A
PRACTICAL TREATISE
ON
THE STRENGTH AND STIFFNESS
OF
TIMBER,

INTENDED AS A GUIDE FOR

ENGINEERS, ARCHITECTS, CARPENTERS,

ETC. ETC. ETC.

IN ESTIMATING THE STRENGTH, STIFFNESS, AND MAGNITUDE OF BEAMS TO BE EMPLOYED IN BUILDINGS AND OTHER WORKS.

TO WHICH IS ADDED,

AN ABSTRACT OF PROBLEMS AND RULES,

WITH TABLES FOR ESTIMATING BY INSPECTION

THE STRENGTH, MAGNITUDE, AND FLEXURE,

OF CAST IRON AND TIMBER BEAMS:

ALSO,

TABLES OF THE PROPERTIES OF TIMBER, METAL, BRICK, CLAY, EARTH, AND STONE.

By WILLIAM TURNBULL,

AUTHOR OF "A TREATISE ON THE STRENGTH OF CAST IRON," AND ASSISTANT ENGINEER IN THE OFFICE OF MESSRS. COTTAM AND HALLEN, WINSLEY STREET, OXFORD STREET, LONDON.

LONDON:

JOSIAH TAYLOR, ARCHITECTURAL LIBRARY,

59, HIGH HOLBORN.

1833.

LONDON:
J. MOYES, CASTLE STREET, LEICESTER SQUARE.

PREFACE.

WHEN writers bring their labours so rapidly before the Public, as scarcely to allow time for turning over the pages of one production before the publication of another is announced, it is usual for them to make some apology for the intrusion,—to describe the plan of their works,—and to explain the motives which prompted their exertions. The Author of the present performance, however, considers it unnecessary to make any apology,—he presumes that the plan and nature of the Work are sufficiently described in the title-page, and his motive for laying it before the Public is briefly as follows.

About the beginning of the present year (1832), there was published, by the same author, a small Treatise on the Strength, Flexure, and Stiffness of Cast Iron Beams and Columns, in which are embodied Tables and Rules for adapting the same principles of calculation to beams of malleable iron and several kinds of timber, such as are most commonly employed in buildings and other mechanical constructions; but on further consideration, the Author conceived, that if a small independent Treatise on Timber were drawn up after the same plan, it would be found of superior utility to practical persons, on account of the ambiguity that may arise in adapting the numbers derived from the properties of one material to the principles and mechanical conditions belonging to another. This was the chief motive that urged him to the undertaking, and if the execution of the work be found in any degree suitable to the object, the wishes of the Author have been fully attained.

No. 2, Winsley Street.
Nov. 1832.

STRENGTH OF TIMBER.

In treating on the *resistance of timber*, there are several properties and terms peculiar to the subject of very frequent occurrence, which it is necessary in the first place to explain. These are,—*Stress*, *Strain*, *Straining force*, *Strength*, *Stiffness*, *Flexure* or *Deflexion*, *Extension*, *Cohesive force*, *Elastic force*, and *Modulus of elasticity*.

DEFINITIONS.

1. *Stress*, is the force excited in the material, or that power, which being applied externally endeavours to produce fracture; it is synonymous with the straining force.

2. *Strain*, is the effect produced by the stress or straining force.

3. *Straining force.* (See *Stress*.)

4. *Strength*, is that property of bodies by which they resist breaking or fracture.

5. *Stiffness*, is that property of bodies by which they resist bending or flexure.

6. *Flexure or deflexion*, is the space through which a body is bent by means of the stress or straining force.

7. *Extension,* is the quantity by which the length of a body is augmented when drawn in the direction of its fibres.

8. *Cohesive force,* is that property by which bodies resist the separation of their parts, and is measured by the force or weight that would tear them asunder.

9. *Elastic force,* is that property by which bodies endeavour to recover their original state when the straining force is withdrawn.

10. *The modulus of elasticity,* is the measure of the elastic force.

PRINCIPLES OF COMPARISON.

The comparative strength of different materials, is expressed by the quotient of their cohesive forces.

The comparative stiffness of different materials, is expressed by the quotient of their moduli of elasticity.

The comparative extensibility of different materials, is expressed by the quotient of their extensions.

The weight of the modulus of elasticity, is equal to the quotient that arises when the cohesive force is divided by the extension.

These things being premised, we shall proceed to consider the nature of the strains to which materials are exposed; these are of four kinds, and are as follow: viz.

The resistance to cross strains, or that force which a body opposes to being broken across.

The resistance to tension, or that force which a body opposes to being torn asunder in the direction of its length.

The resistance to compression, or that force which a body opposes to being compressed in the direction of its length.

The resistance to torsion, or that force which a body opposes to being twisted asunder.

The forms of beams that are most commonly exposed to these several strains, are the *rectangular,* the *square* and *cylindrical;* and to these three forms, shall our inquiries be confined.

Of the Rectangular Beam when exposed to a Transverse or Cross Strain.

The strain which most commonly acts on materials of any kind, is that which tends to break them in a transverse direction; and the writers on the strength of materials have shewn, that when a beam rests horizontally on two supports, the strain is the greatest when the force acts at the middle of the length; our subject, therefore, divides itself into five distinct cases, as follow, viz.

1. *When the beam is supported at the ends, and the straining force applied at the middle of the length.*

2. *When the beam is supported at the ends, and the straining force applied at some intermediate point between the middle and one of the supports.*

3. *When the beam is supported at the ends, and loaded uniformly over the length.*

4. *When the beam projects from a wall, into which one end is fixed, and a load applied at the other.*

5. *When the beam projects from a wall, into which one end is fixed, and loaded uniformly over the length.*

In framing rules for calculating the several conditions of these five cases, we have to observe, that since beams are seldom exposed to strains that break them, it would be inconsistent with safety and the maxims of practice, should our rules bring out the absolute strength; and as there is a point beyond which if beams be not strained, they will always restore themselves to their natural state when the straining force ceases to act, this point is the limit of safety; for it has been shewn, that if beams are strained beyond it, the flexure increases with time if the load be suffered to remain; and a very small additional load will produce fracture. Hence it appears, that rules designed to assist the mechanic in the construction of beams, must, when safety is aimed at, be so constituted as to give results within the elastic power of the material; and such are those which we now proceed to investigate.

The fundamental principle on which the following calculations depend is, that in rectangular bars or beams of any material whatever, whether they be large or small, long or short, if they are supported at the ends and loaded in the middle, *The product of the breadth drawn into the square of the depth both in inches, divided by the length in feet drawn into the load in pounds, must be the same in all.*

Now, if B represent the breadth in inches, D the depth in inches, L the length of bearing or distance between the supports in feet, and W the weight in pounds used in and determined from the experiment; then, putting b, d, l and w for the breadth, depth, length and weight, corresponding to the case for which the calculation is made; we have,

according to our fundamental principle,

$$\frac{BD^2}{LW} = \frac{bd^2}{lw};$$

which, by clearing the equation from fractions, becomes

$$lwBD^2 = LWbd^2. \qquad (A)$$

From the delicate and judicious experiments of the late *Mr. Thomas Tredgold,* it appears, that a bar of English oak of a medium quality, 1 inch broad, 2 inches deep and 24 inches between the supports, will bear 425 pounds at the middle of its length while the strain is just within the elastic power of the material; let these numbers be substituted for B, D, L and W in equation (A), and it becomes, by taking the length in feet and omitting the fraction,

$$lw = 212bd^2. \qquad (1)$$

This equation has been obtained on the supposition that the beam rests horizontally on its supports, and that the strain has been exerted exactly at the middle between them; on this supposition therefore, must the calculation proceed.

By carefully examining the composition of the preceding formula, it appears, that it consists of one constant and four variable factors, any three of which with the constant factor being given, the fourth can always be found; the complete analysis of the equation branches itself into four distinct parts; but since in practical cases, the length of the beam must generally be limited by the circumstances of situation, we shall in what follows, confine our inquiries to three parts only, always supposing that the length is given.

The three combinations of data are as below.

1. Given b, d and l; to find w.
2. Given d, l and w; to find b.
3. Given b, l and w; to find d.

From which we derive the following problems.

PROBLEM I. *In the equation* $lw = 212bd^2$, *there are given, the breadth* b, *the depth* d, *and the length* l; *to find the weight* w.

Here, the required term in combination with the given length, occupies the left-hand side of the equation, and the composition is indicated by multiplication; now, it is a principle in the resolution of equations,

> *That, by whatever process the composition is effected, a process directly the contrary must be employed to accomplish the analysis;*

but division is contrary to multiplication, and the divisor obviously is, the quantity with which the unknown term is combined, viz. the length of the beam, which is here represented by the letter l; let each side of the given equation be divided by l, and we obtain

$$w = \frac{212bd^2}{l};$$

from which the following practical rule is derived.

Rule 1. Multiply 212 times the breadth in inches by the square of the depth in inches, and divide the product by the length in feet, for the weight required in pounds.

Example 1. A beam of oak, of the quality described in the experiment from which our fundamental data have been supplied, is 5 inches broad, 7 inches deep, and 22 feet between the supports; what load will it bear at the middle of its length, supposing its position on the props to be perfectly horizontal, and the strain within the elastic power of the material?

Here, by the rule, we have

$\frac{212 \times 5 \times 7^2}{22} = 2361$ pounds very nearly, for the load required.

It is evident that the load thus obtained, since it measures the straining force, must also include the effect produced by the weight of the beam. Now, the writers on the resistance of solids have shewn, that the whole weight of the beam acting as a uniform force, produces the same effect in augmenting the strain, as if one half that load were collected at the middle point; it may therefore be useful to shew, in what manner this effect is to be estimated.

Mr. Tredgold states in the detail of his experiments, that the specific gravity of oak, of the quality we have chosen is 830, that of water being 1000; but 1000 ounces are exactly $62\frac{1}{2}$ pounds; therefore, we have $1000 : 830 :: 62\frac{1}{2} : 51\frac{7}{8}$, or 52 pounds very nearly, in one cubic foot of oak; hence, to find the weight of the beam,

Multiply the solidity in cubic feet by 52, *and the product will be the weight in pounds.*

In the present example the solidity is $\frac{5 \times 7 \times 22}{144}$ $= 5\frac{25}{72}$ cubic feet; therefore, $5\frac{25}{72} \times 52 = 278$ lbs. and, as we have already stated, this load produces the same effect in straining the beam, as if one half of it, or 139 pounds were collected at the middle; hence we have

$$2361 - 139 = 2222 \text{ lbs.}$$

for the external load with which the beam ought to be strained.

But the formula itself may be so modified as to give the load which the beam will bear, independently of the above operation; and as this may be useful in allowing for the weight of the beam before its dimensions are known, we shall here give the modified equation.

It has been shewn above, that a cubic foot of oak contains 52 pounds very nearly; consequently, a bar one inch square will contain 0·36 pounds: hence, the area of the cross section of the beam in inches, multiplied by 0·36 times the length in feet, must express the weight; that is,

$$0{\cdot}36\,bdl = \text{the whole weight of the beam;}$$

and this is equivalent to $0{\cdot}18\,bdl$ applied at the middle of the length. Now, we have already shewn that the load which the beam ought to sustain, including the effect produced by its own weight, is $w = \frac{212\,b\,d^2}{l}$; hence, we have $w = \frac{212\,b\,d^2}{l} - {\cdot}18\,bdl$ for the load which it ought to sustain excluding such effect; and the modified expression is

$$l\,w = b\,d\,(212\,d - {\cdot}18\,l^2). \qquad (2)$$

Let both sides of this equation be divided by l, and we obtain

$$w = \frac{b\,d\,(212\,d - {\cdot}18\,l^2)}{l};$$

from which the following practical rule is derived.

Rule 2. From 212 times the depth of the beam in inches, subtract 0·18 times the square of the length in feet; multiply the remainder by the breadth drawn into the depth, both in inches, and divide the product by the length, for the weight in pounds that the beam will sustain with safety, excluding its own weight.

Taking the data of the preceding example, we get

$$212 \times 7 = 1484$$

$$22^2 \times 0{\cdot}18 = \underline{\ \ 87{\cdot}12\ \ } \text{ (subtract)}$$

$$\frac{1396{\cdot}88 \times 5 \times 7}{22} = 2222 \text{ pounds}$$

nearly, the same as found above.

PROBLEM II. *In the equation* $lw = 212\,bd^2$, *there are given the depth* d, *the length* l, *and the weight* w; *to find the breadth* b.

In the resolution of the preceding problem, we stated the relation that subsists between the composition of an equation and its analysis, it is therefore unnecessary to repeat the statement. In the present instance, the required term must be disengaged by two successive divisions, by reason of its being combined by multiplication with 212, the constant term in the original formula, and also with d^2, one of those that are variable; but the writers on the theory of numbers have shewn, that to divide successively by two or more quantities, is the same thing as to divide by their continued product; hence we obtain

$$b = \frac{lw}{212\,d^2};$$

from which the following practical rule is derived.

Rule 3. Multiply the length of bearing, or distance between the supports in feet, by the load to be supported in pounds, and divide the product by 212 times the square of the depth in inches, for the breadth required.

Example 2. A rectangular beam of oak, 7 inches deep and 22 feet between the supports, is found to sustain a load of 2361 pounds at the middle of its length, while the elastic force remains perfect; what is its breadth?

Here, by the rule, we have

$$\frac{2361 \times 22}{212 \times 7^2} = 5 \text{ inches, the breadth}$$

required.

This breadth has been obtained on the supposition that the given load includes the effect produced by the weight of the beam; but when it is required to determine the breadth such, that it will make allowance for this effect, we must have recourse to equation (2), where

$$l w = b d (212 d - \cdot 18 l^2).$$

Now, the expression for the breadth as obtained from this equation, is

$$b = \frac{l w}{d (212 d - \cdot 18 l^2)};$$

from which the following practical rule is derived.

Rule 4. Multiply the length of bearing, or distance between the supports in feet, by the load to be supported in pounds, and divide the product by 212 times the square of the depth in inches, diminished by ·18 times the square of the length drawn into the depth, for the breadth required.

Taking the data of the preceding example, we get

$$\frac{2361 \times 22}{212 \times 7^2 - \cdot 18 \times 22^2 \times 7} = 5 \cdot 312 \text{ inches,}$$

the breadth required.

Now, the load that a beam 5·312 inches broad, 7 inches deep, and 22 feet between the supports, will bear at the middle of its length, calculated by the rule to the first problem, is 2508 pounds; and half the weight of a beam of these dimensions is 147 pounds; therefore, $2508 - 147 = 2361$ pounds, the load proposed.

PROBLEM III. *In the equation $lw = 212\,b\,d^2$, there are given the breadth b, the length l, and the weight w; to find the depth d.*

The unknown term being disengaged from those with which it is combined, the expression for its square in terms of the other quantities, is $d^2 = \frac{lw}{212b}$; and by extracting the square root, we get

$$d = \sqrt{\frac{lw}{212b}};$$

from which we derive the following practical rule.

Rule 5. Multiply the length of bearing, or distance between the props in feet, by the load to be supported in pounds; divide the product by 212 times the breadth in inches, and extract the square root of the quotient for the depth required.

Example 3. A rectangular beam of oak, 5 inches broad, and 22 feet between the supports, is found to sustain a load of 2361 pounds at the middle of its length, while the elastic force remains perfect; what is its depth?

Here, by the rule, we have

$$\sqrt{\frac{2361 \times 22}{212 \times 5}} = 7 \text{ inches, the depth}$$

required.

But, as we observed respecting the breadth in the last problem, the above depth has been obtained on the supposition that the given load includes the effect produced by the weight of the beam; now, when it is required to determine a depth that will make allowance for this effect, we must revert to equation (2), where

$$lw = bd\,(212\,d - \cdot 18\,l^2).$$

Here, however, we must remark, that since both the first and second powers of the required term enter the equation, its resolution becomes more difficult, and can only be accomplished by the reduction of an adfected quadratic equation; the equation being resolved according to the rules given for that purpose by the writers on algebra, the expression for the depth in terms of the other quantities is

$$d = \frac{1}{212}\left\{ \cdot 09l^2 + \sqrt{l\left(\frac{212\,w}{b} + \cdot 0081\,l^3\right)} \right\}$$

The above expression for the value of d is very complicated; it does not, however, admit of a simpler arrangement; but the following practical rule may nevertheless be derived from it.

Rule 6. Divide 212 times the given load by the breadth in inches; to the quotient add the cube of the length in feet drawn into the decimal ·0081, and multiply the sum by the length; then, to the square root of the product add ·09 times the square of the length, and divide the sum by 212 for the depth required.

Taking the data of the foregoing example, we get

$$\frac{2361 \times 212}{5} = 100106{\cdot}4$$

$$22^3 \times {\cdot}0081 = \quad 86{\cdot}2488 \quad \text{(add)}$$

$$\sqrt{100192{\cdot}6488 \times 22} = 1484{\cdot}66$$

$$\frac{1484{\cdot}66 + 22^2 \times {\cdot}09}{212} = 7{\cdot}21 \text{ inches very nearly,}$$

the depth required.

Now, the load that a beam 5 inches broad, 7·21 inches deep, and 22 feet between the supports, will bear, calculated by the rule to the first problem, is 2504 pounds, and half the weight of a beam of these dimensions is 143 pounds; therefore, 2504 − 143 = 2361 pounds, the load proposed.

Hence, it appears, that Rules 4 and 6 calculate the dimensions of a beam that will sustain any proposed load at the middle of its length, together with the effect produced by the weight of the beam itself.

What has hitherto been done, applies to uniform rectangular beams of oak, when loaded at the middle of their length, and strained to the full extent of their elastic power; on many occasions however, it may be desirable to know the greatest load that a beam will support without fracture. Now, Mr. Tredgold has shewn, that a bar of oak one inch broad, 2 inches deep, and 24 inches between the supports, broke with a load of 1428 pounds applied at the middle of its length; let these numbers be substituted for B, D, L and W, in equation (A), and we obtain

$$lw = 714\,b\,d^2. \qquad (3)$$

From which we infer, that a force which merely destroys the elasticity, is to the force that produces fracture, as 212 to 714, or as 1 to 3·36. Mr. Tredgold employs the ratio of 1 to 3·3. (*See* his Essay on Cast Iron, Arts. 59 and 261).

The following practical rule determines the resistance to fracture.

Rule 7. Multiply 714 times the breadth in inches by the square of the depth in inches, and divide the product by the length in feet for the weight in pounds that will break the beam. Or thus,

Calculate the load that will destroy the elastic power of the material by the first rule, and multiply that load by 3·36 for the load that will produce fracture.

Example 4. Given the length of an oak beam

16 feet, breadth 15 inches, and depth 18 inches; required its strength, or the weight which, being suspended from the middle, will nearly break it?

Here, by the rule, we get

$$\frac{714 \times 15 \times 18^2}{16} = 216878 \text{ pounds, for the}$$

load that will break the beam.

Otherwise thus,

$$\frac{212 \times 15 \times 18^2}{16} \times 3{\cdot}36 = 216878 \text{ pounds,}$$

the same as before.

This is one of the examples proposed by Mr. John Banks in his "Treatise on the Power of Machines;" his result is 200475 pounds, whence we conclude that he had employed oak of an inferior quality to that from which our constant number has been derived; indeed, he says, in the detail of his experiments, that "the worst or weakest piece of dry heart of oak, 1 inch square and 1 foot long, did bear 660 pounds, though much bended, and 2 pounds more broke it:" from this we infer, that the strength of his specimen is to that of the specimen employed by Mr. Tredgold, as 660 to 714.

We may here observe, that the rules which we have given for beams placed horizontally, answer also for those that are inclined, provided the horizontal distance between the supports be considered as the length of bearing, and used in the rules accordingly.

Let φ be the angle of inclination, and l the distance in feet from the top of one support to the top of the other; then, by Trigonometry, the reduced length, or distance between the supports, becomes $l \cos \varphi$; let this be substituted for l in equation (1), and we have for inclined beams,

$$lw \cos\varphi = 212\,bd^2. \qquad (4)$$

Therefore, when it is required to determine the load that an inclined beam will support at the middle of its length, the inclination and the dimensions of the beam being given, we have only to disengage the required term in the equation from those with which it is combined, and the expression for the weight becomes

$$w = \frac{212\,bd^2}{l \cos \varphi};$$

from which the following practical rule is derived.

Rule 8. Multiply 212 times the breadth in inches by the square of the depth in inches, and divide the product by the length in feet drawn into the natural cosine of the angle of inclination, for the load that the beam will bear.

Example 5. The length of a beam of oak, or the distance from the top of one support to the top of the other is 24 feet, its breadth 12 inches, and depth 20 inches; how much will it bear suspended from the middle, supposing its inclination to the horizon to be 32 degrees, and the strain within the elastic power of oak?

Here, by the rule, we get

$$\frac{212 \times 12 \times 20^2}{24 \times \cdot 848} = 49997 \text{ pounds, the}$$

load required; and the load that will produce fracture is $49997 \times 3{\cdot}36 = 167989$ pounds. Hence it appears, that a beam is stronger when it is supported in an inclined position than when it is horizontal, and the greater the inclination the greater is the increase of strength; for, as the angle of inclination increases, its cosine decreases, and since this is the element that affects the strain, it is obvious that the strength must increase directly as the angle of inclination, till it arrives at the perpendicular, where the inclination is a right angle, in which case it becomes the resistance to compression.

In the next place, we have to inquire what will be the conditions of the strain, when the beam is supported horizontally at the ends and the load or straining force applied at some intermediate point.

By the principles of the maxima and minima of quantities, it is easy to shew that the strain is the greatest when it occurs at the middle of the beam, it must therefore decrease as the point of application approaches either support; and the writers on the strength of materials have shewn, that the ratio of decrease is *as the rectangle of the segments into which the distance between the supports is divided by the direction of the straining force.* Let m and n represent the segments, then we have

$$mn : \frac{l^2}{4} :: \frac{212bd^2}{l} : w = \frac{53bd^2l}{mn};$$ that is,

when the beam is supported horizontally at the ends, and the load or straining force applied at some intermediate point, the equation which involves the several conditions of weight and dimensions, is

$$mnw = 53bd^2l. \qquad (5)$$

This equation consists of seven factors, one constant and six variable, any four of which being given, the fifth can easily be found; we say nothing of the sixth, viz., the length of bearing, which is equal to the sum of the segments m and n, and consequently is always known when they are given separately; but when it is required to find the segments m and n, the length l must be one of the four variable parts; the analysis of the equation, therefore, branches itself into four distinct cases, as follows:

1. Given m, n, and b, to find w;
2. Given m, n, d, and w, to find b;
3. Given m, n, b, and w, to find d;
4. Given b, d, l, and w, to find m and n.

And these cases are resolved in the following manner.

Problem IV. *In the equation* $mnw = 53bd^2l$, *there are given the breadth* b, *the depth* d, *and* m *and* n *the segments of the length; to find the weight* w.

Let each side of the equation be divided by the rectangle of the segments of the length, viz., mn, and the expression for the load, in terms of the other given quatities, becomes

$$w=\frac{53bd^2l}{mn};$$

from which the following practical rule is derived.

Rule 9. Multiply 53 times the breadth in inches by the sum of the segments or the length of bearing in feet drawn into the square of the depth in inches, and divide the product by the rectangle of the segments into which the length of bearing is divided, for the load to be supported in pounds.

Example 6. A uniform rectangular beam of oak, 5 inches broad, 18 inches deep, and 20 feet between the supports, is required to sustain a load at a point 14 feet distant from one support, and 6 feet from the other; how much will it bear, the strain being within the elastic force of the material?

Here, by the rule, we get

$$\frac{53\times5\times\overline{14+6}\times18^2}{14\times6}=20443 \text{ pounds, the}$$

load required.

The preceding load, as calculated by the rule in its present form, includes the effect produced by the weight of the beam, and since this weight, acting as a uniform force, is the same as if one half of it were applied at the middle point, it is obvious, that half the weight of the beam must be taken from the above result, and the remainder is the load that should be applied externally, to make allowance for the effect produced by the weight of the beam, which, in delicate practical cases ought always to be considered. Now, we have shewn elsewhere, that the area of the cross section in inches, multiplied by ·36 times the length of bearing in feet, expresses the whole weight of the beam; therefore, $5 \times 18 \times 20 \times 0{\cdot}36 = 648$ pounds, the half of which is 324 pounds; hence we have $20443 - 324 = 20119$ pounds, for the external load that will destroy the elastic force: but half the weight of the beam is expressed by $0{\cdot}18\,bdl$ (see the first problem); and the present problem gives $w = \frac{53bd^2l}{mn}$; therefore we have

$$mnw = bdl(53d - {\cdot}18mn). \qquad (6)$$

From this formula, the expression for the load becomes

$$w = \frac{bdl\,(53d - 18mn)}{mn};$$

from which the following practical rule is derived,

Rule 10. From 53 times the depth in inches, subtract 0·18 times the rectangle of the segments of the length of bearing in feet; multiply the remainder successively by the breadth, depth, and length, and divide the product by the rectangle of the segments, for the load in pounds.

Taking the data of the preceding example, we get

$$53 \times 18 - 14 \times 6 \times {\cdot}18 = 938{\cdot}88;$$

then, $\frac{938{\cdot}88 \times 5 \times 18 \times 20}{14 \times 6} = 20119$ pounds, the same as before.

And the load that will produce fracture is 20119 $\times 3{\cdot}36 = 67600$ pounds, very nearly.

PROBLEM V. *In the equation* $mnw = 53bd^2l$, *there are given, the depth d, the weight w, and m and n the segments of the length of bearing; to find the breadth b.*

Let each side of the equation be divided by $53d^2l$, and the value of the breadth in terms of the other quantities will be expressed as under, viz.

$$b = \frac{mnw}{53d^2l};$$

from which the following practical rule is derived.

Rule 11. Multiply the load to be supported in pounds by the rectangle of the segments of the length of bearing in feet, and divide the product by 53 times the length or sum of the segments, drawn into the square of the depth in inches, for the breadth required.

Example 7. An oak beam, 18 inches deep and 20 feet between the supports, has a load of 20443 pounds applied at a point, 14 feet from one extremity and 6 feet from the other; what is its breadth, the elastic force remaining perfect?

Here, by the rule, we get

$$\frac{20443 \times 14 \times 6}{53 \times 20 \times 18^2} = 5 \text{ inches, the breadth}$$

sought.

The above breadth supposes the effect produced by the weight of the beam to be incorporated with that produced by the given load; but to find a breadth that will resist both effects, we must recur to equation (6), where

$$mnw = bdl(53d - \cdot 18mn).$$

Let each side of the equation be divided by $dl(53d - \cdot 18mn)$, and we obtain

$$b = \frac{mnw}{dl(53d - \cdot 18mn)};$$

from which the following practical rule is derived.

Rule 12. From 53 times the depth in inches subtract 0·18 times the rectangle of the segments of the length of bearing in feet; multiply the remainder by the sum of the segments, drawn into the depth in inches, and reserve the product for a divisor. Again; multiply the weight in pounds by the rectangle of the segments of the length in feet; then, divide the product by the reserved divisor for the breadth sought.

