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HEAT AND THERMODYNAMICS

HEAT

AND

THERMODYNAMICS

BY

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ERRATA

Page 45, after the equation,

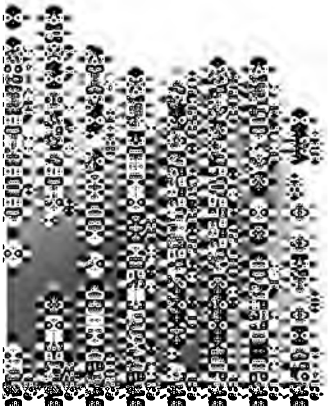
$$p_\tau v_0(1+\beta\tau)=p_0 v_0(1+\alpha\tau),$$

delete remainder of page and read: The foregoing equation follows immediately from *Boyle's Law*, and by solving for τ, we find

$$\tau=\frac{p_\tau-p_0}{p_0\alpha-p_\tau\beta}.$$

Page 239, multiply right hand member of equation (*b*) by J.

Page 283, first line after equation (28), in place of, "equation (27)," read "equation (28)."



PREFACE

WITH so many good works in existence, both on Heat and Thermodynamics, it may perhaps appear presumptuous to publish the following text. The author, however, has long felt the need of a text, in teaching the subject of thermodynamics, which properly covers, without introducing too much material, the fundamental principles of heat measurements. To expect an average student to cull from his text book on physics, or some treatise on heat, no matter how well the subject may have been taught, an introduction to thermodynamics is, in general, expecting somewhat more of him than he can accomplish. But it has been found, by experience, that a short course on the fundamental principles of heat, given as an introduction to the subject of thermodynamics, greatly reduces the difficulties, experienced by most students, in pursuing this subject.

Since it is almost impossible for a student to understand a complex piece of apparatus, unless he can actually examine it, long and tedious descriptions have been purposely avoided. Likewise, for the reason that photographs are seldom, if ever, of any value, all pictorial illustrations are diagrammatic.

It is, of course, impossible to teach the subject of thermodynamics without the application of differential and integral calculus; but the aim has been throughout to keep within the bounds of elementary mathematics. However, a fair knowledge of the calculus, on the part of the reader, has been assumed.

Very few teachers, if any, can present an unbiassed view of a speculative theory; furthermore, before a student has thoroughly mastered the groundwork of any subject, he is not in a position to properly discriminate between the various arguments that may be advanced, either for or against a speculative theory.

It must also be remembered that the average student looks upon his instructor as an infallible authority; and that he accepts a theory on the mere say so of his instructor, no matter how flimsy the arguments upon which it may be based. How frequently one meets those who are in a condition so deplorable that they can talk very glibly about electrons, ionization, etc., and are driven helplessly into a corner by one or two well directed questions. Whether there is or is not such a thing as an atom has nothing to do with the law of definite proportion. Facts will always remain and theories change to fit them. It is for these various reasons that speculative discussions, such as that of the kinetic theory of gases, have been avoided, and that very hypothetical medium—the ether—has found no place in this text. It cannot be too strongly emphasized that before we teach metaphysics to a student we must first give him a thorough training in mathematics and physics.

It is the author's opinion that the best that can be done in any technical course is to thoroughly teach the fundamental principles underlying the subject, and that it is impossible to give a training which makes the student a practical engineer. This part must be learned in practice, and the engineer must keep up to date and in proper touch with his profession by reading the current engineering literature, and by studying individual problems as they arise.

It must not be understood that the author expects this text to supersede such admirable works as Peabody's treatise on "The Thermodynamics of the Steam Engine," Zeuner's "Technische Thermodynamik," etc.; but rather as a proper preparation for the reading of such works. Finally, the author cannot express his feelings too strongly in regard to the pleasure he experienced, as a student, while reading Tyndall's "Heat a Mode of Motion," and Ewing's "The Steam Engine and other Heat Engines."

Thanks are hereby expressed to Mr. Albert Goertz for the care with which he read the manuscript.

F. M. H.

Cooper Union, July, 1911.

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HEAT

CHAPTER I

TEMPERATURE AND THERMAL UNITS

1. The fundamental conception of hotness or coldness is one of bodily sensation. That is, an object is said to be hot or cold depending upon whether it gives us one sensation or another when we are near it or in contact with it; and the more intense the sensation, the hotter or colder the object is said to be. Experience, however, teaches us that the estimates so formed are not accurate; since the intensity of the sensation experienced, in any given case, depends not only upon the condition of the body under consideration, but very largely upon our experience immediately preceding.

Observation shows that, in general, as bodies are heated or cooled they change in volume; and in most cases, other things being equal, bodies increase in volume when heated and decrease in volume when cooled. Observation further shows that there is a continual exchange of heat among bodies; i.e., if in any system of bodies some are gaining heat, others are losing heat. Or, in other words, there is a continual tendency toward equilibrium.

2. Temperature. Assume now, that we are dealing with two bodies, *A* and *B*, and that it is desired to determine which is the hotter. To do this, some test specimen, upon which previous observation has shown a continuous expansion with continued application of heat, may be used in the following manner: The test specimen is put into contact with *A*, and after a suitable interval of time its length, say, is accurately measured; it is then put into contact with *B*, and its length is again measured. If the length is now greater than it was before, i.e., if the test specimen expanded when put into contact with *B*, after having been in contact with *A*, *B* is hotter than *A*; for, by previous observation it was found that the test specimen expanded continually as it became hotter. The body *B*, therefore, was capable of imparting more heat to the test specimen than the body *A* could impart to it, and *B* is said to be at a higher *temperature* than *A*. The *difference of temperature*, then, between two bodies may be measured by the amount of change in dimensions which a test specimen undergoes when, after having been in contact with one of the bodies, it is brought into contact with the other body. Such a test specimen is called a *thermometer;* and, for accurate measurements, the nature of the thermometer must be such that no appreciable change is brought about in the body whose temperature is sought.

If the two bodies, *A* and *B*, of the previous discussion, are now brought into contact, and after a suitable interval of time the test specimen is put into contact with *A* and then with *B*, there will be no change in its dimensions; i.e., the two bodies are in *thermal equilibrium*, or, in other words, at the same temperature. But, the body *B* will have lost heat and the body *A* will have gained heat; hence, *when a body has the capability of imparting heat to another body, it is said to be at a higher temperature.*

Solids, liquids, or gases may be employed in the construction of thermometers; but it is important that the substance used does not change its state during the change of temperature. For, the rate at which a body changes in volume, with respect to change in temperature, depends upon its physical state; i.e., though a substance may exist in the three different states, its rate of expansion will, in general, be entirely different in the various states; being usually the greatest for gases and the least for solids. The rate of expansion, in general, changes abruptly in passing from the solid to the liquid, and from the liquid to the gaseous state.

The three substances most generally employed in the construction of thermometers are: *Mercury*, *alcohol*, and *dry air*; the most convenient and most commonly used being mercury.

There are two standard temperatures, *arbitrarily chosen*, upon which all thermometric scales are based; the one is that of melting ice, and the other that of the vapor of boiling water under a pressure of one standard atmosphere. The pressure of one atmosphere being taken equal to that of a column of mercury, at the temperature of melting ice, whose height is 76 cm., at 45° latitude and sea level, where the acceleration of gravity is 980.60 cm. per sec. per sec.; or, in c.g.s. units, a pressure of 1.01325×10^6 dynes per square centimeter.

THERMOMETRIC SCALES AND THERMOMETERS

3. There are *three* thermometric scales in use: The *centigrade scale*, the zero of which is the melting-point of ice, and the point corresponding to the temperature of the vapor from boiling water under standard conditions, called the boiling-point, is marked 100. Hence there are 100 units, called degrees, for the interval between the melting-point and the boiling-point. The *Fahrenheit scale* is marked 32 for the melting-point and 212 for the boiling-point; hence, there is an interval of 180 degrees between the two fixed points. The *Réaumur scale* is marked 0 for the melting-point and 80 for the boiling-point; hence, there is an interval of 80 degrees between the two fixed points. Fig. 1 is a diagrammatic representation of the relation of the three scales. From the foregoing it is obvious that the value of a degree on the Fahrenheit scale is 5/9 of a degree on the centigrade scale, and the value of a degree on the Réaumur scale is 5/4 of a degree on the centigrade scale. Since both the centigrade and Fahrenheit scales are in common use, it is convenient to have a simple method of conversion from one scale to the other.

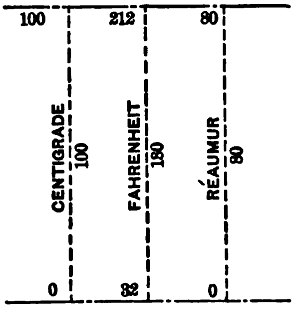

Fig. 1.

Let it be desired to convert the temperature θ on the centigrade scale to the Fahrenheit scale. Since one degree centigrade equals 9/5 degree Fahrenheit, it follows that an interval of θ°C. is equal to an interval of $\frac{9}{5}\theta$°F.; but, since the melting-point on the Fahrenheit scale is marked 32, we must add 32 to give the Fahrenheit reading. Therefore, to convert a temperature on the centigrade scale to the Fahrenheit scale, we must multiply the reading by 9/5 and add 32. And similarly, to convert Fahrenheit to centigrade we must subtract 32 from the reading and multiply by 5/9.

A method of conversion, which is simpler, is to solve for that temperature for which both scales read the same. This temperature is obviously below zero, and is therefore negative. Let θ be that temperature on the centigrade scale; then, $\frac{9}{5}\theta+32$ is the reading on the Fahrenheit scale. But these two readings are by condition equal. Hence,

$$\theta=\frac{9}{5}\theta+32;$$

from which

$$\theta=-40.$$

Therefore, to convert from one scale to the other, the most convenient way is to add 40 to the reading, multiply this by the conversion factor and deduct 40. To convert from centigrade to Fahrenheit, add 40 to the reading, multiply by 9/5 and deduct 40. To convert from Fahrenheit to centigrade, add 40, multiply by 5/9 and deduct 40.

4. Mercurial Thermometer. The mercurial thermometer consists of a capillary tube of, as nearly as obtainable, uniform bore, on one end of which is blown a bulb. The bulb is filled with mercury and heated so as to drive out all the air. When this has been satisfactorily performed, so that nothing but mercury remains in the bulb and tube, the tube is hermetically sealed. The bulb and tube are then immersed in ice from which the water is allowed to drain away, and the point to which the mercury falls marked on the stem. Next, the thermometer is immersed in saturated steam, *under standard pressure*, and the point to which the mercury rises marked on the stem. It is, however, necessary to allow considerable time to elapse between the sealing of the tube and the determination of the fixed points; since glass, after having been heated, does not, upon being cooled, immediately return to its original volume. If the bulb contracts after the fixed points have been placed on the stem, the thermometer will read too high. Joule found that the bulb of a certain thermometer, upon which he had taken observations for twenty years, was still changing slightly at the end of that time.

After the fixed points are determined, a thread of mercury is detached, and, by means of it, the tube calibrated to the desired scale. In this way the units on the scale represent equal volumes, and not necessarily equal lengths. But in good thermometers the tube is of so nearly uniform bore that the lengths of a degree do not differ by any considerable amount over different parts of the scale.

Since glass changes in volume when its temperature changes, it is obvious that the indications of the mercurial thermometer are proportional to the relative changes between the mercury and glass, and do not necessarily indicate the true changes in temperature; i.e., the indicated temperatures between the two fixed points depend upon the substances used in construction. It is readily seen that two mercurial thermometers, if constructed of different qualities of glass, which have not the same rates of expansion with respect to mercury throughout the entire scale, will differ slightly in their readings for some parts of the scale, even though they read alike for the fixed points.

5. Alcohol Thermometer. The alcohol thermometer is constructed in a manner similar to the mercurial thermometer. Its chief advantage lies in the fact that it may be used for temperatures below the melting-point of mercury, which is −39°C. However, on account of its low boiling-point, which is 78.2°C., alcohol cannot be employed for high temperatures; on the other hand, the boiling-point of mercury is 357°C.

The discussion of the air thermometer will be deferred until after the discussion of the laws of gases.

6. Thermo Couple. When the junction of two dissimilar metals is heated, an *e.m.f.* is developed; and since this *e.m.f.* is a function of the temperature, such a combination, called a thermoelectric couple, or simply *thermo couple,* may be employed to indicate temperatures. The thermo couple is, in many cases, where a bulb thermometer cannot be employed, a very convenient device for measuring changes of temperature; and it is particularly valuable in enabling us to estimate changes of temperature above the boiling-point of mercury. By employing proper metals, such as platinum and iridium, very large ranges of temperature can be measured; the melting-point of iridium being about 2500°C. and that of platinum about 1775°C.

7. Resistance Thermometer. The fact that the ohmic resistance of a metal is a function of its temperature makes it possible to estimate changes in temperatures, by noting the changes in resistance of a particular conductor employed for the purpose.

From the foregoing, it is obvious that if any body or combination of bodies manifest some change, which is a function of the temperature and readily measurable, such body or combination of bodies may be employed to indicate temperatures. But it is to be carefully noted that the device employed must be such that the temperature of the body, upon which the measurements are made, is not appreciably altered by the test body; and that, in any case, the changes produced in the test body are peculiar to it, and not necessarily proportional to the changes that would be produced in some other instrument of a different type. Therefore, temperatures must always be referred to some scale chosen as a standard. This will be dealt with more fully in the discussion of *thermodynamics*.

HEAT AS A MEASURABLE QUANTITY

8. If a quantity of water at a temperature τ_1, be mixed with an equal quantity of water at a temperature τ_2, the resulting temperature of the mixture is very nearly the mean between the two initial temperatures. If it requires a certain quantity of heat, q, to raise n grams of water through a given temperature interval, then it obviously requires a quantity of heat, mq, to raise mn grams of water through the same temperature interval. If the water be cooled through the same temperature interval, then there is imparted to the surrounding bodies a quantity of heat which is equal to that absorbed by the water while being raised through that temperature interval.

If the quantities of heat, required to raise a given mass of water through equal temperature intervals throughout the chosen thermometric scale, were all equal, then the resulting temperature obtained when mixing equal masses of water would be the exact mean between the two initial temperatures. This is shown by experiment to be very nearly, but not quite true. Hence, there is not strict proportionality, in the case of water, between change of temperature, according to our thermometric scale, and change of heat.

If equal masses of water and some other substance, say, mercury, be mixed, the resulting temperature will differ considerably from the mean between the two initial temperatures. Experiment shows that if mercury at a temperature τ_1, be mixed with water at a temperature τ_2, the mass of the mercury must be 29.85 times the mass of the water so as to give a resulting temperature equal to $\frac{\tau_1+\tau_2}{2}$.

9. Thermal Capacity. *The thermal capacity of a body is numerically equal to the ratio of change in heat to the corresponding change in temperature produced by it.* The preceding paragraph states that water has, mass for mass, a greater *thermal capacity* than mercury in the ratio of 29.85 : 1; or the thermal capacities of equal masses of mercury and water are to each other as 0.0335 : 1. If copper be compared with water the ratio is found to be as 0.0933 : 1. In general, the ratio is less than unity, has different values for different substances, and varies somewhat with change of temperature. One notable exception is hydrogen gas, where the ratio is found to be, at constant pressure, as 3.409 : 1; and at constant volume, as 2.42 : 1.

To compare different quantities of heat, it is necessary to choose some substance as a standard. On account of convenience, water has been so chosen.

10. The Calorie. The quantity of heat required to raise the temperature of 1 kilogram of water through 1 degree centigrade, is called a *calorie*, and is the unit adopted for heat measurements. But, since this quantity varies slightly for different temperatures it becomes necessary, in making exact measurements, to specify some particular quantity. There are three different definitions for the calorie:

(1) The quantity of heat required to raise the temperature of 1 kilogram of water from 0°C. to 1°C., called the *zero calorie.*

(2) One hundredth part of the heat required to raise the tem-

perature of 1 kilogram of water from 0°C. to 100°C., called the *mean* calorie.

(3) The quantity of heat required to raise 1 kilogram of water from 15°C. to 16°C., called the *common calorie.*

Since it is impossible to realize accurately the first or second of these, on account of the difficulty experienced in working with water at 0°C., the last, or common calorie, is the one most generally used. Furthermore, due to the fact that a great many heat measurements are made in the range between 15°C. and 25° C., no large corrections for change in thermal capacity, due to change in temperature, when the common calorie is employed, are necessitated; hence, this unit is more convenient than the others.

Since, in ordinary heat measurements, masses are generally specified in grams, a secondary unit, called *gram calorie,* having the gram instead of the kilogram for the unit mass, is usually found more convenient than the calorie. In what follows, unless otherwise specified, by calorie is to be understood the quantity of heat required to raise the temperature of 1 kilogram of water from 15°C. to 16°C., and by gram calorie, one thousandth part of the calorie.

11. British Thermal Unit. The thermal unit most commonly employed in engineering practice, in England and America, is the *British Thermal Unit* or B.T.U.; it is the quantity of heat required, at ordinary temperatures, to raise the temperature of 1 pound of water through 1 degree Fahrenheit.

12. Thermal Capacity per Unit Mass and Specific Heat. The ratio of the quantity of heat required to raise the temperature of a given mass of a substance through a given temperature interval, to the quantity of heat required to raise an equal mass of water through the same temperature interval, is called the *specific heat* of the substance. And, since the unit of heat—the gram calorie—is the quantity of heat required to raise the temperature of 1 gram of water through 1 degree centigrade, it follows that the quantity of heat, measured in gram calories, required to raise the temperature of 1 gram of a substance 1 degree centigrade, is numerically equal to the specific heat of the substance; or, in other words, the specific heat of a substance is numerically equal to its *thermal capacity per unit mass.*

13. Water Equivalent. By the water equivalent of a body is understood the mass of water which has a thermal capacity equal to that of the given body, and is numerically equal to the mass of the body multiplied by its thermal capacity per unit mass.

14. Method of Mixtures. Assume a mass of water m_1, at a temperature τ_1, to be mixed with a mass m_2, of some other substance, at a higher temperature τ_2, yielding for the mixture a resulting temperature of θ. Then, if no heat is lost to or gained from the surroundings, during the operation, and the thermal capacities of the water and substance are sensibly constant for the temperature ranges experienced, it follows that, since the heat gained by the water is equal to that lost by the substance, we must have

$$m_1(\theta-\tau_1)=m_2c(\tau_2-\theta); \quad . \; . \; . \; . \; . \quad (1)$$

where c is the thermal capacity per unit mass of the substance, amd m_2c is its water equivalent.

To take a numerical example, assume 268 grams of water at a temperature 10°C. to be mixed with 1000 grams of mercury at a temperature 100°C., giving a temperature of 20°C. for the mixture. Substituting in equation (1), we have

$$268(20-10)=1000c(100-20);$$

from which, the thermal capacity of mercury, per unit mass, is

$$c=\frac{268(20-10)}{1000(100-20)}=0.0335.$$

CHAPTER II

CALORIMETRY

15. As thermometry has for its object measurement of temperatures, so has *calorimetry* for its object the measurement of quantities of heat.

In equation (1), *Art.* 14, it was shown what must be the relation between the masses involved, the changes in temperature, and the thermal capacity per unit mass of a substance, when two substances are mixed and assume a common temperature.

The actual determination of thermal capacities is, however, not so simple. Since, in general, the vessel in which the mixing takes place suffers a change in temperature, its thermal capacity must be taken into consideration. Furthermore, there is usually an exchange of heat between the vessel, in which the experiment is performed, and the surrounding medium during the progress of the experiment. The vessel, specially designed, in which the mixing takes place, is called a *calorimeter.* It therefore follows, from what has just been said, that in making heat measurements it is necessary to know not only the thermal capacity of the calorimeter, but also, the rate at which, for a given difference of temperature, exchange of heat takes place between the calorimeter and the surrounding medium.

16. Law of Cooling. The rate at which a body loses heat to surrounding bodies *is independent of its thermal capacity,* and depends upon the nature and area of its exposed surface, and the difference in temperature between it and the surroundings. For small differences of temperature, i.e., up to a difference of about

15°C., from ordinary room temperature, the rate of cooling is very nearly proportional to the difference of temperature. This is known as *Newton's Law of Cooling.* The foregoing then states that the rate at which a body loses heat at any instant is a function of its surface, and of the difference of temperature between it and surrounding bodies. Newton's law may be stated as follows:

$$\frac{dQ}{dt} = -K\tau; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where Q is quantity of heat, t time, K some constant, depending upon the surface of the body, and τ the difference of temperature. If m is the mass of the body, and c the thermal capacity per unit mass, then equation (1) may be written

$$mc\frac{d\tau}{dt} = -K\tau. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If c is constant, then equation (2) becomes

$$\frac{d\tau}{dt} = -k\tau;$$

where $k = \frac{K}{mc}$. Separating the variables, we find

$$\frac{d\tau}{\tau} = -kdt,$$

from which

$$\log\frac{\tau}{k_1} = -kt;$$

and

$$\tau = k_1\varepsilon^{-kt}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

To determine the constant of integration k_1, assume that we begin to reckon time when $\tau = \tau_1$; i.e., $\tau = \tau_1$ when $t = 0$. Making this substitution, in equation (3), we find $k_1 = \tau_1$; hence, finally

$$\tau = \tau_1\varepsilon^{-kt}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

If m, c, and K are known, thus fixing the value of k, then, by means of equation (4), the difference of temperature, τ, between the body under consideration and the surroundings at any time, t, can be predicted, provided the initial difference of temperature be known, and the surroundings remain constant in temperature.

It is found, by experiment, that bodies having highly polished surfaces, other things being equal, lose heat less rapidly than do those having rough dark surfaces; hence, calorimeters should have their exposed surfaces highly polished. Furthermore, the thermal capacities of calorimeters should be small in comparison with those of the bodies contained in them, and upon which measurements are being made. Also, while the experiment is under progress, the calorimeter should be protected from draughts of air. We are not as yet in a position, nor is it essential, to enumerate all precautions that must be taken to give results of absolute precision.

17. Thermal Capacity of a Calorimeter. Since the calorimeter in which the bodies, upon which measurements are to be made, are contained, always suffers a change in temperature, it is necessary to know its thermal capacity. But, since the thermal capacity of the calorimeter, in general, is small in comparison with that of the bodies upon which measurements are being made, it follows that a small error in the determination of the thermal capacity of the calorimeter will not seriously affect the results obtained for these bodies.

The thermal capacity of a calorimeter may be obtained, though not with absolute precision, in the following manner: The calorimeter containing water has its temperature noted, the water being first stirred to insure uniform temperature, then immediately a quantity of water at some other temperature is poured into the calorimeter, the contents stirred and the resulting temperature is noted. If C is the thermal capacity of the calorimeter and stirrer, m_1 the mass of water originally in it, τ_1 the temperature of calorimeter and contents before mixing, and θ

the common temperature after mixing, then on the assumption that τ_1 is higher than θ, the loss in heat, suffered by the calorimeter and water originally in it, is

$$(m_1+C)(\tau_1-\theta).$$

The gain in heat, by the water poured into the calorimeter, is

$$m_2(\theta-\tau_2);$$

where m_2 is the mass, and τ_2 the temperature of the water poured into the calorimeter. But, if there are no other heat exchanges, the loss on the one side must be equal to the gain on the other, hence

$$(m_1+C)(\tau_1-\theta)=m_2(\theta-\tau_2);$$

from which

$$C=\frac{m_2(\theta-\tau_2)}{\tau_1-\theta}-m_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

There will always be an exchange of heat between the calorimeter and surroundings; but this can be reduced to a small quantity by choosing the masses of water such that the resulting temperature of the calorimeter is as much below the room temperature as was its initial temperature above the room temperature. When it is possible, large differences of temperature, between the calorimeter and room, should be avoided.

18. Cooling Constant of a Calorimeter. When it is impossible to have the initial and final temperatures differ by equal amounts from the room temperature—one, of course, being above and the other below—then, to obtain accurate results, correction must be made, as the case may be, for loss or gain in heat. To do this, the calorimeter is filled with water, at about 15°C. or 16°C. above room temperature, to the same height as it will be when the experiment proper is performed. The temperature is then noted at short intervals of time, the water being continuously stirred to insure a uniform temperature throughout at any instant. If the room temperature has remained constant during the progress of the experiment, then, obviously, equation (4), *Art.* 16, applies and the constant k of this equation is determined. In general, however, better results are obtained by plotting the observations; using times as abscissas and differences in temperatures, between the calorimeter and the room, as ordinates, and passing a smooth curve through the points so found. The slope of the tangent then, to the curve at any point, is the rate of change of temperature at that point; and this slope, divided by the difference of temperature, or in other words, by the ordinate of the point, is, according to Newton's law of cooling, a constant for any point on the curve. Drawing a number of tangents and dividing the slope of each by its ordinate, will give quotients nearly equal; and the mean of these quotients will be a fair value for the rate of change of temperature for unit difference of temperature. The rate so obtained multiplied by the thermal capacity of the calorimeter and contents is numerically equal to the quantity of heat lost, by the calorimeter, per unit time per unit difference of temperature.

19. Determination of Thermal Capacities. Thermal capacities may be determined in various ways; the simplest, though not necessarily the most accurate, and not applicable in all cases, is the method of mixtures. The substance, whose thermal capacity is sought, is heated to a temperature τ_1, which is noted. It is then immediately transferred to a calorimeter of thermal capacity C, containing a mass of water m_2, at a temperature τ_2. If the resulting temperature is θ, and no heat has been lost to or gained from the surroundings during the operation, then the heat lost by the one side must be equal to the heat gained by the other; hence we have

$$m_1c(\tau_1-\theta)=(m_2+C)(\theta-\tau_2); \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where m_1 is the mass of the substance, and c its thermal capacity per unit mass. From equation (6), we find

$$c=\frac{(m_2+C)(\theta-\tau_2)}{m_1(\tau_1-\theta)}; \quad \dots\dots \quad (7)$$

which determines the thermal capacity per unit mass of the substance. If the resulting temperature differs materially from the room temperature, then corrections will have to be made, from the curve of cooling of the calorimeter.

20. Method of Cooling. If heat is generated or absorbed when two substances are mixed, the thermal capacity of a substance cannot be determined by the method of mixtures. As an example, if sulphuric acid is mixed with water, considerable heat is evolved; hence, recourse must be had to some method, other than the method of mixtures, in determining the thermal capacity of sulphuric acid. This may conveniently be done by what is known as *the method of cooling.*

Assume that we have a calorimeter of known thermal capacity, C. Let the calorimeter be filled to a definite height with water, and the time noted which is required for the calorimeter and contents to cool, from a temperature τ_1 to a temperature τ_2, when exposed to a definite and constant room temperature. Next, the calorimeter is filled to the same height with the liquid, whose thermal capacity is sought, and the time which is required to cool from τ_1 to τ_2, when the calorimeter and contents are subjected to precisely the same conditions as when filled with water, is again noted. Then, since the average difference of temperature between calorimeter and contents and the surroundings is the same in both cases, it follows that the thermal capacities, in the two cases, are to each other directly as the times required in cooling through the same temperature intervals. Therefore, if t_1 is the time required for the calorimeter and water to cool from τ_1 to τ_2, and t_2 is the time required for the calorimeter and the substance whose thermal capacity is sought, to cool through the same temperature interval, it follows that

$$\frac{C+m_1}{t_1}=\frac{C+m_2c}{t_2}; \quad \dots\dots \quad (8)$$

where m_1 is the mass of water, m_2 the mass of the liquid, and c its thermal capacity per unit mass. From equation (8), since all quantities excepting c are known, the thermal capacity per unit mass is determined.

It is, however, not necessary to know the thermal capacity of the calorimeter. For, if the calorimeter be first cooled through the temperature interval, $\tau_1-\tau_2$, when empty, then when filled with water, and again when filled with the liquid whose thermal capacity is sought, and the times t, t_1, and t_2 are noted, we shall have

$$\frac{C}{t}=\frac{C+m_1}{t_1}=\frac{C+m_2c}{t_2}; \quad \dots\dots \quad (9)$$

where t is the time required when empty, t_1 when filled with water, and t_2 when filled with the liquid whose thermal capacity is sought, and the other symbols having the same significance as before. Eliminating C, from equation (9), we find

$$c=\frac{m_1(t_2-t)}{m_2(t_1-t)}. \quad \dots\dots \quad (10)$$

A convenient form of apparatus, for the method of cooling, is an alcohol thermometer with its bulb greatly enlarged and having the form of a hollow cylinder. The substance is then placed directly inside of the bulb and the whole thermometer, while cooling, exposed to a constant temperature; this may readily be brought about by placing the thermometer inside of a vessel which is surrounded by melting ice.

21. Mechanical Equivalent of Heat. From mechanics we have the following statement: "The change in kinetic energy that a body undergoes in passing over a given path is equal to the work done in traversing that path." The foregoing statement is very simple, and readily comprehended when considered in the purely mechanical sense. That is, it is merely stated that whenever a given amount of kinetic energy has been destroyed or developed in a system, an equivalent amount of work has been done by the system or, as the case may be, on the system. The statement, however, does not concern itself with the transformation of one form of energy into another.

The most general experience, common to all, is that of the destruction of energy, in the form of mechanical motion, by friction or impact and the simultaneous evolution of heat. However, it was not until 1842 that a clearly formulated statement was made, by J. R. Mayer, to the effect, *that when heat is converted into work, or vice versa, the ratio of the numbers representing the two quantities involved is constant.* Unfortunately, the figures upon which Mayer based his calculations were in error, and consequently the value obtained for the mechanical equivalent was also in error.

Shortly subsequent to Mayer's enunciation, Joule began his series of experiments to determine the mechanical equivalent of heat by direct measurement. Joule's method was essentially as follows: A vessel having fixed vanes, was filled with water, and a paddle-wheel was caused, by means of falling weights, to rotate in the water. The fixed vanes prevented the water from assuming a rotary motion. The heat developed manifested itself by a rise in temperature, and the work done was measured by the weights and distance fallen. A series of experiments was then made, using mercury instead of water. Another series of experiments was made by causing one iron plate to rotate with friction over another iron plate under water.

It is, of course, understood that experiments like these are attended by great difficulties, and various precautions must be taken and corrections made which cannot here be enumerated.

Still, Joule obtained fairly consistent results; and, the figures he finally published were not greatly in error.

The results were expressed in *meter-kilograms of work per calorie;* i.e., Joule found when using water that it required 423.9 *m.kgs.* of work to develop one calorie, 424.7 *m.kgs.* of work per calorie when using mercury, and for the experiment with iron, the number was found to be 425.2. It must be remembered that, if comparisons are to be made between results obtained by experiments, which have been performed in different localities, corrections will have to be made for variations in the value of g.

Rowland varied Joule's method by using a motor drive instead of falling weights. The vessel was suspended and the torque required to prevent rotation measured. This enabled a much more rapid expenditure of energy, and a consequent rapid rise in temperature; thus making the correction due to cooling much smaller. Rowland's experiment covered the range from 5°C. to 36°C. Since the thermal capacity of water varies for different temperatures, a variation was found for the mechanical equivalent of heat. Rowland found, at Baltimore, where $g=980.0$, for the mechanical equivalent of heat, of the common calorie, 427.3 *m.kgs.* This may be taken as being substantially correct.

Anthony modified Rowland's method by having a continuous flow of water through the calorimeter, the temperature of the inflowing water was constant, and its rate of flow was so regulated that the vessel was always at room temperature. The mass of water flowing for a given time was determined by weighing, the number of rotations made by the paddle was recorded on a speed counter, and the torque was noted, together with the temperature of the inflowing and outflowing water. By this method, since the vessel is always at room temperature, no corrections for cooling are required; and furthermore, since the vessel suffers no change in temperature, its thermal capacity need not be known. The values obtained by this method were in practical concordance with those found by Rowland.

In the English system of units, the value for the mechanical equivalent of heat, usually employed, is 778 ft.-lbs. per B.T.U.

Various experiments have been performed, by sending an electric current through a conductor wound upon an insulating support and submerged in water. By noting the current, the applied *e.m.f.*, and the time, during which the current has been flowing, the energy input is readily computed. Results obtained by this method agree almost precisely with those obtained by the methods previously described.

A safe value to use, which if in error is only slightly so, and which is correct for all practical purposes, is 4.195×10^7 ergs per common gram calorie.

It will now readily be seen that, since we can measure electrical quantities with great precision, the best method for obtaining a definite quantity of heat is by sending a steady current for a given time through a given resistance; it being remembered that there are alloys whose resistances are practically independent of temperature.

CHAPTER III

PRODUCTION OF AND EFFECTS OF HEAT

22. SINCE, in a great many cases, those changes which evolve heat during their progress, require the application of heat to bring about a change in the reverse order, it is inadvisable to consider the production of heat and the effects of heat independently.

Whenever a change of a chemical nature takes place there is either an absorption or a liberation of heat. In general, heat is absorbed when a compound is split up into the elements composing it; and heat is liberated when the elements recombine to form the compound. Those compounds which evolve heat, during their formation, are called *exothermic* compounds; and those *rare* compounds which absorb heat, during their formation, are called *endothermic* compounds.

It will now be instructive to consider some particular substance and the various changes which take place with the continuous application of heat. Suppose that we are dealing with a definite mass of ice, under a given pressure, whose temperature is below that of its melting-point for the applied pressure. (The melting-point of ice changes slightly with change of pressure; i.e., the melting-point is *lowered* about 0.0075°C. for each increment in pressure equal to one atmosphere.) A definite amount of heat then, must be applied to raise the temperature of the ice to the melting-point. If heat be then further applied the temperature will no longer change; but, a change of *physical state* takes place together with a continuous absorption of heat until all the ice is melted.

23. Heat of Fusion. The quantity of heat required to convert unit mass of a solid into the liquid state *without change of temperature* is called the *heat of fusion* of the substance. In the case of ice, the heat of fusion is approximately 80 gram calories per gram. If now, after all the ice has been converted into water, heat be continuously applied, the temperature will rise progressively until, if the liquid be under a pressure equal to one standard atmosphere, the temperature of 100°C. is reached. The temperature will then cease to rise, provided the pressure be maintained constant, and a change of *physical state,* namely, *vaporization* at constant temperature, takes place progressively with the continuous application of heat until all the water is evaporated.

The heat of fusion of ice may be determined as follows: Let L, m, and τ_1, respectively, represent the heat of fusion, the mass, and the initial temperature of the ice; and M, and τ_2, respectively, represent the water equivalent of calorimeter and contents, and the initial temperature of calorimeter. Let the ice now be submerged in the water in the calorimeter until it is all melted, and the calorimeter and total contents assume a common temperature θ. The total heat, then, consumed by the ice in having its temperature raised from τ_1 to 0, being converted into water at this temperature, and in raising the temperature of the liquid from 0 to θ, is

$$mc\tau_1+m(L+\theta);$$

where c is the thermal capacity per unit mass of ice. But this quantity of heat must be equal to

$$M(\tau_2-\theta);$$

hence,

$$mc\tau_1+m(L+\theta)=M(\tau_2-\theta).$$

From which

$$L=\frac{M}{m}(\tau_2-\theta)-(c\tau_1+\theta).$$

24. Heat of Vaporization. The quantity of heat required to convert unit mass of a liquid into a vapor *without change of temperature* is called the *heat of vaporization* of the substance. In the case of water, at 100°C., the heat of vaporization is approximately 537 gram calories per gram. The heat of vaporization for most substances becomes less as the temperature rises.

The heat of vaporization of water may be determined by the method of mixtures as follows: Let superheated steam, at a temperature τ_1, be passed into a calorimeter containing water at a temperature τ_2. This is continued until a convenient rise of temperature is obtained in the calorimeter, and a mass of steam m has been condensed. The quantity of heat, given up by the steam, is

$$mc(\tau_1-\tau)+mr+m(\tau-\theta);$$

where τ is the temperature at which the condensation takes place, r the heat of vaporization, c the thermal capacity per unit mass for superheated steam, and θ the resulting temperature. But this quantity of heat must be equal to

$$M(\theta-\tau_2);$$

where M is the thermal capacity of the calorimeter and water initially contained in it. Hence,

$$mr+m\{c(\tau_1-\tau)+(\tau-\theta)\}=M(\theta-\tau_2);$$

from which

$$r=\frac{M}{m}(\theta-\tau_2)-\{c(\tau_1-\tau)+(\tau-\theta)\}.$$

To obtain accurate results by means of calorimetric methods, as previously stated, it is always necessary to take certain precautions and apply proper corrections.

25. Sublimation. Under certain conditions a substance may pass directly from the solid to the gaseous state without passing through the liquid state. Such a change is called *sublimation.* Substances such as camphor and iodine, when gently heated, pass readily from the solid to the gaseous state without melting. Ice, under normal pressure, also sublimes at temperatures lower than the melting-point.

26. Superheating. If now, after all of the liquid has been converted into vapor, heat be further applied, the temperature will rise progressively with continued application of heat, and the vapor will become *superheated.*

All of the foregoing may, instructively, be represented diagrammatically; bearing in mind that the thermal capacities per unit mass of water for the three states are: *Solid* 0.504, *liquid* 1, and *gaseous* 0.481.

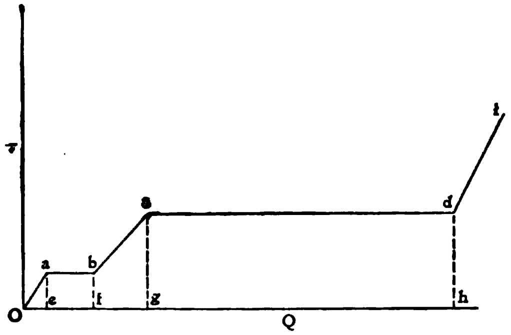

FIG. 2.

Let, as in Fig. 2, the temperatures be taken as ordinates and quantities of heat as abscissas. Then, if we assume some arbitrary zero, such as *O*, for the initial condition of the ice, we have for the application of the quantity of heat *Oe*, the increment of temperature *ea*, to the point of *fusion.* The application of the quantity of heat *ef* then brings about the conversion from the *solid* to the *liquid* state at constant temperature. The further application of the quantity of heat *fg* brings about the elevation of temperature, from the temperature of fusion, to that of *vaporization.* The application of the quantity of heat *gh* brings about complete *vaporziation at constant temperature.* The further application of heat brings about *superheating*, as shown by the line *di*.

In general, the physical history of substances with continued application of heat will be similar to the process just discussed; but, the ratios of the quantities involved will be entirely different for each substance.

Assume now, that the process takes place in the reverse order, step by step; i.e., the steam is first cooled to the point of condensation, then condensation takes place at constant temperature, and so on, step by step, until the initial condition is reached. Then during each change, heat is liberated precisely equal in amount to that which was absorbed when the change was taking place in the opposite direction.

27. Reversible Processes. The changes just described and depicted in Fig. 2 are, however, not the only changes involved. Assume the applied pressure to be maintained constant throughout the entire change, the volume then of the substance will be changing continually; and, in general, will be increasing with the temperature. When the volume is increasing, work is being done *by* the substance in overcoming the applied pressure. When the volume is decreasing, work is being done *on* the substance by the applied pressure. From the initial condition up to 4°C., this being the temperature corresponding to the maximum density of water, work is being done *on* the substance. For all temperatures higher than this, the substance expands continuously with the continued application of heat; and work is being done *by* the substance in overcoming the applied pressure. When the process takes place in the reverse order, then, wherever heat was *absorbed* and work done *by* the substance during the *direct* process, heat will be *liberated by*, and work will be done *on* the substance during the *reverse* process. And if these quantities be mutually equal, and no permanent changes have been made to take place in the surrounding bodies, by this cycle of operations, either process is said to be a *reversible process.* That is, when a system undergoes a change, or a series of changes, the process is said to be reversible if, after it has taken place, a second process can be made to take place, in a manner, such that *when the system is again in its initial condition, there remain, due to these various changes, no changes outside of the system.*

A little consideration will show that vaporization at constant pressure, and hence at constant temperature, provided we could have *perfect insulation and no friction,* would be a reversible process. For, under the assumed conditions, the amount of work done *by* the vapor, during its formation, in overcoming the external pressure, is precisely equal to the amount of work done *on* the vapor during its condensation. Furthermore, the quantity of heat absorbed, during vaporization, from a reservoir of heat at constant temperature, is precisely equal in amount to the quantity of heat rejected, to the reservoir, during condensation. Hence, since all quantities involved balance each other, and no changes have been brought about in the surroundings, the process is reversible.

The cooling and heating of a substance at constant pressure, together with its consequent changes in volume, can be made a reversible process only by the aid of a perfect *regenerator.* The following discussion will make this clear. Assume that a body at a temperature τ_n, which is also the temperature of the first reservoir, cools to a temperature τ_1, by being put, successively, into contact with n reservoirs, perfectly insulated from each other, and each reservoir differing in temperature from the one adjacent to it by an amount equal to $(\tau_n-\tau_1)/(n-1)$. The body then, in cooling, gives up to each reservoir, excepting the first, a definite quantity of heat, and has done upon it, by the constant external pressure, a definite amount of work. Let the process now take place in the reverse order, i.e., the body at a temperature τ_1 is put into contact with the reservoir at a temperature τ_2; a definite quantity of heat will be absorbed, which will be precisely equal to that rejected when put into contact with the reservoir at a temperature τ_1, after having been in contact with the reservoir at a temperature τ_2. Let this be continued until the temperature τ_n is again reached. Now, the work done *by* the external pressure *on* the body, while cooling from the temperature τ_n to the temperature τ_1, is precisely equal in amount to the work done *by* the body, in overcoming the external pressure, while being heated from the temperature τ_1 to the temperature τ_n. This process, however, is not perfectly reversible; since the reservoir at the temperature τ_n has given up heat and received none, and the reservoir at the temperature τ_1 has received heat and given up none. In the limit, however, as the fraction $(\tau_n-\tau_1)/(n-1)$, approaches zero for its value, the process becomes perfectly reversible. But this implies a perfect regenerator; i.e., a series of reservoirs which are perfectly insulated from each other, and still have a continuous variation in temperature throughout the series; but this is practically impossible. Hence, it is obvious that, since in the case of vaporization, we must assume no radiation and no friction to make the process reversible, and in the case of cooling and heating of a body, we must assume a perfect regenerator and no friction and radiation to make the process reversible, the process as described and represented diagrammatically in Fig. 2 is reversible only in an *ideal* sense; i.e., an *ideally reversible process*.

28. Irreversible Processes. If a constant *e.m.f.* be applied to the terminals of a homogeneous conductor, a current will flow which is directly proportional to the applied *e.m.f.*, and inversely to the resistance of the conductor. After a time the conductor will reach a constant temperature; i.e., the rate at which the energy is being converted into heat by the conductor, due to its resistance, will be equal to the rate at which heat, expressed in the same units, is given to the surroundings by the conductor. This, however, does not mean that there is thermal equilibrium. In this case thermal equilibrium can only be brought about by disconnecting the applied *e.m.f.*, and consequently, discontinuing the *dissipation* of energy. This being done, the conductor will finally assume the temperature of the surroundings, and thermal equilibrium will have been established. This process, viz, the conversion of energy, in the form of an electric current, into energy, in the form of heat, differs essentially in one particular feature from the process discussed in *Art.* 27. The former process, which we termed an *ideally reversible process*, can be made to take place, barring various losses, in the reverse order. The latter process, however, cannot be made to take place in the reverse order; i.e., it is absolutely impossible to cause a current to flow in a homogeneous conductor by applying heat to it. Such a process is called an *irreversible process.*

Another example of an irreversible process is that of the conversion of energy, in the form of mechanical motion, into heat by friction. For it is impossible to restore a system of bodies to their initial positions by the application of a quantity of heat to the surfaces, equal to that which was evolved, due to friction, during their displacements. The same is true for the case of impact. When impact takes place between two or more bodies, a certain amount of kinetic energy is always converted into heat. But it is absolutely impossible, by the *direct* application of heat, to restore the kinetic energy which was destroyed during impact. Also, when thermal equilibrium is established by mixing substances, initially at different temperatures, the process is absolutely irreversible.

29. Dissociation. Under *Art.* 26 we discussed the physical history of water with the continued application of heat up to the point of its superheated vapor. If heat be still further applied to the superheated vapor its temperature will continue to rise, up to some very high temperature, when complete *dissociation* takes place; i.e., the vapor splits up into its two constituent elements, viz, *hydrogen and oxygen.* Such a change is called a *chemical* change. If, while the pressure is maintained constant, heat be further applied, the gaseous mixture will rise in temperature and increase in volume progressively with continued application of heat. If now, the process be reversed, i.e., the mixture be cooled, the temperature and volume will diminish until the temperature of dissociation is reached. When this point is reached the two gases recombine to form steam, and precisely the same amount of heat is evolved as was absorbed to bring about decomposition. The quantities of heat, however, which are involved in chemical decomposition and recomposition are very large in comparision with those quantities involved during changes in temperature and changes of physical state. Indeed, our greatest source of supply of energy, in the form of heat, is that due to chemical combination; viz, the combination of carbon, in the form of coal, with oxygen.

It is true that the amount of dissociation is a function of the temperature; i.e., even water at ordinary temperatures has a small percentage of dissociation. This, however, does not invalidate the statement that, the energy absorbed during dissociation is equal to that liberated upon recombination.

To give an illustration of the quantities of heat evolved during chemical combination, we may take as examples the combination of hydrogen and oxygen to form steam, and the combination of carbon and oxygen to form carbon-dioxide. In the former case, 1 gram of hydrogen combining with oxygen to form steam (H_2O), about 34,000 gram calories are evolved, or expressed in mechanical units, 1.43×10^{12} ergs. In the latter case, i.e., when 1 gram of carbon combines with oxygen to form carbon-dioxide (CO_2), about 8000 gram calories are evolved, or, expressed in mechanical units, 3.36×10^{11} ergs.

30. Electrolysis. Dissociation may also, in general, be brought about by *electrolysis*. That is, if an electric current be passed through a chemical compound, in the form of a solution, the compound will be split up into its constituents.

Suppose that we are dealing with a solution of copper sulphate ($CuSO_4$), and that the two electrodes are absolutely inert as regards chemical reactions. Then if an electric current be passed through the solution, copper will be deposited on the negative electrode (*cathode*), and the *radical* SO_4 will be liberated at the positive electrode (*anode*). The SO_4 thus liberated will combine with hydrogen of the solvent to form sulphuric acid (H_2SO_4); and at the same time oxygen will be liberated. The equation, representing this reaction, is

$$CuSO_4+H_2O=H_2SO_4+Cu+O.$$

If an electric current be passed through water, then the water will be split up into its two elements, hydrogen and oxygen; hydrogen being given off at the *cathode* and oxygen at the *anode*.

In the case of dissociation by heat, a definite quantity of heat disappears for a given amount of dissociation; and an evolution of an equal quantity of heat upon recombination. But, since a given quantity of heat represents a definite amount of energy, it follows that dissociation involves storing of energy. Likewise, when dissociation is brought about by electrolysis a definite amount of energy is consumed for a given amount of dissociation, which must necessarily be equal to that consumed when the same amount of dissociation is brought about by the application of heat; since the energy stored is the same in amount for both cases. It is true that a solution becomes heated when conveying a current; but, this has nothing to do with the dissociation. The development of heat being merely due to the resistance of the solution, the same as when any other conductor is conveying a current.

It is immaterial, so far as the foregoing argument is concerned, whether we consider the solution initially partly ionized and the current merely a carrier of the ions, or that the current actually splits up the compound.

31. Faraday's Discoveries. Faraday showed experimentally that the amount of dissociation is directly proportional to the time and the intensity of the current; and furthermore, that the amount of chemical action is the same for all parts of the circuit. The latter part may perhaps be best illustrated as follows: Assume that there are two voltameters connected in series; the first containing a solution of copper-sulphate and the second water. Then upon passing a steady current through the circuit a definite amount of copper will be deposited on the cathode of the first voltameter, for a given interval of time, and a definite amount of hydrogen liberated at the cathode of the second voltameter, during the same interval of time; and *these two quantities will be in the same ratio as their chemical combining numbers.* That is, for every gram of hydrogen set free at the cathode of the second voltameter, 31.59 grams of copper will be deposited on the cathode of the first voltameter; where, if hydrogen be taken as unity, 31.59 is the chemical equivalent of copper in copper-sulphate. During the same time that the 31.59 grams of copper are being deposited on the cathode of the first voltameter, 1 gram of hydrogen must be liberated to combine with the *sulphion* (SO_4), set free to form H_2SO_4.

32. Counter Electromotive Force. Since the amount of dissociation, other things being equal, varies directly as the current, and the amount of energy stored during dissociation depends upon the compound dissociated, it follows that every compound offers a definite *counter e.m.f.* to dissociation. And any applied *e.m.f.* less than this cannot bring about dissociation. To make this clear, the *e.m.f.* necessary to dissociate water will here be calculated. The amount of hydrogen set free, per coulomb of electricity conveyed, is 0.000010357 grams. The number 0.000010357 is called the *electro chemical equivalent* of hydrogen.

Let, in the *c.g.s.* system of units, z be the electro chemical equivalent of hydrogen, I the current, and t the time, then the mass of hydrogen liberated, during the time t, is

$$m = Izt. \qquad \ldots \ldots \ldots \qquad (1)$$

Let h be the heat, expressed in mechanical units, required to dissociate 1 gram of hydrogen, then

$$mh = IEt; \quad \ldots \ldots \ldots \quad (2)$$

where E is the applied *e.m.f.* Substituting in equation (2), the value of m as given in equation (1), we find

$$Izth = IEt;$$

from which

$$zh = E. \quad \ldots \ldots \ldots \quad (3)$$

If now, in equation (3), we substitute for z and h their values, remembering that for 1 gram of hydrogen, combining with oxygen to form steam, $h = 1.43 \times 10^{12}$ ergs; and that z, in the *c.g.s.* system of units, equals 0.00010357 grams per unit quantity of electricity, we find

$$E = 1.43 \times 10^{12} \times 0.00010357 \textit{ c.g.s. } \text{units } \textit{e.m.f.},$$

or,

$$E = \frac{1.43 \times 10^{12} \times 0.00010357}{10^8} = 1.48 \text{ volts.}$$

It must, however, be emphasized that, in general, the *e.m.f.* for most cells is a function of the temperature; and, therefore, to calculate the *e.m.f.* this function must be known. We are not prepared, here, to take up this matter. The counter *e.m.f.* is readily determined by experiment.

33. Junction of Dissimilar Metals. If the junction of two dissimilar metals be heated an *e.m.f.* is developed which is a function of the temperature, and the metals which form the junction. To take a concrete example, assume a junction of *antimony* and *bismuth.* Such a junction, if heat be applied to it, develops an *e.m.f.* which tends to send a current from *bismuth* to *antimony;* and if a current be sent from *bismuth* to *antimony,* by the application of an external *e.m.f.*, there will be a tendency to reduce the temperature of the junction. On the other hand, if a current be sent through the junction from *antimony* to *bismuth*, heat will be developed.

34. As a résumé, we may then state that the most general *effects of heat* are: To change the volumes of bodies; to bring about physical changes of state; in general, to promote chemical dissociation; to develop an *e.m.f.* at the junction of dissimilar metals.

For the *production of heat* we may state the following examples: The production of heat by the mechanical compression of bodies which expand upon the application of heat; when the physical state of a body changes in the reverse order from the change when heat is applied; in general, by chemical combination; at the junction of two dissimilar metals when a current is passed in a direction opposite to that of the developed *e.m.f.* when heat is applied. Also, the production of heat when an electric current is conveyed by a homogeneous conductor; and, in general, when friction is being overcome, and mechanical motion destroyed by impact.

35. The Principle of Energy. The various relations discussed in this chapter may now be summed up and stated quantitatively in a very simple manner. This generalization, known as the *principle of energy*, or conservation of energy, is one of the most extensive of generalizations, and may be stated in substance as follows: *If in any system, from which no energy escapes and into which no energy enters, account be taken of all forms of energy, then no matter what transformations take place within the system, the sum total is a constant quantity.* So far as experience goes, the foregoing statement is consistent with all phenomena; and hence, in all subsequent demonstrations its truth will be assumed.

CHAPTER IV

EXPANSION OF SOLIDS AND LIQUIDS

36. Linear Expansion. It has been previously stated that, in general, bodies expand when the temperature is augmented. It is found by experiment that a body, such as a metal rod, increases in length by approximately equal amounts, between 0°C. and 100°C., for equal increments of temperature. But, even though there is an approximate proportionality between change in length and change in temperature for moderate ranges, such as just specified, it must not be inferred that this is generally true for large ranges or high temperatures.

If a body of unit length expand in length by an amount α for unit increment of temperature, then a body of length l will expand in length by an amount $l\alpha$ for an increment of 1 degree; and for an increment of $\tau°$, between 0°C. and 100°C., the body will expand in length, approximately, by an amount $l\alpha\tau$. Hence, if the length of the body at 0° be denoted by l_0, and at $\tau°$ by l_τ, we have

$$l_\tau = l_0 + l_0\alpha\tau;$$

from which

$$l_\tau = l_0(1+\alpha\tau). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The quantity α is called the *coefficient of linear expansion,* and may be defined as the ratio of the change in length, per unit change in temperature, to the length at zero. The quantity in the parenthesis, viz, $1+\alpha\tau$, is called the *factor of linear expansion.*

37. Voluminal Expansion. Homogeneous *isotropic* bodies will change in amount by like fractional parts of their original dimensions in all directions when the temperature changes. Assume that we are dealing with a rectangular parallelopiped whose three edges at zero temperature are a_0, b_0, and c_0, its volume then, at zero, is

$$v_0=a_0b_0c_0. \qquad (2)$$

If the temperature, now, be changed to $\tau°$, the three edges become: $a_0(1+\alpha\tau)$, $b_0(1+\alpha\tau)$, and $c_0(1+\alpha\tau)$; from which we have, for the volume at $\tau°$,

$$v_\tau=a_0b_0c_0(1+\alpha\tau)^3. \qquad (3)$$

Substituting in equation (3), for $a_0b_0c_0$ the value v_0, as given by equation (2), we have

$$v_\tau=v_0(1+\alpha\tau)^3. \qquad (4)$$

Expanding equation (4), we find

$$v_\tau=v_0(1+3\alpha\tau+3\alpha^2\tau^2+\alpha^3\tau^3). \qquad (5)$$

Now, α is a very small quantity in comparison with the dimensions of most bodies; hence, the two terms containing α^2 and α^3 may be neglected; and we have

$$v_\tau=v_0(1+3\alpha\tau). \qquad (6)$$

Equation (6) is, of course, only approximately true; but, that it is correct for most practical purposes will be evident from an inspection of the following table, which gives the coefficients of linear expansion, per degree centigrade, for a few solids. It must be remembered that for substances like brass and glass the numbers are only approximate; and in any case the value found for the coefficient of expansion will depend somewhat upon the treatment to which the specimen was subjected in its preparation.

COEFFICIENTS OF LINEAR EXPANSION

Platinum	0.899×10^{-5}
Copper	1.678×10^{-5}
Steel (annealed)	1.095×10^{-5}
Zinc	2.918×10^{-5}
Brass	0.187×10^{-4}
Glass	0.083×10^{-4}
Invar (steel containing 36% nickel)	0.087×10^{-5}

It is only necessary to substitute the values of α, as given in the foregoing table, in equation (5) and it becomes evident that equation (6) is approximately true. Hence, we may say that; the coefficient of voluminal expansion is practically equal to three times the coefficient of linear expansion.

38. Non-Isotropic Bodies. There are certain bodies which have different physical properties in different directions. Such bodies are termed *non-isotropic.* A notable example is that of Iceland spar, in which it is found that the coefficient of linear expansion in one direction is 2.63×10^{-5}; whereas, in a direction normal to this it is found to be only 0.544×10^{-5}.

It is also interesting to note that Iceland spar manifests different optical properties in different directions.

39. Expansion of Liquids. Liquids, in general, change more rapidly in volume than do solids for equal changes in temperature. However, since, in the case of liquids, the term linear expansion is meaningless, we deal only with voluminal expansion.

The determination of the coefficient of linear expansion is quite simple; hence, the description of the methods employed in its determination was omitted. The determination of the *coefficient of voluminal expansion* of a liquid is attended by various difficulties; and, since the discussion of the principles involved will prove instructive, a few methods will be described.

Assume that we have a glass flask terminating in a tube of capillary bore, and that the mass of the flask when empty is known. The flask is then filled, to a definite mark on the capillary tube, with the liquid at a temperature τ_1, whose coefficient of voluminal expansion is sought. The mass of the vessel and contents is now determined. The difference between this mass and the mass of the flask gives us the mass of the liquid at the temperature τ_1. If D_1 is the density of the liquid at the temperature τ_1, and V the volume of the liquid in the flask at the same temperature, then

$$D_1=\frac{M_1}{V}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where M_1 is the mass of the liquid in the flask at the temperature τ_1. Let, now, M_2 be the mass of the liquid in the flask, when its temperature is τ_2, the flask being filled to precisely the same mark on the tube as when the temperature was τ_1. The volume of the flask will now be

$$V'=V\{1+\beta(\tau_2-\tau_1)\};$$

where β is the coefficient of voluminal expansion of *the* glass of which the flask is composed. β may be computed from the coefficient of linear expansion, or determined directly by experiment, as will be shown subsequently. The density of the liquid at τ_2, will now be

$$D_2=\frac{M_2}{V'}=\frac{M_2}{V\{1+\beta(\tau_2-\tau_1)\}}. \quad . \quad . \quad . \quad . \quad (8)$$

Dividing equation (7) by equation (8), we find

$$\frac{D_1}{D_2}=\frac{M_1}{M_2}\{1+\beta(\tau_2-\tau_1)\}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

If the coefficient of voluminal expansion of the liquid, between 0° and the other two temperatures under consideration, is approximately constant, we have

$$D_1=\frac{D_0}{1+\alpha\tau_1}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

where D_0 is the density of the liquid at 0° and α the coefficient of voluminal expansion. Similarly, we find

$$D_2=\frac{D_0}{1+\alpha\tau_2}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

from which, by dividing equation (10) by equation (11), we find

$$\frac{D_1}{D_2}=\frac{1+\alpha\tau_2}{1+\alpha\tau_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Finally, by equating the right-hand members of equations (12) and (9), we find

$$\frac{1+\alpha\tau_2}{1+\alpha\tau_1}=\frac{M_1}{M_2}\{1+\beta(\tau_2-\tau_1)\}. \quad . \quad . \quad . \quad . \quad (13)$$

In equation (13), all quantities excepting α are known; hence, its value is determinate.

Another method is as follows: A solid, whose coefficient of voluminal expansion is accurately known, and which does not react chemically with the liquid whose coefficient of voluminal expansion is sought, is weighed in the liquid, first at temperature τ_1, and second at temperature τ_2. The weight of the solid being known, its loss of weight for the two temperatures is known; and, since the volume of the solid for any temperature may, from its known coefficient of voluminal expansion, be computed, the densities of the liquid for the two temperatures τ_1 and τ_2 are readily found, and from these, as previously shown, the coefficient of voluminal expansion.

40. Direct Measurement of Coefficient of Voluminal Expansion. The method about to be described, and the one by which Regnault determined the coefficient of expansion of mercury, depends upon the principle that when communicating columns of liquids are in equilibrium their heights are inversely as their densities. In Fig. 3, *AB* and *CD* are two vertical iron tubes, cross connected by the horizontal tube *BD*. The two horizontal tubes, *AE* and *CF*, terminate in the vertical tubes *EG* and *FI*, which are connected by the inverted glass U-tube *GJI*. The tube containing the stop cock *s*, is connected to the receiver of a compression pump; and after the apparatus has been filled with mercury, air is forced in from the compressor until the mercury in the tubes *EG* and *FI* is at a convenient height. The stop cock *s* is then closed. If, now, the temperature, and consequently the density, of the mercury in the tubes *AB* and *CD* be the same, the columns in *EG* and *FI* will be at the same level. On the other hand, if the temperatures of the two columns *AB* and *CD* be not the same, then the columns in *EG* and *FI* will not be at the same level. For, since there is free communication between *B* and *D*, the pressure must be the same at these two points. Furthermore, the pressure of the air inside the U-tube being everywhere the same, and, since this pressure plus the pressure, due to the column in *FI*, balances the pressure due to the column *DC*, and the same pressure plus the pressure, due to the column in *EG*, balances that of the column *BA*, it follows that if the pressures, due to the two columns *AB* and *CD*, are not the same, the two columns in *EG* and *FI* cannot be at the same level.

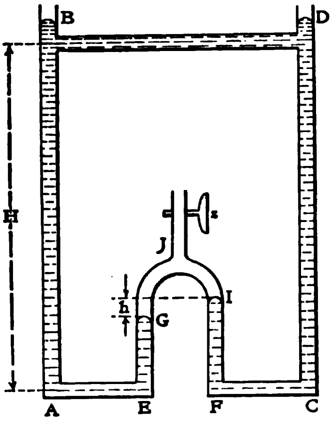

Fig. 3.

Suppose, now, that the column in *EG*, *FI*, and *CD* be maintained at 0°C. and the column *AB* at τ°C., then the column in *FI* must exceed the column in *EG*, in height, by an amount h, such that

$$(H-h)(1+\alpha\tau)=H; \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

where α is the coefficient of voluminal expansion of mercury.

From equation (14) we find

$$\alpha=\frac{h}{\tau(H-h)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

41. If, now, it be desired to determine the coefficient of expansion of some other liquid, it becomes only necessary to take a flask and determine its mass when empty, then the mass of flask and contents when filled to a given mark at various temperatures, first with the liquid whose coefficient is sought, and then when filled with mercury to the same mark for the various temperatures. From the mass of mercury required to fill the flask at various temperatures, and from the known density of mercury for these temperatures, the volume of the flask is readily computed. From the known volume of the flask and the mass of liquid required to fill it at the various temperatures, the densities corresponding to those temperatures are found.

CHAPTER V

FUNDAMENTAL EQUATIONS OF GASES

42. Isothermal Equation. Experiment shows that, between certain limits, for the so-called permanent gases, such as hydrogen, oxygen, nitrogen, etc., the product of pressure and volume is a constant, for constant temperature. Expressed symbolically

$$pv=k; \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

where p is the applied pressure per unit area,* v the corresponding volume of the gas, and k some constant whose value depends upon the units chosen. Equation (1) is usually designated as *Boyle's Law*. But, in any case, the equation which expresses the relation between the pressure and volume of a gas, at constant temperature, is its isothermal equation.

43. Gay-Lussac's Law. As a further result of experiment it is found that all gases which obey *Boyle's Law* have the same *constant temperature coefficient;* i.e., all gases, under constant pressure, expand by the same fractional part of their volumes at zero temperature, for equal increments of temperature. This is known as *Gay-Lussac's Law*. If we denote by α the increment in volume, for a unit volume of a gas, under a constant pressure p_0, when its temperature changes from zero to unity, then the volume of a gas at τ degrees is

$$v_\tau=v_0+v_0\alpha\tau; \quad . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

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* In all subsequent equations, unless otherwise stated, p will be used to denote pressure per unit area.

where v_0 is the volume of the gas at zero temperature, and v_τ the volume, under the same pressure, at τ degrees. Writing equation (2) in another form, we have

$$v_\tau = v_0(1+\alpha\tau). \quad \dots\dots\dots \quad (3)$$

If, after the temperature τ, and the corresponding volume v_τ, under the constant pressure p_0, has been attained, the pressure is augmented, the temperature being maintained constant, until the gas assumes its original volume v_0, we must, from Boyle's law, have the following relation:

$$p_\tau = p_0(1+\alpha\tau). \quad \dots\dots\dots \quad (4)$$

As a matter of fact the experiment is most conveniently performed, by varying the pressure, so as to maintain the volume constant, as the temperature is varied. Equation (4) may therefore be considered as being the expression of experimental results. Multiplying both sides of equation (4), by v_0, we obtain

$$p_\tau v_0 = p_0 v_0(1+\alpha\tau). \quad \dots\dots\dots \quad (5)$$

If, now, while the temperature is maintained constant, the pressure be varied, the volume will vary according to *Boyle's Law;* i.e.,

$$pv = p_\tau v_0 = p_0 v_0(1+\alpha\tau); \quad \dots\dots \quad (6)$$

where p is any pressure and v the corresponding volume at the temperature τ.

Since *Gay-Lussac's Law* also holds for temperatures below zero, α, of course, being a decrement, we have

$$pv = p_0 v_0(1-\alpha\tau). \quad \dots\dots\dots \quad (7)$$

Equation (7) reduces to zero when $\alpha\tau$ equals unity; hence $\tau = 1/\alpha$ is the temperature below zero for which $pv = 0$.

For the centigrade scale $\alpha=0.003665$, very nearly; hence, $\tau=1/0.003665=273$, very nearly. Therefore, if there were no deviation from the relation expressed by equation (7), then at $-273°$C. the product of pressure and volume would become zero. As a matter of fact, all gases liquefy at temperatures above $-273°$C. This point, 273°C. below zero, is called the "*absolute zero*"; and temperatures measured from this zero are called temperatures on the "*absolute scale.*"

Substituting now, in equation (6), for α its value, viz, 1/273, we have

$$pv=p_0v_0\left(1+\frac{\tau}{273}\right)=\frac{p_0v_0}{273}(273+\tau). \quad . \quad . \quad . \quad (8)$$

But, in equation (8), $p_0v_0/273$ is a constant for any particular mass of a gas, and $273+\tau$ is the temperature as measured on the "*absolute scale.*" Replacing the former by R and the latter by T, equation (8) becomes

$$pv=RT. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

The numerical value of R depends, of course, upon the units chosen.

Equation (9) is called the *characteristic equation* of a gas, and shows that the product of pressure and volume is directly proportional to the temperature as measured on the "absolute scale." *

44. Departure from Boyle's Law. For ordinary pressures, Boyle's law is approximately true; i.e., for air the ratio of the product of pressure and volume, when the pressure is one atmosphere, to the product of pressure and volume, when the pressure is two atmospheres, is about 1.002. The ratio of the initial value of pv to the final value of pv becomes greater as the final pressure becomes greater, until certain very high pressures, which are different for the various gases, are reached; after which the value

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* Since all knowledge is relative, the expressions "absolute zero" and "absolute scale" are not well chosen.

of pv increases rapidly with increase of pressure. The values of the pressures for which, at ordinary temperature, the product pv is a minimum, are as follows: 100 meters of mercury for oxygen, 65 meters for air, and 50 meters for both nitrogen and carbon-dioxide. For hydrogen the deviations from *Boyle's Law* are less and in the opposite direction.

