Faculty Working Papers

DYNAMIC THEORY OF FISHERIES ECONOMICS -- II;
DIFFERENTIAL GAME THEORETIC APPROACH

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College of Commerce and Business Administration
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Abstract

A two countries-single species dynamic optimization problem in fisheries is formulated in differential game theoretic framework. The Cournot-Nash solution concept is introduced and two strategies in arriving at the Cournot-Nash solutions are discussed. These strategies are (1) closed-loop catch strategy and (2) open-loop catch strategy.

A quadratic benefit function is then used as the objective function of each country that is to be maximized subject to a linear population-catch dynamics. The solutions are then derived for each catch strategy and their implications are discussed.

The major conclusions are as follows:

(i) The stable optimal catches show that each country must regulate the total catch of her fleet.

(ii) The mesh-size should also be regulated.

(iii) The open-loop optimal catch strategies exist only when two countries' future discount rates are the same.

(iv) The higher the future discount rates, the more fish will be caught now and in the near future, and the slower the convergence of this fish population to the desired level.

October 2nd, 1977

DYNAMIC THEORY OF FISHERIES ECONOMICS--II: DIFFERENTIAL GAME THEORETIC APPROACH*

T. Takayama**

Introduction

Even after the imposition of the two hundred mile territorial waters limit by many countries, there are many fish species being caught in the open international waters. The two hundred mile territorial waters regulation is a revelation of inherent conflicts among a number of countries whose fishing fleets are after one or more fish species of their common interest and need. Of course, the regulation does not solve all the problems of the host country in relation to the desired catch of fish species, but alleviates the intermediate long-run burden of driving the fish population to an undesirably low level close to extinction [2].

The cod war between the United Kingdom and Iceland is another example of international conflicts in the field of fisheries economies. Can they resolve this type of conflict in any manner whatsoever? This is an important question in both theory and politics. In this paper we plan to partially answer this question from a theoretical point of view.

As a natural extension of the dynamic optimization approach related to fisheries economics [1, 10, 11], we apply the differential game theory to a two country-one fish species situation.

In section 1, the fish population dynamics and general properties of the differential game model applicable to fisheries economics are discussed. Quadratic objective functionals and linear fish population dynamics cases are then solved for the Cournot-Nash equilibrium and various implications of the solution are discussed in section 2. In the last section we discussed future research

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directions in this field and conclude with some positive notes for future research in the near future.

1. Differential Game Formulation of Fisheries Economic (Conflict) Problem

The problem the fisheries economicsts face is that of determining and recommending the fishing intensity that will maximize the economic value to the consuming societies and also maximize the producers' surplus at a level of production in perpetuity. In the previous paper, we took the stand that the consuming societies can be looked upon as one society with one market for a fish species and also the producers' supply response can be represented by a single market supply function. In this paper, we deal with two countries or consuming societies pursuing their individual economic value or goal such as a maximum welfare or profit or whatnot. The two producer societies are also separated in the sense that the cost function of each country is expressed by its own currency unit. We also assume in this paper that there is no trade of fresh fish or processed fish products between the two countries.

Thus, country 1 tries to maximize its welfare from the consumption and production (catch) of a fish species. The same or different objective may hold true for country 2. For simplicity's sake, let us write this objective function as a function of the (recruitable) fish population, p(t), at time t (≥ 0) and the intensity of catch by the ith country fleet $x_i(t)$ at time t (≥ 0); that is,

$$(1.1) \qquad W_i = \int_0^t e^{-r_i t} W_i(p(t), x_i(t))dt$$

where

r_i denotes the rate of the future discount by country i, i = 1,2. $W_i(p(t),$ $x(t))$ denotes the welfare or benefit or profit or any other objective that

the ith country tries to maximize, which depends on the fish population (to a lesser extent) and the catch of fish by the country's fleet for its own consuming society.

Thus far, the objective function of each country is completely individualized. This is a clear-cut departure that the differential game theoretical approach makes from the ordinary optimization approach. There is, however, one common object between them that is to be carefully observed; that is, the ocean in which the population of a fish species of their interest grows or declines in relation to the fishing intensities of fishermen of the two countries. The fish population in this paper is considered as a directly observable state variable, and the catch by the fishermen in each country is considered as a control variable; there are naturally two control variables in a two countries-one fish species case.

The fish population (and catch interaction) dynamics is written as

(1.2) $\dot{p}(t) = f(p(t),\ x_1(t), t),\ t \varepsilon [0,\ T)$

where p(t) is defined as before, (recruitable) fish and population at time t, state variable; x(t) denotes total intensity of catch by two countries; t denotes actual time; and T is the terminal time greater than zero and can be finite or infinite. In our two countries model, the total catch intensity x(t) can be divided into two parts as follows:

(1.3) $x(t) = x_1(t) + x_2(t)$

where $x_i(t)$ is defined in relation to (1.1) and denotes the intensity of catch by the ith country fishermen, at time t, i = 1,2.

The simplest dynamics we can consider is the linear first-order dynamics such as

$$\dot{p} = a + bp - x \tag{1.4}$$[1]

where we assume that a and b are constants and most likely

$$\begin{cases} a \le 0 \quad \text{and} \\ b > 0 \; . \end{cases} \tag{1.5}$$

The Figure 1 below shows this catch fish population dynamics.

