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Site Index and Height Growth Curves for Unmanaged Even-Aged Stands of Western Hemlock and Sitka Spruce in Southeast Alaska

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Abstract

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Height growth and site index curves and equations are presented for unmanaged even-aged stands of western hemlock and Sitka spruce in southeast Alaska. Data were mostly collected in stands that developed after logging.

Keywords: Even-aged stands, site index, increment (height), stem analysis, western hemlock, *Tsuga heterophylla,* Sitka spruce, *Picea sitchensis,* southeast Alaska, Alaska (southeast).

Research Summary

Site index and height growth curves and equations for unmanaged even-aged stands of western hemlock *(Tsuga heterophylla)* (Raf.) Sarg.) and Sitka spruce *(Picea sitchensis* (Bong.) Carr.) were derived from stem analysis of data from tree stems on 91 plots in southeast Alaska.

Site index curves can provide estimates of site index for unmanaged stands if total height and age at breast height are known. Height growth curves can provide estimates of total height if site index and age at breast height are known.

The curves are based on measurements of total height and age at breast height for hemlock or spruce on 1/5-acre plots in southeast Alaska. Sample trees were selected in even-aged stands from among trees containing no visible evidence of past attack by insect or disease and having a diameter at breast height near the quadratic mean diameter of the eight largest diameter hemlock and spruce on the plot.

Contents

Introduction

The site index and height growth equations and curves presented here were developed from data collected in natural, well-stocked, even-aged stands of western hemlock *(Tsuga heterophylla* (Raf.) Sarg.) and Sitka spruce *(Picea sitchensis* (Bong.) Carr.) in southeast Alaska. The index age is 50 years at breast height.

The forests of southeast Alaska are part of the hemlock-spruce forest type that occupies a north-south range along the north Pacific coast, extending from near Coos Bay, Oregon, north and west to the Alaska Peninsula, a distance of about 1,800 miles. Within this type, mean site index of Sitka spruce decreases northward at the rate of about 2.6 feet per degree of latitude (Farr and Harris 1979).

The most extensive stands of western hemlock and Sitka spruce occur in southeast Alaska where the type occupies about 12 million acres, about 6 million acres of which is classified as commercial forest land. In Alaska, the type generally consists of virgin stands of western hemlock and Sitka spruce with small percentages of Alaska-cedar *(Chamaecyparis nootkatensis* (D. Don) Spach) and western redcedar (*Thuja plicata* Donn ex D. Don). About 300,000 acres of commercial forest land have been harvested in southeast Alaska since 1900, and 10,000-17,000 acres are now harvested annually. All logging is by clearcutting.

The oldest clearcuts date back to 1910 and are located near sea level adjacent to saltwater. Only since the 1950's have extensive logging roads been built and cutting units placed inland. Abundant moisture and lack of a pronounced summer drought have greatly limited fires, but some older even-aged stands originated after fire and blowdown, mostly at lower elevation.

Even-aged stands of western hemlock and Sitka spruce vary greatly in species composition. Mixed stands are most common. Exceptions are stands of pure hemlock that have developed from a carpet of advance regeneration, or pure stands of spruce that have become established after glacial recession, landslides, fire, or cultural activities.

Taylor's (1934) proportional site index curves for western hemlock and Sitka spruce, based on height at stand age 100 years, have been used in southeast Alaska for 50 years. They are biased at young ages, a feature common to many earlier sets of proportional curves where mean site was not equal over all age classes. Other sets of site index curves have also been prepared for Sitka spruce by Hegyi and others (1979), Stephens and others (1969), and Meyer (1937), and for western hemlock by Hegyi and others (1979), Wiley (1978), and Barnes (1962).

In the late 1960's, dominant and codominant hemlock and spruce were felled and sectioned in 31 plots of blowdown and fire origin in southeast Alaska. Provisional site index curves were then prepared for each species but never published. About the same time Stephens and others (1969) published findings suggesting that stands of blowdown origin may have site qualities much lower than stands on similar soil types originating from logging or fire. Their work suggested that site rejuvenation and available nutrient status may be much poorer after blowdown than after logging or fire and may thus reduce future productivity.

The site index study remained inactive for several years because of these findings and the desire to develop site index curves applicable to stands originating after logging. In 1977, trees on 60 additional plots, located in areas that had been previously logged, were felled and sectioned. A large enough data base was then available for the development of regional site index and height growth equations.

In the intervening years, there have been significant advances in the development of height growth and site index curves. Many nonlinear equation forms have been applied, and differences between equations to predict height growth and site index have been defined (Curtis and others 1974). Site index curves should be used to predict site productivity where present age and total height are known, whereas height growth curves should be used in the development of yield tables to predict future or past height as a function of site index and age.

Data Collection

Two sets of data were used—31 plots taken in 1967 and 1968 and 60 plots taken in 1977. All plots were located in natural, well-stocked, even-aged stands of western hemlock and Sitka spruce (fig. 1). An attempt was made to sample even-aged stands of hemlock and spruce covering a range of sites throughout southeast Alaska. Because of the limited logging history in the region and a preponderance of old-growth timber, it was not possible to sample stands on all the major islands, or at upper elevations. The sample at lower elevations seems to be fairly representative of sites commonly found in southeast Alaska. Several of the 91 plots were later eliminated because they did not have suitable spruce or hemlock site trees; 57 of the plots had suitable hemlock site trees and 71 had suitable spruce site trees (table 1).

The 1967-68 Sample

All plots but one (77 years old) were taken in stands 110 to 180 years of age. All plots were of blowdown or fire origin. Plots were generally one-third to one-half acre in size. Diameter was measured for all trees on each plot, and trees were classified by crown position. Three hemlock and three spruce of quadratic mean diameter among dominants and codominants were felled and sectioned at stump height, at breast height, at 10 feet, and at succeeding 10-foot intervals up the tree.

The 1977 Sample

In 1977, trees on sixty 1/5-acre plots, all of logging origin, were felled and sectioned. Most of the stands were about 50 years old. The oldest was 110. All diameters were measured, and hemlock and spruce representative of the average of the 40 largest diameter trees per acre were felled and sectioned. Trees representative of a fixed number of trees per acre were chosen to avoid the problem of subjective crown classification and to make the tables and equations easier to apply in the field. Use of the average of the 40 largest diameter trees per acre also agrees with procedures used in many areas of the world. It should be noted that this was a change in sampling from that used in 1967 and 1968 where sample trees were representative of the average of dominants and codominants.

All plots were located at elevations below 500 feet, and most were adjacent to saltwater, as most early day logging took place close to the beach. Most stands were 45 to 60 years old at breast height, whereas stands sampled in 1967 and 1968 were 110 to 180 years old.

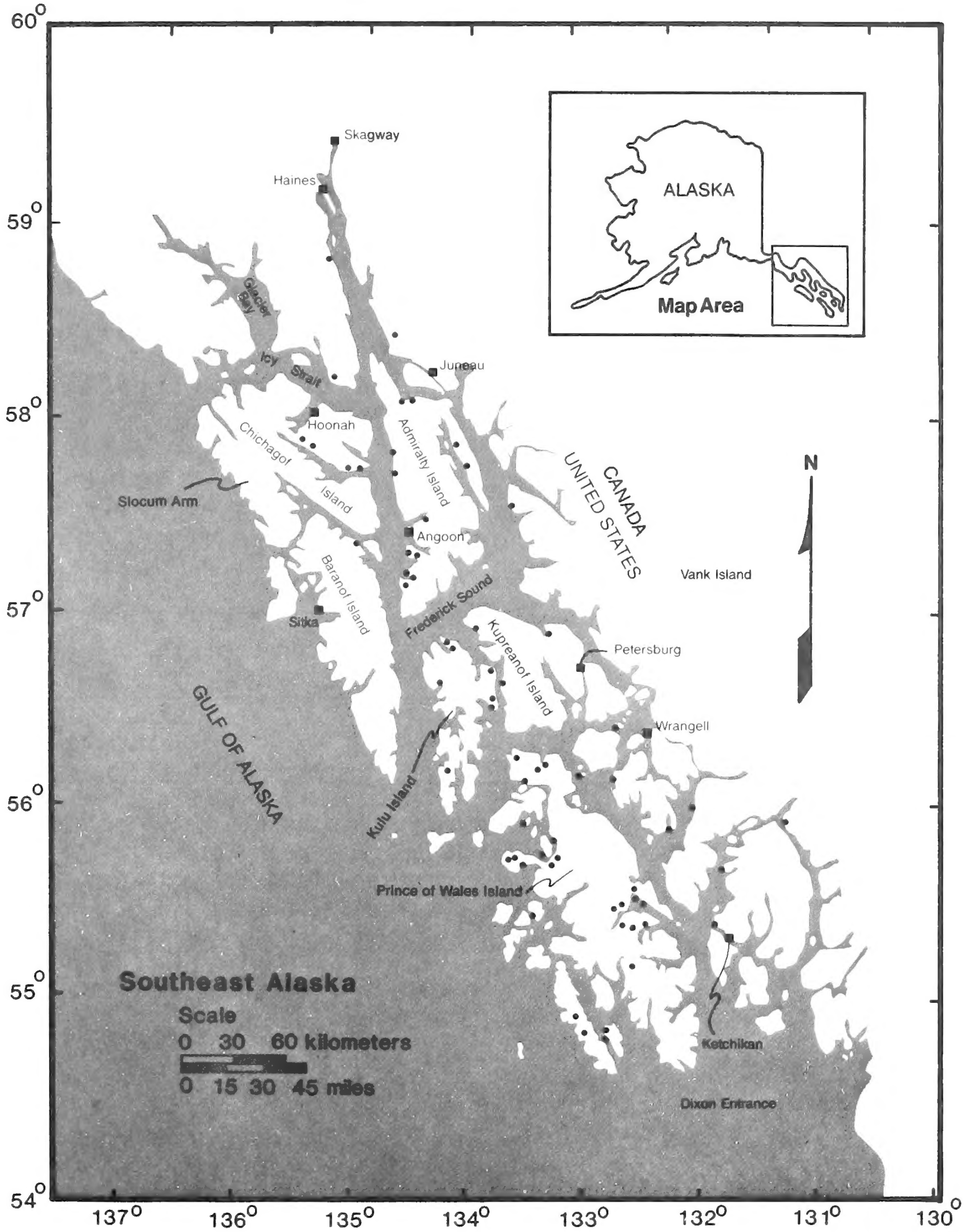

Figure 1.—Locations of site index plots in southeast Alaska.

Table 1—Distribution of plots used to develop the height growth and site index curves for western hemlock and Sitka spruce in southeast Alaska

Range in site index 1/	Western hemlock		Sitka spruce	
	1967-68 sample	1977 sample	1967-68 sample	1977 sample
		Number		
41 - 50	1		1	
51 - 60	2		1	
61 - 70	5	4	5	2
71 - 80	1	8	6	7
81 - 90	6	9	1	7
91 - 100	2	13	9	10
101 - 110		6		16
111 - 120				6
	17	40	23	48
Total plots 2/	57		71	

1/ Index age is 50 years at breast height.
2/ Trees on 91 (1/5-acre) plots were felled and sectioned; 71 of the plots contained suitable hemlock site trees and 57, suitable spruce site trees.