Taking the data of the preceding example, we get

$$\frac{20443 \times 14 \times 6}{18 \times 20(53 \times 18 - \cdot 18 \times 14 \times 6)} = 5 \cdot 08 \text{ inches},$$

for the breadth required.

The load which a beam 5·08 inches broad, 18 inches deep, and 20 feet long will bear, at a point 14 feet from one support and 6 feet from the other, is 20770 pounds, calculated by the rule to Problem IV.; and half the weight of a beam of these dimensions is .327 pounds; therefore, $20770-327=20443$ pounds, the load proposed.

PROBLEM VI. *In the equation* $mnw=53\,bd^2l$, *there are given, the breadth* b, *the weight* w, *and* m *and* n *the segments of the length; to find the depth* d.

Let each side of the equation be divided by $53b\,l$, and we get

$$d^2=\frac{mnw}{53bl}.$$

Therefore, by extracting the square root of both sides of this equation, the value of d, in terms of the rest, becomes

$$d=\sqrt{\frac{mnw}{53bl}};$$

from which the following practical rule is derived.

Rule 13. Multiply the weight in pounds by the rectangle of the segments of the length in feet; divide the product by 53 times the length in feet drawn into the breadth in inches, then the square root of the quotient will give the depth required.

Example 8. A beam of oak, 20 feet long and 5 inches broad, has a load of 20443 pounds applied at a point 14 feet from one support and 6 feet from the other; what is the depth of the beam, the elastic force remaining perfect?

Here, by the rule, we get

$$\frac{20443\times14\times6}{53\times20\times5}=324;$$

and the square root of 324=18 inches, the depth required.

But to find a depth that will make allowance for the effect produced by the weight of the beam, we must again recur to equation (6), where

$$mnw = bdl(53d - \cdot 18mn).$$

The determination of the unknown term in this case, evidently requires the reduction of an adfected quadratic equation, from which we find the expression for the value of the depth in the terms of the rest, to be

$$d = \frac{1}{53}\left\{\cdot 09mn + \sqrt{mn\left(\frac{53w}{bl} + \cdot 0081mn\right)}\right\};$$

from which the following practical rule is derived.

Rule 14. Divide 53 times the given load in pounds by the breadth in inches, drawn into the length of bearing or sum of the segments in feet; to the quotient add the rectangle of the segments, multiplied by the fraction ·0081; multiply the sum by the rectangle of the segments, and extract the square root of the product: to the said square root add the rectangle of the segments in feet, multiplied by the fraction ·09, and divide the sum by 53, for the depth required.

Taking the data of the preceding example, we have

$$\sqrt{14 \times 6 \times \left\{\frac{20443 \times 53}{5 \times 20} + 14 \times 6 \times \cdot 0081\right\}} = 954 \cdot 033;$$

then, $\frac{954{\cdot}033+14\times 6\times{\cdot}09}{53}=18{\cdot}143$ inches, the depth required.

The load that a beam 5 inches broad, 18·143 inches deep, and 20 feet long, will bear, at a point 14 feet from one support and 6 feet from the other, is 20770 pounds, calculated by the Rule to Problem IV; and half the weight of a beam of these dimensions is 327 pounds; therefore, $20770-327=20443$ pounds, the load proposed.

PROBLEM VII. *In the equation* $mnw=53\,b\,d^2l$, *there are given, the breadth b, the depth d, the length l, and the weight w; to find m and n, the segments of the length separately.*

Here, divide both sides of the equation by w, the given weight, and we have

$$mn=\frac{53bd^2l}{w},$$

but $m+n=l$, by the nature of the question; from which it appears that the problem resolves itself into the following, viz.

Given the sum and product of any two quantities; to find those quantities separately. Now, the solution of this problem is easily effected by the following rule.

From the square of the sum of the two given quantities, subtract 4 times their product or rectangle; then, to or from half the sum, add or subtract half the square *root of the difference, and the sum or remainder will give the greater or lesser segment accordingly.*

The expressions for the segments, as determined by this rule, are, in terms of the given quantities, respectively as below, viz.

$$m = \frac{1}{2}\left\{ l + \sqrt{l^2 - \frac{212\,b d^2 l}{w}} \right\};$$

$$\text{and } n = \frac{1}{2}\left\{ l - \sqrt{l^2 - \frac{212\,b d^2 l}{w}} \right\}.$$

Where m is the greater, and n the lesser segment, into which the length of bearing or distance between the supports, is divided at the point of strain; from which the following practical rule is derived:

Rule 15. Multiply 212 times the length in feet by the breadth drawn into the square of the depth, both in inches; divide the product by the weight in pounds, and subtract the quotient from the square of the length in feet; then, to or from the length, add or subtract the square root of the remainder, and half the sum, or half the difference, will be the greater or lesser segment accordingly.

Example 9. Suppose a beam of oak to be 5 inches broad, 18 inches deep, and 20 feet long between the supports, at what point of its length will it sustain a load of 20443 pounds, its elastic force remaining perfect?

Here, by the rule, we get

$$20^2 - \frac{212 \times 20 \times 5 \times 18^2}{20443} = 64;$$

then, $\frac{1}{2}(20 \pm \sqrt{64}) = 14$ feet and 6 feet, for the distances from the supports.

The point which we have just determined, answers to the case where the proposed load includes the effect produced by the weight of the beam; but to find a point that will make allowance for this effect, we must revert to equation (6), where

$$m n w = b d l (53 d - \cdot 18 m n).$$

Now, the expressions for the segments m and n, as determined from this equation, are

$$m = \frac{1}{2}\left\{l + \sqrt{l^2 - \frac{212 b d^2 l}{w + \cdot 18 b d l}}\right\};$$

and $$n = \frac{1}{2}\left\{l - \sqrt{l^2 - \frac{212 b d^2 l}{w + \cdot 18 b d l}}\right\};$$

from which we derive the following practical rule.

Rule 16. Multiply 212 times the length of bearing, or distance between the supports in feet, by the breadth drawn into the square of the depth, both in inches; divide the product by the weight in pounds, added to ·18 times the continued product of the length, breadth, and depth, and subtract the quotient from the square of the length in feet; then, to or from the length in feet, add or subtract the square root of the remainder, and half the sum, or half the difference, will be the greater or lesser segment accordingly.

Taking the data of the preceding example, we have

$$\frac{212 \times 20 \times 5 \times 18^2}{20443 + 20 \times 5 \times 18 \times \cdot 18} = 330 \cdot 7555;$$

then, $400 - 330{\cdot}7555 = 69{\cdot}2444$; therefore,

$\frac{1}{2}(20 \pm \sqrt{69{\cdot}2444}) = 14{\cdot}16$ feet, and $5{\cdot}84$ feet for the distance from the supports.

The load that a beam 5 inches broad, 18 inches deep, and 20 feet long, will bear at a point 14·16 feet from one support, and 5·84 feet from the other, is 20767 pounds, calculated by the rule to Problem IV; and half the weight of a beam of these dimensions is 324 pounds; therefore, $20767 - 324 = 20443$ pounds, the load proposed.

It may sometimes happen, that the load which a beam will sustain at the middle is given, to determine a point in its length where it will support any greater load, or any number of times that which is given. This is apparently near akin to the last problem, but it is not precisely the same; and since, under particular circumstances, it may be useful, we shall here bestow on it a separate solution.

Let w be the load that the beam will sustain at the middle of its length, and W the load which it is required to sustain at some other point: then, since the strength of the beam, or the load which it will bear at any point, is inversely as the rectangle of the segments of the length at that point, we have $mn : \frac{l^2}{4} :: w : W$; that is

$$mn = \frac{l^2 . w}{4W}, \text{ and moreover,}$$

$$m + n = l, \text{ by the nature of the problem;}$$

therefore, by proceeding in the same manner as we did in the preceding solution, we find that

$$m-n=l\sqrt{\frac{W-w}{W}}.$$

Wherefore, the expressions for the segments m and n become

$$m=\frac{l}{2}\left\{1+\sqrt{\frac{W-w}{W}}\right\};$$

$$\text{and } n=\frac{l}{2}\left\{1-\sqrt{\frac{W-w}{W}}\right\};$$

from which we derive the following practical rule:

Rule 17. Divide the difference between the greater and lesser weights by the greater weight; then, to or from unity, add or subtract the square root of the quotient, and multiply the sum or difference by half the length of the beam in feet, for the greater or lesser segment accordingly.

Example 10. A rectangular beam of oak 20 feet long, is found to sustain 17172 pounds, when applied at the middle point; how far distant from each support is the point where it will sustain 20443 pounds?

Here, by the rule, we get

$$\frac{20443-17172}{20443}=\cdot 16;$$

Now, the square root of $\cdot 16=\cdot 4$, and half the length of the beam is 10 feet; therefore, $\overline{1+\cdot 4}\times 10=14$ feet, the distance from one of the supports, and consequently $20-14=6$ feet, the distance from the other. Again, let it be required to find a point where the beam will sustain any number of times the load that it sustains at the middle of its length. Let a be the number of times the load has to be increased; then,

$$mn : \frac{l^2}{4} :: w : aw; \text{ therefore}$$

$$mn = \frac{l^2}{4a}, \text{ and we know}$$

that $m + n = l$; hence, by reduction, we have

$$m - n = l\sqrt{\frac{a-1}{a}}; \text{ therefore,}$$

the expressions for the segments m and n are as under, viz.

$$m = \frac{l}{2}\left\{1 + \sqrt{\frac{a-1}{a}}\right\};$$

$$\text{and } n = \frac{l}{2}\left\{1 - \sqrt{\frac{a-1}{a}}\right\};$$

from which the following practical rule is derived:

Rule 18. Subtract unity from the number that denotes how often the given load has to be increased, and divide the remainder by that number; then, to or from unity, add or subtract the square root of the quotient, and multiply the sum or remainder by half the length of the beam, for the greater or lesser segment accordingly.

Example 11. The length of a rectangular beam of oak is 20 feet; at what point of its length will it bear 4 times as great a load as it is capable of bearing at the middle?

Here, by the rule, we get

$$\sqrt{\frac{4-1}{4}} = \cdot 866;$$

and half the length of the beam is 10 feet; therefore, $\overline{1 + \cdot 866} \times 10 = 18 \cdot 66$ feet, from one support, and $\overline{1 - \cdot 866} \times 10 = 1 \cdot 34$ feet from the other.

We now proceed to the consideration of that case, *when the beam is supported at the ends, and loaded uniformly throughout the length.*

It is a well-established fact in the doctrine of the strength of materials, that a bar or beam is capable of supporting twice the load when uniformly distributed throughout the length, that it can support when applied at the middle point. Now, we have already shewn [*See* Equation (1)], that when a beam is supported at the ends and loaded in the middle, the several conditions of strength and dimensions are expressed by the following equation,

$$lw = 212\, b\, d^2.$$

Therefore, from what we have just stated, when the beam is supported at the ends and loaded uniformly throughout the length, the several conditions must be expressed by

$$lw = 425\, b\, d^2.$$

This equation being precisely of the same form as Equation (1), with the exception of the constant number, it is obvious that the analysis, and the several rules arising from the reduced equations, must be the same also, observing to use the number

425 instead of 212, in all the rules from the first to the sixth, both inclusive. We give the following example for illustration.

Example 12. A beam of oak 12 inches in breadth, and 16 feet between the supports, is found to sustain a load of 25760 pounds uniformly distributed over the length; what must be the depth to make allowance for the effect produced by the weight of the beam, supposing the elastic force of the material to remain perfect?

This example is evidently resolved by Rule 6, taking care to employ 425 for 212 whenever the latter occurs in the rule; or take half the given load and proceed exactly as in the rule, retaining 212 for the constant number. We shall resolve it by both methods, the better to shew the coincidence.

1. By using 425 instead of 212, or rather $212\frac{1}{2}$,

$$\frac{25760 \times 425}{12} = 912333{\cdot}3333$$

$$16^3 \times {\cdot}0081 = \underline{\quad 33{\cdot}1776} \quad \text{(add)}$$

$$\sqrt{912366{\cdot}511 \times 16} = 3820{\cdot}71$$

then, $\frac{3820{\cdot}71 + 16^2 \times {\cdot}09}{425} = 9{\cdot}04$ inches, the depth required.

2. By using half the load and retaining the constant $212\frac{1}{2}$,

$$\frac{12880 \times 212\frac{1}{2}}{12} = 228083{\cdot}3333$$

$$16^3 \times {\cdot}0081 = \underline{\quad 33{\cdot}1776} \quad \text{(add)}$$

$$\sqrt{228116{\cdot}511 \times 16} = 1910{\cdot}35;$$

then, $\frac{1910 \cdot 35 + 16^2 \times \cdot 09}{212\frac{1}{4}} = 9 \cdot 04$ inches, the same as before.

When a beam projects horizontally from a wall, into which one of its ends is firmly fixed, if a load be applied at the other end, the effect of the load so applied is four times as great as when the beam is supported at the ends and loaded in the middle; therefore, the several conditions of strength and dimensions are expressed by the following equation, viz.

$$l w = 53 \, b d^2.$$

Consequently, if the constant number 53 be substituted for 212, the rule above stated will apply here also; and when the beam is fixed at one end, and loaded uniformly over the length, the equation becomes

$$l w = 106 \, b d^2.$$

Hence, if we use the constant 106 instead of 212, the same rules will still apply. We give the following example for illustration.

Example 13. A beam of oak, 9 inches broad, and 16 inches deep, has one end firmly fixed in a wall, and a load suspended from the other end; how much will it bear, supposing the length of projection to be 10 feet, and the strain within the elastic power of the material?

Here, since it is not proposed to make allowance for the weight of the beam, the first rule is applicable, and the process is as follows:

$$\frac{53 \times 9 \times 16^2}{10} = 12211$$ pounds, the load required.

Let the same beam be loaded uniformly over the length, and we have

$$\frac{106 \times 9 \times 16^2}{10} = 24422$$ pounds, the load which the beam will bear uniformly diffused throughout the length.

Before we dismiss the rectangular form, it may perhaps be useful to propose one or two problems of a somewhat different nature from those which we have chosen for illustration; such as the following.

PROBLEM VIII. *Given the breadth b, and the depth d, of a uniform rectangular beam of oak, supported at the ends in a horizontal position; to find its length such that it will just break by means of its own weight.*

We have shewn under the rule to the first problem, that the whole weight of a beam of oak when its form is rectangular, is expressed by $\cdot 36\, bdl$, and, moreover, that this weight produces the same effect in straining the beam, as if one half of it, or $\cdot 18\, bdl$ were collected at the middle point; hence, by the nature of the problem, $\cdot 18\, bdl$ pounds must be sufficient to break the beam.

Now, it is shewn (Equation 3), that in the case of fracture, the conditions of strength and dimensions are expressed by $lw = 714\,bd^2$; which, by dividing both sides by l, becomes

$$w = \frac{714\,bd^2}{l}.$$

Therefore, by the conditions of the problem, we get

$$\cdot 18\,bdl = \frac{714\,bd^2}{l};$$ and this, by multiplication and division, becomes $l^2 = 3966\frac{2}{3}\,d$; and by extracting the square root of both sides of this last equation, the expression for the length becomes

$$l = 63\sqrt{d};$$

from which the following practical rule is derived.

Rule 19. Multiply the square root of the depth in inches by 63, and the product will be the length in feet, of a beam that will just break by means of its own weight.

Example 14. The depth of a rectangular beam of oak is 16 inches; what must be its length that it may just break by means of its own weight?

Here, by the rule, we get

$63\sqrt{16} = 252$ feet, for the length required.

Hence, we infer, that a beam of oak 252 feet long, and 16 inches deep, will break by means of its own weight, whatever its breadth may be: this is evident, from the circumstance of the breadth vanishing in the course of the investigation. Let us suppose that the beam is 4 inches broad; then, half the weight of a beam 4 inches broad, 16 inches deep, and 252 feet between the supports, must be just equal to the weight that will break it, calculated by Rule 7. Now, the weight of the beam is 5806 pounds, its half is 2903 pounds, and the weight that will break it is

$$\frac{714 \times 4 \times 16}{252} = 2902 \text{ pounds.}$$ The coincidence proves the correctness of the principle.

PROBLEM IX. *Given the length of a rectangular beam of oak, that will just break by its own weight; to find the depth.*

It is shewn above, that the expression for the length in terms of the depth, is

$$l = 63\sqrt{d}.$$

Let both sides of the equation be divided by 63, and we get

$$\sqrt{d} = \frac{l}{63};$$

and this, by involution, becomes

$$d = \frac{l^2}{3969};$$

from which the following practical rule is derived.

Rule 20. Divide the square of the given length in feet by the constant number 3969, and the quotient will be the depth of the beam in inches.

Example 15. The length of a beam of oak is 58 feet; what must be its depth to break by its own weight?

Here, by the rule, we get

$$\frac{58^2}{3969} = \cdot847 \text{ inches, the depth}$$

required.

Suppose the breadth to be 4 inches; then, half the weight of a beam 4 inches broad, ·847 inches deep and 58 feet long, should be just equal to the load that breaks it, calculated by Rule 7. Now, half the weight of the beam is 35 pounds, and the weight that will break it is

$$\frac{714 \times 4 \times 847^2}{58} = 35 \text{ pounds nearly,}$$

the same as half the weight of the beam.

PROBLEM X. *Given, the diameter of a cylindrical tree, to find the dimensions of the strongest rectangular beam that can be cut out of it.*

In resolving this problem we must observe, that *that* beam is not the strongest which contains the greatest quantity of material, but that whose breadth drawn into the square of the depth is the greatest; therefore, let b and d represent the breadth and depth of the cross section in inches, D the diagonal of the section or diameter of the tree, and φ the angle which the diagonal makes with the depth; then we have

$$b = D\sin\varphi, \text{ and } d = D\cos\varphi;$$

consequently, $bd^2 = D^3\sin\varphi\cos^2\varphi$; which is to be a maximum; and this happens when φ is such, that $\sin\varphi = \frac{1}{3}\sqrt{3} = \cdot57735$; that is, when $\varphi = 35^\circ\ 16'$.

Hence the expressions for the breadth and depth of the beam, in terms of the diameter of the cylinder, are as follow, viz.

$$b = \cdot 57735\,D,$$

$$\text{and } d = \cdot 81647\,D;$$

from which the following practical rule is derived.

Rule 21. Multiply the given diameter of the cylinder by the natural sine and cosine of 35° 16′ separately, and the products will give the breadth and depth of the beam respectively.

Example 16. The diameter of a round tree is 27 inches; what are the dimensions of the strongest rectangular beam that can be cut from it?

Here, by the rule, we get

$\cdot 57735 \times 27 = 15\cdot 6$ inches nearly, the breadth,
and $\cdot 81647 \times 27 = 22$ inches nearly, the depth.

The area of the cross section of the cylindrical tree is expressed by $\cdot 7854\,D^2$, and the area of the cross section of the strongest rectangular beam that can be cut out of it is expressed by $D^2 \sin \varphi \cos \varphi$; therefore, those areas are to one another as ·7854 to ·4714.

Of Square Beams when exposed to a Transverse or Cross Strain.

In the case of square beams, our fundamental principle is expressed in the following manner.

The cube of the side in inches, divided by the
length in feet drawn into the load in pounds,
is the same in all.

Let S represent the side of the square in inches, L the length of bearing or distance between the supports in feet, and W the weight in pounds that were employed in and determined from the experiment; and let *s*, *l* and *w* be the side of the square in inches, the length of bearing in feet, and the load in pounds, for which the calculation is made; then, by our fundamental principle, we have

$$\frac{S^3}{LW} = \frac{s^3}{lw};$$

which, by clearing the equation from fractions, becomes

$$lwS^3 = LWs^3 \qquad (B)$$

Now, from Mr. Tredgold's experiments we learn, that a bar of oak, 1 inch square and 24 inches between the supports, will just break with a load of 357 pounds applied at the middle of its length; and we have shewn in the third equation preceding, that the load which merely destroys the elastic force, is to the load which produces fracture, as 1 to 3·36; hence we infer, that the load which will destroy the elastic force of a bar 1 inch square and 24 inches long, is 106¼ pounds; let these numbers be substituted for S, L and W in equation (B), and we have, by taking the length in feet and omitting the fraction,

$$lw = 212\,s^3 \qquad (7)$$

The analysis of this equation branches itself into two distinct cases, viz., when the dimensions of the beam are given, to find the load that will destroy the elastic force, and, when the length and the load are given, to find the side of the square section; these cases are expressed as follows:

1. Given l and s; to find w.
2. Given l and w; to find s.

And from these we deduce the following Problems.

PROBLEM XI. *In the equation* $lw = 212s^3$, *there are given the length of the beam* l, *and the side of its cross section* s; *to find the weight* w, *that it will bear while its elastic force remains perfect.*

Let each side of this equation be divided by the length l, and we have

$$w = \frac{212s^3}{l};$$

from which the following practical rule is derived.

Rule 22. Divide 212 times the cube of the side in inches, by the length of bearing or distance between the supports in feet, and the quotient will be the weight in pounds that the beam can support with safety.

Example 17. What weight will a beam of oak, 7 inches square and 16 feet between the supports, sustain at the middle of its length; the strain being within the elastic power of the material?

Here, by the rule, we get

$$\frac{212 \times 7^3}{16} = 4545 \text{ pounds very nearly,}$$

the weight required; and $4545 \times 3 \cdot 36 = 15271$ pounds, the load that will produce fracture.

We have already shewn, that the weight of a uniform beam of oak is expressed by the area of its cross section in inches multiplied by the fraction 0·36, drawn into the length of the beam in feet; and, moreover, we have stated, that the weight of the beam acting as a uniform force, produces the same effect as if one half that load were collected at the middle point; therefore we have, for the weight of the beam, $7^2 \times 16 \times 0 \cdot 36 = 282$ pounds, the half of which is 141 pounds, and this gives $4545 - 141 = 4404$ pounds, for the external load that the beam will bear with safety; and $15271 - 141 = 15130$ pounds, the external load that will produce fracture.

Put $0 \cdot 18\, l s^2 =$ half the weight of the beam; then, Equation (7) modified to make allowance for the effect of this weight, is

$$lw = s^2(212s - 0 \cdot 18\, l^2). \qquad (8)$$

Divide each side of this equation by the length l, and the expression for the weight which the beam will bear, becomes

$$w = \frac{s^2(212s - 0 \cdot 18\, l^2)}{l};$$

from which the following practical rule is derived.

Rule 23. From 212 times the side of the square in inches, subtract 0·18 times the square of the length in feet; multiply the remainder by the square of the side, and divide the product by the length, for the load required.

Taking the data of the preceding example, we get

$$\frac{7^2 \times \overline{212 \times 7 - 0{\cdot}18 \times 16^2}}{16} = 4404 \text{ pounds nearly,}$$

the same as before.

PROBLEM XII. *In the equation $lw = 212s^3$, there are given the length l, and the weight w; to find s, the side of the square.*

Let both sides of this equation be divided by 212, and we get

$$s^3 = \frac{lw}{212};$$

and this, by extracting the cube root of both sides becomes

$$s = \sqrt[3]{\frac{lw}{212}};$$

from which the following practical rule is derived.

Rule 24. Divide the weight in pounds drawn into the length of bearing in feet by 212, and extract the cube root of the quotient for the side of the square in inches.

Example 18. A square beam of oak, 16 feet between the supports, is found to sustain a load of 4545 pounds at the middle of its length while the elastic force remains perfect: what is the side of its cross section?

Here, by the rule, we get

$$\frac{4545 \times 16}{212} = 343;$$

and the cube root of 343 is 7; therefore, the side of the required square is 7 inches. Now, this process has been conducted on the supposition that the given load includes the effect produced by the weight of the beam; therefore, to find the side of a square that will make allowance for this effect, we must revert to equation (8), where

$$lw = s^2(212s - \cdot 18l^2);$$

from which we derive the following cubic equation, involving the value of the side s, viz.

$$s^3 - \frac{\cdot 18l^2}{212}s^2 = \frac{lw}{212}.$$

If in this equation, we substitute the data of the foregoing example, we shall have

$$s^3 - \cdot 217358s^2 = 343.$$

In this instance it would be difficult to give a rule in words at length, by which to determine the value of the required term directly from the data; it would be too operose to be understood, and we are convinced that none but those who are acquainted with the reduction of the several orders of algebraic equations could apply it; we are therefore compelled, although reluctantly, to omit the rule, and refer our readers to Nicholson's or Bonnycastle's Algebra, where the different methods of reducing cubic equations, are pointed out and exemplified. The value of s, as found from the preceding equation, is 7·074 inches very nearly; but the load which a beam 7·074 inches square, and 16 feet between the supports, will sustain at the middle of its length, is 4691 pounds, calculated by Rule 22, and half the weight of the beam is 146 pounds; therefore, $4691-146=4545$ pounds, the very same as the load proposed.

We now proceed to the case where the beam is supported horizontally at the ends, and the load applied at some intermediate point. It was stated in the solution of the same case for the rectangular beam, that the strain is proportional to the rectangle of the segments into which the length of bearing, or distance between the supports, is divided at the point where the load acts; and this principle being applied to the square form gives

$$mnw=53ls^3. \qquad (9)$$

And the several cases of analysis that arise from this equation are as below, viz.

1. Given m, n and s; to find w.
2. Given m, n and w; to find s.
3. Given l, s and w; to find m and n.

From which arise the following problems.

PROBLEM XIII. *In the equation* $mnw=53ls^3$, *there are given the side of the square* s, *and* m *and* n *the segments of the length of bearing; to find* w, *the load that the beam will support at the point of section.*

Divide both sides of the equation by the rectangle of the segments, and we get

$$w = \frac{53 s^3 (m+n)}{mn};$$

where $m+n=l$;

from which the following practical rule is derived.

Rule 25. Divide 53 times the cube of the side in inches, drawn into the sum of the segments of the length by the rectangle of the segments, and the quotient will be the load required.

Example 19. What load will be sustained by a beam of oak 7 inches square and 16 feet between the supports, when the load acts at the distance of 12 feet from one support and 4 feet from the other?

Here, by the rule, we get

$$\frac{53 \times 7^3 \times (12+4)}{12 \times 4} = 6060 \text{ pounds, the}$$

load required.

And $6060 \times 3{\cdot}36 = 20362$ pounds, the load that will produce fracture. But we have shewn in the example to the eleventh problem, that the weight of a beam of oak 7 inches square and 16 feet long, is 282 pounds, and its half 141 pounds; therefore $6060 - 141 = 5919$, and $20362 - 141 = 20221$ pounds for the load that will respectively destroy elasticity, and produce fracture.

Let $0{\cdot}18\, l s^2$ represent half the weight of the beam, then Equation (9) becomes

$$mnw = s^2 (m+n) \{53 s - {\cdot}18\, mn\}. \qquad (10)$$

And the expression for the load is

$$w = \frac{s^2 (m+n) \{53 s - {\cdot}18\, mn\}}{mn};$$

from which the following practical rule is derived.

