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massachusetts
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50 memorial drive
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Efficient Wage Dispersion*

Daron Acemoglu† **Robert Shimer**‡

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Abstract

In market economies, identical workers receive very different wages, violating the Walrasian 'law of one price'. We argue that in the absence of a Walrasian auctioneer to coordinate trade, wage dispersion among identical workers is an equilibrium phenomenon. Moreover, wage dispersion is necessary for an economy to function efficiently. In the absence of wage dispersion, workers have little incentive to gather information, effectively giving monopsony power to firms. This depresses wages and leads to excessive entry of firms and worker nonparticipation. Adding to these inefficiencies, firms curtail irreversible capital investments in these tight labor market conditions, and so the quality of jobs is excessively low. Finally, our theory predicts that if *ex ante* homogeneous firms can choose their production technology, those that offer higher wages will choose better technologies. Therefore, in contrast to previous explanations of wage dispersion based on exogenous technology differences, our approach explains technology dispersion among firms as a result of endogenous wage dispersion and suggests that both of these phenomena may be efficient from a third-best viewpoint.

Keywords: Efficiency, Search, Sorting, Wage Dispersion, Wage Posting, Technology Choice, Technology Dispersion.
JEL Classification: D83, J41, J31.

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†Department of Economics, Massachusetts Institute of Technology. E-mail: `daron@mit.edu`

‡Department of Economics, Princeton University. E-mail: `shimer@princeton.edu`

1 Introduction

In market economies, identical workers receive very different wages (Krueger and Summers, 1988). An important component of this wage dispersion comes from different firms paying different wages for the same jobs in the same line of business (Reynolds, 1951; Abowd and Kramartz, 1994; and Groshen, 1991). One might be tempted to attributed this wage dispersion to unobserved worker and job heterogeneity (Murphy and Topel, 1986). And yet, this does not seem to be the whole story. Worker heterogeneity cannot explain why workers who move from a low wage to a high wage industry receive a wage increase in line with the wage differential between these two sectors (Krueger and Summers, 1988; Gibbons and Katz, 1992). Job heterogeneity cannot explain why high wage jobs attract significantly more applicants (Holzer, Katz, and Krueger, 1991). Thus existing explanations of wage dispersion view it as a failure of market economies. The same commodity, a homogeneous worker, trades at two different prices, distorting the allocation across firms. In other words, an important cornerstone of Walrasian markets, *the law of one price*, fails.

In this paper, we argue that in the absence of a Walrasian auctioneer to coordinate trade, wage dispersion among identical workers is an equilibrium phenomenon. Furthermore, this violation of the law of one price is necessary for the economy to achieve some degree of efficiency. Without wage dispersion, workers have no incentive to gather information about available jobs (*i.e. search*). The resulting imperfect information gives firms monopsony power and depresses wages below workers' social shadow value. Low wages entice firms to enter in large numbers, which is extremely costly, and simultaneously discourage worker participation. Finally, firms, anticipating that they will have trouble hiring workers in this tight labor market, expect a low marginal product of capital. This reduces *ex ante* investments and leads to the creation of many low quality jobs. In sum, an economy without wage dispersion is highly inefficient.

In contrast, when there is wage dispersion, workers optimally expend some effort searching for high wage jobs. This increases the effective competition for labor and induces firms to offer higher wages in order to attract workers. The resulting *average wage* is closer to the social shadow value of labor. Moreover, firms that offer higher wages enjoy longer job queues and choose to create better, *i.e.* more capital intensive, jobs. Thus the efficiency of wage dispersion translates into *efficient technology dispersion*, even though all firms face the same production possibility frontier. Overall, our analysis establishes that equilibrium wage dispersion is necessary for the economy to achieve any degree of efficiency in a number of dimensions.

1.1 Equilibria With and Without Dispersion

We analyze these issues in a general equilibrium search model with fully optimizing agents on both sides of the market. We explicitly build on the partial equilibrium search literature pioneered by Stigler (1961), embedding workers' wage search into a general equilibrium model in which firms make optimal entry, capital investment, and wage offer decisions. Workers choose how much information to gather about the available wages; formally, they sample an optimal number of firms, and learn the wage offered by those firms. Then based on the sampled information, they decide where to apply for a job. An important assumption is that there are decreasing returns to labor at the firm level, so firms do not always hire all the available workers at a given posted wage. As a result, workers have to anticipate that they may not receive the job for which they apply. Thus a fundamental coordination problem arises in the absence of the Walrasian invisible hand: some workers will be unemployed, because they are unlucky and wind up at the back of a long job queue. Recognizing this possibility, in their application decisions, workers will trade off higher wage earnings for lower probabilities of obtaining a high wage job. This encourages firms to 'locate' at different points in the wage distribution. A higher wage reduces profit margins but raises the expected length of job queues, and thus reduces the probability that labor supply falls short of demand.

Workers' sample size or 'search intensity' is a crucial decision in this economy. Previous analyses have typically incorporated this margin by introducing a search effort decision that increases the probability of a 'match' (see Pissarides, 1990). Yet this reduced form modelling ignores important strategic considerations: a more disperse wage distribution tends to increase workers' optimal sample size; and how disperse the wage distribution is in turn depends on workers' sample sizes. The former observation has long been recognized in the partial equilibrium search literature (Rothschild, 1974), although the latter has received less attention. A disperse wage distribution is only possible if workers become *informationally heterogeneous*, *i.e.* some workers learn about many jobs, while others take the first one that comes along. Workers, on the other hand, only choose to become informationally heterogeneous if there is an appropriate degree of wage dispersion, enough so that workers are indifferent about how much information they gather.

It is often claimed that wage dispersion among firms within a narrowly defined sector can be attributed to the firm's technology choice (see Reynolds, 1951, for some case study evidence in line with this view).[1] However, if firm heterogeneity explains wage dispersion, then one should equally like to understand why there is persistent 'technology' dispersion. Our model provides a parsimonious answer to these previously unexplored questions. Even though firms have access to a common production technology, in equilibria with wage dispersion, they will *choose* different capital intensities. This is due to a natural single-crossing property: firms that offer high wages enjoy long job queues and have a high expected marginal product of capital, hence they invest more. Therefore, in contrast to the conventional view, that technology dispersion causes wage dispersion and that technology dispersion is exogenous, this paper argues that wage dispersion causes technology dispersion, and offers a succinct explanation for equilibrium wage dispersion.

[1] Of course, this claim is inconsistent with a competitive labor market, in which identical workers must earn a common wage.

1.2 Efficiency

The focus of this paper is not on the mere existence of equilibria with wage and technology dispersion, but on their relative efficiency. Our main point is easiest to understand by temporarily ignoring firms' investment decision and focusing only on their binary 'entry' margin. When there is no wage dispersion, workers do not search and firms have complete monopsony power over workers who apply for their jobs. Hence firms offer them a wage equal to the disutility of labor. However, this wage is below the social shadow value of the worker, causing inefficiencies. More explicitly, if workers still participate in the market with this low wage, firms will enter in excessive numbers because each employment relation creates high profits. Since firm entry has a positive cost, this method of rent-dissipation is excessively costly. Alternatively, with such low wages, workers may prefer nonparticipation, as in Diamond's (1971) paradox; then the economy collapses. Both sources of inefficiency are avoided with wage dispersion. When wages are disperse, workers search in order to obtain better wages and this forces firms to pay higher wages. Since potential profits are now lower, excessive entry is avoided, and because expected wages are higher, workers are willing to participate.

A second reason that wage dispersion is efficient emerges when we endogenize firms' capital investment (technology) decision. As noted above, we prove the existence of equilibria in which different firms make different *ex ante* investments, and firms that make larger investments offer higher wages. Given that the marginal product of labor differs across firms, it is perhaps not as surprising that there is wage dispersion. However, the *efficiency* of wage dispersion in this environment is extremely surprising. It is well-known that if firms making larger *ex ante* investments must pay higher wages, they will tend to underinvest, because workers will appropriate some of the profits from the investment through this upward sloping wage-investment schedule (Grout, 1984). Still, an equilibrium with a flat wage-investment schedule, *i.e.* with no wage dispersion, is much worse. In an equilibrium without wage dispersion, workers will not gather wage information, and so firms cannot use wages to increase the probability of filling their jobs. Effectively, labor is rationed and the rationing scheme does not depend on the marginal product of labor at particular firms. Free entry then drives firms towards the technology that uses the minimum capital-output ratio, causing severe underinvestment. In other words, in order for firms to make substantial *ex ante* investments, high productivity firms must be able to attract workers at faster rate, a phenomenon that we refer to as *sorting* (the opposite of the rationing scheme without search). Sorting is only possible if workers search, and workers only search if there is wage dispersion. Summarizing this logic, sorting requires firms to offer an upward sloping wage-investment schedule.

Very strikingly, we also find that in the limit in which search costs become very small, the equilibrium with wage dispersion converges to an allocation in which the participation of workers, the entry of firms and firms' *ex ante* investment choices are all constrained efficient. That is, the limit point of the search equilibrium maximizes output in the economy, subject to the fundamental coordination problem caused by workers making independent applications. In traditional search models with wage bargaining, it is not possible to ensure efficiency under any circumstances, and so it is extremely surprisingly that with wage posting, efficiency is achieved under very mild informational requirements. In particular, in this efficient allocation, a fraction of workers sample two wages and the rest sample only one.

We also find that in a static model (Sections 2–4), efficient wage dispersion is equivalent to the statement that an equilibrium in which workers use a fixed sample of size one is much less efficient than an equilibrium with a larger fixed sample size. In a dynamic environment (Section 5), an equilibrium with pure sequential search, in which every worker learns the result of one application before sampling the next, is much less efficient than an equilibrium with a mixture of fixed sample size and sequential search.

To summarize, search and information gathering by workers are necessary for the economy to achieve efficiency in a number of dimensions, and this is only possible if there are rewards to search, namely, wage dispersion.

1.3 Related Literature

Numerous previous papers have found equilibrium wage or price dispersion in search models. See Albrecht and Axell (1984), Robert and Stahl (1993), Salop and Stiglitz (1982), Sattinger (1991) and Stahl (1994), among others. Burdett and Mortensen (1989) generate wage dispersion, because low wages raise the probability of quits; some firms choose to pay higher wages and have lower turnover costs. Butters (1977) and Lang (1991) work with models in which workers receive random amounts of wage information from firms, thereby generating informational heterogeneity and equilibrium wage dispersion. These models do not endogenize informational heterogeneity; and therefore, do not reach our crucial conclusion regarding the link between wage dispersion and efficiency.

Mortensen and Pissarides (1994) obtain disperse wages for identical workers in a model in which firms have exogenously different productivities; workers appropriate some of the profit of high productivity firms through bilateral bargaining. Montgomery (1991) obtains a similar result in a model with wage posting and no sampling costs, because high productivity firms are more eager to attract workers. Neither of these models, however, explain the source of this technology dispersion.

Our paper is most closely related to the important work by Burdett and Judd (1983). In a buyer-seller model of search, they establish equilibrium price dispersion with homogeneous and optimizing agents on both sides of the market, and the reason, as in our paper, is endogenous informational heterogeneity. However, there is a significant difference between Burdett and Judd's model and ours, which makes welfare analysis impossible in their framework. They assume that if a consumer walks into a store, he will purchase the good at the posted price. This is equivalent to constant returns to the variable input: namely, a firm's profit is its markup times its expected queue length. In contrast, we assume that there are decreasing returns to the variable inputs, labor and capital, in addition to a small fixed cost. Thus, as in Peters (1991) and Montgomery (1991), firms always face 'capacity constraints'. This gives us a clear definition of a firm, which enables us to endogenize entry and investment, and to analyze efficiency. Burdett and Judd treat entry, investment, and participation as exogenous, and simply establish equilibrium price dispersion.[2]

1.4 Outline

In Section 2 we describe a static general equilibrium search model; for simplicity, we ignore firms' investment decision, and instead impose that all active firms are *ex post* identical. We characterize the equilibria of this economy in some detail: there will always exist an equilibrium with little sampling and no wage dispersion; and for moderate sampling costs, there will also exist equilibria in which workers sample many jobs and wages are disperse. We provide closed form solutions for variables of interest, including the wage distribution and the number of active firms. Some of the arguments we develop in this section are of independent interest, but we justify our detailed analysis of this simple economy because it facilitates the development of the more complex environments later in the paper. Section 3 demonstrates that equilibria with wage dispersion and sampling Pareto dominate the equilibrium without wage dispersion, thus establishing this paper's title. In Section 4, we endogenize firms' investment decisions, which allows us to address several new issues. We establish the close link between wage and technology dispersion, showing that technology dispersion is an immediate consequence of wage dispersion. We also show that wage dispersion encourages workers' sampling, which in turn enables high productivity firms to attract longer job queues (sorting). Section 5 embeds our results in a dynamic general equilibrium framework. It demonstrates that all the results of the one-period model generalize to the dynamic environment and shows that in this case, equilibrium (and efficient) search is a combination of fixed sample and sequential search. Section 6 concludes.

2 The Static Model

2.1 The Environment

We will first present a one-period economy with no investment. There is a continuum 1 of workers and a larger continuum of firms. Each firm has access to a simple Leontieff technology; it produces 1 unit of output if it employs a worker, and cannot employ more than 1 worker. This captures decreasing returns to labor in a simple

[2]To endogenize entry in Burdett and Judd's model, it would be necessary to close the model with an entry cost. But as search costs disappear, prices converge to the Walrasian level, marginal cost. Firms are then be unable to cover their fixed cost.

form. All agents have linear utility. The disutility of work and unemployment benefit for workers are normalized to 0.

The sequence of events is as follows.

1. Firms decide whether to post a vacancy or not. Posting a vacancy costs γ. We denote the set of firms that post a vacancy by $\mathcal{V}$ and the measure of the set $\mathcal{V}$ by V.

2. Each firm $i \in \mathcal{V}$ posts a wage $w_i \in \mathbb{R}$.

3. Each worker decides how many vacancies to sample (her search intensity). A worker who samples $n \geq 1$ vacancies learns the wage offered by n randomly chosen firms $i \in \mathcal{V}$.

4. Each worker applies to at most one of the firms she has sampled. Workers rationally anticipate that a high wage will also attract other applicants; since each vacancy corresponds to 1 job, the worker will then have a lower probability of actually obtaining a high wage job. The exact form of this trade-off is determined in equilibrium.

5. Finally, each vacancy chooses one of the applicants, and pays the posted wage. Since applicants are homogeneous, the decision of whom to hire is arbitrary. Naturally there may also be some firms without applicants.

The need for sampling comes from the fact that, although workers rationally anticipate the equilibrium *distribution* of wages, they do not know which wage a *particular* firm is offering (see Stigler, 1961). Because all firms appear to be identical before individual wages are observed, a worker randomly samples n of them, and learns the wage offered by the firms that it samples.[3] The cost of sampling the first job, c_1, will be important in determining whether workers participate or not; at this stage, we leave it unspecified. We assume that the cost of sampling the n^{th} job, $n \geq 2$, is $c_n > 0$. We also assume that c_n is weakly increasing, which is equivalent to the cost of search being weakly convex.

Point 4 expresses the fundamental coordination problem in our economy. Even if there are N jobs and N workers, not all workers will be employed nor will all jobs be filled. This is because some workers will apply to the same jobs, and some firms will not receive any applicants (see also Peters, 1991). In contrast, a Walrasian auctioneer would ensure that each job receives one worker, and this coordination problem would not have arisen.

