ELEMENTARY SCIENCE APPLIED TO SANITATION AND PLUMBERS' WORK

ELEMENTARY SCIENCE

APPLIED TO SANITATION AND PLUMBERS' WORK

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SECOND EDITION

GURNEY AND JACKSON
10 PATERNOSTER ROW, LONDON, E.C.
1910

D. VAN NOSTRAND COMPANY,
NEW YORK.

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PREFACE TO SECOND EDITION

In presenting the Second Edition of *Elementary Science applied to Sanitation and Plumbers' Work,* the author desires to express his hearty thanks to all who were kind enough to offer criticisms and express opinions anent the value of the book.

On mature consideration it has been thought advisable to include a varied and extensive series of questions and answers in the mensuration section, which will doubtless be useful to teachers and taught.

The calculations on heating and ventilation have been extended, and several pages added to the mechanics section, in addition to alterations and extensions in other parts of the book.

To the publishers the author's thanks are due for the skill and care bestowed in the production of this edition, and, combined with the extensions and alterations effected in the text, it is hoped the sphere of usefulness of the work will be greatly extended.

A. HERRING-SHAW.

Municipal School of Technology,
Manchester, 1910.

PREFACE TO FIRST EDITION

The object of the author in compiling the matter contained in the following pages, is to produce a book on Elementary Science, fully treated (in its fundamental principles, and application to Sanitation and Plumbers' work) in such a manner as will be readily understood by all students. The price has been fixed at a low figure, to bring the book within the reach of all classes. It will be found useful to students preparing for the Examinations of the City and Guilds of London Institute, the Registered Plumbers' Company, and the Royal Sanitary Institute of Great Britain. The examiners to these institutions, almost without exception, complain of the lack of knowledge of Elementary Science on the part of candidates who take the above examinations.

The author hopes that the contents will be of material assistance in raising the standard of the work done by students of Sanitary Engineering. Should a student find any difficulty in thoroughly grasping any of the principles set forth in this book, the author will be pleased to afford him assistance, if it be in his power to do so.

The sincere thanks of the author are due to Professor J. Radcliffe, M.Sc.(Tech.), and the members of the staff of the Municipal and Sanitary Engineering Department, Municipal School of Technology, Manchester, for valuable suggestions and assistance, and also to various firms for the loan of electros.

A. HERRING-SHAW.

Municipal School of Technology,
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TABLE OF CONTENTS

ELEMENTARY SCIENCE

ARITHMETIC AND MENSURATION

PHYSICS

PHYSICS—*Continued.*

CHEMISTRY

LIST OF TABLES

LIST OF ILLUSTRATIONS AND FIGURES

Problems—

Figures—

Figures—Continued. PAGE

Figures—Continued.

Figures—Continued.

ELEMENTARY SCIENCE APPLIED TO SANITATION AND PLUMBERS' WORK

ELEMENTARY PRACTICAL GEOMETRY

CHAPTER I

DEFINITIONS OF TRIANGLES, CIRCLE, QUADRILATERALS, ETC.

The following has been carefully considered and selected to give the necessary information in a manner that will be easily understood, without previous instruction in the subject, as far as directly concerns **Plumbers' Work.**

INSTRUMENTS

As there is a great benefit derived from the working of the various problems, and later, in the making of both working and finished drawings, it is perhaps advisable to give a list and a description of what is requisite as regards drawing instruments.

A good drawing board, 30 ins. by 22 ins., of yellow pine framed with hard wood, and with its surface perfectly level and the corners true right angles.

A T-square of suitable length, bound with a bevelled hard wood edge. Two set squares, having angles of 45° and 60° respectively—the first 6 ins. long and the latter 10 ins. long.

The compasses should have firmly fastened needle points. Buy just the instruments you require, *i.e.*, a $4\frac{1}{2}$-in. half-set with knee joints and ink and pencil points, and a ruling pen for inking-in lines. It is preferable to use a pricker for marking off distances, etc. This can be bought, or a good one can be made by breaking a sewing needle and forcing the blunt end into the wood of a common penholder, leaving the point projecting half an inch. The foregoing instruments may be carried in a roll of wash leather.

SCALES

These are indispensable. A set of paper scales can be obtained for a few pence.

The **pencils** should be of good quality, and for construction a hard HH or 3H is the best; for lining-in, an H pencil may be used. When sharpened, finish to a fine chisel edge with glasspaper.

A protractor (Fig. 1) is an instrument used for measuring angles already drawn, or for drawing an angle of a given degree (see Definitions). It consists either of a circular or semi-circular disc, made in metal or celluloid. It may also be used as a flat rule. The semi-circular celluloid type is a convenient form; it is very thin, has easily read degrees, and can be used with a fair amount of accuracy. Though perhaps not necessary at first, it is almost impossible to work without one in the later stages.

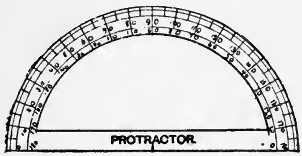


FIG. 1

DRAWING PAPER

For ordinary pencil work smooth cartridge paper is the most suitable; when the drawings have to be inked-in, a better quality is desirable, such as Whatman's cartridge papers.

DEFINITIONS

Axioms and Explanations of Terms, etc.

Def. 1.—A Point denotes position only. It is shown by a dot, or a dot enclosed in a small circle.

Lines

Def. 2.—A Line has length and position, but neither breadth nor thickness. It is indicated by letters placed at its extremities as A, B. Various methods of drawing lines are used, such as thick, thin, dotted, and chain lines.

Def. 3.—A Straight line is the shortest distance between two given points; it is also called a Right line.

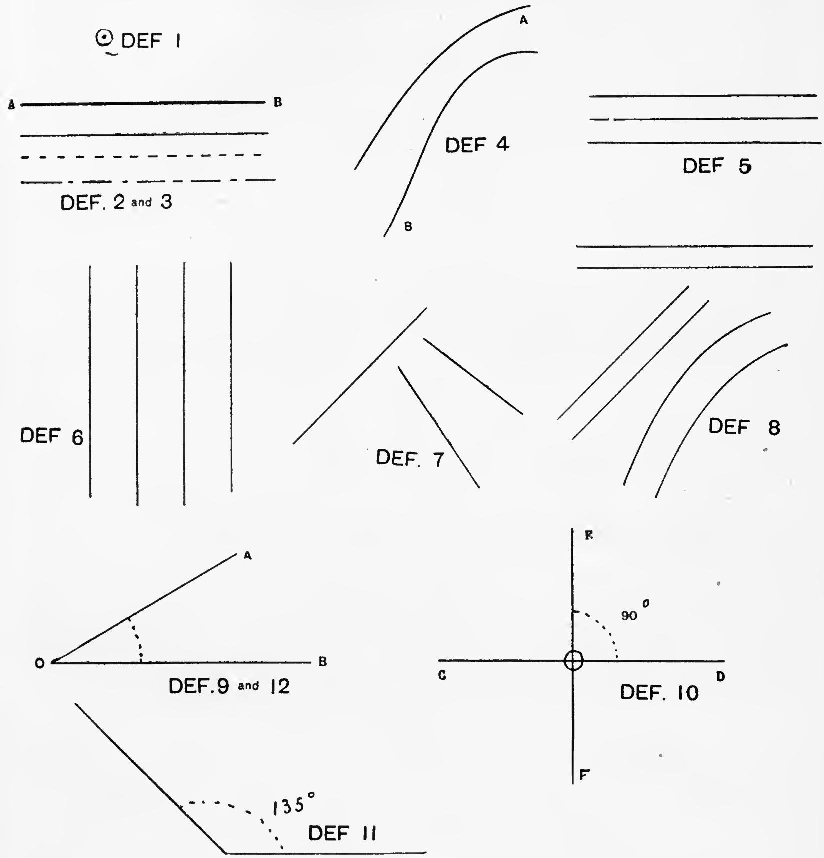
Def. 4.—A Curved line is nowhere straight. There are different kinds of curved lines, such as (*a*) a simple curve, and (*b*) a compound curve.

Def. 5.—A **Horizontal** line is a level line, similar to the surface of still water.

Def. 6.—A Vertical or Perpendicular line is perfectly upright, like a plumb line.

Def. 7.—An Oblique line is neither horizontal nor vertical.

Def. 8.—Parallel lines are the same distance apart, and cannot meet, however far they may be produced.



Angles

Def. 9.—An Angle is the inclination of two straight lines to each other which meet together; when two straight lines meet at a point AOB, the "corner" they form is termed "an angle."

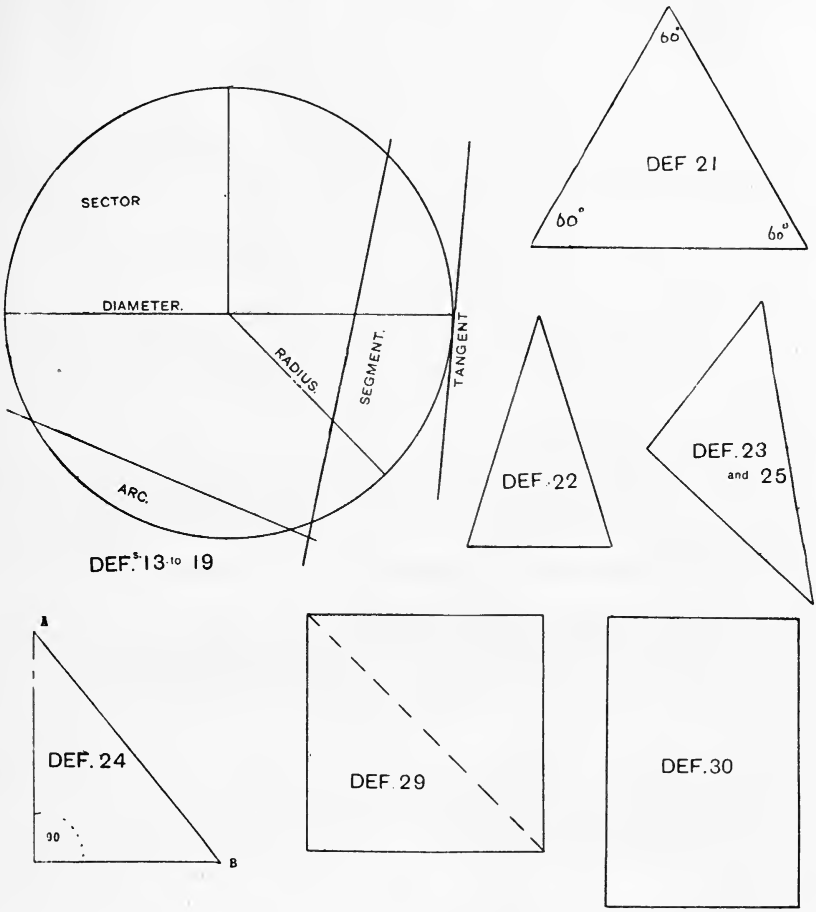
Def. 10.—When a line EO meets another CD at a point O, so that the adjacent angles on each side are equal, then that line is said to be perpendicular, and the angles formed are right angles.

A Right angle is divided into 90 degrees for the purposes of measuring, etc.

In Def. 10, if we continue the line EO below CD, then we have four right angles. Thus it will be seen (if a circle is described with centre O) that a circle contains four right angles, or 360°.

Def. 11.—An Obtuse angle is greater than a right angle.

Def. 12.—An Acute angle is less than a right angle.



The Circle

Def. 13.—A Circle is a plane figure contained by one line called the "Circumference," and is such, that all lines drawn from a certain point within the figure to the circumference are equal to one another.

Def. 14.—This point is called the "Centre" (*Euc.* I., Def. 15, 16).

Def. 15.—A line drawn through the centre and terminating at each end in the circumference, is the Diameter of the circle, and divides the circle into two equal parts, *i.e.*, semi-circles.

Def. 16.—**The Radius** of a circle is the distance from the centre to any point in the circumference, and a line drawn at right angles to this line touching the circumference is called a Tangent to the circle.

Def. 17.—**A Segment** of a circle is the figure contained by a straight line and the portion of the circumference which it cuts off.

Def. 18.—**An Arc** is a portion of the circumference of a circle.

Def. 19.—**A Sector** is any portion enclosed by two radii and an arc, *i.e.*, $\frac{1}{4}$ circle = a Quadrant, $\frac{1}{6}$ = a Sextant, and $\frac{1}{8}$ circle = an Octant.

Triangles

Any figure formed by straight lines is termed rectilineal.

Def. 20.—Trilateral figures, or triangles, are those which are formed by three straight lines.

Triangles are named (1*st*) *from the comparative lengths of their sides to each other :*

Def. 21.—An Equilateral triangle has 3 equal sides,

Def. 22.—An Isosceles „ „ 2 of its sides equal,

Def. 23.—A Scalene „ „ 3 unequal sides;

and (2*nd*) *from the magnitude of their angles :*

Def. 24.—A Right-angled triangle has one of its angles a right angle, *i.e.*, 90°. The side opposite the right angle is called the "Hypotenuse." In Def. 24 AB is the hypotenuse.

Def. 25.—An Obtuse-angled triangle has one obtuse angle.

Def. 26.—An Acute-angled triangle has three acute angles (see Def. 22).

In all triangles the sum of the three angles always equals two right angles, or 180°. Thus, if two angles of a triangle are given, these together, subtracted from 180°, will give the third, or in the case of an isosceles triangle, if the angle at the apex or top be given, the result of this substracted from 180° and afterwards divided by 2 would give the angles at the base :—

$$\frac{180^\circ - \text{Angle at apex (say } 40^\circ)}{2} = 70^\circ \text{ (each angle at base),}$$

for if the three sides of a triangle are equal, the angles are equal; and if the two opposite sides are equal then the two opposite angles are also equal.

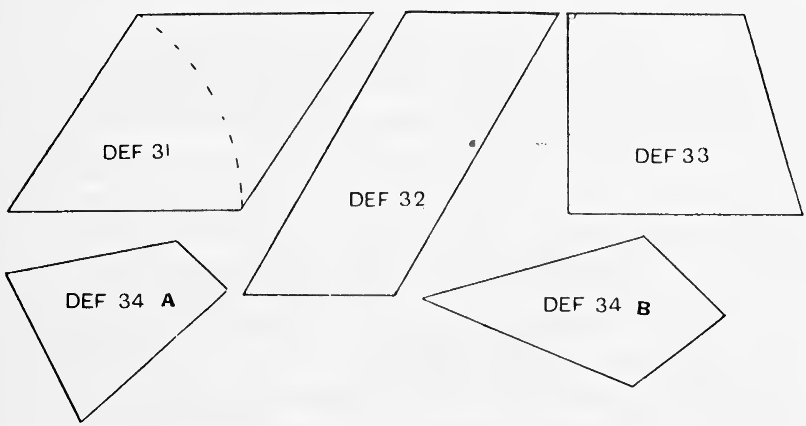
Quadrilaterals

Def. 27.—A Quadrilateral figure or Quadrangle is bounded by four straight lines, the four angles equalling four right angles.

Def. 28.—A Parallelogram is a quadrilateral figure in which the opposite sides are parallel. The Square, Rectangle or Oblong, the Rhombus, and the Rhomboid are parallelograms.

Def. 29.—A Square has all its sides equal and its angles right angles.

Def. 30.—A Rectangle or Oblong has its opposite sides equal and its angles right angles.



Def. 31.—A Rhombus has all its sides equal, but its angles are not right angles.

Def. 32.—A Rhomboid has its opposite sides equal, but its angles are not right angles.

Def. 33.—A Trapezoid has only two sides parallel.

Def. 34.—A Trapezium (A) has none of its sides parallel, but may have two of its sides equal, as (B). It is then termed a Trapezium or kite.

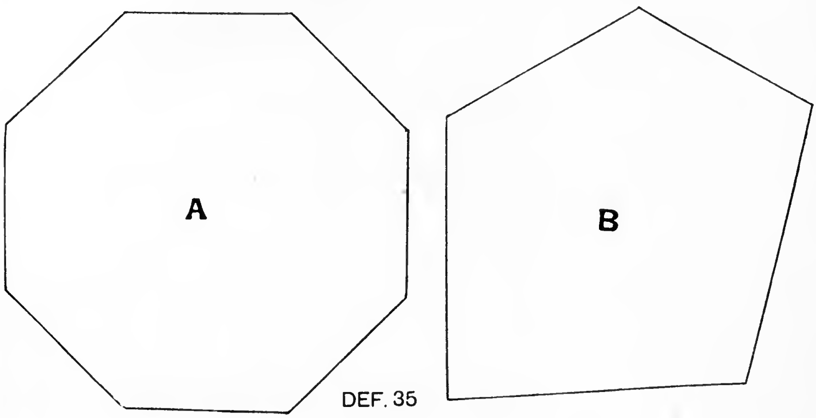
Polygons

Def. 35.—A and B. A Polygon is a plane figure bounded by more than four straight lines.

If the sides are equal it is termed a regular Polygon.
If the sides are unequal it is termed an irregular Polygon.

Polygons are named according to the number of their sides, viz. :—

A Pentagon has five sides.
A Hexagon has six sides.
A Heptagon has seven sides.
An Octagon has eight sides.
A Nonagon has nine sides.
A Decagon has ten sides.
An Undecagon has eleven sides.
A Duodecagon has twelve sides.



DEF. 35

CHAPTER II

PROBLEMS OF TRIANGLES, CIRCLE, QUADRILATERALS, ETC.

LINES AND ANGLES

Problem 1.—**To bisect a given line AB or Arc CD.**

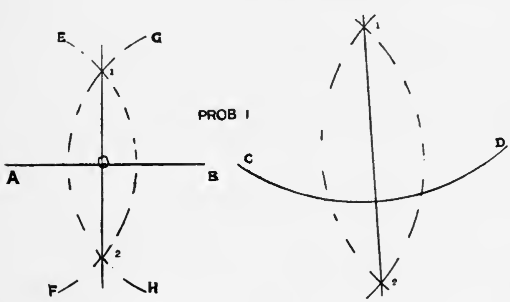
With A as centre and a radius greater than half the line AB, describe the arc EF.

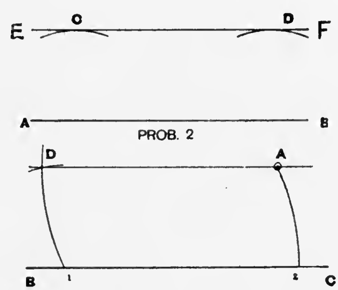
With B as centre and same radius, describe GH, intersecting EF at 1 and 2.

Join 1 and 2, cutting AB at O.

Then AO = OB, and the line AB is bisected, and the line 1-2 is perpendicular to AB.

Treat the arc CD by the same method as shown, and the line 1-2 is perpendicular to the tangent of the arc CD.





Problem 2.—**To draw a line parallel to a given straight line.**

From two centres on line AB draw two arcs C and D of equal radius. Draw EF resting upon, and just touching the arcs C and D. Then EF is parallel to AB.

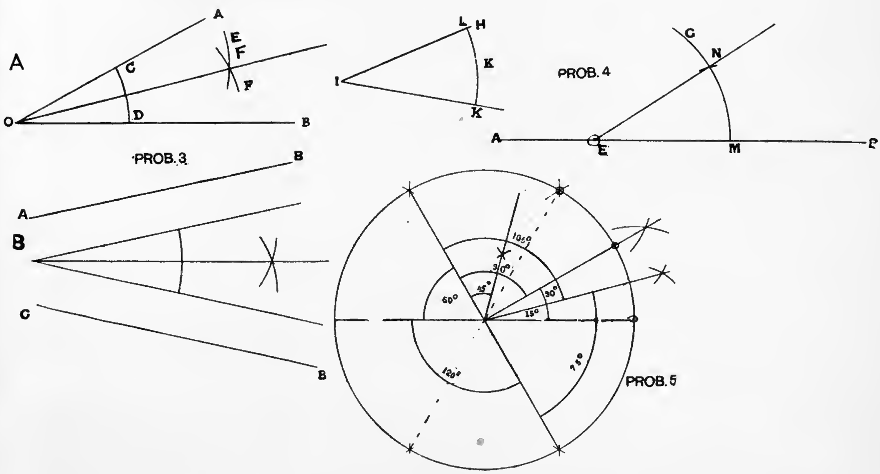
(Another Method.) **To draw a line parallel to a given straight line through a given point.**

Let A be the given point and BC the given line. With any point 1 as centre and radius 1A describe arc A2. With A as centre and **same radius** describe another arc 1D. Cut off 1D equal to A2. Join AD, which will be parallel to BC.

Problem 3.—**To bisect a given angle AOB.**

(*a*) With centre O and a convenient radius describe the arc CD. With C as centre describe arc E, and with the same radius and D as centre cut arc E at F. Join OF. Then the line OF bisects the angle AOB, and AOF = FOB.

(*b*) If the lines could not be produced so as to meet, then draw two lines at equal distances from and parallel to the two lines, *e.g.*, AB and BC.



Problem 4.—**At a given point E in a line AB, to make an angle equal to the given angle HIK.**

With centre I and largest possible radius describe the arc K. Then with centre E describe arc G, using radius IK. With centre M and distance KL cut arc G at N. Draw EN. Then the angle NEM equals the angle HIK.

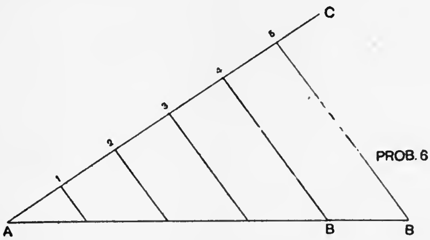
Problem 5.—**To construct an angle containing a given number of degrees.**

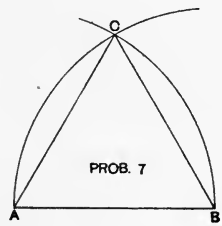
The circumference of a circle is supposed to be divided into 360 equal parts called "Degrees." The radius of a circle may be set off exactly six times round the circumference; hence, if an arc be described, and a portion cut off equal to the radius of the arc, an angle containing 60° will be obtained. By applying this principle a number of angles may be constructed. Construction for angles of 60°, 120°, 30°, 15°, 45°, and 75° are shown. From this other angles may be obtained.

By the protractor (Fig. 1).—When this is used it is merely necessary to place the protractor on the line with its centre point over the required point in the line where the angle is to be formed, and then prick through the paper at the required degree on the edge of the protractor.

Problem 6.—**To divide a straight line AB into any number of equal parts**: *e.g.*, **divide AB into 5 equal parts.**

Draw AC at a convenient angle to AB and of suitable length. Mark off five equal spaces on AC. Join the last, and draw four lines parallel to B5 and cutting AB, thus dividing it into five equal parts.



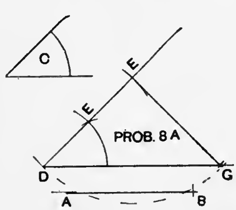


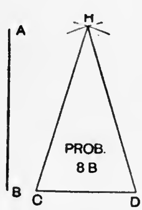
TRIANGLES

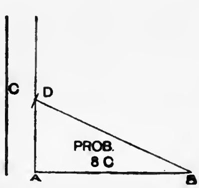
Problem 7.—**To construct an equilateral triangle on a given line AB.**

With centres A and B and radius AB describe two arcs cutting each other at C. Join AC and CB. Then ACB is the required equilateral triangle. (*Euc.* I., 1.)

Problem 8.—(*a*) **To construct an isosceles triangle when each of the equal sides equals AB, and each of the equal angles equals C.**







Draw DG of indefinite length. Make GDE equal to angle C. Cut off DE to length AB. With E as centre and same radius cut DG in G. Join GE. Then DGE is the required isosceles triangle.

(*b*) **To construct an isosceles triangle when the lengths of the two sides and base are given.** Let AB = length of sides, and CD = base. With radius AB and from centres C and D describe two arcs intersecting at H. Join HC and HD. HCD is the required isosceles triangle.

(*c*) **To construct a right-angled triangle, the hypotenuse and base being given.** Let AB be the base and C the hypotenuse. At A erect a perpendicular; with B as centre and radius equal to C cut perpendicular in D. Join DB.

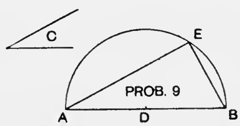
Problem 9.—**To construct a right-angled triangle, the hypotenuse and an acute angle being given.**

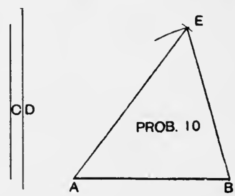
Let AB be the hypotenuse and C one of the acute angles. Bisect AB in D. With centre D describe a semi-circle on AB. At A construct an angle BAE, equal to C. Join BE. Then BAE is the required triangle, the angle in the semi-circle being a right angle. (*Euc.* III., 3.)

Problem 10.—**To construct a triangle, the three sides being given.**

Let AB be the base and C and D the two sides respectively.

With centre A and radius D describe an arc. With centre B and radius C describe an arc intersecting the former arc at E. Join EA and EB. Then ABE is the required triangle. (*Euc.* I., 22.)



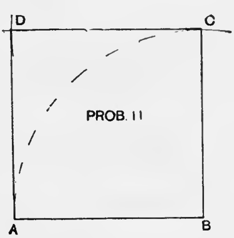


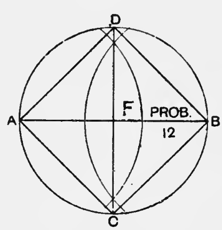
QUADRILATERALS

Problem 11.—**To construct a square, the side being given.**

Let AB be the given side.

At B erect a perpendicular BC and make it equal to AB. With centres C and B and radius AB describe arcs intersecting at D. Draw CD, AD.





Problem 12.—**To construct a square, the diagonal being given.**

Let AB be the diagonal.

Bisect AB. With centre F and radius FA describe circle ADBC. Join AD, DB, BC, and CA.

Note.—The foregoing principles apply in the geometrical construction of rectangles, the only difference being in the lengths of the sides. In Problem 12, for a rectangle, the length of one side would be necessary to complete the problem.

Problem 13.—**To construct a rhombus.**

(*a*) **The side and one angle being given.**

(*b*) **The side and diagonal being given.**

(*a*) Let AB be the given side and C the angle.

At A construct an angle equal to C, and make AD equal to AB. With D and B as centres and radius AB describe two arcs intersecting at E. Draw DE and BE.

(*b*) Let AB be the diagonal and C the side.

With radius C and centres A and B describe four arcs intersecting at D and E. Draw AD, BD, BE, and EA.

THE CIRCLE

Problem 14.—**To find the centre of a circle.**

Draw any two chords AB, BC. Bisect them by lines which intersect at D. This point is the centre of the circle. (*Euc.* III., Cor. 2.)

Problem 15.—**To describe a circle passing through three given points not in the same straight line.**

Let A, B, and C be the three points.

Join AB and BC. Bisect both lines as above. From D, the point where these lines meet, describe the circle.

Problem 16.—**To describe a circle about a triangle.**

Bisect two sides and proceed as Problem 15.

Problem 17.—**To draw a tangent to a circle:**

(*a*) **From a point in the circumference.**

(*b*) **From a point without.**

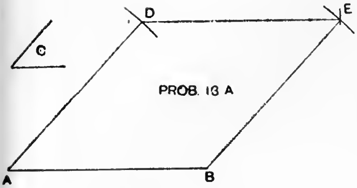
(*a*) Let A be the given point in circumference of circle.

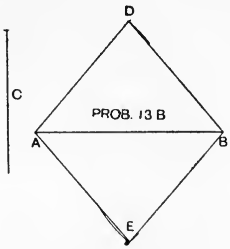
Join this with the centre B and produce to C, making AC equal to AB.

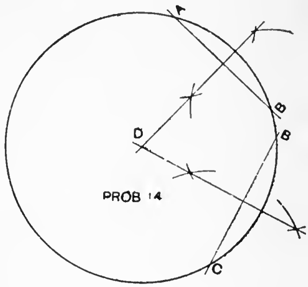
With centres B and C describe two arcs cutting at D. Join DA, which is a tangent of the circle.

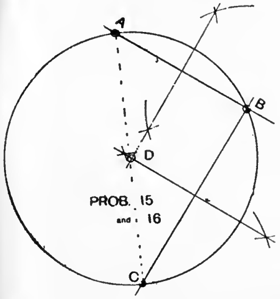
(*b*) Let X be the point without the circle.

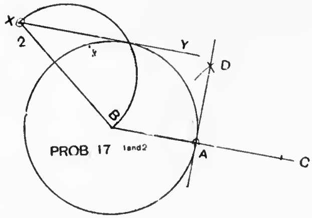
Join BX. Bisect and describe semi-circle cutting circumference of larger circle. Draw XY, the required tangent.

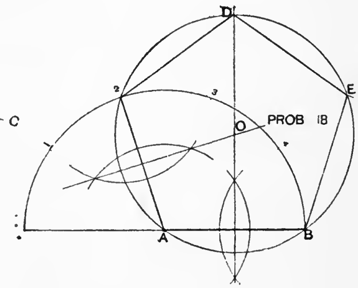


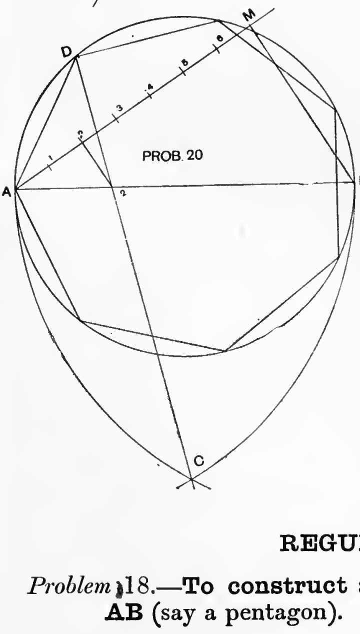


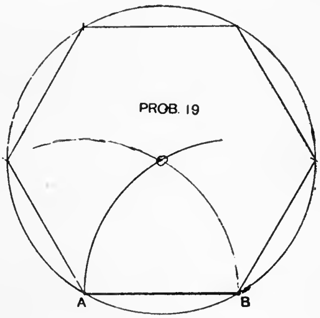












REGULAR POLYGONS

Problem 18.—**To construct any regular polygon on a given line AB** (say a pentagon).

Produce AB, and with centre A and radius AB describe a semi-circle. Divide the semi-circle into as many parts as the polygon has sides (by trial with dividers, or by the aid of a protractor). **Always join A with 2,** giving another side of the polygon, no matter how many sides the polygon required has. Bisect these two sides, also finding the centre of the polygon and of circle containing same. Describe this circle (radius OA). Mark off BE and ED equal to AB. Join BE, ED, and D2, forming the required pentagon.

Note.—The division of the semi-circle must be accurately done, as upon this and the correct finding of centre of figure depends the accuracy of this construction.

Problem 19.—**To construct a regular hexagon on a given line.**

With centres A and B and radius AB describe two arcs cutting at O. With O as centre and same radius describe a circle and set off AB round it. Join these points and form the required hexagon.

The side AB may be obtained by using the 60° set square and the hexagon quickly set up by this means.

Problem 20.—**To inscribe any regular polygon in a circle.**

Draw the diameter AB. If the centre of circle be not given, find same first. Divide AB into seven equal parts. With centres A and B describe two arcs intersecting at C. From C, through the **second** part on AB, rule CD, cutting off AD, one of the sides of the polygon. Set off AD round the circle, and join the points.

Note.—The **greatest care** is necessary in dividing the line and drawing the line from C **exactly** through the **second division in Point 2** on the line AB.

GENERAL NOTES ON POLYGONS

If the angle of two adjacent sides is known, and their length, it is an easy matter to construct the regular polygon by means of the protractor.

If all the angles of a polygon be joined with the centre, there will be formed as many equal isosceles triangles as the polygon has sides, having their vertical angles at the centre equal (*Euc.* I., 15, Cor. 2). Hence, if 360° be divided by the number of sides of the regular polygon, the value of the central angles will be obtained. These may be marked round from readings taken from the protractor.

CHAPTER III

AREAS, SCALE DRAWING, CURVES, THE ELLIPSE

AREAS, GENERAL PRINCIPLES

It is necessary, before working further problems in this subject, that the following elementary principles should be carefully studied:—

(1) **The area of a plane figure** (a sheet of paper, lead, or metal, or the surface of a body) is **the amount of** surface enclosed by its boundary or perimeter. It depends upon both the shape and the perimeter of the figure.

(2) **Parallelograms** upon the same base, or on equal bases, and between the same parallels are equal. (*Euc.* I., 35, 36.) **Thus in Fig. 2**, ABCD = ABDE, FGHJ = FGKL, and MNOP = QRST, because MN = QR, also MNTS = MNOP.

(3) **Triangles** upon the same base, or on equal bases, and between the same parallels, are equal. **Thus in Fig. 3**, ABC = ABD and ABC = DEF.

(4) If a **parallelogram and a triangle** be on the same base, or on equal bases, and between the same parallels, the area of the parallelogram shall be double that of the triangle. **Thus in Fig. 4**, ABCD = twice ABC, and EFGH = twice EFJ.

(5) **The square on the hypotenuse of a right-angled triangle** is equal to the sum of the squares on the other sides. (*Euc.* I., 47.) **Thus in Fig. 5**, the square CBDE = the square ABFG, plus the square AHJC. This is a very important fact for plumbers and engineers, because, coupled with it, all **similar figures** follow the same law. **Thus circles of these diameters, AC and CB, are in area equal to the area of the circle whose diameter is AB.** This is applicable to the comparison of the areas of various sizes of pipes. For example, A and B represent the diameters of two pipes. What will be the diameter of a pipe the area of which will be equal to the two given pipes? *Answer*. Construct a right-angled

triangle ABC, making AB equal to diameter A, and AC equal to diameter B. Measure BC, which is the required diameter.

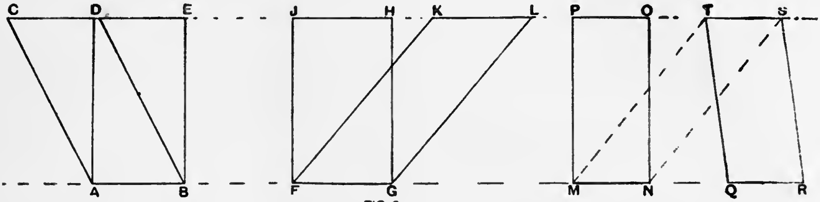


FIG. 2

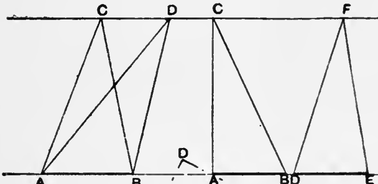


FIG. 3

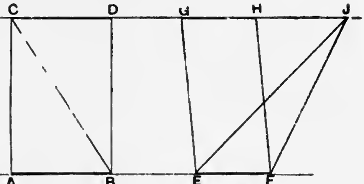


FIG. 4

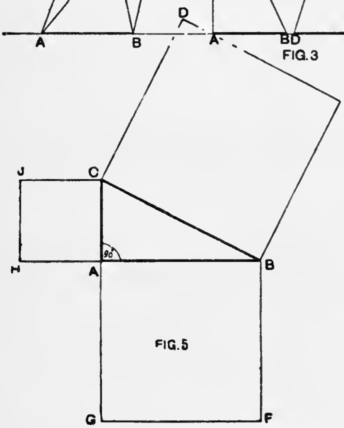


FIG. 5

SCALES AND SCALE DRAWING

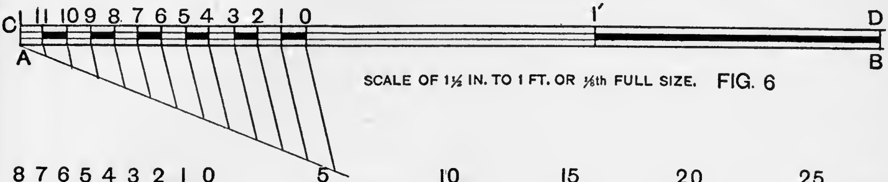
Scales are adopted in order to make diagrams either greater or less than the original diagram or object. They are used in making working drawings, plans, etc., showing methods of construction, etc.

For example, in the plan of a house, it would be very inconvenient to have a drawing full size; but by drawing every measurement to a given reduction (*i.e.*, to scale), we have a drawing having the same relative proportions, and from which, by using the scale, we can take exact full-size measurements.

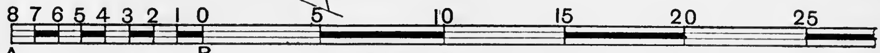
In a scale any given distance may be represented by a measurement decided upon. For example, a scale may consist of 1 in. to the mile—that is, each inch measured on the drawing represents 1 mile; or, in the case of a scale of $\frac{1}{4}$ in. to the foot, each $\frac{1}{4}$ in. on the drawing represents 1 foot full size. In the case of an enlargement, scales such as this may be used (*i.e.*, scale of 1 in. to $\frac{1}{1000}$ of an inch). This time 1 in. measured on the drawing would only represent $\frac{1}{1000}$ of an inch on the object or original drawing. The real distance is called the **Natural Distance,** and the enlarged or reduced length used upon any given scale is the **Artificial Distance.**

Scales to be of use should fulfil the following conditions:—Be divided with great accuracy, and carefully numbered, long enough to measure the principal lines of the drawing.

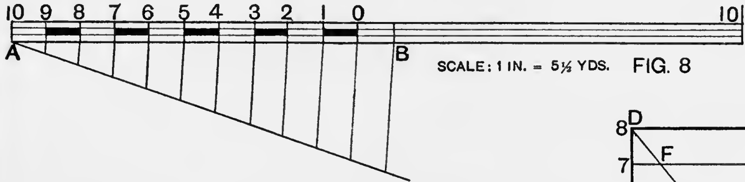
The zero must always be between the unit and its subdivisions.



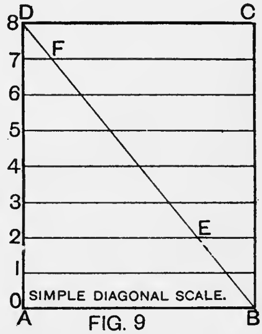
SCALE OF 1½ IN. TO 1 FT. OR ⅛th FULL SIZE. FIG. 6



SCALE OF ⅛ IN. TO 1 FT. OR $\frac{1}{96}$th FULL SIZE. FIG. 7

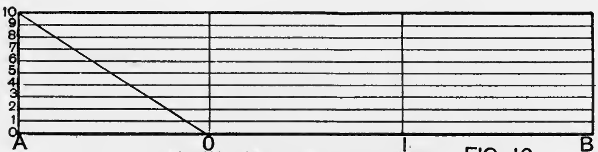


SCALE: 1 IN. = 5½ YDS. FIG. 8

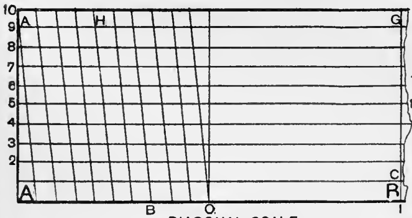


SIMPLE DIAGONAL SCALE.

FIG. 9

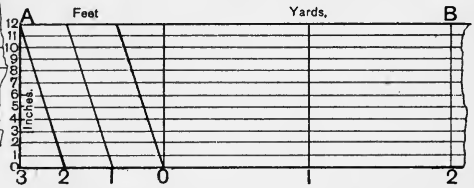


DIAGONAL SCALE, SHOWING TENTHS. FIG. 10



DIAGONAL SCALE,
SHEWING TENTHS AND HUNDREDTHS.

FIG. 11



DIAGONAL SCALE, SHEWING YARDS, FEET AND INCHES.

FIG. 12

To construct any scale (say a scale of $1\frac{1}{2}$ in. to 1 ft.). Draw two parallel lines AB, CD (Fig. 6). Mark off from A, spaces of $1\frac{1}{2}$ in. each. These by scale represent feet. Divide the first space into 12 in., and place figures in the manner shown. This is the most convenient method. If a distance 2 ft. 9 ins. is required, place one leg of dividers on D and open until the other rests on 9 ins.

To construct a scale of 8 ft., to 1 in. or $\frac{1}{96}$, to measure 22 ft.

(Fig. 7). Draw lines as before. Make AB 1 in., and divide into 8 equal parts, and mark off 22 parts, giving the length required.

To draw a scale showing $5\frac{1}{2}$ yds. to 1 in. to measure 20 yds.—If $5\frac{1}{2}$ yds. be represented by 1 in., then 2 ins. will represent 11 yds. (Fig. 8). Draw a line and set off AB 2 ins. Divide this distance into 11 equal parts, and add 9 more to make 20 yards, and complete the scale.

Paper scales generally in use can be bought very cheaply, and are very useful. Those used by builders, etc., are $\frac{1}{8}$, $\frac{1}{4}$, $\frac{3}{8}$, $\frac{1}{2}$, $\frac{3}{4}$, 1, $1\frac{1}{2}$, 2, and 3 ins. to the foot, the latter being for details. All scales divided into inches are called Duodecimal Scales. Those divided into tenths or units of ten are Decimal Scales. All the above are called Plain Scales.

Diagonal scales.—By means of the diagonal scale very minute distances may be measured with great accuracy. The principle of its construction is as follows:—If the rectangle ABCD (Fig. 9) be divided into 8 equal parts by lines parallel to AB, and the diagonal DB be drawn, then 2 to E = 6 parts of AB, and 7 to F = $\frac{1}{8}$ of AB.

To draw a diagonal scale showing inches and tenths (Fig. 10).—Draw a line AB, and mark off inches on the same. At A erect a perpendicular, and set off 10 equal parts to any convenient unit. Draw lines parallel to AB, and draw diagonal from 0 to 10.

Draw a diagonal scale showing inches and hundredths of an inch (Fig. 11).—Draw 11 parallels, and set up verticals. Now divide AO into 10 equal parts, and join first part on AO with 10 at the end of top line. Rule parallels from each of the parts as shown.

Note.—Each part on the line AO shows tenths of an inch, and the distance 9A, on the second line from the top, will be $\frac{1}{10}$ of a tenth—that is, $\frac{1}{100}$—of an inch.

The distance 1B = $1\frac{3}{10}$ or 1·3. If 1·59 in. be required, place one point of dividers on the point G, where vertical 1 meets horizontal 9; open dividers to point H, where diagonal 5 meets horizontal 9.

Draw a scale of $\frac{1}{48}$ to show yards, feet, and inches (Fig. 12).

A scale of $\frac{1}{48}$ would be $\frac{1}{4}$ in. to the foot, or $\frac{3}{4}$ in. to the yard. Draw a line AB, and set off distances of $\frac{3}{4}$ in. Divide the first into three, showing feet. Draw the vertical A3, and set off 12 equal parts. Rule parallels, and complete the scale. To obtain a required distance, say 2 yds. 2 ft. 7 in., place one point of the dividers where vertical 2 meets horizontal 7, on line B2, and extend the other point to the spot where the diagonal 2 intersects horizontal 7.

In making a drawing to scale, first make the scale and then take **all** measurements from it.

CURVES, THE ELLIPSE, etc.

Of the various curves occurring in mechanical and other drawings, the **Ellipse, Parabola,** and **Hyperbola** are termed Conic Sections, because they are formed by plane sections of the cone.

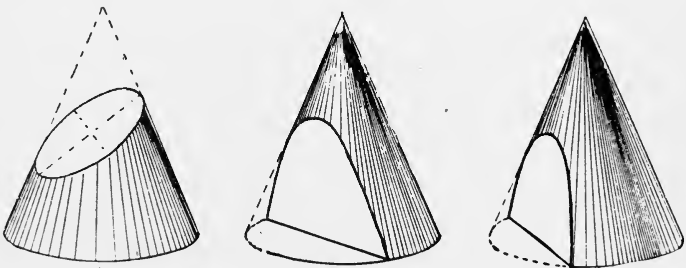
The Ellipse is the section of a cone formed by an oblique plane passing through both sides (Fig. 13, *a*). The longest diameter of an ellipse is Transverse or Major Axis. The shortest diameter of an ellipse is Conjugate or Minor Axis.

Any line passing through the centre and terminating in the curve is a diameter.

There are two points on the transverse axis equidistant from the centre, each of which is a Focus.

If any point in the curve be joined by two lines to the foci, these two lines are, together, equal to the transverse axis.

The Parabola is the curve formed by a plane cutting the cone parallel to its side (Fig. 13, *b*).



A B C

FIG 13

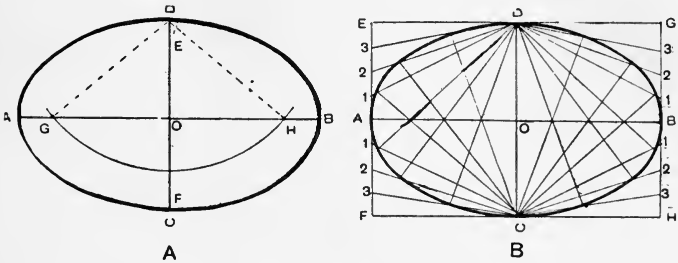
The Hyperbola is the curve formed when the cutting plane makes a greater angle with the base than the side of the cone makes (Fig. 13, *c*).

When the cone is cut by a vertical plane through its vertex, the section is a triangle ; and when the section is parallel to the base, it is a circle.

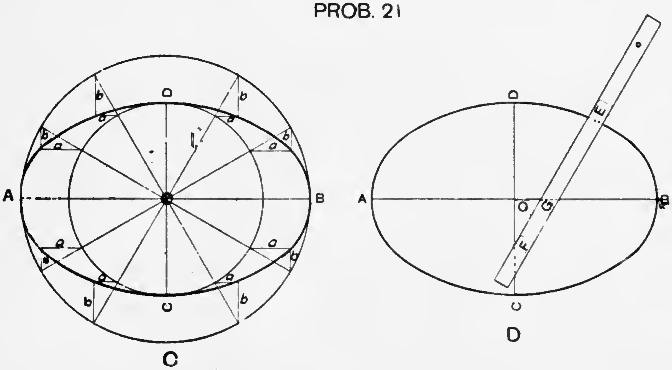
Problem 21.—**To describe an ellipse, the transverse and conjugate diameters being given :**

(*a*) **By means of a piece of thread and two pins.**

Let AB and CD be the two diameters. Bisect AB and CD. Take OE and OF, each equal to half CD. With radius AO and centre E describe an arc cutting AB in G and H. These points are the foci. Take three pins and stick them firmly in the points E, G, and H. Tie a piece of thread round these pins, as shown by the lines EG, GH, and EH. Remove the pin at E, and replace it with a pencil. Move the point of the pencil round, keeping the thread tightly stretched. The curve described by the pencil point will be an ellipse.



PROB. 21



(*b*) **By means of intersecting lines.**

Place the diameters at right angles to each other, and through the ends draw parallels forming a rectangle. Divide AE into any number of equal parts (say 4). Set off these parts on AF, BG, and BH. Join each of these points with C and D. Divide OA and OB into the same number of equal parts. Draw lines from D through 1, 2, and 3 on each side of O on line AB, to meet C1, C2, and C3. In the same manner, draw lines from C through the same points to meet D1, D2, D3. Through the intersections thus obtained draw the curve.

(*c*) **By means of two circles, the diameters of which are equal to the major and minor axis respectively of the ellipse.**

From centre O describe the above-mentioned circles, then AB and CD will be the major and minor axis respectively. From O draw 12 radii 30° apart to cut the circumference of both circles. If horizontal lines be now drawn from each radius where it cuts the circumference of the smaller circle, as AAA, etc., and vertical lines as BBB, etc., be drawn from the points of intersection of the circumference of the large circle with the radii, and produced until they cut the lines AAA, etc., it will be found that the ellipse may be formed by drawing a curve through the intersections of lines A and B.

(*d*) **By means of a paper trammel or straight-edge.**

This is generally adopted in practice. Set up the axes AB and CD as before. Take a piece of paper (or a long flat ruler, if the figure be large) and make EF equal to AO, and EG equal to CO. Place the trammel so that G may be on the transverse and F on the conjugate diameter. Then E will be a point on the curve. By shifting the paper, and always keeping G on the transverse and F on the conjugate diameters, a number of points may be obtained. Draw the curve through the points.

This method is adopted by workmen for marking elliptical structures, but the two axes are constructed in the form of grooves into which oak pegs that are passed through the trammel slide freely, thus allowing the curve to be freely sketched.

CHAPTER IV

SOLID GEOMETRY, PLANS, ETC., DEVELOPMENT OF SURFACES OF SOLIDS

PLANS, ELEVATIONS, AND SECTIONS, AND THEIR PROJECTIONS

The foregoing having dealt with the formation and comparison of **plane** (or flat) figures or surfaces, the application of the same will be now adopted in representing solid objects on paper.

It is necessary now to imagine two planes which, though supposed to be invisible yet always exist. They are the "Vertical and Horizontal Planes," of which, examples may be noted as follows. In an ordinary room the walls represent vertical planes, with the floor or ceiling as horizontal planes.

A piece of paper, folded at right angles and placed on a table or board adjoining a wall, so that half of the paper touches the wall and the other half rests on the board, may be used to illustrate this (Fig. 14). If a simple form of object, such as a box or cube, be placed on this paper, its outline can be traced on the horizontal fold of the paper, and in the same manner on the vertical fold. The length and breadth shown on the horizontal fold, in outline, is the **PLAN** or the representation of what would be seen when looking directly on to the box from a point above it. The length and depth traced on the vertical fold show the outline of the **elevation**, which is a representation of what would be seen when looking directly in front of the box.

If the paper be flattened out, it appears as shown in sketch. It will be seen that a plan and elevation of an object give a more accurate representation of its shape and size than can be shown by any other method. In Fig. 14 (p. 24) the line XY represents the fold of the paper, and is called the intersecting or ground line. It is really the line obtained by the cutting or intersection of the vertical and horizontal planes.

This drawing now obtained is called a Projection, because each point of the object is projected, or thrown upon the part of the **vertical plane** exactly opposite to it.

In this example projection is effected by parallel lines at right angles to the vertical and horizontal planes, and is called Orthographic Projection.

When looking at the front of a building, however, it is impossible to see the elevation as drawn by orthographic projection, because the eye sees only from one point of view at once. In order to represent an elevation as it appears to the eye, it is necessary to adopt what is called Perspective Projection, in which all projectors converge, thus representing all retreating and back lines smaller.

The objects of orthographic projection, therefore, are to show exact sizes and shapes of objects from different points of view, so that measurements can be accurately read off as required.

The following include the names of some simple solid forms:—

A Cube is a solid figure contained by six equal squares.

A Prism has two ends as similar figures (equal) and all its sides are Parallelograms.

A Pyramid has a plane figure for its base, and each of its sides are triangles meeting at a point above the base, called the **Vertex** or **Apex.** A pyramid takes its name from the number of sides, or the figure formed by its base, *e.g.*, a square pyramid has a square for its base and four sides.

A Sphere is formed by the revolution of a semi-circle on its diameter. Any point in its surface is equidistant from its centre.

A Cone is formed by revolving a right-angled triangle about its perpendicular side.

A Cylinder is generated by the revolution of a rectangle about one of its sides.

If the upper part of a pyramid or cone be cut away, the portion left is called the Frustum, and is said to be Truncated.

Problem 22.—**To draw the plan and elevation of a Cube of 3-in. sides.**

(*a*) Standing on the horizontal plane, with one face parallel to the vertical plane.

(*b*) With one face on the horizontal plane, and with a vertical face inclined at 30° to the vertical plane.

(*a*) The **plan** will be a square ABCD, having two of its sides parallel to EF.

The **elevation** is a square of the same size.

(*b*) Place the square ABCD at an angle of 30° to HCF for the **plan.**

From each angle of plan draw projectors. Make DH equal to AB. Draw DB and HF parallel to XY, thus completing the elevation.

THE PYRAMID

Problem 23.—**To draw the plan and elevation of a square pyramid 3 ins. high, in the following position:—**

With one of its triangular faces in the horizontal plane and its base at right angles to the vertical plane.

First, obtain the plan and elevation when the pyramid is standing on the horizontal plane, with one edge of the base parallel to XY. Now turn the elevation so that one **side** rests on XY. Then A, B, E will be the required elevation. For the plan project from A, B, and E to meet parallels from C, D, and E.

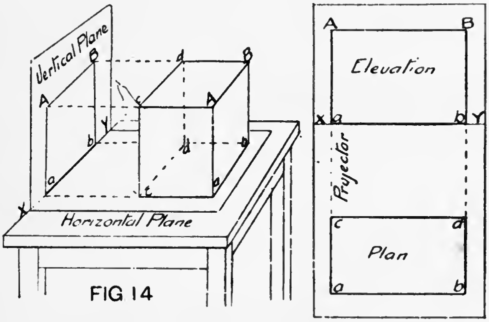
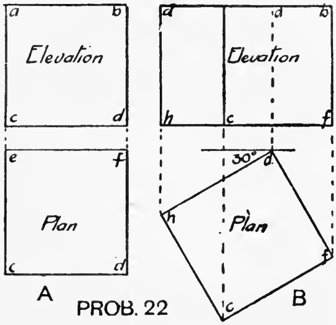
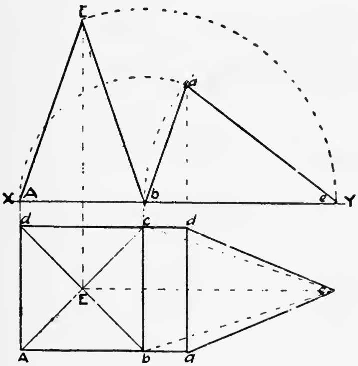


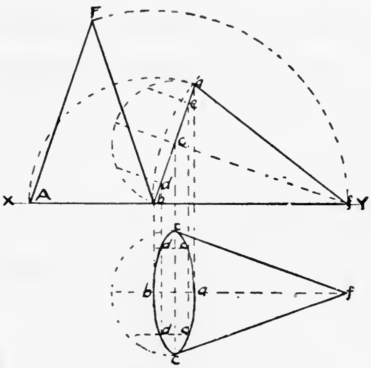
FIG 14



A PROB. 22 B



PROB. 23



PROB. 24

THE CONE

Problem 24.—**To draw the plan and elevation of a cone** lying with its side in the horizontal plane, and its axis parallel to the vertical plane.

Let the diameter of its base be $2\frac{1}{2}$ ins. and its altitude 3 ins.

First, draw elevation ABF when the cone has its axis vertical. Turn this elevation so that BF will be horizontal, then ABF will be the first elevation required. For the plan describe semi-circles on AB and CC, and divide in four equal parts, draw perpendiculars from each division through the diameter of the semi-circle. From B, D, C, E, and A drop projectors and draw centre line BF parallel to XY. Mark off on D, C, and E the same distances or lengths of E, C, and D. Draw the curve through these points. Project F and join C and E with F, completing the plan.

DEVELOPMENT OF SURFACES OF SOLIDS

It is essential that the plumber should have some knowledge of this subject to enable him to readily trace out the required shape of lead, copper, zinc, etc., to fit or to cover the surfaces of structures of various shapes, such as turrets, domes, finials, roofs, cesspools, etc. Several general examples will be given to illustrate the principles.

Fig. 15 shows the development of the curved surface of a cylinder cut by a plane at an angle of 45° with the vertical line (AB).

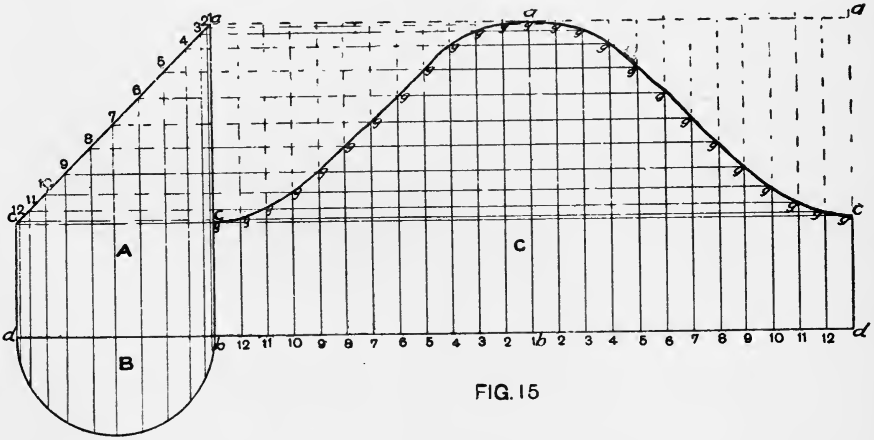


FIG. 15

Method.—Draw the elevation **A** and the half-plan **B**, and divide the circumference of **B** into any number of equal parts (say 12), and project lines, 1, 2, 3, 4, 5, 6, etc., from them to cut the plane AC.

Draw the lines BD and AAA and complete **C** by joining AD and AB. Divide the rectangle **C** into 24 parts by lines 1, 2, 3, 4, etc., each equal to the divisions in **B**. If lines be now projected from the points 1, 2, 3, etc., in the plane AC, to intersect the vertical lines in **C** numbered correspondingly, the intersections at GGG denote the outline of the curve necessary in a plane figure to produce the surface of the cylinder cut by plane AC. This is the method for cutting out curves for the construction of each half of a right-angle elbow.

Fig. 16 shows the development of the curved surface of a cone cut by the plane AB at an angle of 90° with the side BC.

Method.—Draw the elevation **A** and half-plan **B**. Divide the semi-circle CD into 12 equal parts, and project lines perpendicular to the line CD; from the points where these lines cut CD, draw the lines 1, 2, 3, 4, etc., to the apex O, and from their intersections with the cutting plane AB draw horizontal lines to OD. With centre O and

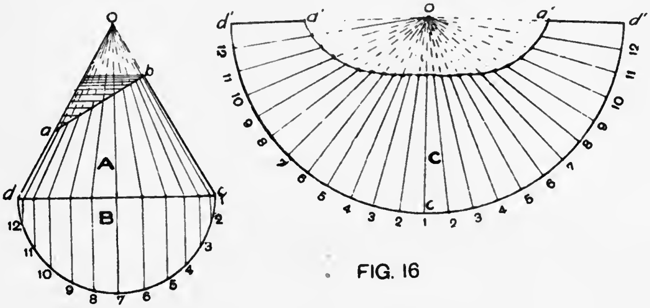


FIG. 16

radius OD describe the arc D^1CD^1, and divide it into 24 parts, each equal to a division of B. Commencing from C, number them 1, 2, 3, 4, etc., respectively right and left, and join each point with O. Fix one point of a pair of dividers on D (**A**), and measure off DA. Then with one point on D^1 in the development, mark off D^1A^1 on each side of O. Repeat the operation, and measure all the intersections of the horizontal lines with the side DO, and by transferring each of these measurements to the lines in the development numbered correspondingly, points will be obtained, through which a curve, if carefully drawn, will give the shape of the surface of the cone **A** cut by plane AB.

Fig. 17 explains the development of the slant surfaces of a frustum of a pyramid with a hexagonal base. The "plan" shows the size of the base and the top, and the elevation shows the vertical height.

Method.—Produce the lines AB and CD until they meet at O, and form a pyramid.

With centre O^1 and radius OA, describe arc A^1, C^1, A^2, and with same centre and radius OB, describe arc B^1, D^1, B^2; join B^1O^1, and from B^1 mark off the length of one side of the base as given at EF in plan. Repeat the operation, marking off 6 divisions, representing the 6 sides, and join the points $E^1E^1E^1$, etc., with O^1. With length GH on plan, mark off 6 divisions on arc $A^1C^1A^2$, join the intersections on both arcs by straight lines, which will be the outlines of the sides of the top and bottom of the frustum respectively, the developments being enclosed by the radii and these lines.

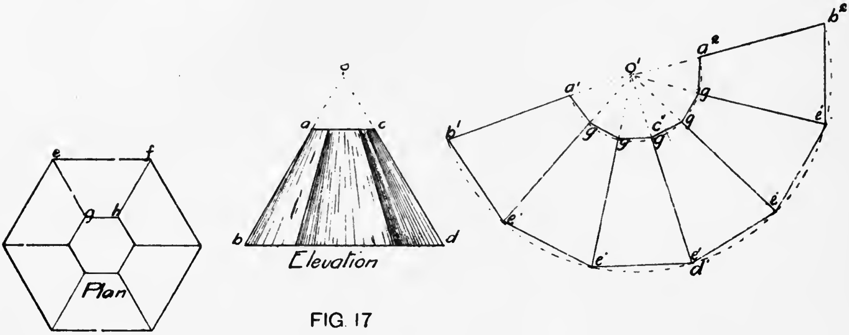


FIG. 17

Fig. 18 shows the plan, elevation, and development of a section of a hemispherical dome which it is assumed has to be

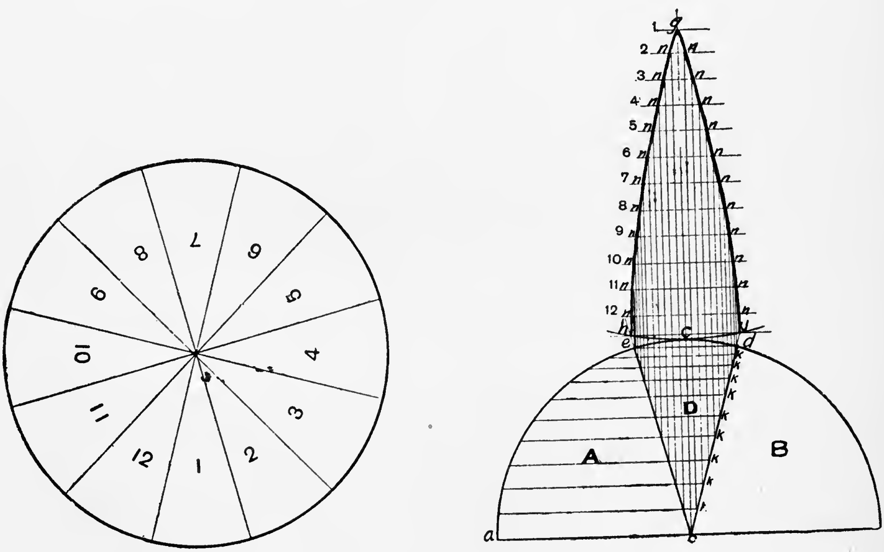


FIG. 18

covered with sheet lead or copper, in sections 1, 2, 3, etc., shown on plan. To find the shape of the plane surface of one of the sections.

Method.—Draw the part plan and elevation **A** and **B**, and divide the arc AC into a number of equal parts (say 12), and draw horizontal lines from these points to cut the radii OC, OD, and OE. The sector ODE represents a plan of one of the sections. Construct the line GC equal in length to the arc AC, and with G as centre describe the arc HJ. Produce the lines OE and OD to HJ. Mark off on the line GC divisions equal to those on arc AC, and draw horizontal lines through these divisions. Project vertical lines from the points KKK, etc., to intersect the lines 1, 2, 3, 4, 5, 6, etc., and through the intersections NNN, etc., draw the curve, denoting the development of the section **D.**

This method will not give the exact width of the section required, owing to the curved surface of the sphere, but for practical purposes it is sufficiently accurate.

Fig. 19 shows the plan, elevation, and development of a

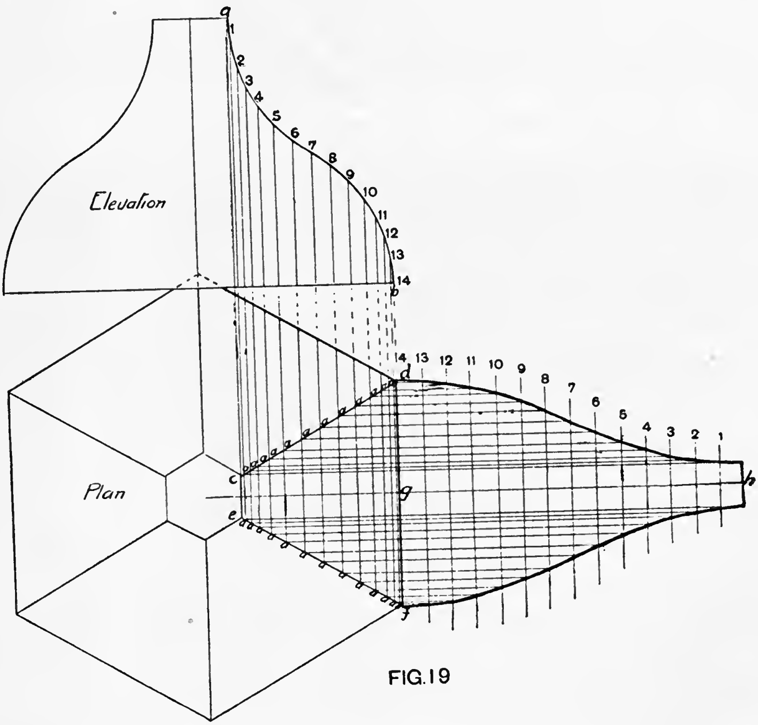


FIG. 19

hexagonal turret, ogee in outline, which requires covering with lead in the six bays.

To find the development of one of the bays.

Method.—From the elevation, which shows two sides, project the plan showing six bays.

Mark off on the curve AB 14 equal parts, and draw vertical lines from each point to intersect the lines CD and EF in plan. Make the line GH equal to AB, and divide it into 14 equal parts by means of vertical lines 1, 2, 3, 4, 5, 6, etc. Project lines from the points A,A,A,A,A, etc., on plan to intersect the vertical lines 1, 2, 3, 4, 5, 6, etc. By carefully drawing a curve through these intersections, the outline of the development is completed.

ARITHMETIC AND MENSURATION

CHAPTER V

MULTIPLICATION, ADDITION, ETC., OF FRACTIONS AND DECIMALS

In most calculations it is impossible to avoid the use of numbers which are of less value than 1. It is therefore necessary to understand the use of fractions and decimals.

A fraction is part of a whole number, such as one-half, one-quarter, three-fifths, etc. Two numbers are required to express a fraction, which are called the Numerator and the Denominator. The numerator is placed above the denominator, with a line between :—

$$\text{Thus,}\quad \text{One-half} = \frac{1}{2}\ \frac{\text{Numerator.}}{\text{Denominator.}}$$

$$\text{One-quarter} = \frac{1}{4}$$

$$\text{Three-fifths} = \frac{3}{5}$$

The denominator denotes how many parts the whole number is divided into, and the numerator states how many of these are considered.

A Proper Fraction is one whose numerator is less than its denominator, its value being always less than 1, as $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{5}$.

An Improper Fraction possesses a numerator equal to, or greater than, the denominator; its value is one or more than one, as $\frac{3}{2}$, $\frac{7}{5}$, $\frac{15}{11}$.

A Mixed Number is a whole number and a fraction united; $5\frac{3}{5}$ is a mixed number, and is equivalent to $5+\frac{3}{5}$: it is read five and three-fifths.

REDUCTION OF FRACTIONS

This is the process of changing their form without altering their value.

Thus, $\frac{6}{8}=\frac{3}{4}$, both denominator and numerator having been divided by 2, the value of the fraction is still the same.

To reduce a mixed number to an improper fraction. Multiply the whole number by the denominator and add the product to the numerator.

Example.—Reduce $9\frac{5}{9}$ to an improper fraction.

$$9\tfrac{5}{9}=\frac{81+5}{9}=\frac{86}{9}$$

To reduce an improper fraction to a mixed number. Divide the numerator by the denominator; the dividend will be the whole number, and the remainder with the denominator forms the fraction.

