Faculty Working Papers

DYNAMIC THEORY OF FISHERIES ECONOMICS - 1*

--OPTIMAL CONTROL THEORETIC APPROACH--

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College of Commerce and Business Administration
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Abstract

The problem the fisheries economists face is that of determining and recommending the fishing intensity that will maximize the economic value to the consuming societies and also maximize the producers' surplus at a level of production in perpetuity.

In this paper the optimal control theoretic approach is employed to theoretically answer this problem within the framework of a one country-one fish species economy.

In order to derive quantitative as well as qualitative results, a quadratic objective function - linear population dynamics model is solved for the optimally controlled catch over time. A general conclusion is that if the initial fish population is smaller than the desired or target population to be determined in the text, the fishing intensity must be curtailed to such a degree that fish population can grow so that in the long run the target catch can be attained at the level of steady population(which will be sustained in perpetuity). A governmental regulation over the total catch ts supported in this case since the social value in the market of a unit catch always exceeds the individual marginal cost of catching the same. Mesh size regulations are also supported due mainly to the fact that the information on the optimal mesh size is external to individual fisherman.

In Appendix detailed derivation procedures of the optimal control are developed. Also a model with quadratic objective function and quadratic population dynamics is traced out for optimal control paths by using a phase diagram.

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September 9, 1977

Dynamic Theory of Fisheries Economics - I*

--Optimal Control Theoretic Approach--

T. Takayama**

Introduction

Fish have been a major source of protein in the diet of a large portion of the world population, and the demand for it has been steadily increasing, while the fish stocks are finite. Contrary to nonrenewable sources such as petroleum and other mineral resources, fish stocks belong to a class of resources for direct human consumption that are renewable. The process and speed of renewal are different among different species. Thus, in order to maintain a steady fish population, the proportion of the catch of one species to the total population of the same species can be quite different among species. Also, interaction among various fish species can be quite an important factor governing the growth of the fish population.

However interesting the population dynamics of fish population may be, fisheries economists should not recognize it as the sole objective of study. The problem the fisheries economists face is that of determining and recommending the fishing intensity that will maximize the economic value to the consuming societies and also maximize the producers' profit or surplus at a level of production in perpetuity.

In this paper, we first discuss some general properties of fish population dynamics in relation to fishing intensity. Then we use this

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as the basic dynamic process the society observes in maximizing an objective function this society reveals in relation with fish consumption and production (catching).

Since we conceive that the fish population follows a typical dynamics, the objective function ("performance function" in optimal control terminology) takes a form of an integration of an instantaneous objective value over time. Therefore, a typical optimal control formulation of fisheries economies results.

After a general formulation of the fisheries economics problem, when the number of species and nations are restricted to only one, we solve a problem of a quadratic (objective functional) — linear (population-catch dynamics) optimal catch control problem. The solution reveals that there exist two solutions, one stable convergent optimal path and one other unstable path. A general conclusion is that if the initial fish population is smaller than the desired or target population to be determined later in the text, the fishing intensity must be curtailed to such a degree that fish population can grow (optimally) so that in the long run the target catch can be attained at the level of steady population (which will be sustained in perpetuity).

Even though there is no problem when the initial fish population is larger than the target population, we will discuss some interesting aspects that may lead us to another exciting topic, that is, "game theoretic formulation" of fisheries economics in which multiple nations (societies) and multiple fish species are involved.

In conclusion, we will investigate some possible implications of our solutions and discuss some future topics of great interest.

1. A Short Review and Dynamics of Fish Population

In his 1964 article [9] in the American Economic Review, Turvey developed a static steady state bionomic equilibrium model to determine optimum weight of catch and fishing effort, and asserted "the fundamental principle that either mesh regulation or the control of fishing effort is better than nothing but that regulation of both is still better" (pp. 73-74).

Boyd in his 1966 article in the same journal [4], following Turvey, developed a comprehensive static model including both the market demand function and the total cost function as a function of total fish catch, and showed how to solve the system for an equilibrium catch.

However, in these models no special effort was made to (1) formulating their problem in its most natural dynamic optimization framework with either finite or infinite time horizon, (2) introducing multi-species in fish population dynamics and/or (3) in market demand and/or supply functions, and finally, (4) introducing multiple nations with either conflicting interests or in collusive or cooperative relationships.

In this paper I plan to pave the way to cover the first three points mentioned above.

Later, in 1967, a simulation model to determine equilibrium paths of fish populations, consumption (= catch), and prices for two fish species was developed by Lampe [7]. The multiple species-one country optimal control modeling results will be reported in the next paper. Another paper will show how to cover all four points above by utilizing "differential game theoretic approach" [8].

In this paper, almost all of the concepts used by Turvey and Boyd (in the works referred to above) will be redefined in a new dynamic environment. "Mesh size" will not be considered as a control variable in the model that follows, but "catch" will be. Later, I will come back to this point and discuss policy recommendations on the mesh size and the catch in a new light.

In optimal control theoretic jargon, fish population can be expressed as "state variable." The fish population dynamics can be written in the following general functional relationship:[1]

(1) $$\dot{p} = f\,(p, x, t), \; t \in [0, T)$$

where p is fish population

x is fishing intensity or catch, control variable,

t is time,

T is the final time at which our plan ceases to become effective,

and

$\dot{p}$ is the time derivative of the fish population.

If we assume the population dynamics (1) is time independent, then we may be able to write it simply as

(2) $$\dot{p} = f\,(p, x), \; t \in [0, T) \,.$$

When we set $\dot{p} = 0$, we can draw the steady state locus (relationship) between p and x as

(3) $$f\,(p, x) = 0, \; t \in [0, T) \,.$$

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[1]For the standard work on the dynamics of fish populations, the reader is referred to [3].

One conceivable dynamic relationship is shown in Figure 1 below, which can be written as

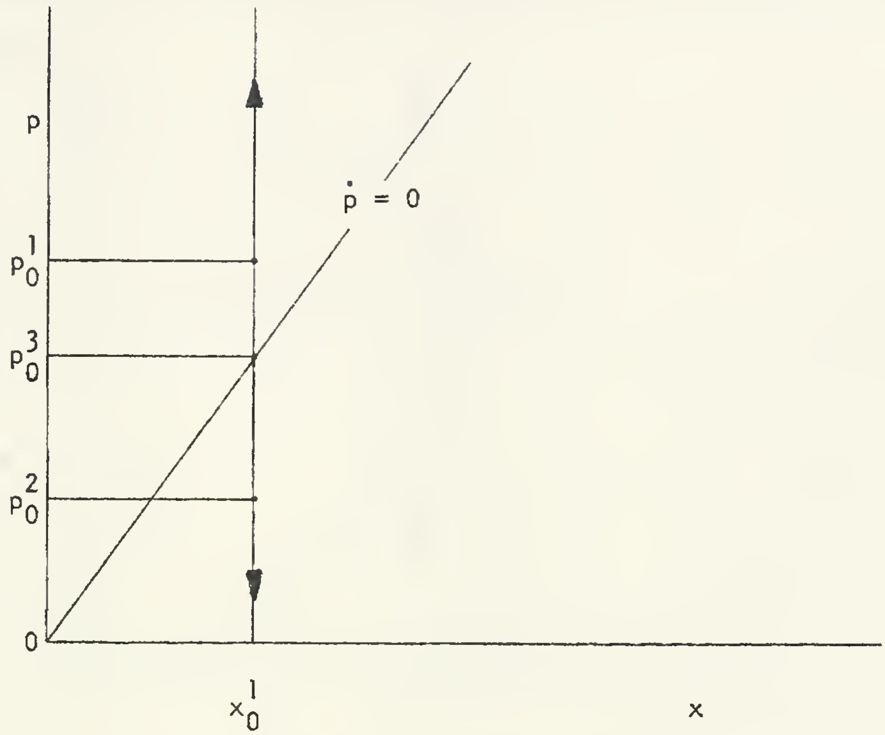

Figure 1. Linear Relationship

$$(4) \qquad \dot{p} = a + b\,p - x,\; t \in [0, T)$$

($a = 0$ and $b > 0$ in the case of Figure 1).

