


MIT LIBRARIES



3 9080 00569723





Digitized by the Internet Archive
in 2011 with funding from
Boston Library Consortium Member Libraries

<http://www.archive.org/details/defaultrenegotia00hart>

HB31
.M415
no. 520

LEWIS

**working paper
department
of economics**

S INST TECH.
AUG 23 1989

DEFAULT AND RENEGOTIATION:
A DYNAMIC MODEL OF DEBT

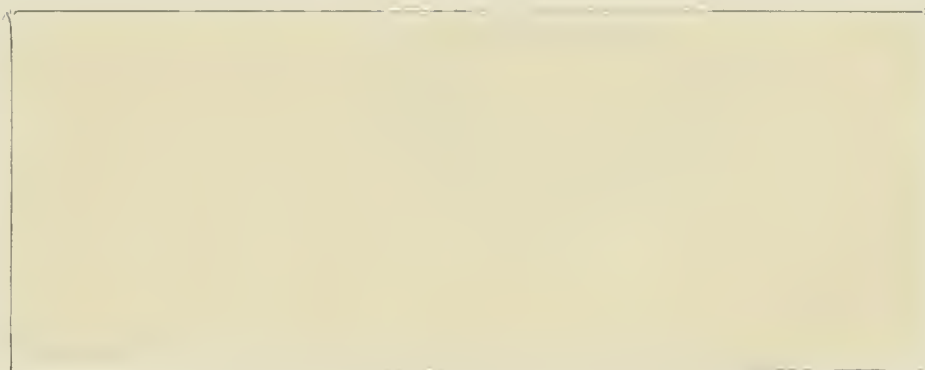
BY OLIVER HART
AND
JOHN MOORE

MAY 1989

No. 520

**massachusetts
institute of
technology**

**50 memorial drive
cambridge, mass. 02139**



DEFAULT AND RENEGOTIATION:
A DYNAMIC MODEL OF DEBT

BY OLIVER HART
AND
JOHN MOORE

MAY 1989

No. 520

DEFAULT AND RENEGOTIATION: A DYNAMIC MODEL OF DEBT

BY

OLIVER HART AND JOHN MOORE*

MAY 1989

* Massachusetts Institute of Technology and Harvard Business School; and London School of Economics, respectively. Helpful discussion with Douglas Baird, David Meerscham, Rick Ruback and Bill Sahlman are gratefully acknowledged. Both authors would like to thank the National Science Foundation and the LSE Suntory-Toyota International Centre for Economics and Related Disciplines for financial assistance. Much of this work was done while the first author was visiting the Harvard Business School as a Marvin Bower Fellow. He would like to thank that institution for its hospitality and financial support, and also to acknowledge financial assistance from the Guggenheim and Olin Foundations and the Center for Energy Policy Research at MIT.

MIT LIBRARIES
AUG 23 1989
RECEIVED

ABSTRACT

This paper considers a situation where an entrepreneur borrows funds from a creditor (e.g. a bank) to finance an investment project. The project will on average generate returns in the future, but these returns accrue in the first instance to the entrepreneur and cannot be allocated directly to the creditor. The role of debt is to provide an incentive for the entrepreneur to transfer some of the future receipts to the creditor. The idea is that if the entrepreneur does not pay his debt, the creditor has the right to seize some fraction of the debtor's assets. We analyze the implications of this right for the form of the long-term debt contract and for the efficiency of the debtor-creditor relationship. We pay particular attention to the fact that, given that the liquidation value of the assets is typically less than their value to the debtor, the creditor will often choose not to exercise her right to seize the assets, preferring instead to renegotiate her loan. However, in some cases, the inability of the debtor to commit credibly to pay a sufficient portion of the assets' future value to the creditor means that liquidation will occur anyway even though it is inefficient. Thus, among other things, the theory provides an explanation of the social costs of default or bankruptcy.

1. Introduction

This paper considers a situation where an entrepreneur borrows funds from a creditor (e.g. a bank) to finance an investment project. The project will on average generate returns in the future, but these returns accrue in the first instance to the entrepreneur and cannot be allocated directly to the creditor. The role of debt is to provide an incentive for the entrepreneur to transfer some of the future receipts to the creditor. The idea is that if the entrepreneur does not pay his debt, the creditor has the right to seize some fraction of the debtor's assets. We analyze the implications of this right for the form of the long-term debt contract and for the efficiency of the debtor-creditor relationship. We pay particular attention to the fact that, given that the liquidation value of the assets is typically less than their value to the debtor, the creditor will often choose not to exercise her right to seize the assets, preferring instead to renegotiate her loan. However, in some cases, the inability of the debtor to commit credibly to pay a sufficient portion of the assets' future value to the creditor means that liquidation will occur anyway even though it is inefficient. Thus, among other things, the theory provides an explanation of the social costs of default or bankruptcy.

Before going into the model's details, we relate it to the literature. Traditional models of a firm's financial structure start from the position that the firm has a fixed, although possibly uncertain, future return stream. The question asked is how should this be divided between debt and equity so as to maximize the firm's market value. Of course, the well-known propositions of Modigliani and Miller tell us that under certain conditions, say if debt is riskless and there are no taxes, the division is irrelevant. Moreover, even if the conditions of the propositions don't hold, e.g. debt is risky, this approach begs the question of why the firm issues debt and equity at all. Why doesn't it issue a richer class of contingent claims so as to obtain the benefits of completing the market? Or if these benefits are small, why doesn't it forget about debt and just issue equity?¹

More recent approaches to financial structure have attempted to resolve these questions by dropping the idea that the firm's return stream is fixed. It is supposed either that management can control return through its actions (moral hazard) or that management has private information about return

(adverse selection). Financial structure is then seen as a bonding or signaling device. High debt constrains management to produce high future profit or signals that management already knows that future profit is high; moreover, this bonding or signaling is credible since low profit in the future would be followed by bankruptcy, an unpleasant event for management.² Unfortunately, this approach also runs into difficulties. First, it is unclear why management cannot signal or bond more directly, and apparently cheaply, by the use of an incentive scheme. For example, management could announce that unless profits reach a certain level, it will take a pay cut or resign. Not only does an incentive scheme avoid the complexities and costs of bankruptcy procedure, but it allows managerial welfare to depend in a more sensitive way on firm performance than does financial structure. Second, the signaling and bonding theories do not explain the nature of bankruptcy costs, simply taking these as given.

In contrast, the approach in this paper emphasizes the control rights associated with debt rather than its incentive properties. The paper takes the point of view that debt refers to any fixed obligation to pay, even if it is contingent. That is, a promise to pay \$100 if the sun shines and \$50 if it rains is as much debt as a promise to pay \$75 whatever happens, to the extent that the creditor has a claim on the debtor's assets in the event of default (e.g. if the sun shines and the debtor doesn't pay \$100; we are assuming that sunshine is a verifiable event). Debt is here to be contrasted with equity where there is no fixed obligation to pay -- an individual equity holder is entitled to her pro-rata share of any dividends the firm chooses to issue, but cannot hold the firm in default if a particular level of dividend is not realized. (In the model to be presented, no dividends are ever paid and there is therefore no role for outside equity in the usual sense.)

The creditor's ability to foreclose on a debtor's assets in the event of default provides the debtor with a powerful incentive to repay his debts if he can. This incentive will be a focal point of the model we consider. That is, debt is seen as a mechanism for releasing funds from a firm ("free cash flow" in Jensen (1986)'s terms). In order to justify the role of this mechanism, however, we must explain why other methods for releasing funds won't work, e.g. the use of an entrepreneurial incentive scheme. Our assumption is that any cash flows received by the firm can be diverted by the entrepreneur for his own purposes on a one to one basis. That is, a dollar

of the firm's receipts can be turned into a dollar of entrepreneurial utility; moreover this utility accrues to the entrepreneur even in default states: the creditor can seize the firm's physical assets, but not the cash flows the entrepreneur has chosen to divert. This of course renders any incentive scheme, in which for each dollar distributed to the creditor the debtor can retain a fraction $\alpha < 1$, useless: the entrepreneur can always do better by diverting the dollar. The assumption that cash flows are totally divertible is a strong one, although perhaps not unrealistic in some cases.³ We believe, however, that a number of our results will generalize to the case where, instead of being able to divert the firm's funds directly, the entrepreneur can invest them in ways which are of interest to him, but not to an outside investor, e.g., in unprofitable pet projects or in prestige-enhancing or empire-building activities;⁴ or where the entrepreneur can threaten to tie the firm's funds up, say by delaying the termination of some profitable project, unless and until the creditors agree to give the entrepreneur an adequate share of return.⁵

The model we use supposes symmetric information between the entrepreneur and creditor both when the contract is written and after uncertainty is resolved. However, many of the variables of interest, such as project return and asset liquidation value, are assumed not to be verifiable by outsiders, e.g. a court; hence contracts cannot be conditioned on these. The symmetry of information between the parties means that renegotiation of the debt contract following default is relatively straightforward to analyze. However, renegotiation does not necessarily lead to first-best efficiency. The reason is that situations can arise where even though the value to the debtor of retaining assets the creditor can foreclose on exceeds the value to the creditor, there is no credible way for the debtor to compensate the creditor for not foreclosing on and liquidating the assets. The point is that the debtor may not have sufficient current funds for such compensation (particularly if his loan default was involuntary), and, while he may promise the creditor a large fraction of future receipts, the creditor may worry that when the time comes, she will not be able to get her hands on these: the debtor will default.

It is worth expanding on this observation, because we believe that it is important. The reason the Coase theorem fails here is that one of the parties is liquidity-constrained and so perfect bribing is impossible (the

same point has been made in a different context by Aghion and Bolton (1988)). As a result, default will generally be followed by a socially inefficient outcome: assets will be sold off even though their value in place exceeds their liquidation value. In our model, this possibility arises because there is no credible way for the entrepreneur to commit himself not to divert cash from the firm in the future. However, we believe that the phenomenon is much more general. For example, take a situation where a firm's future productivity depends on contributions from managers, workers, etc. If complete long-term contracts cannot be written, creditors will realize that the ex-post bargaining strength of these groups will inevitably mean that some of the firm's future return stream will accrue to workers and managers. Hence the creditors may choose to sell off the assets now even though this is socially inefficient.⁶

In our framework, social inefficiency or dead weight loss never arises when the debtor has control of the assets. The reason is that the creditor is not liquidity constrained and hence, given any socially inefficient allocation chosen by the debtor, the creditor can always propose a Pareto superior one and bribe the debtor accordingly.⁷ An implication of this is that an optimal contract will give the debtor control of the assets in as many of the states of the world as possible, subject to the constraint that the creditor recoups her initial investment. A major purpose of the paper is to characterize such a contract. We show that the creditor's expected gross return is increasing in the debt level at any date, and hence reaches a maximum $\bar{\pi}$ when debt is infinite; thus $\bar{\pi} \geq I$, where I is initial investment cost, is a necessary and sufficient condition for the creditor to be willing to finance the project at all. (Infinite debt can also be interpreted as a situation where the creditor is a 100% equity holder, in the sense that she has control of the assets in all states (since the debtor always defaults).) We also suppose that there are many potential creditors at the first date, so that the entrepreneur only needs to offer any one of them I . Thus, assuming that $\bar{\pi} > I$, the question is how to reduce the creditor's return from $\bar{\pi}$ to I so as to maximize the debtor's payoff (or equivalently to maximize social surplus since this is the sum of the debtor and creditor's payoffs).

It turns out that there are some interesting trade-offs. First, in a multi-period setting, reductions in debt level at different dates will have different effects on the debtor's payoff since the social inefficiency caused

by involuntary default followed by seizure of assets soon after the project has started may be quite different from the corresponding social inefficiency at a later date. Hence the analysis can throw light on the optimal path of interest payments over time and in particular on the choice between short and long-term debt. Second, a reduction in debt is not the only way of increasing social efficiency at the expense of the creditor's profit. An alternative is a lump-sum transfer from the creditor to the debtor at the first date; this strengthens the debtor's bargaining position in default states and makes it more likely that renegotiation will lead to the rescheduling of debt rather than to the liquidation of assets. It turns out that this possibility plays an important role in the analysis. In fact an optimal contract will typically be a combination of an up-front transfer and a sequence of future debt liabilities (another interpretation of the upfront payment is that the debtor borrows more than the cost of the investment at the first date; or that the debtor puts in less than his initial wealth into the project).

Our work is related to the recent papers of Aghion and Bolton and Kahn and Huberman. Aghion and Bolton (1988) also analyze debt in control terms, but with two important differences. First, they suppose that project returns are verifiable and can be allocated directly to the creditor, thus ignoring the role of debt as a mechanism for getting a debtor to pay up. Second, they analyze a one period model, in which, although ex-post inefficiencies arise consequent to renegotiation, these are not caused by the inability of a debtor to commit to paying out future cash flows; rather they are due to nonpecuniary costs that the creditor imposes on the debtor when she has control. On the other hand, Kahn and Huberman (1988) investigate the role of debt in encouraging a debtor to repay a loan, but in a context where renegotiation always leads to ex-post efficiency.⁸

Our work also bears a resemblance to the costly state verification models of Townsend (1979) and Gale and Hellwig (1985). As in their work, default is followed by a penalty; however, whereas in their models the penalty consists of being monitored, in our model it consists of having one's assets seized. In addition, we suppose the parties are symmetrically informed, whereas their models rely on asymmetries of information. Finally, and most important in economic terms, they take the cost of bankruptcy to be given (it is the fixed cost of monitoring), whereas we endogenize this.

Last, there is a clear parallel between this paper and the recent, burgeoning literature on sovereign debt (see, e.g., Eaton and Gersovitz (1981), Bulow and Rogoff (1989), Fernandez and Rosenthal (1988) and Froot (1989)). This literature also analyzes default and renegotiation of debt contracts, but under the assumption that there is no third party to enforce contracts. Also, while the models to date have analyzed how much a debtor country can be persuaded to pay back, they have not addressed the issue of ex-post inefficiencies arising from default.

The paper is organized as follows. The basic model is set out in Section 2. Section 3 provides an analysis of the two-period case. The three-period case is discussed in Section 4, and Section 5 provides a summary and conclusions.

2. The Model

We consider a risk neutral entrepreneur who requires I dollars at an initial date 0 to finance a project. For simplicity we take the entrepreneur's initial wealth to be zero. We assume that there is a competitive supply of risk neutral financiers or investors, each of whom is prepared to finance the project as long as she breaks even. The task for the entrepreneur is to design a pay-back agreement that persuades one of them to put up at least I dollars.⁹

We assume that the project lasts three periods, as illustrated in Figure 1, with returns being generated at dates 1, 2 and 3. These returns, which will typically be uncertain as of date 0, are supposed to be specific to this entrepreneur; that is, they cannot be generated without his cooperation. For simplicity, however, we ignore any actions taken by the entrepreneur to generate them; that is, the returns are produced simply by his being in place.

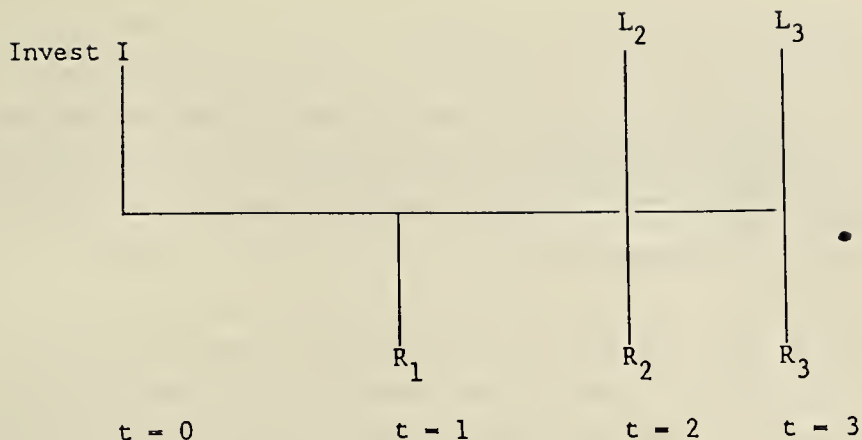


Figure 1

As emphasized in the introduction, we suppose that the project returns R_1 , R_2 , R_3 accrue to the entrepreneur in the first instance. Thus the pay-back agreement must be designed to give the entrepreneur an incentive to hand over enough of these returns to the investor to cover her initial cost I . We take the entrepreneur's and investor's discount rates both to be zero (which is also the market interest rate).

The I dollars of investment funds are used to purchase assets which have a second-hand or liquidation value at each stage of the project. We denote this value by $L_2 + L_3$ at date 1, by L_3 at date 2, and by zero at date 3. The interpretation is that liquidation at date 1 generates a flow return L_2 at date 2 and L_3 at date 3 (as opposed to the project returns R_2 and R_3); liquidation at date 2 generates a return L_3 at date 3 (as opposed to R_3); and the assets are worthless at the end of the project.¹⁰ In contrast to the R 's, however, the present value of the future L 's is received at the moment of liquidation (so liquidation at date 1 yields $L_2 + L_3$ immediately). Like the returns R_1 , R_2 , R_3 , the liquidation values L_2 , L_3 are typically uncertain as of date 0.

We make the major simplifying assumption that all uncertainty is resolved at date 1.¹¹ We suppose that -- as a result of their close post-investment relationship -- both parties learn the realizations R_1 , R_2 , R_3 , L_2 , L_3 at this date (so they have symmetric information). However, these realizations are not verifiable to outsiders, and so date 0 contracts cannot

be conditioned on them.

We make various further assumptions. First, to simplify the analysis, we suppose that the assets always have at least as high a value when operated by the debtor as when liquidated; that is, $R_2 + R_3 \geq L_2 + L_3$ and $R_3 \geq L_3$. In fact, we make the stronger assumption:

$$(*) \quad R_3 \geq L_3 \text{ and } R_2 \geq L_2.$$

Second, we assume that the assets are divisible and production exhibits constant returns to scale once the project is under way.¹² That is, if a fraction $(1-f_2)$ of the assets is sold off at date 1 (so that the entrepreneur retains a fraction f_2 between dates 1 and 2), and a further fraction $(f_2 - f_3)$ is sold off at date 2 (so that the entrepreneur retains a fraction f_3 between dates 2 and 3), the date 2 return equals $f_2 R_2$ and the date 3 return equals $f_3 R_3$; at the same time, the date 1 liquidation receipts are $(1-f_2)(L_2 + L_3)$ and the date 2 liquidation receipts are $(f_2 - f_3)L_3$.