Example.—Reduce $\frac{86}{9}$ to a mixed number.

$$\begin{array}{r} 9\underline{)86} \\ 9\tfrac{5}{9} \end{array}$$

EXAMPLES I

(i.) $\frac{6973}{111}$. (ii.) $\frac{85674}{816}$. (iii.) $\frac{98643}{512}$. (iv.) $\frac{86500}{1100}$. (v.) $\frac{96457}{5000}$

ADDITION OF FRACTIONS

It is necessary to find a number which will contain all the denominators of the fractions to be added without a remainder. This number is called the Least Common Denominator.

To add $\frac{1}{4}, \frac{1}{3}, \frac{1}{9}, \frac{1}{16}$

First, place the denominators in a row, separated by commas, thus:—

$$\begin{array}{r|l} 4) & \underline{4,\ 3,\ 9,\ 16} \\ 3) & \underline{1,\ 3,\ 9,\ \ 4} \\ & 1,\ 1,\ 3,\ \ 4 \end{array}$$

$$=4\times3\times3\times4=144 \text{ (the least common denominator),}$$

and divide by some prime number which will divide at least two of them without a remainder (if possible), bringing down to the row below those denominators which will not contain the divisor without a remainder; the process is repeated until no two numbers remain which may be divided by any prime number.

The divisors and the remaining numbers are all multiplied together, the product being the least common denominator.

The next process is to place the 144 as denominator under a line, divide the denominator of each fraction into it separately, and multiply the numerator in each case by the quotient, placing them above the line as numerators, and, adding them together, the result may be left either as a fraction or reduced to a decimal or a mixed number.

Example continued:—

$$\frac{36+48+16+9}{144}=\frac{109}{144}$$

EXAMPLES II

(i.) $\frac{3}{11}+\frac{29}{22}+\frac{17}{33}$. (ii.) $\frac{3}{5}+\frac{4}{7}+\frac{1}{6}+\frac{4}{9}$. (iii.) $\frac{3}{10}+\frac{13}{15}+\frac{1}{5}+\frac{4}{9}$.

(iv.) $\frac{7}{8}+\frac{7}{12}+\frac{7}{16}+\frac{7}{18}$. (v.) $\frac{11}{70}+\frac{5}{21}+\frac{1}{5}+\frac{17}{42}$.

SUBTRACTION OF FRACTIONS

First, find the least common denominator, and, instead of adding the numerators, subtract them.

Example.—Subtract $\frac{9}{16}$ from $\frac{19}{24}$.

$$\begin{array}{r} 8\,\underline{)\ 16,\ 24} \\ 2,\ \ 3 \end{array} = 48 \text{ L.C.D.} \quad \therefore \quad \frac{38-27}{48}=\frac{11}{48}$$

EXAMPLES III

(i.) $\frac{11}{24}-\frac{1}{6}$. (ii.) $\frac{21}{40}-\frac{6}{47}$. (iii.) $\frac{26}{35}-\frac{7}{20}$. (iv.) $\frac{583}{340}-\frac{73}{102}$. (v.) $\frac{39}{115}-\frac{17}{138}$.

MULTIPLICATION OF FRACTIONS

Multiply the numerators together and the denominators also separately. The results may be stated in mixed numbers, improper or proper fractions, or decimals.

Example.— $$\frac{3}{4}\times\frac{6}{7}\times\frac{7}{9}\times\frac{4}{7}$$

Before multiplying out, it will considerably reduce the amount of work if cancelling is resorted to where possible. This consists of dividing any numerator and denominator together by

$$\frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{4}}}\times\frac{\overset{2}{\cancel{6}}}{7}\times\frac{\overset{1}{\cancel{7}}}{\underset{\underset{1}{\cancel{3}}}{\cancel{9}}}\times\frac{\overset{1}{\cancel{4}}}{\underset{1}{\cancel{7}}}=\frac{2}{7}$$

a whole number, as shown in Example.

EXAMPLES IV

(i.) $\frac{11}{12}\times\frac{8}{15}$. (ii.) $1\frac{2}{7}\times5\frac{5}{6}$. (iii.) $\frac{21}{95}\times\frac{32}{35}\times\frac{38}{45}$.

(iv.) $\frac{7}{24}\times1\frac{11}{28}\times10\frac{10}{11}\times6\frac{10}{13}$. (v.) $3\frac{3}{7}\times\frac{5}{8}\times1\frac{10}{11}\times2\frac{2}{3}\times1\frac{5}{6}\times\frac{7}{16}$.

DIVISION OF FRACTIONS

To divide by a fraction, invert the fraction and proceed as in multiplication of fractions.

Example.—Divide 4 by $\frac{4}{9}$.

$$= \frac{\overset{1}{\cancel{4}}}{1} \times \frac{9}{\underset{1}{\cancel{4}}} = 9$$

Divide $\frac{7}{16}$ by $\frac{3}{4}$.

$$= \frac{7}{\underset{4}{\cancel{16}}} \times \frac{\overset{1}{\cancel{4}}}{3} = \frac{7}{12}$$

EXAMPLES V

(i.) $\frac{32}{75} \div \frac{8}{15}$. (ii.) $14 \div 8\frac{3}{4}$. (iii.) $\frac{7}{8} \div 1\frac{3}{5}$. (iv.) $\frac{100}{109} \div 75$. (v.) $4\frac{53}{55} \div 1\frac{61}{121}$.

When it is necessary to divide two or more fractions, which may be multiplied or added, by one or more fractions which may also be multiplied or added, a line of division is usually placed between the two quantities.

Example.—

$$\frac{\frac{7}{8} \times \frac{4}{5} \times \frac{2}{9}}{\frac{7}{9} \times \frac{2}{5} \times \frac{3}{8}} = \frac{\overset{1}{\cancel{7}}}{\underset{1}{\cancel{8}}} \times \frac{4}{\underset{1}{\cancel{5}}} \times \frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{9}}} \times \frac{\overset{1}{\cancel{9}}}{\underset{1}{\cancel{7}}} \times \frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{2}}} \times \frac{\overset{1}{\cancel{8}}}{3} = \frac{4}{3} = 1\frac{1}{3}$$

As will be seen by the previous example, every fraction in the denominator $(\frac{7}{9} \times \frac{2}{5} \times \frac{3}{8})$ is inverted, the sign changed, and ordinary multiplication proceeded with.

SIMPLIFICATION OF COMPLEX FRACTIONS

Example.—

$$\frac{1\frac{23}{24}}{1\frac{5}{12} + 1\frac{9}{24} + \frac{17}{36}}$$

$$= \frac{\frac{47}{24}}{\frac{17}{12} + \frac{33}{24} + \frac{17}{36}}$$

$$= \frac{\frac{47}{24}}{\frac{255}{72}}$$

$$= \frac{\overset{1}{\cancel{47}}}{\cancel{24}} \times \frac{\overset{3}{\cancel{72}}}{\underset{5}{\cancel{235}}} = \frac{3}{5}.$$

L.C.M. of 12, 24, 36.

$$\begin{array}{r|rrr} =6 & 12, & 24, & 36 \\ \hline 2 & 2, & 4, & 6 \\ \hline & 1, & 2, & 3 \end{array}$$

$$= 6 \times 2 \times 2 \times 3 = \underline{72}$$

$$\frac{17}{12} + \frac{33}{24} + \frac{17}{36}$$

$$= \frac{102 + 99 + 34}{72} = \frac{235}{72}.$$

EXAMPLES VI

(i.) $\dfrac{\frac{3}{5}\times\frac{5}{9}\times\frac{7}{16}\times\frac{4}{21}}{\frac{7}{12}\times\frac{4}{9}\times\frac{7}{8}}$. (ii.) $\dfrac{\frac{5}{13}\times\frac{4}{9}\times\frac{12}{25}\times\frac{7}{4}}{\frac{7}{24}\times\frac{32}{25}}$. (iii.) $\dfrac{\frac{3}{110}\times\frac{5}{16}\times\frac{21}{25}}{\frac{51}{110}\times\frac{1}{5}}$

(iv.) $\dfrac{(16\frac{1}{5}\times 9\frac{1}{8})-(\frac{1}{4}\times 20\frac{1}{5})}{\frac{16}{9}}$. (v.) $\dfrac{\frac{24}{25}\times\frac{5}{6}\times\frac{3}{7}}{\frac{11}{16}-\frac{9}{40}}$.

DECIMALS

Decimals are tenths, hundredths, thousandths, etc., of a unit, or whole number.

The denominator, which is always ten or a multiple of ten, such as 10, 100, 1000, 10,000, is not written as in an ordinary fraction, but is understood. The numerator which forms the decimal is always on the right of the unit figure, and in order to distinguish the decimal from the unit figure, a period or dot (·) is placed between them.

Example.—6·5 = 6 whole numbers + $\frac{5}{10}$.
Example.—7·05 = 7 whole numbers + $\frac{5}{100}$.

Placing a cypher to, or removing one from, the right side of a decimal fraction does not affect its value.

Example.—6·50 = 6 + $\frac{50}{100}$ = 6 + $\frac{5}{10}$.

Placing a cypher in front, or to the left, of a decimal reduces its value to $\frac{1}{10}$ its original amount.

Example.—·05 = $\frac{5}{100}$.
·5 = $\frac{5}{10}$, *i.e.*, 10 times the value of $\frac{5}{100}$.

To convert decimals to vulgar fractions, place in the denominator as many cyphers as there are decimal figures in the numerator and precede them by the figure 1.

Example.—Convert ·731 into a proper fraction.

$$\frac{731}{1000}$$

EXAMPLES VII

Convert the following decimals to proper fractions, improper fractions, or mixed numbers :—

(i.) ·8. (ii.) ·185. (iii.) ·018. (iv.) 17·006. (v.) 100·001.

To convert fractions to decimals.

Divide the numerator by the denominator.

Example.—Convert $\frac{3}{4}$ into a decimal.

$$\frac{3}{4} = 4\overline{)3}$$
$$\cdot 75$$

EXAMPLES VIII

Convert the following fractions to decimals :—

(i.) $\frac{1}{10}$. (ii.) $10\frac{3}{100}$. (iii.) $19\frac{1}{100}$ (iv.) $19\frac{11}{1000}$ (v.) $16\frac{13}{10000}$.

TO MULTIPLY DECIMALS

Proceed as in ordinary multiplication, count the number of decimal places in the multiplier and the multiplicand, and counting the same sum from the right in the product, place a decimal point.

Example.—270·32 × 6·25.

Multiplicand	270·32	Number of decimal places in
Multiplier	6·25	Multiplier and Multiplicand = 4.
	135160	
	54064	∴ Pointing a similar number in
	162192	the product gives
Product	16895000	= 1689·5 *Ans.*

Example.— 95·76 × ·015 (5 decimal figures).

	95·76	
	·015	
	47880	
Pointing off 5 figures	9576	
in product gives =	1·43640	*Ans.*

EXAMPLES IX

(i.) 37·5 × ·8. (ii.) 375 × ·008. (iii.) 1·021 × ·0037.
(iv.) 17·625 × ·0032. (v.) 9·71 × 5·836 × ·001.

DIVISION OF DECIMALS

No attention need be paid to the decimal point until after the division is performed. The number of decimal places in the dividend must equal (or be made equal by annexing cyphers) to the number of decimal places in the divisor. Divide exactly as if you were dealing with whole numbers; then subtract the number of decimal places in the "divisor" from the number of decimal places in the "dividend," and point off as many decimal places in the "quotient" as there are units in the remainder thus found.

Example.—Divide ·625 by 25.

Divisor. Dividend. Quotient.

	25)·625(·025 *Ans.*
	50
	125
	125
Remainder	0

In this example we divide by 25, and as there are no decimal places in the "divisor" and three decimal places in the "dividend," there are therefore 3 − 0, or three decimal places in the "quotient"; one cypher has to be prefixed to 25 to make the three decimal places.

In the following example the "divisor" ·005 contains the same number of decimal places as the "dividend" ·125:—

Divisor. Dividend. Quotient.
·005)·125(25 *Ans.*
10
25
25
Remainder 0

Therefore, dividing by 5, we get 25 for the answer.

It frequently happens that the division will never terminate. In such cases it is necessary to ascertain the degree of accuracy required in the result. Generally, it is sufficient to carry the division to three places of decimals, but where the quotient contains one or more cyphers immediately after the decimal place, as ·0032, then it is advisable to carry the division to six or seven places of decimals.

Example.—Divide ·0025 by 1·15.

1·15)·00250(·002173913 *Ans.*
230
200
115
850
805
450
345
1050
1035
150
115
350
345
5

EXAMPLES X

(i.) 101·6688 ÷ 2·36. (ii.) ·08 ÷ ·008. (iii.) ·0003 ÷ 3·75.
(iv.) ·4 ÷ ·008. (v.) 70307 ÷ ·0025.

It often occurs that one or more numbers will repeat in the quotient no matter how far the division may be carried; these are called recurring decimals.

Example.—Divide 10 by 6.

$$\begin{array}{r} 6)\underline{10{\cdot}000000} \\ 1{\cdot}666666 \end{array}$$

Dividing 6 into 10 we get 1 with 4 as the remainder; placing a decimal point after 1 and annexing a cypher to 4 and dividing by 6 we again get 4 as a remainder; and, as will be readily seen, this repetition of 6 in the quotient would go on indefinitely. When such a case occurs, the number which recurs is called a recurring decimal, and is denoted by a dot placed above the figure to the right, as $1{\cdot}\dot{6}$, spoken of as one point six recurring.

In cases where the whole of the figures after the decimal point repeat, as ${\cdot}\dot{6}2\dot{5}$, it is known as a **Pure recurring decimal**, whereas, if one or more figures do not repeat immediately after the decimal point, as ${\cdot}24\dot{6}2\dot{5}$ (${\cdot}24$ not repeating), it is known as a **Mixed recurring decimal.**

To convert a "pure" recurring decimal to a vulgar fraction, place the figures after the decimal point in the numerator, and in the denominator put as many nines as there are figures after the decimal point; reduce the fraction to its lowest terms.

Example.—${\cdot}\dot{7}\dot{2}$ reduced to a fraction.

$$= \frac{\overset{8}{\cancel{72}}}{\underset{11}{\cancel{99}}} = \frac{8}{11}$$

To convert "mixed" recurring decimals to vulgar fractions, put down all the figures after the decimal point and subtract from them the figures which do not recur, the result being the "numerator" of the required fraction; then place in the denominator as many nines as there are recurring decimal figures, followed by as many cyphers as there are non-recurring decimals, and reduce to the lowest terms.

Example.—

$${\cdot}46\dot{5}8\dot{5} = \begin{array}{r} {\cdot}46585 \\ \underline{46} \\ {\cdot}46539 \end{array} = \frac{\overset{5171}{\cancel{46539}}}{\underset{11100}{\cancel{99900}}} = \frac{5171}{11100}$$

EXAMPLES XI

Convert the following recurring decimals to mixed numbers and vulgar fractions.

(i.) ${\cdot}\dot{5}\dot{4}$. (ii.) $4{\cdot}\dot{9}6\dot{9}$. (iii.) $9{\cdot}\dot{5}0\dot{4}$. (iv.) $27{\cdot}\dot{7}62\dot{6}$.
(v.) $5{\cdot}\dot{7}26\dot{3}$. (vi.) ${\cdot}39\dot{7}\dot{6}$. (vii.) ${\cdot}033\dot{7}\dot{8}$. (viii.) ${\cdot}840\dot{9}$.
(ix.) ${\cdot}219\dot{0}59\dot{4}$. (x.) $3{\cdot}57\dot{4}3\dot{2}$.

ADDITION OF DECIMALS

Place all the numbers to be added together so that the decimal points come over each other, then the units, tens, hundreds, thousands, etc., will be in columns over each other respectively; proceed as with whole numbers.

Example.—Add together 2·503, 75·0032, ·0765.

```
 2·503
75·0032
  ·0765
-------
77·5827
```

EXAMPLES XII

(i.) 7·061 + 82·937 + 5·807 + ·019 + 800·79.
(ii.) 8·097 + 9761·03 + 598 + ·07912 + 52052 + 359·07316.
(iii.) 1·1 + 20·02 + 13 + 2·845 + 1·0001.
(iv.) 31·826 + 3·471 + ·004 + 45 + ·6.
(v.) 7600 + 3·1009 + 473·842691 + ·07 + ·00001 + 1·1.

SUBTRACTION OF DECIMALS

The same rule as applied to addition must also be observed in the subtraction of decimals, and the numbers placed so that the decimal points come immediately over each other.

Example.—Subtract 25·670 from 35·926.

"Minuend"	35·926
"Subtrahend"	25·670
"Difference"	10·256

Subtract ·0278 from 25.

```
25·0000
  ·0278
-------
24·9722
```

EXAMPLES XIII

(i.) 70·304 − 29·30465. (ii.) ·37124 − ·1234567. (iii.) 6340 − ·457034.
(iv.) 1 − ·195176. (v.) 110 − 109·9991.

An explanation is given below of the various signs which are used in Mensuration:—

+	(Plus)	Sign of Addition.	$\sqrt[3]{}$	Sign of Cube Root
−	(Minus)	" Subtraction.	°	" Degree.
×		" Multiplication.	′	" Minute.
÷		" Division.	″	" Second.
=		" Equality.	∴	" Therefore.
∷		" Proportion.	′	" Feet.
$\sqrt[2]{}$		" Square Root.	″	" Inches.

A decimal point (·) placed before a number denotes that all the figures on the right of that number are decimals.

1, 2, or any number placed before a number or series of numbers is called a coefficient as 2 (20-2).

1, 2, or any number placed above and to the right of a number is called an index, and denotes that such number must be multiplied by itself one, two, three, or more times, according to the value of the index, as :—

$$2^2 = 2 \times 2 = 4. \qquad 3^3 = 3 \times 3 \times 3 = 27. \qquad 4^4 = 4 \times 4 \times 4 \times 4 = 256.$$

$\sqrt[2]{\ }$ placed over a number denotes that the square root of such number is required; if the number at the left top corner of the sign is 3, as $\sqrt[3]{\ }$, the cube root is required.

The use of brackets in arithmetic and mensuration requires some consideration.

Where any portion of an expression is enclosed in brackets, as $4(2+8)$, the quantity enclosed by the brackets must be evaluated first, as :—

$$4(2+8) = 4 \times 10 = 40 \quad \textit{Ans.}$$

Again, $$20 - (5 \times 3) = 20 - 15 = 5 \quad \textit{Ans.}$$

Where one portion or quantity in an expression already within brackets is placed within another pair of brackets, the value of the quantity within the inner brackets must be first evaluated, as :—

$$\begin{aligned} &3\{(31-11) \times 6(14+9)\} \\ &= 3\{20 \times 6 \times 23\} \\ &= 8280 \quad \textit{Ans.} \end{aligned}$$

Again, $$\begin{aligned} &4 + 5\{9(6-2) + (12 \times 5)\} \\ &= 9\{36+60\} \\ &= 9 \times 96 \\ &= 864 \quad \textit{Ans.} \end{aligned}$$

CHAPTER VI

INVOLUTION AND EVOLUTION

INVOLUTION

Involution is the process of multiplying a quantity by itself a given number of times. The result of such multiplication is called a **Power** of that quantity, as :—

$5^1 =$	5.	First power.	
$5^2 =$	25.	Second ,,	or square.
$5^3 =$	125.	Third ,,	or cube.
$5^4 =$	625.	Fourth ,,	
$5^5 =$	3125.	Fifth ,,	

Evolution is the reverse process to involution; instead of finding the power of a quantity, we are given the power of the quantity and are required to find the quantity.

This process is generally described as extracting the **root** of a given quantity.

The "square root" of a number is that number which when multiplied by itself will give the original number.

Example.— $\sqrt[2]{25} = 5$; $5 \times 5 = 25$.

To find the square root of a number, we must first mark off our number in periods of two, commencing at the decimal point and working in each direction, as :—

$$\sqrt[2]{275 \cdot 64} = \sqrt{2\overset{\frown}{75} \cdot \overset{\frown}{64}}.$$

Here it will be seen that the first period to the right of the decimal is $\cdot\overset{\frown}{64}$ and the first to the left is $\overset{\frown}{75}$, the second being 2.

Example.—Find the square root of $96 = \sqrt{\overset{\frown}{96}}$.

First mark off the figures in pairs; as there is only one pair we may commence the extraction of the root.

It is evident that the number which is the square root of 96 is something greater than 9 and less than 10, so that the first figure in the answer is 9; after subtracting the square of 9 from 96, we get a remainder of 15, to which must now be affixed two cyphers, making it 1500; having exhausted the whole numbers in the dividend, a decimal point is required after 9 in the root; the next divisor is formed by multiplying the figure in the root by 2, and affixing a number to it which will form the next figure in the root. This is found to be 7; multiplying 187 by 7 and subtracting the product from 1500, a remainder, 191, is obtained, to which must be added two more cyphers, and the figures in the root, *i.e.*, 97 multiplied by 2 to form the next divisor, to which is affixed the next figure in the root, 9, making it 1949, which, when multiplied by 9 and the product subtracted from 19100, leaves a remainder 1559, to which two cyphers are added and the divisor formed as previously described by multiplying the figures 979 in the root by 2, and affixing the next figure in the root to this number. This process may be continued until the root is carried to the requisite number of places, or, as sometimes obtains, until the root works out without a remainder.

```
        96(9·797
        81
    187)1500
        1309
   1949)19100
        17541
  19587)155900
        137109
         18791
```

To find the square root of 786·27 to four significant figures. After dividing the number into periods, working from the decimal point to the right and left, proceed as previously explained.

```
              Root.
     )786·27(28·03
      4
   48)386
      384
 5603)20000
      16809
       3191
```

The root contained in the first period, 7, is 2, with 3 remainder. After dealing with the next period, the whole numbers being dispensed with, place a decimal point after 8 in the root, and carry the work to two decimal places, making an answer, as required, containing four significant figures.

Find the $\sqrt[2]{\cdot 00327184}$ to four decimal places.

In this case, the first period containing two cyphers necessitates a cypher being placed immediately after the decimal point, the next period, *i.e.*, 32, contains the square of 5, with 7 as a remainder, and continuing we get the answer

$$\sqrt[2]{\cdot 00327184} = \cdot 0572.$$

```
     )·00327184(·0572
        25
    107)771
        749
   1142)2284
        2284
```

EXAMPLES XIV

(i.) $\sqrt[2]{296.}$ (ii.) $\sqrt[2]{34512.}$ (iii.) $\sqrt[2]{297\cdot 326.}$

(iv.) $\sqrt[2]{45072\cdot 36.}$ (v.) $\sqrt[2]{603\cdot 6849.}$

EXTRACTION OF SQUARE ROOT BY FACTORS

A number which will divide into another number without a remainder is said to be a Factor of the latter number. Thus when any two or more quantities are multiplied together they constitute factors of their product.

A **Prime factor** is one that cannot again be divided into factors.

The following are examples of prime factors: 2, 3, 5, 7, 11, 13, etc.

$$\begin{array}{r|r} 2 & 11664 \\ \hline 2 & 5832 \\ \hline 2 & 2916 \\ \hline 2 & 1458 \\ \hline 3 & 729 \\ \hline 3 & 243 \\ \hline 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

$$\begin{array}{r} 2\times2\times3\times3\times3=108 \\ 2\times2\times3\times3\times3=\underline{108} \\ 864 \\ 1080 \\ \hline 11664 \end{array}$$

In any quantity the square root of which is a whole number, this may be found by resolving the quantity into its prime factors, and arranging the factors as shown in the following examples:—

Example.—Find by means of factors the square root of 11664.

Method.—Start with the first prime factor that will divide without a remainder, *i.e.*, 2. Try the same factor repeatedly until a number is obtained into which two will not divide. Try the next prime factor, 3, until a final number is obtained, viz., 3. It will be observed that there are ten prime factors—four 2's and six 3's. If these be grouped into two equal quantities, the product of one of these will be the square root required.

Thus, $\sqrt{11664}=108$.

EXAMPLES XV

Find by factors the square roots of the following quantities:—

(i.) $\sqrt[2]{1764.}$ (ii.) $\sqrt[2]{24336.}$ (iii.) $\sqrt[2]{8649.}$ (iv.) $\sqrt[2]{76176.}$ (v.) $\sqrt[2]{50625.}$

CUBE ROOT

In the same manner as in the case of the square root, it is necessary to mark off in periods of **three figures** the quantity whose cube root is required, commencing from the decimal point, and working from thence to the right and also to the left.

Method.—The work is done by means of three columns of figures as follows:—

On the right place the number whose cube root is required, and point off into periods of three figures each. This column will be number 3. Find the largest number whose cube is less than or equal to the first period; in this case it will be 7. Place 7 as the first figure in the root, and also at the head of the column marked 1. Multiply 7 in col. 1 by

7 in the root, and place the product 49 at the head of col. 2. Now multiply the number in col. 2 by 7, the first figure in the root, and write the product 343 under the figures in the first period. Subtract and bring down the next period, obtaining 32741 for dividend. Add the first figure in the root to the 7 in col. 1, obtaining 14, which is called the first correction; multiply this by the first figure in the root, *i.e.*, 7, and add the product, 98, to the number in col. 1, obtaining 147. Add the first figure of the root to the first correction, obtaining 21, which is called the second correction. Annex two cyphers to the number in col. 2, to form the trial divisor, 14700; also annex one cypher to the second correction, 210; dividing the dividend by the trial divisor, 2 is obtained as the second figure of the root. Next add the second figure in the root to the second correction in col. 1, and multiply the sum by 2, adding the product to the trial divisor, which gives 15124 (the complete divisor). This result, multiplied by 2 and subtracted from the dividend,

Col. 1.	Col. 2.	Col. 3. Root.
7	49	375'741'853'696'(7216
7	98	343
14	14700	32741
7	424	30248
210	15124	2493853
2	428	1557361
212	1555200	936492696
2	2161	936492696
214	1557361	00
2	2162	
2160	155952300	
1	129816	
2161	156082116	
1		
2162		
1		
21630		
6		
21636		

$$\therefore \quad \sqrt[3]{375741853696} = 7216$$

gives a remainder of 2493, to which is annexed the third period, thus forming the next dividend, 2493853. Adding the second figure in the root to the new first correction, 214 is obtained as the "new" first correction; this, multiplied by 2 and added to the trial divisor, gives 15552. Adding 2 (the second figure in root) to the first "new" correction, 216 is obtained, which is the second new correction. By annexing two cyphers to the number in col. 2, the new trial divisor 1555200 is obtained, which will go into the dividend 1+. Add one cypher to the second new correction, and then add to this the third figure in the root, *i.e.*, 1; add this correction to the new trial divisor, obtaining 1557361, which, when subtracted from the dividend, gives 936492, to which is annexed the fourth period, and the previous procedure repeated, producing 6 as the last figure in the root.

EXAMPLES XVI

(i.) $\sqrt[3]{1728}$. (ii.) $\sqrt[3]{4096}$. (iii.) $\sqrt[3]{9528128}$.
(iv.) $\sqrt[3]{1906624}$. (v.) $\sqrt[3]{8365427}$.

EXTRACTION OF THE CUBE ROOT BY FACTORS

Extract by factors the cube root of 592704.

$$\begin{array}{r}
2\overline{)592704} \\
2\overline{)296352} \\
2\overline{)148176} \\
2\overline{)74088} \\
2\overline{)37044} \\
2\overline{)18522} \\
3\overline{)9261} \\
3\overline{)3087} \\
3\overline{)1029} \\
7\overline{)343} \\
7\overline{)49} \\
7
\end{array}$$

2 will no longer divide; try next prime factor.

3 will no longer divide, nor will the next prime factor, 5; try next, *i.e.*, 7.

Therefore, there are twelve factors, whose product is 592704. Arrange these in three equal groups, viz.:—

$$\left.\begin{array}{l} 7 \times 3 \times 2 \times 2 = 84 \\ 7 \times 3 \times 2 \times 2 = 84 \\ 7 \times 3 \times 2 \times 2 = 84 \end{array}\right\} \text{ and } 84 \times 84 \times 84 = 592704$$

$$\therefore \quad \sqrt[3]{592704} = 84, \text{ and } 84^3 = 592704.$$

EXAMPLES XVII

Extract by factors the cube roots of the following quantities:—

(i.) $\sqrt[3]{2000376}$. (ii.) $\sqrt[3]{287496}$. (iii.) $\sqrt[3]{99897344}$. (iv.) $\sqrt[3]{373248}$.

CHAPTER VII

RATIO, PROPORTION, PERCENTAGE

RATIO

If it is desired to compare two numbers with respect to their comparative value, the result of the comparison is called the **Ratio** of the two numbers.

The usual method of expressing a ratio is as follows:—

Suppose we desire to compare the two numbers 10 and 2 and express the comparison as a ratio, it would be done as follows:—$10 : 2$, or $\frac{10}{2}$, each of which would be read as "the ratio of 10 to 2."

The **Terms** of a ratio are the two numbers to be compared, as 10 and 2; and when both terms are considered together they are called a **Couplet.**

The first term, as 10 in the above ratio, is called the **Antecedent,** and the second term the **Consequent.** A ratio may be **Direct** or **Inverse.**

The direct ratio of 10 to 2 is $10 : 2$, or $\frac{10}{2}$.

The inverse ratio of 10 to 2 is $2 : 10$, or $\frac{2}{10}$.

The inverse ratio is sometimes called a Reciprocal ratio.

The reciprocal of a number is 1 divided by the number; thus the reciprocal of 10 is $\frac{1}{10}$.

The **Value** of a ratio is the result obtained by dividing the antecedent by the consequent, thus the value of the ratio $10 : 2$ is 5.

Proportion is an equality of ratios, the equality being indicated by the double colon (::) or by the sign of equality (=). Thus, to write in the form of a proportion the two equal ratios $10 : 2$ and $25 : 5$, which have both the same value, 5, we may employ one of the following forms:—

$$10 : 2 :: 25 : 5$$

or, $$10 : 2 = 25 : 5$$

The numbers forming the proportion are called **Terms,** and are numbered consecutively from left to right, thus:—

First.	Second.	Third.	Fourth.
10 :	2 ::	25 :	5

The first and fourth terms are called the **extremes** and the second and third terms the **means**.

A **direct proportion** is one in which both couplets are direct ratios.

An **inverse proportion** is one which requires one of the couplets to be expressed as an inverse ratio; thus, 10 is to 2 inversely as 5 is to 25, must be expressed as 10 : 2 :: 25 : 5.

In any proportion, the product of the extremes equals the product of the means.

Thus, in the proportion 10 : 2 :: 25 : 5,

$$10 \times 5 = 25 \times 2, \text{ since both products} = 50.$$

The products of the extremes divided by either mean gives the other mean.

Example.—What is the third term, x, of the proportion $10 : 2 :: x : 5$?

$$x = \frac{10 \times 5}{2} = 25 \quad \text{(the third term).}$$

The product of the means divided by either extreme gives the other extreme.

Example.—Find the first term of the proportion $x : 2 :: 25 : 5$.

$$x = \frac{2 \times 25}{5} = 10 \text{ (the first term).}$$

When stating a proportion in which one of the terms is unknown, the missing term is represented by the letter x, as in the last examples.

The principle of all calculations in proportion is:—"Three of the terms are always given, the remaining one is to be found."

Examples.—If 4 plumbers finish a piece of work in 24 days, how long will it take 6 plumbers to do the same piece of work.

$$4 : 6 :: x : 24 = \frac{4 \times 24}{6} = 16 \text{ days.}$$

What is the thickness of 8 lbs. sheet lead, if a cubic foot weighs 706 lbs.?

In this example, 1 sq. ft. of lead of a certain thickness weighs 8 lbs., and it is desired to find the thickness by proportion. The thickness of the cube is 12 ins., therefore the thickness of 8 lbs. sheet lead is

$$8 : 706 :: x : 12 = \frac{8 \times 12}{706} = \cdot 1314$$

Thickness of 8 lbs. sheet lead = ·1314 ins.

If a pump discharging 20 galls. per minute fills a tank in 2 hours, how long will it take a pump which discharges 8 galls. per minute to fill the same tank?

$$20 : 8 :: x : 2 = \frac{20 \times 2}{8} = 5 \text{ hours.}$$

Many examples in proportion may be more easily solved by using the principle of "cause and effect." **Cause** may be regarded as that which produces a change or alteration in, or accomplishes something; and change or alteration, or accomplishment, is the **Effect.**

"Like causes produce like effects;" hence, when two causes of the same kind produce two effects of the same kind, the ratio of the causes equals the ratio of the effects.

Thus in this example, 10 men lift 1500 lbs.; how many lbs. can 15 men lift?

We may call 10 and 15 "causes," since they accomplish something. The number of lbs. lifted, 1500 and x, are the "effects." Hence we may write:—

$$\underset{\text{Cause.}}{\overset{\text{1st}}{10}} : \underset{\text{Cause.}}{\overset{\text{2nd}}{15}} :: \underset{\text{Effect.}}{\overset{\text{1st}}{1500}} : \underset{\text{Effect.}}{\overset{\text{2nd}}{x}} = \frac{15 \times 1500}{10} = 2250 \text{ lbs.}$$

EXAMPLES XVIII

(i.) A man works 50 hours for £1, 7s. 1d.; what will he earn in 37 hours?
(ii.) What will $2\frac{1}{2}$ cwt. of lead shot cost if 3 stones cost £1, 1s. $10\frac{1}{2}$d.?
(iii.) If 25 men earn £75 in 13 days, what will they earn in 91 days?
(iv.) 118 men lay a length of water main in 39 days; how long will it take 59 men to do the same work?

COMPOUND PROPORTION

The previous examples consist of cases of "Simple Proportion," *i.e.*, each term consisting of but one number.

There are many instances, however, where the terms have more than one number in them; such cases belong to "Compound Proportion."

Example.—If 20 men earn £800 in 40 days, how much will 16 men earn in 64 days?

In this case, 20 and 40 are the first cause and 16 and 64 the second cause, because in each case they accomplish something; the effects, *i.e.*, those which are accomplished, are £800 and x; hence we express it as:—

$$\begin{array}{ccccccc} \text{1st} & & \text{2nd} & & \text{1st} & & \text{2nd} \\ \text{Cause.} & & \text{Cause.} & & \text{Effect.} & & \text{Effect.} \\ 20 & : & 16 & :: & 800 & : & x \\ 40 & & 64 & & & & \end{array}$$

$$\text{or } (20 \times 40) : (16 \times 64) :: 800 : x = \frac{16 \times 64 \times \overset{1}{\cancel{800}}}{\underset{1}{\cancel{20}} \times \underset{1}{\cancel{40}}} = £1024.$$

EXAMPLES XIX

(i.) If 9 persons spent £147 in 6 months, how many persons spending at the same rate will £130, 13s. 4d. last for 4 months?

(ii.) If 21 men can do a piece of work in 15 days, working 9 hours per day, how many hours per day must 27 men work to do the same amount of work in 10 days?

(iii.) If the cost of carriage of 54 tons for 38 miles be £21, 6s., what should 95 tons cost for 45 miles?

(iv.) What will be the wages of 15 men for 5 months, when 9 men receive £261, 15s. for 4 months?

Percentage is the process of calculating by hundredths. Thus 5% of $180 = 180 \times \frac{5}{100} = 9$.

The **Sign** of per cent. is %, thus 5% reads five per cent.

The **Base** is the number on which the per cent. is computed.

The **Rate** is the number of hundredths of the base to be taken.

The **Percentage** is the result obtained by multiplying the base by the rate.

The **Amount** is the sum of the base and percentage.

The **Difference** is the remainder obtained by subtracting the percentage from the base.

$$\textbf{Percentage} = \frac{\text{Base}}{\text{Rate}}$$

Example.—A pump constructed to deliver 6000 galls. daily, loses 8 per cent. of the above, owing to slow-closing valves; what is the loss per day?

$$\frac{60\cancel{0}\cancel{0} \times 8}{1\cancel{0}\cancel{0}} = 480 \text{ galls. per day.}$$

$$\textbf{Base} = \frac{\text{Percentage}}{\text{Rate}}$$

Example.—A pump loses 480 galls. of water per day, which represents 8 per cent. of its theoretical capacity: find the theoretical capacity.

$$\frac{480}{\frac{8}{100}} = \frac{480 \times 100}{8} = 6000 \text{ galls. per day.}$$

$$\textbf{Rate} = \frac{\text{Percentage}}{\text{Base}}$$

Example.—A pump, constructed to deliver 6000 galls. per day, loses 480 galls.: find the percentage of loss.

$$\frac{48\cancel{0}}{600\cancel{0}} = \cdot 08 \text{ or } \frac{8}{100} \text{ or } 8\%.$$

When base and rate are given, to find the amount:—

$$\textbf{Amount} = \text{base} \times (1 + \text{rate}).$$

Example.—A pump delivering 6000 galls. per day, by repairing valves has an increased efficiency of 5 per cent.: find the amount delivered per day.

$$6000 \times (1 + \tfrac{5}{100}) = 6000 \times 1\tfrac{1}{20} = 6300 \text{ galls. per day.}$$

When the amount and rate are given, to find the base :—

$$\textbf{Base} = \frac{\text{Amount}}{1 + \text{Rate}}$$

Example.—The theoretical discharge of a pump is 6000 galls. per day; this is 20 per cent. greater than the actual discharge: find the actual discharge.

$$\frac{6000}{1 + \frac{20}{100}} = \frac{6000}{1{\cdot}2} = 5000 \text{ galls. actual discharge.}$$

When the difference and rate are given, to find the base :—

$$\textbf{Base} = \frac{\text{Difference}}{1 - \text{Rate}}$$

Example.—A pump delivers 5520 galls. per day, which is 8 per cent. less than the theoretical discharge: find the theoretical discharge.

$$\frac{5520}{1 - \frac{8}{100}} = \frac{5520}{{\cdot}92} = 6000 \text{ galls. theoretical discharge.}$$

To find the **Gain** or **Loss** per cent., divide the difference between the initial and final values by the initial value.

Example.—A pump which should deliver 6000 galls. per day only delivers 4000 galls. per day: find the loss per cent.

$$\frac{6000 - 4000}{6000} = \frac{2\cancel{000}}{6\cancel{000}} = {\cdot}\dot{3}\dot{3} \quad \therefore \text{ Loss} = 33\tfrac{1}{3}\%.$$

EXAMPLES XX

(i.) What is the increase per cent. if 39 be increased to 45?

(ii.) A mixture contains 23 parts gold, 2 parts silver, and 5 parts tin: what is the percentage of silver in this mixture?

(iii.) A certain alloy contains 23·5 per cent. tin, 35·25 per cent. lead, and the rest copper: how many lbs. of copper will be needed to make 175 lbs. of this alloy?

(iv.) A large reservoir loses 16 per cent. of its water by evaporation, etc., and then 14 per cent. of the remainder is drawn off; it is then found to contain 84,000 galls.: what is the capacity of this reservoir?

CHAPTER VIII

MEASURES (TABLES), DUODECIMALS, MENSURATION OF PLANE FIGURES

MEASURES (English System)

A MEASURE is a standard unit established by law or custom, by which quantity of any kind is measured.

Measures are of six kinds :—

1. Extension.	4. Time.
2. Weight.	5. Angles.
3. Capacity.	6. Money or Value.

The measure of extension may be divided into :—

Lineal measure, which denotes length only.

Square or Superficial Measure, which denotes area or superficial surface and has no thickness (unless specifically stated).

Solid or Cubic Measure, which indicates the contents in some unit of measurement.

Lineal Measure

12 inches	= 1 foot		Ins.		Ft.		Yds.		Rd.		Fur.		Mile.
3 feet	= 1 yard	=	36	=	3	=	1						
5·5 yards	= 1 rod	=	198	=	$16\frac{1}{2}$	=	5·5	=	1				
40 rods	= 1 furlong	=	7,920	=	660	=	220	=	40	=	1		
8 furlongs	= 1 mile	=	63,360	=	5,280	=	1,760	=	320	=	8	=	1

Square Measure

144	square inches		=	1 square foot.
9	,,	feet	=	1 square yard.
$30\frac{1}{4}$	,,	yards	=	1 square rod.
160	rods		=	1 acre.
640	acres		=	1 square mile.

Cubic Measure

1728	cubic inches	=	1 cubic foot.
27	cubic feet	=	1 cubic yard.

Measure of Capacity (Liquid Measure)

4 gills	=	1 pint.
2 pints	=	1 quart.
4 quarts	=	1 gallon.

Measure of Angles (or Arcs)

$60''$ (seconds)	=	1 minute.
$60'$ (minutes)	=	1 degree.
90° (degrees)	=	1 right angle or quadrant.
360° (degrees)	=	1 circle.

Duodecimals, also called **Cross Multiplication.**

This method, which is sometimes used for obtaining the superficial measurements of rectangular figures in square feet, consists of expressions of decimals in twelfths of whole numbers, or submultiples of twelfths.

Rule.—Write the multiplier under the multiplicand, feet under feet, inches under inches, and twelfths of inches under twelfths of inches. Then multiply the figures in the multiplicand separately by the terms in the multiplier, carrying one for every 12 in the products, commencing with the feet in the multiplier and following with inches and twelfths, placing each remainder one place farther to the left than the previous one.

Example.—Find the area in square feet of a piece of lead, 2 ft. $6\frac{1}{2}$ ins. by 4 ft. $8\frac{3}{4}$ ins.

After placing feet under feet, etc., we place 6 for $\frac{1}{2}$ inch and 9 for $\frac{3}{4}$ inch under each other in a separate column, and proceed :—

Note.— $\frac{1}{2}'' = \frac{6}{12}$ of an inch.
$\frac{3}{4}'' = \frac{9}{12}$ of an inch.

2	6	6		
4	8	9		
10	2	0		
1	8	4	0	
	1	10	10	6
12	0	2	10	6

12	=	sq. ft.
0	=	1/12th of a sq. ft.
$2'$	=	sq. in. or lines.
$10''$	=	seconds.
$6'''$	=	thirds.

This method is largely used by glaziers in measuring the superficies of glass.

EXAMPLES XXI

Find by duodecimals the areas of the following :—

(i.) 3 ft. $9\frac{1}{4}$ ins. × 2 ft. 6 ins.
(ii.) 25 ft. $6\frac{2}{3}$ ins. × 10 ft. $5\frac{1}{6}$ ins.
(iii.) 71 ft. $5\frac{1}{2}$ ins. × $6\frac{1}{2}$ ins.
(iv.) 20 ft. $\frac{1}{4}$ in. × 15 ft. $7\frac{3}{4}$ ins.

MENSURATION OF PLANE FIGURES OF SUPERFICIES AND SOLIDS

For definitions of figures used or mentioned in the work of this section, see "Geometry," pp. 1 to 29.

Note.—Fractions, decimals, or duodecimals will be used according to their relative facility in the working of the problems.

RECTANGULAR FIGURES

Rule.—Area = Length × Breadth.

Example.—Find the number of square feet in the floor of a room 20 ft. by 10 ft.

$$\text{Area} = \text{L} \times \text{B} = 20 \times 10 = 200 \text{ sq. ft.}$$

To find length when area and breadth are given.

Rule.—Length = Area ÷ Breadth.

Example.—Find the length of a room containing a floor space of 360 sq. ft. if the breadth = 12 ft.

$$\text{Length} = \frac{\text{Area}}{\text{Breadth}} = \frac{360}{12} = 30 \text{ ft.}$$

To find the length of the sides of a square from the given area.

Rule.—Extract the $\sqrt[2]{\ }$ of the area.

Example.—Find the length of the sides of a square which contains 256 sq. ft.

$$\text{Length of side} = \sqrt[2]{256} = 16.$$

To find the weight of metal (say lead) required to cover or line a gutter 12 ft. long by 2 ft. 9 ins. broad, where the lead used weighs 7 lbs. per super. foot.

Rule.—Total weight in lbs. = Area × Weight per square foot.

$$= 12 \times 2\tfrac{3}{4} \times 7 = \frac{\overset{3}{\cancel{12}}}{1} \times \frac{11}{\underset{1}{\cancel{4}}} \times \frac{7}{1} = 231 \text{ lbs.}$$

To find the area of a square when the length of the diagonal is given.

Rule.—Square the diagonal and divide by 2.

Example.—The length of the diagonal of a square bay which has to be covered with sheet lead = 36 ft.: find the area of the square.

$$\frac{\text{Diagonal}^2}{2} = \frac{36^2}{2} = 648 \text{ sq. ft.}$$

This may be explained by the fact that the square of the hypotenuse of a right-angled triangle is equal to the sum of the squares of the base and vertical height. A square is cut into two right-angled triangles by the diagonal passing through its centre and two opposite angles. Since the sides of a square are equal, the area (where the length of the side is given) is obtained by squaring the side; and as the square of the diagonal is equal to the sum of the squares of two sides, it follows that with the diagonal given,

$$\text{Area} = \frac{\text{Diagonal}^2}{2}$$

EXAMPLES XXII

Find the areas of the following :—

(i.) 2 yds. 2 ft. × 3 yds. 1 ft.
(ii.) 13 ft. 7 ins. × 6 ft. 9 ins.
(iii.) 16 ft. 3 ins. × 10 ft. 7 ins.
(iv.) 25 yds. × 2 yds. 1 ft.

Find length of one side :—

(v.) Area 12 sq. yds. 5 sq. ft. 48 sq. ins.; breadth 17 ft.
(vi.) Area 30 sq. yds. 4 sq. ft 12 sq. ins.; breadth 23 ft.
(vii.) Area 177 sq. yds. 5 sq. ft.; breadth 8 yds.

EXAMPLES XXIII

Find the area of squares having the following diagonals :—

(i.) 20. (ii.) 36·4. (iii.) 250·25. (iv.) 6·725. (v.) 200·75

When the thickness only of the metal is given, and you are required to find the weight necessary to cover a given surface, the specific gravity (or the weight of a cubic inch or foot of the metal) may be used in ascertaining the weight of metal required.

Rule.—Multiply the area by thickness, which gives the volume of lead in cubic feet. Then multiply by 62·5 (the weight of a cubic foot of water), which will give the weight in lbs. of a quantity of water equal in volume to the lead; assuming the specific gravity of lead to be 11·36, which means that lead is 11·36 times as heavy as an equal bulk of water, therefore, multiplying by 11·36 we obtain the weight of the lead.

Example.—Find the weight of lead required to cover a dormer top 14 ft. by 12 ft., if the thickness of lead is $\frac{1}{7}$ of an inch.

Note.—In all such calculations the measurements should be either kept in inches or feet and fractions of same. It is obvious, if we multiply the area in feet by $\frac{1}{7}$, that the result will be twelve times greater than the rightful quantity. $\frac{1}{7}$ of an inch equals $\frac{1}{84}$ of a foot; therefore the area must be multiplied by $\frac{1}{84}$ as representing the thickness of lead in the fraction of a foot. It is advisable, for the sake of clearness and rapidity, to place all the terms in one expression :—

$$\underbrace{\frac{\overset{2}{\cancel{14}}}{1} \times \frac{\overset{1}{\cancel{12}}}{1} \times \frac{1}{\underset{\underset{1}{\cancel{7}}}{\cancel{84}}}}_{\text{Cubic contents in feet of lead.}} \times \underbrace{\frac{125}{\cancel{2}}}_{\text{Weight of a cubic foot of water.}} \times \underbrace{\frac{11{\cdot}36}{1}}_{\text{Specific gravity of lead.}} = 1420 \text{ lbs.}$$

Find the weight of lead required to line a tank 3 ft. 6 ins. deep, 4 ft. 6 ins. long, and 2 ft. 4 ins. broad, **not** allowing for laps or passings at the soldered angles and turnover. 7 lbs. lead to be used.

This calculation involves the addition of the areas of several rectangular surfaces.

1st. Area of bottom = 4′ 6″ × 2′ 4″ . $= \frac{\overset{3}{\cancel{9}}}{2} \times \frac{7}{\underset{1}{\cancel{3}}} = 10{\cdot}5$ or $10\frac{1}{2}$

2nd. Area of a piece which will line the sides and two ends . . . $\Big\} = \frac{41}{3} \times \frac{7}{2} = 47{\cdot}83$ or $47\frac{5}{6}$

Length of which = (2′ × 4′ 6″) + (2′ × 2′ 4″).
Depth of which = (3′ 6″).

Total area . . $= \overline{58{\cdot}33}$ or $\overline{58\tfrac{1}{3}}$

Area	=	58·33 sq. ft.
Weight of lead per sq. ft.	=	7 lbs.
Weight of lead .	=	408·31 lbs.

TRIANGLES

To find the area of any triangle where the lengths of the "base" and vertical height are given.

Rule.—Multiply the base by the vertical height, and divide by 2.

Example.—Find the area in square feet of a triangle with a base 7 ft. in length and a vertical height of 10 ft. 6 ins.

$$\frac{\text{Vertical height} \times \text{base}}{2} = \frac{7}{1} \times \frac{21}{2} \times \frac{1}{2} = \frac{147}{4} = 36{\cdot}75 \text{ sq. ft.}$$

When the length of each of the three sides is given, a different rule is necessary to find the area. We will call the three sides, *a*, *b*, and *c* respectively.

Rule.—Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$.

This formula may be explained as follows :—

s = half the sum of the three sides.

a, b, and c = length of each side of the triangle.

Subtract each side separately from half the sum of the three sides, and multiply the differences by each other, and by half the sum of the three sides, finally extracting the square root of the product, which will be the area of the triangle.

Example.—Find the area in square feet of sheet lead required to cover a triangular surface, the three sides of which are 12, 16, and 20 ft. respectively.

$$\text{Half sum of three sides} = \frac{12+16+20}{2} = \frac{48}{2} = 24$$

$$\begin{aligned} \text{Area} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{24(24-12)(24-16)(24-20)} \\ &= \sqrt{24 \times 12 \times 8 \times 4} \\ &= \sqrt{9216} \\ &= 96 \text{ sq. ft.} \end{aligned}$$

$a=12$; $b=16$; $c=20$

```
    )9216(96
     81
186)1116
    1116
```

EXAMPLES XXIV

Find area of the following triangles :—

(i.) Base = 6 ft ; vertical height = 8 ft.
(ii.) Base = 47 ft. ; vertical height = 7 yds.
(iii.) Base = 9 yds. 2 ft. $7\frac{1}{2}$ ins. ; vertical height = 13 ft.
(iv.) Base = 137 yds. ; vertical height = 72 yds.

Find area when three sides A, B, and C are given :—

(v.) When A = 3 ft., B = 4 ft., C = 5 ft.
(vi.) When A = 12 ft., B = 36 ft., C = 40 ft.
(vii.) When A = 48 ft., B = 20 yds., C = 12 yds.
(viii.) When A = 14 ft., B = 26 ft., C = 24 ft.

SOLUTION OF RIGHT-ANGLED TRIANGLE

To find the length of one side of a right-angled triangle, having the other two sides given, we proceed as follows :—

First, naming the base B, the vertical height A, and the hypotenuse C.

To find C when the lengths of A and B are given.

Rule.—Extract the square root of the sum of the squares of A and B.

Formula.—$C = \sqrt{A^2 + B^2}$.

When the lengths of C and A are given, to find B.

Rule.—Extract the square root of the difference of the squares of C and A.

Formula.—$B = \sqrt{C^2 - A^2}$.

Note.—The last formula applies, when it is necessary to find A when B and C are given, by exchanging the position respectively of B and A.

Examples.—(1) Find the length of a ladder which will reach the eaves of a building 32 ft. high, if the foot be placed 8 ft. from the base of the wall.

In this example a right-angled triangle is formed, having a ladder for the hypotenuse, the length of which is required.

Using the formula $C = \sqrt{A^2 + B^2} = \sqrt{32^2 + 8^2} = \sqrt{1088}$, we find the length of the ladder to be 33 ft. (nearly).

(2) Find the height of a wall which may be reached by a ladder 40 ft. long with the foot placed 12 ft. from the base of the wall.

Naming the height of the wall A, the ladder C, and the ground between the foot of the ladder and the wall B.

$$\text{Then A} = \sqrt{C^2 - B^2} = \sqrt{1600 - 144}$$
$$= \sqrt{1456} = 38{\cdot}15 \text{ ft. (height of wall).}$$

EXAMPLES XXV

B = base; A = vertical height; C = hypotenuse.

(i.) Find C if A = 13 and B = 17. (ii.) Find C if A = 7 and B = 23.

(iii.) Find B if A = 101 and C = 105. (iv.) Find A if B = 25 and C = 63.

(v.) Find B if A = 24 and C = 36.

Rhombus and Rhomboid. (See "Geometry," p. 6.)

To find the area of either of the above figures.

Rule.—Multiply the length of one side by the perpendicular distance between the two sides, the product being the required area.

Example.—Find the area of a rhombus, length of one side being 12 ft. 9 ins., and the perpendicular distance between the two sides 10 ft. 6 ins.

$$\text{Then } 12\tfrac{3}{4} \times 10\tfrac{1}{2} = \frac{51}{4} \times \frac{21}{2} = \frac{1071}{8} = 133\tfrac{7}{8} \text{ sq. ft. area.}$$

THE CIRCLE

There are two methods in use for finding the area of a circle; one is to multiply the square of the diameter by ·7854, *i.e.*, area $= D^2 \times {\cdot}7854$, or to multiply the square of the radius by 3·1416. The latter number is generally denoted by the Greek letter π, and will be used where necessary to denote the number 3·1416, which is the ratio between the diameter and circumference of a circle; the latter method of obtaining the area of circles will be used in the following work.

Therefore where R = radius of circle.

$\pi = 3{\cdot}1416$ (correct to four significant figures).

Rule.—Area of circle $= \pi R^2$.

Example.—Find the area in square inches of the cross-section of a pipe 4 ins. diameter.

Area $= \pi R^2 = 3{\cdot}1416 \times 2^2 = 12{\cdot}5664$ sq. ins.

In these calculations π is sometimes taken as $3\frac{1}{7}$, *i.e.*, $\frac{22}{7}$ (which is correct to three significant figures) instead of 3·1416, for convenience of working.

Example.—How many square inches are there in the cross-sections of two 6-in. circular pipes ?

$$\text{Area of pipes} = 2\pi R^2 = \frac{2}{1} \times \frac{22}{7} \times \frac{3}{1} \times \frac{3}{1} = \frac{396}{7} = 56\tfrac{4}{7} \text{ sq. ins.}$$

The relative areas of circles are to each other as the squares of their diameters—that is to say, the areas of circles of different diameters vary as the squares of their diameters.

Thus, if we wish to compare the areas of a 4-in. and 8-in. diameter pipe respectively :—

$$\left.\begin{array}{l}\text{Area of } 8'' \text{ pipe} = \pi R^2 \\ \text{Area of } 4'' \text{ pipe} = \pi R^2\end{array}\right\} = \frac{\pi \times 4 \times 4}{\pi \times 2 \times 2} = \frac{16}{4} = 4$$

As π is a term common to both, we may cancel it, and after squaring the radius of the 8-in. pipe, and also of the 4-in. pipe, and comparing same, it will be seen that an 8-in. diameter pipe possesses an area equal to four times that of a 4-in. diameter pipe.

Example.—(1) How many 1-in. diameter pipes are equal in area to a 6-in. diameter cast-iron water main ?

In this question, the relative areas of the pipes, the diameters of which are given, are required.

$$\left.\begin{array}{l}\text{Area of } 6'' \text{ pipe} = \pi R^2 \\ \text{Area of } 1'' \text{ pipe} = \pi R^2\end{array}\right\} = \frac{\pi \times 3 \times 3}{\pi \times \frac{1}{2} \times \frac{1}{2}} = 36$$

$$\left.\begin{array}{r}\text{or for } 6'' \text{ pipe } D^2 \\ \text{and for } 1'' \text{ pipe } D^2\end{array}\right\} = \frac{36}{1} = 36$$

The cross-sectional area of a 6-in. diameter pipe is equal to the combined cross-sectional areas of thirty-six 1-in. diameter pipes.

If it is required to find the diameter of a pipe equal in area to the total areas of a given number of pipes of various diameters, the same principle holds good.

Example.—(2) Find the diameter of a pipe, the cross-sectional area of which is equal to two 1-in. pipes, four $1\frac{1}{2}$-in. pipes, and three 2-in. pipes.

Rule.—Extract the square root of the sum of the squares of the diameters of the pipes.

$$\left.\begin{array}{lr}2 \times 1^2 \quad . \quad . \quad . & = 2 \\ 4 \times (1\frac{1}{2})^2 = \frac{\cancel{4}}{1} \times \frac{3}{\cancel{2}} \times \frac{3}{\cancel{2}} & = 9 \\ 3 \times 2^2 \quad = \frac{3}{1} \times \frac{2}{1} \times \frac{2}{1} & = 12\end{array}\right\} = \sqrt[2]{23} = 4{\cdot}79 \text{ ins., or rather over } 4\tfrac{3}{4} \text{ ins. diam.}$$

To find the circumference of a circle, multiply the diameter by 3·1416. $D \times \pi = \text{Circumference}$.

Example.—Find the circumference of a circle $10\frac{1}{2}$ ins. diameter.

$$D \times \pi = \frac{\overset{3}{\cancel{21}}}{\underset{1}{\cancel{2}}} \times \frac{\overset{11}{\cancel{22}}}{\underset{1}{\cancel{7}}} = 33 \text{ ins.}$$

From the given area of a circle, to find the diameter, the process must be the reverse of the one for finding the area from a given diameter.

Rule.—Divide the area by π, *i.e.*, 3·1416 or $\frac{22}{7}$, extract the square root of the quotient, and multiply by 2.

Safety-valves are often required on boilers used for hot-water supply purposes to provide for a release of pressure in case of stoppage in the circulation pipes, and preventing explosions thereby. It is necessary in some cases, where dead-weight valves are used, to ascertain the weight required to counterbalance the pressure of the water in the boiler and system tending to force the valve open; when the head of water above the valve and the diameter of the portion of the valve exposed to the pressure are known, the weight may be found by multiplying the pressure in pounds per square inch by the area in inches, or fractions of same, of the exposed portion of the valve.

Example.—Find weight necessary to counterbalance a column of water acting on the $\frac{1}{2}$-in. diameter disc of a safety-valve exposed to the pressure due to 50 ft. head of water. (See "Hydrostatics" for head of water.)

Weight = Pressure per square inch × Area = $50 \times {\cdot}434 \times \pi R^2$.

$$= \frac{50}{1} \times \frac{{\cdot}434}{1} \times \frac{22}{7} \times \frac{1}{4} \times \frac{1}{4} = 4{\cdot}26 \text{ lbs.}$$

This weight would be doubled in practice.

Example.—(1) Find the diameter of a circle, the area of which is 154 sq. ft.

$$D = 2\sqrt[2]{\frac{\text{Area}}{\pi}} = 2\sqrt[2]{\frac{154}{\frac{22}{7}}} = 2\sqrt[2]{\frac{154}{1} \times \frac{7}{22}} = 2\sqrt[2]{49}$$

$$= 2 \times 7 = 14 \text{ ft. diameter.}$$

Example.—(2) What weight of 7 lbs. sheet lead is there in a circular disc 2 ft. 6 ins. in diameter?

$$\textbf{Weight} = \pi R^2 \times 7 = \frac{\overset{11}{\cancel{22}}}{\underset{1}{\cancel{7}}} \times \frac{5}{\underset{2}{\cancel{4}}} \times \frac{5}{4} \times \frac{\overset{1}{\cancel{7}}}{1} = \frac{275}{8} = 34\tfrac{3}{8} \text{ lbs.}$$

EXAMPLES XXVI

Find area if diameter equals :—

(i.) $10\frac{1}{4}$ ins. (ii.) $11\frac{5}{8}$ ins. (iii.) 1 ft. $8\frac{1}{2}$ ins.
(iv.) 2 ft. $5\frac{7}{8}$ ins. (v.) $3\frac{3}{8}$ ins.

Find area if radius equals :—

(vi.) 7 ft. 6 ins. (vii.) 15 ft. (viii.) 30 ft. 6 ins.
(ix.) 35 ft. (x.) 50 ft.

Find area if circumference equals :—

(xi.) 49·95 ins. (xii.) 73·51 ins. (xiii.) 109 ft.
(xiv.) 20·42 ft. (xv.) 56·54 ft.

Find diameter if area equals :—

(xvi.) 27·3397 sq. ft. (xvii.) 141·026 sq. ft.
(xviii.) ·1452 sq. ft. (xix.) ·4418 sq. ft.

The area of a sector of a circle may be found (when the length of the radii, and the angle formed at the centre of the circle by the junction of the radii, are given) by multiplying the total area of the circle (of which the sector forms a part) by the fraction which the angle of the sector is of the number of degrees in a circle (*i.e.*, 360).

$$\text{Area of } \textbf{Sector} = \pi R^2 \times \frac{\text{Degrees in angle of sector}}{360}$$

Example.—Find the area of a **Sector** of a circle, the radii of which are 10 ft. 6 ins., and the angle subtended by the arc 60°.

$$\text{Area of } \textbf{Sector} = \pi R^2 \times \frac{60}{360} = \frac{\overset{11}{\cancel{22}}}{\underset{1}{\cancel{7}}} \times \frac{\overset{\overset{1}{\cancel{3}}}{\cancel{21}}}{2} \times \frac{21}{\cancel{2}} \times \frac{\overset{1}{\cancel{60}}}{\underset{\underset{2}{\cancel{6}}}{\cancel{360}}} = 57{\cdot}75 \text{ sq. ft.}$$

EXAMPLES XXVII

Find the areas of the following sectors, if

D = Diameter of circle.
R = Radius of circle.
A = Angle subtended by the arc cut off by two radii.

(i.) D = 14 ft. A = 110° (ii.) D = 21 ft. A = 25°
(iii.) R = 9 ft. A = 220° (iv.) R = 8 ft. A = 130°

The area of a **Segment** of a circle may be found, where the length of the chord and the height of the segment are given, by dividing the cube of the height by twice the length of the chord, and adding to the quotient two-thirds of the product of the chord and height.

$$\text{Area of } \textbf{Segment} \text{ of circle} = \frac{H^3}{2C} + \left(\tfrac{2}{3}CH\right)$$

Example.—Find the area in square feet of a segment of a circle, the chord of which measures 20 ft. and the height 8 ft.

$$C = \text{chord.} \quad H = \text{height.}$$
$$CH = \text{chord} \times \text{height.}$$

Area of **Segment**

$$= \frac{H^3}{2C} + \left(\frac{2}{3}CH\right) = \left(\frac{\overset{1}{\cancel{8}} \times 8 \times 8}{\underset{1}{\cancel{2}} \times \underset{5}{\cancel{20}}}\right) + \left(\frac{2 \times 20 \times 8}{3}\right) = \frac{64}{5} + \frac{320}{3} = 119\tfrac{7}{15} \text{ sq. ft.}$$

The area of an ellipse may be found by multiplying the product of the major and minor axes by $\frac{\pi}{4}$.

Rule.—Area of **Ellipse** = Longer × Shorter axis × $\frac{\pi}{4}$.

Example.—Find the area of an ellipse, the major axis being 8 ft. and the minor 6 ft.

$$\text{Area} = 8 \times 6 \times \frac{3{\cdot}1416}{4} = 37{\cdot}6992 \text{ sq. ft.}$$

EXAMPLES XXVIII

(i.) Major axis 12; minor axis 8. (ii.) Major axis 1·4; minor axis ·75.
(iii.) Major axis ·84; minor axis ·24. (iv.) Major axis 32·5; minor axis 20·24.

AREAS OF POLYGONS

The areas of polygons may be found by using "constants." Each figure has a separate constant, thus:—

No. of Sides.	Name.	Constant.	No. of Sides.	Name.	Constant.
5	Pentagon . .	1·7205	9	Nonagon . .	6·1818
6	Hexagon . .	2·5981	10	Decagon . .	7·6942
7	Heptagon . .	3·6339	11	Undecagon .	9·3656
8	Octagon . .	4·8284	12	Duodecagon .	11·1962

To find the area of a polygon.

Rule.—Side^2 × Constant.

Example.—Find the area of an octagon, if length of one side = 3·025.

$$\text{Area} = \text{Side}^2 \times C = 3{\cdot}025 \times 3{\cdot}025 \times 4{\cdot}8284 = \underline{44{\cdot}182} \ \textit{Ans.}$$

The area of a hexagon may be found without the aid of a constant. A hexagon may be divided into six equilateral triangles, and if the area of one of these be found, and multiplied by 6, the area of the hexagon will be obtained.

The area of an equilateral triangle may be found by :—

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)} \text{ or } \frac{\text{Side squared}}{4}\sqrt{3}$$

Example.—Find the area of a hexagon, the length of one side of which = 20 ft., using :—

$$\text{Area} = \frac{\text{Side}^2}{4}\sqrt{3} \text{ (for area of equilateral triangle).}$$

$$\text{Then area of hexagon} = \frac{6 \times \text{Side}^2}{4}\sqrt{3}$$

$$= \frac{6 \times 20 \times 20}{4} \times 1{\cdot}732$$

$$= 1039{\cdot}2$$

EXAMPLES XXIX

Find the areas of the following polygons :—

(i.) 5 sides; length of side 27·2.
(ii.) 6 sides; length of side 20·5.
(iii.) 7 sides; length of side ·56.
(iv.) 9 sides; length of side 3·24.
(v.) 10 sides; length of side 10·5.

To find the area of the total surface of a hot-water **cylinder.**

Rule.—Multiply the vertical height by the circumference of the cylinder, and add the areas of the two ends.

$$\textbf{Total area of cylinder} = (2\pi R \times H) + (2\pi R^2) \quad \text{(No. 1.)}$$
$$\text{or, } 2\pi R \times (H + R) \quad \text{(No. 2.)}$$

Example.—(*a*) (Using No. 2.) Find the total area in square inches of a copper hot-water cylinder; height 3 ft. 4 ins., diameter 1 ft. 2 ins.

Area $= 2\pi R(H + R)$; substituting figures for letters, we get :—

$$\frac{2}{1} \times \frac{22}{7} \times \frac{7}{1}\left(\frac{40}{1} + \frac{7}{1}\right) = 44 \times 47 = 2068 \text{ sq. ins.}$$

(*b*) (Using No. 1 for the same question.)

Area $= (2\pi RH) + (2\pi R^2)$

$$= \left(\frac{2}{1} \times \frac{22}{7} \times \frac{7}{1} \times \frac{40}{1}\right) + \left(\frac{2}{1} \times \frac{22}{7} \times \frac{7}{1} \times \frac{7}{1}\right) = 1760 + 308 = 2068 \text{ sq. ins.}$$

If the area of the curved portion only is required, then the height × circumference will give the required area.

When finding the total pressure sustained by the interior surfaces of boilers and cylinders, the height of the column of water is measured from the free surface of the water in the tank to the centre of the boiler or cylinder.

Example.—Find the total pressure on the internal surfaces of a cylinder 1 ft. 8 ins. diameter, 3 ft. 4 ins. high, due to a head of 60 ft.

Total Pressure = Pressure per sq. in. × area in sq. ins.

$$= \{\underset{\text{Area of ends.}}{(2\pi R^2)} + \underset{\text{Area of curved portion.}}{(2\pi RH)}\}\underset{\text{Pressure per sq. in.}}{60 \times \cdot 434}$$

$$= \left\{\left(\frac{2}{1} \times \frac{22}{7} \times \frac{10}{1} \times \frac{10}{1}\right) + \left(\frac{2}{1} \times \frac{22}{7} \times \frac{10}{1} \times \frac{40}{1}\right)\right\} 26 \cdot 04$$

$$= \left\{\frac{4400}{7} + \frac{17600}{7}\right\} 26 \cdot 04 = 81{,}838 \cdot 5 \text{ lbs. total pressure.}$$

EXAMPLES XXX

Find the total area of each of the following cylinders :—

(i.) Diameter = 2 ft. 4 ins., height = 3 ft. 6 ins.
(ii.) Diameter = 1 ft. 2 ins., height = 3 ft. 4 ins.
(iii.) Radius = 7 ins., height = 28 ft.
(iv.) Radius = $10\frac{1}{2}$ ins., height = 10 ft.

