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# Research Note

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A CUBIC-FOOT STAND VOLUME EQUATION FOR LODGEPOLE PINE  
IN MONTANA AND IDAHO

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ABSTRACT

*Presents a total cubic-foot volume equation and table, and merchantable volume conversion factors for lodgepole pine stands in Montana and Idaho.*

Foresters and landowners often have need for quick, reliable estimates of stand volumes. The use of stand volume equations can provide such estimates while eliminating much of the computational work involved in the traditional individual-tree volume approach. In addition, the need for obtaining tree diameter measurements is eliminated when point sampling methods are used.

Stand volumes can be determined rapidly and efficiently from basal area and height data. Spurr (1952) made numerous tests on a number of volume equations and found the product of stand basal area and height to be strongly related to stand volume. Reliability of estimates was adequate for management needs and compared closely with estimates obtained by traditional stand volume sampling methods based on volumes of individual trees. Other workers have since used the basal area-height relationship in developing stand volume equations (Buckman 1961; Brinkman 1967; Myers 1967).

This paper presents a stand volume equation and table, and table of merchantable volume conversion factors for lodgepole pine (*Pinus contorta* Dougl.) in Montana and Idaho, and describes their development and application. The stand volume equation and table provide estimates of total, gross cubic-foot volume per acre and are applicable to even-aged stands. The table of conversion factors provides merchantable cubic-foot volume estimates for specified stump height and minimum upper stem diameters. The equation and tables are suggested for use with point sampling methods.

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## METHODS

A stand volume equation was developed by using plot data from 125 unmanaged lodgepole pine stands located in Montana and Idaho. The stands represented a broad range of stand structures. Basal areas per acre ranged from 30 to 250 square feet and height of dominant trees ranged from 21 to 83 feet. Stand ages ranged from 22 to 125 years.

Cubic-foot volumes for stands were available from 125 permanent plots; merchantable volumes were available from 116 of these plots. Minimum tree sizes included were 0.5 inch d.b.h. for total volume and 4.6 inches d.b.h. for merchantable volume.

Volumes of individual trees were based on an equation developed for forest survey purposes by James Brickell, using stem analysis data from 226 lodgepole pines from western Montana and 606 lodgepole pines from Wyoming and Colorado.<sup>2</sup>

Covariance analysis indicated data from the two geographical areas were not statistically different, so the data were pooled in developing a tree volume equation:

$$V = 0.002782 D^2 H (H^{0.0488} / D^{0.0959})$$

where:

V = Total cubic-foot volume

D = Diameter at breast height, outside bark, in inches

H = Total tree height in feet

Although the nonlinear volume function could have been made linear by logarithmic transformation, iterative fitting of the function was chosen to allow nonlinear experimentation with the ratio of height to diameter as an expression of the cylinder form factor. The final equation was obtained, using Marquardt's (1966) Fortran IV computer program for least squares estimation of nonlinear parameters. The equation was weighted in fitting, using weights inversely proportional to the variance about the regression surface.

Merchantable cubic volumes of individual trees were based on the following volume distribution function for lodgepole pines (Honer 1967):

$$V_m = V_t (0.9658 - 0.1278X - 0.8108X^2)$$

where:

$V_m$  = Volume in merchantable cubic feet

$V_t$  = Volume in total cubic feet

$X = d^2/D^2 (1 + h/H)$

d = Top diameter, inside bark, in inches

D = Diameter at breast height, outside bark, in inches

h = Stump height in feet

H = Total tree height in feet

Merchantable volumes were computed for minimum top diameters of 3 and 4 inches diameter inside bark (d.i.b.) and stump height of 0.5 foot.

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<sup>2</sup>Unpublished data from Idaho and Montana are on file at Intermountain Forest and Range Experiment Station, Moscow, Idaho. Colorado and Wyoming data were furnished by courtesy of Clifford Myers, Rocky Mountain Forest and Range Experiment Station, Fort Collins, Colorado, and were discussed in a previous publication (Myers 1964).

