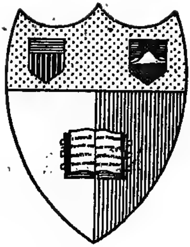

THE
IDEAS OF EINSTEIN'S THEORY

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THE THEORY OF RELATIVITY IN SIMPLE LANGUAGE

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WITH 7 DIAGRAMS AND AN ILLUSTRATIVE CHART

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PREFACE

THE beginning of the twentieth century presented us with a scientific theory which quickly became celebrated all over the world: the Einstein Theory of Relativity. Whoever is interested in the evolution of mental progress will desire to know more of this theory, surrounded though it be by a mailcoat of mathematical formulæ, that presents to every non-mathematician an apparent barrier to further investigation.

Such a truly great idea, however, which contains matter of interest to mankind at large, must be capable of being rendered clear and intelligible, without consisting solely of a maze of mathematical formulæ. This certainly applies to the Theory of Relativity; all the essential traits of the theory can be made clear without the aid of mathematics to those who have a fair amount of geometrical training, and, in point of fact, a number of such popular expositions have already found their way into current literature.

The purpose of the present book is not to give an account of mere details appertaining to the theory, but rather to give a complete and coherent exposition of the whole, at the same time avoiding all mathematical accessories. The reader must not only be able to understand what is meant when we maintain that the space surrounding gravitational masses suffers curvature—he must be made to see how Einstein was bound to arrive at such a conclusion. Hence we must follow up the logical connection of the whole theory, commencing with the Special Principle of Relativity in its most simple and primitive form, and leading up to the far-reaching speculations on the finiteness of the universe, along the path taken originally by Einstein. In order to remain intelligible to the layman, logical operations based on mathematics must be passed over, and it suffices to say that the suppositions *A* and *B* lead us, with the help of mathematical deductions, to the fact *C*, and later on to *D*, and so on. By arguing thus, and inducing conclusions to follow each other in right succession, like the links of a chain, we shall perhaps enable the reader to gain a more lucid view of the matter, than by going deeply into mathematical operations and losing count of what is most essential. The present book, though written primarily for laymen, may also be useful to those who are versed in the theory from the mathematical point of view, but who may find it convenient to supplement their knowledge of the general aspects of the subject.

One thing more must be considered. A serious exposition of the theory will not only have to lay stress on the fact of how very revolutionary Einstein's theory is from the point of view of principle and theory, but must also indicate how very non-revolutionary it appears from a practical point of view. The physical results of the theory which appertain to those phenomena with which we have to do in daily and in technical life diverge so slightly from those of former theories, that these last can be further retained with full justification for all practical purposes. The astronomer, therefore, with few exceptions will continue to calculate according to the Newtonian theory, the man of science will go on using Maxwell's equations, and little will be altered. But the mental foundations of the complete system of physics have been entirely changed. This will be elucidated by numerical examples, so as to dispel wrong and fanciful ideas of the theory on the part of the reader.

J. H. T.

August 1921

The Translator offers her sincere thanks to Dr. R. W. Lawson, of Sheffield University, for his thorough revision. Without his kind help this translation would never have been issued.

PREFACE TO SECOND EDITION

NO alterations have been made in the general plan of the book, but various suggestions from colleagues and critics have been attended to, and this has involved the insertion of several additions and improvements, partially in the form of Supplementary Notes at the end of the book. In particular, in connection with Chapter XVII, I have dealt with an objection which has often been raised but never fully refuted, relative to the appearance of velocities greater than that of light in the rotation of the firmament of fixed stars.

I am indebted to all colleagues whose interest in this little volume has been shown by their kindly and helpful criticism.

J. H. T.

November 1922

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INTRODUCTION

THE Theory of Relativity is a branch of theoretical physics, taking its origin essentially in purely physical experiments. It has led to conclusions of a universal philosophical nature on space and time and the character of the universe—hence the interest surrounding it far exceeds the limited circle of physicists. Just as it is a matter of moment to many more than to geographers and astronomers that the scene of human life is not an extensive plain, but a comparatively small ball circling in space, so it will interest others besides physicists and mathematicians to learn that our usual conceptions of space and time are, in the main, erroneous, although they approximate very closely to the reality. The following chapters are intended to show how, on the basis of physical experiments, such far-reaching conclusions have been arrived at.

The Theory of Relativity was developed in two stages. The first of these is called the Special Theory of Relativity. It was formulated in the year 1905 by the German physicist Albert Einstein, after the way had been prepared notably by the Dutch physicist H. A. Lorentz; two years later the Göttingen mathematician Hermann Minkowski shaped it into its final mathematical form. The theory arises of necessity from physical experience, and its consequences have so magnificently withstood the test of one of the most subtle of physical phenomena, that hardly any doubt can be entertained as to its entire validity. In the years 1907–1915 Einstein built up the daring structure of the General Theory of Relativity, this being at the same time a theory of gravitation, which renders the old Newtonian theory only an approximate one. Einstein's new theory does not contain certain deficiencies of a philosophical and theoretical nature, such as were apparent in Newton's theory, and in its practical applications in the realms of physics and astronomy it leads to formulæ which are, in general, almost identical with those resulting from the old theory. Of course this must be so, because the latter are found to be in accord with experience. There are only two astronomical phenomena in which Einstein's theory and the Newtonian theory of gravitation lead to different results, and in both cases observation decides in favour of Einstein. Nevertheless, in the opinion of the author, the General Theory of Relativity cannot claim the same degree of certainty as the Special Theory of Relativity. But even if the general theory should ultimately be found deficient, it will ever remain a master-stroke of genius. Its insufficiency would only serve to arouse within us feelings of regret, that the real world was not built up in conformity with its laws.

Let it be said emphatically that the Einstein theory is not the capricious product of a mind which finds pleasure in proposing new paradoxical ideas,—it is simply the necessary result of physical experience, followed up with unyielding logic by Einstein.

The genesis of the special theory of relativity can be described as follows. Within the last few decades progressive physical research brought to light two facts with almost absolute certainty, viz.: the principle of relativity and the principle of the constancy of the velocity of light. Now these two principles appeared to be mutually contradictory—if one was right the other must be wrong, and vice versa. In spite of this, all physical experiments and experience led ever and anon to these two principles, so that one was apt to regard the matter as little short of miraculous. It was then that Einstein came to the rescue, when he stated: " We cannot doubt the truth of both principles in question, in as far as we can trust the evidence of our senses at all; nor can any fault be found with the logical thought-process, that proves the antagonism between the two principles. But in the considerations connected with that proof there are certain suppositions concerning the absoluteness and independence of our notions of time and space, which appear to us so self-evident, that up to the present nobody has ever doubted their truth. A more careful analysis of these suppositions, however, shows that they only *appear* to be self-evident, and that they are not absolute conceptual necessities. Furthermore, by suitable modification of these concepts, the antagonism between the two afore-mentioned empirical principles disappears."

This discovery proved a decisive step and induced Einstein to pursue the line of thought in the reverse sense and consequently to derive conclusions arising from the simultaneous validity of both fundamental principles. The sum-total of these conclusions is called the Special Theory of Relativity, and this will be treated in the first part of this book.

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PART I

THE SPECIAL THEORY OF RELATIVITY

CHAPTER I

PRELIMINARY FORMULATION OF THE PRINCIPLE OF RELATIVITY: ITS VALIDITY FOR MECHANICAL PROCESSES

THE Special Theory of Relativity bears this name because it deals with the relativity of a special kind of motion, *i.e.* uniform rectilinear motion. Let us illustrate this idea clearly and without ambiguity by the following example. If a ship is sailing smoothly before the wind in still water, with direct course and constant velocity, and without rolling and pitching, we say that it carries out a uniform rectilinear motion.

As this kind of motion is of great importance in the special theory of relativity, we shall say once and for all, —for the sake of brevity,—that when we talk of motion in the first part of this book, uniform rectilinear motion is referred to. Where any other kind of motion is being considered, for instance curvilinear motion, it will be expressly stated. We shall put forward the following statement concerning this particular uniform rectilinear motion, and call it the "special principle of relativity" in its simplest form. *It is evident that we can only speak of the mutual relative motion of bodies,—we cannot attach any meaning to absolute motion because it cannot be verified. Given any number of observations or measurements made within a closed system* (i.e. *without reference to the surroundings*) *we are unable to ascertain whether or not the system is in motion.*[1]

Now what does this mean? Let us suppose an ocean steamer so perfectly constructed as to suffer no deviation from its course, and no jolting from the engines, so that its motion is really uniform and rectilinear. Are we then in a position to assert that it is in motion, unless we look out of the port-holes and watch the passing waves? Our experience teaches us to negative this question, for in the ship's interior all phenomena would take place just as if it lay in harbour. One could play a game of billiards in a ship that does not jolt, just the same as on land. Even the most sensitive mechanical experiments—weighings, and pendulum observations—would turn out exactly the same as they would in a university laboratory. Now when we look out of the port-holes, we perceive the motion of the ship by the motion of the waves, or, to speak with more exactitude and caution, we perceive that there is a relative motion between the ship and the ocean.

To make our meaning clearer, we shall vary our example a little. One ship lies at anchor, and another passes it with uniform velocity. As mentioned above, all phenomena take place in exactly the same way in

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[1] See Supplementary Note on pp. 165–166.

the interior of both ships; when we stand on the deck of one ship and look at the other, we notice that the intervening distance between the two ships is changing, and that they are moving with respect to each other. We cannot discern more than this by looking at one ship from the other, nor can we determine by mutual observation whether our ship is at rest and the other moving, or vice versa—whether ours is moving and the other at rest. Perhaps the reader will object and say: It is surely reasonable to state that the ship at anchor is at rest, whilst the one under steam is in motion, and not vice versa. Practically speaking, the reader's surmise is correct. But we must not forget that when we say the ship lying at anchor is at rest, we are using only an abbreviation for another and more exact statement: The ship at anchor does not move *relatively to the earth.* We know that the earth is not absolutely at rest, because it rotates on its axis once every day, and describes an orbit of about 150,000,000 km. radius in the course of a year. From our more cautious viewpoint we thus see that the ship at anchor is not actually at rest, and that we do well to formulate the results of our observations more carefully as follows: we detect the relative motion of both ships by looking from one to the other; but we cannot decide whether they move or not by experiments made without reference to the surroundings. (" Move " is used here in the restricted sense already mentioned.) For this reason, our abbreviated form of expression is subsequently justified. We call a ship at anchor, or a solidly founded building on land at rest,—though we know perfectly well that it is not really at rest, but takes part in the earth's motion. Practically speaking, it is immaterial whether the system is absolutely at rest or not, because its motion has no influence on the course of phenomena and experiments taking place within it.

Let us review more critically this assertion, as it constitutes the pith of the problem of relativity. We must again recall that we are talking only of the relativity of uniform and rectilinear motion. As before mentioned, the motion of the earth consists of the daily rotatory motion and the yearly revolution round the sun. We can look upon the latter movement approximately as uniform rectilinear motion, owing to the large radius of curvature of the orbit, but this does not apply to the daily rotatory motion; hence we cannot maintain that the existence of the motion of the earth is not felt in a terrestrial laboratory. On the contrary, daily rotation does influence the progress of physical processes. The best-known experiment of this kind is Foucault's pendulum experiment, with the help of which the daily motion of the earth can be determined without reference to the surroundings (*i.e.* the sun and stars). From our past experience, however, an analogous determination of the yearly revolution is impossible.

Supposing men had been cave-dwellers who had never seen the light of day, but that they had attained in subterranean caverns a degree of culture equivalent to that of the present day. With the help of modern physical apparatus they would have been able to discover the earth's daily rotation, and its angular velocity and direction of axis as well; but the yearly revolution of the earth round the sun would have entirely escaped their observations, and they would have been mightily surprised had they come to the earth's surface at some later date and discovered that motion by means of astronomical observations. We can only maintain our assertion that the earth's motion does not influence laboratory experiments in a restricted sense, admitting that we refer only to the rectilinear yearly component of the motion. Furthermore, we must conscientiously inquire if it is indeed true that all laboratory experiments are uninfluenced by this particular motion. What we can state in the first place is this : According to our trivial personal experience we do not feel the existence of that motion on our own bodies, nor is it at all noticeable in any results on the phenomena of daily life. That, of course, is by no means a sufficient proof for a fundamentally so important matter—for with the help of a delicate physical apparatus we can ascertain many facts that entirely evade our personal observation. The passenger who is smoking his cigar in the smoke-room of an ocean steamer is no more conscious of the waves of wireless telegraphy pervading the ship's hulk, than he would be of the existence of the ship's motion, provided it were actually uniform motion. Yet the telegraphist on the upper deck can verify the existence of these waves without difficulty, and receive the message with the help of his receiver. The reader might perhaps suppose that some sensitive apparatus could be constructed to show whether a ship moves uniformly, and at what rate, without reference to the surroundings (or without coming into contact with them), as for instance with the log, which is used in practice for measuring the speed of a ship.

Such questions imply that we doubt the general validity of the principle of relativity. The problem is therefore as follows : Is the principle of relativity in truth a general principle of nature, having strict validity for all physical processes, or is it merely a rule gained from experience, which tells us that within the limits of our restricted sense-perception the detection of uniform rectilinear motion is impossible without reference to the surroundings ?

This question can only be decided on the basis of detailed theoretical and experimental physical research. We shall divide the question—for certain reasons—into two, and ask first : Does the principle of relativity hold for all mechanical phenomena ? Secondly, is it concerned with, and is it generally valid for all other physical phenomena, and all natural processes ? In the first part of the question we thus have to deal with the following question : Can we decide by operations such as : fall experiments, pendulum experiments, weighings, measurements of elasticity, etc., which take place within a closed system, whether or not that system is moving ? Both theory and experiment answer this question in the negative ; the principle of relativity is thus valid for these phenomena. The fundamental laws of mechanics are essentially built up in such a way that they are the same for processes within a uniformly moving system and in a system at rest. Thus no effect arising from such motion can take place theoretically and never can have taken place during our experience in this domain, which dates back several centuries. Hence we may close this chapter with the assurance that the principle of relativity is valid for mechanical processes.

CHAPTER II

ON THE NATURE OF LIGHT

WE found all in order for mechanical processes; theory and experiment united in telling us that the principle of relativity is valid and that there is nothing contradictory nor doubtful about it. The matter is very different, however, with regard to other physical phenomena, of which those referring to optics are of great importance, because they are accessible by the most sensitive and exact measurements. Here we find the point of conflict that gave rise to the origin of the theory of relativity. Theoretical considerations seemed to indicate that the principle of relativity can have no validity for optical phenomena, whereas experiment teaches us that it has. In order to show how it is that theoretical optics leads to such an assertion, we must insert a short preliminary on the nature of light, which can be omitted by those readers who consider they need no further teaching on the nature of electro-magnetic oscillations.

We all learnt at school that rays of light are waves—and those who know more about it will add—electric waves of very short wave length (*i.e.* about half a thousandth of a millimetre). That is quite correct; but are all those who assert it in a position to give any definite idea as to what it means? What we have been used to call waves in everyday life (*i.e.* water waves) are very different, apart from their wave length, from sound waves and luminous waves. Let us describe what we see on observing water waves. When we look at a small portion of the surface of a lake, we see the surface regularly rising and falling. We find repeated oscillatory motions at regular intervals for a particular place, and we say the phenomenon is periodic with time. If we shut our eyes, open them again for a moment, thereby observing the whole surface of the water, and then close them once more, we do not perceive that the surface is moving, but we see that it is undulated, *i.e.* there are wave crests and wave troughs succeeding each other at equal distances. We say, therefore, the phenomena is spatially periodic. If we open our eyes and gaze at the whole scene freely, we see the united effect of time- and space-periodicity, which gives to the motion of waves their own peculiar character: these waves are apparently advancing, whereas we are fully aware that every single water particle is performing vertical oscillations about a fixed position. Up to the present we have spoken of the surface of water, because what we clearly recognise to be wave motion is the motion of the surface. But we must not forget that in reality the water particles under the surface and the air particles above it participate in the movement. When we speak of wave motion, we are in the habit of imagining a process like the visible phenomenon of water waves advancing along a surface, whereas, in order to apply these ideas to physics, we must train our mind by imagining a process which occupies three dimensions in space. We shall not find it difficult to do so: let us imagine a large number of tiny luminous balls freely suspended under the surface of the water, and likewise a number of tiny balloons floating in the air above it, so that both balloons and balls participate in the motion of their respective media. What we then see, on following the oscillations visually, is the true three-dimensional wave motion, similar to that which plays so great a part in physics. The oscillations of the water particles, and those of the balls suspended in it, take place vertically, whereas the propagation of these oscillations, *i.e.* the transmission of the waves, proceeds horizontally. The direction of the oscillatory motion itself, and of the propagation of oscillations, are perpendicular to each other. Oscillations of this kind are called transversal. But besides these, there are other oscillations called longitudinal ones : these are distinguished from transversal waves by the fact that the directions of their oscillatory motion and of their propagation are identical. This can be simply demonstrated as follows : A number of small weights or lead balls are hung at equal intervals from a long india-rubber tube, fastened at one end to the ceiling. Pull the lowest ball a little downwards and then let it go ; it will oscillate vertically, the motion being propagated upwards along the tube in a vertical direction, and setting all the other weights into vertical oscillations. The direction of the oscillations themselves, and the direction of their propagation, are parallel to each other ; hence we have to deal with longitudinal waves. It is a well-known fact that sound waves are nothing else than longitudinal waves transmitted in air, or other gaseous, liquid or solid medium. The wave length is the distance from one wave-crest to the next. The wave length of sound varies according to the pitch of the note; for example, it amounts to about 130 cm. for the musical tone *C′*, and is smaller for higher and greater for lower notes. Sound waves and water waves have one characteristic in common, viz. they consist of the real motion of a tangible and ponderable substance (air, water, etc.). This cannot be the case with light, because rays of light are propagated through interstellar space (free from ponderable matter) as well as through an artificially created vacuum. Nevertheless, about a century ago, experimental research led to the conclusion that light, as well as sound, is intimately connected with oscillations. Further data were soon obtained on the nature of luminous oscillations. It was discovered that they must be transversal oscillations, their velocity of propagation being about one million times greater than that of sound in air, *i.e.* 300,000 km. per second. *We shall denote that velocity once and for all by the letter* c. On the other hand, the wave length is very small, and is connected with the colour of the light in a similar way as the wave length of sound is connected with the pitch of a note. It is greatest for red light, *i.e.* about $\frac{8}{10,000}$ mm., and it then decreases in the following sequence: red, orange, yellow, green, blue, violet—these last rays having the shortest wave length, *i.e.* about $\frac{4}{10,000}$ mm. The spectrum of visible rays of light corresponds exactly to an octave of tones. All this was soon known with absolute certainty, one item excepted: *What* is it that oscillates in the case of light? As we said before, it cannot be supposed to be a ponderable substance, and hence an unknown hypothetical "something" was introduced, which was termed light-æther or world-æther. All that was known with reference to this æther was that it was neither tangible nor ponderable, that it caused no friction, but that it must be capable of performing very rapid transversal oscillations. These æther oscillations were supposed to be rays of light.

Half a century ago an important advance was made in the theory of light by Maxwell, who founded his electro-magnetic theory of light. At a later date, this was definitely confirmed by the well-known experiments of Heinrich Hertz, and, on further investigation, it led to the invention of wireless telegraphy. According to Maxwell, rays of light belong to an extensive species of electro-magnetic oscillations, which includes the waves of wireless telegraphy and of heat radiation, the chemically active ultra-violet rays, and X-rays. Rays of light are only distinguished from these allied forms by the fact that their wave length falls within the above-mentioned interval, whereas other kinds of electro-magnetic oscillations have other wave lengths characteristic of them. To make the connection clearer, a useful and frequently adopted scheme is indicated in Fig. I, showing how the different kinds of electro-magnetic wave-radiations are distributed over the entire scale of wave lengths. Here we note two gaps in the extensive spectrum of electro-magnetic oscillations, which belong to varieties of rays not yet detected. Thus far we have pointed out the co-ordination of single wave lengths with various kinds of rays. What is missing, however, is that our ideas of electro-magnetic oscillations are not yet fully explained. What is it that oscillates; is it electricity or magnetism? Not exactly; we do not find electricity itself oscillating in an electro-magnetic wave—we find electric force and

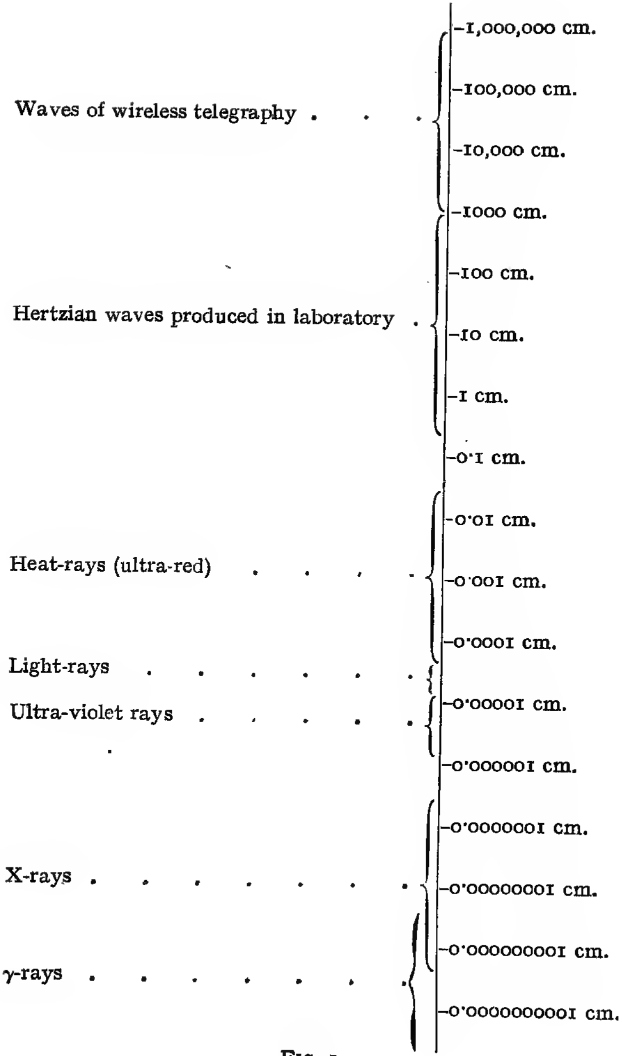
Waves of wireless telegraphy
Hertzian waves produced in laboratory
Heat-rays (ultra-red)
Light-rays
Ultra-violet rays
X-rays
γ-rays
–1,000,000 cm.
–100,000 cm.
–10,000 cm.
–1000 cm.
–100 cm.
–10 cm.
–1 cm.
–0·1 cm.
–0·01 cm.
–0·001 cm.
–0·0001 cm.
–0·00001 cm.
–0·000001 cm.
–0·0000001 cm.
–0·00000001 cm.
–0·000000001 cm.
–0·0000000001 cm.

FIG. 1.

magnetic force. Let us explain what we are to understand by this. Bodies charged with electricity attract or repel each other, according to the unlike or like sign of their charges. To illustrate and to fix this idea in our minds, let us imagine a metal sphere charged with negative electricity, and placed in the close proximity of small particles, for instance pith-balls, which are also charged with electricity. The metal sphere will attract the particles charged with positive electricity, and repel those negatively charged; an electric force acts on the surroundings of the sphere, and we say: the sphere charged with electricity produces an *electric field* around it. The force acting on a small particle carrying a unit of electric charge, at a particular point of the field, is called the electric field intensity or electric force at that particular point of the field. The magnitude and direction of the electric field intensity varies, of course, in various parts of the field. The direction of the field intensity or electric force in the field of a negatively charged ball is always directed towards the centre of the ball, its magnitude diminishing with the distance from the ball. If the ball which produces the electric field is at rest, and if its charge suffers no change, the magnitude and direction of electric force at one particular point of the field remain the same; but as we have said before, it varies for different parts of the field. We say that the field intensity is constant with time, but variable with position. But it is not difficult to imagine an electric field variable *both* with time and position. Let us suppose that the negatively charged ball is shortly to lose its charge and to be gradually recharged with positive electricity. Later the charge of positive electricity is to be again diminished and reduced to zero,—whereupon the ball is again charged with negative electricity, and so forth. What happens then with the ball's electric field? When the charge of the ball is reduced to zero, the electric force will of course be zero, and when the charge of the ball becomes positive, the direction of the electric force will be reversed,—for positively charged particles that were formerly attracted will now be repelled, and vice versa. Thus, the field intensity in a distinct part of the field will gradually vary its magnitude and direction, and in this case we have to deal with a field which varies with position and time. Another matter of importance should be mentioned here: the effect of a change of charge is not transmitted immediately to a distance; on the contrary, it takes some little time to act at a distance. Hence, when the charge of the ball oscillates between positive and negative values, and is reduced to zero at a distinct moment of time, the field intensity at a very great distance will of course also be reduced to zero, though not until a short time later. Now if the changes in the charge of the ball take place very quickly,—say, many million times in a second,—it will happen that at a certain distance from the ball an electric field intensity corresponding to positive charge will be observed, whilst the ball itself is again charged negatively. At double the distance, the field intensity will of course be the one arising from the negative charge the ball possessed one period earlier. At three times the distance, the field intensity will correspond to the last but one positive charge possessed by the ball, and so on. A moment later, when the ball has again changed its charge, the field intensities will again be reversed. Thus we see that the phenomenon bears the character of wave motion, which makes it perfectly justifiable to designate it as an electric wave.