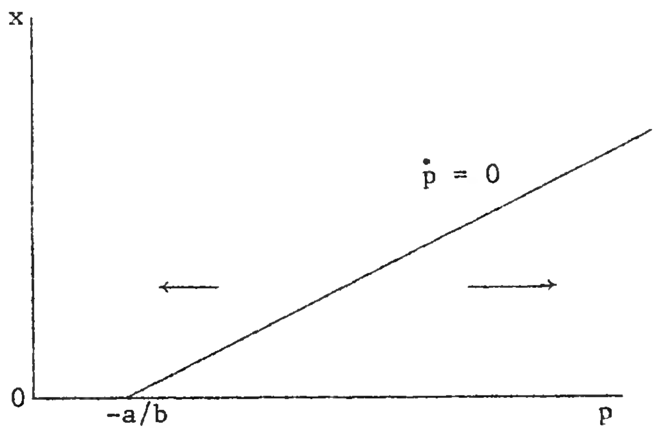

Figure 1: Catch-fish population dynamics; linear case.

A nonlinear dynamic case is also of great theoretical interest. For instance, the Lotka type or Voltera type quadratic dynamics [5] may be important in application. However, we confine ourselves, in deriving some practical inferences, to the linear dynamics case, (1.4), in this paper.

In Figure 1, the steady-state locus of (p, x) combinations is shown by $\dot{p} = 0 = a + bp - x$ equation. When catch is zero, the minimum sustainable population (below which the species become extinct) is $-(a/b)$. To the right of the

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[1] The dynamic identification, t of p(t), x(t), etc., will be omitted, unless otherwise stated in the following development.

line, the population (stock) to catch relationship is favorable to increase in fish population. To the left of the line, due to relatively heavy catches, the fish population tends to be zero.

In formulating the differential game problem within the framework of two countries and one fish species, let us employ as general a formulation as possible. The problem is clearly divided into the following two subproblems:

Fisheries Differential Game Problem (F D G P):

(i) Find $x_1(t)$ that maximizes W_1 in (1.1) subject to (1.2) for $t \in [0, T)$.

(ii) Find $x_2(t)$ that maximizes W_2 in (1.1) subject to (1.2) for $t \in [0, T)$.

This problem looks very much like a dynamic duopoly problem formulated in differential game theoretic framework [7, 8]. The substantial differences between these formulations are: (a) the duopolists are assumed to pursue the same kind of objective, profit in case of [7, 8], while in this case the objectives can be (i) profit in different monetary terms, or (ii) satisfaction, or (iii) any combination thereof, or (iv) any other objective one country claims to attain, and (b) the duopolist control their own supply quantity looking at the market dynamics represented by the price change, while in this case the intensity of catch is controlled by the fishing industry in each country looking at the fish population dynamics.

By carefully checking the objective function, W_i, (1.1), the fish population dynamics, (1.2), and the total and individual country catch relationship, (1.3), we can conclude that the objective function depends only on $x_1(t)$ and $x_2(t)$;

(1.6) $$W_i = W_i(x_1(t), x_2(t)), \ i = 1, 2.$$

As in static duopoly situations there are many different strategies the two parties can employ; there may be equally many or more strategies our dynamic controllers (nations) can assume. In this paper, we confine ourselves to the following two cases: (i) closed-loop strategy, and (ii) open-loop strategy. The close-loop strategy is the one that takes the fish population information $p(t)$ as time goes on and adjusts its catch to it. On the other hand, the open-loop strategy is the one that looks at the initial fish population and then decides the whole course of catch intensity decisions as a function of time t only.

Of course, in optimal control theory, where there is only one controller or decision maker, there is no difference between the outcomes of these two optimally (in some sense) chosen strategies. However, these two strategies are known to bring about two quite different catch intensity histories for these two countries [9], as will be revealed later in the next section.

From the game point of view, we deal only with the so-called "Cournot-Nash" equilirim game [3], that can be defined as a pair of catch histories $(x^*_{1c}(t, p(t)), x^*_{c}(t, p(t))$ for the closed-loop strategy, or $(x^*_{10}(t), x^*_{20}(t))$ for the open-loop strategy that satisfies the following conditions:

$$(1.7.1) \quad \begin{cases} W_1(x^*_{1c}(t, p(t)), x^*_{2c}(t, p(t)) \geq \\ W_1(x_{ic}(t, p(t)), x^*_{2c}(t, p(t)) \text{ and} \\ W_2(x^*_{1c}(t, p(t)), x^*_{2c}(t, p(t)) \geq \\ W_2(x^*_{1c}(t, p(t)), x_{2c}(t, p(t)), \end{cases}$$

or

$$(1.7.2) \quad \begin{cases} W_1(x^*_{10}(t), x^*_{20}(t)) \geq \\ W_1(x_{10}(t), x^*_{20}(t)) \quad \text{and} \\ W_2(x^*_{10}(t), x^*_{20}(t)) \geq \\ W_2(x^*_{10}(t), x_{20}(t)) \,. \end{cases}$$

(1.7.1) states that the pair of catch histories using the closed-loop strategy by both countries, $(x^*_{1c}(t, p(t)), x^*_{2c}(t, p(t)))$ is better or equally as good as any other closed-loop catch strategies that this country can assume when another country sticks to the star-marked closed-loop strategy.

(1.7.2) similarly states that the pair of catch strategies using the open-loop strategy by both countries, $(x^*_{10}(t), x^*_{20}(t))$ is better or equally as good as any other open-loop catch strategies that this country can employ when another country sticks to the star-marked open-loop strategy.

In the sense stated above (1.7.1) and (1.7.2) are similar to static Cournot or Nash equilibrium.

There are other dynamic games such as the dynamic Stackelberg game [3], but due to its operational difficulty we confine ourselves to this Cournot-Nash differential game in this paper.

The necessary conditions for optimality of dynamic Cournot-Nash (hereafter "Cournot" only) catch strategy in closed-loop form, can be derived as follows.

