PRACTICAL

HYDRAULICS

BY

P. M. RANDALL,

AUTHOR OF

"QUARTZ OPERATORS' HAND-BOOK."

PUBLISHED AND SOLD BY

DEWEY & CO.

PROPRIETORS MINING AND SCIENTIFIC PRESS,

SAN FRANCISCO, CAL., 1886.

PREFACE.

The present work is designed as a true exposition of the principles and application of those branches of hydraulics, of which it treats.

The necessity of such a work at this time will be obvious to those who shall have compared the results deduced from the formulas of nearly all our most noted authors on hydraulics, with the results of observation. Thus, the formulas of DuBuat, Eytelwein, Girard, Prony, D'Aubuisson, Neville, Leslie, Pole, Beardmore and Hagen, enjoying the reputation of standard authors, give as by the data at hand, with respect to the flow of water in open streams of medium size, results varying from fifteen to one hundred and twenty-five per cent in excess of the observed results, and in large streams, results varying from thirty to sixty-seven per cent below those observed. These errors are radical. The defectiveness of these and other works heretofore regarded standard on this highly important branch of hydraulics, is well portrayed by the following extracts from an article in an English periodical, "Engineering" of Dec. 31, 1875, entitled "Hydraulic Experiments," viz:

" The tabulated volocities (in Neville's work, based upon DuBuat) " though expressed in hundredths of an inch, are in reality but the " wildest guesses at the actual velocities in irrigation canals of " ordinary dimensions. Colonel Cautley relied upon DuBuat when " he laid out the Ganges Canal, and found him but a rotten reed, " for the water in every instance tore along at an unexpected velo- " city, and the erosion of the bed and destruction of the works " followed in its wake. Du Buat then must be put upon the top " shelf of the book-case, and it will be just as well when the steps "are there to carry up every English work, in which the names of "Brunning, Girard, Bossut, Prony, Eytelwein, or D'Aubuisson are "continually recurring as authorities against whom no action can "be taken. In this general clearance Beardmore, Downing, Box, "and *almost every other hydraulic text book* compiled by English- "men, will with more or less hesitation have been shelved."

Again, "in 1880," says L. D. A. Jackson, in his Hydraulic Manual "the extensive experiments of Captain Allan Cunningham on "the Ganges Canal, have substantiated the truth of Kutter's laws "when applied to very large canals, and dealt the final blow to the "velocity formulas of all the older hydraulicians."

In the main text of the present work it is stated that D'Arcy in 1835, and Bazin, in 1865, published formulas better adapted than any preceding for finding the flow of water in open streams and pipes of medium size; that Humphreys and Abbot published in 1861, formulas suited to the determination of flow of large streams, but not to the flow of small streams, and that the wide gap between the formulas of D'Arcy and Bazin, and those of Humphreys and Abbot were effectually closed up in 1870 by the introduction of Kutter's formula. We will now add, that this achievement with respect to hydraulic science seems to us the masterpiece of the nineteenth century. The Kutter formula applies equally well to small, medium-sized and large streams. Farther experiments may perchance require it to be somewhat modified; but so far as known at present, of all the formulas deduced for like purposes, it seems the nearest approximate to perfection.

The principal tables computed by Herr Kutter, from his formula under consideration, give the coefficients of velocity in terms of *metrical measures*, thereby rendering their application a laborious task in the determination of the velocities themselves in terms of *feet measures.*

To obviate this task, Table 27 in the present work has been computed from the same formula (Kutter's) giving in terms of *feet measures*, the velocities of flow in open streams, differing in regime and in slope, and varying from the size of a small ditch to that of the Mississippi River. The table is nominally for open streams, but is equally well adapted for determining the flow of water in pipes. Table 17 computed for the flow in pipes only, will, for this purpose, be found, however, still more convenient. For this table we are indebted in part to J. T. Fanning's very admirable treatise on "Water Supply Engineering," which indebtedness we hereby respectfully acknowledge. It will be noted however that we have not only considerably enlarged the original table of Mr. Fanning, but, among other things, conferred upon it a new and valuable feature—that of giving the quantity of flow in addition to the velocity. Each result set down in Tables 17 and 27, represents essentially a mean of numerous observed results: hence must necessarily coincide in practice with other results obtained under like conditions. With respect to accuracy, scope of application and ease of reference, these tables seem to meet more fully the requirements of all concerned in this branch of hydraulic engineering, than any others designed for similar purposes.

Tables 28 and 29 will be found very important auxiliaries to Table 27, in the ready determination of the flow of water in beds of different forms.

Tables, two relating to the flow of water in rectangular weirs, four to quadrant weirs, seven to the flow through rectangular orifices, and eight to the different values of the so-called "miner's inch," will also be found of no little value in practice. The simplicity of the *quadrant weir*, its cheapness and the assurances by Prof. Thompson of its superiority over those of different forms, induced the author of the work in hand to compute Table 4.

This form seems peculiarly well adapted to the measurement of the flow of small quantities of water; for example, from two to 20 miner's inches. This table, however, greatly exceeds these limits. The discussion of the subjects of "*maximum work* effected by water on issuing under pressure from pipes," and of "*minimum weight* and consequent *minimum cost* of an inverted siphon," is, so far as the author is informed, new. By the application of the principles here demonstrated, the *greatest economy*, the only proper limit or standard of the *truly practical*, is attained.

The simple plan, pursued in the preparation of the present work consists :—

1st. In demonstrating concisely the principle, or principles, involved in the subject matter, yet in a manner sufficiently ample and clear to be readily followed by the student, or by the practitioner desiring to refresh his mind, or to assure himself of the correctness of the results.

2nd. In expressing in words the simplest rule or rules corresponding to the formula or result of such demonstration.

3d. In applying the rule or rules so derived, to practical examples with full and clear explanations; or in applying the formula direct to the examples, when it is too complex to be well expressed in words.

4th. In providing tables, so far as feasible, to meet the requirements of practice.

By means of these tables and the simple rules given therewith, most of the problems likely to occur with respect to the measurement of water in motion, as through vertical openings, over weirs, in pipes, in open streams and through nozzles; with respect to the quantity of water required for various mining purposes, and for the purposes of irrigation; and with respect to the power of water as a motor, are answered direct, or readily solved by anyone familiar with common arithmetic only, as well as by the skilled engineer.

P. M. RANDALL.

San Francisco, March 17th, 1886.

INDEX.

Formulas, and Formulas and Rules corresponding; "F" representing formula, and "R" rule; page referring to rule:

CONTENTS.

PRINCIPLES OF HYDRAULICS.

OPENINGS.

WEIRS.

MINER'S INCH.

CONTRACTION.

SHORT TUBES.

PIPES.

NOZZLE.

INVERTED SIPHON.

OPEN STREAMS.

MINING.

IRRIGATION.

TABLES.

PRINCIPLES OF HYDRAULICS.

The term hydraulics was originally applied to water pipes, or to the conveyance of water through pipes. It is now used in a wider sense to designate that branch of engineering which treats of water in motion, the means of measuring, conveying, and raising it, and its application to machinery as a prime motor.

The source of this motion is the force of gravity—a force which acts indiscriminately upon every particle of matter, and impresses upon each particle at every instant the same degree of velocity *in vacuo*. The fundamental principles or laws of hydraulics then, are those of uniformly varied motion.

These, with respect to a body falling from rest through a certain hight *in vacuo*, are as follows:

1st—The velocities acquired are proportional to the times elapsed since the beginning of the motion.

2d—The spaces fallen are proportional to the squares of the times elapsed.

3d—These spaces or hights are proportional to the squares of the velocities acquired at the end of each.

4th—The velocity acquired at the end of the first unit of time is equal to twice the distance fallen during this time.

The intensity of the force of gravity varies in different latitudes, but for most purposes in hydraulic calculations, it may be regarded constant.

The velocity which the force of gravity can generate in a second of time at the surface of the earth, is usually designated by g, and is termed acceleration of gravity. Its value, as given in Rankine's Applied Mechanics, p. 485, for Lat. 45° at sea-level, is g.

$$\text{Thus } g = 32.1695 \text{ feet.} \tag{1}$$

For the most part in practice $g = 32.2$.

In case a high degree of accuracy is required, the oblateness of the earth, the latitude and elevation of the place at which the value of the force of gravity is sought, have to be considered.

The formulas involving these elements (the two former deduced from numerous pendulum experiments made in various parts of the world), are given in Rankine's Applied Mechanics, pp. 485–486, as follows:

$$R = 20{,}887{,}540 \text{ feet } \left(1 + 0.00164 \cos 2l\right) \tag{2}$$

$$g = g'\left(1 - 0.00284 \cos 2l\right)\left(1 - \frac{2h}{R}\right) \tag{3}$$

In which R denotes the earth's radius at the locality of observation, l the latitude, g' the force of gravity at sea-level, in Lat. 45°, and g the force of gravity for the elevation above sea-level in the given Lat. l. The factor $\left(1-\frac{2h}{R}\right)$ in the second member of equation (3) is readily obtained from the well-established proposition that the gravity of a body varies inversely as the square of the distance from the center of the earth. If in a given latitude l, g denote the force of gravity at sea-level, and g'' the force of gravity at an elevation, h, there will result:

$$g''=g\ \left(1-\frac{2h}{R}\right) \tag{4}$$

When $h=0$, it is obvious that in equations (3) and (4), the factor $\left(1-\frac{2h}{R}\right)=1$.

When $l=45°$, cos $2l=0$.

Whence, (2), $R=20,887,540$ feet$=3956$ miles. $(2_{,})$

Example 1.—What is the value of the force of gravity at Presidio, San Francisco, in latitude 37°, 47′, 48″, at sea-level?

Calculation.—Employing formula (3).

Find cos 2 (37°, 47′, 48″,)$=0.2488$.

Substitute this value, namely, 0.2488. and the value of $g'=32.1695$, of equation (1) in equation (3), observing that $h=0$, $g=32.1695\ (1-0.00284\times 0.2488)$. Whence, $g=32.1468$ ft.—*Answer.*

Ex. 2.—In latitude 37°, 47′, 48″, as in *Ex.* 1, what is the force of gravity at an elevation of 2 miles?

Cal.—We find from (2) and $(2_{,})$.

$R=3956\ \ (1+0.00164\times .2488)$.

Whence, $R=3957.6$ miles.

Substituting this value of R and the value of g as found from *Ex.* 1, in equation 4, $g''=32.1468\left(1-\frac{2\times2}{3957.6}\right)$ Reducing $g''=32.1144$.—*Ans.*

Ex. 3.—Required the force of gravity at an elevation of two miles above sea-level at the equator.

Cal.—When $l=0$, find $\cos 2l=1$. Substituting this value of $2l$ in eqs. (2) and ($2_{,}$), and reducing, $R=3962.5$. Substituting the value of $R=3962.5$, the value of $\cos 2l=1$, the value of $g'=32.1695$, and the given value of $h=2$ miles in formula (3),

$$g=32.1695\left(1-0.00284\right)\left(1-\frac{2\times2}{3962.5}\right)$$

Whence, $g=32.0457$.—*Ans.*

Remark.—Had we made $R=4000$ in the preceding examples, it would in no case have varied the result to exceed .0003. We may, therefore, without sensible error, regard the radius constant and equal to $R=4000$.

These refinements, with respect to variations in the force of gravity under different conditions, though highly important in establishing a standard of measurement, and in various scientific investigations, yet for the most part are little applied in practice.

Reverting to the subject of the laws of varied motion, it will be noted that the velocity acquired by a falling body at the end of the first second of time, is double the hight which the body has fallen during that time. A body then, *in vacuo*, falls during the first second of time 16.1 feet, or more accurately, 16.085 in Lat. 45°.

Denote in seconds, the times of a falling body *in vacuo* by the consecutive numbers:

1, 2, 3, 4, 5, 6, 7, 8, 9, etc.

Then, according to the 2d law, the hights of the fall are proportional to the squares of these times; thus—1, 4, 9, 16, 25, 36, 49, 64, 81, etc.; and, according to the 3d law, the hights are proportional to the squares of the velocities acquired during these times.

If the hight of fall, as found by law 2d, due any given time, be taken from the hight of fall due this time increased by one second, the remainder will be equal to the space fallen during this increment of one second.

Thus the hights, so fallen in the natural order of times, are 1, 3, 5, 7, 9, 11, 13, 15, 17, etc.

EXAMPLES AND CALCULATIONS.

Ex. 4.—How many feet will a body, as water *in vacuo*, fall during the 5th second of its descent?

Cal.—By the foregoing it will be seen that the fall during the 1st second is 16.1 feet nearly, and during the 5th second is 9 times as much;

Hence, $16.1 \times 9 = 144.9$ feet.—*Ans.*

Ex. 5.—What distance will water fall *in vacuo* in 5 seconds?

Cal.—Note in accordance with law 2d, that a body falls 25 times as far in 5 seconds as it does in 1 second; hence, $16.1 \times 25 = 402.5$ feet.—*Ans.*

In further illustration:

Cal.—Note that the hights fallen respectively during each second of the given time of 5 seconds, are 1, 3, 5, 7, 9. The sum of which is 25, as found in the preceding calculation; hence, $16.1 \times 25 = 402.5$ feet.

Ex. 6.—What will be the velocity of water falling, *in vacuo*, at the end of the 5th second?

Cal.—Observe that, according to the 1st law, the velocity is 5 times as great at the end of the 5th second as it was at the end of the first—that is, $2 \times 5 = 10$; hence, $16.1 \times 10 = 161.$ feet.

Rules with respect to the relations of space, time, velocity, and the force of gravity, involved in the fall of a body, as water, in vacuo.

The velocity given to find the head or distance the water has fallen.

Rule 1.—Divide the square of the given velocity by twice the acceleration of gravity—that is, by 64.4.

Ex. 7.—The velocity is 150 feet per second. What is the head or distance fallen?

Cal.—$150 \times 150 \div 64.4 = 349.4$ feet nearly.—*Ans.*

To find the head or distance water will fall in a given time

Rule 2.—Multiply the square of the time in seconds by 16.1 feet.

Ex. 8.—What distance will water fall in 4 seconds?

Cal.—$4 \times 4 \times 16.1 = 257.6$ feet.—*Ans.*

The time given to find the velocity of water falling freely.

Rule 3.—*Multiply the time in seconds by the acceleration of gravity, namely,* 32.2 *feet.*

Ex. 9.—What velocity does water acquire in falling 7 seconds?

Cal.—$32.2 \times 7 = 225.4$ feet.—*Ans.*

The head, or distance of water fallen freely, given to find the acquired velocity.

Rule 4.—Extract the square root of twice the product of the head and the acceleration of gravity, or multiply the square root of the given head by the square root of twice the acceleration of gravity—that is, by 8.025.

Ex. 10.—What velocity will water acquire by falling freely 196 feet? In other words, with what velocity will it flow under a pressure or head of 196 feet?

Cal.—Square root of 196 feet is 14 feet:

Then $14 \times 8.025 = 112.35$ feet.—*Ans.*

The velocity being given to find the time which a body, as water, has fallen.

Rule 5.—Divide the given velocity by the acceleration of gravity, viz., 32.2 feet.

Ex. 11.—The velocity is 322 feet per second. What is the time fallen?

Cal.—$322 \div 32.2 = 10$ seconds.—*Ans.*

To find the time required for water to fall freely through a given distance.

Rule 6.—Extract the square root of twice the given distance, divided by 32.2, or, in other words, divide the square root of the given distance by the square root of one-half the acceleration of gravity—that is, by 4.012.

Ex. 12.—What time will water require to fall freely a distance of 144 feet?

Cal.—The square root of 144 feet is 12 feet:

Hence, $12 \div 4.012 = 3$ seconds nearly.—*Ans.*

Determination of these laws of hydraulics by analysis.

To determine these laws let h represent the head or distance in feet, through which the water acts, or has fallen in a given time denoted by t; let v represent the velocity acquired at the bottom of this head or distance fallen, and let g denote the acceleration of gravity—that is, the velocity which the force of gravity can generate in a second of time at the surface of the earth.

Then the expressions for velocity and acceleration of gravity will be:

$$v=\frac{dh}{dt} \qquad (5)$$

and

$$g=\frac{dv}{dt} \qquad (6)$$

Eliminate dt from (5) and (6);

$$dh=\frac{vdv}{g} \qquad (7)$$

Integrating (7),

$$h=\frac{v^2}{2g}. \qquad (8)$$

Integrating (6),

$$v=gt \qquad (9)$$

Combining (5), and (9),

$$dh=gtdt \qquad (10)$$

Integrating (10),

$$h=\frac{gt^2}{2} \qquad (11)$$

Combining (9), and (11),

$$h=\frac{vt}{2} \qquad (12)$$

Reduce (7), as to v,

$$v=\sqrt{2gh}=\sqrt{2}\times\sqrt{h} \qquad (13)$$

Divide (9) by g,

$$t=\frac{v}{g} \qquad (14)$$

Reduce (11), as to t,

$$t=\sqrt{h}\div\sqrt{\frac{g}{2}} \qquad (15)$$

By inspection it will be seen that in the text, the given rules and the enunciations termed laws are derived from these formulas, or are but expressions for them; and that the relations of space, time, velocity and force of gravity are more clearly expressed by formula than it was possible to do by words.

To facilitate this inspection, the following references to the different forms of expression, but equivalent in meaning, are given. Thus:

Laws.	Rules.		Formulas.
1st.	3d.	(9),	$v=gt.$
2d.	2d.	(11),	$h=\frac{gt^2}{2}$
3d.	1st.	(8),	$h=\frac{v}{2g}$
4th.		Modification of (12),	$v=2h\div 1$ Sec.
	4th.	(13), reduced from (8),	$v=\sqrt{2gh}$
	5th.	(14), reduced from (9),	$t=\frac{v}{g}$
	6th.	(15), reduced from (11),	$t=\sqrt{\frac{2h}{g}}$

The value of g being substituted in these formulas, every possible question with respect to the free fall of water or other body can be answered.

The formula of most frequent occurrence in hydraulics corresponds to the 3d law, 1st rule, and is denoted in the column of formulas above by (8)—that is, $h=\frac{v^2}{2g}$. This formula, reduced with respect to v, is denoted by (13); $v=\sqrt{2gh}=\sqrt{2g}\times\sqrt{h}=8.025\sqrt{h}$, in which forms it frequently occurs in works in engineering. In this, or these formulas, v is termed the velocity due to a given hight, h, and h the hight due to a given velocity, v—that is, v denotes the distance which water flowing freely under a pressure of h feet in hight, will pass over in one second of time at the bottom of this hight, h, which velocity is the same, as water falling freely through the hight, h, would acquire at its bottom.

FLOW OF WATER THROUGH OPENINGS.

Openings are of two classes—the *submerged* and the *weir*.

An opening having its top, as shown at B, Fig. 1, beneath the water's surface, AD, is termed a submerged orifice; an opening having an open top, as shown at C, the crest, in Fig. 2, is termed a weir. In both, the form of outlet, for the most part in practice, is rectangular. This is more especially true with respect to weirs.

In Fig. 1, representing a vertical section through a

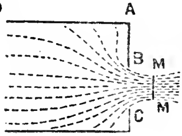


FIG. I.

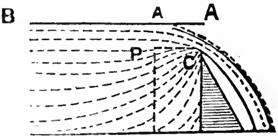


FIG. 2.

rectangular opening, conceive the opening, BC, composed of horizontal fluid layers indefinitely thin.

Let l=horizontal length of opening.

h=AB head with respect to top of opening.

$h_{\prime}$=AC head with respect to bottom of opening.

x=head additional to h, and due any horizontal fluid layer indefinitely thin in the opening BC.

dx=thickness of such fluid layer.

v=velocity due $(h+x)$ per second.

Q=discharge of water in cubic feet per second.

Then by (13), $$v=(2g)^{\frac{1}{2}}(h+x)^{\frac{1}{2}} \quad (16)$$

$$dQ=l\,(2g)^{\frac{1}{2}}(h+x)^{\frac{1}{2}}\,dx. \quad (17)$$

Integrating (17), between the limits of $x=0$, and $x=h_{\prime}-h$=BC.

$$Q=\frac{2l}{3}\left(2g\right)^{\frac{1}{2}}\left(h_{\prime}^{\frac{3}{2}}-h^{\frac{3}{2}}\right) \quad (18)$$

Equation (18), expresses the quantity of water, in cubic feet, which will flow through a submerged orifice under the general conditions imposed. It is, however, alike applicable to weirs.

For, by making $h=0$, there results:

$$Q=\frac{2l}{3}\left(2g\right)^{\frac{1}{2}}h^{\frac{3}{2}} \quad (19)$$

which expresses the quantity of water in cubic feet that will flow over a weir under the general conditions imposed.

The true *mean* velocity of a stream through the submerged orifice under the given conditions, is,

$$v=\tfrac{2}{3}\,(2g)^{\frac{1}{2}}\left\{\frac{h_{\prime}^{\frac{3}{2}}-h^{\frac{3}{2}}}{h_{\prime}-h}\right\} \quad (20)$$

Some hydraulicians assume that the mean head is equal to the distance between the surface and the middle of the submerged orifice; hence, deduce that the mean velocity is by (13),

$$v=(2g)^{\frac{1}{2}}\left(\frac{h_{,}+h}{2}\right)^{\frac{1}{2}} \qquad (21)$$

Comparing (20), and (21), omitting the common factor $(2g)^{\frac{1}{2}}$,

$$v:v_{,}=\left\{\frac{h_{,}+h}{2}\right\}^{\frac{1}{2}}:\frac{2}{3}\left\{\frac{h_{,}^{\frac{3}{2}}-h^{\frac{3}{2}}}{h_{,}-h}\right\} \qquad (22)$$

In refutation of the assumption that the mean velocity of a stream of water flowing from a submerged orifice, takes place at the middle of the opening, the following illustrations are presented. By substituting the respective values of the given heads and vertical widths in equation (22), there result:

When the vertical width is equal to one-half the head above it, the velocity is found one-sixth of one per cent too great; when it is equal to the head above it, the velocity is found one-half of one per cent too great;

When it is equal to twice the head above it, the velocity is found one and one-ninth per cent (.0111) too great;

When it is equal to three times the head above it, the velocity is found one and two-thirds of one per cent (.01645) too great;

And when we pass to the limit of the weir, by making $h=0$, the velocity becomes six per cent too great.

The assumption that the mean velocity takes place at the middle of a rectangular opening, is thus shown erroneous. It may, perhaps, approximate the truth with sufficient accuracy for most practical purposes, when the vertical width of the opening is less than the head above it; but is inadmissible when the width in comparison is greater.

The formula obtained on the assumption that the mean velocity takes place at the middle of the opening, is commended by its simplicity of expression. But its application, in case the ratio of the head to the vertical opening is large, involves the use of a coefficient of flow, varying with that ratio. The formula so encumbered would evidently be more complex and tedious of application than that, namely (20), which it seeks to evade or displace.

Water in motion is subject to resistances, which retard its velocity and diminish its volume of flow. Modifications, therefore, are requisite to be made in the theoretical formulas of hydraulics, so that they shall embrace the measure of these resistances, and thereby more fully meet the requirements of practice. These modifications are effected for the most part by coefficients, whose values have been determined by experiment. A coefficient, as here used, corresponds to that part of the theoretical quantity, as velocity and volume, remaining after it has been diminished in amount equal to the loss due resistances overcome.

