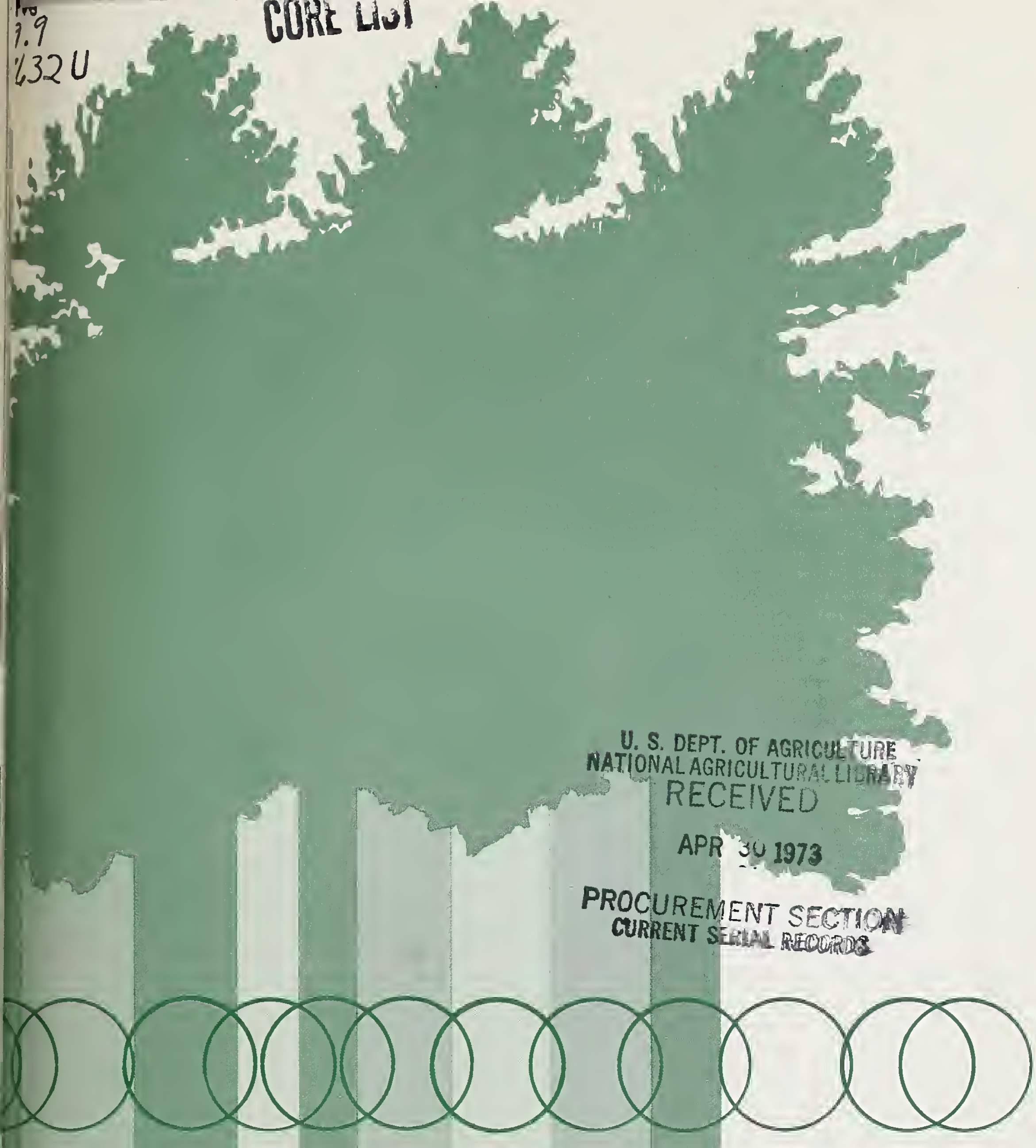


FOREST STOCKING EQUATIONS: Their Development and Application

by Peter F. Ffolliott
and David P. Worley

March 1973

USDA Forest Service
Research Paper RM-102
Rocky Mountain Forest
and Range Experiment Station
Forest Service
U.S. Department of Agriculture

Abstract

Using point-sampling techniques, stocking conditions at a sample point can be described in terms of whether or not the point is stocked to a minimum basal area level corresponding to a particular basal area factor (BAF). Stocking equations relating proportions of a forest stocked to minimum basal area levels corresponding to each BAF used in an inventory can be defined by regression analyses. Stocking equations can be used to help evaluate land treatment potential, determine treatment feasibility on a single management unit, and as a basis for setting operating priorities on a number of management units.

Keywords: Stand density, basal area measurement, forest management, forest surveys.

