ARITHMETIC,

IN ALL ITS PARTS

VULGAR AND DECIMAL:

AS ALSO,

Tables of COINS, WEIGHTS and MEASURES used in different Countries.

WITH

INTEREST and ANNUITIES, SIMPLE and COMPOUND.

Extraction of ROOTS, Mensuration of PLANES and SOLIDS,

AND

Notes on the *Gregorian* KALENDAR.

All digested in the plainest, and most familiar METHODS.

By WILLIAM CRAIGHEAD Schoolmaster at *Monefieth*.

DUNDEE;
Printed, and sold by HENRY GALBRAITH and COMPANY. M.DCC.LVII.

TO THE RIGHT HONOURABLE

***JOHN* LORD GRAY,**

The following SYSTEM of ARITHMETIC

is humbly inscribed by,

His Lordship's

much Obliged,

and most Obedient

humble Servant,

WILLIAM CRAIGHEAD.

THE

PREFACE.

PRefaces to Books are ſo very common, that a Book without one, is thought imperfect: I ſhall therefore ſo far comply with the common cuſtom, as to acquaint the Reader by way of Preface to the following Syſtem, That the Subject has been my favourite Study, as Teaching it has been my principal Buſineſs, for a conſiderable number of years; and it has long been my opinion, that notwithſtanding the many Treatiſes of Arithmetic now extant, ſomething might be done in that way, which would anſwer the purpoſe of Schools better than any I have ſeen.

THIS is what I have attempted in the following Sheets, with what Succeſs muſt be left to others to determine; only this

much

much I may venture to ſay, That they contain more than any Arithmetic Book of the Price that I know of.

I have endeavoured all along, to be as plain and practical as poſſible, and to make this uſeful Science as far as lay in my power, agreeable to the Learner.

THERE is no Rule ommitted in this Syſtem but that of *Progreſſion*, and as the Book has ſwelled beyond what was at firſt intended, I have been prevented from enlarging on *Menſuration;* but I propoſe ſoon to publiſh a compleat Treatiſe on that Subject.

THAT this Book may tend to encourage and promote the Study of this Sceince, ſo univerſally neceſſary, is the ſincere deſire of

Wm. C.

THE

CONTENTS.

I Hope the Reader will meet with few Errors in this Book; but among others that may occur, he is desired to take notice of the following, viz.

Page 148, Queſt. 2d. inſtead of multiplying the Remainder 40 by 36, it ſhould be by 6, and the Anſwer 23 Falls, 5 Ells. Page 170 *Note* 2d, inſtead of dividing by 103, it ſhould be 824 the Inches in a ſolid Gallon.

Vulgar Arithmetic,

IN

Whole Numbers and Fractions.

CHAP. I.

Numeration or Notation.

ARITHMETIC or the Art of numbering, hath for it's subject quantities with respect to their number only; and so is a science, we have recourse unto when we are to consider either of more quantities than one, or how often one quantity is contained in another, which is to find out the true and proper answers to all such questions as demand, *How many*, *what number*, or *multitude* of quantities there are.

ARITHMETIC is divided into three distinct parts.

I. Vulgar arithmetic in whole numbers.

II. The doctrine of vulgar fractions.

III. The artificial part, or decimal arithmetic.

All the rules of arithmetic are comprehended within the due consideration of these five, viz. Numeration (or Notation) Addition, Subtraction, Multiplication and Division: For tho' the rules are many and various, by which numerical operations are performed in all the parts of arithmetic, several of them being formed and raised as occasion requires, when applied to Practice; yet by these five rules only, the whole art is attainable.

Arithmetic, in all its parts, is performed by the various combinations and repetitions of the ten following characters, or figures invented and agreed upon by the greatest part of the known world, viz.

1 One.	2 Two.	3 Three.	4 Four.	5 Five.	6 Six.	7 Seven.	8 Eight.	9 Nine.	0 Cypher.

The first nine of these are called Significant Fgures, to distinguish them from the cypher 0, which of itself, is insignificant; but it serveth to increase or decrease the value of other figures, according as it is placed.

The first of these characters is called Unity, and represents one of any kind of species or quality, as one man, one horse, one cow, &c. and is the beginning of all numbers. That is to say, Number is a multitude of units.

Unity hath this property, viz. That it neither multiplies nor divides, but leaves the number to be multiplied or divided the same.

If the digit 2 be either added to itself, or multiplied in itself, the sum and product are equal; and no square number, how large so ever, can terminate with the said diget.

Be-

Between the digits 5 and 6 there is a secret property; for if you multiply either of them in themselves, the number produced by such multiplications, shall terminate in themselves. And the digit 6 hath another eminent property, for all it's aliquot parts (viz. it's half, it's third and it's sixth) are equal to it's self: And of numbers that have this property very few are to be found.

The digit 9 hath a privilege above all the other figures, for if you take the nines out of the gross sum of any number, the remainder will be the same, as if the nines were taken out of the sum of the figures of that number: Thus, if you take the nines out of 75864, the remainder will be 3. And if they be taken out of the sum of the figures of that number (viz. 30) the remainder will be 3 also. And from hence proceeds the way of proving arithmetical operations, by casting out the nines.

By Numeration, we learn the different value of figures by their different places; and, of consequence to read or write any sum or number.

If you add a cypher or cyphers to, or subtract them from, or place them on the left hand of any whole number, they can neither increase nor diminish that number: But if you place them on the right hand of any whole number, they increase it's value in a ten fold proportion; as you may observe from the following Table.

The TABLE.

9	Units.	1
90	Tens.	12
900	Hundreds.	123
9000	Thousands.	1234
90000	Tens of Thousands.	12345
900000	Hundreds of Thousands.	123456
9000000	Millions.	1234567
90000000	Tens of Millions.	12345678
900000000	Hundreds of Millions, *&c.*	123456789

From

From this Table may be observed:

1. The names of the several places, viz. Units, tens, hundreds, &c. which proceed (increasing by a tenfold proportion) from the right hand to the left.

2. That every figure hath two values; one in itself, and the other from the place it stands in; for a figure when standing alone, or in the units place of any number, has it's simple value, but a figure in the second place, has ten times the value it would have, were it in the first place, or place of units; and a figure in the third place has ten times the value it would have, were it in the second place; and so each place has ten times the value of that immediately preceeding it. That is, in the first place, it is so many units, or ones; in the second place, so many tens; in the third place, so many hundreds; in the fourth, so many thousands; in the fifth, so many ten thousands; in the sixth, so many hundred thousands; in the seventh, so many millions, &c.

Admit this number in the table for explanation, viz. 12345, the five is only five units, or five; but 4 in the second place, is four times ten, or fourty; the 3 in the third place, an hundred times three, or three hundred; the 2 in the fourth place, a thousand times two, or two thousand; and 1 in the fifth place, is ten thousand; thus therefore will the said number be read, twelve thousand, three hundred and forty five.

From this consideration of the simple and local values of the figures, any one may account for the Addition, Subtraction, Multiplication and Division of Numbers; and may find a method of reading and writing Numbers, by assigning names to the several places, as in the foregoing table.

It may also be observed, that the order of places is reckoned from the right hand to the left; but (like that of letters or words) numbers are to be read from the left hand to the right, and so many figures as are placed together without any point, com-

ma,

this, line, or other note of distinction between them, are all but one sum, and must be read as such.

3. From the foregoing table it is very obvious, that if any figure have a cypher or cyphers joined to it's right hand, it's value is increased (as hath been said before) in a ten fold proportion; and that figure still retains the value of it's place, as much as if joined with any other figure or figures in the room of the cypher or cyphers; for instance, 9 standing by itself in the table, represents but nine units, but if a cypher be annexed to it, thus 90, then it becomes ninety, that is ten times nine; for the cypher possessing the place of units, hath thereby removed the 9 into the place of tens; and another cypher more, makes it 900, that is ten times 90, or nine hundred.

From the said notation it is plain, that the only use of cyphers in the arithmetic of integers, is to supply places, so as to augment the value of significant figures, by casting them into higher places; for if you were to write down eighty millions, it would be done thus, 80000000.

Tho' the forgoing table consists but of nine places, yet it might have been extended to twenty, thirty, or more places, at pleasure. As after hundreds of millions, thousands of millions, hundred thousands of millions; then millions of millions, or billions, &c.

In reading a large number, consider first, that every third figure from the place of units, bears the name of hundreds; and so let every third figure be pointed (as in the following number) below the line.

Secondly, observe that the figure on the left hand of each second hundred place is millions, billions trillions, quadrillions, &c. and let these be marked with a point above the line. This being done, if you mind that a figure on the left hand of a hundred point is always thousands, and if in the order of reading

ing after hundreds, you observe that tens always follow, you cannot miss to value and express any sum.

7 5 9 5 8 7 6 7 4 9 8 2 5 6 4 3 7 2 6 7 8 9 2 1 2 3

Thus read,

Seventy five quadrillions, nine hundred fifty eight thousand, seven hundred sixty seven trillions, four hundred ninety eight thousand, two hundred fifty six billions, four hundred thirty seven thousand, two hundred sixty seven millions, eight hundred ninety two thousand, one hundred and twenty three.

Let it be demanded what 8 is in the seventh place by annexing six cyphers on the right hand thus, 6000000; it is eight millions.

What is 6 in the tenth place?

By annexing nine cyphers on the right hand thus, 8000000000; it is six thousand millions.

In placing down any number arithmetically, write down the figures in the same order their values are expressed, beginning at the left hand, and writing towards the right: And if in pronouncing the number any places are omitted, these must be supplied with cyphers.

Write down seven hundred sixty four millions, five hundred eighty six thousand nine hundred and thirty four.

Answer thus, 764586934.

Write down nine millions five hundred and seven.

Answer thus, 9000507.

Write down six hundred billions, five hundred thousand, seventy three millions, seven hundred forty five thousand and thity three.

Answer thus, 600500073745033.

Write down one million wanting one.

Answer thus, 999999.

Write

Write down eleven millions, eleven thousand, eleven hundred and eleven.

Answer thus, 11012111.

Write down eighteen million, eighteen thousand, eighteen hundred and eighteen.

Answer thus, 18019818.

A TABLE of old Roman Numbers.

100	C.
200	CC.
300	CCC.
400	CCCC.
500	D. or IↃ.
600	DC.
700	DCC.
800	DCCC.
900	DCCCC.
1000	M. or CIↃ.
2000	CIↃ. CIↃ. or M. M.
3000	CIↃ. CIↃ. CIↃ. or M. M. M.
5000	IↃↃ.
10000	CCIↃↃ.
50000	IↃↃↃ.
100000	CCCIↃↃↃ or CM.
500000	IↃↃↃↃ.
1000000	CCCCIↃↃↃↃ.
1756	MDCCLVI. or CIↃIↃCCLVI.

Definitions.

1. AN Integer or whole number is that which contains unity, or some number of units, as 6, 44, 172, &c.

2. A

2. A digit is any one of the nine significant figures; for 0 signifies nothing.

3. An even number is such as may be divided into two equal parts: as 24, 36, &c.

4. An odd number is such as cannot be so divided, or that which differs from an even number by unity, as 17, 33, 159, &c.

5. A number is said to measure another number, when it divides that other number witout a remainder; so 6 measures 18 by 3, and 7 measures 28 by 4.

6. One number is said to be prime to another, when no number can be found to measure both precisely, excepting unity; so 11 and 15 are prime to one another; as are 10 and 21, and many more.

7. A prime number is that which unity only measureth, as 7 or 13.

8. A composite number is such as can be measured by some other number than unity, such as 8, 12, 25, &c.

9. Numbers composite betwixt themselves, are such as have one common measure, as 6 and 9, which are both measured by 3.

C H A P. II.

Of Money, Weights and Measures, &c.

Of Money.

ACcompts are kept in Great Britain, in pounds, shillings, pence and farthings, thus mark'd, L. S. D. Q. and divided as follows:

4 Farthings	make	a penny,	either Ster. or Scots
12 Pennys,	make	a shilling,	either Ster. or Scots
20 Shillings,	make	a pound,	either Ster. or Scots

21 Sh. Ster. make a guniea.

10 Sh. 6 d. Ster. make half a guinea.

5 Sh. Ster. make a crown.
2 Sh. 6 d. Ster. make half a crown.
13 Sh. 4 d. make a merk either Ster. or Scots.
12 L. Scots, make 1 l. Ster.
4 Crowns, make 1 l. Ster.
18 Merks Scots, Make 1 l. Ster.
8 L. Scots, makes 1 merk Ster.
3 L. Scots, make a crown.
66 L. 13. s. 4 d. make an hundred merks.

Note. If any quantity or weight of fine gold be divided into twenty four equal parts, and 22 of those parts be mix'd with 2 of the like parts of copper; that mixture (in England) is reckoned standard gold.

And, there, it is also agreed on, that if 11 ounces and 2 penny weight of fine silver, and 18 penny weight of copper be melted together, such mixture, shall be esteemed the true standard for silver coin, called Sterling silver.

Foreign coins computed in value with English coin.

Note, The par of exchange is certain and fix'd, being always like for like, according to the weight and fineness of the coin: But the course or current price betwixt any two countries, rises and falls, according as money is plenty or scarce, or according to the various circumstances and accidents of trade and nations.

FRANCE.

AT Paris, Lyons, Rouen, &c. they keep their accompts in livres, sols, and deniers, and exchange upon the crown, the par of which in Sterling money is 4 s. 6 d.

				l. s. d.
12 Deniers	make	1 Sol	equal to	0:0:0$\frac{9}{12}$
20 Sols		1 Livre		0:1:6
3 Livres		1 Crown		0:4:6

	l.	s.	d.
A Denier is — —	0	0	$0\frac{3}{4}$
A French pistole is equal to —	0	17	6
A Moidore to — —	1	7	0

ITALY.

	l.	s.	d.
THE Livre at Leghorn is equal to	0	0	9
The current Florence crown	0	5	3
Venice Ducal de banco -	0	4	4
A St. Mark - - -	0	2	10
A Julier - - -	0	0	6
The Ducat of Rome - - -	0	7	6
The Crown of Placentia -	0	7	6
The Naples ducat - -	0	5	0
The Crown current - -	0	5	6
The Torri - - -	0	1	0
A Bajocke - - - -	0	0	$0\frac{1}{2}$
A Dollar or piece of eight - -	0	4	6
The Crown at Rome - -	0	7	6

In Genoa, Leghorn, &c. they also keep their accompts in livres, sols, and deniers; and exchange upon the dollar, or piece of eight; the par of which with London is 4s. 6 d. Sterling.

SPAIN.

In Madrid, Seville, &c. they keep their accompts in rials and mervadees, and exchange also by the piece of eight. The par with London 4 s. 6d. Sterling.

34 Mervadies } make { 1 Real
8 Reals } make { 1 Piece of eight

	l.	s.	d.
23 Mervadies are equal to	0	0	$0\frac{1}{4}$
A Real to - - -	0	0	$6\frac{3}{4}$

8 Reals

	l.	s.	d.
8 Reals, or 1 piece of eight	0	4	6
A Piece of eight of Peru	0	4	5
A Doubloon of Castile	0	5	6
A Velentia ducat	0	5	3
A Saragoza ducat	0	5	6
A Barcelona ducat	0	6	0
A Patavoon	0	4	8

PORTUGAL.

IN Lisbon, Oporto, &c. they keep their accompts in rees, and exchange on the mill-ree; the par of which is about 6 s. $8\frac{1}{2}$ d. Sterling.

Note, 1000 rees make a mill-ree.

	l.	s.	d.
A Vintine is equal to	0	0	3
A Real to	0	0	6
A Testoon	0	1	3
An Old crusado	0	5	0
A Mill-ree	0	6	$8\frac{1}{2}$

HOLLAD FLANDERS and GERMANY.

I Take these places together, because their accompts are kept, and their exchange with England made the same way:

For not only in Amsterdam, but also in Anterwerp and Hamburgh, they keep their accompts in pounds, shillings, and pence Flemish; or guilders, stivers, and pennicks; and exchange with England upon the pound, giving for it when at par, 33 s. 4 d. Flemish.

Their pounds, shillings and pence, are divided as ours; viz. their pound into 20 shillings, and their shilling into 12 pence.

Their other money is thus divided,

16 Pennicks } make { 1 Stiver
20 Stivers } make { 1 Guilder

Note,

Note also, that 6 Stivers } make { 1 Shilling } Flemish
6 Guilders } make { 1 Pound } Flemish

	l.	s.	d.
A Dutch stiver is equal to	0	0	$1\frac{1}{5}$
6 Stivers or one skilling Flemish	0	0	$7\frac{1}{5}$
20 Stivers, or 1 guilder	0	2	0
A Flemish pound 33 s. 4 d.	1	0	0
A Zeland, or common dollar	0	3	0
A Ducatoon	0	6	$3\frac{3}{5}$
A Specie dollar	0	5	0
A Cross dollar	0	4	2
Old Philip's dollar	0	5	0
Ferdinando's dollar	0	4	3
Prince of Orange's dollar	0	4	4
Leopold's dollar	0	4	3
Maximilian's dollar	0	4	5
Lyons dollar	0	4	0
Emblem dollar	0	2	$3\frac{1}{5}$
Campen dollar	0	2	$7\frac{1}{5}$
Holland rider	1	5	9
A Rix dollar of the Empire	0	4	$5\frac{2}{5}$
A Guilder of Noremberg	0	7	1
A Mark lubs	0	16	0

POLAND and RUSSIA.

	l.	s.	d.
A Rix dollar is equal to	0	4	6
A Guilder	0	1	6
The Stiver or dollar	0	2	3
The Silver mark.	0	1	6

SWEDEN.

	l.	s.	d.
THE Silver dollar is equal to	0	2	2
The Silver mark, or guilder	0	1	6

Arabia

ARABIA and PERSIA.

	l.	s.	d.
THE Abassis is equal to	0	1	4
A Mamada	0	1	0

ALEPPO and SCANDEROON.

	l.	s.	d.
16 SHekees, or 1 piece of eight	0	4	6
14 Shekees, or 1 Lyon dollar	0	4	0
A Dina	1	10	0
A Soraphat goa	0	4	6

SIAM.

	l.	s.	d.
A Gold coin worth	0	10	7
A Silver coin.	0	2	5

SURAT.

	l.	s.	d.
THE Asper is equal to	0	0	1¼
The Larin	0	4	0
The Sarophim	[illegible]	[illegible]	6
The Rupie	[illegible]	[illegible]	0
The Tenon, or tanam	[illegible]	0	3
A Pagod	0	9	0
A Saltanin	0	8	0

Those that desire to be further satisfied in the common values of foreign coins, may have them at large in the Merchants Map of Commerce, out of which I have collected these few.

Of

Of Weights.

TROY WEIGHT.

THE original of all weights used in England, was a corn of wheat gathered out of the middle of the ear, and being well dried, 32 of them were to make one penny weight, 20 penny weight one ounce, and 12 ounces one pound Troy.

But in latter times it was thought sufficient to divide the aforesaid penny weight into 24 equal parts, called grains being the least weight now in common use; and from thence the rest are computed as in this table.

24 Grains	make	1 Penny weight.
20 Penny weight	make	1 Ounce.
12 Ounces	make	1 Pound.

They are marked thus, Pounds *lb.* Ounces *oz.* Penny weights *p. w.* Grains *gr.*

By these weights are weighed jewels, gold, silver, pearl, electuaries, and all liquors.

The weight of English coin as weighed in air and water. Troy weight.

	In air.		In water.	
	p.w.	gr.	p.w.	gr.
A Guinea weighs	5 :	9	5 :	$1\frac{3}{4}$
A Crown	19 :	$8\frac{1}{2}$	17 :	$11\frac{1}{5}$
Half a crown	9 :	$16\frac{1}{4}$	8 :	$17\frac{3}{5}$
A Shilling	3 :	$20\frac{9}{10}$	3 :	$11\frac{4}{5}$
A Six pence	1 :	$22\frac{1}{2}$	1 :	$17\frac{9}{10}$

The value of gold in Troy weight.

	l.	s.	d.
One Pound weight of gold is worth	48 :	0 :	0
One Ounce of ditto.	4 :	0 :	0
One Penny weight	0 :	4 :	0
One Grain	0 :	0 :	2

The

The value of silver.

	l.	s.	d.	
One Pound weight of silver	3 :	2 :	0	
One Ounce	0 :	5 :	2	
One Penny weight	0 :	0 :	$3\frac{1}{10}$	
One Grain	0 :	0 :	0	$\frac{1}{2}$ a farthing.

A Tun of gold at 4l. the ounce is l. 96000
A Tun of silver at 5s. the ounce is l. 6000

Apothecaries Weights.

APothecaries have their weight deduced from Troy, their pound being the same, viz. 12 ounces; but differently divided, as follows,

20 Grains	make	1 Scruple.
3 Scruples	make	1 Dram.
8 Drams	make	1 Ounce.
12 Ounces	make	1 Pound.

By these weights Apothecaries compound their medicines; but they buy and sell their drugs by Averdupoise weight.

Averdupoise Weight.

BY what law this weight was first settled cannot be found out in the statute books; so that it seems it was first introduced by chance, and settled by custom, viz. from giving large weight to such commodities as are usually weighed by it, which are such as are either very coarse and drossy, or subject to waste; as pitch, tar, rozen, wax, tallow, flax, hemp, copper, tin, steel, iron, lead, tobacco, sugar, fruit, brass, salt; also, flesh, butter, cheese, &c. For it has been found by a very nice experiment, that one pound Averdupois, is equal to 14 ounces, 11 penny weight, and $15\frac{1}{2}$ grains Troy; and one ounce Troy is equal to 1 oz. 1 dram, and something above $\frac{2}{3}$ of a dram Averdupoise.

See

See the following table.

16 Drams	make	1 Ounce.
16 Ounces		1 Pound.
28 Pounds		1 Qr. of an hund. wt;
4 Qrs. or 112 pounds		1 Hundred weight.
20 Hundred weight		1 Tun weight.

They are thus marked, a tun *T.* an hundred weight *C. W.* a quarter *Qr.* a pound *Lb.* an ounce *Oz.*

A pound Averdupoise of gold, is worth	*l.* 58 : 8 : 0
A pound Averdupoise of silver	3 : 15 : 3½

	lb. *oz.*	
100 Pound in gold, weighs	1 : 11¾	Averdupoise.
100 Pound in silver, weighs	26 : 4	

A pound Averdupoise is heavier than a pound Troy; but the ounce Averdupoise is lighter than the ounce Troy; for it is commonly supposed, that 14 pounds Averddupoise are equal to 17 pounds Troy; and the ounce Troy weighs 480 grains, but the ounce Averdupoise doth only weigh 437½ grains.

Averdupoise small Weight.

16 Drops	make	1 Ounce.	*oz.*
16 Ounces		1 Pound.	*lb.*
14 Pounds		1 Stone.	*st.*

The Scots Troy pound (which by the statute 1718 was to be the same with the French) is commonly supposed equal to 15¾ ounces English Troy, or, 7560 grains. By a mean of the standard's kept by the dean of guild of Edinburgh, it is 7599 3/32, or 7600 grains. They who have measured the weights which were sent from London, after the union of the kingdoms, to be the standards by which the weights in Scotland should be made, have found the English Averdupoise pound (from a medium of the several weights) to weigh 7000 grains, the same as Mr. Everard; according to which the Scots, Paris, or Amsterdam

Amsterdam pound will be to the pound Averdupoise, as 38 to 35. The Scots Troy stone contains 16 pounds, the pound 16 ounces, an ounce 16 drops, a drop 36 grains. 20 Scots ounces make a tron pound; but because it is usual to allow one to the score, the tron pound is commonly 21 ounces. Sir John Skeen however makes the tron stone to contain only $19\frac{1}{2}$ pounds.

Note, A boll of meat weighs 8 stone.

A TABLE shewing the Specific Gravity or Weight of a Cubic Inch of various Sorts of Bodies, both in Troy and Adverdupoise Ounces, and Decimal parts of an Ounce.

		Oz. Troy.	*Oz. Aver.*
A Cubic Inch of	Fine gold is,	10,35927	11,305
	Standard gold,	9,96262	10,93042
	Quick silver,	7,38441	8,10175
	Lead,	5,98401	6,55288
	Fine silver,	5,85003	6,41832
	Standard silver,	5,55676	6,09656
	Rose copper,	4,74712	5,20836
	Plate brass,	4,40427	4,83211
	Cast brass,	4,2[illegible]24	4,6303
	Steel,	4,14212	4,5445
	Common iron,	4,05136	4,42297
	Black tin,	3,86151	4,23663
	Fine marble,	1,42941	1,5[illegible]85
	Common glass,	1,36084	1,4[illegible]303
	Dry ivory,	0,96208	1,05554
	Sea water,	0,54274	0,59489
	Common clear water,	0,52745	0,57869
	Red wine,	0,52376	0,57464
	Brandy,	0,48926	0,53679

If the number of Cubic inches contained in any given quantity be multiplied with the tabular weight

 of

of one inch (of the same kind of matter) the product will be the weight of that quantity in ounces.

Divide the given weight of a proposed quantity (it being first reduced into inches) by the tabular weight of one inch of the same kind of matter, and the quotient will be the cubic inches contained in that quantity.

If you subtract the weight of an equal quantity of water (with that of any body mentioned in the table) from the weight of the proposed body, (if it be heavier than water) there will remain the weight of that body when it is immersed or put into water.

Of Long Measure.

A Line, or length to be measured, whether it be distance, depth, or height, is measured by a line less than it.

The least measure of lenth is an inch, and as no other name of any measure below the same is used lesser measures are expressed by the fractions of an inch.

See the following Tables.

12 Inches make a foot.
3 Feet one inch, or 37 inches, a Scots ell.
6 Such ells a fall.
40 Falls a furlong.
8 Furlongs a mile.

The Scots mile is 1184 paces, and so (every pace being accounted at 5 feet) the said mile will consist of $1973\frac{1}{3}$ ells.

These things are according to the statutes of Scotland, notwithstanding which the Glaziers use a foot of only 8 inches.

By an act of parliament made the 16th of June 1685, 37 inches make a Scots ell; yet severals of late have affirmed the Edinburgh ell to be equal to $37\frac{2}{10}$ inches, and from thence conclude that the Edinburgh foot and inch made use of antiently were different

different from those of England; so 185 English feet are equal to 186 Scots ones.

In the year 1749, by order of the dean of guild of Edinburgh, experiment was made of the old standards of weights and measures of Scotland, before several gentlemen; and the proportions (with respect to the measures) are as follow, viz.

The Scots, or Edinburgh old iron ell was found equall to $37\frac{1}{28}$ inches.

The said ell, to 37 such inches as the English ell makes 45.

The Scots foot marked on the old brass ell, to 12 inches or an English foot.

The Scots half ell marked on the old iron ell, to $18\frac{58}{100}$ inches.

The Scots quarter of an ell marked on the said iron ell, to $9\frac{1}{4}$ inches.

The half quarter marked on the said ell, to $4\frac{7}{16}$ inches.

The English brass yard, to sharp 36 inches.

TABLE of English Measures.

12	Inches make a foot.
3	Feet make a yard.
5	Feet make a Geometrical pace.
1056	Such paces, or 1760 yards make an English mile.
2	Yards make one fathom.
3	Miles make a league.
3	Feet 9 inches, or 45 inches make an English ell.
$5\frac{1}{2}$	Yards make a pole.

4 Poles, or 22 yards is the length of Guntier's measuring chain, and that chain is divided into 100 links, each of which is $7\frac{92}{100}$ inches.

The chain for surveying in Scotland, ought to be in length 74 feet, or 24 Scots ells, if no regard is had to the difference of the Scots and English foot, but if regard is had to that difference, the Scots chain ought to consist of $74\frac{2}{5}$ English feet, and being divided into 100 links, each of these is $8\frac{928}{1000}$ inches.

It

If the earth be an exact sphere, the circumference thereof will be very near 24899 English miles.

An Italian mile is 1855½ yards English.

A French toise contains 6 Paris feet.

One degree of a great circle upon the surface of the earth is 69 English miles and 288 yards, and not 60 miles according to the common received opinion of Seamen.

Of Cloth Measure.

4 Nails make a quarter.
4 Quarters, a yard.
5 Quarters, an ell English.
3 Quarters, an ell Flemish.

In the following Table, the most noted Measures are expressed in English Inches and Decimal parts of an inch.

	Inches. Parts
English foot is	12,000
Scots foot	12,065
Paris foot,	12,788
Amsterdam foot,	11,172
Dantzick foot,	11,297
Danish foot,	12,465
Swedish foot,	11,692
Brussels foot,	10,828
Lyons foot,	13,458
Bononian foot,	14,938
Milan foot,	15,631
Rhinland foot,	12,362
Roman Palm used by merchants,	9,791
Roman Palm used by architects,	8,779
Palm of Naples,	10,314
English yard,	36,000
English ell,	45,000
Scots ell,	37,200
Paris Aune used by mercers,	46,786
Paris Aune used by drapers,	46,680
Lyons Aune,	46,570

Geneva

	Inches. Parts.
Geneva Aune, - -	44,760
Amsterdam ell, - -	26,800
Danish ell, - - -	24,930
Swedish ell, - -	23,380
Norway ell, - -	24,510
Brabant or Antwerp ell,	27,170
Brussels ell, - -	27,260
Bruges ell, - -	27,550
Brace of Bononia, - -	25,200
The Brace used by Roman Architects	30,730
Brace used by Roman merchants,	34,270
Merchants Florence Brace,	22,910
Florence geographical brace,	21,570
Vara of Seville, - -	33,127
Vara of Madrid, - - -	39,166
Vara of Portugal, - -	44,031
Cavedo of Portugal, -	27,354
Old Roman foot, - -	11,632
Persian Arish, - -	38,364
Shorter Pike of Constantinople	25,576
Another Pike of Constantinople,	27,920

For this see Dr. Gregorie's Practical Geometry.

Of Superficial Measure.

The smallest superficial measure is a square inch, 144 of which make a square foot. Wrights make use of these in measuring of deals and planks; but the square foot which glaziers use consists only of 64 square inches the other measures are as follow;

36 Square ells make a fall.
40 Falls, a rood.
4 Roods, or 160 falls an acre,

Slaters, Masons and Pavers, use the ell square and the fall. Surveyors of land use the square ell, the fall, the rood, and the acre.

The superficial measures of the English are as follow.

144 Square inches make a foot.
9 Square feet, a yard.
$30\frac{1}{4}$ Square yards a pole.
40 Poles, a rood.
4 Roods an acre.

The pole of $16\frac{1}{2}$ feet is statute measure; but there are some customary measures which are greater; as for fens and wood lands, there are reckoned 18 foot to the pole; and for forrests 21 foot to the pole. Masons measure their hewn work by the English foot, and painters and plaisterers use the yard.

The Scots acre is to the English acre by statute as 100,000 to 78,694, if we have regard to the difference betwixt the Scots and English foot before mentioned. But if the pole consists of 18 or 21 feet, in such cases the acre is greater in the Duplicate Ratio of the number of feet to a pole.

They who measure land in Scotland by an ell of 37 English inches, make the acre less than the true Scots acre by $593\frac{6}{10}$ square English feet, or about $\frac{1}{91}$ part of the acre.

The Arpent about Paris contains 32,400 square Paris feet, and is equal to $2\frac{3}{4}$ Scots roods, or $3\frac{37}{100}$ English roods.

The Actus Quadratus, according to Varro, was a square of 120 Roman feet. The Iugerum was the double of this. It is to the Scots acre as 10,000 to 20,456, and to the English acre as 10,000 to 16,097.

Of Measures of Capacity.

1728 Cubic inches make one cubic foot.
27 Such feet make a cubic yard.

Time and custom have alter'd measures, as they have done weights; for now we have three different measures, viz. one for wine, one for ale or beer, and one for corn.

In treating of these measures, a cubical inch is the smallest measure made use of; and of these 109 make a Scots pint.

The

The accounts of the cubical inches contained in the Scots pint, vary considerably from each other.

Dr. Gregory makes it to contain 109 cubical inches, as just now specified; according to the 7th article of the Union, it contains 99, 8; and according to Mr. Paterson's Scots Arithmetic 102, 3. But the standard jugs kept by the dean of guild of Edinburgh, having been carefully measured several times, and by different persons, the Scots pint, according to those standards, was found to contain about $103\frac{4}{10}$ cubic inches.

The beer or ale gallon (which are both one) is much larger than the wine gallon, it being (as I presume) made at first to correspond with Averdupoise weight, as the wine gallon did with Troy weight.

A TABLE of Wine Measure.

$28\frac{7}{8}$ Cubical inches make a pint English.
231 Such inches, a gallon.
2 Pints a quart.
4 Quarts a gallon.
18 Gallons a roundlet.
$3\frac{1}{2}$ Roundlets, or 63 gallons a hogshead.
$\frac{1}{2}$ An hogshead a barrel.
$1\frac{1}{3}$ Hogshead, or 84 gallons a Puncheon.
$1\frac{1}{2}$ Puncheon, or 2 hogsheads a pipe or butt.
$\frac{1}{3}$ Of a butt a tierce.
4 Hogsheads make a tun.

Tho' the English wine gallon is fixed now at 231 cubical inches, the standard kept in Guildhall being measured before many persons of distinction, May 25, 1688, it was found to contain only 224 such inches.

Nicholas Guitton, general officer in the excise, by experiment found the standard ale quart (kept in the Exchequer) to contain just $70\frac{1}{2}$ cubic inches, consequently the ale gallon must contain 282 such inches, and from thence the following table is computed.

A Table

A TABLE of Beer and Ale Measure.

$35\frac{1}{4}$ Cubic inches make a pint.
282 Such inches make a gallon,
2 Pints make a quart.
4 Quarts make a gallon.
9 Gallons a firkin.
4 Firkins a barrel of beer.
8 Gallons make a firkin of ale.
32 Gallons make a barrel of the same.

By an act of the first of William and Mary, 34 gallons is the barrel, both for beer and ale, in all places in England, except within the weekly bills of mortality.

Scots Denominations.

4 Gills make a mutchkin.
2 Mutchkins a chopin.
2 Chopins, or 103,4 cubic inches, a pint.
2 Pints a quart.
4 Quarts, or 8 pints a gallon.
2 Pints half a gill Scots make 3 English ale quarts.

With respect to the Scots pint it may be further observed, that the several differences amongst those that treat upon weights and measures, might arise from this, viz. that if the edges or brim of a vessel be dry, it will hold a good quantity more than its horizontal content, or than if they should happen to be wet; therefore if some fill the vessel till the water be level with the brim, and others fill it till it run over (the edge having been dry) there must needs be a difference in their computations.

Of Dry Measure.

$272\frac{1}{4}$ Cubical inches make the Winchester corn gallon.
2 Gallons, a peck.
4 Pecks, a bushel.
8 Bushels, a quarter.
21 Scots pints, an English bushel and 14 cubic inches.

The

The above content of the corn gallon is according to the old standard; but by act of parliament in 1697 the round bushel was to be $18\frac{1}{2}$ inches wide, and 8 inches deep, and so to contain 2150,42 cubic inches, and consequently the gallon $268\frac{4}{5}$ such inches; which 2150,42 inches was to be esteemed a legal Winchester bushel, according to the standard in his Majesty's Exchequer.

Scots Denominations.

4 Lippies make a peck.
4 Pecks, a firlot.
4 Firlots or 16 pecks, a boll.
16 Bolls, a chalder.

The wheat firlot contains $21\frac{1}{4}$ pints of the just Stirling jug.

The bear firlot contains 31 pints of the same jug.

The Scots wheat firlot is to the English Winchester bushel, as 100 to 99,642.

A Paris pint is 48 cubical Paris inches, and is nearly equal to an English wine quart. The Boissea contains 644,68 Paris cubical inches, or 780,36 English cubical inches. The Roman Amphora was a cubical Roman foot, the Congius was the 8th part of the Amphora, the Sextarius was one 6th of the Congius. The Medimnus was equal to two Amphoras, that is about $1\frac{1}{2}\frac{1}{5}$ legal English bushels; and the Medius was the third part of the Amphora.

Of Time.

Time only shews the duration or mutation of things, a year being the standard or integer, by which such continuance or change is computed. And a year is that space of time in which the sun (apparently) compleats its revolution from any one point in the Ecliptick to the same point again, which, according to modern observations, is performed in 365 days, 5 hours, 48 minutes, 57 seconds, 21 thirds, &c. But a second being the least part of time that can be truly measured by the motion of any mechanical engine, I begin the following table with seconds.

60 Seconds make a minute.
60 Minutes, an hour.
24 Hours, a day.
365 Days, 5 hours, 48 minutes, 57 seconds make one Solar year.

But the common year, usually called the Julian Year, consists of 365 days 6 hours, and is divided into 12 Kalendar months, whose names and number of days is the subject of every Almanack.

If, after you divide the year of our Lord by 4, 0 remains; that year is called Leap Year, and consists of 366 days.

A Century is 100 years; an Indiction among the Romans, a revolution of 15 years.

Of the Motion of the Heavenly Bodies.

60 Seconds make a minute,
60 Minutes or about 69 English miles, one degree.
90 Degrees, one sign.
12 Signs or 360 degrees, one revolution of the whole Sphere.

Some other things, necessary to be known and of use in Arithmetic.

There are several commodities sold by the dozen, a Table of which follows, viz.

12 Particular things make a dozen.
12 Dozen, a small gross.
12 Small gross, one great gross.

Of Paper and Parchment.

24 Sheets make a quire.
20 Quires, one ream.
10 Reams, one bale of paper.
12 Skins make one dozen.
5 Dozen make one roll of parchment.
4 Inches is a hand in measuring a horse.

4000 Gemetrical paces make a small, and 5000 such paces a great German mile.

A faggot of steel is 120 lb. and a burthen of gad steel 180 lb.

A tun or fodder of lead is $19\frac{1}{2}$ hundred weight. 5 lb. make a stone of glass, and a seam of the same is 24 stone. 10 Hides or skins make a dicker; and 20 dickers a last.

Of Yarn.

Yarn in Scotland, is in some places reeled on a reel 11 quarters, and in others on one of 10 quarters, a Spyndle being the integer, and is thus divided, viz.

120 Threads make a cutt.
2 Cutts, an heer.
6 Heers, an hasp.
4 Hasps, a spyndle.

Note, 12 Sheaves make a stook, and 2 stooks a thrave of corn.

To the foregoing Tables it will not be amiss to give a brief account of such coins, weights and measures as are mentioned in the Scriptures, being taken from the index of a large bible printed Anno 1702, and compared with those used in England by the Bishop of Peterborough.

The Hebrew Weights compared with Troy Weight.

	oz.	p.w.	gr.
A Gerah is equal to, - -	0 :	0 :	$10\frac{1}{2}\frac{9}{8}$
10 Gerahs or a bekah, -	0 :	4 :	$13\frac{1}{2}$
2 Bekahs or a shekel, -	0 :	9 :	3
160 Shekels or meaah. -	45 :	12 :	12

Note, A Shekel is said to be their original weight.

Their Coins compared with English Coin.

	l.	s.	d.
The Silver Menah weighs 60 Shekels, and is equal to,	7 :	1 :	$5\frac{1}{4}$
A Talent of silver weighs 300 shekels, and is equal to,	357 :	11 :	$10\frac{1}{2}$
A Talent of gold is equal to,	5075 :	15 :	$7\frac{1}{2}$
The Gold dram. - - -	1 :	0 :	4

Roman

Roman Money compared with English Money.

A Denarius or silver penny,	7 pence 3 farths.
Asses of copper, - -	3 farths.
Assarium, - - -	1½ farth.
Quadrants, - - -	¾ of a farth.
A Mite. - - -	½ of a farth.

Their measures of Capacity, compar'd with English wine measure.

	gal.	pints.	inches.
A Cotyla measures, - -	0	0½	3,037
A Log, - -	0	0½	9,83
A Cab, - - -	0	3	10,458
A Hin, - - -	1	2	2,5
An Omer, - -	0	6	1,5
A Seah, - -	2	4	5
An Ephah, - -	7	4	15
A Chomer. - - -	75	5	5,625

Their long measure compared with English measure.

	yds.	feet	inch.part.
An hair's breadth, - -	0	0	0,0208
A Finger's breadth, - -	0	0	0,912
An hand's breadth, - - -	0	0	3,648
2 Hand's breadth or least span,	0	0	7,296
3 Hand's breadth or longest span,	0	0	10,944
2 Spans the longest cubit,	0	1	9,888
4 Cubits, a fathom,	2	1	3,552
6 Cubits, Ezekiel's reed,	3	1	11,328
400 Cubits, a stadium,	243	0	7,2
10 Stadiums, a mile,	2432	0	0
3 Miles, a parasang,	7296	0	0

Which is 4 English miles and 256.

I have seen (from no bad authority) their coins, weights and measures otherways accounted for, as follows. The

	l.	s.	d.
The common shekel of silver weighing 2 drams,	0	1	3
The King's shekel 3 drams,	0	1	10½
The Shekel of the Sanctuary 4 drams,	0	2	6
The common talent weighing 6000 drams,	187	10	0
The King's talent 9000 drams,	281	5	0
The Talent of the Temple 3000 shekels of do.	375	0	0
A Silvering, or piece of silver,	0	2	6
The common shekel of gold, 2 drams,	0	15	0
The King's shekel of gold, 3 drams	1	2	6
The Shekel of gold of the Temple, 4 drams,	1	10	0
The common talent of gold,	2250	0	0
The king's talent of gold,	3375	0	0
The Talent of gold of the Sanctuary	4500	0	0

The common cubit was 1½ foot.

The King's cubit 21 inches.

The Cubit of the Sanctuary 1 yard.

The Geometrical cubit 3 yards. And according to this cubit it is thought Noah's ark was built.

A Sabbath day's journey was 600 paces.

A Legion in the time of Romulus consisted of 3000 footmen and 300 horsemen.

In Julius Cesar's time it consisted of 5000 foot and 300 horsemen.

In Augustus and Tiberius Cesars's time it consisted of 6000 foot, and 730 horsemen.

Hesych says, a legion was 6666.

What follows (with respect to the reduction of antient sums to Sterling money) is taken from Prideaux's old and new Testament connected, which may in my opinion, be depended on as very correct.

The Hebrews, Babylonians, Greeks and Romans reckoned their money by talents; and of these talents they had subdivisions, of their Talents into Minas, and

and their minas into drachms. The Hebrews had, besides these, their shekels, and half shekels or bekas and the Romans their denarii, which last were very near of the same value with the drachms of the Greeks. One with attention may come to know the value of an Hebrew talent from Exodus xxxviii. 25. 26. for there 603550 persons being taxed at an half shekel an head, they must have paid in the whole 301775 shekels; and that sum is there said to amount to 100 talents and 1775 shekels over; if therefore you deduct the 1775 shekels from the number 301775, and divide the remainder, viz 300000 by 100, this will prove each of those talents to contain 3000 shekels. Each of these shekels weigh'd about 3s. Ster. and 60 of them, (Ezekiel tells us) made a mina; and therefore 50 of these minas made a talent. As to their drachms, it appears by the Gospel of St. Matthew, that it was the 4th part of a shekel, that is 9 pence Ster. Josephus tells us that a shekel contained 4 Attic drachms, which is not exactly to be understood according to the weight, but according to the valuation in the currency of common payments. The Attic drachm was not so heavy as the Hebrew drachm, but what it fell short of the Hebrew drachm, in weight might be made up in the fineness, and its ready currency in all countries, and so might be made equivalent in common estimation among the Jewes. Allowing therefore a drachm (as well Jewish as Attic) to be equivalent to 9 pence Ster., a bekah or half shekel will be 1 s. 6 d.; a shekel 3 s. a mina 9 l. a talent 450 l. So was it in the time of Moses, Ezekiel and Josephus among that people: for he tells us that one Hebrew mina contained $2\frac{1}{2}$ litras which comes exactly to 9 l. Ster. for a litra being the same with a Roman libra, contained 12 oz. Troy weight, that is 96 drachms, and therefore $2\frac{1}{2}$ litras must contain 240 drachms, which being estimated at 9 pence a drachm, according to the Jewish valuation, comes exactly to 60 shekels or 9 l. Ster.

The

The Alexandrian talent contain'd 12000 Attic drachms, according to the Jewish valuation, which being 12000 9 pences Ster. amount to 450 l. Ster. which is the same value with the Mosaic talent; but it must be observed that tho' the Alexandrian talent amounted to 12000 Attic drachms, yet they themselves reckoned it but at 6000 drachms, because every Alexandrian drachm contain'd two Attic drachms, and therefore computing the Alexandrian money according to the same method in which the Jewish is computed, it will be as follows, viz. one drachm of Alexandria will be in English money eighteen pence; one shekel consisting of 2 drachms of Alexandria or 4 of Attica will be 3 s.; one mina consisting of 60 shekels will be 9 l.; and one talent consisting of 50 minas will be 450 l. which is the talent of Moses and Josephus, for he tells us that an Hebrew talent contain'd 100 Greek or Attic minas; for these 50 minas which here make an Alexandrian talent would be 100 Attic minas in the like method of valuation, the Alexandrian talent being twice as much as the Attic talent, both in whole and all its parts, in whatsoever method both shall be equally distributed.

To make the whole of this matter easier to the reader, I here lay it down before him, as I find it in Prideaux's history.

Hebrew Money.

	l.	*s.*	*d.*
An Hebrew silver drachm is,	0	0	9
Two Drachms made a bekah, or half shekel, which was the tribute money paid by every Jew to the Temple.	0	1	6
Two silver bekahs made a shekel of do.	0	3	0
Sixty silver shekels made a mina,	9	0	0
Fifty silver minas made a talent,	450	0	0
A talent of gold,	7200	0	0

Attic

Attic Money according to Mr. Brerewood.

	l.	s.	d.
A Silver Attic drachm is,	0	0	7½
100 Silver drachms made a mina,	3	2	6
60 Such Minas made a silver talent,	187	10	0
A Talent of gold.	3000	0	0

Babylonish Money according to Mr. Brerewood.

	l.	s.	d.
A Babylonish talent of silver containing 7000 Attic drachms,	218	15	0
A Babylonish gold talent.	3500	0	0

Attic Money according to Dr. Bernard.

	l.	s.	d.
A Silver Attic drachm,	0	0	8¼
100 Such Drams made a silver mina,	3	8	9
60 Such Minas made a silver talent,	206	5	0
A Talent of gold.	3300	0	0

Babylonish Money according to Dr. Bernard.

	l.	s.	d.
A Silver Babylonish talent,	240	12	6
A Babylonish talent in gold.	3850	0	0

Alexandrian Money.

	l.	s.	d.
A Silver drachm of Alexandria containing two Attic drams,	0	1	6
A Silver drachm being 2 Alexandrian drachms, which was an Hebrew shekel,	0	3	0
60 Such Drachms made a silver mina,	9	0	0
50 Such Minas made a silver talent,	450	0	0
A Talent of gold.	7200	0	0

Roman

Roman Money.

	l.	s.	d.
A Denarius or ſilver penny	0	0	$7\frac{1}{2}$
96 Denarii made an Italic mina, which was the ſame with a Roman libra,	3	0	0
72 Roman Libras made a ſilver talent.	216	0	0

Such as want a fuller account of the Money of the Antients may read Biſhop Cumberland on the Jewiſh Meaſures, Weights and Monies; or Dr. Bernard de Menſuris & ponderibus antiquis.

CHAP. III.

Of Addition.

ADDITION is that Rule by which ſeveral numbers or quantities are collected and put together; and that quantity which ariſes or reſults from thence is called the ſum or total amount of theſe quantities.

Addition is of one denomination, or ſeveral.

Addition of one denomination is, when the numbers or ſeveral quantities given to be added are all of one name; that is, all pounds, ells, acres, miles, feet, &c.

In this caſe, two things being carefully obſerved, the work will be eaſily performed.

1. The numbers to be added, muſt be placed in ſuch order under one another, (it matters not which is uppermoſt, the greateſt or leaſt) that units may ſtand under units; tens under tens; hundreds under hundreds; thouſands under thouſands, &c. Then underneath the loweſt rank there muſt always be a

line drawn to separate the given numbers from their sum.

2. The second thing to be observed, is the due collecting or adding together each row of figures that stand over one another of the same value: and that is performed by the following Rule.

RULE.

Always begin your addition at the place of units, adding together all the figures that stand in that place or column, and if their sum be under ten, set it down below the line in its own place; if just ten or any number of tens, set down a cypher; but if above ten or any number of tens, set down the excess, and carry one for every ten to be added to the next column on the left. This second column with what you carried to it for the tens in the row of units being added, place the excess above the ten or tens in its amount under the line in its own place, and carry one for every ten (as before) to the third place or column of hundreds, and so proceed in the same manner to each place or row till all is done; remembering always to set down what the last column amounts to, whatever it is, because you have no place more to carry the tens to. The sum arising from those additions will be the total amount required.

From what is here laid down, I suppose my reader will very soon understand the manner of performing the work of Addition; and the reason of the work appears very evident from this undeniable maxim, viz. That the whole is equal to all its parts, And the method of setting down the total, may easily be accounted for from the nature of Numeration, which explains the different value of places, as they proceed from the right to the left hand: For as 9 is the greatest simple character, or figure; so every number exceeding 9 being compound, must require more places than one to express it. Thus the number 10 can no otherways be expressed in figures but by removing the

the figure 1 into the place of tens, which is done by ſupplying the units place with a cypher: and as it is the ſame with every other column (ten being ſtill the proportion of increaſe) conſequently, when the ſum of any column amounts to 10, or more, (as before mentioned) the units exceeding, if there be any, or a cypher, if none, muſt be ſet under ſuch column, and the ten, or tens of the amount, carry'd on as ſo many units, to the next column on the left.

The following examples will make all plain, ſuppoſe you was deſired to add 960 feet, 58 feet, and 31 feet together, they would be ſet down one under the other as follows,

	feet.		feet.
Thus,	960	or thus,	31
	58		58
	31		960

The numbers being placed as above, draw a line under them, and begin at the loweſt figure on the right hand, being the place of units, ſaying 1 and 8 is 9, which put directly under the line, and juſt below its own rank, viz. under 1 and 8 and 0; and then going to the next column towards the left hand, ſay, 3 and 5 is 8, and 6 is 14; that is 4 over ten, which 4 ſet down alſo under its own proper column, viz. under 3 and 5 and 6; then carry and add one for your ten to the laſt row 9, and the amount is 10 which you muſt place down in order under 9, as you have no other row to carry 1 to for your ten in the amount of that column; and ſo the numbers are added together, and they make 1049, that is one thouſand and forty nine feet in all. See the work.

	Hundreds.	Tens.	Units.
	9	6	0
		5	8
		3	1
Sum 1	0	4	9

	Hundreds.	Tens.	Units.	
		3	1	
		5	8	
	9	6	0	
Sum 1	0	4	9	Feet in all.

What numbers added together, will amount to eleven million, eleven thousand, eleven hundred and eleven, and how will the sum be written and expressed.

11000000	Eleven million.	Numbers required.
11000	Eleven thousand.	
1100	Eleven hundred.	
11	Eleven.	

11012111 Sum. So written, and expressed thus, Eleven million, twelve thousand, one hundred and eleven.

What numbers added together, will make the sum seven hundred and fifty six thousand, four hundred and ninety three ?

700000	Seven hundred thousand.
50000	Fifty thousand.
6000	Six thousand.
400	Four hundred.
90	Ninety.
3	Three
Sum. 756493	

Pounds.	*Ells.*
47696	8745
8457	379
479	2546
67	84
8	327
	51
Sum. 56707 l.	Sum. 12132 Ells.

Here follow some familiar examples, shewing the nature and use of this rule, viz.

Quest. 1. From Edinburgh to Musselburgh are 4 miles; from thence to Haddington 8; thence to Dumbar 8; thence to Old Cambus 8; thence to Berwick 12; thence to Charleton 19; thence to Newcastle

Newcastle 31; and from thence to York 61 miles; how many miles are there between York and Edinburgh?

	Miles.
	4
	8
	8
	8
	12
	19
	31
	61
Anfwer,	151 Miles.

Queft. 2. How many days are there in the 12 calender months, or a year?

	Days.
January hath,	31
February,	28
March,	31
April,	30
May,	31
June,	30
July,	31
Auguft,	31
September,	30
October,	31
November,	30
December,	31
Sum.	365 Days.

Queft. 3. A gentleman hath 6 tenants, A, B, C, D, E, and F, whofe farms contain thefe acres following, viz.

	Acres.
A.	476
B.	79
C.	214
D.	97
E.	8
F.	412

What acres in all? 1286 Anfwer.

Queft. 4

Queſt. 4. What number is that from which if you ſubtract 197, the remainder will be 574?

197
574

771 Number ſought.

Queſt. 5. A farmer ſells 47 bolls of wheat, 97 bolls of peaſe, 187 bolls of barley, and 57 bolls of oats; what bolls will the ſeveral quantities amount to?

47
97
187
57

Anſwer. 388 Bolls.

Queſt. 6. A merchant buys 6 pieces of cloth, each meaſuring as follows, viz.

No.	*Ells.*
1	45
2	9
3	17
4	8
5	27
6	5

Anſwer, 111 Ells.

Queſt. 7. In 587 l. Ster. what l. Scots?

587
587
587

7044 l. Scots.

This is the ſame, as if 587 had been multiplied by 12.

Queſt. 8

Queſt. 8. In 678 guineas what s. Ster.

678
678
678

14238 s. Ster.

This is the ſame, as if 678 had been multiplied by 21.

Queſt. 9. In 6742 l. what merks?

Add the one half of the pounds to themſelves, and the ſum is merks.

6742
3371

10113 Merks.

Queſt. 10. In 588 merks Ster. what merks Scots.

588
588
588

7056 Merks Scots.

This is the ſame as if you had multiplied the merks Ster. by 12.

Queſt. 11. In 547 merks Ster. what l. Scots.

547
547
547
547

2188
2188

4376 l. Scots.

This is the ſame as if you had multiplied the merks Ster. by 8.

Queſt. 12. Suppoſe 4784 guineas lay on one corner

of

of a ſquare table, 588 on another, 779 on another and 84 on the 4th corner; what ſum will they make if gathered together?

4784
588
779
84
Sum 6235 Guineas!

The reaſon for carrying the tens from one row or degree of figures to the next ſuperior degree, will appear from the following method of adding the numbers of the laſt example.

	4784
	588
	779
	84
Sum of the row of units ————	25
Sum of the row of tens ————	31
Sum of the row of hundreds ————	19
4 Thouſand brought down ————	4
	6235

Addition of ſeveral denominations is, when the ſeveral lines conſiſt of divers names, as pounds, ſhillings, and pence; or bolls, firlots, and pecks, &c. And what hath been obſerved as to carrying the tens (the proportion of increaſe) from one column to another in integers may be applied to the numbers of different denominations.

RULE.

Having placed all numbers that are of one denomination exactly under each other, that is (in money) pounds under pounds, ſhillings under ſhillings, pence under pence, &c. obſerving the like in weights and meaſures,

measures, &c. according to their several denominations. Always begin with these figures of the least denomination, and add them together into one sum, then consider how many of the next superior denomination are contained in that sum, for so many units you must carry to the said next superiour denomination, to be added to these figures that stand there; and if any thing remains over or above these units so carried, that overplus must be set down under its own denomination: And so proceed from one denomination to another until all be finished.

Note. In all additions, whether of money, weight, or measure, &c. that denomination towards the left hand (which is the first in setting down, but last in casting up,) must be added as sums of one denomination, for every 10 carrying 1 to the next row: but if it can be brought to a heigher name than what's proposed in the example given, you may do it by the foregoing rule.

This and the former rule extends to all the cases that can fall out in Addition.

Note. That in setting down sums care must be taken, that you do not set down more, or so much in the place of a lesser denomination than makes one of the next greater: for it would be absurd to write down l. 27 : 25 : 17 for l. 28 : 6 : 5. Or, 17 ells 3 qrs. 5 nails for 18 ells 1 nail.

The following examples will make all plain.

	l.	*s.*	*d.*
I borrowed,	45 :	17 :	7
More,——	27 :	18 :	8
More,——	7 :	4 :	10
More.——	9 :	15 :	5
Sum total,	l. 90 :	16 :	6

I here began with the pence being the smallest denomination towards the right hand (as in all additions we must, whether of money, weight, or measure) and said 5 and 10 is 15, and 8 is 23, and 7 is 30 pence, which was 2 s. 6 pence, wherefore I set down

 the

the 6-pence under its own rank, and carryed 2 for the two shillings to the rank of shillings, saying, 2 that I carried and 5 is 7 (for the tens of the shillings are omitted till you come to the top) 7 and 4 is 11, and 8 is 19, and 7 is 26; then I came downwards with the tens, saying, 26 and 10 is 36, and 10 is 46, and 10 is 56 shillings, which was 2 l. 16 s. I set down the 16 under the place of shillings, and carry the 2 l. to the pounds, saying, 2 and 9 is 11, and 7 is 18, and 7 is 25, and 5 is 30; I set down 0, and carried 3 for the 3 tens, saying, 3 and 2 is 5, and 4 is 9, which I set down, and found the whole sum amounted to 90 l. 16 s. 6 d.

In casting up all other sums of money (if they consist of l. s. d. and qrs. for every 4 of the farthings you must carry 1 to the pence, for every 12 of the pence you must carry 1 to the shillings, and for every 20 of the shillings you must carry 1 to the pounds; and the pounds (as was said before) must be cast up as sums of one denomination.

		b.	f.	p.	l.
A farmer sold of	Wheat,	27	3	1	2
	Barley,	19	0	2	1
	Pease,	47	3	3	3
	Oats,	79	2	1	3
	Rye.	8	1	3	2
	Sold in all	183	0	0	3

Here for every 4 of the lippies I carried 1 to the pecks, for every 4 of the pecks I carried 1 to the firlots, and for every 4 of the firlots, I carried 1 to the bolls, and added them up as sums of one denomination.

A collector of excise receives,

	l.	s.	d.
At Dundee,	1724	14	7
Perth,	975	6	8
Forfar,	1984	19	9
Montrose,	975	16	2
Aberdeen.	847	10	3
Total, l.	6508	7	5

Paid

		s.	d.
Paid,	For brandy, eight groats,	2 :	8
	For onions, ſeven farthings,	0 :	$1\frac{3}{4}$
	For wine, ſeventeen pence,	1 :	5
	For beef, fourteen pence,	1 :	2
	For veal, eleven groats,	3 :	8
	For ale, two groats and two pence,	0 :	10
	For tobacco, two and twenty pence,	1 :	10
	For bread, three halfpence.	0 :	$1\frac{1}{2}$
	Sum,	11 :	$10\frac{1}{4}$

From this and the two following examples you may ſee that ſometimes ſums are expreſſed one way, and ſet down another.

	l.	s.	d.
Laid out for coals, thirty ſix ſhillings,	1 :	16 :	0
For cloth, fifty ſeven ſhillings, ——	2 :	17 :	0
For meal, forty eight pence, ——	0 :	4 :	0
For malt, thirty nine pence, ——	0 :	3 :	3
For a calf, fifty two pence. ——	0 :	4 :	4
Sum, l.	5 :	4 :	7

		l.	s.	d.
Lent,	A guinea, ——	1 :	1 :	0
	A crown, ——	0 :	5 :	0
	A half crown ——	0 :	2 :	6
	A merk, ——	0 :	13 :	4
	A moidore, ——	1 :	7 :	0
	A French piſtole, ——	0 :	17 :	6
	Total ſum, l.	4 :	6 :	4

But when ſome of theſe ſums are to ſtand alone, and not in order of pounds, ſhillings, and pence; as in a letter, &c. 'tis better to ſet them down as ſpoken: As 15 pence, 47 ſhillings, &c. rather than 1 s. 3 d. or 2 l. 7 s.

Note.

Note. If your ſums to be added are very large, when you come to carry from the pence to the ſhillings, caſt up the units rank of the ſhillings as ſums of one denomination, for every ten carrying one to the tens of ſhillings, and reckoning them as ſo many ones; if when you come to the top, their ſum is odd, ſet down the odd one in the ten's place of ſhillings, halve the reſt and they are pounds to be carried to the ſame name, as in this example.

		l.	*s.*	*d.*
Bought,	A cow at ———	37	14	8
	A horſe, ———	49	13	7
	A ſaddle and bridle,	1	11	2
	A mare, ———	37	15	8
	A foal, ———	9	14	7
	Four ſheep, ———	12	16	5
	An ox, ———	45	18	8
	An aſs, ———	19	15	3
	Five bolls of meal,	33	13	7
	Four ſtone of iron,	6	14	8
	Twelve lambs, ———	19	16	6
	Three bolls of peaſe,	18	13	5
	Four ſtone of wool.	35	11	8
	Total,	l. 329	9	10

Here I have 82 pence, that is 6 s. 10 d. I ſet down the odd 10 pence, and to the units place of the ſhillings I carry the 6 ſhillings, and the ſum from top to bottom is 59; I ſet down the 9, and to the ten's place of s. I carry 5, they amount to 18 tens, the half of thoſe (being 9 l.) I carry to the pounds, and adding as before, find the total amount is l. 329 : 9 : 10.

A gentleman lets out to a farmer 9 acres of infield land at l. 13 : 17 : 7 per acre, and 5 acres of outfield at l. 6 : 18 : 9 Scots money per acre; what rent did the whole draw?

l.

l.	s.	d.
13	17	7
13	17	7
13	17	7
13	17	7
13	17	7
13	17	7
13	17	7
13	17	7
13	17	7
6	18	9
6	18	9
6	18	9
6	18	9
6	18	9

Total rent, l. 159 : 12 : 0

What cost 32 ells of cloth at 17 s. 7 d. per ell?

s.	d.
17	7
17	7
17	7
17	7
17	7
17	7
17	7
17	7

l. 7 : 0 : 8 Price of 8 ells,
7 : 0 : 8
7 : 0 : 8
7 : 0 : 8

l. 28 : 2 : 8 Price of 32 ells.

In the foregoing example, tho' there were no pounds in the price of the integer, the shillings are added up as before, and the pounds they contain'd set

set down on their left hand. In the said example the price of 8 ells is first found, and if 4 times 8 is 32, 'tis plain that 4 times the price of 8 ells is the value of 32 ells.

What cost 17 ells at 19 s. 9 d. per ell?

```
           s.   d.
           19 : 9
           19 : 9
           19 : 9
           19 : 9
       ---------------
       3 : 19 : 0 Price of 4 ells.
       3 : 19 : 0
       3 : 19 : 0
       3 : 19 : 0
           19 : 9
    -------------------
   l. 16 : 15 : 9 Price of 17 ells.
```

What cost 13 bolls meal at 7 l. 16 s. 5 d. per boll?

```
        l.   s.   d.
        7 : 16 : 5
        7 : 16 : 5
        7 : 16 : 5
        7 : 16 : 5
        7 : 16 : 5
        7 : 16 : 5
       ---------------
       46 : 18 : 6 Price of 6 bolls.
       46 : 18 : 6
            16 : 5
    -------------------
   l. 94 : 13 : 5 Price of 13 bolls.
```

What cost 27 bolls of barley at 6 l. 18 s. 8 d. per boll?

l.

l.	s.	d.	
6 :	18 :	8	
6 :	18 :	8	
6 :	18 :	8	
6 :	18 :	8	
6 :	18 :	8	
6 :	18 :	8	
6 :	18 :	8	
6 :	18 :	8	
6 :	18 :	8	
62 :	8 :	0	Price of 9 bolls.
62 :	8 :	0	
62 :	8 :	0	
l. 187 :	4 :	0	Price of 27 bolls.

I bought 19 pieces of cloth, each 14 ells 3 qrs. 1 $\frac{1}{2}$ nail; how much was in all?

	ells,	qrs.	nails.	
	14 :	3 :	1 $\frac{1}{2}$	
	14 :	3 :	1 $\frac{1}{2}$	
	14 :	3 :	1 $\frac{1}{2}$	
	14 :	3 :	1 $\frac{1}{2}$	
	14 :	3 :	1 $\frac{1}{2}$	
	14 :	3 :	1 $\frac{1}{2}$	
	89 :	0 :	1	In 6 pieces.
	89 :	0 :	1	
	89 :	0 :	1	
	14 :	3 :	1 $\frac{1}{2}$	
Ells	282 :	0 :	0 $\frac{1}{2}$	In 19 pieces.

Here for every 4 nails I carried 1 to the quarters, and for every 4 quarters I carried 1 to the ells, and added them as sums of one denomination.

I bought 5 hogsheads of sugar each 7 c.w. 3 qrs. 17 lb; how much was in all?

c.w.	qrs.	lb.
7 :	3 :	17
7 :	3 :	17
7 :	3 :	17
7 :	3 :	17
7 :	3 :	17
C.w. 39 :	2 :	1 in all.

I bought 17 hogsheads of tobacco for 43 l. 17 s. 9 d. per hhd. and sold the whole so as to gain 7 l. 14 s. 8 d. per hhd. tho' each hhd. cost 3 l. 11 s. 2 d. custom and charges; what did I sell the whole for.

l.	s.	d.	
43 :	17 :	9	Prime cost of 1 hhd.
7 :	14 :	8	Proposed gain on 1 hhd.
3 :	11 :	2	Custom and charges on 1 hhd.
55 :	3 :	7	Price at which 1 hhd. may be sold.
55 :	3 :	7	
55 :	3 :	7	
55 :	3 :	7	
220 :	14 :	4	At which 4 hhds. may be sold.
220 :	14 :	4	
220 :	14 :	4	
220 :	14 :	4	
55 :	3 :	7	
l. 938 :	0 :	11	At which 17 hhds. may be sold.

I bought 40 tun of wine for 589 l. 14 s. 7 d. loading and unloading stood me 9 l. 6 s. 1 d. 1 qr. the charge of a cellar 7 l. 12 s. 3 d. custom 13 l. 6 s. 3 d. how shall I sell the whole to gain 18 l. 1 s. 8 d.?

l.

l.	s.	d.	q.
589	14	7	
9	6	1	1
7	12	3	
13	6	3	
18	1	8	

l. 638 : 0 : 10 : 1 Thing required.

I laid out for a hat,	l. 0	16	1	$1\frac{1}{2}$
For gloves, ———	0	1	2	0
For 8 yards of cloth,	6	17	6	0
For 17 yards of linen,	1	6	5	$2\frac{1}{4}$
For 17 yards of cambrick,	11	17	9	0
For brandy,	9	0	0	0
Total,	l. 29	18	11	$3\frac{3}{4}$

The proof of a ſtack of oats is 2 bolls, 7 pecks, 3 lippies; what's the ſtock when it's equal to 24 times the proof?

b.	f.	p.	l.
2	1	3	3
2	1	3	3
2	1	3	3
2	1	3	3
2	1	3	3
2	1	3	3
14	3	2	2
14	3	2	2
14	3	2	2
14	3	2	2
Stock, 59	2	2	0

A merchant looking over his ſhop book finds that A, owes him l. 45 : 16 : 8, B, 127 : 15 : 7, C, l. 97 : 18 : 5, D, l. 26 : 13 : 8, E, l. 61 : 19 : 3,

 F,

F, l. 7 : 14 : 2; what did they owe among them?

	l.	s.	d.
A,	45	16	8
B,	127	15	7
C,	97	18	5
D,	26	13	8
E,	61	19	3
F,	7	14	2

Answer, l. 367 : 17 : 9

The interest of a certain sum for 1 year at 5 l. per cent. per annum is l. 75 : 14 : 7, what was the sum laid out, when 20 times the interest at that rate is equal to the said principal sum?

l.	s.	d.
75	14	7
75	14	7
75	14	7
75	14	7
75	14	7
378	12	11
378	12	11
378	12	11
378	12	11

l. 1514 : 11 : 8 Pricipal sum.

Bought 10 matts of lint each 7 stone 11 lb. 7 oz. how much was in all, reckoning 14 lb. to the stone and 16 oz. to the lb.

st.	lb.	oz.	
7	11	7	
7	11	7	
7	11	7	
7	11	7	
7	11	7	
39	01	3	In 5 matts
39	01	3	
78	2	6	In 10 matts.

A gentleman

A gentleman lets out to each of 8 tennents 17 acres 3 roods 14 falls 11 ells, how much was in all?

a	*r.*	*f.*	*e.*
17 :	3 :	14 :	11
17 :	3 :	14 :	11
17 :	3 :	14 :	11
17 :	3 :	14 :	11
71 :	1 :	17 :	8
71 :	1 :	17 :	8
142 :	2 :	34 :	16 In all.

I bought 7 pieces of silver, 3 of which weighed each 7 lb. 5 oz. 13 pennyweights, and each of the rest weighed 4 lb. 11 oz. 17 penny weights 14 grains; what did the whole weigh?

Here for every 24 grains carry 1 to the p. wts. for every 20 p. wts. carry 1 to the oz. and for every 12 oz. carry 1 to the lb.

lb.	*oz.*	*pw.*	*gr.*
4 :	11 :	17 :	14
4 :	11 :	17 :	14
4 :	11 :	17 :	14
4 :	11 :	17 :	14
7 :	5 :	13	
7 :	5 :	13	
7 :	5 :	13	
lb. 42 :	4 :	9 :	8 In all.

What will be the length of 4 trees when each measures 17 yards 2 feet 5 inches?

yds.	*f.*	*in.*
17 :	2 :	5
17 :	2 :	5
17 :	2 :	5
17 :	2 :	5
Yds. 71 :	0 :	8 Total length.

I bought

I bought 5 ankers of brandy, each containing 19 pints 1 chopin 5 gills, how much was in all?

Here for every 8 gills carry 1 to the chopins, and for every 2 chopins carry 1 to the pints.

p.	ch.	g.
19	1	5
19	1	5
19	1	5
19	1	5
19	1	5
Pints. 99	0	1 In all.

What period of time will 7 times 14 years 174 days 16 hours 18 minutes make at 365 days to the year?

Here, for every 60 of the minutes carry 1 to the hours, for every 24 of the hours carry 1 to the days, and for every 365 days carry 1 to the years.

y.	d.	h.	m.
14	174	16	18
14	174	16	18
14	174	16	18
14	174	16	18
14	174	16	18
14	174	16	18
14	174	16	18
Years. 101	127	18	6 Period required.

I bought

I bought 14 bundles of tea, each containing 5 lb. 7 oz. 9 drops; how much was in all?

Here, for every 16 of the drops carry 1 to the ounces, and for every 16 of the ounces carry 1 to the lb.

lb	*oz.*	*dps.*	
5 :	7 :	9	
5 :	7 :	9	
5 :	7 :	9	
5 :	7 :	9	
5 :	7 :	9	
5 :	7 :	9	
5 :	7 :	9	
38 :	4 :	15	In 7 bundles.
38 :	4 :	15	
lb. 76 :	9 :	14	In 14 bundles.

I bought 4 quantitys of bottles each 7 grofs 10 doz. and 9; how many were in all?

g.	*doz.*	*par.*	
7 :	10 :	9	
7 :	10 :	9	
7 :	10 :	9	
7 :	10 :	9	
31 :	7 :	0	In all.

If a fhip under fail runs 5 leagues 2 miles 972 yards in an hour, how far will fhe fail in a natural day.

Here, for every 1760 yards carry one to the miles, and for every 3 miles carry 1 to the legues.

I,

l.	m.	yds.	
5	2	972	
5	2	972	
5	2	972	
5	2	972	
5	2	972	
5	2	972	
35	0	552	Sailed in 6 hours.
35	0	552	
35	0	552	
35	0	552	
140	1	448	Sailed in 24 hours, or 1 day.

Proofs of the Work of ADDITION.

Addition can be prov'd 3 different ways.

I. The proof of this rule is usually by a second addition, without the top line; which second sum, if, when added to the uppermost line, it makes the first total, the work is supposed right.

II. Begin at the top and cast it downwards, in the same manner as you did upwards; and if that total agree exactly with the first total, the work is right; otherwise they must be cast upwards and downwards till they correspond.

III. Cast out the nines out of every item or line, place the remainders on the right hand of their respective lines; cast out also the nines out of the total sum and place the overplus on its right hand; if then this last remainder agrees with the overplus when the nines are cast out of the former remainders, the work is supposed right.

I do not much approve of this method, for if a figure in the total shall be transpos'd (as I have very often seen) the remainder over the nines will still be the same, tho' at the same time, by transposing that figure the work is grosly false.

I own

I own indeed, that when any arithmetical operation is performed with attention, that very attention is a presumption that the work is right, and that presumption is heightened to a very great degree of probability, if the proof by casting out the nines succeeds. And thus far only this method is to be depended on.

The following example is prov'd all the 3 ways.

4765	4765	4765	4
7943	7943	7943	5
8567	8567	8567	8
3942	3942	3942	0
25217	25217	25217	8
20452	25217		
25217			

Examples in numbers of divers denominations are prov'd after the same manner.

l. s. d.	l. s. d.	l. s. d.	
47 : 14 : 4	47 : 14 : 4	47 : 14 : 4	4
23 : 13 : 8	23 : 13 : 8	23 : 13 : 8	5
15 : 18 : 2	15 : 18 : 2	15 : 18 : 2	2
73 : 14 : 5	73 : 14 : 5	73 : 14 : 5	8
16 : 13 : 3	16 : 13 : 3	16 : 13 : 3	3
177 : 13 : 10	177 : 13 : 10	177 : 13 : 10	4
129 : 19 : 6	177 : 13 : 10		
177 : 13 : 10			

Here

Here I threw out the nines first out of 47 l. and there remained 2 or 40 s. this I annexed to the left hand of 14 s. and it made 4014, out of which I threw the nines also, and there remained 0, and as there were no nines in the 4 d. I set down the 4 on the right hand of the line as a remainder. Again, for the second line l. 23 : 13 : 8, as 2 and 3 only came to 5, or 100 s. this I annexed to the left hand of the 13 s. and it made 10013 which only came to 5 s. or 60 d. this I annexed to 8 d. and it stood thus 608, out of which I threw the nines and there remained 5, which I placed on the right hand of its line: and so did I manage all the other lines.

The

The Form of a RENTAL.

	Money.			Meal.			Oats.			Barley.			Peafe.			Wheat.			Yarn.		Poultry.	
Names.	L.	S.	D.	B.	F.	P.	B.	F.	P.	B.	F.	P.	B.	F.	P.	B.	F.	P.	Sp.	Ha.	G.	H.
A, pays	47	14	8	9	3	2	14	1	3	7	2	1	29	0	0	18	1	0	2	3	14	5
B,	73	18	0	19	0	1	27	3	0	18	1	3	45	3	2	37	2	1	9	2	36	36
C,	97	14	8	45	1	2	30	2	1	73	2	0	15	2	1	27	3	2	0	0	44	18
D,	67	18	0	27	2	0	14	1	2	48	3	1	00	0	0	18	2	3	12	2	17	6
E,	40	15	6	47	0	0	18	2	0	43	2	2	16	2	1	7	1	0	5	1	9	24
F,	54	00	0	48	2	2	00	0	0	67	0	0	9	2	0	24	2	2	8	2	12	16
G,	39	19	2	27	3	0	19	2	3	44	2	0	5	2	2	18	2	0	4	2	6	24
H,	51	10	4	14	2	2	21	3	0	38	2	2	8	2	0	27	2	2	6	3	8	18
Total,	473	10	4	239	3	1	147	0	1	[illegible]	0	1	130	2	2	180	1	2	49	3	146	147

CHAP. IV.

Of Subtraction.

SUBTRACTION is the taking of a lesser number or sum out of a greater, thereby to find the remainder or difference between the said two numbers; as if you take 6 from 9 the remainder or difference is 3, or this 3 is the excess of 9 above 6.

Subtraction is just the reverse of Addition, for Addition puts numbers together, and Subtraction takes numbers from each other.

The number or sum out of which Subtraction is made is called the Minuend.

The number subtracted is call'd the Subducend.

The Minuend must be greater than (or at least equal to) the Subducend, and is always placed uppermost.

Subtraction is of one denomination, or of diverse.

Subtraction of one denomination, is when the two numbers are both of one kind; that is, both pounds, ells, acres, miles, &c.

In subtraction of numbers of one denomination, the same rule that was given in Addition, of placing figures directly under those of the same value, viz, units under units, tens under tens, and hundreds under hundreds, &c. must be carefully observed; also under the lowest rank of the Subducend there must be drawn a line (as before in Addition) to separate the given numbers from their difference when found. The lesser number being then placed under the greater, and a line drawn under both, the operation may be thus performed.

RULE.

Begin at the right hand figure or place of units (as in Addition) and take or subtract the lower figure in

that

that place from the figure that ftands over it, fetting down the remainder or difference under it's own place: and if the two figures happen to be equal fet down a cypher; but if the upper figure be lefs than the lower, you muft in this cafe add 10 to the upper figure, and then take the lower figure out of the fum (for what you carried in Addition of numbers of one name, you muft borrow in fubtraction) and put the remainder in it's proper place; then for this 10 (which you muft call one that you borrowed) add 1 to the next lower figure, and take it out of that which ftands over it; fetting down the remainder under the line in it's own place; and fo proceed gradually on from one row of figures to another till all the differences be taken, as in the following examples.

Queft. 1. I borrowed l. 42063 whereof I have paid 23427 l. what remains unpaid.

42063 Minuend.
23427 Subducend.

Unpaid l. 18636 Remainder or difference.

Beginning at the place of units, I fay, 7 from 3 I cannot, (but adding 10 to the faid 3, it is made 13) 7 from 13, and there remains 6, which I fet down in its proper place under the line; then I fay, 1 that I borrowed, and 2 (the next figure) is 3, and 3 from 6, there remains 3 which I alfo place under the line in the place of tens; then 4 from 0 I cannot (for I borrowed nothing at the laft fubtraction) but 4 from 10, and there remains 6 which I place alfo under the line in its proper part; then 1 that I borrow'd and 3 (the next figure) is 4, and 4 from 2 I cannot, but 4 from 12 (viz. 10 and 2) and there remains 8 which I alfo fet down under the line in its proper place; then, laftly I fay 1 that I borrow'd and 2 (the laft figure) is 3, and 3 from 4, there remains 1 which I alfo fet down under the line. The work is thus finifhed, and there remains due or unpaid l. 18636.

Queft. 2.

Queſt. 2. The battle of Sherriff Muir was fought in the year 1715; how long is it ſince?

1757
1715

42 Years ſince.

Queſt. 3. A general brought to the field of battle 76854 men, and after the action he had but 47006 alive; how many were killed?

76854
47006

29848 Killed.

By this example you may perceive that cyphers in the ſubducend, or number to be ſubtracted, do not diminiſh the number from whence the ſubtraction is made.

	l.
Queſt. 4. I borrowed,	467
More,	329
More,	75
More,	97
More,	64
Sum,	1032
Paid,	784
l.	248 Remains unpaid.

Queſt. 5. I bought 5000 yards of cloth, and ſold 2745; what remains unſold?

5000
2745

2255 Remains unſold.

Queſt. 6.

Queſt. 6. What's the difference between a piece of timber containing 47 feet, and another piece of 28 feet?

47
28

Anſwer, 19 Feet.

Queſt. 7. If a perſon hath 124 miles to travel, and hath gone 85; how many miles hath he yet to go?

124
85

Anſwer, 39 Miles.

Queſt. 8. I am this preſent year 49 years of age; what year was I born in?

1757.
49

Anſwer in 1708

Queſt. 9. What number added to 497 will make 588?

588
497

91 Number required.

Queſt. 10. What ſeven numbers and all different will make 167?

Set down ſix different numbers at random, taking care that their ſum ſhall not amount to the number propoſed; add them together, ſubtract their total from the number propoſed, and the remainder will be the 7th number which with the other ſix will make 167, as follows.

167 Number proposed.

14
7
18
9
5
3

The six numbers set down at random.

Total, 56 Subtracted from 167.

111 The 7th number.

Quest. 11. How long is it since the Spanish invasion, it being in the year 1588?

