DEPARTMENT OF THE ARMY
CORPS OF ENGINEERS

THE

BULLETIN

OF THE

BEACH EROSION BOARD

OFFICE, CHIEF OF ENGINEERS
WASHINGTON, D.C.

VOL. 6 JULY 1, 1952 NO. 3

DEPARTMENT OF THE ARMY

CORPS OF ENGINEERS

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U. S. Dept. of the Army; Corps of Eng.; Beach Erosion Board. Bulletin.

ERRATA SHEET FOR Vol. 6, No. 3 of THE BULLETIN OF THE BEACH EROSION BOARD

All copies of the above issue dated July 1, 1952, should be marked to indicate the following corrections:

Page	Line	Change
23	21	For "then C_2" substitute "then C_1 is greater than C_2"
25	Equation 4 should be:	$C'' = \frac{C_1' + C_2'}{2} = \frac{1}{2}\left[C_1 + C_2 + \frac{C_1 - C_2}{J} d \sin\alpha + \frac{C_1 - C_2}{J} d \sin(\alpha - \Delta\alpha)\right]$
29	17	For $\tan \frac{\alpha_1 - \alpha_2}{2}$ substitute $\tan \frac{\alpha_1 + \alpha_2}{2}$
32	37	For $\left(\frac{K \mp 2}{K \pm 2}\right)$ substitute $\left(\frac{2 \mp K}{K \pm 2}\right)$
33		In second column heading for $\frac{\Delta L}{L_{av}} = C$ substitute $\frac{\Delta L}{L_{av}} = K$
34	1	For "for proceeding from deeper to shallower water" substitute "where the minus sign in the numerator and the plus sign in the denominator refer to the case of proceeding from deeper to shallower water."

A METHOD OF SEPARATING MULTIPLE SYSTEMS OF OCEAN WAVES FOR DETAILED STUDY OF DIRECTIONS AND OTHER PROPERTIES

by

H. A. WARD
Engineer, Reports and Publications Division

In past studies of water waves, primary attention has been given to the measurements of individual wave trains and to refraction and diffraction theory for a single system of waves. Some attempts have been made to determine the state of the sea for local areas through the use of stereo-photogrammetric methods(1), but aside from this, little has been contributed on the general make-up of the sea patterns.

Various organizations, including the Beach Erosion Board, maintain automatic wave-recording devices that give a record of the change in water surface on a height and time scale as waves pass the gaging point, but such records give no data on the direction of approach, or on the several separate wave systems which may be present at the time. Obviously, the energy value of different systems that pass the gaging point, and their relative directions of approach are of the utmost importance, since they may either augment or oppose each other whereupon the algebraic values of the systems determine their ability to transport material or produce other coastal phenomena of interest.

Where two or more systems of waves are present and the systems are running at angles with each other, the record from the gages is confused and erratic. For a period the waves may be regular, only to be followed by a very irregular series, and occasionally out of a fairly rough sea, a flat of considerable duration will appear. The study of multiple wave systems has shown that data from a single gaging point is not enough to permit evaluation of the forces, and because of lateral components resulting from interference by companion wave trains, clapotis effects and flats will show up on a record where to all appearances only generally uniform systems of waves are present. These lateral components have not been taken into consideration in the theory of harmonic analysis for present methods of wave recording.

Because of the irregularities shown on wave gage records, some have concluded what ocean waves do not conform to classical theory. However, when the separate wave trains of a multiple system are examined, it is found that they do conform to a surprising degree; but the crests must be considered more in the nature of an undulating line whose full behavior

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(1) Methods d'etude des Lames, M. J. Larras, Trauaux, Vol. 21, No. 58, 1937, and Beach Erosion Board Bulletin, Vol. 4, No. 4.

cannot be recorded at a point. It is believed that clapotis groupings and flats will average out to a uniform organization of the various systems composing the state of the sea.

That wave systems do conform more closely to classical theory than one might expect has important implications. Wave refraction theory, which is based on a single system of waves of specific direction and period, is in no way weakened by the knowledge that usually more than one system is present, but instead the use of the theory is strengthened inasmuch as the various systems can be analyzed separately and their net effects evaluated; a step beyond present refraction procedures. This will require a reversal of present processes; and to the betterment of the practice, refraction diagrams may be drawn from the known inshore conditions seaward, rather than from the computed or assumed offshore conditions to sometimes doubtful inshore results.

The development of refraction theory during the last decade is probably the greatest contribution that has been made to coastal engineering, yet little use has been made of refraction diagrams on the east coast of the United States, primarily because the wide continental shelf with its many irregular submarine features produces such complicated ray paths that they go beyond present refraction procedures. Furthermore, the magnitude of some of the diagrams is such that gross errors are introduced even over a fairly uniform bottom. For example, to bring in a long-period diagram from due east to the Long Island and Northern New Jersey coasts it is necessary to go some four-hundred miles offshore to the eastern edge of Georges Bank, and even though the bottom were regular, the errors over this distance would be considerable. Add to this the fact that the bottom topography is most complex, causing the orthogonals to cross and sometimes recross, any computations for values at the shore would be most unreliable.

In view of the foregoing it is sufficient to say that present refraction procedures are inadequate, and that attention should be given to the development of more feasible methods of studying nearshore conditions. To that end, the aerial photographs in the files of the Beach Erosion Board were studied in detail, and as a result, the method of separating the wave systems and determining their direction was developed. If accurate determination of direction alone can be accomplished, (since indications are that even the eye is unreliable in many cases) substantial progress will have been made, but to be able to separate the various systems of waves that are present and determine the directions and at least some of the other properties of each, opens up a wide range of study for both the scientist and the coastal engineer.

There is a wide conception that waves in deeper water are not long crested, uniformly spaced, but are instead a series of short crests which somehow miraculously unite in shallow water to form the breakers usually observed. After examination of many aerial photographs this concept is questioned. It is true that due to interferences of other wave trains the single crests show breaks at the instant of film exposure, such breaks or family groupings are constantly changing position, though in general, the wave systems maintain their organization remarkably well. When a short-crested appearance is found, invariably two or more wave trains are present.

While studying the aerial photographs some very interesting results were obtained through the use of a mechanical aid in the form of a transparent sheet on which parallel lines had been ruled, spaced about ten to the inch. This was used to separate the individual wave systems appearing in the picture and oftimes to the astonishment of the viewer, order appeared out of a turbulent state of the sea that seemed devoid of orderly arrangement.

In making the aid, fairly thin lines were ruled on clear cellophane spaced as previously mentioned. In lieu of the above, a sheet of No. 79 Zip-A-Tone will give good results. In use, the sheet is rotated until one train of waves appear, then aligned generally with the system viewed to disclose the other. The same procedure is used with secondary systems that are running with the primary systems. In some areas, such as the west coast of the United States, long period swells may be running with the lesser systems, but these will usually be obvious without an aid.

Photographs in which waves have only minor irregularities and the image has the appearance of a single system should not be discounted. The smaller the angle the crests make with each other, the more the wave pattern will appear as a single unit, nevertheless, as long as the tell-tale interruption of crests is present, multi-directional groups are indicated.

The accompanying photographs will illustrate the application of the principles. Figure 1-A has two obvious wave trains, one a minor system approaching from the upper left corner, and the other the more prominent system which has the broken-crest characteristics. Figures 1-B and 1-C show the latter to be two trains, each conforming to classical theory remarkably well. Figure 2-A is an example of a turbulent condition of the sea in which, although no wave patterns are clearly shown in the offshore area, two wave systems can be traced into the surf zone as shown in Figures 2-B and 2-C. Note particularly that each maintains its separate identity through the surf, and that each produces littoral currents. In one portion of the picture these currents oppose each other and in another portion they augment each other.

Based on the many pictures examined, the conclusions are reached that single wave trains are rarely found in the nearshore area and that multiple systems are the rule. On the west coast of the United States the secondary systems are often greatly overshadowed by the heavy swells which, of course, are the more significant waves as far as coastal engineering is concerned. Of the heavier swells, some approach from the northwest and concurrently the southerly swells may be running, whereupon the condition is no different than that of other coastal regions, and if net littoral forces are to be determined the result must be the algebraic sum of the various systems. On the east coast many of the pictures show waves that appear similar, which suggests that they are crossed trains resulting from refraction of a single deep water wave.

The use of this simple method of sorting the wave systems opens up possibilities for greater use of aerial photographs as a basis for the study of local areas. It will be noted that once separated, the wave lengths are more measurable and therefore provide a good basis for various computations. In such matters as the investigation of the formation of "finger shoals" or the unusual concentration of wave action at one point, the contributing factors can be traced from known conditions in the shore vicinity.

As a natural consequence of two or three trains running at an angle with each other, they must at intervals synchronize and assume some harmonic arrangement. Disregarding lateral components for the moment, the determination of this arrangement from a single gage record would require a slow, costly analysis of the harmonic contents, which when completed would still give no information on the angle of impingement on the shore. Where aerial photographs are available or perhaps where the wave train directions and length or period can be established by direct observations, or by recording instruments developed for the purpose, a basis will have been provided for a more simplified analysis, for then the major components of the record will have already been set apart. Remembering that the appearence of the individual system is such that a record of any one alone would be fairly regular, the harmonic analysis of the group would be greatly simplified.

More definite knowledge of the complexities of wave patterns points up some of the difficulties that have been encountered in attempts to develop wave direction recorders, in particular, the attempt by the Beach Erosion Board to use the Rayleigh Disk for that purpose(2). Under the influence of multiple forces the disk attempted to align to momentary predominance or resultants, therefore the desired result was not obtained.

At the present time no adequate method exists for the determination of wave directions. It is even questioned whether present visual methods are reliable, for in most cases they are made in connection with dye tests, float runs, or some other activity, and are usually taken from some point low on the beach. Scripps Institution of Oceanography made a study of longshore currents(3) which disclosed unexplained behaviors that may have been due to difficulties in determining the composition of the sea and the direction of approach of the various systems. Commenting on the direction observations the report states, "The field assistants who made most of the observations, did not feel confident in all these observations since many

