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**THE ALLOCATION OF RESOURCES IN
STEADY-STATE UNBALANCED GROWTH**

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University of Illinois at Urbana-Champaign**

FACULTY WORKING PAPERS

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University of Illinois at Urbana - Champaign

January 26, 1972

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T H E A L L O C A T I O N O F R E S O U R C E S I N
S T E A D Y - S T A T E U N B A L A N C E D G R O W T H

By HANS BREMS*

With few exceptions, modern growth models are models of steady-state and balanced¹ growth of homogeneous consumption and capital stock, hence miss imbalance [1], [6] as well as the allocation of resources.

To allow for imbalance, a growth model needs at least two goods. But to allow for the allocation of resources, the two goods cannot be the consumers' good and the producers' good found in the usual [5] two-sector growth models. With only one consumers' good, such models are still models of homogeneous consumption, permitting no substitution among consumers' goods and asking no question, hence offering no answer, concerning the allocation of consumption

expenditure among consumers goods. With only one producers' good such models are still models of homogeneous capital stock, permitting no substitution among producers' goods and asking no question, hence offering no answer, concerning the allocation of investment expenditure among producers' goods.

We wish to build the simplest possible growth model of heterogeneous consumption as well as capital stock, thus allowing for the full allocation of resources. To do that we assume each of our two goods to serve interchangeably as a consumers' or as a producers' good: The physical output of the j th good is X_j where $j = 1, 2$. The j th good is produced from labor L_j and two immortal capital stocks S_{ij} where $i = 1, 2$. There are, then, four capital stocks S_{ij} and four investments I_{ij} in our model. Between two such industries we specify a fourfold interaction:

The two industries compete in their demand for labor. In the

labor market they must pay the same money wage rate w , a parameter. Goods prices P_j are variables, hence the real wage rate w/P_j is also a variable.

The two industries compete in their demand for investment goods. In the market for the j th good they must pay the same price P_j . A firm producing the j th good and setting aside part of its own output for investment I_{jj} should charge itself the price P_j as an opportunity cost.

The two industries compete in their demand for money capital. In the money-capital market the capitalist-entrepreneurs allocate their savings between the two industries such as to maximize the present worth of all their future profits.

The two industries compete in their supply of consumers' goods. In the consumers' goods market the two goods are good, but not perfect, substitutes, and each consumer has a taste for both of them.

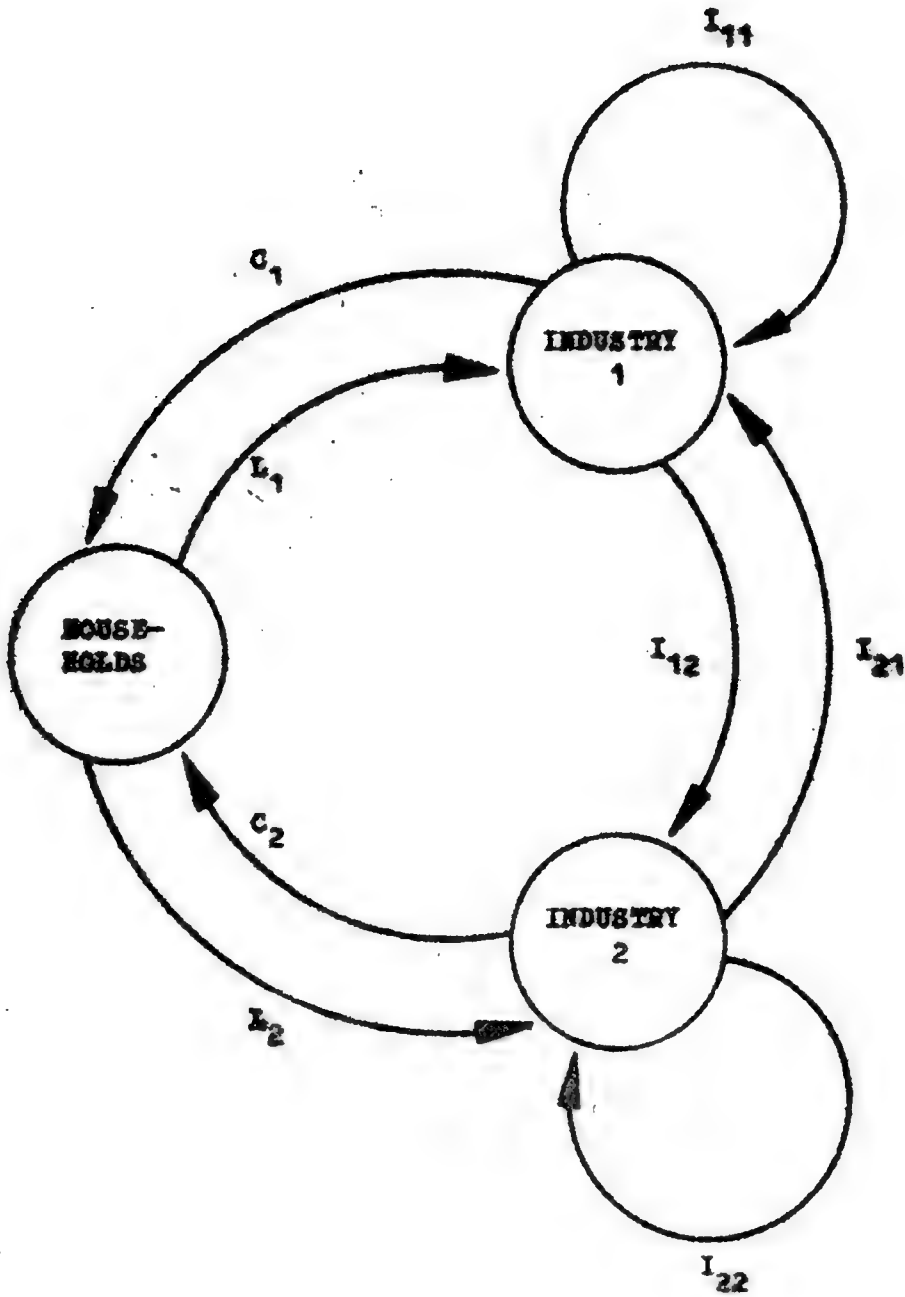


FIGURE 1. THE EIGHT PHYSICAL FLOWS

Figure 1 shows all physical flows in our model. Section I defines variables and parameters. Section II specifies the model mathematically. Section III finds the equilibrium solutions for proportionate rates of growth. Section IV finds the equilibrium solutions for levels of variables. Certain proofs are banished to two appendices.

I. NOTATION

Variables

C \equiv consumption

ϕ \equiv function to be maximized by the Lagrange-multiplier method

g_v \equiv proportionate rate of growth of variable v where $v \equiv C, I, L, P, S, X,$ and Y

I_{ij} \equiv investment of output of i th industry in j th industry

κ_{ij} \equiv physical marginal productivity of capital stock S_{ij}

L \equiv labor employed

- P ≡ price of good
- S_{ij} ≡ jth industry's physical capital stock of ith industry's good
- U ≡ utility
- W ≡ wage bill
- X ≡ physical output
- Y ≡ national money income
- Z ≡ profits bill
- ζ ≡ present worth of all future profits bills

Parameters

- A ≡ exponent of individual utility function
- α, β ≡ exponents of production function
- c ≡ propensity to consume national money income
- e ≡ Euler's number, the base of natural logarithms
- F ≡ available labor force
- g_p ≡ proportionate rate of growth of parameter p where $p \equiv F, M,$

and w

λ \equiv Lagrange multiplier

M \equiv multiplicative factor of production function

N \equiv multiplicative factor of individual utility function

r \equiv discount rate applied by capitalist-entrepreneurs

w \equiv money wage rate

The parameters listed are stationary except F , M , and w , whose growth rates g_F , g_M , and g_w are stationary.

The symbol π_i to be defined in Section II; h_j in Section IV, 2; ρ , m , n , and μ_i in Section IV, 3; v_{ij} and ξ_i in Section IV, 5; and ψ in Appendix I all stand for agglomerations of parameters and variables. Symbols t and τ are time coordinates. Subscripts $i = 1, 2$ and $j = 1, 2$ refer to industry number. All flow variables refer to the instantaneous rate of that variable measured on a per annum basis.

The first part of the document discusses the importance of maintaining accurate records of all transactions. It emphasizes that proper record-keeping is essential for the success of any business and for the protection of the interests of all parties involved. The document outlines the various methods and procedures that should be followed to ensure the accuracy and reliability of the records.

In addition, the document provides a detailed analysis of the financial data and the results of the various operations. It includes a comprehensive review of the income and expenses, as well as a comparison of the actual performance with the budgeted figures. The analysis highlights the areas where the business has performed well and identifies the key factors that have contributed to its success. It also points out the areas where there have been deviations from the budget and discusses the reasons for these deviations.

The document concludes with a series of recommendations and suggestions for the future. It provides a clear and concise summary of the findings and offers practical advice on how to improve the business's performance and profitability. The recommendations are based on a thorough understanding of the business's strengths and weaknesses and are designed to help the business achieve its long-term goals.

