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EQUATIONS AND COMPUTER SUBROUTINES FOR ESTIMATING SITE QUALITY OF EIGHT ROCKY MOUNTAIN SPECIES

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INTRODUCTION

Computer programs for such purposes as the compilation of forest inventory data must contain instructions for operations that were done by hand only a few years ago. At that time, tables, graphs, and alinement charts were indispensable to the forester. The information which was contained therein is still indispensable, but today we must be able to store this information in a computer program in a form readily accessible for use. Computer use of charts or graphs to describe site or yield functions is impractical, but information stored in tabular form could be used. However, this would require either a very extensive table for each species or interpolation within a small table. The core storage required for a number of extensive tables, in addition to the rest of the program and the data being processed, might well be more than is available on smaller computers, while interpolation can lead to inaccuracy.

The existence of an alinement chart or a smooth curve, or tables made therefrom, implies the existence of a mathematical relationship between the variables involved. Sometimes the relationship is known, as when a table is made by repeated solution of an equation. Some charts and tables were derived altogether by graphic methods. When this is the case, an approximate mathematical relationship can usually be derived. However, the resulting approximation equation, complex and sophisticated as it may appear, can never be any better than the hand-drawn curve that it is supposed to represent.

Data collected 40 years ago represent our only source of information on site quality and yield capability for some species. Yield capability, as used by Forest Survey, is defined as mean annual increment of growing stock attainable in fully stocked natural stands at the age of culmination of mean annual increment. Yields may be substantially higher with thinning and other intensive management. Yield capability is measured in cubic feet per acre per year.

Site index, together with associated yield information, provides a means of assessing yield capability or, in other words, productive capacity of the land. However, all variation in yield capability from one site to another cannot be explained by conventional "site index." Short of direct measurement of stand growth itself, it is not generally known what stand parameter(s) could be used to account for variation not explainable by site index.

Despite its shortcomings, site index does provide a means of rating sites in terms of volume yield capacity--a more meaningful expression than site index alone.

Another factor may affect yield capability of a stand--the extent to which an area may be stockable. It may be that available soil is interspersed with areas of rock or gravel that will not support tree growth. Any yield capability figure expressed in this report on a per-acre basis will refer to an acre of tree-supporting soil, not to an acre of land containing some nonstockable area. Where full stocking is not possible, the yield capability estimate should be adjusted. This might be done by multiplying the yield capability estimate by the ratio of productive land area to total land area. The result would be a yield capability estimate for the entire area, including non-stockable portions.

Growing stock is defined, for Forest Survey purposes, as the live, noncull trees of commercial species in the stand 5.0 inches d.b.h. and larger. However, the sources of information from which the material presented in this report was derived used differing size standards, so the definition of growing stock that was necessarily used for each species will be given with the equations for that species.

The equations presented in this report can be used to estimate site index and/or yield capability for each of the following species:

1. Western white pine (*Pinus monticola* Dougl.)

2. Ponderosa pine (*Pinus ponderosa* Laws.)

3. Lodgepole pine (*Pinus contorta* var. *latifolia* S. Wats.)

4. Western larch (*Larix occidentalis* Nutt.)

5. Engelmann spruce (*Picea engelmannii* Parry)

6. Inland Douglas-fir (*Pseudotsuga menziesii* var. *glauca* (Mirb.) Franco)

7. Grand fir (*Abies grandis* (Dougl.) Lindl.)

8. Quaking aspen (*Populus tremuloides* Michx.)

For western white, ponderosa, and lodgepole pines, and for western larch and grand fir, the yield capabilities that are supposed to be equivalent to given levels of site index are shown in table 1. For Engelmann spruce, inland Douglas-fir, or quaking aspen, yield capability equations cannot be given because the necessary information was not available.

Table 1.--Yield capability according to site index for western white pine, ponderosa pine, lodgepole pine, western larch, and grand fir

Site index (50-year base age)	Yield capability				
	Western white pine[1]	Ponderosa pine[1]	Lodgepole pine[2]	Western larch[2]	Grand fir[1]
	Cubic feet per acre per year				
20	49.5	19.4	12.4	--	56.5
25	58.1	24.2	16.4	--	65.2
30	66.8	29.6	21.6	21.8	73.9
35	75.4	35.5	27.8	26.7	82.6
40	84.1	42.1	35.1	33.8	91.3
45	92.7	49.6	43.5	42.5	100.0
50	101.4	57.9	53.0	52.2	108.7
55	110.1	67.3	63.6	62.7	117.4
60	118.7	77.9	75.3	73.8	126.1
65	127.4	89.9	88.0	85.3	134.8
70	136.0	103.4	101.8	97.2	143.5
75	144.7	118.7	116.8	109.3	152.2
80	153.3	136.1	132.7	121.7	160.9

[1]Based on all surviving trees in the stand at the age of culmination of cubic mean annual increment.
[2]Based on all trees larger than 5.0 inches d.b.h.

SITE INDEX AND YIELD CAPABILITY EQUATIONS

The information in this section was derived either from published material or from data in the files of the Intermountain Forest and Range Experiment Station, as will be noted for each particular species. The mathematical details of equation construction or derivation have been omitted because use of the equations or computer subroutines does not require a complete description of the means by which they were derived.

Except for quaking aspen, all site indices are referred to at a base age of 50 years. For the user's convenience, computer programs that contain the calculating procedure are presented whenever the calculations required to obtain site index or yield capabilities for a species involve more than the simplest equation. The programs are written in FORTRAN IV. They have been tested on an IBM 360, Model 67 computer and they can be expected to function on any IBM 360 having the minimum of a FORTRAN IV, Level E compiler available. The programing might not always be the most efficient for the IBM 360 because the programs are designed to allow modification to other versions of FORTRAN. These programs have been written as subroutines because it is expected that their use will be implemented through a larger program.

The user is cautioned that extrapolation of statistically fitted equations beyond the range of the basic data may lead to inaccurate predictions. The probability that this will occur is greater when the equations are more complex. Mistakes in input data, poor selection of site trees, or the sampling of a site quality not represented in the data basic to an equation can prevent the iterative process from reaching a solution. It may well be that this trouble will never be encountered, but it is something of which the user should be aware.