With respect to the velocity of water flowing through orifices, a multiplicity of experiments, during a period of many years, have been made with extreme care by the ablest hydraulicians. The tabulated results of these experiments, differing with respect to head and size of opening, vary from .572 to .795.

If the theoretical velocity of water flowing through a rectangular opening with thin sides or lips be taken as a unit, then will an average of the experiments referred to, closely approximate .62.

This fraction is very generally adopted—that is, for the entire opening, the theoretical velocity is, to the average experimental velocity, as 100 to 62.

Let it not be inferred that the actual velocity of the flowing water is 62 per cent of the theoretical velocity. Water, in flowing from a reservoir, approaches the opening or outlet in convergent lines. This convergence continues a short distance beyond and outside the outlet, as shown at M $M_{\prime}$, Fig. 1, where occurs the minimum cross section of the stream, and where the velocity is nearly equal to the theoretical velocity. The area of the outlet being taken as the unit, the area of this cross section is equal to .637, while the velocity of the stream at this point is equal to .974 of the theoretical velocity. The coefficient of discharge there is equal to the product of these coefficients of cross section and velocity. Let c=this coefficient of discharge;

$$\text{Then } c = .637 \times .974 = .62 \text{ nearly.} \qquad (23)$$

In weir openings, the experiments of J. B. Francis, C. E., make $c=.622$. The top contraction seems to have no separate coefficient in the formula volume. C, the coefficient of discharge, is also the coefficient of average velocity, at and for the entire opening, but not as hitherto remarked for obtaining the actual velocity, as it occurs at M $\text{M}_{,}$, Fig. 1. Introducing this coefficient (.622), or modifier, $(2g)^{\frac{1}{2}}$, into equations (20) and (21), and making $(2g)^{\frac{1}{2}}=8.025$ as found, there results,

$$v_{,}=3.33\left\{\frac{h_{,}^{\frac{3}{2}}-h^{\frac{3}{2}}}{h_{,}-h}\right\} \tag{24}$$

$$v=4.99\left(\frac{h_{,}+h}{2}\right)^{\frac{1}{2}} \tag{25}$$

To determine the velocity of water flowing through a rectangular opening.

Rule 7.—From the square root of the cube of the head of water on the bottom of the opening, subtract the square root of the cube of the head on the top of the opening. Divide the remainder by the difference of these heads, and multiply this quotient by 3.33, the product will be the required velocity.

Rule 7.—Corresponds to equation (24).

Rule 8.—Multiply the square root of one-half the sum of the respective heads on the top and bottom of the opening, by 4.99.

Rule 8.—Corresponds to equation (25).

Ex. 13.—In a rectangular opening, the head on the top of the orifice is 2.25 feet, and on the bottom of the opening is 4 feet. What is the average velocity of the flow of water at the outlet?

Cal. by Rule 7, or formula (24).

Square root of the cube of the head on the bottom of the opening $(4)^{\frac{3}{2}}=8$;

Square root of the cube of the head on the top of the opening $(2.25)^{\frac{3}{2}}=3.375$.

Difference between heads $4-2.25=1.75$.

Thus: $3.33\,(8.-3.375)\div1.75=8.8$ feet.—*Ans.*

Cal. by Rule 8, or formula (25).

Half sum of given heads $(4+2.25)\div2=3.125$; 4.99 times the square root of this quotient.

$4.99\,(3.125)^{\frac{1}{2}}=8.814$ feet.—*Ans.*

Dividing the result obtained under Rule 8 by result obtained under Rule 7.

$8.814\div8.8=1.0016$,

It is seen that in this case, Rule 8, corresponding to formula (25) gives a velocity nearly one-sixth of one per cent too great.

To find the average velocity of water flowing over a weir.

Rule 9.—*Multiply the square root of the head over the crest by* 3.33.

Rule 9 corresponds to formula (24) by making the head on the top nothing.

Rule 10.—Multiply the square root of one-half the head over the crest by 4.99.

Rule 10 corresponds to formula (25) by making the head on the top nothing.

Ex. 14.—In a rectangular outlet, open top, the head on the bottom of the opening is one foot. What is the average velocity of the flow?

Cal.—By Rule 9.

Square root of the given head $(1)^{\frac{1}{2}}=1$.

Then $3.33\times 1=3.33$ feet.—*Ans.*

Cal.—By Rule 10.

Square root of one-half the given head:

$$(.5)^{\frac{1}{2}}=.7072.$$

Then $4.99\times .7072=3.53$ feet.—*Ans.*

Dividing result obtained under Rule 10 by that obtained under Rule 9,

$$3.53\div 3.33=1.06.$$

It is hereby seen that Rule 10 gives a velocity six per cent too great.

Substituting the values, of $(2g)^{\frac{1}{2}}=8.025$, and $c=.622$ of (23) in (18).

$$Q=3.33l\ (h_{,}^{\frac{3}{2}}—h^{\frac{3}{2}} \qquad (26)$$

When $h=0$, (26) becomes

$$Q=3.33l\ h_{,}^{\frac{3}{2}} \qquad (27)$$

Formula (26) is adapted to finding the discharge of a rectangular submerged orifice, and formula (27), the discharge of a weir. In the latter case, when the depth of water on the crest exceeds three inches, and does not exceed two feet, and the length of weir is not less than three times this depth, J. B. Francis, C. E., a most eminent experimentalist, determined by careful experiments made on a large scale, and under the most favorable circumstances, that the loss by means of end contraction, is equal to one-tenth the depth of water over the weir for each such contraction.

Introducing this modification in (27) and there results:

$$Q=3.33\ (l—0.1\ nh_{,})\ h_{,}^{\frac{3}{2}} \qquad (28)$$

In which n denotes the number of end contractions.

For a weir one foot in length with water one foot deep, Nystrom's Mechanics makes the coefficient 3.135 instead of 3.33.

TO FIND THE DISCHARGE OF WATER OVER A WEIR, WITH CORRECTION MADE FOR DEPTH ON CREST.

Rule 11.—Deduct from the length of the weir, one-tenth the depth of water over the crest, for each and every end contraction (usually two); multiply the corrected length so found by 3.33 times the square root of the cube of the depth or head of water on the crest.

Ex. 15.—The length of weir being 3.01 feet, cut in two-inch planks, and the full depth h, over the bottom of the notch 1.023, what is the discharge in cubic feet per second?

Cal.—By Rule 11.

Loss in this case by two end contractions:

$$1.023 \times .2 = .2046.$$

Corrected length: 3.01—.2046=2.8054 feet.

Square root of the cube of the given head:

$$(1.023)^{\frac{3}{2}} = 1.035; \text{ hence,}$$

$Q = 3.33 \times 2.8054 \times 1.035 = 9.67$ cubic feet.—*Ans.*

Working this example by the weir formula given in Weisbach's Mechanics, whence the example is taken, there results:

$Q = 10.22$ cubic feet flow per second, which is five and two-thirds per cent more than obtained by Rule 11, derived from the formula in accordance with experiments of Francis.

This discrepancy would seem to arise mostly from the correction introduced in our rule to compensate for end contraction.

The length of weir being 3.01 feet, and the depth of water on crest .545 feet, the discharge by Rule 11 or formula (28) amounts to 3.887 cubic feet per second, and by Weisbach's formula 4.253, which latter is nine and one-half per cent greater than obtained by Rule 11, or formula 28.

Again, the length of weir being 3.01 feet, and the depth of water on the crest .189 feet, the discharge per second, by Rule 11, amounts to .8133 cubic feet, and by Weisbach's formula .753 cubic feet, which latter is nearly seven and one-half per cent less than the discharge obtained by Rule 11 or formula (28).

In the foregoing examples, the length of weir, the respective depth of water on the crest, and the corresponding coefficient of discharge, are given as the data and results of actual experiments made by W. R. Johnson, editor of the first American edition of Weisbach's Mechanics.

The experiments of Professor Johnson are entitled to great consideration. Those of Mr. Francis, however, from which formula (28), or Rule (11) is derived, were conducted on a large scale with extreme care and with the aid of the most improved mechanical appliances, so as to commend their results as the best authority known at present in weir measurement.

In addition to the variation expressed by the factor $(l—0.1nh)$ in formula (28), the coefficient of discharge is found to vary with the head or depth of water on the crest of the weir.

To compensate for this variation, Table 1 is given, to be used with formula (28).

TABLE I.

WEIR COEFFICIENTS.

Water Supply–Engineering J. T. Fanning.

Depth in in.		1	1½	2	3	4	6	8	10
Depth in feet.		.083	.124	.167	.25	.333	.500	.667	.833
Coefficient......		3.263	3.274	3.285	3.301	3.314	3.329	3.336	3.338
Depth in in.	12	14	16	18	20	24	30	40	48
Depth in feet.	1.000	1.167	1.333	1.500	1.667	2.000	2.500	3.333	4.000
Coefficient......	3.339	3.339	3.340	3.339	3.339	3.338	3.334	3.331	3.317

The mean coefficient, as given in formula (28), is 3.33. The maximum, as seen by Table 1, is 3.34.

The mean coefficient 3.33 corresponds to the depths 523 feet and 3.42 feet.

A comparison shows that the mean coefficient is three-tenths of one per cent (.003) less than the maximum, two per cent greater than that corresponding to a depth of one inch or 0.083 feet; four-tenths of one per cent (.004) greater than that due a depth of four feet; and nine-tenths of one per cent greater than due a depth of one-fourth of a foot. The greatest variation occurring in the coefficient of discharge for different depths, between four feet and one-fourth of a foot, is seen to be below one per cent. An equal variation, for the most part, in practice, is likely to occur from various other causes, and too often elude the observation of the engineer.

Ex. 16.—The length of a weir being six feet, and the full depth of water over the crest two inches, what is the discharge per second?

Cal.—By formula (28) modified by Table 1.

Loss in this case by two end contractions.

2 inches=.167 feet; $.167 \times .2 = .0334$.

$6 - .0334 = 5.9666$ corrected length.

$(.167)^{\frac{3}{2}} = .06824$, square root of cube of given head.

3.285=coefficient as per Table 1, due head of two inches.

$Q = 3.285 \times 5.9666 \times .06824 = 1.307$ cu. feet.—*Ans.*

Ex. 17.—The length of weir being 10.8 feet, and full depth of water over crest four feet, what is the discharge per second?

Cal.—By formula (28) modified by Table 1.

$10.8 - 4 \times .2 = 10$ corrected length of weir.

$(4)^{\frac{3}{2}} = 8$, square root of cube of given head.

3.317=coefficient as per Table 1, due head of four feet.

$Q = 3.317 \times 10 \times 8 = 265.36$ cubic feet.—*Ans.*

TABLE II.

Flow for given depths over each linear foot of a rectangular weir.

Head. Feet.	Flow Cubic Feet.	Head. Feet.	Flow Cubic Feet.	Head. Feet.	Flow Cubic Feet.
.04	.0261	.46	1.0386	1.2	4.3904
.05	.0365	.48	1.1072	1.3	4.9506
.06	.0480	.50	1.1771	1.4	5.5311
.07	.0604	.52	1.2483	1.5	6.1341
.08	.0737	.54	1.3209	1.6	6.7558
.09	.0881	.56	1.3951	1.7	7.3987
.10	.1035	.58	1.4724	1.8	8.0516
.11	.1195	.60	1.5506	1.9	8.7317
.12	.1369	.62	1.6286	2.0	9.4399
.13	.1536	.64	1.7080	2.1	10.1460
.14	.1718	.66	1.7888	2.2	10.8694
.15	.1906	.68	1.8705	2.3	11.6189
.16	.2102	.70	1.9599	2.4	12.3850
.17	.2303	.72	2.0380	2.5	13.1668
.18	.2512	.74	2.1237	2.6	13.9649
.19	.2726	.76	2.2118	2.7	14.7783
.20	.2951	.78	2.2996	2.8	15.6067
.22	.3407	.80	2.3883	2.9	16.4501
.24	.3882	.82	2.4788	3.0	17.3086
.26	.4377	.84	2.5699	3.1	18.1809
.28	.4892	.86	2.6620	3.2	19.0676
.30	.5445	.88	2.7557	3.3	19.9687
.32	.5932	.90	5.8500	3.4	20.7953
.34	.6572	.92	2.9463	3.5	21.7194
.36	.7158	.94	3.0432	3.6	22.6568
.38	.7761	.96	3.1409	3.7	23.6074
.40	.8384	.98	3.2395	3.8	24.5710
.42	.9020	1.00	3.3390	3.9	25.5472
.44	.9717	1.1	3.8522	4.0	26.5360

To obviate the use of a formula so encumbered, Table 2 has been computed. In which the 1st, 3d and 5th columns represent the heads or depths in feet, from the level of still water to the crest; the 2d, 4th and 6th columns represent the quantities discharged per second, for the given depths over each lineal foot of weir. These quantities are the products of the unit length, cubes of the square roots of the given depths, and their respective variable coefficients found in Table 1. If Q_t denote the tabulated quantity, so discharged per lineal foot, $h_{,}$, the given head, and $c_{,}$, the variable coefficient due that head, then we shall have the formula by which Table Q, has been computed, viz.:

$$Q_t = c_{,} h_{,}^{\frac{3}{2}}. \qquad (29)$$

Multiplying (29) by $(l - 0.1\ nh_{,})$, the factor for correcting length of weir as in (28), and putting Q equal discharge over a weir whose length is three or more times the depth of water on crest, and there results:

$$Q = Q_t\ (l - 0.1\ nh_{,}) = c_{,}\ (l - 0.1\ nh_{,}) h_{,}^{\frac{3}{2}} \qquad (30)$$

WHENCE TO FIND THE DISCHARGE OF WATER OVER A WEIR WITH CORRECTIONS MADE DUE VARIABLE COEFFICIENT, AND DEPTH ON CREST.

Rule 12.—Deduct from the given length of the weir one-tenth the depth of water on the crest, for each and every end contraction, and multiply the length so corrected by the quantity in "flow" column opposite the given head in Table 2.

Rule 12 is derived from Eq. 30—middle right hand member employed. The extreme right hand member of the same equation expresses the value of Q in terms of the corrected length, and the head and the variable coefficient as found in Table 1.

This, in fact, is the modified formula of (28), by which examples 16 and 17 were solved.

Ex. 18.—The depth of water being two feet over a crest seven feet in length, what is the discharge?

Cal.—By Rule 12.

$7-2\times.2=6.6$ feet, corrected length.

By Table 2.

9.4299 cubic feet=discharge due head of two feet.

Whence, $9.4399\times6.6=62.30$ cubic feet.—*Ans.*

Cal.—By formula (30), extreme right hand member.

$7-2\times.2=6.6$ feet, corrected length.

$(2)^{\frac{3}{2}}=2.8284$=square root of cube of depth.

3.338=coefficient by Table 1, due head of two feet.

$Q=3.338\times6.6\times2.8284=62.32$ cubic feet.—*Ans.*

The calculation in which Table 2 is employed, is seen to be much shorter and far more simple than that in which Table 1 is employed.

Weirs are usually constructed with horizontal crests and vertical ends, forming a notch, through which the flow of water is measured in its passage from a reservoir, or other storing place; or in its passage over a submerged dam across a flume, canal or natural stream. A weir should be free from vibration. Its crest and ends should be chamfered on the downstream side to an edge—say one-tenth of an inch thick. Its upstream face should be vertical, and its downstream face, so inclined or fashioned as not to resist the flow of water. On the upstream side, the depth of water below the level of the crest should be fully twice as great as the head of water on the crest. The head or depth is the vertical distance, as AC, Fig. 2, between the crest and the level of still water at a point some distance above the weir, as at *a*. In ordinary practice, the head is measured with a common rule or linear measuring scale, from the top of a post, P, in Fig. 2, set level with the crest. If greater accuracy is required, the "Boyden Hook Gauge" should be employed. Care must be taken that the flow over the weir shall not be affected by the approaching stream. The area of the weir opening should not exceed a fifth part of that of the supply stream. With proper care in taking the data, the weir affords very accurate means of measuring the flow of water.

This, taken in connection with the weir's simplicity and facility of construction, cheapness and wide application, renders it of great practical importance, especially to those concerned in the measurement of flowing water, in places of difficult access. A temporary dam, in illustration, built across a natural stream, with a crest board firmly fitted, level and vertically widthwise to the dam's top, makes a good measuring weir. In flumes and canals, measuring weirs can obviously be constructed with equal facility.

The weir crest being about three feet wide and level, with a rising incline to its receiving edge, Mr.

Francis offers the following formula, for approximate measurements, for depths between six and eighteen inches:

$$Q = 3.01208\ l h_{,}^{1.53} \qquad (31)$$

As this formula is somewhat complicated, the writer would present Table 3, computed from it, which will be found far simpler and less tedious of application.

TABLE 3.

Flow for given depths over each lineal foot of weir, with crest three feet wide.

Head.	Flow Cubic Feet	Head.	Flow Cubic Feet	Head.	Flow Cubic Feet
6 inches.	1.043	10.5 inches	2.445	15. inches.	4.238
7.5 "	1.467	12 "	3.012	16.5 "	4.903
9 "	1.939	13.4 "	3.607	18 "	5.601

The depth or head being given.

To find the flow of water over a weir whose crest is three feet wide.

Rule 13.—From "flow" column, opposite the given head, in Table 3, take the number representing the discharge in cubic feet over one lineal foot, which multiply by the given length of the weir.

Ex. 19.—The head being 15 inches, and the length of weir, whose crest is three feet, being ten feet, what is the approximate discharge in cubic feet per second?

Cal.—In Table 3, in "flow" column, opposite 15 inches, given head, find 4.238 cubic feet.

Whence, $4.238 \times 10 = 42.38$ cubic feet.—*Ans.*

TRIANGULAR WEIRS.

A triangular form of measuring weir has been employed with favorable results.

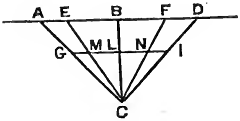


FIG. 3.

To determine the flow of water through a triangular weir:

In Fig. 3 let h=BC represent the head of water in feet, from the apex C to the level, AD, of still water.

p=given ratio between the head, h, and the width of the weir—that is, let ph=AD, EF, or any width taken at pleasure.

c=coefficient of discharge.

Q=quantity of flow in cubic feet.

x=any portion of the head h.

$2g$=force of gravity=32.2.

v=velocity due $(h—x)$.

Then in general by (13) will $v=c\ (2g)^{\frac{1}{2}}(h—x)$ (32)

$$dQ=pc\ (2g)^{\frac{1}{2}}(h-x)^{\frac{1}{2}}x\ dx. \qquad (33)$$

Integrating (33) between the limits of $x=0$ and $x=h$:

$$Q=\frac{4\,pc}{15}\left(2g\right)^{\frac{1}{2}}h^{\frac{5}{2}} \text{ cubic feet.} \qquad (34)$$

Quadrantal Weir.

In a quadrantal weir, that is, a weir in which the angle ACD=90°—a right angle—AD=2BC=$2h=ph$; hence, $p=2$. Making $c=.616$; $(2g)^{\frac{1}{2}}=8.025$.

Substituting these values of pc and $(2g)^{\frac{1}{2}}$ in (34).

$$Q=2.6365\ h^{\frac{5}{2}} \text{ per second.} \qquad (35)$$

If, in (35), h'' representing inches be substituted for h, which represents feet, and the unit of time be made one minute, we shall have the following formula, which is attributed to Professor James Thompson, of Glasgow University:

$$Q=0.317\ h''^{\frac{5}{2}} \text{ per minute.} \qquad (36)$$

If a second be the unit of time, and the head be in inches, we have:

$$Q=.005385\ h''^{\frac{5}{2}} \text{ per second.} \qquad (37)$$

Professor Thompson having experimented satisfactorily with the quadrantal weir, pronounces it more simple and reliable than the rectangular weir, in that the ratio between the head of water and the horizontal width of notch is constant; in that the flow of water through it is less effected by the "depth from the crest to the bottom of the channel of approach," and finally, in that the coefficient of discharge is constant for different depths. In the experiments of Prof. Thompson, "the volumes of water," says J. T. Fanning, C. E., "varied from .033 to .6 cubic feet per second." From this statement it is deducible that the depths of water varied from two inches to 6.6 inches (see Table 4), though the depths given by Mr. Fanning vary from two to four inches.

The simplicity of this weir, its cheapness, and the assurances of its superiority, as stated, have induced the computation of Table 4 for practical use.

TABLE 4.

Flow of Water per Second over a Quadrant Weir.

Head Inch.	Flow Cub. Feet	Head Inch.	Flow Cub. Feet	Head Inch.	Flow Cub. Feet	Head Inch.	Flow Cub. Feet
1.	.0053	4.	.1691	7.	.6852	10.	1.6713
1.1	.0067	4.1	.1799	7.1	.7100	10.1	1.7134
1.2	.0083	4.2	.1911	7.2	.7352	10.2	1.7562
1.3	.0102	4.3	.2026	7.3	.7610	10.3	1.7995
1.4	.0123	4.4	.2146	7.4	.7873	10.4	1.8435
1.5	.0146	4.5	.2270	7.5	.8142	10.5	1.8882
1.6	.0171	4.6	.2398	7.6	.8416	10.6	1.9334
1.7	.0199	4.7	.2531	7.7	.8696	10.7	1.9794
1.8	.0230	4.8	.2668	7.8	.8981	10.8	2 0260
1.9	.0263	4.9	.2809	7.9	.9271	10.9	2.0732
2.	.0298	5.	.2954	8.	.9567	11.	2.1211
2.1	.0338	5.1	.3105	8.1	.9869	11.1	2.1696
2.2	.0379	5.2	.3259	8.2	1.0177	11.2	2.2187
2.3	.0424	5.3	.3418	8.3	1.0489	11.3	2.2686
2.4	.0472	5.4	.3593	8.4	1.0808	11.4	2.3192
2.5	.0522	5.5	.3750	8.5	1.1133	11.5	2.3703
2.6	.0576	5.6	.3922	8.6	1.1463	11.6	2.4222
2.7	.0633	5.7	.4100	8.7	1.1799	11.7	2.4748
2.8	.0693	5.8	.4282	8.8	1.2142	11.8	2.5280
2.9	.0757	5.9	.4468	8.9	1.2489	11.9	2.5818
3.	.0824	6.	.4661	9.	1.2843	12.	2.6365
3.1	.0894	6.1	.4857	9.1	1.3203	13.	3.2205
3.2	.0968	6.2	.5059	9.2	1.3568	17.	6.2979
3.3	.1046	6.3	.5266	9.3	1.3941	19.	8.3167
3.4	.1126	6.4	.5477	9.4	1.4318	23.	13.4089
3.5	.1211	6.5	.5693	9.5	1.4702	29.	23.9369
3.6	.1300	6.6	.5915	9.6	1.5092	31.	28.2800
3.7	.1391	6.7	.6141	9.7	1.5490	37.	44.0128
3.8	.1488	6.8	.6373	9.8	1.5890	41.	56.8896
3.9	.1588	6.9	.6610	9.9	1.6299	43.	64.0830
						47.	80.0420

Table 4 has been computed from formula (37), in which the head is in inches, and the quantity discharged per second is in cubic feet. The tabulated discharges for depths of water over six inches requires the confirmation of experiment; until that shall be had, however, there seems good reason from the data to regard them approximately correct.

Application of Table 4.

Ex. 20.—The depth of water in a quadrantal weir being 2.1 inches, what is the discharge over it in cubic feet per second?

Cal.—In "head" column, Table 4, find the given head 2.1 inches, opposite which, in "flow" column, will be found .0338 cubic feet: the quantity sought.