USDA Forest Service
Research Paper RM-102

March 1973

FOREST STOCKING EQUATIONS: Their Development and Application

by
Peter F. Ffolliott, Associate Silviculturist
and
David P. Worley, Principal Economist
Rocky Mountain Forest and Range Experiment Station[1]

[1]Research reported here was conducted at the Station's Research Work Unit located at Flagstaff, in cooperation with Northern Arizona University; Station's central headquarters maintained at Fort Collins, in cooperation with Colorado State University. Ffolliott is currently associate professor, Department of Watershed Management, University of Arizona, Tucson; Worley is with USDA Forest Service Northeastern Forest and Range Experiment Station's Unit at Columbus, Ohio.

Contents

FOREST STOCKING EQUATIONS: Their Development and Application

Peter F. Ffolliott and David P. Worley

Introduction

Point sampling techniques are widely used for inventorying forests. These inventories provide data on average basal area (or volume, number of trees, and so forth) per acre for forest managers. Such inventories may yield additional information regarding the proportions of a forest stand stocked to minimum basal area levels.

Even-aged stands with regular spacing patterns resulting from plantations or extended periods of management can, possibly, be described by average basal area per acre. With uneven-aged stands of irregular spacing patterns, however, another statistic — the proportion of the stand stocked to a minimum basal area level — would be useful to set realistic limits for forestry practices, judge suitability of an area for management practices, or set priorities for cultural or harvesting operations among forest areas. Such information can be derived from stocking equations, as described here.

The purposes of this paper are to: (1) outline the theory behind the development of stocking equations, (2) illustrate the methodology of stocking equation synthesis, and (3) demonstrate applications of stocking equations in forest management decision-making.

Theory Behind Development of Stocking Equations

The basic theory of point sampling is well known. The number of trees tallied at a sample point, multiplied by the basal area factor (BAF) used, gives an estimate of basal area per acre at that sample point. A sample point is stocked to basal area levels of 50, 70, and 100 square feet per acre on the basis of 5, 7, or 10 trees tallied with a BAF of 10. Estimates from a number of sample points are averaged to estimate the basal area of the forest area.

The use of a single BAF can incorrectly describe the stocking situation at a single point, however, due to irregular spacing patterns and the variety of tree sizes frequently associated with natural timber stands (fig. 1). For example, a sample point would be considered stocked to a basal area level of 75 square feet if three trees were tallied with a BAF of 25. But, it is possible that no trees would be tallied with a BAF of 75, in which case the sample point would not be considered stocked to 75 square feet. Conversely, assume one of the three trees tallied with a BAF of 25 is close enough to the sample point to be tallied with a BAF of 100. The tally with a BAF of 25 would underestimate this stocking condition.

Empirical trials in cutover ponderosa pine stands in Arizona corroborate the above examples. Sample points with a single tree

Figure 1.—Irregular spacing patterns and intermixed size classes in Arizona ponderosa pine stands.

tallied with a BAF of 25 were also stocked with a BAF of 50 and 75 half the time. Sample points stocked with two trees with a BAF of 25 were stocked, with a BAF of 50, only 84 percent of the time, and sample points stocked with three trees with a BAF of 25 were stocked with a BAF of 75 only three-quarters of the time.

If it is not possible to correctly describe stocking conditions at a single sample point with a single BAF, it follows that it may not be possible to describe the proportion of a forest stand stocked to arbitrarily specified basal area levels. A more accurate method for determining the proportion of a stand stocked to different basal area criteria is through the use of stocking equations. The synthesis of stocking equations is based on two assumptions.

First, a sample point is considered stocked to a given minimum basal area level if at least one tree is tallied with a BAF corresponding to that level, or not stocked at that level if no trees are tallied. This concept about stocking conditions has been suggested as a way of determining the proportion of a stand stocked to a single minimum basal area level defined by a specified management objective (Roberts 1964). In the absence of specific management guidelines, an inventory system employing a range of BAF's, allowing sample points to be described in terms of being stocked to a corresponding range of minimum basal area levels, is advantageous.

Secondly, the proportion of a forest stand stocked to a given minimum basal area level can be estimated from the proportion of sample points stocked to that minimum level, provided the sampling of stocking conditions was unbiased. Similarly, relationships can be established between proportions of a stand stocked to minimum basal area levels corresponding to each BAF used in the inventory. These relationships assume mathematical forms which can be defined empirically through regression analyses. The equations describing these regressions are stocking equations, the dependent variable being the proportion of a forest stand stocked to minimum basal area levels within limits dictated by the BAF's used in the inventory.