The Hamil onian of the Fisheries Differential Game Problem (FDGP) can be written as

(1.8) $$H_i^c = W_i(x_1(t, p(t)), x_2(t, p(t))) + \lambda_i(t) f(p(t), x_1(t, p(t)) + x_2(t, p(t)),$$
$$\text{for } i = 1,2.$$

Following Ho [3], Starr and Ho [8], and Simaan and Takayama [7], the following coupled necessary conditions are derived:

Country 1

(1.9) (i)
$$\begin{cases} \dot{p} = f(p(t), x_1(t,p(t)) + x_2(t, p(t))), \; p(o) = p_o \\ \dot{\lambda}_1 = r_1\lambda_1(t) - \lambda_1(t)\left(\frac{\partial f}{\partial p} + \frac{\partial f}{\partial x_2}\frac{\partial x_2}{\partial p}\right) \\ \frac{\partial W_1}{\partial x_1} + \lambda_1(t)\frac{\partial f}{\partial x_1} = 0 \\ \lambda_1(T) = 0 \text{ or } \lim_{t\to\infty} e^{-r_1 t}\lambda_1(t) = 0. \end{cases}$$

(1.9)-continued

(1.9) (ii)

Country 2

$$\dot{p} = f(p(t),\ x_1(t,\ p(t)) + x_2(t,\ p(t)),\ p(o) = p_o$$

$$\dot{\lambda}_2 = r_2\lambda_2(t) - \lambda_2(t)\left(\frac{\partial f}{\partial p} + \frac{\partial f}{\partial x_1}\frac{\partial x_1}{\partial p}\right)$$

$$\frac{\partial W_2}{\partial x_2} = \lambda_2(t)\ \frac{\partial f}{\partial x_2} = 0$$

$$\lambda_2\ (T) = 0 \text{ or } \lim_{t\to\infty} e^{-r_2 t}\ \lambda_2(t) = 0.$$

Here in the second condition for each country we find a term $\frac{\partial f}{\partial x_1}\frac{\partial x_1}{\partial p}$ for the first country and $\frac{\partial f}{\partial x_1}\ \frac{\partial x_1}{\partial p}$ for the second country (these are like "conjectural variation term" in static Cournat case) representing "interdependence" or "interconnectedness" of their actions.

Of course, each country can follow their strategies that ignores this interdependence; that is, each assumes that the other country's strategy is determined by the initial fish population, p_o, and time, t, only. Then, by constructing the Hamiltonian, the following necessary conditions result

(1.10) $$H_i^o = W_i(x_1(t),\ x_2(t)) + \gamma_i(t)f(p(t),\ x_1(t) + x_2\ (t))$$

for i = 1,2.

Country 1

(1.11) (i)

$$\dot{p} = f(p(t),\ x_1(t) + x_2(t)),\ p(0),\ p(o) = p_o$$

$$\dot{\gamma}_1 = r_1\ \gamma_1(t) + \gamma_1(t)\ \frac{\partial f}{\partial p}$$

$$\frac{\partial W_1}{\partial x_1} + \gamma_1(t)\ \frac{\partial f}{\partial x_1} = 0$$

$$\gamma_1(T) = 0 \text{ or } \lim_{t\to\infty} e^{-r_1 t}\gamma_1(t) = 0$$

$$(1.11)\quad (ii)\begin{cases} \dot{p} = f(p(t),\ x_1(t) + x_2(t)),\ p(o) = p_o \\ \dot{\gamma}_2 = r_2\ \gamma_2(t) + \gamma_2(t)\ \frac{\partial f}{\partial p} \\ \frac{\partial W_2}{\partial x_2} + \gamma_2\ (t)\ \frac{\partial f}{\partial x_2} = 0 \\ \gamma_2(T) = 0 \text{ or } \lim_{t\to\infty} e^{-r_2 t}\gamma_2(t) = 0 \end{cases}$$

Here in this open-loop catch strategy case, the interaction term $\frac{\partial f}{\partial x_i}\ \frac{\partial x_i}{\partial p}$ is missing from the second condition for each country. In the next section, we will trace out the consequence of this difference in detail.

2. Quadratic Benefit--Linear Population Dynamics Case

The main purpose of explicibly formulating FDGP in the quadratic benefit-linear fish population dynamics framework is to firmly quantify optimal strategies and resultant fish population dynamics, and consequently to derive policy implications from the results.

In this section we deal with the following two cases:

Case 1: (Competitive Markets--Close-Loop Game)

The social benefit function is defined as the over-time integral of the instantaneous (we omit this term hereafter) consumers' and producers' surplus accruing from the fish market, and the two countries use the closed-loop catch strategy.

Case 2: (Competitive Markets--Open-Loop Game)

The social benefit is the same as defined in Case 1, but the two countries use the open-loop catch strategy.

Another pair of cases dealing with a monopolist controlling the domestic market in each country can be solved easily, but we will not go into them in this paper.

There are other combinatorial possibilities for our case studies of the Cournot game, and we plan to do some exhaustive work later in this area.

2.1. Competititve Markets--Closed-Loop Catch Strategy Differential Game

Following Samuelson [6] and Takayama [10], let us express the consumers' benefit functional as the time integral over [0, T), where T could be infinity, of the integral of the instantaneous demand (felicity) function:

(2.1) Consumers' Benefit in the ith country =

$$CB_i \equiv \int_o^T e^{-r_i t}[\alpha_i x_i - \frac{1}{2}\beta_i x_i^2]dt$$

for $i = 1,2$.

where the instantaneous demand function of the ith country for the fish species is given as

(2.2) $$P_{di} = \alpha_i = \beta_i x_i, \; i = 1,2 \; .$$

We assume in the two cases we handle in this section, that the catch x_i is consumed without any wastage in the process or the consumption is measured in the fresh fish unit.