Finally, we ignore any consumption by the entrepreneur during the relationship. That is, we suppose that any funds not paid over to the debtor are saved (at a zero interest rate) and are available to be used as payments later on (but see Remark 2 in Section 3 and the end of Section 4).

Feasible Contracts.

As mentioned in the introduction, we suppose that the cash flows R_1 can be diverted by the entrepreneur for his own benefit, as they are realized. In contrast, the physical assets generating them (those purchased with the initial investment funds) are assumed to be fixed in place and can be seized by the investor in the event of default (in addition, we suppose that the investor can ensure that these assets are purchased with the funds she puts up, i.e. the entrepreneur cannot take the money and run at date 0).¹³ It follows that the only kind of pay-back agreement which will induce the entrepreneur to part with his funds is a debt contract: the entrepreneur agrees to pay fixed amounts of cash at dates 1 and 2; and if he fails to do so, he is in default and the creditor has a claim on his assets.¹⁴

Note that there is no way to persuade the entrepreneur to pay anything at date 3 since, at that stage the assets are worthless and so the investor has no leverage over him.¹⁵ Note also that in a more general situation, the date 1 and date 2 payments could be made contingent on publicly observable events. In the present model, however, there are no such events (R_i and L_i are not verifiable) and so there is no scope for such an agreement.¹⁶

The above motivates consideration of a debt contract of the following form. At date 0, the investor (henceforth known as the creditor, C) supplies funds equal to at least I : $(I + T)$ say, where $T \geq 0$. In return the entrepreneur (henceforth known as the debtor, D) promises to pay back P_1 at date 1 (just after R_1 is realized) and P_2 at date 2 (just after R_2 is realized). As we shall see, the possibility that $T > 0$ will be quite important.

Suppose that such a contract is in place and a particular realization R_1, R_2, R_3, L_2, L_3 of the return stream and liquidation values occurs at date 1. How will the debtor react? If $T + R_1 > P_1$ he has two choices at date 1: he can make his first debt payment or he can default. In contrast, if $T + R_1 < P_1$, he has only one choice: to default. (Note that D's wealth at date 1 is $T + R_1$ since he carries over T from date 0.) Similarly, at date 2, (assuming he has not defaulted at date 1), D can choose to default or not if $T + R_1 + R_2 - P_1 \geq P_2$; but is forced to default if $T + R_1 + R_2 - P_1 < P_2$.

As we have said above, in the event of default, the creditor has the right to seize at least some fraction of the debtor's assets. However, seizure is only a threat point: in general the liquidation value of the assets may be quite low and the creditor may prefer to renegotiate the debt contract.¹⁷

We will adopt a very stylized form of renegotiation. We will suppose that only the debtor can make offers, while the creditor's sole power is unilaterally to forgive and/or postpone debt payments.¹⁸ We proceed in this way because it makes the analysis tractable and also gives each party a share of the surplus. We are confident, however, that our main results are robust to the form of the renegotiation process.¹⁹ We will also suppose without loss of generality that if the debtor defaults, he pays nothing to the creditor prior to renegotiation; for further comments on this, see Remark 2,

Section 3.

In order to understand the implications of the renegotiation process, we begin by considering the position after a default at date 2, assuming that there has been no previous default at date 1.

The Consequence of Default at Date 2

We suppose the following sequence of events.

- (A) Pursuant to default at date 2, D moves first and makes a single take-it-or-leave-it offer to C. This offer consists of a date 2 cash payment from D to C and a fraction of the assets to be sold off or liquidated at date 2, the proceeds being given to C.
- (B) If C accepts, this new agreement comes into force and is executed.²⁰
- (C) If C rejects, she has the power unilaterally to forgive a portion of the debt -- that is to replace P_2 by $P'_2 \leq P_2$ -- and to foreclose on the rest; in legal parlance D "reduces her claim to judgement and levies on the specific assets in D's possession". At this stage D can choose to make a cash payment r_2 and a sheriff will then sell that fraction of the assets required to realize any remaining debt. That is, the sheriff will sell off assets in the value of $P'_2 - r_2$ if $P'_2 - r_2 \leq L_3$ and all the assets if $P'_2 - r_2 > L_3$.^{21,22}

At date 2 we are effectively in a one period model and it will turn out that stage (A) is irrelevant. In particular, and perhaps surprisingly, C's ability to forgive debt gives her all the bargaining power.²³ This is not the case, however, at date 1.

The Consequence of Default at Date 1

The sequence of events is very similar to that described above. One important difference is that we will suppose that if C forecloses on the assets, all outstanding debts become payable (i.e. the debt is "accelerated"). The timing is therefore as follows:

- (A) Pursuant to default at date 1, D moves first and makes a single take-it-or-leave-it offer to C. This offer consists of a date 1 cash payment from D to C, a promised payment \hat{P}_2 from D to C at date 2 and a fraction of the assets to be sold off or liquidated at date 1, the proceeds going to C.
- (B) If C accepts, this new agreement comes into force and the date 1 part is executed.
- (C) If C rejects, she can unilaterally overlook the default and postpone the current debt; that is she can replace the existing contract by one in which D owes her $P_1 + P_2$ at date 2 (so postponement occurs at a zero interest rate). In this case D continues in possession of all the assets until date 2.
- (D) Alternatively, C can forgive a portion of the outstanding debt (forever) and foreclose on the rest, say $P' \leq P_1 + P_2$. At this stage, D can choose to make a cash payment r_1 , and a sheriff will then sell off that fraction of the assets required to realize any remaining debt. That is, the sheriff will sell off assets in the value of $(P' - r_1)$ if $P' - r_1 \leq L_2 + L_3$ and all the assets if $P' - r_1 > L_2 + L_3$. At this point, all debts have been discharged and any remaining assets belong to D.

The new wrinkle at date 1, apart from acceleration, is that D can propose a contract which promises C a payment in the future. As we shall see, this can provide D with a substantial amount of bargaining power.

In a more general model the decision to accelerate debt would be treated as a choice variable. This seems likely to complicate the analysis without changing its fundamental nature.²⁴ Note that a justification for acceleration can be given (although we are not sure how realistic it is). Suppose that it is difficult to persuade a liquidator to return to the firm frequently to liquidate different parts of it. In particular, assume as an extreme case that there is a constraint that the liquidator can only be called in once. Then some type of acceleration will be required since any post-liquidation promise by D to pay C will not be credible given that the liquidator won't enforce it. Note that one can still imagine different forms

of acceleration from the one we consider; e.g. the contract could specify that default at date 1 leads to a total debt payment of q becoming due, where $q \neq P_1 + P_2$. We doubt, however, that the possibility that $q \neq P_1 + P_2$ would change our results significantly.^{25,26}

3. The Case Where $R_3 = L_3 = 0$

We begin by analyzing the "two period" case where $R_3 = L_3 = 0$. Under these conditions a contract is represented simply by the date 0 transfer T and the promised date 1 debt repayment P_1 . The situation is illustrated in Figure 2.

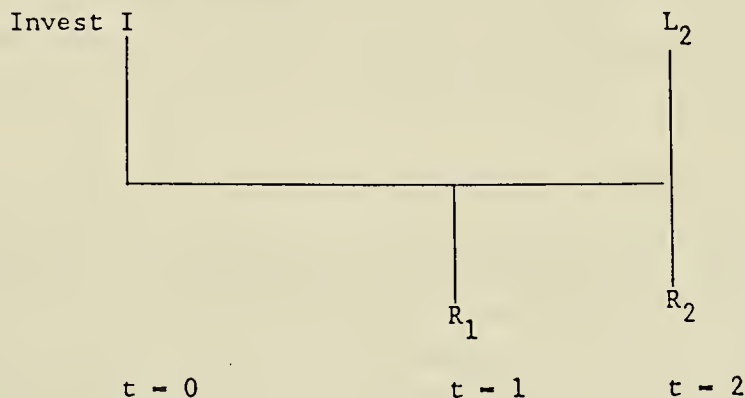


Figure 2

In order to understand how the possibility of default followed by renegotiation influences the debtor's and creditor's payoffs, it is useful to begin with the case where P_1 is infinite (or very large). Under these conditions, default occurs for sure at date 1. Surprisingly, the creditor's ability to forgive debt gives her 100% of the bargaining power in the subsequent renegotiation process in spite of the fact that only the debtor can make explicit offers.

To see why this is, note first that if C could make a take-it-or-leave-it offer to D , it would be the following:

If $T+R_1 \geq R_2$, C charges D R_2 and sets $f_2 = 1$
 (3.1)

If $T+R_1 < R_2$, C charges D $(T+R_1)$ and sets $f_2 = \frac{T+R_1}{R_2}$.

The reasoning is that C will charge D's willingness to pay for the assets -- R_2 per unit -- up to the point where either D has paid R_2 altogether or D has run out of money.

The next step is to note that C can mimic this take-it-or-leave-it offer through debt forgiveness. In particular, suppose that C reduces the debt to

$$(3.2) \quad \bar{P}(T) = \text{Min} \left[R_2, T+R_1 + \left[1 - \left(\frac{T+R_1}{R_2} \right) \right] L_2 \right].$$

(Note that $L_2 \leq \bar{P}(T) \leq R_2$, $\bar{P}(T) = R_2 \Leftrightarrow T+R_1 \geq R_2$ and $\bar{P}(T) > T+R_1 \Leftrightarrow T+R_1 < R_2$ and $L_2 > 0$.) Then, faced with this, D can do no better than to pay over R_2 if he has it, or $T+R_1$ if he doesn't; in the latter case the liquidator will sell off a fraction $\left[1 - \left[\frac{T+R_1}{R_2} \right] \right]$ of the assets to realize the amount still owing to C: $\bar{P}(T) - \left[T+R_1 \right] = \left[1 - \left(\frac{T+R_1}{R_2} \right) \right] L_2$.

To see why it is optimal for D to pay C $\text{Min}(T+R_1, R_2)$, suppose that D pays C the amount $x \leq \text{Min}(R_2, T+R_1)$. Then the liquidator will sell off a fraction $\left[\frac{\bar{P}(T) - x}{L_2} \right]$ of the assets if $\bar{P}(T) - x \leq L_2$ and all the assets otherwise. D's final utility will therefore be

$$(3.3) \quad T+R_1 - x + \text{Max} \left[0, 1 - \left[\frac{\bar{P}(T) - x}{L_2} \right] \right] R_2.$$

It is easy to see that this is increasing in x over the range $\bar{P}(T) - x \leq L_2$ (since $R_2 > L_2$) but decreasing over the range $\bar{P}(T) - x > L_2$. The only question, therefore, is whether D should pay C all the cash he has, up to R_2 , or none of it. It is easy to see that he is indifferent between the two: (3.3) equals $T+R_1$ both when $x = 0$ and when $x = \text{Min}(R_2, T+R_1)$ (we use $L_2 \leq \bar{P}(T)$). Therefore D may as well set $x = \text{Min}(R_2, T+R_1)$. (If C forgives debt

to just below $\bar{P}(T)$, D will strictly prefer to set $x = \text{Min}(R_2, T+R_1)$.)

In words, until D has paid all he owes, every dollar retained by him increases the outstanding debt by one dollar, thus causing the liquidator to sell off one dollar more of assets (assuming the liquidator is not already selling everything). But this is a losing proposition for D since one dollar of assets is worth more than a dollar to him (given that $R_2 > L_2$).

Note that since C effectively charges D R_2 per unit of the assets, D gets nothing out of the renegotiation process and ends up with a payoff of $(T+R_1)$; hence C gets all the surplus, which equals $\bar{P}(T)-T$.

Observe also that the outcome just described ($f_2 = 1$ if $T+R_1 \geq R_2$, $f_2 = 1 - (T+R_1/R_2)$ if $T+R_1 < R_2$) is ex-post Pareto optimal even though it does not generally maximize total surplus. The point is that, although when $f_2 < 1$, some of the assets are liquidated rather than being placed in their highest value use, D cannot bribe C not to liquidate them since D has no remaining date 1 cash; moreover, any promise by D to pay C part of the return R_2 at date 2 is not credible since C knows that D will default at date 2.

Given that the outcome is ex-post Pareto optimal, D's ability to make offers prior to C's forgiveness is irrelevant: C can never lose by turning them down and proceeding to forgive the debt as described above.

Let's turn next to the case $P_1 < \infty$. Not surprisingly, it turns out that P_1 influences the final outcome if and only if $P_1 < \bar{P}(T)$. In particular, if $P_1 \geq \bar{P}(T)$, anticipating that C will forgive the debt to $\bar{P}(T)$ (C can do no better than this since it allows her to extract all the surplus in the subsequent renegotiation), D will default at date 1 even if he can afford to pay P_1 -- and the outcome will be described above. On the other hand, if $P_1 < \bar{P}(T)$, anticipating that C won't forgive the debt (far from wanting to forgive it, C would like to increase it to $\bar{P}(T)$), D will pay C as much as he can until he's paid off the debt; that is, D will pay C the amount $\text{Min}(P_1, T+R_1)$. As before, the reasoning is that every dollar D doesn't pay C causes an extra dollar of assets to be liquidated (unless all the assets are being liquidated anyway); and, since a dollar of assets is worth more than a dollar to D, this hurts D.

Again this outcome is ex-post Pareto optimal ($f_2 < 1 \Rightarrow D$ pays C all the cash he has) and so D's power to make offers is irrelevant.

Note the importance of forgiveness when $P_1 > \bar{P}(T)$. If P_1 were very large and C could not forgive the debt, D would not pay any cash over to C since, regardless of how much he paid over, D would know that all the assets would be liquidated anyway. Given this, C's bargaining power would be much reduced: she would never get more than the liquidation value L_2 .

Lemma 1 sums up the discussion so far.

Lemma 1. Suppose $P_1 < \infty$. Then the following is the unique sub-game perfect equilibrium outcome:

- (1) If $P_1 > \bar{P}(T)$, D defaults (i.e. doesn't pay P_1 at date 1), C ignores any offers from D, C forgives the debt to $\bar{P}(T)$, and D pays C the amount $\text{Min}(T+R_1, R_2)$. The fraction of assets remaining in D's possession satisfies $f_2 = \text{Min}\left(1, \frac{T+R_1}{R_2}\right)$.
- (2) If $P_1 \leq \bar{P}(T)$ and $P_1 \leq T+R_1$, D pays C the amount P_1 and retains all the assets; while if $P_1 \leq \bar{P}(T)$ and $P_1 > T+R_1$, D defaults, C ignores any offers from D, C declines to forgive the debt, D pays C the amount $(T+R_1)$, and a fraction $(P_1 - T - R_1)/L_2$ of the assets is liquidated.
- (3) In both cases (1) and (2), the outcome is characterized by

$$(3.4) \quad f_2(R_2, L_2, P_1, T+R_1) = \begin{cases} 1 & \text{if } T+R_1 \geq R_2 \text{ or } T+R_1 \geq P_1 \\ \text{Max} \left[\frac{T+R_1}{R_2}, 1 - \left(\frac{P_1 - T - R_1}{L_2} \right) \right] & \text{otherwise,} \end{cases}$$

- (3.5) C's payoff (net of the transfer T), denoted by $\gamma(R_2, L_2, P_1, T+R_1)$, satisfies

$$\gamma = \text{Min}(P_1 - T, \bar{P}(T) - T),$$

- (3.6) D's payoff, denoted by $\delta(R_2, L_2, P_1, T+R_1)$, satisfies

$$\delta = R_1 + f_2(R_2, L_2, P_1, T+R_1)R_2 + \left[1 - f_2(R_2, L_2, P_1, T+R_1) \right] L_2 - \gamma(R_2, L_2, P_1, T+R_1).$$

Proof. Part (1) has been established in the text. Part (2) follows from the fact that C will never forgive the debt below P_1 if $P_1 \leq \bar{P}(T)$. The reason is that, by not forgiving, C can induce D to pay $\text{Min}(P_1, T+R_1)$ at the last stage since D's payoff $\left[1 - \left(\frac{P_1 - x}{L_2}\right)\right]R_2 + T+R_1 - x$ is increasing in x , given that $R_2 > L_2$. Note that $P_1 \leq \bar{P}(T) \leq T+R_1 + L_2$ and so when $P_1 \leq \bar{P}(T)$, the combined cash at D's disposal and liquidation value of the assets are enough to repay C's debts fully, i.e. C is repaid what she's owed even when some liquidation occurs.

To establish part (3), note first that if $T+R_1 \geq R_2$, $\bar{P}(T) = R_2$ and so, by part (1), $f_2 = 1$ if $P_1 > \bar{P}(T)$; also, by part (2), $f_2 = 1$ if $P_1 < \bar{P}(T)$. Hence $T+R_1 \geq R_2 \Rightarrow f_2 = 1$. Second, if $T+R_1 < R_2$, $\bar{P}(T) = T+R_1 + \left[1 - \left(\frac{T+R_1}{R_2}\right)\right]L_2$, which implies that $1 - \left[\frac{P_1 - T - R_1}{L_2}\right] \geq \frac{T+R_1}{R_2} \Leftrightarrow P_1 \leq \bar{P}(T)$. Hence, combining parts 1 and 2, we obtain that $T+R_1 < R_2 \Rightarrow f_2 = \text{Min} \left[1, \text{Max} \left[\frac{T+R_1}{R_2}, 1 - \left[\frac{P_1 - T - R_1}{L_2}\right]\right]\right]$. This establishes (3.4).

To establish (3.5), note that we have seen in the proof of part (2) that C's net payoff is $P_1 - T$ when $P_1 \leq \bar{P}(T)$; while we saw in the text that C receives $\bar{P}(T) - T$ if $P_1 > \bar{P}(T)$. This yields (3.5). Finally (3.6) follows from the identities $\delta = T+R_1 - x + f_2R_2$, $\gamma = x + (1-f_2)L_2 - T$, where x is the amount D pays to C. Q.E.D.

Remark 1: Lemma 1 tells us that D defaults voluntarily if and only if $P_1 > \bar{P}(T)$. Since $\bar{P}(T)$ is increasing in R_1 , R_2 and L_2 , this means that voluntary defaults will occur when R_1 , R_2 and L_2 are low. Note that, since $\bar{P}(T) \geq L_2$, voluntary defaults never occur when the debtor is "solvent"; that is, when the value of outstanding debt falls short of asset liquidation value (this is also the case in the three period model of the next section).²⁷

Five observations follow immediately from part (3) of Lemma 1. These are stated (without proof) in Lemma 2.