Rule 26. From 53 times the side of the square in inches, subtract ·18 times the rectangle of the segments of the length of bearing in feet; multiply the remainder by the sum of the segments drawn into the square of the side, and divide the product by the rectangle of the given segments, for the load required.

Taking the data of the preceding example, we get

$$\frac{7^2(12+4)\{53\times 7-{\cdot}18\times 12\times 4\}}{12\times 4}=5919 \text{ pounds,}$$

the same as found above.

PROBLEM XIV. *In the equation* $mnw=53\,ls^3$, *there are given m and n, the segments of the length of bearing, and w the load to be supported; to find s the side of the square.*

Let both sides of the equation be divided by $53\,l$, and we obtain

$$s^3=\frac{mnw}{53\,(m+n)};$$

therefore, by evolution, we have

$$s=\sqrt[3]{\frac{mnw}{53\,(m+n)}};$$

from which we derive the following practical rule.

Rule 27. Multiply the rectangle of the segments of the length by the given load; divide the product by 53 times the sum of the segments, and the cube root of the quotient will be the side of the square required.

Example 20. A square beam of oak 16 feet in length, is found to support a load of 6060 pounds, when applied at a point 12 feet from one support, and 4 feet from the other; what is the side of its cross section?

Here by the rule, we get

$$\frac{12 \times 4 \times 6060}{53(12+4)} = 343,$$

and the cube root of 343 is 7; therefore the side of the square beam is 7 inches. But to determine the side that will be sufficient to support a load of 6060 pounds, and also to resist the effect produced by the weight of the beam, we must have recourse to Equation (10), where

$$m n w = s^2 (m+n) \{53 s - \cdot 18 m n\}.$$

But this reduces to the form

$$s^3 - \frac{\cdot 18 m n}{53} s^2 = \frac{m n w}{53(m+n)};$$

and by substituting the data of the last example, we obtain

$$s^3 - \cdot 163 s^2 = 343,$$

which being reduced by the rules for cubic equations, gives 7·054 inches very nearly, for the side of the square required. Now the load which a beam 7·054 inches square, and 16 feet long, will support under the specified conditions, as calculated by Rule 25, is 6204 pounds, and half the weight of a beam of these dimensions, is 144 pounds; therefore 6204 − 144 = 6060 pounds, the same as the load proposed.

PROBLEM XV. *In the equation $mnw=53ls^3$, there are given, the length of bearing l, the side of the square s, and the load w; to find m and n the segments of the length separately.*

If both sides of the equation be divided by the given load, we have

$$mn=\frac{53ls^3}{w},$$

and $m+n=l$, by the nature of the question.

Hence, by the rule at Problem VII., we obtain

$$m=\tfrac{1}{2}\left\{l+\sqrt{l^2-\frac{212ls^3}{w}}\right\},$$

$$\text{and } n=\tfrac{1}{2}\left\{l-\sqrt{l^2-\frac{212ls^3}{w}}\right\};$$

where m is the greater and n the lesser segment, and from which expressions we derive the following practical rule.

Rule 28. Multiply 212 times the length in feet, by the cube of the side in inches; divide the product by the given load, and subtract the quotient from the square of the length; then, to or from the length, add or subtract the square root of the remainder, and half the sum or half the difference will give the greater or lesser segment accordingly.

Example 21. A beam of oak 7 inches square, and 16 feet between the supports, is found to sustain a load of 6060 pounds at some point of its length while the elastic force remains perfect; how far is the point distant from each support?

Here, by the rule, we get

$$16^2 - \frac{212 \times 16 \times 7^3}{6060} = 64;$$

then $\frac{1}{2}(16 \pm \sqrt{64}) = 12$ and 4 feet, for the respective distances.

But the point which we have just determined will not answer for a load of 6060 pounds applied externally; it is obvious that the effect produced by the weight of the beam must be taken from the given load, and the remainder 5919 pounds is the external load that the beam will bear at that point; it is, however, required to determine the point where the beam will be capable of sustaining 6060 pounds, and for this purpose we must refer to Equation (10), where

$$mnw = s^2 (m+n) \{53\, s - \cdot 18\, mn\}.$$

And the expressions for the segments m and n, as determined from this equation, are

$$m = \tfrac{1}{2}\left\{l + \sqrt{l^2 - \frac{212\, ls^3}{w + \cdot 18\, ls^2}}\right\},$$

$$\text{and } n = \tfrac{1}{2}\left\{l - \sqrt{l^2 - \frac{212\, ls^3}{w + \cdot 18\, ls^2}}\right\};$$

from which expressions the following practical rule is derived.

Rule 29. Multiply the length of bearing in feet by the cube of the side in inches, and again by the constant number 212; divide the product by the given load, increased by ·18 times the length of bearing drawn into the square of the side, and subtract the quotient from the square of the length; then, to or from the length, add or subtract the square root of the remainder, and half the sum, or half the difference, will give the greater or lesser segment accordingly.

Taking the data of the preceding example, we get

$$16^2 - \frac{212 \times 16 \times 7^3}{6060 + \cdot18 \times 16 \times 7^2} = 68\cdot38;$$

then, $\frac{1}{2}(16 \pm \sqrt{68.38}) = 12\cdot13$ and $3\cdot87$ feet, the distances from each support.

The load that a beam 7 inches square, and 16 feet long, will sustain at a point, 12·13 feet from one support, and 3·87 feet from the other, is 6201 pounds, calculated by the rule to Problem XIII. And half the weight of the beam is 141 pounds; therefore 6201 − 141 = 6060 pounds, the same as the load proposed.

In the case *where the beam is supported horizontally at the ends, and the load uniformly diffused over the length,* the formula that involves the conditions of strength and dimensions, is as follows, viz.

$$lw = 425s^3;$$

an equation of the same form as Equation (7), having 425 instead of 212; therefore, if 425 be substituted in Rules 22, 23, and 24 for 212, the rules in every other respect will remain the same.

When the beam is fixed at one end and a load applied at the other, the formula that expresses the conditions is

$$lw = 53s^3;$$

an equation which differs only in the constant number from the preceding; and *when the beam is fixed at one end and loaded uniformly over the length,* the formula is

$$lw = 106s^3.$$

If, therefore, the numbers 53 and 106 be employed in the three rules above specified instead of 212, the rules will not otherwise require to be altered.

Square beams are sometimes strained in the direction of their vertical diagonal, in which case they are considerably weaker than when the direction of the straining force is parallel to the side; this is contrary to the opinion of some eminent authors, but it has been found both from theory and experiment to be the case; the ratio of strength in the two positions is as 1 to $\frac{1}{\sqrt{2}}$, or as radius to the sine of 45 degrees.

Hence, we have 1 : ·7071 : : 212 : 150 very nearly; therefore, the formula for a square beam, supported horizontally at the ends, and loaded in the middle, when strained in the direction of the vertical diagonal, is

$$lw = 150s^3$$

expressed in terms of the side, but when expressed in terms of the diagonal it is

$$lw = 53\,D^3;$$

where D is the diagonal of the square. We now proceed to the consideration

Of cylindrical beams when exposed to a transverse or cross strain.

The transverse strength of a cylindrical beam, is to the strength of its circumscribing square prism, in the ratio of 471 to 800 very nearly; hence we have

$$800 : 471 :: 212 : 125;$$

where 212 is the constant for a square beam placed under the same circumstances as the cylindrical one is supposed to be. Therefore, if d represent the diameter of the cylinder, we have

$$lw = 125 d^3. \qquad (11)$$

And the several cases of the analysis are as below, viz.

1. Given l and d, to find w,
2. Given l and w, to find d.

And from these cases we deduce the following problems.

PROBLEM XVI. *In the equation* $lw = 125 d^3$, *there are given the diameter* d, *and the length of bearing* l, *to find the load* w.

Let each side of the equation be divided by l, and we obtain

$$w = \frac{125 d^3}{l};$$

from which equation the following practical rule is derived.

Rule 30. Divide 125 times the cube of the diameter in inches by the length of bearing in feet, and the quotient will be the load required in pounds.

Example 22. What load will a beam of oak 7 inches diameter, and 20 feet long, sustain at the middle of its length; supposing its elastic force to remain perfect?

Here, by the rule, we get,

$\frac{125 \times 7^3}{20} = 2144$ pounds, the load required.

But the weight of a beam 7 inches diameter, and 20 feet long, is 277 pounds, calculated by the rule given under the first problem, and its half is 139 pounds; therefore, $2144 - 139 = 2005$ pounds, the whole external load that ought to be applied.

To modify the formula for the effect produced by the weight of the beam, we have for the weight of a cylindrical beam $0{\cdot}36 \times {\cdot}7854\, l d^2 = {\cdot}282744\, l d^2$, and its half is ${\cdot}141372\, l d^2$, or for practice ${\cdot}14\, l d^2$ will be sufficiently near the truth; therefore,

$$w = \frac{125\, d^3}{l} - {\cdot}14\, l d^2;$$

or by clearing the equation of fractions, and simplifying the arrangement, it becomes

$$l w = d^2 (125\, d - {\cdot}14\, l^2). \qquad (12)$$

And the expression for the load is

$$w = \frac{d^2 (125 d - {\cdot}14 l^2)}{l};$$

from which the following practical rule is derived.

Rule 31. From 125 times the diameter in inches, subtract ·14 times the square of the length in feet; multiply the remainder by the square of the diameter, and divide the product by the length for the load required.

Taking the data of the preceding example, we have

$$\frac{7^2 \times (125 \times 7 - \cdot 14 \times 20^2)}{20} = 2005 \text{ pounds},$$

the same as before.

PROBLEM XVII. *In the equation* $lw = 125d^3$, *there are given the length* l, *and the load* w; *to find the diameter* d.

Let both sides of the equation be divided by 125, and we get

$$d^3 = \frac{lw}{125};$$

and by extracting the cube root of both sides, the expression for the diameter is

$$d = \sqrt[3]{\frac{lw}{125}}, \text{ or } d = \tfrac{1}{5}\sqrt[3]{lw};$$

from which the following practical rule is derived.

Rule 32. Multiply the length of bearing in feet by the load to be supported in pounds; then, one-fifth the cube root of the product will be the diameter required.

Example 23. A beam of oak 20 feet long, is found to sustain a load of 2144 pounds at the middle of its length while the elastic force remains perfect; what is its diameter?

Here, by the rule, we get

$d = \frac{1}{5}\sqrt[3]{20 \times 2144} = 7$ inches, the diameter sought.

But a beam of 7 inches diameter, and 20 feet long, was found from Example 22, to be capable of supporting only 2005 pounds applied externally, while the elastic force remained unimpaired; the difference is 2144 − 2005 = 139 pounds, which arises from the effect produced by the weight of the beam itself. Hence, to find the diameter such, that it will support any proposed external load, we must recur to Equation (12), where

$$lw = d^2 (125\,d - \cdot 14\,l^2).$$

And this reduces to

$$d^3 - \frac{\cdot 14\,l^2}{125}\,d^2 = \frac{lw}{125},$$

or, by substituting the numbers in the preceding example, we obtain

$$d^3 - \cdot 448\,d^2 = 343,$$

which being reduced by the rules for cubic equations, gives

$d = 7\cdot153$ inches very nearly, for the diameter required.

Now, the load which a beam 7·153 inches in diameter, and 20 feet long, will bear, is 2287 pounds, calculated by Rule 30; and half the weight of a beam of these dimensions is 143 pounds; therefore, 2287 − 143 = 2144 pounds, the same as the load proposed.

In the next place, when the beam is supported horizontally at the ends and loaded at some intermediate point, we have, from the principle already stated,

$$mnw = 31\,ld^3 \qquad (13)$$

From which the following cases of analysis are derived, viz.

1. Given m, n and d; to find w.
2. Given m, n and w; to find d.
3. Given d, l and w; to find m and n.

And from these arise the following problems.

PROBLEM XVIII. *In the equation $mnw = 31\,ld^3$, there are given the diameter d, and $m\,n$ the segments of the length of bearing; to find the load w.*

Let both sides of the equation be divided by the rectangle of the segments of the length, and it becomes

$$w = \frac{31\,d^3\,(m+n)}{mn}; \text{ where } m+n=l;$$

from which we derive the following practical rule.

Rule 33. Divide 31 times the cube of the diameter in inches drawn into the sum of the segments of the length, by the rectangle of the segments, and the quotient will be the load required.

Example 24. A beam of oak 7 inches diameter and 20 feet between the supports, is required to sustain a load at the distance of 16 feet from one support, and 4 feet from the other; how much will it bear, the strain being within the elastic power of oak?

Here, by the rule, we get

$$\frac{31 \times 7^3 \times (16+4)}{16 \times 4} = 3323 \text{ pounds, the}$$

load required.

But it has been shewn, that half the weight of a beam of these dimensions is 139 pounds; therefore, $3323-139=3184$ pounds for the external load.

Now, to modify the formula so as to determine the external load directly by the operation, we have

$$w=\frac{31\,l\,d^3}{m\,n}-\cdot 14\,l\,d^2;$$

and this, by clearing it from fractions and simplifying the expression, becomes

$$m\,n\,w=d^2(m+n)\{31\,d-\cdot 14\,m\,n\}. \qquad (14)$$

Where, as we have shewn above, $m+n=l$. Hence, the expression for the load is

$$w=\frac{d^2(m+n)\{31\,d-\cdot 14mn\}}{m\,n};$$

from which the following practical rule is derived.

Rule 34. From 31 times the diameter of the beam in inches, subtract ·14 times the rectangle of the segments of the length; multiply the remainder by the sum of the segments drawn into the square of the diameter, and divide the product by the rectangle of the given segments, for the load required.

Taking the data of the preceding example, we get

$$\frac{7^2(16+4)\{31\times 7-\cdot 14\times 16\times 4\}}{16\times 4}=3184 \text{ pounds,}$$

the same as before.

Problem XIX. *In the equation $mnw=31ld^3$, there are given, the load w, and m and n, the segments of the length; to find the diameter d.*

Let both sides of the equation be divided by $31l$; or, since $l=m+n$, divide by $31(m+n)$, and we get

$$d^3 = \frac{m\,n\,w}{31(m+n)};$$

and by extracting the cube root of both sides, the expression for the diameter becomes

$$d = \sqrt[3]{\frac{m\,n\,w}{31(m+n)}};$$

from which the following practical rule is derived.

Rule 35. Multiply the rectangle of the segments of the length by the given load; divide the product by 31 times the sum of the segments, and extract the cube root of the quotient for the diameter required.

Example 25. A cylindrical beam of oak 20 feet in the length of bearing, is found to support a load of 3323 pounds, when applied at a point 16 feet from one support and 4 feet from the other; what is its diameter, the strain being within the elastic force of the material?

Here, by the rule, we have

$d = \sqrt[3]{\frac{3323 \times 16 \times 4}{31(16+4)}} = 7$ inches, the diameter required.

But, in order to find the diameter that will make allowance for the effect produced by the weight of the beam itself, we must refer to Equation (14), where

$$m\,n\,w = d^2(m+n)\{31\,d - \cdot 14\,m\,n\};$$

which reduces to the following form, viz.

$$d^3 - \frac{\cdot 14mn}{31}d^2 = \frac{m\,n\,w}{31(m+n)};$$

and by substituting the data of the foregoing example, we get

$$d^3 - \cdot 289\, d^2 = 343;$$

which being reduced by the rules for the reduction of cubic equations, gives

$d = 7{\cdot}098$ inches very nearly, for the diameter required.

Now, a cylindrical beam of oak 7·098 inches in diameter, and 20 feet long, will bear 3464 pounds, at a point 16 feet from one support, and 4 feet from the other; and half the weight of a beam of these dimensions is 141 pounds; therefore 3464 − 141 = 3323 pounds, the same as the load proposed.

PROBLEM XX. *In the equation* $mnw = 31\, l d^3$ *there are given the diameter* d, *the length of bearing* l, *and the load* w; *to find* m *and* n, *the segments of the length separately.*

Divide both sides of the equation by the given load, and we get

$$mn = \frac{31\, l d^3}{w};$$

but $m + n = l$ by the nature of the question.

Hence, by the rule at Problem VII., we obtain for the segments of the length, the following expressions, viz.

$$m = \tfrac{1}{2}\left\{l + \sqrt{l^2 - \frac{125\, l d^3}{w}}\right\},$$

$$\text{and } n = \tfrac{1}{2}\left\{l - \sqrt{l^2 - \frac{125\, l d^3}{w}}\right\};$$

where m is the greater and n the lesser segment; from which the following practical rule is derived.

Rule 36. Multiply 125 times the length of bearing in feet, by the cube of the diameter in inches; divide the product by the given load, and subtract the quotient from the square of the length; then, to or from the length, add or subtract the square root of the remainder, and half the sum, or half the difference, will give the greater or lesser segment accordingly.

Example 26. A cylindrical beam of oak 7 inches in diameter, and 20 feet in length, is found to sustain a load of 3323 pounds, including the effect produced by its own weight; at what point of the length is the load applied?

Here, by the rule, we get

$$20^2 - \frac{125 \times 20 \times 7^3}{3323} = 144;$$

then $\frac{1}{2}(20 \pm \sqrt{144}) = 16$, or 4, the distances from the points of support.

The point just determined answers to the case, when the proposed load includes the effect produced by the weight of the beam; but to find the point where the beam will be able to sustain the proposed load externally, together with the effect produced by its own weight, we must recur to Equation (14), where

$$m n w = d^2 (m + n) \{31 d - \cdot 14 m n\}.$$

And the expressions for the segments of the length, as determined from this equation, are

$$m = \tfrac{1}{2}\left\{l + \sqrt{l^2 - \frac{125ld^3}{w + \cdot 14ld^2}}\right\},$$

$$\text{and } n = \tfrac{1}{2}\left\{l - \sqrt{l^2 - \frac{125ld^3}{w + \cdot 14ld^2}}\right\};$$

from which the following practical rule is derived.

Rule 37. Multiply the length of bearing in feet, by the cube of the diameter in inches, and again by the constant number 125; divide the product by the given load, increased by ·14 times the length of bearing drawn into the square of the diameter, and subtract the quotient from the square of the length; then, to or from the length, add or subtract the square root of the remainder, and half the sum, or half the difference, will give the greater or lesser segment accordingly.

Taking the data of the preceding example, we get

$$\tfrac{1}{2}\left\{20 \pm \sqrt{20^2 - \frac{125 \times 20 \times 7^3}{3323 + 0{\cdot}14 \times 20 \times 7^2}}\right\} = 16{\cdot}16$$

and 3·84 feet, the distances required.

In the case when the beam is supported horizontally at the ends, and loaded uniformly over the length, the equation that involves the conditions of strength and dimensions, is

$$lw = 250\,d^3.$$

When the beam is fixed at one end and loaded at the other, the equation is

$$lw = 31\,d^3;$$

and when fixed at one end, and loaded uniformly over the length, it is

$$lw = 62 d^3.$$

Hence it appears, that if 250, 31, and 62 be substituted for 125 in Rules 30, 31, and 32, the same rules will apply to the cases above specified without further alteration.

Of hollow cylindrical or tubular beams.

Cylindrical beams are sometimes made hollow, in which case they are both stronger and stiffer than solid beams containing the same quantity of material; the expense of boring, however, renders the use of hollow beams less extensive than it otherwise would be; but whenever it is desirable to combine strength and lightness, the tubular form should be resorted to.

Let d be the greater diameter of the tube, and D the lesser diameter, or the diameter of the hollow part: then it is evident, that the strength of the tube, or the strength of the fistular column, must be equal to the difference of the strengths of two cylinders, having the diameters d and D. Now, the strength of a cylinder having the diameter d, is as $125 d^3$, and the strength of another cylinder having the diameter D, is as 125D^3, (*See* Equation 11); therefore, the strength of the tube must be as the difference of these two; that is, as $125 (d^3 - \text{D}^3)$; but $d : \text{D} :: 125 \text{D}^3 : \frac{125 \text{D}^4}{d}$, allowing for the increased resistance of the inner fibres in the tubular form; therefore, when the beam is supported horizontally at the ends, and loaded in the middle, the equation which involves the conditions of strength and magnitude, becomes

$$lw = 125d^3\left(1 - \frac{D^4}{d^4}\right). \qquad (15)$$

And the several combinations of data, or cases of analysis, are as under, viz.

1. Given d, D and l; to find w.
2. Given d, l and w; to find D.
3. Given D, l and w; to find d.

And from these cases we deduce the following problems.

PROBLEM XXI. *In the equation* $lw = 125d^3\left(1 - \frac{D^4}{d^4}\right)$; *there are given the diameters* d *and* D, *and the length of bearing* l; *to find the load* w.

Let both sides of the equation be divided by l, and we get

$$w = \frac{125d^3\left(1 - \frac{D^4}{d^4}\right)}{l}.$$

from which the following practical rule is derived.

Rule 38. Divide the lesser diameter by the greater, and from unity subtract the fourth power of the quotient; then multiply the remainder by 125 times the cube of the greater diameter, and divide the product by the length of bearing, for the load required.

Example 27. A cylindrical tube, or hollow beam of oak, has its greater diameter 8 inches, its lesser diameter 5 inches, and the length of bearing 18 feet; how much will it bear at the middle of its length, including its own weight, while the elastic force remains perfect?

Here by the rule, we get

$$\frac{125 \times 8^3(1-\cdot625^4)}{18} = 3013 \text{ pounds, the}$$

load required.

We have hitherto shewn in what manner the weight of the beam itself is to be estimated, and its effect on the strain allowed for; but in the present instance, on account of the complex fraction that occurs in the equation, it will be better to pass over the effect produced by the weight of the beam, as some of the results would be very complicated; and enough has been done in that way to enable the attentive reader to estimate the effect in any case that may occur in the course of his practice; we therefore pass on to

PROBLEM XXII. *In the equation* $lw = 125d^3\left(1 - \frac{D^4}{d^4}\right)$, *there are given the greater diameter d, the length of bearing l, and the load w; to find* D *the lesser diameter.*

Here, by transposing, and performing the proper reductions, we get

$$D^4 = d\left\{d^3 - \frac{lw}{125}\right\};$$

and by extracting the fourth root of both sides, it becomes

$$D=\sqrt[4]{d\left\{d^3-\frac{lw}{125}\right\}};$$

from which the following practical rule is derived:

Rule 39. Multiply the given length of bearing by the load in pounds, and divide the product by 125; subtract the quotient from the cube of the greater diameter, then multiply the remainder by the said diameter, and the fourth root of the product will give the diameter sought.

Example 28. A cylindrical tube, or hollow beam of oak, has its greater diameter 8 inches, the length of bearing, or distance between the supports, 18 feet, and it is found, while the elastic force remains perfect, to support 3013 pounds at the middle of its length; what is the lesser diameter, or the diameter of the hollow part?

Here, by the rule, we get

$$\sqrt[4]{8\left\{8^3-\frac{18\times3013}{125}\right\}}=5 \text{ inches, the}$$

diameter required.

Problem XXIII. *In the equation* $lw=125d^3\left(1-\frac{D^4}{d^4}\right)$, *there are given, the lesser diameter* D, *the length of bearing* l, *and the load* w; *to find* d, *the greater diameter.*

Here, by transposition and reduction, we get

$$d^4-\frac{lw}{125}d=D^4.$$

Now, it is evident, that in this case the diameter cannot be determined, but by the reduction of an adfected biquadratic equation; and consequently a rule in words at length would be unintelligible to any but algebraists, for which reason we omit it, and refer our readers, as on a former occasion, to Nicholson's, or Bonnycastle's Algebra, where the rules of reduction are exemplified.

If we substitute the numbers of the 28th example in the above equation, it becomes

$$d^4 - 433{\cdot}875\, d = 625.$$

In which the value of d is 8 inches, the diameter required.

When the beam is supported horizontally at the ends, and loaded at some intermediate point, the principle employed in deriving Equation (5), of the rectangular beam, gives

$$mnw = 31\, l\, d^3\left(1 - \frac{D^4}{d^4}\right). \qquad (16)$$

And the several combinations of data, or cases of the analysis, are as under, viz..

1. Given d, D, m and n, to find w;
2. Given d, w, m and n, to find D;
3. Given D, w, m and n, to find d;
4. Given D, l and w, to find m and n.

And from these arise the following problems.

PROBLEM XXIV. *In the equation* $mnw = 31ld^3\left(1-\frac{D^4}{d^4}\right)$, *there are given the diameters d and* D, *the length of bearing l, and m, n, the segments of the length; to find w, the load that the beam will sustain.*

Divide both sides of the equation by mn, the rectangle of the given segments, and we get

$$w = \frac{31ld^3(1-\frac{D^4}{d^4})}{mn};$$

from which the following practical rule is derived:

Rule 40. Divide the lesser diameter by the greater, and from unity subtract the fourth power of the quotient; then, multiply the remainder by 31 times the length of bearing, drawn into the cube of the exterior diameter, and divide the product by the rectangle of the segments of the length of bearing, and the quotient will give the load that the beam will sustain at the point of section, including its own weight.

Example 29. A cylindrical tube, or hollow beam of oak, has its exterior diameter 8 inches, the interior diameter 5 inches, and the length of bearing, or the distance between the supports, 18 feet; what load will it sustain at a point 12 feet from one support, and 6 feet from the other?

Here, by the rule, we get

$$\frac{31 \times 18 \times 8^3(1-\cdot625^4)}{12 \times 6} = 3362 \text{ pounds, the}$$

load required.

PROBLEM XXV. *In the equation* $mnw = 31ld^3\left(1-\frac{D^4}{d^4}\right)$, *there are given, the diameter d, the segments of the length m, n, and the load w; to find the diameter* D.

Here, by transposition and reduction, we have

$$D^4 = d\left\{d^3 - \frac{mnw}{31l}\right\};$$

and by extracting the fourth root of both sides, it becomes

$$D = \sqrt[4]{d\left\{d^3 - \frac{mnw}{31l}\right\}};$$

from which the following practical rule is derived:

Rule 41. Multiply the rectangle of the segments of the length by the given load, and divide the product by 31 times the length or sum of the segments; subtract the quotient from the cube of the given diameter, and multiply the remainder by the said diameter; then, the fourth root of the product will give the diameter required.

Example 30. A cylindrical tube, or hollow beam of oak, whose greater diameter is 8 inches, and length of bearing, or distance between the supports, 18 feet, is found to sustain a load of 3362 pounds, including its own weight, applied at a point 12 feet from one support, and 6 feet from the other; what is the lesser diameter, the elastic force remaining perfect?

Here, by the rule, we get

$$\sqrt[4]{8\left\{8^3 - \frac{12 \times 6 \times 3362}{31 \times 18}\right\}} = 5 \text{ inches, the}$$

diameter required.

Problem XXVI. *In the equation* $mnw=31ld^3\left(1-\frac{D^4}{d^4}\right)$, *there are given, the diameter* D, *the segments of the length m, n, and the load w; to find the diameter d.*

By transposition and reduction, our equation becomes

$$d^4-\frac{mnw}{31l}d=D^4;$$

and by substituting the numbers in the preceding example, we get

$$d^4-433{\cdot}875d=625,$$

which being reduced by the rules for biquadratic equations, gives $d=8$ inches, the diameter required.