1757
1588

169 Years.

Quest. 13. A general after a battle reviewing his army, finds that of 17640 men he brought into the field he had with him 5879 in good spirits, besides 1741 that were sick and wounded whom he had sent into garrisons; now supposing 3972 were slain and 2767 taken prisoners; I demand how many had deserted?

17640

5879
1741
3972
2767

14359 To be subtracted from 17640.

3281 Had deserted.

The reaſon of this rule is evident from the ſame principles as Addition, viz. of the whole being equal to all its parts taken together. That is the number from which the Subtraction is to be made, is underſtood to be the whole, and the number to be ſubtracted is ſuppoſed to be a part of that whole; conſequently if that part be taken from the whole, the remainder will be the other part; and if the ſum of theſe two parts be equal to the whole, it proves the work of Subtraction to be truely performed.

Subtraction of divers denominations is when the two ſums conſiſt of pounds, ſhillings, and pence; or yards, quarters, and nails, &c.

When the ſums to be ſubtracted, are of diverſe denominations, whether they be money, weight, or meaſure, the ſame method muſt be obſerved in ſetting them down, as in Addition; that is, the ſeveral names muſt be ſet juſt under one another; as pounds under pounds, ſhillings under ſhillings, pence under pence; yards under yards, feet under feet, and inches under inches, &c. with points of ſeparation between them; always obſerving, that the greateſt ſum muſt be uppermoſt, as before, in numbers of one denomination. And the work may be performed by obſerving this.

RULE.

Begin with the leaſt denomination towards the right hand, as in Addition; and take the figure (or figures) in that place of the Subducend, from the figure (or figures) that ſtand over them of the ſame denomination; and ſet down the remainder juſt under them below the line. But if that cannot be done, then (obſerving how many of the leſſer denomination make one of the next greater) add theſe to the upper figure (or figures) and from that ſum make the Subtraction; and ſo proceed to the next ſuperior denomination, remembering always to pay the one borrowed, by adding unity to the Subducend in that place, &c. as in whole numbers.

The

The following examples will make all plain.

	l.		*s.*		*d.*	
I borrowed,	46	:	14	:	7	
Whreof I paid	24	:	10	:	5	
	l. 22	:	4	:	2	Remains unpaid.

This first example is self evident, as there is no occasion for borrowing.

	l.		*s.*		*d.*
Lent;	569	:	10	:	6
Received thereof,	389	:	15	:	8
Ballance due,	179	:	14	:	10

Here, because I wanted pence in the minuend, I borrowed 12 pence, or one shilling; and in the shillings, 20 shillings or one pound; and in the pounds I borrowed 10, as in numbers of one denomination; and always paid what I borrowed to the next place, by calling the lower figure one more than it was.

For I began at the place of pence (being here the least denomination) and because I could not take 8 from 6, I borrowed one of the next denomination, viz. 1 s. or 12 d. and added it to the 6 d. which made it 18 d. then I took 8 d. from that 18 d. and there remained 10 d. to be set down under the place of pence; that done, I proceeded to the place of shillings, there I paid the 1 s. saying one that I borrowed and 15 makes 16, this 16 I could not get from 10, therefore I borrow'd one of the next denomination, viz. 1 l. or 20 s. which added to the 10 s. made it 30 s. then I took 16 from 30, and there remained 14 s. to be set down under the place of shillings; I proceeded then to the place of pounds, where paying the 1 I borrowed to 9, it thereby became 10, and as I could not have 10 from 9, I took it from 10 and 9 (viz. 19) and there remained 9 which I set down in it's it's proper place, and so on as in whole numbers until all was finished; and the remainder was 179 : 14 : 10.

This example being a little considered will render all others in this rule easy.

I borrowed from a friend 175 l. whereof I paid at one time l. 24 : 17 : 7, at another time l. 13 : 14, at another time l. 15 : 18 : 8, and at another time l. 27 : 11 : 2; what do I yet owe?

	l.	s.	d.
Sum bor.	175	00	0
	24	17	7
	13	14	0
	15	18	8
	27	11	2
Sum paid.	82	1	5
Ballance l.	92	18	7 yet due.

A, owes to B, four different sums, viz. l. 46 : 17 . 7, l. 39 : 18 : 8, l. 67 : 15 : 5, and l. 84 : 13 : 9; whereof he hath paid l. 55 : 11 : 2, l. 37 : 3 : 8 and l. 16 : 16 : 4; I demand the ballance of their accompts?

	l.	s.	d.	
	46	17	7	
	39	18	8	
	67	15	5	
	84	13	9	
Dr. A to B.	239	5	5	
	55	11	2	
	37	3	8	
	16	16	4	
Paid by A to B	109	11	2	To be subtracted from the former sum.
Ballance, l.	129	14	3	due by A to B,

I

From

From the two foregoing examples it is obvious that ſome queſtions in Subtraction cannot be performed without the help of Addition; for when a ſum is borrow'd, or a debt paid at ſeveral times, then you muſt find the total of the ſeveral ſums borrowed, and alſo, the ſum of the ſeveral payments; and ſubtracting the one from the other, the ballance will come out.

I borrowed l. 76 : 14 : 8, and l. 317 : 12 : 9, and l. 61 : 11 : 5; whereof I have paid l. 179 : 17 : 10 what remains unpaid?

	l.	s.	d.	
	76	14	8	
	317	12	9	
	61	11	5	
Sum bor.	455	18	10	
Sum paid, l.	179	17	10	
Ballance, l.	276	1	0	unpaid.

A collector of the exciſe has received 4053 l. 7 s. $4\frac{3}{4}$ d. and paid into the office by ſeveral remittance l. 2579 : 16 : 8; how much remains in his hands?

	4053	7	$4\frac{3}{4}$
	2579	16	8
l.	1473	10	$8\frac{3}{4}$ Anſ.

What 5 different ſums of pounds, ſhillings and pence, will amount to juſt 70 l.?

l.	s.	d.	
70	00	0	
13	17	7	4 ſums ſet down at random.
9	14	5	
5	18	9	
17	15	2	
47	5	11	To be ſubtracted from the given ſum.
Remains. 22	14	1	The 5th ſum.

How

How oft is l. 67 : 18 : 8 contained in 339 l. 13 s. 4 d.?

339 : 13 : 4			
67 : 18 : 8			
271 : 14 : 8	Once.		
67 : 18 : 8			
203 : 16 : 0	Twice.		
67 : 18 : 8			
135 : 17 : 4	Thrice.		
67 : 18 : 8			
67 : 18 : 8	Four times.		
67 : 18 : 8			

Answer. (0) Five times.

An iron-monger buys 83 tuns 11 c.w. 12 lb. of iron, and hath sold out of the said parcel 59 tuns 17 c.w. 2 qrs. 13 lb. how much remains unsold?

	T.	*c.w.*	*q.*	*lb.*	
	83	11	0	12	
	59	17	2	13	
Answer.	23	13	1	27	Remains unsold.

I have on board of a ship from Jamaica 103 tuns 13 c.w. of logwood, and have received by 3 lighters as follows, viz.

	T.	*c.w.*	*q.*	*lb.*
On board,	103	13	0	00
By one lighter,	15	10	3	17
Another,	24	17	1	14
Another,	39	19	2	18
Total received,	80	7	3	21
Remains on board,	23	5	0	7

A gentleman

A gentleman granted a tack to a farmer at Whitsunday 1744 for the space of 15 years, for the yearly rent of l. 273 : 14 : 8; how many years are past, and to come, and what money is paid, and to pay?

1757	15
1744	13
13 Years past.	2 Years to come.

l.	s.	d.	
273	14	8	
273	14	8	
273	14	8	
273	14	8	
273	14	8	
273	14	8	
1642	8	0	Six years rent.
1642	8	0	Other six which makes 12.
273	14	8	One years rent making 13.
3558	10	8	Paid.

l.	s.	d.	
273	14	8	
273	14	8	
547	9	4	Yet to pay.

A farmer liv'd 6 years on a possession whose yearly rent was l. 375 : 9 : 8, but at the end of that time, it was found that he had paid only 1271 l. 17 s. 9 d. how much was he in arrears?

	l.	s.	d.	
	375	9	8	
	375	9	8	
	375	9	8	
	375	9	8	
	375	9	8	
	375	9	8	
	2252	18	0	Rent 6 years.
Paid,	1271	17	9	
l.	981	0	3	Arrears.

What

What number of pounds, shillings and pence added to 200 merks will make 273 l. 14 s. 8 d.?

l.	*s.*	*d.*	
273	14	8	
66	13	4	Equal to 100 merks.
66	13	4	
133	6	8	To be subtracted from 1st sum.
140	8	0	Number required.

I bought a horse for l. 72 : 17 : 9, and sold him for l. 87 : 16 : 8; what did I gain?

l.	*s.*	*d.*	
87	16	8	
72	17	9	
l. 14	18	11	Gain.

I bought a horse, and sold him for l. 87 : 16 : 8, and so gained l. 14 : 18 : 11; what was the prime cost?

l.	*s.*	*d.*	
87	16	8	
14	18	11	
l. 72	17	9	Prime cost.

What's the difference betwixt 45 l. and 3 farthings?

l.	*s.*	*d.*	*q.*	
45	00	0	0	
			3	
l. 44	19	11	1	Difference.

I bought 5 bolls of oats for l. 27 : 14 : 8, out of which I had $4\frac{1}{2}$ bolls of meal, which I sold at 30 l. 11 s. 6 d. and spent 1 l. 14 s. in buying and selling; what did I gain by the bargain?

	l.	*s.*	*d.*
	30 :	11 :	6 Price of the meal.
	27 :	14 :	8 Price of the oats.
	1 :	14 :	0 Expences.
Sum	29 :	8 :	8 To be subtracted.
	l. 1 :	2 :	10 Gain.

A merchant looking over his shop book, finds he has goods upon credit to the value of l. 288 : 4 : 6, and that A, owes him l. 4 : 15, B, l. 31 : 10 : 6 : 2, C, l. 72 : 16 : 8, D, 152 : 5, E, l. 31 : 13 : 4, F, l. 60 : 16 : 6 : 3, and G, l. 197 : 14; now supposing each of the above persons shall clear him, and that he has l. 188 : 14 : 6 of cash lying by him; what will he have over paying the above sum?

Cash, l. 188 :	14 :	6		
A, 4 :	15 :	0		
B, 31 :	10 :	6 :	2	
C, 72 :	16 :	8		
D, 152 :	5			
E, 81 :	13 :	4		
F, 60 :	16 :	6 :	3	
G, 197 :	14			
740 :	5 :	7 :	1	
288 :	4 :	6		
l. 452 :	1 :	1 :	1 Over.	

A

A gentleman hath l. 4751 : 11 : 2 yearly income, and supposing he lays up l. 987 : 14 : 8 at the year's end; how much doth he expend?

l.	*s.*	*d.*
4751	11	2
987	14	8

l. 3763 : 16 : 6 Expended.

A gentleman hath an estate worth l. 16745 : 3 : 4 per annum, out of which there goes yearly 180 l. 17 s. 8 d. to pay publick burdens, besides 4765 l. 14 s. 4 d. to a jointur'd lady; what will he have over to maintain his pocket and family?

	l.	*s.*	*d.*
	16745	3	4
	180	17	8
	4765	14	4
Subtract,	4946	12	0
	11798	11	4 Over.

I bought 8 hogsheads of raisins, each weighing gross 5 c.w. 1 qr. 11 lb.; and being allowed a deduction of 3 qr. 21 lb. upon each hogshead; what will be the neat weight of the raisins?

c.w	*q.*	*lb.*	
5	1	11	
	3	21	
4	1	18	Neat weight of 1 hhd.
4	1	18	
4	1	18	
4	1	18	
17	2	16	Neat weight of 4 hhds.
17	2	16	
C.w. 35	1	4	Neat weight of 8 hhds.

I bought 7 pieces of cloth, each 15 ells 3 qrs. 3 nails, whereof I have sold 68 ells 3 qrs. 3 nails; what remains unsold?

	e.	q.	n.
	15	3	3
	15	3	3
	15	3	3
	15	3	3
	15	3	3
	15	3	3
	15	3	3
Bought,	111	2	1
Sold.	68	3	3
	42	2	2 Remains unsold.

I bought 123 bolls 1 firlot 1 peck of meal, and sold thereof 87 bolls 2 firlots 1 peck and 3 lippies; what remains unsold?

	b.	f.	p.	l.
	123	1	1	0
	87	2	1	3
Remains	35	2	3	1 Unsold.

I went

I went to a fair, and bought a horse for 7 l. 12 s. 3 d. and sold him for l. 8 : 4 : 9 : 2, and spent 7 d. I bought also 3 cows for l. 5 : 17 : 10 : 2 and sold them for l. 8 : 10 : 6 and spent 10 d. I bought also 46 sheep for l. 7 : 16 : 2 : 3, and sold them for l. 9 : 18 : 5 and spent 11 d.; what did my gain amount to that day?

l.	s.	d.	q.	
8	4	9	2	
8	10	6		
9	18	5		
26	13	8	2	Price at which I sold the whole.
7	12	3		
5	17	10	2	
7	16	2	3	
		7		Expences.
		10		
		11		
21	8	8	1	Prime cost and expences.
l. 5	5	0	1	Gain.

I am a wright to trade worth l. 197 : 14 : 3 of stock, and being to set up in a burgh royal, I find I must lay out l. 137 : 14 : 4 for working tools; how much must I borrow, if I pay l. 108 : 16 : 3 for my freedom?

	l.	s.	d.	
	137	14	4	
	108	16	3	
Sum	246	10	7	for freedom and tools.
	197	14	3	His stock.
	l. 48	16	4	To be borrowed.

I bought 27 lb. 4 oz. 3 drops of saffron, whereof I have sold 17 lb. 6 oz. 9 drops; what remains unsold?

lb.	*oz.*	*dr.*
27 :	4 :	3
17 :	6 :	9
lb. 9 :	13 :	10 Unsold.

Here in the place of drops I borrowed 16, because 16 drops make one ounce, and in the place of oz. 16 also, because 16 oz. make 1 lb.; and the same reason holds in all questions of divers denominations.

I bought 4 casks of brandy, each containing 18 pints 1 chopin 7 gills; whereof I have sold 48 pints 1 chopin 6 gills; what remains unsold.

	p.	*c.*	*g.*
	18 :	1 :	7
	18 :	1 :	7
	18 :	1 :	7
	18 :	1 :	7
Bought,	75 :	1 :	4
Sold,	48 :	1 :	6
	26 :	1 :	6 Unsold.

I should here have given examples of Apothecaries weight, time, long and superficial measures; but if one attentively considers the nature of the foregoing rules laid down for subtracting numbers of divers denominations, he may as readily and easily perform examples of that nature as those before.

I come now to give rules for proving the work of Subtraction.

I. Add the Remainder to the Subducend, and if the

the ſum is equal to the Minuend, the work is certainly right.

II. Subtract the Remainder from the Minuend, and if the difference be equal to the Subducend, the work is right.

III. Caſt out the nines out of the Subducend, and alſo out of the Remainder, and if the overplus of the nines, when caſt out of thoſe two remainders, be equal to what's over the nines when caſt out of the Minuend, the work is ſuppoſed to be right.

Let the work of the following example of integers be prov'd all the three ways.

	l.	*l.*	*l.*	
Borrowed,	7698	7698	7698	3
Paid,	5979	5979	5979	—
				3
Remainder.	1719	1719	1719	0
	7698	5979		3

Here you ſee the overplus of the nines in the Subducend and Remainder comes only to 3, which is equal to what's over the nines in the Minuend, and ſo the work is right.

Let the work of the following example be alſo proved all the three ways.

l. *s.* *d.*	*l.* *s.* *d.*	*l.* *s.* *d.*	
76 : 13 : 7	76 : 13 : 7	76 : 13 : 7	7
27 : 14 : 8	27 : 14 : 8	27 : 14 : 8	—
			5
48 : 18 : 11	48 : 18 : 11	48 : 18 : 11	2
76 : 13 : 7	27 : 14 : 8		7

CHAP.

CHAP. V.

Of Multiplication.

MULTIPLICATION is a Rule by which any given number may be ſpeedily increaſed according to any propoſed number of times, and conſequently ſerves inſtead of many additions; for the product of a Multiplication is only the repetition of the Multiplicand, ſo many times as there are units in the Multiplier.

This rule alſo ſerves to bring great denominations into leſs; as guineas into ſhillings, hours into minutes, pounds into pence, &c.

Multiplication hath three parts to be taken notice of,

I. The Multiplicand, or number to be multiplied; which is generally the greater of the two given numbers.

II. The Multiplier, or number by which you multiply, which is generally the leſſer of the two given numbers.

III. The Product, or number ariſing from the multiplication of the foreſaid two numbers into one another, which is the anſwer; and the firſt example will ſhew to which line each term belongs.

Multiplication is either ſingle or compound.

Single Multiplication is when the Multiplicand or Multiplier conſiſt each but of one figure, and all the examples that can happen in this caſe are performed by the following table.

Compound Multiplication is when the Multiplicand or multiplier, or both conſiſt of more places than one; as if you were to multiply 587 by 76, or 9784 by 273.

The ready performance of this and the following rules depends upon the perfect knowledge of the above mentioned table, which (being ſo eaſy) will need no explanation

The

The TABLE.

2 Times	2	is	4
	3		6
	4		8
	5		10
	6		12
	7		14
	8		16
	9		18
3 Times	3	is	9
	4		12
	5		15
	6		18
	7		21
	8		24
	9		27
4 Times	4	is	16
	5		20
	6		24
	7		28
	8		32
	9		36
5 Times	5	is	25
	6		30
	7		35
	8		40
	9		45
6 Times	6	is	36
	7		42
	8		48
	9		54
7 Times	7	is	49
	8		56
	9		63
8 Times	8	is	64
	9		72
9 Times	9	is	81
10 Times	10	is	100
12 Times	12	is	144

To multiply numbers of one Denomination.

When any number is given to be multiplied by another, ſet the biggeſt uppermoſt, which is the Multiplicand, and under that your Multiplier, in the ſame order as in Addition and Subtraction, viz. units under units, tens under tens, &c. and draw a line below.

Caſe.

Case I. When the Multiplicand consists of several places, and the Multiplier but of one.

RULE.

By that one figure first multiply the units place of the Multiplicand; if that product be less than ten, set it down in its own place of units, and proceed to multiply the next figure or place of tens in the Multiplicand; but if the product be above ten or tens, set down only the overplus (as in Addition) and to the product of the next figure of the Multiplicand add 1 for each ten you had in the last product, and setting down in its proper place what's over the ten or tens in that sum, as before, proceed in the same manner till all the figures of the Multiplicand are multiplied into the Multiplier.

Example.

What will 7 times 7674 l. amount to?

7674 Multiplicand.
7 Multiplier.

l. 53718 Product.

Now the reason of this and all other the like operations, may be easily conceived from what follows.

7674 Multiplicand.
7 Multiplier.

28
490
4200
49000

53718 Product, as before.

Here 7 times 4 is 28, because 4 stands in the units place. Next, it is not 7 times 7, but 7 times 70, because

becauſe 7 ſtands in the place of tens in the Multiplicand. Nor is it here 7 times 6, but 7 times 600, becauſe 6 ſtands in the place of hundreds. And laſtly, it is not 7 times 7, but 7 times 7000, becauſe the 7 in the Multiplicand ſtands in the place of thouſands.

Suppoſe 9 perſons to be placed in a row; I demand how many different ways they may be ſet with reſpect to the change of their places?

```
     1
     2
   ---
     2
     3
   ---
     6
     4
   ---
    24
     5
   ---
   120
     6
   ---
   720
     7
  ----
  5040
     8
 -----
 40320
     9
------
362880 Different ways.
```

To illustrate the foregoing example, suppose only 4 persons, as A, B, C and D, to be so placed, and if it be required how many different ways they can sit; it will appear from the following scheme to be 24.

A	B	C	D
A	B	D	C
A	C	B	D
A	C	D	B
A	D	C	B
A	D	B	C
B	A	C	D
B	A	D	C
B	D	A	C
B	D	C	A
B	C	A	D
B	C	D	A
C	A	B	D
C	A	D	B
C	B	A	D
C	B	D	A
C	D	A	B
C	D	B	A
D	A	B	C
D	A	C	B
D	B	A	C
D	B	C	A
D	C	A	B
D	C	B	A

Case II. When both Multiplicand and Multiplier consist of several places.

RULE.

In this case remember that so many figures as there are in the Multiplier, so many particular products there must be; for all the figures of the Multiplicand must be multiplied with every single figure of the Multiplier; and the sum of all these particular products will be the true product required.

Begin

Begin therefore to multiply all the figures in the Multiplicand by the unit's place of your Multiplier for the firſt product: then by the figure in the place of tens for the ſecond product, and ſo on by all the other places of the Multiplier, ſetting down the ſeveral products ſo under one another as the firſt figure (or cypher) of each may ſtand directly under its multiplying figure.

Or, (if all the figures in the Multiplier be ſignificant) place the firſt figure (or cypher) of the ſecond product directly under the ſecond figure of the firſt product; and the firſt figure of the third product, under the ſecond figure of the ſecond product: and ſo on, as in the next example.

And the reaſon of placing the firſt figure of every particular product in ſuch order, will appear from the ſecond way of working the firſt example; wherein the cyphers are only ſet down, to ſhew the true diſtance of the firſt figure of each particular product, from the units place. And tho' it is not uſual to ſet down cyphers in this manner, yet they are always ſuppos'd to be there, and their places left void.

What's the product when 78094 is multiply'd by 7563?

```
    78094
     7563
   ------
   234282  The firſt particular product by 3.
  468564.  The ſecond product by 60.
 390470..  The third product by 500.
546658...  The fourth product by 7000.
---------
590624922  The total, or true product.
```

Let 76456 be multiply'd by 7005.

```
     76456
      7005
    ------
    382280 The product by 5.
 535192... The product by 7000.
 ---------
 535574280 True product.
```

In the foregoing example, and all others where there are cyphers in the multiplier, you must pass over these, and only work with the significant figures; but take care to place the first figure of each product directly under the figure you multiply by, as already directed.

What's the product when 7645274 is multiply'd by 402003?

```
       7645274
        402003
      --------
      22935822 Product by 3.
   15290548... Product by 2000
 30581096..... Product by 400000.
 -------------
 3073423083822 Answer.
```

What

What number divided by 48, 24, 32, and 63, will leave no remainder?

```
     48
     24
    ---
    192
    96
   ----
   1152
     32
   ----
   2304
  3456
  -----
  36864
     63
 ------
 110592
221184
-------
2322432 Number required.
```

That Multiplication serves instead of many additions, and that consequently the truth of its operation depends upon the same reason, is easily prov'd: for suppose it were required to know the sum of five times eight; by Addition, the work would stand thus.

```
      8
      8
      8
      8
      8
      -
Sum, 40
```

Too tedious a method for practice; being discoverable at once, by the table of Multiplication, to be 40.

ABBREVIATIONS.

ABBREVIATIONS.

To multiply any number betwixt 10 and 20, so as to bring the whole work into one line.

RULE.

Multiply each figure in the Multiplicand by the units figure in the Multiplier, adding to each the figure that stands on the right hand of it, and the work is done.

Let 746 be multiply'd by 17.

746 17 —— 12682 Product.	Here I say 7 times 6 is 42, that is 2 and carry 4; then 7 times 4 is 28 and 4 that I carried is 32 and the back figure 6 makes 38, that is 8 and carry 3; lastly 7 times 7 is 49 and

3 I carried is 52 and the back figure 4 makes 56, that is 6 and carry 5; which 5 (as I have no other figure to multiply) I add to 7 the last figure, and that compleats the product 12682.

To multiply by 1314, or 1619, or by 1517, &c.

RULE.

Multiply first by 14 in one line, and by 13 in the other, as before; and the sum of the two lines is the Product.

Multiply	57846
by	1314
	809844
	751998
	76009644 Product.

Any

Any number may be multiply'd by 10, 100, 1000, &c. only by placing on its right hand one, two, three, or more cyphers: Thus 87 multiply'd by 10 is 870, by 100 is 8700, by 1000 is 87000, &c.

To multiply any number by another consisting only of nines.

RULE.

To the right hand of the Multiplicand annex as many cyphers as your Multiplier has nines, from that sum subtract the given Multiplicand, and the remainder is the Product.

Example wrought both ways.

Let 743 be multiply'd by 999.

743000	743
743	999
742257 Product.	6687
	6687
	6687
	742257 Product.

The reason is plain: for 743000 is equal to 743 multiply'd by 1000, which contains 743 just once more than 743 multiply'd by 999: if therefore from 743000 you deduce 743, you have 999 times 743.

When the Multiplier is such a number, as may be produced by any two numbers in the multiplication table, when multiplied together; multiply the Multiplicand by either of the two numbers, and that product by the other, the last product shall be the answer.

There are, sometimes figures saved by this method, and there is no addition of products.

Let

Let 427 be multiply'd by 56, or by 7 and 8; for 7 times 8 makes 56.

427	427
7	56
2989	2562
8	2135
23912 Product.	23912 Product.

When there is a cypher or cyphers to the right hand either of the Multiplicand or Multiplier, or both; in that case, place the factors as in the following examples, then multiply the significant figures into one another as before; ommitting the cyphers till the particular products are added, and to their sum, or general product on the right hand, annex so many cyphers as are in either, or both the factors.

579	4762000	84300
4200	114	630
1158	66668	2529
2316	4762	5058
2431800	542868000	53109000

I come now to shew the excellent use of Multiplication, in answering all manner of questions that ordinarily occur in bussiness; where we have the price of one thing, and want to know the value of many things at that rate; and that by such easy rules and directions, that any one who understands Addition shall readily perform any question this way: for nothing more is required here than to carry from one denomination to the next, exactly as we do in that rule.

Case I. When the Multiplier consists but of one figure.

RULE.

RULE.

Begin firſt to multiply the leaſt denomination, and ſo proceed from one denomination to another, till you come to the greateſt; ſtill minding (as in Addition) to carry 1 from each denomination to another, for as many digits as make an unit of the next ſuperior order, and to place the overplus of each denomination, orderly under the line.

Examples.

What coſt 8 bolls of meal at l. 5 : 17 : 8 the boll?

5 : 17 : 8
8

Anſwer, 47 : 1 : 4

In the foregoing example, I ſaid 8 times 8 is 64 d. that was 4 odd d. which I ſet down, and carried 5 s. for the 60 d. to the next product; ſaying 8 times 7 is 56 and 5 I carried is 61, I ſet down the 1 and carried 6, ſaying 8 times 1 is 8 and 6 I carried is 14 tens, or, 7 l. then 8 times 5 was 40, and 7 made 47 l. 1 s. 4 d. the price required. And ſo of any other of this nature.

What coſt 7 lb. of tea at 11 s. 7 d. per lb.?

11 : 7
7

l. 4 : 1 : 1 Anſwer.

What coſt 9 ells of cloth at 17 s. 9 d. per ell.

17 : 9
9

l. 7 : 19 : 9 Price.

In

In the foregoing example, I ſaid 9 times 9 was 81 d. that is 6 s. 9 d. I ſet down the 9 d. and carrying the 6 s. I ſaid 9 times 7 was 63, and 6 I carried was 69; I ſet down the 9, and carrying the 6, I ſaid 9 times 1 was 9 and 6 I carried was 15 tens which made l. 7 : 19 : 9 price.

In eight pieces of cloth, each 15 ells 3 quarters 2 nails; what ells?

e. q. n.	
15 : 3 : 2	
8	
127 : 0 : 0	Anſwer.

Caſe II. When the Multiplier is ſuch a number, as any two figures in the multiplication table, multiplied together can produce: then multiply the price or quantity given by one of the numbers, and that product by the other number, and the laſt product will be the anſwer.

What coſt 24 ells of cloth at 17 s. 6 d. per ell?

s. d.	
17 : 6	
6	
5 : 5 : 0	Price of 6 ells.
4	
l. 21 : 0 : 0	Price of 24 ells.

Or, thus.

s. d.	
17 : 6	
3	
2 : 12 : 6	Price of 3 ells.
8	
21 : 0 : 0	Price of 24 ells, as before.

Here

Here, you find that the question can be performed two ways, viz. by 6 and by 4, or by 3 and by 8; for either of them multiply'd together make 24.

What cost 72 bolls of meal at l. 7 : 14 : 8 per boll?

```
        7 : 14 : 8
                 8
      ------------
       61 : 17 : 4 Price of 8 bolls.
                 9
    --------------
 l. 556 : 16 : 0 Price of 72 bolls.
```

What cost 96 hundred weight of sugar at l. 5 : 18 : 8 per c.w.

```
        5 : 18 : 8
                12
    --------------
       71 :  4 : 0 Price of 12 c.w.
                 8
    --------------
 l. 569 : 12 : 0 Price of 96 c.w.
```

Case III. When the Multiplier is such a number as no two figures in the table can be found to answer, then multiply by two such numbers as come nearest to it, and for the overplus multiply the price or quantity given thereby, and adding that product to the other, the total will be the answer.

What cost 34 bolls of malt at l. 9 : 18 : 8 per boll?

```
          9 : 18 : 8
                   8
        ------------
         79 :  9 : 4 Price of 8 bolls.
                   4
       -------------
        317 : 17 : 4 Price of 32 bolls.
         19 : 17 : 4 Price of 2 bolls.
       -------------
  l. 337 : 14 : 8 Price of 34 bolls.
```

In the foregoing example I find the two numbers that multiply'd together come nearest to 34, are 8 and 4, by these I multiply'd as before, and the last product is l. 317 : 17 : 4 the price of 32 bolls; but as the quantity is 34 bolls, I yet want the price of 2 bolls; wherefore I multiply the price of one boll by 2 and it produces l. 19 : 17 : 4 which added to the price of 32 bolls, viz. to l. 317 : 17 : 4 makes l. 337 : 14 : 8 for the anfwer.

What coft 76 c. w. of fhip bifcuit at 13 s. 6d. per c.w.?

13	: 6		
	12		
8 :	2 :	0	Price of 12 c.w.
		6	
48 :	12 :	0	Price of 72 c.w.
2 :	14 :	0	Price of 4 c.w.
l. 51 :	6 :	0	Anfwer.

What coft 55 bolls of barley at l. 7 : 14 : 7 per boll?

7 :	14 :	7	
		9	
69 :	11 :	3	Price of 9 bolls.
		6	
417 :	7 :	6	Price of 54 bolls.
7 :	14 :	7	Price of 1 boll.
425 :	2 :	1	Price of 55 bolls.

In the foregoing example, if I had multiply'd by 7 times 8, which is 56, and fubtracted the price of one boll from the product, the remainder would have been the anfwer, as before.

Thus

Thus by the foregoing examples, it is manifest, that when the price of one thing is given, the price of many (at the same rate) may be found sooner and much easier by Multiplication than by the Rule of Three.

Case IV. When large numbers are to be multiply'd this way, that is, when your Multiplier consists of 1, 2, 3, 4, or more hundreds, always multiply first the price given by 10, and then that product by 10 also, which produces the value of one hundred; then multiply that product by the number of hundreds, whether 2, 3, 4, or 5, &c. and that product is the value of so many hundreds; then for the tens, whether 20, 30, 40, &c. multiply that product which gives the price of 10, either by 2, 3, 4, or 5, according to the number of tens, which place under the last product, without drawing a line; and for the units always multiply the price given by them, whether 2, 3, 4, &c. and set that also under the former products, so that you will have three lines to add together, and the total of them will be the answer.

The following examples will make all the 4 cases easy to be understood.

What cost 743 lb. of tea at 7 s. 8 d. per lb.

	7 :	8	
	10		
3 :	16 :	8	Price of 10 lb.
	10		
38 :	6 :	8	Price of 100 lb.
	7		
268 :	6 :	8	Price of 700 lb.
15 :	6 :	8	Price of 40 lb.
1 :	3 :	0	Price of 3 lb.
284 :	16 :	4	Price of 743 lb.

First

First I multiply 7s. 8d. by 10, and that produces l. 3 : 16 : 8 for the price of 10 lb.; then I multiply l. 3 : 16 : 8 price of 10 lb. by 10, and that produces l. 38 : 6 : 8 for the value of 100 lb. which l. 38 : 6 : 8 I multiply by 7, the number of hundreds, and that product l. 268 : 6 : 8 is the value of 700 lb. Then for the 4 tens in 43, I multiply l. 3 : 16 : 8, the price of 10 lb. by 4, and that product is l. 15 : 6 : 8 the value of 40 lb. And laſtly for the 3 units, I multitiply the the price of one lb. 7s. 8d. thereby, and the product gives the value of 3 lb. And theſe 3 lines being added together give l. 284 : 16 : 4 the price of 743 lb.

And thus may any example of this kind be done, let it be as large as it will, only when the Multiplier conſiſts of thouſands, you have four lines to add together, and when of hundreds, but three.

What coſt 1745 ſtone of hay at 4¾ d. per ſtone?

		d.	*q.*	
		4 :	3	
			10	
	3 :	11 :	2	Price of 10 ſtone.
			10	
1 :	19 :	7 :	0	Price of 100 ſtone.
			10	
19 :	15 :	10 :	0	Price of 1000 ſtone.
13 :	17 :	1 :	0	Price of 700 ſtone.
	15 :	10 :	0	Price of 40 ſtone.
	1 :	11 :	3	Price of 5 ſtone.
34 :	10 :	8 :	3	Price of 1745 ſtone.

Note, The price of 700 ſtone was got by multiplying the price of 100 ſtone by 7; and the price of 40 ſtone was found by multiplying the price of 10 ſtone by by 4; and the price of 5 ſtone was got by multiplying the price of 1 ſtone by 5. And thus you may perform all examples of this kind.

What's the rent of 9 acres of land at l. 10 : 17 : 5 Scots per acre?

l.	s.	d.	
10 :	17 :	5	
		9	
l. 97 :	16 :	9	Anſwer.

What's the rent of 35 acres at 19 s. 10 d. per acre?

	s.	d.	
	19 :	10	
		7	
6 :	18 :	10	Rent of 7 acres.
		5	
l. 34 :	14 :	2	Rent of 35 acres.

What coſt 47 lb. of Bohea tea at 14 s. 5 d. per lb.?

	s.	d.	
	14 :	5	
		6	
4 :	6 :	6	Price of 6 lb.
		8	
34 :	12 :	0	Price of 48 lb.
	14 :	5	Price of 1 lb. ſubtracted.
l. 33 :	17 :	7	Price of 47 lb.

What

What cost 105 yards of fine cambrick at 11 s. 7d. per yard?

s. d.
11 : 7
10

5 : 15 : 10 Price of 10 yards.
10

57 : 18 : 4 Price of 100 yards.
2 : 17 : 11 Price of 5 yards.

60 : 16 : 3 Price of 105 yards.

What principal sum will in 1 year give 767 l. 14 s. 7 d. interest at 5 l. per cent?

Note, 20 times one year's interest of any sum at 5 l. per cent. is equal to the principal sum.

767 : 14 : 7
5

3838 : 12 : 11
4

l. 15354 : 11 : 8 Principal sum.

The proof of a stack of oats is 1 boll 3 firlots 3 pecks 1 lippie, what's the stock, when 24 times the proof is equal thereto?

b. f. p. l.
1 : 3 : 3 : 1
4

7 : 3 : 1 : 0
6

Bolls, 46 : 3 : 2 : 0 Stock.

What's the simple interest of l. 15354 : 11 : 8 for 7 years at 5 l. per cent. per annum.

Multiply the principal sum by the rate of interest, cut off two figures to the right hand of the pounds, and these on the left are the pounds of one year's interest; then multiply the pounds cut off, by 20, taking in, at the same time, the shillings (if any) and cutting off, as before, 2 figures from the right hand of these shillings, the shillings on the left are a part more of one year's interest; lastly multiply the shillings cut off, by 12, taking in the pence (if any) cut off two figures from the right hand (as before) and these on the left are so many pence of one year's interest, and the figures cut off if significant, are so many parts of a penny.

See the Work.

	15354	: 11	: 8	
			5	
l.	767,72	: 18	: 4	
	20			
s.	14,58			
	12			
d.	7,00			

l.	*s.*	*d.*	
767 :	14 :	7	Interest for 1 year.
		7	
l. 5374 :	2 :	1	Interest for 7 years.

Interest at 3, 4, 6, 7, 8, per cent. &c. may be found after the same manner.

A gentleman is to buy an estate of l. 8767 : 18 : 8 per annum at 25 years purchase, what sum will serve for that end?

l.	s.	d.
8767	18	8
		5
43839	13	4
		5
l. 219198	6	8 Answer.

A gentleman granted a tack to a farmer for 19 years commenceing at Whitsunday 1748, for the yearly rent of l. 567 : 14 : 5; what years are past and to come, and what money is paid and to pay, at Whitsunday 1757, supposing all bygone rents paid?

1757   19
1748   9

9 Years past.   10 Years to come.

l.	s.	d.		l.	s.	d.	
567	14	5		567	14	5	
		9				10	
l. 5109	09	9	Paid.	l. 5677	4	2	To pay.

349 Soldiers are to have new cloths, and each coat to contain 3 ells 3 quarters 1 nail; how much cloth will serve?

e.	q.	n.	
3 :	3 :	1	
		10	
38 :	0 :	2	Serves 10 soldiers.
		10	
381 :	1 :	0	Serves 100.
		3	
1143 :	3 :	0	Serves 300 soldiers.
152 :	2 :	0	Serves 40.
34 :	1 :	1	Serves 9.
Ells. 1330 :	2 :	1	Serves 349 soldiers.

A man liv'd 19 years on a farm whose yearly rent was l. 173 : 14 : 9, at the end of which time, it was found that all his partial payments amounted only to l. 1997 : 18 : 8; I demand the sum of arrears?

173 :	14 :	9	
		3	
521 :	4 :	3	Rent due for 3 years.
		6	
3127 :	5 :	6	Rent for 18 Years.
Add. 173 :	14 :	9	Rent for 1 year.
3301 :	0 :	3	Rent due for 19 years.
1997 :	18 :	8	Partial payments.
l. 1303 :	1 :	7	Sum of arrears.

I bought 16 parcels of tea, each 7 lb. 7 oz. 5 drops; how much was in all?

lb.	oz.	dr.	
7 :	7 :	5	
		4	
29 :	13 :	4	In 4 parcels.
		4	
lb. 119 :	5 :	0	In 16 parcels.

7145 Sailors are to be discharged, and each to have 18 months pay; what sum will clear them off, when each hath 27 s. 8 d. a month?

l.	s.	d.	
1 :	7 :	8	
		6	
8 :	6 :	0	Pays one sailor for 6 months.
		3	
24 :	18 :	0	Pays 1 sailor for 18 months.
		10	
249 :	0 :	0	Clears off 10 sailors.
		10	
2490 :	0 :	0	Pays off 100.
		10	
24900 :	0 :	0	Pays off 1000
		7	
174300 :	0 :	0	Pays off 7000 sailors.
2490 :	0 :	0	Pays 100.
996 :	0 :	0	Pays 40.
124 :	10 :	0	Pays 5.
l. 177910 :	10 :	0	Pays off 7145 sailors.

What

What period of time will 14 times 53 years, 192 days 17 hours 11 minutes make, at 365 days to the year?

y.	d.	h.	m.
53 :	192 :	17 :	11
			2
107 :	20 :	10 :	22
			7
Years. 749 :	143 :	0 :	34 Period required.

I bought 9 hogsheads of sugar, each weighing gross 7 c.w. 1 qr. 11 lb. and being allowed a deducti-on of 1 c.w. 2 qr. 13 lb. upon each hogshead; what was the neat weight of the sugar?

	c.w.	q.	lb.	
	7 :	1 :	11	
Sub,	1 :	2 :	13	For allowance on 1 hhd,
	5 :	2 :	26	Neat weight of 1 hhd.
			9	
C.w.	51 :	2 :	10	Neat weight of 9 hhds.

A

A gentleman resting l. 759, and not being able to pay the whole, compounds with his creditors at 14 s. 7 d. per pound; what sum, at that rate, will clear the debt?

	s.	d.	
	14 :	7	
		10	
7 :	5 :	10	Pays 10 l.
		10	
72 :	18 :	4	Pays 100 l.
		7	
510 :	8 :	4	Pays 700 l.
36 :	9 :	2	Pays 50 l.
6 :	11 :	3	Pays 9 l.
l. 553 :	8 :	9	Pays 759 l. the total debt.

What Ster. money may I give in exchange for 27 French crowns at 4 s. 6 d. per piece?

	s.	d.	
	4 :	6	
		9	
2 :	0 :	6	For 9 crowns.
		3	
l. 6 :	1 :	6	For 27 crowns.

What Ster. money may I give in exchange for 16 Spanish patavoons at 4 s. 8 d. per piece?

	s.	d.	
	4 :	8	
		4	
	18 :	8	For 4 patavoons.
		4	
l. 3 :	14 :	8	For 16 patavoons.

What

What Ster. money may I give in exchange for 109 Portugal mill-rees at 6 s. 8 d. per piece?

s. *d.*
6 : 8
10

3 : 6 : 8 For 10 mill-rees,
10

33 : 6 : 8 For 100 mill-rees.
3 : 0 : 0 For 9 mill-rees.

l. 36 : 6 ; 8 For 109 mill-rees.

What's the weight of 46 ingots of silver, each 4 lb. 6 oz Troy?

lb. *oz.*
4 : 6
9

40 : 6 Weight of 9 ingots.
5

202 : 6 Weight of 45 ingots.
4 : 6 Weight of 1.

l. 207 : 0 Weight of 46 ingots.

What quantity of gold must there be to make 72 funeral rings, each to weigh 3 penny weight 12 grains?

p.w. *gr.*
3 : 12
12

2 : 2 : 0 Goes for 12 rings.
6

lb. 1 : 0 : 12 : 0 Goes to make 72 rings.

Proofs

Proofs of Multiplication.

I. Caſt out the nines out of all the particular products, place the remainders on the right hand of their reſpective lines; alſo caſt out the nines out of theſe remainders, as they ſtand; and if the overplus be equal to the remainder when the nines are caſt out of the general product, the work is ſuppoſed to be right.

See the work of the following example ſo proven.

	46758	
	345	
	233790	6
	187032	3
	140274	0
Product.	16131510	0

Or, caſt out the nines out of the Multiplicand, and alſo out of the Multiplier, caſt out the nines out of the product of the two remainders, and if the overplus be equal to the remainder, when the nines are caſt out of the general product, the work is ſuppos'd right; as in this following example.

	58763	2	4
	47	2	
	411341		
	235052		
Product.	2761861	4	

But

But the way of proving Multiplication by casting out the nines is not to be depended on, as I have often experienced: only, this may be said for it, that if the work be rightly performed, it will never appear to be wrong by this way; but it many times makes an operation appear right, when it is utterly false.

II. Multiply the Multiplier by the Multiplicand, and if the Product corresponds with that when the Multiplicand is multiply'd by the Multiplier, the work is right.

	24
487	487
24	168
1948	192
974	96
11688	11688

III. The work of Multiplication is certainly right when the Product divided by the Multiplier quotes the Multiplicand; or divided by the Multiplicand quotes the Multiplier; but this method of proof must be delayed till we come to Division.

I might have here laid down the method of multiplying numbers of divers denominations, by numbers of divers denominations of the same kind; but there being too much division used in that case, I shall defer it till I come to treat of the rules of Practice where it will fall naturally in.

I Come now to that branch of Multiplication call'd Reduction Descending, which is the changing of any propos'd greater denomination, into a lesser name

name required, as pounds into ſhillings, guineas into pence, hundred weights into lb. or bolls into pecks, &c. ſtill preſerving the value, tho' in different terms.

Reduction is no rule of itſelf, and this part of it being performed by Multiplication, ſhould always be taught immediately after the ſame; and before Diviſion, as all the caſes in that rule cannot be pointedly reſolved without this kind of Reduction.

In this part of Multiplication there are three caſes.

Caſe I. When any number or ſum is given to be reduced only to the next inferior name. As ells into quarters, pounds to ſhillings, hours to minutes, or gallons to quarts, &c.

RULE.

Conſider how many of the denomination into which you would reduce it are containd in an unite or integer of the given denomination, and multiply the given number thereby, and the product will be the anſwer to the queſtion.

In 297 l. what ſhillings?

$$\begin{array}{r} 297 \\ 20 \\ \hline 5940 \end{array}$$ Shillings.

Here, I conſidered that in 1 l. are 20 ſhillings, and that the number of ſhillings in 297 l. would be 20 times 297; wherefore I multiply'd 297 by 20, and the product was 5940, and ſo many ſhillings are contain'd in 297 l.

Caſe II. When any ſum or number is given to be reduced, and there is a denomination or denominations between the number given and number required; as if pounds were given to be reduced to pence, ells to nails, acres into ells, or guineas to farthings, &c.

RULE

RULE.

Reduce the given number (as before) into the next inferior denomination, and that product into the next, &c. until you have brought it into the denomination required. As in the following example.

In 576 l. how many farthings?

```
   576
    20
 -----
 11520 Shillings.
    12
 ------
138240 Pence.
     4
 ------
552960 Farthings.
```

Here, I first multiply'd 576 (the given pounds) by 20, and that produced 11520 shillings: then I multiply'd these 11520 shillings by 12, to bring them to pence, and they produced 138240, the pence in 11520 shillings. Lastly I mutiply'd these 138240 pence by 4, to bring them into farthings, and I found the product to be 552960 farthings, which are equal in value to 576 l.

If the 576 pounds had been multiply'd by 960 (the farthings in one pound) they would have produced the same number of farthings.

See the work.

```
   576
   960
 ------
 34560
5184
 ------
552960 Farthings as before.
```

In 425 c.w. what lb.

425
4 The qrs. in 1 c.w.

1700 Qrs.
28 The lb. in a qr.

47600 Lb.

Here I multiply'd the 28 by 1700, and brought the work into one line, as formerly directed. And if the 425 c.w. had been multiply'd by 112, the number of pounds in c.w. the anſwer would have been the ſame as before.

See the work.

425
112

5100
425

47600 Lb. as before.

In 54 yards how many nails?

54
4

216 Qrs.
4

864 Nails.

If you had multiply'd the 54 yards by 16 (the nails in one yard) the same answer would have been produced.

See the work.

54
16
———
864 Nails, as before.

Case III. When the number given to be reduced, consists of divers denominations, as pounds, shillings, pence and farthings; or years, days, hours and minutes, &c.

RULE.

Reduce the highest denomination (as before) into the next inferior, and add thereto the number standing in that denomination to which your highest number is reduced. Then reduce that sum into the next inferior denomination, adding thereto the number standing in that denomination, to which you have reduced the said sum. Do so, until you have brought the given number, into the denomination required.

See the work of the following example.

In l. 87 : 17 : 11 : 3 what ſhil. d. and farthings?

87 : 17 : 11 : 3
20
———
1740
Add 17 ſhillings.
———
1757 Shillings.
12
———
21084
Add 11 pencce.
———
21095 Pence.
4
———
84380
Add 3 farthings.
———
84383 Farthings.

This laſt queſtion, or any other of the ſame kind, viz. where the number given to be reduced conſiſts of ſeveral denominations (whether money, weight, or meaſure) may be more conciſely reſolved thus, Multiply as before, and take in the numbers that ſtand in each inferior denomination, as you reduce the higher to it. That is, when you multiply by the units place of the multiplier, take in the units place of the number ſtanding in the next inferior denomination, and when you come to multiply by the place of tens in your Multiplier, take in the place of tens (if any) of the ſaid number, and by this way you will ſave a good many figures.

Example.

Example.

In l. 54 : 17 : 7 what shil. d. and farthings?

54 : 17 : 7
20

1097 Shillings.
12

13171 Pence.
4

52684 Farthings.

Here, in multiplying by 20, I say 0 times 4 is nothing, but 7 that stands in the units place of shillings, is 7; and because the Multiplier is 0, I go no farther with it; (for if I should the whole product would be 0) but I proceed with 2, the tens place of the Multiplier, saying 2 times 4 is 8, and 1 that stands in the tens place of shillings, is 9, and 2 times 5 is 10, which makes 1097 shillings, in l. 54 : 17. Then when I came to reduce 1097 shillings into pence, I say 12 times 7 is 84, and 7 that stands in the place of pence, is 91, that is 1, and carry 9 to the product of the next figure, and so I go on to multiply till the product comes out to 13171 pence, equal to 54 l. 17 s. 7 d. Lastly, I multiply those 13171 d. by 4, and they produce 52684 farthings, equal to l. 54 : 17 : 7. So the work is finished.

In 38 l. 9 pence, what shillings and pence?

38 : 0 : 9
20

760 Shillings.
12

9129 Pence.

Here

Here you may obſerve, that the 9 pence is not taken in, till you are reducing the ſhillings to pence.

By the foregoing rules, are all Reductions deſcending (that is from greater denominations to leſſer) performed, whether they be money, weight or meaſure, that is, by conſidering how many of the next leſſer make an unit, or one of the greater denomination, as before directed; and then multiplying accordingly, deſcending from one denomination to the next, till the work is finiſhed. And for further aſſiſtance, recourſe muſt be had to the ſeveral tables of Aliquot Parts in money, weight and meaſures, in the II. Chapter of this book.

If any one of an ordinary capacity, conſiders attentively the nature of the foregoing rules, and obſerves the work of the following examples, he cannot miſs to be maſter of Multiplication in all its parts.

In 27 l. 1 farthing, what ſhil. d. and farthings?

```
  20
 ----
 540 Shillings.
  12
 ----
6480 Pence.
   4
 ----
25921 Farthings.
```

In 857 guineas what ſhil. d. and farthings.

```
     21 Shillings in a guinea.
   ------
    857
   1714
   ------
  17997 Shillings.
     12
 -------
 215964 Pence.
      4
 -------
 863856 Farthings.
```

In

In 67 moidores, what shil. Ster,
27 Shillings in a moidore.

469
134

1809 Shillings.

In 157 Crowns, what pence
60

9420 Pence.

In 89 merks Ster. what l. Scots ?
8

712 l. Scots.

In 58 l. Ster. what l. Scots ?
12

696 l. Scots.

In l. 67 : 14 : 8 Ster. what Scots money ?
12

l. 812 : 16 : 0 Scots.

In 546 l. Ster. what crowns ?
4

2184 Crowns.

In 175 merks Ster. what merks Scots ?
12

2100 Merks Scots.

In 64 l. Ster. what merks Scots?
18

1152 Merks Scots.

In 184 crowns, what l. Scots and what shillings Ster?

184	184
3	5
552 l. Scots.	920 Shillings Ster.

In 17 half guineas, what sixpences?
21

17
34

357 Sixpences.

In 524 l. Ster. what half crowns?
8

4192 Half crowns.

In 58 merks, what pennys?
160

9280 Pennys.

In 789 l. Ster. how many groats?
20

15780 Shillings.
3

47340 Groats.

It would have been the same, if you had multiply'd by 60 the groats in 1 l.

789
60

47340 Groats, as before.

In 546 pieces of eight at 4 s. 6 d. what l. Ster.

4 : 6
10

2 : 5 : 0 Value of 10 pieces.
10

22 : 10 : 0 Value of 100.
5

112 : 10 : 0 Value of 500.
9 : 0 : 0 Value of 40.
1 : 7 : 0 Value of 6.

l. 122 : 17 : 0 Value of 546.

In 173 merks, what pounds?

13 : 4
10

6 : 13 : 4 Value of 10 merks.
10

66 : 13 : 4 Value of 100.
46 : 13 : 4 Value of 70.
2 : 0 : 0 Value of 3.

115 : 6 : 8 Value of 173 merks.

In 14 pieces of cloth, each 15 ells 3 quarters, what nails?

15 : 3
4

63 Qrs.
4

252 Nails in 1 piece.
14

3528 Nails in 14 pieces.

In 5 ankers of brandy, each 19 pints 1 chopin, what chopins and gills?

19 : 1
5

Pints 97 : 1 in 5 ankers.
2

195 Chopins.
8

1560 Gills.

In 58 bolls 2 pecks, what firlots, pecks and lippies?

58 : 0 : 2
4

232 Firlots.
4

930 Pecks.
4

3720 Lippies.

In 72 c.w. 17 lb. what qrs. and lb. ?

```
  72 : 0 : 17
   4
  ----
  288 Qrs.
   28
  ----
  2311
  577
  ----
  8081 Lb.
```

In 8 Scots acres, what falls and ells ?

```
   160
     8
  ----
  1280 Falls.
    36
  -----
   7680
  3840
  -----
  46080 Ells.
```

In 57 English miles, what yards, feet, and inches ?

```
     1760
       57
   ------
    12320
    8800
   ------
   100320 Yards.
        3
   ------
   300960 Feet.
       12
  -------
  3611520 Inches.
```

In

In 73 groſs of bottles, how many dozens and ſingle bottles?

73

12

876 Doz.

12

10512 Bottles.

From the creation of the world to this preſent year 1757, by the beſt of prophane hiſtory, it is reckoned 5706 years; how many minutes are there in the ſame?

d. h.

One year is 365 : 6

24

1466

730

8766 Hours in 1 year.

60

525960 Minutes in 1 year.

5706

3155760

368172

262980

3001127760 Minutes deſired

In 18 gallons of wine, what cubic inches?

231

18

4158 Inches.

In

In 48 English gallons of beer, how many cubic inches?

282 Cubic inches in 1 gallon.
48
―――
2256
1128
―――
13536 Answer.

In 67 Scots gallons, what cubic inches?

67
8
―――
536 Pints.
103
―――
1608
536
―――
55208 Inches.

In 84 lb. 7 oz. Troy, what ounces, penny weights and grains?

84 : 7
12
―――
1015 Ounces.
20
―――
20300 Penny weights.
24
―――
812
406
―――
487200 Grains.

In 45 tun of wine, what gallons?

45
4

180 Hogsheads.
63

540
1080

11340 Gallons.

Suppose the circumference of the earth to be 360 degrees, and every degree 60 miles; how many barley corns will go round the same?

360
60

21600 Miles.
1760 Yards in a mile.

1296
1512
216

38016000 Yards.
3

114048000 Feet.
12

1368576000 Inches.
3

4105728000 Barley corns.

In 10 lb. 11 drops, what oz. and drops?
16

160 Ounces.
16

2571 Drops.

In l. 795 : 17 : 8, what half-pence?
20

15917 Shillings.
12

191012 Pence.
2

382024 Half-pence.

In 79 Florins at 3s. 2 d. per piece, what l. Ster.?

s. p.
3 : 2
10

1 : 11 : 8 Value of 10 Florins.
7

11 : 1 : 8 Value of 70.
1 : 8 : 6 Value of 9.

l. 12 : 10 : 2 Value of 79.

In 179 Bitts of Jamaica each 7 d. 2 qr. Ster. what l. Ster.?

		d.	qrs.		
		7 :	2		
			10		
	6 :	3 :	0	Value of 10.	
			10		
3 :	2 :	6 :	0	Value of 100.	
2 :	3 :	9 :	0	Value of 70.	
	5 :	7 :	2	Value of 9.	
l. 5 :	11 :	10 :	2	Value of 179 bitts	

In 799 mill-rees of Portugal, each 5 s. $3\frac{1}{4}$ d. what l. Ster.?

	5 :	3 :	1	
			10	
2 :	12 :	8 :	2	Value of 10.
			10	
26 :	7 :	1 :	0	Value of 100.
			7	
184 :	9 :	7 :	0	Value of 700.
23 :	14 :	4 :	2	Value of 90.
2 :	7 :	5 :	1	Value of 9.
210 :	11 :	4 :	3	Value of 799 mill-rees.

Hundred weights, quarters and pounds may be speedily reduced into pounds, by setting down the hundreds 4 several times, and by taking in the odd weight, as in this following example.

In

In 675 c.w. 2 qrs. 7 lb. what lb.?

675
675
675
675
63 lb. odd. weight.

75663

In 246¾ c.w. of cotton yarn, what lb.?

246
246
246
246
84 odd weight.

lb. 27636

In 49 ells Flemish, what quarters and nails?

49
3

147 Qrs.
4

588 Nails.

In 528 French ells, what qrs. and nails?

528
6

3168 Qrs.
4

12672 Nails.

Reduction descending and ascending do interchangeably prove each other by inverting the question, as shall be shown when we come to that part of Reduction.

CHAP. VI.

Of Division.

DIVISION (which is the reverse of Multiplication) is a Rule by which one number may be speedily subtracted from another, so many times as it is contained therein.

That is, it speedily discovers how often one number is contained (or may be found) in another; As if it were required how often 7 is contained in 56, the answer would be 8 times.

It likewise serveth to work that part of Reduction called Ascending, that is, to bring lesser denominations into greater; as pence into pounds, nails into yards, lippies into bolls, &c. And in this rule, two numbers must be given to find a third.

I. The Dividend, or number to be divided.

II. The Divisor, or number whereby the Dividend is divided.

The number to be found is called the Quotient, and shews how often the Divisor is contained in the Dividend, or into what number of equal parts the Dividend is divided.

And some times there happens to come out a fourth number (after the work is ended) called the Remainder, which is always of the same name or quality with the Dividend; and must be less than the Divisor, if the work be right.

Division is either single or compound.

Single Division is when the Divisor consisteth but of one figure, and the Dividend but of two at the most; and this kind of Division may either be performed by the memory, or by the table of Multiplication: So if it were demanded, How many times 7 is contained in 35, the answer would be found in the table to be 5: Here 35 is the Dividend, 7 the Divisor, and 5 the Quotient, or answer.

That

That Diviſion ſupplies the place of many Subtractions, may be made evident thus.

	35
1	7
	28
2	7
	21
3	7
	14
4	7
	7
5	7
	0

Here you ſee the Diviſor is continually ſubtracted from the Dividend, and accounting an unit for each time it is ſubtracted, the ſum of the units is the Quotient, viz. 5.

Compound Diviſion is when the Dividend conſiſteth of many places or figures, and the Diviſor of one or more. As if 7672 were to be divided by 4, the Quotient would be 1918.

A general Rule for the Work.

First ſeek how oft, then multiply;
Subtract, bring down a new ſupply.
Repeat this work unto the end,
Till all the Dividend you ſpend.

Diviſion is accounted the hardeſt leſſon of Arithmetic; but I ſhall by plain rules, and familiar examples, render it eaſy to the meaneſt capacity; and

ſhew

ſhew its excellent uſe in anſwering many queſtions, which ſeem to require a greater knowledge in Arithmetic. When therefore a queſtion of compound Diviſion is to be wrought, you muſt proceed according to this following rule.

RULE.

Firſt ſet down your Dividend, making a crooked line at each end thereof, that on the left hand to contain the Diviſor, and that on the right hand for the Quotient; and having put your Diviſor in its place, diſtinguiſh with a point ſo many places of your Dividend on the left hand, as are equal to or next exceeding your Diviſor; and aſking how oft your Diviſor is contained in the ſaid ſum, the anſwer muſt be placed in your Quotient on the right hand of the Dividend: then multiply your Diviſor by the figure laſt placed in the Quotient, ſetting the product under your aforeſaid diſtinguiſhed ſum, and drawing a line under both, take the lower from the higher, and to the remainder, point and bring down your next figure of the Dividend, with which proceed as you did with your diſtinguiſhed number; and ſo on till you have pointed and brought down all the figures of your Dividend: and as many points as you have made in your Dividend, ſo many figures will be in your Quotient.

Note, And the times that you take the Diviſor out of the Dividual, never exceed 9: and the Dividual never exceeds the Diviſor above one figure.

All operations in Diviſion begin contrary to thoſe of Multiplication, viz. at the firſt figure to the left hand, or that of the higheſt value, and decreaſe the Dividend by a repeated ſubtraction of each product ariſing from the Diviſor when multiply'd into the Quotient figure.

The only difficulty in diviſion lies in making choice

of

of such a Quotient figure as is neither too much nor too little; and that may easily be obtained by observing the following rule, which hath two cases.

Case I. When the first figure of the Divisor can be taken from the first figure of the Dividend.

RULE.

As often as the first figure of the Divisor is taken from the first figure of the Dividend, so often must the second figure of the Divisor be taken from the second figure of the Dividend when its joined with what remain'd of the first: and as often must the third figure of the Divisor be taken from the third figure of the Dividend, &c.

Case II. When the first figure of the Divisor cannot be taken from the first figure of the Dividend.

RULE.

So often as the first figure of the Divisor is taken from the two first figures of the Dividend, so often must the second figure of the Divisor be taken from the third figure of the Dividend when its join'd with what remain'd of the second: and so often must the third figure of the Divisor be taken from the fourth figure of the Dividend, &c.

That is, the Quotient figure must be such, as being multiply'd into the Divisor, will give a product equal to such a part of the Dividend as is then taken for that operation: but if such a product cannot be exactly found, then the next less must be taken, as in the following example.

Let 53718 be divided by 7.

Dividend. Place for the
Divisor 7) 5 3 7 1 8 (. Quotient when found.

Here, as I cannot take the Divisor 7 from 5 the first

first figure of the Dividend, I consider how often 7 can be taken from 53 the two first figures of the Dividend, and find it may be had 7 times, for 7 times 7 is 49, being the greatest product of 7 into any figure, that can be taken from 53. I therefore place 7 in the Quotient, and thereby multiplying 7 the Divisor, I set down the product under 53, and subtracting it therefrom, the work will stand thus:

```
7)53718(7
   .
  49
  --
  (4)
```

Then I point and bring down 7 (the next figure of the Dividend) to the remainder 4, which will then make 47 for a Dividual. I consider how often 7 may be had in 47, and find it 6 times (for 6 times 7 is 42) I therefore place 6 in the Quotient, and thereby multiplying 7 the Divisor, I set down and subtract the product as before, and the work will then stand thus:

```
7)53718(76
   ..
  49
  --
   47
   42
   --
   (5)
```

For a third operation, I make a point under 1 the next figure of the Dividend, and bring it down as before: then proceeding in all respects as formerly, the work will stand thus:

7)

```
7)53718(767
  ...
  49
  --
   47
   42
   --
    51
    49
    --
    (2)
```

Lastly I point and bring down 8 (the last figure of the Dividend) to the remainder 2, which makes it 28, and finding that 7 can be taken from 28 just 4 times, I place 4 in the Quotient, and the work being finished will stand thus:

```
7)53718(7674
  ....
  49
  --
   47
   42
   --
    51
    49
    --
     28
     28
     --
     (0)
```

Quotient, which is the real 7th part of 53718; and the Multiplicand of the first example in Multiplication. Hence it is evident that Division and Multiplication do mutually prove each other.

The reasons of these operations will be very plain to any one that attentively considers it as follows.

Divisor

Divisor 7) 53718 (7000 first Quotient figure.
Subtract) 49000 (

Divisor 7) 4718 (600 2d. Quotient figure.
Subtract) 4200 (

Divisor 7) 518 (70 3d. Quotient figure.
Subtract (490 (

Divisor 7) 28 (4 4th Quotient figure.
Subtract) 28 (

(0) Sum 7674 Quotient as before.

The first product of the Divisor into the Quotient, you see, is 49000, that is 7 times 7000; the Quotient figure being always of the same value or degree with that figure under which the units place of its product stands.

The second product is here 4200, viz. 7 times 600, not 7 times 6.

Also, the third product is 490, that is 7 times 70, and not 7 times 7, for the foresaid reasons.

And the last product is but 28, viz. 7 times 4, because the 4 stand in the place of units. So the sum of the several quotients is 7674 as before.

If the process of the foregoing example be well considered and compared with the second way of working the first example in Multiplication, it will evidently appear to be only the converse of that example; for the particular products are alike in both, only, that which is last there, is first here; there they are added, here they are subtracted: so that whoever understands the true reason of the one, must needs understand the reason of the other, and then Division will become very easy tho' the Divisor consist of a great many places.

As the preceeding example came under the second case

case of the foregoing rule, take another under the first case of the said rule, as follows.

If 6740 l. is to be equally divided among five persons, what will each have?

Here 'tis plain at sight that the Divisor 5 can be taken from the first figure of the Dividend 6740, and so it comes under the first case of the foregoing rule.

See the work.

5)6740(1348 l. to each.
  ....
  5

  17
  15

   24
   20

    40
    40
    ---
    (0)

Note, If after the work of Division is ended, any thing remains, place such remainder over your Divisor on the right hand of your Quotient, and it will be a fraction of an unit of the said Quotient; as in the following example.

What's the 9th part of 76481?

9)76481(8497$\frac{8}{9}$ Answer.
  ....
  72

  44
  36

   88
   81

    71
    63
    ---
    (8)

The Proof.

To prove the work of Diviſion, multiply the Quotient by the Diviſor, and if the product is equal to the Dividend, the work is right: but obſerve whenever there is a remainder, it muſt be added to the product.

Let the work of the laſt example be proved.

9)76481(8497 Quotient.
9 Diviſor.
Remainder (8)
76481 Dividend.

What hath already been ſaid of dividing by a ſingle figure, I think ſufficient; and therefore ſhall only add ſome contractions, and proceed in the rule.

There can be no ſhorter way, of dividing by a ſingle figure than to omit ſetting down the ſeveral products of your multiplication, and to multiply and ſubtract together mentally as in the following examples.

Let 9654 be divided by 6.
6)9654(1609 Quotient.
(0)

Here, I ſaid ſix in 9, once 6 and 3 over; then 6 in 36, 6 times, and 0 over; and ſix in 5, 0 times; but ſix in 54, 9 times, and 0 remained; and the quotient was 1609.

Let 78456 be divided by 9.
9)78456(8717$\frac{3}{9}$ Quotient.
(3) Remainder.

Let 483 be divided by 7.
7)483(69 Quotient.
(0)

It is also most expeditious to divide by 11 and 12 as by a single figure thus:

11)45678(4152$\frac{6}{11}$ Quotient.

(6) Remainder.

Here I said eleven in 45, 4 times; eleven in 16, once; eleven in 57, 5 times; eleven in 28, twice; and there remained 6; and the Quotient was 4152$\frac{6}{11}$.

12)479674(39972$\frac{10}{12}$ Quotient.

(10)

Here, I said the twelves in 47, 3 and 11 over; then the twelves in 119, 9 and 11 over; the twelves in 116, 9, and 8 over; the twelves in 87, 7, and 3 over; lastly the twelves in 34, 2, and 10 over; and the Quotient was 39972$\frac{10}{12}$.

You may likewise divide by any Divisor that hath cyphers on its right hand, by cutting off the cypher or cyphers with a stroke of the pen, and also as many cyphers or figures towards the right hand of the Dividend; but remember to bring these down (if significant figures) to the right hand of the remainder, if there be any) and that sum is the full remainder; but if 0 remains, then those figures so cut off on the right hand of the Dividend are the only remainder as in the following examples.

11|0)749|6(68$\frac{16}{110}$ Quo.

(16) Remainder,

9|00)819|74(91$\frac{74}{900}$ Quo.

(74) Remainder.

12|00)5279|67(439$\frac{1167}{1200}$ Quo.

(1167) Remainder.

4|0)83200|0(2080 Quo.

(0)

Hence,

Hence, you may observe, that to divide by 10, 100, 1000, &c. is only to cut off so many cyphers or figures from the right hand of the Dividend as there are cyphers in the Divisor: the figures on the left of the stroke in the Dividend is the Quotient, and those on the right hand the Remainder, as in the following examples.

1|0000)76430|0000(76430 Quotient.

(0)

1|00)5867|43(5867 $\frac{43}{100}$ Quotient.

(43) Remainder.

When your Divisor is such a number as is the product of any two digits in the multiplication table, divide your Dividend by any one of the two (it matters not which of them) then divide the Quotient by the other, and that last Quotient is the answer.

This method is more expeditious, and done in fewer figures than dividing by the whole, as in these examples.

Let 4176816 be divided by 56, the product of 7 and 8.

7)4176816

8) 596688

74586 Quotient.

If there happen to be any remainder, either in the first or second division, or both, the Quotient will be the same as if you had divided at once. And to find the true remainder in this case, is to multiply the first Divisor by the last remainder, taking in the first remainder, if any be; as in these examples.

Let

Let 488 be divided by 45, the product of 9 and 5,

$$\begin{array}{l} 9) \\ 5)488(97(10\tfrac{38}{45} \text{ Quotient.} \\ \quad 3. \quad 7. \end{array}$$

Let 8435 be divided by 63, the product of 7 and 9.

$$\begin{array}{l} 9) \\ 7)8435(1205(133\tfrac{56}{63} \text{ Quotient.} \\ \quad 0. \quad 8. \end{array}$$

I now proceed to give some examples wherein the Divisor consists of more figures than one; and as the greatest difficulty in the work of Division arises here, I shall endeavour to illustrate every step of the said work in the process of the two first examples.

Let 7246 be divided by 68.

According to the general rule before laid down, I say, how many times 6 (the first figure of the Divisor) can I have in 7 the first figure of the Dividend, and the answer is once, with 1 remaining; now seeing I can have the second figure of the Divisor, viz. 8 also once out of the sum when the remainder 1 is join'd to the left hand of the second figure of the Dividend, viz. 2, I conclude I can have 68 out of 72 just once, and therefore I set down 1 in the Quotient, thus, 68)7246(1. Then according to the said rule, I multiply the Divisor 68 by 1, and set down the product under 72 in the Dividend; then I draw a line, and subtract 68 from 72, and there remains 4: as in the first step of the work may be seen, standing thus:

$$\begin{array}{l} 68)7246(1 \\ \quad\;\; 68 \\ \hline \quad\;\; (4) \end{array}$$

Then

Hence, you may obſerve, that to divide by 100, 1000, &c. is only to cut off ſo many cyph figures from the right hand of the Dividend as are cyphers in the Diviſor: the figures on th of the ſtroke in the Dividend is the Quotient, thoſe on the right hand the Remainder, as in th lowing examples.

1|0000)76430|0000(76430 Quotient.

(0)

1|00)5867|43(5867 $\frac{43}{100}$ Quotient.

(43) Remainder.

When [illegible] ſuch a number as is the duct of a[illegible] the multiplication t divide yo[illegible] one of the two (it m not whic[illegible] ivide the Quotient by other, a[illegible] it is the anſwer.

This m[illegible] itious, and done in er figures [illegible] whole, as in the amples.

Let 41[illegible] 56, the product and 8.

[illegible]

fi[illegible] either in th[illegible] tient will th[illegible] nd to fi th[illegible] ly the fi D[illegible] ma[illegible]

24922 by 7563.
Divisor 7563 can-
number of figures

second case of the
aken from 59062,
e Divisor may be
s of the Dividend.
w often 7 may be
or 8 times 7 is 56)
, and there remains
the third figure of
out of which I must
isor) so often as I
times : but that
refore 8 is too big
ut ; hence I con-
be taken without
7 in the Quotient,
, setting down the
subtracting it from
e, the work will

point and bring
idend) to the re-
new Dividend ; I
before, and find
ith which I mul-
he product from
ill stand thus :

Then I make a point under the next figure of the Dividend, to wit 4, and bring it down to the right hand of the remainder 4; and then there is 44 for a new Dividend, and the work stands thus:

```
68)7246(1
   68·
   ——
   (44)
```

As I cannot get 68 out of 44, I set down 0 in the Quotient, and I point and bring down 6 (the next and last figure of the Dividend) to the right hand of 44, and the work stands thus:

```
68)7246(10
   68··
   ——
   (446)
```

Now I seek how oft I can have 6 the first figure of the Divisor out of 44, because there is one figure more in the Dividual than in the Divisor, (for there never ought to be more than one) and I find it 7 times, with 2 remaining; but when this remainder 2 is set on the left hand of 6 (the last figure of the Dividual) I cannot have 8 the second figure of the Divisor out of the sum 26 seven times, therefore I take 68 out of 446 a time less, and so finding I can have it 6 times, I place 6 in the Quotient, and multiplying the Divisor thereby, I place the product under 446, and subtracting it therefrom, there remains 38, which remainder I place over the Divisor on the right hand of the Quotient, and the work being finished will stand thus:

```
68)7246(106 38/68 Quotient.
   68··
   ——
    446
    408
    ———
   (38) Remainder.
```

Let

Let it be required to divide 590624922 by 7563.

Here 'tis plain at ſight, that the Diviſor 7563 cannot be taken from 5906, the like number of figures in the Dividend.

Therefore it comes under the ſecond caſe of the foregoing rule, and it muſt be taken from 59062, that ſo the firſt figure (7) of the Diviſor may be taken out of 59 the two firſt figures of the Dividend.

I proceed then, and conſider how often 7 may be had in 59, and find it 8 times, (for 8 times 7 is 56) which I mentaly ſubtract from 59, and there remains 3; to this 3 I mentaly adjoin 0 (the third figure of the Dividend) which makes it 30, out of which I muſt take 5 (the ſecond figure of the Diviſor) ſo often as I took the 7 out of 59, which was 8 times: but that cannot be, for 8 times 5 is 40, therefore 8 is too big a figure to be placed in the Quotient; hence I conclude that 7 (the next leſs) may be taken without any further trial: I therefore place 7 in the Quotient, and with it I multiply the Diviſor, ſetting down the product under the Dividend, and ſubtracting it from thence, as in the former example, the work will ſtand thus:

7563)590624922(7
52941
(6121)

In order to a ſecond operation, I point and bring down 4 (the next figure of the Dividend) to the remainder, and it makes 61214 for a new Dividend; I with this proceed in all reſpects as before, and find the next Quotient figure to be 8, with which I multiply the Diviſor, and ſubtracting the product from the ſaid dividual 61214, the work will ſtand thus:

7563)590624922(78
52941 ··
61214
60504
(710)

To this remainder 710, I point and bring down 9 (the next figure in the Dividend) which makes 7109; and because I cannot have the Divisor 7563 out of the Dividual 7109, I place a cypher in the Quotient, and the work will stand thus:

7563)590624922(780
52941 ···
61214
60504
(7109)

Note, You must never bring down but one figure or cypher at a time out of the Dividend.

Also, For every figure or cypher brought down from the Dividend, in order to a new operation, there must always be either a figure or cypher placed in the Quotient.

Lastly, When you cannot take the Divisor out of the Dividual, you must put a cypher in the Quotient, and take another figure from the Dividend: and if, again, you cannot take it, place another cypher in the Quotient, and, bring down another figure from the Dividend, as in the next example.

But I return to the work of the propos'd example, and to 7109 I point and bring down 2 the next figure of the Dividend, and it becomes 71092, out of which I find I can take the Divisor 9 times; and therefore I set down 9 in the Quotient, and thereby multiplying the Divisor, I subtract the product, as before, and the work will stand thus:

```
7563)590624922(7809
     52941...
     -----
      61214
      60504
      -----
       71092
       68067
       -----
       (3025)
```

To this remainder 3025, I point and bring down 2, the laſt figure of the Dividend, which makes it 30252; then proceeding in all reſpects as before, I find I can have the Diviſor 4 times out of that Dividual; I therefore multiply the Diviſor by 4, then ſubtract their product as before, and find the remainder to be 0, and the work being finiſhed, will ſtand thus:

```
7563)590624922(78094 Quotient.
     52941....
     -----
      61214
      60504
      -----
       71092
       68067
       -----
        30252
        30252
        -----
         (0)
```

Let 76542754 be divided by 764.

764)76542754(100186$\frac{650}{764}$ Quotient
764 . . . . .
———
1427
764
———
6635
6112
———
5234
4584
———
650 Remainder.

Note, When, at any time, after you have subtracted, there remains nothing, and yet there remains a cypher or cyphers in the Dividend, you must put it, or them, in the Quotient as part of it, and the work is done. As in the following example:

Let 76457850000 be divided by 75.

75)76457850000(1019438000 Quotient.
75 . . . .
———
145
75
———
707
675
———
328
300
———
285
225
———
600
600
———
(0)

The

The only difficulty (as I said before) lyes in making choice of a true Quotient figure, which cannot well be done without repeated trials; yet these trials need not be made with the whole Divisor; for by the two or three first figures of the Divisor all the rest are generally regulated: But when you are to take the first figure of the Divisor from the first or two first figures of the Dividend, you must take special notice of the increase that comes by multiplying the third and second figures of the Divisor: As for example, if 221 were given to be divided by 79, you would have the first figure (7) of the Divisor 3 times out of (22) the two first figures of the Dividend; but when you multiply 9 by 3 (which makes 27) the product of 7 by 3 will then be increased by 2 that you carry, and so it will become 23 which is too much; and therefore you can take 79 out of 221 but twice, and you will have 63 remaining; and so of any other.

Note, Division (as well as the foregoing rules) may be proved by casting out the nines, thus:

Cast the nines out of the Divisor, and also out of the Quotient, place the remainders on the right and left sides of a cross; then multiply the said two remainders together, and casting the nines out of their product, suppose what's over to be placed on the left hand of the remainder of the division, cast the nines out of that sum also, and place the remainder on the top of the cross; and if what remains over the nines in the Dividend (which must be placed at the bottom of the cross) corresponds with the top figure, the work may be supposed right, as in the following example:

Let

Let 17964857437 be divided by 587464.

```
587464)17964857437(30580 Quotient.
       1762392····
       -------
        3409374             7
        2937320           7 X 7
        -------             7
         4720543
         4699712
         -------
          208317 Remainder.
```

In the preceeding example, the remainder over the nines in the Divisor is 7, which I place on the left side of the cross; the remainder over the nines in the Quotient is also 7, which I place on its right side; then 7 times 7 is 49, and the remainder over the nines in that product (being 4) I suppose to be placed on the left hand of the remainder of the division, which makes it 4208317, the remainder over the nines in this sum (being 7 also) I place at the top of the cross, and seeing the remainder over the nines in the Dividend is also 7, and so corresponds therewith, the work is supposed right.

How to manage the Remainder in Division.

RULE.

The remainder after Division is ended, is always of the same name and quality with the Dividend; therefore multiply it by the parts of the next inferior denomination, divide the product by the former Divisor, and the Quotient is of the same name with what you reduced your remainder to: Again, if after this second division any thing remains, multiply it by the parts of the next lower denomination, divide the

product

product by the foresaid Divisor, and the Quotient is of the same name with what you reduced that last product to: And thus proceeding till you have brought it as low as can be; place your last remainder (if any) over the Divisor on the right hand of the Quotient as a fraction of an unit of the lowest denomination you brought your remainder to; as in the following examples.

7654 l. is to be equally divided among 48 persons; what will each have?

```
              l.    s.   d.
48)7654(159 : 9 : 2 to each.
   48..
   ----
   285
   240
   ----
    454
    432
    ----
     22
     20
    ----
  48)440(9
     432
     ---
       8
      12
     ---
   48)96(2
      96
     ---
     (0)
```

This serves to shew that Divison cannot be well performed nor understood without the knowledge of Reduction Descending, and is a good reason why it should be taught before Division.

What's

What's the 49 part of 57 guineas?

49)57(1 Guinea, 3 shillings, $5\frac{7}{49}$ or $\frac{1}{7}$ pence.
49
8
21
49)168(3
147
21
12
49)252(5
245
(7)

What's the 789th part of 217 bolls?

217
16
789)3472(4 pecks, $1\frac{475}{789}$ lippie.
3156
316
4
789)1264(1
789
(475)

Here, as the Dividend was less than the Divisor, I was obliged to reduce it to the parts of the next inferior denomination before I could divide.

A

A captain of a 40 gun ſhip, with 160 ſailors, took a prize worth 1440 l. whereof the captain got $\frac{1}{4}$, and the remainder was equally divided among the ſailors? I demand their reſpective ſhares?

1440 Total prize.

4)1440(360 l. captain's ſhare.
(0)

160)1080(6 l. 15 s. to each ſailor.
960
———
120
20
———
16|0)240|0(15
16
———
80
80
———
(0)

Divide 17 hogſheads of tobacco, each 5 c.w. equally among 27 merchants.

17
5 C.w. lib.
27)85(3 : $16\frac{16}{27}$ to each.
81
———
4
112
———
27)448(16
27
———
178
162
———
(16)

A,

A, B, and C, paid 360 l. for a quantity of meal, and in selling it out gain'd 270 l. of which gain as oft as A took up 3 l. B took up 5 l. and as oft as B took up 5 l. C took up 7 l.; what paid each for the meal, and what were their respective parts of the gain?