WESTERN WHITE PINE

Site Index

Site index for even-aged stands of western white pine can be estimated by the equation:

$$S = b_0 H \, [1 - b_1 \cdot \exp\,(b_2 A)]^{b_3}, \text{ where}$$

S = site index at a base age of 50 years

H = average height of dominant and codominant trees

A = age of the oldest dominant tree in the even-aged stand

$b_0 =$ 0.37504453

$b_1 =$ 0.92503

$b_2 =$ -0.0207959

$b_3 =$ -2.4881068.

This equation was obtained by fitting an equation of suitable form to the height/age data in table 24 of Haig's (4) yield tables, then algebraically solving the latter equation for site index. Written in FORTRAN IV (IBM 360) the equation is:

```
S = 0.375045 * H * (1.0 -0.92503 * EXP (-0.020796*A))**(-2.48811).
```

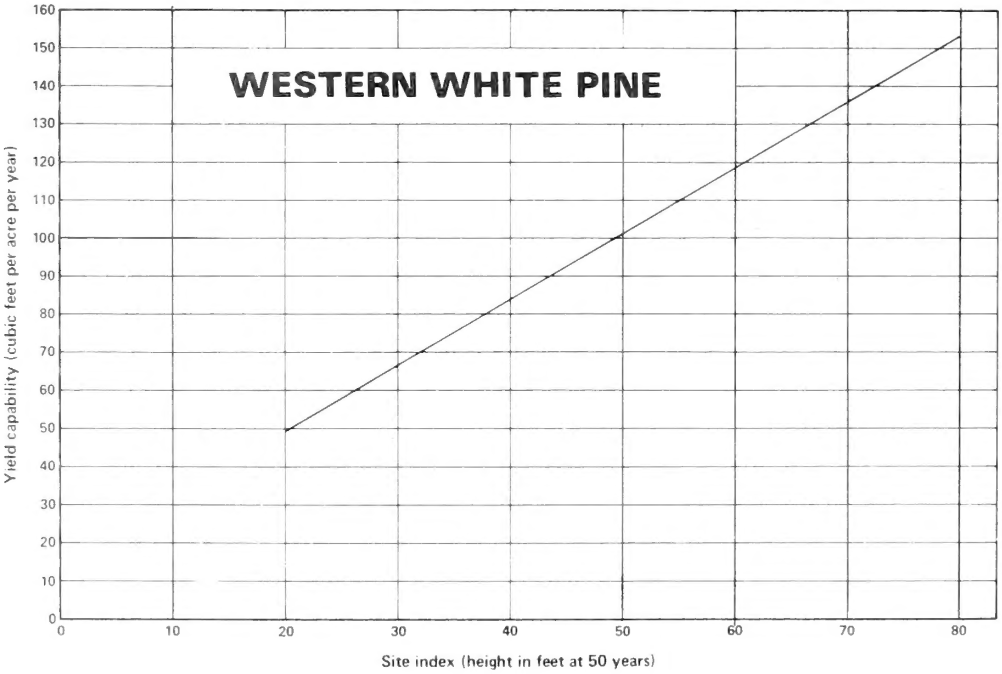

Figure 1.--Relationship between site index and yield capability.

Yield Capability

Yield capability of western white pine can be estimated by the equation:

$YC = b_0 + b_1 S$, where

YC = yield capability in cubic feet per acre of mean annual increment on all trees at the age of culmination of increment

S = site index at a 50-year base age

b_0 = 14.849891

b_1 = 1.7311563.

This equation was derived from table 7 of Haig's (4) yield tables which show average mean annual increment of all trees in so-called normal stands for various ages and levels of site index. For western white pine, cubic mean annual increment culminates at a stand age of about 105 years, regardless of site quality, according to Haig's table. Consequently, this equation expresses the relationship between site index and cubic volume mean annual increment at a stand age of 105 years, as shown in figure 1.

PONDEROSA PINE

Site Index

The estimating procedure for even-aged stands of ponderosa pine is based on the curves developed by Lynch (7). The index base age was changed from 100 years to 50 years and the base of logarithms in the equation was changed so as to use natural rather than common logarithms.[1/] The surface of dominant stand height over age and site index remains unchanged, because the conversion of the curves resulted only in a rescaling of the independent variables. Lynch's system of site index curves takes into account the reduction in height growth due to overdense stocking on lower quality sites. Because one or the other of two equations is to be used, depending on whether or not stocking and site conditions have reduced height growth, the estimation procedure has been put into a computer program subroutine, shown in figure 2.

```
      SUBROUTINE PPSITE(A,H,B,SI50,SD)
      U=1.+0.2432*(100./A-1.)
      PI=(H*10.**(0.6437*(100./A-1.)))**(1./U)
      SD=B*(0.2918+0.0065*H-0.0467*H/A+29.752/A)
      IF(SD-100.0)3,3,1
    1 IF(PI-75.0)2,3,3
    2 Z=(SD-100.)*(75.-PI)
      RZ=EXP(-0.33123E-03*Z+0.276308E-06*Z*Z)
      GO TO 4
    3 RZ=1.
    4 PA=EXP(-1.19222*(100./A-2.))
      WA=0.195623*(100./A-2.)
      SI50=(H/(PA*RZ))**(1./(1.+WA))
      RETURN
      END
```

Figure 2.--Site index estimating subroutine for ponderosa pine.

[1/]Natural or Naperian logarithms are to the base e, where e = 2.71828. Common logarithms are to the base 10. In conventional notation, which will be used in this report, ln X means the natural logarithm of X, while log X means the common logarithm of X.

The mnemonics used in the subroutine are as follows:

A = total stand age

H = average height of trees in the dominant stand

B = stand basal area per acre

SI50 = site index at a 50-year base age

SD = stand density in percent as expressed by Lynch (7), where density is relative to the average for a particular age and site index. That is, when SD = 100 percent the stand is of average density; when SD = 50 percent the stand is half average density, and when SD = 200 percent twice average density is indicated.

