NAVY DEPARTMENT

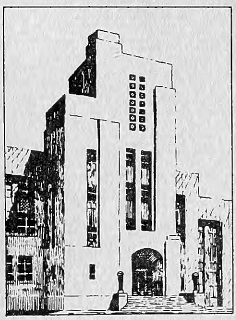
THE DAVID W. TAYLOR MODEL BASIN

WASHINGTON 7, D.C.

APPLICATION OF STATISTICS TO THE PRESENTATION OF WAVE AND SHIP-MOTION DATA

by

Alice W. Mathewson



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Report 813

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NOTATION

a_1 Constant defined by an equation in Appendix 1 and used in evaluating Pearson's Type I Curve

a_2 Constant defined by an equation in Appendix 1 and used in evaluating Pearson's Type I Curve

c Number of distributions required to satisfy conditions assumed in Appendix 2

d Deviation of the mean of a sample from the mean of a population

f Frequency of occurrence

h Wave height measured from crest to trough

h_R Root mean square value of wave heights

p Fraction between 0 and 1

$h^{(p)}$ Mean value of the first pn of n wave heights when arranged in descending order of magnitude

k Constant taken from a student's "t" table

m_1 Constant defined by an equation in Appendix 1 and used in evaluating Pearson's Type I Curve

m_2 Constant defined by an equation in Appendix 1 and used in evaluating Pearson's Type I Curve

N Total number of elements which make up the distribution

n Number of independent observations required to satisfy conditions assumed in Appendix 2.

P Probability

r Constant defined by an equation in Appendix 1 and used in evaluating Pearson's Type I Curve

$\bar{\bar{X}}$ Arithmetic mean of the population

$\bar{X}_i$ ($i = 1, 2, 3, 4$) "ith" arithmetic mean of a sample

x Value of the abscissa in Pearson's Type I Curve

y Value of the ordinate in Pearson's Type I Curve

y_0 Ordinate of the Pearson type curve at the mode

z Deviation of any value of f from the mean value of frequency distribution

β_1 Criterion for the Pearson type curves and defined by an equation in Appendix 1

β_2 Criterion for the Pearson type curves and defined by an equation in Appendix 1

Γ Gamma function

η Variable of integration in the evaluation of $\frac{h^{(p)}}{\bar{h}}$

κ Criterion for the Pearson type curves and defined by an equation in Appendix 1

μ_i (i = 1, 2, 3, 4) "ith" moment about the mean, $\mu_i = \frac{\Sigma f\, z^i}{N}$

ξ Instantaneous wave elevation

σ Standard deviation of the population

σ_s Standard deviation of a sample

DEFINITIONS

Class interval	A grouping of possible values of a variable
Confidence bands	The interval within which the "true" distribution will fall with a certain probability
Distribution	An arrangement of numerical data according to size or magnitude
Frequency	The number of times a value occurs or is observed
Normal distribution	A bell-shaped curve, symmetrical about the mean and defined by the mean and standard deviation
Population	The entire data from which a sample was drawn if all of it were available
Probability	The likelihood of occurrence
Probability density	A quantity which, if integrated over the independent variation, is equal to 1; see probability
Probability level	A number which indicates the degree of confidence that can be placed on a given result, i.e., probability level 0.90 means that 90 times out of 100, a given hypothesis will hold
Random	The method of drawing a sample when each item in the population has an equal chance of selection
Sample	A finite portion of the population
Standard deviation	A special form of the average deviation from the mean, a measure of dispersion, $\sigma = \sqrt{(\Sigma f z^2)/N}$
Standard error	The standard deviation of a distribution of means
Statistic	The estimate of a number describing the numerical property of a population
"t" distribution	The distribution of student's t, defined by $t = (\overline{X}_i - \overline{\overline{X}}) \sqrt{N}/\sigma$ where $\overline{X}_i$ is the mean of a random sample of size N from a normal population with a mean $\overline{\overline{X}}$ and σ is the estimate of the standard deviation of the normal population as estimated from the sample.
Variability	The variation of the data; the lack of tendency to concentrate
Mode	The most frequent or most common value; its value will correspond to the value of the maximum point of a frequency distribution.
Significant wave height	Generally defined as the mean value of the one-third highest waves. Reference 12 and correspondence with the Hydrographic Office indicate that th^ wave heights estimated by observers approximated the "significant" wave heights.

ABSTRACT

Available observations of wave heights have been assembled and evaluated in terms of statistical methods in connection with the study of the service strains and motions experienced by ships at sea. Curves have been fitted to the distribution patterns, and confidence bands, averages, and standard deviations have been computed. Distribution patterns for wave heights observed in different parts of the world are all of the same type with a peak displaced toward the lower wave heights. Pitching motions measured on a ship at sea also follow this same pattern.

INTRODUCTION

The David Taylor Model Basin is making a study of the motions and strains in ships at sea for the purpose of evaluating and improving methods for the design of the ship girder and its structural components. It is probable that the frequency-distribution patterns of strains and motions of ships at sea will be similar to those of wave heights. It is also expected that the year-to-year variability in the distribution patterns of wave heights will be of the same order of magnitude as the year-to-year variability in the distribution patterns of ship motions and dynamic hull-girder stresses inasmuch as the latter are, to a large degree, functions of the wave heights and wave lengths. To verify these expectations, observed wave heights have been obtained from the Weather Bureau and the U.S. Hydrographic Office. These data and data measured by the Model Basin on the USCGC CASCO have been studied to determine (1) the type of distribution pattern, (2) the variation in this pattern over a period of time, and (3) the mathematical function which will best fit these data. The results of the third phase of the study are presented in this report.

PRESENTATION OF DATA

Figures 1, 2, and 3 are frequency distributions of wave heights, that is, depth from crest to trough, obtained from various sources. These distributions are presented in the form of bar-type graphs or histograms. The ordinates of these histograms give the percent of total observations or measured values that fell between given limits of wave height as indicated by the abscissa.