Lemma 2. (1) f_2 is (weakly) decreasing in P_1 and increasing in T . (2) $f_2=1$ (\Rightarrow) $T+R_1 \geq P_1$ or $T+R_1 \geq R_2$. (3) $\gamma(R_2, L_2, P_1, T+R_1) \leq P_1 - T$, (4) $\gamma(R_2, L_2, P_1, T+R_1)$ is (weakly) increasing in P_1 and decreasing in T . (5) If P_1 and T both rise by the same amount, $\gamma(R_2, L_2, P_1, T+R_1)$ falls (or stays constant) and $f_2(R_2, L_2, P_1, T+R_1)$ rises (or stays constant).

It is not surprising that C 's net payoff should be increasing in the debt level and decreasing in the upfront payment. The intuition for the first half of part (5) is that an equal increase in P_1 and T reduces C 's payoff since she loses the upfront payment for sure and is only repaid the debt in some states.

We turn next to the optimal choice of P_1 and T . Since D and C are both risk neutral, an optimal contract will maximize D 's expected return subject to the constraint that C 's expected return is no less than I ; that is, it will maximize $E\delta$ subject to $E\gamma \geq I$. In fact it is easy to see that the constraint will hold with equality since otherwise D 's expected return could be increased by raising T . Given that $E\gamma = I$ at the optimum, maximizing $E\delta$ subject to $E\gamma = I$ is equivalent to maximizing $E(\gamma + \delta) = E \left[R_1 + f_1 R_2 + (1-f_2)L_2 \right] - \text{expected value of social surplus, subject to } E\gamma = I$. That is, an optimal contract solves:

$$\begin{aligned} & \text{Maximize}_{P_1, T} \int f_2(R_2, L_2, P_1, T+R_1) [R_2 - L_2] dG(R_2, L_2, R_1) \\ (3.7) \quad & \text{S.T. } U_C(P_1, T) = \int \gamma(R_2, L_2, P_1, T+R_1) dG(R_2, L_2, R_1) = I, \end{aligned}$$

where G is the joint distribution function of (R_2, L_2, R_1) .

Lemma 3. (1) U_C is (weakly) increasing in P_1 and decreasing in T . (2) If P_1, T rise by the same amount, U_C falls (or stays constant). (3) At an optimum, $P_1 \geq T + I$.

Parts (1) and (2) follow directly from (4) and (5) of Lemma 2. Part (3) follows from (3) of Lemma 2 and the fact that $E\gamma = I$.

Lemma 3 allows us to obtain a necessary and sufficient condition for the project to be financed. Since U_c is increasing in P and decreasing in T , we need only compute $U_c(\infty, 0)$ and see whether it exceeds I .

Proposition 1. A necessary and sufficient condition for the project to be financed is that

$$(3.8) \quad E \text{ Min} \left[R_2, R_1 + \left(1 - \frac{R_1}{R_2} \right) L_2 \right] \geq I.$$

Proof: Follows from the fact that $\gamma = \bar{P}(T)$ when $P_1 = \infty$, $T = 0$ (see (3.5)) and from the formula for $\bar{P}(T)$ (see (3.2)). Q.E.D.

Note that in a first-best world (where there are no diversion problems), the project would be undertaken whenever $R_1 + R_2 \geq I$. Not surprisingly, (3.8) tells us that diversion causes too few projects to be financed.

The interesting case is where (3.8) holds with strict inequality, i.e. $U_c(\infty, 0) > I$. Then the issue is how to lower P_1 from infinity and raise T from zero so as to maximize expected surplus, at the same time driving U_c down to I . It turns out that the instruments P_1 and T have distinct roles and in general an optimal contract will involve the use of both of them, i.e. $P_1 < \infty$ and $T > 0$.

The trade-off can be understood as follows. Suppose $U_c(P_1, T) = I$. Then, from Lemma 3, a decrease in P_1 of 1 must be accompanied by a decrease in T of less than one to keep U_c at I (so $P_1 - T$ falls). From (3.4), this raises f_2 when $f_2 = 1 - \left(\frac{P_1 - T - R_1}{L_2} \right) > \frac{T + R_1}{R_2}$, but lowers f_2 when $f_2 = \frac{T + R_1}{R_2} > 1 - \left(\frac{P_1 - T - R_1}{L_2} \right)$.

In words, a small reduction in P_1 increases social efficiency in the states $T + R_1 < P_1 \leq \bar{P}(T)$, where, anticipating that C won't forgive the debt, D

pays C all the cash he has (it reduces the fraction of assets to be liquidated); while an increase in T raises social efficiency in all states where $f_2 < 1$ since it raises the amount of cash D can use either to pay off the initial debt level P_1 or to buy back the assets at the price of R_2 per unit in the renegotiation process.

In particular cases, we can say more about which instrument will be used. In the deterministic case where R_1, R_2, L_2 are known for sure in advance, there are many ways of supporting the optimum. Specifically, since, given a realization R_1, R_2, L_2 , renegotiation always leads to an ex-post Pareto optimal allocation, any contract that yields C a payoff of I must be optimal in ex-ante terms. One simple contract that does the trick is $P_1 = I, T = 0$ (assuming (3.8) holds).

Proposition 2. Suppose R_1, R_2, L_2 are nonstochastic and (3.8) holds. Then $P_1 = I, T = 0$ achieves the optimum.

Proof. Set $T = 0$. From (3.5), $\gamma = P_1$ when $P_1 \leq \bar{P}(0)$. In addition, by (3.8), $I \leq U_c(\infty, 0) = \bar{P}(0)$. Hence, setting $P_1 = I$ yields $\gamma = I$. Q.E.D.

We can also say quite a lot about the optimal choice of P_1 and T in cases where only one of the variables R_2, R_1, L_2 is stochastic.

Proposition 3. Suppose that only R_1 is stochastic and (3.8) holds. Then it is optimal to set $P_1 = \infty$ (a "pure" transfer contract).

Proof. If $P_1 \geq R_2$, then $P_1 \geq \bar{P}(T)$ for all R_1 . Hence in this case we can set $P_1 = \infty$ without changing anything (see Lemma 1(1)). So assume $P_1 < R_2$. Then, (a) $T+R_1 \geq P_1 \Rightarrow \bar{P}(T) \geq P_1 \Rightarrow \gamma = P_1 - T$ and $f_2 = 1$ (from (3.4)-(3.5)); (b) $T+R_1 < P_1 < R_2 \Rightarrow T+R_1 < \bar{P}(T) = T+R_1 + \left[1 - \left[\frac{T+R_1}{R_2}\right]\right]L_2$ and hence, by (3.4), $f_2 = \text{Max} \left[\frac{T+R_1}{R_2}, 1 - \left[\frac{P_1 - T - R_1}{L_2}\right]\right]$ and $\gamma = T+R_1 + (1-f_2)L_2 - T$ (the expression for γ follows from the fact that the final outcome is ex-post Pareto optimal and so, given that $f_2 < 1$, D ends up with no cash). Combining (a) and (b), we can write

$$(3.9) \quad U_c = E\gamma = E \text{Min} (P_1 - T, R_1) + L_2 - L_2 E f_2 = I.$$

Now raise P_1 to R_2 , at the same time increasing T to keep (3.9) satisfied. By Lemma 3, $(P_1 - T)$ does not fall and so neither does the first term in (3.9). Hence Ef_2 does not fall either. Therefore expected social surplus, given by $Ef_2(R_2 - L_2)$, also does not fall. Therefore the new contract with $P_1 = R_2$ is no worse than the original one with $P_1 < R_2$ (and in general is strictly better). Finally, as remarked at the beginning of the proof, a contract with $P_1 \geq R_2$ can be replaced by an equivalent one with $P_1 = \infty$. Hence we have shown that there is an optimal contract with $P_1 = \infty$. Q.E.D.

It will be helpful to illustrate Proposition 3 with an example.

Example 1

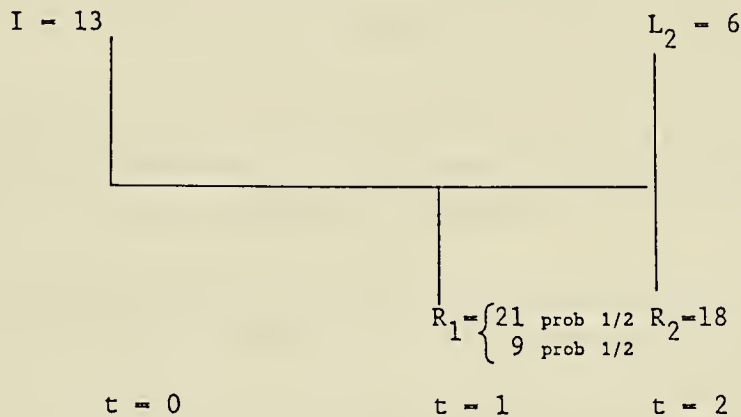


Figure 3

Suppose I , R_1 , R_2 and L_2 are as in Figure 3. Consider first the infinite debt, pure transfer contract which causes C to break even. This is given by $P_1 = \infty$, $T = 3$. To see this, note that with this contract, Lemma 1(1) tells us that D pays 18, $f_2 = 1$ and $\gamma = 18 - 3 = 15$ when $R_1 = 21$ (since $T + R_1 > R_2 = \bar{P}(T)$); while D pays 12, $f_2 = (T + R_1) / R_2 = 2/3$ and $\gamma = 12 + (1/3) 6 - 3 = 11$ when $R_1 = 9$. Hence

$$U_c = 1/2 \cdot 15 + 1/2 \cdot 11 = 13 = I.$$

According to Proposition 3, this contract is optimal. We will not give a complete demonstration of this, but will be content to show that a "pure debt" contract with $T = 0$ does worse. Since U_c is increasing in P_1 , there is

only one pure debt contract that allows C to break even and it is given by $P_1 = 14$. With this contract, Lemma 1(2) tells us that D pays 14, $f_2 = 1$ and $\gamma = 14$ when $R_1 = 21$ (since $R_1 > \bar{P}(0) = R_2 > P_1$); while Lemma 1(1) tells us that C forgives the debt to $\bar{P}(0) = 9 + \left[1 - \frac{9}{18}\right]6 = 12$, D pays 9, $f_2 = 1/2$ and $\gamma = 12$ when $R_1 = 9$. Note that although C breaks even ($U_C = 1/2 \cdot 14 + 1/2 \cdot 12 = 13$), social surplus is strictly less than with the pure transfer contract since f_2 is the same when $R_1 = 21$, but strictly lower when $R_1 = 9$.

The example can help us to understand what drives Proposition 3. When R_1 is uncertain, inefficiency arises in low R_1 states where $T+R_1 < R_2$ and $T+R_1 < P_1$. Reductions in P_1 and increases in T are alternative ways of reducing this inefficiency. At the margin, however, reductions in P_1 will have no influence on the inefficiency in states where $P_1 > \bar{P}(T)$; moreover, since $\bar{P}(T)$ is increasing in R_1 , these are precisely the low R_1 states (c.f. the $R_1 = 9$ state in the example). In contrast, increases in T reduce the inefficiency in every state. Hence T is a more powerful instrument than P_1 when R_1 is uncertain.

We turn next to the case where R_2 is uncertain.

Proposition 4. Suppose that only R_2 is stochastic and (3.8) holds. Then it is optimal to set $T = 0$ (a "pure debt" contract).

Proof. Suppose first that $T+R_1 \geq P_1$. Then $f_2 = 1$ for all R_2 and $U_C = E \text{ Min}(P_1 - T, R_2 - T) = I$. Replace the contract (P_1, T) by one in which $T' = 0$ and P'_1 is such that $E \text{ Min}(P'_1, R_2) = I$. Since $P'_1 \leq P_1 - T$, it follows that $R_1 \geq P'_1$, and so $f_2 = 1$ for all R_2 ; hence social surplus is unchanged.

Consider next the case where $T+R_1 < P_1$. Then if $T+R_1 > R_2$, D pays R_2 to C and $f_2 = 1$ (by Lemma 1(1)); while if $T+R_1 < R_2$, D pays $(T+R_1)$ to C and

$$(3.10) \quad f_2 = \text{Max} \left[\frac{T+R_1}{R_2}, 1 - \left(\frac{P_1 - T - R_1}{L_2} \right) \right].$$

In both cases, $\gamma = \text{Min}(T+R_1, R_2) + (1 - f_2)L_2 - T$, and so

$$(3.11) \quad U_C = E \text{ Min}(R_1, R_2 - T) + L_2 - L_2 E f_2 = I.$$

Now lower T to zero and reduce P_1 to maintain (3.11). Since the first term of (3.11) increases, so does Ef_2 . Also, by Lemma 3, $(P_1 - T)$ falls and, hence by (3.4), f_2 falls when R_2 is low $\left[\text{in which case } \frac{T+R_1}{R_2} > 1 - \left(\frac{P_1 - T - R_1}{L_2} \right) \right]$ and rises when R_2 is high $\left[\text{in which case } \frac{T+R_1}{R_2} < 1 - \left(\frac{P_1 - T - R_1}{L_2} \right) \right]$.

It follows from standard results on stochastic dominance (see, e.g., Rothschild and Stiglitz (1970)) that $Ef_2(R_2 - L_2)$ rises. Hence the reduction in T causes an increase in social surplus. Q.E.D.

It will be helpful to illustrate Proposition 4 with an example.

Example 2

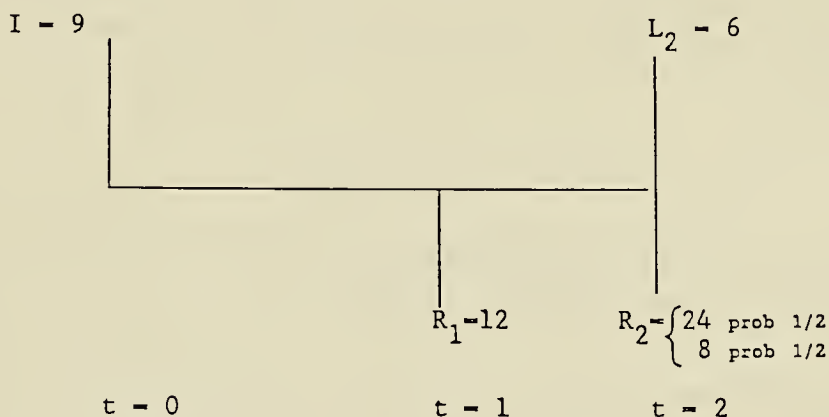


Figure 4

Suppose I , R_1 , R_2 and L_2 are as in Figure 4. Consider first the pure debt contract which causes C to break even. This is given by $P_1 = 10$, $T = 0$. To see this, note that, with this contract, D pays 10 , $f_2 = 1$ and $\gamma = 10$ when $R_2 = 24$ (since $\bar{P}(0) > R_1 > P_1$); while D pays 8 , $f_2 = 1$ and $\gamma = 8$ when $R_2 = 8$ (since $\bar{P}(0) = 8 < P_1$). Hence

$$U_c = 1/2 \cdot 10 + 1/2 \cdot 8 = 9 = I.$$

Note that this contract achieves the first-best since $f_2 = 1$ in both states.

We will be content to show that a "pure" transfer contract (i.e. one with $P_1 = \infty$) does not achieve the first-best. Since U_c is decreasing in T , there is only one such contract which causes C to break even and it is given by $T = 4$. With this contract, when $R_2 = 24$, C forgives the debt to $\bar{P}(4) = 16 + \left[1 - \frac{16}{24}\right]6 = 18$, D pays 16, $f_2 = 2/3$ and $\gamma = 18 - 4 = 14$; while when $R_2 = 8$, C forgives the debt to 8, D pays 8, $f_2 = 1$ and $\gamma = 8 - 4 = 4$. Although C breaks even, this contract does not achieve the first-best since $f_2 < 1$ when $R_2 = 24$.

Again this example can help us to understand what drives Proposition 4. When R_2 is uncertain, inefficiency arises in high R_2 states where $T+R_1 < R_2$ and $T+R_1 < P_1$. Reductions in P_1 and increases in T are alternative ways of reducing this inefficiency. In the present context, a reduction in P_1 is particularly useful since it affects f_2 in those states where $P_1 < \bar{P}(T)$, i.e. when $\bar{P}(T)$ is high, and these are precisely the high R_2 states, where there is inefficiency. (In the example, it is when $R_2 = 24$ that the debt level 10 is binding.) In contrast, an increase in T is a relatively coarse tool since it increases D 's wealth in every state but has no effect on f_2 in the low R_2 states.

Finally, we consider the case where only L_2 is stochastic.

Proposition 5. Suppose that only L_2 is stochastic and (3.8) holds. then it is optimal to set $P_1 = \infty$ (a "pure" transfer contract).

Proof. If $P_1 \geq R_2$, then $P_1 \geq \bar{P}(T)$ for all L_2 and so we can set $P_1 = \infty$ without changing anything. So suppose $P_1 < R_2$. Then by the same argument leading to (3.9) in Proposition 3,

$$(3.12) \quad U_c = E\gamma = \text{Min} (P_1 - T, R_1) + EL_2(1 - f_2) - I,$$

where

$$(3.13) \quad f_2 = \text{Min} \left[1, \text{Max} \left[\frac{T+R_1}{R_2}, 1 - \left(\frac{P_1 - T - R_1}{L_2} \right) \right] \right].$$

Assume first that $P_1 - T \leq R_1$. This implies $f_2 = 1$ by (3.13). Then moving to a new contract in which $P'_1 = \infty$ and $T' = R_2 - (P_1 - T)$ keeps U_c constant at $P_1 - T$ (since $T' + R_1 > R_2 = \bar{P}(T')$), does not alter f_2 in any state, and so does not affect social surplus.

Assume next that $P_1 - T > R_1$. Then (3.12) can be rewritten as

$$(3.14) \quad U_c = R_1 + EL_2(1-f_2) - I.$$

Replace P_1 by $P'_1 = \infty$ and raise T to $T' \geq T$ so as to maintain (3.14). It is clear that $Ef_2 L_2$ stays constant while (3.13) implies that f_2 rises when L_2 is low and falls when L_2 is high. Hence by standard results on stochastic dominance (see, e.g., Rothschild and Stiglitz (1970)), Ef_2 rises and therefore so does $Ef_2 (R_2 - L_2)$.

Thus in all cases setting $P' = \infty$ raises or keeps constant social surplus. Q.E.D.