Exactly the same definitions and explanations as we have used here for the conceptions of electric field, electric field intensity, and electric waves, are also valid for the analogous notions of magnetic field, magnetic field intensity, and magnetic waves. We can repeat them word for word, only substituting magnetic for electric, and the words north-magnetism and south-magnetism for positive and negative electricity respectively. The velocity of propagation of every such action is also exactly the same for electric and magnetic fields, being 300,000 km. in the second; *i.e.* when the ball reverses its charge, the direction of the field intensity will be reversed at a distance of 1 m. after the three-hundred-millionth part of a second. Now this is exactly the velocity *c* of propagation of rays of light, and this coincidence was one of the first indications leading to the conclusion that rays of light are none other than electro-magnetic waves, so that a ray of light can be said to be a temporally and spatially variable electric and magnetic field similar to the one described above. This supposition has developed into absolute certainty in the course of time. It would take us too far to enumerate all the evidence in favour of this point of view, and we shall do better by giving a detailed account of the mechanism of a ray of light. For the purpose of illustration, we shall suppose that we have fictitious apparatus at our disposal, of minute dimensions, which permits us to analyse exactly the electric and magnetic field of a ray of light. We require two tiny test particles, one charged with electricity, the other with magnetism, and a contrivance with the help of which we can measure the force acting on these particles, within a space of time less than a billionth part of a second. We must next provide a source of light, and set it at a distance of several metres in front of us, so that its rays strike us horizontally. We then place the tiny ball charged with electricity, and the tiny magnetic pole straight in front of us and alongside of each other, and we observe with our tiny apparatus what happens. We shall observe that forces are exerted on both particles,—forces which act perpendicularly to each other, and to the direction of propagation of the ray of light. Thus, supposing the ray of light to approach us horizontally, then the electric particle will be pulled vertically upwards, and the magnetic particle horizontally from right to left. The action of the forces in these directions is only maintained for an inconceivably short time; after the thousand billionth part of a second the action is reversed, the electric particle is then pulled downwards, and the magnetic particle from left to right. The next moment all is again reversed, and so forth at the immense rate of about five hundred billions of oscillations per second. In this way we may describe the time-process of this phenomenon for a given point. To ascertain the space dependency, we must imagine other particles to be available besides the first two, these being placed at different points of the ray of light. We should then find that all such particles, which, together with the first pair, are placed at an equal distance from the source of light, will oscillate in the same phase,—*i.e.* all those charged with electricity will be pulled simultaneously upwards, all those charged with magnetism simultaneously to the left, and so on. But if we place a pair of particles slightly nearer to the source of light (at a distance which we shall call $l/2$ nearer the source), they will oscillate in the opposite phase to the original pair, *i.e.* when the electric particle in our standard pair is pulled upwards, the electric particle of the nearer pair is being pulled downwards, and so on. A third pair of particles, nearer to the source of light by an amount l, will oscillate like the first pair, etc. We call the magnitude l the wave length of the light. This varies for different colours; the limits for red and violet were mentioned at the beginning of this chapter. If we add that the magnitude of the electric and magnetic field intensities decreases with the distance from the source of light, we shall then have fully indicated qualitatively their dependency on space and time, and we shall have sufficiently described the inner mechanism of a ray of light. The whole can be briefly recapitulated thus: rays of light are transversal oscillations of electric and magnetic force.

Of course it is quite out of the question for any analysis of a ray of light in the way here indicated to be actually carried out; but there are a sufficient number of indirect proofs, which point to the correctness of the ideas developed above. For the physicist, these ideas thus have almost the same degree of certainty as, for instance, the hypothesis that infectious diseases are transmitted by bacteria has for the medical man.

The preceding description of the mechanism of a ray of light teaches us that in the case of light, quite apart from the far greater velocity of propagation and the smaller wave length, we have to deal with an entirely different kind of oscillatory process from that of water or sound-waves. Sound-waves consist of the movement of a material body (air, water, rock, etc.) which undergoes spatial and temporal periodic changes. In the case of light, however, it is the magnetic and the electric force which change periodically. Nothing material nor concrete oscillates in a ray of light: it is something abstract, a force varying periodically in space and time. As we are quite able to imagine the existence of force *in vacuo*—*e.g.* the gravitational action of the sun reaches through empty space to the remotest planetary orbit and beyond, and the earth attracts a body in an evacuated vessel—so we can readily conceive of the existence of a *variable* force *in vacuo*. We thus see that we are able entirely to dispense with the introduction of a hypothetical substance, *i.e.* that of the æther, as the transmitter of oscillations of light. Formerly the arguments ran thus: Rays of light have been proved to be oscillations, consequently something must exist to carry out these oscillations, for we cannot expect "nothing" to oscillate. That something which oscillates in light, we call "æther." One thing, however, had been overlooked, viz. the unknown something was not necessarily bound to be a concrete substance. We shall find it just as intelligible if we assume that it is something abstract in light which carries out oscillations (periodic variations of direction and intensity), viz. the electric and the magnetic field intensity. Hence we need not talk of æther,—the idea of an electro-magnetic field takes its place.

In spite of this, the word "æther" has been retained in the terminology of modern physics; it designates the very essence of the electric and magnetic field magnitudes. We propose, therefore, for the sake of brevity and simplicity, to make use of the terms æther and æther oscillations in all that follows; the reader will know what is meant after what has already been said.

It should be emphasised that the discussion on the nature of light in this chapter has nothing to do with the fundamental principles of the theory of relativity. It merely serves the purpose of making the subsequent physical developments more readily comprehensible. Essentially, the elements of the theory of relativity could be made clear to a non-physicist, without telling him anything as to what light-rays really are. It appears to me, however, that the knowledge of such a fundamental idea as that of electro-magnetic waves is so important, that anyone who desires to know something of the theory of relativity ought also to have some enlightenment on that idea. Hence the comparatively large space that has been allotted to a subject not immediately connected with our theme.

CHAPTER III

IS THE PRINCIPLE OF RELATIVITY VALID FOR OPTICAL PHENOMENA ?

EQUIPPED with the definite ideas on the nature of light set forth in the preceding chapter, we are now able to approach the solution of the following problem: Is it not possible to detect the annual revolution of the earth, by laboratory experiments connected with the phenomena of the propagation of light ?

On the classical theory, according to which the æther is to be regarded as a real substance, we should expect that such an experiment must be successful. This is easily understood, if we imagine an analogous experiment carried out in the sphere of acoustics.[1] Let us imagine ourselves once more on board an ocean steamer so perfectly constructed that it travels without rolling or pitching, and with straight course and uniform velocity. We shall suppose there is a whimsical rich old gentleman on board, who says : " I'll bet you 10,000 dollars that none of you can satisfactorily demonstrate

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[1] The experiment described in the following could not actually be performed by human observers, because of the infinitesimal time difference involved. It might perhaps be realised with the help of an automatic sound-measuring apparatus, like those in use during the war by the English artillery. But this has nothing to do with the principle of the question.

that this ship is moving, without considering the surroundings." Thereupon all sorts of attempts are made to win the wager,—but all experiments made in the ship's saloons turn out exactly as they would on dry land. Then somebody with more gumption than the others has an idea, and exclaims: "We'll do it after all!" Three passengers are chosen and taken on deck; one is stationed at the bow, a second at the stern, and the third precisely midway between the other two. The one in the middle is provided with a pistol, and told to fire at a particular moment. The other two are provided with stop-watches going at the same rate, and have strict injunctions to stop the watches immediately on hearing the report. The man in the middle fires his pistol, the other two stop their watches, and a subsequent comparison of the two watches shows that the watch at the bow was stopped very slightly later than the watch at the stern. The explanation is simple enough: The atmospheric air that transmits the sound waves does not take part in the ship's motion, hence a current of air from bow to stern is present on board the ship. This causes the sound waves to travel at a quicker rate towards the stern than towards the bow, and explains the difference in time noted. The man responsible for this experiment now explains: "If the ship had not been moving, no difference in time would have been noted between the two watches, because the pistol was fired midway between them. The difference observed is a proof of the existence of motion, and as this was found without considering the surroundings, I have won the wager." The whimsical old gentleman disagrees, however, and adds: "Not so! That is not the way my wager was laid. I admit you haven't visually considered the surroundings, but still your experiment depends entirely on the interaction between the ship and the surrounding air. All you could determine was the relative motion between the two, and your experiment would have furnished the same result had the ship been at anchor, and a wind had blown from bow to stern. Your experiment proves nothing at all, and I shall keep my money." Let us leave them to settle their dispute between them, and think about attempting an analogous experiment ourselves, to prove the existence of the earth's annual revolution. If we look upon the earth as a vast world-ship travelling through space, then the æther is the medium filling space, and corresponding to the air surrounding the ocean liner. Let us interchange terms and replace ship by earth, ship's deck by earth's surface, air by æther, and sound waves by luminous waves. We may then expect that a light-signal sent out from the earth's surface will be transmitted forwards at a slower rate in the direction of the earth's motion than in the opposite direction. Just as a current of air sweeps the deck of the moving ship from bow to stern, so must an æther-drift be present at the earth's surface in a direction opposed to the earth's motion, when the sphere glides through the placid æther. Before we discuss the possibility of actually performing the experiment, it will be as well to consider the meaning of the success or failure of such an attempt. Let us assume the result to have been a positive one, *i.e.* that there exists a slower rate of propagation of light in the direction of the earth's motion than in the opposite direction. That would certainly constitute a new proof of the existence of the earth's annual motion, and furthermore, one accomplished by means of a laboratory experiment, and without astronomical considerations of the sun and stars. Has, then, the principle of relativity, as it was discussed in the first chapter, been violated ? We stated there : " It is impossible to perceive the existence of motion without reference to the surroundings." If we take up the point of view of the whimsical old fellow laying his wager, we can assert in the present case : " This experiment had reference only to the interaction between the apparatus used for our experiment, and the surrounding æther ; the result we finally arrived at was merely the existence of a relative motion between earth and æther, and nothing more. The principle of relativity has thus not been violated." We see that in the end the question amounts to a dispute of words, concerning the meaning of the restriction " without reference to the surroundings." In order to avoid any confusion of words, we shall now bring the principle of relativity into another form, containing no further ambiguities, and one in which experiment will clearly decide one way or the other. For this purpose we shall introduce a new notion, which will also prove itself useful later on. We saw in Chapter I that such statements as " a body in motion " or " a body at rest " need an addition to complete them, namely, the assertion as to what this state of motion or of rest is referred. To describe the position or motion of a body, another body is always required (or at least a fictitious, distinct system of lines in the universe), which can be referred to in stating distances or velocities. In general it is the material structure on which we are situated whilst executing our measurements. If, for instance, a race be run on board a ship, and we state that the victor attained a velocity of 10 m. per second, we understand as a matter of course that this velocity is relative to the ship's deck. (The velocity of the runner with reference to the earth would be 10 m. per second, plus or minus the velocity of the ship, according as the runner travels with or contrary to the direction of motion of the ship.) The body to which the data refer is called the body of reference or system of reference.

In our ordinary laboratory experiments, the earth itself is our body of reference; for the sound experiment mentioned above, the moving ship was the body of reference, and so on. Thus we may state the principle of relativity in the following form: *In different systems of reference moving uniformly and rectilinearly with respect to each other, all natural processes take place in exactly the same way.*

The principle of relativity, as here stated, would certainly be violated if it could be proved that rays of light on the earth are propagated more quickly in one direction than in another. Measured from a system of reference not partaking of the earth's motion, this result would certainly not be obtained; hence physical processes in that system of reference would take place in a different way from that occurring on the earth,—a result contradictory to the statement we have just made. We see, furthermore, that the question of the validity or non-validity of the principle of relativity, in its modified form, includes the problem of the existence of an æther. For if we say the æther is a real substance, we must have the possibility of perceiving it directly or indirectly, even though it need not be tangible or ponderable. Now if its real existence is in any way capable of perception, we must expect that a system of reference relatively to which it is at rest must be distinguished from others relatively to which it is in motion. That, however, would be a contradiction of the principle of relativity. If this is so, then the assumption of a substantial æther is of no use to us. We saw in Chapter II that the æther-hypothesis can be entirely dispensed with for the comprehension of light-processes ; indeed, if the principle of relativity has general validity, the æther conception becomes not only superfluous, but acts directly as a hindrance. We may then use the word æther solely in its abstract meaning, mentioned at the end of Chapter II. On the other hand, the assumption of the existence of an æther as a real substance led us to expect that an experiment of the kind described would give a positive result (*i.e.* permit us to discover an influence of the earth's motion on the propagation of light), if we could only succeed in accomplishing the experiment with sufficiently exact apparatus. Before a final experimental decision had been arrived at, it was only possible to set up conjectures concerning the general validity of the principle of relativity in its latter form. These conjectures may have led the majority of physicists to assume that the principle of relativity was certainly valid for mechanical phenomena, but not for optical and electro-magnetic ones.

After these preliminary considerations, we must now turn to the question of the actual performance of the experiment, concerned with the propagation of light at the earth's surface. A simple calculation shows that a literal analogue of the sound experiment described above cannot produce any satisfactory results in the case of light, since its velocity is a million times greater than that of sound.[1]

We must, of course, chose an experimenting ground of the greatest possible length. Let us station the middle experimentalist *B*, who is to give the light-signal, on a hill visible from all sides. The observer *A* is placed in the direction of the earth's motion 30 km. away from the hill, and the other observer *C* is situated in the opposite direction, also 30 km. from *B*. Both observers have accurate chronometers going at the same rate, which they stop the very instant the light-signal reaches them from *B*. To ensure accuracy we might, in the place of observers, imagine contrivances of a highly sensitive nature which register the exact arrival of the ray of light automatically, and with lightning rapidity. Will a difference be noticed, then, between the two stop-clocks ? The effect to be expected can easily be calculated. The velocity of the earth's revolution around the sun amounts to 30 km. per second, which is, of course, at the same time the velocity of the hypothetical æther-drift. We should expect this æther-drift to influence rays of light in the same way as wind influences sound ; hence light would be propagated with a velocity of 300,030 km. per second in the direction from *B* to *C* relatively to the earth's surface, and with a velocity of 299,970 km. per second in the direction from *B* to *A*.[2]

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[1] The more advanced reader may consider it superfluous to prove numerically the impossibility of a direct experiment, obvious though it may be to every physicist. But there are good reasons for doing so, because we can in this way get a better idea of the minuteness of the effects, *e.g.* contractions of measuring-rods, etc., required by the theory of relativity.

[2] For the sake of simplicity the assumption is made here, that the value of c is *exactly* 300,000 km. per second.

The distances *BA* and *BC* are each 30 km., so that light should require 0·00010001 seconds from *B* to *A*, and 0·00009999 seconds from *B* to *C*. The difference between these times is 0·00000002 seconds, *i.e.* the fifty millionth part of a second. Before we could hope to accomplish this experiment, we should have to construct chronometers surpassing our best clocks in accuracy a millionfold.

The result of our numerical estimate is so discouraging that the reader may well doubt the possibility of an actual experiment of this kind, and will perhaps be satisfied to take it for granted that our present-day technical means are not capable of solving the problem in point. And yet the American physicist *Michelson* performed the experiment, and gained a definite result more than thirty years ago. Later, this became one of the most famous experiments in physics, perhaps on account of its constituting the most important empirical evidence in favour of the principle of relativity. The idea is as follows: A ray of light falls on a glass-plate at an angle of 45°, and is separated into two parts; one part is reflected at the surface of the glass and is propagated perpendicularly to the original direction, whereas the other part traverses the glass plate and travels straight on. Both parts traverse certain distances from their point of origin, and are then reflected in mirrors set up perpendicularly to their respective directions, so that they travel back along the same path and meet again at the glass plate. When a ray of light is split into two rays in this way, and the parts re-unite, certain optical phenomena are observed called interference fringes, and if one of the parts lags behind by an infinitesimally small fraction of a second, this gives rise to shifts of these fringes, which are readily perceptible. In this way, Michelson compared the time taken by one ray of light in travelling to and fro parallel to the direction of the earth's motion, with that of another ray which travelled to and fro in the perpendicular direction. (The experiment was carried out in the following way: The entire apparatus was floated in mercury, so that it was free from vibration, and capable of rotation. A thorough investigation was then made, to decide whether any shifts of the interference fringes occurred, when the apparatus was brought into various positions relative to the direction of the earth's motion.) The results obtained were absolutely negative. The attempts were repeated in subsequent years with more exact and sensitive apparatus, and finally, Morley and Miller tried with contrivances so accurately adjusted, that an effect amounting even to a hundredth part of the computed value must have been detectable. But there was not the slightest trace of a non-uniformity in the propagation of light. Hence the Michelson-Morley experiment decided with certainty in favour of the validity of the principle of relativity for optical processes, as well as for mechanical ones.

It would not, of course, be convincing, if our decision concerning such a fundamentally important problem depended on one experiment only, however exact and conscientious its execution. For it might still be possible that some incidental accessory circumstance, of which nobody had thought, had paralysed the effect of the earth's motion in this one experiment, and that the existence of this motion might still be proved in some other way by laboratory experiments. To begin with, it might for instance have been possible to suppose the moving earth to carry æther along with it, just as a body moving in a liquid carries along liquid particles on its surface owing to friction. In this case there would be only a very small relative velocity between æther and earth at the earth's surface, so that the negative result of the Michelson experiment could readily be understood, without regarding it as decisive for the validity of so important and far-reaching a law of nature. The possibility of such an explanation constituted the theme of close investigation, but the results obtained come into conflict with other facts of experience, so that the assumption that the earth carries æther along with it must be discarded. A number of other experiments quite different from that of Michelson and Morley were devised and carried into execution, but all of them, without any exception, gave negative results. Some of these had nothing to do with the propagation of light, but were concerned with other electro-magnetic processes.

It is of interest to notice that some of the most important and fundamental doctrines of physics and chemistry took their origin in experimental failures. The science of the elements, which forms the foundation of chemistry, arose from the unsuccessful attempts of the alchemists to convert lead and other common metals into gold, and the law of the conservation of energy originated in fruitless efforts to effect perpetual motion. In the same way, Einstein, as a consequence of the negative results of the aforementioned experiments, came to the following conclusion : The dilemma is not due to want of skill on the part of physicists, nor can the insufficient development of our technical knowledge be at fault. The fundamental cause lies rather in the absolute impossibility of determining by laboratory experiments the influence of the rectilinear component of the earth's motion in any physical phenomena whatsoever, because the principle of relativity is valid for all natural processes, and not only for those connected with mechanics, as was formerly supposed.

CHAPTER IV

THE LAW OF THE CONSTANCY OF THE VELOCITY OF LIGHT

THE discovery that the special principle of relativity is valid for the entire range of physics is in itself far more satisfactory than our former assumption, according to which it is valid only for one part of physics, namely, mechanics, and not for any other. We are thus led to ask : Why, then, were we of opinion that it cannot be valid for optical processes ? Without doubt the æther hypothesis was responsible for this. We knew with certainty that rays of light are oscillatory processes, and hence deduced the erroneous conclusion that there must be something concrete and substantial to carry out these oscillations, this "something" being the æther. As long as we accept the notion of an æther, the analogy with the sound experiment on board ship would necessarily appeal to us, and thus lead us to doubt the validity of the principle of relativity for optical processes. If we accept the principle of relativity (and that we must, if we are to understand the negative result of all the experiments mentioned in the last chapter), we must give up the hypothesis of a substantial æther, and consider rays of light to be no more than oscillations of electric and magnetic field intensity. (Chapter II contains a detailed description of what this means.) To complete the description of this process, we must state the velocity of propagation of the waves, and this again has no meaning, unless we indicate the system of reference relative to which the light is propagated with that velocity. Formerly we should have said: "relatively to the æther," in an analogous way, as sound is propagated with a velocity of approximately 330 m. per second relatively to the air. Now the æther, in the former sense of the word, has lost its significance, hence we can no longer refer to it when giving a precise statement of the velocity of light. We must devise a more suitable system of reference, for which our statement of the velocity of light shall be valid.

The experiment of Michelson and Morley taught us that rays of light from a terrestrial source, which takes part in the earth's motion, are propagated in all directions with equal velocity. We might, therefore, be inclined to say: Waves of light are transmitted with a definite velocity as measured from the source of light. The difference between the present statement and the former one, according to which light has a distinct velocity relative to the æther, may be made clear by comparing both alternatives once more with our example of the propagation of sound along the deck of a moving ship. Sound waves have a distinct velocity of propagation relatively to the atmosphere; they are not, therefore, transmitted uniformly in all directions along the moving deck on board ship, but travel more quickly towards the stern than towards the bow of the ship. Thus the velocity of propagation is not the same in every direction relative to that system of reference (the deck), which is at rest relatively to the source of sound.

We find an analogue to this in the former æther hypothesis, the correctness of which would have required a positive result for the Michelson experiment. If, on the other hand, the experiment on board ship had been carried out in such a way that the velocity of the bullet and not the velocity of the sound from the revolver had been measured, whereby the experimentalist, who is stationed in the middle, fires both forwards and backwards, then no difference would have been noticed between the velocities of the two shots. (Neglecting the influence of air resistance.) The bullets move with a definite velocity relative to a system of reference at rest with respect to the marksman. This kind of propagation corresponds to the last proposed hypothesis concerning light, with the difference, of course, that rays of light represent a wave process, and are not, in themselves, material like a bullet; the simile we use characterises only the kind of propagation, but not the nature of the process. The hypothesis that light is transmitted with a distinct velocity with respect to a system of reference, which is at rest relatively to the source of light, was advanced by the Swiss physicist Ritz. The advantage of his theory lies in the fact that it is in complete harmony with the principle of relativity. According to this hypothesis, rays of light from a terrestrial source would always travel with a definite velocity away from it, independently of whether the earth moves or not. It would then be just as impossible to discover the existence of the earth's motion by experiments on the propagation of light-rays from a terrestrial source, as it would be for observers on board ship to determine the existence of the ship's motion by measuring the velocity of bullets fired from the deck of the ship. The negative result of Michelson's experiment would thus be easily understood.

The Ritz theory, however, leads us to another conclusion,—one which conflicts with experience,—and hence we are obliged to give up this hypothesis also. For purposes of illustration, let us revert once more to our simile of the moving ship. We shall suppose it is sailing near the coast, and parallel to it. Bullets fired from the middle of the ship will travel fore and aft with equal speed for observers on the ship. But let us suppose observers posted on land, and provided with means of some kind for measuring the velocity of projectiles fired from the ship. For these observers projectiles fired in the forward direction will travel more quickly than those fired in the opposite direction. If we call the velocity of the bullet relative to the gun q, and the velocity of the ship v, the bullets fired forwards will have the velocity $q+v$, and those fired in the opposite direction $q-v$, for the observer on land. Let us apply this to optical phenomena. According to the Ritz theory, the velocity of rays of light from a star which is approaching the earth must, as measured from the earth, be greater than that from a star which is moving away from us. This deduction has been tested, both for rays of light from radially moving stars, and for those emitted by moving terrestrial sources of light. No dependency of the velocity of light on the state of motion of the source of light could be detected, and therefore the Ritz theory cannot be maintained. On the other hand, a new and important fact based on experience has been obtained: *the velocity of light in vacuo has always the value* $c=300{,}000$ *km. per second, and is quite independent of the state of motion of the* *source of light.*[1] Einstein designates this law as the "Principle of the Constancy of the Velocity of Light,"[2] and regards it as a fundamental principle of nature, equal in importance to the Principle of Relativity. Together, these two principles constitute the foundation pillars of the Special Theory of Relativity.

It is important to notice that these two fundamental principles rest on the safest and surest ground known to exact science; they are supported by the most sensitive optical experiments and the most exact astronomical measurements. If we can in any way trust our own experience, we must have absolute confidence in the validity of these two principles. This point deserves special emphasis, because we shall soon have occasion to doubt their correctness.

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[1] According to our own experience, this is valid when the system of reference is the earth, but owing to the validity of the principle of relativity it holds good also for all systems of reference in uniform rectilinear motion relatively to the earth. On p. 32 we said: "to complete the description of this process, we must state the velocity of propagation of the waves, and this again has no meaning, unless we indicate the system of reference relative to which the light is propagated with that velocity." We are now able to state that light is propagated with the velocity c relatively to *all* the above-mentioned systems of reference which move uniformly and rectilinearly with respect to each other.

[2] For the sake of brevity only the velocity of light is here spoken of. The law mentioned, however, holds good for all kinds of electro-magnetic waves. (Cf. Chapter II.)

CHAPTER V

THE CONFLICT BETWEEN THE TWO FUNDAMENTAL PRINCIPLES

UP to the present everything has been plain sailing. The deduction of conclusions from experiments, and of general laws from our experience, is something quite common in natural science (moreover, in branches of much greater practical importance then the present), and the majority of outsiders rarely trouble their minds in the least about such matters. The fact which distinguishes the theory of relativity and lifts it above the level of everyday experience is this : when we consider the matter more closely, we find that it is quite impossible for both fundamental principles to be valid together, for they contradict each other !

The contradiction between the two is fundamentally the same as that between the Ritz theory and the facts of experience. Let us again develop it. By way of change and more convenient measurement, we shall choose for purposes of illustration a railway train, which moves with constant velocity along a straight track. A light-signal is sent out from the middle of the train at a given moment, and its velocity is to be measured both by observers in the train and others stationed on the embankment. According to the principle of relativity, physical processes must take place in the moving train precisely as they would in a stationary train; hence rays of light, as measured by observers situated in the train, must be propagated both forwards and backwards with the same velocity, just as they would be if the train were stationary. According to the other of the two fundamental principles, however, the velocity of light as measured from the embankment should be the same both in the direction of the motion of the train and in the contrary direction, because it ought to be independent of the state of motion of the source of light. These two demands contradict each other, for if the speed of the train be v and an effect be propagated in the direction of the train's motion with the velocity c (measured in the train), then we must find (as mentioned in the last chapter) the velocity measured from the embankment to be $c+v$, or $c-v$ if propagated in the contrary direction. Sound common sense teaches us that it must be so, and this classical law of the addition of velocities (as we may call it) can be tested and confirmed for all velocities with which we have to deal in everyday life. Let us imagine the roofs of railway carriages joined together in such a way as to form a bicycle track. Then a cyclist, according to the direction in which he traversed the entire length of the train, would naturally possess quite different velocities as judged by an observer on the embankment. We can see no reason why this should not hold for light also, and if we had not made up our minds at the end of the last chapter to trust the two fundamental principles, we should be inclined to say: "As they are so obviously contradictory, one of them must be wrong." (From the point of view of logic, they might of course both be wrong.) On the other hand, if we adhere to our two principles, then we must suppress our wonted mode of thought, and admit that the apparently so obvious analogy of the cyclist on the roofs of the moving railway carriages can have no possible validity for light.

In order to see how this can be possible, we shall analyse more closely the process of measuring the velocity of a light-signal from the train and from the embankment. To measure the velocity of light from the train, we must station an observer at each end of the train, besides the man in the middle who sends out the light-signal, and all three would have to be provided with exact clocks. In the same way, observers on the embankment stationed at certain intervals would have to be provided with exact clocks. All these clocks would have to be timed absolutely alike. At a given moment, say, just as the middle of the train is passing one of the observers stationed on the embankment, the light-signal must be sent out, and all observers must stop their clocks the very instant they see the flash. We could then calculate the velocity of light in each direction, as measured both from the train and from the embankment, by means of the differences in the times and the measured distances.

We saw in Chapter III that our clocks are not nearly exact enough for a direct measurement of the velocity of light, and for the present experiment, conditions are still more unfavourable. On the one hand, the base of our observations is much smaller than in the experiment described there, and on the other hand, the velocity of the train is a thousand times smaller than that of the earth. Hence, the differences of time to be measured are much smaller than they were in that case, so that our clocks would have to be about a billion times more exact than they actually are, to enable us to carry out the necessary measurements. But this does not satisfy our conscience in relation to the logical difficulties which are involved. We cannot suffer two laws of nature to contradict each other, even when the discrepancies are so small as to be imperceptible by modern technical means. We must, therefore, at the outset, take account of the possibility of our having clocks and measuring-rods exact enough to measure the velocity of light with the necessary precision.