OBSERVED WAVE HEIGHTS

Weather Bureau Data

Figure 1 shows yearly and combined wave-height data which were furnished by the U.S. Weather Bureau at the request of the Taylor Model Basin.[1] These data represent wave-height

[1]References are listed on page 13.

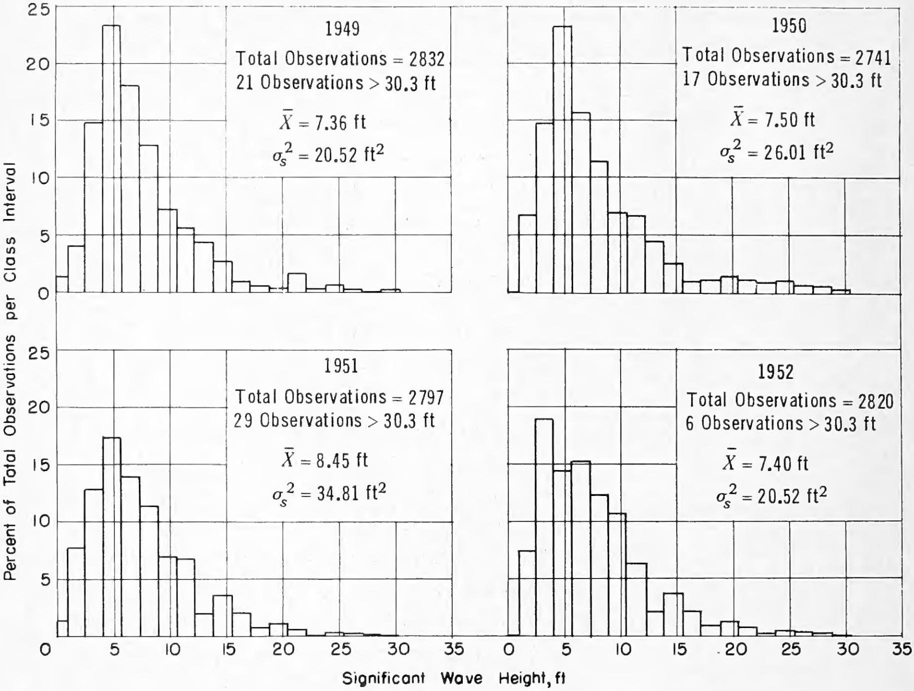

Figure 1a - Frequency Distributions of Yearly Samples

distributions determined from observations made by weather ships at ocean station "Charlie" (52° N, 37° W) in the North Atlantic from 1 January 1949 to 31 May 1953. The observations were made every three hours by trained weather observers in accordance with instructions prescribed by the United States Weather Bureau.[2] The observations are reported as the average of the significant* wave heights. Only one quantitative measurement was recorded each time the sea was observed.

Hydrographic Office Data

Figure 2 shows combined frequency distributions of wave heights for periods of 2, 7, and 40 years tabulated by the U.S. Hydrographic Office[3] at the request of the Model Basin. These observations, also at station "Charlie," were made by German merchant ships from 1901 to 1939. The data are not as reliable as the data presented in Figure 1; because routes

*Generally defined as the mean value of the one-third highest waves; see definitions page vi.

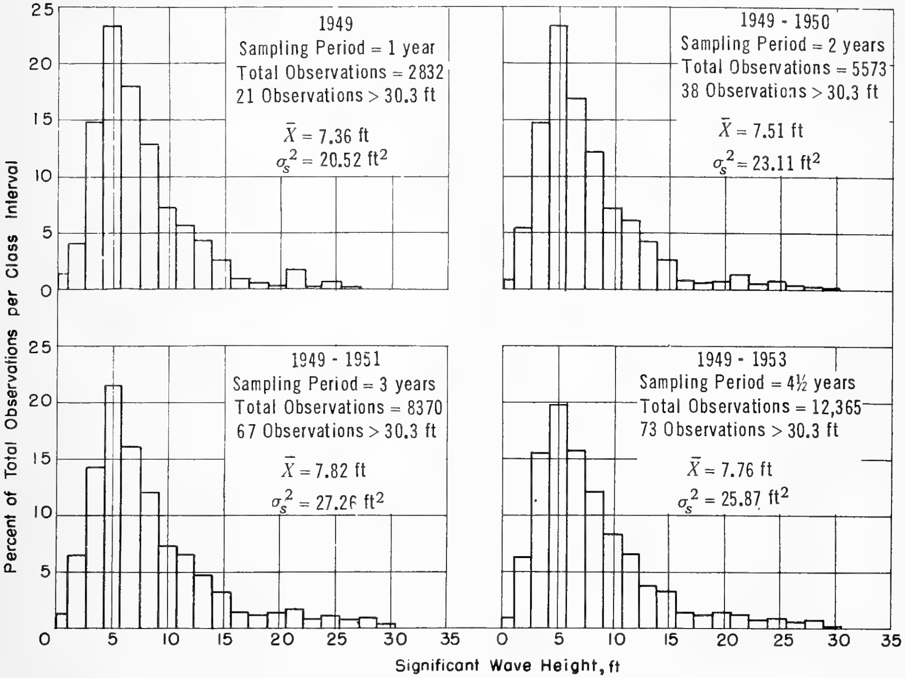

Figure 1b - Combined Frequency Distributions

Figure 1 - Frequency Distributions of Samples of Significant Wave Heights Observed at Ocean Station "Charlie" by U.S. Weather Observers

$\bar{X}$ is the mean and, σ_s^2 is the variance. The observations greater than 30.3 ft were included in the totals given but are not shown on the histograms.

were often avoided at times of high seas. Fewer extreme values were recorded.

MEASURED WAVE DATA

Figure 3a is the frequency-distribution pattern of measured wave heights produced by the wavemaker at the Taylor Model Basin. These were simulated to represent a characteristic confused sea. Only 43 measurements were made.

