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Intermediate (Quartz Monzonitic) Character of the Central and Southern Appalachian Granites

WITH A

Comparative Study of the Granites of New England and the Western United States

BY

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INTERMEDIATE (QUARTZ MONZONITIC) CHARACTER OF THE CENTRAL AND SOUTHERN APPALACHIAN GRANITES, WITH A COMPARATIVE STUDY OF THE GRANITES OF NEW ENGLAND AND THE WESTERN UNITED STATES.*

BY
THOMAS LEONARD WATSON.

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*Read before the Section of Mathematical and Natural Sciences January 18, 1910.

INTRODUCTION

The writer became interested in the plutonic acid igneous rocks discussed in this paper, more than ten years ago, while studying those of Georgia for the State Geological Survey. Since the completion of the Georgia work, he has extended his studies of these rocks at intervals into each of the states south of Pennsylvania. The results of these studies have been published as reports by a number of the State Geological Surveys[1] and in various scientific journals. Within the Appalachian region south of the Susquehanna river, these rocks constitute commercially an important quarrying industry, amounting at present to about 20 per cent. of the total production of the United States.

The principal results thus far obtained from the investigation of these rocks briefly stated are:

(1) Widespread development of intruded acid granular rocks throughout the region of old metamorphic crystalline rocks, which extend northward from Alabama to New England, as extensive, irregular bodies, and a considerable development of more basic intruded rocks, chiefly of the gabbro family, including typically gabbros, diabases, pyroxenites, dunite, and other peridotites. In general chemical character the acid rocks show high silica and alkalies, and usually low lime, magnesia, and iron, while the basic rocks show low silica and alkalies, and high lime, magnesia, and iron. Of the alkalies in the acid rocks, soda is quite generally high, being molecularly equal to or greater than potash in nearly every instance. These rocks have been derived from a common parent body of magma intruded, in most cases, at different times. Some areas strongly suggest differentiation of the magma in place[2], with usually a basic margin and an acid centre. The sequence in igneous activity for the Maryland portion of the region indicates that the intrusions of gabbro were earlier than those of the numerous large bodies of granite[3]. To the south of Maryland, the relations are less plain and are more difficult of interpretation.

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[1]A bulletin on the granites of the Appalachian region south of the Susquehanna river, prepared by the writer for the United States Geological Survey, is in press.

[2]Differentiation prior to intrusion Brögger terms primary or "deep-magmatic," and differentiation subsequent to intrusion, i. e. in place, he terms secondary or "laccolitic" differentiation. Quoted by A. Harker, The Natural History of Igneous Rocks, 1909, p. 133.

[3]Mathews, E. B. Amer. Journ. Science, 1904, Vol. XVII, pp. 141-159.

Granite is used in this paper in its restricted sense to designate a granitoid, igneous rock composed of quartz, potash feldspar (orthoclase or microcline, or both), more or less oligoclase, and the micas (biotite or muscovite, or both). The first three may be combined with either of the micas alone, with hornblende, or with augite. Kemp, J. F., Handbook of Rocks. Third Edition revised, 1906, p. 189.

(2) Metamorphism of the rocks, chiefly recrystallization and schistose structure, is strongly emphasized in the gneisses derived from original massive-granular, acid, intrusive granites. In the massive rocks the most pronounced effects are seen under the microscope in the optical disturbance of some of the minerals, chiefly quartz and to a less extent feldspar. Partial peripheral granulation of these two minerals and irregular fractures are sometimes shown. As a rule, the massive acid rocks do not grade into the more pronounced schistose forms, granite-gneisses, but the two are usually sharply defined from each other. With only few exceptions, the acid gneisses studied are derived from original granites by metamorphism, and have been designated granite-gneisses. The porphyritic and even-granular acid irruptives grade into each other and are regarded as textural variations of the same rock-mass. Potassic feldspar, orthoclase or microcline, is the porphyritically developed mineral in the porphyritic facies of the rocks. The phenocrysts vary in size, may be allotriomorphic or idiomorphic in outline, show good cleavage development, usually without orientation indicated, and contain inclusions of the principal groundmass minerals, especially biotite, and are regarded as having been formed *in place*.

(3) The acid-granular rocks are not of the same age but have been intruded during several different geologic periods. Their exact age is uncertain, but a part of them (granite-gneisses) are pre-Cambrian, while the massive granites are of early or later Paleozoic age.

(4) Closely similar mineral and chemical composition characterize these rocks throughout the entire region. The close similarity in mineral and chemical composition of the rocks is emphasized in subsequent pages of this paper, and need not be repeated here.

(5) Similar textural, structural, and color variations in the rocks are noted but not with the same frequency over all parts of the region. Texturally, variation is from fine- to coarse-grained even-granular to porphyritic rocks, and structurally from massive to completely schistose. Some shade of gray (light, intermediate, or dark) is the usual color, with pink not uncommon in places.

(6) Characteristic contact metamorphic zones frequently observed about the border positions of large granite intrusions are seldom observed. Apophyses or dikes of the same composition as the parent rock frequently project outward from the main granite-masses and cut abruptly into the adjacent rocks. Pegmatites are also found penetrating, as offshoots, the rock masses enveloping the stocks of granite. Incorporated fragments of

other rocks are sometimes observed in the granite about the border portions of the stocks, and basic segregations[1] (schlieren) are common.

(7) Mineralogically unlike materials of marked variation in composition and texture, are observed penetrating the granites in places as veins and dikes, some more frequently than others. These include true quartz veins, pegmatite, aplite, and granite dikes of normal composition and texture, among the acid kinds; and dikes of basic igneous rocks, chiefly diabase, among the basic kinds.

(8) Three sets of joints intersect the granites, a vertical set, a diagonal set, and a horizontal set. The vertical set is usually more strongly developed than the diagonal, and in some of the granite-masses both sets occur. As regards direction of strike these lie mostly in the northeast and northwest quadrants. Subsequent movement in the granite-masses along the joints, resulting in some cases in faulting, is indicated in the frequent development of slicken-sides on the joint surfaces. Joints which are either curved or approach horizontality, being generally parallel with the granite surface, are of frequent strong development and, in most cases, are probably due to variations of temperature, daily and seasonal. In other cases, notably in the granite domes like Stone Mountain and the numerous smaller ones near Lithonia, Georgia, the outer parts of the granite are in a condition of compressive strain, which results in slow exfoliation and the development of joints that run approximately parallel to the surface.

MINERAL COMPOSITION.

Feldspar, quartz, and biotite, in the order named, are the dominant minerals in these rocks throughout the region. The total feldspar varies greatly in amount, but ranges between 43.48 per cent. (Maryland) and 81.06 per cent. (Georgia), with an average for the entire region of 64.42 per cent. of the total rock mass. Averaged for the individual states, a nearly progressive increase in feldspar from north to south (Maryland, 54.53 per cent.; Virginia, 65.21 per cent.; North Carolina, 61.33 per cent.; South Carolina, 66.84 per cent.; and Georgia, 74.17 per cent.) is indicated, the northernmost state, Maryland, showing the minimum, and the southernmost state, Georgia, the maximum amount.

The feldspar composition is a mixture of potassic (orthoclase and microcline, the latter subject to much variation in amount is sometimes in

[1]Very seldom are the schlieren of acid character, consisting wholly or nearly so of quartz and feldspar.

excess of the former) and sodic-lime (usually oligoclase, oligoclase-albite, and some albite) varieties, the sodic-lime varieties being, with few exceptions, either equal to or greater in amount than the potassic feldspar. The molecular ratio of potassic to sodic-lime feldspars ranges from 1:1 to 1:4, with an average for the region as a whole, of 1 to 1.88.

Computing in the usual way the feldspar composition from chemical analyses, the limits of variation in the potassic and sodic-lime feldspars ratio, and the average for each state, may be tabulated as follows:

Limits of variation in ratio of potassic and sodic-lime feldspars in the southeastern Atlantic states granites.

State.	Limits of Variation.	Average.
Maryland	1:1 to 1:4	1:1.7
Virginia	1:1.4 to 1:2.5	1:1.9
North Carolina	1:1.4 to 1:3	1:2.5
South Carolina	1:1.3 to 1:2.6	1:1.7
Georgia	1:1.4 to 1:1.8	1:1.6

The extremes in the feldspar ratio for the entire region are shown in the rocks of Maryland. These are: Port Deposit granite-gneiss (riesenose, quartz-mica diorite), 1 of orthoclase to 3.9 of plagioclase; and Brookville granite (liparose, quartz monzonite), 1 of orthoclase to 0.9 of plagioclase.

Calculating all of the Na_2O in the analyses as albite and the CaO as anorthite, and adding the albite to the anorthite, a sodic-lime feldspar is obtained which ranges from Ab_2An_1 (basic, or andesine oligoclase) to $Ab_{15}An_1$ (oligoclase-albite), with a probable average of Ab_5An_1, acid oligoclase. In some cases the potassic feldspar contains sufficient soda (approximately one-third the total potash) to be designated soda-orthoclase, as shown in the analyses below of carefully selected potassic feldspar phenocrysts from two of the Georgia porphyritic granites of widely-separated localities.[1]

Constituents.	I.	II.
K_2O	11.40	10.00
Na_2O	3.60	3.06
CaO	0.59	0.67

I. Feldspar phenocrysts from porphyritic granite, McCollum Place, Coweta county, Georgia. Bull. No. 9-A, Georgia Geol. Survey, 1902, pp. 92, 322.

II. Feldspar phenocrysts from porphyritic granite, Heggie Rock, Columbia county, Georgia. Bull. No. 9-A, Georgia Geol. Survey, 1902, pp. 235, 318.

[1]Georgia Geological Survey, Bull. 9-A, 1902, p. 239.

Omitting non-feldspathic constituents and allotting the potash to orthoclase, and the soda and lime to albite and anorthite, respectively, the analyses show the following range and average in feldspar composition for the region as a whole.

Name.	Range.	Average.
Orthoclase	8.90 to 33.36	25.4
Albite	16.24 to 42.97	32.7
Anorthite	1.67 to 13.90	8.7

In the quantitative classification[1] of igneous rocks, all analyses of the rocks of the region given on subsequent pages are distributed between rangs 1, 2, and 3, and subrangs 3 and 4 of class I (2 in class II), as follows: Peralkalic, 4; domalkalic, 29; alkalicalcic, 5; sodipotassic, 29; dosodic, 7; between sodipotassic and dosodic, 2.

Microperthite is generally present in the rocks, but anorthoclase, if present, has not been identified.

The free quartz in the rocks has been computed from the complete analyses for each state except Georgia, and the results may be tabulated as follows:

Limits of variation and average percentage of quartz in southeastern Atlantic states granites.

State.	Percentage of quartz.	Average.
Maryland	13.6 to 41.2	30.4
Virginia	23.8 to 32.9	28.0
North Carolina	14.6 to 43.5	32.8
South Carolina	19.3 to 30.8	24.9

Total silica averages 71.18 per cent. for the entire region, with a variation between extremes of more than 11 per cent.; 66.01 per cent. (North Carolina) and 77.19 per cent (North Carolina).

Biotite, subject to variation in amount, is, with few exceptions, a constituent in the rocks of every locality within the region. Muscovite is a frequent associate of biotite in many localities, the two micas appearing

[1]Quantitative Classification of Igneous Rocks. By Cross, Iddings, Pirsson, and Washington. Chicago, 1903.

together with biotite usually in excess of muscovite, about as often as does biotite alone. Muscovite in at least one locality each, in South Carolina, North Carolina, Virginia, and Maryland, is essentially unaccompanied by biotite and occurs alone. In the Stone Mountain granite of Georgia muscovite greatly exceeds biotite in amount. The preponderance of biotite granites over others in this area is noteworthy, and has been commented on by Kemp[1] and others, for the granites occurring in the eastern portion of North America.

The great preponderance of biotite granites over others in the eastern United States is shown in the following tabulation[2], and diagrammatically in figure 1:

Classification of granites in the eastern United States.

State.	I.	II.	III.	IV.	V.	VI.	VII
Georgia	141	13	2	----	----	----	----
South Carolina	34	4	1	----	1	----	----
North Carolina	82	1	6	----	1	1	1
Virginia	31	----	4	----	1	----	3
Maryland	20	4	6	----	----	----	----
New Jersey	----	1	1	12[3]	----	2	----
New York	20	1	12	10	----	----	----
Connecticut	15	3	8	----	----	----	----
Rhode Island	12	----	----	----	----	----	1
Massachusetts	11	5	2	34	----	----	1
Vermont	52	21	----	----	2	----	----
New Hampshire	19	19	5	5	1	2	----
Maine	79	20	2	2	----	----	----
Total	576	92	49	63	6	5	6

I. Biotite granite or gneiss.
II. Biotite-muscovite or muscovite-biotite granite or gneiss.
III. Biotite-hornblende or hornblende-biotite granite or gneiss.
IV. Hornblende granite.
V. Muscovite granite.
VI. Biotite-epidote granite.
VII. Epidote granite.

[1]Kemp, J. F., Bull. Geol. Soc. America, 1899, Vol. 10, p. 377 et seq.

[2]Tabulation is based chiefly on quarry data, and under each state is included every quarry which the writer has been able to find a record of. The sources of information from which the table has been compiled are the Census reports, reports of the States and Federal Geological Surveys on granites, and numerous papers published in scientific journals.

[3]A part of these contain augite with or without hornblende. Annual Report of the State Geologist for 1908. Geol. Survey of New Jersey, 1909, pp. 62-78.

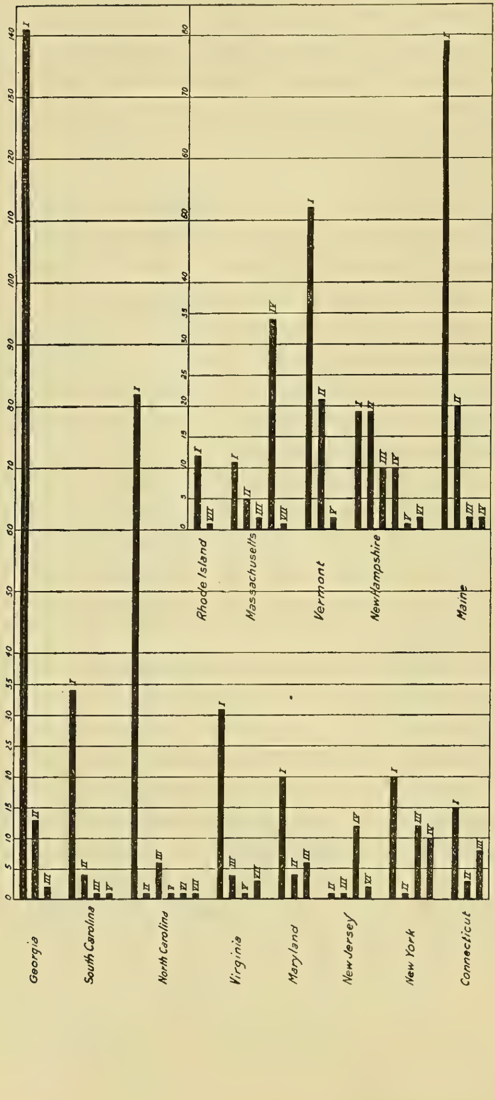

Figure 1.—Digrammatic classification of granites in Eastern United States by states. Roman numerals correspond to those in table on page 7.

On a more generalized basis the seven divisions of the table can be reduced to three as follows: (1) Mica granites, including I, II, III, V, and VI, in which some hornblende (III) and epidote (VI) are associated with the mica (biotite); (2) hornblende granites (IV); and (3) epidote granites (VII). On this interpretation the totals are: Mica granites 728, or 91.3 per cent.; hornblende granites 63, or 8.5 per cent., and epidote granites 6, or less than 1 per cent., which strongly emphasizes the paucity of types other than mica granites.

Hornblende is only occasionally met with and hornblendic varieties of granite are relatively rare. It is noted alone or associated with biotite, in the granites of Maryland at one locality in northeastern Cecil county, and again near Washington in the vicinity of Garrett Park; in Virginia, in a part of the granites south of Falls Church, Fairfax county, and with biotite in the granites near Saxe, in Charlotte county; in North Carolina, in some of the granites of Mecklenburg county; in South Carolina, in the granite occurring one mile south of Winnsboro, Fairfield county, where hornblende is subordinate to biotite; and in Georgia, in the Armour's Mill area, Putnam county, of sheared fine-grained granite, and near Grantville, Coweta county. Though a common constituent in many of the more massive and schistose basic rocks abundantly distributed throughout the crystalline region, hornblende is equally as rare in the acid gneisses as in their massive equivalents. It is associated with biotite in the coarse-textured granite-gneisses around Richmond and Fredericksburg, Virginia.

Epidote occurs in such relations in some of the Maryland granites as to be regarded an original constituent by some students,[1] and is present in sufficient amount for the rocks to be grouped as biotite-epidote granites. It is often present as a secondary mineral, usually in microscopic quantity only, in the granites of the states south of Maryland. In Madison, Page, and Carroll counties, Virginia; Madison county, North Carolina; and Cocke county, Tennessee, epidote is a chief constituent in the epidote variety of granite, known as unakite. In each of these localities the epidote[2] has been shown, from microscopic study, to be a secondary mineral.

Various accessory minerals are known, among which apatite, zircon, iron oxide, and in many localities, more especially in Maryland and South Carolina, titanite, are the commonest ones. Allanite is a frequent accessory

[1]15th An. Rept., U. S. Geol. Survey, p. 685.

[2]Watson, Thomas L. Journ. Geology, 1904, Vol. XII, pp. 395-398; Amer. Journ. Science, 1906, Vol. XXII, p. 248. Phalen, W. C. Smithsonian Misc. Coll., 1904, Vol. XLV, pp. 306-316.

mineral in the Maryland granites, and has been noted in some of the granites of the Carolinas. Chlorite, epidote, and a light-colored mica, muscovite, are the common secondary minerals.

VARIETIES.

Based on mineral composition the biotite type of acid-irruptive rock is the predominant one, with the muscovite-bearing biotite type quite common. Muscovitic and hornblendic types are rare, and epidotic ones are noted in certain localities in Maryland, Virginia, and North Carolina. Structurally, there are three rather well marked phases, namely, granitoid (even-granular granites), porphyritic (porphyritic granites), and schistose (granite-gneisses). The even-granular and porphyritic textures usually grade into each other and are regarded as textural variations of the same rock-mass. The granite-gneisses are derived from original massive granites by metamorphism, and, as a rule, do not grade into the massive granites, but are sharply defined from them. Even-granular and porphyritic textures likewise characterize the schistose type (granite-gneisses).

INTERMEDIATE CHARACTER OF THE ROCKS.

Although these rocks have been described as granites, an examination of the results obtained from their investigation strongly impresses one that they are not normal or true granites, but occupy an intermediate position between granite and quartz-diorite.[1] They differ essentially in mineral composition from the intermediate type of rock of fairly constant character so abundant over parts of the Pacific Coast region of the United States, described and mapped by the geologists of the United States Geological Survey as granodiorite, in the practical absence of hornblende, the composition and ratio of feldspar content, and the frequent presence of muscovite associated with the biotite.

[1]The rhyolite-granite and the trachyte-syenite groups of rocks are characterized by the predominance of alkali feldspars, commonly orthoclase. On the other hand, the andesite-diorite group is characterized by dominant sodic-lime (plagioclase) feldspars. Between these groups of rocks there occur all gradations, with respect to the ratio of potassic and sodic-lime feldspars. Considerations of convenience, therefore, based on the ratio (amounts) of orthoclase and plagioclase of the rocks, have led to the formation of an intermediate group, whose granitoid members are designated as monzonites. (Brögger, W. C. Die Eruptivgesteine des Kristianiagebietes, 1895, II, 21). Quartz monzonite is intermediate between granite and quartz diorite, and monzonite between syenite and diorite. These intermediate rocks contain potassic and sodic-lime feldspars in approximately equal amounts, with or without quartz, and with subordinate amounts of the ferromagnesian minerals. (Clarke, F. W. The Data of Geochemistry, Bull. 330, U. S. Geol. Survey, 1908, p. 380).

Chemically, the difference is very pronounced. In the Sierra Nevada intermediate type of rock (granodiorite)[1] the range in SiO_2 is from 59 to 69 per cent.; CaO, 3 to 6.50 per cent., though the latter is rarely reached; and in total alkalies from 4.92 to 7.13 per cent. and may notably exceed the CaO, but never fall short of it more than one per cent. Lindgren states that in the relation of K_2O to Na_2O, the latter is apt to predominate, ranging from 2.50 to 4.50 per cent.

In the southern and central Appalachians type of intermediate rocks, the range in SiO_2 is notably higher, 66.01 to 77.19 per cent., with an average of 71.18 per cent.; CaO, 0.48 to 4.89 per cent.,[2] with an average of 1.85 per cent. The range in total alkalies is from 4.29 to 10.24 per cent., with an average of 7.69 per cent., always greatly in excess of the CaO, the average ratio of total alkalies to lime being 4 to 1. In the relation of K_2O to Na_2O, the two are approximately equal, the former ranging from 1.48 to 5.63 per cent., with an average of 3.87 per cent.; the latter from 1.95 to 5.09 per cent., with an average of 3.82 per cent.

The plagioclase in the western type is, according to Lindgren,[3] a calcareous oligoclase or an andesine, and is at least double the amount of potassic feldspar. In the eastern type the plagioclase is an acid oligoclase with usually some albite, rarely a calcareous oligoclase, and is usually less than double the potassic feldspar, the average ratio of potassic feldspar to sodic-lime feldspar being 1:1.88. This difference in composition of the plagioclase accounts principally for the difference in lime content, in the rocks of the two areas.

It will be readily observed from these comparisons that on the basis of both mineral and chemical composition the intermediate type of rock for the two regions is strikingly different; the eastern type shows stronger granite affinities and the western type stronger quartz diorite affinities.

The following table makes clear the above relations. It shows the limits of variation and average composition, as indicated in silica, lime, soda, and potash, of granodiorites and quartz-diorites of the western United States, and of the granites of the southern and middle Appalachians.

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[1]Lindgren, W. Amer. Journ. Science, 1900, Vol. IX, p. 275.

[2]Excessively high lime due to large amount of epidote present (Maryland granites) and not reached in any other case; next highest is from Maryland, and is 3.74 per cent.

[3]Lindgren, W. Op. cit., p. 277.

A. Table of partial analyses of granodiorites, quartz-diorites, and granites.

Constituents.	I.	II.	III.	IV.	V.	VI.
SiO_2	59.5 to 65.5	65.1	53.5 to 70.4	62.5	66.01 to 77.19	71.16
CaO	3.1 to 6.5	4.4	3.2 to 7.6	5.0	0.48 to 4.89[1]	1.89
Na_2O	3.3 to 4.1	3.9	2.4 to 4.9	3.9	1.95 to 5.09	3.80
K_2O	1.7 to 3.7	2.5	1.3 to 4.0	2.3	1.18 to 5.63	4.00

I. Limits of variation in granodiorites of Western United States as shown in analyses published by the United States Geological Survey.

II. Average composition of granodiorites as indicated in the analyses included in I.

III. Limits of variation in quartz-mica diorites of western United States as shown in analyses published by the United States Geological Survey.

IV. Average composition of quartz-mica diorites as indicated in the analyses included in III.

V. Limits of variation in granites of middle and southern Appalachians as shown in analyses published by the State Geological Surveys.

VI. Average composition of southern granites as indicated in the analyses included in V.

It will be observed from a study of the table that the differences are slight in the average composition of granodiorites and quartz-diorites, and as Lindgren states[2], "it is often difficult in practice to separate the normal quartz-mica diorite from the granodiorite . . . ."

As is shown in the tables on pages 37 and 38 of this paper the average composition of the Southern, Western, and New England granites is strikingly similar, there being essentially no difference in this type of rock for the three areas, as would be expected.

CHARACTERISTIC MINERAL AND CHEMICAL FEATURES.

The characteristic mineral and chemical features of the acid plutonic rocks of the southeastern Atlantic states are best discussed in the necessary detail by individual states, and the principal facts afterwards summarized for the region as a whole.

[1]This maximum percentage of CaO is contained in one of the Maryland rocks high in epidote. Omitting this and the next highest percentage of CaO, 3.74 per cent., which is likewise an epidote-bearing rock from Maryland, the maximum percentage of CaO shown in the numerous analyses of the southeastern rocks is 2.76 per cent., contained in one of the Virginia rocks.

[2]Op. cit., p. 272. Lindgren restricts granodiorite to the following limits: SiO_2, 59-69 per cent.; Al_2O_3, 14-17 per cent.; Fe_2O_3, 1.5 to 2.25 per cent.; FeO, 1.5 to 4.25 per cent.; CaO, 3 to 6.5 per cent.; MgO, 1-2.5 per cent.; K_2O, 1-3.75 per cent.; Na_2O, 2.5-4.5 per cent.

MARYLAND.

Mineral composition.—The principal minerals in the Maryland acid plutonic rocks are quartz, orthoclase, microcline, plagioclase near oligoclase, biotite, some muscovite and hornblende, epidote, and allanite, together with titanite, magnetite, apatite, zircon, and occasionally some others, as the chief accessory minerals.

Biotite is an important constituent in all the Maryland granites. In some areas biotite is replaced in part by hornblende, in others muscovite, epidote, and allanite are prominently associated with biotite. Near Garrett Park and Rowlandsville hornblende is an important constituent and is about equal in amount to biotite. At Guilford muscovite enters with biotite as an important mineral.

Based on mineral composition the Maryland granites are divisible into the following principal types: (1) Biotite granite, under which the majority of the granites of the State may be grouped. In addition to biotite the granite of some of the Port Deposit, Frenchtown, Woodstock, Dorsey Run Station, Ellicott City, and Ilchester contain considerable epidote and allanite, and may be designated as allanite-epidote-bearing biotite granites; (2) muscovite-biotite granites of which the Guilford area is the only typical representative; and (3) hornblende-biotite granite.

Texturally, there are three rather well marked types of the Maryland acid rocks: (1) Even-granular massive, (2) porphyritic, and (3) banded or schistose granite-gneiss.

Chemical composition.—From the number of analyses that have been made of the Maryland acid rocks the following are selected to show the range of composition in silica, lime, and the alkalies:

B. Table of partial analyses of Maryland and District of Columbia granites.

Constituents.	I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	Average.
SiO_2	73.69	74.87	72.57	71.79	71.45	66.68	70.45	67.22	69.33	70.89
CaO	3.74	0.48	1.65	2.50	1.58	4.89	2.60	1.36	3.21	2.44
Na_2O	2.81	3.06	3.92	3.09	1.95	2.65	3.83	2.00	2.70	2.89
K_2O	1.48	5.36	5.36	4.75	3.28	2.05	3.59	3.26	2.67	3.53

I. Port Deposit, Cecil county, Maryland. Williams, G. H., 15th An. Rept., U. S. Geol. Survey, p. 672; Bascom, F. Maryland Geol. Survey, Cecil County Report, 1902, p. 119.

II. Brookville, Montgomery county, Maryland. Williams, G. H. 15th An. Rept., U. S. Geol. Survey, p. 672.

III. Guilford, Howard county, Maryland. Williams, G. H. 15th An. Rept., U. S. Geol. Survey, p. 672.
IV. Woodstock, Howard county, Maryland. Williams, G. H. 15th An. Rept., U. S. Geol. Survey, p. 672.
V. Sykesville, Carroll county, Maryland. Williams, G. H. 15th An. Rept., U. S. Geol. Survey, p. 672.
VI. Rowlandsville, Cecil county, Maryland. Bascom, F. Maryland Geol. Survey, Cecil County Report, 1902, p. 120.
VII. Dorsey Run Cut, Howard county, Maryland. C. R. Keyes, 15th An. Rept., U. S. Geol. Survey, 1895, p. 697.
VIII. Potomac Stone Company's quarry, District of Columbia. Williams, G. H. 15th An. Rept., U S. Geol. Survey, p. 672.
IX. District of Columbia. Merrill, G. P. Rocks, Rock-Weathering and Soils, New York, 1906, p. 185.

In general, the complete analyses indicate that alumina, ferric iron, and the alkalies vary with the silica, while ferrous iron, lime, and magnesia vary inversely. The percentage of silica (70.89 per cent. average) indicates relatively acid rocks, with a wide range in the amount of quartz shown, from 13.6 per cent. to 41.2 per cent., and an average of 30.25 per cent. Computed from the analyses, the percentages of quartz are: I, 41.2; II, 34.6; III, 28.7; IV, 28.7; IVa[1], 13.6; V, 40.9; VI, 28.4; VII, 26.8; with an average of 30.4.

Computing the mineral composition of the feldspars from the percentages of K_2O, Na_2O, and CaO given in table B, by assigning the potash to orthoclase, and the soda and lime to albite and anorthite, respectively, the analyses show the constitution in table C.

C. Table of feldspar composition of Maryland acid plutonic rocks.

(Numbers correspond to table B.)

	I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.
Orthoclase	8.90	31.69	31.69	27.80	19.46	12.23	21.1	19.46	15.57
Albite	23.58	25.68	33.01	26.20	16.24	22.53	32.0	16.77	23.06
Anorthite	11.15	1.67	8.06	12.51	7.78	7.78	13.1	5.84	(2)
Total plagioclase	34.73	27.35	41.07	38.71	24.02	30.31	45.1	22.61	-----
Total feldspar	43.63	59.04	72.76	66.51	43.48	42.54	66.2	42.07	-----
Ab_nAn_m ratio	2:1	15:1	4:1	2:1	2:1	3:1	2:1	3:1	-----
Or—plag. ratio	1:4	1:1	1:1	1:1	1:1	1:2.5	1:2	1:1	1:2+

It will be observed that the plagioclase molecules as calculated range from nearly pure albite ($Ab_{15}An_1$) to basic oligoclase (Ab_2An_1), with an average of acid oligoclase (Ab_4An_1). It is not improbable that some of the

[1]Dorsey Run, Howard county, Maryland. 15th An. Rept., U. S. Geol. Survey, 1895, p. 722.

[2]Anorthite not calculated because of unknown amounts of epidote and apatite present in the rock.

soda replaces potash in the potassic feldspar, orthoclase, but anorthoclase, if present, has not been identified. Perthitic growths and microcline are usually present.

In three of the analyses (table B), the percentage of Na_2O is greater than that of K_2O, but with only one exception (II), Na_2O molecularly, either exceeds or is approximately equal to K_2O.

Classification.—As is shown in table C, the total feldspar of the rocks varies rather widely, from 42.07 to 72.76 per cent. In five of the analyses, the calculated total feldspar is proportioned in the ratio of approximately 1:1 of orthoclase to plagioclase molecules; in VII and VIII plagioclase is more than double orthoclase, and in I it is approximately four times greater. In the old classification, on the basis of orthoclase-plagioclase ratio, II, III, IV, V, and VII are quartz monzonites, VI and VIII granodiorites, and I closely approaches the transitional position between granodiorite and quartz diorite. Comparing the percentage of CaO in these, I, VI, and VIII, with that of the other five analyses, it is found to be appreciably greater, and falls well within the limits of this constituent for the granodiorites of the Pacific Coast region, as defined by Lindgren.[1]

In the quantitative system of classification, the position of the Maryland rocks has been computed as follows:

Granites of Maryland.

Number (Corresponds to Nos. in tables B and C.)	Class.	Order.	Rang.	Subrang.	Name.
I	I	3	3	3	Riesenose
II	I	4	1	3	Liparose
III	I	4	2	3	Toscanose
IV	I	4	2	3	Toscanose
IVa	I	4	3	4	Yellowstonose
V	I	3	2	3	Tehamose
VI	II	4	3	4	Tonalose
VII	I	4	2	3	Toscanose

IVa. Granite from Dorsey Run cut, Howard county, Md. Not included in tables of Maryland granites discussed in this paper.

Most of the rocks represented in the above table (II, III, IV, V, and VII) are sodipotassic, while the remaining ones (I, IVa, and VI) are dosodic.

[1]Lindgren, W. Op. cit., p. 275.

Virginia.

Mineral composition.—The granitic rocks of Virginia are predominantly of the biotite type. The principal minerals are the same as those enumerated for Maryland on page 13, except that allanite has not been observed in the Virginia rocks. Muscovite in subordinate amount usually accompanies the biotite in the principal areas of granitic rocks in the State, and it is the principal component in the rock of the Hazel Run area west of Fredericksburg. Hornblende is an important constituent in the rocks of the Falls Church area in Fairfax county, and near Saxe in Charlotte county. Epidote is a prominent constituent in the variety of granite known as unakite, occurring in Page, Madison, and Carroll counties.

Based on mineral composition the Virginia granitic rocks may be classified into (1) biotite granite, which usually carries a little muscovite in addition to the biotite, and under which a majority of the granites of the State may be grouped; (2) muscovite granite, of which the Hazel Run area near Fredericksburg is the only typical representative yet known; (3) hornblende-biotite granite, represented by a part of the granites in the Falls Church area, and near Saxe in Charlotte county; and (4) epidote granite, unakite, of which there are only two known localities in the State.

The rocks vary in structure from massive to schistose, and in texture from even-granular to porphyritic. On this basis the three usual types of the rocks are distinguished, even-granular massive, porphyritic, and schistose.

Chemical composition.—Complete analyses of the Virginia rocks show characteristically high silica, alumina, and alkalies, and correspondingly low iron, lime, and magnesia. The difference in silica is slight, but in general the iron, magnesia, and lime show a slight tendency to increase with decreasing silica, indicating proportionately more of the dark minerals and plagioclase, and less quartz. The range in silica, lime, and alkalies is shown in the following table:

D. Table of partial analyses of Virginia granites.

(Wm. M. Thornton, Jr., Analyst.)

	I.	II.	III.	IV.	V.	VI.	VII.	Average.
SiO_2	72.27	71.51	71.19	70.83	69.48	69.44	69.29	70.57
CaO	1.56	1.79	2.04	1.88	2.81	2.11	2.76	2.13
Na_2O	3.46	3.64	3.56	3.49	3.65	3.97	2.89	3.52
K_2O	5.00	4.63	4.45	4.83	3.45	4.25	2.87	4.21

I. Medium-textured and medium gray, biotite granite. Westham granite quarries, 4.5 miles west of Richmond, Chesterfield county.

II. Medium-textured and medium gray biotite granite, Lassiter, and Petersburg Granite Cos. quarries, Petersburg, Dinwiddie county.
III. Fine-grained, dark blue-gray biotite granite. McGowan, Netherwood, and Donald quarries, Chesterfield county, and Mitchell and Copeland quarry, Henrico county, near Richmond.
IV. Medium coarse-textured and medium gray biotite granite. Netherwood, State (Old Dominion) Granite Development Co., Krimm, and Middendorf quarries, Chesterfield county, near Richmond.
V. Fine-grained, dark blue-gray biotite granite. Cartwright and Davis quarries, near Fredericksburg, Spottsylvania county.
VI. Medium-textured and medium gray, biotite granite. McIntosh quarry, Chesterfield county, 5 miles west of Richmond.
VII. Medium coarse-textured, gray, biotite granite-gneiss. Middendorf (Belt Line Railway) quarry, near Manchester, Chesterfield county.

The average percentage of silica in the Virginia rocks, 70.57 per cent., is essentially the same as that in the Maryland rocks, 70.89 per cent. The range in this constituent in the Virginia rocks is from 69.29 to 72.27 per cent. Calculated from the complete analyses the percentages of quartz are as follows: I, 28.44; II, 27.48; III, 28.56; IV, 27.60; V, 27.30; VI, 23.76; and VII, 32.94; average, 28.01.

Of the feldspars present in the Virginia rocks, microcline, which is generally shown in thin sections, is subject to greatest variation, and, as a rule, is much less in amount than either orthoclase or plagioclase. An important feature in the mineral composition of these rocks is the nearly constant large amount of plagioclase present, as would be naturally inferred from the percentages of Na_2O and CaO in the table of partial analyses above. Extinction angles measured against the twinning planes of individual laths of soda-lime feldspar usually indicate a plagioclase near oligoclase. Albite is usually present in subordinate amount, not as separate anhedra but intergrown with the orthoclase as microperthite. Anorthoclase, if present, has not been identified.

The mineral composition of the feldspars, calculated from the percentages of K_2O, Na_2O, and CaO in table D, by allotting potash to orthoclase, and soda and lime to albite and anorthite, respectively, is given in table E.

E. Table of feldspar composition of Virginia granites.
(Numbers correspond to table D.)

	I.	II.	III.	IV.	V.	VI.	VII.	Average.
Orthoclase	29.46	27.24	26.13	28.36	20.57	25.02	17.24	24.86
Albite	29.34	30.92	29.85	29.34	30.92	33.54	24.63	29.79
Anorthite	7.78	8.90	10.01	9.17	13.90	10.56	13.62	10.56
Total plagioclase	37.12	39.82	39.86	38.51	44.82	44.10	38.25	40.35
Total feldspar	66.58	67.06	65.99	66.87	65.39	69.12	55.49	65.21
Ab_nAn_m ratio	4:1	4:1	3:1	3:1	2+:1	3:1	2:1	3:1
Or—plag. ratio	1:1.26	1:1.46	1:1.52	1:1.35	1:2.1+	1:1.76	1:2.2	1:1.86

With the exception of II, V, and VII (table D), which show equal percentages of Na_2O and K_2O, the percentage of Na_2O is slightly less than that of K_2O, but without exception Na_2O, molecularly, either exceeds or is approximately equal to K_2O. The range in plagioclase molecules as calculated is from basic oligoclase (Ab_2An_1) to acid oligoclase (Ab_4An_1), with an average of intermediate oligoclase (Ab_3An_1).

Classification.—Table E shows the amount of total feldspar in the rocks to be strikingly similar, with the exception of VII, in which it is considerably less than for either of the other six. The average in total feldspar, 65.21 per cent., is greater in the Virginia rocks than in those of the other states, except Georgia.

Total feldspar in four of the analyses (II, III, IV, and VI) is proportioned in the ratio of 1 of orthoclase to less than 2 of plagioclase, while in the remaining three plagioclase is somewhat increased and is slightly more than double orthoclase. The percentage of CaO in these last three, which is but slightly increased over that of the other four analyses, is much less than that contained in granodiorites or quartz diorites. With the possible exception of VII, all the properties of the Virginia rocks are those which would group them, in the old scheme of classification, as quartz monzonites. They are characterized by high silica and low lime; oligoclase of intermediate to acid composition, except VII, which contains basic oligoclase; a computed orthoclase content of more than 20 per cent. and less than 30 per cent., except VII, which is less than 20 per cent. (17.24 per cent); and, in most cases, by an orthoclase-plagioclase ratio of 1 to less than 2.

The position of the Virginia rocks in the quantitative system of classification has been computed, and the results may be tabulated as follows:

Granites of Virginia.

Number (Corresponds to Nos. in tables D and E)	Class.	Order.	Rang.	Subrang.	Name.
I	I	4	2	3	Toscanose
II	I	4	2	3	Toscanose
III	I	4	2	3	Toscanose
IV	I	4	2	3	Toscanose
V	I	4	2	3	Toscanose
VI	I	4	3	3	Amiatose
VII	I	4	3	3-4	Between Amiatose and Yellowstonose

It will be observed from the preceding table that all of the Virginia rocks fall in class I, persalane; and order 4, Brittanare. The first five of the seven fall in rang 2, domalkalic; and subrang 3, sodipotassic, and are termed *toscanose*. The other two (VI and VII) fall in rang 3, alkalicalcic; VI falls in subrang 3, sodipotassic, and is termed *amiatose;* VII falls in subrang 3-4, between sodipotassic and dosodic, and is intermediate between *amiatose* and *yellowstonose*.

North Carolina.

Mineral composition.—The North Carolina acid plutonic rocks contain the same essential and accessory minerals as those of Maryland and Virginia. Hornblende as a chief accessory mineral is known to occur in the rocks of only two localities. With the exception of these, the rocks of the State are biotite granites, in which very subordinate muscovite associated with biotite may occur. The rocks of the Dunn's Mountain and Wilson areas are essentially mixtures of feldspars and quartz, with scant biotite as the third principal mineral. Based on texture and structure the usual three types of acid rocks are distinguished.

Chemical composition.—The table of partial analyses below shows the range in silica, lime, and alkalies in the North Carolina rocks.

F. Table of partial analyses of North Carolina granites.

Constituents.	I.	II.	III.	IV.	V.	VI.	Average.
SiO_2	77.19	75.79	73.11	76.89	69.28	66.01	72.64
CaO	0.59	0.62	1.74	1.63	2.20	1.44	1.37
Na_2O	3.15	4.01	4.12	4.42	3.64	5.06	4.14
K_2O	3.15	3.78	3.07	2.50	2.76	3.16	2.97

I. Dunn's Mountain gray granite, near Salisbury, Rowan county, N. C. W. M. Thornton, Jr., analyst.
II. Dunn's Mountain pink granite, near Salisbury, Rowan county, N. C. W. M. Thornton, Jr., analyst.
III. Mt. Airy, Surry county, North Carolina. W. M. Thornton, Jr., analyst.
IV. Granite three miles north of Gold Hill, Rowan county, N. C. A. S. Wheeler, analyst.
V. Raleigh, Wake county, North Carolina. Bull. No. 2, N. C. Geol. Survey, 1906, p. 32.
VI. Mooresville, Iredell county, North Carolina. Bull. No. 2, N. C. Geol. Survey, 1906, p. 84.

The range in silica in the North Carolina rocks is great, from 66.01 per cent. to 77.19 per cent., with a higher average, 72.64 per cent., than for either of the other states. In the complete analyses, decrease of silica is accompanied by increase of iron, magnesia, and lime, showing increasing amounts of the dark minerals and of plagioclase. The percentages of quartz in the rocks as calculated are: I, 43.50; II, 36.66; III, 33.36; IV, 37.44; V, 31.32; and VI, 14.58; average, 32.81.

With only one exception (I), the percentage of Na_2O is in excess of K_2O, and in each one, omitting II, the excess of Na_2O is quite marked. I, the exception noted, shows equal percentages of Na_2O and K_2O. The mineral composition of the feldspars, calculated in the usual way from the percentages of K_2O, Na_2O, and CaO, in table F, is indicated in the following table:

G. Table of feldspar composition of North Carolina granites.

(Numbers correspond to table F.)

	I.	II.	III.	IV.	V.	VI.	Average.
Orthoclase	21.13	22.24	18.35	15.01	16.12	16.12	18.16
Albite	26.72	34.06	34.58	37.20	39.92	42.44	35.82
Anorthite	3.05	3.06	8.62	8.06	10.77	10.84	7.35
Total plagioclase	29.77	37.12	43.20	45.26	41.69	43.28	43.17
Total feldspar	50.90	59.36	61.55	60.27	57.81	69.40	61.33
Ab_nAn_m ratio	9:1	11:1	4:1	5:1	3:1	4:1	6:1
Or—plag. ratio	1:1.4	1:1.6	1:3	1:3	1:2.5	1:3.3	1:2.5

The plagioclase molecules as calculated range from intermediate oligoclase (Ab_3An_1) to acid oligoclase-albite ($Ab_{11}An_1$), with an average of basic oligoclase-albite (Ab_6An_1). The potassic feldspar contains some soda, as is shown by the presence of more or less microperthite. Zonal structure is rarely developed in the feldspars.

Classification.—The total feldspar in the North Carolina rocks ranges from 50.90 per cent. to 69.40 per cent., with an average of 61.33 per cent. The rocks represented by III, IV, V, and VI of table G, fall considerably below the minimum amount of potassic feldspar in quartz monzonite, but are well within the limits of this constituent in granodiorite. In each of these plagioclase is approximately three times that of orthoclase, and although these rocks fall outside the limits of quartz monzonites in orthoclase, their low CaO and generally high SiO_2 are more characteristic of quartz monzonite than of granodiorite. The soda-lime feldspar of these rocks is more acid than that of granodiorite. I and II are quartz monzonites.

The position of the North Carolina rocks in the quantitative scheme of classification has been computed, and is shown in the tabular statement below.

Granites of North Carolina.

Number (Corresponds to Nos. in tables F and G)	Class.	Order.	Rang.	Subrang.	Name.
I	I	3	1	3	Alaskose
II	I	3	1	3	Alaskose
III	I	4	2	4	Lassenose
IV	I	3	2	4	Alsbachose
V	I	4	2	4	Lassenose
VI	II	4	2	4	Dacose

From the position of these rocks in the quantitative classification as tabulated above, it will be observed that I and II are in the peralkalic rang and the sodipotassic subrang, and are termed *alaskose*. III, IV, V and VI fall in the domalkalic rang and the dosodic subrang, but vary in their position either as to class or order. III and V are I.4.2.4., *lassenose;* IV is I.3.2.4., *alsbachose;* and VI is II.4.2.4., and is *dacose.*

South Carolina.

Mineral composition.—The South Carolina granites are biotite granites with or without muscovite. Hornblende associated with biotite is only known to the writer at one locality in the State. The essential and accessory minerals are the same as for the Maryland, Virginia, and North Carolina rocks. The same types, based on texture and structure, are recognized as for the other states.

Chemical composition.—The range in silica, lime, and the alkalies in the South Carolina rocks is shown in table H.

H. Table of partial analyses of South Carolina granites.[1]

Constituents.	I.	II.	III.	IV.	V.	VI.	VII.	VIII.	Average.
SiO_2	68.90	68.70	70.54	68.80	69.79	70.70	67.33	71.77	69.57
CaO	2.66	1.70	1.28	1.64	1.73	2.14	2.65	1.87	1.96
Na_2O	4.76	3.09	3.97	3.45	4.07	3.86	3.95	3.66	3.85
K_2O	3.49	3.36	5.37	4.54	4.45	4.76	4.46	3.70	4.14

I. Clover, York county, South Carolina. Bull. No. 2, Series IV, S. C. Geol. Survey, 1908, p. 254.

II. Moores Creek, Saluda county, South Carolina. Bull. No. 2, Series IV, S. C. Geol. Survey, 1908, pp. 191-192.

[1]Sloan, E. A Catalogue of the Mineral Localities of South Carolina. S. C. Geol. Survey, Bull. No. 2, Series IV, 1908, pp. 174-229, 250-255.

III. Benjamin quarry, Greenwood county, South Carolina. Bull. No. 2, Series IV, S. C. Geol. Survey, 1908, p. 251.
IV. Cold Point, Greenwood county, South Carolina. Bull. No. 2, Series IV, S. C. Geol. Survey, 1908, p. 251.
V. Average 3 analyses, Newberry, Fairfield, and Lancaster counties, South Carolina. Bull. No. 2, Series IV, S. C. Geol. Survey, 1908, pp. 250-255.
VI. Average 2 analyses, Union and Spartanburg counties, South Carolina. Bull. No. 2, Series IV, S. C. Geol. Survey, 1908, pp. 198-200.
VII. Average 4 analyses, Anderson, Laurens-Abbeville, Pickens, and Lexington counties, South Carolina. Bull. No. 2, Series IV, S. C. Geol. Survey, 1908, pp. 250-255.
VIII. Average 3 analyses, Fairfield, Lexington, and Kershaw counties, South Carolina. Bull. No. 2, Series IV, S. C. Geol. Survey, 1908, pp. 250-255.

The range in silica is less than for Maryland and North Carolina, from 67.33 per cent. to 71.77 per cent., with an average of 69.57 per cent. In the complete analyses, the range in alumina, ferrous and ferric oxides, magnesia, and lime is proportionately small, indicating in general but small differences in the amounts of the dark mineral and plagioclase. The percentage of quartz as calculated is: I, 20.94; II, 30.72; III, 22.68; IV, 25.98; V, 23.88; VI, 24.60; VII, 19.26; VIII, 30.84; average, 24.86.

Only one analysis (I) shows the percentage of Na_2O to be greater than that of K_2O. Five (II, V, VI, VII, and VIII) indicate approximately equal percentages of the alkalies, and two (III and IV) show an excess of K_2O over Na_2O of more than 1 per cent. In each case, however, Na_2O is molecularly greater than K_2O. The range in CaO is about that of so-called normal granites.

Thin sections of the rocks show the presence of the potassic feldspars, orthoclase and microcline, the latter subject to greater variation in amount than the former. Microperthite, quite generally present, indicates that the potassic feldspar contains some soda. Zonal structure in the plagioclase is rare, and from optical measurements the soda-lime feldspar corresponds to oligoclase.

Computed from the constituents in the table above, the mineral composition of the feldspars is shown in table J.

J. Table of feldspar composition of South Carolina granites.
(Numbers correspond to table H.)

	I.	II.	III.	IV.	V.	VI.	VII.	VIII.	Average.
Orthoclase	20.57	20.01	31.69	26.69	26 13	27.80	26.13	21.68	25.0
Albite	39.82	25.68	33.54	28.82	35.10	31.96	33.54	30.92	32.4
Anorthite	13.01	8.34	6.12	8.06	7.34	10.56	13.07	9.17	9.4
Total plagioclase	52.83	34.02	39.66	36.88	42.44	42.52	46.61	40.09	41.8
Total feldspar	73.40	53.03	71.35	63.57	68.57	70.32	72.74	61.77	66.84
Ab_nAn_m ratio	3 :1	3:1	5:1	4:1	5:1	3:1	3:1	3:1	
Or—plag. ratio	1:2.56	1:1.65	1:1.25	1:1.38	1:1.6	1:1.5	1:1.77	1:1.8	1:1.7

As calculated the range in plagioclase molecules is from intermediate to very acid oligoclase (Ab_3An_1 to Ab_5An_1).

Classification.—Total feldspar ranges from 53.03 per cent. to 73.40 per cent., with the ratio of potassic feldspar to sodic-lime feldspar less than 1 to 2, except in I where plagioclase is more than double orthoclase. In each case the amount of potassic feldspar is within the limits of the quartz monzonite type, although the minimum is practically reached for this constituent in I, II, and III, and their approach to the granodiorite type is close. The CaO and Na_2O contents of these rocks yield a sodic-lime feldspar too acid in composition to group them with the granodiorites, but in the prevailing (older) classification they would be properly called quartz monzonites, with stronger granite than quartz diorite affinities.

The position of the South Carolina rocks in the quantitative classification has been computed, and is as follows:

Granites of South Carolina.

Number (Corresponds to Nos. in tables H and J)	Class.	Order.	Rang.	Subrang.	Name.
I	I	4	2	4	Lassenose
II	I	4	2	3	Toscanose
III	I	4	2	3	Toscanose
IV	I	4	2	3	Toscanose
V	I	4	2	3	Toscanose
VI	I	4	2	3	Toscanose
VII	I	4	2	3	Toscanose
VIII	I	4	2	3	Toscanose

The remarkable similarity of the South Carolina rocks is shown in the above tabulation. They fall in the same class, order, and rang (domalkalic), and with the exception of I, which is dosodic, they are in the subrang sodipotassic, and are called *toscanose.* I is *lassenose.*

Georgia.

Mineral composition.—The Georgia acid plutonic rocks are prevailingly of the biotite type. Muscovite is frequently associated with biotite, and in several localities, notably Stone Mountain, it greatly exceeds biotite in amount. Hornblende is rare and has not been identified in any of the commercial granites in the State. The principal minerals in the rocks are quartz, orthoclase, microcline, plagioclase (oligoclase chiefly, and some albite), biotite, muscovite, apatite, zircon, and iron oxides. Chlorite and epidote are the most frequent secondary minerals.

Similar variation in texture and structure is shown as for the other states, and on this basis the three types, massive even-granular, porphyritic, and schistose, are strongly developed. Where typically developed, these are usually quite sharply differentiated, but the massive granular and porphyritic facies grade into each other, in practically every locality studied.

Chemical composition.—In chemical composition the three groups of rocks distinguished above show remarkably close agreement, as is indicated in the following analyses:

Analyses of Georgia granites.[1]

Constituents.	I.	II.	III.
SiO_2	69.67	69.28	73.76
Al_2O_3	16.63	16.73	14.52
Fe_2O_3	1.28	1.75	1.03
MgO	0.55	0.72	0.29
CaO	2.13	2.16	1.14
Na_2O	4.73	4.33	4.16
K_2O	4.71	4.59	4.63

I. Average of 21 analyses of massive-granular granites.
II. Average of 10 analyses of porphyritic granites.
III. Average of 12 analyses of granite-gneisses.

Essentially no difference is shown in the figures in I and II which represent facies of the same rock mass, but the gneisses (III) average higher in silica, and lower in alumina, iron, magnesia, and lime, while the alkalies for the three types are very closely similar.

Table K assembles the four constituents, silica, lime, and alkalies, of the 9 leading types of granite, in which the relations between K_2O and Na_2O described above are shown. A few analyses of isolated areas of granite, which strictly conform to one or the other of the 9 types listed, have been omitted from the table. High silica, an average of 71.76 per cent., and low lime, an average of 1.57 per cent., are characteristic features in the composition of the Georgia rocks.

K. Table of partial analyses of Georgia granites.

Constituents.	I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	Average.
SiO_2	72.56	69.86	69.13	69.88	69.14	69.28	76.37	75.07	74.57	71.76
CaO	1.19	2.14	2.04	1.78	1.76	2.16	1.13	0.98	0.97	1.57
Na_2O	4.94	4.73	4.65	4.45	5.09	4.33	4.02	3.87	4.70	4.53
K_2O	5.30	5.06	4.53	5.63	4.94	4.59	3.68	4.87	4.20	4.75

[1]Georgia Geol. Survey, Bull. No. 9-A, 1902, p. 241.

I. Stone Mountain, DeKalb county, Georgia. Bull. 9-A, Ga. Geol. Survey, 1902, p. 241.

II. Average 9 analyses of Oglesby dark blue-gray granite, Elbert and Oglethorpe counties, Georgia. Bull. 9-A, Ga. Geol. Survey, 1902, p. 241.

III. Average 2 analyses of Elberton-Echols Mill light gray granite. Elbert and Oglethorpe counties, Georgia. Bull. 9-A, Ga. Geol. Survey, 1902, pp. 241-242.

IV. Greenville Granite Cos. quarry, Meriwether county, Georgia. Bull. 9-A, Ga. Geol. Survey, 1902, p. 241.

V. Average 3 analyses of medium gray granite, Campbell and Coweta counties, Georgia. Bull. 9-A, Ga. Geol. Survey, 1902, pp. 241-242.

VI. Average 10 analyses of porphyritic granites from the Georgia Piedmont region. Bull. 9-A, Ga. Geol. Survey, 1902, p. 242.

VII. Odessa quarry, Meriwether county, Georgia. Bull. 9-A, Ga. Geol. Survey, 1902, p. 242.

VIII. Average 5 analyses of contorted granite-gneiss from the Lithonia, Georgia, area. Bull. 9-A, Ga. Geol. Survey, 1902, p. 243.

IX. Average 3 analyses of foliated granite, Coweta, Heard, and Meriwether counties, Georgia. Bull. 9-A, Ga. Geol. Survey, 1902, p. 243.

The K_2O and Na_2O approximate nearly equal percentages in a majority of the analyses. Of 23 analyses of massive granular rocks, 13 show a slight excess percentage of K_2O over Na_2O, the excess of K_2O ranging from 0.16 to 1.20 per cent., with an average for the 13 of 0.638 per cent. excess of K_2O. Six of the 10 analyses of the porphyritic rocks show a small excess percentage of K_2O over Na_2O, ranging from 0.11 to 1.43 per cent. with an average of 0.845 per cent. excess of K_2O. Eight of the 12 analyses of the granite-gneisses show a slight excess percentage of K_2O over Na_2O, ranging from 0.11 to 1.92 per cent., with an average of 0.766 per cent. excess of K_2O. With only one exception, a granite-gneiss from Snellville, Gwinnett county, Na_2O molecularly exceeds K_2O in each of the 45 analyses.

The high soda content in these rocks is derived principally from the prevailing large amounts of plagioclase (oligoclase); from the replacement of a part of the K_2O by Na_2O in the potassic feldspar; and from microperthite, which is quite generally present though variable in amount. Optical measurements of the plagioclase individuals indicate an acid oligoclase. Albite occurs as intergrowths with a part of the potassic feldspar. Zonal structure is sometimes developed in the feldspars, but the variation in extinction between the center and periphery is not great.

The mineral composition of the feldspars as calculated from the data in table K, is indicated in table M.

M. Table of feldspar composition of Georgia granites.
(Numbers correspond to table K.)

	I.	II.	III.	IV.	V.
Orthoclase	31.69	30.02	26.69	33.36	29.47
Albite	41.92	39.82	39.30	37.73	42.97
Anorthite	5.84	10.56	10.01	8.90	8.62
Total plagioclase	47.76	50.38	49.31	46.63	51.59
Total feldspar	79.45	80.40	76.00	79.99	81.06
Ab^nAn_m ratio	7:1	4:1	4:1	4:1	5:1
Or—plag. ratio	1:1.5	1:1.6	1:1.8	1:1.4	1:1.75

	VI.	VII.	VIII.	IX.	Average.
Orthoclase	26.68	21.68	28.91	25.02	28.17
Albite	36.68	33.54	32.49	39.82	38.25
Anorthite	10.56	5.56	5.00	4.73	7.75
Total plagioclase	47.24	39.10	37.49	44.55	46.00
Total feldspar	73.92	60.78	66.40	69.57	74.17
Ab_nAn_m ratio	3:1	6:1	7:1	8:1	5:1
Or—plag. ratio	1:1.77	1:1.8	1:1.4	1:1.78	1:1.6

Classification.—The high range in total feldspar for the Georgia rocks is observed, and the average is greater than for any other state in the region discussed. The soda-lime feldspar ranges in amount from 37.49 per cent. to 51.59 per cent., and is an acid oligoclase; potassic feldspar ranges from 21.68 per cent. to 33.36 per cent. This gives an average orthoclase-plagioclase ratio of 1:1.6, which varies between 1:1.4 and 1:1.8, intermediate between quartz monzonite and granodiorite, but more closely allied with the former because of the amount of potassic feldspar always well within the limits of the quartz monzonite type, the more acid character of the soda-lime feldspar, and of the high silica and low lime. Under the old scheme of classification, on the basis of potash-soda ratio, these rocks would be grouped as quartz monzonites.

Bastin[1] has computed the position of the Georgia granites and granite-gneisses, from the writer's[2] analyses, in the quantitative scheme of classification[3]. The results may be tabulated as follows:

[1]Bastin, E. S. Chemical Composition as a Criterion in Identifying Metamorphosed Sediments. Journ. of Geology, 1909, Vol. XVII, p. 451.

[2]Watson, Thomas L. The Granites and Gneisses of Georgia. Bull. 9-A, Geological Survey of Georgia, 1902, 367 pages.

[3]Cross, Iddings, Pirsson, and Washington. Quantitative Classification of Igneous Rocks, 1903, 286 pages.

Granites and granite-gneisses of Georgia.

		Class.	Order.	Rang.	Subrang.	Name.
Normal granites	Average of 21 analyses	I	4	2	3	Toscanose
Porphyritic granites	Average of 10 analyses	I	4	2	3	Toscanose
	4 analyses separately computed	I	4	2	3	Toscanose
Biotite granite-gneiss	2 analyses separately computed	I	4	2	3-4	Between Toscanose and Lassenose
	3 analyses separately computed	I	4	1	3	Liparose

Of the grouping above in the scheme of quantitative classification, it will be observed that the first three (*toscanose*) and the last one (*liparose*) of the Georgia rocks are sodipotassic, with the remaining one (fourth one of table) transitional between sodipotassic and dosodic (*toscanose-lassenose*).

Summary of Conclusions.

The leading facts developed in the study of the acid plutonic rocks, called granites, of the central and southern Appalachian region, may be summarized as follows:

The rocks are closely similar in mineral and chemical composition, and are predominantly mica-bearing, chiefly biotite with usually some muscovite, rarely muscovite alone. Hornblende-bearing varieties are comparatively infrequent. Their range in quartz is great, from 13.5 per cent. to 43.5 per cent. with an average of 29 per cent., but within the limits for this constituent of acid-granular rocks. Total SiO_2 for the region averages 71.18 per cent.

The range in CaO is from 0.48 per cent. to 4.89 per cent. with an average of 1.85 per cent.; and in total alkalies from 4.29 per cent. to 10.24 per cent. with an average of 7.69 per cent. Total alkalies are greatly in excess of CaO, the average ratio of total alkalies to lime being 4 to 1. In the relation of K_2O to Na_2O, the two are approximately equal. K_2O ranges from 1.48 per cent. to 5.63 per cent, with an average of 3.87 per cent., and Na_2O from 1.95 per cent. to 5.09 per cent. with an average of 3.82 per cent. With only one exception (Maryland II), Na_2O molecularly either exceeds or is equal to K_2O.

The feldspar content is a mixture of potassic and sodic-lime varieties, the latter being either equal to or greater than the former. Increase in total feldspar is shown in the granites from Maryland southward to Georgia, except Virginia, in which total feldspar is greater than for North or South Carolina but less than for Georgia. Maryland shows the minimum and Georgia the maximum of this constituent. Orthoclase including microcline is least for North Carolina and greatest for Georgia. Plagioclase is least for Maryland and greatest for Georgia. The ratio between orthoclase, including microcline, and plagioclase (sodic-lime varieties), is greatest for North Carolina and is approximately the same in each of the other four states. Figure 2 illustrates diagrammatically the feldspar composition of the southern granites by states.

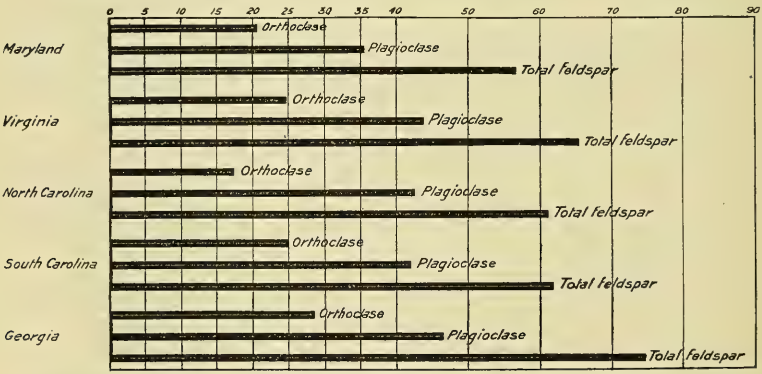

Figure 2.—Diagram illustrating feldspar composition of the southern granites by states.

The molecular ratio of potassic to sodic-lime feldspars is from 1:1 to 1:4, with a general average of 1 to 1.88. In a total of 39 analyses, 28 show an orthoclase-plagioclase ratio of $1<2$, and 11 a ratio of $1>2$. The plagioclase is usually an acid oligoclase with some-albite, rarely a calcareous oligoclase, and as shown above is usually less than double the potassic feldspar.

The above characteristic features of composition, namely, high SiO_2 and low CaO; acid oligoclase; an orthoclase-plagioclase ratio, in most cases, of 1 to less than 2; and a computed orthoclase content usually greater than 20 per cent. and considerably less than 40 per cent.[1], group these rocks in the old system of classification as quartz monzonites. The few exceptions are noted under the discussion of individual states. It is a noteworthy fact that on the basis of orthoclase content (40 per cent. or more on an assumed basis of 60 per cent. total feldspar) there is not a rock represented in the numerous analyses of the entire region that properly would be called granite.

In the quantitative classification of igneous rocks, the rocks represented by the analyses above fall in Class I, persalane (two, Maryland 1, and North Carolina 1, Class II dosalane); in orders 3, columbare, and 4, brittanare, predominantly the latter, with one in order 1, victorare. Four fall in rang 1, peralkalic; 29 in rang 2, domalkalic; and 5 in rang 3, alkalicalcic; 29 in subrang 3, sodi-potassic, 7 in subrang 4, dosodic, and 2 between subrangs 3 and 4. They are dominantly domalkalic and sodipotassic. It is interesting to note that not one of the rocks is perpotassic or dopotassic.

The rocks correspond to the subrang names toscanose (21), lassenose (3), liparose (2), alaskose (2), one each of tehamose, alsbachose, riesenose, amiatose, yellowstonose, dacose, and tonalose, and one each between toscanose and lassenose, amiatose and yellowstonose.

GRANITES OF NEW ENGLAND.

Twenty-one analyses[2] of granites from New England, Massachusetts (6), Maine (6), Vermont (3), New Hampshire (1), Rhode Island (3), and Connecticut (2), show the following limits of variation and average in silica, lime, soda, and potash:

Granites of New England.

Constituents.	Limits of Variation.	Average.
SiO_2	68.40 to 77.08	72.68
CaO	0.31 to 2.80	1.49
Na_2O	1.58 to 5.85	3.62
K_2O	0.95 to 5.78	4.64

[1]Lindgren, W. Op. cit., p. 279.

[2]Selected from publications of State and Federal Geological Surveys.

Omitting nonfeldspathic constituents and assigning potash to orthoclase, and the soda and lime to albite and anorthite, respectively, the twenty-one analyses show the limits of variation and average in composition of the feldspars to be:

Feldspar composition of New England granites.

	Limits of Variation.	Average.
Orthoclase	5.56 to 34.5	27.5
Albite	13.10 to 49.25	30.0
Anorthite	0.5 to 13.90	7.3
Total plagioclase	24.58 to 54.81	37.3
Total feldspar	49.04 to 80.13	64.8
Or—plag. ratio	1:0.76 to 1:8.5	1:1.3

Only one (Bethel, Vermont) of the 21 analyses shows orthoclase to be less than 20 per cent. (5.56 per cent.); in the remaining 20 this constituent ranges from 20.02 to 34.5 per cent.

Examined as to the ratio of orthoclase to plagioclase, 3 have a ratio of 1:<1, ranging from 1:0.76 to 1:0.9, essentially a 1:1 ratio; 11 have a ratio of 1:1; 5, a ratio of 1:<2; and the remaining 2, have a ratio of 1:2.1 and 1:8.5. With the exception of the last two, it will be observed that the New England granites represented by the 21 analyses are, according to the old system of classification, quartz monzonites.

The granites of Sprucehead, Knox county, and Norridgewock, Maine[1]; of Milford and Auburn, New Hampshire[2]; of Bethel, Derby, Dummerston, Groton, Hardwick, Kirby, Randolph, Rochester, Ryegate, and Topsham, Vermont[3], and of Westerly (except "Westerly red") and Niantic, Rhode Island[4], have been described recently by Dale as quartz monzonites.

The position of only 8 of the New England granites has been computed in the quantitative system of classification of igneous rocks. This number is probably sufficient to indicate the general range in position of the granites. These may be tabulated as follows:[5]

[1]Dale, T. N. U. S. Geological Survey, Bull. No. 313, 1907, p. 25.

[2]Idem, Bull. No. 354, 1908, p. 211.

[3]Idem, Bull. No. 404, 1909, p. 120.

[4]Idem, Bull. No. 354, 1908, pp. 188-210.

[5]Washington, H. S. U. S. Geol. Survey, Professional Paper No. 14, 1909.

Granites of New England.

Locality.	Symbol.	Name.
Florence, Mass.	I.4.2.4	Lassenose
Quincy, Mass.	I.4.1.3	Liparose
Rockport, Mass.	I.4.1.3	Liparose
Mt. Ascutney, Vt.	I.4.1.3	Liparose
Near East Clarendon, Vt.	I.4.3.4	Yellowstonose
Millstone Point, Conn.	I.4.2.3	Toscanose
Stony Creek, Conn.	I.4.1.3	Liparose
Conanicut Island, R. I.	I.4.2.3	Toscanose

Of the 21 granites represented, 14 are sodipotassic, 3 (Blue Hill, Maine; Barre and Woodbury, Vermont) are between sodipotassic and dosodic, 2 (gray and red Westerly, Rhode Island) are dopotassic, and 1 (Bethel, Vermont) is persodic.

GRANITES OF NEW JERSEY.

A recent report of the "Building Stones of New Jersey,"[1] by Lewis, affords definite valuable data on the granites of an area lying between that part of the Appalachian region which forms the basis of this paper and the New England region. For convenience of comparison the data relating to New Jersey granites are briefly presented in summary below.

Most of toe New Jersey granites described in the above report are either hornblende-, augite-, or bornblende-augite-bearing rocks. Biotite-, occasionally muscovite-, and epidote-bearing varieties occur.

Chemical composition.—The table of partial analyses below shows the range in silica, lime, and alkalies in the New Jersey rocks.

N. Table of partial analyses of New Jersey granites.[2]

Constituents.	I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	Average.
SiO_2	75.56	75.17	75.15	75.02	74.70	74.36	71.91	69.48	68.60	73.33
CaO	0.84	1.15	0.92	0.88	1.70	0.82	0.70	3.45	1.46	1.32
Na_2O	2.35	3.07	3.60	3.36	4.90	3.44	2.61	4.59	4.82	3.69
K_2O	5.93	4.62	4.71	4.74	2.62	3.76	8.60	1.18	3.52	4.41

[1]Lewis, J. Volney. Building Stones of New Jersey. Annual Report of the State Geologist for the Year 1908. Geological Survey of New Jersey, 1909, pp. 62-81.

[2]Idem, 1909, pp. 62-81.

I. Pinkish granite-gneiss at Charlotteburg, p. 68.
II. Gray granite-gneiss, Kice's quarries, German Valley, p. 77.
III. Gray granite-gneiss, 2 miles north of Waterloo, p. 73.
IV. Gray granite-gneiss, Allen quarry, Waterloo, p. 72.
V. White granite-gneiss, quarry one mile south of Cranberry Lake, p. 74.
VI. Gray granite-gneiss, Di Laura's quarry, near Haskell, p. 67.
VII. Pink granite (pegmatite), quarry at Pompton Junction, p. 65.
VIII. Dark gray gneiss, Malley's quarry near Morris Plains, p. 70.
IX. Gray gneiss, Kice's quarry German Valley, p. 76.

With only two exceptions (I and VII) Na_2O molecularly exceeds K_2O; in II, III, IV, and VI, Na_2O approximately equals K_2O; and in V, VIII, and IX, Na_2O is greatly in excess of K_2O. Assigning potash to orthoclase, and the soda and lime to albite and anorthite, respectively, the analyses show the following composition of the feldspars:

O. Table of feldspar composition of New Jersey granites.

(Numbers correspond to table N.)

	I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.
Orthoclase	35.03	27.24	27.80	27.80	15.57	22.24	51.15	7.23	20.57
Albite	19.91	26.20	30.39	28.30	41.40	28.82	22.01	38.78	40.87
Anorthite	4.17	5.56	4.45	4.45	8.34	4.17	3.61	16.96	7.23
Total plagioclase	24.08	31.76	34.84	32.75	49.74	32.99	25.62	55.74	48.10
Total feldspar	59.11	59.00	62.64	60.55	67.31	55.23	76.77	62.97	68.67
Ab_nAn_m	5:1	8:1	7:1	6:1	5:1	7:1	6:1	2:1	6:1
Or—plag. ratio	1:0.7	1:1.16	1:1.25	1:1.17	1:3.2	1:1.48	1:0.5	1:7.7	1:2.3

The plagioclase molecules as calculated range from a calcareous oligoclase (Ab_2An_1) to acid oligoclase-albite (Ab_8An_1) with an average of basic oligoclase-albite (Ab_6An_1). The orthoclase-plagioclase ratio and the orthoclase molecules of I, II, III, IV, and VI, correspond to quartz monzonite. In IX plagioclase is more than double potassic feldspar, but the acid character of the former and the amount of the latter are more characteristic of quartz monzonites and granites than of quartz diorites. V and VIII show a large excess of plagioclase over potassic feldspar, and both are below (VIII markedly below) the minimum amount of potassic feldspar for quartz monzonites. VIII differs from V very markedly in excessively low orthoclase, very basic plagioclase, and a very large excess of plagioclase over orthoclase, properties more characteristic of quartz diorites than of quartz monzonites and granites. VII is especially noteworthy for its high orthoclase, being double that of plagioclase, and the acid character of the latter.

Under the old system of classification, following Lindgren[1], it is a granite, and the only one of the nine that would be so classified.

Lewis[2] has computed the position of the rocks corresponding to the analyses in table N in the quantitative classification of igneous rocks. The results may be tabulated as follows:

Granites of New Jersey.

Number (Corresponds to Nos. in tables N and O).	Symbol.	Name.
I	I.3.1.23	Alaskose
II	I.3.2.3	Tehamose
III	I.4.2.3	Toscanose
IV	I.4.2.3	Toscanose
V	I.4.2.4	Lassenose
VI	I.3.1.3	Alaskose
VII	I.4.1.2	Omeose
VIII	1.4.3.4	Yellowstonose
IX	I.4.2.4	Lassenose

Of these II, III, IV, and VI are sodipotassic; V, VIII, and IX are dosodic; I is between dopotassic and sodipotassic; and VII is dopotassic.

GRANITES OF THE WESTERN UNITED STATES.

Thirty analyses[3] of granites from the western United States, California (7), Colorado (6), Idaho (2), Michigan (3), Minnesota (3), Missouri (2), Montana (4), and Wisconsin (3), show the following ranges and average in silica, lime, soda, and potash:

Granites of Western United States.

Constituents.	Limits of Variation.	Average.
SiO_2	64.05 to 77.68	71.71
CaO	0.12 to 4.30	1.80
Na_2O	1.76 to 6.42	3.58
K_2O	1.66 to 7.99	4.19

These analyses correspond to the following ranges and average in normative feldspar:

[1] Lindgren, W. Op. cit., p. 279.

[2] Lewis, J. Volney. Op. cit. 1909, pp. 62-81.

[3] Clarke, F. W. Analyses of Rocks. Bull. U. S. Geol. Survey, No. 228, 1904, p. 375. Washington, H. S. Professional Paper No. 14, U. S. Geol. Survey, 1903, p. 495.

Feldspar composition of Western United States granites.

	Limits of Variation.	Average.
Orthoclase	10.0 to 47.3	25.7
Albite	14.7 to 54.0	30.0
Anorthite	0.6 to 23.6	7.9
Soda-lime feldspar	17.5 to 60.2	37.9
Total feldspar	46.8 to 78.4	63.6
Orthoclase—plagioclase ratio	1:0.4+ to 1:5 +	

Seven of the 30 analyses show orthoclase to range from 10 to 17.8 per cent., the normal range in this constituent for granodiorite.[1] In the remaining 23 analyses the range in orthoclase is from 20 to 47.3. Only one of the rocks contains more than 40 per cent. of orthoclase, and according to Lindgren[2] is, under the old system of classification, the only one of the 30 represented that would fall within the limits of a true granite.

When the orthoclase-plagioclase ratio, computed from the normative feldspar for each of the rocks represented by the 30 analyses, is considered, the results are further emphasized and are of considerable interest. Of the 30 analyses, 6 show the ratio of orthoclase to plagioclase to be less than 1:1, ranging from 1:0.45 to 1:0.97; three have a ratio not exceeding 1:0.5, the remaining three being approximately 1:1. Eleven show a ratio greater than 1:1 and not exceeding 1:1.5, six, less than 1:2 and greater than 1:1.5, and the remaining eight range from 1:2+ to 1:5+.

Under the new quantitative classification of igneous rocks[3], the rocks represented by the 30 analyses above[4] correspond to the names found under the subrangs magdeburgose (2), alaskose (4), alsbachose (1), omeose (1), liparose (6), toscanose (8), lassenose (5), and yellowstonose (3). They fall in the class persalane, in the orders columbare and brittanare, chiefly the latter, with one in the order belgare. Although considered to contain orthoclase, as the dominant feldspar, only 2 are prepotassic and 1 dopotassic, 17 being sodipotassic and 10 dosodic.

Comparing the results of the old and new systems a fairly general agreement for the rocks will be observed. The 3 analyses having a ratio not exceeding 1:0.5 are prepotassic (1) and dopotassic (2), containing 47.3,

[1]Lindgren, W. Op. cit., p. 279.

[2]Idem, p. 279.

[3]Cross, Iddings, Pirsson, and Washington. Quantitative Classification of Igneous Rocks, Chicago, 1903.

[4]Washington, H. S. Chemical Analyses of Igneous Rocks. Published from 1884 to 1900 with a critical discussion of the character and use of analyses, U. S. Geol. Survey, Prof. Paper No. 14.

38.4, and 38.9 per cents., of potash respectively, and are granites. Nineteen have a ratio not exceeding 1:2, 17 of which are classified as sodipotassic and are quartz monzonites; the remaining 8 have a ratio exceeding 1:2, are dosodic, and are granodiorites.

QUARTZ DIORITES.

Sixteen[1] quartz diorites from the western United States, showing the range and average in silica, lime, and alkalies, and calculated range and average in feldspar composition, in the table below, have been selected for comparison. Their position in the quantitative classification has been computed[2]. They correspond to the subrang names, adamellose (1), harzose (2), tonalose (7), yellowstonose (4), andose (1), and lassenose (1). Thirteen of these are dosodic, and only three, adamellose (1), harzose (2), are sodipotassic. The range in the percentage of quartz is very wide, from 2.5 in andose (dosodic) to 27.1 in lassenose (dosodic), with a total average of 15.6.

Quartz diorites of Western United States.[3]

Partial Analyses.			Feldspar Composition.		
	Limits of Variation.	Average.		Limits of Variation.	Average.
SiO_2	53.48 to 70.36	62.12	Orthoclase	7.2 to 23.9	13.2
CaO	3.18 to 7.94	5.18	Albite	19.9 to 41.4	32.2
Na_2O	2.35 to 4.91	3.81	Anorthite	11.7 to 30.3	21.1
K_2O	1.23 to 4.01	2.22	Total plagioclase	47.0 to 64.9	53.3
			Total feldspar	54.4 to 73.2	66.5
			Or-plag. ratio	————	1:4

GRANODIORITES.

Eleven[4] representative granodiorites from the western United States have been selected for comparison, and their position in the quantitative system of classification computed. The range and average in silica, lime, soda, and potash, corresponding to the respective range and average in normative feldspar, are shown in the subjoined table. They correspond to

[1]Clarke, F. W. Bull. U. S. Geol. Survey, No. 228, 1904.
[2]Washington, H. S. Prof. Paper, U. S. Geol. Survey, No. 14, 1903.
[3]Average of 16 analyses: California (7), Montana (9).
[4]Clarke, F. W. Op. cit., 1904.

the subrang names[1], harzose (1), tonalose (5), yellowstonose (2), and lassenose (3). With only one exception (harzose, which is sodipotassic) they are dosodic rocks. The range in quartz is less than for the quartz diorites, being from 11.09 per cent. to 25.3 per cent., with a total average of 20 per cent.

Granodiorites of Western United States.[2]

Partial Analyses.			Feldspar Composition.		
	Limits of Variation.	Average.		Limits of Variation.	Average.
SiO_2	59.48 to 68.65	64.95	Orthoclase	11.1 to 21.7	14.9
CaO	3.07 to 6.50	4.53	Albite	27.8 to 40.9	32.2
Na_2O	3.29 to 4.85	3.86	Anorthite	15.0 to 24.5	18.6
K_2O	1.65 to 3.66	2.48	Total plagioclase	45.1 to 56.2	50.9
			Total feldspar	58.8 to 71.2	65.9
			Or-plag. ratio		1:3.4

SUMMARY.

In the tables below are summarized under Southern granites, New England granites, and Western granites, the range and average in silica, lime, soda, and potash; and the range and average in mineral constitution of the feldspar. The tabulations are strikingly similar for the three sections. Of the four oxides, under the column of "average" for the three sections, SiO_2 which is in largest amount, naturally shows the greatest difference, amounting to only 1.52 per cent. between the minimum (Southern granites) 71.16 per cent. and the maximum (New England granites) 72.68 per cent., a difference which must be considered very small. The average in SiO_2 for the New Jersey granites is a fraction of one per cent. higher than for the New England rocks. The difference in range and average of lime (pp. 37 and 38) and the alkalies, Na_2O and K_2O, for the same areas including New Jersey (p. 31) is scarcely appreciable.

Variation in the mineral constitution of the feldspar for the three sections is equally small. The southern granites show the lowest average in silica and the highest average in total feldspar. The New England granites average highest in silica, and average slightly higher in total feldspar than

[1]Washington, H. S. Op. cit., 1903.

[2]Average of 11 analyses: California (9), Washington (2).

the western granites. Similarly small differences in the averages of the different kinds of feldspar molecules (orthoclase, albite, anorthite, and total plagioclase) for the three sections are indicated. Though the difference is very slight, the Southern granites show the smallest average for orthoclase and the largest for total plagioclase of the three sections. For this reason the average ratio of orthoclase to plagioclase is proportionately larger for the Southern granites than for those of New England and the western United States. These relations between the feldspar of the three sections are represented diagrammatically in figure 3.

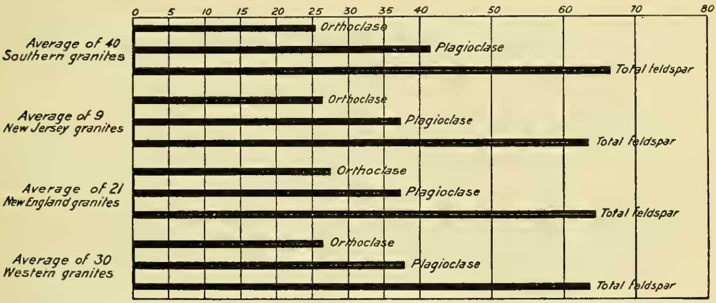

Figure 3.—Diagram illustrating feldspar composition of granites in the United States by sections.

Southern granites.

Partial Analyses.			Feldspar Composition.		
	Limits of Variation.	Average[1]		Limits of Variation.	Average.
SiO_2	66.01 to 77.19	71.16	Orthoclase	8.90 to 33.36	25.4
CaO	0.48 to 4.89	1.89	Albite	16.24 to 42.97	32.7
Na_2O	1.95 to 5.09	3.80	Anorthite	1.67 to 13.90	8.7
K_2O	1.18 to 5.63	4.00	Total plagioclase	22.61 to 52.83	41.4
			Total feldspar	42.07 to 81.06	66.8
			Or-plag. ratio	1:1 to 1:4	1:1.88

[1]Average of 40 analyses distributed as follows: Maryland 9, Virginia 7, North Carolina 7, South Carolina 8, and Georgia 9.

New England granites.

Partial Analyses.			Feldspar Composition.		
	Limits of Variation.	Average		Limits of Variation.	Average.
SiO_2	68.40 to 77.08	72.68	Orthoclase	5.56 to 34.5	27.5
CaO	0.31 to 2.80	1.49	Albite	13.10 to 49.25	30.0
Na_2O	1.58 to 5.85	3.62	Anorthite	0.5 to 13.90	7.3
K_2O	0.95 to 5.78	4.64	Total plagioclase	24.58 to 54.81	37.3
			Total feldspar	49.04 to 80.13	64.8
			Or—plag. ratio	1:0.76 to 1:8.5	1:1.3

[1]Average of 21 analyses.

Western granites.

Partial Analyses.			Feldspar Composition.		
	Limits of Variation.	Average[2]		Limits of Variation.	Average.
SiO_2	64.05 to 77.68	71.71	Orthoclase	10.0 to 47.3	25.7
CaO	0.12 to 4.30	1.80	Albite	14.7 to 54.0	30.0
Na_2O	1.76 to 6.42	3.58	Anorthite	0.6 to 23.6	7.9
K_2O	1.66 to 7.99	4.19	Total plagioclase	17.5 to 60.2	37.9
			Total feldspar	46.8 to 78.4	63.6
			Or—plag. ratio	1:0.4 to 1:1.4	1:1.4

[2]Average of 30 analyses.

For convenience of comparison, partial analyses of three typical quartz monzonites from the western United States are given below:

Partial analyses of quartz monzonites.

Constituents.	I.	II.	III.
SiO_2	68.42	66.83	65.70
CaO	2.60	3.59	2.56
Na_2O	3.23	3.10	3.62
K_2O	4.25	4.46	4.62

I. Idaho-Hailey mine, Hailey, Idaho. 20th An. Rept. U. S. Geol. Survey, 1900, p. 81. Professional Paper No. 14, p. 163.

II. Near San Miguel Peak, Telluride, Colorado. Telluride folio, U. S. Geological Survey, 1899, p. 6. Professional Paper No. 14, p. 165.

III. Nevada Falls Trail, Yosemite Valley, California. Jo. of Geol., 1899, Vol. VII, p. 152. Professional Paper No. 14, p. 167.

The above analyses correspond to normative quartz and feldspar as follows:

Quartz and feldspar composition of quartz monzonites.

	I.	II.	III.
Quartz	25.1	22.2	19.1
Orthoclase	25.6	26.7	27.2
Albite	26.7	26.2	30.4
Anorthite	13.1	14.2	12.1
Total plagioclase	39.8	40.4	42.5
Total feldspar	65.4	67.1	69.7
Ab_nAn^m ratio	2:1	1.8:1	2.5:1
Or—plag. ratio	1:1.55	1:1.1	1:1.56

The position of these three rocks in the quantitative system of classification has been computed by Washington[1], and may be represented in tabular form as follows:

Quartz monzonites of Western United States.

Number.	Class.	Order.	Rang.	Subrang.	Name.
I	I	4	2	3	Toscanose
II	I	4	2	3	Toscanose
III	I	4	2	3	Toscanose

They are sodipotassic, and under the old system of classification they may be regarded on the basis of the potash-soda ratio (1:1) in quartz-feldspar plutonic igneous rocks, as typical quartz monzonites, and were so described by Lindgren, Cross, and Turner.

It is a noteworthy fact that of the large number of analyses of so-called granites, represented in the three areas, only two[2] (western United States 1, New Jersey 1), on the basis of potassic feldspar content, fall under the old system of classification within the limits ascribed by Lindgren for a granite. "The definition of granodiorite would give it, say from 8 per cent. to 20 per cent. orthoclase. In the quartz monzonites I would give this mineral a range from 20 per cent. to 40 per cent., all to an assumed total of 60 per cent. feldspars. The rocks containing more than 40 per cent. orthoclase would then be classed as granites, . . ."[3]

[1]Washington, H. S. Chemical Analyses of Igneous Rocks. Professional Paper, U. S. Geol. Survey, No. 14, 1903.

[2]The maximum percentages of orthoclase computed from the chemical aanlyses of rocks from the various sections are: Three from the Western United States 47.3, 38.9, and 38.4, respectively; one from New Jersey 51.15; one from New England 34.05; and one from the central and southern Appalachians 33.36.

[3]Lindgren, W. Op. cit., p. 279.

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OF THE

PHILOSOPHICAL SOCIETY

Scientific Series, Vol. I, No. 2, pp. 41–44; June, 1910

On Chemical Interaction of Substances in the Solid State

BY

J. W. MALLET

UNIVERSITY OF VIRGINIA
Charlottesville, Virginia, U. S. A.

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SCIENTIFIC SECTION
Vol. I, No. 2, pp. 41–44 June, 1910

ON CHEMICAL INTERACTION OF SUBSTANCES IN THE SOLID STATE.*

BY
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Reference was made to previous investigation in this direction, includin the earlier experiments of Spring, of Roberts-Austen and of Abel, and of those more recently made by Tamman (Chem. Soc. Abstr., 1909, p. 669), Masing (ditto), Cobb (Chem. Soc. Proc., May 20, 1909, p. 165) and Guillet & Griffiths (Comptes rendus, CXLIX, No. 2).

In connection with these experiments attention was drawn to the necessity for taking note of the possible presence of water (or other solvent) in traces, as derived from the atmosphere, and to the possibility of volatilization from the surfaces of solids even at ordinary temperature. As bearing on the question of mobility of the particles of solids themselves notice was taken of the various degrees of evident rigidity or plasticity of substances experimented with, and an interesting paper lately published by Beilby (Roy. Soc. Proc., A. 82, p. 599—and earlier papers in Brit. Assoc. Reports) on "Surface flow in calcite" was referred to.

Some experiments by the author were then described, in which chemical interaction of solids was indicated by change of colour. It was proposed to dry as thoroughly as possible separate parcels of finely pulverized lead nitrate and potassium iodide in a glass tube which should afterwards be sealed at both ends, the contents to be then shaken together, observing whether the yellow colour of lead iodide should appear, and to make a similar experiment with mercuric chloride and potassium iodide, noting the appearance or non-appearance of the scarlet colour of mercuric iodide.

*Read before the Section of Mathematical and Natural Sciences, January 18, 1910.

The materials named underwent a preliminary drying at 100° C., were then rubbed to a fine powder in separate warm porcelain mortars, were again kept at 100° for an hour or two, and were then cautiously introduced from opposite ends into a straight piece of glass tube, of about a centimetre in diameter and 55 or 60 centimetres in length, having a clear interval of about 10 centimetres between the two powders. This tube was in the case of the lead nitrate experiment heated throughout the greater part of its length to 100°, and a slow stream of air was drawn through by means of a jet pump, this air passing first through a train of drying vessels containing in succession sulphuric acid, calcium chloride and phosphorus pentoxide, while a like train of dryers in inverse order was interposed between the tube enclosing the powders and the jet pump. In a first trial of the mercuric chloride it was soon found that at 100° enough vapour was given off from this salt before long to redden perceptibly the surface of the potassium iodide, so that in this case the experiment had to be made at the temperature of the room—about 25° C. In all other respects the drying arrangements were the same for both the lead and mercury experiments. The slow stream of air was kept up in each case for many hours, generally for about 36 hours. At the end of this time the tube was disconnected from the drying trains, its ends instantly closed by corks taken directly from a hot-air drying-bath, and both ends with the least possible delay drawn off and sealed in the blast-lamp flame, slightly slackening the second cork to avoid blowing out of the glass in the final sealing. Great care was taken in handling the tube to keep it in a horizontal position, so that there should be no disturbance of the two parcels of material, and the tubes were laid aside in this position in a locked cupboard for a little more than four months, during which time the atmospheric temperature ranged from about 20° to a little over 32° C.

At the end of this time there was no apparent development of colour, the two parcels of powder in each being quite white as at first. This indicated that change due to volatilization had not occurred. The tubes were then shaken up so as to mix the materials together. On shaking for several minutes a slight reddish tinge became perceptible in the mixture of mercuric chloride and potassium iodide, this colour becoming much more decided on allowing the tube to lie at rest for several hours. No yellow colour was observable in the mixture of lead nitrate and potassium iodide on first shaking, and only after a subsequent rest of several hours could the colour be clearly recognized; after 20 hours the change was very perceptible. In both cases the colours were quite bright after resting for 5 or 6 days, with occasional shakings during this time, but even later than this some increase in depth of colour was probably noticeable after still longer exposure.

When the tubes were first sealed up the powders were thoroughly loose and free from caking together, as could be seen by cautiously revolving each tube about its axis before shaking the powders together, but they became very perceptibly caked after mixture when the tubes had lain at rest for a number of days, and quite vigorous shaking was then necessary to break up again the caked material into loose pulverulent form and allow of further mixture. This tendency to caking was seeemingly more marked in the case of the mercuric than in that of the lead salt.

In other experiments the effect of rapid and violent shaking for as much as an hour was observed in the more speedy increase in depth of colour, and a like effect was obtained by breaking the tube and pressing or rubbing the mixed powder in a dry mortar, with inevitable exposure, of course, to atmospheric moisture, the influence of which was afterwards exaggerated by breathing upon the powder.

In order to test further the extent to which action between the solid materials might be referable to vaporization from the surface of either or both of them, the experiments were repeated in longer tubes by drying as already described, then introducing a parcel of phosphorous pentoxide (lying apart from the other materials), sealing the tube at one end, exhausting from the other with a mercurial pump, sealing up the open end with a fairly good vacuum within, and allowing the tube to lie at rest for several weeks before drawing down the tube between the phosphorus pentoxide and the other materials, sealing up the part of the length containing the latter. No material difference of result was thus obtained.

The general conclusion seems to be justified that the particles of the substances experimented on were, while still in the solid state, brought to a considerable extent within their mutual reach so that chemical interaction took place. The possibility must be admitted that even the means used did not secure *absolute* exclusion of moisture, and the slight differences as to time which were noticed between the production of lead iodide and mercuric iodide suggest the possibility that the greater volatility of mercuric salts as compared with those of lead may not have been entirely without influence. But in the main we seem warranted in referring the effects described to chemical action between solids.

The importance which may perhaps attach to such action in connection with geologic changes, with changes in the materials of engineering and architectural structures, and in general with changes under conditions involving contact between dissimilar solid substances for long periods of time, may well deserve consideration.

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On the Maximum and Minimum Values of a Linear Function of the Radial Co-ordinates of a Point With Respect to a Simplicissimum in Space of n Dimensions.

BY

WILLIAM H. ECHOLS

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ON THE MAXIMUM AND MINIMUM VALUES OF A LINEAR FUNCTION OF THE RADIAL COÖRDINATES OF A POINT WITH RESPECT TO A SIMPLICISSIMUM IN SPACE OF n DIMENSIONS.*

BY
WILLIAM H. ECHOLS.

I.

1. This problem in the particular form, *To find that point the sum of whose distances from the vertices of a triangle is a minimum,* is one of much historic interest. Viviani relates that it was proposed by Fermat to Torricelli and by him handed over as an exercise to Viviani, who gave, in the Appendix to his Treatise *De Maximis et Minimis,* pp. 144, 150 (1659), the following construction for finding the point: Let $A\ B\ C$ be the triangle in which each angle is less than 120°. On $A\ B$ and $A\ C$ describe segments of circles containing angles of 120°. The arcs of these segments intersect in the point required. Viviani's proof that this is the point required is long and tedious. Thomas Simpson in his *Doctrine and Application of Fluxions,* § 36 (1750), gives the following construction for determining the same point: Describe on $B\ C$ a segment of a circle to contain an angle of 120°, and let the whole circle $B\ C\ Q$ be completed. From A to Q the mid-point of the arc $B\ C\ Q$ draw $A\ Q$ intersecting the circumference of the circle in V, which will be the point required. In § 431 Simpson treats the more general problem: Three points being given $A, B, C,$ to find the position of a fourth point P, so that if lines be drawn from thence to the three former, the sum

$$\alpha \,.\, AP + \beta \,.\, BP + \gamma \,.\, CP,$$

where α, β, γ denote any given numbers, shall be a minimum. Both the particular and the more general problem are discussed in *Nova Acta*

*Read before the Mathematical and Natural Science Section, December, 1909.

Academiae . . . Petropolitanae, XI, 235-8 (1798), by Nicolas Fuss in his memoir *De Minimis quibusdam geometricis ope principii statici inventis,* read to the Petersburg Academy of Sciences on 25th February, 1796. In this memoir Fuss gave the values *AP, BP, CP* and the minimum value of their sum for the particular problem in terms of algebraic functions of the sides of *ABC.*

For the foregoing information the writer is indebted to a paper in the Proceedings of the Edinburgh Mathematical Society, Vol. XV, p. 100 (1897), *On the Isogonic Centers of a Triangle,* by Professor J. S. Mackay, in which it is pointed out that the problem of the minimum of the sum of the distances of a point from the vertices of a triangle is closely associated with that of constructing an equilateral triangle of maximum area circumscribing the given triangle $A\ B\ C$. In Gergonne's *Annals de Mathématiques,* I, 384 (1811), the problem: About any given triangle to circumscribe or inscribe the greatest or least triangle similar to a given triangle, was proposed. It was solved by Rochat, Vecten and others in Vol. II, pp. 88-93 (1811 and 1812).

The problem of minimizing the sum

$$r_1 + r_2 + r_3$$

of the distances of a point from the vertices of a triangle $A\ B\ C$ is a favorite one in all the modern treatises on analysis; it is given especially with the view of pointing out the singularity which occurs when an angle in the triangle is equal to 120°. It is discussed by Humbert, *Cours d'Analyse,* I, p. 193 (1903); by Goursat, *Cours d'Analyse,* I, p. 144 (1902); by Vallée Poussin, *Cours d'Analyse,* I, p. 119 (1903). Joachimsthal in his *Diff. and Int. Calculus,* p. 299, devotes a considerable space in the appendix to a geometrical consideration of the problem. In all the treatments of this problem the distances r_1, r_2, r_3 of a point P from the points A, B, C are considered as being absolute values and presenting no ambiguities of sign, a matter to be noticed subsequently.

2. The writer has been unable to find any systematic and conclusive analytical treatment of the problem of minimizing

$$\alpha r_1 + \beta r_2 + \gamma r_3,$$

where α, β, γ are arbitrary constants.

The determination of a critical point in this problem is closely associated with the problems: (a). To construct a triangle of given form whose sides pass through the vertices of a given triangle $A\ B\ C$ and which shall have a maximum area. (b). To construct a point in a plane at which the sides of a given triangle $A\ B\ C$ subtend given angles. This is the familiar coast survey problem of the engineer for mapping harbor soundings. Also it is the problem of orientation by resection of the plane table in terrestrial surveying. The engineer solves this problem without difficulty both graphically and trigonometrically. The well-known constructions for the plane table by Netto and by Bessel can be found in the U. S. Coast Survey papers on the Plane Table. The ordinary trigonometric solution of the engineer adapted to logarithmic computation is very simple and as follows: Let $A\ B\ C$ be the known triangle whose sides a, b subtend known angles α, β respectively at a point P in the plane. Let $x = \angle P\ A\ C$ and $y = \angle P\ B\ C$ be angles to be determined. In any case either $x+y$ or $x-y$ is known. Then

$$PA = b\,\frac{\sin(x+\beta)}{\sin\beta}, \quad PB = a\,\frac{\sin(y+\alpha)}{\sin\alpha}$$

$$PC = b\,\frac{\sin x}{\sin\beta} = a\,\frac{\sin y}{\sin\alpha}.$$

$$\therefore\ \frac{\sin x}{\sin y} = \frac{a\sin\alpha}{b\sin\beta} = \tan k,\ (\text{say}),$$

is known, and by an easy deduction

$$\tan\tfrac{1}{2}(x-y) = \tan\tfrac{1}{2}(x+y).\tan(k-45^\circ),$$

determines x and y and solves the problem. (c). Three concurrent forces (vectors) are of constant magnitudes, their lines of action pass respectively through three fixed points A, B, C. Determine the position of equilibrium. (d). In connexion with the above the problem is directly associated with the reciprocal figures of Graphical Statics.

The solution of the problem gives algebraic solutions of these and similar problems.

II.

3. We now take up the problem proposed and solve it, first, for the plane, in detail along the lines of least resistance, subsequently giving the method which admits of direct generalization to the higher spaces.

Let $A\ B\ C$ be a given triangle and P a point in its plane at distances r_1, r_2, r_3 from A, B, C. We seek the maximum and minimum values of the function

$$s = Lr_1 + Mr_2 + Nr_3, \tag{1}$$

where L, M, N are arbitrary real constants, and r_1, r_2, r_3 are absolute numbers called the radial coördinates of the point P with respect to the fixed triangle $A\ B\ C$.

The problem is simplified by considering L, M, N to either be all positive or only one of them negative. For if they be all negative it amounts to considering $-s$ when they are all positive. If any two of them, say L, M be negative the case is in like manner reduced to the investigation of

$$-(Lr_1 + Mr_2 - Nr_3).$$

Moreover we may always take L, M, N so that their sum shall be unity, for if not so then on dividing both sides of (1) by the sum of L, M, N the case is reduced to this condition, except when the sum of L, M, N may be zero, a condition to be considered subsequently. We therefore consider

$$s = \alpha r_1 + \beta r_2 + \gamma r_3, \tag{2}$$

wherein $\alpha + \beta + \gamma = 1$, and only one of them may be negative. We may consider α, β, γ to be the areal coördinates of a point Q with reference to the triangle $A\ B\ C$.

4. *The necessary conditions.*—With reference to an arbitrarily chosen system of orthogonal cartesian coördinates, let P be x, y and A, B, C be respectively x_1, y_1; x_2, y_2; x_3, y_3. The sign of the radical being taken positive

$$r_1 = \sqrt{(x - x_1)^2 + (y - y_1)^2}, \tag{3}$$

where the equation of $P\ A$ is

$$\frac{x - x_1}{l_1} = \frac{y - y_1}{m_1} = r_1, \tag{4}$$

l_1, m_1 being the direction cosines of $P\ A$. We have like equations for r_2, r_3 on changing the subscripts.

Equating to zero the first partial derivatives of s, in (2), with respect to x and to y, there result the necessary conditions for an ordinary maximum or minimum of s

$$\left.\begin{aligned}\frac{\partial s}{\partial x} &= \alpha l_1 + \beta l_2 + \gamma l_3 = 0,\\ \frac{\partial s}{\partial y} &= \alpha m_1 + \beta m^2 + \gamma m_3 = 0.\end{aligned}\right\} \tag{5}$$

Transposing the last terms to the right, squaring and adding these results

$$\cos\,(r_1 r_2) = -\frac{\alpha^2 + \beta^2 - \gamma^2}{2\alpha\beta}, \tag{6}$$

and in like manner similar values for the cosines of the angles $(r_2\, r_3)$, $(r_3 r_1)$. Eliminating in turn l_2, m_2 then l_3, m_3 there result

$$\frac{\alpha}{\sin\,(r_2\, r_3)} = \frac{\beta}{\sin\,(r_3\, r_1)} = \frac{\gamma}{\sin\,(r_1\, r_2)}. \tag{7}$$

Also directly from (6)

$$\sin^2(r_1 r_2) = \frac{1}{4\alpha^2\beta^2}\,(2\alpha^2\beta^2 + 2\beta^2\gamma^2 + 2\gamma^2\alpha^2 - \alpha^4 - \beta^4 - \gamma^4),$$

$$= \frac{1}{4\alpha^2\beta^2}(\alpha + \beta + \gamma)\ (\alpha + \beta - \gamma)\,(\alpha - \beta + \gamma)\ (-\alpha + \beta + \gamma). \tag{8}$$

We find like expressions for the sines of the other two angles.

These conditions show that it is necessary that the absolute values a', b', c' of α, β, γ must be the sides of a triangle whose sides are parallel to $P\,A$, $P\,B$, $P\,C$. Also (5) shows that segments α, β, γ laid off from P along $P\ A$, $P\ B$, $P\ C$ are in equilibrium, each being directed toward a vertex when positive and away from it when negative.

The equation in areal coördinates

$$(x + y + z)\ \ (x + y\ - z)\ (x - y + z)\ (-x + y + z) = 0, \tag{9}$$

represents four straight lines, the line at infinity and three straight lines passing through the mid points D, E, F of the sides of the triangle $A\ B\ C$. The expression on the left of (9) is positive when the point x, y, z is inside $D\ E\ F$ or in one of the vertical angles D, E, F; it vanishes at any point on the straight lines bounding this region, and is negative at all other points in the plane. Therefore (8) shows that the point Q (α, β, γ) must be in $D\,E\,F$ or one of its vertical angles. Hence α, β, γ must be all positive or only one can be negative.

The triangle $A'\ B'\ C'$ whose sides are a', b', c', the absolute values of α, β, γ, we shall call the reciprocal triangle of $A\ B\ C$.

5. If the triangle $A\ B\ C$ be lettered in the positive direction so that the area bounded is kept on the left in going around it, then the signs of the angles $A\ P\ B$, $B\ P\ C$, $C\ P\ A$ subtended by the sides take care of themselves, counter clockwise being the positive direction. Let x, y, z be the coördinates (areal) of any point P, whose radial coördinates are r_1, r_2, r_3, then

$$\frac{\sin A\,P\,B}{z\,r_3} = \frac{\sin B\,P\,C}{x\,r_1} = \frac{\sin C\,P\,A}{y\,r_2} = \frac{2\Delta}{r_1 r_2 r_3}. \tag{10}$$

The angles have the same signs as those of the corresponding coördinates of P.

Equations (10) show that the segments $x\,r_1$, $y\,r_2$, $z\,r_3$ laid off at P in the directions $P\,A$, $P\,B$, $P\,C$, each toward the vertex if positive and away from it if negative, are in equilibrium, and their absolute values construct a triangle similar to $A'B'C'$. Now if P be the critical point satisfying the necessary conditions of the problem it follows that

$$\frac{xr_1}{\alpha} = \frac{yr_2}{\beta} = \frac{zr_3}{\gamma}. \tag{11}$$

Since the directions of the segments xr_1, yr_2, zr_3 and α, β, γ equilibrated at P must be the same, the signs of the coördinates of P must be the same as the signs of the corresponding coördinates of Q. Multiply each term of the first ratio in (11) by r_1, the second by r_2, the third by r_3; then multiply in the same way by α, β, γ respectively. An easy composition of the resulting ratios gives each member of (11) equal to

$$\frac{\Sigma xyc^2}{s} = \frac{s}{\Sigma xyc'^2}\,xyz, \tag{12}$$

in virtue of the identical relation

$$x\,r_1^{\,2} + y\,r_2^{\,2} + z\,r_3^{\,2} = x\,y\,c^2 + x\,z\,b^2 + y\,z\,a^2, \tag{13}$$

which exists between the radial and areal coördinates of any point. Hence from (12) the value of s at the critical point P is given by

$$s^2 = \frac{\Sigma xyc^2.\ \Sigma xyc'^2}{xyz}. \tag{14}$$

The right member of (13), $\Sigma\,x\,y\,c^2$, we represent by σ; the power of P with respect to the circumcircle of $A\ B\ C$ is $-\sigma$. We shall represent the similar function $\Sigma\,x\,y\,c'^2$ by σ'.

6. The relations between the angles involved in the previous discussion are as follows: Let Θ, Φ, Ψ be the absolute angles subtended at P by the sides a, b, c respectively. Then when Q and P are inside $A\,B\,C$,

$$\begin{aligned}(r_1\,r_2) &= A\,P\,B = \Psi = \pi - C',\\ (r_2\,r_3) &= B\,P\,C = \Theta = \pi - A',\\ (r_3\,r_1) &= C\,P\,A = \Phi = \pi - B'.\end{aligned}$$

When P is outside and in angle C

$$\begin{aligned}(r_1\,r_2) &= A\,P\,B = -\Psi = C' - \pi,\\ (r_2\,r_3) &= B\,P\,C = \quad \Theta = A',\\ (r_3\,r_1) &= C\,P\,A = \quad \Phi = B'.\end{aligned}$$

When P is in vertical angle C

$$\begin{aligned}(r_1\,r_2) &= A\,P\,B = \quad \Psi = \pi - C',\\ (r_2\,r_3) &= B\,P\,C = -\Theta = \quad -A',\\ (r_3\,r_1) &= C\,P\,A = -\Phi = \quad -B'.\end{aligned}$$

Similar results hold when P is outside and in angles B, A or their verticals. Hence in all cases

$$\cot A\,P\,B = \mp \cot C', \cot B\,P\,C = \mp \cot A', \cot C\,P\,A = \mp \cot B', \quad (15)$$

the upper signs being taken when P is inside $A\ B\ C$ and the lower when outside.

7. Directly from the figure

$$\left.\begin{aligned} r_1^2 + r_2^2 - 2\,r_1\,r_2 \cos A\,P\,B &= c^2,\\ r_2^2 + r_3^2 - 2\,r_2\,r_3 \cos B\,P\,C &= a^2,\\ r_3^2 + r_1^2 - 2\,r_3\,r_1 \cos C\,P\,A &= b^2.\end{aligned}\right\} \quad (16)$$

There are also the known relations

$$\left.\begin{aligned} r_1^2 &= c^2\,y + b^2\,z - \Sigma\,x\,y\,c^2,\\ r_2^2 &= c^2\,x + a^2\,z - \Sigma\,x\,y\,c^2,\\ r_3^2 &= b^2\,x + a^2\,y - \Sigma\,x\,y\,c^2,\end{aligned}\right\} \quad (17)$$

expressing the squares of the distances of a point from the corners of the triangle of reference $A\ B\ C$ in terms of the areal coördinates of the point.

Equations (16) are the equations in radial coördinates of the circles $B\,P\,C$, $C\,P\,B$, $B\,P\,A$. Eliminating the radial coördinates between (10), (16), (17) there result the equations to these circles in areal coördinates. Thus for example the equation to the second circle in (16) is

$$0 = \tfrac{1}{2}(c^2 + b^2 - a^2 - 4\,\Delta \cot B\,P\,C)\; x - \Sigma\, x\, y\, c^2, \tag{18}$$

cyclic interchanges give the other two.

The coefficient of x in (18) is known to be the power of A with respect to this circle. Representing this power by p_1, the area of $A'B'C'$ by δ,

$$\begin{aligned} p_1 &= \tfrac{1}{2}(c^2 + b^2 - a^2 - 4\,\Delta \cot B\,P\,C), \\ &= \tfrac{1}{2}(c^2 + b^2 - a^2 \pm 4\,\Delta \cot A'), \\ &= \frac{(c^2 + b^2 - a^2)\delta \pm (c'^2 + b'^2 - a'^2)\Delta}{2\,\delta}, \\ &= 2\,\Delta\,\frac{\sin\,(A' \pm A)}{\sin A' \sin A}, \end{aligned} \tag{19}$$

The upper sign taken when Q is inside $A\ B\ C$, the lower when outside. Cyclic changes of letters give p_2, p_3 the powers of B, C with respect to circles $C\ P\ A$, $A\ P\ B$ respectively.

The radical axes of these three circles are

$$x\,p_1 = y\,p_2 = z\,p_3,\ (= \Sigma\, x\, y\, c^2), \tag{20}$$

and they meet in the point P. Hence the areal coördinates x, y, z of the critical point P are known, and are

$$\frac{\sin A' \sin A}{\sin(A' \pm A)} : \frac{\sin B' \sin B}{\sin(B' \pm B)} : \frac{\sin C' \sin C}{\sin(C' \pm C)}, \tag{21}$$

the upper signs placing P when Q is inside $A\,B\,C$ or α, β, γ are positive, the lower sign when Q is outside $A\,B\,C$ or when one of α, β, γ is negative.

Dividing (20) through by s, each ratio of (20) is equal to each of (11), and therefore

$$\frac{r_1}{\alpha p_1} = \frac{r_2}{\beta p_2} = \frac{r_3}{\gamma p_3} = \frac{1}{s}, \tag{22}$$

whence

$$s^2 = \alpha^2 p_1 + \beta^2 p_2 + \gamma^2 p_3, \tag{23}$$

$$= \tfrac{1}{2}(b^2 + c^2 - a^2)a'^2 + \tfrac{1}{2}(a^2 + c^2 - b^2)b'^2 + \tfrac{1}{2}(a^2 + b^2 - c^2)c'^2 \pm 8\,\delta\,\Delta,$$

the positive sign giving the value of s when Q is inside, the negative when outside $A\ B\ C$. In the above expression for s^2 the numbers a, b, c and a', b', c' are respectively interchangeable. We may observe that the expression

$$\tfrac{1}{2}(b^2 + c^2 - a^2)x^2 + \tfrac{1}{2}(a^2 + c^2 - b^2)y^2 + \tfrac{1}{2}(a^2 + b^2 - c^2)z^2, \tag{24}$$

is equal to the power of the point x, y, z with respect to the self-polar circle of $A\ B\ C$. It will be observed that $s = 0$ when Q is at either of the excenters of $A\,B\,C$, for then $\delta = \Delta$ and the power of each of these points with respect to the self-polar circle is $8\,\Delta^2$. Hence the circle passing through the excenters of $A\,B\,C$ is concentric with the self-polar circle having the ortho-center of $A\ B\ C$ for center. At any point Q on the circumference of this circle, which satisfies the necessary conditions for ordinary maximum, the value of s is zero; it will develop subsequently, however, that only the excenters on this circle satisfy the ordinary conditions.

The values of the radial coördinates of P can now be determined from

$$\begin{aligned} r_1 &= \frac{p_1}{s}\,\alpha, \\ &= \frac{(c^2 + b^2 - a^2)\delta \pm (c'^2 + b'^2 - a'^2)\Delta}{2\delta s}\,\alpha, \qquad (25) \\ &= \frac{2\,\Delta}{s}\,\frac{\sin(A' \pm A)}{\sin A' \sin A}\,\alpha, \end{aligned}$$

interchange of letters giving r_2 and r_3.

The equations thus determined furnish the radial and the areal coördinates of the critical point P and the value of s there for any given real numbers L, M, N in terms of α, β, γ. We now examine the sufficient conditions with the view of determining whether s is a maximum or a minimum.

8. *The sufficient condition.*—The second partial derivatives of s, in (2), with respect to x, y, x and y, are found to be respectively

$$\left.\begin{aligned} \frac{\partial^2 s}{\partial x^2} &= \frac{\alpha}{r_1}\,m_1^2 + \frac{\beta}{r_2}\,m_2^2 + \frac{\gamma}{r_3}\,m_3^2, \\ \frac{\partial y^2}{\partial y^2} &= \frac{r_1}{r_1}\,l_1^2 + \frac{r_2}{r_2}\,l_2^2 + \frac{r_3}{r_3}\,l_3^2, \\ \frac{\partial^2 s}{\partial x \partial y} &= -\frac{\alpha}{r_1}\,l_1 m_1 - \frac{\beta}{r_2}\,l_2 m_2 - \frac{\gamma}{r_3}\,l_3 m_3. \end{aligned}\right\} \qquad (26)$$

On substitution and proper reduction the discriminant

$$\frac{\partial^2 s}{\partial x^2}\frac{\partial^2 s}{\partial y^2} - \left(\frac{\partial^2 s}{\partial x\,\partial y}\right)^2$$

can be written

$$\frac{\alpha\beta}{r_1 r_2}\begin{vmatrix} l_1 & l_2 \\ m_1 & m_2 \end{vmatrix}^2 + \frac{\beta\gamma}{r_2 r_3}\begin{vmatrix} l_2 & l_3 \\ m_2 & m_3 \end{vmatrix}^2 + \frac{\gamma\alpha}{r_3 r_1}\begin{vmatrix} l_3 & l_1 \\ m_3 & m_1 \end{vmatrix}^2,$$

or

$$\sum \frac{\alpha\beta}{r_1 r_2}\sin^2(r_1 r_2). \tag{27}$$

Using (11) and (12) the discriminant may be written

$$\frac{4\delta^2 s^4}{\alpha^2\beta^2\gamma^2\sigma^2}\,\frac{xyz}{\sigma}, \tag{28}$$

otherwise as

$$\frac{4\delta^2}{r_1 r_2 r_3}\,\frac{s}{\alpha\beta\gamma}.$$

When the discriminant is negative s is a maximum, when positive a minimum.

9. We now examine more closely into the positions of P with respect to Q, considering first the case when Q is inside $A\,B\,C$. Here α, β, γ are all positive and Q is confined to the triangle $D\ E\ F$, or on a side or at a vertex of this triangle. Since x, y, z must also be positive or rather cannot be negative P is confined to the interior of $A\ B\ C$ or its boundary. The value of s is always positive when α, β, γ are positive and (29) shows the discriminant to be positive and s a minimum.

Examination of (21) and (25) show that if the position of Q is such that the sum of the corresponding angles of $A'B'C'$ and $A\ B\ C$ is less than π then x, y, z are definite positive numbers, as are also r_1, r_2, r_3, and s has a minimum value at this point P inside $A\ B\ C$.

10. If, however, a pair of these corresponding angles be supplemental, for example

$$A' + A = \pi,$$

then $r_1=0$, $r_2=c$, $r_3=b$, also $x=1$, $y=0$, $z=0$. Hence P is at A. The direction cosines of r_1 in (4) and (5) are indeterminate. The partial derivatives in (5) and (26) are indeterminate. The values of these derivatives depend, therefore, on the path of approach of P to A and the direction in which it attains A. We shall presently show that s is a minimum under the circumstances, but first seek the position or positions of Q when P is at A. Since $\cos A = -\cos A'$ we have

$$\frac{\beta^2 + \gamma^2 - a^2}{2\beta\gamma} = -\cos A.$$

The locus in areal coördinates

$$y^2 + z^2 - x^2 + 2yz \cos A = 0 \tag{29}$$

or as it can be written

$$y + z - x - 2yz(1 - \cos A) = 0$$

is a hyperbola, one branch of which passes through the points E and F. It does not meet $C\ B$. The coördinates of its center, which is on the median through A, are given by

$$-\frac{x}{1 + \cos A} = y = z,$$

the actual value of x being $-\cot^2 \frac{1}{2} A$. To construct the center, draw the diameter of the circumcircle of $A\ B\ C$ through the mid point of side a; join A to J the end of the diameter on the same side of a as A; through the other end of the diameter draw a parallel to $A\ J$ cutting the median in the center of the hyperbola. The lines $E\ D$ and $F\ D$, whose equations are

$$y + x - z = 0,$$
$$x + z - y = 0,$$

meet the hyperbola each in only one finite point, E and F respectively; the asymptotes are therefore parallel to these lines and pass through the previously constructed center. Hence the branch of the hyperbola (29) passing through E and F lies wholly within the region for which δ^2 is positive, and therefore that arc of it interior to $D\,E\,F$ contains all the points α, β, γ at which A' is the supplement of A. In like manner the arcs of the hyperbolae

$$\left.\begin{aligned} x^2 + z^2 - y^2 + 2\,x\,z \cos B = 0, \\ x^2 + y^2 - z^2 + 2\,x\,y \cos C = 0, \end{aligned}\right\}$$

interior to $D\,E\,F$ contain all points α, β, γ for which $B' + B = \pi$ and $C' + C = \pi$ respectively.

Returning to the case when α, β, γ is on the arc of (31) and $A' + A = \pi$, when the critical point is at A, we examine the function s there. Let p be the small distance of a point P' from P.

Indicating the angles which $P'P = p$ makes with the radial coördinates r_1, r_2, r_3 of P, by $(p\,r_1)$, $(p\,r_2)$, $(p\,r_3)$ respectively, we expand the value of s' at P' in terms of s at P and in powers of p by Taylor's theorem and find

$$s' = s + p[\alpha \cos(p\,r_1) + \beta \cos(p\,r_2) + \gamma \cos(p\,r_3)]$$
$$+ \tfrac{1}{2} p^2 \left[\frac{\alpha}{r_1} \sin^2(p\,r_1) + \frac{\beta}{r_2} \sin^2(p\,r_2) + \frac{\gamma}{r_3} \sin^2(p\,r_3)\right] + R. \tag{30}$$

A necessary condition for maximum or minimum at an ordinary point P is, as found before,

$$\alpha \cos(p\,r_1) + \beta \cos(p\,r_2) + \gamma \cos(p\,r_3) = 0,$$

for all directions of p. A sufficient condition is that

$$\frac{\alpha}{r_1} \sin^2(p\,r_1) + \frac{\beta}{r_2} \sin^2(p\,r_2) + \frac{\gamma}{r_3} \sin^2(p\,r_3),$$

shall preserve its sign for all directions of p. If three segments α, β, γ are in equilibrium at a point the sum of the projections of any two on any line is at most equal to the third, as is easily demonstrated.

In particular let P be at A, then the value s' of s in the neighborhood of A is

$$s' = s + p[\alpha + \beta \cos(c\,p) + \gamma \cos(b\,p)]$$
$$+ \tfrac{1}{2} p^2 \left[\frac{\beta}{c} \sin^2(c\,p) + \frac{\gamma}{b} \sin^2(p\,b)\right] + R, \tag{31}$$

by taking, in (30), P' on $A\,P$ then $\sin(p\,r_1) = 0$, $\cos(p\,r_1) = 1$, then moving P to A. Since $(c\,p) + (b\,p) = A$, the greatest value of

$$\beta \cos(c\,p) + \gamma \cos(b\,p) \tag{32}$$

occurs when

$$\beta \sin(c\,p) - \gamma \sin(b\,p) = 0,$$

that is when p is along the diagonal of a parallelogram whose vertex is A and whose sides are β and γ laid off along $A\,B$, $A\,C$ respectively. In fact if v be the critical value of (32), on squaring and adding

$$v^2 = \beta^2 + \gamma^2 + 2\,\beta\,\gamma \cos A, \tag{33}$$

but

$$\alpha^{2\prime} = \beta^2 + \gamma^2 + 2\,\beta\,\gamma \cos A, \tag{34}$$

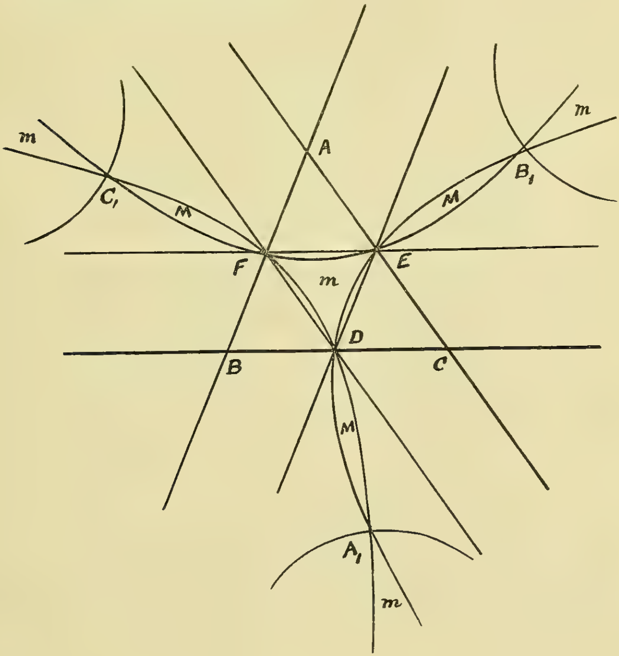
and therefore $v = \alpha$. The second derivative of v at the critical value is $-\alpha$ and since α is positive, v is a minimum.

Hence the coefficient of p in (31) is always positive. It follows that the values of s throughout the neighborhood of A are greater than the value at A which is a minimum. In like manner when Q is any point on the hyperbolic arc $D\ F$ of (29) s has a minimum at B, and when on the hyperbolic arc $D\ E$ of (29) at C.

In particular if Q be at D then $\alpha = 0$, $\beta = \frac{1}{2}$, $\gamma = \frac{1}{2}$; $A' = 0$, $B' = C' = \frac{1}{2}\pi$. Equation (23) gives $s = \frac{1}{2}a$, therefore $r_2 + r_3 = a$ and P must be on side a. In fact it is the projection of A on a, for (25) gives

$$r_2 = h \cot B, \quad r_3 = h \cot C,$$

where h is the altitude from A. This is not a true minimum for s, because s is constant and equal to $\frac{1}{2}a$ for all positions of P on side a; it is clearly less than the value for any point P not on side a. Similar results follow when Q is at E or F.



Therefore, for any position of Q inside the *hyperbolic* triangle $D\,E\,F$ the function s has a determinate ordinary minimum at a point P inside the triangle $A\,B\,C$. When Q is on the boundary of this triangle s has a singular minimum at the corresponding vertex of $A\,B\,C$, and when Q is at a vertex of $D\,E\,F$ s has a pseudo-minimum on the corresponding side of $A\,B\,C$.

11. Suppose now that Q is some point in the triangle $D\,E\,F$ but between the hyperbolic arc $E\,F$ and the chord $E\,F$. Then $A + A' > \pi$. The analytical conditions (5), (11), (12) fail to give a point P of ordinary position at which s has a maximum or minimum. For p_1 is negative, p_2 and p_3 positive as shown by (19). Under the condition that the r's are positive s is positive, while (25) shows r_1 negative and r_2, r_3 positive, also (21) shows x, y, z unlike signed. Hence there is no ordinary point in the plane which fulfils the condition that α, β, γ applied there and directed toward A, B, C shall be in equilibrium. There must, however, be a minimum value of s at a finite point in the plane since s is a one valued positive continuous function which is infinite when the r's are infinite. The method of the latter part of § 10 applies here, for α, β, γ are the sides of a triangle $A'B'C'$. Let $A + A'$ equal $\pi + \theta$, then

$$A' = \pi - (A - \theta) < \pi.$$

and $\theta < A$. Hence (34) becomes

$$\alpha^2 = \beta^2 + \gamma^2 + 2\beta\gamma \cos(A - \theta).$$

Subtract (33), and

$$\alpha^2 - v^2 = 4\beta\gamma \sin(A - \tfrac{1}{2}\theta) \sin \tfrac{1}{2}\theta,$$

is always positive and $\alpha > v$. Hence s is a minimum at the singular point A, as in the previous case. This may also be briefly shown as follows: Let Q be in the position assigned above. The straight line

$$xr_1 + yr_2 + zr_3 = \alpha r_1 + \beta r_2 + \gamma r_3,$$

for any assigned position of P, passes through Q and cuts the hyperbolic arc $E\,F$ in a point x', y', z'. It has been shown in § 10 that

$$x' r_1 + y' r_2 + z' r_3 \geqq \beta\, c + \gamma\, b,$$

hence $s = \alpha r_1 + \beta r_2 + \gamma r_3$ is a minimum at A.

Now let Q be any point on the chord $E\,F$. Then $\alpha = \beta + \gamma$. The point at which these segments directed to A, B, C are in equilibrium is at infinity. However, the coefficient of p in (30) is at once obviously positive and s has a minimum at A.

If Q is any point inside the triangle $A\,E\,F$ then $\alpha > \beta + \gamma$ and no triangle $A'B'C'$ exists, but (30) shows obviously that s has a minimum at A. If Q is on $A\ E$, $\beta = 0$, $\alpha > \gamma$ and (30) shows $s = \gamma b$ at A the minimum. When Q is at A, $s = r_1$ is a minimum there.

In conclusion, when Q is any point inside the hyperbolic triangle $D\ E\ F$ the function s has a minimum at an ordinary point P inside $A\ B\ C$; when Q is any other point inside $A\,B\,C$ s has a singular minimum at a vertex of $A\ B\ C$; when Q is D, F or E s has pseudo minimum along that side; when Q is any other point on the boundary of $A\ B\ C$ s is minimum at the nearest vertex of $A\,B\,C$.

12. In explanation of the singularities encountered above, let s be an ordinate to the plane $A\,B\,C$ at P whose polar coördinates at origin A are ρ, Θ. The nature and character of the singularity of this surface at A and the behavior of the function s in that neighborhood are fully explained by the equation to the surface (31). The slope to the plane $A\ B\ C$ of the tangent line to the surface at A is

$$\alpha + \beta \cos(\rho\, c) + \gamma \cos(\rho\, b) = \tan \Theta,$$

and

$$s = (\beta\, c + \gamma\, b) + \rho \tan \Theta \tag{35}$$

is the equation of a cone of one nappe to the surface at this point. The surface therefore ends in a teat or peak at each vertex of $A\,B\,C$. For all values of α, β, γ that keep this teat pointed down at A there is a minimum of s at A. The reason of this shape is due to the convention which confines r_1, r_2, r_3 to positive values, for if this convention be not observed then s is an 8 valued function symmetrical with respect to the plane $A\,B\,C$. In fact if it be transformed into orthogonal cartesian coördinates and rationalized it will be of degree 8 and of degree 4 in z^2. At the points A, B, C this surface has singular points, one of the sheets of which has an ordinary conical point at a vertex of the second degree, or a quadratic tangent cone there. Each of the four sheets will in general have a maximum or a minimum ordinate. The separate sheets of this surface will have for ordinates respectively

$$\begin{aligned} \alpha r_1 + \beta r_2 + \gamma r_3, \\ \alpha r_1 + \beta r_2 - \gamma r_3, \\ \alpha r_1 - \beta r_2 + \gamma r_3, \\ -\alpha r_1 + \beta r_2 + \gamma r_3. \end{aligned}$$

To change the sign of two of the r's in the first reproduces one of the others with its sign changed; to change all three changes the sign of the first. Hence our convention of keeping the r's positive and changing the signs of α, β, γ, and in fact only one of them, separates the sheets of this surface of degree 8, and will serve to find the maximum or minimum ordinate of each. The complete surface has no maximum or minimum at A, B, C when that point is singular on a sheet, but passes through that point tangent to a cone of second degree there. The convention as to the signs of the r's being positive cuts off one nappe of this cone, giving the singular maximum or minimum of the incomplete surface we have found.

13. We consider now the remaining case when one of α, β, γ is negative, say α, or Q is in the vertical angle D of the triangle $D\,E\,F$.

In the formulæ for the position of P at which an ordinary maximum or minimum may occur, the negative signs are to be taken throughout. Since the r's are positive the point P must occur outside $A\,B\,C$ and in the angle C. Referring to § 6 for the relations between the angles A', B', C' and the angles subtended by the sides of $A\,B\,C$ at P, we recognize that the branches of the three hyperbolas (29) which lie in the vertical angle D are such that the first, whose equation is now

$$y^2 + z^2 - x^2 - 2xy \cos A = 0,$$

contains all points Q for which $A' = A$; the second and third, whose equations are the same as before, contain respectively all points for which $B' = B$ and $C' = C$. It is easy to see, on trial, that these three hyperbolas pass through the point $-a:b:c$ which is the A excenter of $A\,B\,C$, which point we represent by A_1.

(1). If Q is in the closed lenticular space between the two hyperbolas $F\,D\,A_1$ and $E\,D\,A_1$ then

$$A' < A,\ B' > B,\ C' > C,$$

p_1, x, α are negative, σ, s are positive, the r's are positive. The discriminant (28) is negative, and hence s has an ordinary maximum at a determined point P in the segment of the circumcircle of $A\,B\,C$ having $B\,C$ for its chord.

(2). Passing Q through A_1 into the open space between the same hyperbolic arcs,

$$A' > A,\ B' < B,\ C' < C,$$

p_1 is $+$, p_2 and p_3 are $-$, s is negative, r is $-$, y and z $+$, and σ is negative. The discriminant (29) is positive. Hence s is a minimum at a point P in the angle A outside the circumcircle $A\,B\,C$.

(3). Pass Q over the hyperbola $E\,D\,A_1$ into the open space between that hyperbola and the hyperbola $F\,A_1\,E$. Then

$$A' > A,\ B' < B,\ C' > C,$$

p_1, p_3 are positive, p_2 negative. The r's are unlike signed. There is therefore no ordinary maximum or minimum value of s in this case.

(4). In like manner for Q in the open space between hyperbolas $F\,D\,A_1$ and $F\,A_1\,E$

$$A' > A,\ B' > B,\ C' < C,$$

p_1, p_2 are positive, p_3 negative. The r's are unlike signed and there is no ordinary maximum or minimum.

(5). Q in the open space between the straight line $E\,D$ and the hyperbolas $E\,D\,A_1$, $F\,A_1\,E$,

$$A' < A,\ B' > B,\ C' < C,$$

p_1, p_2 are positive, p_3 negative, the r's are unlike signed.

(6). Q in the open space between the straight line $F\,E$, hyperbolas $F\,D\,A_1$, $F\,A_1\,E$,

$$A' < A,\ B' < B,\ C' > C,$$

and similar conditions hold. Hence in this and the preceding case there exists no ordinary maximum or minimum of s.

These cases conclude all cases of the investigation and determination of the ordinary maximum or minimum values of s. For corresponding treatment of the spaces in the vertical angles E and F result in similar conclusions by interchange of letters.

14. We consider now the singular cases of maximum or minimum values of s.

(1). When Q is on one of the hyperbolas (29), P is at a vertex of $A\,B\,C$. The three hyperbolas pass through each of the excenters of $A\,B\,C$. When Q is at the A excenter A_1 then $A' = A$, $B' = B$, $C' = C$. The powers p_1, p_2, p_3 are all zero, and x, y, z are $0/0$, indeterminate; s is zero, σ is zero, and the locus of the indeterminate point P is the circumcircle. In fact the locus is in areal coördinates

$$-a\,r_1 + b\,r_2 + c\,r_3 = 0, \tag{36}$$

Ptolemy's equation of the arc $B\,C$ of the circumcircle in angle C. At each point of this arc the segments $-a$, b, c are in equilibrium, as is easily seen by resolving along r_1. The triangle $A'B'C'$ is similar to $A\,B\,C$. The function s is positive at all other points in the plane and is therefore a pseudo minimum along $B\,C$. Similarly when Q is at excenter B_1, C_1 then s is zero and a pseudo minimum along arc $A\,C$, $A\,B$ respectively.

(2). When Q is on hyperbola $E\,D\,A_1$ in angle A then P is at vertex C. The discussion is the same as if Q were on the arc $F\,E\,B_1$ but in angle B, then Q is at A and the conditions can be examined by making β negative in (30). The same reasoning as in § 10 shows the coefficient of p positive, and therefore s at A is a maximum or minimum according as its value $-b'c + c'b$ is positive or negative, that is according as Q is between E and B_1 or B_1 is between E and Q.

Therefore when Q is on hyperbola $E\,D\,A_1$ between E and A_1 s has a singular maximum at C, when Q is on the prolongation of the arc $E\,A_1$ a singular minimum at C.

(3). If Q is on the branch of the hyperbola $F\,E\,A_1$ in angle A then P is at A and as is obvious from (30) s has a singular maximum at A.

(4). In the areas unconsidered in the vertical angle D the conditions for positive r's fail to give results; these are provided for by the cases already considered by corresponding changes in signs of α, β, γ. In case Q is in angle C outside vertical angle D then one of a', b', c' is greater than the sum of the other two and no ordinary maximum or minimum exists; there will be a singular maximum or minimum at a vertex such that the straight line

$$x\,r_1 + y\,r_2 + z\,r_3 = -a'\,r_1 + b'\,r_2 + c'\,r_3$$

meets a hyperbola in x', y', z' corresponding to which point s has a singularity at that vertex. If we investigate the conditions that the singular maximum of s occurring at the vertices shall be zero, s will be zero at A, B, C respectively when

$$\begin{aligned} c\,\beta + b\,\gamma &= 0, \\ c\,\alpha + a\,\gamma &= 0, \\ b\,\alpha + a\,\beta &= 0. \end{aligned}$$

Representing the circumconic

$$x\,y\,c + x\,z\,b + y\,z\,c = 0$$

by $S = 0$, then $s = 0$ at A when Q is on $\dfrac{\partial S}{\partial x} = 0$ the tangent to $S = 0$ at A. This straight line passes through the B and C excenters. Also s will be zero at B when Q is on $\frac{\partial S}{\partial y} = 0$ the tangent to $S = 0$ at B. It will be zero at both A and B when Q is at the intersection of these two lines, or at the excenter C. The lines

$$\frac{\partial S}{\partial x} = \frac{\partial S}{\partial y} = \frac{\partial S}{\partial z} = 0,$$

the tangents to $S = 0$ at the vertices are the sides of the triangle of the excenters. It is not possible for s to be zero at A, B and C for any position of Q. At no point on any of these lines is there an ordinary maximum. At their intersections only can α, β, γ be in equilibrium satisfying the ordinary necessary conditions.

15. The equation (14) which gives the value of s when an ordinary maximum or minimum at x, y, z shows that x, y, z must make the expression on the right positive. That is P must be inside $A\,B\,C$ or a vertical angle of $A\,B\,C$ or one of the lunules between the circumcircle $A\,B\,C$ and the ellipse $\Sigma\, x\, y\, c'^2 = 0$.

17. The conditions expressing the relations between P and Q are interesting from the point of view of transformations. Thus, using (17)

$$r_1^2 = \frac{\partial \sigma}{\partial x} - \sigma, \qquad r_2^2 = \frac{\partial \sigma}{\partial y} - \sigma, \qquad r_3^2 = \frac{\partial \sigma}{\partial y} - \sigma,$$

and putting σ_1, σ_2, σ_3, for the derivatives of σ with respect to x, y, z, we have the transformations

$$\frac{\alpha}{x\sqrt{\sigma_1 - \sigma}} = \frac{\beta}{y\sqrt{\sigma_2 - \sigma}} = \frac{\gamma}{z\sqrt{\sigma_3 - \sigma}} = \frac{s}{\sigma} = \frac{\sigma\,\sigma'}{x\,y\,z\,s}. \qquad (37)$$

Limiting the radicals to positive sign, this transforms Q into P. Also using (19), equations (20) transform P into Q. Therefore if the equation to the path of either be given that of the other can be written at once. Relations (37) transform the hyperbolic surface, bounded by hyperbolas $D\,E\,F$, inside $A\ B\ C$ into the triangle surface $A\,B\,C$. It transforms the hyperbolic lunule between D and A_1 into the circular segment in angle A with chord $B\,C$. It transforms the hyperbolic triangle and the three lunules into the surface and boundary of the circumcircle. It transforms the points D, E, F into the boundary of $A\,B\,C$; and the points A_1, B_1, C_1 into the circumference of the circumcircle of $A\,B\,C$. The equation of the circumcircle in x, y, z can be written

$$a\sqrt{\sigma_1} + b\sqrt{\sigma_2} + c\sqrt{\sigma_3} = 0,$$

σ_1, σ_2, σ_3 being linear, and ambiguities of sign allowed.

16. There are many interesting theorems closely associated with the problem we have been discussing; we refer to only three of them.

(1). To find the point in a plane at which sides of a given triangle subtend given angles. We have only to take α, β, γ proportional to the sines of the given angles, having regard for the algebraic signs of the angles.

(2). Theorems relating to the groups of triangles each one of which is similar to a given triangle, in — and circumscribed about a given triangle. Special reference may be made to three papers by Mr. John Griffiths, London Math. Soc. XXIV, pp. 121, 181, 369. Mr. Griffiths calls the ratios (21) the *isogonal* coördinates of P.

(3). The maximum and minimum triangles of given form circumscribed and inscribed about a given triangle $A\,B\,C$. The former has its sides perpendicular to r_1, r_2, r_3 at P, and if α, β, γ are the sides of $A'B'C'$ similar to the given form, and M the area of the maximum triangle $A_1B_1C_1$ then

$$a_1 / \alpha = b_1 / \beta = c_1 / \gamma = k = 2\,M / s = s / 2\,\delta, \qquad (38)$$

determines this triangle.

(4). The reciprocal or dual relations existing between the two triangles $A\,B\,C$ and $A'B'C'$ in all the formulæ we have deduced are striking and admit interesting relations.

17. The following algebraic determination of s and P is interesting. Starting with the necessary conditions for an ordinary maximum or minimum

$$\frac{x\,r_1}{\alpha} = \frac{y\,r_2}{\beta} = \frac{z\,r_3}{\gamma} = \frac{\sigma}{s},$$

$$\therefore \qquad \frac{\alpha^2\,\sigma^2}{x^2\,s^2} = r_1^{\,2} = c^2\,y + b^2\,z - \sigma,$$

$$\frac{\alpha^2\,\sigma^2}{s^2} = x(x\,y\,c^2 + x\,z\,b^2) - x^2\,\sigma,$$

$$= x(\sigma - y\,z\,a^2) - x^2\,\sigma,$$

$$= (x\,y + x\,z)\sigma - x\,y\,z\,a^2. \qquad (39)$$

In like manner

$$\frac{\beta^2 \sigma^2}{s^2} = (y z + x y) \sigma - x y z b^2, \tag{40}$$

$$\frac{\gamma^2 \sigma^2}{s^2} = (z x + y z) \sigma - x y z c^2. \tag{41}$$

Divide in turn by x, y, z and add,

$$\therefore \quad s^2 = \frac{\Sigma x y c^2 . \Sigma x y \gamma^2.}{x y z} \tag{42}$$

Eliminate the product $x y z$ from the three equations above,

$$\frac{a^2 \sigma}{a^2 s} - \frac{x y + x z}{a^2} = \frac{\beta^2 \sigma}{b^2 s} - \frac{y x + y z}{b^2} = \frac{\gamma^2 \sigma}{c^2 s} - \frac{z x + z y}{c^2} = - \frac{x y z}{\sigma} \tag{43}$$

Divide by the product $x y z$, there result three independent equations in the reciprocals of x, y, z which in connexion with

$$x + y + z = 1,$$

determine these numbers and also s. Thus

$$\begin{aligned}
&a^2 \frac{1}{x} + \frac{a^2 b^2 - s^2}{a^2} \frac{1}{y} + \frac{a^2 . c - s^2}{a^2} \frac{1}{z} = \\
&\frac{\beta^2 a^2 - s^2}{b^2} \frac{1}{x} + \beta^2 \frac{1}{y} + \frac{\beta^2 c^2 - s^2}{b^2} \frac{1}{z} = \\
&\frac{\gamma^2 a^2 - s^2}{c^2} \frac{1}{x} + \frac{\gamma^2 b^2 - s^2}{c^2} \frac{1}{y} + \gamma^2 \frac{1}{z} = \\
&- a^2 \frac{1}{x} - \beta^2 \frac{1}{y} - \gamma^2 \frac{1}{z}.
\end{aligned} \tag{44}$$

Eliminating x, y, z there results for determining s the equation

$$\begin{vmatrix}
2 a^2 a^2, & a^2 b^2 + \beta^2 a^2 - s^2, & a^2 c^2 + \gamma^2 a^2 - s^2, \\
a^2 b^2 + \beta^2 a^2 - s^2, & 2 \beta^2 b^2, & \beta^2 c^2 + \gamma^2 b^2 - s^2, \\
a^2 c^2 + \gamma^2 a^2 - s^2, & \beta^2 c^2 + \gamma^2 b^2 - s^2, & 2 \gamma^2 c^2,
\end{vmatrix} = 0. \tag{45}$$

In this s is a symmetrical function of a, b, c and a, β, γ, these numbers being respectively interchangeable. The equation is apparently a cubic in s^2 but not so in fact, the term independent of s is zero. To show this observe that

$$\begin{vmatrix} a^2, a^2, t \\ \beta^2, b^2, t \\ \gamma^2, c^2, t \end{vmatrix} \begin{vmatrix} a^2, a^2, 1 \\ b^2, \beta^2, 1 \\ c^2, \gamma^2, 1 \end{vmatrix} = - t \begin{vmatrix} a^2, a^2, 1 \\ b^2, \beta^2, 1 \\ c^2, \gamma^2, 1 \end{vmatrix}^2 .$$

Form the product of the two determinants on the left and in the result put $t = 0$, the result is the determinant in (45) in which $s = 0$. Hence on dividing out s^2 in (45) the result is a quadratic in s^2. The actual solution is effected more simply by reducing the determinant in (45) by subtracting rows from rows and columns from columns in such a manner as to transform it into

$$\begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & c^2 & b^2 & \dfrac{b^2c^2\alpha^2}{s^2} \\ 1 & c^2 & 0 & a^2 & \dfrac{c^2a^2\beta^2}{s^2} \\ 1 & b^2 & a^2 & 0 & \dfrac{a^2b^2\gamma^2}{s^2} \\ 1 & \dfrac{b^2c^2\alpha^2}{s^2} & \dfrac{c^2a^2\beta^2}{s^2} & \dfrac{a^2b^2\gamma^2}{s^2} & 0 \end{vmatrix} = 0. \tag{46}$$

Whence on expansion

$$s^4 - s^2 \Sigma (b^2 + c^2 - a^2)\,\alpha^2 = \Sigma (a^2 + b^2 - c^2)\, c^2\alpha^2\beta^2 - \Sigma b^2c^2\alpha^4.$$

Actually solving this quadratic we find

$$s^2 = \tfrac{1}{2}\Sigma (b^2 + c^2 - a^2)\,\alpha^2 \pm 8\,\delta\,\Delta,$$

the same result previously obtained in (23). In the algebraic work of reduction wherever equations have been multiplied or divided by σ, s or $x\,y\,z$ the assumption is made that these numbers are not zero, and therefore circumcircle solutions have been thrown out. It is not worth while solving (44) for x, y, z.