Average density of Lynch's sample plots was a little less than normal for most ages and sites. The extremes were 83.3 percent and 111 percent of what Meyer (8) called normal.

Yield Capability

The plot data used in Lynch's study are the basis for the equation or procedure. A regression equation was fitted to the relationship between age, site index, stand density, and net cubic foot volume yield of the plots. When stands are assumed to be of average density for the sample (SD = 1), the equation reduces to:

$$\ln Y = b_0+b_1\ln S-b_2A^{-2} -b_3\ln S-b_4 +(b_5-b_6)S-b_7A^{-1}S^{-1},$$

or

$$Y = c_0S^{c_1} \cdot \exp\ (c_2S - b_2A^{-2} - b_7A^{-1}S^{-1}),$$

where

Y = net yield of all trees in cubic feet per acre

S = site index at a 50-year base age

A = stand age

$c_0 = \exp(b_0-b_4) = 13{,}100.281$

$c_1 = b_1 - b_3 = -0.4930327$

$c_2 = b_5 - b_6 =$ 0.26782874E-01[2/]

$b_2 = 467.59461$

$b_7 = 1843.6671$

then, mean annual increment = Y/A.

[2/] According to FORTRAN notation, E±n following a number means that the number is to be multiplied by $10^{(\pm n)}$. In this instance, the E-01 indicates that the decimal point should be moved one place to the left (0.026782874).

On any particular site, the maximum mean annual increment is reached at the age when

$$\partial \text{ m.a.i.}/\partial A = 0.$$

Evaluating the indicated derivative and solving the equation for age, we find that mean annual increment is a maximum when

$$A = \frac{b_7 S^{-1} + \sqrt{b_7^{2} S^{-2} + 8 b_2}}{2},$$

so the age at which mean annual increment is maximized can be expressed as a function of site index. Then,

$$YC = c_0 S^{c_1} \cdot \exp(c_2 S - b_2 A^{-2} - b_7 A^{-1} S^{-1}) \cdot A^{-1},$$

where

A = the age of mean annual increment culmination expressed as a function of site index, as shown above.

These equations have been put into a computer subroutine, which is shown in figure 3, and the relationship between site index and yield capability is shown in figure 4.

```
   SUBROUTINE PPYCAP (S,YCAPP)
   A=(1843.67/S+SQRT(.339911E+7/(S*S)+.374076E+4))/2.
   YCAPP=(13100.281*S**(-0.4930327)*EXP(0.2678294E-01  *S-467.595  /(A
 1*A)-1843.67  /(A*S)))/A
   RETURN
   END
```

Figure 3.--The ponderosa pine yield capability subroutine.

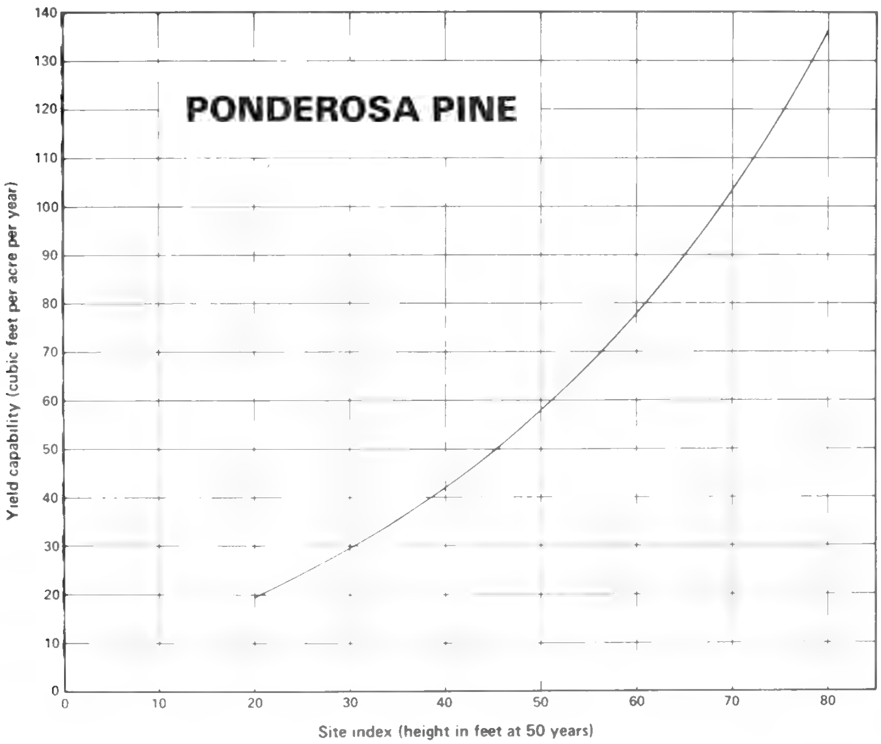

Figure 4.--Relationship between site index and yield capability.

LODGEPOLE PINE

Site Index

The procedure for estimating site index is based on the curves developed by Alexander (1). Like Lynch's curves for ponderosa pine, Alexander's curves for lodgepole pine can be used when stand density is heavy enough to have inhibited height growth of the dominant stand. This is thought to take place at densities where the crown competition factor[3] is greater than 125. Thus, two different procedures have been devised for estimating site index; where CCF is 125 or less a straightforward calculation by means of two equations will suffice.