Let λ_n be the proportion of workers sampling n jobs (or equivalently, the probability that a representative worker samples n jobs). We naturally have $\sum_{n=0}^{\infty} \lambda_n = 1$, and we let $q \equiv 1/V$ be the inverse of the tightness of the labor market. We also denote the cumulative wage distribution by $G(w)$, and let $\mathcal{W}$ be the support of G.

[3]This is an *urn-ball* search technology; each of qN workers throws n balls with equal probability into the N urns representing active firms. We take the limit as N goes to infinity. We work with a continuum of agents, rather than a countable infinity, for notational convenience.

2.2 Defining an Equilibrium

Let $p_n(i; w_1, ..., w_n) \in [0, 1]$ be the probability that a worker who samples n jobs described by the vector $\mathbf{w} = (w_1, ..., w_n)$ applies to the i^{th} job he has located, *i.e.* the job offering w_i. This function will be determined in equilibrium from workers' preferences. At this stage, we impose two restrictions on p. First, since p_n is a probability, $\sum_{i=1}^{n} p_n(i; \mathbf{w}) \leq 1$, where we allow for the possibility that a worker applies to none of the jobs she samples. Second, since there is no inherent order to a sample, we impose the symmetry restriction that for any two vectors $\mathbf{w}_{ij}$ and $\mathbf{w}_{ji}$, identical except that their i^{th} and j^{th} elements have been swapped, $p_n(i, \mathbf{w}_{ij}) = p_n(j, \mathbf{w}_{ij})$. This implies in particular that for some vector $\mathbf{w}$ with $w_i = w_j$, we must have $p_n(i, \mathbf{w}) = p_n(j, \mathbf{w})$.

Building on the preferences embodied in p and the wage distribution G, we define the probability that a worker who samples n random wages from distribution G including one offering w applies to the one offering w as:

$$\hat{G}_n(w) = \int \cdots \int p_n(1; w, w_2, ..., w_n)\, dG(w_2) \cdots dG(w_n) \tag{1}$$

Let us also define $\Sigma(w) = \sum_{n=1}^{\infty} n\lambda_n \hat{G}_n(w)$, which will be the probability that a 'representative' worker applies to a firm offering w, conditional on sampling such a firm. Dividing this by the vacancy-unemployment ratio, or equivalently multiplying this by q, we obtain the expected number of applications that a firm posting w receives. Therefore, the probability that a firm posting the wage w receives at least one application (or equivalently, 1 minus the probability that it receives none) is:[4]

$$P(w) \equiv 1 - e^{-q\Sigma(w)} \tag{2}$$

The gross expected profit of a firm offering wage w is

$$\pi(w) = \left(1 - e^{-q\Sigma(w)}\right)(1 - w) \tag{3}$$

Similarly, the expected return of a worker applying to a job offering w is:

$$\rho(w) = \frac{1 - e^{-q\Sigma(w)}}{q\Sigma(w)} w \tag{4}$$

In words, the return from applying to a vacancy offering w is the probability of obtaining this job times the wage. The fraction in (4) is the probability that a worker applying to w is hired, which is the ratio of the probability that a firm posting w hires a worker, $P(w)$, and the expected queue length or number of applicants for the job, $q\Sigma(w)$.

[4]Since all workers are homogeneous, the firm is indifferent between any positive number of applications. Also, note that workers do not observe how many other workers have sampled a particular firm, and hence cannot condition their applications on this variable.

We also denote the expected return to sampling n wages drawn randomly from G as R_n, $n \geq 1$.

$$R_n = n \int_{\mathcal{W}} \rho(w) \hat{G}_n(w) \, dG(w) - \sum_{i=1}^{n} c_i \tag{5}$$

Also let R_0 denote the return to a non-participating worker, normalized to zero.

Now we can define an equilibrium of this static labor market:

Definition 1 *An equilibrium consists of a measure of active firms V, a distribution of posted wages G with support $\mathcal{W}$, an expected profit function for firms π, expected return functions for workers ρ and R, preference functions $\{p_n\}$, and sample size decisions for workers $\{\lambda_n\}$ such that:*

1. **(Profit Maximization)** $\forall w \in \mathcal{W}$ *and* $\forall w'$, $\pi(w) \geq \pi(w')$
2. **(Free Entry)** $\forall w \in \mathcal{W}$, $\pi(w) = \gamma$.
3. **(Optimal Application)**[5] $\forall n \in \mathbb{N}$, $\forall \mathbf{w} \equiv (w_1, ..., w_n)$,

$$p_n(i; \mathbf{w}) = 0 \text{ if } \rho(w_i) < \max_{j \neq i} \rho(w_j) \text{ or if } \rho(w_i) < 0, \forall i$$

and

$$\sum_{i=1}^{n} p_n(i; \mathbf{w}) = 1 \text{ if } \max_j \rho(w_j) > 0$$

4. **(Optimal Sample)** $\lambda_{n^*} > 0$ *only if* $n^* \in \arg\max_{n \in \mathbb{N}} R_n$.

2.3 Equilibria Without Search

We begin by establishing the existence of a 'no-search' equilibrium, which we define as an equilibrium in which no worker samples more than one job, $\lambda_0 + \lambda_1 = 1$.[6]

Proposition 1 *There exists an equilibrium in which no worker samples more than one job. In any such equilibrium, all active firms offer a zero wage, and the unemployment-vacancy (u-v) ratio is $q\lambda_1 = -\log(1-\gamma)$. If c_1 is negative (positive, zero), then λ_1 is one (zero, free), while $\lambda_0 = 1 - \lambda_1$.*

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[5] These two statements imply that a worker always applies for a wage $w_i > 0$ if it is the most preferred wage in her sample. The statements also allow for the case when workers have multiple 'most preferred wages'.

[6] We follow Stigler (1961) in reserving the term 'search equilibrium' to refer to an equilibrium in which some workers sample multiple jobs, i.e. $\lambda_0 + \lambda_1 < 1$. Stigler writes "A buyer (or seller) who wishes to ascertain the most favorable price must canvass various sellers (or buyers) — a phenomenon I shall term 'search'." (p. 213)

Like all other proofs in this paper, we put the proof of Proposition 1 in the Appendix.

The existence of a no-search equilibrium is not surprising. A worker will not search if there is a degenerate wage distribution. This implies that all firms have full monopsony power over workers who apply to their vacancies and can offer a wage equal to 0. If sampling the first job is pleasant ($c_1 < 0$), then in equilibrium workers will randomly apply to a single firm. However, if it is costly ($c_1 > 0$) then the Diamond (1971) paradox will prevail in equilibrium: anticipating the zero wages, workers will not participate in the labor market, and there will be no production. In the borderline case $c_1 = 0$, some workers may drop out of the labor force, while others sample a single job.

2.4 Equilibria With Search

We now turn our attention to 'search equilibria' in which some workers sample multiple jobs, *i.e.* $\lambda_0 + \lambda_1 < 1$. Characterizing search equilibria is considerably more complex, and we therefore proceed in steps.

2.4.1 Optimal Applications and the Wage Distribution

We begin by characterizing workers' application decision. We find that in equilibrium workers prefer to apply for higher wages, even though high wage jobs are associated with long queues (Lemma 1). We then use this finding to characterize the support of the wage distribution (Lemma 2).

First, combining equation (3) with part 2 of Definition 1 (Free Entry), we obtain that $\forall w \in \mathcal{W}$:

$$\gamma = \left(1 - e^{-q\Sigma(w)}\right)(1 - w) \tag{6}$$

The right hand side is the gross profit level of a firm offering wage w. For all wages offered in equilibrium, this must equal γ.

Next, we define $\tilde{\rho}(w \mid \gamma)$, the return of a worker applying to a wage w, under the (possibly hypothetical) assumption that the firm offering this wage makes gross profits equal to γ. Mathematically, this function is obtained by solving (6) for $q\Sigma(w)$ and substituting in (4). It is the return to a worker applying for wage w, once the probability that other workers applying for this wage is adjusted so as to make this firm's gross profit equal to γ. The expression for $\tilde{\rho}(w \mid \gamma)$ is given as:

$$\tilde{\rho}(w \mid \gamma) = \frac{\gamma w}{(1-w)(\log(1-w) - \log(1-w-\gamma))} \tag{7}$$

It can be verified easily that $\tilde{\rho}$ is a strictly quasiconcave function of w. Also $\tilde{\rho}$ is equal to 0 when $w = 0$ or when $w = 1 - \gamma$, and is maximized at an intermediate point w^* which is the unique solution to:

$$(1 - w^* - \gamma)\left(\log(1 - w^*) - \log(1 - w^* - \gamma)\right) = \gamma w^* \tag{8}$$

If a wage is offered in equilibrium, then $\pi(w) = \gamma$. Therefore, $\tilde{\rho}(w \mid \gamma) = \rho(w)$ whenever $w \in \mathcal{W}$, and so $\tilde{\rho}$ also describes the expected payoff to a worker of applying for any wage offered on the equilibrium path. We use this fact to prove a key result for the rest of our analysis.

Lemma 1 *In equilibrium, ρ is strictly increasing on $\mathcal{W}$.*

This lemma establishes that *in equilibrium*, the fact that there is more competition for high wage jobs does not deter applicants. Workers expect a higher return if they apply for a higher wage, and therefore always apply for the highest wage that they sample. Thus, if $w_i > \max_{j \neq i} w_j$ and $w_i > 0$, then $p_n(i; w_1, ..., w_n) = 1$.

That workers weakly prefer higher wages is not surprising. However, our result is stronger: workers *strictly* prefer higher wages. One might imagine that competition for higher wages is sufficiently severe, so that the decreased chance of being accepted just offsets the increased reward from being hired. Lemma 1 precludes the possibility of indifference by exploiting the properties of $\tilde{\rho}$ and the equivalence of ρ and $\tilde{\rho}$ on $\mathcal{W}$.

Using Lemma 1, we can fully characterize the support of the wage distribution.

Lemma 2 *Assume $\lambda_0 + \lambda_1 < 1$. The support of the equilibrium wage distribution $\mathcal{W}$ consists of a convex non-empty interval $[0, \bar{w}]$ and possibly the point $w^* \geq \bar{w}$. The wage distribution G is atomless on $[0, \bar{w}]$ but may have an atom at w^*.*

This lemma proves that as long as some workers search, only two types of wage distributions are admissible in equilibrium: a continuous distribution on a convex support without any mass; and a distribution that is continuous on a convex support and then has a positive mass at w^*.

Lemma 2 captures Rothschild's (1973) criticism of search models, but turns it on its head: if all firms offer the same wage, why should anyone sample multiple jobs? If *some* workers sample multiple jobs, then the equilibrium wage distribution must not be degenerate. This intuition also relates to the informational externality identified by Grossman and Stiglitz (1980) in the context of a stock market. They show that when prices transmit all the relevant information, no trader will exert effort to find out additional information. Thus for traders to invest in information, there must be a sufficient degree of noise in the system. Similarly, in our economy, for workers to search, there must be a sufficient wage dispersion.

Because the wage distribution is atomless everywhere except at w^*, for any $w \neq w^*$ it is with probability zero that a worker sampling a finite number of firms receives two offers at w. Since in addition workers always apply for the highest wage they sample (Lemma 1), equation (1) implies $\hat{G}_n(w) = G(w)^{n-1}$ for all $w \neq w^*$.[7]

[7] One can also solve for $\hat{G}_n(w^*)$, although the argument is slightly more subtle, and is unnecessary for our analysis. A fraction $1 - G(\bar{w})$ of firms offer this high wage. In a sample of size n, the expected number of firms posting w^* is thus $n(1 - G(\bar{w}))$. The probability that at least one firm posts w^* is $1 - G(\bar{w})^n$. The ratio of these two numbers is the expected number of firms posting w^*, conditional on receiving at least one offer of w^*; and the inverse of this ratio is the probability that the worker applies to any one such firm. Thus $\hat{G}_n(w^*) = (1 - G(\bar{w})^n) / n(1 - G(\bar{w}))$.

Using this, we can rewrite equation (5), the return to sampling n wages, in terms of the equilibrium wage distribution:

$$R_n = n \int_0^{\bar{w}} \rho(w) G(w)^{n-1} dG(w) + \rho(w^*) \left(1 - G(\bar{w})^n\right) - \sum_{i=1}^{n} c_i \tag{9}$$

R_n equals the payoff if the highest wage is drawn from the atomless part of the wage distribution, times the probability of this event; plus the payoff if the sample includes w^*, times the probability of this event; minus search costs.

2.4.2 Optimal Sample Size

In this subsection, we characterize the optimal sample size. The main result is that in equilibrium, no worker will sample more than two jobs. The first step in the proof is showing that there is no equilibrium in which all workers sample multiple jobs.

Lemma 3 *In any equilibrium with* $\lambda_0 + \lambda_1 < 1$, $\lambda_1 > 0$.

In a search equilibrium where some workers sample multiple jobs, *i.e.* $\lambda_0 + \lambda_1 < 1$, it must be the case that some other workers still sample only one job, that is $\lambda_1 > 0$. Intuitively, workers sample multiple jobs only if there is a non-degenerate wage distribution. However, if all active workers sample many jobs, then given the monotonic preferences established in Lemma 1, no worker would apply to the lowest wage firm, and it would have zero gross profits, violating the free entry condition. Thus some workers must always choose a sample size of one.

Next, we show that there are decreasing returns to sample size, n:

Lemma 4 *In equilibrium,* $R_n - R_{n-1} \geq R_{n+1} - R_n \; \forall n \geq 1$, *and strictly if* $\lambda_0 + \lambda_1 < 1$.

This is a well-known result in search models with fixed sample sizes (see for example Stigler, 1961). Intuitively, sampling an additional job is only worthwhile if the job turns out to be superior to the other jobs that a worker has sampled. This becomes less and less likely as the worker samples more jobs. Equivalently, there are decreasing returns to search, because the probability that the worker decides to apply for the last job she has sampled is decreasing in the total number of jobs she samples, and also because marginal search costs are nondecreasing.

Lemmata 3 and 4 imply that some workers sample one job and the remaining workers either all sample zero jobs or else all sample two jobs:

Lemma 5 *In any equilibrium with* $\lambda_0 + \lambda_1 < 1$, $\lambda_1 + \lambda_2 = 1$.

Sampling is costly. For there to be sufficient benefits to search, there needs to be a sufficiently disperse distribution of wages. However, firms will only be willing to offer a disperse wage distribution when some workers take the first job that comes along. Therefore, irrespective of the costs of search, a number of workers will always sample only one job. This will support a distribution of wages which will make it worthwhile for others to search. In other words, in equilibrium some workers must *free-ride* on the sampling of others.

A very convenient implication of this lemma is that no worker samples three or more wages in any equilibrium. Since the no-search equilibrium, i.e. equilibria with $\lambda_0 + \lambda_1 = 1$, are fully characterized by Proposition 1, we focus attention on search equilibria with $\lambda_2 > 0$. We also adopt the notation $z \equiv \lambda_2$, and refer to z as the fraction of workers who search. Also, from Lemma 5, in a search equilibrium $\lambda_1 = 1 - z$.

2.4.3 Characterizing Equilibria

In this subsection, we characterize the wage distribution in search equilibria. We first solve explicitly for $G(w)$ from the zero-profit condition of firms, and then find the tightness of the labor market consistent with equilibrium. Finally, we prove the existence of this equilibrium.