The area of a sphere may be found by multiplying the area of its largest circle by 4.

Rule.—Area of sphere = $4\pi R^2$, it being obvious that the largest circle will have a radius equal to that of the sphere.

Example.—(*a*) Find the area of a sphere 7 ft. diameter.

$$\text{Area of sphere} = 4\pi R^2 = \frac{\overset{1}{\cancel{4}}}{1} \times \frac{22}{\underset{1}{\cancel{7}}} \times \frac{\overset{1}{\cancel{7}}}{\underset{1}{\cancel{2}}} \times \frac{7}{\underset{1}{\cancel{2}}} = 154 \text{ sq. ft.}$$

(*b*) Find the weight of lead, $\frac{1}{11}$ of an inch thick, required to cover a spherical dome 12 ft. diameter (neglecting rolls).

Here, as in previous examples, we must take into consideration the specific gravity of lead, and the weight of a cubic foot of water.

Weight of lead = $4\pi R^2$ × S.G. × weight of a cubic foot of water.

$$\underset{\text{Area of Sphere.}}{\frac{4}{1} \times \frac{22}{7} \times \frac{6}{1} \times \frac{6}{1}} \times \underset{\text{Thickness in ft. of Lead.}}{\frac{1}{132}} \times \underset{\text{Specific Gravity.}}{\frac{11 \cdot 36}{1}} \times \underset{\text{Weight of cub. ft. of Water.}}{\frac{125}{2}} = 2434\tfrac{2}{7} \text{ lbs.}$$

To find the area of a hemispherical dome, it is obvious that only half the above would be taken.

Area of hemisphere $= 2\pi R^2$.

EXAMPLES XXXI

Find the areas in sq. ft. of spheres having diameters as follows :—

(i.) Radius = 10 ins.
(ii.) Diameter = ·21 ins.
(iii.) Radius = 10·5 ins.
(iv.) Diameter = 1·4 ins.

SEGMENT OF SPHERE

The area of the curved portion of a segment of a sphere may be obtained by multiplying the height by the circumference of the sphere.

Rule.—Area of curved portion of segment of sphere $= \pi D \times H$.

Example.—How many square feet of sheet lead will be required to cover the curved portion of the segment of a sphere 12 ft. diameter, height of segment 3 ft. 6 ins. (neglecting rolls and laps).

$$\text{Area} = \pi DH = \frac{\overset{11}{\cancel{22}}}{\underset{1}{\cancel{7}}} \times \frac{12}{1} \times \frac{\overset{1}{\cancel{7}}}{\underset{1}{\cancel{2}}} = 132 \text{ sq. ft. of lead.}$$

To find the area of the curved surface of a cone where the diameter of the base and slant height are given.

Rule.—Half the product of the slant height and circumference of the base = area of curved surface.

(Total area may be obtained by adding area of base.)

Example.—Find the total area in square feet of a right circular cone having a slant height 14 ft., and a base 6 ft. diameter.

Area of curved surface $= \frac{1}{2}$ slant height $\times 2\pi R$

$$\left.\begin{aligned} &= \frac{1}{\underset{1}{\cancel{2}}} \times \frac{\overset{2}{\cancel{14}}}{1} \times \frac{\overset{1}{\cancel{2}}}{1} \times \frac{22}{\underset{1}{\cancel{7}}} \times \frac{3}{1} = 132 \text{ sq. ft.} \\ \text{Area of base} &= \pi R^2 = \frac{22}{7} \times \frac{3}{1} \times \frac{3}{1} = 28\tfrac{2}{7} \text{ sq. ft.} \end{aligned}\right\} \begin{aligned} &\text{Total area} \\ &= 160\tfrac{2}{7} \text{ sq. ft.} \end{aligned}$$

When the **vertical height** and the diameter of the cone are given :—

Rule.—Find the **slant height** by the method adopted for the solution, of triangles, then proceed as above.

Example.—Find the total area of a right circular cone. Diameter of base = 15 ft.; vertical height = 20 ft.

$$\text{Slant height} = \sqrt{15^2+20^2} = \sqrt{225+400}$$
$$= \sqrt{625}$$
$$= 25.$$
$$\text{Total area} = \frac{\text{SH} \times 2\pi\text{R}}{2} + \pi\text{R}^2$$
$$= \left(\frac{25}{2} \times \frac{2}{1} \times \frac{22}{7} \times \frac{15}{2}\right) + \left(\frac{22}{7} \times \frac{15}{2} \times \frac{15}{2}\right)$$
$$= \frac{4125}{7} + \frac{2475}{14}$$
$$= \frac{8250+2475}{14}$$
$$= \frac{10925}{14}$$
$$= 780\tfrac{5}{14}.$$

The area of the curved surface of the frustum of a cone may be found by the following method :—

Rule.—Area of curved portion of frustum of a cone = half the slant height times the sum of the circumferences of base and top.

Example.—Find the area of the curved portion of the frustum of a cone.

Diameter of base	=	10 ft.
,, top	=	6 ,,
Slant height	=	7 ,,

Area of curved portion = $\frac{1}{2}$ slant height × (circumference of base + circumference of top).

$$\frac{1}{2} \times \frac{7}{1}\left(\left\{\frac{10}{1} \times \frac{22}{7}\right\} + \left\{\frac{6}{1} \times \frac{22}{7}\right\}\right) = \frac{7}{2} \times \frac{352}{7} = 176 \text{ sq. ft.}$$

(The total area may be found by adding the areas of top and base.)

EXAMPLES XXXII

Find total area of each of the following frustums of cones :—

(i.) Diameter of base, 7 ft.; diameter of top, 3 ft. 6 ins.; slant height, 12 ft.
(ii.) Diameter of base, 10 ft. 6 ins.; diameter of top, 7 ft.; slant height, 12 ft.
(iii.) Diameter of base, 12 ft.; diameter of top, 10 ft. 6 ins.; slant height, 25 ft.
(iv.) Diameter of base, 1 ft. 6 ins.; diameter of top, 1 ft. 2 ins.; slant height, 1 ft. 9 ins.

To find the area of the surface of a pyramid where slant height and length of sides of base are given :—

Rule.—To area of base add the sum of the areas of the triangles constructed on the sides of the base.

Example.—Find the total area of a pyramid constructed with a slant height of 8 ft. upon a square base with 6 ft. sides.

Total Area = Area of base + area of four triangles

Area of four triangles.

$$= \left(\frac{6\times 6}{1}\right) + \left(\frac{4}{1}\times\frac{8}{1}\times\frac{6}{1}\times\frac{1}{2}\right) = 36+96=132 \text{ sq. ft.}$$

If a pyramid be constructed on a polygonal base, the same rule applies, but the area of the base is found by the use of one of the constants on page 60.

Example.—Find the total area of a pyramid, slant height = 25 ft., length of one side of octagonal base = 4 ft.

Area = area of eight triangles + area of base

$$= \left(\frac{8}{1}\times\frac{25}{1}\times\frac{4}{1}\times\frac{1}{2}\right)+\left(\frac{4}{1}\times\frac{4}{1}\times\frac{4{\cdot}828}{1}\right) = 400+77{\cdot}248$$

$$= \underline{477{\cdot}248} \text{ sq. ft.}$$

The area of the frustum of a pyramid may be found by :—

Rule.—Add to the sum of the areas of the top and bottom the area of the four sides.

Note.—The area of any one side may be found by multiplying half the sum of the lengths of the top and bottom of one side by the slant height.

Example.—Find the total area of the frustum of a pyramid having a slant height of 7 ft., the length of the sides of the squares forming the top and base are 6 ft. and 8 ft. respectively.

$$\text{Area of base} = 8^2 = 64$$

$$\text{,, top} = 6^2 = 36$$

$$\text{Area of four sides} = \frac{4}{1}\times\frac{7}{1}\times\left(\frac{6+8}{2}\right) = 196$$

$$\text{Total area} = \underline{296} \text{ sq. ft.}$$

EXAMPLES XXXIII

Find the total areas of the following pyramids, each having a polygonal base :—

(i.) (Hexagonal base) side = 10 ; slant height = 20.
(ii.) (Octagonal base) side = 14 ; slant height = 21.
(iii.) (Heptagonal base) side = 10 ; slant height = 15.

Find the total areas of the following frustums of pyramids :—

(iv.) (Square base) side of base = 6 ; side of top = 4 ; slant height = 8.
(v.) (Hexagonal base) side of base = 8 ; side of top = 6 ; slant height = 10.

CHAPTER IX

SOLID FIGURES, CAPACITIES OF PIPES, CYLINDERS, TANKS, ETC.

SOLIDS

In the mensuration of solids we desire to find the solid contents and express them in some unit form of measurement, such as cubic inches, cubic feet, cubic yards, gallons, etc.

The cubic contents of a solid, the angles of which are right angles, may be obtained by :—

Rule.—Multiply the length, breadth, and depth together.

Example.—How many cubic feet are there in a tank, length 10 ft., breadth 3 ft. 6 ins., depth 6 ft.?

$$\textbf{Contents} = L \times B \times D = \frac{10}{1} \times \frac{7}{2} \times \frac{6}{1} = 210 \text{ cubic feet.}$$

When length and breadth are given and it is required to find the depth, the cubic contents being given :—

Rule.—Divide the cubic contents by the product of the length and breadth.

Example.—Find the depth of a tank which will hold 210 cub. ft. of water, the length being 10 ft. and the breadth 3 ft. 6 ins.

$$\text{Depth} = \frac{\text{Cub. Contents}}{L \times B} = \frac{210}{10 \times 3{\cdot}5} = 6 \text{ ft.}$$

If any two measurements are given along with cubic contents, the same rule applies.

If the contents and depth are given, and it is required to find the length of the sides of the bottom which form a square :—

Rule.—Extract the square root of the quotient resulting from the division of the contents by the depth.

Example.—Find the length of a side of the bottom of a tank 1 ft. 6 ins. deep, which will contain 2 cub. ft. of water.

$$\text{Length of side} = \sqrt{\frac{2}{1{\cdot}5}} = \sqrt{1{\cdot}\dot{3}} = 1{\cdot}153 \text{ ft.}$$

If it be required to find the number of gallons of water which a tank of given size will hold, the cubic contents multiplied by the number of gallons in a cubic foot will give the necessary answer.

Example.—(*a*) Find the contents in gallons of a w.c. flushing cistern 1 ft. 6 ins. × 8 ins. × 6 ins. (Number of gallons in a cubic foot $= 6\frac{1}{4}$.)

$$\textbf{Contents in gallons} = \frac{3}{2} \times \frac{8}{12} \times \frac{6}{12} \times \frac{25}{4} = 3{\cdot}125 \text{ galls.}$$

The weight of water in a tank may be ascertained by multiplying the number of gallons which the tank holds by 10 (1 gall. of water at 39·2° Fahr. weighs 10 lbs.).

Example.—(*b*) What is the weight of water in a tank 5 ft. 9 ins. × 3 ft. 4 ins. × 2 ft. 3 ins.?

$$\textbf{Weight} = \text{gallons} \times 10 = \frac{23}{4} \times \frac{10}{3} \times \frac{9}{4} \times \frac{25}{4} \times \frac{10}{1} = 2695{\cdot}31 \text{ lbs.}$$

When it is required to find the size of a tank which will store or hold a given number of gallons of water, and no measurements are given, we may assume a measurement for the length and depth and find the breadth, or, if any measurements are given (say length and breadth), we may find the depth.

Rule.—Convert the gallons to cubic feet by dividing the number of gallons by $6\frac{1}{4}$, and divide the quotient thus obtained by the product of the length and breadth.

Example.—(*a*) Find the depth of a tank which will hold 200 galls. of water; the length is 4 ft. 6 ins. and the breadth 2 ft.

$$\textbf{Depth} = \frac{\text{Gallons}}{6\frac{1}{4} \times \text{L} \times \text{B}} = \frac{200}{6\frac{1}{4} \times 2 \times 4\frac{1}{2}} = \frac{200}{1} \times \frac{4}{25} \times \frac{1}{2} \times \frac{2}{9} = 3\tfrac{5}{9} \text{ ft. deep.}$$

Example.—(*b*) Find the depth and breadth of a w.c. flushing tank to hold $2\frac{1}{2}$ galls. of water, the length to be 16 ins. Assuming or fixing the breadth at 7 ins.:—

$$\begin{aligned}\textbf{Depth} &= \frac{\text{Gallons}}{6\frac{1}{4} \times \text{B} \times \text{L}} = \frac{2\frac{1}{2}}{6\frac{1}{4} \times \frac{7}{12} \times \frac{16}{12}} \\ &= \frac{5}{2} \times \frac{4}{25} \times \frac{12}{7} \times \frac{12}{16} = \frac{18}{35} \text{ of a foot.} \\ &= \frac{18}{35} \times \frac{12}{1} = 6\tfrac{1}{5} \text{ ins. (nearly).}\end{aligned}$$

If the bottom of a tank be triangular in shape, the contents may be found by multiplying the area of the triangle by the height of the tank.

The same remark also applies to tanks having polygonal bases. The area of the base is obtained by one of the foregoing rules, and is multiplied by the vertical height.

EXAMPLES XXXIV

(i.) Find the volume in gallons of a tank having a rectangular base—depth = 1 ft. $7\frac{1}{2}$ ins.; length = 2 ft. 9 ins.; breadth = 2 ft. 4 ins.

(ii.) Find the volume in gallons of a tank having a triangular base—length of sides = 4 ft., 3 ft., and 2 ft. 6 ins.; depth = 3 ft. 3 ins.

(iii.) Find the depth of tank which will contain 1000 galls. of water, base of which forms a right-angled triangle, the base and vertical height of the triangle being 9 ft. and 12 ft. respectively.

(iv.) Find the volume in gallons and weight in tons of the water content of a tank —depth = 6 ft.; hexagonal base, length of one side = 10 ft.

To find the contents of a cylinder when the diameter and height are known.

Rule.—Multiply the area of one end by the vertical height of the cylinder.

Example.—Find the contents in gallons of a cylinder—height = 3 ft. 6 ins.; diameter = 1 ft. 6 ins.

$$\textbf{Contents in gallons} = \underset{\text{Area of end.}}{\pi R^2} \times \text{height} \times \underset{\text{Gallons in a cub. ft.}}{6\tfrac{1}{4}}$$

$$= \frac{22}{7} \times \frac{9}{12} \times \frac{9}{12} \times \frac{7}{2} \times \frac{25}{4} = 38{\cdot}67 \text{ galls.}$$

This rule also applies when finding the capacity of pipes of various sizes; pipes are simply cylindrically shaped bodies of varying lengths and diameters.

Thus, if we desire to find the contents in gallons, of 20 yds. of $\frac{1}{2}$-in. pipe (*Note.*—Keep all measurements in feet or fractions of feet):—

Contents in gallons

$$= \pi R^2 \times \text{Length} \times 6\tfrac{1}{4}$$

$$= \frac{22}{7} \times \frac{1}{48} \times \frac{1}{48} \times \frac{60}{1} \times \frac{25}{4}$$

$= {\cdot}51$ gall., or rather more than $\frac{1}{2}$ gall.

Diameter $= \frac{1}{2}''$
Radius $= \frac{1}{4}''$
Radius $= \frac{1}{4} \times \frac{1}{12} = \frac{1}{48}$ of a foot
20 yds. = 60 ft.

If the contents in gallons of a 4-in. diameter drain pipe 25 ft. 6 ins. long are required to be found, then:—

Contents in gallons

$$= \pi R^2 \times L \times 6\tfrac{1}{4} = \frac{22}{7} \times \frac{1}{6} \times \frac{1}{6} \times \frac{51}{2} \times \frac{25}{4} = 13{\cdot}91 \text{ galls.}$$

When the height and contents in gallons of a cylinder are given and the diameter is required:—

Rule.—Multiply by two the square root of the quotient obtained by dividing the gallons capacity by the product of the height, the number of gallons in a cubic foot, and π.

In explanation of the preceding statement, it is clear that it is necessary to reduce gallons to cubic feet by dividing by $6\frac{1}{4}$, and after dividing also by the height of the cylinder, the quotient is the area of one end of the cylinder, which, after dividing by π (3·1416 or $\frac{22}{7}$) and extracting the square root, gives the radius of the cylinder; multiplying this number by two gives the diameter.

This rule also applies when we wish to know the diameter of a pipe of a given length which may be filled with a given number of gallons of water.

Example.—Find the diameter of a cylinder which will hold 50 galls. of water, if the height of cylinder is 3 ft. 6 ins.

Diameter of cylinder

$$= 2\sqrt{\frac{\text{Gallons}}{6\frac{1}{4} \times \text{H} \times \pi}} = 2\sqrt{\frac{50}{\frac{25}{4} \times \frac{7}{2} \times \frac{22}{7}}}$$

$$= 2\sqrt{\frac{50}{1} \times \frac{4}{25} \times \frac{2}{7} \times \frac{7}{22}} = 2\sqrt{\cdot 72}$$

$$= 2 \times \cdot 852 = 1\cdot 7' \text{ or } 1'\ 8\tfrac{1}{2}'' \text{ diameter}$$

If the diameter and contents in gallons of a cylinder are given and the height has to be found:—

Rule.—Divide the number of gallons by the product of the area of one end of the cylinder and the number of gallons in a cubic foot (*i.e.*, $6\frac{1}{4}$).

Example.—(*a*) Find the height of a cylinder which will hold 35 galls. of water, the diameter of cylinder being 1 ft. 3 ins.

$$\textbf{Height} = \frac{\text{Gallons}}{6\frac{1}{4} \times \text{Area of cylinder end}} = \frac{35}{\frac{25}{4} \times \pi \text{R}^2}$$

$$= \frac{35}{\frac{25}{4} \times \frac{22}{7} \times \frac{5}{8} \times \frac{5}{8}}$$

$$= \frac{35}{1} \times \frac{4}{25} \times \frac{7}{22} \times \frac{8}{5} \times \frac{8}{5} = 4\cdot 56 \text{ ft. or 4 ft. } 6\tfrac{3}{4} \text{ ins. high.}$$

Example.—(*b*) Find the length of 4-in. and 6-in. pipe respectively, which may be filled by the discharge from a flush tank of 30 galls. capacity.

$$\textbf{Length of 4-in. pipe} = \frac{\text{Gallons}}{6\frac{1}{4} \times \text{Area of } 4'' \text{ pipe}} = \frac{\text{Gallons}}{6\frac{1}{4} \times \pi \text{R}^2}$$

$$= \frac{30}{\frac{25}{4} \times \frac{22}{7} \times \frac{1}{6} \times \frac{1}{6}} = \frac{30}{1} \times \frac{4}{25} \times \frac{7}{22} \times \frac{6}{1} \times \frac{6}{1}$$

$$= 55 \text{ feet nearly.}$$

$$\textbf{Length of 6-in. pipe} = \frac{\text{Gallons}}{6\frac{1}{4} \times \pi \text{R}^2} = \frac{30}{\frac{25}{4} \times \frac{22}{7} \times \frac{1}{4} \times \frac{1}{4}}$$

$$= \frac{30}{1} \times \frac{4}{25} \times \frac{7}{22} \times \frac{4}{1} + \frac{4}{1} = 24\cdot 4 \text{ feet.}$$

The latter amount could be obtained by multiplying the length of 4-in. pipe filled, by the ratio of the squares of the diameters of the 4-in. and 6-in. pipes.

If x = length of 6-in. pipe, then $4^2 : 6^2 :: x : 55$.

$$x = \frac{16 \times 55}{36} = 24{\cdot}4 \text{ feet of 6-in. pipe.}$$

To find the weight of a circular bar or rod of any substance, given the length, diameter, and specific gravity of the material :—

Rule.—Find the volume and multiply by the specific gravity, and by the weight of a cubic foot of water.

Example.—Find the weight of a cylindrical rod of copper 14 ft. long, $1\frac{1}{2}$ in. diameter. (Sp. gr. of copper, 8·79.)

Weight $= \pi R^2 \times L \times$ S.G. $\times$ Weight of cubic foot of water.

$$= \frac{22}{7} \times \frac{1}{16} \times \frac{1}{16} \times \frac{14}{1} \times \frac{8{\cdot}79}{1} \times \frac{125}{2} = 94{\cdot}4 \text{ lbs.}$$

EXAMPLES XXXV

(i.) Find the volume in gallons of a cylinder, diameter = 2 ft. 11 ins., height = 8 ft.
(ii.) Find the diameter of a cylinder to hold 75 galls., height = 4 ft. 8 ins.
(iii.) Find the height of cylinder which will hold 125 galls., diameter = 2 ft. 4 ins.
(iv.) Find the length of 1-in. water pipe that will hold 175 galls. of water.
(v.) Find the total contents in gallons of a 4-in. drain 28 ft. long, and a 6-in. drain 35 ft. long.
(vi.) Find the weight in pounds of a cylindrical rod of copper, diameter = $4\frac{1}{2}$ ins., length = 64 ft. (Sp. gr. of copper = 8·79.)

The volume or contents of a sphere may be obtained in the following manner :—

Rule.—Multiply the cube of the radius by π and by $\frac{4}{3}$.

Example.—(*a*) Find the volume (in cubic feet) of air contained in a sphere, 7 ft. diameter.

Contents in cubic feet

$$= \tfrac{4}{3}\pi R^3 = \frac{4}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2} = 179\tfrac{2}{3} \text{ cubic feet of air.}$$

(*b*) Find the weight of a solid copper sphere 1 ft. 2 ins. diameter. (Sp. gr. of copper = 8·79.)

Weight of copper sphere $= \frac{4}{3}\pi R^3 \times$ S.G. $\times$ Weight of cubic foot of water

$$= \frac{4}{3} \times \frac{22}{7} \times \frac{7}{12} \times \frac{7}{12} \times \frac{7}{12} \times \frac{8{\cdot}79}{1} \times \frac{125}{2} = 457{\cdot}2 \text{ lbs. (approx.).}$$

The volume of a segment of a sphere may be found by the following formula :—

Volume of segment $= \frac{1}{6}\pi H(H^2 + 3R^2)$ | R = radius of base of segment. H = height of ditto.

Example.—Find the number of cubic feet of water contained by a segment of a sphere the diameter of which is 8 ft. and vertical height 2 ft.

Contents in gallons $= \frac{1}{6}\pi H(H^2 + 3R^2)$

$$= \frac{1}{6} \times \frac{22}{7} \times \frac{2}{1}\left\{\left(\frac{2}{1} \times \frac{2}{1}\right) + \left(\frac{3}{1} \times \frac{4}{1} \times \frac{4}{1}\right)\right\}$$

$$= \frac{22}{21}(4 + 48) = \frac{22}{21} \times \frac{52}{1} = 54{\cdot}47 \text{ cubic feet.}$$

EXAMPLES XXXVI

(i.) Find the volume of a sphere, diameter 20 ins.
(ii.) Find the volume of a sphere, diameter 21 ft.
(iii.) Find the volume of a sphere, diameter 10·5 ins.
(iv.) Find the volume of a sphere, diameter 1·4 ins.

The volume of a cone or a pyramid may be obtained in the following manner :—

Rule.—Multiply the product of the area of the base and vertical height by $\frac{1}{3}$.

Example.—Find the number of gallons contained by a cone possessing a base 10 ft. 6 ins. diameter and a 12 ft. vertical height.

Contents of cone in gallons $= \frac{1}{3}\pi R^2 H \times 6\frac{1}{4}$

$$= \frac{1}{3} \times \frac{22}{7} \times \frac{21}{4} \times \frac{21}{4} \times \frac{12}{1} \times \frac{25}{4} = 2165{\cdot}6 \text{ galls.}$$

If the slant height is given along with the diameter of the base, it will be necessary to find the vertical height, which may be obtained by first considering the isosceles triangle formed by a vertical section through the apex of the cone, and dropping a perpendicular line from the apex to the base, which divides it into two right-angled triangles, and then applying the rule previously given, in which

C = Hypotenuse or slant height.
B = Vertical height.
A = Base (*i.e.*, $\frac{1}{2}$ diameter of base of cone).

$$B = \sqrt{C^2 - A^2}.$$

The volume of a pyramid being worked out on the same principle as the cone, no special example is considered necessary.

EXAMPLES XXXVII

(i.) Find the volume in gallons of a right circular cone, diameter of base = 28 ft., vertical height = 10 ft.

(ii.) Find the volume in gallons of a right circular cone, diameter of base = 10 ft., vertical height = 35 ft.
(iii.) Find the volume in cubic inches of a pyramid, length of side = 4 ft., vertical height = 3 ft.
(iv.) Find the volume in cubic feet of a pyramid having a hexagonal base, length of side = 2 ft., vertical height = 4 ft.

To obtain the **volume or contents of the frustum of a cone or pyramid**:—

Rule.—To the square root of the product of the area of the top and the area of the base of the frustum, add the areas of the base and top, and multiply by one-third vertical height.

Formula $= \frac{1}{3} H(T + B + \sqrt{T \times B})$ | T = Area of top. B = Area of base. H = Vertical height.

Example.—(*a*) Find the number of cubic feet of air contained in a room, the shape forming the **frustum** of a pyramid, on a rectangular base, vertical height 12 ft.

Length of two sides of **base** = 30 ft. and 20 ft. respectively.
" " " **top** 18 ft. and 12 ft. "

Area of base $= 30 \times 20 = 600$
Area of top $= 18 \times 12 = 216$

Contents in cubic feet $= \frac{1}{3}H(T + B + \sqrt{T \times B})$

$$= \frac{1}{3} \times \frac{12}{1}(\overset{\text{Area of top.}}{216} + \overset{\text{Area of base.}}{600} + \sqrt{216 \times 600})$$

$$= \frac{4}{1}(816 + \sqrt{129600})$$

$$= \frac{4}{1}(816 + 360) = 4704 \text{ cubic feet of air.}$$

Example.—(*b*) Find the solid contents in cubic feet of the frustum of a cone.

Vertical height $= 7'\ 0''$
Diameter of base $= 3'\ 6''$
Diameter of top $= 1'\ 9''$

Contents in cubic feet $= \frac{1}{3}H(T + B + \sqrt{T \times B})$

$$= \frac{1}{3} \times \frac{7}{1}\left\{\overset{\text{Area of top.}}{\left(\frac{22}{7} \times \frac{7}{8} \times \frac{7}{8}\right)} + \overset{\text{Area of base.}}{\left(\frac{22}{7} \times \frac{7}{4} \times \frac{7}{4}\right)}\right\} + \sqrt{\left(\frac{22}{7} \times \frac{7}{8} \times \frac{7}{8}\right) \times \left(\frac{22}{7} \times \frac{7}{4} \times \frac{7}{4}\right)}$$

$$= \frac{1}{3} \times \frac{7}{1}\left(\frac{77}{32} + \frac{77}{8} + \sqrt{\frac{77}{32} \times \frac{77}{8}}\right)$$

$$= \frac{7}{3}\left(\frac{385}{32} + \frac{77}{16}\right) = \frac{7}{3} \times \frac{539}{32} = 39{\cdot}3 \text{ cub. ft.}$$

EXAMPLES XXXVIII

(i.) Find the volume of the frustum of a cone, diameter of top $= 10 \cdot 5$, diameter of base $= 14$, vertical height $= 21$.

(ii.) Find the volume of the frustum of a cone, diameter of top $= 2 \cdot 8$, diameter of base $= 3 \cdot 5$, vertical height $= 15$.

(iii.) Find the volume of the frustum of a pyramid square base, length of side of top $= 10$, length of side of base $= 16$, vertical height $= 18$.

(iv.) Find the volume of the frustum of a pyramid hexagonal base, length of side of top $= 10$, length of side of base $= 20$, vertical height $= 36$.

CHAPTER X

CALCULATIONS ON THE HEATING CAPACITIES OF PIPES AND BOILERS, DISCHARGE FROM PUMPS, VENTILATION CALCULATIONS, FLOW OF AIR THROUGH TUBES, ETC.

By a careful study of the examples and methods of working same in the previous chapters, the student should be able to undertake any kind of calculation which may occur in ordinary practice or theory. In this chapter, calculations of various kinds having reference to **Heating, Ventilation, Discharges from Pumps**, etc., will be dealt with.

HEATING

The relative areas of boiler-heating surface and radiating surface, and also of radiating surface and cubic capacity, must be understood before any attempt be made to successfully carry out a system of heating by hot water or steam.

No rule which will rigidly apply in all cases equally has yet been devised, owing to the variety of conditions which obtain in the different systems of heating, and the method of applying same, and also by the influence of local conditions.

In all estimates in connection with heating surfaces, it is advisable to base the calculations upon elementary principles.

The amount of heat that is necessary to maintain the temperature of a room at a given degree, will be exactly the amount that is lost through various channels, as:—

By conduction through walls, floors, ceilings, and windows; also the heat used in warming the incoming air, which is ultimately lost on the escape of the vitiated air from the room.

Numerous experiments have been made in connection with the conductivity of walls, floors, and ceilings of various thicknesses, also of glass surfaces; and coefficients or constants have been devised which state the amount of heat in British thermal units lost by a unit area in a given time under given thermometric conditions.

The units adopted are:—

1 sq. ft., 1 hour, British thermal unit, 1° Fahr. difference between the temperature of the air in the room and that of the outside air.

The following is a table of coefficients :—

Substance.	British Thermal Units, conducted per sq. ft. per hour per degree difference in temp. Fahr.
Brick wall, 9 ins. thickness	·35
Brick wall, 14 ins. thickness	·27
Brick wall, 18 ins. thickness	·23
Brick wall, 22 ins. thickness	·20
Single-glazed windows	1·10
Double-glazed windows	·48
Ceilings (near roof)	·32
Ceilings (good air space between ceiling and roof) .	·13
Floors, 12 ins. concrete and wood	·52
Floors, with wood joists	·17

Note.—1 cub. ft. of air requires ·019 British thermal units to raise its temperature through 1° Fahr.

1 sq. ft. of radiator or pipe surface emits 1·8 British thermal units per hour per square foot, for each degree Fahrenheit difference between the temperature of the water in the pipes and that of the air surrounding the pipes.

1 sq. ft. of boiler surface in radiation contact with the fire will conduct from 2500 to 7000 British thermal units per hour.

Note.—The great variation here recorded is due to difference in boiler designs, the type of fuel and water used, and the height of the chimney to which the boiler is connected.

In all calculations it is advisable to assume a maximum not greater than 5000 British thermal units per square foot per hour, and in some cases it may be necessary to take a smaller quantity.

Example.—Find the radiating surface and boiler area necessary for a single-storied room of the following dimensions and particulars :—

Capacity = 5600 cub. ft. (air to be changed three times per hour).

Area of glass = 65 sq. ft.

Area of 9-in. brick wall = 560 sq. ft.

Area of ceiling (large air space above) = 100 sq. ft.

Area of floor (wood) = 100 sq. ft.

Temperature of air in room to be maintained at 60° Fahr.

In these calculations, the temperature of the outside air is taken as 32° Fahr.

First.—Find the total British thermal units required under given conditions.

Second.—Find the radiating surface required.

Third.—Find the heating surface of the boiler.

It being assumed that the temperature of water in the pipes will average 170° Fahr.

First.—Total British thermal units

	(Three Changes per hour × Difference between Internal and External Air).		British Thermal Units per hour.
Air	$=(5600 \times \cdot 019 \times 3 \times (60-30) = 16800 \times \cdot 019 \times 30$	=	9,576·0
Glass	$=65 \times 1\cdot 1 \times (60-30) = 65 \times 1\cdot 1 \times 30$	=	2,145·0
Wall	$=560 \times \cdot 35 \times (60-30) = 560 \times \cdot 35 \times 30$	=	5,880·0
Ceiling	$=100 \times \cdot 13 \times (60-30) = 100 \times \cdot 13 \times 30$	=	390·0
Floor	$=100 \times \cdot 17 \times (60-30) = 100 \times \cdot 17 \times 30$	=	510·0
	British thermal units required per hour	=	18,501·0

Second.—Mean temperature of water in pipes = 170° F.
Mean temperature of air in room = 60° F.
Difference = 110° F.

If 1 sq. ft. of radiating surface emits 1·8 British thermal units per hour per degree difference between temperature of water and temperature of air, then, with 110° difference, 1 sq. ft. will emit $1\cdot 8 \times 110 = 198\cdot 0$.

$$\text{Therefore, total radiation surface} = \frac{18501}{198} = \underline{93\cdot 3}$$

Third.—Boiler surface, assuming that 5000 British thermal units are absorbed per square foot per hour

$$= \frac{18501}{5000} = \underline{3\cdot 7} \text{ sq. ft.}$$

The average value of the flue area of a boiler varies from $\frac{1}{4}$ to $\frac{1}{12}$ that of the direct heating surface. The foregoing particulars apply to low-pressure hot-water systems only.

The best results are obtained by using a boiler the capacity of which is from 25 per cent. to 50 per cent. greater than the theoretical requirements.

DISCHARGE FROM PUMPS, FORCE REQUIRED FOR WORKING PUMPS

When the discharge in gallons in a given time from a pump (the diameter of the barrel and number of strokes per minute of which are stated) is required.

Rule.—Multiply the area of the cross-section of the barrel in feet by the length of stroke in feet, to give the quantity discharged at each stroke; this multiplied by the number of strokes per hour and $6\frac{1}{4}$ will give the gallons delivered (theoretically) per hour. It is usual to make a deduction of from 10 to 20 per cent. for loss due to friction and leaking valves, etc.

Example.—What is the discharge in gallons per hour from a pump 4 in. diameter, length of stroke 9 in., working 40 strokes per minute, deducting 10 per cent. for loss?

Discharge in gallons

$$= \underbrace{\frac{22}{7} \times \frac{1}{6} \times \frac{1}{6}}_{\text{Area in feet of barrel.}} \times \underbrace{\frac{40}{1} \times \frac{60}{1}}_{\text{Number of strokes per hour.}} \times \underbrace{\frac{25}{4}}_{\text{Galls. in 1 cubic foot.}} \times \underbrace{\frac{9}{10}}_{\text{10\% deduction.}} \times \underbrace{\frac{9}{12}}_{\text{Length of stroke.}}$$

$= 884$ (nearly) galls. per hour.

If the pump is a double-acting one, water is forced from the barrel through the rising main, during both the up-stroke and down-stroke, and the number of strokes must be multiplied by 2 when working out the theoretical delivery.

A much shorter rule is often used for these calculations, which is stated below.

Gallons per stroke = Diameter of pump barrel in inches, squared, multiplied by ·034, and by length of stroke in feet.

Example.—Find the discharge in gallons per hour from a 4-in. diameter pump working 40 strokes per minute, length of stroke 9 ins.

Gallons per hour $= D^2 \times \cdot 034 \times L \times 60 \times \frac{9}{10} \times \frac{40}{1}$

$$= \frac{4}{1} \times \frac{4}{1} \times \frac{\cdot 034}{1} \times \frac{3}{4} \times \frac{60}{1} \times \frac{9}{10} \times \frac{40}{1} = 881 \cdot 28.$$

Note.—This answer practically corresponds with the one above.

This rule may also be used for ascertaining the contents in gallons of circular pipes and wells, by multiplying the square of the diameter in inches by the length of the pipe or depth of the well in feet, and by ·034.

To find the total resistance on the up-stroke of an ordinary lift and force pump (neglecting friction and weight of rods).

Rule.—Multiply the area of the bucket in square inches by the pressure per square inch due to the head of water in the delivery pipe and suction pipe above water-level in well.

Example.—Find the total resistance or weight on the bucket of a $3\frac{1}{2}$-in. diameter pump during the up-stroke, neglecting friction, if the vertical height of delivery pipe above mean water-level in well is 120 ft.

Total weight $= \pi R^2 \times$ Pressure per square inch.

$$= \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} \times \frac{120}{1} \times \frac{\cdot 434}{1} = 501 \cdot 27 \text{ lbs.}$$

Assuming that an ordinary lift and force pump worked by a lever is used for the above, what will be the relative lengths of the portions of the lever (1) from the fulcrum to power, and (2) from fulcrum to weight (*i.e.* pump rod), if the available power is only 50 lbs.? Taking the weight on the bucket as 500 lbs., then:—

$$500 : 50 :: L_1 : L_2 \qquad L_1 = \text{length from P to F.}$$
$$= 10. \qquad L_2 = \text{,, F to W.}$$

The portion between power and fulcrum must be ten times as long as that between fulcrum and weight, neglecting friction. (See Elementary Mechanics section.)

What **Horse-power** will be required to raise, by means of a pump working ten hours per day, 20,000 galls. of water to a storage tank 240 ft. above the water-level in the well?

$$\text{H.P.} = \frac{\text{Foot-lbs. per minute}}{33{,}000} = \frac{\overset{\text{Galls.}}{20{,}000} \times \overset{\text{Weight of 1 gall.}}{10} \times \overset{\text{Height in feet of lift.}}{240}}{33{,}000 \times 10 \times 60} = 2{\cdot}5 \text{ (nearly).}$$

Add to this $\frac{1}{3}$ for friction, then

$$\text{Total horse-power} = \frac{2{\cdot}5}{1} \times \frac{4}{3} = 3\tfrac{1}{3} \text{ horse-power.}$$

VENTILATION: CALCULATIONS

Cubic space.—In **Fig. 20** are three examples of rooms of various shapes, the capacities of which may be required to be found.

A.—In this case, the total area of the floor, multiplied by the height to the eaves, *i.e.* 12 ft., added to the volume of the pryamid formed, as shown in sections by the intersection of the four hips, will give the cubic contents.

Contents of H = Area of floor × 12 ft.

Contents of G = $\frac{1}{3}$ H × Area of base of pyramid.

Area of floor	$= (30 \times 20) + (8 \times 8) =$		644 sq. ft.
Contents of H	$= 664 \times 12$	$=$	7968 cub. ft.
Contents of G	$= \frac{1}{3} \times \frac{6}{1} \times \frac{600}{1}$	$=$	1200
	Total contents	$=$	9168 cub. ft.

B.—This example will be easily worked by finding the total area of the floor and multiplying by the height, 12 ft. 6 ins., and adding the volume of the hemispherical dome.

Contents of K = Area of floor × 12·5

Contents of L $= \dfrac{\frac{4}{3}\pi R^3}{2}$

$$\text{Area of floor} = (18 \times 22) + \left(\frac{1}{2} \times \frac{22}{7} \times \frac{7}{1} \times \frac{7}{1}\right) = \underline{473}$$

$$\text{Contents of K} = 473 \times 12{\cdot}5 = 5912{\cdot}5 \text{ cub. ft.}$$

$$\text{Contents of L} = \frac{4}{3} \times \frac{22}{7} \times \frac{7}{1} \times \frac{7}{1} \times \frac{7}{1} \times \frac{1}{2} = 718{\cdot}6$$

$$\text{Total content} = 6631{\cdot}1 \text{ cub. ft.}$$

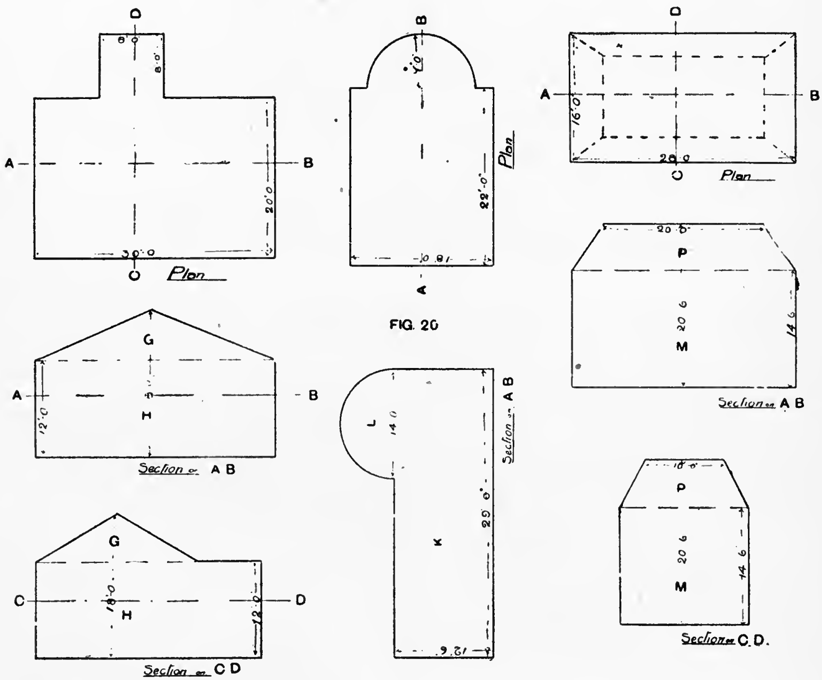


FIG. 20

C.—The cubical contents of M, added to the volume of the frustum of a pryamid formed by the roof portion P, will give the desired answer.

$$\text{Contents of M} = 28 \times 16 \times 14{\cdot}5 = 6496 \text{ cub. ft.}$$

$$\text{Contents of K} = \tfrac{1}{3}H(T + B + \sqrt{T \times B})$$

$$= \tfrac{1}{3} \times \tfrac{6}{1}(200 + 448 + \sqrt{200 \times 448}) = 1894$$

$\left(\begin{array}{l} T = \text{Area of top.} \\ B = \text{Area of base.} \end{array}\right)$

$$\text{Total content} = \underline{8390} \text{ cub. ft.}$$

The composition of the atmosphere varies in different parts of the country, the average being taken as :—

Oxygen . .	209·6
Nitrogen . .	790·0
Carbon dioxide .	·4
in parts per .	1000

An average individual gives off when at rest, ·6 cub. ft. of CO_2 per hour. To maintain the air of rooms or apartments in a healthy state, the quantity of CO_2 should not exceed ·6 cub. ft. per 1000 cub. ft. of air (or ·6 part per 1000). The amount of CO_2 is fixed by some authorities at a limit of ·72, ·8, or even 1 part per 1000.

The quantity of CO_2 in the atmosphere is taken as a gauge or standard of its purity.

Example.—(*a*) What will be the standard of impurity of the air in a room containing 8000 cub. ft. occupied by 24 persons for one hour, at the end of that period?

One person gives off ·6 cub. ft. CO_2 per hour.
$\therefore$ 24 persons give off $\cdot 6 \times 24 = 14{\cdot}4$ cub. ft. CO_2 per hour.
Atmosphere contains ·4 cub. ft. of CO_2 per 1000 cub. ft.
$\therefore$ 8000 cub. ft. contain $\cdot 4 \times 8 = 3{\cdot}2$ cub. ft. CO_2
Impurity present and additional impurity $= 3{\cdot}2 + 14{\cdot}4$
Total impurity $= 17{\cdot}6$ cub. ft. CO_2

$$\therefore \quad \frac{17{\cdot}6}{8} = 2{\cdot}2 \text{ parts of } CO_2 \text{ per } 1000.$$

(*b*) How many times per hour will the air of a room containing 20,000 cub. ft., occupied by 30 persons, require changing, to keep the CO_2 at ·8 part per 1000?

1st.—Amount of CO_2 added per hour $= 30 \times \cdot 6 = 18{\cdot}0$ cub. ft.

Standard . . .	= ·8 cub. ft. of CO_2 per 1000 cub. ft.				
Atmospheric air contains .	= ·4	”	”	”	”
Amount permitted to be added	= ·4	”	”	”	”

$$\text{Amount of air required per hour} = \frac{\text{Added impurity} \times 1000}{\text{Amount permitted per 1000 vols.}}$$

$$= \frac{18 \times 1000}{\cdot 4} = 45{,}000 \text{ cub. ft. per hour.}$$

2nd.—Number of changes per hour $= \dfrac{45{,}000}{20{,}000} = 2\frac{1}{4}$.

(*c*) How much air per head per hour must be supplied to persons occupying a room, so as to keep the impurity at ·72 part of CO_2 per 1000 parts of air?

Added impurity . .	$=\cdot 6$ cub. ft. CO_2 per head per hour.
CO_2 standard . . .	$=\cdot 72$ part per 1000
Present in atmosphere .	$=\cdot 40$,, ,, ,,
∴ quantity to be added .	$=\cdot 32$,, ,, ,,

Amount per head per hour $= \dfrac{\cdot 6 \times 1000}{\cdot 32} = 1875$ cub. ft.

Allowance has also to be made for the products of combustion where oil or gas are used for illumination. Each cubic foot of ordinary coal-gas produces nearly 2 cub. ft. of CO_2, so that a burner consuming 3 ft. per hour, will add, say, 5 cub. ft. of CO_2 to the atmosphere per hour.

It should be distinctly understood, however, that the CO_2—that is the result of the combustion of coal-gas—is not measured by the same standard that obtains in the case of respiratory CO_2, owing to the organic matter which accompanies the latter CO_2 from the lungs.

This organic matter is deleterious to health, and is much more objectionable than the CO_2.

The latter is convenient to use as a standard for estimating the impurity present in the air.

FLOW OF AIR THROUGH TUBES

When the velocity of the air in feet per second or minute is given, the discharge per hour through any given inlets is ascertained by multiplying the area of the openings in square feet by the velocity in feet of the air passing through them.

Example.—How much air will be discharged per hour through two inlets 1 ft. 9 ins. diameter? Velocity of air entering the room $= 160$ ft. per minute.

Quantity of air per hour $= 2\pi R^2 \times 60 \times 160$

$$= \underbrace{\left(\frac{2}{1} \times \frac{22}{7} \times \frac{7}{8} \times \frac{7}{8}\right)}_{\text{Area of inlets.}} \times \underbrace{\left(\frac{60}{1}\right)}_{\text{Minutes in one hour.}} \times \underbrace{\left(\frac{160}{1}\right)}_{\text{Vel. in ft. per min.}} = 46{,}200 \text{ cub. ft. per hour.}$$

When it is necessary to find the **velocity**, an appliance or instrument known as an **Anemometer**, or air meter, may be used, which records automatically the velocity of the air flowing through any inlet or outlet to which it is applied. Six readings are usually taken, each of one minute's duration, and the mean of these represents the velocity per minute.

In systems of natural ventilation where the velocity of the air through the tubes is due principally to the difference in temperature of the air in the room and the outside air, the rate of flow may be found by the following formula, which is based upon the law of falling bodies (known as Montgolfier's Law).

In the Elementary Mechanics section, the velocity of bodies falling from a position of rest without the application of any extraneous force, is found by

$V = \sqrt{2gH}$ in which V = Velocity in feet per second.
g = Acceleration of gravity = 32·2.
H = Distance through which the body has fallen.

Since $\sqrt{2g} = \sqrt{64 \cdot 4} = 8$ approx.

Then $V = 8\sqrt{H}$

But in the case of two bodies of air, the difference in weight due to difference in temperature, causing displacement of the warm by the colder air, has to be taken into account. The formula then becomes :—

$V = 8\sqrt{THC}$ in which V = Velocity of air in feet per second.
H = Height of outlet above inlet.
C = Coefficient of expansion of air, for 1° F. = ·002039 and for 1° C. = ·003665.
T = Difference in temperature of internal and external air.

Example.—Find the velocity in feet per second, through a vertical 16-ft. tube, connected with a room for ventilation purposes, the difference in temperature of the air in the room and the external air = 24° C.

$$V = 8\sqrt{THC} = 8\sqrt{24 \times 16 \times \cdot 003665}$$
$$= 8\sqrt{1 \cdot 40736} = 8 \times 1 \cdot 186 = 9 \cdot 48 \text{ feet per second.}$$

In actual practice, allowance must be made for friction, the amount of which varies according to the size, shape, roughness of the interior surface, and the number and type of bends.

The resistance varies (*a*) directly as the length of the conduit, (*b*) directly as the square of the velocity, (*c*) inversely as the diameter.

One-quarter to three-quarters of the above velocity would be taken, according to local conditions.

Table of cubic space allotted per head. (By various authorities.)

Common Lodging-houses	250 to 300 cub. ft.
Education Act, for Schools :—	
Provinces (8 sq. ft. floor space × 10 ft. height) =	80
In London ,, ,, . =	130
Fever Hospitals . . . . . =	3000
Accident Hospitals . . . . . =	800 to 2000

ANSWERS TO EXAMPLES

EXAMPLES I

(i.) $62\frac{91}{111}$. (ii.) $104\frac{135}{136}$. (iii.) $192\frac{339}{512}$. (iv.) $78\frac{7}{11}$. (v.) $19\frac{1457}{5000}$.

EXAMPLES II

(i.) $2\frac{7}{66}$. (ii.) $1\frac{493}{630}$. (iii.) $1\frac{73}{90}$. (iv.) $2\frac{41}{144}$. (v.) 1.

EXAMPLES III

(i.) $\frac{7}{24}$. (ii.) $\frac{747}{1880}$. (iii.) $\frac{11}{28}$. (iv.) $\frac{1019}{1020}$. (v.) $\frac{149}{690}$.

EXAMPLES IV

(i.) $\frac{22}{45}$. (ii.) $7\frac{1}{2}$. (iii.) $\frac{64}{375}$. (iv.) 30. (v.) $8\frac{3}{4}$.

EXAMPLES V

(i.) $\frac{4}{5}$. (ii.) $1\frac{3}{5}$. (iii.) $\frac{35}{64}$. (iv.) $\frac{4}{327}$. (v.) $3\frac{3}{10}$.

EXAMPLES VI

(i.) $\frac{6}{49}$. (ii.) $\frac{5}{13}$. (iii.) $\frac{21}{172}$. (iv.) $91\frac{3}{5}$. (v.) $\frac{192}{259}$.

EXAMPLES VII

(i.) $\frac{8}{10}$ or $\frac{4}{5}$. (ii.) $\frac{185}{1000}$ or $\frac{37}{200}$. (iii.) $\frac{18}{1000}$ or $\frac{9}{500}$.
(iv.) $17\frac{6}{1000}$ or $17\frac{3}{500}$. (v.) $100\frac{1}{1000}$.

EXAMPLES VIII

(i.) ·1. (ii.) 10·03. (iii.) 19·001. (iv.) 19·011. (v.) 16·0013.

EXAMPLES IX

(i.) 30. (ii.) 3. (iii.) ·0037777. (iv.) ·0405375. (v.) ·005666756.

EXAMPLES X

(i.) 43·08. (ii.) 10·00. (iii.) ·00008. (iv.) 50·00. (v.) 28122800.

EXAMPLES XI

(i.) $\frac{6}{11}$. (iii.) $9\frac{56}{111}$. (v.) $5\frac{807}{1111}$. (vii.) $\frac{5}{148}$. (ix.) $\frac{177}{808}$.
(ii.) $4\frac{323}{333}$. (iv.) $27\frac{2542}{3333}$. (vi.) $\frac{3937}{9900}$. (viii.) $\frac{56}{185}$. (x.) $3\frac{85}{148}$.

EXAMPLES XII

(i.) 896·614. (ii.) 62778·27928. (iii.) 37·9651. (iv.) 80·901. (v.) 8078·113601.

EXAMPLES XIII

(i.) 40·99935. (ii.) ·2477833. (iii.) 6339·54. (iv.) ·804824. (v.) ·0009.

EXAMPLES XIV

(i.) 17·25. (ii.) 185·77. (iii.) 17·245. (iv.) 212·3. (v.) 24·57.

EXAMPLES XV

(i.) 42. (ii.) 156. (iii.) 93. (iv.) 276. (v.) 225.

EXAMPLES XVI

(i.) 12. (ii.) 16. (iii.) 212. (iv.) 124. (v.) 203.

EXAMPLES XVII

(i.) 126. (ii.) 66. (iii.) 464. (iv.) 72.

EXAMPLES XVIII

(i.) £1, 0s. $0\frac{1}{2}$d. (ii.) £7, 5s. 10d. (iii.) £525. (iv.) 78 days.

EXAMPLES XIX

(i.) 12 persons. (ii.) $10\frac{1}{2}$ hrs. per day. (iii.) £44, 7s. 6d. (iv.) £545, 6s. 3d.

EXAMPLES XX

(i.) $15\frac{5}{13}$%. (ii.) $16\frac{2}{3}$%. (iii.) 72·1875 lbs. (iv.) 120,000 galls.

EXAMPLES XXI

(i.) 9′-5-1-6. (ii.) 266′-6-8-5-4. (iii.) 38′-8-5-9. (iv.) 313′-1-10-11-3.

EXAMPLES XXII

(i.) 8 sq. yds. 8 sq. ft.
(ii.) 10 sq. yds. 1 sq. ft.
(iii.) 19 sq. yds. 141 sq. ins.
(iv.) 58 sq. yds. 3 sq. ft.
(v.) 6 ft. 8 ins.
(vi.) 11 ft. 11 ins.
(vii.) 22 yds. 7 ins.

EXAMPLES XXIII

(i.) 200. (ii.) 662·48. (iii.) 31312·53. (iv.) 22·612. (v.) 20150·28.

EXAMPLES XXIV

(i.) 24 sq. ft.
(ii.) 493·5 sq. ft.
(iii.) 192·56 sq. ft.
(iv.) 4932 sq. yds.
(v.) 6 sq. ft.
(vi.) 212·26 sq. ft.
(vii.) 864 sq. ft.
(viii.) 166 sq. ft.

EXAMPLES XXV

(i.) 21·4. (ii.) 24. (iii.) 28·7. (iv.) 57·8. (v.) 26·8.

EXAMPLES XXVI

(i.) Area = 82·51 sq. ins.
(ii.) Area = 106·1 sq. ins.
(iii.) Area = 330 sq. ins.
(iv.) Area = 700·9 sq. ins.
(v.) Area = 8·946 sq. ins.
(vi.) Area = 176·7 sq. ft.
(vii.) Area = 706·86 sq. ft.
(viii.) Area = 2922·4 sq. ft.
(ix.) Area = 3848·46 sq. ft.
(x.) Area = 7854 sq. ft.
(xi.) Area = 198·556 sq. ins.
(xii.) Area = 430·053 sq. ins.
(xiii.) Area = 945·692 sq. ft.
(xiv.) Area = 33·1831 sq. ft.
(xv.) Area = 254·469 sq. ft.
(xvi.) Diameter = 5·9 ft.
(xvii.) Diameter = 13·4 ft.
(xviii.) Diameter = ·43 ft.
(xix.) Diameter = ·75 ft.

EXAMPLES XXVII

(i.) $47\frac{1}{18}$ sq. ft. (ii.) $24\frac{1}{16}$ sq. ft. (iii.) $155\frac{4}{7}$ sq. ft. (iv.) 72·63 sq. ft.

EXAMPLES XXVIII

(i.) 75·39. (ii.) ·8246. (iii.) ·1583. (iv.) 516·6.

EXAMPLES XXIX

(i.) 1272·64. (ii.) 1091·85. (iii.) 1·139. (iv.) 64·87. (v.) 848·26.

EXAMPLES XXX

(i.) 4828 sq. ins. (ii.) 2068 sq. ins. (iii.) 15092 sq. ins. (iv.) 8613

EXAMPLES XXXI

(i.) 1256·6 sq. ins. (ii.) ·1386 sq. ins. (iii.) 1386·00 sq. ins. (iv.) 246·4 sq. ins.

EXAMPLES XXXII

(i.) $246\frac{1}{3}$ sq. ft. (ii.) $360\frac{1}{4}$ sq. ft. (iii.) $1083\frac{39}{56}$. (iv.) $1464\frac{4}{7}$ sq. ins.

EXAMPLES XXXIII

(i.) 859·8. (ii.) 2122·36. (iii.) 888·39. (iv.) 372. (v.) 679·8

EXAMPLES XXXIV

(i.) 65·16 galls. (ii.) 76·0 galls. (iii.) 2·96 ft. (iv.) 9742·8 galls.; 43·49 tons.

EXAMPLES XXXV

(i.) 334·2 galls.
(ii.) 1 ft. $9\frac{3}{4}$ ins. (approx.).
(iii.) 4 ft. $8\frac{1}{10}$ ins.
(iv.) 5131·6 ft.
(v.) 58 galls.
(vi.) 388·48 lbs.

EXAMPLES XXXVI

(i.) 4180 cub. ins. (ii.) 4851 cub. ft. (iii.) 606·37 cub. ins. (iv.) 1·437 cub. ins.

EXAMPLES XXXVII

(i.) $12833\frac{1}{3}$ galls. (ii.) 5729 galls. (iii.) 16 cub. ins. (iv.) 13·856 cub. ft.

EXAMPLES XXXVIII

(i.) 2491·895. (ii.) 117·425. (iii.) 3096. (iv.) 21822·12.

PHYSICAL PROPERTIES AND STATE OF MATTER

CHAPTER XI

PROPERTIES OF MATTER, SOLIDS, LIQUIDS, AND GASES

MATTER

We become aware of the existence of the things which surround us by means of one or more of our senses. In taking a country walk we **feel** the hard ground under our feet, we **see** the trees and the distant hills, we **hear** the rippling brook, we **smell** the aroma of some flowers along the path; we have become aware of all these things by the aid of one or more of our senses: hence they are called material things, or forms of matter. We are now prepared to say that **matter is the name given to all things which we can become aware of by the aid of one or more of our senses.**

PROPERTIES OF MATTER

When we grip a stone, we find that it is hard, or we say that it possesses the property of hardness. Hence, properties are certain effects on the senses, caused by the things which possess them.

The general properties of matter are those properties which are common to all kinds of matter, and are obvious.

(1) **All matter must occupy a certain space.** Thus, a small body occupies a small space, and a large body occupies a large space; hence the space occupied by a body is equal to the size of the body.

(2) **Matter has weight.** We become aware of this property in solids and liquids when we try to lift them. The following experiments will show that gases also have weight.

Experiment I.—Take a large spherical globe, fitted at the neck with a stopcock. By means of an air-pump exhaust the air out of it, and close the stopcock. Place the globe on one pan of the balance, and counterpoise with weights. Open the stopcock. As the air rushes in the pan descends, showing an increase in weight. This increase is due to the weight of the air.

Experiment II.—Take a round-bottomed flask, fitted with a rubber stopper, through which passes a piece of glass tube. On the end fix a piece of rubber tubing fitted with a pinchcock. Put in the flask some water, and boil it for a few minutes so as to drive out all the air (Fig. 21). Close the pinchcock, cool it, and then weigh. Afterwards open the pinchcock. Air rushes in, and on weighing again there is an increase in weight. By this method **the weight of 1 cub. ft. of air at ordinary temperature has been found to be 554 grains.**

(3) Two particles of matter cannot occupy the same space.

(4) **Matter offers resistance** when external forces are applied. This is true for all forms of matter, whether solids, liquids, or gases. In the case of air, this resistance is noticed when the hand is put out of the window of a railway carriage which is travelling at a high speed, by the impact of the particles of air against the hand

(5) **Matter is porous**. We are in the habit of saying that bread and a sponge are porous because they contain small holes or cavities called pores. All bodies possess these small holes, but the latter vary in sizes; hence all bodies are porous, but the degree of porosity varies.

In most solids the pores are so small that they cannot be seen by the naked eye.

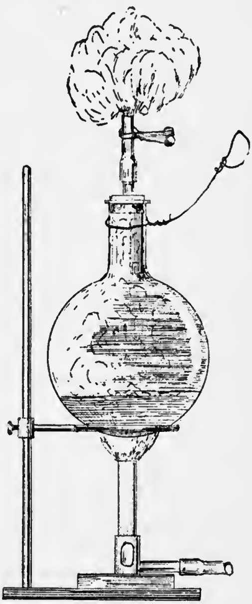


FIG. 21

In a mountainous district water may be seen percolating through the pores of solid rock, thus showing that the rock must be porous.

Experiment III.—Fill one tumbler with water, and one of equal size with sawdust. It is possible to pour practically the whole of the water into the tumbler of sawdust.

Experiment IV.—Take a test tube and half-fill it with water. Carefully pour a small quantity of alcohol over the water, and mark its level. Then shake, and notice that the level is lowered, showing that on mixing alcohol and water there is a shrinking or contraction in volume.

It is possible to reduce the volume of air by exerting pressure on it, and thus the volume of a gas can be reduced to one-half, or quarter, or even less, of its original volume by compression, and finally reduced to a liquid in most cases. The decrease in volume is due to the pores or spaces between the particles of air becoming smaller and smaller.

(6) **Matter is compressible.** In the case of gases it was shown

that compressibility is the natural consequence of porosity. During compression it is the pores between the particles of matter that become smaller and smaller.

This is equally true in the case of solids and liquids, but in a smaller degree. Generally, the pores are very small in liquids and solids, and great pressure is necessary to compress them.

(7) **Matter is elastic.** After the force of compression is released, the pores tend to regain their former size, hence the body returns to its original size. This tendency for a body to return to its former shape when released from compression is called elasticity.

It is easily seen in the case of a gas. If one end of a bicycle pump be closed, and the air inside be compressed, then when the handle is released, the piston is forced outwards to its original place.

STATES OF MATTER. SOLIDS, LIQUIDS, GASES

It is a well-known fact that all material things may be divided into three classes :—Solids, liquids, and gases.

A solid is that form of matter which has a definite shape and a definite size, unless subjected to a considerable force.

A liquid is that form of matter which has a definite volume, but no definite shape. It takes the shape of the vessel into which it is placed.

A gas is that form of matter which has neither definite shape nor definite volume. It will expand indefinitely, and it always fills the vessel containing it. Liquids and gases are included in the term **fluids**.

CHANGE OF STATE

Although solids, liquids, and gases differ in their physical properties, it is possible for the same matter to exist in the three different states.

Experiment V.—Take a piece of ice, notice its definite size and shape. Place it in a beaker, and heat it. It melts, and becomes a liquid—water. Boil the water, and notice that it is changed into steam, which disappears into the room. Thus, by heating, ice has been changed into water, and water into steam.

Lead, which at ordinary atmospheric temperatures is a solid, may be converted into a liquid by the application of sufficient heat, and finally, when at white heat, it vaporises.

In the same way, if all solids could be heated sufficiently, they would be changed into liquids, and similarly, the liquids would be changed into gases.

Experiment VI.—Boil some water, and pass the steam into a cool flask. The steam is condensed to water. On cooling further, the water may be frozen. Thus, by cooling, steam has been changed into water. If further cooled, the water could be frozen.

In the same way, if all gases could be cooled sufficiently, they would be condensed into liquids, and if cooled further the liquids would become solids. The change from one state to another may be sudden or gradual.

When such bodies as ice and paraffin wax are heated, they melt quickly, forming mobile liquids; but when other bodies like sealing-wax are heated, they gradually lose their former shape, form a thick, viscous mass, and then gradually pass into the liquid state.

This shows that there is no hard and fast line between the three states of matter. These intermediate stages are comparatively few, and may be neglected. Some bodies, as iodine and sal ammoniac, when heated, pass from the solid to the gaseous state without becoming liquids. They are said to be volatile.

These physical properties possessed by water should be fully considered and provided for when arranging any system of hot-water supply for domestic purposes. Such consideration would prevent accidents caused by boiler explosions (which may be attended with loss of life) and collapsing of cylinders, causing inconvenience to the occupants of the house.

DISTINCTIVE PROPERTIES OF SOLIDS

(1) **Solids possess elasticity.**

Experiment VII.—Smear a stone slab with oil. From a considerable height drop an ivory ball on the slab, and catch the ball as it rebounds. The whole of the ball that is smeared has been in contact with the slab.

The ball underwent compression, but its elasticity caused it to return to its former shape.

Experiment VIII.—Fix one end of a piece of indiarubber to a stand, and at the other end attach weights. The heavier the weights attached the greater is the stretching. On removing the weights, elasticity causes the rubber to return to its former length.

Experiment IX.—Clamp one end of a long wooden lath to a stand, and to the other end attach weights, Fig. 22. The heavier the weights attached, the greater is the bending. On removing the weights, elasticity causes the lath to return to its former shape.

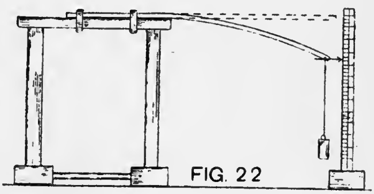


FIG. 22

Experiment X.—Suspend a wire from a stand, and attach a heavy weight to the lower end. Twist the weight, and then the weight twists to and fro by torsion. At the end of each twist, elasticity causes the wire to resume the original position.

(2) **Solids are hard and tenacious,** but these properties vary in degree in different solids.

(*a*) When a solid offers great resistance to being scratched, it is

hard; when it offers little resistance, it is **soft.** **Thus lead** and **tin** are considered **soft metals** on account of the ease with which they may be scratched or marked by the nails of the hand. **Copper, iron, steel, zinc,** are not easily marked, and are therefore classed as **hard metals.**

(*b*) A solid is said to be tenacious when great force is required to tear its particles asunder. It is generally measured when the solid is in the form of a wire, bar, or rod.

In measuring the tenacity, it is necessary to measure the area of cross-section, for the tenacity is directly proportional to the area of cross-section.

(*c*) Some solids are capable of being drawn out into wires, as copper and wrought iron; they are said to be **ductile.**

(*d*) Other solids are capable of being hammered into thin layers, or worked into different shapes, and are said to be **malleable,** as gold, copper, lead, tin, and zinc.

Lead, tin, and **zinc** are not ductile to any appreciable extent.

The great malleability of lead is one of the principal physical properties which greatly enhances its value in positions requiring a metal possessing adaptability in a high degree. Owing to this property, the plumber can work sheet-lead into any shape, and bend lead pipes to adapt them to various requirements.

DISTINCTIVE PROPERTIES OF LIQUIDS

(1) **Flow of liquids.** Some liquids flow rapidly, and are said to be mobile, thus water and mercury are very mobile. Other liquids flow slowly, and are said to be viscous; thus glycerine and treacle are viscous.

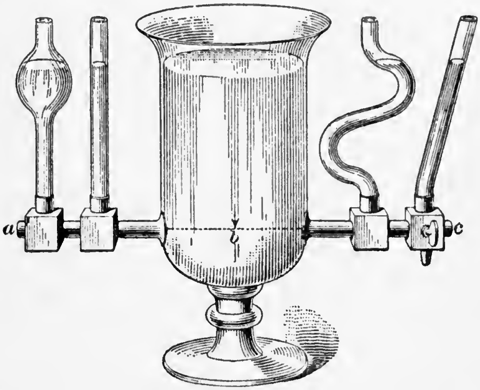


FIG. 23

When a liquid flows, the particles flow over each other. In doing this they stick, and do not flow perfectly. The more they stick together the less mobile they are, and *vice versâ*. The less mobile liquids are sometimes called imperfect liquids.

(2) **Liquids find their own level.** In Fig. 23 it will be noticed that the four tubes have different shapes and are connected at the bottom

to a glass jar. When water is poured down one it rises to the same level in all of them, hence the line joining the surfaces is perfectly horizontal. All houses below the level of water in a reservoir can be served with water from that reservoir, but houses above that level cannot be served, unless the water be pumped to them. Here, again, the same property is made use of.

The same principle applies to the supply of water to sanitary fittings in country houses, where the water is obtained from wells. A daily supply is pumped into a tank or cistern fixed in the upper portion of the house, so as to be well above the highest fitting supplied with water from it.

