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# Definitive Orhit of Comet 1894 IV (ELS Swift). 

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# Definitive Orbit of Comet 1894 IV (E. Swift). 

By firdarick II. Sares.

## 1. Introductory.

Comet 1894 IV was discovered by Edward Swift at Echo Mountain, California, on Nov. 20, 1894 at $8^{\text {h }} 30^{\circ \mathrm{l} \mathrm{\prime}} \mathrm{p} . \mathrm{m}$. The comet was then very faint and had a short tail. The first observations were obtained by Barnard with the 12 inch Equatorial of the Lick Observatory on Nov. 21, 22, and 23, and by Javelle at Nice on Nov. 22 and 23 .

A rapidly increasing deviation from the positions predicted by means of the preliminary parabolic etements gave indication of decided ellipticity which was soon confirmed by later elements.

A great similarity between the elements of the new comet and those of De Vico 1844 I was soon noted and the possibility of identity was suggested by Berberich as early as Nov. 23. On Dec. 1 Tisserand announced that Schulhof had found the two comets to be identical.

Owing to the extreme faintness of the olject observations were obtained with the greatest difficulty as is sufficiently evident from the notes by the various observers, and in many places bad weather made observing quite impossible. This was especially the case at Mt. Hamilton where in spite of constant watchfulness on the part of Barnard the comet was not seen from Nov. 30, 1894 until Jan. 25, 1895 . As a consequence the total number of positions is only 64 for right ascension and 63 for declination.

The first set of elements approximating a definitive solution were by Cbandler from 27 observations grouped into 6 normal places. Basing himself upon this system of elements Chandler undertook a preliminary investigation of the question of identity with De Vico's comet, carrying the perturbations back through the conjunction with Jupiter in
1885. It seemed uscless on account of the necessary indeterminateness of the value used for the mean motion to carry the calculation of the perturbations farther, but enough had been done to accomplish a partial adjustment of the discrepancies between the two comets and to show that the effect of the conjunctions with Jupiter in 1874 and $\mathbf{1 8 6 2}$ would be in the right direction to produce a still better agreement. Notwithstanding the definiteness of the announcement concerning Schulhof's conclusions they were only provisional and were derived from a consideration of the perturbations of De Vico's comet throughout a long series of years. They were substantially the same as those arrived it by Chandler and may be found in A. N. No. 3267. With this encouragement the calculation of the definitive elements of comet Swift was begun, and the results are here presented as a part of an investigation of the question of identity with De Vico's comet which the writer has undertaken.

## 2. Calculation of Ephemeris.

The following elements by Chandler in A. J. No. 338 were used as the basis of the calculation.

$$
\begin{aligned}
T & =1894 \text { Oct. 12.188.17 Gr. M. T. } \\
\prime \prime & =296^{\circ} 344^{\prime} 35.2 \\
& =484437.1 \\
i & =5753.9
\end{aligned} \quad 1894.0
$$

With these was calculated an ephemeris of 1 day intervals from $189+$ Nov. 10.5 to 1895 Jan. 30.5 Berlin M T

Ephemeris for l E erlin Mean Midnight.

| 1894 | $\alpha$ | $\delta$ | $\log \lambda$ | A1). T. | 1894 | \% |  | d | $\log .1$ | Ab. T |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Nov. 19 | $22^{\text {h }} 14^{\text {m }} 9.94$ | -1 $3^{\circ} 34^{\circ} 166^{\prime \prime} 04$ | 0.0075 | $8^{11127} 273$ | Nov. 28 | $22^{11} 40^{\prime \prime \prime} 50.22$ | $-10^{\circ}$ | $29^{\prime} 19.42$ | 0.0384 | $9^{17 .} 46$ |
| 20 | 1710.23 | 131356.29 | 0108 | 312 | 29 | $43+4.85$ | 10 | 835.48 | 0410 | 9.1 |
| 21 | $20 \quad 9.90$ | 125332.38 | 0142 | 35.2 | 30 | $46 \quad 38.82$ | 9 | 4750.70 | 0454 | 13.6 |
| 22 | $23 \quad 8.96$ | 1233467 | 0176 | 39.3 | Dec. | $40{ }^{22.13}$ | 9 | $27 \quad 542$ | 0400 | 18. |
| 23 | $26 \quad 7.40$ | 121233.54 | 0210 | 43.4 | 2 | 5224.77 | 9 | 620.00 | 0525 | 22.8 |
| 24 | $29 \quad 5.23$ | 115159.35 | 0244 | 47.5 | 3 | $\begin{array}{llll}55 & 16.73\end{array}$ | 8 | 453478 | 0501 | 27. |
| 25 | $32 \quad 244$ | 11 3122.48 | 0279 | 51.7 | 4 | 2258801 | 8 | 2450.10 | 0597 | 32.2 |
| 26 | 3259.00 | $\begin{array}{lll}11 & 10 & 4.29\end{array}$ | 0314 | 855.9 | 5 | 23 ○ 58.61 | 8 | $4 \quad 0.28$ | 0634 | 37.0 |
| 27 | $22 \quad 3754.93$ | $\begin{array}{lll}-10 & 50 & 2.15\end{array}$ | 0.0349 | 9 | 6 | $23 \quad 348.52$ | $-7$ | $43 \quad 23.68$ | 0.0670 | 931 |



## 3. Observations and Comparison Stars.

The observations were collected from the usual sources and it is believed that all have been included. The published data of observation were checked wherever possible by independent computation and the parallax factors and reductions to apparent place were recomputed with the constants of the Berlin Jahrbuch.

The comparison stars used in the observations are 47 in number. Their positions have been investigated with considerable care, perhaps with more than is strictly necessary in view of the large probable errors of the observations. It seemed safer however to reduce the errors in the star places as much as possible in order to make them negligible as compared with the errors of observation, especially as the case under consideration is one in which the available material is so scanty as to render difficult the attaimment of that degree of accuracy which is to be desired.

I am indebted to Professor Leuschner for much of the star catalogue data obtained by him from the libraries of various observatories while abroad in 1895-96, and to Professor Schaeberle as acting director of the Lick Observatory for his courtesy in allowing me to use the catalogues at MIt. Hamilton.

