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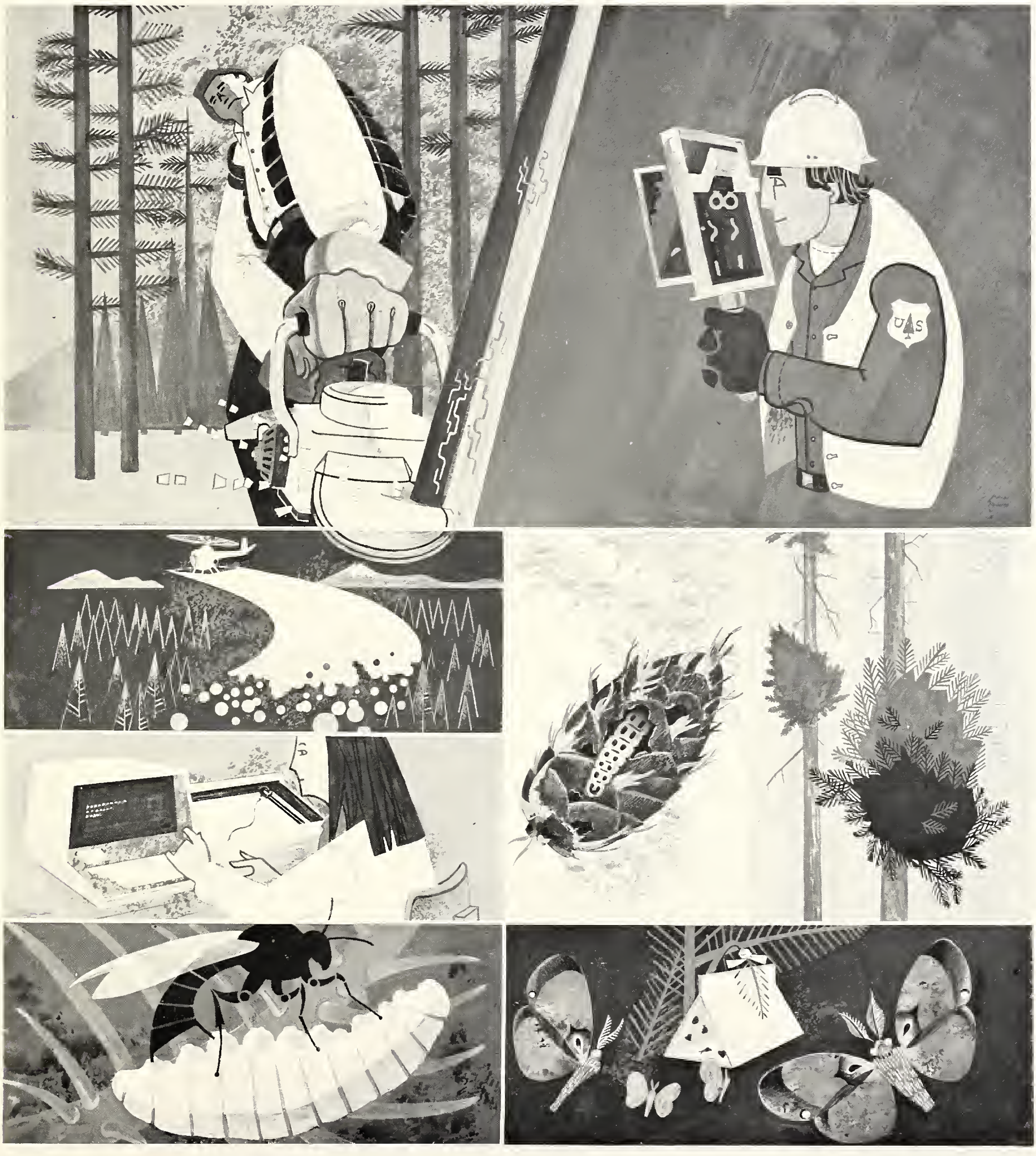
# SAMPLING DESIGNS AND ALLOCATIONS YIELDING MINIMUM COST ESTIMATORS FOR MOUNTAIN PINE BEETLE LOSS ASSESSMENT SURVEYS