With the fraction of workers who sample two wages equal to z, $\Sigma(w)$ can be substituted in equation (3) to give:

$$\pi(w) = \left(1 - e^{-q((1-z)+2\hat{G}_2(w)z)}\right)(1 - w) \tag{10}$$

where $\hat{G}_2(w)$ is the probability that a worker applies to w rather than another wage randomly drawn from G. From the Free Entry condition of equilibrium, $\pi(w) = \gamma$. Using this and inverting (10), we get an expression for $\hat{G}_2$:

$$\hat{G}_2(w) = \frac{\log(1-w) - \log(1-w-\gamma) - q(1-z)}{2qz} \tag{11}$$

It is easily verified that $\hat{G}_2(w)$ is an increasing function of w. In fact, as noted above $\hat{G}_2(w) = G(w)$ for all $w \neq w^*$, thus (11) gives the distribution of wages except at the point of atom, w^*.

Now by Lemma 2, $0 \in \mathcal{W}$, and so by free entry, $\pi(0) = \gamma$. Also, by Lemma 1, $\hat{G}_2(0) = 0$. Then we can immediately solve for the equilibrium u-v ratio (or labor market 'slackness' q) only as a function of the search behavior of workers, z:

$$q = \frac{-\log(1-\gamma)}{1-z} \tag{12}$$

From (12), q is positive and increasing in z. Note that at $z = 0$, this expression corresponds to the equilibrium u-v ratio without search, given by Proposition 1. More generally, with a higher proportion of workers searching, the labor market is less tight. The intuition for this result is simple but instructive: when more workers search, firms are induced to pay higher wages in order to attract workers who now have more options. Hence, rents are shifted from firms to workers, and this discourages entry. A slightly different way of expressing this intuition is as follows.

There are rents in this economy that need to be dissipated. The dissipation of rents either happens by firms entering in larger numbers until the fixed costs of entry (γ) exhaust the rents, or it takes the form of workers searching for high wage jobs, which induces firms to offer higher wages.

With these results, we are now in a position to fully characterize the equilibrium of this economy. We divide search equilibria into two groups, stable and unstable, based on the notion of fictitious play. At an equilibrium allocation, consider a small increase in z, the fraction of workers sampling two wages. If this leads to a decrease (increase) in the marginal return to searching for the 2^{nd} wage, $R_2 - R_1$, we will say that the equilibrium is stable (unstable).

Proposition 2 *$\exists \bar{c}$ such that $\forall c_2 > \bar{c}$, there does not exist a search equilibrium, and $\forall c_2 \in (0, \bar{c})$, there exist at least two search equilibria. One of these equilibria is stable and one is unstable. In each of these equilibria:*

1. *the support of the wage distribution is $[0, \bar{w}] \cup w^*$, where $\bar{w} < w^*$ defined in (8);*

2. *the wage distribution is characterized by*

$$G(w) = \frac{1-z}{2z}\left(\frac{\log(1-w) - \log(1-w-\gamma)}{-\log(1-\gamma)} - 1\right) \tag{13}$$

on $[0, \bar{w}]$ and by a mass μ of firms at w^*, where

$$\mu = \max\left\langle \frac{1+z}{z} - \frac{1-z}{z}\left(\frac{\log(1-w^*-\gamma) - \log(1-w^*)}{\log(1-\gamma)}\right); 0\right\rangle \tag{14}$$

3. *z is less than a cutoff level $\bar{z} < 1$.*

The appendix gives the proof as well as the expression for some of the variables defined in the proposition (*e.g.* $\bar{w}$, $\bar{z}$).

Observe that although the cost of sampling a second job, c_2, is crucial for the existence and characterization of a search equilibrium, search equilibria are unaffected by the cost of sampling a first job c_1, as long as $c_1 \leq c_2$. This is because search equilibria are defined by workers' indifference between sampling one and two jobs. Decreasing returns to search (Lemma 4) implies that whenever workers are indifferent between sampling one and two jobs, they strictly prefer this alternative to nonparticipation.

The proposition establishes that for any moderate cost of sampling a second job, there exists a distribution of wages such that workers are indifferent between sampling one and two jobs. The informational heterogeneity that results from the diversity in workers' search behavior induces firms to offer a distribution of wages. Because there exist some workers (at least a fraction $1 - \bar{z}$) who will take the first job they see, not all firms will offer w^*. Low wage firms will then be trading off a lower probability of hiring against higher net revenues conditional on hiring.

Observe that in response to changes in z, the wage distribution shifts in the sense of first order stochastic dominance. When there is more search, the entire wage distribution function shifts to the right. This again captures the notion of *free-riding* in this model. When a worker decides to sample more jobs, she improves the distribution of wages for all other workers. In this context, the requirement $z \leq \bar{z}$ makes intuitive sense: if a great majority of workers search, then a firm offering a very low wage would have almost no chance of hiring a worker and would therefore make losses. All firms must offer w^*. But a degenerate distribution of wages at w^* would not justify search.

Proposition 2 characterizes *all* search equilibria, and shows that there must be at least two such equilibria when c_2 is not too large. We also conjecture that there are only two equilibria with search in total, and explain why in the appendix; however, we have been unable to prove this result analytically.

2.5 Discussion

Our main result in this section is the existence and characterization of the equilibria. First, there is an equilibrium with no search and a degenerate wage distribution (Proposition 1). Second, there is a (stable) search equilibrium (Proposition 2) with wage dispersion. Some wage dispersion is necessary for there to be a reward for searching, and yet, equilibrium wage dispersion is necessarily limited, because if dispersion were so great that all workers searched, a firm at the bottom of the wage distribution would never hire anyone.

The intuition for the multiplicity of equilibria is worth discussing. Workers want to search if the distribution of wages is disperse. How disperse this distribution is depends on how many workers search. Essentially, it is this general equilibrium interaction that ensures the multiplicity of equilibria. However, there is more to the economics of this result. The dispersion of wages is non-monotonic in the proportion of workers who search. When no one searches, the wage distribution is degenerate at 0. At the other extreme, when at least a fraction $\bar{z}$ of workers sample two or more jobs, there is a single wage at w^*. In contrast in all intermediate cases, the distribution is diffuse. This implies that the incentive to sample is strongest when an intermediate number of other agents search. Put differently, when $z = 0$ — when no one searches — sampling decisions are *strategic complements*: a worker only searches if others do so, because sampling by others induces a distribution of wages. In contrast when z is positive, sampling decisions are *strategic substitutes*; when others search, this limits the dispersion of the wage distribution, and a worker does not have much incentive to sample.

Finally, it is informative to consider the limit point as γ, the cost of creating an additional vacancy, goes to zero. In this limit, there are an infinite number of open vacancies per active worker, and all vacancies expect zero profit. Because the u-v ratio is zero, it is with zero probability that two workers apply for the same job. Therefore, workers do not have to worry about the fundamental coordination problem of our economy. The return to applying to a wage at w is now $\rho(w) = w$. As a result, this limit point corresponds to Burdett and Judd's (1983) model where all workers would obtain jobs. The equilibrium wage distribution is also well-behaved in this limit, and is given by $G(w) = \frac{(1-z)w}{2z(1-w)}$. Thus, our model nests the Burdett-Judd economy. However, as we show in the next section, our efficiency conclusions, even for γ arbitrarily close to 0, differ radically from what one finds with an exogenous number of firms.

2.6 Comparative Statics and Limit Point

What happens when the cost of search, c_2, falls? First, as is usual in models of multiple equilibria, in order to answer this question we concentrate on stable equilibria. As c_2 falls, the no-search equilibrium continues to exist; however small the cost of search may be, the strategic complementarity at the point of no search is sufficiently strong to preserve this equilibrium.[8]

Next consider the stable search equilibrium, in which z is decreasing in c_2. As one would expect, lower search costs lead to more search. As remarked above, the increase in z translates into a shift of G to a new distribution that first-order stochastically dominates the old one. In particular, as c_2 falls and workers search more, the wage distribution becomes increasingly concentrated at w^*, but the lower support remains at $w = 0$. Finally, in the limit point of $c_2 = 0$, the distribution is degenerate at w^*.

We also observe that w^* is exactly the wage that Moen (1995) and Shimer (1996) find as the unique equilibrium wage in a model in which firms post wages and workers observe all the posted wage offers (see also Mortensen and Wright, 1995). This implies an upper hemicontinuity in the set of equilibria as a function of sampling costs c_n in this class of models, and is reassuring.[9] Yet, there are important differences between this paper and Moen's and Shimer's earlier work. First, we have demonstrated that it is not necessary for workers to observe all the posted wages in order to ensure that all firms post w^*. In fact, a much less stringent condition, that at least a proportion $\bar{z} < 1$ of workers observe *two* of the wage offers, is sufficient. Second, this is not the only equilibrium that is a limit point of our model as search costs fall. As noted above, the no-search equilibrium continues to exist at the limit; Moen and Shimer ignore this equilibrium.

Finally, explicitly or implicitly, these papers impose the condition first proposed by Peters (1991) that search strategies should be *anonymous*; by assumption, one worker cannot decide to apply only to 'blue' firms, while an identical worker applies only to 'green' firms. If each worker could for example search for a job that bore her name, the coordination problem would be solved. However, this solution is only

[8] However, at the limit when $c_2 = 0$ — and only at this limit point — this equilibrium involves workers using *weakly dominated* strategies. By searching for two jobs, they have nothing to lose, but a lot to gain if some firms were to post positive wages.

[9] The set of equilibria is however not *lower* hemicontinuous. When $c_2 = 0$, any search behavior with $z \geq \bar{z}$ is an equilibrium. In all of these equilibria, wages are degenerate at w^*.

possible when workers observe all the wage offers, and hence know which job bears her name. In our model, even in the limit with $c_2 = 0$, workers would observe a finite number of wage offers from a continuum (especially if for some $n > 2$, $c_n > 0$). Coordination using names or other non-wage characteristics of jobs would remain impossible. Therefore, we do not require this additional anonymity restriction.

2.7 Employment and Search

Note an interesting implication of our model: as search costs fall, more workers search and average wages increase. This is because, as discussed in Section 2.6, more search raises z, which according to equation (13) leads to a downward shift of $G(w)$ and thus to higher wages. Because the associated reduction in profits attracts entry by fewer firms, the tightness of the labor market and employment decline. This result is stark in our model because more search does not directly increase the number of applications — each worker still applies to one job. Instead, as we have emphasized before, search is a mechanism for sharing rents. Because search transfers rents from firms to workers, it reduces vacancy creation, and thus employment. One might also imagine that higher search intensity increases the number of firms that a worker contacts; yet, the effect of search on rent distribution, ignored in the literature, will always be present when worker search is explicitly modeled. Then, more search will raise employment if the informational benefits of search outweigh the reduction in job creation caused by rent redistribution.

Some simple extensions to our model allow search to have a potentially positive impact on employment. Here we briefly mention three. The first is to assume that there is a distribution of costs of entering the market for workers. Thus effectively worker i has cost of sampling the first job, c_1^i and the distribution of this variable is given by $\Gamma(c_1)$. If we allow for the possibility that $c_1^i > c_2$ for some workers, then high-cost workers may choose not to participate in the labor market, even in a search equilibrium. While all of our other conclusions remain, the implications for employment are quite different. Let R^* be the equilibrium return to participating in the labor market, ignoring the entry cost c_1^i. This is the same for all workers, since in a search equilibrium workers are indifferent between sampling one or two wages, $R^* = \int_0^{w^*} \tilde{p}(w \mid \gamma) dG(w)$. In this case, only a fraction $N \equiv \Gamma(R^*)$ of workers, those with c_1^i less than R^*, actually participate in the labor market. Thus the number of vacancies in the economy, V, will be equal to N divided by the u-v ratio q. For a given q, an increase in the return to worker search will encourage more vacancy creation. And the probability that an average vacancy hires a worker will be given by[10] $\mathcal{E} \equiv \int_0^{\bar{w}} \left(\frac{\gamma}{1-w}\right) dG(w) + \left(\frac{\gamma}{1-w^*}\right) \mu$, independent of workers' participation decision. Total employment is then $N\mathcal{E}/q$. Now a decrease in search costs c_2 increases search intensity and raises wages. By reducing entry, this lowers the probability that an active worker is hired, $\mathcal{E}/q$. However, by raising the return to participation, this

[10]To calculate this probability, note that a firm offering wage w hires a worker with probability $P(w)$ satisfying $P(w)(1-w) = \gamma$. Then integrate over $P(w)$.

raises the number of active workers. The effect on total employment is ambiguous.

Alternatively, we could assume that there is probability χ that the match between a worker and a firm is not successful, and that workers learn about the success of a match after sampling the firm, but before deciding where to apply. Our analysis so far can be considered a special case with $\chi = 0$. With more search, the number of successful matches, and therefore employment, may increase. Finally, we could allow workers to apply to more than one firm among those that they have located. Then with higher search intensity, the number of applications will increase. We conjecture that the last two extensions leave our main results unchanged, although they introduce new complexities. For example, workers may choose to sample three or more jobs.

3 Welfare

We now investigate whether the decentralized equilibria we characterized in the previous section have some desirable efficiency properties. We show that no equilibrium is socially optimal, even in a constrained sense that accounts for the immutable coordination problem. However, any search equilibrium with wage dispersion Pareto dominates the no-search equilibrium with a degenerate wage distribution. Thus wage dispersion is *required* for 'third-best' efficiency in this market economy.

We will use the notion of efficiency that is the usual one in the literature (*e.g.* Hosios, 1990). We compare equilibria to the choice of a 'social planner' who maximizes total output, but is subject to the same search frictions as the decentralized economy. To set the scene for our results, we remind the reader that the standard Diamond-Mortensen-Pissarides search model, in which workers and firms randomly run into each other and then determine wages via Nash Bargaining, achieves efficiency for one particular value of the bargaining parameter (Hosios, 1990). Moreover, Moen (1995) and Shimer (1996) show that the equilibrium is always efficient if firms post wages and workers costlessly observe all the wage offers before applying for a job. We will contrast our results with these findings.

3.1 Social Optimum

Consider a social planner who chooses the number of active firms V and the search intensity of workers $\{\lambda_n\}$ in an effort to maximize output. There is no reason for the social planner to incur search costs, and therefore, assuming c_1 is not so large that he would shut down the economy, he would choose $\lambda_1 = 1$ and $\lambda_n = 0$ for all $n > 1$. Compared to this allocation, a decentralized search equilibrium is inefficient, because from the planner's perspective, search is an inefficient means of transferring wealth across agents. Thus a search equilibrium can never coincide with the constrained efficient allocation. This result relates to our earlier observation that the role of search is not to increase the number of applications, but to change the distribution of rents.

Can the no-search equilibrium described in Proposition 1 be efficient? The planner would set V to maximize total surplus:

$$\max \left\langle 0\,;\, \left(1 - e^{-1/V}\right) V - \gamma V - c_1 \right\rangle$$

She must choose whether to shut down the economy, which gives zero, or to create jobs, which gives the second expression. Since all workers sample and apply for one randomly chosen job, the first term in the second expression is the number of jobs created times the productivity of a job (= 1). The second term is the cost of the vacancies created, and the third term is the 'cost' of workers searching.

One can easily confirm that the planner would not shut down the labor market when $c_1 < 0$. In this case, her objective function is concave (for $V > 0$) and achieves its maximum at V^* satisfying:

$$1 - e^{-1/V^*} - e^{-1/V^*}/V^* = \gamma \tag{15}$$

This gives the optimal 'tightness' of the labor market when the planner can regulate both the entry of firms and the search behavior of workers.