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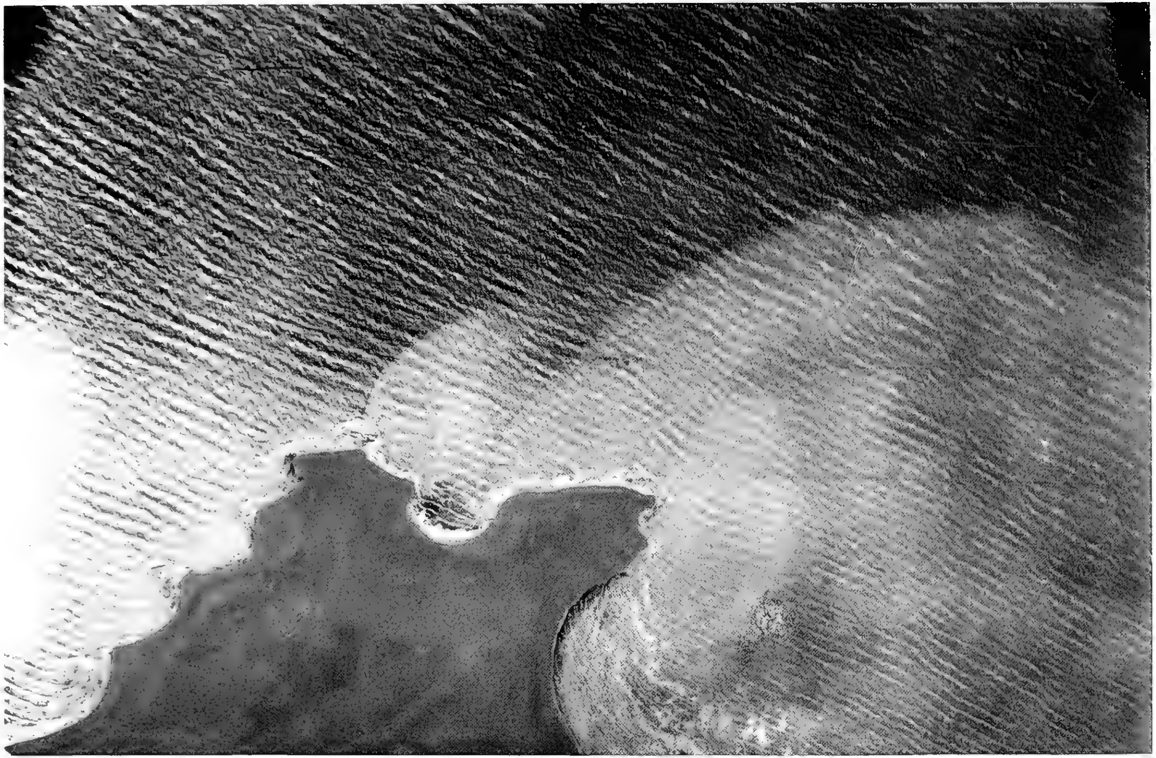
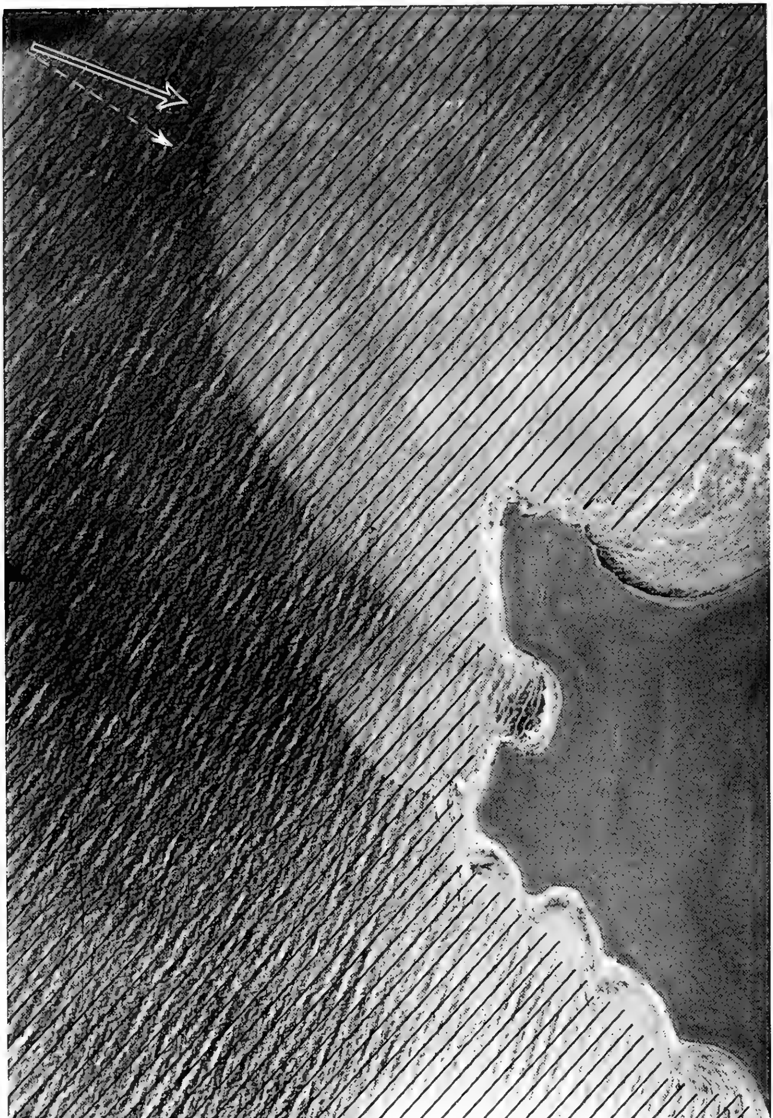
(2) "The Rayleigh Disk as a Wave Direction Indicator", Beach Erosion Board Technical Memorandum No. 18.

(3) "Longshore Current Observations in Southern California", Beach Erosion Board Technical Memorandum No. 13.



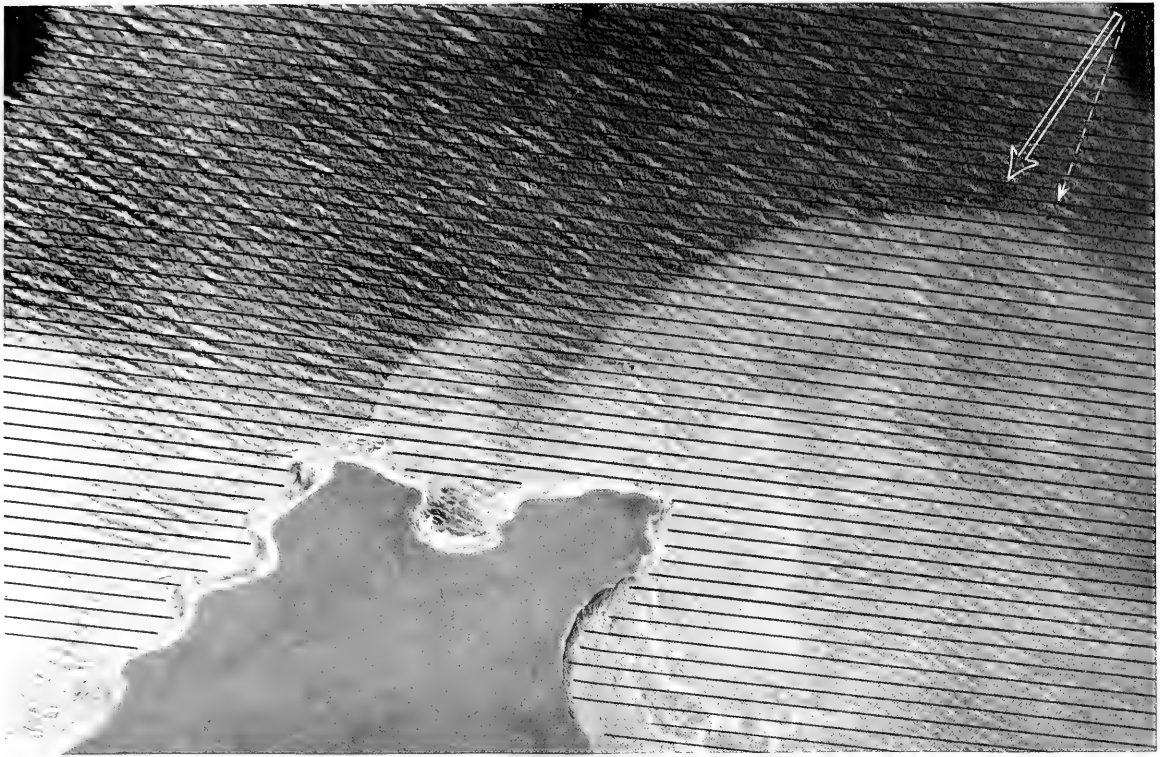
TWO WAVE SYSTEMS—ONE WITH BROKEN—CREST CHARACTERISTICS.

FIGURE—I—A



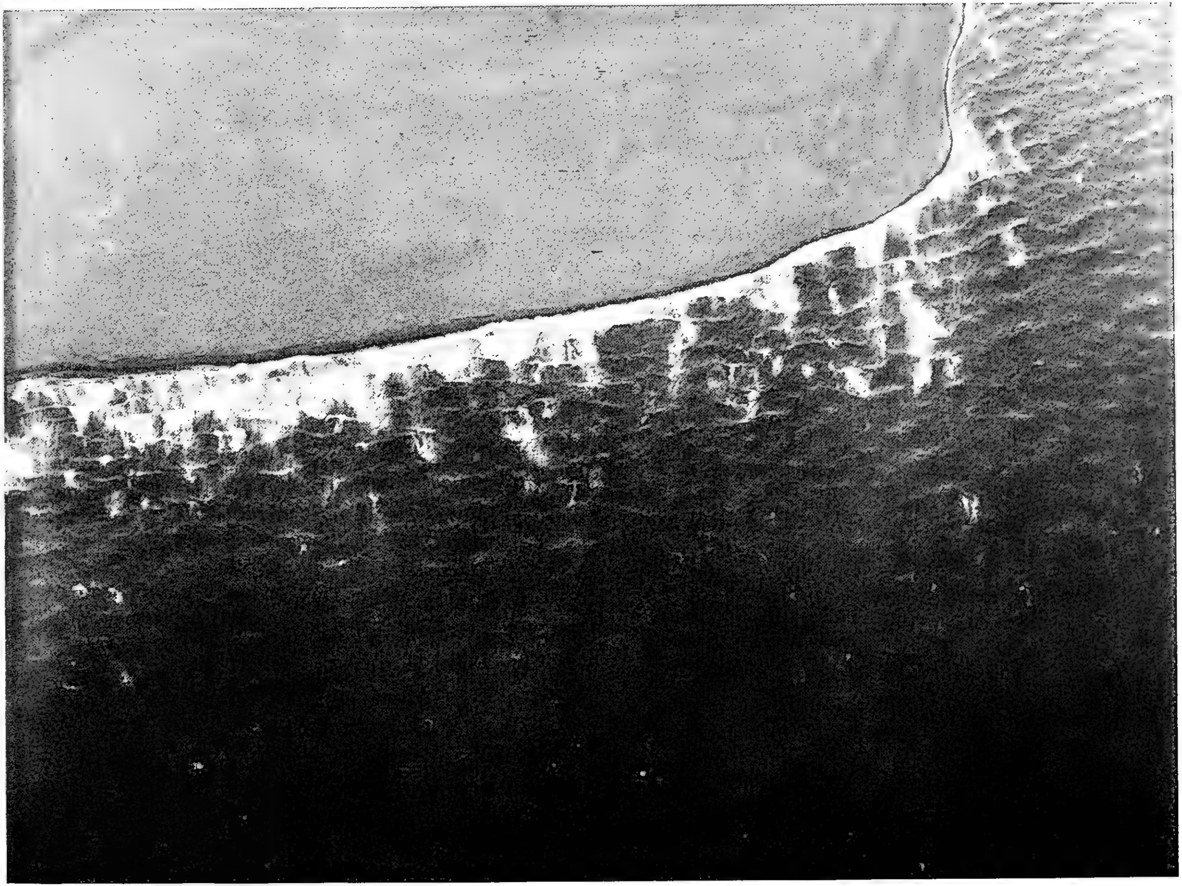
BROKEN—CREST SYSTEM SEPARATED. ARROWS INDICATE DIRECTION OF APPROACH

FIGURE - I-B



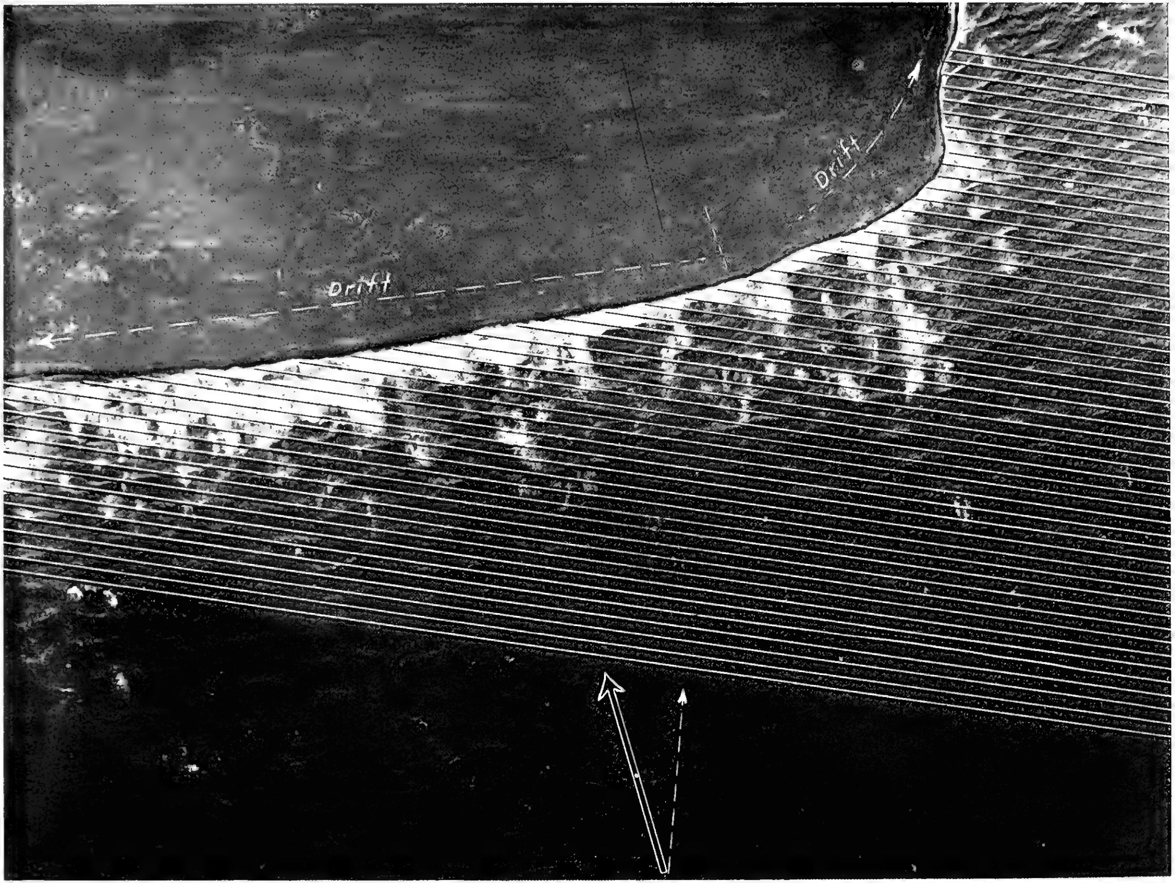
SECOND SYSTEM. COMPARE WITH FIGURE 1-B.

