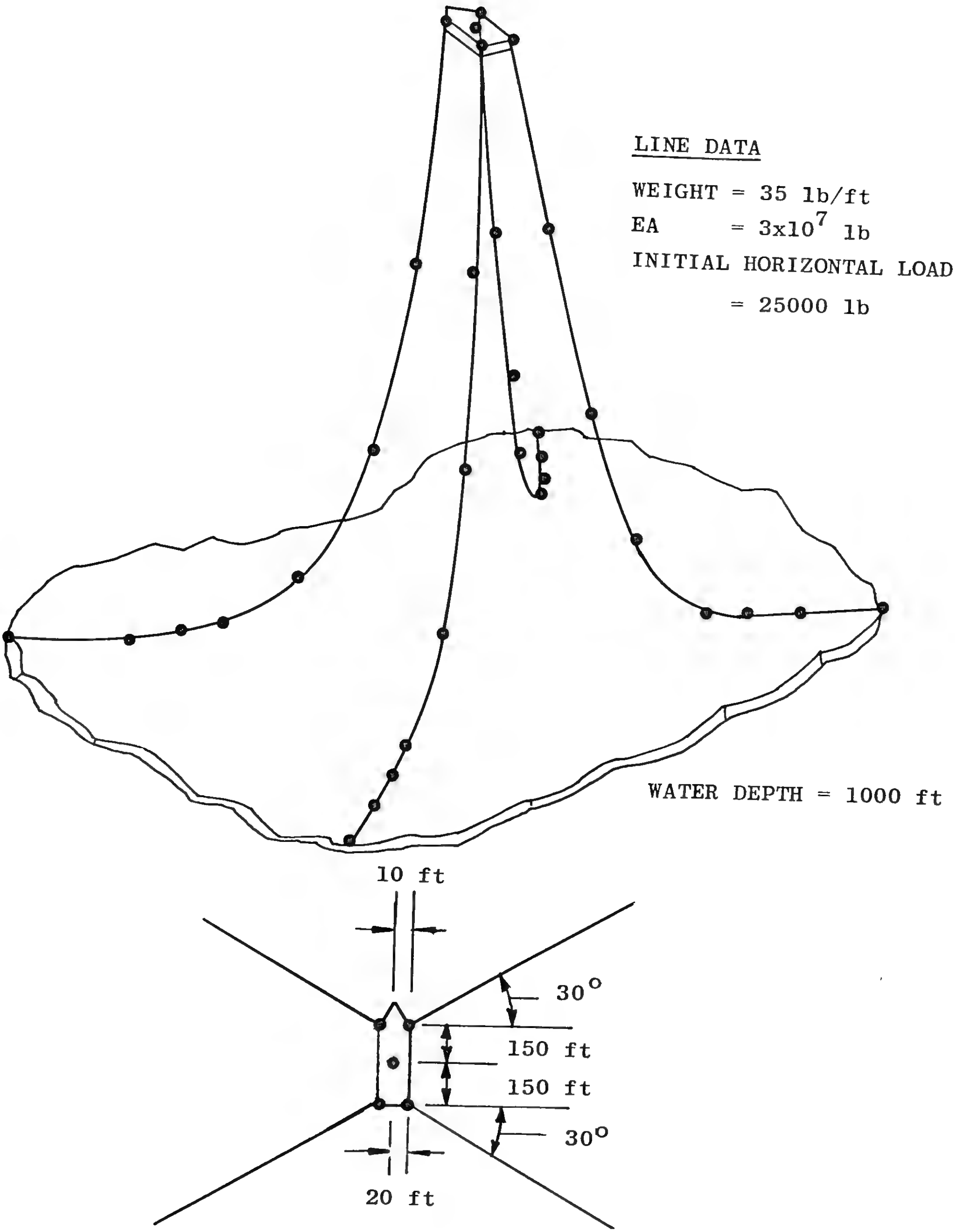

Figure 6. Finite element model of moored ship.

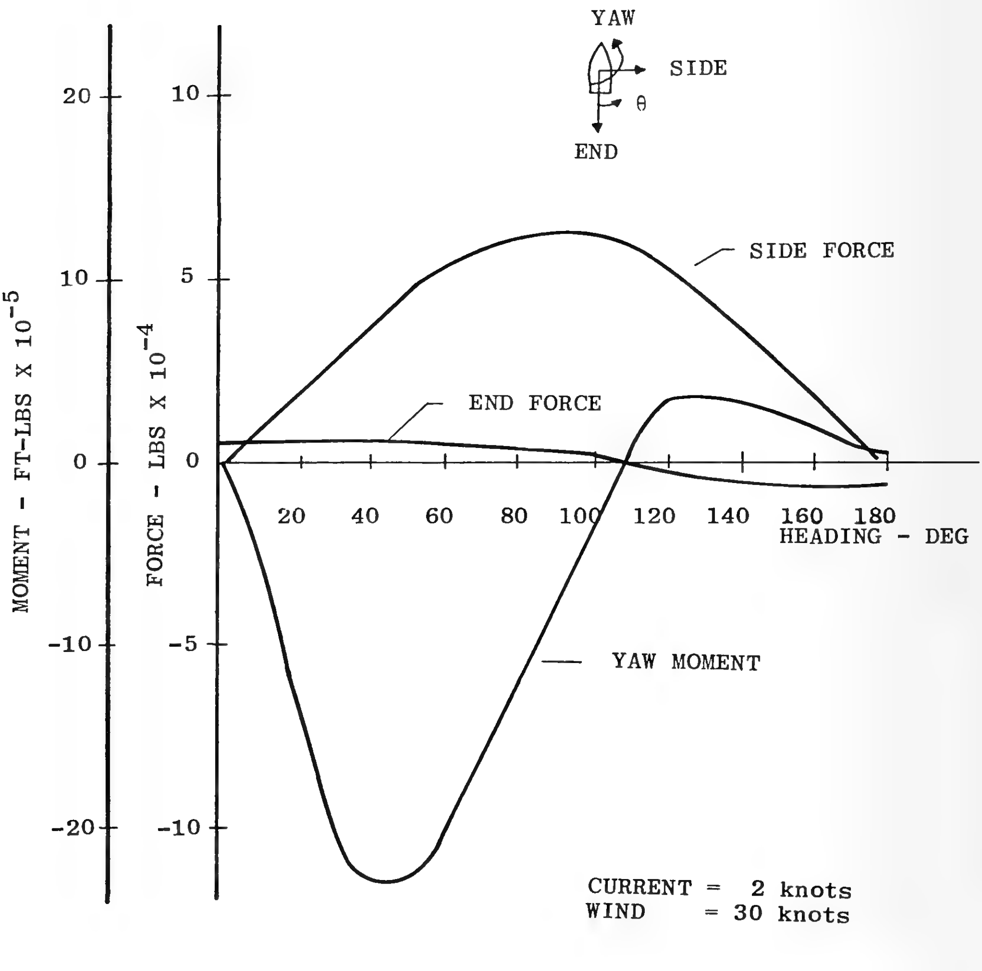

Figure 7. Combined ship loads versus heading.

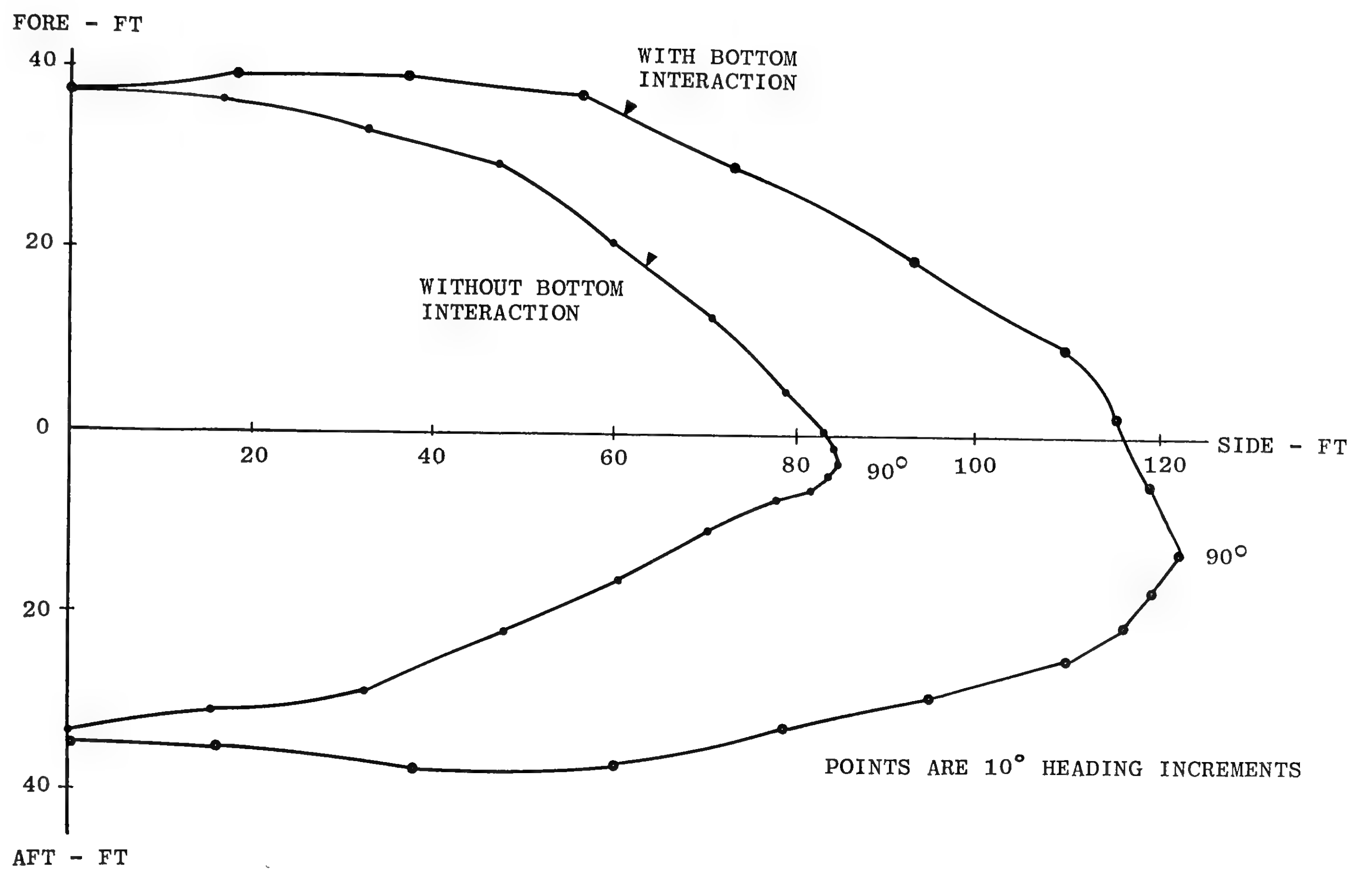

Figure 8. Ship excursion envelopes.

HYDRODYNAMICS OF MOORED VESSELS

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1. INTRODUCTION

The mathematical formulation and solution of the boundary-value problem for the hydrodynamics associated with the motion of a moored vessel in a seaway is rather complex due primarily to the nonlinear free surface boundary condition. Difficulty with the free surface boundary condition has impeded progress on the exact solution for wave/body interaction problems and little progress has been made. Thus, the more fruitful approach has been to pursue linearized solutions as an approximation. The linearized problem is also difficult but computer solutions can be obtained for bodies of practical interest. Moreover, linearization admits the concept of superposition of motions and waves, with which comes the powerful concept of wave excitation spectra and the motion response spectra. Although some rather broad assumptions are made in order to linearize the boundary value problem, linear solutions have been found to give physically realistic results for cases of practical interest.

In addition to the dynamic response of a moored vessel to wave motion at the frequency of the waves, a second-order effect referred to as slowly-varying drift motion also occurs when the vessel is subject to random waves. This is a phenomena which has received a great deal of attention in recent years and is an area of ongoing research.

2. LINEARIZED HYDRODYNAMICS OF FLOATING VESSELS

The theory of the motion of a floating vessel is based on the following assumptions:

(a) Inviscid fluid and irrotational flow.
(b) Small amplitude waves and resulting small amplitude response.
(c) Wave motion and response motion representable by a superposition of regular sinusoids.

The notion of superposition of both the incident waves and the response of the vessel allows one to view the motion of a moored vessel in waves as: (a) the wave interaction with the vessel held fixed and (b) the motion of the vessel oscillating in each of its six degrees of freedom separately in otherwise still water. From consideration of (a), the wave excitation forces and moments are determined, and from (b) the reaction forces and moments resulting from the motion of the vessel are determined. The latter are characterized by use of added mass and damping tensors.

A numerical procedure based on distributed three-dimensional sources has been presented by Garrison (1974) and Faltinsen and Michelsen (1974) for three-dimensional bodies of arbitrary shape.

Bai and Yeung (1974) also have developed a numerical procedure based on the Green's function method (or boundary integral method as it is sometimes called) which utilizes simple sources distributed over the surface of the vessel as well as the free surface, the bottom and an enclosing vertical cylindrical surface far from the vessel. A third numerical method for solution of the three-dimensional free surface problems is referred to as the hybrid-element method. This procedure, which has been applied by Berkhoff (1972), Chen and Mei (1974), Bai and Yeung (1974), Chenot (1975), Yue, Chen and Mei (1977) and Bettess and Zienkiewicz (1977), is based on the finite element method and uses a "super-element" at the outer boundary of the discretized region to infinity. Of the available methods indicated above, the distributed source procedures of Garrison, and of Faltinsen and Michelson is believed to be the most versatile and simplest in application, and has been most widely used in practice.

2.1 Strip Theory

The solution of the three-dimensional boundary-value problem for bodies of arbitrary shape requires computer runs, considerable CPU time, and until recent years numerical methods for solving three-dimensional problems were not available. Thus, it has been common practice to use a strip-theory analysis for elongated (shiplike) bodies in which the hydrodynamic coefficients are determined by subdividing the body into a number of slices or segments and assuming that each segment acts as a two-dimensional body and that segments do not reflect mutual interaction effects. The hydrodynamic coefficients for the complete body are obtained by summing up the coefficients associated with each segment.

Clearly, strip theory represents a valid approximation to a truly three-dimensional hydrodynamic analysis provided the vessel is highly elongated. Of course, one would expect the strip theory approximation to break down as the length to beam ratio decreased and it would be of practical value to know what value of the length to beam ratio might represent a limit on the strip theory approximation. An absolute limit for all vessels does not exist since it is presumably dependent, if only mildly, on the hull shape in addition to the overall proportions, but it appears that few studies comparing three-dimensional theory with strip theory have been made. In fact, the only such comparison known to this writer was made by Migliore and Palo (1979) for rectangular barge configurations. For the series of cases considered, the results indicated that the strip theory analysis tended to breakdown when compared to the three-dimensional theory for length to beam ratios of less than 8. Thus, it would appear that for barges, most cases of practical interest would require the application of three-dimensional theory for predicting hydrodynamic coefficients.

2.2 Comparison with Experiment

Experimental results for hydrodynamic coefficients for three-dimensional bodies are very limited but results of a few studies have been reported. Faltinsen and Michelson (1974) have presenteed experimental results for a model of a simple barge 90 meters by 90 meters by 40 meters draft. In general, although the scatter in the experimental data is large in some cases, the agreement with calculations based on linear theory is good as indicated in Figures 1 and 2.

However, the measured heave damping is substantially greater than the predictions of linear theory. In cases such as this, where a rather large difference occurs between experiment and calculated results, the cause can generally be traced to viscous effects. In the present instance, the bottom surface of the barge acts as the wave generating surface in heave but since it is rather deeply submerged its wave-making ability is diminished. In this connection it may be noted that the damping coefficient in heave is about one-fifth that of surge. Since the wave-making damping in heave is very small the importance of viscosity is rellatively large and this presumably accounts for the experimental values being considerably above the values based on the linear, inviscid theory.

Faltinsen and Michelson (1974) present no pitch data but since motion of the barge in pitch typically produces a very small radiated wave, the damping coefficient predicted by linear theory is normally very small, except in the case of very shallow-draft bodies where the wave-making surfaces are very near the free surfaces. Thus, a similar situation to the above may be expected. In view of the small radiation damping in pitch, theory generally predicts a very large resonance peak which is not observed in reality. However, it is well-known that damping is only important near resonance and, therefore, the motion response is generally in error on this account only near resonance.

Pinkster and van Oortmersen (1977) have also presented experimental results and comparisons with linear theory for excitation loads and response motions of a barge of 150m length, 50m breadth and 10m draft. In general, the linear theory agreed very well with the experimental results. The only significant discrepancy was the rather large resonance peak in roll which is to be expected in view of the above comments regarding roll damping.

3. VESSEL MOTION

There are two rather well-known and commonly applied methods for treating the motion of a vessel in a seaway. These are referred to as frequency-domain and time-domain analyses.

The frequency domain analysis is based strictly on the assumption that all forces acting on the floating body are linear functions of displacement, velocity or acceleration, and as a result the response is directly proportional to the amplitude of the incident wave. For a given frequency, the equations of motion for the floating body would appear as follows:

$$\left(m_{ij} + M_{ij}(\sigma)\right)\ddot{X}_j(t) + N_{ij}(\sigma)\dot{X}_j(t) + K_{ij}X_j(t) = F_i(\sigma) \qquad (1)$$

in which m_{ij} denotes the mass matrix of the body, $M_{ij}(\sigma)$ denotes the added mass matrix, $N_{ij}(\sigma)$ denotes the damping matrix, K_{ij} denotes the restoration force matrix due to buoyancy and elaastic forces, and F_i denotes the wave excitation force.

To examine the difficulty in application of Eq.(1) to random waves it is enough to consider two frequencies, σ_1 and σ_2. It is generally assumed that the response associated with the two

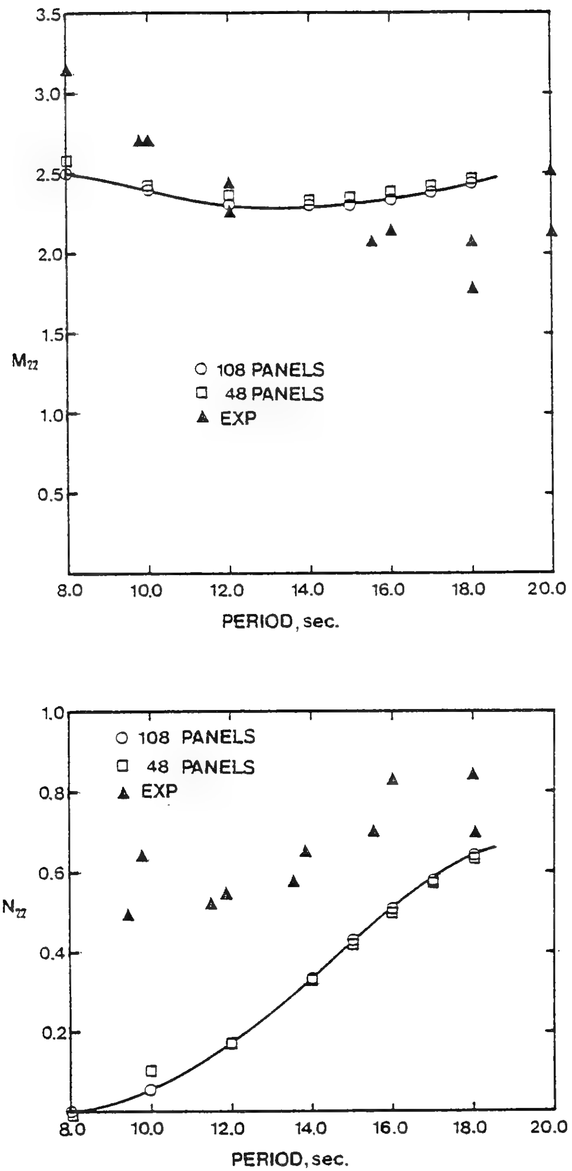
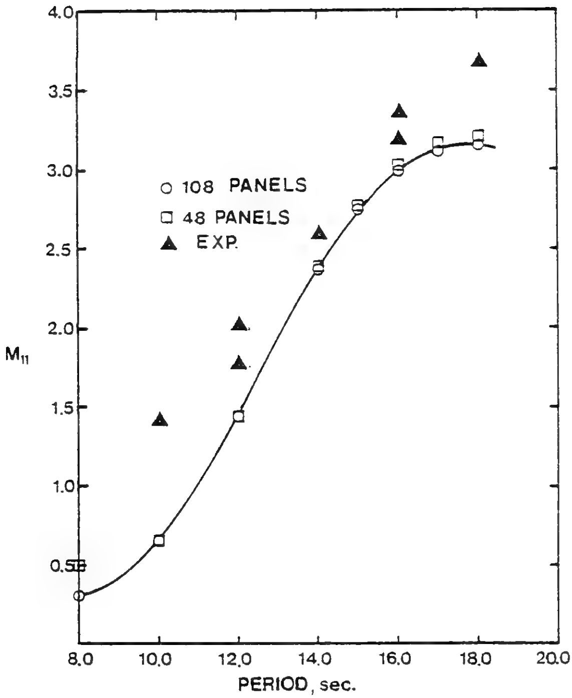


Figure 1 Heave Added Mass and Damping Coefficient for a 90m x 90m x 40m Barge.



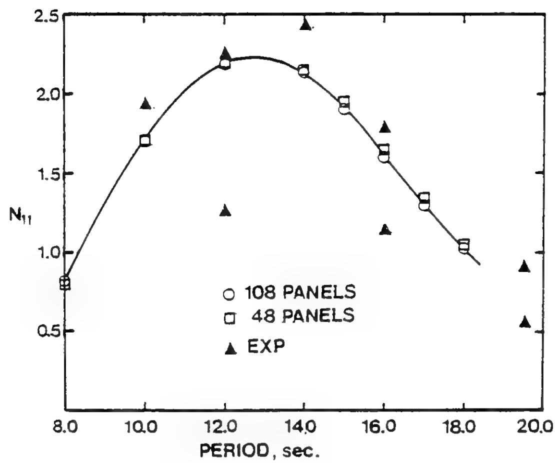


Figure 2 Surge Added Mass and Damping Coefficients for a 90m x 90m x 40m Barge.

wave components, σ_1 and σ_2, is represented by the sum of the response due to σ_1 alone, which we may call $X_j^{(1)}$, and the response due to σ_2 alone, $X_j^{(2)}$. However, as pointed out by Wehausen (1971) it is only in the special case that $M_{ij}(\sigma_1) = M_{ij}(\sigma_2)$ and $N_{ij}(\sigma_1) = N_{ij}(\sigma_2)$ that this could in fact be the case since it is only then that $(X_j^{(1)} + X_j^{(2)})$ could represent a solution to an equation of the form of (1). In spite of this, such equations have been used with some success to describe the motion of a vessel in random waves (see, e.g. Fuchs and Mac Camy (1953), Fuchs (1954), St. Denis and Pierson (1953) in which the values of N_{ij} and M_{ij} were taken as constant at some average value. It appears, however, that it is currently common practice to utilize the superposition discussed above regardless of the difficulty associated with frequency dependent coefficients of mass and damping. Wehausen (1971) has discussed a further method of treating the linearized motion of floating bodies in random seas when the added mass and damping coefficients are frequency dependent as, of course, is the case for large-displacement floating bodies. The procedure outlined is based on the initial-value problem approach.

3.1 Nonlinear Effects

The theory of ship motions is based on linear hydrodynamics. This means that the amplitude of waves and the response to these waves is considered to be small and, therefore, all terms arising in the analysis which are proportional to such amplitudes to the second-power and above are neglected.

If one considers two wave components of frequency σ_1 and σ_2 for instance, and retains terms through the second-order in the small amplitude parameter mentioned above, certain "second-order" forces and moments arise. These forces are oscillatory and components at frequency $(\sigma_1 + \sigma_2)$ as well as at $(\sigma_1 - \sigma_2)$ arise. The force occurring at frequency $(\sigma_1 + \sigma_2)$ is of considerably smaller magnitude than the first-order wave-induced loads and occurs within or above the frequency range of the first-order forces. Thus, these small amplitude, high-frequency forces are of little consequence and are neglected.

The force (or moment) component at frequency $(\sigma_1 - \sigma_2)$ falls below the first-order excitation force frequency range, and although the forces are small compared to the first-order forces, these second-order forces can have significant effects due to the fact that the resonance frequency of the mooring system may lie near these frequencies.

Newman (1974) has developed an approximate method for evaluating the slowly-varying second-order drift-forces in random waves which is based solely on the mean drift-force. The mean drift-force is simply the diagonal terms in a square matrix of coefficients describing the low-frequency force components associated with all combinations of the components associated with a wave spectrum. Newman's approximation renders the evaluation of slowly-varying second-order forces on floating bodies within the realm of the computationally feasible for practical floating body situations. Without the use of this approximation the computational chore would be formidable for three-dimensional bodies.

4. EVALUATION OF THE MEAN DRIFT-LOADS

There are two methods for evaluation of mean drift forces acting on a floating body. It was first pointed out by Haskind (1948, 1959) that the mean drift force could be formulated through application of the integral momentum equation together with a knowledge of first-order potentials. Muro (1960) also independently gave a general treatment of the mean drift-force and Newman (1967) extended Muro's results to include the moment and, in addition, introduced the slender body approximation into the analysis. Faltinsen and Michelsen (1974) applied Newman's formulas to evaluate the mean drift-force using the three-dimensional distributed source procedure. In Newman's notation the formulas for the force components in the horizontal plane and moment about a vertical axis are given by

$$\bar{F}_x = \frac{\rho}{8\pi}\int_0^{2\pi} |H(\theta)|^2 (\cos\beta - \cos\theta)\, d\theta \tag{2a}$$

$$\bar{F}_z = \frac{\rho}{8\pi}\int_0^{2\pi} |H(\theta)|^2 (\sin\beta - \sin\theta)\, d\theta \tag{2b}$$

$$\bar{M}_y = -\frac{\rho}{8\pi k}\, Im\int_0^{2\pi} H(\theta)\, \overline{H'(\theta)}\, d\theta + \frac{1}{2}\frac{1}{k^2}\rho\sigma |A|\, Re[H'(\beta)] \tag{2c}$$

where $H(\theta)$ is the Kochin function. Without giving further formulas, it may be noted that the Kochin function requires a knowledge of the first-order potentials associated with diffraction of the incident wave as well as the radiation potentials for all six degrees of freedom. Thus, the drift-force calculation requires a knowledge of the first-order motions.

A second method for calculation of the drift is based on a straightforward integration of the pressure over the wetted surface of the vessel. This procedure was first presented by Garrison (1974) for three-dimensional floating bodies although a term was left out of the final expression which accounted for the the effect of the displacement of the body. Pinkster and van Oortmerssen (1977) later gave the correct form of the expression for the drift-forces and moments. The force, for instance, is given by

$$\begin{aligned}\vec{\bar{F}} = \rho g\left(\frac{H}{2}\right)\Big\{ & \frac{k}{2}\iint_{S_0} Re[\bar{u}_{T_x} X_0^{(1)} + \bar{u}_{T_y} Y_0^{(1)} + \bar{u}_{T_z} Z_0^{(1)}]\, \vec{n}_0\, ds \\ & + \frac{1}{4}\coth(kh)\iint_{S_0} (|u_{T_x}|^2 + |u_{T_y}|^2 + |u_{T_z}|^2)\, \vec{n}_0\, ds \\ & + \frac{1}{2}\iint_{S_0} Re[\,\overline{(Y_0^{(1)} + k u_T)}\; \vec{N}_0^{(1)}\,]\, ds \\ & - \frac{1}{4}\int_{C_0} Re[(Y_0^{(1)} + k u_T)\overline{(Y_0^{(1)} + k u_T)}]\, \vec{n}_0\, dc\end{aligned} \tag{3}$$

in which u_T denotes the total first-order potential, $X_0^{(1)}$, $Y_0^{(1)}$ and $Z_0^{(1)}$ denote the complex amplitude of the three components of displacement of a point on the hull, $\vec{N}_0^{(1)}$ denotes the first-order correction to the unit normal vector on the mean position of the hull denoted by $\vec{n}_0$. In (3) the first three integrals are carried out over the wetted surface of the hull below the mean waterline and the last term is a line integral along the line defined by the intersection of the hull and the mean free surface. This term accounts for the variation of the wetted surface at the waterline due to run-up of the wave and vertical displacement of the hull.

The direct integration of the pressure has several advantaages over the integral momentum equation formulation. Firstly, all six components of the drift-force can be obtained through this procedure and secondly, it appears that it may be possible to extend the basic development to compute the complete low-frequency second-order forces and moments. The complete second-order force involves the second-order potential, and while the determination of this may be feasible in principle the overall task appears somewhat overwhelming from the viewpoint of computer time. The computation of the low-frequency part of the second-order force is much more within the realm of the attainable since it is strictly dependent only on first-order potentials and motions.

