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# SITE INDEXES FOR LODGEPOLE PINE, WITH CORRECTIONS FOR STAND DENSITY: 

Methodology

## by

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## CONTENTS

Page
Mieasures of stand density ..... 1
Field measurements ..... 2
Developing a CCF equation for lodgepole pine ..... 2
Source of data ..... 3
Control plots ..... 4
Analysis of data ..... 4
Reliability of CCF as a measure of density ..... 4
Preparation of the height-age data for use ..... 6
Determining true site index ..... 6
Effect of density on dominant height ..... 7
Determining the shape of the height-growth curve ..... 8
Determining the combined effects of density and site index on the height-age relationship ..... 8
Testing initial assumption that all plot data were from one population ..... 9
Adjustments of height-growth curves ..... 11
Test of adjusted height-growth curves ..... 13
Field application of site-index curves ..... 16
Literature cited ..... 18

# Site Indexes for Lodgepole Pine, 

# with Corrections for Stand Density: 

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Site indexes are commonly used to express the productive capacity of different forest environments for a particular species. Determination of productivity is basic to the prediction of yields, the establishment of optimum levels of growing stock, and consequently, the level of management that can be profitably applied to any area.

No one measure of site quality has been found to be entirely satisfactory, although many indicators have been tried. The measure of tree growth usually found most independent of stand factors, and therefore the most reliable index of site quality, is the average height of dominant and codominant trees in relation to age. The relationship of height and age, when expressed as dominant and codominant height in feet at some specified reference age, is the familiar "site index" that has come into general use as the conventional measure of site quality.

The height growth of most conifers is independent of stand density over a wide range of stocking. The family of site-index curves developed for those species is simply the expression of the relationship between height growth and the independent variables of site quality and age. The rate of height growth of some species, however, is influenced by density. Height growth is curtailed if stands are either too open or too dense. ${ }^{2}$ For those spe-
${ }^{2}$ Considerable Iiterature is available on the effect of density on height growth, but no attempt will be made to review that work. A good discussion is presented by Lynch (1958). Nomes and dates in parentheses refer to Literature Cited, p. 18.
cies a third independent variable, stand density, must be considered.

The height growth of lodgepole pine (Pinus contorta Dougl.) is probably influenced more by stand density than any other North American conifer. Because it commonly occurs in overly dense stands, and crowding restricts height growth, past attempts to assign productivity classes have failed to estimate the growth potential of lodgepole pine properly.

In 1961, an interregional study of site indexes for lodgepole pine was begun in the Rocky Mountain, Intermountain, and Pacific Northwest regions. The objectives of that study were to:

1. Determine the effect of density on height growth of lodgepole pine;
2. Develop height-growth curves adjusted for different levels of density that could be used to predict site productivity wherever lodgepole pine grows.
The first need was to select a suitable measure of stand density.

## Measures of Stand Density

Stand density in forestry is defined as the relation between number, or number and size, of trees on a specified area, usually 1 acre. Size of trees is expressed by various measures or combinations of measures such as diameter, basal area, and height. Although many measures of stand density have been tried, its expression remains one of the most difficult problems in forestry. Since good
discussions of stocking and stand-density measures are covered by Spurr (1952) and Bickford et al. (1957), only later developments will be discussed here.

A new measure of density, Crown Competition Factor (CCF), was developed by Krajicek et al. (1961). CCF compares growing space available to a tree with that represented by a vertical projection of the average crown area of an open-grown tree of the same stem diameter (Maximum Crown Area, MCA). Because space occupied by a single tree is not easily determined, the comparison is made on a group basis. CCF expresses the sum of MiCA's for a group of trees as a percentage of actual ground area occupied by those trees. It is, therefore, a measure of average growing space per tree, not a measure of crown closure (Krajicek et al. 1961).

According to Spurr (1952), the ideal measure of density should be unrelated to either stand age or site quality. One of the important properties claimed for CCF by Krajicek et al. (1961) was the lack of correlation with either site or age. However, they did point out there was some tendency for high CCF values to be associated with poor sites.

Using methods developed by Krajicek et al. (1961), Vezina $(1962,1963)$ developed CCF equations for three species of conifers in Canada, balsam fir (Abies balsamea (S) Mill.), white spruce (Picea glauca (Moench.) Voss), and jack pine (pinus banksiana Lamb.). He found that CCF, as a measure of density, varied somewhat with stand age and site quality, but no consistent relationship was evident.

Field Measurements
Developing a CCF Equation for Lodgepole Pine
A CCF equation was developed for lodgepole pine by the procedures of Krajicek et al. (1961). Personnel of the Rocky Mountain, Intermountain, and Pacific Northwest Forest and Range Experiment Stations measured the diameter breast height and crown width of open-grown trees to establish regression equations for each region, expressing crown width in terms of diameter. The resulting equations were:

1. Rocky Mountain

$$
\begin{aligned}
\mathrm{CW}= & 3.95+1.4353 \mathrm{D} \\
& \mathrm{r}=0.95 \\
& \text { Basis: } 265 \text { trees }
\end{aligned}
$$

2. Intermountain

$$
C W=2.79+1.4081 \mathrm{D}
$$

$r=0.94$
Basis: 411 trees
3. Pacific Northwest

$$
\begin{aligned}
\mathrm{CW}= & 3.49+1.4373 \mathrm{D} \\
& \mathrm{r}=0.97 \\
& \text { Basis: } 99 \text { trees }
\end{aligned}
$$

where
CW = crown width in feet
$\mathrm{D}=$ diameter in inches
$r=$ correlation coefficient
Analysis of covariance showed that the regression coefficients of the three equations were not significantly different. The regression lines were virtually parallel (fig. 1). The intercept value of the Intermountain equation