Since *Boyle's Law* is not rigidly true, it follows that all equations, which have been based on it, are not rigidly true. However, for ordinary ranges of pressure and temperature the *characteristic equation* is very nearly true. And, for the mathematical discussion of gases, it is very convenient to assume that we are dealing with a gas for which the characteristic equation is *rigidly* true. Such a gas is called a *perfect* or *ideal* gas.

45. Gas Thermometer.* If a gas be confined in such a manner that its volume is maintained constant while the temperature varies, then it follows, from equation (9), that between those limits for which the equation is approximately true, the pressure varies directly as the temperature. All that is necessary then, to enable us to measure temperatures accurately, is a glass bulb of convenient size, filled with a gas, the most convenient being dry air, and some device by means of which the pressure can be regulated and measured. Such an arrangement constitutes a gas thermometer. Fig. 4 is a diagrammatic representation of the arrangement. A is a glass bulb, filled with gas, and connected by an inverted capillary U-tube, to the U-tube abc. B is a vessel, open at the top, partially filled with mercury, and communicates, by means of a flexible tubing, with the U-tube at b. The tube bc being open at the top, the mercury in B and bc must be at the same level. Assume that, when the temperature of A is at zero, the mercury in B, in bc, and in ba is at the same level; then, if the temperature of the bulb A, and consequently that of

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* In all subsequent demonstrations, unless otherwise specified, the symbol T will be employed to designate temperatures as measured by the *ideal gas thermometer*, the zero of which is about $-273\,^{\circ}\mathrm{C}$.

the gas in it, be increased, the gas will expand if the pressure be constant. Hence, to maintain the gas at its original volume, the vessel *B* must be raised so as to increase the column *bc*, in a manner, such that the column *ba*, and hence the volume of the gas. is maintained constant. The difference in the height of the columns *bc* and *ba* measures the increment in pressure; and hence, the increment in temperature may be calculated. A correction must, however, be made for the change in volume, due to change in temperature, for the bulb *A*. This correction is readily applied, provided the coefficient of voluminal expansion for the glass, confining the gas, be known.

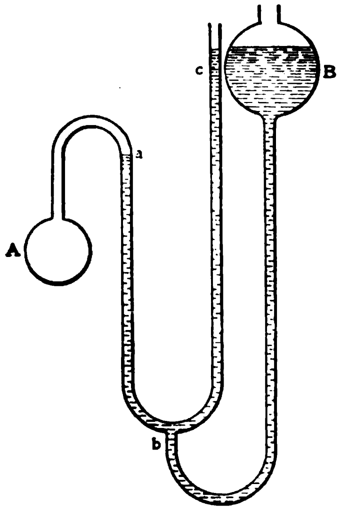

Fig. 4.

Let p_0 and v_0, respectively, represent the pressure and volume of the gas for the temperature at zero, p_τ the observed pressure for the temperature τ, α the coefficient of expansion for the gas, and β the coefficient of voluminal expansion for the glass. The new volume, then, for the gas at the temperature τ, will be $v_0(1+\beta\tau)$; hence

$$p_\tau v_0(1+\beta\tau)=p_0 v_0(1+\alpha\tau).$$

~~Now, $p_\tau=p_0(1+\alpha'\tau)$; where $'\alpha'$ is the apparent coefficient of expansion of the gas. Making this substitution, for p_τ, and eliminating, we find~~

$$(1+\alpha'\tau)(1+\beta\tau)=1+\alpha\tau;$$

~~from which~~

$$\tau=\frac{\alpha-(\alpha'+\beta)}{\alpha'\beta}.$$

See Errata

The foregoing equation follows immediately from Boyle's Law, and solving for "τ", we find

$$\tau=\frac{p_\tau-p_0}{p_0\alpha-p_\tau\beta}.$$

When extreme precision is sought, corrections must also be made for changes in volume of the bulb, due to changes in pressure. Gas thermometers are used only for purposes of standardization.

46. Expansion without Doing External Work. Experiment has shown that there is no energy consumed in the simple expansion of a gas; i.e., when a gas expands in such a manner that no external pressure is overcome, and hence, no external work is being done, no energy is consumed by it. This experiment was first performed by Gay-Lussac (who apparently did not realize its full significance) in the following manner: Two vessels, one of which was exhausted and the other filled with air under a pressure, were placed in a calorimeter and surrounded by water. When the stop cock in the tube, connecting the two vessels, was opened, the air from the one vessel expanded into the other, bringing about an equalization of pressures. During this process the gas increased in volume without doing any work external to the system. That is, work was done by the gas under a high pressure, in the one vessel, in expanding against the increasing pressure of the gas in the other vessel. But, since the temperature of the system after the completion of the process was found to be precisely equal to that of the system before the process began, it follows that the total energy of the system is unchanged by the expansion. If energy were required to bring about an increase in volume, the temperature of the system at the end of the process would necessarily be less than at the beginning of the process. Or, to express the result in another way, since the temperature of the system is unchanged by the change in volume, the work done by the gas in the vessel, initially under the higher pressure, is precisely equal in amount to the work done on the gas in the other vessel. The results just deduced may be embodied in a simple statement; i.e., the *intrinsic energy* of a perfect gas is a function only of the temperature. Or, to put it still in another way, the heat of *disgregation* of a perfect gas, when no external work is being done, is zero.

The foregoing experiment was subsequently repeated, in the most careful manner, by Joule and found to be approximately, though not rigidly, true.

47. Thermal Capacities of Gases. If a gas, under a pressure p, expand by an amount in volume dv, the external work done will be numerically equal to pdv; where p is the pressure per unit area. This is shown as follows: From the definition of work, we have

$$dw = Fds; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

where F is the applied force and ds the displacement. But, since F, the applied force, is numerically equal to the product of p, the pressure per unit area, and the area A, we have

$$dw = pAds. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

But

$$Ads = dv;$$

hence, by substituting in equation (11), we have

$$dw = pdv. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Since, according to the experiments of Gay-Lussac and Joule, when the temperature of a gas is augmented, that part of the heat which is required to elevate the temperature of the gas is practically the same for equal ranges of temperature, no matter whether the volume be varied or maintained constant, it follows that the quantity of heat required to bring about a given elevation of temperature, when the pressure is maintained constant, is greater than the quantity of heat required to bring about an equal elevation of temperature, when the volume is maintained constant. For, in the former case, heat is required, not only to elevate the temperature of the gas, but also to do the external work due to the expansion of the gas; whereas, in the latter case, the heat consumed is only that required to elevate the temperature of the gas.

The ratio of change in heat to the corresponding change in temperature, in a unit mass of gas, when the volume is maintained constant, is a measure of the *thermal capacity per unit mass at constant volume*, and is denoted by C_v. Hence, for a unit mass of gas, we have

$$\left(\frac{\delta Q}{\delta T}\right)_v = C_v. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (13)$$

In a like manner

$$\left(\frac{\delta Q}{\delta T}\right)_p = C_p; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (14)$$

where C_p represents, for a gas, the *thermal capacity per unit mass, under a constant pressure.*

The ratio $C_p/C_v = n$ is practically a constant for all the permanent gases; and, in the case of air, is approximately 1.405. The quantity $(C_p - C_v)$, is evidently a measure of the external work done by the unit mass of gas in expanding against the pressure p, while it is being heated through a range of 1 degree.

Assume that we have a given mass of gas m, whose volume is v at 0°C., confined in a cylinder by a piston of area A, under a pressure p, per unit area. If the piston be perfectly free to move, and heat be applied bringing about an elevation of temperature τ, the pressure being maintained constant during the process, then the volume will be increased by an amount $v\alpha\tau$. The distance through which the piston moves during the expansion is $v\alpha\tau/A$; and the external work done, which is numerically equal to the product of force and displacement, is

$$W = pA\frac{v\alpha\tau}{A} = pv\alpha\tau. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (15)$$

The heat consumed in doing the external work, expressed in mechanical units, is

$$H = Jm(C_p - C_v)\tau; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (16)$$

where J is the mechanical equivalent of heat. But, under the assumed conditions, H and W are numerically equal; hence, from equations (15) and (16), we have

$$Jm(C_p - C_v)\tau = pv\alpha\tau;$$

from which

$$J = \frac{pv\alpha}{m(C_p - C_v)}. \qquad \qquad (17)$$

Equation (17) * enables us to compute the mechanical equivalent of heat from the known constants of a gas. The constants of dry air are as follows: $C_p = 0.2375$, $C_v = 0.1690$, $\alpha = 0.003665$, and the mass of 1 c.c. of air at 0°C., under a pressure of 1.01325×10^6 dynes per sq.cm., is 0.001293 grams. Substituting these values in equation (17), and assuming 1 c.c. for the initial volume, we find

$$J = \frac{1.0132 \times 10^6 \times 0.003665}{0.001293(0.2375 - 0.1690)} = 4.193 \times 10^7 \text{ ergs per gram calorie.}$$

The value of J thus obtained differs by a small percentage from that obtained by the direct conversion of mechanical work into heat. We have here then a complete verification of the numerical relation between heat and work. That is, in the one case, mechanical work is directly converted into energy, in the form of heat, and the numerical ratio of the two quantities involved is determined. In the other case, heat is converted into work, and the numerical relation between the two quantities is again determined; and the difference, between the two values so determined, is well within the limits of observational error.

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* It was by this method that J. R. Mayer first computed the mechanical equivalent of heat. Various writers have attempted to take some of the credit from Mayer, by asserting that it was not then known that no energy is required for the simple expansion of a gas. But Gay-Lussac had performed this experiment, and anyone reading Mayer's original papers will see that he was aware of this and interpreted the experiment properly.

48. Adiabatic Equation. If a gas is compressed, work is being done *on* it and heat is necessarily developed; and since the pressure of a gas, other things being equal, rises with the temperature, it follows that, unless the heat developed by the compression is abstracted from the gas as rapidly as it is developed, the pressure must rise more rapidly, with respect to the amount of compression, than it would during isothermal compression. There are certain processes where compression and dilatation take place so rapidly that there are practically no exchanges of heat between the various parts of the system. Changes, during which no heat enters or escapes, are called *adiabatic changes.*

A good example of adiabatic changes are the compressions and rarefactions which take place in a medium when a sound wave exists in it; the time of compression, or rarefaction, being so small that practically no heat is transferred from particle to particle.

In general, if we are dealing with a unit mass of a perfect gas, we may write

$$dQ=C_v dT+pdv; \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

where all quantities, of course, are expressed in the same units. dQ, expressed in mechanical units, is the quantity of heat absorbed by the gas, or else abstracted from it, $C_v dT$ is the quantity of heat involved in bringing about the change of temperature dT, and pdv represents work done either *by* the gas or *on* the gas.

As a matter of illustration, assume work is being done on the gas in a manner such that its temperature rises and that there is also heat given to the surroundings. If we consider work done by the gas and heat absorbed by the gas positive, then, in equation (18), if applied to this case, both dQ and pdv become negative; on the other hand, $C_v dT$ remains positive.

If, now, during the change in volume, no heat enters or escapes, the process will be adiabatic; and equation (18) becomes

$$C_v dT+pdv=0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

In equation (19), if dv is positive, dT must be negative; i.e., if external work is done by the gas it is done at the expense of the intrinsic energy of the gas, and the temperature must fall. Likewise, if work is done on the gas, since according to our assumption no heat escapes, its temperature must rise. To solve equation (19) we will substitute for dT its value, as found from equation (9), *Art.* 43; i.e., by differentiating

$$pv=RT,$$

we find

$$dT=\frac{pdv+vdp}{R}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

Now, R can be expressed in terms of the two thermal capacities. To find this relation, assume that we are dealing with a unit mass of the gas and allow it to expand under a constant pressure p, while its temperature is increased by unity. If v_1 is the initial and v_2 the final volume, the external work done is

$$p(v_2-v_1)=C_p-C_v.$$

It being understood that all quantities are measured in mechanical units. But, from equation (9), it follows directly, that

$$p(v_2-v_1)=R(T_2-T_1);$$

and, since by the conditions we have T_2-T_1 equal to unity, it follows that

$$R=C_p-C_v.$$

Substituting this value of R in equation (20), and the value of dT so obtained in equation (19), we find

$$C_v\frac{pdv+vdp}{C_p-C_v}+pdv=0;$$

from which

$$\left(\frac{C_p}{C_v}\right)\frac{dv}{v}+\frac{dp}{p}=0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

But, as previously explained, C_p/C_v is practically a constant. Representing this constant by n and substituting in equation (21), we have

$$n\frac{dv}{v}+\frac{dp}{p}=0;$$

from which, by integration,

$$\log v^n+\log p=k_1;$$

where k_1 is the constant of integration. Or, expressed in another form,

$$\log pv^n=k_1;$$

from which

$$pv^n=k; \quad \ldots\ldots\ldots \quad (22)$$

where k is a constant depending upon the units chosen. Equation (22) gives the relation of pressure and volume for a gas, during adiabatic changes, and is known as the *adiabatic equation.*

GENERAL EQUATIONS OF GASES

49. The change of heat involved when a gas suffers a change is a function of the temperature, pressure, and volume; i.e.,

$$Q=f(T, p, v).$$

But, since any two of these quantities may vary independently of the third, we may write

$$Q=f(T, p), \quad Q=f'(T, v), \quad \text{and} \quad Q=f''(p, v).$$

By partial differentiation of these functions, we obtain

$$dQ=\left(\frac{\delta Q}{\delta T}\right)_p dT+\left(\frac{\delta Q}{\delta p}\right)_T dp, \quad \ldots\ldots \quad (23)$$

$$dQ=\left(\frac{\delta Q}{\delta T}\right)_v dT+\left(\frac{\delta Q}{\delta v}\right)_T dv, \quad \ldots\ldots \quad (24)$$

and

$$dQ=\left(\frac{\delta Q}{\delta p}\right)_v dp+\left(\frac{\delta Q}{\delta v}\right)_p dv. \quad \ldots\ldots \quad (25)$$

If it be assumed that we are dealing with a unit mass of gas, then in equation (23), we have

$$\left(\frac{\delta Q}{\delta T}\right)_p dT = C_p dT;$$

since in this case, $\left(\frac{\delta Q}{\delta T}\right)_p$ is the thermal capacity per unit mass at constant pressure. In a like manner,

$$\left(\frac{\delta Q}{\delta p}\right)_T = m$$

is also a unit thermal capacity, and is the ratio of change in heat to change in pressure at constant temperature. Substituting, in equation (23), we have

$$dQ = C_p dT + m dp. \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

In equation (24)

$$\left(\frac{\delta Q}{\delta T}\right)_v dT = C_v dT;$$

since $\left(\frac{\delta Q}{\delta T}\right)_v$ is the thermal capacity per unit mass of the gas at constant volume. In a like manner,

$$\left(\frac{\delta Q}{\delta v}\right)_T = l$$

is a unit thermal capacity, and is the ratio of change in heat to change in volume at constant temperature; hence, by substituting in equation (24), we find

$$dQ = C_v dT + l dv. \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

Again, in equation (25) the two quantities, viz, $\left(\frac{\delta Q}{\delta p}\right)_v$, the ratio of change in heat to change in pressure at constant volume, and $\left(\frac{\delta Q}{\delta v}\right)_p$, the ratio of change in heat to change in volume at constant pressure, are both unit thermal capacities. If we represent the former by j, and the latter by o, then equation (25) becomes

$$dQ=jdp+odv. \qquad (28)$$

Since all of these equations must be true for the particular case when the left-hand members are equal, the right-hand members of equations (26) and (27) may be equated, and we have

$$C_p dT+mdp=C_v dT+ldv. \qquad (29)$$

From the fundamental statement

$$v=f(p, T),$$

we have

$$dv=\left(\frac{\delta v}{\delta p}\right)_T dp+\left(\frac{\delta v}{\delta T}\right)_p dT.$$

Substituting this value of dv, in equation (29), we have

$$C_p dT+mdp=C_v dT+l\left(\frac{\delta v}{\delta p}\right)_T dp+l\left(\frac{\delta v}{\delta T}\right)_p dT;$$

and

$$C_p dT+mdp=\left\{C_v+l\left(\frac{\delta v}{\delta T}\right)_p\right\}dT+l\left(\frac{\delta v}{\delta p}\right)_T dp. \qquad (30)$$

Since this equation is true when the corresponding changes in the two members are equal, it follows that

$$C_p-C_v=l\left(\frac{\delta v}{\delta T}\right)_p, \qquad (31)$$

and

$$m=l\left(\frac{\delta v}{\delta p}\right)_T. \qquad (32)$$

Again

$$p=f(v, T);$$

from which

$$dp=\left(\frac{\delta p}{\delta v}\right)_T dv+\left(\frac{\delta p}{\delta T}\right)_v dT. \qquad (33)$$

Substituting the value of dp as given in equation (33), in equation (29), we have

$$C_pdT+m\left\{\left(\frac{\delta p}{\delta v}\right)_T dv+\left(\frac{\delta p}{\delta T}\right)_v dT\right\}=C_vdT+ldv;$$

from which

$$\left\{C_p+m\left(\frac{\delta p}{\delta T}\right)_v\right\}dT+m\left(\frac{\delta p}{\delta v}\right)_T dv=C_vdT+ldv;$$

hence, by equating like coefficients, we find

$$C_p-C_v=-m\left(\frac{\delta p}{\delta T}\right)_v. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (34)$$

By equating the right-hand members of equations (26) and (28), we find

$$C_pdT+mdp=jdp+odv. \quad . \quad . \quad . \quad . \quad . \qquad (35)$$

From the fundamental statement

$$T=f(v,\ p),$$

we have

$$dT=\left(\frac{\delta T}{\delta v}\right)_p dv+\left(\frac{\delta T}{\delta p}\right)_v dp;$$

and substituting this value of dT, in equation (35), we obtain

$$C_p\left\{\left(\frac{\delta T}{\delta v}\right)_p dv+\left(\frac{\delta T}{\delta p}\right)_v dp\right\}+mdp=jdp+odv;$$

hence, by equating like coefficients, we find

$$o=C_p\left(\frac{\delta T}{\delta v}\right)_p. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (36)$$

From equations (27) and (28), we have

$$C_vdT+ldv=jdp+odv;$$

and, by substituting for dT its value, we find

$$C_v\left\{\left(\frac{\delta T}{\delta v}\right)_p dv+\left(\frac{\delta T}{\delta p}\right)_v dp\right\}+ldv=jdp+odv;$$

from which

$$j=C_v\left(\frac{\delta T}{\delta p}\right)_v. \qquad (37)$$

We then have the following values:

$$l\left(\frac{\delta v}{\delta T}\right)_p=C_p-C_v,$$

$$l\left(\frac{\delta v}{\delta p}\right)_T=m,$$

$$m\left(\frac{\delta p}{\delta T}\right)_v=-(C_p-C_v),$$

$$C_v\left(\frac{\delta T}{\delta p}\right)_v=j,$$

and

$$C_p\left(\frac{\delta T}{\delta v}\right)_p=o.$$

From the characteristic equation

$$pv=RT,$$

we find

$$\left(\frac{\delta v}{\delta T}\right)_p=\frac{R}{p};$$

hence

$$l=\frac{p}{R}(C_p-C_v). \qquad (38)$$

Again

$$\left(\frac{\delta v}{\delta p}\right)_T=-\frac{RT}{p^2};$$

hence

$$m=-\frac{T}{p}(C_p-C_v). \qquad (39)$$

Also

$$\left(\frac{\delta T}{\delta p}\right)_v=\frac{v}{R};$$

hence

$$j=\frac{v}{R}C_v. \qquad (40)$$

Finally

$$\left(\frac{\delta T}{\delta v}\right)_p=\frac{p}{R};$$

and

$$o=\frac{p}{R}C_p. \qquad (41)$$

Substituting in equations (26), (27), and (28) the values of *l*, *m*, *j*, and *o* as just determined, we find

$$dQ=C_p dT-\frac{T}{p}(C_p-C_v)dp, \qquad (42)$$

$$dQ=C_v dT+\frac{p}{R}(C_p-C_v)dv, \qquad (43)$$

and

$$dQ=\frac{v}{R}C_v dp+\frac{p}{R}C_p dv. \qquad (44)$$

These equations, viz, (42), (43), and (44), may be put into different forms; since, from the characteristic equation

$$pv=RT,$$

we have

$$T/p=v/R, \quad \text{and} \quad p/R=T/v.$$

From the assumption, then, that in the fundamental statement

$$Q=f(T, p, v),$$

any two of the variables may vary independently, while the third is maintained constant, we have obtained three distinct equations, viz, equations (42), (43), and (44). It is of further interest to note that if in any one of these three equations the right-hand member is equated to zero, the adiabatic equation is obtained.

Assuming that the change in equation (44) is adiabatic, then $dQ=0$, and we have

$$C_v vdp = -C_p pdv;$$

from which

$$\frac{C_p}{C_v}\frac{dv}{v} = -\frac{dp}{p};$$

and

$$n\int\frac{dv}{v} = -\int\frac{dp}{p};$$

from which

$$n \log v = -\log p + k_1.$$

Or, expressed in another form

$$\log pv^n = k_1;$$

and, finally

$$pv^n = k. \qquad (45)$$

This may also be obtained from the other equations, as may be readily shown. From

$$pv = RT,$$

we have

$$dT = \frac{pdv + vdp}{R}.$$

Substituting this value of dT in equation (43) and equating to zero, we find

$$C_v(pdv + vdp) = -p(C_p - C_v)dv;$$

from which

$$C_v vdp = -C_p pdv;$$

which is identical with the result obtained from equation (44) under the same assumption. Again, substituting in equation (42), for dT its value and equating to zero, we find

$$C_p\frac{pdv+vdp}{R}=\frac{v}{R}(C_p-C_v)dp;$$

from which

$$C_ppdv=-C_vvdp;$$

which is again identical with that previously obtained under the same assumption.

If it be desired to find the temperature of a gas, corresponding to a given pressure and volume, during an adiabatic change, in terms of the initial temperature and pressure and the given pressure, or in terms of the initial temperature and volume and the given volume, we proceed as follows: Let T_1, p_1, and v_1 be, respectively, the initial temperature, pressure, and volume, p and v, respectively, the pressure and volume, for which the corresponding temperature, T, is sought. Then, since the two points are on the same adiabatic, we have

$$p_1v_1^n=pv^n; \quad \ldots \ldots \quad (46)$$

and from the characteristic equation, we have

$$p_1v_1=RT_1, \quad \ldots \ldots \quad (47)$$

and

$$pv=RT. \quad \ldots \ldots \quad (48)$$

Substituting in equation (46), the values of v_1 and v, as found from equations (47) and (48), we find

$$T=T_1\left(\frac{p}{p_1}\right)^{\frac{n-1}{n}}. \quad \ldots \ldots \quad (49)$$

In a similar manner, by substituting in equation (46), the values of p_1 and p, as found from equations (47) and (48), we obtain

$$T=T_1\left(\frac{v_1}{v}\right)^{n-1} \qquad (50)$$

Vapors

50. Vaporization. The gaseous states of bodies, which under ordinary conditions of temperature and pressure are either liquids or solids, are called *vapors;* and the process by which the vapor is formed is called *vaporization.*

In general, vaporization takes place in two distinct ways. In the one process, called *evaporation,* vapor is continually formed at the exposed surfaces of liquids; and in the other process, called *ebullition,* bubbles of vapor are formed in the body of the liquid or at the heated surfaces.

51. Evaporation. If a liquid be enclosed in a space, only part of which is occupied by the liquid, then vapor immediately forms and occupies the space above the liquid. This continues until the vapor has reached a certain density which depends upon the temperature, and is greater as the temperature is higher, but is always the same for the same temperature. In other words, for any given temperature there is a maximum density and hence, a maximum pressure, which the vapor is capable of exerting. When this state is reached the vapor is said to be *saturated.* That is, for the given temperature the space contains the maximum possible amount of vapor. If, after this state has been reached, the temperature be maintained constant and an attempt be made to increase the pressure by the application of an external force the result will be, not an *increment in pressure, but a diminution in volume,* at constant pressure, and a corresponding amount of condensation. In other words, *the pressure for a saturated vapor at constant temperature is constant.* Or, to put it in still another way, the temperature of a saturated vapor is *uniquely* defined by its pressure.

52. Addition of Vapor Pressures. The rate of evaporation depends, of course, upon the rate at which heat is being supplied; but, as has just been stated, the final pressure reached depends merely upon the temperature. Furthermore, evaporation takes place more rapidly in a vacuum than in a space occupied by the vapor of some other substance; however, the final pressure reached by the vapor will be almost, though not quite, as high, when the space is partially occupied by some other gas or vapor, than it would be were the space originally a vacuum, provided always, that the temperature be the same and that there be no chemical action between the vapors. This statement was first made by Dalton, viz, when evaporation takes place in a space filled by another gas, which has no action on the vapor, the final pressure reached by the mixture is equal to the sum of the pressures of its constituents. Careful experiment shows Dalton's statement to be approximately, though not rigidly true.

53. Ebullition. As has been previously stated, when heat is applied to a liquid, the temperature rises progressively with continued application of heat until a certain point, which depends upon the pressure, is reached, when the temperature remains constant. This is the boiling-point for the given pressure; and is that temperature for which the pressure of the vapor is equal to the superimposed pressure. Since the pressure at any point in the liquid, is equal to the pressure at the surface plus the pressure due to the liquid, from the surface to the point under consideration, it follows that, the temperature varies for different depths below the surface of the liquid. Hence, the temperature of the boiling liquid is not a constant throughout; but increases slightly with the depth.

When equilibrium has been attained, i.e., the temperature becomes constant, then all the energy that is supplied, in the form of heat, is consumed in converting the liquid into a vapor. This energy consists of two parts, viz, one part being that energy which is required to overcome the inherent forces, that is, to separate the particles so as to form vapor, and the other part to overcome the external pressure during the augmentation of volume. The former is called the *heat of disgregation* and the latter the *heat of expansion.*

Pressure, however, is not the only factor that fixes the boiling-point of a liquid. As examples, the following may be cited: The nature of the material of the containing vessel has some influence. If the liquid be first carefully freed from the imprisoned air, the temperature may be raised considerably above the temperature at which ebullition ordinarily takes place. Impurities in the liquid influence the boiling-point. And finally, salts dissolved in a liquid always raise the boiling-point. As an example, the boiling-point of a saturated solution of water with common salt is about 109°C. But the temperature of the saturated vapor of a liquid is always the same for the same pressure, no matter what the temperature of the liquid. It is for this reason that the temperature of steam, rather than that of water, under a pressure of one standard atmosphere has been chosen as the boiling-point.

54. Critical Temperature. When a liquid is heated in a closed vessel the vapor accumulates above the liquid and augments the pressure. Up to a certain point, differing for different liquids, there is a sharp definition between the liquid and vapor; but, for every liquid there is reached, finally, a temperature when this definition ceases, and the liquid disappears and is completely converted into vapor, even though the volume occupied by the vapor is but little greater than that occupied by the liquid. The temperature at which this takes place is called the *critical temperature* for the substance. And, it appears that, *for temperatures, higher than this, no matter what the applied pressure, the substance can exist only in the gaseous state.* The following table gives a few substances together with their approximate critical temperatures, and the corresponding pressures:

Substance.	Temperature in Degrees, C.	Pressure in Atmospheres.
Carbon-dioxide	31	77
Sulphur-dioxide	156	79
Ether	194	36
Water	365	195
Oxygen	−118	50
Nitrogen	−146	33
Hydrogen	−234	20

55. It is interesting to note that there apparently is a relation between heat of disgregation of a substance and its critical temperature. Let σ be the specific volume of the liquid; i.e., the volume occupied by unit mass of the liquid, and s the specific volume of the dry saturated vapor, then the increment in volume, when a unit mass of a liquid is converted into vapor, is

$$\mu = s - \sigma; \quad \dots\dots\dots \quad (51)$$

where μ is the increment in volume. The external work, or the heat of expansion, is

$$W = p\mu = p(s - \sigma); \quad \dots\dots \quad (52)$$

where p is the pressure during the evaporation. To express the heat of expansion, during evaporation, in thermal units, we must divide by J, and equation (52) becomes

$$W = Ap(s - \sigma); \quad \dots\dots\dots \quad (53)$$

where $A = 1/J$.

If we represent by r the heat of vaporization, and by ρ the heat of disgregation, equation (53) becomes

$$r - \rho = Ap(s - \sigma); \quad \dots\dots \quad (54)$$

from which

$$\rho = r - Ap(s - \sigma). \quad \dots\dots \quad (55)$$

To illustrate, we will now compute the heat of expansion for water when it is converted into dry steam at 100°C.

Assume that we are dealing with 1 gram of water at 100° C. Its volume will be approximately 1 c.c.; and the volume of its saturated vapor under atmospheric pressure will be about 1670 c.c. Substituting, in equation (54), for $s-\sigma$, A, and p the numerical values, we find

$$r-\rho=\frac{1.013\times10^6\times1670\,^*}{4.195\times10^7}=40.3 \text{ gram calories per gram.}$$

This gives for the heat of disgregation, which is the difference between the heat of vaporization and the heat of expansion, $536.5-40.3=496.2$.

Zeuner gives an empirical equation, for the heat of disgregation for water, which gives results very close to those obtained by equation (55); this equation is

$$\rho=575.4-0.791\tau; \quad . \; . \; . \; . \; . \; . \quad (56)$$

where ρ is in gram calories per gram and τ in degrees centigrade.

In general, the heat of disgregation becomes less as the temperature becomes higher; and the critical temperature appears to be that for which the heat of disgregation becomes zero. In the case of some liquids, very close agreement is found between the values for the critical temperatures, as found by direct experiment, and those values calculated from the empirical equations for the heat of disgregation. In other cases, again, there are discrepancies of considerable magnitude. A notable case is that of water; but, it must be remembered that the critical temperature of water is very high, and therefore, its determination is attended by dif-

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* s not being known definitely to the fourth significant figure, it is immaterial whether we use 1670 or 1669; or, in other words, the volume of the liquid, in this case, is negligibly small in comparison with that of its vapor.

ficulties. Furthermore, the heat of vaporization for water has not been determined for such high temperatures; hence, the empirical equation for such ranges is doubtful.

56. Total Heat of Steam. Before leaving the subject of saturated vapors, we will give, on account of the importance in steam calculations, the empirical equations for the total heat of saturated steam, and the heat of vaporization. By total heat of steam is meant the quantity of heat required to raise the temperature of unit mass of water, from the melting-point of ice, to the temperature under consideration and convert it into saturated steam at that temperature. In the French system the total heat is given by the equation

$$H=605+0.305\,\tau \text{ gram calories per gram. . . .} \qquad (57)$$

In the English system the total heat is given by the equation

$$H=1082+0.305\,\tau \text{ B.T.U. per pound. . . . .} \qquad (58)$$

In equation (57), H equals the quantity of heat, in gram calories, required to raise 1 gram of water from 0°C., to the temperature τ°C., and convert it into a saturated vapor at that temperature. In equation (58), H equals the quantity of heat, in British thermal units, required to raise 1 pound of water from 32°F. to τ°F., and convert it into a saturated vapor at that temperature.

57. Heat of Vaporization for Water. The empirical formula, which gives the heat of vaporization of water, for the various temperatures, is

$$r=1114-0.7\tau \text{ B.T.U. per pound. . . .} \qquad (59)$$

Equation (59) is not quite as accurate as equation (58); but for most steam calculations it is sufficiently precise; since, by means of it the heat of vaporization is found, with an error of less than 1 per cent, between 100°F. and 400°F.

58. Superheated Vapors. Any vapor which, for a given pressure, is at a temperature higher than that corresponding to saturation, for the given pressure, is said to be *superheated*. This is only possible when the vapor is not in contact with its own liquid. Vapors which have been superheated obey Boyle's law approximately; and furthermore, for adiabatic changes, the equation

$$pv^n = k,$$

holds; n having different values, depending upon the vapors with which we are dealing.

59. Hygrometry. Hygrometry has for its object the determination of the state of the atmosphere with respect to the aqueous vapor present. The amount of aqueous vapor present in the atmosphere is a very variable quantity. The effect of the vapor, however, depends not only upon the quantity present, but also upon the temperature. These two facts are included in the single statement, that the effect of the vapor depends upon the *relative humidity*.

By the expression relative humidity, is meant the ratio of the actual density of the vapor, contained by the air, to the density which it would have if there were saturation for the given temperature. Or, if expressed as a percentage, the relative humidity is the *percentage of saturation* for the given temperature.

60. Dew Point. The *dew point* is that temperature at which the vapor present in the atmosphere, begins to condense; i.e., the point of saturation. Assume that a certain portion of the atmosphere is cooled until the vapor present begins to condense. The temperature at which this takes place is readily found by experiment. By referring to a curve giving the relation of temperature and pressure of saturated steam, we can readily find the pressure corresponding to the dew point. Designating this pressure by p_1, and by p_2 the pressure of saturated vapor corresponding to the temperature of the atmosphere, then since non-saturated vapors approximately obey Boyle's law, the density of the vapor present in the atmosphere is to the density the vapor would have were there saturation, very nearly, as p_1 is to p_2. Hence, we have approximately, for the relative humidity

$$h=100\frac{p_1}{p_2} \text{ per cent.}$$

To illustrate further, we will consider a concrete case. Assume the temperature of the atmosphere to be 25°C., and that of the dew point 15°C. From the steam curve we find $p_1=1.278$ cm. of mercury, and $p_2=2.369$ cm. of mercury. From this, the relative humidity, expressed as a percentage, is found to be

$$h=100\times\frac{1.278}{2.369}=53.9 \text{ per cent.}$$

61. Absolute Humidity. The amount of moisture, expressed in grams, contained by a cubic meter of air is called the *absolute humidity.* This is found in a very simple manner. The dew point is determined, giving the temperature of the saturated vapor, and from this, by referring to steam tables, the mass per unit volume is found.

CHAPTER VI

ELASTICITIES AND THERMAL CAPACITIES OF GASES

62. THE adiabatic equation, for gases and vapors, being so frequently employed in the discussion of the theory of heat motors, it is important to say a few words about the determination of the ratio $C_p/C_v=n$. The determination of thermal capacities of gases is attended by far greater difficulties, than those found in the determination of thermal capacities of liquids and solids. This is due to the fact that the thermal capacity of a gas is always very small in comparison with that of the containing vessel.

63. Thermal Capacity at Constant Pressure. Regnault was the first to determine accurately the thermal capacities of gases under constant pressure.* The method is essentially as follows: The gas, whose thermal capacity is sought, is contained in a large reservoir, under a high pressure, from which it is passed through a spiral tube immersed in a bath, the temperature of which is maintained constant. The spiral tube is of sufficient length to insure the gas leaving it to be at the same temperature as the bath. In the tube, connecting the reservoir with the spiral tube, is a valve, by means of which the pressure of the gas is maintained constant. A second spiral tube, through which the gas must pass, is immersed in a calorimeter filled with water; the thermal capacity of the calorimeter and contents being known. The spiral tube, immersed in the calorimeter, is of such length that the temperature of the gas, throughout the progress of the experiment, is reduced to that of the calorimeter before being discharged into

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* For a full description, see Preston's "The Theory of Heat," Chapter IV, Section VI.

the atmosphere. From the initial and final pressure of the gas in the reservoir, together with its temperature, which is maintained constant, by means of a suitable bath, the mass of gas passing through the calorimeter, during the progress of the experiment, is readily found. Furthermore, from the thermal capacity of the calorimeter and contents, together with the initial and final temperatures, the quantity of heat absorbed by the calorimeter during the progress of the experiment, *proper corrections being made for losses*, is determined. And, since the temperature of the gas before entering the calorimeter, as well as the average final temperature, is known, and also the mass of gas which has passed through the calorimeter, the thermal capacity per unit mass is determinate.

64. Thermal Capacity at Constant Volume. In the experiment just described the quantity of gas employed is not limited by any containing vessel; for, the reservoir in which the gas is contained may be of any size whatsoever, without having any influence on the result. Therefore, a large quantity of gas may be used, and consequently, a considerable range of temperature may be obtained in the calorimeter. However, when it is desired to determine the thermal capacity of a gas at constant volume, the quantity of gas upon which we are experimenting, is limited by the containing vessel; and the thermal capacity of the containing vessel is always large in comparison with the thermal capacity of the enclosed gas.

65. Joly's Steam Calorimeter.* In the most primitive form, the steam calorimeter consists of the pan of one side of a beam balance placed in an enclosure with the specimen, whose thermal capacity is sought, supported by the pan. When steam is admitted into the enclosure, condensation takes place until the temperature of the test specimen is equal to that of the steam. The steam then passes through the enclosure without further conden-

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* For a complete description of this apparatus, see Preston's "The Theory of Heat," Chapter IV, Section V.

sation. When this condition has been reached, the balance is counterpoised and the mass of condensed steam, which has been collected by the pan, is noted. From the initial and final temperature of the test specimen, together with the quantity of water collected by the pan, and the heat of vaporization for this particular temperature, the thermal capacity of the test specimen is readily found.

66. Differential Steam Calorimeter. In this form, both pans of the balance, which are made so that they have equal thermal capacities, are suspended in the enclosure. On the one pan is placed a spherical vessel, which has been exhausted, and on the other pan a spherical vessel of like dimensions and equal thermal capacity, filled with the gas whose thermal capacity is sought. When steam is now admitted into the enclosure, the quantity of water which collects in the pan, supporting the vessel containing the gas, is greater than that which is collected in the pan supporting the exhausted vessel. This is necessarily so; since the vessel, together with the contained gas, has a thermal capacity greater than the exhausted vessel. From the excess of condensation in the one pan over that in the other, which is obtained directly by weighing, together with the initial and final temperatures of the enclosure and the mass of gas contained by the one vessel, the thermal capacity per unit mass of the gas at constant volume is readily found. Correction, of course, being made for change in volume of the containing vessel for change in temperature.* It must, of course, always be remembered that it is impossible to obtain absolutely accurate results by this method; since the thermal capacity of the gas is always small in comparison with that of the containing vessel. Still, Dr. Joly, who is the inventor of this method, has obtained fairly good results.

67. Method of Clément and Desormes. In this method the gas, whose thermal capacity is sought, is contained in a large vessel

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* For a complete description, see "The Theory of Heat," by Preston, Chapter IV, Section V.

provided with a delicate manometer. When the contained gas has assumed the temperature of the surroundings, its pressure, which must differ from the atmospheric pressure, is carefully ascertained. When this has been done, a stop cock, having a large orifice, is opened and then closed after a very short interval of time. The time which elapses between the opening and closing of the stop cock must be so small that the change in the gas may be assumed adiabatic. During this change the temperature changes; i.e., there will be, either an elevation of temperature, if the pressure in the flask was initially less than the atmospheric pressure, or else, a diminution of temperature, if the pressure in the flask was initially greater than the atmospheric pressure. After the vessel and contents have again assumed the initial temperature, viz, the temperature of the surroundings, the pressure is again carefully noted.

If, now, we denote by p_1 the initial pressure of the gas, by v_1 the corresponding *volume per unit mass*, by p the atmospheric pressure, which is also the pressure of the gas when the stop cock is open, and by v_2 *the volume per unit mass* after the stop cock is closed, then since the change is assumed adiabatic, we may write

$$p_1v_1^n = pv_2^n. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Also, since the initial and final temperatures are the same, we have

$$p_1v_1 = p_2v_2; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where p_2 is the pressure in the vessel after the temperature of the surroundings has again been assumed.

From equation (1), we find

$$\left(\frac{v_1}{v_2}\right)^n = \frac{p}{p_1};$$

from which

$$n = \frac{\log (p/p_1)}{\log (v_1/v_2)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

From equation (2) we find

$$\frac{v_1}{v_2}=\frac{p_2}{p_1};$$

from which

$$\log\frac{v_1}{v_2}=\log\frac{p_2}{p_1}.$$

Substituting in equation (3) for $\log(v_1/v_2)$, its value, $\log(p_2/p_1)$, we obtain

$$n=\frac{\log(p/p_1)}{\log(p_2/p_1)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Or, expressing this in another form, we have

$$n=\frac{\log p-\log p_1}{\log p_2-\log p_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Since p, p_1, and p_2 are known, n is determinate. In this manner, Roentgen found for dry air the value $n=1.405$.

This method is open to criticism, in so far that when the stop cock is opened, oscillations occur; and it does not necessarily follow that, at the instant of closing, the pressure in the vessel is equal to that obtaining outside.

68. Isothermal and Adiabatic Elasticities. The ratio of the two thermal capacities of a gas is most accurately found by determining the speed of propagation of a disturbance through the gas. We will first show that the ratio of the two thermal capacities is numerically equal to the ratio of the two elasticities.

From the statement of Boyle, we have, the temperature being maintained constant,

$$pv=k_1; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where k_1 is a constant, depending upon the units chosen. By differentiation, we find immediately

$$pdv+vdp=0;$$

from which

$$\frac{dp}{-\left(\frac{dv}{v}\right)}=p; \quad . \ . \ . \ . \ . \ . \ . \ . \quad (7)$$

the minus sign denoting merely that the volume decreases as the pressure increases. Now, the left-hand member of equation (7), is numerically equal to the ratio of change in unit stress to the corresponding change per unit volume; and is, therefore, by definition, the expression for the modulus of elasticity. Hence, for a gas obeying *Boyle's law*, the elasticity is numerically equal to the pressure.

If now, we take the adiabatic equation, viz,

$$pv^n=k_2, \quad . \ . \ . \ . \ . \ . \ . \ . \ . \quad (8)$$

where k_2 is again a constant depending upon the units chosen, and differentiate, we find

$$v^n dp+nv^{n-1}pdv=0;$$

from which

$$vdp+npdv=0;$$

and

$$\frac{dp}{-\left(\frac{dv}{v}\right)}=np. \quad . \ . \ . \ . \ . \ . \ . \ . \quad (9)$$

The left-hand member of equation (9) again expresses, according to definition, the modulus of elasticity. Hence, the modulus of elasticity when no heat is allowed to enter or escape, i.e., for adiabatic changes, is numerically equal to the product of the ratio of the two thermal capacities and the pressure.

Denoting the isothermal elasticity by E_t, and the adiabatic elasticity by E_h, we have

$$\frac{E_h}{E_t}=\frac{np}{p}=n. \quad . \ . \ . \ . \ . \ . \ . \ . \ . \quad (10)$$

From equation (10) we see that the ratio of the two principal elasticities is the same as the ratio of the two principal thermal capacities.

69. Propagation of Wave Motion in an Elastic Medium. To properly appreciate how the ratio of the two elasticities, and hence the ratio of the two thermal capacities, of a gas is found from the speed of propagation of sound in the gas, it is essential to study the character of the motion by means of which sound is propagated in an elastic medium.

Let AB, of Fig. 5, be a prism, of indefinite length and constant cross-sectional area, filled with a homogeneous elastic medium; and let the piston P have impressed upon it a constant acceleration toward the right. If the medium had absolutely no inertia, or were perfectly rigid, then the whole substance, between A and B, would suffer precisely the same displacement in a given interval of time. Due, however, to the inertia, the layer next to the piston will be compressed; the pressure of this layer now being greater than that of the medium in the undisturbed condition, it will react upon the second layer and compress this, which again in turn compresses the third layer, etc. Finally, when every layer throughout the prism, has been compressed by an amount such that its internal pressure is precisely equal to the applied pressure, then the acceleration of each layer will be the same and equal to that of the piston.

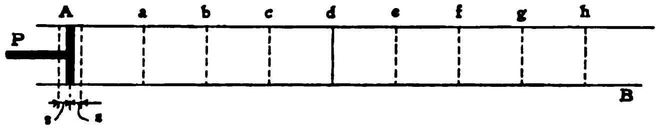

Fig. 5.

Assume now, that the piston P is caused to vibrate periodically, with a small amplitude s. In tracing out a vibration we will begin by assuming the piston in the neutral position, moving toward the right, and the pressure of the medium, throughout, the same as that in the undisturbed condition. As the piston moves toward the right, *condensation* takes place in the medium; the condensation being greatest for the layer in contact with the piston and becoming less as the distance from the piston increases. Suppose now, that when the piston has reached its maximum displacement s toward the right, the *wave of condensation* has reached the section represented by a; i.e., the pressure of the medium at the section a is the same as that in the undisturbed condition, and greater for all portions to the left of a. As the piston now begins to move toward the left, the wave of condensation continues moving toward the right; but, the pressure behind the piston begins to decrease, and by the time the piston has again reached its neutral position, the pressure of the medium directly in contact with the piston is the same as that in the undisturbed condition. The wave of condensation will, in the meantime, have traveled to the section b; the distance ab being equal to Aa. The maximum condensation is now at a, and tapers off to zero from a to b and from a to A. As the piston now continues moving toward the left, the medium behind it becomes rarefied; and a *wave of rarefaction* travels toward the right. By the time the piston has reached its extreme left-hand position, the wave of rarefaction will have reached the section a, and the wave of condensation the section c; where the distances bc and ab are equal. The maximum condensation now exists at b, and the maximum rarefaction at the piston. As the piston now begins its journey toward the right, the pressure behind it begins to rise, until it reaches the neutral position, when the pressure at the piston is equal to that of the medium in the undisturbed condition. In the meantime, the wave of condensation has traveled to the section d, where the distances cd and bc are equal. Hence, during the time required by the piston to complete a period, the disturbance has traveled from A to d; and the conditions now existing are: Maximum condensation at c, maximum rarefaction at a, and at A, b, and d, the pressure is equal to that of the medium in the undisturbed condition.

As the piston now continues moving toward the right, a wave of condensation moves toward the right from A, and also from d; and by the time the piston has completed its second cycle, the disturbance will have traveled to h; where the distances dh and Ad are equal. The condition of the medium between A and d is now at every section precisely the same as it was at the end of the first cycle. Also, the condition of the medium between d and h is precisely the same as it is between A and d; i.e., the condition at e is the same as at a, at f the same as at b, at g the same as at c, etc. At the end of the third cycle the disturbance will have traveled to the right of h, a distance equal to $Ad=dh$; and the condition of this portion will also be the same as the condition of the portions from A to d and from d to h. At the end of N cycles, the disturbance will have traveled through a distance equal to the product of N and Ad.

The distance through which the disturbance travels while the piston goes through one cycle is called a *wave length*, and represented by the letter λ. Or, in other words, *this is the distance a disturbance travels before conditions are beginning to be exactly reproduced.*

From the previous discussion, it is obvious that if there be performed N vibrations per unit time, and λ is the wave length, then the speed of propagation is given by

$$S=N\lambda. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

It is important to note that the distance through which the disturbance travels during a cycle depends upon the time consumed in performing that cycle; i.e., if the *frequency—the number of vibrations per unit of time*—be increased, then according to the discussion, the wave length will be proportionately less, such that the product of wave length and frequency is constant. This is fully verified by experiment for the speed of propagation of sound in gases.

70. Speed of Propagation in Terms of Elasticity and Density. Assume, as in Fig. 6, a cylinder of indefinite length and constant cross-sectional area A, filled with a homogeneous elastic medium whose density is ρ, and pressure per unit area in the undisturbed condition p. Assume further the frictionless piston P, having applied per unit area a pressure $p+\Delta p$; where Δp is a small fractional part of p.

Now, as a matter of convenience, assume that the prism, represented in Fig. 6, is divided into unit lengths, 1, 2, 3, etc., up to N; where N represents the distance the disturbance travels in a time t. The effect of the application of a pressure to the piston, in excess of the pressure of the medium, will be twofold; i.e., the medium will be compressed and also set in motion. It is evident that when any element of the medium in the prism has reached a pressure per unit area equal to $p+\Delta p$, it cannot be further compressed, but will merely serve to transmit the applied stress to the next element.

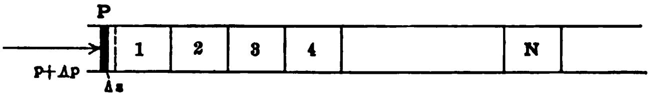

FIG. 6.

Let Δs be the amount of shortening a unit length undergoes while its pressure rises from p to $p+\Delta p$. The total shortening then, that the prism of length N undergoes in being compressed from the pressure p to a pressure $p+\Delta p$, and consequently the distance through which the piston moves during the time this change takes place, is

$$d=N\Delta s;$$

and, since the time consumed to bring about this change is t, the speed with which the piston has been moving is

$$v=\frac{d}{t}=\frac{N\Delta s}{t}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (12)$$

But, at the instant that the pulse has passed through the distance N, all that portion of matter included in the length N, of the prism, is moving with a speed the same as that of the piston as given by equation (12); hence, its kinetic energy is

$$w_1=\frac{mv^2}{2}=\frac{NA\rho}{2}\times\left(\frac{N\Delta s}{t}\right)^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

Since the total change in volume is $AN\Delta s$, and the average resisting pressure per unit area is $p+\Delta p/2$, the work, due to compression, is

$$w_2=(AN\Delta s)\left(p+\frac{\Delta p}{2}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

But the total work done on the system must be equal to w_1+w_2; hence,

$$(p+\Delta p)AN\Delta s=\frac{NA\rho}{2}\times\left(\frac{N\Delta s}{t}\right)^2+\left(p+\frac{\Delta p}{2}\right)AN\Delta s,$$

from which

$$\left(\frac{N}{t}\right)^2=\frac{\frac{\Delta p}{\Delta s}}{\rho}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

Now, $N/t=S$, the speed of propagation; and $\Delta p/\Delta s$, in the limit, represents the ratio of unit stress to unit strain, and hence, is equal to μ, the modulus of elasticity. Substituting, in equation (15), we have finally

$$S=\sqrt{\frac{\mu}{\rho}}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

i.e., the speed of propagation of a disturbance through an elastic medium is numerically equal to the square root of the *ratio* of elasticity to density.

The speed of propagation of sound in air is very readily determined by experiment; and is found to be, at 0°C., very nearly 332 meters per sec. The vibrations in a sound wave take place so rapidly that the changes are sensibly adiabatic; hence μ, in equation (16), will be replaced by the adiabatic elasticity, and we have

$$S=\sqrt{\frac{np}{\rho}},$$

from which

$$n=\frac{S^2\rho}{p}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (17)$$

Substituting numerical values, in equation (17), we find

$$n=\frac{(33{,}200)^2\times 0.001293}{1.0132\times 10^6}=1.406+.$$

The value 1.405 is generally used for dry air; but, for most practical purposes, 1.4 is sufficiently close.

CHAPTER VII

PROPAGATION OF HEAT

71. HEAT is transferred from one place to another in three distinct ways, viz, by *radiation*, by *convection*, and by *conduction*.

72. Radiation. In Chapter II, we dealt with Newton's law of cooling, without considering in what manner the cooling takes place. As a matter of fact, in the cases considered, the cooling was due to two distinct phenomena. To illustrate this, we will consider a concrete case, viz, an incandescent lamp, which consists of a filament inside of a glass bulb; the bulb having been exhausted, so that the filament is practically in a vacuum. The propagation of heat from the filament to the glass bulb, that is, through a vacuum, is called *radiation;* or, in other words, radiation is the propagation of heat through space *without the aid of any material substance.* On the other hand, the dissipation of heat from the surface of the bulb is due, not only to radiation, but also convection; and the propagation of heat from the inner surface of the bulb to the outer surface is due to conduction. The propagation by convection and conduction will be considered later.

73. Theory of Exchanges. Prévost, in 1792, promulgated the theory that there is a continual exchange of heat between bodies, even when they are at the same temperature. Prévost's theory may, perhaps, be best explained by means of the following illustration: Suppose a body suspended in a vessel, which has been completely exhausted. Assume further, that the walls of the enclosure are maintained at a constant temperature, and that the body, when first placed in the enclosure, has a temperature higher than this. The temperature of the body will immediately begin to fall, due to its radiating heat to the walls of the enclosure; and this will continue until the temperature of the body is the same as that of the walls, when it becomes constant, and the body has apparently ceased radiating. If the temperature of the walls be now reduced, by immersing the vessel in a bath of lower temperature, the temperature of the body will fall and it will again be radiating heat. The body then, apparently, ceases to radiate heat when its temperature has fallen to that of the walls of the enclosure, and again begins to radiate heat when the walls are lowered in temperature; and again ceases to radiate heat when its temperature has fallen to that of the walls, and so on indefinitely. If, initially, the temperature of the walls had been higher than that of the body, heat would have been radiated from the walls to the body.

Now, according to the theory of exchanges, the body does not cease to radiate when its temperature has fallen to that of the walls; but, the body and the walls are continually radiating and absorbing heat. That is, it is assumed that, when the body is at a higher temperature than the walls, it is radiating heat more rapidly than it is absorbing heat, when at a lower temperature than the walls, it is gaining heat more rapidly by absorption than it is losing heat by radiation, and when at the same temperature the rates of radiating and absorbing heat are the same.

Without being committed to this theory, it will be interesting to note certain conclusions which must necessarily follow from it.

74. Emissivity. Experiments on radiant heat show that some bodies emit heat, other things being equal, more copiously than others. It is also found that bodies which are good radiators are also good absorbers. The capability which a body has for emitting heat is called its *emissivity*.

Suppose now, that we have two bodies, placed in a space, impervious to heat. Then, according to Prévost's theory of exchanges, they will both radiate and absorb heat, even though they be at the same temperature. If now, one of the bodies absorbs heat more readily than it emits heat, its temperature will rise; this, however, is contradictory to experience. If, on the other hand, one of the bodies radiates heat more readily than it absorbs heat, its temperature will fall; which again contradicts experience. It therefore follows, if Prévost's theory of exchanges holds, that bodies have precisely the same capability for radiating heat that they have for absorbing heat. This appears to be in concordance with experiment.

75. Stefan's Formula. We know that Newton's law of cooling is very limited in its application; i.e., it does not hold when the difference of temperature between the body under consideration and the surrounding medium exceeds 15°C. to 20°C. In other words, it is only an approximate statement. Dulong and Petit performed a number of classical experiments by means of which they endeavored to determine the law of cooling. Their experiments were, however, limited in range of temperature; since, the maximum temperature reached was only about 240°C. From their experiments, they deduced the equation, for the quantity of heat lost per unit time,

$$Q=mk^{\tau}(k^{\theta}-1); \quad . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

where Q is the quantity of heat, τ the temperature of the enclosure, θ the difference in temperature between the enclosure and the radiating body, both measured on the centigrade scale, and m a constant, depending upon the substance and the nature of its surface. For k the value of 1.0077 was found.

Stefan, from an examination of the results obtained by Dulong and Petit, deduced a formula for the loss of heat by *radiation;* i.e.,

$$Q=k(T_1^4-T_2^4); \quad . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

where k is constant, and T_1 and T_2 are the temperatures, as measured on the ideal gas thermometer, respectively, of the radiating body and the enclosure.

Equation (2) appears to give results, in accordance with experiments, up to temperatures of about 1700°C. to 1800°C. It, however, appears from subsequent experiments, that Stefan's formula is not rigidly true; and consequently will require modification. But, for practical purposes, Stefan's formula may be considered correct for the limits of temperature as stated.

Considerable research work is still being done in regard to radiation at high temperatures; and whether, or not, a simple expression will finally be found which will be true for all temperatures, is an open question.

76. Convection. Referring again to the incandescent lamp, and considering the dissipation of heat from the surface of the bulb, we find that part of the heat is absorbed by the atmosphere surrounding the bulb, and the remainder is transferred by radiation. Due to the absorption of heat, the gases in contact with the bulb become heated and therefore change in density. This change in density destroys the equilibrium, in regard to pressure; and hence, currents are established, called *convection currents*, tending to restore equilibrium. In this manner, heat is conveyed from one portion of space to another by currents in the atmosphere; i.e., the particles in contact with the bulb become heated and are replaced by particles at a lower temperature. These particles, in turn, become heated and are replaced by other particles; each particle carrying away a certain amount of heat.

Since, in general, the density of liquids changes with change of temperature, it follows, that when a liquid is not of a uniform temperature throughout, convection currents will be established; and these will, of course, tend to bring about equilibrium. Thus, if a vessel containing a liquid, be heated at the bottom, the liquid in contact with the heated surface becomes less dense, rises, and is replaced by a portion of the liquid of higher density, which in turn becomes heated, is replaced by a denser portion, and so forth.

77. Conduction. If a body, such as a metal rod, be heated at one end, then it is found that the temperature along the rod gradually rises; i.e., *heat is transferred without the displacement of matter*. Or, to put it in another way, heat is transferred from particle to particle, in a manner such that the particles maintain their relative positions. The propagation of heat through a solid is called *conduction*.

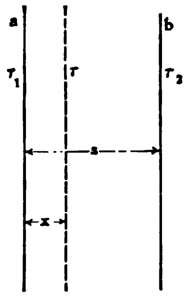

FIG. 7.

Assume that we are dealing with a homogeneous body, bounded by two parallel plane surfaces, indefinite in extent, and that one surface is maintained at a temperature τ_1, and the other at some lower temperature τ_2. Then, after a certain time, steady conditions will be established. Consider now, the simplest case possible, viz, a prism of constant cross-sectional area, normal to the two surfaces, and extending from one surface to the other. Now, since the two surfaces of the body are indefinite in area, we are justified in assuming that there is no lateral flow of heat; i.e., the heat flows through the prism in parallel stream lines, and the quantity of heat absorbed by the surface at a temperature τ_1, for a given interval of time, will be precisely equal in amount to the quantity of heat given off by the other surface, at a temperature τ_2, during the same interval of time. Or, in other words, the flow of heat through the prism will have become uniform; and the quantity of heat passing any section, parallel to the two surfaces, will be the same throughout. Then, as a fundamental principle, verified by experiment, the temperature slope, or the rate of fall of temperature along the prism, is constant. Hence, if the distance between the two surfaces is represented by s, the temperature slope is

$$r=\frac{\tau_1-\tau_2}{s}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where r is the temperature slope, or the rate of fall of temperature. The temperature at any point may be found as follows: Let, as in Fig. 7, a be the surface at a temperature of τ_1, b the surface at a temperature of τ_2, and s the distance between the two surfaces. The rate of fall of temperature between the two surfaces is given by equation (3); and the fall of temperature from the surface a to the plane x, parallel to the two surfaces, is

$$\tau=\frac{x}{s}(\tau_1-\tau_2);$$

hence, the temperature of the plane x, is

$$\tau_x=\tau_1-\frac{x}{s}(\tau_1-\tau_2). \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Theory indicates and experiment verifies that the quantity of heat which is transferred by a prism, such as has just been discussed, is proportional to the area of the exposed surfaces, to the time, and to the temperature slope. Stated symbolically

$$Q \propto Atr; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where Q is the quantity of heat transferred, A the cross-sectional area of the prism, t the time, and r the temperature slope. To make statement (5) an equality, we must introduce a proportionality factor; i.e.,

$$Q=KAtr;$$

from which

$$K=\frac{Q}{Atr}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

K is the ratio of the quantity of heat, passing any section, to the product of the area of the section, the time, and the temperature slope at that section. This ratio is called the coefficient of conductivity of the substance; and, of course, differs for different substances.

From equation (6) it follows that the coefficient of conductivity K, of a substance, is *numerically* equal to the quantity of heat which flows across a section of unit area, in unit time, when the temperature slope is unity. In the c.g.s. system, and using the centigrade scale, the coefficient of conductivity of a substance is *numer-*

ically equal to the quantity of heat, measured in gram calories, which flows across a section 1 sq.cm. in area, in 1 second, when the temperature slope at the section is 1°C. per centimeter.

78. Flow of Heat along a Bar. If a bar be maintained at a constant temperature at one end, and the remainder of the bar be exposed to a space of lower temperature, which is also maintained constant, the fall of temperature along the bar will not be the same as that of the prism previously discussed. For, since the bar is at a higher temperature than the enclosure, it will continually give up heat to the surroundings by radiation and convection. Eventually, heat will be supplied to every portion of the bar, by conduction, as rapidly as it is dissipated by radiation and convection. That is, the temperatures along the bar will finally assume steady values. But, as previously stated, the temperature slope along the bar will not be constant. For since, when a steady condition has been assumed by the bar, *the quantity of heat which passes any section, for a given interval of time, is necessarily equal to the quantity of heat which is dissipated from the bar beyond that section, for the same interval of time,* it follows that the quantity of heat which passes a section of the bar becomes less as the distance from the end, which is maintained at a constant temperature by the application of heat, increases. Hence since, other things being equal, the quantity of heat, which passes any section of the bar, is directly proportional to the temperature slope at that section, it follows that the temperature slope decreases with increase of distance from the heated end.

79. Determination of Coefficient of Conductivity. Since, it is impossible to realize in practice those ideal conditions which were assumed in the discussion of the flow of heat between two parallel walls having areas of indefinite extent, recourse must be had to other methods. A bar maintained at a constant temperature at one end, and having the remainder exposed to a space of constant temperature, furnishes a convenient means for determining the coefficient of conductivity.

To do this, we proceed as follows: After the bar has assumed a constant condition throughout, its temperature is ascertained at a number of definite points along it; this is most conveniently done by means of a thermo couple, which is calibrated by comparing with a standard thermometer. The results are then plotted, differences of temperature between the bar and its enclosure as ordinates and distances along the bar as abscissas. The curve passed through the points so found, shows the difference of temperature between the bar and the enclosure, throughout the length of the bar; and the slope of the tangent, drawn to any point of this curve is numerically equal to the temperature slope at that section. This gives us r for equation (6); and A of this equation, viz, the area of the section, is determined directly from the dimensions of the bar. It now remains to determine Q/t, i.e., the quantity of heat which passes a section per unit time. To do this, a second experiment is necessary. The bar is now heated until its temperature is uniform throughout and slightly higher than the highest temperature on the curve for the rate of fall of temperature along the bar. The bar is then placed in the enclosure, under precisely the same conditions as obtained when the curve for the rate of fall of temperature was determined, and its temperature noted at definite intervals of time. From the data so obtained, a second curve is plotted, differences of temperature between the bar and the enclosure as ordinates and times as abscissas. The curve so obtained is the *curve of cooling;* and the slope of the tangent, drawn to any point of this curve, is numerically equal to the rate of change of temperature of the bar, with respect to time, for the particular difference of temperature between the bar and its enclosure at that time.

Let it now be desired to determine the quantity of heat, which passes in a unit of time, some particular section of the bar, represented by the point a, on the curve A, of Fig. 8. Curve A is the curve representing the temperatures along the bar, and curve B, the curve of cooling. If now, that part of the bar to the right of *a* be divided into elements, such as *ab*, so short, that without appreciable error, the fall of temperature along the element may be considered constant, then the temperature of the element may be taken as the mean of the two temperatures at the points *a* and *b*. If this mean temperature be then projected across to the curve of cooling *B*, and at the point *c*, so found, a tangent be drawn, then the slope of this tangent is numerically equal to the rate of change of temperature with respect to time, for a difference of temperature equal in amount to the difference between that of the mean temperature of the element *ab* and its enclosure. If now, we take the product of the thermal capacity of *the element ab*, and the rate of change of temperature just found, we obtain *q/t*, the quantity of heat lost, per unit of time, by the element *ab* at the instant when its temperature is defined by the point *c*. But, since the temperatures of the various parts of the element *ab* are constant, the mean temperature is a constant, and differs continually from the temperature of the enclosure by an amount precisely equal to the difference of temperature as found from the curve of cooling for the instant when the temperature is represented by the point *c*. Therefore, the element *ab* is continuously losing heat, *at a constant rate*, equal in amount to the quantity just found from the curve of cooling for the temperature represented

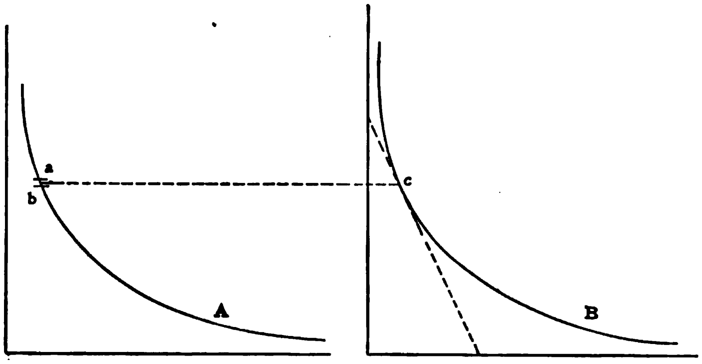

Fig. 8.

by the point c. In a similar manner, the quantities of heat, escaping per unit of time, from the various elements to the right of the element ab, are found. Taking the sum of the quantities of heat so found, for all the elements to the right of ab, the quantity of heat Q/t, of equation (6), which passes the section a, in a unit of time, is found. From which, by substitution, K is found.

Experiment shows, that, in general, the conductivity of solids decreases slightly with increase of temperature.

80. Conductivity in Non-isotropic Substances. If there be a source of heat at a point in an *isotropic* substance, i.e., a substance having like physical properties in all directions, then other things being equal, heat will be propagated with equal speeds in all directions; and the temperatures at equal distances in all directions from the source of heat, at any instant, will be found the same. Or, in other words, the source of heat will be the center of spherical isothermal surfaces. On the other hand, substances which are non-isotropic do not conduct heat with equal speeds in all directions. As an example, the conductivity of Iceland spar is greatest in the direction of the axis of symmetry, and equal in all directions at right angles to this axis. It will be remembered that the coefficient of expansion for Iceland spar is also greatest in the direction of the axis of symmetry, and equal in all directions at right angles to this axis.

81. Non-homogeneous Solids. Tyndall found, by experimenting with cubes of wood, that the speed of propagation of heat is greatest, in the direction of the fibers; i.e., parallel to the length of the tree, and least, parallel to the annual layers. And in a direction normal to both the fibers and the annual layers, i.e., radial to a section of a tree, a value was found for the conductivity slightly greater than that parallel to the annual layers; but, considerably less than that parallel to the fibers. Wood, however, on the whole is a very poor conductor in comparison with metals.

It will be of interest here to note that the speed of propagation of sound through wood is different for the three directions; i.e., the speed of propagation is greatest, parallel to the fibers, least, parallel to the annual layers, and radial to a section of the tree, it is somewhat greater than it is parallel to the annual layers, but considerably less than that parallel to the fibers.