Note, The price was paid in the same manner that the gain was divided.

A. 3
B. 5
C. 7

15)360(24	15)270(18
30	15
60	120
60	120
(0)	(0)
24	18
3	3
72 l. paid by A.	54 l. gain'd by A.
24	18
5	5
120 l. paid by B.	90 l. gain'd by B.
24	18
7	7
168 l. paid by C.	126 l. gain'd by C.

A, B, C, and D entered into a joint adventure, and gained a certain sum, of which A, B, and C, took up up 60 l. B, C and D took up 90 l. C, D and A took up 80 l. and D, A and B took 70 l. what distinct gain did each take up?

```
     60                100
     90                 60
     80               ----
     70                 40 l. D's gain.
    ----              ----
 3)300(100             100
                        90
                      ----
                        10 l. A's gain.
                      ----
                       100
                        80
                      ----
                        20 l. B's gain.
                      ----
                       100
                        70
                      ----
                        30 l. C's gain.
```

Part 1500 acres of land between A, B, and C, and give B 72 acres more than A, and C 112 more than B.

```
      72
       2
    ----
     144
     112
    ----
     256
```

$$
\begin{array}{l}
1500 \\
256 \\
\hline
3)1244(414\tfrac{2}{3} \text{ Acres for A.} \\
(2) \quad 72 \\
\hline
486\tfrac{2}{3} \text{ Acres for B.} \\
112 \\
\hline
598\tfrac{2}{3} \text{ Acres for C.}
\end{array}
$$

A butcher ſent his ſervant with 558 l. to a fair to buy oxen at 30 l. per piece, cows at 24 l. per piece, ſwine at 5 l. per piece, and ſheep at 3 l. per piece, Scots money, and of each a like number; how many of each did he buy?

$$
\begin{array}{l}
30 \\
24 \\
5 \\
3 \\
\hline
62)558(9 \text{ of each.} \\
558 \\
\hline
(0)
\end{array}
$$

A merchant at London buys 64 tun of French wine for 480 l.; the freight from France to London cost 80 l. loading and unloading 12 l. custom 16 l. and charge of a cellar 9 l. how may he sell 1 tun of that wine to gain 116 l. on the whole?

```
      480
       80
       12
       16
        9
      116
     ———      l.   s.   d.   f.
  64)713( 11 : 2 : 9 : 3 Answer.
     64
     ———
      73
      64
     ———
       9
      20
     ———
  64)180(2
     128
     ———
      52
      12
     ———
  64)624(9
     576
     ———
      48
       4
     ———
  64)192(3
     192
     ———
     (0)
```

If

If 60 gallons of water in 1 hour's time fall *into* a ciftern containing 200 gallons, and by a pipe in the faid ciftern there runs out 45 gallons in 1 hour; in what time will it be filled?

```
60
45
——
15)200(13 hours 20 minutes. Anfwer.
  ——
   50
   ——
    5
   60
   ——
15)300(20
    ——
    (0)
```

If the breadth of a board be 9 inches; how much of the length will go to make $2\frac{1}{2}$ fuperficial feet?

```
  144 }
  144 } Inches in 2½ fuperficial feet.
   72 }
  ——
9)360(40 Inches, Anfwer.
  ——
  (0)
```

The content of a piece of land (being a long fquare) is 1760 falls, and length 80 falls; what's the breadth?

```
8|0)176|0(22 falls, the breadth.
     ——
     (0)
```

Divide

Divide 4722 ells of cloth equally among 7 merchants,

```
                e.   q.   n.
7)4722(674 : 2 : 1 1/7 to each.
  ----
     4
     4
    --
  7)16(2
    --
     2
     4
    --
   7)8(1
    --
    (1)
```

The side of a square field of land is 48 falls; how much of the breadth will go to make 7 acres 24 falls?

```
      160
        7
     ----
     1120
       24
     ----
  48)1144(23 falls, 30 ells; Answer,
      96
     ---
      184
      144
     ----
       40
       36
      ---
  48)1440(30
     1440
     ----
      (0)
```

2464 Soldiers are to be ranked, so that the Front shall consist of 88 men; what number must there be in the file?

88)2464(28 Men, Answer.
176
———
704
704
———
(0)

Three merchants A, B, and C, are in company; A owes to B and C, by equal portions l. 96 : 17 : 8, B owes to A and C the same way l. 107 : 14 : 6, and C owes to A and B l. 209 : 13 : 8; how shall their accompts be ballanced?

2)96 : 17 : 8(48 : 8 : 10
———
(0)

2)107 : 14 : 6(53 : 17 : 3
———
(0)

2)209 : 13 : 8(104 : 16 : 10.
———
(0)

The ballance of their accompts will stand thus:

l. 53 : 17 : 3 due by B to A.
104 : 16 : 10 due by C to A.
———
l. 158 : 14 : 1 total due to A.

l. 48 : 8 : 10 due by A to B.
104 : 16 : 10 due by C to B.
———
l. 153 : 5 : 8 total due to B.

l. 48 : 8 : 10 due by A to C.
53 : 17 : 3 due by B to C.
———
l. 102 : 6 : 1 total due to C.

There

There is a piece of ground 170 yards long, and 84 yards broad; how many quicks at 6 inches distance will inclose that piece of ground?

Bring the yards to inches by 36, and divide by the given distance.

$$\left.\begin{array}{r}170\\170\\84\\84\end{array}\right\}\text{The 4 sides.}$$

508 Yards round.
36

3048
1524

6)18288(3048 Quicks.

(0)

If the inner wheel of a corn miln hath 40 cogs, and the trindles 6 rungs; how many times will the upper milnstone have gone round when the inner wheel has turned round 148 times?

148
40

6)5920(986⅓ or ⅔ times, Answer.

(4)

If a hedge-hog in the day time climb 27 feet up a tree 112 feet high, and come down 11 feet at night; in what time will he reach the top of the tree?

27
11

16)112(7 Days, Answer.

(0)

Cut off a cubic half foot from an entire cubic foot, and leave the remainder a cube.

Note, A cubic foot hath 6 sides, and contains 8 cubic half feet, or 1728 inches.

See the work.

```
    6)  144)
8)1728(216(36  (0 36/144  or 1/4 of an inch depth must
 ----  ---  ---           be cut off from each side of the
 (0)   (0)  (36)           cubic foot.
```

That is, if you cut off $\frac{1}{4}$ of an inch from one side of the cubic foot, you will then take off 36 cubic inches, and so from the six sides 216 the cubic inches in a cubic half foot.

In 52684 farthings, how many pounds?

This, with a good many following examples, comes under that branch of Reduction called ASCENDING, and for performing the work of them, this is the rule.

RULE.

Divide the given number by such a number of its own parts as make an unit of the next superior denomination: then divide that Quotient by so many of its own parts as make an unit of the next higher order; and so proceed till you have brought it to the denomination required; always remembering that the remainder of each operation is of the same name and quality with the Dividend.

See the work.

```
          12)     20)   l.   s.   d.
4)52684(13171(1097(54 : 17 : 7  Answer.
  -----   ----  ----
   (0)     (7)  (17)
```

Here,

Here, because 4 farthings make a penny, I divide the farthings by 4, and the Quotient was 13171 pence, 0 farthings remaining: then because 12 pence make a shilling, I divide the 13171 pence by 12, and the Quotient was 1097 shillings with 7 pence remaining: and lastly, because 20 shillings make a pound, I divide the 1097 shillings by 20, and the Quotient was l. 54 with 17 shillings remaining; so the answer was l. 54 : 17 : 7, and so of any other.

Here, and in working the following examples, I have only set down the answer and remainders, the work of Division being left undone for the learner's practice and improvement.

In 6745 pence, what pounds?

20) l. s. d.
12)6745(562(28 : 2 : 1 Answer.
(1) (2)

In 5476 merks Scots, what merks Ster.?

12)5476(456 Merks Ster.
(4) Merks Scots over.

In 5678 l. Scots, what l. Ster.

12)5678(473 l. Ster.
(2) l. Scots over.

In 6400 l. Scots, what merks Ster.?

8)6400(800 Merks Ster.
(0)

In one million of shillings, what pounds?

20)1000000(50000 Pounds.
(0)

In 764 crowns, what l. Ster.?

4)764(191 l. Ster

(0)

In 679 merks Scots, what l. Ster.?

18)679(37 l. Ster.

(13) Merks over.

In 5674 pence, what guineas?

21)

12)5674(472(22 Guineas 10 s. 10 pence.

(10) (10)

In 588 fixpences, what half guineas?

21)588(28 Half guineas.

(0)

In 7840 half merks, what l.?

3)7840(2613 l.

(1) half merk over.

In 5764 lippies, what bolls?

64)5764(90 bolls 4 lippies.

(4)

In 497 pecks, what chalders?

16) *ch.* *b.* *p.*

16)497(31(1 : 15 : 1

(1) (15)

In 584 ſtone weight, what bolls of meal?

8)584(73 Bolls.

(0)

In 56480 lb. what c.w.

c.w. q. lb.

112)56480(504 : 1 : 4

28)32(1

(4)

In 580 penny weights, what lb. Troy.

12)
20)580(29(2 lb. 5 oz. Troy.

(0) (5)

In 74640 gills, what ankers of brandy at 19 pints to the anker?

19)
16)74640(4665(245 Ankers 10 pints.

(0) (10).

In 67392 nails, what pieces of cloth at 13 yards to the piece?

13)
16)67392(4212(324 Pieces.

(0) (0).

In 76745 ſquare ells, what acres of land?

40) 4) *A. r. f. e.*
36)76745(2131(53(13 : 1 : 11 : 29

(29) (11) (1)

In

In 580 inches, what yards long meaſure?

36)580(16 Yards 4 inches.

(4)

In 880000 feet, what Engliſh miles?

1760)
3)880000(293333(166 Miles 1173⅓ yards.

(1) (1173)

In 34560 acres, what ſuperficial miles?

64|0)3456|0(54 Superficial miles.

(0)

In 519 miles, what leagues?

3)519(173 Leagues,

(0)

In 120960 lib. how many tuns?

20)
112)120960(1080(54 Tuns.

(0) (0)

If 6 equal bags of pepper weigh 2418 lb. what's the weight of one bag?

28) 4)
6)2418(403(14(3 C.w. 2 qrs. 11 lib.

(0) (11) (2)

In 923585760 minutes, what years?

365 : 6
24
———
1466
730
———
8766
60
———
525960)923585760(1756 Years.
———
(0)

In 176450 cubic inches, what Scots gallons at 103 cubic inches to the pint?

8)
103)176450(1713(214 Gallons, 1 pint 11 inches.
——— ———
(11) (1)

In 6237 cubic inches, what English wine gallons?

231)6237(27 English gallons.
———
(0)

In 13536 cubic inches, what English beer gallons?

282)13536(48 English beer or ale gallons.
———
(0)

In 4059072 cubic inches, what cubic yards?

27)
1728)4059072(2349(87 Cubic yards.
——— ———
(0) (0)

In

In 87640 pence, what French crowns?

54)87640(1622 French crowns.

(52)pence over.

To divide numbers of divers denominations by Integers.

RULE.

Divide first the greater denomination, if it exceeds the Divisor; but if less, bring it lower, and then divide; and if any-thing remains, reduce it into the parts of the next inferior denomination, always minding to take in the odd parts (if any) then divide as before, and so proceed till you have gone through all the denominations given as in the following examples.

A gentleman is to distribute l.468 : 16 : 8 equally among 9 poor persons; what will each have?

```
                        l.   s.   d.
9)468 : 16 : 8(52 : 1 : 10 2/9 to each.
  45
  --
   18
   18
   --
9)16(1
   9
   --
   7
   12
   --
9)92(10
   --
  (2)
```

In

In 5079 guilders 18 ftivers, what Flemifh pounds?

$$\begin{array}{l} \qquad\qquad\qquad l. \quad s. \\ 6)\underline{5079 : 18}(846 : 13 \text{ Flemifh.} \\ \quad\ 3 \\ \quad\ \underline{20} \\ 6)\underline{78}(13 \\ \quad(0) \end{array}$$

Note, Where the divifor is fmall, the work may be done mentaly, and no figures fet down but the Remainder and Quotient.

In l. 648 : 17 : 7 Scots, what Ster. money?

$$\begin{array}{l} \qquad\qquad\qquad\qquad l. \quad s. \quad d. \\ 12)\underline{648 : 17 : 7}(54 : 1 : 5\tfrac{7}{12} \text{ Ster.} \\ \qquad (7) \end{array}$$

A man bought 2 oxen for l. 77 : 6 : 8 Scots, and one was l. 15 : 13 : 4 dearer than the other; what was the price of each ox?

$$\begin{array}{l} \quad 77 : \ \ 6 : 8 \\ \quad \underline{15 : 13 : 4} \quad l. \quad s. \quad d. \\ 2)\underline{61 : 13 : 4}(30 : 16 : 8 \text{ Price of one ox.} \\ \qquad\qquad\qquad\ \ \underline{15 : 13 : 4} \\ \quad\ (0) \qquad\quad l.\ 46 : 10 : 0 \text{ Price of the other} \end{array}$$

If 9 ftone of beef coft 19 fhillings 6 pence, what coft the ftone?

$$\begin{array}{l} \qquad\qquad s. \quad d. \\ 9)\underline{19 : 6}(2 : 2 \text{ Price of 1 ftone.} \\ \quad (0) \end{array}$$

4 C.w. 3 qrs. of tea is to be equally divided among 7 merchants; what will each have?

q. lb.
4 : 3 (2 : 20 To each.
4
———
7)19(2
———
5
28
———
7)140(20
———
(0)

32 Men drank till the reckoning came to 6 l. what did each pay?

4)6 : 00 : 0
———
8)1 : 10 : 0
———
l. 0 : 3 : 9 Paid by each.

The charge of a feast amounted to l. 32 : 12 : 6, and it was paid by 100 persons; what did each pay?

10)32 : 12 : 6
———
10) 3 : 5 : 3
———
l. 0 : 6 : $6\frac{3}{10}$ Paid by each.

What's

What's the price of 1 lb. of butter at l. 1 : 12 : 8 per hundred weight?

Divide the price of the hund. weight by 7 and 8, which multiplyed together make 56, the half hund. weight) and the half of the last Quotient is the answer.

```
8)1 : 12 : 8
-------------
7)0 :  4 : 1
-------------
2)0 :  0 : 7
-------------
  0 :  0 : 3½ Price of 1 lb.
```

What's the simple interest of l. 769 : 13 : 4 for 7 years at 5 l. per cent. per annum, when at that rate the 20th part of the principal sum is the interest thereof for one year?

```
                   l.   s.  d.
20)769 : 13 : 4(38 : 9 : 8 Interest for 1 year.
   ---                   7
     9              ---------
    20         l. 269 : 7 : 8 Interest for 7 years.
  -----
 20)193(9
  -----
    13
    12
  -----
20)160(8
  -----
   (0)
```

But if the rate of interest be either above or below 5 per cent, find the interest at 5 l. then add or subtract such parts of that Quotient, to or from what it produces at 5 l. as in the following examples.

If l. 1763 : 6 : 8 be foreborn for 9 years at 4½ l. per cent. per annum simple interest; what will it amount to?

20(1763 : 6 : 8

88 : 3 : 4 Interest at 5 l. per cent. for 1 year.
Sub. 8 : 16 : 4 $\frac{1}{10}$ of l. 88 : 3 : 4 for 10 s. over.

79 : 7 : 0 Interest for 1 year at 4½ l. per cent.
9

714 : 3 : 0 Interest for 9 years.

l. 2477 : 9 : 8 Amount of principal and interest.

What's the simple interest of l. 759 : 3 : 4 for 3 years at 6 l. per cent. per annum?

20)759 : 3 : 4

37 : 19 . 2 Interest for 1 year at 5 l.
add 7 : 11 : 10 $\frac{1}{5}$ of l. 37 : 19 : 2 for 1 l. too little.

45 : 11 : 0 Interest at 6 l. for 1 year.
3

l. 136 : 13 : 0 Interest for 3 years.

Before I proceed to the Rule of Three, I shall here lay down (for the benefit and improvement of the learner) some more practical examples on the foregoing rules.

What French crowns must I receive for l. 168 : 10 : 6 Sterling?

All questions of this nature are comprehended under that branch of Reduction which is perform'd by Multiplication and Division together: for by this method we exchange coins, weights and measures, that have not that immediate reference to one another, as those spoken of before; And for working, this is the rule,

RULE,

RULE, Bring both sides of the question into one and the same name, divide the one by the other, and the Quotient is the answer: and the remainder (if any) is of the same name with the Dividend.

OR, Find a third name that is contain'd both in the name given and in the name sought, an equal number of times; reduce the given name to that third name, and also an integer of the name sought to the said third name, divide the one by the other, and the Quotient is the answer.

168 : 10 : 6
20

4—6 3370
2 2

9) 6741 (749 French crowns.

(0)

Here, I reduced the Sterling money to sixpences, which I divided by the sixpences in 1 French crown, and the quotient gave 749 crowns for answer.

How many Portugal mill-rees may I receive for 26 l. Ster.

26
20

6:8 520
12 12

8|0)624|0(78 Mill-rees.

(0)

Here, I reduced both sides to pence.

A

A merchant at Amſterdam is indebted to a merchant at London 642 l. Ster. and would pay it in Spaniſh guilders at 2 ſhillings per piece; how many muſt the Engliſh merchant receive?

Bring your pounds to ſhillings.

```
      642
       20
    ------
  2)12840(6420 Guilders.
    ------
     (0)
```

In 672 Spaniſh guilders, what French piſtoles.
Reduce both ſides to ſixpences.

```
  17:6     672
   2         4
  ---     ----
  35  )  2688(76 Piſtoles, 14 s.
         245
         ---
         238
         210
         ---
         (28) Sixpences or 14 s.
```

In 5764 French crowns, what guineas?
Bring both ſides to ſixpences.

```
  21     5764
   2        9
  --    -----
  42 )51876(1235 Guineas 3 s.
     ---------
     (6) Sixpences or 3 s.
```

Suppoſe

Suppose a bill of exchange were accepted at London for the payment of 400 l. Ster. for the value delivered at Amsterdam in Flemish money at 1 l. 13 s. 6 d. for one pound Ster.; how much Flemish money was delivered at Amsterdam?

```
     1 : 13 : 6
     20
    ----
     33
     12
    ----
    402
     400
   ------ 20)
12)160800(13400(670 l. Flemish.
   ------  -----
    (0)     (0).
```

What Spanish patavoons at 4 s. 8 d. per piece must I receive for 6 l. 6 s. Sterling?

Bring both sides to pence.

```
        6 : 6
        20
       ----
 4:8   126
 12     12
 --    ----
 56 ) 1512(27 Patavoons.
      ------
       (0)
```

In 767 l. what merks? In 1150½ merks, what l.?

Bring both to half merks.

```
  767                    1150½
    3                        2
 -----                   -----
2)2301(1150½ Merks.     3)2301(767 l.
 -----                   -----
  (1)                     (0)
```

In 672 ells English, what yards?

672
5

4)3360(840 Yards.

(0)

In 672 yards, what ells Flemish?

672
4

3)2688(896 Ells Flemish.

(0)

In 6780 l. Ster. what guineas, crowns, shillings, sixpences, half crowns, and half guineas; and of each a like number?

Bring both sides to sixpences.

42
10
2
1
5 6780
21 40

81)271200(3348 of each 6 s. over,

(12) Sixpences or 6 s.

In 720 l. Ster. what guineas?

Bring both to shillings.

720
20

21)14400(685 Guineas 15 s.

(15)

In

In 640 l. Scots, what guineas.

Bring both to pence.

```
21      640
12       20
---    -----
252  )12800(50 Guineas 16 s. 8 pence
      -----
       12)200(16 s.
          ---
          (8) Pence,
```

In 684 guineas, what l. Ster.?

```
        684
         21
       ----
        684
      1368
     ------   l.    s.
   20)14364(718 : 4 Ster.
      -----
       (4)
```

In 540 guineas, what l. Scots?

```
      540
       21
     ----
      540
    1080
    -----
    11340
       12
   ------
20)136080(6804 l. Scots.
   ----
   (0)
```

In

In 527 moidores at 27 s. a piece, what l. Ster. and guineas?

```
 527            527
  27             27
----           ----
3689           3689
1054           1054
----   l.  s.  ----
20)14229(711 : 9 Ster.21)14229(677 Guineas 12 s.
----                   ----
 (9)                   (12)
```

In 711 l. 9 s. what moidores?

```
      711 : 9
        20
    ---------
  27)14229(527 Moidores.
     ------
      (0)
```

In 58 yards of 5 quarters broad cloth, what square yards?

```
       58
       20 Qrs. in one yard.
     ----
  16)1160(72 8/16 or 1/2 square yards.
  ------
       (8) Square quarters, or, 1/2 square yard.
```

There is a stone dyke 7½ quarters high and 320 falls long; what roods of mason work doth it contain?

Bring both to square half quarters, multiply the one by the other, and divide by the sqr. half qrs. in 1 ell.

```
                              320
                                6
                            -----
                             1920 Ells.
                                8 Half qrs. in one ell.
                            -----
                       8    15360 Length in half qrs.
                       8       15 Height in half qrs.
                      --    -----  36)
Sqr.half qrs.in 1 ell 64)230400(3600 (100 Roods.
                          ------  ----
                           (0)     (0)
```

If the height of a stone dyke be 7½ quarters, what length will go to make one rood?

Bring the rood to square half quarters, and divide by the square half quarters upon 1 ell's length.

```
           36
           64 Square half quarters in 1 ell.
        -----
    15    144
     8    216
    --   ----
   120)2304(19 24/120 or 1/5 ells, answer.
        ---
        (24)
```

There is a piece of muir whose length is 200 falls, and breadth 80 falls; how many firrs at five ells distance every way 'twixt tree and tree will plant the said muir?

200
6

5)1200(240 times 5 ells in the length.

(0)

80
6

5)480(96 times 5 ells in the breadth.

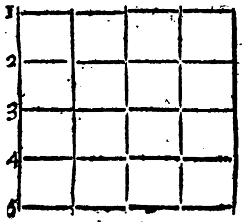
240 more by 1, is 241
96 more by 1, is 97

1687
2169

Anſwer 23377 firrs.

To convince one of the foregoing operation, let it be demanded how many kail plants will plant a ſquare yard of ground at one quarter's diſtance 'twixt plant and plant every way.

The ſide of the yard is 4 quarters, ſo that one would conclude 16 ſhould plant the ground; but to find the number required, you muſt add 1 to the ſide, and the ſquare of the ſum will be 25 plants, as appears by what here is ſubjoin'd.



I am

I am to let out a square piece of ground containing $5\frac{1}{4}$ acres, for sowing lint seed in; I am to allow 12 ells the one way and 10 ells the other for the lippie; how many pecks may be sown therein?

160 Falls in 1 acre.
5

800 Falls in 5 acres.
40 Falls in $\frac{1}{4}$ acre.

840 Falls in $5\frac{1}{4}$ acres.
36 Ells in 1 fall.

12 5040
10 2520

4)
12|0)30241|0(252(63 Pecks, answer

(0) (0)

A merchant hath 8 c.w. 3 qrs. 20 lb. of tea, which he orders his servant to make up into parcels of 3, 4, 5, 6 and 7 lb. per parcell, and of each alike number; how many bundles of each will he have?

8 : 3 : 20
4

3 35
4 28
5
6 280
7 72

25)1000(40 Bundles of each.

(0)

In

In 508 English miles what Scots ones, if 37 English miles are equal to 33 Scots miles?

```
       508
        33
     -----
      1524
     1524
     -----
37)16764(453 3/37 Scots miles.
   -----
     (3)
```

Note, Scots miles are reduced to English ones by multiplying by 37, and dividing by 33.

In 780 Scots ale gallons, what English ale gallons?

```
           780
             8
        ------
          6240 Pints.
           103
        ------
         18720
        6240
        ------
282)642720(2279 English gallons.
    ------
      (42) Cubic inches remaining.
```

Note, If you are to bring English gallons to Scots ones, multiply by 282, and divide by 103.

In 231 guineas, what merks Scots?

Multiply by 189 the 16 penny pieces in a guinea, divide by 10 the 16 penny pieces in a Scots merk, the Quotient is merks, and the remainder so many 16 penny pieces.

231

```
      231
      189
     ----
     2079
    4158
   ------
10)43659(4365 9/10 Merks Scots.
   ------
     9
```

Note, If you are to bring merks Scots to guineas, multiply by 10, and divide by 189.

In 54 guineas, what crowns and merks Ster.?

```
    54                      54
    21                      21
   ---                     ---
    54                      54
  108                     108
 -----                   -----
5)1134(226 4/5 Crowns.    1134
  ----                      12
   4                     -----
                    160)13608(85 Merks Ster.
                        -----
                         8 Pence over.
```

If you are to bring crowns to guineas, multiply by 5, and divide by 21.

If you are to bring merks Ster. to guineas, multiply by 160, the pence in 1 merk Ster. and divide by 252 the pence in a guinea.

In 96 Scots ale pints, what English ale gallons?

```
        103
         96
      -----
        618
       927
      -----
282)9888(35 English ale gallons.
      -----
         18 Cubic inches over.
```

In 745 crowns, what merks Scots?

Bring the crowns to half merks.

```
        745
          9
     ------
   2)6705(3352½ Merks Scots.
     ------
        1
```

In 3352½ merks Scots, what crowns?

```
        3352½
            2
       ------
     9)6705(745 Crowns.
       ------
           0
```

What moidores may I give in exchange for 120 guineas?

```
        120
         21
      -----
  27)2520(93 Moidores 9 shillings.
      -----
        9
```

In

In 370 English acres what Scots acres, when 3844 English acres make 3025 Scots ones?

3025
370
―――
21175
9075
―――
3844)1119250(291$\frac{646}{3844}$ Scots acres.
―――
646

If you are to bring Scots to English acres, multiply by 3844, and divide by 3025.

In 376 French ells, what yards?

376
6
―――
4)2256(564 Yards.
―――
0

In 5766 c.w. 54 lb. of lead, what fodders at $19\frac{1}{2}$ c.w. per fodder?

Bring both to lbs.

19 : 56 5766 : 54
112 112
――― ―――
44 11536
214 63431
――― ―――
2184)645846(295 Fodders.
―――
1566 lb. over.

In

In 18 English miles, what geometrical paces?

Bring both to feet.

1760
18

31680
3

Feet in a pace 5)95040(19008 Paces.

0

A gentleman delivers to a gold smith 16 lb. 3 oz. 14 p.wts. 18 grains of silver, to be made into spoons of 9 oz. 14 p.wts. per piece, salts of 5 oz. 12 p.wts. 6 grains per piece, forks of 4 oz. 9 p.wts. per piece, and into knives of 8 oz. 4 p.wts. per piece, and of each alike number; how many of each will he have?

oz. p.w gr.	lb. oz. p.w
9 : 14 : 0	
5 : 12 : 6	
4 : 9 : 0	16 : 3 : 1
8 : 4 : 0	12
27 : 19 ; 6	195
20	20
559	3914
24	24
2242	15664
1118	7829
13422	) 93954(7 of each.

0

In

In 58 lb. Troy, what lb. Averdupoiſe?

A lb. Averdupoiſe is to a lb. Troy as 60 to 73, therefore multiply by 60, and divide by 73, the Quotient is the anſwer.

$$\begin{array}{r} 58 \\ 60 \\ \hline 73)3480(47\frac{49}{73} \text{ lb. Adverdupoiſe.} \\ \hline 49 \end{array}$$

If you are to bring lb. Adverdupoiſe to lb. Troy, multiply by 73, and divide by 60.

In 9 ounces Troy, what ounces Averdupoiſe?

The ounce Averdupoiſe is to the ounce Troy, as 80 to 73, therefore multiply by 80, divide the product by 73, and the Quotient is the anſwer.

$$\begin{array}{r} 9 \\ 80 \\ \hline 73)720(9\frac{63}{73} \text{ Ounces Adverdupoiſe.} \\ \hline 63 \end{array}$$

If you are to reduce ounces Adverdupoiſe to ounces Troy, multiply by 73, and divide by 80.

In 176 c.w. 2 qrs. 24 lb. of ſugar at Jamaica (the c.w. being 100 lb.) how many c.w. at London, the c.w. being 112 lb.?

```
          176 : 2 : 24
            4
        ------
          706
           25
       -------
         3534
        1414
       --------
112)17674(157 C.w. 3 qrs. 6 lb.
       --------
         90
          4
       ------
112)360(3
       ------
          24
          28
       ------
         192
         48
       ------
  112)672(6
       ------
          0
```

In 56 boxes of ſugar each 2 c.w. 3 quarters, how many c.w.?

Anſwer 154 c.w.

In 46 packs of cloth, each pack 24 pieces, and each piece 42 ells Flemiſh; what Engliſh yards?

Anſwer 34776 yards.

A

A captain of a vessel with 9 sailors, and his cabbin boy, having made a prize, which they sold for 275 l. 14 s. 6 d. agreed to divide it after this manner, (viz.) that the captain should have as much as 4 men (whose shares were to be equal) and every man to have just twice as much as the cabbin boy; I demand their respective shares?

As there is but 1 captain, multiply 1 by 4 men, to the product add 9 the number of sailors, multiply that sum by 2 (because 1 sailor hath twice as much as the boy) to that product add 1 (for 1 boy) and the prize divided by this sum quotes the boy's part, multiply the boy's part by 2, and the product is one sailor's part, which multiplied by 4 gives the captain's part, as followeth.

```
     1
     4
    --
     4
     9
    --
    13
     2
    --
    26
     1
    --               l.    s.    d.
  27)275 : 14 : 6(10  :  4  :  2 24/27 Boy's part.
     ---                          2
       5            ------------------
      20            20  :  8  :  5 21/27 One sailo.'s part.
   27)114(4                       4
      ---           ------------------
       6           81  : 13  : 11 3/27 Captain's part.
      12
      ---
   27)78(2
      ---
      24
```

If

If on a plane the ſhadow of a ſtaff, ſtanding upright 3 feet above ground, makes 5 feet; and the ſhadow of a ſteeple on the ſaid plane makes 90 feet; what's the height of the ſteeple in yards?

3
90
———   3
5)270(54(18 Yards, Anſwer.
——— ——
 0   0

What ſum of Iriſh money muſt be remitted from Dublin for l. 475 Sterling delivered at London, exchange at $8\frac{1}{2}$ per cent. that is when 100 l. Ster. is equal to $108\frac{1}{2}$ l. Iriſh?

475
$8\frac{1}{2}$
———
3800
add 237 : 16 for $\frac{1}{2}$
——————
100)4037 : 10 (40 : 7 : 6 Iriſh advance,
——————   475 : 0 : 0
37   ——————
20   l. 515 : 7 : 6 Iriſh, anſwer.
———
100)750(7
———
50
12
———
100)600(6
———
0

Suppoſe Dublin remitts to London l. 515 : 7 : 6 to receive 100 l. Ster. for every l. $108\frac{1}{2}$; what ſum muſt be receiv'd in London?

Anſwer 475 l.

What

What number deducted from the 26th part of 2262, will leave the 87th part of the same?

26)2262(87 The 26th part of 2262.
26 The 87th part thereof.

61 Number required.

A chapman breaking, owes to A, 48 l. to B, 60 l. to C, 68 l. and to D, 24 l. how much must each have, if he be found but worth 50 l. and what will it be per pound of his debt?

48
60
68 50
24 20

200)1000(5 Shils. per pound, which is but ¼ part.

4)48(12 l. to A. 4)60(15 l. to B. 4)68(17 l. to C.
4)24(6 l. to D.

Divide 140 into two parts, so as the product of these two parts when multiplied into one another, may be equal to the product of 56 multiplied by itself.

140
2)56(28 Lesser part.

0 112 Greater part.

There are two numbers whose sum is 67, and their difference 17; what are these numbers?

67
17
— 67
2)84(42 Greater number.

25 Lesser number.

Two

Two men A and B, are to purchase a house worth 1200 l. A says to B, If you give me $\frac{2}{3}$ of your money, I can buy the house alone; but says B to A, If you give me $\frac{3}{4}$ of your money, I can purchase the house alone; what money had each?

l.
2)1200(600 B's money.
2
1200
3)1200(400.
0 l. 800 A's money.

A wright with his two servants undertook to wainscot a gentleman's house in 52 days; the master was to have 35 pence, the one servant seven pence, and the other 5 pence a day; when the work was finished, every one had just the same sum of money to receive; how many days was each at work?

35 35 52
7 7 7
42 245 364
5 210 5
210 455)1820(4 days the master was at work.
0

35
4
7)140(20 days the journeyman was at work.
0

35
4
5)140(28 days the other servant was at work.
0

A

A captain sends out $\frac{1}{3}$ of his soldiers and 10 more, and then there remain'd with him $\frac{1}{2}$ of them and 15 more; how many soldiers had he?

```
 10      15
  6       6
 ---     ---
 60      90
         60
        ---
        150 Soldiers.
```

There is an army to which, if you add $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ of itself, and take away 5000, the sum total will be 100000; what was the number of the army?

```
   100000      5000
       24        24
  -------    ------
  2400000    120000
   120000
  -------
70)2520000(36000 Number of the army.
   -------
         0
```

Divide 100 into two parts, so that the greater part being divided by 3, and the lesser by 5, the sum of the two Quotients may be equal to 30 or any other number?

```
  5         100
  3           5
 --         ---
 15     5   500
 30     3   450
 ---    ------- 100
 450    2 ) 50 ( 25 Lesser.  }
            --   --          } Answer.
             0   75 Greater  }
```

Divide

Divide 64 into two such parts, so that the greater part being divided by 6, and the lesser by 4, the two Quotients may be equal?

$$
\begin{array}{ll}
6 & 64 \\
4 & 6 \\
\hline
 & 64 \\
10)384(38\tfrac{4}{10} \text{ or } \tfrac{2}{5} & \text{Greater part.} \\
\hline
4 \quad 25\tfrac{6}{10} \text{ or } \tfrac{3}{5} & \text{Lesser part.}
\end{array}
$$

There are three numbers A, B and C, now if to A, you add 209 (or any other number) the sum will be equal to the sum of B and C; and if to B, you add 209, the sum will be equal to double the sum of A and C; and if to C, you add 209, the sum will be equal to triple the sum of A and B; what are the three numbers A, B and C?

Divide the number given to be added to each of the three numbers required by 11, and the Quotient is the number A; divide the Quintuple of the said number by 11, and the Quotient is the number B; lastly divide the Septuple of the said number by 11, and the Quotient is the number C.

11)209(19 A.	209	209
	5	7
0	11)1045(95 B.	11)1463(133 C.
	0	0

There are two numbers, A, the greater, and B, the lesser; now if you multiply A, by 10, and B, by 6, the sum of the two products will be 228; also, if you multiply A, by 4, and B, by 2, the difference of these two products will be 56; what are the two numbers A and B?

10

```
10     4  228     56
 2     6    4     10
--    --  ---    ---
20    24  912    560
      20  560
      --  ---
      44)352(8 Number B.
          ---
           0

    56        228
     6          2
   ---        ---
   336        456
              336
              ---
          44)792(18 Number A.
              ---
               0
```

I bought a certain number of yards of cloth for l. 174 Scots, and found that 3 yards of the same, cost as much above 4 l. as 7 of them cost under 11 l. Scots; how many yards did I buy, and what did I pay for the yard?

```
yds.  l.
  3   11
  7    4
 --   --  l.   s.
 10) 15(1  :  10 Price of 1 yard.
     ---
      5
     20
    ---
 10)100(10
    ---
     0

 l.  s.    l.
 1 : 10   174
 20        20
 --       ---
 30 ) 3480(116 Yards.
      ----
        0
```

A, B and C, owe each a certain ſum of money, ſo that A and B, together owe 210 crowns, B and C, 290 crowns, and C and A, 400 crowns; what did each owe?

210	400
290	290
——	——
500	690
400	210
——	——
2)100(50 Crowns, B's debt.	2)480(240 Crowns C's debt.
——	——
0	0

400
210
——
610
290
——
2)320(160 Crowns, A's debt.
0

There are two numbers (ſuppoſe 15 and 30) and it is required to find a third number, which added to the leſſer of the two given numbers, and the ſum multiplied by the ſaid third number (to be found) the product may be equal to the ſquare of the remainder, when the ſum of the leſſer number and number to be found, is ſubtracted from the ſum of the two given numbers; what is the third number?

30
2
——
60 30
15 30
—— ——
75)900 (12 Number required.
0

A

A gentleman who had several sons, finds that his youngest son was as many years old as he had sons; the common difference of their ages was 3 years, and the eldest was 49 years old; how old was the youngest, or how many sons had he?

3 49
1 3

4)52(13 Years age of the youngest,
and consequently he had 13 sons.
0

Divide 20 (or any other number) into 4 parts, so that the second part may be 2 more than the first, the third 3 more than the first, and the fourth 7 more than the first?

2 20
3 12
7
4)8(2 First or least part, consequently
12 the second will be 4, the third 5,
0 and the fourth 9.

Two merchants A and B, were sharers in a parcel of merchandize, in the purchase of which B, laid out 420 crowns; and they having sold this commodity, find their clear gain to be 854 crowns, whereof A, receives 52 crowns; what did A lay out in the purchase, and what was B's part of the gain?

420
52

854 840
52 2100

B's gain crowns 802)21840(27 $\frac{186}{802}$ or $\frac{93}{401}$ Crowns
laid out by A.
186

If

If two post boys, A and B, (being in the morning 59 miles asunder) set out to meet each other, and A goes 7 miles in 2 hours, and B 8 miles in 3 hours, and B, begins his journey 1 hour later than A; how far will A, have gone before he meets B?

```
                    7       7
                    3       8
                   ---     ---
                   21      56
                   59       1
                   ---     ---
            7     189      56
  8         3     105
  2        ---    ----
 ---       21    1239
 16----16          56
          ---    ----
          37)1295(35 Miles, answer.
             ----
               0
```

If a man gains 30 crowns a week; how much must he spend a week to have 500 crowns together, with the expence of 4 weeks remaining at the year's end?

```
           52
            30
          ----
     52   1560
      4    500
     ---  ----
     56 )1060(18 52/56 or 13/14 Crowns, answer.
          ----
          52
```

A vintner hath two sorts of wine, viz. A and B, which if mixed in equal parts, a pint of the mixture will cost 15 pence; but if they be mixed so as to take 2 pints of A, as often as you take 3 pints of B, a pint of that mixture will cost 14 pence; what was the price of a pint of each wine single?

		15
15	14	2
4	5	
60	70	30
	60	10
		20 Pence, value of a pint of A.
	10 Pence, value of a pint of B.	

Two men have each a sum of money, the one hath four times as much as the other; both their sums put together will not make 100 l. but if they be doubled, and 30 l. be taken from that double, the remainder will be twice as much above 100 l. as their sum before wanted of 100 l.; what sum had each man?

	l. *s.*
100	4)66(16 : 10 : The other's sum.
3	
	2
300	20
30	
	4)40(10
5)330(66l. One's sum.	
	0
0	

A gentleman hires a ſervant, and promiſes him 24l. for a year's ſervice and a livrey coat; at 8 months end the ſervant obtains leave to go away, and for his wages, receives 13 l. and the livery coat, which was his full due for that time; what was the coat valued at?

```
           24       13
            8       12
          ----     ----
   12    192      156
    8    156
  ---   ----
     4 ) 36 (9 l. Value of the coat.
         ---
          0
```

A traveller goes 6 miles a day, and after he had gone 56 miles, another follows him who travelled 8 miles a day; in how many days will he come up to him?

```
        8
        6
       --
       2)56(28 Days, anſwer.
         ---
          0
```

A certain man hires a labourer on this condition, that for every day he wrought he ſhould receive 12 pence, but for every day he was idle he ſhould forfeit 8 pence: when 390 days were paſt, neither of them were indebted to one another; how many days did he work, and how many was he idle?

```
      8    390
     12      8
     --   ----  390
  2|0)312|0(156 Days he wrought.
      ----  ----
        0    234 Days he was idle.
```

A

A general diſpoſing his army into a ſquare, finds he has 284 ſoldiers over and above; but increaſing each ſide with one ſoldier, he wants 25 ſoldiers to fill up the ſquare; how many ſoldiers had he?

284
25
———
309
Sub.—1
———
2)308(154
——— 154
0 ———
616
2310
———
23716
284
———

Anſwer 24000 Soldiers.

What number is that which if added to itſelf, and the ſum multiplied by the ſame (viz. by the number to be found) and the ſame number ſubtracted from the product, and laſtly the remainder divided by the ſame, the Quotient may be equal to any given number ſuppoſe 37?

37
1
——
2)38(19 Number required.
——
0

What

What number is that, to which if you add its double, treble and quadruple, the ſum will be equal to the product, when the ſaid number to be found is multiplied on itſelf?

2
3
4
1

10 Number required.

Suppoſe a clock hath two indices A and B, and that A, is carried 24 times round the circumfrence of the dial in 24 hours, and that B, is carried once round in 12 hours, and that both at once ſhewing the ſame point began to be moved; in what time will they be again conjoined?

24 24
11 12

264)288(1 $\frac{24}{264}$ or 1 $\frac{1}{11}$ hour.

24

Two ſhips A and B, loaden with the ſame ſort of wine, and ſailing by a paſs, were obliged to pay toll according to the quantity of wine they had on board: A had on board 250 hogſheads, out of which ſhe paid 1 hogſhead and 36 ſhillings more: B, had on board 400 hogſheads, out of which ſhe paid 2 hogſheads, and received back 20 ſhillings? what was the wine valued at per hogſhead?

400	250	36	250
1	2	400	20
400	500	14400	5000
	400	5000	

1100) 194100(194 Shillings, value of one hogſhead.

0

A

A father left among five sons an estate worth 500 l. in cash, with 5 bills each of l. 48 : 10 : 6; he ordered 20 l. to be bestowed on his burial, and his debts to be paid amounting to 164 l. Then his free estate to be divided in this manner, viz. the eldest son to have the 3d. part, and the other 4 sons to have equal shares; what is the share of each son?

```
        l.
        164
         20
        ----
        184 Debts and burial charges.
      l.     s.   d.
      48 :  10 :  6
                  5 Bills.
    -----------------
     242 :  12 :  6
     500 :  00 :  0 Cash.
    -----------------
     742 :  12 :  6 Total.
     184 :  00 :  0 Deduced.
   3)558 :  12 :  6 Free estate.
     186 :   4 :  2 Eldest son's portion.
    -----------------
   4)372 :   8 :  4 Remains.
      93 :   2 :  1 The shares of each of
                      the other 4.
```

One spends $\frac{3}{5}$ of his money and 70 crowns more, and he had yet remaining unspent 220 crowns; how many had he at first?

```
        220       70
          5        5
       ----     ----
  5    1100      350
  3     350
  --   ----
  2)1450(725 Crowns he had at first.
    ----
       0
```


What

What number is that which if you multiply by 3, and subtract 5 from the product, and divide the remainder by 2, and add the number to be found to the Quotient, the sum will be 40?

$$\begin{array}{rr} & 40 \\ & 2 \\ \hline 3 & 80 \\ 2 & 5 \\ \hline 5\,) & 85\,(17 \text{ Number required.} \\ & \hline & 0 \end{array}$$

A man bought a piece of cloth from a merchant for 70 shillings, and desiring the merchant to tell him what he gain'd thereby, he said he gain'd $\frac{1}{4}$ of what it cost him; what was the prime cost of that piece of cloth, and what did the merchant gain?

$$\begin{array}{rr} 4 & 70 \\ 1 & 4 \\ \hline & \quad 70 \\ 5\,) & 280\,(56 \text{ Shillings, prime cost.} \\ \hline & 0 \quad 14 \text{ Shillings, the gain.} \end{array}$$

200 l. is to be divided between two men, so as the one may have 73 l. more than the other; what will each man have?

$$\begin{array}{r} 200 \\ 73 \\ \hline \quad 200 \\ 2)273(\,136\tfrac{1}{2} \text{ Greater part.} \\ \hline 1 \quad \text{l. } 63\tfrac{1}{2} \text{ Lesser part.} \end{array}$$

Suppose

Suppoſe a gray hound to be courſing a hare, in ſuch ſort that the hare takes 5 leaps for every 4 leaps of the gray hound, and that the hare is 100 of her own leaps diſtant from the gray hound: now, if 3 of the gray hound's leaps be equal to 4 of the hare's leaps; how many leaps muſt the gray hound take before he catch his prey?

```
 3      4   100
 5      4     3
--     --   ---
15     16   300
       15     4
       --   ---
        1)1200(1200 Leaps, anſwer.
          ----
             0
```

A ciſtern is ſupply'd with water by two pipes A and B, A, alone will fill it in 20 hours, and B, will fill it alone in 30 hours; in what time will both the pipes fill it together?

```
20   20
30   30
--   ---
50)600(12 Hours, time required.
   ---
     0
```

A ciſtern is ſupply'd with water by two pipes A and B, which running together fill the ciſtern in 12 hours, and the pipe A, alone fills it in 20 hours; in what time will the pipe B, alone fill it?

```
20   20
12   12
--   ---
 8)240(30 Hours, anſwer.
   ---
     0
```

CHAP.

CHAP. VII.

Of PROPORTION DISJUNCT; commonly called the GOLDEN RULE.

PROPORTION DISJUNCT is either Direct or Inverse: And these are both single and compound.

This rule is also called the RULE of THREE, because by three numbers given we find a fourth number sought; which, when the proportion is direct, must always bear the same ratio or proportion to the third number, as the second bears to the first. Consequently, the greater the second term is, in respect to the first; the greater will the fourth term be, in respect to the third. As in these.

$$4 : 8 :: 6 : 12.$$

That is, as 8 the second term is twice as much as 4 the first term: so is 12 the fourth term the double of 6 the third term.

Whence it follows, that if 4 numbers are in direct proportion, the product of the first and last will always be equal to the product of the second and third.

Again, the less the second term is in respect to the first; the less will the fourth term be in respect to the third. As in these.

$$48 : 12 :: 8 : 2$$

Here

Here, 'tis plain that 12 the second term is but $\frac{1}{4}$ of 48 the first term; and so is 2 the fourth term $\frac{1}{4}$ of 8 the third term; and consequently are true proportionals.

Note, Any four numbers in direct proportion may be varied several ways. As in these.

viz. 48 : 12 :: 8 : 2 or, 12 : 48 :: 2 : 8.
and 2 : 8 :: 12 48 or, 8 : 2 :: 48 : 12, &c.

But if the proportion be Inverse, then the fourth number must always bear the same ratio to the second as the third does to the first, in an inverse order: That is, the greater the third term is in respect to the first; the less must the fourth be in respect to the second;

Or, the less the third term is in respect to the first; the greater will the fourth term be in respect to the second. As in these.

8 : 12 :: 16 : 6.
16 : 6 :: 8 : 12.

That is (in the first stating) as 16 the third term is double the first term 8; so is 6 the fourth term but half the second term 12.

And (in the second stating) as 8 the third term is but half 16 the first term; so is 12 the fourth term double of 6 the second term.

Whence it follows, that if four numbers are in reciprocal proportion, the product of the first and second will always be equal to the product of the third and fourth.

Note, As in direct proportion, so in reciprocal, any four numbers may be varied several ways. As in these.

viz. 8 : 12 :: 16 : 6 then 12 : 8 :: 6 : 16
and 16 : 6 :: 8 : 12 or, 6 : 16 :: 12 : 8 &c.

These

These variations being well understood, will be of no small use in stating of any question in the Rule of Three.

When three numbers are given, and it is required to find a fourth proportional; the greatest difficulty (if there be any) will be in the right stating the question, or abstracting the numbers out of the words of the question, and placing them down in their proper order.

Observe then that two of the three given terms are only supposed, and assigned to limit the ratio or proportion. The third contains a demand, and moves the question; and the fourth gives the answer.

As for instance, if 4 bolls of meal cost 24 l. what will 10 bolls cost at the same rate or proportion?

Here 4 bolls and 24 l. are two supposed numbers that imply the rate; as appears by the word (if) (then comes the question) what will 10 bolls cost?

RULE for stating and working.

The term which moves the question, or on which the demand lyes, must always be the third in your stating. And the said term is easily known, as it generally hath some of these words before it, viz. *What will? How far? What cost? How many? How long? How much? What time?* &c.

Of the other two numbers, you will always find one of the same quality with the said third, which being made your first in stating, the number left must of consequence fall in the second place, and is always of the same quality with the fourth number sought.

The terms being thus stated, whether the Proportion be Direct or Inverse, you must (if they are not so already) bring your first and third numbers into one name, and your second term (if of several denominations) into its lowest given name; then (if the proportion be direct) multiply your second and third numbers together, divide the product by the first term, and the Quotient is the fourth number and answer to the question in the same denomination you brought

your

your second number to, which (if too low) you may reduce to a higher name; and if any thing remains after your first division, if you judge it can make any thing more, you may manage it as directed in Division. But if the proportion be inverse, then multiply your first and second numbers together, divide the product by the third, and the Quotient is the fourth number, and answer to the question, as before.

Followeth a Rule to know when the Proportion is DIRECT and when INVERSE.

If (after the terms are stated) the third number is more than the first, and requires more; or if it is less, and requires less, the proportion is Direct. But if the said third term is more than the first, and requires less; or if it is less, and requires more, the proportion is Inverse.

I might have here laid down some other Theorems for answering all questions in Direct and Reciprocal Proportion, but as it's improper to confuse the learner with too many rules, I am only to use the immediately prescrib'd one, (it being most general, and usually practised) and so I proceed to illustrate what hath been said on the Rule of Three, in the work of the following examples.

If 4 bolls of meal cost 24 l. what will 10 bolls cost at that rate?

The numbers being ranked according to the directions given, they will stand thus:

b.		*l.*		*b.*
4	:	24	:	10

Here the proportion is certainly direct; for the last term 10 is more than the first term 4, and requires more money; therefore, according to the given Theorem, work as follows.

B.

B.		l.		b.
4	:	24	:	10
		10		

4)240(60 l. Price of 10 bolls.

0

Here (according to rule) I multiplyd the second and third numbers together, viz. 24 and 10, and the product was 240, which I divided by 4 the first number, and the Quotient was 60, which were so many pounds; because the middle number was pounds; and that was the answer to the question.

PROOF.

The proof of all questions in the Rule of Three, when the proportion is direct, may be easily conceived from what hath been already said, viz. That the product of the first and fourth terms, must always be equal to the product of the second and third. Therefore multiply your first by your fourth number found, and comparing the product with that of your second and third terms, if the two products agree, the work is right. Thus, in the foregoing example, the product of 60 (the fourth number) and 4 (the first number) is 240, which is equal to the product of 10 and 24, the second and third numbers.

And from hence arises the invention of the foregoing rule, for finding the fourth number; for if the second number multiply'd by the third, be equal to the first multiply'd by the fourth, it is plain that if the product of the second and third be divided by the first, the Quotient must be the said fourth number; because every Dividend must be equal to the product of it's Divisor and Quotient, the remainder (if any) being taken in, when you multiply.

Questions

Questions in this rule may be also proved by a reverse stating; thus reversing the former example.

If 60 l. buy 10 bolls of meal; how many bolls will 24 l. buy at that rate?

l. *b.* *l.*
60 : 10 :: 24
24

6|0)24|0(4 Bolls, answer.

0

Again, if 10 bolls cost 60 l. what cost 4 bolls?

b. *l.* *b.*
10 : 60 :: 4
4

10)240(24 l. Answer.

0

And, if 24 l. buy 4 bolls; what quantity will 6 l. buy?

l. *b.* *l.*
24 : 4 :: 60
60

24)240(10 Bolls, answer.

0

Note, When your Divisor happens to be an unit, the answer is found by Multiplication only: and if your Multiplier or second number be an unit, the answer will be found by Division only; as in the two following examples.

If 1 ell of cloth cost 7 shillings; what cost 12 ells?

e. *s.* *e.*
1 : 7 :: 12
12

Price 84 shillings, or l. 4 : 4.

If 12 ells cost 84 shillings; what cost 1 ell at that rate?

e. s. e.

The terms are stated thus; 12 : 84 :: 1

And the work will stand thus;

12)84(7 Shillings, answer.
——
0

What will 4 bolls 3 lippies of meal cost; when I pay l. 5 : 12 Scots for 3 firlots 3 pecks of the same meal?

```
 f.  p.   l.  s.      b.  l.
 3—3  :   5—12  ::   4—3
 4         20         4
 —        ——         —
 15       112        16
  4                   4
 —                   ——
 60                  64
                      4
                     ——
                     259
                     112
                     ———
                    3108
                    259
                   ————  20)   l.   s.  d.
             60)29008(483(24 : 3 : 5 36/60 or 3/5
                240      ——   The answer.
                ———       3
                 500
                 480
                 ———
                  208
                  180
                  ———
                   28
                   12
                  ———
                60)336(5
                   300
                   ———
                    36 Remainder.
```

Here,

Here, I brought (according to the given rule) my firſt and laſt terms to one name, viz. both to lippies; and my ſecond term to ſhillings, its loweſt given name: then as the proportion was direct) I multiplied the ſecond and third terms together, and (dividing the product by the firſt) I found the Quotient was 483 ſhil. (for it is always of the ſame name you brought the ſecond term to) and 28 remained. I then divided 483 s. by 20, and they made l. 24, and 3 s. over. Laſtly I multiply'd the remainder 28 by 12, in order to bring it to the next inferior denomination. And, dividing the product by 60 (the former Diviſor) there came out 5 pennys more, with a remainder of 36, which I placed over the Diviſor on the right hand of the 5 pennys, thus $5\frac{36}{60}$, or it's loweſt terms $5\frac{3}{5}$; and ſo the true anſwer to the queſtion was l. 24 : 3 : $5\frac{3}{5}$.

Note, Queſtions in this rule frequently need preparation either by Addition, Subtraction, &c. before the terms can be ſtated in their proper order; as in ſome of the following examples.

If 3 ells of cloth coſt l. 2 : 13 : 6; what coſt 9 pieces of the ſame, when each piece contains 17 ells 3 quarters?

```
                    e.  q.
                    17—3
                        9
e.  l.  s.  d.    ———————
3 : 2—13—6 :: 159—3
4   20             4
—   ——          ————
12  53            639
    12            642
  ————          —————
   642           1278
                2556
               3834
              ———————  12)     20)     l.   s.  d.
          12)410238(34186(2848(142 : 8 : 10 6/12 or 1/2
              ———————  ——————  ——————  The anſwer.
                6.       10.     8.
```

If

If I buy at 8 s. 6 d. and sell at 10 s. 6 d.; what do I gain per cent. or on the 100 l.?

```
                  s.  d.
                  10—6
 s.  d.            8—6
 8—6              ————    l.
 12               2—0 :: 100
————                      240
102                     —————
                        24000
                          2
                 ——————————— 20)  l.  s.
                 102)48000(470(23 : 10 : 7 6/102 or 1/17
                     408      ———— Gain per cent.
                    ————       10
                      720
                      714
                     —————
                        60
                        12
                      ————
                 102)720(7
                     714
                     ————
                        6
```

Three

Three perfons in company paid by equal portions l. 68 : 14 : 6 for 144 ells of cloth, and fold every 6 ells of the fame for l. 3 : 13 : 4; I demand their refpective fhares of the money, and gain?

3)144(48 ells for each.
———
0

l. s. d.
3)68—14—6(22 : 18 : 2 is $\frac{1}{3}$ of the prime coft.
————
0

e. l. s. d. e.
If 6 : 3 —13—4 :: 48
20
——
73
12
——
880
48
——
7040
3520
——— 21) 20) l. s. d.
6)42240(7040(586(29: 6:8 Each man's fhare.
——— ——— ———22:18:2
0. 8. 6. ————
l. 6: 8:6 Each man's gain.

How much 3 quarters broad ferge will be fufficient to line coats for 600 foldiers; when each coat contains 3 yards of 5 quarters broad cloth?

yds.
600
3
q. —— q.
The terms are ftated thus, 5 : 1800 :: 3 and the queftion

question will now run thus; How much ſerge 3 qrs. broad will line 1800 yards of 5 qrs. broad cloth?

Here the proportion is inverſe, for the laſt term 3 is leſs than the firſt term 5, and requires more; for if the ſerge was as broad as the cloth, the ſame yards of ſarge, as of cloth, would line their coats: but here, the ſarge is narrower than the cloth, and conſequently there muſt be more yards of ſerge than of cloth. See the work,

q.	yds.	q.
5 :	1800 ::	3
	5	

3)9000(3000 Yards of ſerge, anſwer.

0

This and any other queſtion that falls under reciprocal proportion, may be ſo ſtated as to have its terms in direct proportion; by only changing the places of the firſt and third terms in the queſtion, thus,

q.	yds.	q.
3 :	1800 ::	5
	5	

3)9000(3000 Yards of ſerge, as before.

0

If I buy 36 bolls of oats for 216 l. and have 32 bolls 3 firlots of meal out of the ſame, and ſpend l. 1 : 7 in buying and ſelling; how ſhall I ſell 4 bolls of the meal to gain l. 16 : 5 on the bargain?

32—3

b. f. l. s. b.
32—3 : 216—0 :: 4
4 16—5 4
1—7
131 16
233—12
20
4672
16
20) l. s. d.
131)74752(570(28 : 10 : $7\frac{67}{131}$, Answer.
655
10
925
917
82
12
131)984(7
917
67

A man bought 120 eggs at 3 for a penny, and 120 more at 2 for a penny, and mixing them all together, he sold them just as he bought them, viz. at 5 for 2 pence; whether did he gain or lose, and how much?

3)120(40 Pence, price of the first 120.
0
2)120(60 Pence, price of the next 120.
0

Pence 100 prime cost of both.

e. d.
5 : 2 :: 240
2
100 Prime cost.
5)480(96 Pence for which he sold them.
0 4 Pence, his loss.

If

If l. 148 , 10 be forborn 7 years at 5 l. per cent. per annum, simple interest; what will it amount to at the 7th year's end?

By the amount, I mean the sum of principal and interest. Here, the 3 terms are all of the same quality, but 2 of them (viz. l.148 : 10 and 100 l.) are principal sums, and the other (viz. 5 l.) is gain, per cent, or per 100 l.

```
          l.
          5
          7
         ---
         35
     add 100
   l.    ---     l.    s.
  100 : 135 : 148—10
   20           20
  ----         ----
  2000         2970
                135
              -----
              14850
              2970
              -------  l.   s.    d.
 200|0)400950|0(200 : 9 1/2 or 6  amount.
              -------
               95
               20
              -------
       2|00)19|00(9
              -------
               1
```

What

What principal ſum forborn 7 years at 5 l. per cent. per annum ſimple intereſt, will amount to 200 l. 9 s. 6 d. at the 7 year's end?

```
     5
     7
    ---
    35
add 100
    ---    l.      l.  s. d.
    135 : 100 :: 200—9—6
     240            20
    ----           -----
    5400           4009
    270             12
    -----          ------
    32400          48114
                     100
                   ------      l.  s.
           324|00)481|14|00(148 : 10 Principal ſum.
                  -------
                   162
                    20
                  -----
              324)3240(10
                  ----
                   0
```

In what time will l. 148 : 10 amount to l. 200 : 9 . 6, if forborn at 5 l. per cent. per annum simple interest?

```
                 l.   s.  d.
                 200— 9—6
                 148—10—0
      l.   s.    ————————     l.
      148—10 :   51—19—6 ::  100
       20         20          20
      ————       ————        ————
      2970       1039        2000
                  12
                 ————
                 12474
                   2000
              ——————————— 12)  20)  5)
        2971|0)2494800|0(8400(700(35(7 years, ansr.
              2376        ———— ———— ———
              ————         0    0    0
              1188
              1188
              ————
                0
```

At what rate per cent. per annum simple interest, will l. 148 : 10 raise a stock of l. 200:9:6 in 7 years?

```
              l.
              200— 9—6
              148—10—0
  l.          ————————         l.
  148—10 :  51—19—6 ::  100
   20        20              20
  ————      ————            ————
  2970      1039            2000
             12
        —————————
        12474
           2000
        ———————— 12)  20)  7)
  2971|0)2494800|0(8400(700(35(5 l. per cent.
        ——————— ———— ———— ———
           0      0    0    0
```

What's

What's the interest of l. 148 : 10 for 7 years at 5 l. per cent. per annum simple interest?

```
              l.
              5
              7
             ---    l.   s.
  100 : 35 : 148 — 10
   20          20
  -----       -----
  2900        2970
                35
              ------
              14850
              8910
              ------ l.  s.   d.
  200|0) 1039|5|0(51 : 19 1/4 or 6 The interest
              1000                    required.
              -----
               395
               200
               ----
               195
                20
               ----
  2|00) 39|00(19
              ----
                1
```

Thus, you have these 5 cases in simple interest performed by the Rule of Three.

If I pay 7 s. 3 d. for $\frac{1}{4}$ of a pound of tea; what will 7 c.w. 3 qrs. of the same cost?

lb. *s.* *d.* *c.w.* *q.*
$\frac{1}{4}$: 7—3 :: 7—3

12 4
87 31
28
248
62
868
4
3472
87
24304
27776
20) *l.* *s.*
12)302064(25172(1258 : 12 Answer.
0 12

If I pay 4$\frac{1}{2}$ pence for 1$\frac{1}{2}$ lb. of chese; what will 364 $\frac{3}{4}$ stone weight of the same cost at 16 lb. to the stone?

lb. *d.* *st.* *lb.*
1$\frac{1}{2}$: 4$\frac{1}{2}$ :: 364—12

2 2 16
3 9 5836
2
11672
9
2) 12) 20) *l.* *s.*
3)105048(35016(17508(1459(72 : 19 Answer.
0 0 0 19

What

What present money will pay a debt of l. 450 due 3 months hence, discompt being allowed at 5 l per cent. per annum ?

Discompt is the abating so much money on a debt paid before 'tis due, as might be gained again by the money receiv'd, if put out to interest at the same rate, and for the same time. So 100 l. present money, would discharge a debt of 105 l. due at a year to come; discompt being made at 5 l. per cent. because 100 l. put out to interest for a year, at the said rate, would regain the 5 l. Therefore in working questions of this nature, your first term will always be 100 l. with it's interest for the time; the second 100 l. alone; and the third the debt, or sum propounded.

See the Work in the following page.

```
l.  s.
1 — 5 Intereſt of 100 l. for 3 months.
100
———————      l.    l.
101 — 5 : 100 :: 450
   20             20
———————        ———————
2025             9000
                    100
               ————————   l.   s.   d.  q.
      2025)900000(444 : 8 : 10 : 2 1350/2025 or 2/3
             8100      Preſent money required.
             ————
             9000
             8100
             ————
              9000
              8100
              ————
               900
                 20
              —————
      2025)18000(8
           ——————
            1800 Remdr.
              12
           ——————
      2025)21600(10
           ——————
            1350 Remdr.
               4
           ——————
      2025)5400(2
           ——————
            1350 Remdr.
```

What's

What's the discompt of 450 l. for 3 months at 5 l. per cent. per annum?

Here, instead of making 100 l. your second term, you must state down for the same it's interest for the time?

l. s.
1—5 Interest of 100 l. for 3 months.
100—0

l. s. l.
101—5 : 1—5 :: 450
20 20 20

25 9000
9000

20) l. s. d. q.
2025)22,000(111(5 : 11 : 1 : $1\frac{675}{2025}$ or $\frac{1}{3}$ Discompt required.

Remdr. 225 11.
12

2025)2700(1

Remdr. 675
4

2025)2700(1

Remdr. 675

What discompt must be allowed on a bill of exchange for 600 l. due July the 10th this being the 29th of April; discompt being made at 5 l. per cent. per annum?

See the work in the next page.

Days.

Days.
2 in April.
31 May.
30 June.
10 July.

days. *l.*
365 : 5 :: 73
5

l.
365)365(1 Interest of 100 l. for 73 days.
0

l. *l.* *l.*
Then, If 101 : 1 :: 600
1

l. *s.* *d.* *q.*
101)600(5 : 18 : 9 : 2 $\frac{98}{101}$ Difcompt, anfwer.
505

95
20

101)1900(18
101

890
808

82
12

101)984(9
909

75
4
101)300(2
202

98

If

If 16 men in 40 days finish a piece of work; how many men will do the same in 32 days?

$$\begin{array}{ccccc} d. & & m. & & d. \\ 40 & : & 16 & : & 32 \\ & & 40 & & \end{array}$$

32)640(20 Men, Answer.
0

A gentleman wants a piece of ground paved before his door with stones 3 foot long and 2 foot broad; the ground is 4 yards broad and 30 yards long; how many stones will serve?

$$\begin{array}{ccccc} & & & & yds. \\ & & & & 4 \\ feet. & & & & 30 \\ 3 & & & & 120 \\ 2 & & ft. & & 9 \\ 6 & : & 1 & :: & 1080 \\ & & & & 1 \end{array}$$

6)1080(180 Stones, answer.
0

How much printed paper will line a room that is 70 yards in circumference and 6 yards high, if the paper be 3 quarters broad?

$$\begin{array}{ccccc} 6 & & & & \\ 4 & & yds. & & qrs. \\ qrs.\ 24 & : & 70 & : & 3 \\ & & 24 & & \end{array}$$

3)1680(560 Yards of paper, answer.
0

I lent my friend 400 l. for 3 months; how much may he lend me at another time for 5 months, to requite my kindneſs?

m. *l.* *m.*
3 : 400 :: 5
3

5)1200(240 l. Anſwer.

0

If I lend my friend 240 l. for 5 months how long muſt I keep 400 l. of his, to requit myſelf?

l. *m.* *l.*
240 : 5 :: 400
240

400)1200(3 Months, Anſwer.

0

A garriſon conſiſting of 1764 men, being beſieg'd, have proviſions for only 12 days; but it being neceſſary they ſhould hold out 3 weeks; how many men muſt be ſent out?

d. *m.* *d.*
12 : 1764 :: 21
12

1764
21)21168(1008 Muſt be kept.

0 756 Muſt be turn'd out.

If

If I pay l. 9 : 12 Scots a year for one acre of land, what will be the yearly rent of 26 acres 32 falls of the same ground?

```
 f.   l.   s.   a.  f.
160 : 9—12 :: 26—32
      20      160
      ———     ————
      192     4192
               192
            ——————
              8384
             79648
           ———————20)  l.   s.  d.
160)804864(5030(251 : 10 : 4 128/160 or 4/5
    ———————   ———— The answer.
Remdr. 64     10
       12
      ———
  160)768(4
      ———
      128
```

If, when the boll of wheat costs 8 l., the two penny loaf weighs 20 ounces; what should it weigh when the boll gives 12 l.?

```
l.  oz.  l.
8 : 20 :: 12
     8
    ———
12)160(13 Ounces 5 1/3 drops, Answer.
   ———
    4
   16
   ———
12)64(5
   ———
    4
```

If I pay 40 l. for 32 yards of cloth; how ſhall I ſell one yard of the ſame to gain 5 l.?

l.
5
40
yds —— *yds.*
32 : 45 :: 1
1
—— *l.* *s.* *d.* *q.*
32)45(1 : 8 : 1 : 2 Anſwer.
——
13
20
——
32)260(8
——
4
12
——
32)48(1
——
16
4
——
32)64(2
——
0

If damage had happen'd to the cloth, and 5 l. were to be loſt on the whole; then the ſaid 5 l. ſhould be ſubtracted from the prime coſt, and the remainder made the ſecond number, as in the following example.

I paid 40 l. for 32 yards of cloth, (which proving worse than I expected) I am willing to sell so as to lose 5 l. on the whole; how shall I sell the yard to lose the said 5 l.?

```
           l.
           40
            5
  yds.     ——     yds.
   32  :   35  ::  1
            1
          ——  l.   s.   d.   q.
      32)35( 1  :  1  :  10  :  2  Answer.
         ——
          3
         20
         ——
      32)60(1
         ——
         28
         12
         ——
      32)336(10
          ——
          16
           4
          ——
      32)64(2
          ——
           0
```

I bought 16 bolls of oats for l. 80 : 10 Scots, out of which I had only 11 bolls 2 firlots of meal; how shall I sell one boll of the meal to gain 8 l. by the bargain?

l. s.
80—10
8

b. f. b.
11—2 : 88—10 :: 1
4 20 4

46 1770 4
4

20) l. s. d.
46)7080(153(7 : 13 : 10 $\frac{44}{46}$ or $\frac{22}{23}$ Answer.

42 13
12

46)504(10

44

I have by me 96 dozen of oranges, which cost me l. 4 : 16, but some damage having happen'd to them, I am willing to lose 24 shil. on the whole; at what rate must I sell them per doz.?

l. s.
4 : 16
1 : 4

doz. doz.
96 : 3—12 :: 1
20

72
12

96)864(9 Pence, answer.

0

How

How long ſhall I be in laying up l. 1000 ſter. if I put by l. 3 : 10 : 6 a week ?

```
 l.  s.  d    w.   l.
 3—10—6  :  1  ::  1000
  20                240
 ———              ———— 52)  y.  w.  d.
  70           846)240000(283(5 : 23 : 4 Anſ.
  12                1692     260
 ———              ————     ———
 846                7080      23
                    6768
                    ————
                    3120
                    2538
                    ————
                     582
                       7 Days in a week.
                   —————
               846)4074(4
                   3384
                   ————
                    690 Remainder.
```

If at 5 s. per ell, I gain 8 l. per cent. by my cloth; what ſhall I gain per cent. if I ſell the ell at 6 s. 3 d. ?

```
 s.   l    s.  d.
 5  : 8 :: 6 — 3
 12        12
 ——        ——
 60        75
            8
          ———
       60)600(10 l. per cent. Anſwer.
          ———
            0
```

I bought 5 pieces of cloth, each containing 56 ells Flemish at 3 s. 2 d. per ell; what will I gain on the whole, if I sell it for 5 s. 8 d. per ell English?

```
        56
         5
       ----
       280 Ells Flemish.
         3
       ----
     5)840(168 Ells English.
       ----
        0

      e.   s.  d.   e.
If    1 : 3—2 :: 280
          12        38
        ----      ----
          38       224
                   84
                ------ 20)  l.  s.  d.
             12)10640(886(44:6:8 Prime cost.
                ------  ----
                    8     6

          e.   s.  d.   e.
Then if   1 : 5—8 :: 168
              12        68
            ----      ----
              68      1344
                     1008
                    ------ 20)  l.  s.
                 12)11424(952(47 : 12
                    ------  ----
                        0    12

l. 47 : 12 — Price I am to sell it at.
   44 :  6 : 8 Prime cost.
   -----------
l.  3 :  5 : 4 Gain.
```

A man owing l. 736 : 10, compounds with his creditors for 7 s. 9 d. per l.; what will that amount to?

```
s.      s. d.     l.   s.
20      7—9 :: 736—10
          12         20
        ----     ------
          93      14730
                     93
                  -----
                  44190
                 13257
              ---------  12)    20)    l. s. d. q.
          20)1369890(68494(5707(285:7:10:2
             -------  -----  -----     Answer.
                  10     10      7
                   4
             20)40(2
                ---
                 0
```

If tobacco which cost 13 pence per lb. be sold for 18 pence per lb.; what is gain'd per cent?

```
         d.
         18
         13
d.      ----    l.
13 : 5    :   100
              240
          -------
            24000
                5
          ---------  12)  20)   l.  s. d. q.
       13)120000(9230(769(38 : 9 : 2 : 3 Answer.
          -------  ----  ----
            30        2      9
          ----
            40
          ------
            10
             4
          ------
         13)40(3
           ----
             1
```

A merchant sends to Spain 1300 pieces of broad cloth, each piece 47 yards, at 15 s. 6 d. per yard, to have returns from thence, the one half in wine at 65 l. per tun, and the other half in oranges at l. 3 : 10 per chest; what quantity of each will he have?

47
1300
———
61100 Yards in all.

yd. *s.* *d.* *yds.*
1 : 15—6 : 61100
12 186
——— ———
186 3666
10998
——— 20) *l.* *s.*
12)11364600(947050(47352 : 10
——— ———
0 10

l. *s.*
2)47352—10 Price of the cloth.
l. *T.* ———
65 : 1 :: 23676—5 Half it's price.
20 20
——— ———
1300 1300)473525(364 Tuns, 1 hogshead of wine.
———
325
4
———
1300)1300(1
———
0

l. *s.* *ch.* *l.* *s.*
3—10 : 1 :: 23676—5
20 20
——— ———
70 70)473525(6764$\frac{45}{70}$ or $\frac{9}{14}$
——— Chests of Oranges.
45

If

If a merchant would mix 8 lb. of tea at 7 s. per lb. with 5 lb. of tea at 10 s. per lb. and with 12 lb. at 14 s. per lb.; what might he sell 1 lb. of the mixture at?

RULE, As the sum of the given quantities of the several simples, is to their total value: so is any quantity of the mixture, to the price thereof.

lb.		*s.*			*s.*
8	at	7	per lb.	is	56
5	at	10	per lb.	is	50
12	at	14	per lb.	is	168
25 Sum of the simples.					274 Total value.

```
         lb.     s.     lb.
Then if 25  :  274  ::  1
                 1
               ------  s.   d.
            25)274(10  :  11 13/25 Answer.
              ------
     Remdr.   24
              12
             ----
          25)288(11
             ----
              13
```

If a man mix three bolls of oats at l. 6 : 10 Scots per boll with 4 bolls of pease at l. 4 : 8 per boll, and with 1 boll of dry malt at l. 9 : 2; what may he sell 3 lippies of the mixture at?

B.		*l.*	*s.*		*l.*	*s.*
3	Oats at	6—	10	per boll is	19 :	10
4	Pease at	4—	8	per boll is	17 :	12
1	Malt at	9—	2	- - is	9 :	2
8	Sum of all the simples.				46 :	4 Total value.

```
              B.  l.  s.   l.
Then, if 8 : 46—4 : : 3
         64   20
              ———
              924
                3
     ——— ———      s.  d.
     512)2772(5 : 4 496/512 or 31/32 Scots, Ansr.
         2560
         ————
          212
           12
         ————
     512)2544(4
         2048
         ————
          496
```

A merchant sent to a tobacconist 5 c.w. 3 qrs. of tobacco in the roll worth 9 pence per lb. to be cut and dryed, and when it came home, it weighed only 4 c.w. 2 qrs., the cutting and drying stood him 32 shillings; how did he sell 1 lb. cut and dry to lose nothing?

wrought in the next page.

lb. d. c.w. qrs.
1 : 9 :: 5—3
4
23
28
184
46
644
9
s.
12)5796(483 Price of the tobacco in the roll.
add 32 for cutting and drying.
0
515

c.w. q. s. lb.
Then, if 4—2 : 515 :: 1
4 12
18 504)6180(12 pence $1\frac{24}{504}$ or $\frac{1}{21}$ farth.
28 504 answer.
504 1140
1008
132
4
504)528(1
504
24

Two merchants, A, and B, barter, or exchange goods; A, hath 5 c.w. 3 qrs. 14 lb. of pepper worth l. 3 : 10 per c.w. and B, hath cotton worth 10 pence per

[illegible]

[illegible], hat [illegible]
d [illegible]
year [illegible]
ad, [illegible]
B, [illegible]
er [illegible]
this, [illegible]
ce [illegible]
[illegible]
us,

[illegible] d.
12
[illegible]
15
11
[illegible]

... ½ of cotton ...

... 86 yards ...

... eight yards of
... and each piece
..., if every piece was
... after the rate of
... demand what the eight
... worth per yard.

per lb.; now, how much cotton muſt B, give to A, for his pepper?

In order to reſolve this queſtion, and all others of this nature, firſt find the true value of that commodity whoſe quantity is given; (which in this queſtion is pepper) and then find how much of the other commodity that ſum will purchaſe at the rate propos'd: for 'tis eaſy to conceive that A, muſt have as much of B's cotton at 10 pence per lb. as ſhall be equivalent to his quantity of pepper at l. 3 : 10 per c.w.

```
lb.    l.  s.     c.w. q. lb.
112 :  3—10  ::   5—3—14
          20        4
         ———      ———
          70       23
                   28
                  ———
                  188
                  47
                 ———
                 658
                  70
                ———        s. d.
          112)46060(411:3 True value of A's
              448                     pepper.
              ———
              126
              112
              ———
               140
               112
               ———
                28
                12
               ———
          112)336(3
              336
              ———
                0
```

See the reſt of the work on the next page.

10

d. lb. s. d.
10 : 1 :: 411—3
12

lb.
10)4935(493 $\frac{5}{10}$ or $\frac{1}{2}$ of cotton, Anſr.

5

A, and B, barter thus; A, hath 86 yards of broad cloth worth 9 s. 2 d. per yard ready money; but in barter he will have 11 s. per yard. B, hath ſhalloon worth 2 s. 1 d. per yard ready money; now how many yards of ſhalloon muſt B, give to A, for his cloth, to make his gain in the barter equal to that of A?

The method of reſolving this, and the like queſtions, differs a little from the laſt caſe; for in this you muſt firſt find what advance B, ought to make per yard upon his ſhalloon, in proportion to what A, hath done upon his cloth. Thus,

s. d. s. s. d.
9—2 : 11 :: 2—1
12 12

110 25
11

110)275(2 s. 6 d. The advanced price for one yard of B's ſhalloon.
220

55
12

110)660(6

0

Then

Then proceed as before in the last example. Thus,

$$\begin{array}{l} \textit{yd.} \quad \textit{s} \quad \textit{yds.} \\ 1 : 11 :: 86 \\ \qquad\quad\ 11 \\ \hline \end{array}$$

946 s. The advanced value of all A's cloth.

$$\begin{array}{lll} & \textit{s. d. yd.} & \textit{s.} \\ \text{Then, if} & 2\text{—}6 : 1 :: & 946 \\ & 12 & 12 \\ & \overline{30} & 30)11352(378\tfrac{12}{30} \\ & & \quad 12 \end{array}$$

$378\frac{12}{30}$ or $\frac{2}{5}$ Yards of shalloon that B, must give to A, for his 86 yards of broad cloth.

A merchant bought 436 yards of cloth for 8 s. 6 d. per yard, and sold the same for 10 s. 4 d. per yard; what did he gain thereby?

$$\begin{array}{l} \qquad\ \textit{s.} \quad \textit{d.} \\ \qquad\ 10\text{—}4 \\ \qquad\quad 8\text{—}6 \\ \textit{yd.} \quad \overline{\qquad} \quad \textit{yds.} \\ 1 : 1\text{—}10 :: 436 \\ \quad\ 12 \qquad\quad 22 \\ \quad \overline{22} \qquad \overline{872} \\ \qquad\qquad\ 872 \\ \qquad\qquad \overline{\quad} \ 20) \quad \textit{l.} \quad \textit{s.} \quad \textit{d.} \\ \qquad 12)9592(799(39 : 19 : 4 \text{ Gain, Ansr.} \\ \qquad\qquad \overline{\quad} \quad \overline{\ } \\ \qquad\qquad\ 4 \quad\ 19 \end{array}$$

A

A draper bought of a merchant eight packs of cloth; each pack had 4 parcels in it; and each parcel contained ten pieces; now, if every piece was twenty six yards, and if he gave after the rate of l. 4 : 16 for 6 yards; I demand what the eight packs came to, and what was it worth per yard?

```
            8 Packs.
            4
          ----
           32 Parcels.
           10
          ----
          320 Pieces.
           26
         -----
         1920
         640
         -----
         8320 Yards.

yds.  l.   s.     yds.
 6  : 4—16  ::  8320
      20          96
     ----      ------
      96        4992
               7488
               ------20)         l.
             6)798720(133120(6656 Price of
               ------  ------      8 packs.
                  0       0

yds.  l.  s.  yd.
 6  : 4—16 :: 1
      20
    -----
    6)96(16 s. Price of 1 yard.
     ---
      0
```

What will 48 oz. 17 p.w. 20 grains of silver plate come to, at the rate of 5 s. 6 d. per ounce?

oz. *gr.* *s.* *d.* *oz.* *p.w.* *gr.*
1 or 480 : 5—6 :: 48—17—20

12 20

66 977

24

3908

1956

23468

66

140808

140808

12) 20) *l.* *s.* *d.* *q.*

480)1548888(3226(268(13:8:10:$3\frac{192}{480}$ or $\frac{2}{5}$ the anfwer.