Practically all of the wisting catalogues were searched. The catalogue positions were reduced to the beginning of the years 1894 and 1895 ly Kreutz's tables (A. N. v. 134)

| 1895 | ${ }^{\prime}$ | $\delta$ | $\log 4$ | Ab. T. |
| :---: | :---: | :---: | :---: | :---: |
| Jan. | $0^{\mathrm{h}} 18^{\mathrm{m}} 4119$ | $+1^{\circ} 31^{\prime} 56{ }^{\prime \prime} 38$ | 0.1710 | $12^{\text {m }} 19 \% 3$ |
| 4 | 2113.13 | 5028.43 | 1747 | 25.6 |
| 5 | $23+4.58$ | 2853.73 | 1784 | 32.0 |
| 6 | $\begin{array}{ll}26 & 15.49\end{array}$ | 2712.17 | 1822 | 38.5 |
| 7 | $28+5.89$ | $2+523.67$ | 1858 | 45.0 |
| 8 | 311581 | $3 \quad 3 \quad 28.12$ | 1895 | 51.5 |
| 9 | $33+5.21$ | 3212546 | 1931 | 1258.1 |
| 10 | $\begin{array}{lll} & 6 & 1+13\end{array}$ | 339915.61 | 1969 | $13 \quad 4.7$ |
| 11 | $38+{ }^{2} \cdot 5^{6}$ | $\begin{array}{llllllllllllllll}3 & 56.51\end{array}$ | 2005 | 11.3 |
| 12 | 4110.52 | 4143408 | 2042 | 18.0 |
| 13 | $43 \quad 38.01$ | $\begin{array}{lll}32 & 2.32\end{array}$ | 2078 | 24.7 |
| 14 | $46 \quad 5.04$ | 44923.19 | 2115 | 31.5 |
| 15 | $483: 63$ | $5 \quad 6 \quad 36.64$ | 2151 | 38.3 |
| 16 | 5057.78 | 52342.66 | 2188 | 45.2 |
| 17 | $53 \quad 23.51$ | 54041.23 | 2224 | 52.1 |
| 18 | 5548.81 | $5 \quad 57 \quad 32.33$ | 2260 | 1359.0 |
| 19 | - $5^{8} 13.71$ | 141597 | 2296 | 146.0 |
| 20 | - $3^{8.22}$ | 63052.09 | 2332 | 13.1 |
| 21 | $\begin{array}{lll}3 & 2.33\end{array}$ | 64720.65 | 2367 | 20.2 |
| 22 | 26.06 | $7 \quad 3+1.63$ | 2403 | 27.2 |
| 23 | 740.42 | 1955.03 | 2438 | 34.3 |
| 24 | 1012.41 | $36 \quad 0.81$ | 2474 | 41.4 |
| 25 | 1235.04 | 75158.94 | 2509 | 48.6 |
| 26 | 1457.32 | 749.39 | 2544 | 1455.8 |
| 27 | $17 \quad 19.24$ | $8 \quad 2332.14$ | 2579 | $15 \quad 3.1$ |
| 28 | 10.40 .82 | 39 7.16 | 2614 | 10.4 |
| 29 | $22 \quad 2.06$ | 85434.44 | 2649 | 17.7 |
| 30 | 12.422 .96 | +9 953.96 | 0.2683 | 1525.0 |

which are based upon Struve's constants. Systematic corrections derived from the introductions to various catalogues, from vol. VII of the Bonner Beobachtungen and from Auwers' papers in A. N. Nos. 3195-96, 3413-14, and 3463, were applied to reduce to the system of the Astronomische Gesellschaft. When systematic corrections could not be found or when they seemed uncertain the simple catalogue position was used. In a few cases however where the total number of observations is large the positions lacking systematic corrections were given zero weight especially if the observations forming the position were old or few in number.

The weiglating of the catalogues for the formation of the final positions is in accordance with the system published ly Inavis in his Declinations and Proper Motions of Fiftysix Stars. This system was derived by a consideration of the probable errors of the catalogues concerned, and its homogeneity has been tested by Dr. Davis in the introduction to his paper Although established primarily for declinations it has been used for right ascensions as well. A few catalogues have been used in the present paper which do not appear in the Davis system. Wherever possible these wete werghted in accordance with the methods used in forming that system.

After the reduction of the catalogue positions to the beginning of the year of observation and the application of systematic corrections and weights, the simple mean by weights was drawn for each comparison star unless there
was some indication of proper motion, in which case the final position of the star and its proper motion were obtained נy a least square solution. The following list includes the computed weights of the coreparison stars, although in the urther computations they were all given equal weight.

Final Comparison Star Positions.


Some of the positions of the comet were referred by steps to the comparison stars of the above list. In such zases the intermediate stars are designated by a,b,c.etc. and their positions referred to the proper comparison stars is determined by the various observers are:

| ${ }^{a}-17$ | $-5^{m} 37^{2} 27$ | + 3'43"5 | $m-47$ | $-4^{m 10} 10.68$ | - $10^{\prime} 37^{\prime \prime} 0$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $b-21$ | 4 56.71 | + 811.0 | $n-\mathrm{I}$ | +-12.29 | $-1036.4$ |
| $i-28$ | +228.69 | + $4^{29.0}{ }^{1}$ | - | +0 22.72 | - 713.3 |
| $1 d-28$ | +249.73 | $\left.+548.0^{2}\right)$ | t-3 | -- 10.93 | + 252.7 |
| $1 d-28$ | +248.89 | + $54^{6.8}{ }^{3}$ | $ף-5$ | -315.10 | - 136.2 |
| $c-35$ | - 29.14 | + 740.9 | $r-4$ | $+228.40$ | - 146.0 |
| $f-36$ | 340.80 | 28.1 | $s-7$ | +-18.12 | + 58.2 |
| $k^{r}-34$ | 349.71 | + 0.7 | $t-8$ | -1 4.36 | + 220.5 |
| $h-37$ | $+325.92$ | + 933 | u- 9 | -2 3.04 | + 117.9 |
| $i-42$ | $4 \quad 5.69$ | - 712.0 | ${ }^{\prime}-10$ | +o 21.19 | -13 26.7 |
| $j-4^{2}$ | 329.18 | -○385 | $\pi^{\prime}-11$ | -0 8.13 | - 650.8 |
| k-44 | ${ }_{6} 10.43$ | + 2476 | $x-13$ | +140.81 | - 241.3 |
| $l-43$ | 450.68 | + $\quad 306$ |  |  |  |

${ }^{1}$ ) Difference in © dircordant. ${ }^{\text {² }}$ ) Measured by Brown. ${ }^{3}$ ) Mea-ured by Howe.
It was not until the calculation had been finished and the results wete being collected for printing that the discordance in the two values for the right ascension of $d-28$ was noticed. Doubtless a reference to the original measures would have revealed an error in one or the other and would have made it possible to somewhat improve the third normal place.

