Procedure for developing a SITE INDEX ESTIMATING SYSTEM from Stem Analysis Data

by FLOYD A. JOHNSON
and NORMAN P. WORTHINGTON

PACIFIC NORTHWEST
FOREST AND RANGE EXPERIMENT STATION
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Philip A. Briegleb, Director
Portland, Oregon

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INTRODUCTION

Site index curves for red alder[1/] were developed from stem analysis data by a procedure which has since been adopted for lodgepole pine in eastern Oregon and for aspen in Alaska. In these three cases, the results have been so encouraging that a detailed description of the procedure seems justified. The red alder data has been used to illustrate the procedure in the description which follows.

THE DATA

Forty-three red alder trees from 16 locations scattered throughout the red alder stands of western Washington were selected, felled, and cut into sections during 1957 for the site index phase of the red alder yield-table project.

Height above ground and tree age corresponding to this height were determined for each point of cutting on every tree. Measurements recorded for tree number 26 have been listed in table 1 to illustrate the nature of the basic data.

Table 1.--Basic data for tree No. 26

Tree age	Tree height	Tree age	Tree height
Years	Feet	Years	Feet
2	8	25	63
3	16	29	68
5	24	34	73
8	32	37	78
12	40	41	83
15	48	52	88
18	53	58	93
22	58	68	96

Tree ages at the time of felling varied: 13 trees were less than 50 years, 3 trees were over 80 years, and the others were somewhere between 50 and 80 years.

Each of the 43 trees was dominant at time of cutting and was considered to have been dominant throughout its entire life. Evidence to the contrary would have been cause for rejection.

1/ Bishop, Daniel M., Johnson, Floyd A., and Staebler, George R. Site curves for red alder. U.S. Forest Serv. Pac. NW. Forest & Range Expt. Sta. Res. Note 162, 7 pp., illus. 1958.

THE PROCEDURE

Step 1

Heights of the various cuts were plotted over corresponding ages for each of the 43 trees, and adjacent points were connected by straight lines. Figure 1 illustrates this step for tree number 26.