II. THE EQUATIONS OF THE MODEL

17 variable growth rates are listed in Section I, i. e. two growth rates of each of C_i , L_i , P_i , and X_i ; four growth rates of each of I_{ij} and S_{ij} ; and one growth rate of Y . To all apply the definition

$$(1) \text{ through } (17) \quad g_v \equiv \frac{dv}{dt} \frac{1}{v}$$

Define investment as the derivative of capital stock with respect to time

$$(18) \text{ through } (21) \quad I_{ij} \equiv \frac{dS_{ij}}{dt}$$

Let the j th industry apply the Cobb-Douglas production function

$$(22) \quad X_1 = M_1 L_1^{\alpha_1} S_{11}^{\beta_{11}} S_{21}^{\beta_{21}}$$

$$(23) \quad X_2 = M_2 L_2^{\alpha_2} S_{12}^{\beta_{12}} S_{22}^{\beta_{22}}$$

where $0 < \alpha_j < 1$; $0 < \beta_{ij} < 1$; $\alpha_1 + \beta_{11} + \beta_{21} = 1$; $\alpha_2 + \beta_{12} + \beta_{22} = 1$; and $M_j > 0$. In each industry let profit maximization under pure competition equalize real wage rate and physical marginal productivity of labor:

$$(24), (25) \quad \frac{w}{P_j} = \frac{\partial X_j}{\partial L_j} = \alpha_j \frac{X_j}{L_j}$$

Physical marginal productivities of capital at time t are

$$(26) \text{ through } (29) \quad \kappa_{ij}(t) \equiv \frac{\partial X_j(t)}{\partial S_{ij}(t)} = \beta_{ij} \frac{X_j(t)}{S_{ij}(t)}$$

Multiply (26) through (29) by price of output of j th industry $P_j(t)$ to find value marginal productivities of capital at time t . Define money profits earned at time t on each physical unit of capital stock $S_{ij}(t)$ as its value marginal productivity. Then multiply by $S_{ij}(t)$ to find money profits earned at time t on capital stock $S_{ij}(t)$. Sum over $i = 1, 2$ and define the outcome as money profits earned at time t on whatever capital stock exists at that time in the entire j th industry:

$$(30), (31) \quad Z_j(t) \equiv \sum_{i=1}^2 \kappa_{ij}(t) P_j(t) S_{ij}(t) = P_j(t) X_j(t) \sum_{i=1}^2 \beta_{ij}$$

Sum over $j = 1, 2$ and define the outcome as money profits earned at time t on whatever capital stock exists at that time in the entire economy:

$$(32) \quad Z(t) \equiv \sum_{j=1}^2 Z_j(t)$$

As seen from the present time τ this profits bill is $Z(t)e^{-r(t - \tau)}$ where e is Euler's number, the base of natural logarithms, and r is the discount rate applied by the capitalist-entrepreneurs. Finally integrate this over $t = \tau$ through ∞ and define the outcome as the present worth of all future profits bills

$$(33) \quad \zeta(\tau) \equiv \int_{\tau}^{\infty} Z(t)e^{-r(t - \tau)} dt$$

Now let capitalist-entrepreneurs use their control variable I_{ij} to optimize the allocation of their capital stock

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... ..

$$f(x) = \sum_{k=1}^{\infty} \frac{1}{k^2} \sin(kx) \quad (1)$$

... ..

$$f(x) = \frac{1}{6} \pi^2 - \frac{1}{2} \sum_{k=1}^{\infty} \frac{\cos(2kx)}{k^2} \quad (2)$$

... ..

S_{ij} within as well as between industries. Within the j th industry they act as stockholders optimizing S_{ij} where $i = 1, 2$ by appointing the right managers. Between industries they act as stockholders optimizing S_{ij} where $j = 1, 2$ by purchasing stock in the right industry. "Optimizing" in what sense? In the sense that

$$(34) \quad \zeta(\tau) = \text{maximum}$$

Under full employment, available labor force must equal the sum of labor employed by the two industries:

$$(35) \quad F = \sum_{i=1}^2 L_i$$

Define the wage bill as the money wage rate times employment:

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$$(36) \quad W \equiv w \sum_{i=1}^2 L_i$$

Define national money income as the sum of the wage bill and the profits bill:

$$(37) \quad Y \equiv W + Z$$

Let all persons have the same utility function. Let the utility function of the k th person be

$$U_k = N C_{1k}^{A_1} C_{2k}^{A_2}$$

where $0 < A_i < 1$ and $N > 0$. Let there be s persons, and let the k th person's money income be Y_k where

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$$\sum_{k=1}^S Y_k = Y$$

Let all persons spend the fraction c , where $0 < c < 1$, of their money income. Then the budget constraint of the k th person is

$$cY_k = P_1 C_{1k} + P_2 C_{2k}$$

Maximize the k th person's utility subject to his budget constraint and find his two demand functions. Then add the s individual demand functions for each good and find the two Graham [2] aggregate demand functions

$$(38), (39) \quad C_i = \pi_i Y / P_i$$

where

- 26 -

$$c = \sum_{i=1}^n \frac{N_i}{n}$$

where N_i is the number of individuals in the i th class, and n is the total number of individuals in the sample.

$$d = \frac{1}{n} \sum_{i=1}^n N_i^2$$

where d is the variance of the number of individuals in the i th class, and n is the total number of individuals in the sample. The variance of the number of individuals in the i th class is given by $d = \frac{1}{n} \sum_{i=1}^n N_i^2 - c^2$.

- 27 -

$$\pi_i = \frac{cA_i}{A_1 + A_2}$$

Industry output equilibrium requires the output of the *i*th industry to equal the sum of consumption and investment demand for it, or inventory would either accumulate or be depleted:

$$(40), (41) \quad X_i = C_i + \sum_{j=1}^2 I_{ij}$$

III. SOLUTIONS FOR PROPORTIONATE RATES OF GROWTH

Our system (1) through (41) possesses the following set of steady-state solutions for its equilibrium proportionate rates of growth:

$$(42), (43) \quad \varepsilon_{C_i} = \varepsilon_{X_i}$$

$$\frac{1}{2} \frac{d}{dt} \left(\frac{1}{2} \frac{d^2 x}{dt^2} \right) = \frac{1}{2} \frac{d^3 x}{dt^3}$$

Let $x(t)$ be the displacement of the mass from its equilibrium position at time t . The force exerted by the spring is $-kx$, and the force exerted by the damper is $-c \dot{x}$. The equation of motion is

$$m \ddot{x} + c \dot{x} + kx = 0$$

$$\frac{d^2 x}{dt^2} + \frac{c}{m} \frac{dx}{dt} + \frac{k}{m} x = 0$$

The characteristic equation is $\lambda^2 + \frac{c}{m} \lambda + \frac{k}{m} = 0$. The roots are $\lambda = \frac{-c/m \pm \sqrt{(c/m)^2 - 4k/m}}{2}$. The general solution is $x(t) = e^{\lambda_1 t} C_1 + e^{\lambda_2 t} C_2$. The initial conditions are $x(0) = x_0$ and $\dot{x}(0) = v_0$. The solution is $x(t) = e^{-\frac{c}{2m} t} \left(C_1 \cos(\omega_d t) + C_2 \sin(\omega_d t) \right)$ where $\omega_d = \sqrt{\frac{k}{m} - \left(\frac{c}{2m}\right)^2}$.

$$x(t) = e^{-\frac{c}{2m} t} \left(C_1 \cos(\omega_d t) + C_2 \sin(\omega_d t) \right)$$

(44) through (47) $g_{Iij} = g_{Xi}$

(48), (49) $g_{Li} = g_F$

(50) $g_{P1} = g_w - \frac{(1 - \beta_{22})g_{M1} + \beta_{21}g_{M2}}{(1 - \beta_{11})(1 - \beta_{22}) - \beta_{12}\beta_{21}}$

(51) $g_{P2} = g_w - \frac{(1 - \beta_{11})g_{M2} + \beta_{12}g_{M1}}{(1 - \beta_{11})(1 - \beta_{22}) - \beta_{12}\beta_{21}}$

(52) through (55) $g_{Sij} = g_{Xi}$

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$$(56) \quad \varepsilon_{X1} = \frac{(1 - \beta_{22})\varepsilon_{M1} + \beta_{21}\varepsilon_{M2}}{(1 - \beta_{11})(1 - \beta_{22}) - \beta_{12}\beta_{21}} + \varepsilon_F$$

$$(57) \quad \varepsilon_{X2} = \frac{(1 - \beta_{11})\varepsilon_{M2} + \beta_{12}\varepsilon_{M1}}{(1 - \beta_{11})(1 - \beta_{22}) - \beta_{12}\beta_{21}} + \varepsilon_F$$

$$(58) \quad \varepsilon_Y = \varepsilon_F + \varepsilon_w$$

To see that it does, the reader should take derivatives with respect to time of all equations (18) through (41) except (26) through (29) and (33), (34). He should then use definitions (1) through (17), insert solutions (42) through (58), and convince himself that each equation is satisfied.

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We defined balanced growth as identical proportionate rates of growth of physical output for all goods. According to our solutions (42) through (58), is our steady-state growth balanced or unbalanced?