FIGURE-1-C



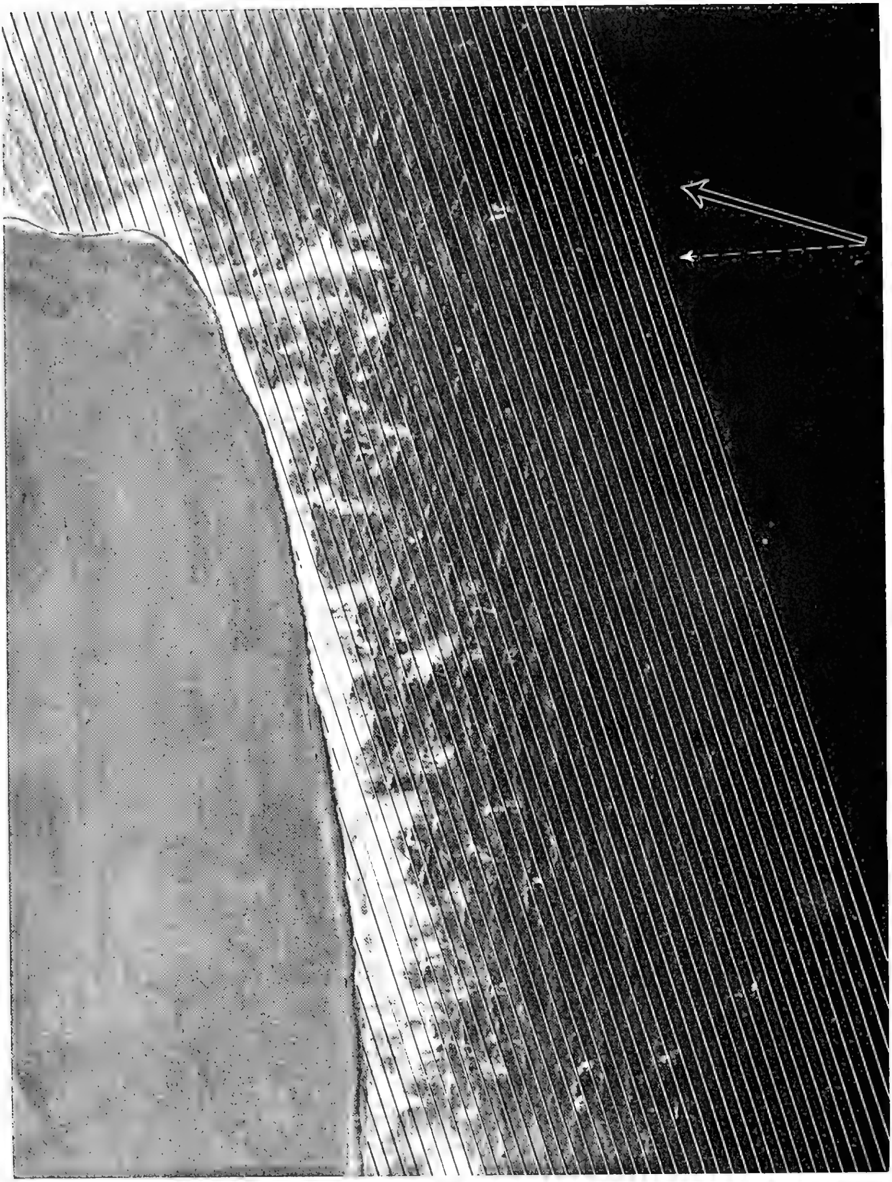
A ROUGH SEA IN THE NEARSHORE AREA. NO DEFINITE WAVE PATTERN.

FIGURE- 2-A



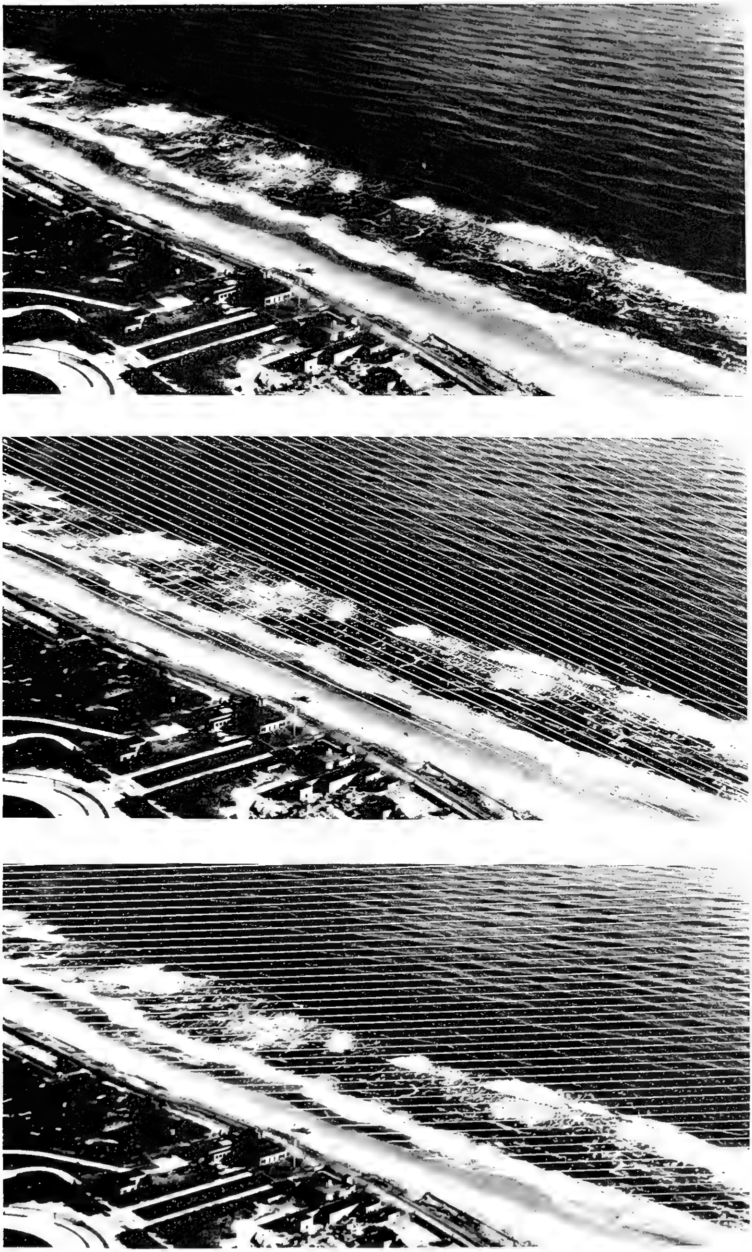
PRIMARILY LEFTWARD LITTORAL DRIFT UNDER WAVE APPROACH SHOWN

FIGURE— 2-B



SECOND SYSTEM PRODUCING DRIFT TO THE RIGHT.

FIGURE - 2 - C



TOP — OBLIQUE PHOTO SHOWING WAVES AT JONES BEACH L.I.
CENTER & BOTTOM — TWO PRINCIPAL WAVE SYSTEMS SEPARATED

FIGURE — 3

of them were made very close to sea level and the waves were often so much confused that it was very difficult to be sure of the direction of approach of the principal wave trains." Regarding unexplained current behaviors it states; "....there appear to be a considerable number of appreciable currents where the approach was normal and even a number of currents which appeared to flow in the wrong direction so far as wave approach was concerned." In view of their experience it is concluded that the determination of the direction of approach of principal wave trains from a point low on the beach is difficult, and that reliable observations for multiple systems is practically impossible. In the subject study, the extent to which unrecognized multiple wave systems may have influenced the results can never be known, but it is noted that dual wave trains from the north were not given recognition in an illustration used (aerial photo, Newport Bay, their Figure 14).

In connection with a study by the Beach Erosion Board at Mission Bay, California, it was found that of the several observational procedures, a satisfactory method of determining wave direction presented the greatest problem. A sighting bar(4) was devised for use with a transit and its use improved the results as far as the predominant waves were concerned. Observations were made from the bluff, and no attempt was made to separate wave systems.

Every test made up to the time of writing indicates that the parallel lines method is the most economical and convenient means yet devised for determining directions, and for studying general sea surface conditions, particularly for those who have a file of aerial photographs available. Were it necessary to fly pictures for wave study alone the expense could scarcely be justified, but since it appears that similar but limited results can be obtained from photographs taken from some high point on the shore, the method comes within the reach of all.

Figure 3 is an oblique photo showing good wave action at Jones Beach, Long Island, New York. In the center and bottom prints, the two principal wave trains have been separated, and it can be seen that each train has a positive direction of approach that can be measured. Note how well the two systems conform to classical theory. Furthermore, when the camera height is known, the wave lengths can be computed and other data derived.

The illustration used has a fairly high oblique angle. Field tests have not been made to determine how low the angle can be and still give satisfactory results, but judging from pictures taken from the pavilion on the Steel Pier at Atlantic City, New Jersey, in connection with stereophotogrammetric wave measurements(5), which were relatively low in oblique angles, the lower limit comes well within the heights of the usual vantage points found on the seashore.

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(4) "A Method of Estimating Wave Direction", D. R. Forest, Beach Erosion Board Bulletin, Vol. 4, No. 2.

(5) "Stereophotogrammetric Wave Measurements", BEB Bulletin, Vol. 4, No. 4.

Deep water wave have been separated in sea-rescue pictures in the files of the Coast Guard which were taken from the Coast Guard Cutter, and were probably lower in oblique angle than those from the Steel Pier. It is of particular significance that deep-water waves as viewed from a vessel respond to the parallel lines method, for now all that is needed to obtain a wide collection of data is a simple method for making observations directly without need for photography. Such a device is under development but must still be tested under dynamic conditions where the wave systems are in constant motion, before it is known whether the present design will be adequate.

DEVELOPMENTS IN THE SCIENCE OF COASTAL ENGINEERING

by

Captain Peter Somers, Executive Officer,
Beach Erosion Board, Corps of Engineers

Geologists tell us that the approximate age of the earth is somewhat more than two billion years, and that during the early part of that time, in response to the pull of the sun the molten liquids of the earth's surface rolled around the globe and that its atmosphere contained much hydrogen. Unlike the sun and her sister planets, our small globe had not enough gravitational pull to retain the light and fast moving hydrogen molecules, with the result that in a few thousand years, most of them had dispersed into space, and the remainder combined with oxygen to form water vapor.

As the atmosphere cooled further the water vapor began to condense, probably at first to be on the night-shaded side of the earth. Further cooling permitted the patches of condensed water to spread and increase in volume. Based on the same theory, it is further believed that when the patches of condensed water became permanent, the first ocean was born.

We are also told that the ocean contains an untold wealth of minerals including silver and gold, which can actually be ours if we can determine how the sea can be made to yield its riches.

This promise of good fortune however is of little comfort to the many states, communities and individuals who find this very same ocean unleashing with terrific fury its damaging forces against their worldy possessions, the land adjoining the ocean and the structures provided to prevent the land from eroding. Erosion is a natural agency which down through the ages has been causing continual changes in land levels and shore lines.