82. Conductivity of Liquids. The determination of the coefficient of conductivity of a liquid is attended by difficulties which are not experienced when dealing with solids. For, in the case of liquids, if we wish to determine the true conductivity, convection currents must be avoided. It is, therefore, necessary to heat the column of liquid from the top. It is impossible here, to consider all the necessary precautions which must be taken to insure accurate results. The principle involved, however, is precisely the same as for solids. That is, to determine accurately the temperature slope along the column and the quantity of heat passing a given section for a definite interval of time.

83. Conductivity of Gases. The determination of the coefficient of conductivity of a gas is still more difficult than is the determination of the coefficient of conductivity of a liquid. For, in a case of a gas, not only must convection currents be eliminated, but radiation must also be taken into account. This makes it extremely difficult to obtain even fairly accurate results.

As a matter of interest, the following coefficients of conductivity for a few substances are given. They are all expressed in the c.g.s. system with the gram calorie as the unit quantity of heat. That is, the numbers in the table represent in each case, the quantity of heat, in gram calories, which passes a section 1 sq.cm. in area, in 1 second, when the temperature slope is 1°C. per cm.

Silver	1.01	Glass	0.002
Copper	0.891	Firebrick	0.0017
Aluminum	0.344	Cork	0.0007
Zinc	0.265	Paraffine	0.0002
Iron	0.167	Water	0.0014
Mercury	0.0152	Ether	0.0003
Ice	0.0057	Hydrogen	0.0004
Granite	0.005	Air	0.000056

The student must always remember that the results given in the tables for the coefficients of expansion and conductivity must be taken as being only approximate. For, the physical properties of a substance depend very largely upon its chemical purity; and, furthermore, the properties any substance may manifest, will depend very largely upon its physical history and composition. This is especially true for alloys, such as brass, organic growths, such as cork, and complex compositions and mixtures, such as glass.

It is interesting to note that, for metals, the order is the same for electrical conductivity as it is for thermal conductivity; i.e., good conductors of heat are also good conductors of electricity, and vice versa. However, there is not, as was at one time supposed, strict proportionality.

THERMODYNAMICS

CHAPTER VIII

FUNDAMENTAL PRINCIPLES

84. First Principle of Thermodynamics. The *first principle of thermodynamics* is merely the application of the principle of energy to the special case of mechanical work and heat; and may be stated as follows: *When heat is converted into work, or work into heat, the ratio of the numbers representing the two quantities involved is a constant.* The foregoing statement is, of course, the result of direct experiment.

85. Second Principle of Thermodynamics. *The second principle of thermodynamics* is stated variously by different authors. Indeed, in some cases, the statement is preceded by discussions which involve almost the whole theory of heat. For our purposes, however, the statement first enunciated by Clausius will suffice. This statement is essentially as follows: *Heat cannot pass from a body of lower temperature to one of higher temperature without the aid of some external agent.* This statement, though not the result of direct experiment, is in conformity with our common experience. As an example, we know from experience that heat passes by conduction and radiation from regions of higher temperature to regions of lower temperature. To illustrate further, assume that we are dealing with two bodies A and B, and that the temperature of the former is lower than that of the latter; then heat may be made to pass from A to B, by applying heat to A, until its temperature is the same as that of B, bringing the two bodies into contact, and by the further application of heat to A, heat will pass from it to B. Heat may also be made to pass from A to B, if work be first done on the former, such as compressing it, until its temperature is the same as that of B; then by bringing the two bodies in o contact and developing, by a further expenditure of work, more heat in A, heat will pass from it to B. But, until there is a tendency to raise the temperature of A above that of B, no heat will pass from the former to the latter. Assume, now, a third body, C, under compression and at the same temperature as A. By bringing the two bodies A and C into contact, and allowing C to expand against the external pressure, thus performing work, its temperature will fall and a certain quantity of heat will flow from A into C. The body C may now be removed from A, and compressed adiabatically until its temperature is equal to that of B, and then by bringing C into contact with B, and by a further expenditure of work on C, heat will flow from it to B. At the end of this process, C may be removed from B and allowed to expand adiabatically, and, if the various ranges have been properly chosen, it will at the end of this cycle of operations be in precisely the same condition as it was at the beginning. But, A now contains less heat, and B contains more heat than it did when the process began; and since C is in the same condition as it was at the beginning, heat has been transferred from a body of lower temperature to one of higher temperature by the aid of an external agent.

86. Heat Motors. *Heat Motors*, or *Heat Engines*, are devices by means of which energy in the form of heat, is converted into energy, in the form of mechanical motion.

All *heat motors* consist of three parts; viz, *a source of heat*, *a working substance*, and *a refrigerator*. Furthermore, all actual heat motors act periodically; i.e., operate on cycles; and for each cycle a certain quantity of heat is abstracted from the *source* by

is converted into work,
etc., is rejected to the

Before making a general
give, as an illustration, a
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reservoir of boiling water
at 0°C., and the working
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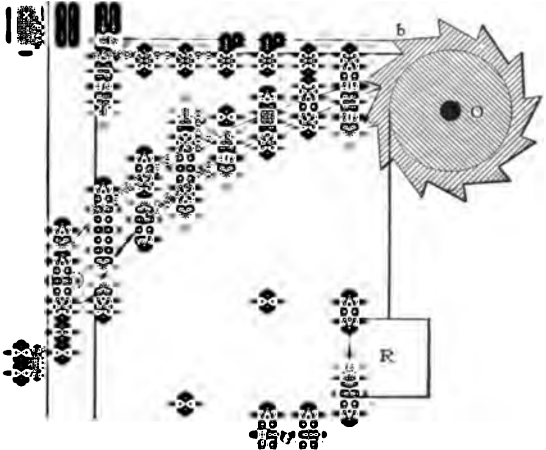

and the other end against
on an axis through *O*. A
engages one of the teeth
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ppose now, that the length
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the disk turns through an
tooth. The resistance *R*,
the pawl will engage the

next tooth. If the rod be now surrounded by a bath of melting ice, it will contract and engage the next tooth; the pawl, in the meantime, holding the disk in position. This process may be repeated indefinitely, until the resistance has been displaced through any desired distance. The *cycle* is then as follows: The rod at the temperature of melting ice is put into position, and surrounded by a bath of boiling water at a temperature of 100°C. In consequence of this elevation of temperature, the rod expands and turns the disk through a certain angle and in this manner does work in overcoming the resistance *R*. The heat taken from the *source* consists of two parts: One part being consumed in elevating the temperature of the rod, and is numerically equal to the product of the mass of the rod, its thermal capacity per unit mass, and the elevation of temperature. The other part consists of the heat equivalent of the work done in displacing the resistance *R*. The rod, now being disconnected and surrounded by melting ice, gives up to the refrigerator, in cooling from 100°C. to 0°C., an amount of heat precisely equal to that absorbed in being heated, without any external work being done, from 0°C. to 100° C. Therefore, the difference between the heat taken from the source and that given to the refrigerator is equivalent to the work done in displacing the resistance *R*. Since this completes a cycle it may be repeated indefinitely without any change in the relation of the quantities involved.

If, now, we represent by Q_1, the quantity of heat absorbed from the source, during a cycle, and by Q_2, the heat rejected to the refrigerator, then the external work done is

$$W=J(Q_1-Q_2); \quad \ldots\ldots \quad (1)$$

where W is the external work done, and J the mechanical equivalent of heat. Since for every cycle the quantity of heat Q_1 has forever disappeared from the source, and only the part Q_1-Q_2 has been converted into work, it follows that, with the contrivance just described, it is impossible to convert all the heat, taken from the source, into *external* work.

88. Heat of Expansion. As explained in *Arts.* 46 and 47, when a gas is heated and expands against an external pressure, the heat required is practically equal to that required to elevate the temperature of the gas, plus the heat equivalent of the external work done; i.e., the external work done, expressed in heat units, is practically equal to the *heat of expansion.* This, however, is by no means the case when a metal rod is heated and expands against an external pressure; for, in this case, the *heat of expansion* consists of two parts, viz, the heat equivalent of the external work done, and the heat required to expand the rod against *its own inherent forces.* The former may be called the *external* heat of expansion, and the latter the *internal* heat of expansion.

In the present state of our knowledge we are unable to assign the proper relative values for the heat consumed in elevating the temperature of a substance and the internal heat of expansion; but, for most substances, the latter is a relatively large quantity.

Since, now, in the cycle discussed in *Art.* 87, the internal heat of expansion is not recovered as work, but is rejected to the refrigerator, it follows that such a contrivance cannot use heat economically.

89. Carnot's Cycle. The first scientific discussion of a periodically acting thermodynamic engine is that due to Sadi Carnot, published in 1824. In this discussion, ideal conditions are assumed; i.e., it is assumed that there are no losses due to radiation and friction. In other words, it was Carnot's object to show that under certain given conditions, assuming ideal processes, a definite fractional part of the heat taken from the source, by a periodically acting engine, is converted into work; and that, for the given conditions, this is the maximal amount of work that may be realized. The following demonstration will make this clear. Assume that we are dealing with any working substance whatsoever, confined in such a manner that it may be put into contact [illegible] igerator, whose temperatures [illegible] cess, and that the conduction [illegible] both the source and refrig- [illegible] ideal conditions, it must be [illegible] bstance such that adiabatic [illegible]

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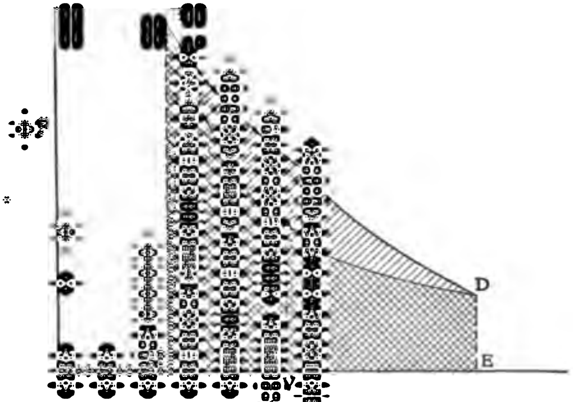

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[illegible] is now insulated and *com-* [illegible] ce of which its temperature [illegible] ued until the temperature of [illegible] hat of the source, and its con- [illegible] olume, is represented by the [illegible] ubstance during this compres- [illegible] nder the curve; i.e., the area [illegible] s now put into contact with [illegible] *isothermally*, by any desired [illegible] ds pressure and volume, at the [illegible] by the point C. During this isothermal expansion a certain quantity of heat Q_1, has been abstracted from the source, and *work* has been done *by* the working substance, represented by the area *BCFK*. The working substance is now again insulated and allowed to expand *adiabatically*, in consequence of which its temperature falls. This expansion is continued until the temperature of the working substance has fallen to that of the refrigerator, and the *work* done *by* it, during this expansion, is represented by the area *CDEF*. The working substance is now put into contact with the refrigerator and compressed *isothermally* until its condition, as regards pressure and volume, is again represented by the point *A*. During this isothermal compression, *work* was done *on* the working substance represented by the area *DEGA*; and, a quantity of heat Q_2, was rejected to the refrigerator.

Since now, the working substance, as regards pressure, volume, and temperature, is in precisely the same condition as it was at the beginning of the cycle, its *intrinsic* energy is also the same; it therefore follows, from the first principle of thermodynamics, that the difference between the heat abstracted from the source and that rejected to the refrigerator, expressed in mechanical units, is equal to the *net work done.* By an inspection of Fig. 10 it is obvious that the *net work done* is represented by the area *ABCD*; and, from equation (1), we have

$$W_1 = J(Q_1 - Q_2);$$

where W_1 is the net work done.

But the heat, expressed in mechanical units, abstracted from the source is

$$W_2 = JQ_1;$$

therefore, the *ideal coefficient of conversion,* or the maximal fractional part of the heat, abstracted from the source, which in an *ideal process* can be converted into work, is

$$\eta = \frac{J(Q_1 - Q_2)}{JQ_1} = \frac{Q_1 - Q_2}{Q_1}. \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The result, just found, has been deduced without making any assumption in regard to the nature of the working substance; it is therefore perfectly general.

Suppose that the working substance suffers a physical change of state during the cycle; the foregoing demonstration still holds. For, since the working substance is in precisely the same condition as regards pressure, volume, and temperature at the end of the cycle as it was at the beginning, it follows that whatever physical changes of state have taken place during any part of the cycle, changes of a like kind must have taken place in the reverse order during some other part of the cycle; and hence, are balanced. Therefore, the difference between the heat taken from the source and that rejected to the refrigerator, expressed in mechanical units, is equal to the external work done.

90. Since the relation, expressed in equation (2), was deduced without considering the properties of the working substance, it must be independent of those properties. There being, however, no other quantities involved in the right-hand member of this equation, excepting quantities of heat, and since these do not depend upon the properties of the working substance, they must be functions of the two temperatures. That is, the quantity of heat taken from the source must be some function of the temperature of the source, and the quantity of heat rejected to the refrigerator must be some function of the temperature of the refrigerator. Just what values are to be assigned to these functions must be determined for some specific case, which is consistent with the demonstration.

91. Carnot's Cycle a Reversible Process. The ideal cycle just described is a reversible process. For, if the working substance, at that part of its cycle when its condition, as regards pressure and volume, is represented by the point A, Fig. 10, and its temperature is the same as that of the refrigerator, is put into contact with the refrigerator and allowed to expand *isothermally* to the point D, it will abstract from the refrigerator, a quantity of heat Q_2, and do an amount of external work, represented by the area $ADEG$. The working substance is then insulated and compressed *adiabatically*, in consequence of which its temperature will rise; let this be continued until its temperature is the same as that of the source, and its condition, as regards pressure and volume, is represented by the point C, and an amount of work, represented by the area $FEDC$, has been done on the working substance. The working substance is now put into contact with the source, and compressed *isothermally* until its condition, as regards pressure and volume, is represented by the point B, a quantity of heat Q_1 being rejected to the source, and an amount of work, represented by the area $FCBK$, has been done on the working substance. The working substance is now insulated and allowed to expand *adiabatically* until its temperature has fallen to that of the refrigerator; its pressure and volume being the same as at the beginning, and the external work, represented by the area $BAGK$, having been done by it. Taking the sum, we find that the work done *by* the working substance is represented by the area $KBADE$; and the work done *on* the working substance is represented by the area $BKEDC$. Finally, the *net* work done *on* the working substance is represented by the area $ADCB$. But, during this process, the quantity of heat Q_2 has been taken from the refrigerator, and the quantity of heat Q_1 has been transferred to the source.

Since now, the working substance is in precisely the same condition as regards temperature, pressure, and volume, as it was initially, it follows that the difference between the heat rejected to the source and that taken from the refrigerator, expressed in mechanical units, is equal to the *net* work done *on* the working substance; i.e.,

$$W=J(Q_1-Q_2).$$

92. It will now be shown that, for a given source and refrigerator, an engine operating on the Carnot cycle, that is, a *reversible* *engine,* converts into work as large a fractional part of the heat taken from the source as is possible under the assumed conditions. To do this, we assume that we have two engines A and B, operating between the same source and refrigerator, the former acting direct and driving the latter, which is running reversed. Let H_a' and H_a'' be, respectively, the heat taken from the source and that rejected to the refrigerator by the engine A during a given interval of time; and likewise, let H_b' and H_b'' be, respectively, the heat transferred to the source and that abstracted from the refrigerator by the engine B, during the same interval of time. Assume, now, that the engine A, which is *non-reversible,* can convert a larger fractional part of the heat taken from the source into work than could the engine B if it were running direct. We then have

$$\frac{H_a'-H_a''}{H_a'} > \frac{H_b'-H_b''}{H_b'}. \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Also, the work done *by* the engine A must be equal to the work done *on* the engine B, since the former is driving the latter; hence, we have

$$W = J(H_a'-H_a'') = J(H_b'-H_b''). \quad . \quad . \quad . \quad . \quad (4)$$

From equation (4) it follows that the numerators of the inequality, expressed by statement (3), are equal; hence

$$H_a' < H_b'.$$

Also, from equation (4), we find

$$H_b'-H_a' = H_b''-H_a'';$$

hence, since H_b' is greater than H_a', H_b'' must be greater than H_a''. But, with a reversible engine, operating on the Carnot cycle between a certain source and refrigerator, the quantities of heat involved are always the same for a definite amount of work, no matter whether the engine is acting direct or reversed;

it therefore follows that under the assumed conditions, the source must be gaining heat, since the quantity of heat H_a' taken from it by the non-reversible engine is less than the quantity of heat H_b' rejected to it by the reversible engine. Likewise, since the quantity of heat H_a'' rejected to the refrigerator, by the non-reversible engine, is less than H_b'', that taken from it by the reversible engine, it follows that the refrigerator is continually losing heat. Hence, under the assumed conditions, we have a system in which heat is being transferred from a body of lower temperature to one of higher temperature, without the aid of an agent external to the system. Since this, however, contradicts the second principle of thermodynamics, we must conclude that the assumption made, viz, that any engine can convert into work a larger fractional part of the heat taken from the source than is possible by means of a reversible engine, operating on a Carnot cycle between the same source and refrigerator, is in error. Therefore, a reversible engine converts into work as large a fractional part of the heat taken from the source as is possible under the given conditions.

We may, however, consider this in another manner. Assume that there is a third engine, operating between the same source and refrigerator, abstracting heat from the source, and rejecting heat to the refrigerator at a rate such that both the source and refrigerator are maintained at a constant temperature. This third engine may then be employed in doing external work, and we have a system which is doing work without the expenditure of energy. This, however, contradicts the principle of energy; hence we must again conclude that the original assumption is in error. Hence the conclusion that, *for any given conditions, no engine can convert into work a larger fractional part of the heat taken from the source than that converted into work by a reversible engine.*

93. Reversible Engine as a Standard. It must be remembered that all processes so far discussed, in this chapter, are *ideal* processes and cannot be realized in practice; in other words, since the cycles of an actual thermodynamic engine, are attended by friction and radiation, they are necessarily irreversible. As a matter of fact, as stated in *Art.* 27, all processes are *irreversible. Reversible* processes are merely *ideal;* i.e., conceptions of perfect operations.

When we speak of the efficiency of a mechanical contrivance, as being the fraction p, we simply mean that the output is the fractional part p of the input. And the closer p approaches unity, the nearer the machine is considered to be to perfection. But here again, our standard is an ideal one; i.e., we are comparing our actual machine with one that is *ideally* perfect. Since it has been shown that a reversible engine, operating on the Carnot cycle, between a given source and refrigerator, converts into work as large a fractional part of heat taken from the source as possibly can be converted into work under the given conditions, we are justified in taking this engine as a standard with which to compare the performance of actual engines.

94. Carnot's Cycle with a Perfect Gas as a Working Substance. Assume that we have confined in a cylinder, by means of a frictionless piston, a perfect gas, and that there is a source of heat at a temperature T_1, and a refrigerator at a temperature T_2. Let the condition of the gas, as regards pressure and volume, be represented by the point A, Fig. 11, when put into contact with the source; pressures being represented by ordinates and volumes by abscissas. We will, furthermore, assume ideal conditions; i.e., perfect conduction for the isothermal processes, perfect insulation so that adiabatic processes may take place, and no losses. Consider now, the four processes as follows:

(1) The cylinder containing the gas at the pressure p_1, volume v_1, and temperature T_1, is put into contact with the source of heat at temperature T_1, and the gas is allowed to expand *isothermally* by any desired amount, say to the point B; its pressure now being p_2 and volume v_2. During this expansion work is done *on* the piston *by* the gas, measured by the area under the curve AB, and a quantity of heat Q_1 is taken from the source. It being

assumed that the temperature of the source during the abstraction of the heat Q_1, remains constant; this may be brought about by supplying heat to it at a proper rate.

(2) The cylinder is now removed from the source, is perfectly insulated, and the gas is allowed to expand *adiabatically*, in consequence of which its temperature falls, due to the fact that the external work is done at the expense of the *intrinsic* energy of the gas. This expansion is continued until the temperature of the gas has fallen to T_2, that of the refrigerator; in the meantime, work has been done *on* the piston *by* the gas, measured by the area under the curve *BC*. The pressure is now p_3 and the volume v_3.

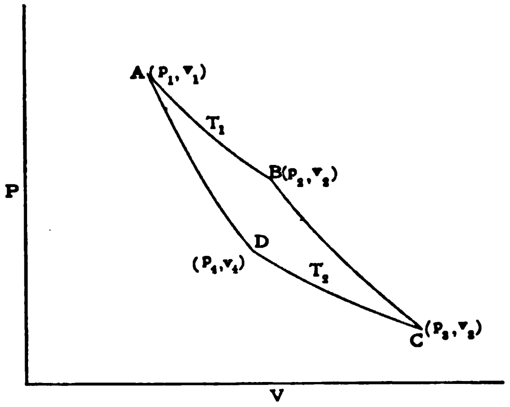


Fig. 11.

(3) The cylinder is now put into contact with the refrigerator and the gas is compressed *isothermally*, until its pressure is p_4, and volume v_4, as represented by the point *D*. During this compression a quantity of heat Q_2 is developed, and work is done *by* the piston *on* the gas, measured by the area under the curve *DC*. It being assumed that the temperature of the refrigerator, during the absorption of the heat Q_2, remains constant; this may be brought about by abstracting heat from it at the proper rate.

(4) The cylinder is now removed from the refrigerator, is perfectly insulated, and the gas is compressed *adiabatically* until

its temperature is T_1, that of the source, and its pressure and volume are, respectively, p_1 and v_1. During this compression work was done, *by* the piston *on* the gas, measured by the area under the curve AD. The gas being now in precisely the same condition as it was initially, its *intrinsic* energy must also be the same.

Since, now, A and B are on the same *isotherm*, and likewise, C and D are on the same isotherm, we have, from the characteristic equation,

$$p_1v_1=p_2v_2=RT_1, \quad \dots\dots\quad (5)$$

and

$$p_3v_3=p_4v_4=RT_2. \quad \dots\dots\quad (6)$$

Also, since B and C are on the same adiabatic, and likewise, A and D are on the same adiabatic, we have

$$p_2v_2^n=p_3v_3^n, \quad \dots\dots\quad (7)$$

and

$$p_1v_1^n=p_4v_4^n. \quad \dots\dots\quad (8)$$

From equation (5) we find

$$p_1=\frac{RT_1}{v_1}, \quad \dots\dots\quad (9)$$

and

$$p_2=\frac{RT_1}{v_2}. \quad \dots\dots\quad (10)$$

From equation (6) we find

$$p_3=\frac{RT_2}{v_3}, \quad \dots\dots\quad (11)$$

and

$$p_4=\frac{RT_2}{v_4}. \quad \dots\dots\quad (12)$$

Substituting the value of p_2 as given in equation (10), and that of p_3 as given in equation (11), in equation (7) we find

$$\frac{RT_1}{v_2}v_2^n=\frac{RT_2}{v_3}v_3^n;$$

from which

$$\frac{v_3}{v_2}=\left(\frac{T_1}{T_2}\right)^{\frac{1}{n-1}}. \quad . \quad . \quad . \quad . \quad . \qquad (13)$$

Again, substituting in equation (8) the values of p_1 and p_4, as given by equations (9) and (12), we find

$$\frac{RT_1}{v_1}v_1{}^n=\frac{RT_2}{v_4}v_4{}^n;$$

from which

$$\frac{v_4}{v_1}=\left(\frac{T_1}{T_2}\right)^{\frac{1}{n-1}}. \quad . \quad . \quad . \quad . \quad . \qquad (14)$$

From equations (13) and (14) it follows that

$$\frac{v_3}{v_2}=\frac{v_4}{v_1};$$

from which

$$\frac{v_3}{v_4}=\frac{v_2}{v_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (15)$$

Equation (15) shows that the volume at C must be to the volume at D, as the volume at B is to the volume at A, so that when the gas is compressed adiabatically from D, it will come to the point A.

Since we are dealing with a perfect gas, its *intrinsic* energy is a function of the temperature only, and is, therefore, independent of pressure and volume. Therefore, the work done *by* the gas in going along the adiabatic from B to C, is exactly equal to the work done *on* the gas in going along the adiabatic from D to A. Hence to obtain the *net* work done during the cycle, and the quantities of heat involved, it is only necessary to consider the two isothermal processes; viz, the heat abstracted from the source, and the external work done, by the gas in going from A to B, along the isotherm T_1, and the heat rejected to the refrigerator, and work done, on the gas, in going along the isotherm T_2 from C to D.

Since the temperature, during the isothermal expansion, from A to B is constant, it follows that the heat abstracted from the source is directly proportional to the external work done. Hence we have

$$Q_1 = A\int_{v_1}^{v_2} p\,dv;$$

where A is the heat equivalent of a unit of work. But p, for any part of this process is equal to RT_1/v; hence

$$Q_1 = ART_1\int_{v_1}^{v_2}\frac{dv}{v}$$

$$= ART_1 \log\frac{v_2}{v_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

By similar reasoning we find, that the heat rejected to the refrigerator, during the isothermal compression, in going from C to D, is

$$Q_2 = A\int_{v_4}^{v_3} p\,dv.$$

But, for any part of this process p is equal to RT_2/v; hence

$$Q_2 = ART_2\int_{v_4}^{v_3}\frac{dv}{v}$$

$$= ART_2 \log\frac{v_3}{v_4}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

The net work done, measured in heat units is, by condition, proportional to the difference between the heat taken from the source and that rejected to the refrigerator; hence

$$AW = Q_1 - Q_2 = AR\left(T_1 \log\frac{v_2}{v_1} - T_2 \log\frac{v_3}{v_4}\right);$$

and the ideal coefficient of conversion, since Q_1 has forever disappeared from the source, is

$$\eta=\frac{Q_1-Q_2}{Q_1}=\frac{AR\left(T_1\log\frac{v_2}{v_1}-T_2\log\frac{v_3}{v_4}\right)}{ART_1\log\frac{v_2}{v_1}};$$

from which, since by equation (15), $v_2/v_1=v_3/v_4$, we find

$$\eta=\frac{Q_1-Q_2}{Q_1}=\frac{T_1-T_2}{T_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

Equation (18) shows that, for a perfect gas operating on a Carnot cycle, the *ideal coefficient of conversion* is the ratio of the difference in temperature of source and refrigerator, to the temperature of the source, as measured on the ideal gas thermometer.

Equation (18) is usually written in the following form:

$$\eta=\frac{S-R}{S}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

where S is the temperature of the source and R that of the refrigerator, both being measured by means of the ideal gas thermometer.

95. A little consideration will show that the foregoing discussion, and result obtained, is perfectly consistent in every way with that of the Carnot cycle using any working substance. We are therefore justified (*Art.* 90) in assuming that even under ideal conditions the maximum quantity of work that can be realized from an engine working between a given source and refrigerator and absorbing the quantity of heat H from the source, is

$$W=JH\left(\frac{S-R}{S}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

Writing equation (19) in another form, we have

$$\eta=1-\frac{R}{S};$$

from which it is obvious that, for η to approach *unity*, R must either approach zero, or S must approach infinity. Experience, however, shows that it is not economical to attempt to maintain the refrigerator at a temperature lower than that of the surroundings. Also, as the temperature of the source is increased, a point is soon reached for which radiation and pressures become excessive, and lubrication becomes difficult. It therefore follows, with conditions such as obtain on the earth's surface, that even a perfect engine can convert only a small fractional part of the heat, taken from a source, into work.

96. Reversible Engine and Refrigeration. An engine operating in a reverse order, i.e., one that is taking heat from a body of lower temperature, transferring heat to a body of higher temperature, and absorbing external work, constitutes *a refrigerating machine.* Let, for any given time, H_1 be the quantity of heat transferred to a body of higher temperature, and H_2 the quantity of heat abstracted from a body of lower temperature, by a perfectly reversible engine; i.e., a perfect refrigerating machine. We will then have the following relation:

$$\frac{H_1-H_2}{H_1}=\frac{S-R}{S}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (21)$$

From equation (20) we have, for the amount of work that must be done, to transfer the quantity of heat H_1, to the body of higher temperature,

$$W=JH_1\frac{S-R}{S}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (22)$$

In general, however, in the case of refrigerating machines, we are concerned principally with the work that must be done to bring about a certain absorption from the body of lower temperature; i.e., the amount of refrigeration. It is therefore advisable to deduce an expression for the amount of work that must be done in terms of H_2, the quantity of heat taken from the body of lower temperature, instead of the quantity of heat H_1, rejected to the body of higher temperature.

From equation (21) we find

$$\frac{H_2}{H_1}=\frac{R}{S};$$

from which

$$H_1=H_2\frac{S}{R}. \quad \text{. . . . .} \quad (23)$$

Substituting in equation (22) the value of H_1, as given by equation (23), we find

$$W=JH_2\frac{S-R}{R}; \quad \text{. . . . . . .} \quad (24)$$

which gives the desired relation.

97. In *Art.* 95, it was stated that it is not economical to attempt to maintain the temperature of the refrigerator lower than that of the surrounding medium. We are now prepared to demonstrate this mathematically.

Let H_1 be the heat taken from the source, at a temperature S, and let R_1 be the temperature of the surroundings. If then the temperature of the refrigerator be also R_1, the work that, under perfect conditions, may be realized is

$$W_1=JH_1\frac{S-R_1}{S}. \quad \text{. . . . . .} \quad (25)$$

Assume now, that the refrigerator, by means of a reversible engine, is maintained at some temperature R_2, lower than R_1. The work that can now be realized, by means of a perfect engine, is

$$W_2=JH_1\frac{S-R_2}{S}. \quad \text{. . . . . .} \quad (26)$$

Subtracting equation (25) from equation (26), member by member, we obtain, due to lowering the temperature of the refrigerator, for the gain in work,

$$W_2-W_1=J\frac{H_1}{S}(R_1-R_2). \quad \text{. . . . . .} \quad (27)$$

To maintain the temperature R_2 we must, by means of a reversible engine, abstract heat from the refrigerator at the same rate that the direct engine is rejecting heat to it, and transfer heat to the surroundings. The heat rejected by the direct engine is

$$H_2=H_1-AW_2=H_1-H_1\frac{S-R_2}{S}=H_1\frac{R_2}{S}. \quad . \quad . \quad (28)$$

The work that must be expended in transferring this quantity of heat from the body of temperature R_2, to the surroundings at a temperature R_1, is

$$W_3=JH_1\frac{R_2}{S}\times\frac{R_1-R_2}{R_2}$$

$$=J\frac{H_1}{S}(R_1-R_2). \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

By comparing equations (29) and (27), it is obvious that, even under ideal conditions, the amount of work that must be done by the reversible engine, to maintain the temperature of the refrigerator, below that of the surroundings, is equal to the gain in work by the direct engine, due to the lower temperature of the refrigerator. It therefore follows that, even without considering losses, there can be nothing gained by attempting to have the temperature of the refrigerator lower than that of the earth's surface. As a matter of fact, if the temperature of the refrigerator is lower than that of the surroundings, heat will continually pass from the surroundings to the refrigerator, and the reversible engine must do an amount of work greater than that given by equation (29). Furthermore, due to imperfections of the engines, the gain in work realized by the direct engine will be less than that specified by equation (27), and the work that must be done on the reversible engine will be greater than that specified by equation (29); hence, there is a decided loss when the refrigerator is maintained at a temperature lower than that of the surrounding media.

98. Thermodynamic Scale of Temperatures. *The thermodynamic scale of temperatures,* which was first proposed by Lord Kelvin, will be made clear by the following considerations. Assume a series of n perfect heat engines arranged in such a manner that the refrigerator of the first engine is the source of the second engine, the refrigerator of the second engine is the source of the third engine, etc., and furthermore, that the heat rejected by any engine is absorbed by the engine next lower in the scale. To show that, if the difference in temperature between source and refrigerator for the various engines is the same, they are all doing the same amount of work.

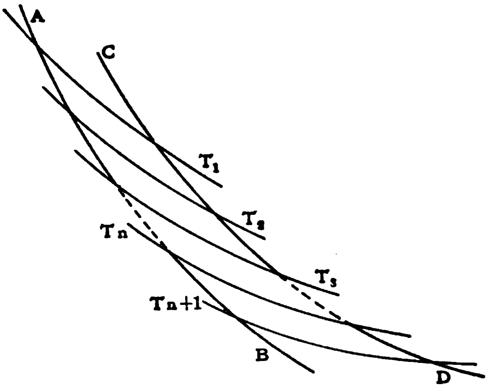


FIG. 12.

Let, as in Fig. 12, the two adiabatics, AB and CD, be cut by the isotherms T_1, T_2, T_3, etc., such that the temperature intervals are all equal, and each equal to τ; i.e.,

$$T_1-T_2=T_2-T_3=T_n-T_{n+1}=\tau.$$

The ideal coefficients of conversion for the various engines, beginning with the first, then are

$$\frac{\tau}{T_1}, \ \frac{\tau}{T_2}, \ \frac{\tau}{T_3}, \cdots \frac{\tau}{T_n}. \quad . \quad . \quad . \quad . \quad . \quad (30)$$

If H is the quantity of heat absorbed by the first engine from its source, during a given interval of time, then

$$H-H\frac{T_1-T_2}{T_1}=H\frac{T_2}{T_1}$$

is the heat rejected to its refrigerator, and absorbed by the second engine, during the same interval of time. In a similar manner, the quantity of heat supplied to the third engine is

$$H\frac{T_2}{T_1}-H\frac{T_2}{T_1}\times\frac{T_2-T_3}{T_2}=H\frac{T_3}{T_1}.$$

The quantities of heat supplied to the various engines, beginning with the first, then are

$$H, \quad H\frac{T_2}{T_1}, \quad H\frac{T_3}{T_1}, \ldots H\frac{T_{n-1}}{T_1}, \quad H\frac{T_n}{T_1}. \quad . \quad . \quad (31)$$

Since, now, the work done by any engine of the series is equal to the product of its ideal coefficient of conversion and quantity of heat, expressed in mechanical units, absorbed by it, it follows from expressions (30) and (31), that all the engines are doing the same amount of work; i.e.,

$$W=JH\frac{\tau}{T_1},$$

is the work done by each engine of the series.

The results just deduced, being independent of the properties of any substance, a thermodynamic scale of temperature may be established in the following manner: Assume a series of n heat engines, working between a given source of temperature T_1, and refrigerator of temperature T_{n+1}, in such a manner that the n engines are all doing the same amount of work, and each engine is absorbing the heat rejected by the engine next higher on the scale. If we then designate the difference of temperature between the source and refrigerator of any one of these ideal engines, as a unit of temperature, we will have a scale of temperatures independent of any substance, and depending only upon the performance of a perfect engine. But, from the discussion just given, we found that by assuming the temperature intervals, as measured on the ideal gas thermometer, equal, the engines were all doing the same amount of work; hence, the thermodynamic scale is identical with that of an ideal gas thermometer; and differs but slightly, for temperatures not exceeding 500°C.; from those as found by means of the ordinary gas thermometer.

CHAPTER IX

STEAM AND STEAM ENGINES

99. The proper design of a heat engine presupposes, on the part of the designer, a knowledge of the construction of mechanical contrivances; i.e., how to construct a machine which shall withstand the stresses imposed upon it in the performance of its duties, with the *lowest cost.* The expression, *lowest cost,* must not be interpreted as meaning lowest *first* cost; but it must be understood to mean that the interest on the capital invested, for both machinery and ground rent, plus depreciation, plus cost of power lost, must be a minimum. This part of the subject comes under the heading of *machine design*; and, properly speaking, has nothing to do, *except in so far as fuel economy is affected by the design,* with the subject of thermodynamics. But, a thorough knowledge of the characteristics of the *working substance* and the changes it undergoes, during its various stages, is fully as important, if not more so, in the designing of an engine, as is a knowledge of machine design. It is for this reason, since steam is so widely used as a working substance, that so much research work has been done, to accurately determine its characteristics.

100. Steam Operating on Carnot's Cycle. Assume that we are dealing with a unit mass of water, at a temperature T_2, which corresponds to that of the refrigerator, and let its condition, as regards pressure and volume, be represented by the point D of Fig. 13. The water is compressed adiabatically until its temperature is T_1, that of the source, and its condition, as regards pressure and volume, is represented by the point A. If the water is now placed into contact with the source, and the pressure is maintained constant, vaporization will take place. Assume this to be continued until all the water has been converted into saturated steam, whose condition, as regards pressure and volume, is represented by the point *B*. The steam is now allowed to expand adiabatically until its temperature has fallen to T_2, that of the refrigerator; its pressure and volume being now represented by the point *C*. During this adiabatic expansion a certain amount of condensation, which will be discussed later, has taken place. The mixture of steam and water is now put into contact with the refrigerator and compressed isothermally until complete condensation has taken place, and its condition, as regards pressure and volume, is again represented by the point *D*. Since, now, the condition of the working substance, as regards temperature, pressure, and volume, is precisely the same as it was initially, its *intrinsic* energy is also the same. Therefore, the net work done during the cycle is measured by the area *DABC*. Furthermore, since the process is *ideally reversible*, the ideal coefficient of conversion is

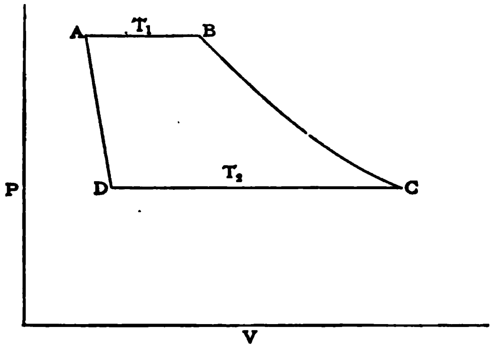


FIG. 13.

$$\eta=\frac{T_1-T_2}{T_1};$$

the same as previously deduced for any working substance.

101. Relation of Temperature and Density of Saturated Steam. It is frequently of prime importance to know the density of saturated steam for a given temperature; and it being difficult to determine this relation by direct experiment, it will be shown how it is found from the relation of pressure and temperature of a saturated vapor, this being easily determined by direct experiment. To show how to determine the relation of temperature and density of the saturated vapor of a substance, it will be assumed that we are dealing with a unit mass operating on a Carnot cycle, as just described, and an indefinitely small difference of temperature, ΔT, between source and refrigerator. This is represented diagrammatically in Fig. 14, where $T+\Delta T$ is the temperature of the source, and T the temperature of the refrigerator. This being a reversible process, the ideal coefficient of conversion is

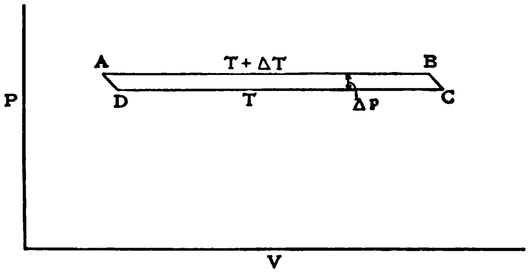


FIG. 14.

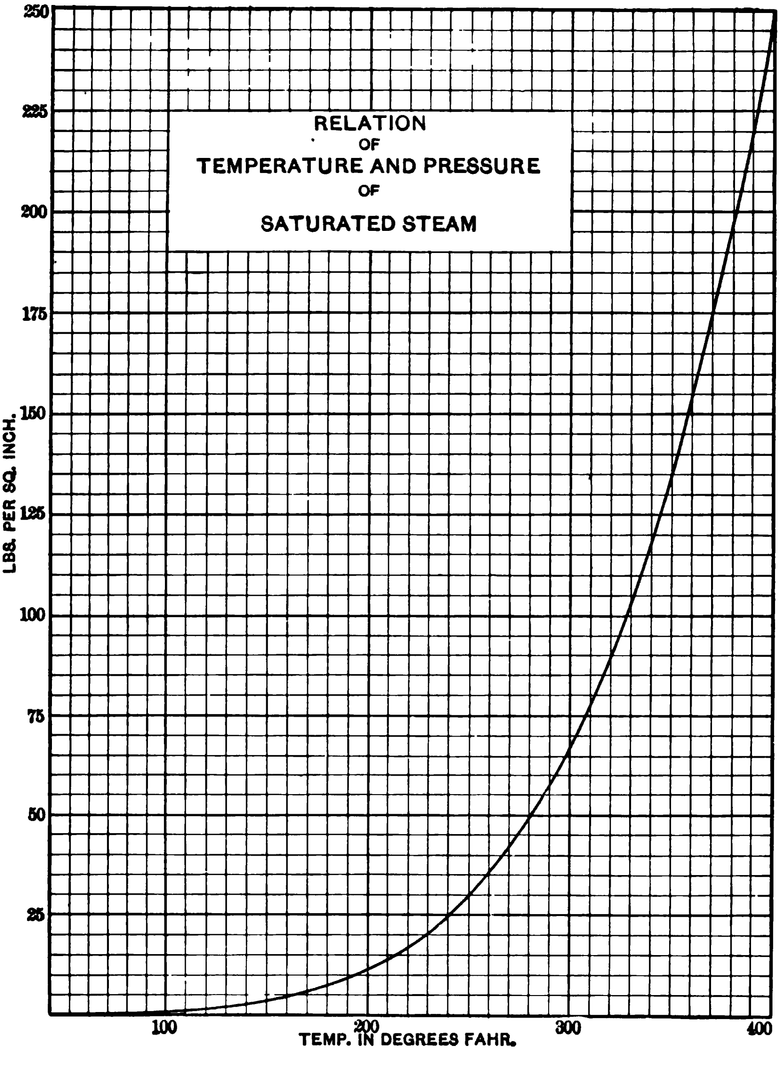
$$\eta=\frac{\Delta T}{T+\Delta T},$$

which, in the limit, becomes

$$\eta=\frac{dT}{T}.$$

If the quantity of heat taken from the source, in going from A to B, is Q, then the work done is

$$W=JQ\frac{dT}{T}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$


RELATION
OF
TEMPERATURE AND PRESSURE
OF
SATURATED STEAM
LBS. PER SQ. INCH.
250
225
200
175
150
125
100
75
50
25
100
200
300
400
TEMP. IN DEGREES FAHR.

The work done during the cycle may also be expressed in terms of the initial and final volumes and the change in pressure dp, corresponding to the change in temperature dT. That is, if σ is the volume of unit mass of the liquid, and s the volume of unit mass of saturated vapor, then the work done, during the cycle, is

$$W=(s-\sigma)dp. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Now the right-hand members of equations (1) and (2) must be equal; since they are expressions for the same amount of work, hence

$$JQ\frac{dT}{T}=(s-\sigma)dp;$$

from which

$$s-\sigma=\frac{JQ}{T}\times\frac{dT}{dp},$$

and

$$s=\sigma+\frac{JQ}{T}\times\frac{dT}{dp}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The quantity Q, in equation (3), represents the quantity of heat required to convert unit mass of the liquid into a saturated vapor at the temperature T, and may be replaced by r, the heat of vaporization; hence, equation (3) becomes

$$s=\sigma+\frac{Jr}{T}\times\frac{dT}{dp}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

In equation (4), σ, the volume of unit mass of the liquid, for the temperature T, is readily found by experiment; and likewise r, the heat of vaporization. dT/dp is found from the curve giving the relation of temperature and pressure of the saturated vapor. Hence, since J, the mechanical equivalent of heat, is known, s is determinate; and the reciprocal of this gives the density of the saturated vapor.

As a matter of interest, the curve showing the relation of temperature and pressure, for saturated steam, is given on page 119.

102. Perfect Steam Engine and Boiler. In the previous discussions it has been assumed that all of the heat is taken in at the highest temperature. This, however, is by no means the case, even under perfect conditions, with a steam engine and boiler.

For the present, we will confine ourselves to the operation of a reciprocating engine, which has supplied to it saturated steam from a boiler. The reciprocating engine consists essentially of the following parts: A source of heat, the *boiler*, where steam is generated under a constant pressure, and hence, at a constant temperature, a *cylinder and piston*, and a refrigerator, or *condenser*, at constant temperature, by means of which the steam, after expanding and doing work against the piston, is converted into water and returned to the boiler. The cycle of operations is as follows: The piston P is at the position as represented in the diagram, Fig. 15, and the condition of the steam, as regards pressure and volume, is represented by the point A, the point of admission. That is, at this point, the valve in the pipe connecting the boiler with the cylinder is opened, and steam is freely admitted. The piston advances to the point B, while vaporization takes place at the temperature T_1. To simplify matters, we will assume that we are dealing with unit mass of water and that complete evaporation has taken place when the volume is represented by the point B. The quantity of heat then, taken from the boiler, is r_1, the heat of vaporization at the temperature T_1. The line of admission, AB, is a straight line and parallel to the axis of volumes, since vaporization has taken place at constant temperature; and hence, at constant pressure. The external work done, during this advance of the piston, is measured by the area $ABFE$. The point B is the point of cut-off; i.e., the *admission-valve* is closed, and the steam is allowed to expand adiabatically until its temperature has fallen to T_2, that of the condenser.

In the meantime, the external work, measured by the area *BCGF*, has been done. The *exhaust-valve* now opens, and the steam remaining in the cylinder, is compressed isothermally, in contact with the condenser, until complete condensation has taken place, and the work represented by the area *DCGE*, has been done by the piston. The condensed steam, at the temperature T_2, is returned to the boiler and heated from the temperature T_2 to that of T_1, thus completing the cycle. The net work done during this cycle is evidently measured by the area *ABCD*, and is necessarily less, as will now be shown, *due to not taking in all the heat at the maximum temperature*, than that which could be realized by a Carnot cycle.

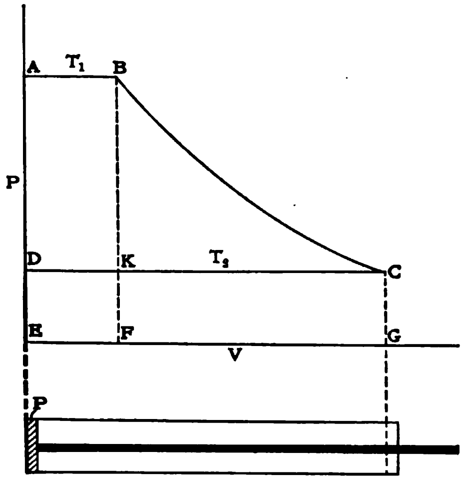


FIG. 15.

The maximum amount of work that could be realized from an engine taking in an elementary quantity of heat dQ, at the temperature T, between the temperatures T_2 and T_1, operating on a Carnot cycle, and rejecting heat to the condenser at the temperature T_2, is

$$dW = JdQ\frac{T-T_2}{T}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

But, dQ is equal to cdT; where c is the thermal capacity and dT the change in temperature. And, since we are dealing with unit mass of water, dQ is practically equal to dT; since for water c is almost constant and equal to unity. Therefore, equation (5) may be written

$$dW = J\frac{T-T_2}{T}dT; \quad . \ . \ . \ . \ . \ . \ . \ . \qquad (6)$$

from which we obtain, for the total work that could be realized, under ideal conditions, from the heat required to elevate the temperature of unit mass of water from T_2 to T_1,

$$\begin{aligned} W' &= J\int_{T_2}^{T_1}\frac{T-T_2}{T}dT \\ &= J\left(T_1 - T_2 - T_2 \log\frac{T_1}{T_2}\right). \quad . \ . \ . \qquad (7) \end{aligned}$$

The work that, under ideal conditions, could be realized from the heat taken from the source during vaporization, since this is absorbed at constant temperature, is

$$W'' = Jr_1\frac{T_1-T_2}{T_1}; \quad . \ . \ . \ . \ . \ . \ . \ . \qquad (8)$$

where r_1 is the heat of vaporization at the temperature T_1.

Adding equations (7) and (8), we obtain for the total work that may be realized, for the given conditions,

$$W_1 = W' + W'' = J\left(T_1 - T_2 - T_2 \log\frac{T_1}{T_2} + r_1\frac{T_1-T_2}{T_1}\right). \quad . \ . \qquad (9)$$

Since the total heat, abstracted from the source, expressed in mechanical units, is

$$J(T_1 - T_2 + r_1),$$

the maximum work which would have been realized, had the operation been on a Carnot cycle, is

$$W_2=J(T_1-T_2+r_1)\frac{T_1-T_2}{T_1}. \quad . \quad . \quad . \quad . \quad (10)$$

Dividing equation (9) by equation (10), we find

$$\frac{W_1}{W_2}=\frac{T_1+r_1-\frac{T_1T_2}{T_1-T_2}\log\frac{T_1}{T_2}}{T_1+r_1-T_2}. \quad . \quad . \quad . \quad . \quad (11)$$

If now, in equation (11),

$$\frac{T_1T_2}{T_1-T_2}\log\frac{T_1}{T_2}>T_2, \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

then W_1/W_2 is less than unity. To prove that the expression, given by the inequality (12), holds, we assume that T_2, the temperature of the condenser, is fixed, and that T_1 is a variable, which may be represented by T; remembering that T is always greater than T_2, and that both are positive. Expression (12) may then be written

$$\frac{TT_2}{T-T_2}\log\frac{T}{T_2}=kT_2; \quad . \quad . \quad . \quad . \quad . \quad (13)$$

where k is a proportionality factor. From this, we find

$$\log\frac{T}{T_2}=k\left(1-\frac{T_2}{T}\right). \quad . \quad . \quad . \quad . \quad . \quad (14)$$

Substituting, in equation (14), for T/T_2, a new variable, x, we have

$$\log x=k\left(1-\frac{1}{x}\right);$$

and, differentiating with respect to x, we obtain

$$k=x=\frac{T}{T_2}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

from which, if T/T_2 equals unity, k equals unity; and, if T/T_2 becomes greater than unity, k must be greater than unity or equation (15) cannot hold. But this means that the left-hand member of equation (13) must be greater than T_2; and hence W_1/W_2 is less than unity. It therefore follows that a steam engine, which rejects condensed steam to a boiler cannot, even under perfect conditions, convert into work as large a fractional part of the heat taken from the source as can an engine operating on a Carnot cycle, between the same limits of temperature.

To illustrate the foregoing, we will deal with a concrete case; i.e., assume the temperature of the entering steam, and of the condenser, respectively, 356°F. and 140°F. This gives: $T_1=816$,* $T_2=600$, and $r_1=865$. Substituting these values, in equation (11), we find

$$\frac{W_1}{W_2}=\frac{816+865-\frac{816\times600}{816-600}\log\frac{816}{600}}{816+865-600}=91.0 \text{ per cent},$$

giving a loss of about 9 per cent due to not taking in all the heat at the maximum temperature.

103. Unresisted Adiabatic Expansion of Steam. If dry saturated steam is allowed to expand adiabatically, from a chamber of given pressure to one of lower pressure, without doing work, the steam becomes superheated. This is due to the fact that, when the steam enters the chamber of lower pressure, eddy currents are developed; and as they subside, the kinetic energy, possessed by them, is converted into heat. Since the process is adiabatic, and no external work is done, the *total heat content*, i.e., the total quantity of heat contained by the steam, will be the same at the end of the process as it was at the beginning. But since, the total heat of steam decreases as the pressure is

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* According to recent experiments, the zero for the thermodynamic scale is 491.65°F. below the melting-point of ice; but, in general, 492 is sufficiently accurate.

decreased, and the final pressure is lower than the initial pressure, the steam must become superheated.

If the steam is not initially dry, then it will become drier by *unresisted* adiabatic expansion. Assume that we are dealing with a unit mass of a mixture of steam and water under a pressure p_1, for which the heat of the water and the heat of vaporization are, respectively, h_1 and r_1. The total heat of the mixture, then is

$$H=h_1+q_1r_1; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

where q_1 is the dryness; i.e., the fractional part of the liquid which is present as steam. After expansion, since the *total heat content* remains the same, we have

$$H=h_2+q_2r_2; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

where h_2, q_2, and r_2 are, respectively, the heat of the liquid, the dryness, and the heat of vaporization for the final pressure p_2.

Equating the right-hand members of equations (16) and (17), we obtain

$$h_1+q_1r_1=h_2+q_2r_2. \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

By *priming* is meant the percentage of moisture present; and if this is low, the steam may become superheated by the unresisted adiabatic expansion, and q_2, in equation (18), becomes unity. It will be shown later how, under certain conditions, advantage may be taken of this, and the initial priming determined experimentally.

104. Resisted Adiabatic Expansion. If steam, the initial priming of which is low, expands adiabatically in such a manner that external work is done, it will become wetter.

If in equation (9), *Art.* 102, it is assumed that complete evaporation has not taken place before the adiabatic expansion begins, then the work, expressed in heat units, yielded per cycle, is

$$W=T_1-T_2-T_2\log\frac{T_1}{T_2}+q_1r_1\frac{T_1-T_2}{T_1}; \quad . \quad . \quad (19)$$

where q_1 is the dryness. The total heat absorbed, in elevating the temperature of the water from T_2 to T_1, and evaporating it to the dryness q_1, is

$$H_1 = T_1 - T_2 + q_1 r_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

And since, under the assumed conditions, the difference between the heat abstracted from the source and that converted into work, must be equal to H', the heat rejected to the condenser, we find, by subtracting equation (19) from equation (20),

$$H' = q_1 r_1 \frac{T_2}{T_1} + T_2 \log \frac{T_1}{T_2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

But, the heat rejected to the condenser, after adiabatic expansion to the temperature T_2, must be equal to the heat liberated during condensation, i.e.,

$$H' = q_2 r_2; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

where q_2 is the dryness and r_2 the heat of vaporization corresponding to the temperature T_2. Equating the right-hand members of equations (21) and (22), we obtain

$$q_2 r_2 = q_1 r_1 \frac{T_2}{T_1} + T_2 \log \frac{T_1}{T_2};$$

from which

$$q_2 = \frac{T_2}{r_2}\left(\frac{q_1 r_1}{T_1} + \log \frac{T_1}{T_2}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

Equation (23) enables us to compute the dryness, during resisted adiabatic expansion, provided the initial dryness be known.

In the next chapter, the relation expressed in equation (23) will be deduced by a much simpler and shorter method.

CHAPTER X

ENTROPY

105. It is obvious that a substance, in going from one isotherm to another, always suffers the same *definite* change in temperature; and furthermore, that this change in temperature is independent of changes in pressure and volume. That is, the change in temperature in going from one isotherm to another is independent of the path pursued during the change. A good analogue of this is the change in potential a body undergoes in going from a surface of potential V_1, to a surface of potential V_2, the change in potential, $V_2 - V_1$, being independent of the path pursued in bringing about the change.

It will now be shown that, in going from one curve to another, both curves representing reversible adiabatic processes, there is *some* definite constant change. Equation (19) of *Art.* 48 specifies for reversible adiabatic processes

$$C_v\,dT + pdv = 0; \quad \dots\dots\dots \quad (1)$$

from which, by substituting for p its value as obtained from the characteristic equation,

$$pv = RT,$$

and separating the variables, we find

$$C_v\frac{dT}{T} + R\frac{dv}{v} = 0. \quad \dots\dots\dots \quad (2)$$

By integrating equation (2), between limits, we obtain

$$C_v \log\frac{T}{T_1} + R\log\frac{v}{v_1} = 0; \quad \dots\dots \quad (3)$$

where T_1 and v_1 are, respectively, the temperature and volume before the change, and T and v, respectively, the temperature and volume after the change. Since equation (3) is equal to zero, no matter what the limits of integration, it follows that there is something which does not change during a reversible adiabatic process. Integrating equation (2) for the primitive, we find

$$C_v \log T + R \log v = k; \quad . \ . \ . \ . \ . \ . \quad (4)$$

where k is a constant of integration. Equation (4) shows that the fundamental differential equation for a perfect gas, yields upon integration for a reversible adiabatic process *a constant.* But since T and v, in equation (4), may have any values whatsoever, provided, always, they are so related that the process is adiabatic, it follows that, no matter what the range, there is some function which remains constant; which conclusion is the same as that drawn from equation (3). Hence, since there is some function which remains constant during a reversible adiabatic change, there must be *some definite constant change* in going from one curve, representing a reversible adiabatic process, to another curve, representing a reversible adiabatic process.

In equation (4), the constant k evidently represents some particular condition for the gas, which remains constant, during an adiabatic process; and its value depends upon the unit of measure and zero chosen. The condition of a gas, as expressed by equation (4), was called by Clausius the *entropy* of the gas; and, as just stated, the numerical value of the entropy depends upon the units chosen and the arbitrary zero from which it is measured.

Equation (4) may now be stated as follows: *The entropy of a substance during a reversible adiabatic change remains constant.*

106. Change of Entropy. If the left-hand member of equation (1) is not equal to zero, i.e., heat is either added or abstracted while the gas changes in volume and temperature, the process is no longer adiabatic, and the equation becomes

$$dQ=C_v\,dT+pdv;$$

from which, by substituting for p its value as obtained from the characteristic equation, we have

$$dQ=C_v\,dT+RT\frac{dv}{v}. \qquad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Dividing equation (5) by T, we obtain

$$\frac{dQ}{T}=C_v\frac{dT}{T}+R\frac{dv}{v}. \qquad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

If we represent the entropy of the gas by φ, equation (4) becomes

$$\varphi=C_v\log T+R\log v;$$

and, if the entropy is variable,

$$d\varphi=C_v\frac{dT}{T}+R\frac{dv}{v}. \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The right-hand members of equations (6) and (7) being equal, it follows that

$$d\varphi=\frac{dQ}{T}; \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

i.e., *for a reversible process, the change in entropy is numerically equal to the ratio of the change in heat to the temperature at which the change takes place;* the temperature being measured on the thermodynamic scale.

The foregoing may be illustrated by equation (18) of *Art.* 94, which states that for a Carnot cycle operating on a perfect gas,

$$\frac{Q_1-Q_2}{Q_1}=\frac{T_1-T_2}{T_1};$$

from which

$$\frac{Q_1}{T_1}=\frac{Q_2}{T_2}. \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Equation (9) shows that the change in entropy in going from the adiabatic AD, (Fig. 16), to the adiabatic BC, is the same whether the change takes place along the isotherm AB or DC; since the ratio of change in heat to the temperature at which the change takes place is the same in both cases. It is obvious that the same ratio holds for any other isotherm cutting the two adiabatics AD and BC. It is, however, not necessary that the change take place along an isotherm. For, assume as depicted in Fig. 16, the irregular path *ef* to be cut by the two adiabatics aa' and bb', which differ by an indefinitely small interval. The change in entropy, in going from g to h, along the irregular path *ef*, may be resolved into the two component changes; i.e., the change in entropy, in going along the isotherm *gi*, which is $d\varphi = dQ/T$; where dQ is the change in heat and T the temperature at which the change takes place. The other component *ih*, being adiabatic, involves no change in entropy; hence, the change in entropy, in going from g to h, is

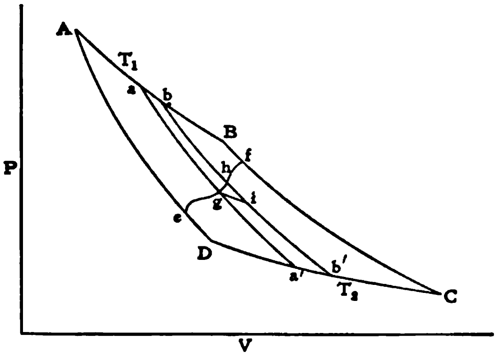


FIG. 16.

$$d\varphi = \frac{dQ}{T}.$$

But since, as has just been shown, the change in entropy, in going from one adiabatic to another is the same for all isotherms, it follows, since *gi* is an isotherm, that the change in entropy in going from g to h is equal to the change in entropy in going from a to b along the isotherm T_1, and also to the change in entropy in going from a' to b' along the isotherm T_2. Similarly, it can be shown that the change in entropy in going along the irregular path *ef*, between any two adiabatics, is equal to the change in entropy in going between the same two adiabatics along either the isotherm T_1 or T_2. It therefore follows that the change in entropy in going from the adiabatic *AD*, to the adiabatic *BC*, is always the same and is independent of the path by means of which the change is brought about.

The foregoing demonstrations establish the fact that we are justified in making the assumption that there is a constant definite change in going from one reversible adiabatic to another; and, this being the case, it follows that during a reversible adiabatic process, some function, which has been termed entropy, must remain constant. It also follows that, since for a Carnot cycle

$$\frac{Q_1}{T_1}=\frac{Q_2}{T_2},$$

the source suffers a diminution of entropy during a cycle, which is precisely equal in amount to the entropy gained by the refrigerator.

107. Universal Increment of Entropy. The conduction of heat, such as discussed in *Art.* 77, is an irreversible process; and if, during a given interval of time, a quantity of heat Q is abstracted from a source at the temperature T_1, then, if steady conditions have been assumed by the prism, an equal quantity of heat will be rejected during the same interval of time, to some receiver at a lower temperature, say T_2. The loss in entropy, of the source, is then

$$\varphi_1=\frac{Q}{T_1};$$

and the gain in entropy of the receiver is

$$\varphi_2 = \frac{Q}{T_2}.$$

The gain in entropy of the system, since that of the prism is unchanged, is

$$\varphi_2 - \varphi_1 = \frac{Q}{T_2} - \frac{Q}{T_1} = \frac{Q(T_1 - T_2)}{T_2 T_1}. \quad . \quad . \quad . \quad . \quad (10)$$

Since the right-hand member of equation (10) is positive, it follows that the entropy of the system, due to conduction, has increased; and further, since the work which would have been realized on a Carnot cycle, for the quantity of heat Q, operating between the same temperature limits, is

$$Q\frac{T_1 - T_2}{T_1},$$

we see that the work, expressed in heat units, which has been irrevocably lost, due to the quantity of heat Q being transferred by conduction from the temperature T_1 to T_2, is numerically equal to the product of change in entropy and temperature of the receiver.

We will now consider this in a wider sense. Assume, first, an engine working direct, which is thermodynamically perfect; i.e., one which maintains, during its operation, the sum of the entropies of source and refrigerator constant. If now, the mechanism upon which the engine does work is perfect and capable at any time of restoring all the energy imparted to it, then the process is perfectly reversible. This, however, is never the case; since all processes are attended by friction and a consequent development of heat, which is imparted, by conduction and radiation, to the surrounding bodies, there is necessarily an increment in entropy. To put it still more broadly, since heat can be only partially converted into work, and all energy, by friction, ohmic resistance, hysteresis, impact, etc., is finally degenerated into heat, it would appear that the entropy of the Universe, such as we know it, is tending toward a maximum. And the most generalized definition we can give, is:[*] *The change in entropy that a system undergoes during a given irreversible process is a measure of the irreversibility of the process.* This is indicated, in a limited way, by equation (10).

108. The concept of entropy has been here introduced, not on account of its great scientific value, in the domain of theoretical physics, but rather because so many of the discussions of practical thermodynamics are simplified so largely by its use. For our purposes, the two most important statements are: *The change in entropy during a reversible process, is numerically equal to the ratio of change in heat to the temperature at which the change takes place*; and *reversible adiabatic processes are also isoentropic.*

109. Temperature Entropy Diagrams. From the equation

$$\varphi=\int\frac{dQ}{T},$$

where φ is the change in entropy, it follows, immediately, that

$$dQ=Td\varphi;$$

and, for a reversible isothermal process, we have

$$Q=T\int_{\varphi_1}^{2}d\varphi=T(\varphi_2-\varphi_1). \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Applying equation (11) to a Carnot cycle, we have for the heat abstracted from the source, during isothermal expansion,

$$Q_1=T_1(\varphi_2-\varphi_1); \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

---

[*] For a comprehensive discussion of entropy, see Planck's "Thermodynamik," and also "Acht Vorlesungen über Theoretische Physik," by the same author.

and during isothermal compression, for the heat rejected to the refrigerator,

$$Q_2 = T_2(\varphi_2 - \varphi_1). \quad \ldots \ldots \quad (13)$$

From equations (12) and (13), we find

$$\eta = \frac{Q_1 - Q_2}{Q_1} = \frac{T_1 - T_2}{T_1}. \quad \ldots \ldots \quad (14)$$

The foregoing may, conveniently, be represented diagrammatically, by plotting the $T-\varphi$ (temperature-entropy) diagram; using temperatures as ordinates and entropies as abscissas. Let,

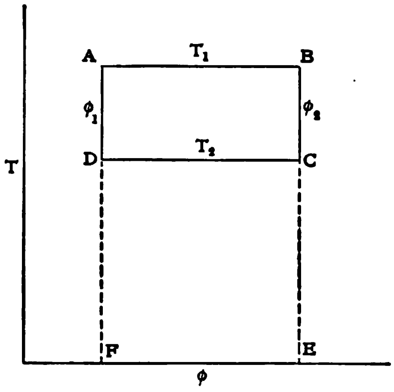


FIG. 17.

in Fig. 17, the point A represent the condition of the working substance, as regards temperature and entropy, at the instant it is put into contact with the source. Expansion now taking place at constant temperature, together with the absorption of the quantity of heat Q_1 from the source, the entropy increases by an amount

$$\varphi_2 - \varphi_1 = \frac{Q_1}{T_1}; \quad \ldots \ldots \quad (15)$$

and, the condition of the working substance, as regards temperature and entropy, is represented by the point B. Since, during this change, the temperature is constant, the line representing the change in entropy is a straight line parallel to the φ axis. Furthermore, since during the adiabatic expansion, the entropy of the substance remains constant and only the temperature varies, this change is represented by the line *BC* parallel to the *T* axis; where the point *C* represents the condition of the substance, as regards temperature and entropy, when put into contact with the refrigerator at the temperature T_2. Compression now taking place at constant temperature, together with the rejection of the quantity of heat Q_2, to the refrigerator, the entropy decreases by an amount

$$\varphi_2-\varphi_1=\frac{Q_2}{T_2}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

and the condition of the working substance, as regards temperature and entropy, is represented by the point *D*. This change in entropy is represented by the line *CD* parallel to the φ axis. Finally, during adiabatic compression, the temperature rises from T_2 to T_1; this change being represented by the line *DA* parallel to the *T* axis.

From equations (15) and (16) we find

$$\frac{Q_1-Q_2}{Q_1}=\frac{T_1-T_2}{T_1},$$

as before, and the Carnot cycle on the $T-\varphi$ diagram is represented by a rectangle; the heat abstracted from the source, during a cycle, being measured by the area

$$FABE=T_1(\varphi_2-\varphi_1),$$

and the heat rejected to the refrigerator is measured by the area

$$FDCE=T_2(\varphi_2-\varphi_1).$$

The difference between these two areas is a measure of the heat converted into work; i.e., the area

$$ABCD=(T_1-T_2)(\varphi_2-\varphi_1)$$

is a measure of the external work done.

