Ultracomputer Note 38

Comments on Lens and Hypertrees - or the Perfect-Shuffle Again

Marc Snir

Computer Science Dept.

New York University

251 Mercer St., New York, N.Y. 10012

Finkel and Solomon introduce in [2] the lens interconnection strategy, and claim it is the only one known performing well with respect to the following five criteria:

1. Small diameter.
2. Fixed degree.
3. Simple routing algorithm.
4. Uniform traffic load.
5. Redundancy.

As it turn out, a network which performs well with respect to all these five criteria is actually presented in another paper in the same issue of the present journal. In [4, p. 930] Goodman and Sequin introduce a network they attribute to de Bruijn (see references there). They claim however that "It is not apparent that algorithms can be derived to take advantage of this unusual network, and it therefore appears to show less promise than Hypertree".

Far from being unusual, this network has been extensively studied in the past. Indeed, the "de Bruijn" network is nothing less that the shuffle-exchange network of Stone [9] in disguise.

Let $\alpha_1 \ldots \alpha_n$ be the binary representation of α. The graph of the shuffle-exchange network consists of $N = 2^n$ nodes labelled $0, 1, .., N-1$ such that edges join $\alpha = \alpha_1 \ldots \alpha_n$ to $S(\alpha) = \alpha_2 \ldots \alpha_n a_1$ (shuffle connection) and to $E(\alpha) = \alpha_1 \ldots \alpha_{n-1} \bar{\alpha}_n$ (exchange connection). In the "de Bruijn" network edges join α to $S(\alpha)$ and to $SE(\alpha_2 \ldots \alpha_n \bar{\alpha}_1)$ (shuffle-exchange connection). Since the shuffle-exchange permutation is a composition of a shuffle and an exchange it is clear that a shuffle-exchange network can simulate a "de Bruijn" network of the same size in constant time. On the other hand, if in the graph of a shuffle-exchange with 2N nodes one merges nodes connected by exchange edges than one obtains the graph of a "de Bruijn" network with N nodes. Thus a "de Bruijn" network with N nodes can simulate in constant time a shuffle-exchange network with 2N nodes. It follows that the variety of algorithms that have been implemented on shuffle-exchange networks (see [7]), and in particular routing algorithms, can be implemented efficiently to run on the "de Bruijn" network. Moreover, as noted in [4] and [8], this network consists essentially of two binary trees, where nodes that are the leaves of one tree are internal nodes of the second tree. Thus, tree algorithms can also be implemented efficiently on this structure. This also implies that the graph of the "de Bruijn" network is biconnected, thereby providing some amount of redundancy.

The relation between the shuffle-exchange network and the "de Bruijn" network has already been observed by several authors, in particular [8], where "de Bruijn's" network is simply called shuffle-exchange and [3] where it is called the 4-pin shuffle. The graph of "de Bruijn" network also provides for the underlying topology of recirculating interconnection networks with shuffle connections (see [1], where this network is again called shuffle-exchange network). The relation of these networks with the shuffle-exchange network of Stone on one hand, and with the multistage Ω-network of Lawrie [5] on the other hand are well known.

Actually the term "shuffle-exchange network" seems to be applied indiscriminately in the literature to any network using "perfect shuffle" connections, multistage networks included [6]. While this terminology may be confusing, it has the merit of underlining the basic similarity between these networks, a similarity which apparently has been overlooked by Goodman and Sequin.

REFERENCES

1. P-Y. Chen, D.H. Lawrie, D.A. Padua and P-C. Yew,"Interconnection networks using shuffles",Computer, Vol. 14, No. 12, December 1981, pp. 55-64.

2. R.A. Finkel and M.H. Solomon,"The lens Interconnection Strategy",IEEE Trans. on Computers, Vol. C-30, No. 12, December 1981, pp. 960-965.

3. J. P. Fishburn and R.A. Finkel,"Quotient networks", Manuscript.

4. J.R. Goodman and C.H. Sequin,"Hypertree: A multiprocessor interconnection topology",IEEE Trans. on Computers, Vol. C-30, No. 12, December 1981, pp. 923-933.

5. D.H. Lawrie,"Access and alignment of data in an array processor",IEEE Trans. on Computers, Vol. C-24, No. 12, December 1975, pp. 1145-1155.

6. D.S. Parker,"Notes on shuffle/exchange -type switching networks",IEEE Trans. on Computers, Vol. C-29, No. 3, March 1980, 213-222.

7. J.T Schwartz,"Ultracomputers", ACM Trans. on Programming Languages and Systems, Vol. 2, No. 4, October 1980, pp. 484-521.

8. D. Steinberg and M. Rodeh, "The dual binary tree: A structure for the pipelined evaluation of tree-like expressions", Tech. Report, Weizman Institute, Rehovot, Israel, 1980.

9. H. Stone, "Parallel processing with the perfect shuffle", IEEE Trans. on Computers, Vol. C-20, No. 2, February 1971, pp. 153-161.