## 4. Comparison of Observations with Ephemeris.



| No. | Date of obs. | Obs. | Cp. | * | $\Delta \alpha$ | Par. | $\mathrm{O}-\mathrm{C}$ |  | $\Delta)^{\prime}$ | Par. | $\mathrm{O}-\mathrm{C}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 37 | 1)ec. 17.62222 | H | 1.1 | $e, f$ | $0^{\text {m1 }} 0: 00$ | +0.13 | -0:44 |  | - 0.0 | $+4: 7$ | + 3'3 |
| 38 | 1765716 | , | 10.10 | $c, f$ | +o 533 | $+0.20$ | $-0.70$ | $+$ | - 39.2 | $+4.7$ | + 0.2 |
| 39 | 18.33499 | J | 6.6 | $3^{8}$ | $-2+1.62$ | $+0.17$ | +0.18 | + | 358.8 | $+5.0$ | $+44$ |
| 40 | 18.64086 | H | 10.10 | ${ }^{5}$ | +0 1.70 | $+0.17$ | $+0.34$ | -- | 256.6 | $+4.6$ | + 1.9 |
| 41 | 18.66399 | W | 8.4 | 34 | $+355.60$ | +0.24 | +0.84 | - | 228.8 | $+5.0$ | $+0.3$ |
| 42 | 1935855 | J | $4 \cdot 5$ | 40 | $-32029$ | $+0.21$ | +0.02 | - | - 12.3 | $+49$ | + 5.9 |
| 43 | 19.58735 | St | 16.- | 39 | -0 10.80 | $+0.21$ | $+0.06$ |  | - | - | - |
| 44 | 19.60218 | * | -. 5 | 39 | - | - | - |  | $4 \quad 6.7$ | $+4.4$ | 1.4 |
| 45 | 19.65045 | H | 14.10 | 40 | -2 33.41 | $+0.19$ | $-0.06$ | $+$ | 534.6 | $+4.5$ | + 2.1 |
| 46 | 20.60650 | - | 2010 | / | +o 3.23 | +0.10 | -0.68 | - | 245.1 | $+4.5$ | + 2.3 |
| 47 | 21.59743 | W | 10.10 | ! | +0 0.70 | +0.08 | $-0.39$ | $+$ | $2 \quad 39.5$ | $+45$ | + 3.5 |
| 48 | 21.68884 | W | 10.6 | j | -0 20.05 | +027 | +0.99 | - | 321.9 | +4.7 | + 4.2 |
| 49 | 22.26650 | P | 55 | $k$ | $-1301$ | $+0.08$ | $-0.10$ | $+$ | - 18.3 | $+5.1$ | + 84 |
| 50 | 22.52831 | Br | 4.- | 41 | +o 503 | $+0.10$ | $-0.23$ |  | - | - | - |
| 51 | 22.54074 | , | -. 5 | 41 | - | - | - | $+$ | 11.9 | $+4.3$ | $+0.5$ |
| 52 | 22.54914 |  | $4-$ | 41 | +o 8.47 | $+0.14$ | $-0.06$ |  | - | - | - |
| 53 | 2260561 | H | 15.10 | $l$ | -0 26.50 | $+0.09$ | $+0.37$ | - | $1+1.7$ | +4.4 | $+77$ |
| 54 | 24.71044 | W | 6.3 | 45 | -4 944 | $+0.29$ | +022 | - | $5 \quad 56.4$ | $+46$ | +281 ${ }^{\text {j }}$ |
| 55 | 25.26631 | P | $5 \cdot 5$ | 45 | $-242.29$ | +0.08 | $-0.39$ | $+$ | $+29.1$ | $+49$ | + 2.0 |
| 56 | 25.60427 | H | -. 5 | m | - | - | - | - | 1473 | $+42$ | + 2.7 |
| 57 | 25.61772 | * | 2 - | m | +0 1774 | $+0.13$ | $+0.23$ |  | - | - |  |
| 58 | 26.34684 | J | 45 | 47 | -1 58.70 | $+0.19$ | +0.13 | $+$ | 20.4 | $+4.4$ | + 0.9 |
| 59 | 27.30877 | C | 10.10 | 46 | $+119.45$ | $+0.16$ | $-0.77$ | - | 98.0 | $+4.3$ | 7.9 |
| 60 | 27.53061 | Mr \& A | -. 4 | n | - | - | - | - | 150.4 | $+4.0$ | $+0.7$ |
| 61 | 27.54977 | Br | 9.- | $n$ | -0 8.02 | +0.15 | $-0.52$ |  | - | - | - |
| 62 | 2758628 | W | 9.5 | 0 | -0 11.01 | +0.12 | +1.18] | - | 48.2 | +4.5 | + 1.9 |
| 63 | 28.35724 | J | 4.4 | 1 | +2 10.88 | $+0.20$ | +0.29 | $+$ | $3 \quad 33.2$ | +4.3 | + 5.2 |
| 64 | 28.61323 | W | 8.6 | 2, p | $+120.21$ | +0.17 | $+0.38$ | $+$ | $+43.4$ | $+4.4$ | -12.3] |
| 65 | 31.59973 | H | -. 5 | I | - | - | - | $+$ | $4 \quad 8.7$ | $+38$ | + 1.6 |
| 66 | 31.61426 | * | 4.- | $q$ | -0 4.22 | $+0.12$ | 0.00 |  | - | - | - |
| 67 | 31.64491 | W | 8.3 | $r$ | -0 1780 | +0.21 | -0.08 | - | 249.4 | $+24$ | $+10.6$ |
|  | 1895 |  |  |  |  |  |  |  |  |  |  |
| 68 | Jan. 18.35415 | J | 7.8 | 6 | $+14.32$ | +0.19 | $-0.13$ | $+$ | - 50 | $+3 \cdot 3$ | $+5.5$ |
| 69 | 19.67310 | H | 1.1 | $s$ | - 0.00 | $+0.21$ | $-0.33$ |  | - 0.0 | $+3.0$ | $+8.7$ |
| 70 | 25.68511 | 11 | $36$ |  |  | $+0.18$ | $+0.07$ | + |  | $+2.6$ |  |
| 71 | 26.67587 |  | 6.6 | " | +o 2.36 | +0.17 | $-0.41$ | - | $2 \begin{array}{lll}2 & 1 & 1.8\end{array}$ | +2.5 | - 3.9 |
| 72 | 27.66756 | * | 5.6 | $7^{\prime \prime} \chi^{\prime}$ | -○ 15.35 | $+0.16$ | $-0.93$ | $+$ | 19.7 | $+2.5$ | $+1.5$ |
| 73 | 28.67994 |  | 4.- | 12 | +0 4.81 | $+0.17$ | +0.61 |  | - | - | $-$ |
| 74 | 2868999 |  | -. 7 | 12 | - | - | - |  | $3 \quad 23.4$ | $+2.5$ | $+0.9$ |
| 75 | 2968100 | * | 4.6 | , | -- 1163 | $+0.17$ | $-0.34$ | $+$ | - 50.8 | $+2.4$ | $-2.0$ |