CHAPTER XI

APPLICATIONS OF TEMPERATURE-ENTROPY DIAGRAMS

110. IN *Art.* 102, it was shown analytically that, even under perfect conditions, a steam engine and boiler cannot convert into work as large a fractional part of the heat taken from a source as can an engine operating on a Carnot cycle. We will now show this by means of the $T-\varphi$ diagram. As a matter of convenience it will be assumed that we are dealing with a unit mass of water, and, furthermore, that its thermal capacity is constant and equal to unity, between the temperature of the condenser and boiler. Under these conditions, the entropy, per unit mass of water, for any temperature T, is

$$\varphi=c\int_{T_0}^{T}\frac{dT}{T}=\log\frac{T}{T_0}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where T_0 is the temperature corresponding to the condition from which the entropy is measured, and c equals unity. Hence, we have a logarithmic curve instead of a straight line for one of the sides of the $T-\varphi$ diagram. Let, as in Fig. 18, the point D represent the condition of the water, as regards temperature and entropy, when it is returned to the boiler. The logarithmic curve DA, then represents the change in entropy with respect to change in temperature as the water is heated from the temperature T_2, that of the condenser, to the temperature T_1, that of the boiler. The water is now evaporated at the temperature T_1, and its change in entropy is represented by the line AB. From the point B, this point representing the condition of the water, as regards temperature and entropy, for complete evaporation, the steam expands adiabatically, its entropy remaining constant, until the temperature has fallen to T_2, that of the condenser; its condition being now represented by the point C. The steam is now compressed isothermally, at the temperature T_2, until complete condensation has taken place, and the initial condition, represented by the point D, is reached.

During the cycle, just described, the net work done, or the heat converted into work, is measured by the area $ABCD$; and the heat abstracted from the source, is measured by the area $FDABE$. Hence, the ideal coefficient of conversion is

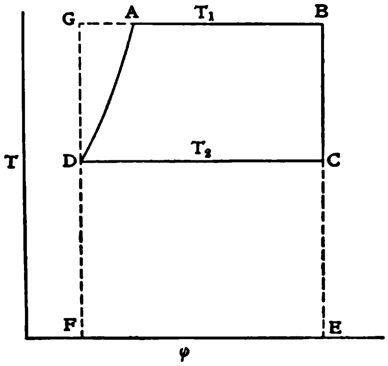


FIG. 18.

$$\eta=\frac{\text{Area } ABCD}{\text{Area } FDABE}. \qquad \qquad (2)$$

Had the process been a Carnot cycle, the ideal coefficient of conversion would be expressed by

$$\eta=\frac{\text{Area } DGBC}{\text{Area } FGBE}. \qquad \qquad (3)$$

The value of η as given by equation (2) is obviously less than that given by equation (3); which is in agreement with the results obtained in *Art.* 102.

111. Change of Dryness during Adiabatic Changes. Equation (23), of *Art.* 104, may be found in a very simple manner from the fact that the entropy of a substance, during reversible adiabatic changes, remains constant. The gain in entropy, for a unit mass of water, in being heated from a temperature T_0 to the temperature T_1 is, by equation (1),

$$\varphi_1=\log\frac{T_1}{T_0}; \quad . \; . \; . \; . \; . \; . \; . \; . \quad (4)$$

and the gain in entropy in evaporating to a dryness q_1, at the temperature T_1, is

$$\varphi_2=q_1\frac{r_1}{T_1}; \quad . \; . \; . \; . \; . \; . \; . \; . \quad (5)$$

where r_1 is the heat of vaporization corresponding to the temperature T_1. Taking the sum of equations (4) and (5), we find, for the total change in entropy

$$\varphi=\varphi_1+\varphi_2=\log\frac{T_1}{T_0}+q_1\frac{r_1}{T_1}. \quad . \; . \; . \; . \; . \quad (6)$$

But for reversible adiabatic changes the entropy remains constant; hence, if, after the condition expressed by equation (6) has been attained, the temperature due to an adiabatic change, which may be either expansion or compression, changes to T_2, and the dryness changes to q_2, we must have

$$\log\frac{T_1}{T_0}+q_1\frac{r_1}{T_1}=\log\frac{T_2}{T_0}+q_2\frac{r_2}{T_2}; \quad . \; . \; . \; . \quad (7)$$

where r_2 is the heat of vaporization corresponding to the temperature T_2. From equation (7), we find

$$q_2=\frac{T_2}{r_2}\left(\frac{q_1r_1}{T_1}+\log\frac{T_1}{T_2}\right);$$

which is the same as equation (23) of *Art.* 104.

112. Dryness by Means of Temperature-Entropy Diagram. The change in dryness which a mixture of water and steam, or any liquid and its vapor, undergoes during adiabatic changes, provided the initial dryness be known, may readily be found by means of the $T-\varphi$ diagram. Let, in Fig. 19, the curve DA be plotted, to proper scale, using various values of $\varphi=\log\frac{T}{T_0}$ as abscissas and the corresponding values of T as ordinates; then this curve represents the relation of temperature and entropy for unit mass of water between the temperatures T_0 and T_1. If now, for various points along the curve DA, between T_0 and T_1, horizontal distances be measured off towards the right, each distance being equal to r/T, where T is the temperature corresponding to the point, and r the corresponding heat of vaporization, the curve of saturation BC is found. By construction, then, at any temperature T, the horizontal distance between the curves AD and BC represents the increase in entropy due to the heat added to bring about complete evaporation at that temperature. Thus, for the temperature T, we have

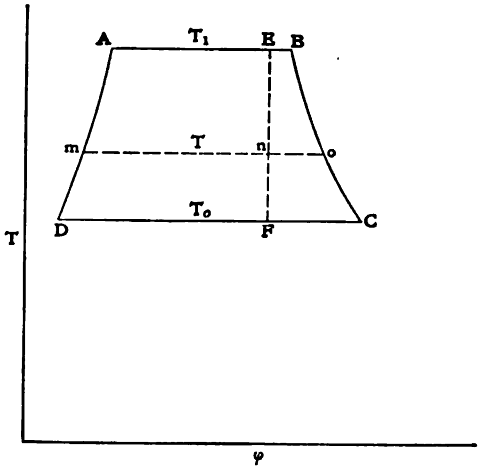
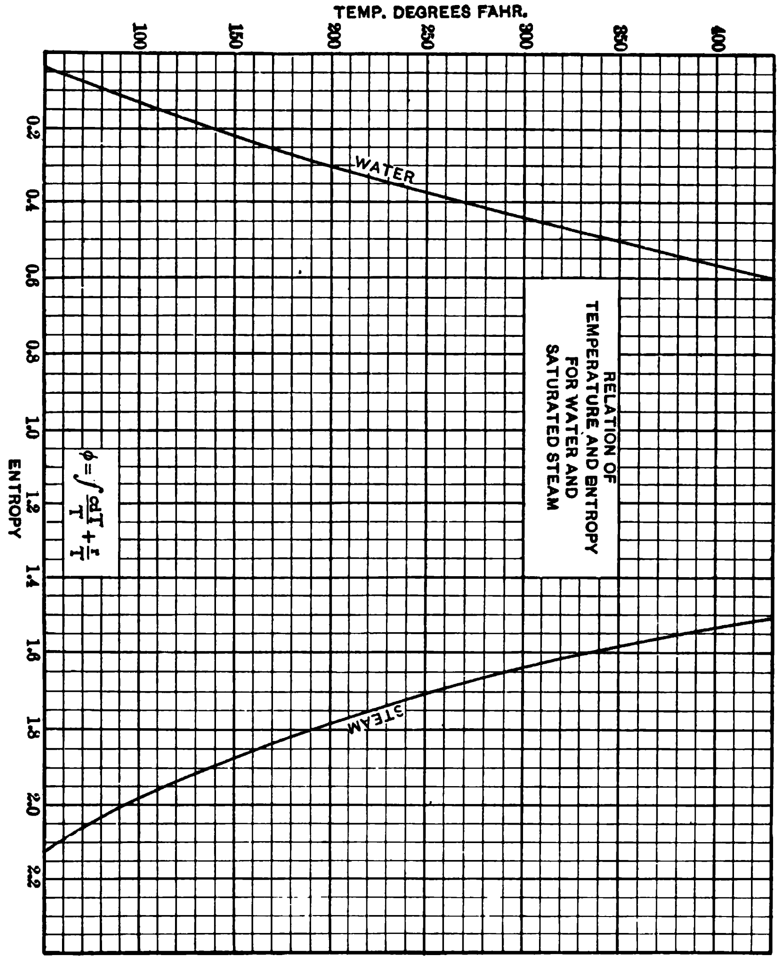


FIG. 19.

$$\varphi=\frac{r}{T}=mo;$$


TEMP. DEGREES FAHR.
100
150
200
250
300
350
400
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
ENTROPY
WATER
STEAM
RELATION OF TEMPERATURE AND ENTROPY FOR WATER AND SATURATED STEAM
$\phi = \int \frac{c\,dT}{T} + \frac{r}{T}$

where r is the heat of vaporization corresponding to the temperature T, and φ the increment in entropy.

Let, now, the initial dryness, at the temperature T_1, be q_1, such that the entropy is represented by the point E. Since the quantity of liquid evaporated, at a given temperature, is directly proportional to the quantity of heat added, and the increment in entropy is also directly proportional to the quantity of heat added, it follows that the increment in entropy is directly proportional to the amount of evaporation. Hence, the initial dryness is

$$q_1=\frac{AE}{AB}.$$

If, now, adiabatic expansion take place, the entropy remains constant, and the vertical line EF represents the relation of temperature and entropy. Hence, the dryness corresponding to the temperature T is given by

$$q=\frac{mn}{mo}. \qquad (8)$$

In a similar manner, provided always the initial dryness be known, the dryness corresponding to any temperature during an adiabatic change, may be found. And it makes no difference whether we are dealing with an expansion or a compression.

It is obvious that if a $T-\varphi$ curve be plotted to a convenient scale, for water, together with the corresponding saturation curve, in a manner as has just been described, between such temperature limits as are likely to occur in practice, we may at once, from such a sheet, provided always the initial dryness be known, determine the amount of dryness at any temperature for adiabatic changes. As a matter of convenience, such a sheet is given on page 141.

113. Zero Curve. Assume that various horizontal distances between the curves AD and BC are all divided into the same number of parts, and each part is the same fractional part of the total distance. If the points, so found, are connected by smooth curves, as shown in Fig. 20, then the dryness along any particular curve is a constant. For, the horizontal distance between any curve, such as *mn*, and the curve *AD*, no matter at what temperature the distance be measured, is always the same fractional part of the total increment in entropy, at that temperature, due to vaporization; and therefore, represents the same fractional part of vaporization. If, now, an adiabatic, such as *ab*, be drawn, it is found to cut the curve *mn*; i.e., the curve *mn* passes to the right of the adiabatic *ab*, as the expansion progresses, and shows that the steam becomes wetter during adiabatic expansion and drier during adiabatic compression. If, on the other hand, an adiabatic, such as *cd*, be drawn, it is found that the curve of equal dryness *op*, as the expansion progresses, passes to the left of it; and shows that the vapor becomes drier during adiabatic expansion and wetter during adiabatic compression. It is thus seen that if the vapor be initially *quite* dry, it becomes *wetter* during adiabatic expansion, and if the dryness is *very low*, it becomes *drier* when expanded adiabatically.

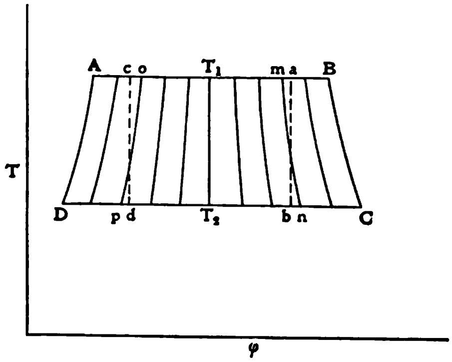


FIG. 20.

If the adiabatic at any point becomes tangent to the curve of constant dryness, then there is at that point, no change in dryness during adiabatic changes. By finding a number of points on the various curves where the tangent is vertical, and joining these points by a smooth curve, then at any point to the right of this curve the dryness is decreased by adiabatic expansion, and at any point to the left of this curve, adiabatic expansion increases the dryness. Such a curve is known as the "*zero curve*"; and for temperatures such as are common in practice, does not lie very far from 50 per cent dryness.

114. Loss of Work Due to Using Steam Non-expansively. Assume, in the first place, that no expansion whatsoever is allowed; but that at the instant of cut-off, the steam is put into contact with the refrigerator, and condensation takes place at constant volume. Then the work lost, due to using the steam non-expansively, is represented by the area *BCK*, of Fig. 15, for the ideal case discussed in *Art.* 102.

As a matter of fact, during the greater part of the eighteenth century, steam was used in this manner; i.e., the steam was not worked expansively, but immediately after cut-off, the steam was condensed by a jet of water, either in the cylinder, or in an adjoining condenser. The result, however, is the same whether the steam be condensed by a jet of water, immediately after cut-off, or allowed to escape to a space of lower pressure; for in either case, there is the same gradual diminution of pressure in the cylinder.

The conditions which obtain, when steam is used non-expansively, are best studied by the aid of the $T-\varphi$ diagram. In Fig. 21, *DA* is the $T-\varphi$ curve for the heating of unit mass of water, *AB* the curve for evaporation at the temperature T_1, *BC* the saturation curve, *BE* the curve for adiabatic expansion, *BnF* the curve of condensation at constant volume, and *FD* the condensation curve at constant temperature. The condensation curve *BnF* is determined as follows: For any temperature *T*, we have to determine a point *n* such that

$$q=\frac{mn}{mp};$$

where q is the dryness corresponding to the temperature T.

The volume of the steam in the cylinder, remaining sensibly constant, since the volume of liquid present is practically negligible, we have

$$qs=s_1; \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (9)$$

where s is the volume of unit mass of saturated steam at the temperature T, and s_1 the volume originally occupied at the temperature T_1. s and s_1, are found from steam tables. From equation (9) we find

$$q=\frac{s_1}{s}. \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (10)$$

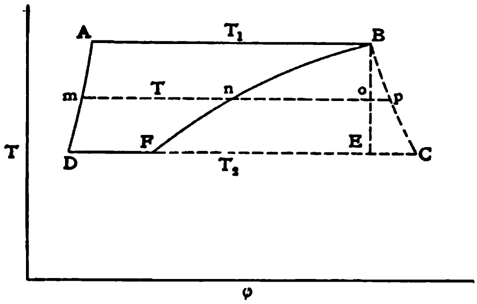


Fig. 21.

But, as has been previously shown, a relation must subsist, such that

$$q=\frac{mn}{mp}; \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (11)$$

hence, by combining equations (10) and (11), we find

$$mn=mp\frac{s_1}{s}. \quad . \; . \; . \; . \; . \; . \; . \; . \quad (12)$$

Finding a number of points in this manner, the curve BnF is determined; and the loss of work, due to using the steam non expansively, is obviously measured by the area BFE.

115. Loss of Work Due to Incomplete Expansion. If there is a partial adiabatic expansion before exhaust or condensation, then the $T-\varphi$ diagram takes the form as depicted in Fig. 22. DA is the $T-\varphi$ curve for the heating of unit mass of water, AB the curve for complete evaporation, at the temperature T_1, BC the saturation curve, BG the curve for adiabatic expansion, to the temperature T', GnF the curve of condensation at constant volume, and FD the curve of condensation at the temperature T_2. To determine the curve of condensation at constant volume, we must find a point n for the temperature T, such that

$$q=\frac{mn}{mp}; \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (13)$$

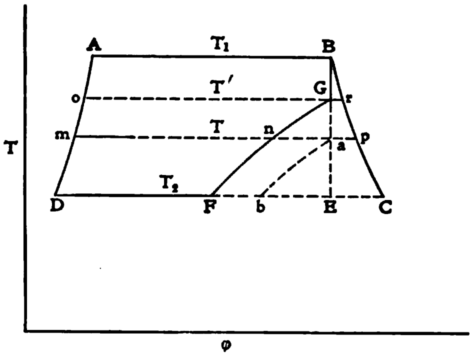


FIG. 22.

where q is the dryness corresponding to that temperature. Now, the volume occupied at the point G, corresponding to the temperature T', is

$$q's';$$

where q' is the dryness at the temperature T', and s' the volume of unit mass of saturated vapor at T'. Also, for constant volume

$$qs=q's'; \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (14)$$

where q is the dryness at the temperature T, and s the corresponding volume for unit mass of saturated vapor. From equation (14), we have

$$q=q'\frac{s'}{s}. \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (15)$$

Finally, combining equations (13) and (15), we obtain

$$mn = q'\frac{s'}{s}mp. \qquad . \ . \ . \ . \ . \ . \ . \qquad (16)$$

Finding a number of points in this manner, the curve *GnF* is determined. If the steam is initially not dry, the curve *GnF* is determined in precisely the same manner; but, the curve *BG* is shifted toward the left by a fractional part of the length *AB*, depending upon the amount of initial priming.

The work lost, due to incomplete expansion, is measured by the area *GFE*; and, by an inspection of the figure, it becomes obvious that the loss of work decreases very rapidly as the expansion is increased. As an example, were the expansion continued up to the point *a*, the loss of work, due to incomplete expansion, would be measured by the small area *abE*.

The greater the amount of expansion, after cut-off, the longer, necessarily, the stroke of the piston; but, the longer the stroke, other things being equal, the higher the first cost of the engine, and the greater the loss of work due to friction. Hence, there must be a point beyond which it is uneconomical to carry the expansion. Besides increasing the friction, there are still other losses introduced, by carrying the expansion too far; these will be considered later. Just how far to carry the expansion so as to give the best economy is a problem far too complex to be solved theoretically. At best, theory can only serve as a guide, and the most economical expansion must be determined experimentally. For a simple engine, the point of cut-off may vary from about one-third to one-sixth of the total stroke; depending upon whether the engine is running non-condensing or condensing. But, it must always be remembered that the ratio of cut-off to length of stroke depends upon various conditions, which will be better understood after we have dealt with the actual behavior of the steam in passing through the cylinder.

116. Gain of Work Due to Superheating. If the steam, after being completely evaporated, be superheated, the ideal coefficient of conversion is increased. But, this must not be understood to mean the same proportional gain in work; for, lubrication and packing become more difficult as the temperature is increased; and when the temperature becomes very high, radiation becomes excessive.

Let, in Fig. 23, DA be the $T-\varphi$ curve for the heating of unit mass of water, AB the curve for evaporation, BE the curve for superheating, EI the adiabatic expansion curve, and ID the curve of condensation. The curve BE, for superheating, is found as follows: The thermal capacity of superheated steam, for temperatures such as are found in practice, is approximately constant; hence, we have

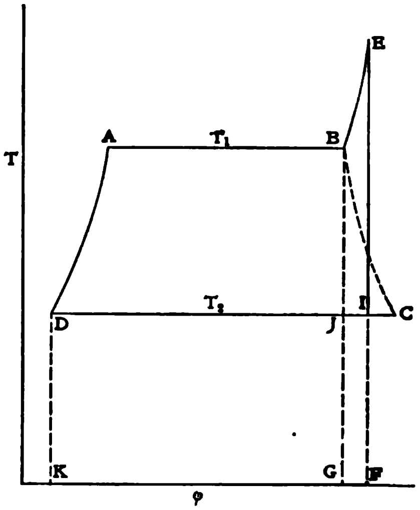


FIG. 23.

$$\varphi = c\int_{T_1}^{T_s} \frac{dT}{T};$$

from which

$$\varphi = c \log \frac{T_s}{T_1}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

where φ is the change in entropy in going from B to E, T_s the temperature to which the steam is superheated, and c the thermal capacity per unit mass of superheated steam. The thermal capacity per unit mass of superheated steam is, as has just been stated, practically constant, and is approximately equal to 0.480.

With no superheating, and expansion along the adiabatic BJ, the ideal coefficient of conversion is,

$$\eta' = \frac{ABJD}{ABGKD};$$

and, with superheating and adiabatic expansion along EI, we find, for the ideal coefficient of conversion,

$$\eta'' = \frac{ABEID}{ABEFKD}.$$

By an inspection of the figure, it becomes obvious that $\eta'' > \eta'$.

If the steam be superheated to a temperature, such that the adiabatic EF passes through the point C, then the steam will be just saturated after it has been expanded to the temperature T_2. To find the amount of superheating that will bring about this condition, it is only necessary to equate entropies, for the points E and C. The change in entropy, in going from D to E, is

$$\varphi' = \log\frac{T_1}{T_2} + \frac{r_1}{T_1} + c\log\frac{T_s}{T_1};$$

and the change in entropy, in going from D to C, is

$$\varphi'' = \frac{r_2}{T_2}.$$

But, in order that the adiabatic EF pass through the point C, φ' must equal φ''; hence,

$$\log\frac{T_1}{T_2} + \frac{r_1}{T_1} + c\log\frac{T_s}{T_1} = \frac{r_2}{T_2},$$

from which

$$\log T_s = \frac{1}{c}\left(\frac{r_2}{T_2} - \frac{r_1}{T_1} - \log\frac{T_1}{T_2}\right) + \log T_1. \quad . \quad . \quad . \qquad (18)$$

By means of equation (18), T_s is readily found.

117. Double-acting Engine. Up to the present, we have been considering matters as though the engine were only single-acting; i.e., admission and exhaust take place only at one end of the cylinder. In general, however, this is not the case. By having proper valve arrangements, admission and expansion take place in one end of the cylinder while release and exhaust take place in the other end. And, the engine is double-acting; thus practically doubling the capacity of the cylinder and giving a more uniform distribution of the work for a rotation of the fly-wheel. Hence, a single-acting engine, to carry its load properly, requires a fly-wheel of greater inertia than does a double-acting engine.

118. Condensing Engine. By a *condensing engine* is meant an engine which exhausts to a receptacle of some kind, called a *condenser*, where the steam is condensed at a comparatively low temperature, and the pressure in the condenser is maintained constant and lower than that of the atmosphere by means of a vacuum pump. In good condensers, the pressure is as low as the equivalent of one inch of mercury. By a *non-condensing engine* is meant an engine which exhausts directly to the atmosphere at practically atmospheric pressure.

Experience shows that, in general, when the supplied steam has a pressure of 100 lbs., or over, despite the fact that a certain amount of power is consumed in operating the vacuum pump, there is a decided gain in economy, when engines are operated condensing. Hence, in general, condensing engines are employed.

CHAPTER XII

ELEMENTARY STEAM AND ENGINE TESTS

119. Before proceeding to discuss the actual behavior of the steam as it passes through the cylinder of an engine, a brief description will be given of the methods pursued in determining the dryness of steam. It is obvious, from the discussions in the preceding chapter, that one of the essentials in studying the performance of a steam engine is a knowledge of the condition of the supplied steam. But, aside from the temperature-entropy diagram, if we wish to plot an adiabatic for steam on the p–v diagram, we must know the initial dryness.

According to calculations by Zeuner, the equation

$$pv^n = k, \quad \dots\dots\dots \quad (1)$$

where k is a constant, may be used for adiabatic changes. But the value of n depends upon the initial dryness. This value of n is given by the empirical equation

$$n = 1.035 + 0.1q; \quad \dots\dots\dots \quad (2)$$

where q is the initial dryness. Equation (2) holds for all values between 70 per cent and 100 per cent dryness. Hence, if an adiabatic for steam is to be plotted, that value of n must be used in equation (1), which is found by means of equation (2), for the given initial dryness.

120. Throttling Calorimeter. It was shown, in *Art.* 103, that steam becomes drier, during unresisted adiabatic expansion; and furthermore, under proper conditions, if the initial priming be low, the steam may become superheated. Depending upon this principle, Professor Peabody designed a calorimeter, by means of which the dryness of steam, provided the priming is low, may be determined. In Fig. 24, S represents the supply pipe, and A a vessel into which the steam expands; the rate of inflow being regulated by the valve V_1, and the outflow by the valve V_2. G_1 is a gauge, indicating the pressure of the steam in the supply pipe, G_2 a second gauge, indicating the pressure of the steam in the vessel A, and m a thermometer, indicating its temperature. The valves V_1 and V_2 are so regulated that the pressure in the vessel A is always considerably less than the pressure in the supply pipe. The vessel A is either well lagged with some non-conducting material, or else highly polished, to reduce radiation to a minimum. After the flow of steam has continued for some time, steady conditions will obtain; and, if the priming of the steam in the supply pipe is low, superheating will take place in the vessel A, and the thermometer m will register a temperature higher than that corresponding to saturated steam, under a pressure as registered by the gauge G_2. Let τ_1 be the temperature of saturated steam corresponding to the pressure indicated by the gauge G_2, and τ_2 the temperature registered by the thermometer; then $\tau_2-\tau_1$ is the amount of superheating. The total heat of unit mass of steam then, in the vessel A, is

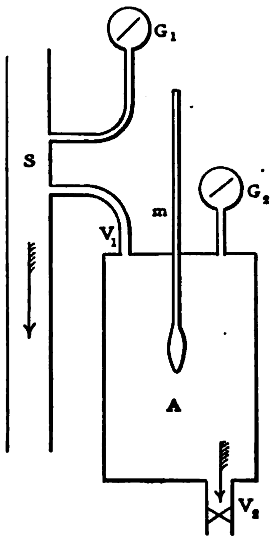


Fig. 24.

$$H'=h_1+r_1+c(\tau_2-\tau_1); \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where H' is the total heat, h_1 the heat of the liquid, corresponding to the temperature τ_1, r_1 the heat of vaporization for this temperature, and c the thermal capacity per unit mass for superheated steam. The total heat, per unit mass, for the steam in the supply pipe, is

$$H''=h+qr; \quad \ldots \ldots \quad (4)$$

where H'' is the total heat, h the heat of the liquid for the temperature τ, which is the temperature of saturated steam for the pressure as registered by the gauge G_1, r the heat of vaporization for this temperature, and q the dryness of the steam in the supply pipe. For an adiabatic flow, however, the total heat for the two conditions is the same; hence, the right-hand members of equations (3) and (4) are equal, and we have

$$h+qr=h_1+r_1+c(\tau_2-\tau_1);$$

from which

$$q=\frac{h_1+r_1+c(\tau_2-\tau_1)-h}{r}. \quad \ldots \ldots \quad (5)$$

In order to obtain reliable results, by the method just described, the two gauges G_1 and G_2 must be accurately calibrated. On the other hand, a slight error in the thermometer does not appreciably alter the result; since the amount of superheating is necessarily small, the quantity of heat involved is small in comparison with the other quantities. But, *the thermometer must be of sufficient accuracy so that we may be assured that there is superheating.*

The amount of moisture that may be removed by throttling depends, of course, upon the difference between the pressure of the steam in the supply pipe and the pressure in the chamber into which it expands. If the pressure in the chamber is equal to that of the atmosphere, and the pressure in the supply pipe is 100 lbs. per square inch, then the dryness must be about 96 per cent so that all the moisture may be removed by throttling. If the pressure in the chamber be reduced by means of a condenser, a greater amount of moisture may be removed. If the initial pressure of the steam be 150 lbs. per square inch, and the pressure in the chamber is atmospheric, then about 5 per cent priming may be removed by throttling.

121. Condensing Calorimeter. The dryness of steam may also be determined by condensation. There are various methods which may be pursued; one is to have a vessel partially filled with water at some low temperature, and passing steam into this until some convenient rise in temperature has been attained. The quantities of water and steam are determined directly by weighing, the initial and final temperatures of the vessel are read from a thermometer immersed in it, and the initial temperature of the supplied steam is determined from the pressure, as indicated by means of a gauge attached to the supply pipe.

Let M be the water equivalent of the vessel and contents, m the mass of the condensed steam, and τ_1 and τ_2, respectively, the initial and final temperatures of the vessel and contents, then if q is the initial dryness and τ the corresponding temperature of the supplied steam, we have

$$M(\tau_2-\tau_1)+mh_2=mqr+mh; \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where h_2 is the heat of the liquid corresponding to the temperature τ_2, r and h, respectively, the heat of vaporization and the heat of the liquid, corresponding to the temperature τ. From equation (6) we find

$$q=\frac{M(\tau_2-\tau_1)-m(h-h_2)}{mr}. \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Equation (7) was deduced on the assumption that there is no radiation during the progress of the experiment. A correction for radiation may, however, be applied by taking a curve of cooling for the vessel. Due to the fact, that the mass of the condensed steam is determined by a difference in weighing, and that this mass is necessarily small in comparison with the mass of liquid initially contained in the vessel, a serious error may be introduced by an inaccuracy in weighing.

A better method than the one just described, is that of passing the steam through a spiral tube, contained in a condenser through which there is maintained a continuous flow of water, in a manner such that the steam is completely condensed and reduced in temperature to that of the outflowing condensing water. After steady conditions obtain, the mass of steam condensed, during a given interval of time, is determined by collecting it in a suitable vessel; and in a similar manner, by collecting in a separate vessel, the mass of water which passes through the condenser, during the same interval of time, is determined. Knowing the initial and final temperatures of the condensing water, together with its mass, the mass of the condensed steam and its initial and final temperatures, then the dryness of the steam is determinate. Let M and m, respectively, be the mass of the condensing water and condensed steam for the same interval of time, τ_1 and τ_2, respectively, the temperature of the condensing water for inflow and outflow, and τ the initial temperature of the supplied steam, then, since the temperature of the outflow and that of the condensed steam is the same, we have

$$M(\tau_2-\tau_1)+mh_2=mqr+mh; \quad . \ . \ . \ . \ . \quad (8)$$

where h_2 is the heat of the liquid corresponding to the temperature τ_2, r and h, respectively, the heat of vaporization and the heat of the liquid corresponding to the temperature τ, and q the dryness. From equation (8), we find

$$q=\frac{M(\tau_2-\tau_1)-m(h-h_2)}{mr}. \quad . \ . \ . \ . \ . \ . \quad (9)$$

Equation (9) was deduced on the assumption that there is no radiation during the progress of the experiment. If the temperature of the vessel differs materially from that of the surroundings, corrections for radiation must be applied for this difference. Radiation may, however, be completely eliminated by regulating the inflow such that the vessel and contents are continuously at room temperature.

122. Separating Calorimeter. Professor Carpenter devised an apparatus by means of which the moisture, present in the steam, is removed mechanically. The steam is passed from the supply pipe into a chamber, where it strikes against a convex surface, surrounded by a wire mesh, through which the escaping steam must pass. When the steam strikes the cup, the water present is separated, passes through the mesh, and is collected in the chamber; the dry saturated steam, meanwhile, passes into an outer jacket, which surrounds the chamber, and escapes from an orifice at the bottom, where it is condensed and collected. The quantity of water collected in the chamber is read directly from a glass gauge, which has been previously calibrated; and the quantity of steam which passes through the calorimeter is determined by condensing and weighing.

In making a determination, the valve in the pipe supplying steam to the chamber is opened, and when steady conditions obtain, a reading is taken on the glass gauge, and simultaneously, the exhaust pipe is passed into the condenser. When the operation has been continued for a sufficient interval of time, the gauge is again read, and at the same instant, the exhaust pipe is removed from the condenser. The mass of steam, passing through the apparatus for the given interval of time, is found directly by the difference in weight of the condenser for final and initial conditions. And this mass compared with the sum of the two masses, i.e., the mass of the condensed steam and the mass of the water collected in the chamber, gives the dryness.

One of the inherent difficulties common to all methods, in determining the dryness of steam, lies in obtaining a sample which is a fair average of the steam supplied to an engine.

123. Clearance. The volume swept out by the piston of an engine, during a stroke, is equal to the product of the area of the piston and length of its stroke. The volume between the piston and cylinder head at the end of the stroke, plus the volume of the supply and exhaust-passages leading to the admission and exhaust-valves, is called the *clearance*. The clearance then is, that part of the volume through which the piston does not sweep, and is readily found by closing the valves and determining the volume of water required to fill the space when the piston is at the end of its stroke. A convenient way of expressing the clearance of an engine is by a ratio; i.e., the ratio of the volume of the clearance, to the volume of piston displacement plus volume of clearance. The ratio of the clearance volume to the piston area gives the *equivalent length* of clearance.

The clearance of different engines varies considerably, depending upon the size of engine and type of valves used; and, other things being equal, the clearance for small engines is relatively larger than it is for large ones. In practice, depending on the type of engine, the clearance may vary from 2 per cent to 10 per cent.

124. Cushion Steam and Cylinder Feed. The mass of steam which remains in the clearance-space at the end of the exhaust-stroke, depends upon the time of closing of the exhaust-valve. Thus, if the exhaust-valve does not close until the exhaust-stroke has been completed, then the pressure of the steam, remaining in the cylinder, is the same as that of the condenser, and the mass of the steam is equal to the product of the clearance-volume and the density of the steam. If, on the other hand, the exhaust-valve closes before the exhaust-stroke has been completed, then the pressure of the steam, remaining in the cylinder at the end of the exhaust-stroke, will be higher than that existing in the condenser; hence, in this case, the mass of steam remaining in the cylinder is greater than that for a later closing of the exhaust-valve. The steam remaining in the cylinder, at the end of the exhaust-stroke, is called the *cushion steam;* and the steam drawn from the boiler, per stroke, is called the *cylinder feed.*

During expansion, both quantities are present, whereas, during compression, the cushion steam alone is present.

125. Wire Drawing. If the exhaust-valve closes late, the pressure of the cushion steam is less than that of the steam in the supply pipe, and a certain quantity of steam must pass into the cylinder, during each stroke, before the maximum pressure is reached. The entering steam, therefore, does not do as great an amount of work on the piston as it would do if the cushion steam had been compressed to the pressure of the incoming steam; in other words, it is a case of imperfectly resisted expansion. It is true that the incoming steam, if dry, becomes superheated, and if partially wet becomes drier, due to the partially unresisted expansion; but the pressure being lower, the heat which is evolved when the eddy currents subside, is applied at a lower temperature, and therefore, the imperfectly resisted expansion constitutes a *thermodynamic drop.*

If the exhaust-valve closes at the proper time, then the pressure of the cushion steam is equal to the pressure of the incoming steam, and the thermodynamic drop, so far as this part of the action of the engine is concerned, is avoided. Furthermore, the work which is done on the cushion steam, in compressing it from the condenser pressure to that of the incoming steam, is precisely equal to the work done by it, in expanding between the same limits of pressure; hence, there is no loss of work involved due to compression. There are, however, other unavoidable losses. The piston advances rapidly, calling for a large supply of steam, and the admission-valve does not open instantaneously, but requires a definite time interval. Hence, due to the resistance offered to the flow of steam, by the supply passages and valves, there is a certain amount of throttling, the same as when the cushion steam is at a pressure lower than that of the supplied steam; causing the pressure in the cylinder, during admission, to be less than that existing in the supply pipe. And furthermore, in general, due to throttling, the pressure in the cylinder gradually decreases as the admission advances. The result of these combined causes, due to which the pressure in the cylinder during admission is lower than that of the supply pipe, or boiler, is known as *wire drawing,* and constitutes a thermodynamic drop.

There is also a loss of work during the exhaust-stroke, due to the fact that the exhaust-passages and valves offer a resistance to the flow of steam, which makes the pressure in the cylinder, during exhaust, always higher than that of the condenser.

126. The Indicator. One of the most important and, at the same time, one of the most delicate pieces of apparatus used in engine testing is the *indicator.* The indicator consists essentially of two parts; the first part being a small piston P fitted accurately into a cylinder and controlled by a helical spring S. The spring may be either inside of the cylinder or, as shown diagrammatically in Fig. 25, outside. The type of indicators having the spring above the cylinder are more convenient; and furthermore, since the springs, in this form, are not subjected to the same fluctuations of temperature, the results obtained are more satisfactory. The cylinder of the indicator, by means of a short supply pipe containing a cock, is tapped onto the cylinder of the engine, over the clearance space, in a manner such that the steam in the engine cylinder exerts its full pressure against the piston of the indicator throughout the entire cycle. If the indicator piston moves freely, i.e., without appreciable friction, and the spring obeys Hooke's Law, then the movement of the piston will be proportional to the fluctuations of the pressure in the cylinder of the engine. To magnify the motion of the piston of the indicator, the end of its piston rod is connected, by means of a system of links, to a lever, in a manner such that a pencil point p, carried by the end of this lever describes, between the limits of travel, practically a right line. The springs are accurately calibrated to a definite scale with respect to the motion of the pencil point. Thus, if a spring is a 60 *lb. spring,* it means that the pencil point moves over a distance of 1 inch for a change in pressure, on the piston, of 60 lbs. per square inch; and a distance of 0.75 inches for a change in pressure of 45 lbs. per square inch, etc.

The second part of the indicator consists of a drum D, controlled by a spring, upon which the *indicator card* is wrapped. The drum has wrapped around its lower part a cord C, which in turn is connected by means of some mechanism, to the cross-head of the engine, in a manner such that the angular displacement of the drum is proportional to the linear displacement of the piston of the engine. The linear displacement of the surface of the drum, however, is less than that of the cross-head; i.e., the motion is reduced by the mechanism through which the

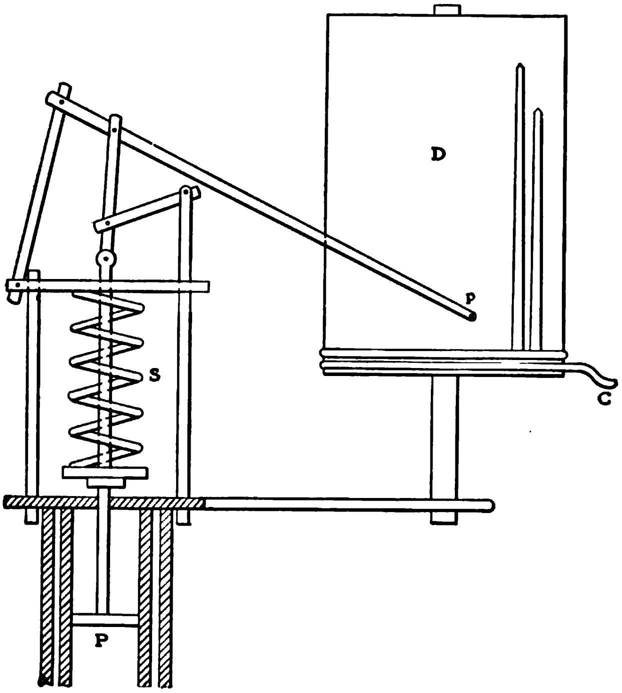


FIG. 25.

cord, operating the drum, is connected to the cross-head. From the foregoing, it is obvious that, if the drum is stationary, and the cock in the supply pipe between the cylinder of the engine and the cylinder of the indicator is open, the pencil point traces a straight line on the indicator card. On the other hand, if the stop-cock is closed and the drum is in motion, the pencil traces a straight line at right angles to the former. This line is the *atmospheric* line, since the stop-cock is so arranged that when the steam is cut off from the indicator cylinder, a vent opens, allowing free access of the atmosphere to the space below the piston of the indicator. If, however, the stop-cock is open, and the drum is moving in unison with the piston of the engine, the position of the pencil point of the indicator, at any part of the cycle, is a measure of the pressure and volume, of the working substance, for that instant. Hence, during a cycle, the pencil point traces out a diagram, which shows to a reduced scale, as regards pressure and volume, the condition of the working substance, for every part of the cycle. The diagram so traced, is the actual *indicator diagram* of the engine.

127. Indicator Diagram and Valve Adjustment. By means of the indicator diagram, the behavior of the working substance may be conveniently studied for the entire cycle; and furthermore, we are enabled by it to judge, whether or not, the valves are properly adjusted, which is very important; since any faulty valve adjustment may seriously affect the efficiency of the engine. Also, as will be shown in this chapter, by means of the indicator diagram, we are enabled to determine the power delivered, by the working substance, to the engine; hence, if the power delivered by the engine be known, the efficiency of the engine, as a mechanical contrivance, is immediately determined.

Fig. 26 is a reproduction of an indicator diagram taken from one end of the cylinder of a 40-H.P. engine, making 300 r.p.m.; the engine working non-condensing; i.e., exhausting to the atmosphere. Fig. 27 is the indicator diagram for the same end of the cylinder when the engine was exhausting to a surface condenser; a partial vacuum being maintained by a pump.

AB is the admission line, *BC* is the expansion line, *C* being the point where the exhaust-valve begins to open, and *D* the point where it is fully open; *DE* is the exhaust line, and at the point *E* compression begins. Just how far the compression will be carried before the admission-valve opens depends upon the set of the valves. In the diagrams here shown, the admission-valve opened at the point *a*. The various points being illy defined is due to the time element involved in the opening and closing of the valves. The line *OH* is the line of zero volume, and is found by taking a distance, to the proper scale, to the left of *FA*, representing the equivalent length of the clearance. The

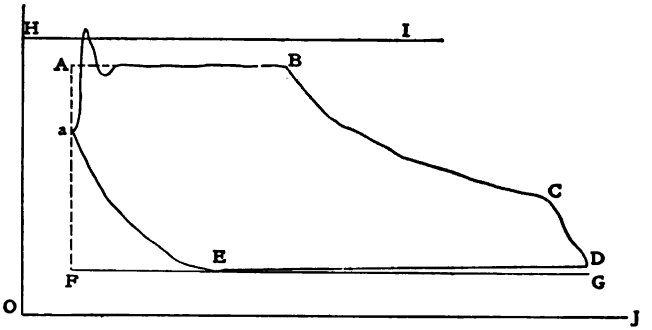


FIG. 26.

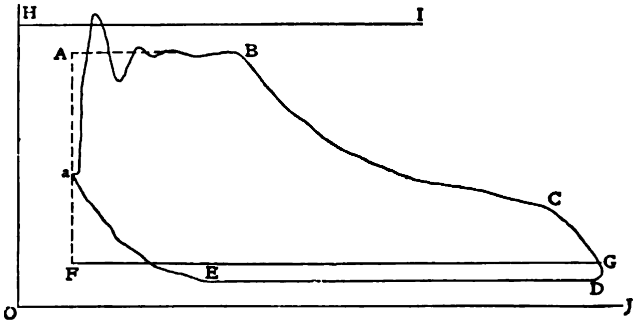


FIG. 27.

line of zero pressure, or vacuum line *OJ*, is found by measuring down from the atmospheric line *FG*, a distance representing the atmospheric pressure at the time the diagram was taken. Finally, *HI* shows, as registered by the gauge, the steam pressure in the supply pipe.

128. Comparison of Theoretical and Actual Curves. If it be desired to compare the expansion or compression curve, with an isotherm or adiabatic, a point on the curve is chosen, preferably about the middle, and the theoretical curve is made to pass through this point. As a matter of convenience, the following method for plotting curves is here given. To plot the curve whose equation is

$$pv^n = k,$$

we proceed as follows: In Fig. 28, *OA* is the line of zero volume, *OB* the line of zero pressure, and *a* a point on the curve. Lay off the line *OC*, making an angle β with the line *OA*, and the line *OD*, making an angle α with the line *OB*, such that

$$1+\tan\beta=(1+\tan\alpha)^n. \quad . \quad . \quad . \quad . \quad . \qquad (10)$$

Draw *ad* parallel to *OB*, and *dh* making an angle of 45° with *OA*. Now draw *af* parallel to *OA*, and through *f*, *fg* making an angle of 45° with *OB*; then the point *b*, which is the intersection of the line *hb*, parallel to *OB*, with the line *gb*, parallel to *OA*, is a point on the curve. For, if we represent, for the point *a*, the pressure and volume respectively, by p_1 and v_1, and similarly for the point *b*, by p_2 and v_2, we have

$$p_1=p_2+p_2\tan\beta=p_2(1+\tan\beta); \quad . \quad . \quad . \quad . \qquad (11)$$

and

$$v_1+v_1\tan\alpha=v_2;$$

from which

$$v_1^n(1+\tan\alpha)^n=v_2^n. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (12)$$

Multiplying equations (11) and (12), member by member, we obtain

$$p_1v_1^n(1+\tan\alpha)^n=p_2v_2^n(1+\tan\beta); \quad . \ . \ . \quad (13)$$

but, by construction, as stated by equation (10),

$$(1+\tan\alpha)^n=1+\tan\beta;$$

hence, equation (13) reduces to

$$p_1v_1^n=p_2v_2^n;$$

and b is a point on the curve. In a similar manner the points c and i are found, etc.

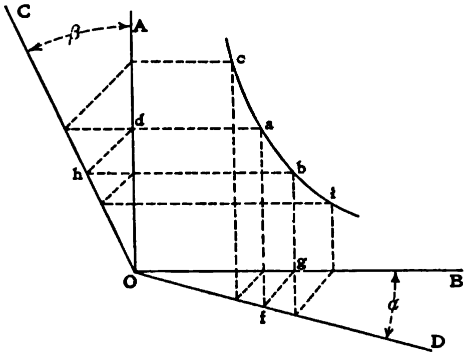


Fig. 28.

The value of n to be used, if we are dealing with steam, is found by means of Zeuner's equation, which is equation (2) of *Art.* 119. And if we wish to plot an isotherm, n in equation (10) is made unity, thus making the angles α and β equal.

In drawing the lines *OD* and *OC*, some convenient value for α, say 15° to 20° is assumed, and the value of β is found by means of equation (10).

In general, the expansion and compression curves, obtained by means of an indicator, for heat motors and compressors conform very closely to the equation

$$pv^n=k;$$

where k and n are constants for a particular process. The value of n, depending upon the nature of the working substance and the condition of operation, may lie anywhere between unity and 1.4.

To find the value of n from the indicator diagram we may proceed as follows: The pressures and volumes corresponding to the points a and b (Fig. 28) are determined by means of a scale, and then, from the equation,

$$p_1v_1^n = p_2v_2^n,$$

a value for n is found. Similarly, values of n are found for a number of points along the curve; and these values will, in general, agree closely among themselves. And the mean of the values so found, compared with the ratio of C_p to C_v, for the given substance, is an indication of how closely the curve, under consideration, approaches an adiabatic.

A method for finding the value of n from the indicator card, which has been found to give satisfactory results and is less laborious than the one just described, is as follows: Lay off the line OC (Fig. 28), making an angle β with the line OA; then choose a point on the curve, such as a, and draw the line ad parallel to OB, and through the point d, a line making an angle of 45° with the line OA, and cutting the line OC at the point h. Through h, now, a second line is drawn, parallel to the line OB, which cuts the curve at some point b; two lines, now, parallel to the line OA, one through a and the other through b, are drawn, and through g, where the line through b cuts the line OB, a line is drawn making an angle of 45° with the line OB and cutting the line, through a, at some point f. Now, this point f lies on a line OD, making some angle α with the line OB. Proceeding in this manner, a number of points, for the line OD, are found which will lie very nearly in a straight line. Drawing a mean line through the points so found, the angle α is determined. And by substituting for β and α, in equation (10), n is found.

It will be noticed that in this case the angle α is determined from the angle β and the curve under consideration; whereas, in the construction first given in this article the curve is determined by means of n and the angles α and β.

129. Behavior of Steam throughout the Cycle. When an engine is first started, the cylinder walls are, of course, at a temperature much lower than that of the steam, and condensation takes place during admission, expansion, and exhaust. After a time, however, permanent cyclic conditions will obtain; i.e., regular periodical fluctuations will have been established, and each cycle, so far as practical conditions permit, will be an exact reproduction of the cycles preceding. When these permanent cyclic fluctuations have been established the incoming steam, during admission, comes into intimate contact with the cylinder walls, which are at a lower temperature, due to the cooling of the lower pressure exhaust steam which has been in contact, just immediately preceding, and condensation takes place. It is true, that, due to wire drawing, a certain amount of drying takes place; but, unless the supplied steam has been superheated, considerable condensation will take place during admission, and will continue during part of the expansion-stroke; and may, in some rare cases, continue throughout the whole of the expansion-stroke. In general, however, during expansion, some point is reached when the temperature of the steam falls below that of the cylinder walls, and reevaporation takes place; i.e., a certain quantity of heat is abstracted, from the steam, by the cylinder walls, during the earlier part of the stroke, and a certain quantity of heat is abstracted, from the cylinder walls, by the steam, during the latter part of the stroke. Even if the quantity of heat abstracted from the cylinder walls were equal to the quantity of heat given up to them, which is never the case, there would still be a thermodynamic loss; for the heat abstracted from the walls is applied at a lower temperature than that absorbed by the walls. The heat abstracted from the cylinder walls, to bring about reevaporation, during exhaust, is completely lost, since it is all rejected to the condenser.*

It is found, by experiment, that the exchange of heat between metal surfaces and perfectly dry gases, is very small even for considerable differences of temperature; hence, the conclusion that, the rapid exchange of heat between the steam and the cylinder walls, in a steam engine, is due to a film of conducting moisture which collects on the surface of the cylinder walls.

130. Change of Dryness during Expansion. To determine the dryness during the expansion-stroke, it is necessary to know the *cylinder feed* and *cushion steam.* To determine the cylinder feed, the exhaust steam, for a given interval of time, is condensed and weighed; and for the same interval of time, the number of working strokes made by the engine, is determined. From this, the mass of steam per stroke, i.e., the cylinder feed is found. The cushion steam is found directly from the indicator diagram.

Let, in Fig. 29, *ABCD* be the actual indicator diagram; *OE* and *OF*, respectively, the axes of zero volume and zero pressure, determined as described in *Art.* 127, and *D* the point where the exhaust-valve has been completely closed and compression begins. If the assmuption be now made that, at the point *D*, the steam is saturated, no serious error is introduced; for, since

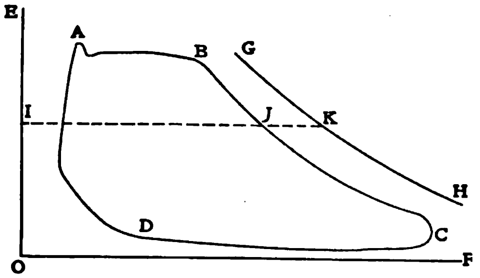


FIG. 29.

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* For a comprehensive discussion of the influence of cylinder walls, see "Thermodynamics of the Steam Engine," by C. H. Peabody.

the mass of the cushion steam is always small in comparison with the total mass of steam present, during the expansion, a small error made in determining the cushion steam, will not appreciably affect the saturation curve. On the assumption then, that at the point D the steam is saturated, the mass of cushion steam is readily found, by means of steam tables; since its volume and pressure are given by the diagram. Taking the sum of the cylinder feed and cushion steam, we have the total mass of steam and water present during expansion; and from this, the saturation curve GH may be plotted. That is, the curve GH gives the volumes, for the various pressures, the steam would have occupied had it been completely saturated. If then, at any pressure such as OI, the horizontal line IK be drawn, the dryness for that pressure is at once found by the relation

$$q=\frac{IJ}{IK}.$$

131. Exchange of Heat, during Expansion, between Steam and Cylinder Walls.

If we plot, from the p–v diagram, a $T-\varphi$ diagram, which is easily done with the aid of steam tables, the transfer of heat, during expansion, between the steam and cylinder walls, is readily found.

In the $T-\varphi$ diagram, Fig. 30, CD is the saturation curve, AC is drawn at a temperature corresponding to the pressure at cut-off, and the point B is so located that the dryness q, for the point of cut-off, as found from the p–v diagram (Fig. 29), is given by

$$q=\frac{AB}{AC}.$$

Taking in this manner the dryness, for various pressures, on the p–v diagram, and transferring to the $T-\varphi$ diagram, the curve of dryness Bnu is found. The curve ux is the curve of condensation at constant volume, and is found as described in *Art.* 115.

The point n, where the vertical line pn becomes tangent to the curve Bnu, is the point of minimum dryness, and is given by

$$q' = \frac{mn}{mo}.$$

It is obvious from the diagram that, during the expansion, up to the point n, the steam is giving up heat to the cylinder walls; and during the remainder of the expansion-stroke, heat is abstracted by the steam, from the cylinder walls. Since the area under the curve is a measure of the heat abstracted, or rejected, it follows that, during the expansion, the heat given to the cylinder walls is to that taken from them as the area $pnBq$ is to the area $pnut$.

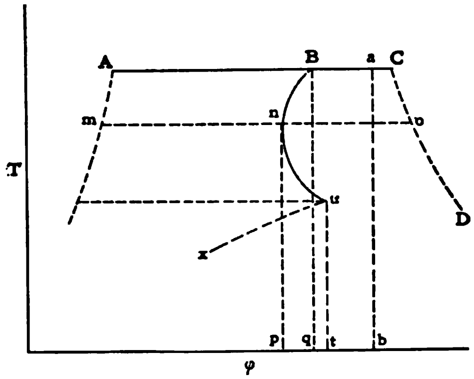

FIG. 30.

As previously explained, the pressure in the cylinder during admission, due to wire drawing, is less than the pressure in the supply pipe; and as stated in *Art.* 125, superheating may occur. In general, however, on account of initial priming, even if there were no condensation during admission, due to the incoming steam coming into contact with the cylinder walls of lower temperature, there would still be present a certain amount of moisture. Most authors assume, in discussing the exchange of heat between the steam and cylinder walls, that the steam is dry at cut-off. This assumption is neither justifiable nor necessary. No prediction can be made unless the dryness of the supplied steam is known. If, however, the dryness of the steam in the supply pipe is known, together with its pressure and the pressure in the cylinder, during admission, the dryness of the steam in the cylinder, during admission, *had there been no condensation*, is readily computed. Let this hypothetical dryness be represented on the $T-\varphi$ diagram (Fig. 30) by

$$q_1=\frac{Aa}{AC}.$$

Since, however, the actual dryness at cut-off, as found from the indicator diagram, is

$$q=\frac{AB}{AC},$$

it follows that an amount of condensation, represented by the change in entropy *Ba*, has taken place during admission. Hence, the heat given up to the cylinder walls by the steam, during admission, is measured by the area *Babq*.

Heat is also given to the cylinder walls during compression; this, however, is not entirely lost. Since, due to this, the temperature of the walls is raised, and the condensation during admission, is partially reduced.

132. Steam Jackets. The fluctuations in temperature of the cylinder walls, as described in *Art.* 129, are the more pronounced the lower the speed of the engine. In other words, the higher the speed of the engine, the smaller the interval of time during which exchanges can take place between the cylinder walls and the steam, and as the speed becomes very high the exchange becomes very small. There is, however, another element to be considered, viz, the cooling of the cylinder, due to the fact that it is always at a higher temperature than the surroundings. This loss of heat must continually be made up by the incoming steam; and hence, increases the condensation. This loss of heat is partially prevented by having the cylinder *jacketed* by some non-conducting material. In some cases, a *steam-jacket* is used, which is maintained full of live steam, taken directly from the supply pipe; and therefore, the pressure of the steam, in the jacket is usually slightly higher than the pressure of the steam, during admission, in the cylinder. There is, therefore, less condensation, during admission, than there would be were the steam jacket absent; and reevaporation begins earlier. On the other hand, the jacket increases the area of the exposed surface; hence, a greater loss of heat, due to radiation. If complete reevaporation takes place before the exhaust-valve opens, the steam during the exhaust-stroke is dry, and very little heat is absorbed by it from the steam in the jacket. The question then is, whether the thermodynamic gain, obtained by applying the heat at a higher temperature, to bring about reevaporation at the earlier part of the stroke, is greater than the energy lost, in the jacket steam, to bring about this reevaporation, plus the greater radiation and heat imparted to the exhaust steam. This question can be answered only by experiment. Experiments performed, on slow and moderate-speed engines, appear to indicate a decided gain in economy, by using a steam-jacket. In a great many cases, however, such discrepant results have been obtained, that it is extremely difficult to say under just what conditions steam jackets are beneficial.

133. Brake Power. The output of an engine of low power, is most conveniently measured by a *friction brake,* which is a device by means of which the power, developed by the engine, is absorbed in overcoming the friction applied to the surface of its fly-wheel; the force required to prevent rotation of the brake, being measured by a balance.

The most common form assumed by the friction brake is depicted in Fig. 31. It consists of a number of wooden blocks fastened by means of bolts, to steel bands, wrapping, approximately, two-thirds of the circumference of the fly-wheel. The wing-nut *w* on the bolt *b* makes it possible to vary the pressure to any desired value. The tie-rod *t*, going from the lower part of the bolt *b* to the lever, is merely to give rigidity to the brake. The rim of the fly-wheel is provided with flanges, so that water may be contained in it, to absorb the heat developed by the work done, in overcoming the friction.

Assume, now, that the fly-wheel is rotating in the direction as indicated by the arrow. Then, due to friction, the brake will tend to rotate in the same direction; and to prevent this, a certain force is applied to the lever, at the point *p*. This force is most conveniently measured by a balance; which may be either a spring balance or a beam balance. Let the fly-wheel be making *N* r.p.m. (rotations per minute), the *net* weight registered by the balance, to prevent rotation, be *W* lbs., and *d* be the horizontal distance between the center of the shaft and point of contact *p*. Then, since power is numerically equal to the product of angular velocity and torque, we have, employing the minute as the unit of time,

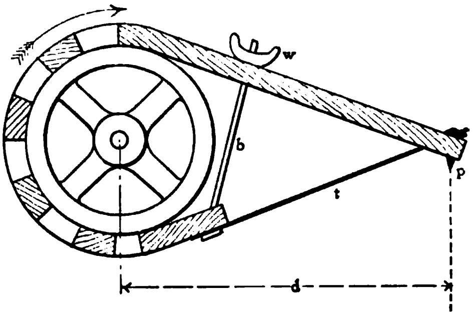

Fig. 31.

$$P = 2\pi NWd \text{ ft.-lbs. per min.};$$

and since *one horse-power* is the equivalent of doing work at the rate of 33,000 ft.-lbs. per minute, we have, for the *brake horse-power*,

$$\text{B.H.P.} = \frac{2\pi N W d}{33,000} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

In the case of very small units, the torque is frequently measured by wrapping a canvas belt around the pulley, and applying tensions to its two free ends. The tensions are then varied, until the machine is loaded to the desired amount, and measured. The torque is then found by taking the product of the difference between the two tensions and the radius of pulley plus one-half the thickness of the belt. In this case, the heat developed by the work done, in overcoming the friction, is also absorbed by water contained in the pulley.

When testing high-power machines, it is neither convenient nor desirable to make friction tests. One method used is that of connecting the engine under test to an electric generator, whose efficiency is known, and by means of an ammeter and voltmeter, or else by a wattmeter, determining its output. From the efficiency of the generator and the power delivered by it, the power delivered to it, by the engine, is readily found.

Another method for determining the power delivered by an engine, is to make the shaft, through which the power is being transmitted, take the place of a transmission dynamometer. This is accomplished by determining the amount of twist, which a definite length of the shaft experiences, when transmitting the given power. Then, from the length and diameter of shaft, its modulus of rigidity, and the angle of torsion, the torque is readily found.

134. Indicated Power. The power expended on the piston of an engine, by the working substance, as found by means of the indicator diagram, is called the *indicated power*. During admission and expansion, work is being done by the working substance on the piston; and during exhaust and compression, work is being done by the piston, on the working substance. Hence, the *net* work done by the working substance, during a cycle, is measured by the area enclosed by the indicator diagram. If then, the area of the indicator diagram be determined and divided by the length of the stroke, reduced to the proper scale, the average ordinate is found. The average ordinate, so found, multiplied by the scale of the spring, used in taking the diagram, gives the *mean effective pressure.* The area of the diagram is most conveniently found by means of a planimeter. There are certain types of planimeters, which are specially designed for determining the mean effective pressure from an indicator diagram. This type of planimeter is very convenient, inasmuch as it is only necessary to set it to the length of the diagram, employing a scale corresponding to the scale of the spring, used in taking the indicator diagram, and following the outline of the diagram with the tracing point of the instrument. The mean effective pressure is then given directly by the reading on the scale.

The mean effective pressure is the average pressure on the piston, during admission and expansion, minus the average pressure during exhaust and compression; hence, it is the *effective* pressure, due to which external work is obtained. If the indicator spring has been calibrated to lbs. per square inch, then the mean effective pressure is also given in lbs. per square inch; and the total effective pressure on the piston is numerically equal to the product of the mean effective pressure and the area, expressed in square inches, of the piston. If we represent by P, the mean effective pressure, in lbs. per square inch, by A the area of the piston, in square inches, by L the length of the stroke in feet, and by N the number of cycles per minute, then the net work done on the piston, per minute, is

$$W = PALN \text{ ft.-lbs.};$$

and the indicated horse-power is

$$\text{I.H.P.} = \frac{PALN}{33,000}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (15)$$

135. Mechanical Efficiency. The indicated power of an engine is always greater than the power delivered by the engine, by an amount which is equal to the power consumed in overcoming the engine friction. *The ratio of the brake horse-power, to the indicated horse-power* gives the *mechanical efficiency*; i.e.,

$$E_m=\frac{\text{B.H.P.}}{\text{I.H.P.}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (16)$$

136. Thermal Efficiency. The *thermal efficiency* of an engine is given by *the ratio of the power delivered by the engine to the power due to the heat taken from the source.* As an example, assume a steam engine to be taking M pounds of steam per minute from a boiler, the total heat of which, per pound, is H. Let the heat of the water in the condenser be h, which we will assume is returned to the boiler without losses. Then the heat, expressed in mechanical units, which is taken per minute from the boiler, is

$$JM(H-h)\text{ ft.-lbs.};$$

and if W represents the number of ft.-lbs. of work delivered per minute by the engine, then the thermal efficiency is

$$E_h=\frac{W}{JM(H-h)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (17)$$

We will now illustrate equation (17) by a numerical example. Assume an engine making 300 r.p.m., doing work against a friction brake whose arm is 5 ft., and which requires a force of 135 lbs., applied at its end, to prevent rotation. If the engine is consuming 16 pounds of saturated steam per minute, under a pressure of 80 lbs., and returns the water without losses directly to the boiler, from the condenser, where the pressure is 2 lbs., what is the thermal efficiency?

Substituting, in equation (17), we find

$$E_h=\frac{2\pi\times300\times135\times5}{778\times16(1182-94.2)}=9.4\text{ per cent};$$

where 1182* is the total heat of steam under a pressure of 80 lbs., and 94.2 the heat of the liquid, corresponding to the temperature of the steam, under 2 lbs. pressure.

137. Commercial Efficiency. The *commercial efficiency* of an engine is given by *the ratio of the power delivered by the engine to the power which a perfect heat engine, working between the same temperature limits, would deliver.* Let the symbols have the same significance as in *Art.* 136, then the work, per minute, which a perfect heat engine would deliver, is

$$JM(H-h)\frac{S-R}{S}\text{ ft.-lbs.};$$

and the commercial efficiency is

$$E_c=\frac{W}{JM(H-h)\frac{S-R}{S}}. \quad . \quad . \quad . \quad . \quad . \quad (18)$$

Substituting in equation (18), the numerical data given as an illustration in the preceding article, we find

$$E_c=\frac{2\pi\times300\times135\times5}{778\times16(1182-94.2)\frac{186}{772}}=39.0\text{ per cent.}$$

This is the proper method of comparison; i.e., comparing the actual performance of the engine with an ideally perfect engine, operating between the same temperature limits.

When an engine exhausts to the atmosphere there is, of course, no heat returned to the boiler by means of the condensed steam, and the heat h, in equations (17) and (18), is lost. It is, however, not proper to charge this entire loss of heat against the engine; since, by proper arrangements part of the heat at least, contained by the liquid, can be returned to the boiler.

There are other methods for rating the performance of engines, which are in certain cases, very convenient. One is, specifying

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* Taken from Peabody's Steam Tables.

the number of pounds of steam per B.H.P. *hour,* consumed by the engine. Another is, specifying the number of B.T.U. per B.H.P. *hour,* or the number of B.T.U. per K.W. *hour* of energy delivered to the bus-bar. The latter is especially expressive; giving, as it does, the rating of the power plant as a whole.

138. Rankine's Cycle. Another important comparison may be made by the aid of Rankine's cycle, the indicator diagram of which is shown in Fig. 32. This indicator diagram is based on the assumption that the cylinder of the steam engine has no clearance and is perfectly insulated. AB represents the admission at constant pressure p_1, BC represents the adiabatic expansion to the pressure p_2, and CD represents the exhaust, at constant pressure p_2.

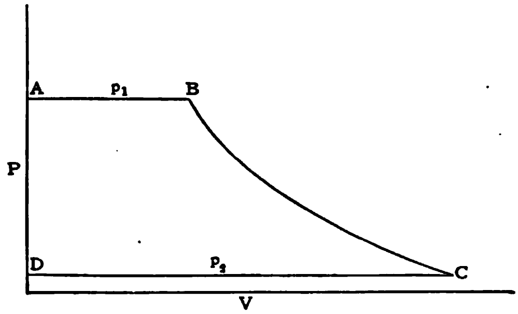

FIG. 32.

Assume now, that we are dealing with a unit mass of liquid, whose specific volume is σ, and that the dryness, during admission, is q_1. If the specific volume of the steam, at the pressure p_1, is s_1, then the volume of the mixture, at the point of cut-off, is

$$v_1 = q_1 s_1 + (1 - q_1)\sigma = q_1(s_1 - \sigma) + \sigma = q_1\mu_1 + \sigma; \quad . \quad . \quad (19)$$

where μ_1 is the increment in volume due to complete evaporation at the pressure p_1. Since the pressure, during admission, is constant, the work done by the steam, on the piston, is

$$p_1 v_1 = p_1(q_1\mu_1 + \sigma). \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

The work done on the piston, by the steam, during the adiabatic expansion, must be equal to the difference between the intrinsic energy of the steam before and after expansion; i.e.,

$$E_1-E_2=J(h_1+q_1\rho_1-h_2-q_2\rho_2); \quad . \quad . \quad . \quad . \quad (21)$$

where E_1, h_1, and ρ_1 are, respectively, the intrinsic energy, the heat of the liquid, and the heat of disgregation, corresponding to the pressure p_1, and E_2, h_2, and ρ_2 are, respectively, the intrinsic energy, the heat of the liquid, and the heat of disgregation, corresponding to the pressure p_2. Or, to put it in another way, the work done on the piston, by the steam, during the expansion, is the difference in the *heat content*, expressed in mechanical units, before and after expansion. The work done on the piston, during exhaust, is

$$-p_2v_2=-p_2(q_2\mu_2+\sigma); \quad . \quad . \quad . \quad . \quad . \quad (22)$$

where q_2 and μ_2 are, respectively, the dryness and increment in volume, due to complete vaporization, at the pressure p_2. Taking the sum of the right-hand members of equations (20), (21), and (22), we find, for the net work done during the cycle,

$$W=J(Ap_1q_1\mu_1+h_1-Ap_2q_2\mu_2-h_2+q_1\rho_1-q_2\rho_2)+(p_1-p_2)\sigma. \quad (23)$$

The second term of the right-hand member of equation (23) is very small in comparison with the other term, and may be neglected; hence, since from equation (55), of Art. 55,

$$r=Ap\mu+\rho,$$

we find, by substituting in equation (23) the proper values,

$$W=J(q_1r_1+h_1-q_2r_2-h_2). \quad . \quad . \quad . \quad . \quad . \quad (24)$$

If p_1 and p_2 are known, the values for h_1, h_2, r_1, and r_2 are found directly from steam tables; and by knowing q_1, the value of q_2 is readily found by means of the $T-\varphi$ diagram.

