DEPARTMENT OF THE ARMY
CORPS OF ENGINEERS

THE BULLETIN OF THE BEACH EROSION BOARD

OFFICE, CHIEF OF ENGINEERS
WASHINGTON, D.C.

VOL. 5 APRIL 1, 1951 NO.2

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VOL. 5 NO. 2

LIMITING BATTER (SLOPE) BETWEEN THE BREAKING AND REFLECTION OF WAVES

by

Ramon Iribarren Cavanilles
and
Casto Nogales Y Olano
(Highway Engineers)

One of the interesting subjects treated in the XVIIth International Congress of Navigation, Lisbon, is the determination of the limiting conditions between breaking and reflection of waves.

This determination is necessary not only for differentiating wave breaking from reflecting embankments or shores, but is also fundamental for the important project of the wave-breaking slopes (rampas rompeolas) or dampers (amortiguadoras) in contrast to reflecting walls of piers (muros de muelles) in the zones in which the plan of wave travel of the interior of the port indicates the existence of incident waves of some importance and whose successive reflections might be disruptive to its needed calmness.

In justified cases these slopes can be covered with open pile piers (muelles en claraboya), but reflecting walls of piers never should be constructed in these zones of agitation in the interior of a port, and especially not in its angles.

In connection with this subject, we submit for those interested in the question, the following quotation from our report, from Section II, 4th Communication, of the cited Congress:

"It is of great interest to determine the limiting batter between the breaking and reflection of the wave, as a function of its characteristics. There exists theoretically a limiting batter such that both phenomena occur simultaneously, and that at the same time separates them in such manner that, for more severe batters, the wave is reflected, while for gentler batters the wave breaks, but in practice, there is a range of batters on which the wave breaks and reflects partially, one or the other phenomenon predominating until one arrives at a batter of total breaking or of total reflection. One cannot properly speak of gentle and severe batters in an absolute sense, but rather this depends on the characteristics of the wave that impinges."

Based on the quadrilateral of approach, which clarifies so many previously inexplicable extremes, we determine in the cited

* This article has been translated by the Waterways Experiment Station, Vicksburg, Mississippi, and published as Translation No. 50-2, 1950

report the expression for slopes (reciprocal of batter) theoretically limiting between breaking and reflection:

$$i = \frac{4}{T}\sqrt{\frac{h}{g}}$$

the wave period being 2T, its height 2h, and g, the acceleration of gravity.

In continuation, we are going to examine whether the degree of approximation, or rather the exactitude, of that simple formula is really satisfactory.

Using the notation indicated in Figure 1, adopted to facilitate the comparisons that we are going to make in our notation, the representatives of the Laboratory of Delft indicated, in one of the sessions of the Congress, their determination that always if

$$\frac{t}{2L} \leqq \frac{1}{4}$$

the reflection was complete; and that always if

$$\frac{t}{2L} \geqq \frac{1}{2}$$

the breaking was complete.

At the first simple view of these results we estimated then and there that they could not be acceptable as general expressions for criteria of breaking or reflection of the waves.

In the first place, it would not be admissible that the conditions of breaking should not be a function of the wave height as well as the length or rather period of the wave; and neither would it seem admissible that the relation t/2L defined said conditions of breaking except in the particular cases in which 2L/H were to be constant, given that the batter, which according to our understanding of the question is the appropriate variable, can be expressed

$$\frac{t}{H} = \frac{t}{2L} \times \frac{2L}{H}$$

The horizontal dimension of the batter, t, can only define the batter, or its inverse the slope, with depth, H, constant.

These logical objections are in accord with our formula

Graph of the Laboratory of Delft

GRÁFICO DEL LABORATORIO DE DELFT.

TALUDES CONTINUOS.

Continuous Batters

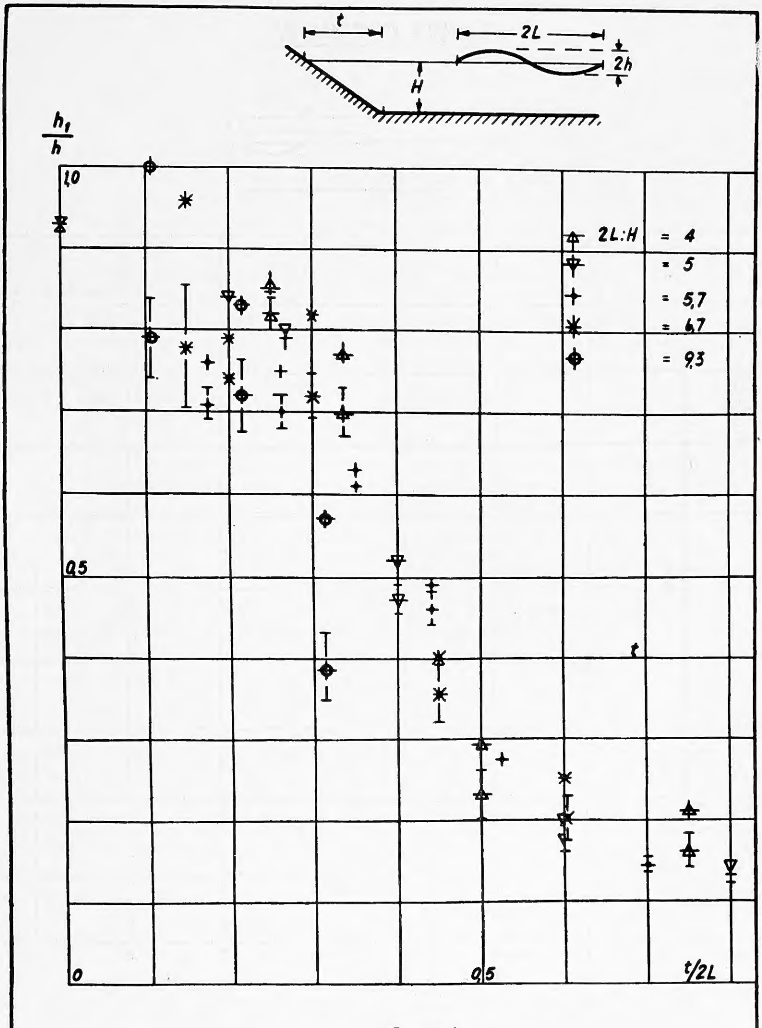

Figura 1.

Graph with revised abscissas $\frac{t}{H}$ (batter)

GRÁFICO CON ABCISAS CORREGIDAS $\frac{t}{H}$ (TALUD)

TALUDES CONTÍNUOS.

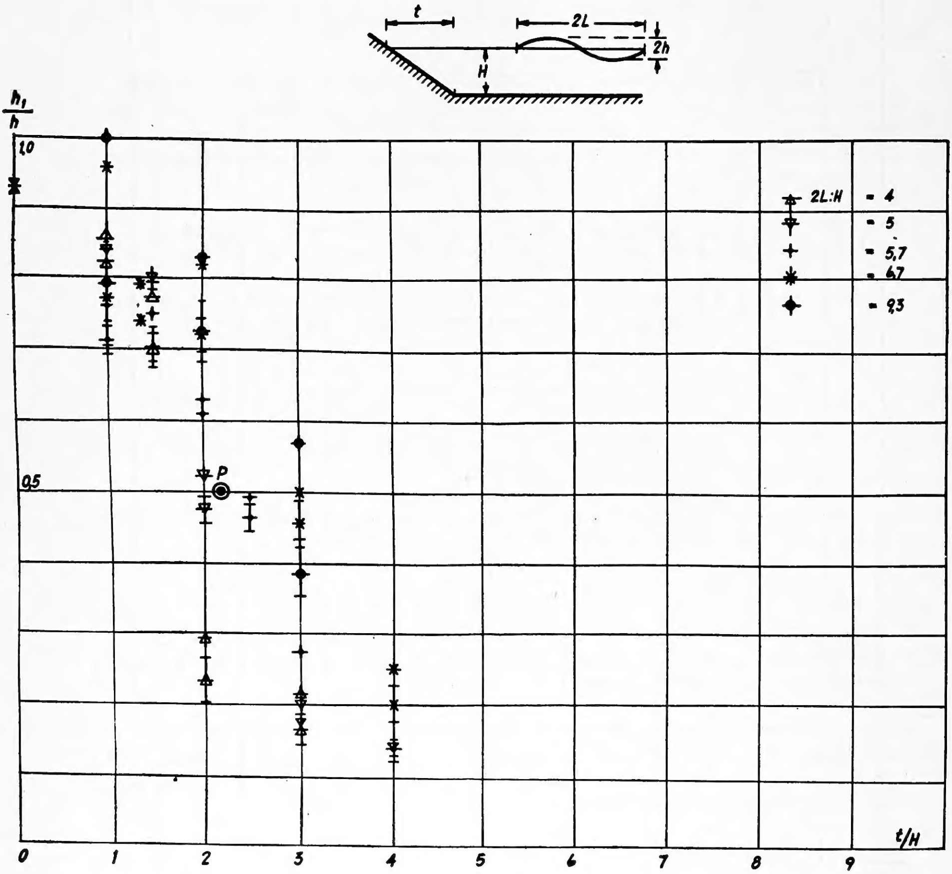

Figura 1'