To determine the flow of water over a quadrantal weir, the head given being equal to the product of two factors, each designating a head of water in Table 4.

Rule 14.—Multiply the product of the discharges due the factor heads by 189.2.

Ex. 21.—The head in a quadrantal weir being 15 inches, what is the discharge per second?

Cal.—Observe that the factors of the given head are three and five: $3 \times 5 = 15$.

By Table 4, flow due 5″ head=.2954.

By Table 4, flow due 3″ head=.0824.

Hence, $.2954 \times .0824 \times 189.2 = 4.6054$ cubic feet.—*Ans.*

Ex. 22.—The head in a quadrantal weir being 42 inches, what is the discharge per second?

Cal.—Observe that the factors of the given head are six and seven: $6 \times 7 = 42$ inches.

By Table 4, flow due 6″ head = .4661.

By Table 4, flow due 7″ head = .6852.

Hence, $.4661 \times .6852 \times 189.2 = 60.4258$ cubic feet.—*Ans.*

The application of Rule 14 to depths below 13 inches will be unnecessary; the rule is given to avoid an extended table.

EQUILATERAL WEIR.

To determine the flow of water over an equilateral weir, ECF, in Figure 3, make $p = \frac{2}{\sqrt{3}}$. That is when h is the hight of an equilateral triangle, its side or width as in ECF, is $\mathrm{EF} = ph = \frac{2h}{\sqrt{3}}$ Substituting this value of p in Eq. (33), there results:

$$Q = 1.52217\ h^{\frac{5}{2}} \text{ per second.} \tag{38}$$

Were the head given in inches, and the quantity of flow required in cubic feet per minute, then would

$$Q = 1.77\ h_{\prime\prime}^{\frac{5}{2}} \text{ per minute.} \tag{39}$$

The heads being equal in the two forms of triangular weirs, thus far considered, then will the discharge in the equilateral form be to the discharge in the quadrantal form as .57735 is to 1. Hence, to find by Table 4, the discharge over an equilateral weir, the head being given:

Rule 15.—Find in Table 4 the discharge due the given head over a quadrantal weir. Multiply the quantity so found by .57735.

Ex. 23.—The head of water in an equilateral weir is eight inches. What is the discharge in cubic feet per second?

Cal.—By Table 4, it is seen that the "flow" due the given head, eight inches, is .9567 cubic feet per second. Hence, $.9567 \times .57735 = .5524$ cubic feet.—*Ans.*

In Eq. (34) any value may be given p, as 1, 2, 3, 4, 5, etc., so as to indicate the relations existing between the hight and width of triangular weirs. But since c, the coefficient of discharge, varies with every different condition imposed, the labor of determining the theoretical flow due any considerable number of such forms would necessarily be barren of practical results.

In general let (34) be reduced to its simplest form:

$$Q = 2.14\, pc\, h^{\frac{5}{2}}. \qquad (40)$$

In which h denotes feet, nd Q cubic feet.

If in (40) we make $p=2$ and $c=.616$, the formula becomes that of the quadrantal weir, as shown by Eq. (35).

If we make $p=\frac{2}{\sqrt{3}}=1.1547$, and $c=.616$, the formula becomes that of the equilateral weir, as shown by Eq. (38.)

If we make $p=2\sqrt{3}=3.4641$, and $c=.616$, the formula becomes that of a weir, whose apex or angle C=120°; Fig. 3.

$$Q=4.5665\ h^{\frac{3}{2}}. \qquad (41)$$

And in this manner may special formulas be deduced from (40), to meet the various requirements of triangular weirs. In theory the results obtained are as they should be; but in fact, experiment best determines in what cases c is constant, and in what variable.

TRAPEZOIDAL WEIRS.

To determine the flow of water over a trapezoidal weir, let, in Fig. 3, ACD represent a triangular weir, in which GI is a line indicating a division with respect to flow of water over the weir.

The flow of the triangular portion, GCI, taken from the entire flow due ACD, there remains that portion of the flow due the trapezoid DAGI. The mean velocity of the water in ACD is evidently greater than the mean velocity is in DAGI, else the flow in the trapezoid could readily be determined from that of ACD by the simple ratio of these areas.

Let, in the present solution, h, p, c, Q, g and v have the same values or functions which they had in the discussion hitherto of triangular weirs. In addition put CL$=nh$;

$$\text{Then } BL=(1-n)\ h, \qquad (42)$$

the depth of the trapezoid DAGI.

Integrating equation (33) between the limits of $x=0$

and $x=nh$, and there results the flow per second due the triangular portion of the weir GCI.

$$Q_{,}=pc\ (2g)^{\frac{1}{2}}\left(-\tfrac{2}{3}\ (1-n)^{\frac{3}{2}}+\tfrac{2}{5}\ (1-n)^{\frac{5}{2}}+\tfrac{4}{15}\right)h^{\frac{5}{2}}. \qquad (43)$$

Deducting (43) from (34), there remains:

$$Q=\tfrac{2}{15}\ pc\ (2g)^{\frac{1}{2}}\left(2+3n\right)\ (1-n)^{\frac{3}{2}}h^{\frac{5}{2}}. \qquad (44)$$

Formula (44) represents the discharge due the trapezoid DAGI.

$$Q=1.07pc\ \left(2+3n\right)\ (1-n)^{\frac{3}{2}}h^{\frac{5}{2}}. \qquad (44)$$

Ex. 24.—The width of a trapezoidal weir being two feet, the depth one-fourth ($\frac{1}{4}$) of a foot, the sides inclining 45° to the horizon, and the coefficient of discharge being .62, required the cubic feet flow over it per second?

Cal.—Since the width is two feet, and the inclination of the sides 45°, were the sides produced downward till they meet, the depth of the triangle so formed would be equal to one-half the given width—that is, $h=1$ foot, and $p=2$.

The given depth of weir being $\frac{1}{4}$, hence, $n=(1-\frac{1}{4})=\frac{3}{4}$.

Making substitution of these values in (44),

$$2+3\times\tfrac{3}{4}=4.25;$$

$$(1-\tfrac{3}{4})^{\frac{3}{2}}=\tfrac{1}{8};$$

(1) $^{\frac{5}{2}}=1$. We have

$$Q=1.07\times2\times.62\times4.25\times\tfrac{1}{8}=7.05 \text{ cubic feet.}—Ans.$$

Ex. 25.—The width of a trapezoidal weir being 4.5 feet, the depth one foot, the sides inclining 45° to the horizon, and the coefficient of discharge being .62, required the cubic feet flow over it per second.

Cal. 1st.—The width being 4.5 feet, and the inclination of the sides 45°, if the sides be produced till they meet, the depth of the triangle so formed will be $2.25=\frac{9}{4}$ feet.

$2.25-1=1.25$; $n=\frac{125}{225}=\frac{5}{9}$; $1-n=1-\frac{5}{9}=\frac{4}{9}$; $p=2$. Substituting these values in formula (44),

$Q=1.07\times2\times6.2\ \left(2+\frac{15}{9}\right)\ (\frac{9}{4})^{\frac{3}{2}}(\frac{4}{9})^{\frac{5}{2}}=10.946$ cubic feet.—*Ans.*

Cal. 2d.—The bottom width $1.25\times2=2.5$ feet.

Regard the trapezoidal weir made up of a rectangular weir, whose length is 2.5 feet and depth one foot, and of a quadrantal weir, whose width is two feet and depth one foot. Then:

The quantities of "flow" are found to be by Table 2, for each linear foot of crest, 3.339 cubic feet; hence, for 2.5 feet, $3.339\times2.5=8.3475$ cubic feet, and by Table 4, for "flow" over a quadrantal weir one foot deep$=2.6365$ cubic feet.

Hence, $8.3475+2.6365=10.984$ cubic feet.—*Ans.*

The discrepancy between the results obtained by the 1st and 2d calculations arises from the different values assigned the coefficients of discharge.

By *Cal.* 1st the coefficient was taken, as proposed in the given problem, at .62.

By *Cal.* 2d, the coefficient employed in computing Table 2, was taken from Table 1, which will be seen to be 3.339. So that the coefficient of discharge employed in Table 2, was, in fact, $3.39 \div \frac{2}{3}(8.025) =$ .6241, instead of .62, as provided in the given proposition. The coefficient of discharge employed in computing Table 4, was .616, which was deduced from the formula given by Prof. Thompson. Weisbach's Mechanics and Engineering state that the coefficient employed by Prof. Thompson, was .619, while J. W. Stone, C. E., author of Hydraulic Formula, states that it was .617.

Cal. 3d.—The top width is given 4.5 feet. The bottom width $= (2.25 - 1) \times 2 = 2.5$ feet. Mean width $(4.5 + 2.5) \div 2 = 3.5$ feet.

By Francis' formula $(3.5 - .1 \times 2 \times 1) = 3.3$, corrected length.

By Table 2, flow over each linear foot, 3.339 cubic feet. Hence, $3.339 \times 3.3 = 11.008$ cubic feet.—*Ans.*

This result differs but little from those obtained from more rigorous solutions.

Flow of water over trapezoidal weirs in which the length of the crest is greater than the top width of the notch.

Let ABCD, Fig. 4, represent a trapezoidal weir in which the length of the crest AB$=b$ is greater than the width of the water surface CD$=t$.

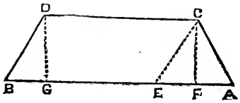


FIG. 4.

Draw CE parallel to DB, and let fall on BA, the perpendicular DG, CF. Then will the rhomboid, ECDB, equal in area the parallelogram FCDG, and the entire weir, ACDB, equal in area FCDG+ACE.

Let h=the head or depth of water between the levels of the crest and still water; p=the ratio of the base, EA, to hight, FC, of the triangle, ACE; c=co-efficient of flow; x=any head not greater than h.

$$\text{Then } dQ=c\ (2g)^{\frac{1}{2}}(t+px)\ x^{\frac{1}{2}}dx. \tag{45}$$

Integrating (45), observing that $Q=0$,
When $x=0$,

$$Q=c\ (2g)^{\frac{1}{2}}(\frac{10}{15}t\ h^{\frac{3}{2}}+\frac{6}{15}p\ h^{\frac{5}{2}}). \tag{46}$$

Reducing (46), observing that $b=3\ (t+ph)$,

$$Q=\frac{2}{15}c(2g)^{\frac{1}{2}}(2t+3b)h^{\frac{3}{2}}. \tag{47}$$

Making $b=ph$, $t=0$, that is, closing the top of the weir, at the level of still water, and equation (46) becomes

$$Q=\frac{6pc}{15}\ (2g)^{\frac{1}{2}}h^{\frac{5}{2}} \text{ cubic feet.} \tag{48}$$

Comparing equations (48) and (34),

$$Q_{48} : Q_{34} :: \frac{6}{15} : \frac{4}{15} :: 3 : 2. \qquad (49)$$

Equation (49) shows that, with respect to two triangular weirs of equal size, the discharging capacity of the weir whose top is closed at the level of

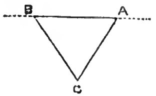


FIG. 5.

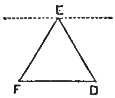


FIG. 6.

still water, as E in Fig. 6, is, to the discharging capacity of the weir whose top is open, as AB in Fig. 5, as 3 is to 2, or 1.5 : 1. Table 4 was computed from an open weir, as ABC, Fig. 5. To render it applicable to a closed weir, as DEF:

Rule 16.—Multiply the tabulated flow due any head given in Table 4 by 1.5.

Ex. 26.—In a trapezoidal weir, the head between the crest and the level of still water being three (3) inches, the length of the crest two (2) feet, and the width of the opening at water level one and three-fourths (1.75) feet, what is the flow in cubic feet per second when the coefficient of discharge is .62 ?

Cal.—Head 3 inches $=\frac{1}{4}$ feet.

By formula (47), $(\frac{1}{4})^{\frac{3}{2}}=\frac{1}{8}$.

Three times bottom width: $2\times3=6$.

Twice top width: $1.75\times2=3.5$.

Whence, $1.07\times.62\ (3.5+6)\ \frac{1}{8}=.7878$ cubic feet.—*Ans.*

Ex. 27.—In a trapezoidal weir, the head being twelve inches, the bottom width three feet, the top width one foot, and the coefficient of discharge .624, what is the flow of water over it in cubic feet per second? By formula (47).

Cal. 1st.—$1.07\times.62\ (1\times2+3\times3)\times1=7.3445$ cubic feet.—*Ans.*

Cal. 2d.—Observe that the weir opening, ACDB, Fig. 4, is resolvable into two parts, to wit: the part CDBE, which is equal to the rectangle CDGF, and the triangular part ACE, whose crest is AE, and whose top is closed at C, at the level of still water. Applying, in Example (27), Table 2, to the rectangular part CDGF, substituted for the part CDBE, and Table 4, to the triangular part ACE:

By Table 2, due one foot head, one foot crest, 3.3390 cubic feet.

By Table 4, due in quadrantal weir with open top, 12-inch head, 2.6365.

By Rule 16, $2.6365\times1.5=3.9547$.

Hence $3.3390+3.9547=7.2937$ cubic feet.—*Ans.*

The discrepancy between calculating 1st and 2d arises from Table 4, in the computation of which .616 on the authority of Prof. Thompson, was employed as the coefficient of discharge, instead of .624, as proposed in the given example.

FLOW OF WATER OVER A RECTANGULAR WEIR, HAVING ITS ANGLES, HORIZONTAL AND VERTICAL, AND ITS UPPERMOST ANGLE ON THE LEVEL OF STILL WATER.

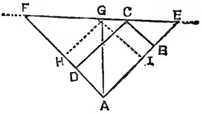


FIG. 7.

Let ABCD, of Fig. 7, represent a rectangular weir, having its vertical angle C, at the level of still water. Through C, draw a horizontal line indefinitely. Produce AB, AD, intersecting this horizontal line in E and F. Draw $AG=h$ perpendicularly to EF, and bisecting A.

Let $AB=m$, and $AD=n$. It is obvious from the imposed conditions, that $AE=AF=m+n$; that $EF=2\,GE=2\,GF=2\,AG=2\,h$; that $FD=DC=AB=m$, and $BE=BC=AD=n$.

Observe that in Fig. 7 are represented three open quadrantal weirs, FAE, FDC and CBE, and that the given rectangle ABCD=FAE—FDG—CBE. (50)

Denote by $Q_{\prime\prime}$ the flow of the given rectangular weir.

By similar triangles, find the heads or depths as follows:

$$PD=\left\{\frac{mh}{m+n}\right\} \qquad (51)$$

$$\text{LB}=\left\{\frac{nh}{m+n}\right\} \tag{52}$$

Substituting these values, and the value of $\text{AG}=h$ in (34), noting that for quadrantal weirs, $p=2$,

$$Q_{\prime\prime}=\tfrac{8}{15}\ c\ (2g)^{\frac{1}{2}}\left\{1-\left\{\frac{m^{\frac{5}{2}}+n^{\frac{5}{2}}}{(m+n)^{\frac{5}{2}}}\right\}\right\}h^{\frac{5}{2}} \tag{53}$$

If in (53), the general formula for the flow of water through a rectangular weir having its uppermost angle vertical at the level of still water, we make m equal n, and substitute the value of $(2g)^{\frac{1}{2}}=8.025$, and $c=.616$,

$$Q_{\prime\prime}=1.70435h^{\frac{5}{2}}. \tag{54}$$

In which case the rectangle becomes a square, as represented in Fig. 7, by AIGN.

Comparing formula (54) with formula (35), which is for the flow of water over a quadrantal weir, we have:

$$Q_{\prime\prime}=\frac{1.70435}{2.6365}=.6464\ Q. \tag{55}$$

To determine the flow of water through a square weir, having its uppermost vertical angle at the level of still water.

Rule 17.—According to formula (54), multiply the square root of the fifth power of the head or depth by 1.70435; or by formula (55), multiply the flow in Table 4 for the given head by .6464.

Ex. 27.—The head being 3 inches $=\frac{1}{4}$ foot in a rectangular weir, having its upper most vertical angle at level of still water, what is the flow in cubic feet per second?

Cal. 1st.—By formula (54).

Fifth power of the square root of $(\frac{1}{4})^{\frac{5}{2}}=\frac{1}{32}$.

$1.70435\times\frac{1}{32}=.05326$ cubic feet. $=$ *Ans.*

Cal. 2d.—By Rule 17, second part.

By Table 4, flow due 3-inch head $=.0824$. Then $.0824\times.6464=.05326$ cubic feet.—*Ans.*

Ex. 28.—The sides of a rectangular weir, with its angles vertical and horizontal being 2 feet and 1 foot, the coefficient of discharge being .62, what is the flow per second?

Cal.—Employ formula (53).

Taking the given data $m=2$, $n=1$,

Then $m+n=3$, and (see Fig. 7),

$\text{AG}=h=\left(\frac{m+n}{}\right)\sqrt{2}=\frac{3}{2}\sqrt{2}=2.12127 h^{\frac{5}{2}}=(2.12127)^{\frac{5}{2}}=$ 6.55376.

By Table 5, $m^{\frac{5}{2}}=(2)^{\frac{5}{2}}=5.657$; $n^{\frac{5}{2}}=(1)^{\frac{5}{2}}=1$.

By Table 5, $(m+n)^{\frac{5}{2}}=(3)^{\frac{5}{2}}=15.590$.

Substituting these values of $m^{\frac{5}{2}}$, $n^{\frac{5}{2}}$, $(m+n)^{\frac{5}{2}}$, $h^{\frac{5}{2}}$, $(2g)^{\frac{1}{2}}=8.025$, and $c=.62$. $Q_{{\prime\prime}}=\frac{8}{15}\times.62\times8.025\left(1-\frac{5.657+1}{15.59}\right)6.55376$.

Whence, $Q_{{\prime\prime}}=9.9644$ cubic feet.—*Ans.*

FLOW OF WATER THROUGH CIRCULAR AND SEMI-CIRCULAR WEIRS.

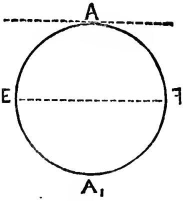


FIG. 8.

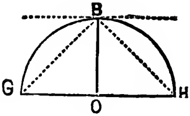


FIG. 9.

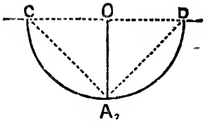


FIG. 10.

Let Figs. 8, 9 and 10, represent respectively the circular and semi-circular weirs, Fig. 8 touching the water surface at A, Fig. 9 in a similar manner at B, and Fig. 10 at its diameter CD.

Let r feet denote the radius with which the several weirs are described, then in both Figs. 9 and 10 will r denote the head, while in Fig. 8 the head (maximum) will be $2r$.

Let x in Fig. 8 denote any portion of the head, and $Q_{\prime}$ the flow in cubic feet, $(2g)^{\frac{1}{2}}$, acceleration of gravity, and c coefficient of discharge.

$$\text{Then } dQ_{\prime}=2c(2g^{\frac{1}{2}}\, 2\, (r)^{\frac{1}{2}} \left(1-\frac{x}{2r}\right)^{\frac{1}{2}} xdx. \qquad (56)$$

Integrating (56) between limits of $x=0$, and $x=2r$, and substituting the value of $(2g)^{\frac{1}{2}}=8.025$,

$$Q_{\prime}=24.2129\ cr^{\frac{5}{2}}. \qquad (57)$$

Let Q_2=the flow in Fig. 9 per second. Integrating (56) between limits of $x=0$, and $x=r$, there results the flow in that portion of Fig. 8 represented by FAE, which is equal to HBG, Fig. 9, the discharge sought, viz.:

$$Q_2=9.2313\ cr^{\frac{5}{2}}. \qquad (58)$$

Again in Fig. 10: Let x denote any portion of the head from A, and Q_3 the flow in cubic feet per second; Then

$$dQ_3=(2g)^{\frac{1}{2}}(r^2-x^2)^{\frac{1}{2}}x^{\frac{1}{2}}dx. \qquad (59)$$

Integrating (59) between limits $x=0$, and $x=r$, and substituting value of $(2g)^{\frac{1}{2}}=8.025$,

$$Q_3=7.6932\ cr^{\frac{5}{2}}. \qquad (60)$$

Comparing equations (60) and (35) and making $c=.616$, and $r=h$,

$$_3=1.79Q. \qquad (61)$$

To find by Table 4 the flow through a semi-circular weir: Open at the top as represented by CD Fig. 10.

Rule 18.—Multiply the flow in Table 4 for the given head or radius by 1.79. See formula (61).

The triangle CA_2D, inscribed in the semi-circular weir, Fig. 10, represents a quadrantal weir whose flow is Q, while the flow of the semi-circular weir CA_2D is Q_3.

Comparing equations (58) and (35) and making $c=.616$, and $r=h$,

$$Q_2=2.1568Q. \qquad (62)$$

To find by Table 4, the flow through a semi-circular weir closed at the top, as represented at B Fig. 9.

Rule 19.—Multiply the flow in Table 4 for the given head or radius by 2.1568 (62).

To find by Table 4 the flow through a circular weir touching the water surface at A as represented in Fig. 8.

Comparing equations (57) and (35) and making $c = .616$, and $r = h$,

$$Q_{\prime} = 5.6566\ Q. \qquad (63)$$

Rule 20.—Multiply the flow in Table 4 for the head or depth equal to the given radius of the circular weir or opening by 5.6566. See formula (63).

Ex 29.—In a semi-circular weir, with open top, as represented by Fig. 10, the head or radius is ten inches. What is the discharge in cubic feet per second?

Cal.—By Table 4 the f w due a head of 10 inches is 1.6713 cubic feet.

By Rule 18 we have—

1.6713×1.79=2.9916 cubic feet.—*Ans.*

Ex. 30.—In a semi-circular weir or opening with closed top, as represented by Fig. 9, the head or radius being ten inches, what is the flow in cubic feet per second ?

Cal.—By Table 4 the flow due head of ten inches is 1.6713 cubic feet.

By Rule 19 there results—

1.6713×2.1568=3.6047 cubic feet.—*Ans.*

Ex. 31.—In a circular weir or opening touching the water surface, as at A; Fig. 8, the radius is eight inches, required the cubic feet flow per second.

Cal. 1st.—By Table 4 the flow due a head of eight inches is .9567 cubic feet.

By Rule 20 we have—

$.9567 \times 56.566 = 5.4117$ cubic feet.—*Ans.*

Cal. 2d.—By formula (57): 8 inches $= \frac{2}{3}$ feet.

By Table 5, $(\frac{2}{3})^{\frac{5}{2}} = \frac{5.657}{15.59} = .3629$ nearly:

$24.2129 \times .616 \times .3629 = 5.4117$ cubic feet.—*Ans.*

Flow of water through parabolic weirs.

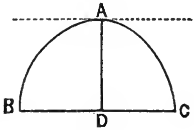


Fig. 11.

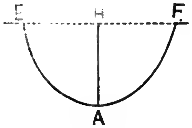


Fig. 12.

Let Figs. 11 and 12 represent parabolic weirs, touching the water surface, Fig. 11, at its apex, A, and Fig. 12 at its inverted base, EF.

Let h, in each weir, as AD or HA, denote the head, and let b denote the base, as BD, DC or EH, FH.

Let c = the coefficient of discharge.

Q_4 = the quantity discharged in cubic feet per second, and $(2g)^{\frac{1}{2}} = 8.025$ due gravity.

Let x = any part of the head, estimated from A, in Fig. 11.