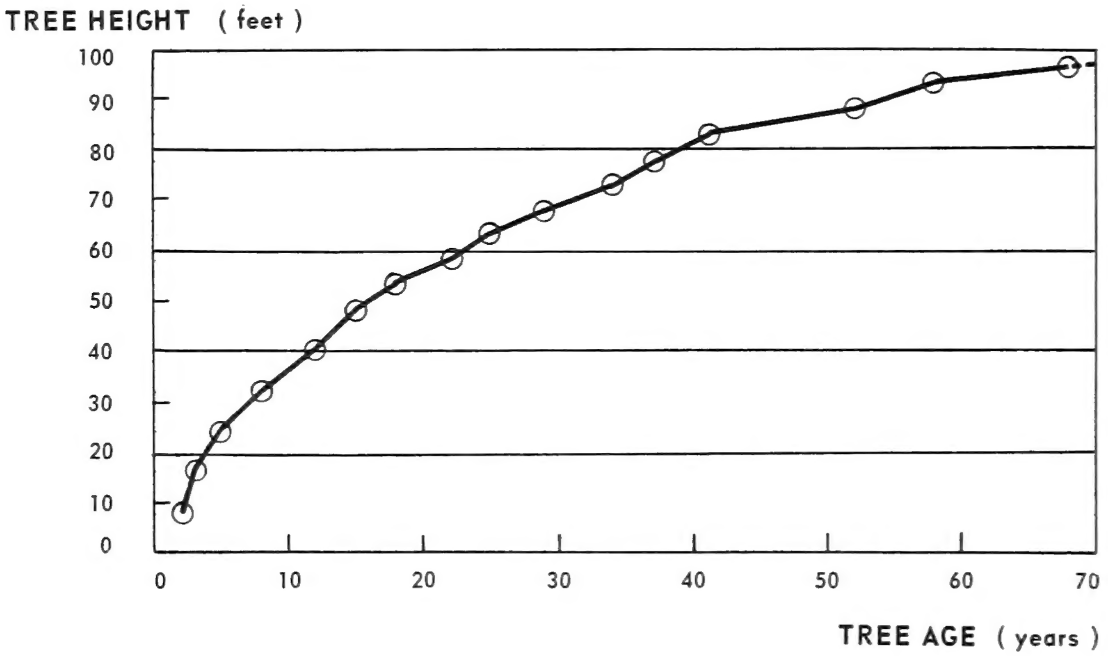

Figure 1.—Height over age for tree No. 26

Step 2

Heights from the segmented curves constructed in step 1 were listed for each of the following ages: 10, 15, 20, 25, 30, 35, 40, 45, 50, 60, and 70. These heights are shown in table 2 for age 30. All 43 trees were at least 30 years old when cut. The uppermost segment of the curve for each of the 13 trees which were less than 50 years old when cut was extended to intercept age 50. Thus, the table for age 50 corresponding to table 2 also contained 43 trees. These 13 extrapolations covered only a few years at the most and were considered entirely acceptable. Tables for ages 60 and 70 had fewer than 43 trees because extrapolations to ages 60 and 70 were not acceptable for all trees. An example of what was considered to be an acceptable extrapolation may be seen in figure 1 as the broken line extension of the last segment to age 70.

Table 2.--Tree heights at age 30 for 43 trees

Tree number	Height at age 30	Tree number	Height at age 30	Tree number	Height at age 30	Tree number	Height at age 30
	Feet		Feet		Feet		Feet
1	81.5	12	53.5	23	94.5	34	82.5
2	78.5	13	50.0	24	80.0	35	80.0
3	80.0	14	58.0	25	72.0	36	83.0
4	68.5	15	59.5	26	69.0	37	80.0
5	69.5	16	60.0	27	66.0	38	74.5
6	71.0	17	61.0	28	68.0	39	73.0
7	67.0	18	79.0	29	63.0	40	81.5
8	51.0	19	88.0	30	57.5	41	86.0
9	63.5	20	82.0	31	57.0	42	70.0
10	59.0	21	86.0	32	47.0	43	73.0
11	48.5	22	89.0	33	53.0		

Step 3

Average height at age 50 as calculated from all trees at a location was arbitrarily called the site index of that location. These site indexes are listed in table 3 for each of the 43 trees. For each of 10 ages (10, 15, 20, 25, 30, 35, 40, 45, 60, and 70), site indexes were then plotted over corresponding heights. This step is illustrated in figure 2 for age 30. Note that site indexes in table 3 were plotted over heights in table 2 to give the scatter diagram in figure 2.