5. CONCLUSIONS, COMMENTS AND RECOMMENDATIONS

1. The hydrodynamic coefficients can be computed fairly efficiently for three-dimensional bodies using the distributed source (Green's function) technique. The available experimental data agrees quite closely with calculations using linear theory except in certain cases where the damping coefficients are rather small. In such cases it is suspected the viscous effects become relatively significant and in order to deal with such cases, some effort should be directed toward development of a "viscous correction" for the damping coefficients which is appropriate for barge-like shapes.

2. Because of the frequency-dependency of the coefficients in the equations of motion of a floating body, superposition of the motion resulting from each component of an incident wave system becomes questionable.

3. Two methods of evaluating the mean drift-forces and moments have been outlined. The pressure integral technique has the advantage over the momentum equation formulation of providing all six components of the load which may be of value in certain cases.

4. It appears that it may be possible to evaluate the complete slowly-varying drift-force using the pressure integral formulation. This could provide a basis for a time-domain simulation of the slowly-varying drift motion of a moored vessel with the high-frequency wave-induced motion superimposed on this motion.

6. REFERENCES

Bai, K.J. and Yeung, R.W.(1974), "Numerical Solution to Free-Surface Flow Problems," Tenth Symposium on Naval Hydrodynamics, ACR-204, ONR.

Berkhoff, J.C.W. (1972), "Computation of Combined Refraction-Diffraction," Proc. 13th Coastal Engineering Conference, ASCE, Vol. I, 471-490.

Bettess, P. and Zienkiewicz, O.C. (1975), "Diffraction and Reflection of Surface Waves Using Finite and Infinite Elements," International Journal for Numerical Methods in Engineering, Vol. 11, 1271-1290.

Chen, H.S. and Mei, C.C. (1974), "Oscillations and Wave Forces in an Offshore Harbor", Tech. Rept. No. 190, Ralph M. Parsons Laboratory for Water Resources and Hydrodynamics, Dept. of Civil Engr., M.I.T., Cambridge, Mass.

Chenot, J.L. (1975), Revue de l'Institut Francais du Petrole, 30: 779-802.

Faltinsen, O.M. and Michelsen, F.C. (1974), "Motion of Large Structures in Waves at Zero Froude Numbers", The Dynamics of Marine Vehicles and Structures in Waves, Paper No. 11, The Inst. of Mech. Engrs.

Fuchs, R.A. and Mac Camy, R.C. (1953), Univ. of Calif., Berkeley, Inst. Engr. Res. Ser. No. 61, Issue No. 2.

Fuchs, R.A. (1954), Proc. 1st. Conf. Ships Waves, Hoboken, N.J., 186-93.

Garrison, C.J. (1974), "Dynamic Response of Floating Bodies", Paper No. OTC 2067, Offshore Technology Conference, Houston, TX.

Haskinds, M.D. (1948), Inzh. Sb., Vol. 4, No. 2, 147-60.

Haskinds, M.D. (1959), Izv. Akad. Nauk SSSR. Otd. Tekh. Nauk, Mekh. Mashinostr., No. 2, 46-56.

Migliore, H. and Palo, P. (1979), "Analysis of Barge Motion Using Strip and Three-Dimensional Theories", OTC 3558, Offshore Tech. Conference, Houston, TX.

Muro, H. (1960), "The Drift of a Body Floating in Waves", J. Ship Research.

Newman, J. N. (1967), "The Drift Force and Moment on Ships in Waves", J. Ship Res.

Newman, J.N. (1974), "Second Order, Slowly Varying Forces on Vessels in Irregular Waves", International Symposium on the Dynamics of Marine Vehicles and Structures in Waves, University College, London.

Pinkster, J.A. and van Oortmerssen, G. (1977), "Computation of the First- and Second-Order Wave Forces on Oscillating Bodies in Regular Waves", Second International Conference on Ship Hydrodynamics, U.C., Berkeley, CA.

St. Denis,, M. and Pierson, W.J., Jr. (1953), Trans. Soc. Naval Arch. and Marine Engr., Vol. 61, 280-357.

Yue, D.K.P., Chen, H.S. and Mei, C.C. (1977), Inst. J. Numerical Methods in Engineering.

Wehausen, J.V. (1971), "The Motion of Floating Bodies", Annual Review of Fluid Mechanics, Vol. 3.

VESSEL EQUATIONS OF MOTION

by

Dr. R. Bhattacharyya

For the prediction of ship motions the so-called "strip theory" is used universally, although there are many versions of this theory depending on the problem formulation, the method of solution as well as the inclusion of the forward speed effect.

The original version as given by Korvin-Kroukovsky and Jacob was developed from an engineering point of view and various terms of the equations of motion for heaving and pitching motions were based on somewhat arbitrary defininition of the relative motion between the vessel and the waves. An improvement of the problem formulation was made by [8] from intuitive point of view whereas Reference [23] developed a strip theory on the basis of mathematically consistent perturbation technique. A major significance of the theoretical methods was the elimination of the original relative motion approach of Korvin-Kroukovsky and instead, as given by Ref [20], the ship motion problem was developed as a sum of radiation and diffraction problems. In other words, the total forces and moments acting on a vessel in a train of regular waves are equal to the sum of the forces and moments acting when the ship is oscillating in calm water, together with the wave forces and moments acting on a restrained vessel. This is quite a significant result, especially from the point of view of model experimentation.

It should be pointed out, however, that although there are differences in various versions of the strip theory it should not be construed that any one particular version is significantly better than the other, except in some particular cases.

There are three different methods for the longitudinal motion as given by:

1) Gerritsma and Beukelman [8]

2) Ogilvie and Tuck [23]

3) Salvesen, Tuck and Faltinsen [26]

For the lateral motions three different approaches are given by:

1) Kaplan, Sargent and Raff [15]

2) Salvesen, Tuck and Faltinsen [26]

3) Grim and Schenzle [12]

Of all the methods above only Ogilvie and Tuck, and Salvesen et al satisfy the Timman-Newman symmetry condition.

Based on two-dimensional case, there are three different ways for the numerical calculation of fluid reactive forces and moments, i.e., added mass or added mass moment of inertia and wave damping.

1) The first and the simplest method is the use of conformal mapping techniques with no-free-surface effects. Apart from the classical extended Joukowski transformation method of [17] which was improved by [16] whereas both [41] and [14] applied the Schwarz - Christoffel transformation in obtaining the added mass.

3) Another method is the use of source distribution over the hull surface which can also be attributed to [35]. The practical use of the source distribution method is due to [7] which is often referred to as "Frank Close fit" method.

As mentioned above, the fluid reactive forces and moments are generally known as added mass/moments of inertia and damping because they act in phase with the acceleration and the velocity of ship motion, respectively. Presently there are four methods available for practical calculation:

1) Infinite frequency approach (no free-surface effect) [16]

2) Lewis transformation [9]

3) Tasia close-fit mapping method [33]

4) Frank close-fit source distribution method [7]

The calculations carried out by [13] indicate that when the frequency parameter $\frac{\omega^2 B}{2g}$ is greater than 4.5, the wave damping becomes negligible and the use of infinite frequency approach is justified. The difference between the Lewis transformation and Tasai close-fit methods is due to the number of terms used in the conformal mapping of the flow around the ship section onto the flow around a circle. Otherwise they both use a series of multipoles.

The Frank close-fit method uses a surface source distribution method and replaces the section of a vessel with segmental singularities.

Application of various methods for the sectional heaving added mass and damping distribution of a destroyer model with a bulbous bow (Davidson A Destroyer Model) is shown in Figs. 1 and 2. [22]

Atlhough it has often been suggested that the accurate representation of the sectional shape is not very important, it is not quite true because the determination of the shearing forces and bending and torsional moments require accurate description for all sections, whereas in motion calculations integration over the length iron out the small errors. Ref. [22] suggests that it is not advisable to use the Lewis transformation method, (that is the two or three parameter family) when transforming the sections near the fore and aft end of the ship except for heaving and pitching predictions of conventional ship forms.

An important consideration for the use of the Frank close-fit method is the condition that the section contour should satisfy the so-called "Lyapunov" regularity conditions [22]. Non-compliance with these conditions throws doubt on the validity of the Frank approach; for example, forward sections of a ship with a bulbous bow do not fulfill these requirements. In fact, for such shapes Frank [1967] has shown that there exist radiationless frequencies. Therefore, Ref. [22] recommends that for these types of geometry a multipole method such as the Tasai close-fit method, should be adopted.

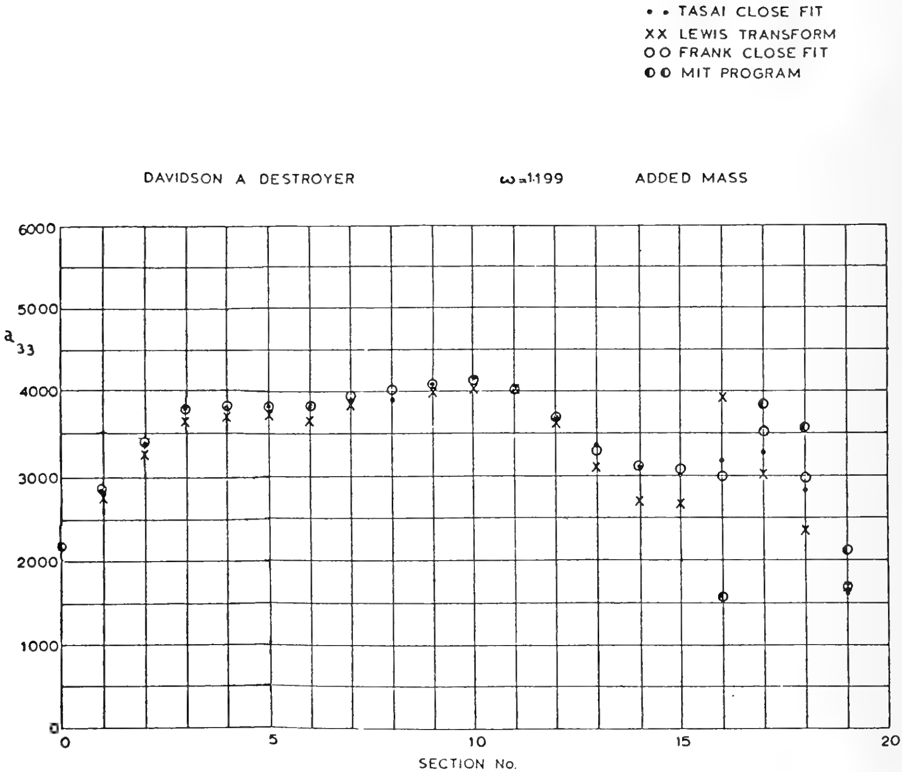


Figure 1

Distribution of Heaving Added Mass

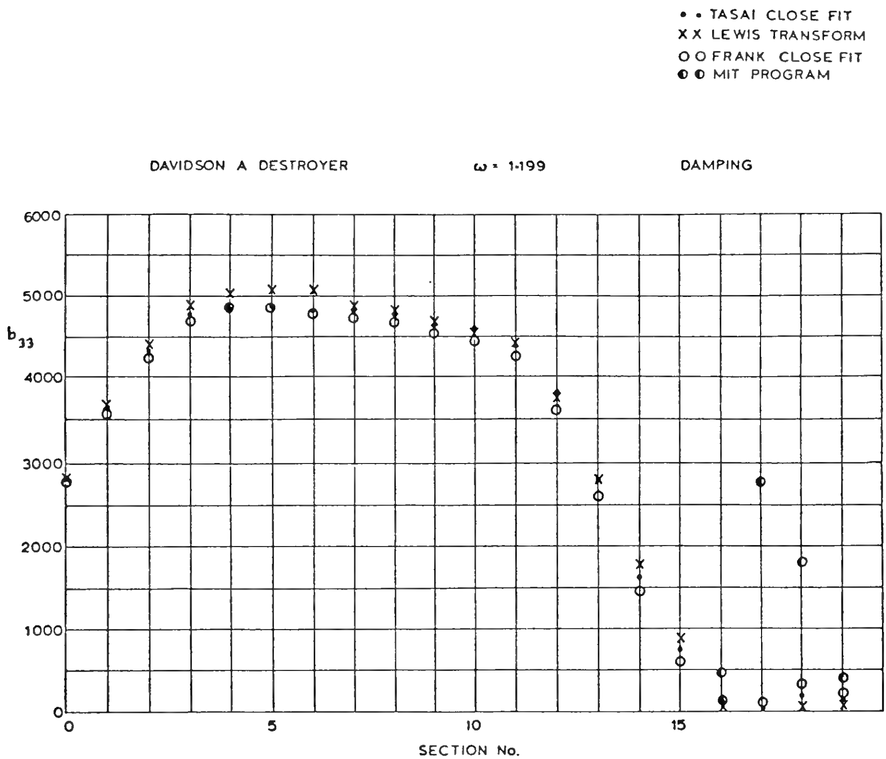


Figure 2

Distribution of Heaving Damping

A problem which has yet to be solved satisfactorily is the heaving added mass at very low frequencies. Although this may appear unimportant, in reality this is not the case because in following waves such low frequencies of encounter are highly probable.

According to the present theory, the heaving added mass tends to infinity as the frequency tends to zero and the ship response in this region is obtained in practice, by extrapolation based on physical arguments. Ref [27] showed that the heaving added mass for a circular cylinder in a finite depth of fluid is finite when the frequency tends to zero, and compared their results with others. This result suggests that a fictitious depth may be applied in order to obtain useful result.

Another problem in relation to the computation of fluid reactive forces and moments arises at the intersection of the hull surface with the free surface. It is necessary that the wetted surface of the ship should be smooth but it also requires this to apply to the closed double-body formed by the ship and its mirror image. But this condition is violated for almost all vessels near the fore and aft ends, thereby necessitating an additional singularity distribution at the intersection of the wetted surface and the undisturbed water surface.

In spite of the above-mentioned limitations, predictions of the sectional fluid reactive forces and moments by the strip theory have been quite successful with the exception of roll damping which has been obtained recently with a significant success by [28] which included additional roll damping moment caused by appendages etc.

Approximation by Lewis Form

A section contour of the forward part of a hull can be approximated reasonably well by Lewis-form but there exist considerable differences for the aft of the vessel. Ref. [29], calculated the hydrodynamic forces and the ship motions approximating the section contour by the three-parameter family and the comparisons between the results thus obtained and those by Lewis form approximation are shown in Figs. 3, 4 and 5. Although the approximations by the three-parameter family is significantly better than those by Lewis-form, there appears to be no large differences in the calculated values of the damping force and of the added mass except for the regions in which the frequency is large or the motion is small. Thus the Lewis-form approximation is considered adequate except for special hull forms.

Forward Speed Effect

In a strictly theoretical sense, strip theory (except the version of [23] cannot be used for a ship with a steady forward motion. This is because the linearized free-surface condition cannot be satisfied on transverse strips.

Theoretical considerations suggest that the strip theory is valid if the forward velocity is small and the frequency of encounter is high.

In practice, of this has led to the formation of a number of different versions of the strip theory. Some representative versions of the fluid reactive terms in the equations of motion for the longitudinal and transverse motions are given in Tables 1 and 2. Although the differences may seem to be significant at first sight, recent computations by [6] and [39] for the longitudinal motions show that the real differences for the speed and the frequency ranges of interest are insignificant except for the coupling terms between heave and pitch, where the results of [23] show a different trend. The evaluation of the integral term of [23] is difficult and therefore this method is impractical. On the other hand the expressions of [37] and [26] give a better prediction than the others. Fig. 6 shows the comparison between the three approaches [2].

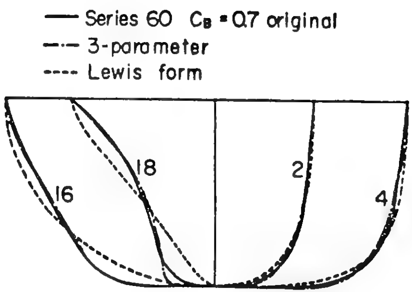


Figure 3

Approximated Body Sections

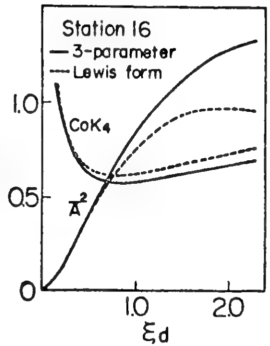


Figure 4

Damping Coefficient

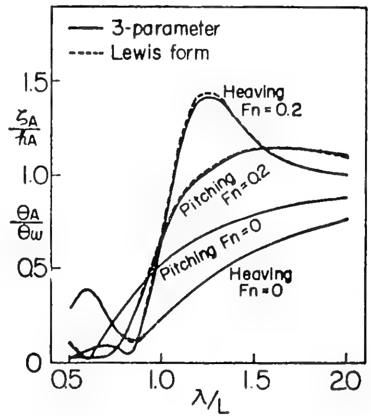


Figure 5

Pitching and Heaving Motions

TABLE 1. **Forward Speed Effect in Various Theories (Vertical Motions)**

Coefficient	Ref [Gerritsma & Beukelman]	Ref [Vugts] Ref [Salvasen, Tuck & Faltinsen]
A_{33}	$\int a_{33}\,dx_1$	$\int a_{33}\,dx_1+\frac{V}{\omega^2}\int\frac{da_{33}}{dx_1}dx_1$
B_{33}	$\int b_{33}\,dx_1-V\int\frac{da_{33}}{dx_1}dx_1$	$\int b_{33}\,dx_1-V\int\frac{da_{33}}{dx_1}dx_1$
C_{33}	$2\rho g\int x_{2w}\,dx$	$2\rho g\int x_{2w}\,dx_1$
A_{35}	$\int a_{33}x_1\,dx_1+\frac{V}{\omega^2}\int b_{33}\,dx_1-\frac{V^2}{\omega^2}\int\frac{da_{33}}{dx_1}dx_1$	$\int a_{33}x_1\,dx_1+\frac{2V}{\omega^2}\int b_{33}\,dx_1-\frac{V^2}{\omega^2}\int\frac{da_{33}}{dx_1}dx_1+\frac{V}{\omega^2}\int\frac{db_{33}}{dx_1}dx_1$
B_{35}	$\int b_{33}x_1\,dx_1-2V\int a_{33}\,dx_1-V\int\frac{da_{33}}{dx_1}x_1\,dx_1$	$\int b_{33}x_1\,dx_1-2V\int a_{33}\,dx_1-V\int\frac{da_{33}}{dx_1}x_1\,dx_1-\frac{V^2}{\omega^2}\int\frac{db_{33}}{dx_1}dx_1$
C_{35}	$2\rho g\int x_{2w}x_1\,dx_1$	$2\rho g\int x_{2w}x_1\,dx_1$
A_{55}	$\int a_{33}x_1^2\,dx_1+\frac{V}{\omega^2}\int b_{33}x_1\,dx_1-\frac{V^2}{\omega^2}\int\frac{da_{33}}{dx_1}x_1\,dx_1$	$\int a_{33}x_1^2\,dx_1+\frac{2V}{\omega^2}\int b_{33}x_1\,dx_1-\frac{V^2}{\omega^2}\int\frac{da_{33}}{dx_1}x_1\,dx_1+\frac{V}{\omega^2}\int\frac{db_{33}}{dx_1}x_1^2\,dx_1$
B_{55}	$\int b_{33}x_1^2\,dx_1-2V\int a_{33}x_1\,dx_1-V\int\frac{da_{33}}{dx_1}dx_1$	$\int b_{33}x_1^2\,dx_1-2V\int a_{33}x_1\,dx_1-V\int\frac{da_{33}}{dx_1}x_1^2\,dx_1-\frac{V^2}{\omega^2}\int\frac{db_{33}}{dx_1}x_1\,dx_1$
C_{55}	$2\rho g\int x_{2w}x_1^2\,dx_1$	$2\rho g\int x_{2w}x_1^2\,dx_1$
A_{53}	$\int a_{33}x_1\,dx_1$	$\int a_{33}x_1\,dx_1+\frac{V}{\omega^2}\int\frac{db_{33}}{dx_1}x_1\,dx_1$
B_{53}	$\int b_{33}x_1\,dx_1-V\int\frac{da_{33}}{dx_1}x_1\,dx_1$	$\int b_{33}x_1\,dx_1-V\int\frac{da_{33}}{dx_1}x_1\,dx_1$
C_{53}	$2\rho g\int x_{2w}x_1\,dx_1$	$2\rho g\int x_{2w}x_1\,dx_1$

TABLE 2. **Forward Speed Effect in Various Strip Theories (Lateral Motions)**

Coefficient	Ref [Kaplan, Sargent & Raff]	Ref [Vugts[49]](Salvasen, Tuck & Faltinsen[50])	Ref [Wang]
A_{22}	$\int a_{22}\,dx_1$	$\int a_{22}\,dx_1+\frac{V}{\omega^2}\int\frac{db_{22}}{dx_1}dx_1$	$\int a_{22}\,dx_1$
B_{22}	$\int b_{22}\,dx_1-V\int\frac{da_{22}}{dx_1}dx_1$	$\int b_{22}\,dx-V\int\frac{da_{22}}{dx_1}dx_1$	$\int b_{22}\,dx_1$
$A_{24}=A_{42}$	$\int a_{24}\,dx_1$	$\int a_{24}\,dx_1+\frac{V}{\omega^2}\int\frac{db_{24}}{dx_1}dx_1$	$\int a_{24}\,dx_1$
$B_{24}=B_{42}$	$\int b_{24}\,dx_1-V\int\frac{da_{24}}{dx_1}dx_1$	$\int b_{24}\,dx_1-V\int\frac{da_{24}}{dx_1}dx_1$	$\int b_{24}\,dx_1$
A_{26}	$\int a_{22}x_1\,dx_1$	$\int a_{22}x_1\,dx_1+\frac{V}{\omega^2}\int\frac{db_{22}}{dx_1}x_1\,dx_1$	$\int a_{22}x_1\,dx_1+\frac{V}{\omega^2}\int b_{22}\,dx_1-\frac{V}{\omega^2}\int I_{22}\,dx_1$
B_{26}	$\int b_{22}x_1\,dx_1-V\int\frac{da_{22}}{dx_1}x_1\,dx_1$	$\int b_{22}x_1\,dx_1-V\int\frac{da_{22}}{dx_1}x_1\,dx_1$	$\int b_{22}x_1\,dx_1-V\int a_{22}\,dx_1-V\int R_{22}\,dx_1$
A_{44}	$\int a_{44}\,dx_1$	$\int a_{44}\,dx_1+\frac{V}{\omega^2}\int\frac{db_{44}}{dx_1}dx_1$	$\int a_{44}\,dx_1$
B_{44}	$\int(b_{44}+b_{44}^*)\,dx_1-V\int\frac{da_{44}}{dx_1}dx_1$	$\int(b_{44}+b_{44}^*)\,dx_1-V\int\frac{da_{44}}{dx_1}dx_1$	$\int b_{44}\,dx_1$
A_{46}	$\int a_{24}x_1\,dx_1$	$\int a_{24}x_1\,dx_1+\frac{V}{\omega^2}\int\frac{db_{24}}{dx_1}x_1\,dx_1$	$\int a_{24}x_1\,dx_1+\frac{V}{\omega^2}\int b_{24}\,dx_1-\frac{V}{\omega^2}\int I_{24}\,dx_1$
B_{46}	$\int b_{24}x_1\,dx_1-V\int\frac{da_{24}}{dx_1}x_1\,dx_1$	$\int b_{24}x_1\,dx_1-V\int\frac{da_{24}}{dx_1}x_1\,dx_1$	$\int b_{24}x_1\,dx_1-V\int a_{24}\,dx_1-V\int R_{24}\,dx_1$
A_{62}	$\int a_{22}x_1\,dx_1$	$\int a_{22}x_1\,dx_1+2\frac{V}{\omega^2}\int b_{22}\,dx_1+\frac{V}{\omega^2}\int\frac{db_{22}}{dx_1}x_1\,dx_1-\frac{V^2}{\omega^2}\int\frac{da_{22}}{dx_1}dx_1$	$\int a_{22}x_1\,dx_1-\frac{V}{\omega^2}\int b_{22}\,dx_1+\frac{V}{\omega^2}\int I_{22}\,dx_1$
B_{62}	$\int b_{22}x_1\,dx_1-V\int\frac{da_{22}}{dx_1}x_1\,dx_1$	$\int b_{22}\,dx_1-2V\int a_{22}\,dx_1-V\int\frac{da_{22}}{dx_1}x_1\,dx_1-\frac{V^2}{\omega^2}\int\frac{db_{22}}{dx_1}dx_1$	$\int b_{22}x_1\,dx_1+V\int a_{22}\,dx_1+V\int R_{22}\,dx_1$
A_{64}	$\int a_{24}x_1\,dx_1$	$\int a_{24}x_1\,dx_1+2\frac{V}{\omega^2}\int b_{24}\,dx_1+\frac{V}{\omega^2}\int\frac{db_{24}}{\partial x_1}x_1\,dx_1-\frac{V^2}{\omega^2}\int\frac{da_{24}}{dx_1}dx_1$	$\int a_{24}x_1\,dx_1-\frac{V}{\omega^2}\int b_{24}\,dx_1+\frac{V}{\omega^2}\int I_{24}\,dx_1$
B_{64}	$\int b_{24}x_1\,dx_1-V\int\frac{da_{24}}{dx_1}x_1\,dx_1$	$\int b_{24}x_1\,dx_1-2V\int a_{24}\,dx_1-V\int\frac{da_{24}}{dx_1}x_1\,dx_1-\frac{V^2}{\omega^2}\int\frac{db_{24}}{dx_1}dx_1$	$\int b_{24}x_1\,dx_1+V\int a_{24}\,dx_1+V\int R_{24}\,dx_1$
A_{66}	$\int a_{22}x_1^2\,dx_1$	$\int a_{22}x_1^2dx_1+2\frac{V}{\omega^2}\int b_{22}x_1\,dx_1+\frac{V}{\omega^2}\int\frac{db_{22}}{dx_1}x_1^2dx_1-\frac{V^2}{\omega^2}\int\frac{da_{22}}{dx_1}x_1\,dx_1$	$\int a_{22}x_1^2\,dx_1+\frac{V^2}{\omega^2}\int a_{22}\,dx_1+\frac{V^2}{\omega^2}\int R_{22}\,dx_1$
B_{66}	$\int b_{22}x_1^2\,dx_1-2V\int a_{22}x_1\,dx_1-V\int\frac{da_{22}}{dx_1}x_1^2\,dx_1$	$\int b_{22}x_1^2dx_1-2V\int a_{22}x_1\,dx_1-V\int\frac{da_{22}}{dx_1}x_1\,dx_1-\frac{V^2}{\omega^2}\int\frac{db_{22}}{dx_1}x_1\,dx_1$	$\int b_{22}x_1^2\,dx_1+\frac{V^2}{\omega^2}\int b_{22}\,dx_1-\frac{V^2}{\omega^2}\int I_{22}\,dx_1$

**This form additionally give* $C_{26}=V\,B_{22}$, $C_{46}=V\,B_{42}$, $C_{66}\doteq V\,B_{62}$ *which are equal to zero in other versions.*

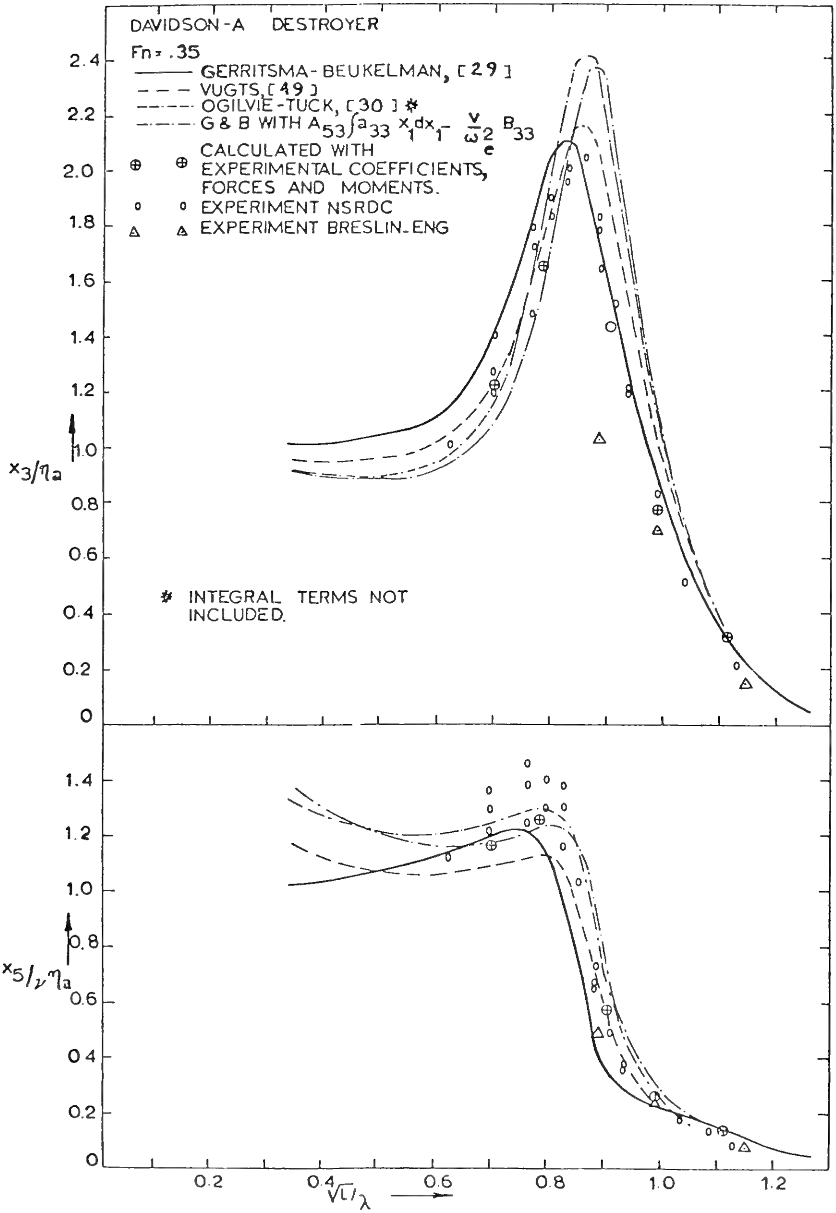


Figure 6

Heaving and Pitching Amplitudes for $F_n = 0.35$

Another point of interest regarding the effect of forward motion is about the value of the parameter $\gamma \frac{\omega V}{g}$.