Analysis

Many nonlinear regression equations have been used to fit stem analysis data to the equation of general form,

Height = f(site index, age);

where: f = function.

Some commonly used equations are:

$$HT = (b_1 + b_2S)\ (1\text{-}e^{-b_3SA})^{b_4},$$

$$HT = (b_1 + b_2S)\ (1\text{-}e^{-[b_3+b_4S]A})^{b_5}, \text{ and}$$

$$HT = b_1S^{b_2}(1\text{-}e^{-b_3A})^{b_4S^{b_5}};$$

where:

HT = total height,
S = site index,
A = age,
e = base of natural logarithms, and
b = least square estimates of the parameters of the equation.

In application, relationships for HT = f(S, A) are generally reversed and used to estimate site index from observed values of height and age because the above equations cannot be used directly to develop site index equations, which are automatically conditioned so that the term $(1\text{-}e^{-bA}) \rightarrow 0$ as $A \rightarrow 0$. In the relationship, S = f(HT, A), site index should not go to zero as age goes to zero.

Curtis and others (1974) and Dahms (1975) presented alternative methods for constructing consistent height growth and site index curves in which they independently fit the relationship, S = f(HT, A).

General procedures used to develop the equations and curves for western hemlock and Sitka spruce were suggested by Curtis and others (1974) and Dahms (1975) and were used by Barrett (1978) and Cochran (1979a, 1979b). Included are height and site curves, both developed around a mean height over age curve, as both height and site equations should predict the same values for the mean curve.

In construction of the site index and height growth curves, the general relationships,

$$S - 4.5 \text{ feet} = a + b\ (HT - 4.5 \text{ feet}), \text{ and}$$

$$HT - 4.5 \text{ feet} = c + d\ (S - 4.5 \text{ feet}),$$

were used; and coefficients a, b, c, and d were determined for each 10-year class. Values for b and d were then smoothed over age by stepwise regression. Each equation was forced through 1 at age 50.

Steps used to construct these equations are given in the appendix.

Results

Estimating Site Index

Separate site index curves were developed for western hemlock (fig. 2) and Sitka spruce (fig. 3) because tests showed that data for the two species could not be pooled. Comparison of figures 2 and 3 shows that the curves are similar at all ages for the higher site classes but are somewhat different at ages greater than 60 for lower site classes.

For more precise estimates of site index, the values in table 2 (hemlock) or table 3 (spruce) can be used to evaluate the equation,

Site index $-$ 4.5 feet $= a_i + b_i$ (height $-$ 4.5 feet);

where:

$i = 1$ for western hemlock and 2 for Sitka spruce.

Equations are also given in the appendix for calculator or computer applications.

Estimation of site index by use of these curves, tables, or equations should be limited to well-stocked, even-aged stands of western hemlock and Sitka spruce.

To apply the site index curves, tables, or equations, use the following procedures:

1. Establish 1/5-acre plots in even-aged, natural stands that are free of residuals from a previous stand and contain no visible evidence of attack by insect or disease.

2. Measure diameter of the eight largest diameter trees (hemlock and spruce only) on each plot, being sure not to include remnants of an earlier stand.

3. Compute quadratic mean diameter of the eight largest diameter trees (hemlock and spruce) per plot (40/acre) and select three hemlock and/or spruce with diameters about that size.

4. Extract an increment core from each tree at breast height to determine age of the tree at breast height.

5. Using the age and total height for each tree, determine site index. Then calculate mean site index for each species.

On the average, dominant hemlock are about 5 years older at breast height than are dominant spruce because small hemlock regeneration is usually present in the understory of old-growth stands. When the old growth is removed, the hemlock has a head start over spruce regeneration that seeds in later. This does not affect estimates of site index as long as site index is estimated from individual tree age and total height.

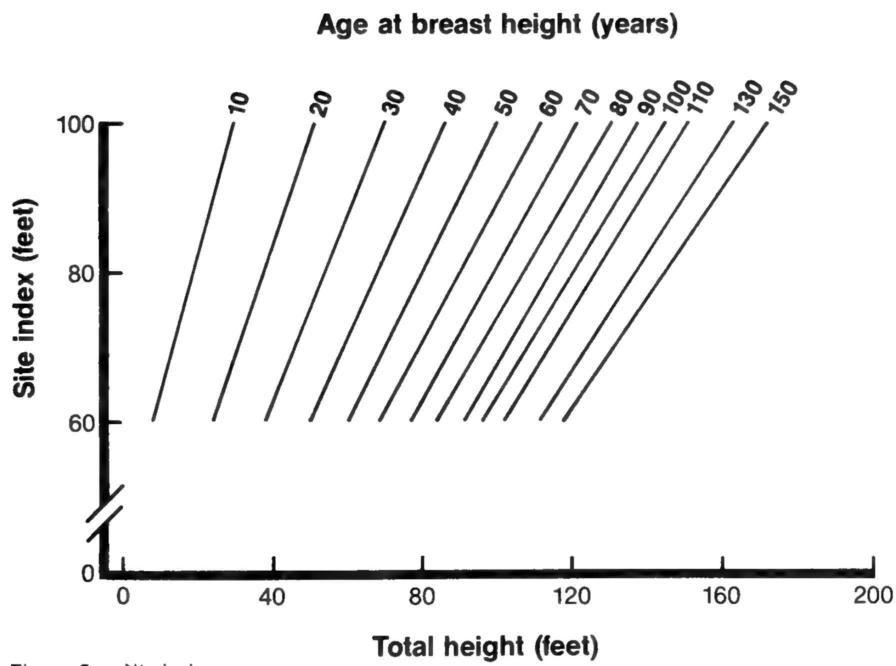

Figure 2.—Site index curves for unmanaged, even-aged western hemlock in southeast Alaska.

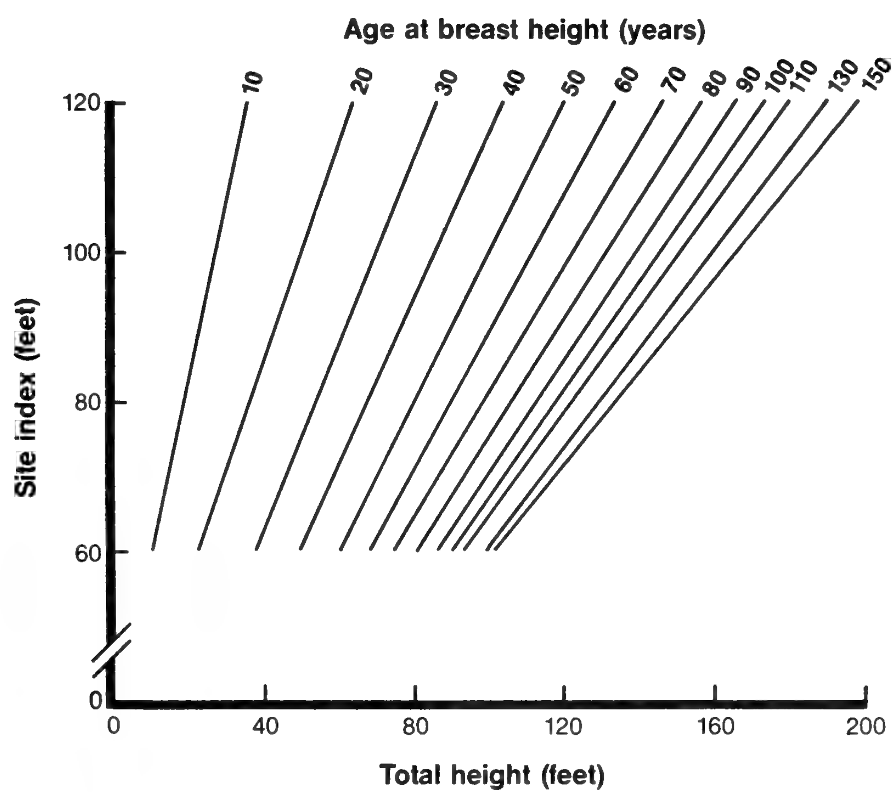

Figure 3.—Site index curves for unmanaged, even-aged Sitka spruce in southeast Alaska.

Table 2—Values for a_1 and b_1 for the family of regressions for estimating site index for western hemlock in southeast Alaska[1/]

Age at breast height	0		1		2		3		4		5		6		7		8		9	
	Years between decades																			
	a_1	b_1	a_1	b_1	a_1	b_1	a_1	b_1	a_1	b_1	a_1	b_1	a_1	b_1	a_1	b_1	a_1	b_1	a_1	b_1
Years																				
10	49.501	1.787	46.488	1.761	43.679	1.732	41.059	1.701	38.616	1.669	36.334	1.636	34.202	1.604	32.206	1.572	30.336	1.540	28.579	1.510
20	26.926	1.480	25.369	1.452	23.899	1.424	22.507	1.398	21.189	1.373	19.936	1.349	18.745	1.326	17.608	1.304	16.523	1.283	15.485	1.263
30	14.489	1.244	13.533	1.226	12.614	1.208	11.728	1.192	10.873	1.176	10.046	1.161	9.247	1.146	8.471	1.133	7.718	1.119	6.987	1.107
40	6.275	1.095	5.582	1.083	4.906	1.072	4.245	1.062	3.600	1.052	2.969	1.042	2.352	1.033	1.747	1.02[illegible]	1.153	1.016	.571	1.008
50	0	1.000	.561	.993	-1.113	.986	-1.655	.979	-2.189	.972	-2.714	.966	-3.231	.960	-3.740	.954	-4.242	.949	-4.736	.943
60	-5.223	.938	-5.704	.933	-6.177	.928	-6.644	.924	-7.105	.919	-7.559	.915	-8.008	.911	-8.450	.907	-8.886	.903	-9.317	.900
70	-9.742	.896	-10.161	.893	-10.574	.889	-10.982	.886	-11.384	.883	-11.781	.880	-12.173	.877	-12.559	.874	-12.940	.871	-13.315	.868
80	-13.685	.866	-14.050	.863	-14.409	.861	-14.764	.858	-15.113	.856	-15.457	.853	-15.796	.851	-16.129	.849	-16.458	.846	-16.781	.844
90	-17.099	.842	-17.412	.840	-17.720	.838	-18.023	.836	-18.321	.833	-18.614	.831	-18.902	.829	-19.185	.827	-19.463	.825	-19.736	.823
100	-20.004	.821	-20.268	.819	-20.526	.817	-20.780	.815	-21.029	.814	-21.273	.812	-21.512	.810	-21.747	.808	-21.977	.806	-22.203	.804
110	-22.424	.802	-22.641	.800	-22.853	.798	-23.062	.797	-23.265	.795	-23.465	.793	-23.661	.791	-23.852	.789	-24.040	.787	-24.224	.785
120	-24.404	.784	-24.580	.782	-24.753	.780	-24.922	.778	-25.088	.776	-25.250	.775	-25.410	.773	-25.566	.771	-25.720	.769	-25.871	.768
130	-26.019	.766	-26.164	.764	-26.308	.763	-26.449	.761	-26.588	.760	-26.725	.758	-26.861	.757	-26.995	.755	-27.127	.754	-27.259	.752
140	-27.389	.751	-27.519	.750	-27.648	.749	-27.777	.747	-27.906	.746	-28.035	.745	-28.164	.744	-28.293	.743	-28.424	.742	-28.556	.741
150	-28.689	.741																		

1/ To estimate site index, determine the quadratic mean diameter of the 8 largest diameter trees per 1/5-acre plot; then measure the height and determine age at breast height of at least 3 hemlock near this mean diameter. For each tree, select appropriate a_1 and b_1 values and substitute these values in the equation (site index - 4.5 feet) = a_1 + b_1 (height - 4.5 feet). For example, for a tree 74 years old at breast height and 105 feet in total height, solve the equation (site index - 4.5 feet) = -11.384 + 0.883 (105 feet - 4.5 feet) for a site index of 81.9 feet. Determine the site index for each hemlock. The mean site index determined is the site index for western hemlock on the 1/5-acre plot.