The equations are:

$$SI = H + b_1 (\ln A - \ln 100) + b_2 (A^2 - 100^2) + b_3 (A^3 - 100^3)$$
$$+ b_4 [(H/A) - (H/100)] + b_5 [(H/A^2) - (H/100^2)] +$$
$$b_6 [(H/A^{2.5}) - (H/100^{2.5})],$$

where

SI = site index at a base age of 100 years	$b_2 = -0.21973907 \times 10^{-2}$
H = height of dominant stand	$b_3 = 0.61670435 \times 10^{-5}$
A = total age of an even-aged stand	$b_4 = -64.32135$
$b_1 = 18.310745$	$b_5 = 9528.3711$
	$b_6 = -34848.289$

and

$$S = c_0 + c_1\, SI,$$

where

S = site index at a 50-year base age

$c_0 = 1.029546$

$c_1 = 0.6297251.$

In stands where CCF exceeds 125 site index must be adjusted upward to compensate for the reduction in height of the dominant stand due to stocking density. Let $g(A)$ be a function of age:

$$g(A) = b_4[(1/A) - 0.01] + b_5[(1/A^2) - 0.0001] + b_6[(1/A^{2.5}) - 0.00001]$$

and

$$k = 0.8188667 \times 10^{-3}\ (CCF-125).$$

Then

$$P = 1-k[g(A) + 1].$$

[3] For an explanation of crown competition factor, see Krajicek, Brinkman, and Gingrich (6).

Within the range of the data on which these equations were based, P will range in value between zero and unity, so dividing SI, as computed above, by P gives the proper adjustment to compensate for the reduction in stand height.

These equations were not based on data including ages over 200 years. Therefore we recommend that ages older than 200 be reduced to 200 years.

```
      SUBROUTINE LPSITE(A,H,CCF,S)
      DIMENSION B(9)
      DATA B/18.310745,0.21973907D-02,0.61670435D-05,64.32135,9528.3711,
     134848.289,0.81886702D-03,1.029546,0.6297251/
      IF(A)2,2,1
    1 IF(H)2,2,3
    2 WRITE(6,201)
      S=0.0
      RETURN
    3 CCFT=CCF-125.0
      FA=B(1)*(ALOG(A)-4.6051702)-B(2)*(A*A-10000.0)+B(3)*(A**3-10.0**6)
      GA=-B(4)*(1.0/A-.01)+B(5)*(1./(A*A)-.0001)-B(6)*(A**(-2.5)-.00001)
      P0=FA+H*(GA+1.0)
      IF(CCFT)4,4,5
    4 CCFT=0.0
    5 EK=B(7)*CCFT
      P1=EK*(GA+1.0)-1.0
      SI=-P0/P1
      S=B(8)+B(9)*SI
      RETURN
  201 FORMAT(62H0EITHER AGE OR HEIGHT OF A LODGEPOLE PINE IS NEGATIVE OR
     1 ZERO.)
      END
```

Figure 5.--The site index estimating subroutine for lodgepole pine.

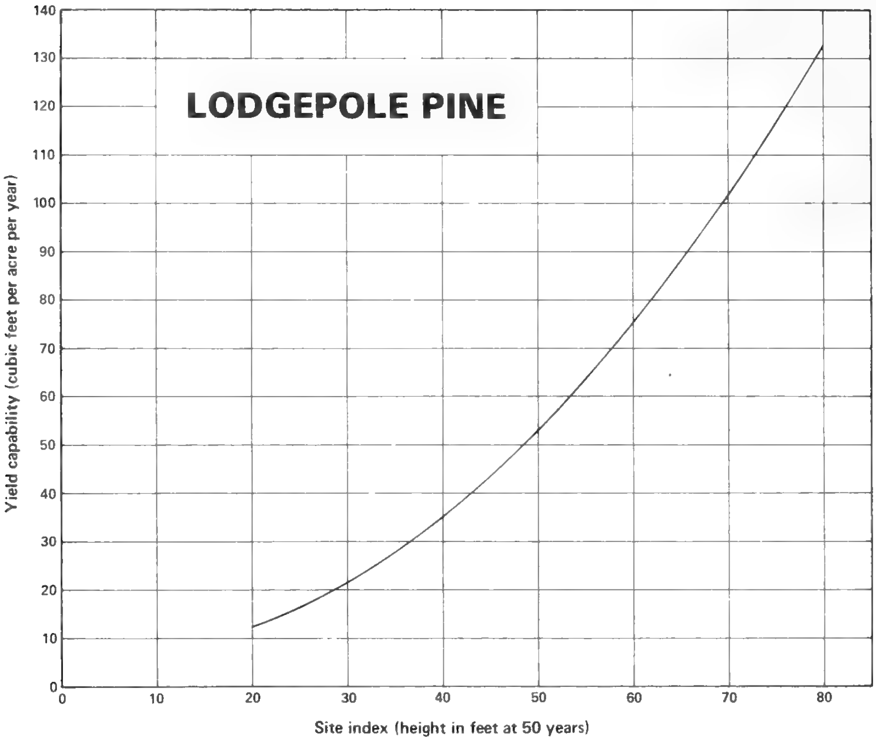

Figure 6.--Relationship between site index and yield capability.

Yield Capability

The equation for lodgepole pine was obtained by projecting yields of hypothetical stands according to the method published by Myers (9). The projection procedure differed from that of Myers in that we maintained a different stocking level for each level of site index. Myers used stocking levels of 80 for poorer sites and 100 for better sites. By 80 and 100 are meant stocking regimes that bring the stand to 80 or 100 square feet per acre of basal area at the time when average[4] stand diameter at breast height (d.b.h.) is 10 inches. Thereafter the stocking level is kept by thinning at 80 or 100 square feet of basal area per acre, even though average stand d.b.h. will increase. In making stand projections to derive a yield capability equation, the levels of stocking used for each site index were a linear function of site index, as shown below.

Site index at a 50-year base age	Stocking level
20	74.0
30	84.5
40	94.8
50	105.0
60	115.2
70	125.7
80	136.0

[4] The d.b.h. of the tree of average basal area.

Site index at a 50-year base age was used rather than at 100 years as in Myers' published example. Stand projection by Myers' method depends on the use of a predicting equation for future d.b.h., which has site index as one of the independent variables. Projections were made for hypothetical stands at higher site index levels than were found on any of the plots that served as the data base for Myers' equation. Nevertheless, the results obtained appeared entirely reasonable, so projections of stands at the higher site index levels were included in the estimates used to derive the yield capability equation.