Growth does spill over from one industry to the other. For example, according to (44) through (47) a more rapidly growing industry i would transmit some of its growth to a more slowly growing industry j investing in the i th industry's good. But the spillover is normally not enough to generate balanced growth. Use (56), (57), and the assumptions that $\alpha_1 + \beta_{11} + \beta_{21} = 1$ and $\alpha_2 + \beta_{12} + \beta_{22} = 1$ to find that

$$g_{X1} \gtrless g_{X2} \text{ implies } g_{M1}/g_{M2} \gtrless \alpha_1/\alpha_2,$$

respectively. Or in English: The first industry's physical output may grow more rapidly than that of the second industry for two and only² two reasons, i. e., first if everything else being equal the

Section 1

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$$\sum_{i=1}^n x_i^2 = \sum_{i=1}^n x_i x_i$$

The following table shows the results of the calculations. The first column shows the value of x_i , the second column shows the value of x_i^2 , and the third column shows the value of $x_i x_i$. The total sum of x_i^2 is 100, and the total sum of $x_i x_i$ is also 100.

first industry has more rapid technological progress g_{M_i} than the second industry, second, if everything else being equal the physical output of the first industry has a lower labor elasticity α_i than that of the second industry: The less labor-sensitive industry is less hampered by the fact that under technological progress labor force is growing less rapidly than physical capital stocks.

It does, however, follow from (50), (51), (56), and (57) that unlike physical outputs X_i , industry revenues $P_i X_i$ will grow at the same proportionate rate $g_P + g_W$.

IV. SOLUTIONS FOR LEVELS

So much for proportionate rates of growth. Let us now turn to the allocation of resources and solve for the allocation of savings between industries; the levels of industry revenues; employments; national money income; physical outputs; prices; physical

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capital stocks and their physical marginal productivities;
consumption; and income distribution.

1. Saving Equals Investment

Use (24), (25), and (36) to see that $W = \alpha_1 P_1 X_1 + \alpha_2 P_2 X_2$, and
(30) through (32) to see that $Z = (\beta_{11} + \beta_{21})P_1 X_1 + (\beta_{12} + \beta_{22})P_2 X_2$,
hence national income equals national output:

$$(59) \quad Y = P_1 X_1 + P_2 X_2$$

Multiply (40) and (41) by P_1 and P_2 , respectively, insert (38),
(39), and (59) and find saving to equal investment:

$$(60) \quad (1 - c)Y = P_1(I_{11} + I_{12}) + P_2(I_{21} + I_{22})$$

Case 2

multivariate normal distribution for β and Σ is given by

$$p(\beta, \Sigma) \propto |\Sigma|^{-1/2} \exp\left\{-\frac{1}{2}(\beta - \mu)^\top \Sigma^{-1}(\beta - \mu) - \frac{1}{2} \text{tr}[\Sigma^{-1} S]\right\}$$

where μ and S are the mean vector and the sum of squares and cross products matrix, respectively, of the observations. The likelihood function for β and Σ is given by

$$L(\beta, \Sigma) \propto |\Sigma|^{-n/2} \exp\left\{-\frac{1}{2}(\beta - \mu)^\top \Sigma^{-1}(\beta - \mu) - \frac{1}{2} \text{tr}[\Sigma^{-1} S]\right\}$$

The log-likelihood function is given by

$$\log L(\beta, \Sigma) = -\frac{n}{2} \log |\Sigma| - \frac{1}{2}(\beta - \mu)^\top \Sigma^{-1}(\beta - \mu) - \frac{1}{2} \text{tr}[\Sigma^{-1} S]$$

2. Present-Worth Maximization

Subject to the constraint (60) let the capitalist-entrepreneurs use their control variable I_{ij} to optimize the allocation of their capital stock S_{ij} within as well as between industries. "Optimize" in what sense? In the sense of maximizing the present worth $\zeta(\tau)$ of all future profits bills in accordance with (34). Using (30) through (33) we write present worth as

$$\zeta(\tau) = \int_{\tau}^{\infty} [(\beta_{11} + \beta_{21})P_1(t)X_1(t) + (\beta_{12} + \beta_{22})P_2(t)X_2(t)]e^{-r(t - \tau)}dt$$

Let it be foreseen by the capitalist-entrepreneurs that prices are growing in accordance with our steady-state solutions (50) and (51), hence

$$P_j(t) = e^{g_{Pj}(t - \tau)} P_j(\tau)$$

and that outputs are growing in accordance with our steady-state solutions (56) and (57), hence

$$X_j(t) = e^{g_{Xj}(t - \tau)} X_j(\tau)$$

Consequently we may take prices and outputs outside the integral sign and write present worth as

$$\begin{aligned} \zeta(\tau) = & (\beta_{11} + \beta_{21})P_1(\tau)X_1(\tau) \int_{\tau}^{\infty} e^{(g_{P1} + g_{X1} - r)(t - \tau)} dt + \\ & (\beta_{12} + \beta_{22})P_2(\tau)X_2(\tau) \int_{\tau}^{\infty} e^{(g_{P2} + g_{X2} - r)(t - \tau)} dt \end{aligned}$$

Since in this expression all variables refer to the same time



Fig. 1. Relationship between number of eggs and number of surviving offspring.

1988) and the number of surviving offspring (Fig. 1).

The relationship between the number of eggs per female and the number of surviving offspring per female is shown in Fig. 1. The number of surviving offspring per female increases rapidly with the number of eggs per female up to about 10^4 eggs per female, after which it levels off at approximately 10^3 surviving offspring per female. This relationship is consistent with the predictions of the R₀ model. The R₀ model predicts that the number of surviving offspring per female should increase with the number of eggs per female up to a point, after which it should level off at a value equal to the inverse of the probability of an egg surviving to become a surviving offspring. In this case, the probability of an egg surviving to become a surviving offspring is approximately 10^{-3} , so the R₀ model predicts that the number of surviving offspring per female should level off at approximately 10^3 surviving offspring per female. The relationship between the number of eggs per female and the number of surviving offspring per female is shown in Fig. 1. The number of surviving offspring per female increases rapidly with the number of eggs per female up to about 10^4 eggs per female, after which it levels off at approximately 10^3 surviving offspring per female. This relationship is consistent with the predictions of the R₀ model. The R₀ model predicts that the number of surviving offspring per female should increase with the number of eggs per female up to a point, after which it should level off at a value equal to the inverse of the probability of an egg surviving to become a surviving offspring. In this case, the probability of an egg surviving to become a surviving offspring is approximately 10^{-3} , so the R₀ model predicts that the number of surviving offspring per female should level off at approximately 10^3 surviving offspring per female.

The relationship between the number of eggs per female and the number of surviving offspring per female is shown in Fig. 1. The number of surviving offspring per female increases rapidly with the number of eggs per female up to about 10^4 eggs per female, after which it levels off at approximately 10^3 surviving offspring per female. This relationship is consistent with the predictions of the R₀ model. The R₀ model predicts that the number of surviving offspring per female should increase with the number of eggs per female up to a point, after which it should level off at a value equal to the inverse of the probability of an egg surviving to become a surviving offspring. In this case, the probability of an egg surviving to become a surviving offspring is approximately 10^{-3} , so the R₀ model predicts that the number of surviving offspring per female should level off at approximately 10^3 surviving offspring per female.

The relationship between the number of eggs per female and the number of surviving offspring per female is shown in Fig. 1. The number of surviving offspring per female increases rapidly with the number of eggs per female up to about 10^4 eggs per female, after which it levels off at approximately 10^3 surviving offspring per female. This relationship is consistent with the predictions of the R₀ model. The R₀ model predicts that the number of surviving offspring per female should increase with the number of eggs per female up to a point, after which it should level off at a value equal to the inverse of the probability of an egg surviving to become a surviving offspring. In this case, the probability of an egg surviving to become a surviving offspring is approximately 10^{-3} , so the R₀ model predicts that the number of surviving offspring per female should level off at approximately 10^3 surviving offspring per female.

τ , we may purge it of τ . Use (50), (51), (56), and (57) to see that $g_{pj} + g_{Xj} = g_F + g_w$. Assume that $g_F + g_w < r$, then integrate:

$$\zeta = \frac{(\beta_{11} + \beta_{21})P_1X_1 + (\beta_{12} + \beta_{22})P_2X_2}{r - (g_F + g_w)}$$

Inserting (30) through (32) into this we find the simple relationship between profits and present worth under steady-state growth:

$$(61) \quad Z = [r - (g_F + g_w)]\zeta$$

Maximizing present worth ζ subject to the constraint (60) is most easily done by using a Lagrange multiplier: Define a new function to be maximized

QUESTION

1. A company has a fixed cost of \$100,000 and a variable cost of \$5 per unit. The selling price is \$15 per unit. How many units must be sold to break even?

$$\begin{aligned} \text{Fixed Cost} &= \$100,000 \\ \text{Variable Cost} &= \$5 \text{ per unit} \\ \text{Selling Price} &= \$15 \text{ per unit} \end{aligned}$$

Let x be the number of units sold. The total cost is $100,000 + 5x$ and the total revenue is $15x$. To break even, total revenue must equal total cost.

$$15x = 100,000 + 5x$$

Solving for x , we subtract $5x$ from both sides: $10x = 100,000$. Dividing both sides by 10, we get $x = 10,000$. Therefore, 10,000 units must be sold to break even.

$$\phi \equiv \zeta + \lambda[(1 - c)Y - P_1(I_{11} + I_{12}) - P_2(I_{21} + I_{22})]$$

What to do with Y? Insert (61) into (37), insert the outcome into ϕ and write the latter

$$(62) \quad \phi = \{1 + \lambda(1 - c)[r - (g_F + g_W)]\}\zeta +$$

$$\lambda(1 - c)W - \lambda[P_1(I_{11} + I_{12}) + P_2(I_{21} + I_{22})]$$

The first four first-order conditions for a maximum ϕ are

$$(63) \quad \frac{\partial \phi}{\partial I_{ij}} = h_j \frac{\partial X_j}{\partial I_{ij}} - \lambda P_i = 0$$

where

QUESTION

1. A particle moves along a straight line with a constant acceleration of 2 m/s^2 . It starts from rest at $t = 0$.

Find the distance travelled by the particle in the first 5 seconds.