Table 3—Values for a_2 and b_2 for the family of regressions for estimating site index for Sitka spruce in southeast Alaska[1/]

Age at breast height	0		1		2		3		4		5		6		7		8		9	
	Years between decades																			
	a_2	b_2	a_2	b_2	a_2	b_2	a_2	b_2	a_2	b_2	a_2	b_2	a_2	b_2	a_2	b_2	a_2	b_2	a_2	b_2
Years																				
10	43.220	2.330	42.069	2.143	40.753	1.998	39.327	1.884	37.827	1.792	36.279	1.716	34.705	1.652	33.120	1.598	31.536	1.551	29.963	1.510
20	28.408	1.474	26.879	1.441	25.380	1.412	23.914	1.385	22.486	1.360	21.098	1.338	19.751	1.316	18.448	1.296	17.189	1.278	15.975	1.260
30	14.806	1.243	13.682	1.227	12.603	1.211	11.569	1.197	10.580	1.182	9.634	1.169	8.731	1.155	7.870	1.142	7.050	1.130	6.271	1.117
40	5.530	1.105	4.826	1.094	4.160	1.082	3.529	1.071	2.932	1.060	2.368	1.050	1.836	1.039	1.334	1.029	.862	1.019	.418	1.010
50	0	1.000	-.391	.991	-.758	.982	-1.102	.973	-1.423	.964	-1.723	.955	-2.003	.947	-2.264	.939	-2.507	.930	-2.732	.923
60	-2.942	.915	-3.136	.907	-3.316	.900	-3.482	.893	-3.635	.885	-3.777	.878	-3.908	.872	-4.028	.865	-4.138	.858	-4.239	.852
70	-4.332	.846	-4.418	.840	-4.496	.834	-4.567	.828	-4.633	.822	-4.692	.817	-4.747	.811	-4.798	.806	-4.844	.801	-4.886	.796
80	-4.925	.791	-4.961	.786	-4.995	.781	-5.026	.777	-5.056	.772	-5.083	.768	-5.110	.764	-5.135	.760	-5.159	.756	-5.182	.752
90	-5.205	.748	-5.228	.744	-5.250	.741	-5.273	.737	-5.295	.734	-5.318	.730	-5.341	.727	-5.364	.724	-5.388	.721	-5.412	.718
100	-5.437	.715	-5.463	.712	-5.489	.709	-5.515	.707	-5.543	.704	-5.570	.702	-5.598	.699	-5.627	.697	-5.656	.694	-5.685	.692
110	-5.714	.690	-5.743	.688	-5.772	.686	-5.801	.684	-5.830	.682	-5.858	.680	-5.885	.678	-5.912	.676	-5.937	.674	-5.961	.672
120	-5.984	.671	-6.005	.669	-6.024	.667	-6.041	.665	-6.055	.664	-6.067	.662	-6.075	.661	-6.081	.659	-6.082	.657	-6.080	.656
130	-6.073	.654	-6.062	.653	-6.046	.651	-6.024	.650	-5.997	.648	-5.964	.647	-5.925	.645	-5.878	.643	-5.825	.642	-5.764	.640
140	-5.694	.639	-5.617	.637	-5.530	.635	-5.435	.634	-5.329	.632	-5.213	.630	-5.087	.628	-4.950	.626	-4.801	.624	-4.640	.622
150	-4.467	.620																		

1/ To estimate site index, determine the quadratic mean diameter of the 8 largest diameter trees per 1/5-acre plot; then measure the height and determine age at breast height of at least 3 spruce near this mean diameter. For each tree, select appropriate a_2 and b_2 values and substitute these values in the equation (site index - 4.5 feet) = a_2 + b_2 (height - 4.5 feet). For example, for a tree 74 years old at breast height and 105 feet in total height, solve the equation (site index - 4.5 feet) = -4.633 + 0.822 (105 feet - 4.5 feet) for a site index of 82.5 feet. Determine the site index for each spruce. The mean site index determined is the site index for Sitka spruce on the 1/5-acre plot.

Comparison of Site Indexes of Western Hemlock and Sitka Spruce

Thirty-one plots in the 1977 sample contained western hemlock and Sitka spruce site trees. For these plots, the relationship between site index of the two species was:

Hemlock site index = 24.41 + 0.674 (site index of Sitka spruce)–where the standard error was 1.22, and the coefficient of determination (r^2) was 0.78;

spruce site index = −6.62 + 1.152 (site index of western hemlock)—where the standard error was 0.94, and r^2 was 0.78.

Estimating Height Growth

Height growth curves define the average pattern of height growth in stands of known site quality. They are appropriately used for constructing yield tables but do not provide optimum estimates of site index from measured height and age in a stand (Curtis and others 1974).

Figures 4 and 5 can be used to make rough estimates of mean height of hemlock and spruce representative of the 40 trees of largest diameter per acre. For more precise estimates of mean total height of the 40 trees of largest diameter per acre, use values in tables 4 or 5 in the equation:

Total height − 4.5 feet = $c_i + d_i$ (site index − 4.5 feet).

Equations in the appendix will be useful for those wishing to use a computer program.

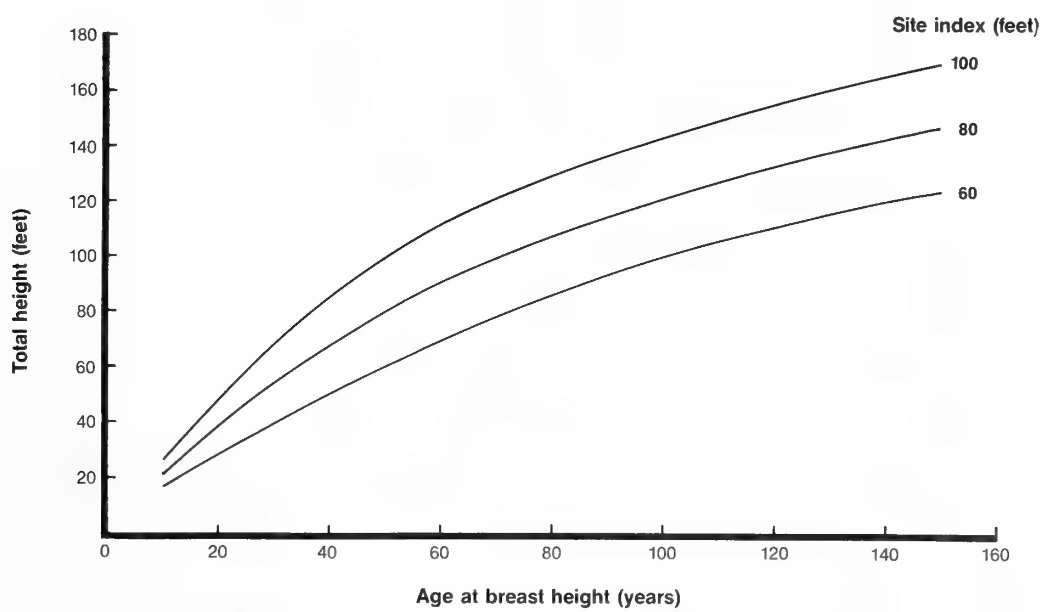

Figure 4.—Height growth curves for unmanaged, even-aged western hemlock in southeast Alaska. Heights represent the mean among the 40 largest diameter trees per acre.

Figure 5.—Height growth curves for unmanaged, even-aged Sitka spruce in southeast Alaska. Heights represent the mean among the 40 largest diameter trees per acre.

Table 4—Values for c_1 and d_1 for the family of regressions for estimating height of the 40 largest diameter western hemlock trees per acre where site index and age at breast height are known, southeast Alaska[1/]

Age at breast height	Years between decades																			
	0		1		2		3		4		5		6		7		8		9	
	c_1	d_1	c_1	d_1	c_1	d_1	c_1	d_1	c_1	d_1	c_1	d_1	c_1	d_1	c_1	d_1	c_1	d_1	c_1	d_1
Years																				
10	-0.983	0.239	-0.888	0.261	-0.865	0.285	-0.907	0.309	-1.004	0.333	-1.148	0.358	-1.331	0.383	-1.546	0.409	-1.786	0.434	-2.042	0.459
20	-2.310	.485	-2.582	.510	-2.853	.535	-3.119	.559	-3.375	.583	-3.616	.607	-3.838	.630	-4.039	.653	-4.216	.675	-4.366	.697
30	-4.486	.718	-4.576	.738	-4.634	.758	-4.658	.777	-4.648	.795	-4.604	.813	-4.524	.830	-4.410	.846	-4.260	.862	-4.077	.877
40	-3.859	.891	-3.608	.905	-3.324	.918	-3.008	.930	-2.662	.942	-2.285	.953	-1.881	.963	-1.449	.973	.990	.983	-.507	.992
50	0	1.000	.530	1.008	1.080	1.015	1.651	1.022	2.240	1.029	2.846	1.035	3.468	1.041	4.104	1.046	4.755	1.051	5.417	1.055
60	6.091	1.060	6.775	1.064	7.468	1.067	8.169	1.071	8.877	1.074	9.591	1.077	10.310	1.079	11.033	1.082	11.760	1.084	12.490	1.086
70	13.221	1.088	13.953	1.089	14.686	1.091	15.419	1.092	16.151	1.093	16.882	1.095	17.611	1.096	18.338	1.096	19.063	1.097	19.784	1.098
80	20.502	1.099	21.216	1.099	21.927	1.100	22.633	1.100	23.334	1.100	24.031	1.101	24.722	1.101	25.409	1.101	26.090	1.102	26.766	1.102
90	27.436	1.102	28.101	1.102	28.761	1.102	29.414	1.103	30.062	1.103	30.704	1.103	31.341	1.103	31.972	1.103	32.597	1.103	33.216	1.103
100	33.830	1.104	34.439	1.104	35.042	1.104	35.639	1.104	36.232	1.104	36.819	1.105	37.401	1.105	37.978	1.105	38.550	1.105	39.117	1.106
110	39.679	1.106	40.236	1.106	40.789	1.106	41.338	1.107	41.882	1.107	42.421	1.107	42.957	1.108	43.488	1.108	44.014	1.108	44.537	1.109
120	45.056	1.109	45.570	1.109	46.081	1.110	46.587	1.110	47.090	1.110	47.588	1.111	48.083	1.111	48.573	1.111	49.059	1.112	49.541	1.112
130	50.018	1.112	50.492	1.113	50.960	1.113	51.424	1.113	51.883	1.113	52.337	1.114	52.786	1.114	53.229	1.114	53.667	1.114	54.099	1.114
140	54.524	1.114	54.943	1.115	55.355	1.115	55.759	1.115	56.156	1.115	56.545	1.115	56.925	1.115	57.296	1.115	57.658	1.114	58.010	1.114
150	58.351	1.114																		

1/ Height at a future age of the average among the 40 largest diameter trees per acre may be estimated on land of known site index by selecting c_1 and d_1 values for the appropriate age at breast height. Substitute c_1 and d_1 values into the equation (height - 4.5 feet) = $c_1 + d_1$ (site index - 4.5 feet). For example, to determine height among the 40 largest diameter trees in a stand of age 85 years on land of known site index 100 feet, solve the equation (height - 4.5 feet) = 24.031 + 1.101 (100 feet - 4.5 feet) for a total height of 133.7 feet.