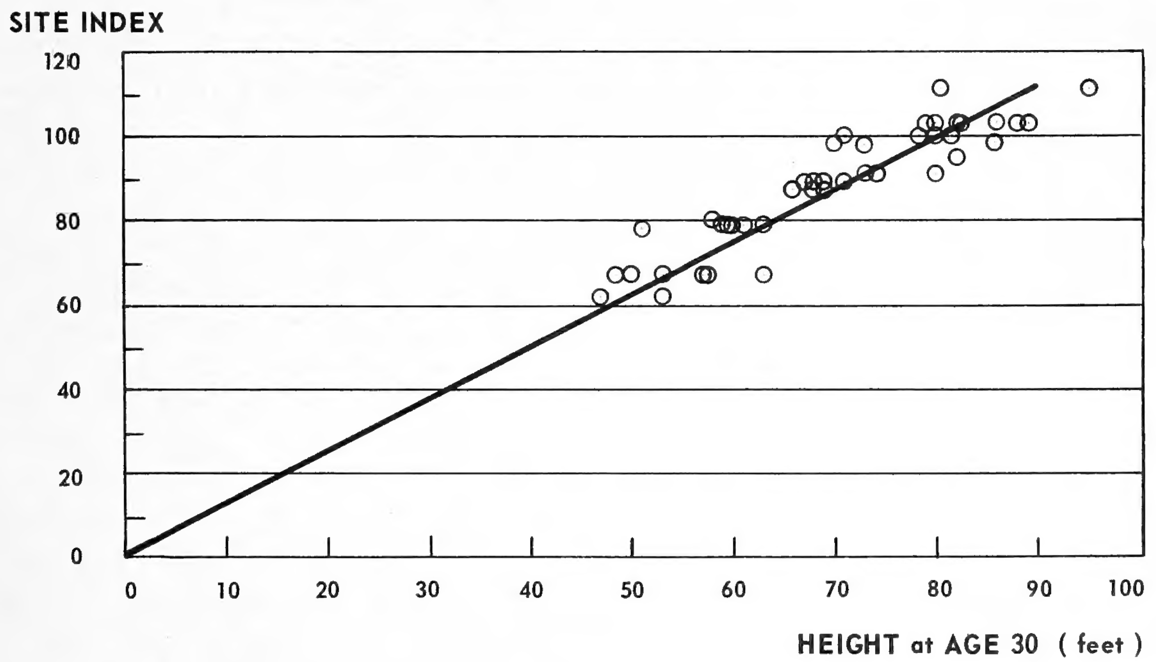

Figure 2.--Scatter diagram. Site index over height at age 30.

Table 3.--Site indexes for 43 trees

Tree number	Site index	Tree number	Site index	Tree number	Site index	Tree number	Site index
1	100.0	12	67.0	23	111.0	34	103.5
2	100.0	13	67.0	24	111.0	35	103.5
3	100.0	14	80.0	25	100.0	36	103.5
4	88.5	15	78.0	26	87.5	37	91.5
5	88.5	16	78.0	27	87.5	38	91.5
6	88.5	17	78.0	28	87.5	39	91.5
7	88.5	18	103.5	29	67.5	40	94.5
8	78.0	19	103.5	30	67.5	41	98.0
9	78.0	20	103.5	31	67.5	42	98.0
10	78.0	21	103.5	32	62.0	43	98.0
11	67.0	22	103.5	33	62.0		

Step 4

A linear least squares line was calculated for the relationship of site index to height with a separate line for each of the 10 ages. The line for age 30 is shown in figure 2. There was no indication from these plottings that anything other than a straight line would have been appropriate. Each line was made subject to the condition that site index was zero when height was zero. Subsequent steps in the procedure proved to be intractable unless this condition was imposed. In addition, the condition was not hard to accept. For each of the 10 scatter diagrams, the calculated line was as obviously acceptable as it was in figure 2.

The mathematical expression for each line was:

$$S = bH \qquad (1)$$

where S = site index

H = height at a given age

b = regression coefficient = $\frac{\Sigma SH}{\Sigma H^2}$

Step 5

The 10 regression coefficients from the 10 analyses (table 4) were plotted over age, and a smooth progression of b values resulted (fig. 3).

Table 4.--Regression coefficients for the relationship of site index on height

Untransformed		Transformed	
Age (A)	Regression coefficient (b)	x = 1/A-1/50	y = b-1
10	2.59	0.080	1.59
15	1.90	.047	.90
20	1.54	.030	.54
25	1.36	.020	.36
30	1.26	.013	.26
35	1.16	.009	.16
40	1.10	.005	.10
45	1.04	.002	.04
60	.95	-.003	-.05
70	.90	-.006	-.10

Step 6

Data in the first two columns of table 4 were transformed as follows to force the condition b = 1.00 when age = 50 (obviously, b must be 1.00 when age is 50 because site index is equal by definition to height at age 50):

$$y = b - 1$$

$$x = 1/A - 1/50$$

where A = age

The transformed values in the last two columns of table 4 were then used to find c for this expression:

$$y = cx$$

$$\text{where } c = \frac{\Sigma yx}{\Sigma x^2}$$

Thus:

$$y = 19.452x$$

and after reconverting y and x:

$$b = 0.611 + 19.452/A \text{-------------------} (2)$$

This curve is shown in figure 3.