Use the equation of motion: $s = ut + \frac{1}{2}at^2$

Given: $u = 0 \text{ m/s}$, $a = 2 \text{ m/s}^2$, $t = 5 \text{ s}$

$$s = 0 \times 5 + \frac{1}{2} \times 2 \times 5^2$$

$\therefore s = 0 + \frac{1}{2} \times 2 \times 25$

$$s = \frac{1}{2} \times 2 \times 25$$
$$s = 1 \times 25$$
$$s = 25 \text{ m}$$

$$h_j \equiv \frac{\{1 + \lambda(1 - c)[r - (g_F + g_w)]\}(\beta_{1j} + \beta_{2j})P_j}{r - (g_F + g_w)}$$

Now according to the production functions (22) and (23), output X_j is a function of capital stock S_{ij} rather than of investment I_{ij} . But according to (1) through (21)

$$(64) \quad S_{ij} \equiv I_{ij}/g_{Sij}$$

where our steady-state growth, as specified by (52) through (57), permits us to express g_{Sij} solely in terms of parameters. Inserting (64) into the production functions (22) and (23) we find

$$\frac{\sum_{i=1}^n x_i^2 + \sum_{i=1}^n y_i^2}{n} = \frac{\sum_{i=1}^n (x_i^2 + y_i^2)}{n}$$

Let X and Y be two independent random variables with means μ_X and μ_Y and variances σ_X^2 and σ_Y^2 . Then the mean and variance of $Z = X^2 + Y^2$ are given by:

$$E[Z] = \mu_X^2 + \mu_Y^2 + \sigma_X^2 + \sigma_Y^2$$

Proof: Since X and Y are independent, we have $E[XY] = E[X]E[Y]$. Also, $E[X^2] = \mu_X^2 + \sigma_X^2$ and $E[Y^2] = \mu_Y^2 + \sigma_Y^2$. Therefore, $E[Z] = E[X^2 + Y^2] = E[X^2] + E[Y^2] = \mu_X^2 + \sigma_X^2 + \mu_Y^2 + \sigma_Y^2$.

$$(65) \quad \frac{\partial X_j}{\partial I_{ij}} = \beta_{ij} \frac{X_j}{I_{ij}}$$

and write the first-order conditions as

$$\begin{aligned} (66) \text{ through } (69) \quad & \beta_{11}(\beta_{11} + \beta_{21})X_1/I_{11} = \beta_{12}(\beta_{12} + \beta_{22})P_2X_2/(P_1I_{12}) \\ & = \beta_{21}(\beta_{11} + \beta_{21})P_1X_1/(P_2I_{21}) = \beta_{22}(\beta_{12} + \beta_{22})X_2/I_{22} \\ & = \frac{\lambda[r - (g_F + g_w)]}{1 + \lambda(1 - c)[r - (g_F + g_w)]} \end{aligned}$$

That the second-order conditions are satisfied is demonstrated

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in Appendix I.

3. Solving for Industry Revenues $P_j X_j$

Use the first-order conditions (66) through (68) to express I_{12} in terms of I_{11} and I_{21} in terms of I_{22} . Insert the results into (40) and (41). Insert (59) into (38) and (39). Insert the results into (40) and (41). Divide (40) by $\beta_{11}(\beta_{11} + \beta_{21})X_1$ and (41) by $\beta_{22}(\beta_{12} + \beta_{22})X_2$, deduct (41) from (40), and again use the first-order conditions (66) through (68). Now define

$$(70) \quad \rho \equiv (P_1 X_1 / (P_2 X_2))$$

rearrange, and write the quadratic

$$(71) \quad \rho^2 + m\rho + n = 0$$

where m and n are the following agglomerations of taste and

technology parameters

$$m \equiv \frac{(\beta_{12} + \beta_{22})[\beta_{12}\pi_2 + \beta_{22}(1 - \pi_1)] - (\beta_{11} + \beta_{21})[\beta_{11}(1 - \pi_2) + \beta_{21}\pi_1]}{(\beta_{11} + \beta_{21})[\beta_{11}\pi_2 + \beta_{21}(1 - \pi_1)]}$$

$$n \equiv - \frac{(\beta_{12} + \beta_{22})[\beta_{12}(1 - \pi_2) + \beta_{22}\pi_1]}{(\beta_{11} + \beta_{21})[\beta_{11}\pi_2 + \beta_{21}(1 - \pi_1)]}$$

The quadratic has the two roots

$$\rho = -m/2 \pm \sqrt{(m/2)^2 - n}$$

We have assumed that $0 < A_i < 1$, $0 < \beta_i < 1$, and $0 < c < 1$, hence $n < 0$. Now regardless of the sign of m , $0 \leq (m/2)^2$, hence

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$$0 \leq (m/2)^2 < (m/2)^2 - n$$

Two things follow. First, from $0 < (m/2)^2 - n$ it follows that both roots are real. Second, from $(m/2)^2 < (m/2)^2 - n$ it follows that regardless of the sign of m , the first root is positive and the second negative. We reject the latter and are left with

$$(72) \quad \rho = -m/2 + \sqrt{(m/2)^2 - n}$$

Use (24), (25), (35), and (36) to find

$$\alpha_1 P_1 X_1 + \alpha_2 P_2 X_2 = wF$$

Take this together with (70) and find

1. Introduction

The purpose of this report is to provide a comprehensive overview of the current state of the market for [Product/Service]. This includes an analysis of the market size, growth trends, and key players. The report also discusses the challenges and opportunities facing the industry and provides recommendations for stakeholders.

2. Market Overview

2.1. Market Size and Growth

2.2. Key Players

2.3. Market Trends

$$(73), (74) \quad P_i X_i = \mu_i w F$$

where

$$\mu_1 \equiv \rho / (\alpha_1 \rho + \alpha_2)$$

$$\mu_2 \equiv 1 / (\alpha_1 \rho + \alpha_2)$$

4. Solving for Employments L_i and Income Y

Use (24), (25), (73), (74) to solve for employments

$$(75), (76) \quad L_i = \alpha_i \mu_i F$$

Insert (73) and (74) into (59) and solve for national money income

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$$(77) \quad Y = (\mu_1 + \mu_2)wF$$

5. Solving for Physical Outputs X_j

Let us begin by finding four investment-output ratios. Again use the first-order conditions (66) through (68) to express I_{12} in terms of I_{11} . Insert the result into (40), insert (59) into (38), and insert the result into (40). Divide (40) by X_1 . Use a similar procedure upon (41) and find the four ratios

$$(78) \quad I_{11}/X_1 = v_{11} \equiv \frac{1 - \pi_1 - \pi_1/\rho}{1 + \beta_{12}(\beta_{12} + \beta_{22})/[\rho\beta_{11}(\beta_{11} + \beta_{21})]}$$

$$(79) \quad I_{12}/X_1 = v_{12} \equiv \frac{1 - \pi_1 - \pi_1/\rho}{1 + \rho\beta_{11}(\beta_{11} + \beta_{21})/[\beta_{12}(\beta_{12} + \beta_{22})]}$$

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$$(80) \quad I_{21}/X_2 = v_{21} \equiv \frac{1 - \pi_2 - \pi_2 \rho}{1 + \beta_{22}(\beta_{12} + \beta_{22}) / [\rho \beta_{21}(\beta_{11} + \beta_{21})]}$$

$$(81) \quad I_{22}/X_2 = v_{22} \equiv \frac{1 - \pi_2 - \pi_2 \rho}{1 + \rho \beta_{21}(\beta_{11} + \beta_{21}) / [\beta_{22}(\beta_{12} + \beta_{22})]}$$

Apply (64) to (78) through (81) and find

$$(82) \quad S_{ij} = X_i v_{ij} / g_{Sij}$$

Insert (82) and our solutions (75) and (76) into the production functions (22) and (23), arrive at two equations in the two unknowns X_j , solve them, and find

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$$(83) \quad X_1 = (\xi_1^{1 - \beta_{22}} \xi_2^{\beta_{21}})^{\frac{1}{(1 - \beta_{11})(1 - \beta_{22}) - \beta_{12}\beta_{21}F}}$$

$$(84) \quad X_2 = (\xi_1^{\beta_{12}} \xi_2^{1 - \beta_{11}})^{\frac{1}{(1 - \beta_{11})(1 - \beta_{22}) - \beta_{12}\beta_{21}F}}$$

where

$$\xi_1 \equiv M_1 (\alpha_1 \mu_1)^{\alpha_1} (v_{11}/g_{S11})^{\beta_{11}} (v_{21}/g_{S21})^{\beta_{21}}$$

$$\xi_2 \equiv M_2 (\alpha_2 \mu_2)^{\alpha_2} (v_{12}/g_{S12})^{\beta_{12}} (v_{22}/g_{S22})^{\beta_{22}}$$

The reader may convince himself that (83) and (84) are indeed growing at the rates (56) and (57) said they should be.