The total cost of catching x_i units of fish is expressed as the over-time integral of the integral instantaneous producers' supply function.

(2.3) $$TC_i \equiv \int_o^T e^{-r_i t}[\mu_i x_i + \frac{1}{2}\Theta_i x_i^2]dt$$

where the instantenous producers' supply function is expressed as

(2.4) $$P_{si} = \mu_i + \Theta_i \, x_i(t)$$

for $i = 1,2$.

The social benefit function of the individual countries can be expressed as the difference between the consumers' benefit, CB_i, and the total cost of catch, TC_i; that is

(2.5) $$SB_i(x_i\,(p(t)) \equiv CB_i - TC_i \equiv \int_0^T e^{-r_i t}[(\alpha_i - \mu_i)x_i - \frac{1}{2}(\beta_i + \Theta_i)x_i^2\,]dt$$

for $i = 1,2$.

Now, let us define our competitive markets—closed-loop catch strategy differential game as follows:

<u>CMCLDG</u>:

Find the dynamic paths of the pair $(x^*_{1c}(p(t)),\ x^*_{2c}(p(t)))$ that satisfy the following Cournot conditions

$SB_1(x^*_{1c}(p(t)) \geq SB_1(x_{1c}(p(t)))$ and

$SB_2(x^*_{2c}(p(t)) \geq SB_2(x_{2c}(p(t)))$

subject to:

(1.4.c) $$\dot{p} = a + b\,p - [x_{1c}(p(t)) + x_{2c}(p(t))].$$

The Hamiltonian for this problem is given as

(2.6) $$H_i^c = (\alpha_i - \mu_i)x_{ic} - \frac{1}{2}(\beta_i + \Theta_i)x_{ic}^2 + \lambda_{ic}\,(a + bp - x_{1c} - x_{2c}).$$

The necessary conditions for optimality of the pair of catches are:

$$(2.7)\begin{cases} \text{(i)} \begin{cases} \text{(a)} & \dot{p} = a + bp - x_{1c} - x_{2c},\ p(o) = p_o \\ \text{(b)} & \dot{\lambda}_{1c} = -(b - r_1 - \frac{\partial x}{\partial p}2c)\lambda_{1c} \\ \text{(c)} & (\alpha_1 - \mu_1) - (\beta_1 + \Theta_1)x_{1c} - \lambda_{1c} = 0 \\ \text{(d)} & \lambda_{1c}(T) = 0 \text{ or } \lim_{t\to\infty} e^{-r_1 t}\lambda_{2c}(t) = 0 \end{cases} \\ \text{(ii)} \begin{cases} \text{(a)} & \dot{p} = a + bp - x_{1c} - x_{2c},\ p\ (o) = p_o \\ \text{(b)} & \dot{\lambda}_{2c} = (-b - r_2 - \frac{\partial x}{\partial p}1)\lambda_{2c} \\ \text{(c)} & (\alpha_2 - \mu_2) - (\beta_2 + \Theta_2)x_{2c} - \lambda_{2c} = 0 \\ \text{(d)} & \lambda_{2c}(T) = 0 \text{ or } \lim_{t\to\infty} e^{-r_2 t}\lambda_{2c}(t) = 0. \end{cases} \end{cases}$$

For the simplicity of computation, in the following we assume that

$$(2.8)\qquad \begin{cases} \alpha_1 = \alpha_2 \equiv \alpha \\ \beta_1 = \beta_2 \equiv \beta \\ \mu_1 = \mu_2 \equiv \mu \\ \Theta_1 = \Theta_2 \equiv \Theta \end{cases}$$

which is equivalent to stating that the two contries possess the same demand and cost structures. In this case, after some calculations involving the solving of Ricatti equations, we get the following solutions.[2] The first pair are

$$(2.9)\qquad x^*_{1c}(p(t)) = x^*_{1c}(p(t)) = \frac{\alpha - \mu}{\beta + \Theta},$$

which are not desirable or stable solutions for these two countries unless the fish population at the initial period was already

$$(2.10)\qquad p^* = \frac{1}{b}\left[\frac{2(\alpha - \mu)}{\beta + \Theta} - a\right]$$

---

[2]The detailed derivation of the solutions is given in the Appendix at the end of this paper.

A pair of optimal stable catch strategies exist and are

$$
(2.11)\quad \begin{cases}
\text{(a)} \quad x^*_{1c}(p(t)) = \dfrac{\alpha - \mu}{\beta + \Theta} - \dfrac{(2b-2r_1+r_2)}{3b}\left[\dfrac{2(\alpha - \mu)}{\beta + \Theta} - a\right] \\
\qquad\qquad + \dfrac{2b-2r_1+r_2}{3}\, p(t) \\
\text{(b)} \quad x^*_{2c}(p(t) = \dfrac{\alpha - \mu}{\beta + \theta} - \dfrac{(2b-2r_2+r_1)}{3}\left[\dfrac{2(\alpha - \mu)}{\beta + \Theta} - a\right] \\
\qquad\qquad + \dfrac{2b-2r_2+r_1}{3}\, p(t) .
\end{cases}
$$

The fish population moves along the following converging path.

$$
(2.12)\qquad \dot{p} = \frac{(b-r_1-r_2)}{3b}\left[\frac{2(\alpha - \mu)}{\beta + \Theta} - a\right] - \left(\frac{b-r_1-r_2}{3}\right)p(t) ,
$$

where we assume

$$
(2.13)\quad \begin{cases}
\text{(i)} \quad b - r_1 - r_2 > 0 \\
\text{(ii)} \quad \dfrac{2(\alpha - \mu)}{\beta + \Theta} - a > 0 .
\end{cases}
$$