408 10 8

4

480)1632(3

192

If

If an acre of land be 4 falls broad and 40 falls long; what must be the length of the acre when it is 9 falls broad?

```
      f.   f.
      4 : 40 :: 9
           4
         ---
      9)160(17 Falls, 4 2/3 ells length answer.
         ---
          7
          6
        ---
     9)42(4
       ---
        6
```

How many dollars at 4 s. 4 d. may I give for 360 guilders at 2 s. 2 d. per piece?

```
        s.  d.    g.     s.  d.
        2—2 : 360 :: 4—4
        12        26    12
        ---      ---    ---
        26      2160    52
                 72
              ------
          52)9360(180 Dollars, answer.
               0
```

If 100 l. principal gain 5 l. interest in 12 months; what principal will gain as much in 5 months?

```
        m.    l.    m.
        12 : 100 :: 5
              12
            ----
          5)1200(240 l. Principal, answer.
           ---
            0
```

How

How many lb. of coffee at 5 s. 9 d. per lb. is equal in value to 426 lb. of tea at 13 s. 4 d. per lb.?

```
 s. d.    lb.     s  d.
 13—4 :  426 :: 5—9
   12    160     12
 ----           --
  160            69
        ------
      69)68160(987 57/69 lb. of coffee, answer.
         621
         ----
          606
          552
          ---
          540
          483
          ---
           57
```

In this, and the three foregoing examples, it was easy to observe that the proportion was inverse, and so will it be in the following one.

If I have 14 c.w carried 136 miles for l. 5 : 5; how many miles may I have 24 c.w, carried for the same money?

```
 c.w.   m.    c.w.
  14 : 136 :: 24
        14
      -----
 24)1904(79 8/24 or 1/3 Miles, answer.
    168
    ---
     224
     216
     ---
       8
```

If

If 37 acres of land give l. 13 : 11 : 4 yearly rent: what is that an acre?

A. l. s. d. A.
37 : 13—11—4 :: 1
20

271
12

12)
37)3256(88(7 s. 4d. Anfwer.

0 4

If I have an eftate of 480 l. per annum; what may I expend daily, and yet lay up yearly 140 l.?

l.
480
140
d. d.
365 : 340 :: 1
20

365)6800(18 s. 7 d. $2\frac{90}{365}$ or $\frac{18}{73}$ farths, anfr.
365

3150
2920

1230
12

365)2760(7
2555
205
4
365)820(2
730

90

If

If 12 yards 3 quarters of cloth coſt l. 3 : 16 : 6 ; what quantity of the ſame may I have for 154 guineas ?

```
 l.  s.  d.  yds. q.   guineas.
 3—16—6 : 12—3 :: 154
 20          4       21
 ——         ——      ———
 76          51      154
 12                 308
 ——                 ———
 918               3234
                     12
                  —————
                  38808
                     51
                  —————
                  38808
                 194040
                ———————  4)
        918)1979208(2156(539 Yards, anſr;
            1836        ———
            ————         0
            1432
             918
            ————
            5140
            4590
            ————
             5508
             5508
            —————
               0
```

If I owe 480 l. and compound with my creditors for 300 l.; what's that in the pound?

$$\begin{array}{l} \quad l. \quad\; l. \quad\; l. \\ 480 : 300 :: 1 \\ \qquad\quad 20 \\ \hline 480)6000(12\text{ s. } 6 \text{ pence, Answer.} \\ \quad 240 \\ \quad 12 \\ \hline 480)2880(6 \\ \hline \quad 0 \end{array}$$

Suppose that six farmers are oblig'd to hire a man in their parish for the king's service, and agree (in proportion to their yearly rents) to give him 39 l. when he sets out; suppose also, that the sum of their rents is 900 l. and if one farmer's rent is 100 l.; what part of the 39 l. must he pay?

$$\begin{array}{l} \quad l. \quad l. \quad l. \\ 900 : 39 :: 100 \\ \qquad\quad 100 \\ \hline \qquad\qquad l.\;\; s.\;\; d. \\ 9|00)39|00(4 : 6 : 8 \text{ Answer.} \\ \hline \quad 3 \\ \quad 20 \\ \hline 9)60(6 \\ \hline \quad 6 \\ \quad 12 \\ \hline 9)72(8 \\ \hline \quad 0 \end{array}$$

After

After this manner (if you had all their particular rents) you could find their respective parts of the 39 l.

I shipt goods for Jamaica to the value of l. 343:1:8, and in return I had 46 c.w. 3 qrs. of sugar at 24s. 6 d. per c.w. and 1570 lb. of indigo at 2 s. 4 d. per lb. what remains due to me of my adventure?

qrs. *s.* *d.* *c.w.* *qrs.*
4 : 24—6 :: 46—3
12 4

294 187
187

2058
5292

12) 20) *l.* *s.* *d.* *q.*
4)54978(13744(1145(57 : 5 : 4 : 2 Price of the sugar.
4
2 4 5
4

4)8(2

0

lb. *s.* *d.* *lb.*
1 : 2—4 :: 1570
12 28

28 12560
314

20) *l.* *s.* *d.*
12)43960(3663(183 : 3 : 4 price of the indigo.
4 3

	l.	*s.*	*d.*	
Exported	343	1	8	
Imported	240	8	8½	Price of sugar and indigo.

Answer, l. 102 : 12 : 11½ Remains due to me.

If

If A, (suppose in Dundee) hath houses and yards to the value of 36 l. yearly rent, and payeth 4 s. 10 d. yearly to the town; what shall B, pay who hath lands there to the value of l. 70 : 10 yearly rent?

```
l.   s.   d.    l.   s.
36 : 4 — 10 :: 70 — 10
 20   12        20
----  ---      -----
720   58       1410
                 58
              -----
              11280
              7050
              -----  12) s.  d.  q.
       72|0)817|0(113(9 : 5 : 2 24/72 or 1/3 Ansr.
        72           ---
        ---           5
         97
         72
        ---
        258
        216
        ---
         42
          4
        ---
     72)168(2
        144
        ---
         24
```

A certain usurer put out 75 l. for 12 months, and receiv'd for principal and interest 81 l. I demand at what rate per cent, he received interest?

```
          l.
          81
          75
   l.    ---
   75 :   6 :: 100
                  6
               ----
             75)600(8 l. per cent, answer.
               ----
                  0
```

A merchant bought 6 packs of cloth, each pack containing 12 pieces, which at 8 s. 4 d. per ell Flemish, cost l. 1080; how many yards were there in each piece?

```
         6
        12
        --      l.     p.
Pieces  72 :  1080 ::  1
                       1
                    ----
              72)1080(15 l. price of 1 piece.
                 ----  20
                    0  --
          s.  d.      300
          8---4        12
          12
         ----       -----
          100 ) 3600(36 Ells Flemish in 1 piece.
                ----    3
                   0  ----
                     4)108(27 Yards in 1 piece, the ansr.
                      ----
                         0
```

A

A merchant bought a certain quantity of ſerge and ſhalloon for l. 226 : 14 : 10; the quantity of ſerge he bought was 48 yards at 3 s. 4 d. per yard; and for every 2 yards of ſerge he had 5 yards of ſhalloon; how many yards of ſhalloon had he, and what did it coſt him per yard?

2)48(24
— 5
0 ——
120 Yards of ſhalloon he bought.

yd. *s.* *d.* *yds.*
1 : 3—4 :: 48
12 40
—— ———— 20) *l.* *s.* *d.*
40 12)1920(160(226:14:10
—— —— 8 - - - Price of ſerge.
0 0 ————
218:14:10 Price of ſhalloon.

yds. *l.* *s.* *d.* *yd.*
120 : 218—14—10 :: 1
20
————
4374
12
————12) 20)*l.* *s.* *d.*
120)52498(437(36(1 : 16 ; 5 $\frac{58}{120}$ or $\frac{29}{60}$ price of 1 yard of ſhalloon.
480 — —
—— 5. 16.
449
360
——
898
840
——
58

A merchant bought 242 yards of cloth for 254 l. 10 s. for 86 yards of the same he paid after the rate of 21 s. 4 d. per yard; I demand how much he gave per yard for the remainder?

yd. *s.* *d.* *yds.*
1 : 21—4 :: 86
12
———
256
86
———
1536
2048
——— 20) *l.* *s.* *d.*
12)22016(1834(254 : 10 : 0
——— ——— 91 : 14 : 8 Price of 86 yards,
8. 14 ———————
l. 162 : 15 : 4 Price of the remainder.

yds.
242
86
——— *l.* *s.* *d.* *yd.*
156 : 162—15—4 : 1
20
———
3255
12
——— 12) *s.* *d.*
156)39064(250(20 : 10 $\frac{64}{156}$ or $\frac{16}{39}$ Price of one yard of the remainder.
312 ———
——— 10
786
780
———
64

If a piece of cloth cost l. 10 : 16 : 8, I demand how many ells English there are in the same, when the ell is worth 8 s. 4 d.?

```
s. d.  e.    l.  s.  d.
8—4 : 1 :: 10—16—8
12           20
——           ——
100          216
              12
            ————
         100)2600(26 Ells English, answer.
             ————
               0
```

A merchant bought 84 pieces of cloth for l. 537 : 12, being at 5 s. 4 d. per yard; I demand how many yards there were in all, and how many ells English were in a piece of the same?

```
s. d. yd.   l.   s.
5—4 : 1 :: 537—12
12          20
——          ——
64         10752
              12
         ————————
       64)129024(2016 Yards in all.
          ——————    4
            0     ———— 5)
              84)8064(96(19 1/5 English ells in 1
                 756   —                  piece.
                 ———   1
                 504
                 504
                 ———
                  0
```

There

There were at a feast 20 men, 30 women, and 15 servants for every 10 s. that a man paid, a woman paid 6, and a servant 2; how much did every man, woman, and servant pay of l. 61 : 10 ?

Multiply 20 by 10, 30 by 6, and 15 by 2; then divide l. 61 : 10 into three parts proportion'd to these products, and you have the total paid by the 20 men, 30 women, and 15 servants: each of which sums being divided by their respective numbers of persons, gives the payment made by each individual.

See the Work.

```
 20          30          15
 10           6           2
----        ----        ----
200         180          30
180
 30
----
410 : 200 :: 61 -10
 20           20
-----       -----
8200         1230
              200
            ------ 20) l. s.
82|00)2460|00(30(1 : 10  Paid  by each
        246          -             man.
      -----          10
          0          20
                   -----
                20)200(10
                   -----
```

410 : 180 :: 61—10
20 20

1230
180

82|00)221 4|00(27 l. Paid by 30 women.
164 20

574 30)540(18 s. Paid by each woman.
574

0 0

410 : 30 :: 61—10
20 20

1230
30

l. s.
82|00(369|00(4 : 10 Paid by all the servants.
328 20

41 15)90(6 s. Paid by 1 servant.
20

82)820(10
82

0

I sold 2 yards of cloth for 11 s. 6 d. and gain'd at the rate of 15 l. per cent; what would I have gain'd per cent, if I had sold them for 12 s.?

This is the same in substance and numbers with Mr. Hill's 8th question of loss and gain; but neither his operation nor answer are the same with this following. (His method of stating the terms being quite wrong.)

11—6

```
s.  d.  l.     s.
11—6 : 115 :: 12
12             12
----           ----
138            144
               115
              -----
              2160
              144
             ------
       138)16560(120
          138   100
          ----  ----
           276  l. 20 gain per cent, the answ.
           276
           ----
             0
```

What's the value of 1 boll of wheat, when I give 28 bolls of the same for 120 pints of hony at 4 s. the pint?

```
          120
            4
   B.    ----   B.
   28 : 480 :: 1
             1
          ----  s. d. q.
     28)480(17 : 1 : 2 24/28 or 6/7 price of 1
        ----                          boll.
          4
         12
        ----
      28)48(1
       -----
         20
          4
       ------
      28)80(2
       ------
        24
```

A man dying bequeathed his goods (which were worth 3600 l.) after this manner. Because his wife was with child, and he yet uncertain whether it was a boy or girl, he made his bequest conditionally, that if it was a son, his wife should have $\frac{1}{3}$ and the son $\frac{1}{2}$ his goods; but if a daughter, then his wife should have $\frac{1}{2}$ and the daughter $\frac{1}{3}$ of them; now it happened she brought forth both a son and daughter; how shall the goods be parted agreeable to the testator's will?

The mind of the testator, here, is to be understood, that such proportion should be between the wife's portion and the son's, as between $\frac{1}{3}$ and $\frac{1}{2}$, that is, the son must have as much and half as much as his mother; and the mother must have the like rate in comparison to her daughter: therefore make choice of any three numbers in such proportion, that the first may be as much as the second and half as much more, and the second to the third in that same proportion, such as 9, 6, and 4; and proceed as follows.

```
 9
 6
 4
——
19 : 9 :: 3600
              9
        ———————   l.   s. d.
        19)32400(1705 : 5 : 3 3/19 Son's portion.
        ———————
              5
             20
        ———————
        19)100(5
        ———————
              5
             12
        ———————
        19)60(3
              3.
```

19 : 6 :: 3600
6

l. *s.* *d.*
19)21600(1136 : 16 : 10$\frac{2}{19}$ Mother's portion.

16
20

19)320(16

16
12

19)192(10

2.

19 : 4 :: 3600
4

l. *s.* *d.*
19)14400(757 : 17 : 10$\frac{14}{19}$ Daughter's part.

17
20

19)340(17

17
12

19)204(10

14

A merchant sells a parcel of jewels (which cost him 250 l. ready money) for 559 l. payable at the end of 6 months; what was his gain worth in ready money, upon rebate of interest at 6 l. per cent, per annum?

			l.
			559
			250
l.	l.		
103	: 100	::	309 gain.
			100

l.
103)30900(300 Answer.

9

There is an island 134 miles in circumference; now at the same time, and from the same place A, and B, begin a journey round about the said island, but they travel towards contrary parts at this rate, viz. A, travels 11 miles in every two days, and B, 17 miles in every three days; I demand in what space of time will they meet, and how many miles each will have travelled?

d.	m.	d.
2 :	11 ::	3
	3	

2)33(16½ Miles that A, will travel in three days.
17 Miles that B, travelled in three days.

33½ Miles travelled by both in three days.

m. d. m.
If $33\frac{1}{2}$: 3 :: 134
2 2
——— ———
67 268
3
———
67)804(12 Days, time in which they will meet.
———
9

d. m. d.
2 : 11 :: 12
12
———
2)132(66 Miles that A travelled.
———
0

d. m d.
3 : 17 :: 12
12
———
3)204(68 Miles that B, travelled.
———
0

There is a square room which is in circuit 60 feet, and 8 feet high; the walls of said room (except a space for a window whose height is 5 feet and wideness 4 feet) are to be furnished with hangings of yard broad stuff at 14 s. 6 d. the yard; how much stuff will serve, and what will the whole cost?

60
8
———
480 Square feet in the room.
20 Content of the window.
———
9)460($51\frac{1}{9}$ Yards of stuff will serve.
———
1

5
4
———
20 Square feet in the window.

yd. s. d. yds. feet.
1 : 14—6 :: 51 : 1
9 12 9
———
9 174 460
174
———
1840
7820
——— 12) 20) l. s. d.
9)80040(8893(741(37 : 1 : 1⅓ or ⅓
——— ——— —— Price of the stuff,
3. 1. 1.

A school-master being asked, How many scholars he had? answered, If I had as many, half as many, and one fourth as many as I just now have, I should then have 132; how many had he?

Suppose he had any number at pleasure, as 8
then as many is 8
½ as many is 4
¼ as many is 2
———
22

If 22 : 8 :: 132
8
———
22)1056(48 Real number required.
———
0

The governor of a garrison being desirous to know how much money the passage of the garrison did amount to in a year, made choice of a loyal servant, and ordered him to receive of every coachman passing with a coach 4 d. of every horseman 2 d. and of every footman ½ d. Now at the year's end, the servant making up his accompt to the governor, gives

gives him l. 94 : 15 : 10, and lets him know that as often as 5 paſs'd with coachs, 9 paſs'd on horſeback; and as often as 9 paſs'd on horſeback, 15 paſs'd on foot; I demand how many coaches, horſemen and footmen paſs'd?

5 Coaches paid 20 d.
9 Horſemen paid 18 d.
15 Footmen paid $7\frac{1}{2}$ d.

```
          C.   l.   s.  d.
45½ : 5 :: 94—15—10
 2          20
 --        -----
 91        1895
             12
           -----
           22750
               2
           -----
           45500
               5
           -----
      91)227500(2500 coaches paſs'd,
         ------
              0

d.   h.   l.   s.   d.
45½ : 9 :: 94—15—10
 2          20
 --        -----
 91        1895
             12
           -----
           22750
               2
           -----
           45500
               9
           -----
      91)409500(4500 Horſemen paſs'd.
         ------
              0
```

d.	f.	l.	s.	d.
$45\frac{1}{2}$	: 15	:: 94	—15	—10
2		20		
91		1895		
		12		
		22750		
		2		
		45500		
		15		

91)682500(7500 Footmen pass'd.

0

A brazen lyon being placed in an artificial fountain, conveys water into a cistern by a stream issuing from his mouth, by two from his eyes, and by another from the bottom of his right foot. Now the pipes through which these streams pass, are of different capacities, in such sort, that by the foot set open alone (the rest of the streams being stopt) the cistern will be filled in 2 hours, by the mouth alone in 12 hours, by the right eye alone in 3 hours, and by the left eye alone in 4 hours; in what time will the cistern be filled, if all these streams be set open at once.

First find how many cisterns will be filled by each pipe in one and the same time; then it will be, As the sum of these cisterns is in proportion to the said time, so is one cistern to the time in which such cistern will be filled by all the streams running together.

Find therefore how many cisterns will be filled by each pipe in 12 hours, which is the time assign'd for the mouth alone to fill 1 cistern in.

If

h. c. h.
If 2 : 1 :: 12
1

2)12(6 cisterns filled by the foot in 12 (hours.
0

h. c. h.
3 : 1 :: 12
1

3)12(4 cisterns filled by the right eye in 12 hours.
0

h. c h.
4 : 1 :: 12
1

4)12(3 cisterns filled by the left eye in 12 hours.
0

Cisterns.
6
4
3
add 1 Filled by the mouth in 12 hours.

14 Will be filled by the 4 pipes running (together.

C. h. c.
Then, if 14 : 12 :: 1
60

14)720(51$\frac{6}{14}$ or $\frac{3}{7}$ Minutes, and in that time the cistern will be filled if all the streams be set open at once;
6

Five

Five merchants A, B, C, D, and E, have gain'd l. 2025, which they divided in such sort, that $\frac{1}{2}$ A's share is equal to $\frac{1}{4}$ of B's share, and to $\frac{1}{5}$ of C's, and to $\frac{1}{6}$ of D's, and to $\frac{1}{8}$ of E's; what was the share of each?

Suppose any number at pleasure (as 12 l.) for A's share, $\frac{1}{2}$ of which is 6 l. then 4 time's 6 is 24 l. for B's share, 5 times 6 is 30 l. for C's share, 6 times 6 is 36 l. for D's share, and 8 times 6 is 48 l. for E's share.

l.
12 A's supposed share.
24 B's
30 C's
36 D's
48 E's

150 : 12 :: 2025
12

15|0)2430|0(162 A's real share, $\frac{1}{2}$ whereof is 81 l.
0

l.
162 A's real share.
4 Times 81 is 324 B's share.
5 Times 81 is 405 C's
6 Times 81 is 486 D's
8 Times 81 is 648 E's

l. 2025 gain propounded.

A gentleman bequeathed to his daughter l. 40000 to be paid her at 8 years end, the executors desired to pay ready money, and to be abated after the rate of 5 l. per cent, per annum simple interest; what ready money will pay this legacie?

By the ready money which they want to pay, is meant such a sum as (being laid out at 5 l. per cent, per annum simple interest) would at 8 years end amount to 40000 l.

5
8

l. 40 Interest of 100 l. for 8 years.
add 100

l.
140 : 100 :: 40000
100

l. s. d. q.
14|0)400000|0(28571 : 8 : 6 : $3\frac{6}{14}$ or $\frac{3}{7}$ ready money, answer.

6
20

14)120(8

8
12

14)96(6

12
4

14)48(3

6

A merch-

A merchant delivers to his factor 500 l. and eſteem'd his perſon at 200 l.; when they made up their accompts, they gain'd 20 l. per cent.; what is the Factors part of the gain?

```
              l.
             500
             200
   l.        ---
If 100 : 20 :: 700
               20
          ------  l.
     7|00)140|00(140 their total gain.
          ------
              0

      l.     l.     l.
  If 700 : 140 :: 200
                200
         -------  140 l.
     7|00)280|00( 40 Factor's part of the gain
          -------  ---
              0   l. 100 Merchant's part thereof.
```

A man dying gave to his eldeſt ſon $\frac{2}{3}$ of $\frac{1}{4}$ of his eſtate, to his ſecond ſon he gave $\frac{1}{5}$ of $\frac{1}{2}$ of his eſtate; and when they had counted their portions, the one had 40 l. more than the other; the remainder of the eſtate was given to his wife and younger children; I demand the value of his eſtate, and alſo their reſpective ſhares of the ſame?

Suppoſe his eſtate to be any ſum you pleaſe, as 120 l. Now $\frac{1}{4}$ of 120 is 30, and $\frac{2}{3}$ of 30 is 20 l. for the eldeſt ſon's portion.

Again $\frac{1}{2}$ of 120 is 60, and $\frac{1}{5}$ of 60 is 12 l. for the ſecond ſon's portion. Now the queſtion ſays, that the one had 40 l. more than the other, but here the one's portion exceeds the other's only by 8 l. therefore ſay;

If

l. l. l.
If 8 : 120 :: 40
40
l.
8)4800(600 Real value of his estate.
0

3) 5) l.
4)600(150(50 2)600(300(60 Second son's portion
2
0 0 0 0
l. 100 Eldest son's portion.

l.
600 Estate.

100 } The two sons portions.
60 }

Sum 160 to be subtracted from 600 l.

Remains 440 l. to the wife and younger children.

A gentleman bought a house with a garden and a good horse in the stable for 6000 l. Now he paid 4 times the price of the hosre for the garden, and 5 times the price of the garden for the house; what did each cost him?

Suppose the horse cost 4 l. then the garden cost 16 l. and the house cost 80 l.

l.
4
16
80

l. l.
If 100 : 4 :: 6000
4

1|00)240|00(240 l. real price of the horse, so the garden cost 960 l. and the house 4800 l.
0

As

A, B, and C, upon a joint adventure gain'd 874 l. 7 s. whereof A, took up a certain sum; B, took up twice as much as A; and C, took up thrice as much as B; what did each take up?

	l.
Suppose A, took up	40
Then B, took up	80
And C, took up	240
	360

l. l. l. s.
If 360 : 40 :: 874—7
20 20
7200 17487
40
l. s.
720|0)69948|0(97 : 3 A's part of the gain: then B's will be l. 194 : 6; and C's will be l. 582 : 18.
108
20
72|0)216|0(3
0

A, hath 100 yards of linen at 4 s. per yard ready money, which he barters with B, at 6 s. per yard, taking yarn at 2 s. per spynle which in ready money is but worth 20 d. per spynle; how many spynles of yarn will pay for the 100 yards of linen, and whether doth A, or B, get the better bargain, and by how much.

yd. s. yds.
If 1 : 6 :: 100
6
l.
20)600(30 Value of A's linen in barter.
0

I bought 20 ton of cheese for 400 l. with which I went into Ireland; the freight and custom came to 50 l. my own charges to l. 16 : 13 : 4; how shall I sell it per lb. to gain 20 l. per cent on the whole.

```
          l.    s.   d.
         400 : 00 : 0
          50 : 00 : 0
          16 : 13 : 4
         ─────────────
100 : 120 :: 466—13—4
240            20
───          ────
24000          9333
                 12
             ──────
             112000
                120
           ───────── l.
24|000)134401|000(560 At which the whole may
           ─────────   be sold, to gain 20 l. per cent.
             0

     T.     l.     lb.
   If 20 : 560 :: 1
      20     20
     ───   ────
     400  11200
     112     12
     ───  ─────
448|00)1344|00(3 d. at which 1 lb. may be sold
        ───────   to gain 20 l. per cent.
```

If

If a quantity of cut tobacco weighing 240 lb. coſt 13 l., what muſt 1 lb. be ſold for, to gain l. 15 : 10 per cent ?

```
lb.    l.    lb.
240 : 13 :: 1
         20
       -----
        260
         12
       -----
24|0)312|0(13 Pence prime coſt of 1 lb.
      -------
         0

 l.      l.   s.      d.
100 : 115 — 10 :: 13
240      20
-----   -----
24000 · 2310
            13
        --------  s.  d.
2400|0)5003|0( 1 : 3 36/2400 or 3/200 at which 1 lb.
        -----      may be ſold to gain l. 15 : 10 per
         603       cent.
          12
         -----
 2400)7236(3
        -----
         36
```

What will 400 l. amount to, forborn 3 years and an half at 5 l. per cent per annum compound intereſt?

To work queſtions of this nature, after having found the firſt year's intereſt, add it to the principal, and find the intereſt of the ſum; and ſo continue to add every year's produce, ſtill accounting the ſum a new principle.

See the Work.

l. l. l.
100 : 5 :: 400 Principal 1st, year.
5
l. 20|00
l.
20
400
l.
100 : 5 :: 420 Principal 2d, year.
5
l. 21|00
l.
21
420
l.
100 : 5 :: 441 Principal 3d, year.
5
l. 22|05
20
s. 1|00
l. s.
22—1
441
l. l.
100 : 5 :: 463—1 Principal 4th year.
5
l. 23|15 : 5
20
s. 3|05
12
|60
4
q. 2|40

Interest

	l.	s.	d.	q.
Intereſt the 1ſt, year - -	20 :	00 :	0 :	0
2d,	21 :	00 :	0 :	0
3d, - - -	22 :	1 :	0 :	0
$\frac{1}{2}$ l. 23:3:0:2,4 for $\frac{1}{2}$ the 4th year's intrſt.	11 :	11 :	6 :	$1\frac{2}{10}$
Total intereſt	74 :	12 :	6 :	$1\frac{2}{10}$
Principal	400 :	0 :	0 :	0
Total amount l.	474 :	12 :	6 :	$1\frac{2}{10}$

The amount may be alſo found thus,

```
 l.      l.      l.
100  :  105  ::  400
         400
       ------
l. 420|00        1ſt, year's amount,

 l.      l.      l.
100  :  105  ::  420
        420
      ------
       2100
       420
     -------
l. 441|00        2d, year's amount.

 l.      l.      l.
100  :  105  ::  441
         441
       ------
          105
         420
        420
       ------
l. 463|05        3d, year's amount,
         20
       ------
      s. 1|00
```

Then

Then for the half year say,

```
   l.   l.   s.    l.  s.
If 100 : 102—10 :: 463—1
    20   20         20
  ————  ————       ————
  2000  2050       9261
                   2050
                 ———————
                 463050
                18522
               ———————— 20)  l.  s. d. q.
        20|00)18985|0510(9492(474:12:6:1 4/20 or 1/5
               ————————  ————  Amount, as before.
                  105     12
                   12
                ——————
         20|0)126|0(6
                ——————
                  6
                  4
                ————
              20)24(1
                ————
                 4
```

What will 1000 l. amount to forborn 3 years at 6 l. per cent per annum compound interest?

```
  l.    l.     l.
 100 : 106 :: 1000
             1000
           ———————
      l. 1060|00   1st, year's amount.
```

l.   *l.*   *l.*
100 : 106 :: 1060
106

6360
1060

l. 112360 2d, year's amount,
20

s. 12|00

l.   *l.*   *l.*   *s.*
100 : 106 :: 1123—12
20   20

2000   22472
106

134832
22472

l. *s.* *d.* *q.*
2000)2382032(1191 : 0 : 3 : $3\frac{72}{200}$ or $\frac{9}{25}$
Amount required.

32
20

640
12

200|0)768|0(3

168
4

200)672(3

72

Now

Now, suppose the foresaid 1000 l. to have been laid out upon simple interest at 6 l. per cent per annum, and forborn 3 years; what's the difference of the amounts?

l.
18 Interest of 100l. for 3 years at 6 per cent,
100

If 100 : 118 :: 1000
118 *l.* *s.* *d.* *q.*
1191 : 0 : 3 : $3\frac{9}{21}$
1100) 1180100 (1180 Amount at simple interest.

0 l. 11 : 9 : 3 : $3\frac{9}{21}$ Difference.

Three partners, viz. A, B, and C, make a joint stock in this manner; A, puts in 32 l. B, 24 l. and C, 40 l. they trade and gain 12 l. what will each man's true part of the gain be?

All questions of this nature are answered by so many several operations as there are partners in the joint stock, if none of their stocks be alike; and this is the Rule,

As the total sum of their particular stocks,
Is to the whole gain or loss:
So is each mans particular stock,
To his particular share of that gain or loss.

See the Work.

l.
A, 32
B, 24
C, 40

l. *l.*
If 96 : 12 : 32
32

96) 384 (4 l. A's part of the gain.

0

Then

$$\begin{array}{l} \quad\quad\quad l. \quad\ l. \quad\ \ l. \\ \text{Then if } 96 : 12 :: 24 \\ \quad\quad\quad\quad\quad\ \ 24 \\ \hline 96)288(3\ l. \text{ B's part of the gain.} \\ \hline \quad\quad 0 \end{array}$$

$$\begin{array}{l} \quad\quad\quad l. \quad\ l. \quad\ \ l. \\ \text{And if } 96 : 12 :: 40 \\ \quad\quad\quad\quad\quad\ \ 40 \\ \hline 96)480(5\ l. \text{ C's part of the gain} \\ \hline \quad\quad 0 \end{array}$$

In queſtions of this nature, if the ſum of their particular parts of the gain or loſs amounts to the total gain or loſs, (as here it will do) the work is true; if not, ſome error is committed which muſt be corrected.

	l.
A's part of the gain	4
B's - - -	3
C's - - -	5
Total gain	12

Three merchants A, B, and C, freight a ſhip with 248 tun of wine: Thus, A, loaded 98 tun, B, 86 tun, and C, 64 tun. By extremity of weather the ſeamen were forced to throw 93 tun of it over board; how much of this loſs muſt each merchant ſuſtain?

T.
A, 98
B, 86
C, 64

———

T. T.
If 248 : 93 :: 98
98

———

744
837

———

248)9114(36 Tun, 189 gallons, A's loss.

———

186
252 Gallons in 1 Tun.

———

372
930
372

———

248)46872(189

———

0

T. T. T.
If 248 : 93 :: 86
86

———

558
744

———

248)7998(32 Tun, 63 gallons, B's loss.

———

62
252

———

504
1512

———

248)15624(63

0

If 248

$$
\begin{array}{l}
\quad\ \ T. \quad\ T. \quad T. \\
\text{If } 248 : 93 :: 64 \\
\qquad\quad\ \ 64 \\
\qquad\quad \overline{\ \ \ \ \ \ } \\
\qquad\quad 372 \\
\qquad\ \ 558 \\
\qquad \overline{\ \ \ \ \ \ \ \ \ } \\
248)5952(24 \text{ Tun, C's loss.} \\
\qquad \overline{\ \ \ \ \ \ \ \ \ } \\
\qquad\quad 0
\end{array}
$$

PROOF.

	T.	g.
A's loss was -	36 :	189
B's - -	32 :	63
C's - - -	24 :	
Total loss	93	

Now, if you were to find how much of the remaining wine that was saved belongs to A, to B, and C, the proportion would be,

As the total sum of their particular losses,
Is to the remaining wine, viz. 155 tun:
So is each mans particular loss,
To his particular share of the remaining wine.

A school master's sallary (being 10 l.) is paid by four heritors A, B, C, and D, in proportion to their valued rents; A's valued rent is 120 l. B's 300 l. C's 1200 l. and D's 2800 l.; how much of the sallary must each heritor pay?

l.

A, 120

B, 300

C, 1200

D, 2800

l. *l.*

4420 : 10 :: 120

20

200

120

s. *d.*

442|0)2400|0(5 : $5\frac{70}{442}$ Paid by A.

2210

190

12

442)2280(5

2210

70

l. *l.* *l.*

4420 : 10 :: 300

20

200

300

s. *d.*

442|0)6000|0(13 : $6\frac{326}{442}$ Paid by B.

254

12

442)3048(6

396

```
 l.     l.      l.
4420 : 10 :: 1200
               10
             ------- l.  s.  d.
        442|0)1200|0(2 : 14 : 3 258/442 Paid by C.
             -------
              316
               20
             -------
         442)6320(14
             -------
              132
               12
             -------
         442)1584(3
             -------
              258

 l.     l.      l.
4420 : 10 :: 2800
               10
             ------- l.  s.  d.
        442|0)2800|0(6 : 6 : 8 160/442 Paid by D.
             -------
              148
               20
             -------
         442)2960(6
             -------
              308
               12
             -------
         442)3696(8
             -------
              160
```

PROOF.

	$l.$
A,	120
B,	300
C,	1200
D,	2800

$$
\begin{array}{l}
\quad\;\; l. \quad\; l. \\
4420 : 10 :: 120 \\
\quad\quad\;\; 20 \\
\hline
\quad\quad\; 200 \\
\quad\quad\;\; 120 \\
\hline
\end{array}
$$

$$44|0)2400|0(5 \; s. : 5\tfrac{70}{442}\; d.$$ Paid by A.

$$
\begin{array}{l}
\quad 2210 \\
\hline
\quad 190 \\
\quad\;\; 12 \\
\hline
442)2280(5 \\
\quad 2210 \\
\hline
\quad\;\; 70
\end{array}
$$

$$
\begin{array}{l}
l. \quad\; l. \quad\; l. \\
4420 : 10 :: 300 \\
\quad\quad\;\; 20 \\
\hline
\quad\quad\; 200 \\
\quad\quad\;\; 300 \\
\hline
\end{array}
$$

$$44|0)6000|0(13 \; s. : 6\tfrac{396}{442}\; d.$$ Paid by B.

$$
\begin{array}{l}
\hline
\quad 254 \\
\quad\;\; 12 \\
\hline
442)3048(6 \\
\hline
\quad 396
\end{array}
$$

l. *l.* *l.*
4420 : 10 :: 1200
10

l. *s.* *d.*
442|0)12000|0(2 : 14 : $3\frac{258}{442}$ Paid by C.

316
20

442)6320(14

132
12

442)1584(3

258

l. *l.* *l.*
4420 : 10 :: 2800
10

l. *s.* *d.*
442|0)28000|0(6 : 6 : $8\frac{160}{442}$ Paid by D.

148
20

442)2960(6

308
12

442)3696(8

160

PROOF.

PROOF,

	l.	s.	d.	Remdrs.
A, Pays	0 :	5 :	5	70
B, -	0 :	13 :	6	- - 396
C, -	2 :	14 :	3	- - 258
D,	6 :	6 :	8	- - - 160
Value of remdrs.		-	2	
Sallary l.	10 :	0 :	0	442)884(2d.
				0

A, B, and C, met together at an inn, and drank till the reckoning came to 20 s. whereof A, paid one half, B, one third, and C, one fourth; what did each pay of the 20 s.?

Here, if you take the natural parts of 20 s. as they are express'd in the question, their sum will exceed 20 s. for $\frac{1}{2}$ 20 s. is 10 : 0
and $\frac{1}{3}$ of it is 6 : 8
and $\frac{1}{4}$ of it is 5 : 0

Sum of the parts 21 : 8

The meaning of the question is, that 20 s. be divided into such proportion as the parts bear to one another; as followeth.

s. d. s. s.
If 21—8 : 10 :: 20
12 12

260 240
10

s. d. q.
26|0)240|0(9 : 2 : $3\frac{2}{26}$ or $\frac{1}{13}$ Paid (by A.

6
12

26)72(2

20
4

26)80(3

2

s. d. s. d. s.
If 21—8 : 6—8 : 20
12 12 12

260 80 240
80

12) s. d. q.
26|0)1920|0(073(6 : 1 : $3\frac{10}{26}$ or $\frac{5}{13}$ Paid (by B.

22 1.
4

26)88(3

10

If 21

```
      s.  d.  s.     s.
If 21—8 : 5 :: 20
    12           12
   ----         ----
   260          240
                  5
               ----  s.  d.  q.
        26|0)120|0(4 : 7 : 1 14/26 or 7/13 Paid by C.
             ----
              16
              12
             ----
         26)192(7
             ----
              10
               4
             ----
          26)40(1
             ----
              14
```

PROOF.

	s.	d.	q.	Remdrs.
A, paid -	9 :	2 :	3	2
B, - -	6 :	1 :	3 - -	10
C, - -	4 :	7 :	1 -	14
Value of the remainders			1	26)26(1 D.
Reckoning	20 :	0 :	0	0

But if A, had paid one third, B, one fourth, and C, one fifth of 20 s. the sum of the parts would have been less than the whole, and the work would have been performed after the same manner as before, viz. thus,

$\frac{1}{3}$ of 20

	s.	s. d.
$\frac{1}{3}$ of	20 is	6—8
$\frac{1}{4}$	is	5—0
$\frac{1}{5}$	is	4—0

```
      s.  d.   s.  d.     s.
If 15—8 : 6—8 :: 20
   12      12      12
          ----    ----
           80      240
          240
       ------- 12) s. d. q.
188)19200(102 (8:6:0 96/188 or 24/47 Paid
                                     (by A.
    ------ ----
       24   6
        4
      ----
  188)96(0
      ----
       96


 s. d.  s.    s.
If 15—8 : 5 :: 20
   12           12
  ----        ----
   188         240
                 5
          ------ s. d. q.
      188)1200(6 : 4 : 2 72/188 or 18/47 Paid by B.
          ----
           72
           12
          ----
      188)864(4
          ----
           112
             4
          ----
      188)448(2
          ----
           72
```

If 15

s. d. s. s.
If 15—8 : 4 :: 20
12 12

188 240
4

s. d. q.
188)960(5 : 1 : $1\frac{20}{188}$ or $\frac{5}{47}$ Paid by C.

20
12

188)240(1

52
4

188)208(1

20

PROOF.

	s. d. q.	*Remdrs.*
A, paid - -	8 : 6 : 0	- - 96
B, paid - - - -	6 : 4 : 2	- - 72
C, paid - - -	5 : 1 : 1	- 20
Value of the remainders	1	
		188)188(1qr.
Reckoning, as before	20 : 0 : 0	
		0

Three farmers A, B, and C, hired a shepherd to keep their sheep at 5 l. per annum; A, committed to his care 400 sheep, B, 600, and C, 1000; what must each pay of the 5 l.?

A, 400

l.
A, 400
B, 600
C, 1000

l. s.
If 2000 :: 5 :: 400
400

2000)2000(1 l. paid by A.

0

s. l. s.
If 2000 : 5 :: 600
600

2|000)3|000(1 l. 10 s. paid by B.

1
20

2)20(10

0

s. l.
If 2000 : 5 :: 1000
5

l. s.
2|000)5|000(2 : 10 paid by C.

1
20

2)20(10

0

This question might have been resolved at one operation; for having found what A, paid, it's very obvious that B, should pay as much and half as much more, because he had as many and half as many more sheep as A: and it's as evident that C, (as he had as many sheep as A, and B, had) should pay as much as they both paid.

A, and B, enter into a joint adventure, and gain'd 120 l. their agreement was that A, should have 10 *l.* per cent gain, and B, 8 l. per cent; what must each have?

Suppose each man's gain per cent to be his stock, and proceed as follows.

```
      l.
    A,10
    B, 8
    ——     l.    l.
    If 18 : 120 : 10
             10
           ———  l.  s.  d.
      18)1200(66 : 13 : 4  A's part of 120 l. which
         ———                taken from 120 l. leaves
          12                l. 53:6:8 for B's part.
           20
          ————
       18)240(13
          ———
            6
           12
         ————
       18)72(4
         ———
          0
```

Two

Two merchants company, A, put in 20 l. and B, put in 144 ducats; they gain'd l. 67 : 10, of which A, took up 30 l.; what's the value of a ducat?

First find a stock for B, equivalent to A's stock, thus.

```
     l.   l.    l.  s.
If  30 : 20 :: 67—10
    20          20
   ----        ----
   600         1350
                 20
              ------ l.
        6|00)270|00(45  B's stock:
        ----------  20
           0       ---- s. d.
              144)900(6 : 3 Value of a ducat.
                  ---
                   36
                   12
                  ---
              144)432(3
                  ---
                   0
```

Two merchants company, A, puts in 36 l. and takes out $\frac{3}{5}$ of the gain; what did B, put in?

If A, took up $\frac{3}{5}$, B, must needs have $\frac{2}{5}$, and seeing the denominators of the fractions are equal, you may work with the numerators, thus;

```
          l.
If 3 : 36 :: 2
          2
        ----
     3)72(24 l. put in by B.
       ----
        0
```

l. l. l.
If 1710 : 684 : 460
460

41040
2736

1710)314640(184 l. Fell due to B.

l. l. l.
If 1710 : 684 :: 500
500

l.
1710)342000(200 Fell due to C.
184 Fell due to B.
300 Fell due to A.

l. 684 Given value of his money (and effects.

The work of this and some of the immediately preceeding examples, (being a very long and tedious process) might be more compendiously managed, if the learner was well acquainted with decimal fractions.

The Rule which I am here to prescribe may well enough answer the end by a Vulgar operation, which is this,

RULE.

Annex a cypher, or cyphers to the second term; (except it be big enough to divide without such cypher or cyphers, which very rarely happens) then dividing it by the first term, multiply each person's particular stock by the Quotient, and the products will be each man's proportional part in the gain or loss;

loss: But you must remember always to strike off to the right hand of the said products (for decimals) so many figures or cyphers as you annex cyphers to the sum to be divided. And for a specimen, let the last example be wrought by this method; Thus:

A, 750
B, 460
C, 590

1710)684,0(4 Common Multiplier.

0

l.
750 Due to A.
4

l. 300,0 A's part of 684 l.

460 Due to B.
4

l. 184,0 B's part of 684 l.

500 Due to C.
4

l. 200,0 C's part of 684 l.

A, B, C, and D, join in company, A, put in a certain ſum, B, put in 72 l. C, 48 l. and D, 56 l ; On this adventure A, gain'd 42 l. and B, C, and D, together gain'd 285 l. ; I demand A's ſtock, and B, C, and D's particular parts of the gain ?

l.
72
48
56

l. —— l.
285 : 176 :: 42
42

352
704

l. s. d. q.
285)7392(25 : 18 : 8 : $3\frac{105}{285}$ or $\frac{7}{19}$ A's ſtock.

267
20

285)5340(18

210
12

285)2520(8

240
4

285)960(3

105

l.   l.   l.
176 : 285 :: 48

```
              48
          ------
            2280
           1140
          ------  l.   s.  d.  q.
176)13680(77 : 14 : 6 : 2 32/176 or 2/11 C's gain.
          ------
           128
            20
          ------
176)2560(14
          ------
            96
            12
          ------
176)1152(6
          ------
            96
             4
          ------
176)384(2
          ------
           32
```

```
 l.    l.     l.
176 : 285 :: 72
       72
     ----
      570
    1995
    -----     l.   s.  d.  q.
176)20520(116 : 11 : 9 : 3 48/176 or 3/11 B's gain.
    -----
     104
      20
    ----
176)2080(11
    ----
     144
      12
    ----
176)1728(9
    ----
     144
       4
    ----
176)576(3
    ---
     48
```

l.	l.	l.

176 : 285 :: 56
56

1710
1425

l. s. d. q.

176)15960(90 : 13 : 7 : $2\frac{96}{176}$ or $\frac{6}{11}$ D's gain.

120
20

176)2400(13

112
12

176)1344(7

112
4

176)448(2

96

A, B, C, and D, made a ſtock of 3000 l. and gain'd as follows, viz. A, B, and C, gain'd 63 l. B, C, and D, 60 l. C, D, and A, 57 l. and D, A, and B, 54 l.; what was each man's ſtock and gain?

ABC, 63
BCD, 60
CDA, 57
DAB, 54

3)234(78

0

```
  78          78          78          78
  60          57          54          63
  --          --          --          --
l.18 A's gain. l.21 B's gain. l.24 C's gain. l.15 D's gain.
  --          --          --          --

     l.
A,  18
B,  21
C,  24
D,  15
    --      l.       l.
   178 : 3000 :: 18
              18
         ------- l.   s.  d.  q.
      78)54000(692 : 6 : 1 : 3 30/78 or 5/13 A's stock.
         -------
           24
           20
          -------
        78)480(6
           ----
            12
            12
           ----
        78)144(1
           ----
            66
             4
           ----
        78)264(3
           ----
            30
```

l. *l.* *l.*
78 : 3000 :: 21
21

78)63000(807*l.* 13*s.* 10*d.* 0 $\frac{48}{78}$ or $\frac{8}{13}$ *q.* B's stock.

54
20

78)1080(13

66
12

78)792(10
12
4

78)48(0

48

l. *l.* *l.*
78 : 3000 : 24
24

l. *s.* *d.* *q.*
78)72000(923 : 1 : 6 : 1 $\frac{66}{78}$ or $\frac{11}{13}$ C's stock.

6
20
78)120(1
42
12
78)504(6
36
4
78)144(1

66

l. l. l.
78 : 3000 :: 15
15

l. s. d. q.
78)45000(576 : 18 : 5 : $2\frac{1}{78}$ or $\frac{2}{13}$ D's stock.

72
20

78)1440(18

36
12

78)432(5

42
4

78)168(2

12

A, lends to B, 1200 l. for 3 years at 5 l. per cent per annum, compound interest; what may A, spend per day, that the total amount may be exhausted at the 3d, year's end?

l. l. l.
If 100 : 105 :: 1200
1200

100)126000(1260 l. 1st, year's amount.

2

If 100

l. l. l.
If 100 : 105 :: 1260
105

6300
1260

100)132300(1323 l. 2d, Year's amount.
0

l. l. l.
If 100 : 105 1323
105

6615
1323

l. s.
100)138915(1389 : 3 3d, Year's amount.

15
20

100) 300(3
0

365 1389—3
3 20

s. d. q.
1095) 27783(25 : 4 : 1 $\frac{969}{1095}$ or $\frac{323}{365}$, Answer.

408
12
1095) 4896(4

516
4
1095) 2064(1
969

A,

A, put into company 560 l. for 8 months, B, 279 l. for 10 months, and C, 735 l. for 6 months; they gain'd 1000 l.; what share of it must each have?

In questions of this nature, multiply each man's stock by the time he puts it in for, and the sum of all the products must be your first number through all your operations; the gain or loss the second (as before) and each man's particular stock multiply'd by its time, the third.

Note, All the particular times (if not so given) must be reduced into one name, that is, all years, all months, all weeks, or all days, &c.

The reason of multiplying the several stocks by the times they are put in for, will appear from the following considerations. First, suppose A, and B, put into trade each 100 l. for 1 month, then certainly their gain, whatever it be, must be equally divided betwixt them. Again, suppose A, puts in 100 l. and B, 200 l. both for the same time; 'tis as plain that B's share of the gain must be twice as much as A's, because, his stock is double: Or if each put in 100 l. but for different times, viz. A, for 1 month, and B, for 2; then as B's stock lyes twice as long as A's, so his gain, as before, must be twice as great. Lastly, suppose A, to put in 100 l. for 1 month, and B, 200 l. for 2 months; then since B's stock is not only double, but the time he leaves it in double too, his share of the profits must be 4 times greater than A's. And so in all other cases.

I shall here leave the work of the foregoing example for the learner's practice, and only set down the answers.

560	279	735
8	10	6
4480	2790	4410

4480 Product of A's stock and time.
2790 B's.
4410 C's.

	l.	s.	d.	q.	Remdrs.
11680 : 1000 : 4480(	383 :	11 :	2 :	3	2080 A's gain.
11680 : 1000 : 2790(	238 :	17 :	4 :	3	800 B's.
11680 : 1000 : 4410(	377 :	11 :	4 :	1	8800 C's.
Value of the remdrs. - - -				1	

Total gain l. 1000 : 00 : 0 : 0 11680) 1680(1 q.
0

A ship's company take a prize of l. 718 : 15, which is to be divided among them according to their pay, and time they have been on board: The officers and midshipmen were 6 months on board, and the sailors 4 months; the officers had each 40 s. per month, the midshipmen 30 s. per month, and the sailors 24 s. a man per month; there were 6 officers, 93 sailors, and 13 midshipmen; what must each have to their share?

		24
		93
40	30	72
6	13	216
240	390	2232
6	6	4
1440 Officers.	2340	8928

1440 Officers.
2340 Mid.
8928 Sailors.

l. s.	l.	s.	d.	q.	Remdrs.
12708 : 718—15 :: 1440(	81 :	8 :	10 :	2	12312 to offirs.
12708 : 718—15 :: 2340(	132 :	6 :	11 :	1	10476 to mid.
12708 : 718—15 :: 8928(	504 :	19 :	1 :	3	2628 to sail.
Value of the remdrs. - -				2	

 Total

Total gain l. 918 : 15   12708)25416(2 q.

0

Note, If you divide the total ſhare of all the officers by their number, the Quotient will be the ſhare of every individual; and ſo will it be with reſpect to the midſhipmen and ſailors.

How many pints of water muſt be mixed with wine at 3 s. per pint, to fill a veſſel of an hundred pints, ſo as a pint may be afforded at 2 s. 6 d.?

$3 : 2\frac{1}{2} :: 100$

$2\frac{1}{2}$

200

50

$3)250(83\frac{1}{3}$ Pints of wine, which taken from 100, leaves $16\frac{2}{3}$ of water.

1

A butcher agrees with a graſier, for the feeding of 20 oxen during the ſpace of 12 months; but at 2 months end the butcher adds 5 oxen more, and $6\frac{3}{4}$ monthes after that he added 10 oxen more; and then it was agreed between them, that the graſier ſhall feed them all ſo long time as will be equivalent to the keeping of the firſt 20 during 12 months; I demand how long he ſhall feed them all, after putting in of the laſt 10?

Conſider

Consider that as he receives more oxen to feed, he ought to feed them the shorter time.

m. ox.
12 20
2 5

ox.
If 20 : 10 : 25
20

25)200(8 Months, and so long may he feed the 25 oxen, which time is equivalent to the feeding of 20 for 12 Months.
0

m. ox.
8 25
$6\frac{3}{5}$ 10

ox.
If 25 : $1\frac{2}{5}$ :: 35
25

25
10

35)35(1 Month, and so long may he feed them all, after putting in of the 10 oxen.
0

A, and B, have entered company, A, put in 500l. and at 4 months end took out a certain sum, leaving the remainder to continue 8 months longer; B, put in 250 l. and at 5 months end put in 300 l. more, and then his whole sum continued 7 months longer. Now at making up of their accompts, A, found that he had gain'd l. 106 : 13 : 4, and B, had gain'd l. 133 : 6 : 8; I demand how much A, took out of the bank at 4 months end?

```
                 250
                 300
      250       ----
        5        550
     -----         7
     1250       ----
                3850
                1250
                ----        l.   s.  d.
If 133—6—8 : 5100 :: 106—13—4
        20                   20
     -------             -------
      2666                 2133
        12                   12
     -------             -------
     32000                25600
                           5100
                         -------
                          256
                        1280
                        ----------
              32|000)130560|000(4080 Product of A's
                    -------             stock and time.
                        0
```

500 l. A's whole stock.
4 Time the whole continued.

2000 Product of his stock and time at 4 months end, which taken from 4080, leaves the product of his stock and time for 8 months, viz. 2080.

8)2080(260 l. that A, continued in bank for 8 (months.
0

l.
500 His whole stock.
260

240 l. That A, took out at 4 months end.

A, and

A, and B, join in partnerſhip upon theſe terms, viz. A, agrees to lay down 100 l. and to employ it in trade 3 months: at the end of which time B, is to lay down his 100 l. and with the whole ſtock of 200 l. they are to trade 3 months more. Now at 6 months end, they find their whole gain to be 21 l.; I demand each mans part of the gain?

100	100
6	3
A, 600	300
B, 300	

l.
If 900 : 21 :: 600
600

9|00)126|00(14 l. A's part of the gain.

0

If 900 : 21 :: 300
300

9|00)63|00(7 l. B's part of the gain.

0

A, B, and C, are in company. A, put in the firſt of March 60 l. B, put in the firſt of May 160 yards of cloth, and C, put in the firſt of June 240 ducats. On the firſt of January following, they accompted their gain; of which A, and B, took up 456 l. B, and C, took up 431 l. and C, and A, took up 375 l. I demand the whole gain, alſo, their particular ſhares, what B, valued a yard of cloth at, and the value of one of C's ducats?

456

l.
456 A, and B.
431 B, and C.
375 C, and A.

2)1262(631 l. The whole gains.
431 B, and C's gain.
0
l. 200 A's real gain.

631 Whole gain.
375 C, and A's gain.

l. 256 B's real gain.

631 Whole gain.
456 A, and B's gain.

l. 175 C's real gain.

Now find the value of a yard of B's cloth; thus,

l. l. l.
If 200 : 60 :: 256
256

20|0)1536|0(76 l. 16 s.

16
20

2|0)32|0(16

0

```
    m.    l.   s.    m.
If 10 : 76—16 :: 8
           20
       ------
        1536
          10
       ------ 16|0)
   8)15|360(19|20(12 s. Value of a yard of
     ------  ------          (B's cloth.
         0       0
```

Then for the value of 1 ducat.

```
      l.    l.     l.
If 200 : 60 :: 175
                 60
              ------
       2|00)105|00(52 l. 10 s.
              ------
                  1
                 20
              ------
               2)20(10
              ------
                  0
```

```
  m.    l.   s.    m.
If 10 : 52—10 :: 7
          20
       ------
        1050
          10
       ------ 24|0)
   7)10500(15|0|0(6 6/24 or 6 s. 3 d. Value of one
     ------  ------                 ducat of C's.
         0       6
```

A, B, and C, company, and put in together 3822 l. A's money was in 3 months, B's 5 months, and C's 7 months: they gain'd 234 l. which was so divided

divided as the $\frac{1}{2}$ of A's gain was equal to $\frac{1}{3}$ of B's gain, and $\frac{1}{3}$ of B's gain was equal to $\frac{1}{4}$ of C's gain; what did each merchant gain, and put in?

Suppose A's gain was 4 l. then B's must be 6 l. and C's 8 l. and you must work thus,

```
    l.
A,  4
B,  6
C,  8
——        l.     l.
   18 : 234 :: 4
              4
          ————
If 18)936(52 l. A's real gain.
          ————
             0
   l.     l.    l.
If 18 : 234 :: 6
             6
         ——————
   18)1404(78 l. B's real gain.
         ———
            0
   l.     l.    l.
If 18 : 234 :: 8
             8
         ——————
   18)1872(104 l. C's real gain.
         ———
            0
```

Now, for their stocks, multiply each man's gain successively by every other man's time; and then say, As the sum of these products is to the whole stock, so will each product be to the partners stock corresponding thereto.

See the work on the next page.

52 A's

52 A's gain.	78 B's gain.	104 C's gain.
5 B's time.	7 C's time.	3 A's time.
260	546	312
7 C's time.	3 A's time.	5 B's time.
1820	1638	1560

1820
1638
1560

		l.	s.	d.	Rem.	Stocks,
If	5018 : 3822 :: 1820(	1386 :	4 :	4	1144	A's
If	5018 : 3822 :: 1638(	1247 :	11 :	11	26	B's
If	5018 : 3822 :: 1560(	1188 :	3 :	8	3848	C's
	Value of the remdrs,			1		

Total gain, l. 3822 5018)5018(1 d.
0

A, B, and C, take a piece of ground for grass, at 46 l. 10 s. in which A, put in 12 oxen for 8 months, B, put in 16 for 5 months, and C, put in 18 for 4 months; what must each pay of the 46 l. 10s.?

12	16	18
8	5	4
A, 96	80	72

B, 80
C, 72

	l.	s.	
If 248 : 46—10 :: 96(	18 :	0	paid by A.
If 248 : 46—10 :: 80(	15 :	0	paid by B.
If 248 : 46—10 :: 72(	13 :	10	paid by C.

A, B, and C, make a stock for 12 months; A, put in at first, 364 l. and 4 months after that, he put in 40 l.; B, put in at first 408 l. and at the end of 7 months he took out 86 l.; C, put in at first 148 l. and 3 months after, he put in 86 l. more, and 5 months after that, he put in 100 l. more; and at the end

end of 12 months, their gain was found to be 1436l. I demand each man's ſhare thereof, in proportion to his ſtock and time?

A, had 364l. for 4 months, and their product is, 1456
then with the 40l. he had 404l. for 8 months. 3232

Sum of the products of A's ſtock and time, 4688

B, had 408l. in 7 months, and their product is, 2856
then took out 86l. ſo he had but 322l. for 5 months, 1610

Sum of the products of B's ſtock and time, 4466

C, had 148l. in for 3 months, and their product is, 444
then with the 86l. he had 234l. in for 5 months, 1170
and with the 100l. he had then 334l. in 4 months, 1336

Sum of the products of C's ſtock and time, 2950
B's, 4466
A's, 4688

Total ſum, 12104

	l.	*l.*	*s.*	*d.*	*Remdrs.*
If	12104 : 1436 :: 4688(	556 :	3 :	6	6192 A's gain.
If	12104 : 1436 :: 4466(	529 :	16 :	9	5496 B's gain.
If	12104 : 1436 :: 2950(	349 :	19 :	8	416 C's gain.

A, gains 5l. in 2 months, B, 15l. in 4 months, and C, 15l. in 3 months, whoſe ſtock is 40l.; what are the ſtocks of A, and B?

40
3

l. 15 : 120 :: *l.* 5
5

2)
15)600(40(20l. A's ſtock.
0

l. 15

l. l.
15 : 120 :: 15
15

4)
15)1800(120(30 l. B's stock.

0

A, B, and C, have a common stock of 90 l. A, gains 5 l. in 2 months, B, 15 l. in 4 months, and C, 15 l. in 3 months; what were their particular stocks?

Divide each partner's gain by his time, and then 90 l. being divided into 3 parts proportion'd to those quotes, gives their particular stocks.

l. s. l. s. l.
2)5(2 : 10 4)15(3 : 15 3)15(5

1. 3 : 15 3 0
5

l. s. l.
11—5 : 2—10 :: 90
20 20 20

225 50 1800
50

20) l.
225(90000(400(20 A's stock

0 0

l. s. l. s. l.
11—5 : 3—15 :: 90
20 20 20

225 75 1800
75

20) l.
225)135000(600(30 l. B's stock.

0 0

l. 15

l. s. l. l.
11—5 : 5 :: 90
20 20

225 1800
5

l.
225)9000(40 C's stock.

0

A, receives of gain 5 l. for 2 months, B, 15 l. for 4 months, and C, 15 l. for 3 months; the sum of the products of their stocks and times is 280; what were their stocks?

l.
A, 5
B, 15
C, 15

l.
35 : 5 :: 280
5

2)
35)1400(40(20 l. A's stock.

0 0

35 : 15 :: 280
15

4)
35)4200(120(30 l. B's stock.

0 0

35 : 15 : 280
15

3)
35)4200(120(40 l. C's stock.

0 0

A, and

A, and B, were in company thus; A, had 50l. of stock in for 10 months, and B, had his stock in for 8 months, and they shared equally of the gain? what was B's stock?

m. l.
If 10 : 50 :: 8
10

8)500(62 l. 10 s. B's stock.

4
20

8)80(10

0

Here the proportion was Inverse, for as they shar'd equally of the gain, and as B's stock was less time in than A's, it behoved to be greater.

A, B, C, and D, made a joint stock of 4020 l.; A's stock was in company for 3 months, B's for 6, C's for 9, and D's for 12 months; they gain'd as follows, viz. A, B, and C, gain'd 72 l.; B, C, and D, 69 l.; C, D, and A, 66 l.; and D, A, and B, 63 l.; I demand each man's stock and gain?

	l.
A, B, C,	72
B, C, D,	69
C, D, A,	66
D, A, B,	63

3)270(90

0

90	90	90	90
69	66	63	72
l. 21 A's gain.	l. 24 B's gain.	l. 27 C's gain.	l. 18 D's gain.

3)21(7 A.	6)24(4 B.	9)27(3 C.	12)18(1½ D.
0	0	0	6

4 B.
3 C.
1½ D.

			l.	s.	d.	Remdrs.	
15½ : 7	::	4020(	1815	9	8	4	A's ftock.
15½ : 4	::	4020(	1037	8	4	20	B's ftock.
15½ : 3	::	4020(	778	1	3	15	C's ftock.
15½ : 1½	::	4020(	389	0	7	23	D's ftock.
Value of the Remainders,					2		
Sum of their ftocks,			4020			31)62)2d.	
						0	

If the fhorter end of the beam of a ballance be 27 inches, and the longeft end from the centre be 36 inches; how much fufpended on the longeft end will equiponderate with 112 lb. weight fufpended on the fhorter end?

	inches.		lb.		inches.
If	27	:	112	::	36
			27		
			784		
			224		

36)3024(84 lb. Anfwer.
0

If the

If the longest end of the beam of a ballance be 36 inches, and the shorter end 27 inches; how much suspended on the shorter end, will equiponderate with 84 lb. on the longer end?

	inc.	*lb.*	*inc.*
If	36 :	84 ::	27
		36	

504
252

27)3024(112 lb. Answer.

0

If the beam of a ballance is 63 inches long, and 84 lb. on the one end weighs 112 lb. on the other; I demand the length of the arms of the said ballance?

lb.
112
84

	inc.	*lb.*
196 :	63 ::	112
		63

336
672

196)7056) 63 Inches, length of both arms.
36 Inches, longest arm.

0 27 Inches, shorter arm.

If I buy at 15s. 10d. and gain by the sale 25 l. per cent; how must I buy to gain by the same sale 35 l. per cent?

(*See the work on the next page.*)

l. s. d. l.
If 125 : 15—10 :: 135
12

190
125

12) s. d. q.
135)23750(175(14 : 7 : 3 $\frac{95}{135}$ or $\frac{19}{27}$, Anſwer.

125 7
4
135)500(3
95

I bought 720 yards of cloth, at 7s. 10d. per yard, 150 whereof were burnt by accident; how ſhall I ſell the remainder per yard, to gain 10l. per cent?

s. d.
7—10
12

94
720

1880
658

720
150

Remainder 570 Yards.

l. l.
If 100 67680 :: 110
110

570) 12) s. d. q.
1100)7444800(74448(130(10 : 10 : 2 $\frac{252}{570}$ or $\frac{42}{95}$ Anſr.

0 348 10
4

570)1392(2
1140

252

A, was

A, was born to a fortune worth 500 l. per annum; we reckon him to begin life at 21 years of age, at which time he had 12 years rent in store, and he spent 1000 l. every year thereafter; I demand how old he was when he commenced a beggar, reckoning his estate worth 20 years purchase?

N. B. The 12 years rent he had in store would serve him 6 years, and the rent of these 6 years (which he had no use for, when he was spending what he had in store) would serve him other 3 years; and then he would be obliged to sell the estate.

500	500	500
12	6	20
6000	3000	10000
		3000
		6000
		19000

year.
If 1000 : 1 :: 19000
1

1|000) 19|000 (19
(21
0
40 Years, Answer.

If 100 l. in 12 months gain 6 l. interest; what will 300 l. gain in 9 months, at the same rate?

Questions of this nature (wherein five numbers are given to find out a sixth proportional) are generally answered by two statings; of which one sometimes happens to be Inverse, and the other Direct; and sometimes both happen to be Direct, or both Inverse; and the stating and working of such questions requires a good deal of consideration.

First then, you must carefully note, that 3 of the five proposed terms are always conditional and supposed;

 ed;

ed; and that the other two move the queſtion. As for inſtance in this example, viz. If 100 l. gain 6 l. in 12 months; theſe three terms are only ſuppoſed or conditional. Then comes the queſtion; What will 300 l. gain in 9 months?

The given numbers (as a preparative for reſolution) muſt be diſpos'd of in due order and place, thus;

That number which is of the ſame quality and denomination with the number ſought, muſt always be the ſecond in your ſtating; and one of the other numbers in the ſuppoſition (it matters not which) muſt be the firſt number; and that number in the demand, which is of the ſame denomination with the firſt, muſt be the third number in your ſtating; which three numbers being thus placed will make one perfect queſtion in the Rule of Three; and the firſt ſtating will then ſtand thus:

	l.		l.		l.
If	100	:	6	::	300

		m.		l.		m.
OR,	If	12	:	6	::	9

```
    l.     l.     l.
If 100 : 6 :: 300
                6
          ---------
1|00)18|00(18 l. Intereſt of 300 l. for 12
          ------            months.
            0
```

There yet remain two numbers to be diſpos'd of, viz. one in the Suppoſition, and another in the Demand; that in the Suppoſition muſt be the firſt in your ſtating, the anſwer to the firſt ſtating is always your middle number, and that other remaining in the Demand, your third; ſo they will ſtand thus;

(See the work on the next page.)

m : l.

m. l. m.
If 12 : 18 :: 9
9
l. s.
12)162(13 : 10 Intereſt of 300 l. for 9 months.
6
20
12)120(10
0

OR, the work may be performed thus;

m. l. m.
If 12 : 6 :: 9
9
l. s.
12)54(4 : 10 Intereſt of 100 l. for 9 months.
6
20
12)120(10
0

l. l. s. l.
If 100 : 4—10 :: 300
20
90
300
20) l. s.
1|00)270|00(270(13 : 10 Anſwer as before.
0 10

I put

I put out 75 l. to receive intereſt for the ſame, and when it had continued 9 months, I received for principal and intereſt, l. 78 : 7 : 6; I demand at what per-cent, per annum I received intereſt?

	l.	s.	d.		
	78	7	6		
	75				
l.					
If 75 :	3	7	6	::	100
			10		
	33	15	0		
			10		
	337	10	0		
	20				

75)6750(90s. Intereſt of 100l. for 9 months.
0

m.		s.		m.
If 9	:	90	::	12
		12		

20)
9)1080(120(6 l. per cent, per annum, the Anſr.
0 0

If a carrier receive 42 ſhillings for the carriage of 3 c.w. 150 miles; I demand how much he ought to receive for the carriage of 7 c.w. 3 qrs. 14 lb. 50 miles, at that rate?

(*See the work on the next page.*)

```
   c.w.  s.   c.w. qrs. lb.
If  3  : 42 :: 7---3---14
   112        4
   ---       ---
   336        31
              28
             ---
             252
             63
             ---
             882
              42
             ---
             1764
            3528
            -----    s.  d.
        336)37044(110--3.  to be paid for the car-
            -----          riage of 7 c.w. 3 qrs.
             84            14 lb. 150 miles.
             12
            -----
        336(1008(3
             0

   m.    s.  d.   m.
If 150 : 110--3 :: 50
         12
       -----
        1323
         50
      ------- 12)  s.   d.
   150)66150(441(36  :  9  to be paid for the car-
      -------  ---          riage of 7 c.w. 3 qrs.
         0     9            14 lb. 50 miles.
```

If 3 c.w. being carried 150 miles, cost 42 shillings; how many c.w. may be carried 50 miles for 36 shillings and 9 pence at that rate?

One

One of the proportions here, will be Inverse, and the other Direct.

```
     s.  c.w.   s.  d.
If  42 : 3  :: 36—9
    12          12
   ----        ----
   504         441
                 3
         ---------- c.w. qrs. lb.
         504)1323(2 : 2 : 14   will be carried 150
         --------              miles for 36 s. 9 d.
              315
                4
         ---------
         504)1260(2
         ---------
              252
               28
           --------
             2016
             504
           --------
           504)7056(14
           --------
                0

     m.   c.w. qrs. lb.     m.
If  150 : 2——2——14 :: 50
          4
        ----
         10
         28
        ----
        294
        150
       ----   28)  4)
    50)44100(882(31(7 c.w. 3 qrs. 14 lb. will be
      ------------- carried 50 miles for 36 s. 9 d.
            0   14  3
```

If 4 masons when the day is 6 hours long, finish a piece of work in 10 days; in how many days will 8 masons perform the same, when the day is 12 hours long?

Here, both proportions will be reciprocal.

```
   m.  d.   m.
If 4 : 10 :: 8
        4
    ---------
    8)40(5 days, time in which 8 masons will do
    ---------   it, when the day is 6 hours long.
       0
   h.  d.  h.
If 6 : 5 :: 12
       6
   ---------
   12)30(2 6/12, or ½ days, time in which 8 masons
   ---------              will do the work, when the
      6                   day is 12 hours long.
```

If 400 pecks of corn, will serve 32 Horses 108 days; what quantity will 500 horses eat in 20 days?

```
   h.    p.     h.
If 32 : 400 :: 500
             500
         ----------
     32)200000(6250 pecks will serve 500 horses for
         ----------      108 days.
            0
    d.     p.     d.
If 108 : 6250 :: 20
                 20
          ------------ pec.   lip.
     108(125000(1157 : 1 68/108, or 17/27 will serve 500
          ------------                horses for 20 days,
             44                       the answer.
              4
        108)176(1
            68
```

If 100

If 100 l. principal in 12 months gain 5 l. interest; what principal sum will gain l. 8 : 15 in 15 months?

m. l. m.
If 12 : 100 :: 15
12

15)1200(80l. will give 5 l. interest in 15 months.

0

l. l. l. s. l.
If 5 : 80 :: 8, 15 : 140 Answer.

In questions of this nature, the five terms may be anged another way, and a solution given by one :ating; thus,

I. Observe (as before) that the given terms are lways five, whereof three are conditional and anecedent, or supposititious, and the other two demand he question, and are consequents answering some of he former antecedents; insomuch, that with the anwer, there will be as many consequents as anteceents, which must match one another in the same enomination exactly.

II. For the right placing of the terms, the three onditional ones are duly to be regarded. Let that 'hich is the principal cause of loss and gain, increase r decrease, action or passion, be put in the first lace; and that which betokeneth the space of time, istance of place, &c. be put in the second place; nd the remaining part in the third. The condition- part being thus stated, the other two terms where- the demand lies, must be placed so under the forer terms, that they may correspond one with anoer.

RULE I.

RULE I.

Then, if the blank or place sought, fall under the third term, multiply the three last terms together for a dividend, and the two first for a divisor, and the quotient gives the sixth term required.

RULE II.

But if the blank fall under the first or second term, multiply the first, second and fifth terms together for a dividend, and the third and fourth for a divisor; the quotient gives the answer.

If 12 roods of ditching be done by 2 men in 6 days; how many roods shall be wrought by 8 men in 24 days?

m.	*d.*	*r.*
2 .	6 .	12
8 .	24	

Now, if (by the first rule) the three last terms, viz. 12, 8, and 24, be multiplied into each other, the last product will be 2304 for a dividend; and the two first terms, viz. 2 and 6 multiply'd together, will give 12 for a divisor, which quotes 192, the answer.

See the work.

```
        24
        12
       ----
6      288
2        8
--   -----
12) 2304 ( 192 Roods, the answer.
      0
```


If 6

If 6 bolls of oats will serve 4 horses 8 days; how many days will 21 bolls serve 16 horses?

h.	*d.*	*b.*
4 .	8 .	6
16 .		21

Here, the blank falls under the second place, therefore you must work by the 2d. rule.

4 the 1st.
8 the 2d.

32
21 the 5th.

the 4th. 16   32
the 3d. 6   64

96)672(7 days, the answer.

0

What principal sum will gain 20l. in 8 months, at 6 l. per cent, per annum?

prin.	*time,*	*gain.*
100 .	12 .	6
.	8 .	20

Here, the blank falls under the first place, therefore the answer must be found by the 2d. rule also.

100 the 1st.
12 the 2d.

the 3d. 6   1200
the 4th. 8   20 the 5th.

48) 24000 (500 l. principal, the answer.
0

The

The proof of ſuch queſtions is beſt performed by varying the queſtion; viz. by ſtating it in another order, as in the laſt example; thus,

If 100 l. principal in 12 months, gain 6 l. intereſt; what will 500 l. gain in 8 months?

prin.	*time,*	*gain.*
100 .	12 .	6
500 .	8	.

Here, the blank falls under the third place, therefore you muſt work by the firſt rule; and the anſwer to the queſtion ſhould be 20 l. if the work of the laſt example was true.

```
                    6   the 3d.
                    8   the 5th.
                  ———
the 1ſt.  12       48
the 2d.   100     500 the 4th.
          ———    ————
        12|00)240|00(20 l. the anſwer.
                 0
```

Suppoſe 100 l. would pay 5 military officers for 22 weeks 6 days; in what time would 12 ſuch officers draw 150 l.?

Offi.	*w.*	*d.*	*l.*
5 .	22—	6 .	100
12 .		.	150

Here, the blank falls under the 2d. place, therefore work by the 2d. rule.

```
                 22—6 the 2d.
                    7
                 ———
                  160 days
                    5 the 1ſt.
                 ————
the 3d. 100       800
the 4th. 12       150 the 5th.
        ——————  ————  7)  w.   d.
      12|00)1200|00(100(14 : 2 the anſwer.
             ————    ——
                 9 — 2
```

If

If when the firlot of wheat is at 3 s. 4 d., the penny loaf weighs 12 ounces; I demand the weight of the loaf worth 9 d. when the firlot ſells at 10 s.

s. d.		d.	oz.	d.
3—4	or,	40 .	12 .	1
10—0	or,	120 .	.	9

Here, the blank falls under the ſecond place, therefore work by the 2d. rule

40 the 1ſt.
12 the 2d.

the 4th. 120 480
the 3d. 1 9 the 5th.

120)4320(36 ounces, the anſwer.

0

There be two rectangular planes of equal content, the length of the one is 20 inches, and of the other 36 inches, the breadth of the one is 10 inches; what's the breadth of the other?

If 20 : 10 :: 36
20

36(200(5 $\frac{20}{36}$ or, $\frac{5}{9}$ inches, anſwer.

20

If two breadths and one length had been given, the unknown length would have been found the ſame way, in an inverted proportion.

A, and B, barter, A, hath 320 ſtone of candles, at 4 s. 6 d. per ſtone; for which B, gives him 30 l. in money, and the reſt in cotton, at 8 d. per lb.; how much cotton did B, give him more than the 30 l.?

ſt. s.

ft. *s.* *d.* *ft.*
If 1 : 4—6 :: 320
12

54
320

1080
162

240)17280 pence price of A's candles.
l. 72 price of ditto.
Subtract 30

42

d. *lb.* *l.*
If 8 : 1 :: 42
20

840
12

112)
8)10080(1260(11 c.w. 1 qr. of cotton, the answer.

0 28)28(1

0

If 285 ſtones each 15 inches long and 10 inches broad be ſufficient to pave 33 yards ſquare; how many ſtones each 24 inches long, and 18 inches broad, will pave 40 yards ſquare?

inc.
15
10

ſt. *yds.*
285 . 150 . 33
18 by 24 is . 432 . 40

Here, the blank falls under the 1ſt. place, therefore, work by the 2d. rule.

the 4th.

the 4th. 432   285 the 1ſt. term.
the 3d. 33   150 the 2d.

1296   42750
1296   40 the 5th.

14256)1710000(119 $\frac{13536}{14256}$ or, $\frac{94}{99}$ ſtones, the anſwer.

13536

If 250 l. in 12 months, at 5 l. per cent, per annum, gain 12 l. 10 s.; what ſum will gain l. 25 : 4 in 9 months at 4 l. per cent, per annum?

l. s. l. l. s.
If 12—10 : 250 :: 25—4
20   20

250   504
250

25200
1008

l.
250)126000(504 principal, that will gain l. 25 : 4 in 12 months, at 5 l. per cent.

0

m. l. m.
If 12 : 504 :: 9
12

9)6048(672 l. will gain l. 25 : 4 in 9 months, at 5 l. per cent, per annum.

0

l. l. l.
If 5 : 672 :: 4
5

4)3360(840 l. Sum required.

0

If 250 l

If 250 l. in 12 months at 5 l. per cent, per annum bring me l. 12 : 10 interest; at what rate per cent, per annum will 850 bring me l. 25 : 4 in 9 months?

	l.	l. s.	l.
If	250 :	12—10 ::	840

```
       20
      ----
      250
      840
     -----
    10000
    2000
    ------
250)210000(840 s. gain'd by 840 l. in 12 months
    ------                     at 5 l. per cent.
        0
```

```
   m.     s.     m.
If 12 : 840 :: 9
            9
         ----
     12)7560(630 s. gain'd by 840 l. in 9 months
        ----           at 5 l. per cent, per annum.
          0
```

```
   s.    l.     l.  s.
If 630 : 5 :: 25—4
               20
              ----
              504
                5
              ----
         630)2520(4 l. per cent per annum, the answ.
             ----
               0
```

If 840 l. in 9 months gain l. 25 : 4 at the rate of 4 l. per cent, per annum; what will 250 l. gain in 12 months at 5 l. per cent per annum?

l. l.

l. l. s. l.
If 840 : 25—4 :: 250
20
504
250
25200
1008

84|0)12600|0(150s. gain'd by 250l. in 9 months at 4l. per cent, per annum.
0

m. s. m.
If 9 : 150 :: 12
12

9)1800(200s. gain'd by 250 l. in 12 months at 4l. per cent.
0

l. s. l.
If 4 : 200 :: 5
5
20) l. s.
4)1000(250(12 : 10 gain'd by 250 l. in 12 months, the answer.
0 10

If 250 l. in 12 months bring me l. 12 : 10 interest at 5 l. per cent, per annum; what will 840 l. bring me in 9 months time, at 4l. per cent, per annum?

l. l. s. l.
If 250 : 12—10 :: 840
20
250
840
10000
2000

250)210000(840s. gain'd by 840 l. in 12 months, at 5 l. per cent, per annum.
0

m. s.

m. l. m.
If 12 : 840 :: 9
9

12)7560(630 s. gain'd by 840 l. in 9 months, at at 5 l. per cent, per annum.

0

l. s. l.
If 5 : 630 :: 4
4

20) l. s.
5)2520(504(25 : 4 gain'd by 840 l. in 9 months at 4 l. per cent, per annum.

0 4

If 72 men in 15 days draw 212 l. wages; in what time will 81 men draw as much?

m. d. m.
If 72 : 15 :: 81
72

81)1080(13 27/81 or 1/3 days, the anſwer.

27

If 315 l. in 12 months bring me l. 15—15 intereſt; in what time will l. 820 bring me as much?

l. m. l.
If 315 : 12 :: 820
12

82[0)378[0(4 50/82, or 25/41 months, the anſwer.

50

If 365 l. pay 18 workmen for 12 months; in what time will 12 workmen draw as much?

men. mo. men.
If 18 : 12 :: 12
18

12)216(18 months, the anſwer.

0

 If

If I pay 35 l. to A, for 9 weeks, and 64 l. to B, for 15 weeks work; for what time will 120 l. pay them both together?

w. l. w.
If 9 : 35 :: 15
15

l. s. d.
9)525(58—6—8 paid to A, for 15 weeks.

3
20

9)60(6

6
12

9)72(8

0

l. s. d.
58—6—8 paid to A, for 15 weeks.
64 - paid to B, for 15 weeks.

w. l.
122—6—8 : 15 :: 120
20 20

2446 2400
12 12

29360 28800
15

w.
2936|0)43200|0(14 $\frac{2096}{2936}$, or $\frac{262}{367}$ Answer.

2096

If I buy at a crown; how must I sell to gain l. 33 : 6 : 8 per cent?

l. s.

l. *s.* *l.* *s.* *d.*
If 100 : 5 :: 133—6—8
240 20

24000 2666
12

32000
5

24|000)160|000(6 s. 8 d. the anſwer.