The equation of the parabola, in which x and y are co-ordinates, is:

$$y^2=2px;\ \text{whence}\ y=(2p)^{\frac{1}{2}}x^{\frac{1}{2}}. \qquad (64)$$

The equation of flow is:

$$dQ_4=c\ (2g)^{\frac{1}{2}}(2p)^{\frac{1}{2}}x\,dx. \qquad (65)$$

Integrating (65) between limits $x=0$, and $x=h$; and substituting the values of $(2p)^{\frac{1}{2}}=\frac{b}{h^{\frac{1}{2}}}$, $(2g)^{\frac{1}{2}}=8.025$;

$$Q_4=4.0125\ c\ b\ h^{\frac{3}{2}}. \qquad (66)$$

Let Q_5=the flow in weir represented by Fig. 12.

$$dQ_5=c\ (2g)^{\frac{1}{2}}(2p)^{\frac{1}{2}}h^{\frac{1}{2}}(1-\tfrac{2}{h})^{\frac{1}{2}}x^{\frac{1}{2}}dx. \qquad (67)$$

Integrating (67) between limits $x=0$, and $x=h$, and substituting the values of $(2p)^{\frac{1}{2}}=\frac{b}{h^{\frac{1}{2}}}$, and $(2g)^{\frac{1}{2}}=8.025$;

$$Q_5=3.1546\ c\ b\ h^{\frac{3}{2}}. \qquad (68)$$

Assume any ratio, n, to exist between the base b, and the hight or head, h:

$$\text{As}\ b=nh. \qquad (69)$$

$$\text{Then}\ Q_5=3.1546\ n\ h^{\frac{5}{2}}. \qquad (70)$$

This formula is adapted to the finding of the flow of water over both shallow and deep weirs. Thus by making n successively equal to 1, 2, 3, 4, 5, 6, 7, etc., the represented flow in (76) becomes correspondingly affected. To accomplish a similar result by the semi-circular weir, would be no easy task, requiring the employment of a very intricate and unwieldy formula or extensive table.

Making in Eq. 70, $n=2$, there results:

$$Q_5=6.3092ch^{\frac{5}{2}}. \qquad (71)$$

In this case, $b=2h$, and hence:

Comparing equations (71) and (35), and making $c=.616$,

$$Q_5=1.4734Q. \qquad (72)$$

TO FIND, BY TABLE 4, THE FLOW OF WATER OVER A PARABOLIC WEIR, WITH AN OPEN TOP—THE WIDTH BEING EQUAL TO TWICE THE DEPTH OR HEAD.

Rule 21.—Multiply the flow in Table 4 for the head or depth, equal the given head, by 1.4734. See formula (72).

Ex. 32.—In a parabolic weir, with open top, the head is 11 inches, and the width 22 inches. What is the discharge of water through it in cubic feet per second?

Cal. by Table 4.—Flow corresponding to head of 11 inches, 2.1696 cubic feet.

Then by Rule 21:

$2.1696 \times 1.4734 = 3.1967$ cubic feet.—*Ans.*

Comparing equations (70) and (35), and making $c = .616$,

$$Q_5 = .73705 Qn. \qquad (73)$$

To find, by Table 4, the flow of water over a parabolic weir, with open top—the width being a given number, n times, the head or depth.

Rule 22.—Multiply the flow in Table 4, for the head or depth, equal to the given head, by .73705 times the ratio between the given head and width. See formula (73).

Ex. 33.—In an open parabolic weir, the head being 10 inches, and the width 50 inches—that is, 5 times 10 inches ($n=5$), required the cubic feet flow per second.

Cal.—By Table 4, flow due 10 inches, 1.6713.

By Rule 22.—$1.6713 \times .73705 \times 5 = 6.1592$ cubic feet.—*Ans.*

Comparing equations (66) and (35), making $b = nh$, as in (69), and $c = .616$,

$$Q_4 = .9375 Qn. \qquad (74)$$

To find the flow of water through a parabolic weir whose apex, A, Fig. 11, is at the level of still water.

Rule 23.—Multiply the flow in Table 4, due the head or depth equal to the given head, by .9375 times the ratio, n, between the given head and width.

Ex. 34.—In a parabolic opening or weir, whose apex reaches the surface of still water, the head or depth being 23 inches, and the width 230 inches—that is, 10 times 23 inches ($n=10$), required the flow in cubic feet per second.

Cal.—By Table 4, flow due 23 inches, 13.4089.

By Rule 23, $13.4089 \times .9375 \times 10 = 125.71$ cubic feet.—*Ans.*

Flow of water through a submerged triangular opening, having its vertex below te base.

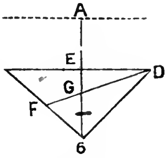


Fig. 13.

Let, in Fig. 13, BCD represent the opening, in which b=BD, the width at top; h=AE, head on the top of orifice; $h_{\prime}$=AC, head on its bottom; c=coefficient of discharge; $(2g)^{\frac{1}{2}}$=acceleration of gravity; Q=discharge in cubic feet per second; x=any part of EC, estimated from E; a=EC, depth of opening.

$$\text{Then } dQ=c\,(2g)^{\frac{1}{2}}(h+x)^{\frac{1}{2}}\left(\frac{a-x}{a}\right)dx. \qquad (75)$$

Integrating (75) between the limits $x=0$, and $x=h_{\prime}-h$,

$$Q=\frac{2c(2g)^{\frac{1}{2}}b}{a}\left\{\frac{2h_{\prime}^{\frac{5}{2}}}{15}-\left(\frac{h_{\prime}}{3}-\frac{h}{5}\right)h^{\frac{3}{2}}\right\} \qquad (76)$$

If in (76), we make $h=0$, and $b=2a$, there results:

$$Q=\tfrac{8}{15}c(2g)^{\frac{1}{2}}h_{\prime}^{\frac{5}{2}}. \qquad (77)$$

Equation (77), derived from the general equation (76), is seen to be identical with equation (35), for the flow of water over a quadrantal weir.

In equation (76), denote the ratio between the head on the bottom and the head on the top of the triangular opening by m; thus $\frac{h}{h_{\prime}}=m$, and substitute the value of $(2g)^{\frac{1}{2}}=8.025$.

$$Q=16.05\frac{cb}{a}\left\{\tfrac{2}{15}-\left(\tfrac{1}{3}-\tfrac{m}{5}\right)m^{\frac{3}{2}}\right\}h_{\prime}^{\frac{5}{2}}. \qquad (78)$$

TO FIND THE FLOW OF WATER THROUGH A SUBMERGED TRIANGULAR ORIFICE, HAVING ITS VERTEX BELOW THE BASE.

Rule 24.—From $\frac{1}{3}$, subtract $\frac{1}{5}$ of the ratio of the given heads on the bottom and top of the orifice, and multiply this difference by the cube of the square root of the same ratio; subtract the product from $\frac{2}{15}$;

multiply the remainder by 16.05 times the product of the ratio between the depth and width of the orifice, the fifth power of the square root of the head on the bottom, and the coefficient of discharge.

Ex. 35.—In a submerged triangular orifice, represented by Fig. 13, the head, AC, on the bottom$=h_{,}=$ 2.25 feet; the head, AE, on the top$=h=$1 foot; the width, BD$=b=$5 feet; the depth EC$=a=$1.25 feet; and the coefficient of discharge $c=.616$. What is the flow in cubic feet per second?

Cal. 1st.—By formula (78) and Rule 24, derived therefrom:

Ratio of heads, $\frac{h}{h_{,}}=m=\frac{1}{2.25}=\frac{4}{9}$;

Difference $(\frac{1}{3}-\frac{m}{5})=\frac{11}{45}$;

Cube of square root of ratio, $m^{\frac{3}{2}}=(\frac{4}{9})^{\frac{3}{2}}=\frac{8}{27}$.

Product of this difference, and the cube of the square root of the ratio of the given heads,

$$\frac{11}{45}\times\frac{8}{27}=\frac{88}{1215}.$$

Difference, $\frac{2}{15}-\frac{88}{1215}=\frac{74}{1215}$.

$Q=16.05\times .616\times\frac{5}{1.25}\times\frac{74}{1215}\times\frac{243}{32}=18.29$ cubic feet.—*Ans.*

Cal. 2d.—Assuming that the effective head is the mean of the given heads on the top and bottom of the orifice, then will the velocity be as per equation (25) or Rule (8).

$v=.616\times 8.025\ (\frac{h_{,}+h}{2})^{\frac{1}{2}}=(\frac{2.25+1}{2})^{\frac{1}{2}}=6.3$ feet per second.

Area orifice$=5\times 1.25\div 2=3.125$ square feet.

Discharge equal to the product of the velocity and the area of the orifice, $Q=6.3\times3.125=19.69$ cubic feet per second.

Cal. 3.—Assume that the true head is on the center of gravity of the opening geometrically considered. In a triangle, the center of gravity is at the intersection of right lines drawn from any two angles and bisecting the opposite sides. Its distance, estimated from an angle, is equal to two-thirds the length of the bisecting line; or estimated from the middle of a side, is equal to one-third the length of the bisecting line.

In Fig. 13, $AG=h'=h+\frac{a}{3}=1+\frac{1.25}{3}=1.4167$.

By formula (13), modified by coefficient $v=.616=8.025\ (1.4167)^{\frac{1}{2}}=5.8836$ feet per second, area of orifice, $5\times1.25\div2=3.125$ square feet.

Discharge equal to the product of the velocity and the area of orifice, $Q=5.8836\times3.125=18.39$ cubic feet per second.

Comparing these results, it is seen that the second is seven and six-tenths per cent (.076) too great, and the third fifty-four one hundredth of one per cent (.0054) too great.

The rule generally adopted is, that "in all cases when the center of gravity of an orifice lies at least as deep below the fluid surface as the figure is high," the depth $h_{\prime}$ (that is, the depth at the center of gravity), of this point may be regarded the head of water. This rule may approximate the truth sufficiently close for ordinary practice, but is not to be employed when a high degree of accuracy is required.

FLOW OF WATER THROUGH A SUBMERGED TRIANGULAR ORIFICE HAVING ITS VERTEX ABOVE THE BASE.

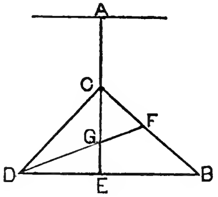


FIG. 14.

Let, in Fig. 14, BCD represent the opening, in which b=BD, the width at bottom; h=AC, head or vertex; $h_{,}$=AE, head or bottom; a=EC, depth of opening; $(2g^{\frac{1}{2}})$=acceleration of gravity; c=coefficient of discharge; Q=flow in cubic feet per second; x= any part of a=EC, estimated from C:

$$\text{Then } dQ=c\frac{(2g)^{\frac{1}{2}}b}{a}(h+x)^{\frac{1}{2}}x\,dx. \qquad (79)$$

Integrating (79) between the limits $x=0$, and $x=a$ $=h_{,}-h$:

$$Q=\frac{2c(g)^{\frac{1}{2}}b}{a}\left\{\frac{2h^{\frac{5}{2}}}{15}+\left(\frac{h_{,}}{5}-\frac{h}{3}\right)h_{,}^{\frac{3}{2}}\right\} \qquad (80)$$

Denote the ratio between the head on the bottom and that on the vertex by m:

$$m=\frac{h}{h_{,}};\ h=mh_{,}.$$

Substitute the value of h in (80), and the value of $(2g)^{\frac{1}{2}}=8.025$.

$$\frac{Q=16.05cb}{a}\left\{\frac{2m^{\frac{5}{2}}}{15}(+\tfrac{1}{5}-\tfrac{m}{3})\right\}h_{,}^{\frac{5}{2}}. \qquad (81)$$

If in the general equation (80), we make $h=0$, and $b=2a$, there results:

$$Q=\tfrac{4}{5}c(2g)^{\frac{1}{2}}h_{,}^{\frac{5}{2}}, \qquad (82)$$

which is identical with formula (48) for the flow of water through a quadrantal weir having its apex at the water's surface.

To find the flow of water through a submerged triangular orifice having its vertex above the base.

Rule 25.—From $\frac{1}{5}$ subtract $\frac{1}{3}$ of the ratio of the head on the bottom to that on the apex of the orifice; add this difference to the $\frac{2}{15}$ part of the fifth power of the square root of the same ratio; multiply this sum by 16.05 times the product of the ratio between the depth and width of the orifice, and the fifth power of the square root of the head on the bottom. Derived from formula (81).

Ex. 36.—In a submerged triangular orifice, represented by Fig. 14, the head, AC, on the apex$=h=1$

foot; the head, AE, on the bottom$=h_{\prime}=2.25$; the width, BD$=b=5$ feet; the depth, CE$=a=1.25$; and the coefficient of discharge, $c=.616$. What is the flow, $Q_{\prime}$, in cubic feet per second?

Cal. 1st.—By Rule 25, or formula (81).

Ratio of head on bottom to that on apex, $m=\frac{h}{h\prime}=\frac{1}{2.25}=\frac{4}{9}$.

Difference $(\frac{1}{5}-\frac{1}{3}\times\frac{4}{9})=\frac{7}{135}$.

$m^{\frac{5}{2}}=(\frac{4}{9})^{\frac{5}{2}}=\frac{32}{243}$; $\frac{2}{15}\times\frac{32}{243}=\frac{64}{365}$.

Sum $\frac{7}{135}+\frac{64}{3645}=\frac{253}{3645}$.

$Q=16.05\times\frac{5}{1.25}\times.616\times\frac{253}{3645}\times\frac{243}{32}=20.84$ cubic feet per second.

Cal. 2d.—Assuming that the effective head is the mean of the given heads on the apex and bottom of the orifice, then will the flow be the same as found by Cal. 2d, for Ex. 35, viz., $Q=19.69$ cubic feet per second.

Cal. 3d.—Assume that the true head is on the center of gravity of the opening, geometrically considered.

The center of gravity, as shown in Fig. 14, is at the intersection of CE and DF; DF bisecting BC in F.

CG : CE :: 2 : 3; CG$=\frac{2CE}{3}$;

AG$=$AC$+$CG$=h'=1+\frac{2}{3}\times1.25=1.8334$.

Area of opening$=5\times1.25\div2=3.125$.

$Q=.616\times8.025\times(1.8334)^{\frac{1}{2}}\times3.125=20.92$ cubic feet flow per second.

By inspection it is seen that the result by Cal, 2d is five and one-half (.055) of one per cent too small, and the result by Cal. 3d is nearly four-tenths (.0039) of one per cent too large. Neither of these empirical methods then would satisfy the requirements of any considerable accuracy.

FLOW OF WATER THROUGH A SUBMERGED CIRCULAR ORIFICE.

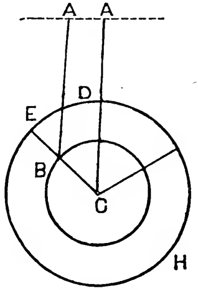


FIG. 15.

Let, in Fig. 15, EDH represent a submerged circular orifice; h=AC, the head on center; n=angle DCE; r=CD=CE, the radius; A=area; c=coefficient of discharge; $(2g)^{\frac{1}{2}}$=acceleration of gravity; Q=flow in cubic feet; x=any part of r, estimated from C. Then, in general,

$$A = r^2 - x^2. \quad (83)$$

$$\text{Differential (83), } dA = -2x dx. \quad (84)$$

$$\text{Head at any point, B} \quad \text{A, B} = h - x \cos n. \quad (85)$$

$$dQ = -2c\tilde{n}(2g)^{\frac{1}{2}}h^{\frac{1}{2}}\, x \left(1 - \frac{x \cos n}{h}\right)^{\frac{1}{2}} dx. \qquad (86)$$

Integrating (86) between the limits $x=0$, and $x=v$:

$$Q = c\tilde{n}(2g)^{\frac{1}{2}}h^{\frac{1}{2}}r^2\left(1 - \frac{r \cos n}{3h} - \frac{r^2 \cos 2n}{16h^2} - \frac{r^3 \cos 3n}{40h^3} - \frac{5r^4 \cos 4n}{384h^4} - \text{etc.}\right) \qquad (87)$$

Observing that the sum of cosines of a complete circle $2\tilde{n}$ is$=0$, and substituting the value of $(2g)^{\frac{1}{2}}=$ 8.025, and $\tilde{n}=3.1416$ in (87):

$$Q = 25.2113ch^{\frac{1}{2}}r^2\left(1 - \frac{r^2}{32h} - \frac{5r^4}{1024h} - \frac{105r^6}{65536h^6} - \text{etc.}\right) \qquad (88)$$

Making in (88), $h=r$, $Q=24.2129c\, r^{\frac{5}{2}}$, (89)
which is identical with (57).

Flow of water through semi-circular orifices.

Let $\tilde{n}^2$=the mean of all the cosines of the first quadrant, and—$\tilde{n}^2$=the mean of all cosines of the second quadrant; then will the mean of the first and second quadrants vanish.

To determine the flow of water through the upper semi-circle of a submerged circular orifice, substitute in one-half of (87), the mean value of $\cos n=\tilde{n}^2$.

$$Q = 12.6056c\, h^{\frac{1}{2}}r^2\Big)1 - \frac{2r}{3\tilde{n}h} - \frac{r^2}{32h^2} - \frac{r^3}{40\tilde{n}h^3} - \frac{5r^4}{1024h^4} - \text{etc.}\Big) \qquad (90)$$

To determine the flow of water through the lower semi-circle of a submerged circular orifice, substitute in one-half of (87), the mean value of cos $n = -\bar{n}^2$.

$$Q = 12.6056c\, h^{\frac{1}{2}} r^2 \left(1 + \frac{2r}{3\bar{n}h} - \frac{r^2}{32h^2} + \frac{r^3}{40\bar{n}h^3} - \frac{5r^4}{1024h^4} + \text{etc.}\right) \quad (91$$

Ex. 37.—The radius of a circular opening is one foot, the head on the center of the opening four feet, and the coefficient of discharge .616. What is the flow in cubic feet per second?

Cal 1st.—Substituting the values of r, h and c in formula (88),

$$Q = 25.2113 \times .616 \times 2 \left(1 - \frac{1}{32 \times 16} - \frac{5}{1024 \times 256} - \frac{105}{65536 \times 4096}\right);$$

$$\left.\begin{cases} \frac{1}{32 \times 16} & = -.0019531 \\ \frac{5}{1024 \times 256} & = -.0000191 \\ \frac{105}{65536 \times 4096} & = -.0000004 \end{cases}\right\} = .0019726.$$

$$1 - .0019726 = .9980274.$$

$Q = 25.2113 \times .616 \times 2 \times .9980274 = 31.00$ cubic feet. —*Ans.*

Cal. 2d.—Assume that the true head is on the center of gravity of the opening, geometrically considered; then will the discharge be:

$Q = .616 \times 8.025 \times 2 \times 3.1416 = 31.06$ cubic feet.—*Ans.*

Ex. 38.—The radius of a circular opening is one foot, the head on the center of the opening four feet, and the coefficient of discharge .616. What is the flow in cubic feet per second in the upper semi-circle?

Cal.—Substituting the values of r, h and c and ñ= 3.1416 in formula (90).

$$Q=12.6056\times.616\times2\left(1-\frac{2}{12\times3.1416}-\frac{1}{32\times16}-\frac{2}{2560\times3.1416}\right)$$

$$\left.\begin{array}{ll} -\frac{2}{12\times3.1416} & =-.0530516. \\ -\frac{1}{32\times16} & =-.0019531. \\ -\frac{1}{2560\times3.1416} & =-.0001245. \\ -\frac{5}{1024\times256} & =-.0000020. \end{array}\right\} =-.0551312.$$

$Q'=12.6056\times.616\times2\times.94487=14.6739$ cubic feet. —*Ans.*

Ex. 39.—The radius of a circular opening is one foot, the head on the center of the opening four feet, and the coefficient of discharge .616. What is the flow in cubic feet per second in the lower semi-circle?

Cal.—Substituting the values of r, h, c and ñ= 3.1416 in formula (91),

$$Q=12.6056\times.616\times2\left(1+\frac{2}{12\times3.1416}-\frac{1}{32\times16}+\frac{1}{2560\times3.1416}\right)$$

$$\left.\begin{array}{llr} +1. & 1.0000000 & \\ +\frac{2}{12\times3.1416} & =+\ .0530516 & \\ -\frac{1}{32\times16} & = & -.0019531 \\ +\frac{1}{2560\times3.1416} & =+\ .0001245 & \\ -\frac{5}{1024\times256} & = & -.0000020 \end{array}\right\}$$

$$.1.0531761-.0019551=1.051221.$$

$Q''=12.6056\times.616\times2\times1.051221=16.3254$ cubic feet.—*Ans.*

The value of Q'' might have been readily found as follows:

$$Q''=Q-Q'=31.0-14.6739=16.326.$$

It will be noted that formula (91) expressed the discharge of water through the lower semi-circle of a submerged lateral orifice, in which the head is on the center; whereas formula (60) expresses the discharge of water through a semi-circular weir, represented by Fig. 10, in which the head OA is on the bottom, care will need be taken that these formulas be not confounded. This fact is rendered more apparent by making in (91), $h=r$, when there results approximately:

$$Q''=14.92cr^{\frac{5}{2}},$$

While with respect to formula (60),

$$Q_3=7.693cr^{\frac{5}{2}}.$$

TABLE 5.

Square Roots, Cubes of Square Roots, and Fifth Powers of Square Roots of Numbers.

No.	Square Roots.	Cubes of Square Roots.	5th Power of Square Roots.	No.	Square Roots.	Cubes of Square Roots.	5th Power of Square Roots.
.1	.316	.032	.003	1.	1.000	1.000	1.000
.125	.353	.044	.006	1.25	1.118	1.397	1.747
.15	.387	.058	.009	1.50	1.225	1.837	2.756
.175	.418	.073	.013	1.75	1.304	2.315	4.051
.2	.447	.089	.018	2.	1.414	2.822	5.657
.225	.474	.107	.024	2.25	1.500	3.375	7.594
.25	.500	.125	.031	2.50	1.581	3.953	9.882
.275	.524	.144	.040	2.75	1.658	4.560	12.541
.3	.548	.164	.049	3.	1.732	5.196	15.590
.325	.570	.185	.060	3.25	1.803	5.869	19.041
.35	.592	.207	.072	3.50	1.871	6.547	22.918
.375	.612	.230	.086	3.75	1.936	7.262	27.232
.4	.633	.253	.101	4.	2.000	8.000	32.000
.425	.652	.277	.118	4.25	2.061	8.761	37.236
.45	.671	.302	.136	4.5	2.121	9.546	42.957
.475	.689	.327	.156	4.75	2.179	10.352	49.174
.5	.707	.353	.177	5.	2.236	11.180	55.901
.525	.724	.380	.200	5.25	2.291	12.029	63.153
.55	.742	.408	.224	5.5	2.345	12.898	70.942
.575	.758	.436	.251	5.75	2.398	13.783	79.281
.6	.775	.465	.279	6.	2.449	14.697	88.181
.625	.791	.494	.309	6.25	2.500	15.625	97.656
.650	.806	.524	.341	6.5	2.550	16.572	107.71
.675	.821	.555	.374	6.75	2.598	17.537	118.37
.7	.837	.586	.410	7.	2.646	18.520	129.64
.725	.852	.617	.448	7.5	2.739	20.539	154.04
.75	.866	.650	.486	8.	2.828	22.627	181.02
.775	.880	.682	.529	8.50	2.915	24.781	210.64
.8	.894	.715	.572	9.	3.000	27.000	243.00
.825	.908	.749	.618	10.	3.162	31.623	316.23
.85	.922	.784	.666	11.	3.317	36.483	401.31
.875	.936	.818	.717	12.	3.464	41.569	498.83
.9	.949	.854	.768	13.	3.606	46.872	609.33
.925	.956	.890	.823	14.	3.742	52.383	733.36
.950	.975	.926	.858	15.	3.873	58.094	871.41
.975	.987	.963	.939	16.	4.	64.000	1024.0

TABLE 6.