None of the solutions of the problem thus far presented serve in a satisfactory way for higher dimensions. We now indicate a process which is applicable in all dimensions, furnishing the solution by means of quadratic equations.

18. We shall make use of some known results, letting the demonstrations subsequently for n dimensions serve for particular cases. The determinant

$$\begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & c^2 & b^2 & r_1^{\,2} \\ 1 & c^2 & 0 & a^2 & r_2^{\,2} \\ 1 & b^2 & a^2 & 0 & r_3^{\,2} \\ 1 & r_1^{\,2} & r_2^{\,2} & r_3^{\,2} & 0 \end{vmatrix}$$

the vanishing of which expresses the mutual relation existing between the distances between any four points in a plane, we represent by

$$\Phi(a, b, c, r_1, r_2, r_3). \tag{47}$$

Lemma. If α, β are any two segments terminating at P and δ is their resultant, the lines of action cut any circle through P in three points A, B, C respectively distant r_1, r_2, r_3 from p, then

$$\delta\, r_3 = \alpha\, r_1 + \beta\, r_2.$$

This relation is undisturbed, if A, B, C are fixed points and P is moved along the circle to coincide with A or B, α, β, δ being constant.

Consider the problem of § 3 as solved. Construct circles $A\,P\,B$, $B\,P\,C$, $C\,P\,A$. Produce $A\,P$, $B\,P$, $C\,P$ to meet these circumferences respectively again in A_1, B_1, C_1. Construct the triangle $A\,B\,C_1$, then by the lemma

$$\alpha\, r_1 + \beta\, r_2 - \gamma \,.\, P\,C_1 = 0\,. \tag{48}$$

Move the point P along the arc first to coincidence with A, then with B, whence

$$A\,C_1 = \frac{\beta}{\gamma}\,c\,,\ B\,C_1 = \frac{\alpha}{\gamma}\,c\,.\text{*}$$

The triangle $A\,B\,C_1$ is completely known. The radial coördinates

$$\frac{\beta}{\gamma}\,c\,,\ \frac{\alpha}{\gamma}\,c\,,\ C\,C_1\,,$$

of C_1 with respect to $A\,B\,C$ satisfy the radial identity

$$\Phi\ (a, b, c, \frac{\beta\, c}{\gamma}\, \frac{\alpha\, c}{\gamma},\ C\,C_1) = 0\,. \tag{49}$$

Hence $C\,C_1$ is determined by a quadratic equation. Also by (22),

$$\alpha \,.\, A\,A_1 = \beta \,.\, B\,B_1 = \gamma \,.\, C\,C_1 = s,$$

therefore s is known.

Thence directly the power p_3 of C with respect to the circle $A\,B\,C_1$ is

$$p_3 = \frac{1}{4\,\Delta_3} \begin{vmatrix} 0 & c^2 & \alpha^2\, c^2/\gamma^2, & b^2 \\ c^2 & 0 & \beta^2\, c^2/\gamma^2, & a^2 \\ \alpha^2\, c^2/\gamma^2, & \beta^2\, c^2/\gamma^2, & 0 & s^2/\gamma^2, \\ b^2 & a^2 & s^2/\gamma^2 & 0 \end{vmatrix}^{\frac{1}{2}}, \tag{50}$$

$\beta\, c/\gamma$, $\alpha\, c/\gamma$, c being the sides of $A\,B\,C_1$, and Δ_3 the area of this triangle is given by

$$-16\,\Delta_3^2 = \begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & c^2 & \alpha^2\, c^2/\gamma^2, \\ 1 & c^2 & 0 & \beta^2\, c^2/\gamma^2, \\ 1 & \alpha^2 c^2/\gamma^2, & \beta^2 c^2/\gamma^2 & 0 \end{vmatrix} \tag{51}$$

Similarly p_2, p_1. Thence x, y, z and r_1, r_2, r_3.

---

*We may observe that α, β, γ, are the coördinates (with respect to ABC_1) of an excenter of ABC, since they are proportional to the sides of that triangle, and (48) is the ptolemaic equation (in radial coördinates with respect to ABC_1) of the arc APB.

III.

19. Let $A\,B\,C\,D$ be a tetrahedron, a, b, c the sides of $A\,B\,C$ and f, g, h the edges from D to A, D, C. We seek the maximum or minimum value of

$$s = \alpha\, r_1 + \beta\, r_2 + \gamma\, r_3 + \delta\, r_4, \tag{52}$$

the r's being the radial coördinates of a point P to the corners A, B, C, D and $x,\ y,\ z,\ w$ its tetrahedral coördinates, $\alpha,\ \beta,\ \gamma,\ \delta$ are taken as the tetrahedral coördinates of an arbitrarily fixed point Q.

The identical relation existing between the content coördinates and the radial coördinates of the same point is

$$\begin{aligned} 0 &= x\, r_1{}^2 + y\, r_2{}^2 + z\, r_3{}^2 + w\, r_4{}^2 \\ &+ x\,y\,c^2 + x\,z\,b^2 + y\,z\,a^2 + x\,w\,f^2 + y\,w\,g^2 + z\,w\,h^2. \end{aligned} \tag{53}$$

The second line of this equation we represent by $\sigma = \Sigma\, xyc^2$, remembering that $-\sigma$ is the power of the point with respect to the circumsphere. The radial identity asserts that the segments $xr_1,\ yr_2,\ zr_3,\ wr_4$ laid off at P in direction toward or from the corner $A,\ B,\ C,\ D$ respectively according as the coördinate $x,\ y,\ z,\ w$ is positive or negative, are in equilibrium at P. Otherwise directly from the definition of these coördinates it follows that each of these segments is proportional to the sine of the solid angle between the other three, which establishes the relation in question. We shall represent the radial identity

$$\begin{vmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & c^2 & b^2 & f^2 & r_1{}^2 \\ 1 & c^2 & 0 & a^2 & g^2 & r_2{}^2 \\ 1 & b^2 & a^2 & 0 & h^2 & r_3{}^2 \\ 1 & f^2 & g^2 & h^2 & 0 & r_4{}^2 \\ 1 & r_1{}^2 & r_2{}^2 & r_3{}^2 & r_4{}^2 & 0 \end{vmatrix} = 0, \tag{54}$$

by

$$\Phi(a\,b\,c\,f\,g\,h\,r_1r_2r_3r_4) = 0. \tag{55}$$

20. Lemma.—A sphere circumscribes a tetrahedron $ABCD$, at a point P on the sphere in the trihedral angle D-ABC four segments $\alpha,\ \beta,\ \gamma,\ \delta$ are in equilibrium, directed to $A,\ B,\ C$ and from D respectively. Then

$$s = \alpha r_1 + \beta r_2 + \gamma r_3 - \delta r_4 = 0, \tag{56}$$

since the ratios of $r_1,\ r_2,\ r_3,\ r_4$ to the diameter through P are respectively the cosines of the angles which the segments make with the diameter, and the sum of the projections on the diameter vanishes. Also

$$\begin{aligned} \frac{xr_1}{\alpha} &= \frac{yr_2}{\beta} = \frac{zr_3}{\gamma} = \frac{wr_4}{-\delta} \\ &= \frac{\sigma}{s} = \frac{s}{\Sigma xyz\delta^2}\, xyzw. \end{aligned} \tag{57}$$

The cubic

$$\Sigma xyz\delta^2 = xyz\delta^2 + xzw\beta^2 + xyw\gamma^2 + yzw\alpha^2,$$

we shall represent briefly by σ'. In (57) both s and σ are zero, hence must $\sigma' = 0$. The cubic surface* $\sigma' = 0$ passes through the six edges of the tetrahedron. It does not enter the tetrahedron, it cuts each face in the edges in that face and nowhere else. The plane passing through an edge which is tangent to $\sigma' = 0$ is tangent all along that edge. The equation to the tangent plane through $x = 0$, $y = 0$ being

$$\frac{z}{\gamma^2} + \frac{w}{\delta^2} = 0,$$

with similar equations of the tangent plane along each other edge. Each edge being repeated twice gives 12 straight lines on the surface. The tangent planes through the opposite edges meet in three straight lines which lie on the surface and in the same plane, for

$$\frac{x}{\alpha^2} + \frac{y}{\beta^2} = 0 \text{ and } \frac{z}{\gamma^2} + \frac{w}{\delta^2} = 0$$

clearly intersect on the surface

$$\frac{\alpha^2}{x} + \frac{\beta^2}{y} + \frac{\gamma^2}{z} + \frac{\delta^2}{w} = 0,$$

and also on the plane

$$\frac{x}{\alpha^2} + \frac{y}{\beta^2} + \frac{z}{\gamma^2} + \frac{w}{\delta^2} = 0.$$

Any plane through an edge cuts the surface also in a conic, in general a hyperbola, which becomes two intersecting straight lines when the plane becomes tangent.

The cubic $\sigma' = 0$ cuts the sphere $\sigma = 0$ in a line of intersection (spherical cubic) passing through P, A, B, C. Hence P can be moved along this spherical cubic to either vertex preserving the relation (56) for constant values of $\alpha, \beta, \gamma, \delta$. The ratios $\alpha:\beta:\gamma:\delta$ must bear a fixed relation to the dimensions of the tetrahedron, since at A, B, C respectively

$$\begin{aligned} \beta c + \gamma b - \delta f &= 0, \\ \alpha c \qquad + \gamma a - \delta g &= 0, \\ \alpha b + \beta a \qquad - \delta h &= 0, \end{aligned}$$

*This surface has been studied by Professor Cayley, *Mémoire sur les Courbes du Troisième Ordre,* Collected Works, I, 183. Journal de Math. Pure et Appliquées (1844), IX, pp. 285-293.

Salmon counts each edge of the tetrahedron as four straight lines on this cubic which together with the other three mentioned above give the 27 lines. An. Geom., p. 501.

and therefore

$$\frac{\alpha}{a(ch+bg-af)}=\frac{\beta}{b(ch+af-bg)}=\frac{\gamma}{c(bg+fa-ch)}=\frac{\delta}{-2abc} \tag{58}$$

21. With reference to an arbitrarily chosen system of cartesian coördinates, the first partial derivatives of s (52) with respect to these coördinates equated to zero give necessary conditions

$$\left.\begin{aligned}\frac{\partial s}{\partial x}&=\alpha l_1+\beta l_2+\gamma l_3+\delta l_4=0,\\ \frac{\partial s}{\partial y}&=\alpha m_1+\beta m_2+\gamma m_3+\delta m_4=0,\\ \frac{\partial s}{\partial z}&=\alpha n_1+\beta n_2+\gamma n_3+\delta n_4=0,\end{aligned}\right\} \tag{59}$$

the l, m, n's being the direction cosines of the radial coördinates of P. These show that the sum of the projections of the segments α, β, γ, δ (laid off along r_1, r_2, r_3, r_4 respectively) on an arbitrary line vanishes. Hence these directed segments are in equilibrium at P, in the same way as are the segments xr_1, yr_2, zr_3, wr_4, and as sides must construct a closed quadrilateral in space. If we solve (59) for the ratios $\alpha:\beta:\gamma:\delta$ there results

$$\sin\omega_1:\sin\omega_2:\sin\omega_3:\sin\omega_4$$

where ω_1 is the solid angle at P formed by r_2, r_3, r_4, etc. The necessary conditions of maximum or minimum s at a point P of ordinary position are represented by the relations

$$\frac{xr_1}{\alpha}=\frac{yr_2}{\beta}=\frac{zr_3}{\gamma}=\frac{wr_4}{\delta}=\frac{\sigma}{s}=\frac{s}{\sigma'}xyzw. \tag{60}$$

Consider P to be at a critical point. Construct the four spheres $DBCP$, $ACDP$, etc. The equations of these spheres are respectively

$$0=xp_1-\sigma,\ 0=yp_2-\sigma,\ 0=zp_3-\sigma,\ 0=wp_4-\sigma;$$

p_1, p_2, p_3, p_4 being the powers of A, B, C, D with respect to the corresponding sphere. Their radical planes

$$xp_1=yp_2=zp_3=wp_4,\ (=\sigma) \tag{61}$$

meet in the point P. Dividing each ratio of (60) by (61)

$$\therefore\quad \frac{r_1}{\alpha p_1}=\frac{r_2}{\beta p_2}=\frac{r_3}{\gamma p_3}=\frac{r_4}{\delta p_4}=\frac{1}{s}. \tag{62}$$

Hence

$$\begin{aligned}s^2&=\alpha^2p_1+\beta^2p_2+\gamma^2p_3+\delta^2p_4,\\ &=\frac{\sigma\sigma'}{xyzw}.\end{aligned} \tag{63}$$

The problem of finding P and s is solved when the p's are known.

Produce the lines through P and the vertices A, B, C, D to meet the four spheres again in points A_1, B_1, C_1, D_1 respectively. Then $p_1 = r_1 . AA_1$, etc. Therefore by (62)

$$s = \alpha . AA_1 = \beta . BB_1 = \gamma . CC_1 = \delta . DD_1. \tag{64}$$

At P on the sphere $ABCP$ the segments $\alpha, \beta, \gamma, \delta$ are in equilibrium, the first three directed toward A, B, C respectively, and δ being directed away from D_1 when P is assumed to be inside $ABCD$ and $\alpha, \beta, \gamma, \delta$ positive numbers. Hence by the lemma, § 20,

$$\alpha r_1 + \beta r_2 + \gamma r_3 - \delta . PD_1 = 0.$$

Move P in succession to A, B, C along the spherical cubic. There result

$$AD_1 = \frac{\beta c + \gamma b}{\delta}, \quad BD_1 = \frac{\alpha c + \gamma a}{\delta}, \quad CD_1 = \frac{\alpha b + \beta a}{\delta}. \tag{65}$$

The tetrahedron $ABCD_1$ is now completely known. If T_4 is its volume then

$$288 T_4^2 = \Phi(a, b, c, AD_1, BD_1, CD_1), \tag{66}$$

$$p_4^2 = \frac{1}{24 T_4^2} \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & c^2 & b^2 & AD_1^2 \\ 1 & c^2 & 0 & a^2 & BD_1^2 \\ 1 & b^2 & a^2 & 0 & CD_1^2 \\ 1 & AD_1^2 & BD_1^2 & CD_1^2 & 0 \end{vmatrix}. \tag{67}$$

Hence the powers p_1, p_2, p_3, p_4 are known, and the coördinates of P and s known, by similar treatment of the points A_1, B_1, C_1. While T_4 and p_4 have been written at once in terms of known numbers, DD_1, etc., might have been determined as a root of the quadratic

$$\Phi(a, b, c, AD_1, BD_1, CD_1, f, g, h, DD_1) = 0,$$

thence s by (64). The radius of this sphere can be found by a quadratic, thence the distance of the center from D by a quadratic, and then p_4 from these values.

22. As to the existence of a maximum or minimum value of s, if $\alpha, \beta, \gamma, \delta$, i. e. Q, is represented by any point inside the tetrahedron, then s at P is uniform, finite and positive at all points in the finite region and infinite and positive at infinity; hence there must exist a minimum, for Q outside it is not so evident.

The six partial derivatives of s of second order are easily found from (59). The two typical ones are

$$\frac{\partial^2 s}{\partial x^2} = \sum \frac{\alpha}{r_1}(1 - l_1^2), \qquad \frac{\partial^2 s}{\partial x \partial y} = -\sum \frac{\alpha}{r_1} l_1 m_1,$$

the others follow on change of letters.

The type discriminant of even order

$$\frac{\partial^2 s}{\partial x^2}\frac{\partial^2 s}{\partial y^2}-\left(\frac{\partial^2 s}{\partial x\partial y}\right)^2$$

is equal to

$$\sum\frac{a^2}{r_1^2}\,n_1{}^2+\sum\frac{\alpha\beta}{r_1r_2}\left\{n_1{}^2+n_2{}^2+(l_1m_2-l_2m_1)^2\right\},$$

cyclic interchanges give the other two. The Hessian of s I have not been able to express in sufficiently simple form to make it worth while writing down. However, we may proceed more simply by expanding s by Taylor's formula. Thus let s be the value at P and s' the value at P' a point distant ρ from P. The expansion is found to be

$$s'=s+\rho\left[\alpha\cos(\rho r_1)+\beta\cos(\rho r_2)+\gamma\cos(\rho r_3)+\delta\cos(\rho r_4)\right]$$
$$+\tfrac{1}{2}\rho^2\left[\frac{\alpha}{r_1}\sin^2(\rho r_1)+\frac{\beta}{r_2}\sin^2(\rho r_3)+\frac{\gamma}{r_3}\sin^2(\rho r_4)\;\frac{\delta}{r_4}\sin^2(\rho r_2)+\right]\quad+R, \tag{68}$$

where R contains higher powers of ρ. This shows s will be a maximum or minimum at P when the sum of the projections of α, β, γ, δ (along the r's) on an arbitrary line vanishes, and the coefficient of ρ^2 retains its sign for all positions of P' in the neighborhood of P. Obviously the conditions for a minimum are satisfied when Q is inside $ABCD$.

23. The segments α, β, γ, δ in absolute values must construct a quadrilateral and therefore no one can be greater than the sum of the other three, for an ordinary maximum or minimum. Disregarding the plane at infinity the four planes

$$\pm x\pm y\pm z\pm w=0$$

passing through the mid-points of the edges of $ABCD$ contain all points such that the sum of three coördinates of Q is equal to the fourth. At all points inside the tetrahedron, whose plane faces pass respectively through the mid-points of the concurrent edges a, b, c, f, g, h, the point Q satisfies these conditions, also when Q is in the opposite diedral angles of this tetrahedron. Not all points in this region, however, satisfy the conditions for an ordinary maximum or minimum.

24. We seek the condition that the critical point P may occur at a vertex, say D. The partial derivatives there are indeterminate, the indetermination arising through the indeterminateness of the direction cosines

l_4, m_4, n_4. Eliminating these from (59) there results the equation of condition between α, β, γ, δ that P may be at D,

$$\alpha^2 + \beta^2 + \gamma^2 - \delta^2$$
$$+ 2\alpha\beta\cos ADB + 2\beta\gamma\cos BDC + 2\alpha\gamma\cos CDA = 0.$$

This equation asserts that there must be at D a direction acting along which δ is the resultant of α, β, γ along DA, DB, DC. Putting λ, μ, ν for the cosines of the face angles ADB, BDC, CDA, the locus of points Q for which P is at D is the hyperboloid of two sheets

$$x^2 + y^2 + z^2 - w^2 + 2xy\lambda + 2yz\mu + 2zx\nu = 0. \qquad (69)$$

This surface passes through the mid-points H, I, J, of f, g, h. It does not meet the plane $w = 0$, since the resultant of any numbers x, y, z along f, g, h can not vanish. It cuts the faces A, B, C respectively in the hyperbolas which have already been constructed for the plane. Corresponding to each vertex there is a similar hyperboloid passing through the mid-points of the edges concurrent there. These hyperboloids serve to map out the regions within which the position of Q gives s an ordinary maximum or minimum value. For a position of Q on the boundary of this region the equation to the surface in the form (68) will show, as was done for the plane, that for Q inside the tetrahedron and on (69) the function s has a minimum.

We go no further into the detailed criticism of the singularities than to observe that the investigations show that the tangent planes at the corners of the tetrahedron to the circumconicoid

$$S = xyc + xzb + yza + xwf + ywg + zwh,$$

are respectively

$$\frac{\partial S}{\partial x} = \frac{\partial S}{\partial y} = \frac{\partial S}{\partial z} = \frac{\partial S}{\partial w} = 0.$$

These planes form a tetrahedron, and it appears that when Q is at a corner of this, then the locus of the critical point P is the spherical cubic whose equations are

$$\Sigma xyz\delta^2 = 0, \ \Sigma xyc^2 = 0,$$

and when Q is at the vertex opposite to that of D, the radial equation of that corresponding part of the spherical cubic on the sphere is

$$\alpha r_1 + \beta r_2 + \gamma r_3 + \delta r_4 = 0,$$

where α, β, γ, δ have the values given in (58). This equation corresponds to the Ptolemaic equation of a circular arc in the plane, and for the position of Q chosen as above is

$$a(ch + bg - af)r_1 + b(ch + af - bg)r_2 + c(bg + fa - ch)\, r_3 = 2abcr_4, \tag{70}$$

and along this line s has a zero value and a pseudo-minimum.

IV.

25. The processes and results of the previous section are true in n dimensions. It is thought not out of place to briefly deduce them for n dimensions, especially as these results can be more simply reached than the writer has seen them attained elsewhere.[1]

A simplicissimum is a figure formed in space of n dimensions by $n + 1$ points, no three of which lie in a straight line, no four in a plane, no five in a hyper-plane, etc. It was thus defined by Sylvester.

Designate the $n + 1$ corners of a fixed simplicissimum (which we shall call briefly the simplex) by $A, B, C, \ldots$. Choose an arbitrary system of orthogonal cartesian coördinates and let

$$x_r, y_r, z_r, \ldots \qquad (r = 1, 2, \ldots, n + 1)$$

be the coördinates of the corners of the simplex.

Let m_r, $(r = 1, 2, \ldots, n + 1)$ be any assigned numbers. Let G be a point of mean position whose coördinates are

$$x = \frac{1}{M}\,\Sigma k_r x_r, \qquad y = \frac{1}{M}\,\Sigma k_r y_r, \qquad z = \frac{1}{M}\,\Sigma k_r z_r, \ldots$$

where $M = \Sigma m_r$. The numbers m_r are called the ratio (mass, charge, barycentric) coördinates of G. Let $k_r = m_r/M$, or what is the same thing let $M = 1$. Then

$$x = \Sigma k_r x_r,\ y = \Sigma k_r y_r,\ z = \Sigma k_r z_r, \ldots$$

Solving for the k's, there result

$$k_r = \begin{vmatrix} x_1, & y_1, \ldots & 1 \\ \cdots & \cdots & \cdots \\ x_{r-1}, & y_{r-1}, \ldots & 1 \\ x_{r+1}, & y_{r+1}, \ldots & 1 \\ \cdots & \cdots & \cdots \\ x_{n+1}, & y_{n+1}, \ldots & 1 \end{vmatrix} \div \begin{vmatrix} x_1, & y_1, \ldots & 1 \\ \cdots & \cdots & \cdots \\ x_n, & y_n, \ldots & 1 \end{vmatrix}. \tag{71}$$

[1]Special reference is made to a paper by Mr. W. J. Curran Sharp, proceedings of the London Mathematical Society, Vol. XVIII, p. 325, read April 7, 1887. On the Properties of Simplicissima (with special regard to the related Spherical Loci).

Let ρ_r be the distances from the arbitrarily chosen origin O to the corners of the simplex, and let $\rho_{i,j}, (i, j = 1, 2, \ldots, n+1)$ be the lengths of the edges of the simplex joining the corners two and two, the number of edges being $\frac{1}{2}(n+1)(n+2)$. Squaring the coördinates $x, y, z, \ldots$

$$x^2 = \Sigma k_r^2 x_r^2 + 2\Sigma k_i k_j x_i x_j.$$

Replace k_r^2 by its equal value

$$k_r - k_r k_1 - \ldots - k_r k_{r-1} - k_r k_{r+1} - \ldots - k_r k_{n+1}.$$

Whence

$$x^2 = \Sigma k_r x_r^2 - \Sigma k_i k_j (x_i - x_j)^2,$$

with precisely similar values for $y^2, z^2, \ldots$

On adding

$$\rho^2 = \Sigma k_r \rho_r^2 - \Sigma k_i k_j \rho_{i,j}^2, \tag{72}$$

where $\rho = OG$, expressing the distance between two points O, G, one given by the radial coördinates, the other by the ratio coördinates with respect to the simplex.[1] The orthogonal coördinates having now disappeared, (72) for ρ and ρ_r constant is the equation in k-coördinates of a spheric whose center is given by the radial coördinates ρ_r and whose radius is ρ. Also for k_r and ρ constant (72) is the radial equation of a spheric with center k_r and radius ρ. If $\rho = 0$, then

$$0 = \Sigma k_r \rho_r^2 - \Sigma k_i k_j \rho_{i,j}^2, \tag{73}$$

is an identical relation existing between the "content" (k_r) and radial (ρ_r) coördinates of any point in space.

If O coincides with the center of the circumspheric of the simplex, $\rho_r = R$ the radius of the circumspheric. Hence the equation in k_r coördinates of the n-dimensional spheric circumscribing the simplex is

$$0 = -\Sigma k_i k_j \rho_{i,j}^2. \tag{74}$$

If K_r are the coördinates of the center, the equation to this spheric in the radial coördinates ρ_r is

$$0 = \Sigma K_r \rho_r^2 - 2R^2. \tag{75}$$

There is an identical relation existing between the mutual distances connecting any $n+2$ points in n-space. Let the corners of the simplex and

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[1]This is a generalization of Stewart's theorem in elementary geometry (1763). Two points A, B on a straight being given, the distance from any point O to a point G in the line (dividing AB in given ratio) is given by

$$OA^2 . BG + OB^2 . GA + OG^2 . AB + AB . BG . GA = 0.$$

G be any $n+2$ points. In (72) move the point O in succession to the corners of the simplex, ρ becomes ρ_r in order and ρ_r becomes $\rho_{i,j}$, while $\Sigma k_i k_j \rho_{i,j}^2$ remains unchanged. Thus when O coincides with the pth vertex,

$$\rho_p^2 = k_1\rho_{1,p}^2 + \ldots + k_{p-1}\rho_{p-1,p}^2 + k_{p+1}\rho_{p+1,p}^2 + \ldots + k_{n+1}\rho_{n+1,p}^2 - \Sigma,$$

thus $n+1$ equations and in addition $\Sigma k_r = 1$. Eliminate the k's and Σ, there results the identical relation

$$\begin{vmatrix} 0 & 1 & 1 & 1 & \ldots & 1 \\ 1 & 0 & \rho_{1,2}^2 & \rho_{1,3}^2 & \ldots & \rho_1^2 \\ 1 & \rho_{1,2}^2 & 0 & \rho_{2,3}^2 & \ldots & \rho_2^2 \\ 1 & \rho_{1,3}^2 & \rho_{2,3}^2 & 0 & \ldots & \rho_3^2 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 1 & \rho_1^2 & \rho_2^2 & \rho_3^2 & \ldots & 0 \end{vmatrix} = 0, \tag{76}$$

there being $n+3$ rows. This is a relation between the lengths of the edges of the simplex and the radial coördinates of any point in space. We refer to this as the *radial* identity, writing it briefly

$$\phi(\rho_{i,j}, \rho_r) = 0. \tag{77}$$

To find the radius R of the spheric circumscribing the simplex, it is only necessary in (75) to move the point ρ_r to each vertex in succession, and eliminate K_r from the resulting equations. Hence

$$R^2 = -\tfrac{1}{2} \begin{vmatrix} 0 & \rho_{1\,2}^2 & \rho_{1\,3}^2 & \ldots \\ \rho_{1\,2}^2 & 0 & \rho_{2\,3}^2 & \ldots \\ \rho_{1\,3}^2 & \rho_{2\,3}^2 & 0 & \ldots \\ & (n+1)\text{rows} & & \end{vmatrix} \div \begin{vmatrix} 0 & 1 & 1 & 1 & \ldots \\ 1 & 0 & \rho_{1\,2}^2 & \rho_{1\,3}^2 & \ldots \\ 1 & \rho_{1\,2}^2 & 0 & \rho_{2\,3}^2 & \ldots \\ & (n+2)\text{rows} & & & \end{vmatrix} \tag{78}$$

If p, p_r, $(r = 1, 2, \ldots, n+1)$ are arbitrary numbers

$$p = \Sigma k_r p_r - \Sigma k_i k_j \rho_{i\,j}^2, \tag{79}$$

is the equation of a spheric in k-coördinates. The coefficients p_r are the powers respectively of the vertices of the simplex and p the power of the point k_r with respect to the spheric

$$0 = \Sigma k_r p_r - \Sigma k_i k_j \rho_{i,j}^2. \tag{80}$$

This last is the equation of the orthotomic spheric of the $n+1$ spherics whose centers are the vertices of the simplex and the squares of whose radii are respectively p_r. The proof of this theorem is as follows. The conditions that (70) shall be a spheric (72) are

$$\frac{\rho_1^2 - \rho^2}{p_1 - p} = \frac{\rho_2^2 - \rho^2}{p_2 - p} = \ldots = \frac{\rho_{n+1}^2 - p}{\rho_{n+1}^2 - \rho^2} = 1. \tag{81}$$

$$\therefore \rho_r^2 - \rho_{r+1}^2 = p_r - p_{r+1}. \tag{82}$$

This is the radial equation of a linear locus perpendicular to the edge $\rho_{r,\, r+1}$. The sum of all such equations vanishes, identically showing that these linear loci meet in a point, the center. In like manner (80) and spheric

$$R^2 = \Sigma k_r \rho_r{}^2 - \Sigma k_i k_j \rho_i{}^2{}_j$$

coincide provided

$$\frac{\rho_1{}^2 - R^2}{p_1} = \ldots = \frac{\rho^2{}_{n+1} - R^2}{p_{n+1}} = 1. \qquad (83)$$

Whence as before

$$\rho^2{}_r - \rho^2{}_{r+1} = p_r - p_{r+1}.$$

The assigned numbers p_r assign the same center as before, independent of p. The equations (81) and (83) show that

$$\rho^2{}_r - \rho^2 = p_r - p \quad \text{and} \quad \rho^2{}_r - R^2 = p_r,$$

hence

$$p = \rho^2 - R^2$$

is the power of k_r with respect to (80). The minimum value of p is when $\rho = 0$ then $p = -R^2$. Also

$$pr = \rho^2 r - R^2,$$

shows that p_r is the power of V_r with respect to (80). Equation (79) is then a spheric concentric with (80), the power of any point on which with respect to (80) is p.

To find the content V of a simplex, we first show[1] that if V' is the content of the simplex in (n-1)-space determined by the vertices but one of the n-space simplex, then $V = pV'/n$. Let p be the perpendicular distance from the $(n+1)$th vertex to the linear locus through the other vertices. Let z be the perpendicular on a parallel linear locus, and v the content of the (n-1)-space simplex determined by the points where the edges from the $(n+1)$th vertex meet the parallel locus. Then, because of similar figures

$$\frac{v}{V} + \frac{z^{n-1}}{p^{n-1}}.$$

$$\therefore \quad V = \int_0^p v\, dz = \frac{V'}{p^{n-1}} \int_0^p z^{n-1}\, dz = \frac{1}{n}\, pV'. \qquad (84)$$

[1]See a footnote, p. 326, in Mr. Sharp's paper.

In 1, 2 and 3-space the contents of the respective simplicissima are known to be

$$\frac{1}{1!}\begin{vmatrix} x_1, 1 \\ x_2, 1 \end{vmatrix}, \quad \frac{1}{2!}\begin{vmatrix} x_1, y_1, 1 \\ x_2, y_2, 1 \\ x_3, y_3, 1 \end{vmatrix}, \quad \frac{1}{3!}\begin{vmatrix} x_1, y_1, z_1, 1 \\ x_2, y_2, z_2, 1 \\ x_3, y_3, z_3, 1 \\ x_4, y_4, z_4, 1 \end{vmatrix}.$$

Let $y_r, z_r, \ldots$ $(r = 2, 3, \ldots, n+1)$ be the orthocartesian coördinates of the n points fixing the base of a simplex in n-space whose vertex is $x_1, y_1, z_1, \ldots$. Assume the content V' of the base in its $(n\text{-}1)$-space to be conformable with the expressions for the content in 1, 2, 3-space, and that

$$V' = \frac{1}{(n-1)!}\begin{vmatrix} y_2, & z_2, & \ldots 1 \\ y_2, & z_3, & \ldots 1 \\ \cdot & \cdot & \cdot \\ y_{n+1}, & z_{n+1}, & \ldots 1 \end{vmatrix}.$$

Then the content V of the simplex in n-space is, by (84),

$$\begin{aligned} V &= \frac{1}{n!}pV' \\ &= \frac{1}{n!}\begin{vmatrix} p & 0 & 0 & \ldots & 1 \\ 0 & y_2 & z_2 & \ldots & 1 \\ 0 & y_3 & z_3 & \ldots & 1 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & y_{n+1} & z_{n+1} & \ldots & 1 \end{vmatrix}. \end{aligned}$$

But this determinant is identical with

$$\Delta = \begin{vmatrix} x_1 & y_1 & z_1 & \ldots & 1 \\ x_2 & y_2 & z_2 & \ldots & 1 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ x_{n+1} & y_{n+1} & z_{n+1} & \ldots & 1 \end{vmatrix}.$$

For Δ is absolutely invariant under any transformation by rotation and translation of orthogonal axes. Choose the linear locus through the n points of the base

$$y_r,\ z_r,\ \ldots \qquad (r = 2, 3, \ldots, n+1)$$

as the coördinate reference locus $x = 0$, and the perpendicular from x_1, y_1, z_1, on it as the corresponding axis. Then $x_1 = p$ and

$$y_1 = z_1 = \ldots = 0, \quad x_2 = x_3 = \ldots = x_{n+1} = 0.$$

Hence Δ is identical with the determinant in question, and for any integer n

$$V = \frac{1}{n!}\begin{vmatrix} x_1 & y_1 & z_1 & \ldots & 1 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ x_{n+1} & y_{n+1} & z_{n+1} & \ldots & 1 \end{vmatrix}. \tag{85}$$

Observe the values of k_r in (71); each is equal to the content of a simplex whose vertex is $x, y, x, \ldots$ and whose base is a simplex face of the n-space simplex, divided by the content of the n-space simplex of reference. Hence the k's are the content coördinates of a point with reference to the simplex of reference.

We shall now abandon cartesian coördinates and in the future use radial and content coördinates as being better suited to the problem before us. We shall represent the coördinates of a point P referred to the simplex of reference by r_i for radial coördinates and by x_i for content coördinates, $r = 1, 2, \ldots, n+1$.

The equation of a sphere in radial coördinates $r_1, r_2, \ldots$, whose radius is R and whose center is given by the content coördinates $x_1, x_2, \ldots$ is

$$0 = \Sigma x_i r_i^2 - \Sigma x_i x_j \rho_i{}^2{}_j - R^2. \tag{86}$$

The square of the distance of any point r_i from x_i is

$$\delta^2 = \Sigma x_i r_i^2 - \Sigma x_i x_j \rho_i{}^2{}_j. \tag{87}$$

The power $\delta^2 - R^2$ of the point r_i with respect to the sphere (86) is therefore

$$p = \Sigma x_i r_i^2 - \Sigma x_i x_j \rho_i{}^2{}_j - R^2. \tag{88}$$

We shall now find the content of a simplex in terms of its edges. Let p be the length of the perpendicular from the $(n+1)$th vertex on the base (the altitude of the simplex), and let x_i, r_i $(i = 1, 2, \ldots, n)$ be the content and radial coördinates of the foot of the perpendicular with respect to the base simplex. Let

$$\sigma = \Sigma x_i x_j \rho_i{}^2{}_j,$$

then

$$\rho_i{}^2{}_{,\,n+1} = r_i^2 + p^2, \quad r_i^2 = \frac{\partial \sigma}{\partial x_i} - \sigma.$$

Eliminate x_i and σ from the $n + 2$ equations

$$\left.\begin{aligned} \Sigma x_i - 1 &= 0, \\ -\sigma + \frac{\partial \sigma}{\partial x_i} - \rho_i{}^2{}_{,\,n+1} + p^2 &= 0, \\ -\sigma + \Sigma x_i \rho_i{}^2{}_{,\,n+1} \qquad - p^2 &= 0. \end{aligned}\right\} \tag{89}$$

The result is

$$p^2 = -\tfrac{1}{2}\,\frac{\phi_{n+1}}{\phi_n},$$

where we write

$$\phi_{n+1} = \begin{vmatrix} 0 & 1 & 1 & \dots & 1 \\ 1 & 0 & \rho_2{}^2{}_1 & \dots & \rho_1{}^2{}_{,\,n+1} \\ 1 & \rho_2{}^2{}_1 & 0 & \dots & \\ \dots & \dots & \dots & \dots & \dots \\ 1 & \rho^2{}_{n+1,\,1} & \rho^2{}_{n+1,\,2} & \dots & 0 \end{vmatrix}.$$

But $V_n = p\,V_{n-1}/n$, therefore

$$\frac{V^2{}_n}{\phi_{n+1}} = \frac{-1}{2\,n^2}\,\frac{V^2{}_{n-1}}{\phi_n}.$$

Also $V_1 = \frac{1}{2}\,\phi_2$, hence

$$V^2 = \frac{(-1)^{n-1}}{2^n(n!)^2}\,\phi_{n+1}. \tag{90}$$

Comparing this with (78) there results

$$V^2\,R^2 = \frac{(-1)^n}{2^n(n!)^2}\begin{vmatrix} 0 & \rho_1{}^2{}_2 & \rho_1{}^2{}_3 & \dots \\ \rho_1{}^2{}_2 & 0 & \rho_2{}^2{}_3 & \dots \\ \rho_1{}^2{}_3 & \rho_2{}^2{}_3 & 0 & \dots \\ & (n+1)\,\text{rows} & & \end{vmatrix} \tag{91}$$

Finally we wish to determine the power of a point, whose radial coördinates are given, with respect to a circumspheric whose edges are given. This is easily done as follows: Let x_i and r_i be the content and radial coördinates of the given point. Then the power of the point with respect to the circumsphere is

$$P = -\Sigma x_i x_j \rho_i{}^2{}_j = -\sigma.$$

Also

$$r_i{}^2 = \frac{\partial\,\sigma}{\partial\,x_i} - \sigma.$$

Eliminating the $n+1$ numbers x_i from the $n+2$ equations

$$\Sigma x_i - 1 = 0,$$

$$\frac{\partial\,\sigma}{\partial\,x_i} - r_i{}^2 + P = 0,$$

and solving the eliminant for P there results

$$P = \frac{\begin{vmatrix} 1 & 1 & 1 & \dots & 1 & 1 \\ 0 & \rho_1{}^2{}_2 & \rho_1{}^2{}_3 & \dots & \rho_1{}^2{}_{,\,n+1} & r_1{}^2 \\ \rho_2{}^2{}_1 & 0 & \rho_2{}^2{}_3 & \dots & \rho_2{}^2{}_{,\,n+1} & r_2{}^2 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \rho^2{}_{n+1,\,1} & \rho^2{}_{n+1,\,2} & \rho^2{}_{n+1,\,3} & \dots & 0 & r^2{}_{n+1} \end{vmatrix}}{\phi_{n+1}}. \tag{92}$$

When $P = 0$ or the point is on the circumsphere, the determinant in the numerator equated to zero is the radial equation of the circumspheric in terms of the edges of the simplex.

We can determine the power P in another way. Let (91) in which we change n into $n+1$ give $V'^2R'^2$ for a simplex in $(n+1)$-space which has the simplex in (91) for base and has a vertex T having radial coördinates $\rho_{i,\,n+2}$ with respect to the points of the base. Let r_i be the coördinates with respect to the base of F the foot of the altitude (equal to h) from T to the base. Produce the altitude to cut the $(n+1)$-space spheric again at a distance δ from the base. Then the power of F with respect to the $(n+1)$-spheric is equal to the power of F with respect to the circumspheric of the base and $P = h\delta$. Also

$$V' = hV / (n+1).$$

$$\therefore \quad \frac{P^2}{V'^2R'^2} = \frac{(n+1)^2}{V^2}\left(\frac{\delta}{R'}\right)^2.$$

Let F be fixed and move T along the altitude to coincide with F, then $V = 0$, $R' = \infty$ and $\rho_{i,\,n+1}$ become r_i, the limit of δ / R' is 2.

$$\therefore P^2 = \frac{(-1)^{n+1}}{2^{n-1}(n!)^2V^2}\begin{vmatrix} 0 & \rho_{12}^2 & \cdots & \rho_{1,\,n+1}^2 & r_1^2 \\ \rho_{21}^2 & 0 & \cdots & \rho_{2,\,n+1}^2 & r_2^2 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \rho_{n+1,\,1}^2 & \rho_{n+1,\,2}^2 & \cdots & 0 & r_{n+1}^2 \\ r_1^2 & r_2^2 & \cdots & r_{n+1}^2 & 0 \end{vmatrix} \qquad (93)$$

Represent the determinant in (93) by Δ and that in (92) by D. Making use of the value of V^2 in (90) there results the relation

$$4\,\Delta = D^2\,\phi_{n+1}, \qquad (94)$$

a remarkable form of the radial identity.

The results of this article furnish the means for attacking the general problem; they have been deduced here because they are not of handy reference, and the writer would not know where to refer to the power equations (92) and (93).

VIII.

26. In seeking the minimum (maximum) of any linear function of the radial coördinates r_i of a point P with respect to the $n+1$ corners of a simplex of n dimensions, the function for examination can obviously be reduced to the study of

$$s = \Sigma a_i r_i, \quad (i = 1, 2, \ldots, n+1) \qquad (95)$$

where $\Sigma\alpha_i = 1$, and α_i are the content coördinates of a point Q whose position is determined by the arbitrary constants of the linear function. If s' be the value of s at a point P' distant ρ from P and ρr_i is the angle which $P'P$ makes with the line joining P to the i th vertex of the simplex, we can expand s' in terms of s and the powers of ρ by Taylor's formula and write

$$s' = s + \rho\Sigma\alpha_i \cos(\rho r_i) + \tfrac{1}{2}\rho^2\Sigma \frac{\alpha_i}{r_i} \sin^2(\rho r_i) + R. \tag{96}$$

The conditions for a min-maximum are first

$$\Sigma\alpha_i \cos(\rho r_i) = 0 \tag{97}$$

for all directions of ρ, or that the sum of the projections of α_i (laid off at P along r_i) on an arbitrary line shall vanish. That is the segments thus constructed must be equilibrated at the critical point of ordinary position. Second

$$\sum \frac{\alpha_i}{r_i} \sin^2(\rho r_i) \tag{98}$$

must keep its sign unchanged for all directions of ρ. This is quite obviously the case when we convene the r's positive and α_i a point inside the simplex. A minimum does exist for the positive one valued function s at some point r_i in the finite space. Our object is to find that point which fulfils the necessary condition (97) and when determined the character of s there can be determined by (98).

Assuming that a critical point P exists in the simplex, let its content coördinates be x_i.

The equations of the $n+1$ spherics, each of which passes through P and n at a time of the vertices of the simplex, are

$$0 = x_m p_m - \Sigma x_i x_j \rho_i{}^2{}_j \tag{99}$$

$m = 1, 2, \ldots, n+1$. The coefficient p_m is the power of the mth vertex of the simplex with respect to the sphere through P and the other vertices. The radical axes of these spheres, two and two are

$$x_1 p_1 = x_2 p_2 = \ldots = x_{n+1} p_{n+1}, \ (= \Sigma x_i x_j \rho_i{}^2{}_j = \sigma) \tag{100}$$

and they meet in P.

The identical relation

$$0 = \Sigma x_i r_i{}^2 - \sigma$$

shows that the segments $x_i\, r_i$ are equilibrated at P in the same way as are the segments α_i. Therefore the relations

$$\frac{x_1 r_1}{\alpha_1} = \frac{x_2 r_2}{\alpha_2} = \ldots = \frac{x_{n+1} r_{n+1}}{\alpha_{n+1}}, \tag{101}$$

and each of these ratios by compounding them is equal to

$$\frac{\Sigma x_i x_j \rho_i{}^2{}_j}{s} = \frac{s}{\Sigma \alpha_i{}^2 / x_i}. \tag{102}$$

$$\therefore \quad s^2 = \Sigma \frac{\alpha_i{}^2}{x_i} \,. \quad \Sigma x_i x_j \rho_i{}^2{}_j\,, \tag{103}$$

Also

$$\frac{r_1}{p_1 \alpha_1} = \frac{r_2}{p_2 \alpha_2} = \ldots = \frac{1}{s}, \tag{104}$$

$$\therefore \quad s^2 = \alpha_1{}^2 p_1 + \ldots + \alpha^2{}_{n+1} p_{n+1}\,. \tag{105}$$

If Q be chosen a point of finite position such that each α is different from zero, (101), (102), (103) show that it is impossible for s to vanish unless $\Sigma x_i x_j \rho_i{}^2{}_j = 0$ and also $\Sigma \alpha_i{}^2 / x_i$ is zero. That is unless P is a point on the circumspheric where the second surface cuts it. We write these surfaces $\sigma = 0$ and $\sigma' = 0$. The surface $\sigma' = 0$ contains the edges and vertices of the simplex. It is a continuous algebraic surface of degree n. We assume that if it cuts the spheric in a point P not a vertex that it will cut the spheric in lines leading to the vertices along the surface of the spheric.

27. If n segments α_i meet at P and have the resultant α_{n+1} and a spheric be drawn through P cutting the $n+1$ segments again at distances $r_1, \ldots, r_n$ and the resultant at ρ from P then

$$\overset{n}{\Sigma} \alpha_i r_i = \alpha_{n+1} \rho\,.$$

The proof follows on dividing both sides by the diameter of the spheric, as in three dimensions.

Let L represent the $(n+1)$th vertex of the simplex. Let α_1 be equilibrated at P. Through P and the other n vertices pass a spheric. Produce LP to cut this spheric again in L_1 and let $PL_1 = \rho$. Then

$$\overset{n}{\Sigma} \alpha_i r_i = \alpha_{n+1} \rho\,. \tag{106}$$

Represent the n common vertices of these two spherics by $V_1, \ldots, V_n$ and their distances from L_1 by $\rho_1, \ldots, \rho_n$.

Move P from the critical point to these n points in succession, preserving the equality (106) at all points of the path. Therefore n equations

$$\left.\begin{array}{l} 0 + a_2\rho_{12} + a_3\rho_{13} + \ldots + a_n\rho_{1n} = \rho_1 a_{n+1} \\ a_1\rho_{21} + 0 + a_3\rho_{23} + \ldots + a_n\rho_{2n} = \rho_2 a_{n+1} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_1\rho_{n1} + a_2\rho_{n2} + a_3\rho_{n3} + \ldots + 0 = \rho_n a_{n+1} \end{array}\right\} \qquad (107)$$

where $\rho_{i,j}$ are edges of the base of the given simplex and are known, ρ_i are the unknown edges of the auxiliary simplex and are determined by (107). Hence the volume, radius, power of a point with respect to this auxiliary circumspheric, can be at once written by § 25. Hence the power of L with respect to it is known, this is the power we called p_{n+1} in (100), (104), (105). In like manner the powers $p_1, \ldots, p_n$ of the remaining vertices of the simplex with respect to their auxiliary spheres are known, and hence s is known by (105), the r's are known by (104) and the x's by (101).

28. Since the segments a_1 must be equilibrated no one can be greater than the sum of the others. The linear loci

$$\pm x_1 \pm x_2 \pm \ldots \pm x_{n+1} = 0, \qquad (108)$$

which pass through the mid-points of the n concurrent edges at a vertex mark off an interior and exterior region in which the point Q must occur. On the boundary of this region $\Sigma a_i = 0$ and the corresponding point P is at infinity. There can be written as in 3-space a hyper-hyperboloid a sheet of which passes through the mid-points of concurrent edges such that when a_i is on the surface P is at a vertex and there s has a hyper-teat minimum or maximum.

29. By putting $S = \Sigma a_i a_j \rho_{ij}$, the condition that s shall vanish at a vertex is

$$\frac{\partial S}{\partial a_i} = 0 \qquad (i = 1, 2, \ldots, n+1). \qquad (109)$$

The discriminant of these $n+1$ equations does not in general vanish. Any n of these equations furnishes a point Q for which $s = 0$ at the corresponding n corners of the simplex. If s vanishes at the first n vertices and the corresponding values of a_i determined in terms of the edges from (109) are A_i, then the equation

$$\sum^{n} A_i r_i = A_{n+1} r_{n+1} \qquad (110)$$

is the radial equation of a path on the spheric surface along which A_i are equilibrated, corresponding to the ptolemaic equation of the arc of the circle, and along which s has a pseudo-minimum value.

East Lawn, March, 1909.

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NEW POSITIONS OF THE STARS IN THE HUYGHENIAN REGION OF THE GREAT NEBULA IN ORION

From Observations Made at the Leander McCormick Observatory

BY

RALPH E. WILSON

UNIVERSITY OF VIRGINIA
Charlottesville, Virginia, U. S. A.

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CONTENTS.

PART I.

INTRODUCTION.

The first survey of the Great Nebula in Orion for star positions was made by Sir John Herschel at the Cape of Good Hope in the years 1834–8. This survey was made for the purpose of plotting the stars on a drawing of the nebula and included 150 stars, some of which were faint. The positions of the stars with reference to Θ Orionis were determined by eye estimates of position within the "skeleton" triangles, the vertices of which were determined by micrometrical measurement. The differences in right

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ascension were estimated to tenths of seconds of time and those in declination to "parts," the value of each "part" being approximately 0."2. Even the brighter stars were liable to an error of at least one or two of the units in which their positions were read, and a relatively large error must be expected for the fainter stars.

In the years 1847–8 Professor W. C. Bond, at Harvard College Observatory, made a series of micrometrical observations of 96 stars in the nebula. [H. C. O. Annals, Vol. V.] In most cases the relative positions of the stars are based on but one or two measures, and consequently have large probable errors.

The work of Lassell, published in the Memoirs of the Royal Astronomical Society, 1854, contains positions of 59 of the brighter stars. The position angles with reference to Θ were read to minutes and the distances to seconds of arc, so that the probable error of all the positions is quite large.

More accurate as well as more extensive were the later surveys of Liapunoff and Struve, St. Petersburg, 1862, and of G. P. Bond, at Harvard College Observatory, 1859-65 [H. C. O. Annals, Vol. V]. The work of Professor Bond was an extensive survey of the whole nebula to determine the positions of all the stars visible in the 15-inch equatorial of the Harvard Observatory. The catalogue contains the positions, relative to Θ Orionis, of 1101 stars, down to approximately the fourteenth magnitude. The positions were for the most part determined by transits for right ascension and mica scale readings for declination. In the cases of the fainter stars, however, these methods were found impracticable and the positions of a large number of these stars were determined from diagrams. In some cases the diagrams were supplemented by one or two micrometer measures, but for all the faint stars there is reason to suspect an appreciable probable error. I was especially surprised to note the number of such determinations in the Huyghenian region of the nebula. Out of 51 stars assigned to this region by Professor E. S. Holden in his "Monograph on the Central Positions of the Nebula in Orion," the positions of eighteen were determined by means of diagrams, while those of eight others were determined from measures with reference to auxiliary stars, the positions of which were no more accurately determined than those of the average of the brighter stars.

The positions of 47 of the bright stars in the nebula were determined heliometrically by Bruno Meyermann at Groningen in 1905. Considering the large probable errors of former work on the faint stars, and having placed at my disposal the 26-inch refractor of the Leander McCormick Observatory, it seemed desirable to apply direct micrometrical measurements to these stars in the Huyghenian region in sufficient numbers to obtain fairly accurate results. It further seemed probable: 1, that some of the brighter stars, the former positions of which ought to be fairly accurate, might in fifty or more years show some proper motion independent of that of the trapezium; 2, that, during the same period of time, new variable stars might have appeared; 3, that some stars fainter than those catalogued by Professor Bond might be seen with a telescope of larger aperture.

The observations were begun in September, 1908, and cover a period of eighteen months. All the observations were made in position-angle and distance. With the view to decreasing the errors arising from the measurement of long distances, I adopted as fundamental stars 628 (Θ Orionis) and 558, 669, and 685, stars forming a triangle about 628. These numbers are the numbers of the stars in Bond's catalogue [H. C. O. Annals, Vol. V], and his designations are used throughout this paper, except in the cases of stars which he did not catalogue. The position-angle of all six possible combinations of the fundamental stars were measured on forty nights. The measures of distances of the same combinations would average about three less. From the average of the observed position-angles and distances, differences in right ascension and declination were determined, and from these the final differential positions with reference to 628 were determined by a least square solution. The positions of the stars within 100″ of the trapezium were determined by measures with reference to 628 directly. The positions of all stars outside of this central group were measured with reference to the nearest fundamental star and through it referred to 628. In some cases exceptions were made to this plan and two sets of measures were made, one with reference to the nearest fundamental star, the other with reference to 628 direct. In such cases both sets have been reduced to differences in right ascension and declination. The first set has then been referred to 628 by means of the definitive position of the fundamental star and the two sets have then been combined, giving each observation equal weight, to form the final position relative to 628. As far as practicable, measures of position-angle and distance were made on ten nights for each star. This plan has been carried out on all but five of the Bond stars and none of these five, save 625, has been observed on less than seven nights. See note on 625 in Part VI.

The pitch of the micrometer screw was determined at different temperatures by transits of polar and equatorial stars and by measures of differences in right ascension of stars in the Pleiades. The observed and adopted values of the pitch are given in the following table:

Temperature.	*Observed.*	*Adopted.*
70°	9.″862	9.″861
50	.855	.856
30	.852	.851

The value of the pitch of the screw for the mean temperature for each set of observations is given by the formula—

$$P = 9.''856 - 0.''00025(50^\circ - t).$$

The values derived from this formula were used for the fundamental stars. For the other stars the mean value 9.″856 was used, the errors arising therefrom being negligible in comparison with the errors of observation.

On the best nights it was my custom to observe the fainter stars and to look for others not catalogued by Professor Bond. Through this search I found five stars, the existence of which is certain. I do not believe that these stars have been catalogued before, and make a special note of their positions in the hope that some one may be interested enough to look them up.

	$\Delta\alpha$	$\Delta\delta$	*Nights.*
1.	— 86.″6	+ 89.″5	4
2.	62.6	— 24.5	4
3.	40.6	+ 77.3	2
4.	23.7	+ 43.8	10
5.	— 22.3	+ 84.7	3

No. 2 is certainly variable. It was not seen at all during the first year of observation, but on November 10, 11, and 12, 1909, it was nearly as bright as 589, from which it is distant about 6″. Upon February 1, 1910, it was considerably fainter. No. 4 must also be variable, since during the fall of 1909 it was almost as bright as 612 and 618 and considerably brighter than several of the Bond stars in the region. Upon February 1, 1910, it also seemed to be much fainter. If either No. 2 or No. 4 had attained a maximum during the period covered by Professor Bond's observations, it is hardly probable that they would have been overlooked by him. The other three stars are very faint and would hardly have been seen by Professor Bond.

In addition to these I obtained measures upon two other stars not catalogued by Professor Bond, the Alvan Clark star within the trapezium and a companion star to Bond 642 which is given by Professor O. Stone in his work on the Nebula in Orion. [Leander McCormick Obs. Publ., Vol. I, Pt. 7.]

The following will serve to explain the tabulation of the observations in Parts II and III:

Column 1. Dates.
2. Seeing on a scale of 5.
3. Mean temperature of the series of observations on the date given.
4. Position-angle.
5. Number of settings in position-angle.
6. Residuals in position-angle.
7. Distance in revolutions of the micrometer screw.
8. Number of settings in distance.
9. Residuals in distance.

In Part II are given the observations on the fundamental stars, followed by the solution for their definitive positions. In Part III are given the observations of the remaining stars, arranged in sets according to the fundamental stars with reference to which the measures were made. After these are added a few miscellaneous measures, most of which were made before the general plan of work was mapped out. These have been given small weight and used in determining the final positions of the stars.

The catalogue of positions given in Part IV will be explained by the following:

Column 1. Number.
2. G. P. Bond's number or other designation, in case the star was not catalogued by Professor Bond.
3. Herschel and Struve number.
4. Herschel's letter.
5. W. C. Bond's number.
6. Lassell's designation.
7. Liapunoff's letter.
8. Magnitude. [L. McCormick Publ., Vol. I, Pt. 7.]
9. Position in right ascension relative to Θ Orionis.
10. Position in declination relative to Θ Orionis.
11. Number of nights observed.

In order to obtain comparisons between my results and those of former observers, all the positions were reduced for precession and proper motion to 1910.0. The proper motion of Θ was taken from Newcomb's Fundamental Catalogue. In computing his precession coefficients Professor Bond used Mädler's proper motion of Θ, which is erroneous by at least 2″ a century, so that the use of his coefficients for reducing observations separated by any considerable period of time will result in a large error.

The precessions for Part V were computed by means of the formulæ,

$$\frac{d(\alpha' - \alpha)}{dt} = n\left\{\sin\alpha'(\tan\delta' - \tan\delta) + 2\sin\tfrac{1}{2}(\alpha' - \alpha)\cos\tfrac{1}{2}(\alpha' + \alpha)\tan\delta\right\},$$

$$\frac{d(\delta' - \delta)}{dt} = n(\cos\alpha' - \cos\alpha),$$

These may be written,

$$\frac{d(\alpha' - \alpha)}{dt} = \left\{\frac{\sin\alpha'}{\cos\delta\cos\delta'}\sin(\delta' - \delta) + 2\sin\tfrac{1}{2}(\alpha' - \alpha)\cos\tfrac{1}{2}(\alpha' + \alpha)\tan\delta\right\},$$

$$\frac{d(\delta' - \delta)}{dt} = 2n\sin\tfrac{1}{2}(\alpha' + \alpha)\sin\tfrac{1}{2}(\alpha' - \alpha).$$

Taking into consideration the small relative distances of the stars, we may simplify the formulæ by putting $\sin\alpha' = \sin\alpha$,

$\tfrac{1}{2}(\alpha' + \alpha) = \alpha$, and $\cos\delta = \cos\delta'$, whence

$$\frac{d(\alpha' - \alpha)}{dt} = n\left\{P\sin(\delta' - \delta) + Q\sin\tfrac{1}{2}(\alpha' - \alpha)\right\}, \qquad \text{(A)}$$

$$\frac{d(\delta' - \delta)}{dt} = R\sin\tfrac{1}{2}(\alpha' - \alpha),$$

where we have put $P = \frac{\sin\alpha}{\cos^2\delta}$, $Q = 2\cos\alpha\tan\delta$, and $R = 2n\sin\alpha$.

Newcomb's value of n, 20.″069, was used, and the position of Θ for 18830, the mean of the epoch of Professor Bond's observations and my own, was taken from Newcomb's Fundamental Catalogue. The following tables of precessions were thus computed:

p. for Dα

$\Delta\alpha$	$\Delta\delta \pm 30''$	$60''$	$100''$	$150''$	$200''$
$\pm 30''$	$\pm 0.''00295$	± 0.00588	± 0.00976	± 0.01462	± 0.01949
60	299	592	980	1466	1953
100	303	596	985	1470	1957
150	309	602	991	1476	1963
200	315	608	997	1482	1969
∓ 30	287	575	960	1440	1921
60	283	570	956	1436	1917
100	280	566	952	1432	1913
150	276	562	948	1428	1909
200	± 0.00272	± 0.00558	± 0.00944	± 0.01424	± 0.01905

p. for Dδ

$\Delta\alpha$	
$\pm 30''$	± 0.00289
60	578
100	964
150	1445
200	± 0.01927

The values of former observations reduced to 1910.0 are given in Part V. Below each set is given the mean of the results obtained by using the following weights:

H.	Herschel	1
W. B.	W. C. Bond	2
Ll.	Lassell	2
G. B.	G. P. Bond	4
L. S.	Liapunoff-Struve	4
N.	Newcomb	4
A. G. C.	Astronomische Gesellschoft	4
Hl.	Hall	4
B.	Burnham	4
M.	Meyermann	4

Below the mean is given my position for each star for the sake of comparison. The comparisons with the weighted means and with the Bond positions are given in the last table. An examination of these results seems to show no evidence of any proper motion in the great majority of the stars. Almost without exception the largest residuals occur in the case of those stars, Bond's positions of which were determined by eye estimates and diagrams, and which are therefore liable to the largest errors. Six of the stars, however, show differences which, I do not think, arise from this cause. They are 612, 618, 622, 636, 686, and 688, and may be studied in pairs. 612 and 618 are a pair of stars, just below the trapezium, of sufficient brightness to permit Professor Bond to make two measures of position in addition to several diagrams. The differences in the case of these two stars would seem to indicate a northward proper motion in declination of between 4″ and 5″ a century.

The differences in right ascension in the cases of 622 and 636 can be largely explained by assuming that these stars are physically connected with the trapezium and share in its motion. This assumption is not improbable, as both the stars are within 30″ of the trapezium. The residuals in right ascension would then reduce from

— 1.″0 to 0.″2 in the case of 622, and
— 1. 3 to 0. 5 in the case of 636.

There still remain, however, small residuals in declination which cannot be explained in this way, and there is need for more accurate measures upon which to base a discussion of this motion.

The cases of 686 and 688 are peculiar. The only basis of comparison in each case is a single estimate of position given by Prof. Bond, from which the following residuals were obtained:

	$\Delta\Delta\alpha$	$\Delta\Delta\delta$
686	$+2''.6$	$+22''.0$
688	-23.8	4.6

Evidently either something is wrong or these stars have large proper motion. The stars themselves are distinctly visible with the 26-inch equatorial on any night with the seeing 4 on a scale of 5. There are no other stars in the immediate vicinity except 671 and 676, and certainly none in the positions given by Professor Bond. These must be the stars which he has catalogued, but unless they are affected by large proper motions, Professor Bond's positions must be erroneous. It may be of interest ten years hence to measure the positions of these two stars in order to see if any great change of position is indicated.

Part VI contains a few notes which may be of interest to any one who is working on the Nebula of Orion.

I wish to thank Professor Stone for the use of the Leander McCormick equatorial and for the many helpful suggestions which he has given me throughout this work.

RALPH E. WILSON.

PART II.

Observations of the Fundamental Stars.

Date			S	T	P	n	Δ	R	n	Δ
					628-685					
1908	Sept.	9	2½	53°	134°.4	4	+0.3	----	----	----
		11	2	55	4.6	4	+0.5	----	----	----
		16	3½	53	3.8	4	—0.3	----	----	----
		25	3½	63	4.1	6	0.0	13.737	8	+0.03
	Nov.	9	2½	44	4.4	4	+0.3	.696	4	—0.01
		15	1½	22	----	----	----	.646	2	—0.06
		16	2½	27	4.3	4	0.2	----	----	----
		18	2	41	4.2	6	0.1	.721	4	+0.01
		20	1	42	4.2	4	+0.1	.685	4	—0.02
		21	1	36	3.8	4	—0.3	.795	4	+0.09
		22	3	37	4.1	4	0.0	.645	4	—0.06
		26	1	55	3.8	4	—0.3	----	----	----
	Dec.	2	1½	30	4.1	4	0.0	----	----	----
		3	2	18	4.2	4	+0.1	.676	4	—0.03
		8	1½	23	3.8	4	—0.3	.696	4	0.01
		14	2	32	4.1	4	0.0	.688	4	—0.02
		17	1	36	3.9	4	—0.2	.794	4	+0.09
1909	Jan.	6	2	45	4.2	4	+0.1	.694	4	—0.01
		10	2	32	4.4	4	0.3	.780	2	+0.07
		21	3	32	4.2	4	+0.1	.677	4	—0.03
		22	2½	42	4.1	4	0.0	.651	4	0.06
		23	3	47	4.0	4	—0.1	.649	4	0.06
	Feb.	4	2	40	4.0	4	0.1	.705	4	0.00
		6	2	35	4.1	4	0.0	.669	2	0.04
		20	2½	50	3.9	4	—0.2	.630	2	0.08
	Mar.	22	1½	40	4.4	4	+0.3	.690	4	—0.02
	Sept.	19	1½	58	4.0	4	—0.1	.751	2	+0.04
		25	2	51	4.1	4	0.0	.700	2	—0.01
	Oct.	1	3	51	3.8	4	0.3	.765	4	+0.06
		2	2	44	4.0	4	—0.1	.683	4	0.08
		8	3	56	4.6	2	+0.5	.769	4	+0.06
		9	2½	55	4.2	4	+0.1	.700	2	—0.01
		12	1	49	3.9	6	—0.2	.664	3	—0.04
		13	1½	35	4.3	4	+0.2	.754	4	+0.05
		17	2	42	4.5	4	+0.4	.773	3	0.07
		20	2	36	4.0	4	—0.1	.731	3	+0.02
		26	1½	38	4.2	4	+0.1	.693	4	—0.01
		27	2	47	4.0	4	—0.1	.727	4	+0.02
	Nov.	10	3	42	4.1	4	0.0	.666	4	—0.04
		11	3½	46	134.2	4	+0.1	13.730	4	+0.02
Mean				42°	134.128	±0.022		13.707	±0.006	

Observations of the Fundamental Stars—Continued.

Date			S	T	P	n	Δ	R	n	Δ
					628-669					
1908	Sept.	16	3½	53°	32°.5	4	0.0	----	----	
		22	3	59	2.6	4	+0.1	----	----	
		25	3½	63	2.4	4	—0.1	----	----	
	Nov.	9	2½	44	2.8	4	+0.3	11.853	4	—0.05
		18	2	41	2.8	4	0.3	.803	4	0.10
		20	1	42	2.6	4	0.1	.861	4	0.04
		21	1	36	2.5	4	0.0	.903	4	0.00
		22	3	37	2.8	4	0.3	.893	4	—0.01
	Dec.	2	1½	30	2.8	4	0.3	----	----	
		3	2	18	2.8	4	0.3	.918	4	+0.02
		8	1½	23	2.6	4	+0.1	.889	4	—0.01
		14	2	32	2.5	4	0.0	.853	4	—0.05
		17	1	36	2.2	4	—0.3	.957	4	+0.05
1909	Jan.	6	2	45	2.8	4	+0.3	.794	4	—0.11
		10	2	32	2.4	4	—0.1	.961	4	+0.06
		21	3	32	2.6	4	+0.1	.896	4	—0.01
		22	2½	42	2.7	4	+0.2	11.972	4	+0.07
		23	3	47	2.3	4	—0.2	12.005	2	+0.10
	Feb.	2	2½	35	2.6	4	+0.1	11.786	2	—0.12
		4	2	40	2.4	4	—0.1	.849	2	—0.05
		6	2	35	2.7	4	+0.2	.995	2	+0.09
		20	2½	50	2.3	4	—0.2	.934	4	+0.03
		25	2	30	2.0	4	0.5	.886	2	—0.02
		26	2½	35	2.2	4	0.3	.941	2	+0.04
	Mar.	22	1½	40	2.2	4	0.3	11.849	3	—0.05
	Sept.	19	1½	58	1.7	4	0.8	12.040	2	+0.14
		24	1½	55	2.2	4	0.3	11.854	2	—0.05
		25	2	51	2.4	4	—0.1	.953	3	+0.05
	Oct.	1	3	51	2.7	4	+0.2	.875	3	—0.03
		2	2	44	1.9	4	—0.6	.920	4	+0.02
		4	3	46	2.3	4	0.2	.963	3	0.06
		8	3	56	2.4	2	—0.1	.985	2	+0.08
		9	2½	55	2.8	4	+0.3	.872	2	—0.03
		12	1	49	2.5	4	0.0	.805	4	—0.10
		13	1½	35	2.2	4	—0.3	.934	4	+0.03
		17	2	42	2.2	4	0.3	.859	3	—0.04
		20	2	36	2.3	4	0.2	.885	3	0.02
		26	2½	38	2.2	4	—0.3	.884	4	—0.02
		27	2	47	2.7	4	+0.2	.916	4	+0.01
	Nov.	10	3	42	32.6	4	+0.1	11.954	4	+0.05
Mean				42°	32.455	±0.029		11.903	±0.007	

Observations of the Fundamental Stars—Continued.

Date			S	T	P	n	Δ	R	n	Δ
					628-558					
1908	Sept.	29	2½	44°	233°.4	3	—0.4	------------	------------	
	Oct.	2	1½	45	4.0	6	+0.2	------------	------------	
		5	2	51	3.9	4	+0.1	------------	------------	
	Nov.	7	3	47	3.7	4	—0.1	------------	------------	
		9	2½	44	4.0	4	+0.2	------------	------------	
		16	2½	27	3.8	4	0.0	------------	------------	
		18	2	41	4.1	4	0.3	20.257	4	+0.02
		20	1	42	3.8	4	0.0	.301	4	0.06
		21	1	36	4.1	6	0.3	.283	4	0.05
		22	3	37	4.1	4	0.3	.262	4	+0.03
	Dec.	2	1½	30	3.9	4	+0.1	.117	4	—0.12
		3	2	18	3.7	4	—0.1	.187	4	—0.05
		8	1½	23	3.7	4	0.1	.335	4	+0.10
		14	2	32	3.5	4	—0.3	.122	4	—0.11
1909	Jan.	10	2	32	4.2	4	+0.4	.184	4	0.05
		21	3	32	3.7	4	—0.1	.155	6	—0.08
		22	2½	42	3.4	4	0.4	.276	4	+0.04
		23	3	47	3.5	4	0.3	.274	4	+0.04
	Feb.	4	2	40	3.5	4	—0.3	.195	4	—0.04
		6	2	35	4.0	4	+0.2	.268	3	+0.03
		20	2½	50	3.5	4	—0.3	.241	2	0.00
		25	2	30	3.6	4	0.2	.225	2	—0.01
		26	2½	35	3.7	4	0.1	.276	2	+0.04
	Mar.	15	2	37	3.6	4	0.2	.208	2	—0.03
		22	1½	40	3.5	4	—0.3	.204	2	0.03
	Sept.	24	1½	55	3.8	4	0.0	.184	3	—0.05
		25	2	51	4.3	4	+0.5	.250	4	+0.01
		28	1½	46	4.0	4	0.2	.273	3	0.04
	Oct.	1	3	51	3.8	4	0.0	.371	3	+0.14
		2	2	44	3.9	4	0.1	.146	2	—0.09
		4	3	46	3.9	4	0.1	.238	2	0.00
		8	3	56	3.8	4	0.0	.221	3	0.02
		9	2½	55	4.1	4	0.3	.118	2	0.12
		12	1	49	4.2	4	0.4	.226	4	—0.01
		13	1½	35	4.0	4	0.2	.260	4	+0.02
		17	2	42	4.0	4	0.2	.348	4	+0.11
		20	2	36	3.8	4	0.0	.215	3	—0.02
		26	1½	38	3.8	4	0.0	.335	3	+0.10
		27	2	47	4.2	4	+0.4	.212	4	—0.02
	Nov.	10	3	42	233.7	4	—0.1	20.251	4	+0.02
Mean				41°	233.830	±0.026		20.236	±0.007	

Observations of the Fundamental Stars—Continued.

Date			S	T	P	n	Δ	R	n	Δ
					685-669					
1908	Nov.	20	1	42°	350°.0	4	0.0	19.878	4	—0.02
		21	1	36	49.7	8	—0.3	20.013	4	+0.12
		22	3	37	50.0	4	0.0	19.847	4	—0.05
		26	1	55	50.1	4	+0.1			
	Dec.	2	1½	30	50.0	4	0.0	.949	4	+0.06
		3	2	18	50.0	4	0.0	.848	4	—0.04
		8	1½	23	50.1	4	+0.1	.825	4	0.07
		14	2	32	49.9	4	—0.1	.803	4	—0.09
		17	1	36	50.3	4	+0.3	.958	4	+0.06
1909	Jan.	6	2	45	50.1	4	0.1	.860	4	—0.03
		10	2	32	50.4	4	+0.4	.963	4	+0.07
		21	3	32	49.7	4	—0.3	19.886	4	—0.01
		22	2½	42	50.1	4	+0.1	20.063	4	+0.17
		23	3	47	50.5	4	0.5	19.899	4	0.00
	Feb.	2	2½	35	50.0	4	0.0	.933	2	0.04
		4	2	40	50.0	4	0.0	.917	2	+0.02
		6	2	35	50.2	4	+0.2	.880	2	—0.01
		20	2½	50	50.0	4	0.0	.931	2	+0.04
		25	2	30	49.9	4	—0.1	.957	2	+0.06
		26	2½	35	49.6	4	0.4	.790	2	—0.10
	Mar.	22	1½	40	49.7	4	0.3	.939	3	+0.05
	Sept.	19	1½	58	50.0	4	0.0	.954	2	0.06
		24	1½	55	49.7	4	0.3	.902	2	+0.01
		25	2	51	49.8	4	—0.2	.893	2	0.00
		28	2	46	50.1	4	+0.1	.883	2	—0.01
	Oct.	1	3	51	50.0	4	0.0	.818	4	—0.08
		2	2	44	49.9	4	—0.1	.930	4	+0.04
		8	3	56	49.8	2	0.2	.909	2	0.02
		9	2½	55	50.0	4	0.0	.911	2	+0.02
		12	1	49	49.9	4	0.1	.779	4	—0.12
		13	1½	35	50.0	4	0.0	.911	3	+0.02
		17	2	42	49.8	4	0.2	.849	4	—0.04
		20	2	36	49.8	5	—0.2	.834	3	0.06
		26	2	38	50.2	4	+0.2	.846	4	0.05
		27	2	47	49.9	4	—0.1	.890	4	0.00
	Nov.	10	3	42	49.8	4	0.2	.872	4	—0.02
		11	3½	46	49.9	4	0.1	.889	4	0.00
		12	3½	52	49.7	4	—0.3	.913	4	+0.02
	Dec.	1	2½	30	50.1	4	+0.1	.867	4	—0.03
		7	2½	32	349.8	4	—0.2	19.875	4	—0.02
Mean				41°	349.962	±0.020		19.894	±0.006	

Observations of the Fundamental Stars—Continued.

Date			S	T	P	n	Δ	R	n	Δ
					685-558					
1908	Nov.	20	1	42°	264.7	4	0.0	26.280	4	+0.01
		21	1	36	4.4	4	—0.3	.388	4	+0.12
		22	3	37	4.8	4	+0.1	.233	4	—0.04
	Dec.	2	1½	30	4.7	4	0.0	.379	4	+0.11
		3	2	18	4.6	4	—0.1	.137	4	—0.13
		8	1½	23	4.6	4	0.1	.207	4	0.06
		14	2	32	4.4	4	—0.3	.157	4	0.11
1909	Jan.	10	2	32	4.9	4	+0.2	.273	4	+0.01
		21	3	32	4.7	4	0.0	.196	4	—0.07
		22	2½	42	4.4	4	—0.3	.371	4	+0.10
		23	3	47	4.6	4	—0.1	.423	4	+0.15
	Feb.	4	2	40	5.0	4	+0.3	.230	2	—0.04
		6	2	35	4.8	4	+0.1	.331	2	+0.06
		20	3	50	4.6	4	—0.1	.190	4	—0.08
		25	2	30	4.5	4	—0.2	.214	3	0.06
		26	2½	35	4.8	4	+0.1	.204	4	—0.07
	Mar.	22	1½	40	4.4	4	—0.3	.361	3	+0.09
	Sept.	19	1½	58	4.9	4	+0.2	.372	3	+0.10
		24	1½	55	4.7	4	0.0	.151	2	—0.12
		25	2	51	4.8	4	+0.1	.391	2	+0.12
		28	1½	46	4.3	4	—0.1	.393	2	0.12
	Oct.	1	3	51	5.0	4	+0.3	.375	3	0.10
		2	2	44	4.8	4	0.1	.419	4	0.15
		4	3	46	5.0	4	0.3	.317	3	+0.05
		8	3	56	4.8	2	0.1	.245	4	—0.02
		9	2½	55	4.9	4	+0.2	.176	4	0.09
		12	1	49	4.7	4	0.0	.197	3	0.07
		13	1½	35	4.7	4	0.0	.196	4	0.07
		17	2	42	4.7	4	0.0	.207	4	0.06
		20	2	36	4.6	4	—0.1	.205	4	0.06
		26	1½	38	4.4	4	—0.3	.215	4	0.05
		27	2	47	4.9	4	+0.2	.259	4	0.01
	Nov.	10	3	42	4.9	4	0.2	.219	3	—0.05
		11	2½	46	5.2	4	0.5	.347	4	+0.08
		12	3½	52	5.3	4	+0.6	.262	4	—0.01
	Dec.	1	2½	30	4.4	4	—0.3	.168	4	0.10
		7	2½	32	4.8	4	+0.1	.248	4	0.02
		8	1	29	4.7	4	0.0			
1910	Jan.	10	2½	25	5.2	4	0.5	.233	4	0.04
	Feb.	1	3	33	264.9	4	+0.2	26.226	4	—0.04
Mean				40°	264°.738	±0.025		26.267	±0.010	

Observations of the Fundamental Stars—Continued.

Date			S	T	P	n	Δ	R	n	Δ
					669-558					
1908	Nov.	20	1	42°	226°.1	4	+0.3	31.526	4	—0.12
		21	1	36	5.6	4	—0.2	.853	4	+0.20
		22	3	37	5.7	4	—0.1	.633	4	—0.02
	Dec.	2	1½	30	6.0	4	+0.2	.611	4	0.04
		3	2	18	5.9	4	+0.1	.558	4	0.09
		8	1½	23	5.7	4	—0.1	.621	4	0.03
		14	2	32	5.8	4	0.0	.537	4	—0.11
		17	1	36	5.6	4	—0.2	.888	4	+0.24
1909	Jan.	10	2	32	6.1	4	+0.3	.572	4	—0.08
		21	3	32	5.9	4	0.1	.671	4	+0.02
		22	2½	42	5.8	4	0.0	.764	4	+0.11
		23	3	47	6.0	4	+0.2	.613	4	—0.04
	Feb.	4	2	40	5.8	4	0.0	.604	2	—0.05
		6	2	35	5.7	4	—0.1	.723	4	+0.07
		20	2½	50	5.4	4	0.4	.539	4	—0.11
		25	2	30	5.5	4	0.3	.635	4	—0.02
		26	2½	35	5.5	4	0.3	.692	4	+0.04
	Mar.	22	1½	40	5.4	4	—0.4	.584	4	—0.07
	Sept.	24	1½	55	5.9	4	+0.1	.561	3	—0.09
		25	2	51	5.8	4	0.0	.693	2	+0.04
		28	1½	46	6.0	4	+0.2	.680	2	0.03
	Oct.	1	3	51	5.8	4	0.0	.695	3	0.04
		2	2	44	5.7	4	—0.1	.728	4	0.08
		4	3	46	5.9	4	+0.1	.730	3	0.08
		8	3	56	5.6	4	—0.2	.652	3	0.00
		9	2½	55	6.0	4	+0.2	.667	3	+0.01
		12	1½	49	5.8	4	0.0	.613	4	—0.04
		13	2	35	6.1	4	0.3	.671	4	+0.02
		17	2	42	6.0	4	0.2	.675	4	+0.02
		20	2	36	5.8	4	0.0	.520	4	—0.13
		26	1½	38	6.0	4	0.2	.688	4	+0.04
		27	2	47	6.1	4	0.3	.715	4	0.06
	Nov.	10	3	42	5.9	4	0.1	.726	4	0.07
		11	3½	46	5.8	4	0.0	.729	4	0.08
		12	3½	52	6.1	4	0.3	.675	4	+0.02
	Dec.	1	2½	30	5.9	4	0.1	.546	4	—0.10
		7	2½	32	6.1	4	+0.3	.581	4	—0.07
		8	1	29	5.6	4	—0.2	.702	2	+0.05
1910	Jan.	10	2½	25	6.2	4	+0.4	.582	4	—0.07
	Feb.	1	3	33	226.2	4	+0.4	31.653	4	0.00
Mean				40°	225.838	±0.024		31.653	±0.009	

The reduction of the mean position-angles and distances for the fundamental stars to differences in right ascension and declination gives the following results:

		$\Delta\alpha$		$\Delta\delta$
628—685	x_1	96."97	y_1	—94."06
—669	x_2	62.95	y_2	98.98
—558	x_3	—160.99	y_3	—117.70
685—669	$x_1—x_2$	34.17	$y_1—y_2$	—193.06
—558	$x_3—x_1$	—257.77	$y_3—y_1$	—23.74
669—558	$x_2—x_3$	223.78	$y_2—y_3$	217.32

A least square solution of these twelve equations gives as definitive positions with reference to 628,

	$\Delta\alpha$	$\Delta\delta$
685	96."96	—94."04
669	62.87	99.14
558	—160.91	—117.87

PART III.

Observations with reference to 628.

Date	S	T	P	n	Δ	R	n	Δ
			567					
1908 Nov. 8	2½	41°	268°.3	2	±1.0			
1909 Feb. 11	3½	35	7.2	4	—0.1	10.030	2	—0.11
17	4	35	6.3	4	—1.0	.049	2	—0.09
20	3	50	8.0	4	+0.7			
25	2	30	6.8	4	—0.5	.147	4	+0.01
26	2½	35	6.2	4	1.1	.136	2	0.00
Sept. 24	2½	35	6.3	4	—1.0	.248	2	0.11
29	3½	46	7.4	4	+0.1			
Oct. 2	4	44	7.4	4	0.1	.154	3	0.02
4	3	46	8.2	4	0.9	.129	4	+0.01
8	3½	56	7.6	4	0.3	.076	3	—0.06
9	2½	55	267.4	4	+0.1	10.268	2	+0.013
Mean		45°	267.26	±0.14		10.137	±0.020	

Observations with reference to 628—Continued.

Date			S	T	P	n	Δ	R	n	Δ
					575					
1908	Nov.	16	2½	27°	255°.9	4	+0.5	9.057	4	+0.08
		18	2	41	6.0	4	+0.6	9.020	2	+0.05
		22	3	37	5.1	4	—0.3	8.914	4	—0.06
	Dec.	3	2	18	5.4	4	0.0	8.820	4	—0.15
1909	Jan.	21	3	32	5.4	4	0.0	9.085	2	+0.11
		23	3	47	5.5	4	+0.1	9.015	2	+0.04
	Feb.	2	2½	35	5.4	6	0.0	8.808	2	—0.17
		6	2	35	5.6	4	+0.2	9.101	2	+0.13
		20	3	50	4.9	4	—0.5	8.988	4	+0.01
	Mar.	22	1½	40	254.9	4	—0.5	8.931	3	—0.04
Mean				37°	255.41	±0.07		8.974	±0.022	
					589					
1908	Nov.	16	3	27°	250°.0	4	+0.1	————	————	
		18	2½	41	50.3	2	0.4	————	————	
		22	3	37	50.0	4	+0.1	6.158	4	—0.05
1909	Jan.	23	3	47	49.8	4	—0.1	.249	2	+0.04
	Feb.	2	2½	35	49.6	4	0.3	.251	2	0.05
		20	3	50	49.8	4	0.1	.402	2	+0.20
		25	2	30	49.5	4	0.4	.130	2	—0.08
		26	2½	35	49.5	4	0.4	.140	2	—0.06
	Mar.	15	2	37	49.7	4	—0.2	.283	2	+0.08
		17	2½	37	50.1	4	+0.2	.174	2	—0.03
	Sept.	29	3	46	49.9	4	0.0	.060	1	—0.14
	Oct.	8	3½	56	250.2	4	+0.3	6.204	3	0.00
Mean				40°	249.88	±0.05		6.205	±0.020	
					595					
1909	Feb.	11	3½	35°	250°.6	4	0.0	5.000	2	0.00
		17	4	35	50.2	4	—0.4	5.030	4	+0.03
	Oct.	7	4	56	49.3	4	1.3	4.950	4	—0.05
		8	4	56	49.3	4	1.3	5.056	4	+0.05
		9	2½	55	50.5	4	—0.1	4.963	4	—0.04
	Nov.	11	3½	46	53.2	4	+2.6	5.000	6	0.00
		12	3	52	52.0	4	+1.4	5.074	3	+0.07
1910	Feb.	1	3	33	249.9	4	—0.7	4.941	4	—0.06
Mean				46°	250.62	±0.31		5.002	±0.012	

Observations with reference to 628—Continued.

Date			S	T	P	n	Δ	R	n	Δ
					601					
1909	Feb.	11	3½	35°	227°.8	4	—1.5	4.460	2	+0.05
		17	4	35	26.0	4	—3.3	.450	2	+0.04
	Oct.	7	4	56	30.9	4	+1.6	.277	4	—0.14
		8	4	56	29.1	6	—0.2	.552	4	+0.14
		9	2½	55	28.8	6	—0.5	.288	3	—0.13
	Nov.	11	3½	46	31.5	6	+2.2	.460	4	+0.05
		12	3½	52	230.7	4	+1.4	4.405	5	—0.01
Mean				48°	229.26	±0.53		4.413	±0.028	
					602					
1909	Feb.	11	3½	35°	208°.2	4	+0.1	7.103	2	+0.02
		17	4	35	8.3	4	+0.2	.047	2	—0.03
	Sept.	9	4	58	7.5	4	—0.6	.199	3	+0.12
		24	2½	55	7.8	4	—0.3	.147	2	+0.07
	Oct.	4	3	46	8.6	4	+0.5	7.043	2	—0.04
		7	4	56	6.4	4	—1.7	6.927	4	0.15
		8	3	56	8.8	4	+0.7	7.062	3	—0.02
	Nov.	10	3	42	8.4	4	0.3	.172	4	+0.09
		11	3½	46	9.3	4	+1.2	.095	4	+0.01
		12	3	52	207.8	4	—0.3	7.014	4	—0.07
Mean				47°	208.07	±0.16		7.081	±0.017	
					608					
1909	Feb.	11	3½	35°	233°.4	4	—1.6	2.829	2	—0.04
		17	4	35	4.9	4	0.1	.847	4	0.02
	Sept.	9	4	58	3.6	4	—1.4	.779	3	0.09
	Oct.	7	4	56	7.0	4	+2.0	.804	3	—0.07
		8	4	56	3.9	5	—1.1	.946	3	+0.07
		9	2½	55	5.0	6	0.0	2.875	3	0.00
		17	2½	42	5.3	4	+0.3	3.048	3	+0.18
	Nov.	10	3½	42	7.6	4	+2.6	2.801	4	—0.07
		11	3½	46	3.5	6	—1.5	.913	3	+0.04
		12	3½	52	235.5	5	+0.5	2.885	4	+0.01
Mean				48°	234.97	±0.30		2.872	±0.016	

Observations with reference to 628—Continued.

Date			S	T	P	n	Δ	R	n	Δ
					612					
1909	Jan.	23	3	47°	326°.4	6	+1.0	3.263	2	+0.02
	Feb.	17	4	35	4.6	4	—0.8	.189	2	—0.06
		20	3	50	5.2	4	0.2	.327	2	+0.08
		26	2½	35	5.2	4	—0.2	.240	4	—0.01
	Mar.	15	2	37	6.1	4	+0.7	.232	3	—0.02
		17	2½	37	6.1	4	+0.7	.279	3	+0.03
		23	3½	40	4.5	4	—0.9	.305	3	+0.06
	Sept.	9	4	58	5.2	4	0.2	.150	2	—0.10
		29	3½	46	4.9	4	—0.5	.228	2	—0.02
	Oct.	1	3	51	325.8	4	+0.4	3.254	2	+0.01
Mean				44°	325.40	±0.15		3.247	±0.011	
					617					
1908	Sept.	9	2½	53°	321°.2	6	—0.4	1.696	4	+0.01
		16	3½	53	2.2	4	+0.6	------	------	------
		22	3	59	2.6	4	+1.0	.655	4	—0.03
		25	3½	63	0.9	4	—0.7	.685	8	0.00
	Nov.	9	2½	44	2.0	4	+0.4	.660	4	0.03
1909	Jan.	21	3	32	1.3	4	—0.3	.666	8	—0.02
		22	2½	42	1.4	4	0.2	.720	4	+0.03
	Mar.	17	2½	37	1.5	4	—0.1	.753	4	+0.06
	Oct.	1	3½	51	2.8	4	+1.2	.682	4	—0.01
		8	2½	56	0.5	4	—1.1	.647	3	—0.04
	Nov.	10	2½	42	321.1	4	—0.5	1.724	4	+0.03
Mean				48°	321.59	±0.14		1.689	±0.007	
					618					
1908	Nov.	22	3	37°	------	------	------	2.873	4	—0.06
1909	Jan.	23	3	47	335.3	4	+0.1	3.079	2	+0.14
	Feb.	17	4	35	5.0	4	—0.2	2.857	3	—0.08
		20	3	50	5.4	4	+0.2	.907	2	0.03
		26	2½	35	5.4	4	0.2	.863	4	—0.08
	Mar.	15	2	37	5.7	4	+0.5	.982	2	+0.04
		17	2½	37	5.2	6	0.0	2.972	4	0.03
		23	3½	40	3.5	3	—1.7	3.033	2	+0.10
	Sept.	9	4	58	5.7	4	+0.5	2.900	2	—0.04
		29	3½	46	5.1	4	—0.1	.905	3	—0.03
	Oct.	1	3	51	335.9	4	+0.7	2.951	3	+0.01
Mean				43°	335.22	±0.14		2.938	±0.017	

Observations with reference to 628—Continued.

Date			S	T	P	n	Δ	R	n	Δ
					619					
1908	Sept.	9	2½	53°	311.6	4	0.0	1.306	4	—0.02
		11	2	55	2.5	6	+0.9	.324	8	0.00
		16	3½	53	2.1	4	+0.5	.309	8	0.02
		22	3	59	1.1	4	—0.5	.317	4	0.01
		25	3½	63	312.3	4	+0.7	1.309	8	—0.02
	Nov.	9	2½	44	311.8	4	+0.2	1.310	4	—0.02
		21	2	36	0.7	4	—0.9	.357	8	+0.03
1909	Jan.	21	3	32	2.6	4	+1.0	.325	8	0.00
		22	2½	42	1.0	4	—0.6	.338	4	0.01
	Mar.	17	2½	37	310.2	4	—1.4	1.354	4	+0.03
Mean				47°	311.59	±0.18		1.325	±0.004	
					621					
1909	Feb.	11	3½	35°	181°.1	4	+0.2	3.979	3	—0.04
		17	4	35	82.1	4	1.2	4.035	2	+0.02
	Mar.	18	3	37	81.1	4	+0.2	3.980	3	—0.04
	Sept.	9	4	58	79.3	4	—1.6	4.056	3	+0.03
	Oct.	1	3½	51	80.6	4	0.3	4.092	3	0.08
		4	3	46	79.0	6	1.9	4.067	4	+0.05
		8	3	56	80.7	6	—0.2	4.005	3	—0.01
	Nov.	10	3½	42	82.0	4	+1.1	3.903	4	—0.11
		11	3½	46	81.8	4	0.9	4.032	7	+0.02
		12	3½	52	181.1	4	+0.2	4.020	4	0.00
Mean				46°	180.88	±0.21		4.017	±0.011	
					622					
1908	Nov.	16	2½	27°	196°.3	6	+0.8	———		———
		18	2	41	5.6	4	+0.1	2.890	4	+0.06
		22	3	37	4.3	6	—1.2	.843	4	+0.01
1909	Jan.	23	3	47	5.7	4	+0.2	———		———
	Feb.	2	2½	35	6.3	4	0.8	.803	2	—0.03
		11	3½	35	5.5	4	0.0	.831	2	0.00
		17	4	35	5.7	4	+0.2	.835	2	0.00
		20	3	50	4.7	6	—0.8	.830	4	0.00
		25	2	30	4.1	4	—1.4	.803	2	0.03
		26	2½	35	5.8	4	+0.3	.813	4	—0.02
	Mar.	15	2	37	195.7	4	+0.2	2.848	2	+0.02
Mean				37°	195.47	±0.13		2.830	±0.006	

Observations with reference to 628—Continued.

Date	S	T	P	n	Δ	R	n	Δ
624								
1908 Sept. 9	2½	53°	342°.6	4	—0.5	1.690	4	—0.02
11	2	55	3.4	6	+0.3	.708	8	0.00
16	3½	53	3.2	4	+0.1	.668	8	0.04
22	3	59	2.9	4	—0.2	.707	4	0.00
25	3½	63	2.7	4	—0.4	.691	8	0.02
Nov. 9	2½	44	3.4	4	+0.3	.692	4	—0.02
21	2	36	2.4	4	—0.7	.734	8	+0.02
1909 Jan. 21	3	32	4.4	4	+1.3	.761	8	+0.05
22	2½	42	2.4	4	—0.7	.700	8	—0.01
Mar. 17	2½	37	343.4	4	+0.3	1.743	4	+0.04
Mean		47°	343.08	±0.10		1.709	±0.006	
625								
1909 Feb. 17	4	35°	178°.5	4		3.064	2	
Sept. 9	4	58	177.0	3		-----------	-----------	
Mean		46°	177.75	-----------		3.064	-----------	
631								
1909 Feb. 11	3½	35°	174°.1	4	+0.9	4.100	2	—0.02
17	4	35	3.4	4	0.2	4.238	2	+0.12
26	2½	35	3.2	4	0.0	3.824	2	—0.30
Mar. 18	3	37	3.8	4	0.6	4.013	3	—0.11
23	3½	40	3.4	4	0.2	.446	4	+0.32
Sept. 9	4	58	3.6	4	+0.4	.176	2	0.06
Oct. 1	3½	51	2.7	4	—0.5	.169	3	+0.05
4	3	46	1.8	6	1.4	.003	4	—0.12
8	3	56	1.7	6	—1.5	.131	3	+0.01
9	2½	55	174.4	4	+1.2	4.113	2	—0.01
Mean		45°	173.21	±0.26		4.121	±0.047	
633								
1908 Sept. 16	3½	53°	119°.5	6	+0.5	-----------	-----------	
22	3	59	19.9	6	0.9	0.407	4	—0.01
25	3½	63	19.2	4	0.2	.414	4	0.00
Oct. 5	2	51	20.2	4	1.2	.399	8	—0.02
Nov. 9	2½	44	19.3	4	0.3	.431	4	+0.01
1909 Jan. 22	2½	42	20.6	6	+1.6	.438	4	0.02
Oct. 1	3½	51	18.7	4	—0.3	.428	8	0.01
8	2½	56	19.5	8	+0.5	.424	4	0.00
Nov. 10	3	42	17.2	8	—1.8	.426	8	0.00
11	3½	46	115.6	8	—3.4	0.425	8	0.00
Mean		51°	118.97	±0.29		0.421	±0.002	

Observations with reference to 628—Continued.

Date			S	T	P	n	Δ	R	n	Δ
					635					
1908	Sept.	16	3½	53°	4.2	4	—0.1			
		22	3	59	4.0	4	—0.3			
		25	3½	63	4.4	6	+0.1			
	Nov.	9	2½	44	4.2	4	—0.1	9.886	4	+0.01
		16	2½	27	3.8	4	—0.5			
		18	2	41	4.4	4	+0.1	.877	4	0.00
		20	1	42	4.4	4	+0.1	.885	4	+0.01
		21	1	36	4.1	6	—0.2	.697	4	—0.08
	Dec.	3	2	18	4.3	4	0.0	.929	4	+0.06
		14	2	32	4.3	4	0.0	.911	4	0.04
1909	Jan.	21	3	32	4.5	4	+0.2	.929	4	0.06
		22	2½	42	4.1	4	—0.2	.994	4	+0.12
		23	3	47	4.3	4	0.0	.841	2	—0.03
	Feb.	2	2½	35	3.7	4	—0.6	9.779	2	—0.09
Mean				41°	4.27	±0.04		9.872	±0.013	
					636					
1908	Nov.	18	2½	41°	139.1	6	—1.4			
1909	Feb.	11	3½	35	43.2	4	+2.7	1.317	2	—0.11
		17	4	35	39.5	4	—1.0	.353	4	—0.07
	Mar.	18	3	37	39.1	4	—1.4	.498	3	+0.07
		23	3½	40	42.6	4	+2.1	.518	3	0.09
	Sept.	9	4	58	40.0	6	—0.5	.433	4	0.01
		24	2½	51	40.8	4	+0.3	.454	3	+0.03
		25	2½	51	41.5	4	+1.0	.412	2	—0.01
	Oct.	1	3	51	38.5	4	—2.0	.376	2	—0.05
		2	4	44	40.4	4	0.1	.448	4	+0.02
		4	3	46	140.4	6	—0.1	1.429	3	0.00
Mean				45°	140.46	±0.29		1.424	±0.012	
					640					
1908	Sept.	9	2½	53°	60.8	6	—0.2	1.353	8	—0.02
		11	2	55	1.0	6	0.0	.348	8	0.02
		16	3½	53	1.0	6	0.0	.376	8	0.00
		22	3	59	1.7	4	+0.7	.376	4	0.00
		25	3½	63	0.9	4	—0.1	.349	8	—0.02
	Nov.	9	2½	44	1.4	4	+0.4	.374	4	0.00
		21	2	36	0.6	8	—0.4	.377	8	+0.01
		22	3	37	1.0	4	0.0	.364	8	—0.01
	Dec.	3	2	18	1.2	6	+0.2	.370	8	0.00
1909	Jan.	21	3	32	0.6	4	—0.4	.352	8	—0.02
		22	2½	42	0.6	4	—0.4	.392	4	+0.02
	Mar.	17	2½	37	61.3	4	+0.3	1.417	4	+0.05
Mean				44°	61.01	±0.06		1.371	±0.003	

Observations with reference to 628—Continued.

Date			S	T	P	n	Δ	R	n	Δ
					642					
1909	Feb.	11	3½	35°	16.4	4	+0.2	4.875	2	—0.10
	Sept.	14	4	60	7.8	4	+1.6	4.853	2	—0.12
	Oct.	7	4	56	5.9	2	—0.3	5.010	2	+0.04
		8	3½	56	6.0	5	0.2	5.038	3	0.06
	Nov.	10	3	42	5.7	4	0.5	5.021	3	0.05
		11	3½	46	5.8	4	0.4	5.041	4	+0.07
		12	3½	52	5.6	4	—0.6	4.937	4	—0.04
1910	Feb.	1	3½	33	16.6	4	+0.4	5.009	4	+0.03
Mean				48°	16.22	±0.16		4.974	±0.020	
					647					
1908	Sept.	16	3½	53°	30°.1	6	—0.1	------------	------------	
		22	3	59	29.7	4	0.5	------------	------------	
	Nov.	9	2½	44	30.2	4	0.0	4.312	4	—0.07
		18	2	41	29.9	4	0.3	.379	4	0.00
		21	1	36	30.0	4	—0.2	.333	4	—0.04
	Dec.	14	2	32	30.3	6	+0.1	.417	4	+0.04
1909	Jan.	21	3	32	29.6	4	—0.6	.323	4	—0.06
		22	2½	42	30.1	4	—0.1	.430	4	+0.05
		23	3	47	30.6	4	+0.4	.491	2	+0.11
	Feb.	2	2½	35	30.8	4	0.6	.382	4	0.00
		25	2	30	30.9	4	0.7	.315	2	—0.06
	Oct.	9	2½	55	30.5	4	+0.3	4.402	2	+0.02
Mean				42°	30.22	±0.08		4.378	±0.012	
					648					
1909	Feb.	11	3½	35°	115.9	4	—0.1	2.739	2	—0.10
		17	4	35	6.9	5	+0.9	.723	2	—0.11
	Mar.	23	3½	40	6.0	4	0.0	.929	3	+0.09
	Sept.	9	4	58	7.8	6	+1.8	.856	3	0.02
		24	3	55	4.8	4	—1.2	.864	4	0.03
		25	2½	51	4.5	4	—1.5	.842	2	0.01
	Oct.	1	3½	51	7.8	4	+1.8	.858	4	+0.02
		2	4	44	5.9	4	—0.1	.821	3	—0.01
		4	3	46	5.5	4	0.5	.850	2	+0.02
		8	3½	56	114.9	4	—1.1	2.868	3	+0.03
Mean				47°	116.00	±0.24		2.835	±0.012	

Observations with reference to 628—Continued.

Date			S	T	P	n	Δ	R	n	Δ
					651					
1909	Feb.	11	3½	35°	32.9	4	+1.1	5.460	2	+0.09
		20	2½	50	2.4	6	+0.6	.296	2	—0.08
		25	2	30	0.6	4	—1.2	.304	4	—0.07
		26	2½	35	2.1	4	+0.3	.403	4	+0.03
	Mar.	17	2½	37	1.8	4	0.0	.458	2	0.08
		23	4	40	2.1	4	0.3	.477	2	+0.10
	Sept.	9	4	58	2.1	4	+0.3	.298	3	—0.08
		29	3	46	1.5	4	—0.3	.318	2	0.06
	Oct.	1	2	51	1.8	4	0.0	.301	4	—0.07
		2	3	44	31.1	5	—0.7	5.425	3	+0.05
Mean				43°	31.84	±0.13		5.374	±0.019	
					654					
1909	Mar.	15	2	37°	74.0	4	+0.4	3.407	2	—0.05
		17	2½	37	4.4	6	0.8	.394	2	—0.06
		18	3	37	4.3	4	0.7	.554	2	+0.10
		22	1½	40	3.8	6	0.2	.474	2	0.02
		23	4	40	4.5	4	+0.9	.493	2	0.04
	Sept.	9	4	58	3.4	4	—0.2	.497	3	0.04
		25	2½	51	2.0	4	1.6	.489	4	0.03
	Nov.	10	3½	42	2.2	4	—1.4	.474	4	+0.02
		11	3½	46	3.6	4	0.0	.421	4	—0.04
		12	3½	52	73.7	4	+0.1	3.379	4	—0.08
Mean				44°	73.59	±0.17		3.458	±0.013	
					671					
1908	Sept.	16	3½	53°	109°.4	4	—0.7	----------	----------	
		22	3	59	09.9	4	—0.2	7.303	4	—0.12
	Nov.	16	2½	27	10.9	4	+0.8	.540	4	+0.12
		18	2	41	10.6	4	0.5	.464	4	+0.04
		21	1	36	10.2	4	0.1	----------	----------	
		22	3	37	10.5	6	+0.4	.429	4	0.00
1909	Jan.	21	3	32	09.7	4	—0.4	.451	2	+0.03
		22	2½	42	10.0	4	—0.1	.341	4	—0.08
		23	3	47	10.4	4	+0.3	.485	2	+0.06
	Feb.	2	2½	35	09.8	4	—0.3	.430	2	+0.01
	Sept.	28	2	46	09.4	4	—0.7	.425	2	0.00
	Oct.	9	2½	55	110.8	4	+0.7	7.374	3	—0.05
Mean				42°	110.13	±0.10		7.424	±0.014	

Observations with reference to 628—Continued.

Date	S	T	P	n	Δ	R	n	Δ
			676					
1908 Sept. 22	3	59°	110.6	4	—0.1			
Nov. 16	2½	27	0.9	4	+0.2	8.685	2	+0.01
22	3	37	0.8	4	+0.1	.761	4	0.09
1909 Jan. 23	3	47	0.4	4	—0.3	.757	2	+0.08
Feb. 11	3½	35	1.2	4	+0.5	.587	2	—0.09
20	2½	50	0.7	4	0.0	.701	4	+0.03
25	2	30	0.7	4	0.0	.637	2	—0.04
Mar. 17	2½	37	1.2	4	+0.5			
Sept. 28	2	46				.686	3	+0.01
Oct. 2	4	44	0.4	4	—0.3	.666	3	—0.01
4	3	46	110.1	4	—0.6	8.585	2	—0.09
Mean		42°	110.70	±0.07		8.674	±0.014	
			686 (Other measures referred to 685.)					
1909 Feb. 17	4	35°	125°.8	4		10.705	2	
			688 (Other measures referred to 685.)					
1909 Feb. 17	4	35°	100°.1	4		13.188	3	
			W 1					
1909 Oct. 8	3½	56°	313°.9	4	—0.2	12.566	2	—0.07
Nov. 11	3½	46	4.2	4	+0.1	.704	4	+0.07
12	3½	52	4.2	4	+0.1	12.634	4	0.00
1910 Feb. 1	3½	33	314.0	4	—0.1			
Mean		47°	314.08	±0.06		12.635		
			W 2					
1909 Nov. 10	3½	42°	248.5	4	—0.1	6.832	4	+0.02
11	3½	46	9.4	4	+0.8	.728	4	—0.09
12	3½	52	8.0	4	—0.6	.869	4	+0.05
1910 Feb. 1	3½	33	248.5	4	—0.1	6.838	4	+0.02
Mean		43°	248.60	±0.20		6.817	±0.021	

Observations with reference to 628—Continued.

Date	S	T	P	n	Δ	R	n	Δ
W 3								
1909 Nov. 11	3½	46°	332°.3	2	0.0	8.875	5	+0.01
12	4	52	332.3	4	0.0	8.848	3	—0.01
Mean		49°	332.3			8.862		
W 4								
1909 Sept. 14	4	56°	332°.4	3	+0.8	4.960	3	—0.10
29	3½	46	1.3	2	—0.3	5.024	2	—0.03
Oct. 2	4	44	1.0	6	0.6	.240	3	+0.18
4	3	46	0.6	4	1.0	.090	3	+0.04
8	3½	56	1.0	4	—0.6	.044	3	—0.01
9	2½	56	2.3	4	+0.7	.020	3	0.04
Nov. 10	3½	42	2.3	4	+0.7	.046	4	0.01
11	3½	46	1.6	4	0.0	.049	4	—0.01
12	3½	52	1.5	4	—0.1	.062	3	+0.01
Feb. 1	3	33	332.4	4	+0.8	5.021	4	—0.03
Mean		48°	331.64	±0.16		5.055	±0.013	
W5								
1909 Feb. 17	4	35°	345°.9	4	+0.7	8.781	2	—0.04
Nov. 11	3½	46	4.2	4	—1.0	.822	5	0.00
12	4	52	345.4	6	+0.2	8.872	4	+0.05
Mean			345.17	±0.38		8.825	±0.018	
Ormond Stone								
1909 Sept. 14	4	56°	10.4	4	+0.9	5.057	3	—0.01
Oct. 7	3½	56	10.3	2	+0.8	.061	2	—0.01
8	3	56	9.5	4	0.0	.117	3	+0.05
Nov. 10	3½	42	8.0	4	—1.5	.116	4	0.05
11	3½	46	9.3	6	0.2	5.094	3	+0.03
12	3½	52	9.4	4	0.1	4.984	4	—0.08
1910 Feb. 1	3	33	9.8	4	+0.3	5.044	4	—0.02
Mean			9.53	±0.19		5.068	±0.012	
Alvan Clarke								
1909 Sept. 9	4	58°	23°.6	4		0.603	4	

Observations with reference to 558.

Date			S	T	P	n	Δ	R	n	Δ
					523					
1909	Oct.	8	3	56°	272.0	4	+0.2	8.346	2	—0.04
		9	2½	55	1.9	4	+0.1	.402	2	+0.01
		12	2	49	1.8	4	0.0	.473	3	+0.08
		13	1½	35	1.7	4	—0.1	.310	3	—0.08
		17	2	42	1.6	4	—0.2	.408	4	+0.02
		20	2	36	1.9	4	+0.1	.449	4	0.06
		26	2½	38	1.7	4	—0.1	.408	3	+0.02
		27	2	47	1.8	4	0.0	.347	4	—0.04
	Nov.	10	3	42	1.9	4	+0.1	.319	4	—0.07
		11	3½	46	271.4	4	—0.4	8.431	4	+0.04
Mean				45°	271.77	±0.03		8.390	±0.012	
					570					
1908	Oct.	2	1½	45°	157.4	4	+0.1	----------	----------	
1909	Oct.	1	1½	51	8.0	4	+0.7	16.943	2	+0.06
		2	2½	44	7.3	4	0.0	.902	4	0.02
		4	3	46	7.1	4	—0.2	.893	4	+0.01
		8	3	56	7.1	3	—0.2	.861	4	—0.02
		9	2½	55	7.4	4	+0.1	.888	3	0.00
		12	2	49	7.4	4	+0.1	.832	4	0.05
		13	1½	35	7.0	4	—0.3	.883	6	0.00
		17	2	42	7.5	4	+0.2	.832	3	0.05
		20	2	36	7.3	4	0.0	.870	3	—0.01
		26	2½	38	157.3	4	0.0	16.933	3	+0.05
Mean				45°	157.34	±0.04		16.884		
					573					
1908	Nov.	16	2½	27°	128°.9	2	+0.5	----------	----------	
		18	2	41	9.3	4	0.9	----------	----------	
1909	Jan.	23	3	47	9.0	4	0.6	9.503	2	+0.01
	Mar.	17	2½	37	8.8	4	+0.4	.448	2	—0.05
		23	3½	40	8.2	4	—0.2	.451	2	—0.04
	Sept.	19	2	58	8.5	4	+0.1	----------	----------	
		24	2½	55	7.7	5	—0.7	.525	3	+0.03
		25	2½	51	7.9	4	—0.5	.519	2	+0.02
		29	2	46	----------	----------		.402	4	—0.09
	Oct.	1	1½	51	8.5	4	+0.1	.554	3	+0.06
		2	2½	44	8.0	4	—0.4	.518	4	+0.02
		4	3	46	8.3	4	0.1	.460	4	—0.04
		8	3	56	127.8	2	—0.6	9.581	2	+0.08
Mean				46°	128.41	±0.10		9.496	±0.012	

Observations with reference to 558—Continued.

Date			S	T	P	n	Δ	R	n	Δ
					575 (Other measures referred to 628.)					
1908	Nov.	16	2½	27	38.8	4		12.264	2	
					581					
1909	Mar.	23	3½	40°	115.3	4	+0.1	9.878	3	+0.02
	Sept.	19	2	58	5.4	4	0.2	.746	2	−0.11
		24	2½	55	5.4	4	+0.2	.884	2	+0.02
		25	2½	51	4.9	4	−0.3	.871	3	+0.01
		29	2	46	4.6	4	0.6	.660	3	−0.20
	Oct.	1	2	51	4.3	4	−0.9	9.908	4	+0.04
		2	2½	44	5.5	4	+0.3	10.006	3	0.14
		4	3	46	5.6	4	+0.4	9.886	4	0.03
		8	3	56	5.1	4	−0.1	.891	2	+0.03
		9	2½	55	115.5	4	+0.3	9.859	3	0.00
Mean				50°	115.16	±0.09		9.859	±0.016	

Observations with reference to 669.

Date			S	T	P	n	Δ	R	n	Δ
					635 (Other measures referred to 628.)					
1908	Nov.	18	2	41°	268.8	4	+0.3			
1909	Jan.	23	3	47	8.4	4	−0.1	5.627	4	−0.03
	Feb.	4	2	40	9.0	4	+0.5	.655	2	0.00
		6	2	35	8.4	5	−0.1	.743	2	+0.09
		20	2½	50	8.4	4	−0.1	.681	4	+0.02
		25	2	30	8.7	4	+0.2	.595	2	−0.06
		26	2½	35	267.6	4	−0.9	5.632	3	−0.02
Mean				40°	268.47	±0.07		5.656	±0.013	
					641					
1909	Jan.	23	3	47°	282.0	4	−1.1			
	Feb.	20	2½	50	3.9	4	+0.8	5.465	2	0.00
	Mar.	18	3	37	3.1	4	0.0	.587	2	+0.12
		23	3½	40	2.9	4	−0.2	.453	4	−0.01
	Sept.	19	2½	58	3.5	4	+0.4	.444	4	0.02
		24	2½	55	2.5	4	−0.6	.350	2	−0.11
		25	2½	51	2.0	4	−1.1	.479	3	+0.02
	Oct.	1	2½	51	3.5	4	+0.4	.488	4	0.03
		4	3	46	3.4	4	0.3	.521	3	+0.06
		8	3	56	3.6	4	0.5	.408	3	−0.05
		9	2½	55	283.6	4	+0.5	5.426	3	−0.04
Mean				48°	283.09	±0.14		5.462	±0.012	

Observations with reference to 669—Continued.

Date			S	T	P	n	Δ	R	n	Δ
652										
1909	Sept.	19	2½	58°	336.6	2	+0.5	7.893	3	+0.05
		24	2½	55	6.6	4	+0.5	.832	2	—0.01
		25	2	51	5.6	6	—0.5	.820	3	0.02
		28	2	46	5.9	4	0.2	.820	2	0.02
		29	2	46	6.1	2	0.0			
	Oct.	1	2	51	6.0	4	0.1	.788	2	—0.06
		2	2½	44	6.1	4	0.0	.863	5	+0.02
		4	3	46	5.8	4	0.3	.851	3	0.01
		7	4	56	5.9	4	—0.2	.898	2	+0.06
		8	3	56	336.3	4	+0.2	7.821	3	—0.02
Mean				51°	336.09	±0.07		7.843	±0.008	
657										
1909	Sept.	19	2½	58°	339.7	2	—0.4	7.012	2	—0.06
		24	2½	55	40.3	4	+0.2	.072	3	0.00
		25	2	51	40.5	4	0.4	.049	2	—0.02
		28	2	46	40.8	4	0.7	.158	3	+0.08
		29	2	46	40.7	4	0.6	.076	2	0.00
	Oct.	1	2	51	40.3	4	+0.2	.086	3	+0.01
		2	2½	44	39.9	4	—0.2	.066	4	—0.01
		4	3	46	39.4	4	0.7	.054	3	—0.02
		7	4	56	39.3	4	0.8	.120	2	+0.05
		8	3	56	339.8	4	—0.3	7.052	2	—0.02
Mean				51°	340.07	±0.12		7.074	±0.007	
663										
1909	Sept.	19	2½	58°	351.4	4	+0.1	4.827	3	—0.01
		24	2½	55	0.9	4	—0.4	.826	4	—0.01
		25	2	51	1.7	4	+0.4	.840	2	+0.01
		28	2	46	1.6	4	0.3	.880	3	+0.05
	Oct.	1	2½	51	1.8	4	0.5	.758	3	—0.08
		2	2½	44	1.8	4	+0.5	.867	4	+0.03
		4	3	46	1.2	4	—0.1	.843	3	+0.01
		8	3	56	0.3	4	1.0	.832	2	0.00
		9	2½	55	1.2	4	0.1	.813	4	—0.02
		12	2	49	350.9	4	—0.4	4.844	3	+0.01
Mean				51°	351.28	±0.10		4.833	±0.006	

Observations with reference to 669—Continued.

681

Date			S	T	P	n	Δ	R	n	Δ
1909	Sept.	19	2½	58°	16.8	2	+0.1			
		24	2½	55	16.7	4	0.0	7.950	2	—0.07
		25	2	51	17.2	4	+0.5	8.107	3	+0.09
		28	2	46	16.3	4	—0.4	8.076	3	+0.06
	Oct.	1	2½	51	16.4	4	—0.3	7.960	4	—0.06
		2	2½	44	16.8	4	+0.1	8.036	4	+0.02
		4	3	46	16.6	4	—0.1	7.996	4	—0.02
		8	3	56	16.9	4	+0.2	8.051	2	+0.03
		9	2½	55	16.8	4	+0.1	7.991	3	—0.03
		12	2	49	16.5	4	—0.2	8.011	3	—0.01
Mean				51°	16.70	±0.05		8.018	±0.012	

Observations with reference to 685.

666

Date			S	T	P	n	Δ	R	n	Δ
1909	Sept.	19	2½	58°	201.7	4	+0.3	10.985	2	—0.03
		24	2	55	1.9	5	+0.5	11.063	3	+0.05
		25	2½	51	1.2	4	—0.2	11.020	2	+0.01
	Oct.	1	2	51	0.7	4	0.7	10.940	3	—0.07
		2	4	44	0.9	4	0.5	10.941	3	—0.07
		4	3	46	1.0	4	0.4	11.027	4	+0.02
		8	3	56	1.2	4	—0.2	11.013	2	0.00
		9	2½	55	2.1	4	+0.7	11.017	3	0.01
		17	2½	42	1.9	4	+0.5	11.016	3	0.00
		26	2½	38	201.0	4	—0.4	11.086	3	+0.08
Mean				49°	201.36	±0.12		11.011	±0.011	

675

Date			S	T	P	n	Δ	R	n	Δ
1909	Feb.	17	4	35°	270.8	4	+0.4	2.445	2	—0.02
	Sept.	14	4	58	69.0	3	—1.4			
	Oct.	2	4	44	71.0	4	+0.6	.448	4	—0.02
		4	3	46	70.5	4	+0.1	.472	4	0.00
		8	3	56	69.4	4	—1.0	.480	3	+0.01
		9	2½	55	71.0	4	+0.6	.579	3	0.11
	Nov.	10	3	42	70.2	4	—0.2	.516	4	0.05
		11	3½	46	69.7	4	0.7	.478	4	+0.01
		12	3	52	70.0	4	—0.4	.374	4	—0.10
1910	Feb.	1	3	33	272.7	4	+1.3	2.431	4	—0.04
Mean				47°	270.43	±0.19		2.469	±0.012	

Observations with reference to 685—Continued.

Date			S	T	P	n	Δ	R	n	Δ
					677					
1909	Sept.	19	2½	58°	190.7	2	—0.3	11.002	4	+0.06
		25	2½	51	89.9	2	1.1	10.916	3	—0.03
	Oct.	1	2	51	90.4	4	—0.6	.962	3	+0.02
		2	4	44	91.3	4	+0.3	.789	3	—0.16
		4	3	46	90.9	4	—0.1	.948	4	0.00
		8	3	56	191.6	4	+0.6	10.943	2	0.00
		9	2½	55	191.9	4	+0.9	10.949	3	0.00
		17	2½	42	1.2	4	0.2	10.932	4	—0.01
		26	2½	38	1.2	4	+0.2	11.080	4	+0.14
		27	2	47	190.7	4	—0.3	10.914	3	—0.03
Mean				49°	190.98	±0.12		10.944	±0.012	
					686					
1909	Feb.	17	4	35°	344.1	4	+0.9	3.440	2	—0.12
	Sept.	19	2½	58	4.5	2	1.3	.456	2	—0.10
	Oct.	2	4	44	3.5	4	+0.3	.754	4	+0.19
		4	3	46	3.1	4	—0.1	.694	4	0.13
		8	3½	56	3.4	4	+0.2	.670	3	+0.11
		9	2½	55	2.4	4	—0.8	.524	3	—0.04
	Nov.	10	3½	42	3.2	6	0.0	.563	3	+0.01
		11	3½	46	2.8	4	0.4	.542	4	—0.01
		12	3	52	2.6	4	0.6	.470	4	0.09
1910	Feb.	1	3	33	342.8	4	—0.4	3.446	4	—0.11
Mean				47°	343.24	±0.14		3.556	±0.026	
					688					
1909	Oct.	2	4	44°	23.6	5	—0.1	7.913	3	—0.05
		4	3	46	4.1	4	+0.4	.957	4	—0.01
		8	3½	56	3.4	4	—0.3	.993	3	+0.03
		9	3	55	3.4	4	—0.3	.979	3	+0.02
	Nov.	11	3½	46	3.8	4	+0.1	.936	4	—0.03
		12	3½	52	3.1	4	—0.6	7.882	4	—0.08
	Feb.	1	3½	33	24.7	4	+1.0	8.082	3	+0.12
Mean				48°	23.73	±0.14		7.963	±0.017	

Observations with reference to 685—Continued.

Date	S	T	P	n	Δ	R	n	Δ
708								
1909 Sept. 9	2½	53°	93.0	4	+0.6	5.352	6	+0.02
16	3½	53	2.2	4	—0.2	.355	8	0.02
22	3	59	2.1	4	—0.3	----------		----------
25	3½	63	2.7	4	+0.3	.427	3	+0.09
Oct. 2	1½	45	2.5	4	0.1	----------		----------
Nov. 9	2½	44	2.8	4	0.4	.315	4	—0.02
18	2	41	2.5	4	0.1	.341	4	+0.01
20	1	42	2.5	4	+0.1	.398	4	+0.06
21	1	36	1.8	4	—0.6	.315	4	—0.02
1909 Jan. 21	3	32	2.2	4	0.2	.343	2	+0.01
22	2½	42	2.2	4	—0.2	.259	2	—0.08
23	3	47	92.4	4	0.0	5.249	2	—0.09
Mean		46°	92.41	±0.06		5.335	±0.011	
709								
1908 Sept. 16	3½	53°	128.5	4	0.0	----------		----------
Oct. 2	1½	45	8.3	4	—0.2	----------		----------
Nov. 18	2	41	8.3	4	—0.2	6.705	4	—0.06
1909 Jan. 21	3	32	8.5	4	0.0	.745	2	—0.02
22	2½	42	8.6	4	+0.1	.783	2	+0.02
23	3	47	8.0	4	—0.5	.725	2	—0.04
Sept. 24	2	55	9.0	4	+0.5	----------		----------
25	2	51	8.4	4	—0.1	.819	3	+0.05
28	2	46	8.7	4	+0.2	.868	2	+0.10
29	2	46	8.7	4	+0.2	.735	4	—0.03
Oct. 8	3	56	128.0	2	—0.5	6.752	2	—0.01
Mean		47°	128.46	±0.06		6.766	±0.012	
724								
1908 Sept. 16	3½	53°	134.0	4	—0.3	----------		----------
22	3	59	4.4	4	+0.1	----------		----------
25	3½	63	4.2	4	—0.1	----------		----------
Oct. 2	1½	45	4.5	4	+0.2	----------		----------
Nov. 9	2½	44	4.5	4	0.2	11.839	4	—0.05
18	2	41	4.6	4	0.3	.887	4	0.00
20	1	42	4.5	4	0.2	.999	4	+0.11
21	1	36	4.4	4	0.1	.908	4	+0.02
1909 Jan. 21	3	32	4.5	4	+0.2	.890	2	0.00
22	2½	42	4.2	4	—0.1	.887	2	0.00
23	3	47	4.6	4	+0.3	.859	2	—0.03
Sept. 28	2	46	4.0	4	—0.3	.898	2	+0.01
Oct. 8	3	56	133.9	2	—0.4	11.805	4	—0.08
Mean		47°	134.33	±0.05		11.886	±0.012	

Observations with reference to 685—Continued.

741

Date			S	T	P	n	Δ	R	n	Δ
1908	Sept.	16	3½	53°	97.5	4	—0.3	----------	----------	
		22	3	59	7.6	4	—0.2	----------	----------	
		25	3½	63	8.1	4	+0.3	----------	----------	
	Oct.	2	1½	45	8.2	4	0.4	----------	----------	
	Nov.	9	2½	44	7.8	4	0.0	13.074	4	+0.02
		18	2	41	8.2	4	0.4	.033	4	—0.02
		20	1	42	7.9	4	+0.1	.129	4	+0.08
		21	1	36	7.7	4	—0.1	.069	4	+0.02
1909	Jan.	21	3	32	7.6	4	—0.2	13.053	2	0.00
		22	2½	42	7.8	4	0.0	12.949	2	—0.10
		23	3	47	7.8	4	0.0	13.043	2	—0.01
	Sept.	28	2	46	8.0	4	+0.2	.129	3	+0.08
	Oct.	8	3	56	97.6	3	—0.2	13.003	3	—0.05
Mean				47°	97.83	±0.04		13.054	±0.012	

Miscellaneous Observations.

640-619

Date			S	T	P	n	Δ	R	n	Δ
1908	Sept.	9	2½	53°	275.8	4	0.0	2.174	4	—0.01
		11	2	55	275.8	6	0.0	2.192	8	+0.01
Mean				54°	275.80	----------		2.183	----------	

640-624

Date			S	T	P	n	Δ	R	n	Δ
1908	Sept.	9	2½	53°	300.4	4	0.0	1.948	4	0.00
		11	2	55	300.5	4	0.0	1.943	8	0.00
Mean				54°	300.45	----------		1.946	----------	

640-633

Date			S	T	P	n	Δ	R	n	Δ
1908	Sept.	22	3	59°	225.6	4		1.195	4	

Miscellaneous Observations—Continued.

Date	S	T	P	n	Δ	R	n	Δ
			619-624					
1908 Sept. 9	2½	53°	31.9	4	—1.0	0.877	6	0.00
11	2	55	33.9	4	+1.0	0.892	8	+0.01
25	3½	63	33.0	4	+0.1	0.876	8	—0.01
Mean		56°	32.93			0.882		
			619-617					
1908 Sept. 22	3	59°	352.8	4	—0.1	0.433	4	+0.02
25	3½	63	353.0	6	+0.1	0.403	4	—0.02
Mean		61°	352.90			0.418		
			624-617					
1908 Sept. 22	3	59°	240.1	6		0.619	4	
			628-570					
1908 Sept. 11	2	55°	199.8	4	+0.1			
25	3½	63	9.5	4	—0.2			
Oct. 2	1½	45	9.8	4	+0.1			
5	2	51	9.7	4	0.0			
Nov. 9	2½	44	199.6	4	—0.1			
Mean		53°	199.68	±0.04				

Miscellaneous Observations—Continued.

Date			S	T	P	n	Δ	R	n	Δ
628-708										
1908	Sept.	11	2	55°	123.1	4	+0.3			
		16	3½	53	2.4	4	—0.4			
		22	3	59	2.6	4	—0.2			
		25	3½	63	2.8	4	0.0	18.175	8	+0.08
	Nov.	9	2½	44	3.1	4	+0.3			
		16	2½	27	2.9	4	0.1			
		18	2	41	3.0	4	+0.2			
		20	1	42	122.5	4	—0.3	18.019	4	—0.08
Mean				48°	122.80	±0.07		18.097	±0.048	
628-724										
1908	Sept.	11	2	55°	134.5	4	+0.2			
		16	3½	53	4.1	4	—0.2			
		22	3	59	3.9	4	—0.4			
		25	3½	63	4.3	4	0.0			
	Nov.	9	2½	44	4.4	4	+0.1			
		18	2	41	134.4	4	+0.1			
Mean				52°	134.27	±0.06				
628-741										
1908	Sept.	11	2	55°	116.6	4	+0.2			
		16	3½	53	6.2	4	—0.2			
		22	3	59	6.2	4	0.2			
		25	3½	63	6.3	4	—0.1			
	Nov.	9	2½	44	6.6	4	+0.2			
		18	2	41	116.6	4	+0.2			
Mean				52°	116.42	±0.06				

PART IV.

Catalogue of Positions.

No.	G.P.B.	H.-S.	H.	W.C.B	Ll.	Li.	Mag.	Δα	Δδ	N.
1	523	45	τ	5	40	l	—	−243″.55	−115″.32	10
2	558	50	—	9	39	υ	10.7	160.91	117.87	120
3	567	51	—	10	—	—	13.3	99.79	4.78	12
4	570	53	σ	13	33	n	—	96.80	271.42	16
5	573	54	—	12	35	n_1	12.4	87.58	−176.01	12
6	W_1	—	—	—	—	—	—	86.63	+89.46	4
7	575	—	—	11	45	—	12.0	85.59	−22.28	11
8	581	ad 54	—	—	—	—	13.1	73.29	159.87	10
9	W_2	—	—	—	—	—	—	62.57	24.52	4
10	589	57	—	15	—	—	12.4	57.42	21.03	12
11	595	—	—	—	43, e	—	13.2	46.57	−16.36	8
12	W_3	—	—	—	—	—	—	40.59	+77.31	2
13	601	—	—	—	—	—	13.6	32.93	−28.36	7
14	602	—	—	—	—	—	13.4	32.84	−61.58	10
15	W_4	—	—	—	—	—	—	23.66	+43.82	10
16	608	—	—	—	f	—	13.1	23.18	−16.22	10
17	W_5	—	—	—	—	—	—	22.26	+84.08	3
18	612	—	—	16	—	—	13.0	18.18	26.35	10
19	618	—	—	19	h	—	12.8	12.14	26.39	11
20	617	64	γ′	—	—	bb_1	10.3	10.34	13.04	11
21	619	65	γ	17	—	b	7.4	9.76	+8.66	10
22	622	II	—	18	—	—	12.0	7.44	−26.88	11
23	621	ad II	—	—	—	—	14.0	6.08	−39.59	10
24	624	67	δ	21	—	d	5.7	4.90	+16.11	10
25	625	ad II	—	—	d	—	—	−1.18	−30.14	2
26	628	69	α	22	—	a	5.7	—	—	—
27	A. Clarke	—	—	—	—	—	—	+2.38	+5.44	1
28	633	71	α′	—	—	—	10.0	3.60	−2.00	10
29	631	—	—	—	—	—	13.6	4.80	−40.32	10
30	635	70	—	23	2	i	9.5	7.20	+97.40	16
31	O. Stone	—	—	—	—	—	14.2	8.27	+49.27	7
32	636	—	—	24	—	—	13.4	8.93	−10.82	11
33	641	III	—	—	—	—	12.8	10.44	+111.33	11
34	640	73	θ	25	—	c	7.1	11.82	6.55	12
35	642	—	—	—	—	—	13.7	13.68	47.03	8
36	647	75	—	26	9, 1	—	11.0	21.72	+37.29	12
37	648	—	—	—	—	—	13.7	25.11	−12.25	10
38	651	ad 75	—	27	—	—	12.3	27.94	+44.99	10
39	652	76	y′	32	—	f″	11.7	31.54	169.80	10
40	654	78	—	31	—	—	13.1	32.69	9.63	10
41	657	80	y″	33	4	f″	11.3	39.10	164.68	10
42	663	84	ω	37	—	f″	12.8	55.65	+146.22	10
43	666	81	—	30	—	—	13.4	57.43	−195.10	10
44	669	87	ν	39	10	k	8.8	62.87	+99.20	120
45	671	88	—	41	18	e″	11.2	68.69	−25.18	10
46	675	—	—	—	a	κ	13.7	72.63	92.21	10
47	677	ad 81	—	34	—	—	13.8	76.42	199.93	10
48	676	ad 88	z	43	k	—	12.8	79.96	−30.22	11
49	681	89	—	—	—	e″	—	85.58	+174.83	10
50	686	91	ε	44	—	—	13.4	95.95	−60.49	11
51	685	93	—	45	26	e	—	96.96	94.04	120
52	688	—	—	—	—	—	—	128.54	22.13	8
53	709	100	G	51	—	μ	12.0	149.17	135.52	11
54	708	101	ξ	50	23	f	6.3	149.48	96.25	20
55	724	104	λ	55	25	h	8.9	180.77	175.92	19
56	741	110	η	61	19	g	7.9	+224.41	−111.57	19

PART V.

Positions of Other Observers Reduced to 1910.0.

Star	$\Delta\alpha$	$\Delta\delta$	Obs.
523	—246.6	—114.6	H
	244.2	115.6	WB
	243.5	116.6	Ll
	243.8	114.8	GB
	—245.8	—115.5	L-S
	—244.6	—115.4	Mean
	—243.6	115.3	W
558	—154.9	—122.0	H
	161.0	119.2	WB
	165.1	115.7	Ll
	160.3	117.8	GB
	162.5	118.2	L-S
	—161.4	118.1	Mean
	160.9	117.9	W
567	—101.8	—10.3	H
	103.7	7.8	GB
	103.1	8.1	Mean
	99.8	4.8	W
570	—96.1	—270.0	H
	96.4	271.9	WB
	93.7	272.6	Ll
	96.9	272.7	GB
	98.4	271.6	L-S
	96.4	272.0	AGC
	96.7	271.8	Mean
	96.8	271.4	W
573	—92.5	—172.0	H
	90.1	180.3	WB
	87.6	174.7	Ll
	89.0	178.5	GB
	90.4	176.2	L-S
	89.3	177.0	Mean
	87.6	176.0	W
575	--------	—21.4	WB
	—85.2	21.4	Ll
	85.7	22.0	GB
	86.4	23.9	L-S
	85.9	22.4	Mean
	85.6	22.3	W

Star	$\Delta\alpha$	$\Delta\delta$	Obs.
581	—77.6	—158.7	GB
	73.3	159.9	W
589	—58.1	—20.1	GB
	58.5	22.3	L-S
	58.3	21.2	Mean
	57.4	21.0	W
595	—47.8	—14.8	GB
	46.6	16.4	W
601	—37±	—30.8	GB
	32.9	28.4	W
602	—33.1	—67.3	GB
	32.8	61.6	W
608	—23.7	—17.9	GB
	23.2	16.2	W
612	—17.0	+24.8	GB
	19.0	21.8	L-S
	18.0	23.3	Mean
	18.2	26.4	W
617	—10.4	+12.8	H
	10.6	13.0	GB
	10.3	12.4	L-S
	10.6	12.8	Hl
	10.3	12.9	B
	10.4	12.8	Mean
	10.3	13.0	W
618	—11.1	+24.7	GB
	13.1	22.7	L-S
	12.1	23.7	Mean
	12.1	26.3	W
619	—10.4	+8.7	H
	9.6	8.4	WB
	9.9	8.7	GB
	9.4	8.4	L-S
	10.0	8.6	Hl
	9.7	8.6	B
	9.8	8.6	Mean
	9.8	8.7	W

Positions of Other Observers Reduced to 1910.0—Continued.

Star	$\Delta\alpha$	$\Delta\delta$	Obs.
621	—8.9	—36±	GB
	8.7	31±	L-S
	8.8	33.5	Mean
	6.1	39.6	W
622	—8.4	—27.8	GB
	8.2	27.6	L-S
	8.3	27.7	Mean
	7.4	26.9	W
624	—5.8	+16.1	H
	3.7	16.4	WB
	4.9	16.1	GB
	4.6	15.9	L-S
	5.0	16.0	Hl
	4.9	16.0	B
	4.8	16.0	Mean
	4.9	16.1	W
625	—4.8	—28±	GB
	1.2	30.1	W
631	+2.2	—4.2	GB
	4.8	40.3	W
633	+4.5	—1.2	H
	3.5	2.1	GB
	2.9	1.9	L-S
	3.4	2.0	Hl
	3.4	2.0	B
	3.4	2.0	Mean
	3.6	2.0	W
635	+4.0	+95.2	H
	5.6	98.5	WB
	9.2	96.6	Ll
	7.9	98.3	GB
	5.8	98.1	L-S
	6.8	97.8	Mean
	7.2	97.4	W
636	+7.6	—8.7	GB
	8.9	10.8	W

Star	$\Delta\alpha$	$\Delta\delta$	Obs.
640	+12.0	+6.7	H
	12.4	6.7	WB
	11.6	6.8	GB
	12.1	6.6	L-S
	11.8	6.4	Hl
	11.9	6.5	B
	11.9	6.6	Mean
	11.8	6.6	W
641	+11.6	+111.2	GB
	7.2	107.2	L-S
	9.4	109.2	Mean
	10.4	111.3	W
642	+12.4	+48±	GB
	12.4	50.2	OS
	13.7	47.0	W
647	+21.6	+42.0	H
	23.4	41.4	Ll
	22.0	37.9	GB
	20.7	39.1	L-S
	21.7	39.3	Mean
	21.7	37.3	W
648	+23.6	—8.8	GB
	25.1	12.2	W
651	+28.8	+47.7	GB
	25.8	42.7	L-S
	27.3	45.2	Mean
	27.9	45.0	W
652	+31.5	+166.2	H
	31.2	168.4	WB
	30.2	171.4	GB
	30.4	171.7	L-S
	30.6	170.9	Mean
	31.5	169.8	W
654	+32.0	+20.2*	H
	32.5	9.8	GB
	33.8	9.5	L-S
	33.0	9.6	Mean
	32.7	9.6	W

Positions of Other Observers Reduced to 1910.0—Continued.

Star	Δα	Δδ	Obs.	Star	Δα	Δδ	Obs.
657	+39.0	+162.0	H	677	+76.7	—201.8	GB
	39.3	160.3	WB		76.4	199.9	W
	32.2*	165.7	Ll	681	84.1	+164.3	H
	39.6	165.0	GB		90.4	172.8	GB
	35.9	164.2	L-S		82.5	172.8	L-S
	38.2	163.9	Mean		85.6	171.7	Mean
	39.1	164.7	W		85.6	174.8	W
663	+56.9	+139.8	H	685	+102.6	—91.8	H
	60.3	148.1	WB		95.9	93.1	WB
	55.4	146.8	GB		95.5	92.4	Ll
	53.7	145.8	L-S		97.1	94.2	GB
	55.8	146.0	Mean		97.0	93.2	L-S
	55.6	146.2	W		97.2	93.9	N
666	+37.9*	—156.0*	H		97.1	94.5	M
	57.9	196.0	GB		97.5	94.4	AGC
	55.0	196.1	L-S		97.1	93.8	Mean
	56.4	196.0	Mean		97.0	94.0	W
	57.4	195.1	W	686	+98.6	—38.5	GB
669	+64.0	+98.6	H		96.0	60.5	W
	63.8	99.3	WB	688	+104.7	—17.5	GB
	60.7	99.5	Ll		128.5	22.1	W
	63.0	99.7	GB	708	+149.6	—96.1	H
	61.6	99.7	L-S		148.4	96.0	WB
	62.4	99.5	Mean		150.2	97.9	Ll
	62.9	99.2	W		150.0	99.3	GB
671	+72.5	—23.6	H		149.4	96.8	L-S
	65.4	35.2*	WB		150.4	97.7	AGC
	66.0	24.8	Ll		149.8	97.6	Mean
	68.7	24.8	GB		149.5	96.2	W
	68.1	24.7	L-S	709	+149.3	—132.9	H
	67.9	24.7	Mean		148.1	136.1	WB
	68.7	25.2	W		151.3	136.9	GB
675	+73.2	—93.8	GB		148.1	134.9	L-S
	72.6	92.2	W		149.4	135.7	Mean
676	+73.3	—37.2	WB		149.2	135.5	W
	77.6	28.0	GB				
	73.6	28.0	L-S				
	75.1	28.9	Mean				
	80.0	30.2	W				

Positions of Other Observers Reduced to 1910.0—Continued.

Star	Δα	Δδ	Obs.	Star	Δα	Δδ	Obs.
724	+181.6	−173.7	H	741	+223.0	−110.3	H
	178.1	174.4	WB		223.5	111.4	WB
	176.7	169.6	Ll		224.1	108.7	Ll
	181.5	176.9	GB		224.6	111.6	GB
	180.0	175.6	L-S		180.0	175.6	L-S
					226.4	112.6	AGC
	179.8	174.7	Mean				
	180.8	175.9	W		224.8	111.4	Mean
					224.4	111.6	W
				Al. Clarke	4.1	6.1	B
					2.4	5.4	W
				O. Stone	3.5	51.8	OS
					8.3	49.3	W

Comparisons with G. P. Bond and with the Mean of Other Observations.

Star	Bond—Wilson		Mean—Wilson	
	$\Delta\alpha$	$\Delta\delta$	$\Delta\alpha$	$\Delta\delta$
523	—0.2	+0.5	—1.0	—0.1
558	+0.6	+0.1	0.5	0.2
567	—3.9	—3.0	—3.3	3.3
570	0.1	1.3	+0.1	0.4
573	1.4	—2.5	—1.7	1.0
575	0.1	+0.3	0.3	0.1
581	4.3	1.2		
589	0.7	0.9	—0.9	0.2
595	1.2	+1.6		
601	4.1	—2.4		
602	0.3	5.7		
608	—0.5	1.7		
612	+1.2	1.6	+0.2	2.9
617	—0.3	0.0	—0.1	0.2
618	+1.0	1.6	0.0	2.6
619	—0.1	0.0	0.0	—0.1
621	2.8	+3.6	2.7	+6.1
622	1.0	—0.9	—0.9	—0.8
624	0.0	0.0	+0.1	—0.1
625	3.6	+2.1		
631	2.6	—1.7		
633	—0.1	—0.1	—0.2	0.0
635	+0.7	+0.9	—0.4	+0.4
636	—1.3	2.1		
640	—0.2	+0.2	+0.1	0.0
641	+1.2	—0.1	—1.0	—2.1
642	—1.3	+1.0	1.3	+2.2
647	+0.3	0.6	0.0	2.0
648	—1.5	3.4		
651	+0.9	2.7	0.6	0.2
652	—1.3	1.6	—0.9	+1.1
654	—0.2	0.2	+0.3	0.0
657	+0.5	0.3	—0.9	—0.8
663	—0.2	+0.6	+0.2	0.2
666	+0.5	—0.9	—1.0	—0.9
669	0.1	+0.5	0.5	+0.3
671	0.0	+0.4	0.8	0.5
675	+0.6	—1.6		
676	—2.4	+2.2	—4.9	+1.3
677	+0.3	—1.9		
681	4.8	2.0	0.0	—3.1
685	0.1	—0.2	+0.1	+0.2
686	+2.6	+22.0		
688	—23.8	+4.6		
708	+0.5	—3.1	0.3	—1.4
709	2.1	1.4	0.2	—0.2
724	0.7	—1.0	—1.0	+1.2
741	+0.2	0.0	+0.4	0.2
Alvan Clarke			+1.7	0.7
O. Stone			—4.8	+2.5

PART VI.

NOTES.

1. In Professor G. P. Bond's catalogue [H. C. O. Annals, Vol. V] I find Herschel 51 identified as Bond 575. The positions reduced to 1910.0 would in that case be—

	$\Delta\alpha$	$\Delta\delta$
Herschel	$-101''.8$	$-10''.3$
Bond	-85.7	-22.0

If, however, we identify Herschel 51 as Bond 567, a variable star which is at times nearly as bright as 575, the positions are much closer.

	$\Delta\alpha$	$\Delta\delta$
Herschel	$-101''.8$	$-10''.3$
Bond	-103.7	-7.8

Therefore I have assumed the latter identification to be the correct one.

