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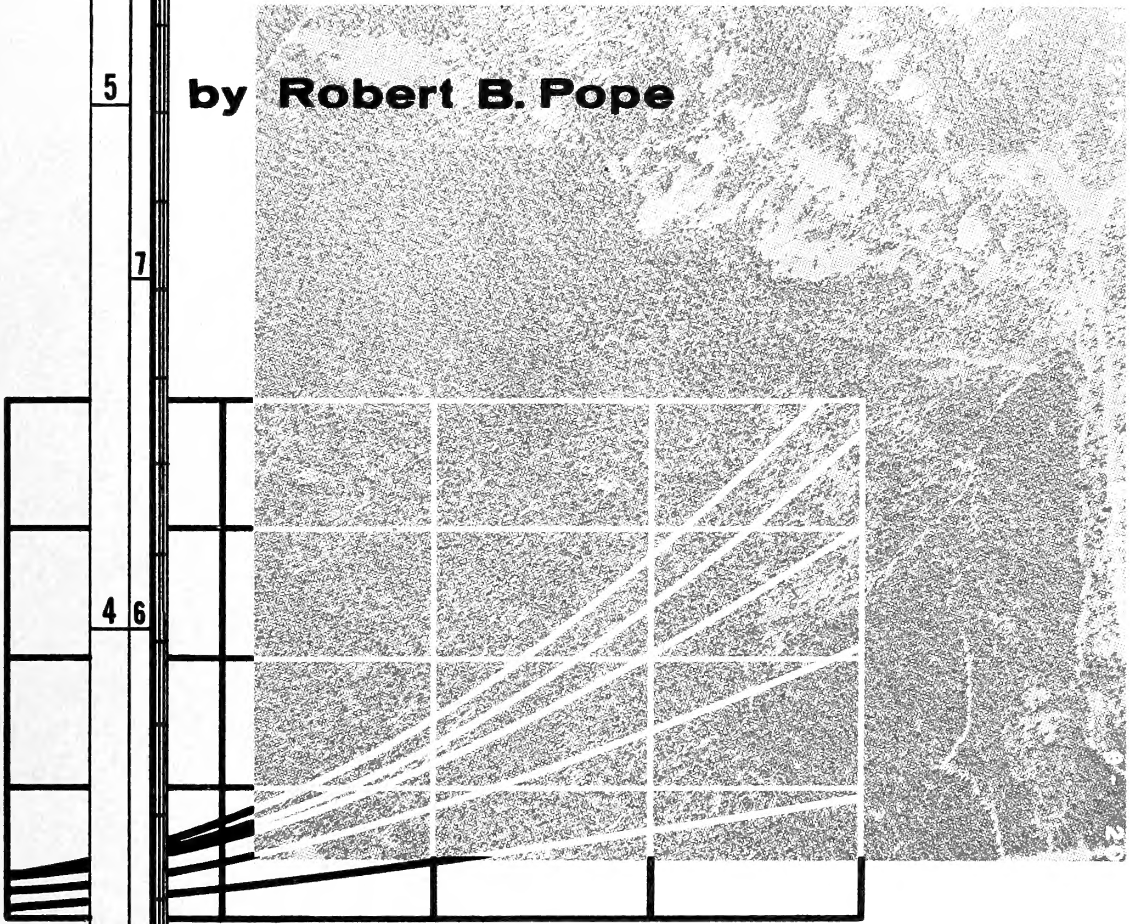
CONSTRUCTING
Aerial Photo Volume Tables

by Robert B. Pope



PACIFIC NORTHWEST
FOREST AND RANGE EXPERIMENT STATION
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INTRODUCTION

Although most foresters are familiar with the use of aerial photo volume tables, little has been written on how to make them. Certain pitfalls in the construction process have either been ignored or only casually mentioned in the existing literature. The forester tackling his first photo volume table is likely to bypass some of the important considerations without being aware of their existence. This does not prevent the tables from being useful, but it is our belief that better tables will result if the major issues are brought out into the open where they can be faced squarely.

Such is the primary purpose of this paper. It describes in detail the methods used to construct an aerial photo volume table for Douglas-fir stands. A special effort has been made to bring out the problems and explain our solutions, giving the assumptions made, the alternatives explored, the results of special investigations, and the reasoning used in making decisions. Routine construction steps are also included in order to make the process complete.

It is our hope that the procedures described will serve as a guide for the construction of aerial photo volume tables, especially for those who are tackling the job for the first time. An additional purpose will be served if others who have met different problems, or have better solutions to the same problems, are encouraged to come forth.

In concept, making an aerial photo stand volume table is quite simple. You must establish a series of sample plots, and from them determine the relation between stand volume (the dependent variable) and certain stand characteristics which can be measured on photos (the independent variables). Then, using these relations, you can construct the tables which show the stand volumes to be expected for given values of the photo-measured stand characteristics.

In making the Douglas-fir volume tables, we chose the procedure of multiple regression analysis as being the most acceptable of several possible methods. For the purpose of describing what was done and the problems that were encountered, the procedure has been divided into the following steps:

(1) Selection of the dependent variables.

(2) Selection of the basic independent variables.

(3) Choice of definitions for variables and methods of measuring the variables.

(4) Collection of data, or measurement of the variables.

(5) Preliminary graphic analysis to determine which independent variables, including the basic ones plus their squares and cross products, should be included in the analysis.

(6) Multiple regression analysis to determine the relation between each dependent variable and all possible combinations of independent variables.

(7) Selection of the regression equations that best predict volume.

(8) Compilation of the volume tables.

(9) Checking and testing the tables.

SELECTING THE DEPENDENT VARIABLES

At first glance, selecting the dependent variables appears to be a simple task having no complications. Volume per acre is obviously what we are after, and it is apparent that this must be measured in the field. It would seem that the only remaining decision is that on merchantability standards. However, there is a problem hidden here--one that apparently has seldom been recognized.

Most aerial photo volume tables are made in terms of both cubic feet and board feet. A logical approach is to make two separate regression analyses, one in which the dependent variable is cubic-foot volume, and the other, board-foot volume. However, experience has shown that when the two tables are made independently, and then compared side by side, strange effects sometimes show up. Due to different shapes of the curves in the two tables, the resulting board-foot/cubic-foot ratios may form an erratic and illogical pattern. A study of existing volume tables shows that this has happened, whether the construction method is freehand curve fitting, alinement chart, or multiple regression analysis.

Such irregularities between the board-foot and cubic-foot volumes of given stands don't prevent the volume tables from being useful. However, we reasoned that it would be desirable to have the board-foot/cubic-foot ratios follow a pattern in close agreement with that of the basic data.

There appears to be a logical way to accomplish this. First, use cubic-foot volume as the dependent variable and construct a cubic-foot table. Then make a second analysis, using board-foot/cubic-foot ratio as the dependent variable. The resulting ratios could then be applied to the cubic-foot table to produce the board-foot table.