$$i = \frac{4}{T} \sqrt{\frac{h}{g}}$$

whose slope, i, or theoretical limiting batter $1/i$, is a function of period 2T, which, for each depth H, fixes the length through the expression

$$T = \sqrt{\frac{\pi}{g} \cdot L \times \mathrm{Cth}\, \pi \frac{H}{L}}$$

but said limiting slope is likewise a function of the height of the wave, 2h.

Lately, there came to our attention a report, presented likewise by the representatives of the Laboratory of Delft, to the third meeting in Grenoble of the International Association of Investigations for Hydraulic Works, in which were described with great detail the experiments made and the results obtained.

In the graph shown in Figure 1, which refers to continuous batters (constant slopes), presented in Grenoble, the ordinates represent the relation h_1/h between the semi-heights h_1 of the reflected wave, and h of the incident, and the abscissas are the relation $t/2L$ between the horizontal dimension t of the batter and the length of the wave 2L.

It is emphasized that this second variable is not adequate, but that what we might call the factor of damping h_1/h would depend, as in our formula, not on the arbitrary relation $t/2L$, but on the slope H/t, or its reciprocal the batter t/H.

Thus, logically it comes out that, passing to this new variable, by means of the simple expression

$$\frac{t}{H} = \frac{t}{2L} \times \frac{2L}{H}$$

we obtain the graph of Figure 1', in which the aggregation of points turns out even better grouped than in Figure 1.

Using the data outlined in the cited report presented in Grenoble, from the extreme observations we obtain the following data and their corresponding results. Intermediate results would come from intermediate conditions.

Waves Classified Short

(a) Data.

Semi-period, T = 0.45 sec.; depth, H = 0.3 m.; length, 2L =

1.2 m.; relation, $\frac{2L}{H} = 4$; relation $\frac{L}{h} = 15$; then, semi-height, $h = \frac{1.2}{2 \times 15}$ $= 0.04$ m.

By application of our formula, we obtain the limiting slope:

$$i = \frac{4}{0.45} \sqrt{\frac{0.04}{9.81}} = 0.568$$

that is, the batter:

$$\frac{t}{h} = \frac{1}{0.568} = 1.76$$

(b) Data.

$T = 0.45$ sec.; $H = 0.3$ m.; $2L = 1.2$ m.; $\frac{2L}{H} = 4$; $\frac{L}{h} = 20$;

$$h = \frac{1.2}{2 \times 20} = 0.03 \text{ m.},$$

we obtain:

$$i = \frac{4}{0.45} \sqrt{\frac{0.03}{9.81}} = 0.492$$

that is, the batter:

$$\frac{t}{H} = \frac{1}{0.492} = 2.03$$

<u>Waves Classified Long</u>

(c) Data.

$T = 0.8$ sec.; $H = 0.25$ m.; $2L = 2.3$ m.; $\frac{2L}{H} = 9.3$; $\frac{L}{h} = 15$;

$$h = \frac{2.3}{2 \times 15} = 0.077 \text{ m.}$$

we obtain:

$$i = \frac{4}{0.8} \sqrt{\frac{0.077}{9.81}} = 0.442$$

that is, the batter;

$$\frac{t}{H} = \frac{1}{0.442} = 2.26$$

(d) Data

$$T = 0.8 \text{ sec.}; \; H = 0.25 \text{ m}; \; 2L = 2.3 \text{ m}; \; \frac{2L}{H} = 9.3; \; \frac{L}{h} = 20;$$

$$h = \frac{2.3}{2 \times 20} = 0.0575 \text{ m.},$$

we obtain:

$$i = \frac{4}{0.8} \sqrt{\frac{0.0575}{9.81}} = 0.383$$

that is, the batter:

$$\frac{t}{H} = \frac{1}{0.383} = 2.61$$

The mean value of the limiting batters obtained is then:

$$\frac{t}{H} = \frac{1.76 + 2.03 + 2.26 + 2.61}{4} = 2.165$$

As has already been indicated in our report, including experimental verification, the theoretical limiting batter should correspond, very approximately, to the mean value between the batter of complete breaking $\frac{h_1}{h} = 0$ or of complete reflection $\frac{h_1}{h} = 1$, that is ought to logically correspond to the value $\frac{h_1}{h} = \frac{0 + 1}{2} = 0.5$.

With this ordinate, $\frac{h_1}{h} = 0.5$, and the abscissa obtained, $\frac{t}{H} = 2.165$, we fix the point P in Figure 1' which, coinciding with great exactitude with the center of the collection of experimental points, demonstrates once again the degree of exactitude of the formula employed.

Even more interesting are the results obtained by analogous procedure on the discontinuous batters, whose graph, as is already apparent in the report presented in Grenoble (Figure 2), is so disordered and its points so dispersed that it is difficult for one to deduce from it reliable or dependable conclusions.

The principal reason for this dispersion is that the variable adopted for the abscissas $\frac{t}{2L}$, is even less acceptable, in these cases of discontinuous batters, by virtue of the real batter of the wave breaking slope being now

$$\frac{t}{z} = \frac{t}{2L} \times \frac{2L}{z}$$

GRÁFICO DEL LABORATORIO DE DELFT.

TALUDES DISCONTINUOS.

Discontinuous Batters

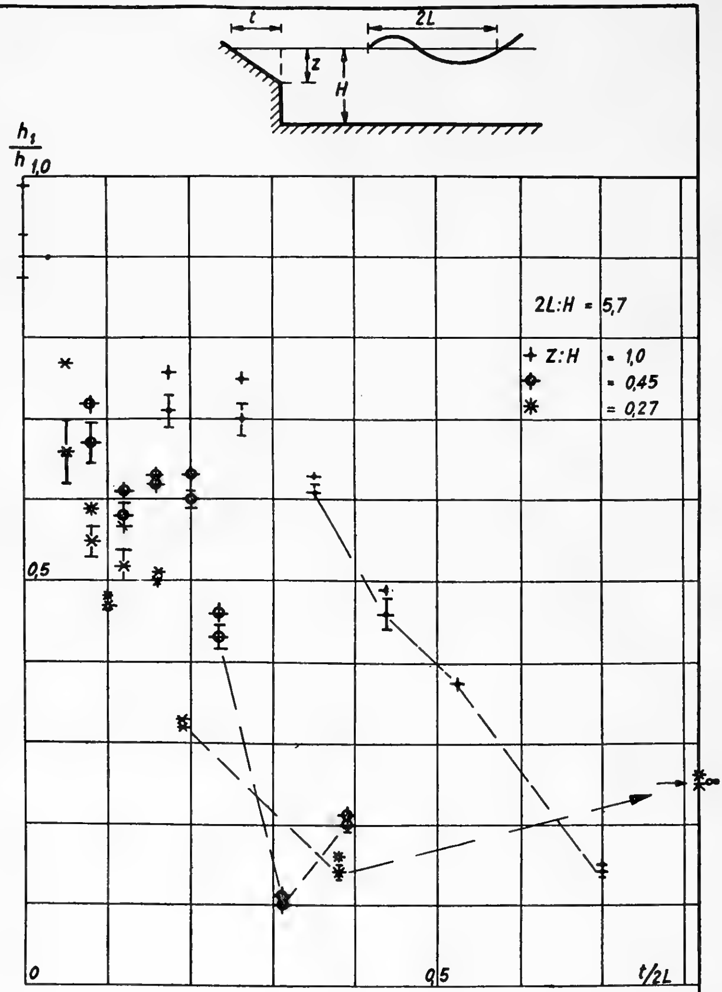

Figura 2.