Problem XXVII. *In the equation* $mnw=31ld^3\left(1-\frac{D^4}{d^4}\right)$, *there are given, the diameters d and* D, *the length l, and the load w; to find m and n, the segments of the length separately.*

Here, by transposition and reduction, according to the rule at Problem VII. we find the expressions for the segments m and n, to be respectively as below, viz.

$$m=\frac{1}{2}\left\{l+\sqrt{l^2-\frac{125l(d^4-D^4)}{dw}}\right\};$$

$$\text{and } n=\frac{1}{2}\left\{l-\sqrt{l^2-\frac{125l(d^4-D^4)}{dw}}\right\};$$

from which expressions the following practical rule is derived:

Rule 42. From the fourth power of the greater diameter, subtract the fourth power of the lesser;

multiply the remainder by 125 times the length of bearing, and divide the product by the greater diameter drawn into the given load, and subtract the quotient from the square of the given length; then, to or from the length, add or subtract the square root of the remainder, and half the sum, or half the difference, will be the greater or lesser segment accordingly.

Example 31. A cylindrical tube, or hollow beam of oak, whose greater diameter is 8 inches, lesser diameter 5 inches, and length of bearing, or distance between the supports, 18 feet, is found to sustain a load of 3362 pounds, including its own weight and the elastic force remaining perfect; at what point of the length is the load applied?

Here, by the rule, we get

$$\frac{1}{2}\left\{18 \pm \sqrt{18^2 - \frac{125 \times 18(8^4 - 5^4)}{8 \times 3362}}\right\} = 12 \text{ or}$$

6 feet, the distances from the supports required.

When the beam is supported horizontally at the ends, and loaded uniformly over the length, the equation that involves the conditions of strength and magnitude, is

$$lw = 250\, d^3\left(1 - \frac{D^4}{d^4}\right).$$

But when the beam is fixed at one end and loaded at the other, the equation becomes

$$lw = 31\, d^3\left(1 - \frac{D^4}{d^4}\right).$$

And when it is fixed at one end and loaded uniformly over the length, we get

$$lw = 62\,d^3\left(1 - \frac{D^4}{d^4}\right).$$

What has hitherto been done applies only to oak timber; but we may here remark, that whatever may be the species of the material, formulæ of the same form and composition will apply to it, if placed under the same conditions as those which we have considered for oak, the only variation being in the constant numbers or co-efficients, which must of course vary according to the cohesive force and extensibility of the material under consideration. Now, since the formulæ undergo no change but in the constant numbers, it is also evident, that the rules derived from them will be the same as those which we have applied to oak, if the respective number for each material be substituted in the rules according to the case in question.

The following tables contain the formulæ with their appropriate constants, for several kinds of wood, corresponding to the several cases of the forms of beams which we have investigated; and from the simple nature of the arrangement, it is presumed, that to practical persons they will be found of considerable utility.

Tables of formulæ, for calculating the strength and magnitude of beams of the several kinds of wood named in the margin, and for the forms of beams specified at the top of each table.

TABLE I.

Name of the wood.	Rectangular beams, strained in the direction of the depth.				
	Case 1.	Case 2.	Case 3.	Case 4.	Case 5.
Ash	$l\,w=196\,b\,d^2$	$m\,n\,w=49\,l\,b\,d^2$	$l\,w=392\,b\,d^2$	$l\,w=49\,b\,d^2$	$l\,w=98\,b\,d^2$
Beech	$l\,w=128\,b\,d^2$	$m\,n\,w=32\,l\,b\,d^2$	$l\,w=256\,b\,d^2$	$l\,w=32\,b\,d^2$	$l\,w=64\,b\,d^2$
Elm	$l\,w=178\,b\,d^2$	$m\,n\,w=45\,l\,b\,d^2$	$l\,w=356\,b\,d^2$	$l\,w=45\,b\,d^2$	$l\,w=90\,b\,d^2$
Fir, red or yellow	$l\,w=255\,b\,d^2$	$m\,n\,w=64\,l\,b\,d^2$	$l\,w=510\,b\,d^2$	$l\,w=64\,b\,d^2$	$l\,w=128\,b\,d^2$
Fir, white	$l\,w=196\,b\,d^2$	$m\,n\,w=49\,l\,b\,d^2$	$l\,w=392\,b\,d^2$	$l\,w=49\,b\,d^2$	$l\,w=98\,b\,d^2$
Oak, English ..	$l\,w=212\,b\,d^2$	$m\,n\,w=53\,l\,b\,d^2$	$l\,w=425\,b\,d^2$	$l\,w=53\,b\,d^2$	$l\,w=106\,b\,d^2$
Pine, American	$l\,w=212\,b\,d^2$	$m\,n\,w=53\,l\,b\,d^2$	$l\,w=425\,b\,d^2$	$l\,w=53\,b\,d^2$	$l\,w=106\,b\,d^2$

TABLE II.

Name of the wood.	Square beams, strained in the direction of the side.				
	Case 1.	*Case* 2.	*Case* 3.	*Case* 4.	*Case* 5.
Ash	$l\,w=196\,s^3$	$m\,n\,w=49\,l\,s^3$	$l\,w=392\,s^3$	$l\,w=49\,s^3$	$l\,w=98\,s^3$
Beech	$l\,w=128\,s^3$	$m\,n\,w=32\,l\,s^3$	$l\,w=256\,s^3$	$l\,w=32\,s^3$	$l\,w=64\,s^3$
Elm	$l\,w=178\,s^3$	$m\,n\,w=45\,l\,s^3$	$l\,w=356\,s^3$	$l\,w=45\,s^3$	$l\,w=90\,s^3$
Fir, red or yellow	$l\,w=255\,s^3$	$m\,n\,w=64\,l\,s^3$	$l\,w=510\,s^3$	$l\,w=64\,s^3$	$l\,w=128\,s^3$
Fir, white	$l\,w=196\,s^3$	$m\,n\,w=49\,l\,s^3$	$l\,w=392\,s^3$	$l\,w=49\,s^3$	$l\,w=98\,s^3$
Oak, English ..	$l\,w=212\,s^3$	$m\,n\,w=53\,l\,s^3$	$l\,w=425\,s^3$	$l\,w=53\,s^3$	$l\,w=106\,s^3$
Pine, American	$l\,w=212\,s^3$	$m\,n\,w=53\,l\,s^3$	$l\,w=425\,s^3$	$l\,w=53\,s^3$	$l\,w=106\,s^3$

TABLE III.

Name of the wood.	Square beams, strained in the direction of the vertical diagonal.				
	Case 1.	*Case* 2.	*Case* 3.	*Case* 4.	*Case* 5.
Ash	$l\,w=138\,s^3$	$m\,n\,w=35\,l\,s^3$	$l\,w=276\,s^3$	$l\,w=35\,s^3$	$l\,w=70\,s^3$
Beech	$l\,w=90\,s^3$	$m\,n\,w=23\,l\,s_3$	$l\,w=180\,s^3$	$l\,w=23\,s^3$	$l\,w=46\,s^3$
Elm	$l\,w=126\,s^3$	$m\,n\,w=32\,l\,s^3$	$l\,w=252\,s^3$	$l\,w=32\,s^3$	$l\,w=64\,s^3$
Fir, red or yellow	$l\,w=180\,s^3$	$m\,n\,w=45\,l\,s^3$	$l\,w=360\,s^3$	$l\,w=45\,s^3$	$l\,w=90\,s^3$
Fir, white	$l\,w=138\,s^3$	$m\,n\,w=35\,l\,s^3$	$l\,w=276\,s^3$	$l\,w=35\,s^3$	$l\,w=70\,s^3$
Oak, English ..	$l\,w=150\,s^3$	$m\,n\,w=38\,l\,s^3$	$l\,w=300\,s^3$	$l\,w=38\,s^3$	$l\,w=75\,s^3$
Pine, American	$l\,w=150\,s^3$	$m\,n\,w=38\,l\,s^3$	$l\,w=300\,s^3$	$l\,w=38\,s^3$	$l\,w=75\,s^3$

TABLE IV.

Name of the wood.	Cylindrical beams.				
	Case 1.	Case 2.	Case 3.	Case 4.	Case 5.
Ash	$lw = 115\,d^3$	$mnw = 29\,l\,d^3$	$lw = 230\,d^3$	$lw = 29\,d^3$	$lw = 58\,d^3$
Beech	$lw = 75\,d^3$	$mnw = 19\,l\,d^3$	$lw = 150\,d^3$	$lw = 19\,d^3$	$lw = 38\,d^3$
Elm	$lw = 105\,d^3$	$mnw = 26\,l\,d^3$	$lw = 210\,d^3$	$lw = 26\,d^3$	$lw = 52\,d^3$
Fir, red or yellow	$lw = 150\,d^3$	$mnw = 38\,l\,d^3$	$lw = 300\,d^3$	$lw = 38\,d^3$	$lw = 76\,d^3$
Fir, white	$lw = 115\,d^3$	$mnw = 29\,l\,d^3$	$lw = 230\,d^3$	$lw = 29\,d^3$	$lw = 58\,d^3$
Oak, English ..	$lw = 125\,d^3$	$mnw = 31\,l\,d^3$	$lw = 250\,d^3$	$lw = 31\,d^3$	$lw = 62\,d^3$
Pine, American	$lw = 125\,d^3$	$mnw = 31\,l\,d^3$	$lw = 250\,d^3$	$lw = 31\,d^3$	$lw = 62\,d^3$

TABLE V.

Name of the wood.	Tubular beams.—Ratio of the exterior to the interior diameter, as 1 to 0·7166.				
	Case 1.	Case 2.	Case 3.	Case 4.	Case 5.
Ash	$lw = 85\,d^3$	$mnw = 21\,l\,d^3$	$lw = 169\,d^3$	$lw = 21\,d^3$	$lw = 42\,d^3$
Beech	$lw = 55\,d^3$	$mnw = 14\,l\,d^3$	$lw = 110\,d^3$	$lw = 14\,d^3$	$lw = 28\,d^3$
Elm	$lw = 77\,d^3$	$mnw = 19\,l\,d^3$	$lw = 154\,d^3$	$lw = 19\,d^3$	$lw = 38\,d^3$
Fir, red or yellow	$lw = 110\,d^3$	$mnw = 28\,l\,d^3$	$lw = 220\,d^3$	$lw = 28\,d^3$	$lw = 56\,d^3$
Fir, white	$lw = 85\,d^3$	$mnw = 21\,l\,d^3$	$lw = 169\,d^3$	$lw = 21\,d^3$	$lw = 42\,d^3$
Oak, English ..	$lw = 92\,d^3$	$mnw = 23\,l\,d^3$	$lw = 184\,d^3$	$lw = 23\,d^3$	$lw = 46\,d^3$
Pine, American	$lw = 92\,d^3$	$mnw = 23\,l\,d^3$	$lw = 184\,d^3$	$lw = 23\,d^3$	$lw = 46\,d^3$

TABLE VI.

GENERAL FORMULÆ FOR TUBULAR BEAMS.

Name of the wood.	*Tubular beams.—Exterior diameter = d, interior diameter = D, and* $\frac{D^4}{d^4} = p^4$.*				
	Case 1.	*Case* 2.	*Case* 3.	*Case* 4.	*Case* 5.
Ash..........	$lw = 115d^3(1-p^4)$	$mnw = 29ld^3(1-p^4)$	$lw = 230d^3(1-p^4)$	$lw = 29d^3(1-p^4)$	$lw = 58d^3(1-p^4)$
Beech........	$lw = 75d^3(1-p^4)$	$mnw = 19ld^3(1-p^4)$	$lw = 150d^3(1-p^4)$	$lw = 19d^3(1-p^4)$	$lw = 38d^3(1-p^4)$
Elm	$lw = 105d^3(1-p^4)$	$mnw = 26ld^3(1-p^4)$	$lw = 210d^3(1-p^4)$	$lw = 26d^3(1-p^4)$	$lw = 52d^3(1-p^4)$
Fir, red or yellow	$lw = 150d^3(1-p^4)$	$mnw = 38ld^3(1-p^4)$	$lw = 300d^3(1-p^4)$	$lw = 38d^3(1-p^4)$	$lw = 76d^3(1-p^4)$
Fir, white	$lw = 115d^3(1-p^4)$	$mnw = 29ld^3(1-p^4)$	$lw = 230d^3(1-p^4)$	$lw = 29d^3(1-p^4)$	$lw = 58d^3(1-p^4)$
Oak, English ..	$lw = 125d^3(1-p^4)$	$mnw = 31ld^3(1-p^4)$	$lw = 250d^3(1-p^4)$	$lw = 31d^3(1-p^4)$	$lw = 62d^3(1-p^4)$
Pine, American	$lw = 125d^3(1-p^4)$	$mnw = 31ld^3(1-p^4)$	$lw = 250d^3(1-p^4)$	$lw = 31d^3(1-p^4)$	$lw = 62d^3(1-p^4)$

* The quantity p^4 is substituted for $\frac{D^4}{d^4}$, for the greater convenience in tabulation, and it is presumed that no ambiguity will arise from it.

The cases, as numbered at the top of the columns, correspond to the five cases of the strain mentioned in a former part of the work, and which, for the sake of clearness and convenience, we here repeat.

1. *When the beam is supported horizontally at the ends, and loaded in the middle.*

2. *When the beam is supported horizontally at the ends, and loaded at some intermediate point.*

3. *When the beam is supported horizontally at the ends, and loaded uniformly over the length.*

4. *When the beam projects horizontally from a wall, into which one end is fixed, and a load applied at the other end.*

5. *When the beam projects horizontally from a wall, into which one end is fixed, and loaded uniformly over the length.*

If the method of calculating these several cases be thoroughly considered, it will be found sufficient for all practical purposes, as every variety that can occur must be an application of one or other of them.

The following general rule will serve for the reduction of the formulæ.

GENERAL RULE. (43)

Substitute the given dimensions of the beam, and their powers, together with the load to be sustained, for the representatives of each in the respective equations, and another equation will arise, involving only one

unknown term, which must be disengaged by multiplication, division, or the extraction of roots, according to the nature of its involution.

Example 32. A cylindrical beam of beech timber 10 feet in length between the supports, is found to sustain a load of 5640 pounds at the middle of its length including its own weight; what is its diameter, admitting the strain to be within the elastic force of beech?

Here the beam is cylindrical and loaded in the middle; this agrees with the first case of the fourth table, where the formula for beech is

$$lw = 75\,d^3.$$

Let the given length of bearing and the load be substituted for l and w in this equation, and it becomes

$$75\,d^3 = 56400;$$

let each side be divided by 75, the member with which the required diameter is combined, and we get

$$d^3 = \frac{56400}{75} = 752.$$

Here, then, we have got the required term to occupy individually one side of the equation, and its cube or third power is known, it being equal to 752; consequently, the diameter itself will become known by extracting the cube root; now, the cube root of 752 is 9·094; therefore, the beam is 9·094 inches in diameter.

And exactly after the same manner may any other case be resolved.

STIFFNESS OF TIMBER.

The formulæ that have hitherto been investigated, and the practical rules deduced from them, are applicable only to the consideration of *strength*, or when the flexure produced by the load is as great as it can be, without destroying a part of the elastic force; but, since in *carpentry* and other departments of practical science, the *comparative stiffness* is of greater importance than the *comparative strength*, we shall, in what follows, endeavour to investigate rules for calculating the dimensions of beams, to bear a given load with a given degree of deflexion.

Now, the principle of deflexion is, that in all similar beams placed under the same conditions, and subjected to a similar strain, whatever may be the form of the beam, *the square of the length in feet, divided by the depth in inches, drawn into the deflexion, is a constant quantity.*

Therefore, if the deflexion for a beam of given dimensions, and anyhow circumstanced, be accurately determined from experiment, the deflexion for a similar beam of any other dimensions and similarly circumstanced, can easily be calculated.

Now, in the experiment that we have chosen as our standard, the deflexion in the middle of the bar, produced by a load of 425 pounds, was 0·112 inches, when the bar was just able to recover itself on the removal of the straining force; therefore, from the above principle, we have the following equations for the deflexion in the several cases of the strain, for the different forms of beams.

1. *When the beam is supported horizontally at the ends, and loaded in the middle,*

$$d\delta = \cdot 056\, l^2.$$

2. *When the beam is supported horizontally at the ends, and loaded at some intermediate point,*

$$d\delta = \cdot 224\, m\, n.$$

3. *When the beam is supported horizontally at the ends, and loaded uniformly over the length,*

$$d\delta = \cdot 07\, l^2.$$

4. *When the beam projects horizontally from a wall, into which one end is fixed, and a load applied at the other,*

$$d\delta = \cdot 224\, l^2.$$

5. *When the beam projects horizontally from a wall, into which one end is fixed, and loaded uniformly over the length,*

$$d\delta = \cdot 28\, l^2.$$

The preceding equations apply to oak beams of any form of section, provided they are uniform throughout the length; it only now remains to adapt them to the different kinds of wood specified in the foregoing tables of formulæ; and this is effected by merely multiplying the constant of each case by the fraction, which expresses the comparative extensibility of the several woods, that of oak being unity.

The following table exhibits the comparative extensibilities, computed by the third principle of comparison mentioned at the outset of the work.

TABLE VII.

Table of the comparative extensibility of woods, that of oak being unity.	
Ash	$\frac{13}{14}$
Beech	$\frac{3}{4}$
Elm	$\frac{29}{28}$
Fir, red or yellow	$\frac{13}{14}$
Fir, white	$\frac{6}{7}$
Oak, English	1
Pine, American	$\frac{29}{28}$

And from the numbers in the above table, the formulæ in that which follows are derived.

TABLE VIII.

Table of formulæ for calculating the deflexion of woods.

Name of the wood.	*Case 1.*	*Case 2.*	*Case 3.*	*Case 4.*	*Case 5.*
Ash	$d\delta = \cdot052\,l^2$	$d\delta = \cdot208\,m\,n$	$d\delta = \cdot0650\,l^2$	$d\delta = \cdot208\,l^2$	$d\delta = \cdot26\,l^2$
Beech	$d\delta = \cdot042\,l^2$	$d\delta = \cdot168\,m\,n$	$d\delta = \cdot0525\,l^2$	$d\delta = \cdot168\,l^2$	$d\delta = \cdot21\,l^2$
Elm	$d\delta = \cdot058\,l^2$	$d\delta = \cdot232\,m\,n$	$d\delta = \cdot0725\,l^2$	$d\delta = \cdot232\,l^2$	$d\delta = \cdot29\,l^2$
Fir, red or yellow	$d\delta = \cdot052\,l^2$	$d\delta = \cdot208\,m\,n$	$d\delta = \cdot0650\,l^2$	$d\delta = \cdot208\,l^2$	$d\delta = \cdot26\,l^2$
Fir, white......	$d\delta = \cdot048\,l^2$	$d\delta = \cdot192\,m\,n$	$d\delta = \cdot0600\,l^2$	$d\delta = \cdot192\,l^2$	$d\delta = \cdot24\,l^2$
Oak, English ..	$d\delta = \cdot056\,l^2$	$d\delta = \cdot224\,m\,n$	$d\delta = \cdot0700\,l^2$	$d\delta = \cdot224\,l^2$	$d\delta = \cdot28\,l^2$
Pine, American	$d\delta = \cdot058\,l^2$	$d\delta = \cdot232\,m\,n$	$d\delta = \cdot0725\,l^2$	$d\delta = \cdot232\,l^2$	$d\delta = \cdot29\,l^2$

Where we have to observe, that in each of the above formulæ, δ is the deflexion in inches.

The rule for calculating the deflexion of beams in the several cases, may be expressed in the following general terms.

Rule 44. Multiply the square of the length, or the rectangle of the segments at the point of strain, in feet, by the constant number for the particular case, and divide the product by the depth of the beam in inches, for the deflexion sought.

Example 33. The depth of a beech plank is 16 inches, and its length 24 feet; what is the deflexion when strained to the extent of its elastic force, supposing the load to be applied at a point 18 feet from one support, and 6 feet from the other?

The formulæ for the second case of beech is

$$d\delta = \cdot 168\, mn. \qquad \text{(See Tab. VIII.)}$$

Now, $m = 18$ feet, $n = 6$ feet, and $d = 16$ inches; hence, by substitution, we get

$$16\delta = \cdot 168 \times 18 \times 6;$$

that is $\delta = 1\cdot 134$ inches, the deflexion required.

Example 34. A square beam of white fir, whose side is 8 inches, and length of bearing 18 feet, is loaded uniformly over the length, and strained to the extent of its elastic force; what is the deflexion?

The formula for the third case of white fir is

$$d\delta = \cdot 06\, l^2. \qquad \text{(See Tab. VIII.)}$$

Now, $d = 8$ inches, and $l = 18$ feet; hence, by substitution, we get

$$8\delta = \cdot 06 \times 18^2;$$

that is $\delta = 2\cdot 43$ inches, the deflexion required.

A cylindrical beam of 8 inches diameter, would be deflected to the same extent, and the same mode of calculation applies to the tubular and other forms.

The deflexions calculated above are those due to the load, when it is the greatest that the beam will bear, without destroying a part of the elastic force; but in practice, when the mechanic is desirous of obtaining uniformity and stiffness to his work, it becomes necessary to limit the deflexion; and it is obvious that this can only be done by increasing the dimensions of the body strained, for in every case it is not possible to diminish the effects or magnitude of the straining force.

Now, the writers on the resistance of solids have shewn, that within certain limits, the deflexion is directly proportional to the strain that produces it, and the deflexions in the foregoing table are those that indicate the extreme limit, being produced by the greatest load that the beams can sustain without injury; taking, therefore, the formulæ that apply to English oak, that being the material which we have chosen as a standard, we shall have, from the first case of Tables I. and VIII.,

$$\frac{\cdot 056\, l^2}{d} : \frac{212\, b d^2}{l} :: D : \frac{212\, b d^3 D}{\cdot 056\, l^3} = \text{the load}$$

that will produce the deflexion D, that being the deflexion beyond which the beam is not to be strained; therefore,

1. *When the beam is rectangular, supported horizontally at the ends, and loaded in the middle,*

$$l^3 w = 3786\, b d^3 D. \qquad (17)$$

Here, then, as in the case of strength, we shall consider the length of bearing always as known, and in addition, we shall assume the load to be borne and the deflexion to be resisted also as given, for in practice this will almost always be the case; the analysis of Equation 17 will therefore divide itself into two parts, as follows:

1. Given d, D, l and w; to find b.
2. Given b, D, l and w; to find d.

And from these cases we derive the following Problems.

PROBLEM XXVIII. *In the equation* $l^3 w = 3786\, b d^3 D$, *there are given, the depth* d, *the deflexion* D, *the length of bearing* l, *and the load* w; *to find the breadth* b.

Let both sides of the equation be divided by $3786\, d^3 D$, and we get

$$b = \frac{l^3 w}{3786\, d^3 D};$$

from which expression the following practical rule is derived.

Rule 45. Multiply the given load by the cube, or third power of the length, and divide the product by 3786 times the deflexion, drawn into the cube of the depth in inches, and the quotient will be the breadth required.

Example 35. What must be the breadth of a rectangular beam of oak, so that it shall not be deflected more than $\frac{3}{4}$ of an inch, by a load of 6632 pounds applied at the middle of its length, the depth being 12 inches, and the length of bearing 18 feet?

Here, by the rule, we get

$$\frac{18^3 \times 6632}{3786 \times 12^3 \times \cdot 75} = 7 \cdot 883 \text{ inches, the}$$

breadth required.

PROBLEM XXIX. *In the equation* $l^3 w = 3786\, b\, d^3 \mathrm{D}$, *there are given, the breadth* b, *the deflexion* D, *the length of bearing* l, *and the load* w; *to find the depth* d.

Divide both sides of the equation by $3786 b \mathrm{D}$, and we get

$$d^3 = \frac{l^3 w}{3786 b \mathrm{D}};$$

and this, by extracting the cube root, becomes, for the depth

$$d = \frac{l^3 w}{3786 b \mathrm{D}};$$

from which the following practical rule is derived.

Rule 46. Multiply the given load by the cube, or third power of the length, and divide the product by 3786 times the breadth drawn into the deflexion; then, the cube root of the quotient will give the depth in inches.

Example 36. What must be the depth of a rectangular beam of oak, that will not be deflected more than $\frac{3}{4}$ of an inch, by a load of 6632 pounds applied at the middle of its length, the breadth being 7·883 inches, and the length 18 feet?

Here, by the rule, we get

$$\sqrt[3]{\frac{18^3 \times 6632}{3786 \times 7{\cdot}883 \times {\cdot}75}} = 12 \text{ inches,}$$

the depth required.

From the second case of Tables I. and VIII. we have for English oak

$$\frac{{\cdot}224mn}{d} : \frac{53lbd^2}{mn} :: \text{D} : \frac{53lbd^3\text{D}}{{\cdot}224m^2n^2} = \text{the load}$$

that will produce the deflexion D; hence we have

2. *When the beam is rectangular, supported horizontally at the ends, and loaded at some intermediate point,*

$$m^2n^2w = 237\,lbd^3\text{D}. \qquad (18)$$

In which expression the cases of analysis are as under.

1. Given d, D, l, m, n and w; to find b.
2. Given b, D, l, m, n and w; to find d.

From these we derive the following problems.

PROBLEM XXX. *In the equation* $m^2n^2w = 237\,lbd^3\text{D}$, *there are given, the depth d, the deflexion* D, *the length of bearing l, the segments of the length m, n and the load w; to find the breadtb b.*

Divide both sides of the equation by $237\,ld^3\text{D}$, and the expression for the breadth becomes,

$$b = \frac{m^2n^2w}{237\,ld^3\text{D}};$$

from which the following practical rule is derived.

Rule 47. Multiply the given load by the square of the rectangle of the segments of the length, and divide the product by 237 times the given length, multiplied by the cube of the depth drawn into the deflexion, and the quotient will be the breadth required.

Example 37. The depth of a rectangular beam of oak is 12 inches, and its length 18 feet; what must be its breadth, so that it shall not be deflected more than $\frac{3}{4}$ of an inch, by a load of 6640 pounds, applied at a point 12 feet from one support, and 6 feet from the other?

Here, by the rule, we get

$$\frac{(12\times6)^2\times6640}{237\times18\times12^3\times\cdot75}=6\tfrac{1}{4}$$ inches nearly, the breadth required.

Problem XXXI. *In the equation* $m^2n^2w=237\,lbd^3\mathrm{D}$, *there are given, the breadth* b, *the deflexion* D, *the length of bearing* l, *the segments of the length* m, n *and the load* w; *to find the depth* d.

Let both sides of the equation be divided by $237\,lb\mathrm{D}$, and we obtain

$$d^3=\frac{m^2n^2w}{237\,lb\mathrm{D}},$$

which, by extracting the cube root of both sides, becomes

$$d=\sqrt[3]{\frac{m^2n^2w}{237\,lb\mathrm{D}}};$$

from which we derive the following practical rule.