$4=$ Anderson, Washington; 26 in. Equ.
$3=$ Barnard, Mt. Hamitton; 12 in. Equ. for observations
$1,3,5,11,15 ; 36$ in for 70 to 75 .
Bi $=$ Bigourdan, Paris; Equ. de la tour de l'Ouest.
$\mathrm{Br}=$ Brown, Washington ; 26 in. Equ.
$Z=$ Cerulli, Teramo; 15.5 in. Equ.
$\mathrm{H}=$ Howe, University Park, Col.; 20 in . Equ.
$\mathrm{Hu}=$ Hulbard, Washington; 26 in. Equ.
$\mathrm{J}=\mathrm{Javelle}$, Nice; 0.76 m Equ.
$\mathrm{K}=$ Kobold, Strassburg ; 18 in . Equ.
$\mathrm{P}=$ Palisa, Vienna; 27 in. Equ. der k. k. Sternwarte.
$\mathrm{s}=$ Searle, Washington; 9 in. Equ.
st $=$ Stone, Charlottessille, Va.; 26 in. Equ.
$W=$ Wilson, Northfield, Minn.; 16 in. Equ.

The dates are in Berlin mean time and have been corrected for aberration.
It will be noted that several positions of the comet are referred to two comparison stars. With the following exceptions both stars are in such cases given unit weight:
in observations 25 and 27 ,
in observations 28, 29, and 30
give $i$ weight $\circ$ in $d$
give 32 weight ${ }^{1} 2$ in a

The bracketed residuals have been excluded from the computation.

## 5. Formation of Normal Places.

The horizontal lines in the list of observations show the grouping for the normal places. As a first approximation all the observations were given unit weight and the simple means were taken. The resulting normal place residuals in the sense observation - ephemeris were found to be

|  | Mean Date | $\Delta \alpha$ | Mean Date | $\Delta 3$ |  |
| :---: | ---: | ---: | ---: | ---: | :---: |
| I | Nov. 23.70 | -0.29 | Nov. 23.70 | -2 .". |  |
| II | 30.68 | -0.29 | 30.68 | +0.1 |  |
| III | Dec. 16.21 | 0.00 | Dec. 16.0 .4 | +3.7 |  |
| IV | 20.62 | +0.10 | 20.49 | +3.3 |  |
| V | 27.70 | -0.05 | 27.92 | +2.0 |  |
| VI | Jan. 1901 | -0.23 | Jan. | 19.01 | +7.1 |
| VII | 27.68 | -0.20 |  | 27.68 | -0.3 |

With these residuals as ordinates and the times as abscissae two curves were plotted which were assumed to represent the deviation of the ephemeris in $a$ and $\delta$ from the observed positions. Then in order to gain some idea
of the relative reliability of the different series of observations each observation residual was corrected by the ordinate of the curve corresponding to the instant of observation, thus forming a new series of residuals which were assumed to represent very approximately the actual errors of the individual observations, the errors of the star places being in general so small as to be negligible as compared with the errors of observation. The observations were then grouped in series according to the observer and the weight computed for each series by means of the formula

$$
h=\frac{\varepsilon_{0}^{2}(u-1)}{\left.\left[z^{\prime},\right]\right]}
$$

where ${ }^{\prime}=$ number of observations in each series, $\varepsilon_{0}=$ mean error of an observation of weight unity.

For this calculation assume

$$
\varepsilon_{0}= \pm 0.26 \text { for } a \text { and } \pm 2.7 \text { for } \delta
$$

The following table shows the results of this calculation and also the weights which were finally adopted.

| Observer | " | Comput. Wt. |  | Adopted Wt. |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Barnard, 12 in. Equatorial | $5 \cdot 5$ | - | - | 0.5 | , 0.5 |
| Barnard, 36 in. Equatorial | $5 \cdot 5$ | - | , 0.5 | 1.0 | , 1.0 ${ }^{1}$ ) |
| Bigourdan | $3 \cdot 3$ | - | , - | 0.8 | , 0.8 |
| Brown, Hubb. and And. | 10.9 | 0.6 | , 2.0 | 0.6 | , 1.02) |
| Cerulli | 1.1 | - | , - | 0.2 | , 0.2 |
| Howe | 12.12 | 0.8 | , 0.8 | 0.8 | , $0.8^{3}$ ) |
| Javelle | 13.13 | 1.0 | , 1.0 | 1.0 | , 1.0 |
| Kobold | 2.2 | - | , - | 1.0 | , 1.0 |
| Palisa | $3 \cdot 3$ | - | , - | 1.0 | , I.O |
| Searle | 2.2 | - | , - | 0.5 | , 0.5 |
| Stone | I. I | - | , - | 0.5 | , 0.5 |
| Wilson | 6.5 | 0.2 | , 0.3 | 0.2 | , 0.3 |
| ${ }^{1}$ ) Excepting observations | 72 and | 73, w | ich give | 0.5 | , 1.0 |
| 2) * * | 29, |  | , . | 0.3 | - |
| $\geqslant$ | 60, |  | $\gamma$ P | - | , 0.5 |
| * | 61, |  | * | 0.3 | , |
| 3) | 38 and | 46 | * | 0.5 | , 0.8 |
| - * | 69, |  | $\rangle$ a | 0.5 | , 0.5 |