GRÁFICO CON ABCISAS CORREGIDAS $\frac{t}{z}$ (TALUD)

TALUDES DISCONTÍNUOS.

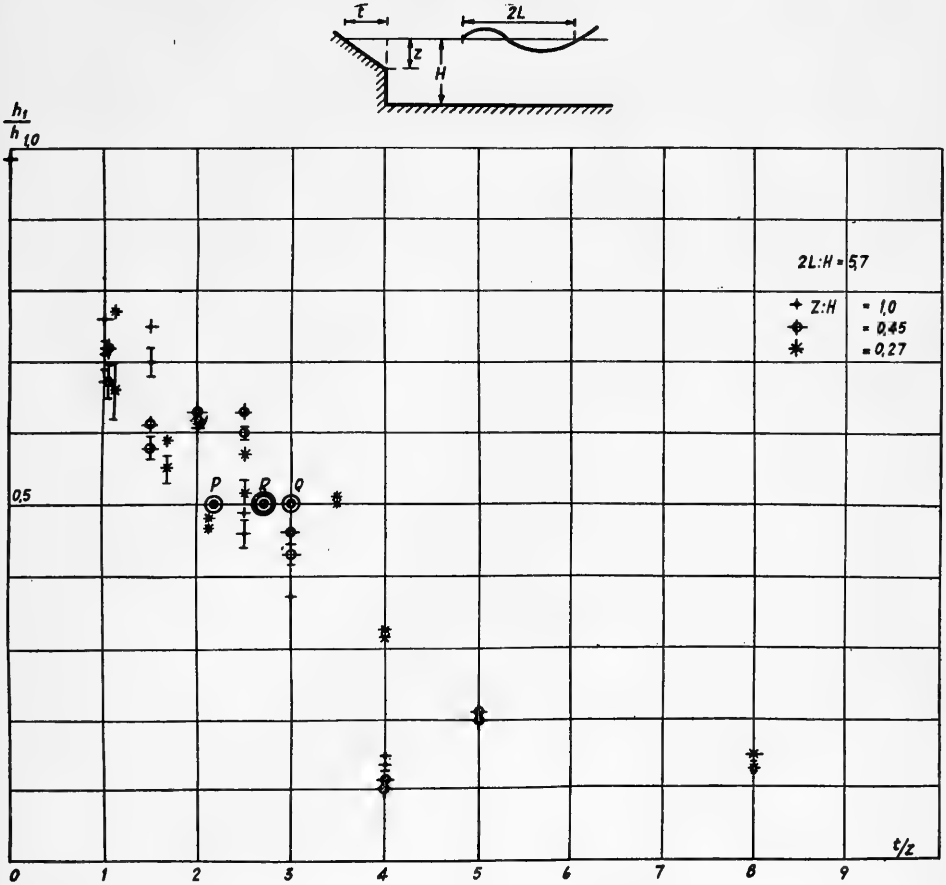

Figura 2.

and the relation

$$\frac{2L}{z} = \frac{2L}{\pi} \times \frac{H}{z}$$

is here essentially variable, principally by $\frac{z}{H}$ being so.

If, by simple change of variable, we adopt as abscissas the values of the batter t/z instead of the arbitrary t/2L we obtain the graph of Figure 2', as ordered and acceptable as that of Figure 1'.

This very interesting result shows once again that the fundamental variable for the study of the breaking or reflection of the waves must be always the batter of the slope t/z, or its inverse the slope, and not t/2L. The data corresponding to the new experimentation, done for these discontinuous slopes shown in the report presented in Grenoble, are the following:

$$\frac{2L}{H} = 5.7; \quad \frac{2L}{2h} = 40; \quad H = 0.316 \text{ m.};$$

from which is deduced:

$$2L = 0.316 \times 5.7 = 1.80 \text{ m.}; \; h = \frac{1.80}{2 \times 40} = 0.0225 \text{ m.};$$

$$K = \text{Cth} \; \frac{H}{L} = 1.166; \; T = \sqrt{\frac{\pi}{g} \cdot L \cdot K} = 0.58 \text{ secs.}$$

In the experiments the relation z/H had the values:

$$\frac{z}{H} = 1; \; \frac{z}{H} = 0.45; \; \frac{z}{H} = 0.27$$

For z/H = 1 we have the case of continuous batters whose experimentation has been utilized here again, and the corresponding point P will be approximated much as that obtained previously.

For:

$$\frac{z}{H} = 0.46; \quad z = 0.45 \times 0.316 = 0.142 \text{ m.} > 2h = 0.045$$

and we obtain the limiting slope:

$$i = \frac{4}{0.58} \sqrt{\frac{0.0225}{9.81}} = \frac{1}{3}$$

that is, the batter:

$$\frac{t}{z} = 3$$

whose point Q is shown likewise in Figure 2'.

For:

$$\frac{z}{H} = 0.27; \quad z = 0.27 \times 0.316 = 0.853 \text{ m.} > 2h = 0.45$$

we obtain the same limiting slope i = 1/3 and the same batter t/z = 3 by which its representative point Q will be the same as that we have just obtained.

The mean abscissa corresponding to the point P and to the points Q, the number of experiments of each of which approximate those of P, will be:

$$\frac{t}{z} = \frac{2.16 + 3 + 3}{3} = 2.72$$

which determines for us the point R perfectly centered likewise in the group of points of Figure 2'.

This shows definitely, we believe, that the formula for the limiting slope:

$$i = \frac{4}{T}\sqrt{\frac{h}{g}}$$

is very acceptable, and even though its application could be refined even more, we estimate that what has been done gives a sufficient approximation for the majority of practical cases. Perhaps it should be repeated once again that in the methods and formulas that we recommend for the maritime technician we do not pretend to utopian theoretical exactitudes, but acceptable practical approximations.

It should be noted here that the experimentation made for the discontinuous batters is acceptable and as has been shown, all of it fulfills the condition determined in our report to the Congress that the depth in the extreme of the batter $H_e = z$ should be equal or greater than the height 2h of the wave, that is $H_e = z \geqq 2h$.

Summarizing, the results of the interesting experimentation of Delft have confirmed ours, contained in the cited report to the Congress, according to which the batters of complete reflection and complete breaking, whose mean is the limiting slope between both, are practically in the relation of two to one, so that having obtained the limiting batter, or its inverse the slope, by means of our formula, it suffices to multiply it or divide it by 3/4 or 3/2 to obtain each one of them.

For the wave-breaking slopes or wave dampers whose effectiveness is somewhat increased by the roughness of the rock fill, it generally suffices to give them the limiting slope $i = T/4 \sqrt{h/g}$ under the condition $H_e = z \geqslant 2h$, because if the height of the reflected wave is reduced to less than half of that of the incident wave its energy is reduced to much less than 1/4 part.

All the foregoing confirms our criteria, according to which in order to get acceptable approximate solutions for the complicated subjects of the maritime technician the enthusiastic and frank collaboration of theorists, technicians and laboratories is necessary.

WAVE DIFFRACTION FOR OBLIQUE INCIDENCE

by
Henri Lacombe
Principal Hydrographic Engineer
Comite Central D'Oceanographie et D'Etude Des Cotes

FOREWORD

The following article was translated from the December 1950 Information Bulletin, Comite Central d'Oceanographie et d'Etudes des Cotes. It is published here as a means of informing interested readers of the interest in and progress being made in foreign countries on ocean wave problems. Numbers in parentheses refer to the bibliography following the article.