Table 1.--Stand volume table for lodgepole pine in Montana and Idaho<sup>1</sup>

Basal area (ft. <sup>2</sup> )	Average height of dominant trees									
	Feet									
	10	20	30	40	50	60	70	80	90	100
----- Stand volumes in cubic feet per acre <sup>2</sup> -----										
30	108	249	390	531	671					
50	202	437	671	906	1,141	1,376				
70	296	625	953	1,282	1,611	1,939	2,268	2,597		
90	390	812	1,235	1,657	2,080	2,503	2,925	3,348	3,770	
110	484	1,000	1,517	2,033	2,550	3,066	3,583	4,099	4,615	5,132
130		1,188	1,798	2,409	3,019	3,629	4,240	4,850	5,461	6,071
150		1,376	2,080	2,784	3,489	4,193	4,897	5,601	6,306	7,010
170		1,564	2,362	3,160	3,958	4,756	5,555	6,353	7,151	7,949
190		1,751	2,643	3,536	4,428	5,320	6,212	7,104	7,996	
210		1,939	2,925	3,911	4,897	5,883	6,869	7,855	8,841	
230			3,207	4,287	5,367	6,447	7,526	8,606		
250			3,489	4,662	5,836	7,010	8,184	9,358		
270			3,770	5,038	6,306	7,573	8,841			
290			4,052	5,414	6,775					

<sup>1</sup>Block indicates extent of basic data.

<sup>2</sup>Includes entire stem volume of all trees larger than 0.5 inch d.b.h.

A stand volume equation was obtained by linear regression of volume per acre in cubic feet on the product of basal area per acre and average height of dominant trees (BH). The equation was weighted in fitting by the quantity  $1/(BH)^2$ , to compensate for nonhomogeneous variance about regression. Average heights of dominant trees were based on 10 dominant trees per plot. Basal areas were computed from plot diameter-class tallies. The equation is:

$$V_T = 0.46952(BH) - 32.79$$

$$r^2 = 0.995$$

$$S_{y.x} = 82.9 \text{ ft.}^3/\text{acre} \text{ (2.4\% of the mean)}$$

where:

$V_T$  = Gross volume in cubic feet per acre of all trees greater than 4.5 feet in height

B = Basal area per acre in square feet

H = Average total height of dominant trees in feet

A volume table based on this equation is presented in table 1.

Factors for conversion of stand volume in total cubic feet to merchantable cubic feet per acre were determined graphically (fig. 1) by plotting ratios of merchantable to total cubic-foot volumes over average stand diameters (diameters of trees of average basal areas). They are presented in table 2.