Square Roots of Numbers.

No.	Square Roots.	No.	Square Roots.	No.	Square Roots.	No.	Square Roots.
1.00	1.000	8.	2.828	19.	4.359	95.	9.747
1.05	1.025	8.1	2.846	19.2	4.382	96.	9.798
1.1	1.049	8.2	2.864	19.4	4.405	97.	9.849
1.15	1.072	8.3	2.881	19.6	4.427	98.	9.899
1.2	1.095	8.4	2.898	19.8	4.450	99.	9.950
1.25	1.118	8.5	2.915	20.	4.472	100.	10.000
1.3	1.140	8.6	2.933	21.	4.583	102.	10.100
1.35	1.162	8.7	2.950	22.	4.690	104.	10 198
1.4	1.183	8.8	2.966	23.	4.796	106.	10.295
1.45	1.204	8.9	2.983	24.	4.899	108.	10.392
1.5	1.225	9.	3.	25.	5.000	110.	10.488
1.55	1.245	9.1	3.017	26.	5.099	112.	10.583
1.6	1.265	9.2	3.033	27.	5.196	114.	10.677
1.65	1.285	9.3	3.050	28.	5.292	116.	10.770
1.7	1.304	9.4	3.066	29.	5.385	118.	10.863
1.75	1.323	9.5	3.082	30.	5.477	120.	10.954
1.8	1.342	9.6	3.098	31.	5.568	122.	11.045
1.85	1.360	9.7	3.114	32.	5.657	124.	11.136
1.9	1.378	9.8	3.130	33.	5.745	126.	11.225
1.95	1.396	9.9	3.146	34.	5.831	128.	11.314
2.	1.414	10.	3.162	35.	5.916	130.	11.402
2.1	1.449	10.1	3.178	36.	6.000	132.	11.489
2.2	1.483	10.2	3.194	37.	6.083	134.	11.576
2.3	1.517	10.3	3.209	38.	6.164	136.	11.662
2.4	1.549	10.4	3.225	39.	6.245	138.	11.747
2.5	1.581	10.5	3.240	40.	6.325	140.	11.832
2.6	1.612	10.6	3.256	41.	6.403	142.	11.916
2.7	1.643	10.7	3.271	42.	6 481	144.	12 000
2.8	1.673	10.8	3.286	43.	6.557	146.	12.083
2.9	1.703	10.9	3.302	44.	6.333	148.	12.166
3.	1.732	11.	3.317	45.	6.708	150.	12.247
3.1	1.761	11.1	3.332	46.	6.782	155.	12.450
3.2	1.789	11.2	3.347	47.	6 856	160.	12.649
3.3	1.817	11.3	3 362	48.	6.928	165.	12.845
3.4	1.844	11.4	3.376	49.	7.000	170.	13.038
3.5	1.871	11.5	3.391	50.	7.071	175.	13.229
3.6	1.897	11.6	3 406	51.	7.141	180.	13 417
3.7	1.924	11.7	3.421	52.	7.211	185.	13.601
3.8	1.949	11.8	3.435	53.	7.280	190.	13.784
3.9	1.975	11.9	3.450	54.	7.348	195.	13.964

TABLE 6.—Continued.

Square Roots of Numbers.

No.	Square Roots.	No.	Square Roots.	No.	Square Roots.	Square Roots.	No.
4.	2.000	12.	3.464	55.	7.416	200.	14.142
4.1	2.025	12.1	3.479	56.	7.483	205.	14.318
4.2	2.049	12.2	3.493	57.	7.550	210.	14.491
4 3	2.074	12.3	3.507	58.	7.616	215.	14.663
4.4	2 098	12.4	3.521	59.	7.681	220.	14.832
4.5	2.121	12.5	3.536	60.	7.746	225.	15.000
4.6	2 145	12.6	3.550	61.	7.810	230.	15.166
4.7	2.168	12.7	3.564	62.	7.874	235.	15.330
4.8	2.191	12.8	3.578	63.	7.937	240.	15.492
4.9	2.214	12.9	3.592	64.	8.000	245.	15.652
5.	2.236	13.	3.606	65.	8.062	250.	15.811
5.1	2.258	13.2	3.633	66.	8.124	260.	16.125
5.2	2.280	13.4	3.661	67.	8.185	270.	16.432
5.3	2.302	13.6	3.688	68.	8.246	280.	16.733
5.4	2.324	13.8	3.715	69.	8.307	290.	17.029
5.5	2.345	14.	3.742	70.	8.367	300.	17.320
5.6	2.366	14.2	3.768	71.	8.426	310.	17.607
5.7	2.387	14,4	3.795	72.	8.485	320.	17.889
5.8	2.408	14.6	3.821	73.	8.544	330.	18.166
5.9	2.429	14.8	3.847	74.	8.602	340.	18.439
6.	2.449	15.	3.873	75.	8.660	350.	18.708
6.1	2.470	15.2	3.899	76.	8.718	360.	18.974
6.2	2.490	15.4	3.924	77.	8.775	370.	19.235
6.3	2.510	15.6	3.950	78.	8.832	380.	19.494
6.4	2.530	15.8	3.975	79.	8.888	390.	19.748
6.5	2.550	16.	4.000	80.	8.944	400.	20.000
6.6	2.569	16.2	4.025	81.	9.000	410.	20.248
6.7	2.588	16.4	4,050	82.	9.055	420.	20.494
6.8	2.608	16.6	4.074	83.	9.110	430.	20.736
6.9	2.627	16.8	4.099	84.	9.165	440.	20.976
7.	2.646	17.	4.123	85.	9.220	450.	21.213
7.1	2.665	17.2	4.147	86.	9.274	460.	21.448
7.2	2.683	17.4	4.171	87.	9.327	470.	21.679
7.3	2.702	17.6	4.195	88.	9.380	480.	21.909
7.4	2.720	17.8	4.219	89.	9.434	490.	22.136
7.5	2.739	18.	4.243	90.	9.487	500.	22.361
7.6	2.757	18.2	4 266	91.	9.539	525.	22.913
7.7	2.775	18.4	4.290	92.	9.592	550.	23.452
7.8	2.793	18.6	4.313	93.	9.644	575.	23.979
7.9	2.811	18.8	4.336	94.	9.695	600.	24.495

Flow of water through vertical, rectangular orifices in thin partitions.

The assumption that the mean velocity of a stream of water flowing through a vertical rectangular orifice is at the middle of the opening, has been shown by equation (22) to be not strictly true. But, owing to its simplicity of application, and its close approximation to the truth, hydraulicians, for the most part, are wont to adopt it, and to correct the error involved by coefficients obtained by experiment.

Table 7, derived from Fanning's Hydraulic Engineering, embraces a wide range of coefficients so determined. Thus, it is suited to heads of water from two-tenths (.2) of a foot to fifty (50) feet, and to orifices one foot wide, whose hights vary from four (4) feet to one and one-half ($1\frac{1}{2}$) inches.

TABLE 7.

Flow of Water per second through rectangular orifices in thin vertical partitions, and the coefficients employed in the computation.

Head on Center.	Coefficient.	4 feet high; 1 ft. wide.	Coefficient.	2 feet high; 1 ft. wide.	Coefficient.	1½ feet high; 1 ft. wide	Coefficient.	1 foot high; 1 ft. wide
0.6	......	......	......	......	......	......	.598	3.72
0.7	......	......	......	......	......	......	.599	4.02
0.8	......	......	......	......	.613	6.60	.600	4.31
0.9	......	......	......	......	.613	7.01	.601	4.57
1.0	......	......	......	......	.614	7 39	.601	4.87
1.25	......	......	.619	11.11	.614	8.26	.602	5.29
1.50	......	......	.619	12.16	.614	9.06	.603	5.92
1.75	......	......	.619	13.13	.615	9.79	.603	6.40
2.00	......	......	.618	14.04	.614	10.45	.604	6.85
2.25	......	......	.618	14.89	.614	10.96	.604	7.27
2.50	.629	31.92	.618	15.67	.614	11.66	.604	7.67
2.75	.628	33.43	.617	16.43	.614	12.24	.605	8.05
3.00	.627	34.75	.617	17.15	.613	12.78	.605	8.41
3.50	.625	37.54	.616	18.49	.612	13.79	.605	9.08
4.00	.625	40.09	.615	19.74	.611	14.71	.605	9.97
4.50	.623	42.39	.614	20.90	.610	15.58	.604	10.29
5.00	.621	44.55	.612	21.98	.609	16.48	.604	10.84
6.00	.616	48.42	.609	23.96	.606	17.88	.602	11.84
7.00	.612	52.23	.606	25.75	.604	19.23	.601	12.76
8.00	.609	55.29	.604	27.39	.602	20.50	.601	13.64
9.00	.606	58.35	.602	28.98	.601	21.72	.601	14.47
10.00	.604	61.26	.602	30.53	.601	22.88	.601	15.25
15.00	.604	75.09	.602	37.42	.601	28.02	.601	18.68
20.00	.605	86.78	.602	43.24	.601	32.37	.601	21.50
25.00	.605	99.06	.603	48.39	.601	36.19	.601	24.12
30.00	.605	106.46	.603	53.34	.602	39.68	.601	26.43
35.00	.606	115.08	.694	57.35	.602	42 88	.601	28.55
40.00	.607	123.13	.605	61.36	.603	45.86	.602	30.53
45.00	.606	130.39	.605	65.14	.603	48.68	.602	32.39
50.00	.609	138.12	.606	67.21	.603	51.36	.602	34.15
	Mean.		Mean.		Mean.		Mean.	
......	.614	......	.610	......	.608	......	.602	......

TABLE 7.

Flow of Water per second through rectangular orifices in thin vertical partitions, and the coefficients employed in the computation.

Head on Center.	Coefficient.	9 feet high; 1 ft. wide. cu. ft.	Coefficient.	6 feet high; 1 ft. wide. cu. ft.	Coefficient.	3 feet high; 1 ft. wide cu. ft.	Coefficient.	1½ feet high; 1 ft. wide. cu. ft.
0.2	......	......	......	......	......	......	.633	.28
0.3	......	......	......	......	.629	.69	.633	.35
0.4	......	......	.614	1.56	.631	.80	.633	.40
0.5	.605	2.57	.615	1.74	.631	.89	.633	.45
0.6	.606	2.83	.616	1.91	.632	.98	.633	.49
0.7	.607	3.06	.616	2 07	.632	1.06	.633	.53
0.8	.608	3.27	.617	2.21	.632	1.14	.633	.59
0.9	.609	3.48	.617	2.35	.632	1.20	.632	.60
1.0	.609	3.67	.617	2.48	.632	1.26	.632	.63
1.25	.610	4.02	.617	2.71	.632	1.39	.631	.69
1.50	.610	4.50	.617	3.03	.631	1.55	.630	.77
1.75	.610	4.86	.617	3.27	.631	1.67	.630	.83
2.00	.610	5.20	.617	3.50	.630	1.79	.629	.89
2.25	.610	5.51	.616	3.71	.629	1.89	.629	.95
2.50	.610	5.81	.616	3 91	.628	1.99	.628	1.00
2.75	.610	6.09	.616	4.10	.627	2.09	.627	1.04
3.00	.610	6.36	.615	4.27	.627	2.18	.627	1.09
3.5	.609	6.36	.615	4.61	.625	2.35	.625	1.17
4.00	.609	7.32	.614	4.92	.624	2.50	.624	1.25
4.5	.607	7.75	.613	5.21	.622	2.65	.622	1.32
5.00	.606	8.16	.611	5.49	.620	2.78	.620	1.39
6.00	.604	8.91	.609	5.98	.615	3.03	.615	1.51
7.00	.603	9.61	.606	6.43	.611	3.24	.611	1.62
8.00	.602	10.25	.603	6.84	.607	3.45	.609	1.71
9.00	.602	10.86	.602	7.25	.605	3.64	.607	1.83
10.00	.601	11.44	.601	7.62	.603	3.83	.606	1.92
15.00	.601	14.01	.601	9.34	.603	4.69	.607	2.36
20.00	.601	16.18	.602	10.80	.604	5.42	.607	2.72
25.00	.602	18.10	.602	12.08	.604	6.06	.608	3.05
30.00	.602	19.84	.603	13.47	.604	6.64	.609	3.35
35.00	.602	21.44	.603	14.31	.605	7.18	.610	3.62
40.00	.603	22.94	.604	15.32	.606	7.68	.611	3.79
45.00	.603	24.35	.604	16.26	.606	8.16	.613	4.12
50.00	.604	25.68	.605	17.16	.607	8.61	.614	4.35
	Mean.		Mean.		Mean.		Mean.	
....	.606	......	.611	......	.620	......	.622	......

An inspection of Table 7 discloses that the coefficient of flow is variable, both with respect to the head of water and form of orifice.

Thus, the orifice being "four feet high," the maximum coefficient .629, is due a head of 2.50 feet; thence the coefficient gradually diminishes to .604, as the head increases to 10 feet; thence is constant to 15 feet; thence gradually increases to .609, with the increase of the head to 50 feet.

In the other given orifices, variations obtain, but to a less extent.

With respect to the variation of coefficients arising from the form of orifice, it will be seen, by running the eye horizontally to the right, from and for any given head, that the values of the coefficients diminish as the hights of the orifices decrease from four feet to one foot, and increase as the hights of the orifice decrease from one foot to one and one-half ($1\frac{1}{2}$) inches. In illustration take several heads, as 3, 10, 25, 50 feet, and the coefficients due the several forms of orifice.

Head. Feet.	$4' \times 1'$ Coef.	$2' \times 1'$ Coef.	$1\frac{1}{2}' \times 1'$ Coef.	$1' \times 1'$ Coef.	$9'' \times 1'$ Coef.	$6 \times 1'$ Coef.	$3'' \times 1'$ Coef.	$1\frac{1}{2}' \times 1''$ Coef.
3	.627	.617	.613	.605	.610	.615	.627	.627
10	.604	.602	.601	.601	.601	.601	.603	.606
25	.605	.603	.601	.601	.602	.602	.604	.608
50	.609	.606	.603	.602	.604	.605	.607	.614

TO FIND THE FLOW OF WATER IN CUBIC FEET PER SECOND THROUGH VERTICAL RECTANGULAR ORIFICES IN THIN VERTICAL PARTITIONS BY TABLE 7, THE HEAE ON CENTER AND SIZE OF OPENING BEING MADE.

Rule 26.—In "head on center" column, Table 7, find the given head, opposite which, in column headed by the given hight of orifice, will be found the flow for an orifice one foot wide, which multiply by the given width in feet.

Ex. 40.—The head being ten feet, and orifice four feet wide and nine inches high, what is the flow per second?

Cal.—In column "9″ high 1 foot wide," opposite 10 feet in "head on center" column, will be found 11.44 cubic feet, which, multiplied by four feet, the given width, gives:

$$11.44\times4=45.76 \text{ cubic feet.—}Ans.$$

The heads, sizes of orifices, and the computed flow of water given in Table 7, will be found highly convenient for ready reference in a great number of cases, but are seen to be too limited to fully meet the requirements of practice. Indeed, a table sufficiently ample for that purpose would be too unwieldly for use.

The general formula for the flow of water per second through vertical rectangular orifices in thin partitions, is:

$$Q=8.025ca\sqrt{h} \qquad (92)$$

In which Q denotes the flow in cubic feet; c, coefficient of discharge; a, the area of the orifice in square feet; and h', the head on the center of the orifice: h' is equal to the half sum of the respective heads on the bottom and top of the orifice, as seen in equation (21).

In case the hight of the orifice and the head on its top are given, then h' is equal to the sum of the given head and half the hight of the opening; or if the hight of the opening and the head on its bottom are given, then h' is equal to the difference between the given head and half the hight of the orifice.

TO FIND THE FLOW OF WATER IN CUBIC FEET PER SECOND, THROUGH VERTICAL RECTANGULAR ORIFICES IN THIN PARTITIONS.

Rule 27.—Multiply 8.025 times the square root of the head on the center of the orifice, by the product of the area of the orifice and the coefficient of discharge.

Rule 27 corresponds to formula (92).

With respect to the "square root of the head," and "the coefficient of discharge," contemplated in Rule 27, it will be remembered that Table 6 gives the square roots of numbers likely to be required, and Table 7, the coefficients of discharge. In finding a proper coefficient of discharge, in case the given hight of orifice is found in Table 7, the coefficient corresponding to that hight and to the given head is to be employed; but in case the given hight of orifice is an intermediate, or lies between the hights contained in the table, its coefficient will need be computed. The tabulated coefficients are, in fact, ordinates of curves, determined by experiment.

In determining these intermediate ordinates or coefficients between any two adjacent hights in Table 7, as 4 feet and 2 feet, 1.5 feet and 1 foot, no appreciable error will occur by substituting a right line for a curve. The determination of the intermediate coefficients will then be effected by arithmetical differences. In illustration: let it be required to find the coefficient due a head of 2.5 feet, and orifice 3.5 feet high.

Now 3.5 is between the adjacent hights, 4 and 2 feet, in Table 7. The respective coefficients due a head of 2.5 feet are .629 in "4 feet high" column, and .618 in "2 feet high" column.

Difference of hights,	4—2=2 feet.
Difference of greater and given hights,	4—3.5=.5 feet.
Quotient of these differences,	2÷.5=4 divisor.
Difference of coefficients,	.629—.618=.011
Arithmetical difference sought,	.011÷4=.003 nearly.
Coefficient due 3.5 feet,	.629—.003=.626

The intermediate coefficients corresponding to 3 feet and 2.5 feet are now readily found. Thus:

Coefficient due 3 feet,	.626—.003=.623
Coefficient due 2.5 feet,	.623—.003=.620

Examples and Calculations.

Ex. 41.—An orifice is 3.5 feet wide, 1.25 feet high and the head on its center is 7 feet. What is the flow in cubic feet per second?

Cal.—By Table 6, square root of 7 feet=2.646.

By Table 7, coefficient due 7 feet; for orifice, 1.5 feet high=.604; for orifice, 1 foot high= .601.

Difference of tabulated hights,	1.5—1=.5
Difference of greater and given hights,	1.5—1.25=.25
Quotient of these differences,	.5÷.25=2 div.
Difference of coefficients,	.604—.601=.003
Arithmetical difference,	.003÷2=.0015
Coefficient due 1.25 feet,	.604—.0015=.6025

Area of orifice, $3.5 \times 1.25 = 4.375$ square feet.

Flow$=8.025 \times .6025 \times 4.375 \times 2.646 = 55.97$ cubic feet.—*Ans.*

Ex. 42.—Given the head on the bottom of a rectangular orifice 12 feet, the head on its top 11 feet, and the width of orifice 4 feet, what is the flow in cubic feet per second?

Cal.—Head on center$=\frac{12+11}{2}=11.5$ feet.

By Table 6, square root of head on center=3.391 feet.

Hight of orifice=12—11=1 foot.

By Table 7, coefficient due head of 11.5 feet, and orifice 1 foot high=.601.

It will be observed that the coefficient is constant for head from 10 to 15 feet, inclusive.

Area of orifice$=4 \times 1=4$ square feet.

Flow$=8.025 \times .601 \times 4 \times 3.391 = 65.42$ cubic feet.—*Ans.*

Ex. 43.—The head on the top of a rectangular orifice 6 inches high and 6 feet wide being 7.25 feet, what is the flow in cubic feet per second?

Cal.—Half the hight of orifice 6″÷2=3″=.25 feet.

Head on center=7.25+.25=7.5 feet.

By Table 7, coefficient due head of 7 feet=.606.

Coefficient due head of 8 feet=.603.

Mean coefficient on that due 7.5 feet=.6045.

By Table 6, square root, 7.5 feet=2.739.

Area of orifice, 6×.5=3 square feet.

Flow=8.025×.6045×3×2.739=29.89 cubic feet.—*Ans.*

Ex. 44.—The head on the bottom of a rectangular orifice 9 inches high and 3 feet wide, being 15.875 feet, what is the flow in cubic feet per second?

Cal.—Half the hight of orifice, 9″÷2=4.5″=.375 feet.

Head on center=15.975—.375=15.6 feet.

By Table 7, coefficient due head of 15.875 feet, and orifice 9″ high=.601.

Observe that the coefficient is constant from 10 feet to 20 feet, inclusive.

By Table 6, square root of 15.6=3.95 feet.

Area of orifice=3×.75=2.25 square feet.

Flow=8.025×.601×2.25×3.95=42.86 cubic feet.—*Ans.*

The preferable unit for measuring the flow of water is 1 cubic foot, but so widely is the "miner's inch," employed in California as a unit of measure, that we cannot well pass it in silence.

"Miner's Inch."

The term "miner's inch" is employed to express that quantity of water which, under a given head or pressure, as 4, 7, 9, etc., inches, will flow through each square inch of a discharge opening; or, in other words, which will flow through each square inch of cross section of a stream of water.

The quantity of water so flowing in a minute, an hour, 24 hours, etc., is designated *minute inch, hour inch, 24-hour inch,* etc., according to the length of time specified.

Statutory Miner's Inch.

Under the head, "Water Rights," the Civil Code of the State of California, Sec. 1415, provides in these words, "That he (the locator) claims the water there flowing to the extent of (giving the number) inches, measured under a four-inch pressure."

On this data, the value of the statutory miner's inch, the mean coefficient of discharge being in practice .6216, is as follows:

For one second (second inch),	0.02 cubic feet.
For one minute (minute inch),	1.20 cubic feet.

For one hour (hour inch),	72.00 cubic feet.
For 24 hours (24-hour inch),	1728 cubic feet.

If a cubic foot be divided by the flow in one second, there will result the number of miner's inches whose discharge is equal to a cubic foot per second. Thus, 1÷.02=50 statutory miner's inches; that is, fifty statutory miner's inches are equal to one cubic foot flow per second.

North Bloomfield, etc., Miner's Inches.

At the North Bloomfield, Milton and Columbia Hill hydraulic mines, the water is measured in its flow through a rectangular orifice 50 inches long, 2 inches wide, and under a pressure of 7 inches on the center of the opening. The flow per square inch of orifice, for 24 hours, due this head, as given me by Hamilton Smith, Jr., C. E., formerly chief engineer of the North Bloomfield Works, and president of the Miners' Association, is 2230 cubic feet; of which the coefficient of discharge is found to be .6064. Mr. Smith's experiments made with a module of equal dimensions under a 7-inch head, at Columbia Hill in 1874, found, as stated by Aug. J. Bowie, Jr., M. E., in an article entitled "Bowie on the Measurement and Flow of Water," found the value of the 24-hour inch to be 2260.8 cubic feet, and the coefficient of discharge to be .616. Mr. Bowie, in the article referred to, gives, in addition, substantially the following data, with respect to the

Smartsville Miner's Inch.

Hight of orifice, 4 inches; head on center, 9 inches; value of 24-hour inch, 2534.4 cubic feet; coefficient of discharge, .6078.

South Yuba Canal Inch.