One other thing we require in our experiment, and that is, that the observers at the ends of the train, as well as those at different stations on the embankment, shall be provided with clocks which not only go precisely, but which are all *timed exactly alike,*—and this is an essential.

Einstein was able to show that a strict analysis of the idea of simultaneity brought the solution of the apparent contradiction between the two fundamental principles.

CHAPTER VI

ANALYSIS OF THE CONCEPT OF SIMULTANEITY

FIRST of all let it be stated that a contradiction between the principle of relativity and the law of constancy of the velocity of light can only be found, if we assume that exactly timed clocks are set up in different places. Two clocks are exactly timed when the hands of the one clock are simultaneously in precisely the same position as the hands of the other clock. If we place both clocks before us on the table, we can readily discern whether the two events—the positions of the hands of one clock at the stroke of twelve, and the corresponding positions of the hands of the other clock—take place simultaneously. The simultaneity of two events in close spatial proximity thus needs no further definition; if I see them at the same time, then they take place simultaneously. But what does it mean when we say that two events in different places occur simultaneously? Let us illustrate by the following drastic example how justified we are in asking this question. On the 21st of February 1901, a new star became visible in the constellation of Perseus, and was called Nova Persei by the astronomers. This star, which had certainly been in existence as a dark mass previously, had been set aglow by some unknown cause and had thus become visible. The flaming-up of the star undoubtedly took place some time before its appearance was discovered by human observation, the delay being equal to the time-interval needed by light to travel from the star to the earth. The question arises as to when this event took place; what date on the earth coincided with the kindling of the star? Let us suppose that it were possible to determine the distance of the star accurately, and that the result be expressed in kilometres. We can then calculate, for instance, that the light took exactly thirty years to reach the earth, and the actual date of the birth of the star would be the 21st of February 1871. The birth of the star and the date, the 21st of February 1871, are supposed to be simultaneous events. Can this be maintained with certainty? If the principle of the constancy of the velocity of light is valid, then our result must undoubtedly be correct; for, according to this principle, the time required by a ray of light to travel from a point *A* to a point *B* will always be equal to the length *AB*, divided by the constant velocity of light *c*, quite independently of whether or not the two points are executing a common motion.

But supposing we knew nothing of this principle, or that we did not believe in it,—how then? Let us take up the point of view of the old æther theory, and assume that our earth, together with the entire visible system of fixed stars, and including the new star, are carrying out a common rectilinear motion in the direction from the earth towards the star. We are then advancing towards the rays of light coming from the star, and hence these will need less time to reach us. Thus the birth of the star did not take place on the 21st of February 1871, but it may, for instance, have taken place on the 21st of

July 1871. Let us suppose, on the other hand, that the common motion of earth and star is in the opposite direction. We are then retreating from the rays of light ; they need longer to reach us, and the event of the new star must have happened sooner, say, for instance, on the 15th of October 1870. Now Michelson's experiment, and the principle of relativity deduced from it, taught us that we could not detect the combined motion of earth and fixed stars. Thus, without the aid of the law of the constancy of the velocity of light, we are never in a position to decide on principle, what date on the earth was simultaneous with the birth of this new star. Without that principle, it is thus quite meaningless to speak of the simultaneity of two events spatially far apart.[1] The philosopher will perhaps take another point of view. He may say : "Never mind not being able to prove simultaneity. If I hit this table with my hand and at the same instant a prominence bursts forth from the star Sirius, then these are simultaneous events, even if I never come to know, all my life, whether the latter event ever happened at all." Now is the philosopher in the right ? He might be, if the inability to prove the simultaneous occurrence of both events was only due to the imperfection of our present-day technical means. But this case is different. It would be fundamentally quite impossible to determine the simultaneity of spatially distant events, without availing ourselves of

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[1] In all this we are assuming that there is no other effect of higher velocity than light to bring us tidings of distant events. In point of fact, according to human experience, there is no such effect. If one, which is propagated with a velocity greater than *c*, should ever be discovered, then the entire structure of the theory of relativity would fall. But this is not likely to happen.

the law of the constancy of the velocity of light; and what cannot essentially be observed, cannot be said to exist. We might perhaps concede to the philosopher the idea of absolute simultaneity of spatially distant events as a pure thought fiction, though it could never be proved. But even then we must discard this idea of absolute simultaneity if it leads to contradictions between the facts of experience, as is actually found in practice.

The matter is different, however, if we retain the principle of the constancy of the velocity of light. If this be valid, then the lapse of time between the birth of the star and its perception on earth *is* perforce equal to the distance earth–star, divided by the velocity of light c, quite independently of whether or not both bodies are executing a common motion. We thus see that it is this principle that defines simultaneity. Hence the notion of the simultaneity of events spatially separated is not given a priori, but is defined by the principle of the constancy of the velocity of light. It may be defined most simply thus: Events A and B, which occur at different places, are simultaneous if observers stationed at equal distances from A and B see the occurrence of both events simultaneously.

Does the reader perceive the far-reaching importance of this principle of the constancy of the velocity of light, and that it means far more than a mere assertion concerning a physical phenomenon? It does much more than acquaint us with a mere property of light—*it defines fundamentally the connection between space and time.* We see, then, that the experiment of measuring the velocity of light from the train and from the embankment cannot lead to any contradiction with the law of the constancy of the velocity of light, for the observers' clocks can be exactly timed only with the help of this law, *i.e.* they are by definition never timed exactly alike, unless measurements performed with them confirm this law.

The discussion in this chapter contains the quintessence of the problem of relativity. Let us recapitulate the matter briefly. Formerly, our minds were trained to the persuasion that the conception of the simultaneity of spatially distant events was given a priori, that it had an absolute meaning, and thus required no previous definition. In the application of this idea of absolute simultaneity, however, we encounter a contradiction between the principle of relativity and the principle of the constancy of the velocity of light, so that at least one of these principles must be wrong, or the absolute idea of simultaneity is to be abandoned. The importance of Einstein's work lies in the fact that, of these alternatives, he gave preference to the two fundamental principles based on our experience, rather than to the apparently self-evident but unproven conception of absolute simultaneity. His idea was as follows: The principles of relativity and of the constancy of the velocity of light are correct, for they have been proved experimentally. Without having regard to our previous habit of thought, we have to modify our conceptions of space and time in such a way that the velocity of light in two or more systems moving uniformly with respect to each other always has the same value c, irrespective of direction. How these modifications are to be carried out, is the subject of the special theory of relativity; it contains all those inferences that can be logically deduced from the simultaneous existence of both fundamental principles.

CHAPTER VII

THE SPECIAL THEORY OF RELATIVITY: A SUM-TOTAL OF THE DEDUCTIONS FROM THE TWO FUNDAMENTAL PRINCIPLES

WE left our observers in the train and on the embankment in the lurch, after we had been fully persuaded that they can in no way upset our conviction of the validity of the two fundamental principles, provided they perform the exact adjustment of their clocks correctly. We must now revert to them again, and let them perform measurements to demonstrate the deductions resulting from the co-existence of both principles. In the first place, it is easy to show that Einstein's rigid definition of the idea of the simultaneity of spatially distant events is not an absolute one, but that it is only relative. Thus, when I say that an event at a given place *A* (say the earth) and another at a place *B* (Sirius) happen simultaneously, this statement is valid only for myself and for those observers at rest relatively to me. Other observers, however, who are in motion relatively to me, will take another point of view, and be quite right in saying: The two events were not simultaneous. Let us proceed to demonstrate how it is that this results from Einstein's definition; but it should be pointed out forthwith that the differences of time-intervals with which we have to deal here are so infinitesimal, that the results of which we shall speak in the following are far from being detectable by our present-day apparatus. We shall suppose only one observer in the train, and him to be stationed exactly in the middle of the train. Electric lamps are set up on the embankment at two places *A* and *B*, their distance apart being equal to the length of the train. They are provided with convenient contact-levers, so that the lamp *A* emits a flash of light the moment the beginning of the train passes it, and the lamp *B* the moment the end of the train passes it. An observer is stationed on the embankment midway between *A* and *B*. The train travels past, the lamps send out their flashes of light, waves of light are propagated with the velocity *c* from both *A* and *B*, and reach the observer on the embankment simultaneously. Thus he sees both events at the same time, and if he has ascertained by measurement that the places *A* and *B* are equi-distant from him, then, according to the definition in Chapter VI, he is quite right in asserting that the flashes took place simultaneously. But the observer in the train has meanwhile travelled a short distance towards *A*, hence rays of light coming from *A* reach him sooner than those coming from *B*, and he maintains quite correctly that the two events did not take place simultaneously. The following objection might be raised: "Is this statement also correct from the point of view of the

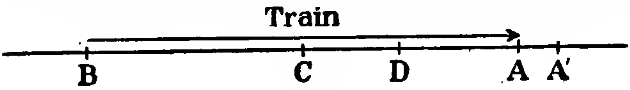

FIG. 2.

Einstein theory? According to definition, two events are simultaneous if an observer standing in the middle sees them at the same time. That is not the case here, for when the rays of light reach him, he is no longer in the middle between *A* and *B*." This latter argument, however, is not correct. The observer was stationed midway between the lamps at the instant they sent out their flashes of light; it is quite immaterial what position the lamps take up with respect to him afterwards. To meet the above-mentioned objection, we can suppose lamps attached to the beginning and end of the train; the first of these sends out a light-signal exactly at the same time as the lamp at *A* on the embankment, and just as it passes *A*, and the second lamp at exactly the same time as the lamp at *B* on the embankment, as it passes this lamp. (In this statement the idea of simultaneity presents no difficulty, because we are only concerned with the simultaneity of two spatially proximate events.) This arrangement does not in the least modify the succession of phenomena seen by both observers. The observer in the train is now, without doubt, situated midway between both lamps; if he observes that they emit their flashes of light at different times, he is quite right from his point of view when he says: "The flashes were not simultaneous."

To mark the difference between assertions with regard to simultaneity which are correct according to our definition, and those which are not, the following may be added: A third observer *D* is supposed to be stationed on the embankment, and to be situated far nearer to *A* than to *B*. He, too, will see the flash of light from *A* sooner than that from *B*, but he cannot maintain that, as a consequence, the events are not simultaneous, for he is not situated midway between the two sources of light. On the contrary, he must take the difference of path between *AD* and *BD* into account, and by doing so he will discover that both events took place simultaneously for him too.

We thus arrive at the conclusion : Two events which take place simultaneously for an observer at rest, do not take place simultaneously for an observer in motion. According to the principle of relativity, the observer in the train is equally justified in considering himself at rest, and the observer on the embankment in motion ; hence, of course, the inverse assertion holds good : Two events which take place simultaneously for an observer in motion, do not take place simultaneously for an observer at rest. The idea of simultaneity is, therefore, a relative idea. The observer's state of motion determines whether or not two spatially distant events occur simultaneously for him.

We can generalise our result slightly, by supposing the contact of the lamp *B* to have a contrivance for delaying it, so that its flash of light will be emitted an instant (say the billionth part of a second) after the end of the train passes it. In this case the flashes of light will not occur simultaneously for the observer on the embankment either, but after a certain time-interval (the billionth part of a second) ; for the observer in motion, however, the time-interval will be still greater, because he is advancing towards the ray of light coming from *A*. This is an extension and generalisation of our former statement. We said then : If the time-interval between two spatially distant events is equal to zero for an observer at rest, it must differ slightly from zero for an observer in motion. In the more general case, we have : When an observer at rest measures the value t for the time-interval between two distant events, an observer in motion will measure a slightly different value t' for the time-interval between the same events. This is called the law of the relativity of time-measurement.[1]

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[1] A more exact mathematical analysis of the foregoing considerations—without going into detail concerning them—leads to the following : Let us suppose K and K' to be two systems of reference in uniform rectilinear motion with respect to each other, *e.g.* two very long platforms gliding past each other along their straight line of separation. In both systems clocks are set up at certain intervals along the line of contact, all of them going correctly. In addition, the clocks of the system K amongst themselves, as well as those of the system K' amongst themselves, are to be timed exactly alike. (Clocks are then going correctly when, with their help and with that of a standard measuring-rod, the measurement of the velocity of light results in the value c.) Furthermore, the clocks of one system amongst one another are then timed exactly alike when the following condition holds good : A light-signal is sent out at a point A at the moment when the clock stationed there shows the time t. It must then arrive at a point B at the moment when the clock stationed there shows the time $t+\tau$, τ being equal to

$$\frac{\text{distance } AB}{c}.$$

An observer at K who possesses a clock which is going correctly with reference to K is now supposed to compare the motion of his clock with the motion of those K'-clocks that he successively passes. (In the same way as a traveller compares his watch with the station-clocks he passes.) What the theory of relativity teaches is then as follows : He will find that the times registered by those clocks and his own watch differ in such a way that the K′-clocks are going more slowly. Similarly, an observer in K' will be able to state that the K-clocks which he passes are retarded with reference to his own watch, so that for him K-clocks are going more slowly than K'-clocks. This result can be briefly summed up in the theory of relativity thus : " Clocks in moving systems go more slowly than clocks at rest."

We find something quite analogous also in the relativity of measurements of length. We can easily deduce from the preceding considerations, that the length of a moving train as measured from the train itself must be different from the length as measured from the embankment. For this purpose we must be quite clear

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This short formulation is a convenient help for the memory, it is true, but on the other hand it must be applied with caution. For, according to the theory of relativity, every observer is fully justified in looking upon his own system as at rest and the other as in motion. If, therefore, C be a clock in the system K, and C' a clock in the system K', C would then have to gain or lose with reference to C', according as to whether the one system or the other were looked upon as in motion—and this was what the antagonists of the theory of relativity naturally regarded as a logical contradiction.

We must consider, however, that it is not sufficient for the comparison of the motion between two clocks to compare the position of their hands only at one particular moment of time—just at the moment, for instance, when both clocks pass each other; this comparison must, on the contrary, be repeated at certain intervals of time. Then, of course, both clocks are spatially separated and not close together, and a comparison of their readings can be carried out in this second moment of time in two different ways—viz. the clock C can either be compared with that clock of the system K' which it happens to be passing at that given moment (timed exactly with C', like all K'-clocks) or C' can be compared with that K-clock which it happens to be passing at that special moment (timed exactly with C, like all K-clocks). The statement of the theory of relativity, " Clocks in moving systems go more slowly," simply means that the comparison of C to C', in the two different ways indicated, leads to different results, in the sense above discussed. That it should be possible to arrive at different results is due to the fact that simultaneity in K is different from simultaneity in K'. K'-clocks, therefore, are timed alike for K'-observers, but not for K-observers, and vice versa. The apparent absurdity of this antithesis is of the same kind as that of the relativity of measurement of length, subsequently to be discussed ; we shall return to this in Chapter VIII.

what is meant by "measured from the train" and "measured from the embankment." If an observer in the train takes a measuring-rod, and begins laying it down repeatedly from the back buffer of the last car along the entire length of the train to the front buffer of the engine, then the figure representing the number of times the measuring-rod was laid down will be the length of the train, as "measured from the train." To measure the length from the embankment, we must determine two points *A* and *B*, so situated on the embankment, that the passage of the beginning of the train past *A*, and of the end of the train past *B*, are simultaneous events for an observer on the embankment. When we know the points *A* and *B*, we can determine their distance apart in the usual way, by the repeated laying down of a measuring-rod. The result of this measurement is the length of the train as "measured from the embankment." In our previous example the lamps *A* and *B* were at those points passed simultaneously by the beginning and by the end of the train (*i.e.* "simultaneous" for an observer posted on the embankment). Hence we must measure the distance between these lamps (to speak more precisely, the distance between the edges of their contact contrivances), which we will suppose to be, say, a hundred metres. We may then state: The length of the train as measured from the embankment amounts to a hundred metres. On the other hand, the flashes of light were not simultaneous events for an observer situated in the train. For this observer, the flash of *A* took place sooner than that of *B*. Hence he must conclude that as the beginning of the train travelled past *A* sooner than the end travelled past *B*, the length of the train must be greater than the line *AB*. For if the length had been the same, the lamps would have been passed simultaneously; if, on the other hand, the train were shorter, the end of it would travel past the point *B* sooner than the beginning past the point *A*. As this is not the case, the train must be longer than a hundred measuring-rods, each of one metre length, laid down on the embankment. To ensure the simultaneous flashing of the lamps for the observer in the train, the lamp at *A* would have to be moved slightly farther forward in the direction of the moving train, *i.e.* farther away from *B* to a point *A'*.[1] For the observer in motion the distance *A'B* will then be equal to the length of the train. Since *AB* is less than *A'B*, we have the following results: (1) The length of the train is smaller for an observer on the embankment (*i.e.* equal to *AB*) than for an observer in the train (who finds it equal to *A'B*). (2) For the observer in the train, the length of the track *AB* is smaller than the length of the train, whereas the observer on the embankment regards *AB* as equal to the length of the train. Hence objects in motion appear shortened to an observer at rest, and objects at rest appear shortened to an observer in motion. (One result necessarily arises from the other, because, according to the principle of relativity, both observers are equally justified in saying: I am at rest and the other is moving.) This contraction of length occurs only in the dimensions of objects lying in the direction of motion; hence only the length of a moving train is shortened, and not its height and breadth. The reason for this lies in the fact that measurements of height and breadth do not involve the detour over the determination of simultaneity. The

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[1] *Cf.* Fig. 2.

observer in the train can determine the gauge between the wheels in exactly the same way as the observer on the embankment can determine the distance between the rails, viz. by the use of his measuring-rod. If the wheels exactly fit the rails, both observers will agree that the gauge between the wheels and the interval between the rails are equal. To measure the height of the carriages from the train we can simply use a measuring-rod; to measure it from the embankment, we might proceed as follows:[1] people in the train could fix a sharp point to project sideways from the top and bottom of the cars. The observer on the embankment erects a large marble slab, the surface of which is placed vertically and parallel to the train's motion, and so close to the rails that the points projecting from the cars scratch two sharp lines in the surface of the marble slab, as the train travels past. The distance between these lines is the height of the cars as measured from the embankment. In a similar way the comparison of a standard measuring-rod in a system at rest (the embankment) with that in a system in motion (the train) could be carried into execution. This process is quite definite, reversible, and can be repeated *ad libitum,* so that there can be no difference of opinion between an observer at rest and an observer in motion as to the lengths of measuring-rods or as to the dimensions of any objects situated normally to the direction of motion.

Let us recapitulate the results of this chapter. State-

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[1] We must remind the reader once more that the experiments of which we are speaking are only conceptual ones. According to our experience concerning the comprehension of the theory of relativity we may even expect to hear an objection of this kind; "The theory of relativity is nonsense, for measurements such as those described above cannot be carried out."

ments of length and of time-intervals have no absolute meaning. We cannot reasonably maintain, for instance, that a pole has such and such a length; we must always state the motion of the object measured relative to the observer. Nor can we say: So many seconds of time elapsed between an event *A* in London and an event *B* in New York. To be exact we must add, "for an observer situated on the earth." This addition is necessary, because the time-interval between the events *A* and *B* for an observer on a passing comet would have a different value. We do not, in this case, mean the apparent interval between the events, *i.e.* the time-interval between the arrival of rays of light or electric waves, which bring the observer tidings of both events. On the contrary, we presume that both observers carry out their measurements correctly, and take into account the time needed by light to reach them from the places where the events happened.

CHAPTER VIII

THE APPARENT ABSURDITY OF THESE CONCLUSIONS

THE conclusions set forth in the last chapter present the quintessence of the special theory of relativity, which brought Einstein great celebrity on the one hand, and many attacks on the other. Seen with the eyes of a philosopher, they are indeed so revolutionary, that only one of two points of view can be accepted: Either it is all nonsense or it is an important forward step in our knowledge.

On the part of some professional philosophers, the objection has been raised against the theory, that it is illogical, and not in itself free from contradictions. That, however, is not true, and only shows that the matter has been misunderstood. It has been said, for instance: " One of the observers comes to the conclusion that the events at *A* and *B* took place simultaneously, the other, however, maintains that they did not take place simultaneously, and according to Einstein both are right. Now when two persons make contradictory statements, they cannot both be right." This argument (advanced even by academic critics) involves the mistake of overlooking the difference between absolute and relative statements. If I say, for instance, " My hand has five fingers," and somebody else tells me, " No, your hand has only four fingers," one or the other must of course be wrong, for my statement concerning the number of fingers on my hand is an absolute statement. But if a man at Cape Town says, " Madagascar is situated on the right-hand side of Africa," and another in Cairo says, " Madagascar lies on the left-hand side of Africa," they are apparently upholding contrary statements, but both are right from their own point of view, because the idea of right or left is relative. According to Einstein, the idea of simultaneity is also a relative one, and has lost its absolute meaning. This relativity, however, does not refer to the position taken up by the observer, as is the case with right and left, but to his state of motion. Certainly, for all practical purposes, we may safely continue to consider the ideas of time and space as absolute, for, as will be shown in Chapter X, the difference between statements of time and length for an observer at rest and for one in motion are, for all terrestrial events, always immeasurably small.

Similarly, it cannot be regarded as contradictory to logic, that the metre measuring-rod which shares the motion of the train is longer for an observer on the train, than the metre measuring-rod at rest on the embankment, whereas for an observer on the embankment his own measuring-rod is longer than the one in the train. We are so familiar with similar apparent contradictions with regard to other relative ideas, that we hardly realise them, as can be shown by the following trivial example : A calf and the old cow are grazing in a field. At some distance from them, another calf and another cow are grazing. The first cow naturally appears to its own calf larger than the other cow, owing to proximity. Hence the first calf says: " My mammy is larger than yours," and the other calf retorts: "No, my mammy is larger than yours." The calf's idea of " magnitude " involves only the angle subtended at its eye by the object seen, and if we accept the word " magnitude " in this sense, then, of course, each calf is perfectly right from its own point of view. When once we get used to accepting the spatial distance of two events as a relative idea, in the same way as the angular magnitude, the contradictions of the theory of relativity will disappear. Let us again point out the difference: The angle at which we see an object depends on the position of the observer, whereas the space- and time-interval between two events depends on his state of motion. That we have not previously noted anything of this relativity, is due to the circumstance that all motions carried out by human means are a million times too small to permit differences of length or time to be observed.

On the other hand, we can of course conceive (and as a matter of fact it is of frequent occurrence in the history of philosophy) of a succession of ideas being logically correct, and yet without use or purpose, these logical deductions having been derived from artificial suppositions of a nature entirely void of importance for our knowledge. We are ready to admit that, considered from a superficial point of view, some of the deductions in the last chapter are liable to raise a similar impression. On the part of antagonists, the theory of relativity has been banteringly termed a mixture of scholasticism and Talmud, and there is no doubt that this remark may appear very plausible to the new-comer, who approaches the theory equipped with the good old traditions of thought (called sound common sense). By way of example, let us take the proof given in the last chapter, that the path *AB* is smaller for the observer in the train than the length of the train itself. That proof is based on the following: The flash of light sent out from the lamp *A* takes place sooner for this observer than the flash of light sent out from *B* (not only appears to do so, for he takes into account the time needed by rays of light to reach him from the beginning and the end of the train); he concludes from this, that the beginning of the train arrives at *A* sooner than the end at *B*, and proceeds to conclude that the length of the train is greater than the path *AB*. Sound common sense, naturally taking the point of view of the simpler absolute theory, will raise the following objection: "In reality, the flashes of light at *A* and *B* were simultaneous. The observer in the train advances towards the rays of light coming from *A*, and therefore they reach him sooner; from this he concludes that the flash takes place at *A* first. He pretends to know nothing of his own movement, and that is where the scholastic hypocrisy of this entire way of thinking comes in." Such thoughts must arise in every reader who follows the matter attentively, so long as the absolute notions of time and space are rooted deeply enough within him. On the other hand, whoever has sufficiently penetrated into the progress of ideas in the theory of relativity will defend the "hypocritical" observer in the train somewhat as follows: That he ignores the fact of his motion is quite all right, for, as we have repeatedly emphasised, according to the principle of relativity, the statements that the embankment is at rest and the train in motion, or that the train is at rest and the embankment in motion, are quite equally justified, for the one thing that matters is the relative motion. If he, too, were an absolutist, the observer in the train might say: " The flash of light was sent out sooner from *A* than from *B* ; the observer on the embankment, however, was moving towards the rays of light coming from *B*, hence the emission of both signals appeared to him to occur simultaneously. That is only because he pretends to know nothing of his own motion." We thus see that both observers can reproach each other, and if we view the matter from a higher point of view, both are right ; however, they must not say that the events were " *in truth* " simultaneous or not simultaneous, but that they were simultaneous or otherwise as seen from their respective systems of reference.

The fundamental difference between the theory of relativity and scholasticism is this, that we are not dealing with subtleties designedly thought of, but with logical results drawn from two experimental facts of nature. Logic never was its own object with Einstein, but only the instrument with which he freed physics from an embarrassing situation.

CHAPTER IX

THE UNION OF SPACE AND TIME; THE MINKOWSKI-WORLD

IN this chapter we propose to consider the results of the special theory of relativity from a new point of view, which makes it all the more plausible to readers gifted with an imaginative faculty for geometry. Those who are not so equipped will perhaps find it difficult to keep up with the following discussion.

We shall commence with considerations that have as yet nothing to do with the theory of relativity, but which are based on the classical theory of absolute space. We shall call events like the flashing of a lamp, which take place at a certain point of space, and at a certain instant of time, "point-events." To determine the place and time of a point-event without ambiguity, we must state certain numbers, the so-called co-ordinates of the point-event. For the statement of time, only one figure is required, *e.g.* the number of seconds which have elapsed between midnight (Greenwich time) of the close of last century, and the occurrence of the event. But to state the place of the event, three numbers are required, because space has three dimensions. We know that to determine any place on the earth, its geographical longitude and latitude must be given. In that way, however, the point is not yet fully determined, because all points that lie vertically above each other have the same geographical longitude and latitude. Hence the height above sea-level must also be stated, and then the point is definitely determined. In this case the earth is the system of reference for our co-ordinates. To determine the positions of stars, of course, other systems of reference are used. On the other hand, in order to fix a particular point in a closed space, for instance in a room, it will be best not to use the geographical longitude and latitude, together with the height above sea-level, but to state the distances from each of two perpendicular walls, together with the height above the floor. To fix a point-event, we must therefore always give four numbers, three space-co-ordinates and one time-co-ordinate. For instance: an electric lamp hanging in a room 2·5 m. from the front wall, 3 m. from the left-hand side wall, and 2 m. above the floor, is to flash out at the time 12 seconds after midnight (chosen as the commencement of counting time). The co-ordinates of the point-event are then 2·5 m., 3 m., 2 m., and 12 seconds. Let us suppose another point-event: a second electric lamp standing on a writing-table in the right-hand corner of the same room is to flash out at the time 8 seconds, its space-co-ordinates being 1 m., 5 m., and 1·5 m. Now let us form the differences of the corresponding co-ordinates of the two events: 2·5 − 1, 3 − 5, 2 − 1·5, and 12 − 8. The difference between the time-co-ordinates 12 − 8 = 4 gives the time elapsing between the flashes of both lamps, the difference of the third space-co-ordinates 2 − 1·5 gives the difference in height of the lamps, and the difference of the other two pairs of co-ordinates indicates how much farther forward one of the lamps is situated than the other, and how much more to the right. According to the classical theory of absolute space and time, the time-interval between the two events is always equal to 4 seconds, quite independently of which system of reference is used. The spatial distance between the two lamps (the length of a thread stretched rectilinearly between them) also has a certain value, quite independently of the choice of the system of co-ordinates. (It is obtainable by a simple mathematical operation from the differences of the spatial co-ordinates, as will be well known to many readers. The result in our example is 2·55 m.) Now let us suppose another room enclosed within the first room, and that its walls are inclined to the walls of the first one. We can, of course, also give the positions of both lamps by means of the co-ordinates (distances from the floor and walls) relative to the second room. Now when the second system of reference is actually inclined to the first, it happens that not only are the co-ordinates of both lamps different from those of the first system of reference, but also the differences of their co-ordinates. But if we calculate the distance between the two lamps from the new differences of co-ordinates, we obtain exactly the same value as before (*i.e.* 2·55 m.). We may summarise these results as follows: The co-ordinates, taken singly, and also the differences between corresponding co-ordinates, are relative magnitudes; they vary according to the choice of the system of reference. The distance between two points, however, and the interval of time between two events are absolute magnitudes; they are independent of the choice of the system of reference. (*N.B.*—We are still speaking from the point of view of the old theory of absolute space and time.)