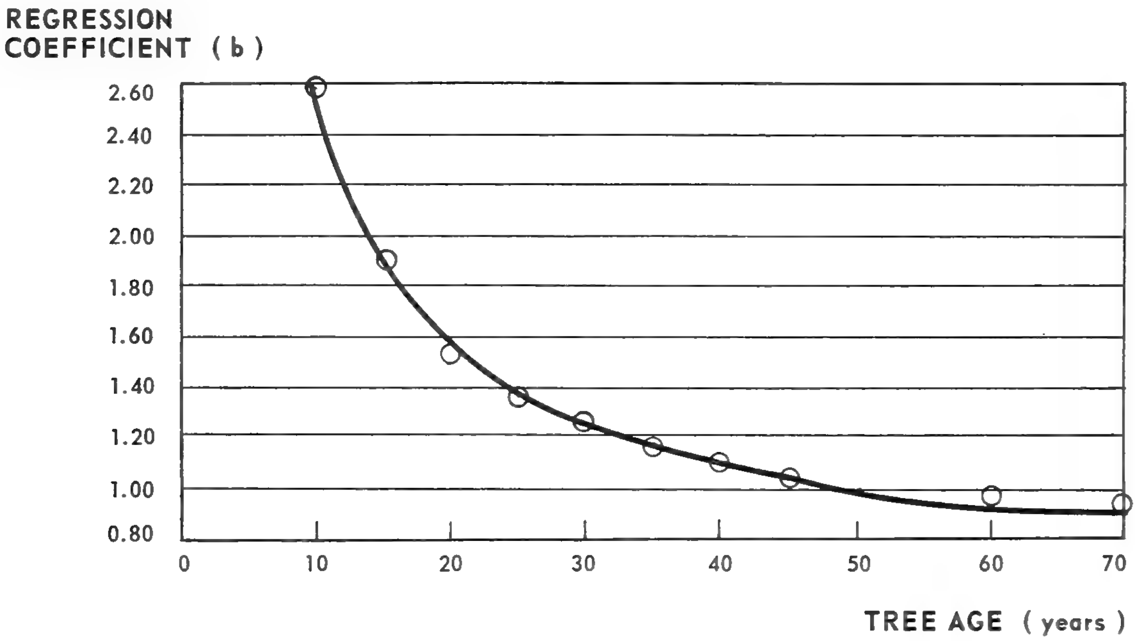

Figure 3.—Regression coefficients for the relationship of site index to height, plotted over age.

Step 7

An equation which generates site index for any combination of age and height resulted when equation 2 was substituted for b in equation 1. Thus:

$$S = (0.611 + 19.452/A)H \text{----------------} (3)$$

DAHMS' MODIFICATION

An evolution of the above procedure is already underway. In developing site curves for lodgepole pine, Dahms[2/] took the height of only the tallest of six trees on a plot at each of several stand ages. Although this left him with only one-sixth the amount of data that would have been available under the procedure described in this report, there is a good chance, at least in his case, that he avoided a serious bias. Except for this one modification, Dahms used the procedure described above.

2/ Dahms, Walter G. Correction for a possible bias in developing site index curves from sectioned tree data. Jour. Forestry 61: 25-27, illus. 1963.

DISCUSSION

Equation 3 is not the same as the comparable equation published in "Site Curves for Red Alder"[3] because a few minor errors were discovered in the original work. Since both equations give almost identical results, these errors were of no practical importance.

When equation 3 is used to develop curves of height over age for each of several site indexes, one curve will be proportional to another in the sense that the ratio of heights for a given age from the two curves will be constant over all ages. This proportionality is caused by the arbitrary restriction on the curves of site index over height (see step 4) which forces them through the zero-zero coordinates. The only justification for the proportionality is the empirical evidence (see fig. 2) which suggests that the restriction is reasonable.