If $\gamma > 0.25$, then the waves generated by the ship oscillation travel more slowly than the vessel and hence are confined in the sector behind the ship, whereas if $\gamma < 0.25$ the waves travel faster than and ahead of the vessel. As indicated by [36] and [37] and recently [38] this feature is not a theoretical anomaly because experimental measurements around the parameter value $\gamma = 0.25$ show some irregularities and scatter.

The computation of the wave excitation is carried out either a) from the Froude-Krylov theory by using the defined relative motion between the ship and the wave, as given in [8], [15], [1] or 2) from the diffraction theory by using Haskind-Newman relationship [23], [37], [26].

Consideration of the forward speed in the coefficients of equations of motion is another source of difference between various strip theories.

Wave Exciting Forces and Moments

There are at present two methods for the calculation of wave exciting forces and moments, namely:

1) Korvin-Kroukovsky type of approach

2) Use of Haskind-Newman formulae

The first method makes use of the relative motion concept and in a way employs Froude-Krylov theory combined with this relative motion definition. Consequently, this approximate method is valid only for the medium range of frequencies; for the short waves the Froude-Krylov hypothesis is not valid whereas for the very long waves the strip theory fails.

Recently [18] experimentally obtained the magnitude and the distribution of wave exciting forces on a segmented tanker model and showed this difference between the theoretical predictions of this type and the experimental measurements for the short wavelengths as illustrated in Figs. 7 and 8. The good

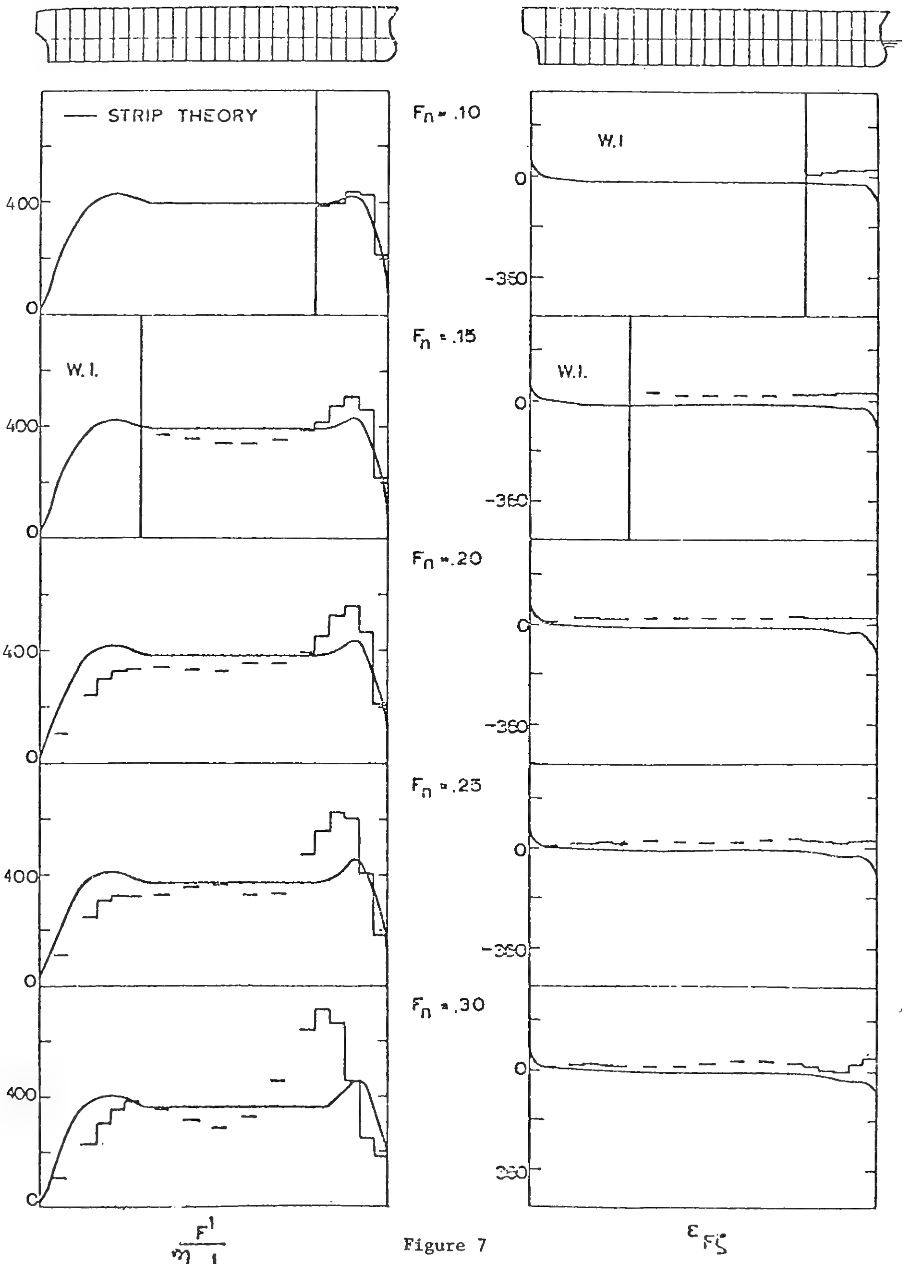


Figure 7

Sectional Waves Forces $\frac{L_w}{L} = 1.00$

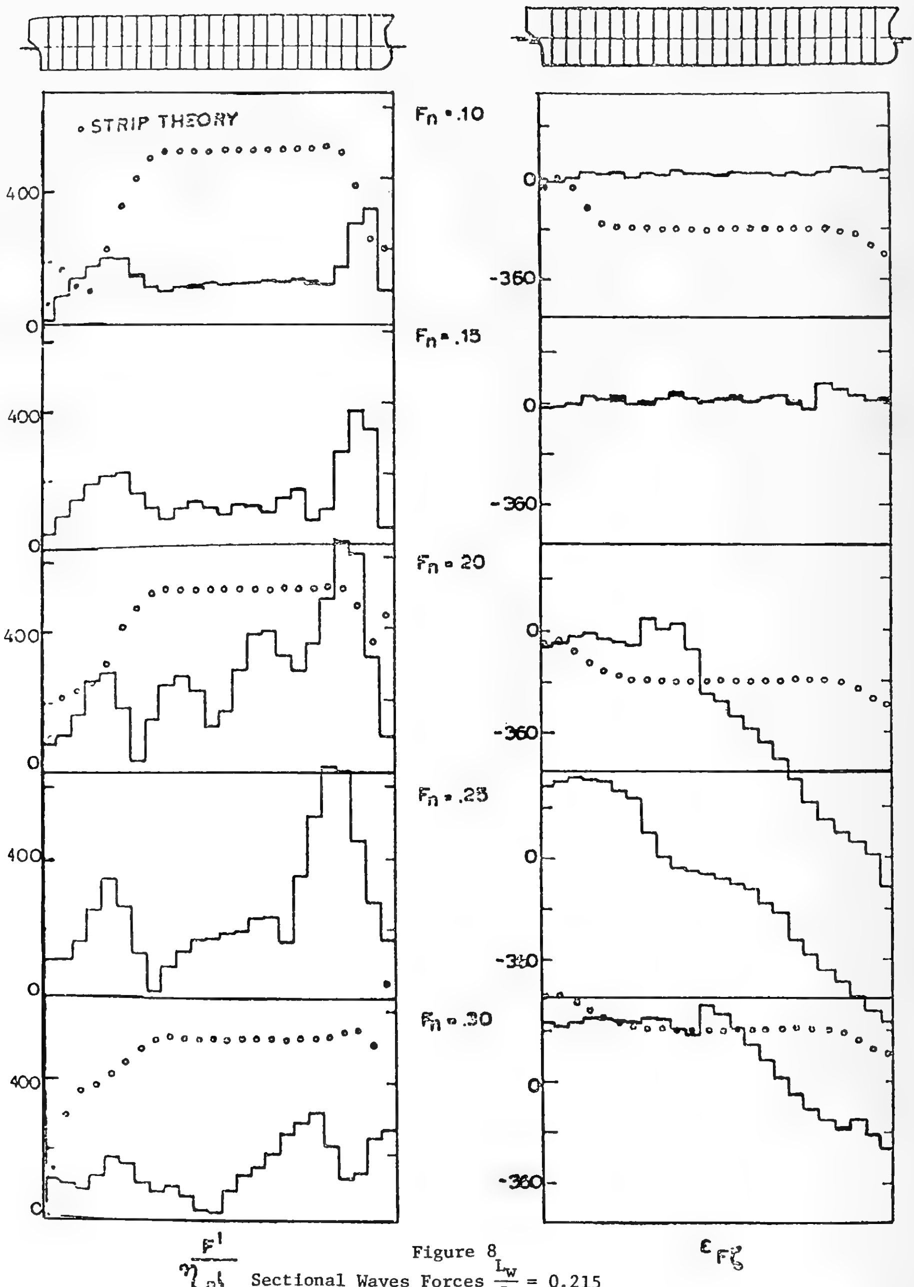


Figure 8

Sectional Waves Forces $\frac{L_w}{L} = 0.215$

agreement is due to the fact that the longitudinal ship motion amplitudes at high and very low frequency ranges are insignificant. But in case of springing and mooring problems, where the high and very low frequencies are involved respectively, this kind of engineering approach may not be satisfactory.

Use of the Haskind-Newman relationship in calculating the wave exciting forces is a useful method so far as avoiding the solution of the diffraction problem, while calculating the forces and moments created by the diffraction of waves. So the approach is, in a way, equivalent to the solution of the wave diffraction problem. The main difference is due to the evaluation of the Haskind-Newman relationship. The original approach requires that in the evaluation of diffraction force (moment) the perturbation potential ϕ^k is the three-dimensional potential satisfying the same state equations and radiation condition as the diffraction potential, whereas in "strip theory" only the two-dimensional potential is available which satisfied different state equations and radiation condition.

Newman [42], however, proved that for the high-frequency range this difference does not cause any significant error.

For longer waves McCreight [43] recently developed a relationship similar to that of Haskind-Newman for the computation of wave exciting forces.

As the numerical evaluation of the wave excitation by the Haskind-Newman relationship is not difficult, this approach should be preferred instead of the previous approach as it eliminates the somewhat arbitrary choice on the relative motion between the ship and the waves.

It should, however, be mentioned that this approach also fails in very long waves because of the breakdown of the strip theory. For such long waves the approach adopted by [11] is preferrable as it includes the effect of wave deformation in an approximate way.

Approximate Calculation of the Wave Force

In the strip method, when calculating the diffraction force, one uses approximations in which the orbital velocity of the regular wave is represented by the value in the mean draft. As to the circular cylinder subjected to transverse waves, there is exact solution of [31] and [19] compared them with the approximate solutions with reference to the force Z_r which is proportional to the orbital acceleration of the wave and the force Z_i which is proportional to the orbital velocity. Fig. 9 shows the comparison whereas Fig. 10 shows the similar calculation for a circular cylinder subjected to longitudinal waves.

In case of transverse waves, there are considerable differences in the regions of high frequency but in case of longitudinal waves there is no noticeable difference. However, in case of longitudinal waves, final conclusion cannot be drawn at present since it includes the problem of the three-dimensional effect.

Critical Review of Strip Theory

It must be mentioned that the usefulness of the strip theory approach, especially for longitudinal motions and associated predictions, has surpassed the imagination of many theorists and engineers.

Predictions for the transverse motions and the associated effects, however, were not so good because of the difficulties arising from the modeling and computation of the roll associated parameters. But recent efforts of [28] have been proven to be very successful.

The difficulties in connection with the transom stern (or more generally blunt-ended) ships have been removed by the inclusion of end effects.

In utilizing the results of strip theory one should always remember that this approach is valid as long as:

1) The vessel is slender, smooth and the geometrical variations in the longitudinal direction are gradual, and not abrupt.

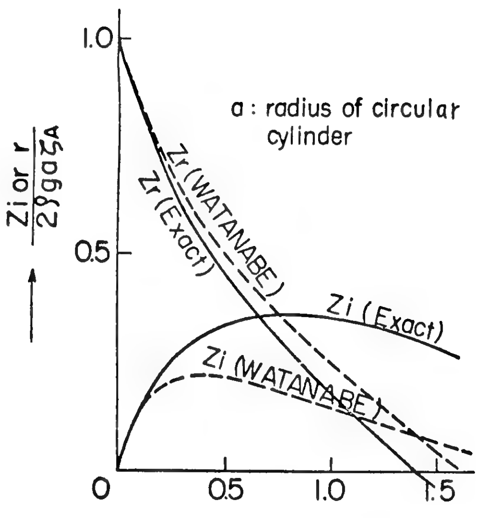


Figure 9

Approximate Calculation of Waves Forces (Lateral)

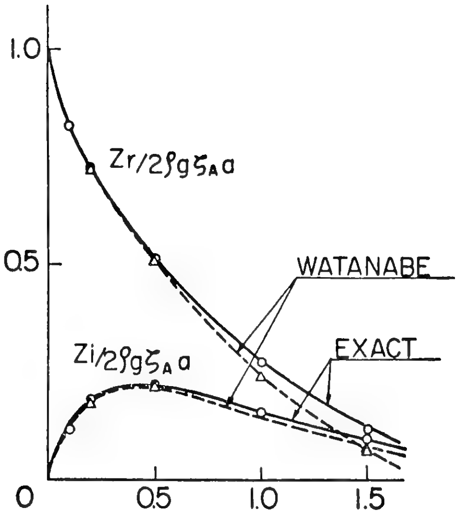


Figure 10

Approximate Calculation of Wave Forces (Longitudinal)

2) Frequencies are high

3) Forward speed (or Froude number) is low.

If these conditions are not satisfied, experimentally-determined transfer functions should be used for the prediction of ship motions in an irregular seaway.

The accuracy of the strip method is to be investigated with regard to longitudinal ship motions, namely, heaving and pitching. The items which should be studied are listed as follows:

1) Three-dimensional effect

2) Non-linear effect

3) Approximation by Lewis form

4) Viscous effect

5) Approximate calculation of wave forces

6) Displacement effect

Thus, from the mathematical point of view the limitations and inaccuracies of the classical methods for the ship wave problems are for the following assumptions:

1) Viscous/wave interactions

The interactions between the viscous effects and the gravity waves are assumed to be small so that potential flow theory can be used for predicting ship motions.

In case of longitudinal motions (i.e. heaving and pitching) the level of wavemaking is higher than in case of rolling motion and it can be justifyably said that the viscous effect caused by the bilge-keel etc. occur seldom except for the region in which the frequency is high [33].

Considering viscous effect another important effect is that of dynamic lift [30], [40].

In case of longitudinal motions there are two cases which can be considered as dynamic lift, namely,

a) One is that proportional to the pitching angle

b) The other is that proportional to the ratio of the velocity of heaving to the advance speed.

The latter may contribute considerably to the damping force.

2) Linearization of free-surface conditions

It is assumed that the wave slopes of the incoming as well as ship generated waves are sufficiently small so that the non-linear free-surface boundary conditions can be replaced by the linearized condition.

Ref [45] dealt with hull-shape non-linearity and showed that the calculated amplitude of motion differs considerably from that of the linear calculation. Ref [44], by calculating the second-order approximation of the diffraction problem regarding the two-dimensional body, showed that when the period of motion is short there are considerable differences from the first-order approximation. Also Ref. [30] discussed about the hydrodynamic force which is proportional to the product of the perturbed velocity due to the forward velocity of the vessel and that due to ship motions and calculated its effect on the motions as shown in Fig. 11 and 12.

Ref. [29] concludes that the ship motion calculations must take into account the non-linearity effect which is extremely important for slamming and deck wetness.

In the strip method, all the perturbed potential which are more than the square are neglected.

In the slender body theory, aside from the effect already stated in the non-linear effect as discussed above, the existence of the effect of the hydrodynamic force which is proportional to the product of the perturbed

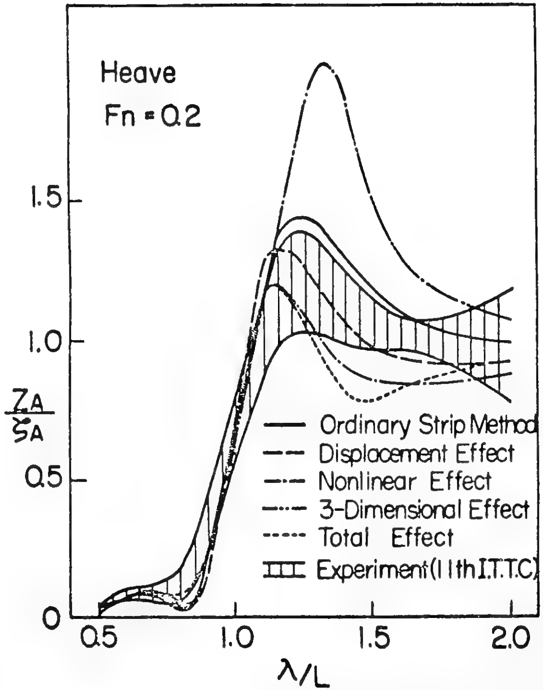


Figure 11

Heaving Motion

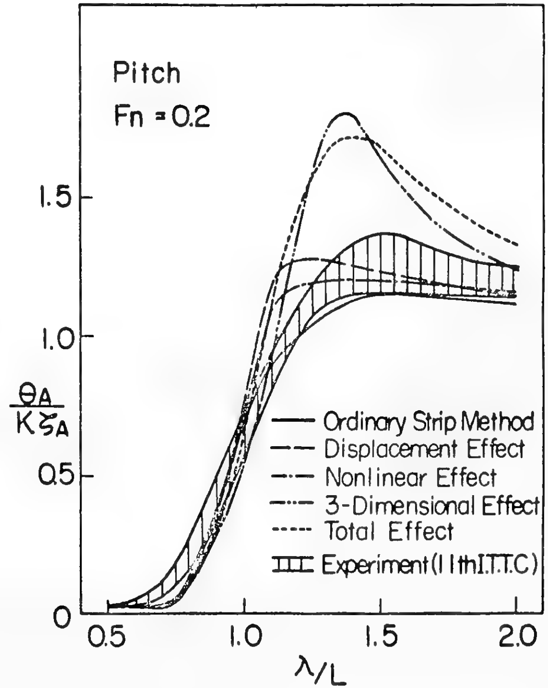


Figure 12

Pitching Motion

velocity of the forward motion, and the oscillatory displacement are considered. Ref. [29] call this as "displacement effect" and calculated the effect on the motions.

Figs. 11 and 12 show the three-dimensional effect, non-linear effect and displacement effect for the motion of series 60, $C_B = 0.70$. The respective effects are significant, but the agreement with the experimental value in the totally corrected calculation is still unsatisfactory.

3) Small amplitude ship motions

Here it is assumed that the unsteady body displacements are small so that the hull boundary condition can be satisfied at the mean position of the ship.

Large-Amplitude Ship Motion

In linear ship-motion theories, it is assumed not only that the free-surface conditions can be linearized, but also that the ship displacements are small relative to the ship dimensions. The exact body boundary condition then can be approximated by satisfying it at the mean position of the hull. However, ship motions are not always small. In fact, they can be on the order of magnitude of the ship dimensions even in typically moderate sea conditions.

So a method should be developed for predicting large amplitude ship motions. This is a difficult non-linear problem both for the boundary conditions at the hull or at the free-surface. Non-linearities resulting from the large amplitude rolling motion influence both the hydrodynamic problem and the equations of motion.

In the hydrodynamic problem, the use of average wetted surface is no longer justified as the geometry of the wetted surface changes significantly during one cycle of motion. This means that the added inertia is a function of the angular position and systematic experiments conducted by [37] indicate that the added inertia of rolling moti n varies with the amplitude of motion.

Recently, [46] used a quasi-steady treatment and calculated the hydrodynamic properties at different angles of heel. His treatment may be useful at very low frequency range.

In the dynamic problem, two additional complications arise:

1) The effect of non-linearity of rolling motion is not confined to the equation of motion of this mode alone, but also makes the coupled sway-roll-yaw equations non-linear.

2) The existence of the position-dependent added inertia gives rise to the existence of additional velocity-dependent terms which may take both positive and negative values. Some of these problems have been considered already by [24], [21].

However, if it is assumed that the frequency of ship motions is sufficiently small (which means that the slope of the body generated waves will also be small) and that the slope of the incident waves is fairly small, then it may be valid to linearize the free-surface conditions even for large body displacements. There are some occasions when the oscillation frequency is low, e.g., ship motions in following and quartering seas, roll motions in beam seas, pitching and heaving in long head waves.

Chapman [5] is developing such a method (JSR vol. 23, No. 1 also).

[3] has developed a three dimensional numerical method for predicting ship motions which solves the complete three-dimensional hydrodynamics problem and satisfies correctly all forward speed effects.

The hydrodynamics problem is solved by distributing three-dimensional oscillating (Kelvin) sources (which satisfy the linearized free-surface boundary condtion) on the wetted hull surface. The strength of these singularities is obtained by solving the hull boundary condition. It is assumed that ship motions are small enough that the hull boundary condition can be satisfied at the mean position of the hull.

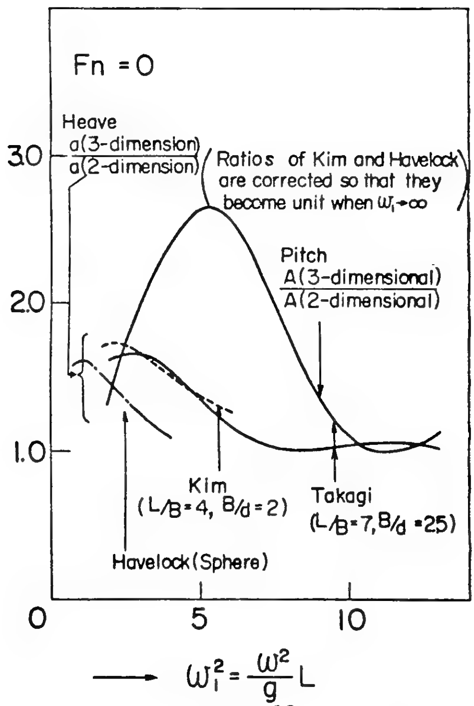


Figure 13

Three-dimensional Effect (Added Mass)

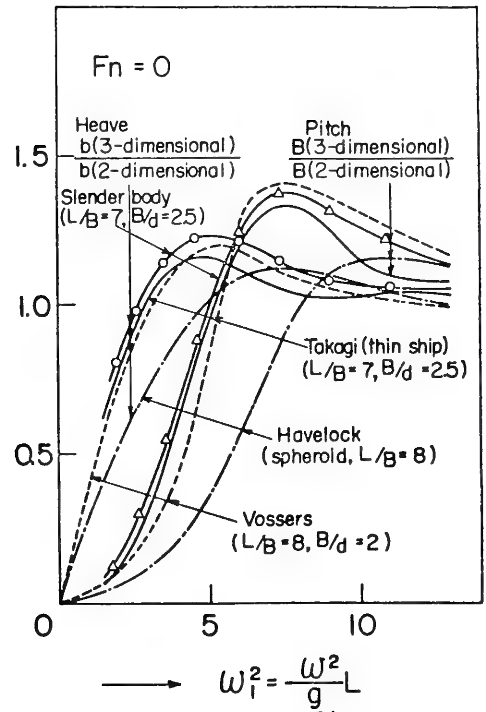


Figure 14

Three-dimensional Effect (Damping Coefficient)

Fig.15a and 15b show some results of added mass and damping coefficients which, next to the exciting forces, are the most important hydrodynamic ingredients needed in predicting ship motions and wave induced loads.

It is seen that Chang's predictions agree well with the experimental results throughout the frequency range whereas the strip-theory results only agree well with the experimental values in the high frequency range.

A complete evaluation of the ship motions by Chang's method is now in progress at DTNSRDC.

Chapman [4] has shown that by applying slender body theory, the three-dimensional problem of a ship oscillating in the lateral modes of motion (sway and yaw) can be reduced to a series of transient unsteady two-dimensional flow problems in the transverse plane.

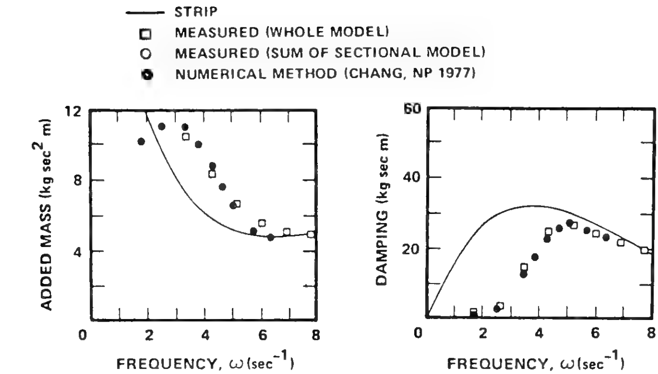
Fig.16 shows some of his results. In some cases Chapman's results are even more accurate than Chang's because Chapman takes into account some non-linear free-surface effects.

4) Hull form approximation

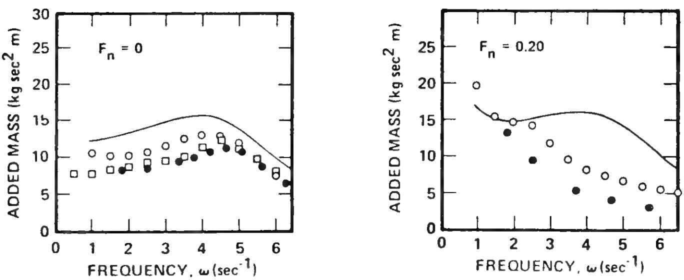
Exact hull boundary condition is replaced by some approximate condition and so the theories are called, thin-ship theory, strip theory, slender body theory, etc.

Three-dimensional effect

In the strip method, the three-dimensional ship hydrodynamics problem is replaced by a summation of two-dimensional sectional problems and the forward-speed effects are only satisfied approximately. The strip theory provides good results for heaving, pitching motions in moderate seas and moderate ship speeds for most conventional hull forms; however, the method gives inadequate results for low frequencies, higher ship speeds, local pressure distributions and for sway and yaw motions. The forward speed limitations is the most severe restriction for naval applications.



a) - Pitch Added-Mass and Damping Coefficients at $F_n = 0$



b) - Yaw Added-Mass Coefficients

Figure 15

Added Mass and Damping Coefficients for Series 60

(B = 0.70)

In case of advancing ship [29] found three-dimensional correction factor for each coefficient of the equations of motion of the ordinary strip method by the thin ship theory of [ ]. In addition they corrected the coefficients of the equations of motions using the assumption of the slender body to which normal ships are subjected. The calculated results of the said three-dimensional effect, together with the results as mentioned above, are illustrated in Fig. 1 and 2 which show that the three-dimensional effect is rather significant. Thus, it is necessary to consider this effect in ship motion calculations.

Also one should be careful in obtaining the longitudinal derivations of the two-dimensional (sectional) added mass and damping coefficients as these may cause significant errors in the calculated force distribution for a ship with forward speed. One should therefore adopt a smoothing procedure before the numerical derivation.

For critical reviews of strip theories one should refer to [30] and [48].

In order to overcome some conceptual and practical shortcomings of the strip theory various attempts have been made to include the effects of three-dimensionality.

However, calculations have shown that these corrections did not provide an improved accuracy. In fact, in most of the cases the predictions become worse when the three-dimensional corrections were applied. Only the technique proposed by [10] may be acceptable. This method proposed an interesting quasi-three-dimensional method which, however, did not receive wide acceptance because of the more complicated calculations needed.