Figure 1.--CCF equation for each region and all regions combined.
was significantly different from those of the Rocky Mountain and Pacific Northwest, but since the magnitude of the difference was so small, the data from the three sources were combined into the following equation, using the common regression coefficient from the analysis of covariance of the combined data:
$C W=3.27+1.423 \mathrm{D}$,
$r=0.94$
Basis: 775 trees
From the combined data, again following the procedures outlined by Krajicek et al. (1961), the following equation was developed:

$$
\begin{align*}
C C F= & 1 / A\left[0.01925 \sum_{i=1}^{k} N_{i}+0.01676 \sum_{i=1}^{k} D_{i} N_{i}\right. \\
& \left.+0.00365 \sum_{i=1}^{k} D_{i}^{2} N_{i}\right] \tag{1}
\end{align*}
$$

where

$$
\begin{aligned}
\mathrm{CCF} & =\text { Crown Competition Factor } \\
\mathrm{A} & =\text { area in acres } \\
\mathrm{N} & =\text { number of trees in each d.b.h. class } \\
\mathrm{D} & =\text { d.b.h. class } \\
\mathrm{k} & =\text { number of d.b.h. classes }
\end{aligned}
$$

CCF was selected as the preliminary measure of density to use. Measurements were taken in the field that would permit alternative measures of density to be substituted for CCF if the latter proved unsatisfactory.

Source of Data
To determine the height-age-density relationship, data were collected in 1961 and 1962 from 262 temporary sample plots in 88 clusters located as follows:

| Area | Plot clusters | Plots |
| :--- | :---: | ---: |
| $--($ Number $)$ | $-\frac{-}{-}$ | 111 |
| Rocky Mountain | 36 | 99 |
| Intermountain | 33 | 52 |
| Pacific Northwest | 19 |  |
| Total | 88 | 262 |

Plots were confined to stands that met the following criteria:

1. Even-aged (not more than 20 years' spread in the age of dominant trees).
2. Thirty years old or older.
3. Free, throughout its life, of fire or other influences that might cause abnormal changes in density.
4. Contained two or more density classes on the same site. Site was considered to be the same if topography, slope, aspect, and soils were similar. Density was considered different if CCF varied by 50 or more from one location to another in the stand.

In the aggregate, plots ranged in age from 35 to 270 years, in site index from 40 to 90 , and in CCF from 60 to 600 plus. Not all density classes were represented, however, in each age-site combination.

On each plot, four trees within the same density class, and as close together as possible, were selected for site determination. Only those trees were selected that were:

1. Dominants and appeared to have been dominant throughout their lives.
2. Reasonably free of dwarfmistletoe or other diseases or injuries that might have reduced height growth.
3. Sound enough for ring counts.
4. Without visible evidence of crown damage, broken, forked, etc., tops.
5. Without erratic or rapidly changing pattern of ring widths from pith to cambium.

The density in which the site trees developed was determined from circular subplots centered on each site tree. Density subplots ranged in size from 0.0025 to 0.2 acre, but only one size subplot was used on a plot. Subplot size was determined from table 1. The subplot size used was the smallest permitted by the height of the site trees, increased where necessary to include at least 10 trees. The four density subplots provided the measurements necessary to compute CCF and basal area for each plot.

The site trees, which were felled and sectioned for stem analysis, supplied the following information:

1. Diameter at breast height.
2. Total height of dominant stand.

Table l.--Sizes of density subplots used for different minimum numbers of trees per acre (l0 per plot) and maximum height of sample trees

| Maximum <br> height <br> of trees <br> (Feet) | Minimum <br> trees <br> per acre | Radius | Plot <br> size |
| :---: | :---: | :---: | :---: |
| Number | $\underline{\text { Feet }}$ | Acre |  |
| Unlimited | 51 | 52.67 | 0.2 |
| 75 | 100 | 37.25 | .1 |
| 53 | 200 | 26.33 | .05 |
| 37 | 400 | 18.67 | .025 |
| 24 | 1,000 | 11.75 | .01 |
| 17 | 2,000 | 8.33 | .005 |
| 12 | 4,000 | 5.92 | .0025 |

3. Age of the dominant stand at ground line, breast height, 10 feet, and every 10 -foot interval thereafter to total height. (Trees less than 30 feet tall were sectioned at 5 -foot intervals starting at 15 feet above ground line.)
4. Radial distance between each 10 annual rings along an average radius at breast height.
5. Preliminary productivity class. Apparent site index was determined from heightgrowth curves developed in the Northern Region of the U.S. Forest Service, from forest survey data.

## Control Plots

To evaluate the effect of density on height, and therefore on site-index determination, an independent and true measure of site quality was required. Since density in lodgepole pine has such a drastic influence on height growth over a wide range of stocking, it was necessary to determine site index from stands where height was unaffected by density.

To investigate the relationship of height growth to CCF, curves were prepared from data published by Parker (1942) and Smithers (1956), and later checked against data collected in the Rocky Mountain Region in 1961. Those curves showed a linear relationship. An increase in CCF resulted in a decrease in height for given age, except that height growth
appeared not to be curtailed by density below a CCF of approximately 150 . CCF 150 or less was therefore the basis for establishing control plots.