Synthesis of Stocking Equations — An Illustration

To illustrate methodology, stocking equations have been developed to describe cutover and virgin ponderosa pine *(Pinus ponderosa* Laws.) stands in north-central Arizona.

Study Areas

Data representing a cutover ponderosa pine stand were collected from watershed 12, encompassing 425 acres, on the Beaver Creek watershed (Brown 1971), 45 miles south of Flagstaff. At the time of measurement, half of the merchantable sawtimber volume had been cut from the area between 1943 and 1950. Sawtimber volume averaged 3,700 board feet per acre, and the site index (Meyer 1961) varied from 45 to 60 feet at 100 years. Soils, derived from basalt parent material, are classified in the Brolliar and Siesta-Sponseller soil management areas (Williams and Anderson 1967). The area was sampled with 197 points arranged in four random starts with four strata (Shiue 1960).

Data from a virgin ponderosa pine stand were collected on the Long Valley Experimental Forest, 65 miles southeast of Flagstaff. At the time of the study, this was one of the few remaining areas in Arizona where a virgin stand could still be found on a good timber-growing site. Timber on the 1,280 acres comprising the Forest was uneven-aged, with different age classes occurring as small, even-aged groups. Sawtimber volume averaged 20,500 board feet per acre, and the site index (Meyer 1961) was 85 to 90 feet. Soils, formed from limestone and sandstone, are classified in the Hogg-McVickers series (Anderson *et al.* 1963). One hundred sixty-six points arranged in an 8-chain by 8-chain grid provided the sample design here.

Methods

Sample points on both study areas were considered stocked or not stocked on the basis of trees tallied with an angle gage corresponding to BAF's of 5, 10, 25, 50, 75, 100, 125, 150, 175, 200, and 250. Diameters (d.b.h.,o.b.) of all tallied trees were recorded to allow subsequent assignment into size classes.

The data were subjected to regression analyses to develop stocking equations describing (a) all size classes and (b) individual size classes on the two study areas. Linear regressions had been used previously to describe the proportions of a cutover ponderosa pine stand stocked to minimum basal area levels between 25 and 75 square feet (Ffolliott and Worley 1965). Here, straight-line prediction mechanisms performed well over a limited range of basal area levels. More complex curvilinear forms were required, however, for regressions that defined stocking conditions near the extremes of a stand population — low or high stocking levels for all timber size class elements within a stand — or for particular size-class components. Furthermore, several scatter diagrams revealed an inflection point near the Y-axis. Consequently, commonly used linear transformations proved unsatisfactory.

A computer program (Jameson 1967) that approximates a 5-parameter transition growth curve describing general sigmoidal relationships (Grosenbaugh 1965) was arbitrarily selected to illustrate the development of stocking equations. Other mathematical models and computer programs designed to produce a sigmoid form might give similarly good results. As a practical matter, the closeness of fit to the data illustrated in figure 2 suggests hand plotting may be adequate for developing curves for many purposes.

Results

Stocking equations describing all size classes of ponderosa pine on the two study areas are illustrated in figure 2. Stocking equations developed for the sawtimber (at least 11.0 inches diameter), pole (4.0 to 10.9 inches diameter), and sapling (less than 4.0 inches diameter) size classes (fig. 3) are summarized as follows:

Beaver Creek (watershed 12)