The history of any shore line is one of continual mobility and change, molded and remolded by the forces in breaking waves. Where shores are undeveloped, variations in the beach are not considered too important to local interests, and recession of the shore line and methods of preventing such recession are of little concern. The problem becomes more acute however, when intense development crowds the shore, and property owners become aware of the forces of nature. When the ocean begins to lap at their doors a loud cry for immediate protection is raised.

The United States possess more than 20,000 miles of tidal shore line (besides the shore of innumerable lakes and rivers) which include a large number of beach areas of more than local significance. Such beach areas are the popular holiday resorts of the nation. The habit of going to the sea shore is so firmly established as a means of healthful recreation that extensive development has resulted on many of the beach areas near population centers.

There appears to be the universal desire to be close to the water. The land closest to the water usually has the greatest value. Very little consideration of the many forces involved in such areas is shown, in fact the dangers are not actually known until the stability of the beach is upset by an attack of wind and waves.

The average engineer finds his interests far removed from the subject of beach erosion until such times as he finds himself confronted with such a problem; however, the engineer whose field of learning includes oceanography, coastal geomorphology, geology, meteorology, hydrology, soil analysis and hydraulics, will find the study of shore phenomena a most facinating and interesting subject.

The accumulated thinking and experiments of many generations to resolve the physical laws that govern the natural forces affecting the loss of the land to the sea have provided a large amount of knowledge to be utilized by the research engineer of today.

The importance of coastal phenomena in connection with studies of beaches and marine structures has long been known to the Corps of Engineers, particularly in the realm of river and harbor works, and it is agreed that while considerable advancement has been noted during the last two decades, the scientific study of beach erosion and shore protection is just now coming into its own.

Beach Erosion Board Problems

The shore problems studied by the Beach Erosion Board fall into three general categories; (1) problems of stabilization and rehabilitation of beach areas (these are the problems of beach loss or damage that occasioned and justified the establishment of the Board originally); (2) problems arising from the effects on adjacent shore lines of the provision of navigation channels from the sea or lake into rivers and inlets; (3) problem of shore control associated with harbor protective works.

The activities of the Board during the past twenty years lead us to believe that the solution to these problems requires adequate knowledge of six cardinal elements of the material supply, energy and economic factors. These are:

1. The sources and character of the shore material;
2. The rates of supply and loss of shore material to and from the problem area;
3. The manner of movement of the shore material from the source to the beach and from the beach to other areas;
4. The feasible methods of increasing the rate of supply to the shore or reducing the rate of loss of shore material from the shore;
5. The design requirements of the feasible methods of modifying rates of supply and loss of shore material;
6. The economic cost of each feasible method of modification.

The development of adequate knowledge of these six elements of any specific shore problem constitutes the beach erosion control study or the study of the shore effects associated with navigational improvement. The determination of how to develop the knowledge required constitutes the general investigation and research activities of the Board.

Coastal Physiography

The shore line of the United States is characterized by a diversity of shore line forms. The New England shore line particularly north of Cape Cod is found to be rather rugged with cliffs and headlands fronting directly on the ocean and with rather short lengths of beaches caught between the headlands.

The Atlantic shore line from Cape Cod south and the Gulf beaches are characterized generally by long uniform beaches with the adjacent topography lying only a few feet above high tide. Frequently, these take the form of barrier beaches, separated from the mainland by salt water marshes or lagoons. The Pacific shore line shows some of the characteristics of both the preceding types of shore line with long sweeping beaches broken by bold headlands.

The geologist tells us that the composition and origin of these beaches and headlands started some two billion years ago when the molten mass of the earth began to solidify and cool into igneous rocks. These original igneous rocks have gone through various changes due to changes in weather, environment, etc., so that the form and characteristics of many of them have greatly changed. After all the weathering, frost action, glaciation, hydration, etc. we find that there are on shores several types of rocks and minerals, the most abundant of which is quartz sand. The quartz reaches the beach environment under the influence of gravity, by wind action, or as water-borne detritus.

Natural Forces Affecting Beach Erosion

Changes of the shore or beach follow from natural physical causes resulting from the interaction of the land, the sea, and the atmosphere at the shore or beach. Changes in the shore line are usually the result of sand movement due to the action of several natural forces. These forces result from wave action, tidal action, and ocean currents, which act on the shore in varying degrees depending on the coastal physiography. The dominant force is the wind-generated ocean wave. These waves after being generated by a wind disturbance, may leave the area of the disturbance and travel for hundreds or even thousands of miles before being interrupted by a land mass. Most of the waves noted along our shores are generated by distant storms.

Tides and Currents

The subject of tides has engaged the attention of mathematicians and engineers for several centuries. The principal effects of the tides in reference to beach erosion are generally agreed to be: (1) they shift the zone of wave attack on the beach and (2) they set up currents that may assist or oppose the movement of the sand particles which compose the beach.

Currents resulting from tidal action are generally noticeable along the coasts only in the vicinity of bays, estuaries, or inlets. The rise and fall of the tide produces a flood and ebb of the current filling and emptying the estuary. This current is usually weak but can reach sufficient magnitude in the vicinity of the estuary to cause erosion by its own action. Where the currents are too small to cause movement of the beach sand alone they can be effective by working in conjunction with the waves breaking on the beach. Waves approaching the shore at an angle will produce a littoral current substantially parallel to the shore.

Sand Movement

The effect of waves on the sand beaches is especially pronounced at the breaker line, where comparatively large quantities of sand are thrown into suspension. An angularity in the direction of wave approach frequently results in a littoral current being set up substantially parallel to the shore. This wave generated littoral current carries the suspended sand alongshore.

The net effect of the action of waves and currents is to produce a beach which may be accreting or eroding, may show cyclic or seasonal changes, and is seldom if ever completely stable. It might be pictured as a river of sand whose direction and velocity are determined by the character of the forces impressed upon it in the form of winds, waves, and currents.

It is well to remember that once a sand grain is put into suspension, the slightest movement of the surrounding water, will produce a corresponding movement of the sand particle. As long as the particle remains in contact with the bottom, it will take currents of appreciable strength to move even the fine sand. However, the almost unceasing wave action on the beaches continually keeps a substantial amount of sand in suspension. When the waves approach normal to the beach, the sand movement would be essentially an oscillation back and forth of the sand particles with little progressive movement parallel to the shore. However, if even a slight littoral current generated by tides or winds is present, the alongshore movement of material can be relatively great.

To avoid confusion we must remember that littoral current is the movement of the water mass while littoral drift is the movement of the solid particles. The coast in the vicinity of Santa Barbara, California, is considered to have a fairly large rate of littoral drift, the direction of drift being from northwest to southeast. Here the average rate of drift is about 300,000 cubic yards of sand per year. A rate of littoral drift ranging from 20,000 to 50,000 cubic yards per year is considered relatively low. Wind blowing over the beach can also be an effective agent in moving the sand and should always be considered when making a study of a specific locality.

Shore Protection Methods

Frequently we find that the natural conditions at a beach are not in stable adjustment and that progressive erosion of the beach is underway. In such cases it is sometimes found that the eroding condition can be arrested by the construction of shore protection structures to check the removal of material from the beach or by increasing the supply of material to the shore.

Presently known feasible methods of modifying supply and loss rates are limited with one exception to various types and placements of structures. We know these by their designations as breakwaters, jetties, bulkheads, seawalls, revetments and groins; artificial nourishment is the non-structural exception.

Breakwaters as the name implies, are energy dissipating or wave interrupting structures employed to break or dissipate water waves and thus prevent their incidence on an area it is desired to protect. Their effect on material movement arises entirely from modification, usually in the form of reduction, of the incident energy of water waves.

Bulkheads, seawalls and revetments constitute a class of structures whose chief function is to substitute non-erodible for erodible material and concurrently provide for energy dissipation. Their effect on the movement of material is direct, by the substitution of materials, and indirect, by the modification, sometimes dissipative, of the energy in incident waves or currents. They are employed widely and are particularly suited for the preservation of a definite land-water boundary at a given location. There is almost always an increase in degradational action associated with their use.

Groins are structures designed and built to intercept and retain material in littoral movement and thus modify rates of supply and loss of material directly. Their influence on the energy pattern is almost entirely indirect, resulting from a reorientation of the shore line in a direction more normal to wave approach. Accordingly, groins should be considered as more than simple barriers to sand movement. Because of this action they may cause an appreciable modification of the entire littoral regimen.

Jetties are multi-purpose structures, whose functions may include those of breakwaters, groins and bulkheads. Although they are usually associated with navigation improvements at inlets, river mouths or harbor entrances, they frequently result in major physiographic changes. Analysis of the effects of jetties on shore lines is one of the most difficult of shore problems to solve in detail, the ease with which general features of the effects can be predicted belying the difficulty of a detailed analysis.

Artificial nourishment is adequately described by its name as a method of supplying beach material additional to that available naturally. It is useful for rehabilitating or rebuilding beaches, for maintaining a desired existing beach condition, or for improving to a large or small extent existing conditions. Although gaining in popularity as a shore control method, little authoritative data on its requirements or performance are available.

In many instances it will be found that alternative solutions of shore problems are indicated by the analysis of the physical problem. In other instances there will be a unique solution. When a choice is possible the problem analysis should include information on the comparative economic cost in sufficient detail to permit final selection of a plan of solution on the basis of economic advantage between otherwise equal plans. It may be found that two or more plans are feasible but not equally desirable from a physical viewpoint. In many cases economic cost may be the controlling item dictating the selection of the plan, even at the expense of desirable physical advantages.

It is recognized by engineers in this field that many aspects of coastal processes are not clearly understood, and several institutions have undertaken research programs to improve present knowledge of the various processes.

Research is conducted in the laboratories of the Beach Erosion Board (and in the field) and is designed to improve knowledge of physical laws pertaining to shore processes and design of shore structures for use of the engineering profession, and to compile general information regarding shores of the United States and the forces acting thereon to aid in the solution of engineering problems at specific locations.

Projects in progress include: (1) a study of the direction and velocity of littoral currents generated by waves, with secondary data on how groins modify littoral currents; and (2) a study of methods of by-passing sand at inlets, which is contributing information on how sand builds up at a jetty, the tidal currents in a jettied inlet, the wave climate in the area, the daily, weekly and bi-monthly changes in the shore line position and bottom topography, the amount of sand transported by waves, and data for relating littoral currents to the generating waves. Laboratory studies include measurements of wave forces against structures with high speed photographs for the study of breaking waves. Another laboratory study is the development of factors that affect the equilibrium profile of beaches. This same data is used to determine the feasibility of using models to study beaches.

Over the course of many years the Board has amassed a large amount of basic data on the character of United States shores. Such information consists chiefly of the geology of the shore, the wave and wind climates, data on the shore material, and the historical record of changes in shore position and condition. It is planned to continue the collection of this background information, which serves as a reservoir of data available for use in beach erosion control studies, thus reducing their cost and the time required for their completion.

NOTES ON DETERMINATION OF STABLE UNDERWATER BREAKWATER SLOPES

BY

Kenneth Kaplan, Engineering Division

The method of determining underwater stone sizes and stable slopes proposed by Iribarren and Nogales (1) contains certain inherent weaknesses: first, the Airy wave relationships commonly used for waves moving up a gradually sloping beach are considered to apply at points over a breakwater slope. Second, it is assumed that waves will break in a depth of water equal to their breaking wave height. Third, and perhaps most important, the cause of stone removal both above and below the water surface is considered the same. That this last does not hold may be seen by referring to the force diagram from which the basic Iribarren equation is derived (2), in which an intermittent "flow-no fluid" is considered to be the basic cause of stone removal above still water level.

It would seem that a more realistic approach would be one in which use is made of the currents caused by wave action. To this end, the relationship derived by Blanchet (3) dealing with the destruction of stone masses by currents may be applied. That is

$$v = K_1 \sqrt{2g \frac{\gamma_s}{\gamma_f}} \sqrt{D} \sqrt{\sin(\alpha - \alpha_0)} \qquad (1)$$

in which v is the critical current velocity for stone mound disintegration;
K is a complex non-dimensional coefficient, but one which should be fairly constant for stones of the same form;
D is a characteristic dimension of the stone;
γ_s and γ_f are the specific gravities of the stone and fluid of immersion respectively;
α is the angle the mound makes with the horizontal; and
α_0 is the angle of repose of the constituent stones.