In the no-search equilibrium with $c_1 \leq 0$, Proposition 1 states that the wage distribution is degenerate at zero and V is given by:

$$1 - e^{-1/V} = \gamma \tag{16}$$

V is always larger than V^*, because compared to (16), (15) has the additional term $-e^{-1/V^*}/V^*$, which captures the *congestion effect*. When the social planner creates an additional vacancy, she takes into account that the reduction in the probability that other firms will be able to hire a worker. In contrast, in the decentralized equilibrium with firms receiving all the output from their match (*i.e.* $z = 0$) this effect is ignored.

What happens when $c_1 \geq 0$? For moderate values of this cost, the no-search equilibrium remains inefficient relative to the planner's choice, although the inefficiency takes a different form. There is no activity in the no-search equilibrium, while the planner prefers to create vacancies and have workers participate. In contrast, for c_1 larger than some positive threshold c^*, the no-search equilibrium is efficient, because the sampling is so costly that the social planner would shut down the economy. The labor markets of interest for us clearly have $c_1 < c^*$.

We summarize this result in the next proposition (proof in the text):

Proposition 3 $\exists c^* > 0$ *such that for all* $c_1 < c^*$, *the decentralized equilibria of Section 2 are inefficient: the no-search equilibrium either has too many vacancies (if* c_1 *is negative) or too little worker participation (if* c_1 *is positive), and in any search equilibrium, workers spend excessive resources sampling jobs. If* $c_1 > c^*$, *then both decentralized economy and the constrained optimum have no production.*

This result is in stark contrast to the efficiency of the decentralized equilibrium without costs of sampling wages. In fact, as noted in the Section 2.6, one of the two limit points of our model with $c_2 \to 0$ is an equilibrium in which $\bar{z}$ workers sample two jobs, and the wage distribution is degenerate at w^*. At this limit, the equilibrium is socially optimal: if all firms offer w^*, free entry implies that there will be V^* vacancies; and since $c_2 = 0$, workers incur no search costs. Comparing our results to this limiting case makes it clear that all the inefficiencies arise because of the sampling decision. Intuitively, by sampling more firms, workers create a number of externalities. As a result, the equilibrium is always inefficient. Either workers incur additional sampling costs that the planner avoids, or there is excessive entry of firms.

It is interesting to note that even though decentralized equilibria are inefficient, there is a government intervention that decentralizes the constrained efficient allocation — a minimum wage at w^*. Then, no firm can offer less than w^*, and no firm would want to offer more (given the form of $\rho(w)$ characterized in Section 2). Workers would save on sampling costs, and the efficient number of firms would enter.

3.2 Efficient Wage Dispersion

Although all decentralized equilibria are inefficient, they can be Pareto ranked. In this section, we prove that any search equilibrium *ex ante* Pareto dominates the no-search equilibrium, in the sense that *ex ante* all workers would strictly prefer to be in a search equilibrium; firms make zero profits in each equilibrium. This is true even though a social planner would never have workers search.

Proposition 4 *Whenever it exists, a search equilibrium* ex ante *Pareto dominates the no-search equilibrium.*

Since no-search equilibria do not involve wage dispersion (Proposition 1) and search equilibria necessarily do (Proposition 2), it follows that wage dispersion is necessary for the economy to achieve *third-best* efficiency.

The intuition for this proposition is very easy to see in the case where c_1 is positive. In the no-search equilibrium, Diamond's paradox applies. This highly inefficient outcome can be avoided if wages were positive, and this is only possible if there is wage dispersion so that workers find it profitable to search. The equilibrium characterized in Proposition 2 does this. With c_1 negative, there is production in the no-search equilibrium, but again the outcome is extremely inefficient. Recall that without search, wages are equal to zero and all the rents go to firms. But, free entry forces all firms to make zero profits, hence rents are dissipated via excessive entry of firms, and costs of vacancy creation are too high.

This result is closely related to the efficiency of search models with bargaining. As shown by Hosios (1990), when firms have too much bargaining power, there will be excessive entry and excessively low unemployment. In our model, the same result obtains in the no-search equilibrium. In contrast, in the search equilibrium with wage dispersion, some rents get transferred to workers. Since worker participation is inelastic, the additional rent dissipation is limited, and welfare is higher.

¿From this discussion it appears that there is some asymmetry between the worker and the firm side. In fact, this asymmetry arises endogenously in our model. On the firm side, when rents are high, all firms want to enter. In contrast, on the worker side, as noted earlier, search intensities are *strategic substitutes*: when other workers are searching a lot, wages are pushed up, and the distribution of wages is concentrated. Therefore, each worker has only weak incentives to exert high search effort. This effect limits the search costs that society has to incur. Thus wage dispersion is the market's means of controlling entry.

Proposition 4 holds for any positive entry cost γ. One might think that in the limit as γ converges to 0 — which we argued in Section 2.5 corresponds to Burdett and Judd's (1983) model — one source of inefficiency in the no-search equilibrium, excessive entry, will disappear. If it did, the no-search equilibrium would dominate the search equilibrium, which would still have excessive search costs. This reasoning is incorrect, however. In the limit, an infinite measure of firms spend arbitrarily little on entry, and total expenditures converges to a finite, positive number, smaller in the search equilibrium than in the no-search equilibrium. As a result, the search equilibrium continues to Pareto dominate the no-search equilibrium. Put differently, when many firms enter, each firm has a small congestion externality on every other firm, but the total external effect does not disappear. Therefore, even when entry costs converge to zero, wage dispersion is socially beneficial.

To conclude, we have established that with homogeneous firms, search is privately a rent-seeking activity. This transfer of rents is nonetheless very useful from an efficiency viewpoint, because (*i*) dissipation of rents by further entry of firms is socially more expensive; and (*ii*) it prevents worker nonparticipation and market breakdown, the well-known Diamond's paradox. Therefore, with homogeneous firms (and no government intervention), wage dispersion is necessary for workers to search, average wages to increase and the society to reach a more efficient allocation.

4 Efficient Technology Dispersion

In reality, wage dispersion appears more common when the marginal product of labor differs across firms. This manifests itself, for example, as a correlation between wages and firm profitability (e.g. Blanchflower, Oswald and Sanfey, 1996). Thus one might be tempted to use productivity differentials to explain wage differentials. Such an answer, however, simply shifts the puzzle one step back. Why do firms opt to use different technologies? To answer this question, we allow firms to make *ex ante* capital investments. We establish the existence of an equilibrium with wage and technology dispersion in this environment. This equilibrium exists because workers will search only if there is wage dispersion; and firms offering higher wages and enjoying longer job queues prefer to make larger *ex ante* investments, a simple single crossing property.

Our main result in this section is that an equilibrium in which firms that make larger *ex ante* investments, choose to pay higher wages, is more efficient than an equilibrium in which the wage distribution is degenerate. This result is surprising; wage dispersion is equivalent to an upward sloping wage-investment schedule, which naturally distorts investment decisions (see Grout 1984; Acemoglu, 1996a). In other words, when workers appropriate some of the returns in the form of higher wages, there will be underinvestment. However, we show that this 'holdup' externality is dominated by another, more important consideration, *sorting*. Firms care about the probability that they are able to hire a worker. In particular, high productivity firms have a larger opportunity cost from not producing, and so want to attract longer queues. They can only do this if workers search, and workers will search only if there is wage dispersion. Thus sorting requires wage dispersion, and if sorting is possible, high productivity firms will choose to offer higher wages. We show that this second effect dominates the traditional holdup problem.

4.1 The Environment

We assume that before contacting a worker, each firm decides what type of job to create. For instance the firm chooses a technology from a production possibility set, or chooses its level of capital investment. Formally, we model this as a choice of 'capital intensity' $k \in [\underline{k}, \infty)$, which is available elastically at a price normalized to 1. $\underline{k} > 0$ represents a minimal investment required to achieve any output, representing perhaps the cost of posting a vacancy (γ). A job using k units of capital will produce $f(k)$ if the firm hires a worker, and nothing otherwise, where $f : [\underline{k}, \infty) \mapsto \mathbb{R}^+$. The rest of the sequence of events is exactly as in Section 2. In particular, each worker simply cares about the wage that a firm offers, not its capital investment. As before, she must sample a firm in order to find out its wage. In keeping with the spirit of this paper, we assume that workers cannot observe a firm's capital intensity without sampling the firm, and hence each worker draws a random sample of the firms that have opened vacancies.To simplify the analysis, from now on we assume that c_1 equals zero, and that all workers participate in the labor market (i.e. $\lambda_0 = 0$). This rules out the Diamond paradox, which we discussed in detail in Section 2, and allows us to focus on the more interesting no-search equilibria, those with worker participation.

Let $H : \mathcal{K} \to [0, 1]$ denote the capital distribution across firms and $G(\cdot \mid k) : \mathcal{W}(k) \to [0, 1]$ denote the conditional wage distribution of a firm using capital intensity $k \in \mathcal{K}$. As in the previous sections, we define $p_n(i; \mathbf{w})$ to be the probability that a worker who samples n wages described by the vector $\mathbf{w}$ applies for w_i. p_n is again determined in equilibrium from worker preferences. Using p_n and the distributions G and H, we construct $\hat{G}_n(w)$, the probability that a worker applies to wage w rather than one of the $n-1$ other wages she samples. Finally, define $\Sigma(w) = \sum_{n=1}^{\infty} n\lambda_n \hat{G}_n(w)$ to be the probability that a representative worker applies to a firm offering w, conditional on contacting such a firm. This allows us to calculate the gross profit of a firm using capital intensity $k \in [\underline{k}, \infty)$ and posting wage

w:

$$\pi(k,w) = \left(1 - e^{-q\Sigma(w)}\right)(f(k) - w) \tag{17}$$

Finally, we impose a number of technical conditions on f. Most importantly, it is continuously differentiable, increasing and strictly concave, with $f(\underline{k}) = 0$. Let $k_0 > \underline{k}$ be the technology that produces the maximum output-capital ratio given a unit of labor input, which is the unique solution to $f'(k_0) = f(k_0)/k_0$. We assume that $f'(k_0) > 1$. If this condition failed, any investment would be unprofitable even if the job filled with certainty at a zero wage. Also, let $k_1 > k_0$ satisfy $f'(k_1) = 1$ (or let $k_1 = \infty$ if $f'(k) > 1$ for all k). These two capital intensities are important, as we prove shortly in Lemma 6: all firms use some capital intensity between k_0 and k_1. Also, define $\phi : [k_0, k_1) \mapsto (0, \infty)$ by $\phi(k) \equiv \log f'(k) - \log(f'(k) - 1)$, a strictly increasing function on its domain. Define $\Phi : [k_0, k_1) \to \mathbb{R}$, with

$$\Phi(k) = \frac{f(k) - kf'(k)}{f'(k)\phi(k)} \tag{18}$$

It is straightforward to check that $\Phi(k_0) = \lim_{k \to k_1} \Phi(k) = 0$ and that Φ is strictly positive on the interior of its support. We assume in addition that Φ is strictly quasiconcave on $[k_0, k_1)$,[11] and explain the importance of this regularity condition in the text.

4.2 Search and No-Search Equilibria

Although the economic issues at hand are quite different, the analysis will have exactly the same technical structure as Section 2. We take advantage of this by stating many of the results without proof. We begin with a natural extension of the definition of an equilibrium.

Definition 2 *An equilibrium consists of a measure of active firms V, a distribution of investment H with support $\mathcal{K}$, wage distributions $G(\cdot \mid k)$ with supports $\mathcal{W}(k)$ for each capital intensity $k \in \mathcal{K}$, a profit function for firms π, return function for workers ρ and R, preference functions $\{p_n\}$, and sample size decisions for workers $\{\lambda_n\}$ such that:*

1. **(Profit Maximization)** $\forall k \in \mathcal{K}$, $\forall w \in \mathcal{W}(k)$, $\forall k'$, *and* $\forall w'$, $\pi(k,w) - k \geq \pi(k', w') - k'$.
2. **(Free Entry)** $\forall k \in \mathcal{K}$, $\forall w \in \mathcal{W}(k)$, $\pi(k,w) = k$.

[11] This is a restriction only on the production function f, and thus can easily be verified for particular production functions. One can prove that Φ is strictly quasiconcave if $kf''(k)$ is increasing and $\phi(k)$ is convex. Also, numerical simulations show that for generalized Cobb-Douglas production functions of the form $f(k) = Ak^\alpha - \underline{k}$ or $f(k) = A(k - \underline{k})^\alpha$, Φ is always strictly concave, and hence strictly quasiconcave. It can also be noted that even if $\Phi(k)$ were multiple peaked (but without flat portions), our analysis would remain essentially unchanged, although there could be multiple atoms in the wage and capital distributions.

3. **(Optimal Application)** $\forall n \in \mathbb{N}_+$, $\forall \mathbf{w} \equiv (w_1, ..., w_n)$,

$$p_n(i; \mathbf{w}) = 0 \text{ } \textit{if } \rho(w_i) < \max_{j \neq i} \rho(w_j) \text{ } \textit{or if } \rho(w_i) < 0, \forall i$$

and

$$\sum_{i=1}^{n} p_n(i; \mathbf{w}) = 1 \text{ } \textit{if } \max_j \rho(w_j) > 0$$

4. **(Optimal Sampling)** $\lambda_{n^*} > 0$ *only if* $n^* \in \arg\max_n R_n$.

We first observe that profit maximization and free entry bound firms' capital intensity.

Lemma 6 *In equilibrium,* $\mathcal{K} \subseteq [k_0, k_1]$.

Next note from part 1 of Definition 2, that all $k \in \mathcal{K}$ and $w \in \mathcal{W}(k)$ jointly maximize $\left(1 - e^{-q\Sigma(w)}\right)(f(k) - w) - k$. The objective is differentiable and strictly concave in k, and so *if* wage w is offered in equilibrium, it must be offered by a firm whose capital intensity k satisfies:

$$\left(1 - e^{-q\Sigma(w)}\right) f'(k) = 1 \tag{19}$$

Since f is strictly concave, (19) defines at most one capital intensity $K(w)$ that is consistent with posting wage w.

Free Entry, part 2 of Definition 2, implies $\left(1 - e^{-q\Sigma(w)}\right)(f(K(w)) - w) = K(w)$. Combining this with the optimal intensity condition (19) implies that $K(w)$ satisfies:

$$w = f(K(w)) - K(w)f'(K(w)) \tag{20}$$

Strict concavity of f implies that $K(w)$ is strictly increasing; therefore, we can invert it to solve for the wage posted by a firm using capital intensity k. We denote this inverse relation between k and w by $W(k)$. This is a strictly increasing function with $W(k_0) = 0$. Thus (proof in the text):

Lemma 7 $\forall k \in \mathcal{K}$, *the equilibrium wage distribution* $G(w \mid k)$ *is degenerate at the point* $W(k) \equiv f(k) - kf'(k)$.

This is an important result: $W(k)$ is independent of all other variables, including workers' sampling behavior and the distribution of capital in the economy. This implies that instead of focusing on wage distributions, as in the earlier sections, we can equivalently analyze the endogenous capital intensity distribution H. Once this distribution is determined, Lemma 7 immediately gives the distribution of wages. Thus we reduce the firm's choice to a single dimension, capital intensity.