Report No. 83-3

3400
November 1982

# SAMPLING DESIGNS AND ALLOCATIONS YIELDING MINIMUM COST ESTIMATORS FOR MOUNTAIN PINE BEETLE LOSS ASSESSMENT SURVEYS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## ABSTRACT


*Tables of optimum sample size for each of three stages are presented for estimating mountain pine beetle loss in ponderosa and lodgepole pine forests in the western United States. These are listed for varying levels of precision and are based on data collected during surveys conducted between 1977 and 1980.*


## INTRODUCTION

Information from previous mountain pine beetle loss assessment surveys has provided the opportunity to improve future sampling designs and allocations. The material for this study was obtained from several surveys of ponderosa and lodgepole pine mortality that were conducted by the Forest Service in the western Regions between 1977 and 1980. These surveys provided a sufficient data base from which variance components were estimated, costs were assessed and various strategies could be evaluated.

The procedures used in the previous surveys and their results have been documented in various reports (Hostetler and Young 1979, Bennett and Bousfield 1978, Bennett et al. 1980, and Lister and Young 1981). The steps involved:

1. Aerial sketchmapping.
2. Stratification based on intensity of mortality per acre as determined from the sketchmapping.
3. Random sample of aerial photos within each stratum (Stage 1).
4. Subsample of aerial photos chosen with probabilities proportional to photo interpreted dead tree counts (Stage 2).
5. Sample of ground plots selected with probabilities proportional to dead tree counts (Stage 3).

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The objectives of this study were to evaluate whether or not stratification is effective, and to find those allocations of sampling units between the three stages which costs the least for a fixed percent standard error. The 20% standard error of the estimate is a requirement specified in the Forest Insect and Disease Information System Implementation Plan (FIDIS) (Ciesla and Yasinski 1980). We also considered the optimum allocation for 10%, 15% and 25% standard errors whenever these were attainable. We were constrained to considering 40 and 90 acre photo plots and 2.5 acre ground plots since all of the usable data available from the previous surveys fell into these categories. We considered the effects of taking one, two or three ground plots per photo plot in the third sampling stage where traditionally two plots have been taken.

## METHODS

We used data from four previous mountain pine beetle surveys (Table 1). Cost factors applicable to the analysis were obtained from data furnished by Dayle Bennett (personal communication) in Region 3 and confirmed by Richard Myhre of the Rocky Mountain Forest and Range Experiment Station, Ft. Collins, Colorado. These costs are in units of the number of person hours to do photo interpretation and obtain ground measurements (Table 2). The cost estimates include plot set-up time and travel time. As expected, the time required to make a ground measurement is significantly higher than to interpret a photo, so that it is evident that a major cost element in sampling is associated with the time spent on the ground.

For each of the previous surveys, the variance components associated with each of the three stages were computed. From these it was possible to estimate the variance which would have resulted for any allocation of sampling units of interest. The estimate of total variance was computed using the following formula:

$$v(y) = (N-n/N)(1/n)v(y_1) + (1/m)v(y_2) + (1/ml)v(y_3)$$

in which

$y$ = the estimate of total mortality

$v(y)$ = the estimate of the variance of the estimate of total mortality

$v(y_i)$ = the estimate of stage i variance

$N$ = the total number of possible photo plots in the stratum

$n$ = the number of photo plots to be sampled at Stage 1

$m$ = the number of photo plots to be subsampled at Stage 2

$l$ = the number of ground plots per photo plot at Stage 3

TABLE 1. Summary of Mountain Pine Beetle data sets used in generating estimates of variances for this study.

| SURVEY | TREE SPECIES | STRATUM | | :number of photos (Stage 1) | :photo plot size (acres) | :number photos subsampled (Stage 2) |
|---|---|---|---|---|---|---|
| 1978 Beaverhead/ Gallatin | LP | L | ≤ 4.9 trees/acre | 94 | 40 | 24 |
| | | M | 5.0-9.9 | 24 | | 16 |
| | | H | ≥ 10.0 | 26 | | 16 |
| 1979 Montana | LP | L | <10.0 trees/acre | 174 | 40 | 18 |
| | | M | 10.0-41.0 | 87 | | 19 |
| | | H | >41.0 | 81 | | 20 |
| 1978 Black Hills | PP | M | | 109 | 90 | 50 |
| | | H | | 77 | | 50 |
| 1979 Colorado Front Range | PP | Contiguous | | 175 | 90 | 20 |