Table 5—Values for c_2 and d_2 for the family of regressions for estimating height of the 40 largest diameter Sitka spruce trees per acre where site index age at breast height are known, southeast Alaska[1/]

Age at breast height	Years between decades																			
	0		1		2		3		4		5		6		7		8		9	
	c_2	d_2	c_2	d_2	c_2	d_2	c_2	d_2	c_2	d_2	c_2	d_2	c_2	d_2	c_2	d_2	c_2	d_2	c_2	d_2
Years																				
10	-2.590	0.244	-3.020	0.275	-3.413	0.304	-3.767	0.333	-4.081	0.361	-4.357	0.389	-4.595	0.416	-4.796	0.442	-4.962	0.468	-5.095	0.492
20	-5.196	.516	-5.268	.540	-5.312	.562	-5.329	.585	-5.321	.606	-5.290	.627	-5.237	.647	-5.164	.667	-5.071	.686	-4.961	.705
30	-4.833	.723	-4.691	.741	-4.533	.758	-4.362	.774	-4.178	.791	-3.981	.806	-3.774	.822	-3.556	.837	-3.328	.852	-3.090	.866
40	-2.844	.880	-2.590	.893	-2.328	.906	-2.059	.919	-1.782	.932	-1.500	.944	-1.211	.956	-.916	.967	-.616	.978	-.311	.989
50	0	1.000	.315	1.010	.635	1.021	.959	1.030	1.287	1.040	1.620	1.049	1.956	1.059	2.296	1.067	2.640	1.076	2.987	1.085
60	3.337	1.093	3.691	1.101	4.049	1.108	4.409	1.116	4.772	1.123	5.139	1.131	5.508	1.137	5.880	1.144	6.256	1.151	6.634	1.157
70	7.015	1.163	7.398	1.169	7.784	1.175	8.173	1.181	8.565	1.186	8.959	1.191	9.356	1.197	9.755	1.202	10.157	1.206	10.561	1.211
80	10.968	1.215	11.377	1.220	11.788	1.224	12.202	1.228	12.618	1.232	13.036	1.235	13.456	1.239	13.878	1.242	14.302	1.246	14.728	1.249
90	15.155	1.252	15.585	1.255	16.016	1.258	16.448	1.260	16.882	1.263	17.317	1.265	17.754	1.267	18.191	1.269	18.630	1.271	19.069	1.273
100	19.509	1.275	19.949	1.277	20.389	1.279	20.830	1.280	21.271	1.282	21.711	1.283	22.151	1.284	22.590	1.285	23.029	1.286	23.466	1.287
110	23.902	1.288	24.336	1.289	24.769	1.290	25.199	1.291	25.627	1.292	26.052	1.292	26.475	1.293	26.893	1.293	27.309	1.294	27.720	1.294
120	28.127	1.295	28.529	1.295	28.926	1.296	29.318	1.296	29.704	1.296	30.084	1.297	30.457	1.297	30.822	1.297	31.180	1.298	31.531	1.298
130	31.872	1.299	32.205	1.299	32.528	1.300	32.841	1.300	33.143	1.301	33.434	1.301	33.713	1.302	33.980	1.303	34.234	1.304	34.474	1.305
140	34.700	1.306	34.911	1.307	35.106	1.308	35.285	1.309	35.447	1.311	35.591	1.313	35.716	1.314	35.821	1.316	35.907	1.318	35.970	1.321
150	36.012	1.323																		

1/ Height at a future age of the average among the 40 largest diameter trees per acre may be estimated on land of known site index by selecting c_2 and d_2 values for the appropriate age at breast height. Substitute c_2 and d_2 values in the equation (height - 4.5 feet) = c_2 = d_2 (site index - 4.5 feet). For example, to determine the average height among the 40 largest diameter trees per acre in a stand of age 85 years at breast height on land with a known site index of 100 feet, solve the equation (height - 4.5 feet) = 13.036 + 1.235 (100 feet - 4.5 feet) for a total height of 135.5 feet.

Metric Equivalents

1 foot = 0.3048 meter
1 mile = 1.6093 kilometers
1 acre = 0.4047 hectare

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Appendix

Height development of two or three sample trees of each species on each plot was plotted for each tree over individual age at breast height. On the average, hemlock were about 5 years older than spruce, probably because hemlock is very tolerant of shade and is commonly found as advance regeneration in the understory of old-growth stands.

Trees were rejected from further analysis if tops had been damaged or if cross sections showed signs of suppression. Straight-line interpolation was used to estimate mean height for each species at 10-year intervals.

Data for each species and plot were then fit to the equation,

$$\text{Height} - 4.5 \text{ feet} = b_1(1-e^{-b_2 age})^{b_3};$$

where: age = age at breast height.

This flexible nonlinear equation fit the typical sigmoid-shaped height over age curves very well. Coefficients b_1, b_2, and b_3 for each species were then plotted and regressed over site index minus 4.5 feet at age 50 to check for significant outliers. This helped to identify plots with height-growth trends obviously different from regional averages by site class. Several plots were eliminated (Daniel and Wood 1971). Fifty-seven of the plots had suitable hemlock site trees and 71 had suitable spruce site trees.

Data sets for western hemlock and Sitka spruce were then analyzed, generally following procedures outlined by Dahms (1975) and used by Barrett (1978) and Cochran (1979a, 1979b).

The two sets of data (1967-68 and 1977) for each species were analyzed separately because the earlier set came from dominant and codominant trees in 110- to 180-year-old stands of blowdown and fire origin, whereas the 1977 data were from trees representative of the average among the 40 largest diameter trees per acre in 50- to 100-year-old stands of logging origin. Extensive testing using several nonlinear equations, plus Dahms' (1975) procedure, was done to see if the data sets could be pooled. If possible, I wanted to pool some or all of the 1967-68 data with the 1977 data so the height and site index curves could be extended to age 150 years. If the two sets of data for each species could not be pooled, then only the 1977 data set from logged over stands would be used and the height and site curves would be extended only to age 100 years.

The analysis showed no significant difference between the two sets of spruce data, so they were pooled. This meant that, although slightly different components of the overstory were sampled in 1967-68 than in 1977, the curve form describing growth patterns could not be shown to differ.

For hemlock at young ages, there were significant differences between the two sets of data, which suggested some initial suppression in the hemlock from stands of blowdown origin. Beyond age 40 years, there was no significant difference in growth trends for trees representative of the same site. In the final analysis for hemlock, the 1977 data and the 1967-68 data were used for ages greater than 40 years. This eliminated any correlation between the residuals and site and age, or between residuals and height and age, and allowed for the extension of the curves to age 150 years.

This analysis showed that the general shape of the curves is essentially the same whether average dominant and codominant trees are used or the average among the 40 largest diameter trees per acre. What is important is consistency in field application.

Construction of Curves for Western Hemlock

Site index curves.—The basic equation used to construct the site index curves for western hemlock was:

$$S - 4.5 \text{ feet} = a_1 + b_1 (HT - 4.5 \text{ feet});$$

where:

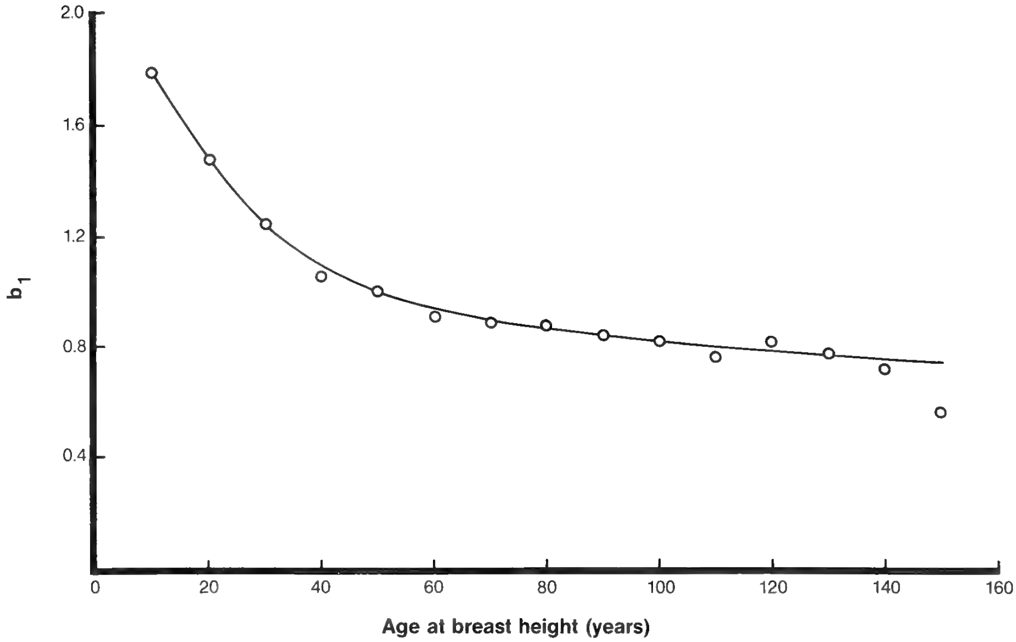
S = site index at age 50 years,
HT = total height, and
a_1, b_1 = least square estimates of the parameters of the equation for western hemlock.

Linear regressions were calculated for each decade from 10 to 150 years. The following estimates were obtained:

Age at breast height	a_1	b_1	r^2	Standard error of the estimate	Observations
(Years)	(Feet)	(Feet)		(Feet)	(Number)
10	49.5115	1.7859	0.43	8.65	40
20	27.0813	1.4827	.72	5.93	40
30	13.9337	1.2503	.88	3.82	40
40	8.1965	1.0605	.96	2.35	40
50	0	1.0000	1.00	0	40
60	-3.2754	.9094	.98	2.08	30
70	-9.4892	.8914	.95	2.94	20
80	-15.8885	.8806	.94	3.18	20
90	-18.4797	.8423	.93	3.78	18
100	-21.9476	.8244	.89	4.69	17
110	-18.9048	.7631	.85	4.88	14
120	-30.9645	.8191	.90	4.29	10
130	-30.5959	.7826	.89	4.64	10
140	-25.1111	.7222	.88	5.18	8
150	-4.2578	.5673	.49	8.58	4

The b_1 values were then smoothed (fig. 6) by fitting a logarithmic equation that described b_1 as a function of age at breast height. The equation was forced through 1 at age 50.