A system was adopted, similar to Lynch's (1958) paired-plot technique, in which clusters of two or more plots, side by side, were sampled. These plots were identical in age, and on the same apparent site, but different in density. The number of plots in a cluster was determined by the number of CCF classes available. Each cluster was to contain one plot--designated the control, located in stands where CCF was 150 or less--that would supply the independent measure of site index. In the field, however, it was difficult to find stands of high CCF in close proximity to or on the same site as stands of low CCF (150 or less). But, because of the apparent relationship between height growth and CCF within site classes, it was possible where necessary to eliminate the need for a control plot in every cluster. That made it possible to sample more intensively in high-density stands. Plots in clusters without a control plot were assigned to apparent site-index classes on the basis of the similarity in height, age, and CCF of the plot of lowest density to plots in clusters which had a control. For example, stand A contained a cluster of three plots, one each with a CCF of 100, 200, and 300; stand B contained a cluster of two plots, one each with a CCF of 300 and 400. The height and age of both plots with CCF 300 were similar. The apparent site index of the plots instand A was, therefore, assigned to the plots in stand B also.

## Analysis of Data

## Reliability of CCF as a Measure of Density

Before CCF was selected as the final measure of density to use, the dependence of CCF on stand age and site index was explored by means of scatter diagrams of CCF-age and CCF-site index (figs. 2,3). No strong relationship between CCF and either age or site index was apparent from those diagrams, although the variance decreased as age increased. Regression analysis also failed to indicate any correlation between CCF and age. There was


Figure 2.--Relationship of CCF to total age.


Figure 3.--Relationship of CCF to site index.
a significant negative correlation between CCF and site index (fig. 3); higher CCF values appeared to be associated with low site indexes. That correlation was not due to density causing a reduction in height growth, because the site-index values used were adjusted for the effects of density (adjustments in site index for the reduction in height growth due to density are explained under "Determining True Site Index'). Although there may be a real relationship between CCF and site index, the correlation coefficient indicated that the regression accounted for only about 6 percent of the total variation in CCF between plots. CCF was accepted, therefore, as a measure of density for lodgepole pine, independent of stand age and site index.

## Preparation of the Height-Age Data for Use

For each of the four site trees on each plot, height-total age curves were prepared graphically from the stem-analysis data. The estimated height of each tree, beginning at age 20 years, for successive 10-year intervals was read from those graphs. Those values were then averaged for each plot to give estimates of mean heights by 10-year intervals. Mean heights, beginning with age 20 years, were plotted by 10-year age classes to obtain the average height-total age curves for each plot (Spurr 1952).

On those plots in age classes 70 to 100 years, where the shape of the height-growth curve was established, the curve was extrapolated to age 100 years. That was necessary to estimate true site index on those plots. None of the height-growth curves were extended beyond age 200 years because of the limited amount of data from older stands. All plot data in age classes less than 70 years old ( 57 plots) were discarded because it was not possible to estimate their height at age 100 years accurately. In addition, eight other plots were eliminated because the height-growth curves showed abnormal trends.

The individual plot height-growth curves were then screened for differences in curve shape, both between Regions and within Regions. The only apparent differences of any consequence was a slower initial height growth
on plots from the Pacific Northwest Region. After age 50 years, however, height-growth curves for Pacific Northwest plots agreed with those of similar site and density from the other two Regions. The height-growth curves from the three Regions were tentatively accepted as being from one population, and age 30 years was chosen as the minimum age to use.

Density data--CCF, basal area, and number of trees--were expressed for each plot on an acre basis.

## Determining True Site Index

The remaining plots (197) were sorted into six site-index classes ranging from 40 to 90 on the basis of the apparent site index of control plots. All plots in a cluster were assigned the same site-index class. Plots in clusters having no control plot were placed in siteindex classes on the basis of similarity in height growth, age, and density of the plot of lowest CCF to plots in clusters with a control.

The average height of the four site trees on each plot, at age 100 years, was then plotted over CCF separately for each apparent siteindex class. The points were screened, and any plots from clusters without a control that appeared to have been assigned to the wrong site-index class were reassigned to the correct site-index class. A balanced line was fitted graphically for each site-index class to determine:

1. Relationship of dominant height at age 100 years to CCF;
2. Value of CCF at which height growth was not curtailed.

The CCF level below which height growth was not curtailed was approximately 125 for all site-index classes. ${ }^{3}$ Furthermore, there was no indication from data available that dominant height was reduced at CCF less than 125.

[^0]The average heights of the site trees at age 100 years were again plotted over CCF separately for each site class. Separate regression lines of the form:
$\mathrm{H}=\mathrm{a}+\mathrm{b}(\mathrm{CCF})$
where
$\mathrm{H}=$ height at age 100 years
$C C F=$ crown competition factor (CCF > 125)
were then computed for all plots with CCF greater than 125. The intercept of each regression line with CCF 125 was true average site index for that class. The heights at age 100 years of the plots with CCF greater than 125 were adjusted to the heights they would have had if density had not curtailed growth by:

1. Adding the difference between their height and the regression line if the plotted points were above the line;
2. Subtracting the difference if the plotted points were below the regression line.

The resulting values for each plot were the first approximation of true site index.

All above data were then combined, and one equation was computed to express the relationship of dominant height at age 100 years to CCF for all site index classes. The resulting equation was:

$$
\begin{align*}
H= & 6.2470+0.0144(C C F)  \tag{2}\\
& +1.2268(\mathrm{SI})-0.0017(\mathrm{CCF})(\mathrm{SI})
\end{align*}
$$

$R=0.99$
Basis: 176 plots
where
$\mathrm{H}=$ height at age 100 years
SI = first approximation of true site index $\mathrm{CCF}=$ crown competition factor ( $\mathrm{CCF}>125$ )

Figure 4, obtained from equation 2 , is the family of curves relating height at age 100 years to CCF for each site class. Height at age 100 years was replotted for each siteindex class and adjusted as above. The resulting values, with very little change from the previous first approximation, were the final approximation of true site index, independent of density. Those site indexes were used in
the development of all subsequent equations. The 21 plots in the range of CCF 125 or less were not used in subsequent equations.