TABLE 2. Cost per plot for photo interpretation and ground measurements (in person hours)

| | | Photo Plot Size | | | Ground Plot Size |
|---|---|---|---|---|---|
| Host Species | Stratum | 40 Acres | 62.5 Acres | 90 Acres | 2.5 Acres |
| LP | L | 0.39 | 0.53 | 0.73 | 10.60 |
| | M | 0.48 | 0.66 | 0.85 | 12.40 |
| | H | 1.04 | 1.11 | 1.40 | 13.10 |
| PP | L | 0.33 | 0.39 | 0.49 | 10.60 |
| | M | 0.40 | 0.51 | 0.63 | 12.40 |
| | H | 0.65 | 0.81 | 1.01 | 13.10 |

The percent standard error of the estimate is computed as 100 v(y)/y. Formulas for computing variance components are given in the appendix.

A computer program used iterative methods to solve for a fixed percent standard error while varying the Stage 2 allocation for numerous values of n and for 1 = 1, 2, and 3. For each solution, the cost was computed with the formula:

$$C = n\,C_{PI} + ml\,C_G$$

in which

$C$ = total cost for this allocation

$C_{PI}$ = cost of interpreting a photo plot

$C_G$ = cost of measuring a ground plot

the minimum cost was then selected from the possible combinations of n, m, 1.

In order to combine strata, weights were used. The weight for each stratum was the ratio of the expected number of plots that would have fallen in the stratum had no stratification been imposed and the actual number of plots that were sampled in the stratum in the previous survey. The variance component for stage i, $v_i$, was computed using the formula:

$$v_i = \sum_{j=1} w_{ij} v_{ij}$$

in which

$v_i$ = the variance component for stage i

k = the number of strata

$w_{ij}$ = the weight for stage i, stratum j

$v_{ij}$ = the variance component for stage i, stratum j

Likewise, since the costs were dependent upon whether the plots were from the light, medium, or heavy stratum, weighted costs were used in the combined strata analysis.

## RESULTS

The results are summarized (Tables 3 through 6) to provide a readily usable tool with which to better plan future surveys. For each host species, those sampling allocations which yielded the minimum cost for the various levels of precision are provided. In addition, the costs for allocations other than the optimum are presented. This is for planning purposes. Often, cost is not the

TABLE 3. Optimal Sampling Allocations for the 1978 Beaverhead-Gallatin Survey in Lodgepole Pine.

| STRATUM | % Std. Error | One Ground Plot Per Stage 2 Plot | | | Two Ground Plots Per Stage 2 Plot | | | Three Ground Plots Per Stage 2 Plot | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | No. of PI Plots | No. of PI Sub-Plots | Cost ($) | No. of PI Plots | No. of PI Sub-Plots | Cost ($) | No. of PI Plots | No. of PI Sub-Plots | Cost ($) |
| L | 10% | 300 | 21 | 339 | 450 | 14 | 471 | 400 | 13 | 569 |
| M | | 200 | 19 | 331 | 250 | 12 | 416 | 250 | 10 | 491 |
| H | | 200 | 18 | 574 | 250 | 16 | 679 | 250 | 13 | 770 |
| TOTAL | | 700 | 58 | 1244 | 950 | 42 | 1566 | 900 | 36 | 1830 |
| NO STRATIFICATION | | 300 | 28 | 517 | 350 | 18 | 626 | 350 | 15 | 733 |
| L | 15% | 150 | 9 | 154 | 150 | 7 | 207 | 200 | 6 | 258 |
| M | | 80 | 8 | 137 | 60 | 6 | 177 | 60 | 5 | 215 |
| H | | 100 | 13 | 274 | 150 | 7 | 339 | 100 | 7 | 379 |
| TOTAL | | 330 | 30 | 565 | 360 | 20 | 723 | 360 | 18 | 852 |
| NO STRATIFICATION | | 140 | 12 | 241 | 140 | 8 | 282 | 150 | 7 | 331 |
| L | 20% | 80 | 5 | 84 | 70 | 4 | 112 | 70 | 3 | 123 |
| M | | 50 | 4 | 73 | 40 | 3 | 93 | 40 | 3 | 131 |
| H | | 60 | 7 | 154 | 60 | 4 | 179 | 70 | 4 | 203 |
| TOTAL | | 190 | 16 | 311 | 170 | 11 | 384 | 180 | 10 | 457 |
| NO STRATIFICATION | | 80 | 7 | 130 | 90 | 4 | 158 | 90 | 4 | 184 |
| L | 25% | 50 | 3 | 51 | 50 | 2 | 62 | 50 | 2 | 83 |
| M | | 20 | 3 | 47 | 30 | 2 | 64 | 20 | 2 | 84 |
| H | | 40 | 4 | 100 | 40 | 3 | 115 | 40 | 2 | 130 |
| TOTAL | | 110 | 10 | 198 | 120 | 7 | 241 | 110 | 6 | 297 |
| NO STRATIFICATION | | 50 | 5 | 84 | 50 | 3 | 102 | 50 | 2 | 120 |