Rule 48. Multiply the given load by the square of the rectangle of the segments of the length, and divide the product by 237 times the length multiplied by the breadth and deflexion; then, the cube root of the quotient will be the depth required.

Example 38. The breadth of a rectangular beam of oak is $6\frac{1}{4}$ inches, and its length 18 feet; what must be its depth, so that it shall not be deflected more than $\frac{3}{4}$ of an inch, by a load of 6640 pounds, applied 12 feet from one support, and 6 feet from the other?

Here, by the rule, we get

$$\sqrt[3]{\frac{(12\times6)^2\times6640}{237\times18\times6{\cdot}25\times{\cdot}75}}=12 \text{ inches nearly,}$$

the depth required.

From the third case of Tables I. and VIII. we have for English oak,

$$\frac{{\cdot}07\,l^2}{d}:\frac{425\,bd^2}{l}::\text{D}:\frac{425\,bd^3\text{D}}{{\cdot}07\,l^3}=\text{the load}$$

that will produce the deflexion D; hence we have

3. *When the beam is rectangular, supported horizontally at the ends, and loaded uniformly over the length,*

$$l^3w=6071\,bd^3\text{D}.$$

From the fourth case of Tables I. and VIII. we have for English oak,

$$\frac{{\cdot}224\,l^2}{d}:\frac{53\,bd^2}{l}::\text{D}:\frac{53\,bd^3\text{D}}{{\cdot}224\,l^3}=\text{the load}$$

that will produce the deflexion D; hence we have

4. *When the beam is rectangular, fixed at one end, and loaded at the other,*

$$l^3 w = 237\, b\, d^3 \mathrm{D}.$$

From the fifth case of Tables I. and VIII. we have for English oak

$$\frac{\cdot 28\, l^2}{d} : \frac{106\, b d^2}{l} :: \mathrm{D} : \frac{106\, b d^3 \mathrm{D}}{\cdot 28\, l^3} = \text{the load}$$

that will produce the deflexion D; hence we have

5. *When the beam is rectangular, fixed at one end, and loaded uniformly over the length,*

$$l^3 w = 379\, b\, d^3 \mathrm{D}.$$

Hence it appears, that by substituting the constant numbers 6071, 237, and 379 respectively, for 3786 in Rules 45 and 46, the same rules apply to the determination of the breadth and depth in each of the three foregoing cases; and when b becomes equal to d, or the beam is a square, $b\, d^3$ becomes d^4, or putting s for the side of the square, we have $s^4 = d^4$; let s^4 therefore, be substituted for $b\, d^3$ in each of the foregoing cases, and the corresponding expressions for a square beam, when the strain is parallel to the side, will be as follows, viz.

$$\begin{array}{ll} 1. & l^3 w = 3786\, s^4 \mathrm{D}. \\ 2. & m^2 n^2 w = 237\, l\, s^4 \mathrm{D}. \\ 3. & l^3 w = 6071\, s^4 \mathrm{D}. \qquad (19) \\ 4. & l^3 w = 237\, s^4 \mathrm{D}. \\ 5. & l^3 w = 379\, s^4 \mathrm{D}. \end{array}$$

And when the strain is in the direction of the vertical diagonal, the formulæ become

1. $l^3 w = 2678 s^4 D.$
2. $m^2 n^2 w = 170 l s^4 D.$
3. $l^3 w = 4285 s^4 D.$ (20)
4. $l^3 w = 170 s^4 D.$
5. $l^3 w = 268 s^4 D.$

From which we infer, that if the proper constants be employed, the same rules will answer for the several cases in both these classes of formulæ; we shall therefore omit the rules here, and pass on to the consideration of the cylindrical beam, where we have, from the first case of Tables IV. and VIII. for English oak

$$\frac{\cdot056\, l^2}{d} : \frac{125 d^3}{l} :: D : \frac{125 d^4 D}{\cdot056\, l^3} = \text{the load}$$

that will produce the deflexion D; hence we have

1. *When the beam is cylindrical, supported horizontally at the ends, and loaded in the middle.*

$$l^3 w = 2232 d^4 D. \qquad (21)$$

Which expression affords the following problem.

Problem XXXII. *In the equation* $l^3 w = 2232 d^4 D$, *there are given, the deflexion* D, *the length* l, *and the load* w; *to find the diameter* d.

Let both sides of the equation be divided by 2232 D, and we obtain

$$d^4 = \frac{l^3 w}{2232 D},$$

and by extracting the fourth root of both sides, it becomes

$$d = \sqrt[4]{\frac{l^3 w}{2232\,D}};$$

from which we derive the following practical rule.

Rule 49. Multiply the given load by the cube, or third power of the length of bearing; divide the product by 2232 times the deflexion, and extract the fourth root of the quotient for the diameter required.

Example 39. In a cylindrical column of oak, there are given, the deflexion ·6 inches, the length of bearing 12 feet, and the load 3360 pounds; what is the diameter?

Here, by the rule, we get

$$\sqrt[4]{\frac{12^3 \times 3360}{2232 \times \cdot 6}} = 8{\cdot}114 \text{ inches, for}$$

the diameter required.

From the second case of Tables IV. and VIII. we have for English oak

$$\frac{\cdot 224 m n}{d} : \frac{31 l d^3}{m n} :: D : \frac{31 l d^4 D}{\cdot 224 m^2 n^2} = \text{the load}$$

that will produce the deflexion D; hence we have

2. *When the beam is cylindrical, supported horizontally at the ends, and loaded at some intermediate point,*

$$m^2 n^2 w = 138 l d^4 D; \qquad (22)$$

from which arises the following problem.

PROBLEM XXXIII. *In the equation* $m^2n^2w = 138\,ld^4\text{D}$, *there are given, the deflexion* D, *the length* l, m *and* n *the segments of the length, and the load* w; *to find the diameter.*

Let both sides of the equation be divided by $138\,l\text{D}$, and we have

$$d^4 = \frac{m^2n^2w}{138\,l\text{D}},$$

and by extracting the fourth root of both sides, it becomes

$$d = \sqrt[4]{\frac{m^2n^2w}{138\,l\text{D}}};$$

from which the following practical rule is derived.

Rule 50. Multiply the given load by the square of the rectangle of the segments of the length; divide the product by 138 times the length drawn into the deflexion, and the fourth root of the quotient will be the diameter required.

Example 40. A cylindrical beam of oak, 14 feet long, is deflected 0·8 of an inch, by a load of 4480 pounds, applied 9 feet from one support, and 5 feet from the other; what is its diameter?

Here, by the rule, we get

$$\sqrt[4]{\frac{(9\times5)^2\times4480}{138\times14\times\cdot8}} = 8\cdot766 \text{ inches, for}$$

the diameter required.

From the third case of Tables IV. and VIII. we have, for English oak

$\frac{\cdot 07\,l^2}{d} : \frac{250\,d^3}{l} :: D : \frac{250\,d^4 D}{\cdot 07\,l^3}$ = the load that will produce the deflexion D; hence we have

3. *When the beam is cylindrical, supported horizontally at the ends, and loaded uniformly over the length,*

$$l^3 w = 3571\,d^4 D.$$

From the fourth case of Tables IV. and VIII. we have for English oak

$\frac{\cdot 224\,l^2}{d} : \frac{31\,d^3}{l} :: D : \frac{31\,d^4 D}{\cdot 224\,l^3}$ = the load that will produce the deflexion D; hence we have

4. *When the beam is cylindrical, fixed at one end, and loaded at the other.*

$$l^3 w = 138\,d^4 D.$$

From the fifth case of Tables IV. and VIII. we have for English oak

$\frac{\cdot 28\,l^2}{d} : \frac{62\,d^3}{l} :: D : \frac{62\,d^4 D}{\cdot 28\,l^3}$ = the load that will produce the deflexion D; hence we have

5. *When the beam is cylindrical, fixed at one end and loaded uniformly over the length,*

$$l^3 w = 221\,d^4 D.$$

Therefore, if 3571, 138, and 221, be respectively substituted for 2232 in Rule 49, the rule for the diameter in the three preceding cases will otherwise remain unchanged.

The following are the formulæ for tubular beams, when the greater diameter is to the lesser in the

Tables of formulæ for calculating the magnitude and stiffness of beams of the several kinds of wood named in the margin, and for the forms of beams specified at the top of each table.

TABLE IX.

	Rectangular beams, strained in the direction of the depth.				
Name of the wood.	***Case 1.***	***Case 2.***	***Case 3.***	***Case 4.***	***Case 5.***
Ash	$l^3 w = 3769\, b d^3 D$	$m^2 n^2 w = 235\, l b d^3 D$	$l^3 w = 6031\, b d^3 D$	$l^3 w = 235\, b d^3 D$	$l^3 w = 377\, b d^3 D$
Beech	$l^3 w = 3048\, b d^3 D$	$m^2 n^2 w = 190\, l b d^3 D$	$l^3 w = 4876\, b d^3 D$	$l^3 w = 190\, b d^3 D$	$l^3 w = 305\, b d^3 D$
Elm	$l^3 w = 3069\, b d^3 D$	$m^2 n^2 w = 194\, l b d^3 D$	$l^3 w = 4911\, b d^3 D$	$l^3 w = 194\, b d^3 D$	$l^3 w = 310\, b d^3 D$
Fir, red or yellow	$l^3 w = 4904\, b d^3 D$	$m^2 n^2 w = 308\, l b d^3 D$	$l^3 w = 7846\, b d^3 D$	$l^3 w = 308\, b d^3 D$	$l^3 w = 492\, b d^3 D$
Fir, white	$l^3 w = 4083\, b d^3 D$	$m^2 n^2 w = 255\, l b d^3 D$	$l^3 w = 6533\, b d^3 D$	$l^3 w = 255\, b d^3 D$	$l^3 w = 408\, b d^3 D$
Oak, English ..	$l^3 w = 3786\, b d^3 D$	$m^2 n^2 w = 236\, l b d^3 D$	$l^3 w = 6071\, b d^3 D$	$l^3 w = 236\, b d^3 D$	$l^3 w = 379\, b d^3 D$
Pine, American	$l^3 w = 3655\, b d^3 D$	$m^2 n^2 w = 228\, l b d^3 D$	$l^3 w = 5862\, b d^3 D$	$l^3 w = 228\, b d^3 D$	$l^3 w = 365\, b d^3 D$

TABLE X.

Name of the wood.	Square beams, strained in the direction of the side.				
	Case 1.	*Case 2.*	*Case 3.*	*Case 4.*	*Case 5.*
Ash	$l^3 w = 3769 s^4 D$	$m^2 n^2 w = 235\, l s^4 D$	$l^3 w = 6031\, s^4 D$	$l^3 w = 235\, s^4 D$	$l^3 w = 377\, s^4 D$
Beech	$l^3 w = 3048 s^4 D$	$m^2 n^2 w = 190\, l s^4 D$	$l^3 w = 4876\, s^4 D$	$l^3 w = 190\, s^4 D$	$l^3 w = 305\, s^4 D$
Elm	$l^3 w = 3069 s^4 D$	$m^2 n^2 w = 194\, l s^4 D$	$l^3 w = 4911\, s^4 D$	$l^3 w = 194\, s^4 D$	$l^3 w = 310\, s^4 D$
Fir, red or yellow	$l^3 w = 4904 s^4 D$	$m^2 n^2 w = 308\, l s^4 D$	$l^3 w = 7846\, s^4 D$	$l^3 w = 308\, s^4 D$	$l^3 w = 492\, s^4 D$
Fir, white	$l^3 w = 4083 s^4 D$	$m^2 n^2 w = 255\, l s^4 D$	$l^3 w = 6533\, s^4 D$	$l^3 w = 255\, s^4 D$	$l^3 w = 408\, s^4 D$
Oak, English ..	$l^3 w = 3786 s^4 D$	$m^2 n^2 w = 236\, l s^4 D$	$l^3 w = 6071\, s^4 D$	$l^3 w = 236\, s^4 D$	$l^3 w = 379\, s^4 D$
Pine, American	$l^3 w = 3655 s^4 D$	$m^2 n^2 w = 228\, l s^4 D$	$l^3 w = 5862\, s^4 D$	$l^3 w = 228\, s^4 D$	$l^3 w = 365\, s^4 D$

TABLE XI.

Name of the wood.	Square beams, strained in the direction of the vertical diagonal.				
	Case 1.	*Case 2.*	*Case 3.*	*Case 4.*	*Case 5.*
Ash	$l^3 w = 2654\, s^4 D$	$m^2 n^2 w = 168\, l s^4 D$	$l^3 w = 4246\, s^4 D$	$l^3 w = 168\, s^4 D$	$l^3 w = 269\, s^4 D$
Beech	$l^3 w = 2143\, s^4 D$	$m^2 n^2 w = 137\, l s^4 D$	$l^3 w = 3429\, s^4 D$	$l^3 w = 137\, s^4 D$	$l^3 w = 219\, s^4 D$
Elm	$l^3 w = 2172\, s^4 D$	$m^2 n^2 w = 138\, l s^4 D$	$l^3 w = 3476\, s^4 D$	$l^3 w = 138\, s^4 D$	$l^3 w = 221\, s^4 D$
Fir, red or yellow	$l^3 w = 3462\, s^4 D$	$m^2 n^2 w = 216\, l s^4 D$	$l^3 w = 5538\, s^4 D$	$l^3 w = 216\, s^4 D$	$l^3 w = 346\, s^4 D$
Fir, white	$l^3 w = 2875\, s^4 D$	$m^2 n^2 w = 182\, l s^4 D$	$l^3 w = 4600\, s^4 D$	$l^3 w = 182\, s^4 D$	$l^3 w = 292\, s^4 D$
Oak, English ..	$l^3 w = 2678\, s^4 D$	$m^2 n^2 w = 170\, l s^4 D$	$l^3 w = 4285\, s^4 D$	$l^3 w = 170\, s^4 D$	$l^3 w = 268\, s^4 D$
Pine, American	$l^3 w = 2586\, s^4 D$	$m^2 n^2 w = 164\, l s^4 D$	$l^3 w = 4138\, s^4 D$	$l^3 w = 164\, s^4 D$	$l^3 w = 258\, s^4 D$

TABLE XII.

Name of the wood.	*Cylindrical beams.* Case 1.	Case 2.	Case 3.	Case 4.	Case 5.
Ash	$l^3 w = 2212\, d^4 D$	$m^2 n^2 w = 139\, l\, d^4 D$	$l^3 w = 3539\, d^4 D$	$l^3 w = 139\, d^4 D$	$l^3 w = 223\, d^4 D$
Beech	$l^3 w = 1786\, d^4 D$	$m^2 n^2 w = 113\, l\, d^4 D$	$l^3 w = 2857\, d^4 D$	$l^3 w = 113\, d^4 D$	$l^3 w = 181\, d^4 D$
Elm	$l^3 w = 1810\, d^4 D$	$m^2 n^2 w = 112\, l\, d^4 D$	$l^3 w = 2897\, d^4 D$	$l^3 w = 112\, d^4 D$	$l^3 w = 179\, d^4 D$
Fir, red or yellow	$l^3 w = 2885\, d^4 D$	$m^2 n^2 w = 183\, l\, d^4 D$	$l^3 w = 4615\, d^4 D$	$l^3 w = 183\, d^4 D$	$l^3 w = 292\, d^4 D$
Fir, white	$l^3 w = 2396\, d^4 D$	$m^2 n^2 w = 151\, l\, d^4 D$	$l^3 w = 3833\, d^4 D$	$l^3 w = 151\, d^4 D$	$l^3 w = 242\, d^4 D$
Oak, English ..	$l^3 w = 2232\, d^4 D$	$m^2 n^2 w = 138\, l\, d^4 D$	$l^3 w = 3571\, d^4 D$	$l^3 w = 138\, d^4 D$	$l^3 w = 221\, d^4 D$
Pine, American	$l^3 w = 2155\, d^4 D$	$m^2 n^2 w = 133\, l\, d^4 D$	$l^3 w = 3448\, d^4 D$	$l^3 w = 133\, d^4 D$	$l^3 w = 214\, d^4 D$

TABLE XIII.

PARTICULAR FORMULÆ.

Name of the wood.	*Tubular beams.—Ratio of the exterior to the interior diameter, as 1 to 0·7166.* Case 1.	Case 2.	Case 3.	Case 4.	Case 5.
Ash	$l^3 w = 1635\, d^4 D$	$m^2 n^2 w = 101\, l\, d^4 D$	$l^3 w = 2600\, d^4 D$	$l^3 w = 101\, d^4 D$	$l^3 w = 161\, d^4 D$
Beech	$l^3 w = 1309\, d^4 D$	$m^2 n^2 w = 83\, l\, d^4 D$	$l^3 w = 2095\, d^4 D$	$l^3 w = 83\, d^4 D$	$l^3 w = 133\, d^4 D$
Elm	$l^3 w = 1328\, d^4 D$	$m^2 n^2 w = 82\, l\, d^4 D$	$l^3 w = 2124\, d^4 D$	$l^3 w = 82\, d^4 D$	$l^3 w = 131\, d^4 D$
Fir, red or yellow	$l^3 w = 2115\, d^4 D$	$m^2 n^2 w = 135\, l\, d^4 D$	$l^3 w = 3385\, d^4 D$	$l^3 w = 135\, d^4 D$	$l^3 w = 215\, d^4 D$
Fir, white	$l^3 w = 1771\, d^4 D$	$m^2 n^2 w = 109\, l\, d^4 D$	$l^3 w = 2817\, d^4 D$	$l^3 w = 109\, d^4 D$	$l^3 w = 175\, d^4 D$
Oak, English ..	$l^3 w = 1643\, d^4 D$	$m^2 n^2 w = 103\, l\, d^4 D$	$l^3 w = 2629\, d^4 D$	$l^3 w = 103\, d^4 D$	$l^3 w = 164\, d^4 D$
Pine, American	$l^3 w = 1586\, d^4 D$	$m^2 n^2 w = 99\, l\, d^4 D$	$l^3 w = 2538\, d^4 D$	$l^3 w = 99\, d^4 D$	$l^3 w = 158\, d^4 D$

TABLE XIV.

GENERAL FORMULÆ.

Name of the wood.	*Tubular beams.—Exterior diameter = d, interior diameter = D, and* $\frac{D^4}{d^4}=p^4$.				
	Case 1.	Case 2.	Case 3.	Case 4.	Case 5.
Ash	$l^3w=2212\,d^4D(1-p^4)$	$m^2n^2w=139\,ld^4D(1-p^4)$	$l^3w=3539\,d^4D(1-p^4)$	$l^3w=139\,d^4D(1-p^4)$	$l^3w=223\,d^4D(1-p^4)$
Beech	$l^3w=1786\,d^4D(1-p^4)$	$m^2n^2w=113\,ld^4D(1-p^4)$	$l^3w=2857\,d^4D(1-p^4)$	$l^3w=113\,d^4D(1-p^4)$	$l^3w=181\,d^4D(1-p^4)$
Elm	$l^3w=1810\,d^4D(1-p^4)$	$m^2n^2w=112\,ld^4D(1-p^4)$	$l^3w=2897\,d^4D(1-p^4)$	$l^3w=112\,d^4D(1-p^4)$	$l^3w=179\,d^4D(1-p^4)$
Fir, red or yellow	$l^3w=2885\,d^4D(1-p^4)$	$m^2n^2w=183\,ld^4D(1-p^4)$	$l^3w=4615\,d^4D(1-p^4)$	$l^3w=183\,d^4D(1-p^4)$	$l^3w=292\,d^4D(1-p^4)$
Fir, white	$l^3w=2396\,d^4D(1-p^4)$	$m^2n^2w=151\,ld^4D(1-p^4)$	$l^3w=3833\,d^4D(1-p^4)$	$l^3w=151\,d^4D(1-p^4)$	$l^3w=242\,d^4D(1-p^4)$
Oak, English ..	$l^3w=2232\,d^4D(1-p^4)$	$m^2n^2w=138\,ld^4D(1-p^4)$	$l^3w=3571\,d^4D(1-p^4)$	$l^3w=138\,d^4D(1-p^4)$	$l^3w=221\,d^4D(1-p^4)$
Pine, American	$l^3w=2155\,d^4D(1-p^4)$	$m^2n^2w=133\,ld^4D(1-p^4)$	$l^3w=3448\,d^4D(1-p^4)$	$l^3w=133\,d^4D(1-p^4)$	$l^3w=214\,d^4D(1-p^4)$

The formulæ which we have just tabulated are of the greatest utility in practical cases, and it is presumed that the following general rule for their reduction will be found sufficient.

A general rule for reducing the formulæ in Tables IX. X. XI. XII. XIII. and XIV.

RULE 51. *Substitute the given quantities and their powers for the representatives of each in the appropriate formula, and an equation will arise involving only one unknown term; then, divide both sides of this new equation by the co-efficient of the unknown term, and extract such a root of the quotient as is denoted by its index, and the result will be the answer.*

One example will suffice.

Example 41. A square beam of ash, whose length is 8 feet, is fixed in a wall at one end, and deflected ·9 of an inch by a load of 928 pounds applied at the other end; what is the side of the square, supposing the direction of the straining force to be parallel to the side?

The formula for the fourth case of square beams, corresponding to ash in Table X., is

$$l^3 w = 235 s^4 \text{D}.$$

Now $l = 8$ feet, $w = 928$ pounds, and $\text{D} = \cdot 9$ inches; let these numbers be substituted for l, w and D, and we get

$$211\cdot 5\, s^4 = 275136;$$

$$\text{that is,} \quad s^4 = 1300\cdot 88,$$

$$\text{or} \quad s = \sqrt[4]{1300\cdot 88} = 6 \text{ inches},$$

the side of the square required.

Thus much for beams strained transversely; and the rules for the *resistance to compression and tension* being so remarkably simple, it will be sufficient to tabulate the formula, as below.

TABLE XV.

Table of formulæ for calculating the resistance to compression and tension.

Name of the wood.	*Rectangular beams.* *Case 1.*	*Rectangular beams.* *Case 2.*	*Square beams.* *Case 1.*	*Square beams.* *Case 2.*	*Cylindrical beams.* *Case 1.*	*Cylindrical beams.* *Case 2.*
Ash	$w=3540\,b\,d$	$w=885\,b\,d$	$w=3540\,s^2$	$w=885\,s^2$	$w=2257\,d^2$	$w=564\,d^2$
Beech	$w=2360\,b\,d$	$w=590\,b\,d$	$w=2360\,s^2$	$w=590\,s^2$	$w=1504\,d^2$	$w=376\,d^2$
Elm	$w=3240\,b\,d$	$w=810\,b\,d$	$w=3240\,s^2$	$w=810\,s^2$	$w=2065\,d^2$	$w=516\,d^2$
Fir, red or yellow	$w=4290\,b\,d$	$w=1073\,b\,d$	$w=4290\,s^2$	$w=1073\,s^2$	$w=2735\,d^2$	$w=684\,d^2$
Fir, white	$w=3630\,b\,d$	$w=908\,b\,d$	$w=3630\,s^2$	$w=908\,s^2$	$w=2314\,d^2$	$w=579\,d^2$
Oak, English ..	$w=3960\,b\,d$	$w=990\,b\,d$	$w=3960\,s^2$	$w=990\,s^2$	$w=2524\,d^2$	$w=631\,d^2$
Pine, American	$w=3900\,b\,d$	$w=975\,b\,d$	$w=3900\,s^2$	$w=975\,s^2$	$w=2486\,d^2$	$w=622\,d^2$

The two cases for which the formulæ are given in the foregoing table, are

1. *When the direction of the straining force coincides with the axis of the beam.*

2. *When the direction of the straining force coincides with the surface of the beam.*

And it appears, that while the strain is within the elastic power of the material, the strength to resist compression or tension is directly as the area of the cross section; therefore, if the area of the cross section be multiplied by the cohesive force of the substance, the product will be the number of pounds it will bear, when drawn or compressed in the direction of the axis, and in this manner was the foregoing table computed.

It only now remains to draw up an abstract of the problems and rules which have been delivered, in order to suit the convenience of those who are unacquainted with the rules of Algebra.

AN ABSTRACT

OF THE

PROBLEMS AND RULES

DELIVERED IN THE FOREGOING PART OF THIS WORK.

OF RECTANGULAR BEAMS.

PROBLEM I. *In a rectangular beam of oak, supported horizontally at the ends, and loaded in the middle of its length; there are given, the breadth and depth, both in inches, and the length of bearing, or the distance between the supports in feet; to find what load the beam will sustain including its own weight, admitting its elastic force to remain unimpaired.*

Rule 1. Multiply 212 times the breadth in inches, by the square of the depth in inches, and divide the product by the length of bearing, or the distance between the supports in feet, for the load in pounds that the beam will support at the middle of its length.

NOTE. To this is added a subsidiary rule for calculating the weight of the beam, as follows.

Multiply the solidity in cubic feet by 52, and the product will be the weight in pounds, whatever may be the form of the beam.

When the beam is of the same form as in the last problem, and placed under like circumstances; to find what load it will sustain, excluding its own weight, the elastic force remaining perfect.

Rule 2. From 212 times the depth of the beam in inches, subtract 0·18 times the square of the length, or distance between the supports in feet; multiply the remainder by the breadth drawn into the depth, both in inches, and divide the product by the length of bearing for the load in pounds that the beam will sustain at the middle of its length, excluding the effect produced by means of its own weight.

PROBLEM II. *In a rectangular beam of oak, supported horizontally at the ends, and loaded in the middle of its length; there are given, the depth in inches, the length of bearing, or the distance between the supports in feet, and the load which it sustains including its own weight; to find the breadth.*

Rule 3. Multiply the length of bearing, or the distance between the supports in feet, by the number of pounds that the beam has to sustain, and divide the product by 212 times the square of the depth in inches, for the breadth required.

When the beam is of the same form, and placed under like circumstances; to find the breadth such as to allow for the effect produced by means of its own weight.

Rule 4. Multiply the length of bearing, or the distance between the supports in feet, by the load to be sustained, and divide the product by 212 times the square of the depth in inches diminished by 0·18 times the depth drawn into the square of the length, and the quotient will be the breadth required.

PROBLEM III. *In a rectangular beam of oak, supported horizontally at the ends, and loaded in the middle of its length; there are given, the breadth of the beam in inches, the length of bearing, or the distance between the supports in feet, and the load which it sustains, including its own weight; to find the depth.*

Rule 5. Multiply the length of bearing, or distance between the supports in feet, by the load to be sustained; divide the product by 212 times the breadth of the beam in inches, and extract the square root of the quotient for the depth required.

When the beam is of the same form, and placed under like circumstances; to find the depth such as to allow for the effect produced by its own weight.

Rule 6. Divide 212 times the given load by the breadth of the beam in inches; to the quotient add the cube of the length of bearing in feet drawn into the fraction ·0081, and multiply the sum by the length; then, to the square root of the product add 0·09 times the square of the length of bearing, and divide the sum by 212, for the depth required.