We explored this possibility but discovered that using a ratio in a regression analysis was subject to a bias. The average of a group of ratios is not the true ratio for the averages. Because of this, we returned to our first approach of making two independent regression analyses--one for board feet and one for cubic feet. But we did so with full realization of the risk involved and of the desirability for scrutinizing the two tables to see that the resulting board-foot/cubic-foot ratios made sense when compared with those from the basic data.

SELECTING THE BASIC INDEPENDENT VARIABLES

The selection of the basic independent variables was straightforward, with no complications. There are only a few stand characteristics which can be measured on aerial photos. Our own past experience, confirmed by that of others, indicated that stand height was the best predictor of stand volume and that crown closure was generally useful. Average crown diameter has occasionally contributed to the prediction of stand volume, but more often it has not. We decided to include it in the regression analysis, since one of our aims was to learn more about how these variables operate.

For this same reason, we introduced another variable--site index. Actually, this is not a practical independent variable, for we don't yet know how to measure it on the photos with acceptable precision. However, some of our past work had suggested that site had an influence on the volume equation, and it deserved to be tested.

DEFINING THE VARIABLES

Now it was necessary to establish a specific definition for each variable, dependent as well as independent. To do this we had to specify exactly what part of the stand was to be measured for each variable, then decide which variables were to be measured in the field and which on photos. This step in the volume table construction procedure has not always been given sufficient attention. The definitions must be established at the time the tables are made. And it is important that they be passed along to the photo interpreter so that in using the tables he can apply the same standards that governed their construction.

STAND VOLUME

One of the immediate uses planned for the Douglas-fir tables was to improve the volume statistics collected by the Forest Survey, a nationwide inventory conducted by the U.S. Forest Service. Therefore, we adopted

Forest Survey standards for the definition of stand volumes. These standards specify the minimum tree size, merchantability limits, log lengths, and tree volume tables to be used. Obviously the measurements had to be made on the ground, and to complete the definition of volume, only one decision remained--whether to make the tables in terms of gross volume or net volume.

There are arguments in favor of both approaches, and neither seems to have a clear superiority. Gross volume tables are especially appropriate when photo interpretation alone is being used for reconnaissance or "quickie" estimates of local areas. The gross volumes thus obtained can be reduced for the defect anticipated in the survey area. Gross volume is also the logical choice whenever the interpreter has a reasonable basis for judging the amount of defect on individual plots. However, it should be remembered that there are situations where net volume tables may be the best approach. For instance, on extensive inventories over large areas, where the interpreter has no basis for judging defect, the simplest and most direct procedure is to let net volume tables take care of the defect automatically. After reviewing the possible uses of the tables, we chose gross volume as being generally more useful than net volume.

STAND HEIGHT

A variety of definitions for stand height have been used in existing photo volume tables. The trees to be used in determining the average stand height have often been either the dominants and codominants or the tallest few on a given size of plot. Some definitions have specified that the heights be measured on the photos, and others that they be measured in the field. Because of this lack of accepted standards, we decided to investigate both the definition of the trees to be measured and the alternate ways of measuring them.

We first examined the height definitions which have been used in the past and selected two that looked promising--the average of the dominants and codominants and the three tallest trees on a 1/5-acre plot. These were then tested to determine which was most likely to accurately predict stand volume.

The basis used for this test was the coefficient of correlation between stand height and stand volume. Using photo measurements as well as field measurements from 282 plots, these correlations were computed for both definitions of stand height. This gave the following results:

	Basis for correlation		
	Field measurements	Photo measurements	
		Interpreter A	Interpreter B
	(Correlation coefficient)		
Stand height definition:			
Average, dominants and codominants	0.84	0.80	0.81
Average, three tallest on 1/5 acre	.87	.79	.80

When field measurements were used, the average height of the three tallest trees was more closely correlated with stand volume, by a slight margin, than was the average of the dominants and codominants. However, in applying aerial photo volume tables, heights must be measured on the photos. Hence, a more realistic appraisal of the two stand height definitions was obtained from the correlation of volume with photo-measured stand height. The above results indicate that, from the practical standpoint of estimating volumes on photos, stand heights based on either the dominants and codominants or the three tallest trees were equally useful. But which should be used? We decided to stick to the one we have successfully used in the past--the average height of the dominants and codominants.

Should these stand heights used in constructing the volume table be measured on the photos or in the field? Since the tables are applied by using photo-measured heights, it seemed that the simple and direct approach was to use photo measurements in making the tables. This procedure would be desirable if photo height could be measured consistently by different interpreters and by the same interpreter on different types of photos. However, there is considerable evidence in the literature that stand height measurement on aerial photos is subject to a bias that varies with interpreter, photography, and various stand characteristics.

To obtain additional data on this question, we analyzed a number of photo height measurements made by five interpreters. These measurements were definitely biased, and this bias differed significantly among interpreters (*5*).[1] This might not create a serious problem if a field sample is used to adjust the photo volume estimates and to remove the bias. However, sometimes it is desirable to make volume estimates from the photos alone, without any field checks. Under these conditions, a photo volume table based on field-measured heights is more useful than one obtained from photo measurements. The interpreter using the table can adjust his height measurements to a field basis, but he cannot adjust to some other interpreter's unknown photo measurements. For this reason, we decided field-measured stand heights were preferable in volume table construction.

CROWN CLOSURE

Crown closure as defined by most existing aerial photo stand volume tables is the proportion of the area occupied by the crowns of the dominant and codominant trees or the major canopy. In a few volume tables, crown closure estimates have been restricted to trees which exceed an arbitrary height, assumed to represent the minimum of merchantability. The first approach makes more sense in even-aged stands. An interpreter faced with such a stand at just about the merchantability limit would surely find it impossible to

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[1] Italic numbers in parentheses refer to Literature Cited, p. 25.

separate the trees just over the limit from those just under it. On the other hand, the concept of merchantability limit seems well suited to uneven-aged stands. Since the Douglas-fir tables were to be for essentially even-aged stands, we decided to base our crown closure estimates on all trees in the major canopy, rejecting occasional trees well below this.

Crown closure is commonly measured or estimated on the aerial photos, although some workers have measured it on the ground. Past experience convinced us that the man on the ground has extreme difficulty trying to estimate what the interpreter is likely to see. Since the field method is also much more costly, there is no incentive for making crown closure measurements this way. Consequently, our crown closure estimates were made on the photos.

AVERAGE CROWN DIAMETER

The trees considered in determining the average crown diameter logically were the same ones used for average stand height, the dominants and codominants. We chose to measure crown diameter on the photos, rather than in the field, for the same reasons used in deciding on crown closure.