Equation (4) exhibits the following fish population dynamics; if the catch is maintained at x_0^1 over time and the initial population of the fish species is at p_0^1, then $\dot{p}$ is positive and the population will steadily increase. However, if the catch level is at x_0^1 and the initial population is at p_0^2, then the fish population will steadily decrease. The only initial population that would not be affected by this catch level is p_0^3. At this level the new recruitment volume bp_0^3 is exactly equal to fishing intensity x_0^1, and the sustained steady state population p_0^3 is attained.

Another conceivable relationship between p and x of (3) is shown below (Figure 2). In this case, the absolute maximum population is reached at $\bar{p}$.

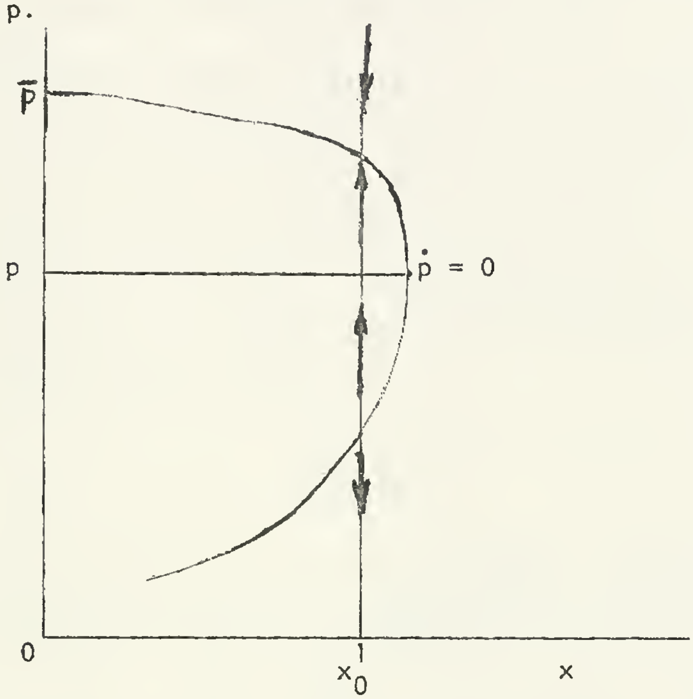

Figure 2. Nonlinear Relationships

This nonlinear case, Figure 1, differs substantially from the linear case. After the population reaches $\bar{p}$, the catch has to be curtailed to make the population grow (Boyd, p. 515). This point will be clarified in a later paper.

The population dynamics, no matter how detailed their construct may be, lacks economic consideration.

In the next section we will introduce economic concepts considered relevant to our fisheries economics.

2. Dynamic Fisheries Optimal Control Model: Formulation in Competitive Market

As was stated at the beginning of this paper, the problem at hand is to determine the fishing intensity, x, that will maximize the economic value to the consuming society and also maximize the producers' profit or surplus at the level of production in perpetuity.[2]

The economic value to the consuming society can be expressed in terms of a welfare gain or an integral under the market demand function. If we write the market demand function as

$$(5) \qquad P_d = D\ (x,\ t),\ t \ \varepsilon\ [0,\ T)$$

where P denotes market price of per unit weight of the fish, the consumer benefit function over time can be written as

$$(6) \qquad G\ (x,\ t) = \int_0^T e^{-rt}\left[\int_0^x D\ (x,\ t)\ dx\right] dt$$

where r is the discount rate of the future in reference to the present.

The industry supply function can be written as

$$(7) \qquad P_s = S\ (p,\ x,\ t),\ t \ \varepsilon\ [0,\ T)$$

and is the horizontal summation of individual marginal cost curves. The total cost function of catching x units of the fish at time t, TC (x, t), can be then expressed as

$$(8) \qquad TC\ (p,\ x,\ t) = \int_0^T e^{-rt}\left[\int_0^x S(p,\ x,\ t)\ dx\right] dt$$

---

[2] V. L. Smith [9], in "Economics of Production from Natural Resources," Am. Econ. Rev., June 1968, 58, developed models similar to this. My views about his approach is partially discussed in the Appendix.

If we also assume that the concept of the "consumers' and producers' surplus" is an operationally valid concept, one can write the value as

$$\text{(9)} \qquad \text{Social Pay-Off } (p, x, t) = \text{SPO } (p, x, t) = G(x, t) - TC\ (p, x, t),\ t \in [0, T)\ .$$

Now the fishery optimal control (FOC) model can be defined as

<u>FOC Model</u>: Find the optimal control $\tilde{x}$ (P_0, t), $t \in [0, T)$, that maximizes SPO (p, x, t) subject to the population dynamics (1) or (2).

The necessary conditions for the optimality of $\tilde{p}$, $\tilde{x}$ may be given by

$$\text{(10)} \qquad \begin{cases} (1)\ \dot{p} = f\ (p, x, t),\ p(0) = p_0 \\ (2)\ \dot{\lambda} = (r - \frac{df}{dp})\ \lambda - \frac{dSPO}{dp} \\ (3)\ \frac{dSPO}{dx} - \frac{df}{dx} = 0 \\ (4)\ \lambda(T) = 0,\ T \text{ finite, or } \lim\limits_{t \to \infty} e^{-rt}\lambda(t) = 0 \end{cases}$$

and the solutions, if they exist, may exhibit stability, instability aspects of the solution paths, as well as comparative dynamics of changing the discount rate, r, and other parameters in (1) or (10) and (9).

The existence of an optimal control (catch) for this problem depends on the following conditions:[3]

The SPO (p, x) must be strictly concave with respect to x (in $R^1_+ = \{\xi | \xi \geq 0\}$) and f (p, x) in (1) must be a concave function of x for each p, etc. (for detailed arguments, see [1]).

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[3]We restrict our argument here to time invariant objective function (8) and time invariant population dynamics (1).

The usual phase diagram analyses and related interpretations will be given later in this paper in specific reference to the quadratic linear case.[4]

In the objective function (9) no condition on the terminal state was specified. Naturally, when the time horizon is finite, policy-decision makers would like to have some fish population target level specified. This can be done easily and will be discussed later.

3. Optimal Catch Control in Competitive Market: Quadratic Benefit and Linear Dynamics Case

In this section we analyze a special but most operational case in which the social pay-off or benefit function, (9), is quadratic and the fish population dynamics is linear, (4).

In constructing the social pay-off function, we make full use of the Marshallian market demand function for the fish species. We also restrict ourselves to the case in which the market demand function is a linear function such as

$$(11) \qquad Pd = \alpha - \beta x \quad ^{5}$$

where Pd denotes the market demand price per unit weight of the fish: α, β are positive constant parameters. The consumers' benefit can be expressed as the integral of (11) over x, that is,

$$(12) \qquad G(x) = \alpha\, x - \frac{1}{2}\beta x^2 .$$

The cost of catching x units of fish is defined as

$$(13) \qquad TC(x) = \mu x + \frac{1}{2}\theta x^2$$

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[4] Interested readers can refer to [1, 8] for further details.

[5] The dynamic identification, t, of x(t), p(t), etc., will be omitted, unless otherwise stated, in the following development.

(14) where μ is a finite constant obeying the restriction that $\alpha - \mu > 0$ and θ is a positive constant or of magnitude $\beta + \theta > 0$.

Therefore, the social benefit function can be written as

$$\text{(15)} \qquad SPO(x) = \int_0^T e^{-rt}[(\alpha - \mu)x - \frac{1}{2}(\beta + \theta)\, x^2]\, dt$$

for $t \in [0, T)$ and $T > 0$.