To be quite certain of being clearly understood, we shall give another example of the relativity of co-ordinate-differences, and for the sake of simplicity, we shall choose it in two dimensions. Fig. 3 shows the profile of a plateau bounded right and left by two hills *A* and *B*. We shall suppose the plateau to be slightly inclined towards the horizontal plane, and a place *O* to be situated at its lowest point. To determine the position of the two hill-tops with reference to this place, we can proceed to give their horizontal distances from *O* and their height above *O*. For that purpose we draw a horizontal straight line *h–h* through *O*, and drop perpendicular lines to it from *A* and *B*, which meet it in *C* and *D* respectively. *OC* and *OD* are then the horizontal distances of the hill-tops from *O*, and *AC* and *BD* their heights with reference to *O*. Hence *OC* and *AC* are the co-ordinates of the point *A*, and *OD* and *BD* the co-ordinates of the point *B* with reference to the system of co-ordinates chosen. Now we can imagine circumstances which make it convenient for the inhabitants of the place *O* not to draw the horizontal straight line *h–h* through *O*, but to draw a straight line *e–e* running parallel to the inclined plane itself, and to use this as the basis for the system of reference. They then define the perpendicular distances of the respective hill-tops

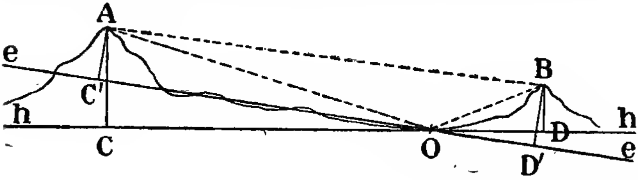

FIG. 3.

from that line *e–e* as their "height." In this case *OC′* and *AC′* are the co-ordinates of *A*, and *OD′* and *BD′* those of *B*. The hills have different "height" and different "horizontal distance" from *O*, as viewed from this inclined system of reference. But what always remain independent of the system of reference are the distances of the hill-tops in the ærial line from *O*, *i.e.* *AO* and *BO*, and the distance *AB* of the hill-tops from each other. Thus, what is really invariable is the distance between two points ; the differences of height and horizontal distances are only projections of this distance on a more or less arbitrarily chosen framework of co-ordinates. They take the part of shadows, and vary in magnitude and shape according to the position of the plane they fall upon. In other words, the concepts of difference of length, difference of breadth, and difference of height are not absolute and independent concepts. They are just the three dimensions, the three components of one single concept—spatial separation.

The reader may now ask : How is all this connected with the theory of relativity ? The reason should now be readily understood. Just as, for instance, the difference of height between two points according to our classic view has no absolute meaning, but depends on the choice of the system of reference, so the spatial distance between two points, measured in the aerial line, and the time-interval between two events, lose their absolute meaning according to the theory of relativity. These magnitudes, too, are liable to variation in value according to the system of reference. What is the result ? In the same way as we said before : The three spatial co-ordinates are only three single dimensions, the components of the notion of distance in space, so now we may say: All *four* co-ordinates of a point-event are neither independent, nor absolute; they are just the four dimensions, the four components of a united idea, which includes space and time simultaneously. " From this day forth, Space taken by itself, and Time by itself are to become mere shadows, and only a kind of union of both is to retain independence." These were the words with which the great German mathematician Hermann Minkowski introduced his lecture before the Association of Natural Scientists at Cologne in 1908,—where he first introduced this view-point of the theory of relativity. According to his proposal, the union between space and time was called " WORLD " by physicists. Furthermore, Minkowski showed that it is possible, by using this notion of " World," and with the help of an ingenious mathematical device, to give the mathematical treatment of the theory of relativity a form of such complete harmony, as had never been achieved previously by any physical theory. The relativistic mode of treatment, although it appears at first sight absurd to the layman, and, even when he has got used to it, at the least very complicated, turns out to be much more simple and more lucid for mathematical treatment. This in itself is a reason in favour of the theory of relativity, which must have weight with the theoretical physicist. For the experimental physicist, however, the deciding circumstance must be that there is no other way of uniting the two fundamental principles repeatedly mentioned, both of which have been proved by experience.

Thus the " world " has four dimensions; but whereas space has three equally justified dimensions, the fourth available dimension (*i.e.* time) is found to play a part of its own. Let us try to explain clearly the difference between equally justified dimensions and particularised dimensions. Let us suppose two lamps fixed in a room exactly above each other, *e.g.* one close to the floor, and the other 1 m. above it. If I contemplate them from a standing position, they appear to me to be *one above the other.* If I lie down horizontally on a sofa close by, and contemplate them from this side position, they appear to me to be situated *alongside of each other,* and if I observe them from above, so that my head is situated in the straight line connecting both lamps, they appear to be *one behind the other.* Thus I can convert "above" into "beside" or "behind" *ad libitum,* by choosing a suitable point of view, quite independently of the position in which, or at what distance, the two contemplated points are situated. In the example of the two hill-tops in Fig. 3, a suitable choice of the plane of reference can always result in the "difference of height" of the two hill-tops (the difference of their perpendicular distance from *e–e*) being equal to zero. How does this work, now, in the theory of relativity? Just as the difference in height and the horizontal distance in the example of Fig. 3 can acquire different values, according to the choice of the system of reference, so the spatial and temporal distances between two point-events can assume different values, when contemplated from systems of reference in different states of motion. But, whereas in the case of the two hill-tops *A* and *B*, a line of reference can always be chosen such that the "difference of height" defined above disappears, in the theory of relativity we may, *under circumstances,* choose a suitable system of reference in motion, such that the difference in time of two point-events is reduced to zero, but we cannot *always* do so. (We shall find more about this in the next chapter.) As shown above, we are able to exchange spatial co-ordinates by a suitable choice of a system of reference, and convert "above" into "alongside of," but we cannot do this analogously with all world-dimensions (*i.e.* we cannot completely exchange spatial distance with temporal succession). That the time co-ordinate plays a particularised rôle in the "world," follows as a matter of course, for our most primitive experience teaches us that time and space are different things. Up to the present it appeared to us that space and time were two absolutely different ideas, quite independent of each other, but the theory of relativity teaches us that this is not the case. The next chapter will show us how it is that they could be maintained for so long as independent ideas, and that for all practical purposes they still justifiably continue to be so.

Let us contemplate the classical Newtonian conception of space and time once more, before we leave it. In the introduction to his world-famed work, *Philosophiæ Naturalis Principia Mathematica*—rightly considered the fundamental pillar of physics and of exact natural science generally—Newton says:

> "I. Absolute, true, and mathematical time, of itself, and from its own nature, flows equably without regard to any thing external, and by another name is called duration.
>
> "II. Absolute space, in its own nature, without regard to any thing external, remains always similar and immovable."

According to Newton, absolute time glides along uniformly like a stream, quite independently of whether events take place in it or not ; and space exists like a large empty vessel, and, according to Newton, it would still exist, even if it contained nothing at all. Long before Einstein and Minkowski, physicists and philosophers (Ernst Mach perhaps with greatest clearness), had pointed out that Newton here makes assertions which go beyond the description of the actual facts of nature. Would anything like time exist if all matter in the universe lay dormant, and executed no movement whatsoever, and if nothing at all were happening ? Could space exist if it contained nothing ? These questions may perhaps appear to us as philosophical subtleties ; but it is necessary that we free ourselves from the conception of time as a stream, gliding along uniformly into eternity, and of the conception of space described above, before we can appreciate the ideas of Einstein and Minkowski, who regard both as single dimensions of a greater whole—the "World." For, according to the theory of relativity, time-intervals and spatial distances vary for observers in different states of motion.

Those readers who have followed with sufficient attention the considerations of this chapter on the Minkowski World, will perhaps, on thinking over all that has gone before, put the following question : According to the classical conception of space and time, the height differences and horizontal distances of two points A and B depend on the system of reference, and are nothing absolute ; but their spatial distance, measured in the aerial line, has an absolute value independent of the system of co-ordinates. In the theory of relativity, the spatial and temporal distances of two point-events take a similar part to those taken by height-difference and horizontal distance formerly.

Is there an absolute magnitude in Minkowski's World, which is geometrically capable of construction from spatial and temporal distance, and which plays the same part as formerly the distance measured in the aerial line, *i.e.* which is independent of the system of reference? This well-justified question must be answered in the affirmative; there is an absolute magnitude of this kind, and it is called the "World-distance" of the two point-events.[1]

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[1] For readers familiar with elementary mathematics, the subject-matter set forth in this chapter can be made clear in a few lines. If the co-ordinate differences of two points in space are x, y, z, we find, with the help of the well-known law of Pythagoras, that the value of the spatial distance of these points is

$$d = \sqrt{x^2 + y^2 + z^2}.$$

If any other co-ordinate system be used to give the position of the points, the value of the co-ordinate differences will in general be different, say x', y', z'. In calculating the spatial distance from these new co-ordinate differences, the same value as before again results; so that

$$\sqrt{x^2 + y^2 + z^2} = \sqrt{x'^2 + y'^2 + z'^2}.$$

Thus the case of classical geometry. In the theory of relativity the matter stands thus: Let us suppose x, y, z and t to be the spatial and temporal co-ordinate differences of two point-events. Now if a new system of reference be introduced, which is in motion relatively to the first co-ordinate system, and in which the co-ordinate differences are given by x', y', z', and t', we find that the equation

$$\sqrt{x^2 + y^2 + z^2} = \sqrt{x'^2 + y'^2 + z'^2}$$

is no longer strictly fulfilled; hence the spatial distance of two point-events (as mentioned repeatedly) has no absolute meaning; its magnitude depends on the choice of the system of co-ordinates. The real absolute magnitude mentioned above, which is fully independent of the co-ordinate system (the "world-distance" of the two point-events) is now given by the expression

$$\sqrt{x^2 + y^2 + z^2 - c^2t^2} = \sqrt{x'^2 + y'^2 + z'^2 - c^2t'^2}$$

where c represents the velocity of light *in vacuo*.

CHAPTER X

NUMERICAL CONSIDERATIONS

IN Chapter VII we drew the following conclusions from the two fundamental principles: (1) The flashing of two lamps *A* and *B* at a distance from each other, although taking place simultaneously for an observer on the embankment, does not take place simultaneously as seen from the train. (2) If the distance between the two lamps *A* and *B* as measured from the embankment is equal to the length of the train, then the corresponding distance, as measured from the train, is smaller than its length.

These conclusions, having been drawn without the aid of mathematics, are of a purely qualitative nature; we have not so far indicated the magnitude of the discrepancies between time- and length-measurements for an observer in the train and for an observer on the embankment. From the numbers quoted in Chapter III the reader will readily imagine that they must be very small, and that is indeed the case, as is shown in the following calculations, which contain numerical deductions resulting from the mathematical formulæ of the theory. The reader will have to accept on trust the statements in this, and in the next chapter, whereas the qualitative conclusions contained in the preceding chapters were of a purely logical nature, and could be readily controlled by the thoughtful reader.

In the first place, it is self-evident that the statements of the two observers will be identical, if the relative velocity between them is equal to zero. Secondly, the differences involved are practically equal to zero for velocities such as can be given to material bodies with the technical means at our disposal. For example: We shall suppose the train in Chapters VI and VII to travel with a velocity of 108 km. per hour, *i.e.* 30 m. a second, and that its length as measured from the train itself is 150 m. If the lamps *A* and *B* emit flashes of light exactly simultaneously for an observer on the embankment, the observer in the train (if he were able to carry out measurements of such exactitude—which he never can in reality!) would state that a time-interval of 0·00000000000005 seconds had elapsed between the two events. Furthermore, the length of the train as measured from the embankment would not be exactly 150 m. but 149·99999999999975 m. The difference in length, therefore, amounts to about the two-hundredth part of the diameter of an atom. If the velocity or the length of the train were smaller, these differences would be reduced still more. It follows that we should perceive no difference in the measurements of the distance- and time-interval between two point-events as seen from the train and from the embankment, even if the precision of our instruments were to be increased a milliard times.

To avoid unnecessary complications, and for all practical purposes, we are therefore quite justified in considering statements of time and space as absolute, so that we may say, for instance: " An iron rod has a length of ten metres," though we ought to add, to be quite correct, "for an observer with such and such a motion," *e.g.* who is at rest relatively to the earth. In point of fact, the length of the rod is different for an observer in motion relatively to it, than for an observer at rest relatively to it, but the difference is a milliard times less than our powers of measurement, and a million times less, for instance, than the changes which the length of the rod undergoes when its temperature is raised by a tiny fraction of a degree by the near approach of a human being.

Whilst the effects demanded by the theory of relativity remain infinitesimally small for those velocities that we have to deal with in practical life, they would assume considerable magnitude if we could succeed in reaching velocities approaching to the velocity of light. If we could travel with a velocity of 244,800 km. per second (this velocity would permit us to run round the equator about six times in a second), and if we were able to carry out an exact measurement of length during this furious journey (both possibilities are, of course, out of the question), the observer on the embankment would find the length of the train to be only half as long as would the observer in the train. But if the train were travelling with the velocity of light, its dimensions contemplated from the embankment would be reduced to zero altogether. Of course the distance of the two lamps *A* and *B* would, on the other hand, also be equal to zero for the observer in the train, since all these relations are reciprocal ones, as already mentioned. The reader may ask: What happens, if the observer moves with a velocity greater than that of light? Let us remind him of what we emphatically declared in

Chapter VI, that the special theory of relativity is based on the supposition that no effect can be transmitted with a velocity greater than that of light; hence no material bodies can move with velocities greater than *c*. According to the theory of relativity, the velocity of light plays the part of a limiting velocity, which cannot be exceeded, and which is never attainable by material bodies. We shall have more to say on this point in the next chapter.

A traveller going along with a velocity approaching that of light, would make other curious observations in connection with the relativity of the notion of time. Let us suppose that by the year five thousand the development of human technics had advanced so far as to admit not only of an inter-planetary service with other planets of our solar system, but also, that we should be able to visit the planets of distant fixed stars, and had established colonies there. Besides this, we shall imagine the introduction of an interstellar time, so that the inhabitants of distant fixed stars could set their watches to agree with terrestrial watches by means of wireless signals. As we know that the idea of time is relative, we shall define interstellar time as the correct time for an observer at rest relatively to our solar system (supposing our earth to preside in the Union of Stars). The world-ships, which carry on the service between the stars are so constructed as to be accelerated more and more after starting from the earth, so that their speed nearly attains to the velocity of light; and they do not slacken speed until they get near the distant planets, where "brakes" are applied to enable them to land slowly. We shall suppose that, in the year 5500, a traveller goes on board one of these world-ships to visit a colony on a planet belonging to a fixed star, at a distance of about one hundred light-years away from the earth. The ship starts with full force, and increases speed until the utmost velocity (equal nearly to c) has been reached. We shall assume that this process of acceleration, according to the watch of the traveller and the ship's chronometers, lasts six months. From the instant the maximum velocity is attained, only a few seconds elapse (according to the statement of the ship's clocks) until they are sufficiently near to the planetary system of the distant fixed star to require to slacken speed again, and thus reduce the velocity of the world-ship to rest, a process which again occupies six months. That part of the journey performed with the maximum velocity, and during which by far the greatest part of the distance is covered, appears to the traveller only a moment. For him the duration of the journey involves only the period of acceleration and the period of retardation, or a year altogether. But when he leaves the ship in the planetary colony, he will find himself in the year 5600 inter-stellar time, and if, after a stay of a few weeks, he then returns to the earth, he will not reach it until the year 5700. Meanwhile generations of men have disappeared, his great-grandchildren are dead and gone, but he himself is little more than two years older. We see that the dream of H. G. Wells' Time-Machine might be realised, if we could succeed in some way in imparting to our means of locomotion velocities approaching the velocity of light.

Let us return to everyday reality. The sober reader will be surprised that exact science places before him such fantastic pictures. The sceptic will say: " No such stuff for me; why should I believe in such nonsense?" And those who are ready to disprove the theory by counter-arguments will exclaim: "That fellow Einstein is a cursed nuisance! He twists matters so that we can't get at him. For all reasonable velocities attainable in practical life, the effects are so small that we can't measure them; and yet we are to believe that the most fanciful results hold in circumstances such as are absolutely unrealisable with our modern technical means, so that we are unable to put them to the test." To this we give the same reply as at the end of Chapter VIII: These conclusions arise with unyielding logic from the two fundamental assumptions (the principle of relativity and that of the constancy of the velocity of light). As long as our experience does not disprove the validity of these two laws, we are compelled to believe deductions made from them, and we must do so all the more when we take account of the fact that small bodies do exist in nature, the velocity of which approaches the velocity of light. Exact measurements made on them confirm the validity of the consequences drawn from the theory of relativity. Of these we shall speak in the next chapter.

Before doing this, we must draw attention to a point of importance. According to the special theory of relativity, it is possible that two events may happen at different points of the earth's surface and at different times for a terrestrial observer, but that they take place simultaneously for an observer in motion relatively to the earth. But we are not to understand this as meaning that a system of reference of this kind could be discovered, from which an event happening in London to-day, and another event happening in

New York to-morrow, would be found to take place simultaneously. This is quite impossible, because two events of such spatial proximity (the distance London–New York is very small compared with astronomical distances) can only be seen simultaneously by observers on a system in motion, if the time-interval between them, as observed from the earth, is very small. Let us state this numerically: Two events which occur at different places can be simultaneous for an observer in rapid motion, only if the interval of time between their detection for a terrestrial observer is smaller, or at most, equal to the time required by light to travel from one place to the other. Thus, if the rectilinear distance between the two places where the events happen is 1000 km., the time-interval between the events for a terrestrial observer must be less than, or at most equal to, $\frac{1}{300}$th second, in order that an observer in motion may detect simultaneity. If we accept the time-interval of $\frac{1}{300}$th second, the moving observer would have to travel with the velocity of light, if the events were to be simultaneous for him. If the time-interval measured by a terrestrial observer were $\frac{1}{600}$th second, the events would be simultaneous for an observer moving with one-half of the velocity of light, whereas they would occur in the reverse order for an observer moving with the velocity of light. What has been said here may serve as illustrative of the statements made near the end of Chapter IX, where the singular part taken by time amongst the four " World " dimensions was under discussion.

There is one other far less obvious, but fundamentally much more important difference between the interchangeability of the spatial dimensions (cf. p. 66), and the dependence of the spatial and temporal distances of two point-events on the state of motion of the system of reference. To return to the example of the two hill-tops, we can arrange, by turning the system of reference, that both the " height-difference " and the " horizontal difference " will be altered. If we investigate the different possible positions of the system of reference, we discover that if the system of reference be turned in such a way as to make the height-difference smaller, then the horizontal-difference will become larger, and vice versa. The case is different, however, in the relativity of space and time. Let us contemplate two point-events from two systems of reference moving relatively to each other. If the time-interval between the two events in the second system is greater than in the first, then the spatial distance will also be greater than in the first, and vice versa. (Compare the numerical example at the beginning of this chapter, where the length of the train as well as the interval of time between the flashing of the two lamps, as measured from the train, is greater than as measured from the embankment.) As we said before, this fact is not so obvious to the non-mathematician, and yet it is just this less apparent circumstance which determines the character of the Minkowski world, and creates the profound difference between space and time.[1]

That two terrestrial events can take place simultaneously for a moving observer only when they occur within a small fraction of a second for an observer at

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[1] Mathematically formulated, this circumstance is due to the fact that the square of the time-difference, t^2, enters into the expression for the " world-distance " (cf. footnote, end of Chapter IX) with the opposite sign to that of x^2, y^2, and z^2.

rest on the earth, is due to the circumstance that the velocity of light is so immense. This magnitude *c* represents a fundamental magnitude in physics, and it is convenient, therefore, to choose the units of length and time in such a way as to make the velocity of light equal to unity. If we retain the second as the unit of time, the unit of length will be 300,000 km. In these "natural" units—as we may call them—the duration of human life is a very long one, for it amounts on the average to many millions of seconds. The spatial scene of our existence, on the other hand, is very limited, for the diameter of our globe measures only about 0·04 units of length. It will perhaps appear to the reader that this statement is without import, because the choice of our units of length and time is quite arbitrary, and we can always arrange matters in such a way that the earth's diameter measured in these units is equal either to a large or to a small number, *ad libitum.* But when we choose the *relation* between the units of length and time so that the velocity of light is equal to unity,—which is physically well-founded, after what has been said above,—the numerical measure (however we choose the single units) of the duration of our life will, in any case, be many millions of times greater than that of the spatial extent of our activity.[1] We will return to this point in the second part of this book.

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[1] This statement implies that all motions carried out by or associated with human beings are very slow as compared with the velocity of propagation of light.

CHAPTER XI

FURTHER CONCLUSIONS AND THEIR EXPERIMENTAL VERIFICATION

IN our analysis of the idea of simultaneity (cf. footnote in Chapter VI), stress was laid on the fact that the Einstein definition of simultaneity, which involves the law of the constancy of the velocity of light, is without purport unless there is no other effect whatsoever that is propagated with a velocity greater than *c*. Hence it is not surprising that this theory subsequently gives rise to results, according to which material bodies can never be given velocities greater than *c* on the one hand, and on the other that absurd results would ensue if we were to assume that any (even if not material) effect could be propagated with a velocity greater than that of light. If an effect of this kind existed, one could devise experiments in which the effect preceded the cause. This is quite contradictory to experience, and so we shall once again have to conclude that there are no such things as effects which are propagated with a velocity greater than that of light. Besides, as mentioned in the last chapter, for an observer at rest the length of a body moving with the velocity of light would be reduced to zero, and furthermore, calculation shows that such an observer would obtain an imaginary number for the length of a body moving with a velocity greater than that of light, a result which, from a physical point of view, would be absolutely unreasonable. Thus we are again confronted with the velocity of light in the theory of relativity, as the upper limit of all velocities. This is in complete accord with our experience, as we know of no effect which travels more quickly than light. At one time it was assumed that gravitation was propagated with a greater velocity than light; but this has proved to be erroneous. It is interesting to follow up the matter in regard to the velocities attainable by material bodies. All velocities connected with human traffic, shooting, and so forth, are so ridiculously small compared with the velocity of light that they do not count at all. Even the much greater velocities which appear in astronomy,—the velocities of planets and comets of the solar system, those of fixed stars and of meteors, the latter of which occasionally shoot through our atmosphere,—are, as a general rule, many thousand times smaller than c. Nevertheless, there are material bodies in nature which move with velocities approaching the value c. These are the atoms of electricity, the so-called electrons, such as pass through the evacuated space of a Röntgen bulb with enormous velocity when it is working. Moreover, some of the rays emitted by radio-active substances (β-rays) consist of electrons, emitted by single atoms of these substances with an incredible velocity. The atoms of the elements themselves, when passing through rarefied gases during electric discharge, attain smaller velocities (though still enormous compared with those of cannon balls or stars), so do also the so-called α-rays of radium, which, as we now know with certainty, are nothing else than electrically charged atoms of the rare gas helium. The velocity of all these particles has been found to be measurable, and it was discovered that it varies according to experimental conditions. Some have relatively very small velocities of only a few hundred km. per second (*i.e.* less than that of many comets), but others have velocities approaching the highest possible velocity of about 300,000 km. per second. If we put all the velocities appertaining to material bodies in nature in a row, we find an unbroken sequence of velocity. From the movement of glaciers, which only amounts to a small fraction of a millimetre per hour, up to the incredibly large velocities of the electrons in the form of β-rays, every possible velocity is represented somewhere in nature; but slightly below the velocity of light the scale ends. From the point of view of the old classical physics, this might be regarded as merely accidental; according to that view, it might be thought possible that a β-ray might also possess a velocity of 310,000 km. per second. From the point of view of the theory of relativity, however, the upper limit of velocity is not accidental, but is a natural law. It is impossible to have velocities greater than that of light.

The theory of relativity is not content in merely laying down the law that velocities greater than c are impossible; on the contrary, with the help of mathematical formulæ, it actually gives a reason why greater velocities cannot exist. To make this clear, we must proceed still further. It was explained in the first chapter that, according to the classical theory of mechanics, the principle of relativity is strictly valid for mechanical processes based on the old ideas of space and time. If we replace these by the new Einstein-Minkowski-world ideas, we find that the principle of relativity of classical mechanics can no longer be valid for mechanical processes. As this principle, however, is supposed to be a general law of nature, the theory of relativity has no other choice than to state that classical mechanics is not strictly valid, and is in need of correction. In point of fact, this is not difficult to accomplish. A very slight alteration in the fundamental equations of mechanics enabled Einstein to make them satisfy the principle of relativity, on the basis also of the new views of space and time. These changes are of such a kind that we can neglect the deviations from classical mechanics which occur for velocities met with in human technics or in astronomy. For velocities, however, approaching the velocity of light, the deviations from the laws of the older mechanics are considerable. They consist in the following: In order to set a body into motion (*i.e.* to accelerate it), we must, as is well known, apply a force to overcome its inertial resistance. According to the elementary Newtonian fundamental law of mechanics, this force is equal to the product resulting from the inertial mass of the body multiplied by the magnitude of the acceleration. If, for instance, a mass of 1 kg. is to be accelerated from the condition of rest so as to attain a velocity of 10 m. per second at the end of 1 second, a certain force must be applied. In the example before us, this force would be about equal to the force exerted by gravity on the kilogram weight. To cause the same body to increase in velocity from 10 m. per second to 20 m. per second in the next second, one would, according to classical mechanics, have to expend the same force, and so on. Thus to increase the velocity of the same kilogram weight within another second from 10,000,000 m. per second, for instance, to 10,000,010 m. per second, exactly the same force would have to be applied. In other words, the quotient resulting from force and acceleration (*i.e.* the inertial mass) is a perfectly definite number for a particular body, and is quite independent of the velocity. This no longer holds good in relativistic mechanics : according to this, it is not quite immaterial whether a body is accelerated from the velocity zero to 10 m. per second, or from the velocity 10,000,000 to 10,000,010 m. per second. The force required in the latter case would be rather greater. In other words, the mass of a body is not constant, but slightly increases with increasing velocity (in contradiction to the statement of the older mechanics made above). This dependence of the mass on the state of motion is not detectable in the small velocities of daily life ; the inertial mass of a railway train of 200 tons weight is, when at rest, only about a hundred thousandth part of a gramme less than if it were to travel at the rate of 100 km. per hour. But the changes in the mass of a body, the velocity of which is approaching the velocity of light, are very considerable, so that the mass of every body would become enormous if its velocity approached that of light. If we accelerate a particle of dust lying on one of our fingers, we cannot feel its inertial resistance at all ; but all the forces stored in our solar system would not suffice to accelerate this self-same particle of dust, if once it had attained the velocity of light. Thus, from the point of view of the theory of relativity, we are able to understand why the manifold velocities of material bodies existing in nature are one and all limited just within the boundary of the velocity of light.