The next example illustrates Proposition 5.

Example 3

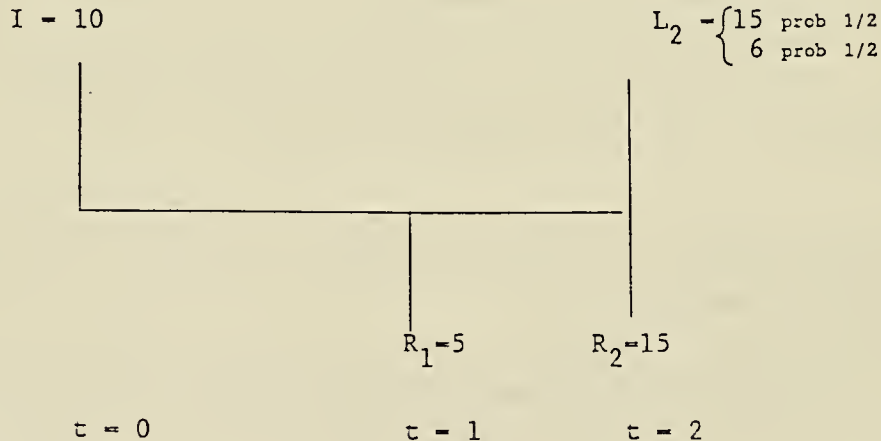


Figure 5

Suppose I, R_1, R_2, L_2 are as in Figure 5. We will show that a pure debt contract cannot do as well as a pure transfer contract. The break-even pure

debt contract is given by $T = 0$, $P_1 = 11$. This leads D to pay 5 and $f_2 = 3/5$ when $L_2 = 15$ (since $\bar{P}(0) = 15 > 11$ when $L_2 = 15$); while D pays 5 and $f_2 = 1/3$ when $L_2 = 6$ (the debt is forgiven to $\bar{P}(0) = 9$ in this state).

To see that a pure transfer contract does better, note that, while infinite debt may reduce f_2 when $L_2 = 15$, this causes no social loss since $R_2 = L_2$; while a transfer will raise f_2 when $L_2 = 6$ (the debt of 11 was not binding here). Hence expected social surplus rises.

The reason a pure transfer works well when L_2 is uncertain is that social inefficiency arises when L_2 is low (from (3.4) this causes f_2 to be low and social loss $(1 - f_2)(R_2 - L_2)$ to be high). A reduction in P_1 may not help since when L_2 is low, so is $\bar{P}(T)$ and hence we may have $\bar{P}(T) < P_1$. In contrast, a transfer reduces social inefficiency in all states since it increases D's ability to repay his debt and to repurchase assets from C in the event of default.

Remark 2. We have assumed that if the debtor defaults, he pays nothing to the creditor prior to renegotiation (he defaults "completely"). It is easy to show, however, that it never pays the debtor to default partially. In particular, suppose the debtor pays $0 < x < P_1$. This is equivalent to an equal fall in T and P_1 of x . By Lemmas 1 and 2, γ falls and f_2 rises and hence, by (3.6), δ falls. In other words, such a strategy can never make the debtor better off. (This argument can easily be shown to generalize to the three period model of the next section.)

Note that another argument shows that allowing the debtor to consume resources prior to renegotiation would not change the two period analysis either. In particular, suppose the debtor can deplete his date 1 wealth by Δ to $w = T + R_1 - \Delta$. It is straightforward to use Lemma 1 to show that $f_2(R_2 - L_2) - \gamma$ falls. Hence by (3.6) the debtor is worse off overall. As we shall see, however, this argument does not generalize to the three period model.

4. The General Case

We turn now to the general case, where R_3 and L_3 can be nonzero. The process of default and renegotiation at dates 1 and 2 was spelt out at the

end of Section 2. The outcome of this process is given in Proposition 6 below.

In order to build up to Proposition 6, let us consider what happens in equilibrium for a particular realisation $(R_1, R_2, R_3, L_2, L_3)$, given some debt contract (P_1, P_2, T) . It is helpful to begin at date 2. (Recall that all the values of R_1, R_2, R_3, L_2, L_3 become known to C and D at date 1.)

Date 2

Suppose that we are date 2, and that D faces a debt of, say, $P > 0$. (If $P = 0$, then D simply keeps possession of all the assets that weren't liquidated at date 1.) There are three histories that might have led up to this point: D could have paid his debt of P_1 at date 1 and he now faces the debt of $P - P_2$ which was specified in the original contract; or D might have defaulted at date 1 and C then postponed P_1 until date 2, in which case D now faces a debt of $P - (P_1 + P_2)$; or D might have defaulted at date 1 and, rather than allowing C to foreclose, D then offered C a new contract specifying, among other things, a new date 2 debt level of $P - \hat{P}_2$, say.

Whatever the history, we can view the situation at date 2 as a "two-period" model (viz. dates 2 and 3), and we can adapt the result of Lemma 1. That is, suppose a fraction f_2 of the assets remain in D's possession following date 1. Then at date 2, given values R_3 and L_3 , if D faces a debt level of P and has an amount of cash Y_2 , the fraction f_3 of the assets which will remain in his possession following date 2 is given by

$$f_3 = \begin{cases} f_2 & \text{if } Y_2 \geq f_2 R_3 \text{ or } Y_2 \geq P \\ \text{Max} \left[\frac{Y_2}{R_3}, f_2 - \left[\frac{P - Y_2}{L_3} \right] \right] & \text{otherwise.} \end{cases}$$

(4.1)

$$= \text{Min} \left[f_2, \text{Max} \left[\frac{Y_2}{R_3}, f_2 - \left[\frac{P - Y_2}{L_3} \right] \right] \right].$$

It can be seen that Lemma 1 has been adapted in a number of minor respects to yield (4.1). Namely, the level of debt at date 2 is P , rather than the date 1 level of P_1 . And the (per unit) return R_2 and liquidation value L_2 have been

changed to R_3 and L_3 respectively -- reflecting the fact that the second period of the "two-period" model is now date 3 rather than date 2. Finally, the largest possible value of f_3 is f_2 (rather than 1) -- reflecting our assumption that if a fraction $1-f_2$ of the assets have been sold off during date 1 liquidation proceedings (at a price $(1-f_2)(L_2+L_3)$), they cannot be repurchased at date 2. (Hence at date 2 the value to D of the remaining assets is f_2R_3 , which is why the first inequality in (4.1) reads $Y_2 \geq f_2R_3$.)

For future reference, we note that in the equilibrium at date 2, D makes C a cash payment equal to

$$(4.2) \quad \text{Min} (f_2R_3, P_2, Y_2) - Z_2, \text{ say.}$$

Date 1

At date 1, unlike at date 2, it is going to matter that there is the possibility of renegotiation. (Recall from Section 3 that in the "two-period" model the liquidation process, combined with C's ability to foregive, led to an ex-post Pareto optimal outcome, so there was nothing to renegotiate.) Our assumption is that only D can make an offer, which gives him the bargaining power in the renegotiation. (However, as we have seen, C's ability to forgive debt in effect gives her a considerable degree of power too.)

To find the outcome at date 1, we first consider the case where D defaults on his debt of P_1 -- either because he is unable to pay (his cash holding $T+R_1$ is less than P_1), or because he chooses not to pay. Then what would happen if D did not make an offer or if C rejected D's offer? Equivalently, what would happen if there were no renegotiation? Since D has the bargaining power in any renegotiation, this will tell us the value of C's payoff after renegotiation.

C has to choose between either overlooking the default and postponing the current debt (P_1), or foreclosing on a portion P' of the outstanding debt (P_1+P_2) and forgiving the rest.

D defaults and C postpones at date 1

If C chooses to postpone the debt, then D continues in possession of all the assets between dates 1 and 2 ($f_2 = 1$). To obtain the outcome at date 2, we apply the formula (4.1), with D's cash holding (Y_2) at $T+R_1+R_2$, and with the debt level (P) at P_1+P_2 (reflecting the postponement of debt). That is,

$$(4.3) \quad f_2 = 1 \quad \text{and} \quad f_3 = \text{Min} \left[1, \text{Max} \left[\frac{T+R_1+R_2}{R_3}, 1 - \left(\frac{P_1+P_2-T-R_1-R_2}{L_3} \right) \right] \right].$$

Bearing in mind that if she postpones the debt she receives nothing at date 1, C's payoff will be entirely determined by what she receives at date 2. It will therefore simply be the "two-period" payoff from Lemma 1, with the return at R_3 , the liquidation value at L_3 , the debt level at P_1+P_2 , and with D's cash holding at $T+R_1+R_2$. That is, C's payoff will be $\gamma(R_3, L_3, P_1+P_2, T+R_1+R_2)$ from (3.5), which using (3.2) equals

$$(4.4) \quad \text{Min} \left[P_1+P_2-T, R_3-T, R_1+R_2 + \left(1 - \frac{T+R_1+R_2}{R_3} \right) L_3 \right] = \gamma^*, \text{ say.}$$

(To keep the notation a little simpler, we don't include the list of arguments of γ^* -- viz., $R_2, R_3, L_2, L_3, P_1, P_2, R_1+T$. The same applies to γ^0 and $\hat{\gamma}$ below.)

D defaults and C forecloses at date 1

If C chooses to foreclose, then by assumption the debt is accelerated. That is, C has the right to foreclose on any amount P' up to P_1+P_2 . (If she chooses $P' < P_1+P_2$ then the difference is forgiven.) Any fraction $1-f_2$ of assets liquidated at this date will fetch $(1-f_2)(L_2+L_3)$. The remaining fraction of the assets, f_2 , will belong to D (so $f_3 = f_2$). It should be clear that at this point the model is essentially the same as a "two-period" model in which the return is R_2+R_3 , the liquidation value is L_2+L_3 , the debt level is P_1+P_2 , and D's cash holding is $T+R_1$. We can therefore borrow the results from Lemma 1. In particular, C's payoff will be $\gamma(R_2+R_3, L_2+L_3, P_1+P_2, T+R_1)$ from (3.5), which from (3.2) equals

$$(4.5) \quad \text{Min} \left[P_1 + P_2 - T, R_2 + R_3 - T, R_1 + \left(1 - \frac{T + R_1}{R_2 + R_3} \right) (L_2 + L_3) \right] = \gamma^0, \text{ say.}$$

Also from Lemma 1 we learn the values of f_2 and f_3 when there is foreclosure:

$$(4.6) \quad f_2 = f_3 = \begin{cases} 1 & \text{if } T + R_1 \geq R_2 + R_3 \text{ or } T + R_1 \geq P_1 + P_2 \\ \text{Max} \left[\frac{T + R_1}{R_2 + R_3}, 1 - \left(\frac{P_1 + P_2 - T - R_1}{L_2 + L_3} \right) \right] & \text{otherwise.} \end{cases}$$

Notice that f_3 equals f_2 , reflecting the fact that once the (accelerated) debt of $P_1 + P_2$ has been discharged at date 1, any remaining assets belong to D.

C will choose between postponement and foreclosure to maximize her payoff:

$$C \text{ will } \begin{cases} \text{postpone} \\ \text{foreclose on} \end{cases} \text{ the debt as } \gamma^* \begin{cases} > \\ \leq \end{cases} \gamma^0.$$

It follows that if D defaults, C's payoff equals $\text{Max} [\gamma^*, \gamma^0]$. This is because, given that D has all the bargaining power in any renegotiation, C's payoff will be the same as if there had been no renegotiation.

Under what circumstances will there be scope for renegotiation? Suppose that, in the absence of renegotiation, f_2 would be equal to 1. This could be either because C would choose to postpone the debt (as per (4.3)), or because C would choose to foreclose on some of the debt but no assets would be liquidated (i.e., $f_2 = 1$ in (4.6)). Then it is not difficult to show that this always leads to a Pareto optimal outcome, and that therefore there is no scope for renegotiation. See Lemma A1 in the Appendix.

This Lemma has a most useful implication. If D pays the debt of P_1 at date 1 (thus far we haven't analysed this possibility), then there is no scope for renegotiation. D will simply retain possession of all the assets

$$Z_1 + (1-f_2)(L_2+L_3) + Z_2 + (f_2-f_3)L_3 - T \geq \gamma^0$$

and subject to certain constraints on Z_1 , f_2 , Z_2 and f_3 :

-- By assumption, f_3 cannot exceed f_2 : $0 \leq f_3 \leq f_2 \leq 1$.

-- Z_1 cannot of course exceed D's current cash holding, $T+R_1$. Indeed, his best choice of Z_1 is the entire amount $T+R_1$, because it is the fact that D is cash-constrained which (in the absence of renegotiation) would lead to f_2 being less than 1. So $Z_1 = T+R_1$.

-- Z_2 cannot exceed D's future cash holding, $T+R_1-Z_1+f_2R_2 = f_2R_2$. And there is no point in D promising to pay C an amount Z_2 more than f_3R_3 , since D would default on the obligation. So $Z_2 \leq f_2R_2$ and $Z_2 \leq f_3R_3$.

The question arises: is there a new debt level \hat{P}_2 for date 2 such that C then ends up getting Z_2 in the form of cash, and $(f_2-f_3)L_3$ in the form of liquidation receipts? That is, can Z_2 and f_3 be implemented? It turns out that the answer is yes; such a \hat{P}_2 does indeed exist.

Rather than spell out this and further details of D's problem of what deal to offer C, we confine ourselves to reporting in Proposition 6 the solution for the various cases. (See the Appendix for details.) We are more concerned that the reader should see the overall picture first, and move quickly on to the numerical examples at the end of this Section.

The final ingredient of the analysis of the equilibrium outcome at date 1 is to consider what happens if D pays his debt of P_1 .

D pays the debt P_1 at date 1

This can of course only happen if D has the cash to pay -- i.e. only if $T+R_1 \geq P_1$.

As already noted, if D pays the debt of P_1 at date 1, then there is no scope for renegotiation. (See Lemma A1 in the Appendix.) D continues in possession of all the assets, and at date 2 f_3 will be as given in (4.1), with D's cash holding (Y_2) at $T+R_1-P_1+R_2$, and with the debt level (P) at P_2 . That is,

$$f_2 = 1 \quad \text{and} \quad f_3 = \text{Min} \left[1, \text{Max} \left[\frac{T+R_1-P_1+R_2}{R_3}, 1 - \left(\frac{P_2-T-R_1+P_1-R_2}{L_3} \right) \right] \right].$$

C's payoff will comprise two parts. First, the debt payment P_1 from date 1. Second, her payoff at date 2, which will simply be the "two-period" payoff from Lemma 1, with the return at R_3 , the liquidation value at L_3 , the debt level at P_2 , and with D's cash holding at $T+R_1-P_1+R_2$. That is, C's net payoff will be $P_1 + \gamma(R_3, L_3, P_2, T+R_1-P_1+R_2)$, which equals

$$(4.8) \quad P_1 + \text{Min} \left[P_2 - T, R_3 - T, R_1 - P_1 + R_2 + \left(1 - \frac{T+R_1-P_1+R_2}{R_3} \right) L_3 \right] - \hat{\gamma}, \text{ say.}$$

Notice that this payoff is never less than the payoff γ^* which C would obtain from postponing the debt (compare the expressions in (4.4) and (4.8)). That is, C cannot be worse off if the debt P_1 is paid when it is due rather than postponed to date 2.

The fact that D has enough cash ($T+R_1$) to meet his debt obligation (P_1) at date 1 does not necessarily mean that he will pay it. Might he be better off defaulting? Bearing in mind that all outcomes are ex-post Pareto optimal, we can assess D's decision on the basis of C's payoff. Specifically, D will act so as to minimize C's payoff. Recall that if D defaults, C obtains the maximum of $\hat{\gamma}^*$ (C's payoff if she chooses to postpone) and γ^0 (C's payoff if she chooses to foreclose). We have just seen that $\gamma \geq \gamma^*$, so if $\gamma^* > \gamma^0$, D will default. If $\gamma^0 > \gamma^*$, though, then $\hat{\gamma}$ D's decision to pay/default will depend on whether γ^0 is greater/less than $\hat{\gamma}$ respectively.

We can now draw together the strands of the preceding analysis, to describe the equilibrium outcome of the general three-period model in full:

Proposition 6 The unique subgame perfect equilibrium outcome of the general model is as follows.