(1) Sawtimber

$$Y = 100 - 89.1\ (1-e^{-0.019x})^{1.00}$$

(2) Poles

$$Y = 100 - 89.2\ (1-e^{-0.019x})^{0.75}$$

(3) Saplings

$$Y = 100 - 103.8\ (1-e^{-0.009x})^{0.25}$$

Long Valley Experimental Forest

(1) Sawtimber

$$Y = 100 - 24.1\ (1-e^{-0.001x})^{0.75}$$

(2) Poles

$$Y = 100 - 96.5\ (1-e^{-0.008x})^{0.25}$$

(3) Saplings

$$Y = 100 - 101.1\ (1-e^{-0.023x})^{0.25}$$

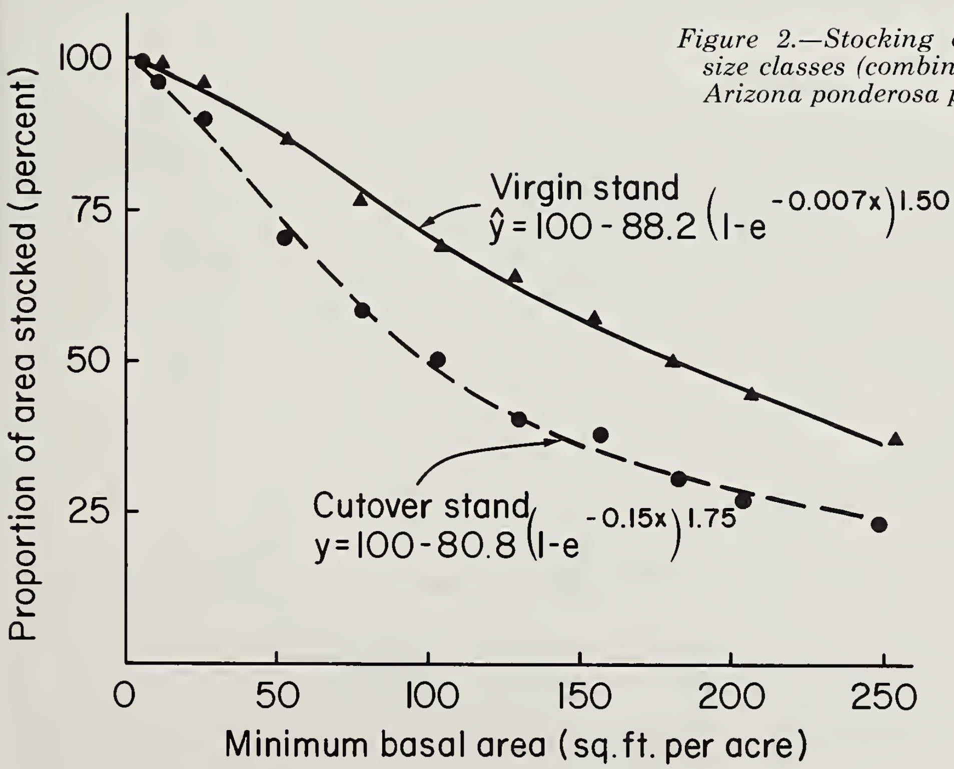


Figure 2.—Stocking equations describing all size classes (combined) in virgin and cutover Arizona ponderosa pine stands.

Summation of the three size-class components at a given basal area level may exceed the stand population stocking at that level, since many sample points were stocked with more than a single size class.

The stocking equations allow us to estimate the proportion of a forest stand stocked by a stand element to minimum basal area levels up to 250 square feet. Solving stocking equations for numerous alternative basal area levels may