Status of the Question in June 1950

In the Information Bulletin of Comite Central d'Oceanographie et d'Etudes des Cotes for June 1949, we discussed the general features of a study on wave diffraction which had been published in considerable detail in the Annales Hydrographique of 1949 (1). We proposed in that article that diffraction for normal incidence could be studied by an almost exclusively graphic method which would not differ greatly from the rigorous solution in the case of semi-infinite breakwaters*. The method was equally applicable to the diffraction caused by vertical obstacles, with the condition that the waves approach the obstacles normally.

An American study by Blue and Johnson (5) discussed the theoretical solution, which was already known, and extended it by means of a certain approximation to the case of openings, but solely for the situation where the waves approached normal to the opening. Photographs obtained in the course of the study on the model show the existence of areas of agitation analogous to those that we had noted, and which are encountered also in the emission pattern of certain submarine acoustic apparatus and radio antenna systems. This study further had the advantage of showing clearly the effect of what one may call wave guides, defining the opening and which are constituted by vertical walls connected to the breakwater and directed toward the exterior of the port in a direction opposite to that of propagation. These guides had the effect of making a little more rigorous the simplified solution of Putnam and Arthur(4)

* Lamb (2) pp. 538-540; Bateman (3) pp. 476-490; Putnam and Arthur (4)

by wave reflected from one part or another of the jetty.

The General Rigorous Solution and Its Practical Complexity

At a meeting of the Section on Fluvial and Maritime Hydraulics of the Societe Hydrotechnique de France in June 1950, Mr. Schoemacher of the Hydraulic Laboratory of the University of Delft, stated that the rigorous solution of the diffraction problem for any incidence of wave and for a limited opening in a breakwater had been discovered for cylindrical waves and reported by P. M. Morse and P. J. Rubenstein (6) in 1938. These latter studies referred to a German study made in 1902 and it is curious to note that neither Putnam and Arthur nor Blue and Johnson apparently had any knowledge of these studies. J. Larras (7) showed in 1942 that the acoustic solution in two dimension was applicable to ocean waves and it followed that the expression given by Morse and Rubenstein entirely solved the problem for swell, at least in theory; but for practical use these authors referred to tables of Mathieu functions which were established by themselves.

The solution utilizes in effect cylindrical elliptic coordinate and the Mathieu function which occurs in various elliptic or hyperbolic problems, noteably those of the vibration of an elliptic membrane (8) and the propagation of electro-magnetic waves in elliptical guides (9). Bateman,(3) on pages 429, 430, and 490, gives supplementary references and adds: "These elliptical coordinates are useful for the treatment of the problem of vibration of an elliptic membrane and the diffusion of electro-magnetic waves by an obstacle having the form of an elliptic or hyperbolic cylinder. A screen containing a rectangular slot of constant width may be considered as the limit of an obstacle having the form of a hyperbolic cylinder." The author did not discuss the solution in detail.

It is a fact that this knowledge which has been unknown theoretically for almost 50 years has not become of current usage. Perhaps the reason may be found in the slight mention made of the Mathieu functions in classical teaching and in the absence of or very slight knowledge of the tables of these functions.

Research Toward a Practical Approximate Solution Based on a Generalization of Huyghen's Principle, Its Justification and Its Expression

Despite the slight theoretical interest of a solution founded on Huyghen's principle, which is necessarily only approximate, we have devised a solution of this type whose practical use is simple and elementary. It is nevertheless necessary to assure oneself that Huyghen's principle, or an adapted Huyghen's principle, is applicable to oblique incidence of the wave. Its approximate justification is made in the case of propagation in three dimensions by classical reasoning due to Kirchoff and discussed notably by Bateman (3), on pages 184-186, as well as by numerous analytical treatises (10) with respect to the propagation of waves. It relates the value of the propagation function at a point P to the values which the function and its normal derivative take on a surface around the point P. Generally one applies the formula obtained to diffraction by means of a certain number of hypotheses which are detailed by DeBroglie (9) on page 91 and following. However, for waves in three dimensions the auxiliary function that must be introduced in addition to the propagation function has a simple expression, it is rigorously /r, where r is the distance from a point source of movement P to a point M situated on the surface surrounding point P. It is not the same in a harmonic movement in two dimensions, where the expression of movement originating in a point source involves Bessel functions of zero order of the first and second types (see Lamb (2) pages 296-297, and 527-529) for which the asmytotic expression occurs in $r^{-\frac{1}{2}}$ and applies only to a distance from point P which is greater than 2 or 3 wave lengths. The first term neglected in the alternate series development of the exact movement is $r^{-3/2}$. However, in employing the exact expression of the movement starting at a point source P one arrives easily at an expression analogous to that obtained by Kirchoff, but which applies to a cylindrical movement and more precisely to a wave or swell. The expression gives the movement at point P if one knows the velocity potential and its normal derivative on a cylinder with normal generatrices in the plane of still water. One deduces from this formula a Huyghen's principle which applies to oblique incidence, but which, containing the asmytotic expression of elementary movement and not the exact expression, involves therefore an approximation and is practically applicable only if the distance PM is greater than 2 or 3 wave lengths.

Referring to Figure 1, the letters CC define a wave crest, let S be the cross section of the cylinder on which one assumes the propagation function and its normal derivative are known. Let α be the angle between the wave crest and the tangent to S, α is the angle of incidence; let θ be the angle between the normal to the crest and the straight line MP, and r the distance from the point P where one seeks to determine the movement, to a very small elementary source situated on S. The depth is supposed constant throughout.

The transposition of the Kirchoff formula, which transposes in a fashion the roles of P and M permits the establishment of a Huyghen's principle of the following form. The movement at P is obtained always in first approximation by considering that the movement is the resultant of elementary movements of the sources, such as M, distributed on the surface S (known movement),

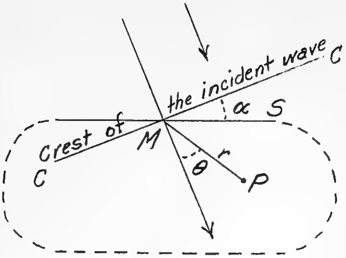

Figure 1.

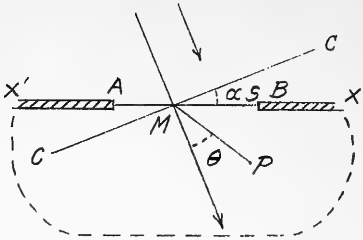

Figure 2.

and which causes a movement at P of amplitude

$$\frac{\cos\theta + \cos\alpha}{2} \cdot \frac{1}{\sqrt{r}} \qquad \text{Eq. 1}$$

that is to say proportional to the inverse of the square root of the distance MP multiplied by the term $\cos\theta + \cos\alpha$, and whose phase at P is equal to that existing at M retarded by $2\pi r$ measuring r in terms of wave length as unity. For normal incidence $\alpha = 0$. One then has $\frac{1+\cos\theta}{2} \cdot \frac{1}{\sqrt{r}}$ which is the mean of the two solutions that we have considered (1), and which are quite similar especially in the neighborhood of the limit of the geometrical shadow.

The Approximations Made

To this point the sole approximations that have been made are due to the fact that the variation of amplitude in $r^{-\frac{1}{2}}$ is only approximate and that in order to obtain equation 1, one has neglected a new term in $r^{-3/2}$. But how does one apply practically the result acquired according to Kirchoff, to the diffraction through a very large opening in a breakwater. It is here that new approximations must be made.

Referring to Figure 2, let A and B be the arms of the breakwater and AB the opening, CC is again the crest of a wave. We will choose a surface S where the velocity potential and its normal derivative are supposed known. This surface is the contour formed by the opening AB, the part X'A and BX of the supposed infinitely long breakwater and a dotted line traced very far from AB toward the interior of the port and on which the agitation is considered to be negligible.