2. The two measures made on 625 are very doubtful. Bond says in his notes that he "suspects" a faint star in the position given for 625. During a series of observations to determine the magnitudes of the stars in this region Professor Ormond Stone was unable to see any evidence of 625, although he looked for it repeatedly. The two measures given in this paper were exceedingly difficult and the object measured may have been simply a condensation in the nebula.

3. During the first six months of observation from September, 1908, to February 26, 1909, the star 654 was invisible. After a period of cloudy weather work was resumed on March 15, at which time 654 was quite bright. The following comparison of brightness was made on that night:

$$647 - 4 - 654 - 1 - 622 - 3 - 618.$$

Throughout the rest of March 654 was about equal in brightness to 647. As Orion was too near the sun for observation after April 1, the star was not seen again until September 9. At that time it was very faint. On November 10, 11 and 12, it was about at the limit of visibility and has not been seen since.

MAP OF STARS IN THE HUYGHENIAN REGION OF THE ORION NEBULA.

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AND

The Losses of Energy Due to Its Motion

BY

WILLIAM H. ECHOLS

UNIVERSITY OF VIRGINIA
Charlottesville, Virginia, U. S. A.

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ON THE FLOW OF WATER IN PIPES, CONDUITS AND OPEN CHANNELS AND THE LOSSES OF ENERGY DUE TO ITS MOTION.*

BY
WILLIAM H. ECHOLS.

I. THEORETICAL CONSIDERATIONS.

1. One of the most interesting chapters in the history of engineering, considered theoretically as well as practically, is that which relates to the flow of water. Its importance from the economic point of view is unquestioned and it has demanded the serious attention of the ablest engineers of every age. The difficulties which beset the problem have long been the despair of the theoretical investigator and the solution has been regarded as well near hopeless by the practical engineer. So many invalid empirical formulæ have been proposed to express the loss of energy due to the effect of the so-called "fluid friction" that the engineer in practice turns aside with impatient irritation at the suggestion of a new one. Almost without exception these formulæ have been essentially and purely empirical, designed without regard for the fundamental mechanical principles involved and which must form the basis of any really serviceable formula. They are, as a rule, mere interpolations designed to fit certain experiments the range of whose data is too limited to give them a dignity of importance above that of mere thumb rule confessedly and grossly approximate save for the narrow range for which they were designed. They serve the purpose of providing provisional estimates in special cases but lack the precision and generality necessary to provide the engineer with a working rule which he can apply with confidence and with a knowledge of the limits of probable error involved. The best formula that has been designed in point of generality is that of Kutter, which is an empirical elaboration of a formula previously given by Darcy, who in his turn generalized one originally due to Chezy. These engineers based their designs upon elaborate

*Read before the Scientific Section of the University of Virginia Philosophical Society at its regular meeting on March 12, 1910.

series of experiments and have furnished the profession with final formulæ which express the flow of water with a certain considerable degree of accuracy. Much quantitative measurement of the flow of water has been made since Kutter designed his formula for the flow of water in open channels, and the investigator has to-day at his command a much larger and more varied list of experimental determinations on which to base his enquiry. If one seeks extreme precision in the testing of any proposed theory one is discouraged at the start by a careful tabulation of the experimental data. Two presumably careful experimentors will differ in the final velocity of discharge of two cast iron pipes of equal lengths, diameters and pressures, frequently from five to ten per cent. The facts in the case are, as every engineer knows, no two pipes, conduits or channels are exactly alike as to roughness of surface and the results show this within certain limits always.

If one seeks a law for the loss of energy in an actual and real flow, one should consider what takes place in a flow under the most perfect of obtainable conditions in a real flow, in order to eliminate complications and the perturbations of all unknown causes of irregularity which obscure the fundamental facts and mislead as to the nature of the causes. Then proceed from the simple to the more complex, knowing that what is true under the best conditions must be true mutatis mutandis under more complex ones. The best condition for investigation is that which takes place in the flow of water in a straight cylindrical pipe whose interior surface is as smooth as possible without joints, and under a constant pressure head. The law of the losses for the rougher surfaces and under less favorable conditions whether in conduits or open channels should differ from these in degree rather than in kind when the surfaces are fairly regular.

2. If we speculate upon what takes place in the course of the flow of water in a straight pipe under constant head, the mean velocity of discharge is constant and we usually consider that after a limited time the flow is in a state of *steady* motion in which the stream lines are assumed as being parallel to the axis of the pipe. It is now known that this condition is far from the truth, save in very smooth tubes of extremely small diameter. The liquid is not in a condition of steady motion but in ordinarily clean pipes is in a condition of more or less *stationary* motion such that about the mean velocity at a given point that of a particle oscillates within certain limits. The lines of flow are not parallel to the axis of the cylindrical pipe but are perturbed and oscillatory with respect to that fixed position.

Given a pipe of constant diameter and straightness and whose inner surface is a homogeneous heterogeneity (that is whose inner surface is such that the roughness over any small area of it is the same all over the surface), the conditions of disturbance of interior stream lines under uniform flow may be assumed symmetrical with respect to the axis of the pipe. It appears that a particle on coming in contact with the inequality of the bounding surface is reacted upon by that surface and is sent back into the interior of the liquid toward the axis with a motion whose direction is transverse to that of the component direction parallel to the axis of the pipe, and the velocity of this particle compounds with those of others which it meets, thereby reducing the resultant velocity of flow in the direction of the discharge parallel to the pipe axis. In the case of the symmetrical pipe, which we are considering, the particles meeting the surface of the pipe simultaneously along the perimeter of a given cross-section are reacted on by the surface with common mean effect such that they form a ring or annulus of particles whose effect on the interior particles is a symmetrical reduction of the effective component of velocity in the direction of the axis and the more rapidly moving particles in the interior moving more in the direction of the axis drive these particles back to the surface of the pipe again whence they are again reflected further on. The result appears to be that something like a vortex ring of particles is rolled along the interior of the pipe surface, the effect of which is to reduce the velocity of flow in a manner analogous to and in effect similar to an actual choking or contraction of the cross-section of the pipe. Grossly exaggerated such a contraction is definitely known to exist in the case of a cylindrical adjutage of length equal to about three diameters. In this case for a flush fitted pipe the actual contraction of the diameter is approximately ten per cent. The converging lines of flow at the circular entrance contract to an effective cross-section, at mid-length of the pipe, whose area is eighty-one per cent. of that of the pipe; the lines then diverge and the flow fills the pipe on exit. In such a flow if the liquid were perfect there would be a gain of energy between the entrance and the contracted section, while between the contracted section and the exit there would be a corresponding loss equal to that gained, and there would be no loss or gain of energy at exit. But if we assume, which is generally admitted to be true, that the energy which would be gained in a perfect liquid is entirely expended in overcoming the internal "fluid-frictional" resistances, the loss of energy from the contracted section to exit would represent the loss of energy of discharge. Assuming the contraction of the diameter at ten per cent. to be approximately correct, v to be the mean velocity in the contracted section and V that of the discharge, the loss of head equivalent to the change of energy is,

$$h = \frac{v^2 - V^2}{2g} = [1 - (.81)^2] \frac{v^2}{2g} = .34 \frac{v^2}{2g}$$

The observed loss of head is actually about thirty-three per cent. of H.

In the common laboratory experiment of the formation of vortex smoke rings, when a chamber containing air and smoke receives an impulse discharging through a short tube a vortex ring is ejected; a number of such impulses in rapid succession expel a number of such rings in close proximity and which become confused under a steady flow and uniform discharge. Something of similar character takes place in the flow through a long pipe: the dragging of the air along the side walls causes a rolling of such rings of disturbance along the rough boundary and a corresponding interference in the lines of steady flow with retardation of the velocity of discharge.

We assume therefore that the loss of energy of water flowing in a pipe or canal is due to the geometrical irregularities of the surface over which the water flows, and to the effect of the internal eddies, vortices, disturbances of stream lines caused by the deflections and reactions of the rough, irregular surface in contact with water. That in a closed pipe under pressure the effect of these disturbances is equivalent in fact to a choking or contraction of the actual cross-section of the pipe.

It is well established that if a single solid annular obstruction be placed in a pipe there is a corresponding loss of energy in any actual flow. The actual loss is computed in every text book on this subject and is formulated on the basis of being measured by the change of momentum caused by the *impact* of an inelastic body of relatively insignificant mass impinging against another of relatively great mass and moving in the same direction—the small mass being the body of water involved in the contraction moving with higher velocity striking the mass of water in the remainder of the pipe moving with slower velocity. See Weisbach, Applied Mechanics, pp. 675, 884, 887; also Rankine, Civil Engineering, p. 677, where in commenting on the loss of energy due to a contraction in a pipe or canal he says, "It appears that all the energy due to the *difference* of the velocities is expended in fluid friction, and consequently there is a loss of head."

3. In the case which we surmise we suppose an indefinite number of equal annular contractions uniformly distributed along the length of the pipe, and that these effectively represent the equivalent of the choking of the pipe by eddies, vortices, regular or irregular as the case may be, whose effect is to produce a loss of energy and corresponding retardation in the flow. But now the volume of water moving in a contracted section is commensurable with that moving in the adjacent uncontracted portion, and hence the loss of energy caused by a single one of these contractions and its adjacent expansion is not to be measured as an impact but directly by the change or difference of the *kinetic energy* of these two masses.

If we fall into the error of assuming the contractions to be infinite in number and close together and each contraction to be infinitesimal, we would proceed as follows. The element of the loss of head at a cross-section of area ω at a distance l from the entrance of the pipe is equal to the differential of the kinetic energy, or

$$dh = \frac{vdv}{2g}. \tag{1}$$

But V being the velocity of discharge and Ω the actual cross-section of the pipe, the law of "continuity" requires

$$v\omega = V\Omega = \text{constant}.$$

$$\therefore \qquad dv = -\frac{v}{\omega}\, d\omega. \tag{2}$$

Let dr be the mean thickness of the obstruction and p a perimeter such that $p\,dr = \omega$. Then

$$dh = -\frac{v^2}{g}\frac{p}{\omega}\, dr. \tag{3}$$

If L is the total length of the pipe, the loss of head due to the resistances inside the pipe is

$$-h = \int_0^L \frac{v^2}{g}\frac{p}{\omega}\frac{dr}{dl}\, dl. \tag{4}$$

If we represent the mean values, throughout the pipe length, of v, p, ω, $\frac{dr}{dl}$ by V, P, Ω, $\frac{dR}{dL}$ the loss of head, dropping the negative sign, is

$$h = \frac{dR}{dL}\frac{P}{\Omega}\frac{V^2}{g}\int_0^L dl, \tag{5}$$

$$= 2\frac{dR}{dL}\frac{PL}{\Omega}\frac{V^2}{2g}. \tag{6}$$

This is Chezy's formula for the resistance head, in which

$$2\frac{dR}{dL},$$

represents his coefficient of resistance and which he assumed to be a constant. But, the results of experiment show that we have fallen into error in passing to the limit in the process of integrating as for a continuum. This perfect condition of affairs does not exist in nature and the result obtained above is only a first or roughly approximate statement of the facts. We proceed therefore more cautiously to the finite summation of the finite differences.

4. Let there be n uniformly distributed annular rings of easy curvature, of equal apertures and area ω, and distant apart ΔL. Let v be the mean velocity parallel to the pipe axis of the water passing through ω, and V that passing through the unobstructed area Ω. The loss of head at a single contraction and adjacent expansion is

$$\Delta h = \frac{v^2 - V^2}{2g} = \left(\frac{\Omega^2}{\omega^2} - 1\right)\frac{V^2}{2g}. \tag{7}$$

If ΔR represents the thickness of the ring and p is a mean perimeter such that $p\,\Delta R = \omega$, then the above loss is represented, after putting

$$\Omega = \omega + p.\,\Delta R,$$

by

$$\Delta h = \left(2\Delta R + \frac{p}{\omega}\cdot\Delta R^2\right)\frac{p}{\omega}\frac{V^2}{2g}. \tag{8}$$

The loss of head due to the n obstructions, where $n\,.\,\Delta L = L$, is

$$h = n\Delta h = \left(2\frac{\Delta R}{\Delta L} + \frac{p\Delta R}{\omega}\frac{\Delta R}{\Delta L}\right)\frac{pL}{\omega}\frac{V^2}{2g}. \tag{9}$$

If we assume that the thickness of the interference ΔR is small, which the experimental results show to be the case for smooth pipes, we may write without serious error P/Ω for p/ω and for either of these the reciprocal of the mean hydraulic radius r. Also putting

$$2\frac{\Delta R}{\Delta L} = k, \tag{10}$$

the resistance head becomes

$$h = \left(k + \tfrac{1}{2}k \frac{\Delta R}{r} \right) \frac{PL}{\Omega} \frac{V^2}{2g}. \tag{11}$$

If we combine into a single symbol

$$\tfrac{1}{2} k\Delta R = t,$$

then

$$h = \left(k + \frac{t}{r}\right) \cdot \frac{L}{r} \frac{V^2}{2g} \tag{12}$$

which is the form given to the value of the resistance head by the distinguished French engineer Darcy, in which Darcy assumed that k and t were certain constants. It is an injustice to Darcy to let it appear, as is so often done in reference to his formula as above presented in (12), that he was not aware of the insufficiency of the expression

$$m = k + \frac{t}{r}, \tag{13}$$

to represent the coefficient of resistance. In fact he proposed the more general expression

$$m = \mu + \frac{\lambda}{r} + \frac{a}{v} + \frac{\beta}{vr^2}$$

for this coefficient, in which a, β, λ, μ were certain positive constants.*

5. Taking up again our expression (9), we observe that if ΔR be considered arbitrarily small then equation (9) degenerates into Chezy's form given in (6), but the result of experiment shows, as was indicated before, that we had no right to pass to the limit of the continuum in the integration but should employ the finite summation and retain the vanishing terms in the form (9).

We shall, as is the conventional custom, term the expression

$$m = 2\frac{\Delta R}{\Delta L} + \frac{\Delta R}{\Delta L} \frac{\Delta R}{R}, \tag{14}$$

the *coefficient of resistance,* and in this expression we shall call

$$2\frac{\Delta R}{\Delta L}$$

*Ency. Brit., Ninth Ed., Hydromechanics, p. 510, Vol. XII.

the *slope of resistance* with respect to the length of the pipe, and

$$\tfrac{1}{2}\Delta R$$

the coefficient of contraction with respect to the cross-section. That $\Delta R/\Delta L$ and ΔR are not constants for a given pipe or channel is quite evident if our theory be sound. It would appear reasonable that the more swiftly the water is flowing with mean motion through the pipe the flatter would be the disturbed lines of flow and the smaller would be the thickness of the disturbing fluid obstruction, which fact is completely established by experiment in all ordinary cases of regular boundaries. It appears also from experiment that

$$\frac{\Delta R}{\Delta L}$$

not only decreases as the velocity increases but also as the mean radius increases. In other words it is a decreasing function of the two variables V and r, and is of the form

$$\frac{1}{f(V,r)}, \tag{15}$$

where f is a positive and increasing function of V and r. Moreover as it appears incredible that the mean slope of the roughness of disturbance $\Delta R/\Delta L$ should become infinite in any ordinary case when V and r become arbitrarily small, we conclude that the function $f(V,r)$ has a positive inferior limit $f(0,0)$. While the form of the function f is certain to be very complex in its complete expression, we feel justified in writing it equal to the first three terms of its expansion in positive powers of the variables V and r. Thus

$$f(V,r) = f(0,0) + AV^{\alpha} + Br^{\beta}, \tag{16}$$

wherein A and B are constants and α and β are positive numerical parameters but slightly dependent on V or r.

On substituting (16) in (15), then dividing the numerator and denominator by the positive constant $f(0,0)$, there results

$$m = \frac{\mu\left(1+\frac{\Delta R}{2r}\right)}{1 + aV^{\alpha} + br^{\beta}}, \tag{17}$$

wherein

$$\mu = \frac{2}{f(0,0)},\quad a = \frac{A}{f(0,0)},\quad b = \frac{B}{f(0,0)}.$$

The experiments show that ΔR decreases as V increases, and perhaps as r increases. We content ourselves by writing tentatively

$$\frac{\mu}{2}\frac{\Delta R}{r}=\frac{\lambda}{r^{p}V^{q}}, \tag{18}$$

wherein λ, p, and q, are constants to be determined by experiment.

The expression for the coefficient of resistance can be finally written

$$m=\frac{\mu+\dfrac{\lambda}{r^{p}V^{q}}}{1+aV^{\alpha}+br^{\beta}}. \tag{19}$$

In this we shall speak of the parameter μ as the *coefficient of roughness with respect to length;* it depends almost entirely on the geometrical configuration of the surface of the natural solid boundary over which the water flows. The parameter λ is a *coefficient of roughness with respect to the cross-section* and again depends almost entirely on the geometrical configuration of the surface of the solid boundary. The coefficients a and b appear to be almost absolute constants, while the parameters α and β are quite constant within certain narrow limits. The parameters p and q appear to be the same under all conditions and each appears to be equal or nearly so to unity. In the case of the constants p and q the experiments are wanting for very precise determination; the sequel will show, however, that as far as our present knowledge goes we are justified in taking each of them equal to unity. The result of comparison with experiments shows that for velocities between zero and twenty feet per second the exponent α of V may be taken as unity.

6. We shall therefore tentatively write

$$m=\frac{\mu+\dfrac{\lambda}{rV}}{1+aV+br^{\beta}}. \tag{20}$$

If H be the total static head, and h_e the head lost at entry, which for a flush fitted pipe is equal to

$$h_e=\tfrac{1}{2}\frac{V^2}{2g}$$

as determined by experiment. then the total head H is equal to the head due to entry plus that due to pipe resistance plus that due to velocity of discharge, or

$$H=\left(1.5+m\frac{L}{r}\right)\frac{V^2}{2g}. \tag{21}$$

Whence the velocity of discharge

$$V = \sqrt{\frac{2gH}{1.5 + m\dfrac{L}{r}}}, \tag{22}$$

$$= \sqrt{\frac{2g}{m + 1.5\dfrac{r}{L}}} \cdot \sqrt{Jr}, \tag{23}$$

wherein $J = H/L$ is termed the static grade or slope in contra-distinction to

$$s = \frac{h}{L}$$

which is called the hydraulic grade or slope, h being the head lost by resistance. If, as is the case in all *long* pipes, conduits and open channels, $1.5r/L$ is so small as to be negligible with respect to m, then J can be taken as equal to s and we write

$$V = c\sqrt{Jr}, \tag{24}$$

where

$$c = \sqrt{\frac{2g}{m}}, \tag{25}$$

is called the coefficient of velocity.

7. Fixing the attention on the equations (20) and (24), when the constants in (20) have been determined and H, L, r are known, the solution of the two equations (20) and (24) furnishes the velocity of discharge. This solution may be, and is generally, effected by choosing a provisional value of V and using it in (20) to compute a provisional value of m, which value substituted in (24) furnishes an approximate value of V. This value of V when used in (20) gives a closer value of m, which in (24) gives a value of V which may be taken as final. Otherwise, on the elimination of m the equations (20) and (24) furnish a quadratic equation in V, the solution of which is easily obtained in a form suitable for logarithmic computation by the ordinary well-known process of solution by means of circular functions. It is, however, preferable to eliminate V between (20) and (24) and determine m from the resulting quadratic equation. The solution is

$$m = \frac{\mu}{1 + br^{\beta}} \tan^2 \tfrac{1}{2}\Theta, \tag{26}$$

where

$$\tan\Theta = \frac{\sqrt{\mu}}{4a} \frac{\sqrt{\frac{1+br^\beta}{Jr}}}{1-\frac{\lambda}{64aJr^2}}, \tag{27}$$

the values of Θ lie between 0 and π.

Whence

$$c = \frac{8}{\sqrt{\mu}} \sqrt{1+br^\beta} \cot\tfrac{1}{2}\Theta, \tag{28}$$

$$V = \frac{8}{\sqrt{\mu}} \cot\tfrac{1}{2}\Theta \sqrt{(1+br^\beta)Jr.} \tag{29}$$

8. In eliminating m between (20) and (24) there results

$$V = \frac{2gJr^2}{\lambda} \frac{1+br^\beta}{1-a+\frac{\mu}{\lambda}Vr}. \tag{30}$$

Hence for capillary tubes, in which br^β and $\frac{\mu}{\lambda}Vr$ are negligible compared with 1, we have

$$V = \frac{2gJ}{\lambda(1-a)} r^2, \tag{31}$$

or the velocity of discharge varies as the square of the diameter and whence the quantity of flow as the *fourth* power of the diameter. This result was established by Poiseuille in a series of experiments conducted with the most elaborate care and precision; see Annales de Chemie, III, *xxi,* 76.

It appears from the above that in tubes of extremely small diameter the velocity of flow is independent of the coefficient of roughness with respect to length, which is the most important factor for pipes of ordinary diameters. The value of m for r very small is

$$m = \frac{\lambda}{1+aV} \frac{L}{rV}. \tag{32}$$

Hence the head lost by pipe resistance is

$$h = \frac{\lambda}{1+aV} \frac{L}{r^2} \frac{V}{2g}, \tag{33}$$

which indicates that the ratio of the lost head to velocity is nearly constant for any given tube. A result verified by the experiments of Reynolds, *Philosophical Transactions* (1883, Part III.).

9. Before proceeding to the determination of the constants we return for a moment to the expression (9) for the value of the resistance head. In passing to the subsequent form of this expression we assumed ΔR to be sufficiently small to justify us in writing ω / p equal to the mean hydraulic radius; we propose now to examine the exact value of (9) expressed in terms of ΔR and the radius R of the pipe. We have

$$\omega = \pi (R - \Delta R)^2.$$

Substituting in (7) we find

$$\Delta h = 2\rho \frac{[1 + (1 - \rho)^2]\,(1 - \tfrac{1}{2}\rho)}{(1 - \rho)^4}\,\frac{V^2}{2g}, \tag{34}$$

where $\rho = \Delta R / R$. Whence the exact value of the resistance head is

$$h = \frac{\Delta R}{\Delta L}\,\frac{[1 + (1 - \rho)^2]\,(1 - \tfrac{1}{2}\rho)}{(1 - \rho)^4}\cdot\frac{PL}{\Omega}\,\frac{V^2}{2g}, \tag{35}$$

giving for m the coefficient of resistance,

$$m = \frac{\Delta R}{\Delta L}\,\frac{[1 + (1 - \rho)^2]\,(1 - \tfrac{1}{2}\rho)}{(1 - \rho)^4}, \tag{36}$$

which, if one prefers, may be written

$$m = \frac{\Delta R}{\Delta L}\cdot\frac{1}{2\rho}\cdot\frac{1 - (1 - \rho)^3}{(1 - \rho)^4}. \tag{37}$$

Tracing the curve, in rectangular coördinates,

$$(1 - x)^4 y = [1 + (1 - x)^2]\,(1 - \tfrac{1}{2}x),$$

we find y is positive between $x = 0$ and $x = 2$, with a vertical asymptote at $x = 1$. The function y begins with the value 2 at $x = 0$ and increases continuously from $x = 0$ to $x = 1$, at which latter value y becomes positive and infinite, as x increases from 1 to 2, y continuously decreases from $+\infty$ to 0. For all other positive values of x the values of y are negative.

In the expression for m previous to this article the value of m continually increases as r or V or both decrease indefinitely, and becomes infinite when either of these variables vanish. If ΔR were the thickness of a solid obstruction it is obvious that the superior limit of ΔR would be R, at which the pipe would be completely closed and necessarily v would vanish. But ΔR is as we know merely a form for an equivalent thickness of a liquid disturbance represented by a contraction of stream lines, possibly representing an oscillatory tranverse vibration of fluid particles. Moreover the form in which it occurs in m is as a mean value of such a thickness or range of disturbance for the entire pipe length. Would it be possible therefore, under any real conditions, for finite values of R and H, for ΔR to be equal to R and thus $m = \infty$ and $V = 0$? Such a condition if even approximately realized could only be attained under small values of R. Furthermore, could it be possible for ΔR to exceed R in value and have as its superior limit $2R$ or D the diameter of the pipe? If so, then for values of the ratio $\Delta R / R$ increasing from 1 to 2 the values of m would diminish, becoming ultimately zero. In this speculation as to the possible behavior of ΔR we have not considered the behavior of the co-factor $\Delta R / \Delta L$ in (36); that for capillary tubes this does not becomes arbitrarily great is assured by the experiments of Poiseuille. It is, moreover, possible that under these circumstances it may become arbitrarily small when its co-factor becomes great and thus the value of m attain a finite superior limit between 0 and 2, in the neighborhood of $\Delta R = R$. It is altogether unlikely that, however $\Delta R / R$ may behave, $\Delta R / R$ can ever in any real case of flow actually attain the value 1 and m thus the value ∞. Some writers on hydraulics point out as a fact that under certain circumstances of relationship of r and s the coefficient m decreases with decrease of r and s; see the remarks on this matter in Hering and Trautwine's translation of Kutter's Flow of Water, with reference to the flow in open channels. The writer has not been able to get access to any reliable experiments on tubes or pipes which exhibit this contrary result in any manner suggestive of law, with the exception of a group of four experiments by Rennie on glass tubes of respective diameters .002; .004; .006; .0083 feet, quoted by J. T. Fanning in his Water Supply Engineering, p. 239. Corresponding to a velocity of about 10 feet per second, Fanning gives as the corresponding values of m for these four pipes respectively about

$$.00060; \quad .0020; \quad .0043; \quad .0043,$$

which are entirely at variance with any formula for m or any experiments known to the writer on pipes.

Granting it to be a fact that for certain values of H and R the values of m decrease as R decreases, could the explanation be that the representative equivalence of the range of the disturbance ΔR becomes greater than R and that under such circumstances there is a reflex interference of the disturbances with each other or transverse vibrations of the liquid particles with the effect that their reducing components of velocity parallel to the pipe axis annul each other and thus cause a relative augmentation of the flow parallel to the axis? The theoretical limit being reached when $\Delta R = 2R$, in which case as it were the waves of disturbance sent out from the boundary actually reach the axis and passing on expand to the boundary again, whence they are again reflected, causing complete interference. Experiments appear to be lacking to either settle this point or to justify further speculation concerning it.

10. In the foregoing we have spoken with special reference to pipes under pressure; there is no change to be made for straight regular conduits flowing part full or regular straight open channels than to replace the annular obstruction by a submerged dam of uniform height along the sides and bottom of the cross-section, under which circumstances p becomes the wetted perimeter and r the mean hydraulic radius or the area of the cross-section divided by its wetted perimeter. H becomes the total fall of the water surface in length L and in this case is the same as h.

II. DETERMINATION OF THE CONSTANTS.

11. The data for the determination of the constants in the formula for the velocity of flow in pipes under pressure consist mainly of experiments made on the sizes of pipes used for service in ordinary engineering practice. The diameters vary from one-fourth of an inch to eight feet, and the pressures are such that the velocities vary from one-tenth of a foot to twenty feet per second. The main mass of experimental data is confined to these limits in authentic and reliable experiments. While the range of the mean hydraulic radius from .005 to 2 feet is ample for pipes, it goes up to 4 or 6 for conduits, and to the extreme limit of 75 for great coastal plain rivers like the Mississippi. The conditions of regularity and normal consistency with the theoretical principles are most nearly realized in the class known in engineering practice as "new, clean, smooth pipes." This is a large class which in its differentiation consists of pipes of tin (tin lined), zinc, lead, glass, galvanized iron, wrought iron or cast iron coated with asphaltum or tar, wrought or cast iron pipes uncoated or lined with cement, riveted wrought iron or sheet iron, coated and uncoated, and pipes of wood, tile and earthenware. With this large variety of inner surfaces, varying character of joints and projections there is a considerable variety of roughness and consequently a considerable degree of discrepancy in the tabulated results of different experiments.

We shall first deal with the general class of "new clean pipes" as a whole for a provisional determination of the constants and subsequently make an analysis of the class differentiating into sub-classes and also a generalization outside of this class. Some investigators and writers prefer to use the coefficient of velocity c, (24), while others use the coefficient m. While to some extent this is a matter of choice and convenience it should be remembered that they vary inversely and that large variations in the one correspond to small variations in the other. To facilitate the computations and comparison of results Table I giving c in terms of m has been computed from (25). It should be remembered that if

$$1.5 \frac{r}{L}$$

is not so small as to be negligible compared with m then the tabulated value of m must be diminished by this amount.

TABLE I.

c	m	c	m	c	m	c	m
10	.6440	43	.0349	76	.01115	109	.00542
11	.5322	44	.0332	77	.01086	110	.00533
12	.4472	45	.0318	78	.01058	111	.00523
13	.3812	46	.0304	79	.01032	112	.00513
14	.3285	47	.0291	80	.01001	113	.00504
15	.2862	48	.0279	81	.00978	114	.00496
16	.2526	49	.0268	82	.00958	115	.00486
17	.2229	50	.0257	83	.00935	116	.00478
18	.1987	51	.0247	84	.00913	117	.00470
19	.1784	52	.0238	85	.00891	118	.00462
20	.1611	53	.0229	86	.00871	119	.00454
21	.1461	54	.0220	87	.00851	120	.00447
22	.1331	55	.0212	88	.00832	121	.00439
23	.1212	56	.0205	89	.00813	122	.00432
24	.1118	57	.0198	90	.00795	123	.00425
25	.1031	58	.0191	91	.00778	124	.00419
26	.0952	59	.0185	92	.00761	125	.00412
27	.0883	60	.01789	93	.00745	126	.00405
28	.0821	61	.01731	94	.00729	127	.00399
29	.0765	62	.01675	95	.00714	128	.00393
30	.0715	63	.01623	96	.00699	129	.00387
31	.0654	64	.01572	97	.00684	130	.00381
32	.0629	65	.01525	98	.00671	131	.00375
33	.0589	66	.01479	99	.00657	132	.00369
34	.0557	67	.01434	100	.00644	133	.00364
35	.0525	68	.01393	101	.00631	134	.00358
36	.0496	69	.01353	102	.00619	135	.00353
37	.0470	70	.01314	103	.00607	136	.00348
38	.0446	71	.01277	104	.00596	137	.00343
39	.0423	72	.01242	105	.00584	138	.00338
40	.0402	73	.01209	106	.00573	139	.00333
41	.0383	74	.01174	107	.05563	140	.00328
42	.0365	75	.01144	108	.00552	141	.00324

Table I—Continued.

c	m	c	m	c	m	c	m
142	.00319	159	.00254	176	.00207	193	.00172
143	.00315	160	.00251	177	.00205	194	.00171
144	.00310	161	.00248	178	.00203	195	.00169
145	.00306	162	.00245	179	.00201	196	.00167
146	.00302	163	.00242	180	.00198	197	.00166
147	.00298	164	.00239	181	.00196	198	.00164
148	.00294	165	.00236	182	.00194	199	.00162
149	.00290	166	.00233	183	.00192	200	.00161
150	.00286	167	.00230	184	.00190	201	.00159
151	.00282	168	.00228	185	.00188	202	.00157
152	.00278	169	.00225	186	.00186	203	.00156
153	.00275	170	.00222	187	.00184	204	.00154
154	.00271	171	.00220	188	.00182	205	.00153
155	.00268	172	.00217	189	.00180	206	.00151
156	.00264	173	.00215	190	.00178	207	.00150
157	.00261	174	.00212	191	.00176	208	.00148
158	.00258	175	.00210	192	.00174	209	.00147

12. The engineer, in the absence of a reliable formula, solves the problem of the flow of water in clean pipes by compiling all the data available in the form of experiments and computes either the coefficient c or m, knowing the observed velocity of flow, from

$$m = \frac{2gHr}{LV^2} - 1.5\frac{r}{L}, \tag{34}$$

$$c = \sqrt{\frac{2g}{m}}.$$

The observations being plotted with respect to coördinate axes for the variables V and m or c, the family of curves corresponding to different values of the parameter D are then interpolated, thus constituting a graphical table which at any time may be entered with two of the three variables m, V, D, and the third read off.

Such tables as these, as for example that of Mr. J. T. Fanning given in his Water Supply Engineering in which m is given as a function of V and D; or that of Mr. Hamilton Smith, Jr., given in his Hydraulics, a small table of which as prepared by Coffin is published in Public Water Supplies by Turneaure and Russell, in which c is given as a function of V and D; may be said to constitute the most reliable, accurate and recent information we possess on this subject. The writer has made use of these tables as well as numerous experiments by Darcy, Weisbach, Fanning, Smith and others, many, indeed most, of which are found tabulated in the 1,200 examples given in Hering and Trautwine's translation of Kutter's Flow of Water in Open Channels,* in determining the constants.

Kutter's formula, built empirically upon Darcy's and designed especially for open channels, in which the conditions are rougher and more irregular than for pipes, is the only one of wide and general scope in which V is given in terms of H, L, and r, which is employed by the profession with more or less acceptable favor. Kutter's formula has not sufficient flexibility, being constructed with only one variable parameter, a coefficient of roughness known as n. The available data for the construction of such a formula at the present day far exceeds that at Kutter's command, and it appears that a more serviceable one can now be designed.

13. Taking the value of m as given in (20) as a basis for such a design and using the general class of "clean pipes" as a preliminary test, the following solutions of the constants have been worked out and studied by the writer.

$$m_1 = \frac{.015 + \frac{.0002}{VD}}{1 + .08V + \sqrt{2D}}. \tag{35}$$

$$m_2 = \frac{.016 + \frac{.0002}{VD}}{1 + \sqrt{1/10\,V} + \sqrt{2D}}. \tag{36}$$

$$m_3 = \frac{.014 + \frac{.0002}{VD}}{1.5 + .08V + \frac{1}{2}D}. \tag{37}$$

$$m_4 = \frac{.015 + \frac{.0002}{VD}}{1 + .08V + 1.2D^{1/3}}. \tag{38}$$

$$m_5 = \frac{.016 + \frac{.0002}{VD}}{1 + \sqrt{1/10\,V} + 1.2D^{1/3}}. \tag{39}$$

$$m_6 = \frac{.01548 + \frac{.00032}{VD}}{1 + \sqrt[3]{1/10\,V} + \sqrt[3]{2D}}. \tag{40}$$

*We shall make such frequent reference to this book that we shall signify it in the future by *H. & T.* In like manner we refer to Turneaure and Russell's book as *T. & R.*

There is but little choice between these six forms for the limits prescribed above; they are however given in the order of respective merit. The constants which enter as exponents have been chosen from the nearest one of 1, ½, ⅓, in order that the tables of square and cube roots may be used in the computations. There appears to be no advantage to be derived from using other numbers than these. In the evaluation of λ the constant .0002 was assigned from experiments on very small tubes of glass, brass and zinc. For pipes of iron the sequel will show that the value of λ should be .0001.

In the Table II are entered under V the observed velocity, D the known diameter, 100 m_0 the observed value of m. In the columns headed 100 m_1, 100 m_2, 100 m_3 are entered the computed values of m from the respective formulæ (35), (36), (37). Under the columns headed $100m_f$ and $100m_s$ are entered the corresponding values of m as taken from the tables of J. T. Fanning and of Hamilton Smith, Jr., as given in T. & R., respectively. From 1 — 18 the experiments are by H. Smith, Jr., on sheet iron asphaltum coated pipes at North Bloomfield, California, taken from Fanning's book, p. 236. From 18 to 46 the experiments are by Darcy, quoted by Fanning, p. 237, as clean cast iron pipes. From 46 to 58 the experiments were by J. T. Fanning (p. 238) on wrought iron cement lined pipe. From 58 to 65, in which the discrepancies are somewhat larger, the experiments are by Couplet on a compound pipe at Versailles, composed partly of stoneware and partly of lead, in fair condition, H. & T. p. 136. From 65 to 79 the experiments were by Weisbach, given in his Mechanics of Engineering, Appendix, Coxe's translation, p. 1100; the first six are glass tubes, the next four of brass and the remainder of zinc. In the Smith-Coffin graphical table used the velocities are tabulated only up to 10 feet, and the curves not generally drawn for velocities less than 0.5 feet. These examples thus tabulated cover a sufficiently wide range of material and velocities as well as diameters to exhibit the general behavior of the formulæ for m and their agreements with each other and a rather miscellaneous collection of experiments, to justify a further and subsequent differentiation into classes. It was not thought worth while to tabulate computations of m_4, m_5, m_6, as these do not agree in general so well as the first three forms.

TABLE II.

Table for comparison of m. Clean pipes.

No.	V	D	100 m_0	100 m_1	100 m_2	100 m_3	100 m_f	100 m_8
1	10.05	.911	.48	.48	.54	.51	.51	.45
2	8.68	.911	.50	.50	.56	.56	.52	.46
3	6.95	.911	.52	.52	.58	.56	.53	.47
4	6.11	.911	.52	.53	.59	.57	.54	.49
5	4.76	.911	.55	.56	.60	.57	.55	.51
6	10.75	1.056	.49	.45	.47	.48	.51	.43
7	8.68	1.056	.50	.48	.48	.50	.52	.44
8	6.98	1.056	.49	.51	.49	.53	.53	.46
9	4.61	1.056	.53	.54	.51	.57	.55	.49
10	12.30	1.23	.42	.43	.44	.47	.48	———
11	10.75	1.23	.42	.44	.45	.47	.48	.41
12	8.52	1.23	.45	.47	.46	.50	.50	.42
13	7.33	1.23	.45	.48	.47	.52	.51	.43
14	6.86	1.23	.46	.49	.47	.52	.51	.43
15	5.00	1.25	.52	.51	.49	.55	.52	.46
16	4.40	1.23	.51	.53	.53	.56	.53	.48
17	20.13	1.416	.37	.36	.39	.37	———	———
18*	.29	.269	1.04	1.02	.96	.97	.78	———
19	1.84	.269	.68	.84	.77	.75	.71	.82
20	2.59	.269	.66	.81	.72	.72	.68	.74
21	5.15	.269	.66	.74	.65	.70	.63	.63
22	8.92	.269	.65	.61	.60	.60	.59	.57
23	10.71	.269	.64	.61	.59	.56	.58	.56
24	.49	.45	.73	.75	.72	.79	.72	———
25	1.60	.45	.59	.72	.70	.76	.67	.75
26	2.50	.45	.55	.70	.66	.73	.65	.69
27	15.39	.45	.51	.57	.50	.47	.54	———
28	.65	.615	.63	.71	.71	.73	.68	.88
29	2.50	.615	.59	.66	.62	.70	.62	.61
30	3.72	.615	.58	.63	.59	.70	.60	.58
31	4.90	.615	.55	.60	.53	.68	.58	.54
32	8.26	.615	.55	.55	.53	.57	.55	.49
33	16.24	.615	.55	.45	.42	.45	.52	———
34	.78	.975	.68	.66	.61	.70	.63	.78
35	2.71	.975	.57	.58	.52	.64	.57	.56
36	3.79	.975	.59	.56	.52	.61	.55	.51
37	5.40	.975	.59	.54	.51	.60	.54	.49
38	7.83	.975	.59	.50	.49	.54	.52	.45
39	10.35	.975	.60	.44	.47	.50	.50	.42
40	1.38	1.64	.63	.52	.50	.58	.54	.57
41	1.47	1.64	.55	.51	.50	.58	.54	.55
42	1.55	1.64	.66	.51	.50	.58	.54	.52
43	2.59	1.64	.47	.50	.48	.55	.52	.48
44	3.66	1.64	.51	.49	.47	.54	.50	.45
45	3.69	1.64	.48	.49	.47	.54	.49	.44
46*	.95	1.67	.68	.58	.52	.59	.55	.59
47	1.45	1.67	.53	.58	.50	.58	.54	.53
48	1.93	1.67	.53	.56	.49	.56	.53	.50
49	2.33	1.67	.51	.55	.49	.55	.52	.49
50	2.60	1.67	.53	.55	.48	.57	.51	.48

Table II—Continued.

No.	*V*	*D*	$\frac{100}{m_0}$	$\frac{100}{m_1}$	$\frac{100}{m_2}$	$\frac{100}{m_3}$	$\frac{100}{m_f}$	$\frac{100}{m_s}$
51	2.87	1.67	.52	.55	.48	.55	.51	.47
52	3.27	1.67	.45	.54	.48	.54	.50	.46
53	3.44	1.67	.52	.54	.47	.54	.50	.45
54	3.74	1.67	.52	.53	.47	.53	.50	.45
55	3.92	1.67	.52	.53	.47	.53	.50	.44
56	4.00	1.67	.52	.53	.45	.53	.50	.43
57	4.04	1.67	.53	.53	.45	.53	.49	.43
58*	.18	.44	1.47	.90	1.10	.95	.74	--------
59	.28	.44	1.22	.86	.90	.90	.73	--------
60	.37	.44	1.04	.83	.80	.87	.73	--------
61	.43	.44	.87	.83	.80	.87	.72	--------
62	.46	.44	.93	.80	.77	.85	.72	--------
63	.47	.44	.93	.80	.76	.85	.72	--------
64	3.48	1.60	.70	.50	.67	.54	.51	.46
65*	.226	.0096	9.15	9.34	8.28	6.85	--------	--------
66	.382	.0135	3.51	4.61	4.93	4.15	--------	--------
67	.579	.0201	1.61	2.53	2.27	2.35	--------	--------
68	.700	.0330	1.04	1.82	1.82	1.52	(1.30)	--------
69	.732	.0325	1.27	1.82	1.82	1.52	(1.30)	--------
70	.575	.0325	1.67	1.92	1.77	1.65	(1.40)	--------
71	.576	.033	1.73	1.92	1.77	1.65	(1.40)	--------
72	.640	.033	1.28	1.87	1.73	1.60	(1.30)	--------
73	.285	.033	3.53	2.86	2.66	2.36	(1.40)	--------
74	.650	.047	1.02	1.08	.99	1.31	1.20	--------
75	1.15	.107	.86	1.38	1.00	.94	.89	.98
76	.788	.0840	1.06	1.23	1.13	1.07	1.00	--------
77	.672	.0833	1.08	1.24	1.13	1.10	1.00	--------
78	.364	.0833	1.30	1.56	1.09	1.42	1.12	--------
79*	.5	2	--------	.50	.50	.55	.54	.62
80	1	2	--------	.49	.49	.54	.52	.54
81	2	2	--------	.47	.47	.52	.50	.47
82	3	2	--------	.46	.45	.51	.48	.43
83	5	2	--------	.44	.43	.48	.47	.47
84	7	2	--------	.42	.41	.46	.45	.38
85	10	2	--------	.40	.40	.43	.44	.35
86	16	2	--------	.35	.37	.37	.43	--------
87	.5	3	--------	.43	.43	.46	.47	.51
88	1	3	--------	.42	.43	.45	.46	.47
89	2	3	--------	.41	.41	.44	.44	.42
90	3	3	--------	.41	.40	.41	.42	.39
91	5	3	--------	.40	.39	.41	.41	.36
92	7	3	--------	.37	.37	.39	.40	.34
93	10	3	--------	.35	.34	.37	.39	.31
94	16	3	--------	.32	.33	.32	.38	--------
95	.5	4	--------	.40	.40	.39	.40	.45
96	1	4	--------	.39	.39	.39	.39	.42
97	2	4	--------	.38	.38	.38	.38	.38
98	3	4	--------	.37	.37	.38	.38	.36
99	5	4	--------	.36	.36	.36	.37	.33
100	7	4	--------	.36	.34	.34	.36	.31
101	10	4	--------	.33	.33	.33	.35	.28
102	16	4	--------	.30	.31	.30	.35	--------

Table II—Continued.

No.	V	D	100 m_0	100 m_1	100 m_2	100 m_3	100 m_f	100 m_s
103	.5	5	--------	.35	.36	.34	.36	.43
104	1	5	--------	.35	.35	.34	.35	.39
105	2	5	--------	.34	.35	.33	.35	.36
106	3	5	--------	.34	.33	.33	.34	.33
107	5	5	--------	.33	.32	.32	.33	.31
108	7	5	--------	.32	.31	.31	.32	.30
109	10	5	--------	.31	.31	.30	.32	.27
110	14	5	--------	.28	.30	.26	.31	--------
111	.5	6	--------	.34	.34	.31	.34	.41
112	1	6	--------	.33	.33	.30	.33	.39
113	2	6	--------	.33	.32	.32	.33	.34
114	3	6	--------	.32	.31	.30	.32	.31
115	5	6	--------	.31	.30	.29	.31	.30
116	7	6	--------	.30	.30	.27	.30	.27
117	10	6	--------	.28	.29	.26	.30	--------
118	14	6	--------	.27	.28	.25	.30	--------
119	.5	7	--------	.31	.32	.29	.32	.38
120	1	7	--------	.31	.32	.28	.31	.36
121	2	7	--------	.31	.31	.27	.31	.32
122	3	7	--------	.30	.30	.26	.30	.31
123	5	7	--------	.29	.29	.26	.29	.28
124	6	7	--------	.29	.29	.26	.29	.27
125	10	7	--------	.27	.28	.24	.28	--------
126	14	7	--------	.25	.27	.23	.28	--------
127	.5	8	--------	.30	.30	.26	.29	.37
128	1	8	--------	.29	.30	.25	.29	.33
129	2	8	--------	.29	.29	.25	.28	.31
130	3	8	--------	.29	.28	.24	.28	.29
131	5	8	--------	.28	.28	.23	.27	.27
132	7	8	--------	.28	.27	.23	.27	--------
133	10	8	--------	.27	.27	.22	.26	--------
134	14	8	--------	.24	.26	.21	.26	--------

14. To give a comparison of the type m_1 with the type m_6, Table III consists of 23 experiments on glass pipes. The first six are by Weisbach as given in the previous table; 7–10 on a $\frac{3}{4}$ inch, 11–14 on a $\frac{1}{2}$ inch glass pipe by H. Smith, Jr., 15–19 by the same on a 1 inch glass pipe, all quoted from Bowie's Hydraulic Mining; 20–23 are by Darcy on a 2 inch glass pipe, H. & T., p. 135. Column V contains the observed velocity, column r the observed mean hydraulic radius. Columns 100 m_0 and c_0 the observed values of m and c respectively. Columns 100 m' and c' contain the computed value of m from

$$m' = \frac{.012 + \dfrac{.00004}{Vr}}{1 + .08V + \sqrt{8r}}, \tag{41}$$

TABLE III.

Glass Pipes.

No.	V	r	100 m_0	100 m'	100 m''	c_0	c'	c''
1	.226	.0024	9.15	7.41	10.5	26	29	25
2	.382	.0034	3.51	3.57	4.73	43	43	37
3	.531	.0050	1.82	2.21	2.64	59	54	49
4	.732	.0081	1.04	1.42	1.62	71	67	63
5	.575	.0081	1.27	1.63	1.74	62	64	61
6	.700	.0132	1.67	1.16	1.28	78	75	71
7	1.40	.0155	.92	.948	.955	84	82	82
8	2.65	"	.77	.832	.865	91	88	86
9	3.62	"	.71	.773	.785	95	91	91
10	4.37	"	.69	.734	.733	96	94	94
11	2.08	.0104	.958	.966	.964	80	81	81
12	2.72	"	.896	.894	.887	85	85	85
13	3.66	"	.834	.830	.828	87	88	88
14	4.44	"	.793	.792	.786	90	90	90
15	1.96	.0191	.805	.771	.830	89	91	88
16	2.95	"	.720	.754	.764	95	92	92
17	3.68	"	.683	.718	.732	97	95	94
18	4.38	"	.653	.689	.710	99	97	95
19	5.01	"	.633	.670	.696	101	98	96
20	.502	.0407	.983	.932	.979	83	83	81
21	1.59	"	.808	.742	.750	89	93	93
22	4.85	"	.642	.629	.635	100	101	101
23	6.92	"	.613	.600	.610	102	103	102

and the corresponding value of c. Columns 100 m'' and c'' contain the values of m and c computed from

$$m'' = \frac{.01548 + \dfrac{.00008}{Vr}}{1 + \sqrt[3]{1/10\,V} + \sqrt[3]{8r}}. \tag{42}$$

15. A further application of the form type m_6 is given in Table IV in which the experimental values of the hydraulic mean radius r, V, m and c as observed for the Sudbury Conduit in Massachusetts and quoted from T. & R., p. 258, a brick-lined conduit described fully in H. & T. There are also entered for comparison the values of m, c and V computed from

$$m = \frac{.01446 + \dfrac{.00035}{Vr}}{1 + \sqrt{1/10\,V} + 2\sqrt[3]{r}}. \tag{43}$$

TABLE IV.

Sudbury Conduit. $J = .0002$.

r	m_0	m	c_0	c	V_0	V
.2	.0072	.0072	95	95	.627	.627
1	.0040	.0042	126	124	1.777	1.75
2	.0034	.0036	132	134	2.740	2.68
3	.0032	.0032	142	142	3.453	3.453
4	.0031	.0030	143	146	4.094	4.12

16. The foregoing results, with many others of similar character, appear to justify a close and exhaustive study of the adaptability of the general form for m with the end in view of seeing whether a differentiated classification can be made of pipes, conduits and canals according to degrees of roughness such that one definite form of m can be utilized for the purpose of constructing a formula which will give results sufficiently accurate to constitute a working formula which the engineer can apply with confidence under a classification so explicit as to remove indefiniteness and guess work in its application. The writer made a long and careful study of some fifteen hundred experiments with different solutions of the general form for m with the result that finally the form of the type

$$m = \frac{\mu + \frac{\lambda}{Vr}}{1 + .08V + \sqrt{8r}}, \tag{44}$$

was selected as having the widest range of applicability, the most generally accurate results with ease of computation. The result of the investigation shows that such a formula with reasonable precision for all *artificial* channels of a fair degree of regularity and construction can be designed. The writer is convinced from his investigations that for natural channels such as rivers whose regimen departs much from the regularity of the artificial channel no such formula can be designed whose service can ever be more than a gross approximation which serves the purpose only of provisional estimation. A study of the observations on the coastal plain rivers like the Mississippi and Missouri, and the torrential streams of the Swiss Alps, will soon convince one that while a special formula may be designed for an individual stream with fair approximation, such streams can not be grouped into classes in such a manner that any other given stream can be definitely classified with reference to them. The United States Government

Hydrographic Survey recognized this by actually gauging the velocity of flow directly in a given cross-section. The Table V consists of 1,085 experiments with the authority from which each is taken. Under the head of 1000 s is entered the observed hydraulic slope, under the heads of V, r, 100 m_0 and C_0 are entered the observed velocity, hydraulic mean radius, observed value of m and corresponding observed coefficient of velocity C respectively. Under 100 m and C are entered the computed values of m and C from the formula (44), in which the values of the coefficients μ and λ are given for the twenty different classes at the head of each class into which the channels of flow have been divided. Some two hundred other observations on natural rivers whose regimen was more irregular were studied with the conclusion reached above, and consequently they were not tabulated. Not because the coefficients of roughness were not found among the classes in the table, but because the laws of flow were so irregular as not to admit of sufficiently precise computation. For the same reason the class of pipes known as old, foul, tuberculated and incrusted pipes have been omitted. First because their imposed obstacles are such as to give no definiteness to their degrees of roughness; second because the irregularity and distribution of the interior projections was such that their surfaces, as those of irregular natural rivers, no longer constitute a homogenous heterogeneity of surface that could be defined or relegated to a definite classification, and consequently the disturbances of stream lines, cross currents and eccentric eddies are such as no longer obey the laws which govern a regular, systematic and symmetrical disturbance.

TABLE V.

Class 1.

$\mu = .010, \ \lambda = .000090.$

No.	1000 s	V	r	100 m_0	100 m	C_0	C	Authority.
1	5.40	1.12	.030	.83	.84	88	88	H. & T. p. 135. Tin Pipe, straight. Bossut, 1771.
2	10.8	1.17	"	.73	.79	94	90	
3	15.1	2.07	"	.67	.71	98	95	
4	26.9	2.95	"	.60	.63	104	101	
5	53.0	4.31	"	.54	.59	109	104	
6	113	6.14	"	.57	.53	106	110	
7	113	6.15	"	.57	.53	106	110	
8	113	6.16	"	.57	.53	106	110	
9	5.30	1.44	.045	.73	.70	94	96	H. & T. p. 137. Tin Pipe, straight. Velocity determined from known volume. Bossut, 1771.
10	10.0	2.11	"	.64	.63	100	101	
11	14.2	2.60	"	.61	.60	103	104	
12	23.9	3.58	"	.53	.56	110	107	
13	46.5	5.23	"	.50	.50	115	113	

Class 2.

$\mu = .011, \ \lambda = .000065.$

No.	1000 *s*	*V*	*r*	100 m_0	100 *m*	C_0	*C*	*Authority.*
14	26.9	2.22	.022	.76	.78	92	91	H. & T. p. 139. H. Smith, Jr., 1886. New wrought iron pipe. Coated with asphalt. Funnel mouth.
15	52.2	3.22	"	.70	.72	96	95	
16	103	4.76	"	.64	.65	100	100	
17	131	5.44	"	.62	.62	102	102	
18	.27	.328	.068	1.09	.80	77	90	H. & T. p. 139. Darcy, 1851. New straight wrought iron riveted pipe, coated with asphalt.
19	2.03	1.17	"	.64	.62	100	102	
20	12.2	3.12	"	.55	.57	108	106	
21	40.7	6.15	"	.47	.50	117	113	
22	106	10.5	"	.42	.43	124	123	
23	156	12.8	"	.41	.40	125	126	
24	.20	.59	.161	.60	.54	104	109	Same. New wrought iron riveted pipe, coated with asphalt, straight. H. & T. p. 141. Darcy.
25	1.29	1.53	"	.57	.50	106	113	
26	5.80	3.53	"	.48	.46	116	118	
27	12.0	5.51	"	.41	.43	125	122	
28	29.7	9.00	"	.38	.39	130	128	
29	122	19.7	"	.32	.30	141	145	
30	.70	1.30	.234	.63	.47	101	117	H. & T. p. 141. Darcy. New straight wrought iron riveted pipe, coated with asphalt.
31	4.33	3.87	"	.43	.41	122	125	
32	11.9	6.67	"	.40	.39	127	128	
33	28.1	10.5	"	.38	.34	130	137	
34	.53	1.23	.319	.73	.70	94	96	H. & T. p. 137. Wooden pipe, rectangular section. Darcy and Bazin.
35	1.07	1.78	"	.69	.68	96	97	
36	1.73	2.28	"	.68	.66	97	99	
37	2.73	2.94	.319	.64	.64	100	100	Weir measurement. 1.57 feet wide, by .98 feet deep.
38	3.87	3.53	"	.68	.61	97	103	
39	6.27	4.35	"	.68	.60	97	103	
40	7.27	4.62	"	.70	.59	96	104	
41	8.80	5.31	"	.64	.57	100	106	
42	.475	1.67	.505	.55	.62	108	102	H. & T. p. 137. Darcy and Bazin. Rectangular wooden pipe, poplar, closely jointed. Weir measurement. 2.62 wide by 1.64 feet deep.
43	1.08	2.52	"	.55	.60	108	104	
44	1.90	3.37	"	.54	.58	109	105	
45	2.91	4.22	"	.53	.56	110	107	
46	4.27	5.07	"	.54	.55	107	108	
47	5.06	5.53	"	.54	.55	109	108	
48	5.76	5.91	"	.53	.53	110	110	
49	6.61	6.37	"	.53	.52	110	111	
50	3.00	2.30	.134	.49	.50	115	113	H. & T. p. 163. Flume in Venezuela, of very hard wood, very smooth. Rectangular. .58 wide, .25 deep.

Class 3.

$\mu = .012, \quad \lambda = .000040.$

No.	1000 s	V	r	100 m_0	100 m	C_0	C	Authority.
51	------	.23	.0024	9.15	7.41	27	29	Weisbach. Mech. of Engineering. Coxe's Trans. of Fourth Ed. Appendix, p. 1100. (51-56) glass tubes, straight. Each experiment the mean of several observations.
52	------	.38	.0034	3.51	3.57	43	43	
53	------	.53	.0050	1.82	2.21	59	54	
54	------	.73	.0081	1.27	1.42	71	67	
55	------	.57	.0081	1.67	1.63	62	63	
56	------	.70	.0132	1.04	1.16	78	75	
57	------	2.08	.0104	.96	.97	80	81	Bowie's Hydraulic Mining, p. 174. H. Smith, Jr., glass tube. Length 11.1, effective head from .7 to 2.5.
58	------	2.72	"	.90	.89	85	85	
59	------	3.66	"	.83	.83	88	88	
60	------	4.44	"	.79	.79	91	90	
61	------	1.40	.015	.92	.94	84	83	Same, p. 174. ¾ inch glass pipe. Length 34.9 feet. Effective head .6 to 4.6. H. Smith, Jr.
62	------	2.65	"	.77	.83	91	88	
63	------	3.67	"	.71	.77	95	91	
64	------	4.37	"	.69	.73	96	94	
65	------	1.96	.019	.80	.77	90	91	Same, p. 174. One inch glass pipe. Length 63.9 feet. Effective head 1.6 to 8.2. H. Smith, Jr.
66	------	2.95	"	.72	.75	95	92	
67	------	3.68	"	.68	.72	97	95	
68	------	4.38	"	.65	.69	99	97	
69	------	5.01	"	.63	.67	101	98	
70	.96	.505	.041	.98	.93	80	83	H. & T. p. 135. Darcy and Bazin. Straight glass pipe.
71	7.71	1.59	"	.81	.74	89	93	
72	57.6	4.85	"	.64	.63	100	101	
73	111.9	6.92	"	.61	.60	102	103	
74	1.5	3.02	.366	.39	.40	129	126	H. & T. p. 161. Darcy, Bazin, No. 24. Test Channel. Open channel, smooth, pure, neat cement, straight uniform grade. Semicircular cross section.
75	"	3.72	.503	.35	.36	136	133	
76	"	4.16	.605	.34	.34	138	137	
77	"	4.60	.682	.31	.32	144	142	
78	"	4.87	.750	.31	.31	145	144	
79	"	5.12	.809	.30	.30	147	146	
80	"	5.29	.867	.30	.30	147	146	
81	"	5.51	.915	.29	.29	149	149	
82	"	5.75	.949	.27	.28	153	152	
83	"	5.91	.992	.27	.28	153	152	
84	"	6.06	1.03	.27	.28	154	152	
85	"	6.11	1.03	.27	.28	155	152	
86	4.9	3.34	.168	.48	.49	116	116	Same, p. 161. Darcy and Bazin, No. 2. Test channel. Open channel, pure, neat cement, smooth. Cross section rectangular.
87	"	4.39	.251	.41	.43	125	123	
88	"	5.04	.322	.40	.40	127	127	
89	"	5.68	.375	.36	.38	132	130	
90	"	6.08	.430	.36	.36	132	133	
91	"	6.51	.474	.35	.35	135	136	
92	"	6.83	.518	.35	.34	135	137	
93	"	7.12	.558	.34	.33	136	139	
94	"	7.41	.559	.34	.32	137	141	
95	"	7.63	.632	.34	.31	137	144	
96	"	7.86	.665	.34	.31	138	144	
97	"	8.07	.696	.33	.30	138	146	
98	.71	.37	.03	.99	1.05	80	80	H. & T. p. 151. Darcy. An old cast iron pipe after being carefully cleaned.
99	1.80	.62	"	.91	.91	84	84	
100	14.4	1.97	"	.71	.77	95	91	
101	39.7	3.39	"	.67	.70	98	96	

Class 4.

$\mu = .013, \quad \lambda = .000030.$

No.	1000 s	V	r	100 m_0	100 m	C_0	C	*Authority.*
102	50.6	2.70	.021	.89	.84	85	88	H. & T. p. 135. Lead pipe in Hamburg (Lager Platz). Iben, 1875.
103	64.6	3.16	"	.85	.82	87	89	
104	112	4.72	"	.66	.75	98	93	
105	216	6.88	"	.60	.68	104	98	
106	348	9.11	"	.55	.62	108	102	
107	.44	.213	.022	1.38	1.38	68	68	H. & T. p. 135. Straight new lead pipe. Darcy, 1851.
108	8.14	1.09	"	.98	.96	81	82	
109	54.4	3.35	"	.69	.79	96	90	
110	146	5.51	"	.69	.71	97	95	
111	.82	.394	.034	1.15	1.00	75	80	Same, p. 137. Straight new lead pipe. Darcy, 1851.
112	7.78	1.40	"	.85	.79	87	90	
113	56	4.23	"	.65	.71	99	95	
114	159	7.56	"	.60	.61	103	102	
115	.017	.20	.21	.57	.60	106	103	H. & T. p. 147. Cast iron main at Stuttgart. About 4 years in use. Asphalted in good condition. Several bends large radius. No valves or branches. Ehmann, 1878-79.
116	.037	.27	"	.57	.60	106	103	
117	.078	.40	"	.64	.58	100	105	
118	.188	.62	"	.64	.57	100	106	
119	.328	.83	"	.63	.56	101	107	
120	.406	.94	"	.61	.56	103	107	
121	.04	.40	.25	.42	.54	123	109	H. & T. p. 147. New cast iron main, Hamburg. Coated with tar. Iben, 1876.
122	.60	1.3	"	.54	.52	109	111	
123	1.49	2.2	"	.51	.50	112	113	
124	7.00	4.8	"	.48	.48	116	116	
125	11.2	6.1	"	.49	.46	114	118	
126	.59	1.6	.343	.53	.47	111	117	H. & T. p. 147. Asphalted cast iron pipe at Dantzig. Straight clean, 5 years old. Lampe, 1870.
127	1.38	2.5	"	.50	.45	114	120	
128	1.63	2.7	"	.49	.45	115	120	
129	1.95	3.1	"	.46	.45	119	120	
130	15.2	1.87	.030	.84	.88	88	86	H. & T. p. 163. Test channel of carefully planed boards, rectangular section. Darcy and Bazin. No. 29.
131	"	2.30	.043	.79	.75	90	93	
132	"	2.68	.053	.72	.71	94	95	
133	"	3.00	.061	.66	.67	99	98	
134	"	3.56	.074	.56	.63	106	101	
135	4.7	.90	.029	1.11	1.09	77	77	Same, p. 163. No. 28. Darcy. Carefully planed boards.
136	"	1.30	.052	.93	.80	83	90	
137	"	1.58	.066	.80	.78	89	91	
138	"	1.74	.075	.75	.70	93	96	
139	"	1.94	.084	.68	.66	98	99	
140	"	2.11	.091	.61	.65	102	100	
141	"	2.16	.093	.60	.66	103	99	
142	.5	1.08	.264	.72	.52	94	111	H. & T. p. 165. Mill race at Schemnitz, smooth wooden trough, semi-octagonal, bottom width .81. Rittenger, 1855.
143	"	1.29	.303	.58	.50	105	113	
144	"	1.64	.371	.44	.46	121	118	
145	"	1.89	.396	.35	.44	134	121	
146	.318	2.62	1.00	.30	.31	147	144	H. & T. p. 149. New cast iron pipe, Sudbury Conduit, asphalted, straight. Stearns, 1885.
147	.711	3.74	"	.33	.32	140	142	
148	1.22	4.96	"	.32	.31	142	144	
149	1.85	6.19	"	.31	.31	144	144	

Class 5.

$\mu = .014, \quad \lambda = .000027.$

No.	1000 s	V	r	100 m_0	100 m	C_0	C	*Authority.*
150	.33	.19	.022	1.28	1.25	71	72	H. & T. p. 139. New straight wrought iron pipe, uncoated. Discharge measured with great accuracy. Darcy, 1851.
151	10.1	1.21	"	.98	.97	81	81	
152	43.5	2.61	"	.89	.89	85	85	
153	106	4.20	"	.85	.83	87	88	
154	310	7.17	"	.85	.72	87	86	
155	.22	.20	.032	1.09	1.20	77	73	Same.
156	3.36	.86	"	.96	.96	82	82	
157	23.9	2.58	"	.75	.82	93	89	
158	123	6.30	"	.64	.70	100	96	
159	224	8.52	"	.63	.64	101	100	
160	.66	1.0	.084	.42	.75	124	93	H. & T. p. 145. New cast iron service pipe at Hamburg, Grindell Alley. Coated with tar. Iben, 1876.
161	5.62	2.1	"	.68	.72	97	95	
162	9.75	2.8	"	.68	.70	97	96	
163	20.5	4.3	"	.58	.65	105	100	
164	33.7	5.5	"	.60	.62	104	102	
165	1.98	1.0	.084	1.03	.75	79	93	Same. Deseniss street.
166	4.11	1.7	"	.76	.72	92	95	
167	6.56	2.1	"	.80	.71	90	95	
168	7.83	2.3	"	.80	.70	90	96	
169	11.1	2.8	"	.78	.70	91	96	
170	.84	.63	.066	.89	.82	85	88	H. & T. p. 151. Coated old cast iron pipe carefully cleaned. Before cleaning m = 1.40. Darcy, 1851.
171	7.23	2.01	"	.75	.75	92	92	
172	15.6	2.83	"	.82	.72	87	94	
173	44.7	5.01	"	.74	.67	93	98	
174	1.5	2.87	.379	.44	.46	120	119	H. & T. p. 161. Test channel. Semi-circular, two-thirds cement and one-third very fine sand. Darcy and Bazin. No. 25.
175	"	3.43	.529	.43	.43	122	122	
176	"	3.87	.635	.41	.40	125	127	
177	"	4.30	.706	.37	.38	132	130	
178	"	4.51	.787	.37	.36	131	133	
179	"	4.80	.839	.35	.35	135	135	
180	"	4.94	.900	.36	.34	135	136	
181	"	5.20	.941	.33	.34	138	137	
182	"	5.38	.983	.33	.33	140	140	
183	"	5.48	1.01	.32	.33	141	140	
184	"	5.55	1.02	.32	.33	142	140	
185	"	5.66	1.04	.31	.32	143	142	

Class 6.

$\mu = .015, \quad \lambda = .000025.$

No.	1000 s	V	r	100 m_0	100 m	C_0	C	*Authority.*
186	.23	.95	.416	.68	.58	97	105	H. & T. p. 155. Wrought iron cement lined force main. No short bend, but two large Y branches, two small blow off branches, and three stop valves. J. T. Fanning, 1880.
187	.44	1.45	"	.53	.58	110	105	
188	.73	1.93	"	.53	.56	111	107	
189	1.04	2.33	"	.51	.55	112	108	
190	1.34	2.60	"	.53	.55	110	108	
191	1.58	2.87	"	.52	.55	112	108	
192	1.99	3.27	"	.45	.54	113	109	
193	2.28	3.44	"	.52	.54	112	109	
194	2.72	3.74	"	.52	.53	111	110	
195	3.00	3.92	"	.52	.53	111	110	
196	3.13	4.00	"	.52	.53	111	110	
197	3.20	4.04	"	.53	.53	111	110	
198	.73	2.00	.417	.49	.50	114	113	H. & T. p. 149. New cast iron force main, Hackensack, N. J. Large number summits, angles, curves, 4 right angles, 10 quadrants of 30 feet radius. Quantity measured at pumps, 5% slip. Head, 165 ft. Brush, 1882-87.
199	.88	2.24	"	.47	.50	117	113	
200	1.03	2.36	"	.49	.49	114	114	
201	1.19	2.52	"	.50	.49	113	114	
203	1.33	2.68	"	.49	.49	114	114	
204	1.49	2.76	"	.53	.49	111	114	
205	1.64	2.92	"	.51	.49	112	114	
206	1.80	3.00	"	.54	.49	110	114	
207	.12	.7	.417	.58	.53	105	110	H. & T. p. 149. New cast iron tar coated service pipe, Hamburg. Iben, 1876.
208	.48	1.6	"	.53	.51	110	112	
209	.76	1.9	"	.54	.51	109	112	
210	1.21	2.5	"	.54	.50	109	113	
211	16.4	12.6	.54	.36	.37	134	132	H. & T. p. 141. H. Smith, Jr. Sheet iron riveted pipe.
212	.27	.67	.154	.60	.70	104	96	H. & T. p. 145. New cast iron pipe. Darcy, 1851.
213	3.68	2.49	"	.60	.64	104	100	
214	22.5	6.34	"	.55	.55	108	108	
215	110	14.18	"	.54	.47	109	117	
216	146	16.17	"	.55	.45	108	119	
217	15.7	3.47	.083	.70	.72	96	95	H. & T. p. 143. New cast iron pipe at Hamburg. Iben, 1875.
218	31.1	4.86	"	.70	.69	96	97	
219	42.1	5.80	"	.67	.66	98	99	
220	48.6	6.17	"	.68	.65	97	100	
221	.38	.73	.166	.74	.69	93	97	H. & T. p. 145. Cast iron main at Stuttgart, asphalted, 4 years old, clean. Ehmann, 1878-79.
222	.85	1.12	"	.71	.68	95	97	
223	1.33	1.45	"	.67	.67	98	98	
224	1.88	1.69	"	.70	.66	96	99	
225	8.5	4.71	.228	.56	.55	107	108	H. & T. p. 141. Sheet iron riveted pipe, North Bloomfield. H. Smith, Jr., 1876.
226	13.3	6.09	"	.52	.52	111	111	
227	16.9	6.93	"	.51	.51	112	112	
228	25.6	8.66	"	.50	.50	113	113	
229	33.1	10.0	"	.49	.48	116	116	

Class 6—Continued.

No.	1000 s	V	r	100 m_0	100 m	C_0	C	*Authority.*
230	6.68	4.59	.264	.54	.53	109	110	Same, coated with asphalt. H. Smith, Jr. Funnel mouthpiece 12 feet long.
231	14.3	6.96	"	.50	.50	113	113	
232	22.2	8.65	"	.50	.48	113	116	
233	33.2	10.7	"	.49	.46	114	118	
234	5.02	4.38	.307	.52	.52	112	111	Same, funnel mouthpiece, 14 feet long.
235	11.0	6.84	"	.46	.48	118	116	
236	12.3	7.31	.307	.45	.47	119	117	
237	16.5	8.46	"	.45	.46	119	118	
238	24.7	10.6	"	.43	.44	122	121	
239	32.3	12.1	"	.44	.43	121	122	
240	.45	1.47	.41	.55	.51	109	112	H. & T. p. 149. New cast iron pipe. Darcy, 1851.
241	1.20	2.60	"	.47	.49	117	115	
242	2.10	3.42	"	.47	.48	116	116	
243	2.60	3.67	"	.50	.48	113	116	
244	66.7	20.1	.354	.37	.35	131	136	H. & T. p. 141. Asphalted sheet iron double-riveted pipe at Texas, Creek. H. Smith, Jr.
245	------	4.5	.231	.56	.55	107	108	T. & R. p. 246. No. 1. Riveted sheet iron pipe, No. 7.
246	------	6.0	"	.53	.53	111	111	
247	------	3.5	.735	.40	.40	127	127	
248	.161	2.53	1.86	.30	.30	146	146	H. & T. p. 163. Sudbury Conduit. Pure cement on brickwork. Fteley and Stearns.
249	.160	2.67	2.05	.29	.29	147	147	
250	.160	2.80	2.11	.27	.27	153	153	

Class 7.

$\mu = .016, \quad \lambda = .0002.$

No.	1000 s	V	r	100 m_0	100 m	C_0	C	*Authority.*
251	11.7	10.8	.607	.39	.39	128	128	H. & T. p. 157. Wrought iron pipe at Cherokee. 5 years old. H. Smith, Jr.
252	66.7	20.1	.354	.37	.37	131	131	Same as No. 244.
253	.29	.31	.083	1.62	1.30	63	70	H. & T. p. 143. Cast iron main at Stuttgart, Neckar St. 9 years old. Ehmann, 1878-79.
254	1.13	.77	"	1.02	1.03	79	79	
255	2.35	1.18	"	.90	.96	84	82	
256	3.76	1.57	"	.81	.92	89	84	
257	6.43	2.06	"	.81	.81	89	89	

Class 7—Continued.

No.	1000 s	V	r	100 m_0	100 m	c_0	c	*Authority.*
258	.066	.18	.111	1.48	1.33	66	69	H. & T. p. 137. Pipe at Versailles. Lead and stoneware. Couplet, 1732.
259	.135	.28	"	1.22	1.11	73	76	
260	.199	.37	"	1.06	1.06	78	78	
261	.250	.43	"	.98	1.01	81	80	
262	.285	.46	"	.94	1.00	82	64	
263	.297	.47	"	.94	1.00	82	64	
264	1.67	.61	.041	1.18	1.50	74	65	H. & T. p. 143. Cast iron asphalted pipe at Hahnwald, 2 years old. Ehmann, 1878-79.
265	24.9	2.43	"	1.11	1.07	76	77	
266	31.3	2.75	"	1.07	1.05	77	78	
267	35.7	2.95	"	1.09	1.04	77	78	
268	3.15	.84	.041	1.17	1.40	74	68	Same. Cast iron asphalted pipe, 2 years old. Many easy curves, several irregularities. Ehmann.
269	6.38	1.22	"	1.13	1.26	75	71	
270	9.74	1.52	"	1.12	1.14	76	75	
271	13.7	1.80	"	1.12	1.10	76	77	
272	15.7	1.93	"	1.09	1.06	76	78	
273	20.2	2.20	"	1.09	1.04	76	78	
274	------	5.0	.315	.52	.53	112	110	T. & R. p. 246. No. 3. No. 4. Riveted sheet iron.
275	------	4.5	.332	.53	.53	110	110	
276	1.5	2.61	.390	.55	.54	108	109	H. & T. p. 165. Test channel of unplaned boards, semicircular cross-section. Darcy and Bazin. No. 26.
277	"	3.23	.537	.50	.49	113	114	
278	"	3.71	.632	.44	.45	121	120	
279	"	4.04	.717	.43	.43	123	122	
280	"	4.25	.796	.43	.42	123	123	
281	"	4.51	.856	.41	.40	126	126	
282	"	4.64	.921	.41	.40	125	126	
283	"	4.87	.964	.39	.38	128	130	
284	"	5.00	1.01	.39	.38	128	130	
285	"	5.18	1.05	.38	.37	130	131	
286	"	5.29	1.10	.38	.37	130	131	
287	"	5.45	1.13	.37	.36	132	133	
288	"	5.54	1.15	.36	.35	134	135	
289	5.9	2.99	.172	.73	.72	94	95	H. & T. p. 169. Test channel of unplaned boards, rectangular section. Darcy and Bazin. No. 10.
290	"	3.98	.255	.61	.61	103	103	
291	"	5.23	.376	.52	.51	111	112	
292	"	6.06	.472	.49	.50	115	113	
293	"	6.69	.554	.47	.47	117	117	
294	"	7.24	.623	.45	.43	119	122	
295	"	7.71	.686	.44	.40	121	126	
296	8.39	3.54	.146	.63	.69	101	97	H. & T. p. 167. Test channel of unplaned boards, rectangular section. Darcy and Bazin. No. 11.
297	"	4.57	.224	.58	.60	105	104	
298	"	6.00	.334	.50	.51	113	112	
299	"	6.89	.424	.48	.47	115	117	
300	"	7.57	.500	.47	.44	117	121	
301	"	8.19	.565	.46	.43	119	122	
302	"	8.74	.621	.44	.41	121	125	

Class 8.

$\mu = .017, \lambda = .0004.$

No.	1000 s	V	r	100 m_0	100 m	C_0	C	*Authority.*
304	11.5	3.36	.125	.80	.82	89	90	H. & T. p. 145. New cast iron riveted main. Iben, 1875.
305	14.2	3.96	"	.71	.76	95	92	
306	4.59	2.00	.125	.96	.86	82	87	H. & T. p. 145. New cast iron street main coated with tar. Iben, 1876.
307	11.6	3.30	"	.85	.82	87	88	
308	16.2	3.90	"	.83	.76	88	92	
309	22.3	4.80	"	.76	.75	92	93	
310	30.3	5.30	"	.85	.72	87	95	
311	3.53	2.30	.125	.52	.85	111	87	Same. Street main, cast iron tar coated. Iben, 1876.
312	14.8	4.30	"	.63	.77	101	92	
313	27.5	5.30	"	.79	.72	90	95	
314	34.2	6.00	"	.76	.71	92	95	
315	50.5	7.10	"	.79	.70	90	96	
316	68.1	8.70	"	.76	.64	95	100	
317	.31	1.47	.625	.58	.52	106	111	H. & T. p. 149. Cast iron force main at Philadelphia. Two years old. Curves 25 feet radius. Four check valves on line. Quantity measured at pumps, 5 % slip. Static head 167.3 feet. Darrach, 1878.
318	.38	1.62	"	.57	.51	106	112	
319	.34	1.76	"	.57	.51	106	112	
320	.50	1.91	"	.55	.51	108	112	
321	.57	2.06	"	.54	.50	110	113	
322	.63	2.20	"	.52	.50	111	113	
323	.70	2.35	"	.50	.50	113	113	
324	.76	2.50	"	.49	.50	115	113	
325	.83	2.64	"	.48	.49	116	114	
326	.89	2.79	"	.46	.49	118	114	
327	.95	2.94	"	.44	.49	120	114	
328	1.02	3.08	"	.43	.49	122	114	
329	1.08	3.23	"	.41	.49	124	114	
330	------	5.0	.798	.57	.58	106	105	T. & R. p. 246. No. 9. Riveted iron pipe.
331	17.0	3.97	.118	.82	.79	89	90	H. & T. p. 165. Mill race at Berne. Rectangular, sawed boards. Kutter.
332	.71	1.07	.198	.78	.81	91	89	H. & T. p. 165. Two small wooden channels. Kutter.
333	.10	1.51	.259	.74	.71	93	95	
334	34.3	8.26	.159	.51	.62	112	102	H. & T. p. 165. Race of Schattberg Stamp Mill. Wooden trough, trapezoidal. Bottom width 1.04 feet. Rittinger.
335	"	8.21	.173	.56	.61	107	103	
336	"	10.1	.237	.51	.53	112	110	
337	"	10.6	.246	.48	.52	116	111	

Class 8—Continued.

No.	1000 *s*	*V*	*r*	100 m_0	100 *m*	C_0	*C*	*Authority.*
338	4.3	2.85	.214	.73	.73	94	94	H. & T. p. 167. Test channel. Unplaned boards. Rectangular. Darcy and Bazin. No. 19. Width 2.65 feet.
339	"	3.47	.299	.68	.62	97	102	
340	"	4.14	.364	.59	.57	105	106	
341	"	4.54	.412	.55	.54	108	109	
342	"	4.91	.461	.53	.52	110	111	
343	"	5.12	.490	.53	.50	111	113	
344	"	4.41	.535	.50	.49	113	114	
345	"	5.60	.563	.49	.48	114	115	
346	"	5.92	.618	.48	.47	115	117	
347	"	6.23	.662	.47	.45	117	119	
348	"	6.48	.700	.46	.44	118	121	
349	4.9	3.37	.235	.66	.66	99	99	Same. Test channel unplaned boards, rectangular section. Width 3.93 feet. Darcy and Bazin. No. 18.
350	"	4.43	.341	.55	.58	108	105	
351	"	5.05	.428	.53	.53	110	110	
352	"	5.54	.498	.51	.50	112	113	
353	"	5.94	.558	.50	.48	114	116	
354	"	6.26	.612	.49	.46	114	118	
355	"	6.50	.661	.49	.45	114	119	
356	4.9	6.76	.703	.48	.45	115	119	
357	"	7.00	.741	.48	.44	116	121	
358	"	7.20	.777	.47	.43	117	122	
359	"	7.42	.808	.46	.43	118	122	
360	"	7.59	.839	.46	.42	118	124	
361	8.24	3.52	.147	.63	.76	101	99	H. & T. p. 169. Test channel, unplaned boards, rectangular. Darcy and Bazin. No. 8. Width 6.53 feet.
362	"	4.42	.231	.62	.65	101	100	
363	"	5.23	.289	.56	.58	107	105	
364	"	5.83	.341	.53	.55	110	108	
365	"	6.24	.393	.53	.53	110	110	
366	"	6.74	.431	.50	.50	113	113	
367	"	7.17	.466	.48	.49	116	115	
368	"	7.44	.506	.49	.48	115	116	
369	"	7.73	.541	.48	.46	116	118	
370	"	8.03	.572	.47	.45	117	119	
371	"	8.26	.604	.47	.44	117	121	
372	"	8.57	.630	.45	.43	119	122	
373	4.9	2.71	.188	.80	.72	89	94	H. & T. p. 169. Test channel of unplaned boards, rectangular section. Width 6.53 feet. Darcy and Bazin. No. 7.
374	"	3.70	.272	.63	.63	101	101	
375	"	4.35	.342	.57	.58	106	105	
376	"	4.85	.402	.54	.54	109	109	
377	"	5.29	.453	.51	.51	112	112	
378	"	5.61	.504	.50	.50	113	113	
379	"	5.93	.547	.49	.48	114	115	
380	"	6.23	.587	.48	.47	116	117	
381	"	6.45	.628	.47	.46	116	118	
382	"	6.71	.662	.46	.44	118	121	
383	"	6.90	.698	.46	.44	118	121	
384	"	7.15	.727	.45	.43	120	122	

Class 8—Continued.

No.	1000 *s*.	*V*	*r*	100 m_0	100 *m*	C_0	*C*	*Authority.*
385	4.9	4.13	.327	.61	.59	103	104	H. & T. p. 71. Test channel, unplaned boards, triangular section, sides sloped 45°. Darcy and Bazin. No. 23.
386	"	5.04	.422	.53	.53	110	110	
387	"	5.56	.494	.50	.50	113	113	
388	"	6.03	.549	.48	.48	116	116	
389	"	6.36	.596	.47	.46	118	118	
390	"	6.59	.643	.47	.45	117	119	
391	"	6.83	.683	.46	.44	118	121	
392	"	7.03	.719	.46	.43	118	122	
393	"	7.23	.752	.45	.42	119	123	
394	"	7.40	.783	.45	.42	119	123	
395	"	7.54	.814	.45	.41	119	125	
396	"	7.75	.839	.44	.40	121	127	
397	4.9	3.58	.257	.63	.64	101	100	H. & T. p. 171. Test channel, unplaned boards. Trapezoidal. One side vertical, the other inclined at 45°. Bottom width 3.1 feet. Darcy and Bazin. No. 22.
398	"	4.71	.361	.51	.56	112	107	
399	"	5.29	.450	.50	.51	113	112	
400	"	5.79	.517	.49	.48	115	115	
401	"	6.25	.570	.46	.47	118	117	
402	"	6.51	.624	.46	.45	118	119	
403	"	6.85	.665	.45	.44	120	121	
404	"	7.05	.707	.45	.44	120	121	
405	"	7.37	.740	.43	.43	122	122	
406	"	7.57	.775	.43	.42	123	124	
407	"	7.76	.807	.42	.41	123	125	
408	"	7.93	.837	.42	.40	124	126	
409	4.9	2.39	.334	.56	.62	107	102	H. & T. p. 173. Test channel, unplaned boards. Sides vertical, bottom polygonal, 3 lines, bottom 3.28 feet, 2 sides sloped 45°. Darcy and Bazin. No. 21.
410	"	2.93	.485	.55	.54	108	109	
411	"	3.35	.586	.50	.50	113	113	
412	"	3.62	.673	.50	.48	114	115	
413	"	3.85	.744	.48	.47	115	117	
414	"	4.03	.809	.48	.44	116	121	
415	"	4.20	.864	.47	.43	117	122	
416	"	4.39	.911	.45	.42	119	123	
417	"	4.51	.959	.45	.41	119	125	
418	"	4.64	1.00	.45	.41	120	125	
419	"	4.76	1.05	.45	.40	120	126	
420	"	4.87	1.10	.45	.39	120	128	
421	.014	.44	1.02	.47	.43	117	122	H. & T. p. 177. Sudbury Conduit, in Mass. Hard brick, smooth surface, with mortar joints well made; surface carefully scraped clear of foreign substances. Bottom slope per 1000 about .16. Length 600. Velocities determined from weir measures with great care. Fteley and Stearns, 1880.
422	.027	.55	.858	.45	.42	120	124	
423	.038	.79	1.01	.40	.40	127	127	
424	.075	1.06	.957	.40	.45	126	120	
425	.098	1.10	.778	.40	.45	126	120	
426	.111	1.24	.850	.39	.46	127	118	
427	.160	1.15	.577	.45	.56	120	107	
428	.163	1.30	.673	.42	.51	124	112	
429	.164	1.08	.493	.45	.58	120	105	
430	.170	1.57	.891	.40	.47	127	117	
431	.171	1.58	.855	.39	.47	128	117	
432	.174	1.42	.762	.42	.50	124	113	
433	.180	1.44	.751	.42	.50	124	113	

Class 8—Continued.

No.	1000 s	V	r	100 m_0	100 m	C_0	C	*Authority.*
434	.193	1.83	1.08	.40	.43	127	122	H. & T. p. 177. Sud. Con. in Mass. Hard brick, smooth surface, mortar joints well made, fairly clean. Bottom slope .189 per m. Fteley and Stearns.
435	.191	2.13	1.37	.37	.38	132	130	
436	.192	2.14	1.38	.37	.38	131	130	
437	.192	2.35	1.62	.36	.36	133	133	
438	.192	2.37	1.63	.36	.36	134	133	
439	.190	2.56	1.84	.34	.34	137	137	
440	.193	2.72	2.05	.34	.32	137	141	
441	.189	2.83	2.19	.33	.32	139	141	
442	.192	2.93	2.33	.33	.32	138	141	
443	.189	1.84	1.07	.38	.42	129	124	Same.
444	.190	2.14	1.37	.36	.38	133	130	
445	.190	2.16	1.38	.36	.37	133	132	
446	.190	2.36	1.62	.35	.35	135	135	
447	.192	2.39	1.62	.35	.35	136	135	
448	.190	2.57	1.84	.34	.34	138	138	
449	.190	2.73	2.04	.33	.34	139	138	
450	.190	2.83	2.19	.33	.33	140	140	
451	.190	2.94	2.33	.33	.32	140	141	
452	.049	1.43	2.18	.34	.32	138	141	Same. Bottom slope .189 per m. Surface slope varies considerably, hence cross sections and flow are not uniform. Values given are averages. First 15 measurements made in lower section 4200 feet long. Second fifteen made in upper section 5294 feet long.
453	.067	1.72	2.28	.33	.32	139	141	
454	.104	1.82	1.76	.35	.35	134	134	
455	.144	1.91	1.49	.37	.39	131	128	
456	.155	2.20	1.73	.36	.34	135	137	
457	.163	2.41	1.91	.34	.34	136	137	
458	.179	2.07	1.40	.38	.40	131	127	
459	.186	2.91	2.34	.33	.32	140	141	
460	.200	2.16	1.37	.38	.42	131	124	
461	.201	2.89	2.18	.33	.32	138	141	
462	.204	2.42	1.60	.36	.36	134	133	
463	.207	2.79	2.01	.34	.33	137	139	
464	.208	2.63	1.81	.35	.34	135	137	
465	.241	3.10	2.10	.34	.32	138	141	
466	.260	3.39	2.29	.33	.31	139	144	
467	.033	1.21	2.36	.35	.32	136	141	Same. Upper section.
468	.049	1.50	2.42	.34	.31	138	144	
469	.062	1.51	1.96	.34	.33	137	139	
470	.095	1.62	1.65	.38	.36	129	133	
471	.116	1.98	1.84	.34	.34	136	137	
472	.136	2.25	1.98	.34	.33	138	139	
473	.147	1.93	1.47	.37	.34	132	137	
474	.179	2.89	2.34	.32	.32	141	141	
475	.210	2.95	2.15	.33	.33	139	139	
476	.239	2.96	1.94	.34	.34	137	137	
477	.255	2.45	1.25	.34	.39	137	128	
478	.258	2.69	1.49	.34	.37	137	132	
479	.260	2.89	1.71	.34	.35	137	136	
480	.460	4.10	1.79	.31	.34	143	138	
481	.491	4.41	2.00	.33	.32	140	141	
482	3.72	10.26	1.50	.34	.32	137	141	H. & T. p. 179. Roquefavour Aqueduct. Bottom neat cement, sides brick smooth. Darcy and Bazin. No. 1.

Class 9.

$\mu = .0185, \ \lambda = .0007.$

No.	1000 s	V	r	100 m_0	100 m	C_0	C	*Authority.*
483	.213	.29	.207	3.38	1.41	44	67	H. & T. p. 147. Cast iron main at Stuttgart, 1 year old. Curves, valves and several bends. Ehmann, 1879.
484	.392	.72	"	1.00	1.00	80	64	
485	.781	1.14	"	.80	.91	90	84	
486	1.34	1.51	"	.78	.86	91	87	
487	2.15	1.90	"	.79	.83	90	88	
488	3.22	2.31	"	.80	.81	89	89	
489	4.60	2.68	"	.85	.79	87	90	
490	.82	1.35	.25	.72	.82	94	89	H. & T. p. 147. Cast iron pipe, some unknown obstruction. Iben.
491	3.10	2.37	"	.88	.76	86	92	
492	4.21	2.77	"	.88	.75	85	93	
493	.39	1.60	.625	.61	.58	103	105	H. & T. p. 149. New cast iron force main, Phila. With turns and curves, also check valves. Darrach.
494	.46	1.74	"	.61	.57	103	106	
495	.53	1.87	"	.61	.57	103	106	
496	.60	2.00	"	.60	.56	103	107	
497	.67	2.14	"	.59	.56	105	107	
498	------	1.00	.294	.85	.82	87	89	T. & R. p. 246. No. 2. Riveted iron pipe.
499	.52	.91	.201	.81	.91	89	84	H. & T. p. 153. Old cast iron pipe after being cleaned. Before cleaned m = 1.14. Darcy.
500	4.98	3.11	"	.66	.78	98	91	
501	20.3	6.25	"	.68	.69	98	97	
502	37.3	8.44	"	.68	.64	98	100	
503	113	14.8	"	.68	.54	98	109	
504	.100	1.15	.984	.48	.49	115	115	H. & T. p. 163. Dhuys Aqueduct. Cement plaster on brick.
505	6.0	3.57	.237	.72	.73	95	94	H. & T. p. 167. Flume of sawed boards, rectangular. Darcy and Bazin. No. 20.
506	"	4.00	.281	.68	.67	97	98	
507	"	4.20	.304	.67	.66	98	99	
508	"	4.23	.317	.68	.65	97	99	
509	"	4.67	.347	.62	.62	102	102	
510	"	4.94	.372	.59	.60	105	104	
511	"	5.10	.393	.58	.60	105	104	
512	"	5.26	.412	.57	.58	106	105	
513	"	5.49	.431	.55	.57	108	106	

Class 9—Continued.

No.	1000 s	V	r	100 m_0	100 m	C_0	C	*Authority.*
514	1.5	1.80	.276	.83	.76	88	92	H. & T. p. 169. Flume of sawed boards, rectangular. Darcy and Bazin. No. 9.
515	"	2.37	.406	.69	.68	96	97	
516	"	3.10	.590	.59	.56	104	107	
517	"	3.63	.720	.53	.51	110	112	
518	"	4.05	.824	.49	.48	115	115	
519	"	4.41	.912	.45	.46	119	118	
520	"	4.66	.998	.44	.44	120	121	
521	4.9	2.75	.192	.80	.86	90	87	H. & T. p. 175. Test channel. Brick work, not smooth. Darcy and Bazin. No. 3.
522	"	3.66	.284	.67	.69	98	97	
523	"	4.18	.365	.66	.65	99	99	
524	"	4.72	.424	.60	.59	104	104	
525	"	5.10	.481	.58	.54	105	109	
526	"	5.33	.540	.60	.53	104	110	
527	"	5.68	.582	.57	.52	106	111	
528	"	6.01	.620	.54	.50	109	113	
529	"	6.15	.668	.56	.49	107	114	
530	"	6.47	.697	.52	.48	111	115	
531	"	6.60	.739	.53	.47	110	117	
532	"	6.72	.779	.54	.47	109	117	
533	2.08	2.08	.240	.74	.78	93	91	H. & T. p. 171. Rectangular flume of sawed boards. Darcy and Bazin. No. 6.
534	"	2.69	.363	.67	.66	98	99	
535	"	3.16	.453	.61	.60	103	104	
536	"	3.53	.528	.57	.53	106	110	
537	"	3.78	.601	.56	.53	107	110	
538	"	4.13	.648	.51	.52	113	111	
539	"	4.34	.704	.50	.50	113	113	
540	"	5.51	.759	.50	.49	114	115	
541	"	4.72	.801	.48	.47	116	117	
542	"	4.88	.846	.47	.46	116	118	
543	"	5.09	.880	.45	.45	119	120	
544	"	5.21	.922	.45	.45	119	120	

Class 10.

$\mu = .020, \ \lambda = .0011.$

No.	1000 s	V	r	100 m_0	100 m	C_0	C	*Authority.*
545	.947	3.46	1.0	.50	.50	113	113	H. & T. p. 151. Cast iron main, 3½ miles long. Asphalted. Bends. Jas. M. Gale, 1869.
546	------	4.00	.798	.54	.53	109	110	T. & R. p. 240. No. 11. Sheet iron riveted pipe.
547	.26	.6	.25	1.17	1.18	74	74	H. & T. p. 153. Old cast iron pipe, 2 years in use, slightly incrusted. Iben.
548	.41	.8	"	.98	1.06	81	78	
549	.81	1.2	"	.89	.94	85	83	
550	1.28	1.6	"	.76	.89	92	81	
551	2.99	2.4	"	.87	.85	86	87	
552	24.6	3.38	.097	1.35	1.11	69	76	H. & T. p. 165. Stamp Mill Race. Wooden trough, rectangular. Width 2.58 feet. Rittinger.
553	"	6.05	.202	.87	.77	86	91	
554	"	9.19	.245	.46	.65	118	99	
555	"	8.76	.277	.57	.64	106	100	
556	"	9.50	.317	.55	.60	108	104	
557	"	10.5	.363	.52	.58	111	105	
558	8.1	.72	.038	3.95	3.88	41	41	H. & T. p. 173. Test channel. Smooth boards covered with stout canvas. Rectangular. Darcy and Bazin, No. 30. Width .31 feet. Depth .05 to .27.
559	"	.89	.046	3.04	2.75	46	48	
560	"	1.11	.055	2.38	2.20	52	54	
561	"	1.33	.067	1.98	1.75	57	61	
562	"	1.51	.078	1.80	1.52	60	65	
563	"	1.88	.102	1.50	1.27	65	71	
564	15.2	.69	.031	6.15	5.00	32	36	H. & T. p. 173. Test channel. Smooth boards covered with stout canvas. Rectangular. Darcy and Bazin, No. 31. Width .31 feet. Depth .04 to .23.
565	"	.82	.040	5.79	4.10	33	40	
566	"	1.19	.051	3.50	2.22	43	54	
567	"	1.25	.054	3.40	2.09	44	56	
568	"	1.55	.066	2.68	2.71	49	48	
569	"	1.62	.067	2.50	1.66	51	62	
570	"	1.91	.079	2.13	1.40	55	68	
571	"	2.12	.089	1.93	1.30	58	70	
572	"	2.23	.095	1.88	1.27	59	71	
573	8.1	5.73	.41	.64	.62	100	102	H. & T. p. 175. Chazilly Canal. Section regular trapezoid. Ashlar masonry smooth. Darcy and Bazin. No. 39.
574	"	7.52	.57	.52	.54	111	109	
575	"	8.19	.68	.53	.51	110	112	
576	"	8.75	.77	.52	.48	111	115	

Class 11.

$\mu = .022, \quad \lambda = .0030.$

No.	1000 s	V	r	100 m_0	100 m	c_0	c	Authority.
577	------	1.0	.882	.70	.68	96	97	T. & R. p. 246. No. 13. Herschel. Kearney extension of East Jersey Water Co., pipe line. Taper joints, new pipe. Coating unusually smooth. Riveted iron pipe.
578	------	1.5	"	.61	.64	103	100	
579	------	2.0	"	.55	.62	108	102	
580	------	2.5	"	.52	.60	111	104	
581	------	3.0	"	.51	.59	113	105	
582	------	3.5	"	.50	.58	113	105	
583	------	4.0	"	.51	.57	113	106	
584	------	4.5	"	.51	.56	112	107	
585	------	5.0	"	.52	.55	111	108	
586	------	5.5	"	.53	.55	110	108	
587	------	6.0	"	.53	.54	110	109	
588	------	1.0	.882	.63	.68	101	97	T. & R. p. 246. No. 14. Same as above on conduit No. 2. New riveted iron pipe.
589	------	1.5	"	.61	.64	103	100	
590	------	2.0	"	.59	.62	104	102	
591	------	2.5	"	.58	.60	105	104	
592	------	3.0	"	.57	.59	106	105	
593	------	3.5	"	.56	.58	107	105	
594	------	4.0	"	.55	.57	108	106	
595	------	4.5	"	.55	.56	108	107	
596	------	5.0	"	.55	.56	108	107	
597	------	5.5	"	.55	.55	108	108	
598	------	6.0	"	.55	.55	108	108	
599	------	1.0	1.01	.63	.63	101	101	T. & R. p. 246. No. 15. Same, on Conduit No. 1; cylinder joints; asphalt coating, new pipe. Riveted iron pipe.
600	------	1.5	"	.58	.61	105	103	
601	------	2.0	"	.54	.59	109	104	
602	------	2.5	"	.52	.57	111	106	
603	------	3.0	"	.50	.56	113	107	
604	------	3.5	"	.50	.55	113	108	
605	------	4.0	"	.50	.55	113	108	
606	------	4.5	"	.51	.54	112	109	
607	------	5.0	"	.51	.53	112	110	
608	------	5.5	"	.51	.52	112	111	
609	------	6.0	"	.51	.52	112	111	
610	------	1.0	1.01	.68	.63	97	101	T. & R. p. 246. No. 17. Portion of same conduit as No. 15, but 4 years later. Riveted iron pipe.
611	------	1.5	"	.63	.61	101	103	
612	------	2.0	"	.60	.59	103	104	
613	------	2.5	"	.58	.57	105	106	
614	------	3.0	"	.58	.56	105	107	
615	------	3.5	"	.58	.55	105	108	
616	------	4.0	"	.59	.55	104	108	
617	------	4.5	"	.60	.54	104	108	
618	------	5.0	"	.60	.54	104	109	
619	------	5.5	"	.60	.53	104	110	
620	------	6.0	"	.60	.52	104	111	

Class 11—Continued.

No.	1000 *s*	*V*	*r*	100 m_0	100 *m*	C_0	*C*	*Authority.*
621	------	1.0	1.01	.68	.63	97	101	T. & R. p. 246. No. 18. Same, on portion of conduit No. 2; taper joints new riveted iron pipe.
622	------	1.5	"	.66	.61	99	103	
623	------	2.0	"	.64	.59	100	104	
624	------	2.5	"	.62	.57	102	106	
625	------	3.0	"	.61	.56	102	107	
626	------	3.5	"	.60	.55	104	108	
627	------	4.0	"	.59	.55	104	108	
628	------	4.5	"	.58	.54	105	109	
629	------	5.0	"	.58	.54	105	109	
630	------	5.5	"	.58	.52	105	111	
631	------	6.0	"	.58	.52	105	111	
632	------	1.0	1.31	.53	.56	110	110	T. & R. p. 246. No. 19. Riveted iron pipe. New pipe, butt joints, asphalt coating. Wing and Hoskins. Ogden, Pioneer Electric Company.
633	------	1.5	"	.52	.51	111	111	
634	------	2.0	"	.53	.50	110	113	
635	------	2.5	"	.55	.49	108	114	
636	------	3.0	"	.58	.48	108	116	
637	------	3.5	"	.53	.47	110	117	
638	------	4.0	"	.52	.47	111	117	
639	------	1.0	.75	.87	.73	86	94	T. & R. p. 246. No. 8. Riveted iron pipe. New pipe. Clemens Herschel. East Jersey Conduit.
640	------	1.5	"	.78	.69	91	97	
641	------	2.0	"	.71	.66	95	99	
642	----	2.5	"	.65	.63	99	101	
643	----	3.0	"	.60	.62	103	102	
644	----	3.5	"	.56	.61	107	103	
645	----	4.0	"	.53	.61	111	103	
646	----	4.5	"	.50	.60	114	104	
647	----	5.0	"	.47	.60	117	105	
648	----	5.5	"	.44	.59	120	106	
649	----	6.0	"	.42	.57	123	106	
650	.427	2.87	1.41	.47	.50	117	113	H. & T. p. 173. Wooden flume near Boston. Straight, 2500 feet long. Square section 6 x 6 feet, plank lengthwise. Principally sewage. Clarke, 1885.
651	.435	2.94	1.45	.47	.49	117	115	
652	.843	4.80	1.50	.35	.44	135	121	

Class 12.

$\mu = .025, \ \lambda = .0055.$

No.	1000 s	V	r	100 m_0	100 m	c_0	c	*Authority.*
653	----	1.0	2.16	.47	.52	117	111	T. & R. p. 246. No. 21. Herschel on Holyoke flume. Cylinder joints; paint coating washed off; rather rusty.
654	----	1.5	"	.50	.51	113	112	
655	----	2.0	"	.53	.50	110	114	
656	----	2.5	"	.54	.49	109	115	
657	----	3.0	"	.55	.48	108	116	
658	----	3.5	"	.56	.48	107	116	
659	----	4.0	"	.57	.47	106	117	
660	----	4.5	"	.58	.46	106	118	
661	1.5	1.65	.302	1.07	1.31	77	70	H. & T. p. 183. Test channel. Boards with wooden laths, 1 x ⅝ inches, nailed crosswise on bottom and sides of flume, ⅝ inch apart. Rectangular. Darcy and Bazin. No. 12, and No. 13.
662	"	2.17	.442	.90	1.18	84	74	
663	"	2.86	.634	.78	.88	91	86	
664	"	3.33	.775	.73	.77	94	91	
665	"	3.68	.889	.68	.67	97	98	
666	"	3.98	.986	.66	.65	99	99	
667	"	4.19	1.08	.66	.64	99	100	
668	5.9	2.50	.205	1.25	1.50	72	65	
669	"	3.34	.302	1.03	1.18	79	74	
670	"	4.40	.442	.87	.87	86	86	
671	"	5.08	.552	.81	.80	89	89	
672	"	6.63	.643	.79	.72	91	94	
673	"	6.14	.716	.72	.69	95	96	
674	"	6.48	.790	.71	.65	95	99	
675	8.9	3.85	.182	1.29	1.82	71	59	H. & T. p. 185. Same as above. Darcy and Bazin. No. 14.
676	"	3.75	.273	1.10	1.10	76	76	
677	"	4.92	.403	.94	.87	82	86	
678	"	5.77	.499	.85	.78	87	91	
679	"	6.38	.582	.81	.73	89	94	
680	"	6.86	.658	.80	.69	90	96	
681	"	7.26	.726	.79	.60	91	102	

Class 13.

$\mu = .032, \quad \lambda = .0080.$

No.	1000 s	V	r	100 m_0	100 m	c_0	c	*Authority.*
682	----	1.0	1.01	1.06	1.02	78	79	T. & R. p. 246. No. 16. Riveted iron pipe. Same pipe as No. 15, but 4 years older.
683	----	1.5	"	.90	.94	84	83	
684	----	2.0	"	.80	.90	89	84	
685	----	2.5	"	.75	.87	92	86	
686	----	3.0	"	.74	.85	93	87	
687	----	3.5	"	.74	.83	93	88	
688	----	4.0	"	.73	.82	94	89	
689	----	4.5	"	.72	.80	94	90	
690	----	5.0	"	.72	.79	94	90	
691	----	5.5	"	.72	.78	95	91	
692	----	6.0	"	.71	.77	95	91	
693	----	1.0	1.31	.87	.83	82	88	T. & R. p. 246. No. 20. Riveted iron pipe. Same pipe as No. 19, two years later.
694	----	1.5	"	.76	.77	92	91	
695	----	2.0	"	.67	.75	98	93	
696	----	2.5	"	.63	.73	101	94	
697	----	3.0	"	.61	.71	102	95	
698	----	3.5	"	.60	.70	103	96	
699	----	4.0	"	.60	.69	104	97	
700	----	4.5	"	.59	.68	104	97	
701	----	5.0	"	.59	.67	105	98	
702	----	5.5	"	.58	.67	105	98	
703	3.88	2.71	.416	1.43	1.33	67	69	H. & T. p. 157. Cast iron force main at Phila. Eleven years old. Darrach, 1878.
704	4.55	3.01	"	1.35	1.34	69	69	
705	5.21	3.31	"	1.28	1.23	71	72	
706	5.88	3.61	"	1.21	1.21	73	73	
707	6.55	3.91	"	1.14	1.18	75	74	
708	7.22	4.21	"	1.09	1.15	77	75	
709	7.89	4.51	"	1.03	1.13	79	76	
710	8.56	4.81	"	.99	1.12	81	76	
711	9.22	5.11	"	.96	1.11	82	76	
712	.305	1.32	.98	1.08	.98	77	81	H. & T. p. 175. Grosbois Canal. Ashlar masonry set in mortar. Sides smoother than bottom. Darcy.
713	.308	1.90	1.29	.71	.80	95	90	
714	.331	2.12	1.49	.70	.75	96	93	
715	.347	2.47	1.60	.58	.61	105	103	
716	37.0	9.04	.424	1.24	1.49	72	66	H. & T. p. 181. Tail race Grosbois Reservoir. Ashlar masonry cement joints. Partly damaged. Sticky, slimy deposit. Darcy.
717	"	11.5	.620	1.13	1.02	76	79	
718	"	13.6	.745	.97	.88	82	85	
719	"	15.1	.852	.89	.84	85	86	
720	.151	2.20	2.52	.50	.59	113	104	H. & T. p. 181. Solani Right Aqueduct. Floor bricks, sides masonry. Rectangular. In fair order. Cunningham, 1880.
721	.145	2.54	2.72	.39	.57	128	106	
722	.200	2.51	2.94	.60	.59	103	104	
723	.208	2.79	2.94	.50	.58	113	105	
724	.253	3.20	2.99	.48	.53	116	110	
725	.473	4.83	3.65	.48	.51	116	112	
726	.025	1.24	4.20	.44	.49	121	114	

Class 13—Continued.

No.	1000 *s*	*V*	*r*	100 m_0	100 *m*	c_0	*c*	*Authority.*
727	1.5	2.17	.454	1.06	1.21	78	73	H. & T. p. 181. Test channel. Semi-circular. Pebbles ⅜ to ⅞ inch, set in cement. Darcy and Bazin. No. 27.
728	"	2.50	.546	.96	1.13	82	75	
729	"	2.69	.619	.96	1.05	82	78	
730	"	2.93	.681	.91	1.01	84	80	
731	"	3.05	.731	.91	.96	84	82	
732	"	3.22	.784	.89	.93	85	83	
733	"	3.33	.826	.91	.91	84	84	
734	"	3.54	.900	.89	.86	85	86	
735	"	3.73	.968	.89	.84	85	87	
736	"	3.95	1.01	.83	.82	88	89	
737	4.9	2.16	.250	1.67	1.47	62	66	H. & T. p. 183. Test channel. Rectangular. Pebbles ⅜ to ⅞ incn, held in place with cement. Darcy and Bazin. No. 4.
738	"	2.95	.375	1.29	1.27	70	71	
739	"	3.40	.450	1.22	1.17	72	74	
740	"	3.84	.520	1.11	1.08	76	77	
741	"	4.14	.588	1.09	1.04	77	78	
742	"	4.43	.644	1.03	.97	79	81	
743	"	4.64	.700	1.01	.94	79	83	
744	"	4.88	.746	.98	.90	81	84	
745	"	5.12	.785	.94	.88	83	85	
746	"	5.26	.832	.94	.85	82	87	
747	"	5.43	.871	.93	.83	83	88	
748	"	5.57	.910	.92	.81	83	89	
749	42.3	9.45	.32	.99	1.04	80	79	H. & T. p. 189. Gontenbach-schale. Dry rubble masonry of large stones. New and well built. Semi-circular. Kutter, 1867.
750	46.4	10.5	.32	.88	1.00	85	80	
751	42.3	9.89	.37	.94	.98	82	81	
752	46.4	11.0	.37	.92	.97	84	82	
753	1.0	1.13	.373	1.88	2.24	58	55	H. & T. p. 191. Mill race at Picbram. Masonry, clay bottom, very regular. Trapezoidal. Rittinger, 1855.
754	"	1.25	.425	1.74	1.63	61	63	
755	.50	1.60	1.01	1.25	1.25	71	71	H. & T. p. 193. Mill race at Freiberg. Dry rubble walls, clay bed. Good condition. Bornemann.
756	3.2	2.89	.403	1.02	1.24	81	72	H. & T. p. 197. Mill race at Magura. Sides of earth, bottom paved with broken stone. Rittinger.
------	"	3.63	.661	1.04	1.06	79	78	
------	"	3.59	.834	1.33	.93	69	83	
------	"	4.02	.918	1.17	.85	74	93	
757	2.0	1.95	.467	1.57	1.50	64	65	H. & T. p. 199. Mill race at Flachau. Shallow ditch in earth. Rittinger.

Class 14.

$\mu = .038, \quad \lambda = .0140.$

No.	1000 s	V	r	100 m_0	100 m	C_0	C	*Authority.*
758	7.66	.80	.124	9.53	8.90	26	27	H. & T. p. 151. Old cast iron pipe at Hamburg. 13 years in use. Foul and heavily incrusted. Iben, 1876.
759	11.3	1.3	"	5.57	6.60	34	31	
760	16.8	1.5	"	5.57	5.40	34	34	
761	19.3	1.7	"	5.57	5.30	34	35	
762	22.2	1.8	"	5.57	4.80	34	37	
763	24.4	1.9	"	5.57	4.47	34	38	
764	2.5	1.26	.289	2.94	2.86	47	47	H. & T. p. 187. Tail race at Staukau. Dry rubble walls, paved. Semi-circular. Rittinger.
765	"	1.49	.359	2.60	2.35	50	52	
766	"	1.64	.419	2.50	2.04	51	56	
767	4.5	1.32	.213	3.51	3.58	43	42	H. & T. p. 187. Dry rubble aqueduct. Rectangular. Rittinger.
768	"	2.40	.439	2.22	1.73	54	61	
769	"	2.43	.486	2.38	1.66	52	62	
770	3.8	1.37	.213	2.79	3.65	48	42	H. & T. p. 187. Dry rubble walls, paved. Rectangular. Rittinger.
771	"	1.83	.344	2.52	2.31	51	53	
772	3.6	1.50	.278	2.84	2.80	47	48	H. & T. p. 187. Dry rubble, paved. Trapezoidal. Bottom 2.5 feet. Rittinger.
773	"	1.93	.351	2.20	2.13	54	55	
774	"	2.10	.403	2.12	1.98	55	57	
775	.243	1.40	1.24	.98	1.07	81	78	H. & T. p. 193. Dry rubble walls, clay bed. Trapezoidal. Bornemann.
776	.520	1.50	.86	1.27	1.30	71	70	Same.

Class 15.

$\mu = .045, \quad \lambda = .020.$

No.	$1000\,s$	V	r	$100\,m_0$	$100\,m$	c_0	c	*Authority.*
777	.92	1.07	.625	3.18	2.30	45	53	H. & T. p. 157. Cast iron force main at Phila. 9 years old. One curve of short radius. Static head 190 feet. Darrach, 1878. 5% slip.
778	1.05	1.20	"	2.92	2.20	47	54	
779	1.18	1.34	"	2.68	2.12	49	55	
780	1.31	1.47	"	2.47	1.93	51	57	
781	1.44	1.60	"	2.20	1.91	54	58	
782	1.58	1.74	"	2.12	1.87	56	59	
783	1.71	1.88	"	1.95	1.82	58	59	
784	1.84	2.01	"	1.82	1.81	59	59	
785	1.97	2.15	"	1.73	1.74	61	61	
786	2.10	2.28	"	1.62	1.71	63	61	
787	.93	1.58	.75	1.48	1.72	66	61	H. & T. p. 157. Cast iron force main at Phila. 7 years old. Curves 25 feet radius. 5% slip. Static head 100 feet. Darrach.
788	1.09	1.74	"	1.73	1.69	61	62	
789	1.25	1.89	"	1.67	1.66	62	62	
790	1.40	2.05	"	1.62	1.62	63	63	
791	1.56	2.21	"	1.52	1.60	65	63	
792	1.72	2.37	"	1.48	1.57	66	64	
793	101	12.3	.324	1.39	1.39	68	68	H. & T. p. 175. Spillway Grosbois Reservoir. Ashlar masonry, with cement joints, not in very good order. Rectangular. Darcy.
794	"	16.2	.467	1.15	1.10	74	77	
795	"	18.7	.580	1.06	1.01	77	79	
796	"	21.1	.662	.96	.93	82	83	
797	.203	1.61	1.95	.85	1.27	87	71	H. & T. p. 181. Solani Right Aqueduct. Floor bricks, sides good masonry. In fair order. Rectangular. Cunningham.
798	.195	2.43	3.26	.69	.75	96	93	
799	.240	3.43	5.00	.66	.61	99	102	
800	.245	3.74	5.43	.61	.58	103	105	
801	.220	3.67	6.14	.64	.55	100	108	
802	.198	3.86	6.63	.57	.55	106	108	
803	.228	3.85	6.88	.68	.53	97	110	
804	4.9	1.79	.291	2.85	3.20	47	45	H. & T. p. 183. Test channel. Lined with pebbles 1¼ to 1½ inches diam., held in place with cement. Rectangular. Darcy and Bazin, No. 5.
805	"	2.43	.417	2.21	2.22	54	54	
806	"	2.90	.510	1.91	1.84	58	59	
807	"	3.27	.587	1.73	1.61	61	63	
808	"	3.56	.656	1.63	1.50	63	65	
809	"	3.85	.712	1.50	1.41	65	67	
810	"	4.03	.772	1.50	1.35	66	69	
811	"	4.23	.823	1.45	1.30	67	70	
812	"	4.43	.867	1.39	1.26	68	71	
813	"	4.60	.909	1.35	1.23	69	72	
814	"	4.78	.946	1.30	1.22	70	73	
815	"	4.90	.987	1.30	1.22	70	73	
816	1.5	1.28	.378	2.22	3.00	54	46	H. & T. p. 185. Test channel. Boards with wooden laths 1 x ⅜ inch, nailed crosswise on bottom and sides, 2 inches apart. Rectangular. Darcy and Bazin. No. 15.
817	"	1.68	.550	1.89	2.01	59	54	
818	"	2.21	.777	1.53	1.55	65	64	
819	"	2.55	.942	1.40	1.33	68	69	
820	"	2.81	1.07	1.31	1.24	70	72	
821	"	2.97	1.20	1.31	1.20	70	73	
822	"	3.11	1.30	1.30	1.14	70	75	

Class 15—Continued.

No.	1000 s	V	r	100 m_0	100 m	c_0	c	*Authority.*
823	5.9	1.91	.264	2.71	2.69	48	49	H. & T. p. 185. Same.
824	"	2.56	.384	2.22	2.22	54	54	
825	"	3.37	.553	1.85	1.66	59	62	
826	"	3.88	.686	1.73	1.45	61	66	
827	"	4.31	.791	1.62	1.43	63	67	
828	"	4.65	.882	1.55	1.22	64	72	
829	"	4.91	.965	1.52	1.20	65	73	
830	8.86	2.21	.232	2.70	3.75	49	42	H. & T. p. 185. Same.
831	"	2.85	.350	2.47	2.24	51	53	
832	"	3.75	.509	2.07	1.73	56	61	
833	"	4.37	.628	1.88	1.50	57	66	
834	"	4.85	.725	1.76	1.37	61	69	
835	"	5.22	.812	1.76	1.26	62	71	
836	"	5.57	.885	1.63	1.20	63	73	
837	3.1	1.11	.272	4.34	4.36	38	38	H. & T. p. 185. Dry rubble, walls paved. Semicircular. Rittinger.
838	"	1.25	.345	4.40	3.91	38	40	
839	2.1	3.30	.212	2.62	2.90	49	47	H. & T. p. 187. Dry rubble masonry conduit. Trapezoidal. Rittinger.
840	"	6.15	.358	1.32	1.72	71	61	
841	"	7.56	.535	1.25	1.28	71	71	
842	12.1	7.58	.88	1.18	1.15	74	75	H. & T. p. 187. Grosbois Canal. Roughly-hammered stone masonry. Darcy and Bazin. No. 1.
843	14.0	8.36	.84	1.07	1.15	77	75	
844	29.0	11.2	.71	1.06	1.11	79	76	
845	60.0	13.9	.62	1.22	1.08	73	77	
846	.648	1.47	.88	1.67	1.61	62	63	H. & T. p. 187. Grosbois Canal. Masonry, in bad order, mud and broken stone on bottom. Darcy and Bazin. No. 46.
847	.671	2.02	1.23	1.31	1.59	70	64	
848	.683	2.34	1.40	1.11	1.13	76	70	
849	.683	2.78	1.50	.85	1.07	87	77	
850	.30	1.12	1.07	1.67	2.16	62	55	H. & T. p. 187. Same.
851	.35	1.69	1.38	1.09	1.21	77	73	
852	.33	1.92	1.57	.91	1.09	84	77	
853	.30	2.18	1.71	.70	1.05	96	78	
854	2.42	1.58	2.42	.93	.91	83	84	H. & T. p. 189. Bed cement, sides rubble masonry. Epper.
855	3.4	6.56	.32	1.62	1.93	63	58	H. & T. p. 189. Dry rubble, large stones. Rectangular. Kutter.
856	2.0	.47	.233	13.4	9.00	22	27	H. & T. p. 191. Race, dry rubble masonry, earth bottom, irregular. Rectangular. Rittinger.
857	"	.85	.296	5.20	4.77	35	37	
858	"	1.14	.385	3.78	3.17	41	45	
859	.120	.89	1.35	1.30	1.42	70	67	H. & T. p. 193. Dry rubble walls, clay bed. Bornemann.
860	.135	.93	1.37	1.38	1.42	68	67	

Class 15—Continued.

No.	1000 *s*	*V*	*r*	100 m_0	100 *m*	C_0	*C*	*Authority.*
861 862 863	4.1 " "	1.32 2.19 2.11	.315 .411 .496	4.75 2.26 2.94	5.94 2.33 2.57	37 53 47	33 52 50	H. & T. p. 199. Mill race in earth, loamy soil. Rittinger.
864	1.3	1.42	.587	2.47	2.10	51	55	H. & T. p. 199. Earth. Habengraben creek. Rittinger, 1855.
865 866 867 868	.250 .275 .246 .275	.89 1.34 1.36 1.47	.96 1.32 1.57 1.78	1.98 1.31 1.35 1.48	2.43 1.27 1.20 1.10	57 70 69 66	52 71 73 76	H. & T. p. 199. Grosbois Canal. Earth, clean. Trapezoidal, bottom width 6.5 feet. Darcy and Bazin. No. 49.
869 870 871	1.75 1.25 1.20	2.82 3.14 3.22	1.04 1.39 1.41	1.48 1.14 1.05	1.27 1.10 1.11	66 75 78	71 76 76	H. & T. p. 199. Mill race, side slopes earth, bed fine gravel. Epper, 1885.
872	1.00	2.20	1.11	1.45	1.31	66	70	
873 874 875	2.7 " "	1.50 1.55 1.93	.315 .492 .596	2.40 3.52 2.75	3.18 2.92 2.04	51 42 48	45 47 56	H. & T. p. 199. Mill race, shallow ditch in sandy soil. Irregular. Rittinger.

Class 16.

$\mu = .052, \ \lambda = .050.$

No.	$\frac{1000}{s}$	V	r	$\frac{100}{m_0}$	$\frac{100}{m}$	c_0	c	*Authority.*
876	112	8.47	.19	1.94	2.80	57	48	H. & T. p. 189. Gerbebach-schale. Dry rubble masonry large stones, well built. Semicircular, width 15.7, depth 6.2, length 500 feet. Kutter, 1867.
877	138	8.91	"	2.19	2.70	54	49	
878	168	9.18	"	2.50	2.60	51	50	
879	185	9.43	"	2.60	2.60	50	50	
880	237	10.1	"	2.91	2.57	47	50	
881	82.8	11.8	.36	1.37	1.83	69	59	H. & T. p. 189. Grunnbach-schale. Dry rubble masonry of large stones, 6 years old. Bed somewhat damaged. Semicircular, width 20.3 depth 6.4, length 1200 feet. Kutter.
882	99.3	13.3	.38	1.38	1.63	68	63	
883	107	13.8	.39	1.40	1.57	67	64	
884	82.8	15.5	.58	1.30	1.27	71	71	
885	99.3	18.3	.61	1.21	1.22	73	73	
886	107	19.2	.65	1.21	1.20	73	73	
887	2.8	1.38	.384	3.65	4.81	42	37	H. & T. p. 191. Mill race at Diosgyor. Dry rubble side-walls, clay bed. Rectangular. Rittinger.
888	"	1.51	.458	3.40	4.06	42	40	
889	"	2.22	.572	2.09	2.02	55	56	
890	4.0	2.40	.487	2.16	2.96	55	47	H. & T. p. 191. Aqueduct at Diosgyor. Dry rubble side-walls, earth bed. Rectangular. Rittinger, 1855.
891	"	2.75	.736	2.50	2.11	51	54	
892	"	3.32	.924	2.15	1.71	55	61	
893	5.0	.78	.242	12.2	11.4	22	24	H. & T. p. 191. Race, masonry side walls, bed sand and gravel. Trapezoidal, bottom width 2.1 feet. Rittinger.
894	"	1.19	.282	6.30	7.40	32	29	
895	"	1.96	.407	4.40	3.87	43	41	
896	"	2.13	.483	4.40	3.31	43	45	
897	"	3.47	.561	1.50	2.27	66	53	
898	.940	1.73	.94	1.91	2.11	58	55	H. & T. p. 193. Mill race at Freiberg. Dry rubble walls, clay bed. First trapezoidal, second rectangular. Bornemann.
899	.755	1.11	.69	2.70	3.30	48	44	
900	.778	1.44	.866	2.12	2.39	55	52	H. & T. p. 199. Hockenbach creek. Earth. Grebneau, 1866.
901	.797	1.46	.879	2.13	2.38	55	52	
902	.036	.449	1.68	1.93	2.44	58	51	H. & T. p. 201. Canal du Jard. Earth, no detritus. Dubuat, 1779.
903	.036	.479	1.94	1.98	2.05	57	56	
904	.046	.607	2.05	1.64	1.53	63	65	
905	.065	1.07	2.58	.94	1.10	82	77	
906	.280	2.69	3.45	.87	.89	86	85	H. & T. p. 201. River Salzach. Earth, regular. Reich, 1855.
907	.290	3.51	4.96	.76	.72	92	94	
908	.348	3.62	3.52	.60	.84	103	88	
909	.410	5.09	5.20	.53	.69	110	97	
910	.607	5.54	5.00	.64	.69	100	97	

Class 16—Continued.

No.	1000 s	V	r	100 m_0	100 m	c_0	c	*Authority.*
911	30.0	7.97	.58	1.79	1.62	60	63	H. & T. p. 191. Saextenbach-schale. Dry rough rubble masonry. Rectangular, bed depressed 16 inches at center. Kutter.
912	.028	1.09	5.83	.88	1.19	86	74	H. & T. p. 201. River Haine.
913	.030	.90	4.83	1.15	1.37	75	69	
914	.156	2.06	5.74	1.35	1.12	69	76	
915	.165	2.39	4.92	.91	1.15	84	76	
916	.45	3.70	5.55	1.15	1.01	75	80	H. & T. p. 201. The river Arve.
917	1.1	.548	.348	8.21	10.6	28	25	H. & T. p. 203. Felso-Banya.
918	1.1	.570	.391	8.62	8.75	27	27	
919	.093	2.52	6.76	.64	.68	100	97	H. & T. p. 211. Ohio river at Pt. Pleasant.

Class 17.

$\mu = .060, \; \lambda = .080.$

No.	$1000\ s$	V	r	$100\ m_0$	$100\ m$	C_0	C	*Authority.*
920	14.2	5.66	.70	2.00	2.08	57	56	H. & T. p. 189. Darcy and Bazin, No. 35. No. 34 after being scraped and cleaned with great care, see Roughly hammered masonry set dry, well preserved.
921	"	7.36	.93	1.57	1.85	64	59	
922	"	8.94	1.23	1.40	1.27	68	71	
923	"	10.1	1.39	1.24	1.26	72	72	
924	"	11.3	1.49	1.06	1.21	77	73	
925	22.9	7.97	.73	2.21	1.80	54	60	H. & T. p. 191. Alpbachschale. Rubble masonry old and very much damaged. Kutter.
926	27.2	8.65	.73	1.74	1.78	61	60	
927	27.6	8.20	.73	1.94	1.79	58	60	
928	32.0	8.01	.69	2.22	1.82	54	59	
929	2.2	.39	.316	29.0	27.7	15	15	H. & T. p. 191. Mill race at Pricbram. Dry rubble walls, earth bottom, very irregular. Trapezoidal. Bottom width 2.1 feet. Rittinger.
930	"	.59	.336	13.4	14.8	22	21	
931	"	.95	.472	7.35	6.42	30	31	
932	"	1.13	.548	5.90	5.48	33	34	
933	"	1.19	.560	5.50	5.40	34	34	
934	"	1.27	.566	4.97	5.40	36	34	
935	.525	1.01	1.00	3.33	3.59	44	43	H. & T. p. 193. Chazilly Canal. Right side wall vertical masonry in mortar, left side wall sloped dry rubble. Darcy and Bazin. No. 42.
936	.450	1.38	1.36	2.05	2.30	56	53	
937	.462	1.58	1.54	1.85	1.63	59	63	
938	.487	1.74	1.67	1.73	2.06	61	56	
939	2.0	3.90	1.66	1.43	1.45	67	66	H. & T. p. 193. River Tauber, banks paved, regular. Ammon. Kutter.
940	1.9	3.79	1.71	1.48	1.45	66	66	
941	0.2	2.01	2.82	.91	1.24	84	72	
942	.64	3.50	3.40	1.15	1.22	75	73	H. & T. p. 195. River Aa. Sides good rubble, bed of gravel. Epper.
943	1.0	4.18	3.15	1.18	1.24	74	72	H. & T. p. 195. River Rhone. Slopes smooth pavements, bed gravel. Epper.
944	.625	5.18	5.44	.81	.79	89	90	H. & T. p. 195. River Aar. Banks and bed paved. Kutter.
945	.50	2.26	2.31	1.45	1.40	66	68	H. & T. p. 201. Marmel Canal, coarse gravel. Nicca.
946	2.0	4.79	2.38	1.33	1.45	69	66	H. & T. p. 201. Gurben Canal, detritus. Kutter.
947	1.15	4.95	3.16	.96	1.01	82	80	H. & T. p. 201. River Lech. Earth, regular. Gumpenberg.
948	.40	3.05	3.94	1.10	.97	77	81	H. & T. p. 201. River Main. Earth, regular. Kutter.

Class 17—Continued.

No.	1000 s	V	r	100 m_0	100 m	C_0	C	*Authority.*
949	.15	3.02	6.95	.73	.73	93	93	H. & T. p. 203. River Reuss. Sandy bed. Kutter.
950	.22	3.98	8.35	.75	.74	93	93	H. & T. p. 203. Solani Emb. New site. Earth, uniform, slopes 1½ to 1. Cunningham.
951	.875	2.07	1.54	2.00	2.30	57	53	H. & T. p. 199. River Saalach, earth with detritus. Roff.
952	1.10	2.24	1.31	1.88	1.88	59	59	
953	1.24	3.08	1.91	1.62	1.39	63	68	
954	1.24	3.38	1.98	1.39	1.39	68	68	
955	3.60	5.47	2.16	1.56	1.24	64	72	
956	.67	1.82	1.46	1.92	2.03	58	56	H. & T. p. 199. Speyerbach creek. Earth. Grabenau.
957	.291	2.82	4.50	1.04	.92	79	84	H. & T. p. 201. Solani Emb. Earth, banks 1:1 and 1½:1; oed very rough. Cunningham.
958	.297	2.79	4.37	1.06	.96	77	82	
959	.304	2.74	4.18	1.05	.96	78	82	
960	.306	2.71	4.07	1.09	.97	77	81	
961	.29	3.41	5.14	.82	.85	88	87	H. & T. p. 203. Linth Canal. Earth, no detritus. Trapezoidal, slightly rounded. Legler, Kutter.
962	.30	3.83	5.93	.78	.76	91	92	
963	.31	4.15	6.48	.75	.75	93	93	
964	.32	4.42	7.12	.75	.70	93	96	
965	.33	4.75	7.52	.70	.67	95	98	
966	.34	4.92	8.09	.73	.66	94	99	
967	.34	5.06	8.28	.71	.65	95	99	
968	.35	5.22	8.62	.71	.64	95	100	
969	.36	5.39	8.87	.70	.63	96	101	
970	.37	5.53	9.18	.71	.62	95	102	
971	.231	3.98	8.64	.81	.73	89	94	H. & T. p. 203. Solani Emb. Old site. Earth, 2½:1. Irregular. Cunningham.
972	.936	1.08	1.05	5.57	5.63	34	34	H. & T. p. 187. Grosbois Canal. Trapezoidal. Stony bottom, one slope rock, very little vegetation. Darcy and Bazin. No. 40.
973	.936	1.37	1.37	4.46	3.80	38	41	
974	.957	1.56	1.52	3.83	3.20	41	45	
975	.964	1.71	1.64	3.49	2.74	43	48	
976	.565	4.98	1.95	1.98	1.91	57	58	H. & T. p. 193. River Aa. Gravel bed, fairly regular. Riprap along shores. Epper.
977	.254	3.77	8.61	.99	.97	80	81	H. & T. p. 197. Elbe, embankment walls. Fischer, Kutter.
978	.363	5.35	13.2	1.09	.89	77	85	
979	.445	.96	1.04	3.18	3.50	45	43	H. & T. p. 205. Grosbois Canal. Earth, stony, but little vegetation. Trapezoidal. Darcy and Bazin. No. 41.
980	.450	1.27	1.38	2.48	2.38	51	52	
981	.455	1.40	1.57	2.38	2.15	52	55	
982	.441	1.51	1.71	2.13	1.91	55	58	

Class 17—Continued.

No.	1000 s	V	r	100 m_0	100 m	C_0	C	*Authority.*
983 984 985 986	.420 .470 .470 .450	.89 1.18 1.13 1.39	1.06 1.41 1.60 1.76	3.65 3.04 2.91 2.68	3.74 2.95 2.27 1.93	42 46 47 49	42 47 53 58	H. & T. p. 207. Grosbois Canal. Earth, covered with vegetation at many points. Trapezoidal. Darcy and Bazin, No. 43.
987	.157	2.96	6.73	.76	.74	92	93	H. & T. p. 215. The Rhine. Slightly irregular. Bed gravel. Epper.
988 989 990 991	.310 .290 .330 .330	.82 1.26 1.30 1.41	1.05 1.42 1.65 1.85	3.18 1.67 2.05 1.98	4.04 2.25 2.11 1.86	45 62 56 57	40 53 55 59	H. & T. p. 205. Grosbois Canal. Earth, some vegetation. Trapezoidal. Bottom width 6.3. Darcy and Bazin. No. 50.
992 993 994 995	.555 .555 .525 .515	.96 1.48 1.57 1.75	.99 1.30 1.56 1.71	3.83 2.13 2.13 1.85	3.60 2.29 2.07 1.86	41 55 55 59	42 53 56 59	H. & T. p. 205. Grosbois Canal. Earth, some vegetation. Near circular arc. Darcy and Bazin. No. 48.
996 997 998 999	.792 .808 .858 .842	1.23 1.67 1.81 2.00	.96 1.20 1.41 1.56	3.18 2.29 2.38 2.13	3.30 2.37 1.98 1.85	45 53 52 55	44 52 57 59	H. & T. p. 205. Grosbois Canal. Earth (stony), but little vegetation. Tapezoidal. Darcy and Bazin. No. 37.
1000 1001 1002 1003	.464 .450 .479 .493	.82 1.32 1.43 1.68	1.09 1.38 1.63 1.71	4.97 2.29 2.48 1.91	4.10 2.40 2.00 1.96	36 53 51 58	40 52 56 57	H. & T. p. 205. Grosbois Canal. Earth, bottom and sides of mud, some vegetation. Trapezoidal. Darcy and Bazin. No. 47.

Class 18.

$\mu = .068, \ \lambda = .130.$

No.	$\frac{1000}{s}$	V	r	$\frac{100}{m_0}$	$\frac{100}{m}$	C_0	C	*Authority.*
1004 1005	3.9	4.98	3.87	1.75	1.72	61	61	H. & T. p. 193. River Aar. Gravel bed, side walls ashlar masonry. Epper.
1006	.678	.91	1.14	5.89	5.17	33	35	H. & T. p. 207. Grosbois Canal. Earth, covered with vegetation. Trapezoidal. Darcy and Bazin. No. 36.
1007	.633	1.28	1.42	3.49	3.15	43	45	
1008	.644	1.45	1.61	3.18	2.68	45	49	
1009	.622	1.65	1.74	2.58	2.34	50	52	
1010	.957	1.24	.96	3.83	4.59	41	38	H. & T. p. 207. Chazilly Canal. Earth (stony); little vegetation. Trapezoidal. Darcy and Bazin. No. 38.
1011	.929	1.70	1.18	2.48	3.16	51	45	
1012	.993	1.80	1.41	2.80	2.67	48	49	
1013	.986	1.96	1.54	2.58	2.39	50	52	
1014	9.65	6.00	1.25	2.16	2.32	55	52	H. & T. p. 207. The Plessur River. Coarse gravel and detritus. La Nicca.
1015	"	9.99	2.33	1.45	1.45	66	66	
1016	"	10.2	3.48	1.45	1.10	66	77	
1017	"	13.6	3.58	1.21	1.03	73	79	
1018	"	13.9	3.59	1.15	1.01	75	80	
1019	"	13.7	4.58	1.50	1.04	65	79	
1020	.38	2.49	3.51	1.39	1.37	68	69	H. & T. p. 213. The Elbe. Coarse gravel pebbles. Changing channel. Harlacher.
1021	.37	3.74	5.18	.88	.96	85	82	
1022	.41	4.95	7.77	.84	.78	88	91	

Class 19.

$\mu = .082,\ \lambda = .180.$

No.	1000 s	V	r	100 m_0	100 m	C_0	C	*Authority.*
1023	5.0	3.51	1.19	3.11	3.07	45	46	H. & T. p. 205. The Emma. Irregular, coarse detritus. Kutter.
1024	3.32	3.21	1.34	2.80	2.53	48	50	H. & T. p. 205. Lutschine. Coarse detritus. Kutter.
1025	2.5	4.02	1.86	1.85	2.10	59	55	H. & T. p. 207. River Isar.
1026	2.5	7.18	6.03	1.88	1.15	58	75	
1027	.254	1.66	2.96	1.75	1.90	60	58	H. & T. p. 209. River Tessin. Coarse gravel and detritus. Grebenau.
1028	.22	3.67	8.66	.91	.92	84	84	H. & T. p. 211. Pannerden Canal. Kutter.
1029	.22	4.20	10.2	.83	.84	88	88	
1030	.04	.564	3.88	3.13	3.19	45	45	H. & T. p. 213. The River Saone. Lévéitté. Darcy and Bazin.
1031	"	.814	4.77	1.85	1.75	59	61	
1032	"	.988	7.06	1.85	1.26	59	71	
1033	"	1.60	8.92	.89	1.01	85	80	
1034	"	1.85	10.9	.81	.91	89	84	
1035	"	1.91	11.6	.82	.84	89	88	
1036	"	1.94	11.8	.81	.83	89	88	
1037	"	2.25	13.3	.67	.80	98	90	
1038	"	2.37	14.6	.67	.76	98	92	
1039	"	2.38	15.8	.72	.72	95	95	
1040	.127	2.09	5.66	1.06	1.18	78	74	H. & T. p. 213. The River Seine at Paris. Fairly regular. Villevert.
1041	.133	2.26	7.08	1.15	1.11	74	76	
1042	.135	2.42	8.43	1.26	.97	72	81	
1043	.140	3.37	9.48	.75	.88	92	86	
1044	.140	3.74	10.9	.70	.86	96	87	
1045	.140	3.82	12.2	.75	.78	92	91	
1046	.140	4.23	14.5	.73	.70	94	96	
1047	.140	4.51	15.2	.67	.69	98	97	
1048	.172	4.68	15.9	.80	.67	89	98	
1049	.131	4.80	16.8	.62	.65	102	100	
1050	.103	4.69	18.4	.56	.62	108	102	

Class 20.

$\mu = .096, \ \lambda = .230.$

No.	1000 s	V	r	100 m_0	100 m	c_0	c	*Authority.*
1051	11.9	3.87	.99	5.05	3.78	36	41	H. & T. p. 205. Lutschine. Coarse detritus. Kutter.
1052	"	5.54	1.20	3.06	2.96	46	47	
1053	"	7.59	1.53	2.05	2.30	56	53	
1054	6.5	4.92	1.82	3.18	4.05	45	40	H. & T. p. 207. Simme Canal. Very coarse gravel and detritus. Wampfler, Kutter.
1055	7.0	5.37	1.87	2.93	2.00	47	56	
1056	11.6	5.49	1.36	3.40	2.69	44	49	
1057	17.0	5.99	1.32	4.06	2.05	40	56	
1058	14.2	2.33	.42	6.72	11.70	30	24	H. & T. p. 207. The Rhine. Coarse gravel and detritus. La Nicca.
1059	"	4.53	.76	3.40	4.35	43	39	
1060	"	6.03	1.21	3.04	2.83	46	48	
1061	.698	2.72	3.66	2.22	1.84	54	59	H. & T. p. 209. Chesapeake and Ohio Canal. In bad order. Humphreys and Abbot.
1062	.698	3.03	3.70	1.82	1.65	60	62	
1063	.662	3.54	3.68	1.25	1.84	72	59	H. & T. p. 209. River Salzach. Detritus. Reich, Kutter.
1064	.940	3.48	3.53	1.74	1.65	60	63	
1065	.940	4.03	4.32	1.57	1.52	64	65	
1066	1.12	5.79	7.39	1.59	1.12	63	76	
1067	1.55	4.10	3.51	2.08	1.63	55	63	
1068	1.55	4.67	4.64	1.41	1.40	67	68	
1069	1.80	4.45	3.87	2.23	1.45	53	66	
1070	1.80	5.15	4.26	1.87	1.50	59	65	
1071	.40	2.30	3.52	1.73	1.96	61	57	H. & T. p. 209. River Zihl. Bed irregular. Mud and fine detritus. Treschel, Kutter.
1072	.46	3.71	5.02	1.08	1.41	77	68	
1073	.81	4.63	5.53	1.35	1.34	69	69	
1074	1.85	5.45	4.35	1.73	1.43	61	67	H. & T. p. 209. Scheuss Canal. Earth, somewhat stony. Kutter.
1075	.787	5.64	6.69	1.01	1.23	80	72	H. & T. p. 211. River Aar. Detritus in small quantities. Treschel, Kutter.
1076	1.27	4.20	3.12	1.45	1.56	67	64	
1077	1.27	6.13	6.10	1.31	1.21	70	73	
1078	.461	2.82	4.22	1.57	1.65	64	63	H. & T. p. 211. River Aar. Irregular bed, detritus. Kutter.
1079	.800	5.15	7.07	1.37	1.15	69	74	
1080	.993	7.51	7.78	.87	1.05	85	78	
1081	.10	2.21	11.5	1.52	1.00	65	64	H. & T. p. 211. River Aar. Irregular bed, sand and mud. Kutter.
1082	.10	3.38	14.9	.83	.90	88	85	
1083	.12	4.23	16.8	.73	.77	94	91	
1084	.928	6.36	6.89	1.02	1.13	80	75	H. & T. p. 215. Rhine at Bale. Coarse detritus, coarse gravel. Grebenau.
1085	1.22	6.38	6.89	1.35	1.13	69	75	

III. CLASSIFICATION.

17. We propose the following formulæ for the computation of the flow of water in pipes, conduits, artificial open channels and natural streams of regular regimen.

$$m = \frac{\mu + \frac{\lambda}{Vr}}{1 + .08V + \sqrt{8r}},$$

$$c = p\sqrt{\frac{1 + .08V + \sqrt{8r}}{1 + \frac{q}{Vr}}}$$

where

$$p = \sqrt{\frac{2g}{\mu}}, \quad q = \frac{\lambda}{\mu}.$$

Provisional values m' and c' of m and c corresponding to $V = 3$ are suggested as follows:

$$m' = \frac{4}{5}\,\frac{\mu + \frac{\lambda}{Vr}}{1 + 2.25\sqrt{r}}.$$

$$c' = 1.118p\sqrt{\frac{1 + 2.25\sqrt{r}}{1 + \frac{q}{Vr}}}$$

Otherwise, should it be preferred to express the final results directly

$$m = \frac{\mu}{1 + \sqrt{8r}}\tan^2 \tfrac{1}{2}\theta,$$

$$c = p\cot\tfrac{1}{2}\theta\sqrt{1 + \sqrt{8r}},$$

$$V = p\cot\tfrac{1}{2}\theta\sqrt{(1 + \sqrt{8r})Jr},$$

where

$$\tan\theta = \frac{\sqrt{\mu}}{.32}\,\frac{\sqrt{\frac{1 + \sqrt{8r}}{Jr}}}{1 - .0195\frac{\lambda}{Jr^2}},$$

$$= \frac{25.06}{p}\,\frac{\sqrt{\frac{1 + \sqrt{8r}}{Jr}}}{1 - .0195\frac{\lambda}{Jr^2}}.$$

In Table VI are recorded the values of the constants μ, λ, p, and q corresponding to each class. While Table V constitutes the best identification of the channels in each class, the descriptions given there of the channels entered are but meager abbreviations of the more detailed and complete descriptions given by the original authorities. It is important in the application to be able to assign to its proper class the given proposed channel for which the velocity is to be determined and this under any circumstances is dependent largely on the judgment and experience of the engineer.

TABLE VI.