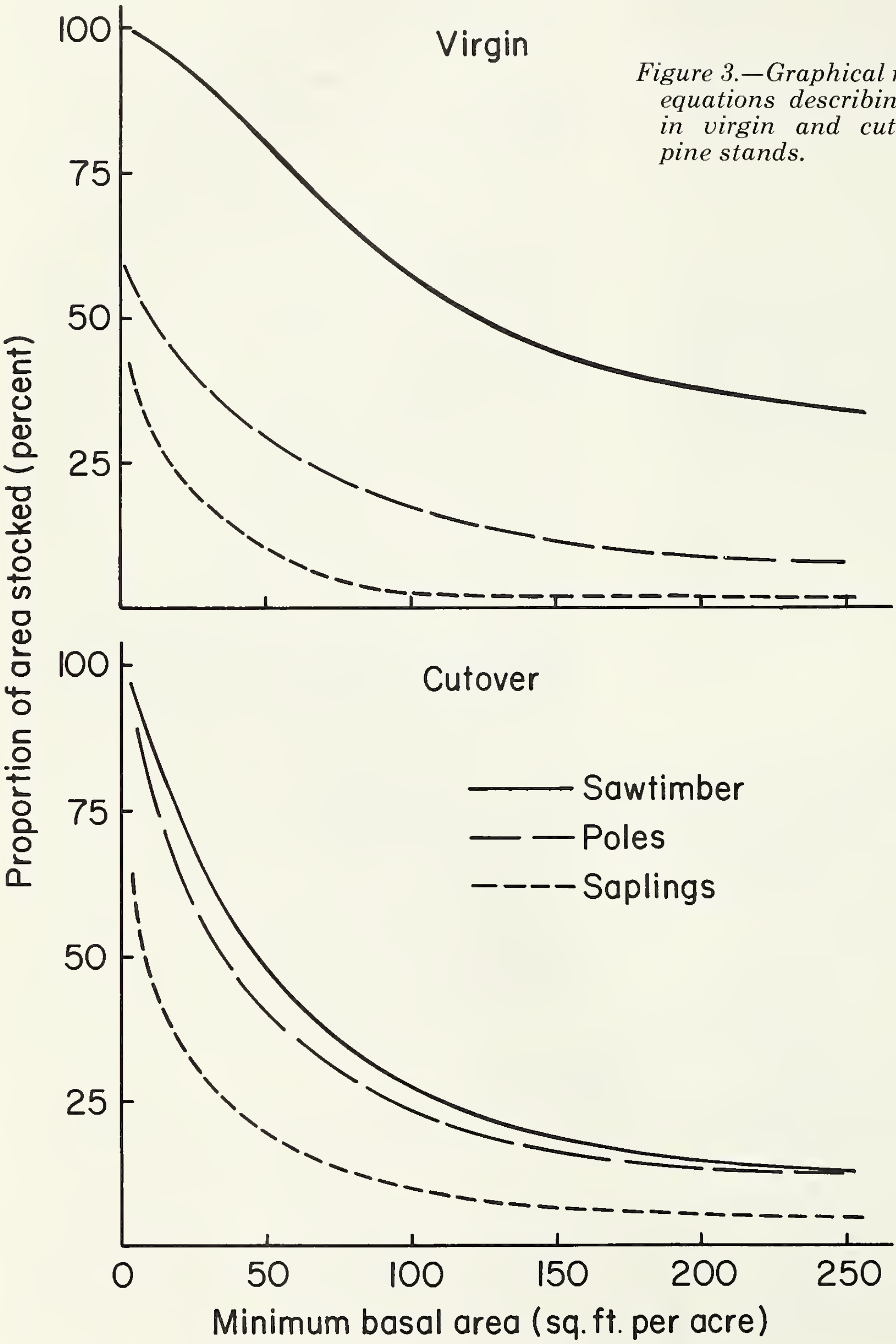


Figure 3.—Graphical representation of stocking equations describing individual size classes in virgin and cutover Arizona ponderosa pine stands.

become time consuming. To ease computations, a supplementary computer program can be written to solve equations in terms of the proportions of a stand stocked to any intermediate basal area. Generally, estimates obtained from a graphical presentation of the stocking equation will suffice.

Applications of Stocking Equations

A truly adequate description of the characteristics of timber on a management unit must answer a variety of questions of management specialists regarding timber production. Stocking equations help provide such answers, by defining the proportion of a forest stand on a management unit stocked to minimum basal area levels dictated by management objectives.

Setting Realistic Limits for Forestry Practices

Stocking equations can help a manager reach a decision as to the feasibility of imposing a treatment (such as harvesting, thinning, and so forth) on a management unit. It is assumed that the proportion of a forest stand stocked to a minimum basal area level which corresponds to the basal area level prescribed by treatment will, subsequently, represent the proportion of the stand that will be placed under treatment.

For example, suppose a silvicultural practice calls for a uniform thinning of all sawtimber in a forest stand to a basal area level of 50 square feet per acre, the assumed "optimum" in terms of a sawtimber management potential. However, a stocking equation developed for the management unit may reveal only 43 percent of the stand could meet the treatment stocking objective (fig. 4). A decision may then need to be made regarding treatment feasibility. Possibly, the original prescription could be discarded in favor of one that would place a larger proportion of the stand on the management unit under treatment. This could be achieved by reducing the uniform thinning treatment to 25 square feet per acre. Unfortunately, thinning to this alternative stocking level may result in a lower sawtimber management potential. Due to the greater proportion of the area treated (fig. 4), however, the outcome could be more favorable in the long run. The final decision must be a compromise between obtaining the maximum management potential,