We suppose further that:

1. On X'A and BX the velocity potential is zero (its normal derivative is necessarily so);

2. Between A and B the movement is the same as though the jetty did not exist.

3. We will suppose that we are able to apply Kirchoff's reasoning, that is to say Green's formula, to the surface S, although the velocity potential and its derivatives are subject to discontinuity in A and B. There is no reason not to believe that one may avoid this discontinuity by taking a different path of integration, avoiding A and B but passing very close thereto (e.g. the ends of the breakwater rounded).

These are without doubt approximations whose effect is difficult to appreciate; even more so since it appears that the second hypotheses is hard to accdpt. In view of the fact that it states that diffraction does not exist at all or at least does not alter the movement at the opening AB, which is certainly not correct. The sole verification possible consists in comparing the results obtained from the use of this method to the results obtained by the exact hydrodynamic solution.

The Resultant Solution

One deduces from Equation 1 the movement in P by an integration from A to B. This movement is practically given by the distance, measured on a spiral analogous to Cornu spirals of optics, between two points characterized by parameters which are related simply to the position of point P in relation to the two arms A and B. The lines of equal value of these parameters are parabolas whose foci are the points A and B and which may be traced in advance by supposing that all horizontal lengths are measured in terms of wave length as the unit of length. In fact if one compares the results obtained thus to the simplified solution of Putnam and Arthur (4) one finds an almost perfect coincidence, the differences not exceeding one hundredth of the amplitude of the incident wave. This is the verification a posteriori of the validity of the hypotheses that have been made.

The form of solution obtained, which is quite similar to that of Putnam and Arthur, leds, for the case of a semi-infinite breakwater (i.e. a very large opening) to a distribution of amplitudes which is in some fashion rigidly bound to the direction of incidence of the wave. If for example, in normal incidence one obtains at a distance y behind the jetty (i.e. at a distance y from the wave coinciding with the breakwater). A certain distribution of movement, then one obtains the distribution in oblique incidence by turning the normal incidence distribution around the breakwater through an angle equal to the angle of incidence. The homolog of the line situated at the distance y from the breakwater in normal incidence is then the line situated at the distance y beyond the crest passing the jetty. The line of equal amplitude are essentially parabolas whose focus is at the breakwater.

For a limited opening the distribution of the amplitudes that one obtains presents "tongues" or "leaves" of the agitation analogous to those noted in normal incidence; and moreover they are more numerous as the opening is made larger with respect to the length of the wave. If one examines the area at quite a distance from the opening the agitation is found to take a very simple form analogous to that which characterizes the emission from radio antenna systems. The maximum agitation is found on the axis of the opening extended parallel to the direction of propagation. If one expresses all the horizontal distances in terms of wave length it is equal to the length of the opening projected on the crests of the incident wave. Very close to the opening maximum agitation approaches the downstream arm of the breakwater, i.e. the one reached last by the incident wave. The maximum amplitude possible is 1.36 times the amplitude of the incident wave.

According to some figures presented in the article of Morse and Rubenstein (6) the exact solution ought to present "tongues" or areas of secondary maximum agitation. In the actual state of the question, development is left to reduced scale model experimentation since verification of the results is required.

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OBSERVATIONS MADE ON KARENTES BEACH

by
Professor W. W. Williams & Miss C.A.M. King

FOREWORD

The following is a technical note appearing in the Information Bulletin of the Comite Central D'Oceanographie et D'Etude Des Cotes of December 1950, reporting work done by Professor W. W. Williams of Cambridge University and his associate Miss C.A.M. King in August 1950. The original technical note has been translated and briefed, the principal omissions being data on the observations made with respect to waves and winds and a daily account of the behavior of the beach. The Beach Erosion Board was fortunate in having the opportunity to be host to Professor Williams for a period of several weeks during the late days of World War II, at which time Professor Williams' interest in offshore bars resulted in many stimulating discussions between himself, the staff of the Board, and Dr. G. H. Keulegan.

Karentes Beach is situated in the Gulf of Lion to the west of Sete. In this region the coast is oriented approximately northeast-southwest, and is constituted by a low beach of fine sand, (median diameter of 0.30 mm), backed by low dunes situated about 300 feet from the mean sea level shore line. The amplitude of the tide is very small and probably does not exceed about $\frac{1}{2}$ foot. On such a beach it should be possible for submarine sand bars to form and they should be of a simple type since they are formed without complications due to tides and or the abnormal tidal currents occurring in the neighborhood of headlands or the mouths of rivers. Currents parallel to the coast are habitually weak and we have been able to observe that at any moment they were uniquely a function of the predominant wind at the site.

Figure 1 shows the profile of the beach, to a depth of approximately 20 feet, as it existed during the last three summers. It will be noted that there is a large bar situated between 630 and 830 feet from the water line and that the form of the bar remains remarkably constant although the bar itself was displaced as much as