Class.	μ	λ	p	q
1	.010	.000090	90.04	.0090
2	.011	.000065	76.52	.0060
3	.012	.000040	73.26	.00033
4	.013	.000030	70.46	.00023
5	.014	.000027	67.82	.000193
6	.015	.000025	65.52	.000167
7	.016	.0002	63.44	.01250
8	.017	.0004	62.98	.02353
9	.0185	.0007	59.00	.03783
10	.020	.0011	56.75	.05500
11	.022	.0030	54.11	.13636
12	.025	.0055	50.75	.2200
13	.032	.0080	44.86	.2500
14	.038	.0140	41.17	.3684
15	.045	.0200	37.83	.4444
16	.052	.0500	35.19	.9615
17	.060	.0800	32.76	1.333
18	.068	.1300	30.77	1.912
19	.082	.1800	28.02	2.195
20	.096	.2300	25.90	2.396

The following is an effort to express in words the nature of the channels belonging to the individual classes.*

Class 1.

Smooth straight tin pipes, straight tin lined pipes under best conditions.

Class 2.

New straight wrought iron pipes coated with asphalt, under best conditions. Wooden pipes, straight and very smooth, closely jointed with hydraulic radius from .3 to .5 feet. Tin lined pipes with easy curvature.

* The writer regrets being unable to place in the proper classification the wooden stave pipes of large diameter now used in the Western United States; he was unable to obtain data of the results of observations on these conduits.

Class 3.

Glass pipes, straight or of very easy curvature. New wrought iron riveted pipes coated with asphalt under good conditions, straight or with easy curvature. Wooden pipes of rectangular section, smoothly planed, well jointed and straight, with r = .3 feet. Straight uniform open channéls lined with neat cement smoothly trowelled of best construction.

Class 4.

Lead pipes, straight and smooth, or clean with easy curvature. New cast iron asphalted pipes, straight or with very easy curves, under best conditions. New wrought iron asphalted pipes in service mains under best conditions. Open channels or flumes of carefully planed boards, smooth joints, straight and of the best construction. Open channels of uniform section and slope lined with neat cement of good construction.

Class 5.

New straight uncoated wrought iron pipe. New straight cast iron pipe, smoothly coated with tar. Clean new cast iron asphalt coated in ordinary service mains under best conditions, with perhaps few very easy curves. Open channels or flumes of planed boards of good construction. Channels lined with cement mortar ⅔ cement, ⅓ fine sand, straight and of best construction.

Class 6.

Clean new straight cast iron pipes uncoated. Straight new wrought iron cement lined pipe. New sheet iron riveted pipe, asphalted, straight, under best conditions, diameters 6 to 18 inches. Channels and conduits of brick work covered smooth with neat cement.

Class 7.

Ordinary service pipes of cast or wrought iron in good condition, coated and uncoated, with easy curves, slight if any irregularities. Flumes of unplaned boards of best construction. Brick conduits covered with cement mortar ⅔ cement, ⅓ sand.

Class 8.

Clean cast iron pipe mains with bends, few valves, slight irregularities of ordinary service. Flumes and channels of sawed boards of good construction. Conduits of brickwork of best construction, hard, smooth brick, smooth mortar joints.

Class 9.

Cast iron and wrought iron mains in use several years, ordinary service pipe mains with some valves, bends, vertical curves, not perfectly clean, but slight deposit uniformly distributed. Channels and conduits of ordinary brickwork of good construction. Flumes of sawed boards of ordinary construction. Channels of uniform section, straight alignment and grade, of the best smooth ashlar masonry.

Class 10.

Old iron pipes not clean, slightly incrusted, with some deposit uniformly distributed. Small flumes and channels whose roughness is like that of stout coarse canvas, with $r = .035$. Channels constructed of good ashlar masonry.

Class 11.

Riveted sheet iron pipes and conduits of various construction, coated with asphalt, clean and new with diameters from 2.5 to 4 feet. Channels of ordinary ashlar masonry.

Class 12.

Riveted sheet iron pipe in ordinary service, coating in fair condition, or but slightly damaged. Channels of roughness equivalent to boards with laths 1x$\frac{3}{8}$ inches nailed crosswise on bottom and sides $\frac{3}{8}$ inches apart.

Class 13.

Old riveted sheet iron pipe in fair condition. Ashlar masonry not smooth, with some deposits, but in fair order. Channels whose roughness is of the character of pebbles $\frac{3}{8}$ to $\frac{7}{8}$ of an inch held in place with cement. Channels of dry rubble of the very best construction. Channels of best dry rubble side walls and clay bottom in good order.

Class 14.

Unclean old iron pipes, somewhat foul, but fine and uniformly distributed deposits. Ordinary dry rubble masonry channels of good construction, with rubble bottom or good clay bed.

Class 15.

Channels in ashlar masonry not in good order. Aqueducts with floor of bricks and sidewalls of good masonry. Channels whose roughness is that of pebbles $1\frac{1}{4}$ to $1\frac{1}{2}$ inches in diameter held in place with cement, or of boards with laths $1 \times \frac{3}{8}$ inches nailed crosswise to sides and bottom, 2 inches apart. Common rubble masonry. Roughly hammered stone masonry. Canals, ditches and channels in earth, of uniform trapezoidal section, straight alignment, in loam or earth with fine gravel.

Class 16.

Channels of dry rubble masonry built of large stones, roughly but well built, with rubble bed or bed of clay and gravel. Creeks or canals in earth of very regular alignment and uniform section. Small rivers or creeks with very regular grade and section, straight reach, in earth and clean.

Class 17.

Rubble masonry irregular and somewhat damaged. Small rivers and canals with side walls of rubble or paved slopes, bed of earth, sand and gravel, slight detritus. Straight canals of trapezoidal cross-section with stony bottom, or earth with little vegetation, sand and mud.

Class 18.

Canals not in good order with vegetation. Rivers in coarse gravel.

Class 19.

Canals in bad order. Rivers with coarse gravel and detritus.

Class 20.

Rivers somewhat irregular, with very coarse gravel and coarse detritus.

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An Investigation of the Value of an Infinite Series on the Boundary of the Region of Convergence

BY

WILLIAM H. ECHOLS

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Charlottesville, Virginia, U. S. A.

UNIVERSITY OF VIRGINIA PUBLICATIONS

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BULLETIN OF THE PHILOSOPHICAL SOCIETY
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AN INVESTIGATION OF THE VALUE OF AN INFINITE SERIES ON THE BOUNDARY OF THE REGION OF CONVERGENCE.*

BY
WILLIAM H. ECHOLS.

1. The object of this paper is to demonstrate that the function represented by an infinite series of functions of a variable has an essential singularity at each point of the boundary of its region of convergence.

In order to do this, however, we shall generalize the definition of an infinite series as follows. Let $u_1(x), u_2(x), u_3(x), \ldots$, be an infinite sequence of functions of the variable x. We represent by the symbol

$$u_1(x) + u_2(x) + u_3(x) + \ldots \tag{1}$$

the value of an infinite series of these functions and define this symbol to mean the limit when, n is infinite, of the sum

$$u_1[x + \varphi(n)] + u_2[x + \varphi(n)] + \ldots + u_2[x + \varphi(n)] \tag{2}$$

wherein $\varphi(n)$ is an arbitrary function of n having the limit zero when n is infinite. At all points of uniform convergence this new definition gives the same results as the definition in current use, at points of non-uniform convergence the results are more general including as special cases the results of the older definition.

Before proceeding to general series we shall illustrate by a few special examples.

Example 1. Consider the geometric series

$$1 + z + z^2 + \ldots$$

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whose boundary of convergence is the circle about the origin with radius 1. The sum of n terms is

$$\frac{1-z^{n+1}}{1-z}=\frac{1-\rho^{n+1}\,e^{i(n+1)\theta}}{1-\rho\,e^{i\theta}}.$$

Let p and q be any assigned integers, then any rational point on the boundary can be represented by $e^{i\frac{p}{q}\theta}$, and any irrational point e^{ia} by a regular sequence of rational points. Let y be an arbitrarily assigned real number and let

$$n+1=mq+r, \qquad (n+1>q,\ 0\leqq r\leqq q) \tag{3}$$

q and r being integers.

Let z proceed, as n becomes infinite, to the point $e^{i\frac{p}{q}\theta}$ on the boundary by the regular sequence of points

$$\left.\begin{aligned} z&=\left(1+\frac{y}{n+1}\right)e^{i\varphi},\\ \varphi&\equiv\frac{\theta}{n+1}+\frac{p}{q}\pi-\frac{rp\pi}{q(n+1)}+\frac{1-(-1)^{mp}}{2(n+1)}\pi\\ \therefore\quad (n+1)\varphi&=\theta+\left[mp+\frac{1-(-1)^{mp}}{2}\right]\pi \end{aligned}\right\} \tag{4}$$

Then $(n+1)\varphi$ is always equal to θ plus an even multiple of π, and the limit of the sum of the n terms, or the value of the series at the point on the boundary is

$$\frac{1-e^{\zeta}}{1-e^{i\frac{p}{q}\pi}}, \tag{5}$$

where $\zeta\equiv y+i\theta$ is an arbitrarily assigned number, y and θ being arbitrary real numbers. Consequently a mode of convergence of z to an arbitrary point on the boundary can always be so chosen as to make the series take at this point, uniquely as a limit, any arbitrarily chosen number N. If we choose $\theta=0$ and

$$y=2b\sin\frac{p\pi}{2q},$$

and make the point z converge along the boundary to the point $z = 1$, then since (5) can be written

$$\frac{1 - e^{y}\ e^{i\theta}}{2 \sin \frac{p\pi}{2q} e^{i\left(\frac{\pi}{2} + \frac{p\pi}{2q}\right)}},$$

the value of the series becomes ib, where b is arbitrary.

Example 2. Consider Tannery's classical example:

$$S_n = z^2 + \frac{z^2}{1+z^2} + \frac{z^2}{(1+z^2)^2} + \cdots + \frac{z^2}{(1+z^2)^n},$$
$$= 1 + z^2 - \frac{1}{(1+z^2)^{n-1}}.$$

First, let z be real, and let z converge to 0 by the regular sequence $z^2 = \alpha^2/(n-1)$, α being an assigned real number. Then the value of the series at 0 is $1 - e^{-\alpha^2}$, or at this point its value is every number from

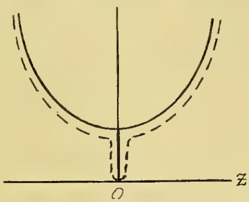

FIG. 1

0 to 1 inclusive. Thus the real function represented by the series has at zero a *shear* (a vertical segment whose lower end is 0 and upper end 1) which is represented by the totality ((0, 1)). This point is an essential singularity when z is complex, for if z converges to zero as

$$z^2 = \frac{\alpha + i\beta}{n-1},$$

the value of the series there is $1 - e^{-(\alpha+i\beta)}$, or the totality $((0, \infty))$.

Numerous examples can be given in illustration, we content ourselves with merely indicating the following familiar ones generally used in the texts on analysis.

Example 3.
$$\sum_1^n \frac{z}{[(n+1)\ z+1]\ (nz+1)} = 1 - \frac{1}{nz+1}.$$

Let $z(=)0$ by the sequence $z = \zeta/n$, ζ arbitrary. The value of the series then is $\zeta/(\zeta + 1)$ or $((-\infty, +\infty))$.

Example 4. $$\sum_1^n \frac{z[n(n+1)z^2-1]}{(1+n^2z^2)[1+(n+1)^2z^2]} = \frac{z}{1+z^2} - \frac{(n+1)z}{1+(n+1)^2z}.$$

When $z(=)0$ by the sequence $\zeta/(n+1)$ the value of the series is $\zeta/(1+\zeta^2)$.

Example 5. $$z + \sum_2^n \left(\frac{1}{z^{2n-1}} - \frac{1}{z^{2n-3}}\right) = \frac{1}{z^{2n-1}}$$

The sequence $z = 1 + \xi/(2n-1)$ gives $S_\infty = e^\xi$.

Example 6. $$\sum_1^n \frac{-2z(1+z)^{n-1}}{[1+(1+z)^{n-1}][1+(1+z)^n]} = \frac{1-(1+z)^n}{1+(1+z)^n}.$$

$z(=)0$ in the manner $z = \zeta/n$ gives $S_\infty = (1-e^\zeta)/(1+e^\zeta)$.

Example 7. $$\frac{1}{1+z} + \sum_1^n \frac{z^n - z^{n+1}}{(1+z^n)(1+z^{n+1})} = \frac{1}{1+z^n}.$$

$S_\infty = 1$ for $|z| < 1$, and $S_\infty = 0$ for $|z| > 1$. If z goes to the boundary in the same way as prescribed in example 1 the value of the series at a definite point on the boundary is $1/(1+e^\zeta)$. If z is confined to real values and goes to the point 1 by the respective sequences

$$z = \pm\frac{t^2}{n}.$$

the value of the series at 1 is $(1+e^{\pm t^2})^{-1}$ or the totality $((0,1))$. The series has there a *shear* represented by a vertical segment joining the value of the series for $|z| < 1$ to its value for $|z| > 1$.

Example 8. A similar example, due to Tannery, having infinitely many poles at roots of unity on the boundary is

$$\frac{1}{1-z} + \frac{z}{z^2-1} + \frac{z^2}{z^4-1} + \frac{z^4}{z^8-1} + \cdots.$$

The sum of its first n terms is $-(z^{2^{n-1}} - 1)^{-1}$. The value of the series is 1 or 0 according as $|z|$ is less or greater than 1. Let z converge to a point on the boundary as in example 1, where in (4) $n + 1$ is replaced by 2^{n-1}. The value of the series at this point is then $1/(e^\zeta - 1)$.

Examples 2 to 6 may be found in Whittaker's Modern Analysis, 7 and 8 in Fiske's Functions of a Complex Variable. These examples illustrate what is meant by considering the value of a series not merely as the limit of a sum as a function of the variable n having x as a variable parameter, but as a function of the two dependent variables x and n. We now proceed to consider the subject more generally.

2. *Taylor's Series.* Let $f(z)$ be a regular function in a region containing a point a and let α be a simple isolated pole of $f(z)$ and the nearest singularity of the function to a. Then inside a circle c with center a and passing through α the generating function $f(z)$ represents the value of its Taylor's series. With a as a center draw a circle C with a radius such that on the boundary C $f(z)$ is regular and within this circle α is the only singularity of $f(z)$. Then on and within C the function $\psi(z) = (z - \alpha) f(z)$ is regular. Let x be a point inside C. The integral

$$\int f(z) \left(\frac{x - a}{z - a}\right)^{n+1} \frac{dz}{z - x} \tag{6}$$

taken around the circle C is equal to the sum of the integrals taken around the three small circles (x), (a) and (α) having x, a and α as centers. The integral (6) taken around C is zero when $n = \infty$ since z is on C and x is a point inside C. The integral (6) taken around the circle (x) is equal to $2\pi i f(x)$ whatever be n. The integral taken around (a) is equal to

$$-2\pi i \frac{(x-a)^{n+1}}{n!} \left(\frac{d}{da}\right)^n \frac{f(a)}{x-a} = -2\pi i \sum_0^n \frac{(x-a)^r}{r!} \cdot f^r(a)$$

The integral (6), which can be written

$$\int \frac{\psi(z)}{z-x} \left(\frac{x-a}{z-a}\right)^{n+1} \frac{dz}{z-\alpha}, \tag{7}$$

taken around (α) is zero or infinite, when $n = \infty$, according as x is an assigned point inside or outside c respectively. Let

$$\frac{x-a}{\alpha-a} = \left(1 + \frac{k}{n-1}\right) e^{i\varphi}$$

where k is any assigned real number and φ is the number given by (4) as in example 1. Then (7) becomes

$$\left(1+\frac{k}{n+1}\right)^{n+1} e^{i\varphi}\int\frac{\psi(z)}{z-x}\left(\frac{\alpha-a}{z-a}\right)^{n+1}\frac{dz}{z-\alpha}$$

or

$$2\pi i\left(1+\frac{k}{n+1}\right)^{n+1} e^{i\varphi}\frac{\psi(\alpha)}{\alpha-x}. \tag{8}$$

Hence at the point $x=a+(\alpha-a)e^{i\frac{p}{q}\pi}$ on the circle c the value of the Taylor's series is

$$f(x)-\frac{\psi(\alpha)}{x-\alpha}e^{k+i\theta}. \tag{9}$$

Since k and θ are arbitrary this can be made to take any assigned value N. If $f(z)$ has n simple poles $\alpha_1, \ldots, \alpha_m$ on c, then the value of the series is

$$f(x)-e^{k+i\theta}\sum_1^m\frac{\psi_r(\alpha_r)}{x-\alpha_r}.$$

For each multiple pole β of order $m+1$ on c there will be a term under the Σ equal to

$$\frac{1}{m!}\left(\frac{d}{d\beta}\right)^m\frac{\psi_r(\beta)}{x-\beta},$$

where in the neighborhood of β, $\psi_r(z)=(z-\beta)^{m+1}f(z)$ is regular. If in (9) x moves along the circle c to α the value (9) will be ∞ unless $k=0$, $\theta=0$, and then has the value $\psi'(\alpha)$.

3. *Laurent's Series.* Let α and β be two isolated simple poles of $f(z)$ and a the center of circles c_1 and c_2 passing through α and β respectively. The point a being a common center, let s_1 be a circle outside α and s_2 a circle inside β such that $f(z)$ is regular on the boundaries s_1 and s_2 and through out the ring between them. With a as a center draw a circle C_2 just outside β and another C_1 just inside α. Draw a small circle (α) around α and connect it by a double line path to a point on s_1, also a small circle (β) around β and connect it by such a path to a point on s_2. Let x be a point in the ring between C_1 and C_2. Taking the integral

$$\int f(z)\frac{dz}{z-x} \tag{10}$$

around C_1 and C_2 in the usual way we have the n-sum of Laurent's series

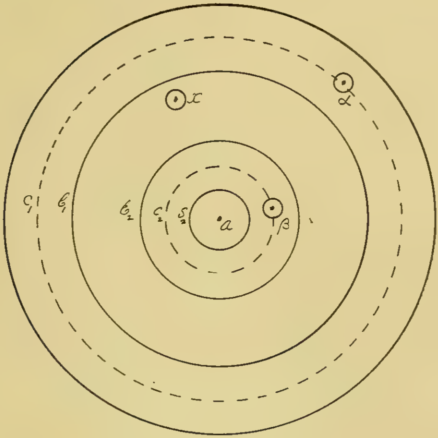

FIG. 2

$$2\pi i \sum_{-n}^{+n} = 2\pi i f(x) - \int_{C_1} f(z) \left(\frac{x-a}{z-a}\right)^{n+1} \frac{dz}{z-x} - \int_{C_2} f(z) \left(\frac{z-a}{x-a}\right)^{n+1} \frac{dz}{z-x}.$$

First, let x converge to a point on the outer boundary c_1 in the manner prescribed as in Taylor's series. When $n = \infty$ the integral about C_2 is zero since $|z - a|$ is less than $|x - a|$. The integral about C_1 is equal to that integral taken around s_1 less the integral around (α). Around s_1 it is zero, when $n = \infty$, since $|x - a| < |z - a|$, while around (α) it is equal to

$$\left(1 + \frac{k}{n-1}\right)^{n+1} e^{i\varphi} \int_{(\alpha)} \frac{\psi_1(z)}{z-x} \left(\frac{\alpha - a}{z-a}\right)^{n+1} \frac{dz}{z-\alpha}$$
$$= 2\pi i \left(1 + \frac{k}{n+1}\right)^{n+1} e^{i\varphi} \frac{\psi_1(\alpha)}{\alpha - x}.$$

Hence when $n = \infty$ and x on c_1 the value of the series is

$$f(x) - e^{\zeta} \frac{\psi_1(\alpha)}{x - \alpha}. \tag{11}$$

Second, let x converge in like manner to a point on the inner boundary of convergence c_2. Then the integral about C_1 is zero, when $n = \infty$, since $|x - a|$ is less than $|z - a|$; while the integral about C_2 is equal to this integral around s_2 less the integral around β. The integral around s_2 is zero, when n is infinite, since $|z - a| < |x - a|$, and the integral around (β) can be written equal to

$$\left(1 + \frac{k}{n+1}\right)^{-(n+1)} e^{-i\varphi} \int_{(\beta)} \frac{\psi_2(z)}{z-x} \left(\frac{z-a}{\beta-a}\right)^{n+1} \frac{dz}{z-\beta}$$
$$= 2\pi i \left(1 + \frac{k}{n+1}\right)^{-(n+1)} e^{-i\varphi} \frac{\psi_2(\beta)}{\beta - x}.$$

Hence the value of the series when $n = \infty$ and x is a point on the i ner boundary of convergence is

$$f(x) - e^{\zeta} \frac{\psi_2(\beta)}{x-\beta}. \tag{12}$$

If there be a number of simple or multiple poles on the boundary terms similar to those in Taylor's series appear.

4. *Teixeira's Series*. Passing over the particular cases, Burmann's, Laplace's, Lagrange's, etc., we proceed at once to point out the slight changes in Laurent's series that are necessary in order to establish the results for the most general form of power series. Let

$$\underset{-n}{\overset{+n}{S}} = \sum_{-n}^{+n} A_r [\varphi(x) - \varphi(a)]^r$$

be the n-term sum in Teixeira's series. The concentric circles in Fig. 2. now become the parametric contour lines

$$|\varphi(x) - \varphi(a)| = \lambda,$$

the different values of the parameter λ corresponding to the radii in the Laurent circles. With these changes we use the same symbols for the respective contours as for the circles respectively. The n-sum is derived from the integral

$$\int f(z) \varphi'(z) \frac{z-x}{\varphi(z) - \varphi(x)} \frac{dz}{z-x}$$

taken around C_1 and C_2, and $2\pi i \underset{-n}{\overset{+n}{S}}$ is equal to $f(x)$ less the sum of the two integrals

$$\int_{C_1} f(z)\,\varphi'(z)\,\frac{z-x}{\varphi(z)-\varphi(x)}\left(\frac{\varphi(x)-\varphi(a)}{\varphi(z)-\varphi(a)}\right)^{n+1}\frac{dz}{z-x},$$

$$\int_{C_2} f(z)\,\varphi'(z)\,\frac{z-x}{\varphi(z)-\varphi(z)}\left(\frac{\varphi(z)-\varphi(a)}{\varphi(x)-\varphi(a)}\right)^{n+1}\frac{dz}{z-x}.$$

When we require x to converge to the outer boundary c_1 by a regular sequence thus prescribed,

$$\frac{\varphi(x)-\varphi(a)}{\varphi(\alpha)-\varphi(a)}=\left(1+\frac{k}{n+1}\right)e^{i\varphi},$$

φ being the number in (4), then when n is infinite it is easily seen, in the same way as in §3, that the value of the series at the point

$$x=\varphi^{-1}\left\{\varphi(a)+[\varphi(\alpha)-\varphi(a)]\,e^{i\frac{p}{q}\pi}\right\}$$

on the outer boundary c_1 is

$$f(x)-e^{\zeta}\,\frac{\psi(\alpha)}{\varphi(x)-\varphi(\alpha)}\,\varphi'(\alpha). \tag{13}$$

When x is on the inner boundary c_2 the sign of ζ is changed. When $f(x)$ has a number of simple or multiple poles on the boundary the corresponding more general forms result as in the simpler series.

Thus in all the forms of the power series the initial theorem appears to be established. We proceed now to consider what is perhaps the most interesting case, that of Fourier's series about the boundary conditions of which much discussion has taken place.

5. *Fourier's Series.* The n-sum in Fourier's series is given by $\pi\,S_{2n+1}$ equal to

$$\int_0^{\frac{\pi-x}{2}} f(x+2z)\,\frac{\sin(2n+1)z}{\sin z}\,dz+\int_0^{\frac{\pi+x}{2}} f(x-2z)\,\frac{\sin(2n+1)z}{\sin z}\,dz. \tag{14}$$

We propose to investigate the limit of this sum when x converges to $+\pi$ in the manner defined by the regular sequence.

$$x=\pi-\frac{2c}{2n+1}, \tag{15}$$

wherein c is an arbitrary positive constant for the present restricted to being not greater than π.

We assume all numbers to be real and $f(z)$ to be a continuous function in the interval $-\pi$ to $+\pi$ inclusive of these numbers.

Substitute the value of x in (15) in the integrals in (14). In the first integral in (14) make the substitution t for $(2n+1)z$, this integral then becomes

$$\int_0^c f\left(\pi - \frac{2(c-t)}{2n+1}\right) \frac{\frac{t}{2n+1}}{\sin\frac{t}{2n+1}} \frac{\sin t}{t}\,dt. \tag{16}$$

Since t lies between 0 and $c \leqq \pi$ the function $\frac{\sin t}{t}$ is always positive and this integral, by the first law of the mean value, can be written equal to

$$\frac{\frac{\zeta}{2n+1}}{\sin\frac{\zeta}{2n+1}} f\left(\pi - \frac{2(c-\zeta)}{2n+1}\right) \int_0^c \frac{\sin t}{t}\,dt. \tag{17}$$

where ζ is some number between 0 and c. Hence when n is infinite this integral has the value

$$f(\pi - 0) \int_0^c \frac{\sin t}{t}\,dt. \tag{18}$$

Making the same substitutions in the second integral of (14), it can be decomposed into the following

$$\int_0^{\pi - \frac{c}{2n+1}} = \int_0^{\pi} - \int_{\pi - \frac{c}{2n+1}}^{\pi} \tag{19}$$

The first of the integrals on the right of (19) can be written

$$\int_0^{\frac{\pi}{2}} + \int_{\frac{\pi}{2}}^{\pi}, \tag{20}$$

of which the first is well known to have the value

$$\frac{\pi}{2} f(+\pi - 0).$$

The transformation $z = \pi - t$ turns the second integral of (20) into

$$\int_0^{\frac{\pi}{2}} f\left(-\pi - \frac{2c}{2n+1} + 2t\right) \frac{\sin (2n+1)t}{\sin t} dt = \frac{\pi}{2} f(-\pi+0),$$

as is well known.

The transformation $z = \pi - t$ makes the second integral of (19) become

$$-\int_0^{\frac{c}{2n+1}} f\left(-\pi - \frac{2c}{2n+1} + 2t\right) \frac{\sin (2n+1)t}{\sin t} dt,$$

which becomes, when $(2n + 1)t = y$, equal to

$$-\int_0^c f\left(-\pi - \frac{2(c-y)}{2n+1}\right) \frac{\frac{y}{2n+1}}{\sin \frac{y}{2n+1}} \frac{\sin y}{y} dy,$$

on applying the first law of the mean this becomes

$$-\frac{\frac{\zeta}{2n+1}}{\sin \frac{\zeta}{2n+1}} f\left(-\pi - \frac{2(c-\zeta)}{2n+1}\right) \int_0^c \frac{\sin y}{y} dy. \qquad 0<\zeta<c.$$

When $n = \infty$ the value of this is

$$-f(-\pi-0) \int_0^c \frac{\sin t}{t} dt.$$

Hence the value of the Fourier series at $x = \pi - 0$ is S_∞ equal to

$$\frac{1}{2}\left\{f(+\pi-0)+f(-\pi+0)\right\} + \left\{f(+\pi-0)-f(-\pi-0)\right\} \frac{1}{\pi} \int_0^c \frac{\sin t}{t} dt. \quad (21)$$

We find approximately

$$\int_0^\pi \frac{\sin t}{t} dt = 1.1796 \frac{\pi}{2}.$$

Therefore at $x = \pi - 0$ the function S has a shear whose lower end is half the sum, and whose upper end is this half sum plus half of 1.1796 times the difference of $f(\pi)$ and $f(-\pi)$.

In the same way we find the value of the series when x converges to $-\pi + 0$ to be S_∞ equal to

$$\frac{1}{2}\left\{f(+\pi-0)+f(-\pi+0)\right\}-\left\{f(-\pi+0)-f(+\pi+0)\right\}\frac{1}{\pi}\int_0^c \frac{\sin t}{t}\,dt. \quad (22)$$

But the series is a periodic function with period 2π. Therefore this last value (22) is the same value which the series takes when x converges to $+\pi+0$. Hence *at* the point π the series has a shear whose upper and lower ends are respectively

$$\frac{1}{2}\left\{f(\pi)+f(-\pi)\right\}\pm\frac{1}{2}\left\{f(-\pi)-f(+\pi)\right\}1.1796. \quad (23)$$

The series S can be made to take uniquely any number of this shear as a limit by properly assigning c in the interval ($(0, \pi)$).

The restriction that $c \leqq \pi$ may be removed, for if $c = m\pi + h$ where h lies between 0 and π, the integral from 0 to c can be decomposed into the partial intervals.

$$\int_0^{\pi} + \int_{\pi}^{2\pi} + \cdots + \int_{(m-1)\pi}^{m\pi} + \int_{m\pi}^{c},$$

in each of which $\sin t/t$ keeps its sign unchanged and therefore to each of which the first law of the mean is, as before, applicable. The integrals

$$\int_{r\pi}^{(r+1)\pi} \frac{\sin t}{t}\,dt$$

are alternately positive and negative and each numerically less than the preceding. Hence as before whatever be the assigned positive number c the value of (16) is given by (18). Since the function of c

$$\int_0^c \frac{\sin x}{x}\,dx$$

is symmetrical with respect to the ordinate axis, c may be any real number whatever. There are two positive values of c one less the other greater than π which give the same value to S, as also there are two corresponding symmetrical negative values. In particular there is a value of c between $\frac{1}{2}\pi$ and π corresponding to $c = \infty$ such that when $x\ (=)\ \pi - 0$ the value of S is $f(\pi - 0)$, or the continuation of the uniformly convergent series for $x < \pi$.*

* In connexion with this subject reference is made to a paper by Professor Bôcher, *Annals of Mathematics*, VII, Second Series, p. 81, §9, 1906. Introduction to the Theory of Fourier's Series. Also *Philosophical Magazine*, Series V, Vol. 45, p. 85, 1898. The Harmonic Analyser of Michelson and Stratton.

6. Differences of opinion in mathematical controversies are often resolved into a difference of interpretation of the conventional definitions upon which repose subsequent reasoning. No generalization of a definition can be accepted which changes any case to which a previously accepted and conventionally established definition applies. But a definition which expressed in more general form and which includes the older definition in all its applications and at the same time embraces and gives definite interpretation to conditions which the older definition is inadequate to reach or properly interpret, is a legitimate definition and permits a true and real extension of analysis.

Thus the definition that (1) is uniformly convergent for all values of x for which (2) has a *unique* limit, includes the older definition of uniform convergence and at the same time brings out clearly Cayley's remark that non-uniform convergence is always associated with discontinuity, and furthermore characterizes such a point as an essential singularity in general. In particular at points of finite discontinuity of a real function it introduces a shear or vertical segment which unites by a continuous path branches which the older definition leaves separated by a vacuum.

If we refer to the function $f(z)$ of a Taylor's, Teixeira's or Fourier's series as the generating function of that series within its region of convergent equality, the definition shows that the series severs its connexion with the function by passing through an essential singularity on the boundary.

The definition of the infinite series given in § 1,

$$S = \mathop{\mathrm{L}}_{n=\infty}^{-y=o} \sum_{\gamma=1}^{\gamma=n} u_\gamma(x+y),$$

as a function of two variables x and y one of which y has the limit zero when $n = \infty$, is in a sense analagous to Weierstrass's generalization in his idea of the continuation of the series

$$f(y) + (x-y)f'(y) + \frac{(x-y)^2}{2!}f''(y) + \cdots$$

over the connected region of the two variables x and y for which the series is uniformly convergent as representing and defining one and the same function $f(x)$.

East Lawn, May, 1911.

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A Contribution to the Geology and Mineralogy of Graves Mountain, Georgia

BY

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SCIENTIFIC SECTION
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A CONTRIBUTION TO THE GEOLOGY AND MINERALOGY OF GRAVES MOUNTAIN, GEORGIA.*

BY
THOMAS L. AND J. WILBUR WATSON.

INTRODUCTION.

To the mineralogist, Graves Mountain, Georgia, has long been known for the occurrence of an interesting association of rather uncommon minerals. Knowledge of the occurrence of the interesting group of minerals at this locality was first made known in a published paper by Professor Charles U. Shepard in 1859. Since the appearance of Professor Shepard's paper in 1859 practically no work, based on field study of the mountain proper, has been published, although the literature contains many references to the description of mineral specimens from this locality.

Very little is known of the geology of the immediate area within which Graves Mountain is located. Mining for gold and copper three and one-half miles north, and for gold ten miles south, of the mountain has been engaged in at frequent intervals since the early 50's. Detailed field studies have not extended beyond the limits of the mines and no maps of any description have been attempted for this part of Georgia. Within recent years several geologists have recorded the results of their studies of the metal mines but the intervening area, more especially that of Graves Mountain and the immediate vicinity, still remains for detailed study and mapping.

The description of the geology of Graves Mountain briefly summarized in this paper is based on several short visits to the area by the senior author since 1900. The last visit was made in March 1911, when a collection of the rocks and minerals was made for laboratory study. The chief object

* Read before the Scientific Section, February 5, 1912.

of this paper is to record the results of field and laboratory (chemical and microscopical) studies of an interesting area that has long been known to the mineralogist but practically neglected by the geologist. The chemical analyses of the specimens collected are the work of the junior author.

GENERAL GEOLOGY.

Location. Graves Mountain lies in the extreme western part of Lincoln County, Georgia, within less than a mile of the Wilkes County boundary; ten miles nearly east from Washington, the county seat of Wilkes

Fig. 1. Map of Middle Northeast Georgia showing Location of Graves Mountain in Lincoln County.

County; six miles southwest of Lincolnton; and about forty miles northwest of Augusta (map, fig. 1). It is within but near the eastern border of the Piedmont Plateau province, a short distance north of the fall line, which marks the passage of the plateau crystalline rocks beneath the Coastal Plain sediments. The area forms a part of the central divide region between the tributaries of Little River on the south and southeast and those of the Savannah River on the east and north. Little River is distant about eight miles south of the mountain and is the nearest stream of large size.

Physiography. The immediate country in the vicinity of Graves Mountain averages about 550 feet above tide. It has gently inclined but broadly undulating surfaces so characteristic of the inner eastern margin of the Piedmont province. The larger streams have carved deep valleys below the plateau surface.

Graves Mountain is a conspicuous ridge (monadnock) of unreduced hard rock, which rises to an altitude of 700 feet above tide level, and several hundred feet above the Piedmont Tertiary base-leveled plain. It is the highest point of land between it and the ocean. It is removed from the major lines of drainage, which fact together with structure and lithologic character of the rock of which it is composed, are the chief factors responsible for its existence. A number of similar low ridges, locally called moun-

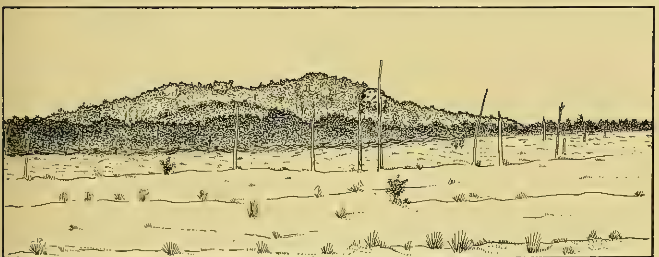

FIG. 2. VIEW OF GRAVES MOUNTAIN, GEORGIA, LOOKING SOUTHEAST. (SKETCH BY J. H. WATKINS, FROM PHOTOGRAPH.)

tains, stand up above the general level of the plateau surface over parts of Lincoln County, but none of these are found in the immediate vicinity of Graves Mountain.

The ridge, locally known as Graves Mountain, has a length of two miles along an approximate northeast-southwest direction, coincident with the general structure of the rocks of the area, is less than a half mile in average width, and rises several hundred (300?) feet above the surface of the surrounding plain. The slopes of the mountain are very unequal. They are less steep on the northwest side which is the direction of dip than on the southeast side. They are steeper near the top of the ridge but become more gradual about half the distance down until merged with the plateau surface at the base. The upper portion of the ridge is greatly scarred and roughened from weathering and low and high cliffs are numerous. Fig. 2, a pen sketch from a photograph taken from a position looking southeast, shows the outline of the mountain. On the crest of the ridge, a fairly pronounced axial depression or sag is noted bounded at either end by higher elevations or peaks which slope away somewhat gradually toward the northeast and southwest until merged into the plateau surface.

Geology of the surrounding areas. The rocks of the region are strongly foliated metamorphosed crystallines of two dominant types,* variable micaceous schists and gneisses. To the north and south of Graves Mountain the schists vary from biotitic to sericitic, and in places are highly quartzitic, feldspar being only sparingly developed and is frequently absent. The gneiss of Little River basin in the vicinity of the gold mines in northern McDuffie County is of granitic composition, usually carrying biotite as the dark silicate mineral, although hornblende has been noted. The dominant rock of the gold belt on Little River is mica schist which encloses the gold veins of milky white quartz containing the sulphides, pyrite, chalcopyrite, and galena. The gneiss of this belt is derived from an original granite and Jones suggests that because of chemical similarity the schists are also of probable igneous origin. Analyses of these rocks are given in columns IV, V, and VI of the table of analyses on page 205.

The geology of the Seminole copper mine and immediate vicinity, three and one-half miles north of Graves Mountain, is more varied, several igneous rock types occurring that have not been observed in the Little River belt on the south. In addition to the mica schists, schistose quartz-albite porphyry occurs, composed of a fine-grained groundmass of quartz and plagioclase feldspar intimately intergrown and exhibiting spherulitic structure, through which are distributed phenocrysts of opalescent quartz, some albite and

* During the field season of 1911, Mr. Otto Veatch of the Georgia Geological Survey made a brief reconnaissance over much of Lincoln County and contiguous parts of the counties to the south. In a personal communication to the senior author, Mr. Veatch states that the belt of metamorphic sedimentary rocks in which Graves Mountain is located seems to extend from the Savannah River in Lincoln County westward to Graves Mountain and Adasburg a short distance south in Wilkes County; thence southwestward through the northern part of McDuffie and Warren counties. A generalized section extending from Lincolnton southeastward to Augusta, a distance of nearly forty miles, prepared by Veatch shows the principal rock types of the area to be schists, slates, and quartzites with an extensive area of granite containing intrusions of serpentine, peridotite and diorite, occupying the central portion of the section. The metamorphic sediments on the northwest side of the granite area dip steeply toward the northwest becoming vertical with slight southeastward inclination near and at Lincolnton. From the southeast margin of the granite to Augusta the metamorphic sediments are steeply inclined to the southeast. As indicated in Veatch's section the metamorphic sediments dip steeply away from the granite on the northwest and southeast sides.

biotite. Numerous dikes of basic igneous rocks, mainly diabasic in character, ranging up to several hundred feet in width, occur. They are clearly intrusive in character since they cut across the foliation of the schists irregularly. Likewise the sulphide ore bodies (chalcopyrite, galena, sphalerite, and pyrite) developed in a shear zone in the sericite schists approximating a northeast-southwest direction and crossing at a small angle the schistosity, are cut across and faulted by some of the dikes of basic rock. The rocks are cut by numerous joints and in places by faults. Columns I, II, and III in the table of analyses below show the composition of some of the rocks near the Seminole mine.

The rocks of the region are deeply decomposed by atmospheric agencies so that relatively few exposures are seen. They are strongly foliated, the general direction of strike being northeast-southwest, with local variation observed from place to place. At Graves Mountain and further northward at the Seminole mine the dip is to the northwest. Southeastward from Graves Mountain about six miles, at Amity Postoffice, and thence southward to Little River the schists likewise dip to the northwest.*

Analyses of porphyry, gneiss, and schist, Lincoln and McDuffie Counties, Georgia.†
(Dr. Edgar Everhart, analyst).

	I	II	III	IV	V	VI
SiO_2	68.07	68.13	69.30	63.82	62.45	61.25
Al_2O_3	15.07	17.49	15.91	15.94	15.91	17.18
Fe_2O_3	1.13	1.19	3.20	2.91	2.07	4.49
FeO	3.42	1.38	0.18	3.87	3.80	2.54
MgO	2.27	1.00	0.21	1.20	1.76	2.21
CaO	0.73	0.75	5.92	2.91	3.70	3.78
Na_2O	6.06	3.50	3.35	2.54	2.33	2.50
K_2O	0.29	3.00	0.14	1.80	0.40	1.60
Ignition	1.25	2.06	1.80	3.70	5.41	3.03
H_2O (100° C.)	0.01	0.03	0.52	0.06	0.03	0.01
TiO_2	0.37	0.74	0.48	0.55	0.82	0.92
P_2O_5	trace	trace	none	0.04	0.13	0.02
S	1.14	0.22	0.03	trace	0.22	0.24
MnO	0.31	0.30	0.11	0.18	0.37	0.27
CO_2				0.21	0.50	
Total	100.12	99.79	101.15	99.73	99.90	100.04

* Veatch, Otto. Personal communication, January, 1912.

† Jones, S. P., *Bull. 19, Ga. Geol. Survey*, 1909, pp. 55–62.

I. Quartz-albite porphyry, near Seminole copper mine, Lincoln County, Georgia. Phenocrysts of quartz, albite, and biotite in a fine-grained groundmass of quartz and plagioclase feldspar with accessory magnetite, apatite, and pyrite. Coarse spherulitic structure shown in groundmass in places.

II. Schistose and altered quartz-albite porphyry, same locality as I. Same as I except that sericite developed from metamorphism.

III. Granite porphyry, 1 mile east of Seminole mine. Rock is considerably altered. Mineral composition essentially the same as I and II. Some biotite and much epidote present.

IV. Biotite gneiss, Bell shaft, Columbia gold mine, McDuffie County, Georgia. Contains feldspar (plagioclase and orthoclase), quartz, and biotite, with accessory magnetite and secondary chlorite, muscovite, and calcite.

V. Mica schist, Columbia gold mine, McDuffie County, Georgia. Contains quartz and sericite, with some plagioclase feldspar, chlorite, calcite, and pyrite partly altered to iron oxide.

VI. Mica schist, Parks gold mine, McDuffie County, Georgia. Composition same as V except that mica is less abundant and much epidote is present.

The norms of the rocks represented by the analyses in columns I, II, III, and IV are as follows:

Norms of quartz-albite prophyry (I and II), granite porphyry (III), and gneiss (IV), Lincoln and McDuffie Counties, Georgia.

	I	II	III	IV
Quartz	25.68	32.88	36.36	34.32
Orthoclase	1.67	17.79	0.56	10.56
Albite	51.35	29.34	28.82	20.96
Anorthite	3.61	3.89	27.52	10.56
Corundum	3.47	7.14	0.10	6.02
Hypersthene	7.98	3.29	1.02	7.09
Hematite			3.20	
Ilmenite	0.76	1.37		
Magnetite	1.62	1.86		4.18
Titanite			1.18	
Pyrite	2.04	0.36		
Apatite				0.93
Calcite				0.50
H_2O	1.26	2.09	2.32	3.76
	99.44	99.62	101.08	100.09

The position of the rocks in the quantitative classification of igneous rocks may be summarized as follows:

NO.	SYMBOL	NAME
I	I–II.4.1.5	Persodic pantellerase; no subrang name.
II	1.3(4).2.4	Alsbachose-lassenose.
III	I.3(4).4.3	No name.
IV	II.3.3.4	Sitkose.

* Classed as of II.

Geology of Graves Mountain. The rock composing Graves Mountain which, as indicated on the map, fig. 1, lies between the group of gold mines along Little River and the Seminole mine but nearer the latter, is a fine-grained quartzite. It varies from moderately thinly foliated to essentially massive in structure. Its position is along the northwest margin of a metamorphic sedimentary series of crystalline rocks. The dip of the quartzite is toward the northwest at a variable angle and the strike is north-northeast. No igneous rocks have been observed at Graves Mountain but acid and basic intrusions occur at other localities within the general area as described above. The basal portion of the mountain and for some distance up the slope on the northwest side the rock is the foliated variety quartzite-schist, while the crest and for an undetermined distance down the slopes the rock is massive quartzite, with only a tendency to schistosity indicated. This latter phase (massive) of the quartzite has been designated by Shepard* the rather rare variety known as itacolumite. On top of the mountain the quartzite is cut by quartz veins, some of the larger ones of which measure several feet in thickness. These do not appear to have been crushed or brecciated and are apparently barren. Smaller quartz veins only a few inches wide contain rutile and iron oxide. The rock from some of the small openings shows fracturing and brecciation.

The quartzite schist is light in color when fresh but is very generally discolored at the surface some shade of red by iron oxide from weathering. It is composed of quartz and less sericite as the essential minerals, with cyanite and occasional small grains of rutile and black oxide of iron as the principal accessory minerals. A partial chemical analysis of this rock gave SiO_2 79.18 per cent; Al_2O_3, 14.14 per cent; and Fe_2O_3, 3.17 per cent.

Under the microscope thin sections of the rock show a fine-grained mosaic of angular quartz, colorless mica (sericite), stout columnar crystals of colorless cyanite with frayed out and ragged ends, usually partly altered to muscovite and red oxide of iron, and frequently containing inclusions of rutile and quartz. In some cases the iron oxide obscures much of the cyan-

*Shepard, Chas. U., *Amer. Journ. Sci.*, 1859, vol. 27.

ite substance and is formed along the cleavage and fracture directions of the mineral. Alteration of cyanite to muscovite is apparent in the hand specimens. The quartz frequently shows optical disturbance and contains inclusions of rutile and other substances of an indeterminate character. Small granules of black oxide of iron, probably magnetite, are usually present, and more or less red oxide of iron is present in all the thin sections. Rutile, as euhedral and anhedral crystals and nonpleochroic, is formed along the boundaries of the quartz grains and as inclusions in the quartz and cyanite.

The massive quartzite which occurs on the crest and higher slopes of the mountain, has essentially the same composition as the quartzite schist. It is compact in texture and light nearly white in color when fresh, but near the surface it is friable easily crumbling into a loose sand under pressure of the hand, and is a buff color, due to the presence of some iron oxide. It is fine-grained in texture, composed largely of fine sugary quartz, some muscovite, and in places contains abundant large and small tables of cyanite of pale green color when fresh, crystals of blue lazulite, and small grains of red rutile. Pyrophyllite is rather common. Cyanite, lazulite, and rutile are usually intimately associated, the first two (cyanite and lazulite) are frequently embedded one in the other, and the rutile occurs as separate grains and as inclusions in the other minerals. The weathered surface of the rock is quite rough and, when examined in detail, the more resistant minerals, lazulite and cyanite, stand out in moderate relief. Alteration of the columnar crystals of cyanite to muscovite is often pronounced. Weathering has partially discolored the cyanite a brownish yellow from free hydrous oxide of iron, also to some extent the lazulite, but the most noticeable effects in the latter mineral are the dulling of luster and lightening of color.

Professor Shepard* remarks that the rock becomes schistose near the southern extremity of the formation, and contains in addition to the minerals mentioned minute crystals of pyrite, occasional drusy cavities nearly filled with massive crystalline barite enclosing perfectly formed minute crystals of quartz, and traces of gold. He adds further that the formation at this point has been worked to some extent for gold, and from its resemblance to the diamond gangue (itacolumite) of Brazil it is worthy of careful examination for the occurrence of diamonds. A partial chemical analysis of the rock collected on top of the mountain gave SiO_2, 69.74 per cent; Al_2O_3, 24.86 per cent; and Fe_2O_3, 0.53 per cent.

* *Op. cit,* 1859.

Microscopically, thin sections of the massive quartzite show a fine granular mosaic of closely interlocking angular quartz, shreds of muscovite, and some cyanite in large crystals. Neither biotite nor feldspar was identified in any of the thin sections of either phase of the rock or in any of the hand specimens. Quartz greatly predominates. Rutile is present in every thin section examined, usually as separate grains and as inclusions in the quartz and cyanite. Most of the quartz anhedra are approximately of the same size but frequently occasional larger individuals occur which show strain shadows and peripheral granulation. The quartz contains liquid and solid inclusions, dustlike particles of an indeterminate character and rutile being the commonest of the solid forms. Alteration of the individual minerals, including lazulite, as shown under the microscope is described below separately under each mineral.

MINERALOGY.

The mineral association at the Georgia locality is somewhat similar to that at Clubb and Crowder Mountains, North Carolina. The minerals include pyrophyllite, lazulite, rutile, cyanite, and hematite, and are described below in the order named.

Pyrophyllite. Pyrophyllite, a hydrous aluminum silicate corresponding to the formula $H_2Al_2Si_4O_{12}$, occurs at Graves Mountain, Georgia, in stellate or radiate aggregates, a less frequent occurrence of the mineral than in foliated or compact massive forms. The radiate form of occurrence has been noted at several localities in the United States,* chief among which are the Chesterfield district in South Carolina; Graves Mountain, Lincoln County, Georgia; Cottonstone Mountain, Mecklenburg County, North Carolina; and the Kellogg lead mine, near Little Rock, Arkansas. Genth† reports the occurrence of lazulite and cyanite with pyrophyllite at the South Carolina locality, a similar association to that of Gaston County, North Carolina, and Lincoln County, Georgia.

Very few good exposures of pyrophyllite were observed on Graves Mountain. The best and largest one occurs some distance up the northwest slope on either side of the road which leads on top of the mountain from the northwest side. Here the mineral is exposed as a solid mass of several feet in thickness in greatly weathered and etched low cliffs. It forms fibrous radiate aggregates averaging about a quarter of an inch in diameter, partially discolored a light dirty brown from infiltrating iron

* Dana, E. S., *A System of Mineralogy*, 1900, 6th ed., p. 692.

† Genth, F. A., *Amer. Journ. Sci.*, 1854, vol. 18, p. 410.

oxide. When fresh the folia are soft and flexible, have pronounced greasy feel and pearly luster, and show basal cleavage. One specimen shows, in addition to the fine sugary quartz of the enclosing rock, clear and transparent quartz in larger grains, which are regarded as indicative of formation from solution. Specimens of pyrophyllite in vein quartz collected from Graves Mountain are on exhibit in the State Museum in Atlanta. A part of the central portion of a second specimen was occupied by a sponge of iron oxide, indicating the presence originally of some mineral other than pyrophyllite removed by weathering and yielding the iron oxide.

In addition to the vein-like occurrence of pyrophyllite, the mineral is frequently distributed through the quartzite in small particles and large masses measuring 8 or 10 inches in thickness, and is sometimes associated more or less closely with cyanite and rutile. Veatch* observed pyrophyllite in the vein quartz breccia but states that it does not show evidence of crushing and must therefore have been formed subsequent to the fracturing of the quartzite.

The exact relations of the pyrophyllite to the surrounding quartzite were not conclusively established, since contacts were largely obscured, but the field evidence suggests formation by probable filling of a fracture or other irregular spacing in the rock. This explanation finds confirmation in a second occurrence of pyrophyllite recently noted by Veatch in the same county about seven and one-half miles east of Lincolnton on the Petersburg road. Mr. Veatch informs us that both the mineral and enclosing rock are very similar in texture and appearance to those at Graves Mountain to the west, and that the pyrophyllite fills fractured quartz veins in the quartzite. Although careful search was made no rutile was found in the veins at this locality.

In thin sections the microscope resolves the pyrophyllite aggregate into a series of extremely minute fibers which are mostly radiate in arrangement, sometimes parallel, with all intermediate relations observed. The mineral is colorless in thin section, has strong birefringence, and contains some small inclusions of rutile and magnetite or ilmenite. Yellowish-brown iron oxide has filtered in along and between the boundaries of the fibers in places, causing discoloration.

An analysis of the pyrophyllite from this locality (column I), compared with one of the same mineral from the Chesterfield district, South Carolina, (column II), is given below.

* Personal communication, January, 1912.

Analyses of pyrophyllite from Graves Mountain, Lincoln County, Georgia, and Chesterfield district, South Carolina.

	I	Ia		II
SiO_2	64.90	1.082	4	66.01
Al_2O_3	26.88	0.264	1	28.52
Fe_2O_3	1.18	0.008		0.87
H_2O	5.69	0.317	1.1	5.22
MgO	0.17	0.004		0.18
CaO	0.74	0.013		0.23
	99.57			101.03
Specific gravity	2.659			

I corresponds to the formula

$$H_2O.Al_2O_3.4SiO_2 \text{ or } H_2Al_2Si_4O_{12}$$

I. Pyrophyllite in yellowish stellate or radiate aggregates collected by Thomas L. Watson from Graves Mountain, Lincoln County, Georgia. J. Wilbur Watson, analyst.

Ia. Ratios from I.

II. Pyrophyllite in radiated aggregates with lazulite and cyanite from Chesterfield district, South Carolina. F. A. Genth, *Amer. Journ. Sci.*, 1854, vol. 18, p. 410. Quoted by Dana, E. S. *A System of Mineralogy*, 1900, p. 692; and Hintze, C. *Handbuch der Mineralogie*, pp. 829, 831.

Lazulite. Lazulite is a rather rare mineral, a phosphate of alumina containing some magnesia and protoxide of iron, corresponding to the formula $(AlOH)_2(Mg_1Fe)(PO_4)_2$ in which the ratio of Fe : Mg(Ca) varies from 1 : 12 to 2 : 3.* The known localities in which the mineral occurs in the United States† are limited to three in North Carolina, and one each in South Carolina and Georgia. The North Carolina localities include Clubb and Crowder mountains in Gaston County, and Sauratown in Stokes County, the South Carolina locality includes the Chesterfield district, and the Georgia locality is Graves Mountain in Lincoln County. The mode of occurrence, including kind of rock and mineral associations, is similar for the localities in Gaston County, North Carolina, and Lincoln Gounty, Georgia, except that corundum which is unknown in the Georgia locality is an associate in the North Carolina localities. According to Genth,‡ the

* Dana, E. S., *A System of Mineralogy*, 1900, pp. 798–799.

† Mr. Frank L. Hess of the U. S. Geological Survey informs the senior author that he has received massive specimens of lazulite from California. No published description of the locality has appeared and, so far as we are aware, nothing is known of the exact occurrence of the mineral.

‡ Genth, F. A., *Amer. Journ. Sci.*, 1854, vol. 18, p. 410.

mineral association in the Chesterfield district of South Carolina is pyrophyllite and cyanite.

The mineral has been noted in a variety of occurrences and mineral associations, both in the massive and crystallized form, from a somewhat large number of widely separated foreign localities.* The occurrences include several localities each in Salzberg and Styria, Austria; several each in Switzerland and Sweden; at Gulabgarh, India; Tijuco in Minas Geraes, Brazil; and near the entrance of Churchill River into Husdon Bay, Keewatin, Canada. From the published descriptions the occurrence and mineral association cf lazulite at Horrsjöberg in the district of Elfsdalen, Sweden, bears striking resemblance to the occurrences in North Carolina and Georgia.

Probably the first occurrence of this rare and interesting mineral in the United States was noted by Dr. H. S. Hunter† in 1822, near Crowder's Mountain in the southern part of Lincoln (now Gaston) County, North Carolina. Specimens were forwarded by Hunter to Professor Olmsted of the University of North Carolina and were noted in the latter's report (second part) to the Board of Agriculture. A few years later the mineral was found in greater abundance near the southern end of Clubb's Mountain about thirty miles northeast of Crowder's Mountain. Hunter reports the occurrence of lazulite in the latter locality in arenaceous and micaceous quartzite, occasionally embedded in compact quartz, and in the triangular cavities of a reddish cyanite. According to this observer the mineral is massive but imperfect crystallizations may be observed in some specimens. Later observers‡ noted the occurrence of the mineral in the Gaston County localities in pale and dark blue crystals and crystalline masses in quartzite associated with rutile, cyanite, pyrophyllite, corundum, quartz, and damourite.

The chief interest in lazulite at present is scientific, although apart from its value as museum specimens, the mineral would probably make an opaque gem or ornamental stone, as the color which is usually lighter than lapis lazuli for which lazulite when first found was mistaken, is often equally as rich.§

At the Georgia locality (Graves Mountain), lazulite occurs in the massive, fine-grained, sugary quartzite (itacolumite) which, according to Shep-

* Dana, E. S., *Op. cit.*, p. 799.

† Hunter, C. L., *Amer. Journ. Sci.*, 1853, vol. xv, pp. 376–377.

‡ Genth, F. A., *Proc. Amer. Phil. Soc.*, 1873, vol. xiii, p. 367. Kunz, G. F., *N. C. Geol. Survey, Bull. 12*, 1907, p. 57.

§ Kunz, G. F., *Op. cit.*, p. 57.

ard* has a thickness of more than 300 feet, and "presents numerous included zones or layers, varying from one to three feet in thickness, in which is found imbedded, masses and crystals of lazulite." The observations of the senior author, which were incomplete from want of time, did not indicate the distribution of lazulite along zones or layers in the quartzite, but rather as numerous crystals in single individuals and aggregates of very irregular distribution. The occurrence of the mineral in places sometimes suggested a tendency to form nests and bunches in the rock.

The lazulite almost invariably occurs as crystals ranging in size up to an inch and more in length, bounded by crystal faces—combination of prisms and pyramids, and in cross section from irregular to squarish and triangular in outline. The crystals are opaque, of deep azure blue color when fresh but of lighter or paler color after prolonged effects of weathering, have uneven fracture and indistinct cleavage, but pronounced vitreous luster. The monoclinic crystals are acute pyramidal in habit, occasionally flattened from extension of the pyramidal faces, and are frequently twinned. Professor Shepard† described and figured five crystals of lazulite from this locality and remarked that a twin (fig. 5) "is by far the most abundant form, equalling in frequency all the others combined."

The lazulite is frequently intimately associated with greenish to colorless, massive, columnar cyanite partially altered to muscovite, and small red crystals of rutile. Frequent small crystals of rutile are observed in the quartzite by aid of the lens. Weathered surfaces of the rock are quite rough showing characteristic pitting from resistant coarse cyanite and lazulite in relief, with the fine-grained sugary quartz falling out from probable granulation, forming the depressions. Scales of colorless mica are conspicuous in the weathered specimens.

In thin sections under the microscope, the lazulite is usually some shade of light blue, with pleochroism varying according to depth of color, it being distinct in the deeper colored sections and weak in the light-colored ones. It has the pleochroism X colorless or nearly so, Y and Z azure blue. The absorption is $Z = Y > X$. The index of refraction is moderate, double refraction strong; cleavage is indistinct but numerous irregular fractures occur. The axial angle is large being $2\ E_r = 111°$ as determined by von Lasaulx. The lazulite contains inclusions of rutile and quartz, and occasionally cyanite and muscovite. Alteration has progressed along the periphery and fractures or cracks, yielding very minute scales and fibers

* *Op. cit.*, 1859.

† *Op cit.*, 1859.

of a light gray nearly white substance, sometimes stained with iron oxide, having high refraction and double refraction, and can probably be referred to hydrargillite as suggested by Rosenbusch.* The common mineral associates of the lazulite in thin sections are quartz, cyanite, rutile, and muscovite. These may be discerned megascopically in the hand specimens.

In the following table are given the analysis of the lazulite from Georgia, and analyses of lazulite from North Carolina, Canada, and Sweden:

Analyses of Lazulite from Graves Mountain, Lincoln County, Georgia, and other localities.

	I	II	IIa		III	IV*	V†
P_2O_5	38.25	40.57	284	1	43.76	42.52	46.39
Al_2O_3	33.92	35.97	353	1.2	31.70	32.86	29.14
FeO	3.99	4.23	58		8.17	10.55	2.09
MgO	9.08	9.63	241	1.3	10.04	8.58	13.84
CaO	3.12	3.31	59			trace	2.83
H_2O	5.83	6.21	350	1.2	5.59	5.30	6.47
SiO_2	6.05				1.07		
	100.24	100.00			100.33	99.81	100.76
Specific gravity	2.958				3.122	2.78	3.045

* MnO—trace.

† Deducted 3 per cent SiO_2.

I corresponds to the formula

$$(Fe, Mg, Ca) O.Al_2O_3.P_2O_5.H_2O$$
$$FeO : Mg(Ca) O = 1 : 5$$
$$MgO : CaO = 1 : 3$$

I. Lazulite collected by Thomas L. Watson from Graves Mountain, Lincoln County, Georgia. J. Wilbur Watson, analyst.

II. I calculated to SiO_2 free basis.

IIa. Ratios from II.

III. Lazulite from Gaston County, North Carolina; average of two analyses. Smith and Brush, *Amer. Journ. Sci.*, 1853, vol. 16, p. 670.

IV. Lazulite from Horrsjöberg, Sweden. (Ingelström, *Journ. pr. Ch.*, 1855, vol. 64, p. 253). Quoted by Dana, E. S., *A System of Mineralogy*, 1900, 6th ed., p. 799.

V. Lazulite from near mouth of Churchill River, Keewatin, Canada. Hoffman, Ch., *Geol. Surv. of Canada*, 1878-79, p. 2.

Rutile. Knowledge of the occurrence of rutile at Graves Mountain was early made known by Professor Charles U. Shepard† who was the first to

* Rosenbusch-Iddings, *Microscopical Physiography of Rock Making Minerals*, 1893, p. 236.

† Shepard, C. U., *Op. cit.*, p. 36.

describe and work it. According to Kunz* the rutile from this locality has realized at least $20,000 for cabinet specimens and has supplied the collections of the world. The rutile was won from the central depression on top of the ridge and along the northwest slope. The openings are of long standing and had so greatly caved at the time of the senior author's visit in March, 1911, that they were largely obscured.

No specimens of coarse rutile were observed by the senior author during his visit but it is plentiful in every specimen collected, chiefly as microscopic inclusions in other minerals and frequently as small crystals and grains visible to the naked eye. It has been observed megascopically in the quartzite, and in intimate association with cyanite, lazulite, pyrophyllite, and hematite. As a microscopic accessory rutile in crystals and formless grains of red and reddish-brown color has been identified as inclusions in the lazulite, cyanite, pyrophyllite, hematite, and quartz, and formed along the boundaries of these minerals. In each case it shows the usual optical properties.

Veatch† observed rutile with quartz and iron oxide in veins not exceeding a few inches in width on top of the mountain. The rock fragments thrown out of the small pits on top of the mountain show fracturing and brecciation, and formation of the vein rutile was probably subsequent to the fracturing of the quartzite but a part of it may have existed in the veins prior to the crushing.

The coarse rutile crystals from this locality observed in the various mineral collections in this country are lustrous black to reddish-brown and red, with brilliant orange red in thin crystals. They vary in size usually up to 5 inches and occur both as single and twin forms. Kunz‡ reports fine single crystals have been found up to four pounds each and with it very interesting hydrous anthophyllite.

The single crystals are usually prismatic with frequently pyramidal terminations shown.

Besides the common forms of rutile crystals this locality has furnished some rather rare and interesting ones, which have been figured and described chiefly by German crystallographers,§ especially Rose, Haidinger, vom Rath, and Mügge. Especially interesting are the beautiful rutile twins figured and described by Rose and others, composed of as many as six- and eight-fold twins commonly known as sixlings, and eightlings;

* Kunz, G. F., *N. C. Geol. Survey, Bull. 12*, 1907, p. 52.

† Personal communication, January, 1912.

‡ Personal communication, February 28, 1910.

§ See Dana, E. S., *A System of Mineralogy;* and Hintze, C., *Handbuch der Mineralogie* for references.

those made up of a less number of parts, such as trillings and fourlings being not uncommon from the Georgia locality.

The three following analyses quoted by Hintze serve to show the essential chemical composition of the Graves Mountain rutile.

Analyses of rutile from Graves Mountain, Georgia.

TiO_2	97.22	97.52	97.64
Fe_2O_3	2.62	2.64	2.61
	99.84	100.16	100.25

While coarse rutile was not observed at this locality by the senior author, careful examination of the dumps alongside the numerous openings on the northwest slope of the mountain from which the mineral was mined, and the presence of rutile in microscopic proportions in thin rock slices, clearly indicate the rutile matrix to be a heavy, dark-colored rock, composed of an aggregate of long bladed and coarsely columnar crystals of cyanite and massive granular hematite, with usually some quartz. Frequently the rock is composed of excessive hematite with less cyanite, and as often cyanite may predominate. In either case the rock presents a more or less porous texture which perhaps is most pronounced in specimens showing dominant cyanite. Professor Shepard's description of the occurrence of rutile in this rock follows. He says:*

The central part of the mountain, to the thickness of 50 feet, is composed of a hematitic rock, which includes in some places an abundance of a ferruginous kyanite, much resembling in appearance the diaspore from the Urals. With the kyanite is found rutile, often in gigantic crystals (weighing upwards of a pound), and possessed of much regularity of crystalline form. The prevailing figure is a square prism with truncated lateral edges, and surmounted at both extremities by an eight-sided pyramid. There is also found a most remarkably perfect twin crystal, in which the geniculation is six times repeated,—producing an hexagonal prism, surmounted at each end by a six-sided pyramid, with a reëntering, six-sided hopper-shaped cavity, at the tips. These crystals are all more remarkable for their symmetry and polish, than any I have ever seen. Some are fully equal in lustre to the brilliant crystals of cassiterite from Cornwall or Bohemia. The most perfect rutiles are generally imbedded in the massive kyanite; and when detached leave behind impressions having a polish and lustre equal to that of their own planes. A little common quartz is also mingled with the kyanite and rutile.

Closely associated with kyanite, rutile and quartz, are considerable masses (8 or 10 inches thick) of a mineral known among the miners of Georgia as steatite, but which is true pyrophyllite,—differing in no respect from that of the Urals, except in the finer stellulations it presents, and in the slight ferruginous stain it exhibits near their centres.

* *Op. cit.*, 1859.

Cyanite. Cyanite is very generally but irregularly distributed through both phases of the quartzite composing the mountain, and is intimately associated with rutile, lazulite, pyrophyllite, coarse crystalline quartz, and hematite. It is a prominent constituent of the rutile-bearing rock, chiefly a mixture of cyanite and hematite with quartz; and it is frequently a conspicuous mineral in the lazulite-bearing quartzite (itacolumite), especially in the lazulite areas. It forms long bladed and coarsely columnar forms up to several inches in length, is usually colorless though green is not uncommon. In the weathered portions of the rock cyanite is partially discolored a yellowish brown from iron stain. Muscovite is a frequent associate and is derived in part at least from the alteration of cyanite.

In thin sections under the microscope the cyanite is colorless and nonpleochroic. It usually shows polysynthetic twinning in broad bands, and good cleavage parallel to the direction of elongation (100) and less often and less well developed a second cleavage parallel to 010. Fracture is common. The cyanite contains inclusions of rutile and quartz, and in some cases it forms inclosures in the lazulite. The rutile inclusions usually show perfect crystal boundaries and the mineral is some shade of red in color. Rutile as formless grains sometimes occurs along the boundaries of the cyanite and other minerals. Concerning the coarse rutiles Professor Shepard* remarks: "the most perfect rutiles are generally imbedded in the massive kyanite; and when detached leave behind impressions having a polish and lustre equal to that of their own planes."

An interesting feature of the cyanite is its alteration to muscovite† and iron oxide. The cyanite of the rutile matrix has been designated by Shepard‡ a ferruginous cyanite. Muscovite in scales is frequently observed on the cyanite by the naked eye and in such relations as to leave no doubt of its derivation from the cyanite by alteration. This is entirely confirmed by the microscope. Thin sections often show the cyanite to have frayed-out or ragged ends altered to muscovite shreds, and at times the cyanite substance is clouded by muscovite scales, not infrequently discolored by reddish brown oxide of iron. Iron oxide has formed along cleavage and fracture directions and, in case of the rutile matrix, it masks much of the cyanite substance.

In the table below is given an analysis of the Graves Mountain cyanite

* *Op. cit.*, 1859.

† Watson, Thomas L. and Watkins, J. H., Association of Rutile and Cyanite from a New Locality. *Amer. Journ. Sci.*, 1911, vol. 32, p. 200.

‡ *Op. cit.*, 1859.

(1), together with analyses of the mineral from North Carolina (II), Sweden (III), and Canada (IV).

Analyses of Cyanite.

	I	Ia		II	III	IV
SiO_2	39.14	0.652	1	37.60	40.02	36.29
Al_2O_3	59.52	0.584	1	60.40	58.46	62.25
Fe_2O_3	1.09	0.007		1.60	2.04	0.55
CaO	0.18	0.004				1.06
MgO	0.40	0.009				0.36
	100.33			99.60	100.52	100.51
Specific gravity	3.282					

I. Cyanite collected by Thomas L. Watson from Graves Mountain, Lincoln County, Georgia. J. Wilbur Watson, analyst.
Ia. Molecular ratios from I.
II. Cyanite, Lincoln County, North Carolina. (Smith and Brush, *Amer. Journ. Sci.*, 1853, vol. 16, p. 371.)
III. Cyanite, Horrsjöberg, Sweden (Ingelström, *Journ. prakt. Chem.*, vol. 64, p. 61).
IV. Cyanite, British Columbia. (Hoffman, *Rept. Geol. Surv. Canada*, 1878–79, p. 1.)

Hematite. Hematite was observed as an important constituent of the rock at all the openings from which rutile had been won. Together with cyanite and less quartz it forms the rutile matrix, which according to Shepard* marks a band 50 feet in thickness. The hematite is massive granular, steel-gray to red in color, has filled in the spaces between the blades of cyanite, and presents a somewhat roughened surface. At times it presents an open or cellular texture due, according to Shepard, to "including the decomposing ferruginous cyanite, particles of pyrophyllite, and even portions of compact rutile." The ratio of hematite to cyanite is variable, sometimes one sometimes the other mineral is in excess. On weathered surfaces cyanite and quartz stand out in relief.

Thin sections show an aggregate of cyanite blades and steel-gray hematite, with some quartz and rutile. Hematite fills the cyanite interspaces and is formed as rims around, and along the cleavage and fracture directions in, the cyanite. Rutile occurs as inclusions in the principal minerals. Cyanite is partly altered to muscovite, and in some hand specimens the cyanite individuals are more or less curved and bent.

* *Op. cit.*, 1859.

Mineral Genesis.

Genesis of the minerals of the unusual association noted at Graves Mountain cannot be considered entirely solved until more detailed work in the field has been accomplished. The quartzite in which the minerals occur affords evidence of both anamorphic and katamorphic changes. Anamorphism is shown chiefly in the production of foliation, the formation of certain characteristic heavy minerals, and recrystallization. Crushing, fracturing, and brecciation of the quartzite, and the occurrence of quartz veins are structures characteristic of the zone of fracture or katamorphism.

No intrusions of igneous rocks of any kind have been observed at Graves Mountain, although granites and basic igneous types are known at several localities some miles away. It is not known whether the igneous rocks are older or younger than the quartzite composing the mountain. The age of the rocks has not yet been determined but reasoning from similarity of lithologic types of widely separated areas it seems not improbable that the Graves Mountain quartzite will prove to be of Cambrian age. No evidence is available at present for regarding the minerals as having formed from the effects of intrusions of igneous rocks.

Dr. Genth regarded the minerals of this locality, except rutile and quartz, as secondary products derived from the alteration of corundum. He says:*

> The same association of cyanite, rutile, pyrophyllite and lazulite in an arenaceous sandrock is found at Graves Mountain, Lincoln County, Georgia, and although as far as I am aware, corundum has never been found at this place, there can be very little doubt that at this locality also all these species except rutile and quartz owe their existence to the former presence and subsequent alterations of corundum.

Not only has corundum never been found at Graves Mountain but the authors have discovered no evidence for regarding the minerals as having been derived from such alteration.

From the evidence already developed in this paper and from the statements which follow below, it seems probable that the minerals at this locality were not all formed at the same time nor under the same conditions. Pyrophyllite, a hydrous silicate of aluminum, belongs to the kaolinite group of minerals, the better known members of which are secondary, the products of hydrous alteration of other species,† chiefly feldspars. They are produced under the conditions of katamorphism. Stellate struc-

* Genth, F. A., Corundum, Its Alterations, and Associated Minerals. *Proc. Amer. Phil. Soc.*, 1873, vol. 13, p. 382.

† Clarke, F. W., *Bull. 491, U. S. Geol. Survey*, 1911, p. 395.

ture of the pyrophyllite and lack of evidence indicating the original mineral from which it was derived, suggest more than the usual processes of residual weathering to explain the formation of pyrophyllite at the Georgia locality. Structure and other conditions of occurrence strongly suggest formation of pyrophyllite at this locality from solution, as the final stage in genesis. Likewise the relations of hematite to the associated minerals, as shown more especially in the study of thin sections, indicate that it was formed in the zone of katamorphism. On the other hand the cyanite and in large part the rutile are regarded from microscopic evidence as products of dynamic regional metamorphism, formed therefore under anamorphic conditions. The very narrow quartz veins which sometimes carry rutile and hematite were formed under different conditions, probably as products of the deep vein zone.

Thus far the rather rare mineral lazulite has been noted chiefly in quartzite of different ages in separate crystals and in pockets and veins. It has been observed in narrow veins in clay slate near Werfen in Salzberg, in massive veins in quartzite in the district of Keewatin, Canada, and in the iron mine at Scania in Sweden. These occurrences clearly indicate formation of lazulite as solution deposits under katamorphic conditions. The occurrence of lazulite at Graves Mountain as observed by the authors was not in veins but as separate and grouped crystals and aggregates distributed through the rock in intimate association with cyanite and rutile. The evidence thus far obtained at the Georgia locality is regarded by the authors as indicating formation of the lazulite under the probable conditions of dynamic regional metamorphism.

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UNIVERSITY OF VIRGINIA PUBLICATIONS

BULLETIN

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An Unusual Occurrence of the Mineral Evansite

BY

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UNIVERSITY OF VIRGINIA PUBLICATIONS

The Proceedings and Transactions of the Philosophical Society of the University of Virginia are published in the form of Bulletins and offered in exchange for the publications of learned societies, institutions, universities and libraries. The Bulletins are issued at irregular intervals in the form of separate papers. These are to be made up into volumes, consecutively paged, of about 400 pages each, for each of the Scientific, Humanistic and Medical Sections. Separate numbers may be purchased from the Distribution Committee, to whom remittances should be made. Communications should be addressed to Dr. Thomas L. Watson, University, Virginia.

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AN UNUSUAL OCCURRENCE OF THE MINERAL EVANSITE.*

BY

JOHN SHARSHALL GRASTY.

The mineral evansite is reported as occurring at so few localities and is so rarely seen that it will be of interest to know something about it and to learn, furthermore, that at last it has been found in America. Until recent years the only known occurrence anywhere in the world was that found in 1885 near Zsetcnik, Hungary, by Brooke Evans of Birmingham, England. Subsequently a mineral identified as evansite was found in the Yoredale Rocks, Ratcliffe Wood, Macclesfield, and described by A. S. Woodward† in 1883. It is thus seen to be one of the very rare minerals.

The next discovery of evansite was made by the writer himself while in charge of some exploratory work in the Coosa coal field of Alabama, where he was assisted by Dr. T. Poole Maynard, now with the Georgia Geological Survey. Specimens of the mineral collected by Dr. Maynard and the writer were given to Mr. Charles Catlett,‡ who published recently a brief paper on its occurrence as an impurity in coal. Mr. Catlett alsc sent one of the specimens presented to him to the U. S. Geological Survey. As a result, a brief statement about it was printed in one of the U. S. Geological Survey bulletins.§ The same bulletin contained a description of the evansite found at another American locality, namely, in Idaho. But, with regard to the Alabama occurrence, it was neither described definitely with reference to location nor as to the character of the coal bed in which it wasfound. It is important, therefore, that these facts be properly recorded.

* Read before the Scientific Section, February 5, 1912.
† *Mining Magazine*, vol. 5, p. 333.
‡ *Bull. 59, A. I. M. E.*, 1911.
§ *U. S. G. S. Bull. 490.*

Evansite* is a hydrated alumina phosphate having approximately the composition expressed by the formula $Al_3(OH)_6PO_4.6H_2O$. It is one of a series of alumina phosphates among which are:

Turquois	$AlPO_4Al(OH)_3+H_2O$
Wardite	$2Al_2O_3.P_2O_5.4H_2O$
Sphaerite	$4AlPO_4.6Al(OH)_3(?)$
Lisheardite	$(Al.Fe)AsO_4.2(AlFe)(OH)_3+5H_2O$
Coeruloloctite	$3Al_2O_3.2P_2O_5.10H_2O$

Other hydrated alumina phosphates whose chemical composition differs from evansite only in the manner in which are proportioned the ingredients characteristic of each are as follows:

Augetite	$2Al_2O_5.P_2O_5.3H_2O$
Berlinite	$2Al_2O_3.2P_2O_5.H_2O$
Trolleite	$4Al_2O_3.3P_2O_5.3H_2O$

To this list might be added also attacolite ($P_2O_5.Al_2O_3.MnO.CaO.H_2O$, etc.) but for its content of manganese oxide, for evansite too, like attacolite, contains a small and varying amount of lime.

The physical appearance of evansite ranges from massive to reniform and botryoidal, and in color from milk-white to dark red and yellow, some specimens being also slightly bluish, translucent and semi-opaque. Its fracture is subconchoidal; and its hardness slightly greater than calcite—the hardness of evansite being about 3.5. According to Forbes,† there is a considerable range in density, the faint yellow varieties being the heaviest. He gives the following table of densities:

Densities of Evansite.

Colorless—translucent	1.822
Colorless	1.872
Faint yellow	2.099
Semi-opaque	1.965
Average density	1.939

Other determinations of density have been made by Smith‡ and also by Kovar.§ In the case of the figure (1.842) for density furnished by the

* *Text Book of Mineralogy*, p. 573, E. S. Dana, 1904.
† *Philosophical Magazine*, vol. 28, p. 341, 1864.
‡ *Proc. Royal Soc. N. S. W.*, vol. 27, p. 382.
§ *Abh. Bohm. Akad.*, vol. 15, p. 1.

former, it will be seen that it is less than the average density as given by Forbes—while according to the determination of this physical property by Kovar, in one instance, namely, for the white variety, it is less (1.874) and for the yellow it is nearly the same as the average density of the several varieties obtained by Forbes as shown in the table above. If we take, however, an average of the higher values and also an average of the lower values for density, the figures obtained (1.93) and (1.86) give approximately the range in specific gravity, due in the denser mineral to about 2 to 6 per cent of ferric oxide, and in the latter to its absence or, if present, to just a trace or to not over 1 per cent. On the other hand a further variation may be charged to the presence or absence of lime. When, therefore, the specific gravity is greater than the average (1.86) for the purer mineral, in general, it is due obviously either to the presence of foreign matter in the form of iron oxide or calcium oxide, generally the former. This relationship of specific gravity or density has been brought out by Schaller* in the following table:

Relation in Evansite of density to iron content.

COLOR OF MINERAL	Fe_2O_3	AVERAGE DENSITY	LIMITS
	Per cent		
Dark red	6.60	2.00	1.990–2.016
Brown	5.45	1.98	1.972–1.990
Yellow	2.15	1.94	1.927–1.947

Since evansite is one of a large group of hydrated alumina phosphates, it is difficult to distinguish it chemically without an ultimate analysis. It possesses so many other properties that obtain in other members of its group that physically it has very few *peculiar* and *distinct* diagnostic features. Probably the mineral that it most closely resembles is coeruloloctite, but it may be distinguished from the latter by inferior hardness and specific gravity. The specific gravity of coeruloloctite ranges from 2.55 to 2.59, while that of evansite never exceeds a density of 2. In fact the chief physical diagnostic feature of evansite is its specific gravity, which, on the whole, is considerably less than that of most of the other minerals of the same group. Consequently if a mineral be obtained which has the following pyrognostic characteristics, namely, decrepitates and yields water when heated in a closed tube; is infusible and leaves a milk-white powder; colors the blow-pipe flame green when moistened with sulphuric acid; gives

* *U. S. G. S. Bull. 490*, p. 94.

an intense blue when heated on charcoal and touched with a cobalt solution; is soluble in sulphuric, nitric, and hydrochloric acids; and in the fluxes may show a trace of iron; while also calcium is frequently detected, but never fluorine—and has a specific gravity under 2, and a hardness of 3 to 3.5—it is very safe to call it evansite. It may be further checked by a chemical analysis when its various constituents will be found to be as follows and close to or within the limits assigned immediately below.

Constituents	*Per cent limits*
Al_2O_3	36.0 – 38.5
P_2O_5	19.0 – 22.0
F_2O_3	0.5 – 6.0
CaO	1.0 – 4.0
MgO	trace – 1.0
Ignition loss (H_2O)	36.0 – 38.5

Evansite is known to occur at but two points in America and these widely separated. It was first discovered in this country, as previously stated, in Alabama and later in Idaho. Specimens were collected from the Idaho locality, situated in the vicinity of Goldburg, and sent to the U. S. Geological Survey by Mr. C. R. Potts. The mineral found there is reported* as occurring in seams and as being "massive amorphous" and "very brittle with a conchoidal fracture." "Its hardness is about 3, and its color is generally brown, though it varies considerably. Some specimens are yellow, or white, or dark red." On the other hand the evansite from Alabama is found as a white to slightly resinous coating with clusters developed here and there in the shape of small batryoidal surfaces.

The Alabama deposit of evansite is unusual because of its being found in association with coal which probably is the only occurrence known where it is found in this association. The mineral was discovered some years also in sinking a shaft on the highly tilted (D. 57° S. E. and S. N. 40° E.) Martin coal seam of the Coosa coal field where it outcrops about one-half of a mile west of Coalville, Alabama, in the N. E. $\frac{1}{4}$ of the N. W. $\frac{1}{4}$ of S. 4, T. 20, R. 1 W. The locality in question is not situated just west of Columbiana as reported by Schaller† and Catlett,‡ but on the contrary is much more north than west of there, since Coalville is 11 miles N. 13° W. from Columbiana, the county seat of Shelby County, Alabama, while the road from the latter (Columbiana) to the Big Narrows (the transverse valley cut by the

* *U. S. G. S. Bull. 490.*
† *U. S. G. S. Bull. 490.*
‡ *A. I. M. E. Bull. 59.*

north fork of Yellow Leaf Creek through Double Mountain) covers a distance of about fifteen miles.

The Martin coal seam in which the Alabama evansite is found as an unevenly distributed coating on the bedding and in the joints and cracks, is unique in being at once the thickest and probably, as to quality, the most inferior coal bed in the South. The section herewith presented not only gives the thickness of this mammoth coal bed but shows also the stratigraphic points in the seam where the evansite is found best developed.

Section.

	Ft.	*In.*
Clay and soft pink colored shales	2	
Thin, soft and reddish sandstones	2	
Yellowish gray sandstone and clay	2	
Coal with evansite coating below	7	1
Blue clay parting—apparently widening down the dip	2	
Coal with evansite coating at top and more or less throughout	8	6
Under clay—dark and thick		

The unusual thickness of coal as shown by the section above appears to be due to a sharp plication accompanied near the outcrop with faulting and the normal thickness may therefore be nearer 8 than 16 feet as now appears to be the case. However, as the coal here was found to be of such inferior quality it was unnecessary to carry the work further in order to determine exact structural relationships. The analysis which follows gives the chemical composition and may serve also as a partial clue to the genesis of the evansite. The results obtained from other analyses made on samples taken foot by foot show very little variation from the percentages given in the bulk sample representing the entire bed, and hence the analysis above referred to not only represents an average composition but practically also the quality of the coal in any foot of the bed that may be selected. In taking the various samples, which was done under great difficulty because of the rapid inflow of water into the shaft, it was noted that the coal had a dominant dull, black, dead and lusterless look but here and there possessed a pavonine glint of a more or less greasy and greenish tone, which, had it been a limonite, would have suggested at once the presence of phosphorus. However, this is seen so often in coal in which phosphorus is entirely absent that but for its actual presence disseminated here through the coal as one of the main constituents of evansite—it would have no real significance. Phosphatic matter supplied by the bones and excreta of organisms, together with the introduction of the mud at the time the carbonaceous material was accumulating, plus the alumina, ferric oxide and silica of the original wood itself, obviously provided all the necessary elements to form evansite and so, perhaps, explains how it happens to be present in this association. The analysis which follows gives the composition of the coal in which it occurs.

Analysis of bulk sample of the Martin coal seam.

	Per cent
Moisture	0.90
Volatile matter	28.90
Fixed carbon	45.00
Ash	25.20
Sulphur	6.43

The argillaceous matter in the Martin coal seam, as indicated by the high content of ash, is finely disseminated through it; partings of slate and other macroscopic evidence of its presence being lacking entirely. The high ash content is, therefore, a defect in its quality for which one is not apt to be all prepared and analyses showing how large a per cent of it is inert, obviously, occasion considerable surprise.

The appearance of the evansite from the opening on the Martin coal seam near Coalville, Alabama, is well shown by the accompanying photograph taken by Holsinger of Charlottesville, Virginia, of the specimen presented by the writer to the Geological Department of the University of Virginia. It is perhaps the only specimen of its kind in any laboratory in the world. Since the coal shaft from which it was obtained has long since been abandoned and is now filled with water it would be both difficult and expensive to duplicate it. Though scarcer than diamond it has, however, no special economic value but it is to be highly prized nevertheless because of its rarity as a mineralogical specimen—especially as associated with coal.

As Mr. Charles Catlett* remarks, in his paper previously referred to, "One form in which phosphorus occurs in coal is evidently as a hydrated phosphate of aluminum; and any coal which shows to the eye the occurrence of a light-colored resinous-looking material should be looked on with suspicion as being high in phosphorus." In other words it is not improbable that a good per cent of the substances found in coal and labelled *resin* may, when their pyrognostic characteristics are more carefully determined, prove to be evansite which some of the resins so closely resemble.

One of the specimens given him (Catlett) by the writer was "purified down to about 0.3 g" and was analyzed by Professor John J. Porter of the University of Cincinnati with the following result:

* *Bull. 59, A. I. M. E.*

FIG. 1. EVANSITE COATING COAL. WHITE, EVANSITE; BLACK, COAL. NOTE BOTRYOIDAL CLUSTERS BELOW THE BLACK SQUARE IN CENTRE.

*Partial Analysis.**

	Per cent
Loss on ignition	37.43
Phosphoric anhydride	10.33
Alumina	36.33

In addition to the above it contained a trace of silica, while lime and magnesia also were observed in "considerable quantity."

To the above should be added also the approximate analysis made by the U. S. Geological Survey in which phosphoric acid was "determined by difference." It is accompanied here, for the purpose of comparison, by the analysis made on the evansite found in the vicinity of Goldburg, Idaho.

Analysis of Evansite from Alabama.†

	Per cent
Al_2O_3	38.33
CaO	1.03
MgO	0.75
Loss on ignition	38.19
P_2O_5 (by difference)	21.70
	100.00

Analysis and loss of water of Evansite from Idaho.‡

	ANALYSIS	TEMPERATURE	LOSS OF WATER
H_2O	36.96		
P_2O_5	19.14	At 107°	20.00
Fe_2O_3	5.49	175°	7.36
Al_2O_3	34.48	255°	3.13
CaO	4.32	290°	.94
MgO	Trace	To low redness	3.90
FeO	None	Blasting	1.61
	100.39		36.94

* *Bull. 59, A. I. M. E.*
† *U. S. G. S. Bull. 490.*
‡ *U. S. G S. Bull. 490.*

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On a Certain Quadratic Form with its Geometric and Kinematic Interpretations

BY

WILLIAM H. ECHOLS

UNIVERSITY OF VIRGINIA
Charlottesville, Virginia, U. S. A.

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ON A CERTAIN QUADRATIC FORM WITH ITS GEOMETRIC AND KINEMATIC INTERPRETATIONS.*

BY

WILLIAM H. ECHOLS.

1. The writer has for years made use of a linear transformation of the quadric in a certain form for the purpose of exhibiting certain geometric and kinematic relations, many of the particular results of which are well known but as the writer has not seen them presented as all being but particular cases of one general permanent form, it appears worth while, on account of its extreme simplicity, to call attention to it in a brief note.

2. The theorem is as follows. Let $x, y, \ldots, z$ be $n-1$ numbers determined by the relations

$$Mx = \Sigma m_i x_i, \quad My = \Sigma m_i y_i, \ldots, \quad Mz = \Sigma m_i z_i, \ldots \tag{1}$$

wherein m_i, $(i = 1, 2, \ldots, p)$, have the relation $\Sigma m_i = M$. Also let $m_i = Mk_i$, whence $\Sigma k_i = 1$. Then it can be easily shown that any homogeneous quadratic function of $x, y, \ldots, z$ such as $Q(x, y, \ldots)$ can be written identically equal to

$$\Sigma k_i Q(x_i, y_i, \ldots) - \Sigma k_i k_j\, Q(\Delta x_{ij}, \Delta y_{ij}, \ldots), \tag{2}$$

or as we shall write more briefly

$$Q = \Sigma k_i\, Q_i - \Sigma k_i\, k_j\, Q_{ij}, \tag{2}$$

where $\Delta x_{ij} \equiv x_i - x_j$, $\Delta y_{ij} \equiv y_i - y_j$, etc., and $i, j = 1, 2, \ldots, p$. The result (2) is obtained by making the substitution (1) in Q and in the result where k_i^2 occurs write k_i times 1 minus the sum of the other k's. It is also

*Read before the Scientific Section, February 18, 1912.

obvious that in making the transformation there may be substituted for any or all of the numbers x, y, . . . their differentials of any orders obtained from (1) regarding the m's constant and the numbers x_i, y_i, . . . as variables, regard being had as to homogeneity of order and degree in the resulting equation. The form (2) and that which it takes when the k's have been replaced by their values in m's

$$\sum m_i Q_i = MQ + \frac{1}{M} \sum m_i m_j Q_{ij}, \tag{3}$$

constitute the general theorem which is the object of this note.

If $p = n$ there is a unique and reciprocal correspondence between the point set x, y, . . . and the point set k_1, k_2, . . . , when the discriminant of the m's in equations (1) does not vanish. In this case the numbers x_i, y_i, . . . may be taken to be the orthogonal cartesian coördinates with respect to arbitrary axes of the n corners P_i of a simplicissimum in $(n - 1)$-space, then the k's are the content coördinates with respect to the simplicissimum of the point P whose orthogonal coördinates are x, y, . . . The m's may be regarded as the mass-charges of the points P_i respectively, in which case P is their center of inertia or mass-center. Otherwise P is a point of mean position which may be said to divide the system of points P_i in the ratio of the numbers $m_1 : m_2 : m_3 : . . .$, the numbers m's are then called the ratio coördinates of P with respect to P_i.

When $p > n$ the correspondence between x, y, . . . and the k's is not unique. However a given set of m's determine P uniquely. In this case there may always be assigned in addition to the equations (1) a sufficient number of additional independent equations in the m's, in which the other numbers are arbitrarily assigned constants such that the variable m's are uniquely determined in terms of the variables x, y, . . . , z. In this condition the point P is confined to an $(n - 1)$-space as a section of a higher space, the relation between the x, y, . . . , m_i or k_i coördinates of P now being $1 : 1$, and the k's being interpretable as content coördinates.

When the configuration of points P_i is displaced to a new configuration P_i', and any point P whose coördinates are k_i with respect to P_i is displaced to a point P' whose coördinates are the same k_i unchanged with respect to P_i', the displacement is homogeneous. We give now some theorems corresponding to different forms of the quadratic Q.

3. Let $Q \equiv x^2 + y^2 + ...$

Represent an arbitrary origin by O. Then (2) gives

$$\rho^2 = \Sigma k_i \rho_i^2 - \Sigma k_i k_j \rho_{ij}^2, \tag{4}$$

Where $\rho_{ij} = P_iP_j$ are the mutual distances of the points P_i, $\rho = OP$, and $\rho_i = OP_i$ are the radial coördiantes of the point O with respect to P_i. The numbers k_i are the mass or ratio coördinates of P and are the content coördinates when the correspondence is one-to-one.

The equation (4) expresses the fact that ρ is the vector sum of $k_i\rho_i$. The points P_i and O being fixed then $k_i\rho_i = \alpha_i$ are the components of OP parallel to OP_i respectively. Omitting any one (OP_n) of OP_i and taking the others as axes of cartesian coördinates, any equation in the k's is transformed into cartesian (α) coördinates by writing for the k's respectively

$$\frac{\alpha_1}{\rho_1}, \quad \cdots, \frac{\alpha_{n-1}}{\rho_{n-1}}, 1 - \sum_1^{n-1} \frac{\alpha_i}{\rho_i}. \tag{5}$$

Geometrically equation (4) is the generalization of Stewart's theorem in elementary geometry. Given in a form similar to (4) by W. J. Curran Sharp, *Proc. Lon. Math. Soc.*, xviii, p. 325, April, 1887.

With reference to a fixed system P_i equation (4) is, if ρ is constant and O fixed, the equation of a circular locus with center O and radius ρ in k-coördinates, the radial coördinates of the center being ρ_i. If the k's are fixed it is a circular locus with center P and radius ρ in radial coördinates ρ_i. If $P \equiv O$ then $\rho = 0$ and (4) expresses the identical relation which exists between the radial and k-coördinates of the same point. It can easily be shown that the equation in k's

$$p = \Sigma k_i\, p_i - \Sigma k_i k_j \rho_{ij}^2, \tag{6}$$

is that of a circular locus whatever be the numbers p and p_i, that p is the power of P and p_i that of P_i with respect to the circular locus.

$$0 = \Sigma k_i p_i - \Sigma k_i k_j \rho_{ij}^2. \tag{7}$$

The p_i may be positive or negative. We say that a real circular locus is orthogonal to one with imaginary radius when the former bisects the boundary of the real circular locus having the same center and radius equal in absolute value to that of the radius of the imaginary locus. The power of a point with respect to a circular locus with imaginary radius is always positive and equal to the square of the distance of the point from the center plus the square of the absolute value of the radius. In case P_i is a simplicissimum, (7) is a circular locus which cuts orthogonally the circular loci having centers P_i and radii $\sqrt{p_i}$.

Replacing the k's by m's (6) becomes

$$\sum m_i \rho_i^2 = M\rho^2 + \frac{1}{M}\sum m_i m_j \rho_{ij}^2. \tag{8}$$

The moment of inertia of a system of masses about a point is equal to the moment of the mass M of the system at the mass-center plus the mass mean of the products of the moments two at a time of the masses about each other. When $\rho = 0$ the last term on the right of (8) is equal to the moment of inertia of the system about its center of inertia. We may speak of it as representing the internal moment of inertia of the system.

4. Let $Q \equiv x\,dy - y\,dx \equiv \rho^2 d\theta$.

If the points P_i move in space, O being arbitrarily fixed and the k's remain constant then the point P is homogeneously displaced with the system P_i. Let the projections of these points on the arbitrary xy-plane describe paths whose elements of sectorial area are represented by $d(P)$, $d(P_i)$. Represent by $d(P_{ij})$ the corresponding area swept over by a radius vector parallel and equal to the projection of P_iP_j on the xy-plane. Then

$$d(P) = \Sigma k_i\, d(P_i) - \Sigma k_i k_j\, d(P_{ij}). \tag{9}$$

If the paths are closed so that the points return to their initial positions then the areas bounded by them are related by*

$$(P) = \Sigma k_i\, (P_i) - \Sigma\, k_i k_j\, (P_{ij}). \tag{10}$$

This is the generalization of Elliott's extension of Holditch's theorem. It is obvious that the areas for any three points and those of their relative motions are all that are necessary for a motion of homogeneous displacement in a plane in order to determine the area for any other point. Thus if P_1, P_2, P_3, the corners of a triangle move in a plane, then any point P, moving in the plane homogeneously with them, will have its path area given by

$$P = x(P_1) + y(P_2) + z(P_3) - xy(P_{12}) - yz(P_{23}) - zx(P_{31}), \tag{11}$$

where x, y, z are the areal coördinates of P. If P be constant the locus of points which describe equal areas lie on a conic, and different values of (P) give a system of similar and similarly situated conics. The areas in

* If the points P_i move completely around the boundary of the same simple closed curve and their mutual distances are so related that $\Sigma k_i, k_j\, (P_{ij}) = 0$ then will the point P trace the same curve.

(11) may have any signs. If we call r the mean radius of a curve as to its area so determined that πr^2 is its area, then (11) can be written

$$r^2 = \Sigma x r_1^2 - \Sigma xy r_{12}^2, \tag{12}$$

in which the r's may be real or imaginary. For constant r's (12) is the equation of a conic with respect to any fixed position of the triangle $P_1P_2P_3$, it is of the ellipse, hyperbola, parabola class according as

$$\delta \equiv s(s - r_{12})\ (s - r_{23})\ (s - r_{31}),$$

where $2s = r_{12} + r_{23} + r_{31}$, is positive, negative or zero. In particular it is a circle when r_{ij} are respectively proportional to the sides of the triangle $P_1P_2P_3$.

The conic is two straight lines when

$$D \equiv \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & r_{12}^2 & r_{13}^2 & r_1^2 \\ 1 & r_{21}^2 & 0 & r_{23}^2 & r_2^2 \\ 1 & r_{31}^2 & r_{32}^2 & 0 & r_3^2 \\ 1 & r_1^2 & r_2^2 & r_3^2 & 0 \end{vmatrix} \tag{13}$$

is zero. The straight lines are imaginary, intersect or are parallel according as δ is positive, negative or zero.

In the case of a closed conic r_{ij} are the sides of a triangle, say UVW, and with respect to this triangle (12) is the equation of a circle, and for different values of r a series of concentric circles. The constants r_1^2, r_2^2, r_3^2 are the powers of U, V, W with respect to the circle

$$0 = \Sigma x r_1^2 - \Sigma xy\, r_{12}^2, \tag{13}$$

and r^2 is the power of x, y, z with respect to (13). When $D = 0$ (13) shrinks to its center becoming the point circle, or two imaginary straight lines intersecting in the real point P whose radial coördinates are r_1, r_2, r_3 and areal are x, y, z with respect to UVW. When the points U, V, W are colinear (12) is a parabola, and in particular two parallel straight lines when r_1, r_2, r_3 are the distances of a point from U, V, W.

If the moving triangle $P_1P_2P_3$ in the xy-plane is a rigid triangle ABC the equation (11) becomes*

$$(P) = \Sigma x(A) - \pi \Sigma xyc^2. \tag{14}$$

* In case the triangle returns to its initial position without complete revolution the last term $\Sigma xy\ (P_{12})$ on the right is zero.

Kempe's theorem, that points describing equal areas lie on a circle (14), and for different values of (P) we have a family of concentric circles is now obvious. Using r, r_1, r_2, r_3 as the mean radii of the areas (P), (A), (B), (C), then the circle.

$$0 = \Sigma x\, r_1^2 - \Sigma xy\, c^2 \tag{15}$$

is the orthogonal circle of the three circles with centers A, B, C, and radii r_1, r_2, r_3. The area enclosed by the path of any point x, y, z as given by (14) has for the square of its mean radius r^2 the power of x, y, z with respect to the circle (15). The minimum area enclosed by the paths of all points in the plane is that for the center of (15) and is equal to the area of that circle.

When in the interpretation of these problems a circle becomes a straight line by reason of its radius becoming infinite, the power of a point with respect to the circle becomes the power of the point with respect to the straight line or the perpendicular distance of the point from the line. The proof of this is general for the circular locus in any space, we give it for the plane.

The equation of a circular locus is

$$p = \Sigma x p_1 - \Sigma\, xyc^2.$$

The power of any point with respect to a circle is equal to the product of its distance from the center d and its distance δ from the polar with respect to the circle, hence

$$\delta = \sum x\,\delta_1 \frac{d_1}{d} - \sum \frac{xyc^2}{d}.$$

when the radius becomes infinite the last term is zero and the limit of d_1/d, etc., is unity. The circle passes over into the line and

$$\delta = \Sigma x \delta_1.$$

Taking up (10) again and replacing k's by m's and dividing by dt, there results

$$\sum m_i \frac{dP_i}{dt} = M\frac{dP}{dt} - \frac{1}{M}\sum m_i\, m_j \frac{dP_{ij}}{dt}. \tag{16}$$

The moment of momentum of a mass system is equal to that of the system concentrated at its mass center plus the internal moment of momentum of the system. If the mass center is at rest the external and internal moments of momenta are equal. The equation (16) was first derived by Laplace (*Méchanique Céleste,* Liv. I, Chap. V, § 22), in his investigation of the invariable plane of a conservative system. Holditch's theorem is therefore the lowest degenerate form of Laplace's beautiful generalization.

5. Let

$$Q \equiv (x' - x'')^2 + (y' - y'')^2 + \dots$$

We are concerned here with a homogeneous displacement (strain) of the points of space.

If the points P_i be displaced in any manner through segments Δs_i to a new position of configuration P_i, the k's remaining constant, the point P is displaced through a segment Δs to a new position P'. The k's are the coördinates of P with respect to P_i and also of P' with respect to P_i'. The segment Δs is the vector sum of the segments $k_i \Delta s_i$ and its value is given by

$$\Delta s^2 = \Sigma k_i \; \Delta s_i^2 - \Sigma k_i k_j \; \Delta s_{ij}^2, \tag{17}$$

where Δs_{ij} is the relative displacement of the points P_i and P_j. We observe also that Δs_{ij} is equal to the vector difference of P_iP_j and $P_i'P_j'$, as well as that of Δs_i and Δs_j.

The *relative* figure is constructed by drawing from a fixed origin segments parallel and equal to Δs_i, the segments joining the ends of these two and two are the segments Δs_{ij} forming a figure with respect to which Δs_i are the radial coördinates of the origin and k_i are the coördinates of P where $OP = \Delta s$. With reference to the relative figure (17) shows that the locus of points corresponding to points of equal displacement is a circular locus, and a concentric family of such when Δs varies. In the original space the locus of points of equal displacement is shown by (17) to be a closed quadric surface and the variation of Δs gives a family of similar and similarly situated such surfaces. There is a point in the original space which has a zero displacement, it is the center of the family of closed quadrics, and its coördinates with respect to P_i or P_i' are the k-coördinates of the fixed origin in the relative figure. In the relative figure if a straight line be drawn through the origin then all points on this straight line have their displacements in the same direction and are equal to their distances from the zero point. Hence given the displacement of three points and the zero point the direction and magnitude of the displacement of an arbitrary point is at once constructed by an easy straight line construction which is quite obvious. If the displacements Δs_i are parallel the right side of (17) is a perfect square and

$$\Delta s = \Sigma k_i \; \Delta s_i.$$

If Δ be changed to d and this equation integrated

$$s = \Sigma k_i \, s_i,$$

which is Bernouilli's theorem connecting the arc lengths of curves traced by points P_i moving in such a manner that at each instant the tangents at the points are parallel.

The whole theory of homogeneous strains admits of very beautiful and symmetrical treatment through this displacement theorem, we follow it no further here, however, than to point out the enunciation of a simple theorem in a homogeneous plane movement.

Three points A, B, C determine the homogeneous movement of the points of a plane in a fixed plane by revolving in circles with equal angular velocities in the same direction about three centers C_1, C_2, C_3 in the fixed plane. The radii of revolution being r_1, r_2, r_3 then any point P whose areal coördinates with respect to ABC are x, y, z describes a circle with the same angular velocity about the point x, y, z with respect to $C_1C_2C_3$ and its radius is given by

$$r^2 = \Sigma x r_1^2 - \Sigma x y r_{12}^2. \tag{18}$$

The segment r_{ij} are constants and are the segment differences of r_1, r_2, r_3, the angles between the latter are constants. The relative figure is a rigid figure rotating about its fixed origin. The locus of points in the original plane which describe circles of equal area is an ellipse and by varying the area a family of similar and similarly situated ellipses. The centers of these circles in the fixed plane lie correspondingly on a similar family of ellipses in the fixed plane, the one family being a homogeneous displacement of the other. They have a common center which is at rest, its x, y, z coördinates are the same as those of the fixed origin in the relative figure. The circles described by all points on the same straight line envelope a conic, the radii of which points at any instant envelope a parabola.

If one of the points, say C, revolves with the same velocity but in the opposite direction the area (C) is negative. Any point (not C or on the straight line AB all of which describe circles) in general whose coördinates are x, y, z with respect to ABC describes an ellipse whose center with respect to $C_1C_2C_3$ is z, y, x and whose radius vector sweeps over equal areas in equal times. Points describing paths of equal area lie on an ellipse, hyperbola or parabola according as the radius r_3 of (C) is less than, greater than or equal to the radius of the minimum circle described by the points of the line AB. In either case by varying the area there results a system of similar and similarly situated conics, all points on one of which describe paths of zero area (straight line segments), points inside and outside this zero conic describe their paths in opposite directions.

The first case is that of the rotational homogeneous movement of the points of a plane, the second is that of the harmonic homogeneous movement.

The object of introducing these particular displacement theorems is to point out the solution of such a problem as, for example, the meteorological one of the wind charts. At a series of points in a territory the direction and velocity of the wind are observed. There are thus formed a network of triangles such as $P_1P_2P_3$. If we assume that the flow takes place under the homogeneous law inside each triangle, then the direction and velocity at each point is determined and interpolated. If we assume that the lines of flow at the observed points are circles either direct or retrograde, which will be more nearly the truth the smaller the triangles, then the lines of flow for all particles can be mapped as continuous curves which are composed of arcs of conics compounded at points where they cross the sides of the triangles.

On replacing Δ by d and k_i by the m's, after dividing by dt^2 there results

$$\sum m_i V_i^2 = M\dot{V}^2 + \frac{1}{M}\sum m_i m_j V_{ij}^2, \tag{19}$$

an expression for the energy of a system of masses. The Σ term on the right representing the mass-mean of the sum of the products of the relative momenta two and two.

Differentiate (19) as to t and there results a corresponding relation in the work of the forces along the paths.

$$\sum F_i\, ds_i = F\, ds + \frac{1}{M}\sum m_i m_j \frac{d^2 s_{ij}}{dt^2} ds_{ij}. \tag{20}$$

6. Let

$$Q \equiv \left(\frac{d^2x}{dt^2}\right)^2 + \left(\frac{d^2y}{dt^2}\right)^2 + \cdots = \alpha^2$$

Similar to the equation in velocities

$$\dot{V}^2 = \Sigma k_i V_i^2 - \Sigma k_i k_j V_{ij}^2 \tag{21}$$

the equation in total accelerations

$$\alpha^2 = \Sigma k_i \alpha_i^2 - \Sigma k_i k_j\, \alpha_{ij}^2 \tag{22}$$

results, and the corresponding force function when the m's are introduced.

Let $Q = dx\, d^2y - dy\, d^2x$. When divided by dt^3 this is $V^2 d\theta$ where $d\theta$ is the element of angle turned through by V. Hence the corresponding equations for moment of momentum in the hodograph. Since here $V^2 d\theta$ is αp, p the perpendicular on the tangent, this becomes the moment of the forces in the hodograph.

If the only forces acting on the system of masses are equal and opposite stresses in the lines of action joining them two and two, then since the sum moments of the resultant stresses about an arbitrary axis is equal to the sum moments of these equal and opposite components and this is zero, then must α be zero, and its tangential and normal components zero. On account of the former V is constant, and on account of the latter the radius of curvature of the mass center infinite. When the mass center is at rest, the equations permit of studying the functions of the relative values instead of the direct values.

7. We conclude these illustrations with an example of a similitudinous homogeneous transformation of the points of a plane. Let the movement be defined by the motion of three masses m_1, m_2, m_3 at the corners of a triangle ABC of constant shape, the masses being projected with certain initial velocities in the plane are subjected to their mutual attractions f_{23}, f_{31}, f_{12} in the sides a, b, c, these attractions being homogeneous functions of the sides of the triangle of the same degree. We consider the motion with respect to the mass-center M at rest. Let λ be the ratio of similarity or that of each side of the triangle to its initial value. We use the subscript i to indicate the corresponding initial value, thus $a = \lambda a_i$, etc. The radial coördinates of the mass-center M with respect to ABC being r_1, r_2, r_3, then

$$\begin{aligned} r_1{}^2 &= \frac{1}{M}(c^2m_2 + b^2m_3) - \frac{1}{M}\sum m_1m_2c^2, \qquad (23) \\ &= \lambda^2 (r_1)_i{}^2, \end{aligned}$$

with similar values for r_2, r_3. The resultant forces F_1, F_2, F_3 acting on m_1, m_2, m_3 are in equilibrium and meet in a point F, the force-center, whose areal coördinates x, y, z, by taking moments about F, are seen to be

$$\frac{x}{a/f_{23}} = \frac{y}{b/f_{31}} = \frac{z}{c/f_{12}} = \frac{1}{\sigma}. \qquad (24)$$

Since f_{23}, etc., are homogeneous functions of the same degree in a, b, c the coördinates of F are constants. Hence the figures $ABCMF$ in any position is a homogeneous similitudinous transformation of any other position.

If p_1 is the perpendicular from M on the direction of F_1, the moment of F_1 about N is, Δ being the area of ABC,

$$p_1 F_1 = \left(\frac{m_3}{z} - \frac{m_2}{y}\right)\frac{2\Delta}{M\sigma}. \tag{25}$$

$$\begin{aligned} r_1^2\omega &= \lambda^2\omega\,(r_1^2)_i, \\ &= (r_1^2\omega)_i = \text{constant}, \end{aligned} \tag{26}$$

with like values for the moments of F_2, F_3. Since any position of the figure can be brought to any other position by a pure rotation followed by a similitudinous projection from M, each line of the figure turns through the same angle in the same time. Let ω be the angular velocity at any instant then,

$$\begin{aligned} \Sigma\, m_1 r_1^2\omega &= \text{constant} \\ \therefore \quad \lambda^2\omega\ (\Sigma m_1 r_1^2)_i &= \omega_i(\Sigma m_1 r_1^2)_i. \end{aligned}$$

Hence

$$\lambda^2\omega = \omega_i = \text{constant}. \tag{27}$$

Therefore

$$\begin{aligned} r_1^2\omega &= \lambda^2\omega(r_1^2)_i, \\ &= (r_1^2\omega)_i = \text{constant}, \end{aligned} \tag{28}$$

with like values for $r_2^2\omega$, $r_3^2\,\omega$. Differentiation with respect to t gives

$$p_1F_1 = p_2F_2 = p_3F_3 = 0.$$

Hence must

$$\frac{x}{m_1} = \frac{y}{m_2} = \frac{z}{m_3},$$

or the force-center must coincide with M the mass-center, and

$$\frac{f_{23}}{m_2 m_3 a} = \frac{f_{31}}{m_3 m_1 b} = \frac{f_{12}}{m_1 m_2 c}. \tag{29}$$

In particular if the mutual forces are proportional to

$$m_2m_3a^n : m_3m_1b^n : m_1m_2c^n,$$

then must $a = b = c$, which is Lagrange's theorem of "*the three bodies*" when $n = -2$.

In this movement

$$\frac{ds_1}{r_1} = \frac{ds_2}{r_2} = \ .\ \ .\ \ .\ = \frac{ds}{r},$$

ds being the element of arc length described by any point distant r from M. The velocities are proportional to the distances of the points from M and their directions make equal angles with their radii vectores from M. The path curve described by any point homogeneously connected to ABC is similar to that of any other point and is known when the path curves of A, B, C corresponding to given mutual attractions are known. Such a movement of the points of a plane is a homogeneous similitudinous attractive movement.

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The Flow of Water in Artificial Channels: Clean Pipes

BY

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BULLETIN OF THE PHILOSOPHICAL SOCIETY

SCIENTIFIC SECTION

Vol. I, No. 10, pp. 243–265 June, 1912

THE FLOW OF WATER IN ARTIFICIAL CHANNELS: CLEAN PIPES.*

BY

WILLIAM H. ECHOLS.

1. In a paper read before this Society in March, 1910, *On the Flow of Water in Pipes, Conduits and Open Channels* and published in the Transactions, Bulletin No. 6, June, 1910, the writer discussed the flow of water and designed a formula for representing the velocity. The design of this formula seems to have been based upon principles which appear in the main features to be correct, but in the determination of the empirical constants in the formula the writer made use of the actual experiments on certain pipes up to 2 feet in diameter and for pipes greater than 2 feet and up to 8 feet in diameter he made use of the hydraulic tables constructed independently by the two distinguished engineers, Mr. J. T. Fanning and Mr. Hamilton Smith, Jr., under the belief that these results represented the best obtainable information. During the two years since the publication of the paper referred to above the writer has more or less continuously studied this question and in the light of more data and closer observation a clearer insight into the principles which underlie the law of flow has been attained and I beg now to present these results. In the first place we observe that for pipes of 4 feet in diameter and larger sizes undue weight has been given to the experiment by Stearns, 1885, on a new cast iron pipe coated with asphalt and very smooth and straight, 4 feet in diameter, for which he found $m = 0.33$ for $V = 4$ feet. Fitzgerald found for a similar pipe $m = 0.42$ and Gale at Loch Katrine Water Works for a 4 foot asphalted cast iron pipe $3\frac{1}{2}$ miles long found $m = 0.50$. The

* Read before the Scientific Section, April 15, 1912.

tables of Fanning and of Smith appear to have been constructed for 4-foot pipes and larger sizes on the basis of the Stearns experiment and which undoubtedly leads to erroneously high velocities for large pipes.

When one takes into consideration the chaotic appearance of the data in the experiments on pipes there is not so large a difference in the results for pipes of large size and of different construction as we are lead to believe by the authorities. The different kinds of pipes of all sizes can be brought under the law of a common formula with quite a satisfactory degree of precision if the degree of that precision be compared with the degree of agreement among the various experiments themselves.

In the previous paper, as referred to, an expression for m, the coefficient of resistance was given as a function of the velocity V and the mean radius r, which expression may be considered as carrying Darcy's formula to the next degree of approximation. In the present paper this expression for m will be carried to a still further degree of approximation which is undoubtedly more nearly the truth. There are two distinct features in the design of such a formula to be considered. In the first place in order to have a proper generality it is necessary to form some idea as nearly as possible of the form of the function of V and r which represents m and this must be done through a theoretical speculation. In the second place the constants or parameters in this function must be determined from actual experiments, this is a problem of great difficulty owing to the scattered experiments and the chaotic nature of their disagreements. It is necessary to consider the data from a large variety of pipes and mass their results in order to fix the law governing the constants.

2. Very little is known of the laws of the so-called fluid friction. It is known however quite definitely that viscosity, in the case of ordinary water, or that property which admits a shearing stress has but little influence in causing a loss of energy in its motion, so little in fact that it may be practically ignored. The principal cause of the loss of energy appears to be that due to cross currents, eddies and vortices caused by particles of water impinging against the rough surface of the channel boundary and by reaction sent back into the body of the liquid in directions transverse to the direction of mean flow thus compounding their velocities with those of other particles with the result of reducing the velocity in the direction of mean flow and thus causing a corresponding loss of effective energy of discharge. This lost energy which disappears under the interference of particles striking each other with transverse velocities and the distruction of motion is undoubtedly dissipated in the production of heat which is conducted away in so elusive a manner as

to render hopeless any direct attempt to determine its mechanical equivalent.* It becomes necessary therefore, as in all such physical problems to speculate upon a law to govern this loss if any attempt is to be made to construct a theory to fit the case. In order to do this it is necessary first that careful qualitative observations be made as to the form of the lines of flow so that as nearly as may be some concrete idea may be had of what actually takes place in the body and near the boundary of water flowing over rigid material surfaces, then any postulate formulated must be repeatedly tested quantitatively by the empirical measurements of actual experiments and thus repeatedly revised and corrected.

It can be readily observed in a channel of flowing water, by introducing visible particles of about the same density as water such as various dyes, particularly permanganate of potash, that the stream lines in the core of the channel are more or less nearly parallel to the axis of the flow, that somewhat nearer to the boundary they are somewhat sinuous and quite near the boundary they are markedly sinuous. For a boundary surface that is regular and uniformly heterogeneous the path of a particle near that surface appears to obey the harmonic law with periodic turning points and changes of curvature. Particles very near the boundary are agitated into paths of short periods, appear to roll and revolve in a whirling motion in vortex rings which extend along the boundary across the cross section and roll with a relatively slow motion of translation along the bottom and sides in the direction of the flow. Water appears to flow over the boundary surface of its channel in ripples which in deep channels with moderate velocity of mean motion are not evidenced on the free surface or at a large distance from the rigid boundary, these ripples are much the same in kind differing only in degree from those that are seen in the free surface of shallow channels. These phenomena are especially noticeable in a stream after a recent shower of rain which discolors the water along the banks showing plainly whirls, eddies and reversed currents near the boundary extending out to the core of the main stream which flows as it were in an undulating channel of liquid boundary. Water flowing in the main channel thus appears to be flowing in a channel of liquid boundary of much slower moving or relatively still water which is a series of undulations or liquid barriers or submerged dams over which

* From a purely physical point of view the existence of a shearing stress is necessary as a causation in the loss of energy. While it does not occur as a measurable cause of loss yet it does come into play as an agent for dissipating the lost energy in heat; i. e., if there were no fluid friction there would be no going over into heat and no creation of vortices.

the main flow rises and falls with expanding and contracting cross sections alternately. It is readily understood that there is no such thing here as a rigid definite or fixed boundary in the liquid itself but quite a definite zone as it were between the main body of the flow in the direction of the axis and the permanent eddies in the neighborhood of the rough boundary.

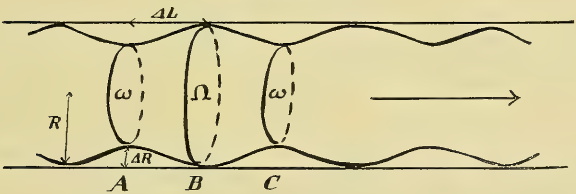
Such a typical case is the condition of flow artificially produced in some of Darcy and Bazin's experiments in which a wooden flume was constructed with laths $1 \times \frac{3}{8}$ inches nailed crosswise on the bottom and sides 2 inches apart. Thus the artificial barriers act as a series of submerged dams over which the water flows alternately rising and falling in sinuous curves of much regularity near the boundary while between these dams the water is whirled in a series of vortex rings. In a natural boundary such as that of the surface of a pipe, brick or cement conduit or smooth earthen canal the same thing takes place with a difference not so much of kind as of degree. In every regular artificial channel, fairly smooth, water flows through a series of more or less uniformly distributed cross sections which act as alternate contractions and expansions of the channel.

The form of this liquid boundary of the main channel flow depends on three things; first the geometrical topography of the rigid material surface boundary which for a given channel is constant, the mean slope of the hachure lines of this surface fixes to some definite extent the mean undulations of the liquid boundary; second the mean velocity with which the water flows in the main channel, the swifter be the main current the closer will the liquid boundary be to the rigid boundary, the more flat will be the lines of flow of the liquid boundary and the less will be the slope of these lines to the axis of flow; third the relative quantities of water flowing in the main channel and outside its boundary. The further the particles of water are from the disturbing influences of the rigid boundary the less their energies are disturbed by these influences and this is measured by some increasing function of the mean hydraulic radius and consequently the mean slope of the liquid boundary is relatively diminished by an increase of the mean hydraulic radius.

3. In view of the above observations we now postulate the hypothetical case of a straight horizontal pipe whose inner boundary is a surface of revolution whose meridian section is a pair of symmetrical sinuous curves, as shown in the figure, such that there are $2n$ cross sections alternately contractions and expansions uniformly distributed throughout the pipe length L at a distance apart ΔL. Let Ω be the area of the expanded section at B of radius R, and ω that of a contracted section at A or C of radius

$R - \Delta R$. Consider the change of energy ΔE in a mass M in passing from A through B to C. If there were no losses from fluid friction, or the fluid were perfect, the mean velocity in passing from A to B would be diminished resulting in a corresponding loss of energy but in passing from B to C the mean velocity would be increased again, under the law of continuity, to the same value which it had at A with a corresponding gain of energy thus restoring exactly the energy lost between A and B, with the final result that there would be neither loss nor gain of energy. But on account of fluid friction there is a resultant loss of energy. Therefore the whole or part of the energy which otherwise would be gained between B and C is consumed in doing the work of overcoming the fluid friction. According to Rankine it is almost wholly consumed. If $-\Delta E$ is the loss of energy in passing from A to B then the gain of energy in passing from B to C is

$$(1-f)\,\Delta E$$



where f is some fraction nearly or quite equal to unity. Hence the whole change of energy in passing from A to C is equal to the loss

$$-f\,\Delta E \tag{1}$$

In order to form some idea of the mechanical equivalent of this loss we make use of the principle of continuity which states that the product of the normal component of mean velocity by the area of the cross section is constant. Let v be the mean velocity at ω and V that at Ω. Let p represent a mean perimeter such that

$$p\,\Delta R = \Omega - \omega \tag{2}$$

The loss of pressure head in passing form one contraction, say at A to an adjacent expansion, at B is for a unit mass

$$\Delta h = \frac{v^2 - V^2}{2g} = \left(\frac{\Omega^2}{\omega^2} - 1\right)\frac{V^2}{2g},$$

$$= \left(2\,\Delta R + \frac{p}{\omega}\,\Delta R^2\right)\frac{p}{\omega}\frac{V^2}{2g}. \tag{3}$$

Therefore the loss of head in passing from the contraction at A to the next contraction at C, disregarding the negative sign, is

$$\Delta h = f\left(2\,\Delta R + \frac{p}{\omega}\,\Delta R^2\right)\frac{p}{\omega}\frac{V^2}{2g}. \tag{4}$$

The loss of head due to the consumption of energy in overcoming the resistances due to fluid friction throughout the length $L = 2n\,\Delta L$ of the pipe, or the so called resistance head h_r is $n\,\Delta h$ or

$$h_r = f\frac{\Delta R}{\Delta L}\left(1 + \tfrac{1}{2}\frac{p}{\omega}\,\Delta R\right)\frac{pL}{\omega}\frac{V^2}{2g}. \tag{5}$$

In ordinary well formed channels with comparatively smooth boundaries we may take n to be large and ΔR to be relatively small compared with R and

$$\frac{p}{\omega} = \frac{P}{\Omega} = \frac{1}{r}, \tag{6}$$

where P is the wetted perimeter of Ω and r therefore the mean hydraulic radius. We take f as unity or regard it as being combined with the factor $\Delta R/\Delta L$. The expression for the resistance head is then

$$h_r = \frac{\Delta R}{\Delta L}\left(1 + \frac{\Delta R}{2r}\right)\frac{L}{r}\frac{V^2}{2g}. \tag{7}$$

The coefficient

$$m = \frac{\Delta R}{\Delta L}\left(1 + \frac{\Delta R}{2r}\right), \tag{8}$$

is then the "coefficient of resistance." If in (8) we assume $\Delta R/\Delta L$ constant and $\Delta R = 0$ there results Chezy's formula. If ΔR be assumed constant and not zero there results the second approximation known as Darcy's formula. Neither $\Delta R/\Delta L$ nor ΔR are constants and we now wish to get some idea of the general forms these functions of r and V should have. There are two terms in the expression for m as given in (8), the first $\Delta R/\Delta L$ we call the mean slope of the resistance, the second ΔR we call the depth or thickness of the disturbance or resistance. In the beginning we can observe that m remains finite for any real channel with real flow. In the case of a pipe flowing under pressure the maximum value which m can attain is the value 0.5 and this occurs when the length of the pipe is equal to three diameters for a flush fitted pipe. For such a pipe m remains practically constant and equal to 0.5 for different velocities and diameters. As the length of the pipe increases the value of m diminishes, rapidly at first and then more slowly until the length of the pipe is about 500 diameters when m assumes the value expressed in (8) for long pipes. This variation in m due to the disturbance at entry dies away in about the length of 500 diameters. If we indicate by M the complete coefficient of resistance for a pipe of length L measured from the end of the first three diameters of length, then

$$M = m + \tfrac{1}{2}\frac{r}{L}.$$

The second term which represents the loss due to entry amounts to 1 in the fourth decimal place for L/D about 1200, but the errors of experiment amount on the whole to 3 or 4 per cent for ordinary pipes, so that pipes from 3 to 500 diameter in length are called *short* pipes, those greater *long* pipes.

4. We consider now the depth of the disturbance ΔR. This from consideration of the stream lines as above varies inversely as the velocity it cannot exceed in an open channel the depth of the water, or at most a certain mean value of the distances of all points in the cross section from the central axis of flow, and in the case of pipes under pressure running full ΔR cannot exceed the radius of the pipe. We assume tentatively the form of the function representing the second term in the parenthesis of (8) to be

$$\frac{\Delta R}{2r} = \frac{p}{rV + q}, \tag{9}$$

in which p and q are constants, the maximum value p/q being some proper fraction or unity.

5. The slope of the resistance $\Delta R/\Delta L$ depends directly on the mean slope of the rigid roughness of the material boundary, and it diminishes as the velocity and mean radius increase. Its greatest value occurs when the velocity and mean radius are least when it approximates the mean slope of the topography of the rigid material boundary. As the velocity and radius increase $\Delta R/\Delta L$ diminishes, but not indefinitely. As the

velocity increases the value of m diminishes becoming more nearly constant as in the case of short pipes, the resistance head becomes more and more nearly proportional to the square of the velocity with increase of velocity.

We write in view of careful consideration and with the effort to use a rational form

$$\frac{\Delta R}{\Delta L} = \frac{C}{1 + \dfrac{aV}{1 + bV + t\dfrac{V}{r}} + \dfrac{\alpha r}{\gamma r + \beta}} \tag{11}$$

6. The complete form of the coefficient of resistance M we assume to be of the type

$$\frac{\mu + \dfrac{\lambda}{Vr + q}}{1 + \dfrac{aV}{1 + bV + t\dfrac{V}{r}} + \dfrac{\alpha r}{\gamma r + \beta}} + \frac{1}{2}\frac{r}{L}, \tag{12}$$

all letters being constants except V, r and L.

In the previous paper referred to at the beginning of this article the writer proposed for the form of m the expression

$$m = \frac{\mu + \dfrac{\lambda}{Vr}}{1 + aV + \alpha r} \tag{13}$$

This has a wider range than Darcy's form and may be considered a next approximation. The form (12) has a much wider range than (13) and it is believed that the sequel will show that it is more precise and comprehensive than any form hitherto proposed.

7. The determination of the constants and the forms of the functions of V and r which occur in m must be carried on hand in hand. The investigator meets with much discouragement over the discrepancies in the reported results of experiments. It is only when large numbers of experiments are massed and tabulated that law and order appears beyond the first crude approximations. There are not as yet available any series of experiments through continuously graduated sizes and velocities for a single particular pipe construction to justify conclusions drawn from this particular style of pipe alone. One has to consider the mass of evi-

dence furnished by all the different styles and conditions in order to cover a sufficiently extensive range to evolve a law.

In determining the constants for pipes the writer has gone through a long and laborious process of plotting results graphically, drawing curves of mean position, adjustments by trial and error and of arithmetical interpolations. Considering the miscellaneous nature of the pipes employed the probable error of the final result is about as small as can be attained with present information. It is believed that the forms of the rational functions in (12) are essentially correct and that it is sufficiently flexible through the variation of the parameters to fit the adjustment to any new conditions that may arise in the future through an increased number of more accurate experiments.

Taking into consideration experiments on pipes of copper, brass, tin, zinc, lead, glass, wood, tile, cast and wrought iron uncoated and coated with tar or asphalt, the riveted iron pipes coated and uncoated, with butt or taper joints, cement lined pipes, several hundred observations, whose diameters vary from 0.1 of an inch to 10 feet, and whose velocities vary from 0.1 of a foot to 50 feet per second, the writer has determined the constants in m to be embodied in the following form

$$m = \frac{0.0094 + \dfrac{0.00003}{Vr}}{1 + \dfrac{0.07\,V}{1 + \left(0.015 + \dfrac{0.002}{r}\right) V} + \dfrac{0.6\,r}{r + 0.1}} \tag{14}$$

Strictly speaking Vr in (14) should be increased by a constant q as in(12) but as q has a value nearly equal to p or 0.00003 it has been neglected as not affecting the product Vr for any known values of V and r. This is the value for long pipes, for short pipes it must be increased by $r/2L$.

The total static head H is

$$H = \left\{1 + \left(m + \frac{r}{2L}\right)\frac{L}{r}\right\}\frac{V^2}{2g}. \tag{15}$$

8. The following table contains about four hundred experiments on pipes arranged according to increasing diameters and velocities, in which are tabulated in columns under $1000s$ the hydraulic slope or loss of head per 1000 feet of length, under V the observed velocity, under $100M_0$ and C_0 the experimental values of these constants. The relation between m and c being $mc^2 = 2g$, a table of corresponding values of m and c was

published in the previous paper. For comparison in the columns $100m$ and c are recorded these constants as computed from the formula (14). The authority for the experiment is given and a brief description of the pipe with a reference to where a more complete description can be found. It will be observed that while the formula is designed for clean pipes, straight or of easy curvature, pipes in actual sevice have frequently been used and in some cases even in use for some years when ever it was necessary to interpolate a diameter for which a new clean pipe was not found. In other instances for sake of comparison pipes obviously not in this class have been included. We do not pretend that the same formula without change of constants will give correct results for pipes of smooth lead, tin and glass as well as for riveted iron or cement lined pipes, but we do not have pipes of tin, glass or lead of several feet in diameter nor do we have riveted iron pipes or cement lined pipes from a $\frac{1}{4}$ to 1 inch in diameter. In fact in actual practice a certain form of construction and material surface goes with certain ranges of diameters, which permits a design for the whole group with a fair approximation. The formula constants have been determined to fit the actual performances of pipes in as large a number as could be accumulated regardless of the materials of construction. The only pipes which overlap in sizes are wooden stave pipes, cast iron coated pipes and the riveted iron pipes. The formula fits the riveted iron class and also all the ordinay clean cast iron pipes of service, but a new clean straight asphalted cast iron pipe is smoother and the velocity from the formula (14) should be increased 5 per cent for such a pipe, but when in use a short time the cast pipes belong properly in the same class as the others. The wooden stave pipes are smoother and the velocity computed from (14) should in general be increased 10 per cent for such pipes.

9. While (14) is complicated in appearance it is easily computable for any particular case being a rational function of r and V. However in order to facilitate its use the writer has computed and carefully drawn the three plates which accompany this paper.*

Plate I gives the corresponding values of m, V and r and is self explanatory.

Plate II gives the corresponding values of c, V, and r.

Plate III gives the corresponding values of V, r, s and k where $V = k\sqrt{s}$. Along the upper border and down the right border of the diagram

*The original drawings, 18 by 24 inches, were constructed with precision. The plates in this paper are photographic reductions of these accurate drawings and, therefore, may be read with a magnifying glass to advantage.

is scaled the values of $1000s$ or lost head in 1000 feet of length. A straight line (preferably a transparent straight edge or black silk thread) through O and a point on the slope scale cuts the diameter curve in a point whose abscissa is at once the velocity read to hundredths, the ordinate of such a point is the value of the coefficient k. In all the plates the curves are drawn for diameters in inches and feet and not for the hydraulic radius.

10. The regulation of the Society regarding the length of its published papers prohibits me at this time from giving the corresponding results for open channels. The improved formula has however been extended to include these cases with corresponding satisfactory results which reduce the number of classes of roughness to one half the number given in the previous paper. This I hope to present at some time in the near future.

11. *Provisional formula.* If Plate III be closely examined, the lines corresponding to given diameters which give the values of the coefficient k computed from (14) are practically straight lines of uniformly increasing slopes. It is easy to design a formula that will give the values of k with practical accuracy in terms of r and V, thus

$$k = (r + 0.4)\,V + 60 + 45r - \frac{6}{r + 0.1}. \qquad (16)$$

Since $V = k\sqrt{s}$, there results for the value of V in terms of r and s the formula

$$V = \frac{60 + 45r - \dfrac{6}{r + 0.1}}{1 - (r + 0.4)\sqrt{s}}\sqrt{s}, \qquad (17)$$

a formula easy to compute and practically accurate.

TABLE OF EXPERIMENTS FOR COMPARISON *

NO.	$\frac{1000}{s}$	V	r	$\frac{100}{m_0}$	$\frac{100}{m}$	C_0	C	AUTHORITY
1		0.24	0.0024	9.15	6.02	27	33	Weisbach. Mechanics of Engineering, p. 1109. Straight glass tubes.
2		0.38	0.0034	3.51	3.09	43	46	
3		0.53	0.0059	1.62	1.80	63	60	
4		0.73	0.0089	1.11	1.27	77	71	
5		0.60	0.014	1.65	1.10	62	78	
6		1	0.0022	1.65	2.10	62	55	E. W. Schoder and A. V. Saph, Cornell University Hydraulic Laboratory, experiments. Drawn brass pipes Cornell Civil Engineer, 1910
7		2	0.0022	1.40	1.39	68	68	
8		3	0.0022	1.25	1.11	72	76	
9		4	0.0022	1.18	1.00	74	80	
10		1	0.0042	1.32	1.40	70	68	
11		2	0.0042	1.12	1.09	76	77	
12		3	0.0042	1.00	0.97	80	91	
13		4	0.0042	0.94	0.90	83	84	
14	89	1.53	0.0052	1.26	1.14	71	75	Burnley and Woods, University Virginia, Mechanical Laboratory experiments, 1911. Straight new un-coated wrought iron pipe.
15	140	1.93	0.0052	1.27	1.09	71	77	
16	279	2.91	0.0052	1.10	0.99	77	80	
17	157	2.57	0.0078	1.16	0.93	75	83	
18	182	3.18	0.0078	0.91	0.90	84	84	
19	350	3.78	0.0078	0.98	0.87	81	85	
20		1	0.0062	1.25	1.25	72	72	Schoder and Saph, Cornell Laboratory. Drawn brass pipes.
21		2	0.0062	1.05	1.01	80	80	
22		3	0.0062	0.95	0.94	82	83	
23		4	0.0062	0.89	0.89	85	85	
24		1	0.0089	1.17	1.10	74	74	
25		2	0.0089	0.99	1.01	80	81	
26		3	0.0089	0.87	0.87	84	84	
27		4	0.0089	0.84	0.84	86	86	
28		0.58	0.0082	1.73	1.52	61	65	Weisbach, Mechanics of Engineering, p. 1190. Brass pipes.
29		0.64	0.0082	1.28	1.31	71	70	
30		0.28	0.0082	3.53	1.98	43	57	
31		0.65	0.0118	1.02	1.17	79	74	
32		2.08	0.0104	0.96	0.93	80	83	Hamilton Smith, Jr., Bowie's Hydraulic Mining, p. 174. Glass tube.
33		2.72	0.0104	0.89	0.89	85	85	
34		3.66	0.0104	0.83	0.84	88	87	
35		4.44	0.0104	0.79	0.82	91	88	

*Throughout this table the abbreviation H. & T. stands for Hering and Trautwine's Translation of Kutter's Flow of Water in Open Channels. T. & R. stands for Turneaure and Russell's Public Water Supplies.