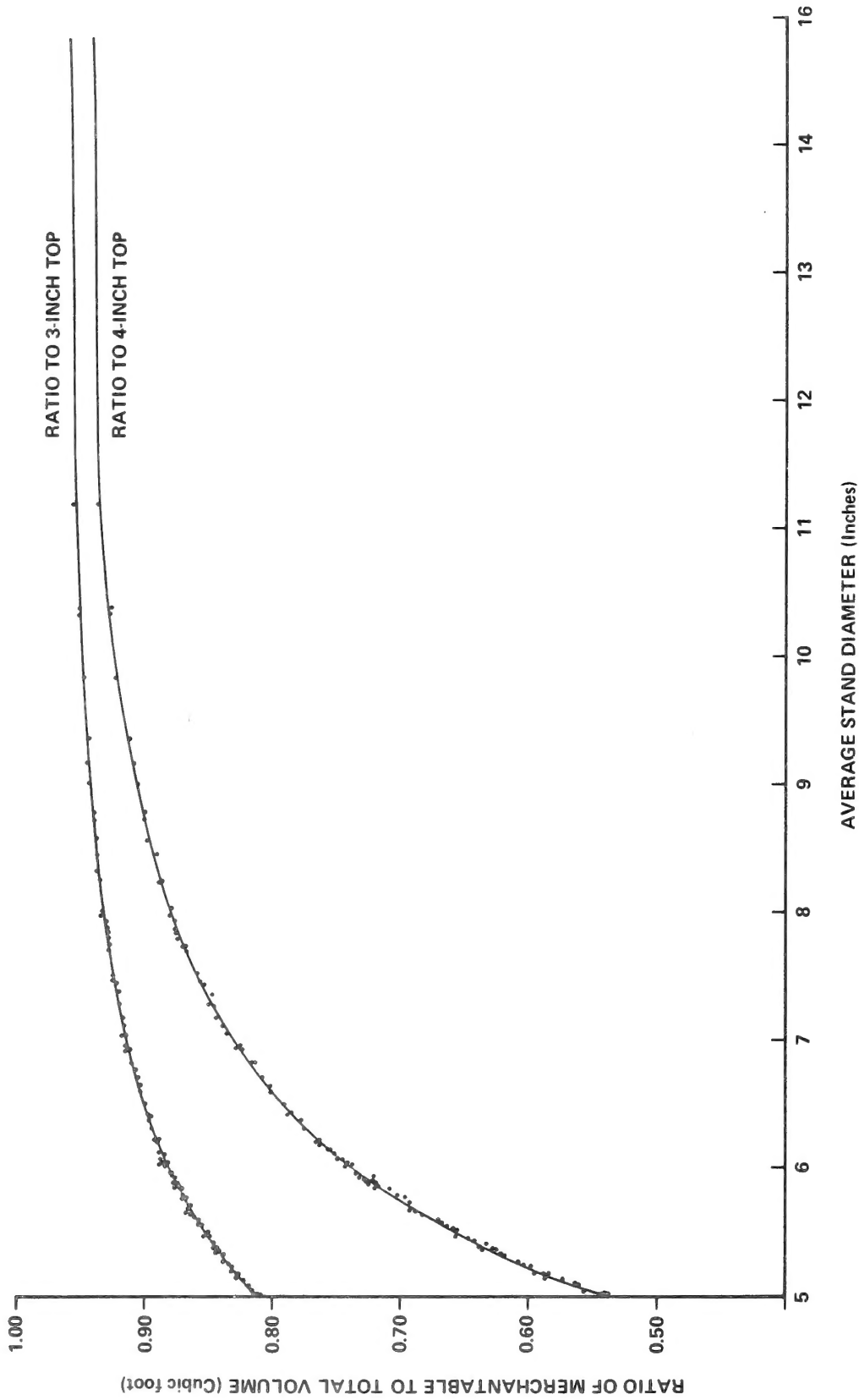


Figure 1.--Ratios of merchantable to total cubic-foot volumes over average stand diameters. (Merchantable volumes were calculated for a stump height of 0.5 foot and minimum top diameters of 3 and 4 inches, inside bark, using Honer's (1967) volume distribution function for lodgepole pines.)

Table 2.--*Factors for conversion of stand volumes in total cubic feet to merchantable<sup>1</sup> cubic feet per acre, lodgepole pine in Montana and Idaho*

Average stand diameter (inches)	Ratio of merchantable to total volume		Average stand diameter (inches)	Ratio of merchantable to total volume	
	3-in. top	4-in. top		3-in. top	4-in. top
5.0	0.810	0.541	8.2	0.933	0.884
5.2	.828	.596	8.4	.935	.890
5.4	.844	.639	8.6	.937	.895
5.6	.857	.675	8.8	.939	.900
5.8	.868	.708	9.0	.941	.904
6.0	.878	.736	9.2	.943	.908
6.2	.887	.762	9.4	.944	.912
6.4	.896	.783	9.6	.945	.916
6.6	.903	.800	9.8	.946	.919
6.8	.907	.815	10.0	.947	.922
7.0	.914	.828	10.5	.949	.928
7.2	.917	.841	11.0	.951	.933
7.4	.921	.852	11.5	.952	.935
7.6	.924	.862	12.0	.953	.937
7.8	.927	.871	13.0	.954	.938
8.0	.931	.878	14.0	.955	.939

<sup>1</sup>Stump height 0.5 foot in trees 4.6 inches d.b.h. and larger.

#### APPLICATION

The stand volume equation and tables presented here should be applicable to any managed or unmanaged lodgepole pine stands that are within the ranges of average dominant height and basal area classes reported in this paper.

Total gross cubic-foot volume of a lodgepole pine stand can be found by obtaining stand basal area and average height of dominant trees and substituting them in the stand volume equation, or by referring to the stand volume table (table 1). Merchantable gross cubic-foot volume is obtained by multiplying the total cubic-foot volume estimate by the appropriate conversion factor from table 2.