Figure 6.—b_1 values for western hemlock in the equation $(S - 4.5 \text{ feet}) = a_1 + b_1 (HT - 4.5 \text{ feet})$ as a function of age. Plotted points are actual b_1 values. Solid line is expressed by the equation $\hat{b}_1 = e^{x_1}$. Coefficient x_1 is defined in the text. Standard error is 0.00071 and r^2 is 0.9922.



Coefficient b_1 turned sharply downward at age 150. Because the logarithmic equation (using natural logs) is very sensitive, the regression curve also turned sharply downward beyond age 140 years. To correct for this, the data were smoothed to remove extreme variation in b_1 over age at ages 140 to 150 years.

The derived equation was:

$$x_1 = \log_e(\hat{b}_1) = -0.5004715 + 1.082291 (\log_e age) - 0.2673998 (\log_e age)^2 + 8.355966 \times 10^{-6} (\log_e age)^8 - 7.751521 \times 10^{-9} (\log_e age)^{12} + 7.142552 \times 10^{-27} (\log_e age)^{36};$$

where: age = age at breast height.

Mean height for each 10-year age at breast height was also calculated and plotted (table 6 and fig. 7). Number of plots and mean site index in each decadal age decreased beyond age 50 years, so mean height at each decadal age beyond age 50 did not represent the height corresponding to mean site index at age 50 years (87.70). Heights were therefore adjusted by use of the c_1 and d_1 coefficients for each age class and mean site index at age 50 in the equation,

$$HT = 4.5 + c_1 + d_1 (87.70 - 4.5).$$

Table 6—Regression coefficients for western hemlock from the equation (height − 4.5) = $c_1 + d_1$ (site − 4.5) by 10-year age class, and corrections for mean height after age 50, southeast Alaska

Age at breast height	c_1	d_1	r^2	Standard error of estimate	Obser-vations	Mean height	Corrected: Mean height 1/	Corrected: Adjusted mean height 2/
Years				Feet	Number	- - - - - - Feet	- - - - - - - -	
10	-0.9758	0.2384	0.43	3.16	40	23.36	23.36	23.36
20	-2.6963	.4873	.72	3.40	40	42.35	42.35	42.35
30	-3.4674	.7075	.88	2.88	40	59.90	59.90	59.90
40	-4.2922	.9016	.96	2.17	40	75.22	75.22	75.22
50	0	1.0000	1.00	0	40	87.70	87.70	87.70
60	5.5180	1.0732	.98	2.26	30	87.90	99.31	99.31
70	14.3348	1.0687	.95	3.22	20	93.00	107.75	107.75
80	22.4161	1.0725	.94	3.51	20	101.35	116.15	116.15
90	27.9638	1.0994	.92	4.32	18	107.83	123.93	123.93
100	35.6033	1.0827	.89	5.37	17	114.65	130.18	130.18
110	39.1424	1.1117	.84	5.89	14	124.00	136.14	136.14
120	46.1402	1.1028	.89	4.98	10	128.50	142.39	142.08
130	49.2752	1.1335	.87	5.58	10	133.80	148.08	147.18
140	47.0255	1.2161	.86	6.73	8	140.00	152.71	151.58
150	80.9291	.8702	.24	10.62	4	157.00	157.83	155.20

1/ After age 50, heights were calculated from the equation height = 4.5 + c_1 + d_1 (87.70 - 4.5) to adjust for decreasing numbers of observations and changing mean site index.

2/ Calculated heights for ages 120 years and older resulted in a poor fit of the combined equation so they were adjusted downward to give the height and site curves a more realistic fit; otherwise, they would turn upward after age 140.

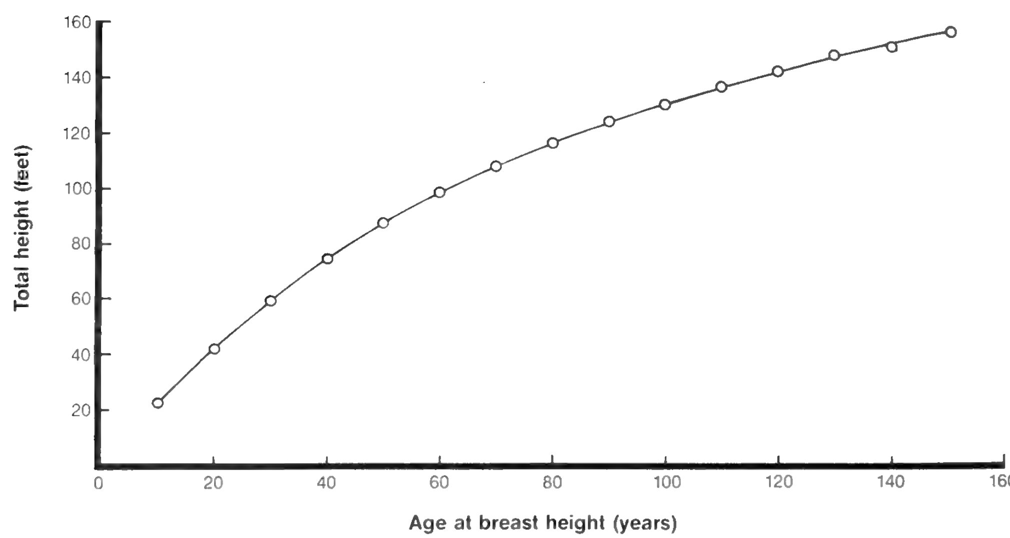


Figure 7.—Mean height of sectioned western hemlock as a function of age at breast height for each 10-year age class. Plotted points are actual mean heights minus 4.5 feet. Solid line is expressed by the equation ($\hat{HT}$ − 4.5 feet) = e^{x_2}, where x_2 is defined in the text. Standard error is 0.374 and r^2 is 0.9993.

Even then, there was a problem at ages greater than 120 years because of few observations. This was corrected by adjusting the mean height over age curve at upper ages (table 6). Adjustments of 1 to 3 feet were made to smooth the curves so as to avoid having curves that turned sharply upward or downward beyond age 120 years. These adjustments were made and tested to avoid any correlation between site index residuals and height and age.

The derived equation was:

$$\begin{aligned} x_2 = \log_e(\hat{HT} - 4.5 \text{ feet}) = {} & 0.3621734 + 1.149181 \,(\log_e \text{age}) \\ & - 0.005617852 \,(\log_e \text{age})^3 \\ & - 7.267547 \times 10^{-6} \,(\log_e \text{age})^7 \\ & + 1.708195 \times 10^{-16} \,(\log_e \text{age})^{22} \\ & - 2.482794 \times 10^{-22} \,(\log_e \text{age})^{30}. \end{aligned}$$

Now that estimating equations are available for b_1 and HT and mean site index, $\bar{S} = 87.70$, is known, the equation is,

$$\bar{S} - 4.5 = a_1 + \hat{b}_1 \,(\hat{HT} - 4.5). \quad \text{Rearranged,}$$

$$\hat{a}_1 = \bar{S} - 4.5 - \hat{b}_1 \,(\hat{HT} - 4.5).$$

The resulting $\hat{a}_1$ values are shown in table 2. The $\hat{a}_1$ and $\hat{b}_1$ values in table 2 can be used to estimate site index for combinations of height and age.

Substituting $\hat{a}_1$, $\hat{b}_1$, and $\hat{HT}$ into the basic equation,

$$S - 4.5 \text{ feet} = a_1 + b_1(HT - 4.5 \text{ feet}), \text{ gives}$$

$$S - 4.5 = \hat{a}_1 + \hat{b}_1 \,(\hat{HT} - 4.5), \text{ and}$$

$$S - 4.5 = [\bar{S} - 4.5 - \hat{b}_1 \,(\hat{HT} - 4.5)] + \hat{b}_1 \,(HT - 4.5), \text{ and}$$

$$S - 4.5 = 83.20 - e^{x_1}e^{x_2} + e^{x_1} \,(HT - 4.5), \text{ and}$$

$$S = 87.70 - e^{x_1}e^{x_2} + e^{x_1} \,(HT - 4.5).$$

The above equation can be used in a calculator or computer to estimate site index for various combinations of height and age.

The final estimating equation fit the basic data well, and there was no correlation between residuals from the equation, and site or age. Deviations of the basic data regression points were evident beyond age 100 (fig. 8).

Height growth curves.—Construction of the height growth curves for western hemlock was similar to that used for the site index curves, except that the roles of total height and site index were reversed to:

$$HT - 4.5 \text{ feet} = c_1 + d_1 \,(S - 4.5 \text{ feet}).$$

Linear regressions of the basic data for western hemlock were calculated for each decade between 10 and 150 years (table 6).

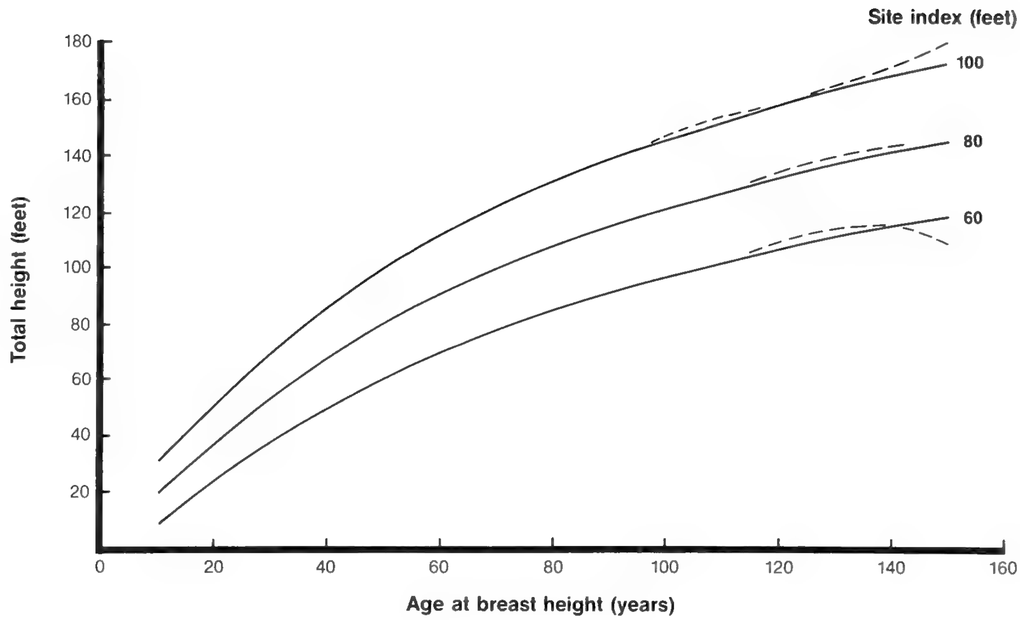


Figure 8.—Site index curves for unmanaged, even-aged western hemlock in southeast Alaska. Dashed lines connect decadal points derived from the unsmoothed basic data regressions of (S − 4.5 feet) = $a_1 + b_1$ (HT − 4.5 feet). Solid lines represent smooth curves from a rearrangement of the estimating equation.