Case 1

$$\text{If } \gamma^* = \text{Min} \left[P_1 + P_2 - T, R_3 - T, R_1 + R_2 + \left(1 - \frac{T + R_1 + R_2}{R_3} \right) L_3 \right]$$

(*)

$$> \text{Min} \left[P_1 + P_2 - T, R_2 + R_3 - T, R_1 + \left(1 - \frac{T + R_1}{R_2 + R_3} \right) (L_2 + L_3) \right] = \gamma^0$$

then D will default on the debt of P_1 at date 1 and C will postpone it until date 2. C's payoff is given by

$$\gamma = \gamma^0 = \text{Min} \left[P_1 + P_2 - T, R_3 - T, R_1 + R_2 + \left(1 - \frac{T + R_1 + R_2}{R_3} \right) L_3 \right];$$

and the fraction of the assets remaining in D's possession after dates 1 and 2 are respectively

$$f_2 = 1 \quad \text{and} \quad f_3 = \text{Min} \left[1, \text{Max} \left[\frac{T + R_1 + R_2}{R_3}, 1 - \left(\frac{P_1 + P_2 - T - R_1 - R_2}{L_3} \right) \right] \right].$$

Case 2

Suppose (*) does not hold. Then if $T + R_1 \geq P_1$ and

$$\begin{aligned} \gamma^0 &= \text{Min} \left[P_1 + P_2 - T, R_2 + R_3 - T, R_1 + \left(1 - \frac{T + R_1}{R_2 + R_3} \right) (L_2 + L_3) \right] \\ &\geq P_1 + \text{Min} \left[P_2 - T, R_3 - T, R_1 - P_1 + R_2 + \left(1 - \frac{T + R_1 - P_1 + R_2}{R_3} \right) L_3 \right] = \hat{\gamma}, \end{aligned}$$

D will pay the debt of P_1 at date 1. C's payoff is given by

$$\gamma = \hat{\gamma} = P_1 + \text{Min} \left[P_2 - T, R_3 - T, R_1 - P_1 + R_2 + \left(1 - \frac{T + R_1 - P_1 + R_2}{R_3} \right) L_3 \right];$$

and the fraction of the assets remaining in D's possession after dates 1 and 2 are respectively

$$f_2 = 1 \quad \text{and} \quad f_3 = \text{Min} \left[1, \text{Max} \left[\frac{T + R_1 - P_1 + R_2}{R_3}, 1 - \left(\frac{P_2 - T - R_1 + P_1 - R_2}{L_3} \right) \right] \right].$$

Case 3

Suppose (*) does not hold and Case (2) does not apply. Then D will default on the debt of P_1 at date 1 and, in the absence of renegotiation, C would foreclose. C's payoff is

$$\gamma = \gamma^0 = \text{Min} \left[P_1 + P_2 - T, R_2 + R_3 - T, R_1 + \left(1 - \frac{T + R_1}{R_2 + R_3} \right) (L_2 + L_3) \right].$$

Let S denote the "shortfall" $\gamma - R_1$ (this may be negative). As a consequence of renegotiation (see Problem (4.7)), the fractions of the assets remaining in D's possession after dates 1 and 2 are respectively:

$$f_2 = f_3 = 1 \quad \text{if } S \leq \text{Min} (R_2, R_3)$$

$$\hat{f}_2 = \hat{f}_3 = \frac{L_2 + L_3 - S}{L_2 + L_3 - R_3} \quad \text{if } R_3 \leq R_2 \text{ and } R_3 < S$$

$$f_2 = 1 \text{ and } f_3 = 1 - \frac{S - R_2}{L_3} \quad \text{if } R_2 < R_3 \text{ and } R_2 < S \leq R_2 + \left[1 - \frac{R_2}{R_3} \right] L_3$$

$$f_2 = \frac{L_2 + L_3 - S}{R_2 \left(\frac{L_2}{R_2} + \frac{L_3}{R_3} - 1 \right)} \text{ and } f_3 = \frac{L_2 + L_3 - S}{R_3 \left(\frac{L_2}{R_2} + \frac{L_3}{R_3} - 1 \right)}$$

$$\text{if } R_2 < R_3 \text{ and } R_2 + \left(1 - \frac{R_2}{R_3} \right) L_3 < S.$$

In all three cases, (1)-(3), D's payoff is given by

$$\delta = R_1 + f_2 R_2 + f_3 R_3 + (1-f_2)L_2 + (1-f_3)L_3 - \gamma.$$

Proof Much of the proposition was proved in the text. In the Appendix, we solve Problem (4.7) for the values of f_2 and f_3 given under Case (3) of the proposition.

A number of observations follow from Proposition 6. These are grouped in Lemma 5.

Lemma 5 (1) $\gamma \leq P_1 + P_2 - T$. (2) γ is (weakly) increasing in P_1 and P_2 and decreasing in T . (3) f_2 and f_3 are (weakly) decreasing in P_1 and P_2 and increasing in T . (4) If P_1 and T both rise by the same amount, or if P_2 and T rise by the same amount, or if P_1 falls by the same amount that P_2 rises, then γ falls (or stays constant) and f_2, f_3 rise (or stay constant). (5) γ is (weakly) increasing in R_2, R_3, L_2, L_3 .

Proof Part (1) of the lemma follows immediately from the fact that γ^* , γ^0 and γ are all bounded above by $P_1 + P_2 - T$.

To demonstrate the various comparative statics results for γ (in parts (2), (4) and (5) of the lemma), first observe that they are true within the three regimes of Proposition 6 (i.e. they are true for γ^* , γ^0 and $\hat{\gamma}$ individually). Moreover, C's payoff γ is continuous w.r.t. P_2 , R_2 , R_3 , L_2 and L_3 . The only possible discontinuity occurs when $\gamma^0 > \text{Max}(\gamma^*, \hat{\gamma})$, at the point where $R_1 + T = P_1$: at this point, as P_1 rises or T falls, D is no longer able to pay the debt at date 1 and C's payoff γ jumps up (from $\hat{\gamma}$ to γ^0). No matter, since this jump up is consonant with parts (2) and (4) of the lemma.

Finally, to demonstrate the comparative statics results for f_2 and f_3 (in parts (3) and (4) of the lemma), notice that whatever the regime which applies in Proposition 6, f_2 and f_3 solve Problem (4.7). (The point being that even in Cases (1) and (2) of the proposition, f_2 and f_3 are ex-post Pareto optimal.) The solution (f_2, f_3) to Problem (4.7) is laid out under Case (3) of the proposition. One can see that either when $R_3 \leq R_2$ or when $R_2 < R_3$, both f_2 and f_3 (weakly) fall as γ ($= -S + R_1$) rises. Hence the comparative statics results for f_2 and f_3 in parts (3) and (4) of the lemma follow from those for γ .

Q.E.D.

Many of these observations in Lemma 5 echo those for the "two-period" model (Lemma 2). The fact that C's payoff decreases if P_1 falls by the same amount as P_2 rises deserves comment. If, in equilibrium, D defaults on P_1 , then there will be no change in γ since the debt is either postponed or accelerated, and in both cases the only debt figure that matters is the sum $P_1 + P_2$ (which hasn't changed). However, if D pays P_1 in equilibrium and P_1 falls by a dollar (say), then even though P_2 rises by a dollar, C's payoff at date 2 may rise by less than a dollar since D may default at date 2. From C's point of view, a bird in the hand is worth two in the bush! Indeed, C's loss may be dramatic; it can change discontinuously as P_1 falls, because D may suddenly have enough cash to pay the debt at date 1. This explains why γ is not necessarily increasing in R_1 (c.f. part (5) of Lemma 5); as R_1 rises, D may have enough cash ($T + R_1$) to pay P_1 , and C's payoff may jump down.

We turn next to the optimal choice of P_1 , P_2 and T . For a particular realisation of $(R_1, R_2, R_3, L_2, L_3)$, let $\gamma = \gamma(R_2, R_3, L_2, L_3, P_1, P_2, R_1 + T)$ denote C's

equilibrium payoff. Also let $f_2 = f_2(R_2, R_3, L_2, L_3, P_1, P_2, R_1+T)$ and $f_3 = f_3(R_2, R_3, L_2, L_3, P_1, P_2, R_1+T)$ denote the equilibrium fractions of the assets which remain in D's possession after dates 1 and 2 respectively. (The values of γ , f_2 and f_3 are given in Proposition 6.) Social surplus is

$$R_1 + f_2 R_2 + (1-f_2)L_2 + f_3 R_3 + (1-f_3)L_3,$$

which means that, as in Section 3, an optimal contract solves:

$$\begin{aligned} \text{Maximize}_{P_1, P_2, T} & \int f_2 [R_2 - L_2] dG(R_2, R_3, L_2, L_3, R_1) \\ & + \int f_3 [R_3 - L_3] dG(R_2, R_3, L_2, L_3, R_1) \\ \text{subject to } U_c(P_1, P_2, T) & - \int \gamma dG(R_2, R_3, L_2, L_3, R_1) = I \end{aligned}$$

where $f_2 = f_2(R_2, R_3, L_2, L_3, P_1, P_2, R_1+T)$, $f_3 = f_3(R_2, R_3, L_2, L_3, P_1, P_2, R_1+T)$,

$$\gamma = \gamma(R_2, R_3, L_2, L_3, P_1, P_2, R_1+T),$$

and G is the joint distribution of $(R_2, R_3, L_2, L_3, R_1)$.

It follows directly from parts (2) and (4) of Lemma 5 that:

Lemma 6 (1) U_c is (weakly) increasing in P_1 and P_2 , and decreasing in T .

(2) If P_1 and T both rise by the same amount, or if P_2 and T rise by the same amount, or if P_1 falls by the same amount that P_2 rises, then U_c falls (or stays constant).

Lemma 6 allows us to obtain a necessary and sufficient condition for the project to be financed. Since U_c is increasing in P_1 and P_2 and decreasing in T , we need only compute $U_c(\infty, \infty, 0)$ and see whether it exceeds I .

Proposition 7 A necessary and sufficient condition for the project to be financed is that

$$E \text{ Max } \left[\text{Min} \left[R_3, R_1+R_2 + \left(1 - \frac{R_1+R_2}{R_3} \right) L_3 \right], \text{Min} \left[R_2+R_3, R_1 + \left(1 - \frac{R_1}{R_2+R_3} \right) (L_2+L_3) \right] \right] \geq I. \quad (4.9)$$

Proof Follows from Proposition 6, putting $P_1 = P_2 = \infty$ and $T = 0$, and noting that case (2) cannot arise. Q.E.D.

In a first-best world (where there are no diversion problems), the project would be undertaken whenever $E(R_1+R_2+R_3) \geq I$. (4.9) tells us that diversion causes too few projects to be financed.

The interesting case is where (4.8) holds with strict inequality, i.e. $U_c(\infty, \infty, 0) > I$. The issue is how to lower P_1 and P_2 from infinity and raise T from zero so as to maximize expected surplus, at the same time driving U_c down to I . As in the "two-period" model, it turns out that the instruments -- P_1 , P_2 and T -- have distinct roles and in general an optimal contract will involve the use of all three of them.

The deterministic case is straightforward, and is analogous to Proposition 2:

Proposition 8 Suppose R_1 , R_2 , R_3 , L_2 , and L_3 are nonstochastic and (4.9) holds. Then the contract $(P_1, P_2, T) = (I, 0, 0)$ achieves the optimum.

Proof Since all outcomes are ex-post Pareto optimal, it is enough to ensure that C gets a payoff of I .

Given that (4.9) holds, it follows from Proposition 6 that $\gamma = I$ for the contract $(P_1, P_2, T) = (I, 0, 0)$.

Q.E.D.

We turn now to the stochastic case.

In Section 2 we examined the interplay between the instruments P_1 and T . P_2 and T have a similar relation to each other, albeit a less direct one.

The crucial new effect in this three-period model, and the one on which we will focus, is the interplay between P_1 and P_2 . Our main concern is with the scheduling of debt. In particular, we draw a distinction between long-term debt and short-term debt. By "long-term" debt, we mean any debt contract (P_1, P_2, T) such that $P_2 > 0$ and for at least one state of nature -- i.e. configuration of R_1, R_2, R_3, L_2, L_3 -- D actually can and will pay the debt of P_1 at date 1. By "short-term" debt, we mean any other debt contract (P_1, P_2, T) -- i.e., one for which either $P_2 = 0$ or one such that D never pays P_1 at date 1. The point being that if D never pays P_1 at date 1, then either C postpones the debt until date 2, or she forecloses at date 1 (unless the contract is renegotiated) -- and in either event the only debt level that matters is the sum $P = P_1 + P_2$. But if this is the case, then the contract need never mention P_2 , or it could be set equal to zero; P_1 can be set equal to P in the first place, and the contract $(P_1 = P, T)$ would be a short-term contract. To state the obvious, though: even though C and D may write a short-term debt contract, this does not mean that they have a short-term relationship. Quite the contrary: they both know that once date 1 arrives, and they learn the state of nature, they may negotiate another (short-term) debt contract specifying, among other things, a debt level \hat{P}_2 for date 2. Since this arrangement of (re-)negotiating short-term debt is sometimes optimal in our model, the model provides a theory of rescheduling (a common arrangement in practice). Another way to think of short-term debt is that it is as if D returns to the capital market every period, to borrow money to 'buy off' his current creditor.

Consider the effects of lowering P_1 and raising P_2 . The last part of Lemma 6 tells us that if P_1 is to be decreased, then P_2 has to be increased by a larger amount in order for C to continue to break even.

The positive effect of lowering P_1 is that fewer defaults will occur at date 1, which is good since default typically entails a certain degree of liquidation -- that is, a certain degree of inefficiency. More specifically, a disadvantage of having a high P_1 (and, certainly, a disadvantage of short-term debt) is that C may have too much control early on; i.e., at date 1. For example, consider a situation where at date 1 the parties learn bad

news about date 3: R_3 is going to be low. (And, by assumption, L_3 must also be going to be low.) Then C can foresee that she will be unable to get much of D's cash at date 2. (D may have quite a bit of cash by then because R_2 may be quite high, but the assets aren't worth much to him.) So C pulls the plug on the project early on, by liquidating at date 1. This may be very inefficient, for the following reason. Although R_3 is low and therefore liquidation at date 2 would entail relatively little social loss (a low value of R_3 implies only a small loss -- no more than $R_3 - L_3$), when C actually chooses to liquidate (at date 1) there may be a very large social loss (up to a maximum of $R_2 + R_3 - L_2 - L_3$). (Example 5 below is based on this sort of idea.)

There are, however, offsetting negative effects from raising P_2 . There are two (related) negative effects, both arising from the fact that P_2 has to be raised by more than P_1 is lowered. First, since $(P_1 + P_2)$ has risen, whenever default does occur at date 1, there will in general be a greater degree of liquidation. Second, there may be a greater degree of liquidation at date 2. Although D may arrive at date 2 with an extra dollar in hand as a result of (say) a dollar fall in P_1 , he nevertheless faces more than a dollar increase in the amount P_2 which he owes. And the more liquidation there is, the larger is the social loss.

Roughly speaking, we understand the trade-off as being between early liquidation at date 1 (incurring a social loss of up to $R_2 + R_3 - L_2 - L_3$), versus late liquidation at date 2 (with a loss of up to $R_3 - L_3$).

It is worth dwelling a little further on Proposition 6, to give some indication of the richness of the three-period model. A useful question to ask is: in which states of nature is there more likelihood of inefficiency -- i.e., of liquidation? In the "two-period" model there is a straightforward answer: namely, liquidation is more likely when R_1 and/or L_2 are low, or when R_2 is high. There are few comparably simple effects in the three-period model.

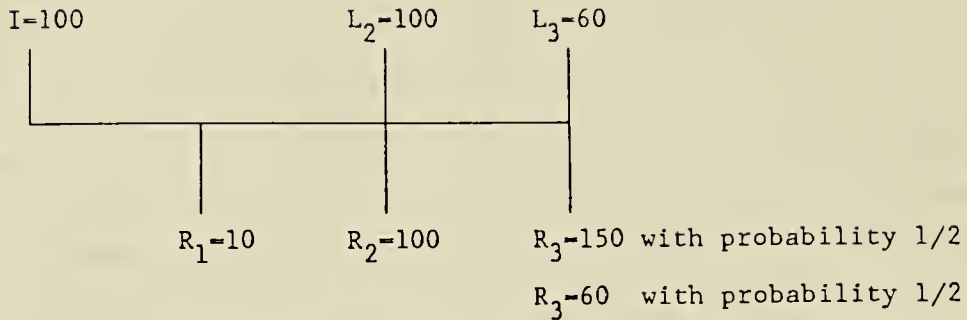
Consider for example the relatively straightforward case of infinite, or large, debt. Certainly, as R_1 rises, f_2 and f_3 rise, as one might expect from the "two-period" model. (D has more cash with which to start date 1, and he therefore can bribe C into allowing him to keep possession of a larger share of the assets.) However as R_2 rises, the effect on f_2 and f_3 can be in

either direction. On the one hand, at date 1 C can charge D more per unit of asset and so D is able to buy less (f_2 and f_3 fall). On the other hand, C can anticipate that a higher R_2 means that D could have commensurately more cash at date 2 if he is allowed to keep possession of a larger fraction of the assets (f_2 rises). Moreover, if the reason f_3 is low is merely that f_2 is low (remember $f_3 \leq f_2$), then this rise in f_2 will have a knock-on effect on f_3 and it too will rise. Next, consider a rise in R_3 . This has an unambiguous effect on f_2 : as the future value of the project rises, early liquidation becomes less desirable and f_2 rises. If the size of f_3 is constrained by f_2 then f_3 will also rise. However, if f_3 is simply the fraction of the remaining assets which D can afford to buy at date 2, then as R_3 rises, C will charge a higher price and f_3 will fall. Finally, one can show that as L_2 and/or L_3 rise, f_2 and f_3 fall -- c.f., the "two-period" model, where f_2 is independent of L_2 . In sum, there is a rich set of possibilities (as a reminder: the discussion in this paragraph is for the case of large debt only). And this is borne of the fact that there are several different forces at work in the three-period model. By the end of this section, we hope to have brought out more clearly what these forces are.

We devote the rest of the section to examples, which illustrate some of these general effects. For the most part, we will provide examples in which long-term debt is optimal.

We should note straight away that it is quite straightforward to think of examples in which short-term debt is optimal. We know that C's payoff is increasing in P_1 and P_2 and decreasing in T ; so if I is high -- say just below the critical threshold given in Proposition 7 -- then there is no alternative but to use short-term debt, because no long-term debt contract (in which D sometimes has enough cash $T+R_1$ at date 1 to pay P_1) will adequately compensate C for her investment I . However, even if the project could proceed using a long-term contract, it may nevertheless be more efficient to use a short-term contract, as Example 4 shows.

Example 4



Short-term Debt

P_2 is irrelevant; only (P_1+P_2) matters, so set $P_2 = 0$. For C to make a return of at least I, we need $P_1 \geq 100$. It turns out that by setting $P_1 = 100$ we can achieve first-best, without a transfer (i.e. set $T = 0$).

Let us proceed to calculate the equilibrium outcome in each state. Rather than simply relying on Proposition 6, we will deliberately flesh out some of the detail. However, we should emphasize that at various points in what follows we are implicitly appealing to the formal analysis which led up to Proposition 6 (for instance, we don't attempt to prove that the offer which D makes C is the best offer that he could make).

Note that D has no hope of meeting the debt of 100, since his cash holding is merely $T+R_1 = 10$. So he must default. (This is a short-term debt contract, par excellence!) It is readily confirmed that C won't postpone the debt (case (1) of Proposition 6 doesn't apply). So, in both states of nature ($R_3 = 150$ or 60), unless D makes an offer which C accepts, C will foreclose on the debt. (If D didn't offer a new contract, in both states C wouldn't forgive any of the debt of 100, since this is less than the liquidation value L_2+L_3 .)

However, when $R_3=150$, D can offer to pay 10 now, and to sign a new debt contract specifying $\hat{P}_2=90$. C will accept this offer, since D will be willing to pay $\hat{P}_2=90$ at date 2 (he will have the cash from R_2 and he values R_3 at 150); and C's payoff will be $\gamma = 10 + 90 = 100$, which is what she would get if she refused the offer and foreclosed on the debt. Hence $\hat{f}_2 = \hat{f}_3 = 1$

and D's payoff $\delta = 10 - 10 + 100 - 90 + 150 = 160$.