The weight of each stone may be written as

$$W = K_2 \gamma_s \omega D^3 \qquad (2)$$

in which ω is the unit weight of fresh water. From this

$$D = \sqrt[3]{\frac{W}{K_2 \gamma_s \omega}} \qquad (3)$$

The maximum current due to wave action which would exist at depths below still water in the absence of the breakwater is given by

$$v' = \frac{\pi a}{T} \qquad \left[a = H \frac{\cosh 2\pi (d-Z)/L}{\sinh 2\pi d/L} \right] \qquad (4)$$

This will not be the current velocity which exists when the breakwater is in place, but if we propose that a measure of this current is the v' of equation (4)., we may call

$$v = K_3 v' = K_3 \frac{\pi a}{T} \qquad (5)$$

Substituting equations (3) and (5) in equation (1) and collecting the coefficients gives

$$\sin(\alpha - \alpha_o) = \frac{Ka}{W^{1/6} T} \qquad \left[K = K_4 \left(\frac{\omega^{1/3} \gamma_f}{g} \right)^{1/2} (\gamma_s)^{1/3} \right] \qquad (6)$$

from which we may find the stable slope for stone of weight W under wave attack of height H and period T at a depth of approximately z (better at a depth, $z - H \frac{\sinh 2\pi\left(\frac{2\pi(d-z)}{L}\right)}{\sinh \frac{2\pi d}{L}}$), in which the constant K (or K_4) must be empirically determined. However if the above development is substantially correct, K should remain fairly constant for any one type of stone.

In the report by Iribarren and Nogales (1) a table of stable slopes is given for the breakwater at Argel measured after wave attach had "shaken down" the slopes. Though the upper face of this breakwater consists of artificial blocks, below the depth of 11 meters the armor stone is natural quarry stone. The maximum wave attack immediately before the structure is given as 6.45 meters in height with a period of 13.75 seconds. The breakwater itself is founded in a depth of 35 meters. At the depth of 11 meters the stone weight is 4 metric tons and the slope is 1 on 1.5. The value of "a" may be found from equation (4) (d - z = 21 Meters) by use of tables of hyperbolic functions (5). Solving equation (6) for K (assuming $\alpha_o = 45°$) a tentative value of K = 1.15 is established.

In an attempt to verify this value the results of model studies performed in 1948 at the Waterways Experiment Station (4) were used. In these tests prototype waves with a height of 15 feet and a length of 270 feet were directed at a breakwater founded in a depth of 60 feet. The tests were continued until the breakwater slopes attained stability. Two depths, 10 feet and 25 feet were chosen at random at which the average stone weights were 10 tons and 1 ton respectively. Stable slopes were then calculated and compared with those found in the model tests. At the 10-foot depth the calculated slope was 1 on 1.46 and at the 25-foot depth the calculated slope was 1 on 1.52. The actual slope at both these depths was 1 on 1.5.

Though these values indicate substantial agreement with the preceding theory, further studies are necessary specifically to determine the value- or range of values - of K. In addition prototype data of existing structures should be analyzed to verify model conclusions.

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A NEW METHOD FOR THE GRAPHICAL CONSTRUCTION OF WAVE REFRACTION DIAGRAMS

by

T. Saville, Jr. and K. Kaplan
Staff, Beach Erosion Board

In connection with the preparation of a design manual for shore structures presently being prepared by the Beach Erosion Board, a reexamination of the methods and validity of the various means of obtaining refraction characteristics became necessary. This reexamination has shown that the error involved in the use of the crestless method for drawing orthogonals devised by Isaacs(1)(2) and in general use throughout the United States, may become quite significant when large angles of incidence or large contour intervals are used -- in extreme cases errors in excess of 5 to 6 degrees in direction and 10 to 15% in refraction coefficient being possible. This error is introduced into the scales devised by Isaacs by the assumption that the change in angle, $\Delta\alpha$, is small in comparison to the angle of incidence, α, so that $\sin(\alpha - \Delta\alpha) \simeq \sin\alpha$.

However, this assumption is not necessary to the derivation of the equation expressing the angle change, and more accurate equations may be derived in a manner similar to that of Isaacs. The basic assumptions for this extension of the original theory remain the same: namely that the velocity varies linearly between contours, that the radius of curvature of the orthogonal between contours is constant (a circular arc), and that $\Delta\alpha$ is small so that $\tan\Delta\alpha \simeq \sin\Delta\alpha \simeq \Delta\alpha$.

Referring to figure 1a, d_1 and d_2, C_1 and C_2, are the depths and velocities respectively at contours 1 and 2. If d_1 is greater than d_2, then c_2, and in moving from A to B an orthogonal will follow a circular arc tangent at A and B to OA and OB respectively. Similarly another orthogonal a differential distance d away from AB is tangent at A' and B' to OA' and OB' respectively. Since d is a differential distance, $AA' \simeq OO' \simeq BB' = d$. If R is the distance of wave advance during a time interval t, then $R_1 = AB \simeq A'B''$, and $R_2 = A'B'$. By construction the angle $B''BB' = \Delta\alpha$, and if $\Delta\alpha$ is considered less than 13 degrees, (for two place accuracy), $\tan\Delta\alpha = \sin\Delta\alpha = \Delta\alpha$, then

$$\Delta\alpha = \tan\Delta\alpha = \frac{B'B''}{BB'} = \frac{R_2 - R_1}{d} \qquad (1)$$

Since the velocity is assumed to vary linearly between contours, the average velocity between A and B is

$$C' = \frac{c_1 + c_2}{2} \qquad (2)$$

and if t is the time interval required for the wave front to move from A A'

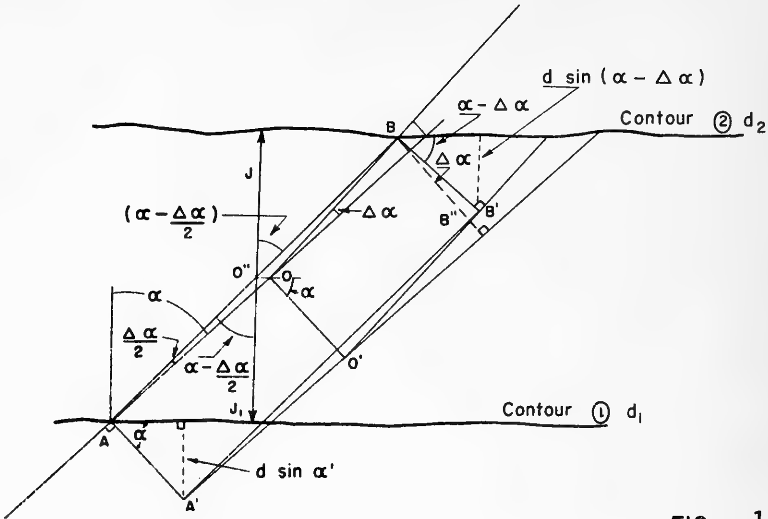

d sin (α − Δα)
α − Δα
Contour ② d_2
B
Δα
J
(α − Δα/2)
Δα
B''
B'
O''
O
α
α
Δα/2
α − Δα/2
O'
J_1
Contour ① d_1
A
α'
d sin α'
A'

FIG. 1a

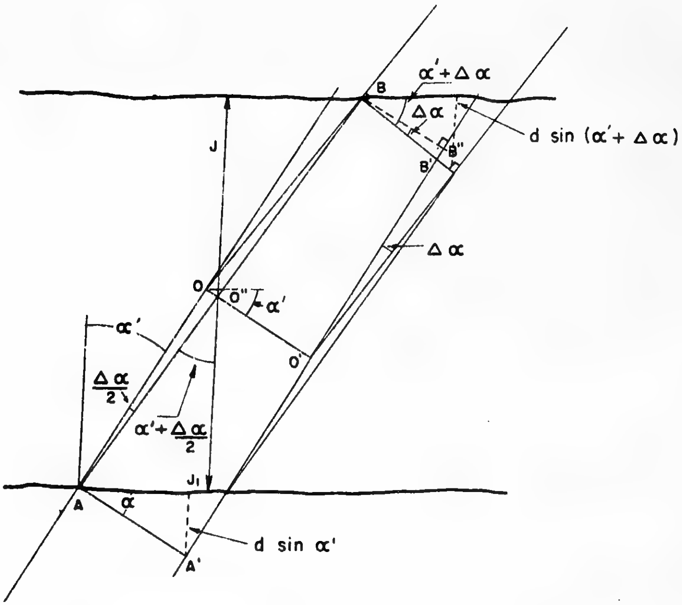

α' + Δα
B
Δα
d sin (α' + Δα)
J
B''
B'
Δα
O
O''
α'
α'
O'
Δα/2
α' + Δα/2
J_1
A
α
d sin α'
A'

FIG. 1b

to B B',

$$R_1 = c't \tag{3}$$

The rate of change of velocity with distance across the contours is $\frac{C_1 - C_2}{J}$ where J is the distance between contours, and therefore the velocity C_1' at A' is greater than that at A by the amount $\frac{C_1 - C_2}{J}$ d sin α ; similarly the velocity C_2' at B' is greater than that at B by the amount $\frac{C_1 - C_2}{J}$ d sin $(\alpha - \Delta\alpha)$. The velocity C" between A' and B' is therefore

$$c'' = \frac{c_1' + c_2'}{2} = \frac{1}{2}\left[c_1 + \frac{c_1 - c_2}{J} d \sin\alpha + \frac{c_1 - c_2}{J} d \sin(\alpha - \Delta\alpha)\right] \tag{4}$$

and, as before

$$R_2 = c''t \tag{5}$$

From equation (1)

$$\Delta\alpha = \frac{R_2 - R_1}{d} = \frac{c'' - c'}{d} t = \frac{c_1 - c_2}{2J} t\left[\sin\alpha + \sin(\alpha - \Delta\alpha)\right] \tag{6}$$

From equation (2) and (3), $t = \frac{R_1}{c'} = \frac{R_1}{\frac{c_1 + c_2}{2}}$ and

$$\Delta\alpha = \left[\frac{c_1 - c_2}{\frac{c_1 + c_2}{2}}\right] \frac{R_1}{2J}\left[\sin\alpha + \sin(\alpha - \Delta\alpha)\right] = \frac{\Delta c}{c_{av.}} \frac{R_1}{2J}\left[\sin\alpha + \sin(\alpha - \Delta\alpha)\right] \tag{7}$$

It may be noted that if $\sin(\alpha - \Delta\alpha)$ is assumed approximately equal to $\sin\alpha$, the equation derived by Isaacs results. However, with the more accurate approximation of $\sin(\alpha - \Delta\alpha) = \sin\alpha \cos\Delta\alpha - \cos\alpha \sin\Delta\alpha \simeq \sin\alpha - \Delta\alpha \cos\alpha$, then $\Delta\alpha$ becomes

$$\Delta\alpha = \frac{\Delta L}{L_{av.}} \frac{R_1}{J}\left(\sin\alpha - \frac{\Delta\alpha \cos\alpha}{2}\right) \tag{8}$$

Now angle O" AO $\simeq \frac{\Delta\alpha}{2}$ and from triangle A O"J

$$\frac{R_1}{J} = \sec\left(\alpha - \frac{\Delta\alpha}{2}\right) \simeq \frac{1}{\cos\alpha + \frac{\Delta\alpha}{2}\sin\alpha} \tag{9}$$

Substituting into equation (8) and reducing terms, then

$$\Delta\alpha = \frac{\Delta L}{L_{av.}} \frac{2\tan\alpha - \Delta\alpha}{2 + \Delta\alpha\tan\alpha} \tag{10}$$

Letting $\frac{\Delta L}{L_{av.}} = K$ and solving for $\Delta\alpha$

$$\Delta\alpha = \frac{-(2+K) + \sqrt{(2+K)^2 + 8K\tan^2\alpha}}{2\tan\alpha} \tag{11}$$

Graphs or overlays may be made up from this equation, or somewhat more easily, from the solution for α :

$$\tan \alpha = \frac{(2+K)\Delta\alpha}{2K - \Delta\alpha^2} \tag{12}$$

One such overlay is shown in Figure 2a.