In this connection the residuals in right ascension of the five observations Nos. 1, 3, 5, 11, and 15 made by Barnard with the 12 in . equatorial at Mt. Hamilton require special attention. It will be noted that in the first normal place these are the only observations giving rise to positive residuals in $a$. The same is true of the second normal place with the exception of observation No. 20 which gives a small positive residual. After applying to the residuals of the Barnard observations the ordinates of the normal place curve in the manner above explained the numbers representing the approximate errors of observation were

$$
+0.22+0.77 \quad+0.55 \quad+0.53+0.48
$$

The prevalence of positive errors of roughly the same order of magnitude would indicate the presence of some systematic difference in these observations as compared with those of the other observers; but the other observations
entering into these two normal places were made by several different observers, and that a systematic error should exist in all these observations is out of the question. Upon request Professor Barnard kindly communicated the original data for his observations and they were rereduced, but without the discovery of any error in the published values. As to the possibility of his having made settings upon a different point from the other observers Prof. Barnard writes:
"The comet was a faint object, and it is perhaps possible to have observed a different point from what others observed. My recollection is that the comet had a faint tail and a faint nucleus, consequently, unless it was well seen - because of its elongated character - one might not observe the precise center of the head, but from the fact that it was very small he could not be far out in bis settings. «

Although no source could be found for the systematic difference the residuals were arbitrarily corrected by $0: 51$ which is the mean of the five quantities given above, and he resulting residuals were given the weight 0.5 . hesula wo

|  | Mean bate |  | $\Delta x$ | Red. | Mean liate |  | $\Delta \geqslant$ | Ked. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I | Nov. | 23.70 | -0.516 | -0:004 | Nov. | 23.70 | -2.26 | -0.08 |
| II |  | 30.68 | $-0.374$ | $-0.004$ |  | 30.68 | +0.37 | -0.06 |
| III | Dec. | 16.21 | -0.036 | +0.005 | Dec. | 16.04 | $+3.68$ | $+0.02$ |
| IV |  | 20.62 | $+0.022$ | -0.001 |  | 20.49 | +3.73 | +0.01 |
| V |  | 27.70 | $-0.001$ | $+0.001$ |  | 27.92 | $+2.11$ | $+0.05$ |
| V1 | Jan. | 19.01 | $-0.197$ | $-0.003$ | Jan. | 1901 | $+6.54$ | 0.00 |
| VII |  | 27.68 | $-0.210$ | +0.001 |  | 27.68 | $-0.48$ | $+0.01$ |

The columns headed Red. give the values for reducing the $\Delta \in$ and $I d$ from the mean date of olsservation to he nearest Berlin mean midnight. The final residuals for the normal places are therefore:


The weights for the normal places are the sums of the veights of the individual observations in the normat place ;roups.

## 6. Computation of Perturbations.

The effect of Jupiter, Saturn, Mars, and the Farth apon the positions of the comet during the period of siibility were computed by the method of variation of contants. The masses used were:


From these the values for the dates of the normal blaces were found by interpolation - the calculated values reing checked by a graphical interpolation from the curves ormed by ploting the perturbations in the elements. The fuantities desired, however, are the effects of the perturrations in $a$ and $\delta$ and these were derived from the perurbations in the elements by means of the differential ormulae given in the following section for determining the lefinitive corrections to the elements. These formulae are he ones given by Schonfeld in A. N. No. 2692.93 and since hey involve the three elements $x, \lambda$ and $r$ in place of the

The calculation was based upon Chandler's elements A. J. No. 338, referred to 1900.0 . They are:

Epoch Dec. 1.0 1894 . Equinox 1900.0 .

| ./1 |  | $22^{\prime}$ | $1{ }^{\prime \prime} 2$ |
| :---: | :---: | :---: | :---: |
| . 7 | 345 | 24 | 13.8 |
| . | 48 | 48 | 52.9 |
| 7 | 2 | 57 | 555 |
| \% |  | 52 | 56.9 |
| , | $605^{\prime \prime}$ | 5 |  |

 intervals beginning with 1894 Now. 1 . The resulting perturbations in the elements were:
usual three $i, \lambda$ and $\omega$, the perturbations in the latter elements must be transformed into perturbations in $\%, \lambda$ and $r$ by means of

$$
\begin{aligned}
& \mathrm{d} x=\mathrm{d} \omega+\cos i \mathrm{~d} \\
& \mathrm{~d} \lambda=\sin \theta \mathrm{d} i-\cos \omega \sin i \mathrm{~d} \\
& \mathrm{~d} \gamma=\cos \omega \mathrm{d} i-\sin \omega \sin : \mathrm{d}
\end{aligned}
$$

This transformation toget eer with the above mentioned interpolation gives the following perturbations in the elements for the dates of the normal places.