When $R_3=60$, D can offer to pay 10 now, to liquidate $3/10$ of the assets ($f_1=7/10$), and to sign a new debt contract specifying $\hat{P}_2=42$. C will accept this offer since D will be willing to pay $\hat{P}_2=42$ at date 2 (he will have the cash from $(7/10)R_2$ and he values $7/10$ of R_3 at 42); and C's payoff will be $\gamma = 10 + (3/10)(100+60) + 42 = 100$, which is what she would get if she refused the offer and foreclosed on the debt. Hence $f_2 = f_3 = 7/10$, and D's payoff $\delta = 10 - 10 + (7/10)100 - 42 + (7/10)60 = 70$.

In sum: C's expected payoff $U_c = (100+100)/2 = 100 = I$; and expected social surplus $U_c + U_d = 100 + (160+70)/2 = 215$. Note that this is first-best since there is no social loss in the state $R_3=60$ even though there is some liquidation (because $R_2=L_2$ and $R_3=L_3$).

Long-term Debt

For this example, the only long-term debt contract for which C breaks even is: $P_1=10$, $P_2=\infty$ and $T=0$. The point is that, in order for D to be able to pay P_1 at date 1, P_1 must be set at no more than $T+10$; and for C the most profitable contract of this form is $(P_1, P_2, T) = (10, \infty, 0)$. Yet even for this contract we will see that C only just recoups her investment I .

In both states, it is readily confirmed from Proposition 6 that D will choose to pay the debt of 10 at date 1; so $f_2=1$.

At date 2, when $R_3=150$, C forgives the debt to 120. D pays 100 in cash and $1/3$ of the assets are liquidated ($f_3=2/3$). Hence D's payoff $\delta = 10 - 10 + 100 - 100 + (2/3)150 = 100$ (which is what he would have gotten had he refused to pay any of his cash), and C's payoff $\gamma = 10 + 100 + (1/3)60 = 130$.

When $R_3=60$ at date 2, C forgives the debt to 60. D chooses to pay this in cash (he values R_3 at 60) and there is no liquidation ($f_3 = 1$). Hence C's payoff $\gamma = 10 + 60 = 70$, and D's payoff $\delta = 10 - 10 + 100 - 60 + 60 = 100$.

In sum: C's expected payoff $U_c = (130+70)/2 = 100 = I$, and expected social surplus $U_c + U_d = 100 + (100+100)/2 = 200$.

Short-term Debt

P_2 is irrelevant; only (P_1+P_2) matters, so set $P_2 = 0$. One can show that the best short-term contract is $(P_1, T) = (\infty, 79)$. [See Footnote 28 below.]

D must default at date 1. It is readily confirmed that C won't postpone the debt (case (1) of Proposition 6 doesn't apply). So, in both states of nature ($R_3 = 60$ or 10), unless D makes an offer which C accepts, C will foreclose on the debt.

If D didn't make an offer in the "good" state $R_3=60$, C would forgive the debt to 122. D would then pay $T+R_1 = 80$ in cash and $7/15$ of the assets would be liquidated. D would get $80 - 80 + (8/15)(90+60) = 80$ (which is what he would have gotten had he refused to pay any of his cash), and C would get net $-79 + 80 + (7/15)(80+10) = 43$. However D can offer to pay 80 now, and to sign a new debt contract specifying $\hat{P}_2=42$. C will accept this offer, since D will be willing to pay $\hat{P}_2=42$ at date 2 (he will have the cash from R_2 and he values R_3 at 60); and C's net payoff will be $\gamma = -79 + 80 + 42 = 43$, which is what she would get if she refused the offer and foreclosed on the debt. Hence $f_2 = f_3 = 1$ and D's payoff $\delta = 80 - 80 + 90 - 42 + 60 = 108$.

If D didn't make an offer in the "bad" state $R_3=10$, C would forgive the debt to 98. D would then pay $T+R_1 = 80$ in cash and $1/5$ of the assets would be liquidated. D would get $80 - 80 + (4/5)(90+10) = 80$ (which is what he would have gotten had he refused to pay any of his cash), and C would get net $-79 + 80 + (1/5)(80+10) = 19$. However D can offer to pay 80 now, to liquidate $1/10$ of the assets ($f_2=9/10$) now, and to sign a new debt contract specifying $\hat{P}_2=9$. C will accept this offer, since D will be willing to pay $\hat{P}_2=9$ at date 2 (he will have the cash from $(9/10)R_2$ and he values $9/10$ of R_3 at 9); and C's net payoff will be $\gamma = -79 + 80 + (1/10)(80+10) + 9 = 19$, which is what she would get if she refused the offer and foreclosed on the debt. Hence $f_2 = f_3 = 9/10$, and D's payoff $\delta = 80 - 80 + (9/10)90 - 9 + (9/10)10 = 81$.

In sum: C's expected payoff $U_c = (43+19)/2 = 31 - I$; and expected social surplus $U_c + U_d = 31 + (108+81)/2 = 125\frac{1}{2}$.²⁸

Long-term Debt

The first-best can be achieved by setting $P_1 = 1$ and $P_2 = 50$, and having no transfer (i.e. set $T = 0$).

In both states, it is readily confirmed from Proposition 6 that D will choose to pay the debt of 1 at date 1; so $f_2 = 1$.

At date 2, when $R_3 = 60$, D chooses to pay the debt of 50 (he has the cash from R_2 and he values R_3 at 60) and there is no liquidation ($f_3 = 1$). Hence C's payoff $\gamma = 1 + 50 = 51$, and D's payoff $\delta = 1 - 1 + 90 - 50 + 60 = 100$.

When $R_3 = 10$ at date 2, C forgives the debt to 10. D chooses to pay this in cash (he values R_3 at 10) and there is no liquidation ($f_3 = 1$). Hence C's payoff $\gamma = 1 + 10 = 11$, and D's payoff $\delta = 1 - 1 + 90 - 10 + 10 = 90$.

In sum: C's expected payoff $U_c = (51+11)/2 = 31 - I$; and expected social surplus $U_c + U_d = 31 + (100+90)/2 = 126$. This is first-best since $f_2 = f_3 = 1$ in both states.

Hence long-term debt (social surplus = 126) dominates short-term debt (social surplus = $125\frac{1}{2}$) in this example. Again, it is again worth considering why.

The drawback of short-term debt is very clear. In the bad state, C will liquidate at date 1; witness the fact that 1/10 of the assets are sold off in this state at social cost $(1/10)[(90+10) - (80+10)] = 1$.

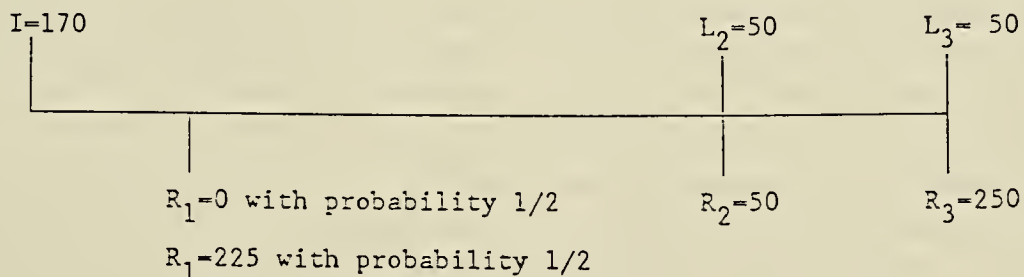
The long-term debt contract, on the other hand, entails no social loss. The reason is that, even in the good state, once D has paid his debt of P_1 at date 1, he receives enough cash from R_2 to be able to meet his debt

obligation of P_2 at date 2. (Indeed, even if P_2 had been higher, R_2 is enough to be able to bribe C into giving him all the assets anyway, since $R_2 \geq R_3$ even in the good state.) For the example, then, we obtain the first-best using a long-term debt contract; and this is unambiguously better than using short-term debt.

We close this section with two further examples, Examples 6 and 7, both showing how long-term debt can dominate short-term debt. (As already noted, it is straightforward to think of examples going the other way; one merely needs to make I sufficiently high.) These two examples are for cases where, respectively, only R_1 and only L_3 are stochastic. (We have examples for the cases where only R_2 and only L_2 are stochastic, but we don't want to overkill!) Examples 6 and 7 have independent interest: they demonstrate that Propositions 3 and 5 do not generalize from the "two-period" model to the three-period model.

As we shall see, Example 6 contains a couple of new features. First, social inefficiency occurs in the same state of nature regardless of whether the contract is short-term or long-term. Second, the nature of the renegotiation is relatively subtle: at date 1, the parties agree on a date 2 debt level which will exceed D 's wealth and hence there will be further liquidation at date 2. (Specifically, when R_1 is low they negotiate to have $f_3 < f_2$ -- as per Proposition 6, case (3) with $R_3 > R_2$.)

Example 6



Short-term Debt

P_2 is irrelevant; only (P_1+P_2) matters, so set $P_2 = 0$. One can show that the best short-term contract is $(P_1, T) = (\infty, 15)$. [See Footnote 29 below.]

D must default at date 1. It is readily confirmed that C won't postpone the debt (case (1) of Proposition 6 doesn't apply). So, in both states of nature ($R_1 = 225$ or 0), unless D makes an offer which C accepts, C will foreclose on the debt.

If D didn't make an offer in the "good" state $R_1=225$, C would forgive the debt to 260. D would then pay $T+R_1 = 240$ in cash and $1/5$ of the assets would be liquidated. D would get $240 - 240 + (4/5)(50+250) = 240$ (which is what he would have gotten had he refused to pay any of his cash), and C would get net $-15 + 240 + (1/5)(50+50) = 245$. However D can offer to pay 240 now, and to sign a new debt contract specifying $\hat{P}_2=20$. C will accept this offer, since D will be willing to pay $\hat{P}_2=20$ at date 2 (he will have the cash from R_2 and he values R_3 at 250); and C's net payoff will be $\gamma = -15 + 240 + 20 = 245$, which is what she would get if she refused the offer and foreclosed on the debt. Hence $f_2 = f_3 = 1$ and D's payoff $\delta = 240 - 240 + 50 - 20 + 250 = 280$.

If D didn't make an offer in the "bad" state $R_1=0$, C would forgive the debt to 110. D would then pay $T+R_1 = 15$ in cash and $19/20$ of the assets would be liquidated. D would get $15 - 15 + (1/20)(50+250) = 15$ (which is what he would have gotten had he refused to pay any of his cash), and C would get net $-15 + 15 + (19/20)(50+50) = 95$. However D can offer to pay 15 now, to liquidate $\hat{1/2}$ of the assets ($f_2=1/2$) now, and to sign a new debt contract specifying $\hat{P}_2=45$. C will accept this offer, since at date 2 C will obtain 25 in cash from D (i.e., all of D's cash, $(1/2)R_2$) and $4/5$ of the remaining assets will be sold ($\hat{f}_3 = 1/10$) for $(4/10)L_3 = 20$ to make up the difference of $(45-25)$. (Check: D is willing to pay 25 to obtain $(1/10)R_3$.) And C's net payoff $\gamma = -15 + 15 + (1/2)(50+50) + 25 + (4/10)50 = 95$, which is what she would get if she refused the offer and foreclosed on the debt. Hence $f_2 = 1/2$ and $\hat{f}_3 = 9/10$, and D's payoff $\delta = 15 - 15 + (1/2)50 - 25 + (1/10)250 = 25$.

In sum: C's expected payoff $U_c = (245+95)/2 = 170 = I$; and expected social surplus $U_c + U_d = 170 + (280+25)/2 = 322\frac{1}{2}$.

Long-term Debt

Consider the long-term contract $(P_1, P_2, T) = (0, \infty, 0)$. (One can show that this is the best long-term contract.)

Since $P_1 = 0$, in both states D retains possession of all the assets until date 2.

At date 2, in the good state ($R_1=225$), C forgives the debt to 250. D chooses to pay this in cash (he has a cash holding of $R_1+R_2 = 275$, and he values R_3 at 250) and there is no liquidation ($f_3 = 1$). Hence C's payoff $\gamma = 250$, and D's payoff $\delta = 225 + 50 - 250 + 250 = 275$.

In the bad state at date 2, C forgives the debt to 90. D pays 50 in cash and 4/5 of the assets are liquidated ($f_3=1/5$). Hence D's payoff $\delta = 50 - 50 + (1/5)250 = 50$ (which is what he would have gotten had he refused to pay any of his cash), and C's payoff $\gamma = 50 + (4/5)50 = 90$.

In sum: C's expected payoff $U_c = (250+90)/2 = 170 = I$, and expected social surplus $U_c + U_d = 170 + (275+50)/2 = 332\frac{1}{2}$.

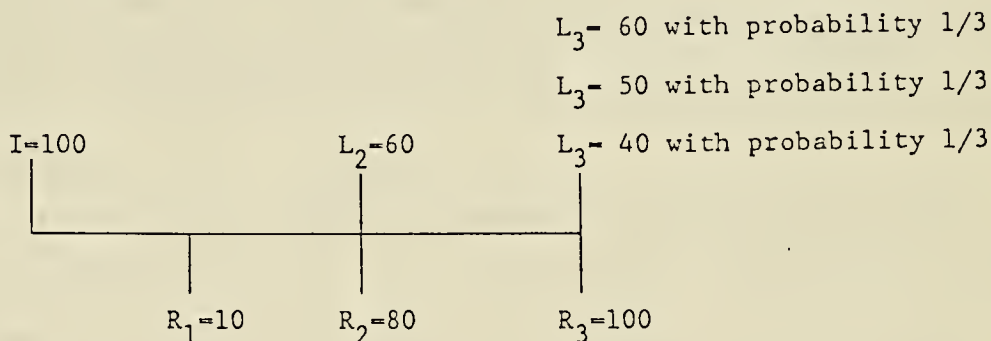
Hence long-term debt (social surplus = $332\frac{1}{2}$) dominates short-term debt (social surplus = $322\frac{1}{2}$) in this example. Again, it is again worth considering why.

Unlike in examples 4 and 5, inefficient liquidation occurs in the same state -- the bad state -- irrespective of whether the debt contract is short or long-term. (As mentioned in the discussion before Example 4, it is not surprising that f_2 and f_3 both rise with R_1 .) From the fact that all outcomes are ex-post Pareto optimal, the only way to reduce this inefficiency is to reduce C's payoff γ in the bad state (and commensurately increase γ in the good state, so that C breaks even in expectation). The point is that C only obtains a higher payoff by virtue of selling off a greater proportion of

the assets during liquidation. Long term debt provides a way of keeping C's payoff in the bad state relatively low, because D retains control at date 1. Put another way: short-term debt gives C too much control in the bad state, with the consequence that too much is liquidated -- albeit that in this example, once the parties have renegotiated it transpires that much of the liquidation is postponed until date 2. Witness the fact that at date 2 (which is the only date at which there is any social loss because $R_2=L_2$ and $R_3 > L_3$ in the example), 9/10 of the assets are liquidated under the short-term debt contract (with social loss 180), whereas only 4/5 of the assets are liquidated under the long-term contract (with social loss 160).²⁹

In passing, notice that we have demonstrated that Proposition 3 -- in which we showed that for the "two-period" model, infinite debt (c.f. short-term debt) is optimal when only R_1 is stochastic -- does not generalise to the three-period model.

Example 7



We will present this example in less detail than before. We believe that it is necessary to have at least three distinct states of nature (i.e., three different possible values of L_3) in order for long-term debt to dominate short-term debt when L_3 is stochastic. On account of this, the example is that much more complex to analyze. In the text, we confine ourselves merely to reporting the nature of the best short-term and the best long-term contracts, leaning heavily on Proposition 6 as we go along.

Short-Term Debt

P_2 is irrelevant; only (P_1+P_2) matters, so set $P_2 = 0$. For C to make a return of at least I, we need $P_1 \geq 100$. The best such contract has infinite debt -- specifically $(P_1, T) = (\infty, 22\frac{8}{11})$. (See the Appendix for further details.) For this contract, denoting C's payoff in state L_3 by $\gamma(L_3)$, it follows from Proposition 6 that

$$\gamma(60) = 108\frac{2}{11}, \quad \gamma(50) = 100 \quad \text{and} \quad \gamma(40) = 91\frac{9}{11}.$$

And in state L_3 , $f_2 = f_2(L_3)$ and $f_3 = f_3(L_3)$ are given by

$$\begin{array}{ll} f_2(60) = 60/77 & \text{and} \quad f_3(60) = 48/77 \\ f_2(50) = 1 & \text{and} \quad f_3(50) = 4/5 \\ f_2(40) = 1 & \text{and} \quad f_3(40) = 21/22. \end{array}$$

[Checking: C's expected payoff U_c is given by

$$\begin{aligned} U_c &= 10 + (1/3) \{ (17/77)60 + (60/77)80 + (29/77)60 \} \\ &\quad + (1/3) \{ \quad 0 \quad + \quad 80 \quad + (1/5)50 \} \\ &\quad + (1/3) \{ \quad 0 \quad + \quad 80 \quad + (1/22)40 \} \\ &= 100 - I. \end{aligned}$$

Expected social surplus $U_c + U_d$ equals

$$\begin{aligned} 10 &+ (1/3) \{ (60/77)80 + (17/77)60 + (48/77)100 + (29/77)60 \} \\ &+ (1/3) \{ \quad 80 \quad + \quad (4/5)100 + (1/5)50 \} \\ &+ (1/3) \{ \quad 80 \quad + \quad (21/22)100 + (1/22)40 \} \\ &= 179\frac{61}{231}. \end{aligned}$$

Long-Term Debt

Consider the long-term debt contract $(P_1, P_2, T) = (10, \infty, 0)$. (One can show that this is in fact the only long-term debt contract for which C breaks even.)

For this contract, it follows from Proposition 6 that

$$\gamma(60) = 102, \quad \gamma(50) = 100 \quad \text{and} \quad \gamma(40) = 98.$$

D can and will pay P_1 at date 1 (so $f_2(L_3) = 1$); and in all three states $f_3(L_3) = 4/5$.

[Checking: C's expected payoff $U_c = 10 + 80 + (1/5)EL_3 = 100 = I$.]
Expected social surplus $U_c + U_d$ equals $10 + 80 + (4/5)100 + (1/5)EL_3 = 180$.

So this long-term contract dominates the best short-term contract.