16
12

24)192(8

0

If by ſelling at 6 s. 8 d. the yard, I gain l. 33 : 6 : 8 per cent; how did I buy?

l. *s.* *d.* *s.* *d.* *l.*
If 133—6—8 : 6—8 :: 100
20 12 240

2666 80 24000
12 80

12)
32000 32|000)1920|000(60(5 s. the anſr.

0 0

I bought 40 packs of cloth, and by ſelling the whole at l. 728, I gain'd 7 d. per ell, or 12 l. per cent; I demand the buying and ſelling prices, and quantity of ells in the 40 packs?

l. *l.*

l. l. d.
If 12 : 100 :: 7
20 20
240 2000
12 7
s. d. q.
2880)14000(4 : 10 : 1 $\frac{1}{2}$ $\frac{96}{288}$, or $\frac{1}{2}$ buying price.
2480
12
288|0)2976|0(10
96
4
288)384(1
96
s. d. q.
4—10—1 $\frac{1}{2}$
7
ell. l.
If 5—5—1 $\frac{1}{2}$: 1 :: 728
12 20
65 14560
4 12
261 174720
3 4
784 698880
3
ells. qrs.
784)2096640(2674 : 1 $\frac{1}{7}$ $\frac{1}{8}$ $\frac{2}{4}$, or $\frac{1}{7}$ in the 40 packs.
224
4
784)896(1
112

If 840

If 840l. in 9 months gain 25l. 4s. at 4l. per cent, per annum; at what rate per cent, per annum will 250l. in 12 months gain l. 12 : 10?

```
       l.   l. s.   l.
If 840 : 25—4 :: 250
            20
        ------
           504
           250
        ------
         25200
         1008
        ------
8410)126000(150s. gain'd by 250l. in 9 months
   -------          at 4l. per cent, per annum.
          0

m.  s.   m.
If 9 : 150 :: 12
           12
      ------
9)1800(200s. gain'd by 250 l. in 12 months, at
  ----              4 l. per cent, per annum.
     0

s.    l.   l.  s.
If 200 : 4 :: 12—10
              20
           ------
             250
               4
           ------
200)1000(5 l. per cent, per annum, answer.
       0
```

If 840 l. in 9 months, gain l. 25—4, at the rate of 4l. per cent, per annum; what sum in 12 months at 5l. per cent, per annum will gain me l. 12 : 10?

l. s.

l. *s.* *l.* *l.* *s.*
If 25—4 : 840 :: 12—10
20 20
————— —————
504 250
840
—————
100
200
————— *l.* *s.* *d.*
504)210000(416 : 13 : 4 fum that will gain l. 12—10 in 9 months, at 4 l. per cent, per annum.
—————
336
20
—————
504)6720(13
—————
168
12
—————
504)2016(4
0

m. *l.* *s.* *d.* *m.*
If 9 : 416—13—4 :: 12
20
—————
8333
12
—————
100000
9
————— *d.*
12)900000(75000 fum that will gain l. 12 : 10 in 12 months, at 4 l. per cent, per annum.
—————
0

l. *d.* *l.*
If 4 : 75000 : 5
4
————— 240) *l.*
5)300000(60000(250 fum that will gain 12 l. 10 s. in 12 months, at 5 l. per cent, per annum, the anfwer.
————— —————
0 0

If 840

If 840 l. in 9 months, at 4 per cent, per annum, gain l. 25—4; in what time will 250 l. gain l. 12 : 10 at 5 l. per cent?

```
     l.      l.   s.    l.
If 840  :  25—4  ::  250
              20
           -------
           504
             250
           -------
            2520
           1008
         --------
84|0)12600|0(150 s. gain'd by 250 l. in 9 months,
   --------              at 4 l. per cent, per annum.
         0
```

```
     s.     m.     l.   s.
If  150  :  9  ::  12—10
                    20
                  ------
                    250
                      9
                  ------
           15|0)225|0(15 months, in which time 250 l.
                  ------   will gain l. 12 : 10, at 4 l.
                    0      per cent, per annum.
```

```
     l.    m.    l.
If   4  :  15  ::  5
            4
         -----
        5)60(12 months, the answer.
         ---
          0
```

If 250

If 250 l. in 12 months gain l. 12 : 10, at the rate of 5 l. per cent, per annum; in what time will 840 l. gain l. 25 : 4, at the rate of 4 l. per cent per annum.

```
    l.     l.   s.    l.
If 250 : 12—10 :: 840
          20
       ------
         250
         840
       ------
        100
       200
      ---------
  25|0)21000|0(840 s. gain'd by 840 l. in 12 months,
      ---------          at 5 l. per cent, per annum.
          0

    s.     m.    l.  s.
If 840 : 12 :: 25—4
                 20
              ------
                504
                 12
              ------
         840)6048(7 168/840, or 1/5 months, time in which
              ----       840 l. would gain l. 25—4,
               168       at 5 l. per cent, per annum.

    l.   m.   l.
If  5 : 7 1/5 :: 4
         5
      -----
      4)36(9 months, the answer.
      -----
        0
```

If 25 masons in 225 days, build a wall 1440 feet long, 51 feet high, and 8 feet thick; in what time will 98 masons build a wall 2160 feet long, 75 feet high, and 9 feet thick?

feet.
1440
51
1440
7200
73440
8

maf. *maf.*
If 25 : 587520 :: 98
98
4700160
5287680
25)57576960(2303078 $\frac{10}{25}$, or $\frac{2}{5}$ cubic feet of work will be done by 98 masons, in 225 days.
10

feet.
2160
75
10800
15120
162000
9

feet. *days.*
If 2303078$\frac{2}{5}$: 225 :: 1458000
5 5
11515392 7290000
225
3645
1458
1458

days. h. m.
11515392)1640250000(142:10:33$\frac{3400704}{11515392}$, or $\frac{70848}{239904}$, the answer.
5064336
24
20257344
10128672
11515392)121544064(10
6390144
60
11515392)383408640(33
3400704

 If I

If I pay 11 s. 3 d. for the English yard of cloth 5 quarters broad; what should I pay for the Scots ell 7 quarters of an English yard wide, when the English yard is to the Scots ell, as 30 to 31?

```
              s.  d.
              11—3
              12
    yds.     ————
    If 30  :  135  ::  31
        5     217       7
     ————    ————    ————
      150     945      217
              135
             270
           ———————— 12)  s.    d.    q.
    150)29295(195( 16  :  3  :  1 3/15, or 1/5  answer.
         —————   ——
            45     3
             4
       15|0)18|0(1
             3
```

If I buy the English yard at 11 s. 2 d.; how may the Scots ell be sold, to gain 40 l. per cent?

```
                s.  d.
       30      11—2       31
       100     12         140
     ——————————————————————————
  If 3000 : 134  ::  4340
                      134
                    ———————
                     1736
                    5642
                   ——————— 12)  s.   d.   q.
       300|0)58156|0(193(16 : 1 : 3 124/300, or 31/75
                                            Answer.
             —————   ——
               256     1
                 4
       300|0)102|4(3
               124
```

shall

I ſhall only add one example more, and leave it undone for the learner's practice.

A merchant hath a ballance, the ſhorter end of whoſe beam is 36 inches, and longer end 37 inches; Now if he ſhould fraudulently buy his goods on the ſhorter end, and ſell them on the longer end of the ſaid beam, and take at the rate of 5 l. per cent, between buying and ſelling; what would his gain be per cent, by ſuch a piece of villainy?

This much concerning the Rule of Three, and I am perſwaded, that by this time the learner may be able to reſolve any queſtion relative to common buſineſs, and pertinent to this rule, if it depends not upon fractions, or geometrical magnitudes; and ſuch as deſire to ſee the demonſtration of this rule, may read Mr. Kerſey's appendix to Mr. Wingates arithmetic; or, the 6th chapter of Mr. Oughtred's clavis mathematica, by whom it is largely demonſtrated, being grounded upon the 16th propoſition of the 6th book of Euclid's elements.

CHAP. VIII.

Of POSITION, or the RULE *of* FALSE.

THIS Rule has not the name of Falſe, from its being in it ſelf really erroneous, but only, becauſe that by the help of falſe ſuppoſed numbers we find the truth: And as it ſerves more for amuſement than for uſe in buſineſs, I ſhall be as brief on the ſame as poſſible.

It is uſually divided into two parts, viz. Single and Double.

SECTION I.

SECTION I.

Of Single Position.

IN Single Position, we make use but of one supposed number, with which proceeding, as if it were the true number sought, according to the conditions of the question; we regulate the result by this proportion, viz.

As the false conclusion,
To the true total;
So the supposed part, or number,
To the true one.

Having prescrib'd several questions in the Rule of Three, that properly belong'd to this part of Position, I shall only here offer a few Examples.

A gentleman bought a chaise, horse, and harness for 50 l.; the horse came to twice the price of the harness, and chaise to twice as much as both horse and harness; what did he give for each?

Suppose the harness cost 4 l.
then the horse cost 8
and the chaise cost 24
36

If 36 : 4 :: 50
50
l. s. d.
36)200(5 : 11 : $1\frac{12}{36}$, or $\frac{1}{3}$ price of the harness.
20
20
36)400(11
4
12
36)48(1
12

If 36

If 36 : 8 :: 50
50

$$\begin{array}{l} \textit{l.} \quad \textit{s.} \quad \textit{d.} \\ 36)400(11 : 2 : 2\tfrac{24}{36}\text{, or } \tfrac{2}{9} \text{ price of the horse.} \\ \underline{} \\ 4 \\ 20 \\ \underline{} \\ 36)80(2 \\ 8 \\ 12 \\ \underline{} \\ 36)96(2 \\ 24 \end{array}$$

A, B, and C, disputing about their ages, A, affirms he is as old as B, and half as old as C. And B, says that his age is equal to three fourths of C's; upon which C, says he is sure that both their ages added to his will make 110 years; how old was each?

Suppose C, was 12 years.
then was B, 9
and A, was 15

If 36 : 12 :: 110
110

$$\begin{array}{l} \textit{y.} \\ 36)1320(36\tfrac{24}{36} \text{ or } \tfrac{2}{3} \text{ C's age.} \\ 24 \end{array}$$

If 36 : 9 :: 110
110

$$\begin{array}{l} \textit{y.} \\ 36)990(27\tfrac{18}{36} \text{ or } \tfrac{1}{2} \text{ B's age.} \\ \underline{} \\ 18 \end{array}$$

If 36 : 15 :: 110
110

$$\begin{array}{l} \textit{y.} \\ 36)1650(45\tfrac{30}{36} \text{ or } \tfrac{5}{6} \text{ A's age.} \\ 30 \end{array}$$

What

What number multiplied by 24 will produce 384?

Suppose 15 to be the number.

```
      24
    -----
If 360 : 15 :: 384
                 15
              -----
          36|0)576|0(16 number required.
              -----
                 0
```

Divide 45 into two parts, so as the greater part may be triple the lesser?

Make choice of any number that will admit of such division, such as 36 which can be divided into 9 the lesser part, and 27 the greater, and proceed as follows,

```
If 36 : 9 :: 45
        45
      -----
   36)405(11 9/36, or 1/4 lesser part of 45.
      -----
         9

If 36 : 27 :: 45
        45
      -----
       135
      108
     ------
   36)1215(33 27/36, or 3/4 greater part of 45.
     ------
        27
```

A certain sum put out at 5 l. per cent, simple interest; at the end of 8 years amounts to l. 112; what was the sum put out?

Suppose 60 l. to be the stock, and then say, If 60 l. with it's interest for 8 years (viz. 24 l.) come from 60 l.; what will l. 112 come from?

```
   l.   l.    l.
If 84 : 60 :: 112
               60
      84)6720(80 l. the answer.
          0
```

A, B,

A, B, and C, bought a ship for l. 9804—18 whereof A, paid a certain sum, B, paid twice as much as A, and C, paid seven times as much as B; what did each pay?

	l.
Suppose A, paid	20
then B, paid	40
and C, paid	280
	340

	l.	l.	l. s.
If	340 :	20 :	9804—18
	20		20
	6800		196088
			20

680|0)392176|0(576 : 15 : $2\frac{8}{68}$ or $\frac{2}{17}$ paid by A.

(l. s. d.)

516
20

68|0)1032|0(15

12
12

68)144(2
8

2

l. 1153 : 10 : $4\frac{16}{68}$ paid by B.

7

l. 8074 : 12 : $5\frac{44}{68}$ paid by C.

One being asked what number of crowns he had about him, answered that the $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$ and $\frac{1}{8}$ of their number added into one sum would make 189 crowns; how many had he?

Suppose he had 60 crowns, $\frac{1}{2}$ whereof is 30
$\frac{1}{3}$ is - - 20
$\frac{1}{4}$ is - - - 15
$\frac{1}{5}$ is - 12
$\frac{1}{6}$ is - - 10
$\frac{1}{8}$ is - $7\frac{1}{2}$
$94\frac{1}{2}$

(See the work next page.)

C. C.

c.	c.	c.
If $94\frac{1}{2}$	60 ::	189
2		2
189		378
		60

189)22680(120 crowns, the answer.

0

I shall now subjoin the two following examples with their answers, and then proceed to the other section of this rule.

A factor would exchange 780 l. Ster. for ducats, dollars, and French crowns, the ducats at 7 s. 6 d. per piece, the dollars at 4 s. 4 d. per piece, and the French crowns at 6 s. per piece; to be in such proportion, that $\frac{1}{2}$ the number of ducats may be equal to $\frac{1}{3}$ the number of dollars, and $\frac{1}{4}$ of the dollars equal to $\frac{3}{16}$ of the number of crowns; how many pieces of each will he receive for his 780 l.

Answer, 600 ducats, 900 dollars, and 1200 French crowns.

A young man received l. $66\frac{2}{3}$ which was $\frac{2}{3}$ of $\frac{1}{2}$ of his elder brother's portion, and $3\frac{1}{2}$ times his eldest brother's portion, was $1\frac{1}{4}$ times his father's estate; what was the elder brother's portion, and value of the father's estate?

Answer, the eldest brother's portion 200 l. and father's estate 560 l.

SECTION II.

Of Double Position.

IN Double Position two false positions are assumed to give a resolution to the question propounded, That is, make choice of any number at pleasure, try whether the said number will answer the conditions

in

in the question, by comparing the number resulting at the end of the work, with the number resulting from the true number sought; if both results be the same, then the number you took by guess, is the true number, or answer to the question.

But, if the number resulting from the number you made choice of, be either greater or less, such excess or defect is called the error of the first supposition.

In likemanner, you must feign a second number, and make trial therewith, as before, and if the result of that work be either greater or less than it ought to be, such excess or defect is called the error of the second supposition.

Having thus made trials with the supposed numbers, set down both your suppositions, and against each of them their respective errors; and for finding the true number, observe the following rule.

RULE, If both errors are alike, that is, both too little, or both too much, multiply the first supposition by the second error, and the second supposition by the first error, subtract the lesser product from the greater, and dividing the remainder by the difference of the errors, the quotient gives the true number answering all the demands of the question.

But if the errors are unlike, that is, one too little, and the other too much, multiply (as before) the suppositions by the errors, and dividing the sum of the two products by the sum of the two errors, the Quotient is the true number or answer to the question.

Examples.

A, B, and C, playing at cards, the money stak'd was 324 crowns; but disagreeing, each seiz'd as many as he could; A, got a certain number, B, as many as A, and 15 more, C, got a fifth part of both their sums added together; how many had each?

(See the work on the next page.)

C.

First suppose A, got 50
then B's share will be 65
and C's 23

Sum 138 which taken from 324, leaves 186 for error first.

C.

Again, suppose A, got 80
then B's share was 95
and C's . . . 35

Sum 210 which taken from 324, leaves 114 for error second, and the work will stand thus;

suppositions.		*errors.*	
50		186	too little.
80		114	too little.
186		50	
14880	Product.	5700	Product.

186 14880 Product. 5700 Product.
114 5700

72) 9180($127\frac{36}{72}$, or $\frac{1}{2}$ crowns A, took up.
15
36
$142\frac{1}{2}$ crowns B, took up.

5)270(54 crowns C, took up.
0

When first the marriage knot was ty'd,
Betwixt my wife and me;
My age did her's as far exceed,
As three times three does three.
But after ten and half ten years,
We man and wife had been;
Her age came up as near to mine,
As eight is to sixteen.
——Now, pray,
What were our ages on the wedding day?

First

	years.		years.
First suppose her to be,	21	then he must be,	63
then adding to each	15	- - - -	15
Her age becomes	36	and his	78

Now, her age should be equal to the half of his, viz. 39, but it wants 3 of being so, therefore the first error is 3.

	years.		years.
Again, suppose her	9	then he must be	27
then adding to each	15	- - -	15
Her age becomes	24	and his	42

Here it's plain, 24 is 3 more than half of 42, therefore, error 2d. is 3 also, and the work will stand thus,

sup.		errors.	
21	- - -	3	too little.
9	- - -	3	too much.
3		21	
27 product.		63 product.	

3 27
3 63

6) 90 (15 years, her age, prov'd thus,

0

She being	15,	he must be	45
add to each	15	- -	15
	30	is just half of	60

What number is that, to which if you add, ½ of it self, and from the sum subtract ⅓ of it self, the remainder will be 252?

Suppose

Suppose 60
$\frac{1}{4}$ of 60 is 15

75

$\frac{1}{5}$ of 60 is 12

63 which taken from 252, leaves 189 for the first error.

Again, suppose 80
$\frac{1}{4}$ of 80 is - 20

100

$\frac{1}{5}$ of 80 is - 16

84 which taken from 252, leaves 168 for the 2d. error, and the work will stand thus

sup.		er.
60	- -	189 too little.
80	- -	168 too little.
189		60

189 15120 product. 10080 product.
168 10080

21) 5040 (240 true number.

0

Three companies of soldiers passing by, the first company takes away from a shepherd the $\frac{1}{2}$ of his whole flock, and $\frac{1}{2}$ of a single sheep more; the second company takes away $\frac{1}{2}$ the remainder of the flock and $\frac{1}{2}$ of a single sheep more; the third company also, takes $\frac{1}{2}$ of what yet remain'd and $\frac{1}{2}$ of a single sheep more; all which was done without killing any sheep, and there remain'd at last 20 sheep with the shepherd; how many had he at first?

Suppose he had 63 at first, then $\frac{1}{2}$ 63, is 31 $\frac{1}{2}$, which with $\frac{1}{2}$ of a sheep more makes 32 that the first company took, and so they left him 31; then $\frac{1}{2}$ 31 is 15 $\frac{1}{2}$

15 ½, which with ½ of a sheep more, makes 16 that the second company took, and so they left him 15: lastly, ½ 15 is 7 ½, which with ½ of a single sheep more, makes 8 which the third company took, and so they left him 7; which (if I had guessed right) should have been 20, but it's less than 20 by 13, therefore error 1st. is 13.

Again, suppose he had at first 87, with which proceed as with the first supposed number, and you'll find that at last there remain'd 10 with him, which is 10 less than 20, therefore the 2d. error is 10, and the work will stand thus;

sup.		*er.*
63	- -	13 too little,
87	- -	10 too little.
13		63

13 1131 product.   630 product.
10 630

3) 501 (167 sheep, real number he had at first.

0

There is a fish whose head is 9 inches long, and his tail is as long as his head and half his body, and his body is as long as his head and his tail; what's the length of that fish?

Suppose it's body to be 18 inches, then ½ 18 added to the length of his head, makes 18 for the length of the tail, now 9 and 18 (the lengths of head and tail) being added together make 27, which should be equal to 18 the suppos'd length of the body, but it exceeds it by 9, therefore error 1st. is 9.

Again, suppose the body to be 20 inches, and proceeding as you did with the first supposition, you'll find the lengths of the head and tail together exceed the

the supposed length of the body by 8, therefore the 2d. error is 8 and the work will stand thus,

sup.		*er.*
18	- -	9 too much.
20	- -	8 too much.
9		18
9 180 product.		144 product.
8 144		

1) 36(36 inches, real length of the body, so the tail was 27 inches, and the whole length 72.
0

I once bought 3 books, whose prices were in proportion as, 12, 5, 1; if the price of the first be doubled, of the second trebled, and of the third quadrupled, the sum of these products will as much exceed 10 crowns, as the sum of the prices of the greatest and middle will be below 5 crowns; what was the price of each book?

Suppose the first book cost 12 l. then the 2d. cost 5 l. and the 3d. 1 l.; the price of the first doubled is 24 l., of the 2d. trebled is 15 l., of the 3d. quadrupled is 4 l., and the sum of these products make 43 l., which exceeds 10 crowns by 13 l., and the prices of the greatest and middle added together make 17 l.; now if this 17 l. had been 2 l. it would have been 13 l. below 5 crowns, as well as the sum of the products is 13 l. above 30 crowns, and so would have answer'd the conditions of the question; but it is 15 l. above 2 l. therefore error 1st. is 15 l.

Again, suppose the 1st. book cost 6 l. then find what the 2d. and 3d. cost, as follows,

If 12 : 6 : 5

5 *l.* *s.*
12)30(2—10 price of the 2d.
6
20
12)120(10
0

If 5 : 2—10 :: 1
20
5)50(10 s. price of the 3d
0

Then

Then, the sum of the prices of the first and middle will be l. 8 : 10, which is l. 6 : 10 below 5 crowns; and the products of the first doubled, the 2d. trebled, and 3d. quadrupled added together make 21 l. 10 s. which should be as much above 10 crowns, as the sum of first and middle was below 5 crowns, but it wants 15 l. of being so, therefore the second error is 15 l. also, and the work will stand thus:

sup.	er.
12 ————————	15 too much.
6 ————————	15 too little.
15	12
15 90	180 product.
15 180	

30) 270 (9 l. real price of the first book.
0

And to find the prices of the 2d. and 3d. do as followeth.

If 12 : 9 :: 5 . If 5 : 3—15 :: 1

5 20

12) 45 (3 : 15 l. s. price of the 2d. 5) 75 (15 s. price of the 3 d.

9 0

20

12) 180 (15

0

Three merchants, A, B, and C, from three different fairs, meet together at an inn, where they reckon up their gains, and find them the sum of 780 crowns. Now, if you add A, and B's gains together, and subtract C's gain from the sum, there will remain A's gain and 82 crowns more; but if you add B, and C's gains together, and subtract A's gain from the sum, there will remain C's gain wanting 43 crowns; what was the gain of each merchant?

Suppose

Suppose C's gain was 218 crowns, which taken from 780, leaves 562 for A, and B's gains; then from 562 subtract 218 (C's supposed gain) and there remains 344 which being made less by 82, makes 262 crowns for A's gain, which taken from 562, leaves 300 for B's gain: Now, (thus supposing A's gain to be 262, B's 300, and C's 218 crowns) I add B, and C's gains together, and they make 518, from which I subtract A's gain, and there remains 256, which should be equal to C's gain wanting 43 crowns, but it is more by 81, therefore error first is 81.

Again, Suppose C's gain was 224 crowns and proceeding as with his first supposed gain, you'll find B's gain to be 306, and A's 250 crowns; then, if you add B, and C's gains together, they make 530, from which subtract A's gain, and there remains 280 which should be equal to C's gain wanting 43 crowns, but it is more by 99, therefore error second is 99, and the work will stand thus:

sup. — *er.*
218 ——— 81 too much.
224 ——— 99 too much.

```
  81        218
 ----      ----
  224      1962
 1792      1962
 -----     -----
18144 product. 21582 product.
        99   18144
        81   -----
        --
        18 ) 3438(191 crowns C's real gain.
```

```
780
191
---
589 A, and B's gains.     589
191                       316
---                       ---
398                       273 B's real gain.
 82
---
316 crowns, A's real gain.
```

A general

A general who had fought a battle, upon reviewing his army, whose foot was thrice the number of his horse, finds that before the battle $\frac{1}{12}$ of his foot wanting 120 had deserted; and that $\frac{1}{12}$ of his horse besides $\frac{1}{4}$ of his whole army were sent into garrisons (reckoning the sick and wounded) and $\frac{3}{8}$ of his army remain'd; the rest who were wanting being either slain or taken prisoners; now if you add 3000 to the number of the slain, the sum will be equal to half the foot he had at the beginning; how many foot and horse had he?

First, suppose he had 6000 horse, then he had 18000 foot; now $\frac{1}{12}$ of 18000 is 1500 from which subtract 120, and there remains 1380 foot that deserted; also $\frac{1}{12}$ of 6000 is 500 which added to 120 makes 620, and 620 added to $\frac{1}{4}$ of 24000 (his whole army) is 6620 that were sent into garrisons: to the deserters 1380 add 6620 that were sent into garrisons: and they make 8000 that were absent in time of battle; to 8000 add $\frac{3}{8}$ of 24000 (the whole army) and the sum is 17000 that remained alive, which taken from the whole army 24000, leaves 7000 that were slain or taken prisoners, to which add 3000, and the sum is 10000 which should be equal to 9000 the half of his foot he had at the beginning, but it is more by 1000, therefore error first is 1000.

Again, suppose he had 12000 horse, then he had 36000 foot; and proceeding with these as with the first supposed numbers, you'll find that the number of the slain or prisoners will be 14000 to which add 3000, and the sum is 17000 which should be equal to $\frac{1}{2}$ 36000 his foot, but it is less by 1000, therefore the second error is 1000, and the work will stand as follows.

Sup.		*Er.*
600		1000 too much.
12000		1000 too little.
1000		600

1000 1200000 product. 600000 product.
1000 600000

27000)18000000(9000 horse, and so he had 27000 foot, which two numbers will answer the conditions of the question.

0

Suppose a crown that shall weigh 60 lb. is to be made of gold, brass iron, and tin mixed together in such proportion, that the joint weight of the gold and brass may be 40 lb. the joint weight of the gold and tin 45 lb. and the joint weight of the gold and iron 36 lb. how much of every one of these 4 metals must be taken?

First suppose 20 lb. of gold to be in the crown, then there must be in it 20 lb. of brass (for 20 and 20 makes 40) and there must be in it 25 lb. of tin (for 25 and 20 makes 45) and there must be in it 16 lb. of iron (for 16 and 20 makes 36) all which quantities added together makes 81, which should be 60 lb. but it exceeds 60 by 21, therefore the first error is 21.

Again, suppose 24 l. of gold to be in the crown, with which proceed as with the first supposed number, and you'll find the sum of the quantities will be 73, which exceeds 60 lb. by 13, therefore the second error is 3, and the work will stand as followeth:

Sup.

sup.	*er.*
20	21 too much.
24	13 too much.
21	20
24	260
48	

21 504 product.
13 260
lb.
8)244($30\frac{1}{2}$ or $\frac{1}{2}$ real quantity of gold.
$9\frac{1}{2}$ of brass.
A $14\frac{1}{2}$ of tin.
$5\frac{1}{2}$ of iron.
60 lb. weight of the crown.

A, B, and C, owe me each a sum of money; I have both forgotten what the total sum of their debts is, and what each man owed me, but I remember that A, and B's debt was 47 l. and A, and C's 71 l. and B, and C's 88 l.; what did each man owe, and what was the total debt?

First, suppose A's debt to be 20 l. then B's must be 27 l. for 20 and 27 make 47; and C's debt must be 51 l. because 20 and 51 make 71 l. now B, and C's debts, viz. 27 and 51 amount only to 78 l. which is less than 88 l. by 10 l. therefore error first is 10.

Again, suppose A's debt to be 24 l. with which proceed as with the first supposed number, and you'll find B, and C's debts together will be but 70 l. which wants 18 l. of 88 l. therefore error second is 18, and the work will stand as followeth:

sup.	*er.*
20	10 too little.
24	18 too little.
10	20
240 product.	18 360 product.
	10 240

8)120(15 l. A's real debt.
8

47	71
15	15
l. 32 B's real debt.	l. 56 C's real debt.
	15 A's.
	32 B's.
	l. 103 total debt.

One being asked, what hour of the day it was? answered, The day at this time is 18 hours long; if therefore $\frac{1}{2}$ of the hours past be added to $\frac{1}{4}$ of the remaining hours to sun setting, the sum will be the hour desired, reckoning from sun rising; what o'clock was it?

First, suppose it to be 10 hours from sun rising, then there will remain 8 hours to sun setting, now $\frac{1}{2}$ 10 is 5, and $\frac{1}{4}$ of 8 is 2, but 2 and 5 make only 7 hours, which wants 3 hours of 10, therefore the first error is 3.

Again, suppose it to be 2 hours after sun rising, and you'll find the result to be 5, which exceeds 2 by 3, therefore error second is 3 also, and the work will stand as followeth:

sup.		*er.*
10	———	3 too little.
2	———	3 too much.
3		10
6 product.		30 product.

3 6 product.
3 30

6)36(6 hours from sun rising, answer.

A person in the afternoon being asked what o'clock it was, answered, That $\frac{3}{4}$ of the time from noon was equal to $\frac{3}{7}$ of the time remaining to midnight; now the time from noon to midnight being 12 hours; what was the present hour of the day.

First, suppose it was 7 o'clock (or 420 minutes) afternoon,

afternoon, then the time remaining to midnight was 5 hours or 300 minutes; now $\frac{3}{4}$ of 420 is 315 minutes, and $\frac{3}{5}$ of 300 is 180 minutes which should be equal to 315, but it is less 135, therefore error first is 135.

Again, suppose it was 8 hours 20 minutes afternoon (or 500 minutes) then the time remaining to midnight will be 3 hours 40 minutes (or 220 minutes) now $\frac{3}{4}$ of 500 is 375 minutes, and $\frac{3}{5}$ of 220 is 132 minutes which should be equal to 375, but it is less by 243, therefore error 2d. is 243, and the work will stand as followeth:

sup. —— *er.*
420 —————— 135 too little.
500 —————— 243 too little.
135 —— 420

67500 product. —— 486
972

243 —— 102060 product.
135 —— 67500

60)
108) 34560(320(5 o'clock after-
(noon.

0 0

A man bought 3 silver cups, A, B, and C, having but one cover of 18 ounces to them all; the cup B, is half the weight of A, and C; now if the cover be put upon A, it will then weigh as much as all the 3 cups; if the cover be put upon B, it will be the weight of C, and twice the weight of itself; and if the cover be put upon C, it will be twice the weight of A, and B; I demand the weight of each cup?

First, suppose the weight of A, to be 15 ounces which with the cover makes 33 ounces the weight of all the 3 cups; and 15 taken from 33, leaves 18 ounces for the weight of B, and C: now, because B, is half the weight of A, and C, it must certainly

weigh

weigh ⅓ of 33, which is 11; and consequently C, must weigh 7 ounces: supposing then that A, weighs 15, B, 11, and C, 7 ounces, if you put the cover upon A, it will weigh all the three cups; if you put it upon B, it will be the weight of C, and twice the weight of itself; but if it be put upon C, it will only weigh 25 ounces which should have been 52 ounces (viz. twice the weight of A, and B) but it wants 27 of being so, therefore error first is 27.

Again, suppose the weight of A to be 3 ounces which with the cover makes 21 ounces for the weight of all the three cups without the cover; and 3 taken from 21, leaves 18 ounces for the weight of B, and C: now, because B, is half the weight of A, and C, it must needs weigh ⅓ of 21, which is 7: and consequently C, must weigh 11: supposing then A, to be 3, B, 7, and C, 11 ounces, if you put the cover upon A, it will weigh all the three cups; if you put it upon B, it will be the weight of C, and twice the weight of itself; but if it be put upon C, it will only weigh 29 ounces which should have been 20 ounces (viz. twice the weight of A, and B, but it exceeds it by 9, therefore error second is 9, and the work will stand as followeth.

	sup.		er.
for A.	15	——	27 too little.
	3	——	9 too much.
	27		15
27	81 product.		135 product.
9	135		

36)216(6 ounces, real weight of A.

0

(See the rest of the work on the next page.)

sup.

	sup.		er.	
for B.	11	————	27	too little.
	7	————	9	too much.
	27		11	
27	189	product.	99	product.
9	99			

36)288(8 ounces, real weight of B.

6

A, 6
B, 8

14
2

28 twice the weight of A, and B.
Sub. 18 the cover.

remains 10 ounces, real weight of C.

There is another Rule for working questions of this nature, which you may use, if you think it preferable to the other; and it is this;

As the difference of the errors, if alike,
Or their sum, if unlike,
To the difference of the suppositions;
So either error to a fourth number,
Which accordingly added to, or subtracted
From the supposition against it, will
Answer the question.

The foregoing examples are sufficient for this rule, but I shall add a few more with their answers, and leave the work of them, for the excercise and improvement of the curious learner; and proceed to Alligation, which is another rule that comes seldom in use.

A man

A man bought 2 cups, A, and B, having but one cover to both, which weighed 5 ounces; which cover being put upon A, makes it's weight double that of B; and if put upon B, it makes it weigh triple to that of A; I demand the weight of each cup?

Anſwer, A, 3 ounces, and B, 4 ounces.

A, B, and C, built a houſe, which coſt them 76 l. whereof A, paid a certain ſum; B, paid as much as A, and 10 l. more; and C, paid as much as A, and B; what did each pay?

Anſr. A, 14 l. B, 24 l. and C, 38 l.

A gentleman dying, left his eſtate to his 3 ſons A, B, and C, as followeth, viz. to A, he left the $\frac{1}{2}$ of it wanting 44 l.; to B, he left $\frac{1}{3}$ of it and 14 l. more, and to C, he left the remainder, which was 82 l. leſs than the ſhare of B; I demand the value of his eſtate, and each ſons portion thereof?

Anſr. His eſtate was 588 l. A's portion 250 l. B's 210 l. and C's 128 l.

One being aſked, what was the preſent hour of the day? anſwered, That the time then paſt from noon, was equal to $\frac{3}{7}$ of $\frac{1}{4}$ of the time remaining to midnight; now (ſuppoſing the time from noon to midnight to be 12 hours) I demand what o'clock it was?

Anſr. 2 o'clock afternoon.

A gentleman aſked his friend, that had 4 purſes A, B, C, and D, in his hand, What money was in each purſe? to whom he anſwered, That in B, there were 8 crowns more than in A, and in C, 8 crowns more than in B, and in D, 8 crowns more than in C, and twice as many as in A; what number of crowns had he in each purſe?

Anſr. in A, 24, B, 32, C, 40, D, 48.

A, and

A, and B, had each a certain number of crowns; quoth A, to B, give me 3 of your crowns, and I shall have as many as you; nay quoth B, to A, give me 3 of your crowns, and I shall have then five times as many as you; how many had each?

Ansr. A, 6 and B, 12.

A man being asked what was the age of his 4 sons, answered, that his eldest was 4 years elder than the second, the second 4 years elder than the third, and the third was 4 years elder than the fourth or youngest, and that the youngest was half the age of the eldest; I demand each son's age? Ansr. the eldest 24, the second 20, the third 16, and the youngest 12 years.

A, B, and C, discourse thus about their ages; quoth B, to A, your age added to mine makes 54 years; quoth C, to B, your age added to mine makes 78 years; and quoth A, to C, your age added to mine makes 72 years; what was each person's age?

Ansr. A, 24, B, 30, and C, 48 years.

Divide 100 into two such parts (viz. A, and B,) that $\frac{1}{3}$ of A, added to $\frac{1}{5}$ of B, may be equal to 30; what are the parts? Ansr. A, 75, and B, 25.

There be two numbers A, and B, whose difference is 4; and the difference of their squares 64; what are these numbers? Ansr. A, 6, and B, 10.

A man seeing a purse in his friend's hand, said, it seems to me you have 100 crowns in your purse; nay quoth he, I have not 100 crowns in it, but if they were increased with $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ of themselves and lastly one crown more, then would there be just 100 in it; how many crowns had he in his purse?

Ansr. $47\frac{13}{25}$ crowns.

A gentleman had two horses of good value, and a sadle worth 50 l. which if set on the back of the first horſe, makes his value double the ſecond; and if ſet on the back of the ſecond, it makes his value treble the firſt; what was the price of each horſe?

Anſr. the firſt 30 l. and the ſecond 40.

A, ſtealing apples, was taken by B, and to appeaſe him gives him half he had, and B, gives him back 10; and going farther meets with C, and was forced to give him half he had left, and he returns him 4; and going farther meets D, and gives him half he had remaining, and he returns him 1; and getting ſafe away, finds he had 13 left; how many had he at firſt?

Anſr. 60.

A, B, and C, bought a ſhip for 200 l. A, ſays to B, give me $\frac{1}{2}$ your money, and I will pay the ſhip; B, ſays to C, give me $\frac{1}{3}$ of your money, and I will pay the ſhip; and C, ſays to A, give me $\frac{1}{4}$ of your money, and I will pay the ſhip; what ſum of money had each? Anſr. A, had 128 l. B, 144 l. and C, 168 l.

A veſſel of 63 gallons was fill'd up with French wine of two ſorts; the one at 2 s. the gallon, and the other at 2 s. 6 d. the gallon; the wine in the hogſhead thus filled up coſt l. 7 : 4; how much was there of each ſort? Anſr. 27 gallons at 2 s. per gallon, and 36 gallons at 2 s. 6 d. per gallon.

A, and B, had been partners, and had in accompt between them 300 crowns, whereof 120 belong'd to the one, and 180 to the other; but in the parting of them they fell at variance, ſo that each catch'd as many as he could: yet afterward being reconciled, they did agree that he who had catched the moſt part of them ſhould lay down $\frac{3}{4}$ of them again; and he that had got leaſt ſhould lay down $\frac{1}{3}$ of them again, and then parting what was laid down into two equal parts, each took one of theſe parts, and ſo had they their juſt portions; what had each gotten by the ſcrambling? Anſr. the one had catch'd 152 $\frac{8}{11}$ and the other 147 $\frac{3}{11}$ crowns.

I ſhall oblige the learner with performing the work of this example.

I ſuppoſe

I suppose then, that the one catched 108 and the other 192 crowns (for 108 and 192 make 300) now he that had the 192 laid down 144 which was $\frac{3}{4}$ of them, and then he had but 48 remaining: also, he that had the 108 laid down 36 which was $\frac{1}{3}$ of them, and then he had but 72 remaining: I add 144 and 36 (being what they laid down) together, and they make 180 which divided into two equal parts, each will have 90 which added to 72 that remain'd to one of them after laying down $\frac{1}{3}$ of what he catched, makes 162, and the said 90 added to 48 that remain'd to the other, after laying down $\frac{3}{4}$ of what he catched, makes 138: now 162 and 138 should have been 180 and 120 if I had guessed right, but 138 exceeds 120 by 18 (for it's to the same purpose, when you compare the lesser result with the least supposed number, as when you compare the greater with the greatest, only, you must mind to make the same comparison in both suppositions) therefore error first is 18.

Again, suppose the one catched 96, and the other 204 (which make 300) proceed with these as before; and comparing the lesser result with the lesser supposed number as you did in the first supposition, you'll find error second will be $23\frac{1}{2}$ and the work will stand as followeth.

sup.		*er.*
108	——————	18
96	——————	$23\frac{1}{2}$
18		108
1728 product.		184
		23
		54
	$23\frac{1}{2}$	2538
	18	1728
	$5\frac{1}{2}$	810
	2	2

11) 1620 ($147\frac{3}{11}$ crowns catched by
3 the one, which taken from 300
leaves $152\frac{8}{11}$ catch'd by the other.

One

One man ſaid to another, I think you had this year 2000 lambs; ſo I had ſaid the other, but what with paying the tithe of them, and then three ſeveral loſſes, they are much abated, for at one time I loſt $\frac{1}{2}$ as many as I now have, and at another time $\frac{1}{3}$ ſo many, and the third time $\frac{1}{4}$ ſo many; how many had he yet remaining? Anſr. 864.

A ſon aſked his father, how old he was; his father anſwered him thus, if you take away 5 from my years, and divide the remainder by 8, the Quotient will be the 3d. of your age: but if you add 2 to your age, and multiply the whole by 3, and then ſubtract 7 from the product the remainder will be the years of my age; I demand both their ages?

Anſr. the father 53, and the ſon 18 years.

A labourer after he had been 40 weeks at work, lays up 28 crowns wanting the pay of 3 weeks; and finds that he had expended 36 crowns and the pay of 11 weeks more; what pay did he receive a week?

Anſr. 2 crowns.

A perſon being aſked, how old he was, anſwered, if I quadruple $\frac{2}{3}$ of my years and add $\frac{1}{2}$ of them and 50 more to the product, the ſum will be ſo much above 100, as the number of my years is now below 100; how old was he? Anſr. 36 years.

A man, his wife, and his ſon's ages make up 96 years, ſo that the father and ſon's years together make the wife's age and 15 years more; but the wife's and the ſon's years make the huſband's age and 2 years more; what was the age of each?

Anſr. the father 47, the wife $40\frac{1}{2}$ and the ſon $8\frac{1}{2}$.

A gentleman gains $\frac{1}{3}$ of his ſtock, and of that ſum (viz. his ſtock and gain) ſpends 8 crowns; after that, he gains $\frac{1}{4}$ of the remainder, and out of this ſum he ſpends 15 crowns; again he gains $\frac{1}{7}$ of that remainder, and out of this laſt ſum he ſpends 5 crowns; at the end of all this, he finds he has 25 crowns more than he had at the beginning; I demand how many he had at the beginning, and how many now?

Anſr. 60 crowns at the beginning and 85 now.

A gold-

A goldsmith being employed to make a crown of pure gold of 8 lb. weight, fraudulently kept back part of the gold, and put in silver for it: now suppose a mass of pure gold of equal weight with the crown to be put into a vessel brimfull of water, and that it expells 3 lb. of water; suppose also, that a mass of silver of the same weight being put into the vessel brimfull shall expell 5 lb. of water; and that the crown being put into the vessel brimfull shall expell 4 lb. of water; I demand how much gold and how much silver the crown was composed of?

Consider that gold is heavier than silver, and therefore a mass of it of the like weight with a mass of silver will take less room.

First, suppose there were 5 lb. of gold in the crown, then there were 3 lb. of silver in it: find therefore how much water each will expell.

If 8 : 3 :: 5
      5

8)15($1\frac{7}{8}$ lb. of water expell'd by 5 lb. of gold.
   7

If 8 : 5 :: 3
      3

8)15($1\frac{7}{8}$ lb. of water expell'd by 3 lb. of silver.
   7

Now $1\frac{7}{8}$ and $1\frac{7}{8}$ added together make $3\frac{6}{8}$ lb. of water the crown would have expell'd if it had been compos'd of 5 lb. of gold and 3 lb. of silver; but it had not been so compos'd, for it expell'd 4 lb. of water which is more by $\frac{1}{4}$, therefore $\frac{1}{4}$ is error first.

Aagain suppose there were 3 lb. of gold and 5 lb. of silver in the crown, find therefore how much water each will expell.

If 8

If 8 : 3 :: 3
3

8)9($1\frac{1}{8}$ lb. of water expell'd by 3lb. of gold.
1

If 8 : 5 :: 5
5

8)25($3\frac{1}{8}$ lb. of water expell'd by 5 lb. of silver.
1

Now $1\frac{1}{8}$ and $3\frac{1}{8}$ added together make $4\frac{1}{4}$ lb. of water the crown would have expell'd had it been compos'd of 3 lb. of gold and 5 lb. of silver; but it had not been so compos'd, for it expell'd only 4 lb. of water which is $\frac{1}{4}$ lb. less than $4\frac{1}{4}$ lb. therefore error second is $\frac{1}{4}$ also, and the work will stand as followeth.

	sup.	*er.*
Gold	5	$\frac{1}{4}$
do.	3	$\frac{1}{4}$
	$\frac{1}{4}$	5
$\frac{1}{4}$	$\frac{3}{4}$ product.	$1\frac{1}{4}$ product.
$\frac{1}{4}$	$1\frac{1}{4}$	

$\frac{1}{2}$) 2 (4 lb. real quantity of gold in the crown, and consequently there were 4 lb. of silver in in the same.
0

A labourer had 576 pence for threshing out 60 bolls of corn, viz. wheat and barley; for the wheat he had 12 pence per boll, and for the barley he had 6 pence per boll; I demand how many of the 60 bolls were wheat, and how many barley?

Ansr. 36 bolls wheat, and 24 bolls barley.

A, B, and C, thus discourse about their ages; quoth A, I am 18 years old; quoth B, I am as old as A, and half C; and quoth C, I am as old as you both; how old was each? Ansr. A, 18, B, 54, C, 72.

CHAP.

CHAP. IX.

Of Alligation.

ALLIGATION is the mixing ſeveral ſorts of ingredients together; as different ſorts of corn, wines, wool, ſpices, or metals; or to compound medicines, &c. according to any required price, or proportion. And it is divided into two branches; called *Medial* and *Alternate.*

SECTION I.

Of Alligation Medial.

ALLIGATION MEDIAL is when having the quantities and prices of the ſeveral ſimples, we find the mean rate of any part of the compoſition.

To find which the proportion is
As the ſum of all the ſimples to be mixed,
Is to their total value:
So is any part of the mixture,
To the price thereof.

Examples.

A grocer would mix 18 lb. of raiſins at 4d. per lb. with 28 lb. at 6d. per lb. and with 12 lb. at 8d. per lb.; at what rate may he ſell 1 lb. of the mixture?

	lb.		*d.*			*d.*
	18	at	5	per lb.	comes to	90
	28	at	6	per lb.	———	168
	12	at	8	per lb.	———	96
Sum of all the ſimples	58				total value	354 d.

Then if 58 : 354 :: 1
1

$58)354(6\frac{6}{58}$ pence rate of 1 lb.
6

A man

A man would mix 10 bolls of oats at l. 6 : 10 per boll, with 5 bolls at 8 l. per boll, and with 7 bolls of beans at l. 4 : 10 per boll, and with 12 bolls of pease at l. 5 : 8; what may he sell 1 lippie of the mixture at?

b.	*l.*	*s.*		*l.*	*s.*
10 at	6—	10	per boll will cost	65—	0
5 at	8—	0	per boll————	40—	0
7 at	4—	10	per boll————	31—	10
12 at	5—	8	per boll————	64—	16
Sum of the simples 34			total value l.	201—	6

b. *l.* *s.* *l.*
Then if 34 : 201—6 :: 1

```
        64      20
        ——
        136
        204
        ——   ———— s.  d.
        2176 )4026(1 : 10 440/2176 or 55/272 Scots money,
              2176                          the answer.
              ————
              1850
                12
              ————
        2176)22200(10
              ————
               440
```

These two examples may suffice. The proof of all operations in these sort of mixtures, is done by comparing the value of all the mixture, being sold at the mean rate; with the total value of all the particular quantities, supposing they had been sold at their respective rates unmixed; if those sums are equal the work is true.

SECTION II.

SECTION II.

ALLIGATION ALTERNATE teacheth what quantities of ſeveral ingredients (the particular rates of every one of them being given) may be mixed, that the compoſition may bear a price propounded. And it admits of three caſes.

CASE I.

When the price of each ingredient is expreſſed, but no quantitie given; and it is required to find how much of each ingredient is requiſite to compoſe the mixture, to ſell any part of the compoſition at a mean price propounded.

In this caſe, you have only to link the ſeveral extremes rightly together, and to find the true difference betwixt them and the mean; for thoſe differences are the quantities ſought.

Example.

A vintner would mix four ſorts of wine, viz. of 12 d. of 18 d. of 24 d. and of 26 d. per pint; what quantity of each muſt he take, to ſell the mixture at 20 d. per pint?

I. Set down the ſeveral given prices (which muſt always be of one name) in a column, one under another; then towards the left hand of theſe draw a line of connection, and on the other ſide of the ſaid line place the mean rate; and the given numbers of the given example will ſtand thus.

$$\text{Mean rate } 20\left\{\begin{array}{l}12\\18\\24\\26\end{array}\right.$$

II. Link them two and two together, always obſerving to join a greater and a leſs than the mean; then

then set down the difference alternatly; that is, against each extreme place the difference betwixt the mean and it's yokefellow, thus.

20	12	4
	18	6
	24	8
	26	2

So the difference betwixt the extreme 12 and 20 the mean being 8, is placed against 24 it's yokefellow; and the difference between 18 and 20 being 2, is placed against it's yokefellow 26; also, the difference betwixt 24 and 20 being 4, is placed against 12; and the difference betwixt 26 and 20 being 6, is set against 18; therefore if he mixes 4 pints at 12 d. with 6 at 18 d. and with 8 at 24 d. and 2 at 26 d. per pint, he may sell the mixture at 20 d. per pint.

The proof of which is easy. For the sum of the differences valued at the mean rate, will be found equal to the amount of the particular differences at their given prices.

For 20 pints (the sum of the differences) at 20 d. per pint is 400 pence; and 4, 6, 8, 2 pints (the particular differences) at 12, 18, 24, 26 pence per pint, will come to the same sum.

Note, As many different ways as the extremes can be combin'd, so many different answers may be given to the question, yet all true.

So the same example being combin'd, thus:

20	12	6	answer.
	18	4	
	24	2	
	26	8	

20	12	4 . 6	10	answer.
	18	4 . 6	10	
	24	8 . 2	10	
	26	8 . 2	10	

gives those different, but true answers, as may be prov'd by the same rule.

In the laſt of theſe on the right hand, the numbers being doubly combin'd (that is) each with two others, make each to have two differences ſet againſt it; for 12 the firſt extreme being link'd both with 24 and 26, muſt have both their differences (viz. 4 and 6) placed againſt it; and ſo the reſt, which double difference muſt be added and placed as you here ſee: and ſo it will take 10 pints of each ſort to make a mixture worth 20d. per pint.

But when in a queſtion there is but one extreme leſs, or but one greater than the mean; ſuch a queſtion will admit of but one anſwer, that ſingle extreme being to be linked with all the reſt; as in this following example.

I would mix tea at 5 s. at 6 s. and at 10s. per lb. ſo as to ſell a lb. of the mixture at 8 s.; what quantity of each muſt I take?

8	5	2	2	anſwer.
	6	2	2	
	10	3 . 2	5	

The numbers being placed and link'd, and differenced according to the foregoing rules, it appears (to ſell my tea at 8 s. per lb.) I muſt mix 2 lb. at 5 s. with 2 lb. at 6 s. and 5 lb. at 10 s.

How much water may be mixed with wines at 3 s. and at 7 s. per pint, that the mixture may be ſold at 6 s. the pint.

6	0	1	1 pint of water.
	3	1	1 pint of wine at 3 s.
	7	6 . 3	9 pints of wine at 7 s.

How many lb. of tea at 5 s. and 8 s. per lb. mixed with tobacco to make it purgative, will make it good for ladies, and to ſell at 6 s. per lb.?

6	0	2	2 lb. of tobacco.
	5	2	2 lb. of tea at 5 s. per lb.
	8	6 . 1	7 lb. of tea at 8 s. per lb.

I would

I would mix oats at 7 s. 6 d. per boll, with oats at 6 s. 9 d. per boll, and with beans at 4 s. 3 d. per boll, and pease at 5 s. per boll; what quantity of each sort must I take, that the mixture may be sold at 6 s. per boll.

In all questions of this kind, wherein it is required to mix 4 things, two of them having their prices greater, and two lesser than the mean rate; you may alligate or compare a greater and lesser price with the mean price, and set down their differences alternately as before.

	d.	
Mean rate 72	90	21 bolls of oats at 7s. 6d. per b.
	51	18 bolls of beans.
	60	9 bolls of pease.
	81	12 bolls of oats at 6s. 9d. per b.

Hence, if 21 bolls of oats at 7s. 6 d. per boll, be mixed with 12 bolls of oats at 6 s. 9 d. per boll, and with 18 bolls of beans at 4 s. 3 d. per boll, and 9 bolls of pease at 5 s. per boll; a boll of that mixture may be sold at 72 pence, or 6 shillings.

	d.	
Or thus, 72	90	12 bolls of oats at 7s. 6d. per boll.
	60	18 bolls of pease.
	81	21 bolls of oats at 6s. 9d. per b.
	51	9 bolls of beans.

Either of these mixtures equally answer the question, which may be easily try'd as before.

The reason of these combinations, and the alternate placing of their differences, will appear from this plain consideration, viz. That thereby whatever is lost upon the quantity sold, whose given price exceeds the mean, is gain'd upon the quantity, whose given price is less than the mean.

CASE II.

WHEN the particular rates of all the ingredients propos'd to be mixed, the quantity of one ingredient, and

and mean rate of the whole mixture are given: thence to find how much of every one of the other ingredients is requisite to compose the mixture.

This is usually called Alligation Partial.

When the numbers are set down, and their several differences found (as before) you must proceed by this proportion.

As the difference against the quantity given,
Is to the rest of the differences one after another:
So is the given quantity,
To the several quantities sought.

How much English brandy at 12 d. and at 8 d. per chopin, may I mix with 40 chopins of French brandy at 16 d. per chopin, that I may sell the mixture at 10 d. per chopin?

$$10\begin{cases}16 & 2 & 2\\ 12 & 2 & 2\\ 8 & 6\ .\ 2 & 8\end{cases}$$

Now it is plain, that were there but 2 chopins of the French brandy, there must be 2 chopins of English brandy at 12 d. per chopin, and 8 of the same at 8 d. to make a mixture to give 10 d. the chopin; but since there are to be 40 chopins of French brandy, say,

c. c. c.
If 2 : 2 :: 40
40

2)80(40 chopins of English brandy at 12 d. per (chopin.
0

c. c. c.
If 2 : 8 :: 40
40

2)320(160 chopins of English brandy at 8 d. per (chopin.
0

So

So that if you mix 40 chopins of French brandy at 16 d. with 40 chopins of English brandy at 12 d. and with 160 chopins of the same at 8 d. per chopin; a chopin of the mixture may be sold at 10 d.

I would with 21 lb. of snuff at 8 s. per lb. mix so much snuff of 4 s. per lb. and so much of 5 s. per lb. that I might afford to sell the mixture at 6 s. per lb.; what quantity of each must I take?

6	8	1 . 2	3
	5	2	2
	4	2	2

lb. *lb.* *lb.*
If 3 : 2 :: 21
21

3)42(14 lb. of snuff at 5 s. per lb.
0

lb. *lb.* *lb.*
If 3 : 2 :: 21
21

3)42(14 lb. of snuff at 4 s. per lb.
0

So that if you mix 21 lb. of snuff at 8 s. with 14 lb. at 5 s. and with 14 lb. at 4 s. per lb.; a lb. of the mixture may be sold at 6 s. per lb.

This case has the same proof as the former.

CASE III.

The particular prices of all the ingredients proposed to be mixed; and the sum of all their quantities, with the mean rate of the compound being given; to find how much of each sort must be taken to make up the quantity propounded.

This

This is usually called Alligation Total, which (after the work of the first case) is performed by this proportion.

As the ſum of all the differences,
Is to the given ſum of all the quantities:
So is every particular difference,
To its particular quantity.

Let it be required to mix 4 ſorts of tea, viz. at 5 s. 6 s. 8 s. and 9 s. per lb. ſo as that the whole quantity may be 87 lb. to be ſold at 7 s. per lb. how much of each muſt be taken?

			lb.
7	5		2
	6		1
	8		1
	9		2

lb. lb.
If 6 : 87 :: 2
2

6)174(29 lb. at 5 s. per lb. and there will be alſo 29 lb. at 9 s. per lb. becauſe the differences are equal.
0

lb. lb. lb.
If 6 : 87 :: 1
1

6)87(14½ lb. at 6 s. per lb. and there will be 14½ lb. at 8 s. per lb. becauſe the differences are equal alſo.
3

The work of this caſe is prov'd by adding the quantities found together, for, (if the work be truly performed) their total ſum will be equal to the given ſum of all the quantities propos'd to be mixed.

Thus have I gone briefly through the rule of Alligation, that I might have ſpace to treat on things of more uſe; I could have inſerted many various examples,

amples, as alſo, the manner of compounding medicines, &c. but I refer ſuch as deſire to ſee more into that buſineſs, to Sir Jonas More's arithmetic, wherein they will find it largely handled.

Tho' Alligation Alternate gives true anſwers to queſtions of that kind, with ſome little variety, according as the ingredients are more or leſs in number; yet it will not give all the anſwers which ſome ſuch queſtions are capable of, nor perhaps thoſe which ſuit beſt with the preſent occaſion: nor can this imperfection be remedied by common arithmetic; but by an Algebraic way of arguing it may; for thereby all the poſſible anſwers to any queſtion in this rule may be clearly and eaſily diſcovered; as will appear from Mr. Wards introduction to the Mathematicks.

I ſhall now proceed to the doctrine of Vulgar Fractions, on which I ſhall be as brief as poſſible, in regard I deſign to treat ſomewhat largely on Decimal Fractions.

CHAP. X.

Of Vulgar Fractions.

SECTION I.

Of Notation.

A Fraction, or broken number, is a part or parts of an unit, and is expreſſed or written by two numbers placed one above the other, with a line drawn between them; as $\frac{3}{4}$, $\frac{75}{87}$, $\frac{411}{712}$, &c.

The number under the line is called the Denominator, becauſe it gives name to the fraction, and alſo ſhews into how many parts the unit is broken; and the number above the line is called the Numerator, becauſe it tells how many of theſe parts are meant by the fraction, and it is always the remainder after Diviſion.

vision. Thus the fraction $\frac{3}{4}$, shews that the unit is broken into 4 parts, and that 3 of those parts are express'd by the said fraction.

I. Note, therefore, that if the Numerator and Denominator of a fraction shall be equal, as $\frac{1}{1}$, $\frac{4}{4}$, the value of such fraction will be exactly an unit or integer; for by the above definition, the Denominator shews into how many parts the unit is broken, and the Numerator expresses how many of those parts are meant by the fraction. Then in the fraction $\frac{4}{4}$, the Denominator 4 declaring the unit to be broke into 4 parts, and the Numerator expressing 4, that is, all of those parts, it is plain the said fraction is equal to an unit or whole number; because the sum of all the parts must be equal to the whole. From which observation it is also plain, that so often as the Denominator is contain'd in the Numerator, so many units or whole numbers are contain'd in such improper fraction. And this note may serve as a reason for the operations of the 1st. 2d. and 3d. sorts of Reduction following.

II. Note also, That as all fractions do indeed arise from the remainders of Division, when the Divisor can no longer measure the Dividend, so every fraction may be looked upon as the two given terms of a Division, the Numerator as the Dividend, and the Denominator as the Divisor; from whence it appears, that if the Numerator and Denominator of a fraction be either multiply'd or divided both by the same number, the products or quotients will still remain in the same proportion, and the new fraction so arising be of the same value with that given. Thus the fraction $\frac{2}{4}$ multiply'd by 2 will produce $\frac{4}{8}$; or divided by 2, will quote $\frac{1}{2}$: all which fractions are of the same value; 4 bearing the same proportion to 8, and 1 to 2, as 2 does to 4. And from hence also may appear the reason of the 1st. 5th. and 6th. sorts of Reduction following.

OF VULGAR FRACTIONS there are 4 sorts, viz.

1st. A proper fraction; whose Numerator is always less than its Denominator, as $\frac{3}{5}$, $\frac{4}{5}$, $\frac{7}{12}$, &c. And this fraction is always less than an unit; That is, it represents a part or parts of any thing less than the whole.

2d. An improper fraction, which is greater than an unit. That is, it represents some number of parts greater than one whole thing; and its Numerator is always greater than the Denominator, as $\frac{6}{4}$, $\frac{9}{7}$, $\frac{12}{2}$, &c.

3d. A compound fraction, or fraction of a fraction; which is a part of a part, consisting of several Numerators and Denominators connected together, and known by the word (of) between them, as $\frac{3}{7}$ of $\frac{4}{5}$ of $\frac{8}{18}$ &c. That is, when an unit (or whole thing) is first divided into any number of equal parts, and each of these parts are subdivided into other parts, and so on; then those last parts are called Compound Fractions, or Fractions of Fractions. As for instance, suppose a l. Ster. (or 20 s.) be the unit or whole; then is 8 s. $\frac{2}{5}$ of it, and 6 s. is $\frac{3}{4}$ of those two fifths, and 2 s. is $\frac{1}{3}$ of those three fourths, viz. 2 s. is $\frac{1}{3}$ of $\frac{3}{4}$ of $\frac{2}{5}$ *of one* pound Ster.

4th. A mix'd fraction; which is a fraction joined with a whole number, as $5\frac{1}{2}$, $13\frac{6}{7}$, &c.

Before fractions can be either added or subtracted, they must be reduced into one denomination, and therefore Reduction must take place, before we proceed to those rules.

SECTION II.

Of Reduction of Vulgar Fractions.

In Reduction of Fractions there are 8 cases.

CASE I. To reduce a whole number to an improper fraction.

RULE, Place 1 for its Denominator, and you have done.

Examples,

Examples, Reduce 4 to a fraction, it makes $\frac{4}{1}$, or 18 and it makes $\frac{18}{1}$.

But if you would assign it any other Denominator, multiply the whole number by the Denominator assign'd, and place the product for a Numerator over the assign'd Denominator.

Examples, Reduce 14 to a fraction, whose Denominator let be 8, it makes $\frac{112}{8}$ or if 9 were to be made a fraction, and its Denominator to be 5, it would become $\frac{45}{5}$,

For the reason of this rule, consider the two foregoing notes.

CASE II. To reduce a mixed fraction into an improper one.

RULE, Multiply the whole number by the Denominator of the given fraction, and to their product add the Numerator, the sum placed over the given Denominator, will be the fraction required.

Examples, Reduce $7\frac{3}{5}$ to an improper fraction, and it makes $\frac{38}{5}$.

$$\begin{array}{r} 5 \\ \hline 35 \\ 3 \\ \hline 38 \text{ Numerator.} \end{array}$$

And so will $15\frac{3}{4}$ when reduced to an improper fraction) be $\frac{63}{4}$.

The reason of this rule is the same as the former, there being no difference in the operation, but the taking in the given Numerator.

CASE III. To reduce an improper fraction into its equivalent whole or mixed number.

RULE, Divide the Numerator by the Denominator, the Quotient gives the whole number contained: but if any thing remains (as in the second example) it must be placed as a new Numerator over the given Denominator.

Examples,

Examples, Reduce $\frac{48}{6}$ to its equivalent whole number.

6)48(8 equivalent whole number.

0

Reduce $\frac{63}{4}$ to its equivalent mixed number.

4)63(15$\frac{3}{4}$ equivalent mixed number.

3

This being only the reverse of the foregoing rule, the same reason still holds.

CASE IV. To reduce a compound fraction to a single one of the same value, that is, a fraction of a fraction to a fraction of an unit.

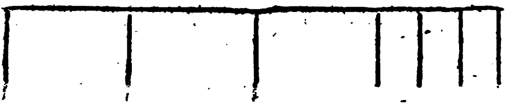
RULE, Multiply all the Numerators together for a Numerator, and all the Denominators for a Denominator.

Examples, Reduce $\frac{3}{4}$ of $\frac{1}{5}$ of $\frac{2}{3}$ to a single fraction of the same value, and it makes $\frac{6}{60}$.

3	4
1	5
3	20
2	3
Numerator 6	Denominator 60

So also $\frac{1}{8}$ of $\frac{2}{11}$ reduced to a single fraction, will become $\frac{2}{88}$. And $\frac{2}{3}$ of $\frac{1}{4}$ will be $\frac{2}{12}$.

The reason of this operation will best appear by representing the unit or whole number by a line, which, according to the last example, must be supposed to be divided into 4 parts, and each of those parts again into 3 smaller parts; thus,



Then,

Then, as $\frac{1}{4}$ of the unit will denote one of the larger divisions, so $\frac{2}{3}$ of that fourth must signify only two of the lesser divisions; consequently if I would express what part of the whole line $\frac{2}{3}$ of $\frac{1}{4}$ is, it will plainly appear to be $\frac{2}{12}$, or had the compound fraction been $\frac{3}{3}$ of $\frac{2}{4}$, the single fraction equal to it had been $\frac{6}{12}$, &c.

CASE V. To reduce a fraction into its lowest terms equivalent to the fraction given.

RULE, Divide the Numerator and Denominator by any figure, so that nothing may remain, and the Quotients will be a new fraction equivalent to the given one. Thus $\frac{25}{35}$ divided by 5, will be $\frac{5}{7}$.

The rule generally given for finding the greatest common measure, or number to divide your given fraction by, is this:

Divide the Denominator by the Numerator; and if any thing remains, divide your Divisor thereby; and if any thing yet remains, then divide your last Divisor by it; so continuing to make the last Divisor your Dividend, and the remainder your Divisor, till nothing remains; then the last Divisor will be your greatest common measure, or number by which you can divide both Numerator and Denominator of the given fraction, and so reduce them both into their lowest terms at one work.

Examples, What is the greatest common measure by which $\frac{228}{304}$ can be divided?

228)304(1

Greatest number 76)228(3

0

Now I divide 228 by 76, and it quotes 3 for a new Numerator; and 304 divided by 76, quotes 4 for a new Denominator; so that $\frac{228}{304}$ in its lowest terms is $\frac{3}{4}$.

So also, $\frac{6048}{8736}$ reduced into its lowest terms by a common measure, will be $\frac{9}{13}$. And $\frac{8081}{20145}$ will be $\frac{13}{15}$. And $\frac{2608}{6184}$ will be $\frac{52}{171}$, &c.

But this way of finding the common measure is too tedious

tedious, often making more work than it saves. Observe therefore these practical directions.

If you cannot at once discover the greatest number you can divide by, and if your Numerator and Denominator are both even numbers, you may always halve them, and that way reduce your fraction to its lowest terms.

Thus $\frac{208}{684}$ (the fraction given in the last example) divided twice by 2, gives, as before, $\frac{52}{171}$

$$2 \mid \frac{208}{684} \mid \frac{104}{342} \mid \frac{52}{171}$$

And $\frac{192}{576}$ may be reduced, by continual halving to $\frac{3}{9}$,

$$2 \mid \frac{192}{576} \mid \frac{96}{288} \mid \frac{48}{144} \mid \frac{24}{72} \mid \frac{12}{36} \mid \frac{6}{18} \mid \frac{3}{9} \text{ or } \frac{1}{3}$$

Also, when both your Numerator and Denominator have cyphers on the right hand, you may abbreviate the fraction, by striking off an equal number of those cyphers from both.

Thus, $\frac{75|00}{96|00}$ will be $\frac{75}{96}$. And $\frac{37|00}{150|00}$ will be $\frac{37}{150}$.

Or, if the right hand figures of your Numerator and Denominator are both fives, or one a five, and the other a cypher, you may always divide them by 5;

Thus $\frac{375}{555}$ divided by 5 is $\frac{75}{111}$.

And $\frac{125}{360}$ divided by 5 is $\frac{25}{72}$.

In the Rule of Three I thus reduced the fraction at the end of the work, to its lowest terms. And for the reason of this rule I refer you to the 2d. Note.

CASE VI. To reduce fractions having unequal Denominators, to fractions of the same value having equal Denominators.

RULE, Multiply all the Denominators together, and the last product shall be the common Denominator. Then multiply each of the given Numerators into all the Denominators except its own;

and

and the laſt product put for a Numerator over the Denominator found as before: ſo this new fraction is equal to that fraction whoſe Numerator you multiplied into the ſaid Denominators. Do ſo by all the Numerators given, and you have your deſire.

Examples, Reduce $\frac{3}{4}$, $\frac{5}{8}$ and $\frac{6}{10}$ to a common Denominator.

3	5	6	4
8	4	8	8
24	20	48	32
10	10	4	10
240	200	192	320
1ſt. Numer.	2d. Numer.	3d. Numer.	Denominator.

So $\frac{3}{4}$ will be $\frac{240}{320}$, and $\frac{5}{8}$ is $\frac{200}{320}$, and $\frac{6}{10}$ becomes $\frac{192}{320}$.

And ſo will $\frac{5}{10}$, $\frac{3}{8}$, $\frac{4}{12}$ and $\frac{9}{15}$ when reduced to a common Denominator become $\frac{72000}{144000}$, $\frac{54000}{144000}$, $\frac{48000}{144000}$, and $\frac{86400}{144000}$, &c.

The reaſon of this 6th caſe of Reduction is evident from the ſecond Note preceeding; for as both the Numerator and Denominator of each given fraction are equally multiplied by all the other Denominators, conſequently the new fractions ariſing thence, muſt be equal to the fraction given.

CASE VII. To find the value of a fraction in the known parts of coin, weight, meaſure, &c.

RULE, Multiply the Numerator by the parts of the next inferior Denomination, and divide the product by the Denominator; the Quotient ſhews the parts ſought, and the remainder becomes a new Numerator to the given Denominator; which muſt ſtill be valued by the ſame rule, proceeding till you have brought it to the leaſt known parts of the integer.

Examples,

Examples, What's the value of $\frac{7}{8}$ of a l. Ster.

7
20

8)140(17 s. 6 d. answer.

4
12

8)48(6

0

What's the value of $\frac{3}{5}$ of a l. Ster.

3
20

5)60(12 s. answer.

0

What quarters and lb. are contain'd in $\frac{7}{12}$ of an hundred weight?

7
4

12(28(2 qrs. $9\frac{1}{3}$ lb. answer.

4
28

12(112(9

4

There will be no difficulty in accounting for this rule, if we consider but the particular working of any one example: Thus, in the 2d. $\frac{3}{5}$ of a l. Ster. are given to be valued: now, as 20 s. make a l. so consequently any part of a pound must be 20 times as great a part of a shilling; therefore $\frac{3}{5}$ of a l. make $\frac{60}{5}$ of a s.; which being an improper fraction, its Numerator is divided

divided by its Denominator, to find the units or whole numbers (which in this case must be shillings) contain'd in it; according to the direction of the 3d. sort of reduction.

By this rule are remainders in the Rule of Three valued.

CASE VIII. To reduce a fraction of one denomination to another of the same value.

This is either Ascending or Descending.

Ascending, when a fraction of a smaller is brought to a greater denomination.

Descending, when a fraction of a greater denomination is brought to a lower.

I. When a fraction is to be brought from a lesser to a greater denomination.

RULE, Make of it a compound fraction, by comparing it with the intermediate denominations between it and that you would have it reduced to; then (by the 4th rule foregoing) reduce your compound to a single fraction, and the work is done.

Examples, What part of a l. is $\frac{5}{7}$ of a d.?

To resolve this, I consider that 1 d. is $\frac{1}{12}$ of a s. and a s. is $\frac{1}{20}$ of a l.; wherefore $\frac{5}{7}$ of a d. is $\frac{5}{7}$ of $\frac{1}{12}$ of $\frac{1}{20}$ of a l. which by the said 4th rule, I find to be $\frac{5}{1680}$ of a l.

What part of a boll is $\frac{3}{4}$ of a lippie?

$\frac{3}{4}$ of a lippie is $\frac{3}{4}$ of $\frac{1}{4}$ of $\frac{1}{4}$ of $\frac{1}{4}$ of a boll, which reduced to a single fraction, makes $\frac{3}{256}$ of a boll the answer.

II. When a fraction is to be brought from a greater to a lesser denomination.

RULE, Multiply the Numerator of the fraction by the parts contain'd in the several denominations betwixt it and the parts you would reduce it to; then place the last product over the Denominator of the given fraction, and you have done.

Examples, Reduce $\frac{3}{4}$ of a l. to the fraction of a d.

To do this I multiply the Numerator 3 by 20 and 12, the product is 720, which I put over the Denominator 4, and it makes $\frac{720}{4}$ of a penny equal to $\frac{3}{4}$ of a l.

SECTION III.

Addition of Vulgar Fractions.

I. If your fractions to be added have a common Denominator, then add all the Numerators together, and their sum will be a new Numerator, under which subscribe the common Denominator; and you have done.

Example, What's the sum of $\frac{7}{36}$, $\frac{11}{36}$, $\frac{12}{36}$ and $\frac{18}{36}$?

The sum of the Numerators will be 48, therefore the sum of those fractions will be $\frac{48}{36}$, or $1\frac{12}{36}$ or $1\frac{1}{3}$.

II. If the fractions to be added have different Denominators, reduce them (according to the 6th sort of Reduction) to a common Denominator, and then add the Numerators together, and put their sum over the common Denominator, and if such fraction shall be improper, reduce it to a whole or mixed number, as in the last example.

Example, What's the sum of $\frac{3}{4}$, $\frac{1}{5}$, $\frac{2}{3}$, and $\frac{2}{7}$?

Those fractions reduced to a common Denominator, will be $\frac{315}{420}$, $\frac{84}{420}$, $\frac{280}{420}$, and $\frac{120}{420}$, and the new Numerators being added, the sum of the fractions will be $\frac{799}{420}$, which, being improper, I reduce to a mixed number, and it is $1\frac{379}{420}$.

III. If compound fractions are given to be added, reduce them (according to the 4th sort of Reduction) to single ones, then reduce these to a common Denominator, and add them as before.

Example, Add $\frac{3}{4}$ of $\frac{5}{8}$ and $\frac{2}{3}$ of $\frac{3}{5}$ and $\frac{5}{8}$ together.

The two compound fractions being reduced to single ones, will be $\frac{15}{32}$ and $\frac{6}{15}$, so you will have these two to add to your $\frac{5}{8}$; reduce them therefore to a common Denominator and they will be $\frac{1800}{3840}$, $\frac{1536}{3840}$, and $\frac{2400}{3840}$, and the Numerators being added the sum of the fractions will be $\frac{5736}{3840}$ or $1\frac{1896}{3840}$ or $1\frac{79}{160}$.

IV. If mixed numbers are given to be added, reduce the fractional parts to a common Denominator, add then the said parts as before; and if their sum be an an improper fraction, reduce it to a mixed number, add its integral part to the integral parts of the given mixed numbers, and the work is done.

Examples, What's the ſum of l. $5\frac{1}{2}$, l. $7\frac{3}{4}$ and l. $3\frac{5}{6}$?

The fractional parts reduced to a common Denominator are $\frac{24}{48}$, $\frac{36}{48}$, and $\frac{40}{48}$ and the ſum of theſe when added together is $\frac{100}{48}$ which reduced to a mixed number is $2\frac{4}{48}$ or $2\frac{1}{12}$ which added to 5, 7 and 3 l. makes l. $17\frac{1}{12}$ the ſum of the mixed numbers.

What's the ſum of $4\frac{1}{2}$, $3\frac{1}{6}$, and $\frac{2}{7}$? Anſr. $7\frac{80}{84}$.

V. If fractions of different denominations be given to be added, reduce them into one denomination, as in caſe 8th of the 2d. ſection, and then proceed as before.

Example, What's the ſum of $\frac{3}{4}$ l. and $\frac{5}{6}$ of a s.?

Of the given fractions here, one is of a pound, and the other the fraction of a ſhilling; and before they can be added together, you muſt reduce $\frac{5}{6}$ of a s. to the fraction of a pound as the other is, and it makes $\frac{5}{120}$ of a pound; then $\frac{3}{4}$ l. and $\frac{5}{120}$ l. being reduced to a common Denominator, and added, as before, their ſum will be $\frac{380}{480}$ or $\frac{38}{48}$ or $\frac{19}{24}$ of a l. when reduced to its loweſt terms by the 5th rule of the laſt ſection.

It would have been the ſame, if by the latter part of the 8th rule in the laſt ſection, you had reduced $\frac{3}{4}$ l. to the fraction of a ſhilling; which you would have found to have been $\frac{60}{4}$ s. which reduced and added to $\frac{5}{6}$ s. as before, would have made $15\frac{20}{24}$ s. which is equal to $\frac{19}{24}$ l. the ſum found before; for by the rule for the 7th ſort of Reduction, the value of $\frac{19}{24}$ l. will be found to be 15 s. 10 d. and ſo will $15\frac{20}{24}$ s. be found juſt the ſame.

SECTION IV.

Subtraction of Vulgar Fractions.

SUBTRACTION of fractions is alſo nothing (after

ter the given fractions are prepared by Reduction) but taking one Numerator from the other.

I. If your fractions to be subtracted have a common Denominator, subtract the lesser Numerator from the greater, and their difference is a new Numerator, under which subscribe the common Denominator (as in Addition) and that new fraction is the difference required. So $\frac{21}{64}$ being given to be subtracted from $\frac{35}{64}$, the difference will be $\frac{14}{64}$ or in its lowest terms $\frac{7}{32}$.

II. But if the fractions to be subtracted have different Denominators, after they are reduced to a common one, subtract (as before) the one Numerator from the other, and place the difference over the common Denominator, so shall that new fraction be the difference sought.

Examples, What's the difference betwixt $\frac{3}{4}$ and $\frac{5}{7}$? The fractions reduced to a common Denominator will be $\frac{21}{28}$ and $\frac{20}{28}$ and their difference is $\frac{1}{28}$.

III. If compound fractions are to be subtracted, reduce them to single ones, and these to a common Denominator, and subtract as before.

Example, What's the difference betwixt $\frac{3}{4}$ of $\frac{1}{8}$ and $\frac{2}{3}$ of $\frac{7}{8}$?

The compound fractions reduced to single ones are $\frac{3}{32}$ and $\frac{14}{24}$, and these reduced to a common Denominator will be $\frac{72}{768}$ and $\frac{448}{768}$, and their difference $\frac{376}{768}$ or in its lowest terms $\frac{47}{96}$.

What's the difference between $\frac{5}{8}$ and $\frac{3}{5}$ of $\frac{1}{8}$?

Reduce the compound fraction to a single one; then proceed as before; and the answer is $\frac{110}{240}$ or in its lowest terms $\frac{11}{24}$.

IV. If a fraction is given to be subtracted from a whole number, subtract the Numerator of the fraction from its Denominator, and put the remainder for a new Numerator over the given Denominator, then subtract an unit (for that you borrowed) from the whole number, and place the remainder on the left hand of the fraction found; which mixed number is the difference sought.

Example,

Example, What's the difference betwixt 24 and $\frac{5}{8}$?

Here if you subtract 5 the Numerator from 8 the Denominator, there remains 3, which put over 8 is $\frac{3}{8}$, and 1 borrowed from 24, rests 23, to which join $\frac{3}{8}$, and it makes 23$\frac{3}{8}$ for the excess.

Subtract $\frac{17}{25}$ from 48, and there remains 47$\frac{8}{25}$.

V. If it be required to subtract a fraction from a mixed number, or one mixed number from another, reduce the fractions to a common Denominator; and if the fraction to be subtracted be less than the other, then subtract the lesser Numerator from the greater, and the difference is a Numerator for the common Denominator: then subtract the lesser integral part from the greater, and the remainder with the remaining fraction annexed thereto is the difference sought.

Examples, What's the difference betwixt 43$\frac{7}{8}$ and $\frac{3}{4}$?

The fractions $\frac{7}{8}$ and $\frac{3}{4}$ reduced to a common Denominator, will be $\frac{28}{32}$ for $\frac{7}{8}$ and $\frac{24}{32}$ for $\frac{3}{4}$, and their difference will be $\frac{4}{32}$, or in its lowest terms $\frac{1}{8}$; and so this mixed number 43$\frac{1}{8}$ is the difference required.

What's the difference betwixt 26$\frac{3}{7}$ and 54$\frac{5}{6}$?

First subtract $\frac{3}{7}$ viz. $\frac{18}{42}$, from $\frac{5}{6}$ viz. $\frac{35}{42}$, the remainder is $\frac{17}{42}$; then 26 from 54, and there remains 28; to which annex $\frac{17}{42}$, it makes 28$\frac{17}{42}$ for the answer.

VI. But if the fraction to be subtracted is greater than the fraction from whence you subtract, then having first reduced the fractions to a common Denominator, take the Numerator of the greatest fraction from the said common Denominator, and add the remainder to the Numerator of the lesser fraction, and their sum is a new Numerator to the common Denominator, which fraction note; then (for the 1 you borrowed) add 1 to the integral part to be subtracted, and taking that sum from the greater number, to the remainder annex the fraction you noted before; so this new mixed number shall be the difference requir'd.

Example, What's the difference betwixt 14$\frac{3}{4}$ and 29$\frac{4}{7}$?