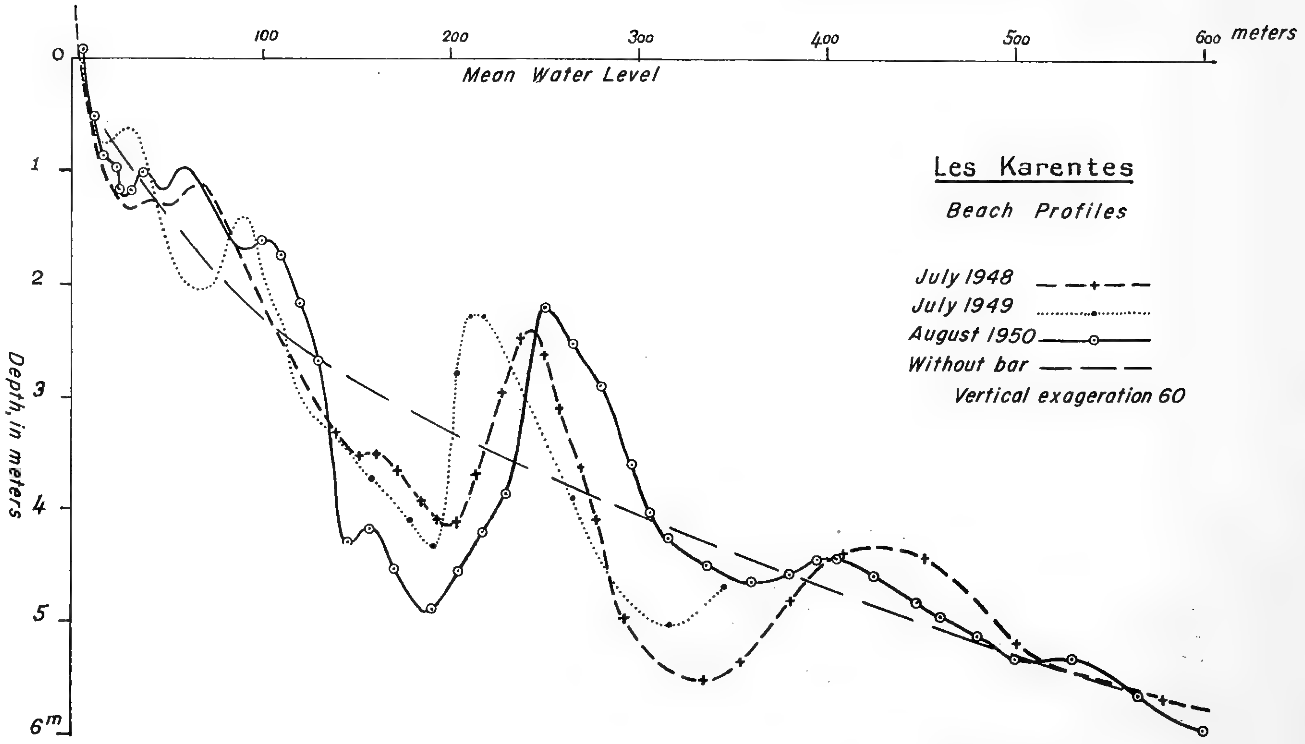
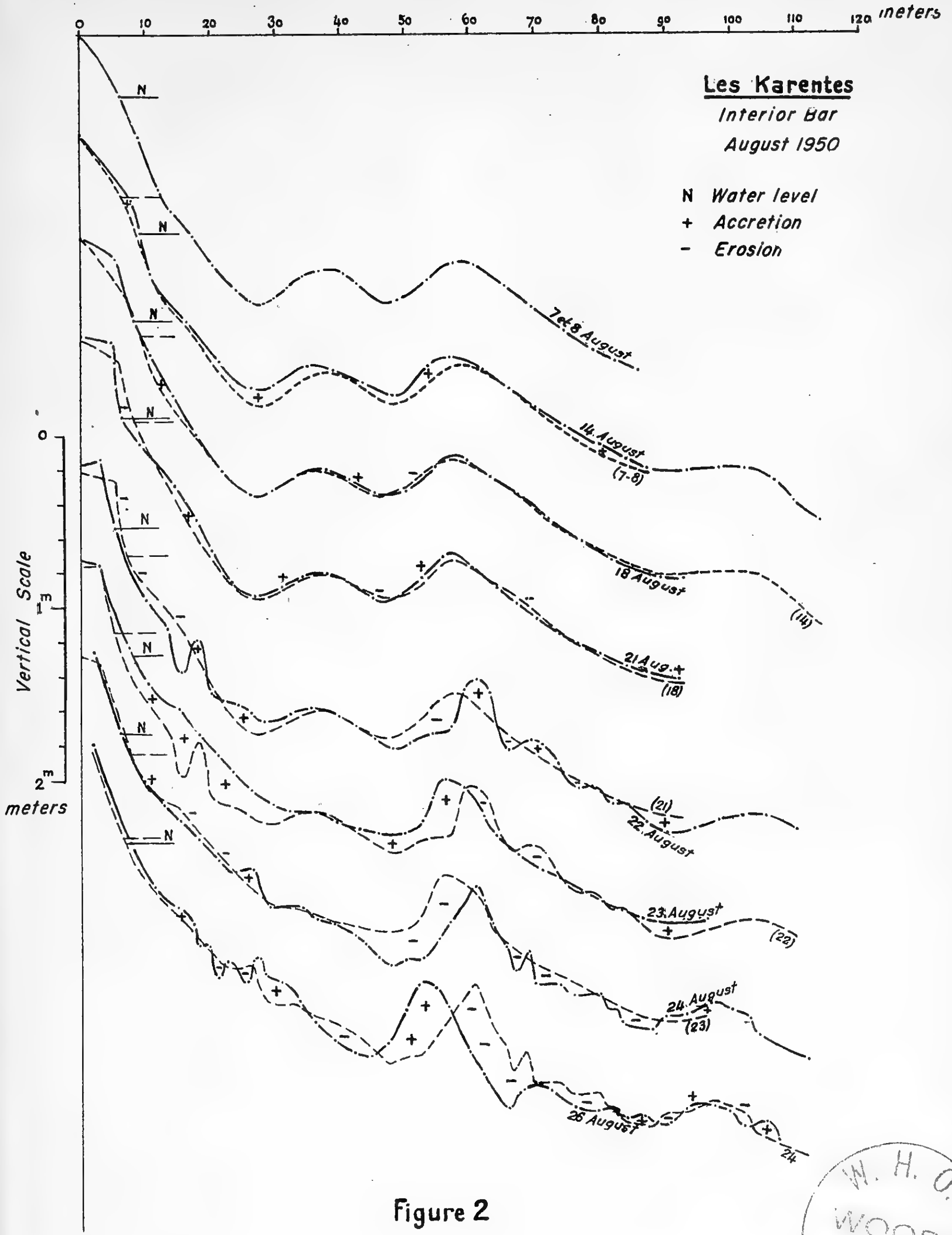

Les Karentes
Beach Profiles
July 1948
July 1949
August 1950
Without bar
Vertical exageration 60
Mean Water Level
0
100
200
300
400
500
600 meters
0
1
2
3
4
5
6m
Depth, in meters

Figure 1



Les Karentes
Interior Bar
August 1950
N Water level
+ Accretion
- Erosion
meters
0
10
20
30
40
50
60
70
80
90
100
110
120
Vertical Scale
0
1m
2m
meters
7 et 8 August
14 August
(7-8)
18 August
(14)
21 Aug.
(18)
(21)
22 August
23 August
(22)
24 August
(23)
26 August
24

Figure 2

125 feet in 2 years. It is possible, in fact probable, that this bar is subjected to displacements which are much larger and more rapid during the course of violent winter storms. At a point about 1420 feet beyond the bar there is another which is less evident. Within 300 feet of the water's edge there are found several bars, which we will call interior bars, which move rapidly as is shown on Figure 2.

On Figure 1 there has been traced a line which I call the "beach profile supposed without bars." We will refer to this later.

The interior bars have been studied in detail. They were measured daily whenever there was any reason to suppose that there had been an appreciable displacement in the bar position. The velocity and direction of the wind and the height and period of the wave also were noted. Available personnel was not numerous and therefore no night observations were made; which is unfortunate, for judging by the noise of the sea at night the height of the waves on certain nights was certainly much greater than the heights which had been noted during the day. Up to a distance of 350 feet from shore the beach profile was determined by leveling with the aid of a stadia board and the heights given are probably correct within about 1 inch. Farther seaward a sounding lead was used to determine depths. The positions of the soundings on the profile were determined by means of a plane table.

Previous research had led to six considerations which were to be confirmed or modified by the observations made on the beach. These considerations are:

A. Bars form to the highest elevation at the breaking point of the wave. At Karentes the exterior bars are formed and displaced by the storm waves of winter. During calm weather they are displaced only slightly, however the interior bar is displaced by very small waves.

B. After breaking the waves transport sand, and by consequence the bars, toward the sea.

C. Before breaking the waves transport sand, and by consequence the bars, toward the beach.

D. If one smooths the profiles of the bars observed on many different occasions, one will obtain a theoretical profile which we have called "beach profile supposed without bars". It appears to be reasonable to assume that the bars are accumulations of sand which are moved freely on the profile, depending upon the action of the waves and as the waves vary. Numerous experiments in the laboratory have shown that there exists a constant ratio between the height of the crest of the bar above the "beach profile supposed without bars" and the depth of the water above the crest of the bar. This ratio is 0.5.

E. The mean ratio between the depths to the crest and to the trough is 1.5; however, the range of the values observed is large and extends from about 1.2 to 1.9.

F. There should exist some relation between the velocity of displacement of bottom material and therefore of the bars, and the energy of the waves.

The last three statements may have a considerable importance in time of war. It was to verify these points on a beach of the simplest type that we made our observations at Karentes. We take this opportunity to express our gratitude to the Comite Central d'Oceanographie et d'Etude Des Cotes and to Mr. Gougenhein in particular, for the authorization to work on the beach and the aid which has been given.

The modifications which occurred on the beach in the course of the month of August are shown on Figure 2.

Conclusions

The following conclusions are reached with respect to the considerations listed above. By reason of the small range of dimensions of the waves and the absence of observations during the night our conclusions may be erroneous, nevertheless it appears to us reasonable to deduce from our work the following results:

Considerations A, B. and C appear to be verified without discussion.

Consideration D. It is probable that the bars are displaced on a relatively easily defined base "profile of a beach supposed without bars", but the ratio $\frac{1}{2}$ observed in laboratory work may not be verified universally in nature. For example, it is false for the deep rudimentary bar found at about 1300 feet from the shore line and shown on Figure 1. It seems equally clear that when the bars are in the course of displacement the crests are somewhat augmented and the depth of the water over them is diminished. Any extended period of calm weather appears to flatten the bars and make them disappear.

Consideration E. More observations are required to reach a conclusion on this point. It is clear that the ratio depends to a great extent upon the factors mentioned above in connection with consideration D.

Consideration F. A study of the use of the energy equation has given some interesting results but the observations made have been too few to produce reliable and useful deductions.