Site index for an individual tree should stay constant over all ages. Thus, if equation 3 were to be applied to the basic data from a single tree, there should be no evidence of a trend when estimated site index is plotted over age. This was done for each of the 43 trees and the results for tree number 26 (see table 1) are shown in figure 4.

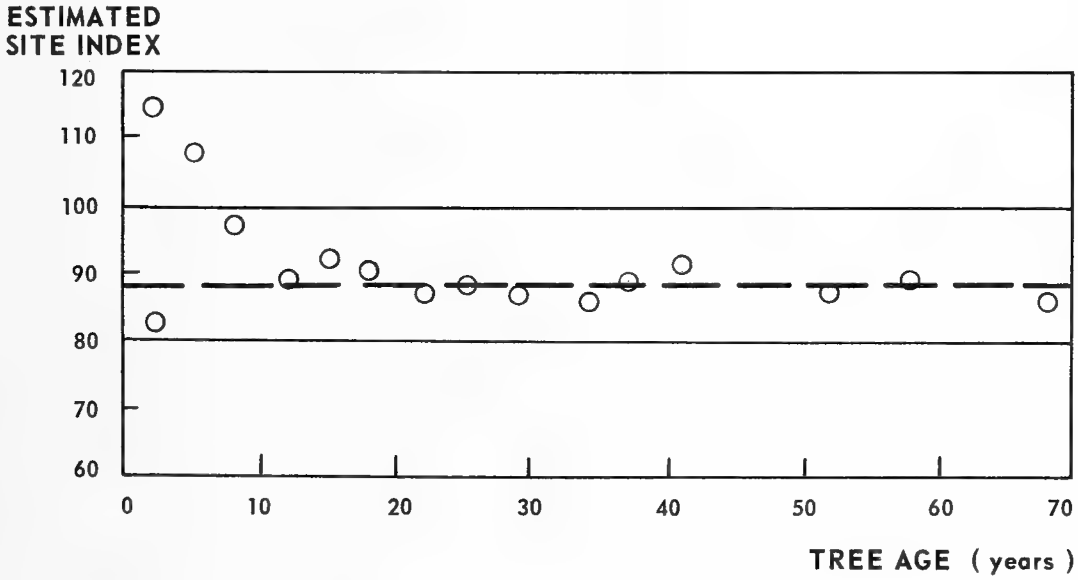

Figure 4.—Estimated site index plotted over age for tree No. 26.

Estimated site index was found from these plottings to be very erratic when age was less than 10 years. Above 10 years, 17 trees gave no evidence indicating a trend of estimated site index on age (tree 26 was one of these 17). A positive trend was apparent on 15 trees, and a negative trend was apparent on the remaining 11 trees. Perhaps this can be taken to mean that equation 3 is acceptable when age is 10 or greater.

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[3] See footnote 1.

Differences between estimated and actual site index, as calculated from all measurements on all trees, were plotted over age in figure 5. Here, a negative bias is evident below 10 years, and a lack of bias is evident when age is 10 or greater. Standard deviations of these differences were found to be 16 site index units for age class 10 to 20 years, 8 site index units for age class 20 to 30 years, 5 site index units for age class 30 to 40 years, and 4 site index units for age class 40 to 50 years.