The fractions reduced will be $\frac{21}{28}$ and $\frac{16}{28}$, viz. $\frac{3}{4}$ equal

equal to $\frac{21}{28}$, and $\frac{4}{7}$ equal to $\frac{16}{28}$: now I should subtract $\frac{21}{28}$ from $\frac{16}{28}$, but I cannot; therefore I subtract 21 from 28, and there remains 7; which added to 16 (the lesser Numerator) makes 23 for a Numerator to 28, and it is $\frac{23}{28}$. Then I come to the integral parts 14 and 29; and say, 1 that I borrowed and 14 is 15, which taken from 29, there rests 14, to which annexing $\frac{23}{28}$, it is $14\frac{23}{28}$ for the remainder or difference betwixt $14\frac{3}{4}$ and $29\frac{4}{7}$.

But in this case I advise the learner to reduce both the mixed numbers to improper fractions (as taught in the 2d. case of Reduction) and then to reduce these to a common Denominator, by the rule given for the 6th case in Reduction: then subtract the one Numerator from the other, place the difference over the common Denominator, and if it be an improper fraction, reduce it to its whole or mixed number as before directed, and so the work is done.

Let the foregoing example be repeated, viz. What's the difference betwixt $14\frac{3}{4}$ and $29\frac{4}{7}$?

The two mixed numbers reduced to improper fractions will be $\frac{59}{4}$ and $\frac{207}{7}$, and these reduced to a common Denominator, are $\frac{413}{28}$ and $\frac{828}{28}$; then 413 subtracted from 828, there remains 415, and so the difference is $\frac{415}{28}$ which reduced to its mixed number makes $14\frac{23}{28}$ the required difference, as before.

VII. If it be required to subtract a fraction of one denomination from a fraction of another name, reduce them both to one denomination, and then to a common Denominator, and subtract as before.

Example, What's the difference betwixt $\frac{1}{3}$ of a pound, and $\frac{3}{4}$ of a penny?

Here, you may either reduce the $\frac{1}{3}$ l. to the fraction of a penny; or the $\frac{3}{4}$ d. to the fraction of a pound; for any of the two will do.

Let $\frac{3}{4}$ d. then be reduced to the fraction of a pound, and it will be $\frac{3}{4}$ of $\frac{1}{12}$ of $\frac{1}{20}$ of a pound, which is $\frac{3}{960}$ of a pound, then $\frac{1}{3}$ and $\frac{3}{960}$ of a l. reduced to a common Denominator will be $\frac{960}{2880}$ and $\frac{9}{2880}$, and $\frac{9}{2880}$

subtracted

subtracted from $\frac{960}{2880}$, there remains $\frac{951}{2880}$ l. or in its lowest terms $\frac{317}{960}$ l. for the difference required.

Or, let $\frac{1}{3}$ of a l. (by rule 2d. for the 8th case in Reduction) be reduced to the fraction of a penny, and it will be $\frac{240}{3}$ of a penny; and $\frac{240}{3}$ and $\frac{3}{4}$ of a penny reduced to a common Denominator, will be $\frac{960}{12}$ and $\frac{9}{12}$; then subtracting $\frac{9}{12}$ from $\frac{960}{12}$, there remains $\frac{951}{12}$ or in its lowest terms $\frac{317}{4}$ of a d. for the difference required, which is precisely equal to $\frac{317}{960}$ of a l. or to 317 farthings.

Note, If two or more fractions be given, to find the greatest, reduce them all to a common Denominator, and that which hath the greatest Numerator is the greatest fraction.

Example, Whether is $\frac{7}{8}$ or $\frac{10}{11}$ the greatest fraction?

When they are reduced to a common Denominator, $\frac{7}{8}$ will become $\frac{77}{88}$, and $\frac{10}{11}$ will be $\frac{80}{88}$; consequently $\frac{10}{11}$ is the greatest fraction, and their difference is $\frac{3}{88}$.

If the examples in the foregoing sections be well understood, the whole business of adding and subtracting Vulgar Fractions will be easy; which is really much more difficult to perform than either Multiplication or Division; as will appear in the next section.

SECTION V.

Multiplication of Vulgar Fractions.

I. If single fractions are given to be multiply'd, multiply the Numerators together for a new Numerator, and the Denominators for a new Denominator, and the new fraction is the product required.

Examples, What's the product, when $\frac{5}{7}$ is multiply'd by $\frac{3}{8}$? Answer $\frac{15}{56}$. For the Numerators 5 and 3 being multiplied, make 15; and the Denominators 7 and 8 being multiplied, make 56.

What's the product when $\frac{9}{11}$ is multiply'd by $\frac{8}{17}$? Answer $\frac{72}{187}$.

II. If

II. If the fractions to be multiply'd be mixed numbers, reduce them to improper fractions, and then multiply them, as before.

Examples, What's the product when $7\frac{5}{8}$ is multiplied by $15\frac{1}{3}$?

When reduced to improper fractions, they are $\frac{61}{8}$ and $\frac{46}{3}$, and when multiply'd together they will produce $\frac{2806}{24}$ and this reduced to a mixed number is $116\frac{11}{12}$ the real product.

What's the product when $24\frac{5}{6}$ is multiply'd by $12\frac{7}{8}$?

Answer $\frac{15347}{48}$, or $319\frac{35}{48}$.

III. If a mixed number is to be multiplied by a fraction, reduce the mixed number to an improper fraction, then multiply the same by the other fraction, as before.

What's the product, when $18\frac{4}{7}$ is multiplied by $\frac{3}{4}$?

$18\frac{4}{7}$ reduced to an improper fraction is $\frac{130}{7}$, and this multiplied by $\frac{3}{4}$ produces $\frac{390}{28}$, which reduced to a mixed number is $13\frac{13}{14}$.

IV. If a compound fraction is to be multiply'd by a single fraction; first reduce the compound fraction to a single one, then multiply the one by the other, as before.

Example, Whats the product when $\frac{15}{19}$ is multiplied by $\frac{1}{4}$ of $\frac{2}{3}$ of $\frac{5}{7}$?

$\frac{1}{4}$ of $\frac{2}{3}$ of $\frac{5}{7}$ reduced to a single fraction is $\frac{10}{84}$, which multiplied by $\frac{15}{19}$ produces $\frac{150}{1596}$, and this reduced to its lowest terms will be $\frac{25}{266}$.

V. And, if the Multiplicand and Multiplier are both compound fractions, reduce them both to single ones, then multiply these new fractions as before, and so you have the product.

Example, What's the product of $\frac{2}{5}$ of $\frac{2}{3}$ by $\frac{3}{7}$ of $\frac{5}{9}$?

$\frac{2}{5}$ of $\frac{2}{3}$ reduced to a single fraction is $\frac{4}{15}$, and $\frac{3}{7}$ of $\frac{5}{9}$ is $\frac{15}{63}$, and $\frac{4}{15}$ and $\frac{15}{63}$ multiplied together produce $\frac{60}{945}$ which in its lowest terms is $\frac{4}{63}$.

VI. If you are to multiply a whole number by a fraction, put an unit under the whole number for a Denominator, whereby it will be an improper fraction; then multiply as before.

Example,

Example. Let 74 be multiplied by $\frac{3}{5}$.

74 reduced to an improper fraction is $\frac{74}{1}$, then $\frac{74}{1}$ multiply'd by $\frac{3}{5}$, will produce $\frac{222}{5}$, and this reduced to a mixed number is $44\frac{2}{5}$.

VII. If a fraction of one denomination is to be multiplied by a fraction of another name, reduce the one to the other's name, and multiply as before.

Example. Let $\frac{3}{4}$ of a boll be multiply'd by $\frac{1}{3}$ of a peck; what's the product?

$\frac{1}{3}$ of a peck is $\frac{1}{3}$ of $\frac{1}{16}$ of a boll, which reduced to a single fraction is $\frac{1}{48}$ of a boll, then $\frac{1}{48}$ and $\frac{3}{4}$ of a boll multiplied together, make $\frac{3}{192}$ of a boll, which reduced to its lowest terms, is $\frac{1}{64}$ of a boll, equal to 1 lippie.

Thus you may see that there is no difficulty in Multiplication: for, if you reduce compound fractions to simple ones, mixed or whole numbers to improper fractions, and fractions of different names to the same denomination, and then multiply the Numerators for a new Numerator, and the Denominators for a new Denominator, you have done: but I advise the learner to be well acquainted with Reduction, before he proceed to any other part of Vulgar Fractions.

Two things require explanation in multiplication of Vulgar fractions.

1st. Why, when the Multiplier is a proper fraction, the product is always less than the Multiplicand.

2d. Why, the Denominators are multiplied as well as the Numerators; whereas in Addition, we only find the sum of the Numerators.

Now the reason why a number multiply'd by a fraction is decreased, will appear by considering that as an unit is no Multiplier, that is, that any number multiply'd by 1, remains still the same, as once 4 is 4, &c. so to multiply any thing by a fraction (which is but part of 1) must consequently produce but such a proportionable part of the Multiplicand as such fraction is of an unit. Thus 4 multiply'd by $\frac{1}{2}$, produces but 2, the half of 4; which 4 being made a fraction by the first rule of Reduction, is $\frac{4}{1}$; so that the numbers resolve themselves into a compound fraction; and the product of 4 multiply'd by $\frac{1}{2}$ is plainly the $\frac{1}{2}$ of $\frac{4}{1}$; which by the rule for reducing a compound fraction, makes $\frac{4}{2}$ or 2: the reason therefore of the work is clear from the explanation of the 4th sort of Reduction.

SECTION VI.

Division of Vulgar Fractions.

WHEN you have made the same preparation of your numbers as in Multiplication, multiply the Denominator of your Divisor by the Numerator of your Dividend, for a Numerator; and multiply the Denominator of your Dividend by the Numerator of your Divisor for a Denominator; and that new fraction is your Quotient.

Examples.

Let $\frac{7}{9}$ be divided by $\frac{3}{8}$. $\frac{3}{8})\frac{7}{9}(\frac{56}{27}$ or $2\frac{2}{27}$ Quotient.

Let $\frac{7}{8}$ be divided by $\frac{2}{3}$. $\frac{2}{3})\frac{7}{8}(\frac{21}{16}$ or $1\frac{5}{16}$ Quotient.

Let 9 be divided by $\frac{3}{4}$. $\frac{3}{4})\frac{9}{1}(\frac{36}{3}$ or 12 Quotient.

Let $7\frac{5}{6}$ be divided by $2\frac{2}{3}$.

$7\frac{5}{6}$ reduced to an improper fraction is $\frac{47}{6}$, and $2\frac{2}{3}$ is $\frac{8}{3}$.

$\frac{8}{3})\frac{47}{6}(\frac{141}{48}$, or $2\frac{45}{48}$ Quotient.

Let $\frac{3}{10}$ be divided by $\frac{3}{4}$ of $\frac{2}{3}$.

The compound fraction $\frac{3}{4}$ of $\frac{2}{3}$ reduced to a single one, is $\frac{6}{12}$.

$\frac{6}{12})\frac{3}{10}(\frac{36}{60}$ or in its lowest terms $\frac{3}{5}$ Quotient.

Let $\frac{2}{3}$ of $\frac{3}{4}$ be divided by $\frac{2}{5}$ of $\frac{5}{6}$.

$\frac{2}{3}$ of $\frac{3}{4}$ reduced to a single fraction is $\frac{6}{12}$, and $\frac{2}{5}$ of $\frac{5}{6}$ is $\frac{10}{30}$.

$\frac{10}{30})\frac{6}{12}(\frac{180}{120}$ or $\frac{18}{12}$ or $1\frac{1}{2}$. Quotient

Let

Let $5\frac{3}{8}$ be divided by $\frac{1}{3}$ of $\frac{3}{4}$.

$5\frac{3}{8}$ reduced to an improper fraction is $\frac{43}{8}$, and $\frac{1}{3}$ of $\frac{3}{4}$ reduced to a ſingle one is $\frac{3}{12}$.

$\frac{3}{12})\frac{43}{8}(\frac{516}{24}$, or $21\frac{1}{2}$ Quotient.

Let $\frac{1}{6}$ of a l. be divided by $\frac{2}{3}$ of a ſhilling.

$\frac{2}{3}$ of a s. is $\frac{2}{3}$ of $\frac{1}{20}$ of a l. or $\frac{2}{60}$ of a l.

$\frac{2}{60})\frac{1}{6}(\frac{60}{12}$, or 5 Quotient.

Let $\frac{3}{5}$ be divided by 8. $\frac{8}{1})\frac{3}{5}(\frac{3}{40}$ Quotient.

Let $\frac{7}{8}$ l. be divided by $\frac{1}{4}$ of $\frac{5}{6}$ of a d.

$\frac{1}{4}$ of $\frac{5}{6}$ of a d. is $\frac{1}{4}$ of $\frac{5}{6}$ of $\frac{1}{12}$ of $\frac{1}{20}$ of a l. which reduced to a ſingle fraction is $\frac{5}{5760}$ of a l.

$\frac{5}{5760})\frac{7}{8}(\frac{40320}{40}$, or 1008 Quotient.

Let $\frac{4}{5}$ of an hundred weight be divided by $\frac{3}{7}$ of a lb.

$\frac{3}{7}$ lb. is $\frac{3}{7}$ of $\frac{1}{112}$ c.w. which reduced to a ſingle fraction is $\frac{3}{784}$ c.w.

$\frac{3}{784})\frac{4}{5}(\frac{3136}{15}$, or $209\frac{1}{15}$ Quotient.

There is nothing in the whole practice of fractions that more requires an explanation, than the manner of performing Diviſion, ſo different from that of whole numbers. Now as the effect of Diviſion is the finding how often one number is contained in another; or, (which is the ſame) what part of the Dividend the Diviſor is; ſo it is plain, that if the Diviſor is unity or 1, the Quotient muſt be equal to the Dividend; conſequently, in what proportion ſoever the Diviſor exceeds unity, in ſuch proportion muſt the Quotient be leſs than the Dividend; and in what proportion ſoever the Diviſor is leſs than unity, by the ſame proportion muſt the Quotient exceed the Dividend. Thus ſuppoſe the number 4 was given for a Dividend, if 1 is propoſed for its Diviſor, the Quotient will be alſo 4; but if we make the Diviſor 2, the Quotient will be but 2, half the former Quotient. Again, let the Diviſor be $\frac{1}{2}$, that is, half 1, and the Quotient will then be $\frac{8}{1}$ or 8, twice as much as the firſt Quotient,

as

as the Divisor here is but half the first Divisor: from whence appears the reason,

1st. Why, dividing a number by a proper fraction gives a Quotient greater than the Dividend: as multiplying a number by a fraction, was before proved to give a product less than the Multiplicand.

2d. Why the operation is to be performed in the manner directed; for, as the greater the Denominator is, the less is the value of the fraction; and consequently that the Quotient must increase in the same proportion, is a clear reason for multiplying the Numerator of the Dividend by the Denominator of the Divisor, for the Numerator of the Quotient: so in like manner, as the increase of the Numerator of a fraction is the increase of the value of such fraction, it is therefore as plain, the value of the Quotient must be diminished in the same proportion, by the increase of its Denominator; which is certainly effected by the multiplying the Denominator of the Dividend by the Numerator of the Divisor. Thus $\frac{5}{6}$ divided by $\frac{1}{4}$, gives $\frac{20}{6}$; but divided by $\frac{2}{4}$, gives $\frac{20}{12}$; the value of the Quotient still decreasing in the same proportion with the increase of the value of the Divisor.

I shall subjoin some few practical examples on the first 4 sections, and so conclude this chapter.

What's the sum of $\frac{11}{12}$ of a pint, $\frac{3}{7}$ of a chopin, [illegible] of a gallon?

$\frac{11}{12}$ of a pint is $\frac{11}{12}$ of $\frac{1}{8}$ of a gallon, or [illegible]; and $\frac{3}{7}$ of a chopin is $\frac{3}{7}$ of $\frac{1}{16}$ of a gallon, or [illegible]; and [illegible], [illegible] and [illegible] reduced to a common Denominator are [illegible], [illegible], and [illegible]; and the sum of these, when added is [illegible], or in its lowest terms [illegible] of a gallon.

I borrowed l. 73, whereof I paid l. 17 : 17 : 7[illegible] what remains unpaid?

l.	s.	d.
73	00	0
17	17	7[illegible]
l. 55	2	4[illegible] unpaid.

What's

What's the product when 2s. 6 d. is multiply'd by 2s. 6d. a pound the integer?

2s. 6d. is $\frac{1}{8}$ l. and $\frac{1}{8}$ multiply'd by $\frac{1}{8}$ is $\frac{1}{64}$ of a l. the anſwer, equal to 3d. 3 farthings.

What's the product when 2s. 6d. is multiply'd by 2s. 6d. a ſhilling the integer?

2s. 6d. is $2\frac{1}{2}$ s. and $2\frac{1}{2}$ s. is $\frac{5}{2}$ s. then $\frac{5}{2}$ multiply'd by $\frac{5}{2}$ produces $\frac{25}{4}$ or $6\frac{1}{4}$, equal to 6s. 3d. the anſr.

What's the product when l. 3 : 6 : 8 is multiply'd by l. 3 : 7 : 6?

l. 3 : 6 : 8 is l. $3\frac{1}{3}$; l. 3 : 7 : 6 is l. $3\frac{3}{8}$; which reduced to improper fractions, are $\frac{10}{3}$ and $\frac{27}{8}$; and their product is $\frac{270}{24}$ or $\frac{90}{8}$ l. which is equal to l. 11 : 5.

What's the product when $\frac{5}{12}$ of a yard is multiply'd by $\frac{2}{7}$ of a Scots ell?

$\frac{5}{12}$ of a yard is $\frac{5}{12}$ of $\frac{36}{37}$, which when reduced is $\frac{180}{444}$ of an ell, and this multiply'd by $\frac{2}{7}$ produces $\frac{360}{3108}$, which in its loweſt terms, is $\frac{30}{259}$ of an ell.

Let $\frac{7}{17}$ of an ell be multiply'd by $\frac{5}{18}$ of a foot.

$\frac{5}{18}$ of a foot is $\frac{5}{18}$ of $\frac{1}{3}$ of $\frac{36}{37}$, which is $\frac{180}{1998}$, and this multiply'd by $\frac{7}{17}$, produces (in its loweſt terms) $\frac{70}{1887}$ of an ell.

Let $3\frac{1}{2}$ Scots acres be multiply'd by $\frac{5}{6}$ of an English acre.

$\frac{5}{6}$ of an Engliſh acre is $\frac{5}{6}$ of [illegible], which reduced, is [illegible] of a Scots one, and this multiply'd by $3\frac{1}{2}$ or $\frac{7}{2}$, produces 1 [illegible] Scots acre.

Divide $\frac{7}{12}$ of a pound Ster. by $\frac{11}{15}$ of a merk Scots.

$\frac{11}{15}$ of $\frac{1}{18}$ is $\frac{11}{270}$) $\frac{7}{12}$ ($\frac{1890}{132}$ or $14\frac{7}{22}$ Quotient.

Divide $\frac{3}{5}$ of a guinea by $\frac{11}{15}$ of a merk Scots.

$\frac{11}{15}$ of $\frac{10}{189}$ is $\frac{110}{2835}$) $\frac{3}{5}$ ($\frac{8505}{550}$ or $15\frac{51}{110}$ Quotient.

Note, 10 guineas are equal to 189 merks Scots.

Let $\frac{2}{3}$ of a merk Ster. be divided by $\frac{11}{15}$ of a merk Scots.

$\frac{11}{15}$ of $\frac{1}{12}$ is $\frac{11}{180}$) $\frac{2}{3}$ ($\frac{360}{33}$ or $10\frac{30}{33}$ Quotient

Let l. 3 : 3 : 4 be divided by l. 5 : 7 : 6.
l. 3 : 3 : 4 is $3\frac{1}{6}$ or $\frac{19}{6}$; and l. 5 : 7 : 6 is $5\frac{3}{8}$ or $\frac{43}{8}$.

$\frac{43}{8})\frac{19}{6}(\frac{152}{258}$ or $\frac{76}{129}$ Quotient.

Divide $\frac{5}{12}$ of a crown by $\frac{11}{19}$ of a guinea.

$\frac{5}{12}$ of $\frac{5}{21}$ is $\frac{25}{252}$

$\frac{11}{19})\frac{25}{252}(\frac{475}{2772}$ Quotient.

If 1 lb. Troy of gold be worth $44\frac{1}{2}$ l.; how many guineas can be made out of the ſame?

$\frac{21}{20})\frac{89}{2}(\frac{1780}{42}$ or $42\frac{8}{21}$ guineas.

If 62 ſhillings are coin'd out of 1 lb. Troy, or 5760 grains; what will be the weight of one ſhilling.

$\frac{62}{1})\frac{5760}{1}(\frac{5760}{62}$, or $92\frac{28}{31}$ grains.

Multiply $\frac{5}{8}$ of a guinea by $\frac{7}{12}$ of a merk Scots.
$\frac{7}{12}$ of a merk is $\frac{7}{12}$ of $\frac{10}{189}$, which is $\frac{70}{2268}$, and this multiply'd by $\frac{5}{8}$, produces $\frac{25}{1296}$ of a guinea.

Multiply $\frac{7}{11}$ of a merk Scots by $\frac{5}{12}$ of a crown.
$\frac{7}{11}$ of a merk is $\frac{7}{11}$ of $\frac{2}{9}$, which is $\frac{14}{99}$, and this multiply'd by $\frac{5}{12}$, produces $\frac{70}{1188}$ of a crown.

Multiply $\frac{2}{11}$ c.w. by $\frac{7}{9}$ of a grain Troy.
$\frac{7}{9}$ of a grain is $\frac{7}{9}$ of $\frac{1}{5760}$ of $\frac{60}{73}$ of $\frac{1}{112}$, which is $\frac{420}{423843840}$, and this multiply'd by $\frac{2}{11}$, produces (in its loweſt terms) $\frac{1}{5550336}$ of an c.w.

Multiply $\frac{5}{8}$ of a ſtone by $\frac{7}{9}$ of a grain Troy.
$\frac{7}{9}$ of a grain is $\frac{7}{9}$ of $\frac{1}{5760}$ of $\frac{60}{73}$ of $\frac{1}{14}$, which reduced to a ſingle fraction is (in its loweſt terms) $\frac{1}{126144}$, and this multiply'd by $\frac{5}{8}$, is $\frac{5}{1009152}$.

Multiply $\frac{5}{12}$ of a Scots mile by $\frac{7}{9}$ of an Engliſh.
$\frac{7}{9}$ of an Engliſh mile is $\frac{7}{9}$ of $\frac{55}{62}$, which is equal to $\frac{385}{558}$, and this multiply'd by $\frac{5}{12}$, produces $\frac{1925}{6696}$ of a Scots mile.

Note, The Engliſh mile is to the Scots as 55 to 62.

To

To these I shall add an example or two on the Rule of Three, and so conclude.

In questions of the Rule of three, the fractions of the first and third terms must be of the same denomination; and having reduced compound fractions to single ones, and mixed or whole numbers to improper fractions, as in the 4 last sections; proceed to a resolution, which is performed the same way as in whole numbers, respect being had to the rules delivered for the working of fractions.

What will $\frac{1}{2}$ lb. of snuff cost, if $22\frac{1}{4}$ of the same come to l. $7\frac{1}{3}$?

The terms prepared, will stand thus.

$$\begin{array}{ccccccc} & lb. & & l. & & lb. \\ \text{If} & \frac{89}{4} & : & \frac{22}{3} & :: & \frac{1}{2} \end{array}$$

Then the second and third terms multiply'd together, produces $\frac{22}{6}$, which divided by $\frac{89}{4}$, quotes $\frac{88}{534}$ equal to 3s. 3d. $2\frac{18}{89}$ farth.

There is another rule which I think better for working questions of this nature, when the proportion is direct; and it is this;

When your terms are stated, and fractions prepared, multiply the Denominator of the first term into the Numerators of the 2d. and 3d. and place the product for a Numerator: then multiply the Numerator of the first term into the Denominators of the 2d. and 3d. and place the product for its Denominator, and this new found fraction is the 4th proportional or answer; which, if it be an improper fraction, must be reduced to a whole or mixed number, as formerly directed.

Take the stating of the preceeding question for an example.

$$\begin{array}{ccccccc} & lb & & l. & & lb. \\ \text{If} & \frac{89}{4} & : & \frac{22}{3} & :: & \frac{1}{2} \end{array}$$

For

For 4, the Denominator of the first number, multiply'd by 22 and 1, the Numerators of the 2d. and 3d. is 88.

And 89, the Numerator of the first term, multiply'd into 3 and 2, the Denominators of the 2d. and 3d. is 534, and these products placed fractionally make $\frac{88}{534}$ as before.

If the proportion shall happen to be inverse, you may either observe the directions given in whole numbers, or work by the following rule.

After the numbers are stated and prepared, multiply the Denominator of the 3d. term into the Numerators of the 1st. and 2d. for a Numerator; then multiply the Numerator of the 3d. term into the Denominators of the 1st. and 2d. for a Denominator; and the fraction so found, is the answer. See both ways in the following example.

How many yards of stuff $\frac{1}{2}$ yard wide are equal to $36\frac{1}{4}$ yards of $\frac{3}{4}$ yard wide?

The numbers prepared will stand thus:

If $\frac{3}{4}$: $\frac{145}{4}$ :: $\frac{1}{2}$

The first and second terms multiply'd together, produce $\frac{435}{16}$, which divided by $\frac{1}{2}$, quotes $\frac{870}{16}$ or $54\frac{3}{8}$ yards of stuff, the answer.

OR, the Denominator of the third number, multiply'd into 145 and 3, the Numerators of the first and second, is 870 for a Numerator. And 1, the Numerator of the 3d. term, multiply'd into 4 and 4, the Denominators of the first and second, is 16 for a Denominator: and these numbers placed fractionally, make $\frac{870}{16}$, as before.

A, keeps $100\frac{1}{6}$ l. of B's for $4\frac{1}{2}$ months; what sum must A, lend B, for $2\frac{1}{2}$ years to require him?

Ansr. l. $14\frac{2}{7}\frac{53}{40}$.

What

What will 82 c.w. of sugar come to, if $\frac{3}{4}$ of $\frac{4}{5}$ of an c.w. costs $\frac{9}{10}$ of a pound? Ansr. l. 262 : 8.

At $1\frac{1}{2}$ d. per ounce of snuff; what will $5\frac{1}{2}$ c.w. come to? Answer l. 61 : 12.

If $\frac{7}{8}$ of a guinea in $\frac{4}{5}$ of a month gain $3\frac{1}{2}$ s.; in what time will $\frac{5}{12}$ of a guinea gain as much?

Answer, in $1\frac{5}{7}\frac{3}{}$ year.

If $\frac{7}{11}$ of a grain Troy cost $\frac{2}{11}$ of a l.; what will $8\frac{5}{11}$ tun Adverdupoise cost? Ansr. l. 12400.

Note, 144 Adverdupoise is equal to 175 Troy; and therefore $\frac{7}{11}$ of a grain Troy is $\frac{3}{44}$ of $\frac{1}{7776}$ of $\frac{144}{175}$ of a tun Adverdupoise.

If $\frac{2}{7}$ of a yard cost $\frac{4}{9}$ of a guinea; what will $\frac{7}{11}$ of a Scots ell cost, when $\frac{2}{7}$ of a yard is $\frac{3}{7}$ of $\frac{4}{5}$ of a Scots ell? Ansr. l. 1 : 1 : 1 : $3\frac{3}{10}$.

If l. $44\frac{1}{2}$ of gold weigh 1 lb. Troy; what will the value of 12 Scots stones of the same metal amount to? Ansr. l. 11273 : 6 : 8.

Note, a Scots stone is $\frac{190}{9}$ or $21\frac{1}{9}$ lb. Troy.

If $\frac{1}{1}$: $\frac{89}{2}$ :: $\frac{190}{9}$ ($\frac{16910}{18}$, or $\frac{8455}{9}$ l. value of a Scots stone.

Then $\frac{8455}{9}$ multiply'd by 12, produces $\frac{101460}{9}$, which is equal to l. 11273 : 6 : 8, the value of 12 Scots stones.

If l. $3\frac{1}{10}$ of silver weigh 1 lb. Troy; what will the weight of 10 Scots stones be worth?

Ansr. l. 654 : 8 : 10 : $2\frac{2}{3}$.

If $\frac{7}{18}$ of a l. in $\frac{1}{6}$ of a year, gain $3\frac{2}{3}$ s.; what will $\frac{1}{3}$ of a guinea gain in that time? Ansr. 6s. $4\frac{1}{2}$ d.

If $1\frac{1}{7}$ lb. of gold is worth $61\frac{1}{7}$ l.; what is a grain worth at that rate? Ansr. $1\frac{1}{2}$ d.

If $\frac{3}{4}$ yard of silk is worth $\frac{3}{4}$ of $\frac{1}{6}$ l.; what's the price of $15\frac{2}{7}$ ells Flemish? Ansr. l. 9 : 12 : 6.

If $\frac{1}{2}$ of $\frac{1}{4}$ of a lb. of tea cost 6s. $2\frac{1}{7}$ d.; what cost the c.w. at that rate? Ansr. l. 69 : 6 : 8.

CHAP. XI.

Of Decimal Fractions.

SECTION I.

Of Notation.

WHEN, or by whom this excellent part of arithmetic was first introduced is uncertain; but it is very compendious, and therefore very useful, especially in calculations of interest, valuing annuities, &c. Some fractions of coin, weight, and measure, cannot be exactly expressed by decimal parts, but that the numbers will often circulate, and therefore it is not always convenient to work with them. Mr. Cunn in his treatise on fractions, hath very curiously shewn how they may be used with least loss; but since such exactness destroys the brevity, I cannot but think, in such cases, Vulgar Fractions preferable.

A Decimal Fraction is an artificial way of setting down and expressing Vulgar Fractions as whole numbers; and in them the integer or whole thing (whether it be time, coin, weight or measure, as one year, one guinea, one pound Ster. one pound weight, &c.) is supposed to be divided into ten equal parts; and every one of those ten parts are supposed to be subdivided into other ten equal parts, &c. *ad infinitum.*

Whereas the Denominators of Vulgar or natural fractions are divers, the Denominators of Decimal fractions are certain: for a Decimal Fraction hath always for its Denominator an unit with a cypher or cyphers annex'd to it, and must therefore be either 10, 100, 1000, 10000, &c. So there is no necessity of writing down the Denominator, when you set down a Decimal fraction; for by inspection it is certainly known, it consisting of an unit with as many cyphers annex'd to it as there are places of figures in the Numerator.

So

So this decimal fraction $\frac{25}{100}$ may be written thus .25 its denominator being known to be an unit with two cyphers, because there are two places or figures in the numerator.

In like manner $\frac{125}{1000}$ may be thus written .125; and $\frac{3575}{10000}$ thus .3575; and $\frac{75}{1000}$ thus .075; and $\frac{65}{10000}$ thus .0065.

But the different value of the several places will more plainly appear from the learned Mr. Oughtred's following table.

Whole numbers,						Decimal parts.					
5	4	3	2	1	0,	1	2	3	4	5	6
&c. Hundreds of thousands.	Tens of thousands.	Thousands.	Hundreds.	Tens.	Units place.	Parts of ten, or $\frac{1}{10}$.	Parts of a hundred.	Parts of a thousand.	Parts of 10 thousands.	Parts of 100 thousands.	Parts of a million, &c.

From whence it is evident.

1st. That as whole numbers increase by a tenfold proportion towards the left hand, from the place of units; so decimal parts decrease in the same proportion towards the right.

Therefore these decimal parts or fractions are really more homogeneal with whole numbers, than Vulgar Fractions; for all plain numbers are in effect but decimal parts one to another.

That is, suppose any series of equal numbers, as 666, &c. the first 6 towards the left hand is ten times the value of the 6 in the middle, and that 6 in the middle is ten times the value of the last 6 to the right of it, and but the tenth part of that 6 on the left, &c.

Therefore all or any of them may be taken either as integers, or parts of an integer: if integers, then they are set down without any separating point betwixt them, thus 666. But if integers, and one part or fraction, they will stand thus 66,6 which signifies 66 whole numbers, and 6 tenths of an unit: Or, if two places of parts be required, they will stand thus 6,66: For decimals are known and distinguished from integers by a point or comma on their left hand or before them, thus .00075 or thus ,00075; and 74.579 or thus 74,579.

From hence (duly compared with the table) it will be easy to conceive that decimal parts take their denomination from the place of their last figure.

That is $\left\{\begin{array}{l} ,5 \text{ is } \frac{5}{10} \\ ,56 \text{ is } \frac{56}{100} \\ ,056 \text{ is } \frac{56}{1000} \end{array}\right\}$ parts of an unit, &c.

2d. That cyphers put on the right hand of whole numbers do increase their value in a tenfold proportion; but being annexed to the right hand of a decimal fraction, do neither increase nor decrease the value thereof: so $\frac{35000}{100000}$ is equivalent to $\frac{35}{100}$ or ,35: and ,50 ,500 or ,5000 are each but 5 tenths of an unit.

3d. That on the contrary, tho' in whole numbers cyphers on their left hand neither increase nor diminish their value; yet cyphers on the left hand of a decimal fraction do diminish its value in a tenfold proportion: for ,25, if you put a cypher before it, becomes ,025 or $\frac{25}{1000}$: and ,125 by putting two cyphers before it, becomes $\frac{125}{100000}$ or ,00125: and ,0005 is 5 parts of ten thousand, &c. Consequently the true value of all decimal parts are known by their distance from the units place: and when you are to write down a decimal fraction whose Denominator hath more cyphers than there are figures in the Numerator, they must be supply'd by placing so many cyphers before the figures of your Numerator; so, if $\frac{19}{1000}$ was to be written down without its Denominator, you must put a cypher before 19, because there are three cyphers in the Denominator, and but two figures in the Numerator, and set it down thus ,019: and $\frac{7}{10000}$ will be set down thus ,0007 &c.

These things understood, the rest is easy: but I advise

vise the learner not to proceed before he understands them.

Place down decimally 729 million, and 209 ten thousand parts of an unit.

It makes ,000729; and ,0209.

Before the learner can Add, Subtract, Multiply or Divide Decimal fractions, he must be throughly acquainted with the three following cases.

CASE I. To change a Vulgar Fraction into a Decimal, and a Decimal into a Vulgar Fraction.

RULE, Place cyphers, at pleasure, on the right hand of the Numerator, then divide by the Denominator, and the Quotient is the decimal equivalent to the Vulgar Fraction given.

But note, that so many cyphers as you annex to the Numerator, and make use of in your division, so many places must be prick'd off in the decimal found; and if there are not so many figures or places in the Quotient, the deficiency must be supply'd by prefixing so many cyphers before the quotient figures as shall make up the number of places; as in some of the following examples.

What decimal Fraction is equivalent to $\frac{5}{8}$ of a l.?

8)5,000(,625 Decimal required.

0

Decimal fractions that come out without any remainder, are called finite. And you must remember that the order of places in decimals is from the left hand to the right; so in this decimal ,625, the figure 6 stands in the first place, and is 6 primes, or 6 tenths of an integer; and 2 the second figure, is 2 seconds, or two hundred parts of an integer, &c.

Bring $\frac{14}{15}$ to a decimal fraction.

15)14,0000(,9333 Decimal.

5

This decimal ,9333 is called an infinite repeating decimal, the remainder being still the same; and so is $\frac{1}{9}$ being ,1111, &c.

Let

Let $\frac{9}{28}$ be reduced to a decimal fraction.

28)9,00000000(,32142857 14 &c. Decimal.

4

This is called an infinite circulating decimal, for in your quote, you have a continual revolution of a certain number of figures always the same; in this example you have 142857, again 142857, and so on infinitely: and so is $\frac{3}{11}$ (equal to ,27,27,27) an infinite circulating decimal having a continued revolution of 27 *infinitum*.

What decimal fraction is equivalent to $\frac{54}{7648}$?

7648)54,00000(,00706 &c. Decimal.

512 Remainder.

Bring $\frac{4}{64074}$ to a decimal fraction.

64074)4,00000000(,00006242 &c. Decimal.

50092 Remainder.

Reduce $\frac{1}{4}$ of $\frac{7}{8}$ to a decimal fraction. $\frac{1}{4}$ of $\frac{7}{8}$ is $\frac{7}{32}$.

32)7,00000(,21875 Decimal.

0

Reduce $\frac{3}{4}$ to a decimal fraction.

4)3,00(,75 Decimal.

0

To reduce a decimal fraction to a vulgar one, find a common measure to the Numerator and Denominator as was taught in reduction of vulgar fractions, and you have done.

Reduce ,75 to a vulgar fraction.

The common measure will be found to be 25.

25)$\frac{75}{100}$($\frac{3}{4}$ Vulgar Fraction.

CASE II. To reduce any part of coin, measure, weight, &c. to a decimal.

RULE, To the number of parts of the lesser denominator nomination given, annex cyphers at pleaſure; divide by the number of ſuch parts as are contain'd in the greater denomination to which the decimal is to be brought, and the quotient is the decimal required.

Or (as was taught in that part of diviſion of whole numbers, called Reduction Aſcending) firſt bring the given part or parts to the next ſuperior denomination, and then that to the next higher, and ſo on 'till you have brought it to the greateſt name required: but remember always (as before directed) to ſupply the deficiency of the quotient, if there be not ſo many places of figures therein as the number of cyphers you made uſe of in the work of diviſion.

Examples.

Reduce $7\frac{1}{2}$ pence to the decimal of a pound.

240)7,50000(,03125 of a l. decimal.

0

20)

Or thus, 12)7,500(,62500(,03125 of a l. as before.

0 0

Reduce 17 s. 6 d. to the decimal of a guinea.

21)17,5000(,8333 &c. of a guinea.

7

What decimal part of $\frac{1}{2}$ crown is $7\frac{1}{4}$ pence?

30)7,25000(,24166 &c. of $\frac{1}{2}$ crown.

20

What decimal part of 6 ells is $7\frac{1}{2}$ quarters?

24)7,5000(,3125 of 6 ells.

0

Reduce 3 falls to the decimal of an acre.

160)3,00000(,01875 of an acre.

0

What

What decimal parts of a gallon are 7 gills?

16
8

128)7,00000(,05468 of a gallon.

96

Reduce $3\frac{1}{4}$ lippies to the decimal of a boll.
64)3,250000000(,05078125 of a boll.
0

What decimal part of a l. is 17 s.?
20)17,00(,85 of a l.
0

Let $3\frac{1}{4}$ inches be reduced to the decimal of a foot.
12)3,25000(,27083 of a foot.

4

Let 9 ounces Troy be reduced to the decimal of a lb.
12)9,00(,75 of a lb Troy.

0

Bring $5\frac{7}{8}$ inches to the decimal of a yard.
36)5,875000(,163194 of a yard.

16

If 7 minutes be brought to the decimal of a year, it will be ,00001331 &c. of a year.

And 11 inches brought to the decimal of an English mile, will be ,000173611 &c of a mile.

And 3 farthings brought to the decimal of a l. Scots, will be ,0375 of a l.

And 3 farthings brought to the decimal of a l. Ster. will be ,003125 of a l.

And $1\frac{1}{2}$ inch reduced to the decimal of a Scots ell, will be ,04032258 of a Scots ell.

Note, If the given parts are of several denominations, reduce them into their lowest name, and proceed according to the foregoing rule.

Examples.

Examples.

Reduce 14 s. 9¾d. to the decimal of a l.

14 : 9 : 3
12

177
4

960)711,0000(,7406 &c. of a l.

240

Reduce 3 qrs. 18 lb. to the decimal of an c.w.

3 : 18
28

112)102,0000(,9107 &c. of an c.w.

16

Reduce 1 foot 7 inches to the decimal of a Scots ell.

37,2)19,000000(,51075 of a Scots ell.

100

Why there are but five places in this quotient, will be accounted for in division of decimals.

Let 9 c.w. 1 qr. 16 lb. be reduced to the decimal of a tun.

9 : 1 : 16
4

37
28

112 302
20 75

2240) 1052,00000(,46964 &c. of a tun.

640

There is another way of reducing parts of different denominations into decimals of the highest integer; thus,

RULE, Bring the lowest parts into the decimals of the next superior denomination; and on the right hand of the decimal found place the given parts of that said next superior denomination; so proceeding till you bring out the decimal parts of the highest integer desired, by still dividing the product by so many of the given parts as make one of the next superior order; for examples, take the first two of that sort before given, viz.

Reduce 14s. 9d. 3qrs. to the decimal of a pound Sterling.

4	3,00
12	9,7500
20	14,812500
	,740625 of a l. as before.

Here the 3 farthings being divided by 4, gives ,75; to which the 9d. being prefixed, it becomes 9,75; which divided again by 12, quotes ,8125; before which the 14s. being also placed, it makes 14,8125; which being lastly divided by 20, the quotient ,740625 is the answer.

What decimal parts of an c.w. are equal to 3 qrs. 18 lb.?

28	7	18,000000
	4	2,571428
	4	3,642857
		,910714 of an c.w. as before.

Here, instead of dividing by 28, which would have been troublesome; I divided by 7 and 4 the component parts of 28.

Thus

Thus you see how easily the decimals answering to any given parts of coin, weight, measure, &c. are found; and after the same manner the learner may calculate tables of these for himself, which will be very useful.

But shillings, pence, and farthings, may more readily be reduc'd thus:

RULE, for the shillings, (if their number be even) set down their half in the first place of decimals, and let the second and third places be fill'd up with the farthings contain'd in the remaining pence and farthings, always remembering to add one, when the number of the said farthings is or exceeds 25; but if the number of shillings be odd, the second place of decimals must also be increased by 5.

Thus the decimal of 8 s. 5½ d. will be ,422, 4 in the first place of decimals standing for 8 s. and 22 farthings being 5½ d.

Again, 14 s. 6½ d. will be ,727; for 1 is added to 26, the number of farthings in 6½ d. because they exceed 25.

But lastly, 9 s. 6½ d. will be thus express'd by decimals, viz. ,477, for 5 is added to the second place, because the number of shillings is odd, the decimal of 1 shilling being ,05

The work will stand thus.	or thus.
2)9,0(,45 decimal of 9 s.	2)8(,4 decimal of 8 s.
—— 27 for 6½ d.	,05 decimal of 1 s.
9 ——	,27 for 6½ d.
,477 of a l.	——
	,477 of a l.

CASE III. To find the value of a decimal in the known parts of money, weight, measure, &c.

RULE, multiply the given decimal by the number of parts in the next inferior denomination, and from the product prick off so many places to the right hand as there were figures in the decimal given; multiply these figures prick'd off by the number of parts in the next inferior denomination, and prick off so

many

many places to the right hand as before: proceed thus till you have brought it into the least known parts of the integer, and the figures standing on the left hand of the separating points, will be the parts required.

Examples.

What shillings, pence and farthings are equal to ,7691 parts of a pound?

```
        ,7691
           20
     --------
  s. 15,3820
           12
     --------
  d.  4,5840
            4
     --------
qrs.  2,3360
```

Here you see that ,7691 of a l. is equal to 15 s. 4 d. 2 farthings and ,336 of a farthing.

What's the value of ,548 of a shilling Sterling?

```
          ,548
            12
         -----
pence    6,576
             4
         -----
farthings 2,304
```

So that the value of ,548 of a shilling is 6 pence 2 farthings ,304 thousand parts of a farthing.

But the decimal parts of a pound may be thus valued at sight.

RULE, The figure standing in the first place of decimals doubled, gives shillings; but if the figure in the second place is or exceeds 5, one more must be added to the number of the said shillings; the second figure (if under 5) or its excess (if above 5) joined with the third, are so many farthings; only remember to abate 1, if their number amounts to 25; or 2 if near 50.

The

The reason of this abatement is, that as 1000 is the denominator of every decimal consisting of three places so consequently by reckoning the figure standing in the said third place as so many farthings, we thereby allow 1000 farthings to the pound; whereas indeed there are but 960: the overplus therefore being 40 in 1000, is certainly 4 in 100, 2 in 50, and 1 in 25, according to the above direction.

Examples.

What's the value of ,375 of a l. Ster.?

Answer 7 s. 6 d.

The 3 doubled is 6 shillings; to which 1 s. being added for the 5 in the second place, makes it 7 shillings; and 2 remaining join'd with the 5 in the third place being accounted 25 farthings, from which 1 being deducted, there remain 24 farthings, or 6 pence.

What's the value of ,846 of a pound?

Answer 16 s. 11 d.

The 8 doubled, makes 16 s. and 2 taken from the 46 farthings, leaves 44 farthings, or 11 pence.

	l.	s	d
So the value of ,725 is	14	:	6
and the value of ,878 is	17	:	6 $\frac{3}{4}$
and the value of ,417 is	8	:	4

What's the value of ,74 parts of a guinea?

Anſr. 15 s. 6,48 pence.

$$\begin{array}{r} ,74 \\ 21 \\ \hline 74 \\ 148 \\ \hline \text{s. } 15,54 \\ 12 \\ \hline \text{d. } 6,48 \end{array}$$

What's the value of ,0674 parts of a crown?

$$\begin{array}{r} ,0674 \\ 60 \\ \hline \text{pence } 4,0440 \end{array}$$

What's the value of ,825 parts of a merk?

$$\begin{array}{r} ,825 \\ 160 \\ \hline \text{pence } 132,000 \end{array}$$

What's

What's the value of ,43569 parts of an c.w.?

```
         ,43569
              4
     ----------
qr.  1,74276
            28
     ----------
        594208
       148552
     ----------
lb.  20,79728
```

What's the value of ,95 parts of a yard?

```
          ,95
           36
        -----
          570
         285
        -----
inches  34,20
```

What's the value of ,375 parts of an acre?

```
         ,375
          160
        ------
falls  60,000
```

What's the value of ,0625 parts of a boll?

```
         ,0625
            16
        -------
peck   1,0000
```

What's the value of ,58 parts of a lb. Troy.

```
           ,58
           112
        -------
ounces    6,96
            20
        -------
p.wts.   19,20
            24
        -------
gr.       4,80
```

What's the value of ,125 parts of a year?

```
          ,125
           365
        -------
          1825
         4380
        -------
days    45,625
            24
        -------
        2500
       1250
        -------
hours  15,000
```

Thus are shown the ways both of finding the decimal answerable to any given parts of coin, weight, measure, &c. or the value of any given decimal in the known parts of its integer.

SECTION II.

SECTION II.

Addition of Decimals.

ADDITION of Decimals is performed after the same manner as addition of whole numbers, only as in whole numbers you place units under units, tens under tens, &c. So in decimals you must take care to place primes under primes, seconds under seconds, &c. and when you have added them as if they were all whole numbers, separate so many decimal parts in the total as the greatest given decimal hath places.

Examples.

How must 6573865 be pointed, and what decimal added will make the sum just 7 integers?

6,573865 thus pointed.
,426135 decimal to be added.

integers 7,000000

What's the sum of .473, .3506, .30751, .29, and .1806 of a pound, and what's the value of the total?

They will stand thus:

,473
,3506
,30751
,29
,1806

l. 1,60171 total sum.
20

s. 12,03420

So the value of the total is l. 1 : 12 .0342.

Add

Add 5,42, and 38,583, and 421,06, and ,8345 together.

5,42	,9999	,00745
38,583	,8742	,00064
421,06	,087	,0075
,8345	,54	,009
	,99	,05
Sum 465,8975	Sum 3,4911	Sum ,07459

	l.	*s.*	*d.*
Borrowed	58 :	18 :	8,25
and	127 :	17 :	7,75
and	9 :	9 :	3,125
and	4 :	14 :	2,875
and	7 :	5 :	5,5
Sum l.	208 :	5 :	3,5

SECTION III.

Subtraction of Decimals.

HAVING plac'd the numbers as directed in Addition, work as if they were all whole numbers. But if the number of places in the decimal to be subtracted be more than in that which you are to subtract from, you must suppose cyphers to make up the number of places.

Examples.

from ,7321	from ,82745	from ,7234
take ,3452	take ,00789	take ,000567
remains ,3869	remains ,81956	remains ,722833
	l.	*ells.*
from ,62	borrowed 734,812	bought 58,003
take ,230734	paid 97,97	sold 37,374
remains ,389266	unpaid 636,842	unsold 20,629

I borrowed l. 74 : 17, and l. 5 : 18, and l. 397 : 19, and l. 49 : 11, whereof I have paid l. 179 : 7 : 6; what remains unpaid?

	l.
	74,85
	5,9
	397,95
	49,55
sum borrowed	528,25
paid	179,375
remains unpaid l.	348,875

SECTION IV.

Multiplication of Decimals.

HERE the numbers are to be set down (without any regard to the value of their places) and wrought in all respects as whole numbers; only observe to separate so many places of decimal parts on the right hand of the product, as were in both Multiplicand and Multiplier; but if it consists not of so many places, then you must supply that defect by placing cyphers to the left hand of the product.

Multiplication admits of 4 cases.

CASE I. To multiply a decimal by a decimal. The general rule serves for all the 4 cases.

Examples.

Multiply	,75	Multiply	,685
by	,25	by	,125
	375		3425
	150		8220
product	,1875	product	,085625

Multiply

Multiply	,0000746	Multiply	,8672
by	,58	by	,0054
	5968		34688
	3730		43360
product	,000043268	product	,00468288

CASE II. To multiply a decimal by a whole number, or a whole number by a decimal.

A piece of ground being 25 falls long, and ,85 parts of a fall broad; what falls doth it contain?

```
 25
,85
----
125
200
----
21,25 equal to 21 falls 9 ells.
```

The length of a piece of timber is 9 feet, and the breadth ,08 of a foot; what part of a superficial foot will it contain?

```
,08
  9
----
,72 of a foot, equal to 103,68 inches.
```

Note, if any whole, or mixed number be multiply'd with a fraction either vulgar or decimal, the product will be less than the multiplicand, in such proportion, as the multiplying fraction is less than an unit or 1.

That is, as the denominator of the fraction is to its numerator; so will the given number be to the product.

Therefore, whenever any number is to be multiply'd with a fraction, whose numerator is an unit, divide that number by the denominator of the fraction, and the quotient will be the answer required. Thus 16 multiply'd by $\frac{1}{4}$ or ,25 produces 4, and divided by $\frac{1}{4}$ or ,25 quotes 4 also. And 16 multiply'd by $\frac{1}{2}$ or ,5 produces 8, and divided by $\frac{1}{2}$ or ,5 quotes 8 also.

CASE III.

CASE III. To multiply a decimal by a mixed number, or a mixed number by a decimal.

Multiply ,0764	Multiply 6,74
by 12,5	by ,86
3820	4044
9168	5392
product ,95500	product 5,7964

Multiply 45,678	Multiply 8,87
by ,0007	by ,99
product ,0319746	7983
	7983
	product 8,7813

CASE IV. To multiply a mixed number by a mixed number, and a mixed number by a whole number, *et e contra.*

Multiply 74,0064	Multiply 748
by 725	by 4,7
3700320	5236
1480128	2992
5180448	product 3515,6
product 53654,6400	

How much linen may I give for 42 yards 3 quarters 2 nails of cambric, when 1 yard of the cambric is equivalent to 4 yards 2 quarters 2 nails of the linen?

3 qrs. 2 nails will be ,875, and 2 qrs. 2 nails will be ,625 parts of a yard.

```
   42,875
    4,625
   ------
   214375
    85750
  257250
 171500                yds.   q.   n.
Answer 198,296875 equal to 198 : 1 : 0,75
```

What's

What's the product when 2 s. 6 d. is multiply'd by 2 s. 6 d., and a pound the integer?

2 s. 6 d. is $\frac{1}{8}$ or ,125 parts of a pound.

$$\begin{array}{r} ,125 \\ ,125 \\ \hline 625 \\ 1500 \\ \hline \end{array}$$

product ,015625 parts of a l. equal to 3$\frac{3}{4}$ pence.

Here you see that 2 s. 6 d. multiply'd on itself produces only 3$\frac{3}{4}$ pence, for fractions multiply'd together become less in the same proportion as integers when multiply'd together become greater.

And questions of this nature admit of a great many answers according to what you make the integer.

What's the product when 2s. 6 d. is multiply'd by 2 s. 6 d. and a shilling the integer?

$$\begin{array}{r} 2,5 \\ 2,5 \\ \hline 125 \\ 50 \\ \hline \end{array}$$

product 6,25 equal to 6 s. 3 d.

Thus you see that the product alters in value, according as you alter the integer.

What's the product when 2 s. 6 d. is multiply'd on itself, and a crown the integer?

2 s. 6 d. is $\frac{1}{2}$ or ,5 tenths of a crown.

$$\begin{array}{r} ,5 \\ ,5 \\ \hline \end{array}$$

product ,25 parts of a crown, equal to 15 pence.

What's

What's the product, when 2s. 6d. is multiply'd on itself, a guinea the integer?

21)2,500(,119 of a guinea.

```
           ,119
  1    ------
          1071
         1309
       ------          d.  f.
product  ,014161 equal to 3 : 2,27 &c.
             21
        ------
          14161
         28322
        ------
   s.  ,297381
            12
        ------
   d. 3,568572
             4
        ------
   f. 2,274288
```

Note, To Multiply any decimal number by 10, 100, 1000, &c. is only to remove the mark of separation so many places towards the right hand as there are cyphers.

Thus, 8,35645 multiply'd by $\left\{\begin{array}{l}10\\100\\1000\end{array}\right\}$ is $\left\{\begin{array}{l}83,5645\\835,645\\8356,45\end{array}\right.$

SECTION V.

Division of Decimals.

DIVISION of Decimals is perform'd after the same manner as division of whole numbers, only the difficulty is, to value the quotient; which you may do by carefully observing the following rule.

RULE,

RULE, When the work of division is ended, count how many places of decimal parts there are in the dividend more than in the divisor, for that excess is the number of places which must be separated in the quotient for decimals: but if there be not so many figures in the quotient as in the said excess, that deficiency must be supply'd with cyphers put before the significant figures of the quotient, towards the left hand thereof, with a point before them; so shall you plainly discover the value of the quotient.

If the divisor and dividend have both the same number of decimal parts, the quotient will be a whole number.

If the divisor consists of more places of decimals than the dividend, there must be a competent number of cyphers annex'd to the dividend to make it consist of as many (so then the quotient, as before, will be a whole number) or more places of decimals than the divisor; for the cyphers added must be reckoned as decimals.

In dividing of whole or mix'd numbers, if there be a remainder, you may bring down more cyphers, and by continuing your division, carry the quotient to as many places of decimals as you please.

Division of decimals being perform'd the same way as in whole numbers, I shall leave the work of the following examples to be done by the learner, and only set down the quotients and remainders, which I think rather an advantage than a loss to the practitioner.

Division of decimals admits of 9 cases, and the foregoing rule serves for them all.

CASE I. To divide a whole number by a whole number, when the dividend is either greater or less than the divisor.

Let 12 l. be equally divided among 48 men.

48)12,00(,25 equal to 5s. for each.
        20
0   s. 5,00

Let

Let 140 l. be equally divided among 25 men.

25)140,0(5,6 equal to l. 5 : 12 for each.

20

0

s. 12,0

CASE II. To divide a whole number by a decimal.

,283)154605,0000000(546307,4264 Quotient.

268

Here, you fee there are four places of decimals in the dividend more than in the divifor, therefore that excefs is feparated for decimals in the quotient.

,000025)45655,000000(1826200000 Quotient.

0

Here, the number of decimal places in the divifor and dividend are equal, fo the quotient muft be all a whole number.

Remark, If any number be divided by a fraction, the quotient will be greater than the dividend by fuch a proportion as unity is greater than the dividing fraction. Thus 12 divided by $\frac{1}{4}$ or ,25, quotes 48, viz. if $\frac{1}{4}$: 1 :: 12 : 48, &c.

CASE III. To divide a decimal by a whole number.

283),1546050000000(,0005463074204 Quotient.

268

25),456550(,018262 Quotient.

0

CASE IV. To divide a mix'd number by a decimal.

,283)1546,050000000(5463,074204 Quotient.

268

,25)4565,50(18262 Quotient.

0

CASE V.

CASE V. To divide a decimal by a mixed number.

28,3),1546050000000(,005463074204 Quotient,

268

2,5),456550(,18262 Quotient.

0

CASE VI. To divide a whole number by a mixed number.

28,3)154605,0000000(5463,074204 Quotient.

268

2,5)45655,0(18262 Quotient.

0

CASE VII. To divide a mixed number by a whole number.

283)1546,050000000(5,463074204 Quotient.

268

25)456,550(18,262 Quotient.

0

CASE VIII. To divide a mixed number by a mixed number.

28,3)1546,050000000(54,63074204 Quotient.

268

2,5)456,550(182,62 Quotient.

0.

CASE IX. To divide a decimal by a decimal.

,283),1546050000000(,5463074204 Quotient.

268

,25),456550(1,8262 Quotient.

0

I shall adjoin some more examples on these cases,

and

and conclude this section with a way of contracting the work of a large multiplication, and division: and then proceed to reduction by such numbers.

,325),400000000(1,230769 Quotient.

75

,042)495,000000(11785,714 Quotient.

12

423),08516438(,00020133 Quotient.

179

6,321)85643,825(13549 Quotient.

596

,081),0000000077200(,00000953 Quotient.

17

209),000030000(,000000143 Quotient.

113

Because in multiplication of decimal parts and mixed numbers, there is no need to express all the figures of the product, but in most cases two, three, or four places will be sufficient; therefore, to contract the work observe this following rule.

RULE, place one of the numbers just as it is given, for a multiplicand; and under that decimal place which you would have to be the last in the product, set the unit's place of your other number; then reversing all the other figures, multiply with them in their order, beginning always with that figure of the multiplicand which stands over the figure you work with, and set down the first figure of each particular product directly one under the other; but a due regard must be had to the increase arising from the figures on the right hand of that number in the multiplicand which you begin with. This will appear more plain in the following examples.

Note, If there be no unit's place in your multiplier, you may supply its place with a cypher.

Note also, The usual allowance for the omitted figures is as follows, viz. If the next figure on the right hand of that you begin with in the multiplicand, multiply'd into the figure of the multiplier you are working with, gives a product betwixt 5 and 15, carry 1; if the product be above 15, and less than 25, carry 2; and if it arise to any number betwixt 25 and 35, carry 3; and betwixt 35 and 45, carry 4; &c.

Multiply 2,735641 by ,54382, and produce only four places of decimals.

```
     2,735641
    28345,0   Multiplier reversed.
    ---------
    13678
     1094
       82
       22
    ------
product 1,4876
```

To prove the certainty of the foregoing rule, let the same example be wrought the common way.

```
       2,735641
         ,54382
    -----------
        |5471282
      21|885128
      82|06923
    1094|2564
   13678|205
   ------------
   1,4876|9628862 product, as before
```

Let

Let 375,13758 be multiply'd by 16,7324, so that the product may have three places of decimals,

```
           375,13758
            4237,61   Multiplier revers'd.
          ----------
            3751376
            2250825
             262596
              11254
                750
                150
     ---------------
product 6276,951
```

Let the same example be repeated, and let the product contain but one place of decimals.

```
          375,13758
         4237,61    Multiplier inverted.
        -----------
           37514
           22508
            2626
             113
               7
               1
      ----------
product 6276,9
```

In division of decimals the common way, when the divisor hath many figures, and it is required to continue the division till the value of the remainder be but small, the operation will sometimes be large and tedious, but may be contracted by the following method.

RULE, First, you must observe that the first figure of the quotient is always to stand in the same place with that figure of the dividend, which answers, or stands over the place of units in the divisor.

Having then considered in what place the said first figure of the quotient ought to stand, and so found its value or denomination, you may easily determine how many

many figures the quotient ſhould neceſſarily conſiſt of, and from thence judge whether any, or how many of the right hand figures of your diviſor may be neglected; ſo that reſerving only ſo many figures of the dividend as are neceſſary for once anſwering the diviſor, cut off the reſt to the right hand as uſeleſs; and having ſet the proper figure in the firſt place of the Quotient, work with it as uſual; then omitting the right hand figure of your diviſor, ſeek how often the other figures thereof are contain'd in the remainder; which figure being enter'd in the quotient and proceeded with as uſual in diviſion, (with regard always to the carriage that muſt be brought from the omitted figures, as, before directed in the contracted work of multiplication) thus continuing to neglect the right hand figure of your diviſor every time you ſeek a new quotient figure, you will ſtill be able to divide the remainder left after the laſt ſubtraction by the diviſor ſo leſſened, till you have brought out the determin'd number of figures in the quotient. The following example, which is done both at length, and the contracted way, will make all clear.

```
2,25743)721,17562(319,467
        677229
      ----------
         43946|6
         22574|3
        --------
         21372|32
         20316|87
        ---------
          1055|450
           902|972
         --------
           152|4780
           135|4458
         ---------
            17|03220
            15|80201
          ---------
             1|23019
```

```
2,25743)721,175|62(319,467
 .....  677 229
        -------
         43946
         22574
         -----
         21372
         20317
         -----
          1055
           903
          ----
           152
           135
           ---
            17
            15
            --
             2
```

And thus are all the figures on the right hand of the perpendicular line in the example wrought at length, sav'd by working after this contracted manner.

Altho' I have given directions for proportioning the divisor and dividend, so as to bring into the quotient what number of decimals you please, yet there is no absolute necessity for it; but you may carry on your division to what degree you please, before you begin to prick off the figures of the divisor, in order to contract the work, as in the following example, where it is not required to prick off any determinate number of decimals, but it may be done at pleasure.

12,34254

```
12,34254) 514,75498 (41,705357
  ......  493 7016
          --------
           2105338
           1234254
           -------
            871084
            863978
            ------
              7106
              6171
              ----
               935
               864
               ---
                71
                62
                --
                 9
                 8
                 -
                 1
```

SECTION VI.

Reduction by Decimals.

Examples.

In 540 guineas, what l. Ster. when 1 guinea is 1,05 l. Ster.

```
          1,05
           540
          ----
           420
          525
          ------
l. Ster. 567,00
```

In

In 567 l. Ster. what guineas ?

1,05)567,00(540 guineas

0

In 580 guineas what l. Scots, when 1 guinea is 12,6 l. Scots ?

```
        12,6
         580
       -----
        1008
        630
       -----
l. Scots 7308,0
```

In 7308 l. Scots what guineas ?

12,6)7308,0 (580 guineas.

0

In 67 crowns what merks Scots, when 1 crown is equal to 4,5 merks ?

```
   67
   4,5
  -----
   335
  268
  -----
  301,5 merks Scots.
```

In 301,5 merks Scots what crowns ?

4,5)301,5(67 crowns.

0

In 54 guineas what merks Scots, when 1 guinea is equal to 18,9 merks ? Ansr. 1020,6 merks Scots.

In 1020,6 merks Scots, what guineas ? Ansr. 54.

In 140 guineas what French pistoles, when 1 guinea is equal to 1,2 pistole ? Ansr. 168,0.

In 168 French pistoles what guineas ? Ansr. 140.

In 48 English ells what English yards, when 1 ell English is equal to 1,25 yard ? Ansr. 60,00

In 60 English yards what such ells ? Ansr. 48.

In

In 54 English yards what Dutch ells, when 1 yard English is equal to 1,33333 Dutch ell?

1,33333
54

533332
666665

71,99982 Dutch ells.

In 72 Dutch ells what English yards?

1,333) 72,000 (54 English yards.

18.

In 720 l. Ster. what French crowns, when 1 l. Ster. is equal to 4,4444 crowns?

Anfr. 3199,9680 French crowns.

In 3199,9680 French crowns what l. Ster.?

Anfr. 720 l. Ster.

In 180 guineas what French crowns, when 1 guinea is equal to 4,66666 crowns?

4,66666
180

839,99880 crowns.

In 839,9988 French crowns what guineas?

4,66666) 839,99880 (180 guineas.

0

What part of a mark is 12s.?

12
12

160) 144,0 (,9 tenths of a mark.

0

SECTION VII.

SECTION VII.

Of the Rule of Three in Decimals.

THE Rule of proportion having been already sufficiently explained, both as to its nature and operation in vulgar arithmetic, 'tis needless to repeat here what has been there said. Only observe, that instead of preparing your numbers, by reducing them into their lowest denomination, you must now bring their fractional parts into decimals. See the following examples.

If I pay l. 3 : 16 for 18½ yards of cloth, what will 32¾ yards of the same cloth amount to?

The fractional parts being reduced to decimals, and the question stated as usual, the work will stand as follows.

```
yds.    l.     yds.
18,5 : 3,8 :: 32,75
                3,8
              -----
              26200
              9825
              -----
     18,5)124,4500(6,727 l. price
              5
```

l. 6,727027 is equal to 6 : 14 : 6 : 1,945 &c. the price of 32¾ yards.

What must the penny loaf weigh, when wheat is at 10s. the boll, if, when at 6 s. 8 d. per boll, the said loaf weighs 5½ ounces?

```
      l.       oz.    l.
If ,333333 : 5,5 :: ,5
               5,5
        -------
        1666665
       1666665
       --------                       oz. p.w. gr.
,5)1,8333315(3,666663 equal to 3 : 13 : 8 fere.
   ---------
      0
```


If

If the carriage of ½ c.w. 40 miles comes to 6 d. what will be the carriage of carrying 16½ c.w. 100 miles?

c.w. l. c.w.
If ,5 : ,025 :: 16,25
,025

8125
3250

,5),40625(,8125

o

m. l. m.
Then, if 40 : ,8125 :: 100
100

40)81,25000(2,03125 equal to l. 2 : 0 : 7½.

o

If the beam of a ballance be 81 inches long, and cheats at the rate of l. 7,5 per cent; how much is the one end shorter than the other?

107,5
100

207,5 : 81 :: 100
100

207,5)8100,00(39,0361445 inches for the shortest end.

1625

207,5 : 81 :: 107,5
81

1075
8600

207,5)8707,5(41,9638554 longest end.
39,0361445 shortest.

450
Inches 2,9277109 answer.

I

I shall here state down some characters with their signification; as also some short decimal tables; and conclude this rule with practical examples on all its different sections, leaving the work of most of them for the exercise and improvement of the learner.

This mark + signifies addition, as 8+5, or 7+3+4 denots that 8 is to be added to 5, and that 7, 3, and 4 are to be added into one sum. This mark — signifies subtraction, as 9—4 signifies that 4 is to be taken from 9. This mark x signifies multiplication, as 7x9 signifies that 7 is to be multiplied into 9. This mark ÷ signifies division, as 18 ÷ 6 signifies that 18 is to be divided by 6. This mark = signifies equality; that is, when placed between numbers, it denotes them to be equal, as 7+5=12, that is 7 added to 5 is equal to 12; and 15—7=8, that is, 7 subtracted from 15, gives a remainder equal to 8. This mark :: is the sign of proportion, or the Rule of Three, it being always placed betwixt the two middle terms in proportion, thus, 4 : 20 :: 6 : 30, to be thus read, as 4 is to 20, so is 6 to 30.

DECIMAL TABLES.

Coin.	Averdupoise.	Troy weight.
1*l.Str.the integer.*	112 *lb. the integer.*	1 *lb. the integer.*
,05=1 *s.*	,25=1 *qr.*	,08333=1 *oz.*
,004166=1 *d.*	,008928=1 *lb.*	,004166=1*p.w.*
,0010416=1 *qr.*	,000558=1 *oz.*	,000173=1 *gr.*
	,000034=1 *dram.*	

Time.	Time.
1 *year the integer.*	1 *day the integer.*
,002739=1 *day.*	,041666=1 *hour.*
,000114=1 *hour.*	,000694=1 *minute.*
,0000019=1 *minute.*	,00001157=1 *second.*

Cloth

CLOTH measure.	LIQUID measure.
1 *yard the integer.*	1 *gallon the integer.*
,25=1 *qr.*	,125=1 *pint.*
,0625=1 *nail.*	,0625=1 *chop.*
	,0078125=1 *gill.*

CORN measure.	LAND measure.
1 *boll the integer.*	1 *acre the integer.*
,25=1 *firlot.*	,25=1 *rood.*
,0625=1 *peck.*	,00625=1 *fall.*
,015625=1 *lippie.*	,0001736=1 *ell.*

The use of these tables will be very evident by the following example.

Let it be required to find the decimal parts equivalent to 17 s. 9 d. 2 farthings.

,05=1 s. Therefore, 17x,05=,85=17s.
& ,004166=1d. Therefore ,004166x9=,037494=9d.
& ,0010416=1f. therefore ,0010416x2=,002083=2f.

Consequently their sum is ,889577=17s 9½ d.

What's the sum when ½ of ¼ of ⅕ of a shilling is added to ⅞ of a guinea, a guinea the integer?

½ of ¼ of ⅕ is $\frac{1}{40}$ which reduced to a decimal is ,025 and ,025 of a s. divided by 21, quotes ,00119 of a guinea and ⅞ of a guinea is ,875.

,875
,00119

Sum ,87619=18s. 4d.

What's the sum of ¾ of 40s. and ⅗ of 30s.

¾ is ,75 and ⅗ is ,6

,75	,6
40	30
30,00	18,0
	30
	Sum 48 shillings

What's

What's the sum of $\frac{7}{8}$ of a crown and $\frac{4}{5}$ of a l. Ster. a l. the integer?

$\frac{7}{8}$ of a crown is ,875 which divided by 4 quotes ,21875 of a l. and $\frac{4}{5}$ of a l. is ,8.

,8
,21875

Sum l. 1,01875=l. 1 : 0 : $4\frac{1}{2}$.

What's the sum of $\frac{7}{8}$ of a crown and $\frac{4}{5}$ of a l. Ster. a crown the integer?

Answer, 4,075=4 crowns $4\frac{1}{2}$ pence.

What's the sum of $\frac{2}{8}$ of a fall and $\frac{1}{5}$ of an acre, an acre the integer? Ansr. ,2046875=32 falls 27 ells.

Reduce $7\frac{1}{2}$ pence to the decimal of a shilling, a crown, a l. Ster., a guinea, an half crown, a merk Ster. and a merk Scots.

Divide (as before directed) the given parts by a number expressing how many such parts make the integer of the decimal fraction required.

What's the sum of $\frac{1}{2}$ l. $\frac{3}{4}$ of a s. and $\frac{5}{8}$ of a penny, a l. the integer? Ansr. ,5401=10s. 9,624 d.

What's the sum of $\frac{4}{7}$ of a lb. and $\frac{1}{5}$ of an c.w. an c.w. the integer? Ansr. ,207133=23,2 lb.

What's the sum of $\frac{5}{4}$ of a peck and $\frac{3}{5}$ of a boll, a boll the integer? Ansr. ,615625 of a boll equal to 9 pecks 3,4 lippies.

What's the difference betwixt $\frac{1}{4}$ of a guinea and $\frac{3}{4}$ of a crown?

$\frac{3}{4}$ is ,75	and $\frac{1}{4}$ is ,25
5	21
3,75	25
	50
	5,25
	3,75

Difference 1,50=1 s. 6 d.

What's

What's the difference betwixt $\frac{3}{5}$ of a penny and l. $5\frac{1}{2}$?

$\frac{3}{5}$=,6 and $5\frac{1}{2}$=5,5

240

220

110

1320,0 pence.

,6

Difference 1319,4=l. 5 : 9 : 11,4.

What's the difference betwixt ,0974 of a yard and ,374 of a nail? Anſr. 1,1844 nail.

What's the difference betwixt ,0312 of an ounce Troy and ,0625 of a lb. Troy, an ounce the integer?

Anſr. ,7188 of a ounce=14 p.w. 9,024 gr.

What's the difference betwixt $\frac{3}{4}$ of a fall and $\frac{4}{5}$ of an acre? Anſr. $127\frac{1}{4}$ falls.

What's the difference betwixt $\frac{7}{8}$ of a peck and $\frac{5}{8}$ of a boll? Anſwer $9\frac{1}{8}$ pecks.

What's the difference betwixt $\frac{3}{8}$ of a pint and $\frac{3}{4}$ of a gill?

$\frac{3}{4}$ of a gill is ,75 and $\frac{3}{8}$ of a pint is ,375.

,375

16 gills in 1 pint.

6,000

,75

Difference 5,25 gills.

What's the difference betwixt $\frac{7}{8}$ of 2 ells and $\frac{1}{5}$ of a fall running meaſure?

$\frac{7}{8}$=,875 and $\frac{1}{5}$=,2

2 6

ell 1,750 1,2 ell.

1,2

ell ,55 difference = 2,2 qrs.

What

What known part of 50 shillings is $\frac{3}{5}$ thereof?

```
        50
        ,6
   s. 30,0
```

What's the product when l. 5 : 12 : 9 is multiply'd by l. 3 : 6 : 1?

```
  s.  d.                          s.  d.
  12 : 9                          6 : 1
  12                              12
  ----                            ----
240)153,0000(,6375 of a l.   240)73,0(,304166 of a l.
    --------                     ------
    0                            160

         3,304166
           5,6375
         --------
         16520830
        23129162
       9912498
      19824996
     16520830
     ----------
product l. 18,6272358250=l. 18 : 12 : 6 : 2,14639.
```

What's the product when 5 c.w. 3 qrs. 7 lb. is multiply'd by 3 qrs. 14 lb. and a tun weight the integer.

5 c.w. 3 qrs. 7 lb.=,29 of a tun, and 3 qrs. 14 lb. =,0437 of a tun, and their product is ,012673 of a tun=lb. 28,38752.

What's the product when $\frac{3}{4}$ of a crown is multiply'd by $\frac{7}{8}$ of a guinea, a guinea the integer?

$\frac{3}{4}$ of a crown is ,1785 of a guinea, which multiply'd by $\frac{7}{8}$ or ,875 of a guinea produces ,1561875 of a guinea = 3,2799 &c. shillings.

What's the product when $\frac{7}{8}$ of a guinea is multiply'd by $\frac{3}{4}$ of a crown, a crown the integer?

Ansr. 2 crowns, 3,78 shillings.

What decimal part of a guinea is $\frac{2}{5}$ of $\frac{1}{4}$ of $\frac{7}{8}$ of a crown?

The

The compound fraction reduced, is $\frac{7}{160}$ = ,04375 of a crown, and this multiply'd by 5, and divided by 21, quotes ,0104119 of a guinea, the anſwer.

How oft is $\frac{3}{5}$ of 30 guineas contain'd in $\frac{7}{8}$ of 3 crowns? $\frac{7}{8}$ of 3 crowns is ,625 of a guinea, by which divide 30 guineas; and the quotient is 48 for the anſwer.

How oft is $\frac{1}{2}$ of a nail contain'd in $\frac{3}{5}$ of a yard? Anſr. 19,2 times.

What's the ſum of $\frac{1}{8}$ of an ell and $\frac{3}{5}$ of half a quarter?

8)
5)3,0(,600(,075 of an ell.
,125 = $\frac{1}{8}$ of an ell.

Sum ,2 tenths of an ell.

What's the product when $\frac{3}{4}$ of a penny Ster. is multiply'd by $\frac{5}{8}$ of a l. Scots, a crown the integer?

$\frac{3}{4}$ of a penny will be found to be ,0125 of a crown, and $\frac{5}{8}$ of a l. Scots will be ,2083 of a crown; and theſe multiply'd together produce ,00260375 of a crown, the anſwer.

What's the ſum of $\frac{1}{4}$ of a merk Scots, $\frac{1}{4}$ of a guinea, $\frac{1}{12}$ of a French piſtole, and $\frac{3}{8}$ of a French crown, a guinea the integer?

,0132275 = $\frac{1}{4}$ merk Scots
,25 = $\frac{1}{4}$ guinea.
,1041666 = $\frac{1}{12}$ piſtole.
,0803571 = $\frac{3}{8}$ crown.

Sum ,4477512 of a guinea = s. 9 : 4 : 3,339.

What decimal part of a merk Scots is $\frac{3}{4}$ of a farthing? Anſr. ,0140625 of a merk Scots.

What decimal part of $\frac{1}{2}$ guinea is $\frac{3}{5}$ of a French crown.

$\frac{3}{5}$ of a French crown is 32,4 d. which divided by 126 the pence in $\frac{1}{2}$ guinea, quotes ,25714285 of $\frac{1}{2}$ guinea, the anſwer.

What decimal part of 3 bolls is $\frac{3}{4}$ of 3 lippies? Anſr. ,008789

What

What decimal part of 9 English miles is $\frac{3}{4}$ of $\frac{5}{8}$ of 11 inches? Ansr. ,0000090422245 of 9 miles.

What number is that whereby if you multiply 188, the product will be equal to the quotient when you divide 188 by 4?

```
   188          4)1,00(,25 number required.
   ,25            ----
  -----             0
   940                       4)188(47 Quotient.
  376                          ----
  -----                          0
  47,00 product.
```

Whether do you account $2\frac{1}{2}$ d. on the shilling or $2\frac{1}{4}$ d. on the Scots merk the greatest gain, when a shilling is to a merk as 9 to 10?

```
9 : 2,5 :: 10
     10
    ----  d.  f.
 9)25,0( 2 : 3 1/9 gain'd by the merk, at the rate of
  ----             2 1/2 d. on the shilling, so that the
    7              greater gain is on the shilling.
    4
  ----
 9)28(3
  ----
    1
```

How much muir ground may I give in exchange for 14 acres 1 rood 10 falls of rich clay ground, when 1 acre of the clay is worth 5 acres 3 roods 20 falls of the muir?

The roods and falls on both sides reduced to decimals of an acre, the numbers to be multiply'd will stand thus,

```
                a.
             14,3125
              5,875
        ----------------  a.  f.
their product is 84,0859375 = 84 : 13 3/4 of muir, ansr.
```

How much linen may I give for 20 yards 1 qr. $3\frac{1}{7}$ nails of cambric, when 1 yard of the cambric is worth 5 yards 3 qrs. 1 nail of the linen?

Ansr. 78 yds. ,59 of a nail of linen.

If the breadth of a ridge of land be 7 ells 3 qrs. how much of the length will go to sow 1 peck of lintseed, at 100 square ells to the lippie?

7,75)400,00(51,6125=51 ells 2,4 qrs. the answer.

25

If the height of a dry stone dyke be $7\frac{3}{4}$ qrs. how much of the length goes to make $3\frac{1}{2}$ roods of work?

ell. ells in $3\frac{1}{2}$ roods. ells.

$7\frac{3}{4}$ qrs. is 1,9375) 126,0000 (65,032258 *length* (rquired.

1250

How oft is $\frac{3}{4}$ of a penny contained in $\frac{3}{4}$ of a guinea?

Ansr. 252 times.

What decimal part of 200 l. is 25 pence? Ansr. ,00052.

What decimal part of 3 tun weight is 8 lb.?

Ansr. ,00119.

What are the decimals of 3, 4, 5, 6, 7, 8 pence, one pound, one guinea, one merk, one crown, one shilling, one moidore, and one French pistole being successively made integers?

As stating the several answers to this example *would* take up too much space, I leave it undone; and proceed to lay down some short practical rules whereby the learner may find the quantity of liquor contain'd in any vessel upon occasion, and thereby improve himself further in decimal arithmetic. And I shall first begin with finding the several Multipliers, Divisors, and Gauge points belonging to the different measures now used in England.

1st. For Rectilineal figures, whether Triangular, Quadrangular, or Multangular.

282)1,0000(,003546 Multiplier for ale gallons.

231)1,0000(,004329 Multiplier for wine gallons.

268,

268,8)1,0000(,0037202 Multiplier for corn gallons.
2150,42)1,000000(,00046502 Multr. for corn bushels.

So if the solid inches in any Rectilineal vessel be multiply'd by the said multipliers, or if they be divided by the divisors 282, 231, 268,8, or 2150,42, the products or quotients will be the content of that vessel in English gallons and bushels in their respective measures.

2dly. For circular vessels, the following Multipliers and Divisors are to be used.

282),785398(,002785 Multiplier for ale gallons.
231),785398(,003399 Multiplier for wine galls,
,785398)282,0000(359,05 Divisor for ale gallons.
,785398)231,0000(294,12 Divisor for wine gallons.
,785398)2150,000(2738 Divisor for corn bushels.