TABLE OF EXPERIMENTS FOR COMPARISON—Continued

NO.	$\frac{1000}{s}$	V	r	$\frac{100}{m_0}$	$\frac{100}{m}$	C_0	C	AUTHORITY
36		1.40	0.0155	0.92	0.93	84	84	H. Smith, Jr., same as above. Glass pipe, $\frac{3}{4}$-inch diameter.
37		2.65	0.0155	0.77	0.83	91	88	
38		3.62	0.0155	0.71	0.80	95	90	
39		4.37	0.0155	0.69	0.78	96	91	
40		1.96	0.0191	0.81	0.82	90	89	H. Smith, Jr., Reference as above. Glass pipe, 1-inch diameter.
41		2.95	0.0191	0.72	0.78	95	91	
42		3.68	0.0191	0.68	0.76	97	92	
43		4.38	0.0191	0.65	0.74	99	93	
44		5.01	0.0191	0.63	0.73	101	94	
45		1	0.011	1.10	1.11	77	77	Schoder and Saph, Cornell Laboratory. Drawn brass pipe, $\frac{1}{2}$-inch diameter.
46		2	0.011	0.92	0.94	84	83	
47		3	0.011	0.84	0.87	87	86	
48		4	0.011	0.78	0.83	91	87	
49		1	0.012	1.05	1.11	78	78	As above. Drawn brass pipe.
50		2	0.012	0.89	0.94	85	84	
51		3	0.012	0.80	0.87	90	87	
52		4	0.012	0.74	0.83	93	89	
53		1	0.017	0.98	1.05	81	78	Schoder and Saph, Cornell Hydraulic Laboratory, 1910 Drawn brass pipe.
54		2	0.017	0.82	0.90	88	89	
55		3	0.017	0.74	0.80	94	94	
56		4	0.017	0.69	0.77	97	92	
57	14.0	1.22	0.0133	0.84	0.96	88	79	Burnley and Woods, University Virginia, Mechanical Laboratory experiments. Straight uncoated wrought iron pipe.
58	23.0	1.47	0.0133	0.91	0.91	84	82	
59	15.0	4.18	0.0133	0.72	0.78	95	89	
60	4.1	0.76	0.0178	0.80	1.07	90	79	
61	7.3	0.97	0.0178	0.87	1.00	86	82	
62	80.0	2.03	0.01	1.29	0.94	71	83	
63	169.0	3.46	0.01	0.91	0.85	85	87	
64	305.0	4.68	0.01	0.89	0.82	85	88	
65	6258	36.1	0.021	0.64	0.62	100	102	Rowland, 1883. Trans. Am. Soc. C. E. Experiments of New York Gaslight Company. Straight wrought iron pipe, 1-inch diameter H. & T., p. 138.
66	8935	43.4	0.021	0.63	0.58	101	105	
67	10742	48.1	0.021	0.62	0.57	102	106	
68	3055	26.7	0.021	0.57	0.60	106	104	
69	4362	32.1	0.021	0.56	0.53	107	110	
70	5244	36.6	0.021	0.52	0.62	111	102	
71	2000	19.9	0.021	0.67	0.67	98	98	
72	2856	24.5	0.021	0.64	0.65	100	100	
73	3433	27.3	0.021	0.62	0.64	102	100	

TABLE OF EXPERIMENTS FOR COMPARISON—Continued

NO.	$\frac{1000}{s}$	V	r	$\frac{100}{m_0}$	$\frac{100}{m}$	C_0	C	AUTHORITY
74		1	0.021	0.93	0.93	83	83	Schoder and Saph, Cornell Hydraulic Laboratory, 1910 Drawn brass pipe.
75		2	0.021	0.78	0.82	91	89	
76		3	0.021	0.71	0.78	96	91	
77		4	0.021	0.66	0.75	99	93	
78		0.36	0.0208	1.30	1.11	70	76	Weisbach, Mechanics of Engineering, p. 1109. Straight Zinc pipes.
79		0.67	0.0208	1.08	1.01	77	80	
80		0.79	0.0210	1.06	0.97	78	81	
81		1.15	0.0270	0.86	0.90	87	84	
82	26.9	2.22	0.022	0.76	0.82	92	89	H. Smith, Jr., 1886. New wrought iron pipe coated with asphalt. Funnel mouth H. & T., p. 139.
83	52.2	3.22	0.022	0.70	0.78	96	91	
84	103.0	4.76	0.022	0.64	0.74	100	93	
85	131.0	5.44	0.022	0.62	0.72	102	94	
86	50.6	2.70	0.021	0.89	0.80	85	90	Iben, 1875. Lead pipe in Hamburg (Lager Platz). H. & T., p. 135.
87	64.6	3.16	0.021	0.85	0.78	87	91	
88	112.0	4.72	0.021	0.66	0.74	98	93	
89	216.0	6.88	0.021	0.60	0.70	104	96	
90	348.0	9.11	0.021	0.55	0.67	108	98	
91	0.44	0.213	0.022	1.38	1.38	68	68	Darcy, 1851. Straight new lead pipe. H. & T. p. 135.
92	8.14	1.09	0.022	0.98	0.91	81	81	
93	54.4	3.35	0.022	0.69	0.77	96	91	
94	146.0	5.51	0.022	0.69	0.72	97	94	
95	0.33	0.19	0.022	1.28	1.43	71	67	Darcy, 1851. New straight wrought iron pipe uncoated Measured with great accuracy. H. & T., p. 139.
96	10.1	1.21	0.022	0.98	0.96	81	84	
97	43.5	2.61	0.022	0.89	0.80	85	90	
98	106.0	4.20	0.022	0.85	0.75	87	92	
99	310.0	7.17	0.022	0.85	0.69	87	96	
100	0.71	0.37	0.03	0.99	1.01	80	80	Darcy. An old cast iron pipe after being carefully cleaned. Before cleaning $c=60$. H. & T., p. 151.
101	1.80	0.62	0.03	0.91	0.93	84	82	
102	14.4	1.97	0.03	0.71	0.79	95	91	
103	39.7	3.39	0.03	0.67	0.73	98	94	
104	5.40	1.12	0.03	0.83	0.85	88	87	Bossut, 1771. Straight tin pipe. H. & T., p. 135.
105	10.8	1.17	0.03	0.73	0.85	94	88	
106	15.1	2.07	0.03	0.67	0.78	98	91	
107	26.9	2.95	0.03	0.60	0.74	104	93	
108	53.0	4.31	0.03	0.54	0.70	109	95	
109	113.0	6.14	0.03	0.57	0.67	106	98	
110	113.0	6.15	0.03	0.57	0.67	106	98	
111	113.0	6.16	0.03	0.57	0.67	106	98	

TABLE OF EXPERIMENTS FOR COMPARISON—Continued

NO.	$\frac{1000}{s}$	V	r	$\frac{100}{m_0}$	$\frac{100}{m}$	C_0	C	AUTHORITY
112		1	0.032	0.84	0.86	88	86	Schoder and Saph, Cornell Hydraulic Laboratory, 1910 Drawn brass pipe.
113		2	0.032	0.70	0.78	96	91	
114		3	0.032	0.64	0.74	100	94	
115		4	0.032	0.60	0.71	104	96	
116	0.22	0.20	0.032	1.09	1.21	77	73	Darcy, 1851. New straight wrought iron pipe uncoated. Measured with great accuracy. H. & T. p., 139.
117	3.36	0.86	0.032	0.96	0.88	82	86	
118	23.9	2.58	0.032	0.75	0.76	93	92	
119	123.0	6.30	0.032	0.64	0.67	100	98	
120	224.0	8.52	0.032	0.63	0.64	101	101	
121	.82	0.394	0.034	1.15	1.10	75	80	Darcy, 1851. Straight new lead pipe. H. & T., p. 137.
122	7.78	1.40	0.034	0.85	0.82	87	89	
123	56.0	4.23	0.034	0.65	0.71	99	95	
124	159.0	7.56	0.034	0.60	0.65	103	100	
125	0.96	0.505	0.041	0.98	0.90	80	85	Darcy and Bazin, Straight glass pipe. H. & T., p. 135.
126	7.71	1.59	0.041	0.81	0.77	89	91	
127	57.6	4.85	0.041	0.64	0.67	100	98	
128	112.0	6.92	0.041	0.61	0.63	102	101	
129	5.3	1.44	0.045	0.73	0.82	94	91	Bossut, 1771. Straight tin pipe. Velocity from known volume. H. & T., p. 137.
130	10.0	2.11	0.035	0.64	0.74	100	93	
131	14.2	2.60	0.045	0.61	0.73	103	94	
132	23.9	3.58	0.045	0.53	0.70	110	96	
133	46.5	5.23	0.045	0.50	0.66	115	99	
134		1	0.042	0.78	0.80	91	89	Schoder and Saph, Cornell Hydraulic Laboratory, 1910. Drawn brass pipe.
135		2	0.042	0.65	0.75	100.	93	
136		3	0.042	0.60	0.71	103	95	
137		4	0.042	0.55	0.68	108	97	
138	0.84	0.63	0.066	0.89	0.80	85	89	Darcy, 1851. Coated old cast iron pipe after being carefully cleaned. Before cleaning m=1.40. H. & T., p. 151.
139	7.23	2.01	0.066	0.75	0.71	92	95	
140	15.6	2.83	0.066	0.82	0.68	87	97	
141	44.7	5.01	0.066	0.74	0.62	93	101	
142	0.27	0.328	0.068	1.09	1.00	77	88	Darcy, 1851. New wrought iron riveted pipe coated with asphalt. H. & T., p. 141.
143	2.03	1.17	0.068	0.64	0.76	100	94	
144	12.2	3.12	0.068	0.55	0.67	108	98	
145	40.7	6.15	0,068	0.47	0.60	.117	104	
146	106.0	10.5	0.068	0.42	0.55	124	110	
147	156.0	12.8	0.068	0.41	0.52	125	112	

TABLE OF EXPERIMENTS FOR COMPARISON—Continued

NO.	$1000\,s$	V	r	$100\,m_0$	$100\,m$	C_0	C	AUTHORITY
148	0.29	0.31	0.083	1.62	0.83	63	89	Ehmann, 1879. Cast iron main at Stuttgart, Neckar Street, nine years old. H. & T., p. 143.
149	1.13	0.77	0.083	1.02	0.74	79	93	
150	2.37	1.18	0.083	0.90	0.72	84	95	
151	3.76	1.57	0.083	0.81	0.70	89	96	
152	6.43	2.08	0.083	0.81	0.68	89	97	
153	15.7	3.47	0.083	0.70	0.64	96	100	Iben, 1875. New cast iron pipe at Hamburg. H. & T., p. 143.
154	31.1	4.86	0.083	0.70	0.60	96	103	
155	42.1	5.80	0.083	0.67	0.59	98	104	
156	48.6	6.17	0.083	0.68	0.58	97	105	
157	0.66	1.0	0.084	0.42	0.73	124	94	Iben, 1876. New cast iron service pipe at Hamburg, Grindell Alley. Coated with tar. H. & T., p. 145.
158	5.62	2.1	0.084	0.68	0.68	97	97	
159	9.75	2.8	0.084	0.68	0.66	97	99	
160	20.5	4.3	0.084	0.58	0.62	105	102	
161	33.7	5.5	0.084	0.60	0.59	104	104	
162	0.27	0.67	0.154	0.60	0.70	104	97	Darcy, 1851. New cast iron pipe. H. & T., p. 145.
163	3.68	2.49	0.154	0.60	0.63	104	102	
164	22.5	6.34	0.154	0.55	0.55	108	109	
165	110.0	14.18	0.154	0.54	0.45	109	119	
166	146.0	16.17	0.154	0.55	0.40	108	126	
167	0.20	0.59	0.161	0.60	0.70	104	97	Darcy, Straight new wrought iron riveted pipe coated with asphalt. H. & T., p. 141.
168	1.29	1.53	0.161	0.57	0.66	106	101	
169	5.80	3.53	0.161	0.48	0.61	116	105	
170	12.0	5.51	0.161	0.41	0.57	125	108	
171	29.7	9.00	0.161	0.38	0.51	130	114	
171	122.0	19.7	0.161	0.32	0.43	141	122	
173	0.38	0.73	0.166	0.74	0.70	93	98	Ehmann, 1879. Asphalted cast iron main at Stuttgart, four years old, clean. H. & T., p. 143.
174	0.85	1.12	0.166	0.71	0.68	95	99	
175	1.33	1.45	0.166	0.67	0.66	98	100	
176	1.88	1.69	0.166	0.70	0.66	96	101	
177	.017	0.20	0.21	0.57	0.72	106	95	Ehmann, 1879. Cast iron main asphalted, four years in use, in good condition, several bends of large radius. Stuttgart. H. & T., p. 147.
178	0.037	0.27	0.21	0.57	0.70	106	97	
179	0.078	0.40	0.21	0.64	0.68	100	98	
180	0.188	0.62	0.21	0.64	0.66	100	99	
181	0.328	0.83	0.21	0.63	0.64	101	100	
182	0.406	0.94	0.21	0.61	0.64	103	100	
183	0.70	1.30	0.234	0.63	0.62	101	102	Darcy. New straight wrought iron riveted pipe coated with asphalt. H. & T., p. 141.
184	4.33	3.87	0.234	0.43	0.56	122	108	
185	11.9	6.67	0.234	0.40	0.51	127	112	
186	28.1	10.5	0.234	0.38	0.46	130	118	

TABLE OF EXPERIMENTS FOR COMPARISON—Continued

NO.	$\frac{1000}{s}$	V	r	$\frac{100}{m_0}$	$\frac{100}{m}$	C_0	C	AUTHORITY
187	8.5	4.71	0.228	0.56	0.54	107	108	H. Smith, Jr., Sheet iron riveted pipe. Funnel mouth piece 8 feet long. North Bloomfield, 1876. H. & T., p. 141.
188	13.3	6.09	0.228	0.52	0.52	111	111	
189	16.9	6.93	0.228	0.51	0.51	112	112	
190	25.6	8.66	0.228	0.50	0.49	113	115	
191	33.1	10.0	0.228	0.49	0.47	116	117	
192		4.5	0.231	0 56	0.55	107	108	Riveted sheet iron pipe. T. & R., p. 246.
193		6.0	0.231	0.53	0.52	111	111	
194	0.04	0.4	0.25	0.42	0.66	123	99	Iben, 1876. New cast iron main, coated with tar. Hamburg. H. T., p. 147.
195	0.60	1.3	0.25	0.54	0.62	109	101	
196	1.49	2.2	0.25	0.51	0.59	112	104	
197	7.00	4.8	0.25	0.48	0.54	116	109	
198	11.2	6.1	0.25	0.49	0.52	114	111	
199	6.68	4.59	0.264	0.54	0.54	109	108	H. Smith, Jr., Sheet iron riveted pipe, coated with asphalt. North Bloomfield. H. & T., p. 141.
200	14.3	6.96	0.264	0.50	0.51	113	113	
201	22.2	8.65	0.264	0.50	0.49	113	115	
202	33.2	10.7	0.264	0.49	0.47	114	117	
203	0.145	0.69	0.293	0.58	0.64	105	100	Trans. Am. Soc. C. E., vol. 40, p. 545. Wooden stave pipe, 14-inch diameter. Angeles Water Supply, California.
204	0.161	0.69	0.293	0.63	0.64	101	100	
205	0.170	0.70	0.293	0.66	0.64	99	100	
206	0.178	0.75	0.293	0.60	0.64	104	101	
207	0.391	1.17	0.293	0.54	0.62	109	101	
208	0.375	1.18	0.293	0.50	0.62	113	101	
209	0.368	0.53	0.293	0.51	0.61	112	102	
210	5.02	4.38	0.307	0.52	0.54	112	109	H. Smith, Jr., Riveted sheet-iron pipe, coated with asphalt. H. & T., p. 141.
211	11.0	6.84	0.307	0.46	0.50	118	113	
212	12.3	7.31	0.307	0.45	0.49	119	114	
213	16.5	8.46	0.307	0.45	0.48	119	116	
214	24.7	10.6	0.307	0.43	0.45	122	119	
215	32.3	12.1	0.307	0.44	0.43	121	122	
216		5.0	0.315	0.52	0.53	112	110	Riveted sheet iron. T. & R., p. 246.
217	0.53	1.23	0.319	0.73	0.62	94	102	Darcy. Wooden pipe, rectangular section. 1.57 by .98 feet. Poplar closely jointed, planed. H. & T., p. 137.
218	1.07	1.78	0.319	0.69	0.59	97	104	
219	1.73	2.28	0.319	0.68	0.58	97	105	
220	2.73	2.94	0.319	0.64	0.57	100	106	
221	3.87	3.53	0.319	0.68	0.56	97	107	
222	6.27	4.35	0.319	0.68	0.56	97	108	
223	7.27	4.62	0.319	0.70	0.56	96	109	
224	8.80	5.31	0.319	0.64	0.53	100	110	

TABLE OF EXPERIMENTS FOR COMPARISON—Continued

NO.	$\frac{1000}{s}$	V	r	$\frac{100}{m_0}$	$\frac{100}{m}$	C_0	C	AUTHORITY
225		4.5	0.332	0.53	0.54	110	109	Riveted sheet iron pipe. T. & R., p. 246.
226	0.59	1.6	0.343	0.53	0.59	111	105	Lampe, 1870. Asphalted cast iron pipe at Danzig. Straight iron pipe at Danzig. Straight, clean, five years old. H. & T., p. 147.
227	1.38	2.5	0.343	0.50	0.57	114	106	
228	1.63	2.7	0.343	0.49	0.57	115	106	
229	1.95	3.1	0.343	0.46	0.56	119	107	
230	66.7	20.1	0.354	0.37	0.38	132	130	H. Smith, Jr., Asphalted sheet iron double riveted pipe at Texas Creek. Easy curves. H. & T., p. 141.
231	1.96	3.6	0.375	0.36	0.55	134	108	Am. Soc. C. E. Trans. Vol. 36, p. 26. Cylindrical wooden stave pipe, 14-inches diameter.
232	0.45	1.47	0.41	0.55	0.60	109	104	Darcy, 1851. New cast iron pipe. H. & T., p. 149.
233	1.20	2.60	0.41	0.55	0.60	117	106	
234	2.10	3.42	0.41	0.47	0.56	116	107	
235	2.60	3.67	0.41	0.50	0.56	113	108	
236	2.5	3.58	0.375	0.47	0.55	117	108	Bidder, 1853. Earthenware, tile pipe. Flowing partly under slight head. H. & T., p. 136.
237	0.23	0.95	0.416	0.68	0.62	97	103	J. T. Fanning, 1880. Wrought iron cement lined force main. No short bend, but two large Y branches, two small blow off branches and three stop valves. H. & T., p. 155.
238	0.44	1.45	0.416	0.53	0.60	110	104	
239	0.73	1.93	0.416	0.53	0.59	111	105	
240	1.04	2.33	0.416	0.51	0.58	112	106	
241	1.34	2.60	0.416	0.53	0.57	110	107	
242	1.58	2.87	0.416	0.52	0.56	112	107	
242	1.99	3.27	0.416	0.45	0.56	113	108	
244	2.28	3.44	0.416	0.52	0.55	112	108	
245	2.72	3.74	0.416	0.52	0.55	111	109	
246	3.00	3.92	0.416	0.52	0.54	111	109	
247	3.13	4.00	0.416	0.52	0.54	111	109	
248	3.20	4.04	0.416	0.53	0.54	111	109	

TABLE OF EXPERIMENTS FOR COMPARISON—Continued

NO.	$\frac{1000}{s}$	V	r	$\frac{100}{m_0}$	$\frac{100}{m}$	C_0	C	AUTHORITY
249	0.73	2.00	0.417	0.49	0.58	114	105	Brush, 1882–87. New cast iron force main. Hackensack, N. J. Large number of summits, curved, 4 right angles, ten quadrants of 10-feet radius. Quantity measured at pump. Static Head, 165 feet. H. & T., p. 149.
250	0.88	2.24	0.417	0.47	0.58	117	106	
251	1.03	2.36	0.417	0.49	0.58	114	106	
252	1.19	2.52	0.417	0.50	0.57	113	106	
253	1.33	2.68	0.417	0.49	0.57	114	106	
254	1.49	2.76	0.417	0.53	0.57	111	107	
255	1.64	2.92	0.417	0.51	0.56	112	107	
256	1.80	3.00	0.417	0.54	0.56	110	107	
257	0.12	0.7	0.417	0.58	0.61	105	102	Iben, 1876. New cast iron tar coated service main. Hamburg. H. & T., p. 149.
258	0.48	0.6	0.417	0.53	0.59	110	104	
259	0.76	1.9	0.417	0.54	0.59	109	105	
260	1.21	2.5	0.417	0.54	0.57	109	106	
261	0.475	1.67	0.505	0.55	0.60	108	106	Darcy and Bazin. Rectangular wooden pipe, poplar closely jointed. Weir measurement. 2.62 feet wide, by 1.64 feet deep H. & T., p. 137.
262	1.08	2.52	0.505	0.55	0.56	108	107	
263	1.90	3.37	0.505	0.54	0.54	109	109	
264	2.91	4.32	0.505	0.53	0.53	110	110	
265	4.27	5.07	0.505	0.54	0.52	109	111	
266	5.06	5.33	0.505	0.54	0.51	109	112	
267	5.76	5.91	0.505	0.53	0.50	110	113	
268	6.61	6.37	0.505	0.53	0.50	110	114	
269	16.4	12.6	0.54	0.36	0.42	134	124	H. Smith, Jr., Sheet iron riveted pipe, asphalted. H. & T., p. 141.
270	11.7	10.8	0.607	0.39	0.43	128	122	H. Smith, Jr., Wrought iron pipe at Cherokee, five years old. H. & T., p. 157.
271	0.39	1.60	0.625	0.61	0.57	103	105	Darrach, New cast iron force main at Philadephia, with turns and curves, also check valves. H. & T., p. 140.
272	0.46	1.74	0.625	0.61	0.57	103	105	
273	0.53	1.87	0.625	0.61	0.57	103	106	
274	0.60	2.00	0.625	0.60	0.57	103	106	
275	0.67	2.14	0.625	0.59	0.56	105	106	
276	0.31	1.47	0.625	0.58	0.60	106	105	Darrach, 1878. Cast iron force main at Philadelphia. Two years old. Four check valves on line. Quantity measured at pumps. H. & T., p. 149.
277	0.38	1.62	0.625	0.57	0.60	106	105	
278	0.34	1.76	0.625	0.57	0.59	106	106	
279	0.50	1.91	0.625	0.55	0.58	108	106	
280	0.57	2.06	0.625	0.54	0.57	110	106	
281	0.63	2.20	0.625	0.52	0.56	111	107	

TABLE OF EXPERIMENTS FOR COMPARISON—Continued

NO.	$\frac{1000}{s}$	V	r	$\frac{100}{m_0}$	$\frac{100}{m}$	C_0	C	AUTHORITY
282	0.70	2.35	0.625	0.50	0.56	113	107	Darrach, 1878. Cast iron force main at Philadelphia. Two years old. Four check valves on line. Quantity measured at pumps. H. & T., p. 149.
283	0.76	2.50	0.625	0.49	0.55	115	107	
284	0.83	2.64	0.625	0.48	0.55	116	108	
285	0.89	2.79	0.625	0.46	0.54	118	108	
286	0.95	2.94	0.625	0.44	0.54	120	109	
278	1.02	3.08	0.625	0.43	0.54	122	109	
288	1.08	3.23	0.625	0.41	0.54	124	109	
289		3.5	0.735	0.40	0.53	127	110	Riveted sheet iron pipe. H. & T., p. 246.
290		1.0	0.75	0.87	0.59	86	104	Clemens Herschel, New riveted iron pipe. East Jersey Conduit. T. & R., p. 246.
291		1.5	0.75	0.78	0.58	91	105	
292		2.0	0.75	0.71	0.57	95	106	
293		2.5	0.75	0.65	0.56	99	107	
294		3.0	0.75	0.60	0.55	103	108	
295		3.5	0.75	0.56	0.54	107	109	
296		4.0	0.75	0.53	0.53	111	110	
297		4.5	0.75	0.50	0.52	114	111	
298		5.0	0.75	0.47	0.51	117	112	
299		5.5	0.75	0.44	0.50	120	113	
300		6.0	0.75	0.42	0.49	123	114	
301		5.0	0.798	0.57	0.51	106	112	Riveted iron pipe, T. & R., p. 246.
302		4.0	0.798	0.54	0.53	109	110	Riveted iron pipe, T. & R., p. 240.
303		1.0	0.882	0.70	0.59	96	104	Herschel, Riveted iron pipe. Kearney extension of East Jersey Water Co. pipe line. New pipe, taper joints. T. & R. p. 246. Coating unusually smooth.
304		1.5	0.882	0.61	0.57	103	106	
305		2.0	0.888	0.55	0.56	108	107	
306		2.5	0.882	0.52	0.55	111	108	
307		3.0	0.882	0.51	0.54	113	109	
308		3.5	0.882	0.50	0.53	113	109	
309		4.0	0.882	0.51	0.52	113	110	
310		4.5	0.882	0.51	0.51	112	111	
311		5.0	0.882	0.52	0.50	111	112	
312		5.5	0.882	0.53	0.50	110	113	
313		6.0	0.882	0.53	0.49	110	114	

TABLE OF EXPERIMENTS FOR COMPARISON—Continued

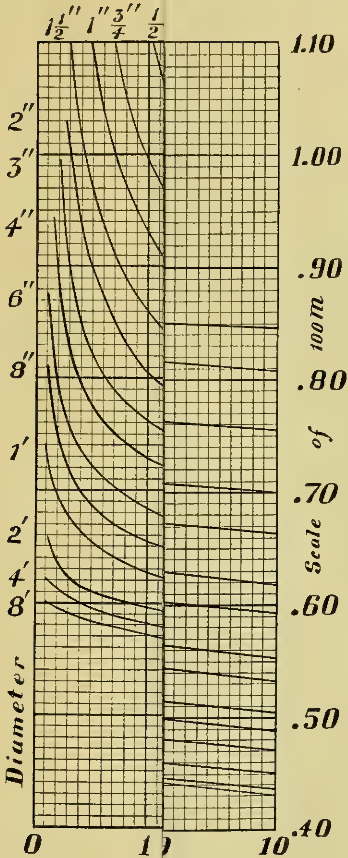
NO.	$\frac{1000}{s}$	V	r	$\frac{100}{m_0}$	$\frac{100}{m}$	C_0	C	AUTHORITY
314		1.0	0.882	0.63	0.59	101	104	Same as above on Conduit No. 2. New Riveted iron pipe.
315		1.5	0.882	0.61	0.57	103	106	
316		2.0	0.882	0.59	0.56	104	107	
317		2.5	0.882	0.58	0.55	105	108	
318		3.0	0.882	0.57	0.54	106	109	
319		3.5	0.882	0.56	0.53	107	109	
320		4.0	0.882	0.55	0.52	108	110	
321		4.5	0.882	0.55	0.51	108	111	
322		5.0	0.882	0.55	0.50	108	112	
323		5.5	0.882	0.55	0.50	108	113	
324		6.0	0.882	0.55	0.49	108	114	
325	0.947	3.46	1.0	0.50	0.52	113	110	Jas. M. Gale, 1869. Cast iron main 3½ miles long. Coated with asphalt, in good condition. Loch Katrine Water Works. H. & T., p. 150.
326		1.0	1.01	0.63	0.58	101	105	Herschel, Kearney extension East Jersey Water Company, Conduit No. 1. Riveted iron pipe, asphalt coating, cylinder joints, new pipe. T. & R., p. 246.
327		1.5	1.01	0.58	0.57	105	106	
328		2.0	1.01	0.54	0.56	109	107	
329		2.5	1.01	0.52	0.55	111	108	
330		3.0	1.01	0.50	0.54	113	109	
331		3.5	1.01	0.50	0.53	113	110	
332		4.0	1.01	0.50	0.52	013	111	
333		4.5	1.01	0.51	0.51	112	112	
334		5.0	1.01	0.51	0.50	112	112	
335		5.5	1.01	0.51	0.50	112	113	
336		6.0	1.01	0.51	0.49	112	114	
337		1.0	1.01	0.68	0.58	97	105	Herschel, Same as Nos. 326–336 on Conduit No. 2, but four years later. Riveted iron pipe. T. & R., p. 246.
338		1.5	1.01	0.63	0.57	101	106	
339		2.0	1.01	0.60	0.56	103	107	
340		2.5	1.01	0.58	0.55	105	108	
341		3.0	1.01	0.58	0.54	105	109	
342		3.5	1.01	0.58	0.53	105	110	
343		4.0	1.01	0.59	0.52	104	111	
344		4.5	1.01	0.60	0.51	104	112	
345		5.0	1.01	0.60	0.50	104	113	
346		5.5	1.01	0.60	0.50	104	114	
347		6.0	1.01	0.60	0.49	104	114	

TABLE OF EXPERIMENTS FOR COMPARISON—Continued

NO.	$\frac{1000}{s}$	V	r	$\frac{100}{m_0}$	$\frac{100}{m}$	C_0	C	AUTHORITY
348		1.0	1.01	0.68	0.58	97	105	Herschel, As above. Riveted iron pipe, new. Taper joints. T. & R., p. 246.
349		1.5	1.01	0.66	0.57	99	106	
350		2.0	1.01	0.64	0.56	100	107	
351		2.5	1.01	0.62	0.55	102	108	
352		3.0	1.01	0.61	0.54	102	109	
353		3.5	1.01	0.60	0.53	104	110	
354		4.0	1.01	0.59	0.52	104	111	
355		4.5	1.01	0.58	0.51	105	112	
356		5.0	1.01	0.58	0.50	105	113	
357		5.5	1.01	0.58	0.50	105	114	
358		6.0	1.01	0.58	0.49	105	115	
359		1.0	1.51	0.53	0.57	110	106	Wing and Hoskins. New riveted iron pipe, asphalt coating, butt joints. Pioneer Electric Company, Ogden. T. & R., p. 246.
360		1.5	1.51	0.52	0.56	111	107	
361		2.0	1.51	0.53	0.55	110	107	
362		2.5	1.51	0.55	0.54	108	108	
363		3.0	1.51	0.58	0.53	108	109	
364		3.5	1.51	0.53	0.52	110	110	
365		4.0	1.51	0.52	0.52	111	111	
366	0.040	0.53	1.5	1.35	0.59	69	104	Am. Soc. C. E. Trans. Vol. 40, p. 471. Wooden stave pipe, 72½-inch diameter. With easy bends and curves. Pioneer Electric Power Company, Ogden, Utah.
367	0.025	0.54	1.5	0.83	0.59	88	104	
368	0.055	0.67	1.5	1.21	0.58	73	105	
369	0.020	0.83	1.5	0.29	0.58	149	105	
370	0.095	1.22	1.5	0.63	0.57	101	106	
371	0.100	1.23	1.5	0.64	0.57	100	106	
372	0.095	1.24	1.5	0.60	0.57	104	106	
373	0.106	1.33	1.5	0.57	0.57	106	106	
374	0.111	1.48	1.5	0.49	0.56	115	107	
375	0.181	1.88	1.5	0.50	0.55	113	107	
376	0.211	1.98	1.5	0.52	0.55	111	108	
377	0.211	2.14	1.5	0.45	0.54	119	108	
378	0.235	2.22	1.5	0.47	0.54	117	108	
379	0.300	2.54	1.5	0.45	0.54	119	109	
380	0.311	2.69	1.5	0.45	0.53	119	109	
381	0.361	2.78	1.5	0.45	0.53	119	109	
382	0.452	3.15	1.5	0.44	0.53	121	110	
383	0.517	3.45	1.5	0.42	0.52	124	111	
384	0.566	3.57	1.5	0.43	0.52	122	111	
385	0.501	3.62	1.5	0.38	0.52	130	111	
386	0.542	3.63	1.5	0.40	0.52	126	111	
387	0.576	3.64	1.5	0.42	0.52	124	111	

TABLE OF EXPERIMENTS FOR COMPARISON—Continued

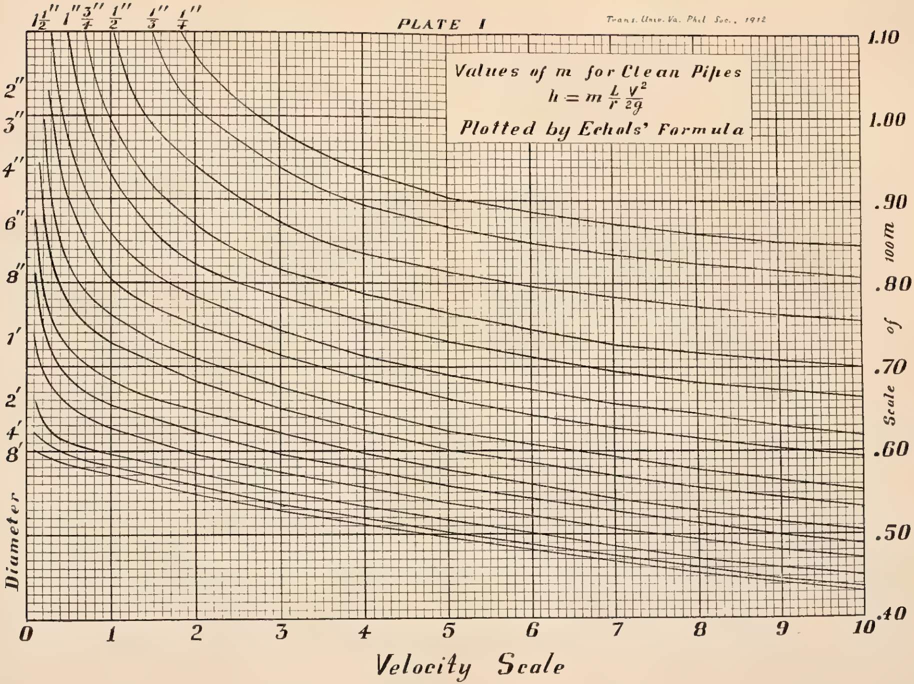
NO.	$\frac{1000}{s}$	V	r	$\frac{100}{m_0}$	$\frac{100}{m}$	C_0	C	AUTHORITY
388	0.032	1.0	2.16	0.47	0.58	117	106	Herschel, Riveted iron pipe. Holyoke flume. Cylinder joints, paint coating washed off, rather rusty. T. & R., p. 246.
389	0.084	1.5	2.16	0.50	0.57	113	107	
390	0.156	2.0	2.16	0.53	0.56	110	108	
391	0.245	2.5	2.16	0.54	0.56	109	109	
392	0.358	3.0	2.16	0.55	0.55	108	110	
393	0.499	3.5	2.16	0.56	0.53	107	111	
394	0.662	4.0	2.16	0.57	0.52	106	112	
395	0.847	4.5	2.16	0.58	0.51	106	113	

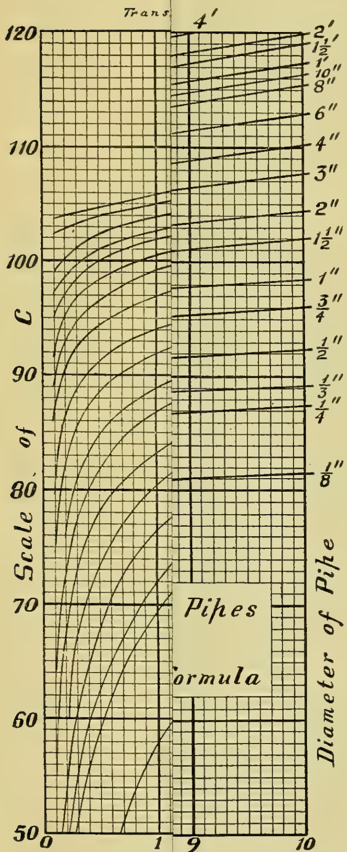


1½"
1"
¾"
½
2"
3"
4"
6"
8"
1'
2'
4'
8'
Diameter
1.10
1.00
.90
.80
.70
.60
.50
.40
Scale of 100m
0
10

Trans. Univ. Va. Phil. Soc., 1912

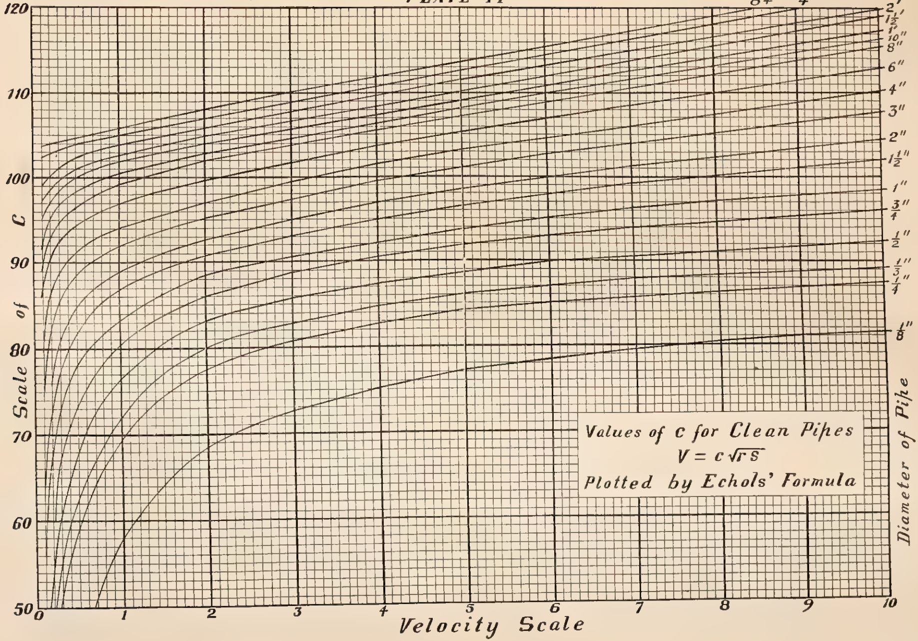
PLATE I



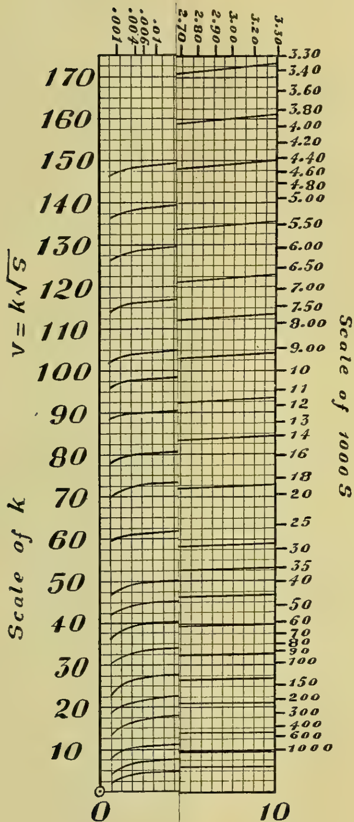


Trans.
4'
120
110
100
90
80
70
60
50
Scale of C
0
1
9
10
2'
1½'
1'
10"
8"
6"
4"
3"
2"
1½"
1"
¾"
½"
⅓"
¼"
⅛"
Diameter of Pipe
Pipes
ormula

PLATE II


Values of c for Clean Pipes
$V = c\sqrt{rs}$
Plotted by Echols' Formula
Scale of c
Velocity Scale
Diameter of Pipe
120
110
100
90
80
70
60
50
0
1
2
3
4
5
6
7
8
9
10
8'+
4'
2'
$1\frac{1}{2}$'
1'
10"
8"
6"
4"
3"
2"
$1\frac{1}{2}$"
1"
$\frac{3}{4}$"
$\frac{1}{2}$"
$\frac{1}{3}$"
$\frac{1}{4}$"
$\frac{1}{8}$"

912

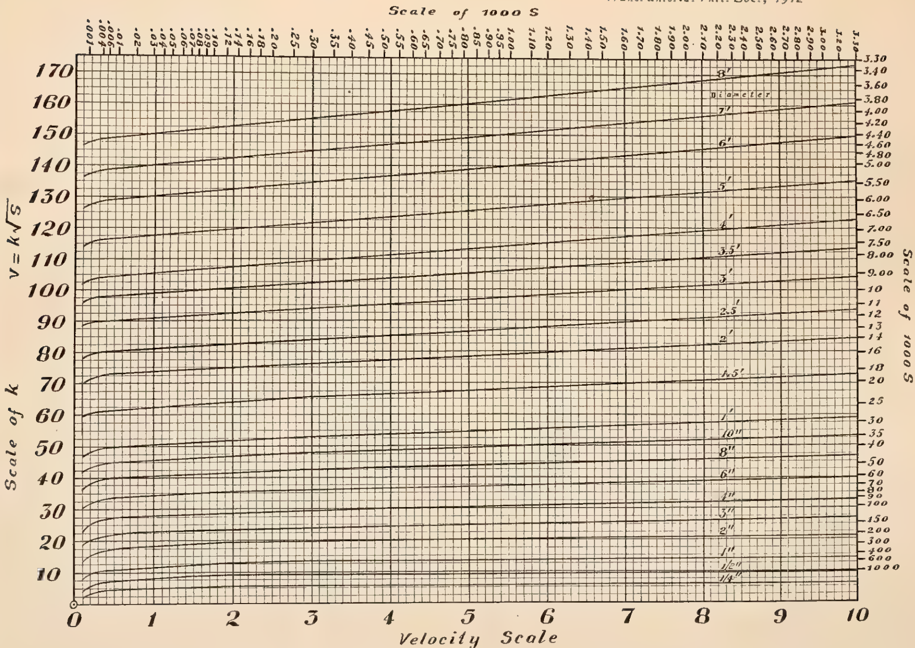


Scale of k
V = k√S
Scale of 1000 S
170
160
150
140
130
120
110
100
90
80
70
60
50
40
30
20
10
0
10
.001
.004
.006
.01
2.70
2.80
2.90
3.00
3.20
3.30
3.30
3.40
3.60
3.80
4.00
4.20
4.40
4.60
4.80
5.00
5.50
6.00
6.50
7.00
7.50
8.00
9.00
10
11
12
13
14
16
18
20
25
30
35
40
50
60
70
80
90
100
150
200
300
400
600
1000

Cormula.

PLATE III

Trans. Univ. Va. Phil. Soc., 1912



Corresponding Values of V, r, S and k for Clean Pipes Plotted by Echols' Formula.

UNIVERSITY OF VIRGINIA PUBLICATIONS

BULLETIN

OF THE

PHILOSOPHICAL SOCIETY

Scientific Series, Vol. I, No. 11, pp. 267–292, July, 1912

Zirconiferous Sandstone near Ashland, Virginia, with a Summary of the Properties, Occurrence, and Uses of Zircon in General

BY

THOMAS L. WATSON AND FRANK L. HESS

UNIVERSITY OF VIRGINIA
Charlottesville, Virginia, U. S. A.

UNIVERSITY OF VIRGINIA PUBLICATIONS

The Proceedings and Transactions of the Philosophical Society of the University of Virginia are published in the form of Bulletins and offered in exchange for the publications of learned societies, institutions, universities and libraries. The Bulletins are issued at irregular intervals in the form of separate papers. These are to be made up into volumes, consecutively paged, of about 400 pages each, for each of the Scientific, Humanistic and Medical Sections. Separate numbers may be purchased from the Distribution Committee, to whom remittances should be made. Communications should be addressed to Dr. Thomas L. Watson, University, Virginia.

Thus far have been published the following papers.

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UNIVERSITY OF VIRGINIA PUBLICATIONS
BULLETIN OF THE PHILOSOPHICAL SOCIETY
SCIENTIFIC SECTION

Vol. I, No. 11, pp. 267-292 July, 1912

ZIRCONIFEROUS SANDSTONE NEAR ASHLAND, VIRGINIA,* WITH A SUMMARY OF THE PROPERTIES, OCCURRENCE, AND USES OF ZIRCON IN GENERAL.†

BY

THOMAS L. WATSON AND FRANK L. HESS.

PREFATORY NOTE.

This paper, as its title indicates, is separable into two parts. Part I describes the discovery of interesting and important deposits of zirconiferous sandstone in the Coastal Plain province along its western margin west of Ashland, Virginia, and points out the probability of finding other like deposits in similar position in the same province. It records, so far as the writers know, the first reported occurrence of zircon in the Atlantic Coastal Plain, which, apart from its scientific interest, probably indicates an easily accessible commercial source of zircon. Part II assembles in systematic form the leading facts recorded in widely scattered literature concerning the properties, occurrence, and uses of zircon.

INTRODUCTION.

In 1910 Mr. August Meyer, of Richmond, Virginia, submitted to one of the writers a specimen of rock obtained about three miles west of Ashland, which was thought to contain rutile. It was a fine-grained friable dark reddish-brown rock, in which grains of ilmenite or some similar black mineral were distinctly visible. The color of the other grains was apparently similar to that of the rutile found 10 to 15 miles farther southwest,

* *Bull. 530-P, U. S. Geol. Survey*, 1912.

† Read before the Scientific Section, May, 1912.

in Hanover and Goochland counties, and under a hand lens no difference in appearance could be distinguished. As the rutile of these counties occurs with a very black ilmenite, it was thought that the specimen might possibly be a fine-grained mass of titanium minerals. Microscopic examination of a thin section, however, showed the rock to be a sandstone composed of very small grains of ilmenite and zircon (zirconium silicate, $ZrSiO_4$), together with a few grains of other minerals, chiefly quartz and silicates, cemented with limonite.

In June, 1911, the writers, in company with Mr. Meyer, visited the locality from which the latter obtained the original specimen, on the farm of Mr. F. B. Sheldon, 3 miles west of Ashland, Hanover County, and about 20 miles north of Richmond.

GENERAL GEOLOGY OF THE AREA.

The area of zirconiferous sandstone forms a part of the western edge of the Coastal Plain, near and along the overlap of the sediments upon the older crystalline rocks of the Piedmont Plateau (see map, fig. 1). Along this edge (the "fall-line") the surface is somewhat roughened from erosion, but to the east it becomes more gently rolling and is essentially flat and featureless. The area lies on the south side of South Anna River, but within its drainage basin and only a short distance southwest of its confluence with the North Anna to form Pamunkey River.

The sandstone outcrops along a low ridge having gently sloping sides and a general direction of N.20°E. At the point where the sandstone seems to be most abundant and perhaps richest in zircon the ridge marks the western edge of the Calvert formation, the lowest formation of the Chesapeake group (Miocene). Within this area and for some distance north and as far south as a point 25 miles north of Petersburg the Calvert formation transgresses the underlying older Coastal Plain sedimentary formations, and its western margin rests upon the crystalline rocks of the Piedmont Plateau.* The Calvert formation in Virginia is about 200 feet thick and consists chiefly of sands, clays, marls, and diatomaceous earth, fine-grained sands being predominant. Diatomaceous earth has not been identified in the Ashland area.

Extending westward from the foot of the west slope of the low ridge mentioned above are the crystalline rocks of the Piedmont Plateau, chiefly granites and gneisses, most of which are of pre-Cambrian age. The contact between the sedimentary formations of the Coastal Plain and the crys-

* *Virginia Geol. Survey, Bull. No. IV,* 1912, p. 126 et seq.

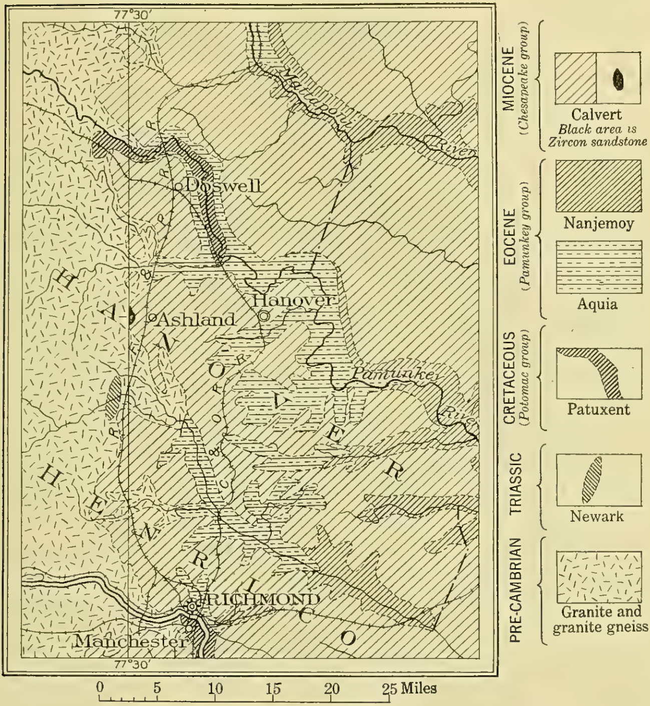

FIG. 1. GEOLOGICAL MAP OF A PART OF MIDDLE EASTERN VIRGINIA.

Showing location of zirconiferous sandstone area west of Ashland, Hanover County. (Geological map of Virginia, Virginia Geological Survey, 1911).

talline rocks of the Piedmont Plateau extends across the State in roughly a general north-south line and in position nearly coincides with the meridian 78°30′.*

In the southern part of the State the Calvert formation is overlain by the St. Mary's formation (middle Miocene), and along the western edge the St. Mary's transgresses the Calvert and rests on the crystalline rocks of the Piedmont Plateau.

DISTRIBUTION AND OCCURRENCE OF THE SANDSTONE.

In the Ashland area the sandstone does not outcrop in a continuous bed. It was seen only in the form of irregular flat fragments lying loose upon the surface. The fragments are of the same reddish-brown to yellow color as the specimen submitted by Mr. Meyer. In size the fragments range from those as large as a man's fist to some measuring 2 feet long, 2 feet broad, and 6 inches thick. There is as much variation in texture as in size, and the rock accordingly ranges from a typical fine-grained sandstone to a typical moderately coarse conglomerate, with intermediate gradations. Much of it is very fine-grained, showing little visible quartz. Other pieces are of varying degrees of coarseness, some containing quartz and quartzite pebbles 2 inches in diameter. Some pieces show cross-bedded structure.

The largest number of the sandstone fragments were seen on a small mound 150 yards southwest of Mr. Sheldon's house, and scattered fragments can be found both to the north and the south for a distance of half a mile. On Mr. J. B. Davis's farm, which adjoins the Sheldon farm on the north, there are many pieces of the sandstone, though most of them are smaller. However, many of the pieces, especially those found farther north, are of lighter color and lower specific gravity than the fragments from the Sheldon farm, though one of the richest specimens collected was from the line between the Thomas Kies and John Boschen farms, half a mile north of the Sheldon farm. The specific gravity is of value in field examination, for specimens having low specific gravity show only a few grains of zircon, whereas those having high specific gravity carry a large percentage of the mineral.

It is probable that the hard lumps of sandstone represent the local cementation of a sandy bed which, in most places, is soft or but slightly consolidated, a characteristic of the Chesapeake group (Miocene). Partly or wholly indurated sands, yielding somewhat highly ferruginous crusts and

* See the geological map of Virginia, Virginia Geological Survey, Charlottesville, 1911.

beds of sandstone, are common in the formations of the Virginia Coastal Plain near its western margin. So far as the authors are aware these ferruginous sandstones have been generally regarded as composed chiefly of quartz grains cemented by iron oxide. At no point beyond the Ashland area, so far as known, have they been tested for zircon or other uncommon heavy minerals.

At the home of Mr. Benjamin Wright, three-eights of a mile southwest of Mr. Sheldon's house, a highly zirconiferous and but slightly consolidated sand bed was cut in the lower part of a well 14 feet deep. This bed is probably the same one from which the indurated or hardened fragments of zirconiferous sandstone have come. Perfectly rounded water-worn quartz and quartzite pebbles, mostly quartz, up to 3 inches in diameter and usually white in color, were taken from this well at a depth of 14 feet. None of the zirconiferous material was found south of Mr. Wright's well, and decomposed granite is exposed in a road 200 yards southwest of his house.

A hundred yards northwest of Mr. Sheldon's house a bed of zirconiferous sand, similar to that cut in the Wright well, was exposed in a shallow prospect hole. The zirconiferous sand was 18 inches thick and was underlain by clay and covered by a few inches of soil.

From the appearance of the float and the sand cut in the prospect hole, the zirconiferous bed is thought to be probably not more than 2 to 3 feet thick. The data at hand indicate that it is probably a narrow lens three-eighths of a mile long and of unknown but probably of less width.

TESTS.

The zircon was separated from six lump samples weighing from 50 to 100 grains each as follows: The lumps were first treated with hydrochloric acid to dissolve the cement of limonite. In two specimens small lumps resisted dissolution and were treated with aqua regia on a steam bath for two days, which resulted in dissolving the cement and disintegrating the sand grains. After washing by decantation the sand was digested with a mixture of sulphuric and hydrochloric acids to remove ilmenite and quartz and then washed. The specimens thus treated yielded zircon as follows:

Zircon obtained from sandstone near Ashland, Virginia.

LOCALITY	GROSS WEIGHT OF SAMPLE	ZIRCON	
	Grams	*Grams*	*Per cent*
Specimen from low hill, F. B. Sheldon's farm.	50	14,955	29.9
Specimen from low hill, F. B. Sheldon's farm.	100	25,375	25.4
Specimen from low hill, F. B. Sheldon's farm.	52	6,280	12.1
Specimen from 100 yards northwest of F. B. Sheldon's house	100	15,890	15.9
Specimen from Wright's well	52	6,815	13.1
Specimen from top of hill on line between Thomas Kies' and John Boschen's land	100	27,230	27.2
Total		96,545	

Accessory heavy minerals in the form of impurities, such as cyanite, garnet, and staurolite, could not be separated from the zircon by the method used, and the results given in the table above are perhaps 2 to 3 per cent too high, though certainly not more. Owing to possible losses through the severe treatment during separation and to the loss of fine zircon in decanting, the tests are as likely to show less as more than the quantity present. The results are not, of course, to be regarded as exact, but the method of selecting random specimens from float rock would not warrant more accurate determinations.

It is not thought that the method used in separating the material introduced appreciable errors, as a blank test run on finely pulverized zircon by treating it with a mixture of sulphuric and hydrofluoric acids, showed at the end of three days no trace of zircon in solution.

The zircon crystals in the material are minute, averaging less than 0.5 mm. in diameter. Out of about 96 grams of zircon separated, a small quantity was caught on a sieve of 60 meshes to the linear inch; possibly 1 per cent would not pass through a sieve of 80-mesh; nearly 17 per cent (16.23 grams) passed through an 80-mesh and was caught on a 100-mesh sieve; 77 per cent (74.15 grams) passed through a 100-mesh sieve and was caught on a 150-mesh sieve; and more than 2 per cent (2.3 grams) passed through a 150-mesh sieve. Most of the accessory minerals (impurities) can be caught on an 80-mesh sieve.

CHARACTER OF THE SEPARATED ZIRCON CRYSTALS.

The zircon crystals, as separated above, are mostly of short, stout form, though they include a smaller number of elongated forms, possibly one and one-half times as long as thick. In mass they are pinkish or pinkish brown, but on heating to redness they become colorless. Under the microscope individual crystals are pink or yellow, but much the largest number are colorless. The crystals in most specimens are very much worn, but the crystals in the specimens from the prospect hole northwest of Mr. Sheldon's house show beautiful crystal form. Though nearly all of the zircon is undoubtedly worn, the wear in general may be in part apparent only, as small zircon crystals formed in place very commonly have outlines that do not show good faces or angles. The difference in the amount of wear of the particles which were caught on a 100-mesh sieve and of those which passed through a 150-mesh sieve is striking (see Pl. I, figs. 1 and 2). The greater mass of the larger crystals small as they are seems sufficient to cause much more fracturing from the force of impact when thrown around by waves and currents.

ASSOCIATED MINERALS.

Associated with the zircon are quartz and a variety of heavy minerals, including garnet (?), ilmenite, rutile, staurolite, cyanite, and an isotropic green mineral which has not been definitely determined but which may be pleonaste or hercynite. Occasional feldspar and pyrite were noted in several thin sections of the rock. As stated above, these are all cemented with limonite, possibly in part siliceous.

Ilmenite is the most abundant mineral in the rich pieces and its grains are of about the same size as those of zircon. The quartz and cyanite grains are generally several times as large. In places the fine-grained zircon and ilmenite surround quartz pebbles an inch long with the other dimensions somewhat smaller.

No magnetite was found in the material.

MICROSCOPICAL PETROGRAPHY.

The petrography of the rock is simple, but the general character of the minerals and their relations to one another and to the cement are more definitely established by microscopic than by megascopic study. Considered as to mineral composition the ten thin sections of the rock studied may be divided into two groups, (1) zircon-ilmenite sandstone and (2) quartz sandstone. The rounding of the ilmenite and zircon grains is pronounced, but the quartz sand is remarkably angular* (see Pls. I and

* This is in accordance with the investigation of Mackie on the rounding of sand grains, who observed that grains of zircon were rounded more readily than those of quartz, due possibly to the greater density of the zircon. See Mackie, Wm., *Trans. Edinburgh Geol. Soc.*, 1897, vol. vii, pp. 298–311.

II). Both are cemented more or less firmly by oxide of iron, chiefly limonite and probably a less hydrous oxide of reddish color, possibly göthite or hematite.

Of the minerals present in the sandstone, zircon, ilmenite, and quartz are the three most abundant, and are described below in the order named. Occasional grains of an unstriated feldspar were noted in one or two of the quartz-rich sections, and red- to yellowish-brown rutile in partially rounded grains of variable size is sometimes present, always in association with ilmenite. Ferromagnesian minerals are entirely absent.

Zircon.

In the thin sections the zircon is mostly colorless, though occasional light yellow and pinkish crystals were observed, and is readily identified by its high refraction and double refraction. The grains usually show rounded outline and many of them are nearly spherical. They range from approximately equidimensional to elongate forms, and the angles or corners of those that exhibit squarish to rectangular cross-sections usually show more or less rounding (Pls. I and II). Crystal outline is frequently observed but as a rule it is modified by rounding from wear. Zircon grains separated from the rock and examined under the microscope usually show rounding from wear and rather dull luster (Pl. I, figs. 1 and 2). The most perfectly rounded grains are apt to exhibit the least luster. The zircon grains average from 0.2 to 0.3 mm. in diameter. Some of the larger elongated grains measure as much as 1.1 mm. in the direction of elongation. Some grains show cleavage and many indeterminable inclusions.

A fairly noticeable feature in many of the zircon grains is an apparent irregular, thin, cloudy and light-colored peripheral zone or border, which appears isotropic or but feebly double refracting. This border probably represents the pitted surface made by pounding against other fragments and possibly to some alteration from hydration.

Ilmenite.

Ilmenite is most abundant in the zircon-rich thin sections. It exceeds zircon in amount and is least in quantity in the quartz-rich sections, and almost absent from some. It is remarkably fresh, in grains of about the same size as the zircon grains, probably most of them a fraction larger, and of irregular though somewhat rounded outline.

Quartz.

Quartz is present in every thin section but varies greatly in amount, from occasional grains in the zircon-ilmenite-rich rock to the dominant and vastly the most abundant mineral in the quartz-rich rock. It is likewise subject to much variation in size and shape of grain. The grains generally range between 0.2 and 1 mm. in diameter, though smaller and larger ones were noted, and in contrast to zircon and ilmenite are mostly angular in outline. Well rounded grains are not numerous.

The quartz grains are of granitic character and some contain abundant liquid and solid inclusions. Many of them show pronounced strain shadows as the result of dynamic forces to which the original rock from which they were derived was subjected. The quartz grains in the same thin section will usually average larger in size and more angular in outline than the zircon. The general character of the quartz grains is shown in Plate II, figure 2.

Cement.

In hand specimens the cement is a decided reddish-brown color. In thin sections it is opaque and generally brown in reflected light, and occasionally transparent and deep red in transmitted light. It is sharply differentiated from the mineral grains, which are remarkably fresh. No gradation from the iron-bearing mineral grains into the cement was observed, such as would be expected if the cement were derived by alteration of those iron-bearing minerals present in the rock.

GENESIS.

The zircon and ilmenite concentration evidently represents an old beach segregation along but within the western margin of the Miocene sediments of the Coastal Plain, of probable Calvert age, and is similar to the black-sand beaches of New Jersey, California, Oregon, New Zealand, New South Wales, and numerous other coasts, and to the gold-bearing garnet (so-called "ruby") sands of the beaches at Nome, Alaska (see fig. 2).

The zircon and other heavy minerals resistant to atmospheric agencies were derived by weathering processes from the crystalline rocks, chiefly granites and gneisses, of the Piedmont Plateau, which extend westward from the Coastal Plain contact. These formed the country rock of the shore, and the zircon and associated minerals derived from them by weathering were accumulated by waters near the mouth of a small stream or behind a sheltered point, while the quartz sand was largely worn and carried away by the currents of the sea.

Zircon is an almost constant minor accessory mineral in the crystalline rocks, especially granites and gneisses, of this old shore and its extension westward, and in places it occurs in large masses. Thin sections of granites occurring immediately west of Ashland and at other places in the Piedmont Plateau of Virginia show the presence of zircon, chiefly as inclusions in the quartz and feldspar. Near Goulden post-office, 10 to 15 miles southwest of the Ashland area, pieces of zircon 3 inches in diameter weathered out of pegmatite dikes have been noted on the surface. Massive zircon without crystal outline, measuring 4 by 6 inches, has been observed in the pegmatites of Amelia County, Virginia. Similar dikes occur in the gneiss-granite complex of the Piedmont Plateau, forming the old shore-line which extends entirely across Virginia from Maryland into North Carolina, roughly coinciding with the meridian of 77° 30″. The zircon in the sandstone was not derived, however, from the pegmatites in which it occurs in comparatively large masses, but from the granites and gneisses which carry it in innumer-

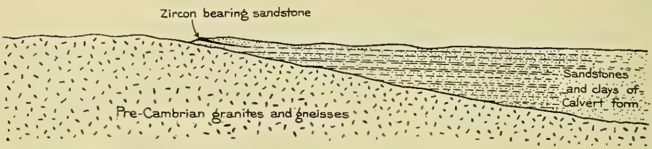

Fig. 2. Generalized East-West Section Across the Fall-Line near Ashland, Virginia.
Illustrating the occurrence of zircon-bearing sandstone.

able very small crystals. It seems probable that similar zircon-rich sandstones may occur at numerous points along this old shore-line. Many zircon-bearing deposits may be covered by later sediments and some may have been removed by erosion, but it is probable that others, which may be richer or poorer, will be discovered along the contact of the granite and gneiss of the Piedmont Plateau with the overlying sediments of the Coastal Plain.

It is probable that some magnetite was present with the ilmenite, and glauconite is abundant at places in the Calvert formation. The alteration of either of these minerals might produce limonite, which forms the cementing material. An occasional pyrite grain was observed in one or two thin sections, and some hand specimens of the rock exhibit cavities which suggest the removal by decay of some previously existing mineral. From microscopic study of thin sections of the rock, it seems more probable, however, that the principal source of the cement was chemical precipitation from iron-bearing waters that percolated or filtered through the sand deposit.

ECONOMIC ASPECTS.

The uses enumerated on pages 291–2 of this paper are largely suggested rather than actual and their practicability is mostly dependent on the cheapness of the zirconia and the quantity available. Böhm* states that large quantities of native zirconia (zirconium oxide) known as baddeleyite are found near São Paulo, Brazil, and that much has been shipped to Germany. This material at the time he wrote, was being furnished at the following prices:

Composition and prices of baddeleyite.

DESIGNATION	ZrO_2	Fe_2O_3	SiO_2	PRICE PER TON (2000 pounds)
	Per cent	*Per cent*	*Per cent*	
Zircon–S–Erz	90–92	1.	8	$151
Zircon–Z–Erz	90–92	7	1	155
Zircon–NS–Erz	98	0.8	1†	215

† Remainder H_2O.

The mineral quoted is in the form of oxide and for most purposes would be more desirable than zircon, which would have to be reduced to the oxide, and should sufficient native oxide be found to supply demands, competition would be difficult for zircon. For ferrozirconium or zirconium carbide the zircon could possibly be used without reduction to the oxide.

Should the demand for zircon and further testing of the Ashland deposit warrant exploitation, operations could be carried on with comparative ease. The rock crushes easily; the zircon and associated heavy minerals could be separated from the quartz by shaking tables, and the ilmenite could be picked out by a magnetic separator.

* Böhm, C. R., Die Technische Verwendung der Zirconerde, *Chem. Zeitung*, Jahr 35, November 14, 1911, pp. 1261–1262.

GENERAL REMARKS ON THE PROPERTIES, OCCURRENCE, AND USES OF ZIRCON.

Elementary Zirconium.

Properties and distribution. The element zirconium derives its name from the mineral zircon, in which Klaproth* discovered the new earth zirconia (ZrO_2) in 1789. It is somewhat closely allied to titanium, is quadrivalent, has an atomic weight of 90.6, and when crystallized has a density of 4.25 and is hard enough to scratch rubies.† The crystallized form of the element‡ somewhat resembles antimony in appearance, does not burn in the air, and is soluble in hot concentrated acids. Amorphous zirconium is a black powder which burns to the oxide when heated in air and is only slightly attacked by acids.§

Zirconium is rather widely diffused in small quantity in igneous rocks, usually in the form of the silicate, zircon. It is also found in minute quantity in certain sedimentary and metamorphic rocks. Professor Clarke¶ gives the average of zirconium in 298 analyses of igneous rocks made in the laboratories of the United States Geological Survey as 0.025 per cent, equivalent to 0.03 per cent of ZrO_2, and estimates the amount of ZrO_2 in the average composition of the lithosphere to be 0.03 per cent. In the distribution of elementary substances in the earth's crust Vogt° estimates the amount of zirconium with seven other elements (P, Mn, S, Ba, Fl, N, and Cl) to range from 0.1 to 0.01 per cent.

Table of zirconium-bearing minerals.

The following zirconium-bearing minerals have been recognized and described. Excepting zircon and baddeleyite the minerals are rare and of scientific interest only. Baddeleyite is known in quantity only in São Paulo, Brazil.

* Roscoe and Schorlemmer, *Treatise on Chemistry,* 1900, vol. ii, p. 656; Ohley, J., *Analysis, Detection and Commercial Value of the Rare Metals,* 1907, p. 173; Browning, P. E., *Introduction to the Rarer Elements,* 1909, p. 75.

† Roscoe and Schorlemmer, *Op. cit.*, p. 657.

‡ Remsen, I., *Inorganic Chemistry,* 1890, p. 413.

§ Browning, P. E., *Op. cit.*, p. 77.

¶ Clarke, F. W., The Data of Geochemistry, *Bull. 491,* U. S. Geol. Survey, 2nd edition, 1911, pp. 27, 32.

° Vogt, J. H. L., *Trans. Amer. Inst. Mng. Engrs.*, 1892, vol. xxxi, p. 128.

*Zirconium-bearing minerals.**

MINERAL	COMPOSITION	INCLOSING ROCK	ZrO_2 Per cent
Oxide:			
Baddeleyite (Brazilite) ...	ZrO_2	Igneous rocks deficient in silica, and in gravels derived from them.	100‡
Zirconates:			
Zirkelite	$(Ca,Fe)O.2(Zr,Ti,Th)O_2$	Magnetite-pyroxenite (jacupirangite).	52.89†
Polymignite ...	$5RTiO_3.5ZrO_3.R(Cb,Ta)_2O_6$	Elaeolite syenite.	29.71†
Silicates:			
Zircon‡	$ZrSiO_4$	Variable, described below.	67.2‡
Cyrtolite	Some cyrtolite is probably hydrated zircon.	Granite, pegmatite.	66.93†
Catapleiite	$H_2(Na_2,Ca)(Zr(OH)_2)(SiO_3)_3$	Elaeolite syenite.	28.8‡
Elpidite	$H_6Na_2ZrSi_6O_{18}$	Elaeolite syenite (?).	20.48†
Eudialyte (Eucolite)	$Na_{13}(Ca,Fe)_6Cl(Si,Zr)_{20}O_{52}$	Elaeolite syenite.	16.88†
Hainite	Related to låvenite, wohlerite, etc.	Phonolite.	Unknown.
Hjörtdahlite ...	$4Ca(Si,Zr)O_3.Na_2ZrO_2F_2$	Elaeolite syenite.	21.48†
Låvenite	$(Na_4,Ca_2,Mn_2,Zr)((Si,Zr)O_3)_2$	Elaeolite- or augite syenite.	31.65†
Lorenzenite ...	$Na_2Si_2(Ti,Zr)_2O_9$	Pegmatite.	11.92
Rosenbuschite (Zircon pectolith)	$Na_2Ca_3((Si,Zr,Ti)O_3)_4$	Elaeolite syenite.	20.10†
Wöhlerite	$(Na_2,Ca)(Si,Zr)O_3.RCb_2O_6$	Zircon syenite.	22.72†

In addition to the tabular list of minerals above, traces of zirconium are reported in the rare minerals melanocerite, caryocerite, and tritonite, silicates of the cerium and yttrium metals, which occur in nepheline syenites in Norway.§ Aeschynite may contain zirconium. Homilite, variety erd-

* Compiled from Dana, E. S., *A System of Mineralogy;* Hintze, C., *Handbuch der Mineralogie;* and Iddings, J. P., *Igneous Rocks*, 1910, vol. i.

† Maximum percentage given in analyses recorded by Dana, E. S., *A System of Mineralogy*, 1900, 6th edition.

‡ Theoretical percentage. The altered forms of zircon, auerbachite, malacon, oerstedite, tachyaphaltite, and calyptolite are not listed as separate species.

§ Iddings, J. P., *Igneous Rocks*, 1910, vol. i, p. 60.

mannite (michaelsonite, 5.44 per cent ZrO_2) contains 3.47 per cent ZrO_2; and sipylite, a niobate of the rare earths contains 2.09 per cent of ZrO_2. Zirconia is found in the titanosilicates in the following amounts: Astrophyllite, 1.0 to 5.0 per cent; johnstrupite, 2.84 per cent; mosandrite, 7.43 per cent; annerodite, 1.97 per cent. A uraninite from Colorado gave 7.59 per cent ZrO_2;* and sometimes a trace is found in xenotime.† Most of the minerals named are rare and occur in nepheline syenites of Norway chiefly, and in less quantity in those of Greenland and Finland.

Zircon.‡

Of the numerous zircon-bearing minerals enumerated above zircon is the most widely distributed and occurs in greatest quantity. Baddeleyite, the native oxide (ZrO_2) found in Montana,§ Sweden, Ceylon, and Brazil, is reported to occur in quantity at São Paulo, Brazil, and since it is already in the form of the oxide it will probably prove more desirable for most purposes than zircon.

Zircon, an orthosilicate of zirconium, corresponds to the formula $ZrSiO_4$ and when pure is composed of 67.2 per cent of zirconia (ZrO_2) and 32.8 per cent silica (SiO_2). Iron is frequently present in small quantity. Zircon crystallizes in the tetragonal system, commonly in square prisms, often elongated, with pyramidal terminations.¶ The crystals sometimes form geniculated twins like those of rutile and cassiterite. The mineral also occurs in formless grains.

Zircon has a hardness of 7.5 and a density of 4.7. It is brittle, has imperfect cleavage, conchoidal fracture, and adamantine luster. It is commonly opaque, and usually some shade of brown, but sometimes colorless, clear yellow, gray, green, red, etc. Molecular weight 183; molecular volume 38.7. It is infusible and insoluble in acids except when acted on in powder with concentrated sulphuric acid.

Two principal varieties of the mineral are usually recognized: (a) ordinary, and (b) gem or hyacinth. The first of these is the commoner and

* Hillebrand, W. F., On the Occurrence of Nitrogen in Uraninite and on the Composition of Uraninite in General, *Bull. 78, U. S. Geol. Survey*, 1891, p. 65.

† Iddings, J. P., *Op. cit.*, p. 60.

‡ Compiled from various authorities.

§ Rogers, A. F., Baddeleyite from Montana, *Amer. Jour. Sci.*, 4th ser., vol. 33, January, 1912, pp. 54–56.

¶ The crystallography of the mineral is simple and numerous studies have been made by mineralogists of zircon crystals in a variety of associations and occurrences from localities in this and foreign countries.

more abundant, and occurs under a variety of conditions and associations. The mineral is one of the most resistant to ordinary atmospheric agents and alters, so far as known, chiefly by hydration when it becomes isotropic and of lower specific gravity. It then has a specific gravity of 3.905 and is known as malacon. It sometimes alters through loss of silica.

Zircon has been produced synthetically by several different methods, as follows: By heating the oxide, zirconia (a) in a current of silicon fluoride, (b) with quartz in the same gas, (c) a mixture of silica, zirconia, and lithium molybdate heated to 800°, and (d) by heating gelatinous silica and gelatinous zirconia under pressure to near redness. Clarke* remarks that Deville's methods of synthesis (a) and (b) above, "indicate a possible pneumatolitic origin for zircon in some instances; the other processes seem to be unrelated to the ordinary occurrences of the mineral."

Zircon is a product (pyrogenetic mineral) of igneous rocks, of regional metamorphism,† of contact metamorphism as it is reported found sometimes in contact limestones,‡ and of pegmatite veins or dikes.§ Palache and Warren¶ have recently described zircon-quartz intergrowths of the Quincy, Mass., pegmatites, which are referred by the authors to pneumatolitic processes.

Alteration.

Very little exact information on the alteration of zircon has been published. This is probably due chiefly to the fact that zircon is one of the most durable and resistant minerals to ordinary atmospheric agents, and to the added fact that it generally occurs in small crystals and grains as a very minor constituent of rocks.

According to Dana° zircon assumes water, becoming isotropic and amorphous, accompanied ultimately by the loss of silica and the addition of iron oxides and other impurities derived from infiltrating waters. Auerbachite, malacon, oerstedite, tachyaphaltite, calyptolite, and cyrtolite

* Clarke, F. W., *Op. cit.*, 1911, p. 336.

† Emmons, W. H., A Genetic Classification of Minerals, *Economic Geology*, 1908, vol. 3, p. 622.

‡ Clarke, F. W., *Op. cit.*, 1911, p. 678.

§ Probably the most noted example is that in Henderson County, N. C. See Pratt, J. H., *Mineral Resources of the United States for Calendar Year 1903*, pp. 1168–1169.

¶ Palache, C. and Warren, C. H., The Pegmatites of the Riebeckite-Aegirite Granite of Quincy, Mass., U. S. A.; Their Structure, Minerals, and Origin, *Proc. Amer. Acad. Arts and Sciences*, 1911, vol. xlvii, p. 131.

° Dana, E. S., *A System of Mineralogy*, 1900, 6th edition, p. 486.

are listed by Dana as probably altered zircon, and 10 analyses of these several altered forms of the mineral are quoted which show the percentage of water to range* from 0.95 to 9.53. Clarke† also states that the only alteration of zircon that has been described is that of hydration, producing hydrous zircon (malacon), which is tetragonal in crystal form and has a specific gravity of 3.905. According to Van Hise‡ the change from hydration involves an increase in volume of 24.05 per cent, and the reaction is written by him as follows:

$$3ZrSiO_4 + H_2O = H_2Zr_3Si_3O_{13}.$$

Kemp§ remarks that the zircons in coarse pegmatites near Port Henry, New York, "have in instances undergone alteration to a greenish outside crust, much more brittle than the fresh mineral."

In 1898 Hussak¶ concluded from his study of the zircon oxide (not baddeleyite) favas° composed of almost pure zirconia (over 97 per cent) and found in large quantities in the gravels of the augite syenite region of the Serra de Caldas Minas Geraes, Brazil, were of secondary origin, derived probably from zircon by loss of silica. After describing the microstructure of the favas Hussak says:

> The whole structure shows that these favas are a secondary product of decomposition, and so far as concerns the occurrence of the great zircon crystals, it is most probable that the mother mineral was zircon which through the loss of SiO_2 had been converted into pure zirconia, as is partly the case in malaconitization (Malaconitisirung). At least, this is a more probable hypothesis than to assume the presence of the very rare silico-zirconiates (Silicozirkoniaten) in the augite syenite mass found on the spot.
>
> As the water content of the zircon favas is very small, they cannot be designated as hydrated (hydratisirte) zirconia, but the product of metamorphosis may at first coincide with the baddeleyite.

Similar zircon favas have been reported from the diamond sands of different localities in Brazil.

Later Hussak and Reitinger⊙ from a very careful chemical investiga-

* Dana, E. S., *Op. cit.*, p. 487.

† Clarke, F. W., The Constitution of the Silicates, *Bull. 125, U. S. Geol. Survey*, 1895, p. 75.

‡ Van Hise, C. R., A Treatise on Metamorphism, *Mon. 47, U. S. Geol. Survey*, 1904, p. 315.

§ Kemp, J. F., *Trans. Amer. Inst. Min. Engrs.*, 1898, vol. 27, p. 200.

¶ Hussak, E., *Tschermak's Min. u. Petr. Mittheil.*, 1898–99, vol. xviii, pp. 339–341.

° A Brazilian word meaning bean-shaped pebbles.

⊙ Hussak, E. and Reitinger, J., *Zeitschr. f. Kryst. u. Min.*, vol. xxxvii, pp. 566–574.

tion of the zircon favas confirmed the secondary origin of the zircon oxide material. The results of the two varieties of zircon favas analyzed may be expressed as follows:

Analyses of zircon favas from Brazil (Hussak and Reitinger).

	BROWN SAMPLE (specific gravity 4.850)	DARK GRAY SAMPLE (specific gravity 5.245)
ZrO_2	81.75	93.18
SiO_2	15.49	1.94
H_2O	0.63	0.47
Fe_2O_3	1.06	2.76
Al_2O_3	0.85	0.64
TiO_2	0.50	0.61
Total	100.28	99.60

Commenting on these results the authors say:

> This seems to confirm the supposition already advanced by Hussak that these favas are a product of the decomposition of the very numerous and large zircon crystals found in the disintegrated augite syenite of the before-mentioned Serra, although up to this time such a complete decomposition of zircon had not been observed.

They state, however, that it would be much simpler to consider the zircon favas as a decomposition product of zircon silicates, such as eucolite, eudialyte, lavenite, rosenbuschite, and wöhlerite, but that these minerals are not known to have been identified in the zircon-rich syenite of the region.

Trueman* notes that when igneous rocks containing zircon break down the zircon becomes lusterless and if subjected to wear from erosion the crystals frequently become rounded. In referring to the work by Trueman, the Winchell† brothers remark that the available data indicate that when zircon-bearing rocks are anamorphosed the zircon crystals on account of small size and durability are modified but slightly.

MODE OF OCCURRENCE.

General statement.

Classified genetically zircon forms as a product of (1) igneous rocks-crystallization from magma, (2) pegmatite dikes, (3) occasionally as a product of contact metamorphism, and (4) dynamo-regional metamorphism.

* Trueman, J. D., *Journ. of Geol.*, 1912, vol. xx, pp. 244–257; see also Derby, O. A., *Proc. Rochester Acad. Sci.*, 1891, vol. 1, pp. 198–203.

† Winchell, H. V. and A. N., *Economic Geology*, 1912, vol. vii, pp. 292, 294.

The mineral has been reported from Essex and Orange counties, New York,* as a constituent of quartz veins associated with pyroxene, scapolite, and titanite in the veins of Orange County; in small brown crystals with tourmaline in "granitic veins" which traverse gneiss on North River in St. Jerome, Terrebonne County, Canada;† of frequent occurrence in fine crystals in the apatite veins of Templeton and adjoining townships in Ottawa County, Quebec;† and in small crystals in a graphitic vein in the township of North Burgess, Lanark County.† Zircon is also an accessory mineral in many iron ore deposits, especially magnetite, and Hintze‡ quotes its reported occurrence in several meteorites.

In many of its occurrences, especially in some pegmatite masses, veins, and contact metamorphism, zircon may have been formed as a product of pneumatolitic processes, and Deveille's methods of synethsis given on page 281, indicate a possible pneumatolitic origin of the mineral in some cases.

Of the several occurrences listed above, (1) and (2) constitute the principal primary sources of zircon. The mineral is found in varying amounts in most classes of sedimentary and metamorphic rocks but, except in rare instances, its original source was from preëxisting igneous masses. Comparing the characteristics of zircons in igneous and secondary (sedimentary and metamorphic) rocks, Trueman§ following Derby¶ points out the use of zircon as a criterion for the identification of the origin of foliated rocks. In the anamorphism of igneous or sedimentary rocks many of the small and stable zircon crystals probably remain essentially unmodified.

Rock Associations.

Igneous rocks. Zircon is a common accessory constituent of nearly all classes of plutonic and volcanic igneous rocks. Its grains are generally of microscopic size though not uncommonly of macroscopic dimensions. It is an ordinary constituent of granites and syenites, and occurs in some diorites and occasionally in gabbros, among plutonic igneous rocks; and in

* Dana, E. S., *A System of Mineralogy,* 1900, 6th edition, p. 485.

† Hoffmann, G. Ch., Annotated List of the Minerals Occurring in Canada (reprinted from *Trans. Roy. Soc. of Canada,* 1889, vol. vii, sec. iii), *Geol. and Nat. Hist. Survey* of Canada, 1888–89, vol. iv, p. 66T.

‡ Hintze, C., *Handbuch der Mineralogie,* 1907, p. 1664.

§ Trueman, J. D., The Value of Certain Criteria for the Determination of the Origin of Foliated Crystalline Rocks, *Jour. Geol.,* 1912, vol. xx, pp. 228–258.

¶ Derby, O. A., On the Separation and Study of the Heavy Accessories of Rocks, *Proc. Rochester Acad. Sci.,* 1891, vol. i, pp. 198–206.

quartz porphyry, trachyte, phonolite, tephrite, dolerite, diabase, and basalt among volcanic igneous rocks. It has also been reported to be present in phonolite and basalt tuffs.*

Zircon is most common in the more alkalic rocks, rich in soda, such as the nepheline syenites, phonolites, tinguaites, and tephrites, and is rare in the basic rocks, especially those rich in lime, magnesia, and iron, but has been reported from a number of localities, chiefly foreign, in gabbro, diabase, basalt, and basalt tuff. It is one of the first minerals to crystallize from the cooling magma, but according to Murgoci† it has formed in riebeckite rocks during the entire period of consolidation of the magma.

The minute crystals of zircon usually present in many igneous rocks, chiefly as inclusions in the principal silicate minerals, and which afford distinct evidence of earlier crystallization from the magma were probably formed under high temperature conditions and possibly without the aid of mineralizers. Harker‡, on the other hand, remarks that when zircon occurs in relatively large crystals of later formation and in abundance, it is probably safe to assume the presence of mineralizing agents.

Zircon has been noted at a number of localities in certain types of igneous rocks in sufficient quantity to give varietal name to the rock. Three occurrences are worthy of note. In the well known augite syenite of Frederiksvaern, Southern Norway, zircon is so abundant as to have given rise to the name zircon syenite.§ Closely similar zircon-rich augite and nepheline syenite have been described by Hussak¶ and von Sachsen-Coburg from Brazil. In 1882 Wadsworth° announced the occurrence of a zircon-rich syenite about Salem harbor (Marblehead), Massachusetts, almost identical with the celebrated zircon syenite of Frederiksvaern,

* For the natural occurrences of zircon see Thürach, H., Ueber das Vorkommen Mikroskopischer Zirkone und Titan-Mineralien in den Gesteinen, *Verhandl. Phys.-Med. Gesell.*, Wurzburg, 1884, vol. xviii, no. 10, 82 pp.; Brögger, W. C., *Zeitschr. Kryst. f. Min.*, 1890, vol. xvi.; Michel-Levy, *Bull. Soc. Geol. de France*, 1883, iii ser., p. 281; Rosenbusch, H., Sulla presenza dello Zircone nelle Roccie, *Atti della R. Accademia della Scienze di Torino*, vol. xvi.

† Murgoci, C. M., On the Genesis of Riebeckite and Riebeckite Rocks, *Amer. Jour. Sci.*, 1905, vol. xx, p. 137.

‡ Harker, A., *The Natural History of Igneous Rocks*, 1909, p. 291.

§ Brögger. W. C., *Zeitschr. Kryst. f. Min.*, 1890, vol. xvi.

¶ Hussak, E., Mineral Notes from Brazil. Part III. *Min. u. Petrog. Mitth.*, 1898–99, vol. xviii, p. 339; see also Recent Occurrence of Zircon in Brazil (Caldas, Province of Minas Geraes) by von Sachsen-Coburg, *Min. u. Petrog. Mitth.*, 1888–89, vol. x, p. 453.

° Wadsworth, M. E., *Proc. Bost. Soc. Nat. Hist.*, 1882 (1883), vol. xxi, p. 406.

Norway. In 1890, Nason and Ferrier* published a brief account of a massive igneous rock, corresponding in composition to hornblende granite, from the Archaean highlands of New Jersey, in which zircon was a prominent constituent of the rock, amounting in some specimens to nearly 20 per cent, with a probable average of 5 per cent. The zircon crystals ranged in size up to 25 mm. in length, with an average of about 5 mm. Mention was made by Cook† in 1868 of a zircon-bearing gneiss at Trenton, New Jersey.

Sedimentary rocks. Thürach‡ found zircon in sands, sandstones, conglomerates, and carbonate rocks (limestone and dolomite) of different ages. He reported it present in every sandstone and in nearly every shale examined. Adams and Barlow§ report large rounded grains of zircon in thin scales of impure limestone in the township of Dudley, Canada, and at Warwick, New York, in limestone and scapolite.¶ Derby° has noted that zircon is essentially absent from argillaceous deposits, and Clarke's⊙ composite analysis of 78 shales shows no zirconium. Its density would probably cause it to settle with the sands and not be transported to the quieter waters in which shales are deposited.

Because of its resistance to atmospheric agencies zircon is a frequent associate in beds of sand and gravel, and in the residual decay derived from the weathering of crystalline rocks in which it originally occurred. In each of the Atlantic States containing areas of crystalline rocks it has been noted in association with many other heavy resistant minerals derived from the rocks by the usual processes of decay.

Numerous references in the literature are made to the abundant occurrence of zircon in the gold sands of the Piedmont region of the Southern Appalachians, especially of the Carolinas. It is a common constituent in the alluvial gold sands of the eastern and western United States, and of foreign countries; in the diamond sands of Brazil; in the monazite sands of the Carolinas; of Sweden, Norway, Australia, India, and the beaches of Brazil, where partial concentration from the action of tides and waves has taken place; in the gem sands of Ceylon; in the northwestern alluvial tin mines of Tasmania; and in the tin placers of the Malay Peninsula.

* Nason, F. L., and Ferrier, W. F., A Notice of Some Zircon Rocks in the Archaean Highlands of New Jersey, *Amer. Asso. Adv. Sci.*, 1890, p. 244 (Abstract).

† Cook, G. H., *Ann. Rept. State Geol. Survey of New Jersey*, 1868, p. 323.

‡ Thürach, H., *Op. cit.*, pp. 18–19.

§ Adams, F. D. and Barlow, A. E., *Memoir 6, Geol. Surv. of Canada*, 1910, p. 216.

¶ Dana, E. S., *Op. cit.*, 1900, p. 485.

° Derby, O. A., *Proc. Rochester Acad. Sci.*, 1891, vol. i, pp. 198–203.

⊙ Clarke, F. W., *Op. cit.*, p. 28.

In the investigation of the black sands of the Pacific slope* by the Federal Survey zircon was reported to be the fifth most abundant constituent in a long list of minerals found.

Metamorphic rocks. Zircon has been reported in nearly all classes of metamorphic crystalline rocks, especially in the feldspar-rich gneisses of probably igneous origin, and in the different types of crystalline schists, particularly the hornblendic,† chloritic and micaceous groups. It is usually present in varying amounts in the quartzites‡ examined. Van Hise§ reports zircon as especially common in marble, and Clarke¶ remarks that it occurs in some limestones in contact positions. According to Hoffmann° reddish-brown crystals of zircon, sometimes a half inch in diameter, occur in abundance in crystalline limestones of the township of Grenville (Argenteuil County), Canada, associated with wollastonite, pyroxene, sphene, graphite, etc. It occurs in small amount in some slates and phyllites.

Iron (magnetite ores). Zircon has been noted as an accessory mineral in iron-ore deposits, especially those of magnetite, in many localities, more particularly in New York, New Jersey, Pennsylvania, and the Unaka Mountains of North Carolina and Tennessee, and at Orendal, Norway. Blake ⊙ reported the occurrence of zircon in considerable quantity in the magnetite ores of the Rees and Wilder tract, Unaka Mountains of east Tennessee and North Carolina. The zircons were described as dark red-brown crystals ranging in size from one-tenth of an inch and less, to five-tenths of an inch in length.

Zircon occurs in the magnetite ores of New Jersey near Franklin Furnace. Palache|| says that some of the minerals associated with zircon

* *Mineral Resources of the United States, Calendar Year 1905*, pp. 1175–1258. Mineralogical examination of these sands at various localities along the Pacific slope has been made by various students, dating back as early as 1853. See Blake, W. P., *Amer. Jour. Sci.*, 1854, vol. 18, p. 156.

† Derby, O. A., On the Separation and Study of the Heavy Accessories of Rocks, *Proc. Rochester Acad. Sci.*, 1891, vol. i, pp. 198–206.

‡ Thürach, H., *Op. cit.*

Trueman, J. D., *Jour. of Geology*, 1912, vol. xx, pp. 244–257.

§ Van Hise, C. R., *Op. cit.*, p. 315.

¶ Clarke, F. W., *Op. cit.*, p. 678.

° Hoffmann, G. Ch., *Geol. and Nat. Hist. Surv. of Canada*, 1888–89, vol. iv, p. 66T.

⊙ Blake, W. P., Note on Zircons in Unaka Magnetite, *Trans. Amer. Inst. Min. Engrs.*, 1879, vol. vii, p. 76.

|| Palache, C., *Folio 161, U. S. Geol. Survey*, pp. 8–10.

are traceable directly to the granite. Bayley* remarks that most of these minerals are found in the ore occurring within the limestone. Zircon has been found in many of the magnetite mines of New Jersey,† usually in small crystals and in sparing quantity.

Zircon is reported by Newland and Kemp‡ along with apatite, titanite, and pyrite as among the less important constituents of the magnetite ore bodies of the Chateaugay mines near the western border of Clinton County, New York.

Pegmatites.

The occurrence of zircon in pegmatite masses, especially those of the granite and syenite families, is recorded by many observers. The mineral occurs both as minute inclusions in the principal rock-forming minerals. chiefly quartz and feldspar, and as a megascopic constitutent. As already noted, it has been observed in close association with riebeckite in the pegmatite§ masses of the Quincy, Massachusetts, granite. The zircons are very abundant in the quartz-rich portions of the pegmatites, occurring in the feldspar and riebeckite, but chiefly in the quartz as quartz-zircon intergrowths. Some of the zircons were of macroscopic size, the grains measuring up to 3 mm. in diameter. Fluorite and ilmenite occur with the zircon and quartz, and the authors state that the "zircon-quartz groups evidently belong to the pneumatolitic period and represent, it is believed, zircon crystals which subsequent to their formation suffered more or less recrystallization, replacement by quartz, and perhaps granulation" (p. 131).

In discussing the genesis of riebeckite and riebeckite rocks Murgoci¶ mentions pegmatitic, micropegmatitic or granophyric structures as characteristic, and remarks that riebeckite is a mineral which requires pneumato-

* Bayley, W. S., *N. J. Geol. Survey*, 1910, vol vii, p. 116.

† Nason, F. L. and Ferrier, W. F., *Amer. Asso. Adv. Sci.*, 1890, p. 244; see also *Ann. Report of the State Geologist*, 1868, p. 323; former reports by Rogers; and report by Bayley, W. S., 1910, vol. vii.

‡ Newland, D. H. and Kemp, J. F., Geology of the Adirondack Magnetic Iron Ores, *Bull. 119, N. Y. State Museum*, 1908, p. 114; see also Kemp, J. F., The Geology of the Magnetites near Port Henry, N. Y., etc., *Trans. Amer. Inst. Min. Engrs.*, 1898, vol. 27.

§ Warren, C. H. and Palache, C., The Pegmatites of the Riebeckite-Aegirite Granite of Quincy, Mass., U. S. A.; Their Structure, Minerals, and Origin, *Proc. Amer. Acad. Arts and Sci.*, 1911, pp. 125–168.

¶ Murgoci, C. M., On the Genesis of Riebeckite and Riebeckite Rocks, *Amer. Jour. Sci.*, 1905, vol. xx, pp. 133–145. References to the literature are cited by the author.

litic conditions for its formation. That much zircon accompanies riebeckite is indicated in the work of Brögger, Washington, Mrazec, Lacroix, Sousa Brandão, and Murgoci.

Zircon is reported from pegmatite in Auburn and Norway, and from Mount Mica, in Paris, Maine.* In the town of Norway near Cobble Hill perfect crystals of zircon, chrysoberyl, and zinc spinel occur in the pegmatite. The zircon crystals one-sixteenth to one-eighth inch long lie upon slicken-sided surfaces, and Bastin says they "were probably formed during the shearing process" (p. 79). Zircons, associated mostly with triphyllite and rarely exceeding one-eighth inch in diameter, are reported in the pegmatite of Mount Mica, Paris, Maine (p. 88).

Kemp† described zircons of two varieties from the coarse pegmatites near Port Henry, New York, some of which were exceptionally fine crystals of dark brown color, and measured up to an inch in length. Bastin‡ reports very perfect crystals of zircon about half an inch in length as abundant in a few places in typical granite pegmatite near Crown Point in Essex County, New York.

Zircon is a constituent of the pegmatites which cut the schists of the New Jersey highlands and are mineralogically like them.§ Bayley refers to zircon, magnetite, fluorite, etc., of the pegmatites as deposited from vapors (p. 152.)

In the well known pegmatites of Amelia County, Virginia, zircon¶ has been noted in small crystals and in masses weighing several pounds, associated with a score or more of other rare and heavy minerals. Near Gouldin post-office, Hanover County, pieces of zircon crystals three inches in diameter which have weathered out of rutile-ilmenite-bearing pegmatite dikes have been found on the surface.

Pratt° and Sterrett⊙ report zircon as one of the minerals found in the

* Bastin, E. S., Geology of the Pegmatites and Associated Rocks of Maine, *Bull. 445, U. S. Geol. Survey*, 1911, pp. 18, 53, 78–79, and 88; Kunz, G. F., On the Tourmalines and Associated Minerals of Auburn, Me., *Amer. Jour. Sci.*, 1884, vol. 27, pp. 303–305.

† Kemp, J. F., *Trans. Amer. Inst. Min. Engrs.*, 1898, vol. 27, p. 200.

‡ Bastin, E. S., Economic Geology of the Feldspar Deposits of the United States, *Bull. 420, U. S. Geol. Survey*, 1910, p. 55.

§ Bayley, W. S., *N. J. Geol. Survey*, 1910, vol. vii, p. 125.

¶ Watson, Thomas L., *Mineral Resources of Virginia*, 1907, p. 282.

° Pratt, J. H., Mining Industry in North Carolina in 1901, *Economic Paper No. 6, N. C. Geol. Survey*, 1902, pp. 40–42.

⊙ Sterrett, D. B., Mica Deposits of Western North Carolina, *Bull. 315, U. S. Geol. Survey*, 1906, p. 407.

pegmatites of granitic composition in North Carolina. According to Pratt* monazite and zircon occur primarily in the highly pegmatized Carolina gneiss of western North Carolina, and are more abundant in the pegmatite than in the gneiss. The principal locality in the United States where zircon has heretofore been found in commercial quantity is in the vicinity of Zirconia, Henderson County, North Carolina.† It occurs abundantly at this locality in a pegmatite dike 100 feet wide and traced for 1½ miles along the direction of strike N.50°E. The pegmatite penetrates Archaean gneisses and is kaolinized to a depth of 40 feet. The zircons are prismatic crystals with pyramidal terminations measuring up to 30 mm. in diameter, of grayish to reddish brown color‡ and occur mostly in the feldspar. Many tons of zircon have been obtained from this locality.

Somewhat similar zircon pegmatites, but very much less weathered, occur in the Wichita Mountains, Oklahoma. Sloan§ reports the occurrence of zircon with monazite in pegmatite bodies in Anderson, Greenville, Spartanburg, York, and Cherokee counties, South Carolina, and loose in the soil derived from the decay of these masses, associated with a variety of heavy minerals.

According to Brögger¶ zircon is a common mineral in the pegmatite dikes of the Christiania, Norway, region. Zircon and polymignite (zircono-titanate) occur here and there in the larvikite pegmatites of the Frederiksvaern district, and the nepheline syenite pegmatites of the Langesundsfjord are reported to be rich in zircon.° The mineral occurs with biotite, ilmenite, fergusonite, monazite, and yttrium spar in the pegmatite of the Riesengebirge, Silesia, and is a frequent constituent of the granite and syenite pegmatites found along the west coast of Greenland,⊙ and in the pegmatites of Basses-Pyrénées.||

Mineral Associates.

The mineral associates of zircon include a very wide range of species. As a product of crystallization from magmas it is associated with most of

* Pratt, J. H., *Economic paper No. 14, N. C. Geol. Survey,* 1907, pp. 109–120, 123.

† Pratt, J. H., *Economic paper No. 8, N. C., Geol. Survey,* 1904, pp. 40–41.

‡ Hidden, W. E. and Pratt, J. H., *Amer. Jour. Sci.,* 1898, vol. vi, pp. 323–336.

§ Sloan, B. E., *Bull. No. 2, S. C. Geol. Survey,* series iv, 1908, pp. 129–142.

¶ Brögger, W. C., *Eruptivgesteine der Kristiania Gebietes,* 1894, vol. i.

° Brögger, W. C., *Syenitpegmatitgänge,* 1890, pp. 121–200; *Die Mineralien der sudnorwegischen Granitpegmatitgänge,* 1906.

⊙ Hintze, C., *Handbuch der Mineralogie,* 1907, p. 1663.

|| Lacroix, *Bull. Soc. Min., 1891,* vol. 14, p. 394.

the common rock-forming silicate minerals. In pegmatites and the few recorded occurrences in veins and contact metamorphic deposits the association frequently comprises minerals usually regarded as having been formed by pneumatolysis, such as tourmaline, fluorite, cassiterite, riebeckite, etc. In alluvial and water sorted sands, the heavy resistant minerals of a variety of mineralogical form occur, such as hematite, ilmenite, magnetite, chromite, rutile, cassiterite, monazite, xenotime, tourmaline, spinel, garnet, staurolite, etc., and sometimes gold and platinum.

USES.

The demand for zircon is now small but, with the probable increased use of zirconia (ZrO_2), it will likely soon become greater.

Böhm* sums up the known and probable uses for zirconium substantially as follows:

Zirconia (ZrO_2) has been used in place of lime and magnesia as the incandescing material in the oxy-hydrogen blowpipe, and a very small quantity of zirconium nitrate is used in making mantles for gas lights. Large quantities of zirconia were once used in the Nernst lamps, a form of incandescent electric lamp in which a small stick of zirconia and yttria is used as a glower, but its consumption is not now so large, owing to the competition of metallic filament lamps. Zirconium carbide has been used in making incandescent electric lamps, but this also has been superseded by metallic filament lamps. The property of incandescence possessed by zirconia has tempted arc-lamp manufacturers to use it in their electrodes, but thus far it has not been used successfully. Zirconia is an excellent insulator for both electricity and heat and when mixed with a conductor can be used for electric heaters. In the Heraeus iridium furnace the iridium may be protected by a glaze made from a zirconium salt in place of the thorium or yttrium salts now used. Zirconia makes an excellent and very refractory crucible, which is manufactured in many sizes by a German firm. Its refractoriness makes zirconia a suitable lining for electric furnaces, and Böhm suggests that it might be used for saggers, but for the ceramic trade it must be free from iron and cheap. He also suggests its use for the walls of furnaces, for the making of molds to withstand high temperatures, and for heat insulation. Owing to its inertness zirconia is suitable for chemical ware, and many forms are manufactured from it. The same property has led to its recommendation for certain medicinal

* Böhm, C. R., *Op. cit.*, pp. 1261–1262.

uses, and in Röntgen ray therapy it is used in place of bismuth nitrate, which has sometimes given bad effects. Zirconia is a beautiful soft white powder which is well adapted for making paints and lacquers, as it is unaffected by gases, acids, or alkalies, and has good covering power. It makes a good opaque glass, but for this use the borate is better than the oxide. It is used for a polishing powder in place of tin oxide. Ferrozirconium is manufactured by one German firm for use in steel. Zirconium carbide is extremely hard and makes a valuable abrasive. Glass 7 mm. ($\frac{1}{4}$-inch) thick is cut with it as readily as with a diamond.

Clear zircons of brownish, orange, or reddish color are cut for gems and are then known as hyacinths. There is no probability of stones sufficiently large for cutting being found at the Ashland locality, but they may be present in some of the pegmatites of the crystalline area.

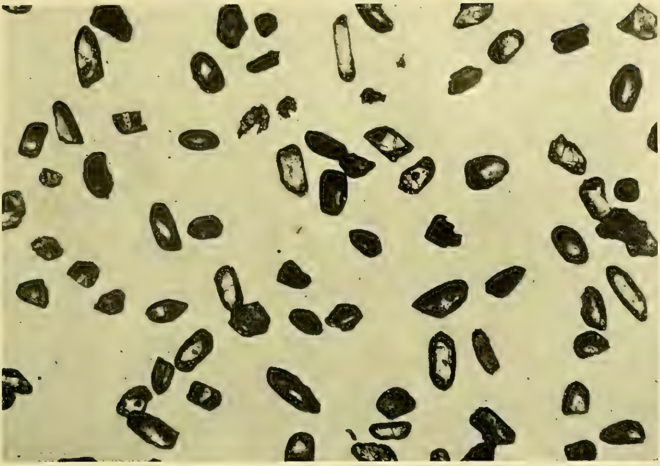

FIG. 1. MICROPHOTOGRAPH OF ZIRCON SEPARATED FROM SANDSTONE OBTAINED THREE MILES WEST OF ASHLAND, VIRGINIA.

Passed through a 150-mesh sieve. Rounding of the grains from wear is shown but in many cases the original crystal outline can be seen. Magnified 97 diameters.

FIG. 2. MICROPHOTOGRAPH OF ZIRCON SEPARATED FROM SANDSTONE OBTAINED THREE MILES WEST OF ASHLAND, VIRGINIA.

Passed through an 80-mesh sieve and caught on a 100-mesh sieve. Rounding of the grains from wear is pronounced. Large prismatic grain to right of center is cyanite. Magnified 97 diameters.

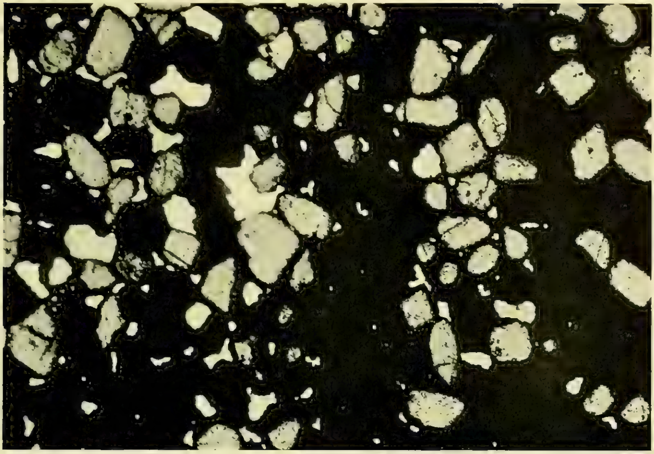

Fig. 1. Microphotograph of a Thin Section of Zirconiferous Sandstone Obtained Three Miles West of Ashland, Virginia.

The grains of high relief showing rounding from wear are zircon; the smaller, angular, white grains are quartz; and the black groundmass is mostly ilmenite; the whole is cemented with limonite. Magnified 97 diameters.

Fig. 2. Microphotograph of a Thin Section of Zircon-Bearing Sandstone Obtained Three Miles West of Ashland, Virginia.

The white, angular grains are mostly quartz with a few of feldspar; the scattered roundish grains of high relief are zircon, and the black groundmass is mostly limonite, which forms the cement, with some grains of ilmenite. Magnified 97 diameters.

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OF THE

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Studies in Human Heredity

BY

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Charlottesville, Virginia, U. S. A.

UNIVERSITY OF VIRGINIA PUBLICATIONS

The Proceedings and Transactions of the Philosophical Society of the University of Virginia are published in the form of Bulletins and offered in exchange for the publications of learned societies, institutions, universities and libraries. The Bulletins are issued at irregular intervals in the form of separate papers. These are to be made up into volumes, consecutively paged, of about 400 pages each, for each of the Scientific, Humanistic and Medical Sections. Separate numbers may be purchased from the Distribution Committee, to whom remittances should be made. Communications should be addressed to Dr. Thomas L. Watson, University, Virginia.

Thus far have been published the following papers.

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1. Intermediate (Quartz Monzonitic) Character of the Central and Southern Appalachian Granites, with a Comparative Study of the Granites of New England and the Western United States. By Thomas L. Watson. Vol. I, pp. 1–40. Price $0.75.
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BULLETIN OF THE PHILOSOPHICAL SOCIETY
SCIENTIFIC SECTION

Vol. I, No. 12, pp. 293-317 July, 1912

STUDIES IN HUMAN HEREDITY.

BY

H. E. JORDAN.

CONTENTS.

INTRODUCTION.

This paper is the outgrowth of several years' effort to collect further data with reference to the inheritance of left-handedness. With respect to left-handedness it represents simply a second contribution from an investigation still in progress. My chief object is to record here my results in the form of pedigree charts, and to give a brief discussion of them in the light of my earlier observations and conclusions, in an attempt to attain to more definite knowledge concerning the causes underlying and the principles governing hereditary left-handedness. Incidentally data have accumulated relative to certain other human characteristics and pathologic conditions, including pulmonary tuberculosis, cancer, hermaphroditism, onyxis, nephritis, melancholia, and thumb-prints. No one of these by itself would perhaps have much import. However, the consistent evidence of the mass seems significant from the standpoint of human inheritance, and its eugenic bearing. If pathologic conditions are determined even in part by hereditary constitutional bases, then methods looking to permanent racial cure, i.e., complete eradication, must reckon more intelligently and widely with the hereditary aspect of disease.

The pedigree charts here given and discussed are for the most part contributed by students of the University of Virginia. I take this occasion to express my best thanks to all who have taken an interest and aided in this work.

LEFT-HANDEDNESS.

In my earlier paper I* showed that left-handedness is hereditary, probably following Mendelian principles of segregation and dominance. The conclusion was arrived at that, fundamentally, the inheritance of left-handedness was the inheritance of a condition of reversed differential cerebral and brachial development (nutrition), consequent upon a variation in the vascular system.† Accordingly, in the last analysis, left-handedness was thought to signify an anatomic variation; and its "determiner" to be the factor underlying a change of structural relationships. In six selected cases the Mendelian expectation was fulfilled, assuming right-handedness to be dominant. Mendelian inheritance was strongly suggested also by the proportion of one to one and a half of left- to right-handed children in childships of four or more individuals. Ambidexterity was tentatively regarded as due to an imperfection of dominance producing functional symmetry.

Major C. C. Hurst‡ has recently reported a number of pedigrees which he interprets in a similar manner. He disagrees, however, with respect to ambidexterity. Hurst regards ambidexterity as left-handedness plus acquired right-handedness. If this is correct, ambidexters are rightly classified as left-handed individuals. This is undoubtedly true of the majority of cases. But an intelligent discussion of this point demands a definition of ambidexterity.

Popularly, a condition where one hand is used for some things and the other for other things is called "ambidextrous." True ambidexterity, however, consists in the ability to do all things equally well and deftly with either hand. Ability to execute halves of bilaterally symmetrical efforts at the same time (e.g., as in drawing) is likewise an aspect of ambidexterity. Even this ability may be to some extent acquired. However, the born ambidexter acquires this skill naturally and more readily. Such are not truly sinistro-manual. True, they may not really be of the nature of imperfect dominants. But a family history in which ambidexterity, as such,

* *American Breeders' Magazine*, 2: 1 and 2, 1911.

† Lueddeckens (Fritz) likewise traces the cause of right-handedness to a condition of asymmetrical embryonic cerebral vascular supply. *Rechts- und links-händigkeit*, Leipzig, 1900.

‡ *The Eugenics Review*, 4: 1, 1912.

appears hereditary would seem to further indicate this. Proximately, ambidexterity signifies cerebral and brachial functional symmetry, probably the result of structural symmetry. The question of ambidexterity will be discussed more fully below.

The following pedigree charts represent the result of an effort to further extend the work in the hope of disproving or more firmly establishing earlier more or less tentative views.

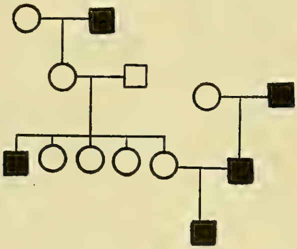
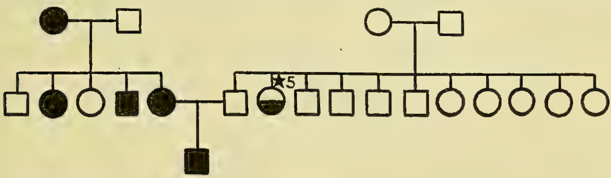


FIG. 1.

Pedigree chart, figure 1,* shows an inveterately left-handed child of five years with left-handedness in both family lines.



★5 Ambidexterity.

FIG. 2.

Chart, figure 2, in general is almost identical. In both instances the last generation may probably be regarded as a pure recessive of a cross between a homozygote recessive (RR) and a heterozygote dominant (DR).

* Explanation of symbols: open square, right-handed or normal male; open circle, right-handed or normal female; solid square, left-handed or affected male; solid circle, left-handed or affected female; half solid character, ambidexterity; character first used in fig. 23, left-handed individual of unknown sex; line without attached character, normal individual of unknown sex; solid small circle (fig. 38), tuberculous individual of unknown sex; open small circle, normal individual of unknown sex.

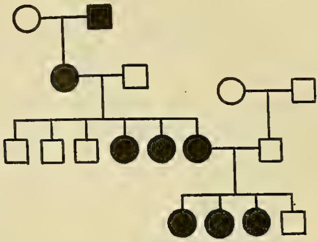


FIG. 3.

Chart, figure 3, gives another four-generation history of left-handedness in the maternal line. Generation three shows the exact Mendelian proportion for an RR×DR cross. But without further assumptions as to the grandparents (generation two) of the paternal line, left-handedness would here seem to be dominant. However, if the father may be regarded as a heterozygote the numerically limited fourth generation is not necessarily contrary to Mendelian expectation. Every possible effort is said to have been made to break the children of the fourth generation of their left-handedness but without success.

A very interesting point in this connection is the fact that in the third and fourth generations all the females were left-handed, all the males right-handed.

FIG. 4.

Chart, figure 4, shows a similar instance of apparently "sex-limited"* heredity of lefthandedness.

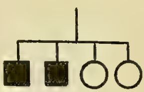


FIG. 5.

In chart, figure 5, on the contrary, all the males are left-handed, all the females right-handed. This family knows of no left-handedness in

* Not in the usual genetic sense.

its history. The left-handed condition of the boys is attributed to the influence of a left-handed nurse. Left-handed nurses are frequently held responsible for the left-handedness of their charges. I have many such complaints. Possibly in some cases of acquired left-handedness this occurs. But usually if careful search is made left-handedness or ambidexterity appears somewhere in the pedigree. In contradiction of the potency of a left-handed nurse's influence is the case given in chart, figure 25, in which among a fraternity of ten, with left-handed predigree, all having had the same left-handed nurse, only one is left-handed.

The succeeding sixteen pedigrees (6 to 21) are of children of six Virginia public schools, kindly secured for me by the respective teachers. The six schools comprised ninety-four (94) females and seventy-two (72) males including three (3) left-handed females and ten (10) left-handed males. Thus among one hundred and sixty-six (166) school children there are thirteen (13) left-handed individuals or about 8 per cent, an abnormally high proportion. A school at Gainesville, Florida, has two hundred and four (204) females and one hundred and ninety (190) males, among whom are one (1) left-handed female and five (5) left-handed males. The proportion here is somewhat less than 2 per cent, which is about normal. In both instances, however, the total number is too small to furnish information of much value respecting actual percentages. The preponderance of left-handed boys over girls is very striking. In my earlier work on left-handedness, representing a much larger canvass, the number of left-handed whites in the school population was 2 per cent, of blacks 4 per cent; and the number of left-handed males and females was approximately equal in both races.

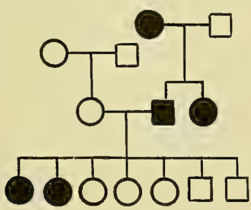


FIG. 6.

Charts like number 6 indicate that in the matter of dexterity* we are dealing with grades corresponding to degrees of dominance. If right-hand-

* i.e., use of hand.

edness is dominant, as is probable, and the mother here is a homozygote, as is indicated, then left-handedness could not appear in the succeeding generation on hereditary assumptions. It would either appear again as a spontaneous variation or by reason of a reversal or low degree of dominance. Of course the mother may be a heterozygote; we have no knowledge of her grandparents; if so, considering the size of the fraternity, the proportion is not greatly at variance with Mendelian expectation.

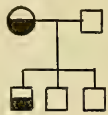


Fig. 7.

Chart, figure 7, is chiefly interesting as suggesting hereditary ambidexterity. If we are really dealing with degrees of dominance in left-handedness (or imperfection of dominance), as some cases indicate, then ambidexterity does not seem to be properly interpreted as due to this condition also. Ambidexterity in the limited sense above defined may of itself be hereditary—frequently obscured, however, by reason of neglect of exercise of ability to use both hands. The anatomic basis would be one of cerebral and brachial symmetry; in inverterate left- and right-handedness the anatomic condition would be one of asymmetry.

This pedigree and that of chart, fig. 34, were so interesting and promising that I made special efforts to secure the most definite and extensive information possible concerning the individuals represented. The result was disappointing. The ambidexterity was in neither case of the theoretical type indicated above. I hesitate to say now that true human ambidexterity as above defined—and as exemplified in infants and the anthropoid apes*—actually exists in any of my own pedigrees, with possibly one or two exceptions. They are not theoretically improbable; but their number is certainly very limited. I am accordingly forced to concur with Hurst that ambidexters as ordinarily found are really left-handed individuals who have by training acquired skill in the use of the right hand.

Chart, figure 8, is interesting as showing a direct four-generation female line of hereditary left-handedness. I have a similar three-generation male left-handed pedigree.

* According to J. Cunningham, *Journ. of Anthropol. Inst.*, G. B., vol. 32, 1902.

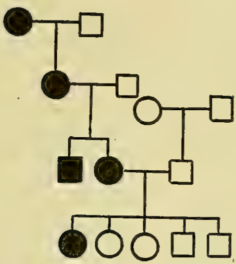


FIG. 8.

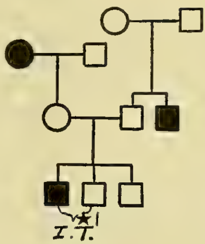


FIG. 9.

Chart, figure 9, includes a pair of identical twins one of which is left-handed. Similar cases were described in my earlier paper. To that discussion should properly be added the following considerations: If the factor for left-handedness is germinal (i.e., if dexterity is already determined in the fertilized ovum) then either identical twins should be both right-handed or left-handed—or, one should always be left-handed and the other right-handed. Although neither combination is the rule, the three varieties probably occur (I have not yet seen a case where both were left-handed). This evidence suggests that dexterity (left-handedness in this case) is not unalterably established during the earliest developmental stages, as is sex, for example. Such instances are disturbing to any hereditary interpretation of left-handedness. It is not improbable, however, when one recalls the several instances of apparent reversal and imperfection of dominance, that the operation of later embryonic influences may be the cause of such reversal and may account also for the dissimilarity in dexterity of identical twins, originally but not unalterably determined.

The ambidexterity of the young infant has its bearing on this point. A baby under my observation handled her playthings indiscriminately and equally well with either hand until the end of the sixth month. At that time she showed a decided preference for the use of the right hand. At the end of the first year she is completely right-handed. We are dealing here apparently with another of those numerous instances where the individual development recapitulates that of the race, primitive man having been perhaps functionally to a large extent ambidextrous* as are the anthropoid apes. Here again we come face to face, the foregoing assumption being granted, with the question of the inheritance of an acquired

* Cunningham, *loc. cit.*

character. What motive first caused man to use the right hand almost exclusively for executions of skill and force, is a problem shrouded in mystery. Moreover, the stimulus may not have been external at all; it may have been an internal stimulus resting upon an anatomic variation of the vascular system, for example. Right-handedness may have been a mutation from a primitive ambidextral condition, left-handedness representing more nearly the primitive ambidextral condition—or a reversed mutation. Further speculation would be fruitless at present. It need merely be pointed out here that dexterity may be represented by a germinal factor which comes to expression only in later stages of development, passing through a more primitive stage of ambidexterity—just as all definitive conditions are the result of a process represented by steps through more primitive conditions. The factor may also be readily responsive to influences of later development, which may change its hereditary tendency, as seems to have been the case in those instances of identical twins where one is left-handed and the other right-handed. If right-handedness is an acquired character it was acquired relatively late in racial history, and in heredity would be expected to show varying degrees and grades of expression. A careful weighing of all the evidence favors more strongly, however, the idea of the appearance of an internal anatomic variation, causing general right-handedness and occasionally (as a reverse variation) left-handedness. This position must further account for the fact that the assumed variation in the vascular supply of the cerebral hemispheres causing left-handedness antedates by several months the expression of the functional condition. This, however, is perhaps only what should be expected, since the altered function should appear only after the anatomic change has had time to work a summation of effects.

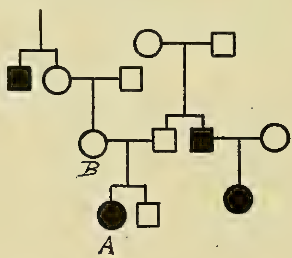
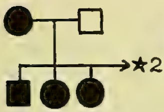


FIG. 10.

Chart, figure 10, is of considerable importance. It illustrates very well how the use of restricted pedigrees may lead to false inferences. If we considered only the direct ancestry, both paternal and maternal, of A (who is inveterately left-handed and even writes with the left hand),—and this is all that is frequently known or given in brief family histories—the left-handed condition here would appear to have arisen spontaneously, and to be without hereditary aspect. The mother has no left-handed brothers or sisters. Nor was the father, or any of the grand-parents on either side, left-handed. The more extensive pedigree chart shows, however, that on both sides there were left-handed relatives. My informant states that there are still other left-handed collateral relatives not here shown. The chart indicates that both parents were probably heterozygous; and left-handedness in some of the children is thus to be expected.



★2 Other children of unknown number, condition and sex.

Fig. 11.

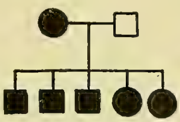


Fig. 12.

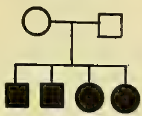


Fig. 13.

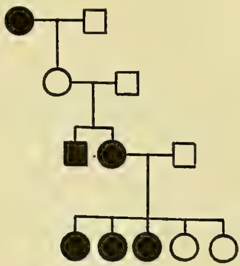


Fig. 14.

Charts, figures 11, 12, 13 and 14, are additional instances of the apparent dominance of left-handedness. In 12 the mother "corrected the left-handedness, making them all ambidextrous." The ambidexterity here alluded to is however more probably simply ability to perform some efforts with the right-hand, others with the left-hand. The final fraternity of chart 14 again indicates an RR × DR cross, which is likely in view of the pedigree of the mother.

Charts, figures 15, 16 and 17, represent an interesting type of left-handed pedigree. In every case neither parent had left-handed brothers or sisters (indicated by character Z). In 15 and 16 the parents are heterozygous dominants; and at least one left-handed offspring was to be expected in a fraternity of four.

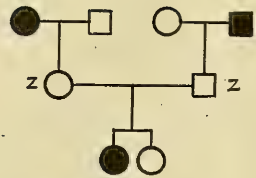


FIG. 15.

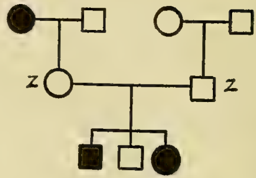


FIG. 16.

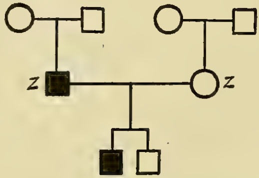


FIG. 17.

Chart, figure 18, is similar in that the mother had no left-handed brothers or sisters. Since her parents are normal, the presumption is that she is a homozygote dominant. The appearance of a left-handed individual in the final childship is difficult to explain except as a variation (spontane-

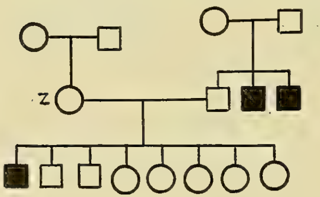


FIG. 18.

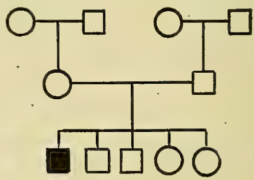


FIG. 19.

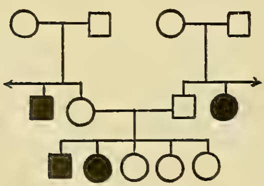


FIG. 20.

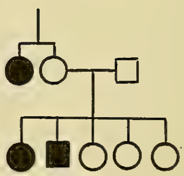


FIG. 21.

ous). The appearance of left-handed uncles indicates inheritance, but the manner or scheme is obscure. Chart, figure 19, belongs to the same obscure category; but here again there are left-handed collaterals,* two first cousins of the mother. Chart, figure 20, again can be explained in terms of heredity only by making the probable assumption that the parents are heterozygous. The boy of this childship writes with the left hand. Chart, figure 21, must be similarly interpreted.

Viewing the pedigree charts as a whole there seems to be too much left-handedness on the assumption that that character is recessive. It must be remembered, however, that the childships are mostly small, and the pedigrees relatively restricted. When a certain character is under investigation, naturally for the most part, only those families come to be included in which children show the characters. The small normal fraternities of affected parents fail of inclusion, although they should be included for accurate percentages and proper testing of hereditary hypotheses. It is a striking fact that the larger the fraternities and the more extensive the pedigrees the more of a recessive nature does the pathologic character assume. The extensive pedigrees of the Davenports† indicate that human albinism is a recessive character; earlier much smaller pedigrees seemed to indicate that it was dominant. When more extensive pedigrees of other pathologic characters, which now appear to partake of the nature of dominance, are known, they too may be found to be Mendelian recessives. The eugenic significance of the point is of the utmost import.

In a recent work on left-handedness in certain gramineae—including two- and six-rowed barley, oats, millet, rye and maize where the character is shown to be not hereditary—R. H. Compton‡ calls attention in a footnote to a pedigree recorded by Cunningham,§ cited from Aimé Péré;¶ which seems to disprove my assumption that right-handedness behaves in heredity as a dominant. It must be admitted that there are a small number of cases which indicate that left-handedness is the dominant character e.g., figures 12 and 14. And this may indeed be the fact in certain human strains. Here we must postulate a condition of reversed dominance. Péré gives two interesting cases. In one both parents were left-handed; of their five children, four were left-handed. This case so closely accords with expectation (total) on Mendelian principles, that the single probable exception among five individuals is not really subversive of the general

* Not shown in chart.

† *Am. Nat.*, 44: 527 and 528.

‡ *Journal of Genetics*, 2: 1, 1912.

§ *loc. cit.*

¶ *Les courbures latérales normales du rachis humain*, Toulouse, 1900.

scheme. The second case is very much more damaging. Here a certain sailor is said to have had seven left-handed brothers and six left-handed sisters. His mother was left-handed, and had three left-handed brothers and three left-handed sisters. The father was right-handed but had a left-handed brother who had five left-handed children. The maternal grandfather was also left-handed. This is a most remarkable family history! As given it undoubtedly indicates the dominance of left-handedness in this stock. In the three fraternities of this family history given by Péré total left-handedness seems to result from a right-hand-left-hand cross. The case weakens somewhat, however, when we notice that in neither case of the right-handed individuals is the ancestry known. But even on the presumption that they are heterozygotes, the case for the last generation, at least, is not materially improved. This fraternity can be brought into line with Mendelian principles on the assumption that the father was really a mild left-handed individual; or on the hypothesis of reversed dominance. Cunningham,* also, believes very strongly in the hereditary nature of left-handedness, but does not discuss its behavior in crossing.

In another set of one hundred and one pupils concerning whom no further data were given, four are left-handed.

In a colored school of Montego Bay, Jamaica, British West Indies, among two hundred (200) pupils (ninety-four (94) females and one hundred and six (106) males) there were six (6) left-handed females and five (5) males.† The percentage, $5\frac{1}{2}$, is slightly higher than among the colored school children of the south, namely, about 4 per cent. Of these eleven left-handed pupils seven had either a left-handed father or mother; the other four had a close relative who was left-handed.

Summarizing the data from the twelve of the foregoing pedigrees with childships of more than four individuals, we see that there are twenty-one (21) right-handed to nineteen (19) left-handed females, and nineteen (19) right-handed to nine (9) left-handed males. In these fraternities the number of left-handed females is twice the number of left-handed males. The number of left-handed is to the number of right-handed as twenty-eight (28) is to forty (40), or as one to one and one-half. *This is the identical result of my earlier study.* On the most legitimate assumption—in view of the fact that the investigation is confined to left-handed fraternities—that the parents are all heterozygous, the proportion does not accord with

* *Loc. cit.*

† These data were collected incidentally during a month's work at the temporary Marine Biological Laboratory of the Carnegie Institution of Washington, during the spring of 1912.

Mendelian expectation. Allowing, however, for quite a number of probably homozygous recessives, and occasional imperfection of dominance, the proportion comes to accord fairly well. Perhaps an equally legitimate assumption respecting the pedigrees in question would be to regard all crosses as between RRs and DRs. The proportion should then be one to one. Allowing then for a number of DR by DR crosses, the proportion should approximate closely to the one obtained.

Professor Hodge, of Winthrop College, South Carolina, has lately had one hundred of my printed questionnaires filled out by left-handed students. Charts, figures 22 to 35, give the data of the most complete of these. These include numerous cases of nine-, ten- and eleven-child fraternities. In this set of pedigrees we have the advantage of large childships. The total number of children in these one hundred families is four hundred and ninety-eight (498); of these eighty-nine (89) are left-handed (sixteen girls, twenty-eight boys, and forty-five of whom the sex was not given). The proportion of left-handed to right-handed is therefore as eighty-nine (89) to four-hundred and nine (409), or as one to four and six-tenths. This is slightly greater than for the DR $\times$ DR cross, but is sufficiently close after due allowance for the preponderance of large childships to be significant as indicating the recessive character of left-handedness.

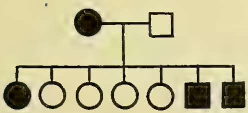
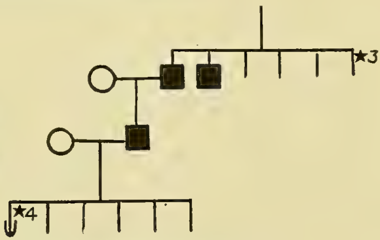


FIG. 22.



★3 Normal children of unknown sex.
★4 Left-handed children of unknown sex.
FIG. 23.

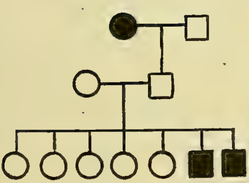


FIG. 24.

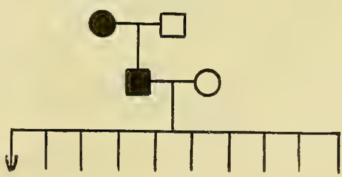


FIG. 25.

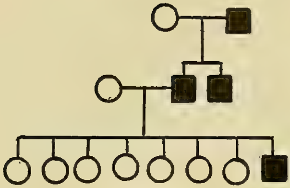


FIG. 26.

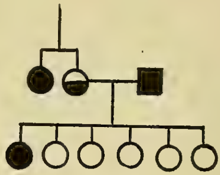


FIG. 27.

The childship of figure 22 indicates an RR × DR cross. Charts, figures 23, 25, 26, 27, 29, 31, 32, 33 and 34 offer obvious difficulties.

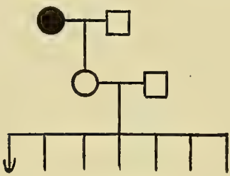


FIG. 28.

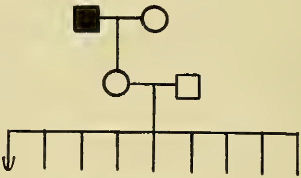


FIG. 29.

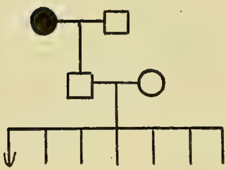


FIG. 30.

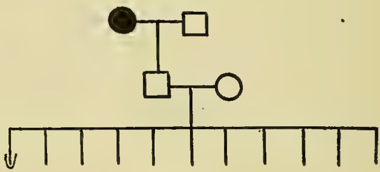


FIG. 31.

The fraternity of chart 25 was mentioned above as an instance where only one child out of ten became left-handed though all had the same left-handed nurse.

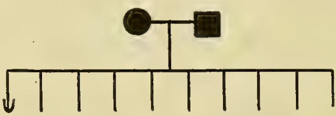


FIG. 32.

The fraternities of charts, figures 24, 28, 30, and 35, are sufficiently close to expectation to suggest a DR × DR cross.

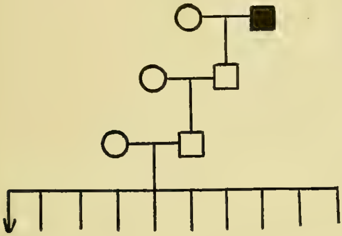


Fig. 33.

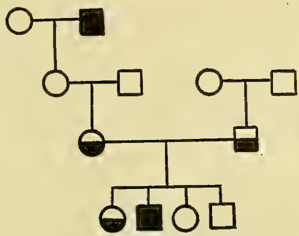


Fig. 34.

Chart, figure 34, moreover again indicates that ambidexterity may perhaps be specifically hereditary. In family history 35, "one grandchild out of each family is left-handed."

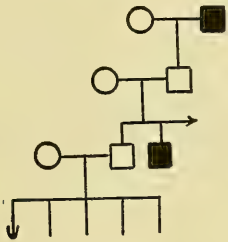


Fig. 35.

In fifty-three instances, among the total of one hundred, neither parent is left-handed; these include fifteen instances where in addition neither grandparent is left-handed; and ten where one or other grandparent is left-handed. In thirty-eight instances no information is given concerning grandparents; these include ten instances where either an uncle or aunt is left-handed.

The cases of total absence of left-handedness in the pedigree (for two previous generations) of a left-handed individual are therefore very rare, if indeed, they exist at all. Actual cases must be interpreted as spontaneous variations, which are probably just as liable to occur now as originally.

The additional evidence on left-handedness here presented establishes beyond doubt, I think, my earlier contention, that left-handedness is hereditary. The practical identity of the proportion of left-handed to right-handed individuals of fraternities of more than four individuals, of my two studies—and the corroborating evidence of Hurst—indicates very strongly that left-handedness behaves in inheritance as a Mendelian recessive. There appears meager evidence that true ambidexterity is inherited as such, representing perhaps a balanced state of dominance and recessiveness.

The questions touching the phylogenetic origin of right-handedness; and the fundamental causes underlying left-handedness; and the nature and complete behavior of the hereditary factor or complex of factors, are approached in an attitude of open-mindedness to all possibly pertinent evidence. The effort, however, it must be admitted, soon leads to speculation, and yields no definite results. Decision halts between the idea of the inheritance of an acquired character (i.e., by tradition and training), and that of a germinal variation causing vascular alterations. The relation of ambidexterity to right-handedness and left-handedness also remains obscure.

PULMONARY TUBERCULOSIS.

With the exception of case, pedigree chart, figure 36, the family histories here considered were secured for me by Dr. Lewis Booker of the North Carolina Tuberculosis Sanatorium, at Montrose.

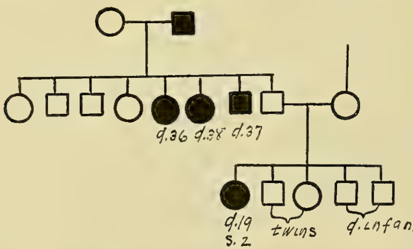


Fig. 36 Pulmonary Tuberculosis.

Case, pedigree chart, figure 36, shows a three-generation family history of tuberculosis. This case suggests the recessiveness of the tuberculosis factor, supposing the first-generation male to be heterozygous dominant and the female homozygous recessive.*

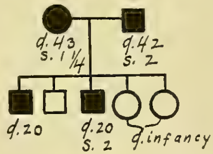


FIG. 37 PULMONARY TUBERCULOSIS.

Case, figure 37, gives a two-generation tuberculosis history, and also indicates that the tuberculosis factor is recessive to the normal condition.

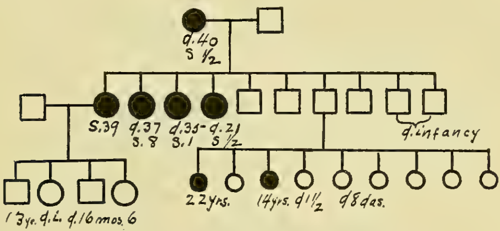


FIG. 38 PULMONARY TUBERCULOSIS.

Case, figure 38, again gives a three-generation history and the same evidence with respect to the recessiveness of the pathologic condition.

Case, figure 39, gives a fairly complete four-generation history. Judging from the progeny we are undoubtedly justified in regarding the mother, C—the cause of whose death is unknown—as also tubercular (pure recessive?). No information could be secured about fraternity A; fraternity B, with one exception, was said to be "living and well." Two individuals of this fraternity have had a child each die of tuberculosis.

Contrary to the opinion frequently heard, no evidence appears among these few cases that the male parent is hereditarily prepotent over the female in the matter of transmitting tubercular diathesis.

* Explanation of symbols of pedigree charts: *d*, died at the age indicated; *s*, sick for the time indicated; *c*, cured for the time indicated; *d.i.*, died in infancy.

While this material does not indicate very definitely the manner of the tubercular heredity, i.e., whether it follows Mendelian principles—it clearly reveals, I believe, a definite hereditary aspect of tuberculosis; and that is the all important matter by reason of its eugenic bearing. In the light of pedigree, figure 39, matings where both parents are tubercular would seem to be ill-advised. If the assumptions indicated are correct, then matings between tuberculars and normals would result in normals; but matings between such "normal" offspring would again be in danger of producing a tubercular among every four offspring. The eugenic bearing is obvious; and restrictive measures clearly indicated in the interests of a final control of the "white plague."

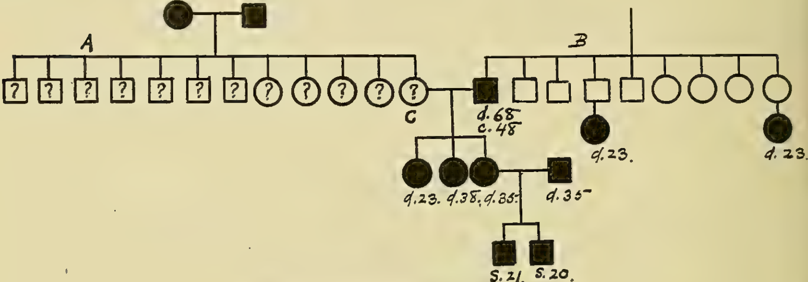


Fig. 39 Pulmonary Tuberculosis.

It will suffice to merely state that by hereditary pulmonary tuberculosis is meant simply the inheritance of a constitutional predisposition or susceptibility or lack of resistance to infection. As to the factors (allelomorphs) concerned we can only speculate. Since the pathologic condition seems more of the nature of a recessive it is more in accordance with present knowledge to regard the tuberbular diathesis as due to the absence rather than the presence of a factor (determiner). Moreover, the facts of the pedigrees strongly favor such interpretation. Two abnormals here as far as is known had no normal offspring.

CANCER.

The following two histories of apparently hereditary cancer were secured for me by Mr. Carrington Williams of the University of Virginia.

Chart, figure 40, gives a four-generation history of carcinoma. Nodules appeared on face, fingers and toes, as indicated. A was married three times, once without issue. With one wife he had five affected and two normal children; with another wife all six children were normal. A's pedigree seems to indicate that cancer is dominant to the normal condition. There are other instances in which a pathologic condition appears dominant to the normal. Susceptibility in some cases seems dominant to immunity—e.g., wheat rust in the plant kingdom. But whether immunity means the presence of an inhibitory factor, or the absence of an inducing factor; likewise whether susceptibility be due to the presence of a predisposing factor, or the absence of an inhibitory factor, remains obscure.

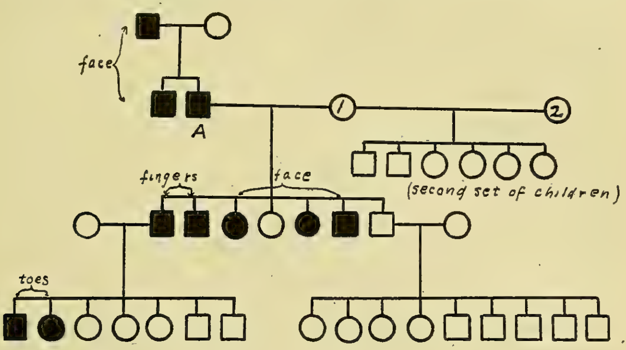


FIG. 40 CARCINOMA.

Moreover, there are probably grades of susceptibility and resistance, as the family history in question indicates. "Grades" probably involves a complexity of factors. The fact that with one wife A had five carcinomatous and two normal, and with the other all six normal offspring, gives further evidence of "grades," or reversal or imperfection of dominance. Furthermore, the affected fraternity of the fourth generation indicates the same. We are dealing here with apparently similar parental germinal constitutions, and with the same number of offspring as in the carcinomatous childship of generation three; but here five out of seven are normal, whereas, there five out of seven were abnormal. The normal fourth generation childship contains nine individuals. What seems clearly indicated in this family history is that carcinoma has a hereditary aspect;

but a Mendelian interpretation apparently demands the assumption of degrees of dominance; which assumption necessarily involves the recognition of a multiplicity of factors.

It is interesting to note that all of the affected families lived on the same farm, the normal families having moved to other regions. The family tradition attributes the cancer to the limestone water of the old farm. We see at once, however, why one of the fourth generation fraternities was wholly normal: its immediate ancestry possessed a germ-plasm from which the cancer-causing factor was eliminated. The other normal fraternity of the third generation offers some difficulties; the most plausible interpretation which suggests itself rests on the assumption of a reversal of dominance. Of further interest is the fact that males and females appear equally vulnerable to the etiologic factor. The specificity of the heredity of the carcinoma is also significant.

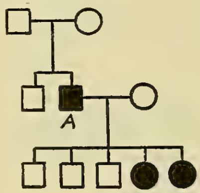


FIG. 41 NEUROFIBROMA.

Further inquiry revealed the fact that A's first wife was his first cousin (a daughter of his paternal uncle). This may explain the greater incidence of carcinoma in the childship of this mating; and suggests grave dangers in human close-breeding.

A second case (fig. 41) is a two-generation history of neuro-fibroma—a comparatively rare tumor condition. An affected father (colored)—the only affected member of his family—had three normal male and two abnormal female children. Here again the abnormality appears partially dominant over the normal.

While the etiology of cancer remains obscure, it is fruitless to speculate further. The indisputable fact remains however, I believe, that the constitutional basis of cancer is as simply hereditary as other better understood human pathologic characters.

HERMAPHRODITISM.

For a photograph and description of the hermaphrodite here under consideration, I am indebted to Mr. G. R. Bell of Denver, Colorado. The case is interesting and of significance in this connection on account of the fact that "it" is one of a family marked by anomalies.

The hermaphrodite dresses in men's attire in public. The absence of facial hair, the presence of large breasts, the general shape of the limbs, and the general female gross appearance of the external genitals, give it female character. A cleft in the rectum is thought to represent a vagina. Menstruation occurs regularly every twenty-eight days. There is present however a penis of about an inch and a half and a fold of skin containing two small bodies, probably testicles. There is no sexual desire.

The hermaphrodite is one of nine children all of whom have some anomalous character. One is a "tallest circus giant;" others have six fingers and toes; tbe mother also has six fingers and toes. This case indicates that there is a general anomaly factor, which may express itself in heredity in one of a number of ways including the extreme and rare anomaly of apparently true hermaphroditism. As such it is contrary to general experience according to which anatomic variations have a specific heredity e.g., polydactylism, left-handedness, onyxis, etc. Dr. Q. I. Simpson of Palmer, Illinois, has written me of a case of hereditary hare-lip dating to pre-revolutionary times. Hermaphroditism in man is probably not hereditary in the strict sense. In its different degrees it would seem to be due to a factor which generally finds specific expression, as in polydactylism* in this case, but more rarely appears as a more extreme anomaly, as hermaphroditism, gigantism, etc.

ONYXIS.

Two cases of hereditary onyxis have come to my notice. Pedigree chart, figure 42, gives the facts in one family kindly secured for me by Dr. H. B. Stone of Baltimore. In the final fraternity of two a boy of five years (B) is abnormal and another of one year is normal. It is commonly believed that ingrowing toe-nails are caused by pressure from tight-fitting shoes. If this were true its heredity would be that of an acquired character.

*Too late to incorporate in the shape of a chart, Mr. Carrington Williams has given me a direct five-generation history of bilateral polydactylism (six digits), including a number of affected collaterals. Both males and females are affected, and the abnormality appears dominant. It is hoped that a more detailed study may be made of this family.

On the contrary the condition is due to the peculiar shape of the hallux nail. The nail arches sharply over the side of the nail-bed (fig. 43), whereas in the usual or normal condition the nail lies more nearly flat (fig. 44). The condition of "ingrowing toe-nails" accordingly is coincident to an anatomic variation in the shape of the nail. In the case of some of the affected members of this family, including both the father (A) and mother

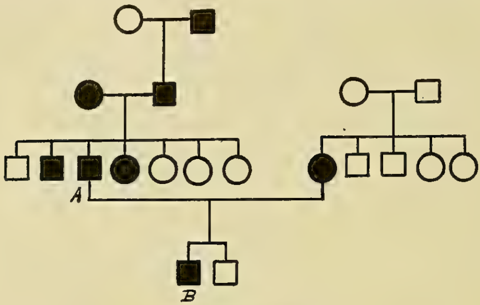


FIG. 42 ONYXIS.

of B the condition appeared to a degree requiring surgical treatment; similarly with respect to B, in whom the condition appeared before the end of the first year, and before walking or wearing shoes. Moreover, in the case of the father (A) there is present a decided tendency to lateral arching of the nail on all fingers and toes.

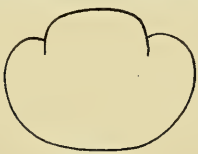


FIG. 43.

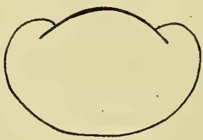


FIG. 44.

Onyxis is evidently recessive to the normal condition; otherwise two normals, as the mother's parents, could not have abnormal offspring. Moreover, it is probably due to the *presence* of a determiner or factor; otherwise two abnormals as the father's parents, could not have any normal offspring. In view of the paternal history—which suggests dominance of the abnormal —a complete explanation of the pedigree involves the assumption of degrees of dominance.

NEPHRITIS.

Abnormalities associated with defective kidney function as, for example, the class of diseases called "Bright's disease," frequently show a tendency to "run in families." An instance of hereditary defective kidney function, causing death by uraemic poisoning is given in chart, figure 45. Only the father is said to have suffered from this defect. Of the seven children four have already died at about the age of forty years, and another is undoubtedly also affected. Only two of the children appear free of the abnormal condition. The character of the factor, whether recessive or dominant, does not very clearly appear. The limited data at hand, however favor more a recessive interpretation; the normals may perhaps be simply relatively "normal."

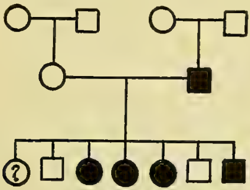


FIG. 45 NEPHRITIS.

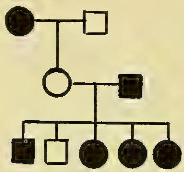


FIG. 46 BRIGHT'S DISEASE.

Mr. C. G. Giddings of Atlanta, Georgia, has given me a very similar pedigree of apparently hereditary kidney defect (fig. 46). Among a childship of five, including two males and three females, one male has just died of uraemic poisoning at the age of forty-two; and the three sisters, close to thirty-five years of age, are all sufferers from chronic "Bright's disease." In the direct ancestry both father and grandmother died of Bright's disease. The two affected males were also "heavy drinkers."

MELANCHOLIA.

The melancholia here considered (fig. 47) may properly be classified among the phenomena of nervous defects. In every instance, with the exception of the young individual of the last generation, the prevalent melancholic temperment, under stress of misfortune usually of a financial nature, developed into a condition of mild insanity. The crisis in each instance came about the age of fifty. At least one individual in the direct line both of the second and third generations were confined in sanatoria for insane. The whole family is "high-strung, nervous and hot-headed." The history in hand includes five generations. The direct line of transmission is from a female, through three males, to a female of the fifth (last)

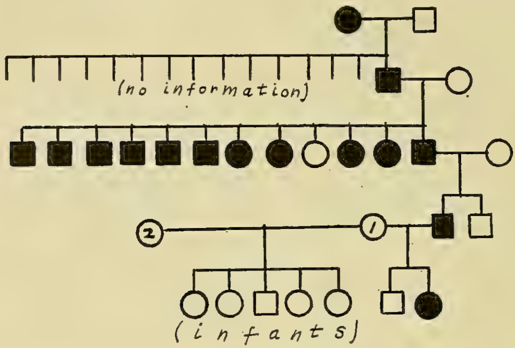


FIG. 47 MELANCHOLIA.

generation. In the third generation the affected male was one of an almost completely abnormal fraternity, including seven males and five females. The entire history indicates dominance of this type of mental defect.

THUMB-PRINTS.

The study of the inheritance of thumb-prints has just been begun. Special value attaches to the case here considered by reason of a distinct bilateral asymmetry of pattern (fig. 48). If types of thumb-prints can be


L. FIG. 48. R.

shown to be hereditary they may be made to serve an important accessory rôle in certain cases of disputed paternity. The pair of thumb-prints illustrated (of individual A) includes a left-hand hooked pattern (L) and a right-hand cone pattern (R). The mother's thumb-prints are of similar pattern, but reversed with respect to hands, i.e., the right thumb has the hooked pattern and the left the cone pattern. The father's thumb-prints are both of the cone type. A's wife has both thumb-prints of the cone type. Their only child, a girl of 14 months, has thumb-prints exactly like the father (A) but again on the reverse hands, like her grandmother.

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The Evolutionary Construction of the Imaginary Power of a Number and its Expression as the Exponential Function

BY

WILLIAM H. ECHOLS

UNIVERSITY OF VIRGINIA
Charlottesville, Virginia, U. S. A.

UNIVERSITY OF VIRGINIA PUBLICATIONS

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BULLETIN OF THE PHILOSOPHICAL SOCIETY
SCIENTIFIC SECTION
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THE EVOLUTIONARY CONSTRUCTION OF THE IMAGINARY POWER OF A NUMBER AND ITS EXPRESSION AS THE EXPONENTIAL FUNCTION.*

BY

W. H. ECHOLS

1. In the preface to a most inspiring book, the first edition of his *Introduction à la Théorie des Fonctions d'un Variable*, M. Jules Tannery remarks:

> On peut constituer entièrement l'Analyse avec la notion de nombre entier et les notions relatives à l'addition des nombres entiers; il est inutile de faire appel à aucun autre postulat, à aucune autre donnée de l'expérience; . . . . Il va sans dire que j'ai dégager la définition des fonctions circulaires de toute considération géométrique.

When he attempts however to evolve the circular functions from the functional law of the addition formulae he finds it necessary to make the additional assumption that the limit of the quotient of sin x to x is unity when x converges to zero, and apparently on account of the necessity for this assumption he abandons the analytical design of these functions in the second edition of his book and resorts as do others to their geometrical definitions, or boldly and artificially assumes them in analytic form.

The circular functions appear as necessarily and naturally born into existence in the construction of the imaginary power in exactly the same way that the hyperbolic functions are created in the construction of the

* This note is extracted from a lithographed edition (1903) of the writer's lectures on the Introduction to Theory of Functions. Much of it is of course not new but it is believed the novelty of treatment is interesting. Presented to the Scientific Sec. Philos. Socy., January, 1913, in its present form.

real power. It is not necessary to make the assumption, as did Tannery, as to the limit of sin x/x for it can be demonstrated as is shown in the sequel.