The fish population converges to

$$
(2.14)\qquad p^*_c = \frac{1}{b}\left[\frac{2(\alpha - \mu)}{\beta + \Theta} - a\right] > 0
$$

Due to (2.12), we find the ultimate fish population as t tends to infinity as

$$
(2.15)\qquad p^*_c = \frac{1}{b}\left[\frac{2(\alpha - \mu)}{\beta + \Theta} - a\right] > 0.
$$

Thus, the fish population history accompanied by the optimal catch (2.11) is given as

(2.16) $$p_c(t) = (p_o - p_c^*)e^{-(\frac{b-r_1-r_2}{3})t} + p_c^* .$$

From (2.11) and (2.14) we get the ultimate optimal catch intensity as

(2.17) $$x_{1c}^* = x_{2c}^* = \frac{\alpha - \mu}{\beta + \Theta} .$$

The following comparative dynamics conclusions can be derived from the above results. The discount rates do not affect the optimal target ($t \to \infty$) catches or the target population; that is

(2.18) $$\frac{\partial x_{ic}^*}{\partial r_i} = \frac{\partial x_{ic}^*}{\partial r_j} = 0 \text{ and } \frac{dp_c^*}{dr_i} = 0, \ i = 1, 2.$$

Given the social discount rate of country 1, r_1, an increase of the discount rate of country 2, r_2, will decrease the catch intensity of country 1 at any level of the fish population less than p_c^* , and increase that of country 2. Similarly, an increase of the discount rate of r_1, given that of country 2, r_2, will increase the catch intensity of country 1, and decrease that of country 2 at any $p(t) < p_c^*$.

In other words, we have

(2.19) $$\begin{cases} \dfrac{\partial x_{ic}(p(t))}{\partial r_j} = \dfrac{1}{3}(p(t) - p_c^*) \\ \dfrac{\partial x_{ic}(p(t))}{\partial r_i} = \dfrac{2}{3}(p_c^* - p(t)) \end{cases}$$

for $i, j = 1,2$.

As a result, if country 1 becomes more conscientious about the future generations than country 2 in relation to fish consumption and production, $r_1' > r_1$, country 1 becomes more conservation-oriented than before in order to attain

the maximum possible social benefit in the long future and in perpetuity.

This conclusion establishes the principle of conservation for renewable resources including fish, deer, forestry, and others.

Also this same principle has a potential element that makes one country, say country 1, legislate the two hundred mile territorial waters limit if she sees foreign vessels exploit her territorial fishing beds to an undesirable extent.

Traditional controversies over whether or not some catch control regulation is necessary can be answered in this two country-one fish species framework.

As in the single country-single species case [10], we can conclude that the total catch for each country stipulated by (2.11) is necessary for both countries to enjoy ultimate maximum social benefit. The reason is rather straightforward. Since, by (2.7) (i,c) and (ii,c), the marginal social value of a unit of fish sold in the market always exceeds the marginal individual cost of catching the same by $\lambda_{ic}(p(t)) > 0$ or $\lambda_{io}(t) > 0$ for $p_o < p^*_c$ or $p_o < p^*_o$, individual fisherman would like to fish more, if left to follow their own individual profit maximization principles, than the socially optimal (2.11). This overcatch represented by (2.9) at each moment of time, when $p(t) < p^*_c$ or p^*_o, must be stopped. An international regulation controlling the catches by both countries must be worked out in this case. How to accomplish this lies outside the scope of this paper.[3]

Mesh-size regulation arguments have been carried out in a static framework in [1, 11], and in dynamic one-country one-fish species framework in [10]. Following [10], one can develop similar arguments for some kind of mesh regulations. The reason is simple enough: the information on the optimum mesh-size for the industry and the consuming societies, if it exists, is external to the individual

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[3] For some arguments in this direction, the reader is referred to [1, 11].

fishermen and the industry concerned in each country.

2.2. Competitive Markets-Open-Loop Catch Strategy Differential Game

If the two countries decide to take the open-loop strategy in the competitive market within their national boundary, the objective functions remain to be the same as (2.5) for $i = 1,2$. The population dynamis is the same as before, (1,4), with

(2.20) $$x \equiv x_{10}(t) + x_{20}(t).$$

The problem can be defined as follows:

CMOLDG (Competitive-Markets-Open-Loop Catch Strategy Differential Game):

Find the dynamic paths of the pair $(x^*_{10}(t), x^*_{20}(t))$ that satisfy the following conditions

$$SB_1(x^*_{10}(t)) \geq SB_1(x_{10}(t)) \quad \text{and}$$
$$SB_2(x^*_{20}(t)) \geq SB_2(x_{20}(t))$$

subject to

(1.4.0) $$\dot{p} = a + bp -(x_{10}(t) + x_{20}(t)).$$

The Hamiltonian for this problem is given as

(2.21) $$H_i = (\alpha_i - \mu_i)x_{io} - \frac{1}{2}(\beta_i + \Theta_i)x_{io}^2 + \lambda_{io}(a + bp - x_{10} - x_{20})$$

for $i = 1,2$.