From equation (24) it is readily seen that the heat converted into work, is the difference between the heat taken in, during admission, and that rejected, during exhaust. This must necessarily follow from the assumption that there are no losses in the engine. But, it must be remembered that in the foregoing discussion, the engine is considered as being independent of the boiler. Hence, by taking the ratio of the work actually performed by the engine, to the work as given by equation (24), a result is obtained which serves as a basis of comparison with other engines operating under similar conditions.

To illustrate, we will take the same numerical values as given in *Art.* 136, excepting that 90 per cent initial dryness will be assumed, instead of complete saturation. The value of r corresponding to 80 lbs. pressure is 899.8, and that corresponding to 2 lbs. pressure is 1021.9. The dryness, after adiabatic expansion, at the pressure of 2 lbs. is found, by means of the $T-\varphi$ diagram, to be approximately, 75.5 per cent. h_1 and h_2 are given, respectively, by 282.2 and 94.2. Substituting these values in equation (24), we have

$$\begin{aligned} W &= 778\times16(0.9\times899.8+282.2-0.755\times1021.9-94.2)\\ &=2{,}817{,}000 \text{ ft.-lbs. per minute.}\end{aligned}$$

Taking the ratio of the work delivered by the engine, to that which would be realized by the Rankine cycle, we find, for the efficiency of the engine,

$$E=\frac{2\pi\times300\times135\times5}{2{,}817{,}000}=45.2 \text{ per cent.}$$

CHAPTER XIII

COMPOUND ENGINES

139. IF a heat engine is to convert a large fractional part of the heat taken from the source, into work, the temperature difference between source and refrigerator must also be large. Hence, other things being equal, for a steam engine to operate economically, it is necessary to have a large range in temperature, or what amounts to the same thing, a large range in pressure.

But, when steam under a pressure of 100 lbs., and upward, is supplied to an engine, the fluctuations of temperature in the cylinder become large; and, consequently, the condensation becomes excessive. To illustrate the fluctuations in temperature, assume an engine receiving steam under a pressure of 100 lbs., and rejecting to a condenser under a pressure of 1 lb. From the steam curve, of *Art.* 101, the temperatures corresponding to these two pressures are, respectively, 328°F. and 102°F.; i.e., a range of about 226°F. Such a large range in temperature means a considerable amount of condensation during the earlier part of the stroke, and a consequent reevaporation during the later part of the stroke; but, as previously explained, this constitutes a thermodynamic drop; i.e., a wasteful application of heat. To obviate this excessive condensation, when high-pressure steam is used, the expansion is made to take place in two or more cylinders; and the engine is said to be a *multiple-expansion,* or *compound engine.* When the expansion takes place in two cylinders, the engine is said to be a *double-expansion* engine, when in three cylinders, a *triple-expansion* engine, etc.

140. Double Expansion. We will consider first the most simple case possible; viz, cylinders without clearance, no losses whatsoever, and a receiver, between the two cylinders, of such volume that the pressure in it, throughout the cycle, is constant. That is, the high-pressure cylinder receives steam from the boiler, which, during admission and expansion, does work on the piston. The steam is then rejected, at constant pressure, to the receiver; the pressure in cylinder and receiver, during the exhaust, being identical. During the same interval of time, that this is taking place in the high-pressure cylinder, the low-pressure cylinder receives an equal mass of steam, from the receiver, which in turn does work, during admission and expansion, on the low-pressure piston. The steam is then expelled, under constant pressure, to a condenser in which a low pressure is maintained.

The indicator diagram, representing the foregoing is shown by Fig. 33. The diagrams, for the two cylinders, are drawn to the same scale and are superimposed. *ABCD* is the diagram for the high-pressure cylinder, and *DCEF* is that for the low-pressure cylinder.

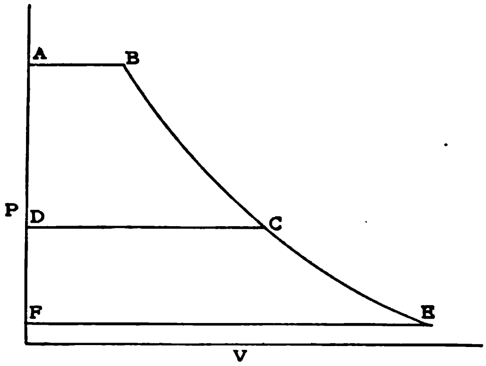

FIG. 33.

It will be noted that the combined diagram *ABEF* is precisely the same as would have been obtained if the expansion, from the initial volume, as represented by *AB*, to the final volume, as represented by *FE*, had taken place in a single cylinder. That is, so far as indicated power is concerned, provided there are no losses, it is immaterial whether the expansion, from the initial volume to the final volume, takes place in one cylinder or a number of cylinders. There is, however, for the case just discussed, due to the fact that the fluctuations in temperature have been reduced, a decided thermodynamic gain. There is also an important mechanical advantage when, other things being equal, expansion takes place in two or more cylinders. For, if the area taken up by the piston rod be neglected, then the stress existing in the rod, at any instant, is proportional to the difference in pressure on the two sides of the piston. This difference is a maximum, while admission is taking place at one end of the cylinder and exhaust at the other. By referring to Fig. 33, it is seen that, for the same ranges in pressure, the maximum difference in pressure for the single-expansion engine is measured by *AF*; whereas, for the double-expansion engine, the maximum differences in pressure for the high and low-pressure cylinders are measured, respectively, by *AD* and *DF*. It therefore follows, that in a compound engine, the piston rods may be considerably reduced in cross-sectional area. as compared with that of a single-expansion engine. And since, for the same initial and final pressures the total work done is the same in either case, it follows that the average thrust on the cranks must be the same for the compound engine as it is for the single-expansion engine. But, in the case of the compound engine, the thrust is more uniformly distributed throughout the cycle; hence, less friction and a consequent smaller amount of wear in the crank bearings and joints. And, further, due to a more uniform thrust, throughout the cycle, there is a smaller fluctuation in speed; hence, for the same uniformity of speed, the fly-wheel for a compound engine need not be as massive as that for a single-expansion engine. On the other hand, the compound engine has a greater number of moving parts; hence, a greater first cost, and additional friction.

It is obvious that the volume of the low-pressure cylinder of a compound engine, for the same initial and final volumes, must be the same as that of a single-expansion engine. The volume of the low-pressure cylinder is therefore fixed by the boiler pressure of the steam, the total expansion, the power to be developed, and the speed of the engine. The volume of the high-pressure cylinder, on the other hand, is a matter of choice; provided always that the ratio of the volume of the low-pressure cylinder to that of the high-pressure cylinder is less than the total ratio of expansion. The point of cut-off, however, for the high-pressure cylinder, as will be shown later, depends upon the ratio of the two volumes.

From a thermodynamic standpoint, the ranges in temperature for the two cylinders should be about equal; since this gives equal fluctuations of temperature in the cylinders. This also gives, very nearly, equal amounts of work done in the two cylinders; which, as will be seen later, is also best mechanically. Since it is advisable to have nearly equal ranges of temperature in the two cylinders, it necessarily follows that the ratio of the cylinder volumes is fixed by the total ratio of expansion. In practice, depending upon the total ratio of expansion, the ratio of the volume of the low-pressure cylinder to that of the high-pressure cylinder may vary from 3 to 5.

141. Tandem Compound Engine with Large Receiver. By a *tandem compound* engine, is meant an engine which has the axes of the two cylinders aligned; and has only one piston rod, which carries both pistons. In an engine of this type, the two pistons, necessarily, have strokes of equal lengths. Assume the volume of the receiver to be so large in comparison with the volume of the two cylinders, that there are, during the cycle, no fluctuations of pressure in the receiver. If, further, there be assumed no clearance and no losses whatsoever, then the indicator diagram will be identical with that depicted in Fig. 33.

Let, in Fig. 34, H represent the high-pressure and L the low-pressure cylinder; and, further, let R be the ratio of the volume of L to that of H. If p_1 be the pressure, during admission, of the steam in H, and p_2 the pressure of the steam in the receiver, which is also the back pressure on P_1, the piston of H, then, neglecting the area of the rod, the stress in the rod, between the two cylinders, due to these two pressures, is

$$S_1=(p_1-p_2)A_1; \quad \dots\dots \quad (1)$$

where A_1 is the area of P_1, and S_1 the stress. In a similar manner, the stress in the rod to the right of P_2, due to the pressures p_2 and p_3, in the cylinder L, is

$$S_2=(p_2-p_3)A_2; \quad \dots\dots \quad (2)$$

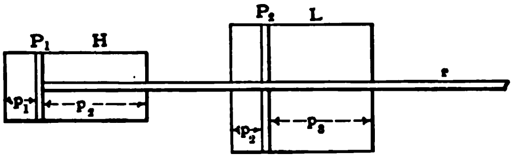

FIG. 34.

where A_2 is the area of P_2, and S_2 the stress. Taking the sum of the right-hand members of equations (1) and (2), we obtain for the stress, in the rod r,

$$S=(p_1-p_2)A_1+(p_2-p_3)A_2. \quad \dots \quad (3)$$

Since now, the cylinders are of equal length, and R is the ratio of their volumes, we have

$$A_1=A_2/R. \quad \dots\dots \quad (4)$$

Substituting the value of A_1, as given by equation (4), in equation (3), we obtain

$$S=(p_1-p_2)\frac{A_2}{R}+(p_2-p_3)A_2. \quad \dots \quad (5)$$

Had the expansion taken place in the low-pressure cylinder between the pressures p_1 and p_3, then the stress in the rod would have been found to be

$$S' = (p_1 - p_3)A_2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Since the relation of pressure and volume of steam is not expressible by a simple equation, it is impossible to eliminate p_2 from equation (5); and, therefore, no general comparison between S, as given by equation (5), and S', as given by equation (6), can be made. But, by assuming particular values for p_1 and p_2, some idea may be obtained in regard to the relation of the stresses S and S'.

To make a comparison, assume p_1 to be 100 lbs., and p_3 1 lb.; and, as a matter of convenience, p_2 to be 20 lbs., then $p_2 = p_1/5$. Substituting this value of p_2 in equation (5), we find

$$S = \left(p_1 - \frac{p_1}{5}\right)\frac{A_2}{R} + \left(\frac{p_1}{5} - p_3\right)A_2. \quad . \quad . \quad . \quad . \quad (7)$$

Again, for the case under consideration, R will have a value of about 3; hence, equation (7) becomes

$$S = \frac{4p_1}{5} \times \frac{A_2}{3} + \frac{p_1A_2}{5} - p_3A_2 = \left(\frac{7p_1}{15} - p_3\right)A_2. \quad . \quad . \quad (8)$$

By comparing equations (6) and (8), it is seen that for the same given initial and final pressures, the maximum thrust for the single-expansion engine is more than double the maximum thrust for the double-expansion engine.

The indicator diagram, of Fig. 33, represents an extreme case; and one that cannot be realized in practice. For, in the first place, to maintain a constant pressure, during the exhaust of the small cylinder and the admission to the large cylinder, requires a receiver of excessive bulk. Secondly, there is always a certain amount of resistance offered to the flow of steam in passing from the first cylinder to the receiver, and from the receiver to the second cylinder. Therefore, the lines representing, respectively, the exhaust for the small cylinder and the admission for the large cylinder, will not coincide. That is, the exhaust line, for the small cylinder, will show a higher pressure than that shown by the admission line for the large cylinder. This drop in pressure is, however, not entirely wasteful; since, due to this partially unresisted adiabatic expansion, part, and in some cases all, of the moisture formed in the first cylinder is removed.

142. Compound Expansion without Receiver. In some engines, usually called *Woolf engines*, the steam passes directly from the one cylinder to the other. The cylinders may be either in tandem or side by side. It is obvious that in engines of this type the two pitsons must begin and end their strokes together. That is, the movements of the two pistons must either be in phase or differ by 180°. The operation is then as follows: Steam is admitted to the small cylinder, up to some desired fractional part of the stroke when cut-off takes place, and then expands to the end of the stroke. At the end of the stroke communication is established between the two cylinders, the steam begins to pass from the small cylinder to the large cylinder and a second expansion takes place. Since, now, the cylinders must remain in communication to the end of the stroke, there can be no cut-off in the low-pressure cylinder; and furthermore, since the pressures in the two cylinders are, at any instant, the same, it follows that the pressure in the small cylinder, during its exhaust, is continually decreasing. Admission and expansion now again take place in the small cylinder, while exhaust is taking place in the large cylinder.

The indicator diagrams, for the cycle just discussed, if it be assumed that there is no clearance and no drop between the two cylinders, will be as represented by Fig. 35. For the high-pressure cylinder, AB represents the admission, BC the expansion, and CD the exhaust; and for the low-pressure cylinder, EF represents the expansion, FG the drop during release, and GH the exhaust. To combine the two diagrams, the diagram *EFGH* must be drawn for a piston area corresponding to that of the diagram *ABCD*, so that equal increments in abscissas, for the two diagrams, measure equal increments in volume. That part of the indicator diagram of the high-pressure cylinder, represented by *ABCI*, will then remain unchanged for the combined diagram. And, to find any point on the expansion curve, for the combined diagram beyond the point *C*, corresponding to the pressure as represented by the point *K*, it is only necessary to draw a horizontal line, such as *KN*, and remember that the total volume of the steam, for this pressure is equal to $KL+MN$. This is represented in Fig. 36, by the line *KN*. In a similar manner, a number of points

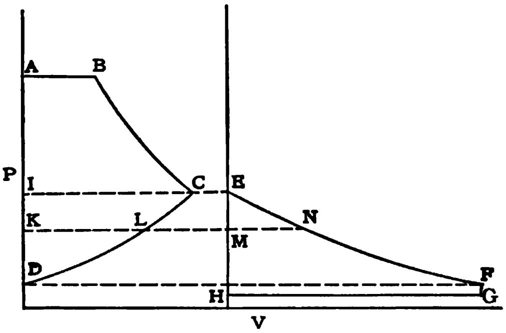

FIG. 35.

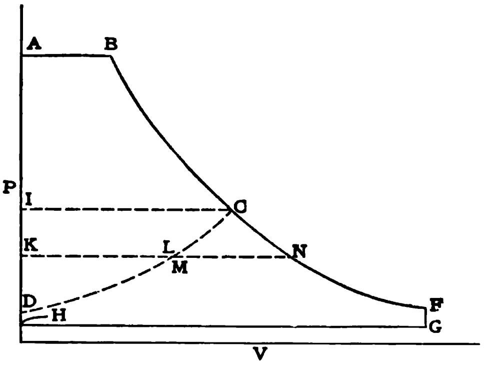

FIG. 36.

are found, and the expansion curve *BCF* is determined. This gives, then, for the equivalent indicator diagram, for the two cylinders, the diagram *ABCFGH*.

143. Tandem Compound and Small Receiver. Since it is impracticable to have a receiver of sufficient volume, so that the pressure in it is constant throughout the cycle, the cut-off, for the large cylinder, must be so chosen that the pressure in the receiver, at the end of the exhaust stroke of the small cylinder is the same as when release occurs, during the next stroke in this cylinder.

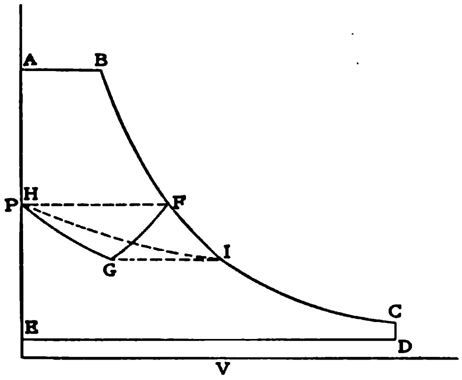

FIG. 37.

Let, in Fig. 37, *ABCDE* be the combined diagram, constructed as described in *Art.* 142; *AB* represents the admission to the small cylinder, *BC* the total expansion, *CD* the drop, during release, in the large cylinder, and *DE* the final exhaust.

Let, now, *F* be the point of release for the small cylinder, then at the same instant admission takes place in the large cylinder; and, since the rate of volumetric displacement for the piston of the large cylinder is greater than that for the piston of the small cylinder, the pressure in the receiver must fall. Let *FG* represent that part of the exhaust curve, for the small cylinder, before cut-off takes place in the large cylinder. After cut-off has taken place in the large cylinder, the steam remaining in the small cylinder is compressed, as represented by the curve *GH*, into the receiver.

If, now, there is to be no drop in pressure when release occurs in the small cylinder, the pressures for the points *F* and *H* must be equal. Since *H* represents the pressure of the steam, when admission begins in the large cylinder, then, by beginning with this point, and taking into account the total volume of the steam for various points, a curve such as *HI*, which shows the falling off in pressure in the large cylinder during admission, is found. And, where this curve, *HI*, cuts the curve of expansion *BC*, will be the point of cut-off for the large cylinder; the pressure at *I* and *G* being identical.

As a matter of convenience, the foregoing discussion has been made with the assumption that the cylinders have no clearance. No new difficulty, however, is introduced by considering the clearance. As previously explained, a small amount of drop is not harmful; hence, absolute precision is not required.

144. Cross-compound Engines. The type of double-expansion engines most frequently used, are those known as *cross-compound engines*. Cross-compound engines are either in twin, i.e., the cylinders are side by side, and the cranks make an angle of 90° with each other, or else, the cylinders make an angle of 90° with each other, and the connecting rods act upon cranks in the same phase. In either case the piston movements are not in phase. That is, there is a phase difference of 90°; therefore, when exhaust begins to take place from the small cylinder, the piston of the large cylinder will not be in a position such that steam can be received. Hence a receiver is necessary.

From a mechanical standpoint, the cross-compound engine is far superior to the tandem-compound; for, in the one case the cranks are actually at right angles, and in the other case, where the cylinders are at right angles, the mechanical effect is precisely the same as though the cranks made an angle of 90° with each other. Therefore, the turning moment, throughout the cycle, is much more uniformly distributed.

The method employed to determine the point of cut-off for the large cylinder, in a cross-compound engine, so that there shall be no drop, is almost precisely the same as that discussed in *Art.* 143. It is only necessary to take into consideration the phase relation of the two piston movements.

In large steam-power plants, the cross-compound engine is the one most commonly employed; boiler pressures as high as 200 lbs. to 250 lbs. being, in some cases, used. Within the last few years, compound engines have been operated in conjunction with low-pressure steam turbines; the turbine operating on the exhaust steam from the low-pressure cylinder.

145. Triple Expansion. Where a more uniform turning moment, than that offered by a cross-compound engine, is desired, *triple expansion* is employed. In the case of triple-expansion engines, the cranks are frequently set so that each crank differs in phase by 120° from the other two; thus giving a good distribution of turning moment. In other cases, however, the triple expansion takes place in four cylinders; i.e., one high-pressure cylinder, one intermediate cylinder, and two low-pressure cylinders. Both of these low-pressure cylinders take steam from the receiver, to which the intermediate cylinder rejects. The engine is equipped with four cranks, with a continuous phase difference of 90°. Triple-expansion engines have been very largely used in marine engineering; but, are now being superseded either by turbines, or else by the combination of triple-expansion engines and low-pressure turbines, which operate on the low-pressure exhaust steam from the reciprocating engines.

From the foregoing, it is obvious that, for cross-compound and triple-expansion engines, it is desirable to have the work done in the various cylinders equal; since this gives the most nearly uniform turning moment for the entire cycle. And, as has been previously stated, the ranges of temperature, for the various cylinders, will be nearly equal, when the work done in the cylinders is equal.

Engines having more than three stages are rare; and, it is doubtful whether they are ever economical. Theory can merely serve as a guide; the final criterion being experiment. In any given case, however, there is a limiting value for the number of stages of expansion; and, in practice, this is fixed when the thermodynamic gain is offset by the interest and depreciation of the extra capital invested, plus the extra mechanical losses

146. Tests of Performance. The tests on compound engines are very similar to those on simple engines. However, when making tests on an engine, for efficiencies, the load should be varied from zero load up to, say, 25 to 50 per cent overload, and a curve plotted, efficiencies as ordinates and loads as abscissas. Or, if it be desired, the number of pounds of water per B.H.P., or else the number of B.T.U. per B.H.P., may be plotted against B.H.P. The curves may also be plotted, using I.H.P. instead of B.H.P. In any case, however, the curve will show the character of performance of the engine for the various loads; and, for what load the best economy is obtained.

If the curve shows that the efficiency of the engine does not decrease rapidly, as the load is decreased, from the normal, then good service will be obtained for variable loads. On the other hand, if the efficiency falls off rapidly with decreasing load, then the engine will give good service only for approximately constant loads.

CHAPTER XIV

INTERNAL COMBUSTION ENGINES AND FUELS

147. WHEREAS, in the steam engine, the combustion of the fuel and the application of heat to the working substance, take place outside of the engine, and in the *internal combustion* engine, as the name implies, combustion of the fuel, and the application of heat to the working substance, take place directly inside of the cylinder, there is, between the two types of heat motors, as regards the manner in which the application of heat takes place, a fundamental difference. Some of the other prominent differences of operation between the two types of heat engines will be discussed later.

The most common fuels which may be used in internal combustion engines are: Coal-oils, alcohol, natural gas, producer gas, blast-furnace and coke-oven gas, city illuminating gas, etc. The fuels most generally used are: Producer gas, natural gas, gasoline, petroleum and alcohol; and in all cases, there must be present a proper amount of air, so that sufficient oxygen is supplied, to bring about complete combustion. Which of these fuels is best depends upon a great many factors; principally upon duty, economy, and convenience.

There are three typical methods for the operation of internal combustion engines; it being the aim to bring about the application of heat to the working substance, for the three different types, respectively, at constant volume, at constant pressure, and at constant temperature; thus, giving three distinct cycles, which will now be discussed in detail.

148. Four-phase Cycle. The *four-phase*[*] or "Otto" *cycle* was first applied to the internal combustion engine by Dr. Otto, in 1876; and is essentially as follows: (1) The piston is at the end of a return stroke with the *exhaust-valve* just closed and the *inlet-valve* open to a chamber where the mixing of the fuel takes place. The mixture consists either of gas and air, or, if a liquid fuel be used, of vaporized liquid and air. The piston then makes an outward stroke, called the *aspirating* stroke, and a charge of mixture is forced, by the external pressure, into the cylinder. (2) The inlet-valve is closed, a return stroke takes place, called the *compression* stroke, and the mixture is compressed until its volume is reduced to that of the clearance. (3) The charge is now ignited, usually by an electric spark, and combustion takes place very rapidly, and practically at constant volume, together with a rapid augmentation of pressure. The gaseous mixture then expands, forces the piston outward and does work on it. This stroke is called the *working stroke* or *power stroke.* (4) When the piston is at the end of the working stroke the exhaust-valve opens; and, during the return stroke, called the *expulsion* stroke, the products of combustion are expelled, and the cycle is completed.

It will be remembered that, in the reciprocating steam engine, every other stroke is a working stroke. Hence, the internal combustion engine, operating on a four-phase cycle, having one working stroke only for every four strokes, must necessarily during the working stroke, other things being equal, store more energy in the fly-wheel. The fly-wheel must, therefore, have a greater moment of inertia in order to carry the load properly during the remainder of the cycle.

The ideal indicator diagram, for a four-phase cycle is represented in Fig. 38. *OF* is the axis of zero pressure, and *AB* represents the aspirating stroke; the pressure in the cylinder and that of

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[*] Ordinarily called "four-cycle"; this, however, is not proper, since there are four phases to a cycle.

the atmosphere being identical. *BC* represents the adiabatic compression of the charge, and the vertical line, *CD*, the combustion at constant volume; *OG* being the axis of zero volume. *DE* represents the adiabatic expansion of the gaseous mixture after combustion, *EB* the drop in pressure to that of the atmosphere, when the exhaust-valve opens, and *BA* represents the expulsion of the products of combustion. The net work done, by the working substance, during the cycle, is represented by the area *DEBC*.

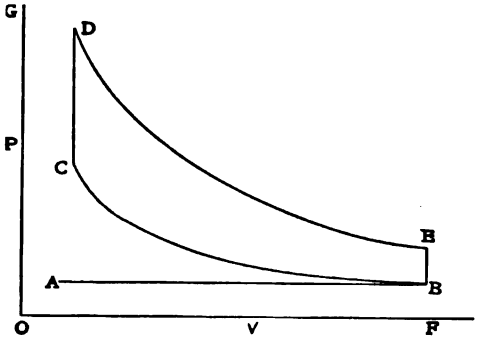

FIG. 38.

149. Two-phase Cycle. In the *two-phase cycle*,* as in the four-phase cycle, it is the aim to bring about the application of heat, to the working substance, at constant volume. In the two-phase cycle engine, however, every other stroke is a working stroke.

Beginning with the ignition of the mixture, there is a rapid rise in pressure, and then expansion of the gaseous products of combustion. In the meantime, a new mixture has been slightly compressed, either by means of an auxiliary compressor, and forced into a subsidiary reservoir, or else compressed in the crank case of the engine. Just before the piston reaches the end of the working stroke, an exhaust-port is opened and then, immediately following this, the inlet-port is opened. Since, at the opening

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* Ordinarily called "two-cycle"; which, however, is an improper designation.

of the exhaust-port, the pressure in the cylinder is slightly in excess of the atmospheric pressure, the gaseous products of combustion immediately begin to flow, out of the cylinder, through the exhaust-port. Immediately after the exhaust-port opens, the inlet-port, on the opposite side of the cylinder, is uncovered by the piston; and, since the mixture has been precompressed to a pressure slightly higher than that obtaining in the cylinder, at the end of the working stroke, a new charge flows into the cylinder. By a suitable arrangement the inflowing gas is directed so as to help expel the exhaust gases. The return stroke now begins; the piston closing first the inlet-port and immediately following the exhaust-port, and the mixture is compressed until its volume is reduced to that of the clearance, when ignition takes place, and the cycle is completed.

From the standpoint of thermal efficiency, the four-phase cycle engine is superior to the two-phase cycle engine; but, on the other hand, the latter engine is far simpler in construction, especially in valve gearing, than the former. And, for the same power output, the two-phase cycle engine always has a smaller mass than the four-phase cycle engine. Hence, for the same power, the two-phase cycle engine requires less space than does the four-phase cycle engine. The thermal efficiency of the two-phase cycle engine is affected by the fact that the products of combustion are more or less imperfectly expelled; and sometimes, due to partial mixing in the cylinder of the new charge and the products of combustion, the exhaust gases may contain unburnt fuel. Furthermore, it may happen that complete combustion has not taken place when the inlet-port is opened, thus causing premature ignition.

150. The Brayton Cycle. In the *Brayton cycle*, it is aimed to bring about the application of heat at constant pressure. In its operation the Brayton engine compresses air in a separate cylinder and stores it in a receiver. This compressed air is admitted through a mass of felt, charged with crude petroleum, to the cylinder of the engine. An auxiliary valve is continuously open, permitting a very small jet of air, charged with petroleum, to flow into a small chamber communicating with the cylinder, where it burns continuously while the engine is in operation. When the main inlet-valve opens, air rushes through the felt, takes up a charge of petroleum, and is ignited by the small flame. Since, now, there is direct communication between the cylinder and the air reservoir, the pressure in the cylinder cannot rise above that in the reservoir; and therefore, combustion takes place at practically constant pressure. The fuel supply is then cut off, and expansion takes place, approximately adiabatically, until the pressure has fallen, depending upon valve adjustment, to any desired value. The exhaust-valve is then opened, and during the return stroke the products of combustion are expelled; hence, this cycle is a two-phase cycle. The Brayton engine has practically been superseded by the one which will be discussed in the next article.

151. The Diesel Cycle. In the Diesel engine the aim is to bring about combustion, and therefore the application of heat, at constant temperature. The Diesel cycle is as follows: During the aspirating stroke a charge of air flows into the cylinder, where it is compressed, approximately adiabatically, to a very high pressure, during the compression stroke. The inlet-valve now opens and the fuel in the form of oil, usually crude oil, is injected and immediately becomes ignited, due to the high temperature of the air, caused by the high precompression. Combustion now takes place at nearly constant temperature, the piston advances, and the expansion of the gases, up to the point when the fuel is cut off, is nearly isothermal. From this point, up to where the exhaust-valve opens, the expansion is approximately adiabatic. The exhaust-valve being fully open, the pressure falls to that of the atmosphere, the expulsion stroke takes place, and the cycle is completed. The Diesel cycle, like the Otto cycle, is a four-phase cycle.

The ideal indicator diagram, of the Diesel engine is shown in

Fig. 39, in which *AB* represents the aspirating stroke, *BC* the adiabatic compression, *CD* the isothermal expansion, *DE* the adiabatic expansion, *EB* the drop in pressure when the exhaust-valve opens, and *BA* the expulsion stroke. Theoretically, the adiabatic expansion *DE* may be carried up to a point when the pressure of the gases has fallen to that of the atmosphere. But, for the same reason as was given in discussing the steam engine, in *Art.* 115, this is not economical.

Theoretically, the Diesel cycle is the most efficient cycle, so far used, in the operation of internal combustion engines; realizing, as it does, the application of heat at a practically constant temperature. Thus approaching, as regards the absorption of heat, the Carnot cycle. The high precompression, however, requires that the engine have a very massive fly-wheel, and subjects it to severe strains. Furthermore, to start the engine, a reservoir of compressed air is required, operating on it very much as steam operates on a steam engine; and when the engine is up to speed, the valves are shifted, so as to disconnect the air, and admit the oil. However, by means of Diesel motors of 300 H.P. capacity, thermal efficiencies as high as 32 to 33 per cent have been obtained.

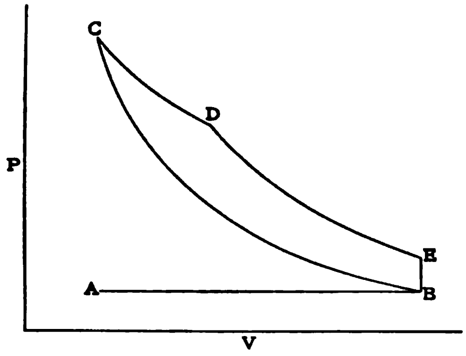

FIG. 39.

FUELS AND FUEL TESTS

152. Before discussing, mathematically, the ideal indicator diagram of the internal combustion engine, it will prove instructive to consider briefly the chemical behavior of the fuel as combustion takes place; and also to compare the volume of the mixture, before combustion, with the volume of the products of combustion. For the volumetric comparison, the same temperature and pressure must, of course, be assumed before and after combustion; and we must know what volume the fuel, in the vaporized or gaseous condition, will occupy. Furthermore, from the chemical constitution of the fuel, we must determine the quantity of oxygen which has to be supplied to bring about complete combustion, and the volumetric changes due to changes in chemical constitution.

153. Chemical Reactions. Experiment shows that, when two or more gases *react chemically* to form a gas or gases of different chemical constitution, the numbers, representing the volumes of the combining gases, are fixed with respect to each other by definite simple ratios; and, likewise, the volume or volumes obtained, after the chemical reaction has taken place, are definitely fixed by the volumes of the combining gases. The foregoing is best illustrated by the consideration of a few concrete cases. As an example, when the gases hydrogen and oxygen react chemically to form steam, then for every given volume of oxygen there is required double the volume of hydrogen; or, to put it numerically, two liters of hydrogen combine with one liter of oxygen to form two liters of dry steam. In the notation adopted by chemists, this chemical action is expressed as follows:

$$H_2+O=H_2O. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The volumetric relations are, of course, only true for identical temperatures and pressures. Likewise, one liter of carbon vapor unites with an equal volume of oxygen to form two liters of carbon-monoxide; and the two liters of carbon-monoxide will combine with one liter of oxygen to form two liters of carbon-dioxide. The reaction is expressed symbolically by

$$CO+O=CO_2. \quad . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

Finally, as another example, six volumes of carbon vapor unite with six volumes of hydrogen to form two volumes of benzene; which is given by

$$C_6+H_6=C_6H_6. \quad . \; . \; . \; . \; . \; . \quad (3)$$

It will be noticed that in all cases, excepting that of two elementary gases of equal volumes, there is, when the gases unite chemically, a reduction in volume; but, in every case, and this is generally so, the volume of the combined gases is two units of the measure of volume chosen. It must, however, not be understood, from the foregoing equations, that in all cases the compound is formed in as simple a manner as here expressed; for, intermediate steps are frequently necessary.

It is of interest to note that, if the density of one of the gases be chosen as unity, and the relative densities of the various gases be known, the density of the compound gas may immediately be found from its chemical formula.* Thus, if the density of hydrogen be taken as unity, which is the most convenient, its density being less than that of any other gas, then the density of oxygen, for the same temperature and pressure, is approximately 16. Hence, we find, from equation (1), since three volumes, having a combined mass of 18, reduce to two volumes, the density of dry steam, with respect to hydrogen equals 9. In a similar manner, from equation (2), since carbon vapor has 12 times the

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* The determination of the chemical constitution of compounds is usually attended by extreme difficulties; but it is here neither possible nor is it essential to describe the various methods used. For complete descriptions and discussions, the student is referred to standard works on physical chemistry.

density of hydrogen, the density of carbon-dioxide is found to be 22; and, from equation (3), the density of benzene is found to be 39.

154. Gasoline. The most volatile of all the fuel oils obtained from petroleum, by fractional distillation, is *gasoline.* The composition of gasoline is somewhat variable; but, its chemical constitution is represented with sufficient accuracy by the formula C_6H_{14}, and its density by 0.7. Assuming the formula C_6H_{14}, then the chemical equation, representing complete combustion, is given by

$$C_6H_{14}+19O=6CO_2+7H_2O. \quad . \quad . \quad . \quad . \quad (4)$$

Since the temperature in the cylinder of an internal combustion engine is of sufficient intensity to insure complete vaporization of the fuel, the gasoline vapor will behave as a gas; and since two volumes of gasoline vapor will yield, upon decomposition, six volumes of carbon and fourteen volumes of hydrogen, nineteen volumes of oxygen, under the same condition, as regards temperature and pressure, must be supplied to bring about complete combustion. Twelve of these volumes of oxygen will combine with the carbon, to form twelve volumes of carbon-dioxide, and seven volumes will combine with the hydrogen to form fourteen volumes of dry steam. Since the average composition of the air, by volume, is 21 per cent oxygen and 79 per cent nitrogen, it follows that for every volume of oxygen supplied there will be present 79/21 volumes of nitrogen. Hence, assuming that the mixture, before combustion, contains two volumes of gasoline vapor, then its total volume is

$$2+19+\frac{79}{21}\times 19=92.5 \text{ volumes (very nearly).}$$

And after combustion the volume, referred to the same temperature and pressure, is

$$12+14+\frac{79}{21}\times 19=97.5 \text{ volumes (very nearly).}$$

Hence, the volume of the products of combustion, referred to the same temperature and pressure, if just enough air be present to bring about complete combustion, is approximately 5.4 per cent in excess of the volume of the mixture. Experience, however, shows that, for an internal combustion engine to operate satisfactorily, the quantity of air supplied to it must be considerably in excess of that required for complete combustion. The excess may vary from 15 to 50 per cent; and this, together with the neutral gases, remaining in the cylinder after the expulsion stroke, reduces the difference between the volumes, before and after combustion, appreciably.

155. Kerosene. The most important of the fuel oils, obtained by the fractional distillation of petroleum, and representing about 50 per cent of the total yield, is kerosene. Kerosene is considerably less volatile than is gasoline, has an average density of about 0.805; and its chemical constitution is represented, with a fair degree of accuracy, by the formula $C_{10}H_{22}$. The chemical equation, representing complete combustion, then is

$$C_{10}H_{22}+31O=10CO_2+11H_2O; \quad . \quad . \quad . \quad . \quad (5)$$

and by the method used, when dealing with gasoline, we find for the volume of the mixture, before combustion,

$$2+31+\frac{79}{21}\times 31=149.6 \text{ volumes (very nearly).}$$

After combustion, the volume, referred to the same temperature and pressure, is

$$20+22+\frac{79}{21}\times 31=158.6 \text{ volumes (very nearly).}$$

Hence, if the quantity of air present carries just a sufficient amount of oxygen to bring about complete combustion, then the volume of the products of combustion is 6 per cent in excess of the volume of the mixture. This percentage difference is, of course, reduced somewhat by an excess of air and neutral gases being present during the cycle.

That the volumes of the gases, before and after combustion, are found to be nearly equal is due to the fact that the volume of inert nitrogen, which is necessarily present, is always large in comparison with the volume of the other gases. In the two cases just discussed, viz, gasoline and kerosene, the volume of the gases after combustion was found to be greater than the volume of the mixture. This, however, is not necessarily the case; for, if we consider carbon-monoxide as a fuel we shall find conditions reversed. The chemical equation, expressing complete combustion for CO, is

$$CO+O=CO_2; \quad . \ . \ . \ . \ . \ . \ . \quad (6)$$

from which we find, respectively, for the volumes of the mixture and products of combustion, approximately 6.8 and 5.8. This gives the volume, after combustion, approximately 14.7 per cent less than the volume of the mixture. CO is a gas of much lower density than is either gasoline or kerosene vapor; and, by considering the two following cases, it becomes manifest that, other things being equal, the higher the density of the fuel gas, the greater the volume of the gases, after combustion, in comparison with the volume of the mixture. Assume the two fuels to be, respectively, C_2H_2 and C_6H_6. For the former we have

$$C_2H_2+5O+\frac{79}{21}\times 5N=2CO_2+H_2O+\frac{79}{21}\times 5N; \quad . \ . \quad (7)$$

and, for the latter,

$$C_6H_6+15O+\frac{79}{21}\times 15N=6CO_2+3H_2O+\frac{79}{21}\times 15N. \quad . \quad (8)$$

From equation (7), we find, for the volumes, before and after combustion, respectively, 25.8 and 24.8; and, from equation (8) we find, for the volumes, before and after combustion, respectively, 73.4 and 74.4. That is, in the latter case, where the density of the fuel is greater, the volume of the gases after combustion, in comparison with the volume of the mixture, is greater than it is in the former, for the less dense fuel. In general, the fuel consists principally of carbon and hydrogen; and the products of combustion consist of CO_2 and H_2O vapors. Hence, if the fuel is comparatively rich in hydrogen, then, since the H_2O vapor is of a much lower density than is CO_2 vapor, it follows that the products of combustion occupy a greater volume than they would, were they produced, from a fuel poor in hydrogen.

156. City Gas. The gases, used for illuminating purposes, in different cities, vary considerably in composition. Not only is there a variation in going from one plant to another, but the gas drawn from the supply main will be found to vary somewhat for different parts of the day. The following table is a fair average for the composition, by volume, of the gas supplied in the Borough of Manhattan:

	Per Cent.
Carbon-dioxide (CO_2)	1.9
Illuminants (practically C_2H_4)	9.9
Carbon-monoxide (CO)	18.2
Methane (CH_4)	22.75
Hydrogen (H_2)	42.
Nitrogen (N_2)	5.25

It is, therefore, impossible to make computations, of any value, with respect to city gas, unless samples of the gas supplied to the engine, are subjected to chemical analysis.

157. Calorific Value of Fuel. In general, to decompose a compound, into its constituents, requires an expenditure of energy in the form of heat; hence, unless the fuel is in a form such that it can combine directly with oxygen, without first being decomposed, the available heat, i.e., the *calorific value of the fuel*, is less than that developed by its combination with oxygen. Furthermore, if the fuel is in the liquid form, heat is absorbed to convert it into a vapor. In general, however, the heat of vaporization is negligibly small in comparison with the other quantities involved.

As a matter of illustration, we will assume that methane (CH_4) is used as a fuel. The chemical equation, for complete combustion, is

$$CH_4+4O=CO_2+2H_2O. \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

In *Art.* 29, it was stated that when 1 gram of H combines with O to form H_2O, about 34,000 gram calories are evolved, and during the combination of 1 gram of C with O to form CO_2, about 8000 gram calories are evolved. Assuming now, as a matter of convenience, 1 gram of CH_4, then since the density of carbon vapor with respect to hydrogen is 12, we will have 0.25 grams of H and 0.75 grams of C. Therefore, the complete combustion of 1 gram of CH_4, if there were no heat required to decompose the compound, would yield

$$0.25\times 34{,}000+0.75\times 8000=14{,}500 \text{ gram calories.}$$

Experiment, however, shows that, when 1 gram of CH_4 is consumed, to form H_2O and CO_2, approximately 13,200 gram calories are evolved. Hence, the difference, viz, 1300 gram calories are absorbed in decomposing the compound into its elements.

It must always be remembered that, the experiments conducted for the purpose of determining heats of combustion and decomposition of a fuel, though simple in operation, may be attended by difficulties of a chemical nature; and therefore, in general, the values found in tables, must be taken as being only approximations.

158. Determination of Calorific Values. The determination of the calorific value of a gaseous fuel is a very simple experiment, provided there is available a modern gas calorimeter. The gas, from the source of supply, is first passed through an accurately calibrated meter, and from this through a pressure regulator which maintains, throughout the progress of the experiment, the pressure of the gas constant. From the pressure regulator, the gas is supplied to a Bunsen burner where combustion takes place. The air has free access to the chamber in which the burner is placed, such that a sufficient quantity of oxygen is supplied to insure complete combustion. From the combustion chamber, the products of combustion pass through a system of tubing, surrounded by circulating water, in a manner such that the temperature of the exhaust gases is reduced to that of the outflowing water, which is practically the same as that of the surroundings, when exhaust takes place. The inflowing water, by means of proper arrangements, is maintained at practically a constant pressure and temperature; and hence, when steady conditions have been attained, the temperature of the outflowing water will also be practically constant.

The exhaust gases will consist mainly of CO_2; since the H_2O, which is formed by combustion, is condensed. After steady conditions have been assumed, i.e., the water due to condensation is flowing at a constant rate, and the temperatures of the water at inflow and outflow are sensibly constant, the outflowing circulating water is caused to flow into a receptacle of known weight, and at the same time the water of condensation is caused to flow into a second receptacle of known weight. Simultaneously with these two operations, a reading is taken on the gas meter and the temperatures of the circulating water at inflow and outflow are noted. The temperatures of the water at inflow and outflow are then noted at suitable intervals of time, until a sufficient quantity of circulating water has been collected. A second reading is then taken on the gas meter, and, simultaneously with this, the collection of water in the two receptacles is discontinued.

From the initial and final readings of the gas meter, the volume of the gas consumed, corresponding to the existing pressure and temperature, is found; and this volume, corrected to a temperature of 62°F.* and a pressure equal to 30 inches of mercury, gives the volume, as regards American gas engine practice, for standard conditions. From the mean of the temperatures for inflow and outflow, together with the quantity of circulating water collected, the quantity of heat, absorbed by the circulating water, is readily found.

Let during the progress of the experiment, τ_1 and τ_2, respectively, be the mean of the temperatures of inflow and outflow, and M the mass of circulating water collected, then the quantity of heat absorbed, by the circulating water, is

$$Q'=M(\tau_2-\tau_1). \qquad \text{. . . . . .} \quad (10)$$

Q′, in equation (10), represents the heat of combustion, plus the heat of condensation. Since, however, the temperature of the exhaust gases, in the case of any internal combustion engine, is always higher than that of condensation, corresponding to the existing pressure, of the H_2O vapor present, it follows that the heat available for doing work in the engine is less, for the given quantity of gas consumed, than is indicated by equation (10). To obtain the available heat, there must be deducted from the heat absorbed by the circulating water, the heat due to condensation, plus the heat given up by the condensed steam in cooling from the temperature of condensation to the temperature of the outflowing water. Let m be the mass of steam condensed, τ the temperature at which condensation takes place, and r the corresponding heat vaporization. The quantity of heat given up to the circulating water, by the steam and condensed water, then is

$$q=m\{r+(\tau-\tau_2)\}. \qquad \text{. . . . .} \quad (11)$$

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* There is no gain by using 62°F. in place of 0°C. For, the temperature is seldom 62°F.; hence, if accurate results are required, a correction for temperature is necessary. It therefore would be more convenient if 0°C. were at all times used, since this is the temperature used by physicists and chemists, as the standard temperature, for which the volumes of gases are specified.

Subtracting equation (11) from equation (10), we find the available heat,

$$Q=M(\tau_2-\tau_1)-m\{r+(\tau-\tau_2)\}. \quad . \quad . \quad . \quad . \quad (12)$$

Dividing by V, the volume of gas consumed, we find the available heat per unit volume; i.e., the calorific value, is

$$H=\frac{M(\tau_2-\tau_1)-m\{r+(\tau-\tau_2)\}}{V}. \quad . \quad . \quad . \quad . \quad (13)$$

The calorific value is usually specified in B.T.U. per standard cubic foot of gas.

The temperature of condensation is approximately the same as the normal boiling-point, since, the pressure in the calorimeter does not vary greatly from that of the atmosphere. However, since q is always small in comparison with Q', no serious error can be introduced in the final result by not estimating the value of τ with absolute precision.

159. Liquid Fuels. In the case of liquid fuels, the calorific value may be determined in precisely the same manner as that of gases. It is only necessary to have an accessory piece of apparatus, by means of which the liquid is converted into a gas or vapor; the quantity of fuel consumed being determined by weighing. In the case of liquid fuels, however, the calorific value is usually specified in B.T.Us. per pound of fuel.

As a matter of interest, a table is here appended, in which is given the lower calorific values of various substances; i.e., that calorific value is given for each substance which would have to be taken if it were considered as a fuel for an internal combustion engine. The calorific values, given in this table, are approximately correct for the first three figures; and the chemical formulæ for gasoline and kerosene, as previously stated, are only approximations. For gaseous mixtures, such as city gas, no figures for the calorific value can be given on account of the variablity in constitution. Actual experiment, by means of a Junker's calorimeter, shows the calorific value of illuminating gas as furnished to the Borough of Manhattan, to be about 590 B.T.U. per cubic foot.

CALORIFIC VALUES

Gas.	Chemical Formula.	Gram Calories, Per Gram.	British Thermal Units. Per Pound.	British Thermal Units. Per Cu. Ft. at 32° F.	British Thermal Units. Per Cu. Ft. at 62° F.
Hydrogen.........	(H_2)	28900	52000	291	273
Methane..........	(CH_4)	11900	21400	957	898
Acetylene.........	(C_2H_2)	11700	21100	1530	1440
Ethylene..........	(C_2H_4)	11400	20500	1600	1510
Ethane...........	(C_2H_6)	11300	20300	1700	1600
Butylene..........	(C_4H_8)	10800	19400	3030	2850
Benzene..........	(C_6H_6)	9610	17300	3770	3540
Carbon-monoxide..	(CO)	2430	4380	343	322
Carbon...........	(C_2)	8110	14600		
Liquid					
Gasoline..........	(C_6H_{14})	10300	18500		
Kerosene..........	($C_{10}H_{22}$)	10100	18200		
Ethyl Alcohol.....	(C_2H_6O)	6560	11800		
Methyl Alcohol....	(CH_4O)	4750	8550		

THERMAL CAPACITIES AND DENSITIES OF GASES

	Chemical Formula.	Grams per c.c. for Standard Pressure at 0° C.	Thermal Capacities, Gram Calories Per Gram. C_p	Thermal Capacities, Gram Calories Per Gram. C_v	Ratio. C_p/C_v
Air..............	....	0.001293	0.2375	0.169	1.405
Oxygen...........	(O_2)	0.001429	0.217	0.154	1.41
Nitrogen..........	(N_2)	0.001255	0.244	0.173	1.41
Hydrogen.........	(H_2)	0.00008955	3.409	2.42	1.41
Carbon-monoxide..	(CO)	0.001250	0.242	0.173	1.40
Carbon-dioxide....	(CO_2)	0.001965	0.217	0.168	1.29
Methane..........	(CH_4)	0.000715	0.593	0.449	1.32
Ethylene..........	(C_2H_4)	0.001251	0.429	0.346	1.24
Ammonia.........	(NH_3)	0.0007616	0.530	0.402	1.32
Superheated steam.	(H_2O)	....	0.480	0.361	1.33

The values for thermal capacities given in the foregoing table are the results obtained from experiments conducted, in general, between the limits of 0°C. and 200°C. In some cases the ranges were very small; hence, the given values are not necessarily true for high temperatures.

The various values given in the two foregoing tables have mostly been taken from Landolt and Börnstein's tables.

CHAPTER XV

IDEAL COEFFICIENT OF CONVERSION AND ELEMENTARY TESTS

160. In *Art.* 102 it was shown what fractional part of the heat abstracted from the source, can be converted into work by a perfect steam engine and boiler, and in *Art.* 138, a perfect engine was considered independently of the boiler. Either of these results may be employed as a standard, depending upon whether we are considering the engine and boiler jointly or the engine alone. Similarly, some ideal standard of performance, by means of which internal combustion engines may be compared, must be assumed. The ideal indicator diagram based upon the four-phase cycle, has been found convenient; since this cycle is the one most generally used.

161. Ideal Indicator Diagram. To deduce an expression for the maximal amount of work that can be realized during a cycle, by means of an internal combustion engine, it is, of course, necessary to assume perfect conditions. The following assumptions will be made: The compression of the mixture is adiabatic, the combustion of the fuel, and therefore the application of heat, takes place at constant volume, the expansion of the products of combustion is adiabatic, the rejection of heat takes place at constant volume, and the thermal capacity of the gas is constant throughout the process. A further assumption, which by a previous discussion, under the heading of fuels, has been shown to be approximately true, will have to be made; viz, that for the same temperature and pressure, the volume occupied by the gaseous mixture, before combustion, is equal to the volume of the products of combustion.

In Fig. 40, *OI* and *OH* are, respectively, the axes of zero pressure and zero volume, and the various parts of the cycle are represented as follows: *AB* is the aspirating stroke at constant pressure, *BC* the adiabatic compression of the mixture, *CD* the combustion and application of heat at constant volume, *DE* the adiabatic expansion of the products of combustion, *EB* the rejection of heat at constant volume, and *BA* the expulsion stroke at constant pressure. Since, during the aspirating and expulsion strokes,

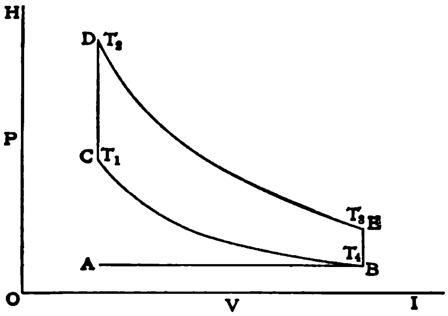

FIG. 40.

the pressures on the two sides of the piston are assumed equal, the net work done, during the cycle, is obviously measured by the area *DEBC*. If, now, we represent by T_1, T_2, T_3, and T_4, the temperature of the working substance for the points *C*, *D*, *E*, and *B*, the heat applied during combustion, while the pressure rises from that represented by the point *C* to that represented by the point *D*, is

$$Q_1 = MC_v(T_2 - T_1); \quad . \; . \; . \; . \; . \; . \; . \quad (1)$$

where *M* is the mass of the mixture, C_v the thermal capacity per unit mass at constant volume, and Q_1 the heat developed during combustion. The heat rejected at constant volume, while the pressure falls from that represented by the point *E*, to that represented by the point *B*, is

$$Q_2 = MC_v(T_3 - T_4); \quad . \; . \; . \; . \; . \; . \; . \quad (2)$$

where Q_2 is the heat rejected by the working substance, while the temperature changes from T_3 to T_4. It is immaterial whether the change in temperature, from T_3 to T_4, since the final result is precisely the same, takes place inside or outside of the cylinder. The conditions are analogous to those discussed, in *Art.* 114, for condensation at constant volume in the steam cylinder.

Since, now, there is no exchange of heat during the adiabatic expansion *DE*, and, likewise, during the adiabatic compression *BC*, and the thermal capacity of the mixture is approximately equal to that of the products of combustion, it follows, from equations (1) and (2), that the heat converted into work, during the cycle, is

$$Q_1 - Q_2 = MC\{_v(T_2 - T_1) - (T_3 - T_4)\}. \quad . \; . \; . \quad (3)$$

And, since the ratio of the heat converted into work to that abstracted from the source, is the ideal coefficient of conversion, we find

$$\eta = \frac{Q_1 - Q_2}{Q_1} = \frac{MC_v\{(T_2 - T_1) - (T_3 - T_4)\}}{MC_v(T_2 - T_1)};$$

from which

$$\eta = 1 - \frac{T_3 - T_4}{T_2 - T_1}. \quad . \; . \; . \; . \; . \; . \; . \; . \quad (4)$$

Since both *DE* and *BC* represent adiabatic changes for the same changes in volume, we have, from equation (50), *Art.* 49,

$$\left(\frac{v_2}{v_1}\right)^{n-1} = \frac{T_3}{T_2} = \frac{T_4}{T_1} = \frac{T_3 - T_4}{T_2 - T_1};$$

where v_1 is the volume of the mixture before compression, and v_2 the volume after compression. Hence equation (4) becomes

$$\eta = 1 - \left(\frac{v_2}{v_1}\right)^{n-1}. \quad . \; . \; . \; . \; . \; . \; . \; . \quad (5)$$

Equation (5) shows that the ideal coefficient of conversion is a function of the ratio of the volume before compression to the volume after compression; and increases with the amount of precompression.

162. Theoretical Temperatures. The temperature which would obtain upon complete combustion, if there were no losses, is readily computed for any given case. That is, the theoretical rise in temperature, viz, $T_2 - T_1$, is numerically equal to the ratio of the heat of combustion to the thermal capacity of the products of combustion. However, it is found to be necessary, in order to have proper lubrication between the piston and cylinder walls, so as to prevent deterioration of material, to abstract heat from the cylinder walls, either by *water jacketing*, or else by *air cooling*. The former, that is *water cooling*, is brought about by having water, at a comparatively low temperature, circulate in a jacket surrounding the cylinder; and the latter, viz, air cooling, is brought about by increasing the surface of the exposed part of the cylinder by means of *ribs*, and having a stream of air playing over it continuously, by means of an air blower of some kind, or else, as is the case in some automobile engines, the circulation of air is brought about by the motion of the car. In any case, the heat abstracted, due to either *water* or *air cooling*, limits the rise in temperature. Therefore, the temperature found in the cylinder of an internal combustion engine, is always less than that predicted from the heat of combustion and the thermal capacity of the products of combustion. Frequently, the actual temperature is found to be only 50 per cent of the theoretical temperature.

163. Standard Diagram. In deducing the expression for the *ideal coefficient of conversion* for the internal combustion engine, *Art.* 161, certain assumptions, in regard to thermal capacities and volumes before and after combustion, were made, which are only approximations. But the errors involved in these assumptions are very small in comparison with the difference between the actual and theoretical temperatures obtaining in the cylinder.

However, the diagram described in *Art.* 161, and the results deduced therefrom, though differing materially from what can be realized in practice, are very convenient as a basis for comparing the performances of internal combustion engines.

ELEMENTARY ENGINE TESTS

164. Brake Power and Indicated Power. The power delivered by an internal combustion engine is determined in precisely the same manner as is that of a steam engine. This has been fully described in *Art.* 133. Furthermore, the indicated power of an internal combustion engine is also found in the same manner as is that of a steam engine, as described in *Art.* 134. But it must be emphasized that N, in equation (15) of *Art.* 134, represents not the number of revolutions per minute of the fly-wheel, but the number of *cycles per minute* in the cylinder under test.

The ratio of *Brake Power* to *Indicated Power* is, of course, in the case of an internal combustion engine, as well as in the case of a steam engine, a measure of the *mechanical efficiency.* It is found, however, that the mechanical efficiency of an internal combustion engine, other things being equal, is always less than the mechanical efficiency of a steam engine. This is principally due to the fact that, owing to the high temperatures existing in the cylinders of internal combustion engines, the lubrication is not as good as that obtained in steam cylinders.

165. Thermal Efficiency of Internal Combustion Engine. The *thermal efficiency* of an internal combustion engine is, of course, the ratio of the power delivered by the engine, to the power due to the fuel consumed. In making a test, the engine is loaded by means of a brake, or some other contrivance, to the desired amount. Then, in the case of a gaseous fuel, the volume of gas consumed is measured by means of a meter. Simultaneously with this, as described in *Art.* 158, the calorific value of the gas is determined. The best results are obtained if continuous tests are made for the calorific value of the fuel; that is, if the supply to the fuel calorimeter is tapped directly onto the main, supplying fuel to the engine, and samples of the fuel are tested, for calorific values, throughout the *entire run.* The ratio, then, of the work done by the engine, during the test, to the work, expressed in the same units, due to the fuel consumed, which is equal to the product of the volume of gas consumed during the run and the mean calorific value of the gas, as found by means of the gas calorimeter, is a measure of the thermal efficiency. Or, if a liquid fuel be used, the work due to the fuel consumed, is found from the product of the mass of liquid consumed, during the run, and the mean calorific value per unit mass. The calorific value, per unit mass of the liquid, is determined as described in *Art.* 159.

166. Actual Indicator Diagram of Internal Combustion Engine. By means of the indicator diagram, taken from an internal combustion engine, the behavior of the working substance may be conveniently studied. The actual indicator diagram differs, of course, from the ideal indicator diagram, as depicted in Fig. 40. Whereas, in the ideal indicator diagram, the line representing the aspirating stroke, is parallel to the axis of zero pressure, in the actual indicator diagram the line representing the aspirating stroke approaches the axis of zero pressure, as represented by *AB* in Fig. 41. This is due to the throttling effect of the inlet-valve, on account of which, the pressure in the cylinder decreases as the piston advances. In Fig. 41, *OI* and *OH* are, respectively, the axes of zero pressure and of zero volume, and *AA'* is the *atmospheric line.* The curve *AB,* as just stated, represents the aspirating stroke; and shows the pressure in the cylinder at the end of this stroke, less than the atmospheric pressure, by an amount *A'B.* The compression of the mixture, which is approximately adiabatic, is represented by the curve *BC.* The combustion of the mixture, and consequent rise of pressure in the cylinder, is represented by the curve *CD,* which is, if the ignition has been properly timed, practically parallel to the axis *OH.* *DE* is the curve representing the expansion of the products of combustion. At *E* the exhaust-valve begins to open, and the pressure decreases rapidly to the end of the stroke *F*. The expulsion stroke then begins, and the pressure continues to decrease rapidly up to the point *G*. At this point, the exhaust-valve is fully open and the pressure decreases gradually as represented by the curve *GA*, to the end of the stroke, where the pressure is practically atmospheric, and the cycle has been completed. The pressure in the cylinder, during expulsion, is higher than that of the atmosphere due to the resistance offered by the exhaust-valve, to the outflow of the products of combustion.

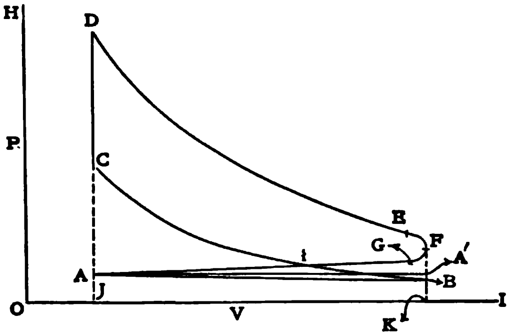

FIG. 41.

That part of the diagram, which represents the effects due to throttling, has been purposely exaggerated.

The curve *DE* is usually, more or less, wavy; this may be due to various causes. If the vibrations appear to be regular and of decreasing amplitude, they are principally due to the inertia of moving parts of the indicator. On the other hand, if the pressure is apparently constant for a time, then suddenly decreases, etc., the waves are due to friction between the piston and cylinder of the indicator. This trouble is readily removed by proper cleaning and lubrication of the piston and cylinder. Furthermore, waves may be established in the mixture in a manner similar to that described in *Art.* 69; i.e., as the piston begins to compress the mixture a wave of compression travels through the medium to the other end of the cylinder, where it is reflected, with change of sign. This reflected wave then travels toward the piston; and when it meets the piston reflection again takes place, etc. In this manner, inequalities in pressure may be established, which under certain conditions may persist throughout the compression and expansion strokes. However, in general, these inequalities will not be manifested to any marked degree on the indicator diagram, since the inertia, of the moving parts of the indicator, will tend to suppress them.

By an inspection of Fig. 41, it is obvious that the work done *on* the piston during the aspirating stroke is measured by the area *JABK*; and, likewise, the work done *by* the piston during the compression stroke is measured by the area *KBCJ*. During combustion, since there is no displacement of the piston, the work done is zero. During expansion the work done *on* the piston is measured by the area *JDEFK*. And, during expulsion, the work done *by* the piston is measured by the area *KFGAJ*. By taking the algebraic sum, we find that the net work done *by* the working substance, during the cycle, is measured by the difference between the areas *CDEFGi* and *AiB*.

Hence, if the mean effective pressure is determined by means of a planimeter, the tracing point of the planimeter, in tracing the area *AiB* must travel in a sense opposite to that pursued in tracing the area *CDEFGi*. That is, if *i* be the starting point, then, to find the difference between the two areas, the tracing point of the planimeter must follow, in order, the path *i*, *C*, *D*, *E*, *F*, *G*, *i*, *A*, *B*, *i*.

The area *AiB* represents the work lost, due to valve throttling, and is, in well designed engines, small in comparison with the area *CDEFGi*. If the power lost, due to valve throttling, is large in comparison with the total indicated power, the valves must be readjusted. In general, the spring which gives good results for measuring the indicated power, has a modulus so high that the part of the diagram, representing the power lost, due to valve throttling, is too small to be accurately measured. But, by using a stop, so as not to injure the spring, a much lower scale spring may be employed. In this manner the power lost, due to throttling, and also the amount of precompression may be accurately determined.

167. Efficiency and Precompression. In *Art.* 161, equation (5), it was shown from theoretical considerations that, other things being equal, the thermal efficiency increases with the amount of precompression. This is found to be so in practice. There are, however, limits, beyond which the precompression may not be carried, due to the severe strains to which the engine is subjected during the explosion of the mixture.

Tests made, in the Cooper Union Laboratories, on a Fairbanks 8 H.P. gas engine, gave the following results:

v_1/v_2	R.P.M.	B.H.P.	Efficiencies.	
			Thermal, Per Cent.	Mechanical, Per Cent.
4.72	368	6.54	13.9	74.0
4.96	395	7.60	15.8	72.0
5.09	414	7.92	16.1	68.0
5.31	478	9.14	19.8	67.0

The value given for the B.H.P. is, in each case, the maximum load the engine would carry for the given precompression. On attempting to carry the precompression higher than that given by $v_1/v_2 = 5.31$, it was found that the vibrations set up in the engine were so violent that satisfactory operation could not be obtained. From the table it is seen that the thermal efficiency increases rapidly with increased precompression. The mechanical efficiency, however, is considerably reduced. The fuel used during these tests was illuminating gas having a calorific value of about 590 B.T.U. per cubic foot. The amount of precompression which, in any case, gives the best results depends, of course, upon the quality of the fuel used.

It must, however, be emphasized that in any case, without considering the severe strains to which the engine is subjected, the amount of allowable precompression depends upon the temperature of ignition for the fuel used. For, if the temperature of the mixture due to the heat developed during the compression, becomes higher than that of ignition, premature explosions will occur, and the engine will not operate successfully.

168. T-φ Diagrams and Internal Combustion Engines. The $T-\varphi$ diagram, very frequently is a material aid in studying the effect produced by a change in the cycle upon which an internal combustion engine operates. As an example, if we plot the $T-\varphi$ diagram for a four-phase cycle, the effect produced by changing the amount of precompression is obvious from an inspection of the figure. Let A, in the $T-\varphi$ diagram, Fig. 42, represent the condition of the mixture, as regards temperature and entropy, at the end of the aspirating stroke. Then, since the compression is assumed adiabatic, the entropy remains constant while the temperature rises from T_4, that before compression, to T_1, that

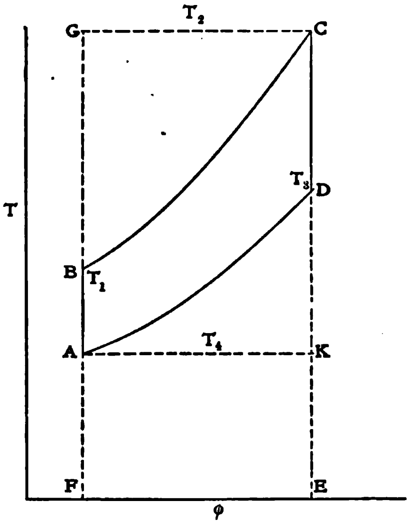

FIG. 42.

after compression; the line representing this being parallel to the T axis, and the condition of the mixture, as regards temperature and entropy, is given by the point B. During combustion there is a rise in temperature, and also, an increment in entropy. The increment in entropy is given by

$$\varphi_1 = M\int_{T_1}^{T_2} C_v \frac{dT}{T}; \quad \ldots\ldots \quad (6)$$

where M is the mass of the gas present, C_v the thermal capacity per unit mass at constant volume, and T_2 the temperature, when complete combustion has taken place. Though the thermal capacities of gases vary somewhat, for the ranges of temperature obtaining in an internal combustion engine, the variations are probably not very large. Hence, so far as the present discussion is concerned, no serious error is introduced by assuming C_v constant, and equation (6) becomes

$$\varphi_1 = MC_v \log \frac{T_2}{T_1}. \quad \ldots\ldots \quad (7)$$

The curve BC, therefore, representing the relation of temperature and entropy, during combustion, is logarithmic. During the expansion, which is assumed adiabatic, the entropy is constant, while the temperature falls from T_2 to T_3. Hence the curve, CD, representing this change, is parallel to the T axis. Finally, heat is rejected, the temperature falls from T_3 to T_4, and the relation of change in temperature to change in entropy is again logarithmic, as represented by the curve DA. By assuming the thermal capacity of the products of combustion constant, while the temperature falls from T_3 to T_4, we find for the change in entropy,

$$\varphi_2 = MC_v \log \frac{T_3}{T_4}. \quad \ldots\ldots \quad (8)$$

From *Art.* 161 we have

$$\frac{T_1}{T_4}=\frac{T_2}{T_3};$$

hence, since T_2 is greater than T_1, T_2-T_3 must be greater than T_1-T_4, and CD on the diagram, must be greater than BA.