Basal area per acre can be determined from a diameter tally of fixed-radius plots, or from variable-radius plots using an angle gage or wedge prism. The latter method is recommended for efficiency.

For variable-plot cruising, an angle factor should be chosen to give about seven "count" trees per point. Generally, an angle factor of 10 will give satisfactory results. Only those trees with diameters larger than the minimum size of interest should be included in the point tally. When merchantable cubic-foot volume is desired, it is also necessary to count trees of the sizes of interest at each sample point to obtain average stand diameter for use with table 2. A 1/20-acre fixed-radius plot superimposed on the variable plot sampling point can be used for sampling trees per acre. A range-finder set for the radius of a 1/20-acre plot (26.33 ft.) allows plot limits to be defined optically by providing a check on borderline trees. Knowing basal area and trees per acre, the corresponding average stand diameter can be found in a standard basal-area table.

Height of average dominant trees should be measured to the nearest foot at each variable-plot sampling point. One or two height measurements per plot will furnish a representative height of dominant trees if sampling design of cruise is adequate for basal area purposes. To avoid overestimates of volume resulting from the tendency to measure outstandingly tall trees in the stand, one should scan surrounding trees and select one or two "average-appearing" dominants for height measurements. These heights will then be comparable to the "10-tree" average dominant heights used in the development of the stand volume equation.

Determination of sample size is a sampling problem dependent on the stand variability and degree of precision desired. Where experience in similar stands is lacking, a preliminary survey to estimate basal area variation is recommended to determine a suitable sample size in the conventional manner.

To illustrate use of the equation and conversion factors, consider the following examples:

- (1) Assume that a stand having diameter classes ranging from 2 to 8 inches is cruised to determine total cubic-foot volume ( $V_T$ ). Stand basal area is 110 square feet per acre and average height of dominant trees is 55 feet. The calculation is made as follows:<sup>3</sup>

$$\begin{aligned}V_T &= 0.46952(BH) - 32.79 \\ &= 0.46952(110 \times 55) - 32.79 \\ &= 2,807.8 \text{ cubic feet}\end{aligned}$$

- (2) Assume that a stand is cruised to determine merchantable cubic-foot volume to a 4-inch minimum top diameter, inside bark. Diameter classes range from 3 to 11 inches; however, minimum size of merchantable trees included in the cruise is 4.6 inches d.b.h. Trees 4.6 inches d.b.h. and larger represent stand basal area of 120 square feet per acre and an average stand diameter of 8.2 inches. Average height of dominant trees is 72 feet. Merchantable cubic-foot volume is determined as follows:

$$\begin{aligned}V_T &= 0.46952(BH) - 32.79 \\ &= 0.46952(120 \times 72) - 32.79 \\ &= 4,023.86 \text{ cubic feet}\end{aligned}$$

From table 2 for an average stand diameter of 8.2 inches, the conversion factor is 0.884. Merchantable stand cubic-foot volume ( $V_M$ ) is computed as:

$$\begin{aligned}V_M &= 0.884 V_T \\ &= 0.884 \times 4,023.86 \\ &= 3,557.09 \text{ cubic feet}\end{aligned}$$

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<sup>3</sup>Alternatively, total cubic-foot volume can be obtained from table 1, interpolating between height and basal area classes as necessary.



## LIMITATIONS

The stand volume equations and tables presented here are intended for estimation of gross stand volumes in Montana and Idaho. No deductions for cull or defect were included in their development. Therefore, stand defect or cull factors (determined by a concurrent survey or past experience) must be applied to the gross volume estimates to obtain net stand volumes.

Limited extrapolations beyond the range of the basic data (as in table 1) seem reasonable; however, no actual tests of the accuracy of such extrapolations have been made.

Where experience shows the equation or table to result in consistently high or low estimates, minor adjustments should be made for the local condition.

## SUMMARY

A stand volume equation is presented for even-aged lodgepole pine stands in Montana and Idaho. The equation gives direct estimates of total gross cubic-foot volume of stands from measurements of stand basal area and average height of dominant trees.

A table of conversion factors is presented to provide estimates of merchantable cubic-foot volume. Based on average stand diameters, use of these factors allows conversion of total gross cubic-foot stand volumes per acre to merchantable gross cubic-foot volumes per acre, for specified utilization limits.

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