(3) **Liquids communicate pressure equally in all directions.**

Experiment XI.—Connect two vessels of the same size as shown in the sketch, Fig. 24. Fit each with a piston. On one side put 1 lb., on the other side must be placed 1 lb. in order to balance it; this experiment shows that liquids communicate pressure.

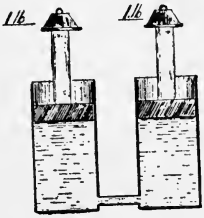


FIG. 24

Experiment XII.—Fit up the apparatus similar to that in Experiment XI., but let one vessel have four times the area of the other (Fig. 25). A weight of 1 lb. on the small piston will balance a weight of 4 lbs. on the large piston. If one is one hundred times the size of the other, 1 lb. on the smaller one will balance 100 lbs. on the larger one.

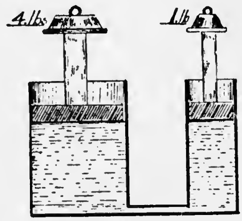


FIG. 25

Experiment XIII.—Take a piece of apparatus as shown in Fig. 26. Fill with water and press down the piston (*a*). The pistons *b, c, d, e, f,* are forced outwards. This shows that liquids communicate pressure equally in all directions.

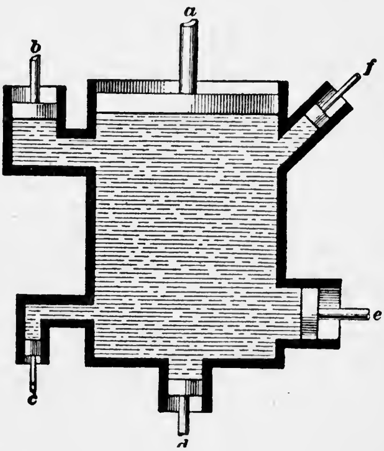


FIG. 26

This is the principle of the hydraulic press or **Bramah press** (Fig. 27). It consists of a cylinder (R) of large diameter, into which the tight-fitting piston (C) works, which communicates by means of the pipe (*d*) with the pump cylinder (A); when the man forces the lever (O) downwards, the small pump piston or plunger (*a*) forces a small quantity of water through the pipe (*d*) into the cylinder (R), and slightly raises the piston (C).

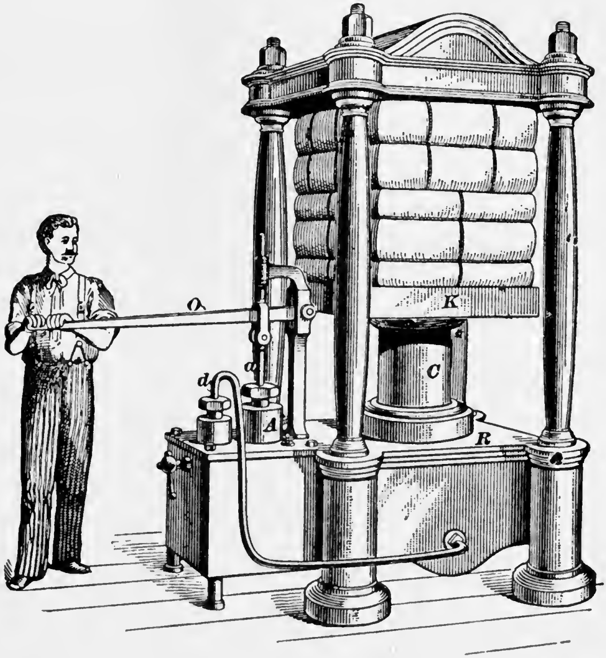


FIG. 27

DISTINCTIVE PROPERTIES OF GASES

Gases are easily compressed, and they expand indefinitely. If the pressure be increased the volume is decreased, and if the pressure be decreased the volume is increased. This may go on indefinitely. Increase in temperature causes increase in volume, and *vice versâ*.

Diffusion of Gases is the name given to the property which all gases possess, of intermingling intimately with each other, and also the power to pass through porous substances.

A common example is the introduction to the atmosphere of such gases as CO_2, SO_2, etc., which, though considerably heavier than atmospheric air, diffuse themselves more or less evenly thoughout the portion of the atmosphere into which they are discharged.

Experiment XIV.—Take a 2-in. glass cylinder 18 ins. long; plug one end securely with plaster of Paris mixed to a paste with water. After the plaster sets and is dry, fill the tube with hydrogen and insert the open end in a trough containing water. In a short time the water will rise inside the tube, proving that the hydrogen has diffused through the plaster plug.

All gases do not diffuse into each other at the same rate or velocity. The law of diffusion discovered by Graham, states :—The diffusive power of a gas varies inversely as the square root of its density.

The following table gives the density and rate of diffusibility of several gases compared with air, which is taken as the standard :—

Gas.	Density.	Square Root of Density.	$1 \div \sqrt{\text{Density}}$, *i.e.*, Rate of Diffusion.
Air	1·0000	1·0000	1·0000
Hydrogen . (H)	0·06926	0·2632	3·7794
Marsh Gas . (CH_4)	0·55900	0·7400	1·3375
Carbon Monoxide (CO)	0·96780	0·9837	1·0165
Nitrogen . (N)	0·97130	0·9856	1·0147
Oxygen . (O)	1·10560	1·0515	0·9510
Carbon Dioxide (CO_2)	1·52901	1·2365	0·8087
Sulphur Dioxide (SO_2)	2·2470	1·4991	0·6671

Diffusion plays an important part in the question of ventilation of houses. It is stated that an ordinary room built of porous bricks and plastered on the inside, having a capacity of 3000 cub. ft. or thereabout, will have the whole of the air which it contains changed by diffusion within one hour.

CHAPTER XII

MEASUREMENT OF VOLUME, SPECIFIC GRAVITIES, BALANCING COLUMNS, HYDROSTATICS, HEAD OF WATER, PRESSURE DUE TO SAME, CAPILLARY ATTRACTION

MEASUREMENT OF VOLUME

IF the solid has a **regular shape** its volume can be found by some rule: thus, if it be a prism, the volume is found by multiplying the area of the base by the height of the prism.

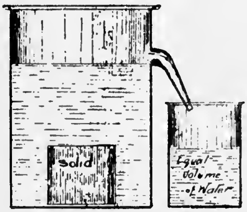


FIG. 28

If the solid has an **irregular shape,** its volume can be found by immersing the solid in water (Fig. 28). (When a solid is immersed in water it displaces a volume of water equal to its own volume); hence the displaced water must be measured in a measuring cylinder.

MEASUREMENT OF MASS

We have already noticed that all material things consist of matter, and in any substance there must be a definite quantity of matter. The quantity of matter in a body is called its **mass.** Masses cannot be compared by placing them alongside each other, because the eye alone cannot judge when two masses are equal. They are usually compared by comparing the pull of the earth on each body—that is, their weights are compared.

The British unit of mass is the Pound Avoirdupois.

THE BALANCE

The instrument used for measuring mass is called the balance (Fig. 29). It consists of a horizontal beam supported in the middle by a pillar. From each end is suspended a pan, and when adjusted, these pans should be in equilibrium. The substance is then placed in the left pan, and weights are placed in the right pan until equilibrium is again produced. The mass of the substance is obtained by finding the sum of the weights in the right pan.

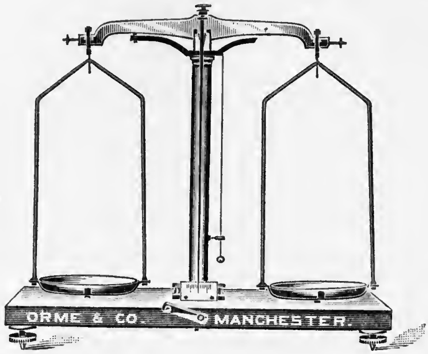


FIG. 29

BRITISH UNITS

As previously stated, the unit of weight is the pound avoirdupois, and from this are obtained the ounce, the quarter, the hundredweight, and the ton, according to the table of weights.

Similarly, the unit of length is the yard, and from it are obtained the other units—the foot, inch, pole, furlong, and mile, according to the table.

For the measurement of volume the unit is the cubic foot. It consists of a cube, each side of which is 1 ft. long.

THE METRIC SYSTEM

The awkward units of the British system led to the construction of another system, the aim being simplicity. The unit of length is the metre, which is rather longer than the yard. All other units of length are either multiples or submultiples of ten of the metre, as shown in the table :—

10 millimetres	=	1 centimetre.
10 centimetres	=	1 decimetre.
10 decimetres	=	1 metre.
10 metres	=	1 dekametre.
10 dekametres	=	1 hectometre.
10 hectometres	=	1 kilometre.

The units of area are obtained by squaring each of the units of length.

The units of volume are obtained by cubing each of the units of length: thus, if a cube has every side one centimetre long, it has a volume of one cubic centimetre (1 c.c.).

The unit of weight is the gramme. It is that mass of pure water, at 4° C., which will fill a cubic centimetre. All other units are multiples or submultiples of ten of the gramme.

Relation between British and Metric Units

I. Length.

1 metre	=	39·37 ins.
	=	1·094 yd.
1 inch	=	2·54 centimetres.
1 cm.	=	0·394 in.
1 kilometre	=	0·621 mile.

II. Volume.

1 c.c.	=	0·061 cub. in.
1 cub. in.	=	16·386 c.c.
1 cub. ft.	=	0·028 cub. metre.

III. Mass.

1 gramme	=	15·4323 grains.
1 kilogramme	=	2·2046 lbs. av.
1 lb.	=	456·3 grammes.

HYDROSTATICS

We have already learned how to find the volume and mass of a body.

If a quantity of water be taken, and its mass obtained in pounds, then its volume in cubic feet, it will be found that 1 cub. ft. of water weighs $62\frac{1}{2}$ lbs., or 1000 ozs. (approximately).

If this experiment be repeated with different volumes of water, the same result will be obtained. Repeat the measurements with a cube of some solid, say copper. It will be found that 1 cub. ft. of copper weighs $548\frac{3}{4}$ lbs.; hence equal volumes of different substances have different masses.

The mass of unit volume of any body is called its Density

In a similar manner the mass of 1 cub. ft. of any solid or liquid could be found. In order to compare the densities of different bodies, it is advisable to compare the density of every solid and liquid with that of water. Thus:—

1 cub. ft. of copper weighs $548\frac{3}{4}$ lbs.
1 cub. ft. of water weighs $62\frac{1}{2}$ lbs.

Hence 1 cub. ft. is $\dfrac{548\frac{3}{4}\text{ lbs.}}{62\frac{1}{2}\text{ lbs.}} = 8{\cdot}78$ times as heavy as 1 cub. ft. of water.

This number is called the **Relative Density**, or **Specific Gravity** of copper.

The specific gravity of any body is obtained by comparing the weight of the body with the weight of an equal volume of water.

$$\textbf{Specific gravity} = \frac{\text{Weight of a portion of substance}}{\text{Weight of an equal volume of water}}$$

To find the **specific gravity of a solid.**

The specific gravity of a solid may be found by one of the following methods:—

(1) If the solid has a regular shape, find its volume in cubic feet by measurement. Then find its weight in pounds, knowing that 1 cub. ft. of water weighs 62·5 lbs. Calculate the weight of an equal volume of water.

$$\text{Specific gravity} = \frac{\text{Weight of substance}}{\text{Weight of equal volume of water}}$$

(2) If the solid has an irregular shape, find its volume by displacement of water, as described in Fig. 28, and then proceed as in the first case.

Calculation.—A ton of chalk occupies a volume of $15\frac{1}{2}$ cub. ft., what is its specific gravity?

$$\text{Mass of an equal volume of water} = 15\tfrac{1}{2} \times 62\tfrac{1}{2} \text{ lbs.}$$

$$\text{Specific gravity} = \frac{\text{Weight of substance}}{\text{Weight of equal vol. of water}} = \frac{2240}{15\frac{1}{2} \times 62\frac{1}{2}}$$

$$= \frac{2240 \times 2 \times 2}{31 \times 125} = 2{\cdot}312 \quad \textit{Ans.}$$

Specific Gravity Bottle

This bottle consists of a small glass flask (Fig. 30), fitted with a conical stopper, through the centre of which passes a fine bore. It is so made that if filled to the top of the bore it will hold exactly the same volume of all liquids with which it is filled. This bottle may be used to find the **specific gravity of a liquid.**

FIG 30

First weigh the bottle empty, and then fill the bottle with the liquid. The difference will give the weight of that volume of liquid in the bottle. Afterwards empty the bottle, fill with water, and weigh again. From the latter weight subtract the weight of empty bottle to obtain the weight of an equal volume of water.

$$\text{Specific gravity of liquid} = \frac{\text{Weight of liquid chosen}}{\text{Weight of equal volume of water}}$$

The specific gravity bottle may also be used to find the specific gravity of a solid when the solid is in small pieces.

Experiment XIV.—Weigh out 2 ozs. of lead clippings. Then weigh the specific gravity bottle full of water **with the lead on the pan.**

Afterwards put the clippings into the bottle, and weigh again. The difference in weight gives the weight of the water displaced.

$$\text{Specific gravity of lead} = \frac{\text{Weight of lead in air}}{\text{Weight of water displaced}}$$

Weight of lead = 2 ozs.

Weight of displaced water = ·1756 oz. $= \frac{2}{\cdot 1756} = 11 \cdot 39.$

Specific gravity of the lead = 11·39.

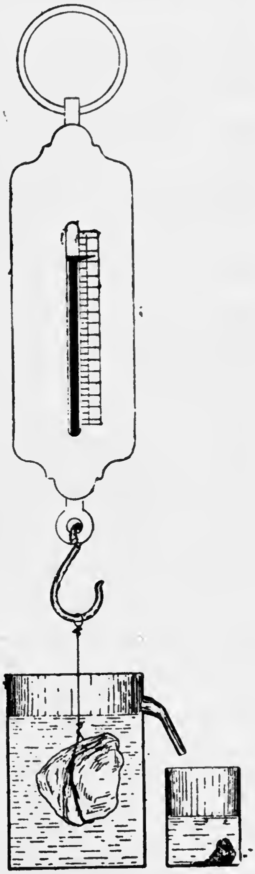


FIG. 31

PRINCIPLE OF ARCHIMEDES

It is a well-known fact that when a solid is immersed in water it loses weight, because the water exerts an upward force of buoyancy which partly counteracts the weight of the body.

Experiment XV.—Suspend the solid from the hook of the spring balance and note the weight. Then immerse the solid in the vessel filled with water to outlet (Fig. 31), and again note the weight. The difference is the loss of weight of the solid in water. The water displaced by the solid has run into the small vessel. Find the weight of the water displaced. If done carefully, the loss of weight of the solid in water should be the same as the weight of the water displaced. From this experiment it follows that when a solid is immersed in water it loses weight equal to the weight of the water displaced. If any other liquid be used instead of water, it would be found that the loss of weight of the solid in the liquid would be the same as the weight of liquid displaced by the solid.

It now follows that **when a solid is immersed in a liquid it loses weight equal to the weight of the liquid displaced.**

This important conclusion was discovered by Archimedes, and it is now known as the "Principle of Archimedes."

The principle of Archimedes may also be established by performing the following experiment :—

Experiment XVI.—From the left hook of the balance (Fig. 32), suspend a bucket (t) and a metal cylinder (a). (The internal capacity of the bucket (t) must be equal to the volume of the cylinder (a).) Add weights to the right pan until equilibrium is produced. Now remove the cylinder from the bucket and immerse the cylinder **only** in the beaker of water. The left side has apparently lost weight. Now pour water into the bucket until equilibrium is restored. It will be found that the water added to restore equilibrium exactly fills the bucket. This experiment shows that the metal cylinder when immersed in the water lost weight equal to the weight of the water displaced.

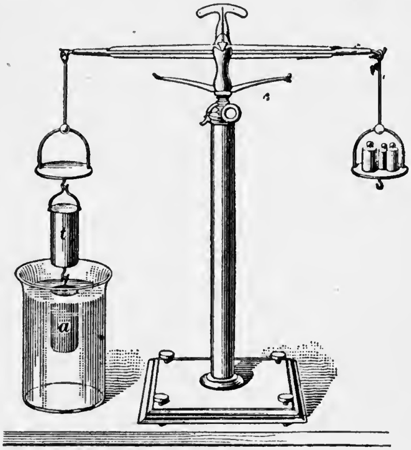


FIG. 32

APPLICATION OF THE PRINCIPLE OF ARCHIMEDES

By making use of this important conclusion, it is possible to find the specific gravities of solids and liquids rapidly and accurately.

Experiment XVII.—Over the left pan of the balance fit up a bridge. On the bridge place a beaker containing water. From the hook above the left pan suspend the solid. First weigh the solid in air, and then weigh the solid immersed in water.

By the principle of Archimedes the loss of weight in water is equal to the weight of the water displaced. Hence :—

$$\text{Specific gravity of solid} = \frac{\text{Weight of solid in air}}{\text{Loss of weight in water}}$$

Calculation.—A piece of lead weighs 90·4 lbs. In water it weighs 82·4 lbs. Find its specific gravity.

$$\text{Specific gravity} = \frac{\text{Weight of substance}}{\text{Weight of equal vol. of water}} = \frac{90\cdot4 \text{ lbs.}}{8 \text{ lbs.}} = 11\cdot3.$$

Experiment XVIII.—Use the same apparatus as in Experiment XVI. Weigh the solid in air and in water, as before. The difference is the loss of weight in water. Now pour out the water, and fill the beaker with the liquid whose specific gravity is required. Find the weight of the solid immersed in the liquid, and subtract from the weight in air. The difference will give the loss of weight of the solid in the liquid.

$$\text{Specific gravity of liquid} = \frac{\text{Loss of weight in liquid}}{\text{Loss of weight in water}}$$

Calculation.—A piece of lead weighs 90·4 lbs. in air, 82·4 lbs. in water, and 83·56 lbs. in methylated spirit. Find the specific gravity of methylated spirit.

Weight in air	= 90·4 lbs.	Weight in air	= 90·4 lbs.
Weight in water	= 82·4 lbs.	Weight in meth. spirit	= 83·56
Loss of weight in water	= 8·0 lbs.	Loss of weight in meth. sp.	= 6·84

$$\text{Specific gravity} = \frac{\text{Loss of weight in meth. spirit}}{\text{Loss of weight in water}} = \frac{6{\cdot}84}{8{\cdot}0} = {\cdot}83.$$

FLOTATION

From the principle of Archimedes it follows that when a solid is immersed in a liquid it loses weight equal to the weight of its own volume of liquid, as shown in Fig. 33 A, B, and C.

(*a*) If the solid be denser than the liquid, the solid will still retain some of its weight, because the downward pull exerted by the earth on the body is greater than the upward thrust of the liquid; hence the solid will sink.

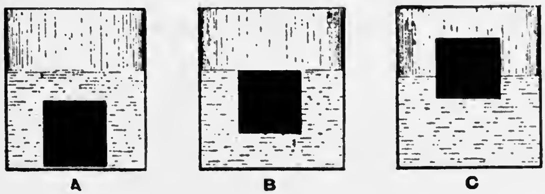


FIG. 33

(*b*) If the solid has the same density as the liquid, the solid will lose all its weight when immersed in the liquid, and it will be at rest under the surface of the liquid.

(*c*) If the solid is less dense than the liquid, a given volume of the solid will weigh less than the same volume of the liquid; hence the upward thrust will be greater than the downward pull. When placed in the liquid, the solid will come to rest partly immersed and part projecting out of the liquid. It is then said to be floating.

In the case of a floating body, the weight of the liquid displaced is equal to the weight of the body.

This fact is made use of in determining the specific gravity, by flotation, of bodies lighter than water, such as the various kinds of wood.

Experiment XIX.—Float a rectangular block of wood on water, and mark the water-line on the wood. Remove the wood, and determine the fraction of the whole block that was immersed in the water. **The fraction of the whole block that is immersed is equal to the specific gravity of the solid.**

Knowing the volume and specific gravity of the wood, it is possible to find the weight of a piece of wood by flotation.

Calculation.—A rectangular block of wood has a volume of 64 cub. ins. When floated, ·45 of its volume sinks below the surface of the water. Find its weight.

Fraction immersed = specific gravity of wood = ·45.

Weight of wood = Volume in cubic feet × specific gravity × $62\frac{1}{2}$ lbs.

$$= \frac{64}{1728} \times \frac{45}{100} \times \frac{62\frac{1}{2}}{1} = \frac{25}{24} = 1\frac{1}{24} \text{ lb.}$$

HYDROMETERS

The Hydrometer is an instrument used for measuring the specific gravity of liquids. It consists of a long, cylindrical tube, inside of which is a scale. At the bottom of the tube are two bulbs. The larger one is filled with air, and the smaller one with mercury. It is so made that when in a liquid, it floats in a vertical position. When placed in a liquid, the hydrometer will displace a weight of the liquid equal to its own weight; hence the less dense the liquid, the greater volume of liquid it will displace, and the deeper the hydrometer will sink. Conversely, the denser the liquid, the less volume of liquid it will displace, and the less it will sink. That is to say, it will float at a deeper level in a lighter liquid than in a heavy liquid.

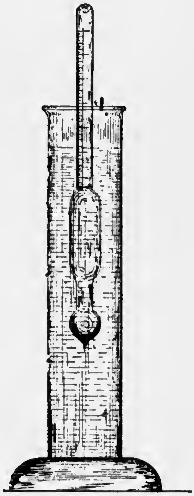


FIG 34

A useful form of hydrometer is one which has its scale graduated as in Fig. 34. When placed in water the scale sinks to the mark (1). This is the specific gravity of water. From 1 to the top of the tube the readings gradually decrease, and denote specific gravities less than 1. From 1 downwards the readings gradually increase and denote specific gravities more than 1.

In general, there are two forms of hydrometers. (1) For finding the specific gravity of liquids lighter than water. On this scale the 1 is at the bottom. (2) For finding the specific gravity of liquids heavier than water. On this scale 1 is at the top.

BALANCING COLUMNS

It has already been stated that liquids find their own level. If any liquid be poured down one arm of a U-shaped tube, when it is open at both ends, it will rise to the same level on the other side.

Experiment XX.—To find the specific gravity of mercury.

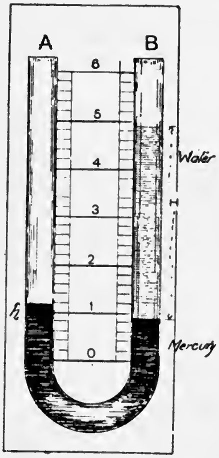


FIG. 35

Fit up the apparatus as shown in the sketch, Fig. 35. It consists of a length of glass tubing bent in the form of the letter U. Between the two vertical glass tubes passes a scale, and the whole is fixed to a board by means of elastic bands or brass clips. Pour mercury down one arm (A). Afterwards pour water down the other arm (B). In B the level of the mercury is pushed down by the column of water above, whilst in A the level of the mercury has risen. Take I as the common level. Then measure the height of mercury above the common level (h), and also the height (H) of the water above the common level. The small column of mercury (h) balances the long column of water (H); hence mercury must be heavier than water; and the specific gravity must be inversely proportional to the heights of the two columns.

$$\text{Specific gravity} = \frac{\text{Height of water column}}{\text{Height of mercury column}} = \frac{H}{h}$$

To find the specific gravity of methylated spirit.

Experiment XXI.—Use the same apparatus as in last experiment. After pouring mercury into the U-tube, half-fill one glass tube with water. The mercury is forced down. Restore the common level by pouring methylated spirit down the other tube, until the mercury in each tube is at the same level. Then measure the height of the water column and the height of the spirit column above the mercury.

$$\text{Specific gravity of spirit} = \frac{\text{Height of water column}}{\text{Height of spirit column}} = \frac{H}{h}$$

Different heights of water could be taken and the corresponding height of spirit determined, and thus several results would be obtained.

In this experiment any liquid may be substituted for methylated spirit, and the specific gravity of the liquid may be determined.

Hare's Apparatus for determination of Specific Gravity.

This apparatus consists of two straight glass tubes connected at the top with a three-way junction by means of rubber tubing, as shown in the sketch, Fig. 36. The lower ends of the tubes dip into beakers containing the liquids to be experimented with.

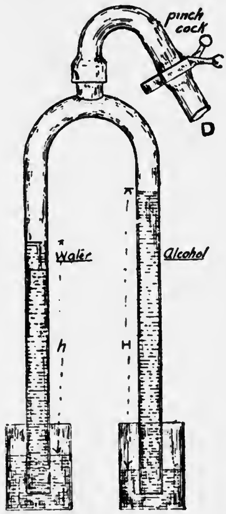


FIG. 36

Experiment XXII.—Fill the two beakers with water, and place the lower ends of the tubes in the beakers. By applying suction at (D) the water is drawn up each tube. When at a convenient height apply the pinchcock. Measure the height of each column of water from the level in the beaker. As before, it will be found that water finds its own level.

Experiment XXIII.—Substitute a beaker of milk for one of the beakers of water. Fix up apparatus as before. Draw up the liquids to a convenient height. Pinch the rubber tubing, and measure the height of each liquid above that in the beaker.

Specific gravity of milk

$$= \frac{\text{Height of water column}}{\text{Height of milk column}} = \frac{H}{h}$$

TABLE OF SPECIFIC GRAVITIES

Solids		Liquids	
Cork	= ·24	Alcohol (absolute)	= ·795
Elm	= ·544	Methylated Spirit	= ·82
Oak	= ·9 to ·99	Turpentine	= ·87
Ice (melting)	= ·92	Olive Oil	= ·92
Ebony	= 1·19	Pure Water at 4° C.	= 1·00
Coal (anthracite)	= 1·33	Sea-water	= 1·026
Sand (dry)	= 1·42	Milk (cow's)	= 1·03
Glass (crown)	= 2·5	Hydrochloric Acid	= 1·64
Zinc	= 7·2	Nitric Acid	= 1·53
Cast Iron	= 7·2	Sulphuric Acid	= 1·854
Wrought Iron	= 7·79	Mercury	= 13·56
Steel	= 7·79		
Copper	= 8·79		
Silver	= 10·47		
Lead	= 11·36		
Gold	= 19·36		
Platinum	= 21·5		

Under the heading Hydrostatics, we have to consider the pressure of fluids at rest, especially water. If we take a piece of wood 1 ft. long and 1 sq. in. section, and place it upright in a copper vessel, and at the top of it we place a pound weight, this same weight will be transmitted by the piece of wood to the area (*i.e.,* 1 sq. in. in the bottom of the copper tank) supporting same. If we substitute for the 1 in. sq. piece of wood a block 4 ins. sq. in section and place the pound weight on top of it, the total weight (neglecting the weight of the wood) would be the same upon the area supporting it as in the former case, but in the latter example the weight is distributed over a larger area; therefore the weight or thrust upon 1 sq. in. will be $4^2 = 16 = \frac{1}{16}$th of a pound.

Pressure is always measured per unit area, which in this country is 1 sq. in.

If a vessel with an aperture at the top containing a tight-fitting piston 1 sq. in. in area, be filled with water and a pound weight be placed upon the piston, the pressure exerted by this weight will be transmitted to the water in the vessel, and, neglecting the height of the vessel, the pressure of the contained water upon each square inch of surface will be 1 lb.

It will be seen from the above, **that pressure exerted anywhere upon a mass of liquid is transmitted undiminished in all directions, and acts with the same force upon all equal surfaces, and in directions at right angles to those surfaces.**

This discovery is due to **Pascal,** who demonstrated the fact by inserting a tube of small bore in the side of a cask filled with water; by obtaining sufficient vertical height of tube and filling it with water, **the pressure due to the height or head of water in the tube** distributed undiminished on each square inch of the interior of the cask was sufficient to burst the cask. This fact is an example of the equal distribution of water-pressure undiminished.

To give a practical example of the above, we will take the case of a copper boiler containing 600 sq. ins. of interior surface, connected to a cistern fixed at such a height as to give a pressure of 6 lbs. per square inch on the boiler, then the total pressure on the 600 sq. ins. of surface $= P \times A = 6 \times 600 = 3600$ lbs.

$P =$ Pressure in pounds per square inch.
$A =$ Area of surface exposed to pressure.

The weight of a cubic foot of fresh water at 4° C. is 62·5 lbs. (approximately). If a tank 1 ft. cube is filled with water at the above temperature, the total pressure exerted upon the bottom of the tank would be equal to the weight it contained, and, as the bottom of the tank contains $12 \times 12 = 144$ sq. ins., the weight sustained by each square inch

$$= \frac{W}{A} = \frac{62{\cdot}5}{144} = {\cdot}434 \text{ lbs.}$$

Therefore we may state that 1 ft. head of water will exert a pressure of ·434 lbs. per square inch.

Again, if it were possible to insert into the 1 ft. cube of water 144 very thin tubes each 1 ft. long and 1 sq. in. in section, and, taking the water out by this means, pile them vertically above each other, the whole of the contents of the tank would be contained in the tube 144 ft. high, and would exert a pressure upon the **square inch** at the foot of the column equal to the weight of the water taken from the tank, *i.e.*, **62·5 lbs.**, therefore the pressure due to 1 ft. $= \frac{62 \cdot 5}{144} = \cdot 434$ lbs. per sq. in.

If the bottom of the tube be enlarged so as to cover an area of 4 sq. ins., the total pressure or thrust upon the bottom would be $P \times A = 62 \cdot 5 \times 4 = 250$ lbs.; thus it will be seen that **the total pressure, or thrust, is increased in direct proportion to the increase in area.**

If a tube $\frac{1}{4}$ in. in diameter is allowed to take the place of the tube 1 sq. in. in section, the pressure at the foot of the column would not be influenced in any way, because the pressure is distributed evenly over the area exposed to it, and would be exactly the same with the $\frac{1}{4}$-in. diameter or any other size of tube. A parallel case has previously been cited where the area exposed to pressure was increased at the foot of the column, and the capacity of the tube forming it in no way altered. From the above it is observed that pressure is due to the height of the column of water, and not to the capacity of the column or volume of water. This perpendicular height is usually termed **head of water, and is always measured vertically.**

The pressure per square inch and the total pressure exerted upon the interior surfaces of boilers, tanks, and cylinders may be readily calculated. It has been ascertained that 1 ft. vertical height or "head of water" exerts a pressure of ·434 lbs. per square inch, and to find the pressure per square inch at the foot of a column 40 ft. high, the head in feet multiplied by pressure exerted by 1 ft. head will give the result in pounds per square inch.

1 ft. exerts ·434 lbs. pressure per square inch.

40 ft. exert $40 \times \cdot 434 = 17 \cdot 36$ lbs. pressure per square inch.

The equal distribution of pressure through a volume of liquid is taken advantage of in many ways; the hydraulic press, used for compressing cloth, etc., intended for shipment, is a common example. There are different types of such machines in everyday use, but in all the principle is the same—namely, the exposure to the pressure of water of a large area in the form of an enclosed cylinder piston surface; the pressure may be obtained by pumping water into accumulators weighted to give a definite pressure per square inch to the water thus stored, or it may be applied in a limited degree from a cistern in the building, or the pressure in the town's main is utilised.

The Hydrostatic Paradox is that a small quantity of water can be made to support a great weight.

One type of hydrostatic press is supplied with a pump worked by hand, as shown in Fig. 27. The large cylinder is filled with water supplied by a small pipe from the pump. When the pump plunger (*a*) is forced down, the pressure is transmitted through the water in the cylinder to the piston face (C); by making the area of the piston (C) very large, and the area of the plunger (*a*) small, a slight downward force applied to the lever (O) will produce a very great upward force at the piston (C), for the pressure of water is the same upon the two surfaces per unit of area (square inch); hence, the thrusts on them are proportional to their areas.

Example.—If the area of the pump plunger (*a*) is 1 sq. in., and the area of the piston (C) 250 sq. ins., then a force of 2 lbs. applied at B will produce a pressure of 2 lbs. per square inch, in the fluid enclosed by the cylinder, and this pressure, acting over the whole area of the piston (C), will produce a total thrust of $250 \times 2 = 500$ lbs.

Thus, by applying a force of 2 lbs. to the pump plunger, a weight or resistance equal to 500 lbs. can be overcome or balanced at (C). In these calculations loss from friction is neglected.

The mechanical advantage obtained by this difference in the areas of the ram piston and the pump plunger is sometimes known as the multiplication of water-power.

CAPILLARY ATTRACTION

If a clean glass rod be placed vertically in water, the water will be drawn up around the rod to a level slightly higher than the normal water surface; if the same rod be placed in mercury, the liquid will be depressed instead of raised. On examination it will be found that water wets the glass, while mercury does not; if the rod be greased and placed in water, it will be noticed that the surface of the water about the rod is depressed. If a clean lead or zinc strip be placed in mercury, the surface of the mercury will be raised about the strip, and on taking it out, it will be coated or "wetted" with mercury.

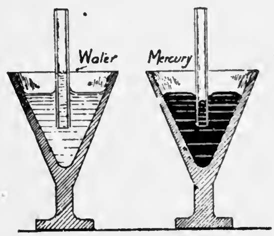


FIG. 37

Generally, it will be found that all liquids that will wet the solids placed in them will be lifted or drawn up, while those that do not will have their surfaces depressed at the point of contact with the solids. These phenomena are called capillary attraction, and may be readily demonstrated by inserting two tubes of fine bore, one in a vessel of water and one in mercury (Fig. 37).

It will be observed that the water in one case rises up the inside of the tube, and is also slightly raised round the outside of the tube, while in the other case, the mercury is depressed around the tube, and does not rise inside to the level of the mercury in the vessel; its surface is convex, and in the former concave.

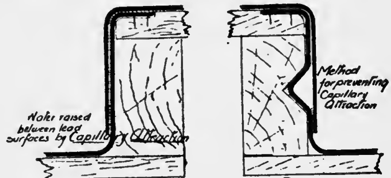
The distance vertically which liquids will travel by capillary attraction in tubes, or be depressed, varies inversely as the diameter of the tube.

Thus, water will rise twice as far in a tube $\frac{1}{16}$ in. diameter as in one $\frac{1}{8}$ in. diameter.

This action is due to the "**attraction**," or "**repulsion**," which solids and liquids have for each other.

When the surface of the solid is wet, it is due to "attraction," and when it is not wet, to "repulsion."

This is a question requiring close consideration on the plumber's part, as it largely affects the efficiency of certain branches of his work, and if not catered for, may bring about disastrous consequences. When laying lead on roofs, laps or passings are of frequent occurrence, consisting of two surfaces of lead, fixed vertically or inclined, in close contact with each other, which produce conditions eminently favourable to "capillarity." If measures are not taken to prevent it, the woodwork which the lead is intended to protect, is gradually saturated with water, and in a short time the stability of that portion of the roof is endangered. Fig. 38 shows sections of drips, one being the method usually employed, permitting capillary attraction to take place; the other shows the method of preventing same by providing an air groove, and dressing the undercloak into it, thus breaking contact of the passings. This method may be successfully employed in most positions in roof-work, for the prevention of capillarity.



Sections of Lead Drips

FIG. 38

This action may also be observed in traps which are not self-cleansing, in connection with soil, waste, or drain pipes. By soakage through the pores of a piece of lint or rag hanging over the outgo of the trap, the water seal of the trap may be entirely removed, thus providing for the escape of foul gases; hence the necessity of having traps self-cleansing.

CHAPTER XIII

MECHANICS: LEVERS, PULLEYS, INCLINED PLANE, WEDGE, SCREW, WHEEL AND AXLE

MECHANICS

UNDER this heading the following simple mechanical powers will be considered :—

I. **The Lever.**
II. **The Pulley.**
III. **The Inclined Plane.**
IV. **The Wedge.**
V. **The Screw.**
VI. **Wheel and Axle.**

THE LEVER

The lever is a rigid bar which can turn about a fixed point, called the **Fulcrum.**

Take a yard scale (Fig. 39), bore a hole through the middle. Pass a stout needle through the hole, and support the needle as shown in the sketch. If the beam is not horizontal, slide a rider along until it is horizontal. Then it is in equilibrium.

A rod supported in this way forms a **lever,** the point at which it is supported is called the **fulcrum.**

Experiment XXIV.—On the left arm place a 2-oz. weight, 10 ins. from the fulcrum. Slide a 2-oz. weight along the other side, and the beam will be found to be in equilibrium when the weight is 10 ins. from the fulcrum. Or replace one of the 2-oz. weights by 4-oz., 5-oz., and 10-oz. weights.

Right Arm.		Left Arm.	
Weight.	Distance of Weight from Fulcrum.	Weight.	Distance of Weight from Fulcrum.
2 ozs.	10 ins.	2 ozs.	10 ins.
2 ,,	10 ,,	4 ,,	5 ,,
2 ,,	10 ,,	5 ,,	4 ,,
2 ,,	10 ,,	10 ,,	2 ,,

When the weight on the right side is doubled, the distance from the fulcrum must be halved, in order to preserve equilibrium.

In every case it will be found that if the beam is in equilibrium :

(1) The product of the weight and the distance on the right side must be equal to the product of the weight and the distance on the left side. If AB is the lever, F the fulcrum, P the weight on left arm, and W the weight on right arm, then $P \times AF = W \times BF$.

(2) The weight on the left arm tends to turn the beam in an opposite direction to that on the right arm, hence the forces must act in opposite directions.

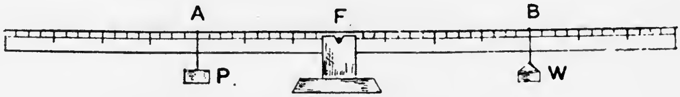


FIG. 39

Orders of Levers.—In using a lever, the body to be lifted is called the weight, and the force which must be applied to the lever in order to produce equilibrium is called the power. The three orders of levers are formed by the different arrangements of the three parts—the weight, the power, the fulcrum.

First Order (Fig. 40).—In this order the fulcrum is situated between the weight and the power. Condition for equilibrium.

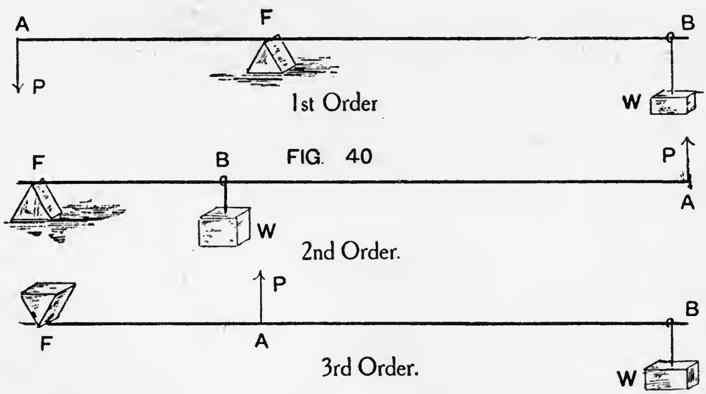


FIG. 40

$$P \times AF = W \times BF$$

$$\therefore \; P = W \times \frac{BF}{AF}$$

Second Order (Fig. 40).—In this order the weight is situated between the fulcrum and the power. Condition for equilibrium.

$$P \times AF = W \times BF$$
$$\therefore \quad P = W \times \frac{BF}{AF}$$

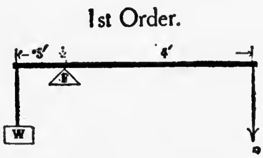
Third Order (Fig. 40).—In this order the power is situated between the fulcrum and the weight.

$$P \times AF = W \times BF$$
$$P = W \times \frac{BF}{AF}$$

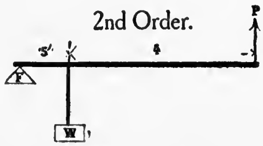
In this case the power must be applied in an upward direction in order to raise the weight.

In every case the power necessary to raise the weight is found by multiplying the weight to be raised by the ratio of the distances of weight and power from fulcrum.

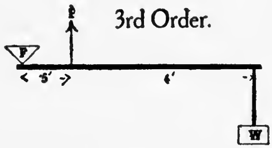
Calculations.—P is 4 ft. and W is ·6 ins. from fulcrum; find P. required to raise 72 lbs. in each order.



$$W \times \tfrac{1}{2}\text{ ft.} = P \times 4\text{ ft.}$$
$$\frac{W}{4 \times 2} = P. \quad \therefore \quad P = \frac{72}{8} = 9\text{ lbs.}$$



$$W \times \tfrac{1}{2}\text{ ft.} = P \times 4\tfrac{1}{2}\text{ ft.}$$
$$P = \frac{72 \times 1 \times 2}{2 \times 9} = 8\text{ lbs.}$$



$$P \times \tfrac{1}{2}\text{ ft.} = W \times 4\tfrac{1}{2}\text{ ft.}$$
$$P = \frac{72 \times 9 \times 2}{2} = 648\text{ lbs.}$$

FIG. 41

Application of the Lever.

First Order.—The steel lever balance A (Fig. 42), generally used by butchers and others, is an example of this order. The substance to be weighed is suspended from the hook, which, it will be observed, is near the fulcrum; the sliding weight which is moved along the graduated lever, to balance the substance being weighed, represents the **power**.

Fig. 42 B shows the application of the lever to the task of moving or raising a heavy roll of sheet-lead. From what has been previously stated, it will be seen that the nearer the **fulcrum** is to the weight, the smaller will be the **power** that is required to raise the lead, and *vice versâ*.

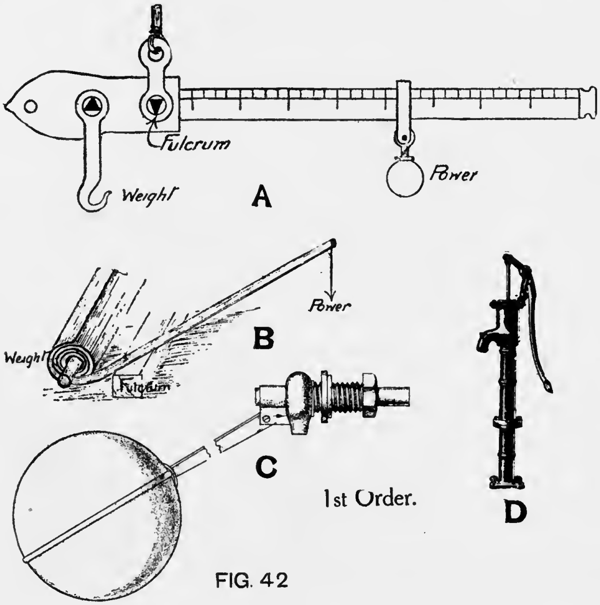


FIG. 42

Fig. 42 C shows another lever of this order, applied to ball taps. The **power** in this case is supplied by the upward thrust of the water upon the hollow copper ball attached to one end of the lever. The **fulcrum** is formed by the pin which passes through the lever and restricts its movements to one plane, and the **weight** is provided by the pressure of water against the surface of the valve in the tap. It will be noticed that the "fulcrum" is situated close to the "weight," thus producing a comparatively great mechanical advantage.

The example (Fig. 42 D) shows the application of the lever to an ordinary "suction pump." The **weight** is the water in the cylinder or barrel above the pump bucket which is attached to one end of the lever, the **fulcrum** is the pin on which the lever works, and the **power** is exerted by the man at the other end of the lever whilst working the pump.

It will be observed that in each of the foregoing examples the **fulcrum** is between the **weight** and the **power**.

Second Order.—This is illustrated in A, Fig. 43, where the crowbar forms the lever required to raise a sheet of lead. In this example, the ground forms the **fulcrum**, and the sheet-lead, which is only several inches from the fulcrum end of the lever, is the **Weight**. The **power** in this case is exerted at the opposite end of the lever in an upward direction.

Fig. 43 B shows this order of lever applied to ball taps. The **fulcrum** is the fixed end of the lever, the **weight** the downward thrust of the water on the valve, and the **power** the upward thrust of the water in the cistern upon the copper ball.

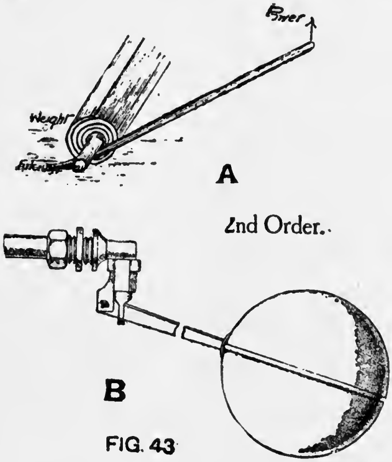


FIG. 43

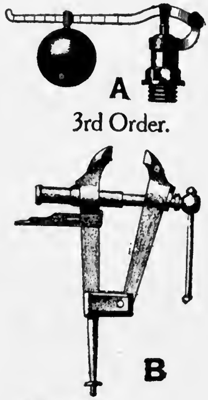


FIG. 44

Third Order.—An excellent example of this order is shown in A, Fig. 44, consisting of a "lever safety-valve." The **power** is exerted between the **fulcrum**, where one end of the lever is attached to the body of the valve, and the **weight** is provided by the sphere of metal, which is made to slide along the lever to adjust the amount of resistance.

The upward thrust of steam, water, or gas, is the **power** tending to raise the **weight**. The further the **weight** is from the **power**, the greater will be the **power** that is required to lift the valve.

Fig. 44 B shows the application of this order of lever to the "vice" used by fitters, blacksmiths, etc. It will be observed that the **fulcrum** is formed by the hinged end of the movable jaw—*i.e.*, the "lever"—and the **power** is applied by the handle of the screw. The **weight** is caused by the resistance of the substance held in the vice, against the movable jaw. Sugar tongs, coal tongs, etc., are also examples of levers of this order.

THE PULLEY

A pulley consists of a circular disc or sheaf the outer edge of which is often grooved, and of a framework or block carrying an axle on which the sheaf turns.

Kinds of Pulleys

(1) **A fixed pulley** (Fig. 45) is one where the axis is fixed, and therefore does not bodily alter its position. The sheaf is capable of turning round its axis, but the pulley as a whole is stationary.

If a cord be passed round a nail driven into a wall, and to one end a weight of 10 lbs. be attached, then, in order to lift the weight, a force much greater than 10 lbs. must be applied downwards. The hindrance is due to the roughness of the nail, which causes friction. If a fixed pulley be substituted for the nail, friction is practically avoided. Still, to raise the 10 lbs. weight, a force of 10 lbs. must be applied. There is no actual gain in power, except that friction is lessened; the advantage is that by using a fixed pulley, there is an **alteration in the direction of the force applied.** To raise a weight without a pulley, a man must exert a force in an upward direction, whereas with the aid of a pulley, he pulls downwards.

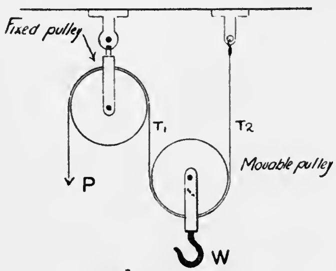


FIG. 45

(2) **A movable pulley** (Fig. 45) is one where the axis is movable, and therefore the pulley itself can be raised or lowered.

Grouping of Pulleys

First System of Pulleys.—In this arrangement there is one or more movable pulleys, and also one fixed pulley.

Let us consider the simplest form.

The weight is attached to the movable pulley, and is held in position by the tension in the cords T_1 and T_2 (Fig. 45). It is clear that the tension in each rope must be one-half the weight to be raised (if the weight of the pulley be neglected).

The fixed pulley merely changes the direction of the force.

Hence the power required to raise the weight is equal to the tension in the rope, which in this case is one-half the weight.

Other combinations, with two or more movable pulleys, may be taken, as Figs. 46, 47, and 48.

In Fig. 47 there are four movable pulleys and one fixed pulley.

Assuming that $W = 64$ lbs., the tension in the two cords $T_1, T_1 = 64$ lbs. and the tension in one cord $(T_1) = \frac{W}{2} = 32$ lbs. Thus, the tension in one of the cords marked $T_2 = \frac{T_1}{2} = \frac{32}{2} = 16$; and again, the tension in the cord $T_3 = \frac{T_2}{2} = \frac{16}{2} = 8$. Hence, the tension in cord $T_4 = \frac{T_3}{2} = \frac{8}{2} = 4$ lbs., and as the movable pulley changes the direction of the force without altering its value, the power required to raise 64 lbs.

$$= T_4 = \frac{W}{16} = \frac{64}{16} = 4 \text{ lbs.}$$

neglecting weight of pulleys and friction.

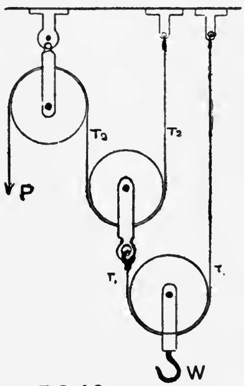


FIG. 46

$$T_1 = \frac{W}{2}$$

$$T_2 = \frac{T_1}{2} = \frac{W}{4}$$

$$P = T_2 = \frac{W}{4}$$

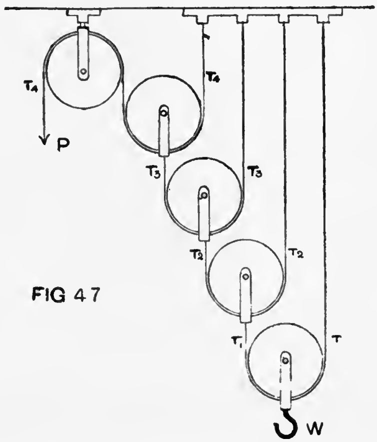


FIG 47

$$T_1 = \frac{W}{2}$$

$$T_2 = \frac{T_1}{2} = \frac{W}{4}$$

$$T_3 = \frac{T_2}{2} = \frac{W}{8}$$

$$T_4 = \frac{T_3}{2} = \frac{W}{16}$$

$$\therefore P = \frac{W}{16}$$

Second System of Pulleys.—The second system consists of an upper and a lower block, each block containing several pulleys, the same cord passing round all. Consider the parts of the cord to be parallel to each other, then we have—

(1*st*) The same tension in all parts of the cord.

(2*nd*) As many pulleys as there are in the upper and lower block together, so many parts of the cord sustain the weight, each part bearing its share of the weight.

Let us suppose that there are three pulleys in each block (Figs. 48 and 49)—that is, six pulleys in all—and the weight is sustained by six cords; hence the tension in each cord is one-sixth of the total weight.

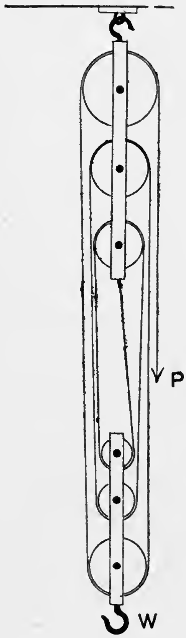


FIG. 48

The tension in the rope to which the power must be applied is thus one-sixth of the weight; hence the power required to raise the weight is only one-sixth of the weight.

In both systems the weight of the movable pulleys has been neglected. If required, the weight of each **movable** pulley in the system and the necessary force to overcome friction, must be added to the weight to be raised.

The pulleys shown in Fig. 49 are of the type generally used for hoisting heavy weights from the ground to the upper portions of buildings.

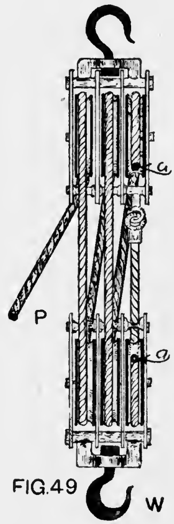


FIG. 49

The number of pulleys or sheafs in each block will depend upon the weight to be raised, and the power available for raising the same; the blocks in this case may be brought more closely together, thus allowing a greater range of action than obtains in the type shown in Fig. 48.

Example.—It is necessary to raise a piece of lead weighing 5 cwt. to the top of a building, 40 ft. high, by using blocks with the same number of sheafs as shown in Fig. 49. (*a*) What length of rope would be necessary to pass round the pulleys when 40 ft. apart, allowing a similar length for applying the power at the ground level? (*b*) What force would have to be applied at P to raise the weight (neglecting friction and weight of pulleys)?

$$a = 7 \times 40 = 280 \text{ ft. of rope.}$$

$$b = \frac{5 \times 112}{6} = 93{\cdot}3 \text{ lbs.}$$

The 93·3 lbs. would only balance the 5 cwt., and additional power—*i.e.*, weight—would have to be exerted to raise the weight to the top of the building, whilst if friction is taken into account, the total power would require to be increased by $\frac{1}{3}$, to overcome this.

There are other more complicated systems of pulleys, but their treatment is beyond the scope of this book.

THE INCLINED PLANE

An inclined plane is a plane inclined to the horizontal. The angle it makes with the horizontal is called **the angle of inclination**; thus ABC is the angle of inclination (Fig. 50).

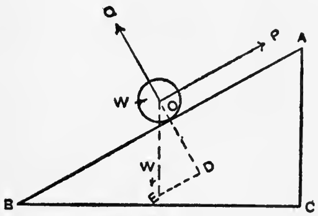


FIG. 50

Let AB represent the inclined plane, and angle ABC the angle of inclination.

If the smooth metal cylinder (O) is supported on the inclined plane AB, and is kept from rolling down by a force P acting parallel to the plane AB, then the cylinder is kept in equilibrium by the three forces.

(1*st*) The weight of the cylinder (W) acting vertically from the centre of gravity.

(2*nd*) The reaction of the plane (Q) due to the weight acting at right angles to AB at the point of contact.

(3*rd*) The force (P) acting parallel to AB.

Since these forces are in equilibrium, they can be represented by the triangle OED.

$$\frac{P}{DE} = \frac{Q}{OD} = \frac{W}{OE}$$

But triangle OED is similar to triangle ABC; hence in the triangle ABC—

(1) W is represented by AB, the length of the inclined plane.

(2) Q „ BC, „ „ base.

(3) P „ AC, the perpendicular height.

$$\text{hence, } P : W = AC : AB$$

$$P = \frac{AC}{AB} \times W$$

If AC, AB and W are known, P can be calculated.

Calculation.—A man wishes to raise a roll of lead weighing 3 cwt. to a height of 3 ft. by rolling it along a plank 14 ft. long. What force does he require to exert (neglecting friction)?

Force required : Weight = Perp. height : length of plane.

$$P : 3 \text{ cwt.} = 3 \text{ ft.} : 14 \text{ ft.}$$

$$P = \frac{3}{14} \times 336 = 72 \text{ lbs.}$$

Experiment XXV.—Fit up the apparatus as shown in the sketch, Fig. 51. The angle of inclination ABC can be increased or decreased by raising or lowering the support H, and the plumb-line AC shows the perpendicular height of the inclined plane AB at A.

The smooth metal cylinder (D) is first weighed, and is then connected to the spring balance, which registers the force required to maintain equilibrium—*i.e.*, P. By altering the angle of inclination, different results may be obtained.

P	AB	P×AB	W	AC	W×AC
$\frac{1}{3}$ lb.	×36 ins.	=12	2 lbs.	× 6 ins.	=12
$\frac{2}{3}$,,	×36 ,,	=24	2 ,,	×12 ,,	=24
1 ,,	×36 ,,	=36	2 ,,	×18 ,,	=36
$1\frac{1}{3}$,,	×36 ,,	=48	2 ,,	×24 ,,	=48

Each set of results shows that :—

$$P \times AB = W \times AC$$

or P : W = AC : AB. Hence $P = \frac{AC}{AB} \times W$

This shows that if AC be decreased P is decreased, if AC be increased P is increased.

$\frac{AC}{AB}$ shows the gradient or slope of the inclined plane.

Instances of the inclined plane are seen very frequently. Heavy loads are put on carts by means of planks sloping from the cart to the ground. Again, the approaches to railway platforms and bridges generally consist of inclined planes. The flight of steps leading to the upper floor of the building is still another example, and others will readily suggest themselves.

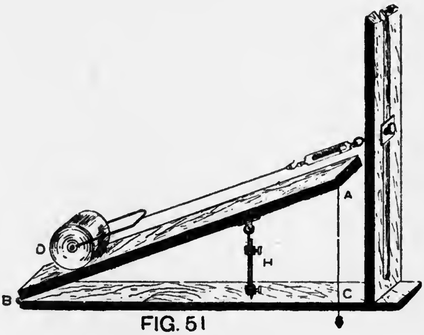


FIG. 51

Practical Illustrations of the Inclined Plane

The Wedge is a modification of the inclined plane. In this case the body is fixed, and the plane is movable, as in Fig. 52 A, B, C, and D.

Let us consider the case of a wedge being driven into an aperture in a piece of wood (Fig. 52 C.).

The driving force (P) is in a direction at right angles to the end of the wedge (BC). The pressure of the wood on the wedge (R) is at right angles to the sides of the wedge. If the wedge is in equilibrium, it is found that :—

$$\frac{P}{R} = \frac{BC}{AC} \text{ or } \frac{P}{R} = \frac{\text{back of wedge}}{\text{one of equal sides of wedge}}$$

$$P = \frac{BC}{AC} \times R$$

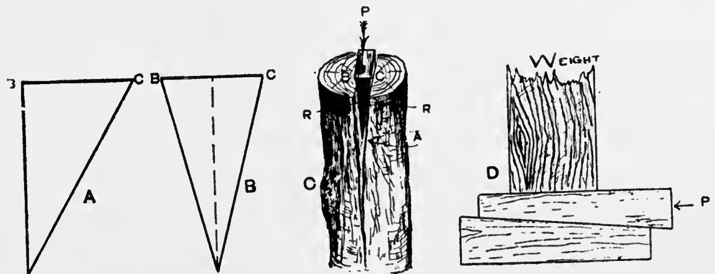


FIG. 52

Hence, the smaller that BC is, compared with AC, the less force is required to drive in the wedge. In other words, it may be said that by making the angle of the wedge more acute, the effectiveness of the wedge is increased.

Instances of the wedge are found in the chisel, the knife blade, and other tools with knife edges. In all cutting tools, the back of the wedge is very small compared with the side of the wedge.

It is also used largely in building operations for tightening the temporary supports of various parts of the structure, as shown in D, Fig. 52, and in a similar way it is also applied to exert a force or pressure on the side timbers of trenches and other excavations, by driving hard wood wedges between the struts and walings.

THE SCREW

The Screw is similar in principle to the spiral staircase, because each consists of an inclined plane winding round a central column.

If a piece of paper be cut into the form of a right-angled triangle and then wound round a cylindrical ruler, the inclined line BC, Fig. 53 A, will mark out a spiral curve, which is called the thread of the screw.

The angle BCD represents the angle of inclination of the thread. It always makes the same angle with base CD. This constant angle is called the angle of the screw.

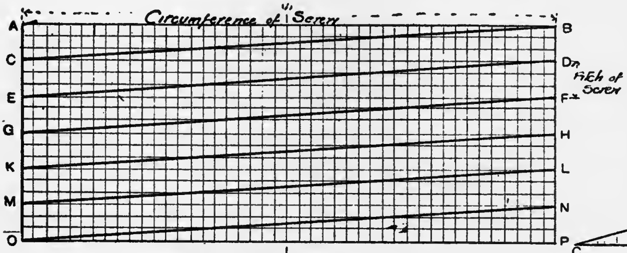


FIG. 53 B

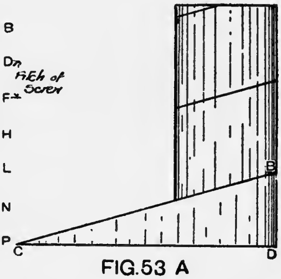


FIG. 53 A

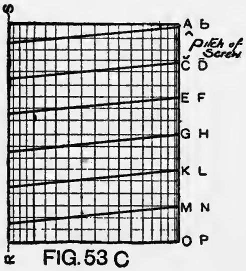


FIG. 53 C

Experiment XXVI.—Take a piece of squared paper similar to ABOP, Fig. 53 B. Draw the oblique lines by joining CB, ED, etc. Now take a cylinder, with circumference equal in length to AB, and paste the paper on it so that AO coincides with BP.

The oblique lines now form one continuous spiral curve round the cylinder, as in Fig. 53 C.

In travelling once round the screw from A to C, we have descended one division AC, twice round would bring us down through two divisions to E, and so on. The distance AC is called the pitch of the screw. It is the vertical distance between two successive parts of the parallel thread.

In turning a screw—

the power : resistance to be overcome = pitch of screw : resistance.

$$P \quad : \quad W \quad = \quad AC \quad : \quad AB$$

$$\therefore \; P = \frac{AC}{AB} \times W$$

It follows that the smaller the pitch of the screw (AC), the less power is required to lift the same weight.

Take for example the "Screw Jack" (Fig. 54), used for lifting heavy weights a short distance.

(1) **The power** is supplied at the end of the lever, which can be fitted into the holes in the nut F. It is the same as using a screw,

having a circumference equal to that traced out by the handle of the lever.

(2) The weight (W) to be lifted, is in position above the top of the "Jack," and by applying force at P the weight is gradually raised—

$$\frac{P}{W} = \frac{\text{Pitch of screw}}{\text{Cir. of circle traced out by end of lever}}$$

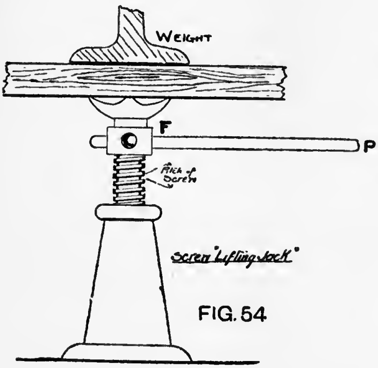


FIG. 54

It now follows that the longer the lever, the greater is the lifting force for the same power exerted.

Example.—What force or power must be exerted (neglecting friction) at the end of the lever P, Fig. 54, to raise a large iron girder weighing 5 tons, length of lever from centre of screw 4 ft., pitch of thread $\frac{1}{2}$ in.?

Circumference traced by lever
$= 2 \times 4\pi = 8 \times 3{\cdot}1416 = 25{\cdot}1328.$

$\therefore$ P : W :: Pitch of screw : Circumference traced by lever.

$$P : 11200 \text{ lbs.} :: \tfrac{1}{2} \text{ in.} : 25{\cdot}1328 \text{ ft.}$$
$$= 18{\cdot}5 \text{ lbs.}$$

which is the force necessary to shift the lever (neglecting friction).

THE WHEEL AND AXLE

This simple machine consists of two cylinders of different radii which rotate on the same axis. The cylinder with the large radius is called the **wheel**, and the one with the smaller radius is called the **axle** (Fig. 55).

Its principle is merely an application of the lever with unequal arms.

The weight is raised by coiling rope on the axle, and the power is supplied at the circumference of the wheel. According to the principle of the lever, the moment of the weight (W) about the axis (F) is equal in amount and opposite in direction to the moment of the power (P) about F.

$$\therefore\ P \times Fc = W \times Fb$$

$$\therefore\ \frac{P}{W} = \frac{Fb}{Fc}$$

$$\therefore\ \frac{\text{Power}}{\text{Weight}} = \frac{\text{Radius of axle}}{\text{Radius of wheel}}$$

It is commonly applied in the windlass, the capstan, the water-wheel crank, etc.

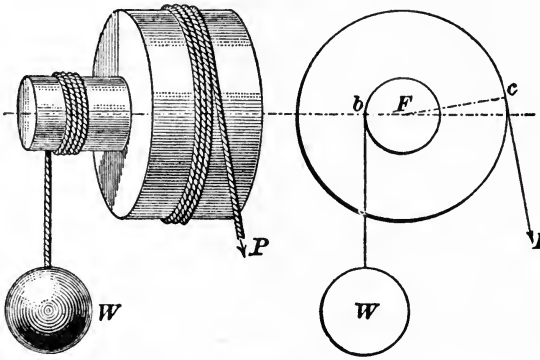


FIG. 55

Example.—A windlass has an axle of 9-in. radius, and handspikes 6 ft. long from axis; find the power required to haul in a load of 1 ton.

$$\frac{\text{Power}}{\text{Weight}} = \frac{\text{Radius of axle}}{\text{Radius of wheel}}$$

$$\frac{x}{2240 \text{ lbs.}} = \frac{9 \text{ ins.}}{72 \text{ ins.}}$$

$$x = \frac{9 \times 2240}{72} \text{ lbs.}$$

$$= \underline{280} \text{ lbs.}$$

CHAPTER XIV

WORK AND ENERGY, FALLING BODIES, FLOW OF WATER THROUGH APERTURES AND PIPES AND OVER WEIRS, HYDRAULIC MEAN DEPTH, BURSTING PRESSURE OF PIPES, THE HYDRAULIC RAM

WORK AND ENERGY

Work is the overcoming of resistance which continually occurs along a path of motion.

Mere motion is not work, but if a body in motion constantly overcomes a resistance, it does work.

The unit of work is the "foot-lb."—*i.e.*, the energy necessary to raise 1 lb. vertically through 1 ft. All work is measured by this standard.

A horse hauling a wagon loaded with goods up a hill, does an amount of work equal to its own weight + the weight of the wagon and contents, + the extra energy due to the effect of friction expressed as equivalent weight, × the vertical height of the hill.

Thus, if the horse weighs 1000 lbs. and the wagon and contents 2200 lbs., and the friction involves extra energy sufficient to add 400 lbs. to the load, and the height of the hill is 120 ft.—

The work done is equal to $(1000 + 2200 + 400) \times 120 = 432{,}000$ foot-lbs.

The total amount of work in this example is independent of time, whether it takes 1 minute or 1 month, but, in order to compare the results of work done with a common standard, time must be considered.

When time is considered, the **unit of time** is always **1 minute,** and the standard unit then becomes 1 foot-lb. per minute.

Since this unit of power is very small, and would necessitate the use of very large numbers in expressing the work done by large machines, the **common standard** to which all work is reduced is the **horse-power.**

One horse-power = 33,000 foot-lbs. per minute.

In other words, it is 1 lb. raised vertically 33,000 ft. in 1 minute, or 33,000 lbs. raised 1 ft. vertically in 1 minute, or any product of weight and vertical height which will give 33,000 foot-lbs. per minute.

Example.—Eight men are employed in raising a sheet of lead weighing 25 cwts., by means of suitable tackle, to the top of a building 60 ft. high; time taken in actually raising the lead = 5 minutes. How much work, expressed by horse-power standard, was done (neglecting friction)?

Work done $= 25 \times 112 \times 60 = 168{,}000$ foot-lbs. in 5 minutes.

$$\therefore \quad \frac{168{,}000}{5} = 33{,}600 \text{ foot-lbs. per minute.}$$

$\therefore$ Work expressed as horse-power, or horse-power developed

$$= \frac{33{,}600}{33{,}000} = 1\tfrac{1}{55} \text{ horse-power (usually written h.p.).}$$

Energy is the term used to express the ability of an agent to do work; the work that a moving body is capable of doing whilst being brought to rest is called the **Kinetic Energy** of the body.

Kinetic energy means the actual energy of a body in motion. The work which a moving body is capable of doing in being brought to rest, is exactly the same as the kinetic energy developed by it when falling in a vacuum through a height sufficient to give it the same velocity.

FALLING BODIES

If a lead ball and a piece of paper are dropped from the same height, the ball will strike the ground first, not because the ball is heavier, but on account of the greater retarding action of the air upon the paper. If we place this same leaden ball and paper in the glass tube shown in Fig. 56, from which all the air has been exhausted, it would be found that when the tube was inverted both would reach the bottom at exactly the same moment, thus proving that it was only the resistance of the air in the former case which caused the lead ball to reach the ground first. The resistance may be nearly equalised by making the two bodies of the same shape and size. For example, if a wooden and an iron ball, having equal diameters, were dropped from the same height, they would strike the ground at almost exactly the same instant, although the iron ball may be ten times the weight of the wooden one.