Figure 3b shows a frequency distribution of wave heights measured at sea by means of a pressure recorder. These data were tabulated on a form that shows the relation between wave heights, lengths, and periods.[4] Since measurements were made for a period of only 30 minutes, it may be assumed that they represent the characteristics of the sea at that time and

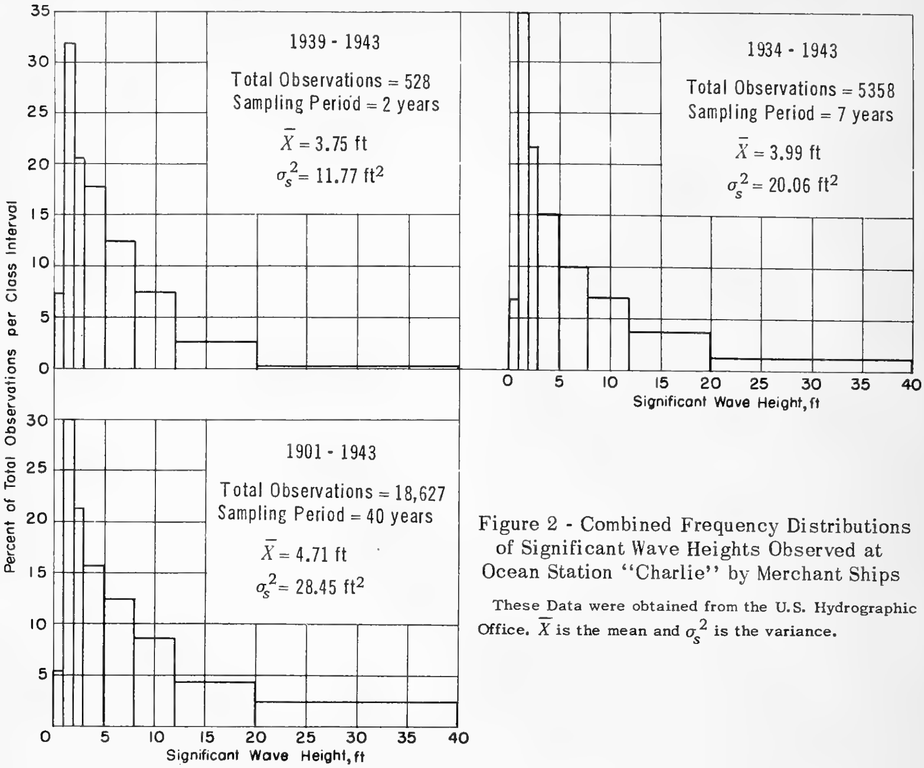

Figure 2 - Combined Frequency Distributions of Significant Wave Heights Observed at Ocean Station "Charlie" by Merchant Ships

These Data were obtained from the U.S. Hydrographic Office. $\overline{X}$ is the mean and σ_s^2 is the variance.

at that particular geographic location. It should be noted that the waves were of very small height (less than 140cm ≈ 4.6 ft).

OBSERVED WAVE HEIGHTS AND MEASURED SHIP MOTIONS

As pointed out in the introduction of this report, wave-height distributions could reasonably be expected to have the same type of pattern as ship motions and stresses. This similarity was evidenced by the weather ship USCGC CASCO.[5] Figure 4 shows the frequency distributions of wave-height observations and pitch-angle measurements made during this test. Both were made at 3-hr intervals over a period of one month at station "Charlie." No waves less than 1 ft nor pitch angles less than 1 deg were recorded.

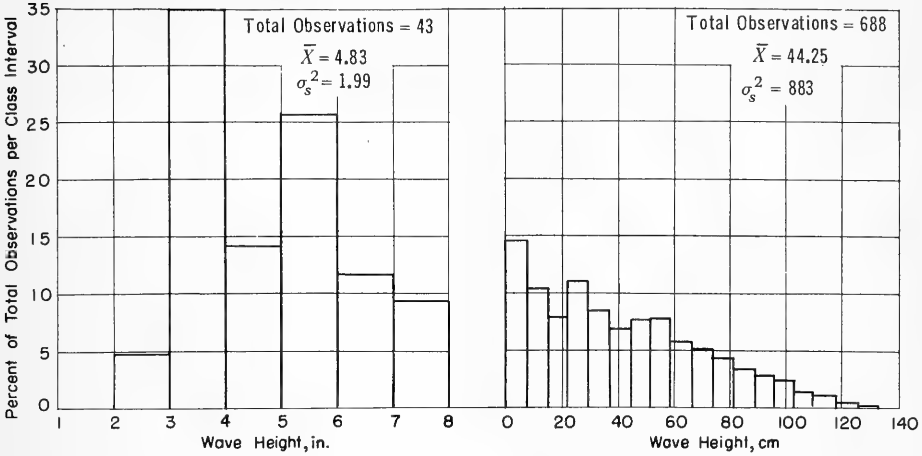

Figure 3a - Frequency Distribution of Measured Wave Heights Produced by TMB Wavemaker to Simulate a Confused Sea

Figure 3b - Frequency Distribution of Wave Heights at Sea as Measured by a Pressure Recorder

All wave heights for a 30-min period are included.