TABLE 4. Optimal Sampling Allocations for the 1979 Montana Survey in Lodgepole Pine.

| STRATUM | % Std. Error | One Ground Plot Per Stage 2 Plot | | | Two Ground Plots Per Stage 2 Plot | | | Three Ground Plots Per Stage 2 Plot | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | No. of PI Plots | No. of PI Sub-Plots | Cost ($) | No. of PI Plots | No. of PI Sub-Plots | Cost ($) | No. of PI Plots | No. of PI Sub-Plots | Cost ($) |
| L | 10% | 500 | 110 | 1360 | 500 | 70 | 1678 | 500 | 56 | 1975 |
| M | | 600 | 177 | 2480 | 700 | 104 | 2913 | 700 | 81 | 3339 |
| H | | 400 | 40 | 944 | 400 | 34 | 1318 | 500 | 29 | 1676 |
| TOTAL | | 1500 | 327 | 4784 | 1600 | 208 | 5909 | 1700 | 166 | 6990 |
| NO STRATIFICATION | | 900 | 114 | 1904 | 1100 | 77 | 2519 | 1200 | 66 | 3113 |
| L | 15% | 250 | 48 | 606 | 300 | 30 | 752 | 250 | 25 | 892 |
| M | | 250 | 81 | 1121 | 300 | 47 | 1315 | 300 | 37 | 1508 |
| H | | 150 | 20 | 421 | 200 | 15 | 594 | 200 | 14 | 754 |
| TOTAL | | 650 | 149 | 2148 | 800 | 92 | 2661 | 750 | 76 | 3154 |
| NO STRATIFICATION | | 400 | 51 | 852 | 500 | 34 | 1127 | 500 | 30 | 1392 |
| L | 20% | 110 | 28 | 339 | 80 | 18 | 413 | 100 | 14 | 484 |
| M | | 50 | 45 | 634 | 150 | 27 | 744 | 200 | 20 | 855 |
| H | | 60 | 11 | 238 | 100 | 9 | 335 | 120 | 8 | 426 |
| TOTAL | | 220 | 84 | 1211 | 330 | 54 | 1492 | 420 | 42 | 1765 |
| NO STRATIFICATION | | 250 | 28 | 480 | 250 | 20 | 636 | 300 | 17 | 785 |
| L | 25% | 70 | 18 | 218 | 70 | 11 | 260 | 90 | 9 | 321 |
| M | | 100 | 29 | 407 | 120 | 17 | 479 | 150 | 13 | 555 |
| H | | 60 | 7 | 152 | 70 | 5 | 215 | 80 | 5 | 275 |
| TOTAL | | 230 | 54 | 777 | 260 | 33 | 954 | 320 | 27 | 1151 |
| NO STRATIFICATION | | 150 | 18 | 307 | 200 | 12 | 408 | 200 | 11 | 503 |