FIG. 56

Acceleration is the rate of change of velocity. If a force acts upon a body free to move, then, according to the first law of motion, it will move for ever with the same velocity, unless acted upon by another force.

Suppose that, at the end of one second, the same force were to act again, the velocity at the end of the second second would be twice as great as at the end of the first second.

If the same force were to act again, the velocity at the end of the third second would be three times that at the end of the first. It will be thus seen, that, if a constant force acts upon a body free to move, the velocity of the body at the end of any time will be the velocity at the end of the first second multiplied by the number of seconds. If a body is dropped from a high tower, the velocity with which it approaches the ground will be constantly increased or accelerated; for the attraction of the earth, or force of **gravity**, is constant, and acts as a **constant accelerating force**, and is generally known as the Acceleration of Gravity.

The velocity acquired at the end of one second is (in this part of the earth) **32·2 ft.,* and the distance fallen = 16·1 ft.** The velocity that a body will acquire in falling through a given height equals the square root of the product of twice 32·2, and the given height; or, $V = \sqrt{2gh}$, in which "g" = Acceleration of Gravity (32·2) and "h" = height in feet through which the body has fallen.

The distance through which a freely falling body will pass in a given time, may be estimated by multiplying the distance through which the body travels in the first second, by the square of the time in seconds during which the body has been in motion.

It has been proved by experiment that a freely falling body starting from rest, will have fallen 16·1 ft. approximately at the end of the first second.

Therefore at the end of the second second it would have fallen through a distance of $2^2 \times 16{\cdot}1 = 64{\cdot}4$ ft., and at the end of the third second $3^2 \times 16{\cdot}1 = 144{\cdot}9$. Fig. 56 A shows this very clearly.

Hydrokinetics, Hydromechanics, or Hydraulics are the names given to the science which deals with water in motion.

The law of falling bodies applies to water as to solids. Thus, if a small aperture be made in the bottom or side of a pipe containing water, the theoretical velocity of the water issuing from the aperture will be

$$V = \sqrt{2gh}$$

g = 32·2

h = Vertical height in feet of the pipe.

This is known as the **Velocity of Efflux.**

Example.—What is the velocity of the water issuing from an aperture in the side of a tank 20 ft. below the level of the water in the tank?

Velocity = $\sqrt{2gh} = \sqrt{2 \times 32{\cdot}2 \times 20} = 35{\cdot}88$ ft. per second.

This is the theoretical velocity, the actual velocity may be obtained by multiplying by ·98.

$\therefore$ The actual velocity = theoretical velocity × ·98 = 35·88 × ·98 = 35·16.

* (Correctly 32·17.)

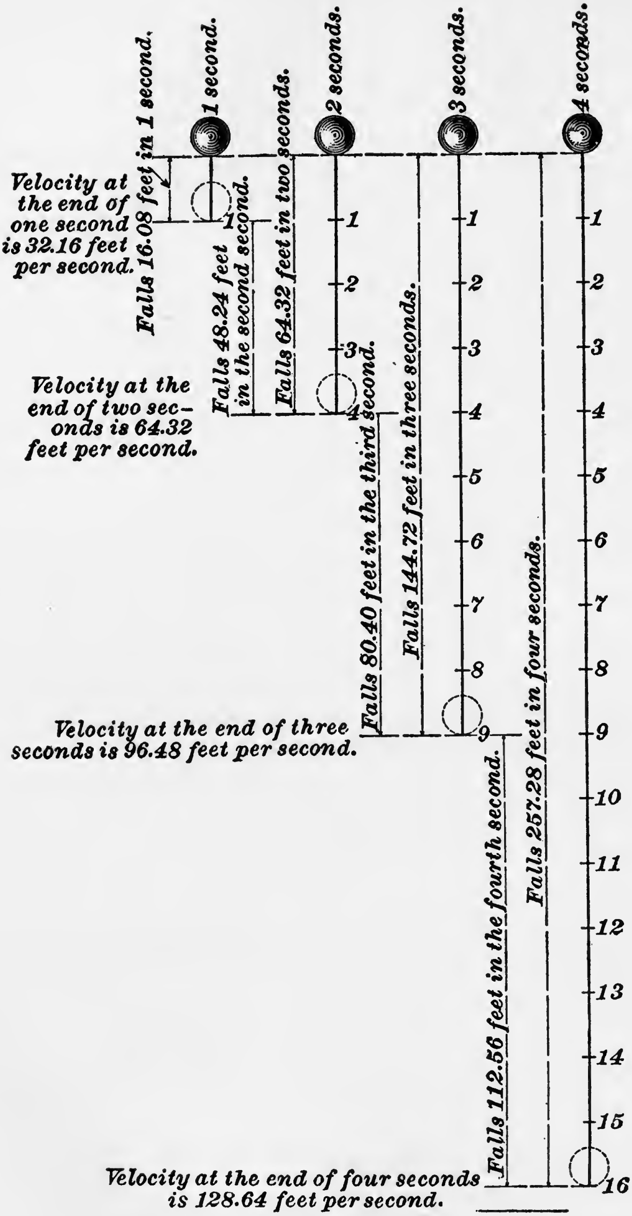

1 second.
2 seconds.
3 seconds.
4 seconds.
Falls 16.08 feet in 1 second.
Velocity at the end of one second is 32.16 feet per second.
Falls 48.24 feet in the second second.
Falls 64.32 feet in two seconds.
Velocity at the end of two seconds is 64.32 feet per second.
Falls 80.40 feet in the third second.
Falls 144.72 feet in three seconds.
Velocity at the end of three seconds is 96.48 feet per second.
Falls 112.56 feet in the fourth second.
Falls 257.28 feet in four seconds.
Velocity at the end of four seconds is 128.64 feet per second.
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16

FIG. 56A

This reduction is due to the following :—

When water issues from an orifice in a thin plate, the stream is contracted at a short distance from the orifice, and expands again to its full size. The point at which the contraction is greatest is equal (in distance from the orifice) to about one-half the diameter of the orifice; in consequence of the contraction and the friction on the edges, also the resistance of the air, the velocity of efflux is slightly reduced to the extent above stated, *i.e.*, ·98. The contraction (Fig. 57) is known as

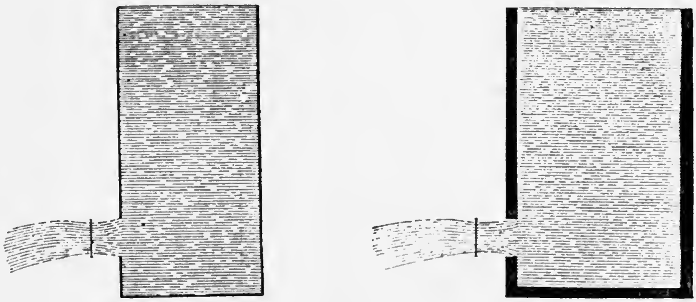


FIG. 57

the contracted vein (*vena contracta*), and in the above example would be about ·8 of the diameter of the orifice, and $\cdot 8 \times \cdot 8 = \cdot 64$ of the area of the orifice.

This, when multiplied by ·98 (the number required for obtaining the actual velocity of efflux), gives the coefficient of efflux, and when it is necessary to find the quantity discharged, the theoretical discharge must be multiplied by the coefficient of efflux to give the actual discharge.

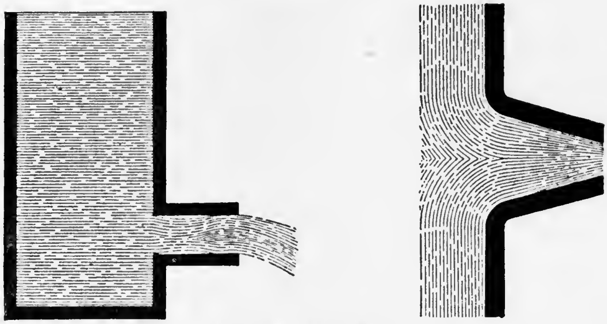
Example.—Find the discharge in cubic feet per second from an aperture 6 in. diameter in the bottom of a tank 30 ft. deep.

Quantity discharged in 1 second = area of orifice $\sqrt{2gh} \times$ coefficient of efflux.

$$= \frac{1}{4} \times \frac{1}{4} \times \frac{22}{7} \sqrt{2 \times 32 \cdot 2 \times 30} \times \cdot 627$$

$$= 5 \cdot 406 \text{ cub. ft. per sec.}$$

If the water discharges through a short tube (Fig. 58) whose length is from one and a half to three times the diameter of the orifice, the discharge is increased. From a large number of experiments, the coefficient of efflux for a short tube may be taken as ·815—*i.e.*, the actual discharge may be taken as ·815 times the theoretical discharge. If the inside edges are well rounded, and the tube conical shaped (Fig. 59), there will be no contraction, and the coefficient of efflux may be taken as ·97.



FIGS. 58 AND 59

WEIRS

The weir is a device generally used for gauging or measuring the quantity of water flowing down a stream (Fig. 60). It consists of a rectangular notch cut into a board placed across the stream at a convenient spot. The width and depth of the notch vary, according to local requirements. A V-shaped notch is also used in some instances.

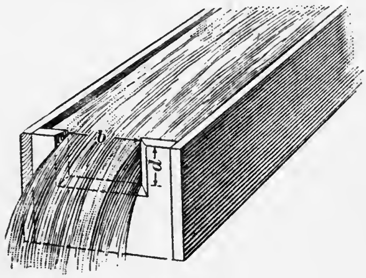


FIG. 60

When the depth of water flowing over the sill is known (this is usually measured some distance up stream from a peg, the top of which is level with the sill of the weir), and the width of the notch also, the volume discharged in a given time may be ascertained by the following formulæ:—

(1) $5{\cdot}15\sqrt{H^3} \times W =$ cubic feet discharged per minute.

(2) $214\sqrt{h^3} \times W =$,, ,,

In the first formula $H =$ height in inches of water flowing over sill.
In the second ,, $h =$ height in feet ,, ,,
In both $W =$ width of notch in feet.

Example.—What is the discharge in cubic feet over a weir, the width of which is 2 ft. and the depth of water flowing over same = 1 ft. 6 ins.?

Amount discharged $= 214\sqrt{h^3} \times W = 214\sqrt{\frac{3}{2} \times \frac{3}{2} \times \frac{3}{2}} \times 2$
(Using No. 2.)

$$= 214 \times 1{\cdot}837 \times 2 = 786{\cdot}23 \text{ cubic feet per minute.}$$

FLOW OF WATER THROUGH PIPES

When water flows through pipes, from reservoirs or tanks, to supply towns or fittings fixed in houses, the velocity of efflux is considerably less than the theoretical velocity due to the head of water in the reservoir or tank. This loss is due to several causes, but is principally the result of the friction of the water against the inside surface of pipes; this friction varies directly as the length of the pipe, and inversely as the diameter; which means that the friction in a pipe 200 ft. long is twice that in a pipe 100 ft. long, and the friction in a pipe 4 ins. diameter is only half that in a pipe 2 ins. diameter of equal length (the velocity being the same in both cases). The friction also varies, nearly as the square of the velocity.

The formulæ used for ascertaining the discharging capacities of various pipes under given heads, have been constructed by the aid of higher mathematics, taking into account the various circumstances which affect the discharge.

The following formulæ have been constructed by a well-known authority on hydraulics, by the aid of which the head, discharge, diameter of pipe, and length of pipe required under various conditions may be ascertained.

In the following formulæ:—

D = Diameter of pipe in inches.
L = Length of pipe in yards.
H = Head of water in feet.
G = Gallons discharged or required per minute.

$$G = \sqrt[2]{\frac{(3D)^5 \times H}{L}}$$

$$H = \frac{G^2 \times L}{(3D)^5}$$

$$D = \sqrt[5]{\frac{G^2 \times L}{H}} \div 3$$

$$L = \frac{(3D)^5 \times H}{G^2}$$

For the easy solution of the above formulæ, it is necessary to use logarithms, and as this part of mathematics has not been dealt with, no examples of the above will be given.

To find the **discharging capacity of sewers or drains**, when the water flowing through them is not under any head, various formulæ have been devised. The following is an example of one in which—

V = Velocity in feet of water per minute.
H = The hydraulic mean depth.
F = The fall in feet per mile.
D = The discharge in cubic feet per minute.
A = Cross-sectional area of drain or sewer in feet.
55 = A constant

(*a*) Velocity in feet per minute $= 55\sqrt{H \times 2F}$

(*b*) Fall in feet per mile $= \left(\dfrac{\frac{D}{A}}{55}\right)^2 \div 2H$

(*c*) $D = V \times A$, or discharge = the velocity in feet per minute multiplied by the area of the cross-section of the pipe.

The term **Hydraulic Mean Depth** is used in these calculations to indicate the depth of a rectangular channel, the width of which is equal to the wetted perimeter of the pipe under observation. It is obtained by dividing the sectional area in feet of water flowing, by the wetted perimeter in feet, or that portion of the pipe's internal surface covered by the flowing water. If the pipe is discharging full bore or only half bore, the hydraulic mean depth will be the same in both cases, and may be found by dividing the diameter of the pipe in feet by 4.

When the velocity has been obtained, the discharge may be found by rule C, which consists of multiplying the sectional area of water flowing by the velocity in feet per minute.

Example.—(1) Find the discharge in cubic feet per minute from a drain 9 ins. diameter, discharging half-full with a fall of 1 in 120.

$$\text{Velocity} = 55\sqrt{H \times 2F}$$
$$\text{and discharge} = V \times A$$
$$\therefore \quad V = 55\sqrt{\cdot 1875 \times 2 \times 44}$$
$$= 55 \times 4\cdot 062 = 223 \text{ ft. per minute.}$$
$$\text{and } D = V \times A = 223 \times \frac{99}{448} = 49\cdot 27 \text{ cub ft. per minute.}$$
$$H = \cdot 75 \div 4 = \cdot 1875$$
$$F = \frac{1760 \times 3}{120} = 44$$
$$A = \frac{\pi R^2}{2} \text{ (\textit{i.e.}, half area of 9-in. pipe in feet)}$$
$$= \frac{22}{7} \times \frac{3}{8} \times \frac{3}{8} \times \frac{1}{2} = \frac{99}{448} \text{ sq. ft.}$$

Example.—(2) What fall in feet per mile will a 9-in. diameter drain require to enable it to discharge 140 cub. ft. of water per minute?

$$\text{Fall in feet per mile} = \left(\frac{D}{A \times 55}\right)^2 \div 2H$$

$$= \left(\frac{140}{\cdot 4419 \times 55}\right)^2 \div 2H = 5{\cdot}76^2 \div {\cdot}375$$

$$= \frac{33{\cdot}1776}{\cdot 375} = 88{\cdot}4 \text{ ft. fall per mile, or 1 ft. in 60 (nearly).}$$

In all cases where water flows through pipes or channels, the quantity discharged is materially affected by the number and type of bends which occur on the pipe or channel; if the bend is constructed to a small radius or in the form of a right angle, it proves very destructive to the velocity; bends, when necessary, should be constructed to as large a radius as possible. These remarks refer equally, whether the pipe or channel is used for hot or cold water supply, soil, waste, vent, or drainage purposes.

BURSTING PRESSURE OF PIPES

It is often necessary to find the bursting stress, or the pressure per square inch, which will fracture a pipe of known thickness and diameter. There is no empirical rule which is applicable to pipes made from all kinds of materials and varying in thickness.

Pipes subjected to an internal pressure do not withstand the stress equally through the cross-section of the pipe; the internal portion is strained considerably more than the external portion; therefore the power to resist internal pressure is not in all cases proportional to the thickness.

The following formulæ, constructed by Barlow, are suitable for thick pipes, such as lead pipes, but are not specially adapted for thin pipes:—

$$(1) \qquad T = \frac{R \times P}{S - P}$$

$$(2) \qquad P = \frac{S \times T}{R + T}$$

$$(3) \qquad S = \frac{(R + T) \times P}{T}$$

In the above

T = Thickness of metal in inches.
P = The internal pressure per square inch.
R = Inside radius of pipe in inches.
S = Tensile strength of the metal in pounds per square inch.

By the above rules, the thickness at which a pipe will fracture, under a given pressure, is obtained, and also the bursting pressure and strength, so that in actual practice the thickness of a pipe necessary to withstand a given pressure must be considerably greater than is necessary to just withstand (or fracture at) a given pressure.

The margin of safety thus allowed is usually from 3 to 10, which means that if a pipe of a given thickness will just withstand (or fracture under) a given pressure, then such pipe should only be subjected in actual practice to from 1/3 to 1/10 (or thereabouts) the bursting stress.

Examples.—(*a*) Find the bursting stress of a 2-in. diameter lead pipe $\frac{1}{4}$ in. thick, taking the tensile strength of lead as 2500 lbs. per square inch. Using No. 2 :—

$$P = \frac{S \times T}{R + T} = \frac{2500 \times \cdot 25}{1 + \cdot 25} = \frac{625}{1 \cdot 25} = 500 \text{ lbs.}$$

Thus, in actual practice, 50 to 180 lbs. would be the maximum pressure per square inch to which the pipe should be subjected.

(*b*) Find the thickness of a 2-in. lead pipe necessary to withstand a pressure of 200 lbs. per square inch (to be laid in actual practice).

Using No. 1 :—

$$T = \frac{R \times P}{S - P} = \frac{1 \times 200 \times 5}{2500 - 1000} = \frac{1000}{1500} = \frac{2}{3} \text{ of an inch.}$$

Note.—The factor of safety has been taken as 5.

A formula which is frequently used, is as follows :—

$$P = \frac{S \times T}{\frac{D}{2}}$$

P = Pressure in pounds per square inch.
S = Ultimate tensile strength.
T = Thickness of pipe wall in inches.
D = Diameter of pipe in inches.

Example.—Find the bursting stress of a 4-in. wrought-iron pipe; thickness of pipe wall $= \frac{1}{4}$ in. Ultimate tensile strength of wrought iron $= 60{,}000$ lbs. per square inch.

$$P = \frac{S \times T}{\frac{D}{2}} = \frac{60000 \times \frac{1}{4}}{\frac{4}{2}}$$

$$= \frac{60000 \times 1 \times 1}{4 \times 2}$$

$$= 7500 \text{ lbs. per square inch.}$$

The following formula is suitable for calculating the thickness of cast-iron pipes used for water and gas supply.

$$T = \frac{(P + 100) \times D}{\cdot 4TS} + \cdot 333\left(1 - \frac{D}{100}\right)$$

in which T = Thickness of metal in inches.
P = Pressure in pounds per square inch.
TS = Ultimate tensile strength in pounds per square inch.
D = Internal diameter or bore of pipe in inches.

Example.—Find the thickness necessary for a 6-in. cast-iron water main to resist an internal pressure of 300 lbs. per square inch (with a reasonable safety margin).

Note.—The previously mentioned formula allows for a safety margin, and the result gives the actual thickness of the metal.

$$T = \frac{(P+100)\times D}{\cdot 4TS} + \cdot 333\left(1 - \frac{D}{100}\right)$$

$$= \frac{(300+100)\times 6}{\cdot 4 \times 18000} + \cdot 333\left(1 - \frac{6}{100}\right)$$

$$= \frac{2400}{7200} + \left(\cdot 333 \times \frac{94}{100}\right)$$

$$= \frac{1}{3} + \frac{313}{1000} = \frac{2}{3} \text{ of an inch nearly.}$$

The ultimate tensile strength of cast iron is taken as 18,000 lbs. per square inch.

TABLE OF TENSILE STRENGTHS OF METALS

Metal.	Average Tensile Strength in lbs. per sq. in.
Copper (cast)	23,500
do. (sheet)	30,000
do. (wire) unannealed	60,000
Gunmetal (Copper and Tin)	39,000
Brass (cast)	23,000
do. (wire) unannealed	80,000
Tin (cast)	4,600
Zinc (cast)	3,700
Lead (cast)	2,000
do. (milled)—*i.e.*, in pipes	2,500
Iron (cast)	18,000
do. (wrought)	60,000
Steel (cast)	80,000
do. (wire) unannealed	156,000

The hydraulic ram, shown in Fig. 61, is an appliance used for raising water automatically to a required height. It consists of a cast-iron body possessing a large valve (*b*) known as the **dash valve,** and also an opening over which is fixed an **air-chamber** (*f*) possessing a small, exceptionally **strong valve** (*d*) **at its base; a small air hole** (*g*) is provided just under the "rising main valve," which plays a most important part in maintaining the continuous working of the ram. Connected to the air vessel is a **delivery pipe**, or rising main, which terminates in the storage tank at the point of delivery; connected to the ram at one end is the **drive or injector pipe** (*a*), controlled by valve (*n*), which supplies the energy, in the form of water, for working the ram. The injector pipe is led at an even gradient to a supply tank or well, 3 or more feet above the ram, and a certain distance from it.

The working of the ram is as follows:—

When the drive pipe is charged with water and the dash valve forced down, a quantity of water escapes, which brings the water in the drive pipe in motion, the friction caused by the water escaping around the valve (*d*), and the power of the moving body of water in

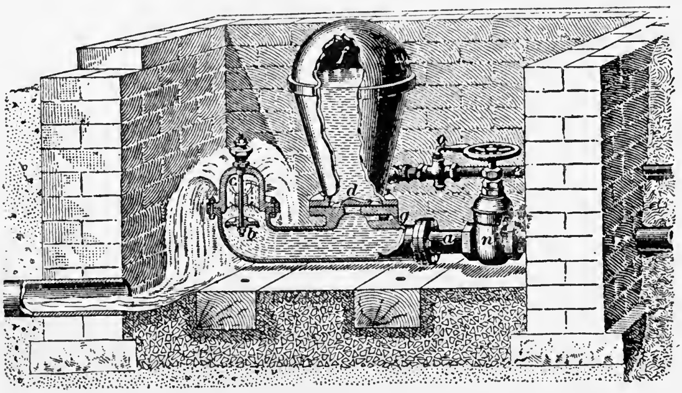


FIG. 61

the ram suddenly closes the dash valve and stops the train of water in the drive pipe, thereby causing a great accumulation of pressure for a moment, which lifts the small rising main valve, and allows a quantity of water to pass into the air vessel, compressing the air, which in turn gradually forces the water along the rising main. The concussion is immediately followed by a rebound, during which the pressure of the water in the ram is momentarily reduced below atmospheric pressure. The large valve (*b*) falls, and a small quantity of air passes through (*g*) and rises into the air-chamber at the next beat of the valve (*b*). The water again escapes past the dash valve and the action is repeated. This continual beating of the valve is only interrupted when the ram is short of water or requires attention.

The small hole "*g*" is provided for renewing the air in the air-chamber, which is absorbed by the water, owing to it being under pressure. **If the air in the chamber became exhausted, the ram would cease to work,** due to the enormous friction which would require to be overcome in forcing the water immediately along the delivery pipe at the same velocity at which it is delivered into the air-chamber past valve "*d*"; the inertia of the water would also absorb a large amount of energy to cause motion at each beat. Moreover, the air acts as a cushion and prevents damage by shock to the ram.

The length of the drive pipe largely influences the working of the ram; if it is too short, the ram will not throw up much water. Its length should be one or one and a quarter times the vertical height to which the water has to be raised.

The amount of water which a ram can deliver compared with the actual quantity passing through the ram, depends upon—

(*a*) The vertical height above the ram to which the water has to be raised.

(*b*) The vertical height above the ram of the inlet of the drive pipe, or the water-level in the tank supplying the ram.

(*c*) The length of the drive pipe.

(*d*) The size of the ram.

The size and total length of the delivery pipe also has some influence upon the quantity discharged, owing to friction.

The minimum head of water on the ram should never be less than 2 ft. 6 in., and it is far better to arrange if possible for six or eight times that amount.

The theoretical effective duty of a ram may be obtained if we know: (*a*) the quantity of water passing through the ram; (*b*) the vertical height or total fall of the water from the tank to the ram, and the height to which the water has to be raised above the ram.

Thus: If a ram, working with 10 ft. head of water, and delivering water at a point 200 ft. above the ram, uses 16,000 galls. per day, what quantity theoretically will be raised?

Total amount passing through the ram per day = 16,000 galls.

Which, falling through the drive pipe a vertical distance of 10 ft., generates a total force of—

$$16000 \times \overset{\left(\substack{\text{Weight of a}\\ \text{gall. of water.}}\right)}{10} \times 10 = 1600000 \text{ foot-lbs.}$$

The height of delivery above ram = 200 feet.

$$\therefore \quad \frac{1600000}{200} = \text{number of pounds raised 200 ft.}$$

$$= 8000, \text{ which, divided by } 10, = 800 \text{ galls. per day.}$$

The actual quantity discharged would be considerably less, owing to friction and leaking valves, and would probably represent $\frac{2}{5}$ of the above, though this would depend upon local conditions.

Taking the actual discharge as $\frac{2}{5} \times \frac{800}{1} = 320$ galls.

which is a low estimate.

The percentage of efficiency may be stated as the ratio between 320 gallons raised 200 feet and 16,000 falling 10 ft.

$$= \frac{320 \times 200 \times 100}{16000 \times 10} = 40 \text{ per cent.}$$

Under some conditions 60 per cent. to 70 per cent. efficiency may be obtained.

CHAPTER XV

PNEUMATICS, BAROMETERS, BOYLE'S LAW, PRESSURE GAUGES, THE SYPHON, PUMPS

PNEUMATICS

The air we breathe envelops the earth to a distance of more than fifty miles above the surface of the earth. Since air has weight, it is evident that this huge mass of air must exert an enormous pressure on all bodies. The following experiments are devised to show the pressure of the atmosphere.

Experiment XXVII.—Take a small can, so made that the neck can be closed with a small cork. Put in some water, and then boil the water for a few minutes. Then close the mouth with the cork, and cool the can. In a short time the sides of the can are forced in. The reason is that the pressure of the steam has been greatly reduced by condensation, whilst the pressure of the air on the outside of the can is not altered; hence, when the difference in pressure between the inside and outside becomes greater than the can is capable of withstanding, the collapse is towards the region of little pressure (*i.e.* inwards).

Experiment XXVIII.—Take a jar, open at both ends. Fit one end over the receiver of an air pump, and close the other end by firmly fastening a sheet of indiarubber over it. On exhausting the air from the jar, the indiarubber is forced inwards, because the pressure of the air inside the jar is reduced below the pressure of the air outside the jar.

Experiment XXIX.—Fill a tumbler completely with water, and carefully cover with a sheet of notepaper. If done carefully, the tumbler can be inverted, and yet the water remains in the tumbler. This is because the pressure of the air upwards is greater than the pressure of the water (its weight) in the tumbler downwards.

In dealing with balancing columns in Experiment XXV., it was shown that a column of one liquid might be balanced by another liquid. By using a modified form of apparatus, it is possible to balance a column of air with another liquid. This is the principle of the barometer.

The Barometer is an instrument used for measuring the pressure of the atmosphere.

Experiment XXX.—To make a barometer, take a glass tube about 36 ins. long and closed at one end (Fig. 62). Pour in mercury. When nearly full close the open end, and invert several times to get rid of air bubbles. Then completely fill with mercury. Close the end with the thumb, and invert it in a basin of mercury. At once the mercury sinks to a certain level and is then steady. The space left is entirely void, and is called a vacuum, and is known as the Torricellian vacuum. The height of the mercury in the tube above the level outside the tube is called the barometric height.

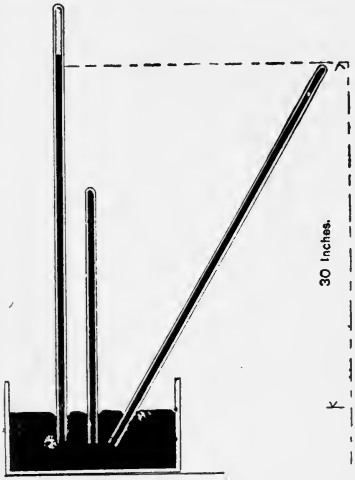


FIG. 62

Whether the tube be held vertically or obliquely, the barometric height at that time is always the same perpendicular height above the outside level, if the tube be sufficiently long to allow it to rise to its own level.

Let us consider the two forces acting upon the open end of the tube:

(1) The weight of the mercury in the tube acting vertically downwards.

(2) The pressure of the atmosphere on the mercury in the tube, which, by means of the mercury in the vessel, acts vertically upwards.

When these two balance one another, then the pressure of the air is equal to the pressure caused by this column of mercury.

The barometric height is generally about 30 ins. or 76 cm. at sea-level, but, of course, it varies from place to place, and from time to time. Thus, on ascending a mountain, the barometric height gradually falls roughly at the rate of 1 in. in 1000 ft. ascent. Again, the presence of water vapour in the air decreases the density of it; hence the barometric height is lowered, and an excess of water vapour causes rain, and *vice versâ*. On this account the barometer is used as a rough indicator of the state of the weather.

Certain kinds of weather are associated with certain levels, thus:—

Very dry	31·0 ins.	Rain	29·0 ins.
Set fair	30·5 „	Much rain	28·5 „
Fair	30·0 „	Stormy	28·0 „
Change	29·5 „		

KINDS OF BAROMETERS

(1) The wheel barometer consists of a syphon tube as in the previous example. On the mercury in the short arm open to the atmosphere a float is arranged, connected to a balance weight by a cord which passes over a small pulley to which is attached the index finger. The variation in atmospheric pressure alters the level of the mercury in the short arm, and raises or lowers the float, which causes the movement of the index round the graduated dial (Fig. 64).

FIG 63

This is the construction of barometer, Fig. 63.

(2) The more modern form is shown in Fig. 65. It is named Fortin's barometer. It is specially constructed for accurate readings, and requires regulating at each observation. It is possible to read variations of 1/500 part of an inch.

It consists of a truly cylindrical tube of glass enclosed by a brass cover. The upper end is sealed and the tube is charged with mercury. The lower end is placed in the mercury held in a small reservoir which is provided with an adjustable base. Before taking a "reading," the mercury in the reservoir is adjusted by the milled screw until its surface makes contact with the point of an ivory peg fixed to the cover of the reservoir. The ivory point denotes the base of the mercury column, and the upper surface is accurately determined by an adjustable cursor, fixed horizontally on the graduated scale. A Vernier attachment renders minute changes easy of determination.

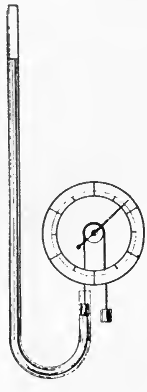


FIG. 64

Mercury is commonly used in barometers because it is the heaviest liquid, but it is possible to make a barometer containing any liquid.

Suppose that water be used instead of mercury. The column of water must be 13·6 times (as long as) the column of mercury, for the specific gravity of mercury is 13·6

$$\therefore \quad \text{Length of water column} = 13{\cdot}6 \times 30 \text{ ins.}$$

$$= \frac{13{\cdot}6 \times 30}{12} \text{ ft.} = 34 \text{ ft.}$$

Thus a column of water 34 ft. high would balance the atmospheric pressure, and it would form a water barometer.

The Aneroid Barometer, shown in Fig. 66, is made in various sizes, ranging from 2 ins. to 10 ins. diameter.

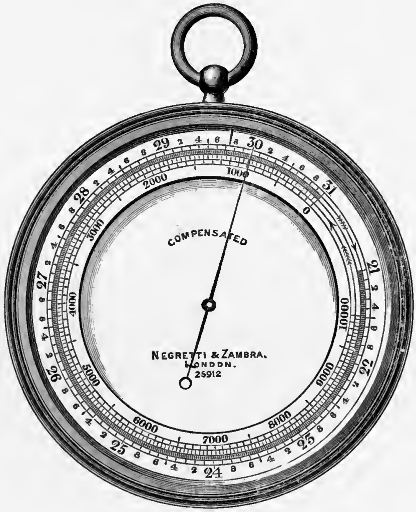


FIG. 66

It consists of a cylindrical box of metal, with a top of thin, elastic, corrugated metal, from which the air is removed. When the atmospheric pressure increases, the top is pressed inwards, and when it is diminished, the top is pressed outwards by its own elasticity, aided by a spring beneath.

These movements are transmitted and multiplied by a combination of delicate levers which act upon an index hand, and cause it to move, either to the right or left of a graduated scale. These barometers are self-correcting for variations in temperature.

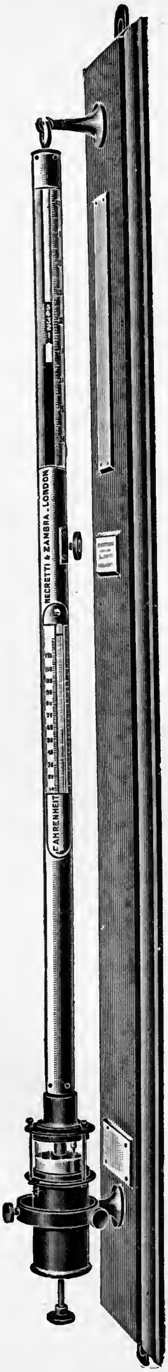


FIG. 65

They are very useful for approximately ascertaining the height of mountains above an observed point.

They are not standard barometers.

BOYLE'S LAW

In experiments with balancing columns it was found that when both tubes were open, water was at the same level in both tubes. Now close one tube by a rubber stopper, and then pour more water down the other tube. The water in the closed tube rises slowly, whilst that in the open tube rises rapidly, and they no longer have the same level. The difference is due to the air in the closed tube being compressed as the water rises; hence the pressure of the air in the closed tube is increased as its volume is decreased.

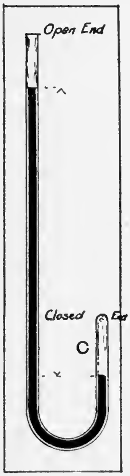


FIG. 67.

Experiment XXXI.—Take a U-shaped tube with a long arm open at the top and a short arm closed at the top (Fig. 67). This is called a Boyle's Law tube. Clamp it vertically and pour a little mercury down the open tube. Then measure the length of the column of air in the closed tube, and also the difference in level of the mercury in the two sides. To get the total pressure at C, add to the difference in level the atmospheric pressure.

If more mercury, in small quantities, be poured into the open end, it will be observed that the volume of air is gradually reduced by the pressure to which it is subjected, when the difference in level of the two columns of mercury is sufficient to give a pressure equal to one atmosphere—*i.e.*, 14·7 lbs. per sq. in.—it will be found that the volume of air in the closed end C is reduced to half its original volume. It is now subjected to the pressure of two atmospheres; if more mercury be poured into the tube until the difference in level of the two columns be twice the previous difference, it will be found that the air is reduced to one-third its original volume. Hence it follows **that the volume of a given quantity of air varies inversely as the pressure to which it is subjected.**

This conclusion is true for all gases when the temperature is not altered, and it is known as **Boyle's Law.**

PRESSURE GAUGES

Experiment XXXII.—To find the pressure of gas in a pipe.

Take a similar apparatus (Fig. 68) to the one used in the experiment with balancing columns. Put in some water, and connect one tube to the gas pipe by means of indiarubber tubing. Then open the tap. If the water in B rises, then the pressure of the gas in the pipes is less than the atmospheric pressure.

If the water-level is the same in both, then the pressure of the gas in the pipe is the same as the atmospheric pressure.

If the water in A **rises** and falls in B (and this is most likely), then the pressure of the gas is greater than the atmospheric pressure. The difference in level gives the excess of pressure in the pipe over that of the air.

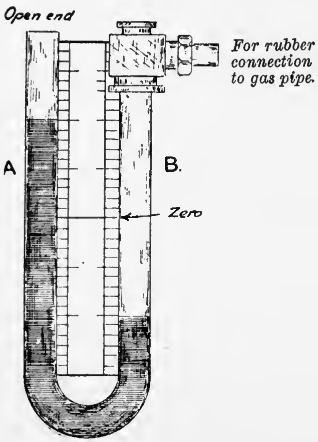


FIG. 68

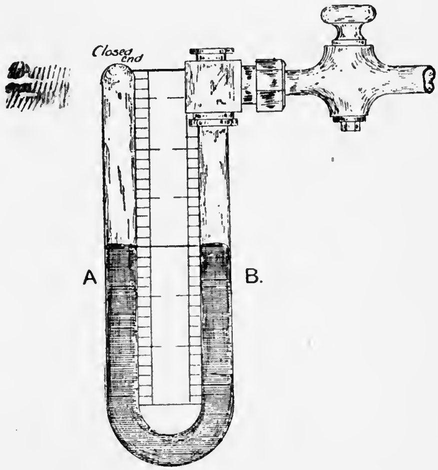


FIG. 69

Experiment XXXIII.—To find the pressure of **water** in a pipe.

Take a piece of stout glass tubing closed at one end and bend it as shown in the sketch, Fig. 69. Pour in some mercury, and adjust it until it has the same level in each side. Then fasten the open end to the water pipe by strong indiarubber tubing. Turn on the tap. The level of mercury will be forced down at B and up at A. The difference in level can be accurately measured by means of the scale, and from this can be obtained the pressure of water in the pipe.

THE SYPHON

The syphon is a bent tube having one arm shorter than the other (Fig. 70).

Experiment XXXIV.—Bend a piece of glass tubing so that one arm is twice the length of the other. Place the short arm in a vessel of water, and by suction fill the syphon with water. Water continues to flow through the syphon as long as the end of the short arm remains below the surface of the water.

The working of the syphon depends entirely upon the pressure of the atmosphere.

Let a = Atmospheric pressure.
b = Perpendicular height of long arm.
c = " " short arm above water-level.

At each end of the pipe the pressure is that of the atmosphere. On ascending each arm the pressure is decreasing, for if the pipe were 34 ft. long there would be **no** pressure at all at the top.

Suppose there is a partition at a, the highest point in the syphon—

Pressure at a in long arm = atmospheric pressure $- ab$.
" " short arm = " " $- ac$.

Hence there is a difference of pressure at a equal to the height bc (height of water-level above end of long arm).

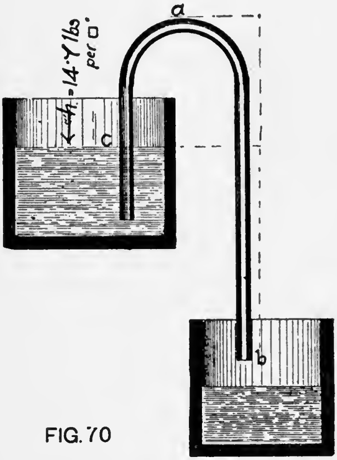


FIG. 70

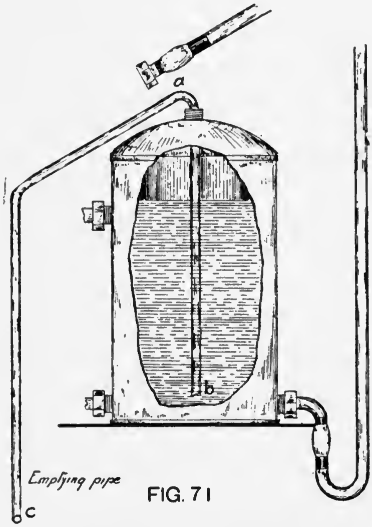


FIG. 71

Since the pressure is greater from the short arm, the water will flow up the short arm and down the long arm with a head equal to bc.

Continuous flow through the syphon can only be maintained as long as—

(1) The shorter arm is kept under the surface of the liquid.
(2) The length (ab) must be greater than ac.
(3) For syphoning water, ac must be less than 34 ft., because under normal atmospheric conditions the pressure of the atmosphere cannot make water rise more than 34 ft.

The principle of the syphon is made use of by the plumber in various ways. Fig. 71 shows its application for the removal of the contents of a hot-water cylinder previous to repairs, where no emptying tap exists.

Syphon Flush Cisterns for water-closets are identical in principle with the above. Fig. 72 shows an example. The flush is commenced by raising the valve in the cistern, which permits a quantity of water to fall down the flush pipe; when the valve is replaced, the falling water tends to create a partial vacuum at *a* and the pressure of the atmosphere on the surface of the water forces the water up the bent tube, when, after passing *a* it falls down the flush pipe, maintaining the partial vacuum caused primarily, until air is admitted by lowering of water-level to C.

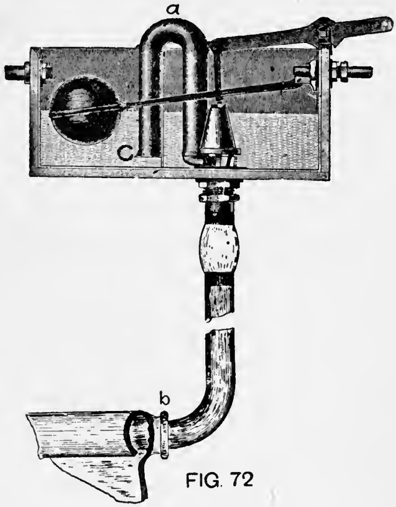
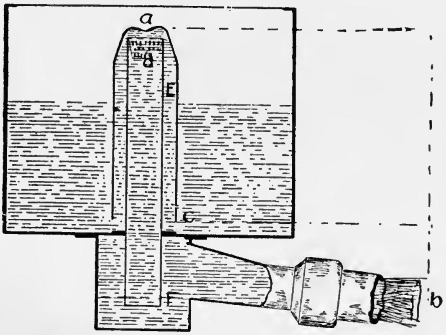


FIG. 72

The Automatic Flush Tank, shown in Fig. 73, is entirely self-acting. As the water rises in the tank, it also rises in the dome (E) at nearly the same rate, the air being forced out through the small water-seal F during filling. When the water-level under the dome reaches the top of the stand-pipe, it is about $\frac{1}{2}$ in. lower than the water-level in the tank around the outside of the dome, the difference being equal to the small water-seal at F. The water now travels slowly at first over the stand-pipe, and is thrown clear of the sides of the pipe by the perforated inverted frustum of a cone (D), the falling water forces some of the air contained in the tube through the water-seal F, thereby reducing the pressure gradually below that of the atmosphere, and causing a continually increasing volume of water to be forced over the stand-pipe, which, falling through F, charges pipe *b*, and thus sets up **syphonage.**



Field's Automatic Flush Tank

FIG. 73

PUMPS

The common pump, Fig. 74, consists of a pipe (P) which passes from the well to the cylinder, at the bottom of which is a valve (v), opening upwards. In the cylinder is a piston containing valves (u), which open upwards only. This piston is moved upwards and downwards by means of the arm or rod, and the water issues from the cylinder at the spout (A). The working of this machine depends upon the pressure of the atmosphere.

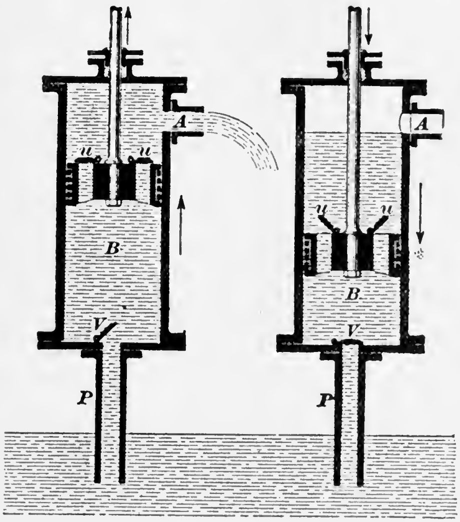


FIG. 74

Let us consider **the action of the pump.**

As the piston moves downwards the air in the lower part of the cylinder is forced out at the upper valves (u). On raising the piston the pressure is reduced, air enters the cylinder from the pipe P through the lower valve (V), and water rises in the pipe. On repeating this operation a few times, all the air is exhausted from the suction pipe and the lower part of the cylinder, and they are filled with water. On lowering the piston, water rushes through the upper valves and fills the space above the piston. On raising the piston, this water is also raised and runs out through the spout A.

In the former portion of this chapter it was shown that the pressure of the atmosphere was equal to the pressure at the base of a column of water 34 ft. high. It will now follow that if all the air in the pipe P and lower part of the cylinder be exhausted by the working of the piston, the water will rise to a height of 34 ft. but no higher, for this column of water has a pressure equal to that of the atmosphere. The essential condition theoretically for working the pump is that the piston must not be placed more than 34 ft.[1] above the surface of the water in the well.

It is found in actual practice that **25 ft. is the greatest height** to which the water will rise and be maintained there by the suction valves, owing to leaking valves and the difficulty of obtaining a perfect vacuum under the suction valve.

[1] This varies according to atmospheric conditions.

When the depth of the mean water-level is greater than 25 ft. below the surface, a pump known as the **Suction Lift Pump** is fixed in the well at a distance of 6 ft. from the water-level, and worked by rods coupled to a crank or engine on the surface of the well. Fig. 75 shows a suction lift pump.

It will be seen that the pump is enclosed at the top and the piston rod passes through a packing gland, a valve (*c*) is provided at the foot of the delivery pipe (P), for retaining the water forced past the valves. This permits greater freedom of action of the bucket. It will be seen that the other portion of the pump is the same as the suction pump. To relieve the stress on the working parts, an **air vessel** (D) is provided just above the valve C, the water being delivered from the pump barrel into the air vessel, and compressing the air, which expands and forces the water practically in one continuous stream through the delivery pipe. The capacity of the air vessel should be not less than the capacity of the pump barrel, and where the pump works at a greater speed than 35 strokes per minute it should have a still greater capacity.

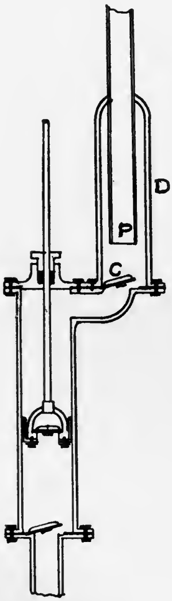


FIG. 75

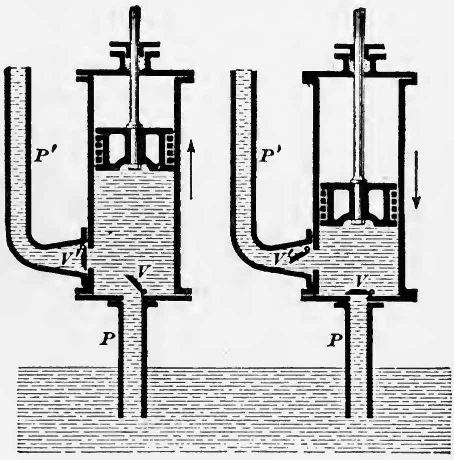


FIG. 76

The Force Pump, shown in Fig. 76, is similar in principle to one which is generally used by plumbers for unstopping pipes which have become blocked. It possesses a solid plunger which, when lifted, tends to create a vacuum, causing the water to flow into the barrel through suction pipe (P), and on the downward stroke the valve (V) closes, and the water is forced past valve (V′) into pipe (P′).

Of the same type as the force pump is the **Plunger or Ram Pump** (Fig. 77), which is generally used for delivering water to great heights, or against a great pressure (as for hydraulic press supplies). It consists of a solid plunger (C) which fits rather loosely in the cylinder, and when moved upwards, after the cylinder is full of water, it tends to create a vacuum, causing water to be lifted by atmospheric pressure through the suction valves at A, which are prevented from opening too far by the two uprights II. On the downward stroke the water is forced past valve B into the delivery pipe. The small pipe D is provided for the escape of air, which if allowed to accumulate under the plunger would eventually stop the delivery of water.

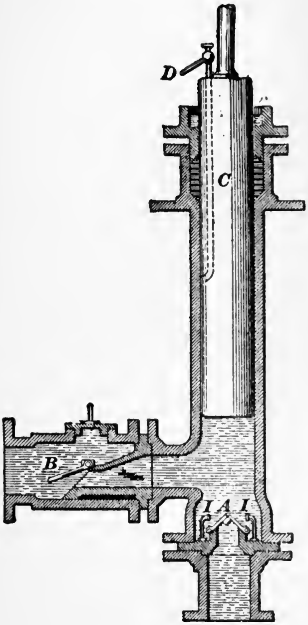


FIG. 77

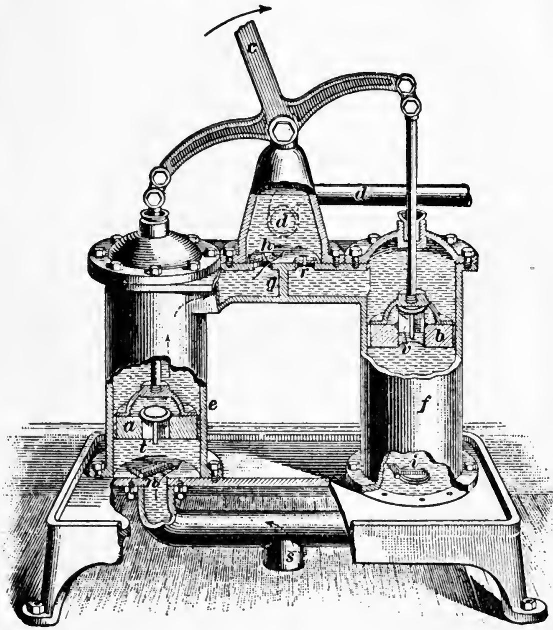


FIG. 78

In order to obtain a larger quantity of water and a more continuous flow, a **Double-Barrelled Pump** is sometimes used, one type of which is shown in Fig. 78.

This pump has one suction pipe (*s*) and one discharge pipe (*d*). The cylinders are separated by the diaphragm (*g*), which prevents communication between them. The piston buckets (*a* and *b*) work alternately up and down, the water passing through the valves (*n* and *i*) and then through *v* and *t*, and finally through *r* and *h* into delivery pipe (*d*).

Centrifugal Pump.—As the name implies, the effects produced by centrifugal force are made use of in this class of pump. It is particularly valuable for raising large volumes of water to great heights. Fig. 79 shows one with part of the casing removed. The centre (*s*) is hollow, and forms the commencement of the suction pipe, to which wings, vanes, or curved arms (*a*) are fixed. These revolve at a high speed in the direction indicated by the arrow, driving the air out through the passage and delivery pipe (*dd*) and causing a partial vacuum in the suction pipe, through which water is forced into the casing by atmospheric pressure. This water is caused to revolve at the same velocity as the vanes, the centrifugal force of the revolving water causes it to fly towards the extremities of the vanes and becomes greater the farther away it gets from the centre; this action compels it to leave the vanes, and finally leave the pump by delivery pipe (*d*). The water, when it is raised, should be delivered with as little velocity as possible; any velocity which the water exhibits at the point of delivery is superfluous, and adds to the cost of working the pump.

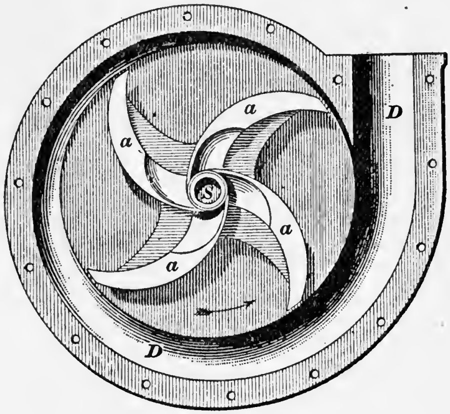


FIG. 79

The correct form and size of the vanes are directly responsible for the percentage of efficiency developed by the pump.

Water Wheels.—This term, generally speaking, may be applied to all rotary machines worked by hydraulic power, but for our purpose will only have reference to machines worked in a rotary manner by the weight of the water supplied to the buckets, causing them to travel at a low velocity, which has practically no relation to the head of water working the wheel.

There are three kinds: Overshot, Breast, and Undershot. The former receives the water near the top; the second named just above the centre, and the latter below the centre.

The degree of efficiency varies with each type.

The	1st	named	will	give	about	75	per cent.
,,	2nd	,,	,,	,,	,,	50	,,
,,	3rd	,,	,,	,,	,,	25	,,

Water wheels are sometimes used where an abundance of water exists, for working pumps for raising water to considerable heights. The pumps are generally coupled to the shaft of the wheel directly.

Turbines.—These are appliances constructed upon similar lines to water wheels, but the vanes or buckets are enclosed by a casing, and the water acts principally by impulse, or reaction upon them, causing them to revolve at a high velocity. There are various types of them, some in which the wheel runs horizontally, having a vertical axis, the outlet of which may or may not be submerged. In another type, the wheel revolves on a horizontal axis, the supply of water in this case is usually delivered at a high velocity in the form of a jet, which impinges upon the cups or blades, setting up a rotary motion of high speed.

CHAPTER XVI

HEAT; THERMOMETERS; EFFECTS OF HEAT UPON SOLIDS, LIQUIDS, AND GASES; CONSTRUCTION OF THERMOMETERS

HEAT

Effects of Heat. Thermometers

Hot and Cold Bodies.—When seated in front of a bright fire on a cold day, one feels the pleasant sensation of warmth. Similarly, if the surrounding objects, as the fixtures or the curb, be touched, a similar sensation is noticed. In both cases this sensation has been caused by the heat from the fire, and we have become aware of it by means of the sense of touch. Again, if on a similar day some outdoor object be touched, one feels the sensation of coldness; hence **heat may be defined as the agent which produces these sensations of warmth and cold.**

The sense of touch is able to distinguish between hot and cold bodies, but it cannot be entirely relied on.

Experiment XXXV.—Select three beakers: fill the first with hot water, the second with tepid water, and the third with cold water. Place one finger in the hot water and another in the cold water. Then place these two fingers in the tepid water. To the former it feels cold, and to the latter it feels warm.

Experiment XXXVI.—Select a piece of flannel, a piece of wood, and a piece of slate. Put them in a hot oven. After a time touch them. The slate feels hotter than the wood, and the wood hotter than the flannel. All have been heated to the same extent, and are as hot as one another.

These experiments show that the sense of touch is not reliable in estimating the intensity of heat in a body.

Temperature

The cold water in the above experiment has a low intensity of heat, and is said to have a low temperature. Similarly, the hot water is said to have a high temperature. **Temperature may be defined as the intensity of heat in a body.**

Effects of Heat

(1) When a body is heated, its temperature rises as long as it remains in the same state; also when a body is cooled, its temperature falls under the same conditions.

(2) Again, when a body is heated, it increases in size or expands. Also, when heat is removed from a body, it decreases in size, or contracts. These facts are true for all states of matter—solids, liquids, and gases.

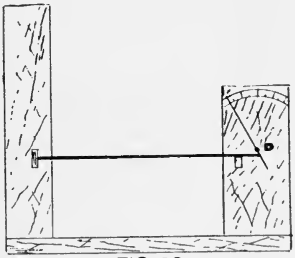


FIG. 80

Experiment XXXVII.—Fit up a knitting needle fastened at one end (Fig. 80). Allow the free end to touch a piece of straw, pinned at D. On heating the needle the straw moves. This is caused by the needle increasing in length or expanding. Allow the needle to cool, and notice the space between the needle and the straw. This space represents the contraction.

Experiment XXXVIII.—Fill a flask with coloured water. Fit into the neck an indiarubber stopper, through which passes a long piece of glass tubing. Place in a jar of hot water, and notice the rise of the coloured water in the tube, Fig. 81. This experiment shows that when water is heated it increases in volume. On cooling, the water in the tube gradually falls, showing that water contracts on cooling.

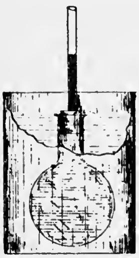


FIG. 81

Experiment XXXIX.—Place a little coloured water in a flask. Put into the neck a stopper through which passes a piece of glass tubing, the lower end passing under the surface of the water (Fig. 82). Place the warm hand on the side of the flask. The water rises rapidly in the tube. This is caused by the air, when heated, increasing in volume, and forcing the water up the tube. On cooling, the water in the tube falls, showing the contraction.

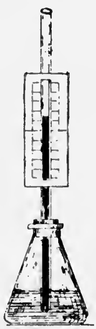


FIG. 82

The above experiments show that gases expand at a much quicker rate than liquids, and liquids than solids.

(3) **Heat sometimes produces a change of state.** If ice be heated, it soon melts and forms water; if water be heated for some time, it boils and forms steam. These changes of state—from solid to liquid, and from liquid to gas—have been brought about by means of heat.

In a similar way, if sufficient heat could be supplied to any solid, it is believed that it would first become a liquid, and afterwards a gas.

Conversely, if heat be taken from steam it is condensed and forms water. If heat be still removed, it finally freezes and forms ice. In a similar way, if sufficient heat could be withdrawn from any gas, it would be possible to liquefy it, and afterwards to solidify it.

Measurement of Temperature

It has been shown that when heat is supplied to a body, an increase in the size of the body accompanies an increase in temperature, and *vice versâ;* hence, when expansion is observed in solids, liquids, or gases, we naturally conclude that there is an increase of temperature, and that heat is being supplied to the body.

The apparatus used in Experiments XXXVIII. and XXXIX. show readily any slight change of temperature. For that reason they are sometimes called **thermoscopes**, the former a water thermoscope and the latter an air thermoscope, a thermoscope being an instrument for detecting any change of temperature.

It is necessary not only to detect any change of temperature; but also to measure that change; hence it is necessary to have an instrument to measure temperature. Such an instrument is called a **thermometer.**

The ordinary thermometer consists of a glass tube, called the stem, with a hollow bulb at one end. From the bulb to almost the top of the stem passes a fine hole, called the capillary bore. The bulb and a portion of the stem contain a liquid, generally mercury or alcohol. On the stem is marked the scale.

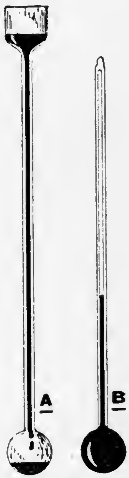


FIG. 83

Construction of a Thermometer

(1) **Filling the tube.**—A suitable tube is first selected. This should contain a bulb, and a long stem with a fine bore that opens out into a cup (Fig. 83 A).

Mercury is then placed in the cup, but as the bore is fine none reaches the bulb. On warming the bulb the air inside expands and some is forced out at the cup. On cooling, the air contracts and some of the mercury is drawn down into the bulb. Heating and cooling are repeated alternately until the bulb and bore are completely filled with mercury. The mercury is then boiled in order to expel all air bubbles. Then it is allowed to cool.

(2) **Sealing of tube.**—Place the bulb and stem in a bath at a temperature slightly higher than the thermometer will be required to indicate. Then seal the stem at the junction of the cup, by means of a small blowpipe flame (Fig. 83 B).

(3) **The Fixed Points.**—Under ordinary conditions, it is found that—

(*a*) Water always freezes at the same temperature. This temperature is called **the freezing-point.**

(*b*) Water always boils at the same temperature when atmospheric pressure is normal. This temperature is called **the boiling-point.**

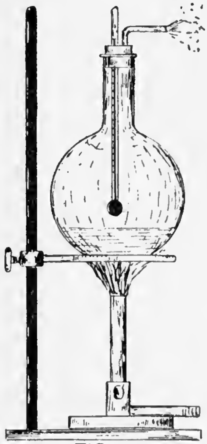


FIG. 84

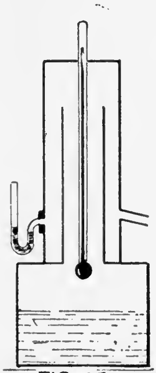


FIG. 85

Since these two temperatures are constant, they are selected as the **fixed points** of the scale, and the intervening space is then divided according to the kind of thermometer scale required.

The Boiling-point.—Place the bulb and the greater portion of the thermometer in a flask fitted up as shown in Fig. 84, and containing water. When the water boils, the steam heats the thermometer, and causes the mercury to expand. After a time the mercury level becomes steady. Then mark its level on the stem, and this will be the **boiling-point.**

For more accurate determinations, the apparatus shown in Fig. 85 is used.

The Freezing-point.—Remove the thermometer from the flask, and cover the bulb with melting ice, using apparatus as shown in Fig. 86. The mercury in the thermometer is cooled, and contracts. After a time it is at the temperature of the melting ice, and then the level is steady. Mark its level on the stem, and so get the **freezing-point.**

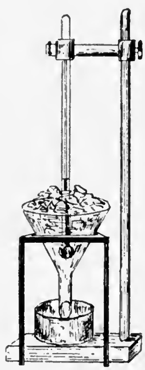


FIG. 86

(4) **Thermometer Scales.**—The distance between the two fixed points is divided into a number of equal parts, called degrees. The number of divisions varies, and thus various thermometer scales are obtained. The following are the chief ones.

(*a*) **Centigrade Scale.**—In this scale the freezing-point is marked 0°, or zero, and the boiling-point 100° C. Therefore the distance is divided into 100 equal parts (Fig. 87 C). This scale is mostly used in scientific work.

(*b*) **Fahrenheit Scale.**—In this scale the freezing-point is marked 32°, and the boiling-point 212°. Therefore the distance is divided into 180 equal parts (Fig. 87 F). It is in common use in England.

(*c*) **Réamur's Scale.**—In this scale the freezing-point is marked 0°, and the boiling-point 80° (Fig. 87 R). Therefore the distance is divided into 80 equal parts. It is in common use in Germany, and is sometimes put on house thermometers.

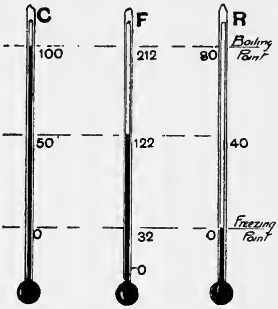


FIG. 87

(5) **Conversion of Scales.**—It is often necessary to convert the reading on one scale to the corresponding reading on another scale. This can be done by calculation or by a graph.

(*a*) **By Calculation**—

$$100^\circ \text{ C.} = 180^\circ \text{ F.} = 80^\circ \text{ R.}$$
$$\therefore \quad 1^\circ \text{ C.} = \tfrac{9}{5}^\circ \text{ F.} = \tfrac{4}{5}^\circ \text{ R.}$$

This means that every degree Centigrade is equivalent to $\frac{9}{5}$° F., or four-fifths of a degree Réaumur.

Example (1).—Convert 50° C. into the corresponding Fahrenheit temperature.

$$\begin{aligned} 50^\circ \text{ C.} &= 50^\circ \text{ C. above the freezing-point.} \\ &= \frac{50}{1} \times \frac{9}{5} = 90^\circ \text{ F. above freezing-point.} \\ &= 90 + 32 = 122^\circ \text{ F.} \end{aligned}$$

(*N.B.*—Since the freezing-point on Fahrenheit scale is 32° above the zero, this number must be added to the above.)

Rule.—To convert Centigrade readings to Fahrenheit readings, multiply the former by $\frac{9}{5}$, and then add 32° to the result.

$$\text{F.} = \tfrac{9}{5}\text{ C.} + 32^\circ$$

Example (2).—Convert 185° F. into the corresponding Centigrade temperature.

$$\begin{aligned} 185^\circ \text{ F.} &= 185 - 32 = 153^\circ \text{ F. above freezing-point} \\ &= \frac{153}{1} \times \frac{5^\circ}{9} \\ &= 85^\circ \text{ C.} \end{aligned}$$

(*N.B.*—Since 1° C. = $\frac{9}{5}$° F., 1° F. must be equal to $\frac{5}{9}$° C.)

Rule.—To convert Fahrenheit readings to Centigrade readings, subtract 32°, and then multiply the result by $\frac{5}{9}$.

$$\text{C.} = (\text{F.} - 32) \times \tfrac{5}{9}$$

The Réaumur scale is seldom used in England, hence the conversions from the other scales to this one and *vice versâ* are of little importance.

(*b*) **By Graph.**—Draw two lines at right angles to each other, OX being horizontal and OY vertical. Each of these lines is called an axis.

From O along OX mark off 100 equal divisions, so that each division represents 1° C. From O along OY mark off 212 equal divisions, so that each division represents 1° F. (Fig. 88).

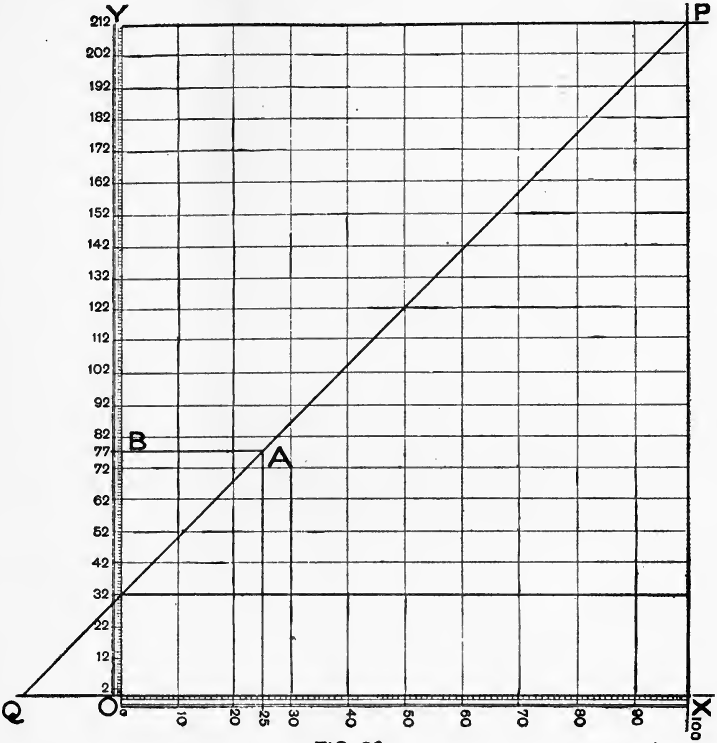


FIG. 88

It has already been shown that 100° C. corresponds to 212° F., so take the point marked 100° C. in OX, and through it draw a vertical line. Next take the point marked 212° F. in OY, and through it draw a horizontal line which cuts the vertical line at P.

Similarly, 0° C. corresponds to 32° F., so that the point Q can be fixed in exactly the same way. The straight line PQ is called the graph.

Use of Graph.—By means of this graph it is possible to convert any Centigrade reading into the corresponding Fahrenheit, or *vice versâ.*

Thus, to convert 25° C. into the Fahrenheit reading, proceed as follows:—

Through the point on OX, which reads 25, draw a vertical line intersecting the graph at A. From A draw a horizontal line which cuts OX at B. Then B is the corresponding reading on the Fahrenheit scale—*i.e.*, 77° F.

In order to convert Fahrenheit readings into Centigrade readings, proceed in a similar way.

First draw a horizontal line through a point on OY denoting the Fahrenheit reading. At the point of intersection of this line and graph, drop a vertical line to cut OX. The latter point will be the Centigrade reading.

The graph can also be used for temperatures below the freezing-point if the graph and the axes OX, OY be produced.

Exercises.—(*a*) Convert the following Centigrade readings into Fahrenheit readings :—

90° C. : 35° C. : 73° C. : 18·5° C.

(*b*) Convert the following Fahrenheit readings into Centigrade readings :—

140° F. : 113° F. : 62° F. : 34·5° F.

Choice of Liquid.—Any liquid may be used in a thermometer until it either freezes or solidifies. The liquids in common use are alcohol and mercury, and they have many advantages over other liquids.

The alcohol thermometer is used for reading very low temperatures, for alcohol only freezes at $-150°$ C. or $-238°$ F.

As it boils at about 80° C. or 176° F., it cannot be used for high temperatures. Most of the house and garden thermometers contain alcohol and register to 120° F.

Mercury is undoubtedly the best liquid to use, for the following reasons :—

(1) It remains in the liquid state through a long range of temperature, for it freezes at $-39·5°$ C. and boils at 350° C.