For each site index, a curve of net yield before thinning was plotted over age. It was assumed that the volumes removed in thinnings would be lost to mortality in natural stands. The age of culmination of net cubic volume mean annual increment was estimated by drawing a line from the origin of coordinates tangent to the yield curve. The point of tangency indicated the age of m.a.i. culmination and the net stand yield at that age. Dividing the yield thus obtained by the culmination age gave the m.a.i. at the age of culmination, or the yield capability associated with that site index. Plotting yield capability over site index resulted in the curve shown in figure 6. This curve is described by the equation:

$$YC = b_0 + b_1 S + b_2 S^2,$$

where

YC = yield capability — $b_0 = 6.9091271$

S = site index at a 50-year base age — $b_1 = -0.16172109$

$b_2 = 0.021683019$

WESTERN LARCH

Site Index

The following equation for height of western larch in even-aged stands was developed by Arthur L. Roe:[5]

$$\log H = \log S - b_1 \, (1/A - 1/50),$$

where

$\log H$ = the common logarithm (to the base 10) of dominant stand height

$\log S$ = the common logarithm of site index at a 50-year base age

A = total stand age

b_1 = 21.036.

This equation can be easily solved for site index:

$$\log S = \log H + b_1 \, (1/A - 1/50)$$

$$S = 0.37956 \, H \cdot \exp \, (48.4372/A)$$

The last equation shown above can be used in computer programs to estimate site index of even-aged stands of western larch.

---

[5] Principal Silviculturist, Intermountain Forest and Range Experiment Station.

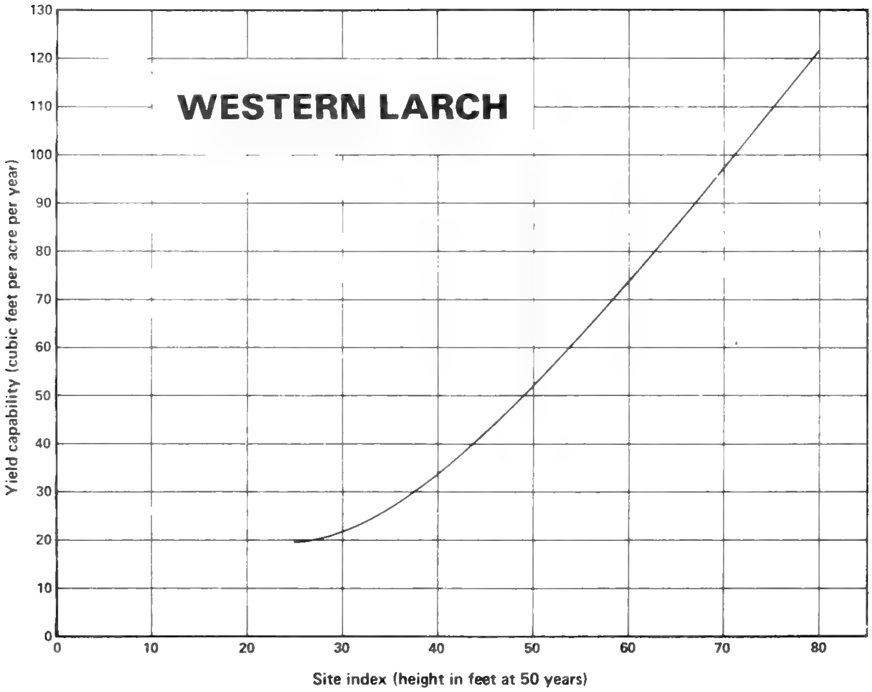

Figure 7.--Relationship between site index and yield capability.

Yield Capability

The yield capability equation for western larch is based on current work being done by Roe and John H. Wikstrom.[6] For cubic volume yield capability in stems 5.0 inches d.b.h. and larger, the equation is:

$$YC = b_0 + b_1 S + b_2/S,$$

where

YC = yield capability in cubic feet per acre per year of mean annual increment

S = site index (50-year base age)

b_0 = -126.05

b_1 = 2.7974081

b_2 = 1919.3157.

This equation cannot be used when site index is less than 26 feet, because the table from which it was made gives no information below site index 30.

[6] Principal Economist, Intermountain Forest and Range Experiment Station.

To derive this equation, the yield curves were plotted for each level of site index. Then a series of lines was drawn, passing through the origin of coordinates and tangent to each yield curve. From the point of tangency the following was read: (a) the age of culmination of mean annual increment, and (b) the yield at that age for each level of site index. The mean annual increment at its culmination age for each site index level was obtained by dividing (a) into (b). The relationship between site index and yield capability was approximated by the least squares fitting of a suitable curve form to the series of points plotted from site index and yield capability. This is shown in figure 7.

ENGELMANN SPRUCE

Site Index

An equation with which site index can be estimated directly from measurements of tree age and tree height is given by Brickell (2). This equation is:

$$S = H + \sum_{i=1}^{11} b_i X_i,$$

where

S = site index at a 50-year base age

H = total tree height

A = total tree age

and

$$\begin{aligned}
b_1 &= 0.10717283 \mathrm{X} 10^{2} & X_1 &= (\ln A - \ln 50) \\
b_2 &= 0.46314777 \mathrm{X} 10^{-2} & X_2 &= [(10^{10}/A^5) - 32] \\
b_3 &= 0.74471147 & X_3 &= H[(10^4/A^2) - 4] \\
b_4 &= -0.26413763 \mathrm{X} 10^{5} & X_4 &= H\,(A^{-2.5} - 50^{-2.5}) \\
b_5 &= -0.42819823 \mathrm{X} 10^{-1} & X_5 &= H\,(\ln A - \ln 50)^2 \\
b_6 &= -0.47812062 \mathrm{X} 10^{-2} & X_6 &= H^2[(10^4/A^2) - 4] \\
b_7 &= 0.49254336 \mathrm{X} 10^{-5} & X_7 &= H^2[(10^{10}/A^5) - 32] \\
b_8 &= 0.21975906 \mathrm{X} 10^{-6} & X_8 &= H^3[(10^{10}/A^5) - 32] \\
b_9 &= 5.1675949 & X_9 &= H^3[A^{-2.75} - 50^{-2.75}] \\
b_{10} &= -0.14349139 \mathrm{X} 10^{-7} & X_{10} &= H^4[\,(100/A) - 2] \\
b_{11} &= -9.481014 & X_{11} &= H^4[A^{-4.5} - 50^{-4.5}].
\end{aligned}$$