Using this insight, the return to a worker who applies for a wage $W(k)$, which is equivalently the return to applying for a job with capital intensity k, is:

$$\rho(k) = \frac{1 - e^{-q\Sigma(W(k))}}{q\Sigma(W(k))} W(k)$$

Eliminating $W(k)$ with its definition in Lemma 7 and $q\Sigma(W(k))$ with its implicit definition in equation (19), yields:

$$\tilde{\rho}(k \mid f) = \frac{f(k) - kf'(k)}{f'(k)\phi(k)} = \Phi(k) \tag{21}$$

$\tilde{\rho}$ is the return to the worker of applying to a job with capital intensity k such that $\pi(k, W(k)) = k$. As in Section 2, $\tilde{\rho}(k \mid f) \leq \rho(k)$, with equality if $k \in \mathcal{K}$.

By assumption (that $\Phi(k)$ as defined in (18) is quasiconcave), $\tilde{\rho}$ is strictly quasiconcave on $\mathcal{K}$. This allows us to prove that, analogous to Lemma 1, in equilibrium a worker always applies to the most capital intensive job she samples. Given the monotonic relationship between capital intensity and wages, she also always applies for the highest wage she samples, *i.e.* $p_n(i, \mathbf{w}) = 0$ if $w_i < \max_j w_j$. Further, in a search equilibrium, *i.e.* when $\lambda_1 < 1$, an analogue of Lemma 2 applies and therefore, the support of the type distribution $\mathcal{K}$ consists of the convex interval $[k_0, \bar{k}]$ and possibly the point $k^* \geq \bar{k}$ that is the unique maximizer of $\tilde{\rho}(k \mid f)$. The type distribution is atomless on $[k_0, \bar{k}]$ but may have an atom at k^*. With a similar reasoning to the one in section 2, we must have that $\hat{G}_n(w) = H(K(w))^{n-1}$ for $K(w) \neq k^*$. In other words, the probability that a worker applies to a firm offering wage w ($\neq W(k^*)$) when she samples $n-1$ other jobs is the probability that all the other jobs offer lower wages, or equivalently use less capital. Thus the return to sampling n jobs is:

$$R_n = n \int_{k_0}^{\bar{k}} \rho(k) H(k)^{n-1} dH(k) + \rho(k^*)\left(1 - H(\bar{k})^n\right) - \sum_{i=1}^{n} c_i$$

which parallels equation (9).

Because workers always apply to the most capital intensive firm that they sample, in a search equilibrium some workers must sample one job; otherwise a firm with capital intensity k_0 would receive no applications and would lose money (Lemma 3). Then decreasing returns to search (Lemma 4) implies that in a search equilibrium, all workers sample either one or two jobs (Lemma 5).

Once again, we let z denote the fraction of workers who sample two jobs, and $1-z$ denote the fraction who sample one. Thus $\Sigma(W(k)) = (1-z) + 2z\hat{G}_2(W(k)) = (1-z) + 2zH(k)$. Using this, invert equation (19) to solve for $H(k)$ in terms of z in a search equilibrium:

$$H(k) = \frac{\phi(k) - q(1-z)}{2qz} \tag{22}$$

Since $H(k_0) = 0$, we obtain the (search equilibrium) labor market tightness in terms of z:

$$q = \frac{\phi(k_0)}{1-z} \tag{23}$$

We can now fully characterize the equilibria of this model:

Proposition 5 *There always exists a no-search equilibrium with the wage distribution degenerate at zero and the capital intensity distribution degenerate at* k_0. *Also,* $\exists\tilde{c}$ *such that* $\forall c_2 > \tilde{c}$, *there does not exist a search equilibrium; and* $\forall c_2 \in (0, \tilde{c})$, *there exist at least two search equilibria, one stable and one unstable. In each of these equilibria:*

1. *the support of the capital distribution is* $[k_0, \bar{k}] \cup k^*$, *for* $\bar{k} < k^*$ *where* $k^* = \arg\max_k \tilde{\rho}(k \mid f)$;

2. *the capital distribution is characterized by*

$$H(k) = \frac{1-z}{2z} \cdot \left(\frac{\phi(k)}{\phi(k_0)} - 1 \right) \tag{24}$$

on $[k_0, \bar{k}]$ *and by a mass* μ *at* k^*, *where*

$$\mu = \max \left\langle \frac{1+z}{z} - \frac{1-z}{z} \cdot \frac{\phi(k^*)}{\phi(k_0)} ; 0 \right\rangle ; \tag{25}$$

3. *a firm with capital* k *offers wage* $W(k) = f(k) - kf'(k)$.

4.3 Discussion

The first point to note is the parallel with the results of Section 2. In particular, there is again a multiplicity of equilibria, and the intuition is the same: search intensity decisions are strategic complements when search intensity is low, and strategic substitutes when search intensity is high. Here, the search equilibria feature not only a distribution of wages, but also capital intensities. This is a very significant result. Even though all firms are *ex ante* identical, the equilibrium is characterized by technology dispersion. As in Section 2, competition between firms to attract workers creates wage dispersion and thus job queues of various expected lengths. Then, a firm with a longer expected job queue will make a larger *ex ante* investment, because it is less likely to be forced to leave its capital idle.

Search has additional consequences in this model with *ex ante* investment (thus with *ex post* heterogeneity). For example, higher search intensity enables socially desirable *sorting*: namely, higher productivity firms enjoy longer queues and a higher probability of filling their vacancy in equilibrium. This is because workers who search are more likely to sample, and therefore, more likely to apply for, high wage jobs. Since it is the high productivity firms that are more willing to pay high wages — recall Lemma 7 — wage dispersion and search increase the relative profitability of high productivity firms. Through this channel, search encourages investment in better technologies and improves the distribution of available jobs. In contrast, in a no-search equilibrium, all firms pay the same wage and all jobs are filled with the same probability—even if they have different labor productivities. This rationing scheme leads to the creation of only jobs using the technology that minimizes the capital-output ratio, that is k_0. This introduces a new reason for search and wage dispersion to be socially desirable.

Finally, note the most interesting comparative static of this section. In the stable search equilibrium, as the cost of sampling a second job, c_2, declines, the proportion of workers sampling two jobs increases as in Section 2. This increases wages and reduces entry as in Section 2, but also improves the investment of firms that do enter. In particular, from (24), when z increases $H(k)$ shifts to the right, or equivalently the investment distribution improves in the sense of first order stochastic dominance.

4.4 Welfare with *ex ante* Investment

How do equilibrium investment decisions compare with the constrained optimum achieved by the hypothetical social planner of Section 3? Proposition 7 below proves that the constrained efficient level of investment is k^*, the highest investment level chosen by any firm in a decentralized equilibrium. Thus if c_2 is positive, firms underinvest in any decentralized equilibrium.

This result and the parallel with those of sections 2 and 3 are not surprising. The basic model can be viewed as a particular (nondifferentiable) limit of this model with *ex ante* investment, say $f(k) = \lim_{x\to\infty} (k-\gamma)^{1/x}$ with $\underline{k} = \gamma$. In the basic model, recall that too many firms participate in any equilibrium with $c_2 > 0$ (Proposition 3). This happens in the current model as well, and the resulting short job queues induce firms to choose low capital intensities.

It is also straightforward to repeat the analysis of Section 3. Again the no-search equilibrium either has no activity (if c_1 is positive) or excessive entry and severe underinvestment in capital (if c_1 is negative); these severe inefficiencies are avoided in the search equilibrium. Thus (proof omitted):

Proposition 6 *If $c_2 < \tilde{c}$, a search equilibrium Pareto dominates the no-search equilibrium.*

This result is more surprising than Proposition 4, the corresponding result without *ex ante* investments. As noted above, in the search equilibrium, there is effectively an upward-sloping wage-investment schedule, thus firms underinvest. In contrast, in the no-search equilibrium, wages are driven to zero, and firms are the full residual claimants of the returns they create. Thus firms might be expected to have the correct investment incentives. But this is not the case. Without wage dispersion, workers do not search and wages are too low. Therefore, there is excessive entry of firms, as in Sections 2 and 3. This reduces the probability that a given firm will contact a worker. Moreover, a no-search equilibrium rules out *sorting*. Recall that firms with higher physical capital would like to offer higher wages and attract longer job queues. This is impossible when workers do not search. Therefore, in the no-search equilibrium, the probability of contacting a worker is low and an individual firm can do nothing to increase it. This implies that firms' physical capital will be left idle with a high probability, and they all choose capital stock, k_0.

The efficiency properties of the search equilibrium are actually much more striking than the above proposition suggests. We previously noted that the limit of the basic model as $c_2 \to 0$ is equivalent to the equilibrium obtained by Moen (1995) and Shimer (1996), and hence constrained Pareto efficient. It is generally appreciated that decentralized efficiency is much harder to achieve when agents must make *ex ante* investments before matching (see Acemoglu, 1996a). Nonetheless, if the cost of sampling a second job disappears, the equilibrium of this model converges to the constrained efficient allocation:

Proposition 7 *In the limit of stable search intensive equilibria with c_2 converging to 0, a fraction $\frac{\phi(k_0)}{\phi(k^*)} \in (0,1)$ workers sample two wage offers while the remaining workers sample two. All firms invest k^* and offer a common wage $W(k^*)$. This limit is the constrained efficient allocation and maximizes the value of net output in the economy.*

This result is quite striking. Despite the numerous externalities, if only a fraction of agents gather more than the most rudimentary information (*i.e.* sample two wages), the equilibrium is efficient. This contrasts, for example, with Acemoglu (1996a), in which firms and workers bargain over wages after matching. In that environment, *ex ante* investments are always distorted. The difference is due to the fact that with wage commitments, firms are the residual claimants on any additional returns generated by a superior technology, after conditioning on workers' application decisions. Interestingly, at the limit $c_2 = 0$, the upward-sloping wage-investment relation (which is the source of inefficiencies in Grout, 1984, and Acemoglu, 1996a) does not disappear. In fact, at this limit we still have $W(k) = f(k) - kf'(k)$, thus a firm which invests less would be able to offer lower wages. But the competition for workers (with a fraction $1 - \phi(k_0)/\phi(k^*)$ of them sampling two wages) is strong enough that this wage gain would not compensate for the reduced probability of finding a worker. The fact that the limit of the search equilibrium with wage and technology dispersion is the constrained efficient allocation reiterates that in our economy, worker search and wage dispersion are fairly efficient ways of allocating workers to firms, and of providing firms with the correct investment incentives.

To summarize, higher search intensity transfers rents from firms to workers and improves social welfare by avoiding excessive entry. There is also a more interesting benefit of higher search intensity: *it leads to better sorting*, as capital intensive firms can offer high wages and raise the probability that they hire a worker. This increases the relative profitability of high productivity jobs and encourages investment.

5 The Dynamic Model

Search decisions are dynamic, and so in this section we demonstrate that all of our results completely generalize to a dynamic environment. Given the parallel to the static results of Sections 2–4, our analysis will be brief.

We consider the following discrete time, infinite horizon economy, a natural generalization of our static model. The agents in the economy are risk-neutral workers and firms, who maximize the expected present value of their income, net of search costs, using a common discount factor $\beta \leq 1$.

At the start of every period, workers are either unemployed or employed; likewise, firms are either inactive, maintain an open vacancy, or have a filled job. In addition, associated with each active firm is a capital intensity $k \in [\underline{k}, \infty)$. During each period, the sequence of events is as follows:

1. Inactive firms decide whether to create a vacancy. Those that do not create vacancies remain idle throughout the period. Those that create vacancies make an irreversible capital investment $k \in [\underline{k}, \infty)$ at marginal cost $\nu > 0$.

2. Job Search:

   (a) Firms with unfilled vacancies from the previous period and those creating vacancies this period post a wage *contract.*

   (b) Unemployed workers choose a sample size $n \geq 1$, and observe n posted contracts, selected independently. For simplicity, employed workers may not search, and unemployed workers must participate (i.e. $\lambda_0 = 0$).

   (c) Each unemployed worker applies to at most one of the sampled firms.

   (d) Each firm that receives an application hires one of the applicants. The firm has a filled job, and the chosen applicant is employed.

3. All firms with filled jobs — new or old — and k units of capital produce $f(k)$ units of output. Firms with unfilled vacancies produce nothing.

4. Each active firm faces an independent and exogenous probability $\delta > 0$ that its capital stock is destroyed. In such an event, at the start of the following period the firm is inactive. In addition, if the firm had a filled job, its employee becomes unemployed. If not destroyed, a firm with capital k remains in the same state (i.e. as a filled or unfilled vacancy).

Note some important features. First, when $\beta = 0$, $\delta = 1$, and $\nu = 1$, this is equivalent to an infinite repetition of the static model in Section 4. Second, as in section 4, firms choose their capital stock before matching with a worker, and this decision is irreversible. Third, this model allows for both fixed sample size (sampling several jobs in a particular period) and sequential sampling (possibly sampling additional jobs next period, depending on the result of this period's search). From an individual perspective, the advantage of sampling multiple jobs in one period is that information is gathered more rapidly; the advantage of sampling jobs sequentially is that the decision about whether to gather additional information can be conditioned on the success of previous efforts. Thus sampling decisions are 'optimal' in the partial equilibrium sense of Morgan and Manning (1985). In our general equilibrium framework, whether workers engage exclusively in sequential sampling is crucial for another reason: as we will show, pure sequential search is the analogyf of 'no-search' in our one period economy. Equilibria with pure sequential search have no wage dispersion and are highly inefficient.

In this dynamic model, we allow firms to post wage *contracts*, not simply flat wages. Of course a wage contract could promise workers a flat wage in every period during which the worker is employed. Yet other possibilities, for instance an upward sloping wage profile, are also possible. This implies that offers by different firms may differ in multiple dimensions. However, these differences are largely irrelevant; since both firms and workers are risk-neutral and have a common discount factor, all they care about is the expected present discounted value of the offer the firm makes. To formalize this observation, note that when a firm hires a worker, the newly created match generates some surplus. This is equal to the amount that the present discounted value of the firm's profits and the worker's wage exceeds what they were before the match, when the worker was unemployed and the firm had a vacancy. The purpose of a wage contract is then to divide that surplus between the worker and the firm; firms effectively compete by offering workers a share of the surplus (see Shimer, 1996). Thus we can reduce the competition between wage contracts to a single dimension, the worker's quantity of surplus. To emphasize the similarity to our previous analysis, we denote the surplus that a worker receives by $w \in \mathbb{R}$.

Next we modify our notation for this dynamic environment. Since we are only interested in steady state equilibria in this section, we suppress time subscripts. Let $H : \mathcal{K} \rightarrow [0,1]$ denote the capital distribution across firms with vacancies and $G(\cdot \mid k) : \mathcal{W}(k) \rightarrow [0,1]$ denote the conditional 'surplus' distribution of a firm using capital intensity $k \in \mathcal{K}$. That is, $G(w \mid k)$ is the fraction of vacant firms using capital k that offer workers surplus less than or equal to w. Again define $p_n(i; \mathbf{w})$ to be the probability that an unemployed worker who samples n offers described by the vector $\mathbf{w}$, applies for w_i. Using p_n and the distribution functions G and H, we construct $\hat{G}_n(w)$, the probability that a worker applies to a firm offering w rather than one of the $n-1$ other firms she samples. Then we define $\Sigma(w) = \sum_{n=1}^{\infty} n\lambda_n \hat{G}_n(w)$ to be the probability that a representative unemployed worker applies to a firm offering w, conditional on contacting such a firm.