2. The synthetic and purely analytical foundation and development of the numbers of ordinary analysis proceeds from defining a group of marks

$$I + I + I + \ . \ . \ . + I \tag{1}$$

as a number called an integer. The direct operations of addition, multiplication and raising to powers follows as shorthand additions resulting, for integers, in the associative, distributive and commutative forms

$$\begin{aligned} a + b &= b + a, \\ a + b + c &= a + (b + c), \\ a + (b + c) &= a + b + c; \\ ab &= ba, \\ a\,(bc) &= (ab)\,c, \\ a\,(b + c) &= ab + ac; \end{aligned} \tag{2}$$

and for exponents

$$a^b a^c = a^{b+c}, \qquad (a^b)^c = a^{bc}, \qquad a^c b^c = (ab)^c. \tag{3}$$

Two fundamental postulates are now made. First, if the symbol x be defined to be a number then is the chain group of elements x

$$x + x + \ . \ . \ . + x \tag{4}$$

defined to be a number. Conversely if the group (4) be defined as a number then is the element x defined as a number. Second, if x be defined as a number then the product group

$$x \times x \times \ . \ . \ . \times x \tag{5}$$

is defined to be a number. Conversely, if the product group (5) be defined as a number then is the element x defined to be a number.

Definition. All numbers constructed from the mark I under the postulates (4) and (5) and which are subjected to obey the distributive, associative, commutative and exponent laws of the integers are thus defined to be *integral* numbers.

The integral numbers, or complex numbers of ordinary analysis, are thus founded on twelve (if we include the definition of I as a number) axiomatic postulates which are necessary and sufficient for the complete design of these numbers.

3. Under the definition follows (for real numbers) the construction of the rational and irrational (by partition and by sequence) and the negative number completing the real number continuum. In the usual way the function a^x (a positive, x real) and the exponential

$$e^x = 1 + x + \frac{x^2}{2!} + \cdots \tag{6}$$

where $e = 2.71 \ldots$, are constructed. The ordinary laws for the convergence of infinite series and products established. The inversion of the square leads to the imaginary iy and thence the complex or integral number $x + iy$. The operations of addition, multiplication, involution and their inverses shown to lead to a complex number. The convergence of series of powers and their products established. Then finally arises the question of the imaginary power of a real number and the necessity for its construction as an integral number if possible. This brings us to the point of constructing a^{ix} or preferably e^{ix} where e is the constant naperian number $2.71 \ldots$ and $i \equiv \sqrt{-1}$.

With respect to the symbol e^{ix} Chrystal says, Algebra, Part II, p. 264:

> It seems to be forgotten by some writers that the e in e^{ix} is a mere *nominis umbra* —a contraction for the name of a function and not 2.71828. . . . . Oblivion of this fact has led to some strange pieces of mathematical logic.

And in the preface (vii) he says:

> Some expounders of the theory of the exponential function of an imaginary argument seem even to have forgotten the obvious truism that one can have no property of a function which has not been defined.

It is in the attempt to remove the "shadow" from this fundamental symbol that this note is undertaken.

4. The construction e^x, when x is real, constructs the real continuum of positive numbers as x varies from $-\infty$ to $+\infty$, this construction being given by (6). The construction e^{ix} is defined as an *integral* number in (3) by being subject, as part of its definition, to the laws of the integer

$$e^{ix}\, e^{iy} = e^{i(x+y)}, \quad e^{ix}/e^{iy} = e^{i(x-y)}, \quad (e^{ix})^{iy} = e^{-xy} \tag{7}$$

The construction of e^{ix} as a possible complex number presents itself as a problem. To find two real number constructions $C(x)$ and $S(x)$ such that under the defining laws of e^{ix}, we shall have

$$e^{ix} = C(x) + i\, S(x). \tag{8a}$$

If two constructions C and S can be found, different from zero, then e^{ix} will have been constructed as a complex number, if on the other hand we find $C = 0$ and $S \neq 0$ then e^{ix} will construct an imaginary number, if $C \neq 0$ and $S = 0$ then e^{ix} is real. If no such functions C and S exist and cannot be constructed then e^{ix} is not a complex number as previously designed but is some new number which must be designed and the complex system extended to include these new numbers.

5. We investigate the construction of the functions $C(x)$ and $S(x)$ under the hypothesis that there exist the two real one-valued continuous functions of x which satisfy (8) subject to the definition of the integral number. Since

$$e^{ix}\, e^{iy} = e^{i(x+y)}$$

there results

$$C(x+y) + iS(x+y) = C(x)\ S(y) - S(x)\ S(y) + i\{C(x)\ S(y) + S(x)\ C(y)\}$$

Equating real and imaginary components,

$$\begin{aligned} C(x+y) &= C(x)\ C(y) - S(x)\ S(y), \\ S(x+y) &= C(x)\ S(y) + S(x)\ C(y). \end{aligned} \tag{8}$$

Also $e^{i(x-y)} = e^{ix}/e^{iy}$ gives in like manner

$$\begin{aligned} C\,(x-y) &= \frac{C\,(x)\ C(y) + S(x)\ S(y)}{C^2(y) + S^2(y)}, \\ S\,(x-y) &= \frac{S\,(x)\ C(y) - C(x)\ S(y)}{C^2(y) + S^2(y)}. \end{aligned} \tag{9}$$

In (8) put $y = 0$ and solve, whence $C(0) = 1$ and $S\,(0) = 0$. In the same equations put $y = -x$, whence*

$$C^2(x) + S^2(x) = \frac{C\,(x)}{C(-x)} = -\frac{S(x)}{S-(x)},$$

$$\{C^2(x) + S^2(x)\}\{C^2(-x) + S^2(-x)\} = 1. \tag{10}$$

Square and add equations (8). Also square and add equations (9), using the second of (10). Whence

$$C^2(x \pm y) + S^2(x \pm y) = \{C^2(x) + S^2(x)\}\ \{C^2(\pm y) + S^2(\pm y)\} \tag{11}$$

upper signs together and lower signs together.

Consider now the two functions $\varphi(x)$ and $\psi(x)$ defined by

* For brevity we write $[C(x)]^n \equiv C^n(x)$, etc.

$$\varphi(x) \equiv \frac{C(x)}{\sqrt{C^2(x)+S^2(x)}}, \qquad \psi(x) \equiv \frac{S(x)}{\sqrt{C^2(x)+S^2(x)}},$$

wherein the absolute value of the radical is taken. Clearly

$$\varphi^2(x) + \psi^2(x) = 1. \tag{12}$$

Divide equations (8) and also (9) by the square roots of the corresponding members of (11). Whence

$$\begin{aligned} \varphi(x \pm y) &= \varphi(x)\varphi(y) \mp \psi(x)\,\psi(y), \\ \psi(x \pm y) &= \psi(x)\,\varphi(y) \pm \varphi(x)\,\psi(y). \end{aligned} \tag{13}$$

Subtract the second from the first in (13), and then add the last two. There result

$$\begin{aligned} \varphi(x + y) - \varphi(x - y) &= -\,2\,\psi(x)\,\psi(y), \\ \psi(x + y) - \psi(x - y) &= +\,2\,\psi(x)\,\varphi(y). \end{aligned} \tag{14}$$

Since $S(0) = 0$ it follows that $\psi(0) = 0$, thence $\varphi(0) = 1$. The first of (14), when $x = 0$, shows that $\varphi(y) = \varphi(-y)$, the second that $\psi(y) = -\psi(-y)$. Hence φ is an even and ψ an odd function of x.

It has been demonstrated for real functions that any function $F(x)$ defined by the relation

$$F(x + y) = F(x)\,.\,F(y)$$

must be A^x where A is some positive constant. Equation (11) shows that $C^2(x) + S^2(x)$ is such a function.

$$\therefore\ C^2(x) + S^2(x) = A^x = e^{2cx}$$

where $2c = \log_e A$. Hence

$$e^{ix} = e^{cx}[\varphi(x) + i\psi(x)]. \tag{15}$$

6. The relations (12) and (13) together with the continuity of φ and ψ will serve to completely construct the numbers we seek. In (13) $y = x$ gives

$$\varphi(2x) = \varphi^2(x) - \psi^2(x), \qquad \psi(2x) = 2\varphi(x)\,\psi(x). \tag{16}$$

These show that if φ and ψ are continuous in any assigned interval $(-h, +h)$ then they must be continuous in the interval $(-2^nh, +2^nh)$ for any assigned integer n. Therefore it was only necessary to assume continuity in the neighborhood of zero.

Let $x = \frac{1}{2}(x'' + x') = x_m$and $y = \frac{1}{2}(x'' - x') = h$, in (14). Then

$$\varphi(x'') - \varphi(x') = -2\,\psi(x_m)\,\psi(h),$$
$$\psi(x'') - \psi(x') = +2\varphi(x_m)\,\psi(h). \tag{17}$$

7. The even function $\varphi(x)$, which can never be greater than 1, is continuous and since $\varphi(0) = 1$ the function is positive in the neighborhood of zero and cannot be equal to zero save for some finite or infinite value of x. Also $\psi(0) = 0$. If $\psi(x)$ remains constant and equal to 0 in the neighborhood of 0 equations (16) and (17) show that $\psi(x)$ is 0 and $\varphi(x)$ is 1 for all values of x which is not possible since then e^{ix} would be constant and equal to 1 which requires $x = 0$. Hence $\psi(x)$ must be different from 0 for some value of x in the positive neighborhood of 0, say at $x = h$, where $\psi(x)$ has either a positive or negative value numerically less than 1, we assume a positive value. Equations (17) show that as x' increases from 0, the interval $x'' - x' = h$ remaining constant, the function φ continues to decrease from 1 and ψ to increase from 0 as long as $\varphi(x_m)$ remains positive. If $\varphi(x)$ remains positive for all values of x the function ψ must continually increase and since ψ can never be greater than 1 the increasing variable $\psi(x)$ must have a superior boundary equal to or less than 1 for some finite or infinite value of x. It is not possible for $\psi(x)$ to increase continually to a superior boundary s when $x = +\infty$, for then φ must attain an inferior boundary

$$c = \sqrt{1 - s^2} \not< 0,$$

and (17) gives

$$0 = c - c = -2\,s\,\psi\,(h),$$

which is not possible since neither s nor $\psi(h)$ is zero. Therefore ψ cannot continue to increase for all values of x but must attain, since it is continuous, a superior boundary for some finite value of x, say at $x = \frac{1}{2}\pi$. Equations (17) give

$$\psi(\tfrac{1}{2}\pi) - \psi(\tfrac{1}{2}\pi - 2h) - 2\varphi(\tfrac{1}{2}\pi - h)\,\psi(h),$$
$$\psi(\tfrac{1}{2}\pi + 2h) - \psi(\tfrac{1}{2}\pi) = 2\varphi(\tfrac{1}{2}\pi + h)\,\psi(h).$$

The first of these shows that $\varphi(\frac{1}{2}\pi - h)$ is greater than zero. The second that $\varphi(\frac{1}{2}\pi + h)$ cannot be greater than zero. Hence the common limit $\varphi(\frac{1}{2}\pi)$, for small values of h, must be zero. Therefore $\varphi(\frac{1}{2}\pi) = 0$ and $\psi(\frac{1}{2}\pi) = 1$.

8. In (13) put $\frac{1}{2}x$ for x and y,

$$\begin{aligned} \therefore \varphi(x) &= \varphi^2(\tfrac{1}{2}x) - \psi^2(\tfrac{1}{2}x), \\ &= 2\varphi^2(\tfrac{1}{2}x) - 1, \qquad (18) \\ &= 1 - 2\,\psi^2(\tfrac{1}{2}x). \\ \psi(x) &= 2\varphi(\tfrac{1}{2}x)\,\psi(\tfrac{1}{2}x). \qquad (19) \end{aligned}$$

$$\varphi(\tfrac{1}{2}\pi) = 0 \quad \text{gives} \quad \varphi(\tfrac{1}{4}\pi) = \psi(\tfrac{1}{4}\pi) = 1/\sqrt{2}$$

also shows

$$\varphi(\tfrac{1}{2}\pi - x) = \psi(x), \qquad \psi(\tfrac{1}{2}\pi - x) = \varphi(x).$$

$x = \frac{1}{2}\pi$, $y = \frac{1}{2}\pi$ in (13) gives $\varphi(\pi) = -1$, $\psi(\pi) = 0$. It can be shown in like manner for n any integer

$$\varphi(n\pi) = (-1)^n, \qquad \varphi\left(\frac{2n+1}{2}\pi\right) = 0,$$

$$\psi(n\pi) = 0, \qquad \psi\left(\frac{2n+1}{2}\pi\right) = (-1)^n.$$

The same equations show by an easy induction

$$\varphi(x + 2n\pi) = \varphi(x), \qquad \psi(x + 2n\pi) = \psi(x),$$

consequently φ and ψ are periodic functions having the common period 2π.

9. *Theorem.* When x converges to zero in any manner the quotient $\psi(x)/x$ converges to a determinate superior limit k.

The number $\psi(x)$ increases continuously from 0 to 1 and $\varphi(x)$ decreases continuously from 1 to 0 as x increases from 0 to $\frac{1}{2}\pi$. In particular $\psi(x)$ increases continuously from 0 to $\frac{1}{2}\sqrt{2}$ and $\varphi(x)$ decreases from 1 to this same value as x increases from 0 to $\frac{1}{4}\pi$. Let n be any arbitrarily great assigned integer and $h = \frac{1}{4}\pi/n$. Then in

$$\psi(x + h) - \psi(x) = 2\varphi(x + \tfrac{1}{2}h)\,\psi(\tfrac{1}{2}h),$$

as x varies from 0 to $\frac{1}{4}\pi$ the left side is positive and continually diminishes. Let $x_0 = 0$ and $x_n = \frac{1}{4}\pi$, then

$$[\psi(x_n) - \psi(x_{n-1})] + \ldots + [\psi(x_1) - \psi(x_0)] = \tfrac{1}{2}\sqrt{2}.$$

Dividing by h there results

$$\frac{\psi(x_n) - \psi(x_{n-1})}{h} + \cdots + \frac{\psi(x_1) - \psi(x_0)}{h} = 2\sqrt{2}\,\frac{n}{\pi}.$$

The quotients on the left are all positive, the least and the greatest are respectively the first and the last, and are

$$\mu = \frac{\psi(x_n) - \psi(x_{n-1})}{h} = \frac{\psi(\frac{1}{4}\pi) - \psi(\frac{1}{4}\pi - h)}{h},$$

$$M = \frac{\psi(x_1) - \psi(x_0)}{h} = \frac{\psi(h)}{h}.$$

$$\therefore \quad n\mu < 2\sqrt{2}\frac{n}{\pi} < nM,$$

or

$$\mu < \frac{2\sqrt{2}}{\pi} < M.$$

Also

$$\mu = \varphi(\tfrac{1}{4}\pi - \tfrac{1}{2}h)\frac{\psi(\frac{1}{2}h)}{\frac{1}{2}h}.$$

$$\therefore \quad \varphi(\tfrac{1}{4}\pi - h)\frac{\psi(\frac{1}{2}h)}{\frac{1}{2}h} < \frac{2\sqrt{2}}{\pi} < \frac{\psi(h)}{h} = \varphi(\tfrac{1}{2}h)\frac{\psi(\frac{1}{2}h)}{\frac{1}{2}h}$$

Whence for all small values of $x = \frac{1}{2}h$,

$$\frac{\pi}{2\sqrt{2}}\varphi(\tfrac{1}{4}\pi - x) < \frac{x}{\psi(x)} < \frac{\pi}{2\sqrt{2}}\varphi(x).$$

Since $\varphi(\frac{1}{4}\pi - x) > \varphi(\frac{1}{4}\pi)$ and $\varphi(x) > 0$, there results

$$\frac{2\sqrt{2}}{\pi} < \frac{\psi(x)}{x} < \frac{4}{\pi},$$

and consequently $\psi(x)/x$ remains finite and positive as x converges to zero. Moreover

$$\frac{\psi(x)}{x} = \varphi(\tfrac{1}{2}x)\frac{\psi(\frac{1}{2}x)}{\frac{1}{2}x}.$$

Since $0 < \varphi(\frac{1}{2}x) < 1$,

$$\frac{\psi(\frac{1}{2}x)}{\frac{1}{2}x} > \frac{\psi(x)}{x},$$

and the sequence of increasing numbers

$$\frac{\psi(x/2)}{x/2}, \quad \frac{\psi(x/2^2)}{x/2^2}, \quad \ldots$$

is a regular sequence assigning a finite positive number k as a superior limit. By repeated application of (20)

$$\frac{\psi\,(x)}{x} = \varphi\left(\frac{x}{2}\right)\varphi\left(\frac{x}{2^2}\right)\cdot\cdot\cdot\varphi\left(\frac{x}{2^n}\right)\cdot\frac{\psi\,(x/2^n)}{x/2^n}.$$

The product of the φ's in this expression can be written

$$\left[1-2\,\psi^2\left(\frac{x}{2^2}\right)\right]\cdot\cdot\cdot\left[1-2\,\psi^2\left(\frac{x}{2^{n+1}}\right)\right].$$

This product is absolutely convergent and different from zero when $n = \infty$ since the series

$$\psi^2\left(\frac{x}{2^2}\right)+\psi^2\left(\frac{x}{2^3}\right)+\cdot\cdot\cdot,$$

(having for its quotient of convergence $\frac{1}{4}$ in virtue of the limit k) is absolutely convergent. Hence when $n = \infty$

$$\frac{\psi\,(x)}{x} = k\,\varphi\left(\frac{x}{2}\right)\varphi\left(\frac{x}{2^2}\right)\cdot\cdot\cdot \tag{20a}$$

in which for $x \neq 0$ the product of the φ's is a positive number greater than 0 and less than 1, since each φ is less than 1. This product continually increases as x converges to 0 since each factor increases, hence it has a superior limit equal to or less than 1, and the product is convergent for $x = 0$. But in any convergent infinite product the limit of the product of all terms after the nth is unity when $n = \infty$. Whatever ϵ be chosen there can be always assigned an n such that

$$1-\varphi\left(\frac{x}{2^{n+1}}\right)\varphi\left(\frac{x}{2^{n+2}}\right)\cdot\cdot\cdot < \epsilon,$$

for any x for which the product is convergent. All the more so is

$$1-\varphi\left(\frac{0}{2^{n+1}}\right)\varphi\left(\frac{0}{2^{n+2}}\right)\cdot\cdot\cdot < \epsilon,$$

and since for any assigned n

$$\varphi\left(\frac{0}{2}\right)\cdot\cdot\cdot\varphi\left(\frac{0}{2^n}\right) \equiv 1$$

$$\therefore\quad 1-\varphi\left(\frac{0}{2}\right)\varphi\left(\frac{0}{2^2}\right)\cdot\cdot\cdot\qquad\epsilon$$

Therefore the product in (21) converges to 1 as x goes to 0. Hence whatever be the way in which x converges to 0 the limit of $\psi(x)/x$ is k.

10. Let $T(x) \equiv \psi(x)/\varphi(x)$, then as x increases from 0 to $\frac{1}{4}\pi$ the function $T(x)$ continuously increases from 0 to 1. When x converges to 0 the limit of

$$\frac{T(x)}{x} = \frac{\psi(x)}{x}\,\frac{1}{\varphi(x)}$$

is clearly k. Whatever be the real number m the limit for $m = \infty$ of

$$\left[\varphi\left(\frac{x}{m}\right)\right]^{m} = \left[1 - \psi^2\left(\frac{x}{m}\right)\right]^{\frac{m}{2}},$$

$$= \left\{\left[1 - \psi^2\left(\frac{x}{m}\right)\right]^{-\frac{1}{\psi^2\left(\frac{x}{m}\right)}}\right\}^{-\frac{x^2}{2m}\left(\frac{\psi(x/m)}{x/m}\right)^2}$$

is $e^{0} = 1$. For any positive integer m the defining law $(e^{ix})^{m} = e^{imx}$ gives

$$[\varphi(x) + i\psi(x)]^{m} = \varphi(mx) + i\psi(mx).$$

The left side can be written in $m + 1$ terms by the binomial formula. On equating the real and the imaginary components

$$\varphi(mx) = \varphi^{m}(x)\,[1 - C_{m,2}T^2(x) + C_{m,4}T^4(x) - \ . \ . \ . \ ],$$
$$\psi(mx) = \varphi^{m}(x)\,[mT(x) - C_{m,3}T^3(x) + \ . \ . \ . \ ].$$

In the first of these put x/m for x, thence $\varphi(x)/\varphi^{m}\left(\dfrac{x}{m}\right)$ is equal to

$$1 - \left(1 - \frac{1}{m}\right)\frac{x^2}{2!}\left(\frac{T(x/m)}{x/m}\right)^2 + \left(1 - \frac{1}{m}\right)\left(1 - \frac{2}{m}\right)\left(1 - \frac{3}{m}\right)\frac{x^4}{4!}\left(\frac{T(x/m)}{x/m}\right)^4 + \cdots$$

Each term of this sum, when m is sufficiently great, is less than the corresponding term of the absolutely convergent infinite series.

$$1 - \frac{x^2}{2!}K^2 + \frac{x^4}{4!}K^4 - \cdots$$

in which K is some assigned positive number greater than k. Therefore when m is infinite the sum is absolutely convergent and

$$\varphi(x) = 1 - \frac{(kx)^2}{2!} + \frac{(kx)^4}{4!} - \cdots \tag{20}$$

In the same way from the expression for $\psi(mx)$ there results

$$\psi(x) = kx - \frac{(kx)^3}{3!} + \frac{(kx)^5}{5!} - \cdots \qquad (21)$$

These series are one-valued absolutely convergent series for all assigned real values of k and x. They define the functions φ and ψ and construct them as was required whenever the constant k is determined.

10. The construction of e^{ix} is therefore

$$\left\{\left(1 - \frac{x^2}{2!}k^2 + \cdots\right) + i\left(kx - \frac{x^3}{3!}k^3 + \cdots\right)\right\} e^{cx},$$

which can be written

$$\left(1 + ikx + \frac{(ikx)^2}{2!} + \cdots\right) e^{cx},$$

or

$$1 + (c + ik)\,x + \frac{(c+ik)^2}{2!}x^2 + \cdots, \qquad (22)$$

a series absolutely convergent for all real values of x, c and k. To determine the constants c and k we observe the defining law of e^{ix} requires

$$(e^{ix})^{iy} = e^{-xy},$$

and in particular $(e^{ix})^i = e^{-x}$ is a real function of the real variable x and is the real exponential function when $e = 2.71\ \ldots$ Furthermore $(e^{ix})^i$ is nothing more or less than the value of e^{ix} at the value ix of x, for this is e^{i^2x} or e^{-x}. The number e^{ζ} is therefore constructed for all values of ζ in the real and in the imaginary continuum. As x varies continuously from x in the real continuum through zero to ix in the imaginary continuum the function (22) varies continuously from its value in (22) to the value

$$1 + (ic - k)\,x + \frac{(ic-k)^2}{2!}x^2 + \cdots$$

which is identical with

$$1 - x + \frac{x^2}{2!} - \cdots$$

for all values of x. Consequently

$$(ic - k)^n = (-1)^n$$

or $ic - k = -1$, whence $c = 0$ and $k = 1$, when $e = 2.71 \ldots$ The functions φ and ψ are therefore

$$\varphi(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$$

$$\psi(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$$

Had we begun with the construction a^{ix} where a is any positive constant other than e we should have found k equal to $\log_e a$.

11. The construction of $e^z = e^x e^{iy}$ is, on multiplying the series

$$e^z = 1 + z + \frac{z^2}{2!} + \cdots$$

for all values for the complex variable z.

The even and odd parts of e^z are named the hyperbolic cosine and sine of z. The even and odd parts of e^{iz} are named the circular cosine and sine of z. In symbols written respectively

$$\cosh z = 1 + \frac{z^2}{2!} + \cdots, \qquad \sinh z = z + \frac{z^3}{3!} + \cdots,$$

$$\cos z = 1 - \frac{z^2}{2!} + \cdots, \qquad \sin z = z - \frac{z^3}{3!} + \cdots.$$

In general for any integral number z

$$e^z = \cosh z + \sinh z,$$
$$e^{iz} = \cos z + i \sin z.$$

Also

$$\cosh iz = \cos z, \qquad \sinh iz = i \sin z,$$
$$\cos iz = \cosh z, \qquad \sin iz = i \sinh z.$$

The theory of these functions can now be analytically developed.

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BULLETIN

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PHILOSOPHICAL SOCIETY

Scientific Series, Vol. I, No. 14, pp. 331-338, February, 1918

Magmatic Names proposed in the Quantitative System of Classification for some new rock types in Virginia

BY

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UNIVERSITY OF VIRGINIA PUBLICATIONS

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BULLETIN OF THE PHILOSOPHICAL SOCIETY
SCIENTIFIC SECTION
Vol. I, No. 14, pp. 331–333 February, 1913

MAGMATIC NAMES PROPOSED IN THE QUANTITATIVE SYSTEM OF CLASSIFICATION FOR SOME NEW ROCK TYPES IN VIRGINIA.*

BY

THOMAS L. WATSON AND STEPHEN TABER.

The purpose of the present note is to place on record at this time the magmatic names proposed by the writers for some new rock types occurring in the Amherst-Nelson counties rutile area in Virginia. The rocks are described in detail in the forthcoming report on the rutile deposits of Virginia by the Virginia Geological Survey. Only the magmatic names suggested for the rocks, their chemical analyses and calculated norms, and their position in the quantitative system of classification of igneous rocks are included in this note; the geology of the area and petrographic descriptions of the rocks being reserved for later publication. The rock specimens yielding the results tabulated below are from Nelson County, Virginia.

Chemical analyses of igneous rocks from Nelson County, Virginia.
(*Wm. M. Thornton, Jr., analyst.*)

	I	II	III	IV
SiO_2	61.51	33.83		0.67
Al_2O_3	23.17	5.19		
Fe_2O_3	0.37	11.38	2.70	2.87
FeO	0.26	15.08	29.14	5.04
MgO	0.26	8.57	0.50	0.15
CaO	5.96	8.22	16.05	12.16
Na_2O	4.64	1.28		
K_2O	3.94	0.50		
H_2O −	0.08	0.45	0.03	0.09
H_2O +	0.32	0.75	none	0.11
TiO_2	0.29	10.00	37.68	9.41
P_2O_5	0.10	4.84	12.48	69.67

* Presented and read before the Scientific Section in February, 1913.

Chemical analyses of igneous rocks from Nelson County, Virginia—Continued

	I	II	III	IV
MnO	trace	0.26	trace	
S		0.25	1.17	0.34
F	none	0.55	1.03	0.70
Cl	none	0.04	trace	trace
CO_2	trace	trace	none	none
	100.90	101.39	100.78	101.21
Excess oxygen		0.30	0.72	0.39
Total	100.90	101.09	100.06	100.82

I, Piedmontose (sodic syenite); II, Roselandose (gabbro-nelsonite); III, Nelsonose (ilmenite-nelsonite); IV, Virginose (rutile nelsonite).

The norms as calculated from these analyses and arranged in the same order are given in the subjoined table.

Norms corresponding to analyses on page 331

Q	6.66		7.38				0.42	0.42
Or	23.35		2.78					
Ab	39.30	98.65	11.00	28.11				
An	28.63		6.95					
C	0.71							
Hy	0.60		22 55				0.40	
Di			1.51					
Il	0.61		19.15		60.65		9.73	
Mt		1.83	16.47	71.52				100.37
Hm	0.48				2.72	100.88	2.88	
Ru					5.76		64.56	
Ap	0.14		11.42		29.57		22.18	
Pr			0.42		2.18		0.62	
Rest	0.40		1.20		0.03		0.20	
	100.88		100.83		100.91		100.99	
Excess F, Cl			0.15					
Less excess CaO					0.39			
			100.98		100 52			

The four rocks represented by the analyses and norms given in tabular form above are new types, and they fall into unoccupied positions in the quantitative system. Since they fall within the limits of the classificatory scheme, and are the first representatives of the types to be described, new magmatic names are proposed for them.

The names proposed by the writers as appropriate for the four new rock-types may be tabulated as follows:

NO.	SYMBOL	NAME	OLD NAME
I	I. 5. 3. I. 5. 3. 4.	*piedmontase* (rang) *piedmontose* (subrang)	Sodic syenite
II	IV. $_{\text{II.}}$ 3. 1. 2. IV. $_{\text{II.}}$ 3. 1. 2. 3.	*roselandase* (rang) *roselandose* (subrang)	Gabbro nelsonite
III	V. $_{\text{II.}}$ 5. V. $_{\text{II.}}$ 5. 5. V. $_{\text{II.}}$ 5. 5. 3. V. $_{\text{II.}}$ 5. 5. 3. 5.	*virginare* (order) *virginore* (section) *nelsonase* (rang) *nelsonose* (subrang)	Ilmenite nelsonite
IV	V. $_{\text{II.}}$ 5. V. $_{\text{II.}}$ 5. 5. V. $_{\text{II.}}$ 5. 5. 4. V. $_{\text{II.}}$ 5. 5. 4. 5.	*virginare* (order) *virginore* (section) *virginase* (rang) *virginose* (subrang)	Rutile nelsonite

In summary No. I (sodic syenite) is a canadare persalane, alkalicalcic and dosodic, for which the rang and subrang names *piedmontase* and *piedmontose* are proposed; No. II (gabbro-nelsonite) is a dofemane, polmitic and perpyric, of the domiric rang and magnesiferrous subrang for which the names *roselandase* and *roselandose* are suggested; Nos. III and IV (ilmenite nelsonite and rutile nelsonite) are perfemanes of the same subclass, order, and section, but differ in the rang and subrang positions. For the order and section the names *virginare* and *virginore*, and the rang and subrang of No. III the names *nelsonase* and *nelsonose*, and of No. IV *virginase* and *virginose*, are proposed.

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BULLETIN

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On the Root of a Monogenic Function Inside a Closed Contour Along Which the Modulus is Constant.

BY

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BULLETIN OF THE PHILOSOPHICAL SOCIETY
SCIENTIFIC SECTION

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ON THE ROOT OF A MONOGENIC FUNCTION INSIDE A CLOSED CONTOUR ALONG WHICH THE MODULUS IS CONSTANT.*

BY

W. H. ECHOLS

1. The basis of Cauchy's theory of functions reposes upon the integral taken around a closed boundary. If the function is regular along and inside this boundary the integral of the function taken around the boundary vanishes. When the closed boundary is a *contour* or a simple closed curve C along which the modulus M of a monogenic function $f(z) = u + iv$ is constant the theory fails owing to the fact that along the contour C

$$\int f(z)\ dz = M \int e^{i\varphi}\ dz,$$

in which $\varphi = e^{i \tan^{-1}\frac{v}{u}}$ is not analytic except when $f(z)$ is a constant. For, in order that the derivative of ϕ with respect to $z = x + iy$ shall be independent of dy/dx it is necessary that

$$i\frac{\partial\varphi}{\partial x} = \frac{\partial\varphi}{\partial y}, \text{ and} \frac{\partial}{\partial y}(u^2 + v^2) = i\frac{\partial}{\partial x}(u^2 + v^2),$$

which can only be true when ϕ and $u^2 + v^2$ are constants.

When we seek the roots of a function $f(z)$ within the contour C through Cauchy's integral taken around C

$$\int d \log f(z) = i\int_0^{2\pi} d\varphi = 2\pi i,$$

*Presented and read before the Scientific Section at its regular meeting in March, 1913.

the theorem fails for the reasons above. In fact the theorem for an ordinary closed boundary (not a contour) on and within which $f(z)$ is regular furnishes a root of $f(z)$ for each pole of $f'(z)/f(z)$ inside the boundary, and if there be n roots the integral is $2\pi in$. Cauchy's integral must therefore be abandoned in the study of the function within the contour when based on the contour integral.

2. The demonstration of the following theorem is so simple that it is almost incredible to believe it new, but as one does not find it in the classic treatises on analysis it is thought worth while to give this note.

Theorem. Every monogenic function $f(z)$ has a root inside of any simple closed contour along which $|f(z)|$ is constant and within which $f(z)$ is regular.

Let the equation of the contour C be

$$|f(z)|^2 = u^2 + v^2 = M^2 = \text{constant}.$$

Consider the surface $\zeta = u^2 + v^2$. This function of x and y is finite, continuous and unlimitedly differentiable at all points inside C. It therefore has an upper or a lower boundary at some point x,y in this region which definite maximum or minimum value ζ must attain since it is continuous. At this point therefore

$$\begin{aligned}
\tfrac{1}{2}\frac{\partial\zeta}{\partial x} &= \quad u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial x} = 0,\\
\tfrac{1}{2}\frac{\partial\zeta}{\partial y} &= \quad u\frac{\partial u}{\partial y} + v\frac{\partial v}{\partial y},\\
&= -u\frac{\partial v}{\partial x} + v\frac{\partial u}{\partial x} = 0.
\end{aligned}$$

On squaring and adding there must result at x, y

$$(u^2 + v^2)\left\{\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2\right\} = 0,$$

or what is the same thing

$$f(z) \,.\, f'(z) = 0.$$

At x,y either the function $f(z)$ is zero, or both $f(z)$ and $f'(z)$ are zero and the theorem is proved, unless $f(z) \neq 0$ and $f'(z) = 0$. At the point x, y must occur a true maximum or minimum of ζ.

3. We will now show that it is not possible for ζ to have a maximum or minimum value where $f(z) \neq 0$ and

$$f^r(z) = 0, \qquad\qquad r = 1, 2, \ldots, n-1,$$

and therefore $f(z)$ must be zero at x, y. The discriminant

$$\frac{\partial^2\zeta}{\partial x^2}\frac{\partial^2\zeta}{\partial y^2} - \left(\frac{\partial^2\zeta}{\partial x\partial y}\right)^2$$

at x, y has the value

$$\tfrac{1}{4}\left\{\frac{\partial(u, v)}{\partial(x, y)}\right\}^2 - \tfrac{1}{2}\left\{u^2\left(\frac{\partial^2 u}{\partial x^2}\right)^2 + v^2\left(\frac{\partial^2 v}{\partial y^2}\right)^2\right\},$$

which is positive when $f(z) = 0$ and $f'(z) \neq 0$, and since then

$$\tfrac{1}{2}\frac{\partial^2\zeta}{\partial x^2} = \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2$$

is positive there is a true minimum at x, y. If $f(z)$ is different from zero and $f'(z) = 0$ the discriminant is negative and there is neither maximum nor minimum at x, y unless also $f''(z) = 0$, in which case the discriminant vanishes and it is necessary to proceed generally and investigate the condition at x, y when $f^r(z) = 0$ for $r = 1, 2, \ldots, n - 1$.

4. Consider the general nth derivative of $f(z)$ as to z.

$$f^n(z) = \frac{d^n(u + iv)}{(dx + i\,dy)^n} = \frac{\left(dx\frac{\partial}{\partial x} + dy\frac{\partial}{\partial y}\right)^n u + i\left(dx\frac{\partial}{\partial x} + dy\frac{\partial}{\partial y}\right)^n v}{(dx + i\,dy)^n}$$

Let $dx = l\,dr = dr\cos\theta$, $dy = m\,dr = dr\sin\theta$. Since $f^n(z)$ is independent of dy/dx or θ there results for all values of θ

$$(\cos n\theta + i\sin n\theta)\left(\frac{\partial^n u}{\partial x^n} + i\frac{\partial^n v}{\partial x^n}\right)$$
$$= \left(l\frac{\partial}{\partial x} + m\frac{\partial}{\partial y}\right)^n u + i\left(l\frac{\partial}{\partial x} + m\frac{\partial}{\partial y}\right)^n v.$$

Hence

$$\left(l\frac{\partial}{\partial x} + m\frac{\partial}{\partial y}\right)^n u = \frac{\partial^n u}{\partial x^n}\cos n\theta - \frac{\partial^n v}{\partial x^n}\sin n\theta,$$
$$\left(l\frac{\partial}{\partial x} + m\frac{\partial}{\partial y}\right)^n v = \frac{\partial^n u}{\partial x^n}\sin n\theta + \frac{\partial^n v}{\partial x^n}\cos n\theta.$$

Now at the point x, y all the derivative of $f(z)$ below the nth vanish, therefore for $p = 1, \ldots, n - 1$,

$$\frac{d^p u}{dr^p} = \left(l\frac{\partial}{\partial x} + m\frac{\partial}{\partial y}\right)^p u = 0, \quad \frac{d^p v}{dr^p} = \left(l\frac{\partial}{\partial x} + m\frac{\partial}{\partial y}\right)^p v = 0.$$

Also in general

$$\frac{d^p\zeta}{dr^p} = 2\left(u\frac{d^p u}{dr^p} + C_{p,1}\frac{du}{dr}\frac{d^{p-1}u}{dr^{p-1}} + \cdots + \frac{du}{dr}\frac{d^{p-1}u}{dr^{p-1}}\right)$$
$$+ 2\left(v\frac{d^p v}{dr^p} + C_{p,1}\frac{dv}{dr}\frac{d^{p-1}v}{dr^{p-1}} + \cdots + \frac{dv}{dr}\frac{d^{p-1}v}{dr^{p-1}}\right)$$

These derivatives vanish at x, y for $p = 1, \ldots, n-1$, while the nth derivative at that point has the value

$$\begin{aligned}
\tfrac{1}{2}\frac{d^n\zeta}{dr^n} &= u\frac{d^n u}{dr^n} + v\frac{d^n v}{dr^n}, \\
&= \left(u\frac{\partial^n u}{\partial x^n} + v\frac{\partial^n v}{\partial x^n}\right)\cos n\theta + \left(v\frac{\partial^n u}{\partial x^n} - u\frac{\partial^n v}{\partial x^n}\right)\sin n\theta, \\
&= +\left(u\frac{\partial^n u}{\partial x^n} + v\frac{\partial^n v}{\partial x^n}\right), \text{ when } n\theta = 0 \\
&= -\left(u\frac{\partial^n u}{\partial x^n} + v\frac{\partial^n v}{\partial x^n}\right), \text{ when } n\theta = \pi \\
&= v\frac{\partial^n u}{\partial x^n}\sin n\theta, \text{ when } u = 0, \frac{\partial^n v}{\partial x^n} = 0,
\end{aligned}$$

and the last value changes sign with $n\theta$. Hence it is impossible for x, y to be a point at which ζ has its minimum. Therefore $f(z)$ must have a root at x, y unless $f(z)$ is merely a constant throughout the region C.

Cor. A polynomial $f(z)$ of degree n cannot be zero for any value $|z| > n\rho$ where ρ is the absolute value of the quotient of the coefficient having the greatest absolute value to that of z^n. Hence for an assigned $\alpha > n\rho$

$$|f(z)| = |f(\alpha)|$$

is a simple closed contour in which $f(z)$ has at least one root, and therefore as easily follows n roots.

5. If at a second point in C the first $n-1$ derivatives of $f(z)$ vanish and there $f(z) \neq 0$, then the nth derivative of $f(z)$ vanishes there for n distinct values of θ and if

$$\zeta = m^2 = u^2 + v^2,$$

$m = |f(a)|$ at this point, this is a curve which has a multiple node at the point with n distinct loops inside C, in each one of these loops $f(z)$ has a root.

6. In case at the critical point x, y both $f(z)$ and $f'(z)$ are zero, then the first derivative of ζ which does not vanish is the fourth, which at the point is given by

$$\frac{1}{6}\frac{d^4\zeta}{dr^4} = \left(\frac{d^2u}{dr^2}\right)^2 + \left(\frac{d^2v}{dr^2}\right)^2,$$

which is positive when $f''(z) \neq 0$, and at the point ζ is a minimum.

In general if at x, y we have $f^r(z) = 0$ when $r = 0, 1, \ldots, n-1$ then the first derivative of ζ which is not zero there is given by

$$\frac{1}{n(n+1)}\ \frac{d^{2n}\zeta}{dr^{2n}} = \left(\frac{d^nu}{dr^n}\right)^2 + \left(\frac{d^nv}{dr^n}\right)^2,$$

which, being positive when $f^n(z) \neq 0$, there results a minimum for ζ at x, y.

7. If a function $f(z)$ is regular at all points of a region S bounded by a simple closed curve s, and if at any point P inside s the modulus of $f(z)$ is less than the least value of the modulus on s then $f(z)$ has at least one root inside s. For if m is the least value of $|f(z)|$ along s then

$$|f(z)| = m$$

is a closed contour in the region S and contains P.

8. The functions u and v being conjugate

$$\frac{\partial^{2n}u}{\partial x^{2n}} = (-1)^n\frac{\partial^{2n}v}{\partial y^{2n}},\quad \frac{\partial^{2n}u}{\partial y^{2n}} = (-1)^{n+1}\frac{\partial^{2n}v}{\partial x^{2n}}.$$

The results of § 4 show that

$$\frac{d^{2n}u}{dr^{2n}} = \frac{\partial^{2n}u}{\partial x^{2n}}\cos n\theta + (-1)^n\frac{\partial^{2n}u}{\partial y^{2n}}\sin n\theta.$$

This changes sign as $n\theta$ changes, and therefore, as is well known, the harmonic function cannot have a maximum or minimum value at any point in the x, y plane.

15. On the Root of a Monogenic Function inside a closed Contour along which the Modulus is Constant. By W. H. Echols. Vol. I, pp. 335–339. Price, $0.25.

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Normal Faulting in the Cambrian of Northern Piedmont, Virginia

BY

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UNIVERSITY OF VIRGINIA
Charlottesville, Virginia, U. S. A.

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NORMAL FAULTING IN THE CAMBRIAN OF NORTHERN PIEDMONT VIRGINIA.*

BY

THOMAS L. WATSON AND JUSTUS H. CLINE

A belt of Lower Cambrian rocks†, roughly paralleling the Blue Ridge and lying just beyond its southeast slope, extends almost entirely across Virginia in a northeast-southwest direction. The western limits of this belt are fairly well known and are delineated with reasonable accuracy; its eastern limits are yet somewhat in doubt and not so accurately drawn because of lack chiefly of detailed field studies. A line connecting the cities of Warrenton, Charlottesville, and Lynchburg gives both the approximate trend and location of this belt. Its width is variable and the belt presumably occupies a synclinal trough in the pre-Cambrian crystallines of the Piedmont Plateau.

What members of the Lower Cambrian are represented in this belt has not been definitely determined. Much of it, in places at least, is the Loudoun formation which has an estimated thickness up to 800 feet. It is composed of material derived from the pre-Cambrian intrusive granitic rocks and extrusive basalts (Catoctin schist). Its basal portions are coarse arkosic and quartzitic sandstones, which grade upward into finer grained sediments, originally shales, in part graphitic intercalated with thin beds of sandstone, and in places a very coarse conglomerate. Limestone lenses of variable thickness and length are common in places throughout the extent of the belt.

In the southern portion of the belt the structure, while not worked out in detail, shows in places steeper dips of a more closed type of folding,

* Read before Scientific Section, April, 1913.

† Watson, Thomas L., Geological map of Virginia, Virginia Geological Survey, 1911.

and the arkoses have been converted into gneisses and the finer material into slates and schists. In Fauquier and Culpeper counties and in other adjacent sections, the structure is more simple. Folding has not been so intense, and metamorphism is correspondingly less, hence it is possible to solve the structural problems presented in this portion (northern) of the belt with less difficulty.

During recent investigations by the Virginia Geological Survey in the Fauquier-Culpeper counties slate district, embracing an area of about 50 square miles, it was found that the conditions were especially favorable for deciphering with reasonable accuracy the structure of the Cambrian rocks. After careful detailed study and mapping of the area it was found that the structural conditions were not those of closed folds and thrust faults but dominantly those of open folds and normal faults, and in the latter respect very similar to the Triassic areas of the eastern United States.

Rocks of both pre-Cambrian and Cambrian age occur in the area. The areal distribution of the different formations is shown on the accompanying geologic map, figure 1. The pre-Cambrian rocks include granite, gneiss, schist, and extrusive basalt (Catoctin schist), the latter being much the most abundant. Schistosity is especially characteristic of the basalts (Catoctin schist). Overlying the pre-Cambrian rocks are Cambrian beds of the Loudoun group forming a northeast-southwest belt that is more or less continuous throughout the central part of the area. The lithologic character of these sediments is remarkably similar to those found throughout the entire extent of the Cambrian belt in western Piedmont Virginia. They are made up of arkoses, sandstones, and shales, more or less foliated, but rarely has the metamorphism been so intense as to entirely obliterate the original bedding. Schistosity occurs but it is not so highly developed as in the basic igneous rocks and is confined for the most part to certain sheared zones. In many places in the basal sandstone (arkosic and quartzitic members), cross-bedding is perfectly preserved; there is no evidence of foliation (schistosity). Small intrusions of basic igneous rocks of the gabbroic type have penetrated the sediments in parts of the district.

The two dominant types of structure occurring in the area are folding and normal faulting. The folding is mostly of the open type with the axes extending in a general northeast-southwest direction. There seem to be the remnants of two anticlinoria partially included within the area. The structure of these has been greatly complicated by numerous faults which cut the rocks of the area in all directions. Minute structures, such as plications which result from intense folding, are limited in occurrence.

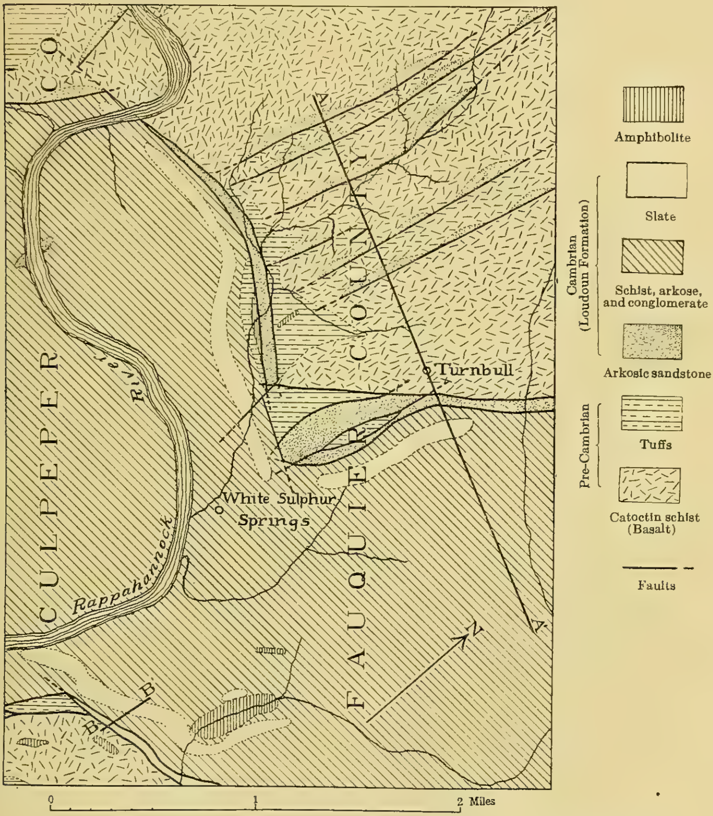

Fig. 1. Geologic Map of a Part of the Fauquier-Culpeper Counties, Virginia, Slate Belt

Both normal and thrust faults occur. Where the latter are developed the thrust is commonly along the bedding planes. Thrusts are also found in the igneous rocks indicated by slickensides. The extent of the thrusts cannot be determined but they have probably produced small displacements only.

Block faulting is now the most conspicuous and interesting structural feature of the area, and the present distribution of the outcrops is largely due to its influence. Normal faults are very common both on a large and small scale. These faults run in all directions and intersect each other at any angle. A fine exhibition of block faulting occurs west of the road leading from Warrenton to White Sulphur Springs (fig. 2 geologic section along line A A' of map, fig. 1). Parallel with this road is a series of belts of isolated units of basal Cambrian sandstone (arkosic and quartzitic) which are undoubtedly the remnants of fault blocks. The overlying shale and slate have been entirely removed by erosion and only a small part of the basal deposits remains. These belts are monoclinal in structure. The sandstone and fine conglomerate of which they are composed exhibit only slight metamorphism, except secondary enlargement. Cross-bedding is one of the common characteristics and it is very perfectly preserved. These remnants lie upon the Catoctin schist and dip toward the southeast at angles of from 20 to 80 degrees, and the strike varies from north to northwest. Erosion has here removed all the interbedded sandstones and shales which were deposited above the basal sandstone, and the structural difficulties have thereby been reduced to a minimum. In other parts of the district similar blocks of the same material dip and strike in various directions, even at right angles to the axis of the folds of the area.

It will be seen by consulting the map, figure 1, that these parallel belts of disconnected sandstone outcrops, separated by Catoctin schist, extend from beyond the border of the area in the extreme northern part to within about one mile of Rappahannock River, and suddenly stop. Between this point and the river the strike is generally in a northwesterly direction. This change in strike, as indicated on the map, is attributed to normal faulting in a transverse direction. It is this that has given rise to the northwesterly trend of one of the slate belts. Because of this structural relation, the slate has not been entirely removed by erosion from this block.

In a small stream entering Rappahannock River near the eastern edge of the Gaines farm a very good structure section across a part of this fault block is observed. Some interesting relations of bedding and cleav-

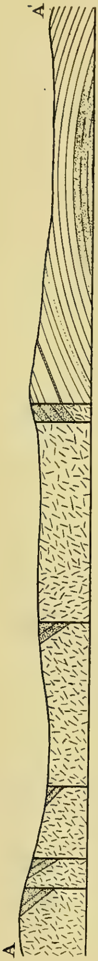

Fig. 2. Structure Section Along Line AA' of Map, Figure 1
Legend same as map, figure 1. Horizontal scale 1 mile = 2.3 inches.

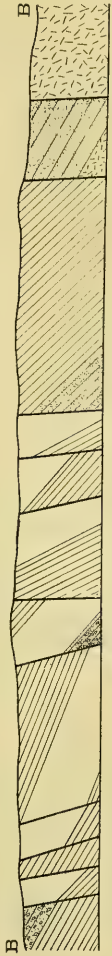

Fig. 3. Structure Section Along Line BB' of Map, Figure 1
Legend same as map, figure 1. Horizontal scale 1 inch = 153 feet.

age exist here, showing that the rocks have been affected at different times by forces applied in different directions. At one point in the bed of this stream the bedding strikes in an east-west direction, while the strike of cleavage is north-south. The slates outcropping on the hill nearby, show that they have been affected by the same movements; the slaty cleavage shows some folding.

South of the river the bedding again assumes its normal northeast-southwest strike, although changes from this direction are common, due either to folding with pitching axes, or to faulting. Near the borders of the pre-Cambrian igneous and sedimentary rocks the faults can be readily recognized. At such places the sedimentaries are almost entirely removed by erosion and comparatively small displacements are readily seen. Where these rocks remain in considerable thickness faulted structure cannot be determined, since the throw is relatively small and beds of sufficiently different character have not been brought into contact. Where the outcrops are favorable for observation they commonly show normal faulting on a small scale. This seems to be especially true of exposures in the vicinity of igneous dikes.

The general structure of the sedimentary belt appears to be anticlinal rather than synclinal, and the preponderance of field evidence gathered by detailed mapping strongly indicates that the preservation of the Cambrian rocks in this area has been due chiefly to normal faulting, which lowered the beds below the plane of erosion.

Normal faults were observed in two different parts of the area. About 1 mile southeast of White Sulphur Springs along the road which cuts across the slate belt, there is a series of fault blocks of small size occurring in a cut by the roadside Figure 3 shows this section (line *BB′* of map, fig. 1) drawn to scale. In a small stream on the north side of Rappahannock River about 1.5 miles northwest of White Sulphur Springs, normal faulting was again observed striking approximately in an east-west direction.

The variations in dip and strike of the Cambrian beds of the area are very great, and many of them are of such character that they can be explained only by normal faulting. When it is borne in mind that this district is only a few miles west of the New York-Virginia Triassic belt, which is known to be cut by normal faults in various directions, characteristic of other Triassic areas in the eastern United States, these structures are normally what would be expected to occur. Since the Cambrian lies on a basement of the same general character as the Triassic, and occurs in a nearby region, it is more than likely that the deformation which affected the latter would likewise affect the former.

In regard to the structure of the Triassic in Connecticut, Davis* says: "The faults must affect not only the Triassic but must penetrate eastward, westward, and downward into the crystalline floor on which the strata rest. . . . . It seems advisable to attach considerable importance to this explicit expansion to this problem of deformation, so that it shall concern not only the Triassic but the crystalline schists as well." Shaler and Woodworth†, in discussing the same question relating to the Richmond, Virginia, Triassic area, remark: "That these basins are due to deformation of the underlying granitic terrane by faulting seems highly probable." Keith‡, in discussing the structure of the Catoctin belt, remarks: "Doubtless Newark structures are present in the Catoctin belt, but the means are not at hand for detecting them. The Catoctin belt is defined by one of the Newark faults along its eastern limit. Otherwise its manifestations are confined to the Newark belt itself."

The results obtained in the Fauquier-Culpeper slate area strengthen the probability of a wider distribution of Newark structures in the Appalachian belt than has formerly been supposed. Since the crystalline rocks of the Piedmont belt are here cut extensively by normal faults in various directions, which are only made recognizable by a thin mantle of well-defined Cambrian strata, and since the writers§ and others have observed similar structures in the Shenandoah group of limestones in the Great Valley province west of the Blue Ridge, it is very likely that further investigations will reveal normal faulting more widely distributed in the Appalachian province.

* Davis, W. M., Triassic Formations of Connecticut, *18th Ann. Rept.*, United States Geological Survey, pt. ii, p. 141, 1896–97.

† Shaler, N. S., and Woodworth, J. B., Geology of the Richmond Basin, Virginia, *19th Ann. Rept.*, United States Geological Survey, pt. ii, p. 487, 1897–98.

‡ Keith, Arthur, Geology of the Catoctin Belt, *14th Ann. Rept.*, United States Geological Survey, pt. ii, p. 355, 1892–93.

§ Watson, Thomas L., and Cline, Justus H., Petrology of a Series of Igneous Dikes in Central Western Virginia, *Bull. Geol. Soc. Amer.*, 1913 (in press).

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Drainage Changes in the Shenandoah Valley Region of Virginia

BY

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Charlottesville, Virginia, U. S. A.

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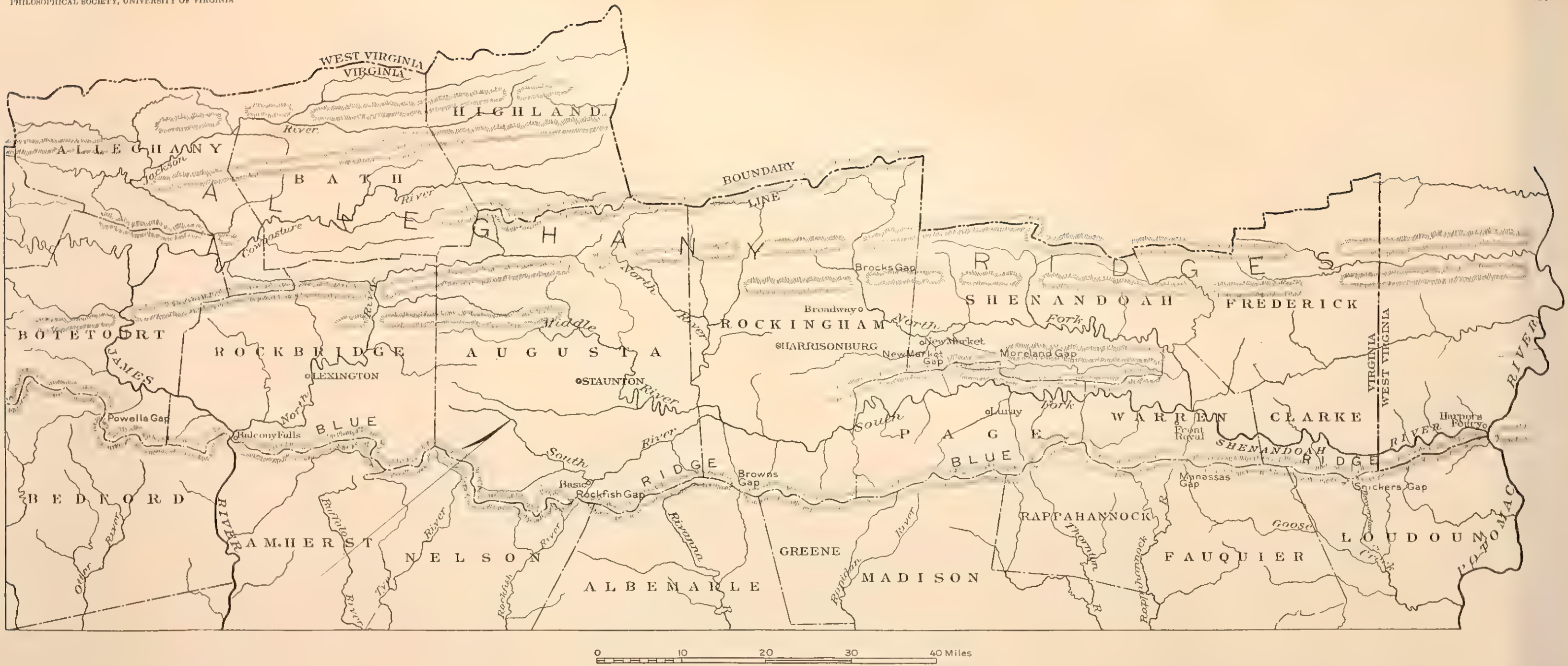

PLATE I. MAP SHOWING PRESENT DRAINAGE OF THE SHENANDOAH VALLEY REGION.

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DRAINAGE CHANGES IN THE SHENANDOAH VALLEY REGION OF VIRGINIA.

BY

THOMAS L. WATSON AND JUSTUS H. CLINE.*

PRELIMINARY STATEMENT.

The Shenandoah Valley in Virginia forms that part of the Great Valley, extending from New York to Alabama, drained by the Shenandoah River and its tributaries. It is not a structural valley, but owes its existence to the more rapid erosion of the limestones of which it is chiefly composed than of the resistant rocks that form the mountains flanking it on the two sides.

It occupies a narrow belt between the Blue Ridge and the Alleghany Ridges in northwestern Virginia and the extreme eastern part of West Virginia which lies east of the Alleghany Ridges (see map, fig. 1). Its trend is northeast-southwest, parallel with the bounding mountains, and it varies in width from 15 miles in the southwestern part along the divide between the James and Shenandoah rivers to 25 miles in the latitude of Harpers Ferry.

The general surface of the Valley is that of a broad undulating plain lying 1000 to 2000 feet below the crests of the bordering mountains, broken in places by low rounded monadnocks which rise 200 to 300 feet above it, or by low longitudinal ridges of greatly subdued character. In the extreme southwestern portion of the divide region between the Shenandoah and James drainage systems, the elevation approaches 2000 feet. From this point the Valley slopes northeastward to an altitude of 400 feet at Harpers Ferry.

* Read before the Scientific Section in May, 1913.

The region has been exposed continuously to erosion since the close of the Paleozoic era. It shows evidence of at least three distinct cycles of erosion with a fourth well inaugurated.

It is the purpose of this paper to trace the major drainage changes which have occurred since the beginning of the first erosion cycle in late Paleozoic or early Mesozoic times.

THE IMPORTANT PHYSIOGRAPHIC FEATURES.

The main physiographic features of the region trend in a northeast-southwest direction in conformity with the strike of the folds of the rocks out of which they have been developed. These are the Shenandoah Valley, the Blue Ridge, and the Alleghany Ridges, which form the main subdivisions of the Appalachian Mountain province.

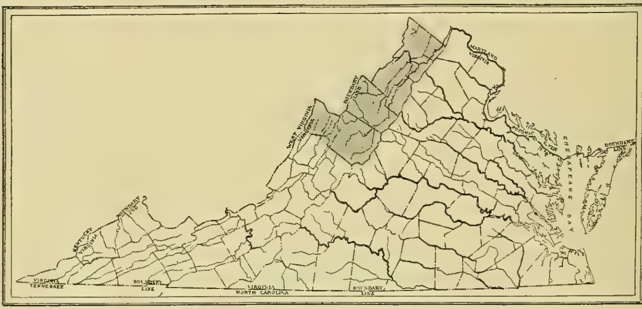

Fig. 1. Index Map showing Position of the Shenandoah Valley Region.

The principal features of the Shenandoah Valley are: (1) Massanutten Mountain (pls. II and III) which divides the valley into two parts between the latitudes of Strasburg and Harrisonburg, (2) the rounded monadnocks and low linear ridges (pl. IV, fig. 1) which mark the cherty beds of the Shenandoah group of limestones, (3) the Shenandoah peneplain (pl. VII), and (4) the valleys of the present cycle of erosion sunk 100 to 200 feet below the Shenandoah surface.

The Blue Ridge consists of a main ridge composed largely of igneous rocks, with an uneven and knobby crest cut at close intervals by numerous wind gaps, and flanked on the northwestern side by ridges of silicious Cambrian sediments (pl. V). The Alleghany Ridges subprovince is characterized by a series of parallel linear ridges separated from each other by narrow valleys which vary in width depending on lithologic and structural conditions. Massanutten Mountain, geographically belonging to the Valley province, is, in respect to lithology and structure as well as topography, an outlier of the Alleghany Ridges (see pls. V and VI).

THE PRESENT DRAINAGE PLAN.

The main stem of the Shenandoah River does not follow the direction of greatest continental slope, but has developed a course parallel with the chief physiographic features of the region. It occupies a position abnormal to the Valley and region which it drains, since it does not follow a central position in the Valley, but maintains a course along its extreme southeastern margin (see map, pl. I.).

The Shenandoah River illustrates an extreme case of an asymmetrical river system. All important tributaries enter from the northwest side; only small streams from the northwest slope of the Blue Ridge enter from the southeast. The system is made up of two distinct stream types, antecedent and adjusted. The main stem is an ideal representative of the latter; while the larger tributaries, North Fork, North River, and Middle River, belong to the former type, except that part of North Fork between New Market and Strasburg, which is of the adjusted type.

PHYSIOGRAPHIC RELATIONS OF THE REGION TO DRAINAGE.

While the main stem of the Shenandoah parallels the chief physiographic features and the strike of the rocks, its tributaries are entirely independent of these features so far as their direction of flow is concerned. The west side tributaries head well back within the Alleghany Ridges in the same manner as the James and the Potomac and other important streams of the Northern Appalachians, except New River which presumably follows a course antecedent to the Appalachian revolution. West of their junction with the Shenandoah, these tributaries are identical in type with those parts of the James, Potomac, and Susquehanna in the Appalachian Mountains province. They traverse the Alleghany Ridges through numerous water gaps in courses transverse to the axes of the ridges and cross the Shenandoah Valley in the same direction regardless of resistant rock barriers, reaching the foot of the Blue Ridge where they join the Shenandoah and turn abruptly in a northeasterly direction (map, pl. I). The larger streams of the Northern Appalachians do not change their course on reaching the Blue Ridge but continue in the same direction through water gaps in the mountain.

Since the headwaters of the streams of the Northern Appalachians which continue across the Blue Ridge are identical in character with those streams which do not, it seems entirely evident that all have had the same initial history, and that the more important streams including the three principal tributaries of the Shenandoah, which followed the direction of greatest continental slope, continued their courses to the Atlantic across the position the Blue Ridge now occupies. There could hardly have been an exception to this in the case of an important stream immediately following the Appalachian revolution.

The relation of North Fork, North River, and Middle River to the Great Valley and the Alleghany Ridges on the one hand, and to the Blue Ridge and Massanutten Mountain on the other, is entirely different (pl. I). Initially the relation of the streams to these physiographic features must have been the same, but owing to geologic conditions chiefly structure and lithologic types, only the more able bodied streams, which made up the original consequent drainage of the region, have been able to maintain their courses across the barriers east of the limestone belt of the Great Valley.

The varying size and distribution of the many wind gaps in the Blue Ridge and the relation of the gaps to existing drainage lines is very significant. It will be observed by consulting the map, pl. I, of the present drainage of the region, that opposite the point where any one of the important tributaries enters the Shenandoah there is a more or less prominent wind gap in the Blue Ridge. In the case of North Fork which enters the Valley through Brock's Gap near Broadway two barriers were encountered, Massanutten Mountain and the Blue Ridge, and in both cases wind gaps afford evidence of its antecedent course across the mountains after which it probably continued to the Atlantic through the Rappahannock. Near New Market this stream changes its course to the northeast opposite a well developed wind gap in Massanutten Mountain (pl. III, fig. 1). If it continued its course through this gap and thence in the same direction to the Blue Ridge it would pass through another similar gap. Instead of following this course it flows in a northeasterly direction along the northwestern foot of Massanutten Mountain to the vicinity of Strasburg where it turns again in an easterly direction and enters the Shenandoah at Riverton opposite the largest and best developed wind gap in the Blue Ridge along the Shenandoah Valley region.

Middle and North rivers, uniting at Mt. Meridian before they join the Shenandoah at Port Republic, likewise flow in a direction which would, if continued, pass through a wind gap in the Blue Ridge.

Rockfish Gap near Waynesboro is the second in size in the Blue Ridge delimiting the Shenandoah Valley region. It seems quite probable that the stream which developed it has been intercepted at a number of points on the Valley side and all evidence of its existence obliterated, except probably that part of Middle River northeast of Staunton.

The relative distribution of the large and small wind gaps in the Blue Ridge is significant not only of ancient drainage lines but of the order in which they were abandoned by their streams. These wind gaps furnish the only evidence of drainage conditions of the region during the various stages of its existence. Figure 2 shows the distribution of wind gaps in the Blue Ridge opposite the Shenandoah Valley between Harpers Ferry and Front Royal, which is more or less typical of the northern Blue Ridge in Virginia.

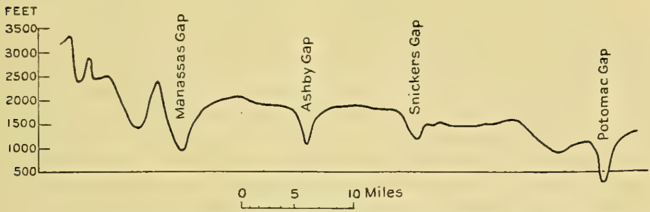

FIG. 2. LONGITUDINAL PROFILE OF THE BLUE RIDGE MOUNTAINS FROM HARPERS FERRY TO FRONT ROYAL, SHOWING POSITION OF GAPS.

After considering these peculiar and significant relations of the Shenandoah system to the physiographic features of the region it becomes strikingly evident that the present drainage lines conform only in part to what they were at the beginning of the first cycle of erosion. This original drainage, reconstructed on the basis of the evidence at hand, is shown in figure 3. The streams are of the consequent type, following courses with their directions determined by the original slope of the region at the close of the Paleozoic era. Only by starting with a condition of this sort can the present drainage and physiography of the region be satisfactorily explained. That the Shenandoah system is the result of the concentration of a large number of small streams which held their courses across the Blue Ridge has long been recognized. The well known case of the capture of Beaver Dam Creek by the Young Shenandoah opposite Snicker's Gap* has become one of the best known instances of

* Willis, Bailey, *The Northern Appalachians*, National Geographic Monographs, vol. i, pp. 169-202.

stream capture, and has been widely quoted in text-books as a type example of the process.

GEOLOGIC INFLUENCES OF THE REGION FAVORING STREAM CAPTURE.

The conditions chiefly responsible for the concentration of the large number of antecedent streams into a single system are attributable to structure and the unequal resistance of the rocks to erosion, especially difference between the lithologic rock types found in the Blue Ridge and the Great Valley.

The sandstones, limestones, and shales deposited during Paleozoic times and forming the region were laid down practically in horizontal position, and their eastern margin was for the lower members of the series at least considerably east of the position now occupied by the Blue Ridge. It is highly probable that even the lower members of the limestone group extended east of the Blue Ridge as the Lower Cambrian silicious sediments evidently did. The uplift and folding following the Paleozoic cycle of sedimentation in the Appalachian Mountains province resulted in reversing the drainage of the region from a northwesterly to a southeasterly direction. This marked the beginning of the consequent system of drainage and the first cycle of erosion.

The uppermost members of this great thickness of Paleozoic sediments were of a silicious character, and must have completely covered the great limestone formation near the base of the column and the igneous rocks of the Blue Ridge. By long erosion of these folds beds of different lithologic character were brought to the surface which varied widely in their ability to resist erosion. The ultimate result was the almost entire removal of the sediments from the crest of the Blue Ridge exposing the core of resistant igneous rocks and a great belt of limestone in the Valley region, with Massanutten Mountain remaining as a monadnock capped with resistant sandstone; and in the Alleghany Ridges province overlying sandstones removed only in narrow belts to a sufficient depth to expose the underlying limestone.

The streams flowing across the region transverse to the axis of folding, subsequently found themselves working on beds of widely varying degrees of resistance in different parts of their courses. East of the Alleghany Ridges the chief barriers of resistant beds were the sandstones of Massanutten Mountain, the cherty beds of the Shenandoah limestone group, and the igneous rocks and Cambrian quartzite of the Blue Ridge. These were the chief and controlling barriers in the path of the southeast flowing streams. The largest of these streams were able to lower their channels across these barriers more rapidly than the smaller and less able bodied ones. As a consequence, this gave the tributaries of the larger streams flowing over soft limestone beds the advantage over the tributaries of the smaller streams flowing at higher levels, and it was therefore only a matter of time till the tributaries of the large streams had captured the small streams.

CYCLES OF EROSION.

Kittatinny cycle. The erosion cycle begun at the time of the Appalachian revolution reduced the entire region to a peneplain, except some minor areas in the Alleghany Ridges and the Blue Ridge regions, which remained as low monadnocks above the general level of the plain. The plain which resulted from this first cycle of erosion, completed during Cretaceous times, has been called the Kittatinny after the mountain of that name in New Jersey. Remnants of this plain in the Shenandoah Valley region are preserved in the crests of the bordering mountains and in Massanutten Mountain (see pls. II, III, and V). The drainage conditions of the region during this entire cycle remained very much as they were at the beginning, as indicated by the large number of wind gaps in the Blue Ridge, below the level of the Kittatinny plain. These were eroded by streams that survived the first cycle of erosion. The abundance of these gaps in the Blue Ridge shows with reasonable certainty that whatever stream captures may have occurred during the Kittatinny cycle of erosion were of minor importance only. The supposed general drainage plan during this cycle is shown in figure 3.

The essential absence of piracy during the first cycle was doubtless due to the more or less uniform lithologic character of the beds on which the streams were working in the region now occupied by the Shenandoah Valley. The overlying sandstone beds which covered the Shenandoah limestones were removed only in part during this cycle of erosion. Instead of a belt of soluble limestone rocks in the Valley district, as at present, there doubtless existed only sandstone beds outcropping during the greater part of this cycle in the region west of the Blue Ridge. In the absence of alternating beds of resistant and nonresistant rocks, structure alone does not seem to greatly favor stream capture.

It is quite probable also that during part of this cycle the belt now occupied by the Blue Ridge was a limestone belt. Since the structure of the Blue Ridge is anticlinal and Cambrian quartzites are known to extend entirely across the mountain in several places it is reasonable to

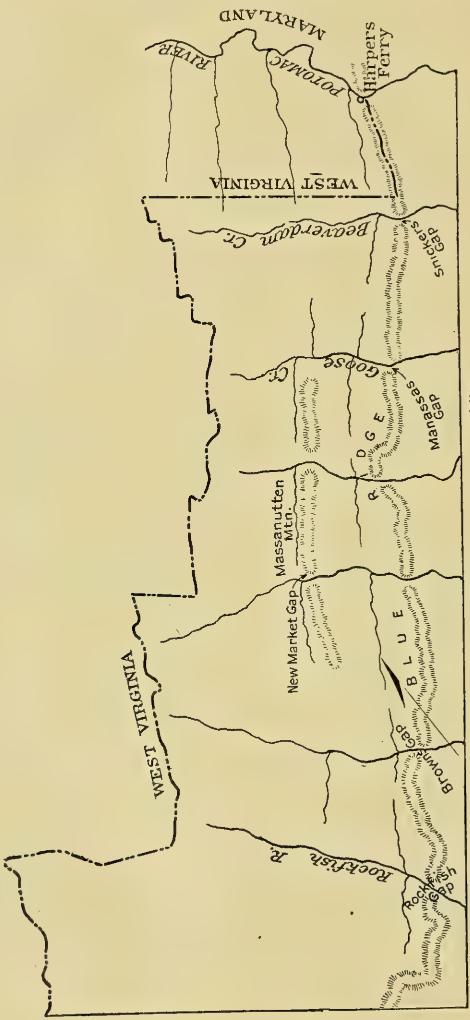

FIG. 3. MAP OF THE SHENANDOAH VALLEY REGION SHOWING THE SUPPOSED GENERAL DRAINAGE PLAN DURING THE KITTATINNY CYCLE, AND INDICATING THE POSITION OF THE BLUE RIDGE AND MASSANUTTEN MOUNTAIN.

suppose that the Shenandoah limestones, which are immediately above the quartzites, were also involved in this anticline, and extended farther eastward. Remnants of limestone found in the Piedmont Plateau are the probable correlatives in age of the lower members of the Shenandoah group of limestones which compose the Great Valley. Assuming this correlation to be correct the limestone was evidently removed from the Blue Ridge province during the Kittatinny cycle of erosion. During this period of removal of the less resistant limestone, the Blue Ridge was a weak belt instead of a barrier as today. If this assumption of the eastward extension of the limestone is correct, its eastern limits extended some distance east of the present location of the Blue Ridge. If this limestone belt east of the Blue Ridge really existed the peculiar change in the course of James River below Lynchburg from a southeasterly to a northeasterly direction and later back to a southeasterly course, is easily explained by resort to capture by a strike stream working on this limestone belt west of a barrier of resistant rock. Like changes in direction in the same relative position also occur both in the case of Roanoke and Rappahannock rivers. The evidence of limestone remnants found in the Piedmont province and the changes in direction of these streams at the same relative positions in their courses, seems to add special weight to this hypothesis.

The second cycle (*Tertiary* (?) *plain*). Uplift of the Kittatinny plain rejuvenated the streams and a second cycle of erosion was inaugurated. After the removal of whatever formations covered the limestones of the Valley, a broad belt of soluble rocks was exposed west of the Blue Ridge which had now become a barrier by offering to erosion the basaltic and granitic igneous rocks of its core, and the resistant Cambrian quartzites which flank it on the northwest side, and in places occupy its crest. The erosion of overlying material resulted in superimposing the streams on limestones in the Valley region and hard resistant rocks in the Blue Ridge; Massanutten Mountain remaining as a barrier by failure of erosion to remove the thickness of hard quartzite which occupies its crest.

During this cycle of erosion most of the stream captures which occurred in the process of drainage adjustment took place. Only one important capture remained for the succeeding cycle. The first step in the interesting series of drainage changes which occurred in the early part of this cycle was the capture of all small streams flowing through shallow gaps in the then low Blue Ridge by tributaries of the major streams. The distribution of the large and small wind gaps in the Blue Ridge argues strongly against capture of all these streams by the young Shenandoah, beginning in order with those nearest Harper's Ferry and thence working backward (southwest) throughout the length of the Shenandoah Valley, capturing one stream after another. On the contrary the early stages of the process resulted in the development of three drainage systems for the entire region. These were, named in order from north to south, the Potomac; Goose Creek, adopted for the stream that once occupied Manassas Gap; and Rockfish named for the stream that once occupied Rockfish Gap. The relative importance of wind gaps, including size, depth, and number, in the Blue Ridge, affords evidence that there was an intermediate stage between the original condition of drainage and this triple system in which there was probably a larger number of smaller systems, but the evident ultimate result was the three systems mentioned above. The drainage of the region at this period as it is conceived to have been is shown in figure 4.

The larger of the two systems south of the Potomac was Goose Creek which left the Valley province through Manassas Gap opposite Front Royal (pl. V, fig. 2). The presence of Massanutten Mountain which rises as an additional barrier in a part of the Valley drained by this system enables us to solve much of the history with ease and reasonable accuracy.

In the Cretaceous cycle, Goose Creek was one of many streams that flowed across the Valley and the Blue Ridge in antecedent courses. Being more able bodied than neighboring streams it lowered its channel across the Blue Ridge barrier faster than the others resulting in its tributaries capturing all the smaller streams in its vicinity, both to the northeast and southwest.

The capture of North Fork by tributaries of Goose Creek affords the most interesting case of piracy in the region, and also shows the details of the process by which the various antecedent streams have been intercepted at a number of points in their courses before adjustment was complete. As stated above North Fork entered the Valley through Brock's Gap opposite Broadway and flowed across the Valley, through New Market Gap (pl. III, fig. 1), in Massanutten Mountain, and thence across Page Valley, and the Blue Ridge through Thornton's Gap. A tributary of Goose Creek first intercepted North Fork on the east side of Massanutten Mountain near Luray; abandoning its course through Thornton's Gap it continued across Massanutten Mountain for a long period through New Market Gap. Another tributary of Goose Creek headed west of Massanutten Mountain with its course through Moreland's Gap in the most westerly ridge of the range, and following Little Fort Valley entered Goose Creek

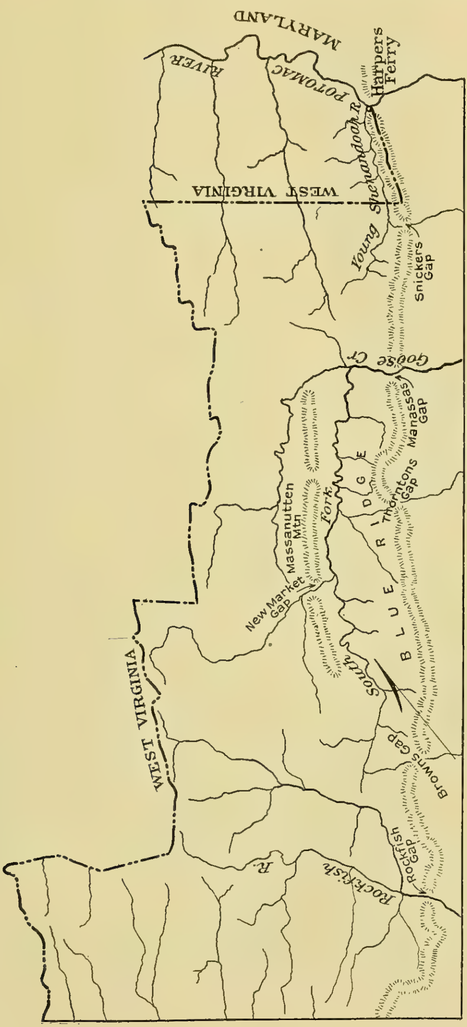

FIG. 4. MAP OF THE SHENANDOAH VALLEY REGION SHOWING THE DRAINAGE PLAN DURING A LARGE PART OF THE TERTIARY CYCLE. NOTE THE THREE STREAM SYSTEMS, ROCKFISH, GOOSE CREEK, AND POTOMAC.

between Riverton and Strasburg. This stream intercepted North Fork on the west side of Massanutten Mountain and diverted its waters through Moreland's Gap. Later a third tributary of Goose Creek which entered it near Strasburg in a course close to the western foot of the mountain, again captured North Fork opposite Moreland's Gap and diverted the stream to its present course.

While the tributaries of Goose Creek were diverting the waters of various neighboring streams to Manassas Gap (pl. V, fig. 2) similar captures were in progress in the southern part of the Shenandoah Valley opposite Rockfish Gap. Since this is the largest and deepest wind gap in the Blue Ridge between the James and Manassas gaps the size of this stream was doubtless large. It evidently was the outlet for the drainage of a large part of the Valley and seems to have included all of the area between Port Republic and the divide between the Shenandoah and James rivers.

Whether or not North River, which joins Middle River at Mt. Meridian and the South Fork at Port Republic, was first captured by a tributary of Rockfish and led through Rockfish Gap, or by a tributary of North Fork before its capture which resulted in abandoning its course across Massanutten Mountain, cannot be answered at present. Since the Rockfish Gap is the nearer of the two to their junction with the Shenandoah, it seems reasonable that they were first directed to the Rockfish, and are so indicated on the accompanying map, figure 4.

Manassas Gap is the only wind gap in the Blue Ridge opposite Shenandoah Valley of lower elevation than the Tertiary peneplain. The approximate position of the Tertiary plain is indicated by the crests of numerous rounded cherty monadnocks (pl. IV, fig 1) in the Valley and by the even crests of low Cambrian ridges of about the same elevation, which flank the Blue Ridge on the northwest side (pl. V, fig. 2). It is evident therefore that Goose Creek maintained its course through the Blue Ridge at Manassas Gap throughout this cycle and for a short period during the Shenandoah cycle. Before the completion of the Tertiary cycle, however, the same tributary which captured North Fork near Luray worked its way back to the vicinity of Waynesboro and captured the Rockfish system, diverting its waters northeastward through Manassas Gap. After the Waynesboro capture the chief outlet of the Shenandoan Valley drainage was through Manassas Gap. The drainage at the close of this cycle is indicated in figure 5.

The Shenandoah cycle. At the close of the Tertiary cycle the streams were rejuvenated by uplift of the region and a new cycle, designated the Shenandoah, was begun, which resulted in the development of the Shenan-

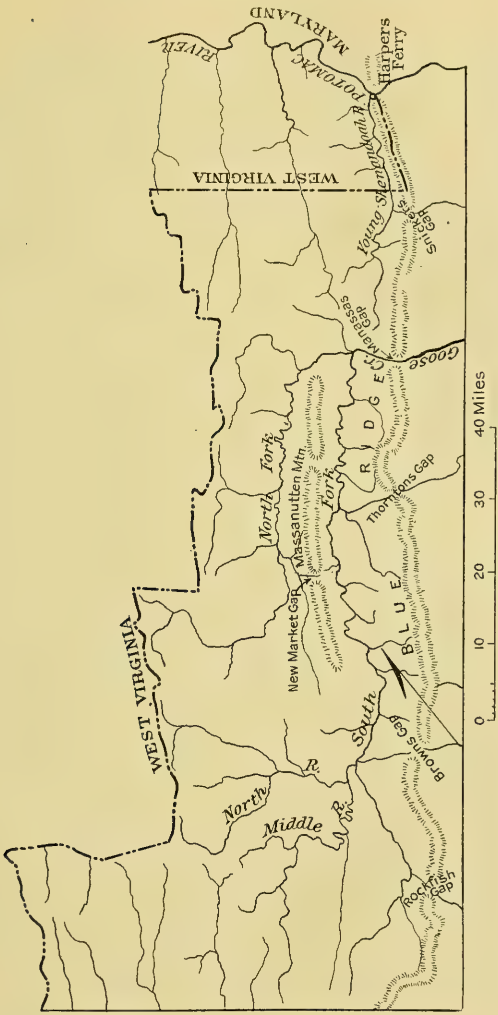

FIG. 5. MAP OF THE SHENANDOAH VALLEY REGION SHOWING THE DRAINAGE PLAN DURING THE LATTER PART OF THE TERTIARY CYCLE AND THE EARLY PART OF THE SHENANDOAH CYCLE.

doah plain in the Valley limestones, completed probably in Lafayette time (see pl. VII). During the early part of the Shenandoah cycle the rival drainage systems were Goose Creek and the Potomac (fig. 5). The Potomac was the more able bodied stream because it was likely the larger of the two, and because the Blue Ridge at Harper's Ferry was, on account of width, a less effective barrier than at Manassas Gap. During the latter part of the Tertiary cycle the young Shenandoah River, a tributary to Potomac River, captured the headwaters of Beaver Dam Creek on the northwest side of the Blue Ridge at Snicker's Gap and possibly other small streams in the northern part of the valley. During the Shenandoah cycle it extended its course headward along the foot of the Blue Ridge and captured the entire Goose Creek system at Manassas Gap, diverting its waters to the Potomac. This was the last important capture in the development of the present Shenandoah system (see pl. I).

The headwaters of Goose Creek were also intercepted by other tributaries of Potomac River in the western part of the Valley and Alleghany Ridges provinces. It is a matter of interest that while several streams in the central part of the Shenandoah Valley region head well back in the Alleghany Ridges province and flow eastward across the ridges for considerable distances to join the Shenandoah, similar antecedent streams are neither found in the southern part of the Valley region in the vicinity of the James River drainage, nor in the northern part in the vicinity of the Potomac.

Similar conquests have been accomplished by tributaries of the Potomac in the Alleghany Ridges district. In many instances a single drainage line has been intercepted a number of times before the present adjustment of the drainage was accomplished.

The Recent cycle. A third uplift of the region brought the Shenandoah cycle of erosion to a close. During the latter part of this cycle there was some deposition of material in the form of gravels and sand derived from the neighboring mountains. This material is now found on well preserved but small remnants of the Shenandoah plain and probably correlates with the Lafayette. The recent cycle of erosion has dissected the Shenandoah plain into an *intaglio* and the streams are now flowing in U-shaped valleys with flood plains which vary in width depending on local lithologic conditions. They may be as much as a mile wide in places, or they may be entirely absent. Temporary base levels are locally in process of development above barriers of resistant material in the streams.

Drainage changes during the recent cycle have been insignificant and of comparatively little importance. Such as have occurred seem to be chiefly of the intercision type* of piracy, where incised meanders of two neighboring streams working on the opposite sides of a divide finally cut through and the larger stream, since it is flowing at a lower level, captures the waters of its neighbor. An interesting case of this kind has occurred in comparatively recent times at Bridgewater. Dry River formerly entered North River about one mile southeast of the town. One mile northwest of the town the valleys of the two streams are separated for a short distance by a prominent chert ridge. Both streams were flowing in meandering courses and each directed its current against opposite points in the ridge. Finally the ridge was cut through and North River being the larger stream and flowing at a lower level took the waters of Dry River. So recent has been this capture that the two streams do not meet at grade and there is a small fall in Dry River a few yards above their junction. Another instance of this type of capture about to occur is near Mt. Crawford where North River is about to intercept another of its tributaries in the same manner about one mile above their present junction.

*An early paper by the writers will be devoted to a description of the *intercision* type of piracy in the Shenandoah Valley Region of Virginia, several cases of which are known and have recently been studied.

FIG. 1. NORTH END OF MASSANUTTEN MOUNTAIN LOOKING WEST FROM THE FOOTHILLS OF THE BLUE RIDGE IN THE VICINITY OF FRONT ROYAL ACROSS THE VALLEY OF THE SOUTH FORK OF SHENANDOAH RIVER. (PHOTO. BY A. N. CARROLL.)

FIG. 2. NORTH END OF MASSANUTTEN MOUNTAIN SHOWING PASSAGE CREEK GAP (VIEW LOOKING SOUTHWEST). (PHOTO. BY A. N. CARROLL.)

Fig. 1. View of New Market (wind) gap in Massanutten Mountain looking East. Note the even Crest Line of the Mountain on the Left (Kittatinny Plain).

Fig. 1. View of extreme North end of Massanutten Mountain looking Southeast. Note the even Crest Line (Kittatinny Plain), and compare with Fig. 1. (Photo. by H. Morrison, Jr.)

Fig. 1. View of the Valley of North River at Bridgewater looking Northwest, showing the Alleghany Ridges in the distance, and the characteristic Monadnocks supposed to represent the Tertiary base-level rising above the Shenandoah Plain. (Photo. by Wm. Dean.)

Fig. 2. McInturf (wind) gap in the westerly ridge of Massanutten Mountain near Edinburg. (Photo. by H. Morrison, Jr.)

FIG. 1. VIEW OF THE BLUE RIDGE MOUNTAINS TAKEN FROM THE FOOT OF THE SOUTH END OF MASSANUTTEN MOUNTAIN IN THE VICINITY OF McGAHEYSVILLE, OVERLOOKING VALLEY OF THE SOUTH FORK OF SHENANDOAH RIVER. RIDGES OF CAMBRIAN QUARTZITE ARE SHOWN PARALLELING BUT SEPARATE FROM THE MAIN HIGHER BLUE RIDGE. (PHOTO. BY WM. DEAN.)

FIG. 2. VIEW OF THE BLUE RIDGE OVERLOOKING FRONT ROYAL SHOWING MANASSAS GAP ON THE LEFT AND CHESTER GAP ON THE RIGHT. (PHOTO. BY A. N. CARROLL.)

FIG. 1. MANASSAS GAP (WIND) AT LINDEN, FOUR MILES EAST IN THE SAME GAP SHOWN IN PLATE V, FIG. 2. (PHOTO. BY A. N. CARROLL.)

FIG. 2. THE PEAK, EXTREME SOUTH END OF MASSANUTTEN MOUNTAIN LOOKING NORTHWEST. COMPARE WITH PLATE III, FIG. 2, VIEW OF EXTREME NORTH END OF MOUNTAIN. (PHOTO. BY WM. DEAN.)

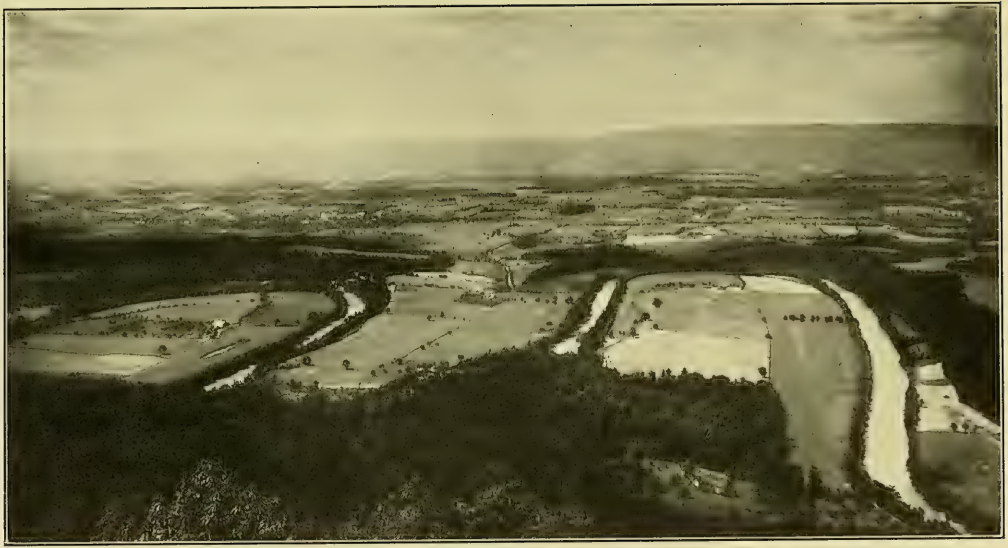

PLATE VII. VIEW FROM MASSANUTTEN MOUNTAIN LOOKING WEST, SHOWING THE SHENANDOAH PLAIN IN THE VICINITY OF WOODSTOCK, AND LITTLE AND GREAT NORTH MOUNTAINS (ALLEGHANY RIDGES) IN THE DISTANCE. NOTE ENTRENCHED MEANDERS OF THE NORTH FORK OF SHENANDOAH RIVER IN THE FOREGROUND. (PHOTO. BY H. MORRISON, JR.)

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BULLETIN

OF THE

PHILOSOPHICAL SOCIETY

Scientific Series, Vol. I, No. 18, pp. 365-371, February, 1914

The Association of Vanadium with Petroleum and Asphalt

BY

R. M. BIRD AND W. S. CALCOTT

UNIVERSITY OF VIRGINIA
Charlottesville, Virginia, U. S. A.

UNIVERSITY OF VIRGINIA PUBLICATIONS

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Continued on page 3 of cover

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BULLETIN OF THE PHILOSOPHICAL SOCIETY
SCIENTIFIC SECTION

Vol. I, No. 18, pp. 365–371 February, 1914

THE ASSOCIATION OF VANADIUM WITH PETROLEUM AND ASPHALT.

BY

R. M. BIRD AND W. S. CALCOTT.

Vanadium is said to be constantly associated in small quantity with high sulphur bearing petroleums and with asphalts. Asphalt deposits are associated with the Peruvian deposits of vanadium sulphide and oxide ores. This investigation seeks a possible cause for such association.

Since vanadium is an almost constant constituent of igneous rocks, and probably is transformed into a soluble vanadate during the weathering of such rocks, we thought it well to examine first the reaction between sodium vanadate and sulphur compounds in the presence of petroleum. The results thus far obtained show:

1. Water solutions of sodium vanadate, ionizable sulphur compounds (H_2S, Na_2S, etc.) and petroleum (both "sulphur bearing" and non-sulphur oils) yield colloidal (apparently) precipitates of vanadium sulphide. The precipitates of vanadium sulphide were usually so finely divided that they would not settle out to any extent even after standing for a very long time, and it was practically impossible to filter them to any degree of completeness. They gave the appearance of being in a colloidal state; although the presence of the excess of oil made the testing of this point so difficult that it was reserved for a future time. The dilution of the oil with kerosene oil did not materially alter the results. The long suspension of the vanadium sulphide certainly permits of its transportation by the oil.

2. The presence of carbon dioxide caused a rapid and almost total precipitation of the sulphide, as V_2S_5.

3. Petroleum containing sulphur in the form usually spoken of as "sulphur bearing oil" could not be made to yield a precipitate of vanadium

sulphide unless conditions were imposed which would probably give rise to the formation of ionizable sulphur compounds. The addition of free sulphur caused no difference in the results obtained. Dilution with kerosene had no effect.

4. Under conditions which give rise to the formation of ionizable sulphides (both from sulphur bearing oil and from added elementary sulphur) precipitates of vanadium sulphide are formed as in (1) and (2) above.

5. Mixtures of petroleum and vanadium sulphide (both the precipitated and finely pulverized ore) became gradually more and more viscous and finally yielded a brittle and asphalt-like mass when exposed to the air or to oxygen gas. This asphalting process required several months when the quantity of oil was great enough to protect the interior portions from the oxygen, but was quite rapid (few days) when oily masses of precipitated vanadium sulphide were left exposed on the filter paper or asbestos mat in a Gooch crucible, and when a paste of natural ore and oil was left in the open.

6. Oily masses of vanadium sulphide ore yielded a liberal quantity of free sulphuric acid when exposed to the air—sufficient to destroy the paper container of the sample.

7. The process of asphalting which the samples of petroleum underwent in this investigation seemed to involve partial oxidation of the petroleum, because no thickening of the oil nor asphalting occurred when oxygen was excluded. In an experiment in which the temperature was slowly raised the mixture exploded and a portion of the resulting mass was seen to be glowing with flameless combustion, strongly suggestive in appearance of the oxidation of carbonatious matter with potassium nitrate.

Along with the sulphide ore in the Peruvian deposits there is much vanadium oxide ore. Strata of vanadium ore and asphalt occur one above another, sometimes in contact and again separated.

Vanadium as an "oxygen carrier" is well known and extensive use is made of this property in some cases, in the manufacture of certain dyes, for example.

Vanadium sulphide is not soluble in water. Small quantities of very finely divided ore (100-mesh) became incorporated or dissolved when mixed and allowed to stand some time in contact with petroleum.

The presence of an alkali sulphide caused part of the vanadium to appear as a water soluble salt of thiovanadic acid, as was expected.

Hence the association of petroleum with vanadium compounds, and of asphalt with vanadium ores may occur in the following way:

Vanadates in solution in ground water coming in contact with oils bearing hydrogen sulphide and yielding vanadium sulphide.

This finely divided vanadium sulphide may travel with the oil, and be deposited by meeting carbon dioxide.

Precipitated vanadium sulphide and atmospheric oxygen may change the accumulating mass of oil into asphalt.

This train of events involves no unusual occurrences of materials or conditions—vanadates and carbon dioxide dissolved in ground waters, oil containing hydrogen sulphide, and conditions permitting the oxidation of accumulating petroleum with the aid of vanadium oxide acting as an oxygen carrier.

We give below some of the experiments from which we have drawn these conclusions. Those not reported presented no contrary results.

PREPARATION OF SODIUM VANADATE.

Unweathered patronite, from the eastern slope of the Andes in Peru, was used in the preparation of the sodium vanadate. The composition of this ore is indicated by the following analyses obtained from the office of the Geological Survey in Washington (by Hillebrand):

	PER CENT	PER CENT	PER CENT
SiO_2	10.88	22.22	6.88
Al_2O_3	3.85	8.32	2.00
Fe	2.45	1.98	2.92
V	16.08	15.36	19.53
MoO_3	0.50		0.18 (Mo)
S (sol. in CS_2)	6.55		4.5
S (combined)	54.06		54.39
S (total)		41.81	
CaO	tr.		0.10
Ni	undetermined	undetermined	1.87
TiO_2	undetermined	undetermined	1.53
C	undetermined	undetermined	3.47
Fe_2O_3	undetermined	undetermined	0.20
Cr	undetermined	undetermined	tr.

The procedure was as follows: A weighed quantity of the unweathered ore was treated with nitric acid, in a small flask on the water bath until the reaction was complete (one and a half to two hours). The filtrate from this was diluted to 100 cc. and made strongly akaline with ammonium hydroxide. The precipitate, after washing, was boiled with successive portions of ammonium carbonate (about half hour each). The filtrate was evaporated to dryness and heated gently, to drive off the excess of ammonium carbonate. The ammonium vanadate thus obtained was dissolved in water and transformed into sodium vanadate by the addition of sodium hydroxide; the necessary quantity being calculated from the known vanadium content of the ore used. This was evaporated until sodium vanadate crystals began to appear, and then diluted to a convenient strength. The vanadium strength was determined by acidifying with sulphuric acid, reducing with sulphur dioxide, and titrating with potassium permanganate (*Bull. U. S. Geological Survey, No. 176*).

EXPERIMENTAL DATA.

The qualitative test for the presence of vanadium in all the following experiments was the formation of ammonium sulpho-vanadate, the color of a solution varying from pink to cherry-red according to the quantity of vanadium present. In this test the solution is acidified with sulphuric acid, made strongly alkaline with ammonia, and hydrogen sulphide gas passed into it.

The precipitation of vanadium sulphide from water solutions of sodium vanadate by hydrogen sulphide and soluble sulphides.

1. *The reaction between hydrogen sulphide and sodium vanadate* was illustrated by diluting 5 cc. of the stock sodium vanadate solution to 50 cc., and passing hydrogen sulphide into it. The solution turned yellow almost immediately and became turbid in ten minutes. After passing the gas for about an hour the precipitation was almost complete. A gelatinous, dark brown precipitate settled out and the solution was colored cherry-red through the secondary action between vanadium sulphide and sodium sulphide. The precipitate was found to contain 63.1 per cent sulphur and 37.2 per cent vanadium, vanadium pentasulphide contains 61.1 per cent S and 38.9 per cent V.

2. *The influence of carbon dioxide on the precipitation of vanadium sulphide.* The influence of strong acids in bringing about the precipitation of vanadium sulphide from solutions of sodium vanadate is known, but it was necessary to determine the effect of carbon dioxide. The water used to dilute the stock solution of sodium vanadate was saturated with carbon dioxide and the experiment conducted as in (1) above. Except that the reaction was a little quicker it was in all respects like that described in (1).

3. One-tenth gram solid sodium vanadate, 0.2 gram solid sodium sulphide, and 15 cc. saturated solution of carbon dioxide were brought together. A bulky, dark brown precipitate formed almost immediately, but it required six weeks standing to effect complete settling. The precipitate was a sulphide of vanadium.

4. Solutions of sodium vanadate and sodium hydroxide, and elementary sulphur were mixed thoroughly. No precipitate formed even on long standing. But on passing carbon dioxide through such a mixture the solution became turbid within ten minutes. The precipitation was very slow and it did not really begin to settle out until after about two days. The product was vanadium sulphide.

5. Sodium vanadate solution was added to a solution of stick sulphur in benzine; no precipitate was formed on standing two days. Carbon dioxide in solution was then added, without a visible reaction taking place. On passing hydrogen sulphide into the mixed solutions a dark brown precipitate formed immediately.

*Reactions involving petroleum.**

6. Five cubic centimeters of the stock solution of sodium vanadate was mixed thoroughly with sulphur bearing petroleum that had been diluted with kerosene oil (2 cc. petroleum to 20 cc. kerosene). The density of the undiluted petroleum made accurate observations very difficult. No change was observed after the mixture had stood for two days. It was then boiled with a reflux condenser for an hour. A slight precipitate was formed, but this contained no vanadium.

7. This experiment was a repetition of (6) except that 0.1 gram finely powdered sulphur was added. No different results were obtained.

8. This was also a repetition of (6) except that 5 cc. of twentieth-normal sodium hydroxide solution was added. No different results were obtained.

(6) $NaVO_3$ aq. + Sulphur bearing petroleum	yielded no precipitate
(7) $NaVO_3$ aq. + Sulphur bearing petroleum + Sulphur	containing vana-
(8) $NaVO_3$ aq. + Sulphur bearing petroleum + NaOH aq.	dium sulphide.

9. Petroleum and sodium vanadate solution were mixed as well as possible and hydrogen sulphide was passed into the mixture. There resulted a very finely divided precipitate that was extremely difficult to filter.

*The oils used in this investigation contained less vanadium than could be detected by the test applied after the reactions.

10. The addition of carbon dioxide was found to cause the precipitated vanadium sulphide to settle out. For example: 10 cc. of oil was mixed with 1 cc. of stock solution of sodium vanadate, and hydrogen sulphide and carbon dioxide gases were passed into the mixture at the same time. The precipitate was filtered off through a Gooch crucible and the vanadium content of the filtered oil was determined as follows: A paste was made with sodium carbonate and heated to drive off the volatile matter. The residue was mixed with potassium nitrate and burned. The product was extracted with water and the extract evaporated to 10 cc. The vanadium content of this solution was determined by the permanganate method given by Hillebrand in *Bulletin 176 of the U. S. Geological Survey.* This analysis showed that some of the vanadium still remained in the oil, 0.0017 gram V_2O_5 per 10 cc. of oil.

Solubility of vanadium sulphide in sulphur bearing petroleum.

11. A water solution of sodium vanadate was treated with hydrogen sulphide and filtered through a Gooch filter. Ten cubic centimeters of sulphur bearing petroleum was added and allowed to stand over the vanadium sulphide for two days. It was then drawn through the sulphide on the filter and its vanadium content determined as in (10) above. The results showed 0.0023 gram V_2O_5 per 10 cc. oil.

12. Vanadium sulphide ore was ground so as to pass a hundred mesh sieve. One-fourth gram of this was mixed with 10 cc. of sulphur bearing petroleum and allowed to stand two days. The vanadium in the filtered oil was determined. Ten cubic centimeters oil showed 0.0010 gram V_2O_5. The relative rapidity of filtering of this oil may have modified its vanadium content, as compared with (10) and (11) above.

The change of petroleum into a mass that resembles asphalt.

The difficulties presented by the filtering of the precipitated vanadium sulphide from oil were so great that a number of experiments were carried out in the effort to obtain results that could be certainly depended upon. It was noticed that the paste left on the filter paper would change in a day or two to a brittle asphalt-like mass.

13. Precipitated vanadium sulphide and the sulphide ore was mixed with petroleum in a number of experiments and the mixtures allowed to stand. When these had access to the air there was a gradual thickening of the oil. In one experiment which was watched closely the oil became very viscous on standing four months. After six months this was brittle

enough to be broken with the hands and strongly resembled asphalt in appearance. When air was excluded no thickening occurred during the time the investigating was going on. In another experiment a thin paste of natural ore and petroleum was left in the open on a watch glass and thickened considerably in a few days.

During the process of asphalting a number of interesting things appeared. These will be more fully investigated later, and at the same time an examination of the asphalt-like product will be made which will be more satisfactory than could be made in the time at our command.

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Locoweed Disease of Sheep

BY

HARRY T. MARSHALL

UNIVERSITY OF VIRGINIA
Charlottesville, Virginia, U. S. A.

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BULLETIN OF THE PHILOSOPHICAL SOCIETY
SCIENTIFIC SECTION

Vol. I, No. 19, pp. 373-436 March, 1914

LOCOWEED DISEASE OF SHEEP*

BY

HARRY T. MARSHALL.

(Published by permission of the Secretary of Agriculture.)

Synonyms: Loco disease, loco, locoism, crazyweed disease, crazy disease, rattleweed disease.

There is an opinion widely current that the live stock ranging over the Western plains and Rocky Mountain regions are remarkably free from disease. The general prosperity of the ranch owners has served in great measure to divert attention from the losses and failures befalling certain of their number and it has taken many years to bring a realization of the menace presented by diseases to the continued success of stock raisers in the West. That parasitic diseases occur in sections of the West has been shown by Curtice, Stiles, Ransom, and other members of the Bureau of Animal Industry. Recently Hall† has shown by charts the present

* The work described in the following report was conducted by me at the request of the U. S. Department of Agriculture, under the supervision of Prof. V. K. Chesnut of the Office of Poisonous Plant Investigation. In addition to his hearty coöperation in many other ways Prof. Chesnut identified the locoweeds used in the experiment and selected the regions over which the loco feeding experiments were conducted. Dr. Charles Wardell Stiles was kind enough to determine the specific identity of the parasites found in my examinations of the sheep during 1903. Mr. B. H. Ransom of the the Bureau of Animal Industry has made a special study describing a new stomach worm of sheep found in the course of my autopsies (Bureau of Animal Industry, 1911, Bull. 127, pp. 62–66). The authorities of the Montana Agricultural College coöperated very courteously with us and moreover performed a valuable and important work in studying the market value of the sheep which were left over from my experiments. Finally, it is a pleasure to acknowledge, even at this late date, my appreciation of the hospitality, interest and assistance everywhere offered us by the ranchmen of Montana.

† U. S. Bureau of Animal Industry, 27th Report for 1910; pub. 1912, pp. 419–461.

known distribution of the parasitic diseases of sheep, but complete systematic studies of the prevailing parasitic diseases of Western live stock have not been made and knowledge upon this subject is incomplete. With regard to poisonous plants it has been shown in a number of publications from the Bureau of Plant Industry such as those by V. K. Chesnut and by Chesnut and Wilcox that there are several widespread and highly poisonous plants on the Western plains which occasionally cause great losses among stock.

The ranchmen themselves attribute great losses to the poisonous action of the locoweed, and this matter has given rise to much writing. The botanical characteristics and distribution of the various locoweeds were studied by Chesnut and others. After Chesnut's report appeared—in which he also described the general symptoms of the locoed animals—I was engaged by the Bureau of Plant Industry to study locoed animals in Montana for the purpose first, of giving a clear description of the symptoms characteristic of locoweed poisoning, and second, of locating and describing the anatomical changes produced in animals from eating the weed. As the result of my findings it proved necessary to conduct a feeding experiment in order to settle the vexed question as to whether it is possible to obtain uncomplicated cases of genuine locoweed poisoning. Inasmuch as other publications have appeared upon the subject of locoweed poisoning since 1903 and 1904, when my studies were made, I conclude this report with a review of the recent literature and with a discussion of the loco problem, as it confronts us today.

The report falls into several divisions:

A. Information obtained from the ranchmen and from literature.

B. The results of examinations of typical "locoed" sheep as they were met with on the ranches.

C. The description of feeding experiments and the results obtained from them.

D. Discussion of the parasitic diseases encountered.

E. A review of recent publications dealing with loco disease.

F. The present status of the "Loco problem."

A. INFORMATION OBTAINED FROM THE RANCHMEN AND FROM THE LITERATURE.

The spread of loco disease. Inasmuch as Marsh* and Crawford† have published elaborate bibliographies and historical reviews of locoweed disease, only special features will be selected for remark here. It seems probable from such information as can be obtained that loco disease was first observed in Mexico or Texas, and thus received its Spanish name of "loco" or "crazy" disease. It was known in the seventies in California; about the same time in Texas and Colorado; a few years later in Wyoming, and apparently it was first observed in Montana about 1884, when Jacob Severence described "loose-teeth lambs." Its general recognition in Montana dates from about 1890. It seems probable that loco disease spread northward from Mexico, the disease becoming notable in each locality ten, twelve, or fifteen years after the ranges were first occupied by live stock. Apparently a very large part of the live stock of the West is descended from old Mexican stock, the animals having been driven further and further north as fresh ranges were needed.

Definition. The general opinion in the West is that locoweed disease is a definite form of drug poisoning produced by the locoweed in animals which have formed the habit of eating the weed. Most ranchmen believe that the whole plant is poisonous, a few hold that the poison resides in the flowers, a large number consider the roots to be the most dangerous part of the plant. Some believe that one variety of locoweed is more dangerous, others, another.

A small number of ranchmen reject the view that the locoweed is poisonous. Most of these men believe that the craving for the locoweed prevents animals from eating a sufficient quantity of nutritious food, and that the symptoms are those of starvation. Others contend that the locoweed is of itself altogether harmless, and that the symptoms are caused by some worm or other parasite residing in the locoweed. A very small number believe that all cases of "locoweed disease" are in reality examples of other diseases, such as starvation, or parasitic infection.

The locoweed. There is some confusion as to the identity of the locoweed. Many other plants have been called locoweed, and there is no doubt that many cases of poisoning by other plants have been looked upon as cases of locoweed disease. The botanical characteristics of the various locoweeds have been given in Bulletins 20 and 26 of the Depart-

* Bureau of Animal Industry, Bull. 112, 1909.

† Bureau Plant Industry, Bull. 129, 1908.

ment of Agriculture, and in "Preliminary Catalogue of Plants Poisonous to Stock," Annual Report B. A. I. for 1898 and 1899, and by Marsh (loc. cit.).

The plants most closely connected with the disease are members of the pea or bean family (Fabaceae or more broadly, Leguminosae). The most widely distributed locoweeds are the *Astragallus mollissimus*, or "woolly locoweed" or "purple locoweed," the *Aragallus spicatus* (Hook) Rydberg, or "white locoweed," and the *Aragallus lamberti*. According to Marsh, who quotes Prof. C. F. Wheeler, *Aragallus spicatus* is identical with *Aragallus lamberti*, but Rydberg separates them as distinct. The *Aragallus spicatus* is the locoweed most abundant in Montana. These locoweeds are found either alone or together, over much of the area in which the disease is reported. Several other closely related plants are also called locoweed. Experiments with the locoweed and its extracts have yielded such inconstant results that they cannot be relied upon. The more recent works of Crawford, Alsberg and Marsh will be discussed later.

Etiology. While the locoweed is generally regarded as the direct cause of the disease there are many factors which are supposed to play a minor part in the development of locoism. Special efforts were made to obtain full information from the ranchmen as to accessory and predisposing factors. The following is a summary of the information obtained in this way from the ranchmen.

1. Species. Horses and sheep are most often the victims, cattle are affected but rarely, especially in Montana (1903–1904). The Angora goats of Montana are said to be quite susceptible. It is questionable whether any other animals (deer, elk, etc.) ever contract the disease.

2. Age. The statement can be obtained from most ranchmen in Montana that adult animals very rarely if ever acquire the disease. The symptoms first appear during the first year of life or more commonly during the second year. In the uncommon cases in which the symptoms first appear in an adult it is assumed that the disease existed but was latent when the animal was younger.

3. Sex. Males and females are equally affected.

4. Food. If animals receive a sufficient supply of nutritious food (grass, alfalfa, etc.), they will rarely if ever fall victims to the "loco habit." The great majority of all ranchmen interviewed united in affirming the close relationship between locoweed disease and an insufficient food supply. An unusually dry season, overstocking of the ranches, or allowing too short a time for a grazed section to recover may reduce the available supply of nutritious grasses, result in partial starvation of the animals and turn them to the locoweed to satisfy their hunger.

5. *Water.* A number of ranchmen hold that there is just as close a relationship between an insufficient water supply and locoweed disease, as an insufficient food supply.

6. *Salt.* It has been stated that where animals are regularly and abundantly salted they are much less prone to become "locoed." The alkali eaten by unsalted animals does not serve as a suitable substitute for salt. Many ranchmen maintain that locoweed disease can be completely averted if attention is paid to this matter.

7. *Lowered vitality.* There is no one point upon which the ranchmen are more thoroughly agreed than in the assertion that *healthy animals never acquire the locoweed disease.* Only one or two ranchmen claim that healthy, fat stock can become locoed. In practically every case the general health of an animal must be lowered before it will form the "loco habit." It is even stated by some that a healthy animal may eat a little of the locoweed along with its regular food, and not form the "loco habit" nor show any bad effects. This, however, is denied by most ranchmen. The general health may be lowered as the result of insufficient food, salt, or water, or as the result of other causes, such as exposure to inclement weather, lack of care on the part of the shepherd, a frail constitution, or any previous disease.

8. *Geographical distribution.* From information received by the Poisonous Plant Investigation it seems that the disease is found wherever the weed occurs, and nowhere else. The answer to circular letters indicate that at present (1904) locoweed disease prevails from Texas to Montana. and from Western Kansas to California. The weed and the disease are both common at an elevation of from 4000 feet to 7000 feet or even 8000 feet above the sea. In Montana the regions most affected appear to be the foot hills and high plateaus around Judith Basin, the Musselshell and the Yellowstone Rivers. In central Montana horses are more affected, in Southern Montana sheep.

9. *Soil.* The locoweed is very hardy and thrives even where the nutritious grasses cannot grow well. A light soil or a broken rocky soil is favorable for the weed. The attempt to prove a direct connection between an alkaline soil and the locoweed disease has not succeeded, even in the cases reported by Crawford.

10. *Incidence.* It is impossible to estimate the numerical loss attributed to locoweed disease. In Montana and Colorado there is probably an average loss of from 10 to 25 per cent of the annual increase from this source. The loss is from sickness and depreciation in value as much as from death of the animals. Individual ranchmen have been met with who have been

forced out of business by this scourge, while other ranchmen in the same neighborhood have escaped loss from the disease almost altogether. Accounts indicated in 1903 that the disease was increasing. According to the ranchmen there are the most remarkable variations in the prevalence of the disease from year to year. A region previously free from the disease may suddenly be devastated by it, only to have it subside or disappear again after a year or two. It could not be ascertained that such outbreaks are coincident with a sudden spread of the locoweed. It is claimed by some, however, that the abundance of the weed varies from year to year.

11. Old ranges. The "loco disease" does not make its appearance when a ranch is first occupied. Thus in Montana the disease was first noticed about ten or twelve years after the plains were settled by the rangers. In Eastern Colorado the evidence was similar. The stock rangers explain the delayed appearance of the disease as the result of a steady decrease of available grazing lands, combined with a rapidly increasing overstocking of the ranches, producing a shortage of grass, and the adoption of the locoweed as food by the hungry animals.*

12. Outbreaks. It is rare for isolated cases to occur, although they are met with. Such cases are most often found in horses. As a rule if one animal on the ranch becomes "locoed" a large proportion of all the young animals will develop the disease. As stated above locoweed disease is frequently enzootic throughout an entire region. Whether the infection attacks all animals or only single species could not be determined. The evidence at hand indicates that the "loco" outbreak is confined to one species, for example, sheep.

13. Seasonal variations. In Montana the first cases of "loco" occur a few weeks after the plant becomes green—that is during April and May. These are regarded as chiefly relapses from the year before. Very few cases develop during the summer. The greatest number of new cases appear in the autumn—November and December. Further south (Colorado) the height of the disease is said to be earlier in the spring and later in the autumn.

The ranchmen hold divergent views as to the relation between the incidence of the disease and the season's rainfall, some holding that more

* Within recent years the number of animals grazed on a given tract of land has risen enormously. One ranger in Montana informed me that 40,000 sheep now graze on a tract formerly ranged over by 6000. The fencing in of the ranges, and the settlement of the public lands, have served to curtail the available ranging lands to a fraction of their former extent, so that at the present time a larger number of animals grazes over a smaller tract than formerly, and returns to the same tract at shorter intervals.

"loco disease" is seen during moist seasons, others that a dry season favors the spread of the disease by decreasing the amount of the nutritious grasses.

Symptoms and course of the disease. The onset is described as being usually gradual and insidious, several weeks after the animal first eats the weed. A rapid onset is described at times, and occasionally symptoms appear within a week after exposure. In Eastern Colorado it seems not unusual for a horse to become "locoed" a day or two after beginning to eat the weed. The symptoms are more definite in the horse. A previously well broken horse becomes unmanageable, bucking, rearing and exhibiting various vicious traits, together with a high degree of nervousness. The animal appears to suffer from defective vision and even from hallucinations. It is subject to tremors of excitement. Most characteristic appears to be the tendency of "locoed" horses to balk at objects in front of them. Thus they will balk at a stick or a rope lying in their path, and if forced to cross it, they will clear it with a leap several feet in the air. It is said that a "locoed" horse cannot be led nor ridden, though it may at times be driven, but that when once started the horse will not stop before it is exhausted. The nervous symptoms continue. Very soon the animal begins to lose strength and weight; emaciation finally becomes extreme, the coat is rough and dull, the ears and head droop, the gait becomes weak and unsteady, the eyes sunken and glassy, and the animal very apathetic. Occasionally one side or one limb may be weaker than the rest of the animal. Stiffness of the hind legs is especially common. Schwartzkopf states that the pupils are widely dilated and that there is a decrease of sensibility to mechanical stimulation. Apparently the animal is in almost a constant state of nervous tremor. The disease is usually chronic, lasting from several months up to two or three years. The horse spends its time searching for the locoweed, which it may even dig up by the roots with its hoof. In the last stages the horse may walk blindly into a tree or rock and stand pushing against it until it drops from exhaustion, or it may fall in the water from which it is drinking and drown. In the chronic disease, horses occasionally exhibit acute maniacal attacks from which they may die suddenly with evidences of great pain. Acute attacks, can be brought on by exciting the animal or by making it exercise until it is hot.

Locoweed disease of sheep and cattle is less fully described. The chief symptoms are the habit of eating the locoweed, nervousness, loss of weight and strength ending in great emaciation, with dull glassy eyes, rough coat, and finally death from exhaustion. Stiffness of the muscles of the neck and hind legs is regarded as quite characteristic in sheep.

Morbid anatomy. No characteristic lesion has been found except the changes described by Marsh (l. c.) and reviewed below in Section E. An increase in the fluid beneath the meninges and in the ventricles of the brain has been described by Stalker and by Schwartzkopf. Scheuchardt states that Schwartzkopf found no bacteria in his cases.

Prognosis. The outlook for locoed animals is usually bad. If the disease is detected in the early stage the animal may be saved if it is fed up on grain, hay, etc., or if sent to a country free from the weed. As a rule the disease is not detected early, and the general attitude of the ranchman is that of non-interference. Even after an apparent cure the animal will relapse if turned out to graze in the former pasturage.

B. EXAMINATIONS OF "LOCOED" SHEEP.

Several hundred "locoed" sheep were seen on various ranches in Montana. No locoed horses could be found. Loco disease of cattle is a rarity in Montana and no cases could be found. Experienced sheep raisers selected a few of the illest and most typically locoed sheep from among the invalids and turned them over to me. Animals selected in this way furnished the materials for the following study.

Six of the illest lambs obtainable were fed for about two weeks on locoweed (*A. spicatus*), gathered fresh every day. *The animals did not like the food*, and at first ate sparingly of it, but more abundantly after a few days. No change in their condition could be detected as the result of this diet, and no acute symptoms were induced (see cases 8, 9 and 11).

A synopsis of the cases examined follows. The anatomical diagnosis was usually confirmed by the microscopic study of tissues fixed in Zenker's fluid or alcohol, sectioned and stained with hematoxylin and eosin. The central nervous system was not studied microscopically, but the brain, cord and membranes were examined at autopsy.

Histories of sheep studied during September and October of 1903.

ANIMAL NO. 1. Sheep, female, about eighteen months old from the Briggs-Ellis Ranch, Big Timber, Montana (Mr. James Vestal), September 24. This sheep was picked out by Mr. Vestal from an invalid band of between fifteen and thirty "locoed" animals, and selected by him as presenting the features of loco most typically and most severely.

When examined, animal was in corral, separated from other sheep; appeared apathetic, not eating nor noticing its surroundings. When alarmed, it ran off at a trot with a stiff gait and stumbling, the legs flying out in an incoördinate manner. The sheep was very weak and tremulous, especially in the hind legs, the tremor worse on movement. There was slight lateral nystagmus. When laid on the ground the animal was hardly able to rise. The hind legs were not able properly to support the animal. The pupils were only moderately dilated. The animal was greatly emaciated and undersized, and stood with head and ears drooping, hardly making a movement. Observed to remain in one position for one hour and thirty minutes and then moved only upon being disturbed. Every two or three minutes a brief spasm contracted the abdominal muscles, less often the muscles of one of the legs.

The vision of the animal was apparently good. She observed movements 10 or 20 feet away. When the hand was waved around slowly at a distance of 2 feet from the animal, she became greatly alarmed and trembled, and shrank against the wall. Hearing apparently acute. If hands were clapped together, even gently, up to a distance of 20 feet from the animal it quivered at the clap but paid no further attention. Tactile sensation apparently not exaggerated. The end of the nose and skin of the body were not unduly sensitive.

September 26 at 12.30 p.m. animal chloroformed. Autopsy No. 1 performed at once. Upon opening the spinal cord and brain, firm adhesions were found between the dura and the base of the skull along the right side extending from the level of the sella turcica posteriorly nearly to the foramen magnum. Upon removing the dura an abscess was opened into between the dura and the base of the skull containing between 2 and 4 drams of thick, viscid, yellow pus with a faint, unpleasant, rather fishy odor.

The brain was symmetrical, convolutions slightly flattened. The vessels not injected except adjacent to abscess where the surface of the brain was roughened and a small amount of exudate had collected in the pia-arachnoid.

Longitudinal section of the head through the nares and roof of mouth presented no abnormality and did not reveal the starting point of the abscess. Incisor teeth were quite loose, and irregular in size, shape, and position.

Subcutaneous and peritoneal fat almost absent. Muscles of the back and abdomen pale, reddish, and translucent.

Pleural cavities dry; left lung adherent at the apex and at two or three points to the diaphragm, being bound by quite firm, fibrous adhesions. Lung was small; generally crepitant, and of a reddish pink color. The bronchi and vessels were clear. The bronchi and trachea pale and dry. Scattered through the lung were 8 to 12 nodules varying from 2 mm. to 5 cm. in diameter, averaging 1.5 to 2 cm., fluctuating, raised, firmer, slightly tense with a yellow center surrounded by a narrow rim of dark red consolidated lung. These areas were the parts adherent to the diaphragm and apex. On section of such a nodule, in the center was found a core of semi-solid, semi-purulent, cheesy, material about two-thirds the size of the nodule, contained in a smooth-walled, cyst-like cavity surrounded by the red, consolidated rim of lung. No miliary nor conglomerate tubercles were found in the consolidated regions. The lymph glands at the hilum of the lung were not large nor caseous.

The right lung presented the same appearance as left, and contained from 8 to 10 nodules.

Heart. Parietal and visceral pericardial layers were normal. A few cubic centimeters (3 to 5) of clear, amber, fluid were present. The subepicardial fat almost absent. The heart valves were clear and delicate. The cavities appeared normal, the muscle was semi-translucent, reddish brown; the heart weighed 35 grams.

Spleen weighed 100 grams, measured 12 x 8 x 2 cm., surface smooth, consistence soft, color dark red; on section uniform in appearance, not opaque and apparently was normal.

Liver weighed 175 grams. Dimensions 18 x 12 x 3 cm. Surface smooth and glistening. Consistence soft, edge sharp. On section the liver was uniformly dark brown, the lobules not distinct. Over the posterior border of the upper right lobe were a few white pin-point areas of thickening in the capsule. A tapeworm was found in the bile duct and adjacent intestine, and was identified as the "fringed tapeworm," or "liver tapeworm," its technical name being "*Thysanosoma actinioides.*"

Left kidney. Capsule stripped readily leaving a smooth, brownish surface with a thick cortex (between 9 and 11 mm.). Glomeruli showed as red points; striae fairly well marked, boundary zone visible. The pyramids and pelvis of kidney appeared normal. Right kidney had the same appearance as left. Combined weight of kidneys 85 grams. The digestive tract presented no abnormality upon opening it except for the presence of tapeworms. The pelvic viscera, uterus, tubes and ovaries were clear except that a soft parasitic cyst was found attached to the right broad ligament.

Microscopic report. Paraffin sections. Stained with hematoxylin and eosin. Heart muscle stained well and appeared clear and normal, both fibrils and striations standing out distinctly. Scattered quite thickly through the heart muscle were deep bluish masses, circular, or oval, from three to fifteen times the size of the heart muscle cell. These were seen to be made up of a colorless, thin capsule in which were tightly packed great numbers of comma-shaped, or crescentic-shaped, deeply blue staining masses. There was apparently no reaction in the muscle outside but here and there one gained the impression that these cysts containing blue chromatin masses were embedded in a heart muscle cell.

Lung. Section through nodule. The cavity was seen to be a dilated bronchiole. The plug had fallen from the center, most of the mucosa had dropped off, and there was left a wall containing abundant loose fibroblastic tissue outside of which were a few compressed lung alveoli, passing rapidly into rather congested lung tissue. A second nodule of lung showed on section a bright pink necrotic structureless center. The mucosa was practically entirely gone or else overgrown by an abundant, loose, fibroblastic tissue which occasionally nearly enclosed a small strand or two of columnar epithelial cells. The new tissue contained, especially near the necrotic border, great numbers of lymphocytes and of eosinophilic cells. Apparently many of these eosinophiles were mononuclear, although the majority were polymorphonuclear. In the region outside of the organized tissue, more or less compressed lung was found with the alveoli partially filled with an exudate composed chiefly of desquamated epithelial cells and polymorphonuclears. Diagnosis—Organizing bronchiectasis and bronchopneumonia. Other nodules from the lung presented much the same appearance. In none of these sections were tubercles found in the consolidated area.

Spleen showed lymphoid hyperplasia and some phagocytic endothelioid cells, but otherwise appeared normal. The liver showed a definite though not very pronounced increase in the cells surrounding the somewhat dilated bile ducts. These cells were in part fibroblasts but chiefly lymphoid cells and a few eosinophilic cells. Kidney capsule not thickened; glomeruli appeared normal; slightly congested. Tubular epithelium slightly cloudy though hardly more so than normally. No increase in the intertubular connective tissue. Lymph gland (from hilum of lung). Capsule not thickened. Germinal centers well marked. A few of them showed lymphatic hyperplasia with large cells near the middle and some evidence of cell divisions in this region. The sinuses toward the center were widely dilated and contained numerous large desquamated endothelial cells with rather abundant pink cytoplasm. Among them were a number of cells showing the nuclear changes of cell division. Many of these desquamated cells contained what appeared to be hemoglobin. In the cell columns also were found evidences of cell division. The peripheral sinus was not dilated. Diagnosis—Lymphoid hyperplasia.

Diaphragm. Section was characterized by the presence of about a dozen cysts like those found in the myocardium, the cyst being enclosed within a muscle cell, although there was no reaction nor cell accumulation around the cyst. A cross section of peripheral nerve presented no abnormality.

Anatomical diagnosis. Extradural basilar abscess; multiple bronchiectatic abscesses in lungs; bronchopneumonia; *Cysticercus tenuicollis* adherent to broad ligaments *Thysanosoma actinioides* infection, biliary hepatitis; emaciation, irregular incisor teeth. *Sarcocystis tenella.*

Case No. 2. Male, yearling sheep on Briggs-Ellis Ranch; Mr. Vestal's house; Big Timber, Montana. September 28, 1903.

This animal was picked out from the same bunch of invalids as No. 1, it being considered about the worst locoed member of the band except No. 1. The sheep had been at Vestal's Ranch for nearly two weeks, having been brought from a distant ranch on account of their sickness. Like the other sheep in the bunch, this one was undersized, thin, emaciated, walked with a weak gait, the hind legs held rather far apart and the legs being used in a slightly clumsy manner, somewhat suggesting the idea that the animal was walking on stilts. Not all the animals had equal difficulty in walking, some going with perfect ease. Some of the weaker animals looked dull and apathetic. None seemed to be nervous, or excitable, except No. 1. The animals did not show any tremor. They fed naturally, and fairly constantly. Rough tests indicated that hearing and sight were normal and no abnormalities of sensation or of reflexes could be detected on superficial examination.

It must be noted that the appearance of No. 1 was markedly different from that of the other members of this flock and that the others showed differences among themselves. The only symptoms which seemed fairly constant were (1) thinness or even emaciation, and (2) weakness shown in the slow, uncertain gait and the awkward posture when standing.

No. 2 being typical of a dozen others in the bunch, was chloroformed, and at the same time the jugular vein and carotid artery were cut. Postmortem examination was made at once at 3 p.m., September 28.

Autopsy No. 2. The yearling measured from tip to tip about one yard. Weight less than 15 pounds. Emaciation extreme. Skin, eyes, nose, head and ears presented no abnormality. Incisor teeth were loose, and irregular in size and location. The molar teeth were very black. The interstices between the teeth were filled with tough, blackened fiber. Subcutaneous and peritoneal fat very scanty. Peritoneal surface smooth and glistening. In peritoneal cavity were found from 50 to 100 cc. of clear fluid, and eight soft, clear, watery parasitic cysts between 2 and 6 cm. in diameter were between the folds of the omentum. The cysts had no relation to vessels or other structures and were filled with a soft, gelatinous, material containing one short, ringed, cylindrical, worm-like structure, 0.25 to 0.5 cm. in length, 0.25 cm. in diameter, from which fine threads reached out into the jelly. At the rounded end of this worm-like structure was a linear depression. Two or three smaller cysts were found elsewhere, two being between the liver and diaphragm, and one forming the center of an adhesion between the gall bladder and the small intestine.

The common bile duct was distended, being as large as a lead pencil. On opening it a flat worm was found completely filling the duct, looped on itself with the head in the lower half of the cystic duct. The rest of the cystic duct was clear. The loops of the worm filled the hepatic duct, extended into several main branches of the hepatic duct and for quite a distance up into the liver and also along the duct of Wirsung for 2 or 3 cm. into the pancreas. Loops of the worm also extended into the intestinal canal.

The liver was of about the same size as that of sheep No. 1. Surface was smooth except for the adherent cyst above described, consistence moderately firm. Just to the left of the median sulcus of the anterior margin was an irregular wedge with the broadest surface toward the edge of the liver, measuring 6 x 8 cm., rough, firm, of scar-like character, pale, yellowish on section and sharply marked from remainder of liver. On section of liver the main branches of the bile ducts were filled with tape-worms.

Spleen measured 10 x 6 x 2 cm. It was smooth, soft, and normal on section. Kidneys were alike, the capsule stripped off readily leaving a smooth surface. On section the cortex measured about 8 mm. The glomeruli were fairly distinct, the pyramids pale, the kidney apparently normal. The adrenals 2 x 1 x 0.75 cm. were bean shaped, with pigment in their surfaces. On section, cortex and medulla were clearly marked, uniform, and apparently normal. Pancreas was soft and friable but presented no abnormality except for the worm in the duct. Bladder was empty and contracted; the mucosa pale. Aorta elastic and apparently normal. Heart. The pericardial sac contained free within it a cyst like those described above. Pericardial fluid not increased. Fat reduced. Surfaces smooth and glistening. The valves were clear and delicate; Foramen ovale closed; myocardium was pale, semi-translucent and brown. The lungs were alike. There was no free pleural fluid. The pleural surfaces were smooth and glistening. The lungs were small, crepitant, cushiony. The bronchi and vessels were clear and pale. On section, the lungs were pink and of uniform appearance except for one or two small patches of emphysema. The brain and cord presented no abnormality. The nares and adjacent sinuses were clear throughout. The trachea and esophagus were normal. The paunch was filled with hay and grass well chewed. The other stomachs were normal with the normal odor of gastric contents. In the duodenum were found several heads of tape-worms apparently four in all, with a great many segments. The rest of the intestine contained normal looking contents. The mesenteric lymph glands were enlarged, soft, pale and apparently normal on section.

Microscopic report. Heart. Section stained well, heart muscle cells for the most part were distinct though slightly granular and a number of the fibers had a wavy contour. Here and there were found cysts, maybe one-half dozen altogether embedded in the muscle and showing no reaction around them. The lung tissue was vesicular and normal. Some of the lung vessels contained an excess of leukocytes.

Liver. The liver cells were large, somewhat vacuolated and granular. The bile ducts were dilated and lined with high columnar epithelium showing abundant goblet cells. The surrounding tissue was slightly edematous and occasionally peculiar cells and debris were found in the lumen of the bile duct, apparently desquamated-epithelial cells. The lobules could be recognized fairly distinctly; the central two-thirds of the lobule stained deeper than the outer third, the outer third showing cells more vacuolated, paler and more granular and containing more yellow pigment than elsewhere in the lobule. In some instances the nuclei in the outer zone of the lobule had entirely disappeared leaving a yellowish pink granular group of cells. In another block from the liver the cells showed the same changes, and, in addition, there was a definite increase in the fibrous tissue around the bile duct and dense accumulations of cells in this region with indications of rather rapid new formation of a fibroblastic tissue. A nodule was found quite sharply marked off, composed for the most part of newly formed fibroblastic cells. A third block from the liver showed again the accumulation of cells around the bile ducts with a beginning fibrosis extending from the ducts into the liver tissue. The degeneration in the outer zone of the lobule was also seen, though not very distinctly. Another section of liver passed through a nodule of very cellular appearance, sending a number of branches into the adjacent liver. In this mass were compressed liver cells and strands of liver cells. Great numbers of fibroblastic cells, chiefly young, with large vesicular nuclei, and dense accumulations of lymphocytes together with a small number of polymorphonuclears occurred. There were also a number of eosinophiles. At the edges of this nodule the infiltrating cells were seen in between the liver cells very clearly, but it was very difficult to decide whether the cells were chiefly in the lumina of the capillaries or packed between the capillary wall and the liver cell. Both situations appeared to be commonly occupied. Here and there a capillary was found which undoubtedly was plugged by a mass of small deeply stained cells whose nuclei had the appearance of lymphoid cells. Among these occassionally was met a large, pale, oval or rounded nucleus.

The kidney showed cloudy swelling and slight congestion. The adrenal showed pigmentation in the capsule with, apparently, islands of adrenal cells here and there in the capsule. The large vacuolated cells of the medulla also contained dots of brown pigment. A few cysts were found embedded in the muscle of the diaphragm.

Anatomical diagnosis. Infection with *Thysanosoma actinioides;* obstruction of common bile ducts, cystic, hepatic, and pancreatic ducts; degeneration (infarction?), of a wedge-shaped section of liver with organization, biliary hepatitis; infection of pericardium, liver capsule and omentum with *Cysticercus tenuicollis;* loose irregular incisor teeth; emaciation, *Sarcocystis tenella* in heart muscle and diaphragm. Cloudy swelling of kidney. Leukocytosis.

Case No. 3. A flock of 2000 sheep, which had just been driven in from the range to be picked over for the winter, was examined. The flock contained 500 to 800 lambs and yearlings. The sheep herder and the owner (Mr. Vestal) estimated that nearly one-half of the lambs and yearlings were locoed. The examination of the sheep in their corral showed that more than one-half, adults and young ones alike, suffered from coryza and bronchitis. At least one-half of the lambs and yearlings were emaciated and moved with a stiff-legged gait. The sheep were more or less sluggish and certainly many of them were very weak. They were not tremulous. They showed no signs of eye or ear defect nor was there any evidence of excitement or mental disturbance. They kept in a bunch and did not tend to wander off to the sides of the corral. Not only the young sheep but many of the older ones were unhealthy looking, one of them being so weak it could not rise to its feet. A lamb about five or six months old which appeared as severely affected as any that could be found was picked out, chloroformed, bled from the carotid, pelted, and examined at once.

Autopsy No. 3. September 29, 1903. Lamb, female. Length about 2½ feet from tip to tip. Weight hardly more than 12 pounds. Subcutaneous fat practically absent.

Muscles pale, translucent, reddish brown. Peritoneal surface smooth and glistening. No excess of fluid, one parasitic cyst found in peritoneal cavity.

Heart. Epicardial fat very slight in amount, surfaces smooth and glistening, valves clear and delicate, foramen ovale closed. Myocardium translucent, reddish brown. The free margin of the left aortic leaflet presented a line of pigmentation. Lungs free from adhesions, pleural surfaces smooth. The color of the lungs in general was pink with here and there small groups of emphysematous cells appearing gray. On the surface of each lung were two or three areas from 0.25 to 1 cm. in diameter which were firmer, irregular, slightly darker red than surroundings and apparently consolidated. On section, the lung elsewhere appeared pink, the bronchi and vessels appeared clear except for the firm dark red areas. Spleen was small, smooth, soft; the structures showed well on section; condition apparently normal. Liver was small, surfaces smooth, the edge moderately firm and sharp; color, brownish red, semi-translucent. On section, appearance was uniform. The lobules were not distinct. On the convexity of right lobe of liver a pin-head sized whitish fibroid nodule was met with. Kidneys appeared pale and normal but under the capsule there were one or two fibroid nodules, 1 or 2 mm. in diameter embedded in the cortex. Pelvis and ureters clear. Bladder empty. Mucosa pale. Ureters, tubes and ovaries small and normal. Pancreas soft, friable, apparently normal. Digestive tract contained normal food in abundance and showed no lesion. Brain and cord apparently normal. No excess of fluid. No adhesions. The sinuses connected with the nose were empty and clear. There was a slight muco-purulent exudate over the turbinate bones. The incisor teeth showed irregularities in position and were loose, otherwise mouth was clear. Mesenteric fat and body fat in general was very greatly reduced. The adrenals were bean-shaped with dark pigmentation over the surface showing the cortex and medulla clearly on section. Hemolymph glands were numerous, pin-head in size.

Microscopic report. Heart. Striations and fibrillations distinct. Muscle cells slightly granular, otherwise apparently normal. Two or three small sarcocysts were found. In two instances the cyst occurred in a Purkinje cell. Lung apparently normal; section from nodule not obtained.

Liver. The section was distinctly cloudy. The liver cells were large, frequently vacuolar and granular. The bile ducts were distended, though only to a slight degree. The blood in the portal vessels contained an excess of leukocytes, quite a number of which were eosinophilic and in addition there was a distinct increase of lymphocytes in the portal blood. The peribiliary tissue was edematous and contained a small number of cells, partly lymphoid, partly eosinophilic.

Spleen, slightly congested, otherwise normal. The kidney tubules were lined by cells which were markedly vacuolated; otherwise the section was normal, except for the nodule in the cortex. A section through one of the white nodules described in the autopsy report showed that the nodule was embedded in the cortex of the kidney. In the center of the nodule was a small focus of coagulation necrosis with fragmenting nuclei surrounded by fibroblasts and mononuclear cells, some of which were lymphoid in character, others larger, and like mononuclears. These large and small mononuclear cells, mingled with fibroblasts, extended quite a distance between and compressed tubules. In the nodule were found three chief foci of coagulation necrosis and two or three smaller ones. The fibroblasts appeared to form a complete capsule immediately around the foci of necrosis. Outside of this infiltration became much more diffuse. One or two large cells were met with at the very edge of the area of coagulation necrosis which had an abundance of pink cytoplasm and contained a considerable number of nuclear fragments. They did not, however, suggest the giant cell of tuberculosis. The fibroblastic tissue did not seem to be especially rich in capillaries. Among the cells were a good many with eccentric nuclei and rather pinkish purple cytoplasm, apparently of the plasma cell type but unusually large for plasma cells. Adrenal, apparently normal; diaphragm showed no cysts. Bone marrow showed an excess of giant cells, and of cells of the myelocyte type. Lymph gland showed hyperplasia of the cell nests with crowding of lymph cells in the outer zone of gland. The sinuses in the central part of gland were widely dilated, and contained large vacuolated endothelial cells having a cloudy, degenerating appearance.

Anatomical diagnosis. Emaciation; irregular incisor teeth; *Cysticercus tenuicollis* in peritoneal cavity; catarrhal inflammation of nares; patches of recent bronchopneumonia. Inflammatory nodules in kidney. *Sarcocystis tenella* in heart. Peribiliary cirrhosis; degeneration of outer zone of liver lobule. Lymphoid hyperplasia.

CASE No. 4. At Mr. Clifford Kelly's ranch, Hunter's Hot Springs, Montana.

The sheep herder brought in a very sick yearling from the range, remarking, "if this is not a locoed animal, I have never seen a case of loco." The animal was thin, weak, stiff-gaited, dull, apathetic, with a rather rough fleece.

Autopsy No. 4. October 14, 1903. Animal bled to death and autopsied at once. Female sheep, greatly emaciated. Feet were apparently normal. There was an extensive area of extravasation and edema over the buttocks. Subcutaneous fat very greatly reduced. Muscle pale, translucent, apparently normal. Peritoneal surfaces in general smooth and glistening except for adhesions. No excess of fluid. Peritoneal fat greatly reduced. No cysts free in peritoneal cavity. Pleural cavities contained no excess of fluid. No adhesions. Right and left lungs were moderately voluminous, pale, pink, crepitant; vessels clear. On opening into the smaller bronchi a number of fine worms were found together with a considerable amount of thick, viscid, mucus. The worms were approximately an inch long, and as thick as medium or coarse, cotton thread, were motile, white with a dark line running spirally from head to tail. Six were removed from one bronchus. On the surface of lung were one or two raised, dark red, firmer areas not very sharply marked off and consolidated. On section a nodule of consolidation was found to communicate with the bronchus containing worms. Pericardial cavity contained small amount of clear, yellow fluid. Pericardial surfaces smooth and shining. Fat very slight in amount. Heart valves clear, delicate. Foramen ovale closed. Myocardium, pale, brown, translucent. Spleen small, surface smooth, consistence soft. On section, dark reddish brown. Malpighian bodies and trabeculae clearly seen. Liver. Between diaphragm and liver, over the whole of the anterior (ventral) surface were firm adhesions in which were cavities containing 50 to 100 cc. of slightly turbid, yellowish, bile-stained fluid together with two or three fairly large, firm, elastic clots 10 or 15 x 1 or 2 cm. in size. This mass of adhesions, etc., was in relation with an ulcerating surface on the liver which showed best on section through the liver. On section, there was found in the middle of the main lobe an ulcer 3 or 4 cm. in diameter, firm, slightly projecting, pale gray, fleshy looking, with irregular margins extending deeply into the lobe. In the center of this area was a necrotic rough-walled cavity, bile stained, about 2 x 3 cm. Elsewhere the liver was brown, moderately soft, and showed little alteration. Gall bladder apparently normal. Bile ducts empty. Stomach contained a normal amount of food. The lining of the stomach was everywhere smooth and pale. In the upper part of the duodenum were found numerous flat worms, the total length of which amounted to some five or ten meters long. These apparently represented three or four worms which were like the liver worms described previously (fringed tapeworm). Rest of the bowel normal, but contained an excess of sticky mucus. The front teeth were set crooked to the gum, and were loose; mouth and esophagus clear. The mucosa of nasal cavities was injected, slightly ecchymotic and covered with a sticky mucus. The ethmoid and frontal sinuses were clear.