The social benefit function defined in (15) can be modified to include the penalty of not reaching a certain target level of fish population in a finite time horizon $T \ll \infty$. However, for the time being, we will ignore it and, instead, analyze the properties of solution paths of the optimal control model when the time horizon extends toward infinity. The finite horizon case will be discussed in the Appendix.

The fishery optimal control problem can now be defined as:

<u>Problem 1</u>: Find the path(s) of optimally controlled intensity of catch, $x^*(p(t), t)$ (as a function of $p(t)$ (and t)), that maximizes (15) subject to (4).

Our task is to find the optimal feedback control

$$\text{(16)} \qquad x^*(p(t), t) = \phi(p(t), t)$$

which observes the dynamic but continuous change in the fish population in the ocean and adjusts the catch to attain the maximum consumer's and producers' benefit in the long run.

The necessary conditions for competitive solutions are

$$(17)\quad \begin{cases} \dot{p} = a + bp - x,\ p(0) = p_0 \\ \dot{\lambda} = (r - b)\lambda \\ \lim_{t\to\infty} e^{-rt}\lambda(t) = 0 \\ (\alpha - \mu) - (\beta + \theta)x - \lambda = 0 \end{cases}$$

There exist two solutions for (17): (i) singular and unstable solution, and (ii) stable solution converging to a finite p^* and $x^*(p^*)$, provided that (14) and

$$(18)\qquad \frac{\alpha - \mu}{\beta + \theta} - a > 0$$

hold.[6]

The singular optimal control is

$$(19)\qquad x_s^* = \frac{\alpha - \mu}{\beta + \theta}$$

which is independent of $p(t)$ or t. Obviously this is unstable unless the fish population is at

$$(20)\qquad p_s^* = \frac{1}{b}\left(\frac{\alpha - \mu}{\beta + \theta}\right).$$

The stable optimally controlled (closed-loop) catch is given by

$$(21)\qquad x^*(p(t)) = \frac{\alpha - \mu}{\beta + \theta} - \frac{(2b - r)}{b}\left(\frac{\alpha - \mu}{\beta + \theta} - a\right) + (2b - r)p(t),$$

Employing this feedback optimal control $x^*(p(t))$, the fish population dynamics (4) turn out to be

$$(22)\qquad \dot{p} = \frac{(b - r)}{b}\left(\frac{\alpha - \mu}{\beta + \theta} - a\right) - (b - r)p(t)$$

[6]For the derivation of the solutions, refer to the Appendix at the end of this paper.

which converges monotonically as long as

(23) $$b - r > 0$$

holds.

The resulting equilibrium fish population (steady-state fish population in perpetuity) is

(24) $$p^* = \frac{1}{b}\left(\frac{\alpha - \mu}{\beta + \theta} - a\right) > 0$$

due to (18).

From (17) one can easily derive

(25) $$\dot{x} = (b - r)\left(\frac{\alpha - \mu}{\beta + \theta} - x\right),$$

which is independent of p.

With (4) and (25) together, and making use of our knowledge about $X^*(p(t))$, we can draw the following phase diagram.

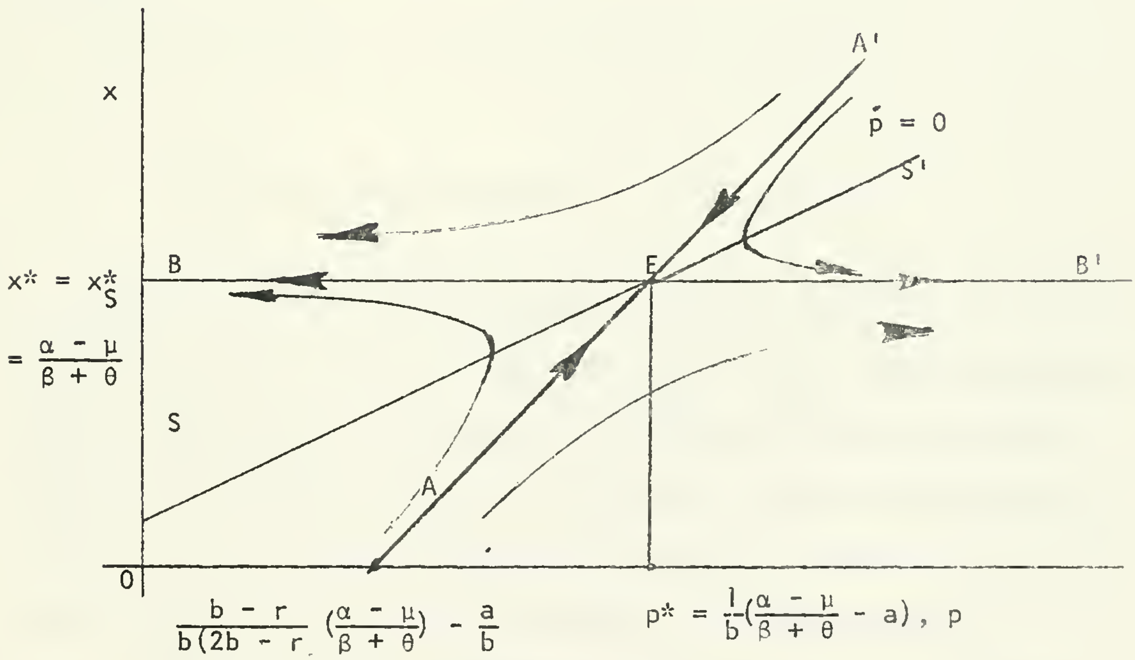

Figure 3. Phase Diagram for Competitive Case

4. Implications of MOdel Solutions: Is Catch Control Necessary?

The trajectory $\vec{AE}$ and $\vec{A'E}$ are the stable optimal-catch control paths that converge to the equilibrium catch E on the steady state fish population dynamics loci SES'($\dot{p} = 0$). $\vec{EB}$ (excluding E) and $\vec{EB'}$ are unstable singular control paths on which the fish population diminishes steadily and increases steadily respectively.

On the stable path $\vec{AE}$, the fish population steadily and asymptotically increases to reach the terminal desired or target population p*, and on $\vec{A'E}$ it decreases steadily and asymptotically to reach p*.

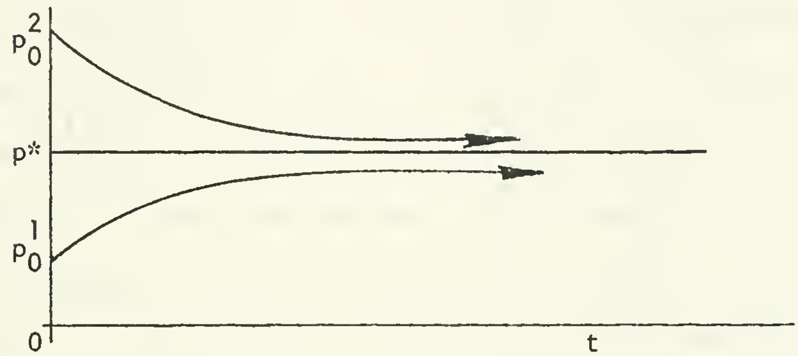

Figure 4. Fish Population Along $\vec{AE}$ and $\vec{A'E}$

When the initial fish population is smaller than p*, it is not advisable to catch fish to the extent that maximizes the instantaneous benefit, that is, x_S^*. The catch that eventually brings the maximum benefit must be smaller than x_S^*. Therefore, the catch should be controlled. But can the catch be controlled? This question will be discussed later in relation to the question of mesh size control.