Figure 3 - Frequency Distributions of Measured Wave Heights

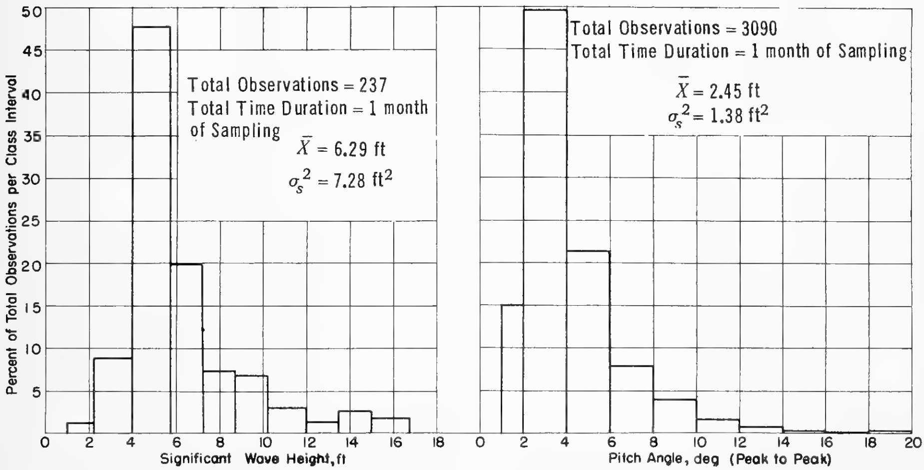

Figure 4 - Frequency of Wave Heights Observed and Pitch Angles Recorded on the USCGC CASCO at 3-hr Intervals over a One-Month Period at Ocean Station "Charlie"

$\bar{X}$ is the mean and σ_s^2 is the variance

ANALYSIS OF DATA

DISTRIBUTION PATTERNS

Examination of Figures 1 through 4 shows that all have a similar type of frequency distribution, that is, distributions peak towards the lower wave heights. Similar distributions presented in Figure 11 of Reference 6 also showed such patterns. The data presented there[6] were compiled from charts of observations made by Japanese merchant ships in the North Pacific during the 15-yr period from 1924 through 1938. These charts[7] are available at the U.S. Hydrographic Office at Suitland, Maryland. Areas of observations were broken down into 2-deg squares, that is, 2 deg latitude by 2 deg longitude. A study of these charts, which present the data in the form of histograms, leads to the conclusion that, in general, these histograms are also peaked in the direction of the lower wave heights.

On the basis of a study of the experimental data thus far available to the author, there is a strong indication that the frequency distributions of wave heights may be approximated by a straight line when plotted on logarithmic probability paper. Figure 5 shows some of the patterns obtained from the data of Figures 1, 2, and 3 and Reference 6. This approximation of the wave-height distributions by a straight line means that they approach a logarithmically normal distribution, that is, if the frequency is plotted as a function of the logarithm of wave height, the distribution will be normal.

FITTING MATHEMATICAL CURVES TO FREQUENCY DISTRIBUTION

In addition to the log-normal curve two other types of curves have been fitted to the Weather Bureau data (Figure 1) in order to find a suitable mathematical function which might be used to represent the observations. The Weather Bureau data were chosen because they appeared to have been obtained by the most reliable and consistent sampling procedure. The fitted curves are shown in Figures 6 and 7. The first is a Pearson Type I Curve whose shape is based on the values of the moments μ_i of the given frequency distribution and whose origin is taken at the mode computed from the measured distribution. The curve is defined by the equation given in Figure 6 and discussed in Appendix 1. The second fitted curve, Figure 7, is of the form known as the "random walk" distribution. It has been shown[8] that if the sea elevation ξ may be represented by a narrow spectrum, the probability that at any fixed location the wave height h lies between h and $h + dh$ is approximately

$$P(h)dh = \frac{2h}{\overline{h^2}} e^{\frac{-h^2}{\overline{h^2}}} dh \qquad [1]$$

where $\overline{h^2}$ is the mean of h^2. If the sea has a narrow spectrum, the elevations ξ of the wave surface have a normal distribution, see Figure 8.

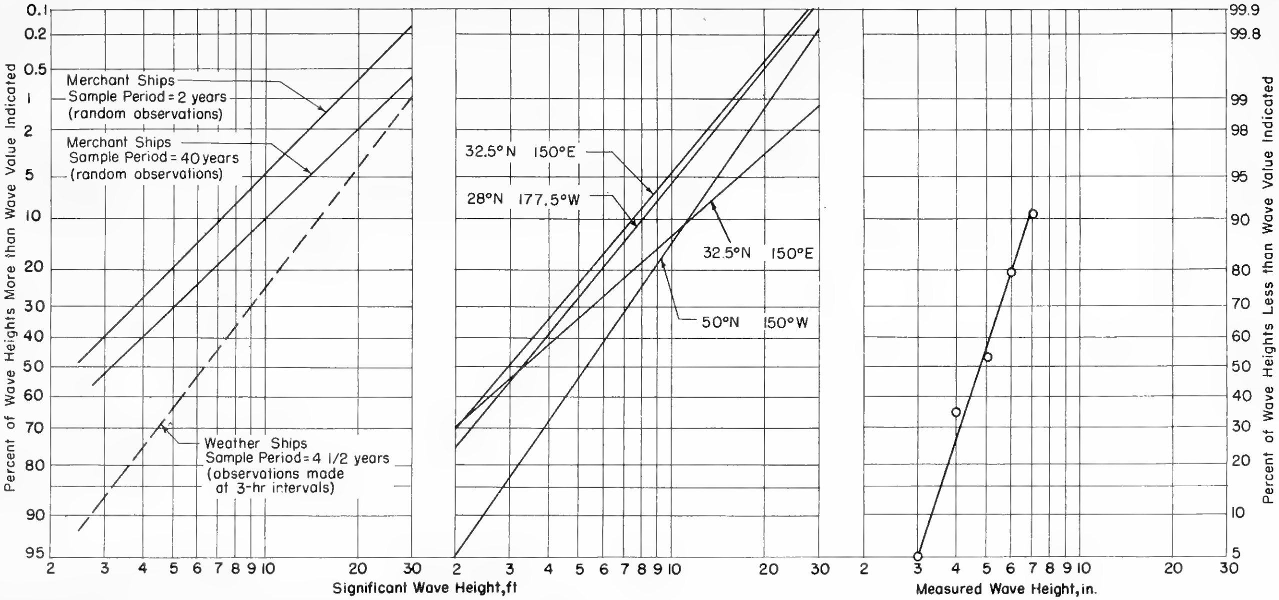

Figure 5a - Probability Distributions of Significant Wave Heights Observed at Ocean Station "Charlie" by Merchant Ships and Weather Observers