Figure 9.—d_1 values for western hemlock in the equation (HT − 4.5 feet) = $c_1 + d_1$ (S − 4.5 feet) as a function of age. Plotted points are actual d_1 values. Solid line is a curve expressed by the equation $\hat{d}_1 = e^{x_3}$, where x_3 is defined in the text. Standard error is 0.0156 and r^2 is 0.9974.

The d_1 values were then smoothed over age (fig. 9). Coefficient d_1 turned upward at age 140 years, then sharply downward at age 150 because there were few observations in these age classes. This led to a distorted curve at upper ages, so an adjustment was made to smooth the curve at ages 140 and 150 years. Otherwise, the final height growth curves would be distorted at the upper ages.

The derived equation was:

$$
\begin{aligned}
x_3 = \log_e(\hat{d}_1) = & -2.146617 - 0.109007\,(\log_e age) \\
& + 0.0994030\,(\log_e age)^3 \\
& - 0.003853396\,(\log_e age)^5 \\
& + 1.193933 \times 10^{-8}\,(\log_e age)^{12} \\
& - 9.486544 \times 10^{-20}\,(\log_e age)^{27} \\
& + 1.431925 \times 10^{-26}\,(\log_e age)^{36}.
\end{aligned}
$$

The resulting $\hat{d}_1$ values are shown in table 4.

Substituting mean site index, ($\bar{S} = 87.70$, into the basic equation,

$$\hat{H}T - 4.5 \text{ feet} = c_1 + \hat{d}_1\,(\bar{S} - 4.5 \text{ feet}), \text{ and rearranging, gives}$$

$$\hat{c}_1 = (\hat{H}T - 4.5 - \hat{d}_1\,(87.70 - 4.5).$$

Values for $\hat{c}_1$ are given in table 4. The $\hat{c}_1$ and $\hat{d}_1$ values in table 4 can be used to estimate at any age total height of the average among the 40 largest diameter trees per acre, on land of known site index.

Substituting $\hat{c}_1$, $\hat{d}_1$, and $\hat{H}T$ into the basic equation,

$$HT - 4.5 \text{ feet} = c_1 + d_1\,(S - 4.5 \text{ feet}), \text{ gives}$$

$$
\begin{aligned}
HT - 4.5 &= \hat{c}_1 + \hat{d}_1\,(S - 4.5), \text{ and} \\
HT - 4.5 &= [\hat{H}T - 4.5 - \hat{d}_1\,(\bar{S} - 4.5)] + \hat{d}_1\,(S - 4.5), \text{ and} \\
HT - 4.5 &= e^{x_2} - e^{x_3}\,(87.70 - 4.5) + e^{x_3}\,(S - 4.5), \text{ and} \\
HT &= 4.5 + e^{x_2} - e^{x_3}\,(83.20) + e^{x_3}\,(S - 4.5).
\end{aligned}
$$

The above equation can be used in a calculator or computer to estimate total height of the average among the 40 largest diameter trees per acre on land of known site index. Height growth curves for hemlock are shown in figure 10.

A graphic comparison between the site index and height growth curves for western hemlock is shown in figure 11.

Figure 10.—Height growth curves for unmanaged, even-aged western hemlock in southeast Alaska. Dashed lines connect decadal points derived from the unsmoothed basic data regressions of (HT − 4.5 feet) = $c_1 + d_1$ (S − 4.5 feet). Solid lines represent smooth curves from the estimating equation. Heights represent the means among the 40 largest diameter trees per acre.

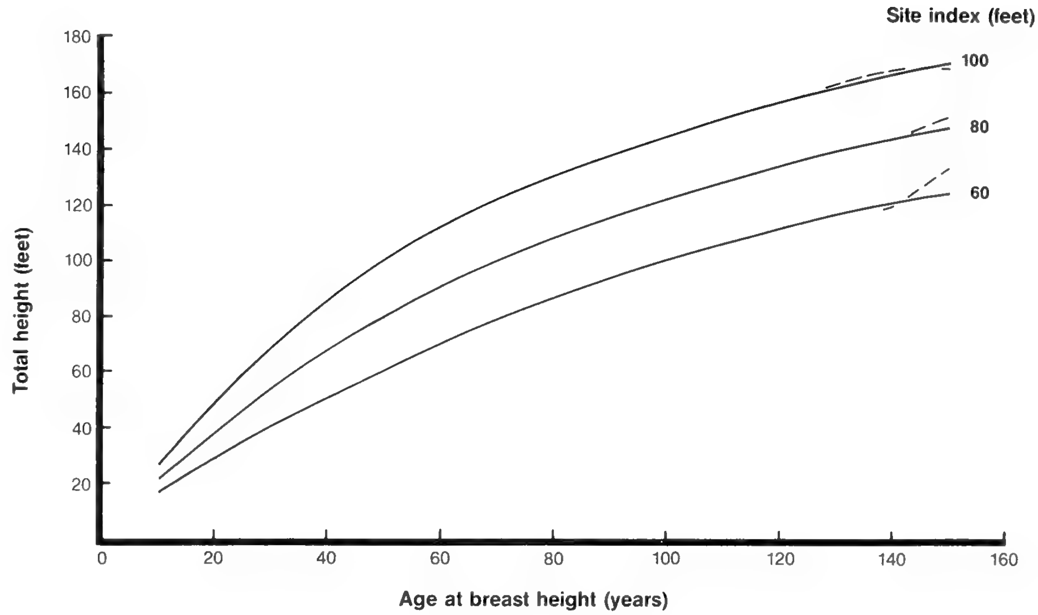
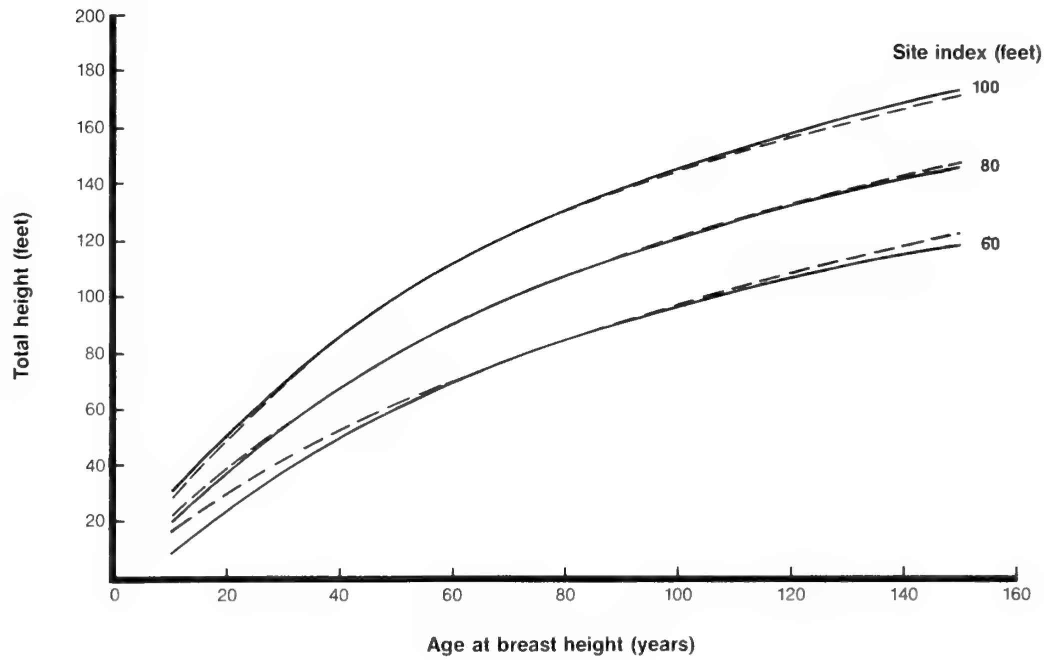


Figure 11.—Site index (solid lines) and height growth curves (dashed lines) for unmanaged, even-aged western hemlock in southeast Alaska.



Construction of Curves for Sitka Spruce

Site index curves.—The same general procedures were used to construct the site index curves for Sitka spruce as were used to construct the site index curves for western hemlock. Linear regressions for the relationship (S − 4.5 feet) = $a_2 + b_2$ (HT − 4.5 feet) were calculated for each decade from 10 to 150 years. They were:

Age at breast height	a_2	b_2	r^2	Standard error of the estimate	Observations
(Years)	(Feet)	(Feet)		(Feet)	(Number)
10	43.3181	2.3233	0.57	10.64	71
20	27.2889	1.5085	.77	7.75	71
30	16.8460	1.2048	.88	5.53	71
40	9.0134	1.0487	.96	3.21	71
50	0	1.000	1.00	0	71
60	-.7774	.8890	.99	1.86	36
70	-2.1011	.8199	.96	2.99	30
80	-4.5038	.7850	.95	3.30	29
90	-6.3957	.7523	.94	3.74	26
100	-7.2216	.7192	.92	4.42	23
110	-6.8901	.6900	.87	4.45	19
120	-8.7211	.6766	.89	4.17	15
130	-8.7727	.6572	.90	4.27	13
140	-5.2298	.6153	.93	3.75	10
150	-1.7375	.5722	.92	4.08	8

Coefficient b_2 dipped downward beyond age 130 (fig. 12). This led to a poor fit of the combined equation at upper ages, so the curve of b_2 over age was adjusted to avoid distorted site index curves beyond 130 years.