SITE INDEX

There were no problems in determining how to define or measure site index. Standard site curves for Douglas-fir are based on the average height of the dominant and codominant trees at 100 years, and this had to be measured on the ground.

COLLECTING THE DATA

Then came the job of collecting the necessary data for the volume table construction. Basically this consisted of establishing a number of plots; visiting each in the field to measure volume, stand height, and site index; and examining each on aerial photos to estimate crown closure and average crown diameter.

Although the procedure seemed simple enough, there were many decisions that had to be made about the details of application.

PLOT LAYOUT

There are no precise answers to the questions of which size, shape, number, and distribution of plots is best for photo volume table construction. Existing plots can be used if they yield the necessary information. In our case special plots were used, which necessitated setting up our own specifications for laying them out.

We chose a circular 1/5-acre plot to use both on the photos and in the field. It was large enough to see on 1:12,000-scale photos and small enough to measure easily in the field. Photo and field plots of different size or shape are a disadvantage because they introduce an additional component of variation. This tends to reduce the variation in field plot volumes that will be accounted for by the independent variables.

We knew of no way to determine how many plots were needed to make a satisfactory photo volume table. Since we wanted to answer several questions as well as to produce a useful volume table, we estimated that between 200 and 300 plots would be desirable. With this goal in mind, in one field season we established 282 plots. No doubt fewer plots would be acceptable if the only purpose was to make a workable photo volume table.

The locations for the plots had to meet several conditions. Basically, we were interested in determining whether one volume table would suffice for all of western Oregon and Washington, or whether several tables were needed. Three logical geographic areas were selected, and an effort was made to get about the same number of plots in each. Within each of these areas, a number of locales were selected for the sample plots. These were limited largely by the availability of recent 1:12,000 photos. Different ages and scales of photography would only have complicated our work.

Within these locales, we picked plots that were in relatively even-aged Douglas-fir stands without logging disturbance. The actual plot locations were chosen subjectively rather than at random or systematically. This gave us a number of important advantages. Our samples could be placed in accessible areas to avoid unnecessary travel time. The plots could be selected to meet our qualifications for type and age structure. Plots could be limited to those positively located on the photos. And a wide range of height and density classes could be sampled without the concentration about the mean that comes with random sampling.

We recognized that along with these advantages of purposive plot selection came some risk of getting a biased sample. However, this chance seemed slight. Within a given height, density, crown diameter, and site index class, it was hard to visualize how we could pick plots with above- or below-average volumes.

FIELD TALLY

Our objectives in collecting field plot data were to determine the gross volume by Forest Survey standards, the average height of the dominants and codominants, and the site index. The following plot procedure met these objectives.

Having selected the plot location and checked to see that it met the required specifications, we marked the plot center on the ground with a temporary stake and on the photo with a pinprick. Then we measured the total height of two dominant trees and bored them to obtain site index. Next, we measured the heights of several other trees, including at least one codominant and one or two in the intermediate or suppressed classes. This provided us with from four to six measured trees to serve as "yardsticks" for the ocular estimation of total height on the remaining trees.

All live sawtimber trees were tallied on the 1/5-acre circular plot, and all live poletimber trees on a 1/20-acre circular plot. The distance to any questionable tree near the plot boundary was measured. Trees were recorded by species, 2-inch d.b.h. class, and 10-foot height class. Small trees were measured with a Biltmore stick and large ones with a diameter tape. Heights were estimated by comparing the tally trees with the measured "yardstick trees." After the tally was complete, the heights of the dominant and codominant trees were averaged and recorded on the plot card.

PHOTO INTERPRETATION

To estimate the crown closure of the major canopy and the average crown diameter of the dominants and codominants, two experienced interpreters examined each plot on the photos. Crown closure estimates were based on standard guides and ocular procedures described in a previous publication (6). Average crown diameter was determined by measuring a number of dominant and codominant trees, using a micrometer wedge or a dot scale.

The crown closure estimates of the two interpreters were within 10 percent of each other 85 percent of the time; their crown diameter estimates were within 5 feet of each other 80 percent of the time. For those plots, the two estimates were averaged for use in the regression analysis. Where the initial estimates differed by more than these amounts, additional interpreters were called in. In some cases, the original interpreters made second estimates. Our whole idea here was to get several estimates close enough to provide a reasonable average.

MULTIPLE REGRESSION ANALYSIS

It is possible to construct aerial photo volume tables by freehand curve fitting or by the alinement chart method. Although these procedures produce useful tables, they are quite subjective, and they provide no ready means for determining the relative contribution of each variable to the estimation of volume.

Mathematical curve fitting by multiple regression analysis essentially eliminates these disadvantages. It has not been used much in the past because of the magnitude of the job when performed on a desk calculator. Recently, however, electronic machine programs have become available so that the analysis can be made quickly and at a reasonable cost. Thus, with former obstacles now removed, multiple regression analysis has become by far the most feasible method of solving problems of relationship, as in aerial photo volume tables.

The first machine program that came to our attention was the Southern Forest Experiment Station's IBM 704 program (4). It accepts up to nine independent variables and produces all 511 possible equations relating one or more of these to the dependent variable. The amount of the total variation in the dependent variable accounted for by each of these equations is also furnished and serves as a basis for selecting the best equations.

Making use of this program for constructing our photo volume table involved these steps: (1) preliminary plotting of the basic data to determine the form of the relationship between the dependent and independent variables, (2) testing for homogeneity of variance, (3) running the data through the electronic computer, and (4) selecting the final volume table equations from the computer output.

DETERMINING THE FORM OF THE INDEPENDENT VARIABLES

We now had all the necessary data on the dependent variable, volume, and the four basic independent variables initially chosen: stand height, crown closure, average crown diameter, and site index. But before conducting a multiple regression analysis, it is generally wise to make a freehand graphic analysis of the data. This answers two important questions which guide the final selection of independent variables.

The first of these questions is: Are each of the basic independent variables worth including in the analysis? Some guesswork went into their initial selection. Now we need to examine the data to see that each independent variable is actually related to the dependent variable. With a machine program that limits the number of variables, we don't want to include any without a reasonable chance of being effective.

The second question is: Is there any evidence of curvilinearity between dependent and independent variables, or of interaction between various independent variables? Past experience has indicated that both of these situations may occur with data of this type. If only the first power of an independent variable is used in the regression analysis, then the assumed relationship with the dependent variable is linear, which may not be in keeping with the true relationship. However, if we add the square of the independent variable, then the relationship can assume the form of a second-degree curve, which may fit the data better. Variation in the dependent variable accounted for by this squared independent variable provides the basis for deciding if the curvature is statistically significant. If there is room for many independent variables, we could add higher powers of the basic ones to provide for more complex curve forms. However, with data of this type such complex curves are seldom more effective than the simpler second-degree curve.