This fact of the limitation of the scale of velocities is a striking circumstance, which undoubtedly tells in favour of the theory of relativity; but, on the other hand, it cannot be taken as a direct proof of its validity. It was found possible, however, to examine Einstein's deduction of the dependency of mass on velocity for the β-rays of radium, and the result proved that the inertial mass of a body really does increase with increasing velocity, and by the amount demanded by the formulæ of the theory.

The theory of relativity has achieved a surprising triumph in recent years, since the German physicist Sommerfeld in Munich succeeded in explaining mathematically the so-called fine structure of spectral lines of hydrogen and helium with the help of this theory. A few remarks on this subject will be useful. If a photograph be taken of the spectrum of a luminous electric discharge in rarefied gases (Geissler-tube), very sharply defined lines appear on the plate, these belonging to light of a distinct colour (*i.e.* of a certain frequency of oscillation). These lines always appear in considerable numbers, series of lines showing on the plate at intervals which, though unequal, nevertheless succeed each other with a certain mathematical regularity. This kind of spectrum has therefore received the name of series-spectrum. Up to a very short time ago our knowledge of the mechanism of the processes in the atom of a luminous gas, emitting these series-spectra, was incomplete. In 1913 the Danish physicist N. Bohr succeeded, with the help of the quantum theory proposed by the German physicist M. Planck, in throwing light on the process of the emission of series-spectra. A discussion of this theory, which is in many respects still more complicated and more mathematical than the theory of relativity, would take us too far. We shall merely mention that each of the atoms of the elements is supposed to be a sort of planetary system. In this the so-called nucleus of the atom takes the part of the sun; it is charged with positive electricity and its mass constitutes almost the entire mass of the atom. Electrons, much lighter in weight than the nucleus (*i.e.* tiny particles, considered to be the atoms of negative electricity) move around it in circular or elliptic orbits, like planets around the sun. The physicist Bohr, by applying the laws holding good for planetary movement in astronomy to the orbits of electrons in the atom, and by combining them with the laws of the above-mentioned quantum theory of Planck, arrived at a theory of the series-spectra of hydrogen and helium which agrees splendidly with experience.[1]

This is not immediately connected with the theory of relativity. A closer analysis of these series, however, gave the following results. The single lines of the spectral-series are not in general simple lines; on the contrary, they are complicated lines consisting of two, three, or more lines very close together, so that when they are investigated with a spectral apparatus of small dispersion, they appear to shrink into one single line. The best known example of this is the *D*-line of sodium, familiar to all who have ever observed a luminous flame containing sodium with a spectral apparatus. The Bohr theory could not explain the appearance of line-doublets, triplets, etc., in the spectral series of hydrogen and helium. At this juncture, in 1916, Sommerfeld showed that if the orbits of electrons in the atom

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[1] In the case of helium, only for those series-lines which occur in the so-called spark spectrum.

(similar to those of planetary orbits) are not calculated according to the laws of classical mechanics, but according to relativistic mechanics, taking into consideration the above-mentioned dependency of the mass on velocity,[1] we obtain not only the general structure of spectral series already elucidated by Bohr, but also the fine structure of the single lines. In addition to this, Sommerfeld was able to predict certain complicated groups of lines in the helium spectrum from calculations based on the theory of relativity, which were subsequently confirmed by Paschen in Bonn by means of very delicate spectral measurements.

Let us survey the results we have so far obtained in experimental confirmation of the theory of relativity. First of all, as shown at the end of Chapter VI, the foundations of the theory — the two fundamental principles—have been supported and strengthened by the most careful and exact experiments, so that we should believe the validity of the theory of relativity even if no further experimental evidence were available. That a striking experimental proof of the correctness of the deductions from the theory of relativity was not immediately forthcoming, which would dispel all doubt, is due to the fact that all deviations from the old laws of mechanics and electro-dynamics, and all the divergencies from our old conceptions of space and time, are immeasurably small for most of the known processes of nature. It is only for the enormous velocities connected with the orbits of electrons in the atom, and for the

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[1] The variability of mass is much more important here than in the case of actual planetary orbits, because the velocity of the electrons in the atom far surpasses that of planets or fixed stars.

β and cathode-rays, that the theory leads to a deviation from the older classical physics, and all experiments on bodies moving with such enormous velocities have, as a matter of fact, decided in favour of the theory of relativity.

Finally, we must refer to those deductions which Einstein stated to be physically the most important result of the theory of relativity. As above mentioned, the mass of a body is greater when it has a velocity v than when it is at rest. Mathematically formulated, the matter can be stated thus: If m_o is the mass of a body at rest and m_v its mass when it has the velocity v, the increase in mass caused by the motion $(m_v - m_o)$ is equal to the kinetic energy possessed by the body with the velocity v, divided by the square of the velocity of light.[1] This last magnitude amounts to an immense figure in the usual physical units of length and time (cm. and sec.), *i.e.* to 900 trillions; thus the increase in mass is immeasurably small for the usual velocities possessed by bodies. If the velocity of the same body be again increased, say from v to $2v$, its mass would again increase by an amount given by the increase of kinetic energy (produced by the increase in velocity) divided by the square of the velocity of light. Hence the increase in mass is proportional to the increase in kinetic energy.

Now Einstein was able to show that not only does an

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[1] We know that the kinetic energy of a mass m with velocity v is given by $\frac{m}{2}v^2$. The increase of mass is therefore $\frac{mv^2}{2c^2}$. This formula is exactly valid only for velocities (v) small compared with c, because, according to relativistic mechanics, the kinetic energy for very large velocities is no longer given by $\frac{m}{2}v^2$.

increase of kinetic energy produce an increase in the mass of a body, but that *every* increase of energy produces such an increase of mass. If, for instance, energy in the form of heat be conveyed to a body, an increase of mass will occur, and this increase of mass will, as in the case of the kinetic energy before mentioned, be equal to the heat energy absorbed, divided by the square of the velocity of light. A direct examination of this law (say by weighing the same body before and after the heating) cannot be performed on account of the infinitesimal smallness of the effect. Certain considerations, however, have led us to suppose that the theory of relativity will perhaps best explain a fundamental problem of chemistry, namely, the problem of the deviation of atomic weights from whole numbers. As is well known, the atomic weights, *e.g.*, of carbon, nitrogen, and oxygen, are almost 12, 14, and 16 times greater, respectively, than the atomic weight of hydrogen. But these numbers do not agree quite exactly; on the contrary, divergencies of somewhat less than 1 per cent. have undoubtedly been established. Now it cannot be regarded as fortuitous that the ratios between the atomic weights of these elements (which succeed each other in the Periodic System of the Elements) and that of hydrogen lie so close to the three successive even numbers 12, 14, and 16. Why these ratios are not exactly equal to the numbers given, has been hitherto quite inexplicable. But the theory of relativity supplies us with a possible explanation. If we suppose, for instance, that a *C*-atom consists of 12 hydrogen atoms, or of 3 helium atoms (atomic weight 4), or of any arrangement of these constituents, then the combination of these constituent parts to a single atomic nucleus is bound to involve the rearrangement of large amounts of electrical energy, from which will ensue the small variations of mass. On the basis of these variations of mass required by the theory of relativity, we can account for the deviations from whole numbers of atomic weight ratios.

At the end of last chapter we mentioned that it is convenient to choose the units of length and time in such a way that the numerical measure of the velocity of light is equal to unity. If we use these natural units in what follows, then the proportionality factor between increase of energy and increase of mass (*i.e.* the square of the velocity of light) will also be equal to unity, and we may then formulate our law in a simpler way. Increase of energy is always accompanied by an equal increase of mass. (This does not in the least alter the facts with regard to our former assertions, because the energies conveyed to a body in the form of heat, etc., if expressed in these natural units, are infinitesimally small.) Now before the introduction of the theory of relativity, it was known that every body, whether hot or cold, always possesses a certain amount of energy. This consists of heat energy, stored within the body, together with the energy of chemical affinity (such as that released in the process of combustion), but probably for the main part of enormous amounts of energy situated in the inside of atomic nuclei, and hitherto not rendered evident except in the case of the radio-active elements. We cannot say, for example, what the total energy contained in a litre of coal gas amounts to, as we can only measure the differences of energy which become free in chemical reactions. By analogy with radioactive substances we might expect that the total energy is very considerable. Now the theory of relativity teaches us that every increase in energy is equivalent to an increase in mass. This leads to the plausible assumption, immediately made by Einstein, that the total mass of a body is equal to the energy stored within it. Mass and energy, according to this view, become identical. And, as a matter of fact, just for these two magnitudes, two fundamental laws of nature of analogous structure are found to have validity; *i.e.* the law of the conservation of mass, and the law of the conservation of energy. Let us suppose a system of bodies surrounded on all sides by an impenetrable envelope, which allows neither radiation nor heat to pass through it. The law of the conservation of mass maintains that the total mass of all the bodies contained in this envelope remains constant, whatever process they may undergo amongst themselves, in the way of chemical reactions, explosions, or combustions, etc. Exactly the same is maintained with regard to energy. Within the envelope, chemical energy may be transformed into thermal energy, and this into mechanical energy; but the total sum of the energies always remains the same. According to the theory of relativity, these two fundamental laws of nature reduce to one law, for mass and energy are one and the same thing.

In order to elucidate the statement of the identity of mass and energy, and to avoid its remaining an empty word, let us analyse the idea of mass. First of all, we must state that we have to deal here with a dual idea, which has been treated in physics as a single one only, owing to an accessory circumstance which plays an important rôle in the second part of this book. In general, the mass of a body is measured with a balance; we determine the force with which gravity pulls it downwards, *i.e.* we compare it with the force exerted by gravity on the unit of mass. The result of this weighing can therefore be designated the gravitational mass of the body. The idea of the inertial mass of a body is somewhat different from this. It is the resistance offered to acceleration, *i.e.* according to the fundamental law of Newtonian mechanics mentioned at the beginning of this chapter, the quotient between force and acceleration. We know from experience that the inertial mass is for all substances always proportional to the gravitational mass; hence, if one body is twice as "inert" as another, it must also be twice as heavy.[1] Now, if we maintain that every form of energy involves mass, we mean that it possesses a certain inertia and a certain weight. If, for example, energy is conveyed to a body in the form of heat, that body will become heavier and more "inert."

We thus arrive at what at first sight appears a startling result, namely, that even an evacuated space which is transmitting energy can be said to have weight and inertia. Thus, if we completely evacuate a vessel (supposing it were possible to remove the last remnants of the gas molecules), the evacuated interior would still be permeated by electro-magnetic radiation (*e.g.* by light-rays, if we are dealing with a glass vessel situated in a lighted room, or if this is not the case, at all events by heat-rays, which are always present, even at the lowest attainable temperatures). But as every kind of radiation transmits energy, so every vessel, even if it

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[1] This is by no means self-evident, and does not follow from the definitions of these notions, as we shall show in detail in the second part of this book.

does not contain any tangible substance, contains energy; consequently even the empty interior of the vessel possesses gravitation and inertia. This result may sound paradoxical, but it is interesting to note that the deceased Viennese physicist Hasenöhrl, before the theory of relativity was developed, and by starting off from completely different considerations, finally arrived at the same result, *i.e.* that inertial mass must be associated with heat radiation in an empty space.

If we leave the "natural" units of measurement, and return once more to the usual C.G.S. system of units, the law in question must be stated thus: The energy contained in a body is equal to its mass, multiplied by the square of the velocity of light, *i.e.* by 900 trillions. This figure is stupendous, and it takes one's breath away to think of what might happen in a town, if the dormant energy of a single brick were to be set free, say in the form of an explosion. It would suffice to raze a city with millions of inhabitants to the ground.[1] This, however, will never happen, because, as we know from radio-active phenomena, these enormous quantities of energy contained in the nuclei of atoms are only liberated with extreme slowness, and are entirely uninfluenced by human agencies.

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[1] It would suffice to lift two millions of battleships of the Dreadnought type to a height of 1000 m.

END OF PART I

PART II

THE GENERAL THEORY OF RELATIVITY

CHAPTER XII

ON INERTIA AND GRAVITATION

NECESSITY led to the origin of the Special Theory of Relativity. If the principle of relativity and the principle of the constancy of the velocity of light are held to be right, there can be no further choice; all those deductions made by Einstein follow by compulsion, as in a mathematical problem.

To proceed with the development of the theory did not seem absolutely necessary from a physical point of view, but its continuation was carried out by Einstein with unexampled perseverance and consistency during the years 1907–1915. The main motive force here at work was Einstein's philosophical perception; he saw clearly that even his new theory, and above all, the Newtonian theory of gravitation (accepted without any modification up to that time) still possessed all those deficiencies of a philosophical nature, that have been clearly and keenly criticised by a number of philosophers in the course of the last half-century—without any of them being able to improve upon the theory. What these deficiencies were, we shall show in the following.

At the beginning of this book we made the assertion: " It is only reasonable to talk of a relative motion between bodies; to speak of absolute motion is unreasonable, because it cannot be proved." When we said " motion," we meant uniform rectilinear motion. For motion in general, however, this statement is not valid, because the existence of a non-uniform motion can be detected quite well, without considering the surroundings, since inertial forces are brought into play. If, for instance, a train be suddenly stopped, we cannot fail to notice this distinctly; in a railway collision the inertial forces called forth by the change of motion may prove absolutely fatal. According to Newtonian mechanics, and also according to the mechanics of the special theory of relativity, these effects do not depend solely on the *relative* accelerations of the bodies to each other, but on their *absolute* accelerations. That is to say: if there were a railway-train in the world and nothing else, *i.e.* if there were nothing relatively to which it moved, the motion itself would not make itself felt, but every change of motion would. In starting or in slowing down of the train, the same phenomena would occur as with a train accelerated relatively to the earth. Now this means neither more nor less than that the idea of absolute space—against which the theory of relativity combated so successfully—comes to the front once more. Uniform motion relative to absolute space is not compatible with reason, and is not perceptible according to the principle of relativity. A change of motion with respect to absolute space is doubtless just as incompatible with reason,

—and yet it is to call forth noticeable effects! However correctly the Newtonian theory, based on such foundations, could explain all astronomical and terrestrial phenomena, it was unable to satisfy the philosophical and scientific frame of mind of a man like Einstein.

The matter had been further aggravated by the special theory of relativity. Formerly one had believed in an æther as taking the place of absolute space. It might have seemed reasonable to physicists and philosophers that accelerated motion relatively to an *æther* (if æther should actually exist) would be able to call forth forces of inertia. But after giving up the idea of a substantial æther, owing to the special theory of relativity, it was not permissible to go on believing that acceleration relatively to " Nothing " should call forth forces of inertia. On the other hand, there is no empirical reason for believing this. In order to make an experiment of this kind we cannot get rid of the earth and the stars. Hence, if our intelligence cannot grasp that forces of inertia should act in an accelerated railway train—supposing it to be quite alone in the world—we are not in any way compelled to believe it! Let us take advantage of this fact and recapitulate for this case what experience tells us on the one hand, and reason on the other: If a body is accelerated relatively to other bodies, inertial forces are called forth; a single body quite alone in the universe would have no inertia.

This last statement is evidently equivalent to the assumption, that the capability of a body to exercise inertial forces (*i.e.* to possess inertial mass) is caused solely by the presence of other bodies in the universe. Hence inertia, according to this mode of thinking, is not something appertaining to every body of itself;

it is caused, on the contrary, by the interaction between it and the other bodies of the universe, just as the weight of a body is caused by the interaction of the body and the earth.

As shown in the foregoing developments, we are driven to these opinions merely on the grounds of the thought process ; it is *more plausible* to imagine inertia to be an interaction of the kind described, than to believe that a single body may possess inertial mass on its own account. One important fact based on experience, which lends additional support to these purely abstract arguments, is the fact of the proportionality between inertial and gravitational mass. At the end of the first part of this book, we explained that inertial mass and gravitational mass are essentially two perfectly distinct notions, but that according to our experience the inertial mass of a body is always proportional to its gravitational mass. This fact of experience occurs in Newtonian mechanics as a perfectly independent law, which has nothing to do with all other laws. The mathematical foundations of classical mechanics would remain completely unchanged if this law did not claim validity.

One could, for instance, imagine a priori that for different substances the ratios between the inertial and gravitational masses differ from each other, just as they differ for the specific gravities of different substances. For instance : A platinum ball is about three times as heavy and three times as inert as an iron ball of the same size. Let us suppose iron and platinum to have different specific gravities, but equal specific inertia, so that two balls of the same size, one of platinum and the other of iron, offer the same resistance to change of motion. This would have been quite possible from the Newtonian point of view, without interfering in the least with the validity of its mechanical fundamental law. This law states: The product of the inertial mass and the acceleration is equal to the force. If the above-mentioned supposition were fulfilled, the gravitational force acting on the platinum ball would be three times as great as that acting on the iron ball; but since the inertial masses are to be equal, the platinum ball would have to fall to earth with three times the acceleration of the iron ball. Thus if the law of the proportionality between inertial and gravitational mass did not hold good, different bodies (even without considering air resistance—*i.e. in vacuo*) would fall with different velocities. But that is not the case, as we can easily convince ourselves with the simple guinea and feather experiment in an evacuated tube. Furthermore, the proportionality between inertial and gravitational mass has been accurately proved by Eötvös' experiments, which were carried out with an exactness of 0·00001 per cent.

This empirical fact being known, physicists took note of, registered and filed it, but no further use was made of it! We can, however, apply it at once to the considerations of this chapter. If inertia and gravitation are so intimately related by the law of proportionality, this will naturally strengthen the conception discussed above, according to which inertia and gravitation are caused by the mutual interaction of bodies. In point of fact, we shall see in the following chapter that, in the hands of Einstein, the empirical fact of the proportionality between inertial and gravitational mass, left unused by physicists for two centuries, became the key to the generalisation of the theory of relativity.

CHAPTER XIII

THE EQUIVALENCE-HYPOTHESIS

WE shall begin the generalisation of the theory of relativity by discussing the question: Can we imagine that the existence of non-uniform motion escapes our observation, and that it is no more detectable than is the case of uniform motion? For the moment this seems hopeless, for any change of motion produces forces of inertia, and these must always reveal to us the existence of such change. How curious then that Einstein says: Inertial forces are, of course, always present in non-uniform motions, but their presence does not compel us necessarily to conclude that a change of motion took place. We simply persuade the observer that these forces are gravitational forces, for he is quite unable to distinguish gravitation and inertia from each other!

Let us illustrate this by the following example: We imagine ourselves in a lift just beginning to move, *i.e.* performing an accelerated rectilinear motion upwards. We notice the acceleration by the fact that the pressure of our bodies on the floor of the lift is slightly greater than usual; a body released suddenly would fall to the ground more quickly; a weight suspended on a spring balance would stretch the spring more, etc. From a physiological point of view, the matter becomes more striking when an accelerated motion is performed downwards. If a lift starts moving downwards quickly, the force of inertia is opposed to the gravitational force, and thus diminishes it. Our body appears to be lighter in weight, and, provided the acceleration is sufficient, we notice a peculiar sensation in the region of the stomach, whilst other phenomena behave differently as compared with the former case: released bodies fall to the ground more slowly; a spring stretched by a weight would be slightly relaxed, etc. But all this would take place in exactly the same way if the lift were at rest and the earth's gravity, for some reason or other, were to become suddenly stronger or weaker. Fluctuations of gravitational intensity do, as a matter of fact, take place at the earth's surface, for we are subject to the simultaneous attraction of the earth, the sun, and the moon, and this combined effect differs at noon and midnight from that in the morning and evening. Since the attraction of the earth far surpasses the other forces, these fluctuations are too small to be felt directly by our bodies, but they are distinctly noticeable indirectly in the phenomenon of the tides.

Let us suppose the earth to be so near the sun as to enable us to notice the daily fluctuations of the gravitational forces acting at the earth's surface, and let us imagine some one waking up after a long sleep in a lift shut off from daylight, but lit up by a lamp in the interior. We suppose the observer to have an exact spring balance by him, with which he can measure the intensity of the force of gravitation at any moment. If he determine that the spring balance shows a small tension, he will say: " Gravitational force is small now, and since I know that this is always the case at noon, it must now be noon."

Another passenger, who has just woken up, says : " This need not necessarily be the case ; the intensity of gravitation may possibly be very great at the present moment, but we may be moving downwards with an acceleration." (We may suppose the lift to be moving in a shaft many kilometres in length, where the accelerated motion can be kept up for some length of time.) We see from the conversation of the two passengers that there can indeed be doubt about the existence of accelerated motion also ; the question is as to whether doubts of this kind arise from an insufficient knowledge of facts, or whether a general principle of nature is again the cause of our inability to decide, without reference to the surroundings, which of the two observers is right. We must pursue the same path as in the first part of this book, when we were discussing the special theory of relativity. We stated there that the existence of rectilinear uniform motion is not discernible by our senses without considering the surroundings ; we then went on to say that, by the most exact measurements and observations in the range of mechanics, we cannot discern the existence of such motion ; and finally, we extended the law of relativity to all physical processes.

That our two lift-passengers are not able to decide by mechanical experiments, viz. by weighings, pendulum- and fall-observations, which of them is right, is owing to the law of proportionality between inertial and gravitational mass, which is the keynote of the general theory of relativity. Let us suppose it is not valid, and assume (a possibility indicated in Chapter XII) that the specific gravity of platinum is three times as large as that of iron, and that their specific inertias, on the other hand, are equal. In that case all doubts could immediately be allayed, as to whether the lift was moving downwards with accelerated motion or not. The observers in the lift would have to replace the iron ball—which we will suppose was originally hanging on the spring balance—by a platinum ball of one-third the volume (*i.e.* of equal weight). If the lift be at rest, only the force of gravity can be taken into account, and the platinum ball will strain the spring exactly the same as the iron ball. But if the lift move downwards with accelerated motion, the effect on the spring consists of the gravitational force, diminished by the value of the inertial force acting upwards. As we have supposed the latter to be weaker for the platinum ball, the tension of the spring would here be greater in consequence. In the same way, pendulum- and fall-phenomena with various substances would turn out differently in an accelerated lift and in a lift at rest.

This, however, is not the case; the law of proportionality between inertial and gravitational mass holds good accurately, and its validity guarantees that the lift passengers cannot possibly decide by mechanical experiments, whether an accelerated motion exists or not. The question now arises (analogous to the problem of the special theory of relativity), whether other physical experiments can be thought of, by means of which a decision can be arrived at. When dealing with the special problem of uniform rectilinear motion, we were obliged to answer the analogous question in the negative, for reasons of an empirical nature; we had a number of experiments before us, all leading to negative results (*e.g.* Michelson's experiment). At the time when Einstein began his theoretical investigations of the more general problem now under consideration, no experimental data were available. Hence, when he assumed that no experiments whatsoever could turn out differently in a lift moving downward with accelerated motion and in one at rest, but situated in a weaker gravitational field, he thereby entered the realm of hypothesis,—whereas the entire structure of the special theory of relativity was pre-eminently a rational working up of empirical facts. Those readers who have followed the foregoing developments carefully will be able to appreciate how very illuminating and plausible this hypothesis must have been to Einstein. Could it be supposed possible that a law of nature (the special principle of relativity) on the one hand, should be valid for all physical processes, and its generalisation, on the other hand,—apparently so necessary from a purely theoretical point of view, and already in demand by many philosophers,—only for mechanical processes, and not for electrical and optical phenomena as well ?

Convinced that laws of nature cannot contain inconsistencies of this kind, Einstein set up his Equivalence-hypothesis (to be explained presently), though not, at that time, under the compulsion of any direct empirical facts. Subsequently, experience proved him to be entirely in the right, as we shall show in the next chapter. For the present, we shall formulate the equivalence-hypothesis. In the second chapter of this book we designated any space in which electric or magnetic forces act at every point as an electric or a magnetic field. Analogously, we shall call every space in which gravitational forces are at work a gravitational field. We understand by a homogeneous field a part of space in every single point of which the gravitational force possesses the same direction and the same intensity.

Every human dwelling on the surface of the earth can be looked upon to a close degree of approximation as a homogeneous gravitational field, since the variation of the magnitude and direction of the gravitational force from point to point within the walls of a house is infinitesimally small. Furthermore we call space, in which inertial forces are present, an inertial field ; in a lift, for instance, which is moving with accelerated motion, an inertial field is present.

Now Einstein maintains that the two passengers in the lift cannot decide by any physical experiments whatever, whether the decreased tension in the spring balance is caused by a momentary diminution of gravity, or by a downward acceleration of the lift. With reference to the ideas of inertial and gravitational field just described, we can formulate this statement as follows : *With respect to all physical phenomena, a homogeneous gravitational field is entirely equivalent to an inertial field produced by a constant rectilinear acceleration.* This assumption was designated by Einstein as the equivalence-hypothesis.

CHAPTER XIV

CURVATURE OF RAYS OF LIGHT IN A GRAVITATIONAL FIELD

THE equivalence-hypothesis is a bridge between the theory of relativity and the theory of gravitation. To find the laws of physical processes in a homogeneous gravitational field, we must calculate how these processes take place in a uniformly accelerated system of reference; according to the hypothesis in question all processes are bound to take place in exactly the same way in both cases.