Figure 5b - Probability Distributions of Significant Wave Heights Observed at Four Localities in North Pacific by Merchant Ships for a 3-yr Period

Figure 5c - Probability Distribution of Measured Wave Heights Produced by TMB Wavemaker

Figure 5 - Probability Distributions of Wave Heights

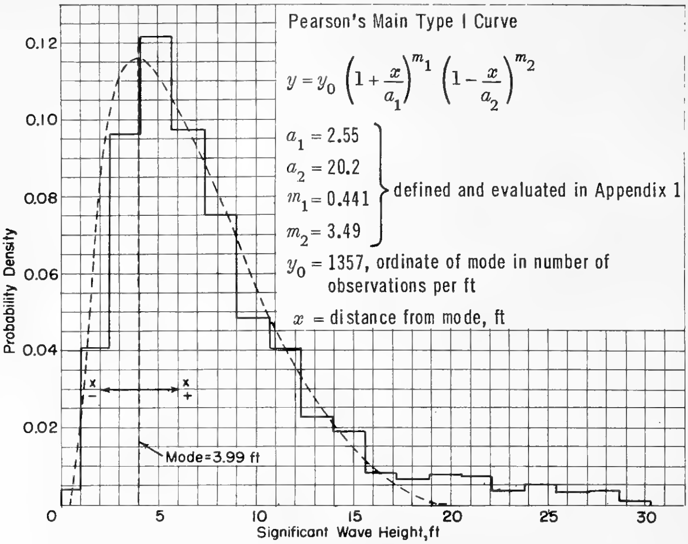

Figure 6 - Pearson's Main Type I Curve[8] Fitted to Probability Density Distribution of Significant Wave Heights Observed at Ocean Station "Charlie" by U.S. Weather Observers from January 1949 to June 1953

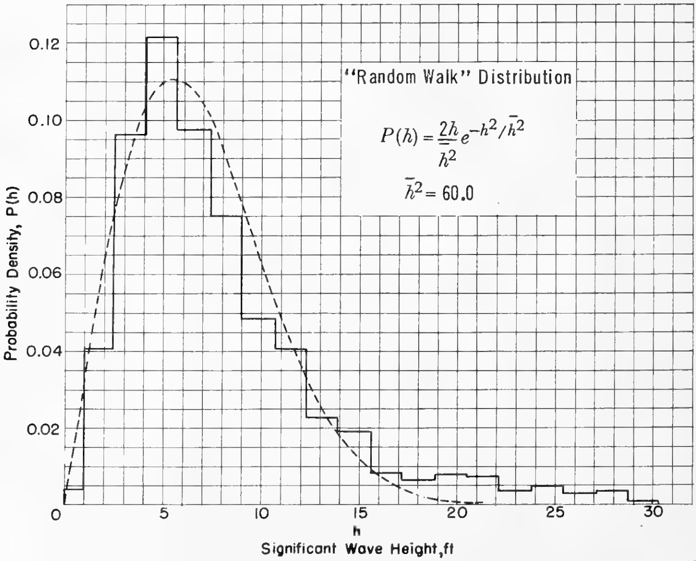

Figure 7 - "Random Walk" Distribution Fitted to Probability Density Distribution of Significant Wave Height Observed at Ocean Station "Charlie" by U.S. Weather Observers from January 1949 to June 1953

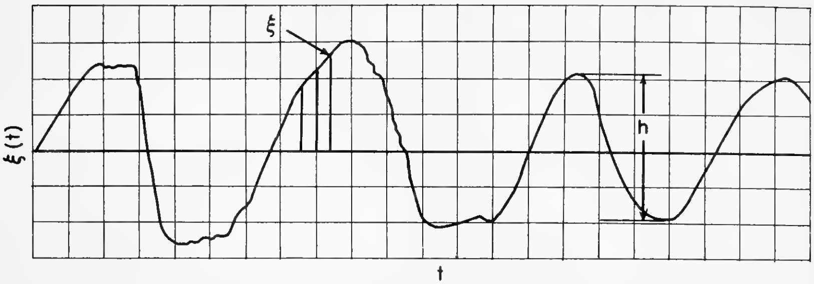

Figure 8 - Wave Record Showing Elevation ξ and Wave Height h

It is not necessarily true that a sea for which the wave heights follow the probability density function [1] will have a normal distribution of $\xi(t)$, where $\xi(t)$ is the instantaneous wave elevation.

Reference 8 gives the ratio

$$\frac{h^{(p)}}{h_R} = \frac{h}{h_R} + e^{h^2/\bar{h}^2} \int_h^\infty e^{-\eta^2/\bar{h}^2} d\frac{\eta}{h_R} \qquad [2]$$

$h^{(p)}$ denotes the mean value of the first pN of the N wave heights when arranged in descending order of magnitude, where p is a fraction between 0 and 1. Thus the average of the "significant waves" is $h^{(1/3)}$. It should be noted that, experimentally, it is difficult to find $\bar{h}$ from measurements of h inasmuch as the average depends to a considerable degree upon the lower limit to which the wave heights are measured; see Reference 4 for a discussion of this effect.

The distribution plotted in Figure 7 is fitted with the mathematical curve given in Equation [1]. The value of $\bar{h}^2$ which gives the best fit is $\bar{h}^2 = 60.0$.* As stated on page 10 the random walk theory holds only if the sea has a narrow spectrum. It may well be that the spectrum of the sea for the wave height distribution shown in Figure 7 will not remain narrow due to the fact that the sampling extended over a period of years.