After investigations of all the topics as mentioned above, [29] made the following observations:

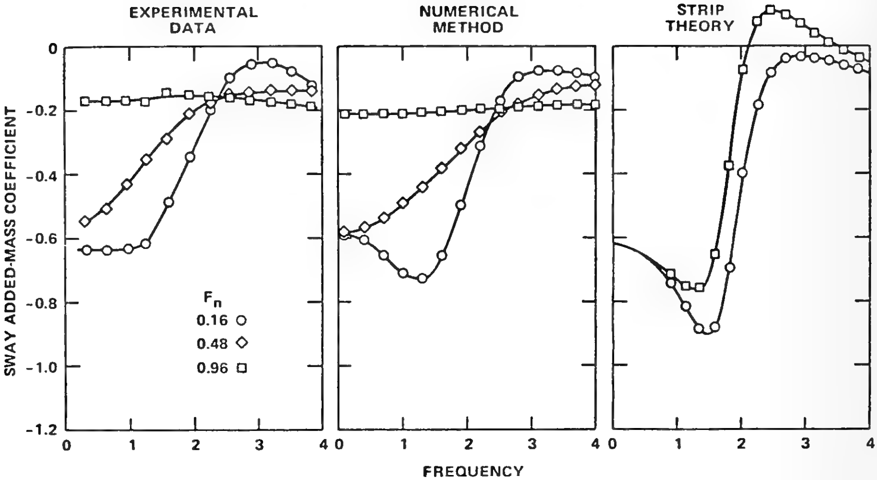


Figure 16

Flat Plate Survey Added Mass Coefficients

1) The study of the three-dimensional correction is found to be rather significant. Therefore practical correction factors must be developed for prediction of ship motions.

2) The effects of the dynamic lift on the hull should be examined experimentally and theoretically.

3) Investigations should be performed with regard to non-linear effect including the displacement effect, etc. Both theoretical and experimental studies should be the basis of this investigation.

Further Investigations

1) Combined action of steady and unsteady excitation:

The equations of motion, which are now in use, are valid in the frequency domain, and therefore, if there is also a steady force, for example, wind, rudder and drift forces, acting on the ship, these equations are no more useful.

2) Low frequency motions:

As it is well-known, even for the heaving motion, the results for low frequencies may not be realistic. A knowledge in the low frequency range is generally very important, especially for the prediction of lateral motions (i.e. sway, roll and yaw) in following waves, because of intact and course-keeping stability of ships. From its basic assumption it is clear that the strip theory may not be suitable for this purpose as three-dimensional effects as well as viscous effects will be significant.

3) Impact pressures:

Determination of pressures during slamming is important in avoiding bow damage and in determining the hull bending moment and springing. The theoretical results are still not satisfactory because of various simplifications made in the problem formulation.

Since a solution which should consider the effects of compressibility, viscosity and three-dimensionality is very difficult one may split the problem in a number of stages such as contact, immersion and cavity formation (i.e. formation of inner free surface and spraying) and then considering each stage with different assumptions.

4) Sea loads on a vibrating ship:

As it is known, the presently available seakeeping theories are valid for rigid body motions. Therefore, as the frequency increases, the wave damping vanishes and the natural frequencies obtained by using so-determined added masses does provide correct results.

5) Wave forces on discontinuous structures

In the present theories it is assumed that the change in the body geometry is gradual. If, however, there are abrupt ends as in the case of a barge, the flow around the ends will be different from a potential flow due to vortex shedding. As a result, the forces exerted by the fluid on the body may differ considerably from the results obtained from potential theory. For these types of forms also the effects of viscosity should be included in the calculations.

6) Interaction problems:

When there is more than one body and each is in close proximity to the other, the flow field around each will differ from the case where the other bodies are not present. Present methods of super-position of the flow fields can provide reasonable approximation, provided the distance between the bodies is large compared to the characteristic dimension of the largest body. For configurations where the bodies are close, interaction effects ought to be considered more carefully.

1 Betts, Bishop, Price, Trans. RINA, 1977

2 Beukelman, W., "Pitch and Heave Characteristics of a Destroyer" ISP, 1970

3 Chang, Poc. 2nd Numerical Hydrodynamics Conference, 1977

4 Chapman, Proc. 2nd Numerical Hydrodynamic Conf. NP, 1977, 75

5 Chapman, J.S.R. (Journal Ship Research)

6 Faltinsen, J.S.R. '74

7 Frank, NSRDC Rep, 1967

8 Gerritsma, J., Beukelman, W., "Distribution of the Hydrodynamic Forces on a Heaving and Pitching Ship Model in Still Water", ISP, 1964

9 Grim, O., Trans. STG, 1953

10 Grim, O., A Method for More Precise Computation of Hearing and Pitching Motions Both in Smooth Water and in Waves. 3rd ONR Symposium on Naval Hydrodynamics, Schereningen, 1960

11 Grim, Takaishi, Schiff und Hafen, 1965

12 Grim, O., Schenzle, Trans. STG, '68, '69

13 Hearn, BSRA, 1977, Singularity, Computations Associated with Three-Dimensional Free-Surface Hydrodynamic Problems", Technical Memorandum 526.

14 Hwang, ISP, '69 , Added Mass of Two Dimensional Cylinders with the Sections of Straight Frames Oscillating Veritcally in a Fee Surface, ISP, '69.

15 Kaplan, Sargent, Raff, SSC Rep. 197, 1969

16 Landweber, Macagno, JSR, 1957, 1959

17 Lewis, F. M., "The Inertia of the Water Surrounding a Vibrating Ship," Trans. SNAME, 1929.

18 Moeyes, '76

19 Motora, Strip wise Calculation of Hydrodynamic Forces due to Beam Series, J.S.R. '64

20 Newman, J. N., "The Exciting Forces on a Moving Body in Waves", J.S.R., 1965

21 Odabasi, A.Y., "Principle of Varying Action and Theory of Ship Motions", T.H. Scheepsbouwkunde, Rep. 365-M, Delft, 1972.

22 Odabasi, Hearn, Trans. NECI, '78 , "Seakeeping Theories: What is the Choice?

23 Ogilvie, T. F., Tuck, E. O., "A Rational Strip Theory for Ship Motions", Dept. of Nav. Arch., & Mar. Eng'r. University of Michigan Rep. 013, 1969

24 Paulling, J. R., Rosenberg, R. M. "On Unstable Ship Motions Resulting from Nonlinear Coupling., J.S.R. '59.

25 Salvesen, NSRDC Report, 1978

26 Salvesen, N., Tuck, Faltinsen, "Ship Motions and Sea Loads", Trans. SNAME, 1970

27 Sayer, Ursell, Proc. 11th ONR Symp. on Nav. Hydrodyn. '76

28 Schmitke, R. T., Trans, SNAME, 1978

29 Takagi, M., Ganno, M., "On the Accuracy of the Strip Method Used for a Calculation of Longitudinal Ship Motions in Waves", 12th ITTC, 1969 , 72, 75

30 Takagi, M., "An Examination of the Ship Motion Theory as Compared with Experiments", International Symposium on the Dynamics of Marine Vehicles and Structures in Waves, London, 1974

31 Tamura, SNAWJ, Vol. 26

32 Tasai, F., 1959,"On the Damping Force and Added Mass of Ships Heaving and Pitching"., Rep. Res. Inst for Appl. Mech., Kyushu Univ,

33 Tasai, F., 1961 , Hydrodynamic Force and Moment Produced by Swaying and Rolling Oscillation of Cylinders on the Free Surface., Rep of Res Inst for Appl Mech Kyushu, Univ.

34 Ursell, 1949, On the Heaving Motion of a Circular Cylinder on the Surface of a Fluid., Quart. J. of Mech. & Appl. Maths.

35 Ursell, Proc. Royal Soc. 1953, "Short Surface Waves Due to an Oscillating Immersed Body", Proc. Royal Suc.

36 Vossers, G., "Behaviour of Ships in Waves" Int. Shipbuilding Progress, 1961

37 Vugts, '70 , The Hydrodynamic Forces and Ship Motions in Waves", Doctoral Dissertation, Uitgeveri, Waltman, Delft.

38 Wahab, R., Vink, J. H., "Wave Induced Motions and Loads on Ships in Oblique Waves", ISP, 1975

39 Wang, JSR, '76

40 Weinblum, G., ISP, '59

41 Wendel, K., "Hydrodynamische Massen und hydrodynamische Massen-tragheitsmomente" Trans. SIG, '50

42 Newman, J., J.S.R., 1970

43 McCreight, MIT, 1973 , "Exciting Forces on a Moving Ship in Waves", Doctoral Dissertation, MIT.

44 Lee, C. H., U. of Cal., '68, "The Second-Order THeory of Heaving Cylinders in a Free Surface" JSR-'68.

45 Parissis, MIT, 1964

46 Kobayashi, "Hydrodynamic Forces and Moments Acting on Two-Dimensional Asymmetrical Bodies", Proc. Int. Conf. on Stab of Ships & Ocean Vehicles,Glasgow, '7

47 Hanaoka, T., et al., Researches on Seakeeping Qualities of Ships in Japan, 60th Anniversary Series, Vol. 8, SNAJ, 1963.

48 Gerristsma, 1976 ,"A Note on the Application of Ship Motion Theory". Schiffstechnik.

49 Hearn, G. E., Alternative Methods of Evaluating the Green's Function in Three-dimensional Ship-Wave Problems. J.S.R., '77

50 Hearn, G.E. A Note on the Determination of the Motion Responses of an Arbitrary Shaped Body, BSRA, TM 519, 1977.

METHODS FOR CALCULATING SLOWLY-VARYING DRIFT FORCES - AN OVERVIEW

by

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for presentation at

CEL Mooring Dynamics Seminar

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INTRODUCTION

A body in waves is subject to not only first order zero mean forces proportional to wave amplitude but also to second order forces which are proportional to the square of the amplitude. In the case of a body free to move, the motions will result in hydrodynamic reaction forces which have both first and second order components.

The second order forces are quite small compared to the first order forces, and in seakeeping work are generally neglected. In some circumstances, however, they can cause significant effects, and must be accounted for. If a force or moment component is not opposed by a corresponding hydrostatic or other force or moment, over a long time span large motions can result, as in surge, sway, and yaw of a ship at zero speed (Maruo, 1960) or heave for a body submerged but near the surface (Newman and Lee, 1971). Added resistance in waves is also a problem for moving ships (Strom-Tejsen et al., 1973). Semi-Submersibles, which have a small water-plane area relative to the volume, are frequently subject to a tilt in waves which is believed to be caused by second order forces (Numata et al., 1976). Moored vessels, with low natural frequencies of the vessel-mooring line system, can be excited at resonance by the low-frequency components of the second order force (see for example, Hsu and Blenkarn, 1970).

In the last decade, major advances in the understanding of and ability to predict these second order wave induced forces have been made. In this paper, developments in the specific area of slowly-varying drift forces will be reviewed.

EXACT REPRESENTATION OF THE SECOND ORDER FORCE

If the seaway is assumed to have a discrete spectrum, the wave height can be represented as

$$\zeta(t) = \mathrm{Re} \sum_m A_m e^{i\omega_m t} \tag{1}$$

where A_m is the complex amplitude (with random phase) of the wave component of frequency ω_m. The force on a body, through second order, can be represented as

$$F(t) = F^{(1)}(t) + F^{(2)}(t) \tag{2}$$

where the first order force is

$$F^{(1)}(t) = \mathrm{Re} \sum_m A_m H^{(1)}(\omega_m) e^{i\omega_m t} \tag{3}$$

and the second order force is

$$F^{(2)}(t) = \mathrm{Re} \sum_m \sum_n A_m A_n H^{(2)}(\omega_m,\omega_n) e^{i(\omega_m+\omega_n)t}$$

$$+ \mathrm{Re} \sum_m \sum_n A_m A_n^* H^{(2)}(\omega_m,-\omega_n) e^{i(\omega_m-\omega_n)t} \tag{4}$$

In these $H^{(1)}$ and $H^{(2)}$ are the first and second order transfer functions respectively and are complex.

For the usual representation of the seaway as a continuous spectrum, there are analogous representations of the forces as single and double integrals over frequency (Neal, 1974). In a digital time domain simulation, the seaway will be represented as a discrete spectrum, and the form used here is appropriate. The continuous form can be derived from a general representation of a nonlinear functional as a Volterra functional series. Neal (1974) gave a discussion of this and further references. An important requirement for the validity of this form is that of the continuity of the functional relationship between the input and the output. This means that such phenomena as hysterisis loops may not be modelled by this form of representation.

Two special cases are of interest. The first is the second order response to two waves of complex amplitude A_1 and A_2 and frequencies ω_1 and ω_2

$$F^{(2)}(t) = \mathrm{Re}\{A_1A_1\ H^{(2)}(\omega_1,\omega_1)e^{2i\omega_1 t}$$

$$+ A_2A_2\ H^{(2)}(\omega_2,\omega_2)e^{2i\omega_2 t}$$

$$+ 2\ A_1A_2\ H^{(2)}(\omega_1,\omega_2)e^{i(\omega_1+\omega_2)t}$$

$$+ A_1A_1^*\ H^{(2)}(\omega_1,-\omega_1) + A_2A_2^*\ H^{(2)}(\omega_2,-\omega_2)$$

$$+ 2\ A_1A_2^*\ H^{(2)}(\omega_1,-\omega_2)e^{i(\omega_1-\omega_2)t}\} \tag{5}$$

It is clear that if we can calculate the response to two waves for all combinations of frequencies, we have all the information required to predict the second order force in a random sea.

The other special case of interest is the second order response to a regular wave of amplitude A and frequency ω

$$F^{(2)}(t) = \mathrm{Re}\{A^2\ H^{(2)}(\omega,\omega)e^{2i\omega t} + A\ A^*\ H(\omega,-\omega)\} \tag{6}$$

The second term gives the well-known steady drift forces in regular waves.

APPROXIMATION OF SLOWLY-VARYING SECOND ORDER FORCE

If we neglect the first term of $F^{(2)}$, which represents the high frequency components, we are left with the low-frequency second order force

$$F_L^{(2)}(t) = \mathrm{Re} \sum_m \sum_n A_m A_n^* H^{(2)}(\omega_m, -\omega_n) e^{i(\omega_m - \omega_n)t} \tag{7}$$

If the wave frequencies are evenly spaced, with

$$\omega_m = m\Delta\omega \tag{8}$$

this can be written the computationally more convenient form

$$F_L^{(2)} = \mathrm{Re} \sum_{m=0}^{M-1} J_m \cdot e^{im\Delta\omega t} \tag{9}$$

where J_m depends on A_m, A_n and $H^{(2)}(\omega_m, \omega_n)$. This expression for the slowly-varying force is in the form of a single summation and can be evaluated more rapidly than the previous form.

Hsu and Blenkarn (1970) proposed an approximate method for calculating the slowly-varying force due to a random seaway. Each successive wave is assumed to apply a force corresponding to a regular wave of the same height and period. They show good comparisons for predicted and measured surge for two cases of moored vessels, which supports this intuitive approach. Pijfers and Brink (1977) have developed a more sophisticated version of this approach, in terms of the square of the wave envelope and the regular wave drift force at a "momentary frequency," which they define.

Newman (1974) proposed an approximation based on the assumption

$$H^{(2)}(\omega_m, -\omega_n) = H^{(2)}(\omega_m, -\omega_m) + O(\omega_m - \omega_n) \tag{10}$$

That is, the contributions to the slowly-varying forces come from wave pairs of nearly the same frequency, and in this case, the second order transfer function can be adequately approximated by the regular wave steady second order force. Newman showed that this was a good approximation to the second order pressure underneath a system of random waves, but that a further approximation to convert the formula into a single summation (but not that given above), did not yield such good results. Loken and Olsen (1979) compared results of this approach with results of the full equation and found generally good results.

CALCULATION OF SECOND ORDER TRANSFER FUNCTIONS

So far the problem of obtaining the second order transfer functions has not been discussed. These may be obtained either from experiment or calculation.

Measurement of drift forces in regular waves is difficult, and two-wave tests are even more difficult. One problem is that the forces involved are small compared to the first order forces. Dalzell (1974) has applied cross-bi-spectral analysis to obtain this data for added resistance. This involves very long test runs and equally lengthy statistical analysis.

Calculation of the steady drift force is relatively easy, since this may be obtained using the first order velocity potentials for the incident, diffracted, and radiated waves. Maruo (1960) and Newman (1967) have derived formulas for the steady drift force and moment based on conservation of momentum.

These results can be applied using potentials obtained by a variety of methods. For example, Faltinsen and Loken (1978) use a strip theory potential including a correct "Helm holz" diffraction potential in Newman's result, Molin (1979) applied Maruo's formula using a potential obtained from a 3-D finite element approach, and Faltinsen and Michelson (1974) used a potential from a 3-D panel method with an intermediate result of Newman (1967). Because the steady drift forces are to be covered elsewhere, we will not consider this topic further here.

To calculate the second order force due to the waves of different frequency, it is necessary to integrate the second order pressure over the hull, taking account of the change in hull shape due to the wave elevation and the motion of the hull. This has been done by Faltinsen and Loken (1978) for beam seas by means of a strip theory. Kim and Dalzell (1979) apply a strip theory to find the result in oblique waves, but omit the second order potential. This is not serious for small difference frequencies, as will be discussed shortly. Pinkster (1979) has used a 3-D panel method for the first order potential and an approximate second order potential. This approximation involves including the second order term in the incident wave potential, and finding the resultant diffraction potential, which can be done quite easily. Second order contributions from the interaction of the incident, diffracted, and radiated waves, and from the hull boundary condition are not included.

Faltinsen and Loken's beam seas calculations, and Pinkster's approximate calculations show that the second order potential contributions go to zero as the difference frequency goes to zero, while contributions obtained from the first order potential become increasingly important. Thus Pinkster's approximation, and Kim and Dalzell's neglect of this term is reasonable, if only the low frequency drift forces are desired.

CONCLUSIONS

Recent developments in predicting slowly-varying forces on a body have been reviewed. An approximation is available for calculating this using only the mean drift forces. Evidence for the usefulness of this approximation is sparse but encouraging. More accurate prediction methods are also available, although the complete second order solution is available only for beam seas acting on a cylinder. It should be noted that all of these developments are for long-crested (uni-directional) seas. Information regarding short-crested seas is not available.

REFERENCES

Dalzell, J.F., "Cross-Bispectral Analysis: Application to Ship Resistance in Waves," Journal of Ship Research, Vol. 18, No. 1, March 1974.

Faltinsen, O.M. and A. Loken, "Drift Force and Slowly-Varying Horizontal Forces on a Ship in Waves," Proceedings of Symposium on Applied Math dedicated to the late Professor Dr. R. Tinnan, Delft, 1978.

Faltinsen, O.M. and F.C. Michelsen, "Motions of Large Structures in Waves at Zero Froude Number," International Symposium on the Dynamics of Marine Vehicles and Structures in Waves, London, 1974.

Hsu, F.H. and Blenkarn, K.A., "Analysis of Peak Mooring Forces Caused by Slow Drift Oscillations in Random Seas," Offshore Technology Conference Paper 1159, 1970.

Kim, C.H. and J.F. Dalzell, "Analytic Investigation of the Quadratic Frequency Response for Lateral Drifting Force and Moment," Stevens Institute of Technology, Davidson Laboratory Report SIT-DL-79-9-2061, May 1979.

Lee, C.M. and J.N. Newman, "The Vertical Mean Force and Moment of Submerged Bodies," Journal of Ship Research, Vol. 15, No. 3, September 1971.

Loken, A.E. and O.A. Olsen, "The Influence of Slowly Varying Wave Forces on Mooring Systems," Offshore Technology Conference Paper 3626, 1979.

Maruo, H., "The Drift of a Body Floating on Waves," Journal of Ship Research, Vol. 11, No. 3, December 1960.

Molin, B., "Computations of Wave Drift Forces," Offshore Technology Conference Paper 3827, 1979.

Neal, Eddie, "Second-Order Hydrodynamic Forces Due to Stochastic Excitation," Tenth ONR Symposium on Naval Hydrodynamics, 1974.

Newman, J.N., "The Drift Force and Moment on Ships in Waves," Journal of Ship Research, Vol. 11, No. 1, March 1967.

Newman, J.N., "Second-Order, Slowly-Varying Forces on Vessels in Irregular Waves," International Symposium on the Dynamics of Marine Vehicles and Structures in Waves, London, 1974.

Numata, Edward, W.H. Michel and Alan C. McClure, "Assessment of Stability Requirements for Semisubmersible Units," Transactions SNAME, Vol. 84, 1976.

Pijfers, J.G.L. and A.W. Brink, "Calculated Drift Forces of Two Semisubmersible Platform Types in Regular and Irregular Waves," Offshore Technology Conference, Paper 2977, 1977.

Pinkster, J.A., "Mean and Low Frequency Wave Drifting Forces on Floating Structures," Ocean Engineering, Vol. 6, pp. 593-615, 1979.

Strom-Tejsen, J., Hugh Y.H. Yeh and David D. Moran, "Added Resistance in Waves," Transactions SNAME, Vol. 81, 1973.

WAVES FOR MOORING SYSTEM DESIGN

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INTRODUCTION

The statistical prediction of responses of ships and ocean structures in a seaway has become common practice in design following the technology developed on the modern probabilistic approach. The prediction of motions and associated forces of a mooring system is not an exception.

The probabilistic prediction of responses of a mooring system can be carried out in either the time domain or the frequency domain. In either case, application of the linear superposition principle which is often used in predicting responses of ships and ocean structures in a seaway may not be applicable for the mooring system because of strong nonlinear behavior of the system in a seaway. Apart from the nonlinear characteristics of the system, wave information is required as an input (excitation) in applying the probabilistic approach to estimate the responses for design consideration.

In most design methodologies of a marine system, in general, fairly heavy emphasis is placed on the prediction of extreme wave heights and the corresponding responses that are expected to occur during the system's lifetime as a result of these waves. Since the response of a system in a seaway is frequency dependent, both wave height and wave frequency (or period) should be considered. This may be of particular importance for the design of a mooring system. More specifically, it is necessary to estimate extreme wave heights for periods that are critical for the mooring system. These estimates can be obtained from the joint probability function of wave height and period. However, before this methodology is applied, we have to consider the moored system to encounter various sea severities which are most commonly expressed in terms of significant wave height. Furthermore, in a given sea condition, the mooring system will encounter an infinite variety of wave conditions which are represented by wave spectral shapes. Hence the variability of wave conditions for various sea severities and the frequency of occurrence of these wave conditions has to be reflected in the prediction.

Thus, for rational design of a mooring system, it is necessary to consider (a) various sea severities (significant wave heights) expected in the area where the system is located, and (b) various wave conditions (spectra) in a given sea severity. Then the responses such as motions and wave-induced forces of the system can be evaluated. The design values can be determined by applying extreme value statistics to the responses. Another way to evaluate the design value is to obtain the responses to extreme wave heights for wave periods that are critical for the system. For either approach, it is highly desireable to estimate the extreme value responses expected to occur over a period of time sufficiently long to cover the desired lifetime of the system.

In general, there are two approaches for estimating the extreme responses expected to occur in the lifetime of the mooring system, i.e., the long-term prediction approach and the short-term prediction approach. The former approach considers responses to all wave heights associated with all sea conditions expected to be encountered by the system irrespective of their magnitude, while the latter approach considers responses to wave heights within a specified sea condition (significant wave height) taking into account the total exposure time during the system's lifetime in that short-term sea condition.

It may appear that estimation through the long-term prediction approach is superior to that through the short-term prediction approach, since it deals with the accumulation of responses to all waves. However, in reality, the method of estimating the extreme value through the long-term approach appears to be insensitive to the frequency of occurrence of mild sea conditions, but is sensitive to the frequency of severe sea. The magnitude of responses will not reach the level critical for the system irrespective of how long time the system is operated in relatively mild seas. While, the magnitude reaches the critical level within a few hours when the system is in severe seas. Hence, the short-term appraoch, which involves only the more severe sea conditions, is appropriate for estimating the extreme values. The approaches for evaluating the extreme responses of the mooring system are summarized in Figure 1.

ESTIMATION OF THE SEVEREST SEA

As mentioned earlier, the sea severity is most commonly expressed in terms of the significant wave height. For estimation of the severest sea condition, it is necessary to prepare statistical information on significant wave height in the are where the mooring system is located. The severest sea condition (the extreme significant wave height) expected to occur during a specified period of time can be estimated by applying order statistics if the probability function which governs the significant wave height is know precisely. Unfortunately, the probability function which is uniquely applicable to significant wave height has not yet been found. Figure 2 shows an example which indicates that neither the Weibull nor the log-normal distribution represents the data obtained in the North Sea. Hence, the extreme significant wave height may best be evaluated by applying the concept of asymptotic distribution of extreme values which is applicable for any probability distribution.

Let us assume that the cumulative distribution function of significant wave height can be expressed in the following form:

$$F(x) = 1 - e^{-q(x)} \tag{1}$$

where, q(x) is a monotomically increasing function. Then, it can be proved that the probable extreme value expected to occur in n-observations, denoted by $\bar{y}_n$, is given for large n as,

$$\bar{y}_n = q^{-1}(\ln n) \tag{2}$$

The recent method is to express q(x) as a combination of an exponential and power of the significant wave height, such as,

$$q(x) = a x^m e^{-p x^k} \tag{3}$$

The constants involved in the function q(x) are determined numerically by applying a nonlinear least squares fitting method for representing the data of significant wave height by the formulation q(x) (Ochi and Whalen, 1980). The cumulative distribution function computed by using the method given in Equation (3) is included in Figure 2. As can be seen in the figure, the probability distribution thus derived represents well the observed data over the entire range of the cumulative distribution function. The magnitudes of significant wave height most likely to occur in 10 years and 50 years are estimated from the distribution function and are shown in Figure 3.

WAVE SPECTRA

The sea condition in a specified sea severity (significant wave height) varies considerably depending on the geographical location, duration and fetch of wind, stage of growth and decay of a storm, and existence of swell. Hence, the question always remains as to how realistic the predicted marine system's responses are if we use commonly available simple spectral formulations which have been developed for some idealized conditions. Since a mooring system encounters a variety of wave conditions even though the significant wave heights are the same, the variability of wave conditions (wave spectra) has to be reflected in the prediction.

One way to cover a variety of spectral shapes which the mooring system may encounter in a sea is to use a series (family) of wave spectra consisting of several members for any specified sea severity.

A significant benefit obtained by using a family of wave spectra for predicting responses of a marine system in a seaway is that, for each sea severity, one of the family members yields the largest response, while another yields the smallest response with a statistical confidence coefficient of 0.95, for example. Hence, by connecting the largest and smallest values obtained in each sea severity, we have the upper and lower bounds of responses. The results of computation made on wave-induced forces of an offshore structure have indicated that the bounds reasonably cover the variation of responses computed by using spectra measured at various oceanographic locations in the world (see Figure 4) (Ochi 1978).

The following are the families of wave spectra developed for open and fetch-limited seas:

1. Two-Parameter Wave Spectra Family

The two-parameter wave spectra family consists of nine members for an arbitrarily specified significant wave height. It is based on the two-parameter spectral formulation given by (Bretschneider 1959),

$$S(\omega) = \frac{1.25}{4} \frac{\omega_m^4}{\omega^5} H_s^2 \, e^{-1.25\left(\frac{\omega_m}{\omega}\right)^4} \tag{4}$$

where, H_s = significant wave height

ω_m = modal frequency

In order to generate a family of spectra, the probability function which governs the modal frequency for a given significant wave height was established from statistical analysis of available data, and a total of nine modal frequencies were derived as a function of significant wave height (Ochi 1978). The values of these modal frequencies are given in Table 1 together with the weighting factor assigned to each frequency. Examples of the family of wave spectra for significant wave height of 3.0 m (9.8 ft) and 9.0 m (29.5 ft) are shown in Figures 5 and 6.

2. <u>Six-Parameter Wave Spectra Family</u>

In order to cover a variety of wave spectra associated with the growth and decay of a storm including the existence of swell, the six-parameter family consisting of eleven members for a specified significant wave height was derived (Ochi 1976). It is given by,

$$S(\omega) = \frac{1}{4} \sum_j \frac{\left(\frac{4\lambda_j + 1}{4} \omega_{mj}^4\right)^{\lambda_j}}{\Gamma(\lambda_j)} \frac{H_{sj}^2}{\omega^{4\lambda_j + 1}} \, e^{-\left(\frac{4\lambda_j + 1}{4}\right)\left(\frac{\omega_{mj}}{\omega}\right)^4} \tag{5}$$

where, j = 1, 2 stands for the lower and higher frequency components respectively

H_s = significant wave height

ω_m = modal frequency

λ = shape parameter

The values of six-parameters, H_{s1}, H_{s2}, ω_{m1}, ω_{m2}, λ_1, and λ_2 are given in Table 2 as a function of sea severity (significant wave height). The weighting factor for the most probable spectrum is 0.50, and is 0.05 for all other spectra of the family. Examples for the family of wave spectra for significant wave height of 3.0 m (9.8 ft) and 9.0 m (29.5 ft) are shown in Figures 7 and 8.