Hight of orifice, 2 inches; head on center, 6 inches.

And with respect to a series of experiments made by himself at La Grange with an orifice 12 inches high, 12.75 inches wide, under a pressure of 6 inches on the top of the orifice, or head of one foot on the center. The mean of which experiments gave as the value of one miner's inch for 24 hours, 2159.146 cubic feet; effective coefficient of efflux, .5905. The flow through this module was assumed equal to 200 miner's inches.

A comparison shows that these coefficients of discharge approximate closely those given in Table 7, obtained on equal data. The results of these experiments also clearly show that the value of a coefficient of discharge depends, among other things, upon the form of the module. In illustration, the module being 50 inches long, 2 inches wide, the coefficient of discharge is found to be .5905. Should we estimate the effect of the difference between the given heads (7 inches and 1 foot) on the coefficients of discharge, there would result .5885 instead of .5905.

The variety of values comprised in the term, miner's inch, as employed in California, is often a source of no little annoyance and confusion. To aid in overcoming this difficulty, Table 8 prepared from Table 7, is given. Each result so obtained is a mean of the experiments of the world's ablest hydraulicians.

TABLE 8.

Flow of water through rectangular orifices due Miner's Inches of different values.

Head on Cent. In.	Orifice. High In.	Orifice. Wide. In.	Coef.	ec. inch Flow. Cub. Feet.	Min. inch Flow. Cub. Feet.	Hour inch Flow. Cub. Feet.	24-hour inch Flow. Cub. Feet.	1 Cub. Ft. Flow per Sec. Miner's In.
3	2	12.	.631	.01758	1.055	63.29	1519.	58.87
4	2	12.	.631	.02030	1.218	73.10	1754.	49.24
5	2	12.	.632	.02274	1.364	81.85	1964.	43.98
6	2	12.	.632	.02490	1.494	89.65	2152.	40.15
7	2	12.	.632	.02690	1.614	96.84	2324.	37.17
8	2	12.	.632	.02876	1.725	103.53	2484.	34.77
9	4	12.	.624	.03011	1.807	108.42	2602.	33.20
10	6	12.	.617	.03139	1.883	113.00	2711.	31.85
11	9	12.	.609	.03249	1.949	116.98	2807.	30.77
*12	12	12.75	.601	.02562	1.537	92.24	2214.	39.03

*The flow due the given opening, 12″ × 12.75″ = 153 square inches, divided by 200, has been proposed, as hereinbefore stated, as the standard miner's inch. Its adoption seems to be but local.

EXAMPLES AND CALCULATIONS.

Ex. 45.—A water right location is made for 6000 miner's inches. What is the equivalent flow in cubic feet per second?

Cal. 1st.—The statutory miner's inch is estimated, as stated, under a 4-inch pressure.

By Table 8, opposite 4 inches in "head on center" column, find 49.24 miner's inches in "1 cubic foot flow per second" column, 6000÷49.24=121.6 cubic feet. —*Ans.*

Cal. 2d.—For the most part in practice, 50 miner's inches measured under a 4-inch pressure, are adopted as equal to a flow of one cubic foot of water per second. This results, as shown in discussing the statutory miner's inch, from taking the mean coefficient .6216 for different heads, instead of the tabulated coefficient .632 for the given 4-inch head.

Whence, 6000÷50=120 cubic feet.—*Ans.*

Ex. 46.—In a water right claim of 5000 miner's inches, measured under a 4-inch pressure, are how many North Bloomfield miner's inches—miner's inches measured under a 7-inch head?

Cal. 1st.—By Table 8, the value of a second inch, under a 4-inch head, is .0203 cubic feet flow; and the value of a second inch, under a 7-inch head, is .0269 cubic feet flow.

Whence, $.0203 \times 5000 \div .0269 = 3773$ miner's inches —*Ans.*

Cal. 2d.—In the discussion of the miner's inch, it has been shown that in common practice the value of the 24-hour inch, under a 4-inch head, is 1728 cubic feet; and under a 7-inch head at North Bloomfield is 2230 cubic feet.

Whence, $1728 \times 5000 \div 2230 = 3874$ miner's inches. —*Ans.*

Ex. 47.—In 2000 miner's inches, through a rectangular opening 2 inches high, and under a 6-inch pressure, as employed at the South Yuba canal, are how many miner's inches flowing through a rectangular opening 4 inches high and under a 9-inch pressure, as adopted at Smartsville?

Cal. 1st.—As the result will be the same, whether the calculation be made in second, minute, hour, or 24-hour miner's inches, let the 24-hour inch be employed; then by Table 8, under a 6-inch pressure through an opening 2 inches high, the value is 2152 cubic feet; and under a 9-inch pressure through an opening 4 inches high, the value is 2602 cubic feet; whence, $2000 \times 2152 \div 2602 = 1654$ Smartsville miner's inches.

Cal. 2d.—Under the heading Smartsville, Bowie "On Measurement and Flow of Water," makes that miner's inch, 2534.4, due coefficient of discharge .6078. instead of .624, as adopted in Table 8.

Whence, $2000 \times 2152 \div 2534.4 = 1698$ Smartsville miner's inches.—*Ans.*

Cal. 3d.—Table 8 shows that the coefficient due a

6-inch head, and opening 2 inches high, is .632, and that the coefficient due a 9-inch head, and opening 4 inches high, is .624. Now, as commonly practiced, the "mean" coefficient .62 would be employed; so that, the result sought would depend upon the square root of the ratio of the given heads, 9 inches=.75 feet, and 6 inches=.5 feet; thus, by Table 5,

$$\sqrt{\tfrac{.5}{.75}}=\tfrac{.707}{.866}=.8165.$$

$.8165\times2000=1633$ Smartsville miner's inches.—*Ans.*

EXAMPLES AND CALCULATIONS.

Ex. 48.—The head being 2.25 feet on the center of a circular orifice .0328 feet diameter, what is the discharge in cubic feet per second?

Cal. 1st.—Rule 27 is equally applicable to rectangular and circular orifices.

By Table 6 the square root of given head 2.25 feet= 1.5 feet.

Area of given orifice .0323 feet diameter is equal to the square of the diameter, multiplied by .7854; $(.0328)^2\times.7854=.000845$ square feet.

By Table 9, coefficient of discharge due a head of 2.25 feet, according to Castel, is approximately equal to .673; then by Rule 27,

$.873\times8.025\times1.5\times.000845=.00685$ cubic feet.—*Ans.*

Cal. 2d.—According to Weisbach, the coefficient due a head of 2.25 feet, and orifice .0328 feet diameter, is approximately .628. Employing this coefficient instead of .673,

$.628 \times 8.025 \times 1.5 \times .000845 = .00639$ cubic feet.—*Ans.*

Weisbach observes, that "for square orifices from 1 to 9 square inches area, with from 7 to 21 feet head of water, according to the experiments of Bossut and Michelotti, the mean coefficient of efflux is $m=.610$; for circular ones of from $\frac{1}{2}$ to 6 inches diameter, with from 4 to 21 feet head of water, $m=.615$."

A mean of the coefficients of Table 9 is equal to .62 nearly.

In ordinary practice this is employed. When greater accuracy is required, reference will need be had to Table 9.

Partial Contraction.

Experiments show that if contraction be suppressed, the flow of water through an orifice will be increased accordingly.

Let n=the ratio between the entire perimeter of an orifice and the part suppressed—that is, if p denote the entire perimeter, and $p_{\prime}$ the part suppressed, then $n=\frac{p_{\prime}}{p}$.

c=coefficient of discharge due perfect contraction.

c_n=coefficient of discharge due partial contraction.

x=a number deduced from experiment, which, being multiplied by the product of the ratio, n, and the coefficient, c, gives cxn, the increase due partial contraction; thus,

$$c_n = c\,(1+xn). \qquad (93)$$

TABLE 9.

Coefficients for the Flow of Water through circular orifices.
Extracts from D'Aubiusson, Fanning and Weisbach.

OBSERVERS.	Diam Feet.	Heads. Feet.	Coeffi- cients.
Mariotti	0.0223	5.8712	.692
"	.0223	25.9120	.692
Castel	.0328	2.1320	.673
"	.0328	1.0168	.654
"	.0492	0.4526	.632
"	.0492	0.9840	.617
Eytelwine.	.0856	2.3714	.618
Bossut	.0889	4.2640	.619
Michelotti	.0889	7.3144	.618
Castel	.0984	0.5510	.629
Venturi	.1345	2.8864	.622
Bossut	.1771	12.4968	.618
Michelotti	.1771	7.2160	.607
"	.2657	7.3472	.613
"	.2657	12.4968	.612
"	.2657	22.1728	.597
"	.5314	6.9208	.619
"	.5314	12.0048	.619
		Mean.	.630
Gen. Ellis	.2	1.7677	.58829
"	.2	5.8269	.60915
"	.2	9.6381	.61530
"	.1	1.1470	.57373
"	.1	10.8819	.59431
"	.1	17.7400	.59994
"	.5	2.1516	.60025
"	.5	9.0600	.60191
"	.5	17.2650	.59626
Weisbach	.0328	2.0000	.628
"	.0656	2.0000	.621
"	.0984	2.0000	.614
"	.1312	2.0000	.607
"	.0328	.8333	.637
"	.0656	.8333	.629
"	.0984	.8333	.622
"	.1312	.8333	.614
		Mean.	.609

An inspection of Table 9 shows that the coefficient of flow for small orifices and for small velocities, is greater than it is for large orifices and for great velocities.

It will also be observed that the results of experiments differ considerably, though the data employed is approximately similar.

Thus Castel finds the coefficient of flow for an orifice .0328 feet diameter, under a head of 2.132 feet, to be .673: while Weisbach finds it, for an orifice .0328 feet diameter, under a head of 2 feet, to be .628.

Bidone's experiments give, for circular orifices, $x=0.128$, and for rectangular orifices, $x=0.125$.

Weisbach's experiments give, for rectangular orifices, $x=0.134$.

Weisbach, however, employs for rectangular orifices the mean between these results—that is,

$$\frac{.152+.134}{2}=0.143.$$

Substituting these values in Eq. (93) there results, for circular orifices:

$$c_n=c(1+0.128n). \qquad (94)$$

And for rectangular orifices:

$$c_n=c(1+0.143n). \qquad (95)$$

To find the coefficient of discharge of partial contraction for circular and for rectangular orifices.

Rule 28.—Case 1st—The orifice being circular, add 1. to .128 times the ratio of the entire perimeter to the part suppressed, and multiply this sum by the coefficient of discharge of perfect contraction.

Case 2d—The orifice being rectangular, add 1 to 0.143 times the ratio of the entire perimeter to the part suppressed, and multiply this sum by the coefficient of discharge of perfect contraction.

Ex. 49.—A rectangular orifice being 1 foot wide, 6 inches high, and the head 10 feet, what is the coefficient of discharge if the contraction at one end be suppressed?

Cal.—By Table 7, coefficient of perfect contraction for the given head and given orifice is $=.601$.

Part suppressed $=6$ inches $=.5$ feet.

Entire perimeter $=1+1+.5+.5=3$ feet.

Ratio of entire perimeter to part suppressed $=\frac{0.5}{3}$—

0.143 times this ratio; $0.143\times\frac{0.5}{3}=.024$.

Sum of 1 and this product $=1.024$.

This sum, multiplied by .601, the coefficient of perfect contraction, $.601\times1.024=.615$, the coefficient of partial contraction.$=$ *Ans.*

Ex. 50.—A rectangular orifice being 1 foot wide, 6 inches high, and the head 10 feet, what is the coefficient of partial contraction if the contraction at both ends be suppressed?

Cal.—By Table 7, the coefficient of perfect contraction, for the given head and given orifice, is=.601.

Part suppressed 6″+6″=12″=1 foot.

Entire perimeter, 1+1+.5+.5=3 feet.

Ratio of entire perimeter to part suppressed=$\frac{1}{3}$, 0.143 times the ratio; 0.143×$\frac{1}{3}$=0.048.

Sum of 1 and this product=1.048.

This sum, multiplied by .601, the coefficient of perfect contraction.

.601×1.048=.630, the coefficient of partial contraction.—*Ans.*

TO DETERMINE THE COEFFICIENT OF CONTRACTION FOR A GIVEN ORIFICE AND GIVEN HEAD OF WATER.

Let a=the hight of orifice; b=the breadth of orifice; h=head of water.

And let c, c_n, n, p, $p_{\prime}$ and x have the same offices as assigned them under the heading, "Partial Contraction;" c_b=the coefficient of contraction due the breadth.

In Table 7, the hights of the orifice vary from 4 feet to 0.125 feet, while the breadth of each orifice is 1 foot. It is evident, if the contraction be suppressed at both ends of any orifice given in Table 7, the contraction due the horizontal lips only, each 1 foot in length, will obtain. Now if the lips be increased any given number of times 1 foot, the contraction will be proportionately increased. This being done, if the contraction due the ends be restored, and the result divided by the length of the elongated orifice, or by the given number of times that the lips were increased in length, the quotient will express the mean contraction due 1 foot breadth of the given orifice.

For an orifice in Table 7, whose hight is a, and whose head of water is h, if the contraction of both ends be suppressed, the ratio $n=\frac{p_l}{p}=\frac{2a}{2+2a}$, and the coefficient of partial contraction:

$$c_n=c+\frac{2cax}{2+2a}. \qquad (96)$$

Multiplying both sides of Eq. (96) by b, the breadth of the given orifice, restoring the end contraction, $\frac{2cax}{2+2a}$, dividing the result by the breadth b, substituting c_b for left hand member, and reducing,

$$c_b=c\ (1+\frac{2ax}{2\ (1+a)})-\frac{2cax}{2b\ (1+a)}. \qquad (97)$$

Substituting in (97) the values of $x=0.143$, and $n=\frac{2a}{2\ (1+a)}$,

$$c_b=c\ (1+0\ 143n)-\frac{0.143cn}{b}. \qquad (98)$$

Rule 29.—Find, as by Rule 28, the value of the coefficient of partial contraction for an orifice of the given hight, 1 foot wide, and having the contraction at both ends suppressed.

From the value so found deduct the quotient arising from dividing 0.143 times the product of the coefficient of perfect contraction, and the ratio of the entire perimeter of the orifice 1 foot wide, to the part suppressed, by the breadth of the given orifice. The remainder will be the coefficient of contraction due the given orifice. Rule 29 is derived from formula (98).

Ex. 51.—A rectangular orifice being 5 feet wide, 3 inches high, and the head of water 3 feet, what is the coefficient of contraction?

Cal.—By Table 9, the coefficient of perfect contraction, for an orifice 1 foot wide, 3 inches high, under a head of 8 feet, is=.607.

Part suppressed=3″+3″=6″=.5 feet.

Entire perimeter=1+1+.5=2.5 feet.

Ratio of entire perimeter to part suppressed=$\frac{.5}{2.5}$=.2; 0.143 times this ratio; 0.143×.2=.0286.

Sum of 1 and this product=1.0286.

This sum, multiplied by .607, the coefficient of perfect contraction, gives the value of the coefficient of partial contraction, when the contraction of both ends is suppressed,

$$c_n=1.0286\times.607=.6244.$$

By Rule 29, 0.143 times the product of the coefficient of perfect contraction, and the ratio of the entire perimeter of the orifice 1 foot wide, to the part suppressed, divided by the breadth of the given orifice, 0.143×.2×.607÷5=.0035;

c_b=.6244—.0035=.621.—*Ans.*

Ex. 52.—A rectangular orifice being 2 feet wide, 4 feet high, and under a head on center of 2.5 feet, what is the coefficient of discharge?

Cal.—By Table 7, the coefficient of perfect contraction, as determined by experiment, for an orifice 1 foot wide, and otherwise conforming to the given conditions, is $=.629$.

Part suppressed (both ends) $4+4=8$ feet.

Entire perimeter, $1+1+8=10$ feet.

Ratio of entire perimeter to part suppressed $=\frac{8}{10}=.8$; 0.143 times this ratio; $0.143\times.8=0.1144$.

Sum of 1 and this product $=1.1144$.

This sum multiplied by .629: $1.1144\times.629=.701$.

By Rule 29, 0.143 times the product of the coefficient of perfect contraction, and the ratio of the entire perimeter of the orifice 1 foot wide, to the part suppressed, divided by the breadth of the given orifice; $0.143\times.8\times.629\div2=0.036$,

$$c_b=.701-.036=.665.\text{—}Ans.$$

Formulas (95) and (98), and Rules 28 and 29, based upon the mean results of the experiments of Bidone and Weisbach, give but approximations to the true coefficients sought. They are, however, sufficiently accurate for most cases occurring in practice. Example 52 is an extreme case. Yet the coefficient .665, determined from its solution, seems practically correct, or not too large, in presence of the fact that the area of the given orifice is twice as great as that of the tabulated orifice whose coefficient of discharge is .629;

while the perimeter or contracting boundary of the former is to that of the latter as 12 is to 10. Still it is to be admitted that, in determining the coefficient for a given orifice, the result is more satisfactory when the hight of the tabulated orifice employed does not much exceed its breadth.

IMPERFECT CONTRACTION.

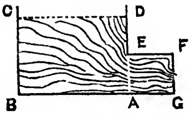


FIG. 16.

In the flow of a stream from an orifice, the head of water being nominally still, and the orifice small, in relation to the side of the vessel in which it lies, the contraction is called perfect; the water arriving with considerable velocity at the orifice, as through a conduit, A G F E, Fig. 16, which cross section varies from 1 to 20 times that of the orifice, the contraction thence is termed imperfect.

Let c=coefficient of perfect contraction; c_n=coefficient of imperfect contraction; n=ratio of the cross section of the conduit, A G F E, Fig. 16, through A E, to the area of the orifice O; A=area of orifice; $A_{,}$= area cross section of conduit.

The values of imperfect contraction given by Weisbach, as determined by his experiments and calculations, are:

1st.—For circular orifices:

$$c_n = c\,[1 + 0.04564\,(14.821^n - 1)\,]. \qquad (99)$$

2d.—For rectangular orifices:

$$c_n = c\,[1 + 0.76\,(9^n - 1)\,]. \qquad (100)$$

Equation (99) for circular orifices is readily resolved into this form:

$$\frac{c_n - c}{c} = 0.04564\,(14.821^n - 1), \qquad (101)$$

And equation for rectangular orifices into this:

$$\frac{c_n - c}{c} = 0.076\,(9^n - 1). \qquad (102)$$

The length of the conduit or adjunct is assumed to be three times its diameter, or not sufficiently great for the flow of water to be sensibly affected by side friction, as occurs in long pipes.

By giving fractional values to $n = \frac{A}{A_{\prime}}$, or values not greater than 1, numerical values corresponding, are found for the expression $\frac{c_n - c}{c}$ in equations (101) and (102).

In illustration: assume the areas of the orifice equal to 1 square foot, and the area of the cross section of the conduit, A G F E, equal to 2 square feet; then $n = \frac{A}{A_{,}} = \frac{1}{2}$.

Substituting the value of n in Eq. 102,

$$\frac{c_n - c}{c} = 0.076\ (9^{\frac{1}{2}} - 1). \qquad (103)$$

Now the $\frac{1}{2}$ power of 9, in other words the square root of 9 = 3; 3—1 = 2; hence $\frac{c_n - c}{c} = 0.076 \times 2 = .152$, correction found for $n = \frac{1}{2} = 0.5$, in Table 2.

In the computation of Tables 10 and 11, different values from $n = .05$—the common difference being .05—to $n = 1$, are employed.

TABLE 10.

Corrections of the Coefficients of Flow for Circular Orifices. Weisbach.

n.....	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
$\frac{n-c}{c}$.	0.007	0.014	0.023	0.034	0.045	0.059	0.075	0.092	0.112	0.134
n.....	0. 55	0.60	0.65	0.70	0.75	0.80	0.85	0.900	0.95	1.00
$\frac{c_{,} - c}{c}$.	0.161	0.189	0.223	0.260	0.303	0.351	0.408	0.471	0.546	0.613

If n has any value not found in Table 10 or Table 11, substitute such value in equation 99 in case the given orifice is circular, or in equation (100) in case the orifice is rectangular, and solve by means of logarithms.

TABLE 11.

Corrections of the Coefficients of Flow for Rectangular Orifices.—Weisbach.

n.....	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
$\frac{c^n - c}{c}$.	0.009	0.019	0.030	0.042	0.056	0.071	0.088	0.107	0.128	0.152
n.....	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00
$\frac{c_n - c}{c}$.	0.178	0.208	0.241	0.278	0.319	0.365	0.416	0.473	0.537	0.608

Examples and Calculations Illustrating the use of Tables 10 and 11.

Ex. 53.—The diameter of a circular orifice being 6 inches=.5 feet, the head on center 9.06 feet, the area of the orifice one-fourth (.25), that of the cross section of the conduit A G F E, Fig. 16, what is the coefficient of discharge?

Cal.—By Table 9, the coefficient of discharge through a circular orifice 6 inches diameter=.5 feet in a thin partition under a head of 9.06 feet of water, nominally still, as observed by Gen. Ellis, is=.60191; say c=.602.

By Table 10, the value corresponding to $n=.25$, the given ratio, is:

$$\frac{c_n-c}{c}=0.045,$$

Solving this equation for c_n,

$$c_n=1.045c.$$

Substituting the value of $c=.602$ in the last equation, there results:

$$c_n=1.045\times.602=.629.\text{—}Ans.$$

Ex. 54.—A rectangular orifice being 9 inches high and 1 foot wide in the end of a conduit, as A G F E, Fig. 16, 1 foot high, 1.25 feet wide, and 3 feet long, under a head of 3.5 feet on center, of water nominally still in tank, B A D C, what is the coefficient of discharge?

By Table 7, the coefficient of discharge due the given orifice and given head of water nominally still is

$$c=.609,$$

Area of orifice, $.95\times1=.95$,

Area of cross-section of conduit, $1.25\times1=1.25$.

Ratio of transverse sections $n=\frac{.75}{1.25}=.6$.

By Table 11, the value corresponding to $n=.6$, the ratio of transverse sections, is $\frac{c_n-c}{c}=0.208$; whence

$$c_n=1.208c.$$

Substituting in last equation the value of $c=.609$.

$$c_n=1.208\times.609=.736.\text{—}Ans.$$

Ex. 55.—An orifice 2 feet square in the end of a conduit, A G F E, Fig. 16, 2.5 feet square, 6 feet long, under a head of 5 feet on center of water, nominally still in tank, B A D C, what is the discharge in cubic feet per second?

Cal. 1st.—By Table 7, the coefficient of perfect contraction applicable to an orifice 1 foot wide, 2 feet high, under a head of five feet of water nominally still, is c=.612.

By Rules 28 and 29:

Part suppressed (both ends) $2+2=4$ feet.

Entire perimeter (tabulated orifice) $1+1+4=6$ feet.

Ratio of entire perimeter to part suppressed$=\frac{4}{6}$, 0.143 times this ratio; $0.143\times\frac{4}{6}=.0953$.

Sum of 1 and this product$=1.0953$.

This sum, multiplied by tabulated coefficient,

$$1.0953\times.612=.670.$$

0.143 times the product of the coefficient of perfect contraction, and the ratio of the entire perimeter of the orifice 1 foot wide to the part suppressed, divided by the breadth of the given orifice,

$$0.143\times.612\times\tfrac{4}{6}\div2=0.029.$$

Coefficient due given orifice (2 feet square) under head of still water:

$$c_b=.670-.029=.641.$$

Area of given orifice, $2\times2=4$ square feet.