1984

1. The first part of the report is a general introduction to the project, which includes a description of the objectives and the scope of the work.

2. The second part of the report is a detailed description of the methodology used in the study, including a discussion of the data sources and the statistical methods employed.

3. The third part of the report is a discussion of the results of the study, which includes a comparison of the findings with the objectives of the project.

4. The fourth part of the report is a conclusion, which summarizes the main findings of the study and provides some suggestions for further research.

5. The fifth part of the report is a list of references, which includes a list of the books, articles, and other sources used in the study.

6. The sixth part of the report is an appendix, which contains some additional information related to the study, such as the raw data and the computer programs used.

6. Solving for Prices P_j

Divide our revenue solutions (73) and (74) by our physical output solutions (83) and (84), respectively, and find

$$(85) \quad P_1 = (\xi_1^{1 - \beta_{22}} \xi_2^{\beta_{21}})^{-1} \frac{1}{(1 - \beta_{11})(1 - \beta_{22}) - \beta_{12}\beta_{21}w\mu_1}$$

$$(86) \quad P_2 = (\xi_1^{\beta_{12}} \xi_2^{1 - \beta_{11}})^{-1} \frac{1}{(1 - \beta_{11})(1 - \beta_{22}) - \beta_{12}\beta_{21}w\mu_2}$$

Similarly the reader may convince himself that (85) and (86) are indeed growing at the rates (50) and (51) said they should be.

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7. Capital Stocks S_{ij} and their Marginal Productivities κ_{ij}

With (83) and (84) inserted into it, (82) will be a solution for physical capital stocks S_{ij} . With (82) inserted into them, (26) through (29) will be solutions for the physical marginal productivities of capital

$$(87) \quad \kappa_{ij} = \beta_{ij} \xi_{S_{ij}} / v_{ij}$$

8. Consumption C_i and Income Distribution W and Z

With (77), (85), and (86) inserted into them, (38) and (39) will be solutions for consumption. With (35) inserted into it, (36) will be a solution for the wage bill

$$(88) \quad W = wF$$

With (73) and (74) inserted into them, (30) through (32) will generate the profits bill

$$(89) \quad Z = [(\beta_{11} + \beta_{21})\mu_1 + (\beta_{12} + \beta_{22})\mu_2]wF$$

With (89) inserted into it, (61) will be a solution for present worth.

9. Properties of Solutions

We have now solved for the levels of all variables. Our solutions (78) through (81) for the investment-output ratios and (87) for the physical marginal productivities of capital are stationary. All other solutions for levels are nonstationary, because they contain one or more of our three nonstationary

The first part of the document discusses the importance of maintaining accurate records of all transactions. It emphasizes that proper record-keeping is essential for the integrity of the financial system and for the ability to detect and prevent fraud.

$$E = mc^2$$

(1)

The second part of the document describes the various methods used to collect and analyze data. It details the procedures for data collection, including the use of surveys, interviews, and focus groups, and discusses the statistical techniques used to analyze the resulting data.

CONCLUSION

In conclusion, the findings of this study indicate that there is a significant relationship between the variables studied. The results suggest that the implementation of the proposed system will lead to improved efficiency and accuracy in the financial reporting process. Further research is needed to explore the long-term effects of the system and to identify potential areas for improvement.

parameters, i. e. available labor force F , the multiplicative factor M_i of the production functions, and the money wage rate w .

Are our solutions real and positive? Section IV, 3 found both roots ρ to be real and found one to be positive, the other negative. All solutions (73) through (89), then, are real. Rejecting the negative root we find solutions (73) through (77), (88), and (89) to be obviously positive. Less obviously, so are solutions (78) through (87), as demonstrated in our Appendix II.

A P P E N D I X I

SECOND-ORDER CONDITIONS FOR A MAXIMUM OF EQUATION (62)

Write the bordered Hessian

$$(90) \quad H \equiv \begin{array}{cccc|c}
 \frac{\partial^2 \phi}{\partial I_{11}^2} & \frac{\partial^2 \phi}{\partial I_{11} \partial I_{12}} & \frac{\partial^2 \phi}{\partial I_{11} \partial I_{21}} & \frac{\partial^2 \phi}{\partial I_{11} \partial I_{22}} & -P_1 \\
 \frac{\partial^2 \phi}{\partial I_{12} \partial I_{11}} & \frac{\partial^2 \phi}{\partial I_{12}^2} & \frac{\partial^2 \phi}{\partial I_{12} \partial I_{21}} & \frac{\partial^2 \phi}{\partial I_{12} \partial I_{22}} & -P_1 \\
 \frac{\partial^2 \phi}{\partial I_{21} \partial I_{11}} & \frac{\partial^2 \phi}{\partial I_{21} \partial I_{12}} & \frac{\partial^2 \phi}{\partial I_{21}^2} & \frac{\partial^2 \phi}{\partial I_{21} \partial I_{22}} & -P_2 \\
 \frac{\partial^2 \phi}{\partial I_{22} \partial I_{11}} & \frac{\partial^2 \phi}{\partial I_{22} \partial I_{12}} & \frac{\partial^2 \phi}{\partial I_{22} \partial I_{21}} & \frac{\partial^2 \phi}{\partial I_{22}^2} & -P_2 \\
 -P_1 & -P_1 & -P_2 & -P_2 & 0
 \end{array}$$

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The first derivatives $\partial\phi/\partial I_{ij}$ have already been taken and were of the form (63). It follows from that form that a good many of the second derivatives contained in our Hessian are zero: After inserting (64) into our production functions (22) and (23) we realize that X_j is a function of neither I_{ii} nor I_{ji} where $i \neq j$, hence

$$(91) \quad \frac{\partial X_j}{\partial I_{ii}} = \frac{\partial X_j}{\partial I_{ji}} = \frac{\partial^2 X_j}{\partial I_{ii} \partial I_{ij}} = \frac{\partial^2 X_j}{\partial I_{ji} \partial I_{ij}} = \frac{\partial^2 X_j}{\partial I_{ii} \partial I_{jj}} = \frac{\partial^2 X_j}{\partial I_{ji} \partial I_{jj}} = 0$$

$$(i \neq j)$$

But X_j is a function of I_{ij} and I_{jj} , hence

$$(92) \quad \frac{\partial^2 X_j}{\partial I_{ij} \partial I_{jj}} = \frac{\partial^2 X_j}{\partial I_{jj} \partial I_{ij}} = \frac{\beta_{ij} \beta_{jj} X_j}{I_{ij} I_{jj}} \quad (i \neq j)$$

QUESTION

1. A particle of mass m is projected from the origin O of a Cartesian coordinate system with an initial velocity u at an angle θ to the horizontal. The particle moves in a parabolic path and reaches a maximum height H and a horizontal range R . Show that $H = \frac{R \tan \theta}{2}$ and $R = \frac{2u^2 \sin \theta \cos \theta}{g}$.



ANSWER

Let the particle be projected from the origin O with an initial velocity u at an angle θ to the horizontal.

$$u_x = u \cos \theta \quad u_y = u \sin \theta$$

$$v_x = u_x = u \cos \theta \quad v_y = u_y - gt = u \sin \theta - gt$$

$$(93) \quad \frac{\partial^2 X_j}{\partial I_{ij}^2} = \beta_{ij}(\beta_{ij} - 1) \frac{X_j}{I_{ij}^2} \quad (i = j \text{ or } i \neq j)$$

Apply (91), (92), and (93) to the Hessian (90). Then try to produce even more zero elements, making the Hessian easier to evaluate. Factor out $\beta_{11}h_1X_1/I_{11}$ from first row; $\beta_{12}h_2X_2/I_{12}$ from second row; $\beta_{21}h_1X_1/I_{21}$ from third row; and $\beta_{22}h_2X_2/I_{22}$ from fourth row, where h was defined as part of (63). Thereby the first four elements of the fifth column become

$$- P_1 I_{11} / (\beta_{11} h_1 X_1),$$

$$- P_1 I_{12} / (\beta_{12} h_2 X_2),$$

$$- P_2 I_{21} / (\beta_{21} h_1 X_1),$$

$$- P_2 I_{22} / (\beta_{22} h_2 X_2)$$

$$\frac{1}{\sqrt{1 - \beta^2}} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma \quad (1)$$

where γ is the Lorentz factor, $\beta = v/c$ is the ratio of the velocity v to the speed of light c . The Lorentz factor γ is defined as:

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (2)$$

The Lorentz factor γ is a function of the velocity v and the speed of light c . It is a scalar quantity that is invariant under Lorentz transformations. It is used to describe the time dilation and length contraction of objects moving at relativistic speeds.

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}}$$

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}}$$

But according to the first-order conditions (66) through (69) those four values are all equal to $-1/\lambda$. Now factor out $1/I_{11}$ from first column, $1/I_{12}$ from second column, $1/I_{21}$ from third column, $1/I_{22}$ from fourth column, and $1/\lambda$ from fifth column.

If to each element of a row is added the corresponding element of another row, the determinant remains unchanged. So factor out (-1) from the first row and add to each element of it the corresponding element of the third row. Factor out (-1) from the second row and add to each element of it the corresponding element of the fourth row.