## Effect of Density on Dominant Height

The effects of density on the dominant height of lodgepole pines (fig. 4) can be summarized as follows:

1. Dominant heights were reduced on all sites as CCF levels increased above 125. No relationship could be determined, however, between CCF levels less than 125 and height growth for any site.
2. Reduction in dominant height was intensified as site quality improved. On very good sites (SI80 and 90), however, actual data indicate little likelihood that very dense


Figure 4.--Relationship of dominant height at age 100 years to CCF greater than 125 for SI classes 40 to 90, from Equation 2. Height growth appeared to be unaffected by density when CCF was 125 or less.
stands (CCF 400 to 600) will occur. Trees on those sites appear able to express dominance and thin themselves effectively at an early age. On poor sites (SI 40 and 50), there is relatively little reduction in height growth in stands of high CCF because there is limited potential for height growth at any density.

## Determining the Shape of the

 Height-Growth CurveThe first step in determining the effect of density and site index on the height-age relationship was to find a mathematical expression of the height-age function.

The curve of height-age is sigmoid, with the point of inflection usually occurring early in the life of the tree. Since the range of ages used in preparing site-index curves is generally above the inflection age, however, the height-age relationship can be described satisfactorily by a smooth curve of positive but decreasing slope.

A number of equations have been used to fit height-age data. Schumacher (1939) suggested the equation form for a sigmoid curve as:

## Log height $=\mathrm{a}-\mathrm{b} /$ age

Some form of that equation generally has been flexible enough to fit similar data (Lynch 1958, Stage 1963).

After screening the height-growth curves on the individual plots, Schumacher's (1939) equation was selected in the first attempt to fit a mathematical expression to the data. The 176 plots that formed the basis of the remaining analyses were sorted into site index-CCF classes. The equation was solved for a number of classes, but the estimated curves did not fit actual data. The equation consistently underestimated actual height beyond age 120 years.

A number of other height-age functions were examined graphically. Of those, a seconddegree curve appeared to best fit the data. The equation:

Height $=a+b$ Age $+c$ Age $^{2}$
was calculated for a number of site indexdensity classes. The curve defined by that equation fitted the actual height-age data closely from age 30 years to age 200 years, although there was a slight tendency for the computed curve to turn down after age 180 .

The use of a polynomial to define the height-age relationship is not a radical departure from accepted procedure (Curtis 1964). The function can be regarded as being of the form:

Height $=\mathrm{a}+\mathrm{b}(\mathrm{f}($ Age $))$
where
$\mathrm{f}($ Age $)=\mathrm{c}($ Age $)+\mathrm{d}\left(\right.$ Age $\left.^{2}\right)$

Determining the Combined Effects of Density and Site Index on the Height-Age Relationship

The next step was to introduce the effect of density and site index into the basic heightage equation. Decadal heights ( 30 to 200 years) obtained from each of the 176 plots were expressed as functions of the several combinations of powers and cross products of the three stand variables, age, site index, and CCF. The coefficients of the following equation were estimated from the data by an IBM 1620 regression program:
$H=b_{0}+b_{1}(A)+b_{2}\left(A^{2}\right)+b_{3}(C C F)$

$$
\begin{aligned}
& +b_{4}(S I)+b_{5}(C C F)(S I)+b_{6}(A)(C C F) \\
& +b_{7}(A)(S I)+b_{8}\left(A^{2}\right)(C C F)+b_{9}\left(A^{2}\right)(S I)
\end{aligned}
$$

where
$\mathrm{H}=$ decadal height at ages from 30 to 200 years
A = decadal age 30 to 200 years
CCF = crown competition factor at total age minus 125
SI = site index at reference age 100 years
$\mathrm{b}_{0}$ to $\mathrm{b}_{9}=$ computed constants
The interaction terms of (A)(SI), (A $\left.{ }^{2}\right)(\mathrm{SI})$, and (CCF)(SI) accounted for 94 percent of the variation in height (table 2). The addition of

Table 2。--Correlation coefficients for nine variable regression analyses of the combined effects on height of age, stand density, and site index

| Number | Variable | $R^{2}$ |
| :---: | :---: | :---: |
| 1 | $($ Age $)(\mathrm{SI})$ | 0.7846 |
| 2 | $1+\left(\mathrm{Age}^{2}\right)(\mathrm{SI})$ | .8905 |
| 3 | $2+(\mathrm{CCF}-125)(\mathrm{SI})$ | .9424 |
| 4 | $3+\left(\mathrm{Age}^{2}\right)$ | .9469 |
| 5 | $4+\left(\mathrm{Age}^{2}\right)$ | .9562 |
| 6 | $5+(\mathrm{CCF}-125)$ | .9577 |
| 7 | $6+\left(\mathrm{Age}^{2}\right)(\mathrm{CCF}-125)$ | .9579 |
| 8 | $7+\left(\mathrm{Age}^{2}\right)(\mathrm{CCF}-125)$ | .9584 |
| 9 | $8+(\mathrm{SI})$ | .9586 |

(A) and ( $A^{2}$ ) raised the amount of variation accounted for to 96 percent. The additional amount of variation accounted for by the addition of all or any one of the remaining four independent variables (A) (CCF), ( $\mathrm{A}^{2}$ )(CCF), (CCF), and (SI) was not important.