Kidneys were alike, and appeared normal. Bladder contracted, empty, apparently normal. Internal genitalia apparently normal. Brain and cord presented no abnormality.

Microscopic report. Heart showed quite a large number of sarco-cysts like those previously described. Most of them were within heart muscle cells. One, however, was found in the space between two groups of muscle bundles. Heart muscle otherwise normal. Lung. The bronchial mucosa stood out clearly and showed no alterations. The bronchial walls were clear. The lung, everywhere vesicular except for one small focus in which possibly as many as a couple of dozen alveoli were filled with red blood corpuscles. Among the corpuscles and in the alveolar walls were a moderate number of polymorphonuclear leukocytes. It appeared that the chief infiltration with leukocytes was into the alveolar wall. One or two alveoli contained a great excess of desquamated epithelial cells. In one alveolus these cells were packed together, well stained, and almost looked as if they were growing in the alveolus.

Liver. Section 1 passed through a necrotic region of liver and showed in the margins great irregular proliferation of bile ducts and a large bile duct with papillomatous changes in the mucosa together with infiltration of the surrounding tissue by large and small mononuclear cells, eosinophiles, and eosinophilic mononucleated cells. Around the dilated duct the necrotic liver tissue exhibited regions in which hemorrhage was abundant, and many regions where very great fibroblast formation was occurring. The section extended from the liver to the diaphragm. Between the liver and diaphragm was necrotic debris in which was considerable fibrin, many polymorphonuclear leukocytes, fragments of nuclei, and yellowish brown material apparently disintegrated red blood cells. Many of the polymorphonuclears took a bright acid stain. Granulation tissue was forming from the diaphragm and extending into the exudate between liver and diaphragm. Throughout the section the replacement of liver tissue by a watery and cellular fibroblastic tissue was marked. In this fibroblastic tissue, atypical bile ducts were abundant. Other sections through liver and diaphragm gave findings practically the same as those described. In addition there were numerous sarcocysts in the diaphragmatic muscle.

Spleen apparently normal, pulp very cellular. Pancreas. The cells were large, very much vacuolated, at first sight reminding one of the adrenal. The definite arrangement into glands and acini was obscured by the tremendous swelling of the cells. There was, however, no evidence of necrosis, the nuclei for the most part were fairly well preserved although they were rather vesicular with dots of chromatin around the margin, and occasionally they were pyknotic. Kidney. Glomeruli unaltered, moderately full, tubules of cortex very cloudy, pale and vacuolated, frequently showing abundant albumin in the lumen. Adrenal apparently normal, capsule pigmented; eosinophilic cells were seen between cortical cells. The capsule and peripheral sinus of the lymph gland were apparently normal, the cortical zone was uniformly composed of dense masses of lymphoid cells except that here and there a germinal center stood out as a pale pink focus in the cortex. All of the central sinuses were very widely dilated and contained free cells. The cell columns were reduced corresponding to the dilatation of the sinuses. Under high power the center of one of the germinal centers showed a moderately large amount of slightly stringy, rather homogeneous deep pink hyaline material in which were only a few cells; around these were a number of large, vesicular nuclei, oval or round, which formed several layers not sharply marked and mixed more and more with lymphoid cells as one passed from the center to the periphery. Small masses of interstitial hyaline were also found in the cortex elsewhere than in the cell nests. The cells in the dilated sinuses appeared to be large desquamated epithelial cells which were often phagocytic. The sinuses also contained a moderate number of polymorphonuclear eosinophiles and lymphoid cells.

Section through a small hemolymph gland about 3 mm. in diameter showed that the peripheral sinus was dilated and packed with red cells; the central sinuses were not nearly so dilated as in the cases previously recorded but most of them were packed with red cells. A distinction into germ centers and cell columns could not be made out, the section appearing to be made of the sinuses distended with red blood cells and of intervening diffuse, cellular tissue in which a light reticulum could be made out. As seen under the high power, the cellular tissue seemed to be chiefly made of cells of a lymphoid type and of variations from this type. Endothelial cells phagocyting red corpuscles were common. Bone marrow showed an increase in polymorphonuclears, many of which were eosinophilic, and an increase in cells of the myelocyte type. Section through the thyroid showed a number of acini containing bluish pink secretion, the acini being lined by vacuolated cuboidal or low columnar cells. There were many small acini.

Anatomical diagnosis. Thysanosoma actinioides infection; abscess of liver with sub-diaphragmatic abscess; fibrosis in liver; vacuolization of pancreas cells and kidney; emaciation; loose and irregular incisor teeth; verminous pneumonia, *Strongylus (Meta-strongylus) filaria*; *Sarcocystis tenella*. Lymphadenitis.

Partial autopsies were performed upon three of the healthier invalid lambs on Mr. Cliff Kelly's ranch. All three died from experimental doses of cavadine (death camus) administered by Professor Chesnut.

Autopsy No. 5. This animal was infected with *Thysanosoma actinioides* and showed a small caseous-cystic cavity in the apex of the right lung.

Autopsy No. 6. The fringed tape-worm was present in this animal also, together with organizing bronchopneumonia, and sarcosporidiosis.

Autopsy No. 7. No worms were found in a partial autopsy of this animal. Microscopically, there were great numbers of sarcocysts in the heart muscle and diaphragm.

Case No. 8. October 15, 1903. Mr. Clifford Kelly's ranch. A number of badly diseased sheep, regarded as cases of "loco" were confined to a pen and fed with locoweed for a short while. One was examined, which had been fed for two weeks on freshly plucked locoweed, occasionally receiving also a little alfalfa. Before autopsy it was observed that there were no special symptoms. Pupils were not dilated nor contracted. The animal presented exactly the same features as the other animals which had not been fed on loco. It ate by preference alfalfa but also ate the loco which was put in the corral. Animal emaciated and stiff. Bled from carotid, skinned, and autopsied at once.

Autopsy No. 8. Very much emaciated animal. Fleece rough and poor. Subcutaneous fat almost absent. Muscles pale, Recti were semi-translucent. Peritoneal cavity contained about 100 cc. of clear yellow fluid. Common bile duct, cystic, hepatic, and pancreatic ducts were filled with tape-worms and dilated to a diameter of about ½ inch. The worms extended a very short distance into the duodenum. Liver was smooth and apparently normal except for the dilatation of the ducts. In the omentum were found several gelatinous cysts and one cyst similar to these was found encapsulated on the surface of the liver. This cyst was more or less dried up into a somewhat cheesy mass. Diaphragm clear. Pericardial cavity contained 10 or 20 cc. of clear fluid. The surfaces were smooth and glistening. Heart valves delicate and normal. Pericardial fat greatly reduced. Myocardium, pale brownish red, semi-translucent. In the substance of the left ventricle was found a caseating nodule similar to that on the surface of the liver. Pleural surfaces smooth and glistening. Lungs were only moderately voluminous, pink, and crepitant throughout. No nodules present. Trachea and bronchi and bronchioles free from mucus and from worms. Vessels at the root of the lungs were clear. On section lungs were normal. Spleen small, surface smooth, consistence soft. On section dark reddish brown; malpighian bodies and trabeculae well made out. Kidneys were alike. Capsules stripped readily leaving smooth, pale surface. On section, the glomeruli and striations were faintly seen. Pyramids were pale. Mesenteric lymph glands were large, soft, and pale, frequently showing on section a rather dark gray medulla. Adrenals appeared normal. Esophagus clear. The first stomach contained about two or three quarts of food in which could be recognized a very small amount of locoweed in a large amount of hay. All four stomachs and intestines were apparently normal showing a slight amount of rather sticky mucus about the middle of the jejunum. No stomach worms were seen nor any worms in the intestine. The sinuses connected with nose appeared clear, the mucosa pale except that in the frontal sinus was found a fly larva. The front teeth were loose, long, and irregular both in length and position. The gums appeared normal, pale and firm. There was no evidence of suppuration nor of foreign material at the roots of the teeth. The brain and its membranes appeared clear throughout. Surfaces of brain and medulla were pale and normal. No excess of fluid. Convolutions prominent. No sign of pressure. No discoloration of brain or medulla. Internal genitália appeared normal. Hemolymph glands stood out plainly, apparently were not enlarged. The intima of aorta was uniform and unchanged.

Microscopic report. Heart, the striations and fibrillations were distinct, especially striations. The undifferentiated central region of heart muscle cells appeared to be unduly large but the pigmentation was not increased. The section contained a large number of cysts located within heart muscle cells, one was found within a large undifferentiated Purkinje cell. In another section of heart, the nodule described in the autopsy was met with. In the center of the nodule was a mass, apparently the cross section of a parasitic worm or embryo. Around this was bluish purple debris with a space between the parasite and the debris. The purple material ended sharply in a zone of fibrillated, pinkish debris, apparently necrotic heart muscle. This passed over rather sharply into relatively normal heart muscle. In the intermediate zone between the debris and the heart muscle there was fibroblastic tissue with large and small lymphoid cells collected here and there. The growth of fibrous tissue was not uniform throughout the periphery of the nodule but could be clearly made out at several points. In one or two regions it was definitely becoming flattened in the direction of forming a capsule around the embryo. The mononuclear and polymorphonuclear infiltration was extremely slight compared with the size of the embryo.

Liver. The structure was not obvious, all of the cells being greatly swollen with corresponding reduction of capillaries, most of which were nearly or quite obscured. The liver cells were swollen, granular, and vacuolated. A second section showed that there was a slight dilatation of the ducts, the duct epithelial cells being large with pale, vesicular, bluish nuclei. Occasionally it appeared that there was a slight degree of edema around the ducts, especially the larger ones, but there was no noticeable fibrosis. In only one or two instances, after searching through two sections, could regions be found in which there was an accumulation of cells around the duct. These cells were chiefly mononuclears of a rather large size with moderately abundant cytoplasm. Spleen apparently normal. Cytolysis of red cells observed. Kidney showed granular epithelium, otherwise normal. Section of lymph gland showed the cortex with very little differentiation between germ centers and cell columns. The deeper part of the gland showed very great dilatation of the sinuses with reduction in the amount of lymphoid tissue between the sinuses, the result being that this part of the gland looked almost like spongy tissue. It could be seen with particular clearness in this section that the sinuses were lined with large, cloudy, pink cells, frequently cuboidal, containing a large, moderately deeply stained but rather vesicular nucleus. These cells were frequently present free in the sinuses, sometimes singly, sometimes in clumps. The sinuses also contained free lymphoid cells. The large cells occasionally contained a large amount of coppery yellow pigment, especially when the cells were free in the lumen. The appearance of the lymph sinuses, owing to this peculiar arrangement of the lining endothelium, suggested at first sight adeno-carcinoma. The blood vessels in the glands were empty or showed only normal contents.

Anatomical diagnosis. Thysanosoma actinioides infection of liver and pancreatic ducts; emaciation; loose and irregular incisor teeth; *Cysticercus tenuicollis* with caseating cysts in capsule of liver and heart; larva (*Oestrus ovis*) in right supraorbital sinus. Hyperplasia and phagocytosis in lymph gland. *Sarcocystis tenella.*

Case No. 9. This was a badly locoed sheep from Mr. Cliff Kelly's invalid band which had been fed on locoweed especially for purposes of experiment but presented no alterations as the result of such feeding, and evidently did not like the weed as a diet. The animal was dull, emaciated, stiff-legged and weak and had a slight coryza. October 15, 1903, was bled from the carotid and examined at once.

Autopsy No. 9. Fleece rough and ragged looking, front teeth loose and irregular. Superficial examination negative. Fat almost absent. About 100 cc. of clear yellow fluid in peritoneal cavity. Peritoneal surfaces smooth and glistening. Several gelatinous cysts with motile embryos were seen in the peritoneal cavity. The common bile duct, the cystic and hepatic ducts were tightly packed with tape-worms. The diaphragm appeared clear, muscles red, uniform, translucent. Pericardial fluid 10 to 15 cc., clear yellow. Pericardial surfaces smooth and glistening, the fat greatly reduced. Myocardium reddish brown, translucent, valves clear and delicate. Foramen ovale closed. Pleural surfaces dry and pale, free from adhesions. Lungs alike, not very luminous, pink, crepitant, and free from nodules. Vessels at the root of the lung were clear. Trachea and bronchi pale, containing a slight amount of sticky mucus. In the finer bronchioles were one or two small thread like structures apparently young worms around which there was very little reaction. Spleen smooth, soft, small. On section, malpighian bodies and trabeculae clearly made out. Liver smooth, moderately soft, dark. On section, negative except for dilated bile ducts. Kidneys alike. Capsule stripped readily leaving a smooth pale surface. On section, the cortex was pale and brownish gray. Glomeruli faintly seen. Appearance on section normal. Adrenals and hemolymph glands were like those in the previous autopsy. The bladder empty and apparently normal. Pancreas soft, apparently normal.

The fourth stomach contained many fine worms which were free on the mucosa. No encysted forms could be made out in the surface of the mucosa. The mucosa of all four stomachs and the intestine was everywhere pale and smooth. The mucosa of the nasal chambers was injected and coated with a slight amount of sticky mucus. The sinuses connected with the nose were apparently clear and normal. The brain and its membranes showed no abnormality.

Microscopic report. Lung. The section showed vesicular lung tissue. Liver cells were greatly swollen, vacuolar, and granular; the alignment less definitely preserved than usual, capillaries greatly reduced or often collapsed as the result of swelling of liver cells. The cells in the outer zone of lobule were possibly slightly more granular than the others; the nuclei in the outer zone being frequently very pale or even lost. The difference between the outer zone and the rest of the liver lobule was not as conspicuous as in preceding autopsies. Bile ducts showed very little alteration and were inconspicuous; they were not notably dilated and there was no increase in the surrounding connective tissue. Kidney showed slight cloudy swelling of convoluted tubules. Spleen normal.

Anatomical diagnosis. Thysanosoma actinioides infection; dilatation of bile ducts; granular and vacuolar degeneration of liver cells; emaciation; loose irregular teeth; infection of stomach with wire worms (*Ostertagia marshalli*); recent infection of lung with *Strongylus* (*Metastrongylus*) *filaria; Cysticercus tenuicollis.*

Autopsies 10 and 11 were only partial autopsies performed October 15 at Cliff Kelly's ranch, one of them (No. 10) upon an animal which had received a plant poison, the other (No. 11) upon a locoed animal which had been receiving special doses of loco, without appreciable effect. The autopsies disclosed the presence of the bile duct tape-worm and the wire worms in the fourth stomach of each animal and the lung worm apparently fresh in the lungs of the "locoed" animal. On microscopic examination of the tissues from case No. 11 sarcosporidia were found in the heart muscle and diaphragm; there was vacuolar and granular degeneration of the outer zone of the liver lobules; the kidney, spleen and stomachs appeared normal.

The sheep just described were studied during the autumn of 1903, and furnished the basis of a report to the Department of Agriculture. During the summer of 1904 examinations were made of a number of sheep selected by the ranchmen from diseased flocks, and regarded as examples of severe loco disease. Descriptions of these cases follow:

Olie Chrest of Howie, Montana selected two "locoed" sheep from his herd for examination. The animals were emaciated, and stiff-legged with ragged fleeces. Their front teeth were loose and irregular. They were regarded as typical locoes. Yearling A was bled to death.

Autopsy No. 20. At autopsy August 23, animal showed absence of fat; there were several young larvae of *Oestrus ovis* crawling over the turbinated bones; there were small calcified cysts in the myocardium. The liver showed dilated and thickened bile ducts with *Thysanosoma actinioides* in the intestine. Worms supposed to be *Strongylus fillicolis* were found tightly attached to the mucosa of the large intestine.

Microscopic report. Heart showed the presence of several sarcocysts. A small structureless mass encapsulated in a fibrous capsule was found in the heart muscle. This cyst, not more than 3 or 4 mm. in diameter was seen at autopsy. Lung apparently normal except for one patch of bronchopneumonia in which there was considerable hemorrhage surrounded by cellular exudate. Liver showed slight vacuolization of the cells in the outer zone of the lobule. No bile ducts appeared in section. Kidney appeared normal.

Yearling B, of Olie Chrest, was shot on August 26 and autopsied at once.

Autopsy No. 21. The animal was thin but not emaciated. The bile ducts were markedly dilated and packed with masses of "fringed tape-worms." The lymph glands of the mesentery were large, soft and grayish brown. Two hair balls about 3.5 cm. in diameter were found in the fourth stomach and in addition many wire worms, none of them firmly attached, were found in the fourth stomach. On opening the kidney two calculi of firm brownish material were found in the calices. The nares contained about two dozen small Oestrus larvae about 2 mm. in length. The mucosa was moderately engorged and swollen and covered with thick tenacious muco-pus.

Microscopic report. Heart unusually thickly studded with sarcocysts of varying size. Lungs clear except for miliary regions in which red blood cells had extravasated into the alveoli. These regions were so numerous as to give a peculiar dotted appearance to the section. No leukocytes appeared with the extravasated reds. The same appearance was seen in sections from several different parts of the lung. Liver appeared practically normal, except for thickening of a large bile duct. The small bile ducts appeared unaltered. Spleen apparently normal, Malpighian bodies conspicuous, pulp cellular. Kidney apparently normal. Voluntary muscle showed a number of sarcocysts, not however nearly so numerous as in heart.

The following cases are interesting as they point to an infectious variety of "loco disease." Two years ago (1902) a ranchman (Mr. T.) failed on account of losses from loco disease, his sheep being at the time on the ranges of B. O. Forsyth, of Busteed, Montana. One year later, another rancher (Mr. V. C.) had heavy losses from loco while using the same range. During this year (1904) Forsyth, using the same ranges, found over 1200 cases of loco disease among his sheep, two year olds and adults suffering chiefly. He has now moved to another range. He sent three of his most severe cases of loco disease for examination.

The sheep were by no means in such wretched condition as many others which had been studied, and one or two other ranchmen considered that they were not severe "locoes." They were thin, but not emaciated; only one walked with the usual stiff-legged gait; there was no cough; the fleece was even and thick, and while one was evidently ill, the general appearance and behavior of the other two was not that of ill animals, though they would not be called large or vigorous.

August 27, 1904, one of Forsyth's sheep was shot and at the same time bled from the carotid.

Autopsy 23. Postmortem at once. The sheep was thin but not emaciated, there was a heavy infection with *Thysanosoma actinioides* in the common and hepatic ducts; over fifty-six small wire worms were found in the fourth stomach. The incisor teeth were long. There was a small amount of black sand in the pelvis of the left kidney. Hemorrhagic spots were found in the lungs and there was a hair ball in the fourth stomach measuring about 4 x 2 x 1.5 cm. On microscopic examination small sarcocysts were found in the myocardium; and pin point areas of extravasation into the alveoli of the lung. The liver cells were distinctly vacuolated, the capillaries being indistinct and compressed. Even the small bile ducts were altered, but the change was more marked in the larger ducts. These were distended, frequently filled with exudate and surrounded by cellular and fibroblastic tissue. The fibroblast formation was only moderate and had invaded the liver tissue to only a slight extent. The cells surrounding the bile ducts were polymorphonuclears, large and small lymphoid cells and fibroblasts. An increased number of polymorphonuclears were present in the portal veins. Occasionally a small duct was found packed tightly with leukocytes so that the wall could hardly be recognized. The spleen showed cytolysis of red blood cells; the kidneys and adrenals were apparently normal. In a section through a lymph gland the sinuses were found dilated, as previously described, and containing eosinophiles, phagocytic endothelial cells and many granules of brownish pigment, like blood pigment. The voluntary muscle contained a few sarcocysts. Section of the fourth stomach showed no definite evidence of lesion from the wire worms; there was no reaction of an inflammatory character in the mucosa or sub-mucosa. Esophagus. Both epithelium and muscle wall appeared to be normal.

Autopsy 24. A two year old ewe from the Forsyth ranch was shot and at the same time bled from the carotid and autopsied at once on August 27. The animal was in fair flesh, the chief findings being that the common bile ducts and hepatic ducts were packed with *Thysanosoma actinioides* which also extended far back into the smaller liver ducts. The bile ducts were thickened and dilated and there were pin point hemorrhages into the mucosa. There was beginning necrosis of a wedge-shaped area on the cephalad surface of the right lobe of the liver. Twenty, or more, wire worms were found in the fourth stomach and Oestrus larvae in the supraorbital sinuses. There were small points of hemorrhage in the lungs. On microscopic examination the heart showed a moderate infection with sarcocysts, one of which was found in a large Purkinje cell. The lung and spleen appeared normal. The liver showed very marked changes especially in the region of the capsule of Glisson. At one place a widely dilated bile duct was cut across showing in the lumen a section through a tape-worm. The structure of the duct wall was practically lost and around the duct there was marked inflammation with necrosis of liver tissue, abscess formation and the evidences of chronic inflammation. Small regions of necrosis were scattered for quite a distance out from the main bile duct and the bile ducts all over the section showed thickening and infiltration around them. The outer zones of lobules were much more affected than the inner two-thirds except near the affected duct described above where lobules were destroyed. Kidney, cloudy. Cross section of lymph gland showed dilated sinuses and hyperplasia, as in previous case.

Jacob Hayem sent from his ranch two sheep which he said were badly locoed. The animals were thin and weak, weighing 25 and 35 pounds respectively. The fleece was good, there was no cough nor nasal discharge. There was only slight stiffness in gait. The incisor teeth were unusually long and widely separated. The field note was made that these animals did not present the usual appearance of the so-called "typical locoes" but would pass for specimens of poor stock which might improve if properly cared for during the winter. Compared with the experimental locoes they were pretty healthy looking animals.

Autopsy 27. At autopsy the smaller animal showed only a very large mass of tape-worm in the middle three-fourths of the small intestine. A field examination indicated that these worms were *Taenia expansa.*

Autopsy 28. The same tape-worm (*Taenia expansa*) was found in large numbers in the intestine of the second sheep in which also the bile ducts contained *Thysanosoma actinioides.* There was also infection with the *Oestrus ovis* together with abscesses in the lung.

Autopsy 29. August 31. The third sheep from Forsyth, a two year old, had been kept in camp for observation for several days. It appeared rather worse than either of the other two (Autopsy 23 and Autopsy 24), and was very thin. It was, however, active, alert, and free from stiffness. It coughed and sneezed very little, did not present the symptoms popularly attributed to the so-called typical loco, showing no twitchings, and having no tremor nor any special nervous symptoms. Hemoglobin 70 per cent by the Tallquist scale, temperature 102°, respiration 16 to the minute, pulse 116. At autopsy it was found to be infected with *Thysanosoma actinioides* causing dilatation and thickening of the bile ducts. There was *Oestrus ovis* inflammation to a slight degree and small wire worms were found in the fourth stomach. *Cysticercus tenuicollis* occurred in the peritoneal cavity. The animal was moderately emaciated.

The eighteen cases described above came from different ranches which were not very close together; the ranchmen were men of experience in sheep raising, and considered themselves, and were considered by their neighbors fully qualified to recognize loco disease. These ranchmen selected from bands of sheep containing many invalids the most typical and pronounced cases of loco disease, and turned them over to me for study. It seems hardly possible that every one of the ranchmen could have fallen into the error of selecting for me invalid sheep which more experienced sheepmen would not have looked upon as locoes. In fact, several opportunities arose to check up the diagnosis of one ranchman by that of another, and only minor differences existed between them. It is, then, reasonable to assume that the ranchmen made few, or no, errors of diagnosis, and that the sheep examined were fairly typical representatives of the armies of locoed sheep in Montana. A study of these sheep ought to reveal the symptoms characteristic of locoweed poisoning, and the autopsies should bring to light any striking anatomical changes produced by the use of the weed if such occur. It must be stated at once that neither of these objects could be accomplished.

The following signs and symptoms were observed:

1. Emaciation, in the majority of the animals selected. The other locoes from the same flock were undernourished, often stunted, not always emaciated.

2. Loose irregular incisor teeth, in nine out of eighteen cases. No note was made in the other cases of the condition of the teeth.

3. Weakness, and stiffness of gait in nearly all the very ill animals but by no means in all animals pointed out by the ranchers as typical locoes.

4. Dullness and apathy in the ill animals, but not in those less severely diseased. The iller animals were apt to wander off from the flock.

5. Tremor and nystagmus in one case (No. 1).

6. Coryza and bronchitis, in some flocks, not in all.

7. Rough, irregular fleece, occasionally.

Of these signs, the condition of the teeth is difficult to explain. The fairly constant findings, such as emaciation and under-development, weakness and stiffness, dullness and apathy are common to so many forms of disease that they have no value for the differential diagnosis of locoism. The coryza, bronchitis, etc., were sufficiently explained by the autopsy findings.

The clearest and most detailed clinical picture which the eighteen sheep allow us to draw is about this: The animals suffer from prolonged and progressive malnutrition; in the case of lambs, the animal is undersized; adult animals become thin or emaciated. As malnutrition becomes severe, the animal loses strength and energy, becomes listless, and does not keep up with the flock. As its strength diminishes the animal begins to walk in an awkward manner, the hind legs especially moving stiffly, as if they were parts of a mechanical toy. With these symptoms at least one-half the cases show loose and irregular incisor teeth. Now it is plain that, aside from the condition of the teeth, the above symptoms may result from any one of many causes which bring about malnutrition.

This forces us to the conclusion that typical and diagnostic symptoms of "Locoism" either do not exist or are so elusive that they escaped both my painstaking examinations and also the observations of the experienced ranchmen who gave their assistance. Inasmuch as the ranchmen were certain that the animals selected for study were severe cases of typical loco disease, there seems to be no escape from the above conclusion. It seems then that it will be either impossible, or exceedingly difficult, to construct the symptomatology of loco disease from animals studied on the ranches. Fondness for the locoweed is commonly regarded as the most constant and characteristic symptom exhibited by locoed animals, but even this was not present in the sheep among which it was especially looked for; these sheep (Cases 8, 9 and 11), preferring other food, (alfalfa), and requiring to be partially starved in order to make them eat locoweed with any freedom.

Of other queer nervous and mental symptoms, commonly ascribed to loco, there was absolutely no trace in the sheep under consideration, except in one sheep with a brain (subdural) abscess.

The anatomical findings, like the clinical study, furnished nothing that helped to establish loco disease as a separate and independent disease. In all of the sheep examined there were evidences of more or less severe starvation. In fourteen cases the "fringed tape worm" (*Thysanosoma actinioides*) was found, and in two others its effects were seen. It caused definite liver lesions in nine instances, of which at least two cases were severe. (Cases No. 4 and No. 24.) Sarcosporidiosis occurred in eight cases out of the nine which were studied microscopically.

Cysticercus tenuicollis occurred certainly five times, and possibly more often.

The lung worm was found three times, the stomach worm seven times. Both were probably overlooked more than once. Pneumonia accompanied the presence of the lung worm once or twice. Small bronchiectatic abscesses in the lung occurred twice. An extradural abscess at the base of the brain was found in Case No. 1; in two instances (Cases Nos. 8 and 20) the remains of an embryo, supposed to be *Cysticercus tenuicollis* was found on the epicardium; an encapsulated focus of inflammation of unknown origin, possibly tuberculous was found in the kidney in one case (No. 3); hyperplasia of the lymph glands with dilated central sinuses was observed frequently and there seemed to be leukocytosis in two instances (Cases 2 and 23). Sheep fly larvae were found six times, none of the infections being severe.

Not one of the conditions observed at autopsy could conceivably be due to locoweed poisoning, with the exception of malnutrition. But it is far from certain that the weed was in any degree responsible for the emaciated condition of the sheep. There are other causes at hand to account for the emaciation and weakness. These causes are sufficient, in and of themselves, to account almost entirely, if not entirely, for the diseases among the sheep examined and also for the diseases among a very large number of sheep in Montana. It is extremely important to understand that the diseases encountered can be clearly explained without any reference to the locoweed whatever, and that the same diseases are known in parts of the world where locoweed does not grow.

First among the causes responsible for the poor condition of our sheep must be placed *insufficient food,* and second *parasitic diseases.* These two causes alone and in various combinations deserve the most careful consideration.

Insufficient food. It is beyond the scope of this article to discuss in detail the subject of sheep feeding. However, there was much evidence acquired during 1903 and 1904 which indicated that overstocking of the ranges was common. Frequently it appeared that there were too many animals on the ranges, and at the same time that the area of available range was becoming reduced. I was told by some men that the supply of natural forage was much less than it had been twelve and fifteen years earlier, and that the range grass was never given time to attain its growth before it was used again for grazing. I was told that it was not generally customary to rely upon alfalfa and other cultivated crops for food for the stock during inclement weather and at times when the range grass was scanty. I also learned that ranchmen raising large crops of alfalfa suffered relatively small losses from loco disease. In one instance I had a chance to observe that one division of a large flock of sheep grew thin and suffered severely from loco disease on the scanty forage of the plains, while another division of the same flock grew fat and prospered on the richer forage of the uplands. This evidence and more like it makes me agree with those ranchmen who hold that there is not enough natural forage to support all the live stock depending upon it, and that, therefore, malnutrition, or even starvation is not infrequent among the stock. In addition to this it was easy to observe, by watching flocks of sheep feeding, that the small and weak members of the flock are at a great disadvantage. In a flock of several thousand, the sheep, when feeding, are always on the move, staying together, several columns deep. The stronger animals keep to the front and get the best forage; the weaker animals at the rear eat what is left—which is poor rations when the range is short.

Parasitic diseases. Several of the parasites found in the "locoed" sheep occasionally produce serious losses. The fringed tape worm (*Thysanosoma actinioides*), the lung worm, and the fly larvae (*Oestrus ovis*) are well known scourges. The wire worm of the stomach is a newly discovered parasite and its exact relation to disease is unknown. Since it is very much like the *Strongylus contortus*, it seems probable that its effects will be similar. The *Strongylus contortus* causes widespread losses among sheep.

That sheep are infected with parasites does not mean that they must necessarily die of the infection. The severity of the resulting disease depends upon a number of factors, such as the *age* of the animal—young animals suffering more than full growns; the *general nutrition* and *health* of the animal—under-nutrition and disease of any kind tending to enhance the severity of the parasitic infection. There are also other influential factors which are practically the same as the accessory etiological factors for loco disease, summarized in Section A. Varying degrees of starvation from underfeeding combined with varying intensities of parasitic infection are sufficient to explain the sheep diseases which I have studied, except for the curious condition of the incisor teeth which is unexplained, but seems to run parallel with the fringed tape-worm infections.

It must be pointed out, that, independent of my autopsy findings, there is strong reason to suspect that the Western ranges are heavily infested with parasitic diseases. The unhygienic conditions of the corrals and watering places, the custom of allowing dead sheep to remain unburied, and the unceasing use of the same grazing grounds year after year, without intermission, offer conditions favorable to the spread of parasitic diseases.

In concluding this section of the work, it may be stated by way of summary, that after careful study of severe and typical cases of loco disease it was not possible to collect a group of symptoms sufficiently constant and characteristic to enable the observer to distinguish loco disease from several other diseases; and there was no single characteristic anatomical change found at autopsy, which could be connected with the locoweed.

Moreover we were forced to the remarkable and paradoxical conclusion that the typical and severely "locoed" sheep, selected for us by various ranchmen, were not really suffering from locoweed poisoning, but from combinations of malnutrition and parasitic infection.

This leads us to the further conclusion that several different diseases pass for "loco disease" on the ranges. Some diseases which pass for "loco disease" have been mentioned above, others will probably emerge upon further study of Western live stock.

A further important conclusion forced upon us is that there is urgent need for a thorough-going technical medical survey of the ranges, to determine the existing forms of parasitic diseases and their extent, and to devise means to combat them.

In view of the findings recorded above, relatively little importance attaches to the answer to the ultimate question as to whether or not the locoweed is capable of producing a disease, which is independent of other diseases. Having found that 100 per cent of severe "locoed" cases examined were suffering from well known diseases other than loco poisoning I suspect that the locoweed has very little to do, directly, with the losses among sheep on the western ranges; but the evidence at hand does not justify an answer to the ultimate question, as to the existence of simple locoweed poisoning.

C. FEEDING EXPERIMENTS.

The campaign of 1903 established the facts that several menacing parasitic diseases are widely spread among the sheep of Montana; that these diseases, either of themselves, or in combination with insufficient food, are named "loco disease" by the ranchmen, regardless of the type of infection, or its severity; and that whatever symptoms the locoweed may cause can not be recognized accurately in sheep suffering from parasitic infection and underfeeding. If the effects of locoweed poisoning can be determined, it must be done in some other way than by the examination of sheep which the ranchmen call "locoes."

The Department of Agriculture directed me to continue the study of the loco problem during the summer of 1904, acting with the advice of Professor Chesnut and in coöperation with Mr. Reese of the Montana Agricultural College. It was decided to conduct a feeding experiment, and hold sheep in corrals where locoweed abounded, while others fed on alfalfa were kept as controls. Professor Chesnut, whose prolonged studies of the locoweed from the botanical side particularly fitted him for the purpose, selected Ten Mile Flat as the site of the experiment, for here the locoweed *Aragallus spicatus* (Hook.) Rydberg was unusually abundant, and other poisonous plants were absent.

Ten Mile Flat is a stretch of public land east of the Crazy Mountains and north of Big Timber, Montana. It has the reputation of being one of the worst locoed districts in this part of Montana. The soil is very poor and dry; there is a moderate amount of alkali, and the streams nearby are alkaline. The forage is quite scanty, a small amount of grass, wire grass and other plants occurring together with large patches of locoweed. From June until autumn the flat is very dry, and is exposed to the full effects of the summer sun. There was no shade where the experiment was conducted. In spite of its bad reputation, large bands of sheep are grazed over this flat every year. In the early spring of 1904, there was a moderate amount of grass to be found over the region, but this was quickly removed by two bands of from three to five thousand sheep apiece, which passed over this region before the middle of June. Another band was taken over soon after, leaving very little nutritious forage behind. Inquiries were made, but it could not be ascertained whether these bands suffered particularly from loco poisoning.

A large area with an abundant growth of locoweed was selected for use during the experiment. The objects of the experiment were: (1) To determine whether sheep can be poisoned by the locoweed when it is used as a food. (2) To determine the signs, symptoms and anatomical changes resulting from loco poisoning, in case such poisoning can be brought about. (3) To determine how soon appearances of poisoning occur after the animals begin to feed on the plant. (4) To determine what diet is preferred by animals which have once learned to eat the locoweed. (5) To determine the relative importance of the various factors which the studies of 1903 had indicated were of influence upon the disease. These factors were age, general health, salt diet, partial starvation, and infections with sheep parasites.

Forty-three yearlings and nineteen ewes with their eighteen lambs were obtained from Mr. Paul VanCleve of Melville, Montana. Only healthy looking animals were taken. They were of medium size for their age, and were of mixed breed. The animals were dosed thoroughly with thymol and creosote in order to free them from intestinal parasites so far as possible; were brought to the experimental camp and placed in corrals, a numbered ear tag being attached to each animal.

Four pounds of alfalfa hay was taken to be a sufficient day's ration for one healthy sheep, and a supply of alfalfa hay was kept with scales beside the corrals, the rations for each corral being weighed out daily. Water was kept in the troughs in the corrals, the troughs being filled twice a day. As often as the forage in a corral was used up, the animals were moved to fresh grazing ground.

The sheep were kept in eight groups which received food as shown in Table I.

TABLE I.

GROUP I (5 yearlings)	GROUP II (5 yearlings)	GROUP III (5 yearlings)	GROUP VII (4 yearlings; 5 ewes; 5 lambs)
Received: alfalfa 4 pounds per head; salt; no fresh forage; no locoweed.	Received: alfalfa, 4 pounds per head; no salt; no fresh forage; no locoweed.	Received: alfalfa, 2 pounds per head; salt; no fresh forage; no locoweed.	Received: no alfalfa; salt; fresh forage and locoweed.
GROUP IV (5 yearlings)	GROUP V (5 yearlings)	GROUP VI (10 yearlings; 9 ewes and 8 lambs)	GROUP VIII (4 yearlings; 5 ewes; 5 lambs)
Received: alfalfa, 4 pounds per head; salt; fresh forage and locoweed.	Received: alfalfa 4 pounds per head; no salt; fresh forage and locoweed.	Received: alfalfa 2 pounds per head; salt; fresh forage and locoweed.	Received: no alfalfa; no salt; fresh forage and locoweed.

Groups I, II and III were kept in corrals in which there was nothing for them to eat except the alfalfa hay which was given to them daily. Groups IV, V and VI had the chance to eat locoweed and other growing plants in addition to the daily ration of alfalfa. Their corrals were moved as fast as the locoweed and other grasses were used up. Groups VII and VIII fed only on the locoweed and grasses to be found in the corrals. These corrals were constantly being moved so as to provide pasturage for their occupants. Groups I, III, IV, VI and VII received salt at frequent intervals.*

The animals were put in the corrals and the experiment started on July 15, 1904 and continued until September 6.†

The animals were closely watched to determine when they first began to eat the loco. On July 18 it was discovered that one ewe and one lamb in Corral VIII had been eating the weed and by evening a considerable quantity had been consumed. All the nutritious forage had been eaten out before the loco had been touched, and it was soon noticed that all the ewes, lambs and yearlings in corrals VII and VIII were eating the loco freely.

The sheep which received hay also, did not take to the loco so readily, and it was July 31 before there was any evidence that the animals in Corrals IV, V, VI were eating the weed. They did not begin to eat the locoweed until they had cleared out all the other green forage in the corrals. After they once began to eat the locoweed they showed quite a preference for it whenever they were subsequently placed in a fresh corral. However, they always ate abundantly of alfalfa, when it was furnished, and also of the other forage in the corrals, never confining their diet to the locoweed, even after they had formed the habit of eating it.

The experiment thus demonstrated perfectly clearly that sheep can be made to eat the locoweed, and that when they once begin eating it, they like it as much as other food, and possibly prefer it. It also appeared that sheep will not touch the weed while they receive a plentiful supply of green forage, but will take it if they are starved, or if they are fed on alfalfa hay and allowed to graze where locoweed is the only fresh plant.

* The assistance of Mr. Reese of the Montana Agricultural College, in caring for the animals, weighing them, moving corrals, etc., was of great value.

† While this experiment was in progress a second feeding experiment was conducted in a distant part of Montana. Five lambs and four ewes, all healthy animals were kept in corrals abounding with locoweed from June 10 to September 16. The experiment was conducted at White Sulphur, Montana, on the ranch of Mr. C. W. Cook from whom the sheep were obtained. The results agreed with those obtained at Ten Mile Flat, but it was impossible to give close continued attention to the animals, and the experiment is therefore not set forth in detail. No result was obtained which in any way conflicted with the results obtained at Ten Mile Flat.

The following field notes indicate the general condition of the animals:

August 7, 1904. Groups I and II. Sheep are not large, but are in fair flesh and appear healthy; one or two have cough, not very severe, with sneeze. If there is any difference between the groups it is hardly noticeable, possibly the unsalted sheep are slightly fatter than the salted. Quite a lot of hay is left on the ground. All fresh plants in corrals have been eaten.

Group III (½ alfalfa, no locoweed) can hardly be distinguished from Group I and II by their appearance. Sheep not quite so fat, but nearly so. Cough is, however, distinctly more pronounced. Not a wisp of hay left in the corral. Three of the five sheep sneeze, cough, and have nasal discharge. Flies seem to be troubling the noses of the sheep a good deal this morning.

Groups IV and V (alfalfa and locoweed). About as much hay left on ground as with I and II. No appreciable difference between the sheep of Group IV and those of Group V. About one-half the bunch have a cough and sneeze, etc., as with I, II and III. Locoweed and other grasses (except wire grass) have been completely eaten off. Animals look as well as or better than I and II.

Group VI (half rations plus locoweed). The yearlings, like those in the other groups, are undersized, and in this corral they are thin and scrawny, but not emaciated. The lambs (May lambs) are hardly one-half the size of June lambs recently seen in a healthy flock. The animals in Group I–VIII are suffering with the heat, and stand panting, with their heads together and near the ground, to avoid flies.

A small amount of alfalfa is still uneaten. The loco and grasses are being eaten. Certainly one-half the bunch have the cough, etc., as above described. Lamb No. 55 in corral VI has very severe cough, etc., and is distinctly weak; would be called a fairly severe "loco" by the average ranchman. Lamb No. 52 in corral VI seems weaker than No. 55, but does not cough, though his nose is filled with mucus. No. 52 and No. 55 do not run off with the rest of the bunch, but stay and let one come up to them. Every lamb in this corral has severe cough, eyes stuffed with mucus, except one larger and sturdier lamb.

Groups VII and VIII (sheep on loco without alfalfa). Sheep are poorer than the others; some are actually emaciated. Lambs even smaller than those in corral VI. No. 46 (lamb) has in addition to cough, etc., an open ulcer on left side of face opposite middle of cheek. In this corral the cough is found in nearly every animal, and is more severe than in the other corrals.

To generalize about the sheep. The day is very hot and the animals are sluggish, staying in groups with heads together or lying in corners and under the drinking troughs. None of the animals are what would be called first class sheep; all are small and scrubby. The yearlings are hardly larger than some lambs recently seen, and the lambs are not more than half size. The yearlings and lambs have grown extremely little. In each corral the animals are coughing, about one-half the animals in Group I, II and III being affected, more in Group IV and in the last two corrals pretty much every animal. In the last two corrals animals *seem about half starved* and the cough is much more severe. Several of the starved animals with this cough are what would be called "locoes" by ranchmen.

The cough, etc., affecting the sheep in Groups VI, VII and VIII became so severe that the animals began to die, and it looked as if the experiment would be upset by this disease killing out the animals in the two most important loco pens—Groups VII and VIII. It was found necessary to build up the general health of the sick animals by abundant rations of alfalfa. This was continued for eight days, by which time the animals had improved sufficiently to allow the experiment to be resumed.

The following notes were made on August 18, the day after the experiment was resumed.

Groups I and II. All animals look well, no stiffness observed on motion; over half of them have occasional cough, and a little mucus from the nose; eyes are clear, and animals seem little troubled by cough.

The stiff stems of alfalfa have remained uneaten. Food given two and a half hours ago is eaten except for the stiff stems. Six sheep—some salted, some unsalted—are still eating. Very little difference apparent between salted and unsalted animals. Possibly unsalted are eating longer.

Group III. Animals are in fair trim, eyes clear, but all have cough and sneeze, which gives some distress. Almost every blade of hay given this morning is eaten, and all five animals are picking at what remains. No hay at all is left from yesterday.

Groups IV and V. Put in new corral of loco after 6 p.m. yesterday. This morning (8.30), in upper part of corral loco is eaten down, seed tops and leaves eaten, occasionally seed stalk left. In lower two-thirds of corral, loco still abundant.

These bands of sheep (IV and V) are the best looking of all, the sheep are in good flesh, eyes are bright; they are but little troubled by the cough, although most of them have it, and are the nimblest and most active of any band. Possibly, as with the sheep receiving full hay, the unsalted are a trifle fatter than the salted. One salted yearling wether is a trifle stiff and is more troubled by the cough than the others.

Group VI. So far as can be determined, the yearlings are as well off as those in pen No. III, both as regards general condition, flesh and cough. The ewes are a trifle thinner than the yearlings. The lambs are pretty badly off, Nos. 48 and 55 and another lamb are especially weakly, their noses stuffed up, eyes bleary and dull. All three lambs are undersized, weak and stiff in gait, and listless. Lamb No. 51 is twice as large as the others, seems better, but suffers more from cough than the yearlings. This lamb has all along eaten about double his share; another lamb is also large and in fairly good condition. Four of the seven lambs are hardly any larger than they were when received; the other three are undersized for their age but much better off than the smaller ones.

After feeding on full hay for eight days in order to relieve the cough, this bunch was turned into a new corral at 11 a.m. yesterday. The sheep had stopped picking the alfalfa which had been fed at 6 a.m. and a good deal still remained on the ground, the sheep being collected in groups with heads together. *When turned into the new corral, all animals at once began to graze eagerly, and continued for over an hour eating grass and loco as it was found.* The ewes and yearlings seemed to prefer the pods and the lambs ate only the leaves of the locoweed. This morning over nine-tenths of all loco is gone from the corral while not one-half of the grass seems to be gone. *Though careful observations were made yesterday and this morning, absolutely no effect could be seen in the sheep or lambs as the result of the weed, in spite of the fact that the* *animals have had more loco than they could eat, all at once, and after eight days of abstinence from the locoweed.*

In addition to grazing, this band has eaten about all of the alfalfa this morning.

Groups VII and VIII. These sheep have improved greatly in the last eight days on their full hay ration. The ewes and yearlings are still the thinnest lot of all, but have improved greatly, being less suffocated, having less bloody mucus from nose, less cough and sneezing, and having gained distinctly in flesh. The animals are still more stuffed up than those in the other corrals. The lambs, without exception, are bunged up with rhinitis; eyes and face swollen, eyes and nose running, eyes dull, gait stiff and slow, most of them very weak and thin.

This band after eight days on hay, was turned into a new corral, about 10 a.m. yesterday, after they had stopped feeding on hay. *At once all began grazing, eating loco by preference, but also other grasses; ewes and yearlings eating the seed tops, lambs the leaves.* After one hour they stopped grazing until evening. *Absolutely no effect could be noted in any sheep as the effect of eating the plant.*

This morning not one loco plant can be found, and in several cases the ewes and yearlings have been seen digging at the root, but apparently they have not eaten the plant below the surface of the ground. The corral still contains about one-quarter to one-third of the grasses and other plants.

The facts of special interest that the experiment afforded up to this time were, that the animals were eating the locoweed, that their general condition was pretty poor, that many of them suffered from cough and rhinitis and that some of the animals with cough would pass as locoed animals with more than half the ranchers. The condition was studied, and the diagnosis was made that the animal was suffering from the sheep fly disease, a diagnosis which was confirmed by autopsy. It was noticed particularly that this disease affected animals in every one of the corrals, those receiving no loco as well as those receiving loco. It was also noticed that the animals receiving the greatest amount of food suffered less severely than animals on insufficient rations; thus the animals receiving four pounds a day of alfalfa and allowed to graze freely on loco stayed in the best condition, the animals receiving four pounds a day of alfalfa alone did almost as well. The animals receiving half rations of alfalfa either with or without loco were more severely diseased. The animals receiving no alfalfa, but scraping a bare subsistence by grazing on the insufficient forage and loco, were the ones most severely diseased. No difference could be seen in the animals receiving salt and in those receiving no salt, as far as this disease was concerned. So severe was the outbreak of sheep fly disease, that the experiment was discontinued for eight days, as noted above, and the animals were kept in their corrals and fed abundantly with alfalfa hay. The symptoms promptly abated, and the experiment was resumed.

It is interesting to note that up to the time of interrupting the experiment, certain groups of animals had been receiving an abundance of locoweed without manifesting any symptoms that could be attributed to the action of the weed.

When the experiment was interrupted, it might have been expected that the animals would show symptoms from being suddenly deprived of the weed, just as morphine habitues, alcoholics and tobacco smokers show symptoms when suddenly deprived of their drugs. This did not occur with the sheep, and they were carefully watched in order to observe any manifestations of this nature. On the contrary, the improvement in the condition of the sheep when their diet was increased was almost immediate and resembled nothing so much as the improvement which takes place in a half starved animal when it is placed upon a proper diet.

When the experiment was resumed, another interesting observation was made: The animals were turned back upon the abundant loco fields after an abstinence from the weed lasting for eight days. It is a common impression that drug users exhibit symptoms when they are suddenly put upon full doses of their drugs after abstinence. This did not occur with the sheep. They were turned into the corrals with loco about an hour after they had finished eating the alfalfa hay which had been given them. They had left some alfalfa hay uneaten, and were lying around in the corners of the corrals. When turned into the corral with fresh forage, they at once began to graze, eating loco and other plants for an hour and a half. At no time did any animal exhibit a single symptom that could be attributed to the weed.

Another very interesting observation was made during this infection with the sheep fly. The animals badly infected with these parasites presented the most typical pictures of loco disease. On turning to the description of sheep fly disease, the Western rancher might well think that the writer of the book was confused and was writing a description of the locoweed disease. It must be remembered, however, that sheep fly disease occurs all over the world, and the symptoms have been described as characteristic for animals which are entirely beyond the reach of locoweed.

The following description taken from Neumann's *Parasites and Parasitic Diseases of the Domesticated Animals*, 1903, pp. 568–570, is inserted in order to make this clear:

"It (the sheep fly) hides in holes and crevices in the walls of the sheepfolds, which it leaves when coupling time has arrived and the temperature is sufficiently high. It then flies in a lively manner to greater elevations, and rests on rocks warmed by the sun. The fecundated female now goes in search of flocks of sheep, which are afraid of its approach, and to avoid it lie down, bury their noses in the dust between their forefeet, or are huddled together with their heads down. According to Bracy-Clark, they raise clouds of dust to deceive their enemy. It is during rumination that the insect finds a particularly favorable time for depositing its progeny. Its small size, gray color, and the rapidity of its flight, do not allow its ovulation to be observed; but there can be no doubt that it does not take place on the nose of the sheep. As soon, in fact, as these animals have been touched by the Oestrus, they become excited, run in every direction, hold down their noses and rub them against the ground or against their feet; often look anxiously around them, sneeze and snort, and seek ditches, furrows and dusty roads. Owing to the repeated rubbings, the nostrils are often abraided and inflamed. . . . . *Symptoms.*—It is usual to find three or four larvae of the Oestrus in the frontal sinuses of sheep, which, during life, had not given any indication of their presence. They rarely occasion any morbid disturbance, unless they are numerous and advanced in development at the commencement of spring. The first sign of their presence is a discharge, often unilateral, at first clear and serous, then thick and mucus, from the nostrils. Then there are frequent sneezings and snortings, accompanied by the expulsion of the mucus, and sometimes of the larvae. Later the animals throw the head upwards, often shake it, rub the nose on the ground, against some part of the body within reach, or with the forefeet. As the malady progresses the sheep hold their heads low, lift their limbs high in movement, as if walking in water—their gait resembling that of horses affected with immobilitie. Sometimes they suddenly throw up the head, carry the nose high, then move it convulsively. From time to time they stagger and are attacked with vertigo, but they do not turn in a circle. In more serious cases there is dyspnoea, the upper air-passage being obstructed by the larvae or the inflammation of the pituitary membrane. The eyes are red and lachrymose. The disease may be more complicated, the animals losing their appetite and their condition; they grind their teeth; foamy saliva flows from their mouth; the eyes pirouette in their orbits; and convulsions set in, then death ensues, sometimes in six to eight days after the appearance of the first symptoms. . . . . The common saying that a whimsical person is 'maggoty,' or has got 'maggots in the head,' perhaps arose from the freaks of sheep affected by these larvae.

"But it is rare that the malady reaches this paroxysm; it continues for a long time, and generally—the larvae being ejected one after another—the symptoms gradually subside, until they disappear altogether."

On September 6 the experiment was closed having lasted fifty-three days. The animals at that time had eaten bare a wide patch of loco. The total area covered by the sheep feeding on loco was: Groups IV and V, 57,894 square feet; Group VI, 289,854 square feet; Groups VII and VIII, 387,084 square feet. The corrals varied from 16,000 to 48,000 square feet in size.

The animals did not thrive on Ten Mile Flat, and did not do nearly so well as the original band (3,200), from which the animals were selected for experiment and which were kept on the mountain side where the forage was fairly abundant, and living conditions better. Even in this band, however, there were one or two hundred animals undersized, stiff-legged and evidently suffering from sheep fly disease.

The following tables show the weights in pounds of the animals when the experiment was begun and the weights at the close of the experiment or at the death of an animal, with the average change in weight for each group.

Group I, receiving daily 4 pounds of alfalfa per head and salted regularly.

TABLE II.

TAG NO.*	INITIAL WEIGHT	FINAL WEIGHT	ALTERATION IN WEIGHT	REMARKS
	pounds	*pounds*	*pounds*	
14	68	71	+ 3	All of group were yearlings.
31	70	74	+ 4	
4	58	59	+ 1	
5	61	73	+ 12	
10	73	73	+ 0	
Total.......	330	350	+ 20	Average increase 6.06 per cent.

* Each animal was numbered with an ear tag.

Group II, receiving daily 4 pounds of alfalfa per head, but no salt.

TABLE III.

TAG NO.	INITIAL WEIGHT	FINAL WEIGHT	ALTERATION IN WEIGHT	REMARKS
	pounds	*pounds*	*pounds*	
6	68	72	+ 4	All of group were yearlings.
34	74	81	+ 7	
24	61	58	− 3	
33	72	84	+ 12	
43	77	81	+ 4	
Total.......	352	376	+ 24	Average increase 6.81 per cent.

Group III, receiving daily 2 pounds of alfalfa per head, and salted regularly.

TABLE IV.

TAG NO.	INITIAL WEIGHT	FINAL WEIGHT	ALTERATION IN WEIGHT	REMARKS
	pounds	*pounds*	*pounds*	
15	66	68	+ 2	All of group were yearlings.
37	66	72	+ 6	
40	67	71	+ 4	
18	74	73	− 1	
36	77	81	+ 4	
Total.......	350	365	+ 15	Average increase 4.28 per cent.

Group IV receiving daily 4 pounds of alfalfa per head, salt and locoweed.

TABLE V.

TAG NO.	INITIAL WEIGHT	FINAL WEIGHT	ALTERATION IN WEIGHT	REMARKS
	pounds	*pounds*	*pounds*	
2	67	73	+ 6	All of group were yearlings.
21	60	71	+ 11	
41	78	83	+ 5	
12	62	70	+ 8	
42	62	68	+ 6	
Total.......	329	365	+ 36	Average increase 10.94 per cent.

Group V, receiving daily 4 pounds of alfalfa per head, locoweed, etc., but no salt.

TABLE VI.

TAG NO.	INITIAL WEIGHT	FINAL WEIGHT	ALTERATION IN WEIGHT	REMARKS
	pounds	*pounds*	*pounds*	
17	70	77	+ 7	All of group were yearlings.
29	73	83	+ 10	
23	73	85	+ 12	
1	70	80	+ 10	
32	57	69	+ 12	
Total.......	343	394	+ 51	Average increase 14.86 per cent.

Group VI, receiving daily 2 pounds of alfalfa per head; salted regularly, locoweed.

TABLE VII

A, yearlings

TAG NO.	INITIAL WEIGHT	FINAL WEIGHT	ALTERATION IN WEIGHT	REMARKS
	pounds	*pounds*	*pounds*	
11	76	89	+ 13	
35	77	84	+ 7	
38	73	76	+ 3	
19	68	61	− 7	
9	75	80	+ 5	
3	60	64	+ 4	
8	68	73	+ 5	
26	76	80	+ 4	
16	75	79	+ 4	
27	71	67	− 4	
Total.......	719	753	+ 34	Average increase 4.72 per cent.

TABLE VII—Continued

B, ewes

TAG NO.	INITIAL WEIGHT	FINAL WEIGHT	ALTERATION IN WEIGHT	REMARKS
70	93	86	− 7	
71	84	97	+ 13	
84	74	78	+ 4	
80	71	81	+ 10	
73	82	80	− 2	
76	98	105	+ 7	
75	77	78	+ 1	
78	68	80	+ 12	
67	79	81	+ 2	
Total.......	726	766	+ 40	Average increase 5.5 per cent.

C, lambs

TAG NO.	INITIAL WEIGHT	FINAL WEIGHT	ALTERATION IN WEIGHT	REMARKS
48	25	20	− 5	Died August 26.
51	27	32	+ 5	
52	22	18	− 4	Died August 7.
53	24	26	+ 2	Killed September 1.
55	17	20	+ 3	Killed September 1.
61	30	29½	− ½	
62	21	21½	+ ½	
63	30	37	+ 7	
Total.......	196	204	+ 8	Average increase 4.08 per cent.

Total initial weight of Group VI..........................1641 pounds
Total final weight of Group VI..........................1723 pounds
Total alteration in weight Group VI................ 82 pounds increase
Average increase in weight, Group VI...............4.99 per cent increase

Group VII, receiving no alfalfa; salted regularly; fed on locoweed.

TABLE VIII

A, yearlings

TAG NO.	INITIAL WEIGHT	FINAL WEIGHT	ALTERATION IN WEIGHT	REMARKS
	pounds	*pounds*	*pounds*	
39	76	74	− 2	
25	(72)			Killed August 23. No weight taken.
20	66	69	+ 3	
22	74	70	− 4	Died September 9.
Total.......	216	213	− 3	Average loss 1.39 per cent.

TABLE VIII—Continued

B, ewes

TAG NO.	INITIAL WEIGHT	FINAL WEIGHT	ALTERATION IN WEIGHT	REMARKS
74	79	79	0	
69	108	97	− 11	
68	74	72	− 2	
66	110	102	− 8	
72	78	77	− 1	
Total.......	449	427	− 22	Average loss 4.9 per cent.

C, lambs

TAG NO.	INITIAL WEIGHT	FINAL WEIGHT	ALTERATION IN WEIGHT	REMARKS
59	22	23	+ 1	Died September 6, weight of 20 pounds.
46	41	37	− 4	Died August 29.
49	30	31	+ 1	
56	31	32	+ 1	
58	25	23	− 2	Killed August 19.
Total.......	149	146	− 3	Average loss 2.01 per cent.

Total initial weight of all animals in Group VII (excluding No. 25).................................. 814 pounds
Total final weight of all animals in Group VII (excluding No. 25)...................................... 786 pounds
Total loss of weight of all animals in Group VII...... 28 pounds
Average alteration of all animals in Group VII........3.43 per cent decrease

Group VIII, receiving no alfalfa and no salt; fed on locoweed, etc.

TABLE IX

A, yearlings

TAG NO.	INITIAL WEIGHT	FINAL WEIGHT	ALTERATION IN WEIGHT	REMARKS
	pounds	*pounds*	*pounds*	
13	77	78	+ 1	
30	74	72	− 2	
28	71	70	− 1	
7	70	71	+ 1	
Total.......	292	291	− 1	Average loss 0.34 per cent.

TABLE IX—Continued

B, ewes

TAG NO.	INITIAL WEIGHT	FINAL WEIGHT	ALTERATION IN WEIGHT	REMARKS
81	72	70	− 2	
82	(76)			Died. Weight not recorded.
77	69	64	− 5	
79	74	84	+ 10	
83	72	79	+ 7	
Total.......	287	297	+ 10	Average gain 3.48 per cent

C, lambs

TAG NO.	INITIAL WEIGHT	FINAL WEIGHT	ALTERATION IN WEIGHT	REMARKS
57	37	37	0	
47	22	15	− 7	Died August 21.
54	31	34	+ 3	
60	30	33	+ 3	
50	29	22	− 7	Died August 29.
Total.......	149	141	− 8	Average loss 5.67 per cent.

Total initial weight of Group VIII (excluding No. 82)......................................	728 pounds
Total final weight of Group VIII (excluding No. 82)..	729 pounds
Total alteration in weight of Group VIII.........	1 pound gain
Average alteration in weight of Group VIII.......	0.0137 per cent increase

A study of these tables corroborates the general impressions derived from daily inspections of the experimental animals, and justifies the following conclusions.

1. The animals were not particularly large, the gains in weight were small even at the best.

2. The animals which increased most in weight were those in Groups IV and V (receiving alfalfa and locoweed, Tables V and VI), their gains being far ahead of those made by any other animals used in the experiment. The animals in Groups I and II (Tables II and III, alfalfa but no locoweed), came next, but they did not gain nearly so much. The animals in Group III (Table IV, one-half alfalfa, no loco) and Group VI (Table VII, one-half alfalfa, and locoweed) were third, with gains which were nearly the same in each group. As might be expected, groups VII and VIII (Tables VIII and IX, no alfalfa, all locoweed), made the worst showing, and presented a total actual loss of weight.

3. The animals receiving half rations of alfalfa, and locoweed (Table VII—A, Group VI) throve better than those on half rations not eating locoweed (Group III, Table IV).

4. The gains in weight were directly proportional to the actual amount of food and also to the amount of fresh forage obtained by the sheep.

5. The animals which throve best were eating abundantly of locoweed, in addition to having an abundance of alfalfa hay together with the rather scanty natural forage of growing plants other than locoweed.

6. Animals receiving no salt (Group II, Table III and Group V, Table VI), gained more in weight than the animals receiving salt (Groups I and IV). The unsalted animals on locoweed alone (Group VIII, Table IX) gained 1 lb., while the salted animals on loco alone (Group VII) lost 28 lbs. It is remarkable that the lambs on locoweed with no salt (Table IX, C) lost more weight than the lambs on locoweed with salt, (Table VIII, C) while with the yearlings on locoweed (Tables IX, A and Table VIII, A), and especially with the ewes (Tables IX, B and VIII, B), the unsalted were better off. Although slightly heavier, the unsalted animals were not more resistant to the sheep fly disease.

7. In general, the larger and stronger animals did better in each corral than the younger and weaker ones; (seen by comparing the average alteration in weight of the ewes, yearlings and lambs, respectively, in Groups VII and VIII, and remembering that the greatest relative increase should be found in growing lambs).

8. The tables show clearly that when enough nutritious food is provided, the locoweed (*A. spicatus*) certainly does not injure the health of sheep, within the limits of time taken for this experiment.

In addition to the inductions just drawn from the tables, the general conclusions from the experiment may be stated in brief;

1. Healthy sheep appear not to eat locoweed if they can easily obtain a plentiful supply of green forage.

2. Sheep can easily be made to eat locoweed by depriving them of other food; by diminishing other food; or by diminishing other green forage available, even though the animals be well fed on alfalfa.

3. Animals which have started to eat locoweed, do not eat it to the exclusion of other food, although they do appear to eat rather more of locoweed than of any other single plant. The lambs eat the leaves, the yearlings and adult sheep eat the stalks and pods. Once in a while an adult sheep will be found rubbing with his forefeet for the root of the weed, but this was rare. The usual cause for sheep rubbing the nose in the ground is the presence of sheep fly larvae in the nose.

4. There was no evidence that the locoweed produced any poisonous effect, clinical or anatomical, in a single sheep during the fifty-three days of this experiment. The evidence indicates that the food value of the weed must be very slight, but if the plant has any narcotic or other action, it is so obscure that it could not be made out by careful and frequent observations of the animals used for this experiment. It is to be noted again that no symptoms developed when the sheep eating the plant were suddenly deprived of it, nor when they were returned to it after a week's abstinence; and that sheep on abundant locoweed and abundant other food were the ones which throve better than any others in this experiment.

5. The sheep used in this experiment did not thrive. This applies to the sheep receiving alfalfa only, as well as to those receiving loco. At the beginning of the experiment the sheep and lambs were average healthy specimens. At the end of the experiment the animals had either gained very little or had lost weight and were evidently in much poorer shape than the members of the flock from which they were selected and which had been kept on the mountain side. The causes for the failure of these sheep to do well were probably confinement, lack of protection against the intense heat of the sun, insufficient green forage, and inadequacy of food in the case of those not receiving alfalfa.

6. During the course of the experiment, sheep fly disease broke out among the animals, giving every appearance popularly attributed to loco disease, and affecting indifferently those eating locoweed and those not eating it. The animals most severely diseased were those receiving the least food.

7. The vermifuges used before starting the experiment did not remove the *Thysanosoma actinioides*.

8. In the course of this rather short experiment, no ill effects were observed in animals deprived of salt.

Soon after concluding the feeding experiment at Ten Mile Flat, the sheep which remained were sent to the Montana Agricultural College where interesting studies were made to determine whether they could be profitably fattened for market. The report of the experiment has been published by Linfield (Bull. No. 59, Montana Agricult. Exp. Sta., 1905). He found that the sheep gained only about half as fast as healthy sheep, and that it was unprofitable to prepare them for market on account of the length of time and cost of the feed required to fatten them.

From time to time during the experiment, a yearling or a lamb died, or was killed, and examined at autopsy. The sheep fly larvae were found in great abundance wandering over the nasal passages and up into the cavities of the head; bronchopneumonia and parasitic worms were also found. A few typical autopsies are detailed below, and the others are summarized.

Autopsy 16. Lamb No. 52, from Corral VI, May lamb of Van Cleve's flock, weight 18 pounds, length 28½ inches from tip of nose to root of tail. Lamb had been in poor condition for some time, suffering with cough, like the rest of the flock, frequently digging its nose into the ground, and occasionally raising its head high in the air. The animal was very weak, walked feebly, ran behind the rest of the bunch of sheep, raising its legs, especially the hind legs, as if wading through water. On August 6 the animal looked very ill, the cough and running from the eyes being very pronounced, the face being swollen and the animal very weak. The lamb died on the evening of August 7 and was autopsied at 9.30 a.m., August 8.

Anatomical diagnosis. Acute bronchopneumonia right and left lungs, with abscess formation, empyema, and bronchitis; acute intense inflammation of mucous membranes of nasal passages and superior orbital sinuses; twenty to forty embryos of *Oestrus ovis* crawling in sinuses, nares and trachea. Emaciation. Adhesions between omentum, ventral wall, intestines and stomach with cysts in the midst of the adhesions containing caseous-purulent contents (*Cysticercus tenuicollis?*); openings from the paunch into four of above-mentioned cysts. Sarcosporidiosis. Irregular incisor teeth.

The lamb was undersized with practically no subcutaneous fat, muscle reddish brown, clear and translucent. Several lymph glands in the femoral region were found large, red and soft; on section being much injected, and having dilated capillaries. The abdominal wall was tightly adherent to the omentum by fibrous adhesions which also bound the omentum to the paunch and intestines. In the adhesions were twenty or thirty small cysts about 1 x ½ cm. with walls approximately 1 or 2 mm. thick. These cysts were smooth on the inside but exteriorly were bound in fibrous tissue. One cyst contained a formless mass of semi-caseous, yellow, purulent material. The cysts were distributed on the under surface of the diaphragm, over the paunch, in the loops of the gut and in the pelvis. The omentum and mesentery were devoid of fat. Mesenteric and retroperitoneal lymph glands large and soft, the retroperitoneal lymph glands were numerous, varying in size from a pin's head to 3 mm. in diameter. The intestines were clear throughout, the stomach contained a moderate amount of soft food. In the first stomach opposite the transverse furrow were three or four round openings 4 or 5 mm. in diameter extending through the stomach wall and communicating directly with the cavity of the caseous cyst described above, pressure upon the cysts forcing pus through the stomach openings.

Spleen 10 x 6½ x 2 cm., small, soft, no adhesions. On section very soft, structures not visible. Liver of fair size, surface smooth. The gall bladder and bile ducts were clear. Kidneys; capsule stripped readily, leaving pale mottled surface with the cortex averaging 11 mm. On section the kidney was gray, and cloudy, the glomeruli were seen indistinctly, the striations easily visible. The pelvic organs and adrenals were normal, the pancreas was soft and showed post-mortem changes.

Thorax. The left lung was tightly bound by fibrous adhesions to the chest wall, the apex and dorsal surface of the lung being free, the rest of it being bound to the chest wall, to the pericardium and the diaphragm. In freeing the lung a cavity was opened into in the lower left lobe and about 100 cc. of thick yellowish gray pus escaped. The entire lung was consolidated except the apex and upper dorsal part of upper lobe.

The cavity occupied two-thirds of the lower lobe on the ventral and inferior sides. On section the cavity had smooth lining and contained pus and semi-caseous yellowish material. The consolidated lung was of a dark red color, not crepitant, rather dry with very little excess of mucus in the bronchi. The bronchi were injected, the bronchial lymph glands greatly enlarged, red and soft. The right lung was also adherent to the diaphragm and pericardium and to the ventral surface of chest wall. There were several abscesses varying from 5 to 8 cm. in diameter in the lower lobe, the consolidated portions resembling the left lung.

Heart. The pericardial and epicardial fat was absent. The surfaces were smooth and glistening. On the anterior (ventral) surface were a few ecchymoses. The valves were clear and delicate, the endocardium clear, the myocardium pale grayish brown, opaque. Just below the larnyx several Oestrus larvae were found crawling around the trachea, which was markedly injected. The mouth was clear, the incisor teeth were loose and twice as long as normal. Nares. The nose was covered with mud. On longitudinal section of the head the mucous membrane over the turbinated bones, septum, etc., was intensely engorged, dark red in color, ecchymotic and covered with thick muco-pus. Great numbers of small Oestrus larvae were found crawling over the mucous membranes as high as the ethmoidal turbinates. Brain and spinal cord presented no abnormality.

Microscopic report. The heart cells were granular, the cross striations not distinct. No sarcocysts were found. Lung. Sections from several parts of lungs showed areas of bronchopneumonia with abscess formation. The pleura was considerably thickened, showing granulation tissue together with large pink mononucleated cells, lymphocytes and polymorphonuclear leukocytes. In a section through the edge of an abscess, the necrotic tissue in the center presented the appearance of a homogeneous débris; passing out from the center there were zones of inflammation becoming less intense to almost normal lung tissue. In the zones of inflammation the predominating cells were large, mononucleated cells with bright pink granular cytoplasm, together with numerous lymphocytes and polymorphonuclear leukocytes. The distribution of the exudate in the lung alveoli was irregular, some alveoli being plugged with exudate, others empty. Desquamated cells and débris appeared in the lumina of the bronchi. Spleen, capsule not thickened, malpighian bodies visible but not conspicuous, showing evidence of hyperplasia in the centers; the pulp congested, it being difficult to distinguish sinuses from pulp. In the pulp were found many red cells and fragments of red cells together with large endothelioid cells which frequently were more or less tinged with blood pigment. A few cells resembling nucleated reds but not positively identifiable were seen in the pulp. Liver, the cells and cell columns were separated, the cells being swollen and vacuolated as if from post-mortem change. Kidney, the epithelium lining the tubules was extremely granular, cloudy and rather vacuolated. Lymph gland, capsule appeared rather edematous, the peripheral sinus only faintly seen. Cell nests not separated from cell columns. The striking feature of the lymph gland was that the periphery of the gland was densely packed with lymphoid cells while the central portions were especially marked by dilated sinuses partly filled with what appeared to be free cells. The cells in the peripheral region varied from small lymphoid cells to large mononuclears, the former predominating. Some of the larger cells had a bright pink non-granular cytoplasm. The dilated sinuses were moderately well filled with large desquamated endothelial cells with vacuolated or pink cytoplasm together with fairly abundant lymphoid cells. The desquamated endothelial cells dominated the appearance in the sinuses. The blood vessels of the lymph gland were markedly congested.

Voluntary muscle. Poorly preserved, showed small bodies which were not very distinct but appeared to be sarcosporidia.

Section through abscess cyst of omentum. Cyst was about 1 cm. x ½. Capsule thin, the outer part being made of laminated connective tissue, the inner portion of young granulation tissue in which there were great numbers of cells resembling large mononuclear leukocytes, and a number of small mononuclears, together with many rather granular and swollen cells of fibroblastic appearance. The inner edge of the wall passed off suddenly into a region of coagulation necrosis in which were found fragmenting nuclei, polymorphonuclear leukocytes and purple débris. This was continuous with a pinkish granular débris filling the center of the cavity. There was no evidence of daughter tubercle formation in the wall of the cyst but the general appearance suggested tuberculosis as a possible diagnosis.

Autopsy 17. Lamb 58, from Corral VII (no alfalfa), had become very ill but had improved markedly when fed on alfalfa, when the experiment was interrupted, and had remained fairly well after being returned to the loco diet; was removed from the corral for the examination. The temperature of the lamb was 101 (rectal temperature); the lamb was emaciated and stiff-legged but was not one of the illest lambs in the corral. It coughed a good deal but seemed in better condition than it had been previous to the diet of alfalfa hay. Its weight was twenty-three pounds. The animal was chloroformed and autopsied at once on August 19.

Anatomical diagnosis. Oestrus ovis infection of nasal passages with muco-purulent inflammation. Multiple pin head bronchopneumonic patches in both lungs. Emaciation. Infarction in liver; *Thysanosoma actinioides* hepatitis. Irregular incisor teeth.

The skin, muscles, and peritoneal cavity were clear; there was a small amount of fat. The spleen was small, soft, translucent, the structures appearing normal. The liver was smooth, semi-translucent excepting for an irregular wedge-shaped area on the upper surface on the left lobe just to the left of midline. This area was not raised, was smooth and had a mottled, grayish red color with fine, pin point dark mottlings. On section the area was wedge-shaped extending 1½ cm. into the liver. The kidneys, pancreas, and bile ducts, stomachs, pelvic viscera, and adrenals presented no abnormality. One small, fringed tapeworm found in the intestines. Heart contained a fair amount of fat; was clear throughout except that muscle was rather gray and opaque.

The lungs were voluminous, surface smooth, for the most part pink, but mottled at irregular intervals with dark red points varying from a pin head to 1 cm. in size. On section the dotted appearance was also seen as if from very numerous, small areas of consolidation. There were no tubercles and no cavities found. The right and left lung were alike. The bronchi and trachea contained frothy, blood-tinged mucus. The turbinated passages were swollen, dark red and covered with muco-pus. A large Oestrus larvae and six small ones were found in the nasal passages. The incisor teeth were unusually long.

Microscopic report. Heart striations and fibrillations distinct, often the clear space around the heart nucleus seemed rather larger than usual. No other noticeable alteration. Lung, showed small regions in which the alveoli were packed with red cells and coagulated albumin, alternating with relatively normal lung. In the region with exudate there were remarkably few leukocytes and only a few desquamated epithelial cells. No fibrin was made out in the H. and E. specimen. The bronchi were practically clear. The alveolar capillaries in general congested. Spleen, malpighian bodies conspicuous, rather dense, and larger than usual, apparently however, a normal spleen.

Liver. A small block showed considerable thickening throughout the portal systems, chiefly around the bile ducts with rather edematous tissue. The central third of the section was devoid of liver tissue and showed a mass of cellular material with two or three areas of necrosis. In this tissue were dilated bile ducts. The cellular tissue was composed of fibroblasts of various size, many of which contained a slight amount of brownish pigment. Among them were mononuclears, polymorphonuclears and eosinophiles. The eosinophiles were abundant and in part appeared to have polymorphous nuclei, in other less frequent instances, single, round nuclei. Compressed and degenerating liver tissue was found at the edge of this cellular accumulation and evidence of the extension into the liver of the newly forming connective tissue was met with. At the edge of the connective tissue there was hemorrhage into the liver. Elsewhere the liver cells were cloudy, vacuolated and granular.

Kidney showed a moderate amount of coagulated albumin in the tubules. The convoluted tubular epithelium moderately cloudy, otherwise the kidney appeared normal. Mucosa of nares; the most striking feature was the very great dilatation of the blood sinuses and bood vessels, the intervening tissue being distinctly edematous. In the edematous tissue were found numerous eosinophiles with polymorphous nuclei, large mononuclears and a few lymphocytes. The eosinophile was the most striking and probably the most abundant infiltrating cell. The epithelium of the glands was very cloudy and full of mucus. Immediately beneath the surface epithelium there were rather dense accumulations of mononuclears, chiefly lymphocytes, the eosinophiles being less abundant in this region. Section of lymph gland, showed widely dilated central sinuses with phagocytic endothelial cells. Voluntary muscle appeared clear, no sarcocysts being found. Epithelium and wall of esophagus appeared normal.

Autopsy 18. Lamb 47 from Corral VIII. The lamb had been very feeble but had improved during the week of alfalfa feedings. Subsequently it fell into worse condition and died on the evening of August 21. The autopsy was performed at 2 p.m., August 22.

Anatomical diagnosis. Oestrus ovis infection; acute mucopurulent inflammation of mucous membrane lining the nasal passages and accessory sinuses; bronchopneumonia with abscess formation. Infection with *Thysanosoma actinioides.* Irregular incisor teeth.

The animal weighed 15 pounds, its wool was ragged, incisor teeth loose and long. Emaciation was extreme. The peritoneal cavity was clear, there was no fat visible, the mesenteric glands were large, pale and soft.

The liver, spleen and kidneys appeared normal. A small fringed tapeworm was found in the small intestine. The stomachs appeared normal. Both lungs were adherent over the ventral and cephalad portions by fibrous adhesions, denser on the right side. In each lung were multiple small areas of consolidation with abscess formation. The bronchi contained bloody mucus. Heart appeared normal, except for absence of fat. On opening the head the engorgement of the mucous membrane of the nasal passages was intense, there being many larvae crawling over the mucous membrane with muco-pus and pin point hemorrhages into the mucosa.

Microscopic report. Heart muscle cloudy, striations indistinct, fibrillations still well marked.

Lung, showed bronchopneumonia, many of the alveoli being packed with a cellular exudate, while adjacent alveoli were relatively free or showed only coagulated albumin and desquamated epithelium. The exudate consisted of desquamated epithelium, coagulated albumin and great numbers of polymorphonuclear cells. In one or two of the alveoli elongated cells were met with of a fibroblast appearance but new capillaries could not be seen. Section did not pass through one of the abscesses. Spleen, capsule not thickened, malpighian bodies not very sharply marked but could be seen. Pulp cellular, the sinuses not being distinct. Liver cells rather large and cloudy, being separated as if there was slight post mortem change, otherwise liver seemed normal. Kidney, normal except for quite marked post mortem change affecting the convoluted tubules. Parotid gland markedly congested, the acini being made up of large purple mucus-containing cells which were not very well preserved. No abnormality could be made out. Lymph gland presented much the features described in previous autopsies, the periphery being packed with lymphoid cells, the blood vessels intensely congested, the central sinuses widely dilated and containing moderate numbers of free cells. Section through esophagus showed normal epithelium and normal muscle wall. Section through vomer showed markedly dilated blood vessels in the mucosa, with advanced mucoid change in the glands. There was also edema of the tissue between the glands. Rather numerous and small lymphoid cells were distributed throughout the mucosa. Section through turbinated bone; the high columnar mucosa was fairly well preserved, beneath which came a markedly edematous tissue containing greatly dilated blood vessels, eosinophiles, lymphoid cells and large mononuclears. Sections through several regions of the nasal mucosa showed similar conditions. In some regions the exudate was slightly more abundant than in others, but the same general features prevailed. The glands in this tissue usually contained a purplish material apparently mucus, sometimes a coagulated pinkish material like albumin.

Autopsy 19. One of the yearlings from Corral VIII had looked rather ill for some time but evidently was convalescing after the use of the alfalfa hay. The animal was still coughing and sneezing; walked stiffly and had a bloody discharge from the nose. On August 23, the animal was chloroformed and autopsied at once.

Anatomical diagnosis. Oestrus ovis infection, subacute. *Ostertagia marshalli* in stomach; *Thysanosoma actinioides* in bile ducts, hepatitis, emaciation. Sarcosporidiosis.

Autopsy 22. Lamb No. 48 was brought from Corral VI, in dying condition. It was seen living on the morning of August 26, but was dead at 2 p.m. and was autopsied at once.

Anatomical diagnosis. Acute *Oestrus ovis* infection. Acute bronchopneumonia. Emaciation.

Autopsy 25. After two months of drouth a heavy cold rain fell on the evening of August 28. On the following morning lambs 46 from group VII, and 50 from group VIII, were dead. Lamb 46 had always been a heavy feeder and looked like one of the strongest in the pen. Autopsy, 8.15 a.m.

Anatomical diagnosis. Acute bronchopneumonia. Acute splenic tumor. Cloudy swelling of myocardium, liver and kidney. Acute catarrhal enteritis of lower ileum and cecum. Acute mucopurulent rhinitis and sinusitis with about a dozen young

Oestrus larvae and one or two older ones in the supraorbital sinus. *Cysticercus tenuicollis* in peritoneal cavity. Sarcosporidiosis. Acute lymphadenitis. Brain and spinal cord normal.

Autopsy 26. Lamb 50, Corral VIII, found dead on morning of August 29, after a cold rain during the previous night.

Anatomical diagnosis. Acute bronchopneumonia. Acute *Oestrus ovis* rhinitis, emaciation. Irregular incisor teeth. Brain normal.

Autopsy No. 30. Lamb No. 55, from Corral VI, was much stunted and emaciated. Fleece was fairly regular and thick. The animal was very weak and uncertain on its feet and walked with a stiff-legged gait. There was marked coughing and sneezing. The respirations were difficult, irregular with many pauses and many short, broken, inspirations. Taking the average of five minutes' count there were 100 respirations per minute. The rectal temperature was 102°. The eyes were gummed with mucus. The eyelids puffy and swollen. The nose was covered with muco-pus and dirt. The lamb was so weak that it could hardly get to its feet when it had been laid on the ground. September 1, 1904, animal was bled to death and autopsied at once.

Anatomical diagnosis. Subacute (convalescent) Oestrus rhinitis and sinusitis with empyema of ethmoidal sinus, cervical lymphadenitis. Bronchopneumonia. Emaciation. Irregular incisor teeth. Red marrow. Brain and spinal cord normal.

Lamb 53, from Corral VI, was selected for examination on September 1. The hemogoblin was 70 per cent (Tallquist), the weight 26 pounds. The lamb was kept till September 5 on alfalfa hay when the weight was 28½ pounds. Respirations were irregular, 46 to the minute, temperature 102°. The paunch of the lamb was distinctly distended; the animal was very weak and thin. When laid on the ground it could hardly recover its feet. It walked with an uncertain stiff-legged gait. The animal at time of examination on September 5 was in better condition than it was a week earlier and for the last four days, between September 1 and 5 improved steadily upon a hay diet. The animal had good vision, it recognized a person bringing alfalfa or water to it. There was no evidence of psychic disturbance. The animal's breathing was distinctly impaired, especially at night and during early morning hours. In the morning, especially, the head was stuffed up, the eyelids being gummed together and there being a discharge from the nose of thick mucus. The animal was bled to death on September 5 and autopsied at once.

Autopsy 31. September 5, 904.

Anatomical diagnosis. Subsiding Oestrus rhinitis and sinusitis; empyema of ethmoidal cells; bronchopneumonia; *Thysanosoma actinioides* dilating common duct, cystic duct and beginning to enter liver ducts; *Cysticercus tenuicollis*; emaciation. Irregular incisor teeth. Brain and cord, and stomachs and intestines normal.

Lamb No. 59 from corral VII was found dying on morning of September 6. Died about 11 a.m. Autopsy at 1.30 p.m.

Autopsy 32. Anatomical diagnosis. Bronchopneumonia, serofibrinous pleurisy on right and left; small purulent cysts in lungs. *Oestrus ovis* in nasal cavities; empyema of ethmoidal turbinatea; *Thysanosoma actinioides;* emaciation. Acute splenic tumor. Necrosis in lymph gland.

D. DISCUSSION OF PARASITIC DISEASES ENCOUNTERED.

During the summers of 1903 and 1904 thirty-two autopsies were performed; twenty-eight are included in the preceding report, one other was upon an experimental "locoed" sheep at White Sulphur, Montana; three autopsies upon cattle have been discarded as they were performed hurriedly and were unsatisfactory. In the twenty-nine autopsies on sheep the principal diseases and parasites observed were:

TABLE X

		Times
Thysanosoma actinioides, Total		29
Thysanosoma with peribiliary hepatitis		14
Thysanosoma with an abscess or infarction of liver		2
Stomach worms (Ostertagia marshalli)		8
Other intestinal worms		5
Lung worms		3
Oestrus ovis larvae		15
Sarcocystis tenella		15
Pneumonia, Total		18
Pneumonia, with lung worms	1	
Pneumonia, with bronchiectatic abscesses, or cysts	4	
Pneumonia, with oestrus ovis	15	
Pneumonia, of unknown origin	3	
Pneumonia, with pleurisy or empyema	2	
Acute enteritis		1
Cysticercus tenuicollis in peritoneal cavity		8
Cysticercus in pericardial cavity		1
Abscess, supposedly around cysticercus, total		5
Abscess, on epicardial surface	2	
Abscess, on liver	1	
Abscess, in peritoneal cavity	2	
Hair balls in stomach		2
Renal calculi		2
Extradural abscess		1
Necrosis in lymph gland		1
Inflammatory nodule in kidney		1
Irregular incisor teeth*		14
Emaciation		16

* No note upon the teeth was made in other autopsies.

The *Thysanosoma actinioides*, or *Taenia fimbriata*, or "fringed tapeworm" has attracted attention in this country chiefly from Curtice, (4th and 5th Annual Reports, Bureau of Animal Industry, 1887–1888; pp. 167,–186; also "The Animal Parasites of Sheep," Washington, 1890; also *Vet.*

Record 1, p. 59). He found that the smallest worms appeared in lambs of about two months age, and occurred in sheep of all ages, and at all seasons except possibly during the winter months. He was unable to remove the parasite by treatment. He concluded that disease resulting from infection with this parasite was commonly called "loco" disease, and he was sceptical regarding the existence of true locoweed poisoning in sheep. He estimated that the losses from death of sheep and depreciation due to this parasite, were enormous, lambs and yearlings suffering chiefly. Practically all Western flocks seemed to be infected but it seemed to be especially common among the descendents of the Mexican or Spanish sheep with which the larger ranches were originally stocked. In all, he found 89 per cent of Western sheep infected.

He saw the worms in the duodenum and bile ducts, the ducts being so tightly distended with them at times that the worms could not be extracted except in pieces. He also saw the worms in the pancreatic ducts. Thickened and dilated ducts were found occasionally with no worms in them. The chief features of the disease produced by the worms were those of progressing malnutrition, or cachexia, with occasional excess of fluid in the serous cavities. In fatal cases death usually was due to starvation, exposure, or intercurrent disease.

My findings corroborate those of Curtice in certain respects and in addition I have been able to demonstrate a serious lesion of the liver, with the appearance of a necrotic, organizing area in the liver, resembling an infarction; while in one case (No. 4), there was a fungating liver abscess in connection with an extensive hemorrhagic subdiaphragmatic abscess. In a number of autopsies the microscopic studies demonstrated the presence of a process of fibrosis, more or less advanced, proceeding from around the thickened ducts and extending out into the liver substance.

In other words, these cases establish a new form of chronic inflammation of the liver, a form distinct from liver fluke disease on the one hand and from the inflammation resulting from gall stone obstruction on the other, for here the tapeworm, *Thysanosoma,* is responsible for the disease.

It may also be remarked that vacuolar changes were present in the pancreatic cells in a case where the pancreatic duct was filled with the tape worm. From the nature of my work it was impossible to determine the clinical course of the disease, but it seemed clear that heavy infections were frequently associated with malnutrition or cachexia.*

* A description of the microscopic changes in the liver together with a demonstration of sections was given by me in an article read before the American Association of Pathologists and Bacteriologists at Philadelphia in April, 1912.

Oestrus ovis infection. Aside from the microscopic studies of the nasal mucous membranes and cervical lymph glands, detailed in the autopsies at the end of Section C, my studies emphasize only two points with regard to *Oestrus ovis* infection; first, that this parasite is a source of danger to Western sheep; second, that the severity of the symptoms produced by the parasite depends most intimately upon the general condition of the animal, and upon the amount of nutritious food which it receives.

It is of interest to observe that the *Sarcocystis tenella* was present in practically every case in which the microscopic examination was made except in the case of lambs only 3 or 4 months old. It is, however, not generally regarded as a dangerous infectious agent, although according to J. Fiebiger, (*Die tierischen Parasiten der Haus-und-Nutztiere*; 1912, p. 113–119) during one year $1\frac{1}{2}$ per cent of sheep at Budapest were condemned as unfit for food on account of infection with this parasite.

No special remarks are required concerning the lung and stomach worms, for parasites of this type are known the world over. It is to be noted however, that this report together with the recent work from the Bureau of Animal Industry shows that the semi-arid condition of the general grazing grounds in the West is no protection against the spread of several dangerous parasites, when the bed grounds, watering places, etc., are allowed to become polluted, and to remain so.

E. REVIEW OF RECENT PUBLICATIONS DEALING WITH LOCOWEED DISEASE.

The office of the Poisonous Plant Investigations in the Bureau of Plant Industry continued the study of locoweed disease, after the conclusions which I drew from the field work of 1903 and 1904 had been submitted. A double-headed campaign was launched, in which Dr. Albert C. Crawford (*loc. cit.*) conducted investigations designed to test the poisonous action of the locoweed under laboratory conditions and to ascertain the nature of the poisons obtained, while Dr. C. Dwight Marsh (*loc. cit.*) carried out experimental studies of the loco problem in the field.

After a scholarly review of the literature, Crawford details his own elaborate experiments, which led him to conclude that the symptoms of loco poisoning can be reproduced in rabbits by feeding them extracts of certain loco plants; that the symptoms in the rabbits which he studied were due to the barium in the locoweed extract; that there may be other poisonous principles in locoweeds from other regions, and that locoweed grown on some soils contains no barium and is not active.

As Dr. Crawford did not test the locoweed grown in the region where my experiments were conducted, it cannot be determined, except by a special study, whether he would have placed these particular locoweeds in the barium-containing, actively poisonous group, or in his inactive group free from barium. But this much is certain; the sheep which ate this locoweed freely for the greater part of the fifty-three days of the experiment presented neither the symptoms, nor anatomical changes which Crawford gives as characteristic of barium poisoning. A recent publication by Alsberg and Black weakens the force of Crawford's conclusions. These writers, (Bureau of Plant Industry, 1912, Bull. 246) as the result of experiments and other considerations which they record, are led to the conviction that the toxicity of locoweeds in laboratory experiments is not due to barium, and Marsh also (Bull. 246) concludes from studies in the field that typical loco poisoning is not produced by barium feeding alone. Up to the present no further laboratory studies have been published, and it now looks as if Crawford's view as to barium cannot be maintained, but that his experiments at least suggest that some locoweeds contain a poison of unknown nature which other locoweeds do not contain.

The second division of the attack upon the loco problem was under the direction of Dr. Marsh who conducted feeding experiments upon a larger and more thorough scale than had ever before been attempted. For three seasons, 1905, 1906, and 1907, horses and cattle were fed upon locoweed in three camps established in Colorado and Nebraska. The animals were observed during life, autopsies were performed on many of them, and many other locoed animals, horses, cattle, sheep, and goats were examined. Accounts of the experiments and of the results obtained appear in several bulletins from the Department of Agriculture, and in reports from the Agricultural Experimental Stations of Colorado and Nebraska which actively coöperated in Marsh's work. From these accounts it is clear that Marsh believes that he has solved the loco problem, that he has proved that the locoweed is poisonous, and that he has established symptoms and anatomical changes which are together equally characteristic of locoweed intoxication. It is, however, difficult for a medical reader studying Marsh's reports to follow him to his conclusions, for the data which he prints in his technical report (Bull. 112), and elsewhere, do not always make it logically necessary for the reader to come to the same conclusions that Marsh reached. This is noticeable between pp. 47 and 72 in his review of cases, where it cannot be seen by the reader that Marsh has always excluded other possible diagnoses with sufficient fulness to justify his conclusions that the animals described owed their maladies to the locoweed. This deficiency may be corrected when he publishes in full his anatomic, microscopic, and bacteriologic studies, which references in Bulletin 112 allow us to expect. It is, however, clear that Marsh's animals on locoweed diet became ill and died, while controls, not eating locoweed remained healthy. The following analysis of Marsh's publications represents an attempt to construct the clearest possible medical description of Marsh's animals, and incidentally indicates the variations which he mentions. Marsh, (Bureau Animal Industry, Bull. 112; 1909, p. 114) claims that "animals eating (the loco plants) succumb sooner or later to their poisonous action." He modifies this statement elsewhere; thus, (Farmer's Bull. 380, p. 10):

During the spring months, before the grass starts, where the white locoweed is abundant practically all animals eat more or less of it. As the grass becomes more abundant, many of these leave the locoweeds and devote themselves entirely to grass. These animals as a rule do not seem to be injured by the habit. Others . . . . continue to eat the locoweed even where there is an abundance of other feed. Whether an animal will become locoed or not is then simply a matter dependent upon the individual. Some cattle and horses will eat locoweeds for a part of the year, for a period of years and suffer no harm. Others . . . . eat this plant almost exclusively, and these will die within a few months, or, in some cases, even within a few weeks.

The same idea occurs also in Bulletin 112 (Case No. 10., etc.)

The symptoms were essentially like those popularly described, a staggering, stiff, and uncertain gait, the hind legs being dragged frequently; a general disturbance of the nervous system which leads in some cases to an apparent partial paralysis of the limbs and to a very distinct lack of muscular coördination. Anaesthesia of the skin may be pronounced.

"The animals eating loco eat more and more of it, although they do not in all cases acquire a passionate love for the weed, and sooner or later lose flesh, and die of starvation." . . . . "The first pronounced symptom (Bull. 246, p. 34) is in the gait, which is stiff with more or less evidence of partial paralysis. There is a lack of muscular coördination, which produces 'high stepping,' rearing, jumping and stumbling." In drinking, the mouth moves in a peculiar way, somewhat as in eating. "The animal is either dull and dejected or in constant motion . . . . It gradually loses flesh, its coat becomes rough, its eyes staring, it becomes profoundly anemic, and eventually it dies of starvation."

Abortion is common among locoed cows. The temperature is usually normal, but varies from subnormal to 108°F. Marsh's colleagues, Peters and Sturdevant, from the Nebraska Agricultural Experiment Station (21st Annual Report, 1908) add to these symptoms, that the horses experimented with showed distinct irregular swellings more or less bilaterally symmetrical, which appeared early on the cheeks and side of the lower jaw as well as "on the place behind the lower lip." Lymphatic enlargement in the intermaxillary space also appeared early. Marsh found (p. 92) that "during the early period of loco feeding there were no symptoms of poisoning. Horses and cattle will eat quite freely of the weed for a considerable period with no apparent ill effects and may even gain considerably in flesh." Marsh frequently speaks of malnutrition among his animals, and of their starving to death, but it is only very recently (Farmers Bulletin 536, issued May 1, 1913), that he has begun to come around to the view expressed in my reports of 1903 and 1905 that underfeeding is one of the main causes of the loss of Western live stock.

Great individual differences were found to exist among animals in regard to susceptibility to the loco poison; in general, the better bred animals succumbed more quickly. Horses were the principal animals attacked in regions where the *Astragalus mollissimus* prevailed. This was found much more toxic than the *Aragalus lamberti* which did more damage to cattle and sheep than to horses.

In most of his experimental animals the symptoms were of sudden onset. The weed was eaten by the animal with no evidence of injury until a relatively short time before death, when symptoms developed rapidly. The interval from the time the animal began to eat locoweed until its death varied from two months and eight days to six months and nine days; with cattle the average interval was about five months, with horses rather less. If the animal ate only moderate amounts of loco, a fatal outcome was indefinitely delayed. Cases of "acute" locoism in lambs were met with in which death resulted in two or three weeks from the first eating of the locoweed. Age was not a factor in the disease, old as well as young animals becoming locoed.

Marsh states (p. 114), that there were certain quite definite anatomical changes found at autopsy.

> The animals were strongly anemic. This anemia was indicated not only by paleness of flesh and actual loss of blood, but by serous deposits in various parts of the body. The blood was found to be poor in hemoglobin and commonly rather rich in leukocytes.

Elsewhere (p. 95) he says that the anemia

> is indicated not only by the emaciation and paleness of the flesh, but by the excess of serous fluids of the body and by the deposits of organized serum in various parts of the body. This is more especially marked at the base of the ventricle of the heart.

Elsewhere (Farmer's Bull. 380, p. 12) Marsh states:

> The postmortem examinations of locoed animals do not in all cases show clearly marked evidence of the progress of the disease. Since in all cases of fatal poisoning the locoed animals die of starvation they are profoundly anemic, as would be expected, and, as a result of this anemia, accumulations of coagulated serum in gelatinous form are found in various parts of the body. These accumulations are particularly prominent about the heart. There is also an accumulation of coagulated serum in the cavity of the spinal column. This is almost always present in cases where the loco poisoning has become a chronic condition.

This remarkable condition in the spinal canal is described as follows by Marsh in the section of his main work devoted to the antomical changes (p. 97–98):

> The central nervous system is generally in a hyperemic or congested condition. In a few cases clots were found in the lateral ventricles of the brain. We have never, however, found clots in the fourth ventricle . . . . The serous exudate in the epidural space is especially abundant, and is more or less organized. Commonly it is particularly abundant about the points of exit of the spinal nerves. This condition is rarely absent in chronic locoes. . . . . In some cases this coagulated serum is especially abundant in the lumbar region . . . .

Later (p. 114) he states that the fluid of the epidural space of the spinal canal is rather unusually abundant, and commonly more or less organized, "so that the spinal canal frequently seems to be filled with a jelly like substance." Peters and Sturdevant, Marsh's colleagues, describe as follows the condition as it was met with in a seventeen months old horse which died under loco treatment (21st Annual Report; Agricultural Experimental Station of Nebraska, 1908). Between the dura mater and the periosteum of the vertebrae enclosing the neural canal all along the spinal cord there was a cherry red transparent organized exudate in great abundance. In another horse the exudate varied from a buff color in the lumbar region to a very dark red in the cervical region. The color of the exudate was due to numerous tiny blood vessels running through in all directions. The exudate "was not adherent to either periosteum or dura but clung to the spinal nerves at their origin, since it ran through and filled the opening between the anterior and posterior nerve roots in each case and was thus held in place by these."

Marsh also autopsied five lambs which died of acute loco disease lasting only two or three weeks (p. 71). All were in good flesh; "all had clots in the lateral ventricles. All had serous coagulum in the spinal canal, and all had congested walls of the fourth stomach. This would seem to confirm our opinion that these lesions are characteristic of the locoed condition, but that in chronic cases they may be more or less masked." There is some contradiction between this and his reiterated statements that these lesions, especially the spinal canal lesions, are peculiarly characteristic of *chronic*, not of *acute*, loco disease. Moreover, as these lambs were affected for only two or three weeks, the anemia alone would hardly have progressed with such rapidity as to cause the serous effusions, in the absence of some more direct cause. From these accounts it appears that this peculiar 'coagulum' was sometimes outside the dura, 'epidural,' at other times in the cavity of the spinal column.

Marsh found also changes in the stomach at autopsy on locoed animals. (p. 114) "A diseased condition of the stomach was a common accompaniment of the locoed condition, this being marked in cattle by ulcers in the fourth stomach." In another place, (p. 97), he says:

> In acute cases the (stomach) walls are very much inflamed. In chronic cases ulcers are commonly present. The ulcers are not so common in the stomach of horses, but are almost invariably present in the fourth stomachs of cattle. In sheep one is apt to find inflamed walls rather than ulcers. In these ulcers a microscopic examination shows that the mucous membrane is entirely destroyed. Sometimes other parts of the alimentary canal may be inflamed or have small ulcers but this is not a usual condition.

In another place (Bull. 246, p. 34) he states, that, as the result of loco feeding, ulcers are found in the stomachs of horses and in the fourth stomachs of cattle and sheep. In addition Marsh found that the hemolymph glands were unduly prominent and very numerous, and ovarian disease was common.

The picture was not uniform in all of Marsh's experimental animals. Two horses developed acute glanders (p. 461). Others, according to Glover, Marsh's colleague (Colorado Agricultural Experiment Station, 18th Annual Report, p. 52), showed unmistakable symptoms of starvation, and on careful autopsy, revealed no characteristic lesion.

Marsh lays great emphasis upon the anemia observed in locoed animals. He found no blood parasites upon examination (p. 92). He found it impossible, however, on account of other duties to secure any very large number of blood determinations. Some counts were made during the second and third summers. The normal number of red corpuscles for healthy cattle was found to be something over 8,000,000. The count was high, as the experiment camp was at an altitude of 5,000 feet above the sea. The hemoglobin (Tallquist), averaged between 90 and 95 per cent. Very severely locoed animals in the last stages of the disease, examined in 1906 gave an average of over 5,000,000 corpuscles, with hemoglobin averaging 70 per cent, while convalescents and those less severely locoed, examined in 1907 averaged over 7,000,000 red corpuscles, with hemoglobin averaging 85 per cent. The hemoglobin estimations on ten healthy sheep gave an average of 87 per cent; on fourteen locoes the average was 78 per cent.

Now these figures given by Marsh show that in none of his cases was the hemoglobin reduced to an extreme degree even in the worst cases recorded, while in the sheep it was relatively insignificant. Although the red corpuscles in some of the severe cases were reduced nearly one-half, it seems peculiar, with the high hemoglobin count, that the anemia alone should be severe enough to explain the remarkable and characteristic serous collections in the heart, spinal canal, and elsewhere.

With regard to the white blood corpuscles Marsh states, (p. 96), that "An average of twelve locoes in 1907 showed 3,735 white corpuscles." This is the only observation which he records upon the white corpuscles, and it cannot be reconciled with his conclusion (p. 114) that the blood was "commonly rather rich in leukocytes." According to Marek (*Klin. Diagnostik der inneren Krankheiten der Haustiere*, pp. 869–887), the white corpuscles of cattle vary between 7,000 and 10,000 during health, so that Marsh's average of 3,735 actually shows a decided reduction and not an increase. According to Marek the red corpuscles of cattle vary in health between 5,000,000 and 7,000,000 per 1 cc., the hemoglobin averaging 65 per cent by the Gowers, or v.Fleischl methods of estimation, which would give a somewhat higher normal reading by the Tallquist method employed by Marsh. In sheep, according to Marek, the normal red count runs from 8,000,000 to 11,000,000; hemoglobin about 55 per cent. Comparing these standards of healthy livestock with blood determinations recorded by Marsh, it is seen Marsh records no cases of severe anemia, and it seems improbable that what he calls "serous accumulations" were the result of anemia.

Marsh studied loco disease in sheep during 1906 and 1907. During 1906, sixty-three yearling bucks and six lambs were studied. All were supposed to be locoed at the time they were received, and some were very weak and in bad general condition (p. 66). Soon after the first lot of bucks were received at the experimental camp, they were fed upon hay mixed with locoweed (*Aragallus lamberti*). *Only twelve of these "locoed" animals would eat the weed at all and even they would only nibble at it. The others did not eat the plant at all, although they were locoed sheep in bad general condition—presumably advanced locoes* (italics are mine). As the season wore on all of them ate more or less locoweed, just how much could not be determined. There were some which ate nothing but locoweed. It is to be observed that these sheep were "*locoed*" when Marsh received them but they did not form the *locoweed habit* until some time later. Thirty-three of the sixty-five sheep were examined at autopsy, disclosing 23 cases of sheep fly disease, more or less severe; 23 cases of infection with *Thysanosoma actinioides*; a serous coagulum in the epidural space of the spinal canal eighteen times; inflammation of the fourth stomach thirteen times; blood clots in the lateral ventricles nine times, etc. Upon these findings, and regardless of the fact that his "locoed" animals did not eat locoweed, until forced to it, Marsh concluded (p. 70) that "The principal difficulty with most of the animals in 1906 was the loco poison, with the effects complicated by parasites." He examined more sheep the next year, however, and, though he gives no details of the second group of sheep, he states; "the chief trouble with most of the sheep in 1907 was caused by the parasites, and that the loco had little if anything to do with their condition." He then goes on to make the significant statement:

> The general appearance of the bands of sheep in 1906 and 1907 was the same, and not only the author, but experienced sheepmen, declared that both bands were locoed. In the majority of cases it was only by postmortem examinations that the diagnosis could be confirmed.

It is perfectly obvious that this statement by Marsh at once knocks the props from under the carefully constructed and elaborate group of symptoms which he gives as characteristic of loco disease. At least in the case of sheep these symptoms are not of differential value, and it is certainly impossible to detect the slightest difference between the photographs of Marsh's locoed sheep, and photographs taken by Professor Chesnut of the sheep studied by me.

Marsh tries to redeem his position by adding, (p. 70):

> If the habits of the sheep are observed there is a marked difference (between locoed sheep and those suffering from parasitic disease). The sheep affected with *Oestrus ovis*, except when they are in very bad condition, keep together like normal animals, and show a preference for good food, although they may at times eat loco. The locoed sheep on the other hand, are more erratic, and develop a solitary habit to a greater or less extent. They show, too, a marked fondness for the locoweed. At the same time, when one is dealing with a considerable number of sheep, it is a matter of much difficulty to separate the locoed animals from those affected with the grub in the head.

Marsh does not even consider the difficulties of differentiating loco from any other prevailing parasitic diseases of sheep except the sheep fly disease. Moreover, the force of his argument is lost when it is observed that his own typical locoed sheep of 1906 ate the locoweed either not at all or only sparingly (p. 67–68), and when it is added that the sheep studied by me, which ate abundantly of locoweed, were not erratic and developed a solitary habit only as starvation and parasitic disease brought them nearer to death. In other words the clinical symptoms given by Marsh do not differentiate locoed sheep from other ill sheep not locoed. In another place, (p. 92) Marsh himself acknowledges that this is the case, for he says, "after some experience we could, by postmortem examinations, distinguish locoes readily, but it is very difficult before death to tell a locoed sheep from one suffering from grub in the head."

If we compare Marsh's findings with the findings in the typically locoed sheep studied by me, it becomes perfectly clear, that while the symptoms were often strikingly similar, not one of my typical locoes presented the anatomical changes which Marsh found to be characteristic of loco disease in horses, sheep and cattle. In other words, sheep suffering severely from what is everywhere called "loco disease" in Montana, differ in important essentials from animals suffering severely from what is called "loco disease" in Colorado and Nebraska. That is to say, Marsh's work serves to emphasize the truth of my conclusion that "loco disease" is not a clinical entity, but is a term used by Western raisers of live stock to designate several widely different forms of stock disease.

The exact and final diagnosis of locoweed appears then to depend chiefly (at least in sheep), upon finding a peculiar coagulum in the epidural space of the spinal canal, or within the spinal canal. Such a lesion in the central nervous system is unusual, except in diseases such as those referred to below. That this material is a serous effusion, as Marsh describes it, is incompatible with its occasionally being found in a state of organization, as described by Marsh (p. 95, p. 97), and by Peters and Sturdevant. The description suggests that the material was an inflammatory exudate. That such an accumulation should occupy, not only the spinal canal, but a position between the dura and the periosteum, is peculiar. That an effusion of such a nature in such a situation, and one capable of undergoing organization should result from eating a poisonous plant is little less than amazing. And yet this lesion is, after all, the most important feature of Marsh's description. It is easy enough to find symptoms like those of his locoes in several other diseases described in veterinary text books; while anatomically the catarrhal condition of the stomach and even the ulcers are not distinctive of loco poisoning, and the other lesions are relatively unimportant. This makes a detailed description of the lesions in the central nervous system essential to the establishment of Marsh's disease. Such a description including microscopic and parasitologic studies has not been furnished.

In summing up Marsh's work, it seems that he has called attention to a disease which develops in some of the animals which eat locoweed. Not all loco eaters get it, but chiefly those which eat locoweed almost exclusively. There are great individual differences in susceptibility among animals to the poisoning. Peculiar nervous and mental symptoms with progressive weakness, paresis, or partial paralysis of the hind quarters mark the progress of the disease, together with an increasing tendency to eat locoweed. The disease develops suddenly two or three months after beginning to eat the weed. The animals die of starvation, and at autopsy show serous effusions; a peculiar exudate in the vertebral canal, either outside the dura or around the spinal cord, with congestion of the vessels of the brain and occasionally hemorrhages into the lateral ventricles. There is congestion, or ulceration, or both, in the stomach of horses, and in the fourth stomach of cattle or sheep. The anatomical condition may vary more or less in regard to the central nervous system and the stomach.

Several diseases have been described which somewhat resemble Marsh's "loco disease." Pica, or licking disease (Nagesucht; Lecksucht), is interesting, as it also bears a relation to the soil conditions, and to certain hays in the diet. This disease, which, according to Hutyra and Marek (*Spezielle Pathologie und Therapie der Haustiere*, 3rd edition, vol. 1, p. 960), affects cattle almost exclusively, when confined to stables for long periods, is produced by certain kinds of hay, depending upon whether the hay was cut before or after the bloom, etc. Hay from certain localities only will produce the disease, and there is apparently an obscure relationship between the disease and the poverty in soda and lime salts, or excess of potash salts in the hay. The symptoms of the disease are like locoism, and the gastric mucosa is inflamed but there is no description of any spinal exudate.

Catarrh of the fourth stomach, chiefly in cattle, would readily pass for loco disease except that no special changes have been described in the central nervous system. It is interesting to note that this disease may follow various kinds of improper food, and that highly nitrogenous foods like vetches may produce it.

Of more interest are the descriptions of meningo-encephalitis, for here the description both of symptoms and of anatomical changes agree rather better with Marsh's account of loco disease. Law (*Veterinary Medicine*, vol. III, 3rd ed. p. 95–6) makes the following statement:

> Among the most common causes of encephalitis in horses is an injudicious dietary. Overfeeding with grain, but especially with grain and seeds that are rich in albuminoids, deserves the first mention. The various leguminous seeds, peas, beans, tares, vetches, and the ripened leguminous fodders, clover, alfalfa and sainfoin, are espe-

cially to be incriminated. These are usually most dangerous when in the stage of advanced ripening and yet not fully matured, evidently indicating the development of narcotic poison at this stage. Such poisons are found habitually in certain species, like the chick vetch (*Vicia cicera*), which produces paralysis if fed to the extent of more than one-twelfth part of the ration. . . . . With sound judgment and in well balanced rations, all such agents can be fed to advantage; it is only when fed exclusively or to excess as the heavy ration that they are to be feared.

The symptoms of this disease are largely those of Marsh's loco disease, though the disease runs a fatal course more rapidly than appears to be usual in Marsh's cases. There is a fairly close resemblance between the anatomical findings so far as can be judged by reading the accounts of the two diseases.

Law does not make the distinction very clear to the reader between meningo-encephalitis and the epizootic cerebro-spinal meningitis of horses. The latter disease is known in Idaho (Law), and both symptoms and anatomic changes are largely those of the loco disease of Marsh.

Hutyra and Marek (vol. 2, p. 624, &c.; and p. 634–645), differ somewhat from Law in their description of the two last described diseases, the chief difference being that they lay little stress upon the food, and attribute each disease to a specific bacterium. In their discussion of the differential diagnosis (p. 632) they outline several diseases which would undoubtedly pass for loco on the ranches. Until Marsh publishes his microscopic and bacteriological studies it will be impossible to decide whether Marsh's loco disease is identical with meningo-encephalitis or with enzootic cerebro-spinal meningitis, but in the meantime it seems rather more probable that Marsh's disease is allied to meningo-encephalitis.

F. THE LOCO PROBLEM.

Crawford's review of the literature sets forth strikingly the state of interminable confusion in which the loco problem has been involved since the very beginning, a state due to the vagueness of the definition of loco disease, and to the discrepancies in the description of symptoms, in the statements of fact, and in the results of experiment. Crawford himself decided that a poison can be extracted from some locoweeds but not from others. This fact may yet be established in spite of Alsberg's contention that the poison is not barium, a contention supported by Marsh. My studies failed to reveal any indications that the animals were poisoned by locoweed, while Marsh is fully satisfied that he has established the poisonous activity of locoweed beyond all doubt. Even Marsh found that eating locoweed does not always produce loco disease, and he lays emphasis upon the persistent, and almost exclusive use of the plant as more or less necessary in order to produce poisoning. There seem to be exceptions even to this rule, due to great individual differences among animals in their toleration for the plant. Many animals actually gain weight on a loco diet.

At the present the work of Crawford, Marsh and myself seems to justify opinions upon several aspects of the loco problem, provided we temporarily concede that Marsh's disease results from the use of the locoweed and is not due to bacteria or parasites.

1. Some locoweeds, including *Aragallus lamberti* and *Astragalus mollissimus*, may exert a deleterious action upon live stock. Apparently not all locoweeds are equally injurious. That the deleterious action is due to a definite poison in the weed, as claimed by Marsh does not seem to be established.

2. There are wide individual variations in the results obtained from feeding live stock upon locoweed. Some animals die; others may actually gain in weight for a certain length of time. Animals may eat the weed as part of their ration for years with no bad results.

3. It requires a large amount of locoweed to produce symptoms. The proportion of locoweed in the diet must be large; in fact in locoism the animals seem to feed chiefly on the weed. Sheep and lambs feeding on the weed as freely as they wanted it were not locoed after eating the weed *A. spicatus* for over forty days, while the closely allied species *A. lamberti* apparently produced locoism in Marsh's cattle and horses after two to five months of feeding.

4. It is an easy matter to make the animals eat locoweed. It is only necessary to reduce the amount of available food, or of available fresh green food, for a short time in order to start sheep eating the weed. They will eat it readily—more readily than other grasses, apparently—but not exclusively, as a rule.

5. The disease described by Marsh as "loco disease" bears a certain resemblance to meningo-encephalitis.

6. There are several other diseases of live stock on the Western ranches which are generally regarded as loco disease.

7. Underfeeding or improper feeding appears to be a very important factor in causing losses among the live stock. Entire flocks of sheep may be underfed if the range is overstocked; or if several flocks in quick succession pass over the same range; or if the season is a poor one for the growth of range grass. The younger and weaker members of a band of sheep will always suffer most from underfeeding, as they cannot keep in the front ranks where the best food is, and they should be particularly cared for, or

even removed, while they are still in good condition, and fed up in a group by themselves.

8. Parasitic diseases, passing as "loco disease" are widely spread among the sheep of Montana, and probably throughout the West. The parasites which have come to light during the loco studies are: 1st, the "fringed tapeworm" (*Thysanosoma actinioides*): 2nd, a wire worm of the stomach (*Ostertagia marshalli*): 3rd, a lung worm; 4th, in sheep which are in poor condition the ubiquitous sheep fly disease may produce serious or fatal illness. The other parasites observed such as the *Sarcocystis tenella* are of less serious moment.

9. The hygenic conditions on the ranches, around the bedding grounds for the sheep, and at the watering places, especially in the latter situation seem peculiarly favorable for the wide dissemination of the parasitic diseases above mentioned. It does not seem at all probable that the loco studies have brought to light all of the diseases masquerading as "loco disease."

The unexpected development of the loco work pointing to underfeeding and parasitic infections leads to problems of immediate importance, so great as to overshadow the original loco problem. It would be very interesting to continue to experiment with the loco plant, but the call of the hour is to combat the losses of live stock by tackling the feeding problem, and by making a thorough medical survey under the direction of trained parasitologists to determine the nature and extent of prevailing parasitic diseases, and to inaugurate measures to combat them. Further investigations of the locoweed at the present time will serve only to postpone the day of relief for the stock raisers. Not until the two problems of underfeeding and infection have been attacked successfully can we hope for a satisfactory settlement of the loco problem.

Even in the case of Marsh's disease it will be necessary to do much work before Marsh's recommendations can be justified. He advises that the locoweed be exterminated by combined action of the individual rancher, the state and the nation. The enormous cost of such an undertaking calls for the most careful work to justify it. It will not be until it has been proved that Marsh's disease itself is not dependent upon some microörganism; until it has been clearly established that it does not exist outside of the locoweed belt, and that only animals eating the weed are victims of the disease; and until the food value of the weed, and its value as part of a ration have been learned—not until these matters are settled can Marsh's recommendations be advocated. If Law is correct in the quotation given above, and if the locoweed is like its relatives in the plant kingdom, it may well be found that the locoweed is a good food if used in moderation and in association with other foods and at certain seasons of the year. Both my animals and Marsh's gained in weight upon a locoweed diet. Disaster followed only the prolonged and almost exclusive use of the weed by his animals. On the other hand, recent work in the Bureau of Animal Industry lends support to my view that the parasitic diseases must be attacked if the Western live stock industry is to be properly conserved. Recently Hall (27th Annual Report Bureau Animal Industry, 1912, p. 419), referring to the parasites of Western sheep, says, "Even relatively light infections are apparently sufficient to cause the death of an animal in adverse weather conditions or during periods when food is scarce." Moreover, evidence is accumulating to support the opinion expressed by Curtice a generation ago, to the effect that the "fringed tape-worm" does more harm to Western sheep than any other internal parasite, (Ransom: Report Bureau Animal Industry, 1911, p. 60). In view of the prevalence of the parasitic diseases when they were looked for, it seems not unreasonable to expect that an attack upon these diseases combined with an attack upon the feeding problem will clear up the majority of the "loco diseases." It seems even more probable that such a campaign will most surely and speedily diminish the huge losses among the Western live stock.

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BULLETIN

OF THE

PHILOSOPHICAL SOCIETY

Scientific Series, Vol. I, No. 20, pp. 437-442, July, 1914

Examples of Intercision Type of Stream Piracy in Western Virginia

BY

THOMAS L. WATSON, JUSTUS H. CLINE, AND THOMAS K. HARNSBERGER

UNIVERSITY OF VIRGINIA
Charlottesville, Virginia, U. S. A.

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Continued on page 3 of cover

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BULLETIN OF THE PHILOSOPHICAL SOCIETY
SCIENTIFIC SECTION
Vol. I, No. 20, pp. 437–442 July, 1914

EXAMPLES OF INTERCISION TYPE OF STREAM PIRACY IN WESTERN VIRGINIA

By
THOMAS L. WATSON, JUSTUS H. CLINE,
AND THOMAS K. HARNSBERGER.

During the progress of systematic field work by the Virginia Geological Survey in western Virginia, a type of stream capture due to the removal of divides by lateral planation was studied by the writers. Three cases have been selected which illustrate different stages of the process and while they involve no new physiographic principle, they illustrate an interesting type of drainage change.

The removal of divides by lateral planation is the last work accomplished by streams in the base leveling process; consequently captures due to lateral shifting of the position of streams under conditions of old age must be common. In the lower Mississippi Valley, for instance, many such captures have occurred or are about to occur. In such regions where streams have passed the stage of maturity the accompanying physiographic results are apt to be slight. In regions of more youthful or mature drainage, stream captures affected by lateral planation are of less frequency, but are likely to be attended by more or less prominent physiographic consequences. In case III described below a water gap has been developed in this way, and in case II a natural bridge may result.

When captures due to lateral planation occur under conditions other than those of old age the disposition to such captures will usually have been inherited from a previous cycle of erosion. In each case described below the stream occupies an entrenched meandering valley characteristic of a rejuvenated region. These meanders existed at the beginning of the present cycle, and therefore a tendency toward lateral planation was probably manifested at a relatively early stage. In a rejuvenated region of meandering streams some lateral planation may occur with the down cutting at an early stage in the cycle. Thus North River near Bridgewater, Rockingham County, has considerably increased the distance between it and Round Hill on one side since the inauguration of the present cycle, and on the other side it has invaded the hill itself—the difference in the two cases being due to the position of the hill with reference to the meanders of the stream.

Three fairly typical examples illustrating different stages in the process of stream capture by lateral planation are described below. By consulting the accompanying topographic maps (figs. 1, 2, and 4), it will be observed that each of these cases is found in a rejuvenated region. The stage of development represented in the present cycle in these cases apparently ranges from late youth to maturity. The first case described is in Dickenson County in the Alleghany Plateau region of southwest Virginia. The capture has not yet taken place, but geologically speaking it is imminent. The second case illustrates the same type of capture in process of taking place, and the third shows that it has already occurred. The last two cases are in Rockingham County in the Shenandoah Valley.

The type of stream piracy illustrated by these three cases is designated by the writers the *intercision* type. The term *intercision* was used by Goldthwait* in describing a peculiar change in drainage near Kenosha, Wisconsin, where the narrow divide between Lake Michigan and the lower part of the meandering valley of Pike River has been removed in places by the combined work of the lake and stream, and the lower part of the stream severed from its upper course.

Case I. The accompanying topographic map (fig. 1) of a small area in Dickenson County, southwest Virginia, shows the existing topographic conditions in this case. The rocks are sandstones and shales and are very gently folded. The region is thoroughly dissected but the streams, which are of the rejuvenated type, are actively engaged in eroding their valleys deeper. The streams have inherited meandering courses from a previous cycle.

It will be observed by consulting the map (fig. 1) that a meander in the valley of Pound River at a point about 3 miles from its junction with Russell Fork brings it very close to that of one of its tributaries, Cane Branch. The minimum distance between the two streams is about 500 feet, and Pound River is flowing at a level more than 100 feet below its tributary (Cane Branch). The divide between the two streams is being

* Goldthwait, J. W., Intercision, a peculiar kind of modification of drainage. School Science and Mathematics, 1908, vol. 8, no. 2, pp. 129-139.

slowly removed chiefly by Pound River which is not only deepening its valley at this point but is also advancing laterally in the direction of Cane Branch by undercutting the divide. Ultimately a breach will be made in the divide at this point with the result that the junction of the two streams will be moved one and a quarter miles up stream and the lower part of the valley of Cane Branch abandoned.

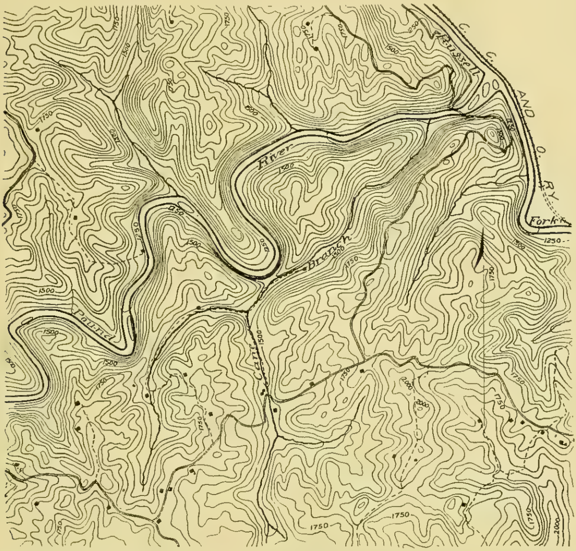

Fig. 1. Topographic Map of a Part of the Northwest Corner of Dickenson County, Virginia, Showing Relations of Pound River to its Tributary, Cane Creek.

Scale 1.318 inches = 1 mile.
Contour interval 50 feet.

Case II. The second example (see map, fig. 2) is at the vil'age of Mt. Crawford, Rockingham County, 18 miles northeast of Staunton. The rocks of the region are chiefly limestones which have been highly folded and faulted. The two streams involved in this case are North Fork of the Shenandoah River and one of its tributaries, Cook's Creek.

Both streams are flowing in entrenched meandering valleys and both have developed flood plains in places. The physiography of the immediate section is shown on the accompanying map (fig. 2). The chief interest in this case is that the capture is in actual process of taking place.

The distance between the two streams at the nearest point is about 150 feet. The divide is occupied by the valley pike which extends from

FIG. 2. TOPOGRAPHIC MAP OF MT. CRAWFORD AND VICINITY, ROCKINGHAM COUNTY, VIRGINIA, SHOWING RELATIONS OF NORTH RIVER AND ITS TRIBUTARY, COOK'S CREEK.

Scale 2.667 inches = 1 mile.
Contour intervals 10 and 20 feet.

Winchester to Staunton. It is composed of more or less massive beds of limestone which dip about 10 degrees southeast, and are jointed but not brecciated. Cook's Creek is flowing at an elevation of 14 feet above the main stream, North River. A part of the water from Cook's Creek has found its way through the divide along joint and bedding planes, so that the two streams are now connected by a subterranean passage. This channel has been enlarged by solution, so that now a sufficient volume of water passes through it from Cook's Creek to North River to cause alarm on the part of the owners of a mill located a short distance below, who depend on the water of Cook's Creek for power. Several attempts have been made to stop the underground passage but without success. The amount of water which flows through the underground channel varies with the stage of water in the two streams, but especially with that in Cook's Creek.

The point where the water emerges on the face of the bluff on the North River side of the divide is about 2 or 3 feet above low water in North River, but at high water the opening is frequently submerged.

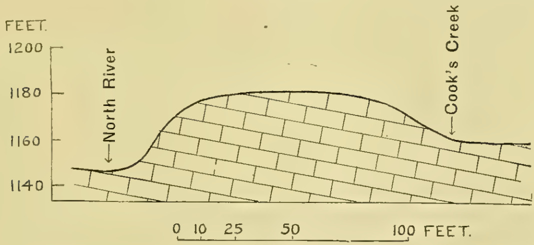

FIG. 3. CROSS SECTION OF DIVIDE BETWEEN NORTH RIVER AND COOK'S CREEK.

The source of the water flowing from the underground channel is determined readily, since the water of Cook's Creek is usually stained very dark by the refuse from a tannery at Harrisonburg. A cross section of the divide between North River and Cook's Creek is shown in figure 3.

Case III. Reference to the map (fig. 4) of Bridgewater and vicinity will show that North River flows in a meandering course through a ridge about 1 mile northwest of the town. This ridge is typical of numerous low linear cherty ridges which characterize the Great Valley in western Virginia. Round Hill is one of the common conical-shaped monadnocks which occur at intervals in these cherty ridges throughout the Valley province. The rocks composing this ridge are for the most part massive-bedded cherty limestones, the most resistant members of the Shenandoah group. The ridge is monoclinal in structure, the beds dipping toward the southeast.

Dry River, a tributary of North River, likewise crosses this ridge about three-fourths of a mile northeast of the junction of the two streams. The point in the ridge where it is crossed by Dry River is less cherty than in the vicinity of North River and therefore Dry River, although the smaller of the streams, occupies the larger of the two valleys.

The peculiar feature in the case of Dry River after passing the cherty ridge is that it has abandoned the lower portion of its valley which was developed in less resistant rocks, flows back again into the cherty ridge area and joins North River more than half a mile above the point where the latter stream leaves the ridge.

The almost level plain south of the Chesapeake and Western Railway is the abandoned southern extension of Dry River valley. This plain is composed of alluvial material, such as sand, gravel, loam, clay, and in places numerous large river boulders. There are also traces of stream channels in the form of linear and meandering depressions. The boundary between this plain and the present flood plain of North River immediately above and below Bridgewater is marked by a distinct terrace with an average difference in elevation of 10 to 20 feet between the two plains. This terrace can be traced all the way from the point where North River emerges from the cherty ridge to the vicinity of Berlinton, a distance of $1\frac{1}{2}$ miles.

The explanation of this interesting physiography is that Dry River formerly entered North River near Berlinton about $1\frac{3}{4}$ miles below their present junction. For some distance above their old junction the two streams had a common flood plain, but at the point of their present junction their valleys were separated by a narrow cherty ridge. Finally a breach was made in this cherty ridge by the two streams which had swung against it at opposite points. Since North River was the larger stream and therefore flowing at a lower level it captured the waters of Dry River. The streams do not meet at grade but there is a fall of 2 or 3 feet in Dry River a few yards above their junction.

The results of the capture were the abandoning of the valley of Dry River between the point of capture and the old junction, and the rejuvenation of that part of North River between these points by increase in volume of water. The result of this rejuvenation has been the marked lowering of the flood plain of North River between the point where it leaves the cherty ridge and Berlinton. The present flood plain of North River is 10 to 20 feet below the level of the common flood plain of the two streams before the capture took place.

1260
1300
1400
1360
1340
1300
1200
1200
1300

FIG. 4. TOPOGRAPHIC MAP OF BRIDGEWATER AND VICINITY, ROCKINGHAM COUNTY, VIRGINIA.

Scale 2.667 inches = 1 mile.
Contour intervals 10 and 20 feet.

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PHILOSOPHICAL SOCIETY

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The Foucault Pendulum as a Lecture Room Experiment

BY

FRANCIS H. SMITH

UNIVERSITY OF VIRGINIA
Charlottesville, Virginia, U. S. A.

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SCIENTIFIC SECTION
Vol. I, No. 21, pp. 443-456 August, 1914

THE FOUCAULT PENDULUM, MADE A USEFUL LECTURE-ROOM APPLIANCE

BY
FRANCIS H. SMITH

The diurnal motion of the earth is best evinced and measured, both as to its great fundamental statement and the minute departures from this, by the apparent daily motion of the fixed stars caused by it. No mere terrestrial apparatus will probably ever release the physicist from dependence on the astronomical observatory as regards the exact value of this rotation.

Yet philosophers have for a long time believed that so great a fact ought to reveal itself by purely terrestrial evidence and many in recent centuries have sought for it.

Newton, in a letter to Hooke, pointed out that a body falling freely ought by the rotation of the earth to be made to deviate to the east of the plumb line. Hooke replied that it should deviate to the south of east. Laplace undertook a mathematical analysis of the falling motion and agreed with Hooke. Gauss followed and deduced from his treatment that the deviation would be to the north of east. Other mathematicians found no deviation at all from Newton's purely eastward deflection. After a long silence the mathematical problem has recently been taken up by Dr. R. S. Woodward who has pushed the analysis a step farther, by taking account of small quantities neglected by Laplace and Gauss, with the interesting result that a heavy body falling in vacuo in our hemisphere, should deviate to the north of east.

The experiments on the deviation of freely falling bodies have been few and unsatisfactory. They require a great vertical distance, a very accurate determination of the vertical itself, and heretofore, the use of a mine shaft, with all the atmospheric troubles which this involves. These conditions put the trial outside of the mathematical researches alluded to. The results may be taken as agreeing with the *eastward* tendency, but seem to give no satisfactory evidence of the meridian disturbance. The experiment of course has no interest for one seeking for a lecture-room demonstration.

The spheroidal figure of the earth is another result and proof of its daily rotation, but like the celestial movement, must be merely described to the class.

In this condition, it is difficult for us at this day to realize the sensation produced in the whole reading world, when at the session of the French Academy of Sciences, February 3, 1851, M. Foucault, a rising young physicist, in a modest note of less than two pages, reported an experiment, demonstrating the rotation of the earth by the use of a pendulum two metres long. He observed that its plane of vibration showed a uniform clock-wise turning at a rate demanded by the daily motion of the earth.* Arago allowed him to repeat the trial with a pendulum 11 metres long in the dome of the observatory. Finally he was permitted to install it on a grand scale in the Pantheon, with a pendulum 67 metres long, having a vibration-time of nearly 8 seconds. The experiment drew throngs of observers and was repeated in the same place at short intervals for months, being at last rudely interrupted by the Coup d'État of Louis Napoleon, Dec. 2, 1851.

As might be expected the apparent facility of the experiment and the extreme simplicity of the apparatus required, caused it soon to be repeated in every land, not only by scientists, but by every earnest student, who could command a metal ball and a string. There were reported repe-

* Foucault's sine formula may be deduced in a number of ways. Here is a simple one.

Let $Pm\ P'\ oP$ be the parallel of latitude at Paris; PT the north and south line there extended to meet the produced rotation axis of the earth at T (fig. 1).

Then by the diurnal rotation the line PT will describe a cone, vertex at T and base equal to the parallel of latitude. The whole daily deviation (counter clockwise) of the line PT *and of every line in the horizontal plane at* P *which makes a fixed angle with* PT will be the cone surface angle at T developed into a plane, as in fig. 2, that is, the angle PTP.

But the angle $PTP = \dfrac{\text{arc } PmP'oP}{rad.\ TP} = \dfrac{2\pi PD}{TP}$
$= 2\pi \sin \varphi$, where $\varphi = ECP =$ latitude of Paris. Hence deviation in radians per mean solar second $= \dfrac{2\pi \sin \text{latitude}}{\text{sidereal day in m. s. seconds}} = \dfrac{6.2832}{86164} \sin \text{latitude}.$
Hence the deviation clockwise of the vibration plane of pendulum *per hour* at Paris is 11°.3; and at the University of Virginia 9°.25.

titions in the Cathedrals of Cologne, Amiens and Rheims, in the tower of St. Jacques and the Conservatoire des Arts et Métiers at Paris, on Bunker Hill Monument, and in the dome of our Rotunda by Prof. Courtenay, though this was never reported. A letter written in 1851 and recently brought to light, tells us that Professor George Tucker, once of this University, tried the experiment from a back window of his residence in Philadelphia, and with success. I was told by one of his successors here, Prof. N. K. Davis, who at the very time was a student in the same city, that he was led to repeat the experiment, and obtained a striking deviation

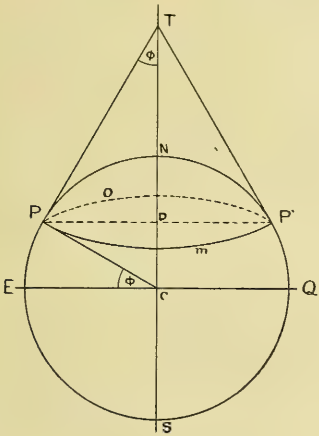

Fig. 1

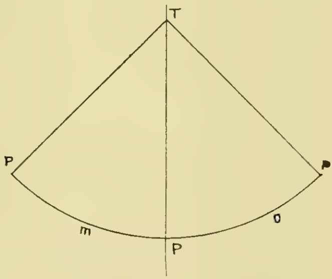

Fig. 2

of the vibration-plane. Upon studying the result closely, Mr. Davis, with characteristic sagacity was led to doubt its being the genuine Foucault disturbance and so stopped his attempts although urged to repeat the swings before the Philadelphia Academy, by some enthusiasts who seemed to think that any deviation would do.

Similar severe scrutiny resulted in the rejection of other trials. Indeed nothing is easier than to get deviations of the vibration-plane of any pendulum. It is hard to avoid elliptical motion of the bob. Any lateral disturbance of the point of suspension, or lack of care in starting the swings, or finally any irregularity in the atmospheric pressure on the two sides of the vib-plane, will cause this motion which, with the amplitude usually employed, will produce a deviation quite different both in amount and sometimes in direction from Foucault's. The experiment requires first *a fixed point of suspension;* next, *avoidance of lateral disturbance in starting* and lastly, *a very small amplitude* of vibration. It is by no means easy to secure all these conditions in ordinary lecture rooms.

It is noticeable that M. Foucault appears soon to *have abandoned the pendulum* for the gyroscope in this problem, and that in the memorable sesqui-centennial repetition of his pendulum-experiment in the Pantheon by M. M. Berget and Flammarion February 26, 1902, the latter astronomer speaks of seeing elliptical movement of the needle. In neither the original trial nor its repetition, can the reader more than guess at the amplitude of the vibrations. The radius of the sand-arc is noted, but for the starting distance we are left to conjecture.

Aftler 1851 attempts were made here, year by year, as doubtless in many other ecture rooms, to exhibit Foucault's pendulum experiment to the class in Physics. There were several difficulties met with. The pendulum was of necessity not a long one, and with ordinary amplitudes it involved a considerable maximum speed of the bob, encountering an air resistance easily becoming unsymmetrical. Then the length of time required was inconvenient, a deviation of 5° calling here for more than half an hour. This duration made it impossible to insure the needed fixedness of the suspension point in a room occupied by a hundred spectators, especially as the result could only be seen by those near the lecture-table and must be taken on faith by the majority. The same troubles must have been fatally common in England, for Dr. Greenhill, of Woolwich, in a laughing fling at the physicist, publicly advised each one to hide a pouch of condensed air in his sleeve, and by a suitable unseen tube force the swinging pendulum to do its duty.

Then matters went on here unsatisfactorily from year to year until 1890. In the summer of 1889, I was engaged with Mr. James Miller, then my Assistant, in a repetition of Bessel's Königsberg experiment to determine the gravity acceleration '*g*' by means of a differential pendulum. The pendulum was swung in the tower of the then unfinished chapel. It had a vibration-time of 4 seconds, and its length enabled us to see amplitudes less than 1°.† The experiment required groups of swings, each about 20

† Foucault's original Pantheon Experiment used an amplitude certainly greater than 5°; and its repetition in 1902 must have involved amplitude greater than $6\frac{3}{4}$° as required by the size of the circles of sand.

minutes in duration, and was continued for two working days. It was necessary to measure the swing-amplitude at the beginning and end of each group and it was soon noticed that the horizontal scale behind the pendulum had to be shifted clockwise an *apparently constant amount* every twenty minutes, to bring it again into parallelism with the vibration-plane. It became apparent that *a heavy spherical pendulum, properly suspended and vibrating in a very small arc*, was quite a different instrument as to constancy of behavior from the same pendulum *swinging in a large arc*. The observation led me, after the work for '*g*' was over, to endeavor to make the Foucault pendulum a reliable lecture-room appliance.

First, *as to the point of suspension*, that which was used for the Bessel experiment proved entirely satisfactory and was much simpler than the devices I had used previously and indeed than that used by Foucault. The fine suspending piano wire was simply passed through a hole of the same diameter in a horizontal metal plate firmly secured, and soldered there.

The suspending wire was the finest steel wire which would certainly support the 'bob.' The 'bob' may be a lead sphere of 10 cm. diameter. In the lecture-room last used, a length of 5 metres for the pendulum was easily obtained. Thus a swing of 17 cm. meant a vib-amplitude less than 1°.

In all our previous trials, the trace of the plane of the swing was made by a needle attached to the bottom of the leaden sphere and made, if possible, to coincide with the prolongation of the suspending wire. This is troublesome to effect and maintain. If slightly in error, any rotation of the bob is fatal to accuracy. It was seen that a needle is both unnecessary and inconvenient. The study of the plane of vibration was transferred from the needle to the suspending wire, which is unaffected by any rotation of the bob, and describes the very surface we have to study.

Next, it was essential to make the change of the vibration-plane visible to the largest assembly, each spectator with his watch, being able to recognize and measure the Foucault deviation. This required the introduction of a lantern and screen.

Finally, the impracticability of insulating the point of suspension and the recording apparatus from disturbance in an ordinary occupied lecture-room, made it essential that the Foucault motion should be *so greatly magnified* in the spot of light on the screen, that the whole experiment could be brought within a few minutes. In this condition even a hundred young fellows can be induced to keep entirely quiet for the short interval required, and no other insulation is needed.

Both of these objects, the projection of the Foucault movement and its great amplification, were secured by the use of a very delicate lever of straw well balanced on its fulcrum, which was a fine needle thrust perpendicularly through the straw and resting its point in a minute glass cup. The lever's length was not less than half the entire swing of the wire at the selected point. It was mounted so as to move in a horizontal plane on the side of the pendulum swing toward which the Foucault motion was expected. It was so placed that the wire in its vibration touched the extremity of the long arm. At its next swing in the same direction, it would

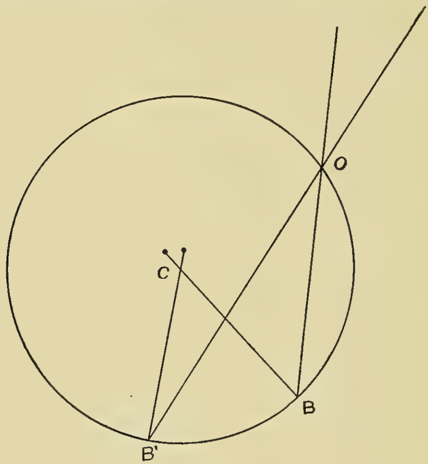

FIG. 3

move the lever slightly, and so from swing to swing always in the clockwise sense in our northern hemisphere. To the straw at the fulcrum was cemented a small plane mirror of thin silvered glass. This reflected a pencil of light from a lantern to a properly placed screen in front of the class. The spot of light on the screen exhibited by its displacement to the entire assembly the fact of the change of vibration-plane.

To *amplify* and *measure* this change, the following adjustment was made. In figure 3 in a horizontal plane, let O be the point through which the wire passes when the pendulum is at rest. Place the straw lever so that its fulcrum C is distant from the stationary wire, a length equal to the longer recording arm of the lever. Thus $CO=CB$. The end B of this arm describes a circle passing through O. Suppose the vibration-plane changes from OB to OB' then the lever turns from CB to CB'. But the angle $BCB'=2BOB'$. Hence the lever and mirror turn twice as fast as the vibration-plane. Now the spot of light turns twice as fast as the mirror. Hence average angular speed of spot = 4 times the Foucault speed.

Then if the screen be graduated into 4-inch intervals and placed 370.9 inches from the mirror the spot of light in this place ought to describe each interval in 60 seconds, and would take three minutes to describe three intervals. Three minutes were found to be a convenient and sufficient space of time for the experiment before the largest class. It is an easy duration to verify with any watch having a second hand.

Or putting the formula into a different shape, sine latitude $=\frac{110.0}{t}$ where t equals the number of seconds the spot takes to move 12 inches. We find t with our watches and may compare the latitude thus found with its known value.

The push of the lever, unfortunately, is rather a *blow* than a pressure. Though it is a very delicate blow, the great magnification of the displacement reveals the fact that the wire at its next swing may not touch the lever. This pause really does not seriously affect the result of a three-minute trial. A more essential point is to keep the lever from recoiling if we try to dampen its motion so as to make it correspond exactly to that of the vibration-plane. After many kinds of dampening were tried, it was found that the only one in which a fatal recoil or back lash was absent, was the friction of a sharp steel needle point on a smooth glass surface. Accordingly a fine needle was thrust through the short arm of the lever so that its point might rest on a small glass plate fastened below. The push of the wire on the lever merely caused this point to scrape lightly on the glass without any recoil which the spot of light could show.‡

The whole contrivance thus described was completed the year after the Bessel experiment, that is in 1890, and it was then shown to the class in Physics. The entire arrangement was kept, like the Atwood Machine, a fixture of the lecture room. It was used regularly from 1890 to 1907,

‡ Outlines of the recording lever (see fig. 4). AB the length of the straw say 12 cm. F the fulcrum needle resting in a small glass cup C. B is the end touched by the swinging wire: m is a counterpoise balancing the lever on C. n is a small needle pressing lightly on the glass plate p. GH is the wooden base. R is the plane mirror.

and found to be a most reliable apparatus. Since 1907, Professor Hoxton, at whose request I have made this report to the Society, has adopted it, with certain excellent improvements—as to mass of the bob, fixity of suspension point, and screening of the ball from accidental currents of air, which I hope he will sometime describe to us.

Allow me a word to acknowledge our debt to the mathematics in this problem.