If the initial fish population is larger than p*, then there may not be any incentive to bring the population down to p*. Thus, purely from the social benefit maximization point of view, the optimal catch

will stop at x^*. The reason that we have this path is that the necessary conditions (16) forced the population to cease growing, thus bringing the population to E along $\vec{A'E}$. Practically however, this path $\vec{A'E}$, may be of some importance in managing private or public lakes. At a certain stage, with a very small amount of catch of say, bass, in a lake, the fish population may be boosted to such an extent that drastic measures may be called for to bring the population back to some ecologically and biologically desirable level. In this case, the path A'E is a desired path to follow. Also, there is a distinct possibility that similar control measure as this may be needed by a small country like New Zealand, for instance, once the two hundred mile territorial water is enforced to exclude the fishing fleet from other nations. This, however, belongs to a class of differential game problems and will be treated fully in the next paper.

Figure 5 shows two (closed-loop) optimal catch control paths corresponding to the initial fish populations of p_0^1 and p_0^2.

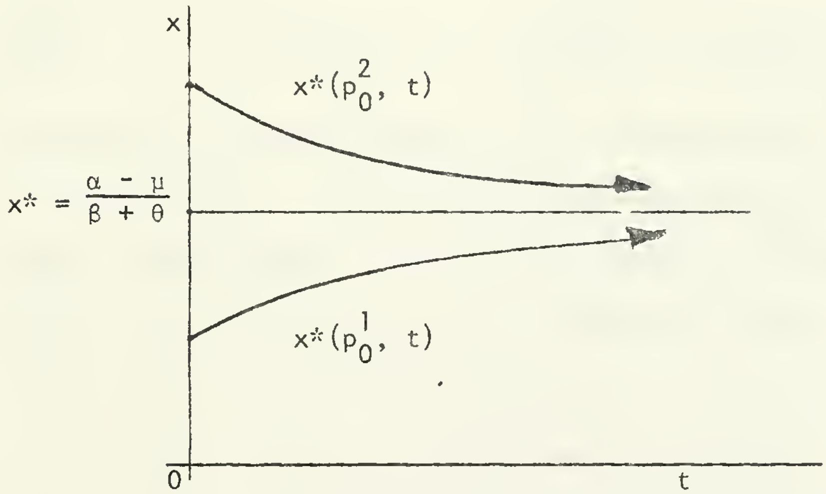

Figure 5. Optimally Controlled Catch Paths

A general conclusion on the control of catch is due. This model clearly shows that in order to attain the maximum social benefit over a long period of time, the catch has to follow the path on $\overrightarrow{AE}$ or $\overrightarrow{A'E}$. Impatient or self-proclaimed optimal catch x_S^* at any moment of time in the most prevalent situation in which fish are scarce, relative to the need and demand of a society, will lead the whole society to predictable misery. Do numerous competitive fishermen know what is the optimal catch to the society and to the industry now and at any time t in the future? It is doubtful that they do. Most likely they expend their utmost effort at any time t to catch as much as they can if there is no governmental regulation on their catch. Impatience and lack of information on the future of the fishing industry by the society, coupled with industriousness of the fishermen, are disasterous to the industry and the consuming society.

One can refer to necessary conditions (17), especially the last condition using the optimal catch $x^*(p(t))$,

(26) $$\alpha - \beta x^*(p(t)) - \mu - \theta x^*(p(t)) = \lambda^*(p(t)),$$ [7]

to explain the situation above in more sophisticated economics jargon.

It is easy to show that the demand price, $\alpha - \beta x^*(p(t))$, is always higher than the supply price, $\mu + \theta x^*(p(t))$, by $\lambda^*(p(t))$ (marginal value product of the fishing banks [4], p. 516), as long as the fish population $p(t)$ is smaller than p^* and vice versa. It can be argued that the marginal social product measured in the demand price diverges by $\lambda^*(p(t))$ from the marginal private product measured by the industry cost (= private marginal cost).

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[7] $\lambda^*(p(t))$ converges monotonically to zero from above when $p_0 < p^*$, and from below when $p_0 > p^*$ (see page 17 for these conclusions).

The abnormal profit when $p(t) < p^*$ may encourage the fishing industry to go for a larger catch, with larger numbers of fishermen participating, due to free entry assumption of a competitive market. This is exactly why governmental regulation is necessary in the fishing industry over the total catch of a fish species. And when $p_0 > p^*$, a governmental subsidy scheme is needed to bring the fish population to the desired level p^* (for static reasoning for this conclusion, see van Meir [11]).

The following additional conclusions can be derived from the results obtained so far.

1. The future discount rate does not affect the terminal or target optimal catch, x^*, or fish population, p^*.

2. For any given initial fish population p_0 smaller than the target population p^*, an increase in r will make the optimal catch larger, and vice versa; a revelation of impatience as the society more heavily discount the future than before, and vice versa. In case p_0 is greater than the target population, p^*, the optimal catch for any p_0 will be smaller than before if r increases.

3. If b is larger, then the target population becomes smaller (provided the demand and cost situations are the same and the two different species exhibit different population dynamics in terms of b only).

4. If r gets larger (but $r < b$), the process of convergence of $p^*(t)$ and $x^*(p(t))$ becomes slower. This is another effect of r similar to (2) mentioned above, expressed this time in terms of the rate of convergence to the desired p^*. Since by (2) the society catches more now or till some finite time t as r increases, the fish population grows at a slower convergence of the optimal control and state. The cases with $r > b$ are discussed in the conclusion of this paper.

5. The market demand price, P_d^*, supply price, P_s^* , and the social and private product discrepancy, $\lambda^*(p(t))$, move along the following paths:

(27)
$$P_d^*(p(t)) = \alpha - \beta x^*_{\frac{1}{2}}(p(t))$$

$$P_s^*(p(t)) = \mu + \theta x^*(p(t))$$

$$\lambda^*(p(t)) = P_d^* - P_s^* .$$

When $p(t) < p^*$, $P_d^*(p(t))$ approaches to $\mu - \theta\left(\frac{\alpha - \mu}{\beta + \theta}\right) = \alpha - \beta\left(\frac{\alpha - \mu}{\beta + \theta}\right)$ from below. The discrepancy is $\lambda^*(p(t))$ and all three are shown in Figure 5 below.

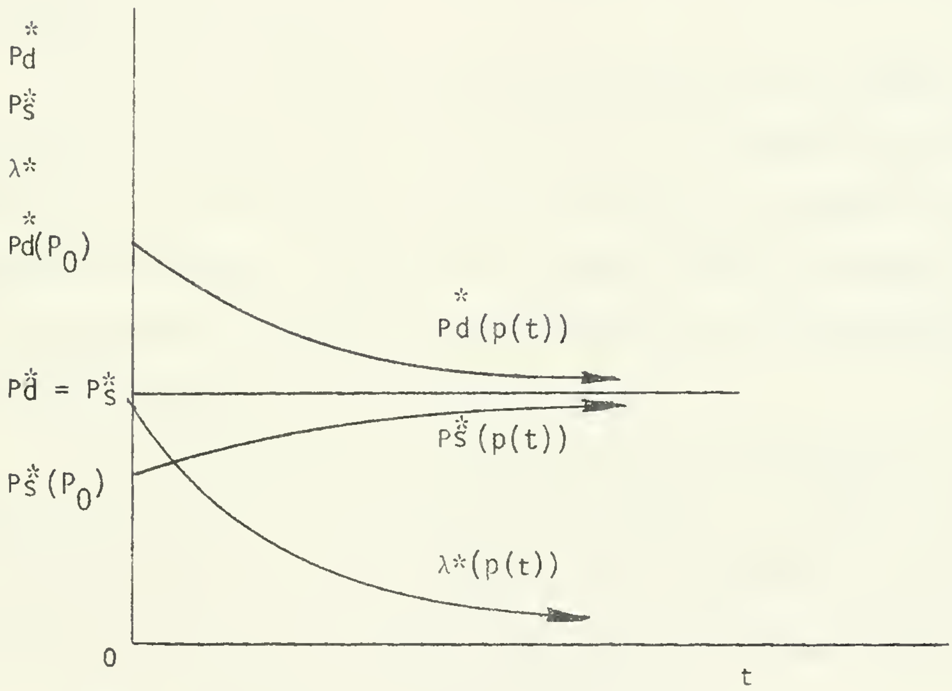

Figure 5. $P_d^*(p(t))$, $P_s^*(p(t))$, and $\lambda^*(p(t))$ Paths; $(p(t) < p^*)$

So far, we have investigated some interesting properties of our quadratic-linear optimal fish catch control model. In the next section some policy issues will be discussed in reference to our modeling methodology.