If to each element of a column is added the corresponding element of another column, the determinant remains unchanged. So add to each element of the third column the corresponding element of the first column. Add to each element of the fourth column the corresponding element of the second column.

By now the Hessian has been transformed into the following very tractable form:

Page 1

The first part of the document discusses the importance of maintaining accurate records of all transactions. It emphasizes that proper record-keeping is essential for the integrity of the financial system and for the ability to detect and prevent fraud. The document also highlights the need for transparency and accountability in all financial dealings.

In addition, the document outlines the various methods used to collect and analyze financial data. It describes the use of statistical techniques to identify trends and anomalies in the data. The document also discusses the importance of data security and the need to protect sensitive information from unauthorized access.

Finally, the document provides a detailed overview of the current state of the financial system. It discusses the challenges facing the system and the steps that are being taken to address these challenges. The document concludes by emphasizing the need for continued vigilance and cooperation from all stakeholders in the financial system.

$$H = \frac{\beta_{11}\beta_{12}\beta_{21}\beta_{22}h_1^2h_2^2X_1^2X_2^2}{I_{11}^2I_{12}^2I_{21}^2I_{22}^2\lambda} \times$$

$$\begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ \beta_{11} & 0 & -\alpha_1 & 0 & -1 \\ 0 & \beta_{12} & 0 & -\alpha_2 & -1 \\ -P_1I_{11} & -P_1I_{12} & -P_1I_{11}-P_2I_{21} & -P_1I_{12}-P_2I_{22} & 0 \end{vmatrix}$$

$$= \frac{\beta_{11}\beta_{12}\beta_{21}\beta_{22}h_1^2h_2^2X_1^2X_2^2}{I_{11}^2I_{12}^2I_{21}^2I_{22}^2\lambda} [\alpha_2(P_1I_{11} + P_2I_{21}) + \alpha_1(P_1I_{12} + P_2I_{22})]$$

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Is our Hessian positive, then? Appendix II will demonstrate that all solutions, including those for $P_i I_{ij}$, are positive. To see if λ is positive, write the fifth first-order condition $\partial\phi/\partial\lambda = 0$ and find it to be the constraint (60). Use (66) through (69) to write

$$P_1(I_{11} + I_{12}) + P_2(I_{21} + I_{22}) =$$

$$\frac{1 + \lambda(1 - c)[r - (g_F + g_w)]}{\lambda[r - (g_F + g_w)]} [(\beta_{11} + \beta_{21})^2 P_1 X_1 + (\beta_{12} + \beta_{22})^2 P_2 X_2]$$

Insert (59) and this into the constraint (60), rearrange, and

The first part of the paper is devoted to the study of the
 asymptotic behavior of the solutions of the system

$$\dot{x} = Ax + B u, \quad x(0) = x_0$$
 where A and B are matrices and u is a control function.
 It is shown that the solutions of this system converge to zero
 as $t \rightarrow \infty$ if and only if the eigenvalues of the matrix
 A have negative real parts.

2. THE CASE OF A CONTINUOUS CONTROL

In the case of a continuous control, the control function
 u is assumed to be a continuous function of time.
 The system of equations is then written as

$$\dot{x} = Ax + B u, \quad x(0) = x_0$$
 where u is a continuous function.

It is shown that the system is controllable if and only if
 the rank of the matrix B is equal to the dimension of the
 state space.

write the latter

$$\lambda = \frac{\psi}{(1 - c)(1 - \psi)[r - (g_F + g_w)]}$$

where

$$\psi \equiv \frac{(\beta_{11} + \beta_{21})^2 P_1 X_1 + (\beta_{12} + \beta_{22})^2 P_2 X_2}{P_1 X_1 + P_2 X_2}$$

It follows from $0 < \psi < 1$ that $\lambda > 0$, hence the Hessian (90) is positive. And now for its principal minors.

1957

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From the Hessian (90) remove successively fourth, third, and second column and row and obtain the bordered 4×4 , 3×3 , and 2×2 principal minors. Their values are respectively

$$- \frac{\beta_{11}\beta_{12}\beta_{21}h_1^2h_2^2X_1^2X_2^2}{I_{11}^2I_{12}^2I_{21}^2\lambda} [(1 - \beta_{12})(P_1I_{11} + P_2I_{21}) + (1 - \beta_{11} - \beta_{21})P_1I_{12}]$$

$$\frac{\beta_{11}\beta_{12}h_1h_2X_1X_2}{I_{11}^2I_{12}^2\lambda} [(1 - \beta_{12})P_1I_{11} + (1 - \beta_{11})P_1I_{12}]$$

$$- \frac{\beta_{11}h_1X_1}{I_{11}^2\lambda} P_1I_{11}$$

The three values are negative, positive, and negative, respectively.

1953

The first of the series of ...

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A P P E N D I X I I

SIGN OF SOLUTIONS (78) THROUGH (87)

Solutions (78) through (87) contain one of the factors v_{ij} . Could those factors be nonpositive? To show that they cannot we prove that our positive root ρ has the following bounds:

$$(94) \quad \pi_1/(1 - \pi_1) < \rho < (1 - \pi_2)/\pi_2$$

Take the first inequality of (94), insert (72), move the term $-(m/2)$ to the other side, and write the inequality

$$\sqrt{(m/2)^2 - n} > m/2 + \pi_1/(1 - \pi_1)$$

Square the inequality, multiply it by $(1 - \pi_1)^2$, and write it

$$- \pi_1^2 - m\pi_1(1 - \pi_1) - n(1 - \pi_1)^2 > 0$$

$$\mathbb{R}^2 \times \mathbb{R}^2 \cong \mathbb{R}^4$$

$$(\mathbb{R}^2)^{\otimes 2} \cong \mathbb{R}^2 \oplus \mathbb{R}^2 \oplus \mathbb{R}^2 \oplus \mathbb{R}^2$$

Let $f: \mathbb{R}^2 \times \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a bilinear map. Then f can be written as

$$f(x, y) = \sum_{i,j,k} a_{ijk} x_i y_j e_k$$

$$= \sum_{i,j} (a_{ij1} x_i y_j + a_{ij2} x_i y_j) e_1 + \sum_{i,j} (a_{ij3} x_i y_j + a_{ij4} x_i y_j) e_2$$

$$= \sum_{i,j} (a_{ij1} + a_{ij2}) x_i y_j e_1 + \sum_{i,j} (a_{ij3} + a_{ij4}) x_i y_j e_2$$

Thus, the bilinear map f is determined by the symmetric bilinear forms

$$A(x, y) = \sum_{i,j} (a_{ij1} + a_{ij2}) x_i y_j$$

$$B(x, y) = \sum_{i,j} (a_{ij3} + a_{ij4}) x_i y_j$$

Conversely, given two symmetric bilinear forms A and B , we can define

$$f(x, y) = \sum_{i,j} (A_{ij} x_i y_j + B_{ij} x_i y_j) e_1 + \sum_{i,j} (A_{ij} x_i y_j + B_{ij} x_i y_j) e_2$$

Now insert the definitions of m and n attached to (71), recall that $\pi_1 + \pi_2 = c$, rearrange, and find

$$(1 - c)[(\beta_{11} + \beta_{21})\beta_{11}\pi_1 + (\beta_{12} + \beta_{22})\beta_{12}(1 - \pi_1)] > 0$$

which it is under our assumptions about β_{ij} and π_i .

Then take the second inequality of (94), insert (72), move the term $-(m/2)$ to the other side, and write the inequality

$$\sqrt{(m/2)^2 - n} < m/2 + (1 - \pi_2)/\pi_2$$

Square the inequality, multiply it by π_2^2 , and write it

$$(1 - \pi_2)^2 + m\pi_2(1 - \pi_2) + n\pi_2^2 > 0$$

Insert the definitions of m and n and find



$$(1 - c)[(\beta_{11} + \beta_{21})\beta_{21}(1 - \pi_2) + (\beta_{12} + \beta_{22})\beta_{22}\pi_2] > 0$$

which it is under our assumptions about β_{ij} and π_i .

Now that we have validated (94), take its first inequality, multiply it by $1 - \pi_1$, divide it by ρ , use the definitions (78) and (79) and find

$$v_{11} > 0$$

$$v_{12} > 0$$

Take the second inequality of (94), multiply it by π_2 , use the definitions (80) and (81) and find

$$v_{21} > 0$$

$$v_{22} > 0$$

We conclude that (78) through (87) are indeed positive.

1980

1. The first part of the paper is devoted to the study of the

properties of the function $f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}$.

It is shown that $f(x) = e^x$ for all x .

The second part of the paper is devoted to the study of the

properties of the function $f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}$.

It is shown that $f(x) = e^x$ for all x .

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It is shown that $f(x) = e^x$ for all x .

The fourth part of the paper is devoted to the study of the

A P P E N D I X I I I

EMPIRICAL MEASUREMENT OF GROWTH IMBALANCE

Yotopoulos and Lau [6] have examined growth imbalance in 65 countries for the periods 1948-53, 1954-58, and 1959-60. In each country, six sectors were distinguished, i. e. agriculture, mining, manufacturing, construction, electricity-gas-water and "others," including transportation and communication, services, etc.