When Foucault made his modest appearance before the French Academy in 1851 his experiment produced a great stir among the mathematicians. Binet immediately followed Foucault with a long and interesting dynamical discussion of it. Poinsot next arose and declared that the Foucault result was not a matter of Dynamics but purely one of Geometry. One after another of the leaders pushed in and for a number of sessions the poor physicist was hidden by the cloud of brilliant theorists. Although

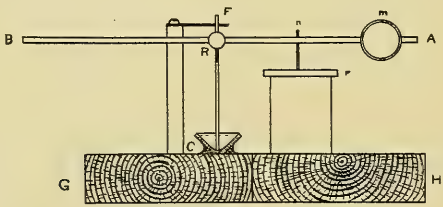

Fig. 4

these debates may have resulted in small help to the experiments, one should not forget that Airy, Bessel, Binet and their followers have in a problem of the greatest mathematical difficulty reached conclusions of signal value to him. Throughout their work they have, as we easily see on review, emphasized the importance of small oscillations. Mr. Airy showed us nearly a century ago that with exceedingly small amplitudes of vibration, friction and atmospheric resistance, so far as it depends on the speed, have no effect on the vibration-time, while Bessel points out the remarkable effect of vibrations not small in causing sympathetic oscillation in the air so that the vibrating mass is virtually increased—a remarkable fore-shadow of electrical inertia in great speeds. Again Binet and Hansen, as reported by Price, have shown that with exceedingly small vibration, the spherical pendulum, to a first approximation, describes an ellipse in same direction as the earth's rotation, and that the *major* *axis of this ellipse turns with the true Foucault motion.* For such amplitudes as we employ in this paper, Binet's formula $\frac{b}{a}=\frac{T\omega sin\varphi}{2\pi}$ shows that with a pendulum 500 cm. long and with an entire swing of 17 cm. the semi minor axis b of the ellipse $= \frac{1}{373}$ mm. It is plainly of theoretical interest only in our experiment.

Sir John Herschel had pointed out that the path of the pendulum would really be a spiral passing in all its motions to and fro through a point o vertically under the centre of suspension (fig. 5). Binet teaches us that

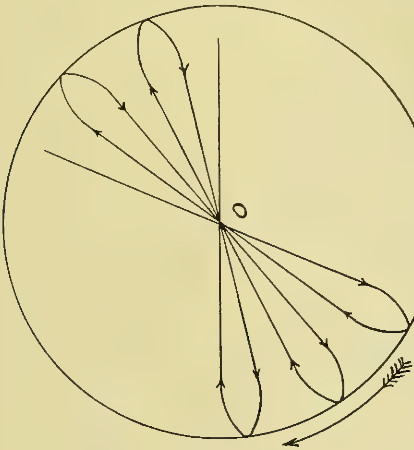

FIG. 5

it is a spiral described by a point executing elliptical harmonic motion in a horizontal ellipse moving uniformly about its centre with an angular speed $=\omega \, sin \, \varphi$ (fig. 6).

The Foucault result remains unmodified.

Note by L. G. Hoxton.

This note has been appended to Professor Smith's article as a result of the suggestion on p. 450 which he has been so kind as to make. The improvements there alluded to are for the most part matters of convenience in detail and hardly deserve attention in themselves. However, a description of the apparatus in use here at the present time may assist some teacher in the task of construction. If so, then this note is justified.

To begin with the modifications introduced since 1907 are briefly: (a) mounting of the apparatus close to the wall, taking advantage of the increased stability offered. This, for instance, permits the lecturer to walk about during the course of the experiment. (b) Further protection against disturbances by surrounding the bob with a box to shield its large surface from air-currents. (c) Employment of a bob filled with mercury

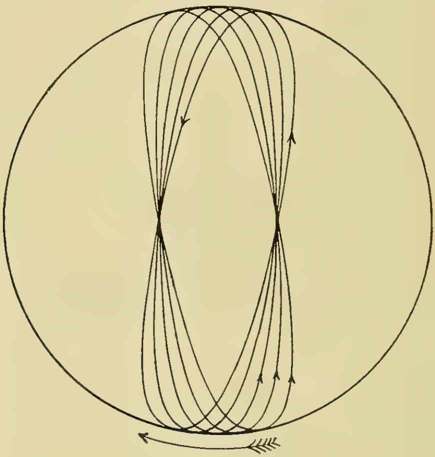

Fig. 6

to damp out the tremors incident to starting the pendulum. (d) Use of a tripod for the recording lever (see below).

Figure 7, reproduced from a photograph, shows the apparatus complete excepting the lantern, screen and point of suspension. The pendulum is suspended after the manner described by Professor Smith using steel piano wires of size no. 24 B & S and 550 cm. long. The amplitude of swing is about 5 cm. (from centre). The bob is made of one of those cast iron shells such as are furnished by apparatus dealers for the experiment of bursting by freezing water. Filled with mercury it weighs 6 kgm. The bob, detachable from the wire as shown, is stored away when not in use leaving the wire coiled up near its suspension. The box likewise is detachable being mounted permanently on a wooden bracket which in turn is

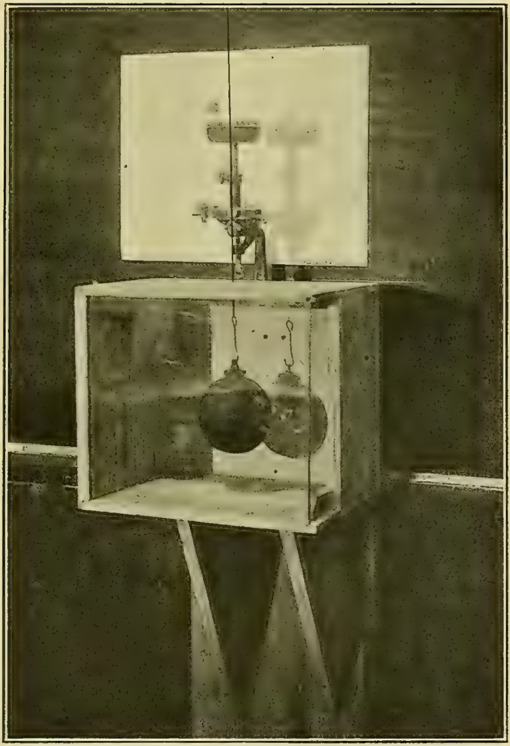

Fig. 7

fastened temporarily by dowel pins and wood screws (hooks would work in place of screws) to a board permanently anchored to the masonry. It is insulated from the blackboard behind. A direct fastening to the wall would be preferred were the blackboard not present. The box is glazed in front to avoid concealing the bob from the class, and is slotted in its top parallel to the wall to permit the passage of the wire swinging in its vibration plane. The width of the slot is about 1 cm., length 12 cm.

The indicating mechanism, which is seen at the top of figure 7 and of figure 8 is supported from the box in the following manner. An inverted L of half-inch round iron screwed to the back of the box rises and bends forward as shown. To this is clamped a "right angled clamp with screw adjustment" such as are furnished by apparatus dealers as parts of standard laboratory supports. This clamp is of course available for other apparatus when the pendulum is not in use. The clamp supports a short rod, on top of which is fastened a wood block and to this is waxed a small glass plate about 8 x 8 cm. forming a smooth horizontal table upon which the indicating lever rests and is damped according to the principle Professor Smith states on p. 449.

The lever is a tripod whose construction is clearly shown in figure 8. One leg consisting of a sewing needle is stepped, point down, into a "glass jewel" or glass cup (p. 449) which is waxed to the plate about 1 cm. from its edge. The other two legs slide over the plate and support the mirror as well as damp its motion. The forward leg is prolonged horizontally and projects over the edge of the plate where its tip is touched by the passing pendulum wire. (The scrap of paper stuck on the left hand corner of the plate is merely for the purpose of showing the extension of this lever in the photograph and does not normally belong there.) It is well to see that the surface of the plate possess no flaws where it is likely to cause the lever to stick. A little water as a lubricant under the sliding legs sometimes helps. The mirror is mounted back of the needle in such a position as to insure stability. This arrangement is easy to make and requires little further attention when once adjusted.

The two glass plates pasted into an L shown in the background of figure 8 is an auxiliary device for use in setting up. Its purpose is to temporarily extend the level area of the glass plate on its front edge when the pendulum is at rest so that the lever may be swung around into position CO, figure 3. Then the adjustment $CO = CB =$ length of the lever (in this case 2.7 cm.) may be nicely finished by means of the screw adjustment of the "right angled clamp." The angle OCB is of the order of magnitude of 120°.

The pendulum is started by the traditional method of pulling aside with a string and, when all is quiet, burning the string. The string passes through a hole in the box to a hook fastened just outside. There the

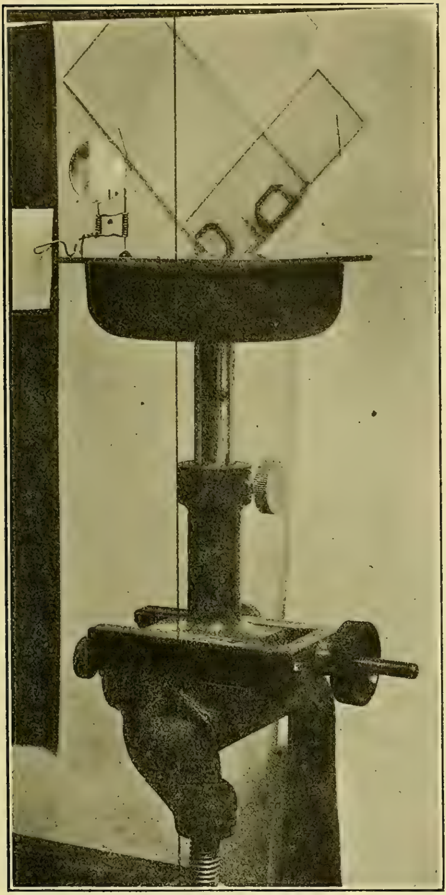

FIG. 8

match is applied. The segment of string, attached to the ball by a hook pointed downward, then drops to the bottom of the box leaving the pendulum perfectly free.

The lantern is placed at a distance from the mirror comparable with that of the screen. It is well to be able to move the lantern about in order to adjust, while the pendulum is working, the zero position of the spot on the screen. The apparatus can be set up in about 15 minutes and give an accuracy of 2 per cent, corresponding to less than 1 degree in latitude here. Further pains would yield greater accuracy than this.

A Foucault pendulum for class room use is said to have been exhibited at the Paris Exposition of 1900. It was a rigid pendulum about 2 meters long suspended from crossed knife edges. The motion was followed by the projection, through a microscope objective (focus, say, 2 inches) parallel to the vibration plane, of a fine wire affixed to the bottom of the pendulum. This wire came in focus momentarily, at the end of each swing and thus the lateral displacement could be traced upon the screen. This form is in use at the Case School of Applied Sciences and is said to be satisfactory.

In the *Physikalische Zeitschrift* several years ago there was described a tripod arrangement similar in principle to that described here: The author, however, actuates it by a point under the bob and so it is subject to the difficulties which Professor Smith has so beautifully avoided in his apparatus.

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BULLETIN

OF THE

PHILOSOPHICAL SOCIETY

Scientific Series, Vol. 1, No. 22, pp. 457-475, August, 1914

Parabolic Orbits of Meteor Streams

BY

CHARLES P. OLIVIER

UNIVERSITY OF VIRGINIA
Charlottesville, Virginia, U. S. A.

UNIVERSITY OF VIRGINIA PUBLICATIONS

The Proceedings and Transactions of the Philosophical Society of the University of Virginia are published in the form of Bulletins and offered in exchange for the publications of learned societies, institutions, universities and libraries. The Bulletins are issued at irregular intervals in the form of separate papers. These are to be made up into volumes, consecutively paged, of about 400 pages each, for each of the Scientific, Humanistic and Medical Sections. Separate numbers may be purchased from the Distribution Committee, to whom remittances should be made. Communications should be addressed to Dr. Thomas L. Watson, University, Virginia.

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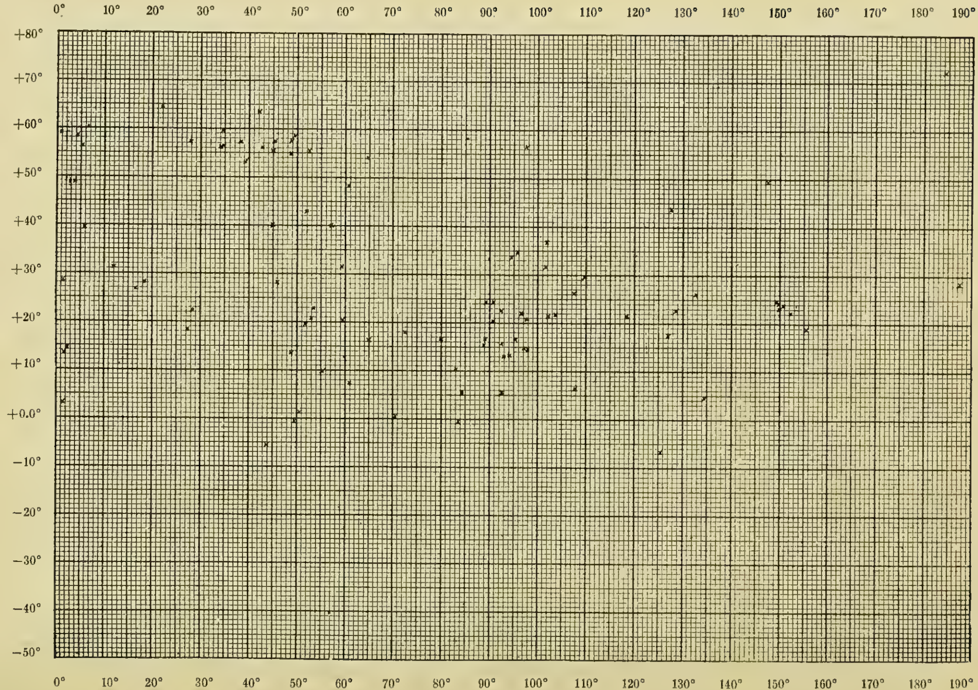
Continued on page 3 of cover

PLATE 1



DISTRIBUTION OF METEOR RADIANTS

Arranged according to right ascension and declination

UNIVERSITY OF VIRGINIA PUBLICATIONS
BULLETIN OF THE PHILOSOPHICAL SOCIETY
SCIENTIFIC SECTION

Vol. I, No. 22, pp. 457–475 August, 1914

PARABOLIC ORBITS OF METEOR STREAMS.

By CHARLES P. OLIVIER.

REPORT OF THE AMERICAN METEOR SOCIETY FOR 1911–1913.

Having been personally interested in meteors for many years, the idea came to the author during 1911 that it might be possible to form an organization whose purpose should be to advance meteoric astronomy in all possible ways by the coöperation of persons interested in their observation. Out of this grew the American Meteor Society. Some of the members have contributed valuable work, while as yet others have sent in little or nothing.

Most especial acknowledgment must be accorded to the Society for Practical Astronomy, because it was through being appointed director of its Meteor Section that I have been able to secure the assistance of most of our best observers, who are also members of that society. It must therefore be understood that all work done by such members (designated thus *) must be accredited first of all to the Society for Practical Astronomy, and it gives me the greatest pleasure to make this acknowledgement. Below is the list of such members as have contributed work to date:

*Nels Bruseth	Silvana, Wash.	+48° 14′	+8h 9m	Station 1
*Nels Bruseth	Index, Wash.	+48° 0′	+7h 50m	Station 2
*Alan P. C. Craig	Corona, Calif.	+34° 10′	+8 6m	Station 3
B. H. Dawson	La Plata, Observatory, Argentine	−34° 54′	+3h 52m	Station 4
P. H. Graham	McCormick Observatory, Virginia	+38° 2′	+5h 14m	Station 5
*Jas. W. Hanahan	Winnsboro, S. C.	+34° 21′	+5h 25m	Station 5
*E. A. McDonald	Isabela, P. R.	+18° 36′	+4h 28m	Station 7
*C. P. Olivier	Decatur, Ga.	+33° 47′	+5h 37m	Station 8
*C. P. Olivier	McCormick Observatory, Va.	+38° 2′	+5h 14m	Station 5
*C. P. Olivier	Yerkes Observatory, Wis.	+42° 34′	+5h 54m	Station 9

J. B. Smith	Hampden Sidney Col., Va.	+37° 14′	+5h 14m	Station 10
*L. J. Wilson	Nashville, Tenn.	+36° 9′	+5h 47m	Station 11

The observers will in general be designated hereafter by their initials and the number of the station will follow, when necessary.

Plan of the Work.

The short life and small membership of our society have made it impossible to carry out the work to date in the ideal way. However, special bulletins, containing general instructions, have been sent out by mail and also published in the *Monthly Register* of the Society for Practical Astronomy, and prepared blanks for recording meteors have been furnished the members. Much the weakest feature of many of the observations sent in has been the small scale and general unfitness of the maps used for plotting the paths. This has somewhat diminished the accuracy of work which otherwise seemed excellent. However, no dues are charged the members, and no funds, therefore, have been available for printing and distributing proper maps. In spite of this drawback, the results on the whole appear good and will easily measure up to the usual standard of meteor observations. The editors of *Popular Astromony and The Monthly Register* have always promptly printed any reports or bulletins sent to them, which has greatly aided the work by keeping up interest and gaining new members.

The plans uniformly followed have been described in full detail in the paper by the author entitled "175 Parabolic Orbits and other Results deduced from over 6200 Meteors" published 1911 in *Transactions of American Philosophical Society,* N.S., vol. xxii, part 1. It is considered useless to copy here these details of observation and readers are referred to that publication. The present paper is practically a continuation of the one just mentioned and to fully understand the results here given the former paper must have been read. The numbers of the radiants in this, beginning as they do at 177, follow serially the radiants given in that publication, which are frequently referred to.

One of the chief problems of meteoric astronomy is that of stationary radiants. Many observers seem convinced that such exist in considerable numbers. On the other hand, many astronomers have attacked their results on theoretical grounds, stating such things as stationary radiants are mathematically impossible, except in a few well known cases. Hence this problem is the most important awaiting solution, and only on new and improved data and methods of reduction can we hope for a final conclusion. To hope to get better data, in other words to observe more

TABLE I.

DATE 1900+			BEGAN		ENDED		TOTAL	METEORS	RATE	FACTOR	CORRECTED RATE	REMARKS	STATION	OBSERVER
y	*m*	*d*	*h*	*m*	*h*	*m*	*m*							
11	5	2	18	15	21	10	175	24	8.2	0.6	13.7	Little cloud	5	CPO
		4	20	18	21	12	54	6	6.7	0.3±	22.3	Cloud and haze	5	CPO
		5	19	31	21	24	115	21	10.9	0.7	15.6	Dawn at 21h	5	CPO
	7	28	16	0	19	10	180	32	10.7	0.8	13.4		5	CPO
	10	17	16	25	20	10	225	51	13.6	0.9	15.1		8	CPO
		19	16	31	20	25	234	69	17.7	0.8	22.1		8	CPO
		23	16	39	21	39	300	110	22.0	1.0	22.0		8	CPO
		23	19	45	21	39	114	29	15.3	1.0	15.3		5	PHG
	10	24	18	6	20	6	120	32	16.0	1.0	16.0		8	CPO
	11	15	20	33	23	3	150	33	13.2	0.8	16.5		8	CPO
		16	18	26	21	38	180	52	17.3	0.8	21.6	Some haze	8	CPO
		16	18	30	20	10	100	17	10.2				10	JBS
12	2	19	17	29	19	27	118	8	4.1	0.9	4.6		8	CPO
	4	18	19	30	21	5	95	14	8.9			Some cloud	10	JBS
		19	16	0	18	0	120	14	7.0				10	JBS
		22	20	18	21	48	90	12	8.0	1.0	8.0		8	CPO
	5	13	18	15	21	35	210	24	6.9	0.7	9.9	Clear except 30m	8	CPO
	7	21	22	5	23	30	85	11	7.7				3	APCC
	8	10	21	20	23	50	120	31	15.5	1.0	15.5	Very clear	3	APCC
		10	15	0	17	40	160	42	15.7	0.9	17.4	Clear	6	JWH
		11	15	0	18	0	150	60	24.0	0.8	30.0	Clear except 30m	6	JWH
		11	19	0	21	40	160	32	12.0	1.0	12.0	Very clear	3	APCC
		11	15	6	21	3	342	120	25.5	0.6	42.5	40 % lost by clouds	5	CPO
		11	16	30	20	55	235	89	22.7	0.8	28.4	Clear except 30m	11	LJW
		12	16	10	21	16	300	112	22.4	0.9	24.7		5	CPO
		13	16	10	18	50	150	37	13.8	0.9	15.3		5	CPO
	10	16	18	9	20	49	220	49	13.4	0.8	16.8		8	CPO
		18	23	15	25	0		16					3	APCC
	11	14	23	0	24	45	105	43	24.6	1.0	24.6	Very clear	3	APCC
		14	19	16	23	4	228	59	15.5	1.0	15.5		8	CPO
	12	9	16	38	20	8	210	36	10.3	0.8	12.9	Slight haze	8	CPO
		13	16	48	20	0	228	45	11.8	0.7	16.9	Some smoke	8	CPO
13	2	6	16	0	16	30	30	8	16.0				1	NB
		7	15	30	16	0	30	4	8.0				1	NB
		9	1	0	1	30	30	5	10.0				1	NB
	3	7	15	35	16	0	25	10	23.8				1	NB
	4	1	20	0	20	35	35	9	15.5				1	NB
		5	17	18	21	18	240	26	6.5	0.8	8.2		8	CPO
		6	16	45	18	0	75	9	7.2	1.0	7.2	Very clear	1	NB
		10	16	45	17	45	60	7	7.0	0.6	11.7	Part cloudy, haze	1	NB
	5	2	21	21	21	55	34	4	7.1	0.5	14.2		8	CPO
		3	20	48	21	49	61	14	13.8	0.7	19.7		8	CPO
		5	22	40	23	55	75	16	12.8	1.0	12.8	Very clear	1	NB
		6	21	35	23	10	95	11	7.0	0.9	7.8		1	NB
		9	20	12	21	39	87	11	7.6	0.6	12.7		8	CPO
		29	21	45	22	45	60	10	10.0	1.0	10.0	Very clear	1	NB
		30	17	30	18	40		9		1.0		Very clear	1	NB
	6	1	20	45	21	45	60	10	10.0	1.0	10.0	Very clear	1	NB
		26	19	0	20	30		14		1.0		Very clear	1	NB
	7	26	18	55	19	35	40	9	13.4				2	NB
		27	16	13	18	35	142	14	5.9	0.9	6.6		9	CPO
		28	21	5	22	5	60	23	23.0				2	NB

TABLE I—Continued.

DATE 1900+			BEGAN		ENDED		TOTAL	METEORS	RATE	FACTOR	CORRECTED RATE	REMARKS	STATION	OBSERVER
y	*m*	*d*	*h*	*m*	*h*	*m*	*m*							
13	7	28	16	10	19	54	224	45	12.1			Moon last half time	9	CPO
		29	22	0	23	30	75	11	8.8	0.8	11.0	Clear except 15m	2	NB
		29	18	0	20	0	120	47	23.5	0.9	26.2	Clear	7	McD
		30	18	45	20	45	120	56	28.0	1.0	28.0	Very clear	7	McD
		31	21	0	23	0	120	52	26.0	1.0	26.0	Very clear	2	NB
	8	1	17	4	20	4	180	52	17.3	1.0	17.3		9	CPO
		1	18	30	20	30	120	15	7.5	1.0	7.5	Very clear	7	McD
		1	14	10	18	35	145	46	19.0	0.9	21.1	Clear	4	BHD
		2	18	0	20	0	120	27	14.5	1.0	14.5	Very clear	7	McD
		2	21	15	22	30	75	22	17.6	0.8	22.0	Faint haze 30m	2	NB
		3	18	0	18	20	20	9	27.0				2	NB
		4	18	50	20	50	120	46	23.0	1.0	23.0	Very clear	7	McD
		4	17	3	19	39	156	50	19.2	0.9	21.3		9	CPO
		8	20	5	21	5	60	25	25.0	0.8	31.2	Mostly clear	2	NB
		9	18	15	18	45	30	3	6.0				7	McD
		10	19	0	21	0	120	17	8.5	0.9	9.4	Clear	7	McD
		10	21	5	23	15	130	71	32.7	1.0	32.7	Very clear	3	APCC
		11	20	0	21	0	60	21	21.0	0.9	23.3	Clear, dawn	7	McD
		12	19	0	20	30	90	24	16.0	0.9	17.8	Clear	7	McD
		12	17	10	20	17	147	39	16.0	0.4±	40.0	Clouds, haze at times	9	CPO
		27	17	45	18	45	60	27	27.0	1.0	27.0	Very clear	2	NB
		28	23	10	24	10	60	9	9.0	0.6	15.0	Clouds	2	NB
		29	20	25	22	25	120	23	11.5	0.8	14.4	Clear above 30°	2	NB
	9	5	18	15	19	15	60	12	12.0	0.9	13.3	Clear	2	NB
		6	18	30	19	55	85	24	16.9	1.0	16.9	Very clear	2	NB
		29	17	35	19	30	115	20	10.9	0.7	15.6	Passing clouds	1	NB
	10	25	17	10	19	56	166	39	14.1	0.9	15.7		8	CPO
		27	19	30	22	30	180	51	17.0	0.9	18.9	Fog 15m	1	NB
		31	16	54	19	54	180	35	11.7	0.8	14.6		8	CPO
	11	2	18	0	20	0	120	42	21.0	1.0	21.0	Very clear	1	NB
	12	1	19	0	20	0	60	16	16.0	1.0	16.0	Very clear	1	NB
		7	23	30	0	35	65	22	20.4	0.8	25.5	Slight haze	1	NB
		8	23	20	1	0	100	36	21.6	1.0	21.6	Very clear	1	NB
		20	17	30	18	0	30	5	10.0	0.9	11.1	Clear	1	NB
		23	19	0	19	30	30	5	10.0	0.9	11.1	Clear	1	NB
Misc.								59						NB
Misc.								43						APCC
Misc.								114						CPO
Misc.								4						

Total.. 2817
Total − Misc... 2597

accurately than many others who have had years of experience, seemed out of the question. But on examination into their methods of reduction, it was at once seen that a great improvement could be made in the results. A few of the main points will be mentioned: The dates were reduced to hundredths of a day in G. M. T.; the basis for comparing the results for various years was made the longitude of the meteoric apex, L, in this way getting rid of the complications leap years introduce into our dates; the positions of the radiants on the maps were measured off to tenths of a degree; frequently two different maps were used on the same night, partly eliminating possible errors of map projection; so far as possible all observers used the same methods in plotting and recording; and finally, the most important rule of all—under no circumstances were meteors seen on different dates or in different years ever combined to secure a radiant. This rule indeed furnishes the first essential step forward in the solution. Because the earth moves forward 1° per day in longitude, while each meteor stream has also its own motion in space, hence it is evident that the radiant for this stream must depend for its apparent position in the sky upon where the orbits of earth and meteor stream intersect. When both are in motion, the apparent radiant (except for a few well-known cases) can not have the same position in the sky for successive nights.

The reader is referred to *Mon. Not. R.A.S.*, for 1878, vol. 38, p. 115 for an article by G. L. Tupman, to *Astr. Nach.* for 1879, vol. 93, p. 209 for one by G. von Niessl, and to *Bulletin Astr.* for 1894, vol. 11, p. 409 for one by M. L. Schulhof, in all of which the question is discussed at length. A later paper by W. H. Pickering, in *Ap. J.* 29, 365, 1909 shows certain much wider conditions under which he concludes theoretically stationary radiants can exist, provided the data on which he bases his article is correct. This data comes from W. F. Denning's *General Catalogue, Memoirs R.A.S.*, vol. 53. Therefore to understand the results we must discuss the data. In my former paper, before referred to, on pages 19, 22 and 23, a careful analysis of Denning's conclusions with regard to the Orionids and α–β Perseids was made. It was there shown that for these streams—and the same would in general be true for most of the others given in the *General Catalogue*—his conclusions for stationary radiation depended mainly upon radiants secured by combining meteors observed on successive dates, anywhere from 2 to 60 nights, and sometimes successive years. Other excellent observers have been influenced into doing the same and hence our meteor catalogues today are filled by many hundreds of fictitious, or at best very rough positions, because the observers would combine the work of many successive dates, totally ignoring the obvious fact that both earth and meteor stream, and hence their intersection point, were all in motion. Given radiants secured thus, and we can prove stationary radiation or anything else. Further he does not hesitate to combine radiants which lie sometimes as much as 10° from one another into a general group. In an area ten degrees in diameter a radiant could have considerable motion and still be "fixed," if we put such wide limits to what we choose to call a radiant. In conclusion our results, reduced separately for each date, show almost no evidence of stationary radiation and so far as they go may be considered to disprove its existence.

TABLE II.

NO.	GMT 1900+			L	PARABOLIC ORBITS										Ω	NO. s →	REMARKS	OBSERVER
					α	δ	l′	b′	l	b	ι	η	q	π				
	y	m	d	°	°	°	°	°	°	°	°	°		°	°			
177	13	4	6.74	285.9	184.3	+72.0	130.8	+61.8	111.7	+24.3	24.4	94.9	0.995	205.8	16.8	5		N.B.
178	13	4	10.76	289.8	217.5	+75.0	130.6	+71.6	113.7	+30.2	30.2	92.6	0.977	205.9	20.8	4		N.B.
179	12	4	12.71	292.0	187.2	+28.4	185.8	+28.4	165.4	+24.3	36.6	136.3	0.479	295.4	22.9	8		N.B.
180	12	4	19.71	298.8	220.1	+98.9	214.6	+23.3	167.1	+17.9	25.6	134.6	0.510	298.9	29.8	4		J.B.S.
181	12	4	19.71	298.8	226.4	+18.1	229.0	−90.6	197.6	− 0.7	3.1	167.8	0.045	5.4	209.8	4		J.B.S.
182	11	5	2.82	311.0	331.7	+ 0.2	333.6	+11.1	350.2	+17.6	157.7	53.7	0.656	149.2	41.8	5	η Aquarids	C.P.O.
183	13	5	3.90	312.5	332.0	− 1.2	333.6	+ 9.6	348.8	+15.6	161.0	56.0	0.696	155.3	43.3	10±	η Aquarids	C.P.O.
184	11	5	4.87	313.0	332.4	− 0.8	334.7	+11.4	350.6	+18.4	157.4	55.3	0.682	154.4	43.8	3	η Aquarids	C.P.O.
185	11	5	5.87	313.9	336.0	+ 0.0	337.8	+ 9.3	354.9	+14.9	160.8	51.4	0.616	147.5	44.7	11	η Aquarids	C.P.O.
186	13	5	5.97	314 5	332.0	− 1.2	333.6	+ 9.6	347.4	+15.8	161.6	59.3	0.745	163.7	45.3	7	η Aquarids	N.B.
187	13	5	6.96	315.5	282.6	+37.2	290.2	+59.8	184.8	+71.8	77.8	76.5	0.954	197.2	46.2	5		N.B.
188	13	5	29.93	337.9	300.2	+21.2	308.2	+40.7	244.5	+67.2	91.6	67.3	0.824	202.9	68.4	9		N.B.
189	13	5	30.75	338.6	229.8	+61.0	178.5	+71.8	69.2	+58.7	90.0	58.7	0.740	186.5	69.2	5–6		N.B.
190	13	7	28.75	35.9	18.3	+27.3	27.5	+18.0	30.6	+40.4	149.6	94.2	1.010	323.9	125.5	3		C.P.O.
191	13	7	28.75	35.9	331.5	− 8.6	330.5	+ 2.9	277.6	+ 3.3	7.1	151.9	0.225	69.6	125.5	4		C.P.O.
192	13	7	29.95	37.0	332.4	− 0.6	334.1	+ 9.9	294.6	+10.8	42.4	163.9	0.078	94.4	126.6	3-4		N.B.
193	13	7	29.95	37.0	341.5	−14.6	337.4	− 6.3	287.4	− 6.8	20.0	159.6	0.123	85.9	300.6	8	Aquarids	N.B.
194	13	7	31.92	39.0	3.1	+ 1.7	3.5	+ 0.3	339.5	+ 0.5	179.0	149.0	0.269	66.5	128.5	3–5		N.B.
195	13	7	31.92	39.0	34.6	+59.4	55.4	+42.4	86.1	+67.1	105.9	73.3	0.931	275.1	128.5	15	Perseids ?	N.B.
196	13	7	31.92	39.0	42.8	+51.7	56.9	+33.5	78.5	+53.8	119.3	67.7	0.869	263.9	128.5	3–10		N.B.
197	13	7	31.92	39.0	331.5	+20.0	329.2	+16.0	282.0	+15.7	36.3	27.2	0.212	182.9	128.5	3–5		N.B.
198	13	7	31.92	39.0	344.2	−11.6	350.0	+ 4.5	304.3	+ 5.3	51.4	173.2	0.010	115.0	128.5	7	Aquarids ?	N.B.
199	13	8	1.72	39.8	44.0	− 5.4	39.8	−21.2	39.8	−35.9	144.1	89.6	1.015	308.4	309.3	4		B.H.D.
200	13	8	1.72	39.8	333.2	+ 0.8	335.4	+11.1	205.1	+11.8	12.2	76.1	0.957	281.6	129.3	7		B.H.D.
201	13	8	1.72	39.8	342.7	−35.7	329.6	−26.0	283.0	−24.9	46.3	144.4	0.344	58.1	309.3	4		B.H.D.

202	13	8	1.72	39.8	346.9	−10.8	343.8	− 4.8	308.0	− 5.8	77.6	174.1	0.097	117.5	309.3	5		B.H.D.
203	13	8	1.72	39.8	359.5	+ 2.4	359.5	+ 2.4	332.5	+ 3.4	171.4	156.5	0.161	82.3	129.3	1	Stationary	B.H.D.
204	13	8	1.77	39.8	22.0	+64.5	51.8	+49.8	97.7	+78.3	96.2	80.1	0.985	289.4	129.3	6–7		C.P.O.
205	13	8	1.77	39.8	28.1	+57.0	49.8	+41.9	69.3	+68.5	103.9	79.4	0.981	288.2	129.3	5–6	Compare 28	C.P.O.
206	13	8	1.77	39.8	28.3	+22.5	34.2	+10.2	30.2	+17.2	162.6	108.8	0.991	346.8	129.3	4	Compare 32	C.P.O.
207	13	8	1.77	39.8	339.2	+23.9	350.9	+30.0	245.9	+12.1	13.4	116.0	0.820	1.3	129.3	4		C.P.O.
208	13	8	2.91	40.9	39.3	+53.3	55.2	+35.8	74.4	+58.1	117.3	72.8	0.914	276.1	130.4	7–8		N.B.
209	13	8	2.91	40.9	348.3	+63.0	31.3	+58.6	357.9	+81.6	96.2	95.7	1.004	320.8	130.4	4–5		N.B.
210	13	8	4.76	42.7	4.1	+48.8	27.5	+42.2	359.2	+67.2	107.0	105.3	0.944	342.8	132.2	3		C.P.O.
211	13	8	4.76	42.7	31.5	+59.5	53.6	+43.2	76.6	+70.2	106.6	101.0	0.977	334.2	132.2	2–5		C.P.O.
212	13	8	4.76	42.7	34.5	+55.9	53.4	+39.3	70.8	+64.4	112.8	78.1	0.971	288.4	132.2	7	Perseids (1)	C.P.O.
213	13	8	8.84	46.7	34.5	+56.5	53.7	+38.9	65.9	+66.0	112.7	82.1	0.995	300.8	136.1	9	Perseids (1)	N.B.
214	13	8	8.84	46.7	338.8	+68.8	36.7	+65.1	243.0	+74.1	74.7	94.6	1.007	325.3	136.1	9–11	Compare 61	N.B.
215	12	8	10.68	48.7	5.4	+56.4	34.6	+48.0	352.4	+75.2	98.4	102.2	0.968	342.4	138.1	3	Compare 222	J.W.H.
216	12	8	10.68	48.7	5.9	+39.5	23.0	+33.5	350.1	+50.6	113.6	122.6	0.720	23.3	138.1	4		J.W.H.
217	12	8	10.68	48.7	12.2	+31.2	24.0	+23.8	2.3	+37.4	47.6	55.3	0.685	248.7	138.1	4		J.W.H.
218	13	8	10.92	48.7	38.3	+57.1	56.4	+39.5	79.7	+72.3	105.2	80.8	0.988	299.8	138.1	15	Perseids (1)	A.P.C.C.
219	12	8	10.94	48.7	45.2	+39.6	54.3	+21.5	59.0	+36.3	143.2	81.2	0.990	300.6	138.1	12		A.P.C.C.
220	13	8	10.92	48.7	52.3	+55.4	64.8	+35.2	85.8	+56.8	117.4	70.4	0.900	279.0	138.1	15	Perseids	A.P.C.C.
221	13	8	10.94	48.9	43.0	+55.7	58.8	+37.2	73.4	+61.4	116.3	78.3	0.972	294.9	138.4	10–13	Perseids (1)	A.P.C.C.
222	12	8	11.69	49.7	4.6	+58.4	35.9	+49.9	347.8	+77.2	96.2	101.2	0.975	341.6	139.1	10	Compare 215	J.W.H.
223	12	8	11.75	49.7	34.0	+55.7	52.9	+39.2	58.3	+65.5	114.2	86.2	1.008	301.6	139.1	8–10	Perseids ?	L.J.W.
224	12	8	11.75	49.7	42.0	+63.2	62.0	+44.3	89.8	+71.2	104.4	77.8	0.968	294.8	139.1	4–5	Compare 70	C.P.O.
225	12	8	11.75	49.7	45.1	+55.0	59.9	+36.1	74.1	+59.6	118.0	77.6	0.967	294.4	139.1	16–17	Perseids (1)	C.R.O.
226	12	8	11.75	49.7	48.6	+57.0	63.0	+37.4	82.5	+60.8	115.0	74.5	0.940	288.1	139.1	18	Perseids (1)	L.J.W.
227	12	8	11.75	49.7	65.0	+54.0	73.0	+32.0	98.5	+50.0	118.6	60.8	0.772	260.8	139.1	4	Perseids	L.J.W.
228	12	8	11.69	49.7	337.4	+58.2	17.4	+59.4	280.5	+67.8	75.7	107.2	0.925	253.4	139.1	4		J.W.H.
229	12	8	11.86	49.8	43.4	+55.9	59.2	+37.3	73.1	+61.6	116.3	78.9	0.976	297.1	139.2	14	Perseids (1)	A.P.C.C.
230	12	8	12.78	50.7	1.2	+28.0	13.0	+25.0	342.8	+35.9	118.0	138.4	0.447	56.9	140.1	4		C.P.O.
231	12	8	12.78	50.7	18.5	+28.0	28.0	+18.6	9.7	+29.7	126.8	55.7	0.678	251.5	140.1	4		C.P.O.

TABLE II—Continued.

NO.	GMT 1900+			L	PARABOLIC ORBITS										Ω	NO. s →	REMARKS	OBSERVER
					α	δ	l′	b′	l	b	ι	η	q	π				
	y	m	d	°	°	°	°	°	°	°	°	°		°	°			
232	12	8	12.78	50.7	45.6	+57.4	61.3	+38.3	77.7	+62.9	114.4	77.8	0.967	295.8	140.1	17	Perseids (1) Average with 234	C.P.O.
233	12	8	12.78	50.7	45.8	+28.0	51.2	+10.3	51.6	+17.6	162.4	88.6	1.012	317.3	140.1	4	Compare 59	C.P.O.
234	12	8	12.78	50.7	48.6	+54.8	62.1	+35.3	77.4	58.1	119.0	76.0	0.954	292.1	140.1	7	Perseids (1) Average with 232	C.P.O.
235	12	8	12.78	50.7	348.9	+19.0	357.7	+21.8	318.8	+26.6	87.4	153.4	0.204	86.8	140.1	3–4		C.P.O.
236	12	8	13.76	51.6	16.6	+26.8	25.8	+18.2	5.5	+28.5	142.6	128.0	0.628	37.1	141.0	2	Compare 58	C.P.O.
237	12	8	13.73	51.6	49.2	+58.3	64.0	+38.5	82.4	+62.2	114.2	76.0	0.953	292.9	141.0	8	Perseids (1)	C.P.O.
238	12	8	13.73	51.6	358.7	+28.3	11.0	+26.3	322.9	+33.1	92.9	146.9	0.302	74.8	141.0	2	Compare 56	C.P.O.
239	13	8	27.76	65.1	344.9	+27.6	358.2	+31.2	309.4	+30.6	54.5	141.3	0.395	76.9	154.3	3		N.B.
240	13	8	28.99	66.3	56.9	+40.0	63.6	+19.5	61.4	+33.1	146.8	93.4	1.006	342.2	155.5	4		N.B.
241	13	8	29.89	67.2	1.1	+58.7	33.9	+51.2	324.3	+65.7	84.6	113.8	0.846	23.9	156.4	6		N.B.
242	13	9	5.78	73.9	2.2	+14.7	8.0	+12.6	326.6	+13.2	39.5	159.0	0.129	121.2	163.0	4	Compare 244	N.B.
243	13	9	5.78	73.9	198.9	+72.5	134.4	+66.0	224.0	+52.2	55.8	72.7	0.918	308.4	163.0	4		N.B.
244	13	9	6.80	74.9	1.5	+13.4	6.8	+11.7	324.9	+11.8	32.6	157.6	0.146	119.2	164.0	3–4	Compare 242	N.B.
245	13	9	6.80	74.9	356.3	+64.2	37.4	+56.8	312.0	+65.2	76.3	110.8	0.880	25.7	164.0	6		N.B.
246	13	9	29.77	95.5	60.8	+48.4	68.7	+27.1	43.3	+41.8	123.8	126.6	0.646	79.7	186.5	4–6		N.B.
247	13	9	29.77	95.5	319.2	+62.2	348.4	+68.9	292.0	+36.2	37.2	102.5	0.956	31.4	186.5	7		N.B.
248	12	10	16.83	114.5	50.6	+ 1.3	48.6	−16.7	5.6	−17.3	45.7	155.5	0.120	174.6	203.6	3	Compare 102	C.P.O.
249	12	10	16.83	114.5	90.4	+20.3	90.4	− 3.2	73.5	− 5.0	173.4	129.9	0.586	103.4	23.6	7–8	Compare 97, 115, 126	C.P.O.
250	12	10	16.83	114.5	94.0	+13.1	94.0	−10.3	78.9	−16.8	159.9	123.1	0.700	89.7	23.6	6–8	Compare 94	C.P.O.
251	11	10	17.72	114.6	55.2	+ 9.6	55.2	− 9.8	32.6	−11.2	127.8	166.8	0.600	101.5	23.7	3		C.P.O.

252	11	10	17.72	114.6	89.1	+16.3	89.1	− 7.2	74.0	−10.8	166.0	174.4	0.603	101.5	23.7	3	Orionids (1)	C.P.O.
253	11	10	17.72	114.6	97.2	+20.3	96.8	− 3.0	84.2	− 4.9	173.4	119.4	0.756	82.6	23.7	3	Compare 269	C.P.O.
254	12	10	18.02	115.7	80.0	+16.5	80.4	− 6.6	55.8	− 9.8	161.5	147.6	0.286	140.0	24.8	3	Compare 109	A.P.C.C.
255	12	10	18.02	115.7	88.8	+15.0	88.8	− 8.4	69.6	−13.3	157.5	143.0	0.361	130.7	24.8	5	Orionids (1)	A.P.C.C.
256	11	10	19.77	116.7	49.5	− 0.8	46.8	−18.4	2.4	−17.9	39.2	150.9	0.236	147.5	25.8	4		C.P.O.
257	11	10	19.77	116.7	53.5	+22.4	56.6	+ 3.1	18.7	+ 3.6	26.9	172.1	0.019	189.9	205.8	4–5		C.P.O.
258	11	10	19.77	116.7	92.8	+13.0	92.8	−10.5	65.4	−16.8	154.7	137.6	0.454	120.8	25.8	13–16	Orionids (1)	C.P.O.
259	11	10	23.78	120.6	72.5	+17.8	73.3	− 4.6	46.8	− 7.4	156.3	161.5	0.101	162.6	29.7	4–5		P.H.G.
260	11	10	23.78	120.6	92.4	+ 5.5	92.5	−17.8	70.6	−27.6	141.4	132.0	0.549	113.8	29.7	4	Compare 134	P.H.G.
261	11	10	23.78	120.6	96.9	+14.5	96.8	− 8.8	79.5	−14.1	161.8	128.8	0.604	107.3	29.7	3	Orionids (1)	P.H.G.
262	11	10	23.80	120.7	49.1	+13.6	50.3	− 4.4	8.3	− 4.3	11.7	158.3	0.136	166.3	29.8	4		C.P.O.
263	11	10	23.80	120.7	53.2	+20.6	55.8	+ 1.4	15.9	+ 1.4	6.1	166.0	0.058	182.9	29.8	4–6	Compare 270	C.P.O.
264	11	10	23.80	120.7	83.2	− 0.5	82.5	−23.8	50.7	−33.8	118.0	140.9	0.395	131.6	29.8	3–4		C.P.O.
265	11	10	23.80	120.7	84.6	+ 5.8	84.4	−17.5	56.6	−25.6	133.3	143.6	0.351	136.9	29.8	6–7		C.P.O.
266	11	10	23.80	120.7	92.6	+15.7	92.4	− 7.7	62.3	−12.0	162.6	136.1	0.479	122.1	29.8	13–16	Orionids (1)	C.P.O.
267	11	10	23.80	120.7	90.8	+24.2	90.7	+ 0.7	70.0	+ 1.1	178.3	130.7	0.416	129.2	209.8	5		C.P.O.
268	11	10	23.80	120.7	102.0	+36.6	99.8	+13.6	84.0	+22.0	153.5	122.8	0.703	95.3	209.8	6–8	Compare 135?	C.P.O.
269	11	10	23.80	120.7	103.6	+21.8	102.6	+ 1.0	90.0	− 1.6	178.1	119.8	0.748	89.4	29.8	5–7	Compare 253	C.P.O.
270	11	10	24.80	121.6	52.4	+19.4	54.9	− 0.4	14.2	− 0.4	1.3	163.5	0.064	177.7	30.8	2	Compare 262	C.P.O.
271	11	10	24.80	121.6	95.5	+16.5	95.4	− 6.8	78.4	−10.9	165.4	131.4	0.559	113.6	30.8	10–11	Orionids (1)	C.P.O.
272	13	10	25.76	123.2	97.8	+14.4	97.6	− 8.8	79.3	−14.0	161.1	131.4	0.560	115.0	32.2	10	Orinoids (1)	C.P.O.
273	13	10	27.88	125.2	61.1	+ 7.0	60.4	−13.5	19.0	−14.3	44.0	20.9	0.126	256.1	34.3	3		N.B.
274	13	10	27.88	125.2	83.0	+ 9.9	82.9	−13.4	52.6	−18.6	132.9	154.1	0.189	162.6	34.3	11–12		N.B.
275	13	10	27.88	125.2	96.8	+21.6	96.3	− 1.7	66.2	− 2.6	175.0	148.0	0.279	150.3	34.3	5–6		N.B.
276	13	10	31.77	129.1	59.0	+20.2	61.1	− 0.2	20.0	− 0.2	0.6	161.7	0.977	181.7	38.2	3–4		C.P.O.
277	13	10	31.77	129.1	70.4	+ 0.4	68.8	−21.6	26.7	−24.1	66.5	153.5	0.198	161.2	38.2	3–4		C.P.O.
278	13	10	31.77	129.1	95.9	+34.5	95.0	+11.1	70.4	+16.6	151.7	144.2	0.340	145.6	38.2	3		C.P.O.
279	13	11	2.79	131.1	65.1	+16.5	66.1	− 4.9	25.8	− 5.2	20.2	164.7	0.069	189.6	40.3	13		N.B.
280	12	11	14.88	143.4	124.8	− 6.8	128.9	−25.6	96.5	− 8.2	168.2	135.6	0.484	143.8	52.6	4		C.P.O.
281	12	11	14.88	143.4	149.3	+24.4	143.0	+11.2	144.1	+19.1	160.9	88.6	0.988	49.9	232.6	8 9	Leonids (1)	C.P.O.

TABLE II—Continued.

NO.	GMT 1900+			L	PARABOLIC ORBITS										Ω	NO. s →	REMARKS	OBSERVER
					α	δ	l′	b′	l	b	s	η	q	π				
	y	m	d	°	°	°	°	°	°	°	°	°		°	°			
282	12	11	14.88	143.4	152.7	+22.0	146.8	+10.0	152.8	+17.1	162.7	80.3	0.960	33.2	232.6	4	Leonids (2)	C.P.O.
283	12	11	14.99	143.5	150.0	+22.6	144.2	+ 9.7	144.8	+16.6	163.4	88.1	0.989	48.9	232.8	4–6	Leonids (1)	A.P.C.C.
284	12	11	14.99	143.5	155.5	+18.6	150.6	+ 7.8	155.7	+13.3	166.3	77.4	0.942	27.6	232.8	4–6	Leonids (2)	A.P.C.C.
285	11	11	15.91	143.6	107.9	+ 7.2	108.4	−15.1	82.1	−22.4	139.8	143.8	0.395	160.6	52.9	3		C.P.O.
286	11	11	15.91	143.6	149.1	+24.4	142.8	+11.1	142.2	+19.0	161.0	90.7	0.989	54.4	232.9	6	Leonids (1)	C.P.O.
287	11	11	16.84	144.6	97.3	+56.7	94.8	+43.4	48.9	+40.8	84.3	138.9	0.427	241.7	233.8	3		C.P.O.
288	11	11	16.84	144.6	102.4	+21.7	101.5	− 1.3	73.8	− 1.8	174.8	160.0	0.116	193.8	53.8	6		C.P.O.
289	11	11	16.81	144.5	118.8	+21.4	116.6	+ 0.5	97.2	+ 0.8	179.1	117.2	0.782	108.2	233.8	5		J.B.S.
290	11	11	16.84	144.6	127.4	+43.8	118.7	+24.0	95.7	+37.5	131.0	126.2	0.811	126.2	233.8	5		C.P.O.
291	11	11	16.84	144.6	128.9	+22.7	125.5	+ 3.9	112.0	+ 6.4	173.4	101.8	0.948	77.4	233.8	4		C.P.O.
292	11	11	16.84	144.6	150.2	+23.8	144.0	+10.9	143.5	+18.6	161.4	90.3	0.989	54.5	233.9	8–9	Leonids (1)	C.P.O.
293	11	11	16.84	144.6	157.4	+49.5	138.4	+36.8	129.3	+61.5	117.7	96.9	0.974	67.6	233.8	4	Compare 142, 148, 157	C.P.O.
294	13	12	1.81	160.0	89.4	+24.0	89.4	+ 0.6	47.3	+ 0.5	1.4	157.8	0.141	205.0	249.5	4	Compare 295	N.B.
295	13	12	9.03	167.3	92.2	+22.5	92.0	− 0.9	48.5	− 0.8	1.8	151.6	0.223	200.1	76.9	6	Compare 294	N.B.
296	13	12	9.03	167.3	107.8	+25.9	106.0	+ 3.4	67.2	+ 3.8	21.9	169.6	0.032	236.1	256.9	7–8	Compare 299	N.B.
297	13	12	9.03	167.3	127.0	+17.1	125.2	− 2.0	97.6	− 2.8	172.3	159.1	0.018	215.1	76.9	11		N.B.
298	12	12	9.77	169.2	94.7	+33.3	94.0	+ 9.9	108.6	+ 9.0	162.4	149.0	0.261	296.9	258.8	4		C.P.O.
299	12	12	9.77	169.2	110.0	+29.7	107.4	+ 7.5	64.0	+ 8.2	29.3	163.1	0.083	225.1	258.8	8	Compare 296	C.P.O.
300	12	12	9.77	169.2	134.5	+ 3.6	135.9	−13.0	227.1	−19.6	145.9	36.8	0.590	332.4	258.8	4		C.P.O.
301	12	12	13.77	173.2	101.5	+31.4	101.6	+ 8.5	19.1	154.4	018.4	154.4	0.184	211.8	262.9	6		C.P.O.
302	12	12	13.77	173.2	132.9	+25.9	128.2	+ 8.0	97.3	+10.9	142.3	162.1	0.093	207.0	262.9	5		C.P.O.

TABLE III.

Uncertain Radiants

1900 + G M T Y	M	D	L	α	δ	s →	REMARKS	OBSERVER
			°	°	°			
13	3	7.66	256.1	96.0	+30.0	3–4		N.B.
13	5	5.97	314.5	258.2	+ 7.9	3		N.B.
13	6	26.82	5.1	294.5	+47.1	3–4		N.B.
13	8	1.72	39.7	47.1	−30.7	4–6		B.H.D.
13	8	1.72	39.7	333.5	− 9.7	5	Compare 191	B.H.D.
13	8	2.91	40.9	36.6	+50.0	7	Perseids	N.B.
13	8	3.9	41.9	39.2	+55.3		Perseids	N.B.
12	8	10.68	48.7	36.6	+57.2	4	Perseids. Map projection very poor in region	J.W.H.
12	8	11.69	49.7	39.7	+55.6	8	Perseids. Map projection very poor in region	J.W.H.
12	8	11.69	49.7	309. ±	+34. ±	3		J.W.H.
12	8	11.75	49.7	9.9	+53.0	5	Compare 48, 57, 173	L.W.W.
12	8	11.75	49.7	12.9	+26.2	2	Compare 58	C.P.O.
12	8	11.75	49.7	355.0	+ 8.5	3		L.W.W.
13	8	29.89	67.2	29.6	+41.9	3		N.B.
13	9	6.80	74.9	8.6	+ 4.6	3–4		N.B.
13	9	27.99	96.4	149.5	+75.7	4		N.B.
12	10	16.83	114.5	79.0	+ 8.4	3	Compare 98	C.P.O.
11	10	19.77	116.7	35.2	+16.0	3–4		C.P.O.
11	10	19.77	116.7	36.1	+ 3.7	4		C.P.O.
11	10	19.77	116.7	36.4	+21.5	3–5		C.P.O.
11	10	23.80	120.7	40. ±	+40. ±	5–8		C.P.O.
11	10	23.80	120.7	69.0	+26.3	3–5	Compare 10U	C.P.O.
11	10	23.80	120.7	82.5	− 4.0	2–4		C.P.O.
13	10	25.76	123.2	60.0	+10.4	3	Compare 131	C.P.O.
13	10	25.76	123.2	70.4	+25.8	3–4		C.P.O.
13	10	25.76	123.2	96.2	+21.8	3	Compare 267??	C.P.O.
13	10	27.88	125.2	45.7	+20.3	3		N.B.
13	10	27.88	125.2	88.2	+12.6	11		N.B.
13	10	27.88	125.2	94.5	+15.4	6		N.B.
13	10	31.77	129.1	95.9	+14.6	2	Orionids??	N.B.
11	11	15.91	143.6	113.	−28.	3		C.P.O.
11	11	16.84	144.6	119.3	+ 9.7	3	Compare 154	C.P.O.
11	11	16.84	144.6	170.0	+61.3	2–3		C.P.O.

General Statement as to Results.

The results depend upon 2800± meteors. Of these perhaps 70 per cent were actually plotted. More or less complete descriptions are available for about 400+ more. The remainder are only useful to determine the hourly rates. Careful study showed that 126 radiants had been probably fixed with enough accuracy to justify the computation of parabolic orbits. These are contained in table II, and form far the most valuable results. Table III contains 22 radiants of whose existence or position there was still some uncertainty. The remaining tables contain other information of interest, each table having a few words of explanation attached to it. Brief sections are devoted to certain meteor streams of special interest.

While all the labor of preparing this paper, including the computation of the orbits etc., has fallen upon the author, it must be clearly understood that its very existance has only been made possible by the enthusiastic and unselfish coöperation of all the other observers. Each one of these has spent many hours of hard work gathering the data and making the observations, and every possible acknowledgement is due to each one.

One further acknowledgement should be made to Dr. F. H. Gaines, President of Agnes Scott College, Decatur, Ga., by whose enlightened policy time was allowed me to prepare this rather laborious paper, through the removal of many of the usual extra duties imposed upon a college professor, while the author was connected with that institution.

CONNECTION BETWEEN HALLEY'S COMET AND THE η AQUARIDS.

Certainly as early as 1868 some guesses were made that the meteors which appeared late in April and early in May might be connected with Halley's Comet. The first paper on the subject which could be found was by Rudolf Falb, written in 1868, and published in A.N. 72, p. 361–5. That his conclusions were totally erroneous, so far as proved by his data, was at once shown by Dr. Edmond Weiss in A.N. 73, p. 41–2. The latter found no such connection. The next attempt to prove this connection was made in 1876 by Prof. A. S. Herschel in M.N. vol. 36, p. 222. But the theoretical radiant is no less than 15° distant from the only observed radiant he gives, which of course was no proof at all, but rather a positive disproof, so far as his data went. He followed this up in May, 1878 by another paper in M.N, vol. 38, p. 379, in which he gives much more data, but still cannot get a closer agreement than 11°, a discordance out of all bounds. It can hardly be claimed therefore that his work made the connection even vaguely probable. Denning in his *General Catalogue* on p. 223, mentions that his group CCLVIII, called by him Aquarids, are probably connected with Halley's Comet. On p. 283 were given the 8 radiants, on which his conclusions must be based. Only two of these were deduced from observations of a single night; the others therefore are really useless for the question involved. Whether he made any computations or not is not stated, but the presumption is that he is repeating the conclusion of Prof. A. S. Herschel. In any case were orbits computed for these eight radiants they would give hopelessly discordant results, and hence could prove nothing.

Various attempts had been made here previous to 1910 to observe these meteors but with poor success due to bad weather, moonlight, etc. It was not until, 1910, May 4, when the author was at tbe Lick Observatory, that a good radiant was secured for the η Aquarids.

Another excellent radiant was obtained in 1910, May 11, and the parabolic elements were at once computed and the results published in the *Publications of the Astronomical Society of the Pacific* for June, 1910. The elements of the comet's orbit were placed side by side with those of the meteors, and the connection between them first definitely proved in this paper, so far as is known to the author.

Further papers have been published, as more data were acquired, the last and most important appearing in A. J. No. 640, p. 128–130.

In table IV of the present paper the elements based on all eight radiants, secured from 1910 to 1913 inclusive, are given. They are calculated in two ways: First, by assuming the meteors had the parabolic velocity, which is the assumption made for all the orbits in table II, secondly that they have the same major-axis as Halley's comet, and are moving in elliptical orbits. In spite of the fact that the comet's orbit never comes nearer than 4,000,000 miles of the earth's orbit and that consequently the orbits of meteors we meet can not be absolutely identical with the comet's path, yet the agreement of the elements are so close for both cases that no doubt can remain that these meteors were originally intimately connected with the comet.

It is of great interest to see how far from the comet's orbit some of these meteors actually move—namely as much as 11,000,000 miles for those of 1910, May 11. It should likewise be noted that the meteors were still coming in 1913 in nearly as great numbers as in 1910, when the comet was nearest the earth. We can have no better example of the process of slow dissolution of a large comet into a meteor stream, and hence meteor observers are urged in future years to pay more attention to these meteors, which seem so strangely neglected until very recently.

TABLE IV.

η Aquarid Meteors.

	PARABOLIC ORBITS					ELLIPTICAL ORBITS								
No.	ι	q	π	Ω	π−Ω	ι	q	π	Ω	π−Ω	log e	log a	Observer	Wt.
	°		°	°	°	°		°	°	°				
Halley						162.2	0.587	169.0	57.3	111.7	9.986	1.254		
182	157.7	0.656	149.2	41.8	107.4	157.6	0.646	147.5	41.8	105.8	9.984	1.254	C.P.O.	1
183	161.0	0.696	155.3	43.3	112.0	161.3	0.712	157.0	43.3	113.7	9.982	1.254	C.P.O.	3
184	157.4	0.682	154.4	43.8	110.6	157.2	0.673	152.1	43.8	108.4	9.983	1.254	C.P.O.	1
166	166.2	0.677	154.1	44.0	110.0	166.1	0.669	152.1	44.0	108.0	9.984	1.254	C.P.O.	2
185	160.8	0.616	147.5	44.7	102.8	160.6	0.605	145.8	44.7	101.1	9.985	1.254	C.P.O.	3
186	161.6	0.745	163.7	45.3	118.4	161.2	0.741	162.5	45.3	117.2	9.982	1.254	N.B.	1
167	163.1	0.608	147.6	45.9	101.7	163.0	0.598	145.4	45.9	99.6	9.985	1.254	G.H.	3
168	166.7	0.630	155.1	50.9	104.3	166.6	0.621	153.2	50.9	102.3	9.985	1.254	C.P.O.	2
Mean	161.8	0.664	153.4	45.0	108.4	162.0	0.658	152.0	45.0	108.1	9.984	1.254		
Mean Weighted	162.3	0.654	152.3	43.9	107.2	162.3	0.649	151.1	43.9	106.0	9.984	1.254		

TABLE V.

Orbits of Main Perseid Stream.

No.	G.M.T. 1900+			L	α	δ	l′	b′	ι	q	π	Ω	$\xrightarrow{s}$	Observer
	y	m	d	°	°	°	°	°	°		°	°		
212	13	8	4.76	42.7	34.5	+55.9	53.4	+39.3	112.8	0.971	288.4	132.2	7	C.P.O.
213	13	8	8.84	46.7	34.5	56.5	53.7	39.8	112.7	0.995	300.8	136.1	9	N.B.
218	13	8	10.92	48.7	38.3	57.1	56.4	39.5	105.2	0.988	299.8	138.1	15	A.P.C.C.
221	13	8	10.94	48.9	43.0	55.7	58.8	37.2	116.3	0.972	294.9	138.4	10–13	A.P.C.C.
225	12	8	11.75	49.7	45.1	55.0	59.9	36.1	118.0	0.967	294.4	139.1	16–17	C.P.O.
226	12	8	11.75	49.7	48.6	57.0	63.0	37.4	115.0	0.940	288.1	139.1	18	L.J.W.
229	12	8	11.86	49.8	43.4	55.9	59.2	37.2	116.3	0.976	297.1	139.2	14	A.P.C.C.
½(232–4)	12	8	12.78	50.7	47.1	56.1	61.7	36.8	116.7	0.960	294.0	140.1	17+7	C.P.O.
237	12	8	13.73	51.6	49.2	58.3	64.0	38.5	114.2	0.953	292.9	140.1	7	C.P.O.
Average.....									114.1	0.969	294.5	138.0		

TABLE VI.

Orionid Meteors.

NO.	DATE 1900+ G.M.T.			L	α	δ	l′	b′	ι	q	π	Ω	$\xrightarrow{s}$	Observer
	y	m	d	°	°	°	°	°	°		°	°		
252	11	10	17.72	114.6	89.1	+16.3	89.1	−7.2	166.0	0.603	101.5	23.7	3	C.P.O.
255	12	10	18.02	115.7	88.8	+15.0	88.8	−8.4	157.5	0.361	130.7	24.8	5	A.P.C.C.
258	11	10	19.77	116.7	92.8	+13.0	92.8	−10.5	154.7	0.454	120.8	25.8	13–16	C.P.O.
261	11	10	23.78	120.6	96.9	+14.5	96.8	−8.8	161.8	0.604	107.3	29.7	3	P.H.G.
266	11	10	23.80	120.7	92.6	+15.7	92.4	−7.7	162.6	0.479	122.1	29.8	13–16	C.P.O.
271	11	10	24.80	121.6	95.5	+16.5	95.4	−6.8	165.4	0.559	113.6	30.8	10–11	C.P.O.
272	11	10	25.76	123.2	97.8	+14.4	97.6	−8.8	161.1	0.560	115.0	32.2	10	C.P.O.
Average.....				119.0					161.3	0.516	115.8	28.1		

TABLE VII.

Leonid Meteors (1) and (2).

NO.	DATE 1900+ G. M. T.			L	α	δ	l′	b′	ι	q	π	Ω	⟶	Observer
	y	*m*	*d*	°	°	°	°	°	°		°	°		
281	12	11	14.88	143.4	149.3	+24.4	143.0	+11.2	160.9	0.988	49.9	232.6	8–9	C.P.O.
283	12	11	14.99	143.5	150.0	+22.6	144.2	+ 9.7	163.4	0.989	48.9	232.8	4–6	A.P.C.C.
286	11	11	15.91	143.6	149.1	+24.4	142.8	+11.1	161.0	0.989	54.4	232.9	6	C.P.O.
292	11	11	16.84	144.6	150.2	+23.8	144.0	+10.9	161.4	0.989	54.5	233.8	8–9	C.P.O.
Average.....				143.8	149.6	+23.8			161.7	0.989	51.9	233.0		
282	12	11	14.88	143.4	152.7	+22.0	146.8	+10.0	162.7	0.960	33.2	232.6	4	C.P.O.
284	12	11	14.99	143.5	155.5	+18.6	150.6	+ 7.8	166.3	0.942	27.6	232.8	4–6	A.P.C.C.
Average.....				143.4	154.1	+20.3			164.5	0.951	30.4	23.7		

EXPLANATION OF TABLES.

Table I.—This table contains the dates, intervals, number of meteors seen, etc., for all observations made. The factor column gives an arbitrary constant for each date which is based on the notes as to condition of sky and which is supposed to show the ratio of the meteors seen to what would have been seen on a clear, moonless night. The corrected rate gives this latter estimate. (Rev. E. A. McDonald only counted meteors during the periods of his observations. Therefore his rates per hour should be at least one-third greater than for the others, all of whom plotted and recorded most of the meteors seen.)

Table II.—This table contains 126 parabolic orbits, the method of their computation being taken from "Die Bahnbestimmung der Himmelskörper" by J. Bauschinger, Chapter 35. The symbols used have the following meanings: α, δ, l′, b′ refer to the apparent right ascension, declination, longitude and latitude of the radiant, l, b to the true longitude and latitude. The elements proper ι, q, π, Ω have the usual significance, that is inclination of orbit plain, perihelion distance, longitude of perihelion and longitude of ascending node. An auxiliary angle η is also given, since it serves as a useful check upon the other elements. Under the 'Remarks' column will be found notes calling attention to the confirmation of a given radiant from other sources, especially radiants formerly determined by the author.

Table III.—These radiants are considered uncertain from various causes, but still as probably existent. Poor projection of maps, too small scale, the radiant being over the edge, poor observation of some of the meteors, etc., are the principal causes for the inclusion of a radiant in this table.

Table V.—This table contains 10 orbits which seem certainly to belong to the main Perseid stream. They fully confirm the connection of these meteors with comet 1862 III, or Tuttle's Comet.

Table VI.—This table contains 7 orbits which resemble each other sufficiently and whose basic radiants show an approximately increasing longitude, from date to date, to permit their being placed together. It is believed that to the unprejudiced reader there can no longer be any doubt about the fact that the Orionid radiant does slowly shift its position, as this table shows. As this group of meteors has been held up as the typical example of stationary radiants, the results here given are quite instructive. This opportunity is taken to again point out the very close resemblance of the elements of these Orionids and the η Aquarids.[1]

Table VII.—This contains 6 orbits of the Leonid Meteors. On the two dates in 1912, for both A. P. C. C. and C. P. O., there can be no doubt that two distinct radiants are shown on the maps of each. No possible errors of observation could get all the meteors observed to conform to one radiant for either observer. Hence the resulting orbits have been divided into two groups: Leonids (1) the main stream, and Leonids (2) a sub-stream which was clearly shown in 1912 on November 14. Of course the elements confirm the well-known connection of these meteors with Temple's Comet—1866 I.

Table VIII.—This table shows the magnitudes of the meteors seen, as observed by those of the members who sent in the larger lists.

Table IX.—The percentages of meteors of a given magnitude to the totals seen are here given for the two observers who have sufficiently large numbers of observations to make these data useful.

Table X.—The first part of this table gives the number of meteors of a given color and duration. The second part gives the number of meteors of a given magnitude and duration, but whose colors were *not* recorded. As is to be expected few bright meteors come into the second part, and of course few very faint meteors were included in the first part of this table. Again the colors of red, yellow and orange meteors are more easily detected than green or white. Hence the latter part of the table is mostly made up of these last. Besides if a meteor was white, i.e., had no detectable color, it was seldom recorded in the color column at all, which explains why so few white meteors appear. These results are based on the

[1] See Transactions of American Philosophical Society, vol. xxii. p. 11.

TABLE VIII.

Magnitudes of Meteosr.

YEAR(S)	>0	0	1	2	3	4	5	6	<6	TOTAL	OBSERVER
1912–3	3		42	98	139	150	81	5		518	N.B.
1912–3	16	12	56	34	16	30	20	9	11	204	A.P.C.C.
1913		5	3	8	7	5	9	5		42	B.H.D.
1911	0	9	16	51	136	181	93	16	2	504	C.P.O.
1912	13	12	30	77	147	209	70	7		565	C.P.O.
1913	14	6	14	41	86	152	44	7		364	C.P.O.
1912			6	9	6	3	6	2		31	L.J.W.
	46	44	167	318	537	730	323	51	13	2229	

TABLE IX.

Magnitude of Meteors. Percentage.

YEARS	>0	0	1	2	3	4	5	6	<6	BASED UPON:	OBSERVER
1912–3	0.6		8.1	18.9	26.8	29.0	15.6	0.1		518	N.B.
1911–3	1.9	1.9	4.2	11.7	25.6	37.6	14.4	2.1	0.1	1433	C.P.O.

TABLE X.

Relation of Duration to Color and Magnitude (C.P.O.).

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.5	1.6	2.0	2.5	3.0	3.5	4.0	5.0
Red		4	29	33	20	12		3		6		1	1	1		1			
Orange		3	7	20	7	8	1	3	1	2		2		1		1		1	
Yellow	1		21	60	21	18	1	4	1	4				1					
Green		3	7	11	12	4	8	3	1	2		2		1		1			
Blue	1			1	2	6	1			1									
Purple		1			1	1				1	1	2		1					
White			2	5	2	1						1		1					
1 magn.				1	3	2													
2		2	4	13	5	2				1									
3	2	3	38	75	44	8	2			2									
4	4	4	109	233	91	19	7	2		3	5				1				
5		3	45	109	24	7		2											
6			6	8	5	3				1									

TABLE XI.

Meteor Durations (*C. P. O.*). *Meteor Durations* (*N. B.*)

	ALL 1911–1913	NO.	s < 1.1 1911–1913	NO.				
	s		s					
Red..............	0.508	111	0.451	107				
Orange............	0.633	57	0.463	52				
Yellow............	0.475	132	0.465	130				
Green.............	0.617	55	0.518	51				
Blue..............	0.567	12	0.567	12				
Purple............	0.888	7	0.575	4				
White.............	0.625	12	0.420	10				
					s		s	
1 magn............	0.517	6	0.517	6	1.202	38	0.691	26
2	0.431	26	0.431	26	0.661	88	0.615	84
3	0.432	177	0.416	174	0.425	126	0.413	125
4	0.421	461	0.409	455	0.349	136	0.349	136
5	0.399	189	0.399	189	0.315	68	0.315	68
6	0.469	22	0.469	22	0.334	5	0.334	5

work of C. P. O. alone. None of the other observers has as yet sent in enough results to make a similar table useful.

Table XI.—This contains a summary of the results of Table X. Further it contains a summary of the results of N. B., based on all the meteors, colored or uncolored, observed by him, for which durations were recorded. The results of these tables are of considerable interest, showing as they do how the durations shorten as the magnitude grows fainter—a result of course foreseen but which serves as a most useful check upon the accuracy of the estimated durations.

NOTES OF SPECIAL INTEREST.

(1) By B. H. Dawson; written 1913, Jan. 28, of Perseid shower 1907, August 11, from memory. On Pike's Peak, Col., alt. 12,000 feet ±. "We had beautifully clear weather, and I was greatly impressed by the immense number of meteors seen. While stopping to rest at an altitude of 12,000 feet ±, I watched them especially, and my impression is that scarcely an interval of 5 seconds went by in which I did not see one or more, while many times I saw two at the same time, and many others, they would follow one another as fast as one could count. They were all small—the

average I would put about 4.5 magn., with few brighter than 3, and the duration scarcely ever exceeding a half second. . . . . Many seen next evening, but not nearly so many as on 11th."

(2) By A. P. C. Craig; written 1912, Dec. 21. Of Perseid shower, 1910, August 13, from notes. While camping out, lying on ground, "I awoke about 2 a.m. and during the time I watched which was about 1 hour, 200 to 300 meteors must have fallen. . . . . average the 2 magn. Probably more fainter ones fell, but I did not notice very many as I was intent on brighter ones. It was the richest shower I have ever seen."

(3) By Latimer J. Wilson. Effect of passage of large meteor on seeing, from original observations. " . . . On 1911, May 17, during an observation of Jupiter, a meteor, fully as bright as Jupiter, passed 35′ above J., moving E to W, and the train quickly disappeared. May 17, at 10h 40m C.S.T., was one night when seeing was perfect, and as soon as I looked through the telescope after the passage of the meteor, the air was in such violent motion that only the coarsest features of Jupiter could be seen. The air vibrations were at first very rapid, less so as the seeing improved, until finally after a few ripples the seeing became normal. The duration of the disturbance was between 4 and 5 minutes. Color of meteor that of Jupiter, path 30° ± long."

In conclusion it may be added that many interesting descriptions of peculiar and exceptional meteors, sent in by the different observers, have been omitted, as the length of this paper is limited. Also no attempt has been made to draw general conclusions, for 2800 meteors are rather too few on which to base them.

However, with steady growth in our membership, it should not be a great while before many thousand observations have been sent in from which general conclusions can be safely drawn.

LEANDER MCCORMICK OBSERVATORY,
UNIVERSITY OF VIRGINIA.
July 13, 1914.

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BULLETIN OF THE PHILOSOPHICAL SOCIETY
SCIENTIFIC SECTION

Vol. I, No. 23, pp. 477–512 October, 1914

INFLUENCE OF THE CONCENTRATION OF POTASSIUM IODIDE ON THE RATE OF DIFFUSION OF IODINE IN POTASSIUM IODIDE SOLUTION.*

BY

STERLING H. DIGGS.

OUTLINE.

The purpose of this work was to discover, if possible, why the concentration of potassium iodide should affect the rate of solution of metals in iodine, the concentration of the latter being constant.

This fact, that the concentration of potassium iodide does influence the reaction, had been established by the work of Van Name and Edgar, and was in a way a serious objection to the "Diffusion Theory of Reaction Velocity," which their work in all other respects confirmed.

The following is a brief outline of the present work:

PART I.

The diffusion theory of reaction velocity, What it is and why it was advanced.

PART II.

Determination of the rate of diffusion—Apparatus used by Stefan and Öholm—Modification used by Edgar—Apparatus used in this work—Advantages and disadvantages.

Mathematics involved—Solution of the differential equation.

Complete solution for the present case.

PART III. EXPERIMENTAL DATA.

Diffusion of iodine in solutions of potassium iodide of varying concentrations—Viscosity of potassium iodide solutions—Density of potassium iodide solutions Curves.

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* A dissertation presented to the faculty of the University of Virginia as a part of the requirements for the Degree of Doctor of Philosophy. Read before the scientific section of the Philosophical Society at its regular meeting March 23, 1914.

PART I.

THE DIFFUSION THEORY OF REACTION VELOCITY IN HETEROGENEOUS SYSTEMS.

Work of Noyes and Whitney, 1897. In 1897 Noyes and Whitney carried out a series of experiments for the purpose of discovering, if possible, the laws governing the rate of solution of salts in water and other solvents. From the results of their work, and those of others, the theory has been advanced by Nernst that reaction velocity in heterogeneous systems is controlled largely, and in some cases entirely, by the rate of diffusion of the reacting or dissolving substances through a thin *unstirred* layer of the saturated solution adhering to the surface of the solid (1) (2) (3). That is, we conceive a thin film of saturated solution adhering to the dissolving solid somewhat as a film of water adheres to the inside of a pipette, this film is always saturated on the inside, because the rate of solution is very rapid in comparison with the rate of diffusion, which latter in liquids is very slow.

Thus, if we assume the thickness of the layer to be approximately constant, as seems reasonable with uniform stirring, it should follow that the solution, or reaction velocity, varies directly as the rate of diffusion. This latter is usually assumed to be proportional to the difference in concentration on the opposite sides of the thin unstirred layer. This theory was suggested by Noyes and Whitney, and further elaborated by Nernst.

The rate of reaction for first order reactions in homogeneous systems is expressed by the "Unimolecular formula,"

$$\frac{dx}{dt} = k \cdot (a - x)$$

where k is the reaction velocity constant, that is, a number depending on the so-called "affinity" of the reacting particles for each other. (a is the amount of reacting material originally present, and x the amount transformed).

The rate of reaction in heterogeneous systems is expressed by the formula,

$$\frac{dx}{dt} = k.O.\ (a - x)$$

where O is the surface of the reacting solid. It would seem natural to suppose that k had the same significance as in the first case, but according to the "Diffusion Theory" this is not true. k is a constant depending on the rate of diffusion of the reacting, or dissolving, substance and is independent of the "affinity" of the reacting substances. (k is also independent of the order of the reaction).

Numerous investigations have been made which seem to support this hypothesis, and it has been accepted as a working basis by many writers, but the criticism of it has been vigorous by others. In testing its validity we should choose such reagents as will react to produce only soluble bodies under the conditions of the experiment. Thus we should not expect the theory to hold for metals reacting with acids to liberate hydrogen, or for metals reacting with soluble salts to yield insoluble salts. These facts should be remembered in evaluating the criticisms advanced by Marc, Wildermann, and others (4) (5) (6) (7) (8) (9).

Let us now briefly survey the chief of these objections that have been advanced against the diffusion theory of reaction velocity.

I. That it is unnecessary, as the same equation for reaction velocity in heterogeneous systems can be, and have been (9) deduced from other and better established premises.

II. That the temperature coefficient of reaction velocity is found by experiment to be very different from the temperature coefficient for diffusion in many cases (4).

III. That if the diffusion theory were correct metals having widely different solution pressures should dissolve in a given solution at the same rate; likewise, that such substances as marble, dolomite, and magnesite should dissolve in acids at the same rate; which conclusions, it is urged, are manifestly not in accord with the well known facts.

The above list of objections, while not by any means exhaustive, embody the most important arguments against the theory.

A type of reaction that seemed to lend itself well to this investigation was the rate of solution of certain metals in a solution of iodine in potassium iodide. The advantages were: (1) that no gaseous product was formed which might disturb the "unstirred" layer; (2) the ease with which the materials might be obtained pure and in the desired form; (3) the ease and accuracy with which iodine can be estimated in solutions of all concentrations. A little work along this line had been done as early as 1891 by Schükarew, who worked with various metals in solutions of halogens (10). (This work was done about six years before the diffusion theory of reaction velocity had been advanced by Noyes and Whitney.)

In 1905 Brunner worked with zinc in solutions of hydrochloric acid, and also in iodine solutions (11). This work, while on the whole confirming the theory of reaction velocity given above, is very far from satisfactory. The most systematic and definite work thus far done along this line was done by Van Name and Edgar in 1910.

Work of Van Name and Edgar. In 1910 R. G. Van Name and Graham Edgar undertook a series of experiments for the purpose of determining more exactly reaction velocities between metals and dissolved halides for the purpose of testing the validity of the diffusion theory of reaction velocity. Having a very definite end in view Van Name and Edgar took all possible precautions against such sources of error as were likely to invalidate in any way their work for the particular end in view. All of their work was done at the temperature of 45°; the stirring was thoroughly and uniformly accomplished by means of an eighth horse power motor. Samples of iodine solution were withdrawn at desired intervals and titrated. The velocity constant k could be determined as many times as desired from the same materials. The following from Van Name and Edgar will make clear the method of calculation (12).

From the point of view of the diffusion theory the mechanism of the reaction is as follows: the weight of iodine which reaches and reacts with the surface of the metal in the time interval dt is the amount which can diffuse through the adherent layer of liquid in that time, that is according to Fick's law, it is proportional to the concentration fall across the layer. Owing to the rapidity of chemical reaction the concentration of the iodine at the surface of the metal is always practically zero. At the outer surface of the layer it is equal to c, the concentration of the main solution. Hence the concentration fall is c, and if m is the total weight of available halogen in the solution,

$$-\frac{dm}{dt} = -\frac{d\,(cv)}{dt} = K\,.\,c.$$

where K is the velocity constant. Integration for constant volume gives

$$K = \frac{v}{t_2 - t_1}\log.\frac{c}{c_2}.$$

Van Name and Edgar worked with Hg, Cu, Zn, and Ag. Though most of their work was done in iodine solution, some was also done in bromine solution. Some attention was paid to the effects of stirring at different rates, and to temperature effects.

As was to be expected the rate of reaction was increased in all cases by increasing the rate of stirring. This is in perfect accord with the diffusion theory, for we would expect the unstirred layer to be thinner when the stirring was rapid than when slow.

The following sample table taken from the work of Van Name and Edgar will serve to illustrate the high degree of accuracy of their work.

Rate of Solution of Hg *in Iodine.*
At 25°.

c	Δt	V	K
0.0381		500	
0.0356	5.0	480	6.51
0.0328	6.0	460	6.30
0.0304	5.3	440	6.36
0.0279	5.7	420	6.25
0.0257	5.0	400	6.55
0.0236	5.0	380	6.59
Average K.	. . . .	. . . .	6.43

100 grams KI to the litre, rate of stirring 170.

Collecting all of the constants obtained by Van Name and Edgar for a given rate of stirring (say 240) we have the following table:

	K	
Hg 400 grams KI per litre	10.48	
Cu	9.98	mean, 10.13
Ag	99.93	
Hg 200 grams KI per litre	9.55	
Cd	9.56	mean, 9.58
Zn	9.64	
Hg 100 grams KI per litre	8.81	
Cd	8.69	mean, 8.71
Zn	8.64	

In the above tables c represents the concentration, Δt the time interval, v the volume of the solution, and k the reaction velocity constant. There are three points to be particularly noted:

1. That the results are of high order of accuracy for this kind of work, and that therefore we can put more reliance in the results than we can in the rather uncertain results of previous investigators.

2. That the reaction velocity constant is approximately the same for all metals used, and that this "constant" *increases very decidedly with increase of the potassium iodide.*

3. That the reaction velocity constant increases, as was to be expected, with the rate of stirring.

The fact that the reaction velocity increases so markedly with increasing concentration of potassium iodide at once aroused interest, for it was not to be expected, and it did not accord with the diffusion theory of reaction velocity in so far as known facts went. (This point gives us the starting point for our present work.)

When we remember how far apart the metals in question are in the potential series it becomes impossible to ascribe the foregoing results to purely local cell effects, as has been done by Wildermann and other critics of this theory. We should bear in mind that the close duplication of results constantly shown by the data of Van Name and Edgar remove the possibility of mere accidental coincidence, which has been claimed to be the explanation of some data given by previous investigators to prove the same theory. Van Name and Edgar thus summarize their work:

1. "The rates of solution of the metals Hg, Cd, Zn, Cu, and Ag, in aqueous iodine solutions containing a large excess of potassium iodide have been measured at 25° and shown to be practically equal, a slight difference observed with copper and silver being in all probability due to accumulation of the solid iodide at the contact surface."

2. "The temperature coefficient for 10° (between 25° and 35°) is about 1.3."

3. *"An increase in the concentration of the potassium iodide produces a marked acceleration of the reaction."*

4. "Mercury dissolves in bromine in the presence of potassium bromide slightly faster than in iodine, but in cupric bromide much more slowly. . . "

5. "The reaction was found to be proportional, on the average, to the $\frac{4}{5}$ power of the rate of stirring."

6. "So far as can be decided from the data at present available, the diffusion theory of Noyes, Whitney, and Nernst, gives a satisfactory explanation of the results obtained."

It will be noted that the fact hardest to reduce to terms of the diffusion theory is that an increase in the concentration of the potassium iodide causes marked acceleration in the rate of the reaction.

Work of Van Name and Bosworth (13). Later Van Name and Bosworth worked with iron, nickel, and cobalt, in addition to those metals used by Van Name and Edgar. The fact that these metals also react with iodine at very nearly the same rate as those previously discussed adds greatly to the probability of the diffusion theory. The following quotation from the work of Van Name and Bosworth is of interest here.

The agreement between the metals Cd, Fe, Ni, and Co, is very striking, and clearly proves that under like conditions these metals dissolve in iodine at the same rate. In the earlier investigations a like result was obtained with the five metals, Hg, Cu, Ag, Zn, and Cd. Eight metals in all have, therefore, been shown to possess the same rate of solution in iodine, a result for which there seems to be no satisfactory explanation other than that furnished by the diffusion theory.

Work of Van Name and Hill. Recently Van Name and Hill (14) have measured the effect of added non-electrolytes on the reaction velocity, using metallic cadmium and iodine in potassium iodide solution as the reacting substances. The results obtained by them were not in accord with the formula deduced by Arrhenius (15); this they attribute to the increase in the thickness of the diffusion layer caused by greater viscosity of the solution. For our purpose the most important point in the work of Van Name and Hill is that they have shown that the reaction velocity varies approximately as the fluidity varies when the fluidity change is caused by the addition of a non-electrolyte. This, as we shall see later, certainly does not hold for the case where the decrease in fluidity is caused by increasing the concentration of potassium iodide beyond a certain limit.

Attention has been called to the fact that the reaction velocity increases as the concentration of the KI increases. Nernst deduces the following theoretical expression for the rate of diffusion of any electrolyte:

$$D = 2\frac{uv}{u+v}.\, g.RT.10.^{-9}$$

Where D is the diffusion constant, u the velocity of migration for the cation, v that of the anion, and g is a constant. According to this simple formula we should not expect the addition of a common ion (K) to have any effect on the rate of diffusion. (Or on the reaction velocity.) This apparently unexplained deviation from the results naturally expected from the diffusion theory of reaction velocity, becomes of great importance when we note that it is the only such deviation observed in the very careful and convincing work of the investigators quoted above. In fact it seemed that if this exception could be cleared up in a satisfactory manner we would be in a position to definitely look upon the diffusion theory of reaction velocity as established in those cases—typical of reaction in hetergeneous systems—studied by Van Name, Edgar, and Bosworth.

At the suggestion of Dr. Graham Edgar and in every way aided and guided by him, the writer undertook to measure directly the rate of diffusion of iodine in solutions of KI of various concentrations.

PART II.

DIFFUSION OF IODINE IN POTASSIUM IODIDE.

The work previously quoted showed plainly that, "An increase in the concentration of the potassium iodide produces a marked acceleration of the reaction." As no data bearing upon the rate of diffusion of iodine in KI solutions of varying concentrations were available, a few rough determinations were made by Van Name and Edgar. These, however, owing to the experimental difficulties encountered and to the lack of time to push further a side investigation, only gave a rough confirmation of the hypothesis. During the winter of 1911–1912 Dr. Graham Edgar made a few determinations of the same kind at the University of Virginia. The results of these preliminary investigations showed that it was very probable that the original hypothesis was correct as to iodine diffusing more rapidly in strong than in weak KI solutions, also that the value of the diffusion constant was somewhere about 1.1. It also developed that experimental difficulties were much greater than has been anticipated.

As previously stated Nernst has deduced an expression for the rate of diffusion of electrolytes in terms of the velocity of migration of the ions formed. This expression was deduced on purely theoretical grounds, making the assumption that the electrolyte was completely dissociated. The complete expression is:

$$D = 0.4485 \frac{uv}{u+v}. \left\{1 + 0.0034\,(t - 18)\right\}$$

where D is the diffusion constant, u and v the transport numbers, and t is the temperature on the Centigrade scale.

Making the assumption that the KI is completely dissociated, and that the ions would migrate at the same velocities in the KI solution as in water, and using the values given by Nernst for the K ion, and that given by Bredig for the I_3 ion we obtain for the diffusion constant 1.21, (16) (17) (18). This value is in all probability too great for the reason that the triiodide can not be completely dissociated. In fact the theoretical values thus obtained for a very large number of salts are in nearly all cases much larger than the results of actual experiments performed by Scheffer and by Öholm (16) (19) (20). In this connection it is of interest to note that the theoretical constant similarly calculated for KI is 1.47 and that actually found by Öholm using a solution of 0.01 normal was 1.46. This solution was about five times as dilute as the solution of KI_3 used by Van Name and Edgar, to say nothing of the immense excess of KI always present, the effect of which would certainly be to drive the dissociation still further back (21).

In the case of acetic acid Öholm found that the constant was 0.93 as against 1.37 theoretical, using 0.01 N solution. (Acetic acid is, of course, a very weakly dissociated acid.) This shows the effect of incomplete dissociation to be very considerable. Of course too, we have no *a priori* right to assume that the rate of migration of the ions is the same in KI as in water, or that it is the same, in a weak as in a concentrated solution of KI.

Molecular condition of the dissolved iodine. Le Blanc and Noyes concluded from freezing point experiments, that when iodine is added to a solution of KI it enters into combination to form a complex ion, probably I_3. The fact that iodine thus added does not lower the freezing point of the KI solution is excellent evidence that the total number of molecules is not increased, but does not show whether the resulting compound is KI_3, KI_5, KI_7 or some higher iodide (22).

In 1877 Johnson actually obtained crystals having the definite composition KI_3, by volatilization over concentrated sulphuric acid. This, while very strong evidence in favor of the KI_3 solution theory, yet left room for question, for the existence of a solid compound does not necessarily prove that it exists in solution (23).

The first really conclusive work was done by Max Roloff, who made use of the principle of partition coefficients between dissimilar solvents (24). Roloff did not work with iodine in KI solution, but with bromine in KBr solution. The other solvent used was CS_2. Roloff showed very conclusively that practically all of the bromine was present in KBr solution as KBr_3, and not as any higher bromide. His method was to assume that the bromine formed a given bromide, KBr_3, KBr_5, or what ever seemed reasonable; and then from the known laws of partition between non-miscible solvents to deduce a function of concentration in each solvent that theoretically should remain constant with varying concentrations. Roloff then tested this hypothesis by actually determining the concentration over as large range as seemed practical. Thus he showed that the bromine in a KBr solution is present as KBr_3 almost exclusively, and by analogy iodine in KI should be present as KI_3. For a full discussion of the theory involved and for the data the reader should consult the original article (24).

Later (1896) Jakowkin, using the same methods as those used by Roloff, showed that the reaction was strictly analogous for other bases and for other halogens; in particular for iodine in KI solution. Jakowkin goes into much more detail than does Roloff (25).

Two years later A. A. Noyes showed that the amount of free iodine (I_2) in solutions of KI far more dilute than any with which we worked is negligible (26).

Still more recently Osaka showed that the freezing point of a solution of KI was actually *raised* by the addition of iodine. In fact there was a small rise in the freezing point as found by Le Blanc and Noyes, but they put this down as experimental error. However, Osaka showed that though small it was easily measurable, and further that it was greater for HI than KI. From this he concludes that KI_3 and HI_3 must be somewhat less dissociated than KI and HI. He does not make quantitative computations because he feels that the data is not sufficiently accurate for this, though undoubtedly sufficient for a qualitative statement as made above (27).

Work of the writer under the direction of Dr. Edgar. In October, 1912, the writer, under the direction of Dr. Graham Edgar, began a series of experiments having the end in view to determine if possible the rate of diffusion of iodine in solutions of KI of various concentrations. In all cases it was decided to have the solutions of normal strength or some simple multiple thereof. The iodine was assumed to be present as KI_3, and the calculated amount of KI was added *extra* to the solution containing the iodine. That is to say, the concentration of the KI *exclusive* of that forming triiodide was the same in both solutions (layers).

The apparatus used was modeled after that of Stefan, utilizing the four layer scheme and making use of Stefan's tables (28) (29). As the values given in these tables required considerable interpolation, a set of curves was drawn on a suitable scale from which the values for X could at once be read.

The apparatus used by Edgar consisted of a flat-bottomed glass cylinder of about 100 cc. capacity, carefully selected to have uniform diameter and almost flat bottom. A mercury bottom could not be used because the mercury would be attacked by the iodine; nor was any liquid known suitable to use for this purpose. Three portions of KI solution of about 25 cc. each were first run into the cylinder by means of a separatory funnel having a thin walled capillary tube for a stem. This capillary was set within the cut-off stem of the funnel by means of hard paraffine, great care being taken to prevent any paraffine getting on the inside of the capillary tube. This tube extended down to within 0.2 to 0.3 mm. of the bottom of the cylinder, thus causing the inflowing liquid to spread out into a thin sheet as it entered the cylinder. If the layer containing the iodine was slowly and carefully run in there was *no* mixing of the layers so far as could be detected by the eye. When the last layer had been run in there was a space of about 0.5 cm. between the top of the liquid and the bottom of the cork. The cork was slightly hollowed on the under side, and covered with a thin coat of paraffine to protect it from the iodine. The delivery tube was so placed that it tapped the highest point of the under side of the cork (see fig. 1). When it was desired to remove the layers for titration, a very concentrated solution of KI (always having a higher specific

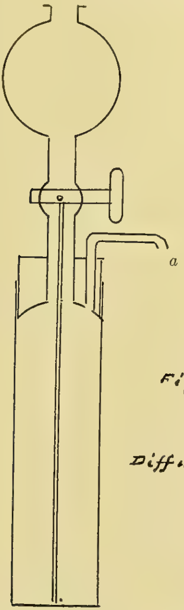

Figure 1.

Diffusion Cylinder

gravity than that used in the experiment) was slowly run in through the funnel till the air space was exactly filled. Then the stopcock was turned and all excess of the displacing fluid was carefully removed from the funnel; first with a pipette, and then with a rolled bit of filter paper. Next there was introduced into the funnel, and slowly run into the cylinder, the same amount of displacing fluid (strong KI solution) that was in each layer. This was measured in a pipette of the desired size. Thus, layer by layer the liquid was removed after diffusion had proceeded so far as seemed desirable. Experience showed that a period of from six to eight days was required to give a satisfactory quantity of iodine in the upper layers. All of these experiments were performed in a gas heated thermostat that kept constant temperature to within $\pm 0.1°$ *if* the gas pressure was constant.

The first experiments made by the writer were made with the apparatus just described; except that an electrically heated thermostat was substituted for the one heated by gas. This thermostat was very sensitive and would hold the temperature constant to within about 0.2° for a week if the temperature of the room did not vary more than 5 or 6°—and if the current was not cut off in the interval. As we had no cellar at our disposal, the temperature frequently *did* vary far more than this, and in consequence we frequently lost set after set of experiments. Also the current was frequently interrupted by various accidents to the city power plant—repairing, rewiring, and so forth.

New apparatus used. It was decided to use a cylinder much shorter than before, and to have only two, instead of four, layers. That is, equal quantities of iodine solution and of pure KI solution were used. The cylinder was of the same general type as before (see fig. 1) but somewhat smaller in diameter and very much shorter. The length of the new cylinders actually used was about 7.5 cms., and the pipette used to measure each layer held about 15 cc. The best results can be obtained with this apparatus when the time is from three to four days. The advantages of this shortening of the time is obvious and very great, but it is not attained without some disadvantage. The four layer system gives theoretically three independent equations, from which to evaluate the diffusion constant. It is true that in general some of the values are uncertain for mathematical reasons, i.e., the value of $\frac{h^2}{k \cdot t}$ corresponding to a given titration figure changes very little for quite large changes in the titration figure. In spite of this the four layer system usually gives more than one independent value for "k" in a single experiment. In the two layer system, the only check is to run separate experiments, either at different times or with different cylinders. This of course means more work, yet we found that the results were much more satisfactory than with the four layer system.

The method of procedure in this case was the same as for the four layer system, the iodine solution forming the bottom layer. In general four cylinders were filled at the same time. After a period of from three to four days the top layer was removed as in the four layer case. As there is no check in the two layer system there was no advantage in removing the second layer. (Titration being very much more accurate than the other factors involved.) Some of the original iodine solution was always placed in the bath along with the cylinders, and a pipette full was removed at the same time that the samples were removed from the cylinders. A small porcelain dish was used to receive the iodine solution as it came drop by drop from the delivery tube (*a*, fig. 1). As soon as a sample was removed it was transferred to a small glass stoppered flask, a pipette full of the original iodine solution being also put into a similar flask. When all four samples had been removed, the iodine in the flasks was titrated with sodium thiosulphate (approximately N/75). Starch paste was used as an indicator. Thiosulphate as weak as this will change strength from day to day; but this introduces no error, as a fresh sample of the original iodine solution is titrated every time a determination is made. The equation used to determine k is:

$$k = \frac{r^2}{t} \cdot \frac{2.30}{\pi} \log \frac{8}{\pi^2} \frac{V_0}{V_0 - V_1}$$

(The derivation of this formula will be given later.) Here k is the diffusion constant, t the time in days, V_0 the total amount of iodine in the cylinder, V_1, the amount of iodine in the top layer after time t has elapsed, and r is the length of the total liquid column, i.e., both layers. It will be noted that the quantity V, occurs to the first power in both denominator and numerator of our fraction (and there only); hence, *it does not matter in what unit V is measured.* This is of great practical value, for it allows us to be indifferent to the exact strength of thiosulphate used.

In a few cases, instead of placing the cylinder in the thermostat it was placed in a Dewar flask, being securely held in place by a well fitted cork. This was possible only in summer, and at such times as the temperature of the room was nearly at 25° and fairly constant. The flask was filled with water at as nearly room temperature as possible, the empty cylinder and thermometer inserted, and the whole left for a day or so, in order that the system might be in thermal equilibrium. Then the experiment was carried out as in the other cases. The temperature recorded for the experiment was the mean observed (4 times a day) during the run. Constants thus obtained were reduced to 25° by means of the formula given on a previous page (16).

It was only rarely that conditions permitted of the use of this method, and as it offered no special advantages over the use of the thermostat, it was only used a few times. The one interesting thing learned from this method was that if the temperature changes be slow and steady, no perceptible mixing is caused by rise or fall of 0.5°, while experience showed that a temperature change of only 0.1° caused by a break in the heating circuit of the thermostat was always fatal to results. Of course, in the latter case the fall was comparatively rapid.

MATHEMATICS INVOLVED IN THE PROBLEM.

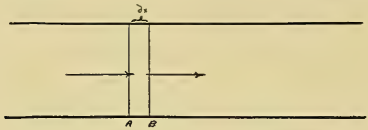

Figure 2.

Consider a substance diffusing in the direction indicated by the arrow in figure 2. Let u be the concentration at A, and $u-du$ the concentration at B. If now we make the thickness of the layer *very* small ∂u becomes du. Hence we have the expression.

$$(u + du) - u$$

for the difference in concentration at A and B. Differentiating this expression we obtain for the *change in concentration* corresponding to dx, the thickness of the layer.

$$\frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial x} = \frac{\partial^2 u}{\partial x^2}$$

The change in concentration with respect to time is, $\frac{\partial u}{\partial t}$, hence we can write,

$$\text{(I)} \qquad \frac{\partial u}{\partial t} = k \cdot \frac{\partial^2 u}{\partial x^2}.$$

Any solution of equation (I) must be a function of x and t. Put

$$u = f\ (x,\ t) = \cos nx \cdot e^{-kn^2t},$$

differentiating with respect to t and we have,

$$\frac{\partial u}{\partial t} = -\ k \cdot n^2 \cdot \cos nx \cdot e^{-kn^2t},$$

differentiating with respect to x we have,

$$\frac{\partial u}{\partial x} = -n \cdot \sin nx \cdot e^{-kn^2t},$$

differentiating again with respect to x we have,

$$\frac{\partial^2 u}{\partial x^2} = -n^2 \cdot \cos nx \cdot e^{-kn^2t}.$$

Substituting in equation (I) we have an identity; hence, $u = \cos nx \cdot e^{-kn^2t}$, is a solution. In exactly the same way we can show that $u = \sin nx \cdot e^{-kn^2t}$ is a solution; and hence,

$$\text{(II)} \qquad u = \sum_{n=0}^{n=\infty} (a_n \cos nx + b_n \sin nx) \cdot e^{-kn^2t}$$

is a solution. If this is to satisfy our conditions we must have for $t = 0$, $u = f(x)$ *known* i.e. u_0, and when $x = 0$ or $x = r$

$$\frac{\partial u}{\partial x} = 0, \text{ at all times.}$$

(Here r is the total length of the column, i.e. both layers.)

Differentiating equation II, and putting $x = 0$, we have:

$$\frac{\partial u}{\partial x} = \sum_{n=0}^{n=\infty} -n\,(a_n \sin nx - b_n \cos nx) \cdot e^{-kn^2t} = 0.$$

For this case $\sin nx$ is identical with zero; but as e^{-kn^2t} *can not* be zero, b_n must be zero *for all values of n*. If now we put $x = r$ the above equation becomes,

$$\text{(III)} \qquad \frac{\partial u}{\partial x} = \sum -n \cdot a_n \sin nr = 0$$

Hence, nr must be some multiple of π, i.e., the values of nr must be

$$0,\ \pi,\ 2\pi,\ 3\pi,\ 4\pi,\ \cdots$$

Or n must be,

$$0,\ \pi/r,\ 2\pi/r,\ 3\pi/r,\ 4\pi/r,\ \cdots$$

Equation (II) now becomes,

$$u = a_0 \cos 0 \cdot e^0 + a_1 \cos \pi x/r \cdot e^{\frac{-\pi^2kt}{r^2}} + a_2 \cos 2\pi x/r \cdot e^{\frac{-4\pi^2kt}{r^2}} + a_3 \cos 3\pi x/r \cdot e^{\frac{-9\pi kt}{r^2}} + a_4 \cos 4\pi x/r \cdot e^{\frac{-16\pi^2kt}{r^2}} + \cdots$$

It now remains to evaluate the coefficients $a_0, a_1, a_2, \cdots$

Put $t = 0$, then will the above equation become,

$$u = f\ (x) = a_0 \cos 0 + a_1 \cos \pi x/r + a_2 \cos 2\pi x/r + \cdot \cdot \cdot$$

Multiply both sides by $\cos n\pi x/r \cdot dx$, and integrate between the limits $x = 0$ and $x = r$.

$$\int_{x=0}^{x=r} f\ (x) \cdot \cos n\pi x/r \cdot dx = \int_{x=0}^{x=r} a_0 \cdot \cos n\pi x/r \cdot dx +$$

$$\int_{x=0}^{x=r} a_1 \cos \cdot \pi x/r \cdot \cos n\pi x/r \cdot dx + \int_{x=0}^{x=r} a_2 \cos 2\pi x/r \cdot \cos n\pi x/r \cdot dx$$

$$\cdot \cdot \cdot \int_{x=0}^{x=r} a_n \cos^2 n\pi x/r \cdot dx + \cdot \cdot \cdot$$

Make use of the relation that,

$$2 \cos A \cdot \cos B = \cos\ (A + B) + \cos\ (A - B)$$

The first term may be written,

$$\int_{x=0}^{x=r} a_1 \cos \pi x/r \cdot \cos \pi n x/r \cdot dx = \tfrac{1}{2}\, a_1 \left\{ \int_{x=0}^{x=r} \cos \frac{x\ (\pi + n\pi)}{r}\, dx + \int_{x=0}^{x=r} \cos \frac{x\ (\pi - n\pi)}{r} \cdot dx \right\}$$

Evaluating the above integral between the assigned limits we obtain zero. In the same way all terms except the *nth* fall out.

Making use of the relations,

$$\sin^2 A = \tfrac{1}{2}\ (1 - \cos 2A), \text{ and } \cos^2 A = \cdot\ 1 - \sin^2 A, \text{ and hence,}$$
$$\cos^2 A = \tfrac{1}{2}\ (\cos 2A + 1), \text{ and putting } n\pi x/r = A,$$

and substituting in the *nth* term we have,

$$ar/n\pi \int_{x=0}^{x=r} \cos A \cdot dA = ar/2n\pi \int_{x=0}^{x=r} \cos 2A \cdot dA + ar \cdot A/2n\pi =$$

$$ar/4n\pi \cdot \sin A + ar \cdot A/2n\pi.$$

Within the assigned limits the sine of A is always zero (being multiple of π): Hence, replacing A by its value $n\pi x/r$, and solving for a_n, we have,

$$a_n = 2/r \cdot \int^{=} f\ (x) \cdot \cos n\pi x/r \cdot dx$$

For this special case, when $t = 0$, $u = 0$, from $x = 0$ to $x = r/2$; $u = 0$ from $x = r/2$ to $x = r$, hence,

$$a_n = 2u_0/r \int_{x=0}^{x=r/2} \cos n\pi x/r \cdot dx = 2u_0/n\pi \cdot \sin n\pi/2$$

Hence: $a_1 = 2u_0$, $a_2 = 0$, $a_3 = 2u_0/3$, $a_4 = 0$, $a_5 = 2u_0/5 \cdot \cdot$

(Note that we can not obtain a_0 from the above formula because it takes the indeterminate form 0/0. To determine a take our equation before integrating and put $t = \infty$. $f(x)$ or u now clearly becomes $\frac{1}{2}u_0$, and all other terms on the right except the first or "zero" term vanish; hence, $a_0 = \frac{1}{2}u_0$.

We can now write the complete solution for our case, which is:

$$\text{(IV)} \int_{x=0}^{x=r/2} f(x) \cdot dx = \int_{x=0}^{x=r/2} u \cdot dx = ru_0/4 + 2u_0/\pi \cdot r\pi \cdot \sin \pi/2 \cdot e^{\frac{-\pi^2 kt}{r^2}} + \\ 0 + 2u_0/3\pi \cdot r/3\pi \cdot \sin 3\pi/2 \cdot e^{\frac{-9\pi^2 kt}{r^2}} + 0 \cdot \cdot \cdot$$

Note that all of the even terms fall out because they contain the expression $\sin m\pi$, where m is an integer. Also as the series stands we see that the sines are alternately $+1$ and -1, hence we can write,

$$\text{(V)} \quad \int_{x=0}^{x=r/2} f(x) \cdot dx = 2u_0 r \left\{ 1/8 + 1/\pi^2 \cdot e^{\frac{-\pi^2 kt}{r^2}} - 1/9\pi^2 \cdot e^{\frac{-9\pi^2 kt}{r^2}} + \right. \\ 1/25\pi^2 \cdot e^{\frac{-25\pi^2 kt}{x^2}} - 1/49\pi^2 \cdot e^{\frac{-49\pi^2 kt}{r^2}} \cdot \cdot \cdot$$

Numerical considerations. For our case,

$r = 50$ approximately
$k = 1.2$ "
$t = 3$ "

The third term in (V) becomes therefore:

$$X = 1/89 \cdot e^{\frac{-9\pi^2 \times 1.5 \times 3}{50}} = 1/89 \cdot e^{-8}$$

Put $e^{+8} = N$, then will $\log_e N = 8$; and hence,

$$\text{Log } N = .4343 \times 8 = 3.47 \text{ (approximately)}$$

Finding the number corresponding to this logarithm from tables, we have;

$$N = 3000 \text{ (approximately)}$$

Hence, $$X = \frac{1}{89 \times 3000} = 3.75 \times 10^{-6} = .00000375$$

This value is obviously too small to have any significance; and as all succeeding terms are very much smaller and alternating in sign, we can with perfect safety discard all after the first two.

Computing the amount of diffusing substance in either layer after lapse of time t. Let V_0 be the amount of iodine (say) originally put into the cylinder, V_2 the amount left in the lower layer after time t, V_1 that found in the upper layer after time t. Obviously,

$$V_2 = \sigma \int_{x=0}^{x=r/2} u \cdot dx = 2\sigma \cdot u_0 \cdot r \left\{ 1/8 + 1/\pi^2 \cdot e^{\frac{-\pi^2 kt}{r^2}} \right\}$$

(Where σ is the cross section.) Also

$$V_0 = \tfrac{1}{2} \sigma \cdot u_0 \cdot r$$

Therefore,

$$V_2 = 4V_0 \left\{ 1/8 + 1/\pi^2 \cdot e^{\frac{-\pi^2 kt}{r^2}} \right\}$$

Or

$$e^{\frac{-\pi^2 kt}{r^2}} = \left\{ \frac{V_2}{4V_0} - \frac{1}{8} \right\} \pi^2$$

Put $V_2 = V_0 - V_1$, and solve for k

$$k = -\frac{r^2}{\pi^2 t} \cdot 1n_e \left\{ \frac{V_0 - 2V_1}{V_0} \frac{\pi^2}{8} \right\} = \frac{r^2}{\pi^2 t} \cdot 1n_e \left\{ \frac{V_0}{V_0 - 2V_1} \cdot \frac{8}{\pi} \right\}$$

$$= \frac{r^2}{\pi^2 t} 2.30 \times \log_{10} \left\{ \frac{8}{\pi^2} \frac{V_0}{V_0 - 2V_1} \right\}$$

$$= \frac{r^2}{\pi^2 t} 0.233 \times \log_{10} \left\{ 0.8106 \frac{V_0}{V_0 - 2V_1} \right\}$$

This last is the formula used in all cases for computing k by the two layer method.

PRELIMINARY TABLE.

Diffusion of iodine in potassium iodide.

Values obtained by Dr. Graham Edgar alone *previous* to the author's first work. This work was done with apparatus involving the four layer system, and each value given is the mean of the most concordant values for two, three, or four layers.

100 grams KI per litre	K = 1.093
200 grams KI per litre	K = 1.191
400 grams KI per litre	K = 1.278

Translating into terms of normal solutions the above becomes:

0.60 normal	K = 1.093
1.20 normal	K = 1.191
2.40 normal	K = 1.278

These values are used in future tables, due note being made of the fact in each case.

TABLE I.

Diffusion of Iodine in Potassium Iodide Solution.

Normality of KI_3, $\frac{1}{20}$. Normality of KI, 1.

$$K = \frac{l^2}{t} 0.233 \times \log_{10} \left\{ 0.8106 \frac{V_0}{V_0 - 2V_1} \right\}$$

DATE 1913	*	l	l^2	t DAYS	V_0	V_1	TEMP.	K	K @ 25°
July									
15–19	a	7.54	56.9	3.72	26.20	8.18	25.0°	1.20	1.20
15–19	b	7.56	57.2	3.73	26.20	8.18	25.0°	1.19	1.19
15–19	c	7.60	58.0	3.74	26.20	7.90	25.0°	(1.12)	
15–19	d†	7.57	57.3	3.75	26.20	7 95	22.5°	1.12	1.19
15–19	e	7.57	57.3	3.74	26.20	8.40	25.0°	(1.26)	
21–25	a	7.54	56.9	4.01	26.50	8.51	24.8	1.175	1.18
21–25	b	7.56	57.2	4.02	26.50	8.54	24.8°	1.183	1.19
21–25	c	7.60	58.0	4.02	26.50	8.52	24.8°	1.194	1.20
21–25	d†	7.57	57.3	4.01	26.50	8.25	23.0°	1.167	1.207
21–25	e†	7.57	57.3	4.02	26.50	8.25	23.0°	1.165	1.205
Mean*									1.204

* Letters in this column were distinguishing marks on cylinders, scratched with a diamond.

† These cylinders were not in thermostat, but in a vacuum flask filled with water at room temperature.

Bracketed values for K are not counted in mean.

(Work done under guidance of Dr. Graham Edgar.)

TABLE II.

Diffusion of Iodine in Potassium Iodide Solution.

Normality of KI_3, $\frac{1}{20}$. Normality of KI, 0.5.

All cylinders in thermostat at 25°.

DATE 1913	*	l	l^2	t	V_0	V_1	K
September							
27–30	a′	7.48	56.2	3.02	38.50	11.08	(1.21)
27–30	c	7.60	58.0	3.02	38.50	11.20	(1.28)
27–30	d	7.57	57.3	3.02	38.50	12.30	(1.49)
27–30	e	7.57	57.3	3.02	38.50	11.50	(1.46)
Sept. and Oct.							
30–4	a′	7.48	56.2	3.68	39.00	11.80	1.10
30–4	c	7.60	58.0	3.68	39.00	13.20	
30–4	e	7.57	57.3	3.69	39.00	11.50	1.09
30–4	d	7.57	57.3	3.69	39.00	13 00	
October							
4–8	a′	7.48	56.2	3.75	39.40	12.20	(1.14)
4–8	c	7.60	58.0	3.75	39.40	11.88	(1.12)
4–8	d	7.57	57.3	3.75	39.40	11.70	1.07
4–8	e	7.57	57.3	3.75	39.40	11.70	1.07
16–21	a′	7.48	56.2	3.73	42.00	16.40	(1.135)
16–21	c	7.60	58.0	3.73	42.00	12.40	1.07
16–21	d	7.57	57.3	3.73	42.00	12.50	1.08
16–21	e	7.57	57.3	3.73	42.00	12.50	1.08
Mean*......	....	....	....	....	....	....	1.08

Bracketed values or uncomputed values of K not counted in mean.
(Work of the author under the direction of Dr. Graham Edgar.)

TABLE III.

Diffusion of Iodine in Potassium Iodide Solution.

Normality of KI_3, $\frac{1}{20}$. Normality of KI, 0.25.

All cylinders in thermostat at 25°.

DATE 1913	*	l	l^2	t	V_0	V_1	K @ 25°
October							
25–29	a′	7.48	56.2	3.70	49.05	14.50	(1.065)
25–29	c	7.60	58.0	3.70	49.05	14.40	1.083
25–29	d	7.57	57.3	3.70	49.05	14.50	1.080
25–29	e	7.57	57.3	3.70	49.05	14.50	1.080
Mean							1.08

(Work of the author under the direction of Dr. Graham Edgar.)

TABLE IV.

Diffusion of Iodine in Potassium Iodide Solution.

Normality of KI_3, $\frac{1}{20}$. Normality of KI, 2.

All cylinders in thermostat at 25°.

DATE 1913	*	l	l^2	t	V_0	V_1	K @ 25°
September	a′	7.48	56.2		35.32	10.20	1.25
	c	7.60	58.0		35.32	10.15	1.26
	d	7.57	57.3		35.32	10.26	1.27
Mean	....	....	....	....	....	....	1.26

(Solely the work of Dr. Graham Edgar, done early in September, the author being sick at that time.)

TABLE V.

Diffusion of Iodine in Potassium Iodide Solution.

Normality of KI_3, $\frac{1}{20}$. Normality of KI, 3.

All cylinders in thermostat at 25°.

(Work of the author under the direction of Dr. Graham Edgar.)

DATE 1913	*	l	l^2	t	V_0	V_1	K @ 25°
Oct. and Nov.							
30–1	a′	7.48	56.2	3.10	41.30	12.54	(1.283)
30–1	c	7.60	58.0	3.10	41.30	14.45	
30–1	d	7.57	57.3	3.10	41.30	14.70	
30–1	e	7.57	57.3	3.10	41.30	20.20	
November							
4–7	a′	7.48	56.2	3.20	48.30	14.30	(1.220)
4–7	c	7.60	58.0	3.21	48.30	14.30	1.255
4–7	d	7.57	57.3	3.22	48.30	14.20	(1.223)
4–7	e	7.57	57.3	3.22	48.30	16.00	
8–12	a′	7.48	56.2	3.74	48.60	15.85	1.285
8–12	c	7.60	58.0	3.74	48.60	15.75	1.300
8–12	d	7.57	57.3	3.74	48.60	15.50	1.250
8–12	e	7.57	57.3	3.74	48.60	18.00	
14–17	a′	7.48	56.2	3.06	48.90	14.45	1.275
14–17	c	7.60	58.0	3.06	48.90	14.30	1.286
14–17	d	7.57	57.3	3.06	48.90	14.30	1.275
14–17	e	7.57	57.3	3.06	48.90	14.30	1.205
24–27	a′	7.48	56.2	2.92	51.00	14.95	1.268
24–27	c	7.60	58.0	2.92	51.00	14.50	
24–27	d	7.57	57.3	2.92	51.00	Broken	
Mean.........	....	....	....	....	....	....	1.277

TABLE VI.

Diffusion of iodine in potassium iodide solution.
Normality of KI_3, 1/20. Normality of KI, 4.5.
All cylinders in thermostat at 25°.

DATE, 1913	*	l	l^2	t	V_0	V_1	K @ 25°
December							
5–8	a′	7.48	56.2	3.73	33.50	13.60	
5–8	c	7.60	58.0	3.73	33.50	10.60	
5–8	d	7.57	57.3	3.73	33.50	12.40	
10–13	a′	7.48	56.2	3.15	44.75	13.50*	(1.29)
							1.28
10–13	c	7.60	58.0	3.15	44.75	13.20	1.27
10–13	d	7.60	57.3	3.15	44.75	13.30	1.27
Mean.........	....	....	....	....	....	....	1.28

(Work of the author under the direction of Dr. Graham Edgar).

* In this case 2 drops too much were drawn off with the layer titrated. Making an approximate correction, we obtain the unbracketed value for K given above.

This table and the one preceding one were especially difficult to obtain owing to the exceedingly small difference between the specific gravity of the KI solution and that containing the iodine.

TABLE VII.

Diffusion of iodine in comparison with viscosity and density of the KI *solution at* 25°.

NORMALITY	TIME OF FLOW	Z REL. 25°	Z′ REL. 0°	η ABSOLUTE	F FLUIDITY	ρ DENSITY	K DIFFUSION
0.125	44.4	0.992	0.496	0.00897	1.010	1.0125	
0.250	43.8	0.997	0.499	0.00898	1.010	1.0275	1.080
0.500	41.8	0.976	0.488	0.00883	1.025	1.0580	1.080
0.600							1.090*
0.750	40.2	0.967	0.483	0.00874	1.035	1.0890	
1.000	38.6	0.952	0.476	0.00861	1.050	1.1180	1.200
1.200							1.194
1.500	36.2	0.940	0.470	0.00849	1.060	1.1760	
2.000	34.2	0.993	0.467	0.00843	1.070	1.2360	1.260
2.400							*1.278
2.500	32.6	0.932	0.466	0.00842	1.070	1.2950	
3.000	31.4	0.937	0.469	0.00847	1.070	1.3520	1.277
3.500	*	0.943	0.472	0.00852	1.060	1.4165	
4.000	*	0.960	0.480	0.00858	1.042	1.4680	
4.500	*	0.978	0.489	0.00884	1.020	1.5260	1.278
5.000	*	1.015	0.508	0.00918	0.996	1.583	

Z, Viscosity relative to water at 25°.

Z, Viscosity relative to water at 0°.

η, Absolute viscosity, C.G.S. units.

F, Fluidity, i.e., reciprocal of Z.

* Obtained by Dr. Edgar alone.

(Except as noted, work of Dr. Edgar and the author).

TABLE VIII.

Ordinates for curves on the next page and the diffusion constant computed from these curves.

NORMALITY	y (F−1) 100	y′ (ρ−1) 20	y″ (K−1.02) 50	y_1'' FROM CURVE	K′ CURVE	K EXPERIMENT	K′− K *per cent*
0.125	1.0	0.25					
0.250	1.0	0.55	3.0	1.8	1.056	1.080	2.2
0.500	2.5	1.20	3.0	3.7	1.094	1.080	1.3
0.600			3.6	4.4	1.108	1.090	1.6
0.750	3.5	1.80		5.3	1.126		
1.000	5.0	2.40	9.0	7.3	1.166	1.200	2.8
1.200			9.0	8.4	1.188	1.194	0.5
1.500	6.4	3.50		9.9	1.220		
2.000	7.0	4.70	12.0	11.7	1.254	1.260	0.5
2.400			12.9	12.6	1.272	1.280	0.5
2.500	7.0	5.90		12.8	1.276		
3.000	6.7	7.00	12.9	13.7	1.294	1.280	1.3
3.500	6 0	8.20		14.2	1.304		
4.000	4.2	9.40		13.7	1.294		
4.500	2.0	10.50	12.9	12.5	1.270	1.280	0.6
5.000	0.4	11.70		11.3	1.246		

y″, is ordinate for the dotted curve, made by adding the density and fluidity curves.

TABLE IX.

A comparison of reaction velocity constants with diffusion, fluidity, and density. Also ordinates for reaction velocity curve.

NORMALITY	FLUIDITY	DENSITY	DIFF.	A	B	y	y′Hg
0.125	1.01	1.0125					
0.250	1.01	1.0275	1.080				
0.500	1.025	1 058	1.080				
0.600			1.090	8.71	8.81	1.05	1.55
0.750	1.035	1.089					
1.000	1.05	1.118	1.200				
1.200			1.194	9.58	9.55	5.40	5.25
1.500	1.06	1.176					
2.000	1.07	1.236	1.260				
2.400			1.280	10.13	10.48	8.15	9.90
2.500	1.07	1.295	1.280				
3.000	1.07	1.352					
3.500	1.06	1.4105					
4.000	1.055	1.468					
4.500	1.02	1.526	1.280				
5.000	0.996	1.583					

Reaction velocity curves taken from the work of Van Name and Edgar.

A is reaction velocity for Zn, Cu, Cd, and Hg, averaged.

y = (A − 8.50) .5 = ordinate for velocity constant "average."

y′ = (B − 8.50).5 = ordinate for velocity constant for Hg.

TABLE X.

Densities and viscosities of solutions of lithium chloride and sodium chloride.
(For comparison with potassium iodide.)

NORMALITY	DENSITY NaCl	VISC. 25° NaCl	DENSITY LiCl	VISC. LiCl
0.125	1.0057	1.0126	1.0018	1.0116
0.250	1.0112	1.0239	1.0047	1.0314
0.500	1.0210	1.0471	1.0109	1.0665
1.000	1.0426	1.0973	1.0231	1.1423
1.500	1.0643		1.0355	
2.000	1.0862		1.0481	
2.500	1.1082		1.0597	
3.000	1.1313		1.0715	
3.500	1.1552		1.0836	
4.000	1.1781		1.0912	
4.500	1.2040		1.1100	
5.000			1.1222	
	*	†	††	§

* Van Nostrand's Chemical Annual. (1913.)
† Landolt & Börnstein (4 Auflage.)
†† Landolt & Börnstein (4 Auflage), page 273.
§ Wagner, Zeit für Phys. Chemie, v, 31.
(Transformed from per cent to normal data by the writer.)

Determination of viscosity and density of KI *solutions.*

The apparatus used in determining the density consisted of a very accurate set of hydrometers, graduated to three decimal places and readily estimated to the fourth place. The viscosity determinations were made with the usual Ostwald-Poiseulle apparatus. The one used for the first nine values was a commercial viscosity meter having a very short period of delivery (about 45 seconds at 25°). The last four determinations were made with a home made viscosity meter made by Dr. Edgar; which had a period of about 100 seconds. For our purposes the results found by using either apparatus are sufficiently accurate.

The formula used to obtain the relative viscosity is: $z/z_1 = t/t_1$ (30, 31). In table VII z at 25° for water is taken to be unity, the viscosity at 25° is taken to be 0.50 of that at zero, and the viscosity of water at zero is taken to be 0.018086 C.G.S. units (32) (33). For our immediate purpose the unit chosen is of no importance, yet in view of the fact that no satisfactory table of viscosities for potassium iodide solutions is known, it seemed worth while to give these values in terms of all commonly used units. The best data that could be found on this subject was that of

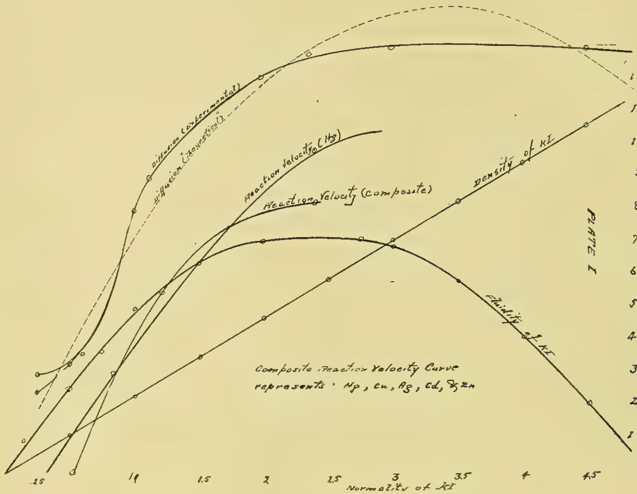
PLATE I
Diffusion (experimental)
Diffusion (theoretical)
Reaction velocity (Hg)
Reaction Velocity (composite)
Density of KI
Fluidity of KI
Composite Reaction Velocity Curve
represents · Hg, Cu, Ag, Cd, & Zn
Normality of KI
.25
1
1.5
2
2.5
3
3.5
4
4.5
1
2
3
4
5
6
7
8
9
10
11
12

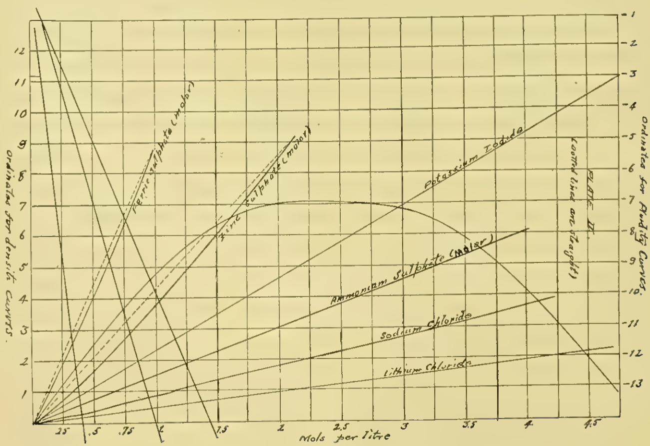
PLATE II
(Dotted lines are straight)
Ordinates for Fluidity Curves.
Ordinates for density Curves.
Ferric sulphate (molar)
Zinc Sulphate (molar)
Potassium Iodide
Ammonium Sulphate (molar)
Sodium Chloride
Lithium Chloride
Mols per litre
.25
.5
.75
1
1.5
2
2.5
3
3.5
4
4.5

Taylor and Rankin (1903). Their results, so far as comparable, agree fairly well with ours; but as he worked with only three concentrations his data was not sufficient for our purposes. Getman (1906–1908) also determined the viscosity of KI solutions of different concentrations; but his work was done at 18°.

In drawing the curves for fluidity, density, and diffusion, the axes were so shifted as to give a set of curves that could all be drawn on the same sheet for the sake of comparison. A full discussion of these curves will be found in the proper place.

DISCUSSION OF RESULTS.

General confirmation of the work of Van Name and Edgar. If we examine table IX, we see that there is no apparent discrepancy between the relative increase in the rate of reaction velocity with increasing concentration of KI and that of diffusion under the same conditions. This becomes clearer if we examine the curves of these quantities. In fact we may say that within experimental error the curves for diffusion and for reaction velocity are parallel. (It is to be noted that the scale on which these curves are drawn magnifies the discrepancy.) It is to be regretted that the work done by Van Name and Edgar, and by Van Name and Bosworth, do not give us any points on our curve beyond a concentration of 2.5 N, and hence, we can only guess as to the probable form of the curve at higher concentrations. It would be very interesting and instructive to obtain a curve for reaction velocities covering all concentrations covered by the diffusion or viscosity data. As it is, there is no way to know if the reaction velocity curve continues to rise or like the diffusion curve falls or becomes flat at higher concentrations. In the absence of evidence the latter seems the more probable, for two reasons; (a) the velocity curve though still rising, is rising much more slowly near the end, (b) the reaction velocity curve has approximately the same form as the diffusion curve so far as it is known, and hence it seems reasonable to assume that it will probably continue to have the same form.

General theory of diffusion. The phenomenon of diffusion may be defined in three ways according to the point of view, viz.:

1. As an empirical fact.
2. As a consequence of the kinetic theory.
3. As a result of the second law of thermodynamics, i.e., from the view point of energy relations.

1. Diffusion may be defined as the mixing which takes place when two dissimilar but miscible liquids are left in contact for some time under

conditions precluding mechanical motion or convection. Or diffusion may be defined as the mixing taking place in a system that is isolated, thermally and mechanically.

2. Diffusion may be defined as mixing due to random motion of molecular particles, in distinction from mixing due to directed motion of masses of the substance, e.g., mechanical mixing or convection.

3. Diffusion may be defined as mixing due to the decrease of the free energy of the system (increase of the entropy) in accordance with the second law of thermodynamics.

It should be noted that just as diffusion may be defined from the viewpoint of the kinetic theory, so by a reciprocal process we may say that the facts of diffusion furnish the most striking, and to many minds the most convincing, proof of the kinetic theory.

In general it would follow as a consequence of the kinetic theory that the rate of diffusion should be a function of the temperature, of the size and nature of the molecule, and the medium through which diffusion takes place. (The mathematical treatment of diffusion when the temperature, kind of molecule, and medium, are constant, has been taken up in detail for our particular case on a previous page.) Nernst has developed a very complete theory of diffusion for electrolytes which has already been mentioned in this article. In so far as an electrolyte is dissociated the separate ions will have independent motions *within a certain narrow range*—this range is very sharply limited by the large electrostatic forces that must come into play as soon as an accumulation of one kind of charge is perceptible in any part of the solution (34).

On the above theory as a basis, Nernst deduces the expression already given for the velocity of diffusion of an electrolyte, viz.:

$$K = \frac{2\,uv}{u + v}\, g\, RT\, 10^{-9}$$

where u and v represent the ionic mobilities of the cation and anion respectively, and g is a constant depending on the unit in which K is given. It is assumed that conditions during diffusion are the same as when u and v are measured. This is not the case for us, and there is no way in which we can do more than guess the ionic mobilities under conditions of varying concentrations of KI.

Theory of the "Reststrom." If a very low potential difference be maintained between two electrodes immersed in a conducting solution it is found that a weak current flows, even when this potential difference is far below the "decomposition potential" for the solution. Further, this current ("Reststrom") does not obey Ohm's law, even approximately, for it remains almost constant through changes in the e.m.f. of several hundred per cent, say from 0.1 to 0.3 volt. Nernst and Merriam have shown that with rapid stirring at a given rate the "Reststrom" is practically constant over a very large range of potential differences, and is closely proportional to the rate of diffusion of the depolarizer. They make the assumption that this "Reststrom" is due to the diffusion of the ion (or depolarizer as the case may be) through the thin unstirred layer which they assume to cover the electrodes (35).

Work of Harry Heymann. In 1912 Heymann made use of the "Reststrom" as a means of further establishing the "Diffusion theory of reaction velocity in heterogeneous media." He worked with platinum catalysis of hydrogen peroxide, iodine in KI, and the solution of copper in I_2 + KI. In the preface to this article Heymann calls attention to the fact that Van Name and Edgar had covered much the same ground, using various metals in I_2 + KI solutions. Using a solution of normal KI in which iodine was dissolved, Heymann determined the "Reststrom" using potentials varying from 0.1 to 0.45 volt. The "Reststrom" within these limits was practically constant as it was for Nernst and Merriam.

Heymann found that the thickness of the "unstirred layer" decreased with more rapid stirring as did Van Name and Edgar. Heymann made a single determination of the diffusion constant for iodine in normal KI directly, by a method practically similar to that used in this work. Using his value and exterpolating by Nernst's formula we obtain for the diffusion constant at 25° 1.19. This is in close agreement with our observed data. Exterpolation is hardly fair however over so great a temperature interval—for Heymann worked at about 8°. In the second part of his article Heymann discusses the work of Van Name and Edgar at length, and quotes much of their data to show that it is in line with the deductions made by Nernst, Merriam, and himself from experiments with the "Reststrom." He closes by saying,

> Aus an grossen Anzahl von Reststrombestimmingen fand ich, dass der Diffusionkoeffizient des Jods mit Steigender KJ-Konzentration stark wächst, und zwar ziemlich analog den von Van Name und Edgar gemessenen Auflösungsgeschwindigkeiten von Metallen in gleich Konzentrierten Jodkaliumjodidlösungen (36).

This, of course, is in accord with the results of the present writer also.

Discussion of some of the probable factors involved.

For our case as the temperature was held constant and the iodine content (free) was also constant, the only *independent* variable was the concentration of the KI. It is not so easy however to be sure that we have under consideration all of the dependent variables, and even if so it is in some cases difficult to determine what function such variables are of the concentration. The following factors will be discussed.

1. Density.
2. Fluidity (or reciprocal viscosity).
3. Dissociation of the KI_3 molecule into ions.
4. Effect of the common ion (K).

These factors will be taken up and discussed separately, and so far as possible the relation existing between them and the concentration of the KI will be determined.

Density and concentration. A glance at the density curve for KI solutions (Plates I and II) will show how *exactly* density and concentration go together for this salt. A careful scrutiny of table VII will show the same. Density curves for some other solutions will be found on plate II. While these curves are approximately straight lines, all have some perceptible curve except that for KI, which is a straight line *exactly*, at least so far as our measurements show.

All curves on Plate II are drawn on the same scale as those on Plate I, and therefore are fairly comparable. The data was obtained from Van Nostrand's Annual, Biedermann's Chemiker-Kalender, Landold-Börnstein-Roth's Tabeln, or from original sources. In most cases laborious computations had to be made to transform by interpolation values given in percent, into values for normal or molar solutions. This suggests the need for a set of tables giving the various physical properties of solution for normal and molar concentrations instead of the usual percent concentrations. Such tables should cover the fullest possible range, and not merely values below and up to normal, as is the case for those few tables that use normality as the concentration unit.

The previously noted fact, that the density of KI solutions is directly proportional to the concentration of the KI prevents density from having any distinct or separate value as a variable; that is, we can not distinguish between the effects of density, as such, and of concentration as such.

Fluidity and concentration. By fluidity is meant the reciprocal viscosity. For our purpose it matters little what unit be used, but for convenience I have always taken the viscosity of pure water at 25° as 1, except when specifically stated otherwise. As in the case of density, viscosity is usually given in terms of percent concentrations, and has to be changed to normal or molar terms before it is of real use for comparison. The only data of interest for us that could be found when this work was being done was that of Taylor and Rankin, who measured the viscosity of KI solutions at the following concentrations, 1 N, 2 N, and 3 N (37).

In view of the meagerness of the available data we determined the viscosity of various KI solutions (see table VI), and from this data the curves on Plates I and II are drawn. If we examine Plate II, we see that for most salts ($ZnSO_4$, LiCl, and NaCl are shown) the fluidity *rapidly* falls as the concentration increases, that both density curves and fluidity curves are almost straight lines; and hence the one varies inversely as the other. For KI the fluidity curve is peculiar, it first *rises* and then falls, being fairly symmetrical and having a maximum at about 2.5 normal concentration.

If we examine the diffusion curve (Plate I) in comparison with the fluidity curve we see that it rises, as was to be expected, with the fluidity curve; but *much more rapidly.* Further, at about the concentration at which the fluidity curve shows a maximum, the diffusion curve becomes almost flat. Thus we see that diffusion can not depend on viscosity alone for our case, as was thought probable when the work was begun. It is interesting to note that so long as the fluidity curve rises, the diffusion curve rises more rapidly than either the fluidity curve or the density curve; and that as these two curves begin to go apart the diffusion curve ceases to rise and becomes flat. This seems to indicate some relation between the density and the rate of diffusion similar to that between fluidity and the rate of diffusion. (Of course here density and concentration go hand in hand.)

Degree of dissociation of KI_3. Osaka has shown that KI_3 is probably less dissociated than KI (27). Hence, its dissociation would be very much driven back in strong KI solution. This would mean a far larger proportion of undissociated KI_3 molecules in such solutions. If now we *assume* that the effects of increasing viscosity is less on KI_3 than on the ions of the same salt, we have a possible explanation of the observed facts. This explanation is, of course, very far from satisfactory, for we do not know that the effects of increasing viscosity on the KI_3 and its ions is as indicated (*Argumentum ad ignorantium*). Yet in the absence of something better it may serve as a hint of a possible explanation.

In general it is supposed that ions migrate faster than the corresponding undissociated salt. This would seem probable *a priori* as the undissociated molecule must be larger, *unless* the ion is hydrated to a greater extent than the molecule. Öholm has made a special study of the rate of diffusion of electrolytes, comparing the diffusion constant calculated from Nernst's formula with experimental values. In most cases the observed rate of diffusion was less than the calculated, and the discrepancy was far greater for weak than for strong electrolytes. This is easiest explained by assuming that the undissociated molecule migrates more slowly than the ions, hence, the diffusion constant calculated for complete dissociation will always be too great unless the dissociation be complete.

These observations are not in accord with the experimental facts for our case, and they apparently contradict the possible explanation suggested some pages back; however, the theory of Nernst and the experiments of Öholm have value for water solutions only and need not hold for KI solutions.

Influence of an added electrolyte having a common ion. It has been shown by Abegg and Bose (38) that the effect of an added electrolyte having an ion common with the diffusing electrolyte is to make the rate of diffusion approach that of the ion that is not common. The following scheme, taken from the article by Abegg and Bose, will make clear why this should be so, and as a matter of experimental fact it is so.

Let us consider two diffusion systems. In the first let there be two layers of the same electrolyte. $A.B.$ having in one layer the concentration c and in the other layer the concentration $c + dc$. In the second system let there be the same electrolyte having the same concentration, and also the added electrolyte $A.X$ having the same concentration C in both layers: C being very great in comparison with c. Then will the scheme given below represent the concentration of all kinds, supposing the dissociation to be complete. We see that in the second system, the common ion A has practically the same concentration in both layers, and hence it diffuses very slowly. This leaves the *uncommon* ion as the only factor of importance in the rate of diffusion. Of course in practice these conditions are not realized, but the rate of diffusion approaches that of the uncommon ion as the concentration of the added electrolyte increases in comparison with that of the original diffusing electrolyte.

Scheme copied from Abegg & Bose (38).

I

A (c)	A $(c + dc)$
B (c)	B $(c + dc)$

II

A $(c + C)$	A $(c + C + dc)$
B (c)	B $(c + dc)$
X (C)	X (C)

For our case this would mean that the rate of diffusion would approach nearer and nearer to that of the I_3 ion as the concentration of the KI increased. As the migration velocity for I_3 is certainly less than that for K we should expect the rate of diffusion to fall off with increasing concentration of KI. This it does not do. (According to Bredig the migration number for I_3, is 44.2, according to Crotogino it is 60, that for K is well known to be about 65.3 (39) (40) (41). The above forces us to conclude that other and more potent factors are working in the opposite direction.

Discussion of Plate I. A careful review of the factors just discussed leads us to the consideration of the only two independent variables (or variables not *known* to be simply related) having important and traceable effect on the diffusion. These are the *fluidity* and the *density* of the solution. The effect of the fluidity is as one would theoretically expect it to be, at least up to about 2 normal; but it is not clear why the rate of diffusion should increase with the density, though it certainly does as is shown by the curves on Plate I.

The simplest assumption that can be made as to the effects of viscosity, and density, respectively, is that the rate of diffusion varies directly as the density and fluidity. This assumption however can not be even approximately true, for it would necessitate a very much more rapid increase in the rate of diffusion than is shown by experiment. However, the writer has deduced an empirical expression, by means of the three curves in question, which gives a very satisfactory numerical relation between the diffusion constant and the fluidity and density. It is,

$$50\,(\mathrm{K} - 1.02) = 100\,(F - 1) - 20\,(\rho - 1)$$

or

$$\mathrm{K} = 2F + 0.4\rho - 1.42$$

K calculated according to this equation gives the dotted curve for diffusion on Plate I. The difference between the diffusion constant thus calculated and that actually found by experiment is given in the last column of table VIII. The maximum variation is 2.8 per cent. This is hardly greater than the probable error, and the average variation is certainly within the probable error of experimentation. While it is not to be expected that this equation expresses the exact relation, it is probable that it is a fairly close approximation. It is of intereest to note that the curve thus calculated shows a maximum at a concentration of about 3.5 normal—a point at which we have no experimental data—and falls to very nearly the observed value at 4.5 normal, the next point at which we have an experimental value for the diffusion. The writer hopes at some time in the near future to find by experiment whether or not such a maximum really exists at about this concentration. Also it would be interesting to see if the diffusion curve falls, as does the theoretical curve, at concentrations higher than 4.5 normal. All that we can say at present is that the empirical equation given above in all probability gives a fair approximation to the truth.

The following empirical formula also gives a fair approximation to the experimental value for diffusion constants,

$$K = F\sqrt{\rho} + A$$

where A is a constant (about 6). This formula seems to have a somewhat more rational basis than the one previously given, but it does not agree quite so well with the experimental facts. Table XI gives the values so calculated, and also those obtained by the former method, in comparison with the experimental data.

In any case it is necessary to assume that the density is an important and direct factor in the rate of diffusion. (Of course we can not distinguish between density and concentration.)

TABLE XI.

A comparison of diffusion as calculated from the formula

$$\text{(I)}\ K = 2F' + 0.4p - 1.42$$

and the formula,

$$\text{(II)}\ K = F\sqrt{\rho} + A \qquad (\text{Where } A = 6)$$

and the experimental values.

NORMALITY	DIFFUSION I	DIFFUSION II	DIFFUSION, EXPERIMENTAL
0.125			
0.250	1.060	1.08	1.08 (1.065)
0.500	1.090	1.09	1.08
0.600			1.09
0.750	1.130	1.14	
1.000	1.170	1.17	1.20
1.200			1.194
1.500	1.220	1.21	
2.000	1.250	1.25	1.26
2.400			1.28
1.500	1.280	1.28	
3.000	1.295	1.30	1.28
3.500	1.300	1.32	
4.000	1.295	1.34	
4.500	1.270	1.32	1.28
5.000	1.250	1.31	

SUMMARY.

1. It has been shown that the rate of diffusion of iodine in KI solution increases with the concentration of the potassium iodide, as was anticipated from the results of reaction velocity experiments performed by Van Name and Edgar.

2. Determinations of the density and viscosity of KI solutions were made for the following concentrations, 0.125 N, 0.25 N, 0.50 N, 1 N, 1.50 N and so on up to 5 N.

3. Curves have been drawn showing the relation of fluidity (viscosity), density, and diffusion, to concentration of potassium iodide, and to each other.

4. A comparison has been made with some other well known salts, and curves to illustrate this have been drawn. Also an empirical expression has been found to express approximately the relation between the fluidity, density, and diffusion, for our present case, i.e., iodine in KI solution.

5. The diffusion theory of reaction velocity in general, and the work of Van Name and Edgar in particular, has been completely confirmed, in so far as the disturbing effect of increased concentration of KI is concerned.

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University of Virginia.

March 18, 1914.

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