Again, when the beam is of the same form, and placed under like circumstances; to determine the load that will produce fracture.

Rule 7. Multiply 714 times the breadth of the beam in inches by the square of the depth in inches, and divide the product by the length of bearing in feet, for the load in pounds that will break the beam, including the effect produced by means of its own weight. Or thus,

Calculate the load that will destroy the elastic force, by the first rule, and multiply that load by 3·36 for the load that will produce fracture.

When the beam is of the same form, but instead of being placed horizontally, it is inclined to the horizon at a given angle; to find how much it will sustain including its own weight, the elastic force remaining perfect.

Rule 8. Multiply 212 times the breadth by the square of the depth, both in inches, and divide the product by the distance between the tops of the supports in feet, drawn into the natural cosine of the given inclination, for the number of pounds that the beam will sustain.

Problem IV. *In a rectangular beam of oak, supported horizontally at the ends, and loaded at some intermediate point; there are given, the breadth* *and depth, both in inches, and the respective distances of the point where the force acts from each support; to find what load the beam will sustain at that point, including its own weight, and the elastic force remaining perfect.*

Rule 9. Multiply 53 times the breadth drawn into the square of the depth, both in inches, by the length of bearing in feet, and divide the product by the rectangle of the segments, into which the length is divided at the point where the load is applied, and the quotient will be the number of pounds that the beam will sustain at that point.

When the beam is of the same form, and placed under the like circumstances; to find how much it will bear, excluding its own weight.

Rule 10. From 53 times the depth of the beam in inches, subtract 0·18 times the rectangle of the segments of the length of bearing, or distance between the supports in feet; multiply the remainder successively by the breadth, depth, and length of the beam, and divide the product by the rectangle of the segments, for the load in pounds that the beam will support.

PROBLEM V. *In a rectangular beam of oak, supported horizontally at the ends, and loaded at some intermediate point; there are given, the depth of the beam in inches, the respective distances of the load from each support, and the load which the* *beam sustains, including its own weight; to find the breadth.*

Rule 11. Multiply the load to be supported in pounds, by the rectangle of the segments of the length of bearing in feet, and divide the product by 53 times the length or the sum of the segments, drawn into the square of the depth in inches, and the quotient will be the breadth required.

When the beam is of the same form, and placed under the same circumstances; to determine the breadth such that the effect produced by the weight of the beam itself may be taken into the account.

Rule 12. From 53 times the depth in inches, subtract 0·18 times the rectangle of the segments of the length of bearing in feet; multiply the remainder by the sum of the segments drawn into the depth in inches, and reserve the product for a divisor.

Multiply the given load by the rectangle of the segments of the length, then divide the product by the reserved divisor, and the quotient will be the breadth required.

Problem VI. *In a rectangular beam of oak, supported horizontally at the ends, and loaded at some intermediate point; there are given, the breadth of the beam in inches, the respective distances of the load from each support, and the load which the beam sustains at that point, including its own weight; to find the depth.*

Rule 13. Multiply the given load by the rectangle of the segments of the length of bearing; divide the product by 53 times the sum of the segments, drawn into the breadth in inches; then, the square root of the quotient will give the depth required.

When the beam is of the same form, and placed under the same circumstances; to determine the depth such, that the effect produced by the weight of the beam may be taken into the account.

Rule 14. Divide 53 times the given load by the breadth of the beam in inches, drawn into the length of bearing or the sum of the given segments; to the quotient add the rectangle of the segments, multiplied by the fraction 0·0081; multiply the sum by the said rectangle, and extract the square root of the product; to the square root just found, add the rectangle of the segments of the length multiplied by the fraction 0·09, and divide the sum by 53 for the required depth.

PROBLEM VII. *In a rectangular beam of oak, supported horizontally at the ends, and loaded at some intermediate point; there are given, the breadth and depth, both in inches, the length of bearing, or the distance between the supports in feet, and the load, including the weight of the beam; to find the point, or the distance from each support where the load is applied.*

Rule 15. Multiply 212 times the length of bearing in feet, by the breadth drawn into the square of the depth, both in inches; divide the product by the given load, and subtract the quotient from the square of the length; then, to or from the length, add or subtract the square root of the remainder, and half the sum, or half the difference, will give the greater or lesser segment accordingly.

When the beam is of the same form, and placed under the same circumstances; to determine the point where the load must be applied, so that the effect produced by the weight of the beam may be taken into account.

Rule 16. Multiply 212 times the length of bearing, or distance between the supports in feet, by the breadth drawn into the square of the depth, both in inches; divide the product by the load in pounds, added to 0·18 times the continued product of the breadth, depth, and length of bearing, and subtract the quotient from the square of the length; then, to or from the length, add or subtract the square root of the remainder, and half the sum, or half the difference, will give the greater or lesser segment accordingly.

Note. It sometimes happens, that the load which a beam will support at the middle is given; to find the point where it will support any greater load, or any number of times the given load. The two following rules apply to this case.

Rule 17. Divide the difference between the greater and lesser load by the greater load; then, to or from unity, add or subtract the square root of the quotient, and multiply the sum or difference by half the given length of bearing, for the greater or lesser segment accordingly.

Rule 18. Subtract unity from the number that denotes how often the given load has to be increased, and divide the remainder by that number; then, to or from unity, add or subtract the square root of the quotient, and multiply the sum or remainder by half the length of bearing, for the greater or lesser segment accordingly.

PROBLEM VIII. *In a uniform rectangular beam of oak, supported horizontally at the ends; there are given, the breadth and depth, both in inches; to find the length such, that it will just break by means of its own weight.*

Rule 19. Multiply the square of the given depth by 63, and the product will be the length required.

PROBLEM IX. *Given the length of a rectangular beam of oak that will just break by means of its own weight; to determine the depth.*

Rule 20. Divide the square of the given length in feet by the constant number 3969, and the quotient will be the depth required.

Problem X. *Given the diameter of a cylindrical tree; to find the dimensions of the strongest rectangular beam that can be cut out of it.*

Rule 21. Multiply the diameter of the tree by the natural sine and cosine of 35 degrees 16 minutes separately, and the products will be the breadth and depth respectively.

OF SQUARE BEAMS.

Problem XI. *In a square beam of oak, supported horizontally at the ends, and loaded in the middle, there are given, the length of bearing in feet, and the side of its cross section in inches; to find what load it will bear, including its own weight, the elastic force remaining perfect.*

Rule 22. Divide 212 times the cube of the side in inches, by the length of bearing, or distance between the supports in feet, and the quotient will be the load required.

When the beam is of the same form, and placed under like circumstances; to determine the load that the beam will bear, excluding its own weight.

Rule 23. From 212 times the side of the square in inches, subtract 0·18 times the square of the length in feet; multiply the remainder by the square of the side, and divide the product by the length of bearing for the load required.

PROBLEM XII. *In a square beam of oak, supported horizontally at the ends, and loaded in the middle, there are given, the length of bearing, or distance between the supports in feet, and the load which the beam will bear, including its own weight; to find the side of its cross section, the elastic force remaining perfect.*

Rule 24. Divide the given load drawn into the length of bearing, by 212, and the cube root of the quotient will be the side required.

PROBLEM XIII. *In a square beam of oak, supported horizontally at the ends, and loaded at some intermediate point, there are given, the side of the cross section in inches, and the respective distances of the load from each support; to find how much the beam will bear, including its own weight, while the elastic force remains perfect.*

Rule 25. Divide 53 times the side of the cross section in inches, drawn into the sum of the distances of the load from each support, by the rectangle of the said distances, and the quotient will be the load required.

When the beam is of the same form, and placed under like circumstances; to determine the load that the beam will bear, excluding its own weight.

Rule 26. From 53 times the side of the cross section in inches, subtract 0·18 times the rectangle of the distances of the load from each support; multiply the remainder by the sum of the distances drawn into the square of the side of the cross section, and divide the product by the rectangle of the given distances, or segments of the length, for the load required.

PROBLEM XIV. *In a square beam of oak, supported horizontally at the ends, and loaded at some intermediate point, there are given, the segments of the length of bearing, or the distances of the load from each support, and the load which the beam can sustain, including its own weight; to find the side of its cross section.*

Rule 27. Multiply the rectangle of the segments, or distances of the load from the supports, by the given load; divide the product by 53 times the sum of the segments, and the cube root of the quotient will be the side of the square required.

PROBLEM XV. *In a square beam of oak, supported horizontally at the ends, and loaded at some intermediate point, there are given, the length of bearing, or distance between the supports in feet, the side of the cross section in inches, and the load that the beam will sustain, including its own weight; to find the point where the load is applied, the elastic force remaining perfect.*

Rule 28. Multiply 212 times the length of bearing in feet, by the cube of the side in inches; divide the product by the given load, and subtract the quotient from the square of the length; then, to or from the length, add or subtract the square root of the remainder, and half the sum or half the difference will give the greater or lesser segment accordingly.

When the beam is of the same form, and placed under like circumstances; to find the point where the load is applied when the effect produced by the weight of the beam is taken into the account.

Rule 29. Multiply 212 times the length of bearing in feet, by the cube of the side in inches; divide the product by the given load, added to 0·18 times the length of bearing drawn into the square of the side, and subtract the quotient from the square of the length; then, to or from the length, add or subtract the square root of the remainder, and half the sum, or half the difference, will give the greater or lesser segment accordingly.

OF CYLINDRICAL BEAMS.

PROBLEM XVI. *In a cylindrical beam of oak, supported horizontally at the ends, and loaded in the middle, there are given, the diameter in inches, and the length of bearing in feet; to find how much the beam will bear, including its own weight, the elastic force remaining perfect.*

Rule 30. Divide 125 times the cube of the diameter in inches by the length of bearing, or distance between the supports in feet, and the quotient will be the load that the beam will bear, including its own weight.

When the beam is of the same form, and placed under like circumstances; to find the load that the beam will bear, excluding its own weight.

Rule 31. From 125 times the diameter in inches, subtract 0·14 times the square of the length in feet; multiply the remainder by the square of the diameter, and divide the product by the length for the load required.

Problem XVII. *In a cylindrical beam of oak, supported horizontally at the ends, and loaded in the middle, there are given, the length of bearing, or distance between the supports in feet, and the number of pounds that the beam can sustain, including its own weight; to find the diameter of the beam, its elastic force remaining perfect.*

Rule 32. Multiply the length of bearing by the given load, and divide the cube root of the product by 5, for the diameter required.

Problem XVIII. *In a cylindrical beam of oak, supported horizontally at the ends, and loaded at some intermediate point, there are given, the diameter of the beam in inches, and the segments of the length, or the distances of the point where the load is applied* *from each support; to find how much the beam will sustain at that point, including its own weight, the elastic force remaining perfect.*

Rule 33. Divide 31 times the cube of the diameter in inches drawn into the sum of the segments of the length, by the rectangle of the segments, and the quotient will be the load required.

When the beam is of the same form, and placed under like circumstances; to find the load that the beam will bear, excluding its own weight.

Rule 34. From 31 times the diameter of the beam in inches, subtract 0·14 times the rectangle of the segments of the length; multiply the remainder by the sum of the segments drawn into the square of the diameter, and divide the product by the rectangle of the given segments, for the load which the beam will sustain.

PROBLEM XIX. *In a cylindrical beam of oak, there are given, the distances of the point where the load is applied from each support, and the number of pounds that the beam sustains, including its own weight; to find the diameter, the elastic force remaining perfect.*

Rule 35. Multiply the rectangle of the given distances of the load from the points of support, by the load which the beam sustains; divide the product by 31 times the sum of the segments, and the cube root of the quotient will be the diameter required.

PROBLEM XX. *In a cylindrical beam of oak, supported horizontally at the ends, and loaded at some intermediate point; there are given, the diameter of the beam in inches, the length of bearing in feet, and the load to be supported in pounds, including its own weight; to find the distance of the strained point from each support.*

Rule 36. Multiply 125 times the length of bearing in feet, by the cube of the diameter in inches; divide the product by the given load, and subtract the quotient from the square of the length; then, to or from the length, add or subtract the square root of the remainder, and half the sum, or half the difference, will give the greater or lesser distance accordingly.

When the beam is of the same form, and placed under like circumstances, to find the distances, when the weight of the beam is not included in the given load.

Rule 37. Multiply the length of bearing in feet by the cube of the diameter in inches, and again by the constant number 125; divide the product by the given load, increased by 0·14 times the length of bearing drawn into the square of the diameter, and subtract the quotient from the square of the length; then, to or from the length, add or subtract the square root of the remainder, and half the sum, or half the difference, will give the greater or lesser distance accordingly.

PROBLEM XXI. *In a hollow, cylindrical, or tubular beam of oak, supported horizontally at the ends, and loaded at the middle of its length; there are given, the interior and exterior diameters of the tube, and the length of bearing, or distance between the supports; to find how much it will bear, including its own weight, supposing the elastic force to remain unimpaired.*

Rule 38. Divide the lesser diameter by the greater, and from unity subtract the fourth power of the quotient; then multiply the remainder by 125 times the cube of the greater diameter, and divide the product by the length of bearing, for the load in pounds that the beam will sustain.

PROBLEM XXII. *In a hollow cylindrical, or tubular beam of oak, supported horizontally at the ends, and loaded at the middle of its length, there are given, the exterior or greater diameter in inches, the length of bearing, or distance between the supports in feet, and the number of pounds that the beam can sustain, including its own weight; to find the interior or lesser diameter.*

Rule 39. Multiply the length of bearing by the load to be supported in pounds, and divide the product by 125; then, subtract the quotient from the cube of the given diameter, and multiply the said diameter by the remainder; then, the fourth root of the product being extracted will give the lesser diameter required.

PROBLEM XXIII. *In a hollow cylindrical, or tubular beam of oak, supported horizontally at the ends and loaded at the middle; there are given, the interior or lesser diameter in inches, the length of bearing, or distance between the supports in feet, and the number of pounds that the beam can sustain, including its own weight; to find the greater diameter.*

We have stated that the rule for resolving this problem depends on the solution of an equation of the fourth degree, and therefore cannot easily be expressed in words. (*See* the solution of Problem XXIII. page 66).

PROBLEM XXIV. *In a hollow cylindrical, or tubular beam of oak, supported horizontally at the ends and loaded at some intermediate point, there are given, the greater and lesser diameters, and the respective distances from each support of the point where the load is applied; to find how much the beam will bear, including its own weight, the elastic force remaining perfect.*

Rule 40. Divide the lesser diameter by the greater, and from unity subtract the fourth power of the quotient; then, multiply the remainder by 31 times the length of bearing drawn into the cube of the exterior diameter, and divide the product by the rectangle of the given distance, and the quotient will give the load that the beam will support at the point of section, including its own weight.

PROBLEM XXV. *In a hollow cylindrical, or tubular beam of oak, supported horizontally at the ends, and loaded at some intermediate point, there are given, the greater diameter, the distances of the load from each support, and the number of pounds that the beam can sustain; to find the lesser diameter.*

Rule 41. Multiply the rectangle of the given distances, or segments of the length, by the load to be supported in pounds, and divide the product by 31 times the sum of the distances; subtract the quotient from the cube of the given diameter, and multiply the said diameter by the remainder; then, the fourth root of the product will give the diameter required.

PROBLEM XXVI. *In a hollow cylindrical, or tubular beam of oak, supported horizontally at the ends, and loaded at some intermediate point, there are given, the lesser diameter, the distances of the load from each support, and the magnitude of that load; to find the greater diameter.*

We have already stated, that the rule for resolving this Problem depends on the solution of an adfected biquadratic equation. (*See* the solution of Prob. XXVI).

PROBLEM XXVII. *In a hollow cylindrical, or tubular beam of oak, there are given, the greater and lesser diameters, the length of bearing, and the load to be sustained; to find the distances of the point where the load is applied from the supports.*

Rule 42. From the fourth power of the greater diameter subtract the fourth power of the lesser; multiply the remainder by 125 times the length of bearing, and divide the product by the greater diameter drawn into the given load, and subtract the quotient from the square of the length; then, to or from the length, add or subtract the square root of the remainder, and half the sum, or half the difference, will give the greater or lesser distance accordingly.

NOTE. The following is a general rule for reducing the formulæ in Tables I. II. III. IV. V. and VI. when applied to calculate the strength and magnitude of beams constructed of those timbers specified in the margin.

Rule 43. Substitute the given dimensions of the beam and their powers, together with the load to be sustained, for the representatives of each in the respective equation, and another equation will arise, involving only one unknown term, which must be disengaged by multiplication, division, or extraction of roots, according to the nature of its involution.

GENERAL RULE FOR THE DEFLEXION.

Rule 44. Multiply the square of the length, or the rectangle of the segments into which the length is divided at the point of strain, by the constant number for the particular case, and divide the product by the depth, or diameter of the beam in inches, for the deflexion required.

PROBLEM XXVIII. *In a rectangular beam of oak, supported horizontally at the ends, and loaded in the middle, there are given, the depth of the beam and the deflexion in inches, the length of bearing in feet, and the load in pounds that produces the given deflexion; to find the breadth.*

Rule 45. Multiply the given load by the cube, or third power of the length, and divide the product by 3786 times the given deflexion drawn into the cube of the depth in inches; then, the product will be the breadth required.

PROBLEM XXIX. *In a rectangular beam of oak, supported horizontally at the ends, and loaded in the middle, there are given, the breadth of the beam and the deflexion in inches, the length of bearing in feet, and the load in pounds that produces the given deflexion; to find the depth.*

Rule. 46. Multiply the given load by the cube, or third power of the length, and divide the product by 3786 times the breadth drawn into the deflexion; then, the cube root of the quotient will give the depth required in inches.

PROBLEM XXX. *In a rectangular beam of oak, supported horizontally at the ends, and loaded at some intermediate point, there are given, the depth of the beam and the deflexion in inches, the distances of the point where the load is applied from each support, and the load in pounds that produces the given deflexion; to find the breadth.*

Rule 47. Multiply the given load by the square of the rectangle of the segments of the length, and divide the product by 236 times the sum of the segments, multiplied by the cube of the depth drawn into the given deflexion, and the quotient will be the breadth required.

PROBLEM XXXI. *In a rectangular beam of oak, supported horizontally at the ends, and loaded at some intermediate point, there are given, the breadth of the beam and the deflexion in inches, the distances of the point where the load is applied from each support, and the load in pounds that produces the given deflexion; to find the depth.*

Rule 48. Multiply the given load by the square of the rectangle of the segments of the length, and divide the product by 236 times the sum of the segments multiplied by the breadth and deflexion;

then, the cube root of the quotient will give the depth required.

PROBLEM XXXII. *In a cylindrical beam of oak, supported horizontally at the ends, and loaded at the middle, there are given, the deflexion in inches, the length of bearing in feet, and the load in pounds; to find the diameter.*

Rule 49. Multiply the given load by the cube, or third power of the length, and divide the product by 2232 times the deflexion; then, extract the fourth root of the quotient for the diameter required.

PROBLEM XXXIII. *In a cylindrical beam of oak, supported horizontally at the ends, and loaded at some intermediate point, there are given, the deflexion in inches, the distances of the load from each support, and the load in pounds; to find the diameter.*

Rule 50. Multiply the given load by the square of the rectangle of the given distances; divide the product by 138 times the sum of the distances drawn into the deflexion, and the fourth root of the quotient will be the diameter required.

TABLES

FOR ESTIMATING, BY INSPECTION,

THE

STRENGTH, MAGNITUDE, AND FLEXURE

OF

CAST IRON AND TIMBER BEAMS,

DESIGNED AS BEARERS AND SUPPORTS IN BUILDINGS AND OTHER MECHANICAL CONSTRUCTIONS.

THE following Tables have been computed from formulæ investigated by the late Mr. THOS. TREDGOLD: they are intended to facilitate the process of estimating the *strength*, *magnitude*, and *deflexion* of *cast iron* and *timber* beams, when employed as bearers or supports in buildings and other mechanical constructions. They were originally computed for the author's own immediate use, and have been employed by him for the above purpose several years: the aid thus afforded is very great, and a desire to promote, as far as lies in his power, the interest of the mechanic and engineer, has induced the author to revise his operations and submit them to the public. The explanation and use of the tables will appear from what follows.

TABLE A exhibits the greatest weight that a beam of cast iron will bear in the middle of its length, when it is just able to restore itself if the load be removed; if loaded beyond that point, its elastic force is destroyed, and it takes a permanent set.

The numbers at the top of the columns denote the depth of the beam in inches, and those in the left-hand marginal column denote the length of bearing, or distance between the supports in feet; the other columns contain the weight in tons that the beam will bear with safety, the breadth being one inch; consequently the numbers found in the table must always be multiplied by the given breadth, to obtain the entire load.

Table B contains the deflexion in inches produced in the middle of the beam by the load in Table A, the arguments being the same. The black lines that run across the pages, mark the point where the depth has arrived at that proportion of the length, when the beam becomes too rigid for bearing purposes, if exposed to any degree of impulsive force.

Tables C, D, and E contain data that are exceedingly useful on many occasions: they have been compiled and arranged from Mr. Tredgold's Alphabetical Table, given at the end of his valuable Essay on Cast Iron.

Table F is useful for adapting the numbers in Table A to the several sorts of timber specified in the margin, the column marked *b* applies to the breadth of the beam, column *d* to the depth, and column *s* to the strength.

A few examples will render the use of the tables manifest.

Example 1. A cast iron beam, 2 inches broad, 18 inches deep, and 42 feet long, is placed horizontally on two supports exactly 40 feet asunder; how much will it bear suspended from the middle of its length, the elastic force remaining perfect?

In Table A, under 18 inches at top, and opposite 40 feet in the left hand column, stands 3·07 tons: this is the load that a beam one inch broad of the given dimensions will bear with safety; but the proposed beam is two inches broad, and the strength increases directly as the breadth; therefore, 3·07 × 2 = 6·14 tons, the entire load.

Example 2. A cast iron beam, 18 inches broad and 40 feet between the supports, is found to bear 6·14 tons at the middle of its length, while the elastic force remains perfect; what is the breadth?

In TABLE A, under 18 inches at top and opposite 40 feet in the left hand column, stands 3·07 tons for the load that a beam one inch broad will bear; therefore

$\frac{6{\cdot}14}{3{\cdot}07} = 2$ inches, the required breadth.

Example 3. A cast iron beam, 2 inches broad and 40 feet between the supports, is found to bear 6·14 tons at the middle of its length, while the elastic force remains perfect; what is the depth?

Divide 6·14 by 2, then opposite 40 feet in the left-hand column look for the quotient 3·07 in the body of the page, and above it, at the top of the column, will be found 18 inches, the required depth.

So much then, for the application to cast iron; we shall next proceed to shew how the numbers are to be adapted to beams of timber.

Example 4. A beam of *yellow fir*, 5 inches broad, 22 inches deep and 38 feet long, is placed horizontally on two supports exactly 36 feet asunder; how much will it bear at the middle of its length, the elastic force remaining perfect?

In Table A, under 22 inches at the top of the columns and opposite 36 feet in the margin, stands 5·1 tons for a beam one inch broad, and the strength is directly as the breadth; therefore $5 \cdot 1 \times 5 = 25 \cdot 5$ tons for the strength of a cast iron beam of the given dimensions; but the proposed beam is yellow fir, therefore in Table C, opposite *yellow fir*, in the column entitled *comparative strength*, or in Table F, in the column marked *s*, we get 0·3; then, $25 \cdot 5 \times 0 \cdot 3 = 7 \cdot 65$ tons for the load required.

Example 5. A beam of *yellow fir*, 22 inches deep and 36 feet between the supports, is found to bear 7·65 tons at the middle of its length, while the elastic force remains perfect; what is its breadth?

In Table A, under 22 inches at the top of the columns, and opposite 36 feet in the margin, stands 5·1 tons for the strength of a beam one inch broad; therefore, $7{\cdot}65 \div 5{\cdot}1 = 1{\cdot}5$ inches for the breadth of a cast iron beam of equal strength; then, in Table F, opposite *yellow fir* in the column marked *b*, we find $3\frac{1}{3}$; hence $1{\cdot}5 \times 3\frac{1}{3} = 5$ inches for the breadth required.

Example 6. A beam of *yellow fir*, 5 inches broad and 36 feet between the supports, is found to bear 7·65 tons at the middle of its length, while the elastic force remains perfect; what is its depth?

Divide 7·65 by 5, then, in Table A, opposite 36 feet the given length, the nearest number to 1·53 is found under 12 inches at the top of the columns, which is the depth of a cast iron beam of equal strength; then, in Table F, opposite *yellow fir* in the column marked *d*, we have 1·825; therefore, $1{\cdot}825 \times 12 = 21{\cdot}9$, or 22 inches nearly, for the depth required.

And exactly after the same manner is Table B to be applied for the deflexions.