The necessary conditions for optimality of the pair of catches are:

(2.22) (i)
$$\begin{cases} \text{(a)} & \dot{p} = a + bp - x_{10} - x_{20},\ p(o) = p_o \\ \text{(b)} & \dot{\lambda}_{10} = -(b - r)\lambda_{10} \\ \text{(c)} & (\alpha_1 - \mu_1) = (\beta_1 + \Theta_1)\, x_{10} - \lambda_{10} = 0 \\ \text{(d)} & \lambda_{10}(T) = 0,\ \lim_{t \to \infty} e^{-r_1 t}\, \lambda_{10}(t) = 0 \end{cases}$$

(2.22)--continued

$$
\text{(ii)}\begin{cases}
\text{(a)} \quad \dot{p} = a + bp - x_{10} - x_{20}, \; p(o) = p_o \\
\text{(b)} \quad \dot{\lambda}_{20} = -(b-r)\lambda_{20} \\
\text{(c)} \quad (\alpha_2 - c_2) - (\beta_2 + \theta_2)x_{20} - \lambda_{20} = 0 \\
\text{(d)} \quad \lambda_{20}(T) = 0, \; \lim_{t\to\infty} e^{-r_2 t}\lambda_{20}(t) = 0
\end{cases}
$$

As in the previous case, Case 1, we assume for simplicity's sake, that (2.8) holds; the two countries are the same in their demand and supply structure.

In this case, however, there is no optimal stable (convergent) open-loop catch strategies unless the following condition holds:

(2.23) $\quad r_1 = r_2$ (= r hereafter).

The implication of this condition is quite interesting and suggestive. If the two countries with the same demand and supply (cost) structure engaging fishing the same fish species in the common ocean (fishing banks), and if one country's future discount rate is different from the other, these countries cannot find any reasonable catch strategy that will eventually bring them to their target catch and fish population defined and discussed in the previous subsection.

What makes the open-loop catch strategy case so diagonally different from the closed-loop catch strategy game? In the case of the closed-loop case, the parties involved observe carefully the history of fish population over time, p(t). This makes one party's catch responsive to the other party's catch through the observed fish population. Thus, each party responds to each other sensitively, and this interaction brings about an optimum catch strategy to each party (as long as (2.13) holds). However, in the case of the open-loop strategy game, each party decides that she can determine her own optimal strategy on the basis of the initial fish population and time. Thus, unless complete symmetry exists

in their environment, they cannot come up with any optimal catch strategy.

After superimposing (2.8) and (2.23), we get the following results. A pair of singular solutions exist and are:

$$(2.24) \qquad x_{10}^{*}(t) = x_{20}^{*}(t) = \frac{\alpha - \mu}{\beta + \Theta} \quad .$$

They are not desirable or stable solutions unless the fish population is already at the level represented by (2.10). Optimal and stable, open-loop catch strategies exist and are

$$(2.25) \qquad x_{10}^{*}(t) = x_{20}^{*}(t) = \frac{\alpha - \mu}{\beta + \Theta} - \left(\frac{2b-r}{2b}\right)\left[\frac{\alpha - \mu}{\beta + \Theta} - a\right] + \left(\frac{2b - r}{2}\right)p(t).$$

The fish population dynamics using the open-loop catch strategies is then

$$(2.26) \qquad \dot{p} = \left(\frac{b-r}{b}\right)\left[\frac{2(\alpha - \mu)}{\beta + \Theta} - a\right] - (b-r)p$$

$$(2.27) \qquad \begin{cases} \text{(i)} & b-r > 0 \\ \text{(ii)} & \dfrac{2(\alpha - \mu)}{\beta + \Theta} - a > 0 \ . \end{cases}$$

This population converges to

$$(2.28) \qquad p_{o}^{*} = \frac{1}{b}\left[\frac{2(\alpha - \mu)}{\beta + \Theta} - a\right]$$

which is exactly the same as the closed-loop desired or target population, (2.14).

The ultimate optimal catch in perpetuity is given by

$$(2.29) \qquad x_{10}^{*} = x_{20}^{*} = \frac{\alpha - \mu}{\beta + \Theta}$$

as expected.

The optimal time profile of the fish population in this case is given as

$$(2.30) \qquad p_{o}(t) = (p_{o} - p_{o}^{*})e^{-(b-r)t} + p_{o}^{*}.$$

As a consequence, the optimal open-loop catch strategy brings a much faster convergence to the common target fish population

$$\frac{1}{b}\left[\frac{2(\alpha - \mu)}{\beta + \Theta} - a\right].$$

At the same time, for a given initial fish population, the optimal open-loop catch is always smaller than the optimal closed-loop counterpart as the following computation shows:

$$(2.31) \qquad x^*_{ic}(p(0)) - x^*_{io}(0) = -\frac{(2b - r)(p_o - p^*_c)}{b} > 0,$$

$$\text{for } p(0) < p^*_c = p^*_o = p^*$$

which is shown in Figure 2 below.

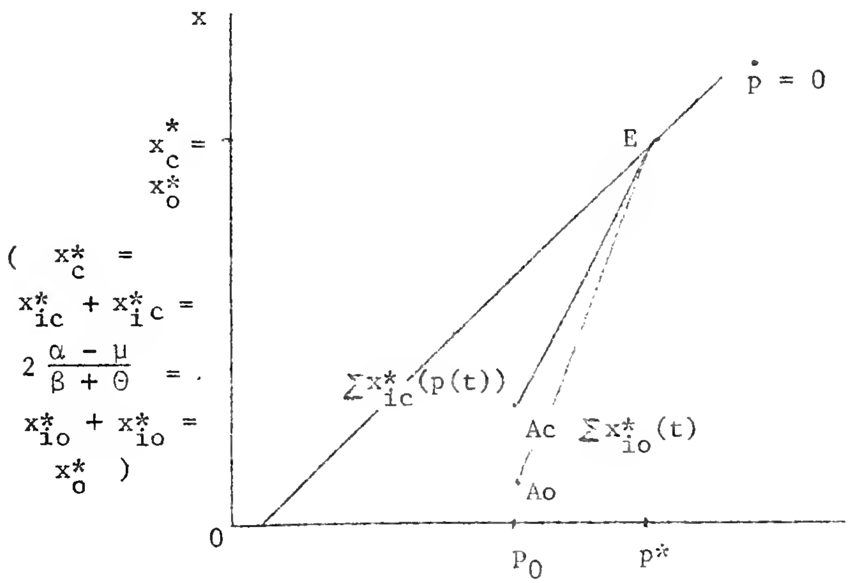

Figure 2. Optimal catch strategies $\sum x^*_{ic}(p(t))$ and $\sum x^*_{i0}(t)$.