The standard error of estimate (S_{yx}) for this equation is 0.69 foot of site index units. An example of how this equation might be programed in FORTRAN IV (IBM 360) is shown in figure 8.

```
      SUBROUTINE ESSITE(A,H,SI)
      DIMENSION X(11),B(11)
      Q=A
      P=H
      DATA B/10.71728,0.4631478E-2,0.7447115,-26413.76,-0.428198E-1,-0.4
     1791206E-2,0.4925434E-5,0.2197591E-6,5.167595,-0.1434914E-7,-9.4810
     214/
      SI=0.0
      Q2=Q*Q
      Q3=Q2*Q
      Q4=Q2*Q2
      Q5=Q2*Q3
      RECQ5=1.0/Q5
      P2=P*P
      P3=P2*P
      P4=P2*P2
      X(1)=ALOG(Q)-3.912023
      X(2)=1.0E10*RECQ5-32.0
      X(3)=P*(1.0E4/Q2-4.0)
      X(4)=P*(SQRT(RECQ5)-5.656854E-5)
      X(5)=P*X(1)*X(1)
      X(6)=P*X(3)
      X(7)=P2*X(2)
      X(8)=P*X(7)
      X(9)=P3*(SQRT(RECQ5*SQRT(1.0/Q))-2.127318E-5)
      X(10)=P4*(100.0/Q-2.0)
      X(11)=P4*((1.0/Q4)/SQRT(Q)-2.262742E-8)
      DO 1 I=1,11
    1 SI=SI+B(I)*X(I)
      SI=SI+H
      RETURN
      END
```

Figure 8.--A site index estimating subroutine for Engelmann spruce.

If a shorter but less precise equation is desired, the following is recommended:

$$S = H + \sum_{i=1}^{5} k_i Z_i,$$

where

$k_1 = 0.32158242$	$Z_1 = H\ [(10^4/A^2) - 4]$
$k_2 = -0.98468901X10^4$	$Z_2 = H\ (A^{-2.5} - 50^{-2.5})$
$k_3 = -0.12253415X10^{-2}$	$Z_3 = H^2[(10^4/A^2) - 4]$
$k_4 = 1.0662061$	$Z_4 = H^3[A^{-2.75} - 50^{-2.75}]$
$k_5 = -0.80894818X10^{-8}$	$Z_5 = H^4[(100/A - 2]$.

For this equation, $S_{yx} = 1.22$ feet.

These equations are valid for trees between the ages of 20 and 200 years and for site indices ranging from 10 to 95. A site index estimate should be computed for each sample tree. The average of the individual tree estimates will be the average site index for the stand. If trees older than 200 years must be used as site trees, estimate the site index as if tree age were 200. Above that age, height increases very little.

INLAND DOUGLAS-FIR

Site Index

A site index equation, similar to the one given for Engelmann spruce, has been derived for inland Douglas-fir by Brickell (3). The equation is:

$$S = H + \sum_{i=1}^{18} b_i X_i,$$

where

S = site index at a 50-year base age

H = total height of a sample tree in the dominant stand

A = total age of a sample tree

and

$b_1 = 40.984664$	$X_1 = (A^{-1/2} - 50^{-1/2})$
$b_2 = 4521.1527$	$X_2 = (A^{-2.5} - 50^{-2.5})$
$b_3 = 123059.38$	$X_3 = (A^{-3.5} - 50^{-3.5})$
$b_4 = -0.5332868X10^{-8}$	$X_4 = (A^4 - 50^4)$
$b_5 = 0.37808033X10^{-10}$	$X_5 = (A^5 - 50^5)$
$b_6 = 216.64152$	$X_6 = H(A^{-1.5} - 50^{-1.5})$
$b_7 = -158121.49$	$X_7 = H(A^{-4} - 50^{-4})$
$b_8 = 1894030.8$	$X_8 = H(A^{-5} - 50^{-5})$
$b_9 = -0.10230592X10^{-9}$	$X_9 = H(A^4 - 50^4)$
$b_{10} = -6.0686119$	$X_{10} = H^2(A^{-2} - 50^{-2})$
$b_{11} = -25351.090$	$X_{11} = H^2(A^{-5} - 50^{-5})$
$b_{12} = 0.33512858X10^{-4}$	$X_{12} = H^2(A - 50)$
$b_{13} = 0.17024711X10^{-2}$	$X_{13} = H^3(A^{-1} - 50^{-1})$
$b_{14} = 398.36720$	$X_{14} = H^3(A^{-5} - 50^{-5})$
$b_{15} = -0.88665409X10^{-8}$	$X_{15} = H^3(A^{1.5} - 50^{1.5})$
$b_{16} = 0.40019102X10^{-14}$	$X_{16} = H^3(A^4 - 50^4)$
$b_{17} = -0.46929245X10^{-8}$	$X_{17} = H^5(A^{-1/2} - 50^{-1/2})$
$b_{18} = -0.16640659X10^{-20}$	$X_{18} = H^5(A^{4.5} - 50^{4.5})$.