Interaction between independent variables means that the relationship between the dependent variable and one of the independent variables changes with different values of a second independent variable. Past experience has indicated that this is likely with photo volume table data. For instance, we would expect the amount of stand volume added by a 10-percent increase in crown closure to be much greater for 200-foot stands than for 100-foot stands. Variation in stand volume due to such an interaction can often be accounted for by introducing the product of the two interacting variables as an additional independent variable in its own right.

Our objective, then, in making a graphic analysis of the basic data, was to select the nine independent variables most likely to be effective from among the four basic variables, their squares, and possible cross products. Since we suspected that stand height and crown closure were the most important variables, we began by sorting the basic data into stand-height and crown-closure classes. Then we plotted stand volume over these variables as shown in figure 1.

The definite slope to the lines substantiates our original assumption that both stand height and crown closure have an influence on volume and should be included in the regression analysis. The tendency of volume over height to curve suggests that the square of height should be included as the third independent variable. Evidence on the curvilinearity of volume over crown closure was inconclusive, but, because of the importance of this variable, we decided to include the square of crown closure as the fourth independent variable. The changing slope of the lines in each graph of figure 1 causes them to fan out and is evidence of the interaction between stand height and crown closure. Therefore, the product of these two basic variables became our fifth independent variable.

We also attempted to learn in advance about the effect of crown diameter and site index on volume, within height and density classes. The data became rather weak when divided into so many classes, and results were somewhat erratic. However, there was some evidence that site index, and possibly crown diameter, was influencing volume. Hence, we felt justified in using these two variables in the regression analysis in their simple linear form. It did not seem worth while to include their squares.

There was little scientific evidence to guide us in selecting the remaining two independent variables. We decided to include height (squared) times crown closure because Avery and Myhre (1) had found this variable useful. As our final variable, we chose a similar one, crown closure (squared) times height.

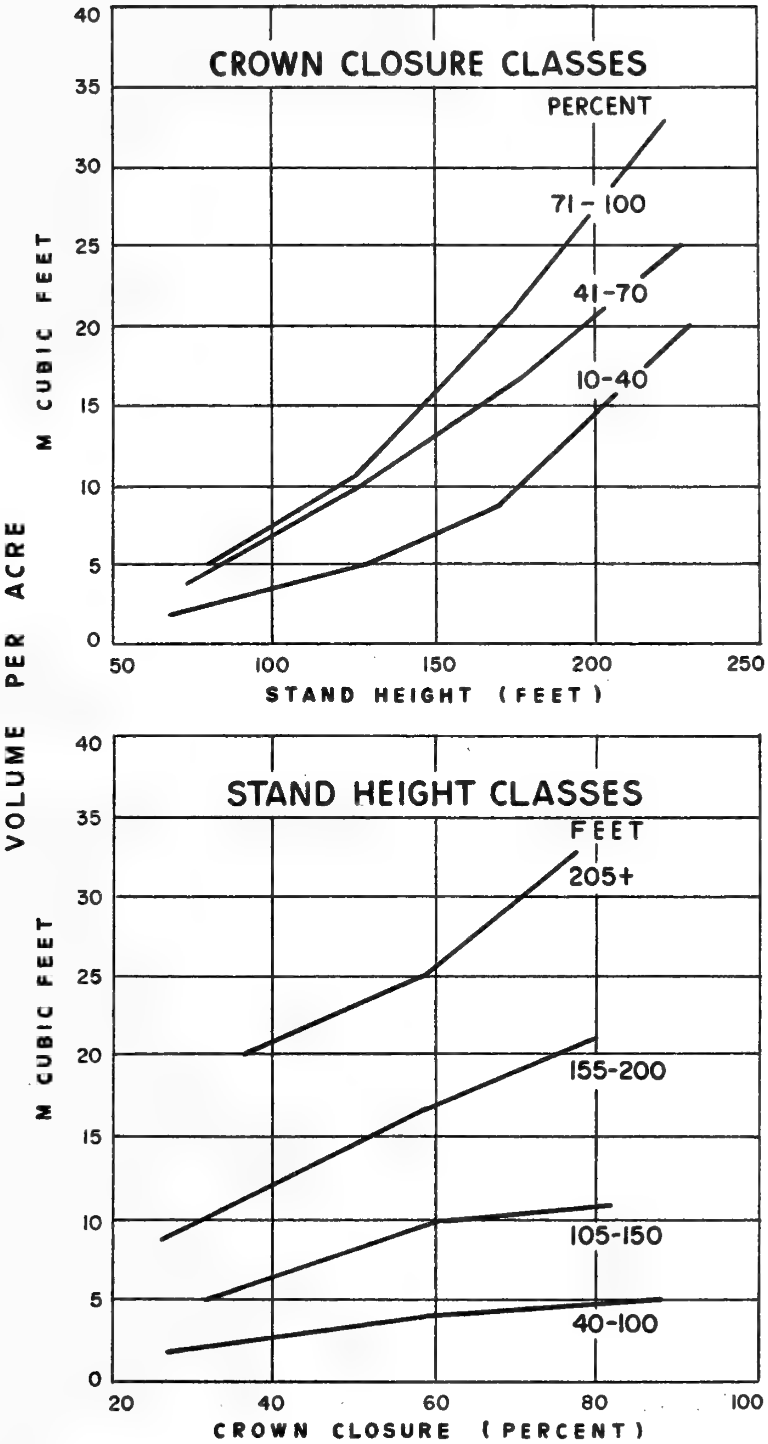

Figure 1.--Relation of stand volume to stand height and crown closure (plotting of basic data).

TESTING THE HOMOGENEITY OF VARIANCE

The general process of least squares curve fitting gives equal weight to each observation on the assumption that the variance of the dependent variable is constant for all values of individual independent variables. If this variance is not homogeneous, then a proper curve fit requires that the observations be weighted, with more weight being given to those classes having low variance.

This aspect of regression analysis is commonly ignored, and has apparently never been considered in connection with aerial photo volume table construction. Our past experience with regression analysis for producing local volume tables had shown us that volume data seldom have homogeneous variance. Failure to heed this required condition can lead to errors in curve fitting. It seemed worth while to investigate this problem in connection with photo volume tables, even though it might cause difficulty.

To check on the homogeneity of variance, we computed the volume variance for each of the stand-height and crown-closure classes into which the data had been sorted. Then we plotted this volume variance over stand height and over crown closure to see if there was a relation. There was no indication that volume variance was influenced by crown closure, but the data shown in figure 2 leave no doubt that volume variance is not homogeneous with respect to stand height. The variance of the taller stands is over 150 times that of the shorter stands!