The equation with the first five independent variables in table 2 was, therefore, the most practical. However, since that equation accounted for only slightly more variation than equations containing the first three and four independent variables in table 2 , the three equations were tested to determine which equation best fitted the actual data as follows:

1. The basic data from one plot in each cluster, randomly selected to cover the available range in age, site, and CCF, were substituted individually into each of the three equations. Heights estimated by each equation were then plotted separately and compared with the actual height-growth curve for each plot.
2. The equations with three and four independent variables did not fit actual data satisfactorily. Both equations underestimated heights at ages below 100 years and overestimated heights at ages greater than 100 years, for all site classes and CCF's sampled.
3. Height-growth curves estimated by the equation with five independent variables closely approximated actual height-growth curves, or could be made to fit with a minimum of adjustment. Accordingly, the equation for CCF $\geq 125$ :

$$
\begin{aligned}
\text { Height }= & 9.89331-0.19177(\mathrm{~A}) \\
& +0.00124\left(\mathrm{~A}^{2}\right) \\
& -0.00082(\mathrm{CCF})(\mathrm{SI}) \\
& +0.191387(\mathrm{~A})(\mathrm{SI}) \\
& -0.00005\left(\mathrm{~A}^{2}\right)(\mathrm{SI}) \\
R= & 0.98 \\
\text { Basis: } & 704 \text { trees }
\end{aligned}
$$

was selected to provide the basic heightgrowth curves needed for site-index determination.

Height-growth curves can be developed from equation 3 that will estimate site index for all combinations of height, age, and CCF available from original data. Density appears in equation 3 only once, as the interaction term of (CCF)(SI). For stands with a CCF of 125 or less, (CCF)(SI) is set equal to zero, and there is no reduction in height growth due to density. Height-growth curves for CCF 125 or less developed from equation 3 for all siteindex classes are shown in figure 5.

For stands where the CCF is greater than 125 , an increase in density results in a reduction in height growth that is constant for all ages within any site index class. As site quality improves, the reduction in height growth due to an increase in density becomes greater. An increase in CCF, therefore, reduces the differences in height growth between site index classes (fig. 6).

## Testing Initial Assumption That All Plot Data Were From One Population

Preliminary screening of the raw heightage data from the three Regions had indicated that, on the basis of (1) similarity of curve shape, and (2) reduction in dominant height due to density, all data could be considered as from one population. That assumption served


as the basis of the analyses to this point. Before making adjustments in the height-growth curves computed from equation 3 to obtain the final curves, it was necessary to test whether there were any important differences in the height, age, site index, and CCF relations between Regions. Although this reverses the usual procedure, subsequent analyses established the reasonableness of originally grouping the data from the three Regions.

The assumption was made that if all data were from one population, then separate equations of the form of equation 3 could be solved independently for each Region, and the resulting equations would not differ substantially from each other or equation 3. To test the hypothesis, the following equations were computed for each Region:

## 1. Rocky Mountain Region--

$$
\begin{aligned}
\mathrm{H}= & 12.21947-0.21121(\mathrm{~A}) \\
& +0.00115\left(\mathrm{~A}^{2}\right)-0.00077(\mathrm{SI})(\mathrm{CCF}) \\
& +0.01351(\mathrm{~A})(\mathrm{SI})-0.00004\left(\mathrm{~A}^{2}\right)(\mathrm{SI})
\end{aligned}
$$

$R=0.98$
Basis: 356 trees
2. Intermountain Region--
(3b)

$$
\begin{aligned}
\mathrm{H}= & 9.72443-0.23733(\mathrm{~A}) \\
& +0.00160\left(\mathrm{~A}^{2}\right)-0.00091(\mathrm{SI})(\mathrm{CCF}) \\
& +0.01490(\mathrm{~A})(\mathrm{SI})-0.00005\left(\mathrm{~A}^{2}\right)(\mathrm{SI})
\end{aligned}
$$

$R=0.98$
Basis: 244 trees
3. Pacific Northwest Region--
(3c) initial growth was more apparent as density increased. The differences in height growth between Regions indicated that data from Pacific Northwest sources may not be from the same population as Rocky Mountain and Intermountain data. Differences in height growth were largely associated with the younger age classes, however, and since young trees do not provide as reliable an estimate of site index as older trees, the data from Pacific Northwest plots were included in the analyses.

Adjustments of Height-Growth Curves
Two adjustments of the height-growth curves computed from equation 3 were necessary to obtain the final height-growth curves. Those adjustments were first made to the curves of each site-index class for CCF 125 or less.

1. None of the height-growth curves from equation 3 for any site-index class passed through the reference height at the index
$\mathrm{R}=0.99$
Basis: 104 trees
Height-growth curves for all available combinations of age, site, and CCF were computed from each of the equations and compared with height-growth curves computed from equation 3. (Height-age curves for site index 60 and CCF 125 or less are shown in figure 7.) There were no important differences between heightgrowth curves estimated by the Rocky Mountain and Intermountain equations and equation 3 for any combination of site index and CCF.

The height-growth curves estimated by the equation for the Pacific Northwest was of the same general shape as those estimated by the other equations, but height growth was slower in the younger age classes, and the slower
保别

$$
\begin{aligned}
\mathrm{H}= & 3.08159-0.22528(\mathrm{~A}) \\
& +0.00214\left(\mathrm{~A}^{2}\right)-0.00108(\mathrm{SI})(\mathrm{CCF}) \\
& +0.01608(\mathrm{~A})(\mathrm{SI})-0.00006\left(\mathrm{~A}^{2}\right)(\mathrm{SI})
\end{aligned}
$$