5. Control of Catch and/or Mesh Size

In the previous section I concurred (though for different reasons) with Turvey's contention that governmental catch control regulation is unavoidable in the fishing industry. In this section I want to advance an argument and modeling framework that would help find an optimal mesh size as well as the level of catch consistent with the mesh size, which should also be regulated.

Boyd [4], following Turvey [10], developed a static model that relates the recruitment (catch) of the fish as a function of both mesh size and the size of the annual catch. Since there is a danger of misinterpreting static model assumptions in dynamic modeling framework, I would rather choose to present my model assumptions in introducing mesh size in my model developed in the previous sections.

It is a well-known fact that there exists a minimum fish size below which the fish may not be commercially handled. It stands to reason that the mesh size must be large enough to catch commercially disposable fish. Therefore, we can safely assume that there is a minimum mesh size, say mo.

Now, I advance the following hypothesis:

For any mesh size, m equal to or greater than mo, there corresponds a fish population dynamics, that is,

$$\dot{p} = a(m) + b(m)\, p(t) - x(t), \text{ for } t \in [0, T) \text{ and } m \geq mo. \tag{28}$$

On the supply side, the cost of catching $x(t)$ fish with mesh size m can be written as

(29) $$S(x|m,\ t) = \mu(m) + \theta(m)x(t).$$

On the demand side, there is no change since most likely the consumers do not know anything about mesh sizes with which the fish they consume are caught.

Following the development and conclusions of the previous section, we can derive the following conclusion:

For each mesh size (m) there exists a stable optimal catch control path for any initial fish population, provided that conditions

(18') $$\frac{\alpha - \mu(m)}{\beta + \theta(m)} - a(m) > 0$$

(23') $$b(m) - r > 0$$

for any $m \geq mo$ hold.

After a sufficiently long time period, the instantaneous benefit can be very close to

(3) $$SPO(m) = (\alpha - \mu(m))x^*(m) - \frac{1}{2}(\beta + \theta(m))(x^*(m))^2$$

where

(31) $$x^*(m) = \frac{a - \mu(m)}{\beta + \theta(m)} \text{ for any } m \geq mo.$$

Naturally we have,

(32) $$p^*(m) = b^{-1}(m)\left(\frac{a - \mu(m)}{\beta + \theta(m)} - a(m)\right)$$

It is not known, in general, whether the social pay-off or benefit function SPO(m) in (28) is a concave function or not. However, this will most likely be the case.

If (30) is a concave function with respect to $m(\geq mo)$, then there will be the maximum SPO(m) at say m^*—m^* may not be unique. However, we choose one m^*.

One may be able to conclude that the mesh size m^* is the most desirable size.

Following the same line of logic as we employed in section 3, we can conclude that as long as $p_o < p^*(m^*)$ and $m^* \geq mo$, there must be a regulation limiting the total catch. The most desirable mesh size (which could consist of a mixture of different size meshes $\geq mo$) can be found as m^* ($\geq$ mo). However, this information is external to the fishermen, and the mesh size m^* should be recommended, from the social value point of view, as the one to be used by all the fishermen. Whether or not this recommendation should be instituted as a regulation is a difficult question to answer, in general, and should be considered in a rather specific and practical context.

In one of the papers to follow this, I plan to investigate a model which reflects the size of the fish population in the cost function (12). This observation has been prompted by the paper by Van Meir [11][8] and incorporated in my general formulation (7). This will complicate the solution procedures, but may not change the general properties of the solution.

5. Quadratic-Linear Optimal Catch Control Model for a Monopolist

In the non-centrally planned countries it is not likely that the fishery industry is run by a monopolist. However, for theoretical completeness, a case of optimal catch control by a monopolist will be developed

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[8] See Chart 2 of [10].

here. Also, to save the paper space, only a quadratic profit functional and linear dynamics case will be presented below.

If we assume that the market demand function remains the same as (11) after the monopolist takes over the market, the cost structure is assumed to be

$$(33) \qquad TC(x) = \mu x + \theta x^2$$

with parameters satisfying (14).

Ignoring the terminal cost or penalty function, the monopolist profit function can be written as

$$(34) \qquad \pi(x) = \int_0^T e^{-rt}[(\alpha - \mu)x - (\beta + \theta)x^2]dt$$

for $t \varepsilon [0, T)$ and $T > 0$.

The monopolist is considered to observe the fish population dynamics (4) while trying to maximize his profit over the entire time horizon, (34). This last assumption is a standard monopolist assumption but in a dynamic situation, such as in fishery industry, may turn out to be an impractical one as we will eventually discuss later in this section in conjunction with a catch control regulation.

Let us now define our monopolist optimal catch control problem as

Problem 2: Find the path(s) of optimally control of intensity of catch, $\overline{x}(p(t), t)$, that maximizes (34) subject to (4).

The optimal catch in the feedback form can be given by[9]

$$(35) \qquad \overline{x}(p(t)) = \frac{\alpha - \mu}{2(\beta + \theta)} - \frac{(2b - r)}{b}\left(\frac{\alpha - \mu}{2(\beta + \theta)} - a\right) + (2b - r)p(t).$$

The terminal or targer catch is given as

[9] See Appendix at the end of this paper.

(36) $$\bar{x} = \frac{\alpha - \mu}{2(\beta + \theta)}$$

and the target fish population will be

(37) $$\bar{p} = \frac{1}{b}\left(\frac{\alpha - \mu}{2(\beta + \theta)} - a\right).$$

The process of convergence of (35) follows the mode of that of p which is given as

(38) $$\dot{p} = \frac{(b - r)}{b}\left(\frac{\alpha - \mu}{2(\beta + \theta)} - a\right) - (b - r)p(t).$$

For these solutions to be economically meaningful, they have to satisfy the following conditions:

(39) $$\begin{cases} \alpha - \mu > 0 \\ \dfrac{\alpha - \mu}{2(\beta + \theta)} - a > 0 \\ b - r > 0 \end{cases}.$$

There exists a singular control

(40) $$\bar{x}s = \frac{\alpha - \mu}{2(\beta + \theta)}$$

which is unstable except at $p_o = \bar{p}$.

All the properties of our solutions for the competitive market case apply to this case.

It is quite natural to conclude that the monopolist has every reason to follow the optimal catch rule laid down by (35) in order to attain the maximum long-run profit. However, it is also equally conceivable that the maximum profit motive may have very little to do with the idea that the monopolist has to stay in the industry forever. If it plans to maximize the profit in, say, 10 years, it may most likely choose a different optimal

catch control rule. It may even catch a larger quantity of fish (very close to $\bar{x}_s$ if this is at all possible) in a few years' time than the infinite horizon optimal catch suggests, and may eventually make an abrupt exit from the industry, leaving the consuming society without the fish species on their table. As private ethic and public ethic can be different, unless the society acts to prevent the monopolist from abandoning the industry after quickly exploiting the fish stocks, there could be a socially undesirable situation that I discussed above.

Therefore, in the monopolist case, as well as the competitive market case, catch regulation must be followed or, more strongly, enforced.

The optimal determination of the mesh size in the monopolist case can be more easily accomplished by the monopolist since it is in his own interest to choose the profit maximizing mesh size, say m^*.