3. JONSWAP Wave Spectra Family

The JONSWAP wave spectra family consisting of five members for a specified significant wave height was derived for evaluating reponses of a marine system operating in fetch-limited seas (Ochi 1979 b). It is based on the following JONSWAP wave spectral presentation:

$$S(\omega) = \alpha \frac{g^2}{\omega^5} e^{-1.25\left(\frac{\omega_m}{\omega}\right)^4} \gamma^{e^{-\frac{(\omega-\omega_m)^2}{2\sigma^2\omega_m^2}}} \tag{6}$$

where,
$\alpha = 0.076\ (\bar{X})^{-0.22}$

$\omega_m = 2\pi(3.5)\frac{g}{U}\ (\bar{X})^{-0.33}$

$\bar{X}$ = fetch length

U = wind speed

γ = peak shape parameter

$$\sigma = \begin{cases} 0.07 & \text{for } \omega \leq \omega_m \\ 0.09 & \text{for } \omega > \omega_m \end{cases}$$

From a statistical analysis of the measured data, five values of the peak shape parameter, γ, and the associated weighting factors are determined as given in Table 3. It is noted that the JONSWAP spectral formulation is given as a function of wind speed. Hence, for the design of mooring system, it is necessary to estimate the extreme value of wind speed expected in the area where the system is located. However, for a location where the effect of fetch length as well as the effect of bottom topography on wind-generated sea may exist, statistical information of both wind and significant wave height are required. In this case, the extreme value of significant height should be estimated first, then evaluate the equivalent wind speeds for deep water by the following formulation:

$$U = k X^{-0.615} H_s^{1.08} \tag{7}$$

where,
k = constant given in Table 4

H_s = significant wave height

As an example, a family of wave spectra for fetch length X = 150 NM (185 km) and for significant wave height of 4.0 m (13.1 ft) is shown in Figure 9.

It is of interest to see a comparison of spectral shapes for three different families. Since these families of wave spectra consist of several members, it is not convenient to compare them by assembling all members into one figure. Hence, a comparison will be made on three members taken from each family. These three represent (i) the spectrum which is most likely to occur, and (ii) the spectrum which has the smallest modal frequency, and (iii) the one which has the largest modal frequency.

Figure 10 shows a comparison made for a significant wave height of 8.2 m (26.9 ft) in the North Sea. The fetch length is 250 NM for the JONSWAP family. As can be seen in the figure, the shapes of the JONSWAP family are significantly different from those of the families for open seas. That is, the range of modal frequencies is much smaller and the shapes are much sharper for the JONSWAP family than those of the open sea. This may cause a significant difference in the magnitude of responses of mooring systems in a seaway; and therefore, serious consideration must be given to this for the design of mooring systems located in fetch-limited areas.

EXTREME WAVE HEIGHT CRITICAL FOR THE SYSTEM

It was stated in the Introduction that the concept of estimating extreme wave height along with its associated period is highly desirable for design consideration. This is because if the wave period is either sufficiently long or short in comparison with the natural motion periods of a mooring system, then the system may not be in danger even though the wave height is large. Hence, for more rational design of a mooring system, it is necessary to estimate extreme wave height for periods critical for the system. These estimates can be achieved through the use of a joint probability function of wave height and period. That is, we estimate the extreme wave height under the condition that wave periods fall into a certain range critical for the system's behavior in a seaway.

Longuet-Higgins (1975) has derived the following probability density function of wave height and period:

$$f(\xi, \eta) = \frac{\xi^2}{\sqrt{2\pi}} e^{-\frac{\xi^2(1+\eta^2)}{2}} \tag{8}$$

where, ξ and η are non-dimensional wave amplitude and period, respectively, given by

$$\xi = \frac{a}{\sqrt{m_0}} \tag{9}$$

$$\eta = \frac{T-\bar{T}}{\nu\bar{T}}$$

where, a = wave amplitude

T = wave period

$\bar{T}$ = mean wave period = $2\pi(m_o/m_1)$

$\nu = \sqrt{m_o m_2/m_1^2 - 1}$

m_k = k-th moment of the wave spectrum

Let us consider the conditional probability that the wave amplitude will exceed a certain value, ζ, given that wave periods fall into a certain range of values, η_1 and η_2. It is given by

$$Pr\{\xi > \zeta \mid \eta_1 < \eta < \eta_2\} = \frac{\int_{\eta_1}^{\eta_2}\int_{\zeta}^{\infty} f(\xi, \eta)\, d\xi\, d\eta}{\int_{\eta_1}^{\eta_2}\int_{0}^{\infty} f(\xi, \eta)\, d\xi\, d\eta} \tag{10}$$

Here, ζ is the extreme wave amplitude in non-dimensional form and is unknown at this stage. The values of non-dimensional period, η_1 and η_2 in Equation (10) are for the range of wave periods for which the system's behavior in a seaway becomes critical. They may be chosen in practice as the lower and upper bound period for which the response characteristics of the system are large. Using the joint probability density function given in Equation (8), Equation (10) becomes (Ochi and Whalen 1979a),

$$\Pr\{\xi > \zeta \mid \eta_1 < \eta < \eta_2\}$$

$$= \left[e^{-\frac{\zeta^2}{2}} \left\{ \Phi(\zeta\eta_2) - \Phi(\zeta\eta_1) \right\} + \frac{\eta_2}{\sqrt{1+\eta_2^2}} \left\{ 1 - \Phi\left(\zeta\sqrt{1+\eta_2^2}\right) \right\} \right. \tag{11}$$

$$\left. - \frac{\eta_1}{\sqrt{1+\eta_1^2}} \left\{ 1 - \Phi\left(\zeta\sqrt{1+\eta_1^2}\right) \right\} \right] \div \frac{1}{2} \left(\frac{\eta_2}{\sqrt{1+\eta_2^2}} - \frac{\eta_1}{\sqrt{1+\eta_1^2}} \right)$$

where,

$$\Phi(u) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{u} e^{-\frac{u^2}{2}} du$$

Thus, the conditional probability of wave height, ξ, for a given spectrum can be evaluated by Equation (11). Since we use a family of spectra, the conditional probability has to be weighted. For this, let us write the non-dimensional wave height, ξ, given in Equation (11) as ξ_j identifying that it is applicable for the j-th member of the spectral family. Then, the conditional probability applicable for the short-term is given by,

$$\Pr\{\xi > \zeta \mid \eta_1 < \eta < \eta_2\} = \frac{\sum_j n_j p_j \Pr\{\xi_j > \zeta \mid \eta_1 < \eta < \eta_2\}}{\sum_j n_j p_j} \tag{12}$$

where, p_j = weighting factor assigned to the j-th wave spectrum

n_j = number of wave cycles per unit time for the j-th spectram in a specified sea

m_k = k-th moment of the wave spectrum

Finally, the unknown non-dimensional extreme wave height in Equation (12) can be evaluated by equating Equation (12) with the probability 1/N, where N is the total number of wave cycles in a specified sea, that is,

$$N = \sum_j n_j p_j \, T \times (60)^2 \int_{\eta_1}^{\eta_2} \int_0^{\infty} f(\xi, \eta) \, d\xi \, d\eta.$$

$$= \sum_j n_j p_j \, T \times (60)^2 \, \frac{1}{2} \left(\frac{\eta_2}{\sqrt{1+\eta_2^2}} - \frac{\eta_1}{\sqrt{1+\eta_1^2}} \right) \tag{13}$$

where, T = total time considered for a specified sea in hours.

As a numerical example of estimating the extreme height with specified periods, computations are carried out using the six-parameter wave spectra family for a sea of a significant wave height 7.5 m (24.6 ft) with various ranges of period as shown in Table 4. The time T in this sea is estimated from the available data as 78.8 hours in 50 years. As can be seen in the table, the magnitudes of the conditional extreme wave height decreases substantially for the range less than 7 sec. in this particular sea. This implies that if a system is designed such that the ranges of periods critical for the system's behavior is less than 7 sec., then the extreme wave height for design consideration may be reduced in this sea condition.

REFERENCES

Bretschneider, C. L. (1959), "Wave Variability and Wave Spectra for Wind-Generated Gravity Waves", Beach Erosion Board, U. S. Army Corps of Engineers, Tech. Memo No. 118.

Longuet-Higgins, M. S. (1975), "On the Joint Distribution of the Periods and Amplitudes of Sea Waves", Journal of Geophysical Res., Vol. 80, No. 18, pp 2688-2694.

Ochi, M. K. and Hubble, E. N. (1976), "On Six-Parameter Wave Spectra", Proc. 15th Coastal Eng. Conf. Vol. 1, pp 301-328.

Ochi, M. K. (1978), "Wave Statistics for the Design of Ships and Ocean Structures", Trans. Soc. Nav. Arch. and Mar. Eng., Vol. 86, pp 46-76.

Ochi, M. K. and Whalen, J. E. (1979a), "Estimation of Extreme Waves Critical for the Safety of Offshore Structures", Offshore Tech. Conf. OTC 3596, Vol. 3, pp 2085-2090.

Ochi, M. K. (1979b), "A Series of JONSWAP Wave Spectra for Offshore Structures Design", Conf. Behavior Offshore Structures, Vol. 1, pp 76-86.

Ochi, M. K. and Whalen, J. E. (1980), "Prediction of the Severest Significant Wave Height", will appear in Proc. 16th Coastal Eng. Conference.

Table 1 Modal frequencies for the two-parameter wave spectra family (ω_m in rps, H_s in meters)

ω_m - value	Weighting factor	
$0.048(8.75 - \ln H_s)$	0.0500	
$0.054(8.44 - \ln H_s)$	0.0500	
$0.061(8.07 - \ln H_s)$	0.0875	
$0.069(7.77 - \ln H_s)$	0.1875	
$0.079(7.63 - \ln H_s)$	0.2500	(Most probable extreme value)
$0.099(6.87 - \ln H_s)$	0.1875	
$0.111(6.67 - \ln H_s)$	0.0875	
$0.119(6.65 - \ln H_s)$	0.0500	
$0.134(6.41 - \ln H_s)$	0.0500	

Table 2 Values of six parameters as a function of significant wave height (H_s in meters)

	H_{s1}	H_{s2}	ω_{m1}	ω_{m2}	λ_1	λ_2
Most Probable Spectrum	$0.84\ H_s$	$0.54\ H_s$	$0.70\ e^{-0.046\ H_s}$	$1.15\ e^{-0.039\ H_s}$	3.00	$1.54\ e^{-0.062\ H_s}$
0.95 Confidence Spectra	$0.95\ H_s$	$0.31\ H_s$	$0.70\ e^{-0.046\ H_s}$	$1.50\ e^{-0.046\ H_s}$	1.35	$2.48\ e^{-0.102\ H_s}$
	$0.65\ H_s$	$0.76\ H_s$	$0.61\ e^{-0.039\ H_s}$	$0.94\ e^{-0.036\ H_s}$	4.95	$2.48\ e^{-0.102\ H_s}$
	$0.84\ H_s$	$0.54\ H_s$	$0.93\ e^{-0.056\ H_s}$	$1.50\ e^{-0.046\ H_s}$	3.00	$2.77\ e^{-0.112\ H_s}$
	$0.84\ H_s$	$0.54\ H_s$	$0.41\ e^{-0.016\ H_s}$	$0.88\ e^{-0.026\ H_s}$	2.55	$1.82\ e^{-0.089\ H_s}$
	$0.90\ H_s$	$0.44\ H_s$	$0.81\ e^{-0.052\ H_s}$	$1.60\ e^{-0.033\ H_s}$	1.80	$2.95\ e^{-0.105\ H_s}$
	$0.77\ H_s$	$0.64\ H_s$	$0.54\ e^{-0.039\ H_s}$	0.61	4.50	$1.95\ e^{-0.082\ H_s}$
	$0.73\ H_s$	$0.68\ H_s$	$0.70\ e^{-0.046\ H_s}$	$0.99\ e^{-0.039\ H_s}$	6.40	$1.78\ e^{-0.069\ H_s}$
	$0.92\ H_s$	$0.39\ H_s$	$0.70\ e^{-0.046\ H_s}$	$1.37\ e^{-0.039\ H_s}$	0.70	$1.78\ e^{-0.069\ H_s}$
	$0.84\ H_s$	$0.54\ H_s$	$0.74\ e^{-0.052\ H_s}$	$1.30\ e^{-0.039\ H_s}$	2.65	$3.90\ e^{-0.085\ H_s}$
	$0.84\ H_s$	$0.54\ H_s$	$0.62\ e^{-0.039\ H_s}$	$1.03\ e^{-0.030\ H_s}$	2.60	$0.53\ e^{-0.069\ H_s}$

Table 3 γ-value and weighting factor for the family of JONSWAP spectra

γ-Value	Weighting Factor
1.75	0.081
2.64	0.256
3.30 (Mean JONSWAP)	0.326
3.96	0.256
4.85	0.081

Table 4 k-value to evaluate equivalent wind speed for a given fetch and significant wave height

		k-Value	
		X in NM U in Kts	X in Km U in M/Sec
γ-Value	1.75	128.1	96.2
	2.64	117.6	88.3
	3.30	111.4	83.7
	3.96	106.6	80.1
	4.85	101.7	76.4

Table 5 Extreme wave heights for various ranges in wave period. Significant wave height = 7.5 m (24.6 ft). Total time T = 78.8 hours in 50 years.

Period range in sec.	17.0 - 9.0	9.0 - 5.5	7.0 - 4.5
Frequency range in rps.	0.37 - 0.70	0.70 - 1.14	0.90 - 1.40
Number	15.38×10^3	10.86×10^3	4.86×10^3
Extreme wave height in m.	16.46	16.25	13.59

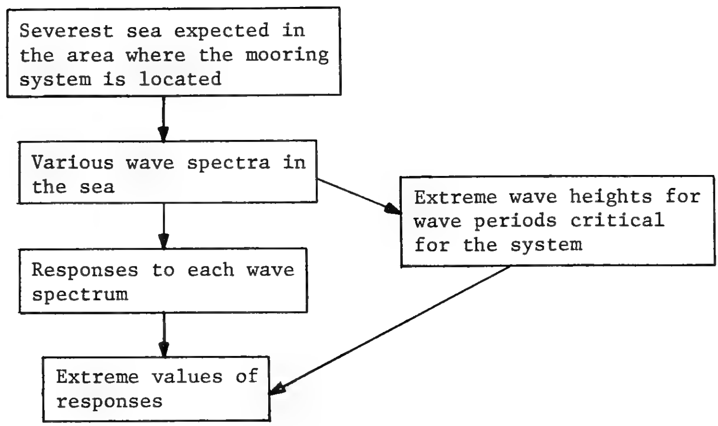

Figure 1 Approaches for evaluating the extreme value of mooring responses in random seas

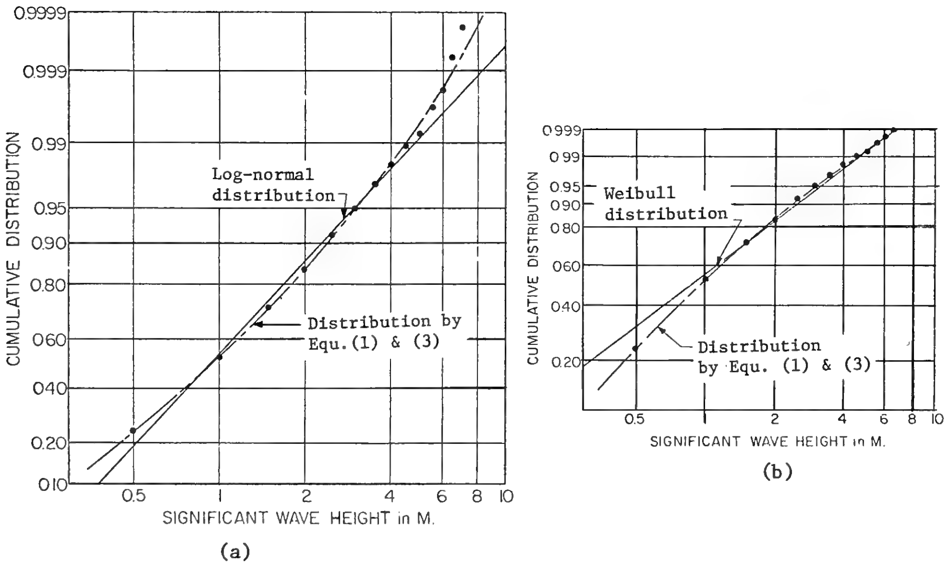

Figure 2 Comparison of distribution for North Sea data

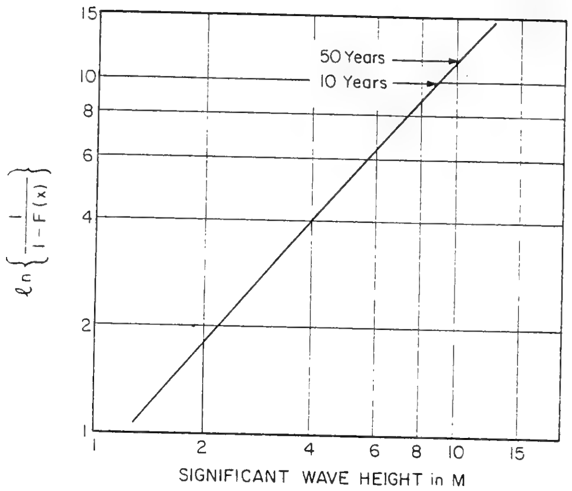

Figure 3 Evaluation of the severest significant wave height for North Sea data

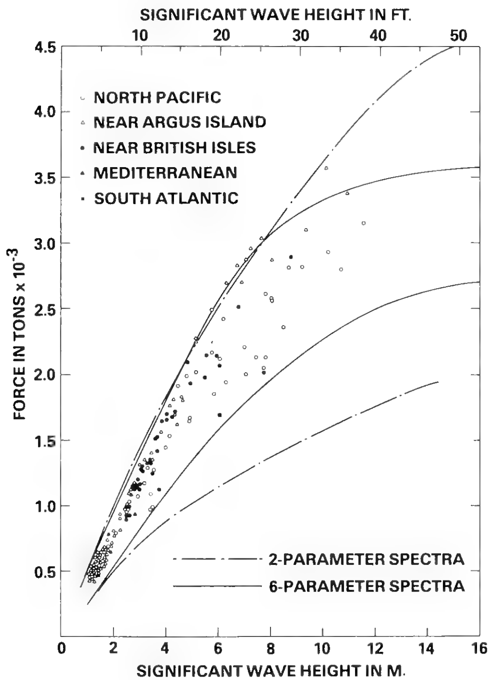

Figure 4 Extreme values of the wave-induced transverse force on a semi-submersible platform in a seaway. Comparison between the upper and lower bounds of the forces and those computed using the worldwide measured spectra.

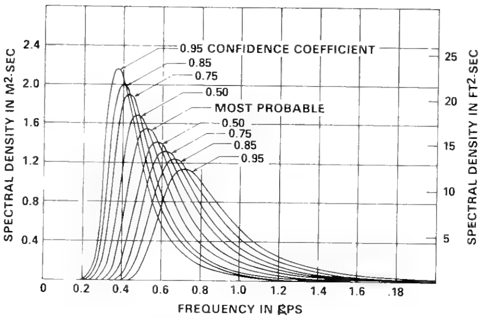

Figure 5 Family of two-parameter wave spectra for significant wave height 3.0 m (9.8 ft)

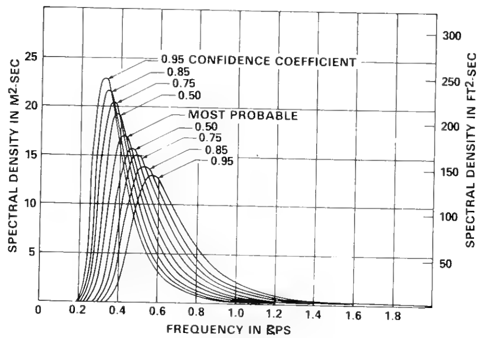

Figure 6 Family of two-parameter wave spectra for significant wave height 9.0 m (29.5 ft)

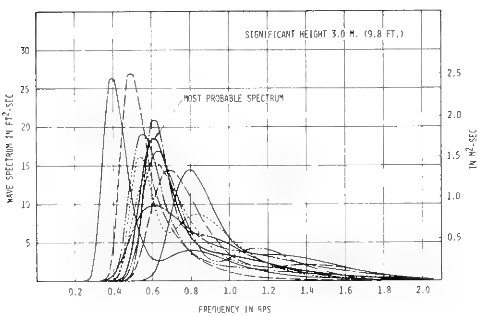

Figure 7 Family of six-parameter wave spectra for significant wave height 3.0 m (9.8 ft)

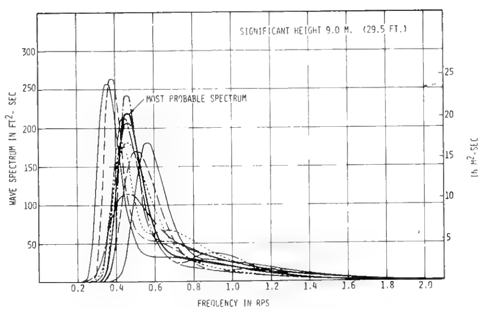

Figure 8 Family of six-parameter wave spectra for significant wave height 9.0 m (29.5 ft)

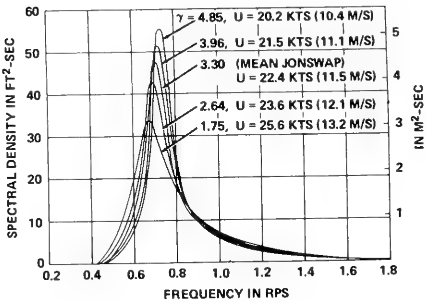

Figure 9 Family of JONSWAP spectra for fetch length 150 NM (185 Km) and significant wave height 4.0 m (13.1 ft)

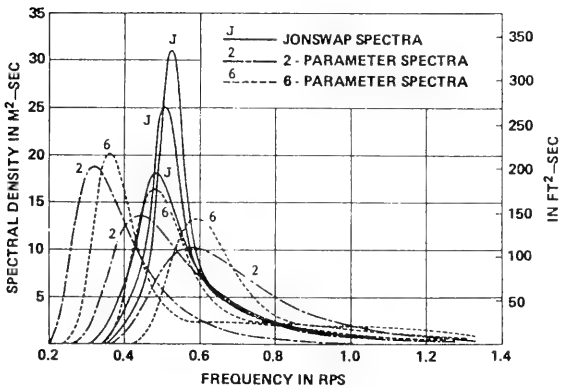

Figure 10 Comparison between families of JONSWAP, 2-parameter, and 6-parameter wave spectra

SPECTRAL ANALYSIS TECHNIQUES FOR SYSTEM IDENTIFICATION

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SPECTRAL ANALYSIS TECHNIQUES FOR SYSTEM IDENTIFICATION

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1. Introduction

In the past few years, new analytical procedures have been developed for optimum linear system identification using spectral analysis techniques [1-3]. These techniques apply to:

(a) single input/single output problems.

(b) single input/multiple output problems.

(c) multiple input/single output problems.

(d) multiple input/multiple output problems.

The key to carrying out this work is the implementation of new practical computational algorithms showing how to decompose output records from input records. Data are allowed to be realistic stationary random or transient random records with arbitrary correlation properties between the records.

This problem was previously treated in [4] and in other books where the general solution is derived by involved matrix computations or equivalent algebraic operations that are difficult to carry out and interpret. Complicated formulas were given for desired multiple coherence functions and partial coherence functions which did not provide significant engineering insight to inherent relationships of interest. Straightforward engineering interpretations are now being obtained by the new procedures.

The purpose of this paper is to outline some special features in these recent developments to help engineers conduct this analysis. Many applications exist today where these results can identify acoustics and vibration sources, and can predict their separate and combined effects at any output point in a general evnironment. In particular, differences can be detected due to system changes between excitation points and response points indicating system failures. These techniques can also be used to quantify overall nonlinear system features that may be present at different frequencies.

When input noise sources are not related or when mechanical systems are not structurally connected, then ordinary coherence functions and associated coherent output spectra can provide many useful answers as discussed in [5]. However, when multiple noise sources are measured on structurally connected systems, the ordinary coherence function will not separate out the effects of the various sources or distinguish between the possible transmission paths. Use of the ordinary coherence function by itself in these situations will give erroneous results and lead to wrong interpretations. For these physical cases, correct results can be obtained only by using partial coherence functions and multiple coherence functions as employed in [6].

Material to follow discusses multiple input/output models for given arbitrary input records and for derived ordered sets of conditioned input records. Iterative computational formulas are explained to compute conditioned spectral density functions, partial and multiple coherence functions. Results are then illustrated for a general three input/output model.

2. Multiple Input/Output Models for Arbitrary Inputs

As shown in Figure 1, the input records are assumed to pass through physically realizable constant parameter linear systems described by frequency response functions $\{H_i(f)\}$, $i = 1, 2, \ldots, q$. The output record $y(t)$ is assumed to be the sum of the individual outputs due to passage of the individual inputs $\{x_i(t)\}$, plus an unknown independent noise record $n(t)$ which accounts for all unknown nonlinear operations as well as extraneous noise effects.

Note in Figure 1 that $q!$ different configurations are possible depending upon which record is chosen as $x_1(t)$, which is then selected as $x_2(t)$, and so on. The analysis to follow is based on choosing a particular ordering of the inputs and sticking with this order. Similar results apply to any other desired ordering. Special attention will be given in this paper to the case of three inputs since the formulas can be listed for this case without difficulty and it is representative of the general case.

Figure 2 gives the result for the total output spectral density function $S_{yy} = S_{yy}(f)$ in terms of other quantities for the three input case, where the dependence upon frequency f has been omitted in all these terms for simplicity in notation. This will also be done in later equations of this paper. Note that S_{yy} can be either a power or an energy spectral density function depending upon whether the data is either stationary random data or transient data. The output noise spectral density function S_{nn} represents the difference between S_{yy} and results predicted from x_1, x_2 and x_3 by passage through any linear systems, H_1, H_2 and H_3. Because of the cross-terms between inputs, it is not clear here how much of the output is due to any particular input.

Optimum linear systems are defined by least-squares prediction techniques as those systems which produce minimum mean square system error. This will occur if S_{nn} is minimized as a function of H_i for all $i = 1, 2, \ldots, q$, leading, in general, to a set of complicated equations with many interacting input terms. However, for mutually uncoherent inputs, these equations simplify greatly since each optimum linear system can be determined from its own particular input independently of the other inputs.

For the three input case of arbitrary inputs, the three optimum frequency response functions will satisfy the equations listed in Figure 3. The terms shown in the numerators and denominators are conditioned (residual) spectral density functions found by computational algorithms developed in [1,2].

In Figure 3, the particular conditioned quantities in H_3 are defined as follows with similar definitions for H_1 and H_2.

$S_{33\cdot 1,2}$ = spectral (power or energy) density function of $x_3(t)$ when the linear effects of both $x_1(t)$ and $x_2(t)$ are removed from $x_3(t)$ by optimum least-squares prediction techniques.

$S_{3y\cdot 1,2}$ = cross-spectral density function between $x_3(t)$ and $y(t)$ when the linear effects of both $x_1(t)$ and $x_2(t)$ are removed from either $x_3(t)$ or $y(t)$, or from both, by optimum least-squares prediction techniques.

3. Multiple Input/Output Models for Conditioned Inputs

Figure 4 shows a multiple input/output model for conditioned inputs $x_1(t)$, $x_{2\cdot 1}(t)$ and $x_{3\cdot 1,2}(t)$, and so on, which are obtained from the original inputs $x_1(t)$, $x_2(t)$ and $x_3(t)$, and so on, shown in Figure 1. These conditioned inputs are defined in the following ordered way:

(1) The first input $x_1(t)$ is left alone.

(2) The second input $x_2(t)$ is replaced by $x_{2\cdot 1}(t)$ obtained by removing the linear effects of $x_1(t)$ from $x_2(t)$ by optimum least-squares prediction techniques.

(3) The third input $x_3(t)$ is replaced by $x_{3\cdot 1,2}(t)$ obtained by removing the linear effects of both $x_1(t)$ and $x_2(t)$ from $x_3(t)$ by optimum least-squares prediction techniques.

This procedure can be extended to any number of inputs.

The systems in Figure 4 are denoted by $\{L_i(f)\}$ instead of by $\{H_i(f)\}$ as in Figure 1 to distinguish these two distinct types of models. The terms $n(t)$ and $y(t)$ are exactly the same in both models. Relationships between these systems are derived in [1,2]. Note that the set of conditioned inputs in Figure 4 will be mutually uncoherent. Optimum frequency response functions for the three input case will now satisfy the equations listed in Figure 5. The systems L_1 and L_2 are simpler to compute then H_1 and H_2 in Figure 3, while L_3 is the same as H_3.

4. Conditioned Spectral Density Functions

Conditioned spectral density functions can be obtained by the iterative computational formulas shown in Figure 6. Observe that the formula for $S_{2y\cdot 1}$ includes $S_{22\cdot 1}$ and $S_{yy\cdot 1}$ as special cases. This formula also gives $S_{23\cdot 1}$ and $S_{y2\cdot 1} = S^*_{2y\cdot 1}$. Similarly the formula for $S_{3y\cdot 1,2}$ includes $S_{33\cdot 1,2}$ and $S_{yy\cdot 1,2}$ as special cases, and gives $S_{y3\cdot 1,2} = S^*_{3y\cdot 1,2}$. When there are only three inputs, as assumed here, the term $S_{yy\cdot 1,2,3} = S_{nn}$, the spectral density function of the output noise $n(t)$.