The standard error of estimate for this equation is 0.24 of site index. A computer program in FORTRAN IV (360) to evaluate this equation is shown in figure 9.

```
      SUBROUTINE DFSITE(A,H,SI)
      DIMENSION B(18),X(18)
      Q=A
      P=H
      DATA B/40.98466,4521.153,123059.4,-5.332868E-9,3.780803E-11,216.64
     115,-158121.5,1894031.,-1.023059E-10,-6.068612,-25351.09,3.351286E-
     25,1.702471E-3,398.3672,-8.866541E-9,4.00191E-15,-4.692924E-9,-1.66
     34066E-21/
      SI=0.0
      P2=P*P
      P3=P2*P
      P4=P2*P2
      P5=P4*P
      Q12=SQRT(Q)
      Q15=Q*Q12
      Q2=Q*Q
      Q4=Q2*Q2
      X(1)=1.0/Q12-0.14142132
      X(2)=1.0/(Q2*Q12)-5.656854E-5
      X(3)=1.0/(Q2*Q*Q12)-1.131371E-6
      X(4)=(Q2-2500.0)*(Q2+2500.0)
      X(5)=Q*Q4-3.125E8
      X(6)=P*(1.0/Q15-2.828426E-3)
      X(7)=P*(1.0/Q4-1.6E-7)
      X(8)=P*(1.0/(Q4*Q)-3.2E-9)
      X(9)=P*X(4)
      X(10)=P2*(1.0/Q2-0.0004)
      X(11)=P*X(8)
      X(12)=P2*(Q-50.0)
      X(13)=P3*(1.0/Q-0.02)
      X(14)=P*X(11)
      X(15)=P3*(Q15-353.5534)
      X(16)=P2*X(9)
      X(17)=P5*X(1)
      X(18)=P5*(Q4*Q12-0.4419416E8)
      DO 1 I=1,18
    1 SI=SI+B(I)*X(I)
      SI=SI+H
      RETURN
      END
```

Figure 9.--A site index estimating subroutine for inland Douglas-fir.

Although it provides less precise results, a shorter equation is:

$$S = H + \sum_{i=1}^{5} k_i Z_i,$$

where

$k_1 = 101.08708$ $\quad$ $Z_1 = (A^{-1/2} - 50^{-1/2})$

$k_2 = 122.04763$ $\quad$ $Z_2 = H\ (A^{-1.5} - 50^{-1.5})$

$k_3 = \quad 0.14082397 \mathrm{X} 10^{-12}$ $\qquad Z_3 = H\ (A^5 - 50^5)$

$k_4 = -13{,}140.717$ $\qquad Z_4 = H^2(A^{-5} - 50^{-5})$

$k_5 = \quad 0.13492191 \mathrm{X} 10^{-4}$ $\qquad Z_5 = H^5(A^{-3} - 50^{-3})$.

For this equation the standard error of estimate is 0.90 foot.

These equations are valid for ages from 20 to 200 years and for site indices from 10 to 110. The site index of trees older than 200 years should be estimated as if age were 200.

GRAND FIR[7]

Site Index

Information on productivity of grand fir has been developed using a direct index of relative productivity (Q) rather than site index. Procedures for calculating Q from stand age and height have been published (10). Figure 10 shows a FORTRAN program RPGF.

```
      SUBROUTINE RPGF( Q,H,T,CR,IER )
      DOUBLE PRECISION A,B,C,D,X,BT,DET,TOL,VAL,BRAC,ALGCR
      DIMENSION C(5)
      DATA C /-10.29862, 2.0, 0.018167, -2.7 , 2.0 /
      DIV = 0.5
      IER = 0
      IGT = 1
      COR = 0.
      I = 0
      BT =-C(3)*(T + C(4))
      IF ( T - 90. ) 91,91,92
   91 TOL = 0.0D0
      GO TO 95
   92 TOL = ALOG( 0.8*H/T )
   95 ALGCR = ALOG(H-4.5) - (ALOG(CR)-3.747474-8.218305/T)*18.786/T
      TS = TOL
   99 IF ( TOL .GT. 20.) GO TO 101
      DET = BT*DEXP( TOL )
      IF ( DET ) 991,100,100
  991 BRAC = 1.0D0 - DEXP(DET)
      IF (BRAC) 100,100,992
  992 TOL = C(1) + C(2)*(ALGCR - C(5)*DLOG(BRAC) )
      I = I + 1
  996 GO TO ( 21,22,23),IGT
   21 X = TOL
      A = X - TS
```

(con. next page)

[7]The section on grand fir was contributed by Albert R. Stage, Principal Mensurationist, Intermountain Forest and Range Experiment Station.

```
      B = -A
      IGT = 2
      GO TO 99
   22 VAL = X - TOL
C        START ITERATION LOOP
      IF ( VAL) 1,7,1
C        EQUATION IS NOT SATISFIED BY X
    1 B = B/VAL - 1.0D0
      IF(B) 2,8,2
C        ITERATION IS POSSIBLE
    2 A = A/B
      X = X + A
      B = VAL
      TOL = X
      IGT = 3
      GO TO 99
   23 VAL = X - TOL
C        TEST ON SATISFACTORY ACCURACY
      D = DABS(X)
      TOL = 0.0008*D
    4 IF ( DABS(A)-TOL) 5,5,6
    5 IF(DABS(VAL)-1.0D0*TOL) 7,7,6
    6 IF ( I - 20 ) 1,1,10
C        END OF ITERATION LOOP
    7 Q = DEXP(X)
      RETURN
C        ERROR RETURN IN CASE OF ZERO DIVISOR
    8 IER = -2
      RETURN
C        NO CONVERGENCE AFTER 20 ITERATION LOOPS. ERROR RETURN
   10 IER = I
      RETURN
C        ERROR CONDITION ON INITIAL ITERATION
   12 IER = -3
      RETURN
   14 IER = -4
      RETURN
  100 GO TO (12,12,102),IGT
  101 GO TO ( 14,14,102),IGT
  102 X = X - A
      A = DIV*A/B
      DIV = DIV*DIV
      I = I + 1
      GO TO 2
      END
```

Figure 10.--FORTRAN subroutine to calculate relative productivity.

The variables in the calling sequence are:

Q = the measure of relative productivity

H = height in feet

T = age in rings at 4.5 feet above ground

CR = ratio of live crown to total height in percent

IER = an integer used to flag possible errors in calculation. (Should be zero if no errors.)

If required, site index can be approximated from Q by the FORTRAN equation:

SI = 4.5 + SQRT(Q) * 172.32 * (1. - EXP(-.6905*Q))**2.