The square root of the divisor is the Gauge-point.

The Gauge-point for squares in	ale measure	is 16,79
	wine measure	is 15,19
	malt bushel	is 46,36

The gauge-point for circular figures in -	ale measure,	is 18,95
	wine measure,	is 17,15
	malt bushel,	is 52,32

Note, When the word *area* is mention'd, you must understand it to be the content of the vessel upon 1 inch deep.

PROBLEM I.

To find the content of any right lined vessel in ale or wine gallons.

RULE, Find the area in inches, as shall be immediately directed, then bring it to gallons, by dividing it by 282 for ale, or by 231 for wine, or by 2150,42 for bushels; or else by multiplication, by ,003546 for ale, or by ,004329 for wine, or by ,00046502 for corn bushels, and those quotients or products, multiply'd by the depth of the vessel give the contents in the respective measures.

Examples.

Suppose a back or cooler in form of a Parallelogram, or long square, to be 250 inches long, 84,5 inches broad,

broad, and 7 inches deep; how many gallons of worts doth it contain?

Multiply 250 by 84,5 and the product is 21125 the area in inches, which divided by 282, quotes the area in English ale gallons, viz. 74,9, and this multiply'd by the depth 7, gives 524,3 gallons of ale, the answer.

Note, If you had multiply'd 21125 by ,003546, and that product by 7, the content would have been the same.

Or, If 74,9 had been multiply'd by 7, and the product divided by 282, the content would have been the same.

And, if 21125 had been divided by 231, or multiply'd by ,004329, and both quotient and product multiply'd by 7, each would have given 640,15 gallons of wine for the content.

To find the true gauging place in a cooler.

RULE, Supposing its bottom to be cover'd all over with worts, dip it in 8 or 10 several places as remote and equally distant from each other as you can well do, noting down the wet inches and decimal parts of every dip.

Divide the sum of all those dips by the number of places you dip'd in, and the quotient will be the mean wet of all those dips.

Lastly, find out such a place by the side of the cooler (if you can) that just wets the same with that mean dip, and make a mark there for the true and constant dipping place of that cooler. Then if any quantity of worts (which covers the whole cooler) be dipped or gauged at that place, and the wet inches so taken, be multiply'd into the area of the back in gallons, the product will shew how many gallons of worts are in that cooler at that time, provided the sides stand at right angles with the bottom.

PROB. II.

To find the content in ale or wine gallons or corn bushels of any Prism, what form soever its base is of.

A Prism is a solid contain'd under several planes, having its bases alike, equal, and parallel,

Suppose

Suppose a tun or cistern whose base is a right angled Parallelogram or long square, its length being 49,3 inches, its breadth 36,5 inches, and depth 42,6 inches; what's its content in ale and wine gallons, and in bushels?

RULE, The length, breadth, and depth, being multiplied into one another produce 76656,57; and this divided by 282, quotes 271,83 ale gallons; and divided by 231, quotes 331,84 wine gallons; and divided by 2150,42 quotes 35,65 corn bushels that such a cistern will hold.

PROB. III.

To find the content of a tun, whose bases are alike and parallel, but unequal, being the Frustum of a Pyramid.

RULE, Find the area of each base, and a mean proportional between them, then multiply the sum of those three by $\frac{1}{3}$ of the depth, and the product is the content.

Suppose then, a tun whose bases are Parallelograms, the length of the greater is 100 inches, and breadth thereof 70 inches; the length of the lesser base 80, and its breadth 56; and the depth of the tun 42 inches; What's the content in ale and wine gallons?

Multiply 100 by 70, the product is 7000, the area of the greater base in inches; and 80 multiply'd by 56, produces 4480 inches for the area of the lesser base; then multiply the two areas into each other; and the product is 31360000, whose square root (which you'll hereafter learn to extract) is 5600, a geometrical mean proportional.

See the work.

The greater area	7000
The lesser area	4480
The mean proportional	5600
	17080
$\frac{1}{3}$ of the depth	14

282)239120(847,94 ale gallons.
231)239120(1035,15 wine gallons.

PROB. IV.

PROB. IV.

To find the quantity of malt in a ciſtern, or upon a floor.

RULE, Firſt find the area of the baſe in buſhels, by multiplying the length by the breadth, and dividing the product by 2150,42 (or by 2738, if the baſe be circular) and that area multiply'd by the mean depth, gives the content in buſhels.

Note, You muſt take the mean depth, as directed in Problem I.

There is a ciſtern, whoſe length is 84 inches, and breadth 54 inches, and the mean depth 43,6 inches; what's the content in buſhels?

84 multiply'd by 54, produces the area in inches, viz. 4536. which divided by 2150,42 quotes 2,1093 buſhels, the area upon 1 inch deep, which multiply'd by 43,6 the mean depth, produces 91,98 buſhels, the content of that ciſtern.

Suppoſe a quantity of malt upon a floor, whoſe length is 245 inches, and the breadth 184 inches, and the mean depth 5,6 inches; how many buſhels are there?

Multiply 245 by 184, and the product is 45080, which divided by 2150,42 quotes 20,96 buſhels the area at 1 inch deep, and this multiply'd by 5,6 the mean depth, produces 117,376 the content in buſhels.

PROB. V.

The diameter of a circle being given in inches, to find the area and content in ale or wine gallons.

RULE, Multiply the ſquare of the diameter by ,002785 for ale, or by ,003399 for wine; or if it be divided by 359,05 for ale, or by 294,12 for wine, the products or quotients will be the reſpective ale or wine gallons at 1 inch depth, which multiply'd by the depth of the veſſel, gives the content required.

Suppoſe then the diameter at the baſe of a circular veſſel be 32,6 inches, and depth of the veſſel 38 inches; what's the content in ale or wine gallons?

The

The ſquare of 32,6 is 1062,76.

Then, 359,05)1062,76(2,9599 area in ale gallons.
And, 294,12)1062,76(3,6133 area in wine gallons.
Or, 1062,76 x ,002785=2,9598 area in ale gallons.
And, 1062,76x,003399=3,6133 area in wine gallons.
Again, 2,9599 x 38=112,4762 content in ale gallons.
And, 3,6133 x 38=137,3 content in wine gallons.

PROB. VI.

The longeſt and ſhorteſt diameters of an Elliptical or Oval veſſel being given, to find its area and content in wine or ale gallons.

RULE, Multiply the two diameters together, and for the area and content, do with their product as with the ſquare of the circle's diameter.

Admit then, that the longeſt diameter at the baſe of an oval veſſel is 81,4 inches, and ſhorteſt diameter 54,6 inches, and depth of the veſſel 45 inches; what's the area and content in ale or wine gallons?

The product of the two diameters is 4444,44.

Then, 359,05)4444,44(12,38 area in ale gallons.
And, 294,12)4444,44(15,11 area in wine gallons.
Or, 4444,44 x ,002785=12,38 area in ale gallons.
And, 4444,44 x ,003399=15,11 area in wine gallons.
Again, 12,38 x 45=557,1 content in ale gallons.
And, 15,11 x 45=679,95 content in wine gallons.

PROB. VII.

To find the content of a tun, whoſe baſes are parallel and circular, being the Fruſtum of a Cone, which you'll have occaſion to ſee deſcrib'd afterwards.

RULE, To the product of the greater and leſſer diameters add $\frac{1}{3}$ part of the ſquare of the difference of the diameters, and that ſum is the ſquare of a mean diameter, with which proceed to work in all reſpects as with the ſquare of the circle's diameter, and you'll have the content in ale or wine gallons.

Suppoſe then, the greater diameter to be 80 inches, and the leſſer diameter 71 inches, and the depth of the tun 34 inches; what's the content in wine or ale gallons.

80

$80 \times 71 = 5680$, And $80 - 71 = 9$, and $9 \times 9 = 81$, and $81 \div 3 = 27$. Then $5680 + 27 = 5707$ and this is the square of a mean diameter.

Then, 359,05)5707,00(15,894 area in ale gallons.
And, 294,12)5707,00(19,4 area in wine gallons.
Again, $15,894 \times 34 = 540,396$ content in ale gallons.
And, $19,4 \times 34 = 659,6$ content in wine gallons.

PROB. VIII.

To compute the content of any close cask.

To do this, the three following dimensions must be truly taken.

viz. { The bung diameter / The head diameter / The length of the cask } within the cask.

In taking these dimensions, it must be carefully observed.

I. That the bung hole be in the middle of the cask; and that the bung staff and the staff opposite to the bung hole are both regular, and even within.

II. That the heads of the cask are equal, and truly circular; if so, the distance between the inside of the chine to the outside of its opposite staff, will be the head diameter within the cask very near.

III. Take the shortest distance or length between the outsides of the two heads, from that length subtract $1\frac{1}{2}$ inch (more or less, according to the largeness of the cask) for the thickness of the head: the remainder will be the length of the cask within. But if the cask be empty, you may take the length, by putting a strait rod in at the top hole, and allow for the thickness of the head.

Now, by these dimensions, one would think the content of the cask was perfectly limited; but the diameters and length of one cask may be equal to those of another, and yet one of those casks may contain several gallons more than the other.

Therefore, there cannot be one general rule given whereby the content of all sorts of casks can be found. And for that reason, officers of excise do usually suppose

pose every cask to be in some of the following forms.

I. The middle Frustum of a Spheroid, when the staves of the cask are very much curved.

II. The middle Frustum of a Parabolick Spindle, when the staves between the bung and head are something less curved.

III. The lower Frustums of two equal Parabolick Conoids, when the staves between the bung and head are very little curved.

IV. The lower Frustums of two equal Cones, joining together (as the last) upon one common base; and having the staves strait between head and bung.

RULE, The shortest and most practical way of finding the content of each of those forms, is to find such a mean diameter as will reduce the cask to a cylinder or an exact round cask. Thus, multiply the difference of the bung and head diameters by ,7 for a Spheroid; by ,65 for the second Form; by ,6 for the third Form; and by ,55 for the fourth Form; and the product added to the head diameter, gives a mean diameter; with which proceed to work in all respects as you did with the diameter of the circle, and so you'll find the content in wine or ale gallons as required.

Suppose then, the bung diameter be 32 inches, the head diameter 24 inches, and length 40 inches; the content in each variety is required.

```
  32
  24
  ---
   8 difference.
  ,7
  ---
 5,6
 24
 ---
29,6 mean diameter.
29,6
----
1776
2664
592
------
876,16 square of the mean diameter.
```

359,05)876,16(2,44 area in ale gallons.
294,12)876,16(2,975 area in wine gallons.
876,16 x ,002785=2,44 area in ale gallons.
876,16 x ,003399=2,978 area in wine gallons.
Then 2,44 x 40=97,6 content in ale gallons.
and 2,975 x 40=119 content in wine gallons.

Now for the 2d. form.

8 difference of the diameters.
,65

5,20
24 head diameter.

29,2 mean diameter.
29,2

584
2628
584

852,64 square of the mean diameter.

359,05)852,64(2,374 area in ale gallons.
294,12)852,64(2,898 area in wine gallons.
Then 2,374 x 40=94,96 content in ale gallons.
And 2,898 x 40=115,92 content in wine gallons.

The content of the third Form will be found to be 92,4 ale gallons, and of the fourth 89,85 such gallons.

The contents of those 4 casks may be easily found by the sliding rule thus,

For the first form.

Set the gauge-point 18,95 upon D, to the length 40 upon C; then against the mean diameter 29,6 upon D, is 97,4 ale gallons, the content upon C.

For the second form.

Against 29.2 the mean diameter on D, is 92,4 ale gallons upon C.

For

For the third form.

Againſt 28,8 the mean diameter on D, is 92,4 ale gallons upon C.

For the fourth form.

Againſt 28,4 the mean diameter on D, is 89,85 ale gallons upon C.

Note, I ſhould have here laid down rules for gauging a copper, for finding the fall of a tun, with a table of the ſegments of a circle for finding the ullage, or quantity of liquor remaining in a caſk with its axis, either parallel or perpendicular to the Horizon; but as the book is already ſwell'd to more than I at firſt intended, and as there are ſeveral books on that art done by perſons of better experience, I recomend ſuch to the curious learner, and proceed to conclude this chapter with a few more very uſeful and entertaining examples.

If the firkin of butter in England coſt l. 1 : 1; how may I ſell the lb. in Scotland to gain 50 l. per cent?

N. B. A firkin of butter is 64 lb. Engliſh weight.

lb.	*l. s.*		*lb.*
59	: 1—1	::	1
100	240		150
5900	: 252	::	150
	150		

d. f.

59|00)378|00(6,4067 pence=6 : $1\frac{4}{5}\frac{7}{9}$ anſwer.

47

A man in the beginning of a fair ſold cloth at 11 s. 6 d. per ell and gain'd thereby 15 l. per cent, but in the end of the mercat, he ſold the ſame ſort of cloth at 12 s. per yard; what did he gain per cent, on the 2d. ſale?

See the work on the next page.

s. l. s.
11,5 : 115 :: 12
12

11,5)1380,0(120
100
0

20 l. per cent, anſwer.

A merchant buying at 8s. 8d. made 7 l. per cent. on the ſale; what would he have made per cent, had he bought at 7s. 3d.?

s. l. s.
8,5 : 107 :: 7,25
8,5

535
856

7,25)909,50(125,44827
100
425

l. 25,44827=l. 25 : 8 : $11\frac{1}{2}$, anſr.

A, B, and C, creditors to D, have reſpectively claims againſt him to the extent of l. 48, l. 57, and l. 91 A's money was due 7 months hence, B's 12 months, and C's 18 months hence: the poor unfortunate gentleman D, had only 100 l. to pay with all; what will fall due of the 100 l. to each creditor, and what will it be per pound, at 5 l. per cent?

m. l. m.
12 : 5 :: 7
7

12)35,000(2,916
8

m. l. m.
12 : 5 :: 12
12

12)60(5
0

m. l. m.
12 : 5 :: 18
18

12)90,0(7,5
0

2,916

2,916
100
102,916 : 48 :: 100(46,639

5
100
105 : 57 :: 100(54,285

7,5
100
107,5 : 91 :: 100(84,651
Sum 185,575

185,575)100,000(,5388

Rem. 121900.

l.
,5388 x 46,639=25,1290932 A's share.
,5388 x 54,285=29,248758 B's share
,5388 x 84,651=45,6099588 C's share.
for proof add ,121900 remainder.

l. 100

l. *l.* *s.*
Then, if 185,575 : 100 : 20
20 20

2000
20

s. *s. d. f.*
3711,500)40000,000(10,7773=10:9:1,3
(per l.

5105

In 1659 shillings Ster. what merks Scots?
1659
9
10)14931(1493,1 merks Scots.
1

A,

A, B, C, D, and E, made a joint ſtock, and gain'd 540 l. their ſtock in common was l. 7200, whereof A, put in 7 l. for B's 6 l. and B, 8 l. for C's 7, and C, 5 l. for D's 12, and D, 3 l. for E's 7; and they agreed that A's ſhare of the gain ſhould be at the rate of 8 l. per cent. B, at the rate of 9 l. C, at the rate of 11 l. D, at the rate of 14 l. and E, at the rate of 10 l. per cent. What was each perſon's ſtock and gain?

Aſſume for A's portion any number you pleaſe, ſuch as 7, then multiply it by 6 and divide by 7 for B's; again, multiply it by 6 and divide by 8 for C's, and multiply by 9 and divide by 5 for D's; then laſtly, multiply by 21 and divide by 5 for E's.

Then ſay, As the ſum of theſe quotients is to 7200, ſo each of the quotients to the ſtocks required.

A,	7
B,	6
C,	5,25
D,	12,6
E,	29,4
Sum	60,25

The work may be ſhortly performed thus,

60,25)7200,000000(119,50207 common multiplier.

Rem. 2825

119,50207 x 7 = 836,51449 A's ſtock.
119,50207 x 6 = 717,01242 B's ſtock.
119,50207 x 5,25 = 627,3858675 C's ſtock.
119,50207 x 12,6 = 1505,726082 D's ſtock.
119,50207 x 29,4 = 3513,360858 E's ſtock.
for proof add - - 2825 remainder.

l. 7200 common ſtock.

Then,

	l.		l.	
Then, if 100 :	836,51449	::	108(	903,43564
100 :	717,01242	::	109(	781,54353
100 :	627,3858675	::	111(	696,3983129
100 :	1505,726082	::	114(	1716,527733
100 :	3513,360858	::	110(	3864,696943
			Sum	7962,6021589

7962,6021589)540,000000000000000(,06781702

Remainder 5013783552

	l.	
,06781702 x 903,43564 =	61,2683128	A's gain.
,06781702 x 781,54353 =	53,0019532	B's gain.
,06781702 x 696,3983129 =	47,2276583	C's gain.
,06781702 x 1716,527733 =	116,4097955	D's gain.
,06781702 x 3864,696943 =	262,0922298	E's gain.

Proposed gain very near. l. 539,9999496

A, B, C, and D, made a joint stock of 720 l. A's money was in company for 3 months, B's for 4, C's for 5, and D's for 6 months; and they gained as follows, viz. A, B and C, gain'd 45 l. B, C and D, 42 l. C, D and A, 39 l. and D, A and B, gain'd 36 l.; I demand each man's stock and gain, and what they gain'd per cent?

	l.
A, B, C,	45
B, C, D,	42
C, D, A,	39
D, A, B,	36

3)162(54

0

l.			
54	54	54	54
42	39	36	45
12 A's gain.	15 B's gain.	18 C's gain.	9 D's gain.

(3

3)12(4
4)15(3,75
5)18(3,6
6) 9 (1,5

12,85)720,00000(56,031
Rem. 165

l.

56,031 x 4 =224,124 A's ſtock.
56,031 x 3,75=210,11625 B's ſtock.
56,031 x 3,6 =201,7116 C's ſtock.
56,031 x 1,5 = 84,0465 D's ſtock.
for proof add - 165 remainder.

Sum of their ſtocks l. 720,00000 as given.

720 : 54 :: 100
100

l.
720)5400(7,5 their gain per cent.
0

A, B, C, and D, traded together, A, put in a certain ſum for 8 months, B, 48 l. for 10 months; C, 54 l. for 15 months; and D, put in 21 l. for 40 months; A, got 50 l. of the gain, B, C, and D, got 225 l. amongſt them: what were B, C, and D's gains, and A's ſtock?

B, 48 x 10=480
C, 54 x 15=810
D, 21 x 40=840

2130)225,0(,105633
Rem. 171

l.

,105633 x 480=50,70384 B's
,105633 x 810=85,56273 C's gain.
,105633 x 840=88,73172 D's
for proof add - 171 remainder.

l. 225 B, C, and D's gains.

225 : 2130 :: 50
50
——— 8)
225)106500,0(473,33333(59,16666=l.59 : 3 : 4 A's (stock.
75 5

What present money will pay a debt of l. 1 : 13 : 4 due $\frac{7}{12}$ of a month hence, rebate being allowed at $4\frac{3}{5}$ per cent per annum, simple interest?

m. l. m.
12 : 4,6 :: 583333(,2236109833
l. l. l. l.
100,2236109833 : 1,66666 :: 100(1,66294 present money equal to l. 1 : 13 : 1,1

A gentleman left 8000l. to his wife, 5 sons, and 4 daughters, and ordered that each son should have thrice as much as each daughter, and every daughter twice as much as his wife; pray what were their respective portions?

Suppose any number for the widow's part, as 1 l. then 2 l. will be each daughter's part, and 6 l. each son's part, according to the question: consequently 4 daughters will have 8 l. and 5 sons 30 l. which with the mother's 1 l. makes 39 l. but as the sum amounts not to 8000 l. I say,

l. l. l.
If 39 : 1 : 8000
1
———
39)8000(205,1282=l. 205 : 2 : $6\frac{1}{13}$ mother's (part.
2
———
410 : 5 : $1\frac{7}{13}$ to each (daughter.
3
———
1230 : 15 : $4\frac{8}{13}$ to each son.

There is a piece of squared timber 12 feet long, and the side at the end is 8 inches; how many solid inches

 must

must be taken off the ſame, in order to convert it into a round Priſm or Cylinder, and how many ſolid feet will this cylinder contain?

8
8
——
64 ſquare of the diameter.
,7854 multiplier.
——
50,2656 inches, area at the baſe.
Mult. by 144 length in inches.
——
7238,2464 ſolid inches in the cylinder.

8
8
——
64 inches, area at the baſe.
Mult. by 144
——
9216 ſolid inches in the ſquare piece.
7238,2464 content of the cylinder.
——
1977,7536 ſolid inches to be taken off.

1728)7238,2464(4,1888 ſolid feet in the cylinder.
——
0

To find the burthen of any ſhip, whoſe length, breadth, and depth is given in feet, the common rule is,

RULE, Multiply the length of the keel by the length of the mid ſhip beam, then multiply that product by the depth of the hold; divide this laſt product by 100, and the quotient gives you the tuns: but when there is no allowance for guns, divide the laſt product by 95.

What will be the burthen of a ſhip whoſe keel is 80 feet, beam 32, and depth of the hold 14,1 feet?

See the work on the next page.

```
   80
   32
 ----
 2560
 14,1
 ----
 2560
35840
-----
```

100)36096,0(360,96 tuns, anſwer.

But if there be no allowance made for guns, divid the laſt product 36096,0 by 95, and in this caſe he burthen will be found to be 379,95, that is 380 tuns very near.

A landed man two daughters had,
And both were very fair,
He gave to each a piece of land,
One round, the other ſquare;
At 20 l. the acre juſt,
Each piece its value had,
The ſhillings which encompaſs'd each
For each exactly paid;
If croſs a ſhilling be an inch,
As it is very near,
Who had the better portion
That had the round, or ſquare?

RULE, Multiply 5760 (the ſquare ells in one acre) by 1383,84 (which is the ſquare of 37,2 the inches in a Scots ell) and the product is 7970918,4 which multiply by 16, and the product is 127534694,4 which divided by 160000 (the ſquare of the number of ſhillings in 20 l.) quotes 797,09184 the acres in the ſquare piece.

Multiply ,0795774715377 (which is the area of a circle whoſe circumference is unity) by 160000 (the ſquare of the ſhillings in 20 l.) and the product is 12732,39544602 by which divide the ſquare inches in in one acre, viz. 7970918,4 and the quotient is 626,0344672599 the acres in the round, So she that had the square had the better portion.

For proof.

Find the 4 sides of the square in inches, find also the circumference of the round in inches, and if both correspond with their respective values in shillings, the work is right.

CHAP. XII.

SECTION I.

Of Simple Interest.

SIMPLE INTEREST, is that which is paid for the loan of any sum of money lent out for some time, at any rate per cent, agreed on betwixt the borrower and lender.

There are several ways of answering questions about simple interest, but as they are too tedious to be repeated here, I shall only shew how they may be advantageously work'd by decimals, and lay down such rules as will suit with all cases. In order then to that,

Let:
- P = the principal sum put to interest.
- R = the ratio of the rate.
- T = the time the principal is out at interest.
- A = the amount of principal and interest.
- I = the interest alone.

By the ratio of the rate, I mean the simple interest of one pound for one year at any given rate per cent, which is thus found, by dividing the rate of interest by 100.

100)5,00(,05=the ratio of the rate at 5 l. per cent, per annum.

100)4,50(,045 the ratio at $4\frac{1}{2}$ l. per cent, per annum, &c.

In simple interest there are four cases, or variety of questions.

CASE I. When P, T, and R, are given to find I, and A.

That

That is, If any principal, with the time of its being at interest, and the rate of interest per cent, per annum are given; to find the interest, and the amount. This question I take to be of the most general use, of any that occurs in the whole business of simple interest, and may be performed, thus:

RULE, Multiply the principal, the time, and the ratio of the rate, all three together, and the last product is the interest, to which add the principal, and the sum is the amount.

Example, How much interest will be due on 564 l. 17 s. 6 d. if it be forborn for 3 years, and what will principal and interest amount to in that time, at 5 l. per cent, per annum?

564,875 principal.
,05 ratio of the rate.
28,24375 interest for 1 year.
3
84,73125 interest for 3 years.
564,875
649,60625 amount = l. 649 : 12 : 1½.

Note, The odd shillings and pence (if there be any) in the principal sum, must be reduced to decimals; as in this example.

From the foregoing work you may also observe, that if any principal sum be multiply'd by the ratio of the rate, the product will be the interest of that sum for one year; and if that product be multiply'd by 2, 3, 4, &c. it will give the interest of the given principal sum for 2, 3, 4, or more years.

CASE II. When A, T, and R, are given, to find P.

RULE, Multiply T, by R, and to their product add 1: then divide the proposed amount by the sum, and the quotient is P, or the principal required.

Example, What sum of money put to interest for 3 years will amount to (or raise a stock of) 649 l. 12 s. 1½ d. at 5 l. per cent, per annum?

Or,

Or, it may be propofed otherways, thus;

Suppofe, a debt of l. 649 : 12 : 1½ were not to be paid until 3 years hence; what would it be worth in ready money, the creditor allowing difcompt to the debtor at the rate of 5 l. per cent, per annum?

3
,05
———
,15
add 1
———
1,15)649,60625(564,875 = l. 564 : 17 : 6 = prin. (or ready money) required.
0

CASE III. When P, T, and A, are given to find R.

RULE, Multiply the principal by the time, by that product divide the difference between the propofed amount and the principal; and the quotient will be the ratio of the rate of intereft required.

Example, At what rate of intereft per cent, per annum will l. 564 : 17 : 6 raife a ftock, or amount to l. 649 : 12 : 1½ in 3 years time?

564,875 　 649,60625
3 　 564,875
———
1694,625) 84,73125(,05 ratio of the rate.
00 　 100
———
l. 5,00 rate per cent.

CASE IV. Having P, R, and A, given; to find T.

RULE, Multiply P, by R, by that product divide the difference between A, and P; and the quotient will fhew the time required.

Example, In what time will l. 564 : 17 : 6 amount to (or raife a ftock of) l. 649 : 12 : 1½ at 5 l. per cent, per annum?

564,875 　 649,60625
,05 　 564,875
———
28,24375) 84,73125(3 years.
0

It must needs be easy to conceive, that what is here done at 5 l. per cent, may be done at any other rate of interest, by forming the ratio of the rate accordingly.

Example, What's the interest and amount of 312 l. 18 s. for 7 years at $4\frac{1}{2}$ per cent, per annum?

312,9 principal sum.
,045 ratio of the rate.

14,0805 interest for 1 year.
7

98,5635 interest for 7 years.
312,9 principal sum.

411,4635 amount = l. 411 : 9 : 3,24

If the work of the (1st 4 examples, and the rules whereby they are performed, be well understood, they will be sufficient (tho' there is really but one question, only, it is varied according to the several cases) to shew how any question of the like kind may be truly resolved, at any proposed rate of simple interest, and for any assigned time, especially if the time given (or sought) does consist of whole or compleat years.

But if the time given (or sought) be but an even part of a year only (as $\frac{1}{2}$, $\frac{1}{4}$, or $\frac{3}{4}$) or years and any even part of a year, then that odd time must be reduced into decimal parts of a year, as in the two following examples.

What will l. 438 : 16 amount to in $\frac{1}{4}$ year at 5 l. per cent, per annum?

438,8 principal.
,05 ratio of the rate.

21,940 interest for 1 year.
,25 time of continuance.

109700
43880

5,48500 interest for $\frac{1}{4}$ year.
438,8 principal sum.

444,285 amount = l. 444 : 5 : 8,4

Or,

Or, you might have divided the year's interest by 4, thus:

$$\begin{array}{r} 438,8 \\ ,05 \\ \hline \end{array}$$

4)21,940 (5,485 interest for $\frac{1}{4}$ year.
     0   438,8
              l. 444,285 amount as before.

What will l. 475 : 10 amount to in $4\frac{3}{4}$ years at 6 l. per cent, per annum?

```
      475,5
        ,06
   --------
     28,580 interest for 1 year.
       4,75
   --------
     142650
    199710
   114120
   --------
   135,51750 interest for 4¾ years.
   475,5     principal sum.
   --------
   611,0175  amount = l. 611 : 00 : 4,2
```

If the time given (or sought) is not an even part of a year, then the best way will be to reduce the odd time into days, and then work with the decimal parts of a year that are equivalent to those number of days. And for the ready finding the decimal parts of a year that are equal to those given days, I have here inserted a small table as follows.

A Table

A Table for the ready finding the decimal parts of a year, equal to any number of days.

Days.		Decimal parts.	Days.		Decimal parts
1	=	,00274	30	=	,082192
2	=	,005479	40	=	,109589
3	=	,008219	50	=	,136986
4	=	,010959	60	=	,164383
5	=	,013699	70	=	,191781
6	=	,016438	80	=	,219178
7	=	,019178	90	=	,246575
8	=	,021918	100	=	,273973
9	=	,024657	210	=	,547945
10	=	,027397	300	=	,821918
20	=	,054794			

If the propoſed number of days can be exactly found in the table (under days) their decimal parts are found alſo by inſpection only.

But if the true number of days cannot be exactly found there, then both they, and their decimal parts muſt be collected out of the table at twice, or thrice, according as their number requires.

Example, What decimal parts of a year are equal to 175 days?

	Days.		*Decimal parts.*	
Thus	100	=	,273973	add all theſe together.
	70	=	,191781	
	5	=	,013699	

Hence 175 = ,479453 the decimal parts required.

What then is the intereſt of l. 684 for 175 days at 5 l. per cent, per annum?

684
,05

34,20 intereſt for 1 year.
,479453 time of continuance.

16,39729260 intereſt = l. 16 : 7 : 11,35

 What

What will the interest and amount of 144 l. for 7 years 50 days at l. 4 per cent, per annum.

```
                  144
                  ,04
                 5,76 interest for 1 year.
             7,136986 time of continuance.
          41,10903936 interest for 7 years 50 days.
          144         principal.
         185,10903936 amount = l. 185 : 2 : 2,16
```

Or, the interest of a sum of money may be found for any number of days, thus, viz. divide the year's interest by 365, and the quotient multiply'd by the proposed number of days, gives the interest required.

Suppose then (as in the last example except one) that l. 684 lys out upon interest at 5 l. per cent, per annum for 175 days; what interest will be due at that time?

```
            684
            ,05
       365)34,20 ( ,0937 interest for 1 day.
                    175 days, time given.
            5
       product l. 16,3975 = the interest as before.
```

But for the more ready casting up the interest of money for days, it will be necessary to have the interest of 1 l. for 1 day, at all rates ready calculated, as a table or standard, thus:

The interest of 1 l. for 1 day.

At	1	*per cent, is*	,00002739726
	2		,00005479452
	3		,00008219178
	4		,00010958904
	5		,0001369863
	6		,00016438356
	7		,00019178082
	8		,0002191 7808
	9		,00024657534
	10		,0002739726
	$4\frac{1}{2}$		,00012328767

The

The interest of 1 l. for one day is thus found; divide the given rate of interest per cent, per annum by 100, and the quotient is the interest of 1 l. for one year; which divided by 365, gives the interest of 1 l. for one day; which when multiply'd into both the number of days, and the principal sum, produces the interest for the time required.

Example, What's the interest of 560 l. for 60 days, at 5 l. per cent, per annum?

,000136986
560
82191780
6849315
,076712328
60
4,60273968 = l. 4 : 12 : 0,65

Having now gone through all the general cases of simple interest, I design'd to have concluded here; but because the business of Rebate or Discompt of money paid before the time it becomes due, comes often into practice upon several occasions, and being but just touch'd upon by an example in case 2d. of simple interest; lest I should be thought too remiss in so useful a part of interest as that is, I shall proceed a little further, and lay down particular rules for that purpose.

CASE, I. To find the true discompt of any sum at any rate of simple interest.

RULE, Multiply the given time by the ratio of the rate of interest, and to the product add 1, by which sum divide the product of the time, ratio of the rate, and sum to be discompted for when multiply'd together; and the quotient is the discompt required.

Example, what discompt must be allowed on a bill of 500 l. paid 20 days before it is due, rebate being allowed at 5 l. per cent, per annum?

See the work on the next page.

,054794 time, from the table.
,05

,054794 ,00273970
,05 500

1,00273970) 1,36985000(1,3661 l. discompt.

Or, according to Mr. Hatton's new method.

RULE, To the product of 365 by 100, add the product of the days multiply'd into the rate of interest per cent, per annum, by which sum divide the product of the rate of interest, sum to be discompted for, and given days when multiply'd together; and the quotient is the discompt required.

Let us then keep still the same example.

20	365	500
5	100	5
100	36500	2500
	100	20

366100) 500100(1,3661 l. discompt as before.

74

Though the rule in case 2d. of simple interest be sufficient for finding the present worth of any sum, at any given rate, due at the end of any given interval of time, yet I shall resume the example in foresaid 2d. case, and lay down Mr. Hatton's new method of resolving it; which is this:

RULE, to the product of 365 by 100, add the product of the days multiply'd into the given rate of interest per cent, per annum; by which sum divide the product of the days in a year, the sum to be discompted for, and 100, when multiply'd together; and the quotient is the discompt required.

What present sum of money will pay a debt of l. 649 : 12 : 1½ due 3 years hence, rebate being allowed at 5 l. per cent, per annum?

See the work on the next page.

365	365	
100	3	
36500	1095	649,60625
	5	365
	5475	237106,28125 product.
	36500	100

l.

41975) 23710628,12500 (564,875 ready money as before.

And if it were required to find the whole discompt of several sums due at the end of several intervals of time, it is but computing them at so many several operations; and then the sum of all those particular discompts being taken from the total sum of all the debts, will leave the present money that shall pay the whole debt.

Suppose then, that A, is indebted to B, 750 l. to be paid at three several payments, after this manner, viz. 250 l. at the end of $1\frac{1}{2}$ year, 100 l. to be paid at the end of 2 years, and 400 l. at the end of 4 years; the question is, To find how much of the 750 l. B, ought to rebate at 6 l. per cent, per annum simple interest, to have the whole debt paid by A, in present money?

According to the Data in this question, the particular discompts (found by the rules laid down for that purpose in last case) will be these following.

	l.		l.
The discompt of	250	for $1\frac{1}{2}$ year is	20,6423
	100	for 2 years is	10,7142
	400	for 4 years is	77,4193
		Total discompt l.	108,7758

which taken from 750 l. the whole debt, leaves precisely 641,2242=l.641 : 4 : $5\frac{3}{4}$ the true sum that A, must give to B, in present money, to be discharged of the debt.

And from hence naturally follows the following method of finding the true equated time, wherein several sums, due at several intervals of time, may be paid at one intire payment, without any loss either to debtor or creditor.

CASE II.

CASE, II. To determine the equated time of payments.

The usual rule is this, Multiply every single sum of money with the time it becomes due, divide the sum of those products by the total debt, and the quotient will be the true time at which all the money ought to be paid. But when any number of payments are to be paid off at one entire sum, in order to determine the equated time for the payment of that sum, I advise that you first find the particular discompts of all the proposed payments (as before directed) whose times are to be equated according to the given intervals of those payments, at any rate as shall be agreed on by the parties concerned, and then proceed according to this rule.

RULE, Multiply the present worth by the ratio of the same rate of interest, by which those discompts were computed, by that product divide the sum of all the discompts, and the quotient is the true equated time at simple interest.

Let it then be required to find the true equated time wherein the three aforesaid sums due from A, to B, (as in the last instance) may be safely paid without loss to either. Here the sum of all the particular discompts at 6 l. per cent, was found to be 108,7758, which taken from 750, leaves 641,2242 the present worth, or sum to be immediately paid.

Then by the last rule, the work will stand thus:

```
641,2242
     ,06                         y.   d.
38,473452)108,775800(2,8273=2 : 302 the equated
                    time for paying the whole debt.
```

See the work of the same example the common way.

```
250     100     400
1,5       2       4
375,0   200    1600
                200
                375      y.   d.
        750)2175(2,9=2 : 328½ which is
              0          undoubtedly false.
```

A TABLE

A TABLE of INTEREST, contrived after a most comprehensive method, serving for all rates, and for any number of days, from one day to ninety million of days; or for any number of pounds, from one pound to ninety million of pounds.

Below *p*, in the Table, are placed tenth parts of a penny.

10,000,000.				1,000,000.				100,000.				10,000.				1000.				100.				10.				1.				
L.	s	d	p	L	s	d	p	L	s	d	p	L	s	d	p	L	s.	d	p	L	s	d	p	L	s	d	p	L	s	d	p	
1369	17	3	1	136	19	8	7	13	13	11	7	1	7	4	8	00	2	8	9	00	00	3	3	00	00	00	3	00	00	00	0	1
2739	14	6	2	273	19	5	4	27	7	11	3	2	14	9	5	00	5	5	8	00	00	6	6	00	00	00	7	00	00	00	0	2
4109	11	9	4	410	19	2	1	41	1	11	0	4	2	2	3	00	8	2	6	00	00	9	9	00	00	1	0	00	00	00	1	3
5479	9	0	5	547	18	10	8	54	15	10	7	5	9	7	0	00	10	11	5	00	1	1	2	00	00	1	3	00	00	00	1	4
6849	6	3	6	684	18	7	6	68	9	10	4	6	16	11	8	00	13	8	4	00	1	4	5	00	00	1	6	00	00	00	1	5
8219	3	6	7	821	18	4	3	82	3	10	0	8	4	4	6	00	16	5	3	00	1	7	7	00	00	1	9	00	00	00	2	6
9589	00	9	8	958	18	1	0	95	17	9	7	9	11	9	4	00	19	2	1	00	1	11	0	00	00	2	2	00	00	00	2	7
10958	18	1	0	1095	17	9	7	109	11	9	4	10	19	2	1	1	1	11	0	00	2	2	3	00	00	2	6	00	00	00	2	8
12328	15	4	1	1232	17	6	4	123	5	9	0	12	6	6	9	1	4	7	9	00	2	5	6	00	00	3	0	00	00	00	3	9

The interest of 1 l. for 100 days, is equal to the interest of 100 l. for 1 day; consequently, if you multiply any sum by the number of days for which the interest is sought, the interest of that product

duct for 1 day answers your question. Thus, to find the interest of 15 l. for 600 days, or of 600 l. for 15 days (being all one) I multiply 15 by 600, and the product is 9000; then opposite to 9 on the right hand of the table, and under 1000, I find l. 1 : 4 : 7,9 which is the interest desired at 5 l. per cent, and so for any other sum and time. And for any other rate the rule is, Multiply one fifth of the product of the sum and time by any rate of interest, and take the interest of that product out of the table. Thus, the interest of 200 l. for 50 days may be found at 3 l. per cent, I multiply l. 200 by 50 days, and the product is 10,000; then I take a fifth of that which is 2000, and multiplying the same by 3, the product is 6000, then opposite to 6 on the right hand, and under 1000, I find s. 16 : 5,3 which is the interest desired at 3 per cent, and so for any other rate, sum or time.

If the sum consists of pounds, shillings and pence, take the decimals of such shillings and pence. Thus, the interest of l. 60 : 9 : 6 for 104 days is found, by multiplying 60,475 by 104, which makes 6289,4, the interest of which (the fraction being of no account) I find in the table to be 17 : 2,8

	l.		s.	d.
Thus,	6000	- - -	16 :	5,3
	200	- - -	00 :	6,6
	80	- - -	00 :	2,6
	9	- - -	00 :	0,3
Principal	6289	Interest	17 :	2,8

Note, in calculating interest by days, as here; a leap year ought to be supposed to contain only 365, and not 366 days.

SECTION II.

SECTION II.

Of ANNUITIES or PENSIONS in arrears; computed at simple interest.

Put { U=Annuity, pension, or yearly rent.
R=Ratio of the rate, as before.
T=the time of its being unpaid.
A=the amount of the annuity and its interest.

This admits of 4 cases.

CASE I. Having U, T, and R, given; to find A.

RULE, Multiply the time, the time less by 1, and and half the ratio of the rate all three together, to their product add the time, and that sum multiply'd by the annuity gives the amount required.

E*xample,* A farmer binds his son apprentice to a merchant for 6 years, and at the same time letts an annuity of 800 l. run to the expiration of the said time, that it might be a stock for his son; I demand what his stock will be, accompting simple interst at 5 l. per cent. per annum?

5=the time less by 1.
6=the time.

30
,025 half the ratio ,05.

,750
add 6,000 the time.

6,750
800 annuity.

l. 5400,000=the amount, his stock required.

What will l. 250 yearly rent amount to if forborn, or unpaid 7 years, at 6 l. per cent. for each payment as it becomes due? Ansr. 2065 l.

CASE II. When A, T, and R, are given; to find U.

 RULE,

RULE, Multiply the time, the time less by 1, and half the ratio of the rate, all three together; and to their product add the time (as before) then divide the propos'd amount by that sum, and the quotient is the annuity, or yearly rent required.

Example, What annuity, yearly rent, or pension forborn 6 years, will amount to (or raise a stock of) 5400 l. allowing 5 l. per cent. for every payment as it becomes due?

5 = the time less by 1.
6 = the time.
30
,025 half ,05 the ratio.
,750
add 6,000 the time
l.
6,750)5400,000(800 answer.
0

CASE III. When U, A, and R, are given; to find T.

RULE, Subtract the ratio of the rate from 2; then divide the remainder by twice the ratio, and call the quotient X.

Next multiply X, on itself, and call the product Z. Then multiply the annuity by the ratio, and by that product divide the double of the proposed amount, to that quotient add Z, then extract the square root of that sum, and from that root subtract X, the remainder is the time sought.

Example, In what time will an annuity of 800 l. raise a stock of 5400 l. allowing 5 l. per cent. &c. for the forbearance of the payments as they become due?

,05   2,00
2   ,05 ratio of the rate.
,10) 1,95(19,5 = X.
19,5
975
3705
380,25 = Z.

See the rest of the work on the next page.   8000

800 5400
,05 2
40,00) 10800(270
380,25=Z to be added.
650,25(25,5
4 19,5=X to be subtracted.
6 years answer.
45)250
225
505)2525
2525
0

CASE IV. If U, T, and A, are given; to find R.

RULE, Multiply the annuity with the time, and multiply that product with the time again, from this last product, subtract the product of the annuity with the time, and reserve half the difference of those two products for a divisor. Next, Subtract the product of the annuity with the time from the proposed amount, divide the remainder by the aforesaid reserved divisor, and the quotient will be the ratio of the rate of interest required.

Example, Suppose 800 l. yearly rent, being forborn or unpaid 6 years, shall amount to l. 5400; what rate of interest per cent. &c. must be allowed for every payment as it becomes due?

800 annuity.
6 time.
4800
6 time again.
28800
Sub. 4800 product of A, and T.
2)24000(12000 divisor.
0

5400 propos'd amount.
Sub. 4800 product of A, and T.
12000)600,00(,05 ratio of the rate, which multiply'd by 100, gives 5 l. per cent. per annum?

Thus

Thus you have all four cases relating to annuities or rents, &c. in arrears, with their examples in yearly payments; but if the annuities or rents, are to be paid by half yearly or quarterly payments, Then

I. Instead of the ratio of the given rate of interest, you must take the $\frac{1}{2}$ of that ratio for half yearly payments, and the $\frac{1}{4}$ of it for quarterly payments.

II. And you must take $\frac{1}{2}$ the yearly rent for half yearly payments, and $\frac{1}{4}$ of it for quarterly payments.

III. But instead of the proposed number of years, you must take twice that number for half yearly payments, and four times that number for quarterly payments. As in the two following examples.

Suppose 800 l. per annum annuity, payable every half year, were forborn or unpaid 6 years; what would all those arrears ammount to, at the rate of 5 l. per cent. per annum?

Now here it will be U=400, viz. $\frac{1}{2}$ 800 l. ratio =,025 the $\frac{1}{2}$ of the ratio at 5 per cent. and T=12, viz. 6 x 2 the number of half years in 6 years; thence to find the amount per rule in case first of annuities.

12 =time. 2),025(,0125
11 =time—1.

132
,0125 = $\frac{1}{2}$ ratio.

660
1584

1,6500
add 12 the time.

13,65
400=U.

---

l. 5460,00= amount of arrears.

Hence

Hence it may be obſerved that half yearly payments are more advantageous than yearly; for l.5460 is more than l.5400 by 60 l. conſequently quarterly payments are more advantageous than half yearly payments.

Suppoſe an annuity of 800 l. to be paid every quarter, what would it amount to if forborn 6 years at 5 l. per cent. &c. for every payment as it becomes due?

Here is given U=200, viz. the $\frac{1}{4}$ of 800l. and R=,0125=$\frac{1}{4}$ of ,05 the ratio of 5 per cent. and T=24, viz. 6 x 4 the number of quarters in 6 years; thence to find the amount as before.

```
          24=time,          2),0125(,00625
          23=time—1.
        ------
          72
         48
        ------
         552
       ,00625=½ the ratio.
       ------
        2760
       1104
      3312
      ------
      3,45000
  add 24        =the time.
      ------
      27,45
        200=U.
      ------
   l. 5490,00= amount of arrears.
```

You may now obſerve,

That, an annuity of 800l. for yearly payments in 6 years at 5 per cent. amounts only to l. 5400
But half yearly payments amount to - 5460
And quarterly payments amount to - - 5490

If the two laſt examples be well conſidered and underſtood, it will be eaſy to conceive how to compute any queſtion in the other three caſes, when the payments

payments are either half yearly, or quarterly; and therefore I shall insert no examples, but shall proceed to the next section.

SECTION III.

The present worth of annuities, pensions, &c. computed at simple interest.

I shall here make use of these letters to denote the several parts of the question.

viz. { U, the annuity or rent. T, the time of its continuance. R, the ratio of the rate. } as before.

And, P, the present worth of the annuity.

This section admits of 4 cases.

CASE I. Having U, T, and R, given; to find P.

RULE, Multiply the time, the time less by 1, and half the ratio of the rate, all three together, and to their product add the time; then multiply that sum with the annuity, and reserve the product for a dividend. Then multiply the ratio into the time, to the product add 1, divide your reserv'd dividend by this sum, and the quotient shews the present worth required.

Example, Suppose an annuity of 20 l. per annum is to be sold for 7 years; what is it worth in ready money, interest being computed at 6 l. per cent. per annum?

,06 ratio. 7 time.
7 time. 6 time—1.

,42+1=1,42 divisor. 42
,03=½ ratio.
1,26
add 7 time
8,26
20 annuity.
1,42)165,20(l. 116,338 ready money,
4 (answer.

CASE II.

CASE II. When P, T, and R are given; to find U.

RULE, Multiply the time, the ratio of the rate, and the present worth all three together, to their product add the present worth, and make that sum a dividend. Then multiply the time, the time less by 1, and half the ratio of the rate all three together, to their product add the time, and your aforesaid dividend divided by this sum, quotes the annuity or rent required.

Example, What annuity or rent to be paid yearly, and to continue 7 years, may be purchased for l. 116,338 at the rate of 6 per cent.

7 T.	116,338 P.	
6 T—1	7 T.	
42	814,366	
,03=½ R.	,06 R.	
1,26	48,86196	
7= T.	116,338	
		l.
8,26)	165,19996 (	19,9999 annuity required.

CASE III. When U, P, and R, are given; to find T.

RULE, To the present worth add half the annuity, multiply that sum with the ratio, and subtract the product from the annuity; then divide the remainder by the product of the annuity multiplied with the ratio, and call the quotient X.

Next multiply X into itself, and call the product Z. Then divide the present worth by the product of the annuity into the ratio, to the double of that quotient add Z, then from the square root of that sum subtract the number called X, and the remainder is the time required.

Example, In what time will 20l. per annum pay off a debt of l. 116,338 by yearly payments, allowing the creditor 6l. per cent. interest for his money, until the debt be discharged?

Or (which is the same thing) If I lay out l. 116,338 ready money for an annuity of 20l. per annum, to

be

be paid yearly; how long may I enjoy that annuity, to be allowed 6 l. per cent, per annum for my money?

Anſwer 6,999 years.

CASE IV. When U, P, and T, are given; to find R.

RULE, To twice the preſent worth add the annuity, from their ſum ſubtract the product of the annuity with the time; then multiply the remainder with the time, and reſerve the product for a diviſor.

Next, multiply the annuity with the time, from the product ſubtract the preſent worth, and if you divide twice that remainder by the aforeſaid diviſor, the quotient will be the ratio required.

Example, Suppoſe l. 116,338 were given for an annuity of 20 l. per annum, to be paid yearly, and to continue 7 years; what rate of intereſt per cent. &c. is allowed the purchaſer?

Anſr. ,06, ſo the rate will be at 6 l. per cent.

Theſe four examples may be ſufficient to ſhew how any other of the like kind may be reſolved for whole years, and alſo for thoſe of half yearly or quarterly rents, if you duly conſider what was ſaid anent the working of ſuch queſtions in the laſt ſection; provided the purchaſed rent, leaſe, or annuity is to commence immediately.

But if it be required to find the preſent value or worth of any rent, or annuity in reverſion, that is, when it is not to be entered upon until after ſome time or number of years paſt; then you muſt work by this rule.

RULE, Firſt (as in caſe firſt of this ſection) find what the propoſed annuity or rent would be worth for the given time of its continuance, as if it were immediately to be entered upon.

And then find (by the rule for caſe 2 d. in ſimple intereſt) what principal or ſum being put out to intereſt, and forborn during the time of that reverſion, (viz. the time wherein the annuity is not to be in poſſeſſion) would amount to the foreſaid value, and that principal will be the ſum which ſhould be paid for the propoſed annuity in reverſion.

Example,

Example, There is the reversion of a lease of 175 l. per annum, to be lett for 11 years, which are to commence after 9 years are expired; I demand the present worth of that lease, if the purchaser be allowed 6 l. per cent. for his ready money?

11=T. ,06=R.
10=T−1 11
——— ———
110 ,66+1=Divisor.
,03=½ R.
———
3,30
add 11=T.
———
14,3
175=U.
———
715
2431
———
1,66)2502,50(1507,53 present worth, if the lease were to commence immediately.

,06
9
———
,54+1=1,54)1507,53(978,9162=l.978 : 18 : 3,88 the true present worth of that lease in reversion as was required.

If it be required to find what annuity in reversion may be purchased for any proposed sum, and at any given rate of interest per cent. when the time the annuity is not to be enter'd upon, and the time of its continuance are both given.

Then, first find what sum proposed to be laid out in the reversion, would amount to in that time wherein the annuity is not to be in possession, as if it were forborn at interest during that time at any given rate per cent. (by the rule for case 1st. in simple interest.)

And then find what annuity that amount will purchase for the time of continuance (as in case 2d. of this section) and that will be the answer to the question.

Example, What annuity in reversion to continue 11 years, and to commence 9 years hence, may I purchase for l. 978,9162, if I be allowed 6 l. per cent. for my ready money ? Ansr. l. 175 annuity.

These two are the most general and useful questions that relate to purchasing annuities, or leases in reversion: not but that, if there be occasion, either the time, or rate of interest may be found by a due application of their respective rules.

I shall only add a few more examples for the reader's exercise, and conclude this rule.

A, hath an annuity of 20 l. per annum to continue 7 years; B, hath an annuity of l 5 : 10 to continue 21 years; now these two persons would exchange annuities, and allow each other simple interest at 6 l. per cent. I demand who must pay money, and how much!

Find the present worth of both annuities, subtract the one from the other, and the remainder is l. 34 : 11 : 7¼ that B, must pay to A.

A gentleman hath 160 l. which he would lay out to purchase an annuity of 20 l. per annum; how many years must the said annuity continue, simple interest being computed at 6 l. per cent?

Here, annuity, rate, and present worth are given, to find the time of continuance, which will be 10 years, 1 month, 5 days.

A gentleman hath 1000 l. which with the interest thereof (suppose at 5 l. per cent.) he is to spend in 20 years, so as to spend equally each year, and to exhaust the whole sum and interests at the end of the said time; what must he spend yearly according to those regulations?

Here, the present worth, time, and rate are given, to find the annuity, viz. l. 67 : 15 : 11$\frac{1}{10}$.

A gentleman has an estate of 240 l. per annum payable yearly by responsible tenants, which he makes over to another for l. 3755 ready money, till the estate

clears

-clears itself; how long time will that other gentleman enjoy the annuity, if he be allowed 6 l. per cent. simple interest for his ready money, the yeary payments being duly made?

Here, are given the annuity, present worth, and rate; to find the time of continuance.

A merchant is indebted l. 360, the creditor is to receive the same at 10 equal yearly payments, the debtor allowing for the forbearance of the same money at the rate of 6 l. per cent. simple interest; I demand what these yearly payments ought to be?

Here are given the present worth, time, and rate, to find the annuity. And the answer will be l. 45 : 7 : 1.

A gentleman bequeathed l. 1500 to his daughter, to be paid her at 14 years end; what ready money must the executors pay, to be abated after the rate of 5 l. per cent. simple interest?

$$\begin{array}{r} 14 \\ 5 \\ \hline 70 \\ 100 \\ \hline \end{array}$$

If 170 : 100 :: 1500 (882,3529, l. answer.

CHAP. XIII.

Of Compound Interest.

COMPOUND INTEREST is that which is produced from any principal and its interest put together, as the interest of that principal becomes due: that is, at every payment, or rather at the times when the payments become due, there is still created a new principal by the increase of the growing interest; and therefore it is called, Interest upon Interest, or Compound Interest.

In all computations of this kind, instead of the ratio of the given rate of interest, we make use of the amount

mount of one pound for one year; and that amount of 1 l. is no more but the ratio of the given rate added to unity.

The amount of 1 l. for 1 year at any given rate of interest per cent. may be found by this proportion;

viz. As 100 : 105 :: 1 : 1,05 amount of 1 l. at 5 per cent. and so of any other given rate of interest.

These things being understood, we may proceed to the resolving of questions about money forborn at compound interest; and as it is not lawful to lett out money at interest upon interest, and that these things are best performed by Logarithims (which I am not here to treat of) I shall only lay down the most useful cases; and (for the rest) refer the reader to Mr. Ward's Mathematician's Guide.

I shall here make use of the same letters to denote the several parts of the question, as before, except R, only.

viz. { P, the principal put to interest.
T, the time of its continuance.
A, the amount, or principal and its interest.

And R, the amount of 1 l. for 1 year.

This admits of the same 4 cases, as in simple interest.

Case I. If P, T, and R, are given; to find A.

Rule, multiply the amount of 1 l. for 1 year at the given rate so often into itself, as are the number of years proposed, wanting one; and the last product multiply'd by the principal given, will give the amount required.

Example, What's the amount of 500 l. forborn 4 years at 5 l. per cent. per annum?

See the work on the next page.

1,05 amount of 1 l. for 1 year.

```
       1,05
      -----
       525
      105
     ------
     1,1025
       1,05
    -------
      55125
    11025
    -------
    1,157625
        1,05
    --------
     5788125
   1157625
   ----------
   1,21550625
          500 principal sum.
   ----------
   607,75312500
```

607,75312500= l. 607 : 15 : 0 : 3 answer, and if from this you subtract the principal, there remains the interest = l. 107 : 15 : 0 : 3.

CASE II. If A, T, and R, be given; to find P.

RULE, Divide the amount continually so many times by the amount of 1 l. for 1 year as are the number of years proposed, and the last quotient is the answer.

Example, What principal sum will in 4 years time amount to l. 607 : 15 : 0 : 3 at 5 l. per cent. per annum?

Or (which is the same) what present sum of money will pay a dept of l. 607 : 15 : 0 : 3 due 4 years hence, rebate being made at 5 l. per cent. per an.? Ansr. l. 500.

CASE III. When P, A, and T are given; to find R.

RULE, Divide the amount by the principal, then from the quotient extract the root of that power correspondent to the time of forbearance, and it is the amount of 1 l. for 1 year; then as 1 l. is to its interest, so is 100 l. to the rate of interest required.

Example,

Example, If 500l. laid out upon compound interest and forborn 4 years, amounts to l. 607,753125 at the 4th year's end; I demand at what rate per cent. per annum? Ansr. at 5 l. per cent.

As the time here is 4 years, you must extract the Biquadrate root of the quotient.

400 l. forborn 8 years at compound interest, at the 8th year's end amounts to l. 590,98217515625, I demand at what rate per cent. per annum?

As the time here is 8 years, you must extract the root of the 8th power, and the answer will be 5 l. per cent.

Of Annuities.

CASE I. Annuitie, rate and time given; to find the amount.

RULE, Multiply the first yearly payment by the amount of 1 l. for 1 year, to the product add the 2d. yearly payment, and the sum is the amount in 2 years; which multiply'd again by the amount of 1 l. for 1 year, the product with the addition of the 3d. yearly payment is the amount for 3 years, &c.

Example, What will a yearly pension of 30l. amount to, being forborn 4 years at 5 l. per cent. per annum?

```
      30
    1,05
   -----
   31,50
   30
   -----
   61,5   amount in 2 years.
    1,05
   -----
    3075
   615
   -----
   64,575
   30
   -----
   94,575 amount in 3 years.
     1,05
   -----
  472875
  94575
  -------
  99,30375
  30
  -------
l. 129,30375 amount in 4 years, answer.
```

CASE II.

Case II. Annuity, rate, and time given; to find the present worth.

Rule, Divide the annuity continually so many times by the amount of 1l. for 1 year, as there are years proposed, and the sum of the particular quotients is the present worth required.

Example, What's the present worth of an annuity of 50l. to continue 4 years, rebate at 5l. per cent. per annum?

l.		
50,	divided by	47,61904
47,61904	1,05 gives	45,35147
45,35147		43,19187
43,19187		41,13512

Total l. 177,2975 the present worth.

Case III. Amount, rate, and time given; to find the annuity.

Rule, Suppose an annuity at pleasure, then, as 5 is to 100, so is the supposed annuity to a 4th number which call X, from the amount of which (found as in case first of compound interest) subtract X, by the remainder divide the product of the given amount with the supposed annuity, and the quotient is the annuity required.

Example, An annuity forborn 4 years upon compound interest at 5 l. per cent. amounts to l.129,30375; I demand the annuity?

Suppose 6l. to be the annuity.

then, if 5 : 100 :: 6
6

5)600(120=X.
0

See the rest of the work on the next page.

```
           1,05
           1,05
          ------
           525
          105
         -------
          1,1025
            1,05
        --------
          55125
         11025
        ---------
         1,157625
             1,05
        ---------
         5788125
        1157625
       -----------
       1,21550625
              120=X.
      --------------
      145,86075000
Sub.  120=X
      ------------
      25,86075

               129,30375=the amount.
                       6=the supposed annuity.
               -------------
      25,86075)775,82250(30 l.=the true annuity.
               ---------
                   0
```

CASE IV. Annuity, rate, and amount given; to find the time.

RULE, First, find a correspondent principal, thus,

```
         If 5 : 100 :: 30
                   30
               --------
               5)3000(600 correspondent principal.
               --------
                  0
```

Add

Add the correspondent principal to the given amount, divide that sum by the correspondent principal, and the quotient by the ratio (or amount of 1 l. for 1 year) and that quotient again by the said ratio, and so on till the ratio itself come out in the last quotient, and as many divisions as you make, so many years hath the annuity been forborn.

Example, How many years shall an annuity of 30 l. be forborn upon compound interest at 5 l. per cent. that it may amount to l. 129,30375? Ansr. 4 years.

I shall subjoin 2 or 3 examples more, and so conclude this rule.

A merchant hath owing to him l. 10000 to be paid in 5 years, viz. l. 2000 at the end of every year; and agrees with his debtor, that if he will pay him 5000 l. ready money, he will take the remainder in 21 years by an equal annual payment, compound interest being computed at 6 l. per cent. to which his debtor agrees; now what will this annual payment be?

Ansr. l. 291,11725.

Find the present worth of 2000 l. per annum to continue 5 years, and it is l. 8424 : 14 : $6\frac{1}{2}$, from which subtract 5000 l. and there remains l. 3424 : 14 : $6\frac{1}{2}$; then find what annuity to continue 21 years that remainder will purchase, and it will be l. 291,11725, the answer.

What is an estate of 200 l. per annum to continue for ever, worth in ready money, allowing the purchaser 5 l. per cent. per annum compound interest?

,05)200,00(4000 l. answer.

O

Suppose 4000 l. were proposed to be laid out in the purchase of a freehold estate; what annual rent would it buy, allowing the purchaser 5 l. per cent. per annum?

4000
,05
---
l. 200,00 annual rent.

Suppose a freehold estate of 200 l. per annum cost 4000 l. what rate of interest per cent. is allowed the purchaser?

4000)200,00(,05
100
0
l. 5,00 per cent.

What difference is there between the interest of 500 l. at 5 per cent. compound interest for 12 years, and the discompt of the same sum at the same rate, and for the same time?

What present money will discharge a debt of 7500 l. to be paid at the end of 10 years, rebate being made at l. 5 per cent. per annum compound interest?

What is the present worth of a reversion of a lease of 500 l. per annum to continue 20 years, but not to commence till after the end of 5 years, allowing the purchaser 6 l. per cent. per annum simple interest?

First find the present worth of the annuity for the whole time both of possession and reversion, and then deducting the present worth of the possession; the remainder will be the present worth of the reversion.

CHAP. XIV.

Rules of Practice.

THIS is of excellent use among merchants, tradesmen, &c. being a most compendious method of casting up their accompts, when the first term in the rule of three is unity: and for its quick and elegant dispatch of business, and frequent use, is called the Rule of Practice. In order for working, the following tables are to be well understood, and perfectly got by heart.

The

The even parts of a Shilling.			The even parts of a Pound.			
d.			s.	d.		
1	is	$\frac{1}{12}$	1	0	is	$\frac{1}{20}$
$1\frac{1}{2}$	is	$\frac{1}{8}$	1	8	is	$\frac{1}{12}$
2	is	$\frac{1}{6}$	2	0	is	$\frac{1}{10}$
3	is	$\frac{1}{4}$	2	6	is	$\frac{1}{8}$
4	is	$\frac{1}{3}$	3	4	is	$\frac{1}{6}$
6	is	$\frac{1}{2}$	4	0	is	$\frac{1}{5}$
			5	0	is	$\frac{1}{4}$
			6	8	is	$\frac{1}{3}$
			10	0	is	$\frac{1}{2}$

This rule admits of 4 cases.

CASE I. When the price of the integer is such a number of pence as make an even part of a shilling.

RULE, Consider what part of a shilling the price of the integer is, divide the quantity of goods thereby, and the quotient is the price of the goods in shillings, and those divided by 20, quotes the price in pounds.

Note, If any thing remains after your first division, each one of that remainder must be counted at the price of the integer, and is part of the price of the goods.

Examples.

What cost 611 doz. of pears at 1 d. per dozen?

Here it is obvious to any intelligent person, that (as the pears are at 1 d. the doz.) every 12 doz. of them must cost 12d. or 1s. and the 11 odd doz. counted at 1 d. the doz. will draw 11 d.: and so of any other example, when the price of the integer is in pence, and those pence an even part of a shilling.

See the work at large.

12)611(50s. price of 600 doz.

Rem. 11   11 price of 11 doz.

l. 2 : 10 : 11 price of 611 doz.

What cost 517 doz. of herrings at $1\frac{1}{2}$ d. per doz.?

Here

Here (as 1 doz. costs $1\frac{1}{2}$d. and as $1\frac{1}{2}$ is $\frac{1}{8}$ of a shilling) every 8 doz. must of consequence draw 1 shilling, and the remaining dozens must be counted at $1\frac{1}{2}$ per dozen: therefore I say 8 in 51, 6 times, and 3 over; then 8 in 37, 4 times, and 5 doz. over, which at $1\frac{1}{2}$d. per doz. draw $7\frac{1}{2}$ pence; consequently the price of 517 doz. is 64s. $7\frac{1}{2}$d.

517

s. 64 : $7\frac{1}{2}$ price.

l. 3 : 4 : $7\frac{1}{2}$ price also.

What cost 412 doz. of eggs at 2 d. per doz.?

s. 68 : 8 price.

l. 3 : 8 : 8 price also.

What cost 329 oranges at 3d. per piece?

s. 82 : 3 price.

l. 4 : 2 : 3 price also.

What cost 191 lb. of soap at 4d. per lb.?

s. 63 : 8 price.

l. 3 : 3 : 8 price also.

What cost 975 ells of ribbon at 6d. per ell?

s. 487 : 6 price.

l. 24 : 7 : 6 price also.

If there be an even part of the integer given, as $\frac{1}{2}$, $\frac{1}{3}$ $\frac{1}{4}$, $\frac{1}{8}$, &c. take the same part of the price of the integer for the price of that part, which added to the price of the given integers, the sum is the total price.

What

What cost 587 ells 1 quarter of brown cloth at 6 d. per ell?

587 : 1

s. 293 : 6 price of 587 ells.
 $1\frac{1}{2}$ price of $\frac{1}{4}$ ell.

l. 14 : 13 : $7\frac{1}{2}$ total price.

What cost $87\frac{1}{8}$ ells at 4 d. per ell?

s. 29 : 0 price of 87 ells.
 $0\frac{1}{2}$ price of $\frac{1}{8}$ ell.

l. 1 : 9 : $0\frac{1}{2}$ total price.

Again, If $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, &c. of the integer be given, as in this example;

What cost $89\frac{7}{8}$ lb. of raisins at 4 d. per lb.?

After you have got the price of the whole numbers, take $\frac{1}{8}$ of the price of the integer for its price, which state under the price of the integers; then, in your mind, multiply the price of that $\frac{1}{8}$ by 6, and the product is the price of $\frac{6}{8}$, which place under the price of $\frac{1}{8}$, add all up together, and the sum is the total price. *See the work.*

$89\frac{7}{8}$

s. 29 : 8 price of 89 lb.
 0,5 price of $\frac{1}{8}$ lb.
 3 price of $\frac{6}{8}$ lb.

l. 1 : 9 : 11,5 total price.

What cost $137\frac{5}{6}$ lb. at 3 d. per lb.?

s. 34 : 3 price of 137 lb.
 0,5 price of $\frac{1}{6}$ lb.
 2 price of $\frac{4}{6}$ lb.

l. 1 : 14 : 5,5 total price.

Lastly; When parts of parts of the integer are given, as if it was required to find the price of 158 ells 3 qrs. $3\frac{1}{2}$ nails of any kind of cloth at 6 d. per ell.

Here, find first (as before) the price of 158 ells, then for the price of 2 qrs. take $\frac{1}{2}$ the price of the integer, and for 1 qr. take $\frac{1}{2}$ the price of 2 qrs.; and (because 2 nails is the $\frac{1}{2}$ of 1 qr.) take $\frac{1}{2}$ the price of 1 qr. for its price, and $\frac{1}{2}$ the price of 2 nails for the price of 1 nail, and $\frac{1}{2}$ the price of 1 nail for the price of $\frac{1}{2}$ nail; and so of any other like example; as in the following work.

185

s. 79 - -	price of 158 ells.
0 : 3	price of 2 qrs.
1,5	price of 1 qr.
0,75	price of 2 nails.
0,375	price of 1 nail.
0,1875	price of $\frac{1}{2}$ nail.

l. 3 : 19 : 5,8125 total price

There is one thing more I would have the learner to take notice of, and it is this; When the quantity of goods is given in a higher name than the integer whose price is given, bring the goods to the same name with the given integer, and for the total price work as before.

What cost 16 gross of bottles at $1\frac{1}{2}$ d. per piece.

144
16

2304 bottles.

s. 288 price.

l. 14 : 8 price.

CASE II. When the price of the integer is pence, and no even part of a shilling.

RULE,

RULE. Divide the pence into even parts, take the price of the goods (as before) at each part, add the several prices together, and their sum is the total price required.

Examples.

What cost 734 ells of brown linen at 7 d. per ell?

734

s. 244 : 8 price at 4 d.
183 : 6 price at 3 d.

l. 21 : 8 : 2 total price.

What cost 673 lb. of sugar at $11\frac{1}{2}$ d. per lb.?

s. 336 : 6 price at 6 d.
224 : 4 price at 4 d.
84 : $1\frac{1}{2}$ price at $1\frac{1}{2}$ d.

l. 32 : 4 : $11\frac{1}{2}$ total price.

What cost 74 bolls 3 pecks $3\frac{1}{2}$ lippies of meal at $8\frac{1}{2}$ d. per peck?

b. *p.* *l.*
74 : 3 : $3\frac{1}{2}$
16

pecks 1187

s. 593 : 6 price of 1187 pecks at 6 d.
98 : 11 their price at 1 d.
148 : 4,5 their price at $1\frac{1}{2}$ d.
4,25 price of 2 lip.
2,125 price of 1 lip.
1,0625 price of $\frac{1}{2}$ lip.

l. 4 : 4 : 1 : 4,9375 total price.

When the price of the integer is pence and farthings, then work for the pence as before; and for the farthings, observe what part they make of the parts taken

taken before, which take out of any one of the lines of which the farthing or farthings make an even part, and add all together.

What cost 856 yards of ribbon at 5¾d. per yard?

s. 285 : 4 price at 4d.
107 : 0 price at 1½d.
17 : 10 price at ¼d.

l. 20 : 10 : 2 total price.

When the price of the integer is any number of pence above a shilling, and not two shillings, let the given quantity of goods stand as shillings, and take even parts for the odd pence, and set the prices underneath in proper order, without drawing a *line*, and add those to the given number, and the total will be the answer in shillings.

What cost 541 yards of cloth at 17d. per yard?

180 : 4 price at 4d.
45 : 1 price at 1d.

l. 38 : 6 : 5 price at 17d.

When the price of the integer is such a number of pence as make more shillings than one, multiply the quantity of goods by the number of shillings contain'd in those pence, and work for any odd pence as before.

What sterling money must be paid in London to receive in Paris 438 crowns; at 56d. per crown?

438
4

s. 1752 for 4s.
219 for 6d.
73 for 2d.

20)2044(102 l. 4s. Ster.

4

How

How much sterling shall I receive in London, if I pay in Genoa 820 dollars; exchange at 51d. per dollar, or piece of eight?

820
4
s. 3280 for 4 s.
205 for 3 d.
20)3485(174 l. 5 s. Ster.
5

Another table of the parts of a shilling.

d.			
10	is	$\frac{5}{6}$	of a shilling.
9	is	$\frac{3}{4}$	
8	is	$\frac{2}{3}$	
$7\frac{1}{2}$	is	$\frac{5}{8}$	
$4\frac{1}{2}$	is	$\frac{3}{8}$	

If the price of the integer be at any of the rates in this table, multiply the given quantity by the numerator, and divide by the denominator, and the quotient will be the answer in shillings.

What cost 746 lb. of soap at $4\frac{1}{2}$ (which is $\frac{3}{8}$ of a shilling) per lb.?

746
3
s.
8)2238(279,75 price.
l. 13 : 19 : 9 price.

What cost 671 foot of glass at $7\frac{1}{2}$ d. (which is $\frac{5}{8}$ of a shilling) per foot?

671
5
s.
8)3355(419,375 price.
l. 20 : 19 : 4,5 price.

 What

What cost 9 pieces of cloth, each 15 ells 3 qrs. 1½ nail at 7½ d. per ell?

15 : 3 : 1½
9

142 : 2 : 1½ total quantity.

71 - - price of 14 ells at 6 d.
17 : 9 - their price at 1½ d.
3,75 price of 2 qrs.
,46875 price of 1 nail.
,234375 price of ½ nail.

l. 4 : 9 : 1,453125 total price.

What cost 197 stone of hay at 5½ d. per stone?

65 : 8 price at 4 d.
24 : 7,5 price at 1½ d.

l. 4 : 10 : 3,5 total price.

CASE III. When the price of the integer is such a number of shillings, or shillings and pence as make an even part of a pound.

RULE, Take that part of the quantity of goods that the price of the integer is of a pound, and it is the price in pounds; and each one remaining must be counted at the price of the integer, as before.

Examples.

What cost 147 bolls of oats at 10 s. per boll?

l. 73 : 10 price.

What cost 631 ells of cambric at 6 s. 8 d. per ell?

l. 210 : 6 : 8 price.

What cost 1797 ells of cloth at 3 s. 4 d. per ell?

l. 299 : 10 price.

When

When at any time the price of the integer is 2 s. the answer may be known at sight; for 'tis but doubling the place of units for shillings; and all the other figures towards the left hand are pounds.

What cost 5768497 ells at 2 s. per ell?

Ansr. *l*. 576849 : 14

What cost 4 c.w. of tea at 4 s. per lb.?

112
4
---
448 lb.
---
l. 89 : 12 price.

What cost 311 spynles of yarn at 1 s. 8 d per spynle?

---
l. 25 : 18 : 4 price.

Case IV. When the price of the integer is such a number of shillings; or, shillings, pence and farthings, as make not an even part of a pound.

Rule, Divide the price into even parts, take the price of the goods at each part, and the sum of the several prices will be the total price. Thus, 7 s. 6 d. is compos'd of 5 s. and 2 s. 6 d. and 11 s. 8 d. of 10 s. and 1 s. 8 d. &c.

Or, multiply the quantity of goods by the shillings in the price, and take parts for the rest, and add all together.

Examples.

What cost 87 yards of fine linen at 7 s. 7 d. per yard?

7
---
s. 609 price at 7 s.
43 : 6 price at 6 d.
7 : 3 price at 1 d.
---
l. 32 : 19 : 9 total price.

What

What cost 5 c.w. of tea at 11 s. 11 d. per lb.?

112
5

lb. 560

280 - - price at 10s.
46 : 13 : 4 price at 1 s. 8 d.
140 : 0 price at 3 d.

l. 333 : 13 : 4 total price.

What cost 58 bolls of meal at 13 s. 4 d. per boll?

19 : 6 : 8 price at 6 s. 8 d.
2

l. 38 : 13 : 4 price at 13 s. 4 d.

What cost 143 gallons of spirits at 17 s. 7 d. 1 farthing per gallon?

143
17

2431 - price at 17 s.
71 : 6 price at 6 d.
11 : 11 price at 1 d.
2 : 11 : 3 price at 1 farth.

l. 2435 : 6 : 4 : 3 total price.

What cost 45 ankers of brandy, at 19 s. 11 d. per anker?

l. 45 price of 45 ankers at 20 s.
Sub. 3 : 9 for 1 d. too much.

l. 44 : 16 : 3 real price.

When the price of the integer is an even number of shillings, and it is required to know what quantity of any thing may be bought for so much money; annex

a

a cypher to the money, divide by half the propos'd price of the integer, and the quotient is the quantity required.

How many pounds of indigo may I purchase for 54 l. at 4s. per lb.?

,2)54,0(270 lb. answer.

3

What quantity of cloth at 12 s. per yard may be bought for l. 127?

,6)127,0(211,666 yards, answer.

4

When the price of the integer is an even number of shillings, multiply the quantity by half the price, and in multiplying, double the first figure of the product, and set it a part for shillings; and the other figures to the left hand will be pounds.

What cost 7976 yards of Spanish cloth at 14 s. per yard?

7976
,7

l. 5583 : 4 price.

What cost 18 packs of brown linen, each pack 14 pieces, and each piece $15\frac{1}{2}$ ells at 8 s. Scots per ell?

$15\frac{1}{2}$
14

217
18

3906 ells.
4

l. 1562 : 8 price.

What cost 487 lb. of tea at 16s. per lb.?
8

l. 389 : 12 price.

When

When the price is an odd number of shillings, work for the even part, as just now, and for the odd shilling take $\frac{1}{20}$ of the given number.

What cost 216 lb. of China silk at 17s. per lb.?

216
8

172 : 16 price of 216 lb. at 16s.
10 : 16 price at 1 s.

l. 183 : 12 total price.

When the price of the integer is any number of shillings above a pound, and not two pounds, let the quantity of goods stand as pounds, and take even parts for the odd shillings and pence; set the prices down in proper order, and the sum total is the price required.

What cost 13 c.w. of tobacco at l. 1 : 14 : 8 per c.w.?

13 price at 1 l.
9 : 2 price at 14s.
6 : 6 price at 6 d.
2 : 2 price at 2 d.

When the price is such a number of shillings as make more pounds than one, multiply the quantity of goods by the number of pounds contained in those shillings; and work for any odd shillings and pence as before.

What cost 44 bolls 3 firlots of meal at l. 6 : 7 : 6 Scots per boll?

44 : 3
6

264 - - - price of 44 bolls at 6 l.
11 - - - price at 5 s.
5 : 10 - price at 2 s. 6 d.
3 : 3 : 9 price of 2 firlots.
1 : 11 : 10½ price of 1 firlot:

l. 285 : 5 : 7½ total price.

In

In working the two preceeding examples, you might have multiply'd the price of the integer by the component parts of the quantity, and have taken the price of the odd parts of the quantity as before.

How much Ster. money may I give for 84½ pieces of l. 3 : 12 per piece?

3 : 12	
7	
25 : 4	value of 7 pieces.
12	
302 : 8	value of 84 pieces.
1 : 16	value of ½ piece.
l. 304 : 4	answer.

What Ster. money must be paid in London to receive at Paris 438 crowns, exchange at 4s. 6d. per crown?

438	
87 : 12	value of them at 4s.
10 : 19	value at 6d
l. 98 : 11	answer.

What Ster. money may I draw for in London, if I pay in Venice 512 ducats, exchange at 4s. 7½ d. per ducat?

512	
102 : 8	value at 4s.
12 : 16	vale at 6d.
3 : 4	value at 1½
l. 118 : 8	answer.

What

What Ster. money may I give for 48009 rees, exchange at 6s. 5d. per mill-ree?

48009

9601 : 16 at 4s.
4800 : 18 at 2 s.
800 : 3 at 4 d.
200 : 0 : 9 at 1 d.

1000)15402 : 17 : 9(l. 15 : 8 : 0½ answer.

What cost 16 c.w. 2 qrs. 17 lb. of tea at l. 28 : 6 : 8 per c.w.?

28 : 6 : 8
4

113 : 6 : 8 price of 4 c.w.
4

453 : 6 : 8 price of 16 c.w.
14 : 3 : 4 price of 2 qrs.
3 : 10 : 10 price of 14 lb.
10 : 1,428 price of 2 lb.
5 : 0,714 price of 1 lb.

l. 471 : 16 : 0,142 total price.

If I pay in London l. 492 Ster. what may I draw my bill for to Amsterdam; exchange at 34s. 5d. Flemish per l. Ster.?

492
34

1968
1476

16728 at 34 s.
164 at 4 d.
41 at 1 d.

20)16933(846l. 13s. answer.

New

Now, if you multiply the said l. 846 : 13 Flemish by 6, it will produce guilders and stivers; thus,

846 : 13
6

5079 G : 18 S.

What will the yearly rent of 57 acres 3 roods 13 falls of land amount to in 7 years at l. 9 : 11 : 2 per annum Scots money?

9 : 11 : 2
8

76 : 9 : 4 rent of 8 acres for 1 year.
7

535 : 5 : 4 rent of 56 acres for 1 year.
9 : 11 : 2 rent of 1 acre.
4 : 15 : 7 rent of 2 roods.
2 : 7 : 9,5 rent of 1 rood.
11 : 11,375 rent of 10 falls.
2 : 11,6875 rent of 2 falls.
1 : 5,84375 rent of 1 fall.

l. 552 : 16 : 3,40625 rent for 1 year.
7

l. 3869 : 13 : 11,84375 rent for 7 years.

Here, you may observe that when I had got the rent of 57 acres for 1 year, I took ½ the rent of 1 acre for 2 roods, and ½ the rent of 2 roods for 1 rood, and ¼ the rent of 1 rood for 10 falls, and ⅕ the rent of 10 falls for 2 falls, and ½ the rent of 2 falls for 1 fall; and so you may work any example of this nature.

Suppose 15 c.w. 2 qrs. 13 lb. tare were allowed on 456 c.w. 1 qr. 19 lb. of tobacco; what would be the neat weight?

Tare is an allowance made by the seller to the buyer, either (as here) at so much on the whole; or at so much on the cask, box, bag, &c. or at so much per c.w.

If on the whole, subtract the allowance from the gross weight, and the remainder is the neat weight.

See the work.

c.	q.	lb.	
456	1	19	gross.
15	2	13	allowance.
440	3	6	neat weight.