5. Partial and Multiple Coherence Functions

Definitions for partial coherence functions are stated in Figure 7. Specific formulas follow by substituting the particular conditioned spectral density functions computed from Figure 6. Note that partial coherence functions are really ordinary coherence functions of conditioned variables and hence are bounded between zero and unity.

A general definition for the multiple coherence function is given at the top of Figure 8 that applies both to arbitrary inputs as in Figure 1 and to ordered conditioned inputs as in Figure 4. Formulas for cases of one, two or three inputs are listed in Figure 8 which reveal new relationships between multiple and partial coherence functions.

6. Decomposition of Three/Output Model

The preceding analysis yields the frequency domain decomposition of the conditioned three input/output model as shown in Figure 9 where the $\{L_i\}$ are the optimum frequency response functions of Figure 5. It follows that the total output spectral density function S_{yy} is decomposed here into four distinct terms where:

$\gamma^2_{1y} S_{yy}$ = spectral output at $y(t)$ due to linear effects of $x_1(t)$ as $x_1(t)$ passes through L_1;

$\gamma^2_{2y\cdot 1} S_{yy\cdot 1}$ = spectral output at $y(t)$ due to linear effects of $x_{2\cdot 1}(t)$ as $x_{2\cdot 1}(t)$ passes through L_2;

$\gamma^2_{3y\cdot 1,2} S_{yy\cdot 1,2}$ = spectral output at $y(t)$ due to linear effects of $x_{3\cdot 1,2}(t)$ as $x_{3\cdot 1,2}(t)$ passes through L_3;

$S_{yy\cdot 1,2,3} = S_{nn}$ = spectral output at $y(t)$ due to unknown independent terms $n(t)$.

This procedure can be extended to any number of inputs.

7. Conclusion

This paper has outlined some procedures to follow to analyze and predict useful frequency relationships in measured multiple input/output data. Many important engineering applications can be solved with the help of these techniques. To interpret results properly, since the data are statistical in nature and available records are limited in both duration and number, these applications also require appropriate statistical error analysis of any measured results as discussed in [7]. Physical interpretations for all of these concepts with engineering examples and extended details for recommended data analysis procedures will soon be available in a new book [8].

References

1. J. S. Bendat 1975 Journal of Sound and Vibration 44, 311-325. "Solutions for the multiple input/output problem."

2. J. S. Bendat 1976 Journal of Sound and Vibration 49, 293-308. "System identification from multiple input/output data."

3. C. J. Dodds and J. D. Robson 1975 Journal of Sound and Vibration 42, 243-249. "Partial coherence in multivariate random processes."

4. J. S. Bendat and A. G. Piersol 1971 RANDOM DATA: Analysis and Measurement Procedures. New York: John Wiley and Sons, Inc.

5. W. G. Halvorsen and J. S. Bendat 1975 Sound and Vibration (U.S.) 9 (8), 15-24. "Noise source identifivaion using coherent output power spectra."

6. S. Barrett 1979 Shock and Vibration Bulletin 49, 43-58. "On the use of coherence functions to evaluate sources of dynamic excitation."

7. J. S. Bendat 1978 Journal of Sound and Vibration 59, 405-421. "Statistical errors in measurement of cohernece functions and input/output quantities."

8. J. S. Bendat and A. G. Piersol 1980 Engineering Applications of Correlation and Spectral Analysis. New York: John Wiley & Sons, Inc.

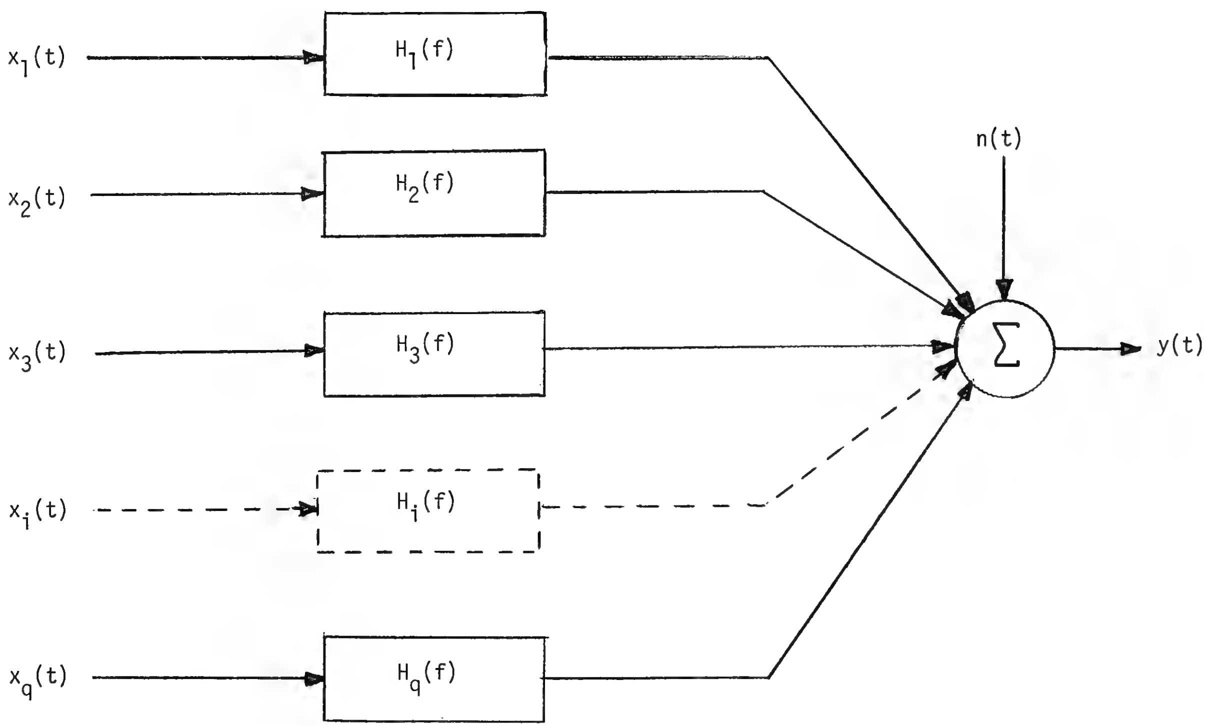

FIGURE 1. MULTIPLE INPUT/OUTPUT MODEL FOR ARBITRARY INPUTS

FIGURE 2

OUTPUT TERMS FOR ARBITRARY INPUTS

Three Input Case

$$S_{yy} = |H_1|^2 S_{11} + H_1 H_2^* S_{21} + H_1 H_3^* S_{31}$$

$$+ H_2 H_1^* S_{12} + |H_2|^2 S_{22} + H_2 H_3^* S_{32}$$

$$+ H_3 H_1^* S_{13} + H_3 H_2^* S_{23} + |H_3|^2 S_{33} + S_{nn}$$

FIGURE 3

OPTIMUM FREQUENCY RESPONSE FUNCTIONS

FOR ARBITRARY INPUTS

Three Input Case

$$H_1 = \frac{S_{1y \cdot 2,3}}{S_{11 \cdot 2,3}}$$

$$H_2 = \frac{S_{2y \cdot 1,3}}{S_{22 \cdot 1,3}}$$

$$H_3 = \frac{S_{3y \cdot 1,2}}{S_{33 \cdot 1,2}}$$

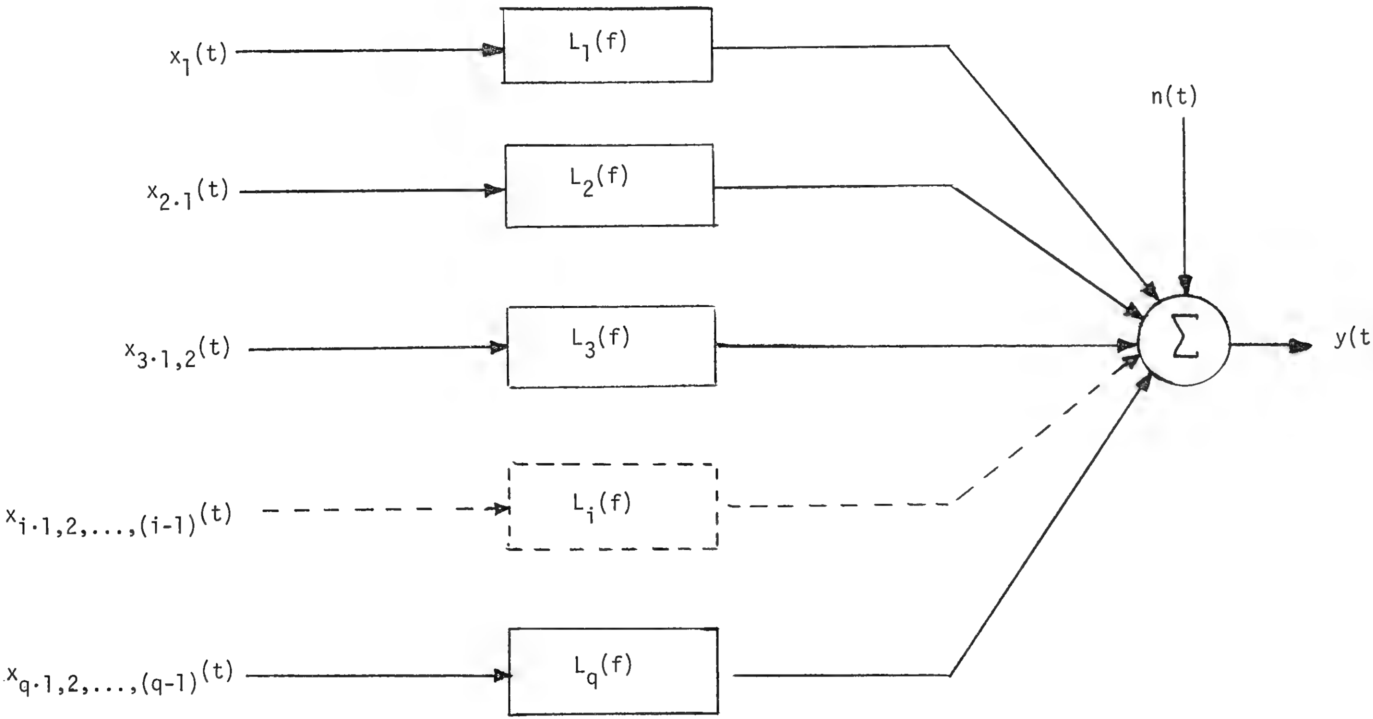

FIGURE 4. MULTIPLE INPUT/OUTPUT MODEL FOR CONDITIONED INPUTS

FIGURE 5

OPTIMUM FREQUENCY RESPONSE FUNCTIONS

FOR CONDITIONED INPUTS

<u>Three Input Case</u>

$$L_1 = \frac{S_{1y}}{S_{11}}$$

$$L_2 = \frac{S_{2y \cdot 1}}{S_{22 \cdot 1}}$$

$$L_3 = \frac{S_{3y \cdot 1,2}}{S_{33 \cdot 1,2}}$$

FIGURE 6

CONDITIONED SPECTRAL DENSITY FUNCTIONS

$$S_{2y \cdot 1} = S_{2y} - \left(\frac{S_{1y}}{S_{11}}\right) S_{21}$$

$$S_{3y \cdot 1,2} = S_{3y \cdot 1} - \left(\frac{S_{2y \cdot 1}}{S_{22 \cdot 1}}\right) S_{32 \cdot 1}$$

$$S_{yy \cdot 1,2,3} = S_{yy \cdot 1,2} - \left(\frac{S_{3y \cdot 1,2}}{S_{33 \cdot 1,2}}\right) S_{y3 \cdot 1,2}$$

FIGURE 7

PARTIAL COHERENCE FUNCTIONS

$$\gamma_{1y}^2 = \frac{|S_{1y}|^2}{S_{11}\ S_{yy}} = \gamma_{y1}^2$$

$$\gamma_{2y\cdot 1}^2 = \frac{|S_{2y\cdot 1}|^2}{S_{22\cdot 1}\ S_{yy\cdot 1}} = \gamma_{y2\cdot 1}^2$$

$$\gamma_{3y\cdot 1,2}^2 = \frac{|S_{3y\cdot 1,2}|^2}{S_{33\cdot 1,2}\ S_{yy\cdot 1,2}} = \gamma_{y3\cdot 1,2}^2$$

FIGURE 8

MULTIPLE COHERENCE FUNCTIONS

$$\gamma_{y:x}^2 = \frac{S_{yy} - S_{nn}}{S_{yy}}$$

Single Input $\gamma_{y:x}^2 = 1 - \left[1 - \gamma_{1y}^2\right] = \gamma_{1y}^2$

Two Inputs $\gamma_{y:x}^2 = 1 - \left[\left(1 - \gamma_{1y}^2\right)\left(1 - \gamma_{2y\cdot 1}^2\right)\right]$

Three Inputs $\gamma_{y:x}^2 = 1 - \left[\left(1 - \gamma_{1y}^2\right)\left(1 - \gamma_{2y\cdot 1}^2\right)\left(1 - \gamma_{3y\cdot 1,2}^2\right)\right]$

For 3 uncorrelated inputs, $\gamma_{y:x}^2 = \gamma_{1y}^2 + \gamma_{2y}^2 + \gamma_{3y}^2$

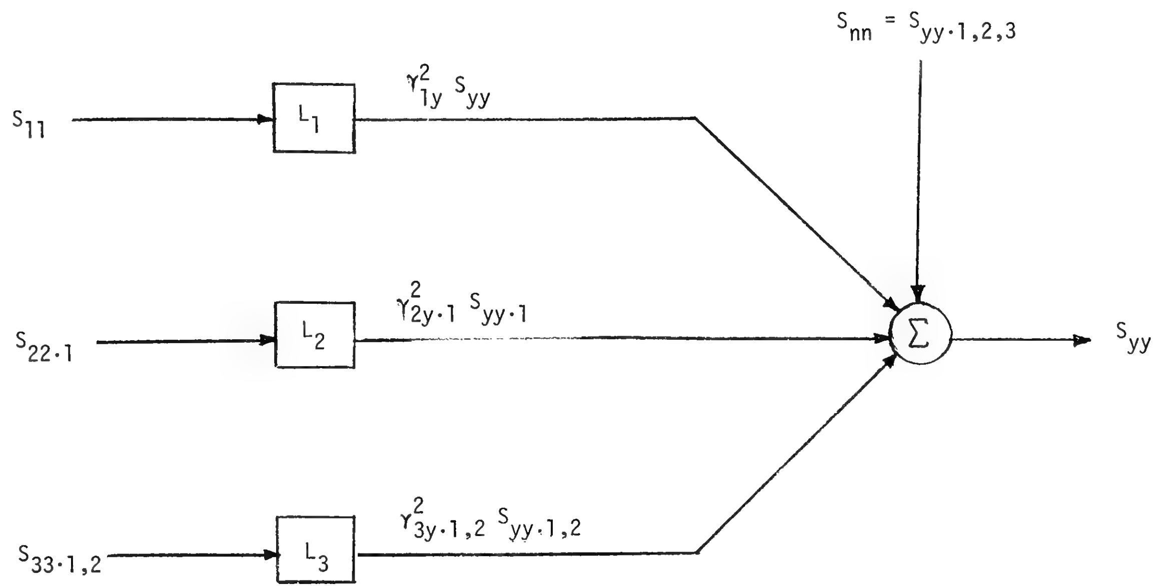

FIGURE 9. DECOMPOSITION OF THREE INPUT/OUTPUT MODEL
(Similar results follow for any number of inputs)

Civil Engineering Laboratory, Port Hueneme, California
Mooring Dynamics Seminar, January 10-11, 1980

THE RESPONSE OF MOORED FLOATING PLATFORMS TO OCEAN WAVES

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I. INTRODUCTION

The principal loads which are exerted on a floating structure by the sea depend on the relative motion between water and platform, thus, both the motion of the sea and the motion of the platform must be known in order to completely determine these fluid forces. Since the motion of the platform constitutes its response to the total external force system, it is seen that simultaneous solutions to the loads and motions problems must, in general, be performed. In addition to the fluid loads from waves, currents and platform motion, there are additional forces from external phenomena such as wind and those which result from the mooring or positioning system which is used to maintain the platform in its mean position.

The hydrodynamic forces caused by wave and platform motion, in general, have a complex dependence on the platform geometry and on the motions of fluid and platform. In order to develop a practical procedure for estimating these forces, it is invariably necessary to make some retreat from reality. The usual simplifying assumptions in this regard involve restrictions to either special geometric configurations of the platform or restrictions on the amplitudes of the fluid and platform motions. In the former category, one procedure has been developed which assumes that the platform configuration consists of a space frame assembly of slender cylindrical members. A second widely used procedure is restricted to platforms whose main elements consist of one or more slender ship-like hulls floating either at the water surface or submerged.

Several types of mooring systems or devices may be used to maintain the platform in its mean position. Examples are:

(1) Vertical or slanted taut cables.

(2) Catenary chain moorings, sometimes with buoys or additional clump weights.

(3) Dynamic positioning.

In each of these cases, it is usually necessary to represent the mooring device by a somewhat simplified analytical model in order to develop a practical analysis procedure. As an example, if the platform motions are expected to be small, the nonlinear force/displacement characteristics of a catenary chain mooring line may be replaced by a constant coefficient equal to the derivative of the force versus displacement graph at the mean position of the platform.

In the case of the fluid forces, it is frequently possible to consider the motion of both fluid and platform as small. In this case, the forces caused by the waves can be computed independently of the forces caused by platform motions and the latter are found to be proportional to the motion variables and their derivatives. This forms the basis for the linear spectral technique of determining the platform response to a random wave system by superposition of the responses to a number of different elementary regular wave systems. The procedure has found wide application to such diverse problems as the prediction of sea state imposed limitations on platform operations, or the long term cumulative structural damage due to fatigue.

In some cases, however, information concerning the effect of specific nonlinear phenomena is required and the simplifications noted above are not possible. Examples of such cases involve the motion response of the platform to an extremely high wave, possibly involving capsize or other hazard, or the effect on motion response of a nonlinear dynamic position keeping system. It is seldom possible to obtain a complete and exact nonlinear representation of platform response including all relevant effects. Instead, it is usually necessary to formulate the problem in such a way that the nonlinear effects thought to be of importance to the phenomenon of interest are included, and other, less important quantities are represented only approximately. It is clear that such an approach depends heavily on the insight and experience of the analyst for its success. Model experiments and full scale observation of similar structures may contribute much to the understanding required for the formulation of this approach.

II. THE EQUATIONS OF MOTION

The general procedure followed in analyzing the dynamic response of a floating platform to waves is based upon the assumption that it behaves as a rigid body having six degrees of motion freedom, and that any effects of the elastic deformation of the platform are negligible. The external forces acting on the platform include those which result from the relative motion with respect to the water, those exerted by the mooring or positioning system and other external effects such as wind.

In deriving the equations of motion, it is first necessary to define two coordinate systems which are shown in Figure 1. The first is labelled OXYZ and is assumed fixed in the platform with its origin located at the center of gravity. The second, oxyz, is fixed in space and its location is defined with some reference to the first system. For example, if we are analyzing the wave-induced oscillatory motion of the platform about its mean position it may be convenient to define the space coordinate system in such a way that it occupies the mean position of the platform system. In the case that there is no mean position of the platform, the space coordinate system might coincide with the initial position of the platform system. In general, the equations of motion are formulated in such a way as to describe the time-varying position of one coordinate system with respect to the other.

The complete equations of motion are given in the Appendix where it is noted that the equations of rotational motion contain nonlinear terms involving products of the angular velocities and trigonometric functions of the angles which relate the position of one coordinate system to the other. If the angular motions of the platform are small so that the products of small quantities may be neglected in comparison to the first order terms, all of the nonlinear terms noted above may be neglected. Next, recall that the external forces and moments which form the right hand side of the equations of motion depend on the incident waves and on the motion of the platform. As noted earlier, if the motions of the platform and the waves are small, the motion-dependent forces are decoupled from the wave-dependent forces and the equations of motion may be rewritten in the form of a set of linear differential equations:

$$(m_{ij} + a_{ij})\ddot{x}_j + b_{ij}\dot{x}_j + c_{ij}x_j = F_j(t) \tag{1}$$

Here a_{ij}, b_{ij}, and c_{ij} are coefficients in the expressions for the motion-dependent forces termed "added mass", "damping" and "restoring constant" and $F_j(t)$ is the wave-induced time-dependent exciting force.

In the case of a platform containing an appreciable portion of its volume near the water surface, the damping and added mass coefficients are found to be frequency dependent and equation (1) can be considered strictly applicable only for the case of excitation due to a system of sinusoidal regular waves. In this case, the system of differential equations is reduced to an equivalent system of algebraic equations in the amplitudes of the motion responses for the given wave frequency. The principle of linear superposition may still be utilized in obtaining the response to random seas by first decomposing the sea into its regular wave components and solving for the response to each. These are then superimposed to obtain the total response. Such "frequency-domain" analysis procedures are well established in other fields such as control system design.

In the case of nonlinear motions analysis, the equations of motion must usually be integrated in the time domain using a step-by-step procedure. If a complicated relationship is assumed between the fluid forces and the motion variables or if a simulation of an extended time period of platform operation is required in order to obtain, e.g., the response statistics in random seas, this procedure may become quite demanding of computer resources.

III. THE EXTERNAL FORCES

The external force system acting upon the structure usually originates in four sources:

(1) The incident waves and currents.

(2) The motions of the structure itself.

(3) The anchoring or other position keeping system.

(4) Wind.

The first two categories comprise fluid forces resulting from the relative motion between the water and the structure. In computing these forces, several simplifying assumptions may be made, which depend upon the geometry of the structure, the expected severity of the motion, and the nature of the computational process to be used in the motion analysis.

Structure made up of slender members. As an example of the simplification noted above, consider a structure comprising a space frame made up of slender cylindrical members. In this case, it is possible to obtain a good estimate of the total fluid force by computing the force which would act on each member individually in the absence of any hydrodynamic interference between individual members, and then taking the sum of such forces for all members. Examples of platform motion analyses using this procedure may be found in papers by Burke (1969), Paulling (1970) and Hooft (1971).

It is assumed that a variation of Morison's formula (Morison, et al (1951)) may be applied to the computation of the fluid forces on a slender cylinder oriented at an arbitrary inclination to the flow direction. The fluid force is assumed to be dependent upon the pressure gradient in the flow field, and the components of relative fluid velocity and acceleration which are normal to the centerline of the cylindrical member, this is illustrated in Figure 2.

$$\vec{F} = -\iint p\,\vec{n}\,ds + \int C_D \vec{u}_n\,|u_n|\,dl + \int C_a \vec{a}_n dl \qquad (2)$$

Note that the first term, which is sometimes called the Froude-Krylov force, was included with the added mass term in the original Morison paper, and this is approximately correct for a slender stationary cylinder. In the case considered here of a moving body, however, the relative acceleration term includes components due to both body motion and fluid motion while the Froude-Krylov term is dependent upon fluid motion alone.

The pressure, p, is determined from Bernoulli's equation and the velocity potential appropriate for the wave motion. For infinitisimal, deep water waves the potential function is given by

$$\phi = \frac{ga}{\omega} e^{ky} \sin(kx - \omega t), \qquad (3)$$

and the pressure by

$$p = -\rho g \eta - \rho \frac{\partial \phi}{\partial t} + \frac{1}{2}\rho(\phi_x{}^2 + \phi_y{}^2 + \phi_z{}^2) \ . \qquad (4)$$

In a linearized motion analysis procedure, the last term in the expression for the pressure is neglected since it involves the square of the (small) wave-induced fluid velocity.

The second term in equation (2) involves the square of the relative velocity between fluid and member and is chosen by analogy to the conventional representation of the fluid force on a body immersed in a steadily flowing fluid. In order to obtain a linearized formulation of the problem, this quadratic drag force is sometimes replaced by an equivalent linear drag force, $C_D' u_n$. The equivalent linear drag coefficient C_D' is chosen in such a way that the temporal mean square error between the linear drag and the "exact" quadratic drag is minimized. In regular waves, this results in

$$C_D' = \frac{8}{3\pi} C_D u_{n0} \qquad (5)$$

where u_{n0} = amplitude of the relative velocity.

In random waves,

$$C_D' = \sqrt{\frac{8}{\pi}}\, C_D\, \overline{u}_n$$

where $\overline{u}_n$ = RMS value of the normal relative velocity.

Structure having ship-like hulls. In the case of a platform having one or more ship like hulls floating either at the surface or submerged, a technique termed "strip theory" has been developed principally in connection with the prediction of the wave induced motions of surface ships. In this procedure, the solution of the potential flow problem for the pressure distribution and, therefore, the fluid loading on the three-dimensional ship hull is obtained from the solution for the flow about two-dimensional bodies having cross sections similar to the transverse cross sections of the ship. The two-dimensional problem is relatively easy to solve and several methods have been developed which are suitable for either single or multiple hulls. The damping, added mass and wave exciting forces are first obtained for two-dimensional shapes similar to the transverse sections of the ship, for the wave-frequencies of interest, and these two-dimensional forces are then integrated over the length to obtain the three-dimensional forces and moments. The usual procedures are based upon assumptions of small motion amplitude and inviscid irrotational fluid theory and yield results suitable for inclusion in a linear motions analysis procedure.

An example of the two-dimensional forces, expressed as damping and added mass coefficients, is shown in Figure 3. In this case the geometry consists of twin circular sections having proportions typical of some twin-hulled semisubmersible platforms. The figure illustrates two features of the behavior of these forces. First, the dependence of damping and added mass on the motion frequency is clearly seen. Second, in the lower part of the figure, an additional viscous damping is shown for comparison with the wave damping which is predicted by the two-dimensional potential flow theory. The viscous damping is assumed proportional to the square of the velocity and, therefore, in this nondimensional plot, depends on the amplitude of motion. Several different combinations of drag coefficient and amplitude of motion are shown in order to illustrate the relative importance of viscous and wave damping for such a configuration.

Second order, slowly varying wave forces. In addition to the wave-frequency forces described above, there will be a system of forces which depend upon wave reflection and interference between the ship motions and the incident wave system. These forces are proportional to the squares and products of wave height and ship motions, and consequently are neglected in a linearized motion analysis.

For a moored ship or platform, they may be of considerable importance to the mooring response and must usually be included. In regular waves, the wave reflection force is constant and merely causes a mean offset. In random waves, however, this effect gives rise to a slowly varying force having important frequencies equal to the frequencies of the envelope of the wave time history. These low frequency forces may excite resonance of the platform/mooring system which in turn may lead to high loads in mooring lines.

The computation of the low frequency wave loads is beyond the scope of this paper, but a paper by Newman (1974) discusses the important fluid dynamics aspects.

IV. APPLICATION OF ANALYSES PROCEDURE

There are a number of design or operational problems to which the hydrodynamic loading and motions analysis procedures described in the preceeding section may be applied. We shall describe several of them here.

Performance prediction. A vital element in any design procedure is the ability to predict the performance of the system being developed at an early phase of the design process. Two performance parameters of great importance in the design of offshore platforms are the motions of the platform and the forces in the mooring system. Figures 4 and 5 illustrate two different types of stable platforms with which we are concerned here. The first type is the conventional twin-hulled semisubmersible, and Figure 4 depicts a somewhat simplified version of an actual design. The second, Figure 5, is a tension leg platform which is an innovative concept currently attracting considerable interest as a candidate for deep water drilling and production. The tension leg platform illustrated here is a small experimental platform which has been tested at sea off the coast of California. In the case of both of the platforms depicted here, model test data have been published for structures of similar although not identical configuration to those depicted. For the semisubmersible, data for the pitch and heave motions in head seas are shown in Figures 6 and 7. The computed response is shown by the solid line and the correlation with experimental data may be considered typical of that obtainable using the twin-hull strip theory procedure. In the case of the tension leg platform, data are shown in Figure 8 for the surge motion and in Figure 9 for the wave induced tension variations in an anchor line. In these figures, the theoretical predictions were performed using the slender member-space frame procedure. Here, an excellent prediction of the motion is obtained, but the mooring tension prediction is not quite in so good agreement with experiments. This behavior has been observed rather generally and indicates that the mooring system may be more sensitive to nonlinear effects that are the motions.

Effects of high waves. It was noted earlier that a numerical integration of the nonlinear equations of motion may be performed in order to investigate certain specific nonlinear effects. Such an effect, which may be of importance in the operation of a tension leg platform, is the mean offset from the initial position caused by high waves. To visualize the effect in question, refer to Figure 10 and recall that the vertical tensioned mooring system will effectively suppress the heave motion of the platform, one column of which is shown in this figure. It is clear, that when a wave crest is centered at this column, the length of member immersed is much greater than it is in a trough. If the wave is moving from left to right in the figure, the result will be that a considerably greater drag force will act on the member when in a wave crest, than when the member is in the trough, solely because of the difference in the immersed length. In the wave crest, the fluid velocities, thus the drag force is to the right, while in the trough. the velocity and drag force are directed to the left. As a result, an average force directed to the right will act on this member as well as others comprising the platform, causing a mean offset of the platform in the downwave direction. This effect is clearly seen in Figure 11 which gives the results of a nonlinear time-domain integration of the equation of surge motion. The effect is found to be closely related to the wave steepness. The mean force is a nonlinear function of the wave height, but in this case, unlike the wave reflection force discussed earlier, the force depends on viscous drag.