The reason why long-term debt can dominate short-term debt is more subtle than in Examples 5 and 6. L_3 takes three possible values, and in all three states one may have inefficient liquidation. The ideal arrangement would be to keep C's payoff $\gamma(L_3)$ low in all three states (because a lower γ means less liquidation) -- but of course on average $E\gamma(L_3)$ cannot fall below $I = 100$ otherwise C would not break even. Short and long-term debt span different sets of triples $(\gamma(60), \gamma(50), \gamma(40))$. In particular, consider the triple $(102, 100, 98)$ which is achieved by the above long-term debt contract $(P_1, P_2, T) = (10, \infty, 0)$. This leads to a reasonable degree of balance of inefficiency across the three states -- viz., in all three states there is no liquidation at date 1, and 1/5 of the assets are liquidated at date 2. And the point is that the same degree of balance cannot be achieved by any short-term debt contract.

In passing, note that we have demonstrated that Proposition 5 -- in which we showed that for the "two-period" model, infinite debt (c.f. short-term debt) is optimal when only the liquidation value is stochastic -- does not generalise to the three-period model.

We end this section by remarking on another respect in which the three-period model differs from the "two-period" model. With three periods, D might have an incentive to consume early on if this were possible.

For example, suppose $T=0$ and the debt level is high. Consider the state of nature $R_1=1, R_2=R_3=2$, and $L_2=L_3=1$. Following default at date 1, unless D

makes an acceptable offer, C will forgive the debt to $2\frac{1}{2}$. (D would pay C 1 at date 1 and 3/4 of the assets would be liquidated.) In fact D can offer to pay C 1 at date 1 and to sign a new debt contract specifying $\hat{P}_2 = 1\frac{1}{2}$ -- so in equilibrium $f_2=f_3=1$, and D's payoff equals $R_1 - 1 + R_2 - 1\frac{1}{2} + R_3 = 2\frac{1}{2}$.

However, suppose instead that D consumes R_1 at date 1 (prior to renegotiation). Then C can only hope to get $L_2+L_3 = 2$ from foreclosing at date 1, since D no longer has any cash. D can therefore offer to sign a new debt contract specifying $\hat{P}_2 = 2$ -- so in equilibrium $f_2=f_3=1$, and D's final payoff equals the $R_1=1$ that he consumed early plus $R_2 - 2 + R_3$ (totalling 3). In this example, then, D is better off consuming early.

We believe that the broad nature of our findings would be unchanged by allowing D to consume early. And anyway there is certainly a range of circumstances in which it makes good sense to assume that D cannot do so; he may only be in a position to stash away money for future consumption.

5. Summary and Conclusions

We have developed a theory of entrepreneurial finance based on the idea that the distinguishing feature of debt is that default by a debtor gives a creditor the right to seize some of the debtor's assets. In our model, a contract between an entrepreneur-debtor and an investor-creditor is characterized by a lump-sum transfer at date 0 from the creditor to the debtor (over and above the initial investment cost I) and promised repayments from the debtor to the creditor at future dates. We showed that the creditor's payoff is increasing in the debt repayments and decreasing in the transfer, reaching a maximum $\bar{\pi}$ when debt is infinite and the transfer is zero. High debt levels and low transfers, however, lead to dead-weight losses in default states: in particular, the inability of the debtor to commit to pay a sufficient portion of future cash flows to the creditor may cause inefficient asset liquidations. Under the assumption that $\bar{\pi} > I$ and there is a competitive supply of creditors, the task for the debtor is to raise the lump-sum transfer from zero and reduce the debt payments from infinity in such a way as to minimize the dead-weight losses of default, at the same time as driving the creditor's payoff down from $\bar{\pi}$ to I .

The mechanisms by which increases in the transfer and reductions in debt reduce dead-weight losses -- or, equivalently, raise social surplus -- differ in important ways. Low debt levels reduce the likelihood of default and also the degree of liquidation in the event that default occurs. High transfers also reduce the probability of default, but they have an additional role: by increasing the debtor's wealth they strengthen his bargaining position in post-default states and make it easier for him to repurchase assets from the creditor.

While transfers are thus a more powerful instrument for increasing social surplus in gross terms, they are also more costly: a one dollar increase in transfer reduces the creditor's payoff by more than a one dollar reduction in debt (the creditor loses the transfer in all states, but only loses the debt in non-default states). In net terms, neither instrument will dominate the other, and each will typically be used in an optimal contract. In particular cases, however, we found that one instrument will be favored. For example, in a two period model we showed that an optimal contract consists of a pure transfer when there is uncertainty only about early project returns or asset liquidation values; while an optimal contract consists of pure debt when there is uncertainty only about late project returns. The reason is that in the former two cases inefficient liquidations occur in states where the debtor is likely to default for a wide range of debt levels and where therefore a reduction in debt is unlikely to be effective in reducing inefficiency; whereas the reverse is true when there is uncertainty about late project returns: inefficient liquidations occur precisely in states where the debtor would pay low debt levels if offered them.

An interesting new trade-off arises as we move from a two period model to a three period one. The debtor and creditor now have a choice whether to have low debt repayments early in the relationship (at date 1, say) and high debt repayments later in the relationship (at date 2, say), or vice-versa. The trade-off is as follows. A reduction in the date 1 repayment by one dollar must be matched by an increase in the date 2 repayment by more than a dollar to keep the creditor's payoff constant ("a bird in the hand is worth two in the bush"). The positive effect of lowering the date 1 repayment is that fewer early defaults will occur, and thus there will be fewer inefficient liquidations at date 1. There are, however, two offsetting

negative effects from raising date 2 repayments. First, since total debt has risen, whenever date 1 defaults do occur, they will involve a greater degree of liquidation and hence more inefficiency. Second, more defaults will occur at date 2, and thus there will be more inefficient liquidations at this date.

We used these ideas to understand the costs and benefits of short and long-term debt. Short-term debt is an extreme case of a large early repayment and a small late repayment. It is as if the debtor had to return to the capital market each period to refinance his project. The disadvantage of short-term debt is that it can give the creditor too much control over the project's future at an early stage. (In some cases, short-term debt will be preferred to long-term debt in spite of this disadvantage.) For instance, in Section 4, Example 5 we considered a situation where early on in their relationship the debtor and creditor receive news that while project returns in the medium-run will be high, in the longer-run they will be low. The creditor will foresee that she will be unable to capture a large fraction of the medium-run returns since, by the time these are earned, the debtor will prefer to default rather than make large repayments (the assets are not worth much to him once the medium-term is over). So, the creditor will take the opportunity that a short-term contract provides to pull the plug on the project as soon as news about the project's long-term prospects arrives. (Other creditors in the capital market will feel exactly the same way.) However, this outcome may be very inefficient given that the sum of the medium-run and long-run returns may far exceed the project's liquidation value at the time the news arrives.

There are numerous ways in which our work could be extended. Some obvious ones are: to increase the number of periods from three; to allow for further investment expenditures during the course of the debtor-creditor relationship; to allow for the possibility that the project's liquidation value exceeds its value in place in some states of the world; to relax the assumption that the debtor can divert funds on a one-to-one basis; and to introduce multiple investors.

We believe that the last two extensions are particularly fruitful. As we have noted, the assumption that the debtor can divert cash costlessly is strong and it would be desirable to see how our results would change if this were replaced by the assumption that the debtor can use project funds in less

extreme, but still discretionary ways, e.g. to engage in empire-building activities. Also, many new issues are introduced when there are several investors. One is whether it is still reasonable to suppose that renegotiation is a costless process. For example, with a large number of creditors, hold-out problems may prevent debts from being renegotiated even when there are gains from trade from doing this (bankruptcy law may also be relevant in such a situation). Note that while this possibility may decrease the flexibility of debt finance, it may also provide some advantages: the debtor will be less likely to default voluntarily if the probability of rescheduling debt is low (in other words, large numbers of creditors may act as a kind of commitment device).

Perhaps the most interesting issue concerning multiple investors involves the conditions under which it is optimal for different investors to hold different kinds of securities issued on a project or firm. In practice, corporations issue a variety of financial instruments, e.g. debt, equity, preferred shares, etc. and these appear to be held by disparate investing groups. A challenging direction for future research is to extend the model to include flexible claims like equity as well as debt (see Section 1). The hope is that eventually a model like this can be used to provide a theory of optimal capital structure based on the control rights of different investor groups. Among other things, such a model would explain the basic, but still poorly understood, observation that equity holders are granted voting rights, while creditors cannot vote, but, of course, do instead have the right to foreclose on the firm's assets.³⁰

FOOT NOTES

1. Of course, one reason for issuing debt is to reduce taxes. We will ignore the tax motivation for debt in this paper on the grounds that, while taxes may be an important element in financial decisions, it is very unlikely that they can explain them entirely. In particular, as many people have remarked, debt was important before taxes were. For a discussion of debt and taxes, see King (1976), Miller (1977) and Scholes and Wolfson (1989).
2. See Leland and Pyle (1977), Ross (1977), Myers (1977), Myers and Majluf (1984), Grossman and Hart (1982) and Jensen (1986), among others. The related work of Stiglitz and Weiss (1981) should also be mentioned.
3. There are a number of ways in which a manager can divert funds in practice, short of outright theft. One is to use the firm's resources to pay high salaries or to purchase managerial perks, such as expensive plane trips, vacations, meals, accommodation, offices, secretaries. Another is to contract to sell output of this firm to another firm the manager or a friend or relative has an interest in at an artificially low price. For a discussion indicating that diversion can be a problem in practice and that monitoring it is costlier for fungible assets like cash than for physical assets, see White (1984).
4. Such investments presumably pay off only prior to or in the absence of default.
5. In fact, if the entrepreneur's bargaining power is large, he may realize such a large share of return that it is almost as if he could divert the firm's funds. A related phenomenon to the tying up of funds has been modelled in a recent paper by Shleifer and Vishny (1988).
6. This suggests that the indirect costs of bankruptcy may far exceed the direct costs measured by Warner (1977).
7. In fact the particular version of the model we study assumes that the assets are always worth more to the debtor than to the creditor and so any allocation where the debtor has control of the assets is automatically efficient. However, as the argument given in the text indicates, the conclusion holds more generally.
8. The work of Green and Shoven (1982) and Allen (1983) should also be mentioned. Green and Shoven consider a model of corporate debt which is similar to ours in some respects (the parties are symmetrically informed, certain key variables are not verifiable, etc.). However, Green and Shoven do not analyze the role of debt in encouraging a debtor to repay a loan, and also take the cost of default or bankruptcy to be exogenous. On the other hand, Allen (1983) studies a model in which the penalty for not repaying a loan is future exclusion from the capital market. Allen focusses on inefficiencies with respect to the initial size of the project, however, rather than on control issues or the social costs of default.
9. For simplicity we ignore agreements with several investors. But see the remarks in Section 5.

10. For simplicity we suppose that no liquidation is possible before date 1. The period from 0 to 1 can be interpreted as one during which assets are being assembled and when, in consequence, liquidation would be very disruptive.
11. In future work, it would be interesting to investigate the case where uncertainty is resolved gradually during the course of the relationship.
12. However, we suppose that the project is indivisible at date 0. That is, the full amount of I dollars is required at date 0 to generate any return.
13. In practice there is no clear division between physical assets and cash as types of collateral for a loan since there is a risk of diversion in both cases. It does seem reasonable, however, to suppose that it is harder to monitor cash diversion than diversion of physical assets, such as buildings. For comments supporting this idea, see White (1984). We are simply considering the polar case where it costs nothing to monitor physical asset diversion and an infinite amount to monitor cash diversion. For alternative interpretations of cash diversion as representing perquisite-producing investments or the consequence of the entrepreneur's ability to threaten to tie up funds, see the introduction.
14. So, the debtor's assets are a hostage in the sense of Williamson (1983).
15. We ignore reputational motives for the repayment of debt.
16. It is conceivable that P_1 and P_2 could be conditioned on verifiable messages that the parties send each other or to a third party. Allowing for this possibility would complicate, but, we suspect, not substantially change, the results to be presented below.
17. For an early discussion of the implications of renegotiation for debt contracts, see Hellwig (1977).
18. For other models of debt forgiveness, see Fernandez and Rosenthal (1988).
19. One can also imagine situations where the parties can use the initial contract to design an appropriate renegotiation process; e.g. by insisting on a particular sequence of offers by the debtor and creditor and penalizing any party which does not follow this sequence (see Aghion, Dewatripont and Rey (1989) for a discussion of this). Our feeling is that this would complicate, but would not substantially change, the analysis. Also the following should be noted in this regard. First, constraining the renegotiation process may be difficult in practice given the ability of parties to make informal or furtive offers outside any prescribed process. Second, in the present context, it may be particularly hard to limit the debtor's bargaining power since the worst penalty that the creditor can exact against the debtor if he contravenes any agreement is to seize the debtor's assets; which is a punishment the debtor must face anyway in the event that he has defaulted. Finally, increasing the debtor's bargaining power has disadvantages as well as advantages. In particular, although, as will

become apparent in later sections, it may increase the ex-post efficiency of the relationship, it will also reduce the creditor's return and make it less likely that the project will be financed in the first place.

20. We assume that there are no enforcement problems with this execution. In other words, on-the-spot diversion by D of the funds realized by asset liquidation can be prevented.
21. We rule out the possibility that the debtor can use funds that he has diverted to bid for the assets at the liquidation stage. This can be justified on the grounds that making a bid would expose the debtor to the risk that the existence of these funds would become public information, thus leading to their confiscation.
22. The procedure described here for the settling of unpaid debts is in a formal sense an out of bankruptcy one. Such a procedure seems most appropriate in the present context, since, at least in the U.S., the courts are unlikely to permit a bankruptcy filing when there is a single creditor. Note, however, that the outcome under Chapter 7 bankruptcy is unlikely to be very different from the one described here; and that even under Chapter 11 bankruptcy the final settlement will be sensitive to what the creditor would get in the event of asset liquidation. For a detailed discussion of out-of bankruptcy and bankruptcy procedures, see Baird and Jackson (1985).
23. For another model with this property, see Fernandez and Rosenthal (1988).
24. An alternative assumption to acceleration is that only the current part of the debt becomes due; that is, if D owes \$100 at date 1 and \$200 at date 2, default at date 1 gives C the right to liquidate \$100 worth of assets (rather than \$300 worth). This introduces a new feature: in deciding whether to liquidate, C must consider how this would affect the project's future profitability and in particular the likelihood of her being paid \$200 at date 2. More generally, a debt agreement could be characterized by partial acceleration: e.g., $x\%$ of the future debt could be brought forward in the event of default and $(100-x)\%$ left in place, where x is a choice variable in the original contract.
25. It is worth noting that in practice most debt agreements apparently do include acceleration clauses. Also acceleration is the rule in bankruptcy proceedings, at least in the U.S.
26. Two further assumptions implicit in our analysis should be mentioned. First, we have ignored the possibility that the debtor can go to the capital market at dates 1 and 2 to refinance the debt. Allowing this would not change anything, however, since, given our assumption that only the debtor can make offers, he can do as well by bargaining with the existing creditor as by approaching a new one. (We are supposing that the creditor has the power to prevent the debtor from using the collateral for her original loan to take out a new loan, if she has not been fully paid off.)

Second, we have ruled out contracts that state that D must give up a certain (physical) fraction of the assets at dates 1 or 2 rather than owe a particular monetary amount. This can be justified on the

grounds that (a) the assets may not be truly homogeneous, so that people may disagree about what, say, 10% of the assets really means; (b) it may be hard to describe the assets in detail in advance of their acquisition at date 0 and hence it may be difficult to specify the appropriate parts to be sold off. In contrast, a statement of the form that D owes C \$100, and, if he doesn't pay, \$100 of the assets should be sold off is relatively unambiguous.

27. For (casual) evidence suggesting that defaults on automobiles do indeed occur when liquidation value is low, see New York Times, February 16, 1989, p1.
28. Note that any short-term debt (P_1, T) contract for which $T < 79$ -- and $P_1 < \infty$ is commensurately low so that on average C breaks even -- would be worse than the contract $(P_1, T) = (\infty, 79)$. The reason is that C's payoff, $\gamma(10)$ (say), in the bad state would be higher for a lower value of T ($\gamma(10)$ is unaffected by the fact that P_1 is lower since C will be forgiving some of the debt anyway). And a higher $\gamma(10)$ corresponds to a greater fraction of the assets being liquidated, and therefore to a greater social loss.
29. We can also now see why the best short-term debt contract has the largest possible transfer ($T=15$) commensurate with C breaking even. Any other short-term debt contract will raise C's payoff, $\gamma(0)$ (say), in the bad state. To understand why, first note that for the above short-term contract, $\gamma(0) = 95$. Now consider lowering T, and correspondingly lowering P_1 . The fact that P_1 is lower doesn't affect $\gamma(0)$; C will be forgiving some of the debt anyway in this state. But the fact that T is lower raises $\gamma(0)$ above 95. And as we have just discussed, this increases the extent of liquidation and reduces welfare.
30. For a recent analysis of this issue, see Harris and Raviv (1988).

Appendix

Lemma A1 Suppose D retains control over all the assets after date 1 (i.e. $f_2 = 1$), and at date 2 the agents play according to subgame perfect equilibrium strategies. Then there is no Pareto improvement possible at date 1.

Proof Suppose that the subgame perfect equilibrium outcome at date 2 has D keeping possession of a fraction f_3 of the assets. If $f_3 = 1$, then the first-best is being achieved (no liquidation at either date 1 or 2), and so there can be no possible Pareto improvement. Assume, then, that $f_3 < 1$. Since the agents are presumed to play equilibrium strategies at date 2, D must be paying C all his available cash at that date -- for otherwise he would use it to reduce the fraction $(1-f_3)$ of assets which is being liquidated. That is, D's payoff δ only comes from the return $f_3 R_3$ at date 3 (all previous receipts $T + R_1 + R_2$ having been paid to C):

$$\delta = f_3 R_3.$$

C's payoff γ is therefore given by

$$\begin{aligned}\gamma &= T + R_1 + R_2 + (1-f_3)L_3 - T \\ &= R_1 + R_2 + (1-f_3)L_3.\end{aligned}$$

Now suppose that, contrary to the Lemma, some Pareto improvement is possible. In this new allocation, let the fractions of the assets remaining in D's possession after dates 1 and 2 be respectively \hat{f}_2 and \hat{f}_3 . Divide D's payoff $\hat{\delta}$ into two components: the return $\hat{f}_3 R_3$ which he gets at date 3 from possessing the fraction \hat{f}_3 of the assets between dates 2 and 3, plus the amount of cash, Y_3 (say), which he carries over from date 2:

$$\hat{\delta} = \hat{f}_3 R_3 + Y_3.$$

C's payoff $\hat{\gamma}$ is then given by

$$\begin{aligned}\hat{\gamma} &= T + R_1 + (1-\hat{f}_2)L_2 + \hat{f}_2R_2 + (1-\hat{f}_3)L_3 - Y_3 - T \\ &= R_1 + (1-\hat{f}_2)L_2 + \hat{f}_2R_2 + (1-\hat{f}_3)L_3 - Y_3.\end{aligned}$$

If this is to represent a Pareto improvement, then

$$\hat{\gamma} - \gamma = - (1-\hat{f}_2)[R_2 - L_2] + (f_3-\hat{f}_3)L_3 - Y_3$$

must be nonnegative -- implying that $\hat{f}_3 \leq f_3$.