The derived equation was:

$$x_4 - \log_e(\hat{b}_2) = 6.396816 - 4.098921\,(\log_e age) + 0.7628741\,(\log_e age)^2 - 0.00244688\,(\log_e age)^5 + 2.445811 \times 10^{-7}\,(\log_e age)^{10} - 2.022153 \times 10^{-22}\,(\log_e age)^{30}.$$

Mean height minus 4.5 feet for each 10-year age at breast height was also calculated (table 7 and fig. 13). Number of plots and mean site index in each 10-year age decreased beyond age 50, so mean height beyond age 50 did not reflect the correct height for mean site 90.93. Heights were then adjusted by use of the c_2 and d_2 coefficients for each age class and mean site index at age 50 in the equation,

$$HT = 4.5 + c_2 + d_2\,(90.93 - 4.5).$$

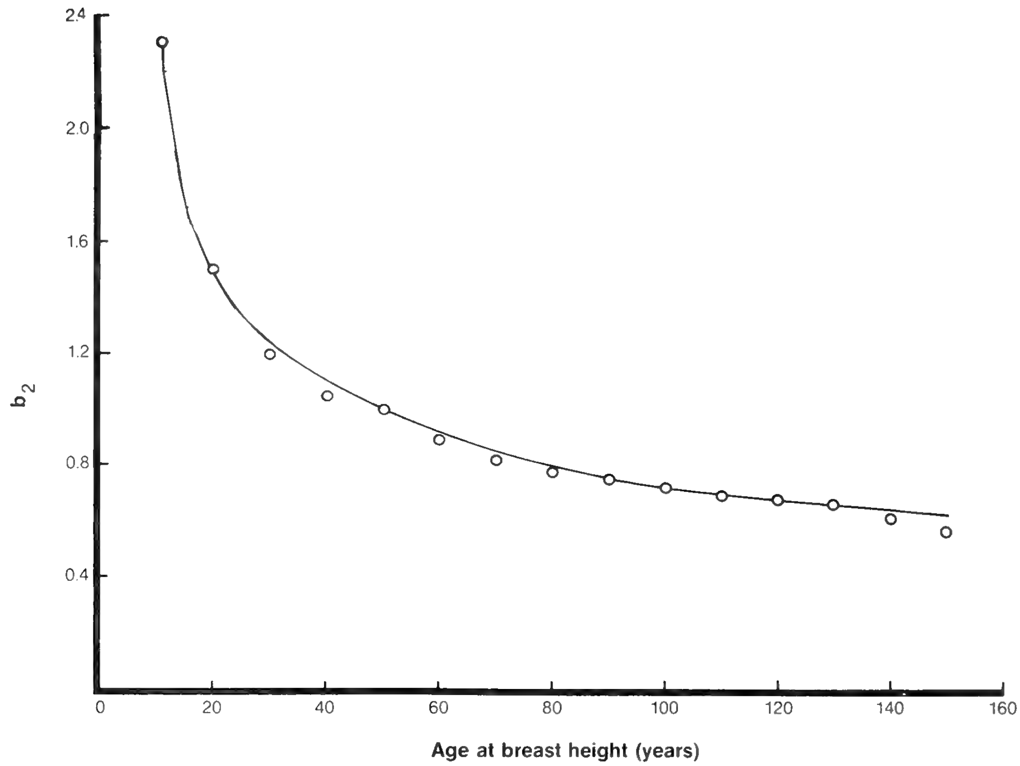


Figure 12.—b_2 values for Sitka spruce in the equation $(S - 4.5 \text{ feet}) = a_2 + b_2 (HT - 4.5 \text{ feet})$ as a function of age. Solid line is expressed by the equation $\hat{b}_2 = e^{x_4}$. Coefficient x_4 is defined in the text. Standard error is 0.0008 and r^2 is 0.9959.

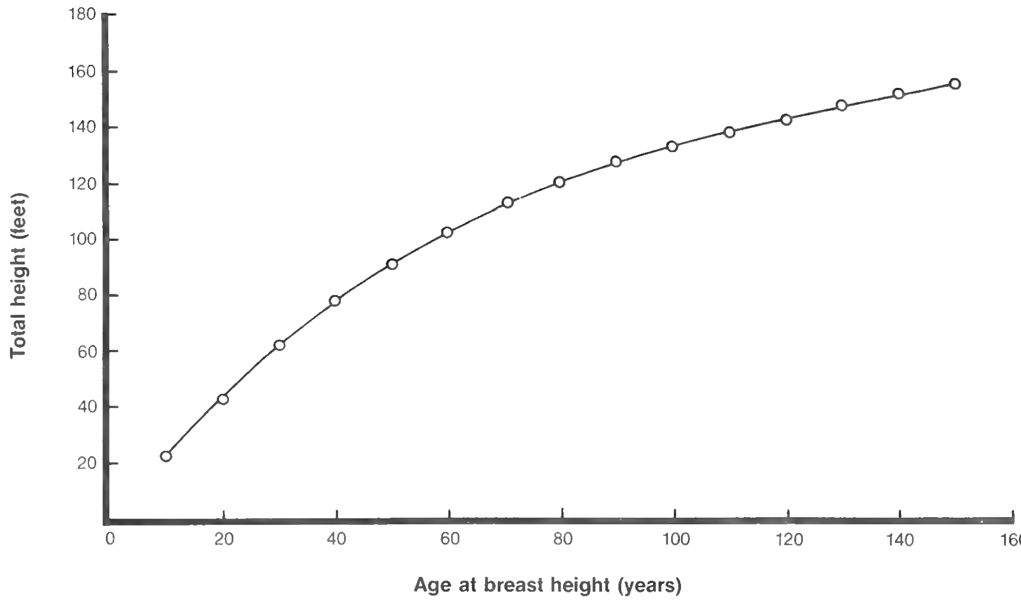
Table 7—Regression coefficients for Sitka spruce from the equation (height − 4.5) = $c_2 + d_2$ (site − 4.5) by 10-year age class, and corrections for mean height after age 50, southeast Alaska

Age at breast height	c_2	d_2	r^2	Standard error of estimate	Obser-vations	Mean height	Corrected: Mean height 1/	Corrected: Adjusted mean height 2/
Years				Feet	Number	- - - - - - -	Feet	- - - - - - -
10	-2.5912	0.2447	0.57	3.45	71	23.06	23.06	23.06
20	-4.9658	.5110	.77	4.51	71	43.70	43.70	43.70
30	-5.6386	.7334	.88	4.31	71	62.25	62.25	62.25
40	-5.3580	.9161	.96	3.00	71	78.32	78.32	78.32
50	0	1.0000	1.00	0	71	90.93	90.93	90.93
60	2.1197	1.1090	.99	2.08	36	93.61	102.47	102.47
70	6.4509	1.1680	.96	3.57	30	98.90	111.90	111.90
80	10.6176	1.2088	.95	4.10	29	105.59	119.59	119.59
90	14.4174	1.2503	.94	4.82	26	112.69	126.98	126.98
100	18.4553	1.2779	.92	5.90	23	118.43	133.40	133.40
110	24.5582	1.2631	.87	6.02	19	127.95	138.23	138.23
120	29.3460	1.2537	.89	5.49	15	130.07	142.20	144.90
130	24.8464	1.3724	.90	6.17	13	135.08	147.96	148.73
140	18.1998	1.5044	.93	5.86	10	143.50	152.73	151.99
150	14.9222	1.6006	.92	6.82	8	148.88	157.76	154.75

1/ After age 50, heights were calculated from the equation height = $4.5 + c_2 + d_2 (90.93 - 4.5)$ to adjust for decreasing numbers of observations and changing mean site index.

2/ Values actually used to compute coefficients in the height over age equation. They were derived by fitting the basic data to a nonlinear equation for HT = f(site, age), then solving for mean site index 90.93 to estimate smoothed mean heights for ages 120 to 150 years.

Figure 13.—Mean height of sectioned Sitka spruce as a function of age at breast height for each 10-year age class. Plotted points are actual mean heights minus 4.5 feet. Solid line is expressed by the equation $(\hat{H}T - 4.5 \text{ feet}) = e^{x_5}$, where x_5 is defined in the text Standard error is 0.094 and r^2 is 0.99995.



Even then there were problems at upper ages because of scarcity of observations. This caused a biased fit of the combined equation at 120 to 150 years. The problem was later corrected by fitting the basic data to a combined nonlinear equation for HT = f(S, A), then solving for mean site index 90.93 to estimate smoothed mean heights for ages 120 to 150. This avoided any correlation between the residuals and site and age.

The derived equation was:

$$
\begin{aligned}
x_5 = \log_e(\hat{H}T - 4.5 \text{ feet} = & - 0.2050542 + 1.449615 \, (\log_e \text{age}) \\
& - 0.01780992 \, (\log_e \text{age})^3 \\
& + 6.519748 \times 10^{-5} \, (\log_e \text{age})^5 \\
& - 1.095593 \times 10^{-23} \, (\log_e \text{age})^{30}.
\end{aligned}
$$

Now that estimating equations are available for b_2 and HT and mean S = 90.93 is known, the equation is,

$$\bar{S} - 4.5 = a_2 + \hat{b}_2 \, (\hat{H}T - 4.5). \text{ Rearranged,}$$

$$\hat{a}_2 = \bar{S} - 4.5 - \hat{b}_2 \, (\hat{H}T - 4.5).$$

The resulting $\hat{a}_2$ values are those shown in table 3. The $\hat{a}_2$ and $\hat{b}_2$ values in table 3 can be used to estimate site index for combinations of height and age.

Substituting $\hat{a}_2$, $\hat{b}_2$, and $\hat{H}T$ into the basic equation,

$$S - 4.5 \text{ feet} = a_2 + b_2(HT - 4.5 \text{ feet}), \text{ gives}$$

$$S - 4.5 = \hat{a}_2 + \hat{b}_2 (\hat{H}T - 4.5), \text{ and}$$

$$S - 4.5 = [\bar{S} - 4.5 - \hat{b}_2 \ (\hat{H}T - 4.5)] + \hat{b}_2 (HT - 4.5), \text{ and}$$

$$S - 4.5 = 86.43 - e^{x_4}e^{x_5} + e^{x_4} (HT - 4.5), \text{ and}$$

$$S = 90.93 - e^{x_4}e^{x_5} + e^{x_4} (HT - 4.5).$$

The above equation can be used in a calculator or computer to estimate site index for various combinations of height and age. The site index curves are shown in figure 3.

The final estimating equations fit the basic data regression points well. Deviations were evident beyond age 120 (fig. 14).

Height growth curves.—Construction of the height growth curves for Sitka spruce was similar, except the roles of total height and site index were reversed to,

$$HT - 4.5 \text{ feet} = c_2 + d_2 (S - 4.5 \text{ feet}).$$

Linear regressions of the basic data for Sitka spruce were calculated for each decade between 10 and 150 years (table 7).

The d_2 values were then smoothed over breast high age (fig. 15). Coefficient d_2 turned sharply upward beyond age 120 as there were few observations in the upper age classes. This led to a poor fit of the combined equation beyond age 120, so the curve of d_2 was, by trial and error, subjectively smoothed for ages greater than 120 years so that the height growth curves could be extended to age 150.

The derived equation was:

$$\begin{aligned} x_6 = \log_e(\hat{d}_2) = \ & - 5.6118790 + 2.418604 (\log_e age) \\ & - 0.259311 (\log_e age)^2 \\ & + 1.351445 \times 10^{-4} (\log_e age)^5 \\ & - 1.701139 \times 10^{-12} (\log_e age)^{16} \\ & + 7.964197 \times 10^{-27} (\log_e age)^{36}. \end{aligned}$$

The resulting $\hat{d}_2$ values are shown in table 5.

Substituting mean site index $\bar{S} = 90.93$ into the basic equation,

$$\hat{H}T - 4.5 \text{ feet} = c_2 + \hat{d}_2 (\bar{S} - 4.5 \text{ feet}), \text{ and rearranging, gives}$$

$$\hat{c}_2 = \hat{H}T - 4.5 - \hat{b}_1(\bar{S} - 4.5).$$

Figure 14.—Site index curves for unmanaged, even-aged Sitka spruce in southeast Alaska. Dashed lines connect decadal points derived from the unsmoothed basic data regressions of (S − 4.5 feet) = $a_2 + b_2$ (HT − 4.5 feet). Solid lines represent smooth curves from a rearrangement of the estimating equation.