Yield Capability

The yield equation also already has been published (11). The maximum m.a.i. for average stocking corresponding to this equation can be solved by the FORTRAN function CUPGF illustrated in figure 11. The single argument to the function is Q obtained from the previous subroutine RPGF in figure 10. Figure 12 shows the linear approximation of the relationship between yield capability and site index.

```
FUNCTION CUPGF(S)
B = 89.761/S - 22.701
A = B *(0.5 + SQRT(0.25 + 12./(S*B)))
CUPGF = 22228.*EXP(-B/A)/(A + 12./S)
RETURN
END
```

Figure 11.--The function subprogram for grand fir yield capability.

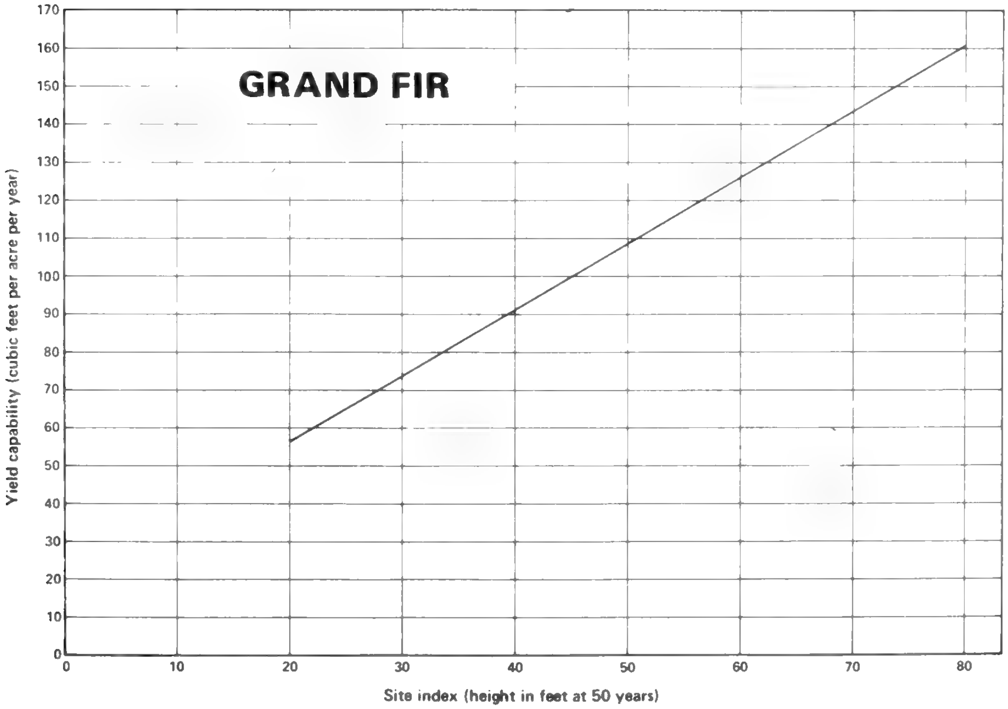

Figure 12.--Relationship between site index and yield capability.

QUAKING ASPEN

Site Index

A table published by Jones (5) gives site index of quaking aspen according to stand age and dominant height. Age at breast height is used, and it is important that average dominant height not be estimated from trees that are all of the same clone. In Jones' table a site index base age of 80 years is used. Equations were fitted by regression methods to the site index values in Jones' table, using stand age and height as independent variables. The best of several equations fitted is:

$$S = H + \sum_{i=1}^{10} b_i X_i,$$

where

S = site index at an 80-year base age

A = stand age at breast height

H = average height of the dominant stand

$b_1 = -8.7810841$	$X_1 = [(100/A) - 1.25]$
$b_2 = -5.1824560$	$X_2 = (\ln A - \ln 80)$
$b_3 = -40.260849$	$X_3 = H[((\ln A)/A) - ((\ln 80)/80)]$
$b_4 = 1.8589039$	$X_4 = H[(100/A) - 1.25]$
$b_5 = 0.34567436X10^{-11}$	$X_5 = H[A^5 - 3{,}276{,}800{,}000]$
$b_6 = -0.16454828X10^{-6}$	$X_6 = H[A^3 - 512{,}000]$
$b_7 = -11{,}647{,}641.0$	$X_7 = H[\exp(-A) - \exp(-80)]$
$b_8 = 0.19235397X10^{-3}$	$X_8 = H^2[\sqrt{A} - \sqrt{80}]$
$b_9 = 0.30310790X10^{-14}$	$X_9 = H^2[A^5 - 3{,}276{,}800{,}000]$
$b_{10} = -0.84256893X10^{-3}$	$X_{10} = H^2[(10{,}000/A^2) - 1.5625]$.

The standard error of estimate for this equation is 0.55 foot. The equation is valid for breast high ages between 20 and 160 years, and site indices (base age 80) between 20 and 90. Figure 13 shows how this equation might be programed in FORTRAN IV (360). This equation will not be valid for ages beyond 160 years. Older trees should be treated as if age were 160 years.

```
      SUBROUTINE ASPSIT(A,H,S)
      DIMENSION X(10),B(10)
      DATA B/-8.781084,-5.182456,-40.26085,1.858904,0.3456744E-11,-0.164
     154828E-6,-1.164764E1,1.92354E-4,3.031079E-15,-0.8425689E-3/
      S=0.0
      Q=A
      P=H
      P2=P*P
      RECQ=100.0/Q
      Q2=Q*Q
      Q3=Q*Q2
      QLN=ALOG(Q)
      X(1)=RECQ-1.25
      X(2)=QLN-4.382027
      X(3)=P*(QLN/Q-0.5477533E-1)
      X(4)=P*X(1)
      X(5)=P*(Q2*Q3-32768.0E5)
      X(6)=P*(Q3-512.0E3)
      X(7)=P*(((1.0E3/EXP(Q/2.0))**2)-1.804851E-29)
      X(8)=P2*(SQRT(Q)-8.944272)
      X(9)=P*X(5)
      X(10)=P2*(1.0E4/Q2-1.5625)
      DO 1 I=1,10
    1 S=S+B(I)*X(I)
      S=S+P
      RETURN
      END
```

Figure 13.--A site index estimation program for quaking aspen.

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