What this means is that data for the shorter stands, having less variance, are more reliable and should be given more weight than data for the taller stands. A method for accomplishing this is to weight each observation by the inverse of the volume variance for its height class. The electronic computer program we planned to use had no provision for such weighting. Without this, we faced a choice of two other methods for weighting the data. One was to do it by hand, multiplying the value for each variable on a plot by the appropriate weight for that plot. The other method was to duplicate extra punch cards for each plot in proportion to its weight. Neither way looked appealing. The hand weighting was laborious and the extra punch cards would increase our original 282 observations to over 7,000, exceeding the capacity of the computer and requiring a two-stage analysis. Was it worth the extra effort and expense?

In an effort to partially answer this question, we took a portion of our data and fitted second-degree curves of volume over height, both unweighted and weighted by the inverse of variance. The two curves were not very different. Hence, we went ahead with an unweighted solution to the regression analysis, feeling reasonably sure that we were not getting into serious trouble. However, this problem cannot be dismissed, and in future photo volume table studies we plan to carry our investigation further. A solution to the problem is now in sight, since at least one program for electronic computation of multiple regression analyses now provides for weighting observations.

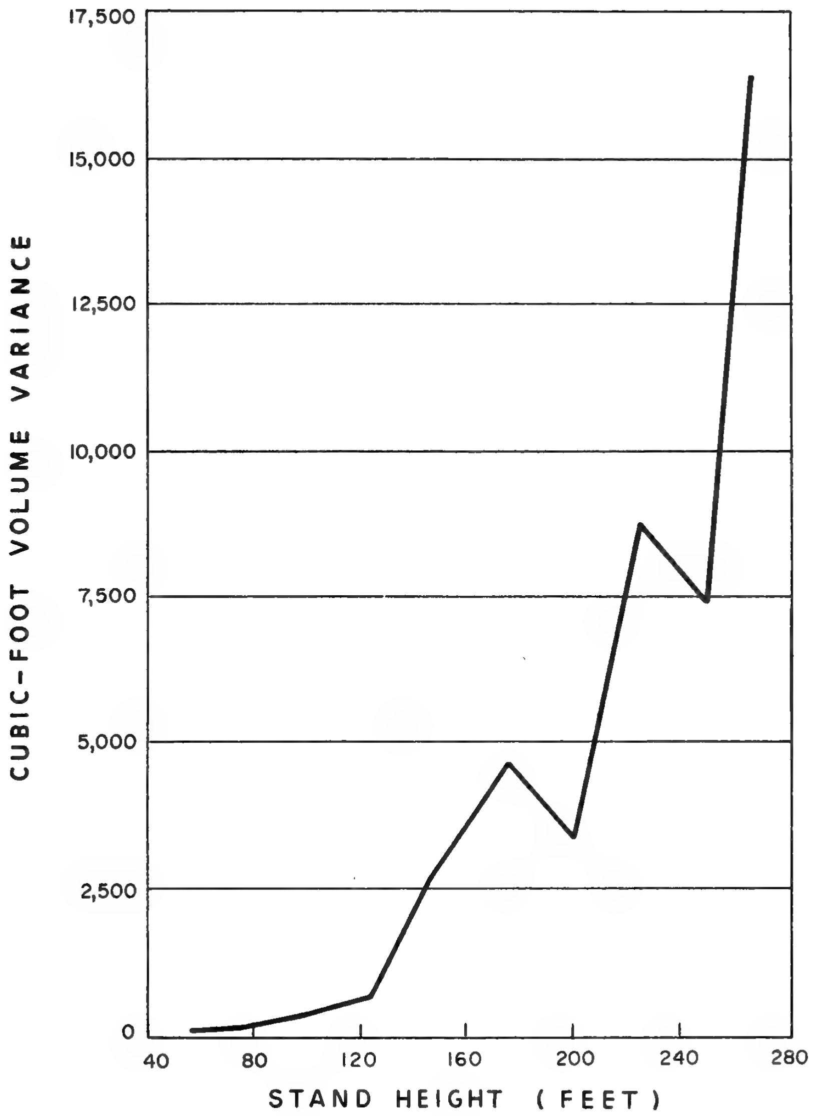

Figure 2.--Relation of volume variance to stand height.

One more question needed answering before the regression analysis could proceed. The data had been collected in three geographic areas. Should we make separate volume tables for each or pool all data to make only one? The elegant solution to this problem was covariance analysis. However, available machine programs for such an analysis would not give us the complete solutions for all possible equations--information we considered important.

Therefore, an empirical approach was used to answer this question. We simply took our basic data for each height and density class, sorted it into geographic areas, and compared mean volumes. Since there was no indication that the areas had any influence on volume, all data were pooled to make one set of tables. The validity of this decision was further verified after the volume tables had been constructed, and the differences between tabular and field volumes were sorted by geographic areas. Again, there was no evidence that a particular area tended to have volumes different from the average of all areas.

SELECTING THE BEST EQUATION

All decisions necessary for the multiple regression analysis had now been made. Therefore, we put the values for the variables on punch cards, one card to a plot, and sent them off for processing. We were aware that the strict mathematical model for this analysis had been violated in several respects. However, investigations of these discrepancies had given us reasonable assurance that the results would still be meaningful; alternative procedures seemed to present intolerable difficulties at this time.

The results of the analysis were returned to us as two rather imposing sheets of figures, each 22 feet long--one sheet for board-foot volumes and the other for cubic-foot volumes. Each sheet contained 511 formulas for predicting stand volume. These formulas covered all possible sets that could be made from the nine independent variables. There were 9 formulas with a single independent variable, 36 formulas with two variables, and so on, up to 1 formula involving all nine variables.

Also shown on these output sheets was the amount of volume variation accounted for by the particular set of independent variables in each of the 511 formulas. This provided the basis for selecting the best prediction equation and for judging the usefulness of each independent variable.

While the process for selecting the best equation is supposed to be well known, we were unable to find a clear statement of it. So we adopted our own approach which follows a series of logical steps. First, for each number of independent variables, we selected the equation which accounted for the most variation. This gave us nine equations: one with a single variable, one with two, and so on, up to the one with all nine variables. From these we chose as "best" the one with the largest number of independent variables which was significantly better than the one with one less independent variable. In other words, we started out with the nine-variable solution and tested to see if a significant loss occurred in dropping to the best eight-variable solution. This process of dropping variables was continued until a significant loss occurred. The previous equation then became the one selected as "best."

This procedure was relatively simple and led rather quickly to the one "best" equation from the 511 solutions. However, there were several practical considerations which made it advisable to modify the method. Photo measurements of the different independent variables are not all made with equal ease or expense. Crown closure is quickly and easily estimated on aerial photos. Stand height and average crown diameter are more time consuming, requiring the determination of photo scale and a number of measurements which increase cost. On the other hand, some of the independent variables are free. For instance, if height is used as an independent variable, then the square of height can be used as a second variable at no extra cost.