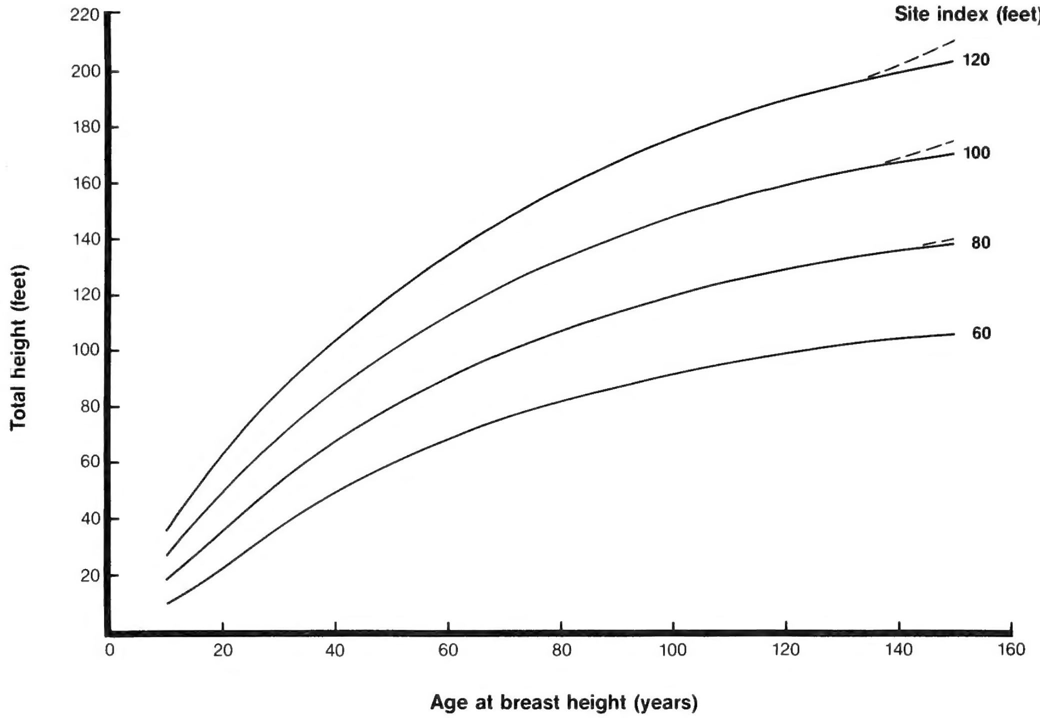
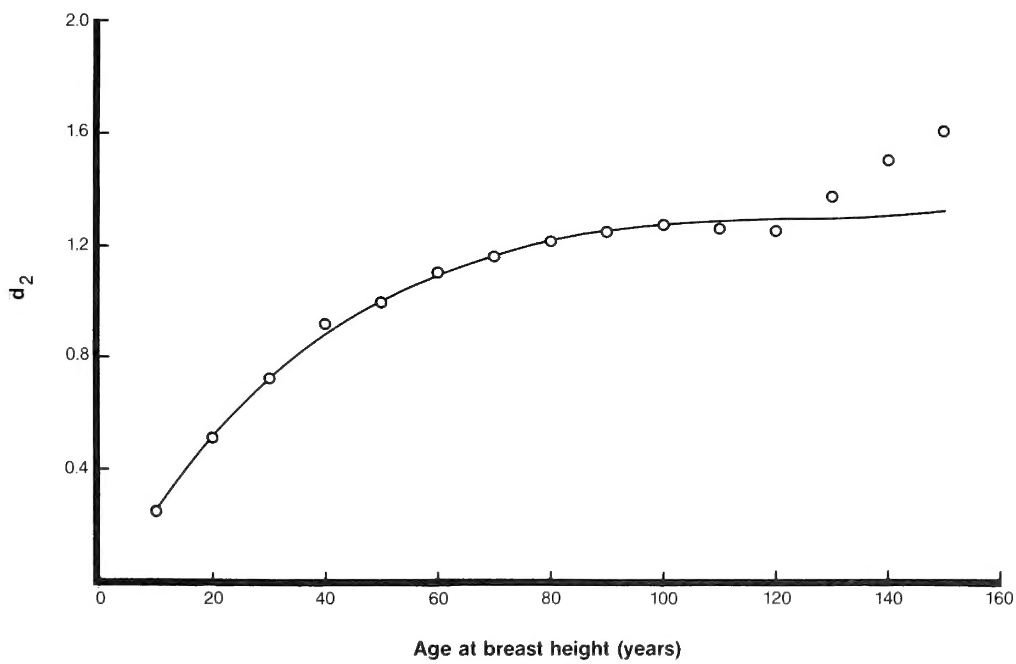


Figure 15.—d_2 values for Sitka spruce in the equation (HT − 4.5 feet) = $c_2 + d_2$ (S − 4.5 feet) as a function of age. Plotted points are actual d_2 values. Solid line is a curve expressed by the equation $\hat{d}_2 = e^{x_6}$, where x_6 is defined in the text. Standard error is 0.008 and r^2 is 0.9921.



Values for $\hat{c}_2$ are given in table 5. The $\hat{c}_2$ and $\hat{d}_2$ values in table 5 can be used to estimate at any age total height of the average among the 40 largest diameter trees per acre on land of known site index.

Substituting $\hat{c}_2$, $\hat{d}_2$, and $\hat{H}T$ into the basic equation,

$$HT - 4.5 \text{ feet} = c_2 + d_2 (S - 4.5 \text{ feet}), \text{ gives}$$

$$HT - 4.5 = \hat{c}_2 + \hat{d}_2 (S - 4.5), \text{ and}$$

$$HT - 4.5 = [\hat{H}T - 4.5 - \hat{d}_2 (\bar{S} - 4.5)] + \hat{d}_2 (S - 4.5), \text{ and}$$

$$HT - 4.5 = e^{x_5} - e^{x_6} (90.93 - 4.5) + e^{x_6} (S - 4.5), \text{ and}$$

$$HT = 4.5 + e^{x_5} - e^{x_6} (86.43) + e^{x_6} (S - 4.5).$$

The above equation can be used in a calculator or computer to estimate total height of the average among the 40 largest trees per acre on land of known site index. Height growth curves for Sitka spruce are shown in figure 16.

A graphic comparison between the site index and height growth curves for Sitka spruce is shown in figure 17.

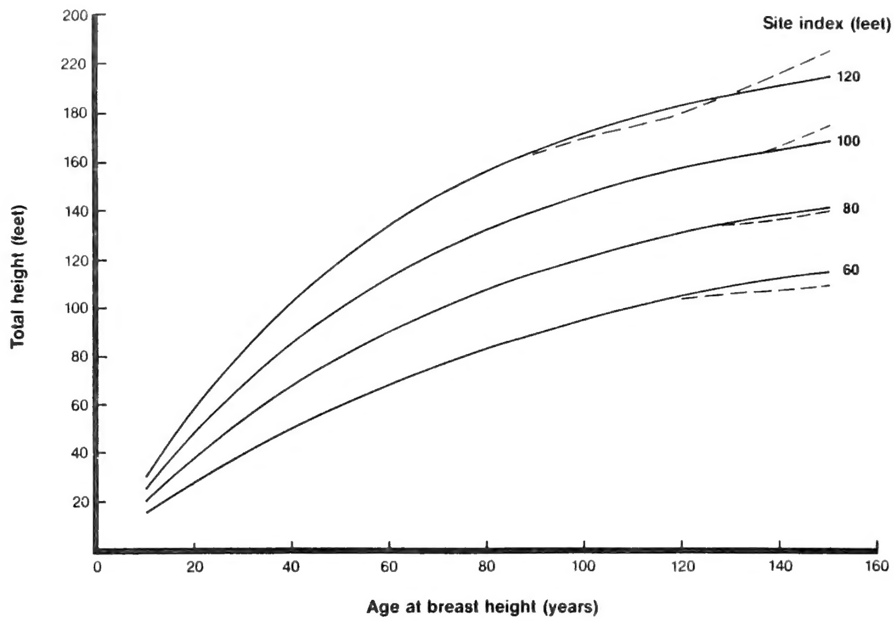


Figure 16.—Height growth curves for unmanaged, even-aged Sitka spruce in southeast Alaska. Dashed lines connect decadal points derived from the unsmoothed basic data regressions of $(HT - 4.5 \text{ feet}) = c_2 + d_2 (S - 4.5 \text{ feet})$. Solid lines represent smooth curves from the estimating equation. Heights represent the mean among the 40 largest diameter trees per acre.

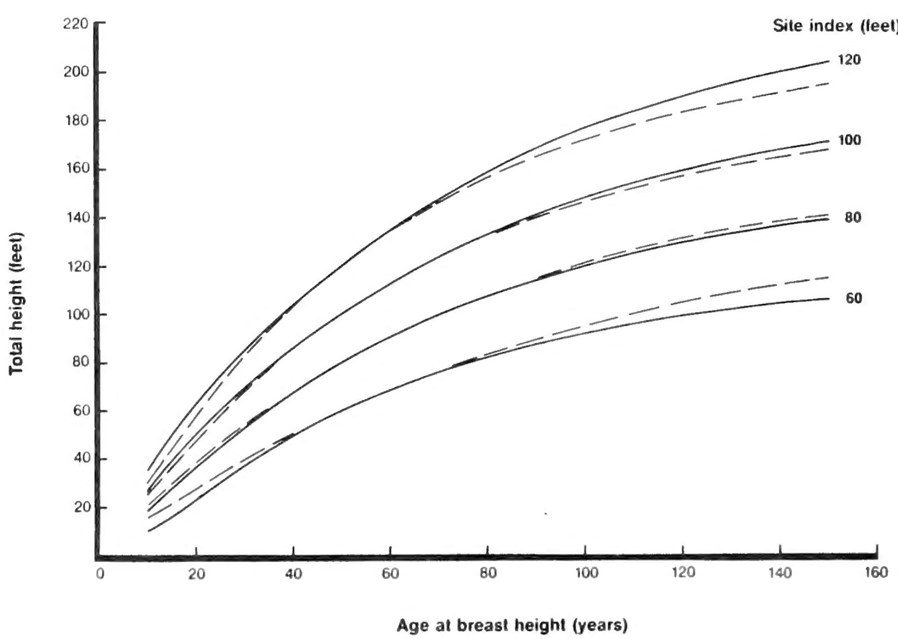


Figure 17.—Site index (solid lines) and height growth curves (dashed lines) for unmanaged, even-aged Sitka spruce in southeast Alaska.

Farr, Wilbur A. Site index and height growth curves for unmanaged even-aged stands of western hemlock and Sitka spruce in southeast Alaska. Res. Pap. PNW-326. Portland, OR: U.S. Department of Agriculture, Forest Service, Pacific Northwest Forest and Range Experiment Station; **1984.** 26 p.

Height growth and site index curves and equations are presented for unmanaged even-aged stands of western hemlock and Sitka spruce in southeast Alaska. Data were mostly collected in stands that developed after logging.

Keywords: Even-aged stands, site index, increment (height), stem analysis, western hemlock, *Tsuga heterophylla,* Sitka spruce, *Picea sitchensis,* southeast Alaska, Alaska (southeast).

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