MEANS AND STANDARD DEVIATIONS

Means and standard deviations were computed for the distributions shown in Figures 1 and 2 and are given in Table 1. It will be observed that the average of the data obtained from merchant-ship observations is much lower than the average of the Weather Bureau data. From the standard error of the mean (the standard deviation of the distribution of the means of samples) of the four yearly Weather Bureau samples, it may be stated that there are 99.7

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*The best fit was determined by a Chi square test.

TABLE 1

Means and Standard Deviations for Frequency Distributions of Wave Heights

Sample	Number of Observations	Mean ft	Standard Deviation ft	Average of Means ft	Standard Deviation of Means ft
Computed from Weather Bureau Data (Figure 1a)					
1949	2811	7.36	4.53		
1950	2724	7.50	5.10	7.69	0.45
1951	2768	8.45	5.90		
1952	2814	7.40	4.53		
Computed from Weather Bureau Data (Figure 1b)					
1 yr	2811	7.36	4.53		
2 yrs	5535	7.51	4.81		
3 yrs	8303	7.82	5.22		
4½ yrs	12,272	7.76	5.07		
Computed from Hydrographic Office Data (Figure 2)					
2 yrs	528	3.75	3.43		
7 yrs	4830	3.99	4.48		
40 yrs	18,627	4.71	5.33		

chances out of 100 that the average mean computed, 7.69 ft, will be no further away from the true mean than 1.35 ft.[9] ($3\sigma = 1.35$ ft) for the period 1949 to 1952.

CONFIDENCE BANDS

Figure 9 shows confidence bands fitted to the probability density distribution of the Weather Bureau data. These confidence bands, computed according to Kolmogorov's statistic,[10] show the interval within which the "true" distribution will fall at a probability level of 99 percent, that is, in 99 cases out of 100 random sampled distributions, the distribution will fall within these bounds. The requirement for the use of Kolmogorov's statistic is that the sampled wave heights be random and that the distribution of wave heights be continuous.

A plot of the data on probability paper is shown in Figure 9a. The encircled points were computed from the observed wave heights and the solid line represents a logarithmically normal distribution. In this figure the confidence bands were fitted to the observed points. The curve fitted to the probability density distribution shown in Figure 9b was obtained by taking the average probability density of the class intervals at their centers and fairing a curve through these points to make the area under the curve equal to the area under the

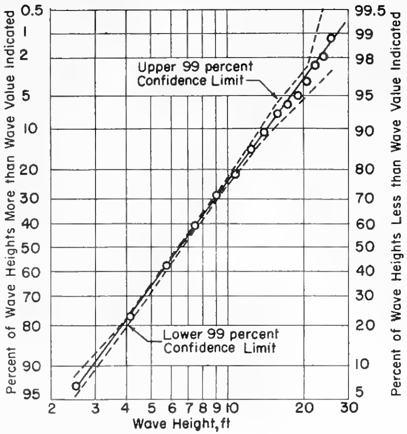

Figure 9a - Confidence Bands for Wave Height Probability Distribution

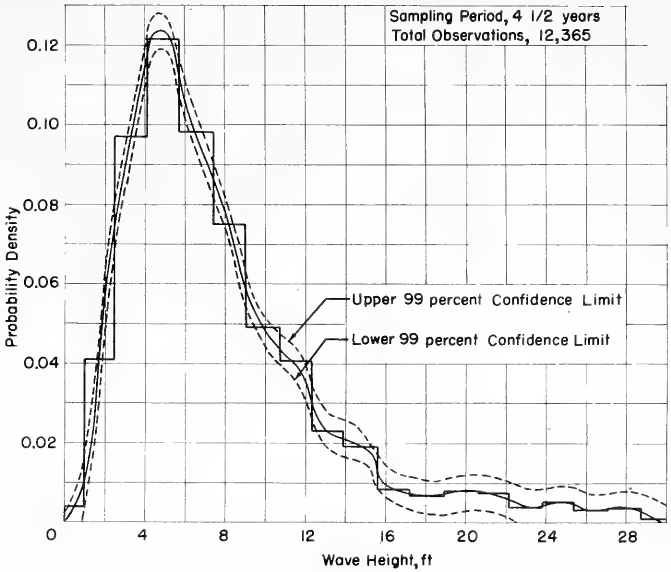

Figure 9b - Confidence Bands for Probability Density Distribution

Figure 9 - Ninety-Nine Percent Confidence Bands for Distribution of Significant Wave Heights Observed at Ocean Station "Charlie" by U.S. Weather Observers from January 1949 to June 1953 (also shown in Figure 1b)

histogram. The confidence bands were computed for this curve, utilizing Kolmogorov's statistic

CORRELATION BETWEEN WAVE HEIGHTS AND PITCH ANGLES

The scatter diagram* of Figure 10 shows the pitch angles measured on the CASCO plotted as a function of wave heights. Except for scattered observations, this diagram indicates a correlation between pitch angle and wave height which may be approximated by a straight line. The figure also indicates that the most probable combination is that corresponding to about 3-deg pitch angle and 5-ft significant wave height.

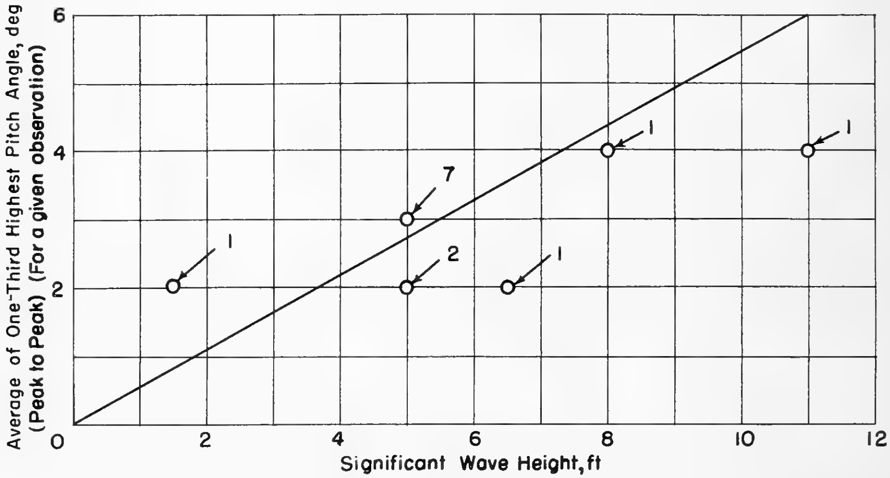

Figure 10 - Scatter Diagram of Pitch and Wave Height Data of Figure 4

A straight line was faired by using pitch angle measurements when ship was headed into the waves; these are indicated by O; the numbers give the number of observations.