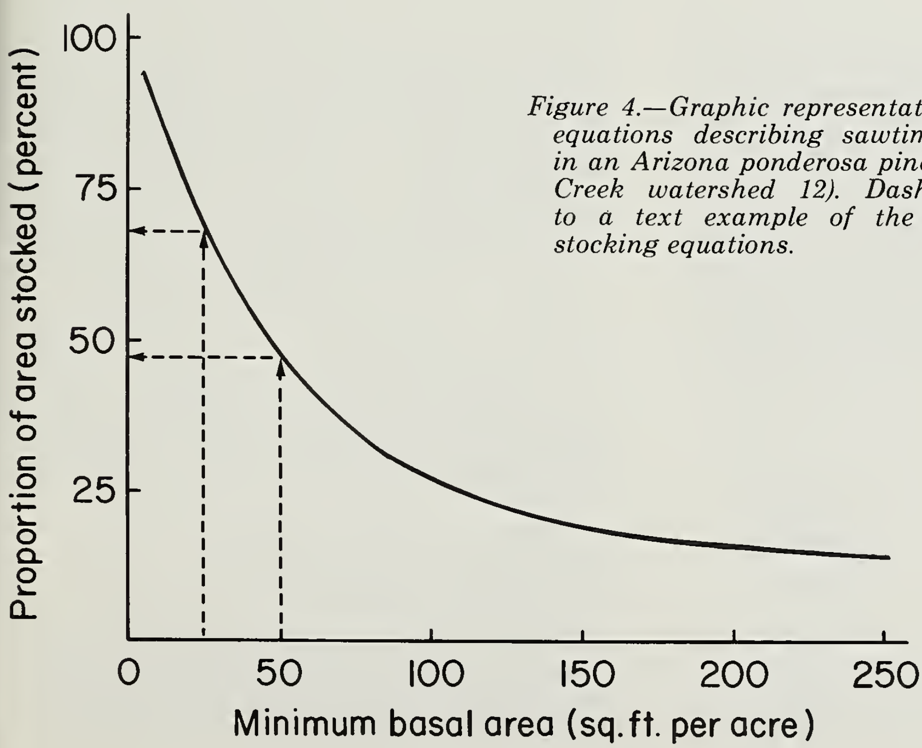


Figure 4.—Graphic representation of stocking equations describing sawtimber size class in an Arizona ponderosa pine stand (Beaver Creek watershed 12). Dashed lines refer to a text example of the application of stocking equations.

as prescribed by treatment, and extending the treatment to the largest possible proportion of the stand on the management unit.

Regardless of what a specific land treatment is to accomplish, the application of stocking equations will help evaluate treatment potential and prescribe treatment feasibility. A range specialist may ask "What proportion of a management unit is stocked in excess of a given basal area level considered maximum to allow acceptable forage production for allotment management?" An economist interested in costs might ask "How much of a management unit needs to be treated, and to what intensity does the treatment need to be applied, to bring the tract to a prescribed stocking level?" A timber manager might need data describing the extent of merchantable sawtimber to a basal area level considered the minimum for profitable harvesting.

Decisionmaking at Different Levels of Interest

If the "optimum" basal area level criterion for sawtimber management potential is 50 square feet per acre, we saw (fig. 4) that 43 percent of the management unit described above would meet the treatment stocking objective. This elementary type of statistic might be called primary, or at the first level of interest.

If the sampling intensity is great enough, other levels of interest can be exploited from stocking equations. Our timber manager might ask "If 43 percent of the management unit meets the sawtimber treatment prescription, how much of this latter area will be stocked with residual trees of submerchantable size?" This is an example of a secondary level of interest. Source data from the proportion of the management unit that meets the treatment prescription can be subjected to regression or graphic analysis. For our example, the proportion of the management unit that meets the sawtimber treatment prescription stocked with submerchantable ponderosa pine at various minimum basal area levels is:

Basal area of submerchantable ponderosa pine (Sq. ft./acre)	**Cutover area stocked** (Percent)
20	82
40	69
60	59
80	50
100	42

The above information could provide the basis for scheduling planting or determining site preparation costs. For instance, if 60 square feet of basal area per acre is judged satisfactory stocking for advanced regeneration after cutting, 59 percent of the proportion of the management unit that meets the sawtimber treatment prescription is already stocked, and reproduction measures need be planned for 41 percent.

Setting Operating Priorities

The output of stocking equations — the proportion of a management unit stocked by a stand element to a specified criterion — can be used with other information to set management priorities. This can be illustrated by an example.

Ten Beaver Creek watersheds, similar to watershed 12, were inventoried so that stocking equations, and timber volume information, could be developed. Let us rank these watersheds according to the desirability of harvesting ponderosa pine sawtimber to achieve the "best release" of pole-sized ponderosa pine trees and a minimum release of a timber "weed" species, Gambel oak *(Quercus gambelii* Nutt.). Direct information, obtained from the stocking equations, is the proportion of each watershed stocked with pole-sized ponderosa pine, and the proportion of each watershed stocked with Gambel oak. Selected criteria are: (a) a minimum sawtimber cut of 1,000 board feet per acre, (b) at least 25 percent of the watershed stocked with pole-sized ponderosa pine at a minimum basal area level of 50 square feet per acre, and (c) no more than 25 percent of the watershed stocked with Gambel oak at a minimum basal area level of 50 square feet per acre. This information is arrayed in table 1.

The application here combines the area information — the output of stocking equations — with sawtimber volume estimates to determine cutting priorities. Individual watersheds are eliminated from consideration when they do not meet one or more criteria. Those remaining are ranked on a sawtimber volume basis. They could be ranked on any combination of the above three criteria which could be shown to maximize benefits or minimize costs.