COMPARISON OF OBSERVED WAVE DIRECTION WITH A REFRACTION DIAGRAM

by
Donald R. Forrest

During the course of routine wave observations at Mission Bay, San Diego, California, by the Field Research Group. there occurred recently a comparatively rare combination of visibility and wave condition which made possible a comparison of observed wave direction with a refraction diagram. Observations have been made twice daily for more than a year and the usual case is that the region on the sea surface where swell can be clearly defined from the observation point is limited. On 19 January 1951, however, visibility was exceptionally good and even though the swell was less than one foot high it was possible to measure its direction of travel over a relatively large area.

Directions were measured by the transit-sighting bar method described in Beach Erosion Board Bulletin, Volume 4, Number 2, 1 April 1950. The 536-foot elevation of the instrument at the observing point, U. S. Coast and Geodetic Survey triangulation station "Jolla", permitted direction measurements to be made up to about five miles distant.

The most seaward of a group of fifteen observations was plotted on a hydrographic chart and an orthogonal drawn seaward to deep water. This depth was the 72 fathom contour for the 13-second wave observed. The deep water azimuth thus established was 292°, a change in direction of two degrees from that observed at 38 fathoms. Other orthogonals were drawn, resulting in the accompanying refraction diagram. The method of plotting orthogonals was that developed by Johnson, O'Brien and Isaacs (Hydrographic Office Publication No. 605). The remaining fourteen observations were then plotted on the refraction diagram. These are shown as short heavy lines.

On the whole, agreement is good. However, directions observed close inshore show a little less curvature than does the refraction diagram, while those observed at the 15 to 20-fathom region tend to indicate a little more curvature. The largest difference, about ten degrees, seems to be that of the most northerly observation, close to shore. The observing technique dictates that this particular measurement should not be in error by more than about two degrees. The reason for the difference probably lies in the fact that the bottom in this area is shallow, very rocky and irregular. This portion of the refraction diagram, based on rather sketchy hydrographic information, is probably erroneous.

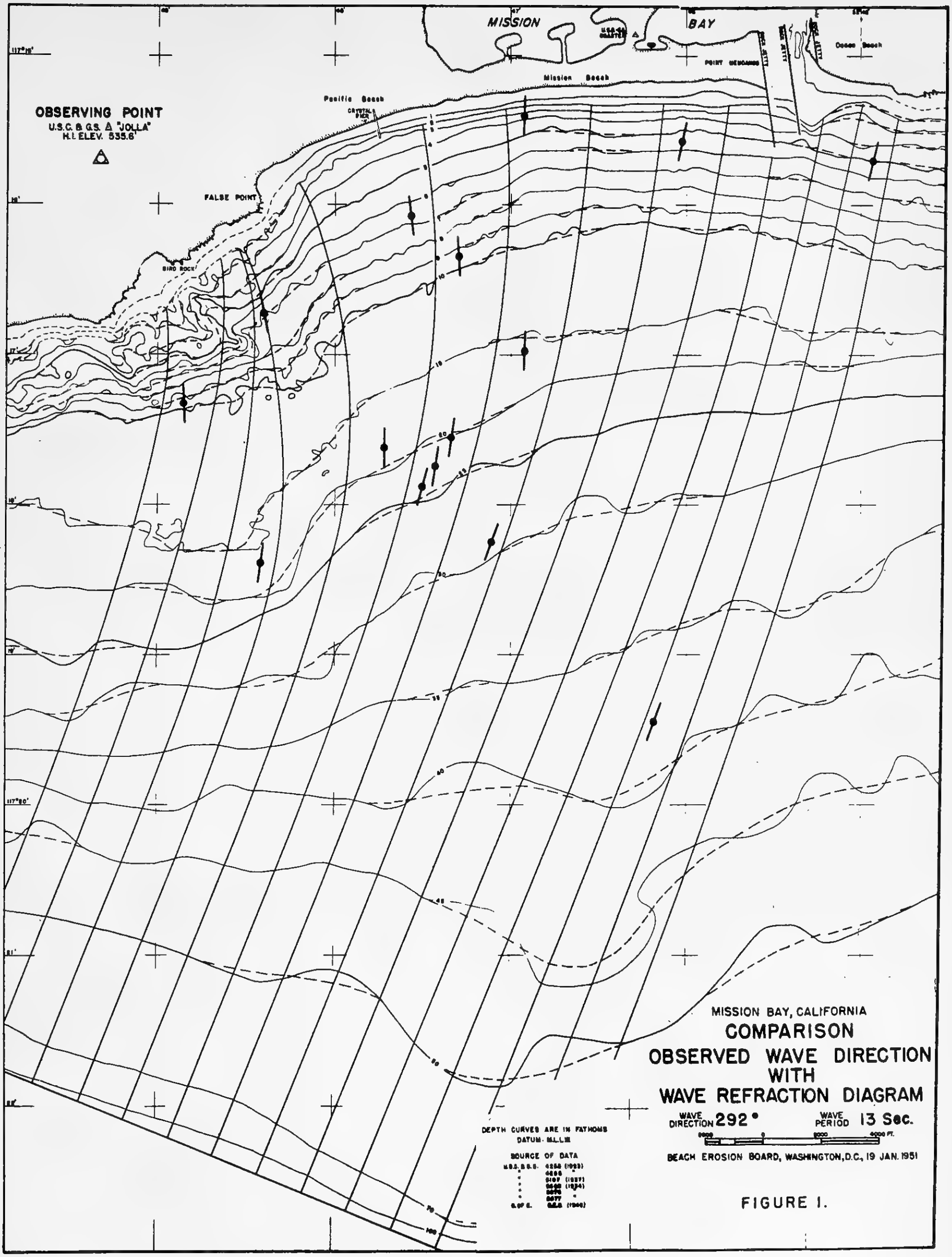

MISSION
BAY
Mission Beach
Pacific Beach
Ocean Beach
OBSERVING POINT
U.S.C. & G.S. Δ "JOLLA"
H.I. ELEV. 535.6'
FALSE POINT
BIRD ROCK
MISSION BAY, CALIFORNIA
COMPARISON
OBSERVED WAVE DIRECTION
WITH
WAVE REFRACTION DIAGRAM
WAVE DIRECTION 292°
WAVE PERIOD 13 Sec.
DEPTH CURVES ARE IN FATHOMS
SOURCE OF DATA
BEACH EROSION BOARD, WASHINGTON, D.C., 19 JAN. 1951

FIGURE 1.

BEACH EROSION STUDIES

The principal types of beach erosion control reports of studies at specific localities are the following:

a. Cooperative studies (authorization by the Chief of Engineers in accordance with Section 2, River and Harbor Act approved on 3 July 1930).

b. Preliminary examinations and surveys (Congressional authorization by reference to locality by name).

c. Reports on shore line changes which may result from improvements of the entrances at the mouths of rivers and inlets (Section 5, Public Law No. 409, 74th Congress).

d. Reports on shore protection of Federal property (authorization by the Chief of Engineers).

Of these types of studies, cooperative beach erosion studies are the type most frequently made when a community desires investigation of its particular problem. As these studies have greater general interest, information concerning studies of specific localities contained in these quarterly bulletins will be confined to cooperative studies. Information about other types of studies can be obtained upon inquiry to this office.

Cooperative studies of beach erosion are studies made by the Corps of Engineers in cooperation with appropriate agencies of the various States by authority of Section 2, of the River and Harbor Act approved 3 July 1930. By executive ruling the cost of these studies is divided equally between the United States and the cooperating agency. Information concerning the initiation of a cooperative study may be obtained from any District Engineer of the Corps of Engineers. After a report on a cooperative study has been transmitted to Congress, a summary thereof is included in the next issue of this bulletin. A summary of a report transmitted to Congress and a list of cooperative studies now in progress follow:

SUMMARY OF REPORT TRANSMITTED TO CONGRESS

The area studied is located in the Town of Hull, Massachusetts, 12 miles southeast of the City of Boston. It consists of the 1-mile length of beach in the publicly owned Metropolitan District Commission Nantasket Beach Reservation adjacent to the mainland base of a $3\frac{1}{2}$-mile long tombolo which is extensively developed as an amusement area. The reservation consists of a wide beach, a seawall, a highway, and a public area 125 to 150 feet wide which contains parking space, pavilions, a hotel, a bathhouse, and sanitary buildings. The beach is wide and flat and is composed of hard-packed sand in foreshore and offshore areas, and soft sand and stones in backshore areas. The beach is extensively used for recreation.