As a consequence, the open-loop catch strategy, assuming of course that the two countries use the same future discount rate, brings a much faster convergence to the target fish population,

$$\frac{1}{b}\left[\frac{2(\alpha - \mu)}{\beta + \Theta} - a\right].$$

Thus, declaring that both countries employ the same future discount rate and stipulate and monitor the optimal history of catch each country is to follow, may be a faster way to reach the target population and catch.

It is also easy to conclude that along the stable convergent linear paths, $\vec{A_c E}$ and $\vec{A_o E}$, the catches and the fish population increase till they reach the long-run equilibrium point, the Cournot-Nash equilibrium point, if these two countries employ the closed-loop and open-loop catch strategy, respectively (provided $p_o < p_c^* = p_o^*$).

The future discount rate has no effect on the target catch or fish population. However, as r increases, the optimal intial catch will increase due to the fact that

$$(2.32) \quad \frac{\partial x_{io}^*(o)}{\partial r} = \frac{1}{2} \; (p_o^* - p(o)) > 0, \; i = 1,2,$$

as long as $p(o) < p_o^*$.

This is consistent with the larger future discount rate or the assertion of the "more now, less later" attitude of a society.

The same conclusions as those for the closed-loop case apply to this case as to the catch control and mesh-size issues.

Conclusion

In this paper, we have formulated the fisheries economies problem involving two countries and one fish species in a differential game framework. A quadratic social benefit and linear fish-catch dynamics problem is then solved for the Cournot-Nash differential game solutions. Obviously, there are at least two strategies to play within this game framework, and the solutions are derived

for the closed-loop catch and open-loop catch strategies.

We find that, if the demand and (cost) supply structures of the two countries are exactly the same, the closed-loop catch strategy generates a pair of optimal catch paths to follow for any future discount rate of individual countries as long as $b - r_1 - r_2 > 0$, while the open-loop counterpart produces a pair of twin catch paths to follow only if the future discount rates of individual countries are exactly the same. This is an outstanding feature of this renewable resource differential game.

In both cases, the increasing appreciation of the present over the future increases the optimal catch at present, slows down the process of convergence of the fish population to the target.

In the case of the closed-loop catch strategy, an increase of country 1's future discount rate, r_1, increases the initial optimal catch of country 1, while country 2 accepts the fact that r_1 has increased and curtails the optimal catch accordingly (see (2.19)). Whether country 2 is willing to follow this course of action may not be an issue at all. Rather, that r_1 and r_2 are historically given and accepted as such must be the basic framework of our model.

Of course, if the sum of the two countries' discount rates, $r_1 + r_2$, exceed the rate of increase of the fish population, b, there will not be any economically meaningful solution for our closed-loop strategy game. While the open-loop strategy game is much less restrictive in this respect since that the common rate of future discount, r, must be less than the growth rate of the fish population, b, is the only requirement. However, since the condition for the existence of optimal catches depends on the equality of the two rates, the open-loop catch strategy may be considered too restrictive.

A dynamic theory of fisheries economies we have developed so far has established a conservation principle that a scarce renewable resource such as a fish

species for human consumption requires a total catch control or a control over the catch by the fishing fleets of every individual country.

Also, we argue that some workable mesh-size regulation must be enforced since individual fishermen have no way of knowing what mesh-size is optimal from the consuming societies' point of view (as well as their own in the long-run), that is, this information is external to both consumers and fishermen.

There are many directions to go and many topics to cover in the future research in the area of renewable resources economics. A natural extension of our two countries-single species formulation is a two countries-single species formulation with different demand and supply structures. Also, finite time horizon problems should be solved for more practical problems. A system of matrix Ricatti equations must be solved to obtain any meaningful quantitative results from a multiple species-two countries formulation. In this direction, computer-based algorithms are absolutely necessary, and many diverse and practical problems may be solved efficiently.

Another extension is to solve more than two countries-one species model by effectively utilizing our model developed here along with the concept of coalitions.

An ultimate extension is to formulate and solve multiple countries-multiple species problems in generality, but quantitatively.

Fish population-catch dynamics can be made nonlinear to attain generality. One observation dealing with a quadratic population dynamics case is already discussed elsewhere [10], and may shed some new light on the predator-prey dynamics [5] widely accepted by the economics profession for some time.

Econometrically, estimation of parameters in population-catch (or forestry growth and harvesting) dynamics is a challenging field. This is a field in which various interdisciplinary activities will prove most productive. Cooperative

efforts in these directions and topics stated above by fisheries specialists, optimal control and differential game specialists, economists, and environmentalists will make regulations and control economically and politically viable and sound.

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Appendix

In this appendix only Case 1 problem is solved in detail. The reader will find it easy to solve Case 2 problem by following the development below.