If on the cask hogshead, &c. multiply the lb. tare by the number of casks, &c. and subtract as before.

What's the neat weight of 7 hogsheads of tobacco each 5 c.w. 1 qr. 11 lb. gross, tare 20 lb. per hogshead?

20
7

total tare 140 lb. = 1 c.w. 1 qr.

c.w	q.	lb.	
5	1	11	
		7	
37	1	21	gross.
1	1	00	tare.
36	0	21	neat.

And if at so much per c.w. then take such part or parts of the gross weight as the allowance is of an c.w.

What's the neat weight of 246 c.w. 3 qrs. 12 lb. gross of sugars; tare 14 lb. per c.w.?

8)246 : 3 : 12
30 : 3 : 12 total tare.

216 : 0 : 0 neat.

What's the neat weight of 428 c.w. 1 qr. 19 lb. gross; trett 4 lb. per 104 lb.?

Trett

Trett being always 4 lb. per 104, the method of finding it, is by taking the $\frac{1}{26}$ part of the line it is to be deducted from, 4 times 26 being 104.

26)428 : 1 : 19	
16 : 1 : 25$\frac{1}{2}$	trett.
411 : 3 : 21$\frac{1}{2}$	neat.

What will be the neat weight of 5647 c.w. 3 qrs. 13 lb. gross; allowing for clough 2 lb. for 3 c.w.?

As tare is an allowance for the cask, chest, bag, &c. that contains the commodity; and trett for the waste, motes or dust; so cloff or clough is an allowance of 2 lb. for every 3 c.w. for the turn of the scale, and is found by taking $\frac{1}{168}$ part of the line it is to be deducted from, 2 lb. being the $\frac{1}{168}$ part of 336 lb. pr 3 c.w.

168)5647 : 3 : 13	gross.
33 : 2 : 13	clough.
5614 : 1 : 0	neat.

What's the neat weight of 3 hogsheads of tobacco weing gross 15 c.w. 3 qrs. 20 lb. tare 7 lb. per c.w. trett 4 lb. per 104 lb. and clough 2 lb. for 3 c.w.?

Note, what I mean by gross weight, is the whole weight, before any allowances are made; when part is deducted the remainder is called suttle; and when all is taken from it, what's left, is called Neat.

c.w.	q.	lb.	
15	3	20	gross.
	3	27$\frac{1}{2}$	tare.
14	3	20$\frac{1}{2}$	suttle.
	2	8$\frac{1}{4}$	trett.
14	1	12$\frac{1}{4}$	suttle.
		9$\frac{1}{2}$	clough.
14	1	2$\frac{3}{4}$	neat.

In

In this preceeding example, the allowances are found by the foregoing rules, and tho' many more examples might be offered, I think those already given are sufficient for the improvement of any ingenious person: I now proceed to shew how to multiply numbers of divers denominations by numbers of divers denominations; and this I shall do by illustrating each step in the process of the following example.

What will be the product when l. 24 : 14 : 8 is multiply'd by l. 6 : 18 : 8?

24 :	14 :	8	
6 :	18 :	8	
148 :	8 :	0	product of 24 : 14 : 8 by 6.
12 :	7 :	4	half the top line for 10s.
8 :	4 :	$10\frac{2}{3}$	is $\frac{1}{3}$ of said line for 6 s. 8 d.
2 :	9 :	$5\frac{3}{5}$	is $\frac{1}{10}$ of said line for 2 s.
l. 171 :	9 :	$8\frac{4}{15}$	product.

Here I multiply'd the top line by 6 at the bottom, and the product was l. 148 : 8, then for 10 s. at the bottom I took half the top line; and for 6 s. 8 d. I took $\frac{1}{3}$ of the said line; and lastly, for 2 s. at bottom I took $\frac{1}{10}$ of the top line; and the sums being added, made l. 171 : 9 : $8\frac{4}{15}$ or, 266, and so of any other.

If I had reduced the shillings and pence in both factors to vulgar or decimal fractions, and multiply'd them as mixed numbers, the answer would have been the same.

Or, if I had reduced both factors to pence, and divided their product by the square of (240) the pence in a pound, the quotient would have been the pound, and the remainder (if any) parts of a pound, to be managed as taught in Division.

See the work this way.

5936 pence in l. 24 : 14 : 8
1664 pence in l. 6 : 18 : 8

l. s. d.
sqr. of 240=57600)9877504 product.(171:9:8,266 as (before.

Rem. 27904
20

57600)558080(9

Rem. 29680
12

57600)476160(8,266

Rem. 3840

	l.	s.	d.	
Mul.	43	11	7	
by	9	11	11	
	479	7	5	product of 43 : 11 : 7 by 9.
	21	15	9,5	for 10s. at bottom.
	2	3	6,95	for 1 s. at bottom.
	1	1	9,475	for 6 d. at bottom.
		14	6,3166	for 4 d. at bottom.
		3	7,57915	for 1 d. at bottom.
	l. 505	6	8,82075	product.

Here, as before, I multiply'd the top line by the pounds in the bottom line (beginning first to multiply the pence) then for 10 s. at bottom, I took half the top line; and for 1s. at bottom, I took $\frac{1}{20}$ part of the top line, or $\frac{1}{10}$ of that at 10s.; and for 6d. at bottom, I took the half of that at 1s.; and for 4 d. I took $\frac{1}{3}$ of that at 1s.; and then for 1 d. I took the $\frac{1}{4}$ of that at 4d. and those sums added made l. 505 : 6 : 8,82 for the product required.

There

There is a stone 8 feet 7 inches 8 parts long, and 3 feet 5 inches 3 parts broad, how many square superficial feet doth it contain?

f.	i.	p.	
8 :	7 :	8	
3 :	5 :	3	
25 :	11 :	0	product of 8 : 7 : 8 by 3 feet.
2 :	10 :	6,666	for 4 inches at bottom.
	8 :	7,666	for 1 inch at bottom.
	2 :	1,916	for 3 parts at bottom.
29 :	8 :	4,248	product.

Here, I multiply'd the top line by the 3 feet at the bottom, (beginning at the parts, 12 of which make an inch) then for 4 inches at bottom, I took $\frac{1}{3}$ of the top line; and for 1 inch at the bottom I took $\frac{1}{4}$ of that at 4 inches; and for 3 parts at bottom, I took $\frac{1}{4}$ of that at 1 inch; and the product was 29 : 8 : 4,248.

And, if I had squared 144 (the parts in a foot) and by that square divided the product of the factors in parts, the answer would have been the same.

How much linen may I give for 18 yards of cambric, when 1 yard of the cambric is worth 3 yards 3$\frac{1}{2}$ of the linen?

y	q	n	
18 :	0 :	0	
3 :	3 :	2	
54 :	0 :	0	product of 18 by 3 yds.
9 :	0 :	0	is $\frac{1}{2}$ of top for 2 qrs at bottom.
4 :	2 :	0	for 1 qr. at bottom.
2 :	1 :	0	for 2 nails at bottom.
69 :	3 :	0	linen, answer.

What quantity of pease may I give for 15 bolls 7 pecks 3 lippies of wheat, when 1 boll of the wheat is worth 2 bolls 1 firlot 1$\frac{1}{2}$ lippie of the pease?

See the work on the next page. 15

b. f. p. l.
15 : 1 : 3 : 3
2 : 1 : 0 : 1½

30 : 3 : 3 : 2 product of top line with 2 bolls.
3 : 3 : 1 : 3,75 = $\frac{1}{8}$ the top line for 1 firlot.
3 : 3,484 = $\frac{1}{16}$ of that $\frac{1}{4}$ for 1 lippie.
1 : 3,742 = $\frac{1}{2}$ of that $\frac{1}{16}$ for ½ lippie.

35 : 0 : 3 : 6,976 product, of pease.

How much suggar may I give for 14 c.w. 3 qrs 16 lb. of tea, when 1 c.w. of the tea is worth 5 c.w. 2 qrs. 20 lb. of the sugar?

c.w. q. lb.
14 : 3 : 16
5 : 2 : 20

74 : 1 : 24 product of top line with 5 c.w.
7 : 1 : 22 = $\frac{1}{2}$ the top line for 2 qrs.
1 : 3 : 12,5 = $\frac{1}{4}$ of that $\frac{1}{2}$ for 14 lb.
1 : 1,7857 = $\frac{1}{7}$ of that $\frac{1}{4}$ of 2 lb.
2 : 3,5714 = to twice $\frac{1}{7}$ for 4 lb.

84 : 2 : 7,8571 sugar.

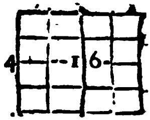
CHAP. XV.

Extraction of the Square Root

TO extract the square root, is to find the side of a square figure; or numerically speaking, it is to find out such a number, which multiply'd into itself, will produce the number given. Thus the square root of 64 is 8, 8 times 8 making 64.

What

What a square is, may be seen by the figure here annexed; which being divided every way into 4 equal parts, its whole content or square is 16, and its side or root is 4.



Square numbers are either single or compound.

A single square number is always less than 100, being produced by the multiplication of some one single figure by itself; as 36 from 6, &c. so that the root of any single square may be found in the following table, always taking the root of the next less square, for any number not there inserted; as, for 37 take 6, or 3 for 10, &c.

Squares	1	4	9	16	25	36	49	64	81
Roots	1	2	3	4	5	6	7	8	9

A compound square number being composed by the multiplication of two or more figures by themselves, always exceeds 100; as 144, which is 12 times 12; or 441, which is 21 times 21, &c.

If the root therefore is express'd by two figures, its square must at least consist of three; for the least root express'd by two figures is 10, whose square is 100. And if the root has three figures, its square must at least have five. If four, the square has seven, &c. So that you cannot augment the root one figure, but you increase the square two.

The root of any compound square number may be found by observing the following directions.

1st. You must point your given number, that is, make a point over the units place, another upon the hundred's, and so upon every second figure throughout. And so many points as happen, so many places will the root consist of.

2dly. Then seek the greatest square number in the first point towards the left hand, placing the square

number

number under the first point, and the root thereof in the quotient, and subtracting the said square number from the first point, to the right hand of the remainder, bring down the next point, and call that the Resolvend.

3dly. Then double the quotient, and place it for a divisor on the left hand of the resolvend; and seek how often the said divisor is contain'd in the resolvend (reserving always the units place) and put the answer in the quotient; and also on the right hand of the divisor, then multiply by the figure last placed in the quotient, and subtracting the product from the resolvend, (as in common division) bring down the next point to the remainder (if there be any more) and proceed as before.

Note, If the divisor multiply'd by the quotient figure, gives a product greater than the resolvend, 'tis false and must be rectified by a smaller quotient figure.

Note also, If there be any remainder after the work is ended, and if you have a mind to find the value of the same, you may annex cyphers by two at a time to the remainders, and so prosecute the work to what number of decimal parts you please; and as many parts of cyphers as are added, so many places of decimal parts will there be in the root.

Examples.

Let 106929 be a number given, and let the square root thereof be required.

```
     106929(327 root
      9
   62)169 resolvend.
      124
  647)4529 resolvend.
      4529
         0
```

Here, I first pointed the given number, as before directed, putting a point upon the units, hundreds,

 and

and tens of thousands; then I sought what was the greatest square number in 10, (the first point) which by the little table I found to be 9, and three the root thereof; I placed 9 under 10, and 3 in the quotient, then I subtracted 9 from 10, and there remained 1, to which I brought down 69 the next point, and it made 169 for a resolvend; I then doubled the quotient 3, and it made 6, which I plac'd on the left hand of the resolvend for a divisor; then I sought how often 6 in 16, the answer was twice; I put 2 in the quotient, and also on the right hand of the divisor making it 62. Then I multiply'd 62 by the 2 I put in the quotient, and the product was 124, which I subtracted from 169 the resolvend, and there remain'd 45, to which I brought down 29 the next point, and it made 4529 for a new resolvend: then I doubled the quotient 32, and it made 64, which I placed on the left hand of the said new resolvend for a divisor, and seeking how oft 64 in 452, I found it 7 times, I then put 7 in the quotient, and also on the right hand of the divisor, making it 647, which I multiply'd by the 7 in the quotient, and it made 4529, and subtracting the same from the resolvend, there remained nothing: so 327 is the square root of the given number 106929 which root multiply'd into itself, that is 327 by 327, will produce the number 106929, and proves the work to be right. And so of any other

What's the square root of 547?

```
       547(23,388 root very near
       4
     43)147
        129
    463)1800
        1389
   4668)41100
        37344
  46768)375600
        374144
   Rem.   1456
```

Now

Now, if you square 23,388, or multiply it on itself, and to the product add the remainder, the sum will be equal to 547 the given number.

To extract the square root of a mixed number.

RULE, make the number of decimal places even, that is, 2, 4, 6, 8, &c. that so there may a point fall upon the units place of the whole numbers, and proceed to work as if they were all integers, only you must put a separating point betwixt the integral and the fractional part of the root.

What's the square root of 751417,57456?

```
        751417,574560(866,843
        64
       166)1114
           996
      1726)11817
           10356
     17328)146157
           138624
    173364)753345
           693456
   1733683)5988960
           5201049
      Rem. 787911
```

To extract the square root of a vulgar fraction.

RULE, Find the root of the numerator, for a numerator, and let the root of the denominator be its denominator; thus the square root of $\frac{9}{64}$ is $\frac{3}{8}$, and of $\frac{16}{25}$ is $\frac{4}{5}$, and $\frac{49}{81}$ is $\frac{7}{9}$.

The root of a mixed number is also found after the same manner, being first reduced into an improper fraction; thus $6\frac{1}{4}$ reduced, makes $\frac{25}{4}$, the root of which is $\frac{5}{2}$ or, $2\frac{1}{2}$. But if either a proper fraction or a mixed number

be

be incommensurable to its root; to extract it, reduce the fraction to a decimal of an even number of places, and extract the root as if it were a whole number; thus $\frac{7}{8}$ reduced to a decimal, will be ,87,0 the square root of which is ,935 &c. but had it been $4\frac{7}{8}$, which reduced is 4,8750, the root would be 2,20794, &c.

What's the square root of $\frac{3}{960}$?

960)3,00000009(,0031250(,0559 root.
25
0 105)625
525
1109)10000
9981
Rem. 19

Here you may observe, that because the first point consisted of cyphers, I put a cypher first in the quotient.

The work of extracting the square root of a decimal fraction differs nothing from that of whole numbers, only mind to point your given number aright; for (as was before directed) the number of places must always be made even, and then begin to point at the right hand, as in whole numbers.

Let ,125 be a decimal fraction given, whose square root is required; and let it be required to have 4 places of decimals in the root.

,12500000(,3535 root.
9
65)350
325
703)2500
2109
7065)39100
35325
Rem. 3775

Some

Some more examples for practice.

2268741(1506,23 root.

Rem. 121871

To prove this rule square the root, and to the product add the remainder (as was before directed) and if the sum be equal to the given number, the work is right.

Let us then prove the work of the last example.

Mult. 1506,23
by 1506,23

2268728,8129 square.
add 12,1871 rem.

2268741,0000 given number.

What's the square root of 7596796?

7596796(2756,228 root.

Here I took down three pairs of cyphers, and the remainder was 3212016.

7514175745(866,84 root.

Rem. 59889

656714,375120(810,379 root.

Rem. 251479

,007150(,084 root.

Rem. 94

582169(763 root.
0

322442265(56784 root.
0

C H A P. XVI.

CHAP. XVI.

Extraction of the Cube Root.

TO extract the cube root is to find the side of a solid figure, whose length, breadth and depth are equal; or, numerically speaking, it is to find what number multiply'd twice into itself will produce the number given; thus the cube root of 27 is 3, for 3 times 3 is 9, and 3 times 9 is 27.

What a cube is, may be easily conceived, by attentively considering the dimensions of a square solid foot, for it is 12 inches in length, 12 in breadth, and 12 in height; so that 1728 is the cube (for 12 times 12 is 144, and 12 times 144 is 1728) and 12 is the *root*.

Cube numbers are either single or compound.

A single cube number is always less than 1000, being produced by the multiplication of one single figure, first by itself, and then by its product; as 216 from 6, &c. so that the root of any single cube may be found in the annexed table; always remembering to take the root of the next less cube for any number not there inserted, as for 736 take 9, or 5 for 128, &c.

Cubes	1	8	27	64	125	216	343	512	729
Squares	1	4	9	16	25	36	49	64	81
Roots	1	2	3	4	5	6	7	8	9

A compound cube number, being composed by the multiplication of 2 or more figures, first by themselves, and then by their product, always exceeds 1000, as 2197 from 13, or 10648 from 22, &c.

If therefore the root is expressed by 2 figures, its cube must at least consist of four; for the least root express'd by two figures is 10, whose cube is 1000;

if

if the root has 3 figures, its cube must at least have seven, &c. so that you cannot augment the root one figure, but you increase the cube three.

Therefore, to find the root of any compound cube number, as suppose 15625.

1st. Distinguish it into single cubes by placing points over every third figure, beginning from the right hand, thus, 15625

And so many points as happen, so many places there will be in the root; which in this example are two.

2dly. Draw a crooked line on the right hand of your number, seek the greatest root of 15 your first single cube, which by the preceeding table you will find to be 2, which place in the quotient, and the cube thereof viz. 8 place under 15, subtract the one from the other, and there remains 7, as you see, if you observe the work. This is your first work, and no more to be repeated.

```
15625(2
 8
----
 7
```

3dly. To your remainder 7 bring down your next and last single cube, and it will make 7625 for a dividend; and the work will stand thus;

```
15625(2
 8
-------
 7625 dividend.
```

4thly. Square your quotient 2, and it makes 4, which multiply by 300, and the product is 1200 for a divisor; then the work will stand thus;

```
              15625(2
               8
Divisor 1200) 7625 dividend.
```

5thly. Seek how oft 1200 in 7625; answer but 5 times, because of the increase that will come from the quotient; and the work will stand thus;

See the next page. 15625

15625(25
8

Divisor 1200)7625 dividend.

6thly. Draw a line under the dividend, then multiplying your divisor 1200 by 5 (the figure last placed in the quotient) the product is 6000, which place orderly under the dividend; and the work will stand thus;

15625(25
8

Divisor 1200)7625 dividend.

6000

7thly. Proceed to find the increase coming from the quotient; thus, square 5 your last quotient figure, and it makes 25, which multiply by 2 the rest of the quotient, the product is 50, multiply this by 30, and it makes 1500, which place orderly under your last number 6000; and the work will stand thus;

15625(25
8

Divisor 1200)7625 dividend.

6000
1500

8thly. Cube 5 (the figure last placed in the quotient) it makes 125, which place orderly under your last number 1500, and add those 3 subducends (for so many you have in every operation after the first) into one sum, and they make 7625, which you must subtract from the dividend; and seeing they are equal, and no more periods, or single cubes to bring down, you see the work is finished, and that the given number is a right cube number whose root is 25.

See

See the whole work.

15625(25 root.
8

Divisor 1200)7625 dividend.

$$\left.\begin{array}{r}6000\\1500\\125\end{array}\right\}\text{subducends.}$$

7625 from dividend sub.

0

Note, If there had been more places in the number given, the next single cube should have been brought down to the remainder for a new dividend; and the work of the 4th, 5th, 6th, 7th, and 8th rules must be repeated as often as you form a new dividend.

Note also, If the sum of the subducends is greater than the dividend, the work is false, and must be rectified by placing a lesser figure in the quotient.

I might have here laid down other rules for extracting the cube root, but this being the easiest and most concise method by natural numbers, I advise the learner to hold by the same, till he comes acquainted with the way of doing it by the logarithms, which is this;

For the square root, $\frac{1}{2}$ the logarithm of the given number is the logarithm of the root.

For the cube root, $\frac{1}{3}$ the log. of the given number is the log. of the root.

For the biquadrate root, $\frac{1}{4}$ the log. of the given number is the log. of the root.

And $\frac{1}{5}$ the log. of the given number is the log. of the 5th power, &c.

But for the satisfaction of the curious and such as understand not logarithmical arithmetic, I shall by and by shew the reader how the roots of some of the higher powers may be found by natural numbers.

If your number to be extracted hath a remainder, and the exact root cannot be discovered by art, you must add to the remainder in every operation 3 cyphers, and so work as far as you will, according to the foregoing rules.

What's the cube root of 282?

282(6,557 root.
216

Divisor 10800)66000 dividend.

54000
4500 } subducends
125

58625 from dividend sub.

Divisor 1267500)7375000 dividend.

6337500
48750 } subducends.
125

6386375 from dividend sub.

Divisor 128707500)988625000 dividend.

900952500
962850 } subducends.
343

901915693 from dividend sub.

Remainder 86709307

In extracting the cube root of a mixed number, always observe to make the decimal part consist of either three, six, nine, &c. places, that is, always to consist

of

of even points, that a point may fall upon the units place of the whole numbers; as in this example.

What's the cube root of 93759,57507?

93759,575070(45,42 root.

Rem. 5918698 2

If you are to extract the cube root of a decimal fraction, it is done the same way as in whole numbers; only, as was before directed, the decimal must always consist of three, six, nine, &c. places, and if it be not so, it must be made so, by annexing of cyphers.

What's the cube root of ,0001416?

,000141600(,052 root.

Rem. 992

What's the cube root of ,401719179?

,401719179(,737 root.

Rem. 1403626

If a mixed number or vulgar fraction be commensurable to its root, then extract the cube root of the numerator for a numerator, and the cube root of the denominator for a denominator; and if it be a mixed number, reduce it to an improper fraction, and manage it the same way: so the cube root of $\frac{27}{64}$ will be $\frac{3}{4}$, and the cube root of $\frac{216}{27}$ will be $\frac{6}{3}$, and the cube root of $\frac{125}{8}$, or $15\frac{5}{8}$ will be $\frac{5}{2}$, or $2\frac{1}{2}$, and so of any other.

But if your fraction or mixed number be incommensurable to its root, reduce the single fraction to a decimal, and extract the cube root as before; reduce also the fractional part of the mixed number to a decimal, annex it to the integral part, and extract the cube root as before; always minding to make your decimal to consist of three, six, nine, &c. places, as formerly directed.

What's

What's the cube root of $\frac{3}{4}$, to three places of decimals?

,750000000(,908 root.
729

Divisor 2430000)21000000 dividend.

19440000
172800 } subducends.
512

19613312 from dividend subtract.

Rem. 1386688

What's the cube root of $\frac{5}{276}$?

276)5,000000000(,018115942(,226 root.

What's the cube root of $25\frac{5}{8}$?

25,265000000000(2,9481 root.

What's the cube root of 311665752?

311665752(678 root.

What's the cube root of 259697989?

259697989(638 root.
rem. 3917

The proof of this extraction, is by multiplying the root found, first by itself, and that product again by the root; adding the remainder if there be any, to the last product; and if that sum be equal to the given number, the work is right.

Let then the last example be thus prov'd

638
638

Square 407044
638

Cube 259694072
add 3917 rem.

259697989=the given number.

I shall here shew how to find the the square and cube roots of an irrational number near, without the use of decimal fractions; and it is thus,

I. For

I. For the ſquare root. After you have found the integral part of your root; to its quadruple add unity for the denominator of the fractional part, and the remainder doubled is the numerator: ſo the root of 166 will be $12\frac{32}{49}$, and thus of any other.

II. For the cube root. After you have found the integral part of the root, to the treble thereof add unity, and that ſum added to the treble ſquare of the ſaid root is the denominator to which the remainder is a numerator; ſo the cube root of 30 will be $3\frac{25}{27}$ near enough for ordinary practice.

For finding the biquadrate root, or root of the 4th power, extract the ſquare root twice; that is, the ſquare root of the ſquare root of the given number is the root of the 4th power.

The cube root of the ſquare root of the given number is the root of the 6th power.

The ſquare root of the ſquare root of the ſquare root of the given number is the root of the 8th power.

The cube root of the cube root of the given number is the root of the 9th power.

Extract the cube root of the given number, and then the biquadrate root of that cube root for the root of the 12th power.

The ſquare root of the ſquare root of the ſquare root of the ſquare root of the given number is the root of the 16th power.

The cube root of the cube root of the ſquare root of the given number is the root of the 18th power.

The cube root of the ſquare root of the ſquare root of the ſquare root of the given number is the root of the 24th power.

Extract the cube root of the given number thrice for the root of the 27th power.

Extract the ſquare root of the given number five times for the root of the 32d. power.

Extract the biquadrate root of the given number, and then twice the cube root of that root for the root of the 36th power.

Extract

Extract the square root of the given number four times, and the cube root of the last square root is the root of the 48th power.

Extract the square of the cube root of the given number, and then extract the cube root of that square root twice, for the root of the 54th power.

Extract the square root of the given number 6 times for the root of the 64th power.

The best and easiest way of extracting the root of the 5th, 7th, 10, &c. powers not here mentioned, is by the Logarithms; or you may read Mr. Ward's Mathematician's Guide.

Note, If you divide the amount of any sum at compound interest by the given principal, and from the quotient extract the root of that power correspondent to the time of forbearance, you will have the amount of 1 l. at the first year's end; then as 1 l. is to its interest, so is 100 l. to the rate of interest per cent. per annum.

l. 155 : 10 laid out upon compound interest at the end of 18 years amounted to the sum of l. 374,22929083896356537733740043640136718 75; I demand at what rate per cent. per annum?

Here, after division, you must extract the root of the 18th power; and the answer will be 5l. per cent.

The use of the Square and Cube Roots.

PROBLEM 1. To find a mean proportional between any two numbers given.

The square root of the product of the given numbers is the mean proportional sought.

The length of a long square is 30, and breadth 24; what's the side of a square equal?

```
    24
    30
   ----
   720(26,8328 mean proportional or side
                     of a square equal.
```

The

The perpendicular of a right angled triangle is 30, and base 40; what's the side of a square equal?

```
     2)40(20
          30
          ----
          600(24,4948 mean proportional, or
          4                side of a square equal.
        ----
       44)200
          176
      484)2400
          1936
    4889)46400
         44001
         -----
          2399 &c.
```

PROB. 2. To find the side of a square equal in area to any given superficies whatsoever.

The square root of the content of any given superficies is the side of the square equal.

A gentleman has a triangular garden of 164 falls, and wants a new one that shall be a right square; what will be its side?

164(12,80624 falls, side.

PROB. 3. The sum of the squares of two numbers is 3161, and the square of their half difference is 20,25; I demand the two numbers?

From $\frac{1}{2}$ the sum of the squares subtract the square of their half difference, and the square root of the remainder is the half sum of the two numbers; to this square root add the square root of the square of their half difference, the sum is the greater number, and if the said square root of the square of their half difference be subtracted from the first square root, the remainder is the lesser number.

2)3161(1580,5 = $\frac{1}{2}$ sum of their squares.

20,25 = the square of $\frac{1}{2}$ their difer.

1560,25(39,5 their half sum.

20,25(4,5 square root of the square of $\frac{1}{2}$ their dif.
add 39,5

44 greater number sub. 39,5 − 4,5 = 35 lesser number.

PROB. 4. If the sum of the squares of half the sum, and half the difference of two numbers be 1580,5, and one of the said numbers be 35; what is the other number?

From 3161 the double sum of the squares, subtract 1225 the square of 35 the given number, the remainder is 1936 whose square root is 44 the other number.

PROB. 5. To divide a number given, by extreme and mean proportion.

Multiply the square of the given number by 5, divide the product by 4, from the square root of the quotient subtract $\frac{1}{2}$ the given number, and the remainder is the greater portion, which subtracted from the whole gives the lesser.

Let the given number be 12, and you will find the two numbers will be $7\frac{22}{53}$ and $4\frac{31}{53}$.

PROB. 6. Let there be an army of 32400 men given to be form'd into a square battle, what will be the number either in rank or file?

The square root of 32400 is 180 the number in rank, or in file.

PROB. 7. A gentleman has a garden of 120 falls, and wants a square one that shall contain thrice as much; what will be the side of the square one?

Multiply 120 by 3, and the product is 360, whose square root is 18,9734=18 falls 5,8 ells.

PROB. 8. There are two numbers, to the lesser of which if you add 4, and multiply that sum by the greater wanting 7, the product will be 168; and on the contrary, if from the lesser you subtract 7, and multiply the remainder by the greater added to 4, the product will be 69; what are the 2 numbers?

To the greatest given product 168 add the product of of 7 and 4 (viz 28) and multiply that sum by 4; also to the least given product 69, add the product of 7 and 4, multiply the sum by 7; to which product add the former product, divide the sum by the sum of 7 and 4, and reserve the quotient for use.

To 69 add the product of 7 and 4, from that sum subtract the square of 7; also to 168 add the product of 7 and 4, and from that sum subtract the square of 4; divide the difference of these two remainders by the sum of 7 and 4, and square the quotient; then to $\frac{1}{4}$ of that squar'd quotient add the reserved quotient, to the square root of that sum add $\frac{1}{2}$ the squar'd quotient, and that sum is 19, the greater of the two numbers.

Multiply 19 (thus found) by 4, subtract the product from the sum of 168 and the product of 7 and 4, divide the remainder by the difference betwixt 19 and 7, and the quotient is 10 the lesser number.

PROB. 9. 8450 soldiers, and it is required to place them so, as there may be twice as many in rank as in file; what number will be in each?

Extract the square root of $\frac{1}{2}$ 8450, and it is the number in file, which doubled gives the number in rank. Answer 65 in file and 130 in rank.

PROB. 10. It is required to order 4900 into a quadruple batallia of men, that is, that shall have four times as many in rank as in file. How many must be placed in each? Answer 35 in file and 130 in rank.

The square root of $\frac{1}{4}$ 4900 is the number in file, which quadrupl'd is the number in rank.

PROB. 11. 6075 soldiers are to be ordered into three equal square battalions; what number will there be in the side of each battalion? Answer 45.

The square root of $\frac{1}{3}$ 6075 is the answer.

PROB. 12. 5400 soldiers are to be so ordered that their distance in file may be 6 foot and in rank 4 foot; and it is required to order them into a square battalia of ground; how many must be in rank and how many in file, that the ground they stand on may be a true square? Answer 60 in file and 90 in rank.

Multiply 5400 by 4, divide the product by 6, and the ſquare root of the quotient is the number to be placed in file, by which divide 5400, and the quotient is the number to be placed in rank.

PROB. 13. 1800 ſoldiers are to be ſo ordered, that there may be twice as many in rank as in file; and if they ſtand at cloſe order (which is $1\frac{1}{2}$ foot) I demand how many ſquare foot of ground they'll occupy?

The ſquare root of $\frac{1}{2}$ 1800 is 30 the number to be placed in file, which doubled gives 60 the number to be placed in rank; now multiply the number in file by $1\frac{1}{2}$, the product is 45; and multiply alſo the number in rank by $1\frac{1}{2}$, it makes 90; then if 45 be multiply'd by 90, the product will be 4050 feet, the anſr.

PROB. 14. What number is that which being multiply'd by 13, and the product ſubtracted from the ſquare of the number to be found, the remainder will be 388,3125? Anſwer 27,25.

Square 13, add $\frac{1}{4}$ of its ſquare to 388,3125, to the ſquare root of that ſum (20,5) add $\frac{1}{2}$ 13, and the ſum is the number ſought.

PROB. 15. What number is that which being multiply'd by 7, and the product added to the ſquare of the number to be found, the ſum will be 408?

Add $\frac{1}{4}$ of the ſquare of 7 to 408, from the ſquare root of the ſum ſubtract $\frac{1}{2}$ 7, and the remainder is 17, the number ſought.

PROB. 16. Divide 140 into two ſuch parts, ſo that the product of thoſe parts may be equal to the ſquare of 56. Anſwer 28 and 112.

From $\frac{1}{4}$ of the ſquare of 140 ſubtract the ſquare of 56, and the ſquare root of the remainder ſubtracted from $\frac{1}{2}$ 140 leaves 28 the leſſer number which taken from 140 leaves 112 the greater.

PROB. 17. A ſet of boon companions dining at an inn, their reckoning came to 175 ſhillings; but before the bill was paid off, 2 of them ſlunk away; and then the club of thoſe that remained came to 10 ſhillings a man more; how many were there in company? Anſwer 7.

Multiply

Multiply 175 by 2, divide the product by 10, to the quotient add $\frac{1}{4}$ of the ſquare of 2, and the ſquare root of the ſum added to $\frac{1}{2}$ 2 is the number in company.

PROB. 18. A company of men drank at an inn till the reckoning came to 17s. 6d. 1f. how many were in company, and what did each pay, ſuppoſing they paid equally? Anſwer 29.

Reduce the reckoning to farthings, and the ſquare root of theſe farthings is both the number in company, and the number of farthings paid by each.

PROB. 19. A gentleman diſtributes a number of pence in geometrical proportion continued, among 4 poor perſons; ſo that if their portions were multiply'd continually together, the laſt product would be 109395,5625, and the two leaſt numbers multiply'd together would produce 36,75; I demand the whole ſum, and each perſons part thereof? Anſwer 140 pence the whole ſum, the 1ſt perſon's part $3\frac{1}{2}$, the 2d's 10,5, the 3d's 31,5, and the 4th's 94,5 pence.

Multiply 109395,5625 by the ſquare of 36,75, and the ſquare root of the ſquare root of the ſquare root of the product is the 2d perſon's part, viz. 10,5

Divide the ſquare of 36,75 by the ſquare of 10,5, and the ſquare root of the quotient is 3,5, the firſt perſon's part.

Divide the ſquare of 10,5 by 3,5, and the quotient is 31,5 the third man's part.

Divide the cube of 10,5 by the ſquare of 3,5, and the quotient is 94,5, the laſt perſon's ſhare, and the ſum of their reſpective ſhares is 140 the whole ſum be diſtributed.

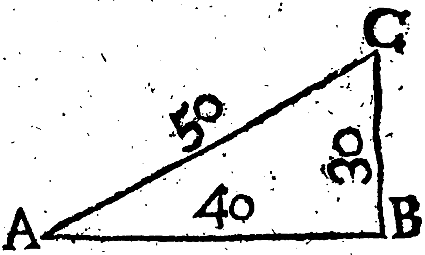
PROB. 20. To find two numbers in the proportion of 2 to 3, whoſe product if they be multiply'd into one another, ſhall be equal to 54.

Divide the double of 54 by 3, and the ſquare root of the quotient is 6 the leſſer number, and conſequently the greater will be 9.

PROB. 21. To find two numbers whoſe product ſhall be 240, and the triple of the greater divided by the leſſer ſhall be 5.

Triple 240, then divide its ſquare root by 5, and the quotient is 12 the leſſer number, by which divide 240, and the quotient is 20 the greater.

PROB. 22. In the following right angled triangle A, B, C, let the baſe A, B, = 40 yards repreſent a ditch, and let the perpendicular B, C, = 30 yards repreſent a city wall; what will be the length of the Hypothenuſe or ſcaling ladder, that will reach from the extremity of the ditch at A, to the top of the wall at C? Anſwer 50 yards.



The ſquare root of the ſum of the ſquares of the baſe and perpendicular is the length of the hypothenuſe.

PROB. 23. The hypothenuſe A, C, and baſe A, B, being given, to find the perpendicular B, C.

The ſquare root of the difference of the ſquares of the baſe and hypothenuſe is the length of the perpendicular.

PROB. 24. Two ſhips X, and Z, ſail from one and the ſame port B: X, ſails 40 leagues eaſt to A, I demand how far ſouth Z, had ſailed to C, when they were 50 leagues aſunder? Anſwer 30 leagues.

PROB. 25. The hypothenuſe A, C, and perpendicular B, C, being given, to find the baſe A, B.

The ſquare root of the difference of the ſquares of the hypothenuſe and perpendicular is the length of the baſe.

PROB. 26. A tree 80 feet high growing on a plain broke by a blaſt of wind in a certain point, the broken

piece

piece A, C, was 50 feet, the ſtanding piece B, C, 30 feet; I demand how far the top in falling ſtruck the ground from the root of the tree, or what was the diſtance A, B ? Anſwer 40 feet.

PROB. 27. The perpendicular B, C, and hypothenuſe A, C, being given in one ſum, and the baſe given, to find the perpendicular and hypothenuſe.

Divide the ſquare of the baſe by the double ſum of the hypothenuſe and perpendicular, to the quotient and ½ their ſum, and that ſum is the hypothenuſe A, C, which taken from their given ſum, leaves the perpendicular B, C.

A tree 80 feet high growing on a plain, breaks in a certain point as at C, and ſtrikes the ground with its top end 40 feet from the root; I demand how much ſtood and how much fell ?

Anſwer 30 feet ſtood and 50 fell.

PROB. 28. In any right angled triangle, ſuppoſe the foregoing one A, B, C, there is given the perpendicular B, C, =30, with the area and baſe A, B, in one ſum =640, to find the hypothenuſe A, C.

To the perpendicular add 2, by the ſquare of that ſum divide the quadruple of the ſquare of the ſum of the baſe and area, to that quotient add the ſquare of the perpendicular, and the ſquare root of that ſum is 50=the hypothenuſe.

PROB. 29. The baſe A, B, is given=40, with the area and perpendicular B, C, in one ſum=630, to find the hypothenuſe A, C.

To the baſe add 2, by the ſquare of that ſum divide the quadruple of the ſquare of the ſum of the perpendicular and area, to that quotient add the ſquare of the baſe, and the ſquare root of that ſum is 50=the hypothenuſe.

PROB. 30. There are two towers on a plain, at 360 feet diſtance, the one is 240 feet high, and the other 180 feet, a ladder is to be ſet upon the line of diſtance at ſome point, of ſuch a length, as from thence it may reach the top of both the towers; I demand the point in the line of diſtance, and alſo the length of the ladder?

Square

Square 360 the line of distance, to its square add the square of 240 (the height of the highest tower) from that sum subtract the square of 180 the height of the lowest tower) divide the remainder by the double of the line of distance, and the quotient is the distance from the foot of the lowest tower (viz. 215 feet) where the ladder must be placed that from thence it may reach the top of both towers.

To the square of the height of the lowest tower add the square of 215, and the square root of the sum is the length of the ladder.

PROB. 31. A gentleman is to lay out 600 square falls of ground for a garden, and wants it to be an equilateral triangle; pray what will be its side?

Multiply the square of the area (600) by 16, divide the product by 3, and the biquadrate root of the quotient is 37,2247 falls the side required.

Or multiply the area by 120, divide the product by 51,96, and the square root of the quotient is the side, as before.

Or, as 43,3 is to 100, so is 600 (or any other area) to a 4th number whose square root is the side, as before.

PROB. 32. The difference between the diagonal of a square and its side is given=6,21 feet; what's the side of the square? Answer 14,99 feet.

To the double square of the given difference add the said difference, and the square root of the sum is the side of the square.

PROB. 33. If a right angled paralellogram whose breadth is 6 falls, be added to a square whose side is equal to the length of the paralellogram and the content of both be equal to 160 falls; what's the side of the square? Answer 10 falls.

To $\frac{1}{4}$ of the square of the breadth of the paralellogram add the given area of the compound rectangle, from the square root of that sum subtract $\frac{1}{2}$ the breadth of the paralellogram, and the remainder is the side of the square.

PROB.

PROB. 34. To find two numbers whose ratio is to one another as 4 to 5, and the sum of their squares is 2624.

Multiply the given sum of their squares by the square of the lesser term 4, divide the product by the sum of the squares of the two terms of the ratio, and the square root of the quotient is 32 the lesser number; and then as 4 is to 32, so is 5 to 40, the greater number.

PROB. 35. 969 soldies are to be drawn up into an oblong form, so that the sum of the two sides may be 74; what number will be in each side?

From $\frac{1}{4}$ the square of the sum of the two sides subtract the number of soldiers, and to the square root of the remainder add $\frac{1}{2}$ the sum of the sides, and that sum is the number in the greater side, viz. 57, which taken from the sum of the sides, leaves the number in the lesser side, viz. 17.

PROB. 36. 969 soldiers are to be drawn up in an oblong form, so that the difference of the two sides may be 40; what number will be in each side?

Answer 57 and 17.

To the number of soldiers add $\frac{1}{4}$ the square of the difference of the sides, from the square root of that sum subtract $\frac{1}{2}$ the difference of the sides, and the remainder is the number in the lesser side, which added to the difference of the sides gives the number in the greater side.

PROB. 37. There be two numbers the sum of whose squares is 125, and the difference of their squares 75; what are the two numbers?

To the sum of their squares add the difference of their squares, and the square root of $\frac{1}{2}$ that sum is the greater number 10, whose square subtracted from the sum of their squares, leaves a number whose square root is 5 the lesser.

PROB. 38. There be two numbers, the greater of which hath such proportion to the lesser as 2 to 1, and the difference of their squares is 75; what are the two numbers?

Multiply

Multiply the difference of their squares by the square of the greater term of the ratio, divide the product by the difference of the squares of the two terms of the ratio, and the square root of the quotient is the greater number 10, from whose square subtract the difference of their squares, and the square root of the remainder is the lesser number 5.

PROB. 39. The product of two numbers is 63, and the sum of their cubes is 1072; what are the two numbers?

To the square root of the difference between the cube of their product and $\frac{1}{4}$ of the square of the sum of their cubes, add $\frac{1}{2}$ the sum of their cubes, and the cube root of that sum is 9 the greater number, by which divide their product, and the quotient is 7 the lesser.

PROB. 40. The base of a right angled triangle is 20, and the hypothenuse with the area in one sum is 175; what's the perpendicular?

From the square of the base subtract 4 for a denominator: then multiply the sum of the hypothenuse and area by the base, and quadruple the product for a numerator, so will you have this improper fraction $\frac{14000}{396}$ which call C.

Next square the base, and also the sum of the hypothenuse and area; quadruple the difference of those two squares, and let that be a numerator to your former denominator, so will you have the improper fraction $\frac{120900}{396}$ which call D.

Now, multiply the numerator of D, by its denominator, and placing the product over the square of the denominator, it will be $\frac{47876400}{156816}$, which subtract from $\frac{49000000}{156816}$ (being $\frac{1}{4}$ of $\frac{196000000}{156816}$ the square of C) and the remainder will be $\frac{1123600}{156816}$ whose square root ($\frac{1060}{396}$) being subtracted from $\frac{1}{2}$ C, ($\frac{7000}{396}$) leaves the improper fraction $\frac{5940}{396}$, and this reduced to its equivalent whole number gives 15 the perpendicular required.

PROB. 41. The perpendicular of a right angled triangle is 15, and the hypothenuse with the area in one sum is 175; what's the base? Answer 20.

This is resolved after the same manner as the last.

PROB. 42. If the area of a right angled triangle is 600, and the sum of the three sides 120; what are the sides?

Divide the area by $\frac{1}{2}$ the sum of the sides, subtract the quotient from $\frac{1}{2}$ the sum of the sides, and the remainder is the hypothenuse=50. From the square of 50 (thus found) subtract the quadruple of the area, to the square root of the remainder add the difference between the sum of the sides and the hypothenuse, and $\frac{1}{2}$ that sum is the base=40, which added to 50, and taken from the sum of the sides, leaves 30 for the perpendicular; and so of any other.

PROB. 43. A gentleman wants a rectangular garden whose length from east to west shall be 22 falls, and breadth from north to south 10 falls, and a walk on the north and west sides to be made in a situation parallel to the said sides, that shall contain $\frac{1}{4}$ the area of the whole garden; what will be the breadth of the walk?

To the length add the breadth, from $\frac{1}{4}$ the square of that sum subtract $\frac{1}{4}$ the product of the length into the breadth, and the square root of this remainder subtracted from the sum of length and breadth, leaves the breadth of the walk=3,9 falls.

PROB. 44. A merchant bought two sorts of linen for 30 crowns, one sort fine and the other coarse; an ell of the best cost as many crowns as he had ells thereof, and he bought 28 ells of the worst at such a price, that 8 ells thereof cost as many crowns as one ell of the finest; how many ells of the finest did he buy, and what price did he give for the ell of both sorts? Answer 4 ells best, at 4 crowns per ell.

Divide $\frac{1}{4}$ the square of 28 by the square of 8, to the quotient add 30; from the square root of that sum subtract $\frac{1}{2}$ the quotient when 28 is divided by 8, and the remainder is the number of ells of the finest; square the ells of the finest, and that is the price of its whole quantity, viz. 16 crowns, which taken from 30, leaves 14 crowns, and this divided by 28, quotes $\frac{1}{2}$ crown, price of an ell of the coarsest.

PROB. 45. The semidiameter of the earth being 3984,58 miles, and the perpendicular height of a mountain $6\frac{1}{2}$ miles; how far will it be seen at sea, or on plain ground, supposing the eye of the spectator to be on the surface of the ground or water?

Answer 227,687 miles.

To the earth's semidiameter add the height of the mountain, and from the square of that sum subtract the square of the earth's semidiameter; the square root of that remainder is the distance required.

But if the eye be higher than the surface of the water or ground, then to the earth's semidiameter add the heigh of the eye above the water or ground, from the square of that sum subtract the square of the earth's semidiameter, and the square root of the remainder added to the former square root gives the answer.

PROB. 46. The earth's semidiameter being 3984,58 miles, and if a mountain be seen at sea 217,687 miles off; what's the height of the mountain? Ans. $6\frac{1}{2}$ miles.

To the square of the earth's semidiameter add the square of 227,687, from the square root of that sum subtract the earth's semidiameter, and the difference is the height of the mountain.

PROB. 47. If a bullet of brass of 8 inches diameter weigh 72 lb. what shall a bullet of brass weigh whose diameter is 4 inches? Answer 9 lb.

As the cube of the given diameter is to the weight thereof: so is the cube of the diameter sought to the weight thereof.

PROB. 48. If a ship of 100 tun be 44 foot long at the keel, of what length shall the keel of a ship be of 220 tun? Answer 57,225 feet.

Multiply the cube of 44 by 220, divide the product by 100, and the cube root of the quotient is the answer.

PROB. 59. There is a cubical vessel whose side is 12 inches; what will be the side of another such vessel that shall contain 3 times as much? Ansr. 17,3 inches.

Multiply the cube of the given side by 3, and the cube root of the product is the side required.

PROB.

PROB. 50. If the content of a globe, cylinder, cone, or such like be 15625 square solid inches; what's the side of a cube equal in capacity thereto? Ansr. 25 inches.

The cube root of the solid content of any solid body given, is the side of a cube of equal solidity.

PROB. 51. I want two mean proportionals between 6 and 162, or any other two numbers.

Divide the greater extreme by the less, multiply the less extreme by the cube root of the quotient, and the product is 18 the lesser mean, which multiply'd by the said cube root gives 54 the greater mean sought.

PROB. 52 If ,43 lb. of gun powder be sufficient to charge a gun whose concave diameter is $1\frac{1}{2}$ inch; how much of equal strength will suffice to charge a gun whose concave diameter is 7 inches?

Multiply the cube of 7 by the given quantity of powder ,43 lb. divide the product by the cube of $1\frac{1}{2}$, and the quotient is the quantity required.

PROB. 53, Three men bought a grindstone 40 inches diameter, which cost 20 shillings; of which sum A, paid 9 shillings, and B, 6 shillings, and C, 5 shillings: I demand how much of the stone each must wear down, proportionable to the money he paid?

All circles are in duplicate reason of their diameters.

Square the semidiameter, which makes 400.

Then 20 : 400 :: 9 (180 square of the semidiameter of the circle belonging to A.

And 20 : 400 :: 6 (120 square of the semidiameter of the circle belonging to B.

And 20 : 400 :: 5 (100 square of the semidiameter of the circle belonging to C.

Then from 400 (the square of the semidiameter of the stone) subtract 180, and there remains 220, whose square root is 14,83 inches which subtracted from 20 inches (the semidiameter) there remains 5,17 inches for the breadth of the ring to be ground down by A.

Then from 220 subtract 120, and there remains 100, whose square root is 10; subtract that from 14,83, and there remains 4,83 inches for the part of the stone to

be

be ground down by B, after A, had ground down his part, and C, must grind down the remainder which is 10 inches, the square root of 100.

PROB. 54. There is a round bushel whose diameter is $18\frac{1}{2}$ inches, and depth 8 inches; what will the diameter of another equal thereto be, when its depth must be $7\frac{1}{2}$ inches? Answer 19,107 inches.

Square 18,5 the given diameter, multiply its square by ,7854, and that product multiply'd by 8, gives the solid inches in the given bushel; which divide by 7,5, and the quotient is 286,72336, divide this by ,7854, and the square root of the quotient is the answ.

PROB. 55. What length of a cord, the one end tied at my horse's mouth, and the other end fixed to a stake in the ground, will allow him the liberty of eating an acre of grass and no more? Answer 42,818 ells.

Multiply 5760, the ells in an acre by 1,2732, and the square root of the product is 85,636 ells the diameter of a circle whose content is an acre, the half whereof is 42,818 ells, length of the cord.

PROB. 56. By the content of a circle to find the circumference. If the content of a circle be 160 falls, or 5760 ells; what's the circumference? Answer 269,039 ells.

Multiply the content of any circle by 12,56637, and the square root of the product is the circumference.

PROB. 57. There is a wall containing 18225 cubic feet whose height is 5 times its breadth, and its length 8 times its height; what's the length, breadth and height of that wall?

Suppose the breadth 2, then the height must be 10, and the length 80; which three numbers multiply'd together produce 1600, and the cube of 2 is 8; therefor say,

1600 : 8 :: 18225 (91,125 whose cube root is 4,5 feet the breadth; then 5 times 4,5 is 22,5, and 8 times 22,5 is 180 feet the length.

PROB. 58. There is a stone 20 inches long, 15 inches broad, and 8 inches thick, which weighs 217

lb;

lb. I demand the length, breadth, and thickness of another of the same kind and shape which weighs 1000 lb?

The cube of 20 is 8000, therefore say,

217 : 8000 :: 1000(36870,645 whose cube root is 33,28 inches, length required.

20 : 33,28 :: 15(24,96 breadth required.

20 : 33,28 :: 8(13,312 thickness required.

PROB. 59. The circumference of a cable is 5 inches, what will be the circumference of one 10 times its strength?

10(3,16 square root of 10, which being multiply'd by 5, gives 15,8 inches for the answer.

```
  10(3,16
   9
 ------
 61)100
     61
 ------
626)3900
    3756
    ----
     144
```

PROB. 60, If a merchant should fraudulently gain 40 l. per cent. by an unequal beam of 6 inches long, by buying on the shorter and selling on the longer end; what are the different ends of the beam?

Multiply always the gain per cent. by 100 and add to the product the square of 100; then from the square root of that sum take 100, divide the remainder by the proposed gain per cent. and the quotient multiply'd by the length of the beam gives 2,748 inches for the shorter end, which taken from the length of the beam, leaves 3,252 inches for the longest end.

PROB. 61. The length of a pendulum given in inches, to find the time it takes to make one, or any number of vibrations or swings.

Multiply the length in inches by the constant number ,025553, and the square root of the product is the time in seconds.

Note, The pendulum's length must always be in inches.

In what time will a pendulum 130 inches long make one vibration, or swing?

,025553
130
——— root.
3,321890(1,8226 second, time required.

PROB. 62. How many vibrations will a pendulum 5 inches long make in one hour, or 3600 seconds?

,025553
5
——— root.
,127765(,3574 of a second, time wherein it will make one vibration.

,3574)3600,0000(10072,74 vibrations required.

PROB. 63. In what time will a pendulum 5 inches long make 10072,74 vibrations?

,025553
5
——— root.
127765 (,3574 of a second, time wherein it will make one vibration.

10072,74
,3574
———
3599,997276 product in seconds=1 hour, or 60 minutes, time required.

PROB. 64. To find the distance of remote objects by light and sound.

Find the time in seconds as before, between seeing and hearing the flash of lightning or smoke of a gun, and the approach of the sound, by the constant number, ,191868, and the product will be the distance in Scots miles; or by ,216, and the product will be the distance in english miles.

Between seeing the lightening and hearing the thunder, were measured 28 vibrations of a pendulum 28 inches long; I demand the observer's distance from the thunder?

See the work on the next page. ,025553

,025553
28

204424
51106 *root.*

,715484(,8458 second, time of 1 vibration.
28

67664
16916

Seconds 23,6824 time of 28 vibrations.
,191868

1894592
1420944
1894592
236824
4499656

4,5438947232 Scots miles, the answer.

PROB. 65. A ship at sea sees a privateer fire a gun the interval of the sound and smoak was measured by 112 vibrations of a pendulum 20,7 inches long; I demand the observer's distance from the privateer?

,025553
20,7

178871
51106

root.

,52894710(,7272 second, time of 1 vibration.
112

87264
7272

Seconds 81,4464 time of the 112 vibrations
,191868

6515712
4886784
6515712
814464
15474816

15,6269578752 Scots miles, the answer.

Many

Many more examples, to ſhew the uſe of the ſquare and cube roots might be added, but let theſe ſuffice: I now proceed to the menſuration of planes and ſolids, and then ſhall conclude.

CHAP. XVII.

SECTION I.

Of Superficial Meaſure.

THE ſeveral kinds of meaſuring are three, viz. 1ſt. Lineal, by ſome called Running Meaſure, and is taken by a line, and reſpects length without breadth. Cornice, freeze, cloth, &c. are thus meaſured.

2dly. Superficial or flat ſquare meaſure, is that which reſpects length and breadth.

3dly. Solid, or cube meaſure, which reſpects length, breadth, and depth, or thickneſs.

Note, As the price of the book could not bear the expence of copperplates for the geometrical figures, I preſume the reader will excuſe that defect: and as I intend very ſoon to publiſh a complete treatiſe of menſuration, I ſhall be as ſhort (but as plain as poſſible) on this ſubject; and ſhall begin firſt with

LAND Meaſuring.

I am only here to lay down ſuch practical rules as may be uſeful to country gentlemen and farmers, whereby they may find the true content of any piece of land, and that by the chain only.

The Scots chain conſiſts of 74 feet or 24 Scots ells, and is divided into 100 links.

The Engliſh chain conſiſts of 66 feet or 22 yards, and is alſo divided into 100 links.

They who meaſure land in Scotland by an ell of 37 Engliſh inches, make the acre leſs than the true Scots acre by 593,6 ſquare Engliſh feet.

Every ell of our Scots chain ought to be 37,2 inches long.

Any

Any number of links in the chain is so many 100 parts of the whole chain, so 35 links are 35 hundred parts, and 6 links are ,06=six hundred parts of the whole chain, &c.

4 Poles make an English chain, and each pole is $5\frac{1}{2}$ yards long.

The Scots chain is 4 falls, and each fall is 6 Scots ells in length.

An English statute mile is 80 English chains.

A Scots mile is $1973\frac{1}{3}$ English yards.

The square of the chain is 10000 square links; ten squares of the chain or 100000 square links give an acre: therefore, if the area be expressed by square links, divide by 100000, or cut off five decimal places to the right hand, and the quotient shall give the area in acres and decimals of an acre: multiply those decimals by 160, cut off 5 figures of that product to the right hand (as before) and those on the left are falls: multiply those 5 figures cut off by 36, cut off other 5 figures (as before) from that product, and those on the left are ells: Lastly multiply the decimals of the ell or ells by 9,5, cutting of 5 figures on the right hand of the product, and these on the left hand are feet in Scots measure.

If you suppose the Scots ell equal to 37 inches, multiply the English acres by ,795471, and the product gives the Scots acres, and decimal parts of an acre; or divide the Scots acres by ,795471, and the quotient gives the English acres, and decimal parts of an acre.

If you suppose the Scots ell equal to 37,2 inches, multiply the English acres by ,7869407, and the product gives the Scots acres, and the decimals of an acre; or divide the Scots acre by ,7869407, and the quotient gives the English acres, and decimal parts of an acre.

When any thing is to be measured, you must consider what form it is of; and then it must be measured according to the several rules for each figure.

If it be a geometrical square, then multiply the side into itself, and the product is the content in the same name with the given side.

What's the content of a square garden whose side is 4 chains 5 links, or 16,2 falls?

See the work both ways.

4,05	16,2
4,05	16,2
2025	324
1620	2592
Acre 1,64025	160)262,44(1,64025
160	160
	0
Falls 102,44000	Falls 102,44000
36	36
264	264
132	132
Ells 15,84	Ells 15,84
9,5	9,5
420	420
756	756
Feet 79,80	Feet 79,80

The content of the garden by this method is 1 acre, 102 falls (or 2 roods 22 falls) 15 ells 79,8 feet.

Content 1 acre 2 roods 22 falls 15 ells 79,8 feet, as before.

If it be a long square, then multiply the length by the breadth, and the product is the content in the same name the demensions are given.

There is a piece of land 97,4 falls long and 80 falls broad; what's the content in acres.

97,4
80

7792,0 Content in falls.

a. r. f.
160)7792(48 : 2 : 32 Content.
40)112(2
32

The

The content of a long square being given, or of any other plain figure; to find the perpendicular of a right angled plain triangle (its base being given) that shall be equal in content to the said given figure.

Divide the content of the given figure by the base of the triangle, and the quotient doubled is the perpendicular required.

To lay out any number of acres in form of a long square that shall be 2,3,4,5, &c. times as long as broad.

Divide the given acres by the proportional number (be it 2, 3, 4, 5, 6, &c.) and the square root of the quotient is the breadth, which multiply'd by the proportional number gives the length.

A gentleman hath two gardens each a geometrical square, the side of the one is 70 ells, and the side of the other 60 ells, but he wants a square one equal to both; what will be the side thereof?

The square root of the sum of the squares of the sides of the two old ones is the side thereof, viz. 92,1952 ells.

If it be a field bounded with straight lines, and broader at the one end than the other, then take either the breadth in the middle, or add the measure of both ends together, and take half the sum for the mean breadth, which multiply by the length, and the product is the content.

There is a field 8 chains 24 links at the one end, and 10 chains 34 links at the other end, and 20 chains 8 links long; what's the content in acres?

See the work on the next page.

ch. l.
8 : 24
10 : 34

2)18 : 58(9 : 29 mean breadth.
20 : 08

7432
1758

Acres 18,65432
4

Roods 2,61728
40

Falls 24,69120
36

414720
207360

Ells 24,88320
9,5

441600
794880

Feet 83,90400

a. r. f. e. f.
Content is 18 : 2 : 24 : 24 : 83,9

If a field be very irregular in its breadth, take the breadth in 6 or 8 places, add the several breadths together, divide the sum by the number of places, and the quotient will be the mean breadth, which multiply'd by the length gives the content very near.

If you are to measure 1, 2, 3, 4, &c. ridges in a field by themselves, cast up the content at every 6 falls end (taking the breadth at both ends and in the middle

middle of every cast) and when you have in this order run the length of the ridges, add the several contents into one sum, and the total is the content of the whole.

If you are to take any quantity of acres or falls off the side of a field, take first the length of the field as truly as possible, by which length divide the quantity to be taken off (which must be in the same name with the length) and the quotient is the breadth that goes for the said quantity.

If the field be triangular, you may find the content without the help of a perpendicular, thus; Take the 3 sides of the triangle, and add them into one sum, from the half of which sum let the sides be separately subtracted, that 3 differences may be found betwixt the foresaid half sum and each side; then let these 3 differences and the half sum be multiplied into one another, and the square root of the last product gives the area of the triangle, in the same name with the sides.

The 3 sides of a triangular piece of land are 30, 40, and 50 falls; what's the content in acres?

30				60
40				30
50				1800
2)120(60	60	60	60	20
	50	40	30	36000
	10	20	30	10

160) a. r.
360000(600(3 : 3 Content.
36 480
0 40)120(3
0

If the field has 4 unequal sides, place a pole at every corner, and divide the field into two triangles from one corner to its opposite; this line of division will be a side to both triangles, and their contents being found separately (as just now directed) will, if added give the content of the field.

If

If there be any offsets without the bounds of the triangle or any other figure you measure, these must be carefully taken notice of, and the sum of their contents added to the content of the regular figure.

There is another way of finding the content of a field of 4 unequal sides, and it is this,

Add the opposite sides together, multiply the half of these two sums upon one another, and the product is the content: but I advise the practitioner to measure by the last rule, tho' the process be some what tedious.

If a field consists of 5 sides or more, and the whole can be view'd from each corner, divide it by poles into triangles, find the content of each (as formerly directed) and the sum of their severeal contents is the total content of the field, always minding to measure the offsets (if any) and to add their contents to the total content of the triangles.

But if you cannot from each corner view the whole field, set up a pole in some high place in the field from whence you can view each corner, and by taking lines from the several corners to that pole, you can divide the whole field into triangles, which being measured (as before) and their several contents added, give the total content.

To measure an oval or ellipsis, multiply the longest diameter by the shortest, and that product by ,7854, this last product is the content of the oval.

To make an oval (as a garden plot or table) having both length and breadth given.

First lay down the length with a line (if a garden plot) and cross it in the middle with the breadth so as breadth and length may be at right angles; this done, take half the length with a line apply one end of that extent to any end of the breadth, and where the other end touches the length on one side of the centre, there drive in a stake; and where it touches the length on the other side of the centre, drive in another stake; then putting a line about both stakes, make fast the two ends at such an exact length, that stretching

it

it by the two stakes, the bent of the line may exactly touch all the ends of the given length and breadth; so moving the line still round, it will describe an exact oval.

Of a Circle.

A circle is a plain figure contain'd under one line called the circumference, unto which all lines drawn from a point in the middle of the figure, called the centre, are equal the one to the other; and any one of those lines is called a Semidiameter of the circle.

The circle contains more space than any plain figure of equal compass.

Every circle is equal to a long square whose length is equal to half the circumference, and the breadth to half the diameter.

1. By the diameter to find the circumference.

Multiply the diameter by 355, divide the product by 113, and the quotient is the circumference.

2. By the circumference to find the diameter.

Multiply the circumference by 113, divide the product by 355, and the quotient is the diameter.

If the diameter of a circle is 22,6 inches; what's the circumference? Answer 71.

If the circumference of a circle is 71; what's the diameter? Answer 22,6.

3. To find the content of any circle.

Multiply the semicircumference by the semidiameter, and the product is the content.

If the diameter of a circle is 22,6 inches, and circumference 71, what's the content? Ans. 401,15 inches.

The diameter of a circle is a line drawn cross the circle through the center, and its half is called the semidiameter.

4. By the diameter of a circle to find the content.

Multiply the square of the diameter by ,7854, and the product is the content.

If the diameter of a circle is 22,6 inches; what's the content? Answer 401,15 inches.

5 By

5. By the circumference to find the content.

Multiply the square of the circumference by ,07958, and the product is the content.

If the circumference is 71, what is the content?

Answr 401,16.

6. By the diameter of a circle to find the side of a square equal in content to that circle.

Multiply the diameter of the circle by ,8862, and the product is the side of the square.

7. By the side of a square, to find the diameter of a circle equal in content to that square.

Multiply the side of the square by 1,128, and the product is the diameter of the circle.

8. By the content of a circle to find the diameter.

Multiply the content by 1,2732, and the square root of the product is the diameter required.

9. By the content of a circle to find the circumference.

Multiply the content by 12,56637, and the square root of the product is the circumference.

10. By the side of a square to find the diameter of the circumscribing circle.

Multiply the side of the square by 1,4142, and the product is the diameter.

11. By the side of a square, to find the circumference of the circumscribing circle.

Multiply the side of the square by 4,443, and the product is the circumference.

12. By the diameter to find the side of the greatest square which may be inscrib'd in that circle.

Multiply the diameter by ,7071, and the product is the side of the inscrib'd square.

13. By the circumference of a circle, to find the side of a square equal.

Multiply the circumference by ,2821, and the product is the side of the square equal.

Note, The diameter of the greatest circle that can be inscrib'd in a square and that touches all the 4 sides of the square, is equal to the side of that square in which it is inscrib'd.

A.

A man had 5 sons, to whom he left a piece of square ground, each side whereof was 68 falls long; to the eldest he left the inscrib'd circle which toucheth the square on each side, and to each of the other 4 he left a corner without the inscrib'd circle; what were their respective parts of that ground?

Subtract the content of the circle from the content of the square, divide the remainder by 4, and the quotient is the portion of each of the 4 youngest sons.

14. How to divide a circle according to any proportion, by a line concentric with the first.

Divide the square of the semidiameter by the proportional number, and the square root of the quotient gives the semidiameter of the circle that shall be concentric with the first, which take from your scale, and upon the same centre draw the circle, and, then the first circle will be divided according to the proportion given.

Thus, if a circle whose semidiameter is 64 falls was to be divided equally between two men, the square of 64 is 4096, the half of 4096 is 2048, whose square root is 45,254 falls the semidiameter required, and if upon the same center, and with the extent of the said semidiameter, you draw a circle, the first circle is thereby divided into two equal parts.

15. How to make a triangle that shall contain any number of acres, being confined to a certain base.

Bring the acres to falls, divide them by half the base, and the quotient is the perpendicular of the triangle.

What will be the perpendicular of a triangle that shall contain 100 acres of land, if its base be given equal to 160 falls in length?

160
100

$\frac{1}{2}$160=80) 16000(200 falls the perpendicular.

0

16. To meaſure a ſemicircle or quadrant.

Multiply the ſemidiameter by half the arch line, and the product is the content.

17. To meaſure the ſector of a circle which is a part thereof comprehended under two ſemidiameters not making a right line, and part of the circumference, and it may be either leſs or greater than a ſemicircle.

Multiply half the arch line by the ſemidiameter, and the product is the content.

18. To meaſure the ſegment of a circle.

A ſegment of a circle is a part terminated by a right line leſs than the diameter call'd a chord, and by a part of the circumference.

Multiply the chord line by $\frac{2}{3}$ of the perpendicular height of the ſegment from the middle of the chord to the arch, and the product is the content very near.

19. To meaſure a regular pollygon, which is a figure of more than four ſides, all the ſides and angles thereof being equal.

Multiply the ſum of all the ſides by half the diſtance between the center and the middle of one of the ſides, and the product is the content.

20. To find the ſuperficial content of a Parabola which is a curvilineal figure made by the ſection of cone, being cut by a plane parallel to one of its ſides.

Multiply the baſe by the perpendicular height, then multiply that product by 2, and that laſt product divided by 3, quotes the area of the parabola.

Note, If you are to parcel out ground for ſowing lintſeed in, a chain of 6 ells, and each ell divided into qarters, is the moſt proper one for that purpoſe. Take the breadth of the ridge always about the middle of that part you are to ſet off for 1, 2, 3, or 4 lippies, and thereby dividing 100 ſquare ells for 1, lippie, 200 for 2 lippies, 300 for 3, and 400 for 4 lippies, the quotient gives the length that goes for the reſpective quantities.

The breadth of the ridge will always be either ells, or ells and 1, 2, or 3 quarters, or ells and ,25, 5, or 75, of an ell. If

If the breadth of a ridge be 11,75, what length goes for 3 lippies of lintseed?

```
11,75) 300,00 (25,53
       2350      4
       ----    ----
       6500    2,12
       5875
       ----
       6250
       5875
       ----
       3750
       3525
       ----
  Rem. 225
```

Here, you see that the length required, is 25 ells 2 quarters and a thing of little value more; and so you may calculate for yourself a table for measuring lintseed ground from 1 ell to any number of ells you please.

If the breadth of a ridge be 5 ells; what length goes for 2 lippies ground?

```
5)200(40 ells, the answer.
    0
```

Note, Falls multiply'd by ells, and the product divided by 6, quote square falls And if feet be multiply'd by inches, and the product divided by 12, the quotient is square feet.

Glaziers work is measured by the foot square. If the windows be square, multiply the length by the breadth and the product is the content in the same name you take the demensions.

But if you take them in inches you must divide the product by 144 for the content in feet.

If the windows are arched, take the dimensions from the highest part of the arch to the bottom of the window, and find the content as if it was square, for no allowance is made by reason of the extraordinary trouble, and waste of time and glass in such cases.

painters

Painters work and wainſcotting is meaſured by the yard ſquare.

The mouldings in both, are to be meaſured by themſelves, and to be added to the content of the room.

The dimenſions in both are taken in feet and inches, and the content being ſo caſt up, it is divided by 9 to bring it to yards.

There are divers ſorts of wright work, ſuch as cornice, guttering, &c. that are meaſured by lineal or running meaſure; and doors and door caſes, balcony doors, columns and pilaſters, &c. are valued by the piece ſometimes and othertimes not.

There is a room of wainſcot 129,5 feet in circumference, and 16,75 feet high, (being girt over the mouldings) there are two windows each 7,25 feet high, and the breadth of each from cheek to cheek 5,5 feet; the breadth of the ſhutters of each is 4,5 feet; the cheek boards, top and bottom boards of each window taken together, is 24,5 feet, and their breadth 1,75 foot; the door caſe 7 feet high and 3,5 feet wide; the door 3,25 feet wide; I demand how many yards of wainſcot are contain'd in that room?

Anſwer 253 yards 5 feet.

The door and ſhutters are at work and half.

129,5	7,25
16,75	45
6475	3625
9065	2900
20720	
2169,125 content of the room.	32,625
	16,3125 half.
	48,9375
	2
	97,875 ſhutters at work and half.

3,25

3,25	7,25
7	5,5
22,75	3625
11,375 half.	3625
34,125 door at work and half.	39,875
	2
24,5	79,75 two windows.
1,75	
1225	3,5
4165	7
42,875	24,5 door case.
2	add 79,75 two windows.
85,75 cheek boards, &c.	104,25 to be deducted.

Content of the room	2169,125
Shutters at work and half	97,875
Door at work and half	34,125
Cheek boards, &c.	85,75
	2380,875
Deduct window lights and door case	104.25

Answer 253,625 yards=feet 2282,025

Board is measured by multiplying the length by the breadth, and if the dimensions be taken in inches, divide their product by 144, and the quotient is the content in feet. Or, if the breadth be taken in inches, and length in feet, divide their product by 12, and the quotient is the content in feet, and the remainder (if any) parts of a foot.

If a board be 19 inches broad and 14 feet long, what feet doth it contain?

19
14
12)266(22,166 feet.
  2

If

If a board be 19 inches broad, how many inches in length will make a foot?

19)144(7,58 inches very near.

For this purpoſe there is a line upon moſt ordinary joint rules, with a little table placed upon the end of all ſuch numbers as exceed the length of the rule, as in this little table annexed.

0	0	0	0	5	0	$8\frac{1}{2}$	6
12	6	4	3	2	2	1	1
1	2	3	4	5	6	7	8

Here you ſee, if the breadth be 1 inch the length muſt be 12 feet; if 2 inches the length is 6 feet, and if 5 inches broad, the length is 2 feet 5 inches, &c.

The reſt of the lengths are expreſs'd in the line, thus: if the breadth be 9 inches, you will find it againſt 16 inches, counted from the other end of the rule; if the breadth be 11 inches, then a little above 13 inches will be the length of a foot, &c.

The dimenſions for ceiling, plaiſtering and paving are taken in feet, and the content found in yards by dividing by 9.

How many yards are there in a piece of plaiſtering or pavement 47 feet 3 inches long, and 18 feet brood?

$$\begin{array}{r} 47,25 \\ 18 \\ \hline 9)850,50(94,5 \text{ yards, content.} \\ 0 \end{array}$$

Of Maſons Work.

Hewn work is meaſur'd by the ſquare foot, and the dimenſions taken in feet and inches. The hewn work of doors and windows ought to be meaſured into the extremity of the chack.

All laid work is meaſur'd by the ſquare rood of 36 ſquare ells, and the dimenſions taken in Scots ells and inches.

If you take the dimenſions in feet, divide by 9,5 for the content in Scots ells.

To meaſure a vent, add the circumference at the

top

top to the circumference at bottom, multiply ½ that sum by the perpendicular height, and the product is the content.

To measure a chimney head, multiply the depth by the circumference, and the product is the content.

To measure a gavel above the level, drop a line from the top to the middle of the breadth at the level, multiply half that line by the said breadth at the level, and the product is the content of one gavel, which multiply'd by their number, gives the content of all.

There are other things, such as level, corners scuncions, &c. measured commonly to masons, but I advise gentlemen and others to give, and masons to take a higher price for the rood of laid work only, and to pass these altogether.

The walls of a house are commonly measured thus. Take the height from the foundation to the level in different places, add the several heights into one sum, which divide by the number of places, and that quotient being multiply'd by the circumference of the walls without, gives their content.

But I think the better way is, to add the circumference of the walls without, to their circumference within doors, and to multiply half that sum by their height for the content. But in this case the tradesman should have allowance for corners

Slater work is also cast up into square roods at 36 square ells to the rood, and the dimensions taken in Scots ells.

They are always allowed 18 inches for the eve.

Solid Measure.

Solid measure is that of timber, stone, digging, and liquids.

If the dimensions are taken in inches, and the content cast up in that name, divide by 1728, and the quotient is the content in cubic feet.

To measure a cube (which is a square solid comprehended under six geometrical squares) multiply its side into itself, and that product again by the side, and the last product is the solid content.

A

A Parallelopipedon or ſquare Priſm, repreſenting a cheſt, or ſquare piece of timber or ſtone is thus meaſured: multiply the ſquare of the ſide at the end by the length, and the product is the content.

If the end of the ſquar'd ſolid, is broader on the one ſide than the other, multiply the one ſide by the other, and the product is the area at the end, which multiply'd by the length, gives the ſolid content.

If a piece of ſquared timber be 25 inches broad and 9 inches deep, and 25 feet long; how many ſolid feet are contained therein?

If you multiply inches by feet in ſolid meaſure, divide by 144, and the quotient is the ſolid content in feet.

```
      25
       9
     ---
     225
      25
    ----
    1125
    450
144)5625(39,0625 ſolid feet.
     0
```

How many feet of braſs wyre ,005 parts of an inch in circumference, will be equal to one ſquare ſolid foot of braſs?

Multiply the ſquare of the circumference of the wyre by ,07958, and the product is the area at the end, which multiply'd by 12, gives the ſolid content of one foot of the wyre, by which divide 1728, and the quotient is the length of the wyre in feet, as was required, viz. 72379994,9736 feet.

If a piece of timber be 22 inches deep, and 15 inches broad; how much in length will make a ſolid foot?

```
   22
   15
330)1728(5,236 inches in length
    120
```

There is a line for this purpoſe upon moſt ordinary rules, with a little table at the end of all ſuch numbers

as

as exceed the length of the rule, such as this annex'd.

0	0	0	0	9	0	11	3	9	Inches.
144	36	16	9	5	4	2	2	1	Feet.
1	2	3	4	5	6	7	8	9	side of the sq.

Here you see, if the side of the square be 1, the length must be 144 feet; if two inches be the side of the square, it must be 36 feet in length to make a solid foot, &c. If the side of the square be not in the little table, you will find it upon the line; thus if the side of the square be 16 inches, you will find it against 6,7 inches, counted from the other end of the rule.

If you multiply the area of the base of any prism (whether square, triangular or multangular) by the length or height, the product is the solid content.

Of a Pyramid.

A pyramid is a solid figure, whose base is a pollygon, and whose sides are plain triangles, their several tops meeting together in one point.

To measure a pyramid, multiply the area of the base by a third part of the perpendicular height, and the product is the solid content thereof.

If the base be quadrangular, triangular, or in form of a regular pollygon, you may find its area by the rules already delivered for measuring such figures.

Of a Cylinder.

A cylinder is a round body having its bases circular equal, and parallel, in form of a rolling stone.

To find its solid content, find the area at the end, (as you did the content of a circle) which multiply by the length, and the product is the solid content,

All equally round timber is measured as a cylinder.

But if the bases of a piece of round timber are unequal, the usual way to measure it, is to take a fourth part of the girth in the middle of the piece for the side of a mean square, which fourth part being multiply'd into itself, and that product into the length gives the solid content.

Of a Cone.

A cone is a solid heaving a circular base, and grows smaller and smaller till it ends in a point, and may be nearly represented by a sugar loaf.

To find the solid content of a cone, find the area of the base (as you did the content of a circle) which multiply by one third part of the perpendicular height, and the product is the solid content thereof.

All round straight tapering timber ending in a point is measured after this manner.

Of the Frustum of a Pyramid.

A frustum of a pyramid is the remaining part when the top is cut off by a plain parallel to the base, and its solid content is thus found: Multiply the areas of the two bases together, and to the square root of their product add the two areas; that sum multiply'd by one third of the perpendicular height gives the solid content of any frustum square or multangled.

All squared timber that is thicker at one end than at the other should be measured after this manner.

Of the Frustum of a Cone.

A frustum of a cone, is that part which remains when the top end is cut off by a plain parallel to the base. And it is measured by the same rule as the frustum of a pyramid.

All round timber whose bases or ends are unequal is measured after this manner.

Of a Globe.

A globe is a round solid body, every part of whose surface is equally distant from its centre.

To find the solid content of a globe, multiply the diameter by the circumference, the product is the superficial content, which multiply'd by a sixth part of the diameter, gives the solid content.

Note, If the diameter of a globe be 1, then the circumference and superficial content are equal, viz. 3,141592. and if the diameter be 6, then the solidity and superficial content are equal, viz. 113,097312.

To measure a segment or piece cut off less than half the

the globe, multiply the triple height of the segment by the square of half the chord (or half the greatest wideness at the base) to which add the cube of the segment's height, and this sum multiply'd by ,5236, gives the solidity of the segment.

CHAP. XVII.

The Gregorian Kalendar.

1. To find the golden number.

The golden number is the space of 19 years, the time of one revolution of the moons nodes (that is the two points where her orbit cuts the ecliptick) round the ecliptick.

Add one to the given year of our Lord, divide the sum by 19, and the remainder is the golden number for that year; but if 0 remains, then 19 is the golden number.

What will be the golden number for the year 1758?

1758
1

19)1759(92
11 golden number.

2. To find the epact, by the golden number.

The epact is the space of 11 days, the excess of the solar above the lunar year; the former containing 365, and the latter 354 days.

Take 1 from the golden number, multiply the remainder by 11, the product (if less than 30) is the epact; but if it exceeds 30, divide by 30, and the remainder is the epact.

What

What will be the epact for the year 1758 ?

$$
\begin{array}{rl}
11 & \text{golden number for 1758.} \\
1 & \\
\hline
10 & \\
11 & \\
30)110(3 & \\
\hline
20 & \text{epact.}
\end{array}
$$

3. To find the dominical letter for any year.

To the year of our Lord add its fourth (not regarding the remainder, if there is any) divide the sum by 7, and the remainder (after that division) substracted from 7, leaves the number of the letter, reckoning A 1, B 2, C 3, D 4, E 5, F 6, G 7.

What will be the dominical letter for the year 1758 ?

$$
\begin{array}{rl}
1758 & \\
439 & \text{its 4th} \\
\hline
7)2197(313 & \\
\hline
\end{array}
$$

Rem. 6 Which taken from 7 leaves 1 and consequently the dom. letter for that year will be A.

4. To find the age or change of the moon.

Rule, Add the epact for the year to the number of the month, and the sum subtracted from 30 gives the day of the moon's change in that month.

To the number of the month add the epact and day of the month proposed, and the sum if below 30, is the moon's age; but if the sum exceeds 30, substract 30 from it, and the remainder is her age.

Each

Each month hath its number, as here.

Jan.	*Feb.*	*Mar.*	*Apr.*	*May.*	*June.*	*July.*
0	2	1	2	3	4	5

Aug.	*Sep.*	*Oct.*	*Nov.*	*Dec.*
6	8	8	10	10

What day of the month of October 1757 did the moon change on; the epact for that year being 9?

8 No. of the month. 30
9 Epact. 17

17 13th day, answer.

How old is the moon this 21st of October 1757?

8 No. of the month.
9 Epact.
21 day of the month.
38
30

The 8th day after her change.

5. To find the time of the moon's coming to the meridian.

Multiply the moon's age by 4, divide the product by 5, the quotient is hours, and each one remaining is 12 minutes, and both are the time of her southing.

What time is the moon south when she is 9 days old?

9
4 h. m.
5) 36 (7 : 12 the answer.
1

If to the moon's southing, you add the time of high water on the change day at any port, the sum will be the time of high water at that port. So at London high water at the change is at 3; therefore on the 9th day of the moon it will be high water there 12 minutes after 10 o'clock.

6. To