Table 3. --Heights of dominant trees at CCF levels of 125 or less for site index classes 30 to 100 by decadal ages 30 to 200 years

| Total age (Years) | Site index class |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
|  |  |  |  |  |  |  |  |  |
| 30 | 16 | 20 | 24 | 28 | 32 | 36 | 40 | 45 |
| 40 | 18 | 23 | 28 | 34 | 39 | 44 | 49 | 55 |
| 50 | 20 | 26 | 32 | 39 | 45 | 51 | 58 | 64 |
| 60 | 22 | 29 | 36 | 44 | 51 | 58 | 65 | 72 |
| 70 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 |
| 80 | 26 | 35 | 44 | 52 | 61 | 70 | 79 | 88 |
| 90 | 28 | 37 | 47 | 56 | 66 | 75 | 85 | 94 |
| 100 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
| 110 | 32 | 42 | 52 | 63 | 73 | 84 | 94 | 104 |
| 120 | 34 | 44 | 55 | 66 | 76 | 87 | 98 | 108 |
| 130 | 35 | 46 | 57 | 68 | 79 | 90 | 101 | 111 |
| 140 | 37 | 48 | 59 | 70 | 81 | 92 | 103 | 114 |
| 150 | 39 | 50 | 61 | 72 | 83 | 94 | 105 | 116 |
| 160 | 40 | 51 | 62 | 73 | 84 | 96 | 107 | 118 |
| 170 | 42 | 53 | 64 | 75 | 86 | 97 | 108 | 119 |
| 180 | 43 | 54 | 65 | 76 | 87 | 99 | 110 | 120 |
| 190 | 45 | 56 | 67 | 78 | 89 | 100 | 111 | 122 |
| 200 | 46 | 57 | 68 | 79 | 90 | 101 | 112 | 123 |

age, although they were very close. It was necessary, therefore, to either raise or lower the height curves slightly. That was done by adjusting each height-growth curve proportionately, based on the difference between height estimated at age 100 years and the reference height at age 100 years.
2. The height-growth curves of the higher site-index classes had a tendency to turn down in the upper portion. That was due in part to the lack of data beyond ages 120 to 150 years in those site-index classes, and in part to the natural shape of the seconddegree curve. In site-index classes 40 to 60 , which contained all the plots 200 years old, the curves in the upper portion overestimated actual heights. To adjust the upper portions of the site curves to provide a better fit, the height-growth curves beyond age 150 years for site-index classes 40 to 60 were fitted to actual data. The interval between the curves was then adjusted proportionately. The shape of those curves and the interval between them was then used to determine the shape of the curves beyond age 150 years for site-index classes 70 to 90.

Table 3 and the curves in figure 8 are the adjusted height-growth curves for stands where height growth is unaffected by density (CCF $\leq 125$ ). The height-growth curves for site-index classes 30 and 100 were added by extrapolation. To obtain the reduction in height growth due to density within each site-index class, table 4 was prepared. The values in table 4 were obtained to the nearest 0.5 foot

Table 4.--Reductions in height due to density for each site index class

| SI | CCF |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | 200 | 300 | 400 | 500 |
| - - - Feet - - - - |  |  |  |  |
| 30 | 2.0 | 4.5 | 7.0 | 9.0 |
| 40 | 2.5 | 6.0 | 9.0 | 12.5 |
| 50 | 3.0 | 7.0 | 11.0 | 15.5 |
| 60 | 3.5 | 8.5 | 13.5 | 18.5 |
| 70 | 4.5 | 10.0 | 16.0 | 21.5 |
| 80 | 5.0 | 11.5 | 18.0 | 24.5 |
| 90 | 5.5 | 13.0 | 20.0 | 27.5 |
| 100 | 6.0 | 14.5 | 22.5 | 31.5 |



Figure 8.--
Adjusted height growth curves for CCF 125 or less, all SI classes.
from the 0.00082 (CCF)(SI) term in equation 3. Decadal heights for CCF 200, 300, 400, and 500 for ages 30 years to 200 years were then determined for each site-index class by subtracting the appropriate value in table 4 from each decadal height of that site-index class in table 3. The height-growth curves for CCF 200, 300, 400, and 500 for site-index classes 30 to 100 are presented in tabular form in tables 5, 6, 7, and 8 .

## Test of Adjusted Height-Growth Curves

The following procedure was used to determine how well the adjusted height-growth curves estimated true site index for all combinations of site, age, and density:

1. True site index at age 100 years was estimated from the adjusted height-growth curves for each of the 176 plots used in developing equation 3. The CCF of each plot determined the appropriate table to enter. Site index was then estimated from the measurements of height at age 100 years in the conventional manner.
2. True site index at ages 30 years to actual age by 10 -year intervals was then estimated for all plots in a similar manner.
3. The differences between site index estimated at age 100 years and site index
estimated at ages 30 to 90 years and ages 110 to 200 years were recorded for each decadal age.
4. The mean differences from site index at age 100 years for all plots by decadal ages were computed (fig. 9).

The mean differences in site index estimated at age 100 years and site index estimated at ages 110 to 200 years were not greater than 1 foot. Site index estimated at ages 40 to 90 years differed from site index estimated at age 100 years by not more than 2 feet, but at age 30 years the mean difference was more than 4 feet. The substantially greater difference at age 30 years was due largely to the slow early growth of trees on plots from the Pacific Northwest Region. Since young trees seldom provide an accurate measure of site index, however, and there was no pattern in the differences associated with site or density, the adjusted curves were accepted as the final curves. In the Rocky Mountain and Intermountain Regions, site index can be estimated from the heightgrowth curves in figure 8 or from tables 3,5 , 6,7 , and 8 for all combinations of height, age, and CCF indicated. In the Pacific Northwest Region, site index can be reliably estimated only if the "site trees" are 50 or more years old.