With the application of these corrections and the new system of weights, the residuals were again combined to form normal place residuals with the following results:

|  |  | $\Delta x$ |  | $\Delta M_{0}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | - | 3."002 |  | 1.035 |
|  | 30.5 | - | 1.888 |  | 0.731 |
| Dec | 16.5 |  | 1.115 | - | 0.558 |
|  | 20.5 |  | 1.970 | - | 1.028 |
|  | 27.5 |  | 3.600 | - | 2.057 |
| 1895 Jan | 19.5 |  | 0.267 |  | 7.624 |
|  | 27.5 |  | 3128 |  | 10.546 |

These quantities were then substituted into the differential formulae whose coefficients are given below and the corresponding perturbations in $\alpha$ and $\delta$ were found to be:

Observation - Undisturbed Position.

|  | Date$189+$ Nov. |  | $\Delta a \cos \delta$ | $\Delta \delta$ |
| :---: | :---: | :---: | :---: | :---: |
| I |  | 23.5 | - 0.347 | -0.147 |
| II |  | 30.5 | $-0.017$ | -0.008 |
| III | Dec. | 16.5 | - 0.266 | $-0.128$ |
| IV |  | 20.5 | - 0583 | $-0.281$ |
| V |  | 27.5 | - 1439 | $-0.701$ |
| VI | 1895 Jan. | 19.5 | - 7.009 | -3340 |
| VII |  | 27.5 | - 10.000 | $-4.668$ |

Applying these perturbations with the reversed sign to the normal place residuals, after the right ascensions of the latter have been multijlied by the cosines of the declinations we derive the residuals Undisturbed Position minus Ephemeris. These are the absolute terms of the equations of condition used in determining the definitive osculating elements.

Undisturbed Position-Ephemeris.

| Date |  |  |  | $\Delta c \cos \delta$ |
| :--- | ---: | ---: | ---: | :---: |
| I | 1894 Nov. | 23.5 | -7.28 | -2.109 |
| II |  | 30.5 | -5.57 | +0.32 |
| III | Dec. | 16.5 | -0.20 | +38.3 |
| IV |  | 20.5 | +0.90 | +4.02 |
| V |  | 27.5 | +1.44 | +2.86 |
| VI | 1895 Jan. | 19.5 | +4.03 | +9.88 |
| VII |  | 27.5 | +6.90 | +4.20 |


| $\Delta \mu$ | $\Delta y$ | $\Delta \lambda$ | $\Delta V$ |
| :---: | :---: | :---: | :---: |
| +0.0090 | -0.741 | -0.013 | +0.017 |
| +0.0064 | -0.544 | -0.009 | +0.011 |
| -0.0049 | +0.415 | +0.008 | -0.008 |
| -0.0086 | +0.761 | +0.015 | -0.014 |
| -0.165 | +1.484 | +0.029 | -0.026 |
| -0.0531 | +4.874 | +0.109 | -0.076 |
| -0.0706 | +6.415 | +0.152 | -0.095 |

The residual in $\delta$ for the normal place of Jan. 19.5 appears to be discordant when compared with those of the other normal places. That this is actually the case becomes more certain when it is noted that all of the normal places except this one depend upon from 5 to 13 observation while this is based upon only 2, Nos. 68 and 69, and the latter of these depends upon an assumed coincidence betweer comet and comparison star. It was suspected that it woulc be impossible to pass through the normals an orbit whick would give a good representation for the declination of this date, and a preliminary solution proved this to be the case Although the errors of the positions forming this norma are not larger than those occurring in a number of othel observations they are of the same sign, thus preventing com pensation. A consideration of all the data led me to be lieve that the retention of these observations as a separate normal place would add nothing to the accuracy of the results. Nor did it seem advisable to combine them wit\} the normals of Dec. 27.5 or Jan. 27.5 on account of the magnitude of the intervening intervals. The declination: were therefore excluded from the calculation while the righ ascensions, not presenting any special discordance, wert retained and given a small weight.

## 7. Differential Formulae and Least Square Solution for Definitive Elements.

Transforming the ephemeris positions of the come for the dates of the normals to the equinox of 1900.0 , whict has been choosen for the calculation, they become:

| $\delta$ | $\log d$ |
| :---: | :---: |
| $-12^{\circ} 10^{\prime} 55^{\prime \prime} 96$ | 0021015 |
| $9+6 \quad 11.10$ | 0.045424 |
| - 41647.57 | 0.103769 |
| 2563025 | 0.118683 |
| - 0397.03 | -144884 |
| +61548.73 | 0.229572 |
| + 8251.24 | 0.257886 |

by assigning arbitrary variations to the elements and deter mining the resulting changes in $a$ and $d$ both by the differ ential formulae and by the ordinary ephemeris formulae.

The equations of condition thus derived are:

These coordinates together with Chandler's elements referred to the equinox of 1900.0 form the basis for the calculation of the differenti.n formulae, which, as has already been stated, was carried ut according to the method of Schönfeld. The computation of these coefficients was checked
$\log 1$
0.3854
0.5167
0.4286
0.4722
0.3702


The coefficients are logarithmic and the last column contains the logarithm of the square root of the weights of the equations of condition. Applying these weights and introducing new unknowns defined by the relations

$$
\begin{array}{ll}
x=[0.4900] \mathrm{d} x & t=[0.5200] \mathrm{d} r \\
y=[1.1600] \mathrm{d} H_{0} & \|=[0.5200] \mathrm{d} \lambda  \tag{A}\\
z=[2.8000] \mathrm{d} \mu & \pi^{\prime}=[0.4200] \mathrm{d} \mu
\end{array}
$$

and further choosing 1.2700 as the logarithm of the unit of error, there resulted the following weighted homogeneous :quations of condition (logarithmic coefficients):


The usual least square method gave as normal equations (numerical coefficients):
$1)+4.2893 x+4.0257 y-2.7810+3.8095 t+0.1734 u+02153 \pi^{1}+0.2722=0$
$2+4.0257+3.8040-2.7452+3.4000+0.1372+0.1669+0.5226=0$
$3)+2.7810-2.7452+2.5497-1.4801+0.0256+0.0375-1.6937=0$
$4)+3.8095+3.4000-1.4801+4.7055+0.3146+0.3909-1.7799=0$
$5)+0.1734+0.1372+0.0256+0.31 .46+3.5163+3.8867-1.6398=0$
$6)+0.2153+0.1669+0.0375+0.3909+3.8867+4.8210-2.0085=0$

The similarity of the coefficients of the first and second ind of the fifth and sixth equations indicated that one or nore of the unknowns would be affected with considerable
be indeterminate. Rewriting the normals so that these unknowns appeared last in the solution the following elimination equations were found (logarithmic coefficients): incertainty, and a preliminary solution showed $x$ and $y$ to