Nantasket Beach has for many years been relatively stable. Where erosion has occurred, the beach has subsequently been rebuilt by natural forces. The seawalls have withstood direct wave attack and adequately serve their purpose. The only beach problem existing at Nantasket Beach is that caused by stones on the beach which interfere with its recreational use.

The division engineer considered the desires of the cooperating agency and studied the character, sources, and movement of beach material, the existing structures, the changes in the shore lines and offshore bottom, and the effects of winds and storms. He concluded that improvement of the composition of the beach is a problem of periodic beach maintenance which is the responsibility of local authorities and that this maintenance may be accomplished by: (1) covering the stones with sand, or (2) burying stones in trenches or pits dug in the backshore area, or (3) removing stone deposits and replacing with equal volumes of suitable sand. He recommended that the cooperating agency continue maintaing the beach in a suitable condition for recreational use, expanding its maintenance methods as necessary to bury and cover the stone deposits more completely or to remove the stones and replace them with equal volumes of sand.

The Board carefully considered the report of the division engineer and concurred in his conclusions and recommendations. In compliance with existing statutory requirements the Beach Erosion Board stated its opinion that:

a. It is not advisable for the United States to adopt a project for the locality at this time;

b. The recommended maintenance measures are in the public interest, but since the policy established by Public Law 727, 79th Congress does not include a provision for Federal participation in maintenance costs;

c. No share of the expense should be borne by the United States.

COOPERATIVE BEACH EROSION STUDIES IN PROGRESS

NEW HAMPSHIRE

HAMPTON BEACH. Cooperative Agency: New Hampshire Shore and Beach Preservation and Development Commission.

Problem: To determine the best method of preventing further erosion and of stabilizing and restoring the beaches, also to determine the extent of Federal aid in any proposed plans of protection and improvement.

MASSACHUSETTS

PEMBERTON POINT TO GURNET POINT. Cooperating Agency: Department of Public Works, Commonwealth of Massachusetts.

Problem: To determine the best methods of shore protection prevention of further erosion and improvement of beaches, and specifically to develop plans for protection of Crescent Beach, The Glades, North Scituate Beach and Brant Rock.

CONNECTICUT

STATE OF CONNECTICUT. Cooperating Agency: State of Connecticut (Acting through the Flood Control and Water Policy Commission).

Problem: To determine the most suitable methods of stabilizing and improving the shore line. Sections of the coast will be studied in order of priority as requested by the cooperating agency until the entire coast is included.

NEW YORK

JONES BEACH. Cooperating Agency: Long Island State Parks Commission

Problem: To determine behavior of the shore during a 12-month cycle, including study of littoral drift, wave refraction and movement of artificial sand supply between Fire Island and Jones Inlets.

NEW JERSEY

OCEAN CITY. Cooperating Agency: City of Ocean City.

Problem: To determine the causes of erosion or accretion and the effect of previously constructed groins and structures, and to recommend remedial measures to prevent further erosion and to restore the beaches.

VIRGINIA

VIRGINIA BEACH. Cooperating Agency: Town of Virginia Beach.

Problem: To determine the methods for the improvement and protection of the beach and existing concrete sea wall.

SOUTH CAROLINA

STATE OF SOUTH CAROLINA. Cooperating Agency: State Highway Department.

Problem: To determine the best method of preventing erosion, stabilizing and improving the beaches.

FLORIDA

PINELLAS COUNTY. Cooperating Agency: Board of County Commissioners.

Problem: To determine the best methods of preventing further recession of the gulf shore line, stabilizing the gulf shores of certain passes, and widening certain beaches within the study area.

LOUISIANA

LAKE PONTCHARTRAIN. Cooperating Agency: Board of Levee Commissioners, Orleans Levee District.

Problem: To determine the best method of effecting necessary repairs to the existing sea wall and the desirability of building an artificial beach to provide protection to the wall and also to provide additional recreational beach area.

TEXAS

GALVESTON COUNTY. Cooperating Agency: County Commissioners Court of Galveston County.

Problem: To determine the best method of providing a permanent beach and the necessity for further protection or extending the sea wall within the area bounded by the Galveston South Jetty and Eight Mile Road.

To determine the most practicable and economical method of preventing or retarding bank recession on the shore of Galveston Bay between April Fool Point and Kemah.

CALIFORNIA

STATE OF CALIFORNIA. Cooperating Agency. Division of Beaches and Parks, State of California.

Problem: To conduct a study of the problems of beach erosion and shore protection along the entire coast of California. The initial studies are being made in the Ventura-Port Hueneme area, the Santa Monica Bay area and the Santa Cruz area.

WISCONSIN

RACINE COUNTY. Cooperating Agency: Racine County.

Problem: To prevent erosion by waves and currents, and to determine the most suitable methods for protection, restoration and development of beaches.

KENOSHA. Cooperating Agency, City of Kenosha.

Problem: To determine the best method of shore protection and beach erosion control.

OHIO

STATE OF OHIO. Cooperating Agency: State of Ohio (Acting through the Superintendent of Public Works).

Problem: To determine the best method of preventing further erosion of and stabilizing existing beaches, of restoring and creating new beaches, and appropriate locations for the development of recreational facilities by the State along the Lake Erie shore line.

TERRITORY OF HAWAII

WAIKIKI BEACH:
WAIMEA & HANAPEPE, KAUAI. Cooperating Agency: Board of Harbor Commissioners, Territory of Hawaii.

Problem: To determine the most suitable method of preventing erosion, and of increasing the usable recreational beach area, and to determine the extent of Federal aid in effecting the desired improvement.

BEACH EROSION LITERATURE

There are listed below a collection of papers presented at the Institution on Coastal Engineering, University of California, Long Beach, California. Copies of these papers can be obtained on a two-week loan by interested official agencies.

Munk, W. H., "Origin and Generation of Waves"

Wiegel, R. L., "Elements of Wave Theory"

Mason, M. A., "Transformation of Waves in Shallow Water"

Shepard, F. P., "Nearshore Circulation"

Vanoni, Vito A., "Harbor Surging"

Arthur, R. S., "Wave Forecasting and Hindcasting"

Studds, R.F.A., "Coast and Geodetic Survey Data - An Aid to the Coastal Engineer"

Fleming, R. H., "The Engineering Application of Sea and Swell Data"

Elliott, D. O., "The Beach Erosion Board"

Handin, John W., "The Geological Aspects of Coastal Engineering"

Einstein, H. A., "Estimated Quantities of Sediment Supplied by Streams to a Coast"

Krumbein, W. C., "Littoral Processes in Lakes"

Eaton, R. O., "Littoral Processes on Sandy Coasts"

Blackman, Berkeley, "Dredging at Inlets on Sandy Coasts"

Reilly, G. P., "Dredging at Coastal Inlets - The Sea-going Hopper Dredge"

Simmons, H. B., "Contribution of Hydraulic Models to Coastal Sedimentation Studies"

Peel, K. P., "Location of Harbors"

Rawn, A. M., "Factors Influencing and Limiting the Location of Sewer Ocean Outfalls"

Kenyon, E. C., Jr., "Case History of Ocean Inlets Los Angeles County Flood Control District"

Hickson, R. E., "Case History of Columbia River Jetties"

McQuat, H. W., " Case History of the Los Angeles Harbor"

Wicker, C. F., "Case History of the New Jersey Coastline"

Hudson, R. Y., "The Hydraulic Model as an Aid in Breakwater Design"

Morison, J. R., "Design of Piling"

Schauffle, H. J., "Erosion and Corrosion on Marine Structures"

Ayers, J. R., "Seawalls and Breakwaters"

Hickson, R. E., "Design and Construction of Jetties"

Kaplan, "Design of Breakwaters"

Streblow, A. G., "Breakwater Construction"

Horton, D. F., "The Design and Construction of Groins"

Caldwell, J. M., "By-Passing Sand at South Lake Worth Inlet"