Table A.—*The strength of cast iron beams*—*Breadth one inch.*

Len. in feet	Depth in inches. —— $w = \frac{850\,bd^2}{2240\,l}$.								
	1	2	3	4	5	6	7	8	9
	Tons	*Tons*	*Tons*	*Tons*	*Tons*	*Tons*	*Tons*	*Tons*	*Tons*
1	0·38	1·51	3·41	6·07	9·48	13·66	18·59	24·28	30·74
2	0·19	0·76	1·70	3·03	4·74	6·83	9·29	12·14	15·37
3	0·13	0·50	1·13	2·02	3·16	4·55	6·19	8·09	10·25
4	0·09	0·38	0·85	1·51	2·37	3·41	4·64	6·07	7·68
5	0·07	0·30	0·68	1·21	1·89	2·73	3·71	4·85	6·14
6	0·06	0·25	0·57	1·01	1·58	2·27	3·10	4·04	5·12
7	0·05	0·22	0·49	0·86	1·35	1·95	2·65	3·47	4·39
8	0·04	0·18	0·42	0·76	1·18	1·70	2·32	3·03	3·84
9	0·04	0·17	0·38	0·67	1·05	1·51	2·06	2·69	3·41
10	0 03	0·15	0·34	0·61	0·95	1·36	1·85	2·42	3·07
11	0·03	0·14	0·31	0·55	0·86	1·24	1·69	2·20	2·79
12	0·03	0·13	0·28	0·50	0·79	1·13	1·55	2·02	2·56
13	0·03	0·12	0·26	0·46	0·73	1·05	1·43	1·86	2·36
14	0·03	0·11	0·24	0·43	0·68	0·97	1·32	1·73	2·19
15	0·02	0·10	0·23	0·40	0·63	0·91	1·24	1·62	2·05
16	0·02	0·09	0·21	0·38	0·59	0·85	1·16	1·51	1·92
17	0·02	0·09	0·20	0·36	0·55	0·80	1·09	1·42	1·81
18	0·02	0·08	0·19	0·33	0·53	0·75	1·03	1·35	1·71
19	0·02	0·08	0·18	0·32	0·50	0·72	0·98	1·27	1·62
20	0·02	0·07	0·17	0·30	0·47	0·68	0·93	1·21	1·54
21	0·02	0·07	0·16	0·29	0·45	0·65	0·88	1·15	1·46
22	0·02	0·07	0·15	0·27	0·43	0·62	0·84	1·10	1·39
23	0·01	0·06	0·15	0·26	0·41	0·59	0·81	1·05	1·33
24		0·06	0·14	0·25	0·39	0·57	0·77	1·01	1·28
25		0·06	0·14	0·24	0·38	0·54	0·74	0·97	1·23
26		0·06	0 13	0·23	0·36	0·52	0·71	0·93	1·18
27		0·05	0·12	0·22	0·35	0·50	0·69	0·90	1·14
28		0·05	0·12	0·21	0·34	0·49	0·66	0·86	1·09
29		0·05	0·11	0·21	0·33	0·47	0·64	0·84	1·06
30		0·05	0·11	0·20	0·31	0·45	0·62	0·81	1 02
31		0·05	0·11	0·19	0·30	0·44	0·60	0·78	0·99
32		0·05	0·10	0·19	0·29	0·43	0·58	0·76	0·96
33			0·10	0·18	0·29	0·41	0·56	0·73	0·93
34			0·10	0·18	0·28	0·40	0·55	0·71	0·90
35			0·10	0·17	0·27	0·39	0·53	0·69	0·88
36			0·09	0·17	0·26	0·38	0·51	0·67	0·85
37			0·09	0·16	0·25	0·37	0·50	0·65	0·83
38			0·09	0·16	0·25	0·36	0·49	0·64	0·81
39			0·08	0·15	0·24	0·35	0·48	0·62	0·79
40			0·08	0·15	0·24	0·34	0·46	0·60	0·77
41				0 15	0·23	0·33	0 45	0·59	0·75
42				0·14	0·22	0·32	0·44	0·58	0·73
43				0·14	0·22	0·32	0·43	0·56	0·71
44				0·14	0·21	0·31	0·42	0·55	0·69
45				0·13	0·21	0·30	0·41	0·54	0·68
46				0·13	0·20	0·29	0·40	0·53	0·66
47					0·20	0·29	0·39	0·51	0·65
48					0·19	0·28	0·39	0·50	0·64
49					0·19	0·28	0·38	0·49	0·63
50					0·19	0·27	0·37	0·48	0·61

Table A.—*The strength of cast iron beams—Breadth one inch.*

Len. in feet	*Depth in inches.——* $w = \frac{850\,b\,d^2}{2240\,l}$.								
	10	11	12	13	14	15	16	17	18
	Tons	*Tons*	*Tons*	*Tons*	*Tons*	*Tons*	*Tons*	*Tons*	*Tons*
1	37·95	45·91	54·64	64·13	74·38	85·38	97·14	109·67	122·94
2	18·97	22·96	27·32	32·07	37·19	42·69	48·57	54·83	61·47
3	12·65	15·31	18·22	21·37	24·79	28·46	32·38	36·55	40·98
4	9·49	11·48	13·66	16·03	18·59	21·34	24·29	27·42	30·73
5	7·59	9·18	10·93	12·83	14·87	17·07	19·48	21·93	24·59
6	6·32	7·65	9·11	10·68	12·39	14·23	16·19	18·28	20·49
7	5·42	6·56	7·81	9·16	10·62	12·20	13·88	15·66	17·56
8	4·74	5·74	6·83	8·02	9·29	10·67	12·14	13·71	15·37
9	4·21	5·10	6·07	7·12	8·26	9·48	10·79	12·18	13·66
10	3·79	4·59	5·46	6·41	7·44	8·54	9·71	10·96	12·30
11	3·45	4·17	4·97	5·83	6·76	7·76	8·83	9·97	11·18
12	3·16	3·82	4·55	5·34	6·19	7·11	8·09	9·14	10·25
13	2·92	3·53	4·20	4·93	5·72	6·57	7·47	8·43	9·46
14	2·71	3·28	3·90	4·58	5·31	6·10	6·94	7·83	8·78
15	2·53	3·06	3·64	4·27	4·96	5·69	6·47	7·31	8·19
16	2·37	2·87	3·41	4·01	4·65	5·33	6·07	6·85	7·68
17	2·23	2·70	3·21	3·77	4·37	5·02	5·71	6·45	7·23
18	2·11	2·55	3·04	3·56	4·13	4·74	5·39	6·09	6·83
19	1·99	2·42	2·88	3·37	3·91	4·49	5·11	5·77	6·47
20	1·89	2·29	2·73	3·20	3·72	4·26	4·85	5·48	6·14
21	1·80	2·18	2·60	3·05	3·54	4·06	4·62	5·22	5·85
22	1·72	2·08	2·48	2·91	3·38	3·88	4·41	4·98	5·58
23	1·65	1·99	2·37	2·78	3·23	3·71	4·22	4·76	5·34
24	1·58	1·91	2·27	2·67	3·09	3·55	4·04	4·57	5·12
25	1·52	1·83	2·18	2·56	2·97	3·41	3·88	4·38	4·92
26	1·46	1·76	2·10	2·46	2·86	3·28	3·73	4·21	4·73
27	1·40	1·70	2·02	2·37	2·75	3·16	3·59	4·06	4·55
28	1·35	1·64	1·95	2·29	2·65	3·05	3·47	3·91	4·39
29	1·31	1·58	1·88	2·21	2·56	2·94	3·35	3·78	4·24
30	1·26	1·53	1·82	2·13	2·48	2·84	3·23	3·65	4·09
31	1·22	1·48	1·76	2·07	2·39	2·75	3·13	3·53	3·96
32	1·18	1·43	1·71	2·00	2·32	2·66	3·03	3·42	3·84
33	1·15	1·39	1·66	1·94	2·25	2·58	2·94	3·32	3·72
34	1·11	1·35	1·61	1·88	2·18	2·51	2·85	3·22	3·61
35	1·08	1·31	1·56	1·83	2·12	2·44	2·77	3·13	3·51
36	1·05	1·27	1·52	1·78	2·06	2·37	2·69	3·04	3·41
37	1·02	1·24	1·48	1·73	2·01	2·31	2·62	2·96	3·32
38	0·99	1·21	1·44	1·68	1·95	2·24	2·55	2·88	3·23
39	0·97	1·17	1·40	1·64	1·91	2·19	2·49	2·81	3·15
40	0·95	1·15	1·36	1·60	1·86	2·13	2·42	2·74	3·07
41	0·92	1·12	1·33	1·56	1·81	2·08	2·37	2·67	2·99
42	0·90	1·09	1·30	1·52	1·77	2·03	2·31	2·61	2·92
43	0·88	1·07	1·27	1·49	1·73	1·98	2·26	2·55	2·86
44	0·86	1·04	1·24	1·45	1·69	1·94	2·21	2·49	2·79
45	0·84	1·02	1·21	1·42	1·65	1·89	2·15	2·43	2·73
46	0·82	0·99	1·19	1·39	1·61	1·85	2·11	2·38	2·67
47	0·81	0·98	1·16	1·36	1·58	1·81	2·06	2·33	2·61
48	0·79	0·96	1·14	1·33	1·55	1·78	2·02	2·28	2·56
49	0·77	0·94	1·11	1·31	1·51	1·74	1·98	2·24	2·51
50	0·76	0·92	1·09	1·28	1·48	1·71	1·94	2·19	2·46

TABLE A.—*The strength of cast iron beams—Breadth one inch.*

Len. in feet	Depth in inches.—— $w=\frac{850bd^2}{2240l}$.								
	19	20	21	22	23	24	25	26	27
	Tons	*Tons*	*Tons*	*Tons*	*Tons*	*Tons*	*Tons*	*Tons*	*Tons*
1	136·98	151·78	167·34	183·66	200·74	218·58	237·18	256·53	276·63
2	68·49	75·89	83·67	91·83	100·37	109·29	118·59	128·26	138·31
3	45·66	50·60	55·78	61·22	66·91	72·86	79·06	85·51	92·21
4	34·25	37·95	41·83	45·91	50·18	54·64	59·29	64·13	69·16
5	27·40	30·36	33·47	36·73	40·15	43·71	47·43	51·30	55·33
6	22·83	25·30	27·89	30·61	33·46	36·43	39·53	42·75	46·11
7	19·57	21·68	23·91	26·24	28·68	31·22	33·88	36·65	39·52
8	17·12	18·97	20·92	22·96	25·09	27·32	29·65	32·06	34·58
9	15·22	16·86	18·59	20·41	22·30	24·28	26·35	28·50	30·74
10	13·70	15·18	16·73	18·36	20·07	21·85	23·72	25·65	27·66
11	12·45	13·80	15·21	16·70	18·25	19·87	21·56	23·32	25·15
12	11·42	12·65	13·95	15·31	16·73	18·21	19·76	21·37	23·05
13	10·54	11·67	12·87	14·12	15·44	16·81	18·24	19·73	21·28
14	9·78	10·84	11·95	13·12	14·34	15·61	16·94	18·32	19·76
15	9·13	10·12	11·16	12·24	13·38	14·57	15·81	17·10	18·44
16	8·56	9·48	10·46	11·48	12·55	13·66	14·82	16·03	17·29
17	8·05	8·93	9·84	10·80	11·81	12·86	13·95	15·09	16·27
18	7·61	8·43	9·29	10·20	11·15	12·14	13·17	14·25	15·37
19	7·21	7·99	8·81	9·66	10·56	11·50	12·48	13·50	14·56
20	6·85	7·59	8·36	9·18	10·04	10·93	11·86	12·83	13·83
21	6·52	7·22	7·96	8·74	9·56	10·41	11·29	12·22	13·17
22	6·22	6·90	7·60	8·35	9·12	9·93	10·78	11·66	12·57
23	5·95	6·60	7·27	7·98	8·72	9·50	10·31	11·15	12·03
24	5·71	6·32	6·97	7·65	8·36	9·10	9·88	10·69	11·53
25	5·48	6·07	6·69	7·34	8·03	8·74	9·48	10·26	11·06
26	5·27	5·85	6·43	7·06	7·72	8·40	9·12	9·86	10·64
27	5·07	5·62	6·19	6·80	7·43	8·09	8·78	9·50	10·25
28	4·89	5·42	5·97	6·56	7·17	7·80	8·47	9·16	9·88
29	4·72	5·23	5·77	6·33	6·92	7·54	8·17	8·84	9·54
30	4·56	5·06	5·57	6·12	6·69	7·28	7·90	8·55	9·22
31	4·42	4·89	5·39	5·92	6·47	7·05	7·65	8·27	8·92
32	4·28	4·74	5·23	5·74	6·27	6·83	7·41	8·01	8·64
33	4·15	4·60	5·07	5·56	6·68	6·62	7·18	7·77	8·38
34	4·03	4·46	4·92	5·40	5·90	6·43	6·97	7·54	8·13
35	3·91	4·33	4·78	5·24	5·73	6·24	6·77	7·33	7·90
36	3·80	4·21	4·65	5·10	5·57	6·07	6·58	7·12	7·68
37	3·70	4·01	4·52	4·96	5·42	5·90	6·41	6·93	7·47
38	3·60	3·99	4·40	4·83	5·28	5·75	6·24	6·75	7·28
39	3·51	3·89	4·29	4·71	5·14	5·60	6·08	6·57	7·09
40	3·42	3·79	4·18	4·59	5·01	5·46	5·93	6·41	6·91
41	3·34	3·70	4·08	4·48	4·89	5·33	5·78	6·25	6·74
42	3·26	3·61	3·98	4·37	4·78	5·20	5·64	6·10	6·58
43	3·18	3·53	3·89	4·27	4·66	5·08	5·51	5·96	6·43
44	3·11	3·45	3·80	4·17	4·56	4·96	5·39	5·83	6·28
45	3·04	3·37	3·71	4·08	4·46	4·85	5·27	5·70	6·14
46	2·97	3·30	3·63	3·99	4·36	4·75	5·15	5·57	6·01
47	2·91	3·23	3·56	3·91	4·27	4·65	5·04	5·45	5·88
48	2·85	3·16	3·48	3·82	4·18	4·55	4·94	5·34	5·76
49	2·79	3·09	3·41	3·74	4·09	4·46	4·84	5·23	5·64
50	2·74	3·03	3·34	3·67	4·01	4·37	4·74	5·13	5·53

Table A.—*The strength of cast iron beams—Breadth one inch.*

Len. in feet	*Depth in inches.* —— $w=\frac{850\,bd^2}{2240\,l}$.								
	28	29	30	31	32	33	34	35	36
	Tons	*Tons*	*Tons*	*Tons*	*Tons*	*Tons*	*Tons*	*Tons*	*Tons*
1	297·51	319·12	341·52	364·68	388·56	413·25	438·65	464·85	491·80
2	148·75	159·56	170·76	182·34	194·28	206·62	219·32	232·43	245·90
3	99·17	106·37	113·84	121·56	129·52	137·75	146·22	154·95	163·93
4	74·38	79·78	85·38	91·17	97·14	103·31	109·66	116·21	122·95
5	59·50	63·82	68·30	72·93	77·71	82·65	87·73	92·97	98·36
6	49·58	53·19	56·92	60·78	64·76	68·87	73·11	77·47	81·96
7	42·50	45·59	48·79	52·09	55·51	59·03	62·67	66·41	70·25
8	37·19	39·89	42·69	45·58	48·57	51·65	54·83	58·11	61·47
9	33·05	35·46	37·95	40·57	43·17	45·92	48·74	51·65	54·64
10	29·75	31·91	34·15	36·47	38·86	41·32	43·87	46·48	49·18
11	27·05	29·01	31·05	33·15	35·32	37·57	39·88	42·26	44·71
12	24·79	26·59	28·46	30·39	32·38	34·44	36·56	38·74	40·98
13	22·88	24·55	26·27	28·05	29·89	31·79	33·74	35·76	37·83
14	21·25	22·79	24·39	26·05	27·76	29·52	31·33	33·20	35·13
15	19·83	21·28	22·77	24·31	25·91	27·55	29·24	30·99	32·79
16	18·59	19·95	21·34	22·79	24·29	25·83	27·42	29·05	30·74
17	17·50	18·77	20·09	21·45	22·86	24·31	25·80	27·34	28·93
18	16·53	17·73	18·97	20·26	21·59	22·95	24·37	25·82	27·32
19	15·66	16·80	17·97	19·19	20·45	21·75	23·09	24·46	25·88
20	14·87	15·96	17·08	18·23	19·43	20·66	21·93	23·24	24·59
21	14·17	15·20	16·26	17·36	18·50	19·68	20·89	22·14	23·42
22	13·52	14·51	15·52	16·58	17·66	18·78	19·94	21·13	22·35
23	12·93	13·88	14·85	15·86	16·89	17·97	19·07	20·21	21·38
24	12·40	13·30	14·23	15·19	16·19	17·22	18·28	19·37	20·49
25	11·90	12·77	13 66	14·58	15·54	16·53	17·55	18·59	19·67
26	11·44	12·27	13·14	14·03	14·94	15·90	16·87	17·88	18·92
27	11·02	11·82	12·65	13·51	14·39	15·31	16·25	17·22	18·21
28	10·62	11·40	12·20	13·02	13·88	14·76	15·67	16·60	17·56
29	10·26	11·00	11·78	12·57	13·40	14·25	15·13	16·03	16·96
30	9·91	10·64	11·38	12·16	12·95	13·77	14 62	15·50	16·39
31	9·59	10·29	11·02	11·76	12·53	13·33	14·15	14·99	15·86
32	9·29	9·97	10·67	11·40	12·14	12·92	13·71	14·53	15·37
33	9·01	9·67	10·35	11·05	11·77	12·52	13·29	14·09	14·90
34	8·75	9·38	10·04	10·73	11·43	12·15	12·90	13·67	14·46
35	8·50	9·12	9·75	10·42	11·10	11·81	12·53	13·28	14·05
36	8·26	8·86	9·48	10·13	10·79	11·48	12·18	12·91	13·66
37	8·04	8·62	9·23	9·85	10·50	11·17	11·86	12·56	13·29
38	7·83	8·39	8·98	9·59	10·23	10·87	11·54	12·23	12·94
39	7·63	8·18	8·75	9·35	9·96	10·59	11·25	11·92	12·61
40	7·43	7·97	8·53	9·11	9·71	10·33	10·96	11·62	12·29
41	7·25	7·78	8·33	8·89	9·47	10·08	10·70	11·34	11·99
42	7·08	7·59	8·13	8·68	9·25	9·84	10·45	11·07	11·71
43	6·92	7·42	7·94	8·48	9·03	9·61	10·20	10·81	11·44
44	6·76	7·25	7·76	8·29	8·83	9·39	9·97	10·56	11·18
45	6·61	7·09	7·59	8·10	8·63	9·18	9·74	10·33	10·93
46	6·46	6·93	7·42	7·92	8·44	8·98	9·53	10·11	10·69
47	6·33	6·79	7·26	7·75	8·26	8·79	9·33	9·89	10·46
48	6·19	6·64	7·11	7·59	8·09	8·61	9·13	9·68	10·25
49	6·07	6·51	6·97	7·44	7·93	8·43	8·95	9·48	10·04
50	5·95	6·38	6·83	7·29	7·77	8·26	8·77	9·29	9·83

TABLE B.—*Deflexion of cast iron beams.*

Len. in feet	*Depth in inches.*——— $\delta = \frac{\cdot 02\, l^2}{d}$.											
	1	2	3	4	5	6	7	8	9	10	11	12
	In.	*In.*	*In.*	*In.*	*In.*	*In.*	*In.*	*In.*	*In.*	*In.*	*In.*	*In.*
1	0·02	0·01										
2	0·08	0·04	0·03	0·02	0·01	0·01						
3	0·18	0·09	0·06	0·04	0·03	0·03	0·02	0·02	0·02	0·02	0·02	0·01
4	0·32	0·16	0·11	0·08	0·06	0·05	0·04	0·04	0·04	0·03	0·03	0·03
5	0·50	0·25	0·17	0·12	0·10	0·08	0·07	0·06	0·06	0·05	0·04	0·04
6	0·72	0·36	0·24	0·18	0·14	0·12	0·10	0·09	0·08	0·07	0·06	0·06
7	0·98	0·49	0·33	0·24	0·19	0·16	0·14	0·12	0·11	0·09	0·09	0·08
8	1·28	0·64	0·43	0·32	0·25	0·21	0·18	0·16	0·14	0·13	0·11	0·10
9	1·62	0·81	0·54	0·40	0·32	0·27	0·23	0·20	0·18	0·16	0·14	0·13
10	2·00	1·00	0·67	0·50	0·40	0·33	0·28	0·25	0·22	0·20	0·18	0·16
11	2·42	1·21	0·81	0·60	0·48	0·40	0·34	0·30	0·27	0·24	0·22	0·20
12	2·88	1·44	0·96	0·72	0·57	0·48	0·41	0·36	0·32	0·29	0·26	0·24
13	3·38	1·69	1·13	0·84	0·67	0·56	0·48	0·42	0·38	0·34	0·31	0·28
14	3·92	1·96	1·31	0·98	0·78	0·65	0·56	0·49	0·44	0·39	0·35	0·33
15	4·50	2·25	1·50	1·12	0·90	0·75	0·64	0·56	0·50	0·45	0·41	0·37
16	5·12	2·56	1·71	1·28	1·02	0·85	0·73	0·64	0·57	0·51	0·46	0·43
17	5·78	2·89	1·93	1·44	1·15	0·96	0·82	0·72	0·64	0·58	0·52	0·48
18	6·48	3·24	2·16	1·62	1·29	1·08	0·92	0·81	0·72	6·65	0·59	0·54
19	7·22	3·61	2·41	1·80	1·44	1·20	1·03	0·90	0·80	0·72	0·65	0·60
20	8·00	4·00	2·67	2·00	1·60	1·33	1·14	1·00	0·89	0·80	0·73	0·66
21	8·82	4·41	2·94	2·20	1·76	1·47	1·26	1·10	0·98	0·88	0·80	0·74
22	9·68	4·84	2·23	2·42	1·93	1·61	1·38	1·21	1·08	0·97	0·88	0·80
23	10·58	5·29	3·53	2·64	2·11	1·76	1·51	1·32	1·18	1·06	0·96	0·88
24		5·76	3·84	2·88	2·30	1·92	1·64	1·44	1·28	1·15	1·04	0·96
25		6·25	4·17	3·13	2·50	2·08	1·78	1·57	1·39	1·25	1·13	1·04
26		6·76	4·51	3·38	2·70	2·25	1·93	1·69	1·50	1·35	1·23	1·13
27		7·29	4·86	3·64	2·90	2·43	2·08	1·82	1·62	1·45	1·32	1·21
28		7·84	5·23	3·92	3·12	2·61	2·24	1·96	1·74	1·57	1·42	1·30
29		8·41	5·61	4·20	3·36	2·80	2·40	2·10	1·87	1·68	1·53	1·40
30		9·00	6·00	4·50	3·60	3·00	2·57	2·25	2·00	1·80	1·63	1·50
31		9·61	6·41	4·80	3·84	3·20	2·74	2·40	2·13	1·92	1·75	1·60
32		10·24	6·83	5·12	4·10	3·41	2·92	2·56	2·27	2·05	1·86	1·70
33			7·26	5·44	4·36	3·63	3·11	2·72	2·42	2·18	1·98	1·81
34			7·71	5·78	4·62	3·85	3·30	2·89	2·57	2·31	2·10	1·92
35			8·17	6·13	4·90	4·08	3·50	3·07	2·72	2·45	2·22	2·04
36			8·64	6·48	5·18	4·32	3·70	3·24	2·88	2·59	2·35	2·16
37			9·13	6·85	5·48	4·56	3·91	3·43	3·04	2·74	2·49	2·28
38			9·63	7·22	5·78	4·81	4·12	3·61	3·21	2·89	2·62	2·40
39			10·14	7·60	6·08	5·07	4·34	3·80	3·38	3·04	2·76	2·53
40			10·67	8·00	6·40	5·33	4·57	4·00	3·55	3·20	2·91	2·66
41				8·40	6·72	5·60	4·80	4·20	3·73	3·36	3·05	2·80
42				8·82	7·06	5·88	5·04	4·41	3·92	3·53	3·21	2·94
43				9·25	7·38	6·16	5·28	4·63	4·11	3·69	3·36	3·08
44				9·93	7·74	6·45	5·53	4·97	4·30	3·87	3·52	3·22
45				10·13	8·10	6·75	5·78	5·06	4·50	4·05	3·68	3·37
46				10·58	8·46	7·05	6·04	5·29	4·70	4·23	3·84	3·52
47					8·84	7·36	6·31	5·52	4·91	4·42	4·01	3·68
48					9·22	7·68	6·58	5·76	5·12	4·61	4·19	3·84
49					9·60	8·00	6·86	6·00	5·33	4·80	4·36	4·00
50					10·00	8·33	7·14	6·25	5·55	5·00	4·54	4·16

Table B.—*Deflexion of cast iron beams.*

Len. in feet	Depth in inches.—— $\delta = \frac{\cdot 02\, l^2}{d}$.											
	13	14	15	16	17	18	19	20	21	22	23	24
	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.
1												
2												
3	0·01	0·01										
4	0·02	0·02	0·02	0·02	0·02	0·02	0·01	0·01	0·01			
5	0·04	0·03	0·03	0·03	0·03	0·03	0·02	0·02	0·02	0·02	0·02	0·02
6	0·05	0·05	0·05	0·04	0·04	0·04	0·04	0·03	0·03	0·03	0·03	0·03
7	0·07	0·07	0·06	0·06	0·06	0·05	0·05	0·04	0·04	0·04	0·04	0·04
8	0·10	0·09	0·08	0·08	0·07	0·07	0·07	0·05	0·06	0·05	0·05	0·05
9	0·12	0·11	0·11	0·10	0·09	0·09	0·08	0·08	0·07	0·07	0·07	0·06
10	0·15	0·14	0·13	0·12	0·12	0·11	0·10	0·10	0·09	0·09	0·08	0·08
11	0·18	0·17	0·16	0·15	0·14	0·13	0·13	0·12	0·11	0·11	0·10	0·10
12	0·22	0·20	0·19	0·18	0·17	0·16	0·15	0·14	0·13	0·13	0·12	0·12
13	0·26	0·24	0·22	0·21	0·20	0·19	0·18	0·17	0·16	0·15	0·14	0·14
14	0·30	0·28	0·26	0·24	0·23	0·22	0·21	0·19	0·18	0·17	0·17	0·16
15	0·35	0·32	0·30	0·28	0·26	0·25	0·24	0·22	0·21	0·20	0·19	0·18
16	0·39	0·36	0·34	0·32	0·30	0·28	0·27	0·25	0·24	0·23	0·22	0·21
17	0·44	0·41	0·38	0·36	0·34	0·32	0·30	0·29	0·27	0·26	0·25	0·24
18	0·49	0·46	0·43	0·40	0·38	0·36	0·34	0·33	0·31	0·29	0·28	0·27
19	0·55	0·51	0·48	0·45	0·42	0·40	0·38	0·36	0·34	0·32	0·31	0·30
20	0·61	0·57	0·53	0·50	0·47	0·44	0·42	0·40	0·38	0·36	0·35	0·33
21	0·68	0·63	0·59	0·55	0·52	0·49	0·46	0·44	0·42	0·40	0·38	0·37
22	0·74	0·69	0·64	0·60	0·57	0·54	0·51	0·48	0·46	0·44	0·42	0·40
23	0·81	0·75	0·70	0·66	0·62	0·59	0·56	0·53	0·50	0·48	0·46	0·44
24	0·88	0·82	0·76	0·72	0·67	0·64	0·60	0·57	0·54	0·52	0·50	0·48
25	0·96	0·89	0·83	0·78	0·73	0·69	0·66	0·63	0·59	0·56	0·54	0·52
26	1·04	0·96	0·90	0·84	0·79	0·75	0·71	0·67	0·64	0·61	0·59	0·56
27	1·12	1·04	0·96	0·91	0·85	0·81	0·77	0·72	0·69	0·66	0·63	0·60
28	1·20	1·12	1·05	0·98	0·92	0·87	0·82	0·78	0·74	0·71	0·68	0·66
29	1·29	1·20	1·12	1·05	0·98	0·93	0·88	0·84	0·80	0·76	0·73	0·70
30	1·38	1·28	1·20	1·12	1·06	1·00	0·94	0·90	0·86	0·81	0·78	0·75
31	1·48	1·37	1·28	1·20	1·13	1·06	1·01	0·96	0·91	0·87	0·83	0·80
32	1·57	1·46	1·36	1·28	1·20	1·13	1·08	1·02	0·97	0·93	0·89	0·85
33	1·67	1·55	1·45	1·36	1·28	1·21	1·15	1·09	1·03	0·99	0·94	0·90
34	1·78	1·65	1·54	1·44	1·35	1·28	1·22	1·15	1·10	1·05	1·00	0·96
35	1·88	1·75	1·63	1·53	1·43	1·36	1·29	1·22	1·16	1·11	1·06	1·02
36	1·99	1·85	1·73	1·62	1·52	1·44	1·36	1·29	1·23	1·17	1·12	1·08
37	2·10	1·95	1·83	1·72	1·60	1·52	1·44	1·37	1·30	1·24	1·19	1·14
38	2·22	2·06	1·93	1·80	1·69	1·60	1·52	1·44	1·37	1·31	1·25	1·20
39	2·34	2·17	2·02	1·90	1·78	1·69	1·60	1·52	1·45	1·38	1·32	1·26
40	2·46	2·28	2·13	2·00	1·88	1·77	1·68	1·60	1·52	1·45	1·39	1·33
41	2 58	2·40	2·24	2·10	1·97	1·86	1·77	1·68	1·60	1·52	1·46	1·40
42	2·71	2·52	2·35	2·20	2·07	1·96	1·86	1·76	1·68	1·60	1·53	1·47
43	2·84	2·64	2·46	2·32	2·17	2·05	1·96	1·84	1·76	1·68	1·61	1·54
44	2·98	2·76	2·58	2·48	2·27	2·15	2·04	1·93	1·84	1·76	1·68	1·61
45	3·11	2·89	2·70	2·53	2·37	2·25	2·13	2·02	1·92	1·84	1·76	1·68
46	3·25	3·02	2·82	2·64	2·48	2·35	2·23	2·11	2·01	1·92	1·84	1·76
47	3·39	3·15	2·95	2·76	2·59	2·45	2·32	2·21	2·10	2·00	1·92	1·84
48	3·54	3·29	3·07	2·88	2·69	2·56	2·42	2·30	2·19	2·09	2·00	1·92
49	3·69	3·43	3·20	3·00	2·81	2·66	2·53	2·40	2·29	2·18	2·09	2·00
50	3·84	3·57	3·33	3·12	2·92	2·77	2·63	2·50	2·38	2·27	2·17	2·08