Another practical consideration that influenced our selection of the best equation was our desire to make sure that the two volume tables, one in cubic feet and one in board feet, were coordinated. That is, we didn't want the board-foot/cubic-foot ratios to jump around irrationally, as they might if the board-foot and cubic-foot equations were selected independently.

An examination of the processed data showed that site index, as measured on the ground, was indeed an influential variable for the prediction of stand volume. It was present in the equations selected as "best" for both cubic feet and board feet by the strict statistical procedure which ignores the cost of the variables. In the cubic-foot equations, it was present in the "best" from each group of variables down to and including the two-variable equation. In the board-foot solutions, site index was present in each "best" equation down to and including the four-variable equation.

Although it was apparent that precise estimates of site index would improve our estimates of per-acre volume, previous studies (2, 7) had shown that such estimates based on the photo measurement of physiographic features were not precise enough to be useful for this purpose. We could see only two other choices for getting site index--ocular estimates or the use of existing site maps.

Smith and Bajzak (7) had shown that an ocular estimate of site index by an interpreter intimately familiar with the area was moderately successful. But because of the subjectivity of this approach, we decided against it for the time being.

Small-scale generalized site maps existed for our region, and these could be used to furnish an estimate of site index for each plot or stand sampled. To test this relatively crude approach, we made two aerial photo volume tables--one using site index as a variable and one without it. Both of these tables were used to estimate volumes on the 282 plots used in the table construction. The volume estimates were based on photo measurements of stand height and crown closure by two interpreters, and a site classification was made from the site maps. The photo volume estimates were no better using site as a variable than they were without it. Hence, we eliminated site index from further consideration and selected our final equations from those without it.

When we examined the remaining equations, containing only the customary variables of stand height, crown closure, and crown diameter, plus combinations of these, several interesting facts emerged. Most equations accounted for a substantial amount of the variation in plot volumes. Furthermore, there was a relatively small difference in the volume variation accounted for between the equation containing all eight variables and the best single-variable equation, which was height (squared) times crown closure (H^2C).

	Variation accounted for	
	Cubic feet	Board feet
	(Percent)	(Percent)
Equation:		
Best single-variable (H^2C)	80.8	80.0
All eight variables	82.0	84.7

The large percentage of volume variation accounted for by these equations was encouraging. The multiple correlation coefficients ran between 0.90 and 0.92. It was apparent that these equations would predict stand volume with a relatively high degree of precision. However, it must be remembered that stand height was measured in the field, and crown closure and crown diameter were averaged from several interpreters. Use of an individual interpreter's photo measurements as independent variables would probably result in lower correlation coefficients.

The fact that the best of the single-variable equations accounted for nearly as much variation as using all eight variables suggested an interesting possibility. A single-variable regression analysis is easy to do on a desk calculator. Thus, it is possible that others who are interested in constructing aerial photo stand volume tables, but don't want to make a multiple regression analysis, could obtain a satisfactory solution by using the single variable we found best--H^2C.

Perhaps this particular variable was useful only for our Douglas-fir data. In order to see how well it worked on a completely different forest type, we tried it on Gingrich and Meyer's (3) data for upland oak in Pennsylvania. These workers constructed an aerial photo volume table by multiple regression analysis. They concluded that the best equation contained two variables, crown closure and the product of height and crown closure. This solution produced a multiple correlation coefficient of 0.85 for the cubic volume in trees

5 inches and larger and 0.87 for trees 7 inches and larger. Since Gingrich and Meyer reported their basic observations on the 93 plots they used in their multiple regression analysis, we were able to make another analysis employing the single variable, H^2C. When we did this, the multiple correlation coefficients were 0.86 for both utilization standards. Hence, this single-variable equation predicts oak volume to the same degree of precision as the equation selected by Gingrich and Meyer. While further testing needs to be done, this provides some encouragement that a single-variable regression, using H^2C, might prove to be a quick and easy way to construct aerial photo stand volume tables.

Further examination of the output data from our multiple regression analysis showed us the relative importance of the three basic variables which are measured or estimated on the photos--stand height, crown closure, and crown diameter. The following tabulation shows the percentage of volume variation accounted for by the best equations based on only a single stand characteristic, any two such characteristics, and on all three:

	Variation accounted for	
	Cubic feet (Percent)	Board feet (Percent)
Best equation containing:		
Stand height only	72.0	78.9
Crown closure only	12.7	13.4
Crown diameter only	22.9	30.4
Stand height and crown closure	81.8	84.3
Stand height and crown diameter	73.6	79.1
Crown closure and crown diameter	47.7	48.6
Stand height, crown closure, and crown diameter	82.0	84.7

From this it can be seen that if we desired to base a photo volume table on only one stand characteristic, stand height is likely to be the best by a great margin. This margin is exaggerated somewhat because height was measured in the field. When measured on the photos, the inevitable measurement errors would result in height being somewhat less useful than shown by the above figures. However, this difference shouldn't be very much, for the coefficients of correlation between photo and field heights ran 0.93 and 0.95 for two interpreters' measurements.

If we wished to base our volume table on two stand characteristics, the obvious choice would be stand height and crown closure. And this combination is substantially better than stand height alone. The addition of crown diameter to stand height and crown closure adds a negligible amount to the usefulness of the prediction equation, although this amount is statistically significant in the case of board-foot volumes. It costs money to measure average crown diameter on the photos, and the small gain would not be worth the expense on most forest inventories.

Hence, our volume tables were based on stand height and crown closure only. But how many of the independent variables containing combinations of height and crown closure should be used? Altogether there were seven: H, H^2, C, C^2, HC, H^2C, and HC^2. An analysis showed that only a few of these contributed significantly to the prediction of stand volume. However, even the nonsignificant variables should be considered since their inclusion in the volume table would not add anything to the cost of using the tables. The amount of volume variation accounted for, whether we used only the few significant variables or all seven, was not very much different. Thus, for all practical purposes, we could take our choice from among dozens of possible equations containing various combinations of these variables and get about the same result. After trying several likely ones for both board feet and cubic feet and comparing them for rational board-foot/cubic-foot ratios, we selected the following two equations to express the photo volume table relationships:

$$V_c = 0.9233HC + 0.0070H^2C - 0.0086HC^2 - 179$$

$$V_b = 0.9533C^2 + 3.2313HC + 0.0716H^2C - 0.0883HC^2 - 3285$$

Where:

V_c = Gross per-acre volume, cubic feet

V_b = Gross per-acre volume, board feet

H = Average height of dominants and codominants, feet

C = Crown closure of major canopy, percent

CONSTRUCTING THE VOLUME TABLES

With the regression equations chosen, the volume table construction was a simple job of solving the equations for selected intervals of the independent variables. We chose 10-foot height classes and 10-percent crown-closure classes, intervals commonly used in existing photo volume tables. Height classes ranging from 5 to 20 feet and crown-closure classes between 5 and 15 percent can probably be used without any practical effect on the usefulness of the tables. Tables 1 and 2 show volumes resulting from the chosen regression equations.