Area of cross section of conduit, $2.5 \times 2.5 = 6.25$ square feet.

Ratio of transverse sections, $n = \frac{4}{6.25} = .64$.

By Table 11, the value corresponding to $n = .65$ (nearest to .64), the ratio of transverse sections is $\frac{c_n - c}{c} = .241$.

By interpolation between values corresponding to $n = .60$, and $n = .65$, there results approximately:

$$\frac{c_n - c}{c} = .234; \text{ whence,}$$

$$c_n = 1.234c.$$

Substituting the value of $c_b = .641$, as before found, for c in the last equation, and there results:

$$c_n = 1.234 \times .641 = .791.$$

By Table 6, square root of head $\sqrt{5} = 2.236$.

By Rule 27, $Q = .791 \times 8.025 \times 4 \times 2.236 = 56.78$ cubic feet.—*Ans.*

Cal. 2d.—By Table 7, the discharge found for an orifice 1 foot wide, 2 feet high, under a head of 5 feet, is $= 21.98$ cubic feet per second.

If it be assumed, that for practical purposes, the discharge through the given orifice 2 feet square, in Example 55, will be proportionate to the tabulated discharge, there will result:

$Q = 21.98 \times 2 = 43.96$ cubic feet per second.

By comparison, it is seen the result by Cal. 1 is nearly 30 per cent greater than that obtained by Cal. 2d.

This discrepancy seems to illustrate the necessity of careful investigation in essaying the determination of problems of practical hydraulics, however tedious the process may be.

COEFFICIENT OF THE FLOW OF WATER THROUGH A VERTICAL RECTANGULAR ORIFICE, UNDER A HEAD IN MOTION.

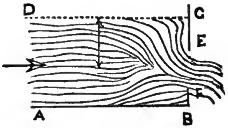


FIG. 17.

The case in which the head of water is in motion, occurs for the most part in open channels. In Fig. 17, A B C D represents a vertical section lengthwise of a stream of water in an open channel. B C a dam across the stream, in which is an orifice, E F in hight. The dam is assumed to act as a restraint, but not sufficient to sensibly affect the mean velocity of the stream of water above it.

Let c=coefficient of discharge under a head of water, nominally still.

c_n=coefficient of discharge under a head of water in motion, and dependent for its value on the ratio n.

A=area of orifice.

$A_{\prime}$=area of cross section of canal or channel.

$n=\frac{A}{A_{\prime}}$ ratio of these areas, not exceeding $\frac{1}{2}$.

Weisbach gives as the result of his experiments, the head being measured 1 meter=3.28 feet above the dam.

$$\frac{c - c}{c} = 0.641 \left\{\frac{A}{A_{\prime}}\right\}^2 = 0.641\ n^2. \qquad (104)$$

Whence,

$$c = (1 + .641n^2)\ c. \qquad (105)$$

To Find the Coefficient of Discharge Under a Head of Water in Motion.

Rule 30.—Add 1 to .641 times the square of the ratio of transverse sections—that of the canal to that of the orifice—and multiply this sum by the coefficient of discharge due the given orifice and given head as though it were in still water.

Rule derived from formula (105).

TABLE 12.

Corrections of the Coefficients of Flow Through Rectangular Orifices Under a Head of Water in Motion.—Weisbach.

n....	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
$\frac{c_n - c}{c}$	0.002	0.006	0.014	0.026	0.040	0.058	0.079	0.103	0.130	0.160

Ex. 56. A dam containing a rectangular orifice 5 feet wide, 1 foot high, put across a flume 6 feet wide, raises the water 5 feet in hight above the bottom of the flume, and 3.5 feet above the lower edge of the orifice. What is the discharge in cubic feet per second ?

Cal. 1st.—Half hight of orifice=.5 feet.

Head on center 3.5—.5=3 feet.

Area of orifice $5\times1=5$ square feet.

Cross section of flume, $6\times5=30$ square feet.

Ratio of transverse sections $\frac{5}{30}=\frac{1}{6}$.

Square of ratio $(\frac{1}{6})^2=0.0278$.

By Table 7. Coefficient of perfect contraction for an orifice 1 foot wide, 1 foot high, under a head of 3 feet is=.605.

By Rule 28.--Part (both ends) suppressed $1+1=2$ feet. Entire perimeter (tabulated)=4 feet. Ratio of entire perimeter to part suppressed, $\frac{2}{4}=.5$; 0.143 times this ratio; $.143\times.5=.0715$. This sum multiplied by .605, the coefficient of perfect contraction, gives the value of the coefficient of partial contraction when the contraction of both ends is suppressed.

$$c_n=1.0715\times.605=.648.$$

By Rule 29.—0.143 times the product of the coefficient of perfect contraction and the ratio of the entire perimeter of the orifice 1 foot wide, to the part suppressed, divided by the breadth of the given orifice;

$$0.143\times.5\times.605\div5=.009.$$

Whence,

$$c_b = .648 - .009 = .639.$$

Substitute the value of $c_b = .639$ for c, and the value of the square of the ratio; $(\frac{1}{6})^2 = .0278$ in formula (105) or employ Rule 30.

$$c_n = (1 + .641 \times .0278) \times .639 = .650.$$

By Table 6:

Square root of given head of 3 feet=1.732.

By Rule 27:

$Q = .650 \times 8.025 \times 5 \times 1.732 = 45.17$ cubic feet per second.—*Ans.*

Cal. 2d. By Table 7: The discharge found for an orifice 1 foot wide, 1 foot high, under a head of 3 feet, is 8.41 cubic feet per second. If it be assumed that for practical purposes the discharge through the given orifice, 5 feet wide, 1 foot high, in Example 56, will be proportionate to the tabulated discharge, there will result:

$$8.41 \times 5 = 42.05.\text{—}Ans.$$

A discrepancy of 3.12 cubic feet, or $7\frac{4}{10}$ per cent.

Flow of Water Through Short Tubes.

Short tubes or adjutages are cylindrical, conical or compound in form.

Cylindrical Tubes.—The length of a cylindrical tube being from 2.5 to 3 times its diameter, the mean coefficient of flow through it as determined by the experiments of Bidone, Eytelwine, D'Aubuisson and Weisbach, is .815, while under otherwise similar circumstances, the mean coefficient of discharge through an orifice in a thin plate is .615. The ratio of .815 to .615 is 1.325; that is, the discharge through a short tube of the given proportions (2.6 to 1), is 1.325 times as much as the discharge through an orifice of equal diameter in a thin plate. For practical purposes this ratio may be assumed general in its application without material error.

Hence, to find the coefficient for a short tube, having given the coefficient of an orifice in a thin plate, of equal diameter, and under an equal head.

Rule 31.—Multiply the given coefficient of the orifice by 1.325.

Ex. 57.—The diameter of a short tube—length to diameter as 2.6 to 1—being 6 inches, and the head 9.06 feet, what is the coefficient of discharge?

Cal.—Given diameter 6″=.5 feet.

By Table 9, coefficient due orifice, .5 feet diameter, under 9.06 feet head=.602.

By Rule 31, $.602 \times 1.325 = .798$.—*Ans.*

In case there are no experiments on which to rely, the mean coefficient .815 is to be employed.

Let d=diameter in feet of a cylindrical tube whose length is from 2.5 to 3 times the diameter.

h=head of water on center.

c=.815, coefficient of discharge.

$a=.7854d^2$, area of cross section of tube.

Q=discharge in cubic feet per second.

Substituting the values of a, c and $h=h_{,}$ in equation (92).

$$Q=.815\times8.025\times.7854\, d^2 \sqrt{h}. \qquad (106)$$

Whence,

$$Q=5.137\, d^2 \sqrt{h}. \qquad (107)$$

To find the flow of water through a cylindrical tube whose length is from 2.5 to 3 times the diameter.

Rule 32.—Multiply the square root of the head of water on center by 5.137 times the square of the diameter of the tube.

Rule 32 derived from equation (107).

Ex. 58.—A tube being 3 inches in diameter and 8 inches long, and the head of water in the center being 5 feet, what is the discharge in cubic feet per second?

Cal.—Diameter 3″=.25 feet.

Square of diameter $.25\times.25=.0625$ square feet.

By Table 6: Square root of head, $\sqrt{5}=2.236$.

$Q=5.137\times.0625\times2.236=.718$ cubic feet.=*Ans*.

In case the proportion of length to diameter is much changed, as 1 to 1, the coefficient of flow is nearly the same as that for a thin plate, or if the length be much increased over three times the diameter, the coefficient .815 becomes diminished according to the occurrence of friction of the sides of the lengthened tube, which is termed a pipe.

Conical Tubes.—Conical tubes are convergent or divergent. The outer orifice being smaller than the inner, the tube is convergent; but if larger, the tube is divergent.

Convergent Tubes.—Extensive experiments have been made by D'Aubuisson and Castel on the flow of water through convergent tubes. These were made with tubes of various sizes and proportions; but mostly with those .61 inches diameter at the discharging end, 1.59 inches at the inlet end, and under a head of water 9.84 feet. The results of their experiments, as stated by Weisbach, are given in the following table:

TABLE 13.

Coefficients of discharge and velocity for flow through conically convergent tubes.

Smaller diameter=.61 inches.

Angle of Convergence	Coefficient of Flow.	Coefficient of Velocity.	Angle of Convergence	Coefficient of Flow.	Coefficient of Velocity.
0° 0'	0 829	0.829	13° 24'	0.946	0.963
1° 36'	0.866	0.867	14° 28'	0.941	0.966
3° 10'	0.895	0.894	16° 36'	0.938	0.971
4° 10'	0.912	0.910	19° 28'	0.924	0.970
5° 26'	0.924	0.919	21° 0'	0.919	0.972
7° 52'	0.930	0.932	23° 0'	0.914	0.974
8° 58'	0.934	0.942	29° 58'	0.895	0.975
10° 20'	0.938	0.951	40° 20'	0.870	0.980
12° 4'	0.942	0 955	48° 0'	0.847	0.984

Ex. 59. The smaller diameter of a conically convergent tube being 6 inches, the angle of convergence

5° 26′ and the head of water on center 9.06 feet, what is the flow of water in cubic feet per second?

Cal.—Diameter 6 inches=.5 feet.

By Table 9 the coefficient corresponding to the given diameter and head=.602, and coefficient corresponding to .61 inches on which Table 13 is based =.618.

By Table 13 the coefficient corresponding to the given angle of convergence 5° 26′ is=.924.

Ratio of coefficients .602÷.618=.974.

Then coefficient of flow due the given diameter .924×.974=.900.

And cross section of tube .5×.5×.7854=.1963 square feet.

By Table 6, square root of head=$\sqrt{9.06}$=3.01 nearly.

By Rule 27, Q=.900×8.025×.1963×3.01=4.27 cubic feet.—*Ans.*

Divergent Tubes.—Experiments show that the flow of water through a short divergent tube is similar to that in a thin plate, the coefficients of which are given in Table 9. In ordinary practice .62 is employed.

In case a vacuum is formed in a divergent tube, the flow is greatly increased, so that it may then even exceed the theoretical flow due the force of gravity, through an orifice in a thin plate, whose diameter is equal to that of the smaller diameter of the divergent tube; in other words, its coefficient of flow becomes greater than unity. The conditions effecting this result are a high velocity of flow in a tube of small divergence, and whose length is several times its smaller diameter. Thus, the smaller diameter of a divergent tube being 1.32 inches, the length 9 times this diameter=11.88 inches, the included angle of the tube equal to 5° 6′, and the head of water 2.89 feet, Venturi found the coefficient of flow equal to 1.46, or 2.4 times that of an equal orifice in a thin plate. If the entry end of an otherwise similar tube be bell-mouthed in form, the coefficient of flow estimated for the smaller diameter will evidently exceed that obtained by Venturi. The principle of the formation of a vacuum by flowing water at a high rate of velocity, through a divergent tube, and thereby greatly increasing the volume of discharge, was known to the ancients. D'Aubuisson states that the application of the principle, at a distance less than 52.5 feet from the public conduits of Rome, by Roman citizens having grants of water, was prohibited by Roman law.

TABLE 14.

Coefficients of the flow of water through divergent tubes.

Angle.	Length of Tube. Feet.	Coefficient.	Angle.	Length of Tube. Feet.	Coefficient.
3° 30′	0.364	0.93	5° 44′	.193	.82
4° 38′	1.095	1.21	10° 16′	.865	.91
4° 38′	1.508	1.21	10° 16′	.147	.91
4° 38′	1.508	1.34	14° 14′	.147	.61
5° 44′	0.57	1.02			

Ex. 60.—In a divergent tube the smaller diameter being .61 of an inch, the length 1.508 feet, the angle included between its sides 4° 38′, and the head on center 2.89 feet, what is the volume of flow in cubic feet for 24 hours?

Cal.—Diameter .61 inches=.0508 feet.

By Table 14, mean coefficient of discharge (1.21+1.34)÷2=1.275.

Area of cross section of tube, .0508×.0508×.7854 =.002027 square feet.

By Table 6, square root of head $\sqrt{2.89}=1.7$.

By Rule 27, volume of discharge per second,

$Q=1.275\times8.025\times.002027\times1.7=.03525$.

In 24 hours are 86,400 seconds; hence, .03525× 86400=3046.05 cubic feet.—*Ans.*

Ex. 61.—In a compound tube, Fig. 18, the cylindrical part, P, is .0853 feet in diameter, 2.0605 feet in length; the convergent part, C, .2559 feet long; the divergent part, D, .7667 feet in length; and the head 2.3642 feet. What will be the discharge in 10 hours?

Cal.—By Table 15, the coefficient of flow due C P D=.905.

Compound Tubes.—Compound tubes are of various forms. Eytelwine, as stated by J. T. Fanning, after experimenting with cylindrical tubes of uniform diameter and different lengths, placed between them and the reservoir convergent tubes of the form of the contracted vein, and renewed the experiments; then added to the discharge end a divergent tube with 5° 6′ angle. Fig. 18.

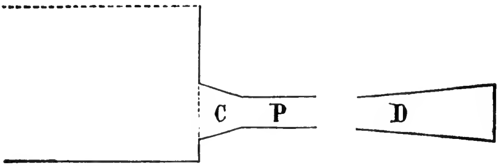


FIG. 18.

In Fig. 18, C represents the conically convergent part of the tube of the form of the contracted vein; P, the cylindrical part of uniform diameter, but of different lengths, and the conically divergent part with 5° 6′ angle. The results obtained are given in the following table:

TABLE 15.

Coefficients of the flow of water through compound tubes.

Head. Feet.	Diameter of P Feet.	Length of P in Diameter.	Length of P in Feet.	Coefficient for P.	Coefficient for C P.	Coefficient for C P D.
2.3642	0.0853	0.038	0.0033	0.62	......	....
2.3642	0.0853	1.000	0.0853	.62	.967	....
2.3642	0.0853	3.000	0.2559	.82	.943	1.107
2.3642	0.0853	12.077	1.0302	.77	.870	.978
2.3642	0.0853	24.156	2.0605	.73	.803	.905
2.3642	0.0853	36.233	3.0907	.68	.741	.836
2.3642	0.0853	48.272	4.1176	.63	.687	.762
2.3642	0.0853	60.116	5.1479	.60	.648	.702

Area of cross-section of tube P, $.0853 \times .0853 \times .7854 = .005761$ square feet.

By Table 6, the square root of head $\sqrt{2.3642} = 1.54$ nearly.

By Rule 27, $Q = .905 \times 8.025 \times .0057 \times 1.54 = .06367$ cubic feet per second.

In 10 hours are $3600 \times 10 = 36{,}000$ seconds; hence, $.06367 \times 36000 = 2292.07$ cubic feet.—*Ans.*

Divergent and Compound Tubes.—These tubes seldom find a place in practice. The lessons, however, which they teach, are of interest, and serve to stimulate the vigilance of the engineer, lest irregularities occurring from design or otherwise, shall elude his observation in matters of importance.

FLOW OF WATER THROUGH PIPES.

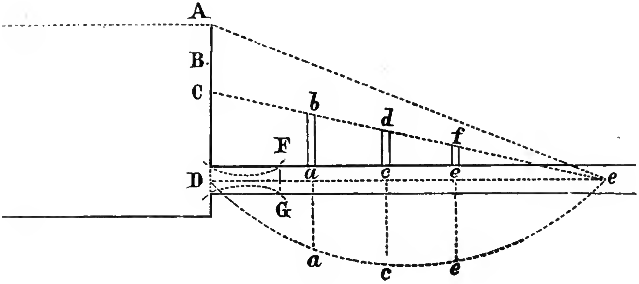


FIG. 19.

The flow of water through a pipe is estimated to begin at a point where the stream, after contraction, expands so as to fill the pipe, as at F G, Fig. 19.

The part, D F G, performing the office of a short tube, is, as hereinbefore shown, from 2.6 to 3 times the diameter of the pipe.

The total head, A D, consists of three parts: A B, which generates the velocity; B C, which overcomes the resistance of entry; and C D, which overcomes all resistances in the pipe, F G E. C E is termed the hydraulic gradient, and is such, if the pipe is running full, the water will rise to this grade through tubes, as *a b*, *c d* and *e f*.

Short and Long Pipes.—A pipe, exclusive of the tube portion described, in case its length does not exceed a thousand times its diameter, is termed a *short pipe*, and in case its length exceeds a thousand times its diameter, is termed a *long pipe.*

Let, in Fig. 19:

h=A D, the total head.

h_v=A B, the velocity head, or head necessary to generate the velocity v.

h_e=B C, the entry head, or head necessary to overcome the resistances of entry.

$h_i=h_v+h_e$=A C, the inlet head, or head necessary to generate the velocity, v, in the pipe, and to overcome the resistances of entry.

h_f=C D, the head necessary to overcome the resistances within the pipe.

v=the measured velocity of discharge.

v_i=the theoretical velocity due the head h_i=A C.

c=the coefficient of flow in a short tube (length to diameter as 3 to 1), as determined by experiment.

$c_v=c$, the coefficient of velocity, as the stream, after contraction, fills the pipe.

c_e=the coefficient of entry.

c_f=a variable coefficient for the resistances within pipes, as determined by experiment.

d=internal diameter of pipe.

p=perimeter or internal contour of pipe.

a=area of cross section of pipe.

l=length of pipe.

s=sine of slope.

r=hydraulic mean radius.

f=amount of resistances to flow in the pipe.

w=weight of water discharged during the time of resistance to its flow.

Then there results:

Equation of total head, $h=h_v+h_e+h_f$. (108)

Equation of entry head, $h_e=h_i-h_v$. (109)

By equation (8), velocity head, $h_v=\frac{v^2}{2g}$. (110)

By equation (8), inlet head, $h_i=\frac{v_i^2}{2g}$. (111)

Equation of theoretical velocity due inlet head,

$$v_i=\frac{v}{c_v}. \qquad (112)$$

Substituting the value of v_i of (112) in (111),

$$h_i=\frac{v^2}{2g\,c_v^2}. \qquad (113)$$

Substituting the values of h_i of (113), and of h_v of (110) in (109),

$$h_e=\left\{\frac{1}{c_v^2}-1\right\}\frac{v^2}{2g}. \qquad (114)$$

Putting $c_e=\left\{\frac{1}{c_v^2}-1\right\}$. (115)

Substituting c_e for $\left\{\frac{1}{c_v^2}-1\right\}$ in (114),

$$h_e=\frac{c_e\,v^2}{2g}. \qquad (116)$$

The work performed by the weight, w, falling vertically by the force of gravity through the distance, h_f, in one second, is

$$F=w\,h_f, \text{ “foot pounds.”} \qquad (117)$$

Experiments show that the amount of resistances occurring from friction of the internal surfaces of a pipe, and from other causes, varies nearly as the square of the velocity, v.

Experiments also show that the amount of resistances increases directly as the length, l, of the pipe, and inversely as its diameter, d, or hydraulic mean radius, $r=\frac{a}{p}$.

The work performed by the force of gravity, in overcoming these resistances, so as to effect the discharge of the weight, w, of water with the velocity, v, per second, as first proposed by Chezy, and subquently adopted by most authors on hydraulics, is:

$$F=\frac{w\,c_f\,p\,l\,h_v}{a} \text{ “foot pounds,”} \qquad (118)$$

in which c_f is a variable coefficient whose values are determined by experiment.

Substituting the value of $h_v = \frac{v^2}{2g}$ of (110) in (118), and equating (117) and (118),

$$w\,h_f = \frac{w\,c_f\,p\,l\,v^2}{2g\,a}. \qquad (119)$$

Dividing (119) by w, $h_f = \frac{c_f\,p\,l\,v^2}{2g\,a}$. (120)

Substituting the values of h_v of (110), h_e of (116), and h_f of 120, in (108),

$$h = \left\{ \frac{v^2}{2g} + \frac{c_e\,v^2}{2g} + \frac{c_f\,p\,l\,v^2}{2g\,a} \right\} \qquad (121)$$

Factoring (121), $h = \left\{ 1 + c_e + \frac{c_f\,p\,l}{a} \right\} \frac{v^2}{2g}$. (122)

Transposing (122) with respect to v,

$$v = \left\{ \frac{2\,g\,h}{1 + c_e + \frac{c_f\,p\,l}{a}} \right\}^{\frac{1}{2}} \qquad (123)$$

Hydraulic radius, $r = \frac{a}{p} = \frac{ñ\,d^2}{4\,ñ\,d} = \frac{d}{4}$. (124)

Substituting $\frac{1}{r}$ for $\frac{p}{a}$ of (124) in (123),

$$v = \left\{ \frac{2\,g\,h}{(1 + c_e) + \frac{c_f\,l}{r}} \right\}^{\frac{1}{2}} \qquad (125)$$

Factoring (125),

$$v = (2\,g\,h)^{\frac{1}{2}} \left\{ \frac{1}{(1 + c_e) + \frac{c_f\,l}{r}} \right\}^{\frac{1}{2}} \qquad (126)$$

Under the heading, "flow of water through short tubes," the value of the mean coefficient of discharge has been shown to be $c = .815$. But $c = c_v = .815$; that is, the coefficient of flow at the inlet orifice of a short tube, is equal to the coefficient of velocity in the pipe, estimating the beginning at that point, where the stream, after contraction, expands so as to fill the pipe.

Substituting the value of $c_v = .815$ in equation (115),

$$c_e = .505. \qquad (127)$$

Substituting the value of $c = .505$ in (125),

$$v = \left\{ \frac{2\,g\,h}{1.505 + \frac{c_f\,l}{r}} \right\}^{\frac{1}{2}} \qquad (128)$$

In determining the velocity of flow in a pipe, whose length exceeds a thousand times the diameter, the value of $(1+c_e)$ (in 125), being small in comparison with the value of $\frac{c_f\,l}{r}$ is usually omitted as insignificant; or, more direct, let equation (120) be transposed with respect to v, and $\frac{1}{r}$ be substituted for $\frac{p}{a}$:

$$v = \left\{ \frac{2\,g\,r}{c_f} \times \frac{h_f}{l} \right\}^{\frac{1}{2}} \qquad (129)$$

Substitititing $s = \frac{h_f}{l}$, the sine of slope, C E D, in (129),

$$v = \left\{ \frac{2\,g}{c} \right\}^{\frac{1}{2}} \left\{ r\,s \right\}^{\frac{1}{2}} \qquad (130)$$

Hydraulicians have given different empirical formulas for the determination of the values of c_f. Thus: Weisbach, assuming that the resistance of friction increases simultaneously as the square, and as the square root of the cube of the velocity finds as follows:

$$z=(4\,c_f)=0.01439+\frac{0.017155}{\sqrt{v}}. \qquad (131)$$

This formula, as claimed by its author, agrees more accurately with observations than do those of the older hydraulicians. In the experiments from which it was derived, the velocity varied from 0.14 feet to 15.25 feet per second, and the pipes from 1.06 inches to 5.31 inches in diameter.