If the orthogonal is being drawn from shallow to deep water (e.g. past a shoal, or from shore seaward), α' may be defined as the angle between the wave crest and the first contour crossed (at which the velocity C_1 will be less than C_2). In this case $\Delta\alpha$ will be turned in the opposite direction, and a derivation similar to that above gives

$$\Delta\alpha = K\frac{2\tan\alpha' + \Delta a}{2 - \Delta\alpha \tan\alpha'} = \frac{(2-K) - \sqrt{(2-K)^2 - 8K\tan^2\alpha'}}{2\tan\alpha'} \tag{13}$$

and

$$\tan\alpha' = \frac{(2-K)\Delta\alpha}{2K + \Delta\alpha^2} \tag{14}$$

An overlay based on this solution is shown in Figure 2b. This overlay should be used whenever the orthogonal is being drawn from shallower to deeper water.

If a derivation by the Isaacs' method is made for an orthogonal being drawn from shallow to deep water the form of the solution for $\Delta\alpha$ is

$$\Delta\alpha = K\tan\alpha'$$

For a particular orthogonal, $\alpha' = \alpha - \Delta\alpha$ but since Isaacs' original derivation gave for $\Delta\alpha$:

$$\Delta\alpha = K\tan\alpha$$

this could never hold.

Indeed, a measure of the accuracy of the new derivation is that for equation (11) and (13), or (12) and (14) α' very nearly equals $\alpha - \Delta\alpha$, and, to the limit of accuracy of the overlays, α' does equal $\alpha - \Delta\alpha$.

The solutions presented will be applicable in all ordinary cases. However, the application of these solutions is limited by the necessity of keeping $\Delta\alpha$ small so that the distance between contours becomes small as α approaches 90 degrees. Equation (8) also represents a valid solution, however, and, for practical reasons of workable distances between contours, it is desirable to use this equation wherever α exceeds about 80 degrees. For large values of α, sin α and cos α do not vary appreciably and the values for $\alpha = 85^\circ$ may be chosen as representative of the range between 80° and 90°. Equation (8) may be solved to give

$$\Delta\alpha = \frac{2K\frac{R}{J}\sin\alpha}{2 + K\frac{R}{J}\cos\alpha'} \tag{15}$$

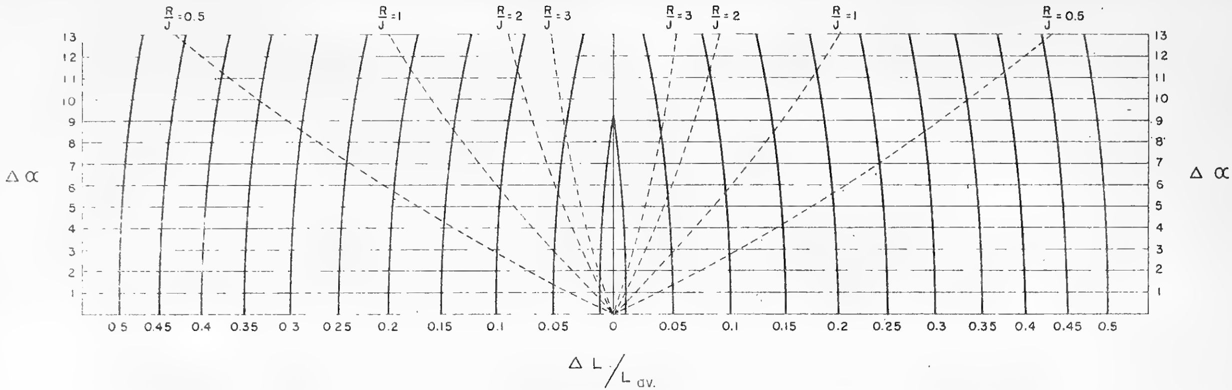
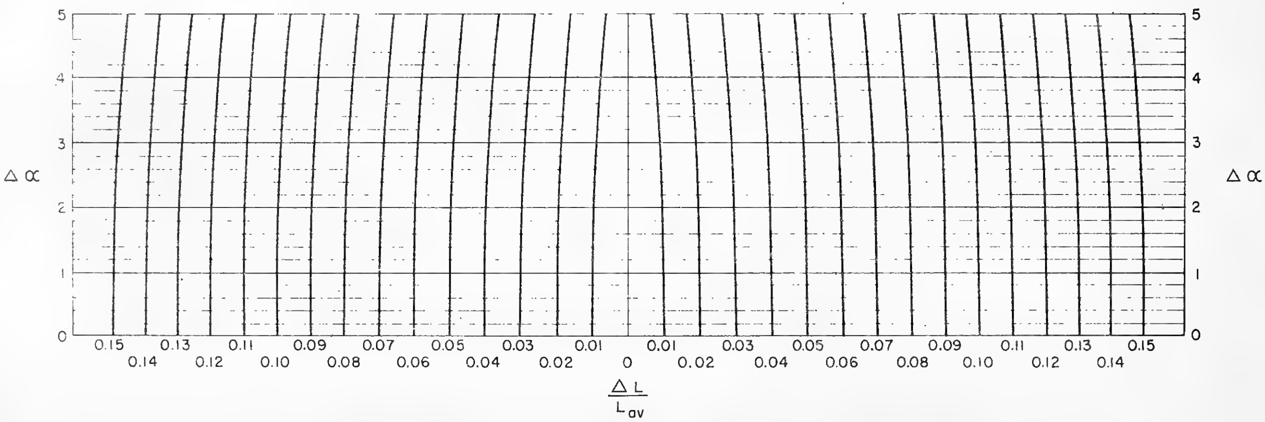


DIAGRAM TO BE USED WHEN DRAWING ORTHOGONAL FROM DEEPER TO SHALLOWER WATER



ENLARGEMENT OF PORTION OF DIAGRAM USED FOR PROCEEDING TOWARD SHALLOWER WATER

FIG. 2a

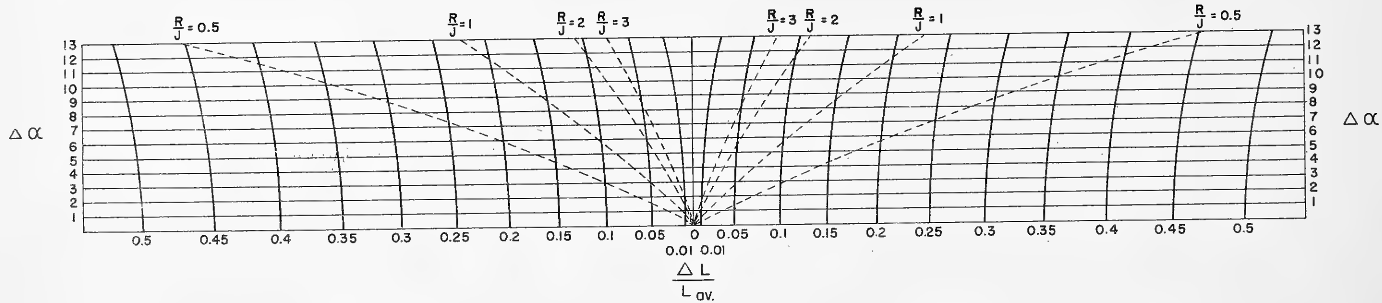
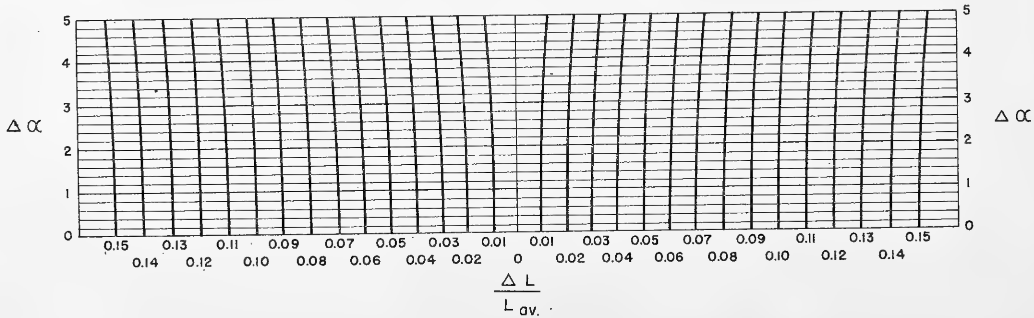


DIAGRAM TO BE USED WHEN DRAWING ORTHOGONAL FROM SHALLOWER TO DEEPER WATER



ENLARGEMENT OF PORTION OF DIAGRAM USED FOR PROCEEDING TOWARD DEEPER WATER

FIG. 2b

Curves showing this variation of $\Delta\alpha$ with $\frac{\Delta L}{L_{av.}}$ and $\frac{R}{J}$ for $\alpha=85°$ are also shown in the overlay in figure 2a.

When either the wave direction, or the direction in which the orthogonal is being drawn, is from shallow to deeper water, the direction of turning is opposite and equation (15) becomes

$$\Delta\alpha = \frac{2K\frac{R}{J}\sin\alpha'}{2-K\frac{R}{J}\cos\alpha'} \qquad (16)$$

Curves showing this solution are included in the overlay in figure 2b.

These two overlays may be used either as ordinary graphs, or in conjunction with a drafting machine in the same method as the original Isaacs' overlay. In the past, the turning point for the change in angle ($\Delta\alpha$) has been assumed to be at the mid-contour. However, the correct orthogonal (circular arc) is not tangent to two straight lines that meet at the mid-contour, but to two lines that meet some small distance above (toward deeper water from) the mid-contour (see figure 3). The ratio of the distance of this point from the deeper contour to the full distance between contours is given by

$$\frac{\tan\frac{\alpha_1-\alpha_2}{2}-\tan\alpha_2}{\tan\alpha_1-\tan\alpha_2}$$

If, for example, $\alpha_1=60°$ and $\alpha_2=55°$, this ratio is 0.48 instead of 0.5, and if $\alpha_1=60°$, $\alpha_2=50°$, this ratio is 0.42 instead of 0.5. The magnitude of the error in the distance along the contours is given by the difference between the tangent of the average angle (the exact solution) and the average tangent of the two angles, (i.e. $\tan\frac{\alpha_1+\alpha_2}{2} - \frac{\tan\alpha_1+\tan\alpha_2}{2}$)

The accuracy is improved considerably by taking the turning point not at the mid-contour, but at a point toward deeper water, such that, in Figure 3, the distance AO = OB. This point may be estimated by eye with sufficient accuracy.

A direct construction of R_1, may be made with a drafting machine by changing the direction of the orthogonal by $\frac{\Delta\alpha}{2}$ once at the first contour and again at the second contour, rather than once by the full value of $\Delta\alpha$ at the mid-contour. The angles being correct, this construction will define the position and direction of the orthogonal at the second contour.

However, it is not felt that the increase in accuracy which would result from this procedure warrants a change in the standardized method of constructing orthogonals using one angle change. Rarely are contours defined accurately, and the modified procedure suggested, in requiring two angle changes instead of one, complicates the mechanics of orthogonal construction. It is thought that sufficient accuracy is obtained by making the one angle change at a point estimated by eye such that the distances OA and OB are equal.

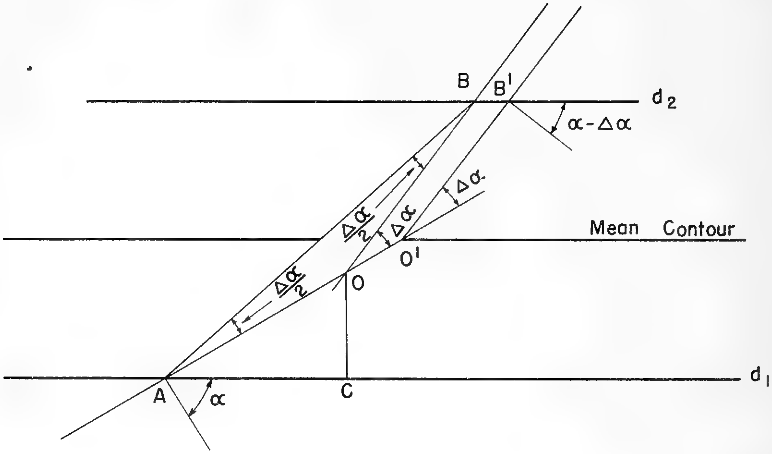


Figure demomstrating inaccuracy involved in using the mid-contour (O') as the turning point rather then the correct turning point O, such that OA = OB. The distance OC is given by the expression,

$$\left[\frac{\tan\left(\alpha - \frac{\Delta\alpha}{2}\right) - \tan\left(\alpha - \Delta\alpha\right)}{\tan\alpha - \tan\left(\alpha - \Delta\alpha\right)}\right]\Delta y$$

where Δy is the distance between the contours d_1 and d_2.