Development of Distribution and Density Functions

The relationships defined by stocking equations are exceedance curves, which describe the proportion of a stand stocked to minimum basal area levels. Distribution functions, which

Table 1.--Priorities for harvesting large sawtimber

Watershed	Sawtimber volume	Area stocked at minimum of 50 square feet of--		Feasibility limits (a,b,c)[1] and priority rankings (1,2,3)
		Pole-sized ponderosa pine	Gambel oak	
	Bd.ft./acre	- - - - Percent - - - - -		
A	2,800	29	16	2
B	3,110	38	23	1
C	830	26	13	Insufficient volume
D	1,660	22	26	Insufficient poles; too much oak
E	1,230	29	31	Too much oak
F	2,410	29	22	3
G	1,470	18	38	Insufficient poles; too much oak
H	2,290	16	16	Insufficient poles
I	1,830	27	28	Too much oak
J	570	46	13	Insufficient volume

[1] Limiting criteria: (a) a minimum sawtimber cut of 1,000 board feet per acre; (b) at least 25 percent of the watershed stocked with pole-sized ponderosa pine at a minimum basal area level of 50 square feet per acre; (c) no more than 25 percent of the watershed stocked with Gambel oak at a minimum level of 50 square feet per acre.

describe cumulative frequency, can be readily developed from these relationships if desired. With distribution functions, it would be possible to derive density functions, which define the probabilities of obtaining a small interval of forest stocking considered prerequisite to imposing a land treatment. Estimates of these probabilities can be of value in decisionmaking at the first level of interest.

Summary and Conclusions

1. Using point sampling techniques, stocking conditions at a sample point can be described in terms of whether or not the point is stocked to a minimum basal area level corresponding to a particular BAF.
2. Mathematical relationships between proportions of a forest stand stocked to minimum basal area levels corresponding to each BAF used in an inventory can be defined through regression analyses. The equations describing these regressions are stocking equations; the dependent variable is the proportion of a forest stand stocked to a given minimum basal area level, and the independent variable is the minimum basal area level.
3. Stocking equations describing cutover and virgin ponderosa pine stands in Arizona were developed by means of a computer program that approximates a sigmoidal relationship. These equations define the proportion of these two stands stocked to minimum basal area levels up to 250 square feet per acre.
4. Stocking equations can be used to help evaluate land treatment potential, to determine treatment feasibility on a single management unit, and as a basis for setting operating priorities on a number of management units.
5. To apply this technique, the land manager must have a multiple BAF inventory made for the management unit in question, then prepare stocking equations or graphs similar to those described here.

Literature Cited

Anderson, T. C., A. A. Love, L. D. Wheeler, and J. A. Williams.
1963. Soil management report for Long Valley Ranger District, Coconino National Forest. 97 p. U.S. For. Serv., Albuquerque, N. Mex.

Brown, Harry E.
1971. Evaluating watershed management alternatives. Am. Soc. Civil Eng., J. Irrig. and Drain. Div. 97(IR1): 93-108.

Ffolliott, Peter F., and David P. Worley.
1965. An inventory system for multiple use evaluations. U.S. For. Serv. Res. Pap. RM-17, 15 p. Rocky Mt. For. and Range Exp. Stn., Fort Collins, Colo.

Grosenbaugh, L. R.
1965. Generalization and reparametrization of some sigmoid and other nonlinear functions. Biometrics 21: 708-714.

Jameson, Donald A.
1967. The relationship of tree overstory and herbaceous understory vegetation. J. Range Manage. 20: 247-249.

Meyer, Walter H.
1961. Yield of even-aged stands of ponderosa pine. U.S. Dep. Agric. Tech. Bull. 630, 59 p. (slightly revised).

Roberts, Edward G.
1964. A new insight to point sampling. J. For. 62: 267-268.

Shiue, Cherng-Jiann.
1960. Systematic sampling with multiple random starts. For. Sci. 6: 42-50.

Williams, John A., and Truman C. Anderson, Jr.
1967. Soil survey of Beaver Creek area, Arizona. 75 p. U.S. Dep. Agric., Wash., D. C.

Ffolliott, Peter F., and David P. Worley.
1973. Forest stocking equations: their development and application. USDA For. Serv. Res. Pap. RM-102, 8 p. Rocky Mt. For. and Range Exp. Stn., Fort Collins, Colo.

Using point-sampling techniques, stocking conditions at a sample point can be described in terms of whether or not the point is stocked to a minimum basal area level corresponding to a particular basal area factor (BAF). Stocking equations relating proportions of a forest stocked to minimum basal area levels corresponding to each BAF used in an inventory can be defined by regression analyses. Stocking equations can be used to help evaluate land treatment potential, determine treatment feasibility on a single management unit, and as a basis for setting operating priorities on a number of management units.

Keywords: Stand density, basal area measurement, forest management, forest surveys.