DURATION OF SAMPLE

An estimation of the period throughout which samples must be taken in order to permit a statistically valid prediction is often necessary. The details of the computation involved for two such methods are given in Appendix 2.

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*A scatter diagram[9] is a method of showing the relationship between two associated variables. In this form the independent variable is placed along the abscissa while the dependent variable is placed along the ordinate. It is obvious that if the relationship between the two variables were perfect, every given value on the abscissa would indicate a value of the ordinate. If there is a direct simple relationship between the variables plotted, the points will tend to fall on some curve, possibly a straight line.

CONCLUSIONS

1. Frequency distributions of wave heights are not normal but tend to peak toward the lower wave heights. They do, however, have a pattern which is approximated by a logarithmically normal distribution.

2. The frequency distribution of pitch angle for the USCGC CASCO has the same general form as that shown to be applicable for the wave heights. It is reasonable to expect that this pattern will also hold for other ships.

3. The frequency distribution patterns of the pitch angles measured on the USCGC CASCO show a correlation with those of the wave height observations.

4. The Pearson Type I Curve may be fitted to frequency distributions of wave heights.

ACKNOWLEDGMENTS

The studies made in conjunction with this report were done under the supervision and guidance of Mr. N.H. Jasper. Members of Statistical Engineering Laboratory of the National Bureau of Standards, namely Mr. I. Richard Savage, Mr. Marvin Zelen, and Dr. Edgar King made suggestions as to treatment and presentation of data. The wave height data presented in various figures was obtained with the cooperation of members of the Division of Oceanography, U.S. Hydrographic Office and Aerology Branch, CNO. Finally, the author is indebted to Dr. George Suzuki for review of the statistical mathematics in this report and the suggestion of Appendix 2.

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APPENDIX 1

PEARSON-TYPE DISTRIBUTION CURVES

A set of curves that may be fitted to different frequency distributions was compiled by the statistician Karl Pearson. The theoretical derivation and calculations necessary for fitting these curves are described in Reference 11. The curve type which best fits a frequency distribution may be identified from criteria calculated on the basis of the values of the moments μ_i.

The steps for identifying and computing the constants for the fitting of the curve shown in Figure 6 are given here. The numerical values are those for the frequency distribution of the Weather Bureau data for the 4½-yr period, Figure 1.

The moments measured about the mean value of the distribution are:

$$\mu_1 = \frac{\Sigma f z}{N} = 0 \quad [3]$$

$$\mu_2 = \frac{\Sigma f z^2}{N} = 13.68 \quad [4]$$

$$\mu_3 = \frac{\Sigma f z^3}{N} = 40.27 \quad [5]$$

$$\mu_4 = \frac{\Sigma f z^4}{N} = 593.6 \quad [6]$$

where z represents the deviation of the actual value f from the mean,

f is the frequency, and

N is the total frequency of the sample.

The criteria β_1, β_2, and κ computed from the preceding moments are:

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = 0.6336 \quad [7]$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = 3.173 \quad [8]$$

$$\kappa = \frac{\beta_1(\beta_2 + 3)^2}{4(4\beta_2 - 3\beta_1)(2\beta_2 - 3\beta_1 - 6)} = -0.3596 \quad [9]$$

Since the criterion κ is negative, it identifies Pearson's Main Type I Curve as the most suitable one. This curve is defined by the equation

$$y = y_0 \left(1 + \frac{x}{a_1}\right)^{m_1} \left(1 - \frac{x}{a_2}\right)^{m_2} \quad [10]$$

where y_0 is the ordinate at the mode,*

x is the distance from the mode, and

m_1, m_2
$a_1, \& a_2$ are computed from the equations which follow and

$$\frac{m_1}{a_1} = \frac{m_2}{a_2} \quad [11]$$

First a parameter r must be evaluated

$$r = \frac{6(\beta_2 - \beta_1 - 1)}{(6 + 3\beta_1 - 2\beta_2)} = 5.934 \quad [12]$$

$$a_1 + a_2 = \frac{1}{2}\sqrt{\mu_2 \{\beta_1 (r + 2)^2 + 16(r + 1)\}} = 22.71 \quad [13]$$

$$m_1, m_2 = \frac{1}{2}\left\{(r - 2) \pm r(r + 2)\sqrt{\frac{\beta_1}{\beta_1 (r + 2)^2 + 16(r + 1)}}\right\} \quad [14]$$

When μ_3 is positive, m_2 is the root corresponding to the plus sign; if μ_3 is negative, m_2 is the root corresponding to the minus sign.

For our numerical example

$$m_1 = 0.441$$

$$m_2 = 3.493$$

Finally

$$y_0 = \frac{N}{a_1 + a_2} \cdot \frac{m_1^{m_1} m_2^{m_2}}{(m_1 + m_2)^{m_1 + m_2}} \cdot \frac{\Gamma(m_1 + m_2 + 2)}{\Gamma(m_1 + 1)\,\Gamma(m_2 + 1)} \quad [15]$$

Tables of the gamma function are given in the reference 11. With the use of logarithms, y_0 was computed to be 1404, and the mode of the distribution was found to be at 3.99 ft. Equation [10] becomes:

$$y = 1357 \left(1 + \frac{x}{2.55}\right)^{0.441} \cdot \left(1 - \frac{x}{20.2}\right)^{3.49} \quad [16]$$

This gives the frequency distribution in terms of a class interval of unit length. Therefore for a class interval of 1.6 ft, the frequency would be 2171. Since the probability density distribution was desired, y was divided by N.