Modifying the Yotopoulos-Lau notation slightly to make it consistent with our own, let us define

- E_i \equiv income elasticity of demand for output of i th sector
- G \equiv proportionate rate of growth of gross domestic product in constant prices
- g_{Xi} \equiv proportionate rate of growth of output of i th sector in constant prices
- w_i \equiv share in gross domestic product of value added by i th sector

Section 1

Section 1.1: Introduction

The first part of the course is an introduction to the subject.

The course is designed to provide a comprehensive overview of the field. It covers the basic principles and concepts, as well as the latest developments and research. The course is suitable for students who are new to the subject, as well as those who have some prior knowledge. The course is taught by a leading expert in the field, who will provide a clear and concise explanation of the subject matter. The course is divided into several sections, each covering a different aspect of the subject. The first section is an introduction to the subject, followed by a section on the basic principles and concepts. The third section covers the latest developments and research, and the final section is a summary of the course.

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Yotopoulos-Lau now applied two different concepts of imbalance. First, an index of Samuelson-Solow-von Neumann imbalance defined as

$$(95) \quad V^* \equiv \frac{1}{G} \sqrt{\sum_{i=1}^n \omega_i (g_{Xi} - G)^2}$$

or, in English, the reciprocal of the national real growth rate times the square root of the weighted sum of the squared deviations of sectoral real growth rates from the national real growth rate.

For their entire sample of 65 countries, Yotopoulos-Lau found a rather strong negative correlation between the Samuelson-Solow-von Neumann index of imbalance and the national real growth rate; The coefficient of correlation was -0.322. They also found the most highly developed countries to have have the lowest index of

... and the

$$\frac{1}{2} \left(\frac{1}{1+x} + \frac{1}{1-x} \right) = \frac{1}{1-x^2}$$

...

... ..

$$\text{INDEX OF SAMUELSON-SOLOW-VON NEUMANN IMBALANCE } V^* \equiv \frac{1}{G} \sqrt{\sum_{i=1}^n \omega_i (g_{Xi} - G)^2}$$

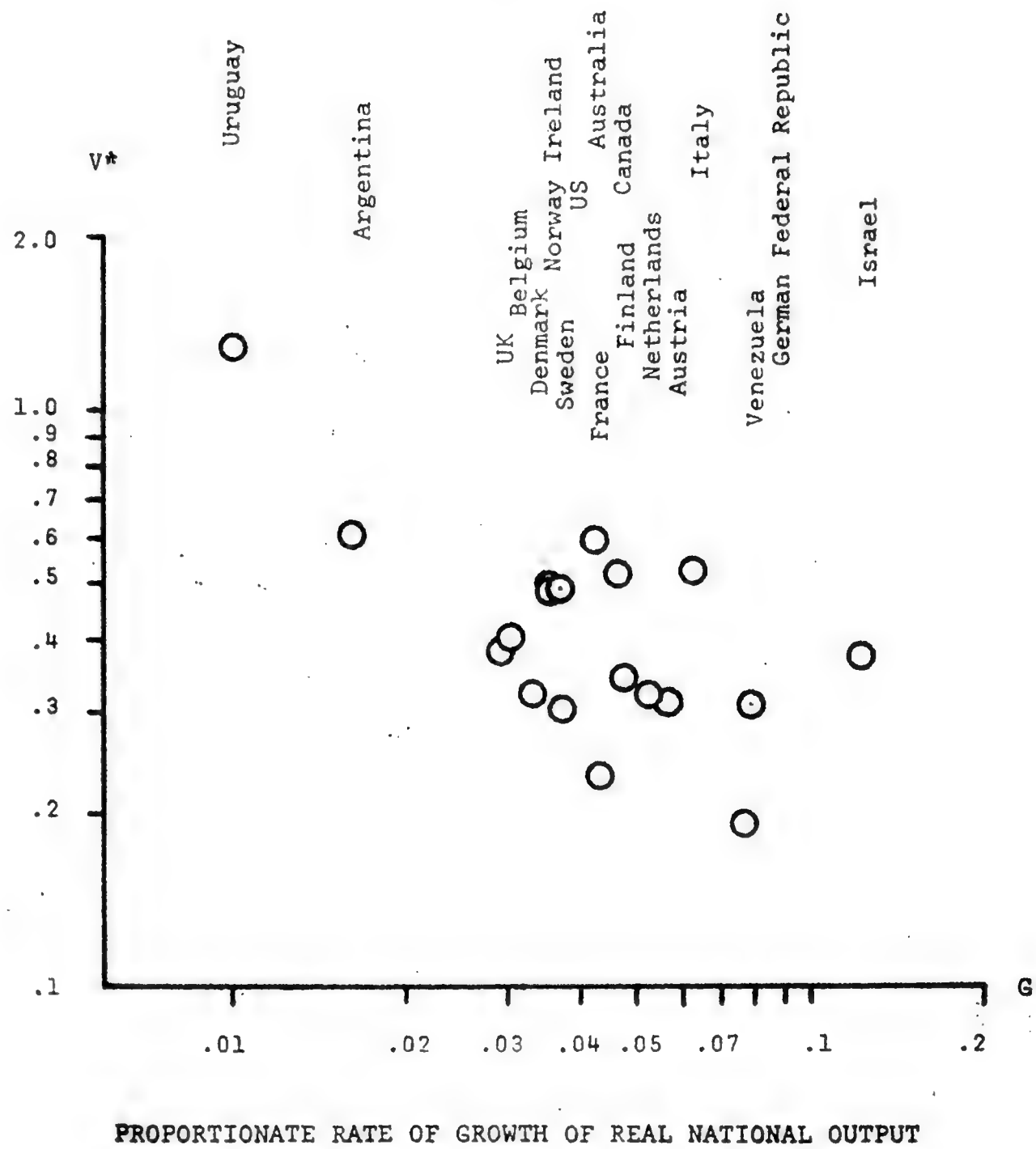


FIGURE 2. SAMUELSON-SOLOW-VON NEUMANN IMBALANCE IN 19 COUNTRIES 1950-60

$$\text{INDEX OF NURKSE IMBALANCE } V' = \frac{1}{G} \sqrt{\sum_{i=1}^n \omega_i (gX_i - E_i G)^2}$$

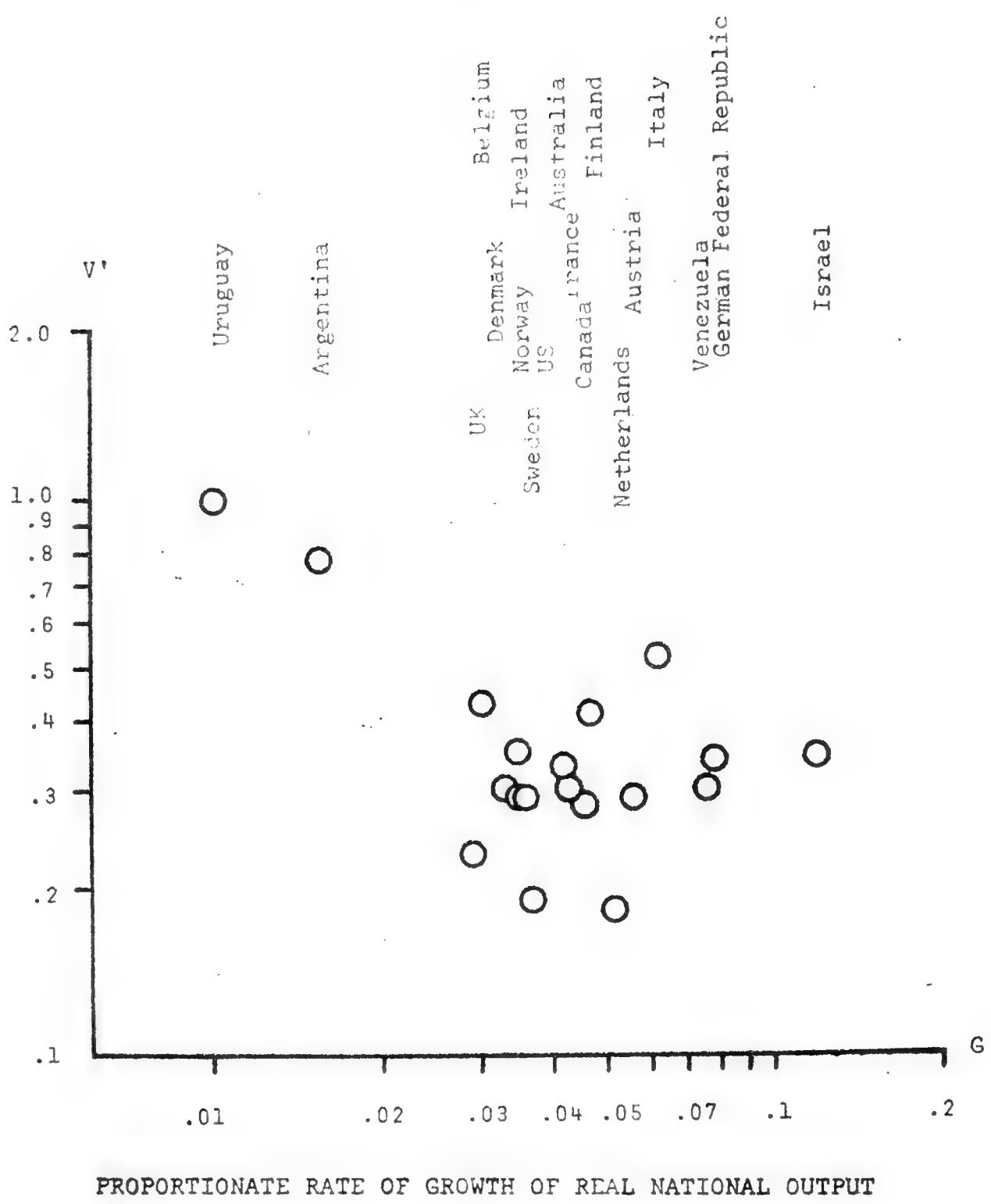


FIGURE 3. NURKSE IMBALANCE IN 19 COUNTRIES 1950-60

imbalance.