Let $\pi(k,w)$ denote the expected present discounted value of a vacant firm using k units of capital and offering workers a quantity of surplus w. Also let $S(k)$ denote the expected value of *total* surplus in a match using k units of capital, as defined above. Next, let $\rho(w)$ be the expected *flow* value of a worker who has applied for a job offering surplus w, and R_n be the expected *total* value of an unemployed worker who samples n jobs in a particular period. For convenience we also define J to be the supremal value of R_n, $n \in \{1, 2, ...\}$. Thus a worker's 'reservation surplus', the amount of money that makes her just willing to forego the opportunity to search while unemployed, is $(1-\beta)\, J$ per period. Finally, we make the convenient normalization that the marginal cost of a unit of capital is $\nu \equiv (1-\beta(1-\delta))^{-1}$. Equivalently, the expected 'rental rate' of capital is normalized to unity. A firm's reservation profit is therefore identical to its capital stock.

We can now define a steady state equilibrium for this economy.

Definition 3 *A steady state equilibrium consists of a measure V of firms that maintain vacancies each period; an unemployment rate U; a distribution of investment H with support $\mathcal{K}$, surplus distributions $G(\cdot \mid k)$ with supports $\mathcal{W}(k)$; a value function for firms with vacancies π; return function for unemployed workers ρ, R, and J, preference functions $\{p_n\}$, and sample size decisions for unemployed workers $\{\lambda_n\}$ such that;*

1. **(Profit Maximization)** $\forall k \in \mathcal{K}$, $\forall w \in \mathcal{W}(k)$, $\forall k'$ *and* $\forall w'$, $\pi(k,w) - \nu k \geq \pi(k',w') - \nu k'$.

2. **(Free Entry)** $\forall k \in \mathcal{K}$, $\forall w \in \mathcal{W}(k)$, $\pi(k,w) = \nu k$.

3. **(Optimal Application)** $\forall n \in \mathbb{N}$, $\forall \mathbf{w} \equiv (w_1, ..., w_n)$,

$$p_n(i; \mathbf{w}) = 0 \text{ if } \rho(w_i) < \max_{j \neq i} \rho(w_j) \text{ or if } \rho(w_i) < 0, \ \forall i$$

   and

$$\sum_{i=1}^{n} p_n(i; \mathbf{w}) = 1 \text{ if } \max_j \rho(w_j) > 0.$$

4. **(Optimal Sample)** $\lambda_{n^*} > 0$ *only if* $R_{n^*} = J \equiv \sup_n R_n$.

5. **(Steady State)** U *and* V *are constant over time.*

To characterize a steady state search equilibrium, first consider the surplus in a match using $k \in \mathcal{K}$ units of capital, $S(k)$. If the pair produce this period, they create $f(k)$ units of output. Additionally, unless the firm's capital stock is destroyed, the following period, the pair will obtain the same surplus $S(k)$ — since we are in steady state. During a match, however, the worker and firm forego search in the next period, and so we subtract the reservation profit flow k and reservation wage $(1-\beta)\,J$ from this value. Solving the resulting Bellman equation for $S(k)$ yields:[12]

$$S(k) = \nu\,(f(k) - \beta(1-\delta)\,(k + (1-\beta)\,J)) \tag{26}$$

[12] There are a number of alternative ways to derive this equation. For example, if a worker and firm match, they produce $f(k)$ units of output in every period until the firm's capital is destroyed, at which point the firm has value 0 and the worker has value J. The infinite sum of payoffs is:

$$\left(\sum_{t=0}^{\infty} (\beta(1-\delta))^t\right)(f(k) + \beta\delta J) = \nu\,(f(k) + \beta\delta J)$$

where discounting reflects both impatience and match destruction. On the other hand, if the match never occurred, the worker would have value J next period and the firm would have value vk next period, unless its capital were destroyed. Thus the value of not matching is $\beta\,(J + (1-\delta)\nu k)$. Subtracting this from the value of matching yields the surplus in equation (26).

Next consider the value of a firm using k units of capital and offering surplus w. If the firm manages to hire a worker, it enjoys a capital gain $S(k)-w$. Additionally, unless its capital stock is destroyed, it has a steady state continuation value $\pi(k,w)$ the following period. Solving this standard Bellman equation for $\pi(k,w)$ yields:

$$\pi(k,w) = \nu\left(1-e^{-q\Sigma(w)}\right)(S(k)-w) \tag{27}$$

where $q = U/V$. As in Section 4, when firms create vacancies and buy capital, they simultaneously determine the surplus that they will offer. If a firm is to offer surplus w, we can therefore use equation (27) and the free entry condition to determine the capital stock it will employ. This leads to an analogue of Lemma 7; a firm using capital stock k offers workers surplus:

$$W(k) \equiv S(k) - kS'(k) \tag{28}$$

We can now eliminate the worker's share of surplus from equation (27) and write the net present value of firm profits as:

$$\pi\left(k, W(k)\right) = \nu\left(1-e^{-q\Sigma(W(k))}\right)kS'(k) \tag{29}$$

We have once again reduced the firm's problem to a single variable, the choice of capital intensity, k.

Next one can solve for the return to a worker who applies for a job offering surplus $w = W(k)$, which from (28) is equivalently a firm with k units of capital. This expected flow return is given as:

$$\rho(k) = \frac{1-e^{-q\Sigma(W(k))}}{q\Sigma(W(k))}W(k) \tag{30}$$

If hired, the worker obtains a capital gain equal to $W(k)$ (due to the surplus offered by the firm). We can once more use equation (27) and the free entry condition $\pi(k,W(k)) = \nu k$ to eliminate $q\Sigma(W(k))$ from (30). This yields the return to applying for a job posted by a firm using k units of capital, when job queues are adjusted so that the expected value of the firm in question is zero:

$$\tilde{\rho}(k \mid f) = \frac{S(k)-kS'(k)}{S'(k)\left(\log S'(k) - \log\left(S'(k)-1\right)\right)}$$

This is a dynamic version of equation (21), and as before, for $k \in \mathcal{K}$, $\rho(k) = \tilde{\rho}(k \mid f)$. Now, use equation (26) to write this expression in terms of model primitives and the value of an unemployed worker J.

$$\tilde{\rho}(k \mid f) = \frac{f(k)-kf'(k)-\beta\left(1-\delta\right)\left(1-\beta\right)J}{\left(f'(k)-\beta(1-\delta)\right)\phi(k)} \tag{31}$$

where $\phi(k) \equiv \left(\log\left(f'(k)-\beta(1-\delta)\right) - \log\left(f'(k)-1\right)\right)$, a generalization of the definition in Section 4 to allow for matches that survive multiple periods and agents who look towards the future. Observe that if $\beta = 0$, *i.e.* agents are completely myopic, (31) is identical to (21). Parallel to our assumption in section 4, we assume that $\tilde{\rho}$ is a strictly quasiconcave function of k.

Under these conditions, workers always apply to the firm offering the largest share of surplus and using the most capital (Lemma 1). Let k_0 be the capital intensity associated with offering workers zero surplus, so $f(k_0) - k_0 f'(k_0) = \beta(1-\delta)(1-\beta)J$. From (28), $W(k_0) = 0$. Note that k_0, and in fact the entire equilibrium capital distribution depends on J, which is itself an endogenous variable. When workers have a higher reservation wage, production is less profitable.

It follows that in a steady state search equilibrium, the support of the capital distribution is a convex interval $[k_0, \bar{k}]$ and possibly the point $k^* \geq \bar{k}$ which is the unique maximizer of $\tilde{\rho}$ in (31); and that the capital distribution is atomless on $[k_0, \bar{k}]$ but has an atom at k^* (Lemma 2). This once again implies that the return to sampling n firms when unemployed is:

$$R_n = n \int_{k_0}^{\bar{k}} \rho(k) H(k)^{n-1} dH(k) + \rho(k^*) \left(1 - H(\bar{k})^n\right) - \sum_{i=2}^{n} c_i + \beta J$$

analogous to equation (9).

Continuing on, in a steady state equilibrium, some unemployed workers sample only one firm per period; otherwise no firm would be willing to offer a very small share of the surplus (Lemma 3). Since there are decreasing returns to a worker's sample size (Lemma 4), we obtain that in a steady state search equilibrium all workers must sample one or two firms in every period, $\lambda_1 + \lambda_2 = 1$ (Lemma 5). We can now define $z \equiv \lambda_2$ as the proportion of unemployed workers who sample two firms in a given period (search), and $\lambda_1 = 1 - z$. We can then use (29), the free entry condition, and the fact that $\hat{H}_2(k) = H(k)$ over $[k_0, \bar{k}]$ to obtain the distribution of capital among vacant firms:

$$\hat{H}_2(k) = H(k) = \frac{\phi(k) - q(1-z)}{2qz} \tag{32}$$

over $[k_0, \bar{k}]$. This is clearly the analogue of equation (11) in Section 2. Now, using the fact that $H(k_0) = 0$, we immediately obtain $q = \phi(k_0) / (1-z)$. Then:

Proposition 8 *There always exists a steady state no-search equilibrium with the wage distribution degenerate at zero, the capital intensity distribution degenerate at k_0, and the value of an unemployed worker $J = 0$. Also, $\exists \hat{c}$ such that $\forall c_2 > \hat{c}$, there does not exist a steady state search equilibrium; and $\forall c_2 \in (0, \hat{c})$, there exist at least two steady state search equilibria. In these equilibria:*

1. *The support of the capital distribution is $[k_0, \bar{k}] \cup k^*$, for $\bar{k} < k^*$, where $k^* = \arg\max_k \tilde{\rho}(k \mid f)$, and k_0 satisfies $f(k_0) - k_0 f'(k_0) = \beta(1-\delta)(1-\beta)J$;*

2. *the capital distribution is characterized by*

$$H(k) = \frac{1-z}{2z}\left(\frac{\phi(k)}{\phi(k_0)} - 1\right) \tag{33}$$

on $[0, \bar{k}]$ *and by a mass* μ *at* k^*, *where*

$$\mu = \max\left\langle \frac{1+z}{z} - \frac{1-z}{z} \cdot \frac{\phi(k^*)}{\phi(k_0)}\,;\, 0 \right\rangle \tag{34}$$

3. *a firm with capital* k *offers surplus*

$$W(k) = \nu\left(f(k) - kf'(k) - \beta\,(1-\beta)\,(1-\delta)\,J\right) \tag{35}$$

4. *the value of an unemployed worker satisfies*

$$(1-\beta)\,J = \int_{k_0}^{k^*} \tilde{\rho}(k \mid f)\, dH(k) \tag{36}$$

5. *the steady state unemployment rate is:* $U = \dfrac{\delta}{\delta + (1-\delta)\,\mathcal{E}/q}$, *where* $\mathcal{E}$ *is the average probability that a vacancy turns into a job, given by:*

$$\mathcal{E} = \int_{k_0}^{k^*} \left(\frac{1-\beta\,(1-\delta)}{f'(k) - \beta\,(1-\delta)}\right) dH(k). \tag{37}$$

The steady state equilibria of the dynamic economy are very similar to the equilibria of the static economy, and the comparative statics and welfare analysis immediately generalize to this environment. The steady state equilibria with search and wage dispersion Pareto dominate the no-search equilibrium. Again, wage dispersion is necessary for the market economy to function without a Walrasian auctioneer, because without wage dispersion workers do not gather enough information, *i.e.* do not "search". Interestingly, "not searching" in the sense we (and Stigler, 1961) use is equivalent to pure sequential search; each unemployed worker observes one wage each period. Morgan and Manning (1985) argue that such a rule is generally not optimal in a partial equilibrium setting, because sequential search is more time-consuming than is fixed sample size search. Our results demonstrate that in a general equilibrium setting, if pure sequential search is strictly better than fixed sample size search, firms will exploit their resulting monopsony power. This leads to entry by an excessive number of firms, each of which use very little capital.

6 Conclusion

This paper builds a general equilibrium search model along the lines of the partial equilibrium literature pioneered by Stigler (1961). Therefore our model closely resembles the actual practice of job search in an economy with uncoordinated trade.

Workers engage in optimal search for the best wage offer. They sample a number of firms, and then, anticipating the probability of being hired, decide for which to apply. Firms, anticipating the search and application decisions of workers, make different investments and offer different wages. Despite the small number of parameters — essentially only the production and sampling technologies — the careful microfoundations deliver a number of predictions.

The main implication of our analysis is that in an economy without the invisible hand, wage dispersion among identical workers is ubiquitous but also desirable. Without wage dispersion, workers will not search, and the wage rate falls below the shadow value of labor. This causes excessive entry, short job queues, and investment in technologies with a high average and marginal product of capital. Hence the title of the paper: *Efficient Wage Dispersion*.

The links between wage dispersion, search and efficiency are stark in our model because of the simplifying assumptions. In particular, there is no search without wage dispersion. In more general models in which workers have heterogeneous preferences over jobs' non-wage characteristics, there may be job search with a degenerate wage distribution. However, the general thrust of results will remain: without a Walrasian auctioneer, search is socially beneficial and workers will search more when there is more wage dispersion.

Even though our analysis is based on explicit microfoundations and maximization on both sides of the market, the model admits analytical solutions. This permits a number of interesting extensions. First, such a model could open a middle ground for the study of the decentralized allocation of heterogeneous workers and firms. In particular, our approach applied to this problem would avoid the criticism of most search models that meetings are completely random; but it would also not coincide with the Walrasian allocation in which a worker is automatically assigned to the firm at which her marginal product is highest. This middle ground is likely to generate a number of insights about decentralized allocations in the presence of heterogeneity. Secondly, incorporating risk-aversion into this framework is relatively straightforward, and this extension enables the analysis of optimal unemployment insurance, and the interaction between unemployment, wages and savings. Thirdly, the model can be generalized to allow firms to hire multiple workers, thus enabling an analysis of the size distribution of firms and of firm level job creation and job destruction dynamics.

The analysis of this paper also invites a number of observations of empirical interest. Most importantly, small variations in the costs of sampling may have significant effects on wages, inequality, and unemployment. A reduction in the cost of sampling two jobs increases search and raises the wage distribution in the sense of first order stochastic dominance. This will lead to higher wages, more investment, less entry — and therefore higher unemployment — and typically to less wage inequality. As previously commented (Section 2.7), the impact of sampling costs on unemployment is extreme, because there is no countervailing increase in the number of matches resulting from the improved information. Nevertheless, it is instructive of the general equilibrium connection between search, wages, and employment. If Gary Burtless is correct in observing that (1987, p. 149): "*compared with government employment services in Europe, the U.S. is relatively ineffective in aiding and monitoring the search for jobs*," then European workers who face lower search costs will optimally sample more firms (as is in line with casual empiricism). This will naturally have important effects on wage dispersion and the composition of jobs, as described in this paper (see Acemoglu, 1996b). Overall, the links between search, employment, and the dispersion of wages and investment appears a fruitful and under-researched area for future work.