VII. APPENDIX

The rigid-body equations of motion. The two coordinate systems, one fixed in space and one fixed in the platform are illustrated in Figure 1. The objective of the motions analysis, simply stated, is to describe the time varying position of one system with respect to the other. The translatory position of the origin of the platform coordinates is given by the three coordinates (x_1, x_2, x_3) measured in the directions of the three space axes. The angular motion is expressed in terms of the three components of the platforms angular velocity $(\omega_1, \omega_2, \omega_3)$ resolved in the directions of the axes fixed in the platform. The details of the derivation of the equations of motion of a rigid body having six degrees of freedom are given in standard textbooks on dynamics and need not be repeated here. In vector form the equations may be written as follows:

$$[m]\{\frac{d^2x}{dt^2}\} = \{F\}$$

$$[I]\{\frac{d\omega}{dt}\} + \{\omega\} \times \{I\omega\} = \{M\} \qquad (7)$$

Here $\{F\}$, $\{M\}$ = vectors of external forces and moments respectively. $\{F\}$ is expressed in the space coordinate system and $\{M\}$ in body coordinates.

$[m]$ = 3 x 3 diagonal matrix in which the three nonzero terms are the platform mass.

$[I]$ = 3 x 3 matrix containing the moments and products of inertia of the platform.

$\{\frac{d^2x}{dt^2}\}$ = vector of the second derivatives.

$\{\omega\}$ = angular velocity vector expressed in coordinates along the body axes.

In order to express the angular position of the structure with respect to the fixed inertial coordinate system, it is necessary to utilize a set of three Euler angles. These are defined as follows; let the body axes initially coincide with the axis system fixed in space, Figure 1. The structure first rotates in yaw through angle θ_2 about oy, then through the pitch angle, θ_3, about the new position of oz, and finally, through the roll angle θ_1 about the final position of ox. The relationship between the two coordinate systems is now:

$$\begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix} = \begin{bmatrix} \cos\theta_2\cos\theta_3 & \sin\theta_3 & -\sin\theta_2\cos\theta_3 \\ \sin\theta_1\sin\theta_2 - \cos\theta_1\cos\theta_2\sin\theta_3 & \cos\theta_1\cos\theta_3 & \cos\theta_1\sin\theta_2\sin\theta_3 + \sin\theta_1\cos\theta_2 \\ \sin\theta_1\cos\theta_2\sin\theta_3 + \cos\theta_1\sin\theta_2 & -\sin\theta_1\cos\theta_3 & \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2\sin\theta_3 \end{bmatrix} \begin{Bmatrix} x-x_1 \\ y-x_2 \\ z-x_3 \end{Bmatrix} \quad (8)$$

Now, if $\{\omega\} = (\omega_1, \omega_2, \omega_3)$ are the components of the instantaneous angular velocity along the body axes, the relationship between $\{\omega\}$ and the time derivatives of the Euler angles is

$$\{\omega\} = [B]\{\dot{\theta}\}$$

where

$$[B] = \begin{bmatrix} 1 & \sin\theta_3 & 0 \\ 0 & \cos\theta_1\cos\theta_3 & \sin\theta_3 \\ 0 & -\sin\theta_1\cos\theta_3 & \cos\theta_1 \end{bmatrix} \quad (9)$$

In the case of small angular motions, we see that $\sin\theta_i \approx \theta_i$ (small) and $\cos\theta_i \approx 1$, so

$$[B] \approx \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix} \quad (10)$$

Thus, for small angular motions, the angular velocity components are approximately equal to the time derivatives of the Euler angles. The equations of motion, (7) may be written in a more compact matrix form in this case:

$$\{F\} = [m]\{\ddot{x}\} \tag{10a}$$

where F = vector of all six eternal forces and moments.

$[m]$ = 6 x 6 matrix containing mass, moments of inertia and products of inertia of the platform.

$\{x\}$ = vector containing three translational displacements of the platform center of gravity and three rotations about the coordinate axes.

If the vector $\{F\}$ may be expressed in terms of linear functions of all relevant motion variables, equations (10a) form the basis for a linearized motion response computation.

In general, however, the force system contains nonlinear terms in such quantities as the motions (displacements, velocities and accelerations) and incident wave height. The latter may be of importance in computing the large amplitude motions in high waves. In this case, it is convenient to introduce a new variable,

$$\{v\} = \{\frac{dx}{dt}\}$$

which may be combined with equations (7) and (9) to yield the complete nonlinear motion equations as

$$\begin{aligned} \{\frac{dv}{dx}\} &= [m]^{-1}\{F\} \\ \{\frac{dx}{dt}\} &= \{v\} \\ \{\frac{d\omega}{dt}\} &= [I]^{-1}\ \{\{M\} - \{\omega\} \times \{I\omega\}\} \\ \{\frac{d\theta}{dt}\} &= [B]^{-1}\{\omega\} \end{aligned} \tag{11}$$

Equations (11) form a system of twelve first-order differential equations in state variables $\{v\}$, $\{x\}$, $\{\omega\}$ and $\{\theta\}$ which express the velocities and position of the body as functions of time. Nonlinear terms of two types are retained in this system of equations:

(1) The $\{\omega\} \times \{I\omega\}$ and $[B]^{-1}$ terms

(2) Nonlinear terms in the relationships between the external forces and either the platform motions or the wave amplitudes.

The Mooring System. A cable or chain mooring device is usually thought of as exerting a force on the moored ship or platform which depends only on position. The dynamic characteristics of the line itself and fluid forces exerted on it by virtue of its motion relative to the water are usually ignored. For moderate water depths, line tensions, and wave motion, this assumption is reasonably good. In many realistic cases, however, the fluid and dynamic effects become important in just the cases of greatest interest, i.e., the case of survival of ship and mooring in extreme sea conditions. The following development is based upon the static model of the mooring, but the procedures may be easily extended to more complete models.

Assume that the force exerted by the mooring line on the platform at the point of attachment may be expressed by three components in the global oxyz coordinate system. In a linearized motion analysis, these will be identical to the components in the platform coordinate system. The coordinates of the point of attachment are given by (X_c, Y_c, Z_c) in the platform system. Now, assume that the force versus displacement characteristics of this end are also known. For a linear motions analysis, the forces may be related to the displacements of the mooring line by a 3×3 stiffness matrix $[k_a]$.

$$\{F_A\} = [k_a] \cdot \{x_c\} \quad . \tag{12}$$

$\{F_a\}$ = xyz - forces by mooring line on platform

$[k_a]$ = stiffness matrix.

$\{x_c\}$ = xyz - displacements of end of mooring line.

The three motions of the attachment point are related to the motions of the platform CG by the inverse of equation (8). If the two coordinate systems coincide initially, the displacements of the point are

$$\{x_c\} = \{x_1\} + ([A]^{-1} - [I])\{X_c\} \tag{13}$$

Here $\{x_1\}$ = translations of the CG = (x_1, x_2, x_3).

$[A]$ = coordinate transformation defined by (8).

$\{X_c\}$ = coordinates of anchor line end, (X_c, Y_c, Z_c).

$[I]$ = unit matrix.

For a linear motions analysis, we retain only first order terms in (13) and replace the trigonometric functions by the small angle approximation, $\sin\theta \approx \theta$, $\cos\theta \approx 1$. Noting that (8) constitutes an orthogonal transformation, therefore the inverse of [A] equals the transpose, and after multiplying and rearranging, we obtain

$$\{x_c\} = \{x_1\} + \begin{bmatrix} 0 & Z_c & -Y_c \\ -Z_c & 0 & X_c \\ Y_c & -X_c & 0 \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{Bmatrix} \tag{14}$$

Equation (14) may be rewritten as

$$\{x_c\} = [D] \cdot \{x\} \tag{15}$$

where

$\{x\}$ = vector of six platform motions = $\{x_1, x_2, \ldots, \theta_3\}$

$$[D] = \begin{bmatrix} 1 & 0 & 0 & 0 & Z_c & -Y_c \\ 0 & 1 & 0 & -Z_c & 0 & X_c \\ 0 & 0 & 1 & Y_c & -X_c & 0 \end{bmatrix} .$$

The linearized mooring line force is then given, in terms of platform motions by

$$\{F_A\} = [k_a][D]\{x\} \quad . \tag{16}$$

The three forces $\{F_A\}$ are transformed back to three centroidal forces and three moments about xyz by the transpose of [D]

$$\begin{aligned} \{F_{AO}\} &= [D]^T\{F_A\} \\ &= [D]^T \cdot [k_a][D]\{x\} \quad . \end{aligned} \tag{17}$$

This enables us to define the centroidal 6 x 6 mooring line stiffness matrix

$$[k_o] = [D]^T \cdot [k_a] \cdot [D], \tag{18}$$

from which,

$$[F_o] = [k_o]\{x\} \quad . \tag{19}$$

The mooring line spring constant matrix $[k_a]$ is defined according to the type of mooring. A simple example will be described here, a vertical taut cable or "tension leg" mooring line. The nomenclature for such a mooring is shown in Figure 12.

The initial length of the cable in the vertical position is L and the initial tension is T. In the displaced position, the cable is inclined to a small angle β and stretched by a small amount δL. The new tension is given by

$$T_o + \delta T = T_o + k\delta L, \tag{20}$$

where k = elastic constant of the line.
Typically, $k = \frac{AE}{L}$,

where A = cross sectional area of the line,
E = Young's modulus.

The forces, in the horizontal and vertical directions, exerted on the platform by the mooring line are

$$F_x = -(T_o + \delta T)\sin\beta \ ,$$

$$F_v = T_o(1 - \cos\beta) - \delta T\cos\beta \quad . \tag{21}$$

For small β and small $\delta L \approx \delta y$, these become,

$$F_x = -T_o\beta = \frac{T_o}{L}\delta x \ ,$$

$$F_v = -\delta T = \frac{AE}{L}\delta y \quad . \tag{22}$$

The 3 x 3 spring constant matrix $[k_a]$ may, therefore, be written for the vertical tension leg as

$$[k_a] = -\frac{1}{L}\begin{bmatrix} T_o & 0 & 0 \\ 0 & AE & 0 \\ 0 & 0 & T_o \end{bmatrix} \tag{23}$$

VIII. REFERENCES

Burke, B. G. "The Analysis of Motions of Semisubmersible Drilling Vessels in Waves, OTC 1024, Houston 1969.

Hooft, J. P. "A Mathematical Method of Determining Hydrodynamically Induced Forces on a Semisubmersible" Trans. SNAME, v. 79, 1971, 28-70.

Horton, E. E.; McCammon, L. B.; Murtha, J. P. and Paulling, J. R. "Optimization of Stable Platform Characteristics", OTC 1553, Offshore Technology Conference, Houston 1972.

Kim, C. H. and Chou, F. "Motions of a Semisubmersible Drilling Platform in Head Seas", Marine Technology, April 1973, pp. 112-123.

Morison, J. R., et al "The Force Exerted by Surface Waves on Piles", Pet. Trans., AIME, v. 189, 1950, pp. 149-154.

Paulling, J. R. "Wave Induced Forces and Motions of Tubular Structures", Eigth Symposium on Naval Hydrodynamics, Pasadena, ONR ACR-179, pp. 1083-1110.

Paulling, J. R. and Horton, E. E. "Analysis of the Tension-Leg Stable Platform", OTC 1263, Offshore Technology Conference, Houston 1970.

Paulling, J. R. "Elastic Response of Stable Platform Structures to Wave Loading", Proc. Intl. Symp. on Dynamics of Marine Vehicles and Structures in Waves, Inst. of Mech. Engrs., London 1974.

Paulling, J. R. "Time Domain Simulation of Stable Platform Motions with Application to the Tension Leg Platform", Proc. SNAME STAR Symposium, Paper T6-2, San Francisco 1977.

Paulling, J. R.; Hong, Y. S.; Stiansen, S. G. and Chen, H. H. "Analysis of Semisubmersible Catamaran Type Platforms", OTC 2975, Offshore Technology Conference, Houston 1977.

Paulling, J. R.; Hong, Y. S.; Stiansen, S. G. and Chen, H. H. "Structural Loads on Twin-hulled Semisubmersibles", OTC 3246, Offshore Technology Conference, Houston 1978.

Newman, J. N. "Second order Slowly-Varying Forces on Vessels in Irregular Waves", Intl. Symp. on Dynamics of Marine Vehicles and Structures in Waves", Univ. College, London 1974.

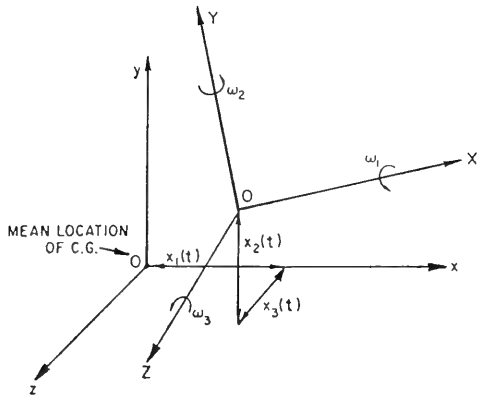

Figure 1. Coordinate systems

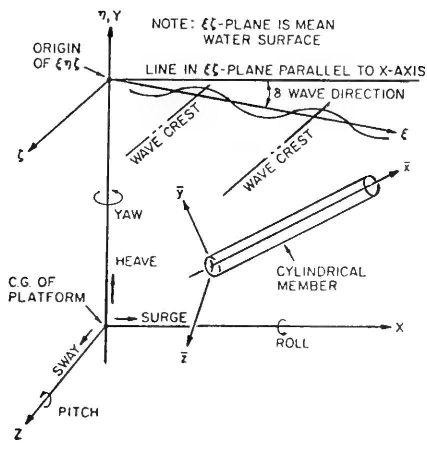

Figure 2. Nomenclature for cylindrical member

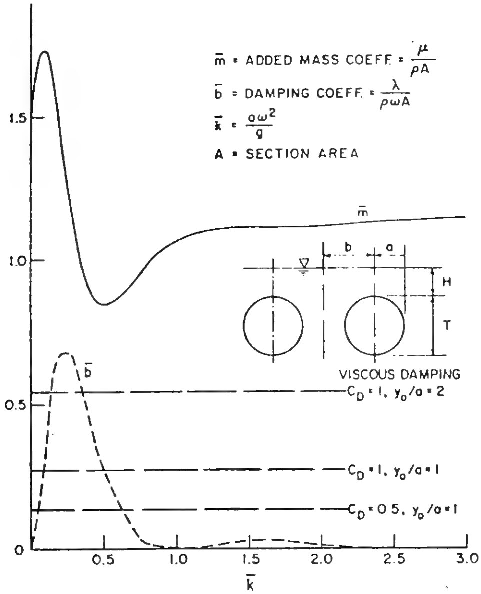

Figure 3. Added mass and damping for twin circular cylinders
$b/a = 1.25$, $H/T = 1.25$

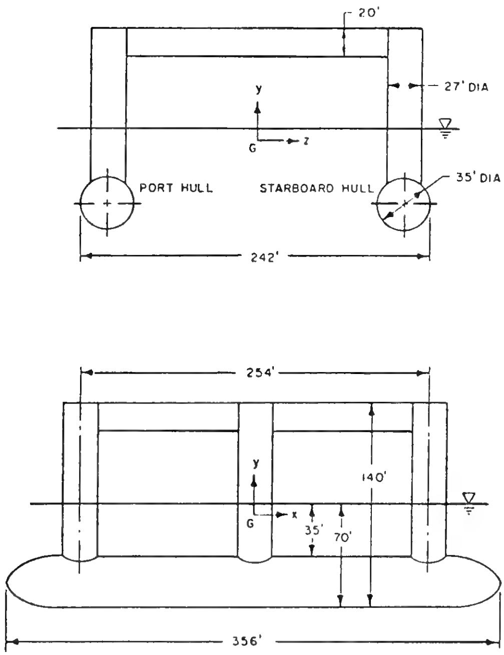

Figure 4. Schematic twin-hulled semisubmersible

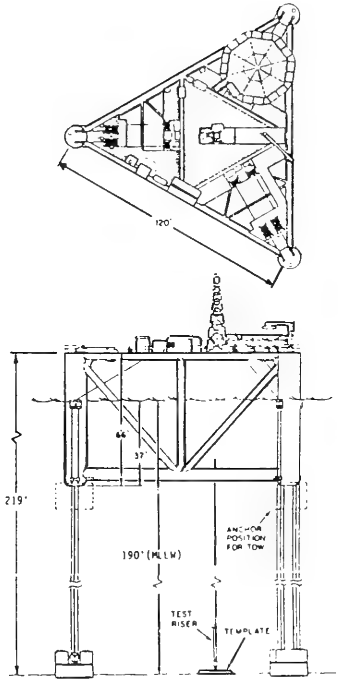

Figure 5. Tension leg platform "Deep Oil X-1"

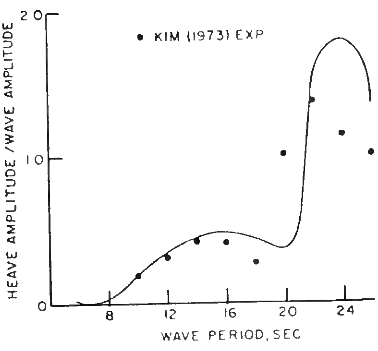

Figure 6. 'Heave motion of semisubmersible

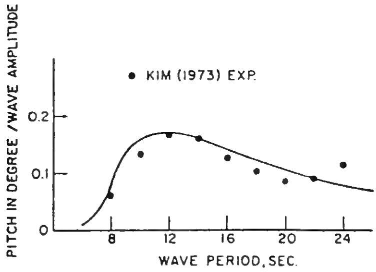

Figure 7. Pitch motion of semisubmersible

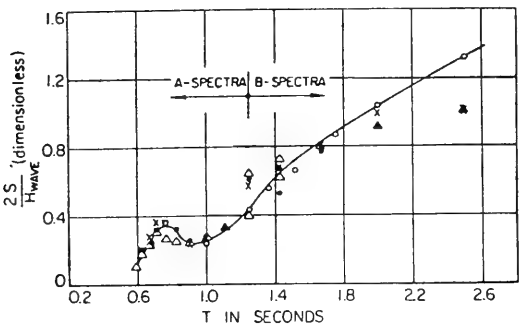

Figure 8. Surge motion for tension leg platform

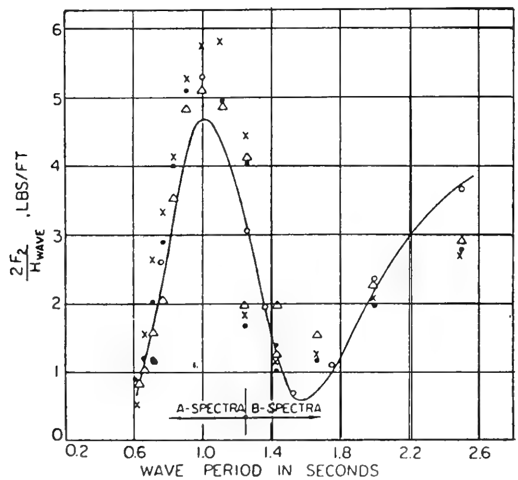

Figure 9. Tension variation in anchor cable for TLP

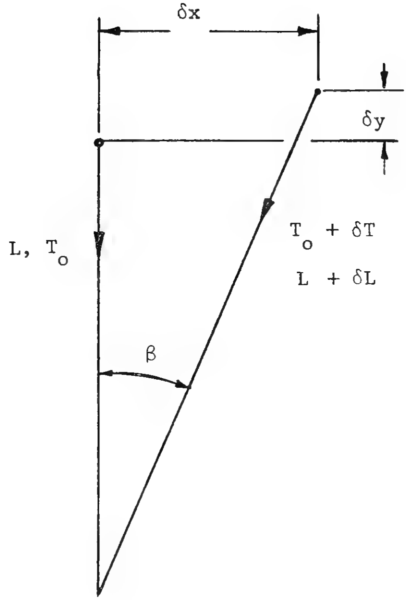

Figure 12: Nomenclature for vertical taut cable mooring line.

MOORING DYNAMICS SEMINAR

By
Dr. S. Calisal

INTRODUCTION

A computer program to obtain a computer solution to sizing and locating mooring tackle based on moored ship dynamics at sea is under development at the U.S. Naval Academy. The important features of this program are that it should require a minimum of input data and that the output should be easily understood by users who are not necessarily specialists in the field of ship or cable dynamics.

A survey of the expected input variables was first done. To our surprise, most of the variables required for input for a classical ship dynamics problem can at best be based on "guessed" quantities. This fact forced us to study possible sets of assumptions not normally required for the solution of a ship dynamics program. During the early stage of the model development, the major problem we faced was selecting the assumptions that could be accepted. For a successful model, the assumptions had to be:

a. realistic
b. consistent

A flow diagram of the calculation can be represented as in Figure 1. The assumptions in each block had to have the same implication and same order of importance. The following is an attempt to point out some of the assumptions observed in the literature and used to build the total model.

SHIP GEOMETRY AND WEIGHT DISTRIBUTION

The first question encountered was: "how accurately should the ship impermeable boundary be represented?" In Naval architecture, a ship is defined with a table of offsets and continuous curves; surfaces are assumed to exist between the defined points. Ship surface definition is still a continuous field of research and development, and Figure 2 shows one such effort from the Abkowitz paper (1966). The purpose is to have a mathematical function to define the stations. Van Oortmerssen (1976), on the other hand, used plane surface elements, while Raichlen (1965) and Bomze (1974) used "equivalent displacement" rectangular blocks for frequency and time domain solutions.

For wind-induced resistance, the geometry of the ship above water is certainly important. The general trend is to have a representation based on projected area and the centroid of this area only.

For simplicity of input requirements, and noting that the coefficients in the dynamic equations require integration and moments of distributed quantities and also the good correspondence reported between experimental and theoretical values, an equivalent box representation is adopted. This is not a limitation, however, as the program has a modular design and this subroutine can be replaced by more sophisticated ones.

FLOW OF INFORMATION FOR MOORING FORCES

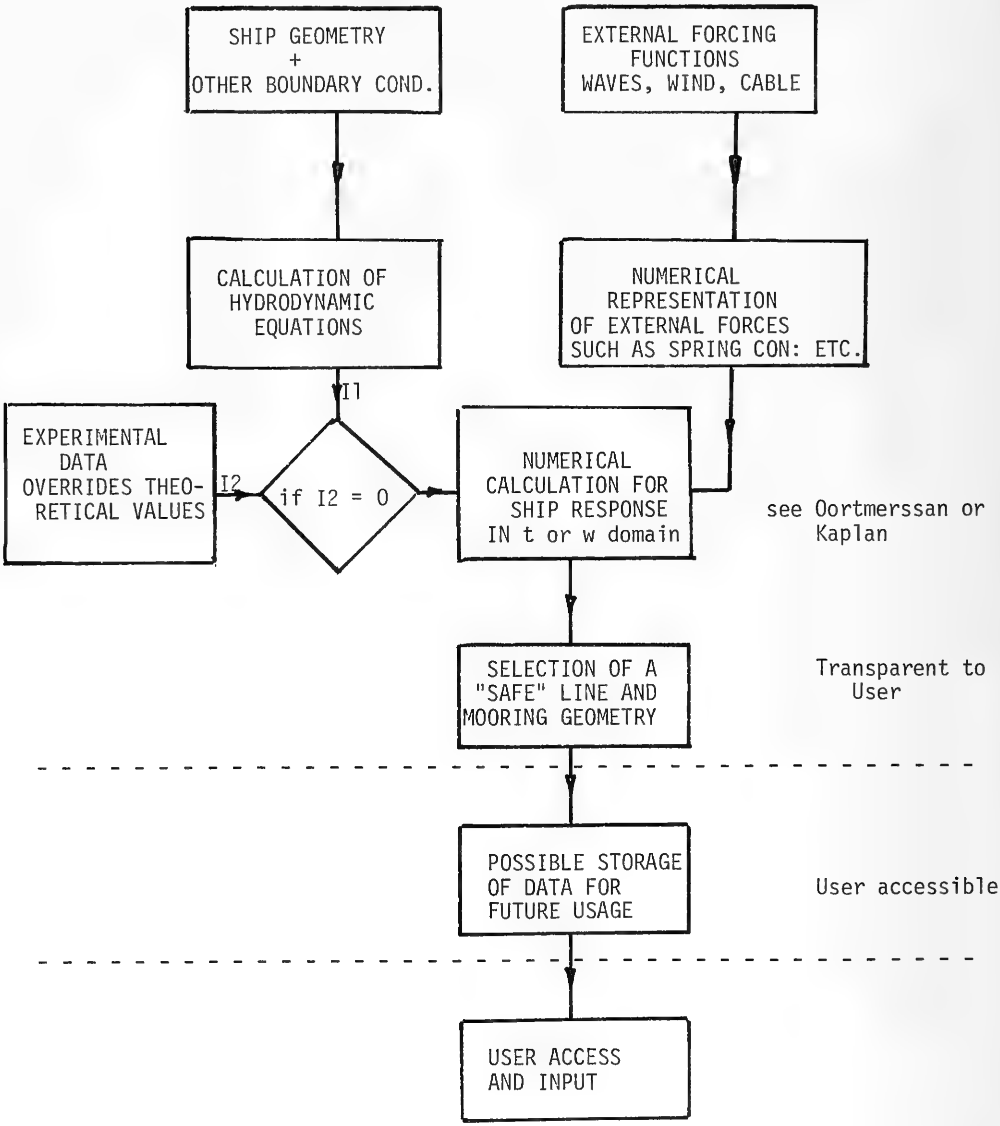

Figure 1

ASSUMPTIONS ON DYNAMIC EQUATION

The dynamic equations require an inertial frame, while the ship geometry is constant in the ship frame. The study of ship dynamics requires two sets of reference frames. Two frames coincide at the undisturbed conditions and the origin is taken at the ship center of gravity. As $(d/dt)_{inertial} = (d/dt)_{ship} + wx_{ship}$, and neglecting the flexibility of the ship, the equations of motion for translation can be written as:

$$F_i = M(\ddot{X}_i + \dot{X}_j w_j - \dot{X}_k w_k)$$

If all the terms in the parentheses are of order ε, then one can simplify the above equation and obtain an uncoupled linear equation as:

$$F_i = M \ddot{X}_i$$

This assumption should be valid outside the range of resonance or large motion amplitude. This, in turn, implies that the solution will not be accurate at or close to resonance. Similarly, for moment equilibrium, one can write:

$$M_i = I_{ii} \alpha_i$$

Even though the exclusion of nonlinear terms cannot be easily justified, they are adopted for simplicity and linearization required in the other building blocks of the model. The major question is how to interpret a large value of motion. The formulation adopted is the one due to Kaplan (1970).

HYDRODYNAMIC FORCES AND MOMENTS

The force acting on the ship can be decomposed into:

$$F = F_{pressure} + F_{viscous} + F_{restoring}$$

$F_{pressure}$ should, in general, represent the effect of fluid flow about the ship. This force is usually computed for the ship in equilibrium and fixed, assuming that the ship is transparent to incoming waves (i.e, "Froude-Krylov hypothesis"). The integration of pressure is usually done on the still-water-wetted surface or its projection. This calculation avoids harmonics that one can obtain by integrating up to the calculated wave height. Motion-induced pressure forces are represented under the group of forces added mass and damping coefficients. In their usage it is possible to see terms like

$$F_I = M^*(\ddot{u} - \dot{x})$$

where M^* is the added mass, $\ddot{u}$ is the volume-averaged water particle acceleration, and $\ddot{x}$ is the acceleration of the ship. This brings no complication to linearized equations except that it seems inconsistent with the definition of added mass, but it is probably not incorrect. We decided to ignore this possible correction term.

Viscous terms are normally expressed with the help of a viscous resistance coefficient and wetted surface area. In the application, one can find the usage of net velocity as:

$$F_{viscous} = \frac{\rho}{2} c_f \; s(u - \dot{x})(u - \dot{x})$$

or in linearized form

$$F_{viscous} = \frac{\ell}{2} c_f^* \; s(u - \dot{x})$$

These definitions exclude separation and form drag. Dynamic viscous effects are neglected in the present calculations in comparison to wave pressure loads.

Added mass and damping coefficient values for shiplike shapes are available for periodic motions for high frequencies (Lewis form) and for variable frequencies to the first or higher orders. These values are then assembled according to the relevant theory, such as strip theory. End effects are, therefore, usually neglected, and L is assumed to be very high compared to B and D. The real limitation from these calculations is that the values obtained make sense only for periodic motion.