Ffolliott, Peter F., and David P. Worley.
1973. Forest stocking equations: their development and application. USDA For. Serv. Res. Pap. RM-102, 8 p. Rocky Mt. For. and Range Exp. Stn., Fort Collins, Colo.

Using point-sampling techniques, stocking conditions at a sample point can be described in terms of whether or not the point is stocked to a minimum basal area level corresponding to a particular basal area factor (BAF). Stocking equations relating proportions of a forest stocked to minimum basal area levels corresponding to each BAF used in an inventory can be defined by regression analyses. Stocking equations can be used to help evaluate land treatment potential, determine treatment feasibility on a single management unit, and as a basis for setting operating priorities on a number of management units.

Keywords: Stand density, basal area measurement, forest management, forest surveys.

Ffolliott, Peter F., and David P. Worley.
1973. Forest stocking equations: their development and application. USDA For. Serv. Res. Pap. RM-102, 8 p. Rocky Mt. For. and Range Exp. Stn., Fort Collins, Colo.

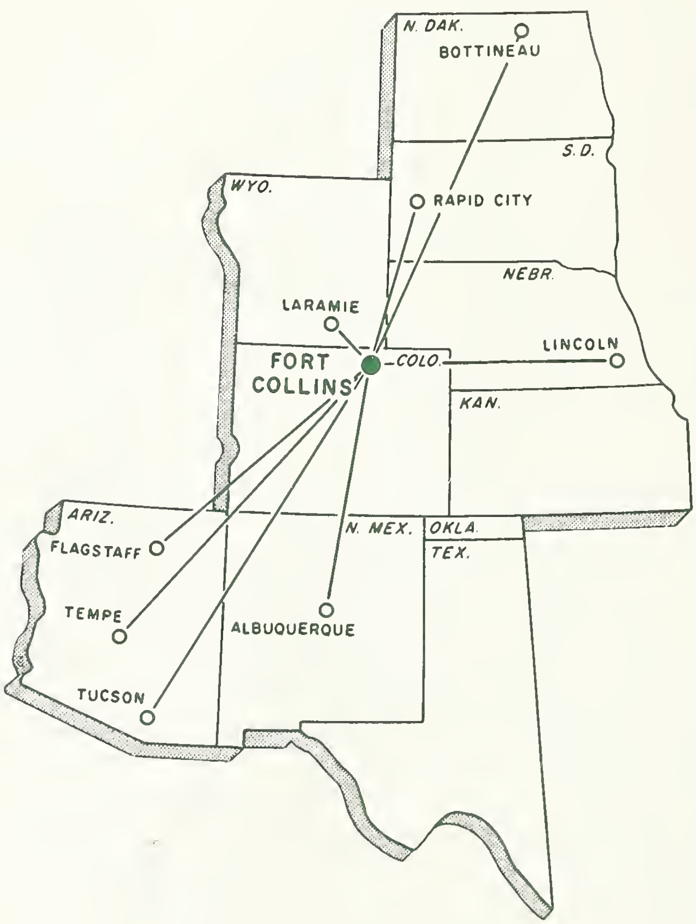
Using point-sampling techniques, stocking conditions at a sample point can be described in terms of whether or not the point is stocked to a minimum basal area level corresponding to a particular basal area factor (BAF). Stocking equations relating proportions of a forest stocked to minimum basal area levels corresponding to each BAF used in an inventory can be defined by regression analyses. Stocking equations can be used to help evaluate land treatment potential, determine treatment feasibility on a single management unit, and as a basis for setting operating priorities on a number of management units.

Keywords: Stand density, basal area measurement, forest management, forest surveys.

Ffolliott, Peter F., and David P. Worley.
1973. Forest stocking equations: their development and application. USDA For. Serv. Res. Pap. RM-102, 8 p. Rocky Mt. For. and Range Exp. Stn., Fort Collins, Colo.

Using point-sampling techniques, stocking conditions at a sample point can be described in terms of whether or not the point is stocked to a minimum basal area level corresponding to a particular basal area factor (BAF). Stocking equations relating proportions of a forest stocked to minimum basal area levels corresponding to each BAF used in an inventory can be defined by regression analyses. Stocking equations can be used to help evaluate land treatment potential, determine treatment feasibility on a single management unit, and as a basis for setting operating priorities on a number of management units.

Keywords: Stand density, basal area measurement, forest management, forest surveys.


N. DAK.
BOTTINEAU
S.D.
WYO.
RAPID CITY
NEBR.
LARAMIE
LINCOLN
FORT COLLINS
COLO.
KAN.
ARIZ.
FLAGSTAFF
N. MEX.
OKLA.
TEX.
TEMPE
ALBUQUERQUE
TUCSON