Since, by *Art.* 109, the area under the curve, BC, is proportional to the heat absorbed, and the area under the curve, AD, is proportional to the heat rejected, it follows that the ideal coefficient of conversion is

$$\eta=\frac{\text{Area } FBCE-\text{Area } FADE}{\text{Area } FBCE}=\frac{\text{Area } ABCD}{\text{Area } FBCE}. \quad . \quad . \quad (9)$$

From equation (9), and by an inspection of Fig. 42, it is obvious that the ideal coefficient of conversion increases with increased precompression. Thus, if the precompression had been such that the temperature at the end of the compression were T_2, as represented by the point G, such that combustion takes place at the constant temperature T_2, the ideal coefficient of conversion would be

$$\eta=\frac{\text{Area } AGCD}{\text{Area } FGCE};$$

which is obviously greater than that specified by equation (9). This is the principle upon which the Diesel motor operates; i.e., an attempt is made to bring about the application of heat at constant temperature. Again, if after complete combustion has taken place, the expansion be continued until the temperature, as represented by the point K, has been reached, the ideal coefficient of conversion is still further increased, and is given by the relation

$$\eta=\frac{\text{Area } AGCK}{\text{Area } FGCE};$$

which brings us back to the Carnot cycle.

But, as has been previously explained, in *Art.* 115, this requires a stroke of greater length than is consistent with economy.

169. Actual p-v and T-φ Diagrams of Internal Combustion Engine. The quantity of heat which a gas absorbs, or liberates, during a given temperature change, depends upon whether the change takes place at constant pressure or at constant volume. The change, however, usually takes place in a manner such that neither the pressure nor the volume remains constant. When both pressure and volume vary, the change in entropy is readily found from equations (42), (43), and (44), of *Art.* 49. Equation (42) states that

$$dQ = C_p dT - \frac{T}{p}(C_p - C_v)dp.$$

Assuming the process reversible, then, dividing through by T, we have

$$\frac{dQ}{T} = d\varphi = C_p\frac{dT}{T} - (C_p - C_v)\frac{dp}{p};$$

and

$$\int_{\varphi_1}^{\varphi_2} d\varphi = C_p\int_{T_1}^{T_2}\frac{dT}{T} - (C_p - C_v)\int_{p_1}^{p_2}\frac{dp}{p};$$

from which

$$\varphi_2 - \varphi_1 = C_p \log\frac{T_2}{T_1} - (C_p - C_v)\log\frac{p_2}{p_1}; \quad . \quad . \quad . \quad (10)$$

where φ_1, T_1, and p_1 represent, respectively, the initial entropy, temperature, and pressure, and φ_2, T_2, and p_2 represent, respectively, the final entropy, temperature, and pressure.

Equation (43), of *Art.* 49, states that

$$dQ = C_v dT + \frac{p}{R}(C_p - C_v)dv;$$

from which, by substituting for p/R its value, T/v, and dividing through by T, we find

$$\frac{dQ}{T} = d\varphi = C_v\frac{dT}{T} + (C_p - C_v)\frac{dv}{v};$$

and

$$\int_{\varphi_1}^{\varphi_2} d\varphi = C_v\int_{T_1}^{T_2}\frac{dT}{T}+(C_p-C_v)\int_{v_1}^{v_2}\frac{dv}{v};$$

from which

$$\varphi_2-\varphi_1=C_v\log\frac{T_2}{T_1}+(C_p-C_v)\log\frac{v_2}{v_1}. \quad . \quad . \quad . \quad (11)$$

Again, equation (44), of *Art.* 49, states that

$$dQ=\frac{v}{R}C_v dp+\frac{p}{R}C_p dv;$$

from which, by substituting for v/R and p/R, respectively, T/p and T/v, and dividing through by T, we find

$$\frac{dQ}{T}=d\varphi=C_v\frac{dp}{p}+C_p\frac{dv}{v};$$

and

$$\int_{\varphi_1}^{\varphi_2} d\varphi=C_v\int_{p_1}^{p_2}\frac{dp}{p}+C_p\int_{v_1}^{v_2}\frac{dv}{v};$$

from which

$$\varphi_2-\varphi_1=C_v\log\frac{p_2}{p_1}+C_p\log\frac{v_2}{v_1}. \quad . \quad . \quad . \quad . \quad (12)$$

Equations (10), (11), and (12), then give the change in entropy, respectively, in terms of the change in temperature and pressure, the change in temperature and volume, and the change in pressure and volume. Equation (12), however, is usually the most convenient; since, by means of a scale, after laying off on the indicator diagram, the axis of zero pressure and zero volume, the pressures and volumes corresponding to various points of the diagram are readily found; and from these, by choosing a suitable point for zero entropy, the entropy corresponding to the various points is easily computed. Dividing, equation (12), by C_v, we have

$$\frac{1}{C_v}(\varphi_2-\varphi_1)=\log\frac{p_2}{p_1}+n\log\frac{v_2}{v_1}; \quad . \quad . \quad . \quad . \quad (13)$$

and since the scale employed, in plotting the $T-\varphi$ diagram is arbitrarily chosen, we may drop the factor $1/C_v$ and employ the equation

$$\varphi_2-\varphi_1=\log\frac{p_2}{p_1}+n\log\frac{v_2}{v_1}. \quad . \quad . \quad . \quad . \qquad (14)$$

Fig. 43 is a reproduction of an indicator diagram taken from an 8 H.P. Fairbanks gas engine, operating on illuminating gas. The ratio of the volume before compression to that after compression, was 5.31; and the scale of the spring used, in taking the card, was 200 lbs. per square inch.

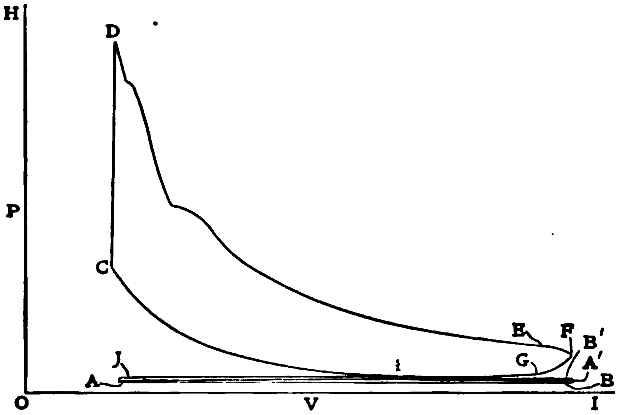

FIG. 43.

OI is the axis of zero pressure, obtained by measuring down, to proper scale, from the atmospheric line, *AA'*, a distance representing the instantaneous barometric reading. *OH* is the axis of zero volume, obtained by measuring, to proper scale, to the left of the point *A*, a distance representing the equivalent length of the clearance volume. *AB* represents the aspirating stroke, *BB'* measures the rise in pressure, while the admission-valve is closing, *B'C* represents the compression stroke, *CD* represents the combustion, at practically constant volume, *DE* represents the expansion of the products of combustion, *EFG* represents the change from the time the exhaust-valve begins to open until it is fully open, and *GJ* represents the remainder of the expulsion stroke. At the end of the expulsion stroke the exhaust-valve closes, the admission-valve opens, the pressure falls from *J* to *A*, and the cycle is completed.

Fig. 44 is a representation of the $T-\varphi$ diagram, plotted from the p–v diagram, as shown in Fig. 43. Assuming the temperature at the point *B*, Fig. 43, to be equal to that of the atmosphere, and employing the characteristic equation, the temperatures corresponding to the various points were computed. An arbitrary value of entropy for the substance, corresponding to the point where the curves, representing compression and expulsion, intersect, was assumed. Then by substituting in equation (14), the values of pressure and volume, as found by scale, from the p–v diagram, for the various points, the entropy, corresponding to these points, was readily computed. The value of n employed in making these computations, was a mean value, computed from the constitution of the mixture, before combustion, for the compression curve, and from the constitution of the exhaust gases, for the expansion curve.

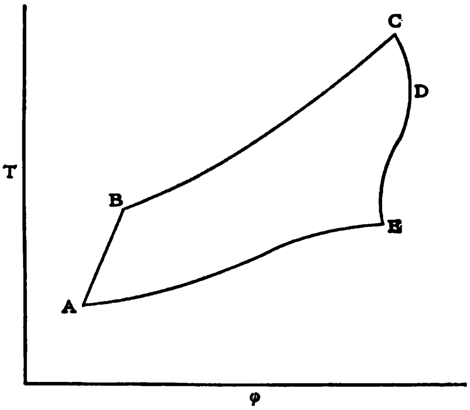

FIG. 44.

The point *A*, Fig. 44, represents the condition of the working substance, as regards temperature and entropy, at the point where the compression curve and expulsion curve intersect; and, the curve AB represents the relation between temperature and entropy from this point to the end of compression. BC is the curve, representing the relation of temperature and entropy, for combustion, at practically constant volume. CDE represents the relation of temperature and entropy for expansion, and EA the relation of temperature and entropy for the abstraction of heat at, practically, constant volume.

Though, as has been previously stated, the thermal capacities of gases are not known for high temperatures, and therefore the diagram does not rigidly represent exact conditions, it still appears that, during compression, in this particular case, since the entropy is increasing as the compression advances, the cylinder walls are giving up heat to the mixture. From C to D the entropy of the substance appears, from the diagram, to be increasing, which is probably due to after burning; i.e., complete combustion has not taken place when the power stroke begins. From D to E the entropy first decreases and then increases, which appears to indicate that for a time, while the temperature of the products of combustion is high, heat is being given up by the gases to the cylinder walls, and later, as the temperature falls, heat is abstracted by the gases from the cylinder walls. In this manner, even though the $T-\varphi$ diagram may not represent exact conditions, conclusions may still be drawn, in regard to exchange of heat between the cylinder walls and the working substance.

By assuming the compression adiabatic and computing the temperature for the point B, a value was found which was considerably less than that obtained, from Fig. 43, by means of the characteristic equation. This also shows that, during compression, heat has been abstracted, by the gases, from the cylinder walls.

170. Multi-cylinder Engines. Due to the fact that in a four-phase cycle there is only one power stroke for every four strokes, engines operating on this cycle are usually built with, at least, two cylinders; but more frequently, especially for automobiles, with four, six, and sometimes, even with eight cylinders. The explosions are so timed that the turning moment on the shaft throughout a rotation of the fly-wheel is, as nearly as possible, uniform.

171. Double-acting Cylinders. For power plants, internal combustion engines are, at the present time, frequently constructed so as to be double acting; i.e., explosions properly timed, are brought about in both ends of the cylinder. However, for a well distributed thrust there must be two such cylinders, which may operate either tandem or twin.

172. General Outline of Test. As stated in *Art.* 167, actual tests show that the thermal efficiency of an internal combustion engine is a function of the amount of precompression. Hence, in making a complete test of an engine, it is necessary to determine what precompression yields the best results. To do this, various methods may be employed; one, conveniently carried out, is that of fastening, by means of machine screws, disks of various thicknesses, and diameters equal to that of the cylinder bore, to the end of the piston, thus changing the clearance. In general, it will be found that, for every change in the amount of precompression, an adjustment of the governor is necessitated, so that the load is properly carried. At the same time, the calorific value of the fuel is determined, as well as the chemical constitution of the exhaust gases. If, by a chemical analysis, it is found that the exhaust gases show incomplete combustion, the amount of air admitted, during the aspirating stroke, must be changed, by a change in the admission valves, until an analysis of the exhaust gases shows complete combustion. Proceeding in this manner, step by step, it will be found that for every engine, there is a definite precompression which yields a maximal thermal efficiency; and, when this has been determined, the various losses are readily found.

To determine the various losses, the load on the engine is maintained constant, for a given run, and noted. This gives the output of the engine. From the quantity of fuel consumed, samples of which are tested for calorific values, from time to time, during the run, the total energy consumed is found. The difference between energy consumed, during the run, and the work delivered by the engine, during the same interval of time, constitutes the combined losses. The losses are chiefly: Mechanical losses, heat carried away by exhaust gases, heat carried away by the cooling water, heat lost by incomplete combustion, and heat lost, from the surface of the engine, by radiation and convection.

The mechanical losses are determined by taking the difference between the indicated work and the work delivered. It is, of course, necessary to take a number of indicator diagrams, during the run, so as to obtain an average value for the indicated work; and furthermore, the number of power strokes should be determined from the number of explosions, rather than from the number of rotations, made by the fly-wheel.

To determine the heat, carried away by the exhaust gases, the temperature of the gases, at the exhaust port, is determined by means of a pyrometer. And, from the constitution of the exhaust gases, as found by analysis, the mean thermal capacity per unit mass is found. Again, from the constitution of the fuel, the mass of fuel consumed, and the constitution of the exhaust gases, the total mass of the exhaust gases is readily computed.

Finally, by taking the product of the difference in temperature between the exhaust gases and the room, the mass of the exhaust gases, and the mean thermal capacity per unit mass, the heat carried away by the exhaust gases is found. It must, however, be remembered that the thermal capacities for gases, as given in tables, are, in general, the results obtained by experiments conducted between the limits of 0°C. and 200°C.; and, it is not at all certain that these values are correct for high temperatures.

The heat carried away by the jacket water is determined directly from the difference in temperature, between inflow and outflow, and the mass of water flowing through the jacket during the run. The temperatures are found by means of ordinary thermometers; and, the mass of water, by collecting in a suitable vessel and weighing.

The heat lost, due to incomplete combustion, which should be very small, is computed from the constitution of the products of combustion.

There finally remains, then, the heat lost by radiation and convection. This cannot be found directly, but is assumed to be equal to the difference between the input and the sum of the other losses plus energy delivered.

Having found the various losses, a comparison may be made between the engine under test and other engines; and conclusions drawn therefrom in regard to making changes in the design or the method of operation.

CHAPTER XVI

COMPRESSED AIR AND COMPRESSORS

173. COMPRESSED air is used extensively and for a variety of purposes. It is used in tunneling; in mining, where, after it has done work upon an air motor, it may be employed for ventilating purposes; in general, for power transmission; for air brakes on trains; etc. Hence, it is important that the compression be brought about in the most economical manner possible. To do this, in attempting the design of an *efficient air compressor*, it is necessary to consider the underlying principles of thermodynamics, as well as those of machine design.

However, before developing the formula representing the least amount of work that must be done in compressing a given mass of air, from one pressure to another, we will, as a matter of convenience, first determine the constants for air, and discuss briefly isothermal and adiabatic compression.

174. Air Constants. The density of air, under a pressure of one standard atmosphere (1.01325×10^6 dynes per square centimeter) and at a temperature of 32°F., is 0.001293 *grams per cubic centimeter.* Converting this to *pounds* and *cubic feet,* we find for the density, 0.08072 *pounds per cubic foot;* and, from this, a volume of 12.39 *cu.ft. per pound.* The pressure of one standard atmosphere, expressed in gravitational units, is, approximately, 2115 *lbs. per square foot;* and, this corresponds, very nearly, with a pressure of 14.7 *lbs. per square inch.* Taking the product of pressure and volume, we find

$$pv = 2115\times12.39 = 26{,}200 \textit{ lbs. per sq.ft.}\times \textit{cu.ft.}$$

Since 32°F. corresponds, approximately, to 491.6 on the thermodynamic scale, we find, from the characteristic equation, for 1 pound of air,

$$R=\frac{26,200}{491.6}=53.29.$$

The foregoing constants have all been given to the nearest figure in the fourth place; but, in general, results of sufficient accuracy will be found by rounding off to the nearest figure in the third place. For, in designing compressors, some assumption has to be made regarding the average annual temperature of the atmosphere; and, frequently, it is assumed that this average temperature is 62°F. This, however, is not the proper value for all cases and localities. Assuming 62°F. to be the proper temperature to be employed, then we shall have, for the product of pressure and volume,

$$p_a v_a = RT_a = 53.3 \times 522 = 27,800;$$

where T_a is the temperature on the thermodynamic scale, corresponding to 62°F., p_a is the atmospheric pressure, and v_a the corresponding volume, for 1 pound of air at 62°F.

175. Compression and Expansion. Assume the purpose, for which the compressed air is employed, to be that of driving an air motor; the construction of an air motor, and the cycle upon which it operates, being very nearly the same as that of a steam engine. Let it be further assumed that the compression, in the air compressor, takes place in a manner such that the heat developed, by the compression, is immediately absorbed by the surroundings; i.e., the compression is isothermal. The intrinsic energy of the air, then, at the end of the process, is precisely the same as at the beginning. Again, if it be assumed that the expansion in the air motor takes place in a manner such that the heat required during the expansion, in doing external work, is immediately supplied from the surroundings, as required, then the intrinsic energy of the air, at the end of the process, is precisely the same as at the beginning. And under these assumed conditions, the work done on the air, while being compressed, is precisely equal to the work done by it, while expanding, between the same limits of pressure. It is thus seen that a vessel, containing compressed air, at room temperature, does not constitute a reservoir of energy; but, the air is merely in a condition such that it can absorb energy, in the form of heat, from the surroundings, and convert it into mechanical work. On the other hand, if the compression is adiabatic and the vessel, in which the compressed air is stored, is perfectly insulated, such that the heat developed during compression is retained, then the vessel does constitute a reservoir of energy; for, by an adiabatic expansion, the work done on the air, during compression, may again be recovered; and the air at the end of the expansion is in precisely the same condition as it was at the beginning, without any exchange having taken place between the air and the surroundings. This, of course, assumes no other losses. The receiver, then, in the latter case, containing the compressed air, constitutes a reservoir of energy, not because it contains compressed air, but merely because heat has been stored which may be reconverted into mechanical work.

In general, however, when air is compressed, the compression is very nearly adiabatic; and the air is stored in a receiver, where, in a very short interval of time, the heat developed during compression is, by means of conduction and radiation, given up to the surroundings, and is irrevocably lost. Hence, when expansion takes place in the motor, which is again, approximately, adiabatic, the external work is done at the expense of the intrinsic energy of the air; and, accordingly, the temperature falls. Hence, since the intrinsic energy of the air is practically a function of the temperature only, it follows that the intrinsic energy at the end of the expansion, in the motor, is less than it was initially in the compressor, at the instant when compression began.

176. Isothermal Compression and Expansion. The results just deduced, in *Art.* 175, are best illustrated by means of the ideal *p–v* diagram; first, by considering isothermal processes, and then by considering adiabatic processes. Let, in Fig. 45, *OI* and *OH* be, respectively, the axes of zero pressure and zero volume; and, for the present discussion, it will be assumed that the compressor has no clearance. The work done, then, *on* the piston *by* the air, during the aspirating stroke, as represented by the line *AB*, is measured by the area *OABE*. During the isothermal compression, represented by the curve *BC*, the work done *by* the piston, *on* the air, is measured by the area *EBCF*. At the point *C*, the air is under a pressure equal to that obtaining in the receiver, the exhaust-valve opens, and the air under a constant pressure, as represented by the line *CD*, is forced into the receiver, and the cycle is completed. The work done *by* the piston *on* the air during expulsion, is measured by the area *FCDO*. Finally, the net work done by the compressor, during the cycle, is obviously measured by the difference between the areas *EBCDO* and *EBAO*; i.e., the area *ABCD*.

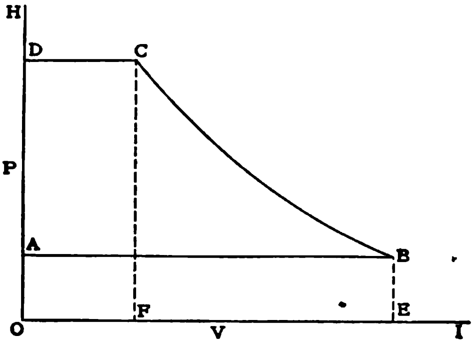

FIG. 45.

Representing the pressure and volume of the air, corresponding to the point *B*, respectively, by p_a and v_a, and likewise, for the point *C*, by p_1 and v_1, then, by considering the work done *on* the piston negative, and the work done *by* the piston positive, we have for the work done, during the aspirating stroke,

$$W_1 = -p_a v_a. \qquad (1)$$

The work done, during the compression, BC, is

$$W_2 = -\int_{p_a}^{p_1} p dv; \qquad (2)$$

and the work done, during expulsion, is

$$W_3 = p_1 v_1. \qquad (3)$$

Finally, the net work done, during the cycle, is the algebraic of W_1, W_2, and W_3; i.e.,

$$W = -p_a v_a - \int_{p_a}^{p_1} p dv + p_1 v_1; \qquad (4)$$

where W is the net work done. But, since the compression is isothermal,

$$p_a v_a = p_1 v_1;$$

and equation (4) becomes

$$W = -\int_{p_a}^{p_1} p dv. \qquad (5)$$

Substituting now, in equation (5), for dv its value, as found from the equation

$$pv = p_a v_a,$$

we find

$$W = p_a v_a \int_{p_a}^{p_1} \frac{dp}{p} = p_a v_a \log \frac{p_1}{p_a}. \qquad (6)$$

If we are dealing with 1 pound of air, and assume the temperature, during the aspirating stroke, to be 62°F., then we may substitute, in equation (6), the value of $p_a v_a$ as found in *Art.* 174, and we find

$$W = 27{,}800 \log \frac{p_1}{p_a} \text{ ft.-lbs. per pound.} \quad . \quad . \quad . \quad (7)$$

That is, the right-hand member of equation (7) is the expression for the amount of work in ft.-lbs., that must be done in taking in, under the assumed conditions, 1 pound of air at a pressure p_a, compressing it to a pressure p_1, and forcing it against this pressure into a receiver. It is obvious that the ratio p_1/p_a is independent of the unit of measure chosen; but, it is necessary, in order to obtain the work in *ft.-lbs.*, that the numerical coefficient, in equation (7), be deduced by expressing the pressure in *lbs. per square foot* and the volume in *cubic feet.*

Assume, now, that the process takes place in the reverse order, step by step, in a manner such that the diagram is traced in the order, *DCBA*. By assuming a perfect regenerator, such as was discussed in *Art.* 27, the process becomes ideally reversible, and the net work done, on the motor, is precisely equal in amount to that done by the compressor, as given by equation (6). And since this is the best that can be done in any case, the indicator diagram, representing isothermal compression, is the one chosen as a standard, for the comparison of the performance of air compressors.

177. Adiabatic Compression. For the same initial and final pressures, the work done, during compression, and also during expulsion, will not be the same for an adiabatic process as it is for an isothermal process. On the other hand, the work done during the aspirating stroke is the same in either case.

Let in Fig. 46, the line *AB* represent the aspirating stroke, the curve *BE* the adiabatic compression, and the line *ED* the expulsion stroke at the pressure of the receiver. The net work done by the piston is measured by the area *ABED*. Had the compression been along the isotherm *BC*, the net work done would be measured by the area *ABCD*; hence, the excess of work done, during a cycle, when the compression is adiabatic, over that done when the compression is isothermal, is measured by the area *BCE*.

The work done during the aspirating stroke is again

$$W_1 = -p_a v_a; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

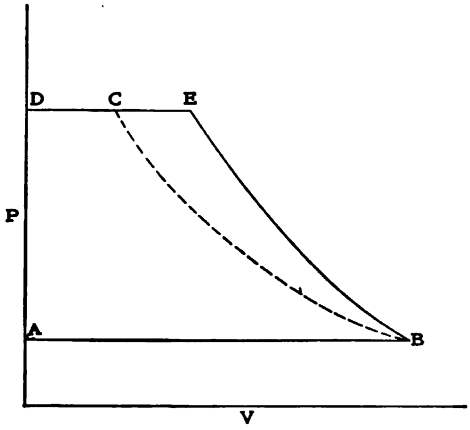

FIG. 46.

where the symbols have the same significance as in *Art.* 176. The work done, during adiabatic compression, is

$$W_2 = -\int_{p_a}^{p_1} p dv; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

but, since we are now dealing with an adiabatic change, the value of *dv* to be substituted in equation (9) must be deduced from the equation

$$pv^n = p_a v_a^n.$$

Solving for *v* we find

$$v = p_a^{\frac{1}{n}} v_a p^{-\frac{1}{n}},$$

from which,

$$dv = -\frac{1}{n} p_a^{\frac{1}{n}} v_a p^{-\frac{1+n}{n}} dp.$$

Substituting in equation (9), we find

$$W_2=\frac{p_a^{\frac{1}{n}}v_a}{n}\int_{p_a}^{p_1}p^{-\frac{1}{n}}dp$$

$$=\frac{p_a^{\frac{1}{n}}v_a}{n-1}(p_1^{\frac{n-1}{n}}-p_a^{\frac{n-1}{n}})$$

$$=\frac{p_av_a}{n-1}\left\{\left(\frac{p_1}{p_a}\right)^{\frac{n-1}{n}}-1\right\}. \quad . \quad . \quad . \quad . \quad . \qquad (10)$$

Since now the volume, at the end of the adiabatic compression, is

$$v_1=p_a^{\frac{1}{n}}v_ap_1^{-\frac{1}{n}},$$

the work done, during expulsion, is

$$W_3=p_1v_1=p_a^{\frac{1}{n}}v_ap_1^{\frac{n-1}{n}}=p_av_a\left(\frac{p_1}{p_a}\right)^{\frac{n-1}{n}}. \quad . \quad . \qquad (11)$$

Taking the algebraic sum of the right-hand members of equations (8), (10), and (11), we find, for the net work done by the compressor,

$$W=-p_av_a+\frac{p_av_a}{n-1}\left\{\left(\frac{p_1}{p_a}\right)^{\frac{n-1}{n}}-1\right\}+p_av_a\left(\frac{p_1}{p_a}\right)^{\frac{n-1}{n}};$$

which reduces to

$$W=\frac{n}{n-1}p_av_a\left\{\left(\frac{p_1}{p_a}\right)^{\frac{n-1}{n}}-1\right\}. \quad . \quad . \quad . \qquad (12)$$

It was stated, in *Art.* 176, that the cycle having isothermal compression may be taken as a standard cycle. The ratio of the work done, during a cycle, when the compression is isothermal, to that when the compression is adiabatic, may be termed the *theoretical efficiency of compression.* Dividing equation

(6) by equation (12), we find, for the theoretical efficiency of compression,

$$\eta=\frac{\log\frac{p_1}{p_a}}{\frac{n}{n-1}\left\{\left(\frac{p_1}{p_a}\right)^{\frac{n-1}{n}}-1\right\}}. \quad . \quad . \quad . \quad . \quad . \quad (13)$$

As a matter of convenience a table is here given, which was obtained by computing the theoretical efficiencies, by means of equation (13), on the assumption that n has the value of 1.4.

$\frac{p_1}{p_a}$	η Per Cent.	$\frac{p_1}{p_a}$	η Per Cent.
1.5	94.3	6	76.6
2	90.4	7	74.8
3	85.1	8	73.2
4	81.5	9	71.9
5	78.8	10	70.7

It will be noted that, when the ratio $p_1/p_a=3$, the theoretical efficiency of compression is approximately 85 per cent, and when $p_1/p_a=4$, it is approximately 81.5 per cent. Hence, when the ratio of p_1 to p_a is greater than 3 or 4, the losses, from a thermodynamic standpoint, become excessive. Therefore, if it be desired to operate economically, it becomes necessary to limit the ratio of final to initial pressure to a value between 3 and 4.

Equation (12) may be transformed so as to express the work done by the compressor, in terms of the initial and final temperatures. Since

$$\frac{T_1}{T_a}=\left(\frac{p_1}{p_a}\right)^{\frac{n-1}{n}}$$

where T_a is the initial and T_1 the final temperature, and since

$$p_av_a=RT_a,$$

we find, by substituting in equation (12),

$$W=\frac{n}{n-1}RT_a\left(\frac{T_1}{T_a}-1\right). \quad . \quad . \quad . \quad . \quad . \qquad (14)$$

And finally, by substituting, in equation (14), for n and R, respectively, the equivalent values, C_p/C_v, and $J(C_p-C_v)$, we find

$$W=JC_p(T_1-T_a). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (15)$$

That is, equation (15) shows that the net work consumed by an air compressor, per cycle, per pound of air, when the compression is adiabatic, is precisely equal in amount to the heat, expressed in mechanical units, consumed in elevating the temperature of the air, at constant pressure, from that before compression to that after the compression is completed.

Equation (15) may be deduced in an entirely different, though very simple, manner. The work done by the piston, during admission, is

$$W_1=-p_av_a=-RT_a=-J(C_p-C_v)T_a. \quad . \quad . \quad . \qquad (a)$$

And, during adiabatic compression, since the intrinsic energy of the air is a function of its temperature only, the work done by the piston is

$$W_2=C_v(T_1-T_a)J. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (b)$$ See Errata.

The work done by the piston, during exhaust, is

$$W_3=p_1v_1=RT_1=J(C_p-C_v)T_1; \quad . \quad . \quad . \quad . \quad . \qquad (c)$$

taking the sum of W_1, W_2, and W_3, as given by equations (*a*), (*b*), and (*c*), we find

$$W=JC_p(T_1-T_a).$$

178. Multi-stage Compression. Various methods, in which an attempt is made, to bring about compression, approaching an isothermal process, have been tried; but, it appears impossible to reduce the exponent, n, to a value approaching unity. The various methods used are: Water-jacketing, playing a jet of water into the cylinder while compression is taking place, and spraying, by means of an atomized jet of cold water, the air while it is being compressed. Water-jacketing appears, so far as the results of investigations show, to give very little, if any gain in economy. That is, when recourse is had to jacket cooling, the heat developed by compression is absorbed so slowly that the compression is practically adiabatic. When the cooling is attempted by means of a jet of water, played into the cylinder, the exponent, n, may be reduced to a value of about 1.35; and when the cooling is brought about by an atomized spray, the value of n may be reduced to about 1.25. Hence, at best, the compression is far from approaching an isothermal process; and, for efficient operation, when the ratio of the final pressure to the initial pressure is greater than 4, recourse must be had to *multi-stage compression.*

We will consider, first, a two-stage compressor. That is, during the aspirating stroke, a certain quantity of air, at a pressure p_a, flows into the cylinder, which on the return stroke is compressed to some pressure p_1; the relation of pressure and volume being given by the equation

$$pv^n = k;$$

where the exponent n, depending upon the method of cooling applied, may have a value ranging from about 1.25 to 1.4. At the end of this compression stroke, when the pressure p_1 has been attained, the air is expelled into a receiver under a constant pressure p_1. The receiver has a jacket through which water is circulated in a manner such that the heat developed, during compression, is removed; and the product of pressure and volume, after cooling in the receiver, is equal to the product of pressure and volume at the instant the compression began.

In other words, the condition of the air, as regards pressure and volume, in the receiver, is the same as though the compression had been isothermal; i.e.,

$$p_1v_1 = p_av_a.$$

Let, in Fig. 47, the line AB represent the aspirating stroke, for the low-pressure cylinder, and the pressure and volume, corresponding to the point B, be given, respectively, by p_a and v_a. The compression then takes place approximately adiabatically, as represented by the curve BE. At the point E the exhaust-valve opens and expulsion takes place, to the receiver, at a constant pressure p_1. The volume of the receiver being large in comparison with that of the first cylinder, the pressure in it is sensibly constant; and the volume of air, in cooling from T_1, the temperature at the end of the compression BE, to T_a, the atmospheric temperature, shrinks from the volume, represented by FE, to that represented by FG, such that

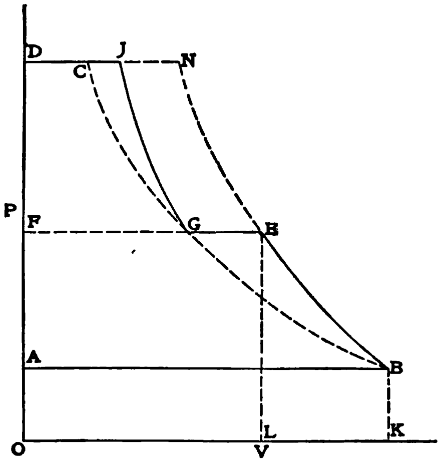

FIG. 47.

$$p_1v_1 = p_av_a;$$

and the point G is on the isotherm through the point B. For the same quantity of air, then, the line FG represents the aspirating stroke for the second, or high-pressure, cylinder. The curve *GJ*, represents the compression, which is practically adiabatic, to the pressure p_2, existing in the reservoir in which the air is stored. Finally, the expulsion stroke to the reservoir is represented by the line *JD*.

The various quantities of work involved during the cycle are as follows: During the aspirating stroke, for the low-pressure cylinder, the work done *on* the piston is measured by the area *ABKO*; and during compression, to the pressure p_1, the work done *by* the piston is measured by the area *KBEL*; and during expulsion, to the receiver, the work done *by* the piston is measured by the area *LEFO*. Hence, the net work done by the piston, in the low-pressure cylinder is measured by the area *ABEF*. In a similar manner, we find that the net work done, by the piston in the high-pressure cylinder, is measured by the area *FGJD*. The total net work done, therefore, by the two-stage compressor, during the cycle, is measured by the area *ABEGJD*. Had the compression taken place in a single-stage compressor, between the same limits of pressure, the net work done, by the piston, would be measured by the area *ABEND;* where the curve *BEN* represents an adiabatic through the point *B*. Hence, the saving in work, neglecting losses, by employing a two-stage compressor, is measured by the area *ENJG*. And the work done by the piston of this two-stage compressor, in excess of that which would have been done had the compression been isothermal, is measured by the sum of the areas *GBE* and *CGJ*, where the curve *BGC* represents an isotherm.

By equation (12), we have for the work done in the low-pressure cylinder, per cycle, when 1 pound of air is taken in at a pressure p_a, is compressed adiabatically to a pressure p_1, and expelled at this pressure to a receiver,

$$W_1=\frac{n}{n-1}p_a v_a\left\{\left(\frac{p_1}{p_a}\right)^{\frac{n-1}{n}}-1\right\}. \quad . \quad . \quad . \quad . \quad (16)$$

In a similar manner, the work done in the high-pressure cylinder, when 1 pound of air is taken in at a pressure p_1, is compressed adiabatically to a pressure p_2, and expelled at this pressure to a receiver, is

$$W_2=\frac{n}{n-1}p_1v_1\left\{\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}-1\right\}. \quad . \quad . \quad . \quad . \quad (17)$$

Since, however, the air in the intermediate receiver has its temperature reduced to the initial value, we must have

$$p_1v_1=p_av_a;$$

and equation (17) becomes

$$W_2=\frac{n}{n-1}p_av_a\left\{\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}-1\right\}. \quad . \quad . \quad . \quad . \quad (18)$$

Taking the sum of equations (16) and (18), we find that the total work done, during a cycle, by the two-stage compressor, is

$$W=W_1+W_2=\frac{n}{n-1}p_av_a\left\{\left(\frac{p_1}{p_a}\right)^{\frac{n-1}{n}}+\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}-2\right\}. \quad (19)$$

Now, p_a is a constant and so is p_2; since, for any particular case, p_2 is the desired final pressure. p_1, however, is a variable; and the value of W obviously depends upon the value chosen for p_1. Since, the only variable in the right-hand member of equation (19) is p_1, the value found for W is a minimum when the expression,

$$\left(\frac{p_1}{p_a}\right)^{\frac{n-1}{n}}+\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}-2,$$

is a minimum. Differentiating this expression, with respect to p_1, equating to zero, and solving for p_1, we find

$$p_1=p_a^{\frac{1}{2}}p_2^{\frac{1}{2}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

Substituting the value of p_1 as given by equation (20), in equations (16) and (18), we find

$$W_1=\frac{n}{n-1}p_av_a\left\{\left(\frac{p_2}{p_a}\right)^{\frac{n-1}{2n}}-1\right\}; \quad . \quad . \quad . \quad . \quad (21)$$

and,

$$W_2=\frac{n}{n-1}p_av_a\left\{\left(\frac{p_2}{p_a}\right)^{\frac{n-1}{2n}}-1\right\}. \quad . \quad . \quad . \quad . \quad (22)$$

From equations (21) and (22) it is seen that, if the work done, during a cycle, by a two-stage compressor, is to be a minimum, it must be equally divided between the two cylinders. Taking the sum of the right-hand members of equations (21) and (22), we find the net work done, when employing the most efficient compression possible, by a two-stage compressor, in taking air under a pressure p_a and expelling to a receiver, under a pressure p_2, is

$$W=\frac{2n}{n-1}p_av_a\left\{\left(\frac{p_2}{p_a}\right)^{\frac{n-1}{2n}}-1\right\}\text{ ft.-lbs. per pound.} \quad . \quad (23)$$

If the compression is brought about by three stages, the final pressure being p_3, and the pressures of the intermediate receivers, respectively, p_1 and p_2, then, on the assumption that, in the two intermediate receivers, the temperature is reduced to that of the atmosphere, the work done in the first, second, and third cylinders is given, respectively, by

$$W_1=\frac{n}{n-1}p_av_a\left\{\left(\frac{p_1}{p_a}\right)^{\frac{n-1}{n}}-1\right\}, \quad . \quad . \quad . \quad . \quad (24)$$

$$W_2=\frac{n}{n-1}p_av_a\left\{\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}-1\right\}, \quad . \quad . \quad . \quad . \quad (25)$$

and

$$W_3=\frac{n}{n-1}p_av_a\left\{\left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}}-1\right\}. \quad . \quad . \quad . \quad . \quad (26)$$

Taking the sum of W_1, W_2, and W_3 we find, for a cycle, the net work done by the three-stage compressor, is

$$W=\frac{n}{n-1}p_av_a\left\{\left(\frac{p_1}{p_a}\right)^{\frac{n-1}{n}}+\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}+\left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}}-3\right\}. \qquad (27)$$

The right-hand member of equation (27) is a minimum when the expression included in the brace is a minimum. Differentiating this expression, first, assuming p_1 variable, and p_a, p_2, and p_3 constant, equating to zero, and solving for p_1, we find

$$p_1=\sqrt{p_ap_2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (28)$$

Equation (28) gives the relation of p_1 to p_a and p_2 such that the process in going from p_a to p_2 shall involve a minimum amount of work. Differentiating again, this time, however, assuming p_1 and p_3 constant, and p_2 variable, in order to obtain the relation p_2 must bear to p_1 and p_3 such that a minimum amount of work is involved, while the process takes place from p_1 to p_3, we find

$$p_2=\sqrt{p_1p_3}. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (29)$$

By elimination we find, from equations (28) and (29),

$$p_1=\sqrt[3]{p_a^2p_3}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (30)$$

and

$$p_2=\sqrt[3]{p_ap_3^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (31)$$

Substituting, in equations (24), (25), and (26), the values of p_1 and p_2, as given by equations (30) and (31), we find

$$W_1=\frac{n}{n-1}p_av_a\left\{\left(\frac{p_3}{p_a}\right)^{\frac{n-1}{3n}}-1\right\}, \quad . \quad . \quad . \quad . \qquad (32)$$

$$W_2=\frac{n}{n-1}p_av_a\left\{\left(\frac{p_3}{p_a}\right)^{\frac{n-1}{3n}}-1\right\}, \quad . \quad . \quad . \quad . \qquad (33)$$

and

$$W_3=\frac{n}{n-1}p_av_a\left\{\left(\frac{p_3}{p_a}\right)^{\frac{n-1}{3n}}-1\right\}. \quad . \quad . \quad . \quad . \quad . \qquad (34)$$

Equations (32), (33), and (34), again show that, for the most economical compression, the work must be equally divided between the cylinders; and the net work, for the three-stage compressor, is

$$W=\frac{3n}{n-1}p_av_a\left\{\left(\frac{p_3}{p_a}\right)^{\frac{n-1}{3n}}-1\right\}\text{ft.-lbs. per pound.} \quad . \quad (35)$$

In a similar manner, for the most economical four-stage compressor, we find

$$W=\frac{4n}{n-1}p_av_a\left\{\left(\frac{p_4}{p_a}\right)^{\frac{n-1}{4n}}-1\right\}\text{ft.-lbs. per pound.} \quad . \quad (36)$$

By a comparison of equations (12), (23), (35), and (36), it is seen that in each case the coefficient $n/(n-1)$, is multiplied by the number of stages, and the exponent $(n-1)/n$, is divided by the number of stages; and in all other respects, the equations are identical. We may then write a general equation for the net work done by a multi-stage compressor, in taking in 1 pound of air under a pressure p_a, compressing it, by means of S stages, to a pressure p, and expelling it, at this pressure to a reservoir. This net work done is given by

$$W=\frac{Sn}{n-1}p_av_a\left\{\left(\frac{p}{p_a}\right)^{\frac{n-1}{Sn}}-1\right\}\text{ft.-lbs. per pound.} \quad . \quad (37)$$

It is obvious that for any two given initial and final pressures, the greater the number of stages, the smaller the areas, representing the difference in work, between adiabatic and isothermal compression, become; and hence, the nearer the compression approaches an isothermal process. And, in the limit, as S in equation (37) becomes indefinitely large, the compression becomes isothermal. However, increasing the number of cylinders, increases the bulk and first cost of the compressor, as well as the loss of work due to friction and imperfect valve action. On the other hand there are also certain mechanical advantages in a multi-stage compressor, similar to those discussed, in *Arts.* 141 and 144, for the compound engine. That is, for a two-stage compressor the stresses in the moving parts are less than in a single-stage compressor operating between the same limits of pressure; and, if the compressor be cross-compound, the thrust on the crank bearings is more uniform. But, in any given case, there is a limiting value for the number of stages; practically when the thermodynamic gain is offset by the interest on the extra capital invested, plus depreciation and mechanical losses. For final pressures of about six atmospheres, two-stage compressors are usually employed.

179. Clearance. The clearance of an air compressor is very small in comparison with the clearance of a steam cylinder. Still, account must be taken of the clearance in designing a compressor; since, due to it, the effective volumetric displacement of the piston is less than the actual volumetric displacement. Let, in Fig. 48, *AB* represent the displacement of the piston, *BC* the compression, *CD* the expulsion, and *EA* the clearance. Then, since the volume of air, *EA*, remaining in the cylinder, when the exhaust-valve closes, is under a pressure p_1, expansion must take place to atmospheric pressure, as represented by the curve *DF*, before the inlet-valve opens and a new supply of air flows into the cylinder. Hence, the effective piston displacement is measured by *FB*.

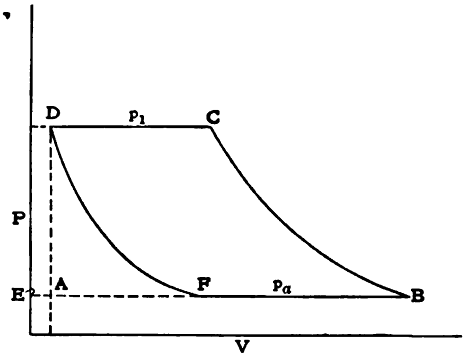

FIG. 48.

Let K * be the ratio of the actual piston displacement, AB, to the clearance, EA, then

$$EA=\frac{AB}{K}. \qquad \ldots \ldots \quad (38)$$

For the expansion along the curve DF, we may write

$$p_1v_1{}^n=p_av_a{}^n; \qquad \ldots \ldots \quad (39)$$

where v_1 and v_a are the volumes, respectively, as represented by EA and EF. From equation (39), we find

$$v_a=v_1\left(\frac{p_1}{p_a}\right)^{\frac{1}{n}},$$

and, by substituting for v_1 and v_a, respectively, the value of EA, as given by equation (38), and EF, we obtain

$$EF=\frac{AB}{K}\left(\frac{p_1}{p_a}\right)^{\frac{1}{n}}. \qquad \ldots \ldots \quad (40)$$

Now, the effective displacement is given by

$$FB=AB-(EF-EA); \qquad \ldots \ldots \quad (41)$$

hence by substituting, in equation (41), the value of EF, as given by equation (40), and the value of EA, as given by equation (38), we find

$$FB=AB\left\{1-\frac{1}{K}\left(\frac{p_1}{p_a}\right)^{\frac{1}{n}}+\frac{1}{K}\right\}. \qquad \ldots \ldots \quad (42)$$

The effective piston displacement not being equal to the actual piston displacement, does not affect the expression deduced for the work done on an air compressor; for, the air remaining in the cylinder, at the end of the expulsion stroke, does an amount of work

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* K usually has a value of about 50.

on the piston, in expanding, which is practically equal to that which was done on it while being compressed. The effect of the clearance, then, is merely to reduce the capacity of the cylinder.

180. Throttling and Other Imperfections. The capacity of a cylinder of an air compressor is very frequently more seriously affected by other causes than it is by clearance. In the first place there is always, due to valve friction, a certain amount of throttling, which causes the pressure in the cylinder, during the aspirating stroke, to be less than atmospheric. Further, due to imperfect valve action, i.e., the valves not opening or closing at the proper time, the capacity is reduced. And, finally, the temperature of the cylinder walls is usually higher than that of the incoming air, which again tends to reduce the capacity of the cylinder. These combined causes may reduce the apparent capacity, depending upon the speed of the machine, from 5 to 20 per cent.

181. Adiabatic Expansion in Motor. The cycle of an *air motor* is practically the reverse of that of an air compressor. The admission-valve opens and air from the mains, under a practically constant pressure, forces the piston forward to the point of cut-off, and the work done *on* the piston, per pound of air is

$$W_1 = p_1 v_1; \quad \dots\dots\dots \quad (43)$$

where p_1 is the pressure in the main, and v_1 the volume of one pound of air at cut-off. The expansion is then practically adiabatic, and the work done in expanding from the pressure p_1, to p_a, that of the atmosphere, is

$$W_2 = \int_{p_1}^{p_a} p\,dv = -\frac{1}{n} p_1^{\frac{1}{n}} v_1 \int_{p_1}^{p_a} p^{-\frac{1}{n}}\,dp = \frac{p_1 v_1}{n-1}\left\{1 - \left(\frac{p_a}{p_1}\right)^{\frac{n-1}{n}}\right\}. \quad (44)$$

The exhaust-valve then opens, the air is expelled under a pressure p_a, and the work done, by the air, is

$$W_3 = -p_a v_a = -p_1^{\frac{1}{n}} v_1 p_a^{\frac{n-1}{n}}. \quad \dots\dots \quad (45)$$

Taking the sum of the right-hand members of equations (43), (44), and (45), we find, for the net work done on the motor, per pound of air,

$$W=\frac{p_1v_1}{n-1}\left\{1-\left(\frac{p_a}{p_1}\right)^{\frac{n-1}{n}}\right\}+p_1v_1-p_1^{\frac{1}{n}}v_1p_a^{\frac{n-1}{n}}$$

$$=\frac{n}{n-1}p_1v_1\left\{1-\left(\frac{p_a}{p_1}\right)^{\frac{n-1}{n}}\right\}\text{ft.-lbs.}\quad . \ . \ . \ . \quad (46)$$

Since the temperature in the mains is practically atmospheric, p_1v_1 is the product of pressure and volume, for one pound of air under ordinary conditions, and may be replaced by the constant 27,800. Hence, equation (46) becomes

$$W=27{,}800\frac{n}{n-1}\left\{1-\left(\frac{p_a}{p_1}\right)^{\frac{n-1}{n}}\right\}.\quad . \ . \ . \ . \quad (47)$$

The indicator diagram for the preceding discussion is shown in Fig. 49, in which *AB* represents the admission, *BC* the expansion, and *CD* the expulsion. The work done *on* the piston during admission and expansion, is measured by the area *ABCGF*; and the work done *by* the piston, during exhaust, is measured by the area *CGFD*. Hence, the net work done, by the air, is measured by the area *ABCD*.

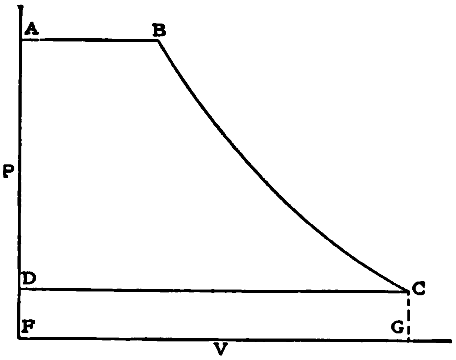

Fig. 49.

Equation (46) may be put into another form, by substituting for p_1v_1, its value RT_1, and for $(p_a/p_1)^{\frac{n-1}{n}}$, its value T_a/T_1. Making these substitutions, we find

$$W=\frac{n}{n-1}RT_1\left(1-\frac{T_a}{T_1}\right);$$

from which, for adiabatic processes, since in that case $n=C_p/C_v$, we obtain

$$W=\frac{C_p}{C_p-C_v}R(T_1-T_a).$$

And, since

$$R=J(C_p-C_v),$$

we have finally

$$W=JC_p(T_1-T_a). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (48)$$

It must be noted that, in equation (48), T_1 is the temperature in the mains, which is practically that of the atmosphere, and T_a, the temperature of the air after expanding adiabatically from the pressure p_1, that existing in the mains, to p_a, that of the atmosphere.

182. Reheating. When air at atmospheric temperature, and under a high pressure p_1, expands to atmospheric pressure p_a, the corresponding temperature, T_a, will be very low. As an example, if air under a pressure of five atmospheres, and at atmospheric temperature T_1, expands adiabatically to a pressure of one atmosphere, its temperature becomes approximately,

$$T_a=T_1\left(\frac{p_a}{p_1}\right)^{\frac{n-1}{n}}=522\left(\frac{1}{5}\right)^{\frac{2}{7}}=330=-130°\text{F.};$$

n having been assumed to have the value 1.4. Temperatures as low as this, due to the fact that the moisture present in the air freezes, makes lubrication difficult and clogs the valves, are undesirable at the exhaust of an air motor. To prevent this, recourse must be had to reheating; i.e., the air from the mains is passed through a heater before being admitted to the motor.

In being heated at constant pressure, the volume of the air is increased, and the ratio of the two volumes is given by T/T_1; where T is the temperature of the air after heating. Hence, equation (46), giving the work done, per pound of air, on the air motor, becomes

$$W_1 = \frac{T}{T_1} \times \frac{n}{n-1} p_1 v_1 \left\{ 1 - \left(\frac{p_a}{p_1} \right)^{\frac{n-1}{n}} \right\}. \quad . \quad . \quad . \quad . \quad (49)$$

If, now, T_a' is the temperature at the end of the adiabatic expansion, to the pressure p_a, and since $n = C_p/C_v$, $p_1 v_1 = RT_1 = J(C_p - C_v)T_1$, and $(p_a/p_1)^{\frac{n-1}{n}} = T_a'/T_1$, we find, by substituting in equation (49),

$$W_1 = \frac{T}{T_1} \times \frac{C_p}{C_p - C_v} J(C_p - C_v) T_1 \left(1 - \frac{T_a'}{T} \right). \quad . \quad . \quad (50)$$

And further, since the ratio of final to initial pressure is the same whether there be reheating or not, the ratio of final to initial temperature must also be the same for both cases; hence,

$$T_a' = T\frac{T_a}{T_1};$$

where T_a is the final temperature when there is no reheating. Substituting this value of T_a' in equation (50), and simplifying, we find

$$W_1 = JC_p \frac{T}{T_1}(T_1 - T_a). \quad . \quad . \quad . \quad . \quad . \quad (51)$$

Equation (51) is the expression, in terms of the three temperatures, with reheating, for the work done per pound of air, on the air motor. Equation (48) is the expression for the work done per pound of air without reheating. Taking the difference between equations (51) and (48), we find, due to reheating, for the gain in work

$$W' = JC_p \frac{T}{T_1}(T_1 - T_a) - JC_p(T_1 - T_a) = JC_p(T_1 - T_a)\frac{T - T_1}{T_1}. \quad (52)$$

The heat consumed, expressed in mechanical units, in raising the temperature of 1 pound of air at constant pressure, from T_1 to T, is

$$JC_p(T-T_1);$$

and the work which could be realized from this quantity of heat, by means of a Carnot cycle, is

$$W''=JC_p(T-T_1)\frac{T-T_1}{T}. \quad . \quad . \quad . \quad . \quad . \quad (53)$$

Taking the ratio of W', as given by equation (52), to W'', as given by equation (53), we find

$$\frac{W'}{W''}=\frac{T}{T_1}\times\frac{T_1-T_a}{T-T_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (54)$$

In equation (54), the first factor, viz, T/T_1, is always greater than unity, and for any given case, T_1-T_a is a constant. Hence the ratio, W'/W'', is greater than unity until the air is reheated to a temperature such that

$$\frac{T}{T_1}=\frac{T-T_1}{T_1-T_a}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (55)$$

And, for reheating to a temperature higher than this, the ratio becomes less than unity. Solving equation (55), for T, we find

$$T=\frac{T_1^2}{T_a}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (56)$$

Hence, for reheating to temperatures lower than that given by equation (56), there is a thermodynamic gain; i.e., the gain in work, due to the heat applied in reheating the air, is greater than that which could be realized if an equal quantity of heat were utilized on a Carnot cycle for the same limits of temperature.

To illustrate, we will assume a particular case and solve for T. Let p_1, the pressure in the mains, be six atmospheres, T_1 be 522, and p_a, the final pressure, be one atmosphere; then

$$T_a=T_1\left(\frac{p_a}{p_1}\right)^{\frac{n-1}{n}}=522\left(\frac{1}{6}\right)^{\frac{2}{7}}.$$

From equation (56), we find

$$T=\frac{T_1^2}{T_a}=\frac{(522)^2}{522\left(\frac{1}{6}\right)^{\frac{2}{7}}}=871+.$$

Giving a temperature for reheating, above that existing in the mains, of practically 349° F., which is higher than ever employed in practice.

The heat consumed in reheating is applied to much better advantage than in the case of a steam engine and boiler. Furthermore, since the fuel used in reheaters may be of a much lower grade than that ordinarily employed for heat motors, there is a saving in cost of fuel.

183. Loss of Head in Transmission Pipes. When a liquid flows in a pipe, there is always, due to friction between the liquid and the surfaces with which it comes into contact, a resistance to be overcome; and on account of this, there is a loss in pressure. That is, when friction is taken into account, *Bernouilli's Theorem,* which states that, *for the steady flow of a liquid in parallel stream lines without friction, the pressure head plus the velocity head plus the static head is a constant for any section under consideration,* no longer applies. The energy consumed in overcoming friction, is manifested by eddy currents. These eddy currents in turn subside; and the energy, possessed by them, is converted into heat, which in turn is lost by being dissipated to the surroundings. The loss of energy thus experienced by a given mass of the liquid is usually expressed by a loss *of head.* That is, the loss of head, experienced by a *unit* mass of the liquid, is numerically equal to the vertical height through which it would have to fall to do an amount of work equal to that consumed in overcoming the friction.

From a great number of experiments upon the flow of liquids in pipes, the following facts have been adduced: The loss of head is very nearly proportional to the square of the speed of flow, varies directly as the length of the pipe, as the wetted perimeter, and inversely as the cross-sectional area of the stream.

These relations may be stated symbolically as follows:

$$H=f\frac{s^2LP}{2gA}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (57)$$

where f is an experimental constant depending upon the nature of the liquid and inner surface of pipe, and H, s, L, P, and A are, respectively, the loss in head, speed of flow, length of pipe, wetted perimeter, and area of stream. For any particular cross-section the ratio of A to P is a constant; which is termed the *hydraulic radius*, and may be replaced by the symbol K. Hence, equation (57) may be written

$$H=f\frac{s^2L}{2gK}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (58)$$

Since the temperature of air, flowing in a pipe of any considerable length, is sensibly constant, the product of pressure and volume is also practically constant; and hence, as the pressure falls the speed of flow must increase. Therefore, since equation (58) assumes a constant speed, it is not directly applicable to the flow of air, or any other gas. In the limit, however, we have

$$dH=f\frac{s^2}{2gK}\,dL. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (59)$$

And, since the loss of head is numerically equal to the work done by a unit mass of the substance, we have

$$dH=pdv; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (60)$$

where p is the pressure, and dv the change in volume, per unit mass, for the section under consideration. From equations (59) and (60), we find

$$pdv=f\frac{s^2}{2gK}\,dL. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (61)$$

There is, of course, due to change in speed, also a change in kinetic energy; but, in general, this is so small in comparison with the total loss of head that it may be neglected.

Substituting, in equation (61), for dv its value as obtained from the equation

$$pv=RT,$$

we find

$$\frac{RT}{p}dp=-f\frac{s^2}{2gK}dL. \quad . \quad . \quad . \quad . \quad . \quad . \quad (62)$$

Under steady flow the mass of air passing any section, for a given interval of time, is a constant throughout the entire length of pipe. Hence, we have, for the speed of flow,

$$s=\frac{Mv}{A}=\frac{MRT}{pA}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (63)$$

where M is the mass passing any section per unit time, v the volume per unit mass, and A the cross-sectional area of the pipe. Substituting the value of s, as given by equation (63), in equation (62), we find

$$pdp=-f\frac{M^2RT}{2gKA^2}dL;$$

from which

$$\int_{p_1}^{p_2}pdp=-f\frac{M^2RT}{2gKA^2}\int_0^L dL; \quad . \quad . \quad . \quad . \quad (64)$$

where p_1 and p_2 are, respectively, the initial and final pressures, and L the length of the pipe. Finally, integrating, as indicated in equation (64), we find

$$p_1^2-p_2^2=f\frac{M^2RTL}{gKA^2}. \quad . \quad . \quad . \quad . \quad . \quad (65)$$

From equation (63) we have

$$p_1^2=\frac{M^2R^2T^2}{s_1^2A^2}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (66)$$

where s_1 is the initial speed. Dividing equation (65) by equation (66), member by member, we find

$$\frac{p_1^2-p_2^2}{p_1^2}=f\frac{s_1^2L}{gKRT}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (67)$$

Solving equation (67), respectively, for p_2, s_1, and f, we find

$$p_2=p_1\left(1-\frac{fs_1^2L}{gKRT}\right)^{\frac{1}{2}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (68)$$

$$s_1=\left(\frac{p_1^2-p_2^2}{p_1^2}\times\frac{gKRT}{fL}\right)^{\frac{1}{2}}, \quad . \quad . \quad . \quad . \qquad (69)$$

and

$$f=\frac{p_1^2-p_2^2}{p_1^2}\times\frac{gKRT}{s_1^2L}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (70)$$

By means of equations (65), (66), (68), and (69), the necessary calculations, for any given case, may be made; and, by means of equation (70), the coefficient f may be found for a given set of observations.

The ratio A/P is, for cylindrical pipes, a function of the diameter only; i.e.,

$$K=\frac{\pi D^2/4}{\pi D}=\frac{D}{4}.$$

We may substitute, then, in equation (68), the following constants: $g=32.2$, $K=D/4$, and $R=53.3$, and find

$$p_2=p_1\left(1-\frac{fs_1^2L}{32.2\times53.3\times\frac{D}{4}T}\right)^{\frac{1}{2}}=p_1\left(1-\frac{fs_1^2L}{429DT}\right)^{\frac{1}{2}}. \qquad (71)$$

In a similar manner, the various equations may be simplified.

Equation (71) is, perhaps, best illustrated by assuming a concrete case, and solving for the terminal pressure. As an example, let it be required to find the final pressure, for the case when the initial pressure is six atmospheres, the temperature

62°F., the quantity of air required 1200 cu.ft. per minute, the length of pipe 5 miles, and the diameter of the pipe is 1 ft. First of all, from equation (71), it is obvious that the pressure may be specified in any units whatsoever.

From a series of observations made by Riedler and Guttermuth upon the compressed air system of Paris, extending over a distance of about 10 miles, the diameter of the cast-iron pipe being very nearly 1 ft. (exactly 300 mm.), Professor Unwin deduced for the coefficient *f*, in this particular case, the value of 0.0029.* It must be remembered that this is not the coefficient for a straight piece of cast-iron piping; including as it does, bends and joints, and also a small amount of leakage. Though transmissions to such distances are unusual, the value just quoted for the coefficient is probably a good average value to use for a practical case for the same diameter of piping. That is, in any practical case, for piping of an equal diameter, we should probably find the average losses per given length, approximately the same.

From the conditions we have $p_1=88.2$ lbs. per square inch, $T=522$, $L=26{,}400$ ft., $D=1$ ft., and $s_1=(1200/60)/\frac{\pi D^2}{4}=25.5$ ft. per second. For *f*, we will use the value 0.0029. Substituting these values, in equation (71), we find

$$p_2=88.2\left(1-\frac{0.0029\times\overline{25.5}^2\times 26{,}400}{429\times 522}\right)^{\frac{1}{2}}=77.8 \text{ lbs. per sq. in.}$$

Thus giving a loss in pressure of about 11.8 per cent. It must, however, not be understood from this, that the percentage loss of power in transmission is also 11.8 per cent. The efficiency of transmission is found by taking the ratio of the work which the air motor can do in expanding adiabatically from the pressure p_2 to that of the atmosphere, to that which would have been obtained had adiabatic expansion taken place before transmission.

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* "On the Development and Transmission of Power," by W. C. Unwin.

That is, the efficiency of transmission is

$$\eta=\frac{\frac{n}{n-1}p_1v_1\left\{1-\left(\frac{p_a}{p_2}\right)^{\frac{n-1}{n}}\right\}}{\frac{n}{n-1}p_1v_1\left\{1-\left(\frac{p_a}{p_1}\right)^{\frac{n-1}{n}}\right\}}=\frac{1-\left(\frac{p_a}{p_2}\right)^{\frac{n-1}{n}}}{1-\left(\frac{p_a}{p_1}\right)^{\frac{n-1}{n}}}. \quad . \quad . \quad (72)$$

Substituting in equation (72), for p_a, p_1, and p_2, respectively, 14.7, 88.2, and 77.8, we find, for the efficiency of transmission,

$$\eta=\frac{1-\left(\frac{14.7}{77.8}\right)^{\frac{2}{7}}}{1-\left(\frac{14.7}{88.2}\right)^{\frac{2}{7}}}=0.945;$$

where n is assumed equal to 1.4.

It is thus seen that, though the loss in pressure is about 11.8 per cent, the loss in power, due to this loss in pressure, is only about 5.5 per cent.

The efficiency of transmission may also be defined, depending upon the point of view, as the ratio of the work that could be realized, before transmission, by allowing the air to expand isothermally, to that which would be realized by means of isothermal expansion after transmission. In any case, for pressures such as are ordinarily employed, the value found, for the efficiency of transmission, by this comparison will not differ materially from that found by means of equation (72). If we make the computation for this particular case, we find, by assuming isothermal processes,

$$\eta'=\frac{\log\frac{77.8}{14.7}}{\log\frac{88.2}{14.7}}=0.930;$$

which differs approximately, only 1.5 per cent from the value found by comparing adiabatic processes.

In the case of water, the coefficient f, other things being equal, is a constant for all diameters. This, however, is not the case for gases. In the case of air, the coefficient f is some function of the diameter. Various empirical formulæ have been proposed, for cast-iron piping, by means of which f is found, in terms of the diameter. None of them, however, are true for all diameters. As an example, the following formula, proposed by Professor Unwin, may be cited. According to this formula, the coefficient is

$$f=0.0027\left(1+\frac{3}{10D}\right).$$

However, by computing the coefficient, for various diameters, by means of this formula, and comparing with the values, as found by actual experiments, it is found that there is considerable discrepancy, as the following table will show:

D. In Feet.	Coefficient.	
	By Experiment.	By Formula.
0.492	0.00449	0.00435
0.656	0.00377	0.00393
0.980	0.0029	0.00351

For the two smaller diameters there is very close agreement; but, in the case of the one of 0.98 ft. diameter, the discrepancy is considerable. Professor Unwin has proposed the value 0.003 for all diameters of 1 ft. or over.

184. Composite Diagram. We are now prepared to show, by means of the p–v diagram, the losses for the compressor, the line, and the motor. Let, in Fig. 50, OI and OH represent, respectively, the axes of zero pressure and zero volume; and the line AB the aspirating stroke. Assume further that the compressor is one working on two stages, compressing first adiabatically, in the low-pressure cylinder, from the pressure p_a, as represented by the point B, to a pressure p, as represented by the point C. At the point C, the exhaust-valve of the low-pressure cylinder opens and the air is expelled to the receiver at the constant pressure p. In the receiver, the temperature falls to its initial value, and the volume shrinks by an amount represented by CD; the point D being on the isotherm BF. The condition of the same mass of air now, as regards pressure and volume, at the end of the aspirating stroke, in the high-pressure cylinder, is represented by the point D. Compression now takes place adiabatically from the point D, to the point E, to a pressure p_1. When the pressure p_1 has been attained, the exhaust-valve opens and expulsion takes place under constant pressure, as represented by the line EG. In the reservoir, the temperature of the air falls to its initial value, and the volume shrinks by an amount EF; the point F being on the isotherm BDF. During isothermal transmission, the pressure falls by an amount represented by GK; and at the end of admission in the air motor, i.e., at the point of cut-off, the condition of the air, as regards pressure and volume, is represented by the point L. The point L is again on the isotherm BDF. From the point of cut-off, L, the air expands adiabatically, as represented by the curve LM. At the point M release occurs; and the expulsion stroke is represented by the line MA.

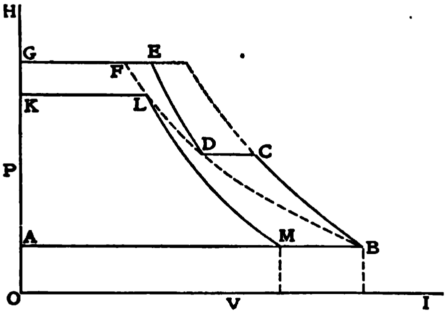

FIG. 50.