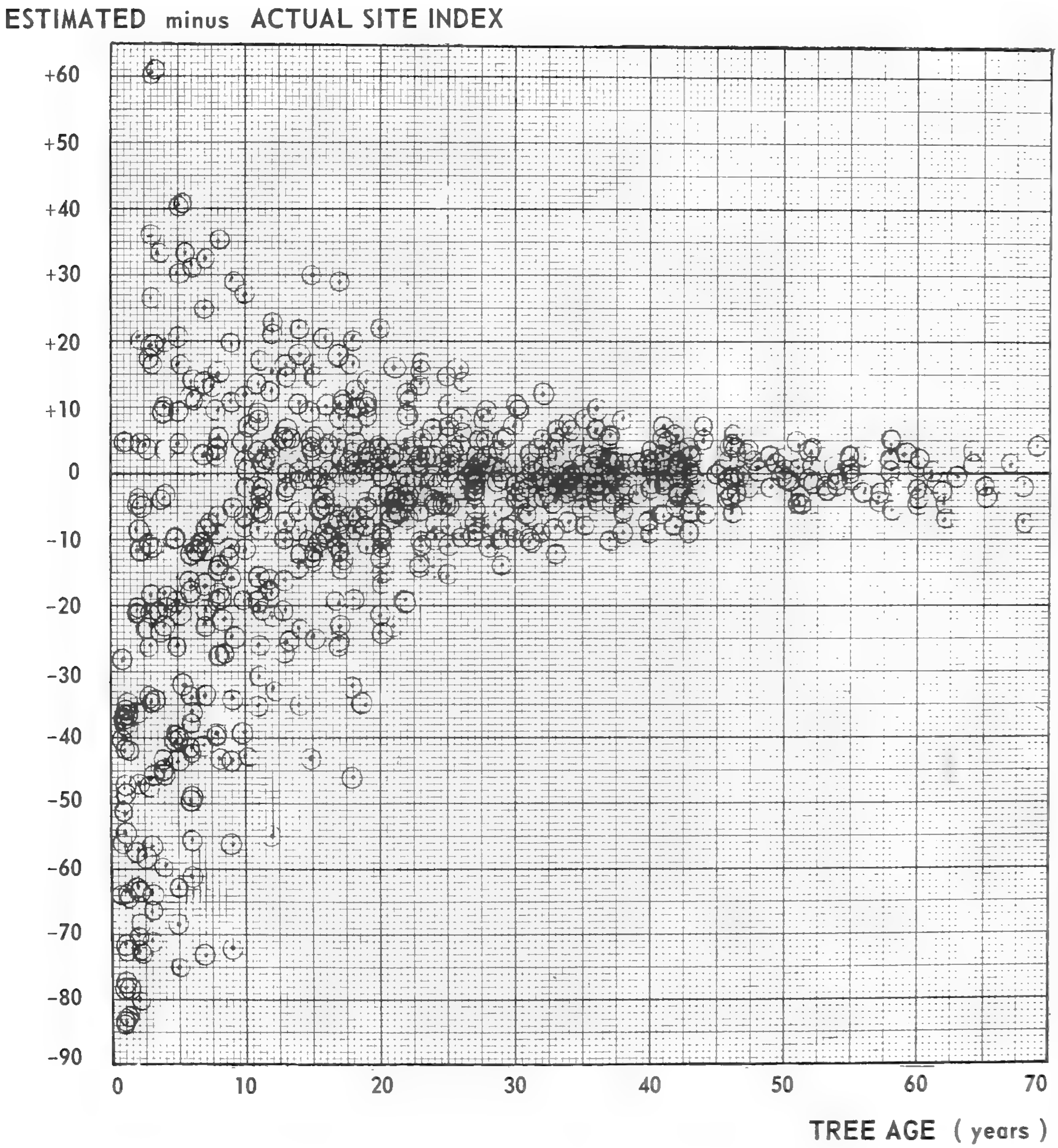

Figure 5.—Differences between estimated and actual site index plotted over tree age (43 trees).

The complete set of red alder data has been made available in table 5 for the benefit of anyone who might wish to experiment further with procedures for developing a site index estimating system.