TABLE 5. Optimal Sampling Allocations for the 1978 Black Hills Survey in Ponderosa Pine*

| STRATUM | % Std. Error | One Ground Plot Per Stage 2 Plot | | | Two Ground Plots Per Stage 2 Plot | | | Three Ground Plots Per Stage 2 Plot | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | No. of PI Plots | No. of PI Sub-Plots | Cost ($) | No. of PI Plots | No. of PI Sub-Plots | Cost ($) | No. of PI Plots | No. of PI Sub-Plots | Cost ($) |
| M | 10% | 1400 | 62 | 1646 | 1400 | 37 | 1797 | 1500 | 27 | 1944 |
| H | | 400 | 52 | 1084 | 400 | 32 | 1246 | 500 | 23 | 1407 |
| TOTAL | | 1800 | 114 | 2730 | 1800 | 69 | 3043 | 2000 | 50 | 3351 |
| M | 15% | 700 | 35 | 870 | 800 | 18 | 953 | 800 | 14 | 1027 |
| H | | 200 | 25 | 530 | 200 | 15 | 607 | 200 | 12 | 687 |
| TOTAL | | 900 | 60 | 1400 | 1000 | 33 | 1560 | 1050 | 26 | 1714 |
| M | 20% | 400 | 22 | 528 | 500 | 11 | 577 | 500 | 8 | 621 |
| H | | 110 | 15 | 309 | 120 | 9 | 355 | 130 | 7 | 397 |
| TOTAL | | 510 | 37 | 837 | 620 | 20 | 932 | 630 | 15 | 1018 |
| M | 25% | 300 | 13 | 347 | 300 | 8 | 378 | 300 | 6 | 411 |
| H | | 70 | 10 | 201 | 80 | 6 | 230 | 80 | 5 | 260 |
| TOTAL | | 370 | 23 | 548 | 380 | 14 | 608 | 380 | 11 | 671 |

*Strata were not combined since no data were available for the light condition.

TABLE 6. Optimal Sampling Allocations for the 1979 Colorado Survey in Ponderosa Pine.

| STRATUM | % Std. Error | One Ground Plot Per Stage 2 Plot | | | Two Ground Plots Per Stage 2 Plot | | | Three Ground Plots Per Stage 2 Plot | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | No. of PI Plots | No. of PI Sub-Plots | Cost ($) | No. of PI Plots | No. of PI Sub-Plots | Cost ($) | No. of PI Plots | No. of PI Sub-Plots | Cost ($) |
| | 10% | 1400 | 352 | 5240 | 1600 | 228 | 6654 | 1800 | 186 | 8044 |
| | 15% | 600 | 165 | 2421 | 800 | 104 | 3079 | 900 | 85 | 3725 |
| | 20% | 400 | 91 | 1378 | 400 | 61 | 1763 | 500 | 49 | 2135 |
| | 25% | 200 | 62 | 894 | 200 | 35 | 894 | 300 | 38 | 1130 |

only consideration in allocation of resources. Scheduling, amount of training required, travel restrictions, etc., often play a part. Therefore, it is important to be able to know how much will be sacrificed in having a design which is suboptimum in some respect.

Some of the results were consistent over all of the previous survey data sets. In no case was stratification beneficial. This is probably because in variable probability sampling the measurement upon which the variance depends is the ration of the next stage measurement to the previous stage measurement. This would not be necessarily more homogeneous within strata defined by intensities of mortality. In every case, sampling with only one ground plot per photoplot in the final stage was best.

Results for ponderosa pine are somewhat limited in that for the Black Hills, data from only two strata were available and for the Colorado Front Range survey, only one stratum was usable. The results are summarized in Tables 5 and 6. Since data were incomplete, we were not able to do the analysis for no stratification.

## DISCUSSION

The variances in any two surveys will not be the same. Much depends upon the geographic location of the survey, the quality of the photos, the skill of the interpreters, and the inherent variability in the population being surveyed. Consequently, the results presented here should be used conservatively as a guideline, not as an absolute rule.

It is felt that the results derived from these analyses are highly dependent on the cost information used. If more precise answers are to be obtained, more effort should be directed in the future to obtain and maintain cost data. For each survey, a good estimate of costs could be obtained if the total person hours spent doing photo interpretation and the total number of photos interpreted were tallied, as well as the total person hours spent doing ground work plus the number of plots measured on the gorund were recorded.