One can argue about relative magnitudes of target catch size and fish population size between competitive and monopolist market structures, and I leave these theoretical exercises to the reader (see pp. 206-209 of [11] for an interesting discussion on this point).

6. Future Research Directions

The dynamic optimal control formulation of fisheries economics, developed above, opens up various avenues to future research in this field.

A natural extention of our single fish species-single country formulation is a multiple fish species-single country formulation which I will present shortly.

With multiple, say n (> 1) species formulation, a system of market demand functions goes into the society benefit function along with the industry cost function.

The fish population dynamics expand into a system of n fish population dynamics. Even in its most operational formulation, namely, a quadratic-linear version, careful pereliminary investigations on existence of a solution, controllability, observability, and stability properties of the system need to be carried out. Aside from theoretical scrutiny of the problems at hand, there are many practical, exciting problems related to ecology, environmental protection, etc.

Tuna and dolphin create a serious problem to our society. Here is a problem of inherent interactions between two species in their habitat and between two groups in the society; consumers seriously concerned about the possibility of extinction of a lovely and intelligent fish species and the tuna fishermen struggling for their existence fishing tuna (and possibly some dolphins). Mesh size control does not solve this problem. Catch control has proved almost impossible. Is it possible for the tuna industry to breed dolphin and release them to replace the unfortunate ones that died in the fishermen's nets?

These interactions can be traced out in a multiple species formulation.

Another natural extension is toward solving single and then multiple species-multiple nations problems in a fisheries economy. Here come conflicts of interest among nations over their territorial sea. The cod war between Iceland and the United Kingdom in recent years is one typical example. Unless all the nations involved in fishing activities agree to cooperate and use a common currency, the multiple nation formulation cannot be reduced to a simple optimal control problem. Therefore, we have to step outside the realm of familiar optimal control theory into the so-called "differential game theory."

In the next paper, I plan to show some interesting features of a two-country/one-species fisheries economics model in the differential game theoretic framework and discuss some implications of the model results.

Naturally, it is more to the satisfaction of many theorists if the quadratic-linear form can be generalized and still rich quantitative conclusions can be obtained. One can strive for it, and here we have a great need for interdisciplinary work ahead of us. Fisheries technicians, optimal control and differential game theorists and practioners, economists specializing in this and related field can work together to make regulations and controls economically and politically viable and sound.

7. Conclusions

In this paper a dynamic theory of fisheries economics within one nation-one species framework has been developed in the optimal control theoretic framework. Conclusions from the quadratic benefit and linear dynamics case are quite interesting. First of all, static analysis concentrates on the ultimate target catch or population only and ignores the processes along which optimal catch leads the industry and the consuming society to the target, if it exists. The characteristics of the processes are important, and not the target state or control value per se.

Secondly, the catch must be regulated if the fish stocks are not abundant already, due to the clear-cut discrepancy between the social value product of a unit of the fish and the private value product explained at the end of section 4 of this paper.

Thirdly, the optimal mesh size should be studied carefully in relation to fish population dynamics and the resultant fishing cost through fishing effott. Proper recommendations should be made on the

optimal mesh size. Irrespective of the need for finding the optimal mesh size, the minimum mesh size for each species should be stipulated as a governmental regulation, if there is only one species of fish we are concerned about. A multiple species case will be dealt with in another paper.

Fourthly, the future discount rate, r, of social benefit is not a determining factor of target population or catch. However, it affects the rate of convergence of the optimal catch and the fish population to the desired levels or targets. The larger the future discount, the greater the present catch and thus the slower the fish population growth. However, as long as the catch is on the optimal path, it will lead itself to the target catch and population no matter what the discount rate may be. Figure 6 below shows this relationship.

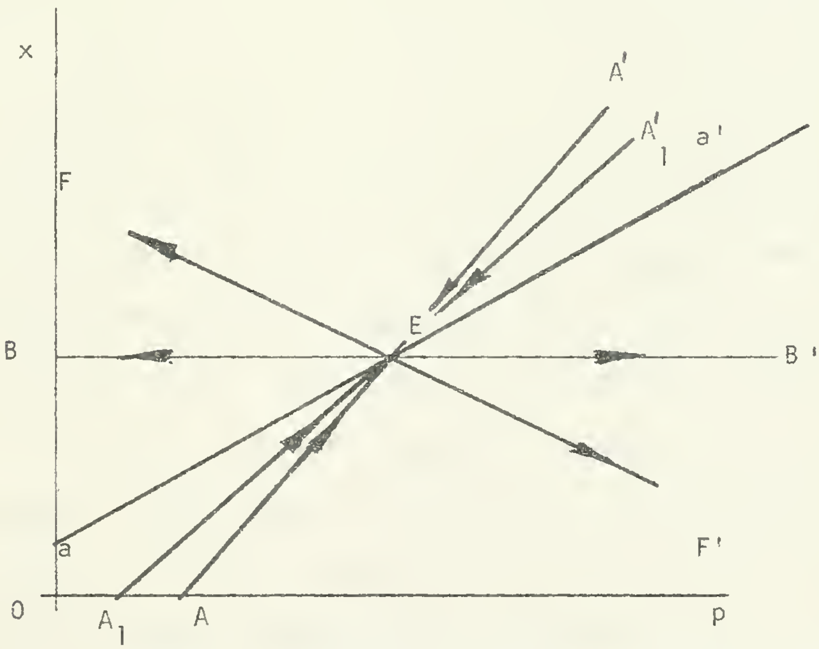

Figure 6. Different Discount Rates and Corresponding Optimal Catch and Population Paths

When $b - r > 0$, but r increases from r to $r_1 (r_1 > r)$, then the optimal catch path moves upward from $\vec{AE}$ to $\vec{A_1E}$ (downward from $\vec{A'E}$ to $\vec{A_1'E}$).

When $b = r$, the optimal control disappears. The only solution is the singular solution x_s^* which does not render any guidelines to maximizing the society benefit over the long run.

When $b < r$, and $2b > r$, or $b < r < 2b$, the pattern of convergence reverses itself in the BE a and B'Ea' domains, and given the initial fish population p_o (except $p_o = p^*$) every optimal catch control path will move away from E.

If $r = 2b$, the singular, unstable control x_s^* is the only catch control law we have. On the other hand, if the society discount the future more severely than this case, and we assume $r > 2b$, then we have unstable, and explosive paths $\vec{EF}$ and $\vec{EF'}$.

These cases with $r > b$ are not our concern at this stage, even though there is a lesson we can learn from this on how important the way we, the present generation, weigh the value of our future generations in determining the future course of the fishing activities and fish populations.

In this paper, I agree completely with Dr. Burkenroad's view on the principle of fisheries management: "The management of fisheries is intended for the benefit of man, not fish; therefore, effect of management of fish stocks cannot be regarded as beneficial per se" [5]. Thus, I constructed a dynamic model to handle the fisheries economics or management problems. I hope this model and model conclusions serve as extensions of models and theories advanced by our precursors in this field, such as Crutchfield and Zelloner [6], Turvey, Boyd, Bell [2], Burkenroad [5], van Meir, Smith, and others.

In the next paper, I will deal with two country/one fish species problems in the differential game theoretic framework and derive some interesting noncooperative game type conclusions (along with cooperative game type ones).

References

[1] M. Athans, and P. L. Falb. Optimal Control. New York: McGraw-Hill, 1966.

[2] F. W. Bell, and J. E. Hazleton (Eds.) Recent Developments and Research in Fisheries Economics, Oceana Publications, Inc., 1967.

[3] R. J. H. Beverton and S. J. Holt. On the Dynamics of Exploited Fish Populations, London, 1975.

[4] J. H. Boyd. "Optimization and Suboptimization in Fishery Regulation: Comment." Am. Econ. Rev, June 1966, 56, 511-517.

[5] M. D. Burkenroad. "Theory and Practice of Marine Fishery Management." Journal du Conseil Permanent International pour l'exloration de la Mer, January 1953, 13, No. 3.