Let us demonstrate an application of the equivalence-hypothesis by a simple example. We will suppose a lift moving upwards with constant velocity, and imagine a ray of light to move horizontally outside the lift and enter it through a hole in the wall. During the minute time-interval required by the light to traverse the chest, the latter moves a short distance upwards, so that the light-ray strikes the opposite wall at a point slightly lower than the hole. To the passengers of the lift the path of the light-ray thus appears to be inclined downwards and not to be horizontal. (This phenomenon has long been known to astronomers as aberration.) The inclination of the path naturally increases with the velocity of the chest. Now if the chest possesses a uniform acceleration, its velocity will increase with time, and the inclination of the ray of light after traversing the chest will be greater than at entrance, so that it describes a curved path. The inference is clear: Rays of light describe curved paths in an accelerated system, and since an accelerated system is equivalent to a system at rest in a gravitational field, it follows that light will suffer curvature in a gravitational field.[1] Rays of light will be curved downwards, *i.e.* towards the attracting mass; the path of a ray of light is, therefore, similarly curved to the path of a bullet,—only the curvature is so infinitesimal owing to the enormous magnitude of the velocity of light, that we cannot determine the deviation from rectilinearity in the earth's gravitational field. The matter is different, however, in the far greater gravitational field of the sun. Einstein calculated that a ray of light travelling just past the sun's limb would suffer a deflection of $1{\cdot}7''$.[2]

The way this circumstance influences our astronomical observations is illustrated in Fig. 4, in which the stellar distances are immensely reduced, whilst the deflection of rays of light is exceedingly magnified. *E* is the earth, *S* a star, and *H* and *H'* respectively the sun in two different positions. As long as the sun is sufficiently distant from the connecting line *ES*, rays of light travel practically in a straight line, but when the sun arrives at the position *H'*, they are transmitted in the slightly curved line *SPE*, and an observer situated on the earth sees the star as if it were situated at *S'*.

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[1] The foregoing considerations only result in the curvature of light-rays in a homogeneous gravitational field. Calculations teach us, however, that this is the case in any gravitational field, whether homogeneous or not.

[2] In Chapter XVIII a supplementary note will be added on this point.

Hence, in order to verify if Einstein is right, we should, for instance, have to photograph a zodiacal constellation of stars at the time the sun is situated in it, and then again at another period of the year, when the sun is in another part of the sky. The positions of the stars on the two photographs will not quite coincide, but will, according to Einstein's hypothesis, show small mutual displacements. Though these displacements are very small indeed (they amount to only about $\frac{1}{50}$ mm. on the plates used in Eddington's expedition, where the effect was finally observed), the accuracy of astronomical measurements is great enough to determine the effect in question with certainty. The difficulty lies elsewhere; in general the starred heavens in the vicinity of the sun cannot be photographed at all, because the glaring sunlight would cause a complete blurring of the negatives during the long time of exposure necessary to receive a picture of the stars on the plate. It was therefore necessary, in order to carry out observations, to wait for a total solar eclipse, during which star-photographs can be taken just as they can by night. The first total solar eclipse after Einstein's prophecy of the bending of light-rays in the sun's gravitational field took place in August 1914, just after the war broke out, and owing

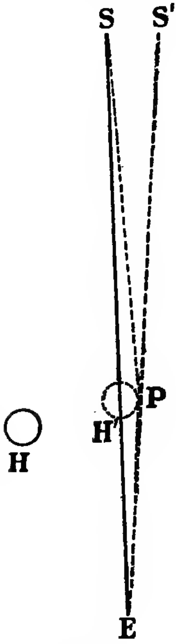

FIG. 4.

to this it had to be passed over unused, though full preparations had been made. The next eclipse took place on the 29th of May 1919. Two British expeditions were equipped, under the leadership of Eddington, to take the necessary photographs ; one went to Sobral in Brazil, the other to the Principe Islands near the west coast of Africa. In both places the photographs taken during the solar eclipse were successful. A few months later, when the sun had moved on sufficiently far in the sky, control plates of the same stars were made with the same instruments, and then the necessary measurements could be performed. The results confirmed the deflection of rays of light by the amount predicted by Einstein.

The reader will readily judge what the result of this successful prophecy meant for the Einstein theory ; it signified the last link in the chain of proofs for the validity of a general theory of relativity, and of Einstein's conception of gravitation. Let us recall the development once more : First of all we had to do with the special theory of relativity which referred only to rectilinear uniform motion, but which was valid for all processes of nature. Then, for reasons of a theoretical nature, we were obliged to demand a generalisation of this principle for arbitrary motions also ; we could state this generalisation for mechanical processes with certainty in the form of an equivalence-principle, since we were supported in this by the empirical fact of the proportionality between inertial and gravitational mass. The extension of the equivalence-principle to all physical processes was, at first, a hypothesis only ; there was no actual experience to compel an assumption of that kind. Since the solar eclipse of 1919, however, we do possess empirical knowledge in this matter ; we find a natural phenomenon which is not only explicable on the basis of the equivalence-hypothesis, but which exactly coincides with Einstein's predictions!

It is true, those opposed to the theory of relativity tell us that the deflection of rays of light in the sun's gravitational field might be otherwise explained, as for instance by the refraction of rays of light in the atmosphere of the sun. Of course it is always possible to explain an established natural fact subsequently by some hypothesis invented *ad hoc,* but Einstein's original explanation, based as it is on the compulsion of profound thought, will be given preference. Moreover, the explanation of the deflection as due to the atmosphere of the sun is to be completely rejected for other reasons. If the sun possessed an atmosphere of such immense magnitude and density as would be needed to cause the observed deflection of light, certain other phenomena would be observable, but this is not the case.

CHAPTER XV

THE RELATIVITY OF ROTATORY MOTION

BEFORE proceeding, we shall find it necessary and convenient to take up the former train of thought once more, so as not to lose the general survey of logical connections. The theory of relativity tends to banish as an empty fiction the idea of absolute space from physics, hence it is necessary to eliminate as meaningless the idea of absolute motion, both uniform rectilinear motion and accelerated motion. This leads to the further conclusion that inertial forces can only appear for accelerations which are relative to other bodies of the universe, and not for " absolute " accelerations; in other words, the inertial mass of a body is caused similarly to the gravitational mass by its interaction with all other bodies. These considerations lead us to the principle of equivalence. Its validity for mechanical processes was guaranteed from the beginning by the empirical fact of the proportionality of inertial to gravitational mass, and its validity for optical processes was proved subsequently by the successful prediction of the deflection of light-rays in the sun's gravitational field.

Without waiting for the results of observations made during the solar eclipse, Einstein continued to work with unbounded confidence in the truth of his theory, so that it was completed four years before Eddington's splendid confirmation followed. The fundamental structure of Newton's theory had to be pulled down to make room for the new edifice. The beginning of Einstein's train of thought has been made clear already; how it was continued will be best made clear by contemplating the problem of the relativity of rotatory motion.

We know that centrifugal forces appear in rotatory motion, whence Newton concluded that in the case of rotation the idea of an absolute motion is reasonable. He devised a well-known experiment to show this empirically: A bucket of water is set into quick rotatory motion. Owing to inertia the water does not partake of the motion immediately, but is gradually set rotating by friction with the sides of the bucket, until the whole mass of water rotates with the same velocity as the bucket. As soon as this is the case the effect of centrifugal force becomes apparent: the surface of the water does not remain flat but becomes curved in the form of a concave mirror; water particles rise up the sides of the bucket under the influence of centrifugal forces. At first, when the sides of the bucket rotate, but not the water, the surface of the water remains quite even (as ascertained by Newton),—this being a sure proof that no centrifugal forces are then at work. He argued thus: At the beginning of the experiment the relative motion between the sides of the bucket and the water is greatest, and yet no effect is perceptible. Afterwards, however, when there is no relative motion between the bucket and the water owing to the water partaking of the rotation, centrifugal forces appear, hence they must depend on *absolute* rotatory motion and not on *relative* rotatory motion. This conclusion seems plausible enough at first, but when submitted to rigid criticism it cannot be maintained, as stated distinctly by Mach. He says: "Newton's experiment with the rotating water bucket teaches us that noticeable centrifugal forces do not appear for the rotatory motion of the water relative to the sides of the bucket, but they appear as the result of rotation relative to the mass of the earth and the heavenly bodies. Nobody can say how the experiment would turn out if the sides of the bucket were made increasingly thicker and more massive, up to, say, several miles thick. Only this one experiment has been performed, and we must make it consistent with the other known facts of nature and not with our arbitrary fictions."

We now proceed to those phenomena arising from the earth's rotation, and considered by Newton to be proof of the absolute existence of this rotation. In this case centrifugal forces are so minute, owing to the small angular velocity (one revolution per day), that they cannot be perceived on our own bodies, but with refined instruments they can be proved without doubt. Moreover, their effect appears in the fact of the earth's oblateness. These centrifugal forces act on bodies *at rest* on the earth's surface. There is another type of force, also caused by the earth's rotation, but acting on bodies *in motion* relatively to the earth's surface. These are called "Coriolis-forces," and are manifested in deflections suffered by bodies moving freely along the earth's surface; these deflections are to the right in the direction of motion on the northern hemisphere, and to the left on the southern hemisphere. If a projectile, for instance, be shot due southwards, it will not travel quite accurately southwards, but will be deflected very slightly to the right, *i.e.* in a westerly direction, for the earth goes on rotating eastwards during the time the bullet is travelling. Other effects of the Coriolis-force are the following: the north-easterly trade-winds on the northern hemisphere, the greater wear and tear (in the direction of motion) of the right-hand rails of railway tracks, the greater wearing down of right-hand river banks, the rotation of the pendulum-plane in Foucault's pendulum experiment, and so on. Considered from the Newtonian point of view, all these phenomena prove that the statement, "the earth rotates," has an absolute and real significance, and that it would be wrong to suppose the earth at rest and the fixed-star system rotating round it.

Let us again hear what Mach has to say on the subject: "Let us consider the point on which Newton appears to lean with full justification, concerning the distinction between relative and absolute motion. If the earth performs an absolute rotation about its axis, centrifugal forces will make their appearance, it will become flattened, the acceleration of gravity near the equator will be diminished, the plane of Foucault's pendulum will be turned, etc. All these phenomena will disappear if the earth be at rest, and the other heavenly bodies rotate round it absolutely, in such a way that the same *relative* rotation takes place. It is so, if we start from the idea of absolute space. But if we keep to the basis of facts, we can only speak of *relative* space and *relative* motion. All motions in the Universe are the same relative to each other, both according to the Ptolemaic and the Copernican system, if we take no notice of the unknown and unconsidered medium of the Universe.[1] Both systems are equally *right*, only the latter is simpler and more practical. The Universe has not been given us *twice* over, with an earth at rest and with an earth in rotation, but only *once*, and with its relative motions which are alone determinable. Hence we cannot say how it would be if the earth did not rotate. We can interpret the actual case in different ways, but if we interpret it in contradiction to experience, our mode of interpretation will be wrong. The fundamental principles of mechanics might be regarded in such a way that centrifugal forces result also for relative motions."

Thus the conflict between the Ptolemaic system (the earth at rest) and the Copernican system (rotating earth) is, according to Mach, irrelevant; both theories maintain nothing essentially different—they are merely different interpretations of one and the same fact. In this Mach clearly sets up that programme which was turned into account about thirty years later by Einstein.

In order to accomplish this, it was necessary to discard Newton's mechanics, with its ideas of absolute acceleration, etc., and his theory of gravitation. According to Newton, the Ptolemaic system is not only more inconvenient than the Copernican system, but is indeed quite impossible. The physicist who maintains the point of view of the Newtonian theory questions as follows: " How can we reconcile the Ptolemaic system with the fact that centrifugal forces and Coriolis-forces act at the earth's surface and do not act on the stars

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[1] Mach means by this the light-æther, which has already been discarded, however, by the special theory of relativity.

(if they really rotate round the earth), scattering them into space ?" A relativistic mechanics and theory of gravitation must give us the following answer: (1) No perceptible centrifugal forces appear in the fixed stars, for acceleration relatively to "nothing" no more calls forth inertial forces than rotation relatively to "nothing" can call forth centrifugal forces,[1] and the mass of the earth is indeed a mere nothing compared with all the masses in the universe. (2) If we assume the earth to be at rest, then centrifugal and Coriolis-forces on the earth must be looked upon as gravitational forces exerted by the revolving celestial bodies.

With the first answer we overthrow Newtonian mechanics (which we were obliged to modify in view of the special theory of relativity), and with the second answer we overthrow his theory of gravitation, for according to Newton's law of gravitation,[2] the gravitational forces between bodies acting on each other depend on the masses of the bodies and their mutual distances apart, but not on the state of their motion. Thus fixed stars considered as revolving round our earth would act, according to Newton, with no other force than fixed stars regarded as being at rest, *i.e.* with no force at all, for the fixed stars on the average are uniformly dis-

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[1] Since rotation is only a special case of non-uniform motion, and centrifugal forces are again only a special case of inertial forces.

[2] It runs thus: The gravitational force acting between two bodies (sun and earth, for instance) is proportional to the product of their masses and inversely proportional to the square of their distance apart. Hence, if the earth were twice as far distant from the sun as is actually the case, the attracting force would be only one-quarter of that actually existing; if it were three times as far distant, the force would be one-ninth, and so on.

tributed round our solar system, and their forces thus neutralise each other.

A truly relativistic theory of gravitation must be so constructed, however, that according to its formulæ the rotating system of fixed stars must produce a gravitational field which is equivalent to that of the centrifugal and Coriolis-forces. Furthermore, a truly general relativistic law of motion in mechanics must be constructed in such a way as to admit of inertial forces only for *relative* accelerations, rotations, and so on.

CHAPTER XVI

THE NOTION OF SPACE-CURVATURE AND OF WORLD-CURVATURE

IN the last chapter we have explicitly pointed out the mark aimed at by Einstein's speculations, which was finally attained by him. A more detailed presentation of the theory which realises the demands here set up, can only be given exactly with the help of higher mathematics.[1] Without the help of any mathematics, some, at least, of the most characteristic traits of the theory can be elucidated, and this is best carried out by the application of the special theory of relativity (as far as is reasonably possible) to the problem of the relativity of rotatory motion discussed in the last chapter. In the place of our solar system let us suppose a huge circular disc freely poised in space, and so thin that it exerts only very small gravitational forces. Immediately above this disc and concentric with it, we suppose a second disc of equal magnitude.[2] The centres of both discs are supposed to be connected by an axis, round which

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[1] In the general theory of relativity the demand for mathematical accessories exceeds the amount of knowledge formerly acquired by mathematical physicists.

[2] For the sake of simplicity and clearness we speak of "top-" and "bottom-" disc, though we are aware, of course, that it is meaningless to talk of " top " and " bottom " in the Universe.

both discs can rotate. The lower disc is to be at rest relatively to the system of fixed stars, and the upper one is to rotate relatively to it. We are to imagine both discs inhabited by intelligent beings, provided with all sorts of physical apparatus, including, of course, measuring-rods and clocks. For the sake of brevity we will call the inhabitants of the top disc the " Reds " and those of the lower disc the " Whites." The Reds will observe the occurrence of centrifugal and Coriolis-forces on their disc; if they have acquired the relativistic mode of thought explained in the last chapter, they will know that the existence of these forces can be interpreted in two ways,—either as inertial forces, if they consider their disc in motion, or as gravitational forces exerted by the revolving firmament of fixed stars, if they consider their own disc to be at rest. (If, indeed, their mode of thought in physics had gone through a different development from ours, which led via Galilei and Newton, they would perhaps not be able to realise that there exist two different interpretations. Perhaps their theory does not recognise the difference between inertial and gravitational mass. This by the way.) Let us further suppose them to be in communication with the Whites, and always to compare their time- and length-measurements with those of their lower neighbours. The Reds living in the immediate vicinity of the axis will have a small velocity relatively to the Whites situated below them, and in consequence the effects of length-contraction and difference of clock-motion will be immeasurably small. For those living at the centre of the disc, therefore, the measuring-rods and clocks of both Whites and Reds will practically agree. But it is different for those living near the periphery of the disc. To speak more plainly and concretely, we shall assume that the diameters of the discs are approximately equal to the diameter of the earth's orbit (300,000,000 km.), the angular velocity corresponding to one revolution per week. The velocity of a point at the edge of the top disc relatively to the lower disc will be about 5,000,000 or 6,000,000 km. per hour, or 1500 km. in the second. With these velocities the contractions in length demanded by the special theory of relativity (although they still amount to less than $\frac{1}{100}$ per cent.) will be just measurable and the different motions of the clocks will also be noticeable, if the results of the special theory of relativity (which were derived expressly only for rectilinear uniform motions) are appliable at all to the case of rotatory motion. A transference of the results of the special theory to non-uniformly moving systems can only be permitted if we limit the contemplation to very small world-elements, *i.e.* to small spaces and short times. Thus, if we contemplate a relatively small region near the edge of the upper disc (say about the size of the earth's surface) and follow its movement for a short time, *e.g.* a few minutes, the motion of this region relatively to a corresponding region on the lower disc will be practically quite uniform and rectilinear. Hence we can apply the results of the special theory of relativity with a clear conscience.

According to this, the measuring-rods, like all other objects belonging to the marginal regions of the top disc when considered from the lower stationary disc, will appear shortened in the direction of motion, and in the same way the motion of the clocks of the Reds must be slower as compared with the motion of clocks belonging to the Whites.[1] Measuring-rods of the Whites among themselves being perfectly equal, and in agreement with those of the Reds who inhabit the vicinity of the disc's centre, it follows that the lengths of measuring-rods and the motions of clocks will be different for the Reds inhabiting the margin and for the Reds inhabiting the centre. We know that contraction takes place only in the direction of motion, hence only those measuring-rods of the Reds will be shorter which are laid down tangentially (parallel to the disc's edge); those laid down radially (perpendicularly to the disc's edge) agree with the corresponding rods of the Whites. When therefore the Reds and Whites come to measure the diameters of their discs, both will arrive at the same result (in our example 300,000,000 km.). But if they measure the circumference of their discs, they will arrive at different results, for the Reds are measuring with shortened measuring-rods and will have to lay them down oftener to get round the disc than the Whites. For them, the resulting figure for the circumference of their disc will be greater than for the Whites.[2]

[1] Cf. footnote on p. 49, Chapter VII. An explanatory note will be added in Chapter XVIII, p. 153.

[2] Against this conclusion the following objection has often been raised: "Not only will measuring-rods laid down tangentially suffer contraction, but also the entire circumference of the disc, and in the same proportion as the measuring-rods, for it runs tangentially in the direction of motion. Hence the resulting figure for the length of the circumference must be the same for the Reds as for the Whites." This objection, however, does not hold good, because one cannot draw conclusions from the special theory of relativity about the circumference of the disc as a whole. As mentioned above, the results of this theory can only be applied to cases of rotatory motion, when very small space-time elements are dealt with. See Supplementary Note on p. 166.

Consequently the ratio between the circumference and the diameter of the circle will be a different one for the Reds than for the Whites. The ratio—and this is remarkable—will be different for circles of different magnitudes. If the Reds living at the centre of the top disc draw a relatively small circle (with a radius of only a few kilometres) and then determine the ratio between circumference and diameter, they will find the same number as the Whites (*i.e.* the well-known Ludolph-number 3·14159265 . ., designated once and for all by the Greek letter π), for their measuring-rods are only slightly contracted. If the inhabitants of an intermediate zone of the top disc draw a circle, the diameter of which is about equal to half the disc-diameter, they will find a somewhat larger number than π, and the inhabitants of the periphery of the disc will find a still greater number, since the measuring-rods laid down by them tangentially will be contracted most of all.

The inhabitants of the top disc will, according to their experiments and their geodesic measurements, arrive at a different geometry from those of the lower disc. Whereas the ratio between circumference and diameter of circles will always be equal to π for the Whites, quite independently of the magnitude of the circle (just as we learned at school), this law holds good for the Reds only approximately, and agrees best for those circles of dimensions small as compared with the disc on which they live. Deviations from this law are greatest for circles comparable with the magnitude of the disc itself. The objection might be raised that the Reds are wrong in their measurements, because they use contracted measuring-rods at the periphery; but if reproached with this, they would be justified in replying: "According to a general principle of relativity we are fully justified in taking up the point of view that we are at rest, whilst the lower disc and the fixed stars move around us. Hence for us there need be no such thing as a contraction of measuring-rods; such as they are, they give us correct measurements, and the results derived from these measurements lead us to that geometry, which is the right geometry for us, because it fits in correctly with our experience."

We shall be able better to appreciate this mode of thinking if we leave the Whites and Reds for a moment and return to the earth. Let us turn to fiction again and suppose men to be beings of only two dimensions,[1] who can neither raise themselves from the earth's surface nor penetrate into the earth—the idea of a third dimension or the mere possibility of it never presents itself to them. To begin with, it would never occur to them that the earth's surface was not plane but curved,—the idea of a curved surface would be entirely strange to them and inconceivable, though the notion of curved and straight *lines* would, of course, be familiar to them. All great circles on the earth's surface (for instance, the meridians and the equator) would be straight lines to them, *i.e.* lines continuing in the same direction. A line, however, which first ran in a north-south direction, and then turned more and more to the west, would be a curved line. This could be readily determined by these two-dimensional inhabitants of

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[1] The missing third dimension is, of course, height; we should have to imagine these flat beings somewhat like infinitesimally thin leaves of paper in a horizontal position gliding along the earth's surface.

the earth, for the two dimensions, north-south and east-west, are at their disposal. Surface curvature, however, is a different matter. The curvature of our earth's surface can be described thus: We imagine a tangential plane at that point of the terrestrial globe in which we are situated (a horizontal plane); then we touch this plane, whilst the earth's surface vanishes beneath the plane (a ship receding from us on the ocean disappears below the horizon). Now, if the ideas of "above" and "below" are absolutely unknown and strange to two-dimensional beings, this latter statement has no significance at all to them, and the curvature of the earth's surface would be quite inconceivable, or at least not obvious. Nevertheless, two-dimensional beings would be able, given sufficient progressive development, to arrive by abstract mathematical reasoning at the conclusion of attributing "curvature" to the surface inhabited by them. To understand this we will suppose them to begin measuring the ratio between circumference and diameter for circles drawn on the earth's surface. Here, again, for circles small compared with the circumference of the terrestrial globe, the well-known number π would result, but the ratio would be less than π for circles of greater magnitude. This is plausible enough: Let us imagine the circle in question to be the circle of latitude 60°. The real diameter of the circle for us three-dimensional beings is the chord AB (Fig. 5), *i.e.* the connecting line through the earth's interior between two diametrically opposite points A and B of the circle. The ratio between the circumference of this circle of latitude and the chord is, of course, π. The notion of the earth's interior cannot exist, however, for two-dimensional beings. They only know of the earth's surface, and according to their perception the straight connecting line between two points of a circle of latitude which lie diametrically opposite to each other is the part *ANB* of the meridian going through the two points. The length of this meridian-line is greater than that of the corresponding chord *AB*; hence the ratio of the circumference of the circle to the line *ANB* is smaller than the ratio of the circumference of the circle to the path *AB*, considered by us to be the "real" diameter. If two-dimensional beings were to determine the ratio between the circumference of the equator and its diameter, the resulting number would be only two. Geometry resulting from their experience would therefore contain the following law concerning the ratio between the circumference and the diameter of circles: "This ratio depends on the magnitude of the circle; for small circles it reaches the limit π, and for larger circles it decreases, reaching the value 2

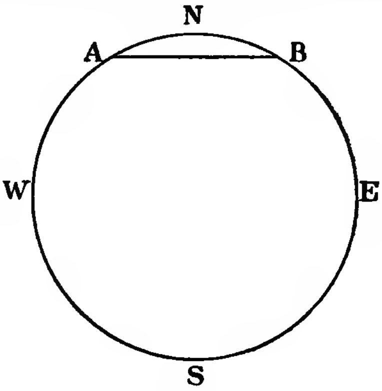

FIG. 5.

for a circle with a diameter of 20,000 km." Mathematicians amongst the two-dimensional beings would be able to show that it is possible to imagine a fictitious surface for which the ratio between the circumference of a circle and its diameter is equal to π for all circles, independently of their magnitude, and, furthermore, that surfaces could be imagined for which this ratio would vary more than on the earth, *e.g.* so as to be equal to 2 for a diameter of 1 km., and so on. Lastly, they might show that surfaces are imaginable for which this ratio *increases* with increasing diameter of the circle, *i.e.* becomes larger than π.[1] Mathematicians would go on to discover that the quality distinguishing these various surfaces from each other can be conveniently designated as "curvature," for they could determine with the help of science and simply by calculation, what we three-dimensional beings know merely from observation: On an uncurved surface (plane) the ratio between the circumference of a circle and its diameter is constant and equal to π; with curved surfaces, however, this ratio varies for circles of various magnitudes, the more so the greater the curvature (supposing, of course, that the diameter is always measured along the surface itself).

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[1] Saddle-surfaces are surfaces of this kind. Let us imagine a closed curved line on an ordinary riding-saddle, and drawn in such a way that the shortest distance measured along the saddle-surface from the centre of the saddle is exactly the same for all points on this line. Such a line would be a circle for two-dimensional beings on the saddle-surface, and for these " circles " the ratio between circumference and diameter is greater than π. Surfaces where the ratio between circumference and diameter of " circles " exceeds the value π are called, in mathematics, surfaces with negative curvature.

These considerations lead to the following: It is owing to the circumstance that we are, in point of fact, three-dimensional beings that we have been able at a comparatively early stage (in times of antiquity) to acquire the knowledge of the earth's spherical shape. This circumstance, however, was not a necessary condition for the definite attainment of this knowledge. Mankind would have been able to attain to it even if he had been a two-dimensional being, provided that great mathematicians like Gauss, for instance, had been at his disposal, in order to take the necessary geometrical measurements. But in this case the knowledge of the curvature of the earth's surface would never have penetrated the public mind, for it would have been inconceivable and unintelligible. It would have remained more or less the sole property of the mathematically educated.

We find it reasonable now to ask the question: "How about our three-dimensional space, embracing the solar system and all the stars?" Concerning it, we are in a similar position to the fictitious two-dimensional beings in relation to the earth's surface. We cannot depart from it, hence we are not able to imagine a fourth dimension, and we are not able to judge directly whether our space possesses curvature or not, *i.e.* whether space is in three dimensions what a curved surface or a plane is in two dimensions. According to the foregoing explanations, it is clear that only mathematicians and geometers can give an answer to this question. It will be as follows: If the laws of our school-geometry (designated as Euclidean geometry) are exactly valid in the universe for geometrical figures of any magnitude, we must then say that space possesses no curvature. (Mathematicians designate such spaces as Euclidean spaces.) On the other hand, space is called non-Euclidean or " curved " space, if deviations from these laws can be discovered, *e.g.* if the ratio between circumference and diameter, for instance, is slightly different from π for very large circles, or the sum of the angles of very large triangles is not exactly equal to 180°.