7 Appendix: Proofs

Proof of Proposition 1: If $\lambda_n = 0$ for $n \geq 2$, then equation (3) states $\pi(w) = \left(1 - e^{-q\lambda_1}\right)(1 - w)$, decreasing in w. Thus part 1 of Definition 1, Profit Maximization, implies that any active firm offers a zero wage. Then since $\rho(0) = 0$, equation (5) implies that the return to sampling n jobs is $R_n = -\sum_{i=1}^{n} c_i$. Since $c_n > 0$ for $n \geq 2$, this is maximized either with a sample size of 0 or 1, depending on whether c_1 is greater or less than zero. Thus part 4 of Definition 1, Optimal Sample, establishes that no worker samples multiple jobs when all firms offer a zero wage, and that λ_1 equals zero (one) when c_1 is positive (negative), and lies between zero and one when $c_1 = 0$. Finally, $q\lambda_1$ is determined from part 2 of Definition 1, Free Entry. Since all active firms offer a zero wage, one can solve $\pi(0) = \gamma$ for $q\lambda_1$. □

Proof of Lemma 1: First, we claim that ρ is nondecreasing on $\mathbb{R}$. Assume to the contrary that there is a $w_1 < w_2$, with $\rho(w_1) > \rho(w_2)$. From part 3 of Definition 1 and the definition of $\hat{G}_n$ in equation (1), $\hat{G}_n(w_1) \geq \hat{G}_n(w_2)$ for all n. Thus by definition $\Sigma(w_1) \geq \Sigma(w_2)$. Then equation (4) implies $\rho(w_1) \leq \rho(w_2)$, a contradiction.

Next, $\tilde{\rho}(w \mid \gamma) \leq \rho(w)$ for all w, with equality if $w \in \mathcal{W}$. If the inequality were reversed, a firm offering wage w would earn gross profits in excess of γ, which is inconsistent with Free Entry, part 2 of Definition 1.

Finally, we prove that ρ is strictly increasing on $\mathcal{W}$. Since for all $w \in \mathcal{W}$, $\rho(w) = \tilde{\rho}(w \mid \gamma)$, it is sufficient to prove that $\tilde{\rho}$ is strictly increasing on $\mathcal{W}$. Since $\tilde{\rho}$ is strictly quasiconcave, this is true if $\mathcal{W} \subseteq (-\infty, w^*]$. Suppose not. Take $w > w^*$, with $w \in \mathcal{W}$. Then since $w \in \mathcal{W}$ and $\{w^*\} = \arg\max \tilde{\rho}(w \mid \gamma)$, it follows that $\rho(w) = \tilde{\rho}(w \mid \gamma) < \tilde{\rho}(w^* \mid \gamma) \leq \rho(w^*)$, violating weak monotonicity. □

Proof of Lemma 2: From Lemma 1, we know that $\mathcal{W} \subseteq (-\infty, w^*]$. In fact, since the return to applying for a negative wage is negative (equation (4)), $\Sigma(w) = 0$ if $w < 0$, and so $\pi(w) = 0 < \gamma$. Hence the support of the wage distribution is a subset of $[0, w^*]$.

We now prove that if there is an atom in the wage distribution, it must occur at w^*. If a wage $w \in [0, w^*)$ is offered by a positive mass of firms, then $\rho(w') \geq \tilde{\rho}(w' \mid \gamma) > \tilde{\rho}(w \mid \gamma) = \rho(w)$ for all $w' \in (w, w^*]$ implies that any higher wage offer would attract discretely more applicants. A sufficiently small increase in the wage above w would be nearly costless, and hence any w' slightly larger than w would be strictly more profitable.

Next observe that a positive measure of firms must offer some wage besides w^*. Suppose not. Then, $R_1 > R_n$ for all n since sampling multiple firms would offer no benefit, and yet would be costly. Therefore, $\lambda_n = 0$ for all $n \geq 2$ by Optimal Sample, part 4 of Definition 1, contradicting our starting assumption.

Now we prove that $\mathcal{W}$ consists of a convex interval $[0, \bar{w}]$ and possibly the point $w^* \geq \bar{w}$. Here we exploit the fact that the support of a distribution is closed.

Let $\bar{w}$ be the highest wage offered in equilibrium except w^*. Suppose in order to find a contradiction that there are points $0 \leq w_1 \leq w_2 \leq \bar{w}$, with $w_2 \in \mathcal{W}$ but $(w_1, w_2) \in \mathcal{W}^c$, the complement of $\mathcal{W}$. Then a firm offering w_2 could cuts its wage to w_1. It would lose no applicants, but it would save $w_2 - w_1$ on labor costs if it hires a worker. Then $\pi(w_1) > \pi(w_2) = \gamma$, contradicting the free entry condition. □

Proof of Lemma 3: By the way of contradiction, suppose $\lambda_0 < 1$ and $\lambda_1 = 0$. Lemma 2 applies, and so $[0, \bar{w}] \in \mathcal{W}$. Since $\Sigma(0) = 0$ when $\lambda_1 = 0$, and Σ is continuous in w on $[0, \bar{w}]$, the continuous part of the wage distribution, there is a w offered by some firm with $\Sigma(w) < -\log(1-\gamma)/q$. But from simple algebra, it follows that $\pi(w) < \gamma$ for such a wage, contradicting the free entry condition. □

Proof of Lemma 4: As this result is well-known in the partial equilibrium search literature, we include it only for completeness. Perform integration by parts on the expression for R_n in equation (9) and use the fact that $\rho(w) = \tilde{\rho}(w \mid \gamma)$ for $w \in \mathcal{W}$:

$$R_n = \tilde{\rho}(\bar{w} \mid \gamma) - \int_0^{\bar{w}} G(w)^n \tilde{\rho}'(w \mid \gamma) dw + (1 - G(\bar{w})^n)\left(\tilde{\rho}(w^* \mid \gamma) - \tilde{\rho}(\bar{w} \mid \gamma)\right) - \sum_{i=1}^{n} c_i$$

Thus the marginal return to sampling the $n + 1^{\text{st}}$ wage is

$$R_{n+1} - R_n = \int_0^{\bar{w}} G(w)^n \left(1 - G(w)\right) \tilde{\rho}'(w \mid \gamma) dw$$
$$+ G(\bar{w})^n \left(1 - G(\bar{w})\right)\left(\tilde{\rho}(w^* \mid \gamma) - \tilde{\rho}(\bar{w} \mid \gamma)\right) - c_{n+1} \qquad (38)$$

Assume $\lambda_0 + \lambda_1 < 1$. Then Lemma 2 states $\bar{w}$ is strictly positive, and so the first term is strictly decreasing in n. The second term is decreasing in n if $G(\bar{w}) < 1$, and constant otherwise. The third term is nonincreasing in n by assumption. Hence $R_{n+1} - R_n < R_n - R_{n-1}$ if $\lambda_0 + \lambda_1 < 1$.

If $\lambda_0 + \lambda_1 = 1$, then the first two terms may be identical to zero, but one still obtains a weak inequality. □

Proof of Lemma 5: If $\lambda_0 + \lambda_1 = 1$, the result is immediate. Otherwise, Lemma 3 implies $\lambda_1 > 0$. Then according to part 4 of Definition 1 (Optimal Sampling), $R_1 \geq R_n$ for all n, and in particular $R_1 \geq R_2$. Then Lemma 4 implies $R_2 - R_1 > R_3 - R_2$. Since the left hand side of this inequality is nonpositive, $R_2 > R_3$. A simple induction argument establishes that $R_1 > R_n$ for all $n \geq 3$, and thus Optimal Sampling implies that $\lambda_n = 0$ for all such n.

Finally, suppose $\lambda_2 > 0$, so $R_1 = R_2$. Then Lemma 4 implies $R_1 - R_0 > R_2 - R_1 = 0$, or $R_1 > R_0$, and so Optimal Sampling implies $\lambda_0 = 0$, thus establishing $\lambda_1 + \lambda_2 = 1$. □

Proof of Proposition 2: We will first construct the equilibrium, and then prove that it exists.

1. Construction: First use equation (12) to eliminate q from (11). This gives

$$\hat{G}_2(w) = \frac{1-z}{2z}\left(\frac{\log(1-w)+\log(1-\gamma)-\log(1-w-\gamma)}{-\log(1-\gamma)}\right) \tag{39}$$

Recall that $G(w) = \hat{G}_2(w)$ along the atomless portion of the wage distribution $[0, \bar{w}]$, and so this yields (13).

For a certain distribution of wages to be an equilibrium, we require that $\tilde{\rho}$ is strictly increasing on the support of the wage distribution $\mathcal{W}$ (Lemma 1). Thus there is an atomless distribution of wages in equilibrium if $G(w^*) \geq 1$, where G is defined by (13) and w^* by (8), or equivalently iff:

$$z \leq \frac{\log(1-w^*)+\log(1-\gamma)-\log(1-w^*-\gamma)}{\log(1-w^*)-\log(1-\gamma)-\log(1-w^*-\gamma)} \equiv \underline{z} \in (0,1). \tag{40}$$

If $z > \underline{z}$, a mass μ of firms will offer wage w^*, and these firms must still earn profit γ. Since $p_2(1; w^*, w) = 1$ if $w < w^*$ (part 3 of Definition 1, Optimal Application) and $p_2(i; w^*, w^*) = 1/2$ (symmetry restriction on p), it follows that $\hat{G}_2(w^*) = 1 - \mu/2$. Substitute this into (39) to obtain (14); the 'max' operator takes care of the case when the wage distribution is atomless.

The proportion of workers sampling two jobs, z, is pinned down by the requirement that $R_1 = R_2$ in a search equilibrium. According to equation (38), this imposes:

$$\int_0^{w^*} G(w)(1-G(w))\tilde{\rho}'(w \mid \gamma)dw = c_2 \tag{41}$$

Also, observe that since $R_1 = R_2$ in the candidate equilibrium, Lemma 4 implies that $R_1 > R_n$ for $n \notin \{1, 2\}$, and thus no worker wishes to change her sample size. In particular, all workers are willing to participate in the labor market, $R_1 > 0$.

Next, the top of the convex part of the wage distribution, $\bar{w}$, must satisfy $G(\bar{w}) = 1 - \mu$. Thus from (13):

$$\bar{w} = 1 - \frac{\gamma}{1-(1-\gamma)^{(1-z(2\mu-1))/(1-z)}} \tag{42}$$

Finally, to see that z can never exceed some threshold $\bar{z}$, impose the restriction that $\mu < 1$ in equation (14). If this restriction failed, then the wage distribution would be degenerate at w^*, violating Lemma 2. The restriction is equivalent to:

$$z < \frac{\log(1-w^*)+\log(1-\gamma)-\log(1-w^*-\gamma)}{\log(1-w^*)-\log(1-w^*-\gamma)} \equiv \bar{z} \tag{43}$$

One can easily confirm that $\bar{z} \in (\underline{z}, 1)$.

2. Existence and Stability: Observe that the left hand side of equation (41) is directly and indirectly a continuous function of the endogenous variable z. Moreover,

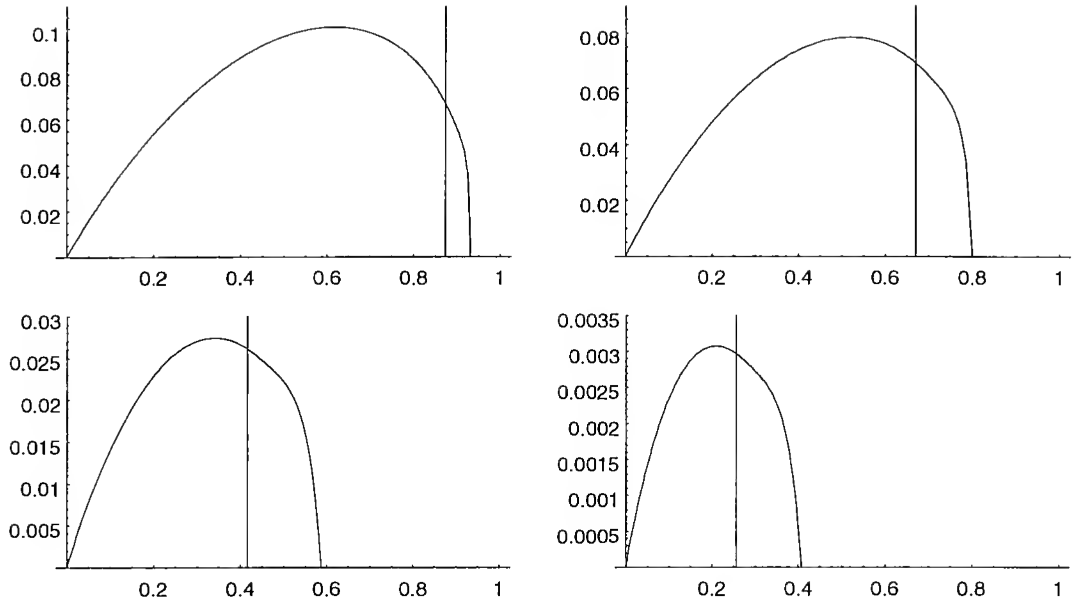

Figure 1: These panels plot the gross return to searching for a second job as a function of the equilibrium value of $z \leq \bar{z}$, for four different values of γ. The vertical lines show $\underline{z}$. In equilibrium, this gross return must equal the cost of searching for a second job, c_2; see equation (41). The top row shows the limit as γ converges to 0, and $\gamma = 0.1$ while the bottom row shows $\gamma = 0.5$ and $\gamma = 0.9$. The general shape of this figure is remarkably independent of γ.

the left hand side of equation (41) is nonnegative, and evaluates to 0 at $z = 0$ and $z = \bar{z}$ (see Figure 1). Therefore, this expression must have an interior maximum, whose value we denote by $\bar{c}$. For all $c_2 > \bar{c}$, there can be no solutions to (41), and hence no equilibria with search. For all $c_2 \in (0, \bar{c})$, continuity implies that there must be at least two solutions to (41), and thus at least two equilibria with search. At one intersection, the left hand side of equation (41) is decreasing in z. A small increase in the number of workers searching for two jobs z reduces the return to searching for two jobs by a small amount (since the function is decreasing). Hence, equilibria where the left hand side of equation (41) is decreasing in z are 'stable'; conversely, equilibria where this expression is increasing in z are unstable. □

Figure 1 draws the left-hand side of (41) for illustration.

Proof of Proposition 4: In any equilibrium, firms make zero net profit, from the Free Entry condition, part 2 of Definition 1. Thus it is sufficient to show that workers are better off in a search equilibrium. In a no-search equilibrium, the return to a worker is the maximum of $R_0 = 0$ (if she does not participate) and $R_1 = -c_1$ (if she participates), since the wage distribution is degenerate at 0 by Proposition 1. Now consider a search equilibrium, which by Proposition 2 exists if $c_2 < \bar{c}$. The return to a worker is $R_1 = R_2 > 0$. To complete the proof, we must show that if $c_1 < 0$, $R_1 > -c_1$ as well. But this is trivial algebra, as workers have a chance of receiving a strictly positive wage in addition to 'paying' the search cost c_1. □

Proof of Lemma 6: Again, we include this proof only for completeness. First suppose in equilibrium some firm uses $k \in [\underline{k}, k_0)$ units of capital and pays a wage $w \geq 0$. Consider alternatively using k_0 units of capital and paying w. Its net profit would be:

$$\begin{aligned}\left(1-e^{-q\Sigma(w)}\right)(f(k_0)-w)-k_0 &> \left(1-e^{-q\Sigma(w)}\right)\left(\frac{k_0}{k}f(k)-w\right)-k_0 \\ &= \frac{k_0}{k}\left[\left(1-e^{-q\Sigma(w)}\right)\left(f(k)-w\cdot\frac{k}{k_0}\right)-k\right] \\ &\geq \frac{k_0}{k}\left[\left(1-e^{-q\Sigma(w)}\right)(f(k)-w)-k\right]\end{aligned}$$

The first inequality follows because $f(k_0)/k_0 > f(k)/k$ by the definition of k_0. The equality is simple algebra. The second inequality follows because $wk/k_0 \leq w$ if $k < k_0$. Since the expression in brackets is $\pi(k, w) - k = 0$ by Free Entry, part 2 of Definition 2, we obtain that $\pi(k_0, w) > k_0$, contradicting Free Entry.