Table 5. --Heights of dominant trees at CCF 200 for site index classes 30 to 100 by decadal ages 30 to 200 years

| Total age (Years) | Site index class |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
|  |  |  |  |  |  |  |  |  |
| 30 | 14 | 18 | 21 | 25 | 28 | 32 | 35 | 38 |
| 40 | 16 | 21 | 25 | 30 | 35 | 39 | 44 | 48 |
| 50 | 18 | 24 | 29 | 35 | 41 | 46 | 52 | 58 |
| 60 | 20 | 27 | 33 | 40 | 47 | 53 | 60 | 66 |
| 70 | 22 | 30 | 37 | 45 | 52 | 59 | 67 | 74 |
| 80 | 24 | 32 | 41 | 49 | 57 | 65 | 73 | 81 |
| 90 | 26 | 35 | 44 | 53 | 62 | 70 | 79 | 88 |
| 100 | 28 | 37 | 47 | 56 | 66 | 75 | 84 | 94 |
| 110 | 30 | 40 | 49 | 59 | 69 | 79 | 88 | 98 |
| 120 | 32 | 42 | 52 | 62 | 72 | 82 | 92 | 102 |
| 130 | 33 | 44 | 54 | 64 | 74 | 85 | 95 | 105 |
| 140 | 35 | 45 | 56 | 66 | 77 | 87 | 98 | 108 |
| 150 | 37 | 47 | 58 | 68 | 79 | 89 | 99 | 110 |
| 160 | 38 | 49 | 59 | 70 | 80 | 91 | 101 | 112 |
| 170 | 40 | 50 | 61 | 71 | 82 | 92 | 103 | 113 |
| 180 | 41 | 52 | 62 | 73 | 83 | 94 | 104 | 114 |
| 190 | 43 | 53 | 64 | 74 | 84 | 95 | 105 | 116 |
| 200 | 44 | 54 | 65 | 75 | 86 | 96 | 106 | 117 |

Table 6.--Heights of dominant trees at CCF 300 for site index classes 30 to 100 by decadal ages 30 to 200 years

| Total age <br> (Years) | Site index class |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
|  |  |  |  |  |  |  |  |  |
| 30 | 12 | 14 | 17 | 20 | 22 | 25 | 28 | 30 |
| 40 | 14 | 17 | 21 | 25 | 29 | 33 | 36 | 40 |
| 50 | 16 | 21 | 25 | 30 | 35 | 40 | 45 | 50 |
| 60 | 18 | 24 | 29 | 35 | 41 | 47 | 52 | 58 |
| 70 | 20 | 26 | 33 | 40 | 46 | 53 | 59 | 66 |
| 80 | 22 | 29 | 36 | 44 | 51 | 59 | 66 | 73 |
| 90 | 24 | 32 | 40 | 48 | 55 | 64 | 72 | 80 |
| 100 | 26 | 34 | 43 | 51 | 60 | 69 | 77 | 86 |
| 110 | 27 | 36 | 45 | 54 | 63 | 72 | 81 | 90 |
| 120 | 29 | 38 | 48 | 57 | 56 | 75 | 85 | 94 |
| 130 | 31 | 40 | 50 | 59 | 69 | 78 | 88 | 97 |
| 140 | 33 | 42 | 52 | 61 | 71 | 81 | 90 | 100 |
| 150 | 34 | 44 | 54 | 63 | 73 | 83 | 92 | 102 |
| 160 | 36 | 46 | 56 | 65 | 75 | 84 | 94 | 103 |
| 170 | 38 | 47 | 57 | 66 | 76 | 86 | 95 | 105 |
| 180 | 39 | 49 | 58 | 68 | 77 | 87 | 97 | 106 |
| 190 | 40 | 50 | 59 | 69 | 79 | 88 | 98 | 108 |
| 200 | 41 | 51 | 61 | 70 | 80 | 90 | 99 | 109 |

Table 7. --Heights of dominant trees at CCF 400 for site index classes 30 to 100 by decadal ages 30 to 200 years

| Total age <br> (Years) | Site index class |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
| $\ldots \ldots$ - - - - - - |  |  |  |  |  |  |  |  |
| 30 | 9 | 11 | 13 | 15 | 17 | 18 | 20 | 22 |
| 40 | 11 | 14 | 17 | 20 | 23 | 26 | 29 | 32 |
| 50 | 13 | 17 | 21 | 25 | 29 | 33 | 37 | 41 |
| 60 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
| 70 | 17 | 23 | 29 | 35 | 40 | 46 | 52 | 58 |
| 80 | 19 | 26 | 32 | 39 | 45 | 52 | 59 | 65 |
| 90 | 21 | 28 | 36 | 43 | 50 | 57 | 64 | 72 |
| 100 | 23 | 31 | 39 | 46 | 54 | 62 | 70 | 77 |
| 110 | 25 | 33 | 41 | 49 | 57 | 66 | 74 | 82 |
| 120 | 27 | 35 | 44 | 52 | 60 | 69 | 77 | 86 |
| 130 | 28 | 37 | 46 | 54 | 63 | 72 | 80 | 89 |
| 140 | 30 | 39 | 48 | 56 | 65 | 74 | 83 | 92 |
| 150 | 32 | 41 | 49 | 58 | 67 | 76 | 85 | 94 |
| 160 | 34 | 42 | 51 | 60 | 69 | 78 | 86 | 95 |
| 170 | 35 | 44 | 53 | 61 | 70 | 79 | 88 | 97 |
| 180 | 36 | 45 | 54 | 63 | 72 | 81 | 89 | 98 |
| 190 | 38 | 47 | 55 | 64 | 73 | 82 | 91 | 99 |
| 200 | 39 | 48 | 57 | 65 | 74 | 83 | 92 | 101 |