1) $0.40649:+0.17030_{1} t+8.40909 u+8.57461 z+0.44420_{n} x+0.43858 \mathrm{n} y+0.22884 \mathrm{n}=0$
2) 
3) 
4) 

$$
\begin{aligned}
0.58504+951786+9.61563 & +0.34145+0.25682+0.44140_{\mathrm{n}}
\end{aligned}=0
$$

By successive substitution $\pi^{\prime}, u, t$ and : were expressed as functions of $x$ and $y$ through (log. coefficients):

$$
\begin{align*}
& z^{\prime \prime}=8.05865 \mathrm{n} x+7.66967 \mathrm{n}+9+47543 \\
& \prime \prime=7.9444^{8}+7.35411+8.82905 \\
& t=9.75604_{\mathrm{n}}+9.67150_{\mathrm{n}}+9.83286  \tag{B}\\
& z=9.88069+9.90540+0.02295
\end{align*}
$$

and substituting these into the original homogeneous weighted equations of condition the folowing series was found for he determination of $x$ and $y$ (logarithmic coefficients) :

| 1) | $8.3414 x$ | $+7.5119 y$ | $+8.5587 \mathrm{n}$ | - 0 |
| :---: | :---: | :---: | :---: | :---: |
| 2) | 7.5563 | + 7.1139 | $+8.8817$ | - |
| 3) | $8.2455 n$ | $+7.4771{ }^{1}$ | $+8.6702_{n}$ | - 0 |
| 4) | $8.2900{ }_{n}$ | + $7.55{ }^{6} 3 \mathrm{n}$ | $+8.862 \mathrm{In}_{\mathrm{n}}$ | - |
| 5) | 8.0755 u | $+7.3802 \mathrm{n}$ | +7.7993 | - |
| 6) | 8.0792 | +7.3222 | +90328 | - |
| 7) | 8.5198 | +7.7559 | $+8.501 \mathrm{~m}_{\mathrm{n}}$ | - |
| 8) | 8.0043 | + 7.1139 | $+92750$ | - |
| 9) | 7.3424 | $+6.6990$ | $+7.7672$ | 0 |
| 10) | 7.9777 n | $+7.1614 \mathrm{n}$ | +9.3228 | - |
| 11) | 7.9395n | $+7.1761_{11}$ | +9.1638 | - |
| 12) | $7.681 \mathrm{z}_{11}$ | +6.9777n | +9.1126 | - |
| $13)$ | 8.2553 | + 7.4914 | +9.2653 | - |
| Check | [ $n n \cdot 4$ ] | $=0.1829$ | $\left[n^{\prime} n^{\prime}\right]=$ | 1829 |

New unknowns defined by

$$
\begin{equation*}
x^{\prime}=[8.5200] x \quad y^{\prime}=[7.7600] y^{\prime} \tag{C}
\end{equation*}
$$

were introduced to secure homogeneity and the resulting series was solved by least squares. The normal equations for $x^{\prime}$ and $y^{\prime}$ were (numerical coefficients):

$$
\begin{aligned}
& +2.9052 x^{\prime}+2.84901^{\prime}+0.2745=0 \\
& +2.8490+2.8292+0.2601=0
\end{aligned}
$$

Here again the similarity in coefficients denoted uncertainty in the solution, but as $y^{\prime}$ appeared to be the more uncertain of the two, $x^{\prime}$ was expressed in terms of $y^{\prime \prime}$ giving (logarithmic coefficients) :

$$
\begin{equation*}
x^{\prime}=9.99^{152 n} y^{\prime}+8.97544 n \tag{D}
\end{equation*}
$$

This value for $x^{\prime}$ substituted into the equations of cundition for $x^{\prime}$ and $y^{\prime \prime}$ gave the following series for the determination of $y^{\prime}$ (numerical coefficients):

| 1) | $-0.0851$ | - 0.0988 | = |
| :---: | :---: | :---: | :---: |
| 2) | +0.1193 | +0.0659 | = |
| 3) | -0.0001 | $-0.0034$ | - |
| 4) | -0.0482 | $-0.0172$ | = |
| 5) | -0.0647 | +00970 | $=$ |
| 6) | $+0.0095$ | $+0.0736$ | = |
| 7) | $+0.0104$ | -0.1261 | = |
| 8) | $-0.0732$ | +0.1595 | = |
| 9) | $+0.0217$ | -0.0004 | = |
| 10) | $+0.0293$ | $-0.1832$ |  |
| 11) | $-0.0031$ | $-0.1210$ |  |
| 12) | $-0.0230$ | +0.1433 |  |
| $13)$ | $+0.005^{6}$ | +0.1328 | = |

Check $[n n \cdot 5]=0.1570 \quad\left[n^{\prime \prime} n^{\prime \prime}\right]=0.1569$
A new unknown $y^{\prime \prime}$ was introduced such that

$$
10 y^{\prime \prime}=y^{\prime}
$$

and the series solved for ..$^{\prime \prime \prime}$ giving
$\log _{1^{\prime \prime}}{ }^{\prime \prime}=8.40572$
whence

$$
\log y^{\prime}=9.40572
$$

The residuals for the normal places were found by substituting $y^{\prime}$ into the above equations of condition. When squared and added
while from the elimination, as a check,

$$
[n n \cdot 6]=0.1546
$$

Then by successive substitution of $y^{\prime}$ into (D), of $x^{\prime}$ and $y^{\prime}$ into (C), and finally of $x$ and $y$ into (B) the most probable values of the unknowns were found to be

$$
\begin{array}{ll}
\log x=1.0167 n & \log t=1.1509 n \\
\log y=1.6457 & \log u=8.8806 \\
\log z=1.4584 & \log w=9.3244
\end{array}
$$

Restoring the original unknowns by (A) and reintroducing the second of arc as the unit of measurement the following corrections to the elements chosen for the calculation were obtained.

$$
\begin{array}{ll}
\log \mathrm{d} \chi=1.7967 \mathrm{n} & \log \mathrm{~d} \varphi=1.9009 \mathrm{n} \\
\log \mathrm{~d} M_{0}=1.7557 & \\
\log \mathrm{~d} \lambda=9.6306 \\
\log \mathrm{~d} \mu=9.9284 & \\
\log \mathrm{~d} \nu=0.1744
\end{array}
$$