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*The mode is the most frequent or common value; it will correspond to the maximum ordinate of the frequency distribution.

APPENDIX 2

DURATION OF SAMPLE

Assume in the first approach that for the specific locale indicated, one of the wave-height distributions has a mean

$$\bar{X} = 7.76 \text{ ft}$$

and

$$\sigma_s = 5.07 \text{ ft}$$

Then, by standard statistical procedure, the sample size necessary to obtain a sample mean which differs from the true mean by no more than 5 percent with a confidence coefficient of 90 percent can be obtained by solving for n in the equation

$$\sqrt{n} = \frac{k\ \sigma}{0.05\ \bar{\bar{X}}} \qquad [17]$$

where $\pm k$ is the particular abscissa on the "t" distribution with n defined such that the area under the "t" distribution between $\pm k$ is 90 percent. By substitution

$$n = \left[\frac{1.67\ (5.07)}{0.05\ (7.76)}\right] \approx 467$$

that is approximately 467 indcpendent and random observations are necessary.

Weather observations are characterized by the lack of independence in successive observations when the time interval between observations is relatively short. The duration of interval necessary to insure independence cannot be determined. If one independent observation can be obtained every 10 days, then by the above calculations, more than 13 years are necessary to obtain a sample fulfilling the stipulated conditions. If 7 days are sufficient, then about 9 years are necessary.

As another approach, suppose that the means of the wave heights obtained for each year (Table 1, page 10) represent independent observations. In this treatment the statistical "population" is the totality of these independent observations. Basing the following computations on the observed mean values, it is found that $n = 7$. This implies that 7 years of rather extensive observations are necessary to fulfill the conditions imposed.

Problem:

Find the number of samples which are required to make

$$\left|\frac{\bar{\bar{X}}}{\bar{X}_i} - 1\right| \leq 0.05 \qquad [18]$$

with a probability of 0.90, where $\bar{\bar{X}}$ is the mean of the population and $\bar{X}_i$ is the mean of the ith sample.

Procedure:

1. Find the n means of n distributions, $\bar{X}_1$, $\bar{X}_2$, $\bar{X}_n$.

2. Assume the mean of the population to equal the average of these means and compute

$$\bar{\bar{X}} = \frac{\bar{X}_1 + \bar{X}_2 + \ldots . \bar{X}_n}{n} \qquad [19]$$

3. Assume that the standard deviation of the means of n yearly distributions is equal to the standard deviation of the population of means of yearly distributions and compute

$$\sigma = \sqrt{\frac{\Sigma d^2}{n-1}} \qquad [20]$$

where d is the distance of the mean of the sample from the mean of the population.

4. The mean of any one distribution of c yearly means may take on a range of values

$$\bar{X}_i = \bar{\bar{X}} \pm \frac{k\,\sigma}{\sqrt{c}} \qquad [21]$$

$\bar{\bar{\bar{X}}}$ is the mean value of the population,
σ is the standard deviation of the population,
$\bar{X}_i$ is a sample of c means of the population,
k is taken from a student's "t" table at probability level 0.90 and $(c - 1)$ degrees of freedom, and
c is the number of distributions required; in other words the number of independent observations.

5. From Equations [18] and [21] obtain

$$\left|\frac{k\,\sigma}{\sqrt{c}}\right| \leq 0.05\ |\bar{X}_i|$$

Thus, solving for $\sqrt{c}$, gives

$$\sqrt{c} \geq \frac{k\,\sigma}{0.05\,\bar{X}_i} \qquad [22]$$

6. Values of c are assumed until Equation [22] is satisfied.

Example:

Assume each Weather Bureau yearly distribution to represent one sample. Then, from Figure 1 (with $n = 4$)

$\bar{X}_1 = 7.36$ (year 1949)
$\bar{X}_2 = 7.50$ (year 1950)
$\bar{X}_3 = 8.45$ (year 1951)
$\bar{X}_4 = 7.40$ (year 1952)

Therefore

$$\bar{\bar{X}} = \frac{30.71}{4} = 7.69$$

$$\sigma = \sqrt{\frac{\Sigma\, d^2}{n - 1}} \qquad [23]$$

$$\sigma = \sqrt{\frac{0.10 + 0.03 + 0.59 + 0.08}{3}} = 0.52$$

and

$$\sqrt{c} > \frac{0.52\,k}{0.05\ (7.36)} = 1.4\,k \text{ from Equation [22]}$$

when $\bar{X}_1$ is used.

As a first approximation, assume $c = 5$. The value $k(t)$ from the "t" table at a 0.90 probability level and four degrees of freedom is 2.13

$$\sqrt{5} > 1.4 \cdot 2.13$$

As a second approximation, assume $c = 6$

$$\sqrt{7} \approx 1.4 \cdot 2.02$$

Therefore $c = 7$, indicating that measurements would have to be made over a period of 7 yr to establish a distribution pattern which would be valid at a probability level of 0.90 such that Equation [19] is satisfied, providing that no year-to-year bias exists in the data.

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Available observations of wave heights have been assembled and evaluated in terms of statistical methods in connection with the study of the service strains and motions experienced by ships at sea. Curves have been fitted to the distribution patterns, and confidence bands, averages, and standard deviations have been computed. Distribution patterns for wave heights observed in different parts of the world are all of the same type with a peak displaced toward the lower wave heights. Pitching motions measured on a ship at sea also follow this same pattern.

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2. Water waves - Height - Frequency measurement
3. Girders (Marine) - Stresses - Frequency measurement
4. Correlation functions
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