Table 1.--Aerial photo stand volume table for even-aged Douglas-fir in the Pacific Northwest

(In hundred cubic feet per acre)[1]

Stand height[2] (feet)	Crown closure (percent)[3]								
	15	25	35	45	55	65	75	85	95
40	5	8	11	13	14	15	15	14	13
50	7	11	15	18	20	21	22	21	20
60	9	15	20	24	27	29	30	29	28
70	12	19	25	31	34	37	39	39	38
80	14	24	31	38	43	46	48	49	49
90	17	28	38	45	52	56	59	61	61
100	21	33	45	54	61	67	72	74	75
110	24	39	52	63	72	79	85	88	90
120	28	45	60	73	83	92	99	103	106
130	31	51	68	83	95	106	114	120	124
140	35	57	77	94	108	121	130	138	143
150	40	64	86	105	122	136	148	157	163
160	44	71	96	118	136	153	166	177	185
170	49	79	106	130	152	170	185	198	208
180	54	87	117	144	168	188	206	220	232
190	59	95	128	158	184	208	227	244	257
200	64	104	140	173	202	228	250	269	284
210	70	113	152	188	220	249	274	295	313
220	75	122	165	204	239	271	298	322	342
230	81	132	178	220	259	293	324	351	373
240	87	142	192	238	280	317	351	380	406
250	94	152	206	256	301	342	379	411	439
260	100	163	221	274	323	367	408	443	474

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[1] Gross volume, in trees 5.0 inches and larger, from stump to top limit of 4.0 inches d.i.b. Volume tables from U.S. Dept. Agr. Handb. 92.

Equation for table: $V_c = 0.9233HC + 0.0070H^2C - 0.0086HC^2 - 179$

Where: V_c = volume, cubic feet per acre; H = stand height in feet; and C = crown closure in percent
Multiple correlation coefficient (R) = 0.904
Standard deviation around regression or standard error of estimate = 3,777 cubic feet per acre, or 29.2 percent of the mean plot volume.

Based on 282 1/5-acre plots, largely in western Oregon.

[2] Average height of dominants and codominants, as measured in the field.

[3] Includes all trees in the major canopy (occasionally excluding small trees definitely below the general canopy); average photo estimate of several experienced interpreters.

Table 2.--Aerial photo stand volume table for even-aged Douglas-fir in the Pacific Northwest

(In thousand board feet per acre)[1]

Stand height[2] (feet)	Crown closure (percent)[3]								
	15	25	35	45	55	65	75	85	95
50	1	3	4	5	5	4	3	1	--
60	2	5	7	8	8	8	6	4	--
70	4	8	10	12	13	12	11	8	4
80	6	11	14	17	18	18	16	14	10
90	8	14	19	22	23	24	23	20	16
100	10	18	23	28	30	31	30	28	24
110	13	22	29	34	37	39	39	37	34
120	16	26	34	41	46	48	49	48	44
130	19	31	41	49	54	58	60	59	56
140	22	36	48	57	64	69	72	72	70
150	25	41	55	66	75	81	85	86	85
160	29	47	63	76	86	94	99	101	101
170	33	53	71	86	98	107	114	118	118
180	37	60	80	97	111	122	130	135	137
190	41	67	89	108	125	138	148	154	158
200	46	74	99	121	139	154	166	174	179
210	50	82	109	134	154	172	185	196	202
220	55	90	120	147	170	190	206	218	226
230	60	98	132	161	187	209	228	242	252
240	66	106	143	176	205	230	250	267	279
250	71	116	156	192	223	251	274	293	308
260	77	125	168	208	242	273	299	320	337

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[1] Gross volume, Scribner Decimal C, in trees 11.0 inches and larger. Trees 11.0 to 20.9 inches scaled in 16-foot logs to top d.i.b. of 50 percent of the scaling diameter of butt log. Trees 21.0 inches and larger scaled in 32-foot logs to top d.i.b. of 60 percent of the scaling diameter of the butt log. Volume tables used: Mason, Bruce, and Girard, based on total height and form class.

Equation for table: $V_b = 0.9533C^2 + 3.2313HC + 0.0716H^2C - 0.0883HC^2 - 3285$

Where: V_b = volume, board feet per acre; H = stand height in feet; and C = crown closure in percent
Multiple correlation coefficient (R) = 0.918
Standard deviation around regression or standard error of estimate = 26,312 board feet per acre, or 34.8 percent of the mean plot volume.

Based on 282 1/5-acre plots, largely in western Oregon.

[2] Average height of dominants and codominants, as measured in the field.

[3] Includes all trees in the major canopy (occasionally excluding small trees definitely below the general canopy); average photo estimate of several experienced interpreters.

CHECKING AND TESTING THE TABLES

The only remaining job was to check the volume tables. As we saw it, there were two phases to this job: (1) a check to see that the volume tables fit the basic data and made sense and (2) a practical test to see how useful the tables might be under operational conditions.

Often there is a tendency to accept output data from a machine program with blind faith, but, though the machine may be infallible, the people that punch the cards and run the machines do make mistakes. As a check, we plotted the volume tables as graphs (fig. 3) and then compared these with our graphs of the basic data (fig. 1) to make sure the two were in accord.

This graphing process also gave us a visual picture of the volume table and how stand height and crown closure were related to stand volume. The plotting of the final volume-table curves is desirable no matter what process is used to construct the tables. It calls attention to irregularities or illogical curve shapes, and the investigation of these may lead to the uncovering of errors.

While checking our volume tables for reasonableness, we were puzzled by one feature of the board-foot table. The volume of short stands increased with crown closure up to the middle classes but dropped off again with higher crown closure. Does this make sense? We aren't sure, but we have two possible explanations.

Perhaps this drop in board-foot volume at the high stocking levels is caused by a decrease in average stem diameter associated with these overstocked stands. Short stands of Douglas-fir usually consist of young trees right around the diameter limit for sawtimber. When such stands are open grown, as in the low and medium crown-closure classes, many of the trees are of sawtimber size and contribute to the board-foot volume. However, densely stocked stands of the same height tend to have smaller diameters. Even though there are more trees in these stands, there may be fewer of sawtimber size and therefore less board-foot volume.

Or, this reverse trend of volume in the high crown-closure classes may be a result of the curve fitting process rather than an expression of the actual relationship. The multiple regression analysis provides a family of curves that, on the average, fit the basic data best. There is no guarantee that they fit every part of the data, and this may be a case where they don't.

It seemed to us that tests to see how well the tables worked under operating conditions should be in two parts. The first consisted of applying the tables to plots other than those used in their construction. The second part consisted of testing photo measurements made by individual interpreters rather than the field heights and average crown closures used in making the tables.