FIGURE — 3

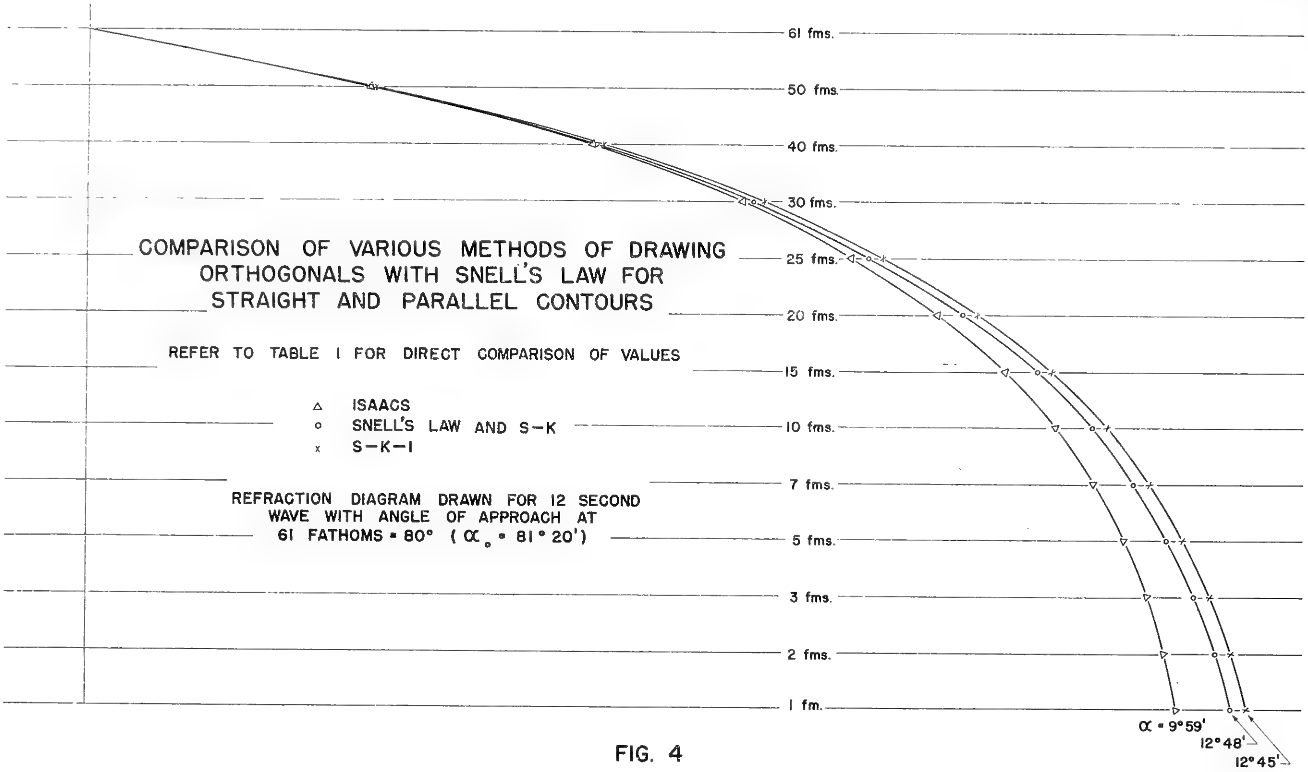

COMPARISON OF VARIOUS METHODS OF DRAWING ORTHOGONALS WITH SNELL'S LAW FOR STRAIGHT AND PARALLEL CONTOURS
REFER TO TABLE I FOR DIRECT COMPARISON OF VALUES
△ ISAACS
○ SNELL'S LAW AND S-K
× S-K-I
REFRACTION DIAGRAM DRAWN FOR 12 SECOND WAVE WITH ANGLE OF APPROACH AT 61 FATHOMS = 80° (α_o = 81° 20')
61 fms.
50 fms.
40 fms.
30 fms.
25 fms.
20 fms.
15 fms.
10 fms.
7 fms.
5 fms.
3 fms.
2 fms.
1 fm.
α = 9°59'
12°48'
12°45'

FIG. 4

A hypothetical case of refraction over straight, parallel contours is shown in Figure 4. The parametric equations for the orthogonal given by Arthur and Munk (3) are

$$\frac{dx}{dt} = c \cos\beta ; \ \frac{dy}{dt} = c \sin\beta ; \ \frac{d\beta}{dt} = \sin\beta \frac{\partial c}{\partial x} - \cos\beta \frac{\partial c}{\partial y}$$

These may be solved by simple separation of variables for a velocity field which is a function of y alone. (Here $\beta = 90^\circ - \alpha$). In particular, for a field which varies linearly with y, $c = c_0(1 - ay)$ the solutions for x and y are

$$x = \frac{1}{a \sin\alpha_0}(\cos\alpha - \cos\alpha_0) \ \text{ and } \ y = \frac{1}{a\sin\alpha_0}(\sin\alpha_0 - \sin\alpha)$$

which are the parametric equations of a circle of radius

$$r = \frac{\sqrt{\sin^2 2\alpha_0 + 4}}{a \sin 2\alpha_0} \quad \text{and center at} \quad x = -\frac{2}{a\sin 2\alpha_0}, \ y = \frac{1}{a}$$

The solution for y may be put in the form

$$\sin\alpha = \sin(\alpha_0 - \Delta\alpha) = \left(1 - \frac{\Delta c}{c_0}\right)\sin\alpha_0$$

which is Snell's law.

From these, exact values of $\Delta\alpha$ and x at any point in the field may be found.

The values for the particular case chosen are compared with values obtained from Isaacs' equations and the ones contained herein. This comparison is shown in Figure 4, and also in Table 1, where both values of the change between contours and cumulative values are given. It may be seen that angular values obtained from the new equations agree almost exactly with those determined from Snell's law, and are considerably more precise than those obtained from the original Isaacs' equation. Distances along the contours are still somewhat in error if the mid-contour is used as the turning point (as in the Isaacs' case), though not nearly as greatly in error as obtained from the Isaacs' method (see column S-K-I). If the turning point is selected so that the segments of the tangent lines within the contours are equal, then the result is almost identical with Snell's Law. This is shown under "S-K".

It is concluded that these new protractors, or overlays represent a considerable advance over the original Isaacs' protractor, in that they approach much more closely exact values.

It may be noted that a paper by Arthur, Munk and Isaacs recently presented at the May 5 meeting of the American Geophysical Union, but not yet published, has considered the equations of the wave rays, or orthogonals. Solutions of these equations using essentially the same assumptions as used in the geometrical derivation herein, give for

$$\Delta\alpha = \sin^{-1}\left[\left(1 + \frac{\Delta L}{L_1}\right)\sin\alpha\right] - \alpha = \sin^{-1}\left[\left(\frac{K \mp 2}{K \pm 2}\right)\sin\alpha\right] - \alpha$$

TABLE 1

Comparison of Angles and Distances as Orthogonal Crosses Specific Contours for the Various Methods of Drawing Orthogonals. Wave Period is 20 seconds

d (fathoms)	$\frac{\Delta L}{L_{av}} = C$	Snell's Law	S-K	Isaacs	Snell's Law	S-K-I	Isaacs	S-K
61		80°	80°	80°	0.00	0.00	0.00	0.00
	.0076	(2° 13')	(2° 13')	(2° 29')	(5.0926)	(5.1448)	(5.0941)	(5.0929)
50		77° 47'	77° 47'	77° 31'	5.0926	5.1448	5.0941	5.0929
	.018	(4° 5')	(4° 6')	(4° 42')	(3.9382)	(4.0170)	(3.8753)	(3.9323)
40		73° 42'	73° 41'	72° 49'	9.0308	9.1618	8.9694	9.0252
	.044	(7° 0')	(7° 0')	(8° 9')	(2.7842)	(2.8682)	(2.6731)	(2.7750)
30		66° 42'	66° 41'	64° 40'	11.8150	12.0300	11.6425	11.8002
	.040	(4° 51')	(4° 49')	(4° 53')	(2.0715)	(2.0959)	(1.9147)	(2.0760)
25		61° 53'	61° 53'	59° 47'	13.8865	14.1259	13.5572	13.8762
	.062	(5° 49')	(5° 52')	(6° 4')	(1.6605)	(1.6772)	(1.5396)	(1.6608)
20		56° 4'	56° 1'	53° 43'	15.5470	15.8031	15.0968	15.5370
	.094	(7° 3')	(7° 4')	(7° 21')	(1.3061)	(1.3145)	(1.2051)	(1.2981)
15		49° 1'	48° 57'	46° 22'	16.8531	17.1176	16.3019	16.8351
	.155	(8° 45')	(8° 45')	(9° 19')	(.9876)	(.9967)	(.9014)	(.9850)
10		40° 16'	40° 12'	37° 3'	17.8407	18.1143	17.2033	17.8201
	.150	(6° 29')	(6° 28')	(6° 30')	(.7538)	(.7564)	(.6776)	(.7526
7		33° 47'	33° 44'	30° 33'	18.5945	18.8707	17.8809	18.5727
	.150	(5° 11')	(5° 12')	(5° 4')	(.6042)	(.6052)	(.5334)	(.6041)
5		28° 36'	28° 32'	25° 29'	19.1987	19.4759	18.4143	19.1768
	.237	(6° 26')	(6° 26')	(6° 28')	(.4752)	(.4750)	(.4106)	(.4732)
3		22° 10'	22° 7'	19° 01'	19.6739	19.9509	18.8249	19.6500
	.194	(4° 04')	(4° 04')	(3° 50')	(.3667)	(.3661)	(.3080)	(.3656)
2		18° 06'	18° 03'	15° 11'	20.0406	20.3170	19.1329	20.0156
	.335	(5° 18')	(5° 18')	(5° 12')	(.2765)	(.2761)	(.2237)	(.2755)
1		12° 48'	12° 45'	9° 59'	20.3171	20.5931	19.3566	20.2911
						+.2760	-.9605	-.0260
						+1.36%	-4.73%	0.13%

for proceeding from deeper to shallower water. These will be recognized as a rearrangement of Snell's Law, and would result in protractors identical with those contained herein for angle changes up to about 13°.

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BEACH EROSION STUDIES

The principal types of beach erosion control studies of specific localities are the following:

a. Cooperative studies (authorization by the Chief of Engineers in accordance with section 2, River and Harbor Act approved 3 July 1930).

b. Preliminary examination and surveys (Congressional authorization by reference to locality by name).

c. Reports on shore line changes which may result from improvements of the entrances at the mouths of rivers and inlets (Section 5, Public Law No. 409, 74th Congress).

d. Reports on shore protection of Federal property (authorization by the Chief of Engineers).

Of these types of studies, cooperative beach erosion studies are the type most frequently made when a community desires investigation of its particular problem. As these studies have greater general interest, information concerning studies of specific localities contained in these quarterly bulletins will be confined to cooperative studies. Information about other types of studies can be obtained upon inquiry to this office.

Cooperative studies of beach erosion are studies made by the Corps of Engineers in cooperation with appropriate agencies of the various States by authority of Section 2 of the River and Harbor Act approved 3 July 1930. By executive ruling the cost of these studies is divided equally between the United States and the cooperating agency. Information concerning the initiation of the cooperative study may be obtained from any District Engineer of the Corps of Engineers. After a report on a cooperative study has been transmitted to Congress, a summary thereof is included in the next issue of this bulletin. A summary of a report transmitted to Congress since the last issue of the Bulletin and lists of authorized and completed cooperative studies follow:

SUMMARY OF REPORT TRANSMITTED TO CONGRESS

STATE OF CONNECTICUT-CONNECTICUT RIVER TO HAMMONASSET RIVER

The area studied comprises the shore of Long Island Sound between the mouths of Connecticut River and Hammonasset River. It includes the shores of the Towns of Old Saybrook, Westbrook, and Clinton, a total length of about 12.5 miles. This shore area is about 30 miles east of New Haven Connecticut, and about 100 miles east of New York City. It is extensively developed as a resort and residential area, with improvements ranging from cottages to small estates. The permanent population of the three towns is about 6,500. The summer population is more than 3 times as great. A number of small town owned beaches are included in the area.