Table 8. --Heights of dominant trees at CCF 500 for site index classes 30 to 100 by decadal ages 30 to 200 years

| $\begin{gathered} \text { Total age } \\ \text { (Years) } \end{gathered}$ | Site index class |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
|  | - - - - - - - Height in feet $\ldots$ - $-\ldots \ldots$ |  |  |  |  |  |  |  |
| 30 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 40 | 9 | 11 | 13 | 15 | 17 | 20 | 22 | 24 |
| 50 | 11 | 14 | 17 | 20 | 24 | 27 | 30 | 33 |
| 60 | 13 | 17 | 21 | 25 | 29 | 33 | 38 | 42 |
| 70 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
| 80 | 17 | 23 | 28 | 34 | 40 | 45 | 51 | 57 |
| 90 | 19 | 25 | 32 | 38 | 44 | 51 | 57 | 63 |
| 100 | 21 | 28 | 35 | 42 | 48 | 55 | 62 | 69 |
| 110 | 23 | 30 | 37 | 44 | 52 | 59 | 66 | 74 |
| 120 | 24 | 32 | 40 | 47 | 55 | 62 | 70 | 78 |
| 130 | 26 | 34 | 42 | 49 | 57 | 65 | 73 | 81 |
| 140 | 28 | 36 | 44 | 52 | 59 | 67 | 75 | 83 |
| 150 | 29 | 37 | 45 | 53 | 61 | 69 | 77 | 85 |
| 160 | 31 | 39 | 47 | 55 | 63 | 71 | 79 | 87 |
| 170 | 33 | 41 | 49 | 57 | 65 | 73 | 81 | 89 |
| 180 | 34 | 42 | 50 | 58 | 66 | 74 | 82 | 90 |
| 190 | 35 | 43 | 51 | 59 | 67 | 75 | 83 | 91 |
| 200 | 36 | 44 | 52 | 60 | 68 | 76 | 84 | 92 |

## Field Application of Site-Index Curves

Detailed instructions for field application of the site-index curves have been published (Alexander 1966).

In addition to the conventional measurements of height and age, a measure of stand density, expressed as CCF, is needed to determine site index for lodgepole pine. CCF can be computed directly, either from equation 2 or from the sum of MCA values for each diameter class as shown in table 9. However, since CCF contains both the sums of diameters and the sums of diameterssquared, the following equation was computed so that CCF could also be estimated indirectly from measurements of basal area and average
diameter (fig. 10):
$\mathrm{CCF}=50.58+5.25(\mathrm{BA} / \overline{\mathrm{D}})$

$$
\begin{equation*}
r=0.98 \tag{4}
\end{equation*}
$$

where
CCF = crown competition factor
$\mathrm{BA}=$ basal area
$\mathrm{D}=$ average diameter

Once CCF has been determined, site index can be estimated in the conventional manner from measurements of height and age by entering the appropriate height-growth curve or table. Total age was used in the development of the site-index curves, but total age can be approximated from age at breast height by adding 9 years to the latter.

Table 9. --MCA values for each diameter class, and sample computations of total MCA and CCF for a lodgepole pine stand

| Diameter <br> class | MCA ${ }^{1}$ per tree | Number of trees | Total MCA | Sample computations |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 0.040 |  |  | Plot size $=0.8$ acre |
| 2 | . 067 |  |  |  |
| 3 | . 102 |  |  |  |
| 4 | . 145 |  |  |  |
| 5 | . 194 |  |  | Total MCA $=159.6$ |
| 6 | . 251 |  |  |  |
| 7 | . 315 | 23 | 7.245 | $\mathrm{CCF}=\frac{\text { Total MCA }}{\text { Area in acres }}$ |
| 8 | . 387 | 41 | 15.867 |  |
| 9 | . 466 | 37 | 17.242 |  |
| 10 | . 552 | 47 | 25.944 | $C C F=\frac{159.6}{0.8}=199.5$ |
| 11 | . 645 | 50 | 32.250 |  |
| 12 | . 746 | 30 | 22.380 |  |
| 13 | . 854 | 24 | 20.496 |  |
| 14 | . 969 | 12 | 11.628 |  |
| 15 | 1.092 | 6 | 6.552 |  |
|  |  |  | 159.604 |  |
| 16 | 1.222 |  |  |  |
| 17 | 1.359 |  |  |  |
| 18 | 1.504 |  |  |  |
| 19 | 1.655 |  |  |  |
| 20 | 1.814 |  |  |  |

${ }^{1}$ Computed from equation, $\mathrm{MCA}=0.00365 \mathrm{D}^{2}+0.01676 \mathrm{D}+0.01925$


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Res. Paper RM-29, $18 \mathrm{pp}$. . illus. Rocky Mountain Forest and Range Experiment Station, Fort Collins, Colorado, 80521.

Methodology used to develop height-age curves for estimating คə estimating site index corrected for stand density are also presented. Site indexes adjusted for stand density were developed from data collected in Colorado, Wyoming, Utah, Idaho, Montana, and eastern Washington and Oregon. Instructions for field use were published in U. S. Forest Service Research Paper RM-24. Alexander, Robert R., Tackle, David, and Dahms, Walter G. 1967. Site indexes for lodgepole pine, with correction stand density: Methodology. U. S. Forest MounRes. Paper RM-29, 18 pp., ilis. Station, Fort Collins, Colorado, 80521.

Methodology used to develop height-age curves for estimating site index of lodgepore fand density are also presented. estimating site index corrected for stand density are also presented. Site indexes adjusted for stand density were developed from data ern Washington and Oregon. Instructions for field use were published in U. S. Forest Service Research Paper RM-24.



[^0]:    ${ }^{3}$ The CCF's of 21 plots were below 125. Those plots were, therefore, the only actual control plots in the study. Eighteen additional plots in the CCF range of 125-150 were also used to assign true site index to plots of greater density.