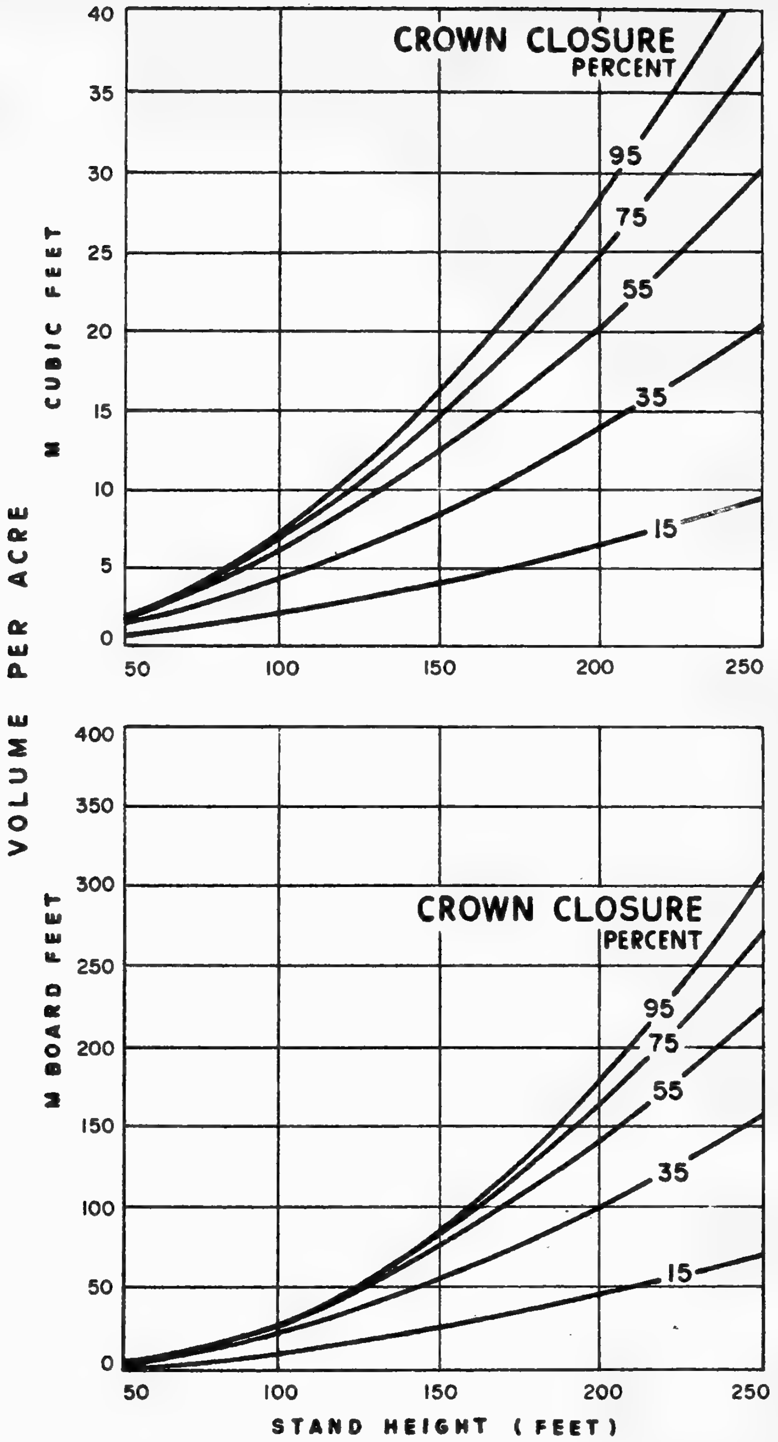

Figure 3.--Relation of stand volume to stand height and crown closure (prediction equations).

Testing the tables on new plots is aimed at determining whether or not the original plots were truly representative, and we had serious doubts about the value of this. The wide distribution of our original plots insured that there was little risk they were not representative. Moreover, the job of testing for this presented some difficulties.

One way to test volume tables on new plots is to withhold part of the original data for testing, making the tables from the remainder. We had decided against this, believing that this additional data would be of more value in improving the original tables than in serving as a check on weaker tables.

A second way to check the volume tables on new plots is to collect data, make the tables, then collect new data to test the tables. We rejected this approach, reasoning that the effort of collecting extra data and the consequent delay in table publication were out of line with the value to be gained from such a test. The large number and wide distribution of plots used gave us confidence that the tables would be useful and that the testing might better be conducted by a variety of users in different areas over the next few years.

However, the second part of the practical test did require investigation. Our only measure of how well the tables would predict stand volume was based on stand height as measured in the field and crown closure as averaged from several interpreters. How much would the precision suffer when the volume estimates were based on photo measurements of height and density by an individual interpreter? As an approximate answer to this question, we used the original photo measurements of the two interpreters as a basis for estimating plot volumes. The following tabulation shows that the loss in precision is not very great, and gives us reasonable assurance that the tables will be useful when applied under operational conditions.

	Correlation coefficient	
	Cubic feet	Board feet
Basis for correlation:		
Multiple regression equation (using field heights and average crown closure)	0.90	0.92
Interpreter A	.87	.88
Interpreter B	.84	.86

SUMMARY AND RECOMMENDATIONS

This paper describes in detail the step-by-step procedure used to construct an aerial photo stand volume table. It brings out a number of problems which have not been adequately covered in existing literature. We have presented our solutions to these problems, giving the assumptions made, the alternatives explored, the results of special investigations, and the reasoning used in making decisions.

The procedures described here are recommended as a guide for those constructing aerial photo volume tables. This is not to imply that the methods are perfect. Occasionally we were forced to sidestep an issue because the alternatives were intolerable. And some of our solutions are stopgaps which can undoubtedly be improved by additional research. Perhaps other workers have better solutions to some of the problems.

However, we believe that the methods used are essentially sound and logical. If followed, they should produce aerial photo volume tables that are objective and useful. The methods should provide a firm basis for future improvement.

Following are the highlights of the recommended procedures:

1. Use multiple regression analysis. Available machine programs place this objective method within the reach of anyone.

2. Define both dependent and independent variables completely, and pass these on to the volume table user. This involves describing what trees were measured and how they were measured.

3. Make a preliminary graphic analysis of the basic data to detect evidence of curvilinearity (suggesting inclusion of the square of the independent variable) or of interactions between independent variables (suggesting inclusion of their cross product).

4. Test the variance of the dependent variable for homogeneity. With photo volume table data this variance is not likely to be uniform, and a special weighting of data is desirable.

5. Select the final regression equation with regard not only to statistical significance but also to the cost or convenience of measuring the various independent variables.

6. Plot the selected volume table equation as a graph to make sure it fits the basic data.

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