From (2.7) and assuming (2.8) we have the following necessary conditions

$$
\text{(A.1)}\begin{cases}
\text{(i)}\begin{cases}
\text{(a)}\quad \dot{p} = a + bp - x_{1c} - x_{2c},\ p(o) = p_o \\
\text{(b)}\quad \dot{\lambda}_{1c} = -(b - r_1 - \frac{x_{1c}}{dp})\lambda_{1c} \\
\text{(c)}\quad (\alpha - \mu) - (\beta + \Theta)x_{1c} - \lambda_{1c} = 0 \\
\text{(d)}\quad \lim e^{-r_1 t}\lambda_{1c}(t) = 0
\end{cases} \\
\text{(ii)}\begin{cases}
\text{(a)}\quad \dot{p} = a + bp - x_{1c} - x_{2c},\ p(o) = p_o \\
\text{(b)}\quad \dot{\lambda}_{2c} = -(b - r_2 - \frac{\partial x_{1c}}{\partial p})\lambda_{2c} \\
\text{(c)}\quad (\alpha - \mu) - (\beta + \Theta)x_{2c} - \lambda_{2c} = 0 \\
\text{(d)}\quad \lim e^{-r_2 t}\lambda_{2c}(t) = 0
\end{cases}
\end{cases}
$$

By setting

$$\text{(A.2)}\qquad \lambda_{ic} = K_i p + E_i,\ i = 1,2,$$

we have, from (A.1) (i,c) and (ii,c),

$$\text{(A.3)}\qquad x_{ic} = \frac{\alpha - \mu}{\beta + \Theta} - \frac{K_i}{\beta + \Theta}p - \frac{E_i}{\beta + \Theta},\ i = 1,2.$$

By differentiating (A.2) with respect to t, and equating the results to (A.1) (i,b) or (ii,b) respectively, one gets

(A.4) $$\{K_i + K_i(b + \frac{K_1}{\beta+\Theta}\frac{K_2}{\beta+\Theta}) + (b - r_i + \frac{K_i}{\beta+\Theta})K_i\}\ p$$
$$+\{K_i(a - \frac{2(\alpha-\mu)}{\beta+\Theta} + \frac{E_1}{\beta+\Theta} + \frac{E_2}{\beta+\Theta}) + (b - r_i + \frac{K_i}{\beta+\Theta})E_i + \dot{E}_i = 0$$

for i = 1,2.

Equality in (A.4) should hold for all variations of p. Thus, we get

(A.5) $$\begin{cases} \dot{K}_i + K_i\ (b + \frac{K_1}{\beta+\Theta} + \frac{K_2}{\beta+\Theta}) + (b - r_i + \frac{K_i}{\beta+\Theta})\ K_i = 0 \\ K_i(a - \frac{2(\alpha-\mu)}{\beta\quad\Theta} + \frac{E_1}{\beta+\Theta} + \frac{E_2}{\beta+\Theta}) + (b - r_i + \frac{K_i}{\beta+\Theta})E_i + \dot{E}_i = 0 \end{cases}$$

for i = 1,2.

Assuming that $\dot{K}$ and $\dot{E}$ converge to zero as t tends to infinity, we get

(A.6) $$\begin{cases} (2b - r_i + \frac{K_i}{\beta+\Theta} + \frac{K_2}{\beta+\Theta} + \frac{K_i}{\beta+\Theta})\ K_i = 0 \\ K_i(a - \frac{2(\alpha-\mu)}{\beta+\Theta}\ \ \frac{E_1}{\beta+\Theta} + \frac{E_2}{\beta+\Theta} + (b - r_i + \frac{K_i}{\beta+\Theta})E_i = 0 \end{cases}$$

for i = 1,2.

One set of signular solutions is obtained as:

(A.7) $$K_1 = K_2 = 0$$
$$E_1 = E_2 = 0.$$

The other set of stable solutions is obtained after some algebraic manipulations

(A.8)
$$K_1 = \frac{(2b - 2r_1 + r_2)(\beta+\Theta)}{3}$$
$$K_2 = \frac{(2b - 2r_2 + r_1)(\beta+\Theta)}{3}$$
$$E_1 = \frac{(2b - 2r_1 + r_2)\ [2(\alpha-\mu) - a(\beta+\Theta)]}{3b}$$
$$E_2 = \frac{(2b - 2r_2 + r_1)[2(\alpha-\mu) - a(\beta+\Theta)]}{3b}$$

By substituting (A.8) into (A.2), (A.3) and (A.1) (ii,a) or (ii,a) in that order, we get the desired results.

The two other unstable pairs of strategies as other solutions of the Ricatti equations are

(A.9)
$$\begin{cases} \text{(a)} \quad x^*_{1c} = \dfrac{\alpha - \mu}{\beta + \Theta} \\ \text{(b)} \quad x^*_{2c}(p(t)) = \dfrac{\alpha - \mu}{\beta + \Theta} - \left(\dfrac{2b - r_2}{2b}\right)\left(\dfrac{\alpha - \mu}{\beta + \Theta} - a\right) + \dfrac{2b - r_2}{2} p \end{cases}$$

and

(A.10)
$$\begin{cases} \text{(a)} \quad x^*_{1c} = \dfrac{\alpha - \mu}{\beta + \Theta} - \left(\dfrac{2b - r_1}{2b}\right)\left(\dfrac{\alpha - \mu}{\beta + \Theta} - a\right) + \dfrac{2b - r_1}{2} p \\ \text{(b)} \quad x^*_{2c} = \dfrac{\alpha - \mu}{\beta + \Theta} \quad . \end{cases}$$

Corresponding to these strategies, the fish population moves along the paths stipulated by the following dynamics

(A.11)
$$\dot{p} = -\frac{r_i}{2b}\left(\frac{2(\alpha - \mu)}{\beta + \Theta} - a\right) + \frac{r_i}{2} p \quad \text{for } i = 1,2,$$

which are unstable due to the fact that $r_i > 0$ for $i = 1,2$.

It is obvious that these two solutions do not really satisfy (2.7) (i, d) or (ii, d). Therefore, even though these are solutions of the Ricatti equations, they are not those of (2.7).

similar results as above can be obtained for the open-loop strategy case.