[6] J. Crutchfield and A. Zellner. "Economic Aspects of the Pacific Halibut Fishery." Fishery Industrial Research, April 1962, 1, No. 1.

[7] H. C. Lampe. "The Interaction Between Two Fish Populations and Their Markets--A Preliminary Report." in [1], 179-195.

[8] M. Simaan and T. Takayama. "An Application of Differential Game Theory to a Dynamic Duopoly Problem with Maximum Production Constraints." Automatica, 1977, forthcoming.

[9] V. L. Smith. "Economics of Production from Natural Resources." Am. Econ. Rev., June 1968, 58.

[10] R. Turvey. "Optimization and Suboptimization in Fishery Regulation." Am. Econ. Rev., March 1964, 54, 64-76.

[11] L. W. van Meir. "A Study of Policy Considerations in Managing the Georges Bank Haddock Fishery," in [1], 197-209.

APPENDIX

COMPETITIVE MARKET

The necessary conditions for optimal competitive solutions are:

$$\text{(A.1)}\quad \begin{cases} (1) \quad \dot{p} = a + bp - x \\ (2) \quad \dot{\lambda} = (r - b)\lambda = -(b - r)\lambda \\ (3) \quad \alpha - \mu(\beta + \theta)x - \lambda = 0 \\ (4) \quad \lambda(T) = 0 \text{ or } \lim_{t\to\infty} e^{-rt}\lambda(t) = 0 \end{cases}$$

Let us set

$$\text{(A.2)}\qquad \lambda = Kp + E$$

where K and E are Recatti coefficients and time dependent in general.

By using (A.2), we can write the optimal catch x as the function of p (closed-loop or feedback control) via (A.1.3):

$$\text{(A.3)}\qquad x = \frac{1}{\beta + \theta}(\alpha - \mu - Kp - E).$$

Differentiating (A.2) with respect to time t, we get

$$\text{(A.4)}\qquad \begin{aligned} \dot{\lambda} &= \dot{K}p + K\dot{p} + \dot{E} \\ &= (\dot{K} + bK + \frac{K^2}{\beta + \theta})p + aK - \frac{(\alpha - \mu)}{\beta + \theta}K. \end{aligned}$$

By (A.1.2) and (A.2), we get

$$\text{(A.5)}\qquad \dot{\lambda} = (r - b)(Kp + E).$$

Setting (A.4) and (A.5) equal to each other and factoring out, we have:

$$\text{(A.6)}\qquad \{\dot{K} + bK + \frac{K^2}{\beta + \theta} + (b - r)K\}\, p + \{aK - \frac{(\alpha - \mu)}{\beta + \theta} K + \frac{1}{\beta + \theta} KE + (b - r)E + \dot{E}\} = 0, \text{ for all } p.$$

For the finite time horizon T, we have to solve

$$\text{(A.7)}\qquad \begin{cases} \text{(i)} & \dot{K} = -bK - \dfrac{K^2}{\beta + \theta} - (b - r)K,\ K(T) = 0 \\ \text{(ii)} & \dot{E} = \dfrac{(\alpha - \mu)}{\beta + \theta} K - aK - \dfrac{1}{\beta + \theta} KE - (b - r)E,\ E(T) = 0 \end{cases}$$

backward from time T as the boundary conditions are known to be zero.

When the time horizon tends towards infinity, then it can be shown that K and E will converge to some constants ($\dot{K} = 0$, $\dot{E} = 0$). Therefore, in this case, our task is to solve

$$\text{(A.8)}\qquad \begin{cases} \text{(i)} & (2b - r - \dfrac{K}{\beta + \theta})K = 0 \\ \text{(ii)} & \dfrac{(\alpha - \mu)}{\beta + \theta} K - aK - \dfrac{1}{\beta + \theta} KE - (b - r)E = 0. \end{cases}$$

One set of solutions is

$$\text{(A.9)}\qquad \begin{cases} K = 0 \\ E = 0 \end{cases}$$

which give rise to the singular solution

$$\text{(A.10)}\qquad x_s^* = \frac{\alpha - \mu}{\beta + \theta} \quad \text{for} \quad t \geq 0.$$

The other set of solutions of (A.8) is

$$\text{(A.11)}\qquad \begin{cases} K = -(2b - r)(\beta + \theta) \\ E = \dfrac{(2b - r)}{b}\{(\alpha - \mu) - a(\beta + \theta)\} \end{cases}$$

From (A.11) and (A.3), we finally get

$$\text{(A.12)} \quad x^*(p(t)) = \frac{\alpha - \mu}{\beta + \theta} - \frac{(2b - r)}{b}\left(\frac{\alpha - \mu}{\beta + \theta} - a\right) + (2b - r)p(t),$$

and

$$\text{(A.13)} \quad \dot{p} = \frac{(b - r)}{b}\left(\frac{\alpha - \mu}{\beta + \theta} - a\right) - (b - r)p(t)$$

which shows that, as long as $(b - r) > 0$, the population converges to the target or desired population. The target population then is found as

$$\text{(A.14)} \quad p^* = \frac{1}{b}\left(\frac{\alpha - \mu}{\beta + \theta} - a\right)$$

and the target catch will be

$$\text{(A.15)} \quad x^* = \frac{\alpha - \mu}{\beta + \theta} .$$

(A.13) can be integrated out to be

$$\text{(A.16)} \quad p(p_o, t) = (p_o - p^*)e^{-(b-r)t} + p^*.$$

By using (A.16) we can write (A.12) as

$$\text{(A.12')} \quad x^*(p_o, t) = \frac{\alpha - \mu}{\beta + \theta} + (2b - r)(p_o - p^*)e^{-(b-r)t}$$

which is a monotone increasing (decreasing) function of t when $p_o < p^*$ ($p_o > p^*$), and converges to $\frac{\alpha - \mu}{\beta + \theta}$ as $t \to \infty$.

The discrepancy between the social value and the private cost will be

$$\text{(A.17)} \quad \lambda^*(t) = \frac{(2b - r)}{b}\{(\alpha - \mu) - a(\beta + \theta)\} - (2b - r)(\beta + \theta)p(t) = -(p_o - p^*)e^{-(b-r)t},$$

which shows

$$\text{(A.18)} \quad \begin{cases} \lambda^*(t) \geq 0 \text{ for all } t > 0 \text{ if } p_o < p^* \\ \lambda^*(t) \leq 0 \text{ for all } t > 0 \text{ if } p_o > p^* \\ \text{and } \lambda^*(t) \text{ convergest uniformly to zero as } t \to \infty. \end{cases}$$

Now the market demand price and supply price can be written as the functions of $p(t)$:

$$\text{(A.19)} \quad P_d = \alpha - \beta x^*(p(t))$$

$$= \alpha - \beta\left[\frac{\alpha - \mu}{\beta + \theta} - \frac{(2b - r)}{b} \quad \frac{\alpha - \mu}{\beta + \theta} - a) + (2b - r) \; p(t)\right]$$

$$= \alpha - \beta(\frac{\alpha - \mu}{\beta + \theta}) - \beta(2b - r)(p_o - p^*)e^{-(b-r)t}$$

$$\text{(A.20)} \quad P_s = \mu + \theta x^*(p(t)) = \mu + \theta\left[\frac{\alpha - \mu}{\beta + \theta} - \frac{(2b - r)}{b} (\frac{\alpha - \mu}{\beta + \theta} - a) + (2b - r)p(t)\right]$$

$$= \mu - \theta(\frac{\alpha - \mu}{\beta + \theta}) - \theta(2b - r)(p_o - p^*)e^{-(b-r)t}$$

From (A.12') we can derive the following result:

$$\text{(A.21)} \quad \frac{\partial x^*(p_o, t)}{\partial r} = (p_o - p^*)e^{-(b-r)t} \cdot \{(2b - r)t - 1\}.$$

Thus, the optimal catch is greater as r increases before the time reaches $1/(2b - r)$, after which it becomes less, other things being equal.