(2) It expands regularly and does not stick to the glass.

(3) It is opaque and can easily be seen in the bore.

(4) It rapidly takes the temperature of the substance surrounding it.

Maximum and Minimum Thermometers.—It is often necessary to register the highest and lowest temperatures indicated during a certain period. These can be obtained by special forms of thermometers, called **maximum and minimum** thermometers.

A common type of maximum thermometer consists of an ordinary mercury thermometer with **the bore constricted near the bulb** (Fig. 89). The stem is fixed horizontally. When the temperature rises, the mercury expands and forces its way through the narrow part, and along the bore; but when the temperature falls, the thread of mercury in the bore remains stationary. Therefore the end of the mercury column farthest from the bulb registers the maximum temperature for that period.

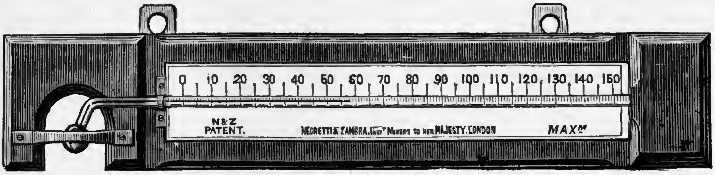


FIG. 89

A common type of minimum thermometer consists of an ordinary alcohol thermometer with **a double-headed pin or index** immersed in the alcohol contained in the stem, which is fixed horizontally (Fig. 90).

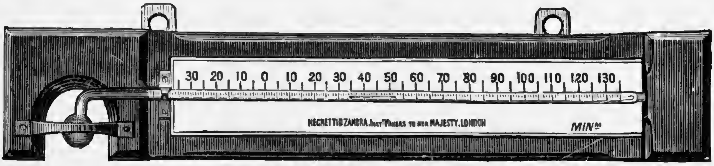


FIG. 90

When the temperature falls, the alcohol contracts, and the alcohol surface pulls the index towards the bulb. When the temperature rises, the alcohol expands and leaves the index behind. Therefore, the end of the index furthest from the bulb registers the minimum temperature for that period.

CHAPTER XVII

EXPANSION OF SOLIDS; PRACTICAL APPLICATION

Expansion of Solids

I. **Linear Expansion.**—It has already been shown that when a solid is heated it expands. If the solid is in the form of a bar, it increases in length (as shown by the movement of the straw touching the free end of the knitting needle, Fig. 80). The increase in length is called the **linear expansion.**

Experiment XL.—Take a compound bar, consisting of strips of iron and brass riveted together, and hammer it until it is perfectly straight (Fig. 91). Then heat it over a Bunsen, and it will be curved with the brass on the outside of the curve. On cooling, it gradually becomes straight again. The cause of the bending was the unequal expansion of the brass and iron, the brass being on the outside of the curve because it expanded at a quicker rate. In a similar way it can be shown that different solids have different rates of expansion.

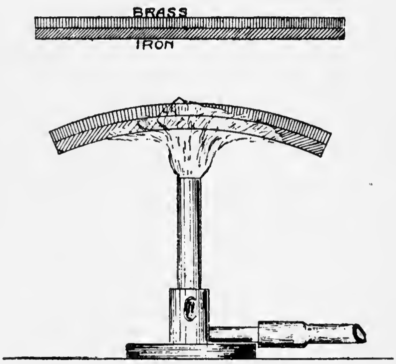


FIG. 91

Coefficient of Linear Expansion. — In order to compare the rates of expansion of different solids, it is first necessary to find the fractional increase in length when the solid is heated through 1° F. This fractional increase is called the **coefficient of linear expansion,** and is defined as that fraction of the original length which a solid expands when heated through 1° F. In the case of solids, this quantity is very small, and great care is required in order to accurately determine it.

Coefficients of Expansion (Linear, Surface, and Cube) of solids, mercury, and alcohol, for use with the Fahrenheit scale :—

Name of Substance.	Linear Expansion.	Surface Expansion.	Cubic Expansion.
Cast Iron	·00000617	·00001234	·00001850
Copper	·00000955	·00001910	·00002864
Brass	·00001037	·00002074	·00003112
Silver	·00000690	·00001390	·00002070
Wrought Iron . .	·00000686	·00001372	·00002058
Steel (untempered) .	·00000599	·00001198	·00001798
Steel (tempered) . .	·00000702	·00001404	·00002106
Zinc	·00001634	·00003268	·00004903
Tin	·00001410	·00002820	·00003229
Mercury	·00003334	·00006668	·00010010
Alcohol	·00019259	·00038518	·00057778
Lead	·00001650	·00003111	·00004666
Platinum	·00000494	·00000998	·00001482
Glass	·00000494	·00000998	·00001482

It has already been shown in previous experiments, that when a solid bar of metal is heated it expands regularly; the amount of expansion depends upon :—

(1) The length of the bar.
(2) The extent to which it is heated (*i.e.*, the rise in temperature).
(3) The substance itself (*i.e.*, the linear expansion of the solid).

In order to work numerical examples, it is necessary to consider each of these items.

Example.—An iron bar at 45° F. is 4 ft. long. Find its length at 245° F.

Coefficient of linear expansion for iron = ·0000068.

∴ Increase in length of

1 ft. for 1° F. = ·0000068 ft.
4 ft. „ 1° F. = ·0000272 ft.
4 ft. „ (245 − 45)° F. = ·0000272 × 200 = ·00544 ft.

Total length = Original length + Expansion
= 4 ft. + ·00544 ft.
= 4·00544 ft. *Ans.*

It will be seen that the total expansion is the continued product of the original length, the rise in temperature, and the coefficient of expansion.

In general terms it may be expressed as follows :—

$$L = l + lat.$$

where L = final length ; l = original length ; a = coefficient of linear expansion ; t = rise in temperature.

If any three terms in the above equation be given, the fourth can be calculated.

The contraction of a bar may be calculated in a similar manner :—

Example.—A copper bar is 24 ft. long at 110° F. Find its length at 30° F.

Decrease in length of

1 ft. for 1° F. = ·0000095 ft.
24 ft. „ 1° F. = ·0000095 × 24 ft.
24 ft. „ 80° F. = ·0000095 × 24 × 80 = ·01824 ft.

Final length = Original length − Contraction
= 24 ft. − ·01824 ft.
= 23·98176 ft.

As the above equation gives in general terms the final length when expansion takes place, the following one expresses it when contraction takes place :—

$$L = l - lat.$$

Applications of Expansion.—*Experiment XLI.*—The iron bar A, Fig. 92, is heated strongly in a furnace, and then placed in the stand D. One end is wedged by the iron wedge B (a ¼-in. file). The end N is then screwed up tightly, so that the bar is fixed at both ends, and as the bar cools the force of contraction is powerful enough to break the iron file B.

The forces of expansion and contraction are so powerful that allowance must be made in various structures because of the difference in temperature between summer and winter.

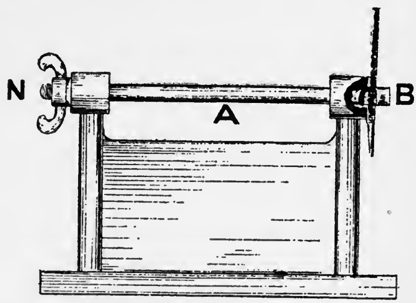


FIG. 92

In the construction of railways, the rails are laid a short distance apart to allow for expansion. If carefully examined, it will be seen that the space is much smaller in summer than in winter.

In the building of large bridges, such as tubular bridges, one end is generally fixed, whilst the other is free to move on rollers.

In winter the telegraph wires appear to be stretched tightly from post to post, but in summer they hang loosely or "sag," due to the expansion caused by the rise in temperature.

The wheelwright makes use of this property of expansion in fixing the iron rim to the wooden framework of the wheel. The iron rim is made a little smaller in diameter than the wooden framework. The former is then heated until the latter fits inside; and then the rim is cooled and is firmly fixed on the wooden framework.

II. **Superficial Expansion.**—*Experiment XLII.*—Take a metal rod which has a square face and at ordinary temperature just fits into the hole made in a piece of metal. Heat the rod, and the face will be found too large for the hole. This experiment shows that when a solid is heated the area of cross-section increases.

The coefficient of superficial expansion may be defined as the ratio of the increase in area to the original area when the temperature is raised 1°.

$$\text{Coefficient} = \frac{\text{Increase in area for } 1^\circ}{\text{Original area}}$$

The value of this coefficient for any substance is approximately **twice the coefficient of linear expansion.**

This fact can be shown graphically in the following manner:—

Let ABCD, Fig. 93, represent one square face of any substance.

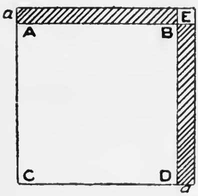


FIG 93

On raising the temperature 1° F., the length is increased by a (the coefficient of linear expansion) and becomes $(1 + a)$ feet; similarly for breadth, so that the shaded portion represents the superficial expansion. This consists of two rectangles each having an area of a sq. ft., and the small square E, which is so minute that it can be ignored.

Hence the coefficient of superficial expansion

$$= 2 \times a = 2 \times \text{coefficient of linear expansion.}$$

In numerical exercises, first convert the linear coefficient into the superficial coefficient, and then proceed as shown in the former examples.

III. **Cubical Expansion.** *Experiment XLIII.*—Take a brass ball and suspend it by means of a chain from the stand, as shown in Fig. 94. When cold, the brass ball is just able, in any position, to pass through the ring. Heat the ball, and it will be found that the ball will not pass through the ring until it has cooled again.


FIG 94

This experiment shows that when a solid is heated it increases in size. The increase in size is called the cubical expansion.

With a simple change of terms the **coefficient of cubical expansion** may be defined in a similar manner to those of linear and superficial expansion.

$$\text{Coefficient} = \frac{\text{Increase in volume for } 1^\circ \text{ F.}}{\text{Original volume}}$$

The value of this coefficient for any substance is approximately **three times the coefficient of linear expansion.**

Consider a cube of some substance, each side measuring 1 inch, then the volume of the cube is 1 cubic inch. On heating through 1° F. it expands.

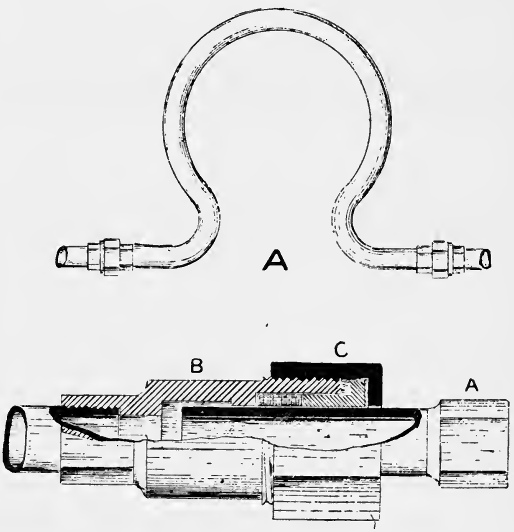
It now becomes $(1+a)$ inches in length, breadth and height, if the coefficient of linear expansion is a.

$$\begin{aligned} \text{The volume then is} \quad & (1+a)(1+a)(1+a) \text{ cub. ins.} \\ &= (1+3a+3a^2+a^3) \text{ cub. ins.} \\ \therefore \quad \text{Cubical expansion} &= (1+3a+3a^2+a^3)-1 \text{ cub. ins.} \\ &= 3a+3a^2+a^3. \end{aligned}$$

The values of $3a^2$ and a^3 are so minute that they can be neglected.

$\therefore$ Coefficient of cubical expansion $= 3 \times a = 3 \times$ coefficient of linear expansion.

In numerical exercises, first obtain the coefficient of cubical expansion from the linear one, and then proceed as shown in the former examples.



B

FIG 95

It will be noticed on reference to the table of coefficients of expansion that each metal has its own coefficient (excepting glass and platinum) differing from the others. This fact should be borne in mind when such pipes as "**lead and earthenware,**" and "**lead and iron**" **require joining together,** or in the jointing of cast-iron pipes required for conveying hot water, either as waste pipes or heating pipes.

In the former cases, it is usual to solder a brass ferrule to the lead pipe before connecting it to the iron or earthenware, and in the latter case, lead is not a suitable jointing material. Portland cement or cast-iron borings (*i.e.*, rust cement), preceded by a small quantity of yarn, proves highly satisfactory, as the coefficients of expansion of these substances are practically the same as that of cast iron.

This property has also to be considered when fixing long lengths of copper or iron pipes, for conveying hot water or steam, especially where branches are taken from the pipe.

An appliance, shown in Fig. 95 A, allows an increase or decrease of the length of pipe to which it is attached to take place during variation in temperature of the water passing through it.

Fig. 95 B shows an appliance used for the same purpose, and known as an **expansion joint**. The part A is free to move in B, and the connection between the movable parts is made watertight by a packing gland, C, containing greased hemp or some similar substance.

CHAPTER XVIII

EXPANSION OF LIQUIDS AND GASES, MAXIMUM DENSITY, ABSOLUTE TEMPERATURES FOR GASES, NORMAL TEMPERATURE AND PRESSURE

Expansion of Liquids and Gases

I. **Expansion of Liquids.**—It has already been shown that when a liquid is heated it expands or increases in volume. This increase is called the expansion.

Experiment XLIV.—Take the water thermoscope as shown in Fig. 81, and after marking the level of the liquid in the tube, plunge it into hot water. The level at first falls, then becomes stationary for a moment, and afterwards it gradually rises until the water becomes as hot as its bath.

The apparent contraction of the water at first was due to the expansion of the glass flask immediately it was immersed. The glass first became hot and expanded before the heat reached the water; hence the water had to fill the flask, causing the fall, the fall showing the expansion of the flask on immersion. When the water receives heat it expands also, and rises up the tube. This shows that the water is expanding more rapidly than the glass; for if not, the water would either remain stationary or continue to fall.

The observed expansion shows the amount that the liquid has expanded more than the glass, and is called the **apparent expansion**.

In order to obtain the **real** or total expansion, the expansion of the vessel must be added to the apparent expansion.

Real Expansion of Liquid = Apparent Expansion + Expansion of Vessel containing it.

Unequal Expansion of Liquids

Experiment XLV.—Fill three thermoscopes of exactly the same size, one with water, C, another with turpentine, B, and the last with alcohol,

A. Adjust the stoppers so that the liquid in each tube is at a common level. Then place them in hot water for a minute. Remove them and mark their levels. The alcohol rises rapidly, and the water very slowly.

This experiment shows that all liquids do not expand at the same rate.

In order to compare the rates of expansion of different liquids, it is first necessary to establish a coefficient, which is the **coefficient of cubical expansion.** It is that fraction of the original volume which a liquid expands when heated through 1°.

$$\text{Coefficient} = \frac{\text{Increase in volume of liquid for } 1^\circ}{\text{Original volume}}$$

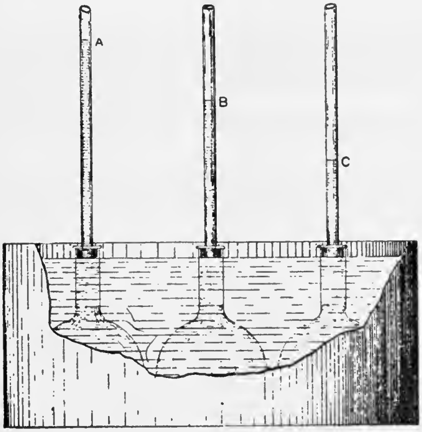


FIG. 96

In the case of liquids this value is very small, but is much larger than that of solids, as may be expected from the above experiment.

The following values are the coefficients of cubical expansion (for 1° C.) for the corresponding liquids :—

Alcohol . .	·0013
Turpentine . .	·0011
Glycerine . .	·00053
Mercury . .	·00018
Water (not the same for each degree increase of temperature), about .	·00049

Peculiar Behaviour of Water

Experiment XLVI.—Take a small flask and put in a small quantity of mercury, equal to about one-seventh the capacity of the flask. Then fill with coloured water, and in the mouth fix a rubber stopper (through which passes a piece of glass tubing having a very fine bore) and a thermometer, and arrange the apparatus so that the water shows in the tube. Place the flask in a freezing mixture of ice and salt. Slowly the level falls for a time, then it becomes steady, afterwards it begins to rise, slowly at first and afterwards very rapidly.

This experiment shows that when water is cooled it contracts until it reaches a certain temperature (4° C.). On further cooling, it expands until the water begins to freeze, and then it expands rapidly.

If the experiment be continued, and heat applied to the flask, it will be observed that, during the increase in temperature from 0° C. to 4° C. the water in the glass tube gradually sinks until 4° C. is reached, when a gradual expansion takes place until boiling-point (*i.e.*, 100° C.) is reached. If the height of the column of water in the tube is measured at each degree increase in temperature, it will be observed that the amount of expansion is not the same for each degree. It may therefore be stated that water expands at a varying rate between 4° C. and 100° C.

Maximum Density of water. Fig. 97 shows the apparatus generally used to show the temperature of the maximum density of water. The central column A is filled with water, the temperature of which is shown near the top and bottom by two thermometers, B and C, inserted through the side of the glass cylinder. The outer casing which surrounds the central column is filled with a freezing mixture. As the water is cooled it becomes denser and sinks. This continues until all the water is at 4° C. On further cooling, the water expands, becomes less dense, and floats near the surface. This causes the water at the surface to fall in temperature, until finally it freezes, whilst that at the bottom remains at **4° C.** (as shown by the thermometer).

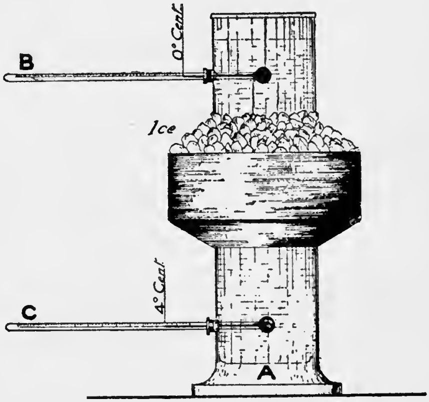


FIG. 97

From this experiment it follows that a definite weight of water will have the least volume at 4° C.; hence at this temperature it will have its greatest or **maximum density.**

In nearly all cases liquids contract until they solidify.

Expansion on Freezing.—Experiment XLVI. showed that water expands on freezing. As this property is of great importance to plumbers, the following experiments are described to illustrate the same property.

Experiment XLVIII.—Place some ice in a small flask and fill up with cold water. Fit in a stopper with tube attached, allowing the water to rise in the tube. Gently heat the ice, and notice that the level of the liquid falls, showing that contraction is taking place as the ice changes into water; hence the water must have expanded on freezing.

Experiment XLIX.—Fill a test tube with cold water and tightly seal the mouth. Then place it in a freezing mixture. On solidifying, the test tube bursts.

In a similar way, if an iron shell ($\frac{1}{4}$ inch thick) be filled with cold water, screwed up, and put in a freezing mixture, the force of expansion exerted by the water on freezing is powerful enough to burst the iron shell.

Application.—Water pipes are occasionally fixed in exposed positions. In winter the water contracts until it reaches its maximum density. Then it begins to expand slowly. When it freezes it expands rapidly, and as the pipes are full of water, the force of expansion is sufficient to burst the pipe; but there is seldom a leakage of water until the thaw melts the ice. It is usual to cover the water pipes near the outside walls with a bad conductor of heat, as felt, slag wool, or straw, to protect them in winter. As water in motion is not so liable to freeze as water at rest, some extravagant people, in winter, leave the taps running at night whilst the frost lasts.

FIG. 98

If ice be placed in water at 0° C. it is found to float with ten parts below to one above the surface. This shows that it is lighter than water, being only nine-tenths as dense; hence considerable expansion must have taken place in the change from water to ice.

Temperatures.—The diagram Fig. 98 shows the behaviour of water in passing through each state.

The rectangle represents the volume of ice at 0° F. On heating, the ice expands regularly until it begins to melt at 32° F. Then there is a sudden contraction, and this is followed by gradual contraction to 39·2° F. This is the maximum density of water (*i.e.*, 4° C.). On further heating, expansion takes place until the water boils at 212° F. Then it expands rapidly to form steam, which occupies a volume 1700 times that of the water.

If we commence with steam, the diagram may be read as follows:—

At 212° F., big contraction caused by steam condensing to form water.

212° to 39·2° F., gradual contraction of water.

39·2° to 32° F., gradual expansion of water.

At 32° F., sudden expansion on freezing.

32° to 0° F., gradual contraction of solid ice.

II. Expansion of Gases

It has already been shown that when gases are heated they expand, and when cooled they contract.

Experiment L.—Fit up the apparatus as shown in Fig. 99. The cold flask is filled with air and fitted with the delivery tube, which is placed under the measuring cylinder. The flask is then placed in boiling water, and air bubbles pass into the cylinder. The water displaced in the cylinder represents the expansion of air in flask due to rise in temperature.

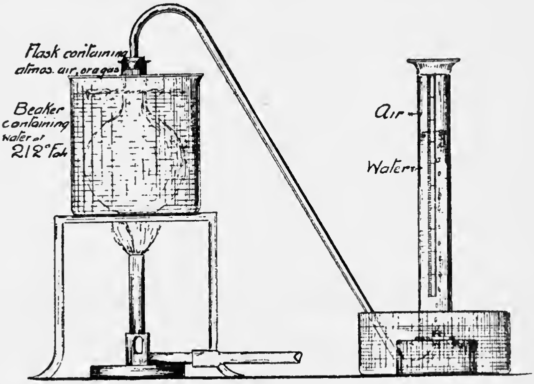


FIG. 99

It is found that 27·3 cub. ins. of air at 0° C. raised to 80° C. expand 8 cub. ins.

Experiment LI.—Now allow the flask to cool and then fill it with coal-gas. Repeat the same experiment, and it will be found that the amount of expansion of coal-gas is the same as that of air; hence these two gases expand at the same rate.

In the same way it may be shown that **all gases expand at the same rate.**

In the above experiment:—

27·3 cub. ins. of air raised 80° C. expand 8 cub. ins.

$\therefore$ 1 cub. in. " 80° C. " $\dfrac{8}{27{\cdot}3}$ "

$\therefore$ 1 " " 1° C. " $\dfrac{8}{27{\cdot}3 \times 80} = \dfrac{1}{273} = {\cdot}00366.$

Hence the coefficient of expansion for all gases is ·00366. In words it may thus be stated:—**All gases expand $\frac{1}{273}$ part of their volume at 0° C. for every rise in temperature of 1° C.**

This law was established independently by the scientists Dalton, Gay-Lussac, and Charles, and it is sometimes named after each of them, as "**Charles' Law**," etc.

Let us consider the volume of a mass of gas whose volume at 0° C. is unity, remembering that the above law holds good for contraction on cooling.

The last result shows that **if a gas could be cooled to $-273°$ C.,**

and if the coefficient remained the same, its volume would be reduced to zero.

Temperature.	Expansion.	Volume.
0° C.	...	1 or $\frac{273}{273}$
1° C.	$\frac{1}{273}$	$1+\frac{1}{273}=\frac{274}{273}$
10° C.	$\frac{10}{273}$	$1+\frac{10}{273}=\frac{283}{273}$
100° C.	$\frac{100}{273}$	$1+\frac{100}{273}=\frac{373}{273}$
273° C.	$\frac{273}{273}$	$1+\frac{273}{273}=\frac{546}{273}=2$
− 1° C.	$-\frac{1}{273}$	$1-\frac{1}{273}=\frac{272}{273}$
− 273° C.	$-\frac{273}{273}$	$1-\frac{273}{273}=0$

Absolute Scale of Temperatures.—This temperature of − 273° C. is called **the absolute zero,** and if a scale of temperature be made with this as the zero, these temperatures would be called **absolute temperatures.**

To convert a Centigrade temperature to the corresponding absolute one, add 273 to it.

Let us again consider the volume of the mass of gas whose volume at 0° C. was unity.

Centigrade Temperature.	Absolute Temperature.	Volume of Gas.
0° C.	273° A.	1 or $\frac{273}{273}$
10° C.	283° A.	$\frac{283}{273}$
273° C.	546° A.	$\frac{546}{273}$
− 10° C.	263° A.	$\frac{263}{273}$
= 273° C.	0° A.	0

From the above table it follows that **the volume of a given mass of any gas is directly proportional to its absolute temperature.**

Example.—At 25° C. the volume of a gas is 1000 cub. ins.; find its volume at 0° C.

Convert the temperatures into the absolute scale.

$$25° \text{ C.} = 25 + 273 = 298° \text{ A.}$$
$$0° \text{ C.} - 0 + 273 = 273° \text{ A.}$$

Volume of gas at 298° A. = 1000 cub. ins.

" " 273° A. = $1000 \times \frac{273}{298}$ = 916 cub. ins. (approx.)

or 298 : 273 :: volume at 15° C. : volume at 0° C.

298 : 273 :: 1000 x

$$x = \frac{273 \times 1000}{298} = 916 \text{ cub. ins. (approx.).}$$

Normal Temperature and Pressure.—It is now evident that the volume of a given mass of gas depends upon its pressure and temperature.

As stated in **Boyle's Law, the volume of a given mass of gas at constant temperature is inversely proportional to the pressure exerted upon it.**

As shown in the last paragraph, the volume of a mass of gas is directly proportional to its absolute temperature.

By means of these two laws it is possible to find the volume of a gas at any temperature, if its volume is known at a certain temperature and pressure. In practical work it is often necessary to convert a known volume of a gas at known temperature and pressure to normal temperature and pressure (*i.e.*, 0° C. and 30 ins. or 760 mm. barometric pressure).

Example.—At a pressure of 28 ins. and temperature 15° C., the volume of a gas is 600 cub. ins.; find its volume at normal temperature and pressure. (N.T.P.)

First.—Consider pressure.
Volume is inversely proportional to pressure.

Volume at pressure of 28 ins. = 600 cub. ins.
∴ " " 30 ins. $= (600 \times \frac{28}{30}) = 560$ cub. ins.

Second.—Consider temperature.
Volume is directly proportional to absolute temperature.

$15°\text{ C.} = 15 + 273 = 288°\text{ A.};\quad 0°\text{ C.} = 273°\text{ A.}$

Volume at 288° A. = 560 cub. ins.
∴ Final volume at 273° A. $= 560 \times \frac{273}{288} = 531$ cub. ins. (approx.).

Combining two stages.

$$\text{Final volume} = 600 \times \overset{\text{Press.}}{\frac{28}{30}} \times \overset{\text{temp.}}{\frac{273}{288}} = 531 \text{ cub. ins.}$$

In general.

Let V = original volume of gas at P ins. pressure and $T°$ C.
and V_1 = final " " P_1 ins. " $T_1°$ C.

Then
$$V_1 = V \times \frac{P}{P_1} \times \frac{(273 + T_1)}{(273 + T)}$$

or
$$\frac{V.P.}{(273 + T)} = \frac{V_1 P_1}{(273 + T_1)}$$

CHAPTER XIX

SPECIFIC HEAT, TEMPERATURE OF FUSION, LATENT HEAT OF FUSION, AND VAPORISATION

Specific Heat

Heat and Temperature.—These two terms are often confused, but they are not the same, and should be distinguished.

At the beginning of this section it was shown that heat was an **agent**, capable of doing certain things, whilst temperature simply showed the **state** of the body, as to whether it was hot or cold. The relation between the two terms is that if heat be put into a body, the temperature is raised, and *vice versâ*. Again, if a hot body be placed in contact with a cold body, heat passes from the former to the latter until both have the same temperature.

Quantities of Heat.—If heat be supplied to a body until its temperature is raised 1°, we do not know how much heat has been supplied unless other particulars be known. This shows that temperature alone is not sufficient to measure heat.

For instance, if a bucket and a small beaker, both filled with water, be heated through 1°, it is clear that the former will require more heat than the latter, and conversely, in cooling through the same range of temperature, the former will give out more heat than the latter.

Again, if the same **quantity of heat** be supplied to the water in the bucket as to that in the beaker, the latter will rise in temperature much more quickly than the former, because there is less mass to be heated.

In considering measurement of the quantity of heat, it is necessary to consider mass as well as temperature. In order to measure heat some unit must first be chosen. The **British unit** for the measurement of heat is defined as **that quantity of heat required to raise the temperature of 1 lb. of water through 1° F.** It is called the pound-degree, or the British thermal unit.

The following experiments should be carefully studied:—

Experiment LII.—1 lb. of hot water at 90° F. is mixed with 1 lb. of cold water at 50° F., and the temperature of the mixture is found to be about 70° F.

Rise in temperature of cold water $= (70° - 50°) = 20°$
Fall „ „ hot „ $= (90° - 70°) = 20°$
Units of heat given out by hot water $= 20 \times 1 = 20$ units
„ „ taken in by cold „ $= 20 \times 1 = 20$ „

Experiment LIII.—1 lb. of hot water at 160° F. is mixed with 3 lbs. of cold water at 40° F., and the temperature of the mixture is found to be 70° F.

Rise in temperature of cold water $= 70° - 40° = 30°$ F.
Fall „ „ hot „ $= 160° - 70° = 90°$ F.
Units of heat given out by hot water $= 90 \times 1 = 90$ units
„ „ taken in by cold „ $= 30 \times 3 = 90$ „

These results show that in each case the hot water gives out the same quantity of heat as the cold water receives.

This fact is universally true whenever hot and cold bodies are mixed without any outside loss; hence:—

Heat gained by cold body = Heat lost by hot body.

Thermal Capacity

Experiment LIV.—1 lb. of copper at 200° F. is mixed with 1 lb. of cold water at 50° F., and the temperature of the mixture is found to be 63° F.

Rise in temperature of cold water $= 63° - 50° = 13°$ F.
Fall „ „ copper $= 200° - 63° = 137°$ F.

These results show that 1 lb. of copper in cooling 137° only gives out sufficient heat to raise the same weight of water 13°.

(*N.B.*—When equal quantities of water were mixed, the "rise" in temperature was equal to the "fall.")

Hence it follows that the quantity of heat which raises 1 lb. of water through 13° will raise 1 lb. of copper through 137°.

Therefore, water requires $\frac{137}{13} = 10\dot{}5$ as much heat as copper to raise it 1°.

In a similar manner it can be shown that water requires more heat to raise its temperature through 1° than the same weight of any other substance does, whether it be solid or liquid.

Experiment LV.—Take four small metal balls, say of iron, copper, zinc, and lead, all of the same size. Keep them for a time in a hot oil bath until all have the same temperature. Then drop the iron one on a slab of paraffin wax. As the former cools it gives heat to the latter, which it melts, and the ball sinks down. Then drop the other balls on the wax in turn, and they will sink to different depths, showing that different quantities of heat have been given out in cooling. The iron sinks the deepest, and the lead the least (Fig. 100), showing that in cooling through the same temperature the iron gives out most heat in this case, and lead the least; hence they must have taken in different quantities when heated in the oil bath.

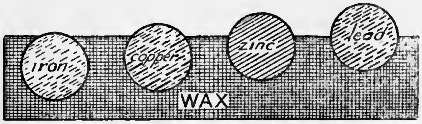


FIG. 100

These experiments show that different substances require different quantities of heat to raise them through the same temperature; hence they are said to have different capacities for heat, or different **thermal capacities.**

The thermal capacity of a body is the quantity of heat required to raise the temperature of the body through 1° F.

Specific Heat

In order to compare the thermal capacities of different substances, it is necessary to compare equal weights of these substances.

In Experiment LIV. it was found that:—

1 lb. of copper cooling 137° raised 1 lb. of water 13° F.

1 „ „ 1° would raise 1 lb. of water $\frac{13}{137} = \cdot 095°$ F.

Therefore, to raise 1 lb. of copper 1° F. requires ·095 times the amount of heat required to raise 1 lb. of water 1° F.

This number is called the "**specific heat**" of copper.

The specific heat of a substance is the ratio of the heat required to raise its temperature through 1° to the heat required to raise the temperature of an equal mass of water through 1°.

$$\text{Specific heat} = \frac{\text{Heat required to raise substance } 1°}{\text{Heat required to raise equal weight of water } 1°}$$

If the unit of mass be the pound in each case, then the specific heat will be expressed in a **fraction of the unit of heat, thus:**—

To raise 1 lb. of water 1° F. requires 1 unit of heat
∴ „ 1 lb. of copper 1° F. requires ·095 unit of heat.

In these terms, **the specific heat of a substance is that fraction of a British thermal unit** required to raise 1 lb. of it through 1° F.

Methods of finding Specific Heats

I. Method of Mixtures.

Experiment LVI.—To find the specific heat of lead shot. Weigh out $\frac{1}{4}$ lb. of lead shot, and place it in a test tube, which is then placed in a can of boiling water (212° F.). Then weigh out 1 lb. of cold water, place it in a beaker, and take its temperature (70° F.). When the lead has had sufficient time to be heated to 212° F., pour the lead shot rapidly but carefully into the cold water, and find the temperature of the mixture (71·1° F.).

Results.—Weight of lead shot = $\frac{1}{4}$ lb.
Weight of cold water = 1 lb.
Temperature of lead shot = 212° F.
Temperature of cold water = 70° F.
Temperature of mixture = 71·1° F.
Rise in temperature of cold water = 1·1° F.
Fall in temperature of hot lead = 150·9° F.

∴ $\frac{1}{4}$ lb. of lead cooling 150·9° F. raises 1 lb. of water 1·1° F.
∴ 1 „ „ 150·9° F. „ 1 „ „ 4·4° F.
∴ 1 „ „ 1° F. „ 1 „ „ $\frac{4\cdot4}{150\cdot9} = \cdot032°$ F.

∴ Specific heat of lead is ·032.

In a similar manner, the specific heat of any solid or liquid may be determined, if the solid will not melt or the liquid boil in the hot water. In the latter case the water must not be boiled, but heated to a lower temperature.

II. Method of Cooling.

Experiment LVII.—Place equal weights of water and methylated spirit in each of two beakers of the same shape. Raise the temperature of each to 160° F. Then allow each to cool to 120° F., and note the time taken in each case.

Thus:— Methylated spirit cooled . 40° in 246 secs.
Water cooled . . 40° in 400 „

$$\therefore \text{ Specific heat of methylated spirit } = \frac{246}{400} = \cdot61.$$

Both liquids are giving out the same amount of heat in the same time; hence the water gives out more heat than the spirit. Since they have cooled through the same range of temperature, it follows that the spirit only required ·61 times the heat that the water required to raise it through the same temperature.

In a similar manner, the specific heat of any solid or liquid can be found. The solids compared should have same shape and weight.

The following values are the specific heats of the corresponding substances :—

Solids

Copper	0·0951	Cast Iron	0·1298
Gold	0·0324	Lead	0·0314
Wrought Iron	0·1138	Platinum	0·0324
Steel (soft)	0·1175	Silver	0·0570
Zinc	0·0956	Tin	0·0562
Brass	0·0939	Ice	0·5
Glass	0·1937		

Liquids

Water	1·0000	Benzene	0·450
Alcohol	0·7000	Sulphuric Acid	0·335
Mercury	0·0333	Oil of Turpentine	0·426

Applications.—It will be noticed that mercury has a low specific heat. This is another reason in favour of its use in thermometers. When placed in a hot liquid it requires very little heat to raise it to the temperature of the liquid; also, it rapidly reaches this temperature.

Again, water has a very high specific heat. This partly accounts for the sea around our shores keeping cooler than the land in the day-time, and causing the sea-breeze. On the contrary, at night, the sea cools much more slowly than the land, because it contains more heat, and this gives rise to the **land breeze**.

In the hot-water system for heating buildings, water is the best liquid to use, for it has the highest specific heat, and therefore contains the most heat for the same temperature. For this reason, hot water is used to fill foot-warmers and hot-water bottles.

Change of State.—It has already been shown that the same form of matter can exist in three states. Thus, if water be heated, its temperature rises until it begins to boil, and changes into steam. Again, if water be cooled, its temperature falls until it freezes, and becomes ice.

Fusion.—The change of state from solid to liquid is called "fusion." In most cases the change is well marked, and can be readily seen. It always takes place at a definite temperature, which is called "the melting-point" of that substance. When the liquid is cooled, it solidifies at the same temperature (if kept in motion).

Thus the melting- and solidifying-points of a substance are at the same temperature.

In finding the freezing-point on the thermometer (Fig. 62), we really found the melting-point of ice. When the melting-point of the solid is below the temperature of the room, the former can be determined in the same way.

When the melting-point is below the boiling-point of water, it can be found in the following manner :—

Experiment LVIII.—To find the melting-point of paraffin wax. In a small glass tube, closed at one end, put a small quantity of paraffin wax, and fasten the tube to the bulb of a thermometer. Fix them in a beaker of water. Heat the water until the wax begins to melt, then remove the lamp, and read the temperature (60° C.). Then allow it to cool, and read the temperature at which it begins to solidify (54·5° C.). When solid again, heat the water very slowly, waiting after a rise of half a degree until it begins to melt. Read the melting-point (55·3° C.). Allow it to cool, and read the solidifying-point (54·8° C.).

Take the mean of these two results, and obtain the melting-point of wax (55° C.).

When the melting-point is above the boiling-point of water, as in the case of metals and alloys, the apparatus must be modified.

Experiment LIX.—**To find the melting-point of Solder.** The iron vessel (Fig. 101) is cylindrical in shape, and contains a narrow

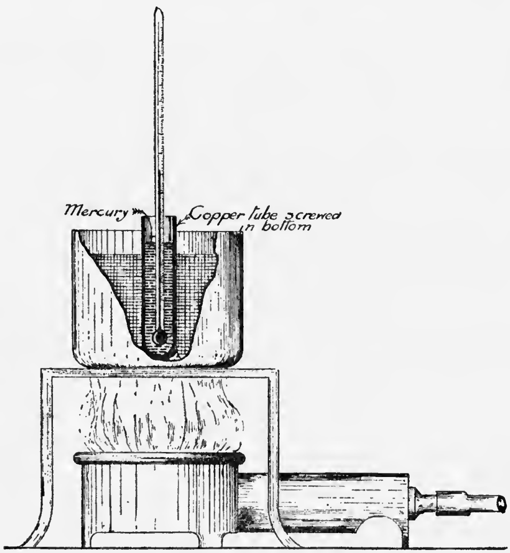


FIG. 101

copper cylinder inside. The outer cylinder is filled with the alloy, and in the inner one is placed a suitable thermometer in a mercury bath. The apparatus is fitted on a tripod, and the experiment is performed as in the previous one.

Temperatures of fusion (melting-points) and **of vaporisation**, and their respective **latent heats** in the Fahrenheit scale:—

Substance.	Temperature of Fusion. (Melting-point.)	Temperature of Vaporisation.	Latent Heat of Fusion.	Latent Heat of Vaporisation.
	Degrees Fahr.	Degrees Fahr.		
Water	32	212	142·65	966·6
Mercury	– 37·8	662	5·09	157·0
Sulphur	228·3	824	13·26	...
Tin	446	...	25·65	...
Lead	617	...	9·67	...
Zinc	680	1900	50·63	493
Alcohol	Unknown	173	...	372
Copper	2100	...	...	...
Cast Iron	2192	3300	...	...
Wrought Iron	2912	5000	...	...
Steel	2520	...	...	...
Platinum	3632	...	...	...

Vaporisation or **Boiling.**—The change of state from liquid to gas is called Vaporisation. The temperature at which the liquid boils and changes into vapour is called "the boiling-point."

If the atmospheric pressure be constant, the boiling-point of any liquid is constant. In fixing the boiling-point of water on the thermometer (Fig. 84), we determined the "boiling-point of water."

By using a suitable thermometer, the same apparatus might be used for finding the boiling-points of most liquids.

Experiment LX.—To find the boiling-point of methylated spirit.

The apparatus consists of a flask containing methylated spirit. Through the stopper passes a suitable thermometer, and a tube passing to a cool flask, which condenses the spirit.

Heat gently, until the spirit boils. When the thermometer is steady, take a reading every half-minute for five minutes.

The correct boiling-point will be the mean ot the ten results.

Table of Boiling-points

	Fahrenheit.	Centigrade.		Fahrenheit.	Centigrade.
Methylated Spirit	172·4°	78°	Turpentine	312·8°	156°
Benzene	177·4°	80·8°	Glycerine	554°	290°
Nitric Acid (pure)	186·8°	86°	Mercury	662°	350°
Water	212°	100°	Paraffin	698°	370°

Latent Heat of Fusion

Experiment LXI.—Half fill a beaker with water, and take its temperature (70° C.). Add melting ice (32° F.) and stir the mixture until the temperature falls to 32° F. It will be noticed that some of the ice has melted. The water has cooled 38°; hence it must have given heat to the ice, but there is no rise in temperature. The ice has simply changed from ice at 32° F. to water at 32° F.; hence the heat given out by the water has been used in melting the ice, without change of temperature.

The heat thus used is called the latent heat. When the water freezes, to form ice, this heat is given out.

Experiment LXII.—$\frac{1}{4}$ lb. of ice is mixed with 2 lbs. of water at 70° F. When the ice has melted, the temperature of the mixture is 50° F.

2 lbs. of water have cooled (70 to 50) = 20° F.
∴ Heat given out . . = 40 units of heat.

This heat had served to melt the ice, and also to raise the temperature of melted ice from 32° F. to 50° F.

∴ Heat taken in by melted ice $= \frac{1}{4} \times 18 = 4\frac{1}{2}$ units.

∴ Heat used to melt the ice without change of temperature is $40 - 4\frac{1}{2} = 35\frac{1}{2}$ units.

∴ $\frac{1}{4}$ lb. of ice requires $35\frac{1}{2}$ units of heat to melt it.
∴ 1 " " 142 " " "

This number shows that every pound of ice requires 142 units of heat to melt it without change of temperature. It is called the **latent heat of fusion of ice.**

The latent heat of fusion of ice is the amount of heat required to melt 1 lb. of ice without change of temperature.

Experiment LXIII.—Cooling of paraffin wax.

Place some wax in a test tube, melt it, and put a thermometer in it. Allow it to cool, and read the temperature every 30 seconds. Whilst liquid it cools rapidly, whilst solidifying its temperature is steady for a few minutes, although it is giving out heat all the time.

The heat given out during the latter period produces no change of temperature.

It is the latent heat which was required to melt the wax, and which is set free as the wax solidifies.

When a liquid is cooled to solidifying-point, it does not solidify until this latent heat has been removed. This accounts for the length of time taken for a pond to freeze, after the water is cooled to freezing-point.

Latent Heat of Vaporisation

In finding the boiling-point of water, it was noticed that when the water boiled the temperature was steady and did not rise, although the water was being heated. In this case heat was being supplied to the water, which produced a change of state but no change of temperature.

The same occurred in the boiling of methylated spirit. (Ex. LX.)

The following statement is true for all liquids :—

When a liquid boils, its temperature remains steady (*i.e.*, the boiling-point) as long as any liquid remains, and all further heat supplied is used in converting the liquid into gas, without change of temperature. This heat is called the **latent heat of vaporisation.**

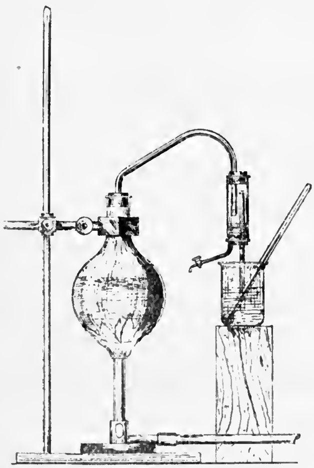


FIG. 102

When the vapour is condensed into a liquid, this latent heat is liberated.

Experiment LXIV.—Place 97 cub. ins. of water in a vessel, and take its temperature (72° F.). Boil water in a flask, and allow the steam to bubble through the cold water (Fig. 102). It will be found that the water rises in temperature. Allow the steam to pass through until the water is raised to boiling-point (212° F.), and then measure the water again. It will be found to occupy about 111 cub. ins. The cold water has received sufficient heat from the steam to raise its temperature 140°, yet the steam at 212° F. has only changed into water at 212° F.; hence it follows that when 14 cub. ins. of steam are condensed, sufficient heat is given out to raise 97 cub. ins. through 140° F.

This heat is called the "latent heat of vaporisation."

Suppose 1 cub. in. of water weighs 1 lb. Then—

Heat received by cold water $= 97 \times 140$ units of heat.

$\therefore$ Heat given out by 14 lbs. of steam $= 97 \times 140$ units of heat.

$\therefore$ Heat given out by 1 lb. of steam $= \dfrac{97 \times 140}{14} = 970$ units.

The correct result is 966·6 units.

It means that every pound of steam at 212° F. gives out 966·6 units of heat in condensing, to form water at 212° F., and it is called "the latent heat of vaporisation of water."

The latent heat of vaporisation of water is that amount of heat required to convert 1 lb. of water at 212° F. into steam at the same temperature.

Examples on units of heat.

(1) How many units of heat are required to convert 1 lb. of ice into steam?

1 lb. of ice at 32° F. to water at 32° F. requires 142·7 units.

1 lb. of water at 32° F. to water at 212° F. requires (1 × 180) = 180 units.

1 lb. of water at 212° F. to steam requires 966·6 units.

Total = 142·7 + 180 + 966·6 = **1289·3 units of heat.**

(2) How many units of heat are required to convert 1 lb. of methylated spirit at 70° F. into vapour?

(Specific heat = ·61; latent heat = 372; boiling-point = 172° F.)

1 lb. of spirit raised from 70° F. to 172° F. = 1 × ·61 × 102 = 62·2 units.

1 lb. of spirit raised from 172° F. into vapour = 1 × 372 = 372.

Total = 62·2 + 372 = **434·2** units of heat.

One pound of coal, when completely burnt, evolves from 8000 to 14,000 heat-units; but the whole of this is not effectively utilised in any system for heating water, owing to loss due to various causes.

One pound of petroleum contains about 20,000 heat-units.

The latent heat of steam is taken advantage of in the heating of buildings, as a pound of steam at 212° when condensed to water at the same temperature, releases 996 heat-units, or, in other words, sufficient heat to raise the temperature of 996 lbs. of water through 1° F.

It has also been largely adopted in recent years for heating water for domestic purposes, either by discharging the steam into the water directly or passing it through a coil of pipes fixed in a tank or cylinder, the latent heat from the steam passing by conduction to the water in contact with the outer surface of the coil. Where steam under pressure or the "exhaust" from engines is available, this method of heating water is an economical one.

Variation of the boiling-point temperature of water. It has been previously mentioned that water boils at 212° F. when the atmospheric pressure is normal—*i.e.*, 14·7 lbs. per square inch; but if the pressure be increased or reduced, the boiling-point of water is raised or lowered respectively. Instances of this variation are illustrated in the enclosed vacuum pans, used for evaporating water from substances (such as milk, etc.) which cannot be subjected to the normal boiling-point temperature of water without destroying some of their essential constituents. Again, in the boilers used for generating steam, the boiling-point of the water varies, according to the pressure of the steam upon the water in the boiler. The **high-pressure heating apparatus** is also another example. In this case the system is entirely enclosed; the water is heated, and by its expansion, the pressure inside the apparatus is gradually raised, thus raising the boiling-point temperature. In this system the water is frequently heated to from 300° to 400° F., and the pressure generated is often more than $\frac{1}{2}$ ton per square inch.

CHAPTER XX

TRANSMISSION OF HEAT: CONDUCTION, CONVECTION, RADIATION; TABLE OF CONDUCTORS AND NON-CONDUCTORS; NATURAL AND MECHANICAL VENTILATION; CONDUCTION AND CONVECTION OF LIQUIDS AND GASES; HUMIDITY

TRANSMISSION OF HEAT

Conduction—Convection—Radiation

Conduction.—*Experiment LXV.*—Take a copper rod, warm it, and then cover it with a thin layer of beeswax and let it cool. Heat one end, and notice that the wax gradually melts along the rod from the heated end until all of it is melted. This shows that heat must have passed along the rod from the heated end to the colder end.

Each particle of copper has played its part, for each one has passed on the heat to the next, and thus the heat is said to have passed through the rod by **conduction**.

Conduction is one process by which heat passes from the hotter to the colder parts of the same body. In conduction the heat passes from particle to particle, and the particle itself does not relatively alter its position.

The body through which the heat passes is called a **conductor** of heat.

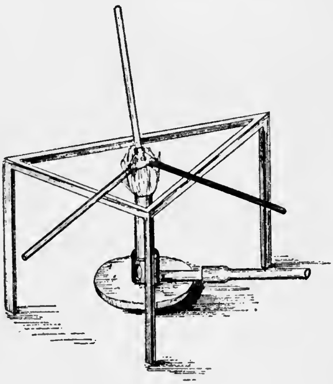


FIG. 103

Good and Bad Conductors.—*Experiment LXVI.*—Cover a copper rod, an iron rod, and a glass rod of the same length, with beeswax. Place them on a tripod (Fig. 103), and heat them at one end equally. The wax on the copper rod melts rapidly; on the iron rod it melts slowly, whilst on the glass rod it scarcely melts at all.

This shows that the heat travels quickly along the copper, fairly quickly along the iron, and scarcely at all along the glass; hence copper may be said to be a good conductor of heat, iron a fairly good conductor, and glass a bad conductor of heat.

On examining the heated ends, the glass will be found to be white hot, the iron red hot, and the copper its natural colour, showing that at this end the copper is cooler than the glass.

This, again, shows that copper is a good conductor of heat, and glass a bad one.

Action of Good Conductors on Flames.—*Experiment LXVII.*—Take a piece of copper wire and wind it into a narrow coil. Lower the coil slowly over the flame of a candle. The flame goes less, and gradually goes out, because the copper cools the flame so much that the candle cannot burn.

Experiment LXVIII.—Take a piece of iron wire gauze with fine meshes, and lower it on the flame of a Bunsen lamp. The flame remains below the gauze, and appears to be cut off (Fig. 104). That portion of the flame which ought to be above the gauze has been put out by the cooling action of the iron. If a light be applied above the gauze, the coal-gas burns here as well.

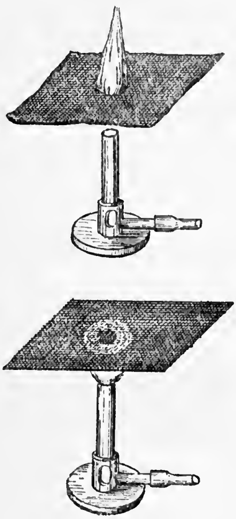


FIG. 104

A similar experiment can be performed by having the flame above the gauze.

In each of the above experiments, the metal lowers the temperature of the flame below the ignition point of the gases given off; hence they do not burn beyond the gauze.

This is the principle of the miners' safety lamp (Fig. 105). The flame of the oil lamp is surrounded by iron wire gauze of fine meshes. The inflammable gases in the mine may pass through the gauze and burn inside, but as long as the gauze remains cool, no flame passes out of the lamp, and thus, as long as the lamp is closed, the lamp can burn, without fear of causing an explosion by igniting the gases in the mine.

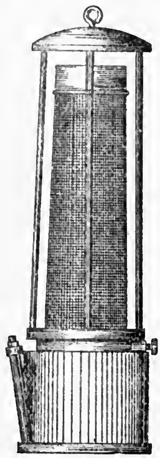


FIG. 105

Liquids are bad conductors, as a rule, the only exception being mercury.

Experiment LXIX.—Take a test tube three-quarters full of water, and, holding it near the bottom, **heat the water near the surface,**

(*N.B.*—If liquids are heated at the bottom they are set in motion.)

Soon the water at the surface begins to boil, but that near the bottom is quite cold. This shows that water is a bad conductor of heat. Similarly, it may be shown that nearly all liquids are bad conductors of heat.

Gases are even worse conductors than liquids.

Relative Conductivity.

That power of conducting heat which a body possesses is called the conductivity of the body. The following experiment will show how the conductivity of different substances can be compared.

Experiment LXX.—Take six similar bars of various substances, as:—copper, iron, tin, lead, slate, and wood. Cover them with beeswax, and fit them in one side of a metal trough (Fig. 106), so that one end of each rod is inside the trough. Pour hot water into the trough, and notice that the wax gradually melts along each bar for a time, and then stops. Then the flow of heat along the bars is steady. If the length of wax melted along each bar be measured and then compared, a rough determination of the relative conductivity is obtained.

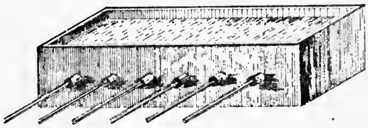


FIG. 106

The following table gives conductivities of different substances as compared with water, or it gives the relative conductivity with water as the standard:—

Table of Relative Conductivities

Silver	786	Mercury	10·7
Copper	743	Glass	1·2
Brass	171	Water	1·00
Iron	143	Slate	·57
Tin	107	Flannel	·021
Lead	57		

Table of Non-Conductors

Useful for covering steam, hot- and cold-water pipes (felt being taken as unit—*i.e.*, 1·00).

Substance.	Relative Value.	Remarks.
Loose Wool	3·35	Very good
Feathers	1·10	,,
Felt	1·00	,,
Charcoal (cork)	·87	,,
Silicate Cotton (slag wool)	·85	,,
Straw Rope (loose)	·75	Very fair
Cork	·71	,,
Sawdust	·68	,,
Wood	·50	,,
Asbestos	·50	,,
Sand	·17	Poor
Stone	·02	Bad

Applications.—Iron is a good conductor of heat, and for this reason the hot-water pipes for heating rooms are made of iron. Copper is largely used for hot-water boilers for domestic purposes.

Flannel is a bad conductor, and for this reason it is worn in articles of clothing, especially during the winter. Again, ice is often preserved in hot weather by wrapping it in flannel or packing in sawdust, because the external heat does not penetrate the bad conductors.

Many metal tools which have to be heated, such as soldering bits, have a wooden handle, which is a bad conductor. Hence, when the tool is heated strongly, the handle does not get uncomfortably hot. Again, the iron cylinders of engines are generally encased in a layer of some non-conducting substance, and by this means the heat is kept in the cylinder.

Water pipes also, which may be used for conveying hot or cold water, when fixed in exposed positions, are usually covered with such non-conducting substances as, straw rope, felt, sawdust, silicate cotton, etc., to prevent freezing of the water in the pipes in winter.

Steam pipes are also covered with a paste made of chaff, asbestos, and clay.

Pipes conveying cold water, which pass through rooms containing a moist, warm atmosphere, may cause considerable annoyance, owing to condensation of the moisture in the air around the pipe, and the continual dropping of the water. This may be prevented by coating the pipe with cork dust immediately after applying a coat of black varnish.

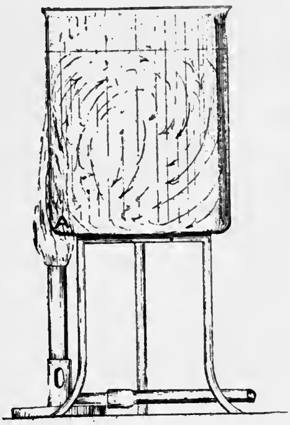


FIG. 107

Convection

(1) **In Liquids.**—*Experiment LXXI.*—Take a large beaker and nearly fill it with water. Fit it up in a retort stand, and heat it on one edge by means of a small flame (Fig. 107). In a short time drop in a small piece of solid colouring matter, and the motion of the water is shown by the streams of coloured water to be in the directions indicated by the arrows.

The water at the bottom is heated, it expands, and becomes less dense; hence it is displaced by the colder water, which moves in from the opposite side of the beaker to take its place.

The warmer water rises to the surface, and then moves towards the sides of the beaker, and down again to the place it started from.

In this way all the water in the beaker is set in motion, and definite currents are set up.

By this method all the water becomes heated, by each particle receiving its heat at A, and then carrying it away.

When fluids are heated in this way, they are said to be heated by **convection**, and the movements are called "**convection currents.**"

Convection may be defined as **the method of heating liquids and gases by the actual bodily movement of their particles.**

Experiment LXXII.—Take a large glass or metal trough. Partly fill with water, and at one end put a small block of ice. Heat the under surface at the other end. The water above the flame expands and rises, whilst that under the ice contracts, becomes denser, and sinks; hence convection currents are set up in the direction shown in the sketch, Fig. 108, and may be clearly indicated by pouring a few drops of sodium hydrate at one side, and several drops of phenolphthalein at the other.

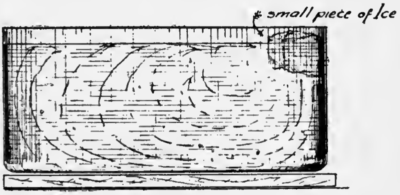


FIG. 108

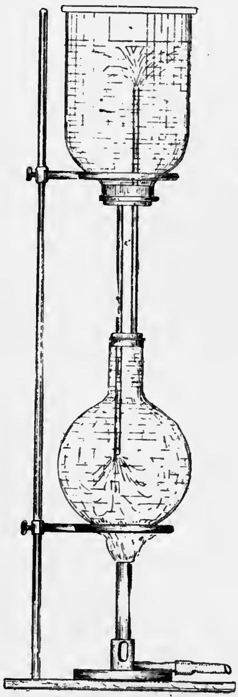


FIG. 109

Circulation of Water.—*Experiment LXXIII.*—Fill a flask with water, and into the neck fit a stopper, through which passes two pieces of tubing arranged as in Fig. 109. The pieces of tubing pass into a large gas jar open at the top. Fill the apparatus with water and then heat the flask. After a time, pour into the jar some indigo solution to colour the water. It will be noticed to travel **down** the tube which passes half-way down the flask and descend to the bottom of the flask. Then it rises up the flask, and last of all, **up** the other tube into the open jar again. In this way the water circulates throughout the system.

This experiment, again, illustrates the heating of water by convection. The water in the flask first expands and rises to the top of the flask, and through the straight tube to the surface of the water in the jar.

As the hot water rises from the bottom of the flask, cold water moves in from the jar above.

Applications

(*a*) **Warming Buildings by Hot Water.**—The above experiment is a simple illustration of the method of warming buildings by hot water, and heating water for domestic purposes. In the former case the flask is replaced by a boiler fixed in the basement of a building. The pipes are usually of iron, because they are good conductors of heat (Fig. 110).

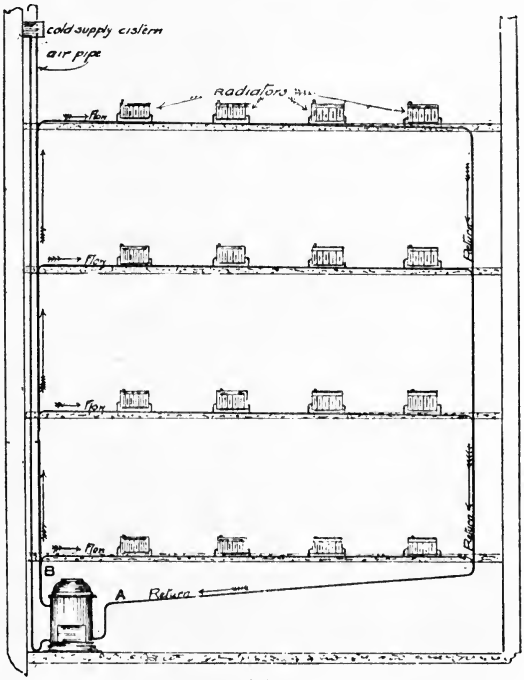


FIG 110

The hot-water pipe, or flow-pipe, B, leaves the **top** of the boiler and goes to the highest floor of the building. The tube A forms the return pipe, which passes through each room, and which has the coils or "radiators" fitted to it. It enters the boiler near the bottom, and brings back the cold water to the boiler.

This is only one example of the various methods in vogue, which all depend upon the previously mentioned principles for their efficiency.

(*b*) **Domestic Hot-water Supply Systems.**—These form typical examples of the property of convection, which is taken advantage of for heating and storing large quantities of water for domestic use.

There are two principal systems: (1) tank; (2) cylinder. In each case a boiler, of either copper or iron, is fixed behind the kitchen fire, so as to be in direct contact with the fire, or an independent boiler is used where large quantities of hot water are required at all times during the day.

The **tank system** consists of two tanks fixed at corresponding levels and connected together by a pipe possessing a dip; to one of the tanks is connected the cold-water supply from the main, and the other tank communicates with the boiler by means of two pipes, one of which passes into the boiler and terminates about 2 ins. above the bottom, and is called the "return pipe," and the other, which terminates at the top of the boiler, is called the "flow-pipe."

The water in the boiler, when heated, flows along this pipe, owing to the displacement caused by the colder water, which travels down the return pipe. Thus circulation is set up, and continues so long as there is any difference in temperature of the water in the tank and the boiler.

Hot water may be either drawn direct from the tank or from the flow-pipe.

In the **cylinder system**, the hot-water storage tank is replaced by a cylinder, which should be fixed as near to the fire as possible, so as to reduce the length of the circulating pipes, which are similar in arrangement to the circulating pipes in the tank system. The flow-pipe should always be taken near to the top of the cylinder, so that hot water may be obtained soon after the fire has been lighted, without warming the whole of the contents of the cylinder. By referring to Fig. 111 it will be seen that the cylinder is fed by a pipe from a cold-water tank, fixed in a high part of the house; the hot-water supply being taken from the top of the cylinder for two reasons: (1) To obtain hot water as long as there is any in the cylinder; (2) to prevent the cylinder being emptied during the stoppage of the cold supply by frosts or other causes. From the top of the cylinder an air pipe is taken, which terminates either above the cold supply cistern, or passes through the roof to permit air or steam, which may be generated, to escape.

In all systems of hot-water supply, safety-valves should be provided, and fixed directly on the boiler, to guard against explosion of the boiler, should the circulating pipes become choked and the passage of water from the cylinder to the boiler entirely stopped in both pipes.

The common causes of explosion are:—

(1) Fixing of stopcocks on both circulation pipes.
(2) Stoppage by ice (during frost) of the circulating pipes.
(3) Stoppage of pipes by deposit of lime from temporarily hard water.
(4) Sudden discharge of water into a boiler from which the water has previously been evaporated and the plates heated to redness.

No. 3 is a doubtful cause. There are no authentic records of an explosion due to this cause. The deposit of lime does not occur equally in the flow and return pipes; the flow-pipe is more rapidly "furred" up than the return pipe.

If one pipe—*i.e.*, the flow-pipe—becomes blocked, steam will be generated in the boiler, and, escaping through the return pipe, will cause such unpleasant and startling noises as will compel the householder to have the matter attended to, long before there is any chance of the return pipe being stopped with deposit.

No. 4 is never responsible for an explosion, as the water delivered against the red-hot plates will assume a spheroidal form, and be held

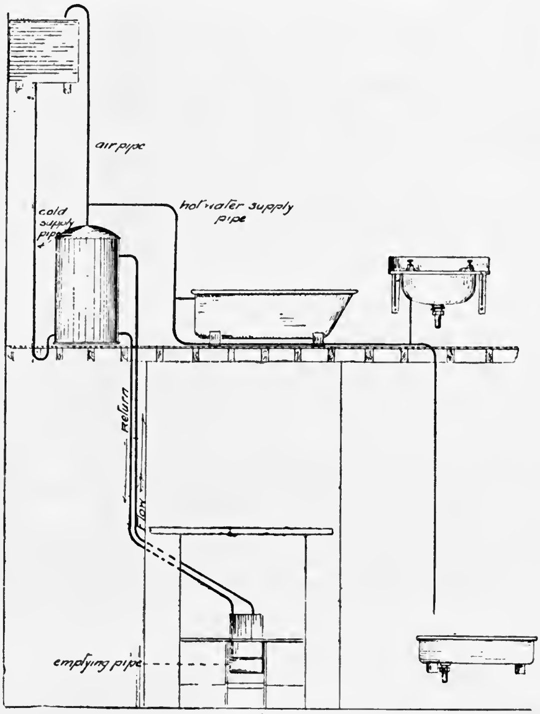


FIG. 111

from direct contact with the plate by a cushion of steam which is immediately formed between the water and the plates; again, the specific heat of iron or copper is so low compared with water, that sufficient heat-units, required to convert the water in the boiler suddenly into steam in sufficient volume to burst it, could not be obtained from the red-hot plates of an ordinary size of boiler, whilst an outlet is provided at the point where the water enters.

If the boiler be of cast iron, it will probably be fractured, owing to unequal contraction of its surfaces; but a fractured boiler is not necessarily an exploded boiler.

Consideration should be given to prevent collapsing of cylinders which become empty, principally during frosty weather, owing to stoppage of the cold supply. The water in the boiler is partly evaporated, and the cylinder consequently filled with steam; the expansion pipe in the meantime becomes frozen over, usually at the point where it terminates on the roof. Condensation takes place, and when we consider that 1 cub. in. of water will form 1 cub. ft. of steam approximately (at normal atmospheric pressure), the vacuum thus formed is apparent. To prevent this a vacuum valve is fixed directly on the cylinder, which admits air to the cylinder immediately the internal pressure is lower than the external. Cylinders are sometimes corrugated to minimise the risk of collapsing.

(2) **Convection in Gases.**—*Experiment LXXIV.*—Take a cigar box and bore two large holes through the lid. Over each hole fit a lamp glass. Under one hole place a lighted candle, and then fasten down the lid (Fig. 112). Hold a piece of smouldering paper over the right-hand one. The smoke travels down this one and up the left-hand one.

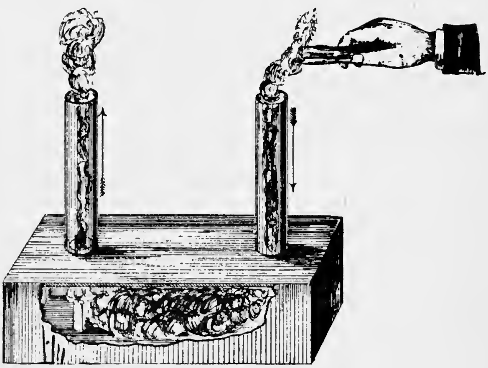


FIG. 112

Since the smoke moves with the air-currents, it shows that the heated air ascends the left funnel, and the cold air descends the right funnel.

These air-currents are convection currents, caused by the cold air displacing the lighter and warmer air.

The air in a room is generally in motion. If the door be slightly opened and a lighted candle be held near the floor, the flame is blown inwards; at the middle the flame is steady, showing that the air at that point is not in motion; at the top the flame is blown outwards.

The room is generally warmer than the air outside; hence this experiment shows that the cold fresh air enters at the bottom, and the impure, warm air leaves at the top. In this way the air in a room may be renewed, or the room may be **ventilated.**

Ventilation is the process by which impure air is removed from rooms and enclosed spaces, and replaced by a supply of fresh air.

The ventilation of buildings is a most important factor which has a direct effect upon the health of the people who inhabit or frequent houses or other buildings. During respiration, human beings discharge into the atmosphere impurities of various kinds, the chief of which are carbon dioxide (CO_2) and particles of living and dead organic matter from the lungs. The CO_2 is formed by the combustion of the carbon compounds partaken of in foods of all descriptions. This combustion in the body produces the necessary energy for accomplishing internal or external work, and also maintains the body at an even temperature. The organic matter consists of waste tissue, and in the case of persons suffering from diphtheria, phthisis, and other diseases, micro-organisms are generally present in their expired breath, which, if breathed into the lungs of any unaffected persons, may cause them to develop similar diseases.

The removal of oxygen from the air, and the addition of CO_2, along with organic impurities, to the air, renders it necessary that all enclosed spaces occupied by human beings should be ventilated, so as to dilute and gradually remove the added impurities and provide for a fresh supply of air.