The corrections to $i, \omega$ and $\delta$ were derived from $\mathrm{d} x, \mathrm{~d} \lambda$ and $\mathrm{d} \nu$ by

$$
\begin{aligned}
\mathrm{d} i & =\cos \omega \mathrm{d} v+\sin \omega \mathrm{d} \lambda \\
\sin i \mathrm{~d} z & =\sin \omega \mathrm{d} v-\cos \omega \mathrm{d} \lambda \\
\mathrm{~d}(\Omega+\omega) & =\mathrm{d} x+\operatorname{tg} 1 / 2 i \sin i \mathrm{~d} \Omega \\
\mathrm{~d}(\Omega-\omega) & =-\mathrm{d} x+\operatorname{ctg}{ }^{1 / 2} i \sin i \mathrm{~d} \Omega \sigma
\end{aligned}
$$

As thus determined the final corrections to Chandler's elements are

$$
\begin{array}{ll}
\mathrm{d} \mu_{0}=+57.0 & \mathrm{~d} \delta z=-29.5 \\
\mathrm{~d} u=+0.8479 & \mathrm{~d} \pi=-62.7 \\
\mathrm{~d} \varphi=-79.6 & \mathrm{~d} \omega=-33^{\prime \prime} .2 \\
\mathrm{~d} i=+0.3 & \mathrm{~d} L=-5.7
\end{array}
$$

whence the definitive osculating elements:
Epoch 1894 Dec. 1.o. Osculation 1894 Dec. 10.0 .

$$
\begin{aligned}
& M_{0}=8^{\circ} 22^{\prime} 58.12 \pm 4^{\prime \prime 2} \\
& \pi=345^{2311.1} \pm 4.4 \\
& \delta 8=\begin{array}{c}
48 \\
48 \\
\hline
\end{array} \frac{23.4 \pm 27.7}{} 1900.0 \\
& i=257558 \pm 1.4 \\
& \boldsymbol{f}=345 \mathrm{I} 37.3 \pm 7.1 \\
& \mu=605.9999 \pm 0.0665
\end{aligned}
$$

The appended quantities are the mean errors anc are based upon the standard value for a single observation of unit weight

$$
\varepsilon_{0}= \pm \mathbf{2 !} 8
$$

computed from the residuals of the equations of condition
To test the accuracy of the least square solution thi definitive corrections were sulstituted into the origina equations of condition; the resulting residuals were squared multiplied by the proper weight, and added with the resul

$$
[\gamma, \gamma]=54 \%
$$

The value of $[n, 6]$ from the least square solution was 0.1546 . Expressed in seconds of are

$$
[n n \cdot 6]=53: 6
$$

The agreement is satisfactory in view of the fact that only four places of decimals have been used in the solution. The reduction in the sum of the squares of the weighted esiduals is from

$$
13499^{\prime \prime} 3 \text { to } 54^{\prime \prime \prime}
$$

Finally the definitive elements were used to compute
the undisturbed positions of the comet for the dates of the normal places. To these the perturbations were applied and the results compared with the observed positions. The outstanding differences, in the sense obs. - comput., resulting from a six place calculation are tabulated below, together with the residuals obtained by direct substitution of the definitive corrections to the elements into the equations of condition.

| Date |  |  | $\Delta \alpha \cos \theta$ |  | $\Delta 3$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Def. Elem. | Diff. Form | Def. Elem. | Dill. Form. |
| 1894 | Nov. | 23.5 | +0.09 | +0.07 | -0."9 | -1. I |
|  |  | 30.5 | -0.04 | $-0.03$ | 0.0 | 0.0 |
|  | Dec. | 16.5 | $+0.01$ | 0.00 | $+1.2$ | $+1.0$ |
|  |  | 20.5 | 0.00 | $+0.01$ | $+0.8$ | $+0.8$ |
|  |  | 27.5 | $-0.05$ | $-0.04$ | -1.2 | -1 1 |
| 1895 | Jan. | 19.5 | $-0.03$ | $-0.08$ | - | - |
|  |  | 27.5 | $+0.07$ | $+0.07$ | -1.3 | -1.1 |

In order to determine the effect of small variations in $\mathrm{d} \mu$ upon the sum of the squares of the weighted residuals the values of the increments to the other elements were substituted into the weighted observation equations and the numerical terms were summed. The resulting equations of condition for $\mathrm{d} \mu$ were found to be:

| ) | $2.7522_{\mathrm{n}} \mathrm{d} \mu+\mathrm{r} .4085=$ |  |  |
| :---: | :---: | :---: | :---: |
| 2) | 2.7929 n | +1.4527 |  |
| 3) | $2.4405 n$ | + 1.0990 |  |
| 4) | 2.3979 n | +1.0549 |  |
| 5) | 2.1099 n | +0.774 |  |
| 6) | 1.4056 | $+0.034$ |  |
| 7) | 1.9086 | +0.581 |  |
| 8) | $2.3516_{11}$ | +1.0159 |  |
| 9) | 2.4048 n | +1.0634 |  |
| 10) | 2.1304 n | $+0.7761$ |  |
| 11) | 1.9944n | +0.6405 |  |
| $12)$ | r.6494n | $+0.3368$ |  |
| 3) | -. 7506 | +0.3860 |  |

in which the coefficients are logarithmic, and the logarithm of the unit of measurement is 1.2700 .

The definitive value for $\log \mathrm{d} \mu$ was found by the least square solution to be 9.928 . The variations

$$
+0.0100+0.0020-0.0020-0.0100
$$

were successively applied to this logarithm and the resulting values were substituted in the above series of equations. The residuals found by each substitution were squared and added. The following table exhibits the relation between the sums and the variations assumed for $\mathrm{d} \mu$.

| $\log \mathrm{d} \mu$ | $\Delta \mathrm{d} \mu$ |  |
| :---: | :---: | :---: |
| 9.938. | +0.0199 | $455^{\prime \prime}$ |
| 9.9304 | +0.0041 | 75 |
| 99284 | 0.0000 | 54 |
| 9.9264 | -0.0037 | 68 |
| 9.9184 | -0.0192 | 427 |

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Friderik H. Siares.

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\cos ^{1}
$$

 $c$ $\square$

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A

# $\cdots$ 