TABLE B.—*Deflexion of cast iron beams.*

Len. in feet	*Depth in inches.* —— $\delta = \frac{.02 l^2}{d}$.											
	25	26	27	28	29	30	31	32	33	34	35	36
	In.	*In.*	*In.*	*In.*	*In.*	*In.*	*In.*	*In.*	*In.*	*In.*	*In.*	*In.*
1												
2												
3												
4												
5	0·02	0·02	0·02									
6	0·03	0·03	0·03	0·02	0·02	0·02	0·02	0·02	0·02	0·02	0·02	0·02
7	0·04	0·03	0·03	0·03	0·03	0·03	0·03	0·03	0·03	0·03	0·03	0·02
8	0·05	0·05	0·04	0·04	0·04	0·04	0·04	0·04	0·03	0·03	0·03	0·03
9	0·06	0·06	0·06	0·05	0·05	0·05	0·05	0·05	0·04	0·04	0·04	0·04
10	0·08	0·07	0·07	0·07	0·07	0·06	0·06	0·06	0·06	0·06	0·05	0·05
11	0·09	0·10	0·09	0·08	0·08	0·08	0·07	0·07	0·07	0·07	0·06	0·06
12	0·11	0·11	0·10	0·10	0·10	0·09	0·09	0·09	0·08	0·08	0·08	0·08
13	0·13	0·13	0·12	0·12	0·11	0·11	0·11	0·10	0·10	0·10	0·09	0·09
14	0·16	0·15	0·15	0·14	0·13	0·13	0·12	0·12	0·11	0·11	0·11	0·11
15	0·18	0·17	0·16	0·16	0·15	0·15	0·14	0·14	0·13	0·13	0·13	0·12
16	0·20	0·19	0·19	0·18	0·17	0·17	0·16	0·16	0·15	0·15	0·14	0·14
17	0·23	0·22	0·21	0·20	0·20	0·19	0·18	0·18	0·17	0·17	0·16	0·16
18	0·26	0·24	0·24	0·23	0·22	0·21	0·21	0·20	0·19	0·19	0·18	0·18
19	0·29	0·27	0·26	0·25	0·25	0·24	0·23	0·22	0·21	0·21	0·20	0·20
20	0·32	0·30	0·29	0·28	0·27	0·26	0·26	0·25	0·24	0·23	0·23	0·22
21	0·35	0·34	0·32	0·31	0·30	0·29	0·28	0·27	0·26	0·26	0·25	0·24
22	0·38	0·37	0·36	0·34	0·33	0·32	0·31	0·30	0·29	0·28	0·27	0·27
23	0·42	0·41	0·39	0·37	0·36	0·35	0·34	0·33	0·32	0·31	0·30	0·29
24	0·46	0·44	0·42	0·41	0·39	0·38	0·37	0·36	0·34	0·33	0·32	0·32
25	0·50	0·48	0·46	0·44	0·43	0·41	0·40	0·39	0·37	0·36	0·35	0·34
26	0·54	0·52	0·50	0·48	0·46	0·45	0·43	0·42	0·41	0·39	0·38	0·37
27	0·58	0·56	0·54	0·52	0·50	0·48	0·47	0·45	0·44	0·42	0·41	0·40
28	0·63	0·60	0·58	0·56	0·54	0·52	0·50	0·49	0·47	0·46	0·45	0·43
29	0·67	0·64	0·62	0·60	0·58	0·56	0·54	0·52	0·51	0·49	0·48	0·46
30	0·72	0·69	0·66	0·64	0·62	0·60	0·58	0·56	0·54	0·53	0·51	0·50
31	0·77	0·74	0·71	0·68	0·66	0·64	0·62	0·60	0·58	0·56	0·55	0·53
32	0·82	0·78	0·75	0·73	0·71	0·68	0·66	0·64	0·62	0·60	0·58	0·56
33	0·87	0·83	0·80	0·77	0·75	0·72	0·70	0·68	0·66	0·64	0·62	0·60
34	0·92	0·89	0·85	0·82	0·79	0·77	0·74	0·72	0·70	0·67	0·66	0·64
35	0·98	0·94	0·91	0·87	0·84	0·81	0·79	0·76	0·74	0·71	0·70	0·68
36	1·04	0·99	0·96	0·92	0·89	0·86	0·83	0·81	0·78	0·76	0·74	0·72
37	1·09	1·05	1·01	0·97	0·94	0·91	0·88	0·86	0·83	0·80	0·78	0·76
38	1·16	1·11	1·07	1·03	0·99	0·96	0·93	0·90	0·87	0·84	0·82	0·80
39	1·22	1·17	1·12	1·08	1·05	1·01	0·98	0·95	0·92	0·89	0·87	0·84
40	1·28	1·23	1·18	1·14	1·10	1·06	1·03	1·00	0·97	0·94	0·91	0·88
41	1·34	1·29	1·24	1·20	1·16	1·12	1·08	1·05	1·02	0·98	0·96	0·93
42	1·41	1·35	1·31	1·26	1·22	1·17	1·13	1·10	1·07	1·03	1·01	0·98
43	1·47	1·42	1·37	1·32	1·27	1·23	1·19	1·16	1·12	1·08	1·05	1·02
44	1·55	1·49	1·43	1·38	1·33	1·29	1·25	1·21	1·17	1·13	1·10	1·07
45	1·62	1·55	1·16	1·44	1·40	1·35	1·30	1·26	1·22	1·18	1·16	1·12
46	1·69	1·62	1·56	1·51	1·46	1·41	1·36	1·32	1·28	1·24	1·21	1·17
47	1·77	1·69	1·63	1·57	1·52	1·47	1·42	1·38	1·34	1·29	1·26	1·22
48	1·84	1·77	1·71	1·64	1·59	1·53	1·48	1·44	1·39	1·34	1·31	1·28
49	1·92	1·84	1·77	1·71	1·65	1·60	1·55	1·50	1·45	1·40	1·37	1·33
50	2·00	1·92	1·55	1·78	1·72	1·66	1·61	1·56	1·51	1·46	1·43	1·38

TABLE C.—*The properties of Timber.*

Name of the wood.	Specific gravity	Weight of a cubic foot (lbs.)	Weight of a bar one foot long and one inch square (lbs.)	Will bear without permanent alteration (lbs.)	Extension in parts of the length	Comparative strength (Cast iron being unity)	Comparative extensibility (Cast iron being unity)	Comparative stiffness (Cast iron being unity)
Ash	0·76	47·5	0·33	3540	0·00215	0·23	2·6	0·089
Beech	0·696	45·3	0·315	2360	0·00175	0·15	2·1	0·073
Elm	0.544	34·0	0·236	3240	0·00241	0·21	2·9	0·073
Fir, red or yellow ..	0·557	34·8	0·242	4290	0·00217	0·3	2·6	0·1154
Fir, white	0·47	29·3	0·204	3630	0·00198	0·23	2·4	0·1
Larch	0·56	35·0	0·243	2065	0·00192	0·136	2·3	0·058
Mahogany, Honduras	0·56	35·0	0·243	3800	0·00238	0·24	2·9	0·087
Oak, English	0·83	52·0	0·36	3960	0·00232	0·25	2·8	0·093
Pine, Amer. yellow	0·46	26·75	0·186	3900	0·00241	0·25	2·9	0·087

TABLE D.—*The properties of Metals.*

Name of the metal.	Specific gravity	Weight of a cubic foot (lbs.)	Weight of a bar one foot long and one inch square (lbs.)	Will bear without permanent alteration (lbs.)	Extension in parts of the length	Comparative strength (Cast iron being unity)	Comparative extensibility (Cast iron being unity)	Comparative stiffness (Cast iron being unity)
Brass, cast	8·37	523	3·63	6700	0·00075	0·435	0·9	0·49
Bronze, or gun-metal	8·153	509·5	3·54	10000	0·00104	0·65	1·25	0·535
Copper	8·75	549	3·81					
Iron, cast	7·207	450	3·2	15300	0·00083	1	1	1
Iron, malleable ..	7·6	475	3·3	17800	0·00071	1·12	0·86	1·3
Lead, cast	11·352	709·5	4·94	1500	0·00208	0·096	2·5	0·0385
Steel	7·84	490	3·4					
Tin, cast	7·291	455·7	3·165	2880	0·00063	0·182	0·75	0·25
Zinc, cast	7·028	439·25	3·05	5700	0·00024	0·365	0·5	0·76

TABLE E.—*The properties of Brick, Clay, Earth, Slate, and Stone.*

	Specific gravity	*Weight of a cubic foot* lbs.	*Cohesive force of a square inch* lbs.	*Crushed by, on a square inch* lbs.	*Absorbs of its weight of water*	*Extension in parts of the length*
Brick	1·841	115	275	562	0·0666	
—— *work*		117				
Clay	2·000	125				
Earth, common	1·76	110				
Granite, Aberdeen	2·625	164		10910		
Marble, white	2·706	169	1811	6060		000·0717
Porphyry, red	2·871	179		35568		
Slate, Welsh	2·752	172	11500			0·00073
—— *Westmoreland*			7870			0·00061
—— *Scotch*			9600			0·000608
Stone, Portland	2·113	132	857	3729	0·0625	0·000559
—— *Bath*	1·975	123·4	478		0·0769	
—— *Craigleith*	2·362	147·6	772	5490	0·0159	
—— *Dundee*	2·621	163·8	2661	6630	0·0196	
—— *work*		160				

Several other particulars necessary to be known may be stated as follows:—

Bridges. The greatest extraneous load on a square foot is about 120 pounds.

Floors. The least load on a square foot is about 160 pounds.

Roofs. If covered with slate, on a square foot, 51¼lbs.; if tiled, 56¼ pounds.

TABLE F.—*For adapting the numbers in* TABLE A *to the calculation of Timber.*

Name of the wood.	*b*	*d*	*s*
Ash	4·35	2·085	0·23
Beech	6·6̇6	2·582	0·15
Elm	4·76	2·182	0·21
Fir, red or yellow	3·3̇3̇	1·825	0·3
Fir, white	4·35	2·085	0·23
Larch	7·35	2·712	0·136
Mahogany, Honduras	4·17	2·041	0·24
Oak, English	4·00	2·000	0·25
Pine, American, yellow	4·00	2·000	0·25

The numbers in Table G have been obtained at the expense of much labour; they were computed independently from the equation given with the argument at the top of the page, the form of which renders it inconvenient for differential summation,—hence the necessity of an independent process for each number. Great attention has been paid to the accuracy of the results, and this will insure a very near approximation to the real strength of the columns in question; the formula is that which considers the direction of the straining force to coincide with the surface of the column, which limit has been taken to secure sufficient strength; for if the direction of the force should fall within the column nearer to the axis, the strength will be augmented in proportion as it approaches the centre; therefore, columns constructed from the numbers given in the Table will always be sufficiently strong.

The Table is adapted for oak and fir only, but as these are the only timbers used in supporting heavy loads, its application is sufficiently extensive; it may be applied to other timbers, but the results will not be so accurate.

To use this Table.

Enter with the diameter in inches at the top, and the length in feet in the left-hand margin; then in the body of the Table, under the diameter and opposite the length, is the load in cwts. that the column will support with safety.

To find the diameter.

Enter with the given length in the margin, and run the eye across the page till the weight be found; then at the top of the column is the diameter in inches.

TABLE G.—*The strength of oak and fir columns.*

Len. in feet	Diameter in inches.—— $w = \frac{308\cdot75 d^4}{56 d^2 + 7 l^2}$.							
	1	2	3	4	5	6	7	8
	Cwt.	Cwt.	Cwt.	Cwt.	Cwt.	Cwt.	Cwt.	Cwt.
1	19·60	21·38	48·94	87·53	137·15	197·80	269·46	352·17
2	14·70	19·60	47·01	85·54	135·13	195·76	267·43	350·12
3	10·38	17·21	44·11	82·42	131·90	192·47	264·09	346·76
4	7·35	14·70	40·60	78·41	127·63	188·04	259·56	342·16
5	5·34	12·38	36·83	73·80	122·52	182·63	253·96	336·43
6	4·01	10·38	33·08	68·85	116·81	176·43	247·43	329·68
7	3·09	8·71	29·52	63·80	110·71	169·62	240·14	322·04
8	2·45	7·35	26·27	58·31	104·42	162·39	232·24	313·65
9	1·98	6·24	23·35	54·03	98·10	154·91	223·90	304·66
10	1·63	5·35	20·77	49·52	91·89	147·33	215·25	395·20
11		4·61	18·51	45·35	85·88	139·76	206·44	285·41
12		4·01	16·54	41·51	80·14	132·32	198·40	275·40
13		3·51	14·82	38·02	74·71	125·08	188·77	265·29
14		3·09	13·33	34·85	69·61	118·11	180·10	255·17
15		2·75	12·03	31·99	64·86	111·43	171·64	245·14
16		2·45	10·89	29·41	60·45	105·08	163·43	235·24
17		2·20	9·89	27·08	56·37	99·07	155·51	225·55
18		1·98	9·02	24·98	52·61	93·40	147·91	216·10
19		1·79	8·25	23·09	49·14	88·08	140·64	206·95
20		1·63	7·57	21·38	45·95	83·09	133·71	198·10
21		1·49	6·96	19·84	43·01	78·41	127·13	189·57
22		1·37	6·43	18·45	40·30	74·05	120·89	181·39
23		1·25	5.94	17·19	37·81	69·97	114·98	173·55
24		1·16	5·51	16·04	35·53	66·16	109·40	166·05
25		1·07	5·12	15·00	33·41	62·61	104·13	158·90
26		1·00	4·77	14·04	31·47	59·30	99·16	152·07
27			4·46	13·18	29·67	56·21	94·47	145·58
28			4·17	12·38	28·01	53·32	90·05	139·40
29			3·91	11·65	26·48	50·63	85·89	133·53
30			3·67	10·98	25·06	48·12	81·97	127·95
31			3·45	10·37	23·74	45·77	78·27	122·65
32			3·26	9·80	22·52	43·57	74·79	117·62
33			3·07	9·28	21·39	41·51	71·50	112·84
34			2·91	8·79	20·33	39·59	68·41	108·31
35			2·75	8·34	19·34	37·78	65·49	104·01
36			2·61	7·93	18·43	36·09	62·74	99·92
37			2·48	7·54	17·57	34·50	60·14	96·05
38			2·35	7·18	16·77	33·00	57·68	92·36
39			2·24	6·85	16·02	31·60	55·36	88·86
40			2·13	6·53	15·31	30·28	53·16	85·54
41			2·03	6·24	14·66	29·03	51·09	82·38
42			1·94	5·97	14·04	27·86	49·12	79·37
43			1·86	5·71	13·46	26·75	47·26	76·52
44			1·78	5·47	12·91	25·70	45·49	73·80
45			1·70	5·24	12·39	24·71	43·82	71·21
46			1·63	5·03	11·90	23·78	42·23	68·74
47			1·56	4·83	11·44	22·89	40·72	66·39
48			1·50	4·64	11·01	22·05	39·28	64·15
49			1·44	4·46	10·60	21·26	37·92	62·02
50			1·39	4·29	10·21	20·50	36·62	59·98

Table G.—*The strength of oak and fir columns.*

Len. in feet	*Diameter in inches.*——$w = \frac{308{\cdot}75d^4}{56d^2 + 7l^2}$.							
	9	10	11	12	13	14	15	16
	Cwt.	*Cwt.*	*Cwt.*	*Cwt.*	*Cwt.*	*Cwt.*	*Cwt.*	*Cwt.*
1	445·90	550·65	666·43	793·24	931·07	1079·9	1239·8	1410·7
2	443·85	548·60	664·38	791·18	929·01	1077·9	1237·8	1408·7
3	440·47	545·21	660·98	787·77	925·60	1074·3	1234·3	1405·3
4	435·83	540·53	656·28	783·05	920·86	1069·7	1229·6	1400·5
5	430·00	534·63	650·33	777·06	914·84	1063·7	1233·5	1394·4
6	423·08	527·60	643·20	769·87	907·68	1056·4	1216·2	1387·0
7	415·19	519·52	634·98	761·54	899·17	1047·9	1207·6	1378·5
8	406·45	510·50	625·75	752·04	889·65	1038·3	1197·9	1368·7
9	396·97	500·65	615·61	741·77	879·09	1027·8	1187·1	1357·7
10	386·88	490·08	604·66	730·51	867·59	1015·8	1175·2	1345·7
11	376·32	478·91	593·00	718·46	855·22	1003·2	1162·4	1332·7
12	365·39	467·24	580·73	705·71	842·07	989·73	1148·6	1318·7
13	354·21	455·18	567·97	692·36	828·23	975·49	1134·0	1303·9
14	342·88	442·85	554·79	678·49	813·79	960·55	1118·7	1288·1
15	331·49	430·31	541·30	664·20	798·82	945·02	1102·7	1271·7
16	320·12	417·68	527·60	649·58	783·42	928·96	1086·1	1254·6
17	308·85	405·03	513·74	634·70	767·67	912·45	1068·9	1236·9
18	290·95	392·41	499·83	619·65	751·64	895·57	1051·3	1218·6
19	286·81	379·91	485·91	604·50	735·40	878·39	1033·3	1199·9
20	276·13	367·56	472·06	589·31	719·03	860·99	1015·0	1180·8
21	265·74	355·42	458·32	574·14	702·59	843·42	996·40	1161·4
22	255·64	343·51	444·75	559·05	686·13	825·74	977·64	1141·6
23	240·27	331·88	431·38	544·08	669·72	808·02	958·75	1121·7
24	236·43	320·55	418·25	529·28	653·39	790·31	939·78	1101·6
25	227·33	309·52	405·38	514·69	637·20	772·65	920·79	1081·4
26	218·57	298·83	392·81	500·33	621·17	755·09	901·83	1061·2
27	210·16	288·41	380·54	486·23	605·35	737·67	882·93	1040·9
28	202·09	278·45	368·59	472·42	589·77	720·42	864·13	1020·7
29	194·35	268·78	356·98	458·91	574·44	703·37	845·48	1000·5
30	186·94	259·45	345·70	445·71	559·39	686·56	827·01	980·53
31	179·86	250·47	334·77	432·85	544·64	670·00	808·74	960·65
32	173·08	241·82	324·18	420·32	530·20	653·71	790·69	940·95
33	166·60	233·50	313·94	417·63	516·08	637·72	772·11	921·46
34	160·41	225·50	304·04	396·24	502·29	622·03	755·39	902·19
35	154·51	217·81	294·47	384·77	488·84	606·67	738·16	883·17
36	148·86	210·44	285·24	373·61	475·73	591·63	721·23	864·42
37	143·47	203·35	276·33	362·79	462·97	576·92	704·61	845·95
38	138·33	196·56	267·74	352·31	450·55	562·56	688·32	827·78
39	133·42	190·04	259·45	342·16	438·48	548·53	672·36	809·92
40	128·73	183·78	251·47	332·34	426·74	554·85	656·74	792·38
41	124·25	177·78	243·78	322·84	415·35	521·52	641·46	775·17
42	119·98	172·02	236·37	313·65	404·28	508·53	626·52	758·29
43	115·89	166·50	229·24	304·77	393·55	495·88	611·93	741·75
44	111·99	161·21	222·37	296·18	383·13	481·35	597·68	725·56
44	108·26	156·11	215·76	287·88	373·04	471·59	583·77	709·70
45	104·70	151·26	209·40	279·87	363·25	459·94	570·20	694·19
47	101·29	146·58	203·27	272·12	353·76	448·62	556·98	679·03
48	98·03	142·10	197·36	264·64	344·57	437·61	544·08	664·21
49	94·91	137·79	191·68	257·42	335·48	426·91	531·52	649·72
50	91·93	133·66	186·21	250·44	327·04	416·52	519·28	635·58

TABLE G.—*The strength of oak and fir columns.*

Len. in feet	Diameter in inches.——— $w = \frac{308 \cdot 75 d^4}{56d^2 + 7l^2}$.							
	17	18	19	20	21	22	23	24
	Cwt.	Cwt.	Cwt.	Cwt.	Cwt.	Cwt.	Cwt.	Cwt.
1	1592·7	1785·7	1989·7	2204·7	2430·7	2667·8	2915·9	3175·0
2	1590·6	1783·6	1987·6	2202·6	2428·6	2665·1	2913·8	3173·0
3	1587·2	1780·2	1984·2	2199·2	2425·2	2662·3	2910·4	3169·5
4	1582·4	1775·4	1979·4	2194·4	2420·4	2657·5	2905·6	3164·7
5	1576·3	1769·3	1973·3	2188·3	2414·3	2651·4	2899·5	3158·6
6	1568·9	1761·9	1965·8	2180·8	2406·8	2643·9	2892·0	3151·1
7	1560·3	1753·2	1956·7	2172·1	2398·0	2635·2	2883·2	3142·3
8	1550·5	1743·3	1947·2	2162·1	2388·0	2625·1	2873·1	3132·2
9	1539·4	1732·2	1936·0	2150·9	2376·8	2613·8	2861·8	3120·9
10	1527·3	1720·0	1923·9	2138·5	2364·3	2601·3	2849·3	3108·3
11	1514·1	1706·7	1910·3	2125·0	2350·8	2587·6	2835·5	3094·5
12	1499·9	1692·3	1895·8	2110·4	2336·0	2572·8	2820·6	3079-5
13	1484·8	1677·0	1880·3	2094·8	2320·3	2556·9	2804·6	3063·4
14	1468·9	1660·8	1863·9	2078·1	2303·4	2539·9	2787·5	3046·1
15	1452·1	1643·7	1846·5	2060·5	2285·6	2521·9	2769·4	3027·9
16	1434·5	1625·8	1828·3	2042·0	2266·9	2503·0	2750·2	3008·6
17	1416·3	1607·2	1809·3	2022·7	2247·9	2483·2	2730·2	2988·3
18	1397·5	1587·9	1789·6	2002·6	2226·9	2462·4	2709·2	2967·1
19	1378·2	1568·0	1769·2	1981·8	2205·7	2440·9	2687·4	2945·0
20	1358·4	1547·5	1748·2	1960·3	2183·8	2418·6	2664·7	2922·1
21	1338·1	1526·6	1726·7	1938·3	2161·2	2395·6	2641·3	2898·2
22	1317·6	1505·3	1704·7	1915·6	2138·1	2372·0	2617·3	2873·9
23	1296·7	1483·6	1682·2	1892·5	2114·3	2347·7	2592·5	2848·7
24	1275·6	1461·6	1659·4	1868·9	2090·1	2322·9	2567·2	2822·9
25	1254·3	1439·3	1636·2	1845·0	2065·5	2297·6	2541·3	2796·4
26	1232·9	1416·8	1612·8	1820·7	2040·4	2271·9	2514·9	2769·4
27	1211·4	1394·2	1589·2	1796·2	2015·0	2245·7	2488·0	2741·9
28	1189·9	1371·5	1565·4	1771·4	1989·3	2219·2	2461·0	2714·0
29	1168·4	1348·7	1541·5	1746·4	1963·1	2192·3	2433·1	2685·6
30	1146·9	1326·0	1517·5	1721·3	1937·2	2165·2	2405·1	2656·8
31	1125·5	1303·2	1493·4	1696·0	1910·9	2137·9	2376·9	2627·7
32	1104·3	1280·5	1469·4	1670·7	1884·4	2110·4	2348·4	2598·3
33	1083·2	1257·9	1445·3	1645·4	1857·9	2082·7	2319·7	2568·7
34	1062·2	1235·4	1421·4	1620·1	1831·3	2055·0	2290·8	2538·8
35	1041·5	1213·1	1397·5	1594·8	1804·7	2027·2	2261·9	2508·8
36	1021·0	1190·9	1373·8	1569·7	1778·2	1999·3	2232·8	2478·6
37	1000·8	1168·9	1350·3	1544·6	1751·7	1971·5	2203·7	2448·3
38	980·80	1147·2	1326·9	1519·6	1725·2	1943·6	2174·6	2418·0
39	961·10	1125·8	1303·7	1494·8	1698·9	1915·9	2145·5	2387·6
40	941·69	1104·5	1280·8	1470·2	1672·8	1888·2	2116·4	2357·2
41	922·59	1083·6	1258·1	1445·8	1646·7	1860·7	2087·4	2326·9
42	903·80	1062·9	1235·6	1421·7	1620·9	1833·3	2058·5	2296·6
43	885·34	1042·6	1213·5	1397·7	1595·3	1806·0	2029·8	2266·3
44	867·20	1022·6	1191·6	1374·1	1569·9	1779·0	2001·1	2236·2
45	849·41	1002·9	1170·0	1350·7	1544·8	1748·1	1972·7	2206·2
46	831·95	983·48	1148·7	1327·5	1519·9	1725·5	1944·4	2176·3
47	814·84	964·43	1127·7	1304·7	1495·2	1699·1	1916·3	2146·6
48	798·07	945·71	1107·1	1282·2	1470·9	1673·0	1885·5	2117·1
49	781·64	927·34	1086·8	1260·0	1446·8	1647·1	1860·8	2087·8
50	765·56	909·31	1066·8	1238·1	1423·0	1621·5	1833·5	2058·8

LONDON:
J. MOYES, CASTLE STREET, LEICESTER SQUARE.

6