Examples of polluted air, owing to inadequate ventilation, are common in the compartments of railway carriages, theatres, churches, lecture halls, bedrooms, etc., and are indicated by a stuffy smell only observed by persons entering the room, etc., after being for some time in the fresh air. When the quantity of CO_2 in the air is excessive, the people breathing this air experience a depressed feeling, accompanied by lassitude and severe frontal headache. This is not due entirely to the excess of CO_2, but to the organic matter which accompanies the CO_2 given off from the lungs.

This may be proved by the air in aerated-water manufactories, which contains a large amount of CO_2, frequently as much as 1·2 parts of CO_2 per 1000 parts of air, having no deleterious effect upon the workers in the buildings. If the pollution be due to respiration, the quantity of organic matter in the atmosphere will vary with any variation in the amount of CO_2 in the atmosphere.

The addition of CO_2 to the atmosphere may be caused by the combustion of carbon compounds, such as coal, wood, oil, candles, coal-gas, etc.

The latter—*i.e.* coal-gas—which is largely used in dwellings and public buildings, has a direct bearing upon the amount of air required to be discharged into a room in a given time.

It has been determined that 1 cub. ft. of coal-gas gives off about 2 cub. ft. of CO_2 during its combustion; and though it is possible to have a large quantity of pure CO_2 present in the air without producing any appreciable ill effect upon the health of the persons breathing it, the removal of the same is essential, to obtain a thoroughly satisfactory condition of the air of rooms and enclosed spaces.

The amount of CO_2 present in the atmosphere is stated by various authorities as ·2 to ·4 part per 1000 parts of air, and the amount of CO_2 given off by an adult at rest = ·6 of a cubic foot per hour. The limit of impurity considered permissible is ·6 part of CO_2 per 1000 parts of air.

As the most important impurities in the air of inhabited rooms are CO_2 and the organic matters which accompany it during respiration, the quantity of CO_2 is taken as the standard in fixing the limit of impurity consistent with reasonably-healthy conditions. This standard, as previously mentioned, is ·6 part of CO_2 per 1000 parts of air.

To maintain this, each person requires 3000 cub. ft. of air to be delivered per hour into the room he is inhabiting.

Humidity.—The condition of the atmosphere with relation to the amount of water vapour which it holds in suspension, is spoken of as the **Humidity** of the atmosphere. The **Actual Humidity** is the amount of water vapour actually present in a given volume of air.

The term **Relative Humidity** is used to indicate the relation between the amount of water vapour actually present and the quantity that would be required to saturate the atmosphere at a given temperature and pressure.

The amount of water vapour which the atmosphere is able to hold varies with different temperatures. Thus at 41° F. air is saturated if it contains 3 grains of vapour per cubic foot, whilst at 83° F. it would require 12 grains per cubic foot to saturate it.

When the point of **Saturation** is reached, part of the water vapour falls in the form of **dew** or **rain**. It will be seen that if the air contains 10 grains of vapour per cubic foot at 83° F., and it is suddenly cooled to say 56° F., the excess over the quantity required to saturate the air at 56° F. would be deposited in the form of rain.

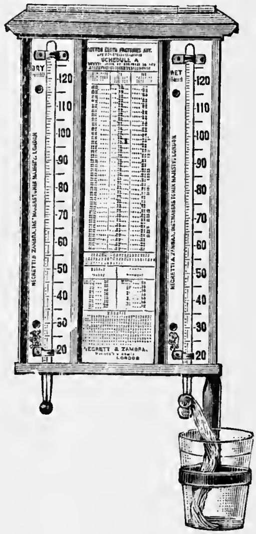


FIG. 113

The **Humidity of the Atmosphere** is generally determined by the aid of an instrument known as a **Hygrometer** (shown in Fig. 113). It consists of two thermometers, fixed on a frame side by side. To the bulb of one of them is attached a piece of cotton, which entirely surrounds it and is continued into a small glass vessel immediately below, containing water. The water travels up the cotton by capillary attraction, and keeps the bulb of the thermometer constantly wetted; from this wetted surface evaporation is continually going on, which extracts heat from the **wet bulb**, and causes it to record a lower temperature than the **dry bulb.** The difference between the readings of the "wet and dry" thermometers will be greater when the air is comparatively dry, than when it is near the point of saturation, owing to evaporation taking place more rapidly in a dry than in a moist air.

The percentage of humidity which is most agreeable is between 70 and 80 per cent. of saturation for temperatures between 55° and 70° F.

Table of Dew-points, with weight of vapour required for saturation for each degree Fahrenheit from 10° to 100°:—

Degrees Fahrenheit.	Weight of Vapour in grains per cubic foot.	Degrees Fahrenheit.	Weight of Vapour in grains per cubic foot.	Degrees Fahrenheit.	Weight of Vapour in grains per cubic foot.
10	0·84	40	2·86	70	8·01
11	0·88	41	2·97	71	8·27
12	0·92	42	3·08	72	8·54
13	0·96	43	3·20	73	8·82
14	1·00	44	3·32	74	9·10
15	1·04	45	3·44	75	9·39
16	1·09	46	3·56	76	9·69
17	1·14	47	3·69	77	9·99
18	1·19	48	3·82	78	10·31
19	1·24	49	3·96	79	10·64
20	1·30	50	4·10	80	10·98
21	1·36	51	4·24	81	11·32
22	1·42	52	4·39	82	11·67
23	1·48	53	4·55	83	12·03
24	1·54	54	4·71	84	12·40
25	1·61	55	4·87	85	12·78
26	1·68	56	5·04	86	13·17
27	1·75	57	5·21	87	13·57
28	1·82	58	5·39	88	13·98
29	1·89	59	5·58	89	14·41
30	1·97	60	5·77	90	14·85
31	2·05	61	5·97	91	15·29
32	2·13	62	6·17	92	15·74
33	2·21	63	6·38	93	16·21
34	2·30	64	6·59	94	16·69
35	2·39	65	6·81	95	17·18
36	2·48	66	7·04	96	17·68
37	2·57	67	7·27	97	18·20
38	2·66	68	7·51	98	18·73
39	2·76	69	7·76	99	19·23
...	...	...	...	100	19·84

To find the relative humidity, or percentage of saturation, of the atmosphere, it is necessary to find the **dew-point**—*i.e.*, the temperature at which the atmosphere would be saturated, with the amount of vapour it contained at the time of observation. To find the dew-point it is necessary to use factors, which have been carefully worked out by Glaisher, and are given in the following table :—

Reading of Dry Bulb. Fahr. degrees.	Factor.	Reading of Dry Bulb. Fahr. degrees.	Factor.	Reading of Dry Bulb. Fahr. degrees.	Factor.
10	8·78	40	2·29	70	1·77
11	8·78	41	2·26	71	1·76
12	8·78	42	2·23	72	1·75
13	8·77	43	2·20	73	1·74
14	8·76	44	2·18	74	1·73
15	8·75	45	2·16	75	1·72
16	8·70	46	2·14	76	1·71
17	8·62	47	2·12	77	1·70
18	8·50	48	2·10	78	1·69
19	8·34	49	2·08	79	1·69
20	8·14	50	2·06	80	1·68
21	7·88	51	2·04	81	1·68
22	7·60	52	2·02	82	1·67
23	7·28	53	2·00	83	1·67
24	6·92	54	1·98	84	1·66
25	6·53	55	1·96	85	1·65
26	6·08	56	1·94	86	1·65
27	5·61	57	1·92	87	1·64
28	5·12	58	1·90	88	1·64
29	4·63	59	1·89	89	1·63
30	4·15	60	1·88	90	1·63
31	3·70	61	1·87	91	1·62
32	3·32	62	1·86	92	1·62
33	3·01	63	1·85	93	1·61
34	2·77	64	1·83	94	1·60
35	2·60	65	1·82	95	1·60
36	2·50	66	1·81	96	1·59
37	2·42	67	1·80	97	1·59
38	2·36	68	1·79	98	1·58
39	2·32	69	1·78	99	1·58
...	...	...	...	100	1·57

To use Glaisher's Factors for finding the dew-point.

Rule.—Subtract the reading of the wet bulb thermometer from the reading of the dry bulb. Multiply the difference by the **factor** opposite the dry bulb in the preceding table, and subtract the product from the reading of the dry bulb, and the result will be the dew-point.

Example.—Reading of Dry Bulb = 68.
Reading of Wet Bulb = 60.
Factor opposite reading of Dry Bulb = 1·79.

$$\text{The dew-point} = 68 - ((68 - 60) \times 1{\cdot}79)$$
$$= 68 - 14{\cdot}32$$
$$= 53{\cdot}68^\circ \text{ Fahr.}$$

Having found the dew-point, the actual amount of water vapour present in a known volume of the atmosphere may be obtained from the table previously given, of weights of vapour constituting saturation for each degree Fahrenheit.

The **Relative Humidity** will then be the ratio between the amount of vapour actually present and the amount required to saturate the atmosphere at the temperature indicated by the dry bulb.

It is generally expressed as a percentage of saturation as follows:—

	Grains.
Amount of vapour per cubic foot required to saturate at	53·68 = 4·65
" " "	68·00 = 7·51

Therefore Relative Humidity

$$= \frac{4{\cdot}65 \times 100}{7{\cdot}51} = 62 \text{ per cent. of saturation, or } 62 \text{ per cent. of humidity.}$$

Ventilation of Rooms

Everyone has noticed the draught up the chimney when a fire burns in the grate, and also felt the draught caused by the cold air moving near the floor from the door to the fire-grate. The heat from the fire sets up convection currents in the air, and thus the room is ventilated.

Diffusion as well as **convection**, plays a very important part in the natural ventilation of rooms. It has already been stated that the air in rooms is hotter than that outside; hence, in the walls, if the walls be porous, diffusion between inside and outside air will take place according to Graham's Law, with the result that the air of a room is, under some conditions, changed once per hour by this method of ventilation alone.

There are **two methods,** generally speaking, for effecting the ventilation of buildings: 1st, **Natural**; 2nd, **Mechanical.**

The **Natural system** depends upon the forces in nature for its efficiency, such as the difference in weight of bodies of air of different temperatures, diffusion, and the action of air-currents or winds.

In the application of natural systems of ventilation, provision is made for the admission of fresh air in one or more parts of the room, and an exit, or exits, are provided in positions as far removed as possible from the inlets. Considering the fact that **the heated, impure air of a room is lighter, bulk for bulk, than the cold fresh air entering the room at a lower temperature,** the air outlets are usually fixed near the top of the room, and often in the ceiling, whilst the air inlets are inserted either 2 ft. above the floor line, or a bent tube is carried up to a height of 6 ft. above the floor, and

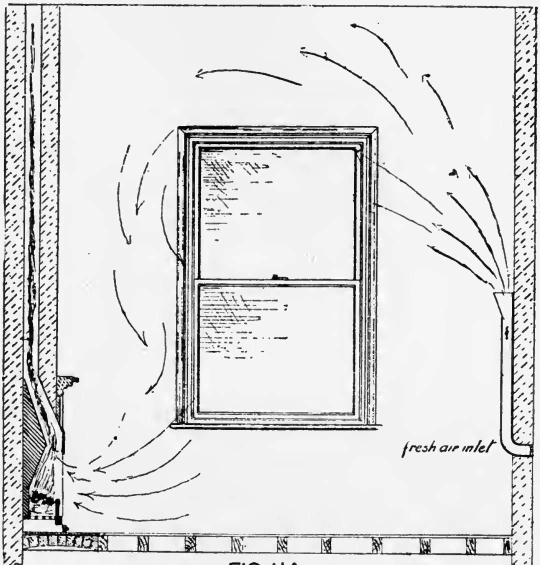


FIG. 114

thus discharges the air in an upward direction, the inlet to the tube is arranged about 1 to 2 ft. from the floor. This appliance is known as Tobin's Tube.

In ordinary living rooms this tube may be used for admitting air, the outlet in such cases is generally the chimney, which, when there is a fire in the grate, proves a very powerful exhaust shaft, owing to the warming of the shaft and the air passing through it, by the heat of the fire. Fig. 114 shows this arrangement.

The air may also be admitted by other methods, as shown in Fig. 115. This consists of a specially constructed window, possessing a deep bead inside the frame which allows the lower sash to be opened 2 or 3 ins., without causing an aperture between the bottom sash and the window sill; the air passes into the room between the top and bottom sash in an upward direction. This is known as the **Hinckes-Bird window ventilator.**

Arrangements are sometimes made to warm the incoming air, as shown in Fig. 115A. The cold, fresh air in this illustration passes through the tubes of the radiator, and is thereby warmed, and does not give rise to the unpleasant consequences resulting from the delivery of cold air into inhabited rooms, causing draughts; it also assists ventilation.

FIG. 115

The outlet arrangements are often provided in the ceiling, irrespective of the chimney. This is especially the case where a collection of gaslights on one or more pendants is used (Fig. 116). A tube is fixed above the light, and conveys the heated and impure air through the roof into the open atmosphere. The heat evolved from the combustion of gas has a valuable aspirating effect in the removal of the vitiated air from the room, thereby greatly aiding ventilation.

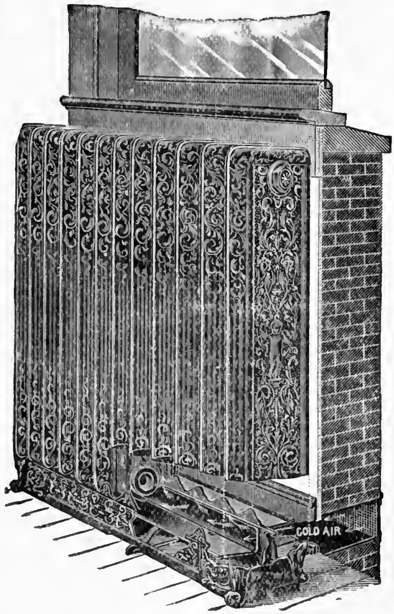


FIG. 115A

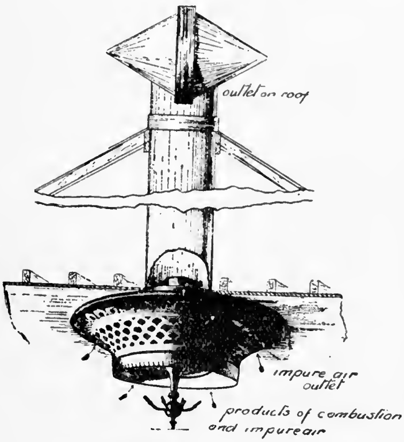


FIG. 116

Mechanical Ventilation depends for its efficiency upon mechanical forces set in action by man, and consists essentially of forcing air into rooms, or extracting air therefrom by means of fans or propellers.

In most mechanical systems of ventilation the air is warmed to the required temperature, by passing it over pipes heated by hot water or steam, before it is delivered into the various rooms. This arrangement combines the heating and ventilation of the building in one plant, and, if properly arranged, with air-propellers of sufficient capacity, and ducts of suitable sizes, it proves a reliable and easily controlled method of ventilation, especially suited to the requirements of large buildings.

Ventilation is an essential feature in all **drainage, soil-, and waste-pipe** arrangements, the products of decomposition are diluted and removed by the air-currents passing through the pipes. In the case of drains, the air is usually admitted at the point where the drain leaves private land, and the air outlet or vent pipe is fixed at the head or highest point in the drainage system, and is carried up the wall of the building and terminated above the eaves or ridge of the roof, so as to discharge the foul air at a high level, well above the point where the air is utilised for breathing purposes.

In the case of **Waste Pipes** from baths, lavatories, etc., air is admitted at the foot of the waste pipe, and escapes at its termination, which should be arranged as previously described in the example of a drain vent pipe.

The air in drains and waste pipes is usually warmer than the surrounding atmosphere, and there is consequently a displacement of the long column of warm air in the vent, drain, and waste pipe caused by the colder atmosphere outside the pipes, which takes the place of the displaced air, and is in turn warmed and displaced; thus giving rise to convection currents as long as there is any difference in temperature of the air inside and that on the outside of the pipes.

Radiation

If a person be seated in the front of a hot fire, he feels the heat on his face. If he lifts a screen to his face he no longer feels it, but if he makes a hole in the screen, and holds the latter near his face, he again feels the heat on that part of his face near which the hole is situated, thus showing that the heat is travelling in straight lines.

In this case, the heat from the fire does not reach him by conduction or convection.

In the first part of this chapter it was stated that air was a bad conductor of heat. Also, it was shown that when a fire was lighted in a room, the hot air rises up the chimney and the cold air moves towards the fireplace by convection; hence a fire does not heat a room either by conduction or convection.

The heat passes from the fire in straight lines (as shown by the above screen) travelling through the space between the particles of air, and not through them. When heat passes from one point to another in this way, it is said to be transmitted by **radiation**.

Heat is transmitted by radiation when it passes from one point to another in straight lines, without heating the medium through which it passes.

The heat of the sun reaches us by radiation. Most of the space between the sun and our earth does not contain any form of matter, hence, the heat which reaches us cannot have come either by conduction or convection.

Applications

In hot-water systems for heating buildings the heat from the fire passes by conduction through the boiler and heats the water, and the hot water circulates through the pipes by convection. The heat from the hot water passes through the iron pipes by conduction; and from the hot pipes the heat is mostly radiated throughout the room. The air in contact with the pipes is heated, and thus gives rise to convection currents. Hence, in this system, we have examples of the three methods for the transmission of heat—*i.e.*, conduction, convection, and radiation.

Similarly, we have examples of conduction and convection in the hot-water supply systems.

CHEMISTRY

CHAPTER XXI

ELEMENTS AND COMPOUNDS, CHEMICAL AND PHYSICAL CHANGES, METALS AND NON-METALS, SYMBOLS, ATOMIC WEIGHTS, MIXTURES AND COMPOUNDS

ELEMENTS AND COMPOUNDS

Physical and Chemical Changes

It has already been shown that water may be changed from one state, as solid, to another state, as liquid or gas, by an alteration of temperature. Whilst in the liquid state, water possesses the properties common to liquids; in the solid state it possesses those common to solids; and as a gas, the gaseous properties.

The properties for each state are quite distinct, yet the composition of water in all the states is the same. Under ordinary circumstances, a piece of steel is not a magnet. If magnetised, it becomes a magnet, and has properties quite different from those of the original steel, as attracting iron filings; but again the composition remains unaltered.

Usually a glass rod does not attract small pieces of paper, but if bed with a piece of silk it is electrified, and then it does so. Again, the properties of the glass have been altered, but the composition remains the same.

In all similar cases as the water, steel, and glass rod, the properties of the substance may easily be altered again, and made like the original substance. Such changes are called **physical changes.**

A physical change takes place in a substance when there is a change of properties, but no change of composition.

If the above piece of steel be exposed to the damp air for a few days it will become rusty. The brownish flakes of rust have quite different properties from the original steel, and if the whole be weighed before and after rusting, there will be found to be an increase in weight, due to rusting. In fact, the "iron rust" has not the same composition as the steel; hence a change of properties has been accompanied by a change of composition.

Again, if a piece of bread be toasted, it is slightly charred, and there is a change of composition as well as a change of properties.

In both cases it would be extremely difficult to restore the original substances from the final result. Such changes are called **chemical changes.**

A chemical change takes place in a substance when the change of properties is accompanied by a change of composition.

Elements

It has been possible to split up some substances which occur in the earth's crust into other substances simpler than themselves. Chemists have strenuously worked at these substances for many years, until at the present time there exist only about seventy which have not been further decomposed.

These bodies are the simplest forms of matter, and are called **elements.**

An element is the simplest form of matter, and it cannot be split up into anything simpler than itself.

If future research should prove that any one of the above seventy substances can be split up into other substances simpler than itself, it can no longer be regarded as an element according to the above definition.

Metals and Non-Metals

These elements may be grouped according to certain properties. Most of them are solids, and possess a bright, shining surface or lustre, especially when freshly cut. They are also good conductors of heat and have a high specific gravity. Such elements are called **metals.**

To the other class belong all liquids (except mercury) and gases, and also those solids which have not a bright lustre, and have a comparatively low specific gravity, as well as being bad conductors of heat. Such bodies are called **non-metals** or **metalloids.**

The distinction between the two classes is so marked, that out of all these elements only a few possess properties common to both classes, arsenic being the chief one.

Symbols

For the sake of convenience, a system of symbols has been introduced, to represent the different elements. In this system each element is represented by a different symbol. This is usually the first letter of its Latin name, which is generally the same as the English one; but where two elements have the same initial letter, another letter is added.

Thus C represents carbon; Ca, calcium; Cu, copper (L., cuprum); Co, cobalt.

Atomic Weight

In a later chapter it will be shown that all gases have not the same density.

Thus it will be found that one gas, hydrogen, is lighter than air, whilst another gas, carbon dioxide, is much heavier than air.

In order to compare the densities of gases, it is necessary to select a standard.

For this reason the densities of all gases are compared with hydrogen, which is the lightest gas known.

The weight of a known volume of any gas is compared with the weight of the same volume of hydrogen, and the result gives the number of times that the gas is heavier than hydrogen.

The **atomic weight** of an element shows the number of times that the atom of an element is heavier than an atom of hydrogen.

Table of Elements

The following list contains the principal common elements, along with their symbols and atomic weights. The non-metals are printed in **italics**, and the metals in **small capitals** :—

Table of Common Elements

Elements.	Symbol.	Atomic Weight.
ALUMINIUM	Al	26·9
ANTIMONY	Sb (Stibium)	119·4
ARSENIC	As	74·4
BARIUM	Ba	136·0
BISMUTH	Bi	206·4
Boron	B	10·7
Bromine	Br	79·4
CADMIUM	Cd	111·3
CALCIUM	Ca	39·7
Carbon	C	11·9
Chlorine	Cl	35·5
CHROMIUM	Cr	51·9
COBALT	Co	58·6
COPPER	Cu	63·0
Fluorine	F	18·9
GOLD	Au (Aurum)	195·7
Hydrogen	H	1·0
Iodine	I	125·9
IRON	Fe (Ferrum)	55·6
LEAD	Pb (Plumbum)	206·0
MAGNESIUM	Mg	24·2
MANGANESE	Mn	54·6
MERCURY	Hg (Hydrargyrum)	199·0
NICKEL	Ni	58·6

Elements.	Symbol.	Atomic Weight.
Nitrogen	N	14·0
Oxygen	O	16·0
Phosphorus	P	30·8
PLATINUM	Pt	193·3
POTASSIUM	K (Kalium)	39·0
SILVER	Ag (Argentum)	107·1
Silicon	Si	28·2
SODIUM	Na (Natrium)	22·9
Sulphur	S	31·8
TIN	Sn (Stannum)	118·0
ZINC	Zn	65·0

Compounds

In dealing with elements, it was stated that chemists had succeeded in splitting up most bodies into simpler bodies. Those bodies which can be decomposed into simpler bodies are called compounds.

Experiment LXXV.—Take a small quantity of mercuric oxide, and put it into a test tube. Heat gently, and notice that a silvered mirror of mercury collects on the cooler parts of the tube, while a gas is given off which rekindles a glowing chip: this gas is oxygen.

Mercuric oxide has been split up into mercury and oxygen; hence it must be a compound. This change may be represented by the following equation:—

Mercuric Oxide = Mercury + Oxygen.

Substances can be formed by the combination of two or more elements. Thus if iron be allowed to rust, it increases in weight, and the rust has totally different properties from the original iron. In fact, the rust is a compound formed by the union of the iron and a gas in the air called oxygen, and is called iron oxide.

Iron + Oxygen = Iron Oxide.

Again, if a piece of bright, shining copper be heated in the air, it first tarnishes and then darkens in colour, and the surface is covered with black scales. If weighed before and after heating, there will be found to be an increase in weight, due to heating. The black scales are a compound, called copper oxide, formed by the union of copper with the oxygen of the air. Thus:—

Copper + Oxygen = Copper Oxide.

Similarly, it is found that whenever two elements unite, they form a **compound.**

A compound is a body formed by the chemical combination of two or more elements, and out of which two or more different elements can be obtained.

As elements are represented by symbols, so it is possible to represent compounds.

Thus mercuric oxide is represented by the letters HgO, which is called its **formula.** This means that it consists of the elements mercury (Hg) and oxygen (O).

It also shows the proportion in which they are present. For, according to the tables, the atomic weight of mercury is 199, and that of oxygen 16; then 16 lbs. of oxygen would unite with 199 lbs. of mercury to form 215 lbs. of mercuric oxide.

Similarly, for copper oxide:—

Copper	+	Oxygen	=	Copper oxide
Cu	+	O	=	CuO
63	+	16	=	79

It will now be clear that a chemical formula shows:—

(1) The elements of which the compound is composed.
(2) Its exact composition, as shown by the proportion in which the elements are represented in the compounds.

The molecular weight of a compound is obtained by finding the sum of the atomic weights of the elements composing it, thus:—

Copper oxide $(CuO) = 63 + 16 = 79$.
Mercuric oxide $(HgO) = 199 + 16 = 215$.
Potassium chlorate $(KClO_3) = 39 + 35{\cdot}5 + (16 \times 3) = 122{\cdot}5$.

Mixtures and Compounds

Experiment LXXVI.—Take some finely powdered sulphur, and intimately mix it with fine iron filings. Scatter some on water: the sulphur floats, whilst the iron sinks. Next pass a magnet over the remaining contents. The filings are attracted, but the sulphur is not; hence the iron is again separated from the sulphur. In a similar manner, sulphur and iron may be mixed in any proportions, but no change takes place, and the ingredients may easily be separated again. Such a mixture is called a **mechanical mixture.**

Experiment LXXVII.—Place some of the above mixture into a test tube, and heat it until the sulphur melts and the iron begins to **glow** or appear red hot. Then stop the heating. The iron still glows for a time, showing that great heat is being produced. Now break the test tube. The black substance which remains does not resemble either iron or sulphur, and these are no longer separated, either by scattering on water or by a magnet. It is a different substance with different properties, and it has been formed by the union of the iron and sulphur in a definite proportion:—

Iron	+	Sulphur	=	Iron sulphide
Fe	+	S	=	FeS
56	+	32	=	88·0

Such a substance is called a **chemical compound.**

Experiment LXXVIII.—Mix powdered sulphur and copper turnings, and heat the mixture in a test tube until the copper begins to glow. Stop the heating, and note the heat produced, as shown by the glow of the copper. At the end examine the residue. The green substance which remains is copper sulphide, and it is quite different from either the copper or the sulphur:—

Copper	+	Sulphur	=	Copper sulphide
Cu	+	S	=	CuS
63	+	32	=	95

These numbers show that 63 parts by weight of copper unite with 32 parts by weight of sulphur to form 95 parts of copper sulphide.

If an excess of sulphur be added, it is either driven off as vapour or burnt, for only the above proportion of sulphur unites with the copper.

The following table shows the chief differences between mechanical mixtures and chemical compounds:—

Mixtures	Compounds
(1) The components are simply mixed. The particles exist side by side, and can easily be separated.	Every particle of one component is chemically united with one or more particles of the other, and they cannot be easily separated.
(2) The components may be mixed in any proportion.	The components unite in a definite proportion to form a chemical compound.
(3) There is no change of temperature on mixing.	The chemical union of the components is accompanied by change of temperature generally caused by the production of heat.
(4) The properties of the mixture are similar to those of the components.	The properties of the compound are totally different from those of its components.

Indestructibility of Matter

It is sometimes assumed that if a substance disappears from view it has been destroyed. Thus if a candle be allowed to burn away, we at first think that the candle has been destroyed. Similarly, if any substance be burnt it is changed, and we think that it has been destroyed.

Again, if water be boiled, it gradually diminishes in volume as the vapour disappears.

Experiment LXXIX.—Place a pound of water in the glass retort

(Fig. 117) fitted with the condensing arrangement. Evaporate the water, and weigh the amount condensed in the flask. If carefully done, they should be the same.

This experiment shows that when water is converted into steam and back again, there is no loss of weight; hence it is not destroyed.

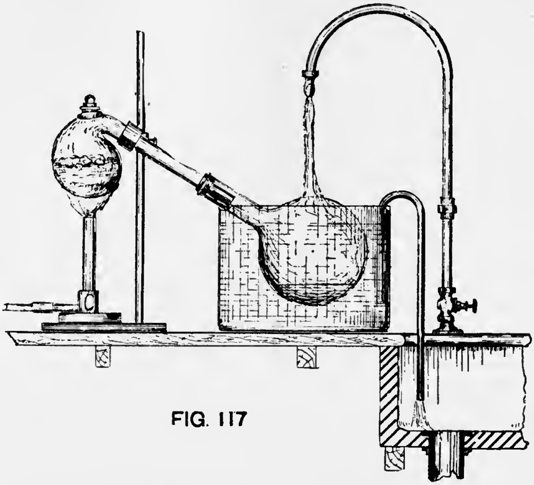


FIG. 117

Similarly, if any liquid be evaporated, it becomes a gas, but is not destroyed.

Experiment LXXX.—Place a piece of phosphorus in a small flask, and close the latter with a tightly fitting stopper. Weigh the flask and then warm it, until the phosphorus burns, giving off dense white fumes. When the phosphorus has disappeared, allow the flask to cool, and weigh again.

There is no change in weight, although the phosphorus has gone, and the flask is full of dense white fumes.

The phosphorus has chemically united with a portion of the air to form phosphorus pentoxide, but it has not been destroyed.

Experiment LXXXI.—Take a large gas jar, and allow a candle to burn in it for a time. At first the sides of the jar are perfectly clear, but they gradually become covered inside with moisture. When the candle goes out, remove it; put into the jar some lime water, and shake. The latter is turned milky, which proves that a gas called carbon dioxide has been produced.

Neither the moisture nor the carbon dioxide were present in the jar before the candle was lighted; hence we conclude that when a candle burns it forms water and carbon dioxide.

The next experiment is devised to collect all these gases as the candle burns, and to show that although the candle itself burns away, other products are formed, and there is no loss in weight.

FIG. 118

Experiment LXXXII.—Take an ordinary lamp glass, and fit into the base a perforated cork, on which is fastened a small candle (Fig. 118). Near the top place a layer of calcium chloride (to absorb all moisture produced), and then a layer of potassium hydrate (to absorb all carbon dioxide). Fasten this apparatus to the left arm of a balance and weigh it. Then light the candle.

As the candle burns away, the left arm descends, showing an **increase in weight** (which will be explained later).

Hence although the candle has disappeared, it has not been destroyed.

These experiments thus illustrate the universal truth that "**matter is indestructible.**"

CHAPTER XXII

COMPOSITION OF AIR, PROPERTIES, ETC., OXIDATION (COMBUSTION), OXYGEN AND NITROGEN, CARBON DIOXIDE

Air and its Constituents

THE air we breathe surrounds our earth to a height of at least 50 miles. Although it is invisible, and, when pure, tasteless and without smell; we become aware of it whenever the wind blows.

It has already been shown that air is a gas, and possesses those properties common to gases. It has also been shown that the air has weight, and exerts a pressure at sea-level of about 15 lbs. to the square inch.

Composition of the Air by Volume

Experiment LXXXIII.—Fix a candle on a cork, and float it in a trough of water. Place a bell jar over the candle. The candle burns for a time, but at length goes out, and the water rises to some extent up the inverted jar. The gas which remains puts out the light. Hence, when a candle burns it takes a portion of the air, and the remaining gas will not support combustion.

Experiment LXXXIV.—Moisten the inside of a large gas jar with water and scatter inside some iron filings. Invert over water, and leave for a few days. The iron rusts, and the water again rises about one-fifth. Test the remaining gas and see that it will not support combustion.

This experiment shows that when iron rusts, it removes from the air the same substance that a candle does when it burns; in fact, the rusting action is a slow form of combustion.

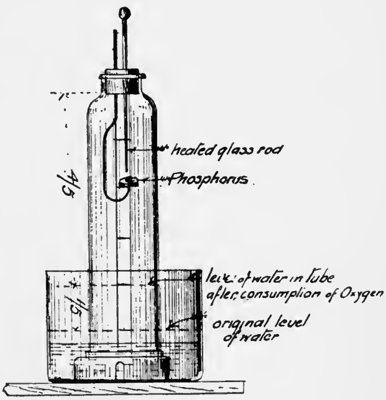


FIG. 119

Experiment LXXXV.—The apparatus shown in Fig. 119 consists of a vessel partly inverted in water. Through the stopper passes a deflagrating spoon containing a small piece of phosphorus and a pointed glass rod, which can be heated, and thus be used to set fire to the phosphorus. The latter burns vigorously, and forms dense white fumes. At length it goes out, and the fumes dissolve in the water. Then it will be noticed that the water has risen exactly one-fifth of the way up the jar. The remaining gas will extinguish a light, and is called **Nitrogen.**

The part used has chemically united with the phosphorus to form the compound phosphorus pentoxide. This part is called "Oxygen," and forms the active constituent of the air.

It has thus been shown that air is composed of

4 parts of nitrogen to 1 part of oxygen.

Air a Mixture, not a Compound

(1) When oxygen and nitrogen are mixed in the above proportion, the mixture has the same properties as pure air, and there is no change in temperature on mixing.

(2) Again, the elements in a chemical compound are always present in the same definite proportions, but the composition of the air is found to vary slightly in different places.

(3) Oxygen is more soluble in water than nitrogen; hence if air dissolved in water be examined, it is found to contain 1 part of oxygen to 2 parts of nitrogen, which is not the ordinary composition of the air. This could not be the case if air were a chemical compound.

From the above and other proofs, it is correct to state that air is a mechanical mixture and not a chemical compound.

Constituents of the Air

I. **Oxygen** (symbol, O; atomic weight, 16).

Occurrence.—Oxygen occurs not only in a free state in the air, but also in a combined state in water, in plants and animals, and in most of the rocks which form the earth's crust. In fact, one-half the total weight of the earth is composed of oxygen, free or chemically united with other substances.

Preparation.—It is usually prepared, on a small scale, by decomposing potassium chlorate ($KClO_3$), because on heating, the latter is split up into potassium chloride (KCl) and oxygen (O).

Experiment LXXXVI.—Fit up the apparatus as shown in Fig. 120. Into the flask put a mixture of 4 parts of potassium chlorate to 1 part of manganese dioxide, the latter making the potassium chlorate liberate the oxygen more readily and rapidly. On heating gently, the oxygen is liberated and passes down the leading tube. It is collected in the jar by the displacement of water.

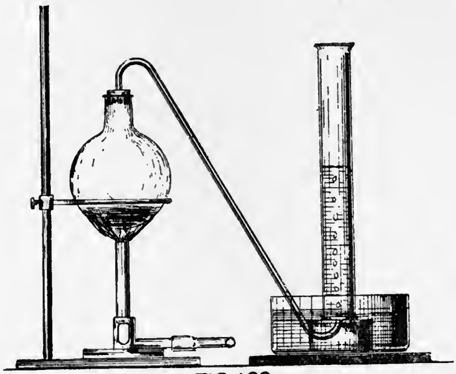


FIG. 120

By this method, several jars of the gas may be collected, and its properties shown by the following experiments:—

Equation—

Potassium chlorate	=	Potassium chloride	+	Oxygen
$KClO_3$	=	KCl	+	$3O$
122·5	=	74·5	+	3×16

These numbers show that when 122·5 lbs. of potassium chlorate are entirely decomposed by heat, 48 lbs. of oxygen are liberated.

Properties—

If the gas collected be examined, it will be found to be invisible, tasteless, and odourless.

Since it was collected by the displacement of water, it must either be insoluble or only very slightly soluble in water.

Combustion in Oxygen

Experiment LXXXVII.—Place a piece of wood **charcoal** on a deflagrating spoon, heat it until it becomes red hot at one corner, and then plunge it in a jar of oxygen. The charcoal immediately becomes white hot and rapidly wastes away, while sending out a shower of sparks or scintillations. Pour into the jar a little lime water. It is turned milky, showing that **carbon dioxide** (CO_2) is formed when carbon or charcoal burns in oxygen:—

Carbon	+	Oxygen	=	Carbon dioxide
C	+	O_2	=	CO_2

O

Experiment LXXXVIII.—Similarly, burn **sulphur**, first in air and then in oxygen. In the former it burns with a small flame, but in the latter it burns with a bright, blue flame, producing in both cases the obnoxious gas, **sulphur dioxide** (SO_2).

$$\begin{array}{ccccc} \text{Sulphur} & + & \text{Oxygen} & = & \text{Sulphur dioxide} \\ S & + & O_2 & = & SO_2 \end{array}$$

(These fumes dissolve in water, forming sulphurous acid, which turns blue litmus red.)

Experiment LXXXIX.—Similarly, burn phosphorus in air and in oxygen. In the latter it burns much more brilliantly, forming dense white fumes of **phosphorus pentoxide** (P_2O_5).

$$\begin{array}{ccccc} \text{Phosphorus} & + & \text{Oxygen} & = & \text{Phosphorus pentoxide} \\ 4P & + & 5O_2 & = & 2P_2O_5 \end{array}$$

(These fumes dissolve in water to form phosphoric acid, which turns blue litmus red.)

From these experiments it will be seen that substances which burn in air burn much more violently in oxygen; hence oxygen is said to be a good **supporter of combustion.**

Experiment XC.—Fill a bell jar with oxygen, and put into the cork a watch spring, at the end of which burns a piece of sulphur. The iron burns brilliantly, and the **iron oxide** (Fe_3O_4) falls into the water:—

$$\begin{array}{ccccc} \text{Iron} & + & \text{Oxygen} & = & \text{Iron oxide} \\ 3Fe & + & 2O_2 & = & Fe_3O_4 \end{array}$$

In all these cases it will be noticed that the compound formed consists of oxygen, chemically united with the combustible body; such compounds are called **oxides.**

Oxides

Nearly all the elements unite with oxygen to form oxides. The metals, sodium and potassium, unite with the oxygen of the air at ordinary temperatures. On heating, magnesium ribbon burns brilliantly in air or oxygen, to form the white oxide of magnesium, MgO, commonly called magnesia. The common metals, as copper, iron, lead, zinc, and tin, tarnish on heating in air and become coated with an oxide of the metal. Gold and platinum are not oxidised when heated in the air, but their oxides can be formed by indirect methods. If the above common metals be exposed to damp air, they are found to "rust," or become coated with a thin covering of the oxides more or less readily. In this way, iron readily rusts, and forms the familiar brown oxide of iron.

This rusting action is most marked in the case of cast-iron cisterns used for storing water in houses, the insides of the tanks are exposed alternately to air and water, owing to the variation of the water-level whilst the water in the tank is being used.

It also occurs in cast- and wrought-iron pipes used for distributing water, especially if the water is pure or soft and well aerated (*i.e.*, containing dissolved air).

Wrought iron is more readily acted upon than cast iron.

To prevent this action, or minimise it, iron goods are coated with various substances, which are not readily oxidised, and which tend to prevent contact between the iron and the water or air.

Painting, galvanising, tarring, glass enamelling, and coating the iron with a special mixture known as Dr Angus Smith's solution (consisting of boiled pitch and 5 per cent. linseed oil) are used; the latter, when properly applied, gives satisfactory results. Another coating, applied by a special process known as "The Bower-Barff Process," which forms a thin film of black or magnetic oxide of iron on the surface of the iron, is sometimes used; but this is expensive, and difficult to apply.

(*N.B.*—**This rusting will not take place, either in dry air or pure water.**)

Certain oxides, on heating, give up all or a portion of their oxygen. Thus: (1) The red oxide of mercury (HgO) when heated in a test tube gives off oxygen, and a mirror of mercury collects on the cool parts of the tube.

Mercuric oxide	=	Mercury	+	Oxygen
HgO	=	Hg	+	O
215	=	199	+	16

(2) The red oxide of lead (Pb_3O_4) or red lead, when heated, gives off a portion of its oxygen.

(3) Manganese dioxide (MnO_2), if perfectly dry when heated, also gives off a portion of its oxygen.

Test for Oxygen

When a glowing splinter of wood is plunged into oxygen, it immediately bursts into flame, owing to the rapid combustion and oxidation of the constituents of the splinter, carbon and hydrogen.

II. **Nitrogen** (symbol, N; atomic weight, 14).

Occurrence.—Nitrogen occurs not only in the free state in air, but also in a combined state in the bodies of plants and animals, and in the class of salts called nitrates, the chief of which is saltpetre or nitre (KNO_3).

Preparation.—It has already been shown how the oxygen may be removed from the air, leaving nitrogen.

If required pure, the air must be passed over red-hot copper, the latter being oxidised by the oxygen, and the nitrogen passes over and may be collected as shown in Fig. 121.

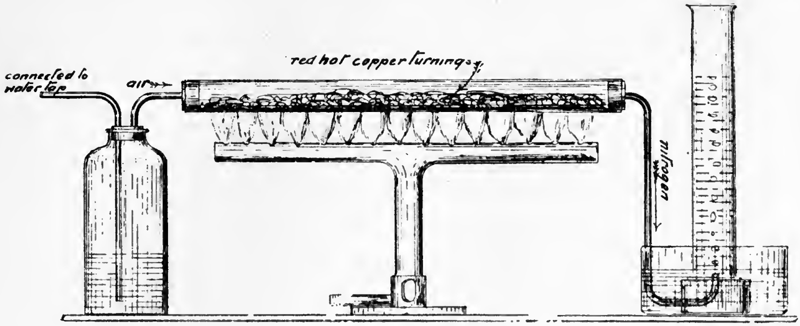


FIG. 121

Nitrogen may also be prepared by heating a strong solution of ammonium **nitrite** ($NH_4 . NO_2$) according to the equation:

$$\text{Ammonium nitrite} = \text{Nitrogen} + \text{Water}$$
$$NH_4 . NO_2 = N_2 + 2H_2O$$

By using an apparatus similar to the one employed for the preparation of oxygen (Fig. 120), the gas may be collected by the displacement of water.

Properties.—It is an invisible gas without taste or smell. Since it was collected by water displacement it must be practically insoluble in water. If a burning substance be plunged into nitrogen, it is immediately extinguished, showing that nitrogen will not support combustion. When small animals are put in nitrogen they die, not from poisoning, but from the want of oxygen. In all these cases it will be noticed that nitrogen is inactive, and it is said to be an **inert** gas. Its presence in the air in such a large proportion acts merely as a diluent to the oxygen, which otherwise would be too active, having evil effects on combustible bodies and animals.

III. Other Constituents

Besides oxygen and nitrogen, other substances are present in the air as impurities, the most important being water vapour, which is present in varying amounts (about 4 per cent.), carbon dioxide (about ·04 per cent.), and ammonia in minute quantities.

Water Vapour

Clouds and mists are composed almost entirely of water vapour. Again, when an excess of water vapour is present in the air, and the latter is cooled, the vapour is condensed and falls as rain.

Such substances as strong sulphuric acid and calcium chloride have the power of absorbing moisture when exposed to the air. In this way the presence of water vapour in the air can be shown by exposing a weighed quantity of either substance to the air for a time, and noting the increase in weight. As already mentioned, the presence of water vapour in the air is probably the cause of iron rusting when exposed to the atmosphere.

Carbon Dioxide

If clear lime water be exposed to the air for a time, a white film is formed on the surface, thus showing the presence of the above gas in small quantities. Its source is easily explained, for whenever a substance containing carbon, as coal or wood, is being burnt, carbon dioxide is being formed. Again, the oxygen in our lungs is used in oxidising the waste tissues, which consist chiefly of carbon, to form carbon dioxide, and this is exhaled into the atmosphere. Since oxygen is being used in such large quantities by animals and combustible bodies, the supply would soon run out were it not for the work of plants. These have the power of taking in the carbon dioxide, and of splitting it up, keeping the carbon to build up their tissues, and giving the oxygen to the air again. Thus plants in the daytime are constantly purifying the atmosphere.

Composition of Air by Weight

When air is slowly passed over red-hot copper, the oxygen unites with the copper to form copper oxide, and the nitrogen passes over.

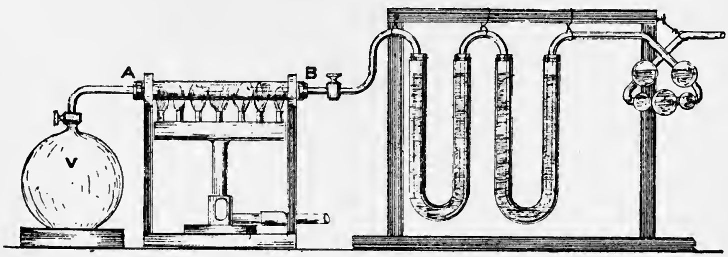


FIG. 122

For this experiment the apparatus shown in Fig. 122 is required. The globe V is exhausted of air and closed by a stopcock. It is then connected to the glass tube AB, containing metallic copper, which is fixed in the furnace. To the other end of the tube are attached tubes to purify the air: one contains calcium chloride to absorb the water vapour, and the other contains potassium hydrate solution to absorb the carbon dioxide. On opening the stopcock slightly, the purified air passes over the red-hot copper, and pure nitrogen enters the globe. If weighed before and after the experiment, the increase gives the weight of nitrogen. The weight of oxygen is obtained by weighing the glass tube AB before and after the experiment. If carried out with great care, 100 g. of pure air are found to contain 23 g. of oxygen to 77 g. of nitrogen.

Composition of Pure Air

	By Weight.	By Volume.
Oxygen . . .	23	20·8
Nitrogen . .	77	79·2
	100	100·0

CHAPTER XXIII

HYDROGEN AND WATER, COMPOSITION AND PREPARATION AND PROPERTIES; THE VOLTAMETER, SOLVENCY, SOLUBILITY OF SALTS, SOLUBILITY OF GASES, NATURAL WATERS, IMPURITIES IN SAME, FILTRATION OF WATER, HARD AND SOFT WATERS

HYDROGEN AND WATER

Hydrogen

Occurrence.—This gas does not occur in the free state in nature, but combined with other elements; it is found in water, and in all acids

Preparation.—The preparation of hydrogen consists of extracting the gas from either water or acid by suitable means.

I. From Water.—(1) Sodium and potassium have the power to displace a portion of the hydrogen from water, liberating it as a gas, and forming another compound.

Experiment XCI.—Place a small piece of sodium in a wire cage, and hold it under a tube of water inverted in a trough also containing water (Fig. 123). As the bubbles of hydrogen rise, collect them in the tube. Collect a tube of the gas and test it (as mentioned later). Notice that the liquid left is soapy to the touch, and is really a solution of sodium hydrate or caustic soda.

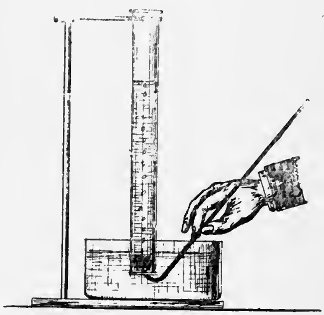


FIG. 123

$$\begin{array}{ccccccccc} \text{Sodium} & + & \text{Water} & = & \text{Sodium hydrate} & + & \text{Hydrogen} \\ 2Na & + & 2H_2O & = & 2NaOH & + & H_2 \end{array}$$

A similar reaction takes place in the case of potassium. In this case the action is so violent that the hydrogen is set on fire.

(2) When an electric current is passed through acidulated water, hydrogen is liberated at one pole and oxygen at the other. If each

pole be covered with a separate tube (Fig. 124), the two gases may be collected separately. Such an apparatus is called a **voltameter.**

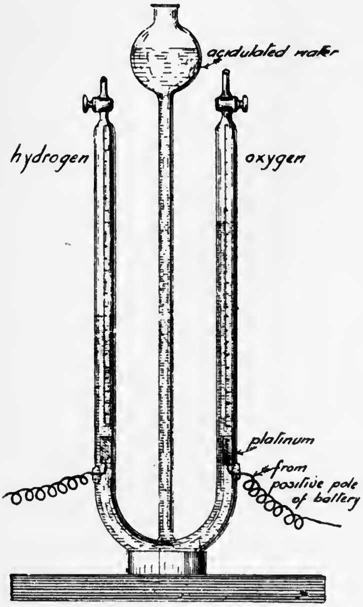


FIG. 124

(3) When steam is passed over red-hot iron, the iron splits up the steam, uniting with the oxygen to form iron oxide, and the hydrogen passes over and may be collected by water displacement (Fig. 125).

$$\text{Iron} + \text{Steam} = \text{Iron oxide} + \text{Hydrogen}$$
$$3Fe + 4H_2O = Fe_3O_4 + 4H_2$$

II. **From Acids.**—Certain metals, as zinc and iron, will dissolve in some acids, as sulphuric acid (H_2SO_4) and hydrochloric acid (HCl), and liberate hydrogen.

Experiment XCII.—Charge the apparatus shown in Fig. 126 with dilute sulphuric acid and granulated zinc. The zinc is placed in the generating chamber A, and the diluted acid, in the proportion of 8 parts water to 1 part acid, is poured into the chamber B; on opening the tap C, the air escapes from the chamber A, and allows the acid to come in contact with the zinc. If tap C be now closed it will be noticed that bubbles of hydrogen gas are given off, and the chamber A is gradually filled with hydrogen gas, which forces the acid back again into B, leaving the zinc free from contact with the acid. When gas is discharged by opening C, the acid descends through D into A, and the generation of hydrogen again takes place. It will be observed that this apparatus is so constructed that when no gas is required the acid is forced clear of the zinc, and the generation of hydrogen ceases. It is called the "Kipp" apparatus.

Take a small quantity of the liquid from the compartment D, after the apparatus has been in use for some time, and evaporate to dryness; the salt which remains is called **zinc sulphate.**

The chemical action may be thus represented:—

$$\text{Zinc} + \text{Sulphuric acid} = \text{Zinc sulphate} + \text{Hydrogen}$$
$$Zn + H_2SO_4 = ZnSO_4 + H_2$$

This apparatus is the same in principle as that used for generating **hydrogen for lead-burning purposes,** several modifications being introduced; the apparatus is usually made of lead.

Properties.—Hydrogen is an invisible gas without odour or taste. Since it is collected by water displacement, it must be insoluble in water.

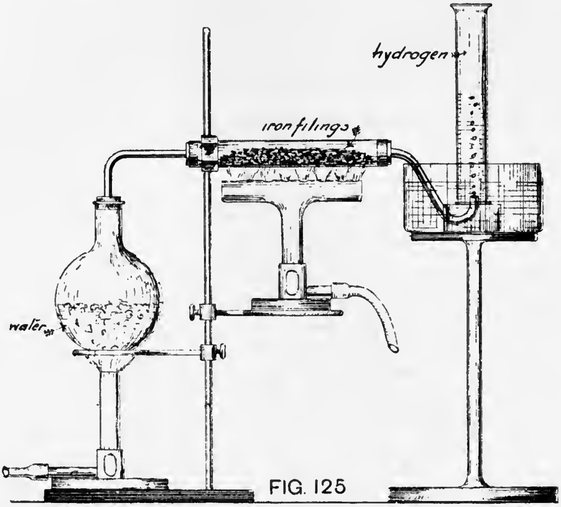


FIG. 125

It may also be collected by upward displacement of air; hence it must be lighter than air. This property can be shown by many experiments: thus hydrogen can be poured upwards from one jar to another; also, soap-bubbles filled with hydrogen rise in the air.

Experiment XCIII.—From the left pan of a balance suspend an inverted beaker, and counterpoise it with weights on the other pan. Carefully fill the beaker with hydrogen, and notice the decrease in weight, showing that hydrogen is lighter than air.

In fact, hydrogen is the lightest gas known, and for this reason it is taken as the standard in finding the density of other gases.

Experiment XCIV.—Holding a jar of hydrogen mouth downwards, plunge a lighted taper into it. The hydrogen burns with a blue flame at the mouth, but the taper is immediately extinguished. At the end notice that the inside of the glass is covered with moisture.

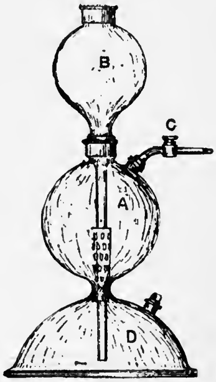


FIG. 126

This experiment shows that hydrogen is an inflammable gas which burns to form water, and also that it will not support combustion.

If a jet of hydrogen be allowed to burn under a cooled surface, drops of water form on the latter, and may be collected.

$$\begin{array}{ccccc} \text{Hydrogen} & + & \text{Oxygen} & = & \text{Water} \\ 2H_2 & + & O_2 & = & 2H_2O \end{array}$$

Hydrogen will also unite with oxygen to form water under the following conditions.

Experiment XCV.—Pass a stream of dried hydrogen over heated copper oxide, using the apparatus shown in Fig. 127. The copper oxide

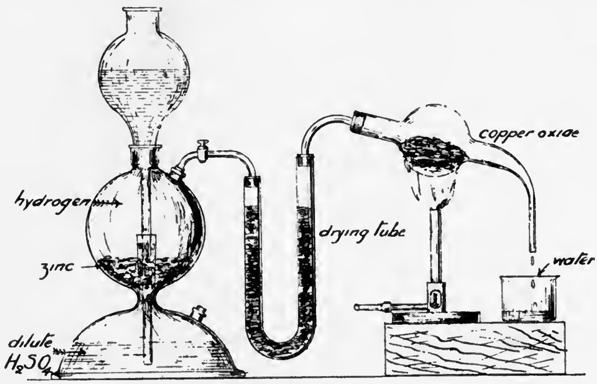


FIG. 127

is reduced to copper, and the oxygen unites with the hydrogen to form water, which condenses, and may be collected in a flask.

The chemical reaction may be represented as follows:—

$$\begin{array}{ccccccc} \text{Copper oxide} & + & \text{Hydrogen} & = & \text{Copper} & + & \text{Water} \\ CuO & + & H_2 & = & Cu & + & H_2O \end{array}$$

If hydrogen and oxygen be mixed in the proportion of 2 of H to 1 of O, and a light be applied, they explode with violence, forming water.

Test for Hydrogen.—Hydrogen burns with a bright blue flame in either air or oxygen, to form water.

Water (symbol, H_2O; molecular weight, 18).

Occurrence.—Water is one of the commonest substances on the earth. It occurs in large quantities as fresh water, sea water and rain, in the liquid state, and as the water vapour of the air in the gaseous state.

Composition of Water

I. **By Volume.**—It has already been stated that water can be decomposed into the elements hydrogen and oxygen by passing an electric current through acidulated water; and its composition can be shown in this way.

Experiment XCVI.—The best apparatus is the Hofmann voltameter (Fig. 124). The bulb and straight tube are filled with acidulated water. Open the taps at the top of the side tubes until the water fills them, and then close. Each side tube contains a piece of platinum foil, which forms an electrode and is connected to a terminal of an electric battery. Bubbles rise from each foil, and collect at the top of each tube. Mark the level of each gas, and then test them separately for hydrogen and oxygen. It will be found that the volume of hydrogen is twice that of oxygen; hence water is composed of two parts of hydrogen to each part of oxygen.

It has also been stated that hydrogen and oxygen unite with explosive violence, to form water.

Experiment XCVII.—Fill an eudiometer tube (Fig. 128) with 2 parts of hydrogen and 1 part of oxygen. Clamp it down to an indiarubber pad immersed in a trough of mercury. Pass an electric spark through the mixed gases, and after the explosion lift the tube from the pad. The mercury rises until there remains at the top of the tube, above the mercury, a small quantity of **water**, being the product of the combustion of the gases hydrogen and oxygen.

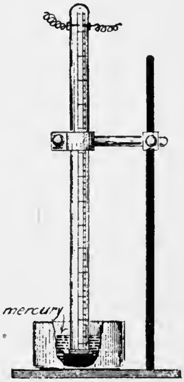


FIG 128

In 1781 Cavendish proved that water was a compound and not an element, and found its composition by this method.

From these experiments it follows that water is a chemical compound, which is composed of the elements hydrogen and oxygen, united in the proportion of 2 parts by volume of hydrogen to 1 part by volume of oxygen.

II. **By Weight.**—It has already been shown in Fig. 127, Exp. XCV., that water is formed when hydrogen is passed over red-hot copper oxide.

If the copper oxide and tube be weighed before and after the experiment, the **loss** in weight would be due to the oxygen removed.

If the water formed be collected in drying tubes, which are weighed before and after the experiment, the **increase** in weight is due to the water formed.

The difference between these two results will give the weight of hydrogen, which has united with the oxygen to form the water.

In 1843, Dumas found the composition of water by this method. He carefully dried and purified the hydrogen, and then carefully absorbed, by means of drying tubes, **all the water formed.**

In taking the mean of nineteen separate experiments, he found that 840·16 grammes of oxygen were used in the production of 945·44 grammes of water. From these results the percentage composition of water by weight is :—

Oxygen	=	88·86
Hydrogen	=	11·14
		100·00

Since the density of oxygen is sixteen times that of hydrogen, this composition agrees with the composition of water by volume.

Properties

The physical properties of water have been considered in the section on Physics, and in the present chapter the solvent power of water and the characteristics of natural waters will be dealt with.

Water as a Solvent

If a small quantity of common salt be put in water, and stirred, it will dissolve and form a solution of common salt, water being the solvent. Water is capable of dissolving a larger number of substances than any other liquid. Not only do many solid substances dissolve in water, but many liquids and nearly all gases dissolve to a greater or less extent, the liquid in each case being called an aqueous solution of that substance.

Solubility of Salts

Saturated Solutions.—*Experiment XCVIII.*—To a definite quantity of water add potassium nitrate, little by little. At first it readily dissolves, but after a time no more will dissolve, however well it be stirred. The solution is then said to be saturated with potassium nitrate at that temperature, and the amount of salt dissolved may easily be determined.

If the experiment be repeated, using different quantities of water, it will be found that the maximum amount of a substance which can be dissolved in a known quantity of water at the same temperature is fixed and definite, and in each case the solution is saturated.

All solids do not dissolve to the same extent in water, as may be shown by adding to water the same quantity of each of the following substances :—potassium nitrate or nitre, sodium chloride or common salt, and potassium chlorate. The first dissolves rapidly, while the last one dissolves very slowly, and is only slightly soluble in water.

Effect of Temperature

Experiment XCIX.—Take 1 lb. of water, and make a saturated solution of potassium nitrate. Note the temperature and find the weight of salt dissolved. Raise the temperature, and notice that it is capable of dissolving more salt. This continues as the temperature is increased. Take temperature readings and weighings as in first case, and obtain results. It has been found by experiment that 1 lb. of water at a temperature of

10° C.	dissolves	·2 lb.	of potassium nitrate
20° C.	„	·31 lb.	„
30° C.	„	·45 lb.	„
50° C.	„	·85 lb.	„
60° C.	„	1·09 lb.	„

From these results it is clear that **the solvent power of water increases as the temperature is raised,** except in the case of $CaCO_3CO_2$.

As the temperature of water is increased, it is capable of dissolving more of a substance, until a saturated solution at a higher temperature must contain much more of the dissolved substance than a saturated solution at a lower temperature. Then, if a saturated solution be cooled, the excess of dissolved salt cannot remain in solution, and it separates out in the form of crystals.

Thus, if the above saturated solution of potassium nitrate be cooled from 60° C. to 10° C., ·89 lb. of the salt would separate out as crystals.

Water of Crystallisation

Very often, when salts crystallise from aqueous solutions, they retain a definite quantity of water in the crystal, the whole forming a definite chemical compound. This water is called the water of crystallisation, and is present in the following crystalline salts:—

Sodium carbonate, or washing soda .	$Na_2CO_3 : 10H_2O$
Magnesium sulphate, or Epsom-salt .	$MgSO_4 : 7H_2O$
Zinc sulphate, or white vitriol . .	$ZnSO_4 : 7H_2O$
Copper sulphate, or blue vitriol . .	$CuSO_4 : 5H_2O$

Solubility of Gases

It is found that all gases dissolve in water to a greater or a less extent. Such gases as ammonia (NH_3) and hydrochloric acid (HCl) are very soluble; others, such as carbon dioxide (CO_2) and chlorine (Cl) are fairly soluble; whilst many gases, as hydrogen, oxygen, and nitrogen dissolve but slightly.

The Law of Henry states that at the same temperature and pressure, water always dissolves the same volume of a gas; thus if a larger quantity has to be dissolved, it can be done by increasing the pressure. In this way large quantities of carbon dioxide are dissolved in water to form the well-known aerated waters, the escape of a large quantity of the gas being evident when removing the stopper of a bottle containing a quantity of the water.

Effect of Temperature

As the temperature increases, the solubility of a gas decreases, until, at the boiling-point of water (100° C.), all dissolved gases are driven from solution, unless the gas be chemically united with the water, as: hydrochloric acid solution. This behaviour of gases is opposite to that of solids.

Natural Waters

These waters may be divided into the following classes, **sea water** and **fresh water**, the latter including rain water, spring water, well water, river water, and the water in fresh-water lakes.

All the above kinds contain impurities to a greater or less extent, and these impurities may be arranged as follows:—

(1) Suspended solid impurities, which may be allowed to settle, or be separated by filtration.

(2) Dissolved solid impurities, which generally consist of the carbonate, chloride, and sulphates of calcium, sodium, and magnesium.

The kind of impurity, and the amount of it depends upon the district in which the rain falls.

The following table shows the amount of **dissolved impurity in 1000 parts** of the various samples of water:—

Rain water	·030
River water and lakes	·097
Spring water	·282
Deep well water	·438
Sea water (English Channel)	35·255

In sea water the chief impurity is sodium chloride or common salt (27·06 parts).

(3) Dissolved gases, chiefly air.

Experiment C.—Fill a flask with water. Then fill the leading tube with water, and fit it into the neck of the flask, so that the other end passes under a graduated cylinder immersed in water (Fig. 129). Heat the water in the flask, and collect the air-bubbles in the cylinder. At the end the amount can be measured and the gases tested. It is dissolved air, but it contains more oxygen than ordinary air, because oxygen is more soluble in water than nitrogen.

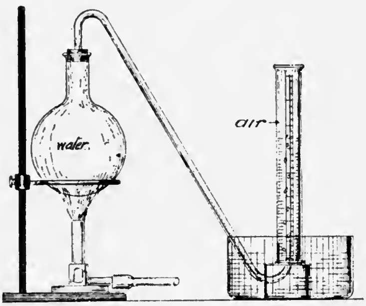


FIG. 129

The refreshing taste of fresh water is due to the air dissolved in the water; fishes live on it, and the dissolved oxygen is capable of oxidising impurities, which would otherwise pollute the water.

Again, the action of ordinary water on such metals as iron is principally due to the dissolved air.

The impurities in natural waters may be divided into two classes, viz.: **Organic** and **Inorganic.**

The former are by far the most deleterious to health, especially the portion in suspension. It usually consists of living and dead particles of organic matter; the living organisms may be disease germs of a specific kind, which, when taken into the human system, are liable to produce diseases or ailments of various kinds.

The organic matter in solution may consist of an acid, which is generated in vegetation during decomposition by the aid of two different kinds of micro-organisms. This action is especially marked in the decaying vegetation of peaty moorland surfaces, and water obtained from such gathering grounds is usually slightly acid. This is not a disadvantage as far as the dietetic value of the water is concerned, but it is found that **such waters rapidly corrode lead pipes,** and the water carries along with it (in solution and suspension) the lead salt, thereby **causing lead-poisoning to the consumers.** To remedy this, the acid may be neutralised by the addition of an alkali, such as lime, or tin-lined lead pipes may be used as service pipes. In some cases lead pipes are abolished, and wrought-iron, tin-lined, or plain wrought-iron pipes are used instead. The water acts upon the iron, but the dissolved iron salt has no harmful action upon the consumer, although in many cases the water is discoloured.

Filtration of Water

The removal of the impurities in water is very often necessary when the water is required for drinking purposes. The matter in solution can only be removed by chemical treatment, and in most cases this is not necessary, as the dissolved substances are more often beneficial than otherwise. It is the **suspended matter** which requires removal. The inorganic portion of this usually sinks to the bottom if the water remains stationary for say twenty-four hours, but the organic particles, especially the living ones, float about in the water, and require subjecting to the screening action of a fine filter to remove them. **Filters are screens of varying degrees of fineness, according to the material used.**

Experiment CI.—Take a beaker of water and add some salt—*i.e.* sodium chloride—to it, and pass it through a filter paper; afterwards taste the water, it will be found that the salt has not been removed.

Experiment CII.—Add some finely divided sand or clay to a beaker of water, and stir up until it becomes turbid, then pass it through filter paper; it will be observed that the sand or clay particles have been deposited on the filter paper, the water being practically clear again.

By the above it will be seen that **only matters in suspension can be removed by filtration. Matters in solution cannot** be removed by filtration.

The filters used in dwelling-houses are of various kinds, but the types which give the greatest efficiency are the **Pasteur-Chamberland,** the **Berkefeld,** and the **Doulton** porcelain tube.

The former is shown by Fig. 130; it consists of a tube of fine, hard, unglazed porcelain, through which the water has to pass. The rate of **filtration is very slow,** but its **efficiency** is very **high.**

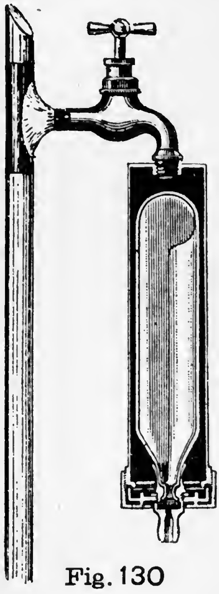


Fig. 130

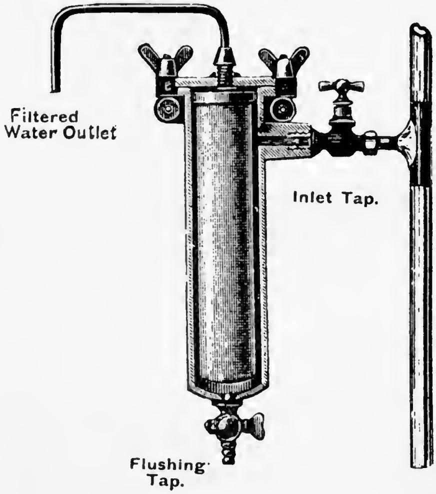


Fig. 131

The Berkefeld filter (Fig. 131) is similar to the previous one in principle, but the porous tube or filtering medium consists of a kind of diatomaceous earth; it is more porous than the Pasteur medium, and the rate of filtration is greater, but its efficiency is not so high.

These filters should be cleaned every two or three weeks, and sterilised by boiling them for half an hour.

Hard and Soft Waters

Experiment CIII.—Prepare carbon dioxide (CO_2) as shown later, and pass it through lime water. The latter is turned milky; the milkiness is due to the formation of calcium carbonate ($CaCO_3$), which is insoluble in ordinary water.

Continue with the carbon dioxide, and the milkiness gradually disappears, until finally a perfectly clear liquid is again obtained.

The carbon dioxide dissolved in the water, and the solution was then able to dissolve the calcium carbonate.

Hence a solution of carbon dioxide in water will dissolve calcium carbonate.

Experiment CIV.—Take equal quantities of the above liquid and ordinary water (rain water preferred), and shake up each with soap. The rain water produces a lather readily, but a scum forms on the other liquid, and a lather is only formed with difficulty.

The rain water is called **soft water,** because it readily lathers with soap; and the other liquid is called **hard,** because of the difficulty to produce a lather.

From these experiments it follows that the term **hardness** is applied to water with reference to its behaviour with soap, and this hardness is due to dissolved solid impurities, chiefly calcium carbonate ($CaCO_3$).