This must be looked upon as the definition of " curved space "—an attempt to give us three-dimensional beings a clear conception thereof would be as futile as the analogous attempt to give fictitious two-dimensional beings a conception of the curvature of the surface on which they live. Hence, when we talk of space-curvature in the following, we mean nothing more concrete than that certain deviations from Euclidean geometry will appear, if sufficiently exact measurements of the space be taken.

The idea that deviations of this kind might be found on measuring the universe had been foreseen nearly a century ago by mathematicians like Gauss and Riemann. It had become clear to them that the validity of the laws of Euclidean geometry could not be accepted like a sort of divine dogma ; on the contrary, they saw that other systems of geometrical laws could be set up, free from logical contradictions and differing from Euclidean geometry. Finally, they recognised that experience must teach us which of these is suitable to describe the geometrical properties of the universe in which we live. In point of fact, Gauss performed direct measurements on a large triangle with sides many miles in length, to determine experimentally whether the sum of the angles actually amounts to 180° in large triangles; but no deviation was detectable.

Every such negative attempt, of course, could only teach us that no deviations from Euclidean geometry appear within the limits of exactitude of our measuring instruments, or in other words, that the curvature of space, if it exist at all, must be very slight. It certainly can in no way be maintained that such a curvature does not exist. It might be determinable at some later date with more perfect instruments, or by measurements on much larger geometrical figures. If our fictitious two-dimensional beings inhabited only a small part of the earth's surface (say the size of a few square kilometres), the aforementioned deviations from Euclidean geometry would certainly elude their measurements. Now, as we pointed out at the end of Chapter X, the spatial extent of the scene of our activity is given by a comparatively small number in the " natural " units of space, and as a matter of fact, the portion of space inhabited by terrestrial beings is an infinitesimally small part of the visible stellar universe. Hence it is quite possible that we are here in the same position as the fictitious two-dimensional beings whom we supposed to inhabit only a small part of the earth's surface; we cannot discern the curvature of the space we live in, because all our experience and measurements refer only to a very small portion of the whole universe.

In the discussion of the preceding paragraphs we seem to have digressed from our theme, and the reader may perhaps ask: What has all this to do with the relativity of motion and with gravitation? This will be made clear in the following statement: The Red inhabitants of the disc which is rotating relatively to the fixed-star system arrive at a geometry through careful geodesical measurements of the space they live in, which diverges from Euclidean geometry. Hence, for them, the case long ago considered possible by mathematicians has been realised. The space in which they live does not partake of the character of Euclidean space, but of curved, non-Euclidean space.

This does not mean that the *surface* of their circular disc is to be thought of as curved upwards or downwards like a shallow bowl. That this is not the case they could readily determine, for if they be three-dimensional beings themselves they can use the third dimension (above and below) perpendicular to their disc for their measurements. (Besides, if the disc's surface were curved like a bowl, the ratio between the circumference of the surface and its diameter would be smaller than π and not greater than π, as observed by the Reds.) On the contrary, the results of their measurements turn out as if the whole three-dimensional space in which they work and carry out their measurements were embedded in a four-dimensional space (which we cannot imagine) in which it is curved, just as the two-dimensional earth's surface is embedded and curved in three-dimensional space in a manner which to us is both visible and imaginable.

Let us proceed to consider the following: All those phenomena, by which processes on the top disc are distinguished from those on the lower disc (the appearance of centrifugal and Coriolis-forces, the existence of space-curvature), are caused by the fact that the top disc rotates relatively to the firmament of fixed stars, whilst the lower disc does not. In the foregoing chapter we saw clearly, in the sense of the Mach-Einstein conception, that the appearance of those forces could be interpreted in a double way: Either as inertial forces, if we consider the disc as moving, or as gravitational forces exerted by the rotating fixed stars, if the disc be considered at rest. The same can be said of the curvature of space; this too may be an effect of the rotation of the disc, or an effect of a gravitational field due to the rotating fixed stars.

We have now got to the point towards which we were steering: *We see that a gravitational field of a special kind* (*that due to rotating fixed stars which create centrifugal and Coriolis-forces*) *causes curvature of space.* We chose the example of this special kind of gravitational field, because in this case the appearance of space-curvature can be made plausible without the help of higher mathematics. The mathematical formulæ of the theory, however, teach us more than this; not only the special gravitational field dealt with here, but *every* gravitational field causes curvature of space. The gravitational field of our sun, of the earth, and of every body in the universe causes a certain curvature of space, characteristic of the field in question. This curvature, however, is so slight, that it could not be determined hitherto by our available means of measurement.

Thus the prediction of our great mathematicians has been fulfilled in accordance with the Einstein theory, though somewhat differently from what had been expected. What they conceived was approximately the following: The universe in itself has a very slight curvature (in the sense of the definition aforementioned), in a similar way as the surface on which we live possesses a slight curvature. That the presence of gravitating matter (the fixed stars and their planets) had anything to do with this curvature was hardly dreamt of by anybody before Einstein.[1] According to the general theory of relativity the matter lies thus: At a great distance from all gravitational masses, space is almost exactly Euclidean space; in the vicinity of gravitating masses, however, it is curved, the curvature depending on the gravitational force exerted by the masses in question. To make it clearer, we shall imagine the universe for a moment as two-dimensional, *i.e.* as a surface. Our picture would then be somewhat as follows: In the wide regions lying between fixed stars, the surface of the universe would be almost exactly plane, but in the vicinity of every single star there would be a slight shallow convexity, in the mid-point of which the star would be situated. But as the curvature of space even near the largest stars is very small, these convexities would be too slight to be discovered with the naked eye, if we had a true-to-nature model of this " world-surface " before us.

The considerations of this chapter referred to the curvature of *space*, called forth by a gravitational field; there was no question of *time* in connection with them. Now Minkowski showed that according to the special theory of relativity, space itself plays only the part of a shadow. Just as the shadow of a body is different in magnitude according to the surface on which it falls, so the space taken up by any object is different in size according to the state of motion of the system of reference from which it is seen. The following more geometrical formulation is equivalent in meaning to this

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[1] With one exception perhaps: Riemann seems to have foreseen a connection of this kind.

statement: Just as a surface is only a two-dimensional part of three-dimensional space, so space itself is not an independent whole, but only a three-dimensional part of the four-dimensional world. (For those readers who do not quite grasp the sense of this short statement we refer once more to the discussion of Chapter IX.) In explaining the idea of the curvature of space we said: "The results of measurements turn out as if the whole three-dimensional space in which they (the Reds) work and carry out their measurements were curved and embedded in a four-dimensional space (which we cannot imagine), just as the two-dimensional earth's surface is curved and embedded in a three-dimensional space, in a way which to us is both visible and imaginable." In comparing this sentence with the aforementioned statement of Minkowski, the following assumption appears plausible enough: Do the ideas of "space" and "world," in a certain sense, play a similar part in the theory of relativity, to the ideas "earth's surface" and "space" in classical physics and geometry? Let this be explained: In pre-relativistic times it was assumed that the earth's surface was a two-dimensional curved manifold, embedded in a non-curved (Euclidean) three-dimensional manifold, *i.e.* space. By analogy, is it not permissible now to say that the space in the vicinity of gravitating masses is a curved three-dimensional manifold embedded in a non-curved four-dimensional manifold, *i.e.* the world?

This last sentence is correct in all but one word; the attribute "non-curved" must be omitted, as applied to the four-dimensional world. According to the results of Einstein's calculations, not only space, but the entire space-time-entity, called by Minkowski the "world," must be regarded as curved. What this signifies is, of course, still more difficult to define than the significance of space-curvature itself, because one fails to see any connection between the ideas of "time" and "curvature." Hence we will confine ourselves to a brief suggestion. Four co-ordinates are required (as shown in Chapter IX) to determine events in nature without ambiguity: the three spatial co-ordinates relating to the place of the event, and one co-ordinate which states the point of time at which the event happened. As there explained, the spatial distance between two point-events can be calculated by means of the well-known Pythagorean law (47th proposition of the first book of Euclid), from the differences of the three spatial co-ordinates (difference of height, length, and breadth). Formerly this spatial distance was supposed to be an absolute magnitude, independent of the system of reference. According to the theory of relativity this is not the case, but it is possible to calculate a magnitude with the help of a generalised Pythagorean law, and using all four differences of co-ordinates. This new magnitude is designated the "world-distance" of point-events, and it has a real absolute significance. This generalised Pythagorean law (given in a footnote at the end of Chapter IX for the case of the special theory) is contained in "world-geometry."[1] Now if the laws contained in this "world-geometry" are analogous to those of plane-geometry, for the simplest description of physical phenomena,

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[1] Just as we talk of plane-geometry, which deals with corresponding problems in two dimensions, and of space-geometry (stereometry) for three-dimensional problems, so we can, of course, talk of "world-geometry" in the case of four dimensions.

and if they are also analogous to those of the stereometry of Euclidean space,[1] we then say that the world is Euclidean. When this is not the case, we say that the world is non-Euclidean (curved). According to the special theory of relativity the laws of the world-geometry are still Euclidean, hence the Minkowski-world is Euclidean. But according to the general theory, this is no longer valid for parts of the world surrounding gravitational masses, hence world-curvature exists there, in the above-mentioned sense of the word.

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[1] Such a law is that used as an example before: The ratio of the circumference of a circle to its diameter is always equal to π, quite independently of the magnitude of the circle.

CHAPTER XVII

THE NEW THEORY OF GRAVITATION

THOSE readers who, before the perusal of this book, knew the general theory of relativity to be at the same time a theory of gravitation, may perhaps be disappointed and will ask: Where do we find the explanation for gravitation in this; why do all bodies attract each other according to Einstein? Before answering this question we must consider the following: To explain a phenomenon means to trace it back to a simpler and more general phenomenon. If we were to explain this other phenomenon, which was given as the cause of the first, we should have to reduce it to a third phenomenon, and so on. But by continuing in this manner, we finally arrive at a point where no further answer can be given. If a child ask us, for instance: " Why do I fall on my nose when I jump off a moving tramcar? " we may answer: " Owing to your inertia; your body retains its motion after leaving the steps of the car, whereas your feet are suddenly brought to rest by friction with the ground. That is why you tumble down." If the child goes on questioning: " Why does the body retain its motion? " we can give no further reason, but only say that this is a fundamental law of nature.

There are certain ultimate facts which admit of no further explanation; they simply exist. Amongst these is gravitation: All bodies attract each other. This fact cannot be explained, and needs no explanation; it is simpler than any other phenomenon to which it might be reduced. This leads us to the conclusion that we cannot expect a theory of gravitation to give us an *explanation* of the phenomenon. We expect the theory, however, to *describe* the phenomenon of gravitation. This description must be quantitative, *i.e.* it must provide us with the possibility of calculating exactly the motion of a body, for instance of a planet, under the gravitational action of other masses. The Newtonian theory of gravitation has done this in a very simple and unequivocal manner, and there would be no reason to depart from this theory, did it not show those peculiar theoretical defects referred to at the beginning of the second part of this book.

To the defects of a philosophical nature there mentioned, we must add a further defect of a physical nature. According to Newton, gravitation possesses an infinite velocity of propagation. The meaning of this will be made clear in what follows. The gravitational force acting on a planet is known to consist of the attraction of the sun (forming by far the greatest part of the entire force) and that of the other planets. These forces depend on the mutual distances of the celestial bodies, and will be constantly changing in the course of time, in accordance with the positions of the planets. Now Newton maintained that the force acting at a given moment on a planet—say Jupiter—is to be calculated from the *instantaneous* constellation of the attracting masses. If, however, gravitation possesses a finite velocity of propagation, the calculation of the forces acting at a certain moment on Jupiter will have to be performed differently: we must not insert the distances between the planets at that precise moment in our formula, but rather other distances corresponding to an earlier instant, *i.e.* as much earlier as gravitation needs to travel from those planets to Jupiter.

On the part of physicists, great doubt was formerly cast on the conception of forces propagated to a distance with infinite velocity. Even Newton himself on one occasion admitted that he could not believe in action of that kind at a distance. Seen from the point of view of the theory of relativity, this conception is utterly impossible. For, as expounded in Chapter V, one of the fundamental assumptions of the special theory of relativity consists in the statement: No effects can be propagated with greater velocity than the velocity of light. For these reasons, science, with its progressive development, was at last obliged to advance beyond the Newtonian theory; but this theory will ever remain an immortal work for all time. It was the first theory which enabled mankind to obtain an exact treatment of problems belonging to natural sciences, and in the future it will remain for all practical purposes almost the exclusive instrument as a theory of approximation for the physicist and the astronomer. Since the Newtonian theory of gravitation and Newtonian mechanics can be looked upon as the model type of a mathematical description of natural phenomena, we will first of all use it to exemplify the nature of a description of this kind, and then show how the corresponding description is given by the Einstein theory.

The Newtonian theory supplies the mathematical apparatus by means of which we can calculate the gravitational force due to any given configuration of attracting masses, at any point in the neighbourhood of these masses. This accessory is called Newton's law of gravitation (cf. footnote at the end of Chapter XV). The description of the motion executed by a body under the action of force is further given by the Newtonian fundamental law of mechanics, which states: If no forces act on a body, that body will persist in a state of rest or of uniform rectilinear motion. If, however, a force act on it, acceleration will ensue. The direction of the acceleration will be parallel to the direction of the force, and the magnitude of the acceleration will be equal to the quotient of the force and the inertial mass of the body.

These laws were brought by Newton into the form of differential equations, and in point of fact, by their aid it is possible to calculate the motion carried out by a body under the action of given forces, or vice versa, to calculate the forces necessary to impart a certain state of motion to a body. How great the efficiency of this theory was, is shown by the history of the discovery of the planet Neptune—a discovery which is looked upon with full justification as one of the greatest triumphs of human science. The French astronomer Leverrier had observed that the combined forces of the hitherto known planets did not suffice to fully explain the orbit of the planet Uranus. There remained a small discrepancy between the motion calculated, and that actually performed. Leverrier assumed that a new undiscovered planet might be the cause of these deviations, and calculated by means of Newton's theory *where* this planet ought to revolve in order that the forces exerted by it would just be sufficient to explain the disturbances observed. As a result of this mathematical analysis he was able to predict that this hypothetical planet would be visible at a certain time, and at a certain position in the heavens, and, as a matter of fact, the planet (which was afterwards called Neptune) was discovered by the Berlin astronomer Galle in the position and at the time indicated.

We now propose to show the way in which the description of gravitational phenomena is given in the Einstein theory, and must for this purpose introduce a notion which is necessary for the comprehension of what follows. The *position* of a point in space is given mathematically, as shown in Chapter IX, by three numbers—its three co-ordinates. Further, the *motion* of a point is described unequivocally, if its position is stated for every possible moment of time. Expressed in mathematical terms, this means that the value of the three space-co-ordinates must be given for every value of the time-co-ordinate. The mathematical formula representing the solution of a problem of motion must therefore be a certain mathematical device, which shows us how to calculate the three space-co-ordinates for any value of the time-co-ordinate. This device may also be given in a graphical way instead of by calculation, *i.e.* the description of a body's motion may be given by a diagram. This is the case, for instance, in the so-called graphical time-charts, which enable railway officials, by a glance at a drawing, to survey the instantaneous position of all the trains on a railway line at every moment of time. Let us illustrate the graphical description of motion by a simple example, which shows the motion of a mass-point (particle) along a vertical straight line. We draw a horizontal straight line *OX* (Fig. 6) and a vertical straight line *OY*. The straight line *OX* is divided into equal parts representing seconds of time, and the straight line *OY* into equal parts representing cms. We then draw vertical straight lines through the division points of *OX* and describe the motion of a particle as follows : A mark is made on the straight line going through the division-point 1 second, at that distance from *OX* traversed by the body during the first second after motion began. Another mark is made on the straight line going through the division point 2 second, at that distance from *OX* traversed by the body during the first 2 seconds, and so on. If we suppose the drawing more accurately made, for instance, by marking the position for every tenth or every hundredth of a second, and further, if we imagine all the marks connected together, we obtain a line which represents the motion of the particle in a vertical direction. If the particle move

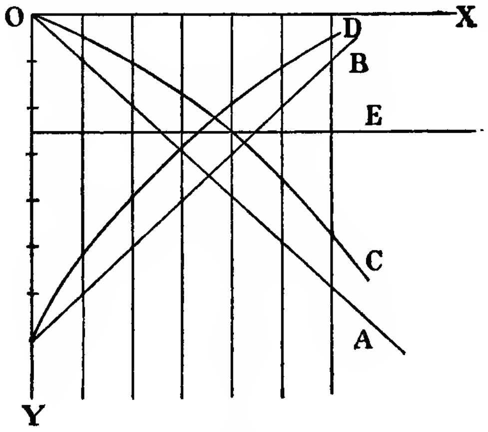

FIG. 6.

with uniform velocity, the path traversed during the first second will be equal to the path traversed in the next second, and so on. The corresponding line will then be a straight line. But if the motion does not proceed with uniform velocity, then the line will be curved. In Fig. 6 the line *A* represents motion with uniform velocity downwards, and the line *B* a similar motion upwards. *C* shows us accelerated motion downwards and *D* retarded motion upwards. The straight line *E*, on the other hand, represents a particle at rest. These lines represent the story of motion of a particle or a material point, and are designated the "world-lines" of that particle. The individual marks in the drawing that make up the world-lines are called world-points.[1] Any special problem, say in celestial mechanics, can be considered to be solved, if the world-lines of the planet or comet in question are known.

The world-lines can only be represented in a plane if the motion of the particle in question takes place in one dimension only (*e.g.* in a straight line). In the case of the two-dimensional motion of a particle (*e.g.* motion in a circle), a plane drawing does not suffice to represent the world-line, and a three-dimensional model is needed. Let us take, for instance, the example of the motion of a particle moving in a circle with constant velocity. Its world-lines can be constructed thus: The circle described by the particle is drawn on a sheet of paper lying horizontally on the table. At a height of 1 cm. above the surface of the paper a mark is made exactly above that point of the circle which contains the particle at the time 1 second. Then a mark is put 2 cm. high above the

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[1] The world-point is nothing else than the graphical representation of a "point-event." (Cf. Chapter IX.)

position in which the particle is situated at the time 2 seconds, etc. The connecting line of all these marks then becomes a screw-line, which resembles a spiral spring. Now when the particle runs along a three-dimensional curve (a screw-line, for instance), the world-line becomes a four-dimensional curve, and can no longer be represented by a model. In this case it must suffice to describe the motion by means of mathematical formulæ only, but even so, the corresponding values of the three space-co-ordinates and one time-co-ordinate are designated as "world-points," and the sum total of the world-points belonging to the motion of one particle, as a "world-line."

A theory supplies an exact description of gravitational processes if it contains unambiguous rules that enable us to calculate the world-line of bodies under the influence of gravitational masses. This is just what the Einstein theory does, and in what is fundamentally a very simple manner, by a suitable generalisation of the above-mentioned Newtonian law concerning the motion of a body not acted upon by any forces. This law states that a body left to itself persists in a state of rest or of uniform rectilinear motion. The world-lines of a body at rest or in uniform rectilinear motion are straight lines; translated into the terminology of "world-geometry," the Newtonian law of inertia would therefore run thus: The world-lines of a moving body not acted on by any forces are straight lines. When gravitational masses are present, however, a body never can move free of forces, because it is always being acted upon by gravitational force; hence there can then be no straight world-lines. That agrees very well with Einstein's assertion, according to which the world in the neighbourhood of gravitational masses is curved. There are no straight lines in a curved manifold. It is impossible, for instance, to draw a straight line on a spherical surface, for an exact straight line would touch the sphere only in one point, whereas all the other points of the straight line would lie outside the spherical surface.

But every surface has certain types of unique lines, which, though not exactly straight lines, can nevertheless be termed " straightest " lines, since they show the least deviation from a straight line, as compared with all other lines on that surface. We mentioned in the last chapter that the meridians and the earth's equator would appear to be straight lines to fictitious two-dimensional earth-inhabitants, because for them these lines would always run in the same direction. Now what distinguishes these lines, and what have they in common with real straight lines ? To answer this question, we shall consider what follows : If two points are given on a plane, an infinite number of lines can be drawn on that plane from one point to the other. But of all these, the one *straight* connecting line is distinguished by being the shortest. The idea of a straight line can be defined directly in this way : It is the shortest connecting line between two points of a plane, or, more generally, of a Euclidean manifold. Now if, on the other hand, two points are given on a curved surface, they cannot always be connected by a straight line which lies completely in the surface, because straight lines cannot, in general, be drawn on a curved surface. But here again, of the numerous lines that can be drawn on a surface between two points, one line will always be the shortest, and that is the line designated above as the " straightest."

In mathematics, lines of that kind are called geodesic lines. On a spherical surface, the geodesic lines are the great circles (*i.e.* circles, the diameters of which are equal to the diameter of the sphere, such as the meridians and the equator in the case of the earth); on a plane the geodesic lines are, of course, straight lines.

By means of this notion of geodesic lines, Einstein established a remarkably simple law for the motion of a body under the influence of gravitational force. It runs thus: *The world-line of a body situated in a gravitational field is a geodesic line.* It is obvious that this law includes the Newtonian law of inertia as a special case. For in places where no forces are acting and where accordingly no gravitational field exists, the world is Euclidean (not curved). In that case the geodesic lines are straight lines, hence the world-lines become straight lines, and that is, as before mentioned, the Newtonian law of inertia in the terminology of world-geometry. Wherever a gravitational field is present, however, the world is curved, and the geodesic lines will be curved lines like the curves *C* and *D* in Fig. 6.

It is, of course, necessary for a complete description of gravitational phenomena to establish another law besides the fundamentally very simple law of motion mentioned. This law must tell us in what way the world becomes curved by the presence of gravitational masses, for geodesic lines naturally vary according to the kind of curvature. This second law, however, can only be expressed in mathematical formulæ and not in words. These formulæ were designated by Einstein the "Field-equations" of Gravitation. With the advent of these field-equations the structure of the new theory of gravitation became complete. It can be characterised in two sentences (intelligible only in connection with the foregoing discussion) thus: Owing to the presence of gravitational masses the world suffers curvature, the nature of which depends on the distribution of the gravitational masses and can be calculated by means of the field-equations. Bodies move in this curved world in such a way that their world-lines are geodesic lines. In Einstein's theory the field-equations play the same part as the law of gravitation in Newton's theory; the law of the geodesic line, however, corresponds to the law of motion of Newtonian mechanics.[1]

In the developments of the last chapters we discussed gravitation almost exclusively, and hardly spoke at all of the problem of relativity, although the considerations which led to the idea of curvature of space and to that of world-curvature took their origin in the relativity of rotatory motion. The following question is therefore justifiable: Is the new theory, here outlined, free from the defects of the Newtonian theory as presented in Chapter XII, and does it fulfil the demands set up in connection with Mach's considerations at the end of Chapter XV?

This question must be answered entirely in the affirmative. Those notions we objected to in the Newtonian theory (*e.g.* absolute acceleration) do not

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[1] The law of the geodesic line is strictly valid only for the motion of material points; that suffices, however, for the purposes of astronomy, for the stars are always dealt with as mass-points in celestial mechanics. It would take us too far to discuss the laws of mechanics which hold exactly for spatially extended bodies according to the general theory of relativity.

appear at all in the Einstein theory. This theory does not deal with forces or accelerations at all, but only with geodesic lines and with world-curvature. By introducing these new ideas (certainly with the disadvantage of their not being obviously intelligible) the Einstein theory complies with a far more general principle of relativity than that of the special theory exactly formulated in Chapter III. We said there: "In different systems of reference moving uniformly and rectilinearly with respect to each other, all natural phenomena take place in exactly the same way." It is true we cannot generalise this sentence and say: "In different systems of reference moving *arbitrarily* with respect to each other, all natural processes take place in exactly the same way." For if *A* and *B* are two different systems of reference moving relatively to each other with an acceleration (*e.g.* rotating with respect to each other), physical processes will take place in different ways in *A* and in *B*. In the former example of the two rotating discs, a smooth ball which receives an impact will continue to roll on with rectilinear uniform velocity with reference to the lower disc, but not with reference to the top one, for, owing to the centrifugal force in this case, the ball will have an outward acceleration.

It is still possible, however, to take into account the fact that only relative motion (whether uniform or not) has any physical significance. That can be done by putting the laws of motion into a form in which they are valid for all systems of reference. With the Newtonian laws this was not the case. The law, "A body not acted upon by any force retains its state of rest or of uniform rectilinear motion," is valid only if "rest" or " motion " are spoken of relatively to certain distinguished systems of reference (called " inertial systems " in physics), as, for instance, to the lower disc of our example. If the " rest " or " motion," however, refer to the top disc, the above-mentioned law is no longer valid, and must be replaced by another.

This is not so in Einstein's theory. The law of motion of the geodesic line is universally valid for all systems of reference, and in the same way the field-equations, by which the curvature of the world may be calculated for a given distribution of gravitating masses, retain their form for systems of reference moving arbitrarily with respect to each other. Furthermore, the laws of electricity, optics, heat, etc., can be accommodated to the new notions of world-curvature, and thus be brought into a form that is valid for any systems of reference. In this way the new theory fulfils a general principle of relativity which can be expressed thus: *Laws of nature can be brought into a form which does not alter, even when the motions of the bodies are referred to any systems of reference whatsoever.*

Furthermore, we set up the following demand at the end of Chapter XV: " A truly relativistic theory of gravitation must be so constructed that, according to its formulæ, the revolving firmament of fixed stars produces a gravitational field which is equivalent to that of the centrifugal and Coriolis-forces. Also, a truly general relativistic mechanical law of motion must be constructed in such a way that inertial forces appear only for *relative* accelerations, rotations, and so on." These demands are also fulfilled, as has been proved[1] by direct calculation. The problem of establishing the complete system of physics on a new basis, which is more satisfactory than the old system from a philosophical point of view, can therefore be said to have been successfully solved by the Einstein theory.[2]

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[1] To satisfy these claims completely, it is necessary, however, to accept the views on the finiteness of the universe formulated in Chapter XIX.

[2] See Supplementary Note on pp. 166–167.

CHAPTER XVIII

DEDUCTIONS FROM THE GENERAL THEORY

THE new theory of gravitation, as already mentioned, is completely different from the old Newtonian theory in its essential traits. Entirely different notions are introduced ; world-lines take the place of uniform or accelerated motion, and world-curvature takes the place of forces—in short, we have to deal with a fundamentally different description of nature.