From an inspection of the figure it is evident that the net work done, per cycle, by the compressor, is measured by the area

ABCDEG; and, the net work recovered by the air motor, per cycle, is measured by the area *KLMA*. Hence, the total loss of work is measured by the area *BCDEGKLM*. The work lost, per cycle, due to the compression and expansion not being isothermal, i.e., in the compressor and motor, is measured by the area *BCDEFLM*; and, the work lost in transmission is measured by the area *FGKL*. In general, the thermodynamic loss in the compressor and motor is large, in comparison with the loss, due to friction, during transmission.

185. Theoretical Efficiency of System. It will now prove instructive to assume a concrete case and make, without considering other losses, a comparison between the three losses; that is, the thermodynamic loss, due to the compression, in the compressor, being adiabatic in place of isothermal, the loss in transmission, and the thermodynamic loss, due to the expansion in the motor, being adiabatic in place of isothermal.

Let it be required to compress the air to a pressure of six atmospheres by means of a two-stage compressor. The work required, per pound of air, according to equation (23), will be

$$W_1 = \frac{2n}{n-1} p_a v_a \left\{ \left(\frac{p_2}{p_a} \right)^{\frac{n-1}{2n}} - 1 \right\}$$

$$= \frac{2.8}{0.4} \times 27{,}800 (6^{\frac{1}{7}} - 1) = 56{,}800 \text{ ft.-lbs. per pound.}$$

If the expansion now take place isothermally, after cooling, the work recovered will be

$$W_2 = p_a v_a \log \frac{p_2}{p_a} = 27{,}800 \log 6 = 49{,}800 \text{ ft.-lbs. per pound.}$$

This is a loss of about 12.3 per cent.

Assume, now, that the transmission line has the same constants as that discussed in *Art.* 183. Then the pressure at the end of the line will be 77.8 lbs. per square inch; and the work that can now be recovered, due to isothermal expansion, will be

$$W_3=27,800 \log \frac{77.8}{14.7}=46,300 \text{ ft.-lbs. per pound.}$$

This is a loss of about 6.2 per cent of the total work.

If the expansion now takes place adiabatically, the work done on the air motor is

$$W_3=27,800\times\frac{1.4}{0.4}\left\{1-\left(\frac{14.7}{77.8}\right)^{\frac{2}{7}}\right\}=36,900 \text{ ft.-lbs. per pound.}$$

This gives a loss in the air motor of about 16.6 per cent of the total work done.

We have then, the following:

	Per Cent.
Loss in compressor	12.3
Loss in transmission	6.2
Loss in motor	16.6
Efficiency of system	64.9
Total	100

From the foregoing computations, it is obvious that the efficiency of the system is low, not due to the loss in transmission; but on account of the combined losses in the compressor and motor.

Assume, now, that the air is reheated to a temperature 275°F. above the surroundings. The work which the air will now do on the motor is

$$W_4=\frac{T}{T_1}W_3=\frac{797}{522}\times 36,900=56,300 \text{ ft.-lbs. per pound;}$$

where T is the temperature to which the air is heated before being admitted to the motor. This gives, for the gain in work, for the same quantity of air consumed, by the motor, approximately 52.6 per cent.

To make a comparison now, between the work done on the motor and that done on the compressor, it will be necessary to add to the work done on the compressor, the work due to the heat consumed in reheating the air. The heat consumed in elevating the temperature of 1 pound of air from the temperature T_1 to the temperature T, is

$$Q = C_p(T - T_1) \text{ B.T.U.}$$

And the work which would be realized on a Carnot cycle, is

$$W_5 = JC_p(T - T_1)\frac{T - T_1}{T}.$$

Substituting the various values, we find

$$W_5 = 778 \times 0.238 \times 275 \times \frac{275}{797} = 17{,}600 \text{ ft.-lbs.}$$

Taking the ratio now, of W_4 to the sum of W_1 and W_5, we find, for the efficiency of the system, 75.7 per cent, as against 64.9 per cent, obtained without reheating. Commercially, however, the gain is greater than that indicated by the computations. For, as previously stated, a low-grade fuel may be employed, and the motor operates better, especially so if a small percentage of water is injected into the heater. This water is evaporated in going through the heater, and condensed in going through the motor. There is involved in this operation a small thermodynamic loss; but otherwise, the effect is good, since the water present helps to prevent leakage. Finally, reheating has the effect of increasing the capacity of both the compressor and line.

It must be emphasized that in no case are efficiencies obtained as high as those indicated by the foregoing computations. Due to imperfect valve action, leakage, and mechanical losses, in both the compressor and motor, the efficiency of the system may be reduced by 10 to 15 per cent below that indicated by the computations.*

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* For actual tests on air transmission systems, see Unwin, "On the Development and Transmission of Power."

CHAPTER XVII

REFRIGERATION

186. The object of *refrigeration* is to maintain the temperature of some body, or aggregation of bodies, at some point lower than that of the surroundings. This may be done in two ways. One method is to abstract heat directly, by means of a refrigerating machine, from the medium surrounding the bodies. The other method is to bring about the desired lowering of temperature by means of ice. The ice employed, to bring about the desired refrigeration, may be harvested, during the cold season, from rivers and lakes, or else, the so-called " artificial ice," produced by means of refrigerating machines, may be used.

Since the putrefaction of organic growths, such as foodstuffs, is retarded with lowering of temperature, and, in general completely prevented when the temperature becomes sufficiently low, the prime object of refrigeration is not the maintaining of low temperatures, but rather the effects due to such low temperatures; i.e., the preservation of foodstuffs during storage and shipment, and, in general, the promotion of health and comfort.

187. Commercial Refrigerating Machines. Refrigeration may be brought about in various ways. But commercially successful refrigerating machines are restricted to two types; viz, refrigerating machines in which air is the working substance, and machines in which some volatile liquid, such as ammonia, or carbon-dioxide, is employed as a working substance. For ammonia machines, there are again two distinct methods of operation; viz, *compressor machines*, and *absorption machines*. These various types will be discussed subsequently under separate headings.

All commercial refrigerating machines operate as reversed engines; but, it must not be understood from this that the machine is reversible. The working substance abstracts heat from a body of, relatively, low temperature, called the *refrigerator*, consumes energy either in the form of mechanical work or heat, and rejects heat to a *condenser* or *cooler*. The heat rejected to the cooler, barring various losses, is equal to the heat taken from the refrigerator plus the heat equivalent of the energy consumed in bringing about the transfer.

Equation (24), of Chapter VIII, states that, for an engine operating reversed, on a Carnot cycle,

$$W=JH_2\frac{S-R}{R}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where W is the energy consumed in bringing about the transfer, H_2 the heat abstracted from the refrigerator, S the temperature of the source, and R the temperature of the refrigerator. The source, in the case of a reversible engine, corresponds to the cooler of a refrigerating machine.

In discussing the Carnot cycle, it was found that, other things being equal, the greater the range in temperature, the greater the amount of work realized for a given quantity of heat abstracted from the source. On the other hand, equation (1) clearly indicates that, other things being equal, for a given quantity of heat H_2, abstracted from the refrigerator, the work which must be done by the compressor decreases as the difference of temperature between the cooler and refrigerator is decreased. Hence, the range in temperature between refrigerator and cooler, for refrigerating machines, should be as small as possible.

188. Air Refrigerating Machine. The *air refrigerating system* consists essentially of four parts; viz, a cold storage room, a compression cylinder, an expansion cylinder, and a cooler. The cycle is as follows: During the aspirating stroke, of the compressor piston, air flows into the cylinder, from the cold storage room, which during the return stroke is compressed, practically adiabatically, to the desired pressure, and expelled to the cooler. The cooler, usually, consists of a series of pipes in which the air is cooled by water circulating through the tank in which the pipes are placed. From the cooler the air passes into the expansion cylinder, where it does work on the piston, expanding practically adiabatically, and is finally exhausted, at a low temperature, to the cold storage room. The work done in the expansion cylinder is utilized in helping to drive the compressor. Hence the work, barring mechanical losses, which must be supplied to the compressor by means of some motor, is the difference between that done in the compression cylinder and that done in the expansion cylinder.

189. Ideal Coefficient of Performance. To make a mathematical discussion, of the cycle just described, it will be necessary to assume ideal conditions. Let T_2 be the temperature of the air entering the cooler, at the end of the adiabatic compression, T_a its temperature as it leaves the cooler and is admitted to the expansion cylinder, T_0 its temperature at the end of the adiabatic expansion as it enters the cold storage room, and T_1 its temperature as it leaves the cold storage room and enters the compressor. It will now be assumed that the pressures in both the cooling pipes and cold storage room are constant throughout the cycle, and the machine is mechanically perfect.

By equation (15), *Art.* 177, we have for the work done per pound of air, on the piston of the compressor,

$$W_1 = JC_p(T_2 - T_1). \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

By equation (48), *Art.* 181, we have for the work done per pound of air, on the piston, in the expansion cylinder,

$$W_2 = JC_p(T_a - T_0). \quad . \quad . \quad . \quad . \quad . \quad . \qquad (3)$$

The work which must be supplied, to make the process take place, is the difference between W_1 and W_2; i.e.,

$$W_3 = JC_p\{(T_2 - T_1) - (T_a - T_0)\}. \quad . \quad . \quad . \quad . \quad (4)$$

Since, according to the assumed conditions, the ratio of the ranges in pressures for the two cylinders are the same, we find

$$\frac{T_0}{T_a} = \frac{T_1}{T_2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Substituting the value of T_0, as given by equation (5) in equation (4) we find

$$W_3 = JC_p(T_2 - T_1)\frac{T_2 - T_a}{T_2}. \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The heat per pound of air, expressed in mechanical units, taken from the refrigerator, is

$$W_4 = JC_p(T_1 - T_0).$$

Substituting again, for T_0, its value, we find

$$W_4 = JC_p\frac{T_1}{T_2}(T_2 - T_a). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Taking the ratio of W_4 to W_3 we find, for the *ideal coefficient of performance*,

$$\eta = \frac{W_4}{W_3} = \frac{T_1}{T_2 - T_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Equation (8) again shows that, the smaller the difference in temperature between refrigerator and cooler, the larger will become the ratio of the work equivalent of the heat abstracted from the refrigerator, to the work supplied. It is, of course, obvious that, due to the fact that it is practically uneconomical to construct cooling pipes of sufficient volume, such that the pressure throughout the cycle in the cooler is constant, and further, since the pressure in the cold storage room varies, the ratio, as expressed by equation (8), cannot be realized in practice. Furthermore, due to various losses, which must be experienced, in the case of an actual refrigerating machine, this ratio is still further reduced. Solving equation (8), for the work that must be supplied to a perfect machine, we find

$$W_3 = W_4 \frac{T_2 - T_1}{T_1}. \quad \ldots \ldots \ldots \quad (9)$$

The *commercial efficiency* of a refrigerating machine may be defined as the ratio of the work which would have to be done, for the given range of temperature and given quantity of heat removed from the refrigerator, on a perfect machine, to that actually required. If W_5 is the work actually required, then the commercial efficiency is

$$\frac{W_3}{W_5} = \frac{W_4}{W_5} \times \frac{T_2 - T_1}{T_1}. \quad \ldots \ldots \ldots \quad (10)$$

The heat which must be carried away by the circulating water, in the cooler, per pound of air, is

$$H_1 = C_p(T_2 - T_a). \quad \ldots \ldots \ldots \quad (11)$$

The cycle of an air refrigerating machine may be conveniently represented by means of the $T-\varphi$ diagram. By equation (12), *Art.* 169, the change in entropy, when both the pressure and volume vary, is

$$\varphi_2 - \varphi_1 = C_v \log \frac{p_2}{p_1} + C_p \log \frac{v_2}{v_1}. \quad \ldots \ldots \quad (12)$$

In the cycle just discussed it was assumed that the pressure, during the absorption and rejection of heat, remains constant. Hence, equation (12) becomes

$$\varphi_2 - \varphi_1 = C_p \log \frac{v_2}{v_1} = C_p \log \frac{T_2}{T_1}; \quad \ldots \ldots \quad (13)$$

and the heating and cooling curves, on the $T-\varphi$ diagram, are logarithmic. Let, in Fig. 51, the point A represent the condition of the air, as regards temperature and entropy, at the instant when it enters the compressor at the temperature T_1. During the adiabatic compression the entropy remains constant and the temperature changes from T_1 to T_2, as represented by the line *AB*. The cooling then takes place, as represented by the curve *BC*, at constant pressure, to the temperature T_a. The adiabatic expansion, from the temperature T_a to the temperature T_0, is represented by the constant entropy line *CD*. Finally, the rise in temperature, in the refrigerator, at constant pressure, from T_0 to T_1, is represented by the curve *DA*; and the cycle is completed. The heat abstracted from the refrigerator is measured by the area *FADE*, the heat rejected to the cooler is measured by the area *FBCE*, and the work done, on the compressor, is measured by the area *ABCD*. Finally, the ideal coefficient of performance is given by

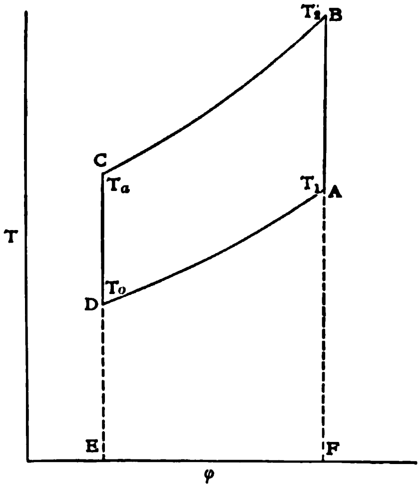

FIG. 51.

$$\eta = \frac{\text{Area } FADE}{\text{Area } ABCD}.$$

Since the rejection of heat to the cooler, and the abstraction of heat from the cold storage room, both take place at constant pressure, equation (8) may be deduced in a very simple manner. If the heat abstracted from the cold storage room, for a given interval of time, is

$$H_2=C_p(T_1-T_0),$$

then the heat rejected to the cooler for the same interval of time, is

$$H_1=C_p(T_2-T_a)\cdot$$

Therefore, the ideal coefficient of performance is

$$\eta=\frac{H_2}{H_1-H_2}=\frac{C_p(T_1-T_0)}{C_p(T_2-T_a)-C_p(T_1-T_0)}=\frac{T_1}{T_2-T_1};$$

which is the same as previously found.

It must be emphasized that the equations deduced, in this article, do not represent conditions as found in actual practice. For the pressure, in the cooling pipes, of any actual refrigerating machine will vary considerably throughout the cycle. Hence, the actual coefficient of performance, even when all other losses are neglected, will be less than that indicated by equation (8).

Due to the fact that air has a low thermal capacity, air refrigerating machines are necessarily bulky, and therefore, commercially uneconomical. However, there are certain places, as for example on board of ships, where it is inadvisable to use machines employing a volatile liquid. For, in the first place, there are possible dangers from injurious escaping gases. But, even if the escaping gas is not injurious, there is always the possibility of a large leak, and consequently a total loss of the working substance, which cannot be replaced until the end of the trip. This, however, means a complete disablement of the plant. Hence, air machines are used only as a matter of expedience and not economy, in place of refrigerating machines employing a volatile liquid as a working substance.

190. Compression Machines Using Volatile Liquids. *Compression refrigerating machines*, using a volatile liquid for the working substance, consist essentially of the parts as represented diagrammatically, in Fig. 52. *A* is the compression cylinder where the vapor is compressed, and then expelled into coils immersed in water in *B*; *B* being the condenser, or cooler. If the vapor is just saturated as it leaves the refrigerating coils, superheating may take place, during compression; this however is usually very small in comparison with the heat of condensation. Due to the high pressure in the condenser, and the low temperature, maintained by the circulating water, the vapor condenses, gives up the superheat and heat of condensation, which is carried away by the water, and the liquid flows into *C*, the storage tank.

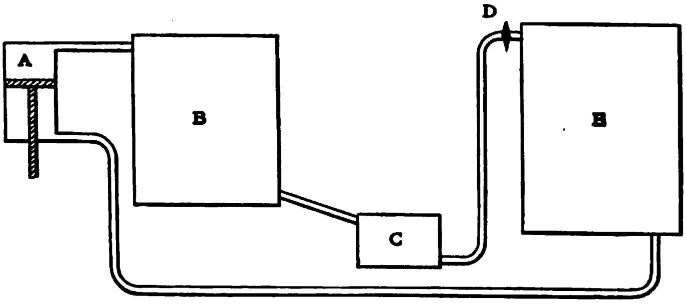

FIG. 52.

In the tank *C*, the liquid is under a pressure corresponding to that of its vapor, for the existing temperature; the temperature of the liquid in the storage tank, usually does not differ materially from that of the surroundings. As an example, if the liquid employed be ammonia and the temperature in the tank is 75°F., then the pressure of the vapor is approximately 140 lbs. per square inch. Due to this high pressure, under which the liquid is in *C*, it flows, through the expansion valve *D*, into the coils in the refrigerator *E*. The pressure in the coils, due to the aspirating action of the compressor, is low. By regulating the expansion valve, or the speed of the compressor, or both, the pressure in the refrigerator coils may be varied at pleasure. Since, when the liquid passes through the expansion valve, the process is adiabatic, and no work is being done, the total heat content remains the same. Therefore, for thermal equilibrium to obtain, when the pressure falls from p_1, that existing in the storage tank, to p_2, that existing in the refrigerating coils, there must take place a certain amount of evaporation, such that

$$h_1 = h_2 + qr_2; \quad . \ . \ . \ . \ . \ . \ . \ . \quad (14)$$

where h_1 and h_2, respectively, are the heats of the liquid corresponding to the pressures p_1 and p_2, q the amount of dryness, and r_2 the heat of vaporization at the pressure p_2. From equation (14), we find

$$q = \frac{h_1 - h_2}{r_2}; \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (15)$$

and the remainder of the liquid can, then, if completely vaporized, take from the surrounding medium the quantity of heat

$$H_2 = (1 - q)r_2. \quad . \ . \ . \ . \ . \ . \ . \ . \quad (16)$$

In order that heat may flow from the medium in E, into the coils it is, of course, necessary that the temperature of the medium be higher than that of the liquid, in the coils, corresponding to the pressure p_2. If the difference of temperature is sufficient, the liquid will be completely vaporized; and the quantity of heat, as expressed by equation (16), will be removed from the refrigerator. If the difference of temperature be greater than this, the vapor becomes superheated; and the quantity of heat removed from the refrigerator will be greater than that indicated by equation (16). The ideal p–v diagram, of the cycle just discussed, is represented in Fig. 53. The point A represents the condition, as regards pressure and volume, of the vapor at the beginning of the aspirating stroke, and the point B represents the condition at the end of the aspirating stroke; the line AB, therefore, represents the volume, due to complete vaporization under constant pressure. The curve BC represents the compression, which is nearly adiabatic, *CD* represents the expulsion, and also the condensation, under constant pressure, in the condenser, and *DE* the change in pressure, and consequent change in volume, due to partial evaporation in passing through the expansion-valve. Therefore, the net work done, during the cycle, is measured by the area *ABCD*. Finally the ideal coefficient of performance is given by the ratio of the work equivalent of the heat removed from the refrigerator to the work equivalent of the area *ABCD*.

The refrigerating coils, in which the vaporization takes place, may be placed directly in a cold storage room, in the form of pipes, or else placed in a tank containing a solution of some salt, called brine. The freezing point for the brine must, of course, be lower than the temperature in the coils. The brine may then be employed, by circulating it through pipes, to bring about refrigeration in some place remote from the plant, or else, to produce ice, by abstracting heat from water, contained in tanks, immersed in the brine.

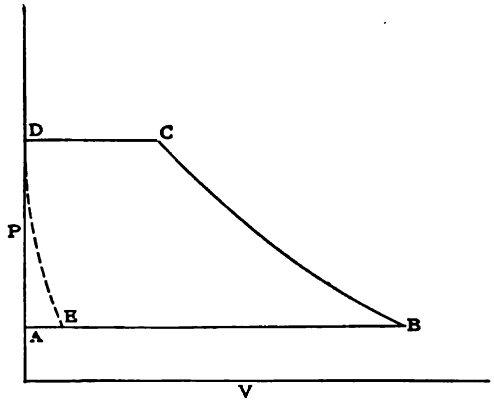

FIG. 53.

The cycle of a compressor refrigerating plant, using a volatile liquid as a working substance, is most instructively represented by the $T-\varphi$ diagram. However, before plotting the $T-\varphi$ diagram, it will be necessary to deduce an expression for the change in entropy, for the substance, when passing through the expansion valve. To do this, let T_1 be the temperature of the liquid in the storage tank, then by assuming some arbitrary temperature, say T_0, from which the entropy is measured, the entropy of a unit mass of the liquid, before passing through the expansion valve, is

$$\varphi_1=c\int_{T_0}^{T_1}\frac{dT}{T}=c\log\frac{T_1}{T_0}; \quad . \quad . \quad . \quad . \quad . \quad (17)$$

where c is the thermal capacity, of the liquid, per unit mass. Assume some temperature T, in the refrigerating coils; T being, of course, less than T_1. The entropy, then of a unit mass of liquid and vapor, measured from the same zero, is

$$\varphi_2=c\int_{T_0}^{T}\frac{dT}{T}+\frac{qr}{T}=c\log\frac{T}{T_0}+\frac{qr}{T}; \quad . \quad . \quad . \quad . \quad (18)$$

where q is the amount of dryness, and r the heat of vaporization corresponding to the temperature T. Taking the difference between equations (18) and (17), we find, for the change in entropy in passing through the expansion valve,

$$\varphi=\varphi_2-\varphi_1=c\log\frac{T}{T_0}-c\log\frac{T_1}{T_0}+\frac{qr}{T}. \quad . \quad . \quad . \quad (19)$$

But,

$$qr=h_1-h=c(T_1-T);$$

substituting this value of qr in equation (19), and simplifying, we find

$$\varphi=c\left(\log\frac{T}{T_1}+\frac{T_1}{T}-1\right). \quad . \quad . \quad . \quad . \quad . \quad (20)$$

Differentiating equation (20) with respect to T, T_1 being assumed constant, we find

$$\frac{d\varphi}{dT}=-\frac{c}{T}\left(\frac{T_1}{T}-1\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

Equation (21) shows, since $T_1>T$, that as the temperature increases, the entropy decreases, and vice versa. Hence, the entropy of the substance is increased by passing through the expansion valve. This is necessarily so, since the process is irreversible.

The $T-\varphi$ diagram, Fig. 54, indicates the various parts of the cycle. *BC* is the constant entropy line for the adiabatic compression of the vapor, from the temperature T_2 to T_3; if the vapor be just saturated, as shown, when the compression begins, it will become superheated during compression. *CK* represents the cooling of the vapor to the temperature of condensation, T_1; and *KD* represents the condensation of the vapor, in the condenser, at the constant temperature T_1. *DA* represents the temperature entropy curve for the cooling of the liquid, without expansion, from the temperature T_1 to T_2. Had evaporation taken place, without expansion, after cooling along the curve *DA*, the quantity of heat removed from the refrigerator would be measured by the area *FABH*. Due to expansion, however, through the expansion valve, the entropy of the substance increases in changing from the temperature T_1 to T_2, as indicated by the curve *DE*. The curve *DE* is determined by assuming various values of temperature, between T_1 and T_2, and solving, by means of equa-

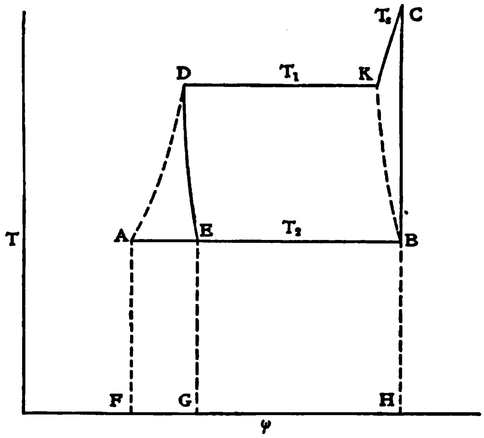

FIG. 54.

tion (20), for the corresponding entropy. Hence the heat that it is now possible to remove from the refrigerator, in bringing about complete vaporization, is measured by the area *GEBH*. Consequently, the amount of refrigeration that is lost, due to the change in entropy, in passing through the expansion valve, is measured by the area *FAEG*. Had there been superheating in the refrigerating coils, the quantity of heat removed from the refrigerator would be increased; but, due to this superheating, the vapor at the end of the compression will, likewise, be superheated by an additional amount.

Since the evaporation, which takes place while the liquid passes through the expansion valve, has no refrigerating value, but merely brings about thermal equilibrium, by reducing the temperature of the liquid to that existing in the coil, it follows that the change in entropy, along the curve *DE*, depends upon the ratio of the heat of vaporization to the thermal capacity of the liquid. The higher the ratio of the heat of vaporization to thermal capacity of liquid, the smaller the amount of vaporization required, for a given difference of temperature, to reduce the temperature of the liquid to that existing in the coil; and consequently the smaller will be the area *FAEG*. Therefore, since the work done by the compressor is independent of the amount of vaporization that takes place, along the curve *DE*, it follows that a liquid for which the ratio, of heat of vaporization to thermal capacity, is high, is best suited, from an economical standpoint, for refrigerating purposes.

191. Absorption Machines. The principle of operation of an *absorption refrigerating machine* is based on the fact that the volume of ammonia vapor that can be absorbed by a given volume of water, other things being equal, depends upon the temperature, and decreases rapidly as the temperature is increased. Hence if water, at a low temperature, is saturated with ammonia vapor, then, to drive off the vapor, heat must be absorbed by the water. Likewise, if ammonia vapor be passed into water at a low temperature, absorption will take place with a consequent development of heat.

An absorption refrigerating machine is represented, diagrammatically, in Fig. 55. *A* is a storage tank containing ammonia from which expansion takes place through the valve *B*, into refrigerating coils in *C*, where refrigeration takes place. From the coils in *C*, the ammonia vapor passes into the liquid in the *absorber D*. The liquid in *D* is a solution of ammonia in water, of slight concentration and, relatively, low temperature. The liquid in the absorber being at a low temperature and only slightly concentrated, the incoming vapor is readily absorbed. The liquid of high concentration is removed from the bottom of *D*, by means of the pump *P*, and forced, at *a*, into the generator *F*. In the generator is placed a heating coil *H*, by means of which heat is supplied to the highly concentrated solution, and raises its temperature. Due to the high temperature, part of the vapor is expelled from the solution, under a high pressure, and passes into the *condenser G*. The condenser is maintained at a, relatively, low temperature by means of circulating water. Due to this low temperature and the high pressure, the vapor condenses and flows into the storage tank *A*. By means of the valve *I*, the pressure in *G* and *A* is regu-

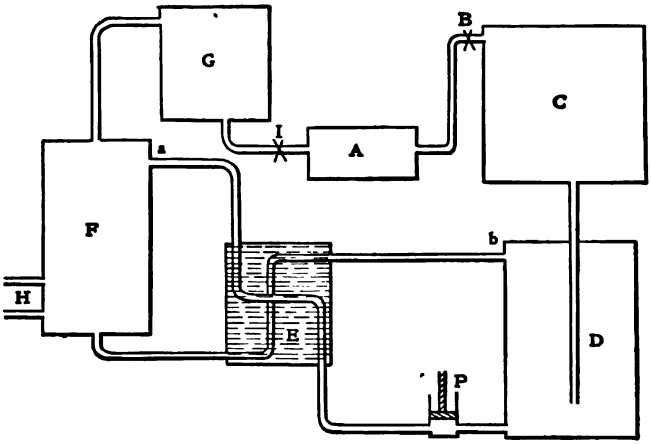

Fig. 55.

lated. Finally, the solution of low concentration, at the bottom of the generator *F*, is forced, due to the high pressure subsisting in the generator, into the absorber, at *b*. The pipes which convey the highly concentrated solution into the generator at *a*, and the solution of low concentration into the absorber at *b*, both pass through the *heat exchanger E*. In the heat exchanger the solution at a low temperature, going from the absorber to the generator, takes up heat from the high temperature solution, going from the generator to the absorber.

The cycle is, then, as follows: The absorption, in the absorber, corresponds to the aspirating stroke of the compressor, as represented by *AB* of Fig. 56. The change in pressure in going from the absorber through the generator is represented by the curve *BC*, and corresponds to the compression curve of the compressor. The line *CD* represents the condensation at constant pressure, the same as in Fig. 53. Finally, the curve *DE* represents the fall in pressure and consequent increment in volume, due to partial evaporation of the liquid, in passing through the expansion-valve *B*. There is, of course, in this cycle, as well as in the compressor cycle, due to a partial evaporation when the liquid passes through the expansion-valve, a loss in refrigeration. Furthermore, there is, due to the fact that the ammonia vapor when distilled in the generator, carries with it a certain amount of aqueous vapor, an unavoidable loss.

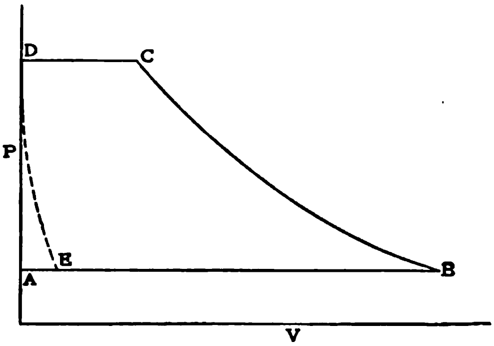

FIG. 56.

Thermodynamically speaking, the ideal coefficient of performance of the absorption machine, as well as that of any other refrigerating machine, is given by

$$\eta = \frac{R}{S-R};$$

where S is the temperature of the condenser, and R the temperature in the refrigerator. On the other hand, the commercial efficiency is given by the ratio of the work which would have to be done on a perfect engine, to bring about the given transfer of heat, to the energy actually consumed. That is, if H_2 is the quantity of heat abstracted from the refrigerator, during a given interval of time, H_1 the quantity of heat supplied to the generator, and W the work done on the pump, during the same interval of time, the commercial efficiency is given by

$$E_c = \frac{JH_2\frac{S-R}{R}}{JH_1+W}. \quad \ldots \ldots \quad (22)$$

The following table, taken from "Landolt and Börnstein," is given to show how the coefficient of absorption for ammonia vapor, under normal pressure, varies with the temperature:

τ	K	τ	K
0	98.7	15	60.6
1	92.7	16	59.1
2	87.7	17	57.6
3	83.6	18	56.1
4	79.9	19	54.7
5	77.3	20	53.5
6	75.6	21	51.9
7	73.9	22	50.6
8	72.3	23	49.6
9	70.6	24	48.6
10	68.9	25	47.6
11	67.2	26	46.5
12	65.5	27	45.5
13	63.7	28	44.4
14	62.1	29	43.4

where τ is the temperature in degrees centigrade, and K is the number of grams of ammonia vapor absorbed per 100 c.c. of water.

By *heat of dilution* of a substance is meant the quantity of heat which is evolved when a unit mass of the substance is diluted to an extent such that practically no more heat is evolved upon further dilution.

According to experiments by Berthelot, when 1 gram of liquid ammonia has been dissolved in n grams of water, and this solution is then fully diluted, the heat evolved is as given in the following table:

n	Gram Calories.	n	Gram Calories.
1.04	75.6	3.18	22.6
1.06	74.4	3.76	18.8
1.13	68.8	6.11	12.3
1.98	40.0	10.1	0.12

The results given in the table were obtained from experiments conducted at temperatures of 14°C.

By inspection it is seen that very little heat is evolved after the dilution is greater than 10 to 1. It has been proposed to employ the empirical equation

$$H=\frac{h}{n}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

for the heat of dilution when a solution, of 1 gram of ammonia dissolved in n grams of water, is fully diluted; H being the heat evolved, and h some constant. If equation (23) be applied to the values as given in the table, the value 78 be assigned to h, and the values of H computed and compared with the observed values, an idea will be obtained as to how closely the empirical equation conforms to the actual experimental results.

n	Observed.	Computed.	n	Observed.	Computed.
1.04	75.6	75.0	3.18	22.6	24.5
1.06	74.4	73.6	3.76	18.8	20.7
1.13	68.8	69.0	6.11	12.3	12.8
1.98	40.0	39.4	10.1	0.12	7.7

The foregoing table shows that, when the initial dilution is not greater than 6 to 1, equation (23) gives fairly consistent results; but, for initial dilutions greater than this, the equation breaks down completely. Furthermore, since the lowest initial dilution in Berthelot's experiments was 1.04 to 1, equation (23) is necessarily doubtful for initial dilutions lower than this.

Experiment shows that, if 1 gram of ammonia vapor is absorbed by water and completely diluted, there is evolved a quantity of heat equal to 496 gram calories; hence, if m grams of ammonia are absorbed, and complete dilution take place, there will be evolved $496m$ gram calories. Therefore, if we assume equation (23) to hold, there will be evolved, when m grams of ammonia vapor are absorbed by n grams of water,

$$H_1 = mQ - \frac{mh}{n/m} = m\left(Q - \frac{m}{n}h\right); \quad . \quad . \quad . \quad . \quad (24)$$

where n/m is the number of grams of water per gram of ammonia, and Q the quantity of heat evolved when 1 gram of ammonia vapor is absorbed by water and completely diluted. If, now, $m+k$ grams of ammonia be absorbed by n grams of water, the number of grams of water per gram of ammonia will be $n/(m+k)$. Therefore, the quantity of heat

$$H_2 = (m+k)\left(Q - \frac{m+k}{n} \times h\right), \quad . \quad . \quad . \quad . \quad (25)$$

will be evolved. Taking the difference between the right-hand members of equations (25) and (24), for the quantity of heat evolved, when a solution containing m grams of ammonia to n grams of water, absorbs k grams of ammonia, we find

$$H=H_2-H_1=(m+k)\left(Q-\frac{m+k}{n}\times h\right)-m\left(Q-\frac{m}{n}h\right)$$
$$=k\left\{Q-\frac{h}{n}(2m+k)\right\}. \quad . \quad . \quad (26)$$

Substituting for Q and h the numerical values, we obtain

$$H=k\left\{496-\frac{78}{n}(2m+k)\right\}\text{gram calories.} \quad . \quad . \quad (27)$$

Equation (27) may be reduced to English units as follows:

$$H=k\left\{893-\frac{140}{n}(2m+k)\right\}\text{B.T.U.} \quad . \quad . \quad . \quad . \quad (28)$$

That is, equation ~~(27)~~ 28 is the expression for the heat, in B.T.U., which is evolved when a solution containing m pounds of ammonia to n pounds of water, absorbs k pounds of ammonia vapor.

See Errata

As previously stated, the foregoing equations are empirical and are true only between certain limits of initial dilution; furthermore, since the heat of absorption and dilution varies with the temperature, the results obtained by means of these equations are to some extent doubtful. The equations have been deduced merely to show the method of attack.

Under ideal conditions the heat developed in the absorber is equal to that required in the generator; but, since the temperature of the generator must be higher than that of the absorber, it is impossible to utilize the heat developed in the absorber. Hence, to maintain the process, heat must be supplied, by means of some independent source, to the high-temperature generator, and heat must be abstracted from the low-temperature absorber.

In the case of a compression machine the energy consumed varies directly as the difference of temperature between the condenser and refrigerator. This, however, is not so in the case of an absorption machine; hence, for a wide range in temperature, the absorption machine is thermodynamically superior. A further advantage, which is mechanical, is that no compressor is required. In certain cases the heating in the generator is brought about by means of exhaust steam, from engines, which is again economical. Finally, the power consumed by the pump in an absorption machine is small in comparison with the other quantities involved.*

192. Comparison of Air and Ammonia Machines. It was stated in *Art.* 189 that, due to the low thermal capacity of air, refrigerating machines employing air as a working substance are necessarily bulky. It is impossible to make a general comparison; but a rough estimate may be obtained by assuming a concrete case. Let it be assumed that the temperature of the refrigerator is 32°F., and that the range in temperature of the air in passing through the refrigerator is 100°F. One pound of air will then remove, from the refrigerator,

$$C_p(T_1-T_0)=0.238\times100=23.8 \text{ B.T.U.};$$

and to do this, the compressor must take in 12.4 cu.ft. The volume, per B.T.U. removed from the refrigerator, then is

$$12.4/23.8=0.521 \text{ cu.ft.}$$

Assume, now, an ammonia compression machine, with a temperature of 70°F. for the reservoir. The dryness after passing through the expansion valve will be

$$q=\frac{h_1-h_2}{r_2}=\frac{42\dagger}{540}=0.078;$$

and the quantity of heat that can be removed, by complete vaporization taking place at 32°F., is

$$(1-q)r_2=540(1-0.078)=498 \text{ B.T.U. per pound.}$$

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* For a comprehensive discussion of absorption machines see "Modern Refrigerating Machinery" by Hans Lorenz.

† From Peabody's Steam Tables.

The specific volume of ammonia vapor, at 32°F., is approximately 4.74 cu.ft. per pound. Hence we find, for the volume, per B.T.U.,

$$4.74/498=0.00952.$$

Taking the ratio of the volume for air, to that for ammonia, we find

$$0.521/0.00952=54.7.$$

This shows that for the assumed conditions, other things being equal, the bulk of the compression cylinder of an air refrigerating machine is very large in comparison with that of an ammonia machine; but, further than this, the air machine must also have an expansion cylinder. For lower temperatures in the refrigerator, the ratio of the two volumes becomes somewhat less. Assume the temperature of the refrigerator 15°F., then the dryness, after passing through the expansion-valve, is

$$61/554=0.110;$$

and the quantity of heat that can be removed, by complete vaporization, is

$$554(1-0.110)=493 \text{ B.T.U. per pound.}$$

The specific volume for the vapor of ammonia at 15°F. is 6.68 cu.ft. per pound; hence, we find for the volume, per B.T.U.,

$$6.68/493=0.0135 \text{ cu.ft.}$$

The volume of air that the compressor must now take in, at the temperature of 15°F., is approximately 12 cu.ft.; hence the volume of air per B.T.U., is

$$12/23.8=0.504 \text{ cu.ft.}$$

Taking the ratio of the volume for air, to that for ammonia, we find

$$0.504/0.0135=37.3.$$

The ammonia machine is also superior from the thermodynamic standpoint. By considering the two cycles, it is obvious that the cycle of the ammonia refrigerating machine, approaches the Carnot cycle much more closely than does the cycle of an air machine. For, in the ammonia cycle, the greater part of the heat is abstracted and rejected, respectively, during vaporization and condensation; i.e., at constant temperature. On the other hand, in the case of the air cycle, both the abstraction and rejection of heat take place with continuously varying temperature. The foregoing may be illustrated roughly as follows: As previously shown, the work done by a compressor per cycle, if the compression is adiabatic, is

$$W_1 = JC_p(T_2 - T_1) \text{ per pound.}$$

The heat, expressed in mechanical units, removed by 1 pound of ammonia, from the refrigerator, is

$$W_2 = Jr(1-q);$$

and the ideal coefficient of performance is

$$\eta' = \frac{W_2}{W_1} = \frac{r(1-q)}{C_p(T_2 - T_1)}. \quad \ldots \ldots \quad (29)$$

It was shown, in *Art.* 189, that the ideal coefficient of performance of an air refrigeration machine is

$$\eta'' = \frac{T_1}{T_2 - T_1}. \quad \ldots \ldots \ldots \quad (30)$$

In equations (29) and (30), T_1 and T_2 are, respectively, the temperatures before and after adiabatic compression. If we assume, now, that the ranges in temperature for the two machines are equal, we find, for the ratio of the performance for the two processes,

$$\frac{\eta'}{\eta''} = \frac{r(1-q)}{C_p T_1}. \quad \ldots \ldots \ldots \quad (31)$$

If we are dealing with ammonia, and conditions are such as ordinarily obtain in refrigerating plants, then, in equation (31), the numerator and T_1 will be practically equal. But, C_p for ammonia vapor is approximately 0.53; hence, the coefficient of performance for the ammonia machine, as expressed by equation (29), is approximately double that for the air machine, as expressed by equation (30).

From the foregoing discussion it is obvious that, due to its enormous bulk, and consequent mechanical losses, together with its thermodynamic inferiority, the air refrigerating machine is very uneconomical, both from the standpoint of first cost and operation.

193. The Kelvin Heating Machine. Before leaving the subject of refrigerating machines, it will be interesting to consider a heat engine running reversed as a warming machine. This was suggested as early as 1852 by Lord Kelvin. To illustrate this, let it be desired to maintain the temperature of a room higher than that of the surrounding atmosphere. This may be brought about by the direct application of heat, or else by a heat engine running reversed. Assume the heating to be brought about by an air refrigerating machine, such as discussed in *Art.* 188, then during the aspirating stroke a charge of air flows into the compression cylinder at a temperature T_a. This charge is now compressed to a temperature T_1 and expelled into pipes, placed in the room which it is desired to heat, where heat is abstracted. After cooling, the air does work in the expansion cylinder and is expelled to the atmosphere. For a reversible engine, the heat rejected to the room is equal to the heat taken in from the atmosphere plus the heat equivalent of the work done on the air. If H_1 is the heat rejected to the room, and H_a the heat taken in from the atmosphere, then

$$H_1 = H_a + AW;$$

and

$$W = JH_1\frac{T_1 - T_a}{T_1}.$$

If, now, $T_1 - T_a$, the required range, be small, then the heat equivalent of W will be a small fractional part of H_1.

To illustrate further: Assume a situation where it is impossible to obtain fuel of any kind, but that there is available energy in the form of an electric current. Heating may then be brought about in two ways. That is, heat may be developed by passing the current through a suitable resistance, or else, the energy may be consumed in driving an electric motor, which in turn drives some form of reversed heat engine. In either case, the energy spent per unit time, due to the current consumed, is given by the product of *e.m.f.* and current. To make a simple comparison it will be necessary to assume certain conditions. Let the temperature of the atmosphere be 0°F., and that required in the heating coils, so as to maintain the room at a proper temperature, be 165°F. Now, to bring about equal heating effects, the heat dissipated per unit time must be the same in each case. Let H be the heat required per unit time, then

$$AEI_1 = H; \quad . \; . \; . \; . \; . \; . \; . \; . \quad (32)$$

where E is the applied *e.m.f.* and I_1 the current consumed, when the heating is brought about by means of resistances. Assume now, a perfect electric motor driving a perfect warming machine. The power consumed to bring about the same heating effects, for the given temperatures, is

$$AEI_2 = H\frac{T_1 - T_a}{T_1} = \frac{33}{125}H, \quad . \; . \; . \; . \; . \quad (33)$$

where I_2 is the current consumed by the motor. Solving by means of equations (32) and (33), for I_2, we find

$$I_2 = \frac{33I_1}{125};$$

showing that for a commercial efficiency even as low as 26.4 per cent, the warming machine is thermodynamically equal to the direct method. And for efficiencics higher than 26.4 per cent, the warming machine is thermodynamically superior.

CHAPTER XVIII

STEAM TURBINES

194. The detailed descriptions of the various types of *steam turbines* and the attendant mathematical discussions require an extended treatise. For such a treatise the reader is referred to Dr. A. Stodola's classical work, "Die Dampf-turbinen."* No attempt will here be made to do anything further than lay down the most elementary principles, so as to enable the student to take up reading matter, on the subject, of an advanced nature.

In steam turbines, as well as in water turbines, there are *impulse turbines* and *reaction turbines*. However, in the case of water wheels, the types most frequently used are single stage; i.e., one stationary part, which carries the *guides*, by means of which the water is given the proper direction before entering the wheel, and one rotating part, carrying a number of *curved blades*, by means of which the energy stored in the water due to pressure and velocity, is absorbed. On the other hand steam turbines must be multi-stage, i.e., consist of a number of fixed parts called *guides*, and a number of rotating wheels, called *rotors;* otherwise the speed would be impracticably high. Fig. 57 is a diagrammatic representation for two stages of a multi-stage turbine.

195. Impact on Curved Surfaces. Before proceeding to make any mathematical discussions for steam turbines, it will be well to consider a few cases for a non-expansive fluid, such as water, impinging on curved surfaces. Let, in Fig. 58, *abc* represent the section of a curved blade, having impinging upon it a stream of water with a velocity, relative to the earth's surface, represented

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* Translated by L. C. Loewenstein.

in magnitude and direction by *ha*. This velocity is briefly designated as *absolute* velocity. The line *ad* represents in magnitude and direction the velocity of the blade. Now, while a particle of water starting from *a* moves to *e*, a distance equal to V_1, the tip of the blade *a* has suffered a displacement v_1, as represented by *ad*. Hence the velocity of the water V_r, relative to the blade, is given by *de*. Therefore, the absolute velocity at entrance is equal to the *vector sum* of the velocity of the tip of the blade, at entrance, and the relative velocity.

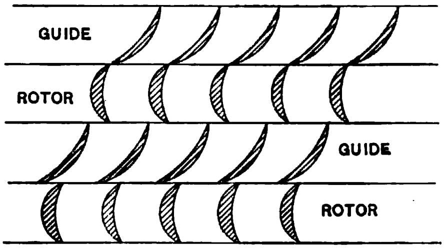


FIG. 57.

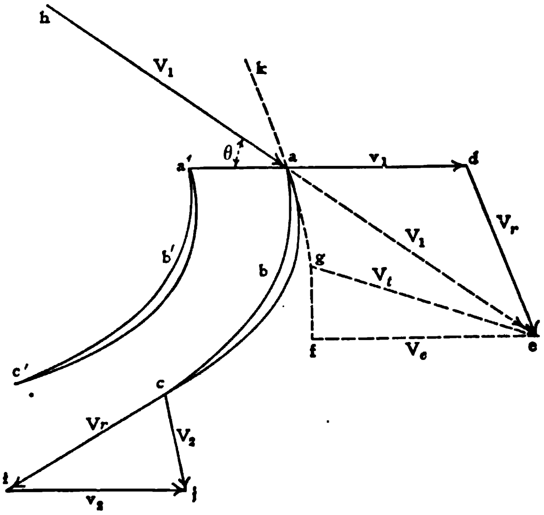


FIG. 58.

If the water is to *glide onto* the blade, so that there shall be no *shock*, the tangent, to the tip of the blade at *a*, must be parallel, as shown by *ka*, to the relative velocity V_r. If, now, there is experienced no friction by the water as it *glides along* the blade, and there is no sudden change in direction, then the magnitude of V_r will not change; and the water will leave the blade, parallel to the tip at exit, as shown by *ci*. If v_2 is the velocity of the tip at exit, then the *vector sum* of V_r and v_2, gives for the absolute velocity at exit, V_2, as shown by *cj*. We then have, respectively, for the triangles of velocities at entrance and exit, *ade* and *cij*. If, from *a* we draw *ag* equal and parallel to V_2, and close the triangle by *eg*, we find V_t, the vector difference between V_1 and V_2; i.e., the total change of absolute velocity. Resolving V_t into two components, one normal to the motion of the blade and the other parallel to the motion, we find V_e, represented by *ef*, the total change of absolute velocity in the direction of motion. V_e may be called the effective component, since this is the one producing the motion. From the diagram, it is obvious that the absolute velocity at entrance cannot be parallel to the motion of the blade, but must make some angle θ with it; otherwise entrance into the channel, included between *abc* and *a'b'c'*, cannot take place. Similarly, at exit, there must be a normal component to carry the water away, so as not to interfere with the following blade. Furthermore, it is essential that the two surfaces of the blade tips, both at entrance and exit, come to a point and have, practically, a common tangent, parallel to the relative velocity, so that the following blade, *a'b'c'*, may *glide into* the stream without shock. Unless this is so, there will be a loss in efficiency; for, whenever a stream of water impinges upon a surface with shock, there are developed eddy currents which, when subsiding, develop heat and the energy thus consumed is dissipated to the surroundings.

To find the theoretical efficiency of a system of blades, as depicted in Fig. 58, it is only necessary to take the ratio of change in kinetic energy, in passing through the channel, to the kinetic energy at entrance. Let M be the mass of water, per unit time, passing through the channel; then the kinetic energy, at entrance, is

$$W_1=\frac{MV_1^2}{2}. \quad \ldots \ldots \quad (1)$$

The kinetic energy at exit is

$$W_2=\frac{MV_2^2}{2}; \quad \ldots \ldots \quad (2)$$

and the energy given up to the system is

$$W_1-W_2=W_3=\frac{M}{2}(V_1^2-V_2^2). \quad \ldots \quad (3)$$

Hence, the efficiency is given by

$$\eta=\frac{W_1}{W_3}=\frac{V_1^2-V_2^2}{V_1^2}. \quad \ldots \ldots \quad (4)$$

Equation (4) shows that V_2 should be as small as possible; which means that it must be normal to the direction of motion of the blades and just sufficient to carry the required quantity of water away from the channel.

We may consider this in another manner. Since force is numerically equal to rate of change of momentum, the effective force, in producing motion, is

$$F=MV_e; \quad \ldots \ldots \quad (5)$$

where V_e is the change of absolute velocity in the direction of motion. And the power developed, since power is numerically equal to the product of force and speed, is

$$P_2=MV_ev_1; \quad \ldots \ldots \quad (6)$$

where P_2 is the power developed. Equation (6) assumes v_1 and v_2 numerically equal. The power of the stream before impinging, since M is the mass of water which passes through the channel per unit time, is

$$P_1=\frac{MV_1^2}{2}; \quad \ldots \ldots \quad (7)$$

where P_1 is the power delivered by the stream. Taking the ratio of P_2 to P_1, we find for the efficiency,

$$\eta=\frac{P_2}{P_1}=\frac{2V_e v_1}{V_1^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (8)$$

Equation (8) again shows, that if V_1 and v_1 are fixed, the efficiency is a maximum when V_e is a maximum; i.e., the velocity component normal to the direction of motion is as small as possible.

196. The Pelton Cup. Pelton wheels may be taken as representing impulse turbines in the case of hydraulic motors. One of the cups, as used in this type of wheel, is represented diagrammatically in Fig. 59. V_1 is the absolute velocity of the entering jet, and v_1 the velocity of the cup, whose section is represented by *abc*. The velocity of the water relative to the cup is given by

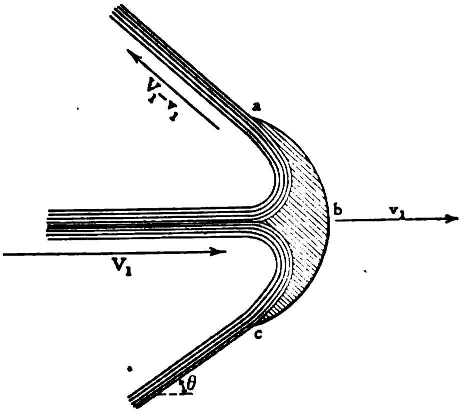


FIG. 59.

$$V_r=V_1-v_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (9)$$

Since the direction of motion of the water at exit makes an angle θ with the direction of motion of the cup, the component of the velocity of the stream at exit, parallel to the motion of the cup, is given by

$$V_e=(V_1-v_1) \cos \theta. \quad . \quad . \quad . \quad . \quad . \qquad (10)$$

And the absolute velocity, parallel to the direction of motion of the cup, at exit, is

$$V_2' = v_1 - (V_1 - v_1) \cos \theta. \quad . \quad . \quad . \quad . \quad . \quad (11)$$

The change in absolute velocity in the direction of motion, therefore, is

$$V_a = V_1 - V_2' = V_1 - \{v_1 - (V_1 - v_1) \cos \theta\}$$
$$= (V_1 - v_1)(1 + \cos \theta). \quad . \quad . \quad (12)$$

If M is the mass of water, per unit of time, impinging on the cup, then, the force moving the cup is numerically equal to the product of change in velocity and mass; hence we find for the force acting,

$$F = M(V_1 - v_1)(1 + \cos \theta). \quad . \quad . \quad . \quad . \quad . \quad (13)$$

Finally, since power is numerically equal to product of force and speed, we have, by multiplying both sides of equation (13) by v_1, for the power developed by the cup,

$$P_2 = Fv_1 = Mv_1(V_1 - v_1)(1 + \cos \theta). \quad . \quad . \quad . \quad . \quad (14)$$

It is obvious, from Fig. 59, that the motion of the water cannot be completely reversed; i.e., the direction of motion of the water leaving the cup must be inclined to the direction of motion of the cup. For, otherwise, the stream at exit will interfere with the forward motion of the following cup.

By assuming, in equation (14), the power and the velocity of the cup variable, and the other quantities constant, we find, by differentiating for a maximum,

$$\frac{dP}{dv} = M(V_1 - 2v)(1 + \cos \theta) = 0;$$

from which

$$v = \frac{V_1}{2}.$$

That is, for maximum power, the velocity of the cup must be one-half the velocity of the stream.

Substituting in equation (14), this value for v_1, we find, for the maximum power developed by the cup,

$$P_2 = M\frac{V_1}{2}\left(V_1 - \frac{V_1}{2}\right)(1+\cos\theta) = \frac{MV_1^2}{4}(1+\cos\theta). \quad . \quad (15)$$

Since the energy of the impinging jet is

$$P_1 = \frac{MV_1^2}{2},$$

the efficiency of the cup becomes,

$$\eta = \frac{P_2}{P_1} = \frac{V_1^2(1+\cos\theta)}{2V_1^2} = \frac{1+\cos\theta}{2}. \quad . \quad . \quad . \quad (16)$$

We may deduce the expression for the efficiency of the cup by considering the kinetic energy at entrance and exit. By an inspection of Fig. 59, it is obvious that the absolute velocity at exit is the vector sum of $(V_1 - v_1)$ and v_1. Designating the absolute velocity at exit by V_2, we find

$$\begin{aligned} V_2^2 &= (V_1 - v_1)^2 + v_1^2 - 2(V_1 - v_1)v_1 \cos\theta \\ &= V_1^2 - 2(V_1v_1 - v_1^2)(1+\cos\theta). \end{aligned}$$

And the efficiency becomes, since it is given by the ratio of the kinetic energy absorbed by the cup, to the kinetic energy of the stream,

$$\eta = \frac{2(V_1v_1 - v_1^2)(1+\cos\theta)}{V_1^2}.$$

Assuming V_1 and θ constant, and the velocity of the cup and the efficiency variable, the expression for the efficiency becomes,

$$\eta = \frac{2(V_1v - v^2)(1+\cos\theta)}{V_1^2},$$

and solving this for a maximum, we find

$$\frac{d\eta}{dv} = \frac{2(V_1 - 2v)(1+\cos\theta)}{V_1^2} = 0;$$

from which

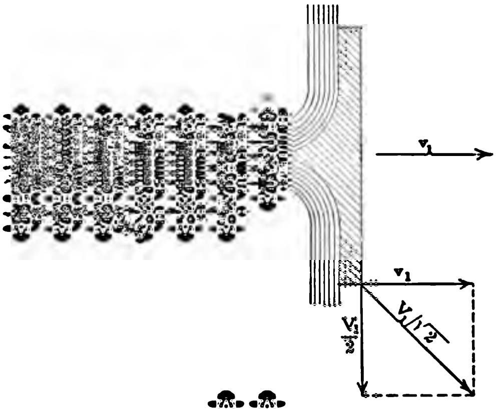
$$v = \frac{V_1}{2}.$$

[illegible] equation for efficiency, we find [illegible]

[illegible] θ [illegible];

[illegible]).

[illegible] angle θ equals zero, so that the [illegible] reversed, the efficiency equals [illegible] necessarily so, since for max- [illegible] water is brought to rest; and [illegible] before impact, is completely [illegible] as previously stated, it is



[illegible]

[illegible] to zero, on account of inter- [illegible] Assume the angle θ to vary [illegible] and 90°. Then, as just stated, [illegible] unity; and for the angle $\theta=90°$, [illegible] equals 90°, the result is equiv- [illegible] shock, as represented in Fig. 60. [illegible] impact, then, since for maxi- [illegible] cup equals $V_1/2$, the absolute [illegible], is equal to $V_1/\sqrt{2}$. Therefore, since the efficiency is equal to the ratio of the kinetic energy absorbed by the cup to the kinetic energy of the stream, before impact, we find

$$\eta=\frac{V_1^2-V_1^2/2}{V_1^2}=\frac{1}{2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

Hence, theoretically, the efficiency of an impulse turbine may vary between 50 and 100 per cent; depending upon the value of the angle θ. In practice it is attempted to have the angle θ just sufficiently large so that the stream, leaving the cup, does not interfere with the following cup. In properly designed impulse wheels the actual efficiency may be, considering all losses, as high as 90 per cent. The results deduced clearly indicate, that in any case, it is essential, if a high efficiency is to be realized, to reduce the absolute velocity of the impinging stream, in going through the turbine, as nearly as possible, to zero. And this, in the case of *steam turbines*, is just as necessary a prerequisite for high efficiencies, as it is in the case of water turbines.

The question, why is it possible, in the case of hydraulic motors, to convert so large a fractional part of the theoretical energy, due to the difference in topographical level, into actual work, and in the case of heat motors, so small a fractional part of the energy, due to the difference in "temperature level," naturally suggests itself. The answer is obvious. Every heat motor must act periodically. Even though the identical working substance is not used in the succeeding cycle, the result is just the same. For, the condition of the working substance, for the best results, must be at the beginning of each cycle the same as it was at the end of the preceding cycle. This is equivalent to cyclic operation. In the case of the hydraulic motor, however, the cycle is only partially completed. That is, the water, after having performed work, in falling through a certain height, is restored to its original condition by the action of the sun, which completes the cycle automatically. In other words, the water at the height H_1, of the headrace, falls to the height H_2, of the tailrace, and performs, theoretically, the amount of work

$$w(H_1-H_2);$$

where H_1 is the height of the headrace, H_2 the height of the tailrace, and w the weight of water. But, to complete the cycle, the water must again be raised from the level H_2 to H_1. This is done at the expense of the radiant heat from the sun, by means of which the water from streams, lakes, the oceans, etc., is evaporated and carried, by means of convection currents, to higher elevations, where condensation takes place, and the difference in elevation, H_1-H_2, is again established.

197. Flow of Fluids in Pipes of Varying Section (De Laval Nozzle). The flow of a gas or vapor, under steady conditions, through a pipe of varying cross-section, is very similar to the flow of a liquid under similar conditions; but, the flow of a gas differs materially from that of a non-compressible liquid in two respects. That is, in general, the weight of the gas, or statical head, is negligibly small in comparison with the pressure head and velocity head; but, on the other hand, account must be taken of the expansion.

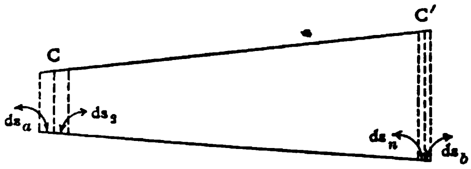


FIG. 61.

Consider a pipe CC', such as is represented in Fig. 61, and assume a steady flow, i.e., the mass of gas entering the section at C, for a given interval of time, is equal to the mass leaving the section at C', during the same interval of time. And further, that the gas flows, without friction, in straight stream lines. Let m be the mass of gas that enters the channel at C, per unit time, with a speed s_a, and pressure p_a, then an equal mass will leave the channel at C', during the same interval of time, with some speed s_b, and pressure p_b. Assume the pipe CC', to be divided into an indefinitely large number of sections, such that the thickness of each section is indefinitely small. Let the pressures on the left-hand side of the various sections, be, respectively,

$$p_a, p_2, p_3, \ldots p_{n-1}, p_n;$$

and on the right-hand side of the sections,

$$p_2, p_3, p_4, \ldots p_n, p_b.$$

Representing the respective cross-sectional areas by

$$A_a, A_2, A_3, \ldots A_n, A_b,$$

and the corresponding thicknesses of the elements by

$$ds_a, ds_2, ds_3, \ldots ds_n,$$

then the work done by the positive pressures, during the time that the displacement ds_a takes place at C, and the displacement ds_n takes place at C', is

$$W' = p_aA_ads_a + p_2A_2ds_2 + \cdots + p_{n-1}A_{n-1}ds_{n-1} + p_nA_nds_n. \quad . \quad (18)$$

Likewise, the work done by the back pressures is

$$W'' = -p_2A_2ds_2 - p_3A_3ds_3 - \cdots - p_nA_nds_n - p_bA_bds_b. \quad . \quad . \quad (19)$$

Taking the sum, of equations (18) and (19), we find, for the net work done, due to change in pressure,

$$W_1 = W' + W'' = p_aA_ads_a - p_bA_bds_b. \quad . \quad . \quad . \quad (20)$$

Changing, in equation (20), the subscripts a and b to 1 and 2, we have

$$W_1 = p_1A_1ds_1 - p_2A_2ds_2. \quad . \quad . \quad . \quad . \quad . \quad (21)$$

If ds_1 is the distance passed through, at C, in the time dt, and ds_2 the distance passed through at C' in the same interval of time, then

$$ds_1 = s_1 dt,$$

and

$$ds_2 = s_2 dt;$$

where s_1 and s_2 are, respectively, the initial and final speeds. Substituting, in equation (21), these values of ds_1 and ds_2, we obtain

$$W_1 = (p_1 A_1 s_1 - p_2 A_2 s_2) dt. \quad . \; . \; . \; . \; . \quad (22)$$

Since, m is the mass of gas, flowing per unit time, we have, for steady flow,

$$m = \frac{A_1 s_1}{v_1} = \frac{A_2 s_2}{v_2};$$

where v_1 and v_2 are, respectively, the volumes per unit mass of the gas corresponding to the pressures p_1 and p_2. Substituting in equation (22), we obtain

$$W_1 = (p_1 v_1 - p_2 v_2) m dt. \; . \; . \; . \; . \; . \; . \quad (23)$$

The change in kinetic energy is

$$W_2' = \frac{s_1^2 - s_2^2}{2} m dt;$$

and expressed in engineer's units, this becomes

$$W_2 = \frac{s_1^2 - s_2^2}{2g} m dt. \quad . \; . \; . \; . \; . \; . \; . \; . \quad (24)$$

The work due to expansion is,

$$W_3 = (m dt) \int_{v_1}^{v_2} p dv; \quad . \; . \; . \; . \; . \; . \quad (25)$$

and for the assumed conditions,

$$W_1 + W_2 + W_3 = 0.$$

Hence,

$$(p_1v_1-p_2v_2)mdt+\frac{s_1^2-s_2^2}{2g}mdt+(mdt)\int_{v_1}^{v_2}pdv=0;$$

from which

$$\frac{s_2^2-s_1^2}{2g}=p_1v_1-p_2v_2+\int_{v_1}^{v_2}pdv. \quad . \quad . \quad . \quad . \quad (26)$$

But,

$$p_1v_1-p_2v_2+\int_{v_1}^{v_2}pdv=\int_{p_2}^{p_1}vdp;$$

hence,

$$\frac{s_2^2-s_1^2}{2g}=\int_{p_2}^{p_1}vdp. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

If, now, the process be such that the equation

$$pv^n=k$$

holds, then $v=k^{\frac{1}{n}}p^{-\frac{1}{n}}$; and by substitution, equation (27) becomes

$$\frac{s_2^2-s_1^2}{2g}=k^{\frac{1}{n}}\int_{p_2}^{p_1}p^{-\frac{1}{n}}dp=\frac{n}{n-1}k^{\frac{1}{n}}(p_1^{\frac{n-1}{n}}-p_2^{\frac{n-1}{n}}). \quad . \quad (28)$$

Substituting for $k^{\frac{1}{n}}$ its value, we find

$$\frac{s_2^2-s_1^2}{2g}=\frac{n}{n-1}p_1^{\frac{1}{n}}v_1(p_1^{\frac{n-1}{n}}-p_2^{\frac{n-1}{n}})=\frac{n}{n-1}p_1v_1\left\{1-\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right\} \quad (29)$$

Or, since

$$k^{\frac{1}{n}}=p_1^{\frac{1}{n}}v_1=p_2^{\frac{1}{n}}v_2,$$

equation (28) may be simplified by dividing by $k^{\frac{1}{n}}$, and multiplying the first term in the parenthesis by $p_1^{\frac{1}{n}}v_1$, and the second term by $p_2^{\frac{1}{n}}v_2$. Performing this operation we find

$$\frac{s_2^2-s_1^2}{2g}=\frac{n}{n-1}(p_1v_1-p_2v_2). \quad . \quad . \quad . \quad . \quad (30)$$

If the pipe is curved, then the pressure on the convex surface is less than on the concave surface. However, unless the change in direction is considerable, the difference in pressure is very small.

The foregoing conclusions are the result of a modification of of Bernouilli's theorem; i.e., applying the theorem to a compressible fluid of negligible weight.