As the rain falls through the air the carbon dioxide is dissolved, and as it soaks through the soil, containing decaying vegetable matter, which liberates carbon dioxide, it becomes a stronger solution of this gas. When layers of limestone or chalk, which are forms of calcium carbonate, exist below the soil, the water percolates through the rock, dissolving as much as possible, and forming hard water.

Softening Processes

When the calcium carbonate is brought out of solution, the soap readily lathers in it, and the water has been **softened.** This may be done by any of the following methods:—

I. **By Boiling.**—*Experiment CV.*—Take some of the "hard" water made in Experiment CIII., and boil it. The milkiness slowly reappears, and the white calcium carbonate settles to the bottom.

The reason is that **on boiling, the dissolved carbon dioxide was driven off,** and the calcium carbonate became insoluble in the ordinary water. Most natural waters possess **hardness** to a greater or less extent, hence, when boiled, **the carbonate forms a coating on the inside of the vessel.** In this way **incrustations are formed** inside boilers, and are called "boiler crust." This is also the cause of "furring" in kettles, especially in districts where the water is hard, as in Derbyshire.

II. **Addition of Lime.**—When lime (CaO) is added to hard water it unites with the carbon dioxide (CO_2) to form more calcium carbonate, and the carbonate previously dissolved is precipitated again; thus the water is softened. This process was first introduced by Dr Clarke, and is known as "**Clarke's Softening Process.**"

The following equations will represent the reactions :—

$$\underset{\text{Soft water.}}{H_2O} + CO_2 = \underset{\text{Solution of carbon dioxide.}}{H_2CO_3}$$

$$CaCO_3 + H_2CO_3 = \underset{\text{Hard water.}}{Ca(HCO_3)_2}$$

Clarke's process :—

$$\underset{\text{Hard water.}}{Ca(HCO_3)_2} + \underset{\text{Lime.}}{CaO} = 2CaCO_3 + \underset{\text{Soft water.}}{H_2O}$$

This property of dissolving carbonate of lime, owing to the presence of CO_2 in the water and the subsequent precipitation of the same on boiling, requires consideration when arranging systems for domestic hot-water supply purposes. **The boiler, cylinder, and pipes should be so arranged as to be easily accessible,** to allow the scale or deposit to be removed periodically. The greatest amount of deposit is found in the boiler, and as this acts as a non-conductor, the heating efficiency of the boiler is gradually reduced; the deposit also occurs in the pipes between the boiler and the cylinder, gradually reducing the bore and eventually giving rise to unpleasant noises.

Temporary and Permanent Hardness

If the hardness can be removed by the above methods, it is principally due to the presence of dissolved calcium carbonate ($CaCO_3$), and is called **temporary hardness.**

In some cases hardness is due to dissolved calcium sulphate ($CaSO_4$), and since this cannot be removed by the above methods, it is called **permanent hardness.**

When not required for drinking purposes, water having permanent hardness can be softened by the addition of washing soda (Na_2CO_3; $10H_2O$, really crystals of sodium carbonate); thus :—

$$\underset{\text{Sodium sulphate causing permanent hardness.}}{CaSO_4} + Na_2CO_3 = \underset{\text{Insoluble in ordinary water.}}{CaCO_3} + \underset{\text{Dissolved in water.}}{Na_2SO_4}$$

CHAPTER XXIV

ACIDS, BASES, AND SALTS: PREPARATION AND ACTION UPON METALS OF HYDROCHLORIC, NITRIC, AND SULPHURIC ACIDS

ACIDS, BASES, AND SALTS

Acids

THE non-metals, carbon, sulphur and phosphorus, were found to burn in the air, and better in oxygen to form carbon dioxide (CO_2), sulphur dioxide (SO_2), and phosphorus pentoxide (P_2O_5) respectively. Each oxide dissolved in water, and formed a solution which turned litmus red, denoting the presence of an acid.

$$H_2O + CO_2 = H_2CO_3, \text{ Carbonic acid.}$$
$$H_2O + SO_2 = H_2SO_3, \text{ Sulphurous acid.}$$
$$3H_2O + P_2O_5 = 2H_3PO_4, \text{ Phosphoric acid.}$$

In most cases an acid may be regarded as an aqueous solution of some oxide of a non-metal, thus, nitric acid (HNO_3) may be regarded as a solution of an oxide of nitrogen; thus:—

$$H_2O + N_2O_5 = 2HNO_3$$

Again, sulphuric acid (H_2SO_4) may be regarded as a solution of an oxide of sulphur, higher than sulphur dioxide—

$$H_2O + SO_3 = H_2SO_4$$

All acids contain hydrogen, and this hydrogen is generally liberated when zinc is put in the acid, leaving a salt; thus:—

$$\begin{array}{ccccccccc} \text{Zinc} & + & \text{Sulphuric acid} & = & \text{Zinc sulphate} & + & \text{Hydrogen} \\ Zn & + & H_2SO_4 & = & ZnSO_4 & + & H_2 \end{array}$$

If zinc sulphate is regarded as a salt of zinc, then sulphuric acid may be regarded as the sulphate of hydrogen, or hydrogen sulphate. Hence **an acid is a substance containing hydrogen, which may be replaced by a metal to form a salt.**

Acids turn blue litmus red, and generally have a sour taste.

Bases

Quicklime or oxide of calcium (CaO) is slightly soluble in water, and forms lime water or calcium hydrate; thus:—

$$CaO + H_2O = Ca(OH)_2$$

Lime water will turn red litmus blue.

Sodium hydrate ($NaOH$) and potassium hydrate (KOH) have the same action on litmus. The latter bodies may be regarded as the oxides of sodium and potassium dissolved in water; thus:—

$$\begin{array}{lccccc} & \text{Sodium oxide} & & & & \text{Sodium hydrate.} \\ & Na_2O & + & H_2O & = & 2NaOH \\ \text{likewise, Potassium oxide} & K_2O & + & H_2O & = & 2KOH \text{ Potassium hydrate.} \end{array}$$

The oxides of metals are called **bases.**

When soluble in water they form hydrates which turn red litmus blue, feel soapy to the skin, and neutralise acids to form salts. Such hydrates are called **alkalies**; hence **an alkali is a base which will dissolve in water.**

Salts

In most cases salts have no action on litmus; hence they are said to be **neutral** to litmus.

Since alkalies and acids have opposite effects on litmus, it is possible to mix them so that the mixture will not affect litmus. Then the mixture will be **neutral,** and the acid and alkali are said to have **neutralised** each other.

Experiment CVI.—To a small quantity of sodium hydrate solution ($NaOH$) add dilute hydrochloric acid (HCl) until the mixture does not affect litmus. The acid has then neutralised the alkali. Evaporate the liquid to dryness. A white salt remains, which tastes like common salt. It is, indeed, common salt, or sodium chloride ($NaCl$), and is an example of a salt formed by the neutralisation of an alkali with an acid.

The experiment could be reversed, so that the alkali would neutralise the acid, and the same result would be obtained.

$$\begin{array}{ccccccc} NaOH & + & HCl & = & NaCl & + & H_2O \\ \text{Sodium hydrate} & + & \text{Hydrochloric acid} & = & \text{Common salt} & + & \text{Water} \\ & & & & \text{(Sodium chloride)} & & \end{array}$$

Similarly, other salts may be formed by varying the acid and alkali.

$$\begin{array}{ccccccc} 2NaOH & + & H_2SO_4 & = & Na_2SO_4 & + & 2H_2O \\ \text{Sodium hydrate} & + & \text{Sulphuric acid} & = & \text{Sodium sulphate} & + & \text{Water} \end{array}$$

$$\begin{array}{ccccccc} KOH & + & HNO_3 & = & KNO_3 & + & H_2O \\ \text{Potassium hydrate} & + & \text{Nitric acid} & = & \text{Pot. nitrate} & + & \text{Water} \end{array}$$

In general, the equation may be stated:—

$$\text{Alkali} + \text{Acid} = \text{Salt} + \text{Water.}$$

Since lime water is alkaline, and carbon dioxide, which has an acid reaction, dissolves in water and forms (H_2CO_3), the salt formed by neutralising one with the other must be calcium carbonate; thus :—

$$Ca(OH)_2 + H_2CO_3 = CaCO_3 + 2H_2O$$

The lime water is turned milky because the calcium carbonate will not dissolve in the water.

Although all bases will not dissolve in water to form alkalies, yet all bases will dissolve in acids. In each case the base neutralises the acid and forms a salt; thus :—

$$\begin{array}{ccccccccc} MgO & + & H_2SO_4 & = & MgSO_4 & + & H_2O \\ \text{Magnesium oxide} & + & \text{Sulphuric} & = & \text{Magnesium} & + & \text{Water} \\ \text{(base)} & & \text{(acid)} & & \text{(sulphate salt)} & & \end{array}$$

$$\begin{array}{ccccccc} CuO & + & 2HCl & = & CuCl_2 & + & H_2O \\ \text{(Copper oxide)} & & & & \text{(Copper chloride)} & & \end{array}$$

In every case it will be noticed that the salt consists of two parts, the first part coming from the base (the metallic portion), and the other part coming from the acid (the acid radicle). Hence **a salt may be defined as the compound formed when the hydrogen of an acid is replaced by a metal.**

The next few pages will deal with the preparation and common properties of the mineral acids.

Hydrochloric acid (HCl).
Nitric acid (HNO_3).
Sulphuric acid (H_2SO_4).

I. **Hydrochloric Acid** (symbol, HCl; molecular weight, 36·5).

Preparation.—This acid is prepared by the action of strong sulphuric acid (H_2SO_4) on common salt ($NaCl$).

Experiment CVII.—Fit up the apparatus shown in Fig. 132. Put the sodium chloride into the flask, and pour the acid down the funnel. After a time, warm gently, and collect the gas by downward displacement.

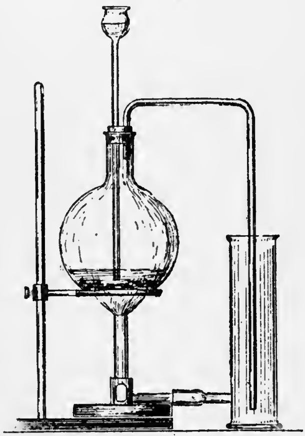


FIG. 132

The chemical reaction takes place as follows :—

$$\begin{array}{ccccccc} NaCl & + & H_2SO_4 & = & NaHSO_4 & + & HCl \\ & & & & \text{(Sodium hydrogen} & & \\ & & & & \text{sulphate, fumes)} & & \end{array}$$

In the manufacture of this acid the sulphuric acid and common salt are mixed in a large furnace. On heating strongly, further decomposition takes place; thus:—

$$NaHSO_4 + NaCl = Na_2SO_4 \text{ (Sodium sulphate)} + HCl$$

or

$$2NaCl + H_2SO_4 = Na_2SO_4 + 2HCl$$

The hydrochloric acid gas is passed up large towers, down which trickles water. Being very soluble in water, a strong solution of hydrochloric acid runs out at the bottom of the towers.

Properties.—It is a colourless gas with a suffocating smell. In dry air it is colourless, but in moist air it fumes, because the gas unites with the water vapour. Its density is 18·2 compared with hydrogen; hence it is heavier than air (density 14·4), and can be collected by downward displacement.

The gas is exceedingly soluble in water, and it is found that one volume of water at 15° C. will dissolve over 450 volumes of hydrochloric acid, forming a 42-per-cent. solution, which has a specific gravity of 1·21.

Experiment CVIII.—Fill a large globe with hydrochloric acid gas, through the stopper pass a glass tube. Invert over water coloured **blue** with litmus, as shown in Fig. 133. Cool the globe until the water rises up the tube. A beautiful fountain then begins, and continues until the globe is filled with water now coloured **red.**

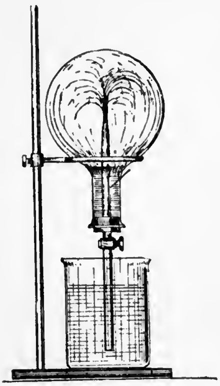


FIG. 133

The gas probably unites with water to form a compound. If a dilute solution be boiled, water is driven off until the strength is about 20 per cent. and, on further boiling, the strength remains the same.

Again, if a strong solution be boiled, hydrochloric acid gas is driven off until the solution contains about 20 per cent. of gas, and again the strength remains constant. This strength is expressed by the formula $HCl : 8H_2O$.

Action on Metals.—A solution of hydrochloric acid acts violently on most of the common metals, as iron, zinc, and tin, dissolving the metal, liberating the **hydrogen** (thus, $Zn + 2HCl = ZnCl_2 + H_2$), and forming a **chloride** of the metal. On lead it has practically no action when cold, but when boiled with lead, in the presence of air, lead chloride ($PbCl_2$) is formed.

On copper, mercury, silver, gold, and platinum it has no action whatever.

Gold and platinum will dissolve in "**aqua regia,**" a mixture of hydrochloric and nitric acids, to form the **chloride** of each metal.

II. **Nitric Acid** (symbol, HNO_3; molecular weight, 63).

Preparation.—This acid is prepared by acting with sulphuric acid (H_2SO_4) on nitre (KNO_3), or on Chili saltpetre ($NaNO_3$).

Experiment CIX.—Fit up the apparatus as shown in Fig. 117. Put equal parts of nitre and acid in the glass retort, and then heat carefully. The nitric acid distils over and collects in the flask cooled by the running water.

The chemical reaction takes place as follows:—

$$KNO_3 + H_2SO_4 = \underset{\text{(Pot. hydrogen sulphate)}}{KHSO_4} + HNO_3$$

In the manufacture of this acid circular iron retorts are used, and Chili saltpetre is used, because it is cheaper. On heating strongly complete decomposition takes place, and the nitric acid is condensed in large earthenware vessels; thus:—

$$2NaNO_3 + H_2SO_4 = \underset{\text{Sodium sulphate}}{Na_2SO_4} + \underset{\text{Nitric acid}}{2HNO_3}$$

(*N.B.*—The reaction is similar to the one for the preparation of hydrochloric acid.)

Properties.—Nitric acid, thus prepared, has a yellowish tint, but on boiling, reddish-brown fumes of the oxides of nitrogen are liberated, and the colourless liquid which remains is pure nitric acid. As its formula shows, nitric acid is rich in oxygen, containing 76 per cent.; hence it acts as a powerful oxidising agent. Thus, if nitric acid be poured on sawdust, the latter will burst into flames, caused by the rapid oxidation of the carbon in the sawdust, and dense red fumes are emitted. A similar reaction takes place with turpentine.

Action on Metals.—Nitric acid acts violently on most of the common metals, especially copper and zinc, the action on iron and tin being increased by slightly diluting the acid. (Pure HNO_3 has no action on pure iron; hence iron boilers are used in the manufacture of nitric acid.)

In all cases, reddish-brown fumes of the oxides of nitrogen are evolved, and a **nitrate** of the metal remains as the salt; but no hydrogen is given off, as in the case of hydrochloric and sulphuric acids.

In order to explain these results, let us consider the action of nitric acid on zinc.

When the zinc dissolves, it displaces the hydrogen from the acid, and forms zinc nitrate; thus:—

$$\begin{array}{ccccccc} \text{Zinc} & + & \text{Nitric acid} & = & \text{Zinc nitrate} & + & \text{Hydrogen} \\ Zn & + & 2HNO_3 & = & Zn(NO_3)_2 & + & 2H \end{array}$$

This hydrogen is very active or **nascent**, and immediately takes oxygen from the remaining nitric acid to form water, splitting up the former into water and oxides of nitrogen, which are liberated as reddish-brown fumes:—

Hydrogen (nascent) + Nitric acid = Oxides of nitrogen + water.
(red fumes)

Nitric acid is without action upon either gold or platinum. Strong nitric acid has no action upon lead, but the dilute acid dissolves it.

III. Sulphuric Acid (symbol, H_2SO_4; molecular weight, 98).

Preparation.—This acid can be prepared by dissolving sulphur trioxide (SO_3) in water; thus:—

$$SO_3 + H_2O = H_2SO_4$$

The sulphur trioxide may be prepared by passing a mixture of pure sulphur dioxide and oxygen over heated platinised asbestos:—

$$2SO_2 + O_2 = 2SO_3$$

It has already been shown that sulphur dioxide (SO_2) dissolves in water to form sulphurous acid (H_2SO_3):—

$$H_2O + SO_2 = H_2SO_3$$

Under suitable conditions it is possible to oxidise this sulphurous acid into sulphuric acid by using nitrogen peroxide as the oxidising agent.

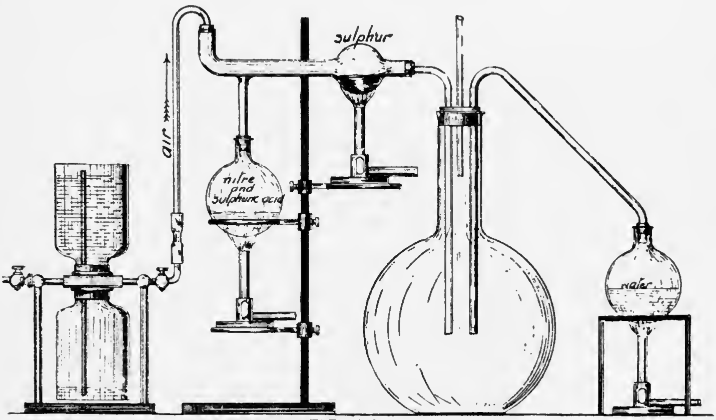


FIG. 134

Experiment CX.—Into a large globe (Fig. 134) are passed steam, sulphur dioxide formed by burning sulphur in a stream of air, and nitrogen peroxide.

These unite as shown in the chemical equation :—

$$H_2O + SO_2 + NO_2 = H_2SO_4 + NO \text{ (Nitric oxide).}$$

The sulphuric acid collects in the globe, and the nitric oxide easily takes up more oxygen ; thus :—

$$2NO + O_2 = 2NO_2,$$

and it thus acts as a carrier of oxygen.

The same process is used in the manufacture of this acid. Sulphur dioxide made from burning iron pyrites is passed along with steam and nitrogen peroxide into a huge leaden chamber, where the commercial sulphuric acid collects, and is afterwards purified.

Properties.—Sulphuric acid, when pure, is a colourless, heavy, oily liquid, commonly known as "oil of vitriol."

At 0° C. it has a specific gravity of 1·85, and at this temperature colourless crystals of pure sulphuric acid separate out, which melt at 10·5° C. It boils at 338° C., and it distils over unchanged. Its powerful affinity for water has already been shown in Chapter XXII., where it was used to absorb moisture from the air. It acts violently on organic bodies containing the elements of water, removing the water and leaving a charred mass of carbon.

Experiment CXI.—Make a strong solution of cane sugar, add a little strong sulphuric acid. The mixture turns black, and froths over the beaker containing it. The black mass is charcoal. In a similar way it chars or blackens wood.

When this acid and water are mixed, much heat is produced, and probably the acid and water chemically unite to form another compound ($H_2SO_4 : 2H_2O$).

Action on Metals.—**Dilute** sulphuric acid dissolves such metals as zinc, iron, and magnesium, liberating hydrogen, and leaving in each case a **sulphate** of the metal dissolved in the water present; thus :—

$$Zn \quad + \quad H_2SO_4 \quad = \quad \underset{\text{(Zinc sulphate)}}{ZnSO_4} \quad + \quad H_2$$

Concentrated sulphuric acid has quite a different action. In the cold it has little or no action on most metals ; but when heated, it acts upon zinc, copper, tin, silver, and most common metals, forming the sulphate of the metal in each case, and liberating, instead of hydrogen, a heavy, suffocating gas called sulphur dioxide.

The action is said to take place in two stages, and the change is probably due to the hydrogen set free being in the **nascent state,** as in the action of nitric acid on metals ; thus :—

$$Zn + H_2SO_4 = ZnSO_4 + 2H.$$

In the first stage the ordinary reaction takes place, but as soon as the nascent hydrogen is set free, it probably splits up the sulphuric acid, taking out oxygen, and the sulphur dioxide is liberated; thus:—

$$2H + H_2SO_4 = 2H_2O + SO_2$$

In the preparation of sulphur dioxide in the laboratory, copper is generally used, but the action is similar.

The two reactions may be included in one equation, as follows:—

$$Cu + 2H_2SO_4 = CuSO_4 + 2H_2O + SO_2$$

Gold and platinum are unacted upon by sulphuric acid in any form.

CHAPTER XXV

CARBON AND SULPHUR, WITH THEIR COMPOUNDS: CHARCOAL, CO_2, CO, H_2S, SO_2; DETECTION OF LEAD IN WATER

CARBON AND SULPHUR, WITH THEIR COMPOUNDS

Carbon

Experiment CXII.—(*a*) Place a few wooden chips in a test tube, incline the tube so that the mouth is a little below the horizontal, and heat the closed end. The fumes given off are inflammable, and burn at the mouth of the tube. The cooler parts of the tube become moist, and a liquid drops from the mouth. Collect it and test. In smell and taste it resembles vinegar or acetic acid; it turns blue litmus red, and is called **pyroligneous acid**. The sides of the tube are brown, due to **pitch** from the wood, and a **charred** or black residue remains. In shape and form this mass resembles the original splinter. On heating, it no longer burns with a flame, but glows and leaves behind a grey ash. The charred residue is called **charcoal,** which is a form of carbon.

(*b*) Place some powdered coal in a hard glass tube and heat. At first a gas is given off, which turns red litmus paper blue; this gas is **ammonia,** and in the manufacture of coal-gas it is a valuable by-product. Coal-gas is then given off, which can be burnt at the mouth of the tube. The sides of the latter become coated with tar, and the residue which remains is coke, another form of carbon.

Experiment CXIII.—Hold any cold object, as a plate, in a luminous gas flame. It becomes smoked, due to a covering of soot or charcoal. In the same way, when any substance as petroleum, turpentine, or camphor, etc., burns with a smoky flame, the soot or lampblack may be collected, and produces one of the purest forms of carbon. Lampblack is largely used in the manufacture of printers' ink.

Carbon forms an essential part of every organic substance, animal or vegetable, and if the substance be heated out of contact with air, the substance is charred, and the charred mass is named animal charcoal or vegetable charcoal, according to the body heated.

Wood Charcoal

Large quantities of wood charcoal are made by the charcoal burner, who first cuts the trees into logs, and then builds them loosely to form a hemispherical dome, leaving space for a limited circulation of air and a central chimney. The whole is covered with clay, and then the fire is kindled and the logs are allowed to **smoulder** until the whole mass is charred or carbonised, and is then ready for use.

Wood charcoal is very porous, and possesses the remarkable property of absorbing large quantities of gases, especially obnoxious gases (thus charcoal will absorb 172 times its own volume of ammonia). For this reason it is largely used in sick-rooms to remove bad smells, and formerly by butchers to preserve the meat.

Bone Charcoal

Bone charcoal is made by heating bones out of contact with air in cast-iron retorts (*i.e.* by the carbonisation of bones).

Boneblack possesses the property of removing any vegetable colouring matter from solution during filtration.

Thus, if litmus solution be filtered through boneblack for a few times, the liquid is decolorised. Because of this property boneblack is used in the purification of sugar.

If any form of charcoal be heated strongly, the carbon is oxidised, and passes off as the gas carbon dioxide, leaving the mineral ash behind.

All forms of charcoal are without regular shape or crystalline form; hence they are said to be **amorphous**.

Two other forms of carbon occur naturally, which have definite crystalline forms, and are the diamond and graphite.

Diamond

This is the purest form of carbon, and is found only in very small quantities in a peculiar rock in Brazil and South Africa. When pure, it occurs in colourless crystals of octahedral form. These crystals are remarkable for their hardness, and are thus used in cutting and boring instruments, and also for polishing other precious stones. As a gem the diamond is valued for its brilliance.

When heated very strongly in contact with oxygen it glows brightly, gradually diminishes in size, and finally disappears, leaving behind a small ash.

The diamond has been oxidised, and carbon dioxide has been formed by the oxidation. Its presence can be shown by adding lime water.

When strongly heated out of contact with air, the diamond changes in form, and becomes the other crystalline form—graphite.

Graphite occurs naturally in many localities as a greyish-black substance, having a metallic appearance, and being soft and greasy to the touch. It generally occurs in laminated masses, and sometimes in flat, hexagonal crystals.

It is largely used in writing-pencils, and also as a substitute for oil in lubricating machinery, because of the above properties.

When heated strongly in oxygen, graphite burns more readily than diamond to form carbon dioxide.

If equal weights of diamond, graphite, and pure charcoal be burned in oxygen, they form equal weights of carbon dioxide only; hence it follows that these three substances are chemically the same substance—*i.e.* **carbon**—although their physical properties are entirely different. For this reason diamond, graphite, and charcoal are called the **allotropic forms** of carbon.

Occurrence of Carbon

It has already been shown that carbon occurs in the natural state as diamond and graphite in the combined state in all organic bodies and in the carbon dioxide of the atmosphere.

Carbon is also present in that group of minerals called carbonates, the chief of which is calcium carbonate ($CaCO_3$), which is found as limestone, chalk, marble, coral, and shells.

Carbon Dioxide (symbol, CO_2; molecular weight, 44)

Occurrence.—In Chapter XXII. it was shown that carbon dioxide was present in the air as an impurity. The sources of this gas were explained, and its removal by plants was referred to. Carbon dioxide is always evolved from decomposing organic matter; hence it is sometimes found at the bottom of caverns, mines, and coal-pits, where it is known as **choke-damp** or after-damp (formed from fire-damp).

Preparation.—Carbon dioxide can be obtained from any carbonate by heating, or by the action of an acid on it.

Experiment CXIV.—Put some copper carbonate in a test tube and heat it. The colour changes from green to black, and the heavy gas given off turns lime water milky. It is carbon dioxide, and the black residue is copper oxide:—

$$CuCO_3 = CuO + CO_2$$

Calcium carbonate ($CaCO_3$) does not part with its carbon dioxide so readily, but if heated in a muffle furnace the carbon dioxide is driven off, and leaves calcium oxide or quicklime (CaO).

Quicklime is manufactured in this manner by "burning" limestone in lime kilns:—

$$\underset{\text{(Limestone)}}{CaCO_3} = \underset{\text{(Quicklime)}}{CaO} + CO_2$$

In fact, if any carbonate be sufficiently heated, carbon dioxide is evolved, and the oxide of the metal remains.

When required for experimental purposes, it is better to prepare it by the action of an acid on a carbonate.

Experiment CXV.—Fit up the apparatus as shown in Fig. 126. Put into the generating chamber A a few pieces of marble, and add dilute hydrochloric acid to the chamber B. An effervescence takes place when the acid passes through D to A and comes in contact with the marble, and the carbon dioxide liberated is collected by downward displacement of air, through tap C.

The reaction may be represented as follows :—

$$CaCO_3 + 2HCl = \underset{\text{(Calcium chloride)}}{CaCl_2} + H_2O + CO_2$$

At the end of the experiment, pour into an evaporating basin some of the liquid remaining in the generating bottle, and evaporate to dryness. Expose the white salt to the air for some time. It becomes damp, showing that it has taken up or absorbed moisture from the air; this salt is called calcium chloride.

Properties.—Carbon dioxide is a colourless gas with a slightly acid taste. It was collected by downward displacement; hence it must be heavier than air. Its formula, CO_2, denotes that its density is 22 compared with hydrogen; hence it is one and a half times as heavy as air. This property can be shown in many ways; thus :—

(1) If a beaker be placed in one pan of a balance and counterpoised. Then carefully pass in carbon dioxide, and an increase in weight on the same side is shown.

(2) The gas can be poured downwards.

(3) Soap-bubbles filled with carbon dioxide sink in the air, and soap-bubbles filled with air float in a jar containing carbon dioxide.

This gas dissolves in its own volume of water and forms a solution possessing properties of a weak acid; hence the solution is sometimes called carbonic acid—

$$H_2O + CO_2 = H_2CO_3$$

The properties of this solution have been shown in the section dealing with "hard" and "soft" waters.

If a burning candle be plunged in carbon dioxide it is immediately extinguished, and a lighted candle may be used for testing when the jar is full of the gas. This gas is also frequently present in wells which have not been opened for some time, and as it will not support combustion its presence should be determined by lowering a lighted candle into the well, before any attempt be made to enter; if the candle goes out, a few buckets of water may be thrown into the well to effect the displacement of the air therefrom.

Experiment CXVI.—Collect a jar full of carbon dioxide. Kindle a piece of magnesium ribbon and plunge it into the jar. It then burns less brilliantly, but now makes a crackling noise, whereas in air it burns silently. At the end white masses of magnesia are formed, and also black specks of carbon appear on the sides of the jar.

The carbon dioxide has been split up by the magnesium, the oxygen having been used, and the carbon deposited; thus:—

$$CO_2 + 2Mg = 2MgO + C.$$

Carbon dioxide is also produced by human beings and the lower animals during respiration, owing to the combustion of the carbon contained in the foods partaken of, with oxygen taken into the lungs. The burning or oxidation of the carbon in the body is the principal source of heat and energy to human and animal systems. The CO_2 formed during this process is an important impurity which has to be considered in ventilation arrangements for buildings. It is present in the atmosphere in varying quantities. In towns the amount on an average equals ·4 part of CO_2 per 1000 parts of air. In country districts the quantity is as low as ·2 part per 1000.

Test for Carbon Dioxide.—*Experiment CXVII.*—Bubble the gas through lime water. If carbon dioxide be present, the liquid will first turn milky, and will gradually become clear again. This clear liquid is **"hard" to soap.**

Sulphur (symbol, S; atomic weight, 32).

Occurrence.—Sulphur or brimstone is found in the native state in the neighbourhood of active and extinct volcanoes, as around Mount Etna, Sicily (and its origin will be explained later in this chapter). It also occurs united with metals to form a class of bodies called **sulphides,** as in the minerals, Galena (PbS), Cinnabar (HgS), Zinc Blende (ZnS), Iron Pyrites (FeS_2).

Again, sulphur occurs in **sulphates**: thus, Gypsum ($CaSO_4$, $2H_2O$), Heavy Spar ($BaSO_4$), Glauber Salt (Na_2SO_4, $10H_2O$), Green Vitriol ($FeSO_4$, $7H_2O$).

Extraction.—The native sulphur is found mixed with earthy matter, from which it must be separated.

In Sicily the crude sulphur is made into a kind of kiln and allowed to burn with a limited supply of air. The melted sulphur leaves at the bottom, and is run into moulds.

In this country it is purified by distillation. The sulphur is boiled in an iron retort, and the fumes pass into a brick chamber.

If flowers of sulphur be required, the walls are cooled, and the sulphur solidifies on the walls in small crystals. If not, the sulphur fumes condense to form a liquid, which can be run out at the bottom of the chamber into moulds.

Allotropic Forms.—Sulphur, like carbon, exists in three distinct forms, which have the same chemical composition, but different physical properties; hence they are the allotropic forms of sulphur.

I. Common purified sulphur consists of transparent rhombic crystals, which have a specific gravity of 2·06 and melt at 114° C. These crystals are very stable.

Experiment CXVIII.—Dissolve some sulphur in carbon bisulphide. Suspend a small crystal of sulphur in it. A larger rhombic crystal of sulphur gradually grows.

II. Prismatic sulphur consists of long, transparent, needle-shaped crystals, which have a specific gravity of 1·96, and melt at 120° C.

They are formed by allowing melted sulphur to cool slowly, but they are unstable, and below 97° C. they gradually change into the rhombic form.

III. Plastic sulphur is formed when boiling sulphur is poured in a thin stream into cold water. It forms a brown, soft mass, which is unstable and gradually becomes yellow, forming the first variety.

Properties.—Sulphur is a yellow solid body, which is very brittle. It is a bad conductor of heat, for when common roll-sulphur is clasped in the hand it crackles, and soon falls in pieces.

Behaviour on Heating.—When ordinary sulphur is heated to 114° C. it melts and forms a pale yellow mobile liquid. On further heating it, the liquid darkens and becomes more viscous, until at 230° C. it appears as a dark red viscid mass. On raising the temperature still further, it retains its dark colour, but becomes more mobile, until at 448° C. it boils, and is converted into a red-coloured vapour, which condenses as pure sulphur.

On allowing the boiling liquid to cool, it passes through the reverse order to the above, until it solidifies.

When sulphur is heated strongly it burns in the air with a small flickering flame; in oxygen it burns much more violently. In both cases the product of combustion is sulphur dioxide (SO_2), a heavy gas with a suffocating odour. Sulphur is insoluble in water, but it dissolves in carbon bisulphide (CS_2), from which it will crystallise in rhombic crystals.

Action on Metals.—In Experiment LXXVI. a mixture of iron filings and powdered sulphur was heated, and in Experiment LXXVIII. a mixture of copper turnings and powdered sulphur was heated. In both cases powerful combustion took place, and much heat was produced. The residue which remained was found to be an entirely different substance, iron sulphide (FeS) in the former case, and copper sulphide (CuS) in the latter one, both being definite chemical compounds.

Experiment CXIX.—Boil some sulphur in a flask, and fit into the neck a large copper spiral. The copper burns in the sulphur vapour, and the green copper sulphide (CuS) drops to the bottom of the flask.

In a similar manner sulphur will combine with many other metals, as silver, sodium and potassium, in a definite proportion in each case, to form a **sulphide.** Thus :—

Silver	+	Sulphur	=	Silver sulphide
2Ag	+	S	=	Ag_2S
214	+	32	=	246

These numbers show that 214 ozs. of silver will unite with 32 ozs. of sulphur to form 246 ozs. of silver sulphide.

Compounds of Sulphur

Sulphur Dioxide, Sulphuretted Hydrogen.

Sulphur dioxide (symbol, SO_2; molecular weight, 64).

It has already been shown that when sulphur burns in the air this compound is formed; but in this case it is mixed with the air or nitrogen, and it is necessary to prepare it in another manner in order to show its properties. This gas is present in the mixed gases emitted from a volcano during eruption.

Preparation.—This gas is evolved when metals are heated with strong sulphuric acid.

Experiment CXX.—The apparatus required is similar to that required for the preparation of hydrochloric acid (Fig. 132). Put some copper turnings into the flask, and cover them with strong sulphuric acid.

On heating, sulphur dioxide is given off, and is generally collected by downward displacement of air.

The reaction may be represented as follows :—

$$Cu + 2H_2SO_4 = \underset{\text{(Copper sulphate)}}{CuSO_4} + 2H_2O + SO_2$$

Properties.—Sulphur dioxide is a colourless gas with a pungent and suffocating odour. Its density is 32 as compared with hydrogen; hence it is more than twice as heavy as air.

It is very soluble in water, as shown in Experiment LXXXVIII.; hence it cannot be collected by water displacement. Like other gases, its solubility varies with temperature; thus :—

1 vol. of water at		0° C.	dissolves	80 vols. of SO_2		
1	„ „	20° C.	„	39	„	„
1	„ „	40° C.	„	19	„	„
1	„ „	100° C.	„	0	„	„

This solution is strongly acid, and is called sulphurous acid :—

$$H_2O + SO_2 = H_2SO_3$$

This is a definite chemical compound, formed by the union of the sulphur dioxide and the water, and not merely a solution of the gas in water. Like other acids, it may be neutralised by bases to form a class of salts called Sulphites.

In the presence of water this gas possesses bleaching properties, because it extracts oxygen from the water to form sulphuric acid, and the nascent hydrogen set free bleaches vegetable colouring-matter.

$$SO_2 + 2H_2O = \underset{\text{(Sulphuric acid)}}{H_2SO_4} + 2H.$$

Under ordinary conditions, combustible bodies will not burn in it, but many metals take fire when heated in the gas. Thus, when a stream of this gas is passed over finely divided iron which is being heated, the latter glows, and forms a mixture of iron oxide and sulphide.

Also, if sulphur dioxide be passed over lead peroxide (PbO_2) it glows, and forms lead sulphate by direct combustion ; thus :—

$$PbO_2 + SO_2 = PbSO_4, \text{ Lead sulphate.}$$

Sulphuretted Hydrogen (symbol, H_2S ; molecular weight, 34).

Occurrence.—This gas is present along with sulphur dioxide in the mixed gases emitted from volcanoes during eruption, and they react on each other to form sulphur ; thus :—

$$2H_2S + SO_2 = 2H_2O + 3\mathbf{S} \text{ (Sulphur).}$$

This reaction accounts for the large quantities of crude sulphur which occur in the neighbourhood of volcanoes.

It is also found dissolved in certain mineral waters, called sulphur mineral waters, and it is present in coal-gas before purification.

Preparation.—This gas is usually prepared by the action of a mineral acid on a metallic sulphide.

Experiment CXXI.—Take the apparatus shown in Fig. 126, which is the same as that used for the generation of hydrogen. In the chamber (A) place a few small pieces of iron sulphide (FeS), and add dilute sulphuric acid to chamber B ; by opening the tap C, the acid passes into D, and from thence into A, where it comes into contact with the FeS, and sulphuretted hydrogen is liberated in an impure state. The reaction is as follows :—

$$FeS + H_2SO_4 = FeSO_4 + H_2S.$$

Properties.—Sulphuretted hydrogen is a colourless gas with a very disagreeable odour resembling rotten eggs. Its density is 17, and it is thus slightly heavier than air, hence it may be collected by downward displacement.

It is soluble in water, one volume of water dissolving three of this gas to form a solution which may be used instead of the gas in many reactions.

This gas is inflammable, and burns with a blue flame to form water and sulphur dioxide; thus :—

$$2H_2S + 3O_2 = 2H_2O + 2SO_2$$

Action on Metals.—This gas acts on many metals with the formation of sulphides. Thus, when lead, silver, tin, etc., are exposed to it, they readily tarnish, due to the formation of a sulphide of the metal on its surface.

Sometimes the air in towns, especially in the neighbourhood of acid works, contains this gas as impurity, and then the articles of silver readily tarnish, and the pictures darken, if lead be present in the paint.

Use.—Sulphuretted hydrogen is used to detect the presence of any compounds of lead which may be dissolved in water.

Experiment CXXII.—Dissolve a little lead nitrate in water, and add sulphuretted hydrogen, either in the form of a gas or a solution. A **heavy black precipitate of lead sulphide** sinks to the bottom.

The reaction is as follows :—

$$Pb(NO_3)_2 + H_2S = \underset{\text{(Lead sulphide)}}{\mathbf{PbS}} + 2HNO_3$$

Experiment CXXIII.—To a beaker of water add a few drops of lead nitrate solution. Then add the sulphuretted hydrogen. A brown coloration indicates the presence of lead in small quantities. This is a delicate test.

If tap-water is suspected of containing dissolved lead, take a sample, and add sulphuretted hydrogen. A brown coloration denotes the presence of lead, and the amount may be estimated from the intensity of the coloration.

CHAPTER XXVI

THE COMMON METALS, IRON, LEAD, ZINC, TIN, COPPER; ORES OF SAME, EXTRACTION, PURIFICATION, PROPERTIES, USES; WHITE AND RED LEAD

COMMON METALS

Iron, Lead, Zinc, Tin, Copper

Iron (symbol, Fe (Ferrum); atomic weight, 55·6).

Occurrence.—In the metallic form iron is found in Greenland and in meteorites. The chief ores are:—

Magnetite, Fe_3O_4, is found in Sweden and North America.

Red hæmatite, Fe_2O_3, occurs in Belgium and the Furness district of Lancashire.

Brown hæmatite, $Fe_2O_3, 3H_2O$, is found in Spain and South Wales.

Besides the oxides, iron is found in the form of carbonate, and sulphide; thus:—

Spathic iron ore is the carbonate $FeCO_3$, and iron pyrites the sulphide FeS_2.

Extraction.—The ores used for the production of iron are the oxides and the carbonate, because they are fairly pure, and the oxide can be reduced to the metallic iron by heating with carbon; thus:—

For oxide . $Fe_2O_3 + 3C = 2Fe + 3CO$ (Carbon monoxide)
For carbonate . $FeCO_3 = FeO + CO_2$ (Iron oxide reduced by carbon)

Pig Iron.

The ore is first calcined, or heated with a limited supply of air, in order to drive off certain impurities and render the ore more porous. It is then smelted in a blast furnace. The charge consists of the ore, along with limestone and coke, and a good blast of air. When the mass is molten the lime forms a slag with the earthy matter of the ore, and the crude iron sinks and is run off into moulds or "pigs."

Kinds of Iron.—**Cast Iron** is a purified form of pig iron. It contains carbon, which when combined with the iron, forms **white cast iron,** but when present as small particles of graphite, it forms **grey cast iron.**

It contains about 92 per cent. of iron, 5 per cent. of carbon, and 3 per cent. of other impurities, chiefly silicon.

Wrought Iron is made by the purification of cast iron. The latter is put into the **puddling furnace**, and so worked that the impurities are oxidised and removed.

It is the purest form of iron, and only contains about 0·1 per cent. of carbon.

Steel consists of **pure** iron combined with about 0·7 per cent. of carbon. It is made from wrought iron and from cast iron.

Properties.—Iron is a fibrous and tough metal possessing a tensile strength of over 25 tons to the square inch.

The physical properties of iron are dependent upon the impurities. Thus pure iron is very malleable and difficult to fuse, whilst the addition of carbon makes it more brittle and lowers the melting-point, which varies from 1600 to 1800° C. The specific gravity varies from 7·2 to 7·8.

Chemical Properties.—Iron is unacted upon, either by pure **dry** air or by pure water; but if exposed to moist air or water containing dissolved air, it corrodes and becomes coated with rust or iron oxide. In an experiment previously given, iron filings were exposed in moist air in order to show roughly the composition of the air.

When steam is passed over red-hot iron, the iron splits up the water to form iron oxide (Fe_3O_4) and hydrogen is liberated.

$$4H_2O + 3Fe = Fe_3O_4 + 4H_2$$

In this way hydrogen was prepared from water.

The action of acids on iron has already been studied in a previous chapter.

Hydrochloric acid and dilute sulphuric acid dissolve iron with the evolution of hydrogen. Strong sulphuric acid has no action in the cold, but on heating, sulphur dioxide is liberated (as shown in the preparation of sulphur dioxide). Dilute nitric acid dissolves iron with the evolution of red fumes (oxides of nitrogen).

Strong nitric acid has no solvent action on iron, and for this reason iron retorts are used in the manufacture of nitric acid.

Lead (symbol, Pb (Plumbum); atomic weight, 206).

Occurrence.—The chief ore of lead is **galena,** or lead sulphide (PbS). It is widely distributed in the older rocks as limestone and quartz, and is worked in Cornwall, Cumberland, Flint, and also in Spain and other parts of the world. Other lead ores occur, but are less in quantity, and are less worked, as cerusite or carbonate of lead ($PbCO_3$), and anglesite or sulphate of lead ($PbSO_4$).

Extraction.—The extraction of lead from its ore depends upon the following action:—

$$PbS + 2PbO = 3Pb + SO_2$$

When a mixture of lead sulphide and oxide is heated, sulphur dioxide is liberated and metallic lead sinks to the bottom.

For the richer ores, the galena is placed on the hearth of a reverberatory furnace (Fig. 135), and heated at a moderate temperature until the ore is partly oxidised; thus:—

$$2PbS + 3O_2 = 2PbO + 2SO_2$$

The oxide and sulphide are then thoroughly mixed, and the temperature is raised. Double decomposition takes place, and the metallic lead is run off.

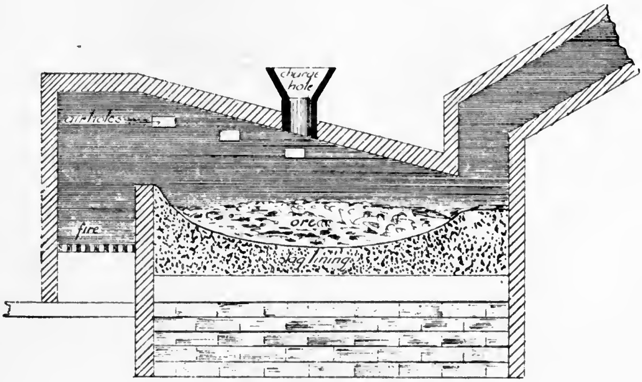


FIG. 135

The poorer ores are reduced in blast furnaces. The ore is roasted, and converted into the oxide (PbO), which is then reduced by carbon, as in the extraction of iron from its ores; thus:—

$$PbO + C = Pb + CO \text{ (Carbon monoxide).}$$

Purification.—The lead obtained from the above processes contains small quantities of zinc, tin, copper, and silver, which render it hard. It is **softened** by heating in a reverberatory furnace until the oxides of the above metals (excepting silver) form a scum on the top. Most of the silver is removed by other processes.

Properties.—Lead is a soft, bluish-grey metal with a bright metallic surface when freshly cut. It is very malleable, but possesses little tenacity. The specific gravity is 11·36, and it melts at **325° C.**

Chemical Properties.—*Action of Air.*—The bright surface of lead readily tarnishes or oxidises when exposed to the air, due to the formation of a thin black film of the suboxide of lead (Pb_2O).

Action of Water.—Lead is unacted upon by pure water, free from dissolved air, and out of contact with it; but in contact with air, lead hydroxide, $Pb(OH)_2$, is formed, and in time covers its surface. This hydroxide is slightly soluble in certain kinds of water, and when present renders it unfit for drinking purposes.

If carbon dioxide be present, either in the water or in the air in contact with it, the action is much quicker. The hydroxide becomes a basic carbonate of lead ($2PbCO_3$, $Pb(OH)_2$), which is insoluble in water, and settles down as a white precipitate.

The action of water on lead may also be due to the presence of dissolved salts in water. If the water be **soft,** and contains traces of dissolved ammonium nitrate, or carbon dioxide, the lead is dissolved. When such water is allowed to stand in leaden pipes, the lead is dissolved, and if used for drinking purposes, may give rise to lead-poisoning. On the other hand, if the water be **hard,** and contain calcium carbonate, there is practically no action on the lead.

Action of Heat.—When lead is heated until it melts (325° C.) a black film forms on the surface; this is the suboxide of lead (Pb_2O). When more strongly heated it forms the yellow oxide of lead, or litharge (PbO). If this oxide be heated for a long time at 430° C. it gradually turns red, and forms the red oxide of lead, or **minium** (Pb_3O_4). The commercial red lead, which is used as a pigment, may be made in this way.

If red lead be heated at a temperature above 450° C. it gives off oxygen, and leaves the lower oxide, litharge (PbO); thus:—

$$2Pb_3O_4 = 6PbO + O_2$$

Action of Acids.—Hydrochloric acid has no action on lead when cold, but if boiled for a time in the presence of air, the lead slowly dissolves, and forms plumbic chloride ($PbCl_2$).

Sulphuric acid has little or no action on lead. For this reason the chambers used in the manufacture of sulphuric acid are lined with lead.

Dilute nitric acid dissolves lead, and forms lead nitrate ($Pb(NO_3)_2$), but strong nitric acid does not attack it.

White Lead.

This well-known substance forms the most important basic carbonate of lead, and it has the composition $2PbCO_3$, $Pb(OH)_2$.

It is manufactured chiefly by the **Dutch process.** The sheet-lead is coiled and placed on a ledge in an earthenware pot with acetic acid at the bottom, as shown in Fig. 136, and the pots are stacked and covered with decaying vegetable matter. The heat, generated by decay, vaporises the acetic acid, and it then acts on the lead, to form basic lead acetate. The carbon dioxide liberated from the organic matter acts upon this salt to form basic lead carbonate, or white lead, which is scraped from the coil and collected.

FIG. 136

Uses.—White lead is a heavy, **white,** amorphous powder, which is insoluble in water; it is largely used as a white pigment by painters for preserving and adorning woodwork. It is also used to some extent in the jointing of various kinds of iron pipes.

It is poisonous, and it is blackened by sulphuretted hydrogen (H_2S) when used in the form of white paint for coating woodwork.

Zinc (symbol, Zn; atomic weight, 65).

Occurrence.—The chief ores of zinc are zinc blende, or zinc sulphide (ZnS), and calamine, or zinc carbonate ($ZnCO_3$).

Extraction.—The ore, whether it be sulphide or carbonate, is first heated or calcined until all the zinc present is in the form of zinc oxide (ZnO); thus:—

$$\underset{\text{(Calamine)}}{ZnCO_3} = \underset{\text{(Zinc oxide)}}{ZnO} + CO_2$$

$$\underset{\text{(Zinc blende)}}{2ZnS} + 3O_2 = 2ZnO + 2SO_2$$

The calcined ore (ZnO) is then mixed with coal, and roasted in iron retorts.

The zinc oxide is reduced to metallic zinc, as shown in the following equation:—

$$ZnNO + C = Zn + CO.$$

The zinc is more volatile than any of the impurities; hence it distils over, and is collected in a pure state.

Properties.—Zinc is bluish-white in appearance, crystalline, and brittle at ordinary temperatures. Between **100° and 150° C.** it becomes ductile, and can be drawn into wire, and rolled into sheets or foil. After rolling, the zinc retains its malleability, and is no longer brittle.

If heated to **300° C.** it loses its cohesion, and can be easily powdered in a mortar.

At 420° C. it melts, and if other metals, after special preparation, be dipped into molten zinc, they are galvanised or coated with zinc.

At 905° C. zinc boils, and gives off a vapour which burns with a bluish-white flame to form zinc oxide (ZnO).

If the molten zinc be slowly poured into cold water it solidifies in the form of **granulated zinc**, which is used in the preparation of hydrogen.

Chemical Properties.—Zinc is unacted upon by dry air at ordinary temperatures, but its surface readily tarnishes if exposed to moist air. This is due to the formation of a thin film of zinc oxide (ZnO), which is insoluble in water, and it thus acts as a protective covering. Because of this, iron goods, which are exposed to the air and water, such as iron rain-water pipes, cisterns, W.C. brackets, soil pipes, corrugated roofing, are generally coated with zinc—*i.e.*, are galvanised.

Ordinary commercial zinc is readily acted upon and dissolved by hydrochloric acid, and by dilute sulphuric acid.

When heated with strong sulphuric acid it gives off sulphur dioxide, and leaves zinc sulphate ($ZnSO_4$).

The action of either hydrochloric acid or sulphuric acid on zinc is probably due to the impurities in the zinc, for if pure zinc be used, these acids have little or no action on it.

Nitric acid dissolves it, red fumes of oxides of nitrogen being evolved, and zinc nitrate $Zn(NO_3)_2$ remaining in solution.

The plumber uses zinc to "kill" spirits of salt (hydrochloric acid) in order to obtain "killed spirits of salt" (or zinc chloride), for use as a flux in soldering.

Zinc is occasionally used for roofwork on account of lightness and cost, but in towns where the atmosphere contains acid impurities it is readily destroyed. On the other hand, in the open country it will last for a considerable time, owing to the almost total absence of acid gases from the atmosphere.

Tin (symbol, Sn (Stannum); atomic weight, 118).

Occurrence.—The chief ore is tin-stone, or cassiterite (SnO_2), which occurs in large quantities in Cornwall and Devon, in Saxony, in Tasmania, and in the Straits Settlements.

Extraction.—The tin-stone is first crushed and washed in order to remove the earthy matter. The ore is then calcined in order to remove such volatile impurities as sulphur and arsenic. The tin oxide (SnO_2) is then reduced by heating the ore in a reverberatory furnace with anthracite coal and lime to act as a flux—

$$SnO_2 + 2C = Sn + 2CO.$$

The molten tin is run off from the dross, and is afterwards purified by stirring with green wood, in order to bring the remaining slag to the surface.

Properties.—It is a white malleable metal, which is harder than lead but softer than zinc.

From ordinary temperatures to 100° C. it is soft and malleable, and may be readily beaten out into **foil**. If heated to a temperature of about 200° C. it becomes very brittle and may be powdered.

It then becomes crystalline in structure, and forms "grained tin," which is composed of interlacing crystals of tin. The peculiar crackling sound made when tin is bent is caused by the friction between the crystals of tin. It melts at 230° C.

Chemical Properties.—It is unacted upon by either dry or moist air; hence it keeps its bright, shining surface on exposure. For this reason it is used in tinning sheet-iron to form tin-plate.

When strongly heated in the air metallic tin burns to form stannic oxide (SnO_2).

Hydrochloric acid has little action on tin when cold, but on heating it dissolves, slowly forming tin chloride ($SnCl_2$) and liberating hydrogen; thus:—

$$Sn + 2HCl = SnCl_2 + H_2$$

With **strong sulphuric acid** it behaves like zinc and copper, tin sulphate is formed and sulphur dioxide is liberated; thus:—

$$Sn + 2H_2SO_4 = SnSO_4 + SO_2 + 2H_2O.$$

Concentrated nitric acid is almost without action upon it, but the dilute acid dissolves it readily, forming tin nitrate, and liberating the oxides of nitrogen (red fumes).

Copper (symbol, Cu (Cuprum); atomic weight, 63).

Occurrence.—Copper is found in the native state in large quantities in the neighbourhood of Lake Superior; and it occurs in smaller quantities in Cornwall and the Ural Mountains. In the combined state it is widely distributed in the form of the oxide, sulphide, and carbonate.

The chief ores are:—

Cuprite, or cuprous oxide (Cu_2O), is mined in Cornwall, South America, and Australia.

Copper glance (Cu_2S) is the sulphide of copper.

Copper pyrites (Cu_2S, Fe_2S_3) contains the sulphides of iron and copper.

Malachite and **Azurite** are the basic carbonates of copper, which occur in the native state..

Extraction.—For the ores which contain no sulphur, such as the oxide and the carbonate, the extraction is simple, and the process is similar to the reduction of iron and tin. The ore is mixed with coal or coke and smelted down in a blast furnace, when reduction takes place as follows:—

$$Cu_2O + C = Cu + CO \text{ (Carbon monoxide).}$$

For the mixed ores containing sulphur the process is a long one, and consists of many stages. The preliminary stages are to remove the impurities, and obtain the ore in the form of the sulphide (Cu_2S). It is then raised to a moderate temperature, so as to partially oxidise it. The mixture of oxide and sulphide are then mixed and heated strongly in a reverberatory furnace. The following reaction takes place:—

$$Cu_2S + 2Cu_2O = 6Cu + SO_2$$

This process is similar to the one used in the reduction of lead.

Purification.—The impure metal is refined by heating the metal until all the impurities, as iron, lead, arsenic, or sulphur pass off as vapour, or combine with the flux to form a slag, which is removed.

Properties.—Copper is a reddish-brown metal, with a bright, shining surface. It is a soft but tough metal, and is capable of being drawn into fine wire, and hammered into thin leaf. When heated strongly and cooled quickly it becomes very brittle, and can be reduced to a powder; thus it resembles zinc and tin. But if heated and cooled very slowly it is exceedingly soft, malleable, and ductile.

Copper is a good conductor of heat and electricity, and it is for this reason used for boilers, cylinders, and hot-water fittings. In the form of wire and cable it is extensively used in telegraphy and electric lighting. Copper has a specific gravity of 8·93, and it melts at 1050° C.

Chemical Properties.—Dry air is without action on copper at ordinary temperature, but on heating, the surface tarnishes, and becomes coated with the black cupric oxide (CuO), which easily leaves the surface in black scales. For this reason copper was used in finding the composition of air by weight. In the presence of moisture and carbon dioxide it quickly corrodes, and becomes coated with the green basic carbonate of copper. To prevent this action, copper pipes are **tinned** inside and outside with a thin film of tin.

Action of Acids.—Dilute hydrochloric and sulphuric acids have no action on copper out of contact with the air. But in the presence of air, the copper is slowly attacked.

Strong hydrochloric and sulphuric acids have no action in the cold, but on heating, the copper is dissolved.

Strong hydrochloric acid, when boiled with finely divided copper, dissolves it slowly, liberating hydrogen, and forming cuprous chloride; thus:—

$$2Cu + 2HCl = \underset{\text{(Cuprous chloride)}}{Cu_2Cl_2} + H_2$$

Strong sulphuric acid, when heated with copper, forms copper sulphate ($CuSO_4$) and liberates sulphur dioxide (SO_2); thus:—

$$Cu + 2H_2SO_4 = CuSO_4 + 2H_2O + SO_2$$

Nitric acid, strong or dilute, readily attacks copper, dissolving it, and forming copper nitrate, and oxides of nitrogen.

The complicated reaction may be represented as follows:—

$$3Cu + 8HNO_3 = \underset{\text{(Copper nitrate)}}{3Cu(NO_3)_2} + 4H_2O + \underset{\text{(Oxide of nitrogen)}}{2NO}$$

Tearing or Tensile Strength in pounds per square inch.	Metal.	Specific Gravity.	Melting-point.	Specific Heat.	Atomic Weight.	Symbol.
			Degrees Cent.			
16,350	Cast Iron . . .	7·2	...	...	...	...
50,700	Wrought Iron . .	7·79	...	...	...	...
...	Steel	7·79	1,600	·112	55·6	Fe
2,159	Lead	11·2	325	·032	206·0	Pb
6,720	Zinc	7·01	420	·094	65·4	Zn
4,800	Tin	7·3	231	·056	118·0	Sn
33,000	Copper . . .	8·95	1,050	·095	63·0	Cu

CHAPTER XXVII

ALLOYS, MAKING OF SAME; MELTING-POINTS AND COMPOSITION OF VARIOUS ALLOYS, INCLUDING HARD AND SOFT SOLDERS, AMALGAMS, FLUXES; THEIR USES

Alloys

Experiment CXXIV.—Take equal weights of tin and lead. Melt the lead and slowly add the tin. Stir well, until all the tin is melted, and the whole is a homogeneous mass. Allow it to solidify, and examine the mass carefully. In appearance and properties it differs from lead and tin. In fact, it is a new substance, and it is called an **alloy** of tin and lead.

What is an Alloy ?—At first it would be said that an alloy consists of two or more metals intimately mixed together.

We have already learned that complex substances may be divided into mechanical mixtures and chemical compounds. To which class do alloys belong?

In most cases the alloy prepared from two or more metals is quite different in appearance and properties, both physical and chemical, from its constituents. Thus melting-point is below that of either constituent. Specific gravity is altered.

Again, all metals do not "mix" to form alloys; thus if lead and zinc be melted and intimately mixed, the lead gradually sinks and the zinc rises to the surface.

Again, heat is developed in forming an alloy, but it has not been definitely settled whether this heat is due to chemical combination merely, or to a change of physical condition.

On the other hand, it is found that certain metals will "mix" in almost all proportions, and also other metals may be added to form fresh alloys. Since this fact is true for mixtures and not compounds, and since there is only small evidence that alloys are chemical compounds, we shall in future refer to them as **mechanical mixtures.**

An alloy may be defined as **an intimate mixture of two or more metals, which possesses properties quite different from those of its constituents.**

To make an Alloy.—First weigh out the amount of each metal in the proportion required to form the alloy. Then select the metal with the highest melting-point, and fuse it under a layer of charcoal, to prevent oxidation. Then put in the other metals, and thoroughly mix them. Allow the mass to cool slowly and solidify. When metals which volatilise readily, as sodium, potassium, magnesium, and zinc are used, they should be added last of all.

Melting-points of an Alloy.—Different alloys of the same metals have different melting-points, but the same alloy always melts at the same temperature, which is called the melting-point.

Experiment CXXV.—Take the apparatus shown in Fig. 101. It consists of an outer vessel of iron or copper, inside which is fixed a narrow tube of copper, used for holding the thermometer. Fill the outer vessel with the above alloy, and allow it to solidify. Into the inner vessel put a thermometer, and surround the bulb with mercury. Place the apparatus on a tripod, and heat until the alloy begins to melt. Read the temperature indicated by the thermometer. Allow the alloy to solidify, and repeat the experiment several times with care, and get the average of the results.

Remember that the **melting-point** is the **lowest** temperature at which the alloy melts.

Kinds of Alloys

I. Solders.—These are mixtures of lead and tin, chiefly in proportions varied according to the class of work for which they are required. They are used chiefly by the plumber for joining metals of various kinds. In all solders it is essential that the solder must have a lower melting-point than either metal to be joined, and also it must set hard, and be durable.

Solders may be classified as follows :—

With lead, tin will mix in all proportions, and many alloys are in common use, consisting of these two metals. In all cases the alloys are white, or have a silvery lustre.

Soft Solders, Alloys of Tin and Lead

Name.	Constituents.		Melting-point.	Uses.
	Tin.	Lead.		
			Degrees Fahr.	
	$4\frac{1}{4}$	1	370	
Common Pewter	4	1	365	
Common Pewter	$3\frac{1}{2}$	1	360	
	3	1	356	
	$10\frac{1}{2}$	4	350	
	2	1	340	Blowpipe joints.
Fine Solder	3	2	334	Plumbers', zincworkers', and tinsmiths' copper-bit work.
Copper-bit Solder	1	1	370	Plumbers', zincworkers', and tinsmiths' copper-bit work.
Plumbing Solder	1	2	441	Plumbers' wiped joints.
	1	3	482	Coppersmiths' wiped joints.

Tin melts at 442° F. and lead at 617° F.; hence it will be noticed that all the above alloys except the last one, melt at temperatures lower than either constituent. Again, the melting-point is lowered as the proportional quantity of **tin** is decreased, until a certain point is reached, after which the melting-point increases with a further proportional decrease of tin.

II. **Very Fusible Alloys.**—The metal bismuth (melting-point 507° F.) readily forms alloys with other metals, and when mixed with the soft solders (alloys of lead and tin) it lowers the melting-point, and imparts hardness to the alloy ; thus :—

Name.	Lead.	Tin.	Other Metal.	Melting-point.	
				Degrees Fahr.	
	26	24	8 of Bismuth	320	
Solder for Tin Pipes .	16	8	8 ,,	300	
	13	8	8 ,,	280	
	9	8	8 ,,	260	
	8	5	8 ,,	240	
	7	3	8 ,,	220	
	5	3	8 ,,	212	Melt in boiling water.
	4	4	8 ,,	203	
	2	2	8 ,,	201	
Woods' Fusible Metal	2	1	4 ,, 1 of Cadmium	149 ...	

Antimony (melting-point 840° F.) is used in certain alloys, because when solidifying it expands. This property is imparted to the alloy containing antimony, and it gives it the valuable quality of taking very fine and sharp castings. Antimony is also used in alloys in the construction of appliances where it is necessary to resist the action of strong acids. It also produces hardness in the alloy. The most important are :—

Name.	Lead.	Antimony.	Tin.	Uses.
Type Metal . .	75	20	5	
Stereotype Metal .	112	18	3	
	Copper.			
Britannia Metal .	3	9	140	Spoons, teapots, etc.

Hard Solders are used for joining metals which melt at a higher temperature, as iron and copper. They melt at a higher temperature, and are much stronger than soft solders. As a rule, the hard solder should contain some of the metal to which it is to be joined, so that the alloy will mix with the surface of the metal and be alloyed with it.

In fact, it is usual to form an alloy of the metal to be joined, to which is added some other metal to make it more fusible; thus :—

Name of Solder.	Constituents and their Melting-points.			Uses.
	Copper, 1996° F.	Zinc, 773° F.	Silver, 1830° F.	
Spelter—				
Hardest . .	2	1	...	For joining iron, copper, gunmetal.
Hard . .	3	2	...	
Soft . . .	1	1	...	For ordinary brasswork.
Fine . . .	2	2	$\frac{1}{4}$	For fine brasswork.

In practical work a metallic substance is often required which must have properties not possessed by any one metal alone. These properties can be obtained by introducing certain proportions of other metals, thus forming an alloy. It has been found by experiment that certain metals produce definite properties in the alloy; thus :—

Mercury, bismuth, tin, and cadmium are used to give fusibility to alloys.

Zinc gives hardness and durability to the alloy, but if over 50 per cent. of zinc be used, the alloy is very brittle.

Tin gives hardness and tenacity when used in considerable quantity.

Lead and iron give hardness, and lead makes an alloy more fluid and cast better.

Arsenic and antimony render the alloy brittle.

Aluminium increases the tenacity, and it resists corrosion.

III. Brasses and Bronzes, etc. Copper Alloys

(1) **Brasses** are alloys of Copper and Zinc.

Name.	Copper.	Zinc.	Uses.
Common Brass . .	1	1	Ordinary brasswork.
English Brass . .	2	1	Very tough, and is used for wire and foil.
Muntz Metal . .	3	1	Unacted upon by sea-water; hence it is used in coating ships.
Dutch Brass, or Tombac	5	1	Suitable for hot-water and high-pressure work.
Dutch Metal . .	11	2	Very malleable.

In the so-called German silver, or nickel silver, the nickel is introduced to the alloy of copper and zinc; thus :—

Copper 2 : Zinc 1 : Nickel 1.

In Belgium, Germany, and the United States the small coinage consists of the following alloy :—

Copper 3 : Nickel 1.

Alloys of copper and zinc (brasses) which contain over 80 per cent. of copper are red or reddish-yellow, and those containing over 50 per cent. of zinc are white and brittle, as already stated.

(2) **Bronzes** are alloys of copper and some other metals, chiefly tin, copper in every case being the most important metal.

Name.	Copper.	Tin.	Other Metals.	Remarks.
Bronzes—				
(1) For Coinage.	95	4	1 of zinc	The zinc increases the hardness and durability.
(2) Gun Metal .	90	10		Tough and tenacious, used for water fittings, where water would destroy yellow brass.
(3) For Bells .	78	22		
(4) For Cymbals	80	20		
Specula Metal .	67	33		Suitable for telescopes. It is very hard and brittle.
Bell Metal . .	70	30		
Aluminium Bronze	90	...	10 of aluminium	Tensile strength equal to steel, golden colour, and takes high polish.

(3) Other alloys containing copper :—

Silver and gold are too soft to be used alone for coinage, jewellery, etc., hence they are alloyed with copper, in variable proportions, to increase the hardness; thus :—

Gold Alloys.

(1) For coinage. Gold 90 parts, copper 10 parts.

(2) For jewellery and plate. Gold 75 to 92 parts, copper 25 to 8 parts.

Silver Alloys.

(1) For coinage. Silver 90 parts, copper 10 parts.
(2) For vessels. Silver 95 parts, copper 5 parts.
(3) For jewellery, etc. Silver 80 parts, copper 20 parts.

IV. Other Alloys—Amalgams

Mercury alloys with most metals, forming an amalgam. Most amalgams are readily fusible, and in some cases they have a definite composition.

Sodium amalgam and tin amalgam are used to produce the bright reflecting surface of mirrors.

Aluminium produces many alloys with different metals, and many are valuable for their colour, their tensile strength, and their resistance to corrosion.

The alloy, aluminium 90 per cent., tin 10 per cent., possesses the valuable properties of brass, and it is also brighter and non-corrosive.

When present in iron or steel in very small quantities, it lowers the melting-point, increases the fluidity, and is thus more easily worked.

Fluxes

These are substances used during soldering operations to prevent the formation of an oxide on the surface of the metals which are being soldered, and also to assist the solder to form a surface alloy with the metals. It will be noticed that most metals when heated, after the surface has been cleaned, gradually become covered with a film of the oxide of the metal, which has no affinity for the solder, and will not unite with it. Hence, to prevent this, fluxes are necessary in all soldering operations, except lead burning.

List of Fluxes

Fluxes.	Uses.
Tallow	Joining lead to lead, copper or brass with plumbers' solder.
Resin and Tallow . . .	Joining lead to lead, copper or brass with fine solder.
Killed Spirits (Zinc Chloride)	For soldering zinc, copper, iron, and brass.
Gallipoli Oil	For soldering tin and pewter.
Borax or Sal-ammoniac . .	For hard soldering—*i.e.*, iron and steel.

INDEX

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