It is not necessary to take more than one plot in the final stage of sampling in order to estimate the standard error of the estimate of total for any single survey; however, it is impossible to evaluate the variance component in the final stage for the optimization of future surveys if only one plot is sampled. For this reason, it is often desirable to consider only those designs with at least two ground plots per photo plot in the final stage even though this may not be the most cost effective. Also, in choosing a viable alternative for a particular survey, some provision should be made to allow for missing or unusable data. This is another reason why it may be better to use more than one ground plot per cell in the final sampling stage.

## REFERENCES

Bennett, D.D., and W.E. Bousfield. 1979. A pilot survey to measure annual mortality caused by the mountain pine beetle in lodgepole pine on the Beaverhead and Gallatin National Forests. USDA For. Serv., Northern Region, Rep. No. 79-20. 13 pp.

Bennett, D.D., and W.E. Bousfield, M.D. McGregor, and K.E. Gibson. 1980. Evaluation of multistage sampling techniques to measure lodgepole pine mortality caused by mountain pine beetle in Montana, 1979. USDA For. Serv., Northern Region, Rep. No. 80-13. 11 pp.

Ciesla, W.M., and F.M. Yasinski. 1980. Forest insect and disease information systems (FIDIS) implementation plan. USDA For. Serv., FIDM/MAG Rep. No. 80-3. 10 pp.

Hosteller, B.B., and R.W. Young. 1979. Estimation procedures for determining annual tree mortality caused by the mountain pine beetle. USDA For. Serv., Rocky Mtn. Region, Tech. Rep. R2-20. 25 pp.

Klein, W.H., D.D. Bennett, R.W. Young. 1979. A pilot survey to measure annual mortality of lodgepole pine caused by the mountain pine beetle. USDA For.Serv., FIDM/MAG Rep. No. 78-4. 15 pp.

Lister, C.K., and R.W. Young. 1981. 1979 Colorado forest range mountain pine beetle survey. USDA For.Serv., Rocky Mtn. Region, Tech. Rep. R2-22. 19 pp.

# APPENDIX

The variance components of the unbiased estimator for the number of dead trees are defined in this section. Let $V(Y_1)$ be the variance component associated with the first stage sampling, then $V(Y_1)$ is as follows:

$$V(Y_1) = \frac{N(N-n)}{n} \quad \frac{\sum_{i=1}^{N} x_i^2 - N'\bar{x}^2}{n-1}$$

Let $V(Y_2)$ be the variance component associated with the second stage sampling, the $V(Y_2)$ is as follows:

$$V(Y_2) = \frac{1}{m}\left(\frac{N}{N'}\right)^2 \quad \frac{\sum_{j=1}^{M'} \frac{v_j^2}{P_j^2} - M'\hat{v}^2}{M'-1}$$

Let $V(Y_3)$ be the variance component associated with the third stage sampling, the $V(Y_3)$ is as follows:

$$V(Y_3) = \left(\frac{N}{N'}\right)^2 \frac{1}{m\ell}\left\{\frac{N'}{M'}\sum_{j=1}^{M'} \frac{1}{P_j} \quad \frac{\sum_{k=1}^{L'} \frac{v_{jk}^2}{P_{jk}^2} - L'\hat{v}_j^2}{L'-1}\right\}$$

The definitions for the variables applicable to the above formulas as follows:

| | | |
|---|---|---|
| $N$ | = | total possible number of PI plots |
| $N'$ | = | total number of PI plots taken |
| $x_i$ | = | total PI for plot i. |
| $x$ | = | mean of Pi values for all photos |
| $v_{jk}$ | = | ground measurement for photo j and ground plot k |
| $p_{jk}$ | = | probability of ground plot k within photo plot j |
| $V_j$ | = | $V_j = \bar{V}_j = \frac{1}{L'}\sum_{k=1}^{L'} \frac{v_{jk}}{P_{jk}}$ |

L' = number of ground plots samled per photo plot

L = total possible number of ground plots/photo plot

$p_j$ = probability for selecting photo plot j

M' = number of photo plots sub-sampled during second stage in original survey

m = number of photo plots to sub-sample during second stage for next time (parameter to be optimized)

l = number of ground plots/photo plot to sample next time (parameter to be optimized)

n = number of photos in first stage next time (parameter to be optimized)