Table 5.--Basic red alder stem analysis data

Location	Tree No.	Heights and ages for 43 red alder trees																					
1	1	H	10	20	30	40	50	60	70	75	80	85	90	95	100								
		A	6	8	9	11	16	17	19	26	29	32	38	43	50								
	2	H	2	10	20	30	40	50	60	70	75	80	85	90	95	100							
		A	1	4	5	8	10	13	16	22	27	31	36	40	46	50							
	3	H	10	20	30	40	50	60	70	75	80	85	90	95	100	102							
		A	9	12	15	18	21	23	24	27	30	32	36	41	48	51							
2	4	H	2	4	10	20	30	40	50	60	65	70	75	80	85	90	96						
		A	2	5	6	7	11	15	20	23	25	32	34	37	47	50	58						
	5	H	10	20	30	40	50	55	60	64	68	74	78	80	84	88	94	98					
		A	5	6	8	15	17	22	24	25	29	33	35	37	41	46	52	58					
	6	H	2	4	10	20	30	40	50	65	70	75	80	85	90	100	105	108					
		A	2	3	5	6	10	12	13	22	29	34	38	43	60	72	79	83					
	7	H	4	10	20	30	40	50	55	60	65	70	75	80	85	90	95	100	105				
		A	3	6	8	12	14	15	18	24	29	31	34	37	51	56	62	64	76				
3	8	H	2	6	11	18	24	30	35	40	45	50	55	60	65	68							
		A	1	4	7	11	13	17	18	20	24	29	33	37	40	43							
	9	H	3	8	16	24	32	40	45	50	55	60	65	70	74								
		A	2	3	6	9	11	14	16	18	22	26	32	38	42								
	10	H	1	8	16	22	32	40	45	50	55	60	65	70	71								
		A	3	4	9	11	14	17	19	22	27	31	36	39	40								
4	11	H	3	8	16	24	32	40	45	50	55	60	64										
		A	2	4	8	10	16	21	27	31	33	43	46										
	12	H	2	8	16	24	32	40	45	50	55	60	65	65									
		A	1	4	8	13	17	21	23	27	31	38	41	42									
	13	H	1	4	8	16	24	32	40	45	50	55	60	65									
		A	1	5	8	14	17	20	24	28	30	38	43	47									
5	14	H	2	8	24	32	40	44	48	52	56	60	64	68	72	76	80						
		A	1	3	6	11	17	19	23	25	29	31	35	38	40	42	48						
6	15	H	2	8	16	24	32	36	40	48	52	56	64	68	78								
		A	1	2	4	7	10	11	16	21	24	27	34	37	46								
	16	H	2	8	16	24	32	36	40	42	48	52	56	60	64	68	71	79					
		A	2	3	4	6	11	13	16	18	21	24	26	30	34	36	39	46					
	17	H	2	8	16	24	32	40	44	48	52	56	60	64	68	73	79						
		A	1	2	3	5	9	11	13	15	18	22	27	36	39	42	44						
7	18	H	3	8	16	24	32	40	44	48	52	60	64	68	72	76	82	88	92	96	100	104	107	
		A	2	3	6	7	8	10	11	15	16	17	19	21	24	25	32	37	38	43	51	58	62	
	19	H	3	12	28	40	48	52	56	64	70	80	84	88	92	96	100	104	108					
		A	2	3	8	11	13	14	16	17	19	25	29	30	34	35	38	50	55					
	20	H	2	8	16	24	32	40	48	56	64	68	72	76	80	84	88	92	96	100	104	108	112	116
		A	1	4	6	10	12	14	17	19	21	22	24	27	28	32	34	36	40	41	44	54	61	67
	21	H	2	12	24	32	40	48	52	56	60	68	76	80	84	88	92	96	100	104	108			
		A	1	2	3	5	7	9	12	15	18	20	22	26	27	33	36	41	45	53	58			
	22	H	2	12	24	32	40	48	52	56	66	72	80	84	88	90	96	100	104	108	112			
		A	1	2	3	5	6	10	11	14	17	20	26	27	29	34	40	46	47	55	65			