From the Yotopoulos-Lau sample of 65 countries our own Figure 2 has selected, for the period 1950-60, a much smaller sample consisting of the 19 capitalist countries which had, in 1958, a per capita income of \$500 or more per annum. Figure 2 shows that even those countries still had a substantial Samuelson-Solow-von Neumann index of imbalance: Their square root of the weighted sum of squared deviations ranged from 0.19 (Venezuela) to 1.26 (Uruguay) of the national real growth rate, with the majority of the countries lying between 0.30 and 0.55 of that rate.

Could imbalance be explained by nonunitary sector income elasticities? Here it occurred to Yotopoulos-Lau to define a second index of imbalance removing from the imbalance concept those deviations which are caused by nonunitary sector income elasticities. That index they called a Nurkse imbalance index and defined it as

Dear Sir,
 I have the pleasure to inform you that your application for the position of Junior Engineer in the Public Works Department has been received and is being considered. The details of the examination are as follows:

Name of the Department: Public Works Department
 Name of the Post: Junior Engineer
 Name of the Exam: Written Examination
 Date: 15/11/2018
 Time: 10:00 AM to 12:00 PM
 Venue: Government College, Bangalore
 Fee: Rs. 1000/-
 Age Limit: 18 to 25 years
 Education: B.E. or B.Tech. in Civil Engineering
 Experience: None
 Relaxation: None
 Reservation: None
 Language: English
 Medium: English
 Number of Posts: 05
 Selection: Written Examination
 Duration: 1 hour
 Marking Scheme: 100 Marks
 Date of Issuance: 10/11/2018
 Issued by: Mr. Ramesh Kumar
 Designation: Assistant Engineer
 Office: Public Works Department, Bangalore

$$(96) \quad V' \equiv \frac{1}{G} \sqrt{\sum_{i=1}^n \omega_i (g_{Xi} - E_i G)^2}$$

or, in English, the reciprocal of the national real growth rate times the square root of the weighted sum of the squared deviations of sectoral real growth rates from the product of sector income elasticity and national real growth rate.

Now suppose that imbalance were fully explained by nonunitary sector income elasticities. Then the output of the i th sector would always be growing at the rate $g_{Xi} = E_i G$, consequently according to (96) $V' = 0$. In other words, Nurkse imbalance would be zero.

Applying to the same period and the same countries as Figure 2, our Figure 3 shows that Nurkse imbalance is far from zero. The

THE HISTORY OF THE
CITY OF BOSTON

The first settlement in the city of Boston was made in 1630 by a group of Puritan settlers from England. They came to the city in search of a place where they could practice their religion freely and without the interference of the English government. The city was founded on a small island in the harbor, and the settlers built a fort to protect themselves from the Indians who lived in the surrounding area. The city grew rapidly, and by 1639 it had a population of about 1,000 people. In 1644, the city was incorporated as a town, and in 1688 it became a city. The city has since become one of the most important and prosperous cities in the United States.

The city of Boston has a rich history and a unique character. It is a city of many firsts, and it has played a major role in the development of the United States. The city is known for its many historical landmarks, including the Freedom Trail, the Old State House, and the Boston Common. The city is also known for its many cultural institutions, including the Boston Symphony Orchestra and the Boston Public Library. The city is a city of many firsts, and it has played a major role in the development of the United States.

Nurkse imbalance in Figure 3 is almost as substantial as the Samuelson-Solow-von Neumann imbalance in Figure 2. The Nurkse range has the same floor but a slightly lower ceiling than the Samuelson-Solow-von Neumann range: The square root of the weighted sum of squared Nurkse deviations ranges from 0.19 (the Netherlands) to 1.0 (Uruguay) of the national real growth rate, with a majority of the countries lying between 0.25 and 0.50 of that rate. We conclude that the Nurkse index has removed precious little imbalance from the Samuelson-Solow-von Neumann index.

How come, so little? Suppose all sector income elasticities were unity, then the Samuelson-Solow-von Neumann index would become equal to the Nurkse index: If $E_i = 1$ it follows from (95) and (96) that $V^* = V'$. And indeed the income elasticities used by Yotopoulos-Lau differed very little from unity:

QUESTION

1. The following table shows the number of people who attended a concert in each of the five years from 2010 to 2014. The number of people who attended the concert in each year is given in the table below.

Year	Number of people
2010	120
2011	150
2012	180
2013	210
2014	240

2. The following table shows the number of people who attended a concert in each of the five years from 2010 to 2014. The number of people who attended the concert in each year is given in the table below.

Year	Number of people
2010	120
2011	150
2012	180
2013	210
2014	240

ANSWER

1. 120

Agriculture	0.952
Mining	0.892
Manufacturing	1.044
Construction	1.035
Electricity-gas-water	1.045
Others	0.999

These sector income elasticities were estimated from cross sections of some of the countries examined but applied to all countries.

From the Yotopoulos-Lau measurements we conclude three things. First, that growth imbalance is a rather ubiquitous phenomenon. Second, that in highly developed countries it is not strongly correlated with the national real growth rate. Third, that nonunitary sector income elasticities play a minuscule role in explaining real-world growth imbalance.

F O O T N O T E S

*For a reflective fall semester of 1970 as an associate at the University of Illinois Center for Advanced Study, the author is indebted to the Graduate College of the University of Illinois. For careful checking of the mathematics and for valuable suggestions, he is indebted to Mr. Bojan Popovic, a graduate student at the Department of Economics and the Coordinated Science Laboratory at the University of Illinois.

¹We define, as Hahn and Matthews [3] did, steady-state growth as stationary proportionate rates of growth of physical outputs. We define, as Solow and Samuelson [4] did, balanced growth as identical proportionate rates of growth of physical output for all goods.

²Our Graham-type demand functions (38) and (39) have unitary income elasticities. In our model, then, possible growth imbalance must have causes other than nonunitary income elasticities. From Yotopoulos-Lau [6] one may conclude that nonunitary sector income elasticities play a minuscule role in explaining real-world growth imbalance. This conclusion is derived in Appendix III.

UNIT 10

1. The first part of the unit is a reading passage about the history of the world.

The passage is divided into three sections. The first section is about the beginning of the world, the second section is about the development of the world, and the third section is about the future of the world. The passage is written in a simple and clear style, and it is easy to understand. The passage is a good example of a reading passage for a student who is learning English.

2. The second part of the unit is a listening exercise about the same topic.

The listening exercise is a short audio recording of a person talking about the history of the world. The person is speaking in a clear and slow voice, and it is easy to hear. The listening exercise is a good example of a listening exercise for a student who is learning English.

3. The third part of the unit is a writing exercise about the same topic.

The writing exercise is a short paragraph about the history of the world. The student is asked to write a paragraph about the history of the world. The writing exercise is a good example of a writing exercise for a student who is learning English.

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GREEK LETTERS USED

α alpha
 β beta
 ζ zeta
 κ kappa
 λ lambda
 μ mu
 ν nu
 ξ xi
 π pi
 ρ rho
 Σ sigma
 τ tau
 ϕ phi
 ψ psi
 ω omega

MATHEMATICAL SYMBOLS USED

{ } brace
[] bracket
| | determinant
= equal to
> greater than
 \equiv identically equal to
 \int integral of
< less than
 \neq not equal to
() parenthesis
 ∂ partial derivative of
 $\sqrt{\quad}$ square root of

QUESTION

1. A company has the following data:

Revenue	1000
Cost of Sales	600
Operating Expenses	200
Interest Expense	50
Income Tax Expense	100
Depreciation Expense	100
Change in Accounts Receivable	50
Change in Accounts Payable	20
Change in Inventory	10
Change in Prepaid Expenses	5
Change in Deferred Revenue	10

2. The company's net income is:

Revenue	1000
Cost of Sales	(600)
Operating Expenses	(200)
Interest Expense	(50)
Income Tax Expense	(100)
Net Income	50

3. The company's cash flow is:

Net Income	50
Depreciation Expense	100
Change in Accounts Receivable	(50)
Change in Accounts Payable	20
Change in Inventory	(10)
Change in Prepaid Expenses	(5)
Change in Deferred Revenue	10
Cash Flow	110

4. The company's free cash flow is:

Net Income	50
Depreciation Expense	100
Change in Accounts Receivable	(50)
Change in Accounts Payable	20
Change in Inventory	(10)
Change in Prepaid Expenses	(5)
Change in Deferred Revenue	10
Free Cash Flow	110

5. The company's operating cash flow is:

Net Income	50
Depreciation Expense	100
Change in Accounts Receivable	(50)
Change in Accounts Payable	20
Change in Inventory	(10)
Change in Prepaid Expenses	(5)
Change in Deferred Revenue	10
Operating Cash Flow	110



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