Long Island Sound is a tidal arm of the Atlantic Ocean. Tides are semi-diurnal, the mean range increasing gradually from 3.5 feet at Saybrook to 4.7 feet at Clinton. Spring ranges are respectively 4.2 and 5.5 feet at these locations. Maximum tide of record at Saybrook was 9.9 feet above mean high water. Tides 3 feet or more above mean high water occur about once a year. With a tidal stage of 3 feet above mean high water, the maximum height of breakers landward of the low water line is about 5 feet at the east end of the study area and 6 feet at the west end. Larger waves can reach the shore only during infrequent higher tides.

Due to the limited size of Long Island Sound, local storms are the sole cause of important wave action. Ordinary short storm waves cause littoral movement and offshore loss of beach material. Absence of swells probably precludes the possibility of return of material from offshore by wave action. The greater fetch and wind movement from the west and southwest account for the general predominance of eastward and northward littoral drift depending on shore alignment. Waves caused by easterly storm winds cause reversals of drift direction. Where sections of the shore are protected by islands or structures from waves from the west, westward littoral drift is predominant.

The study area is characterized by headlands of unconsolidated glacial material with some rock outcrops, between which wave-built bars have been formed and the landward areas generally have filled and become marshy. The headlands formerly supplied ample material to the intervening beaches, but the headlands are now generally protected by seawalls and revetments. The supply of material is thus reduced or eliminated and consequently the beaches have slowly deteriorated. Groins have been found to be capable of causing minor accretion areas and stabilizing a narrow band along the upper portion of the beach, but the natural supply of material is insufficient for the formation of adequate protective beaches. The building and maintenance of adequate beaches may be accomplished by artificial placement of sand. The prospective low rates of loss of beach material, based on past experience of shore line recession, are insufficient to warrant the construction of groins, except where necessary to prevent the shoaling or closing of inlets or drainage channels.

The Board concluded that the best methods of protection and improvement of beaches within the study area are as follows:

a. Borough of Fenwick (West Part) - Construction of a dumped riprap wall along the high water shore line;

b. Plum Bank Beach - Direct placement of a protective sand beach in front of the sea walls and cottages, and construction of an impermeable groin at the north limit of the fill;

c. Great Hammock Beach - Direct placement of a protective sand beach or dune in front of the cottage development.

d. Saybrook Manor, Chalker Beach and Chapman Beach - Direct placement of a protective sand beach in front of the cottages or residential developments;

e. West Beach - Direct placement of a protective sand beach in front of the sea wall along the west end of the public beach and in front of the cottage development west of and adjacent to the sea wall;

f. Grove Beach - Construction of an impermeable groin at the east end of the beach.

It is also concluded that projects for Plum Bank, Great Hammock, Saybrook Manor, Chalker and Grove Beaches appear to be justified by evaluated benefits. The Board also believed that the public ownership and interest in the projects are insufficient to warrant Federal aid under the policy established by Public Law 727, 79th Congress. The Board recommended that local authorities consider adoption of projects for protection and improvement of these beaches at local expense, substantially in accordance with the foregoing plans. The Board considered it advisable, however, for local interests to make independent evaluations of prospective benefits from these projects in determining justification for their construction at local expense.

In accordance with existing statutory requirements the Board stated its opinion that:

a. It is inadvisable for the United States to adopt projects authorizing Federal participation in the cost of protecting and improving the shores within the area studied;

b. The public interest involved in the proposed measures for these shores is small, and

c. No share of the expense should be borne by the United States.

COMPLETED COOPERATIVE BEACH EROSION STUDIES

Location	Completed	Published in House Doc.	Congress
MAINE			
Old Orchard Beach	20 Sep 35		
NEW HAMPSHIRE			
Hampton Beach	15 Jul 32		

MASSACHUSETTS

South Shore, Cape Cod (Pt. Gammon to Chatham)	26 Aug 41		
Salisbury	26 Aug 41		
Winthrop Beach	12 Sep 47	764	80
Lynn-Nahant Beach	20 Jan 50	134	82
Revere Beach	12 Jan 50	167	82
Nantasket Beach	12 Jan 50		
Quincy Shore	2 May 50	145	82

RHODE ISLAND

South Shore (Towns of Narragansett, South Kingstown, Charlestown & Westerly)	4 Dec 48	490	81

CONNECTICUT

Compo Beach, Westport	18 Apr 35	239	74
Hawk's Nest Beach, Old Lyme	21 Jun 39		
Ash Creek to Saugatuck River	29 Apr 49	454	81
Hammonasset River to East River	29 Apr 49	474	81
New Haven Harbor to Housatonic River	29 Jun 51		
Connecticut River to Hammonasset River	28 Dec 51		
Pawcatuck River to Thames River	31 Mar 52		

NEW YORK

Jacob Riis Park, Long Island	16 Dec 35	397	74
Orchard Beach, Pelham Bay, Bronx	30 Aug 37	450	75
Niagara County	27 Jun 42	271	78
South Shore of Long Island	6 Aug 46		

NEW JERSEY

Manasquan Inlet & Adjacent Beaches	15 May 36	71	75
Atlantic City	11 Jul 49	538	81
Ocean City	15 Apr 52		

VIRGINIA

Willoughby Spit, Norfolk	20 Nov 37	482	75
Colonial Beach, Potomac River	24 Jan 49	333	81
Virginia Beach	25 Jun 52		

NORTH CAROLINA

Fort Fisher	10 Nov 31	204	72
Wrightsville Beach	2 Jan 34	218	73
Kitty Hawk, Nags Head & Oregon Inlet	1 Mar 35	155	74
State of North Carolina	22 May 47	763	80

SOUTH CAROLINA

Folly Beach	31 Jan 35	156	74
Pawleys Island, Edisto Beach & Hunting Island	24 Jul 51		

GEORGIA

St. Simon Island	18 Mar 40	820	76

FLORIDA

Blind Pass (Boca Ciega)	1 Feb 37	187	75
Miami Beach	1 Feb 37	169	75
Hollywood Beach	28 Apr 37	253	75
Daytona Beach	15 Mar 38	571	75
Bakers Haulover Inlet	21 May 45	527	79
Anna Maria & Longboat Keys	12 Feb 47	760	80
Jupiter Island	13 Feb 47	765	80
Palm Beach (1)	13 Feb 47	772	80

MISSISSIPPI

Hancock County	3 Apr 42		
Harrison County - Initial	15 Mar 44		
Harrison County - Supplement	16 Feb 48	682	80

LOUISIANA

Grand Isle	28 Jul 36	92	75

TEXAS

Galveston	10 May 34	400	73
Galveston Bay, Harris County	31 Jul 34	74	74

CALIFORNIA

Santa Barbara - Initial	15 Jan 38	552	75
Supplement	18 Feb 42		
Final	22 May 47	761	80
Ballona Creek & San Gabriel River (partial)	11 May 38		
Orange County	10 Jan 40	637	76
Coronado Beach	4 Apr 41	636	77
Long Beach	3 Apr 42		
Mission Beach	4 Nov 42		

(1) A cooperative study of experimental steel pile groins was also made, under which methods of improvement were recommended in an interim report dated 19 Sep 1940. Final report on experimental groins was published in 1948 as Technical Memorandum No. 10 of the Beach Erosion Board.

Point Mugu to San Pedro Breakwater	27 Jun 51		
Carpinteria to Point Mugu	4 Oct 51		

PENNSYLVANIA

Presque Isle Peninsula, Erie			
(Interim)	3 Apr 42		
(Final)	23 Apr 52		

OHIO

Erie County - Vicinity of Huron	26 Aug 41	220	79
Michigan Line to Marblehead	30 Oct 44	177	79
Cities of Cleveland and Lakewood	22 Mar 48	502	81
Chagrin River to Fairport	22 Nov 49	596	81
Vermilion to Sheffield Lake Village	24 Jul 50		
Fairport to Ashtabula	1 Aug 51		
Ashtabula to Pennsylvania Line	1 Aug 51		

ILLINOIS

State of Illinois	8 Jun 50		

WISCONSIN

Milwaukee County	21 May 45	526	79
Racine County, Wisconsin	5 Mar 52		

PUERTO RICO

Punta Las Marias, San Juan	5 Aug 47	769	80

AUTHORIZED COOPERATIVE BEACH EROSION STUDIES

NEW HAMPSHIRE

HAMPTON BEACH. Cooperating Agency: New Hampshire Shore and Beach Preservation and Development Commission

Problem: To determine the best method of preventing further erosion and of stabilizing and restoring the beaches, also to determine the extent of Federal aid in any proposed plans of protection and improvement.

MASSACHUSETTS

PLUM ISLAND. Cooperating Agency: Department of Public Works

Problem: To devise effective means of preventing further erosion of the shore.

PEMBERTON POINT TO GURNET POINT. Cooperating Agency: Department of Public Works.

Problem: To determine the best methods of shore protection prevention of further erosion and improvement of beaches, and specifically to develop plans for protection of Crescent Beach, The Glades, North Scituate Beach and Brant Rock.

CONNECTICUT

STATE OF CONNECTICUT. Cooperating Agency: State of Connecticut (Acting through the Flood Control and Water Policy Commission).

Problem: To determine the most suitable methods of stabilizing and improving the shore line. Sections of the coast are being studied in order of priority as requested by the cooperating agency until the entire coast has been included.

NEW YORK

JONES BEACH. Cooperating Agency. Long Island State Parks Commission

Problem: To determine behavior of the shore during a 12-month cycle, including study of littoral drift, wave refraction and movement of artificial sand supply between Fire Island and Jones Inlets.

NEW JERSEY

STATE OF NEW JERSEY. Cooperating Agency: Department of Conservation and Economic Development.

Problem: To determine the best method of preventing further erosion and stabilizing and restoring the beaches, to recommend remedial measures, and to formulate a comprehensive plan for beach preservation or coastal protection.

NORTH CAROLINA

CAROLINA BEACH. Cooperating Agency: Town of Carolina Beach.

Problem: To determine the best method of preventing erosion of the beach.

FLORIDA

PINELLAS COUNTY. Cooperating Agency: Board of County Commissioners.

Problem: To determine the best methods of preventing further recession of the gulf shore line, stabilizing the gulf shores of certain passes, and widening certain beaches within the study area.

LOUISIANA

LAKE PONTCHARTRAIN. Cooperating Agency: Board of Levee Commissioners, Orleans Levee District.

Problem: To determine the best method of effecting necessary repairs to the existing sea wall and the desirability of building an artificial beach to provide protection to the wall and also to provide additional recreational beach area.

TEXAS

GALVESTON COUNTY. Cooperating Agency: County Commissioners Court of Galveston County.

Problem: To determine the best method of providing a permanent beach and the necessity for further protection or extending the sea wall within the area bounded by the Galveston South Jetty and Eight Mile Road.

To determine the most practicable and economical method of preventing or retarding bank recession on the shore of Galveston Bay between April Fool Point and Kemah.

CALIFORNIA

STATE OF CALIFORNIA. Cooperating Agency: Division of Beaches and Parks State of California.

Problem: To conduct a study of the problems of beach erosion and shore protection along the entire coast of California. The current study covers the Santa Cruz area.

WISCONSIN

KENOSHA. Cooperating Agency: City of Kenosha.

Problem: To determine the best method of shore protection and beach erosion control.

OHIO

STATE OF OHIO. Cooperating Agency: State of Ohio (Acting through the Superintendent of Public Works).

Problem: To determine the best method of preventing further erosion of and stabilizing existing beaches, of restoring and creating new beaches, and appropriate locations for the development of recreational facilities by the State along the Lake Erie shore line. Sections of the coast are being studied in order of priority as requested by the cooperating agency until the entire coast has been included.

TERRITORY OF HAWAII

WAIKIKI BEACH
WAIMEA & HANAPEPE, KAUAI. Cooperating Agency: Board of Harbor Commissioners, Territory of Hawaii.

Problem: To determine the most suitable method of preventing erosion, and of increasing the usable recreational beach area, and to determine the extent of Federal aid in effecting the desired improvement.

Problem: To correct a ... of ... [illegible] ... protection along ... [illegible]
The ... [illegible]

[illegible]

KENTUCKY - ... [illegible]

Problem: ... [illegible]

[illegible]

STATE OF OHIO - ... [illegible]
Superintendent of Public Works [illegible]

Problem: ... [illegible]

[illegible]

[illegible]

Problem: ... [illegible]