Alternatively, suppose some firm uses $k > k_1$ units of capital and pays a wage $w \geq 0$. Consider a firm using k_1 units of capital and paying wage w. Its net profit would be:

$$\begin{aligned}\left(1-e^{-q\Sigma(w)}\right)(f(k_1)-w)-k_1 &> \left(1-e^{-q\Sigma(w)}\right)(f(k)+(k_1-k)-w)-k_1 \\ &\geq \left(1-e^{-q\Sigma(w)}\right)(f(k)-w)-k\end{aligned}$$

The first inequality follows by the fact that a concave function lies below its tangent line and that $f'(k_1) = 1$. The second inequality is simple algebraic manipulation and use of the fact that $k_1 < k$. Thus $\pi(k_1, w) - k_1 > \pi(k, w) - k$, contradicting Profit Maximization, part 1 of Definition 2. □

Proof of Proposition 5: The proof once again follows Propositions 1 and 2, and so we proceed quickly.

First, if no worker samples more than one job, then all firms offer workers their reservation wage, 0. Since $K(0) = k_0$, this is firms' capital intensity. On the other hand, if all firms offer a zero wage, then workers will sample zero (c_1 positive) or one (c_1 negative) jobs. This establishes the existence and characterization of a no-search equilibrium.

Turn now to the search equilibria. Substitute q back into (22), giving the equilibrium type distribution (24). Also, use the condition that $\hat{G}_2(W(k^*)) = 1 - \mu/2$

to obtain condition (25). The fraction μ lies between 0 and 1 if $z \in (\underline{z}, \bar{z})$, defined by:

$$\underline{z} = \frac{\phi(k^*) - \phi(k_0)}{\phi(k^*) + \phi(k_0)} \qquad \bar{z} = 1 - \frac{\phi(k_0)}{\phi(k^*)}$$

Equilibria with $z < \underline{z}$ have atomless type distributions, while there are no equilibria with $z > \bar{z}$. Finally, the supremum of the atomless portion of the capital distribution, $\bar{k}$, satisfies $H(\bar{k}) = 1 - \mu$.

To close the system of equations for a search equilibrium, use the workers' sample size indifference condition, $R_1 = R_2$:

$$\int_{k_0}^{k^*} H(k)(1 - H(k))\tilde{\rho}'(k \mid f)dk = c_2 \tag{44}$$

Existence for moderate values of c_2 follows from continuity of the left hand side of (44) in z. □

Proof of Proposition 7: The characterization of the limiting equilibrium follows from Proposition 5. All firms use capital intensity $k = k^*$. The fraction of workers searching is $z = \bar{z}$, defined in the previous proposition. Substituting this into the equation for labor market tightness (23) yields $q = \phi(k^*)$.

We now characterize the constrained efficient allocation. It is no longer trivial that the planner would like all workers to sample one job. In particular, if it is optimal for different firms to use different technologies, then the planner might want capital intensive firms to enjoy longer queues, which is only possible if workers search. However, since the output of a firm is concave in its expected queue length q and its capital intensity k, $(1 - e^{-q}) f(k) - k$, it can never be optimal to have different firms with different queue lengths and different capital intensities. Thus we must simply ask about the capital intensity and queue length — or the u-v ratio — at a representative firm.

The planner chooses k and $q = 1/V$ to maximize output at a representative firm times the number of firms.[13]

$$\frac{(1 - e^{-q}) f(k) - k}{q}$$

This objective is a concave function of k, but is not jointly concave in q and k. However, we can find the optimal pair q and k through a simple trick. For fixed q, the first order condition with respect to k is necessary and sufficient for a maximum, and yields $(1 - e^{-q}) = 1/f'(k)$ or equivalently $q = \phi(k)$. Use this to eliminate q from the objective. The planner then simply chooses k to maximize:

$$\frac{f(k) - kf'(k)}{f'(k)\phi(k)} = \tilde{\rho}(k \mid f)$$

[13] Since we are concerned with the limit as c_2 converges to zero, and $c_1 \leq c_2$, we do not consider the possibility that the planner would shut down the economy. This would only happen if c_1 is very large.

from equation (21). Thus it follows from the definition of k^* that the optimal capital intensity is k^*, and therefore that $q = \phi(k^*)$. □

Proof of Proposition 8:

1. Construction. The characterization of no-search equilibria is straightforward. To construct a search equilibrium, we begin by substituting our expression for q into (32) immediately yields (33). The Free Entry condition for firms using capital k^* then determines the mass of firms at this point, given by (34). We established that the wage equation (35) applied in equilibrium in the text. The value of an unemployed worker equals R_1, since some workers choose to sample only one job.

To determine the steady state unemployment rate, observe that the measure of firms that hire a worker in any given period, $\mathcal{E}$, can be calculated by integrating the probability that a vacant firm using k units of capital hires a worker in any period, $1 - e^{-q\Sigma(W(k))}$, times the 'density' of vacant firms using capital k, $dJ(k)$. Since $\left(1 - e^{-q\Sigma(W(k))}\right) kS'(k)$ is equal to the flow or rental value of a firm with capital k, which is normalized to k, we find that the probability that a vacant firm with k units of capital hires a worker is $1/S'(k)$. Equation (37) then follows immediately from the derivative of S in (26). Finally, the steady state unemployment rate satisfies a 'job creation equals job destruction' equation, $(1-\delta)\, U\mathcal{E}/q = \delta(1-U)$, which is easily solved for U.

2. Existence. This proof is slightly more complex than the preceding existence arguments. We first show that for J in an interval $[0, \bar{J}]$, we can use the workers' Bellman equation to define a search intensity $z = Z(J)$ for each J, where Z is a continuous function. Then using this function, the condition that workers are indifferent between sampling one and two jobs pins down the equilibrium value of an unemployed worker. In proving this result, one must keep in mind that the capital distribution H depends on search intensity z, and does so continuously from (33). Also, k_0, k^*, $\tilde{\rho}$, and H all depend on J, but given the strict quasiconcavity of $\tilde{\rho}$ and the strict concavity of f, they do so continuously.

First, let $\bar{J}$ solve

$$(1-\beta)\, \bar{J} = \tilde{\rho}(k^* \mid f)$$

To see that this equation uniquely defines $\bar{J}$, observe that the workers' Bellman equation (36) and the definition of k^* imply $(1-\beta)\, J \geq \tilde{\rho}(k^* \mid f)$ for all J. Since $\tilde{\rho}(k \mid f)$ is a continuously decreasing function of J for fixed k, so is its maximum, $\tilde{\rho}(k^* \mid f)$. Thus there is a unique solution, $\bar{J}$, to the above equation.

Next define $Z : \left[0, \bar{J}\right] \to [0,1]$, where for $J \neq \bar{J}$, $Z(J)$ is the unique z that solves the Bellman equation (36) for a given value of J; and $Z(\bar{J}) \equiv 1 - \phi(k_0)/\phi(k^*)$. We now establish in three steps that $Z(J)$ is a well-defined and continuous. *First,* observe that $\int_{k_0}^{k^*} \tilde{\rho}(k \mid f) dH(k)$ is a continuous function of z, equal to zero when z is zero and equal to $\tilde{\rho}(k^* \mid f) \geq (1-\beta)\, J$ when $z = 1 - \phi(k_0)/\phi(k^*)$, hence for all $J \in [0, \bar{J}]$, there is some $z \in [0,1]$ that solves (36). *Second,* since the capital distribution H improves in the sense of first order stochastic dominance in response to an increase in z (see (33) and (34)), $\int_{k_0}^{k^*} \tilde{\rho}(k \mid f)dH(k)$ is also an increasing function of $z \in [0, 1-\phi(k_0)/\phi(k^*)]$, and so there is in fact a unique z that solves (36) for a given value of J. Therefore, $Z(J)$ is a well defined function. *Finally*, continuity of $Z(J)$ follows from continuity of $(1-\beta)\, J - \int_{k_0}^{k^*} \tilde{\rho}(k \mid f)dH(k)$ in z and J.

Next, we claim that for $J \in (0, \bar{J})$, we have $Z(J) \in (0, 1-\phi(k_0)/\phi(k^*))$, and therefore, the capital distribution is nondegenerate. To prove this, note that our usual arguments imply that the capital distribution can only be degenerate at k_0 or k^*. If it is degenerate at k_0, then since $W(k_0) = 0$, (36) implies $J = 0$. On the other hand, if it is degenerate at k^*, then $(1-\beta)\, J = \tilde{\rho}(k^* \mid f)$, which by definition implies $J = \bar{J}$.

Now finally turn to the condition that workers are indifferent between sampling one or two jobs, $R_1 = R_2$:

$$\int_{k_0}^{k^*} H(k)\,(1 - H(k))\,\tilde{\rho}'(k \mid f)dk = c_2$$

The left hand side of this expression is directly and indirectly (through $Z(J)$) a continuous function of J. It is strictly positive for $J \in (0, \bar{J})$, since the capital distribution is nondegenerate, but evaluates to 0 for $J \in \{0, \bar{J}\}$. Thus it obtains a maximal value $\hat{c} > 0$ at some $J \in \left(0, \bar{J}\right)$. Then using standard arguments, for all $c_2 < \hat{c}$, there are at least two values of J satisfying the workers' indifference condition. These correspond to search equilibria.

To complete the proof, we claim that a search equilibrium does not exist when $c_2 > \hat{c}$. Observe that the previous argument implies that a search equilibrium with $J \leq \bar{J}$ does not exist for such c_2. Also, there is no z that solves (36) for $J > \bar{J}$. □

References

[1] **Abowd, John and Francis Kramartz, 1994,** "High Wage Workers and High Wage Firms," National Bureau of Economic Research Working Paper, No. 4917

[2] **Acemoglu, Daron, 1996a** "A Microfoundation for Social Increasing Returns in Human Capital," *Quarterly Journal of Economics*, 111, 779–804.

[3] **Acemoglu, Daron, 1996b** "Good Jobs versus Bad Jobs: Theory and Some Evidence," MIT Working Paper.

[4] **Albrecht, James W. and Bo Axell, 1984** "An Equilibrium Model of Search Unemployment," *Journal of Political Economy*, 92, 824–840.

[5] **Blanchflower, David, Andrew Oswald and Peter Sanfey, 1996** "Wages, Profits and Rent-sharing" *Quarterly Journal of Economics*, 111, 227-253.

[6] **Bulow, Jeremy and Lawrence Summers, 1986** "A Theory of Dual Labor Markets with Applications to Industrial Policy, Discrimination and Keynesian Unemployment," *Journal of Labor Economics*, 4, 376–414.

[7] **Burdett, Kenneth and Kenneth Judd, 1983** "Equilibrium Price Dispersion," *Econometrica*, 51, 955–969.

[8] **Burdett, Kenneth and Dale Mortensen, 1989** "Equilibrium Wage Differentials and Employer Size," Mimeo, Northwestern.

[9] **Burtless, Gary, 1987** "Jobless Pay and High European Unemployment," in *Barriers to European Growth* eds Robert Lawrence and Charles Schultze, Washington DC: Brookings Institution.

[10] **Butters, Gerard, 1977** "Equilibrium Distribution of Sales and Advertising Prices," *Review of Economic Studies*, 44, 465–491.

[11] **Diamond, Peter, 1971** "A Model of Price Adjustment," *Journal of Economic Theory*, 3, 156–168.

[12] **Diamond, Peter, 1982** "Wage Determination and Efficiency in Search Equilibrium," *Review of Economics Studies* 49:2, 217–27.

[13] **Gibbons, Robert and Lawrence Katz, 1992** "Does Unmeasured Ability Explain Inter-industry Wage Differentials," *Review of Economic Studies*, 59, 515–35.

[14] **Groshen, Erica, 1991** "Sources of Wage Dispersion: How Much Do Employers Matter," *Quarterly Journal of Economics*, 106, 869–84.

[15] **Grossman, Sanford and Joseph Stiglitz, 1980** "On the Impossibility of Informationally Efficient Markets," *American Economic Review*, 70, 393–408.

[16] **Grout, Paul, 1984** "Investment and Wages in the Absence of Binding Contracts: A Nash Bargaining Approach," *Econometrica*, 52, 449–460.

[17] **Holzer, Harry, Lawrence Katz and Alan Krueger, 1991** "Job Queues and Wages," *Quarterly Journal of Economics*, 106, 739–768.

[18] **Hosios, Arthur, 1990** "On the Efficiency of Matching and Related Models of Search and Unemployment," *Review of Economic Studies*, 57, 279–298.

[19] **Krueger, Alan and Lawrence Summers, 1987** "Reflections on the Inter-Industry Wage Differentials," in *Unemployment and the Structure of Labor Markets*, edited by Kevin Lang and Jonathan Leonard, Basil Blackwell.

[20] **Krueger, Alan and Lawrence Summers, 1988** "Efficiency Wages and the Inter-Industry Wage Structure," *Econometrica*, 56, 259–93.

[21] **Lang, Kevin,1991** "Persistent Wage Dispersion and Involuntary Unemployment," *Quarterly Journal of Economics*, 106, 181–202.

[22] **Moen, Espen, 1995** "Competitive Search Equilibrium," SNF-Oslo Mimeo.

[23] **Montgomery, James 1991** "Equilibrium Wage Dispersion and Interindustry Wage Differentials," *Quarterly Journal of Economics*, 106, 163–179.

[24] **Morgan, Peter and Richard Manning 1985** "Optimal Search," *Econometrica*, 53, 923–944.

[25] **Mortensen, Dale and Christopher Pissarides 1994** "Job Creation and Job Destruction in the Theory of Unemployment," *Review of Economic Studies*, 61, 397–416.

[26] **Mortensen Dale, and Randall Wright, 1995** "Competitive Pricing and Efficiency in Search Equilibrium" Mimeo.

[27] **Peters, Michael, 1991** "Ex Ante Price Offers in Matching Games: Non-Steady States," *Econometrica*, 59, 1425–1454.

[28] **Pissarides, Christopher, 1990** *Equilibrium Unemployment Theory*, Oxford: Basil Blackwell.

[29] **Reynolds, Lloyd. 1951** *The Structure of Labor Markets: Wages and Labor Mobility in Theory and Practice*, New York, Harper.

[30] **Robert, Jacques and Dale Stahl, 1993** "Informative Price Advertising in a Sequential Search Market," *Econometrica*, 61, 657–86.

[31] **Rothschild, Michael, 1973** "Models of Market Organization With Imperfect Information: A Survey" *Journal of Political Economy*, 81, 1283–308.

[32] **Rothschild, Michael, 1974** "Searching for the Lowest Price When the Distribution of Prices is Unknown" *Journal of Political Economy*, 82, 689–711.

[33] **Salop Steven and Joseph Stiglitz, 1982,** "A Theory of Sales: A Simple Model of Price Dispersion with Identical Agents," *American Economic Review*, 72, 1121–1130.

[34] **Sattinger, Michael, 1991** "Consistent Wage Distributions," *Quarterly Journal of Economics*, 106, 277–288.

[35] **Shimer, Robert, 1996** "Essays in Search Theory," PhD dissertation, Massachusetts Institute of Technology: Cambridge, Massachusetts.

[36] **Stahl, Dale 1994** "Oligopolistic Pricing and Advertising," *Journal of Economic Theory*, 64, 162–77.

[37] **Stigler, George, 1961** "The Economics of Information," *Journal of Political Economy*, 69, 213–225.