But now consider the change in social surplus:

$$\begin{aligned}(\hat{\gamma} + \delta) - (\gamma + \delta) &= - (1-\hat{f}_2)[R_2 - L_2] - (f_3-\hat{f}_3)[R_3 - L_3] \\ &\leq 0.\end{aligned}$$

That is, there is no Pareto improvement after all.

Q.E.D.

The Solution to Problem (4.7)

Substituting $T+R_1$ for Z_1 , and denoting by S the shortfall in C 's payoff after D has paid her all his cash at date 1,

$$S = \gamma^0 + T - (T+R_1) = \gamma^0 - R_1,$$

we can write Problem (4.7) more succinctly:

$$\begin{aligned}\text{Maximize} & \quad f_2R_2 + f_3R_3 - Z_2 \\ & f_2, Z_2, f_3\end{aligned}$$

$$\begin{aligned}\text{subject to} & \quad (1-f_2)L_2 + (1-f_3)L_3 + Z_2 \geq S & \text{(i)} \\ & \quad Z_2 \leq f_2R_2 & \text{(ii)} \\ & \quad Z_2 \leq f_3R_3 & \text{(iii)} \\ \text{and} & \quad 0 \leq f_3 \leq f_2 \leq 1. & \text{(iv)}\end{aligned}$$

It is clear that (i) must always bind, for otherwise there would be no lower bound on Z_2 . We split the analysis into two cases:

Case 1: $R_3 \leq R_2$

In this case, constraint (ii) can be ignored, since it follows directly from (iii), (iv) and $R_3 \leq R_2$.

If $S \leq R_3$, then D can achieve the first-best, $f_2 = f_3 = 1$, by offering $Z_2 = S$. (Constraint (iii) is satisfied.) From (4.1) and (4.2) it is readily confirmed that these values of f_3 and Z_2 can be implemented by setting $\hat{P}_2 = S$.

Conversely, if $S > R_3$, constraint (iii) must bind: $Z_2 = f_3 R_3$. Substituting this value for Z_2 into D's payoff and into (i) gives

$$\begin{aligned} &\text{Maximize} && f_2 R_2 \\ & && f_2, f_3 \\ & \text{subject to} && (1-f_2)L_2 + (1-f_3)L_3 + f_3 R_3 \geq S && \text{(i)} \\ & && \text{and} && 0 \leq f_3 \leq f_2 \leq 1. && \text{(iv)} \end{aligned}$$

It is optimal to raise f_3 to equal f_2 , in order to slacken (i). Hence, using the fact that (i) binds at an optimum,

$$f_2 = f_3 = \frac{L_2 + L_3 - S}{L_2 + L_3 - R_3}.$$

From (4.1) and (4.2) it is readily confirmed that these values of f_3 and Z_2 can be implemented by setting

$$\hat{P}_2 = \left(\frac{L_2 + L_3 - S}{L_2 + L_3 - R_3} \right) R_3.$$

Case 2: $R_2 < R_3$

If $S \leq R_2$, then D can achieve the first-best, $f_2 = f_3 = 1$, by offering $Z_2 = S$. (Constraints (ii) and (iii) are satisfied.) From (4.1) and (4.2) it is readily confirmed that these values of f_3 and Z_2 can be implemented by setting $\hat{P}_2 = S$.

If $R_2 < S$, consider ignoring constraint (iii). In this relaxed program constraint (ii) must bind (otherwise f_2 and f_3 would be set equal to 1 and Z_2 would be set equal to S , violating (ii)): $Z_2 = f_2 R_2$. Substituting this value for Z_2 into D's payoff and into (i) gives

$$\begin{aligned} &\text{Maximize} && f_3 R_3 \\ & && f_2, f_3 \end{aligned}$$

$$\text{subject to } (1-f_2)L_2 + (1-f_3)L_3 + f_2 R_2 \geq S \quad (\text{i})$$

$$\text{and } 0 \leq f_3 \leq f_2 \leq 1. \quad (\text{iv})$$

It is optimal to raise f_2 to 1, in order to slacken (i). Hence, using the fact that (i) binds as an equality at an optimum,

$$f_2 = 1 \quad \text{and} \quad f_3 = 1 - \frac{S-R_2}{L_3}.$$

Now for these values of Z_2, f_2, f_3 , the missing constraint (iii) reads

$$R_2 \leq \left[1 - \frac{S-R_2}{L_3} \right] R_3$$

which is satisfied if and only if $S \leq R_2 + \left[1 - \frac{R_2}{R_3} \right] L_3$.

Hence for $R_2 < S \leq R_2 + \left[1 - \frac{R_2}{R_3} \right] L_3$, Z_2 equals R_2 , f_2 equals 1, and f_3 equals $1 - \frac{S-R_2}{L_3}$. From (4.1) and (4.2), it is readily confirmed that these values of f_3 and Z_2 are implemented by setting $\hat{P}_2 = S$.

Conversely, for $S > R_2 + \left(1 - \frac{R_2}{R_3}\right)L_3$, constraint (iii) must bind: $Z_2 = f_3R_3$. Substituting this value for Z_2 into D's payoff and into (i) gives

$$\begin{aligned} &\text{Maximize } f_2R_2 \\ &f_2, f_3 \end{aligned}$$

$$\text{subject to } (1-f_2)L_2 + (1-f_3)L_3 + f_3R_3 \geq S \quad (\text{i})$$

$$f_3R_3 \leq f_2R_2 \quad (\text{ii})$$

$$\text{and } 0 \leq f_3 \leq f_2 \leq 1. \quad (\text{iv})$$

Constraint (ii) must bind also, for otherwise it would be optimal to raise f_3 to f_2 (in order to slacken (i)), which violate (ii). Knowing that in this case constraints (ii) and (iii) both bind enables us to solve directly for the optimal f_2 and f_3 :

$$f_2 = \frac{L_2 + L_3 - S}{R_2 \left[\frac{L_2}{R_2} + \frac{L_3}{R_3} - 1 \right]} \quad \text{and} \quad f_3 = \frac{L_2 + L_3 - S}{R_3 \left[\frac{L_2}{R_2} + \frac{L_3}{R_3} - 1 \right]}$$

[The two numerators are nonnegative, the two denominators are positive, and both fractions are less than unity. This follows from the fact that

$$R_2 + \left[1 - \frac{R_2}{R_3}\right]L_3 < S \leq L_2 + L_3.]$$

From (4.1) and (4.2), it is readily confirmed that these values of f_3 and Z_2 are implemented by setting $\hat{P}_2 = S$.

That completes the analysis of Problem (4.7). The above results are summarised in Proposition 6, Case (3).

Further details of Example 7

It is straightforward, but rather tedious, to confirm that the best short-term debt contract is $(P_1, T) = (\infty, 22\frac{8}{11})$:

Briefly, if C is to break even from some short-term debt contract (P_1, T) , then T must lie in the range $0 \leq T \leq 22\frac{8}{11}$. In this range, social surplus is piecewise-linear in T, with regime changes at

$$T = 8, 9\frac{4}{5}, 11\frac{3}{5} \text{ and } 18\frac{1}{8}.$$

(The corresponding values of P_1 are $108, 110\frac{3}{10}, 112\frac{3}{5}$ and $120\frac{15}{16}$.) In fact, as a function of T, social surplus is constant for $T \in (0, 8)$, then rises for $T \in (8, 9\frac{4}{5})$, rises again for $T \in (9\frac{4}{5}, 11\frac{3}{5})$, then falls for $T \in (11\frac{3}{5}, 18\frac{1}{8})$, and finally rises for $T \in (18\frac{1}{8}, 22\frac{1}{11})$. Social surplus therefore achieves a local maximum when $(P_1, T) = (112\frac{3}{5}, 11\frac{3}{5})$. For this contract, denoting C's payoff in state L_3 by $\gamma(L_3)$, it follows from Proposition 6 that

$$\gamma(60) = \gamma(50) = 101 \text{ and } \gamma(40) = 98.$$

And in state L_3 , $f_2(L_3)$ and $f_3(L_3)$ are given by

$$\begin{array}{ll} f_2(60) = 1 & \text{and} \quad f_3(60) = 49/60 \\ f_2(50) = 19/20 & \text{and} \quad f_3(50) = 19/25 \\ f_2(40) = 1 & \text{and} \quad f_3(40) = 4/5. \end{array}$$

[Checking: C's expected payoff U_c is given by

$$\begin{aligned} U_c &= 10 + (1/3) \{ 0 + 80 + (11/60)60 \} \\ &+ (1/3) \{ (1/20)60 + (19/20)80 + (6/25)50 \} \\ &+ (1/3) \{ 0 + 80 + (1/5)40 \} = 100 - 1. \end{aligned}$$

Expected social surplus $U_c + U_d$ equals

$$\begin{aligned} &10 + (1/3) \{ 80 + (49/60)100 + (11/60)60 \} \\ &+ (1/3) \{ (19/20)80 + (1/20)60 + (19/25)100 + (6/25)50 \} \\ &+ (1/3) \{ 80 + (4/5)100 + (1/5)40 \} \\ &= 179\frac{2}{9}. \end{aligned}$$

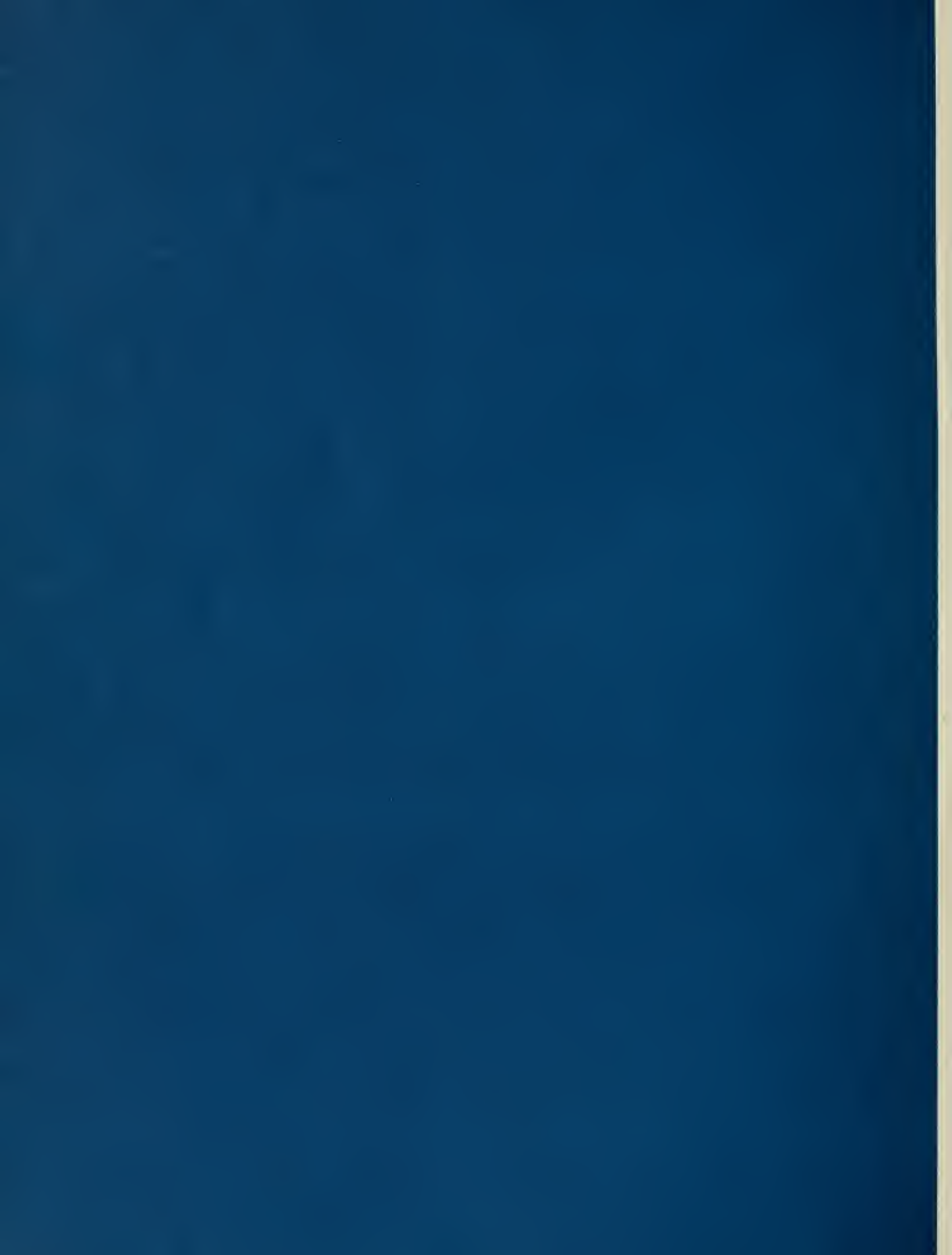
This is less than the expected social surplus of $179\frac{61}{231}$ which we calculated (in the text) for the short-term contract $(P_1, T) = (\infty, 22\frac{8}{11})$.

REFERENCES

- Aghion, P. and P. Bolton (1988), "An 'Incomplete Contracts' Approach to Bankruptcy and the Financial Structure of the Firm", mimeo.
- Aghion, P., M. Dewatripont and P. Rey (1989), "Renegotiation Design Under Symmetric Information", mimeo.
- Allen, F. (1983), "Credit Rationing and Payment Incentives", Review of Economic Studies, Vol.L(4):639-646.
- Baird, D. and T. Jackson (1985), Cases, Problems and Materials on Bankruptcy, Little Brown and Co., Boston, MA.
- Bulow, J. and K. Rogoff (1989), "A Constant Recontracting Model of Sovereign Debt", Journal of Political Economy, Vol.97(1):155-178, February.
- Eaton, J. and M. Gersovitz (1981), "Debt with Potential Repudiation: Theoretical and Empirical Analysis", Review of Economic Studies, 48 (April).
- Fernandez, R. and R. Rosenthal (1988), "Sovereign Debt Renegotiations: A Strategic Analysis", NBER Working Paper #2597, May.
- Froot, K. (1989), "Buybacks, Exit Bonds, and the Optimality of Debt and Liquidity Relief", International Economic Review, Vol.30(1), February.
- Gale, D. and M. Hellwig (1985), "Incentive Compatible Debt Contracts: The One-Period Problem", Review of Economic Studies, Vol. LII(4):647-663.
- Green, J. and J. Shoven (1982), "The Effects of Financing Opportunities and Bankruptcy on Entrepreneurial Risk Bearing", in Entrepreneurship (J. Ronen, ed), Lexington, MA: Lexington Books.
- Grossman, S. and O. Hart (1982), "Corporate Financial Structure and Managerial Incentives", in J.J. McCall, ed., The Economics of Information and Uncertainty, University of Chicago Press.
- Harris, M. and A. Raviv (1988), "The Design of Securities", University of Chicago, mimeo.
- Hellwig, M. (1977), "A Model of Borrowing and Lending with Bankruptcy", Econometrica, Vol.45(8):1879-1906.
- Jensen, M. (1986), "Agency Costs of Free Cash Flow, Corporate Finance and Takeovers", American Economic Review, 76:323-329.
- Kahn, C. and G. Huberman (1988), "Default, Foreclosure, and Strategic Renegotiation", University of Chicago, mimeo.
- King, M. (1977), Public Policy and the Theory of the Corporation, (New York: Wiley).
- Leland, H. and D. Pyle (1977), "Informational Asymmetries, Financial Structure, and Financial Intermediation", Journal of Finance, 32:371-88.
- Miller, M. (1977) "Debt and Taxes", Journal of Finance, 32:261-75.

- Myers, S. (1977), "Determinants of Corporate Borrowing", Journal of Financial Economics, 5:147-175.
- Myers, S. and N. Majluf (1984), "Corporate Financing and Investment Decisions When Firms Have Information That Investors Do Not Have", Journal of Financial Economics, 13.
- Ross, S. (1977), "The Determination of Financial Structure: The Incentive-Signalling Approach", Bell Journal of Economics, 8:23-40.
- Rothschild, M. and J. Stiglitz (1970), "Increasing Risk: I. A Definition", Journal of Economic Theory, Vol.2(3):225-243.
- Scholes, M. and M. Wolfson, Taxes and Business Strategy: A Global Approach, Prentice-Hall, forthcoming, 1990.
- Shleifer, A. and R. Vishny (1988), "Managerial Entrenchment: The Case of Manager-Specific Investments", mimeo, Chicago.
- Stiglitz, J. and A. Weiss (1981), "Credit Rationing in Markets with Imperfect Information", American Economic Review, 71:393-410.
- Townsend, R. (1979), "Optimal Contracts and Competitive Markets with Costly State Verification", Journal of Economic Theory, 21:265-293.
- Warner, J. (1977), "Bankruptcy Costs: Some Evidence", Journal of Finance, XXXII(2):337-347.
- White, M. (1984), "Bankruptcy Liquidation and Reorganization", Chapter 35 in Handbook of Modern Finance, E. Logue (ed.), Boston: Warren, Gorham and Lamont.
- Williamson, O. (1983), "Credible Commitments: Using Hostages to Support Exchange", American Economic Review, 83:519-540.

5960.002



Date Due

APR 19 1990	MAY 23 1990
DEC 26 1990	JAN. 08 1997
APR 26 1991	MAR. 19 1997
MAY 30 1991	SEP. 02 1997
JUN 17 1992	MAR. 05 1998
JUL 11 1995	
OCT. 11 1995	
AUG. 31 1995	
AUG. 31 1995	
OCT. 12 1995	
FEB. 24 1996	

MIT LIBRARIES DUPL 1



3 9080 00569723 7