In deriving the phase diagram, the knowledge about $\dot{x} = h(p, x)$ functional form is needed. One can, however, easily derive for this case as

$$\text{(A.22)} \quad \dot{x} = (b - r)(\frac{\alpha - \mu}{\beta + \theta} - x),$$

which is independent of p. Clearly

$$\dot{x} \geq 0 \text{ when } x \leq x_s^* = \frac{\alpha - \mu}{\beta + \theta} = x^*, \text{ and } \dot{x} \leq 0 \text{ when } x \geq x_s^*.$$

MONOPOLY MARKET

The monopoly case can be solved in exactly the same manner as the competitive market case, and we will not repeat the procedures here.

V. SMITH MODELS

In his 1968 paper [9], Smith advanced his dynamics in line with the stability arguments of the general equilibrium theory. Assumption (1.3) is a typical one. We argue [1, 801-802] that the dynamics on the control is inherent in the system and can be derived from the system of the necessary optimality conditions (10). This difference makes a significant difference in investigating the path of convergence or divergence of optimal controls if they exist.

Without going into a detailed analysis, let us derive paths below in Figure A.1 for a more general fish population dynamics, given by a quadratic function

$$(A.23)\quad \dot{p} = a + bp - cp^2 - x = f(p,\ x)$$

where

$a, b, c > 0.$

For simplicity's sake, let us assume that the social benefit function is also quadratic, but in this case takes the following form:

$$(A.24)\quad W = \int_0^T e^{-rt}\{\alpha x - \frac{1}{2}\beta x^2 - (\mu - wp)x - \frac{1}{2}\theta x^2\}dt,$$

where μ, w, $\theta > 0$, and fishing industry enjoys the external economy from the increasing fish population, p, that is, the industry supply function is written as

$$P_s = (\mu - wp) + \theta x. \tag{A.25}$$

Now, our competitive market optimal control problem is defined as

<u>Problem A.1</u>:

Find optimal catch control path(s) that maximizes (A.24) subject to (A.23) for the time horizon [0, T), where T may be infinity.

After a simple manipulation of the necessary optimality conditions (see Simaan and Takayama [1], pp. 801-802), one can get the following $\dot{x} = h(p, x)$ functional form:

$$\dot{x} = \frac{1}{\beta + \theta}\Big[aw + (b-r)(\alpha - \mu) + \{(2b - r) - 2c(\alpha - \mu)\}\, p - 3cwp^2 + 2c(\beta + \theta)px - \{(b - r)(\beta + \theta) + 2w\}x\Big] = h(p, x) \tag{A.26}$$

which is strictly concave, quadratic function of p for any level of x. Therefore, we have the following general phase diagram. We draw only one diagram in which convergent paths exist.

In general, the intersections of $f(p, x) = 0$ (A.23), and $h(p, x) = 0$ (A.26), produce at most, three solutions with respect to p. Figure A.1 below shows the case with two positive real solutions and another solution at a negative value of p and/or x. The two solutions are E_1 and E_2. Stable converging optimal catch paths are $\overrightarrow{A_1E_1}$, $\overrightarrow{A_1'E_1}$, and $\overrightarrow{A_2E_2}$. Unstable paths are $\overrightarrow{E_1B_1}$ and $\overrightarrow{E_1B}$ (eventually coincides stable path $\overrightarrow{A_2E_2}$).

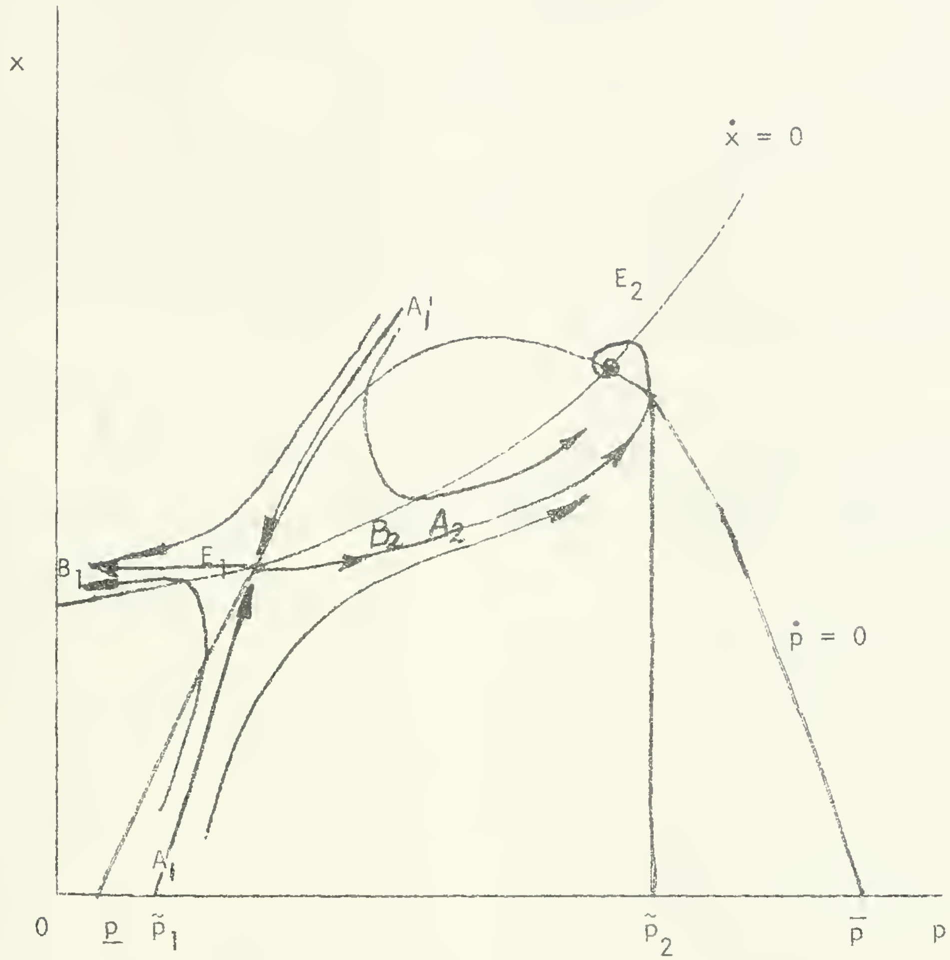

Figure A.1. A Nonlinear Case with Two Equilibria (E_1 and E_2).

In case the initial fish population is at $\overline{p}$, there is no optimal control for this system, contrary to the Smith reasoning[10] (he considers K as a state variable as well as X? In the optimal control framework, clear-cut understanding of state variables and control variables is absolutely necessary). The path Smith claims as optimal starting at ($\overline{p}$, 0) must be a suboptimal path in our framework (see Figure A.1). For the fish population ranging between $\tilde{p}$ and $\tilde{p}_2$ we have optimal control catch path along which the social benefit will be maximized in the long run.

Which equilibria between E_1 and E_2 the society would like to reach is a completely open question at this stage.

We plan to take up nonlinear models in the near future and investigate some important issues and problems in fisheries economics in a new model framework.

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[10] In my model the total industry catch is the sole control variable. The number of firms N(t) in the industry can then be written as

$$N(t) = k\,\dot{\lambda}$$

where $\dot{\lambda}$ is similar to that in my (10.2), where λ is the marginal discrepancy between market demand price and average private cost of the fish. This way N(t) can be traced outside of themodel

Reference to Appendix

[1] M. Simaan and T. Takayama, "Optimum Monopolist Control in a Dynamic Market," IEEE Transactions on Systems, Man, and Cybernetics December 1976, SMC-6, No. 12, 799-807.