On the other hand, it was clear from the beginning that, with reference to numerical results, any new theory could differ from the old Newtonian theory only to a very slight extent. For all calculations performed on the basis of the latter theory agree with experience with almost absolute precision. If, therefore, the results of any new theory were to diverge much from the old theory, they would contradict experience and have to be discarded from the beginning. Hence Einstein, in establishing his field-equations, bore in mind that the resulting laws concerning the motion of bodies in gravitational fields must necessarily agree approximately with those of the Newtonian theory.[1] As regards the

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[1] Subsequently it turned out that the field-equations of the general theory which lead to formulæ agreeing approximately with those of Newton's theory, are at the same time just the

degree of approximation, the matter is similar to that of the special theory of relativity. Here, too, the deviations from the laws of classical mechanics and electricity are exceedingly small; they are apparent only when very swiftly moving bodies are dealt with. When, therefore, the motion of a falling stone, or the trajectory of a projectile under the influence of gravitational force is calculated, the results obtained according to the new theory differ from those obtained from the Newtonian theory by such infinitesimal amounts, that it is absolutely impossible to detect the difference even with the very finest of instruments. Only in strong gravitational fields are the differences between the results of the old and new theories within the possibility of measurement.

So far we know of three phenomena that are bound to turn out differently according to the Einstein theory than they would according to the older theories. One of them has already been discussed, and concerns the deflection of light-rays in the sun's gravitational field. To this we must add a supplementary remark. In Chapter XIV we began our considerations by stating that a ray of light moving horizontally outside a vertically accelerated chest, and entering it by a small hole, will describe a curved path with reference to the chest. If, according to the equivalence-hypothesis, we assume the path of a ray of light in a corresponding gravitational

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ones which, for formal mathematical reasons, alone call for consideration. This circumstance tells in favour of Einstein's theory; it shows that this theory is not composed of hypothesis invented *ad hoc*. On the contrary, starting from the considerations dealt with in the last chapters, and putting them into a mathematical form, we are led necessarily to formulæ which agree as well or better with experience than those drawn from the Newtonian theory.

field to be curved in the same way, then this indicates (as can be easily seen) that a ray of light describes the same path under the influence of gravitation as any material body travelling with the velocity of light, or, in short, the light-ray *falls* in a gravitational field. Now for bodies in very rapid motion, the law of motion resulting from Einstein's theory differs considerably from the Newtonian law; in the present case we find that rays of light suffer twice the deflection in a gravitational field according to Einstein's theory that they would according to the Newtonian theory, assuming, of course, that in this case, too, rays of light fall like material bodies. Hence the matter stands thus: Conformably to Maxwell's theory of light, no influence of the sun's gravitational field on the propagation of light is to be expected. The deflection would have to be equal to zero according to that theory. However, if (contrary to Maxwell's theory) we assume that rays of light fall in a gravitational field, the result arising from Newton's theory of gravitation is a deflection of the light-rays passing the sun's limb amounting to 0·85,″ whereas a deflection of 1·7″ results from Einstein's theory of gravitation. The observations of both British expeditions proved the latter value to be the right one.

Another phenomenon suited to the experimental examination of the Einstein theory is planetary motion. We know the orbits of planets to be represented with good approximation by Kepler's first law, which states that planets describe ellipses, of which the sun is a focus. This law, which was first empirically established by Kepler, was subsequently deduced theoretically by Newton from his theory of gravitation. This was the first great triumph of his theory, and an historical fact in the development of physics. As we have already mentioned, the fundamental basis of the exact mathematical treatment of natural sciences by means of the infinitesimal calculus was then laid down. Now this law of Kepler does not exactly agree with experience, and also from Newton's theory it only follows with exactitude when solely the attraction of the sun is taken into account in calculating the motion of the planets. But all planets are acted upon not merely by the attraction of the sun, but also by that of all the other planets of the solar system, and when all these are taken into account, certain slight deviations from the elliptic orbit result, which are termed *disturbances* of the orbit. One of these disturbances is the so-called *motion of the perihelion* of planets. The ellipse described by a planet does not maintain its position relatively to the system of fixed stars, but rotates slowly in its own plane. In Fig. 7 is shown the elliptic orbit of a planet, one focus of which contains the sun S. (For the sake of clearness the eccentricity of the ellipse is exaggerated.) The planet does not describe this orbit with exactitude, but only approximately, so that a second orbit does not

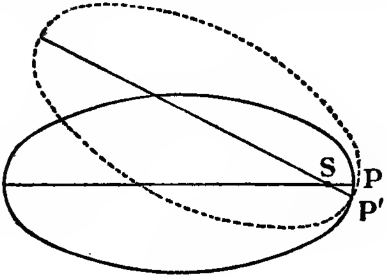

FIG. 7.

coincide completely with the first, being slightly displaced with reference to it, and a third orbit deviates from the second, and so forth. After a time, *i.e.* after some thousands of revolutions, the elliptic orbit is turned through a certain angle from its initial position, as illustrated by the dotted curve in Fig. 7. The vertex of the ellipse which lies nearest the sun (called the *perihelion*) has then changed its position from P to P'. The motion of the ellipse in its plane is therefore designated as the motion of the perihelion.

Perihelion motions of this kind occur more or less in all planetary orbits, and can be explained according to the Newtonian theory by the disturbing forces due to other planets. A discrepancy exists only for the planet Mercury, the observed motion of the perihelion deviating from that calculated (from the disturbing forces) by an amount equal to 43″ per century.[1] Now if planetary motion be calculated from Einstein's theory, we obtain an elliptic orbit with perihelion motion, even if the gravitational action of the sun only be taken into account. For Mercury this effect amounts exactly to the observed value of 43″; for the other planets it is too small (compared with the disturbances from the remaining planets) to be observable. Mercury's orbit is the one nearest the sun; the gravitational force acting upon it is very strong, and hence the Einstein effect of perihelion motion is noticeable only in the case of this particular planet.

The new theory, therefore, gives us the right result on that very point where the old theory failed. Wherever the old theory, on the other hand, was found correct within the limits of accuracy of our measurements, Einstein's theory leads to the same results.

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[1] See Supplementary Note on p. 167.

In order to understand the last of the three above-mentioned deductions drawn from the general theory of relativity, we must revert once more to the example of the rotating disc used in Chapter XVI. We mentioned that the clocks of the peripheral inhabitants of the top disc go slightly slower than the clocks of the inhabitants at the centre. Let us elucidate the meaning of this statement (without giving reasons for it) and formulate it more precisely. We assume that the inhabitants of the centre and the inhabitants of the periphery each set up a standard clock which goes correctly. This must be carried out in such a way that the measurement of the velocity of light with the help of these clocks and by use of a standard measuring-rod, must result precisely in the value c. When the inhabitants of the rim of the disc give light signals or wireless signals at equal time-intervals (say time-intervals of exactly 1000 seconds according to their clocks), the inhabitants of the centre, on receiving these signals, will state that the time-intervals between the signals, according to *their* clocks, are not exactly 1000 seconds, but slightly more. This is the meaning of our statement that the clocks of the peripheral inhabitants go more slowly than those of the central inhabitants. If we are asked: " What is the reason for the different motion of these clocks ? " we must answer : The cause is the same as that for the contraction of measuring-rods and the appearance of centrifugal forces ; all these phenomena are results of the relative rotation between the disc and the firmament of fixed stars, or in other words, they are results of gravitational forces exerted by distant revolving fixed stars. Hence, by our example of the rotating disc, we become acquainted with another effect of the gravitational field: clocks stationed at different points of the field go at different rates. Mathematical treatment further teaches us that this is not only the case in this particular gravitational field, but quite generally, in every gravitational field. Furthermore, with the help of our example we can discern the manner in which this influence of a gravitational field takes place. As we travel from the centre of the disc to the edge, our movement will be assisted by the centrifugal forces; it is as though we were going downhill. If, on the other hand, we were travelling from the periphery towards the centre of the disc, the action of the centrifugal forces would have to be overcome, and the feeling would be that of going uphill. In the first case we should be travelling into regions where clocks go more slowly, and in the second case into regions where they go more quickly. Now the equations of the Einstein theory teach us that this rule holds generally. Thus if we go uphill, a watch is bound to be accelerated (neglecting all other influences). This effect, however (like most of the effects of the theory of relativity) is many million times too small to be perceptible. Even if one could succeed in accomplishing a mountain tour of superhuman dimensions, *e.g.* from the sun's surface against its attractive force to the distance of the earth's orbit, the effect would be far smaller than the usual daily fluctuations of our best chronometers. In spite of this, its observation is not impossible. We pointed out in Chapter XI that the atoms of luminous gases emit rays of light of quite definite colour, which appear as single lines (spectral lines) in the spectrum of the luminous gas. Now a ray of light of a particular spectral colour is nothing else (according to the considerations of Chapter II) than an electromagnetic wave of a perfectly definite frequency. Hence we can consider an atom which emits sharp spectral lines to be a kind of clock, producing alternate positive and negative electric fields in its surroundings at regular time-intervals. Now if an atomic clock of this kind goes more slowly, *i.e.* emits slower oscillations than another atom of the same substance, the colour of its spectral lines will be shifted towards the red end of the spectrum as compared with the corresponding lines of the other atom. (This is because the red rays are the slowest oscillations in the visible spectrum, whilst the violet rays are the quickest). On the other hand, we know that the frequency and colour of light are connected with the wave length, red rays having the longest wave length, and violet rays the shortest. Hence we can say that retarded atoms emit light of greater wave length. In agreement with what we said above, clocks on the sun will go more slowly than on the earth, because one would have to go " uphill " against the attraction of the sun in order to get from the sun to the earth. Hence the atoms on the sun's surface, if they act as correct clocks (which can be presupposed for very good reasons), will emit light of longer wave length than the corresponding atoms on the earth. The result of the new theory of gravitation concerning the motion of clocks can therefore be verified by comparing the wave length of solar spectral lines with the wave length of corresponding lines of terrestrial sources of light.[1] The effect is so small, however, that it lies

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[1] For the benefit of the physicist we must add that the lines of the solar spectrum with which we have to deal here are not emission lines, but absorption lines; this does not, however,

at the very limit of exact measurement. The difference of wave length between sunlight and corresponding terrestrial light amounts only to about 80 billionths of a centimetre for the spectral lines examined. Nevertheless, the delicacy of our optical measuring methods is just sufficient to permit us to perceive the difference. Owing to the smallness of the effect, these measurements are extremely difficult ; hence it has not been possible to arrive at final results up to the present. Whereas some observers maintain they have detected the effect, others maintain that it does not exist, so that we must regard this point as not yet finally settled.

If we survey the results of the experimental examinations of the new gravitational theory hitherto obtained, it must be said, in view of the verification of two of its results, that the probability for the truth of the theory is very great ; still it would seem premature to consider it as confirmed beyond doubt.

alter the matter essentially, since the period of oscillation of absorbing atoms is influenced in the same way as that of emitting atoms.

CHAPTER XIX

THE HYPOTHESIS OF THE FINITENESS OF THE UNIVERSE

THE recognition that space is not Euclidean but curved (though only slightly) opens up a new possibility concerning the conception of the universe. As long as we were convinced that space is Euclidean, we were necessarily compelled to assume that our universe is infinite. But now we are no longer bound to believe this. This can be best explained by a two-dimensional example. Let us again suppose two-dimensional beings on a smooth sphere (Chapter XVI), and let us imagine them to inhabit only a small part of the sphere, so that their measurements would not yet have revealed to them the existence of curvature. They thus believe the scene of their activity to be a plane surface. If they were to be asked whether the surface of their world is finite or infinite, they would answer with conviction: "It must be infinite; our conceptions do not permit us to assume an ultimate limit. Beyond every boundary the world-surface must go on extending." If at a later date they had arrived at a knowledge of the spherical shape of the earth by measurement or by voyages round the world, they would have gained knowledge of something they were quite unable to grasp previously, namely, the geometrical fact that a surface can be finite without being bounded. This is the case with the surface of a sphere; it is nowhere bounded. It is possible to travel over it for any length of time and in any direction without arriving at a boundary, and yet it is not infinite. Such surfaces, which are unbounded but not infinite, are designated as closed surfaces. We can imagine quite a number of other surfaces possessing this property, as, for instance, egg-shaped surfaces, ring-shaped surfaces, etc. On the other hand, the following are unclosed surfaces: planes, cylinder-surfaces, surfaces of cones, paraboloids, etc.[1] Only curved surfaces can be closed; a plane has either an edge, or it runs on into infinity. Geometry teaches us (and this was known long before Einstein) that the same holds good for three- and more-dimensional space. A curved space can therefore be closed, *i.e.* it can be finite, without having any limit.

Now since our "world-space" is curved, we must take into account the possibility that it is a closed space, *i.e. our universe is perhaps finite, although it is certainly unbounded.*

According to Einstein, this assumption, which revolutionises all our views concerning the universe, is not only possible, but probable. The reason for this is that the idea of an infinite universe presents certain difficulties, independently of whether the Newtonian theory of gravitation or that of Einstein is looked upon as right. Concerning the infinite universe, the

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[1] There is an analogy in the case of one-dimensional figures; here too a distinction can be made between closed and unclosed curves. The former are finite, but unbounded (a snake biting its own tail is an instance); the circle and the ellipse belong to them.

following two alternatives are possible a priori: (1) The entire infinite universe is filled with fixed stars in such a way that the average density of distribution is about as great or greater than in those parts of the heavens visible to us. (2) Those stars, nebulæ, and Milky Way systems visible to us represent a kind of solitary island in the universe, whilst in the infinite regions beyond visible space the density of distribution of the stars gradually decreases to zero. Now amongst other things, the observed motions of the fixed stars are unfavourable to the first alternative. (As a matter of fact, these do not retain really absolute fixed positions in the heavens, but rather travel to and fro like the individuals of a swarm of flies. It is true this wandering about takes place at a relatively slow rate, so that even after centuries a change in the form of constellations is hardly perceptible to the naked eye.) We must conclude, from the slowness of motion of the stars, that the gravitational forces exerted on each other by fixed stars are very feeble. This could not be the case, however, if the universe were everywhere filled with an equal or greater mean density of attracting masses than in our surroundings.

Other objections tell against the second alternative. If the whole system of fixed stars were to exist as an island in the infinite universe, this state of things could not continue to exist to all eternity. Rather would the stars disperse gradually into space. After æons the starry sky would no longer be visible in the surroundings of our sun; every star would pursue its own solitary path, severed from its neighbours by distances of inconceivable magnitude. Even the mutual attractions of the stars would not prevent them from dispersing, as can be shown by calculation. Now it is true that we have no physical proof against the possibility of such a dissolution of the universe after trillions of years; but we are instinctively driven to repudiate this eventuality. Hence it cannot be said that the island-hypothesis concerning our universe cannot be upheld scientifically, but we shall gladly revert to some other expedient if it can be found. Such an expedient is the above-mentioned Einstein hypothesis. According to Einstein, the first of the two alternatives is the right one: on the whole, the universe is filled with a uniform average density of stars. But it is not infinite; it is a closed space in the sense explained at the beginning of this chapter. Hence the above-mentioned counter-arguments against the first alternative can be dismissed. We can only appreciate the idea of a closed universe by translating the whole matter into two dimensions, as we did before at the end of Chapter XVI. We said there: "In the extensive regions that lie between fixed stars, the surface of the universe would be almost exactly plane, but in the vicinity of every single star there would be a slight shallow hump at the centre of which would be the star itself." We must now supplement our picture as follows: The world's surface taken as a whole is a spherical surface of immense extension, and is studded with many small shallow humps, having the stars as their centres. (This is no contradiction to what was said before, for the average distances between neighbouring fixed stars are very small compared with the girth of the universe, and hence those parts of the world's surface between neighbouring stars can, in point of fact, be looked upon as almost plane.) In such a way, we might imagine a two-dimensional *picture* of the universe; the *real* universe would be, according to Einstein, its counterpart in three dimensions. Geometry designates this kind of curved space as *spherical space,* for it is analogous to a spherical surface.[1]

The reasons developed above against the idea of an infinitely extended universe had nothing to do with the problem of relativity in itself. There is another argument, however, in favour of the idea of a finite universe, which is most intimately connected with the idea of relativity. It is as follows: The considerations developed in Chapter XII, according to which inertia is not possessed by a body in itself, but, like gravitation, is caused by the interaction of bodies, guided Einstein in his task of establishing the equations of motion and the field equations. These comply with the general principle of relativity formulated in Chapter XVII, and with Mach's demands mentioned there concerning the relativity of rotational motions. A mathematical analysis, however, shows that we must not necessarily interpret these now completed equations, subsequently, as meaning that the inertia of a body is actually caused only by the interaction between it and the other masses of the universe. There are many physicists who consider the mathematical formulation of the new theory correct, but who do not agree with the above-mentioned conception of the nature of inertia.

The theory would then be robbed of its most profound

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[1] The general theory of relativity has been repeatedly objected to, because three-dimensional spherical space of that kind (as well as curved space generally) cannot be grasped by our imagination. But it is only fair to say that it is not Einstein's fault that our powef of imagination fails in this point.

conceptual nucleus. If it is to be in truth a completely consistent theory of relativity, and not merely a mathematical description of astronomical facts of the greatest possible accuracy, the inertia of bodies must be interpreted by it in the way indicated. Now Einstein showed mathematically that this interpretation is only possible if we assume universal space to be a closed spherical "world." Hence if the radical relativistic Mach-Einstein point of view be accepted, we shall have to believe the universe to be unbounded, but finite.

CONCLUDING REMARKS

THE purpose of this book is to elucidate the connections between the fundamental ideas of the theory of relativity. It will be useful, therefore, if we collect together in the form of a genealogical table the genesis of the ideas discussed in detail in the text. This has been done in the table on p. 167, which hardly needs any further explanation after what has already been said. It is intended to serve as a map of the regions of the special and general theory of relativity, for those who have lost their bearings in the mist of mathematical and geometrical difficulties.

After faithfully following the discussions of this book throughout, the reader may judge for himself as to the value or otherwise of the theory of relativity. If the author were to express his own opinion, it might perhaps appear too exuberant and cause mistrust. Still, we shall add a few objective words concerning the criticisms with which the theory of relativity, thanks to its reputation, has been more richly endowed than any other physical theory.

If critics take the point of view : " The truth of the theory is far from being sufficiently proved to merit our ranking its creator alongside Galileo or Newton," nothing can be said against it, unless it be that a structure of ideas can be admirable even if it has nothing to do with the reality of things.

But there are many who declare the whole to be logical nonsense. These people either do not understand the theory at all, or they are not clear as to the limits of the concept of "logic." It has been said, for instance: "The statement that a ray of light possesses the same velocity *c* with reference to two systems moving rectilinearly and uniformly with respect to each other is logically wrong." In this case the critic, though understanding what the theory of relativity is intended to convey, is nevertheless unaware of what belongs to the realm of logic. In point of fact, the statement quoted has nothing at all to do with logic; it merely upsets the traditional ideas of space and time. That cannot be denied.

A third critic reproaches the theory of relativity with being the most confused and mathematically difficult theory ever set up. That is to be explained thus: The average layman generally has not much to do with higher mathematics, and he is quite happy about it. If, in attempting to find the way to an understanding of the theory of relativity, he becomes entangled in mathematical problems, he is then surprised at the difficulties and becomes baffled. Of course he is unable to judge that these are not greater than in any other branch of mathematics. The theory of numbers, algebra, the theory of functions, etc., contain many a chapter which is far more difficult than those parts of differential geometry and absolute differential calculus that form the basis of Einstein's calculations. Mathematics is verily no child's play!

On the other hand, the theory of relativity is frequently overrated, as regards the *extent* of its importance. It supplies us with a new view of the world in respect of geometry, physics, and perhaps also of philosophical science. But it has nothing to do with what we call "World-conception" in the general human sense of the word. The man Einstein may be interesting from this point of view; his theory, however, must not be mixed up with this.

The aim of the theory is solely to approach the ideal of the rational description of physical processes as nearly as possible; and as far as can be judged at present, that purpose has been fulfilled.

SUPPLEMENTARY NOTES

NOTE TO PAGE 2

In accordance with the method of treatment here used, and guided by the desire for brevity and simplicity, we have already unhesitatingly ranged two statements alongside of each other, which may perhaps arouse the criticism of philosophically trained minds. For those (and only for those) who feel the need of a stricter formulation of the ideas involved, we shall add the following supplementary note: By the motion of a body (in general, and not only in the above-mentioned restricted sense) we understand the alteration of its position, and since the position of a body is only given by its distance from other bodies, the concept of motion is in its essence a relative one. Thus the first of the above statements does not express anything new; it is an analytical expression of opinion that characterises a property already belonging to the idea of motion in virtue of its definition. This significance of the word "motion" has been called the *phoronomic* (kinematic) conception of motion. But we can regard the idea of motion in yet another way, in that we understand by the "motion" of a body a physical condition, the presence of which might under certain circumstances be established also without reference to other bodies. For instance, if a body is in a state of rotation, the existence of this state may be recognised by the occurrence of centrifugal forces, without consideration of the surroundings. This *physical* idea of motion in Newtonian mechanics is thus not of the nature of a relative one. The second statement mentioned above has reference (with the limitation there made) to the physical conception of motion, and is therefore not self-evident, but a statement of a physical fact. From the discussion of the second half of this book it will be recognised that the tendency of the theory of relativity is to weld the phoronomic and the physical conceptions of motion into one, in such a manner that we can only speak of a physical state of motion of a body when it also carries out a phoronomical motion (*i.e.* relative to other bodies).

NOTE TO PAGE 119

We must take into account, as previously emphasised, that the results of the special theory of relativity can only be applied to a case of non-uniform motion, if very small world elements are being considered. We may infer, therefore, that a contraction of individual measuring-rods takes place; but it is not possible, on the basis of the special theory of relativity, to draw conclusions about the circumference of the disc as a whole. On the other hand, any concentric circle drawn on the top disc will always coincide with a corresponding concentric circle on the lower disc; the circumference of the top disc, for instance, will permanently run along the circumference of the lower disc, *i.e.* one will cover the other completely. But it is obviously reasonable to regard two figures which cover each other as being equal. (Cf. the transference of measuring-rods, situated normally to the direction of motion, Chap. VII, p. 53.) In this sense it is thus quite reasonable to say that each single measuring-rod contracts, but that the circumference of the disc as a whole does not contract.

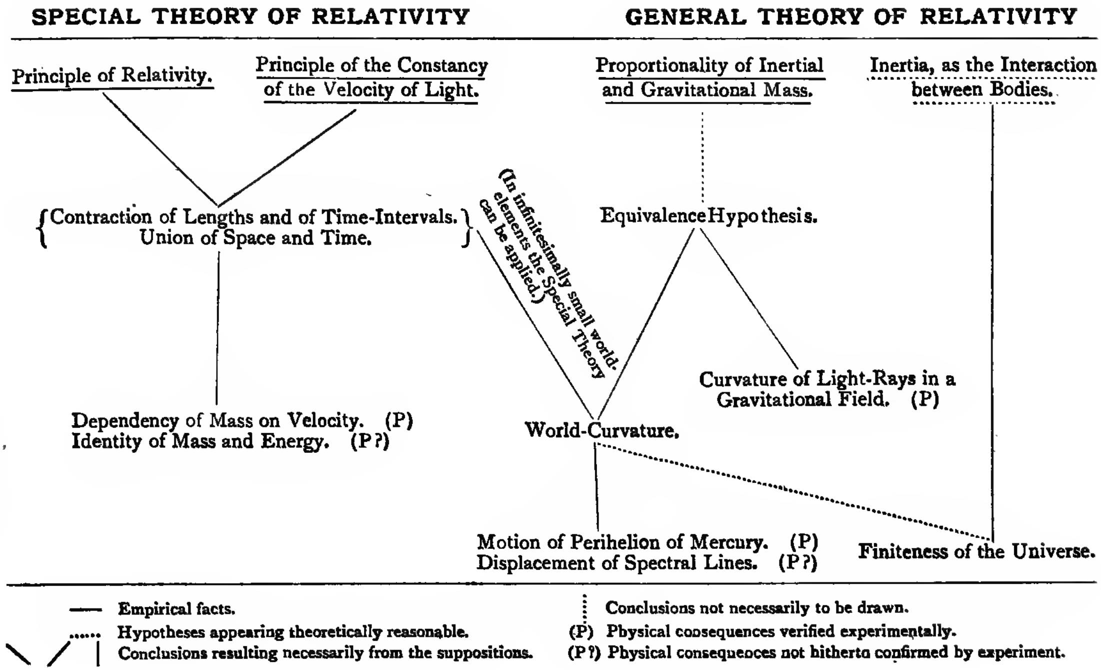
NOTE TO PAGE 147

By fulfilling the requirements of Mach mentioned in Chapter XV, Einstein's theory represents a reconciliation, as it were, of the Ptolemaic and Copernican world-systems (the latter, as pointed out by Mach, will always be the most useful one for all practical purposes). We can only say that the earth and the firmament of fixed stars carry out a rotatory motion relatively to each other, and there is no point in maintaining that " in truth " only one of the two is in motion, and the other at rest.

It was believed that one argument derived from the special theory of relativity had been found, which indicated that only one of the above-mentioned assertions can be correct, viz. " the earth rotates and the firmament of fixed stars is at rest." As explained in Chapter XI, it necessarily follows from the special theory of relativity that no material body can move with a greater velocity than that of light. The opponents of the general theory of relativity thus argue as follows: If the earth were at rest, and the firmament of fixed stars were revolving round it, even the nearest fixed stars would attain a velocity greater than that of light, and very distant ones must be revolving with velocities many million times greater than *c*, in order that they may cover their enormous orbits round the earth in a single day. To this we reply that the theory of relativity deals only with *relative motions*. According to the special theory of relativity it is quite out of the question, for instance, that at any time an outside cosmic body could traverse the solar system with a greater velocity than that of light. On the other hand, nothing prevents us from imagining a system of reference moving, say, in the direction of the earth's axis from north to south with a velocity of 400,000 km. per second. The earth and all heavenly bodies would then have velocities, relative to this system of reference, which would be greater than the velocity of light, without thereby violating the above-mentioned law of the special theory of relativity—for there would be no *relative velocities* greater than *c*. These appear only with reference to the fictitious co-ordinate axes of our peculiarly chosen system of reference. The case of a co-ordinate system with the earth at rest in it and the firmament of fixed stars rotating relatively to it is quite analogous. Here, too, we have no *relative velocities* greater than *c*. The distances of the fixed stars from each other, as well as from the centre of the earth or any other point of the globe, do not alter with velocities greater than *c*; these higher velocities appear only relatively to the axes of that co-ordinate system which is fixed to the earth—*i.e.* purely conceptual structures—and that is no more a violation of the special theory of relativity than the example aforementioned.

NOTE TO PAGE 152

Quite recently doubts have been raised by astronomers concerning this result, because a numerical error has been discovered in the fundamental work of the American astronomer Newcomb on the orbital Elements of the Four Inner Planets, whence this statement had been taken. It is possible, therefore, that the wonderfully striking coincidence between the value of the perihelion motion of Mercury worked out by Einstein and the observed value was accidentally caused by an error in the reduction of observations by Newcomb. Be that as it may, Einstein's calculations of the orbit of Mercury seem to agree better with current observations than those performed according to the Newtonian law. A full explanation of this question will only be possible, after Newcomb's great work on the Elements of the Four Inner Planets has been revised.



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