If the initial speed is negligible, as is the case when discharge takes place from a comparatively large vessel, then s_1, in equation (29), may be omitted, and we have

$$\frac{s_2^2}{2g}=\frac{n}{n-1}p_1v_1\left\{1-\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right\}. \quad . \quad . \quad . \quad . \quad . \qquad (31)$$

Solving for s_2, we find

$$s_2=\left[\frac{2gn}{n-1}p_1v_1\left\{1-\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right\}\right]^{\frac{1}{2}}. \quad . \quad . \quad . \qquad (32)$$

Now, the mass of fluid conveyed, per unit time, through any section is

$$m=\frac{As}{v};$$

where A is the area of the section, s the speed, and v the volume per unit mass. But for steady flow this is a constant throughout the pipe. Hence we have

$$m=\frac{As}{v}=\frac{A_2s_2}{v_2}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (33)$$

where A_2 is the area corresponding to the pressure p_2, and v_2 the corresponding volume per unit mass. By combining equations (32) and (33) we find

$$m=\frac{A_2s_2}{v_2}=\frac{A_2}{v_2}\left[\frac{2gn}{n-1}p_1v_1\left\{1-\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right\}\right]^{\frac{1}{2}}. \quad . \quad . \qquad (34)$$

Assuming again, the flow to be such that the equation

$$pv^n=p_1v_1^n. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (35)$$

holds, we find

$$v_2 = \left(\frac{p_1}{p_2}\right)^{\frac{1}{n}} v_1.$$

Substituting this value of v_2 in equation (34), we find

$$m = A_2\left[\frac{2gn}{n-1}\frac{p_1}{v_1}\left\{\left(\frac{p_2}{p_1}\right)^{\frac{2}{n}} - \left(\frac{p_2}{p_1}\right)^{\frac{n+1}{n}}\right\}\right]^{\frac{1}{2}}. \quad . \quad . \quad (36)$$

Since, as previously stated, m is constant, equation (34) may be written

$$m = \frac{As}{v} = A\left[\frac{2gn}{n-1}\frac{p_1}{v_1}\left\{\left(\frac{p}{p_1}\right)^{\frac{2}{n}} - \left(\frac{p}{p_1}\right)^{\frac{n+1}{n}}\right\}\right]^{\frac{1}{2}}; \quad . \quad (37)$$

where A is any section, p the corresponding pressure, and v the corresponding volume per unit mass. Since As/v is a constant, s/v must be a maximum, when A is a minimum. But, s/v becomes a maximum, and hence, A a minimum, when the value of the expression included in the bracket, of equation (37), becomes a maximum. Hence, we may write

$$\frac{s}{v} = Kx = \left(\frac{p}{p_1}\right)^{\frac{2}{n}} - \left(\frac{p}{p_1}\right)^{\frac{n+1}{n}};$$

and

$$K\frac{dx}{dp} = \frac{2}{n}\frac{p^{\frac{2-n}{n}}}{p_1^{\frac{2}{n}}} - \frac{n+1}{n}\frac{p^{\frac{1}{n}}}{p_1^{\frac{n+1}{n}}} = 0;$$

from which

$$p = p_1\left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

The value of p, as given by equation (38), is that value which makes s/v a maximum, and, therefore, A a minimum. Substituting this value of p, for p_2, in equation (32), and reducing, we find

$$s = \left(\frac{2gn}{n+1}p_1v_1\right)^{\frac{1}{2}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (39)$$

Finally, by means of equations (35), (38), and (39), we find

$$m=\frac{As}{v}=\frac{A}{v}\left(\frac{2gn}{n+1}p_1v_1\right)^{\frac{1}{2}}=A\left\{\frac{2gn}{n+1}\,\frac{p_1}{v_1}\left(\frac{2}{n+1}\right)^{\frac{2}{n-1}}\right\}^{\frac{1}{2}}. \quad (40)$$

Equating the right-hand members of equations (36) and (40), we find

$$\frac{A_2}{A}=\left[\frac{\frac{n-1}{n+1}\left(\frac{2}{n+1}\right)^{\frac{2}{n-1}}}{\left(\frac{p_2}{p_1}\right)^{\frac{2}{n}}-\left(\frac{p_2}{p_1}\right)^{\frac{n+1}{n}}}\right]^{\frac{1}{2}}; \quad \dots\dots \quad (41)$$

which gives the ratio of the area, for the pressure p_2, to minimum area.

Solving equation (38), on the assumption that the fluid is saturated steam, for which n equals 1.135, we find

$$p=0.577p_1.$$

By means of equation (32), the final speed may be determined, when the initial and final pressures are known; and with the aid of equation (37) any one of the three quantities, viz, m, A, and p, may be found, if two of them are given.

Since, in deducing the foregoing equations, we equated work, expressed by the product of pressure and volume, against energy, expressed by the product of mass and square of the speed, we must in substituting numerical values, in these equations, use the same system of units. As an example, if in equation (39), s is to be given in feet per second, p_1 must be given in lbs. per square foot, and v_1 in cubic feet. Reducing equation (39), so that p_1 is expressed in lbs. per square inch, and substituting for g and n the proper values, we find

$$s=\left(\frac{2\times32.2\times1.135}{2.135}\times144p_1v_1\right)^{\frac{1}{2}}=70.2(p_1v_1)^{\frac{1}{2}}.$$

For steam under a pressure of 100 lbs. per square inch, the volume per pound is approximately 4.43 cu.ft.; hence, by substitution, we find

$$s=70.2(443)^{\frac{1}{2}}=1478 \text{ ft. per sec.}$$

If the pressure be 200 lbs. per square inch, for which the volume, per pound, is approximately 2.29 cu.ft., we find

$$s=70.2(458)^{\frac{1}{2}}=1503 \text{ ft. per sec.}$$

These two computations show that the variation in speed is small when compared with the variation in pressure.

Reducing equation (40) in a similar manner, we find

$$m=\frac{A}{144}\left\{\frac{2\times 32.2\times 1.135}{2.135}\left(\frac{2}{2.135}\right)^{\frac{2}{0.135}}\times 144\frac{p_1}{v_1}\right\}^{\frac{1}{2}}$$

$$=0.30A\left(\frac{p_1}{v_1}\right)^{\frac{1}{2}} \text{ pounds per sec.};$$

where A is now in square inches.

198. Two Principal Types of Turbines. From the previous discussions on the flow of steam through pipes it is obvious that the speed of flow, for any considerable difference in pressure, is very high; and that if any single-stage turbine, i.e., a turbine consisting of a set of nozzles and only one rotating part, were to utilize practically all the kinetic energy of the steam, due to its speed at nozzle exit, the speed of the turbine would have to be abnormally high. As an example, some of the De Laval turbines, which were single stage, had speeds as high as 40,000 r.p.m. Though the efficiencies of the De Laval turbines, from the standpoint of steam consumption, were not exceptionally low, the enormously high speeds were a serious disadvantage. For, in no other mechanical contrivance, not even dynamo electric machines, which are operated at relatively high speeds in comparison with other machines, are such high speeds ever approached.

Therefore, to utilize the power developed by a single-stage turbine it is necessary to employ a reduction gear. But, a reduction gear means an additional first cost, and a lowering of mechanical efficiency. Furthermore, proper lubrication becomes exceedingly difficult when machines are operated under speeds such as are attained by single-stage turbines.

The difficulties, however, stated in the preceding paragraph, were overcome by the introduction of *multi-stage turbines.** That is, by allowing the steam to act successively upon the rotors of a multi-stage turbine, its speed is gradually reduced, and the speed of the turbine need not be abnormally high. The turbine must of course be so designed that the steam expands, and the temperature is reduced continuously to the lowest possible value at exit. That is, the kinetic energy of the steam at exit must be, as nearly as possible, equal to zero.

There is then the choice of the following types of multi-stage turbines: Combined impulse and reaction, and impulse.

199. The Parsons Turbine. *The Parsons turbine*, at the present time, represents one of the commercially successful types of turbines; and may be considered a combined impulse and reaction turbine of the parallel-flow type. That is, the steam passes through the first set of guide blades approximately parallel to the shaft of the turbine, and has given to it the proper direction so that it may enter into the channels of the first rotor without shock. At exit from the first rotor, the steam enters a second set of guide blades, where it is again directed so as to properly enter the channels of the second rotor, etc. In this way the steam reacts, expands, and falls in pressure continuously as it travels, from

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* There appears to be considerable confusion in regard to the meaning of the word "stage." In some cases authors designate a turbine, as an n-stage turbine when there are n rotors, which is consistent with the nomenclature employed in the case of hydraulic turbines. In other cases, however, namely the Curtis turbine, by number of stages is meant the number of sets of expanding nozzles.

entrance to exit, through the turbine. Since the steam is continually expanding, the length of the blades and spacing, for both guides and rotors, must be increased so that the ratio of steam speed and blade speed, upon which the efficiency of the turbine depends, is maintained constant.

200. The Curtis Turbine. The *Curtis turbine* is of the impulse type. The steam expands in a set of nozzles, where the pressure head is converted into velocity head, and then impinges on the curved blades of a rotor. Part of the kinetic energy of the steam is absorbed by the first rotor; the steam then at reduced speed passes through a set of guide blades where it is directed so as to properly enter the channels of a second rotor, where the speed is still further reduced, etc., until, finally, the speed is very low. The steam is then expanded through a second set of nozzles, and again passes through a series of rotors and guides, precisely as in the first stage. This is continued until the pressure of the steam has been reduced to the desired exhaust pressure. The number of stages, other things being equal, depends, of course, upon the range in pressure. Due to the fact that the speed of the steam is reduced in each rotor, the passages traversed by the steam must be continuously enlarged. This is brought about by reducing the curvature of the blades as well as lengthening them.*

201. Comparison of Parsons and Curtis Turbines. Since the speed of the steam entering a Parsons turbine is moderately low, and for high efficiency its absolute velocity at exit must approach zero in value, it follows that the relative velocity must be high. That is, the relative velocity at exit being, approximately, the vector difference between the absolute velocity at entrance and the velocity of the wheel, it follows that, since the velocity of the steam at entrance is low, the relative velocity at exit will be high, and therefore, the velocity of the wheel must be high

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* For a comprehensive discussion on the design and testing of turbines see "The Marine Steam Turbine" by J. W. Sothern.

in order that the absolute velocity at exit may be low. On the other hand, in the case of an impulse turbine, the velocity of the blade, for the best efficiency, is approximately one-half that of the entering steam. Hence it is obvious that, other things being equal, the Parsons turbine is inherently a higher speed prime mover than is the Curtis turbine.

202. Turbines and Reciprocating Engines. No matter how operated the steam turbine is inherently a high-speed prime mover; and since power is proportional to the product of torque and angular velocity, it follows that for equal output, the steam turbine, with its high rotative speed, will be of smaller dimensions than a reciprocating engine. Furthermore, where rotative motion is required, which is usually the case, the turbine needs no connecting rod and crank, as does the reciprocating engine. Again, where electric generators are direct connected, as in power plants, high speeds, up to a certain point, are desirable. Since, the power developed is equal to the product of *e.m.f.* and current, high-speed generators, for equal output, will have a lower first cost and occupy less floor space than low-speed generators. Finally, the turbine has the further mechanical advantage of having a uniform turning moment. On the other hand there are certain cases where low speeds are essential either to successful operation or economy; under such conditions the reciprocting engine is superior. As an illustration of this we may consider present conditions in marine engineering. As previously stated, the turbine, using high pressure steam is, for high efficiencies, inherently a high-speed prime mover; on the other hand the propeller of a ship, is, for high efficiencies, inherently a low-speed mechanism. On passenger liners, the increased rates, which passengers are ready to pay for a reduction of time in transit, more than pay for the increased cost of operation. However, on freight steamers, such is by no means the case; and it appears that for such steamers the reciprocating engine combined with a low-pressure turbine, as regards economy, is at least equal if not superior to the turbine.

Thermodynamically, the steam turbine is far superior to the reciprocating engine. For, in the turbine there is no alternate heating and cooling of the surfaces with which the steam comes into intimate contact. In other words, in the case of a turbine, very shortly after starting, steady conditions will prevail; and the incoming steam, therefore, does not come into contact with surfaces which have been previously chilled by the low-temperature exhaust steam. That is, the steam changes gradually in pressure and temperature from admission to exhaust. And this means that there is no condensation excepting that due to radiation. Hence, in a turbine, condensation is largely eliminated in comparison with a reciprocating engine; and herein lies one of the great factors that makes the turbine thermodynamically superior to the reciprocating engine. Another important factor is the fact that in a turbine a good vacuum is utilized to much better advantage. This is illustrated by Fig. 62. Let, in the figure, *ABCDE* be the indicator diagram of a reciprocating engine operating between the pressures as indicated by the points *A* and *E*. Then the net work done by the engine is measured by the area *ABCDE*; and the net work realized by means of a turbine, for the same limits in pressure, is measured by the area *ABCFE*. If, now, the back pressure be reduced, from that as represented by the line

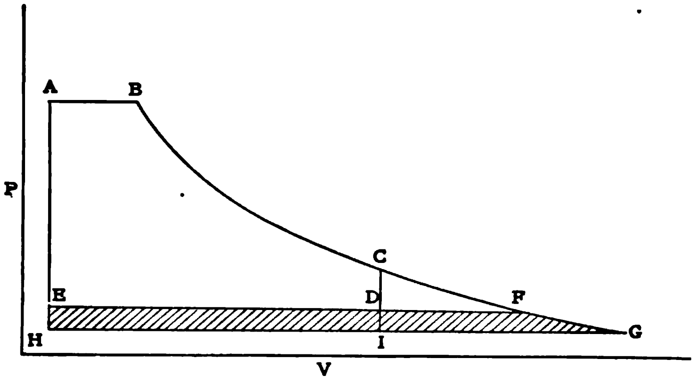

FIG. 62.

ED, to that represented by the line *HI*, the net gain in work, by means of a reciprocating engine, is measured by the area *EDIH*, and that, in the case of a turbine, by the area *EFGH*. That is, due to mechanical considerations, the length of stroke of the reciprocating engine is fixed; and hence, full expansion cannot be realized. But, in the turbine full expansion is realized and the *toe* of the indicator diagram is utilized in doing useful work. This gain in work, in the case of low-pressure turbines, is quite appreciable.

203. Turbine Tests. It has been found impossible, up to the present time, to devise any method by means of which to determine the indicated power of a turbine, in the same manner that the indicated power of a reciprocating engine is determined. There is, however, no difficulty experienced in determining the output, or brake power. The output is determined in precisely the same manner as described in *Art.* 133; and the thermal efficiency is determined as described in *Art.* 136. If it be desired to determine the commercial efficiency, the method of procedure is precisely the same as that described in *Art.* 137. The comparison frequently made is that between the actual output of the turbine, and that which would have been obtained on a Rankine cycle, as discussed in *Art.* 138.

204. Reciprocating Engines and Low-pressure Turbines. In 1906, H. G. Stott presented a paper* before the American Institute of Electrical Engineers on " Power Plant Economics," which gave a complete analysis of the various losses, from the coal bunkers to the bus-bars, of the Interborough Power Plant, located at Fifty-ninth Street and Eleventh Avenue, New York City. The prime movers employed at that time were of the Manhattan type compound Corliss engines; two engines being connected to one generator of 7500 K.W. maximum capacity.

The following quotation is an extract from Mr. Stott's paper,

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* "Power Plant Economics," Transactions of the A.I.E.E., Vol. XXV.

which is one of the most complete and instructive analyses that has ever been made of a power plant:

" Three years ago the steam-power plant for the generation of electricity had apparently settled down to an almost uniform arrangement of standard apparatus in which one power plant differed from another only in details of construction of engines, generators, and auxiliaries. As only about twenty years had then elapsed since the first central station was put in operation on a commercial basis, this uniformity of design seemed to indicate that in the near future it would only be necessary to purchase a standard set of power-plant drawings, and make the necessary changes in size of units in order to have a station of the best type known to the art.

" The internal combustion or gas engine had from time to time been brought forward as a candidate for the position of prime mover, with every prospect of improved economy in fuel consumption; but with the exception of a few special instances it was not looked upon with favor; as shown by the almost universal use of the steam engine.

" After a long period of development a new factor in power-plant design; namely, the steam turbine, was placed on the market in commercial sizes. It is safe to say that during the last three years no other piece of apparatus has had so stimulating an effect upon the power plant. Its effect upon the entire plant has been most beneficial, for it has revived the apparently moribund superheater. This has now been so developed and improved that superheat of 200° or 300° fahr. can be safely and economically obtained. With the development of the superheater further study of the problem of combustion has improved the efficiency of the furnace; and this most important subject is apparently susceptible of still further development.

" One other important result of the steam-turbine development has been the development of condensing apparatus to such a point of efficiency that a vacuum within one inch of the simultaneous barometer reading can now be maintained without difficulty.

" Another change in the power plant has been the reversion to high-speed generators, resulting in decreased cost of the generator and its foundations, as well as saving in floor space.

" Last but not least the steam turbine has put the reciprocating engine and the gas engine on the defensive and has actually been unkind enough to throw out hints in regard to the application of Dr. Osler's proposed methods to the treatment of older apparatus.

" The reciprocating engine and internal combustion engine have not been slow in accepting this challenge; they have responded by showing so improved an economy (especially in the gas engine) that the situation has become most interesting to the power-plant designer. It is safe to say that the developments of the next ten years will show very marked improvement in power plant efficiency.

" In regard to this development the author wishes to direct attention to the basic fact that in power plants one should not look merely for increased efficiency in the prime mover, but should also investigate and analyze the entire plant from the coal to the bus-bars: first, in regard to efficiency; secondly, in regard to the effect of load-factor upon investment; and thirdly, the effect of the first and second upon the total cost of producing the kilowatt-hour, which is the ultimate test of the skill of the designer and operator.

" EFFICIENCY.

" In Table 1 will be found a complete analysis of the losses found in a year's operation of what is probably one of the most efficient plants in existence to-day and, therefore, typical of the present state of the art.

" Table No. 1

ANALYSIS OF THE AVERAGE LOSSES IN THE CONVERSION OF ONE POUND OF COAL INTO ELECTRICITY.

	B.T.U.	Per Cent.	B.T.U.	Per Cent.
1. B.T.U. per pound of coal supplied	14150	100.0		
2. Loss in ashes			340	2.4
3. Loss to stack			3212	22.7
4. Loss in boiler radiation and leakage			1131	8.0
5. Returned by feed-water heater	441	3.1		
6. Returned by economizer	960	6.8		
7. Loss in pipe radiation			28	0.2
8. Delivered to circulator			223	1.6
9. Delivered to feed-pump			203	1.4
10. Loss in leakage and high-pressure drips			152	1.1
11. Delivered to small auxiliaries			51	0.4
12. Heating			31	0.2
13. Loss in engine friction			111	0.8
14. Electrical losses			36	0.3
15. Engine radiation losses			28	0.2
16. Rejected to condenser			8524	60.1
17. To house auxiliaries			29	0.2
	15551	109.9	14099	99.6
	14099	99.6		
Delivered to bus-bar	1452	10.3		

" Discussion of Data in Table 1

" *Item 1. B.t.u. per Pound of Coal Supplied.* The thermal value of the coal used is evidently of prime importance, as it affects the cost efficiency of the entire plant. The method of purchasing coal used in the plant from which this heat balance is derived is that of paying for B.t.u. only, with suitable restrictions on the maximum permissible amount of volatile matter, ash, and sulphur.

" A small sample of coal is automatically taken from each filling of the weighing hoppers, so that the final sample represents a true average of a boat-load of coal. This final average sample is then pulverized and tested for heat value in a bomb calorimeter, after which a proximate analysis is made of another portion of the sample. This method of purchasing coal has been in use for two years, with highly satisfactory results.

"*Item 2. Loss in Ashes.* It is doubtful whether a further saving in this item can be made, as the extra care and labor necessary to accomplish any improvement would in all probability offset the saving in coal.

"*Item 3. Loss to Stack.* This is one of the most vulnerable points to attack, as the loss of 22.7 per cent. is very large. Recent investigations show that promising results may be obtained by the use of more scientific methods in the boiler room. In practically all cases it will be found that this loss is due almost entirely to admitting too much air to the combustion chamber, resulting in cooling of the furnace. This result is usually produced by " holes " in the fire; these " holes " may be due to several causes, but usually are due to carelessness on the part of the fireman.

"Fortunately, a very valuable piece of apparatus has been placed upon the market in the shape of a CO_2 recording instrument. The results of a series of tests made with this instrument are shown in Figs. 63 to 66.

"Fig. 63 shows the average condition of a furnace using small sizes of anthracite, with forced draught. The conditions are such that approximately 40 per cent. of the thermal value is being lost.

"Fig. 64 shows what improvement may easily be obtained by watching the CO_2 record, and indicates a saving of about 19 per cent. over the previous case.

"In the combustion of the small sizes of anthracite it is necessary to use a draught of not less than 1.5 in. of water; this breaks the crust of the fire in the thin spots, allowing the air to come through in such volumes that an enormous amount of heat is wasted in raising the temperature of the surplus air and at the same time causing inefficient combustion in the entire furnace.

"Fig. 65 shows a record taken from a stoker boiler whilst the recorder was covered up to prevent the fireman from seeing the record.

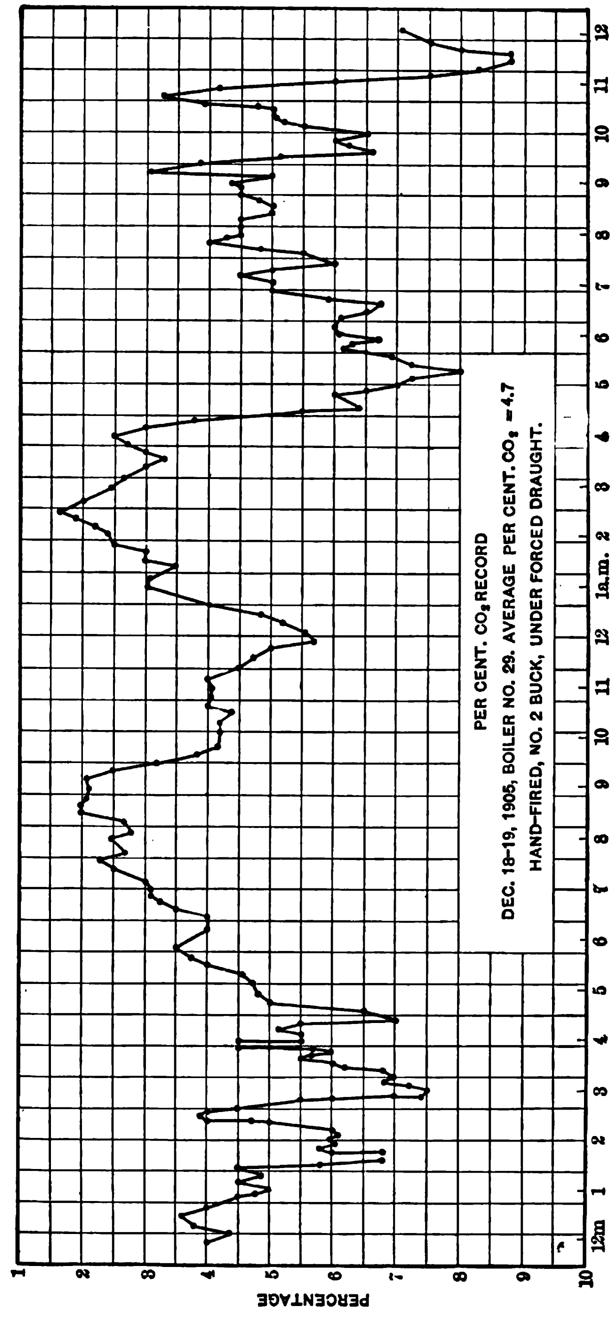
PER CENT. CO_2 RECORD
DEC. 18-19, 1905, BOILER NO. 29. AVERAGE PER CENT. CO_2 =4.7
HAND-FIRED, NO. 2 BUCK, UNDER FORCED DRAUGHT.
TIME
12m
1
2
3
4
5
6
7
8
9
10
11
12
1a.m.
2
3
4
5
6
7
8
9
10
11
12
PERCENTAGE
1
2
3
4
5
6
7
8
9
10

FIG. 63.

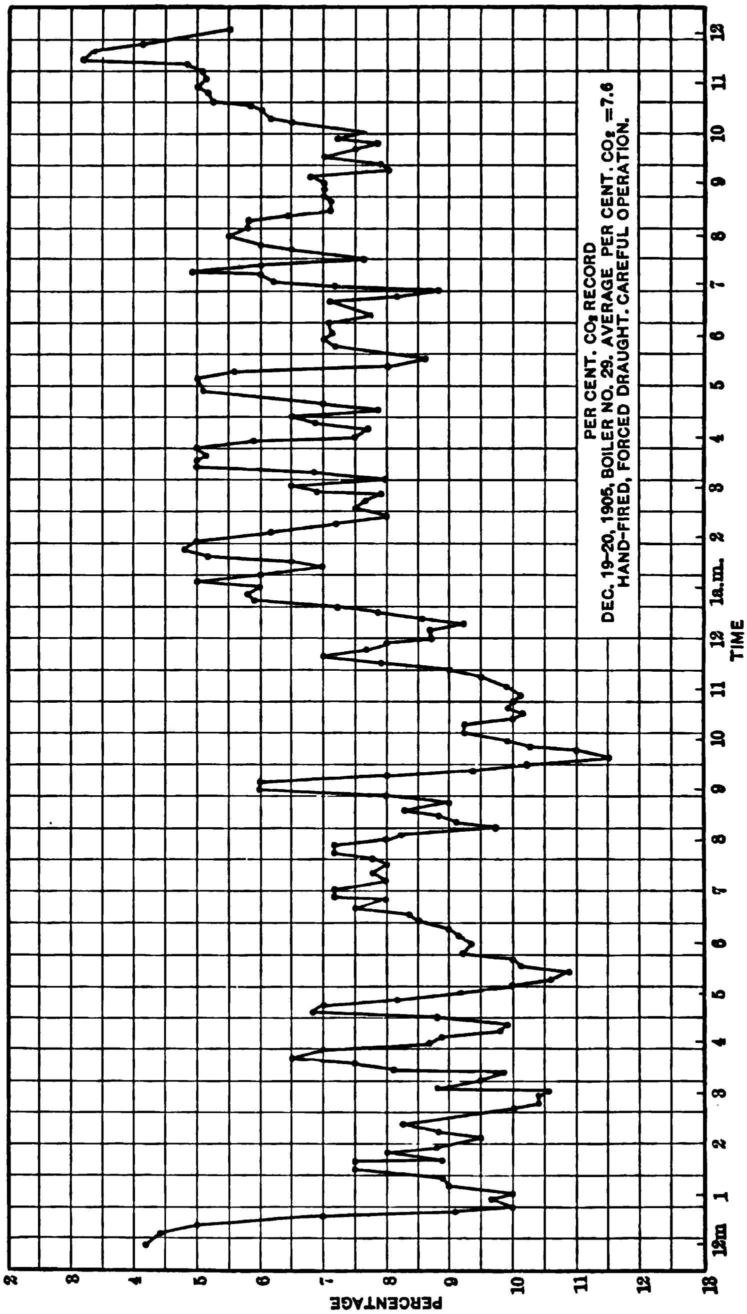
PER CENT. CO_2 RECORD
DEC. 19–20, 1905, BOILER NO. 29. AVERAGE PER CENT. CO_2 =7.6
HAND-FIRED, FORCED DRAUGHT. CAREFUL OPERATION.
PERCENTAGE
2
3
4
5
6
7
8
9
10
11
12
13
TIME
12m
1
2
3
4
5
6
7
8
9
10
11
12
1a.m.
2
3
4
5
6
7
8
9
10
11
12

FIG. 64.

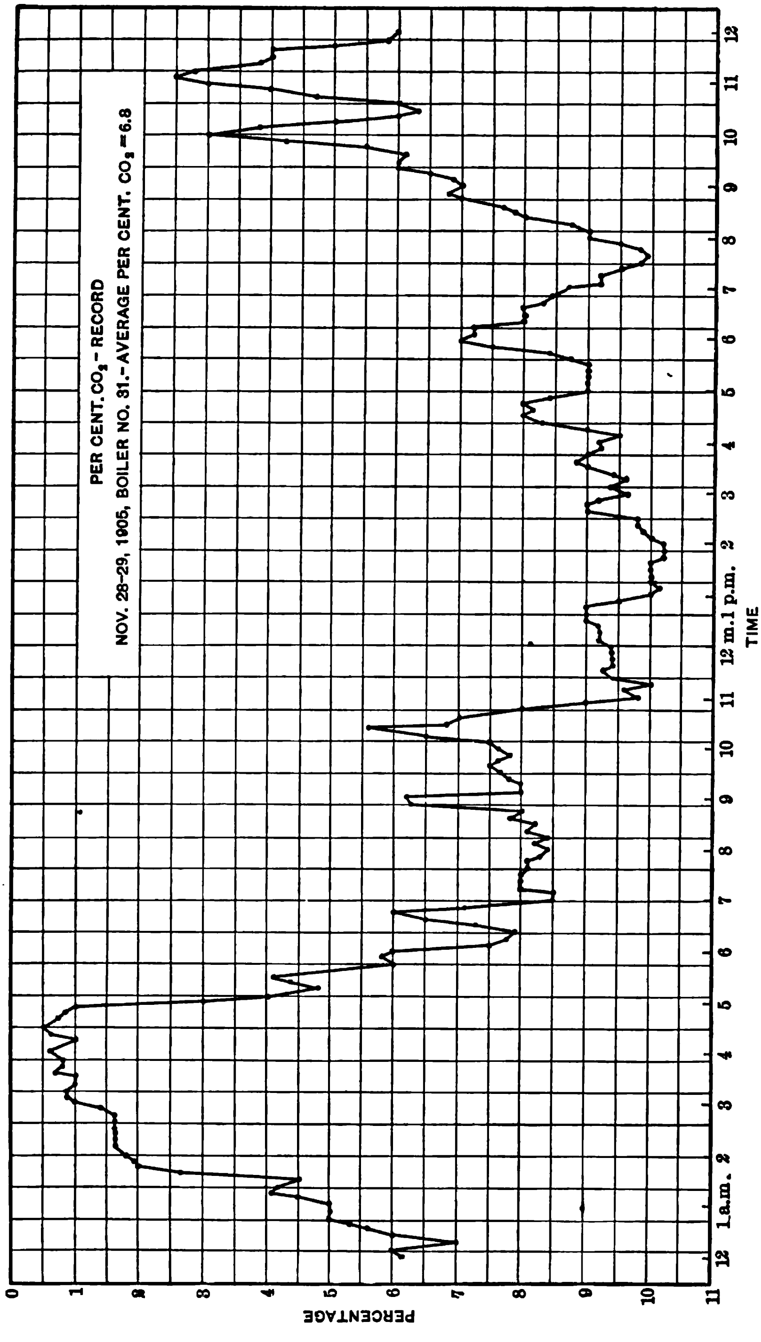
PER CENT. CO_2 - RECORD
NOV. 28-29, 1905, BOILER NO. 31.-AVERAGE PER CENT. CO_2 =6.8
PERCENTAGE
0
1
2
3
4
5
6
7
8
9
10
11
TIME
12
1 a.m.
2
3
4
5
6
7
8
9
10
11
12 m.
1 p.m.
2
3
4
5
6
7
8
9
10
11
12

FIG. 65.

"Fig. 66 shows a record taken from the same stoker boiler with the fireman watching the CO_2 indications, resulting in a saving of over 12 per cent. Later records show that even better results than an average of 11.4 per cent. of CO_2 can be obtained.

"Fig. 67 shows the calculated losses in fuel corresponding to various percentages of CO_2 for three different temperatures of flue gases.

"From a consideration of the above tests it seems reasonable to assume that the 22.7 per cent. loss to stack can, by scientific methods in the fireroom, be reduced to about 12.7 per cent. and possibly to 10 per cent.

"Before the installation of the CO_2 recorder a long series of evaporative tests was made to determine the most economical draught to carry when a high-grade semi-bituminous coal was burning on the automatic stokers. The results shown in Fig. 68 were so remarkable that they were repeated under different conditions in order to confirm them. Since the installation of the CO_2 recorder, however, the explanation is apparent; as the draught giving maximum evaporation per pound of combustible corresponds to the point of maximum CO_2, illustrating the inherent difficulty of maintaining efficient conditions in the combustion chamber with high draught. This is well illustrated by Fig. 69, showing the draught, per cent. of rating, and percentage of CO_2.

"*Item 4*. The loss in boiler radiation and leakage, amounting to 8 per cent., is largely due to the inefficient boiler setting of brick which, besides permitting radiation, admits a large amount of air by infiltration. This infiltration will increase with the draught, thus tending to exaggerate the maximum and minimum points on Fig. 68. The remedy for this radiation and infiltration loss is evidently to use new methods of boiler setting, such as an iron plate air-tight case enclosing a carbonate of magnesia lining outside the brickwork.

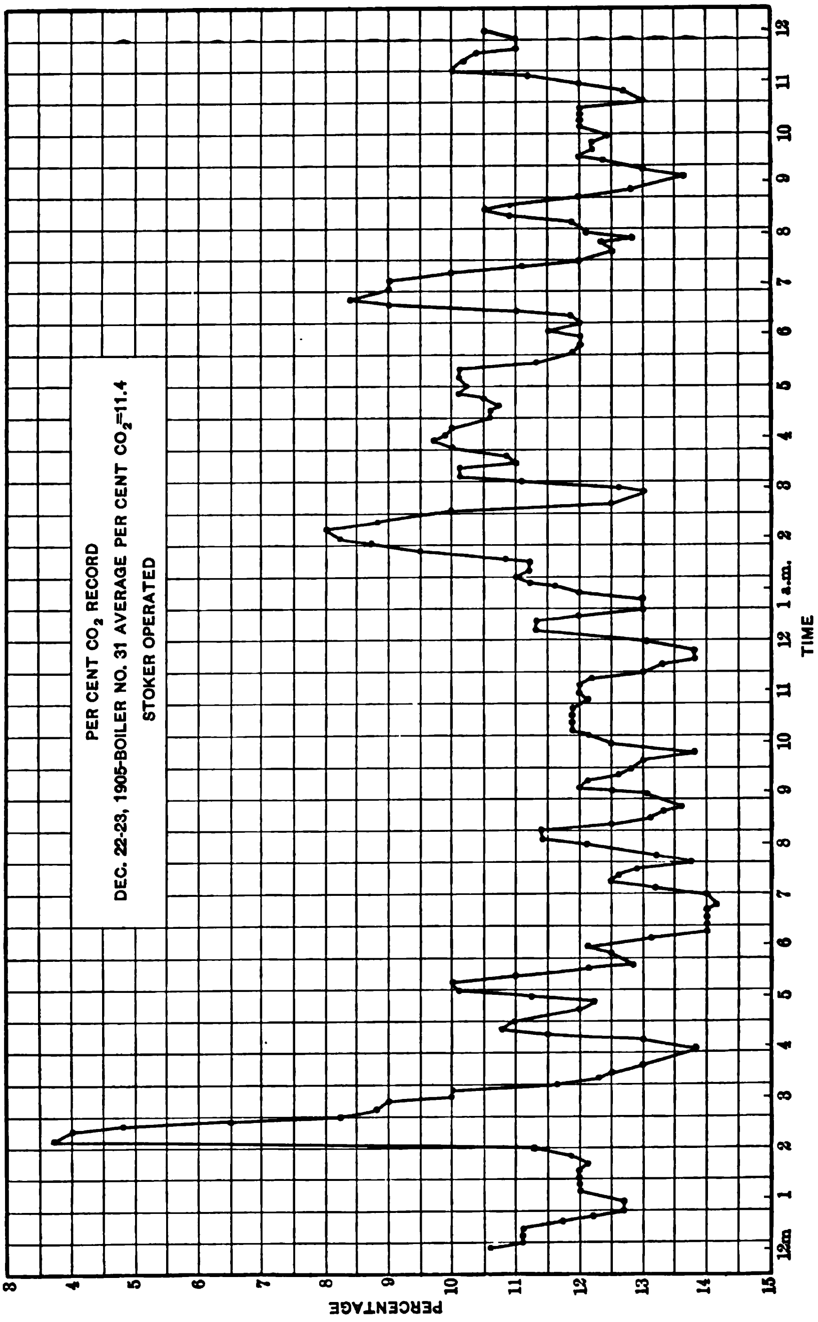
PER CENT CO$_2$ RECORD
DEC. 22-23, 1905-BOILER NO. 31 AVERAGE PER CENT CO$_2$=11.4
STOKER OPERATED
PERCENTAGE
8
4
5
6
7
8
9
10
11
12
13
14
15
TIME
12m
1
2
3
4
5
6
7
8
9
10
11
12
1 a.m.
2
3
4
5
6
7
8
9
10
11
12

FIG. 66.

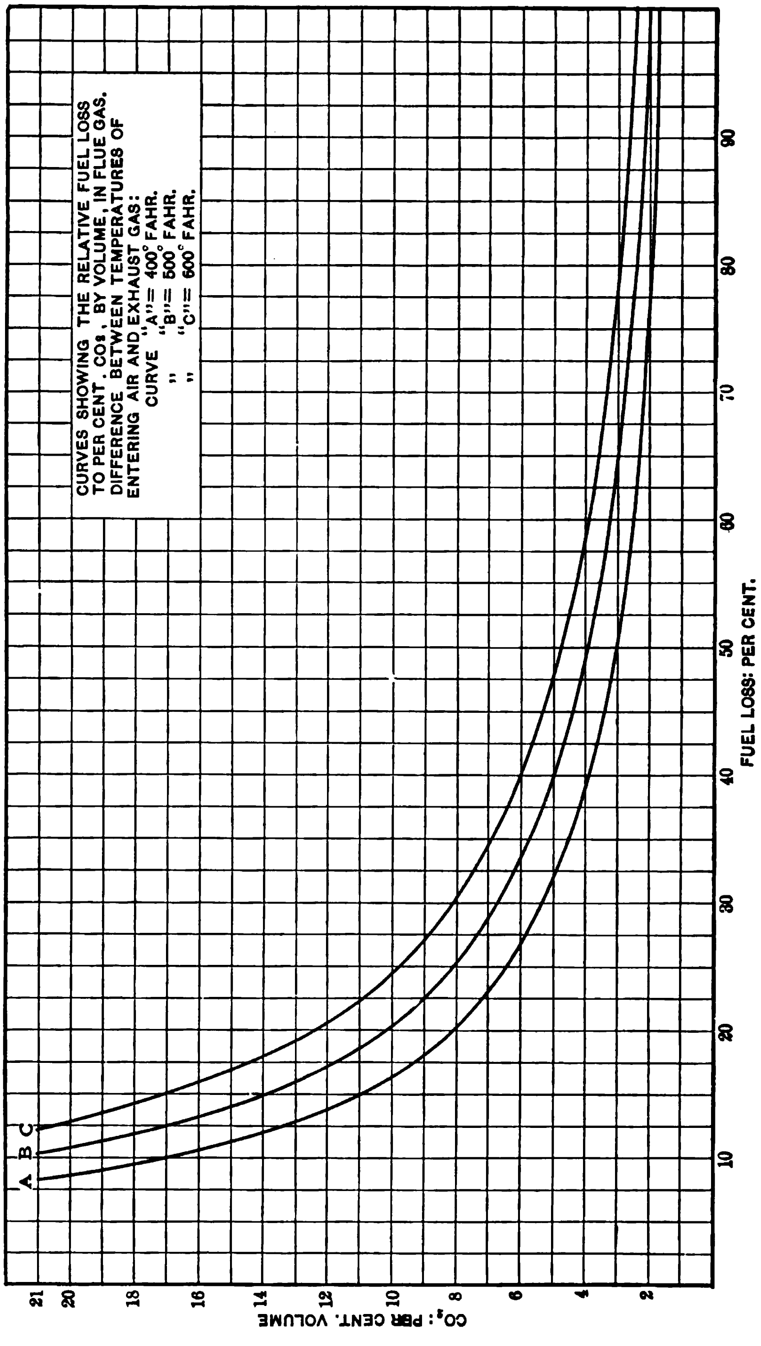
CURVES SHOWING THE RELATIVE FUEL LOSS
TO PER CENT. CO₂, BY VOLUME, IN FLUE GAS.
DIFFERENCE BETWEEN TEMPERATURES OF
ENTERING AIR AND EXHAUST GAS:
CURVE "A"= 400° FAHR.
" "B"= 500° FAHR.
" "C"= 600° FAHR.
A
B
C
21
20
18
16
14
12
10
8
6
4
2
CO₂: PER CENT. VOLUME
10
20
30
40
50
60
70
80
90
FUEL LOSS: PER CENT.

FIG. 67.

"Mr. W. H. Patchell,* of London, who recently visited us, has introduced very large boilers, assembling two in one setting; each boiler has a normal evaporation of 33,000 lb. per hour and in this way has cut down to a minimum the radiating surface per square foot of heating surface. He has also introduced the iron case with magnesia lining, and with good results.

"The question of boiler leakage is one in which the choice of the lesser of two evils is necessary; for in the tubular or cylindrical boiler the leakage will undoubtedly be less than in the water-tube type, owing to the smaller number of joints in the water space. But these two advantages are offset by the increased difficulty of construction, and the danger of using large boilers of the tubular type, especially with high-pressure steam.

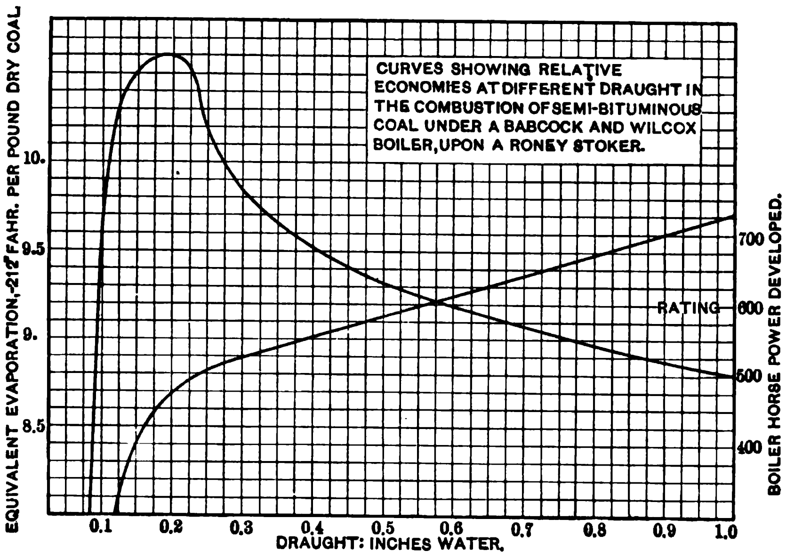

Fig. 68.

"It is now generally admitted that there can be no more difference in the efficiency of different types of boilers under

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* See paper read December 7, 1905, before the Institution of Electrical Engineers, by W. H. Patchell.

similar conditions than there can be in electric heaters, press agents to the contrary notwithstanding.

"*Item 5. Returned by Feed-water Heater.* The importance of getting the feed water to the maximum temperature obtainable is generally recognized, and would seem to indicate that all auxiliaries should be steam driven so that their exhaust may be utilized in the feed-water heater; in this way the auxiliaries may operate at about 80 per cent. thermal efficiency.

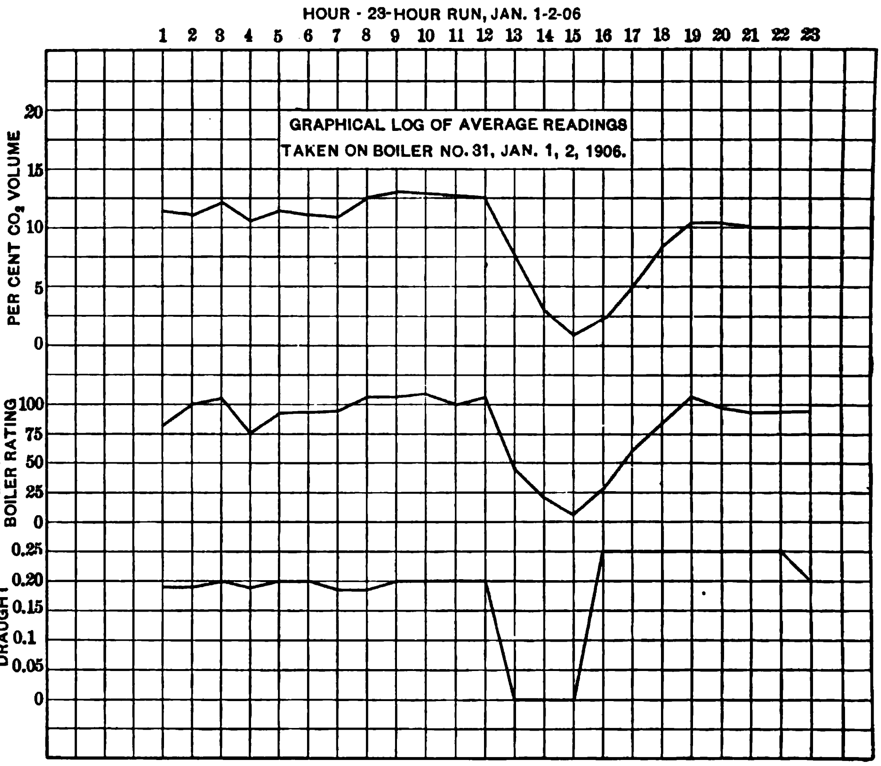

Fig. 69.

"*Item 6.* Owing to the difficulty of pumping water at temperatures above 150 degrees fahr., when under pressure, it becomes necessary to install economizers for the purpose of increasing the feed-water temperature to 200 or 250 degrees fahr.

As this increase of temperature is obtained from the waste gases at no expense for fuel, it only becomes necessary to consider the load-factor, as will be shown later, in order to decide whether economizers should be installed or not. In practically all cases where the load factor exceeds 25 per cent. the investment will be justified.

" In deciding upon the size of economizer to be installed it is important to consider first, the influence of the economizer upon the available draught due to the obstruction it offers and also due to the reduced stack temperature; the second important consideration is to equate the interest and depreciation charges against the saving in fuel, and so determine the amount of investment justified in each particular case.

" *Item 7. Loss in Pipe Radiation.* By the use of two-layer pipe covering, each layer being approximately 1.5-in. thick, and sections put on in such manner that all joints are broken, the radiation losses have become practically negligible.

" *Items 8 and 9. Heat Delivered to Circulating and Boiler-Feed Pumps.* As these auxiliaries may be either electrically driven or steam driven it is interesting to note that the thermal efficiency of the electrically-driven pumps would be equal to the thermal efficiency of the plant, multiplied by both the efficiency of conversion from the alternating to direct current and by the motor efficiency. In this case, there would be a net thermal efficiency of $10.3 \times 0.93 \times 0.90 = 8.63$ per cent., whereas the thermal efficiency of the steam-driven auxiliary discharging its exhaust into a feed-water heater at atmospheric pressure would be approximately 87 per cent.

" *Item 10. Loss in Leakage and High-Pressure Drips.* The loss in leakage should be infinitesimal, and the high-pressure drips can be returned to the boilers, so that practically all the loss under this heading is recoverable.

" *Items 11, 12, and 17* are probably unavoidable and of so small a magnitude as not to merit much consideration.

"*Item 13. Loss in Engine Friction.* Recent tests of a 7500-h.p. reciprocating engine show a mechanical efficiency of 93.65 per cent. at full load, or an engine friction of 6.35 per cent. As this forms only 0.8 per cent. of the total thermal losses it is relatively unimportant. Attention is called to the method of lubricating all the principal bearings by what is known as the flushing system, whereby a large quantity of oil is put through all the bearings by gravity feed from elevated oil reservoirs common to all the units; after passing through the bearings the oil is returned by gravity to oil filters in the basement and then pumped up to the reservoir tanks again. About 200 gallons per hour are put through each engine, and of this quantity only about 0.5 per cent. is lost. This method of oiling undoubtedly contributes to the general results.

"*Item 14.* As large electrical generators can now be obtained which give from 98 to 98.5 per cent. efficiency, it would seem as if the limit in design had been reached and that hereafter the problem of design is to be merely one of altering dimensions to suit varying sizes and speeds. While this is true as far as the efficiency is concerned, other problems are continually arising, such as the design of generators for an overload capacity of 100 per cent. to meet the demand for apparatus capable of taking care of great overloads economically for short periods, corresponding to peak loads of a railroad or lighting plant.

"*Item 15. Engine Radiation Losses.* This source of loss has evidently been reduced to a negligible quantity by the use of improved material and methods of heat insulation.

"*Item 16. Rejected to Condenser, 60.1 per cent.* This immediately introduces the thermodynamics of the steam engine, a subject so broad that it will be impossible to do more than touch upon some of the most important points in considering steam-engine efficiency.

"The efficiency * of any heat engine can be expressed by the

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* Defined as ideal coefficient of conversion, *Art.* 89.—AUTHOR.

ratio of $E=\frac{T_1-T_2}{T_1}$ where T_1 is the absolute temperature of the steam entering the engine and T_2 the absolute temperature of the steam leaving the engine. Thus in the engine whose steam-consumption curve is given in Fig. 70, if the initial pressure is 175 lb. gauge and the vacuum at the low-pressure exhaust nozzle is 28 in., then the maximum thermal efficiency is $\frac{837-560}{837}$ $=33$ per cent. This would be true for any form of engine or turbine working between the same temperature limits.

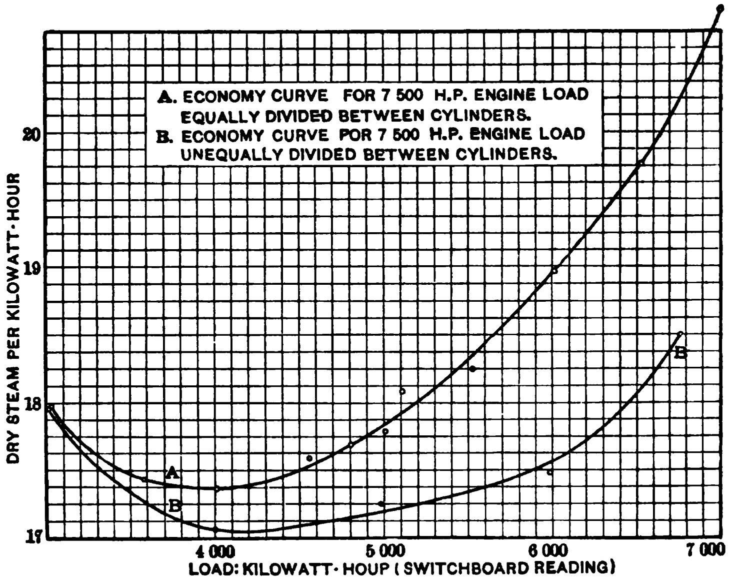

FIG. 70.

"In Fig. 70, however, it is seen that the point of maximum economy shows a steam consumption of approximately 17 lb. per kilowatt-hour, which is equivalent to 20,349 B.t.u. per hour. One kilowatt-hour is equal to 3412 B.t.u. per hour, so that the actual efficiency of the steam engine and generator is $=\frac{3412}{20349}=16.7$ per cent. As the generator efficiency at this load is approximately 98 per cent. the net engine thermodynamic efficiency * is $\frac{16.7}{0.98}=17$ per cent.

"The difference between the theoretical efficiency and the actual is then $33-17=16$ per cent., of which 0.8 per cent. has already been accounted for in engine friction, so that the balance of 15.2 per cent. is due to cylinder condensation, incomplete expansion, and radiation.

"As the engine friction in a two-bearing engine with high-pressure poppet valves and low-pressure Corliss valves has by careful design been reduced to less than 0.8 per cent. gain cannot be expected here, so attention must be centered on the loss due to cylinder condensation, etc., amounting to 15.2 per cent., in order to effect any improvement.

"Superheated steam is the only remedy at hand and with it we can probably effect an improvement of 5 or 6 per cent. by using such a degree of superheat in the boilers that dry steam will be had at the point of cut-off in the low-pressure cylinder.

"Any greater amount of superheat than this will merely result in loss to the condenser; for it should be remembered that the cylinder losses increase with the difference in temperature between the steam and exhaust portions of the cycle; in other words, the greater the thermal range of temperature the greater the condensation loss. This would seem to point to the use of more cylinders; but this involves additional first cost and friction as well as more space and higher maintenance charges.

"Fig. 71 shows what may be gained by reducing the temperature at the end of the cycle by means of increased vacuum, but in the case in point the maximum vacuum obtainable in practice was used so that no additional economy can be expected in this way.

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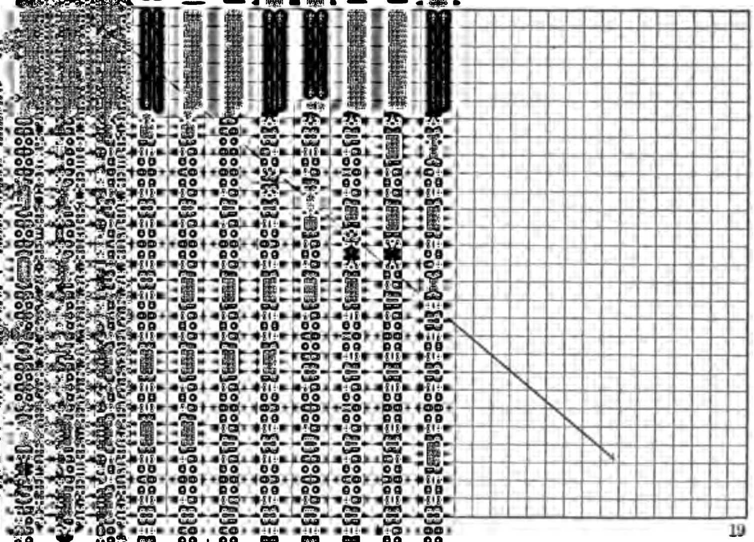
* Defined as thermal efficiency under *Art.* 136.—AUTHOR.

[illegible] HEAT BALANCE.

[illegible] ing reciprocating engines [illegible]

[illegible] 12%
[illegible] leakage. . . . 5%
[illegible] e of superheat 6%



[illegible] OUR-LOAD 4 000 KILOWATTS

[illegible] efficiency of the entire [illegible] he total thermal efficiency [illegible]

[illegible] apacity of the plant was [illegible] o take care of the increas- [illegible] raffic; and it was finally [illegible] o operate on the exhaust [illegible] ines.

The following quotation * indicates the all-around gain by this combination.

"During the year 1908 it became apparent that owing to the ever-increasing traffic in the New York subway, it would be necessary to have additional power available for the winter of 1909–1910.

"2. The power plant of the Interborough Rapid Transit Company, which supplies the subway, is located on the block bounded by 58th and 59th Streets, and by 11th and 12th Avenues, adjacent to the North River; it contains nine 7500-kw. (maximum rating) engine units, besides three 1250-kw. 60-cycle turbine units which are used exclusively for lighting and signal purposes.

"3. The 7500-kw. units consist of Manhattan-type compound Corliss engines, having two 42-in. horizontal high-pressure cylinders and two 86-in. vertical low-pressure cylinders. Each horizontal high-pressure cylinder and vertical low-pressure cylinder has its connecting rod attached to the same crank, so that the unit becomes a four-cylinder 60-in. stroke compound engine with an overhanging crank on each side of a 7500-kw. maximum rating 11,000-volt, three-phase, 25-cycle generator. The generator revolving field is built up of riveted steel plates of sufficient weight to act as a flywheel for the two engines connected to it. This arrangement gives a very compact two-bearing unit. The valve gear on the high-pressure cylinders is of the poppet type, and on the low-pressure of the Corliss double-ported type.

"4. The condensing apparatus consists of barometric condensers, arranged so as to be directly attached to the low-pressure exhaust nozzles, with the usual compound displacement circulating pump and simple dry-vacuum pump.

"5. These engine and generator units are in general probably the most satisfactory large units ever built, as five years' experience

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* "Tests of a 15,000-KW. Steam-Engine-Turbine Unit," by H. G. Stott and R. J. S. Pigott. Transactions of the A.S.M.E., Vol. XXXII.

with them has proved; their normal economic rating is 5000 kw., but they operate equally well (water rate excepted) on 8000 kw. continuously.

"6. In considering the problem of how to get an additional supply of power, every available source was considered, but by a process of elimination only two distinct plans were left in the field.

"7. The electric transmission of power from a hydraulic plant was first considered, but owing to the high cost of a double transmission line from the nearest available water power, and the impossibility of getting reliable service (that is, service having a maximum total interruption of not more than ten minutes per annum) from such a line, further consideration of this plan was abandoned.

"8. The gas engine, while offering the highest thermo-dynamic efficiency, at the same time required an investment of at least 35 per cent more than ordinary steam-turbine plant with a probable maintenance and operation account of from four to ten times that of the steam turbine.

"9. The reciprocating-engine unit of the same type as those already installed, was rejected in spite of its most satisfactory performance, on account of the high first cost and small range of economical operation. Reference to Fig. 72 will show that the economic limits of operation are between 3300 kw. and 6300 kw.; beyond these limits the water rate rises so rapidly as to make operation undesirable under this condition, except for a short period during peak loads.

"10. The choice was thus narrowed down to either the high-pressure steam turbine or the low-pressure steam turbine. There was sufficient space in the present building to accommodate three 7500-kw. units of the high-pressure type, or a low-pressure unit of the same size on each of the nine engines, so that the questions of real estate and building were eliminated from the problem.

"11. The first cost of a low-pressure turbine unit is slightly lower than that of a high-pressure unit, due to the omission of the high pressure stages and the hydraulic governing apparatus, but the cost of the condensing apparatus would be the same in both cases. The foundations and the steam piping in both cases would not differ greatly. The economic results, so far as the first cost is concerned, would then be approximately the same, if we consider the general case only; but in this particular instance the installation of high-pressure turbines would have meant a much greater investment for foundations, flooring, switchboard

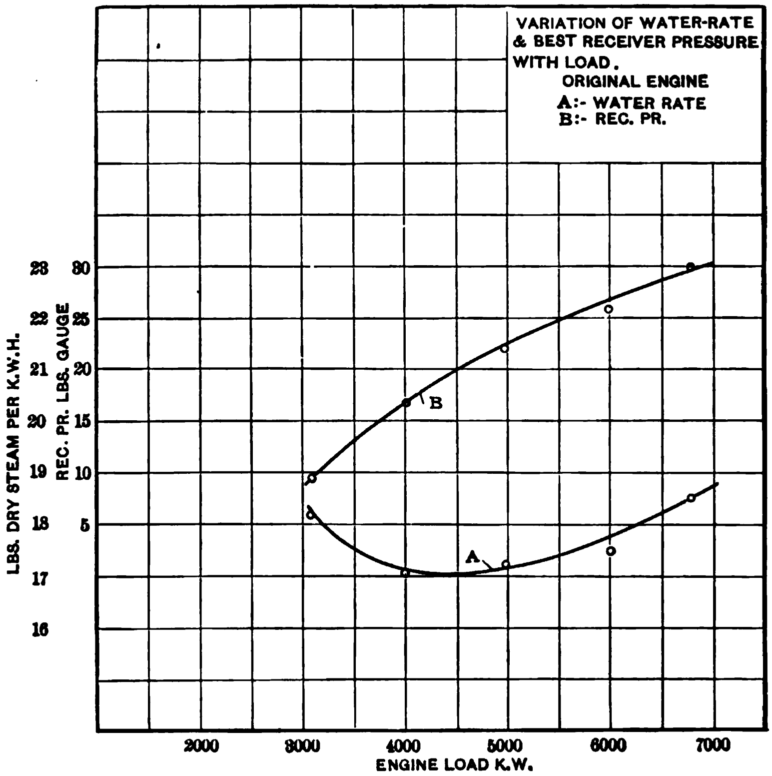

Fig. 72.

apparatus, steam piping and water tunnels, amounting to an addition of not less than 25 per cent to the first cost.

" 12. The general case of displacing reciprocating engines and installing steam-turbine units in their place was also considered. The best type of high-pressure turbine plant has a thermal efficiency approximately 10 per cent better than the best reciprocating-engine plant, but the items of labor for operation and for maintenance, together with the saving of about 85 per cent of the water for boiler-feed purposes and the 10 per cent of coal, reduce the relative operating and maintenance charges for the steam-turbine plant to 80 per cent, as compared to 100 per cent for the reciprocating-engine plant.

" 13. Assuming that the reciprocating engine plant is a first-class one and has been well maintained, about 20 per cent of its original cost (for engines, generators and condensers) may be realized on the old plant and so credited to the cost of the high-pressure turbine plant. But on the other hand, if the high-pressure turbine installation is to receive credit for the second-hand value of the engines, it must also have a debit charge for 100 per cent of the original reciprocating-engine plant which it displaced. The relative investments, therefore, upon this basis would be approximately equal for the high-pressure or the low-pressure turbine; but 80 per cent of the cost of the original engine plant would have to be charged against the high-pressure turbine plant, as against an actual increase in value (to the owner) of the engine by reason of its improved thermal efficiency, due to the addition of the low-pressure turbine.

" 14. The preliminary calculations, based upon the manufacturers' guarantees for the low-pressure and high-pressure turbines, showed that the combined engine-turbine unit would give at least 8 per cent better efficiency than the high-pressure turbine unit, so that it was finally decided to place an order for one 7500-kw. (maximum rating) unit, as by this means we would not only get an increase of 100 per cent in capacity, but at the same time give the engines a new lease of life by bringing them up to a thermal efficiency higher than that attained by any other type of steam plant.

"15. The turbine installed is of the vertical three-stage impulse type having six fixed nozzles and six which can be operated by hand, so as to control the back pressure on the engine, or the division of load between engine and turbine. An emergency overspeed governor, which trips a 40-in. butterfly valve on the steam pipe connecting the separator and the turbine and at the same time the 8-in. vacuum breaker on the condenser, is the only form of governor used. The footstep bearing, carrying the weight of the turbine and generator rotors, is of the usual design supplied with oil under a pressure of 600 lb. per sq. in. with the usual double system of supply and accumulator to regulate the pressure and speed of the oil pumps.

"16. The condenser contains approximately 25,000 sq.ft. of cooling surface arranged in the double two-pass system of water circulation with a 30-in. centrifugal circulating pump having a maximum capacity of 30,000 gal. per hr. The dry vacuum pump is of the single-stage type, 12-in. and 29-in. by 24-in., fitted with Corliss valves on the air cylinder. The whole condensing plant is capable of maintaining a vacuum within 1.1 in. of the barometer when condensing 150,000 lb. of steam per hr. when supplied with circulating water at 70 deg. fahr.

"17. The electric generator is of the three-phase induction type, star-wound for 11,000 volts, 25 cycles and a speed of 750 r.p.m. The rotor is of the squirrel-cage type with bar winding connecting into common bus-bar straps at each end. This type of generator was chosen as being specially suited to the conditions obtaining in the plant.

"18. With nine units operating in multiple, each one capable of giving out 15,000 kw. for a short time, operating in multiple with another plant of the same size, it is evident that it is quite possible to concentrate 270,000 kw. on a short circuit. If we proceed to add to this, synchronous turbine units of 7500-kw. capacity, which, owing to their inherently better regulation and enormous stored energy, are capable of giving out at least six times their maximum rated capacity, the situation might soon become dangerous to operate, as it would be impossible to design switching apparatus which could successfully handle this amount of energy. The induction generator, on the other hand, is entirely dependent upon the synchronous apparatus for its excitation, and in case of a short circuit on the bus-bars would automatically lose its excitation by the fall in potential on the synchronous apparatus.

" 19. The absence of fields leads to the simplest possible switching apparatus, as the induction generator leads are tied in solidly through knife switches, which are never opened, to the main generator leads. The switchboard operator has no control whatever over the induction generator, and only knows it is present by the increased output on the engine generator instruments.

" 20. The method of starting is simplicity itself—the exciting current is put on the engine generator *before* starting the engine, and then the engine is started, brought up to speed and synchronized in exactly the same way as before. While starting in this way, the induction generator acts as a motor until sufficient steam passes through the engine to carry the turbine above synchronism, when it immediately becomes a generator and picks up the load. Three of these 7500-kw. low-pressure turbine units have been installed and tests run on Nos. 1 and 2. No. 3, having been just started, has not yet been tested.

" 21. Instead of inserting in this paper the enormous accumulation of data incident to these tests, we have divided the paper into two parts in the hope that it would thus be more accessible for reference, the first part giving the reasons for adopting this particular type of apparatus, with a brief description of the plant and a summary of the results obtained, and the second part containing all the principal data acquired during the tests, with sufficient explanation to make their meaning clear without reference to the text." . . . .

"24. The net results obtained by the installation of low-pressure turbine units may be summarized as follows:

"*a*. An increase of 100 per cent in maximum capacity of plant.
"*b*. An increase of 146 per cent in economic capacity of plant.
"*c*. A saving of approximately 85 per cent of the condensed steam for return to the boilers.
"*d*. An average improvement in economy of 13 per cent over the best high-pressure turbine results.
"*e*. An average improvement in economy of 25 per cent (between the limits of 7000 kw. and 15,000 kw.) over the results obtained by the engine units alone.
"*f*. An average unit thermal efficiency between the limits of 6500 kw. and 15,500 kw. of 20.6 per cent."

205. Summary. The two preceding quotations are self-explanatory; hence no comment is necessary. But, before concluding, it must be remarked that the internal combustion engine and the steam turbine are still in the experimental stage; and that it is impossible to predict what the final adjustment will be. It is true that the internal combustion engine has a higher thermal efficiency than has any other heat motor. But, due to complexity of construction, the internal combustion engine has a higher first cost; and furthermore, its regulation is inherently inferior to a reciprocating engine or turbine. Due to this, in spite of the fact that the reciprocating engine has a lower thermal efficiency, it still holds its place, on account of its simplicity and ***high over-load capacity***; the latter being especially important in most power plants where it is necessary to take care of large "peak loads."

It must be further remarked, that the installation of every power plant is finally affected by the economy of transmission.

Whether power can be developed economically at any locality depends upon whether the cost of power for the particular locality is greater or less if developed at this particular point, or developed at some other point and transmitted to the point under consideration. This, of course, depends largely upon the economy of transmission.

At the present time electrical engineers are giving considerable attention to the subject of high-tension transmission. And if it develop that methods can be devised by means of which corona losses can be eliminated, or partially avoided, for potential differences far in excess of those employed at present, the subject of power plant economics will require revision. For, if corona losses can be avoided, the cost of power for any particular locality will be materially changed. And hence, the cost for the production of power will, likewise, be changed.

To illustrate concretely: Assume that it becomes possible to transmit with a potential difference of 300 kilo-volts instead of 125 or 150 kilo-volts. Under these conditions the economy of transmission is considerably increased; and the distances to which coal can be transported, to compete with the increased efficiency of transmission, is considerably reduced. This, however, is not the only governing factor. Ground rent also influences the choice. That is, when the saving in transmission and the saving in ground rent, by locating the plant at the coal fields, is balanced against the hauling of the coal, and the ground rent for a large city, it may develop that it is more economical to locate the power plant where the coal is mined. A similar argument, of course, applies to water-power plants. That is, the initial cost of a water-power plant is high, and therefore the distance, for a given potential difference, over which power can be profitably transmitted is limited; and, of course, the lower the cost of transmission, the greater the area over which profitable transmission may take place. Hence, as the potential difference, which may be employed in transmission, is increased, the smaller, relatively, due to high ground rent, becomes the economy of a localized plant. Therefore, if it develop, that potential differences, far in excess of those employed at the present time, may be used, power plants in large cities, where ground rent is high, will disappear; and the future power plant will be located at the point where the raw material, for the development of power, is found.

INDEX

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