Calculation of hydrostatic restoring forces is done using the well-known ship parameters TPI, GM, GM', etc. Again, pitch and roll interaction effects have to be neglected due to linearization. The assumption that the mean position of the ship is the upright one is not always valid as external effects, operation conditions, and damaged ships might have a large trim or heel, and a correction term is therefore necessary for these calculations. A complete, tested program is being coupled to the main program for the computation of hydrodynamic forces.

ASSUMPTIONS ON EXTERNAL CONDITIONS

It is assumed that the wind speed and its direction are constant and the boundary layer effect is neglected. Similarly, current speed and direction are taken as constant. Incoming waves are assumed to be periodic, small in amplitude, and consistent with linearizations so far introduced. Superposition is accepted in the form of known spectral densities (ϕ), such as Pierson-Moskowitz or Bretschneider, of the form:

$$\phi = \frac{A}{W^5} \exp\left(-\frac{B}{W^4}\right)$$

with the possible fetch effects. Different spectra are available for a specific selection. An important question here is that these spectra are verified mainly for the North Atlantic conditions. Solitary waves cannot and should not be used for the calculation even though they might generate a more serious mooring condition.

Wind and current force and moments are approximated by functions of the form:

$$F = A \sin \beta \qquad \text{and} \qquad M = B \cos 2 \beta$$

using the experimental data in DM-26 for Naval ships.

MOORING SYSTEM LOADS

These forms are observed to be more complex to describe than the previous ones. They are usually linearized, and the major new assumptions are the symmetry in the mooring system and continuity in mooring load. Elastic characteristics of the mooring line are reported to be of the form:

$$\frac{\sigma}{\sigma_{ult}} = R\, \varepsilon^{m} \qquad m \cong 1.7$$

R and m values are usually assumed to be constant. R is reported to change for wet, dry, new, and old mooring lines except for chains. Fenders are not always preloaded and cannot be studied by these equations as, in general, their responses are nonlinear and discontinuous. Such systems, on the other hand, have very important dynamic responses as they generate jumps and subharmonics (Wilson, 1973).

Lines are assumed to be pretensioned, and a geometric "catenary" description is used to find the "quasi-static" tension in the lines. For fixed anchor points, the maximum horizontal distance from anchor points to line connection points on the ship is calculated using the solution of dynamic equations. Line vibration stress is obtained by multiplying the quasi-static stress by a dynamic load factor.

Based on the survey of the above assumptions and built in the model, one can claim that a mooring system load calculation can be successfully computed by this computer program if:

1. The mooring system is symmetric and pretensioned.
2. The ship is in an environment where the sea can be represented by a spectrum, and the wind and current are constant in magnitude and direction.
3. The influence of other boundary and initial conditions is not significant, such as very shallow water or other ships in proximity.

4. Angular and translational velocities are moderate.

5. Viscous forces are small compared to other forces.

It seems that for other cases, calculations should be based on time domain solutions.

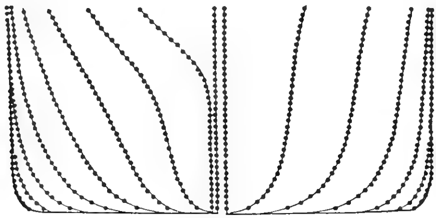

Figure 2

BIBLIOGRAPHY

Abkowitz, M. A., L. A. Vassilopoulis, and F. H. Sellar (1966). Trans. Soc. Naval Arch., Mar. Engr., vol 74, pp 194-259.

Bomze, H. (1974). "Analytical determination of ship motions and mooring forces," Offshore Technology Conference, 1974.

Kaplan, P. (1970). "Hydrodynamic analyses applied to mooring and positioning and vehicles and systems in a seaway," 8th Symposium on Naval Hydrodynamics, Aug 1970.

Raichlen, F. (1965). Wave induced oscillation of small moored vessels, W. M. Kech Laboratory of Hydraulics and Water Resources, California Institute of Technology, Report KH-R-10. Pasadena, Calif., Oct 1965.

Van Oortmerssen, G. (1976). The motion of a moored ship in waves, Doctoral thesis. Wageningen, 1976.

Wilson, B. W. (1973). "Progress in the study of ships moored in waves," in Proceedings of NATO Adv. Study Inst. on Berthing and Mooring of Ships, Mallingford, England, 1973.

SEMINAR SUMMARIES

1980 CEL Mooring Dynamics Seminar

Evaluation of

Mooring Dynamics Seminar

Civil Engineering Laboratory
Port Hueneme, California

January 10, 11, 1980

by

Bruce J. Muga

There were a number of discussions and points made during this seminar that were of specific interest. They are discussed below in somewhat random order.

1. The presentation by Dr. Ochi was particularly interesting since he has provided a rational explanation for what has been known but for which there has been no systematic development, explanation or description. This has substantial implication for design of systems to be deployed on a short term basis, and for scheduling of the deployment. It also clearly shows the site dependence of the climatic conditions. On a large global scale, this is to be expected. A case in point is the trade winds in the lower latitudes. On a smaller scale, sometimes these site dependent effects are obscured by the extreme variability of other processes.

2. The discussion concerning the second order wave force determination was an important part of the seminar. Unfortunately, a number of crucial details were not available.

   A number of methods for calculating the mean wave drift force were discussed. Essentially, they are:

Method	Hull Surface Description	Boundary Evaluation	General or Specific	Comparisons (Accuracy)
Newman (1967)	Strip	Far Field	All Wave Directions	None
Faltinsen & Michelsen (1974)	3-D Source Sink (Diffraction)	Far Field	All Wave Directions	Appropriate for Barges (Demonstrate that strip theory is not suitable for barges)

Method	Hull Surface Description	Boundary Evaluation	General or Specific	Comparisons (Accuracy)
Pinkster & Van Oortmerssen (1977)	Strip	Near Field	Beam waves only	Limited
Kim & Dalzell (1979)	Strip	Near Field	All wave Directions	Limited to Slendership
Molin (1979)	3-D Source Sink (Diffraction)	Far Field	-	Similar to Faltinsen & Michelsen

It appears that for determination of the mean drift force, the method of Faltinsen and Michelsen (1974) has the most general and immediate applicability, whereas, the method of Kim and Dalzell (1979) offers significant advantages for the future.

For the problem of predicting the slowly varying forces, the method of Kim and Dalzell (1979) is a very rigourous and sophisticated approach from which the mean drift force may also be obtained. The unique feature of this approach is the "near field" evaluation of the boundary condition where the pressures are integrated over the hull surface. This approach discloses the effects of the response motions and other body properties, which are obscured and/or averaged out with the "far field" approach.

The mean drift force was shown to be composed of five terms which are a function of:

a. (Relative wave height)^2

b. Integral (around the hull surface) of the square of the velocity. In other words, the Bernoulli quadratic.

c.-d. Two terms related to the radiation potential: One term is due to gyroscopic coupling, and the other is a force rate x displacement

e. Second order potential.

Kim and Dalzell (1979) have shown that (in at least one case) the first two terms are dominant and that the second order potential term may be neglected. It should be remarked that the procedures reported by Kim and Dalzell and by Pinkster and Van Oortmerssen both make use of the "weak scattering" assumption introduced by Salvesen. This is not quite the same as the slender body assumption although for certain wave directions, the assumptions have the same effect.

In summary, the method developed by Kim and Dalzell (1979) appears to be an accurate, rigorous method which could be extended to include 3-D source representations. This was indicated as being possible by Dr. J. Bendat. Moreover, such a procedure when combined with an appropriate mechanical model would yield the response statistics directly.

Following another approach of a general deterministic nature, there seemed to be general agreement that the Newman approximation (1974), in which only the diagonal elements are employed for determination of the time varying drift force from the mean drift force, is a suitable and appropriate engineering tool.

However, there did not seem to be clear and definitive agreement on the relative importance of the radiation potential as compared with the diffracted potential in determination of the mean drift force for either inertia or restoration force dominated systems.

In order to avoid the difficulties associated with determination of the second order force (which depends upon the first order motions), several possibilities were discussed. It was pointed out that the second order force could be evaluated by employment of a double convolution integral corresponding to that contribution generated by the radiation potential.

It is believed that this discussion could have benefited from more available time. For example, consider the case of an inertia dominated system such as a tanker. For such a system, the ship is very unresponsive to the exciting waves. Therefore, the motions at frequencies corresponding to the exciting wave periods are very small. As a result, although the ship has a large surface area, the wave system radiated (by the ship motions) is small as compared with the incident and/or diffracted wave systems. Therefore, one might conclude that that part of the second order drift force attributable to the radiation potential could be approached by an iterative method (as suggested, and, incidentally, as it has been done).

On the other hand, for a restoration-force dominated system, one might arrive at a completely different conclusion.

All in all, there were a number of aspects of this topic that should have been explored in more detail.

3. In summary, I believe that it was realized that the problem could be approached in two fundamentally different ways.

    a. A deterministic (SEADYNE/DSSM) approach in which the output statistics are obtained from multiple simulations (i.e., essentially "Monte Carlo").

b. A stochastic approach making use of cross bi-spectral analysis for determination of the exciting forces (both first and second order) and multiple input-output analyses for determination of system properties. In this approach, the mechanical model (linear or non-linear) is geared toward capability of handling stochastic input and output.

It is speculated that the latter method is potentially very rewarding and that the additional effort that is required in order to apply it to problems of interest more than justify its development. In addition, there are a number of parallel studies that should be carried out. These are:

(1) Delineation of inertia and restoration dominated systems.

(2) Evaluation of relative importance of forces due to radiation potential for

(a) inertia dominated system,

(b) restoration force dominated system.

(3) Evaluation of criteria for including dynamic cable effects.

(4) Evaluation of criteria for determination of linear, weakly non-linear, moderately non-linear, and strongly non-linear systems.

All of the above would be useful studies resulting in enhancements to the essential development effort.

4. I believe that the seminar probably achieved its purpose but in a round about way that was somewhat inefficient and redundant. In retrospect, I believe the format may have been too structured and the agenda not geared toward achieving the goals. For example, a clear statement of how the Navy breaks the problem down (into the various tasks), how the tasks are related and their priorities, what results are expected, how accurate the results should be, etc., etc., should have preceeded the presentation of papers.

All-in-all, the discussions could have benefited from more decisive stimulation with summaries of each topic by the moderator (or someone designated by the moderator) at the conclusion of each discussion period.

MOORING CABLE DYNAMICS SUMMARY

By

Dr. R. L. Webster

The two-day mooring dynamics seminar sponsored by CEL on 10 and 11 January 1980 covered various disciplines related to mooring vessels and platforms in unprotected waters. The participants were well chosen to provide broad coverage to the problem from practical as well as theoretical aspects. Attempts at ranking the presentations appear futile since they covered such diverse topics and points of view. The presentation by Bruce Muga pointed out some categories of approach to or features of mooring analysis that are useful. His classifications included (not necessarily in his order):

Ship Dominated	vs.	Mooring Dominated
Linear	vs.	Nonlinear
Frequency Domain	vs.	Time Domain
Stochastic	vs.	Deterministic

Muga further pointed out that the majority of mooring analyses presume ship-dominated linear systems that are treated either in the time or frequency domain with essentially deterministic methods. Treatment of nonlinearities requires time domain methods at present. Probabilistic data about the mooring responses are obtained from the time domain data by statistical evaluation of the output using a knowledge of the statistical nature of the input. Frequency domain solutions typically generate response spectra from superposition of discrete responses for waves selected from a wave spectrum or set of spectra.

Webster's paper emphasized the role of the mooring lines and their effect on the system response. It was suggested that ship-dominated systems are much less common than they are assumed to be. Both Paulling's and Webster's papers emphasized the importance of the geometric stiffening effect of the preloads in the mooring lines. The common assumption that mooring line dynamics have little consequence in ship-dominated systems may be justified in situations where small excursions occur that do not significantly reorient or stretch the lines. However, even in these situations, the effect of the mooring line preload on motions transverse to the line must be accounted for in the forces applied to the vessel. It is not correct to represent a mooring line as a single-force member (simple spring) with stiffness in only one line of action.

The major focus of the seminar was on the methods available for treating the mooring-dominated situation (be it in shallow or deep water) where the dynamics and nonlinearities of the lines cannot be ignored. Dynamic effects in the lines refer to mass- and stiffness-related phenomena such as resonances in the lines themselves as well as in the coupled system. Nonlinearities include material nonlinearities and slack/snap phenomena, bottom interaction, and geometric nonlinearities (large displacements).

All present appeared to agree that realistic analysis of mooring-dominated responses of a moored vessel or platform is a very formidable problem. The major difficulty is the development of the equations of motion for the vessel or platform and the expressions for the wave-induced loads. A particularly troublesome aspect of this is the evaluation of the effects of the second order wave-induced drift forces. These forces have two components: one at high frequency and one at low frequency. The high-frequency component is generally of sufficiently high frequency and low amplitude that it can be neglected. The low-frequency component cannot be ignored since it can produce large-amplitude, low-frequency excursions of the system, thereby involving the nonlinearities of the mooring. Calculation of these second order forces is greatly complicated by the fact that they depend on the motion of the moored body, which in turn depends on the characteristics of the mooring, which change significantly as the system moves. Although it was emphasized that the major interest was in designing and evaluating the adequacy of the mooring system and not in the specific motion of the moored object, it is apparent that you cannot get one without the other.

The SEADYN/DSSM approach was discussed briefly. The setting of the presentation (following an extensive discussion of possible approaches) precluded much critical discussion of the DSSM approach. In general, the feeling appeared to be that the approach was sound but did not go far enough. Questions were raised about the spectra used, the nature of the statistical calculations, and that it does not deal with the unsteady part of the second order drift forces.

Three general approaches to the problem emerged from the discussions. There was not time in the seminar, nor was it the appropriate place, to explore them enough to completely define them, but some rough outlines were developed. These three approaches are briefly described below.

APPROACH A – EXTEND THE PRESENT DSSM SOLUTION

1. Calculate the initial static reference state with steady components of wind, current, workloads, etc.
2. Solve frequency domain dynamics of a coupled system with appropriate wave spectra to estimate the steady part of second order drift forces.
3. Adjust the reference state for steady drift forces.
4. Repeat the dynamic solution to evaluate changes (repeat step 3 if needed) and estimate the unsteady second order drift forces.
5. Perform large displacement time domain solutions to get the response to the unsteady drift forces. The small displacement dynamics represented by the frequency domain solution are neglected in this step.
6. Locate the extreme states from step 5, and repeat the frequency domain solution at each of these states to identify the worst combined conditions and make statistical estimates.

Changes required in SEADYN:

- New wave spectra form

- Improved calculations of steady drift forces
- More reliable static solution method
- Calculation of unsteady drift forces
- New capability in DYN solution to deal with rigid-body dynamics including slave/master constraints
- Improved statistical evaluations
- General logic development to tie the steps together

APPROACH B – TIME DOMAIN SOLUTION

1. Calculate the initial static reference state with steady components of wind, current, workloads, etc.
2. Generate the retardation function for the moored body in the reference state.
3. Generate the time sequence of wave loads representing the wave spectra.
4. Solve the large displacement time domain equations until significant motion occurs.
5. Repeat steps 2 through 4 until the time span is completed.
6. Make statistical evaluations of the output.

Changes required in SEADYN:

- Completely new coding representing steps 2, 3, and 6
- New capability in DYN to deal with the convolution equation form and the slave/master constraints
- A reasonable algorithm for estimating when significant movement occurs (step 4)
- General logic development to tie the steps together

APPROACH C – STOCHASTIC METHODS

This is a very loosely defined approach that would rely on the generation of second order transfer functions for the coupled mooring system. The fact that second order terms are contained in the transfer functions makes it possible to include the second order drift forces directly. It is obvious that a static reference state must be obtained first as in the other approaches, but the procedure beyond that point is not clear. The second order transfer function depends on the static reference state since it is based on a Taylor Series expansion relative to that state. The procedure for dealing with large shifts in the reference state is undefined. Work is also needed to define the second order terms for the mooring line stiffness, damping, and mass. The second order approximations for the drag effects on the lines would also need to be developed. It is also probably true that the second order terms would have to be developed for ships and platforms.

Much more study is required before any of these approaches can be clearly defined and compared. Some things that may help in this study could be obtained through sensitivity studies on the present DSSM computer program. Consideration might be given to the importance of mass and damping of the mooring lines, changes in the reference state, etc.

Comments Regarding the CEL Seminar Discussion on Mooring Dynamics - C. J. Garrison

The discussion was primarily directed toward the various mathematical models and assumptions which might be appropriate to the mooring of ships or barges in random seas. Very quickly the discussion converged on the question of the modelling of slowly varying drift-forces and resulting motion.

Provided one dealt with both the high-frequency motion and slowly varying drift forces within the realm of the frequency-domain, no difficultics should arise. However, while the mooring lines can generally be adequately treated by use of a linear approximation for relatively small oscillations about a mean position, this is not the case for the large displacement oscillations resulting from the slowly varying drift-forces. Thus, it seems obvious that a time-domain analysis should be used to treat the slowly varying motion while a frequency-domain analysis could be used for the high-frequency motion. The difficulty in this approach is that considerable interaction between the high-frequency motion and the slowly varying motion exists. The frequency response of the vessel is dependent on the mooring line tension which in turn is dependent on the slowly varying drift. On the other hand the slowly varying drift-force is strongly dependent on the high-frequency response. It appeared that by the conclusion of the meeting this difficulty remained and no obvious solution seemed to be forthcoming.

SUMMARY
MOORING DYNAMICS SEMINAR

McCreight

The principal question considered during the discussion of the assembly and evaluation of consistent mooring models concerned inclusion of slowly-varying drift forces into the time-domain model. It was quickly agreed that direct evaluation of the double convolution integral model is far too expensive computationally for use in a practical model. Newman's single summation approximation using the steady drift force data appears to be a good model for time-domain simulation of the slowly-varying drift force.

There are several problems in applying this model, aside from obtaining the steady drift force data in the first place. The principal difficulty is that the coefficients $H^{(2)}(\omega_n,-\omega_n)$ depend on the first-order motions of the ship, which in turn are affected by the drift force which causes the stiffness and geometry of the mooring system to change as the ship moves in response to the drift forces. The variation of the phase of the wave components with position must also be accounted for. Muga stated that in his experience the drift motions do not change the first-order oscillatory motions sufficiently to significantly affect the results. He attributes this to the relative importance of the diffraction and radiation potentials for the slowly-varying drift force. In general, however, we must allow for the possibility of these effects. There is some data from Stevens Institute for a Series 60 hull which shows quite large differences between drift forces for the fixed and free hull cases, and consequently if the first-order motions are affected by the drift motions there will be a problem if this effect is neglected.

Methods of including this effect were discussed. One approach is an iterative calculation in which the drift force in a given iteration is calculated based on the first-order motions from the previous iteration. This would be a quite expensive procedure, and convergence is an open question.

Pauling proposed a method in which time histories of the slowly-varying drift force are precomputed for a grid of ship positions (surge, sway, and yaw). During the actual run, the slowly-varying drift force would be interpolated using these precomputed time histories. This approach would also avoid the difficulty of extracting amplitude and phase for each component of the linear response, which is the form in which the first-order responses are actually required as these responses would be computed in the frequency domain using the existing linear frequency-domain model. A possible drawback is that an excessively fine grid may be required for accurate interpolation, which would increase the number of time histories to be precomputed. It is otherwise a rather attractive approach.

A direct method requiring neither iteration nor multiple precomputed drift force time histories would be very desirable for those cases in which the drift motions do affect the first-order motions.

The possibility of a second-order frequency domain simulation as an alternative to the time-domain simulation was discussed. If it is possible to extend the linearized cable dynamics model to compute the second-order response to two simultaneous sinusoidal incident waves, this could be combined with the second-order ship motions and drift force model to obtain the corresponding second-order system response. From such a model various statistics of the system responses can be calculated, as discussed in several of the references to the presentation. Development of this approach would be a very ambitious project, but could yield an accurate, efficient model taking into account the most important nonlinear effects, and should be considered.

SUMMARY NOTES ON MOORING DYNAMICS SEMINAR

Michel K. Ochi

Comments and remarks on some important subjects associated with mooring dynamics are summarized as follows:

1. Waves for Mooring System Design

(a) Since a mooring system encounters a variety of wave conditions even though the sea severity (significant wave height) are the same, and since the system responds strongly to low frequency components of wave spectra, it is highly recommended to use a series (family) of wave spectra consisting of several members for any specified sea severity.

(b) It is recommended to use the fetch-limited wave spectra for evaluating responses of a mooring system located in the area where the fetch length has to be considered. The shapes of the fetch-limited wave spectra are significantly different from those for open sea spectra.

(c) For mooring in coastal zones including the continental shelf, the wave spectra should be modified taking into account the effect of water depth on spectra. This can be done by developing a computer program to modify the wave spectra from deep to shallow water area.

(d) The estimation of extreme values, such as the maximum tension of the mooring lines, etc., has to be precise based on extreme value statistics. The method of evaluating the maximum tension loads given in the available literatures appears to be inadequate.

(e) It is recommended to consider the directional wave spectra for more accurate prediction of responses of a mooring system in a seaway although the computation would be extremely complicated.

2. Nonlinear Behavior of Mooring System

It is apparent that the nonlinear dynamic response has to be considered for a mooring system in a seaway. Many nonlinear dynamic response problems which appeared in naval and ocean engineering have been solved by applying either the equivalent linearization technique or the perturbation method. However, the complexity involved in this particular nonlinear dynamic system has not been fully explored, and hence it is somewhat difficult to recommend the approach which is most appropriate to solve this particular problem. Some approach suggested during the Seminar appears to be highly desirable, but the approach may not be feasible in practice. For example, the nonlinear response of a system may be expressed as follows:

$$y(t) = y_1(t) + y_2(t)$$

$$= \int h(\tau)\, x(t-\tau)\, d\tau + \iint h(\tau_1, \tau_2)\, x(t-\tau_1)\, x(t-\tau_2)\, d\tau_1\, d\tau_2$$

where, the first term represents the linear response, and $h(\tau)$ can be evaluated from the frequency response function of the system without any difficulty. On the other hand, $h(\tau_1,\tau_2)$ involved in the second term can be evaluated only through the cross bispectrum of the system. It is unknown, in general, unless the results of either model experiments or full scale trials carried out on the individual mooring system are available. This results in the approach shown in the above equation appears to be not feasible, in practice.

Judging from the information available on the nonlinear mooring dynamics, the iteration method in the frequency domain appears to be a promising approach.

RECOMMENDATIONS FROM CEL MOORING DYNAMICS SEMINAR

J. S. Bendat

A two-day seminar on mooring system dynamics was held at CEL on 10-11 January 1980 where various aspects of this field were discussed. The first day was devoted to separate presentations by nine invited participants on topics dealing with mooring systems analysis and cable models, mean and second-order drift forces, vessel equations of motion, random wave characteristics, and spectral analysis techniques for system identification. The CEL cable dynamics/moored vessel model was outlined that employs finite-element techniques to account for various linear and nonlinear system properties.

Group discussions on the second day concerned desired results from any mooring system model and the particular results obtained by the CEL model. A major open problem was pointed out on the need to obtain actual experimental input/output data to provide detailed evaluations and comparisons between model predicted results and measured results. It was also clear from this discussion that a strong requirement exists to be able to determine the <u>overall</u> linear and nonlinear features of the total mooring system that has been included in the CEL model from input points to output points, as well as the <u>specific</u> linear and nonlinear system properties between certain measurement points.

Recommendations

1. The first recommendation is that computer simulation studies be performed to determine the <u>overall</u> linear and nonlinear features in the present CEL model and in later revisions of this model. Presently available coherence analysis techniques can be used for this work. In particular, a coherent output spectrum calculation can decompose the total measured output spectrum at any frequency into two parts representing (a) the overall optimum linear features and (b) the remaining uncorrelated noise due to nonlinear effects as well as other causes.

2. The second recommendation is that an investigation be undertaken to develop new techniques to determine from measured input/output data specific linear and nonlinear system properties between input wave spectra and output response spectra. Practical digital computer procedures are desired where the analysis is conducted in the frequency domain rather than in the time domain.

   (a) Linear systems should be found based on analysis of autospectral and cross-spectral density functions using current known procedures.

   (b) Nonlinear systems should be found based on analysis of appropriate bispectral and cross-bispectral density functions by developing new methods that can provide useful engineering interpretations.

   (c) Special nonlinear coherence functions should be defined to determine the validity of proposed nonlinear models.

   (d) Output spectra should be decomposed into linear components, nonlinear components, and remaining uncorrelated noise. (These results will provide the basis for future evaluations and comparisons of measured results with predicted results from CEL models.)

3. This participant believes it is now feasible to carry out the above work to identify nonlinear system properties and their effects from measured input/output data, where the practical procedures to be developed can be direct extensions of current methods used to obtain optimum linear systems from such data. Recommended tasks are as follows:

   1. Study of past published work on bispectra analysis.
   2. Development of new detailed analytical procedures.
   3. Writing of digital computer programs.
   4. Verification by computer simulation studies.

J. S. Bendat

J. S. Bendat

SUMMARY OF THE RESPONSE OF MOORED FLOATING PLATFORMS TO OCEAN WAVES

By

Dr. J. R. Paulling

I appreciate being given the opportunity to participate in the Mooring Dynamics Seminar on January 10-11 and feel that it was an effective and worthwhile meeting. I believe that you were able to bring together an enthusiastic group of people, representing all of the scientific and engineering disciplines which are of importance in dealing with mooring problems, and all participants seemed to have some significant contributions to the discussions which took place. One feature which made the meeting especially worthwhile was the small size of the group, which eliminated any feeling of formality and made possible some very spontaneous and stimulating discussions.

In reviewing my notes on the discussions, there are two points which I think should be considered in directing your future work, and I have outlined them below.

1. The methodology for solving the response of a moored ship involves the simultaneous solution for linear and nonlinear motion effects. This is because the wave frequency ship motions are reasonably well predicted by linear methods (e.g., strip theory), while the slow drift forces, which are important to the mooring dynamics, depend in part on two nonlinear effects: the wave reflection, which is proportional to the square of the wave amplitude, and the nonlinear interaction between the waves and ship motion, which is neglected in strip theory.

The simultaneous solution might be approached in either of two ways, and the optimum is not completely clear at this point. The first is to use a sort of "brute force" direct numerical integration of the complete equations of motion, which include both the linear and nonlinear effects. The potential drawbacks to this procedure are the computer time requirements for direct integration and, second, the complication of including the frequency-dependent hydrodynamic forces on the ship. The latter can be overcome by a convolution integral technique, and has been applied to ship steering problems by a student of mine, Dr. Leo Perez. The second procedure would involve several steps as follows:

a. Solve for the linear response of the moored ship.

b. Compute the nonlinear slow drift forces using this response combined with the assumed wave spectrum.

c. Solve for the nonlinear response of the moored ship to the slow drift forces of b.

d. If the slow drift large amplitude motion results in sufficient change in the mooring configuration to appreciably affect the linear response, repeat steps a-c. Several iterations may be required before the process converges.

This procedure has the possible advantage of breaking the total problem down into two somewhat simpler ones which are then solved separately. I do not know of its having been tried in a similar context, and it is not certain that convergence will be assured.

The first procedure described above is essentially already being used in your programs developed by Dr. Webster except, as I understand, the wave frequency dependence of the forces is not included.

2. The method of bispectral analysis was suggested as a promising means of obtaining the slow drift forces and other nonlinear effects from experimental data either on model scale or full size. I would certainly agree that this method should be explored and its full potential evaluated, but I have one serious reservation. This concerns our ability to measure the input (i.e., the waves with the required precision). In full-scale measurements especially, but also in model scale, the presence of the ship significantly distorts the input waves by diffraction. It is, therefore, not possible to measure the "pure" input signal, although the output or ship response can be measured quite precisely. In the laboratory, we can approach this pure signal by first recording the waves with the model removed and then replaying exactly the same control signal through our wave generator with the model in place. There is still a source of error due to imperfect synchronization and other imperfections in the apparatus. In full scale at sea we can only measure waves in the near vicinity of the ship and accept the distortion. I think, therefore, that these problems must be addressed simultaneously with further exploration of the bispectral approach in order to fully evaluate the accuracy of any results which might be obtained in that way.

SUMMARY OF CEL MOORING DYNAMICS SEMINAR
By
Dr. S. Calisal and Dr. R. Bhattacharyya

The Mooring Dynamics Seminar at the Civil Engineering Laboratory brought together specialists related to the field. The presentations and discussions during the first day indicated the necessity of close cooperation between different fields contributing to the formation and solution of the problem. The discussions during the second day indicated to this participant that:

1. Most computer programs that are available today seem to have a weak link in their procedures.
2. A hierarchy of computer programs should be available to CEL.
3. A reliability factor should be attached to computer programs not only by numerical tests but also by comparing predicted results with full-scale or model tests.
4. Second order effects should be included for calculations that require special accuracy either in magnitude or phase of physical variables.

We would like to thank CEL for giving use a chance to explain the set of assumptions used in the mooring program under development at the U.S. Naval Academy and other participants for their constructive criticism.