Table 5.--Basic red alder stem analysis data--Continued

Location	Tree No.	Heights and ages for 43 red alder trees																					
8	23	H	4	12	24	32	40	48	54	60	66	74	80	84	88	92	96	100	104	108	112	116		
		A	2	3	4	6	7	8	10	12	14	15	17	20	23	26	33	34	40	43	46	52		
	24	H	3	8	20	32	48	52	56	60	64	68	72	76	80	84	88	92	96	100	102	108	112	116
		A	2	5	6	8	13	15	17	18	20	21	22	25	30	32	35	40	42	44	46	51	62	68
9	25	H	3	8	16	24	32	40	48	54	60	64	68	72	76	80	84	88	92	100	104	108		
		A	2	7	9	10	18	19	20	22	23	25	28	30	31	32	37	42	46	50	55	60		
10	26	H	8	16	24	32	40	48	53	58	63	68	73	78	83	88	93	96						
		A	2	3	5	8	12	15	18	22	25	29	34	37	41	52	58	68						
	27	H	2	24	32	40	48	53	63	68	73	78	83	88	93	98								
		A	1	8	13	16	20	23	27	34	38	43	47	53	65	73								
	28	H	2	8	16	24	32	40	48	53	58	63	68	73	78	83	88	93	98	103	107			
		A	1	2	7	11	12	15	18	20	21	24	30	34	37	42	46	55	64	69	79			
11	29	H	2	8	16	24	32	40	44	48	52	56	60	64	68	72								
		A	1	3	4	5	6	9	13	15	17	20	25	31	41	57								
	30	H	2	8	16	24	32	40	44	48	52	56	60	64	68									
		A	1	3	5	7	11	12	15	16	18	25	38	42	56									
	31	H	2	8	16	24	32	40	44	48	52	56	60	64	69									
		A	1	4	5	10	12	18	20	23	26	28	36	42	62									
12	32	H	1	8	16	24	32	36	40	44	48	52	56	60	69									
		A	1	4	7	9	15	18	20	23	33	36	41	49	59									
	33	H	1	8	16	24	32	36	40	44	48	52	56	60	63									
		A	1	5	7	12	15	16	19	23	25	29	32	37	47									
13	34	H	1	8	17	25	33	41	49	57	65	69	75	80	85	88	93	97	101	105	106			
		A	1	2	3	5	7	10	11	14	17	18	23	27	33	36	42	46	51	57	60			
	35	H	1	8	17	25	33	41	49	57	65	69	75	80	85	89	93	100	104	108				
		A	1	2	3	4	7	11	13	15	17	19	25	30	33	36	42	46	49	60				
	36	H	1	9	17	25	33	41	49	57	65	70	75	80	85	90	93	100	105	112				
		A	1	5	6	8	9	11	14	16	19	21	23	27	32	35	42	45	49	60				
14	37	H	2	8	16	24	32	40	48	56	64	72	76	80	85	90	94							
		A	1	3	4	5	6	9	12	15	18	23	25	30	36	42	52							
	38	H	2	8	16	24	32	40	48	56	60	64	68	72	76	80	84	88	92					
		A	1	3	4	7	8	11	16	19	20	23	25	28	31	35	39	43	52					
	39	H	2	8	16	24	32	40	48	56	64	68	72	76	80	84	88	94						
		A	1	2	5	9	11	13	17	21	24	27	29	33	40	41	46	54						
15	40	H	2	8	16	32	40	48	64	72	76	80	84	88	92	96	102							
		A	1	2	3	8	10	13	19	22	26	28	33	37	43	54	63							
16	41	H	1	9	17	24	37	49	58	66	74	78	82	86	90	94	98							
		A	1	2	5	7	8	13	14	17	21	23	26	30	32	36	42							
	42	H	1	9	21	33	43	51	59	67	71	75	79	84										
		A	1	5	6	7	10	16	18	27	31	33	38	43										
	43	H	1	9	17	25	33	49	57	62	66	70	74	82	86	90	92							
		A	1	3	6	8	11	14	19	20	21	24	27	35	37	41	43							

Johnson, Floyd A., and Worthington, Norman P.
1963. Procedure for developing a site index estimating system from stem analysis data. U.S. Forest Serv. Res. Paper PNW-7, 10 pp., illus.

The procedure presented in this report has been applied successfully to three tree species. It is simple, direct, and apparently statistically sound. Perhaps others, faced with the problem of developing site curves from stem data, will find it useful.

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