H. Darcy's formula for velocity, resolved with respect to this coefficient, gives:

$$c_f=.00497554+\frac{.00010433}{r}. \qquad (132)$$

This formula was deduced from very extensive experiments. In these the variation, with respect to velocity, was from 0.29 feet to 16.24 feet per second, and with respect to diameters of pipes, from 3 inches to 20 inches nearly.

Weisbach remarks of this formula, that it "is not sufficiently accurate for small velocities."

J. T. Fanning's "Series of Coefficients of Flow (m)" [$m=c_f$] "of water in clean pipes, under pressure, at different velocities, and in pipes of different diameters"—from which the following table is extracted—seem more simple and comprehensive on this subject than can well be rendered in a single formula. These coefficients are deduced from experiments, in which the variation in velocities was from 0.18 feet to 46.7 feet per second, and in which the diameters of the pipes were from ½ inch to 3 feet.

TABLE 16.

Coefficients of resistance to the Flow (c_f) of Water in Clean Pipes. Extracts from Fanning.

Velocity. Feet per Second.	Diameters.							
	½-inch Coef.	1-inch Coef.	3-inch Coef.	6-inch Coef.	12-inch Coef.	24-inch Coef.	48-inch Coef.	96-inch Coef.
.1	.0150	.0119	.0080	.0073	.0067	......	......	......
.2	.0143	.0116	.0079	.0072	.0066	......	......	......
.3	.0137	.0113	.0078	.0072	.0066	.0055	......	......
.4	.0133	.0110	.0078	.0071	.0065	.0054	......	......
.5	.0128	.0107	.0077	.0071	.0065	.0054	.0040	......
.6	.0124	.0104	.0077	.0070	.0064	.0054	.0040	.0029
.7	.0120	.0102	.0076	.0070	.0064	.0053	.0040	.0029
.8	.0116	.0100	.0075	.0069	.0063	.0053	.0040	.0029
.9	.0113	.0097	.0075	.0069	.0063	.0053	.0040	.0029
1.0	.0110	.0095	.0074	.0068	.0062	.0053	.0040	.0029
1.1	.0107	.0093	.0074	.0068	.0062	.0052	.0039	.0029
1.2	.0104	.0091	.0073	.0067	.0062	.0052	.0039	.0029
1.3	.0101	.0090	.0073	.0067	.0061	.0052	.0039	.0029
1.4	.0099	.0088	.0072	.0067	.0061	.0051	.0039	.0028
1.5	.0096	.0087	.0072	.0066	.0061	.0051	.0039	.0028
1.6	.0094	.0085	.0072	.0066	.0060	.0051	.0039	.0028
1.7	.0092	.0084	.0071	.0066	.0060	.0051	.0039	.0028
1.8	.0090	.0083	.0071	.0065	.0060	.0051	.0039	.0028
1.9	.0088	.0082	.0070	.0065	.0060	.0050	.0039	.0028
2.	.0086	.0081	.0070	.0065	.0059	.0050	.0038	.0028
2.25	.0084	.0079	.0069	.0064	.0059	.0050	.0038	.0028
2.5	.0080	.0077	.0068	.0063	.0058	.0049	.0038	.0028
2.75	.0078	.0075	.0068	.0063	.0058	.0049	.0038	.0028
3.	.0075	.0073	.0067	.0062	.0057	.0048	.0038	.0028
3.5	.0073	.0071	.0066	.0061	.0056	.0048	.0037	.0028
4.	.0072	.0070	.0065	.0061	.0055	.0047	.0037	.0027
5.	.0070	.0068	.0064	.0059	.0054	.0047	.0037	.0027
6.	.0069	.0067	.0062	.0058	.0053	.0046	.0036	.0027
7.	.0068	.0066	.0061	.0057	.0053	.0045	.0036	.0027
8.	.0066	.0065	.0060	.0056	.0052	.0045	.0036	.0026
9.	.0065	.0064	.0059	.0056	.0051	.0045	.0036	.0026
10.	.0064	.0063	.0058	.0055	.0051	.0044	.0035	.0026
12.	.0063	.0061	.0058	.0054	.0050	.0044	.0035	.0026
14.	.0062	.0061	.0057	.0053	.0049	.0043	.0035	.0026
16.	.0062	.0060	.0057	.0053	.0049	.0043	......	......
18.	.0062	.0060	.0057	.0053	......	......	......	......
20.	.0062	.0060	.0057	......	......	......	......	......

COMPARISON OF THE VALUES OF THE COEFFICIENT OF RESISTANCES, c_f, AS FOUND BY WEISBACH, DARCY AND FANNING.

Ex. 62.—The velocity, in a clean pipe $\frac{1}{4}$ foot diameter, being 1 foot, what is the coefficient of resistances?

Cal. 1st.—By Weisbach's formula (131), square root of given velocity:

$$\sqrt{v}=1.$$

Substituting the value of $\sqrt{v}$ in (131), $z=4\,c_f=0.01439+\frac{0.017155}{1}$; whence,

$$c_f=.0079.\text{—}Ans.$$

Cal. 2d.—By Darcy's formula (132), hydraulic mean radius $r=\frac{d}{4}=\frac{1}{16}$.

Substituting value of r in (132), $c_f=.0066$.—*Ans.*

Cal. 3.—By Table 16, from Fanning's series of coefficients, $c_f=.0074$.—*Ans.*

Ex. 63.—The velocity in a clean pipe 2 feet diameter, being 4 feet per second, what is the coefficient of resistance?

Cal. 1st.—By Weisbach, formula (131), square root of given velocity:

$$\sqrt{v}=\sqrt{4}=2.$$

Substituting value of $\sqrt{v}$ in (131), $z=4\,c_f=0.01439+\frac{0.017155}{2}$; whence,

$$c_f=.0057.\text{—}Ans.$$

Cal. 2d.—By Darcy's formula (132), hydraulic mean radius $r=\frac{d}{4}=\frac{2}{4}=\frac{1}{2}$.

Substituting value of r in (132),

$$c_f=.0052.\text{—}Ans.$$

Cal. 3d.—By Table 16, Fanning's:

$$c_f=.0047.\text{—}Ans.$$

Ex. 64.—The velocity in a clean pipe 4 feet diameter, being 9 feet per second, what is the coefficient of resistance?

Cal. 1st.—By Weisbach's formula (131), square root of velocity,

$$\sqrt{v}=\sqrt{9}=3.$$

Substituting value of $\sqrt{v}$ in (131), $z=4\,c_f=0.01439+\frac{0.017155}{3}$; whence,

$$c_f=.0050.\text{—}Ans.$$

Cal. 2d.—By Darcy's formula (132), hydraulic mean radius $r=\frac{d}{4}=\frac{4}{4}=1$.

Substituting value of r in (132),

$$c_f=.0051.\text{—}Ans.$$

Cal. 3d.—By Table 16,

$$c_f=.0037.\text{—}Ans.$$

Ex. 65.—The velocity in a clean pipe 8 feet diameter, being 9 feet per second, what is the coefficient of resistance?

Cal. 1st.—By Weisbach's formula (131), square root of velocity,

$$\sqrt{v}=\sqrt{9}=3.$$

Substituting value of $\sqrt{v}$ in (131), $z=4$ $c_f=0.01439 + \frac{0.0171 5 5}{3}$; whence,

$$c_f=.0050.—Ans.$$

Cal. 2d.—By Darcy's formula (132), hydraulic mean radius $r=\frac{d}{4}=\frac{8}{4}=2$.

Substituting value of r in (132),

$$c_f=.0050.—Ans.$$

Cal. 3d.—By Table 16, from Fanning's series of coefficients,

$$c_f=.0026.—Ans.$$

Weisbach states that his formula for the coefficient of resistance (z) "is founded upon the assumption that the resistance of friction increases at the same time with the square and with the square root of the cube of the velocity." He further says "that the values from newer experiments show that the coefficient of resistance (z) for the friction of water in tubes decreases not only as the velocity (v) increases, but also, although more slowly, as the diameter of the pipe becomes greater." He omits, however, to amend his formula so as to embrace this element.

The coefficient of resistance (c_f in our notation) as deduced from Darcy's formula for velocity, decreases as the diameter of the pipe increases.

An inspection of the results obtained by Darcy's formula for Examples 64 and 65 would show that this diminuition practically ceases, when the diameter exceeds 4 feet. Thus, for a pipe 4 feet diameter, the coefficient found is .0051, and for a pipe 8 feet diameter, it is .0050. The results for large pipes, by Darcy's formula, are not in conformity with those obtained by later experiments.

By Table 16, from Fanning's series, the coefficients under the imposed conditions, as seen, are .0037 for a 4-foot pipe, and .0026 for an 8-foot pipe.

It is to be noticed that, in general, the results of later experiments with respect to the velocity of water in large pipes closely approximate those tabulated by Mr. Fanning in terms of the coefficient of resistance. Thus F. P. Stearns, M. Am. Soc. C. E., in a paper read before the American Society of Civil Engineers, October 1, 1884, states that three experiments made with the "Sudbury Conduit," a cast-iron pipe 4 feet diameter and 1747 feet in length, coated with a coal tar preparation, and in good condition, resulted as follows:

Mean velocity,	4.966 feet.
Mean coefficient of velocity,	142.11 feet.
Mean value of $r\,i$,	0.001221 feet.

Substituting the values of the mean velocity here given, $v=4.966$ feet, and the value of $r\,i=r\,s=0.001221$, in equation (130) and resolving with respect to the coefficient of resistance,

$$c_f=0.003188.$$

Referring to Table 16, we find the coefficient of resistance corresponding to a velocity of 5 feet (nearest approximate to 4.966 feet) in a pipe 4 feet diameter,

$$c_f = 0.0037.$$

Substituting the value of $c_f = 0.0037$ in Eq. (130), and resolving with respect to the coefficient of velocity, c, as employed by Mr. Stearns, there results: $c = 131.9$, as compared with 142.11.

Mr. Stearns further states, in the paper referred to, as follows:

"The experiments of Hamilton Smith, Jr., M. Am. Soc. C. E. (transactions for April, 1883), give curves of coefficients for pipes up to 30 inches diameter. Extending these curves would give for a 48-inch pipe, with velocity of 5 feet per second, a coefficient of 128, as compared with 142.1 above."

"The experiments of S. N. Tubbs, M. Am. Soc. C. E., on pipes of 2 and 3 feet diameter, would give by extending the curves a coefficient about the same as that in the average above."

"The experiments of Dr. Lampe, at Danzig, give results somewhat higher than those of Mr. Hamilton Smith, Jr."

The formulas applied to Example 3, to find the values of the coefficient of resistance for a 3-inch pipe, give by

Weisbach, $c_f = 0.0079$.
And by Darcy, $c_f = 0.0066$.
The value by Fanning is $c_f = 0.0074$.

We thus perceive concerning the values of this coefficient for *small pipes*, that Fanning differs less than 7 per cent from Weisbach, whose experiments were confined, as hitherto shown, to this class of pipes, while Darcy differs nearly 20 per cent from him. It must not be understood, however, that these differences obtain to the same extent in estimating the velocity. Reference to Eq. (130) shows that the coefficient of velocity depends not upon the direct value of the coefficient of resistance, but upon the square root of it. So that in estimating the velocity or quantity of flow, the real difference, in the case cited, between the results of Weisbach and Fanning would amount to only about $3\frac{1}{3}$ per cent.

Interpolation in Table 16.

The general formula of Darcy furnishes a simple means of interpolation with respect to the coefficients of resistance, when the difference between the extremes is not large. Thus the general formula is:

$$c_f = m + \frac{n}{r}, \qquad (133)$$

In which r represents the mean hydraulic radius m and n auxilliary numbers whose values are determined by substituting the tabulated values of r and c_f in (133) between which latter intermediate values are sought corresponding to different values of r'. The values thus found for m and n are employed as constants in the determination of these intermediate values of c_f.

For example, the velocity of flow being 3 feet per second, let it be required to determine the values of c_f due the diameters 30, 36, 42 and 44 inches, between 24 and 48 inches.

The hydraulic mean radii of 24 inches and 48 inches, are respectively $\frac{2}{4}=\frac{1}{2}$ and $\frac{4}{4}=1$. By Table 16, the value of c_f due a pipe 24 inches diameter is .0048, and that due a pipe 48 inches diameter is .0038. Substituting these values of r and c_f in Eq. (133), there results:

$$m+2n=.0048, \qquad (134)$$

$$\text{and } m+n=.0038. \qquad (135)$$

Subtracting (135) from (134),

$$n=.0010, \qquad (136)$$

Substituting value of n of (136) in (134),

$$m=.0028. \qquad (137)$$

The hydraulic mean radii corresponding to the given intermediate diameters, are respectively $\frac{5}{8}$, $\frac{3}{4}$, $\frac{7}{8}$ and $\frac{11}{12}$.

Substituting the values of $m=.0028$, $n=.0010$, and $r=\frac{5}{8}$, $\frac{3}{4}$, $\frac{7}{8}$ and $\frac{11}{12}$ in (133), there results for the given diameters:

$$30'', c_f=.0028+\frac{8}{5}(.0010)=.0044; \qquad (138)$$

$$36'', c_f=.0028+\frac{4}{3}(.0010)=.0041; \qquad (139)$$

$42''$, $c_f = .0028 + \frac{8}{7}(.0010) = .0039$; (160)

$44''$, $c_f = .0028 + \frac{12}{11}(.0010) = .0039$; (141)

to be interpolated as required.

Thus, by interpolation, may Table 16 be completed with the approximate values of the coefficient (c_f) of resistance.

To find the velocity of water flowing through short pipes.

Rule 33.—Extract the square root of 64.4 times the given head (total head) in feet, divided by 1.505, increased by the product of the length of the pipe in feet, and the coefficient of resistance—coefficient found in and computed from Table 16—due the given diameter, divided by the hydraulic mean radius.

Rule 33 corresponds Eq. (128).

Examples and calculations with respect to the flow of water through short pipes.

Ex. 66.—A pipe being 1 foot in diameter, 10 feet long, and the head of water being one hundred feet, what will be the velocity of flow per second?

Cal.—As the pipe is very short, it is evident that a large portion of the head will be expended in generating the velocity. Let it be assumed that not less than one-half the total head will be so expended. In which case the velocity will exceed 50 feet per second. Turning to Table 16, we perceive that the coefficient of resistance due a velocity of 16 feet in a pipe 1 foot diameter is .0049, and that the decrease of the preceding coefficients in the 1 foot diameter column, corresponding to the increase of velocity, is very small. Let, then, the least coefficient in column be taken, $c_f = .0049$.

Hydraulic mean radius $r=\frac{1}{4}$.

Substituting these values of $c_f=.0049$, $r=\frac{1}{4}$; also, the values of $2g=64.4$, $h=100$ feet, and $l=10$ feet in Eq. (128); or, in other words, apply Rule 33:

$$v=\left\{\frac{64.4\times100}{1.505+.0049\times10\div\frac{1}{4}}\right\}^{\frac{1}{2}}$$

Whence, $v=61.53$ feet.—*Ans.*

It will be remembered that equation (128), or Rule 33, is to be employed in finding the velocity when the given length of the pipe is less than one thousand times its diameter.

Ex. 67.—The head being 10 feet, what will be the velocity of water flowing through a pipe 2 feet in diameter and 1000 feet long?

Cal.—Assume by way of trial that the velocity will be 9 feet per second.

By Table 16, the coefficient of resistance due 9 feet velocity in a 2-foot pipe is $c_f=.0045$.

Hydraulic mean radius $r=\frac{2}{4}=\frac{1}{2}$.

Substituting the values of $c_f=.0045$, $r=\frac{1}{2}$, $h=10$,

$l=100$, and $2g=64.4$, in equation (128); or applying Rule 33:

$$v=\left\{\frac{64.4\times 10}{1.505+.0045\times 1000\div\frac{1}{2}}\right\}^{\frac{1}{2}}$$

Whence, $v=7.83$ feet, trial result.

Turning again to Table 16, we find that, corresponding to 8 feet velocity nearest approximate to our trial result 7.83 feet, the coefficient of resistance is $c_f=.0045$, the same as employed in our calculation.

Therefore we have:

$$v=7.83 \text{ feet.—} Ans.$$

Had the tabulated coefficient of resistance corresponding to the velocity found by trial, not have been equal to, or closely approximate to, that employed in our calculation, it would have been necessary to repeat the operation, using this new coefficient.

Ex. 68.—The head being 20 feet, the pipe 4 feet diameter, and 4,000 feet long, what is the velocity of flow per second?

Cal. 1st.—Assume the velocity for the purpose of trial 9 feet. Then:

By Table 16, $c_f=0.035$.

Hydraulic mean radius $r=\frac{4}{4}=1$.

Substituting the values of $c_f=.0036$, $r=1$, $h=20$ feet, $l=4{,}000$ feet, and $2g=64.4$ in Eq. (128); or applying Rule 33:

$$v=\left\{\frac{64.4\times 20}{1.505+.0036\times 4000\div 1}\right\}^{\frac{1}{2}}$$

Whence, $v=8.999$ feet.—*Ans.*

Cal. 2.—Employing formula (130), which is for a *long pipe*, there results:

$$v = \left\{ \frac{64.4 \times 20}{.0036 \times 4000 + 1} \right\}^{\frac{1}{2}}$$

Whence, v=9.428 feet.—*Ans.*

Comparing these results, the difference is seen to be .429 feet velocity per second. It appears quite evident, then, that 1.505, the first term in the denominator of the right hand member of Eq. (128)—an equation for velocity of water in *short pipes*—cannot be omitted, where accuracy is required, even if the length of the pipe, as in the given example, is 1,000 times the diameter.

Ex. 69.—The head being 12 feet, the pipe 15 inches diameter, 1,200 feet long, let it be required to find:

The velocity of flow per second;

The discharge in cubic feet per second;

The loss of head due velocity;

The loss of head due resistance of entry;

The head expended in overcoming the resistances within the pipe;

The sine of slope;

The fall per mile due resistances in pipe;

And the total fall per mile.

Cal.—Assume by way of trial the velocity of flow= 6 feet per second.

By Table 16, the coefficients of flow due a velocity of 6 feet in a 12-inch pipe, are c_f=.0053; and in a 24-inch pipe, c_f=.0046.

The hydraulic mean radii are as follows:

For 12″, $r=\frac{1}{4}$;

And for 24″, $r=\frac{2}{4}=\frac{1}{2}$.

Substituting the values of $c_f=.0053$, and of $r=\frac{1}{4}$, in Eq. (133); also, the values of $c_f=.0046$, and of $r=\frac{1}{2}$ in the same equation, there results:

$$m+4n=.0053. \qquad (a)$$

$$m+2n=.0046. \qquad (b)$$

$$\text{Whence, } n=.00035. \qquad (c)$$

Substituting value of (n) in Eq. (a),

$$m=.0039. \qquad (d)$$

Substituting the values of m of (d), and of n of (c) in Eq. (133),

$$c_f=.0039+\frac{.00035}{r}. \qquad (e)$$

The hydraulic mean radius due 15″ $r=\frac{5}{16}$. Substituting this value of $r=\frac{5}{16}$ in Eq. (e),

$$c_f=.0039+.00035\div\tfrac{5}{16}=.0050. \qquad (f)$$

Substituting the values of $c_f=.0050$, $r=\frac{5}{16}$, $h=12$ feet, $l=1{,}200$ feet, and $2g=64.4$, in Eq. (128), or applying Rule 33,

$$v=\left\{\frac{64.4\times12}{1.505+.0050\times1200\div\frac{5}{16}}\right\}^{\frac{1}{2}} \qquad (g)$$

$$\text{Whence, } v=6.11 \text{ feet.—}Ans. \qquad (h)$$

The value of v of Eq. (h) is so near the assumed value, 6 feet, it will be unnecessary to repeat the operation for finding a nearer approximate to the true value.

Area of cross-section of given pipe,

$$a=(\tfrac{15}{12})^2\times .7854=1.227 \text{ square feet.} \qquad (i)$$

Quantity of flow is equal to the product of the area of cross-section and the velocity; hence,

$$Q=av=1.227\times 6.11=7.5 \text{ cubic feet.—}Ans. \qquad (j)$$

To find the loss of head due velocity, substitute the value of v of Eq. (h), and the value of $2g=64.4$, in Eq. (110),

$$h_v=\frac{(6.11)^2}{64.4}=.58 \text{ feet.—}Ans. \qquad (k)$$

To find the loss of head due the resistance of entry, substitute the value of $c_e=.505$, of Eq. (127), and the value of $h_v=\frac{v^2}{2g}=.58$, of Eq. (k), in (116),

$$h_e=.58\times .505=.29 \text{ feet.—}Ans. \qquad (l)$$

To find the head expended in overcoming the resistances within the pipe, substitute the values of $h_v=$ .58 of Eq. (k), of $h_e=.29$ of Eq. (l), and of $h=$ 12 feet, the total or given head in Eq. (108), and transposing:

$$h_f=12-.58-.29=11.13 \text{ feet.—}Ans. \qquad (m)$$

To find the sine of slope, divide the head, h_f, expended in overcoming the resistances in the pipe by the length of the pipe. Thus, the sine of the angle of slope, C E D, Fig. 19, is in the given example:

$$s=\frac{h_f}{l}=\frac{11.13}{1200}=.009275.\text{—}Ans. \qquad (n)$$

To find the fall per mile:

Let F = fall in feet per mile:

$$1 \text{ mile}=5280 \text{ feet}. \qquad (o)$$

$$\text{Then } 1 : .009275 :: 5280 : \text{F}. \qquad (p)$$

$$\text{Whence, F}=48.972 \text{ feet.—}Ans. \qquad (q)$$

When 1.505, the leading term in the denominator of Eq. (128), or, in other words, when Eq. (130) is employed instead of Eq. (128), it is assuming that in Fig. 19, A E is the slope or hydraulic gradient instead of C E, which is erroneous.

But when the pipe is very long in comparison with its diameter, the error is insignificant.

Let F_m = the entire fall per mile.

Substitute the values of $h_v=.58$ of Eq. (k), and $h_e=.29$ of Eq. (l) in Eq. (109),

$$h_i=.58+.29=.87. \qquad (r)$$

Then $F_m=F+h_i=48.972+.87=49.842$ feet.—*Ans.*

Flow of water in long pipes.

For a given or determined velocity, the inlet head h_i, remains constant, regardless of the length of the pipe.

In general, if F denote the fall per mile—fall requisite to overcome the resistances within the pipe—(n) the number of miles, length of the pipe, and F_t the total fall, then will

$$F_t = n\,F + h_i = h_f + h_i. \qquad (a)'$$

In the computation of the following table for the velocity and quantity of flow in long pipes, Eq. (130), in which the value of $s=\frac{h_f}{l}$ is given, has been employed. The inlet head, h_i, if required, will be determined from the velocity.

TABLE 17.

Velocities and Quantities of Flow in Clean Iron Pipes due given Slopes and Diameters.

Fall per Mile. Feet.	Sine of Slope. $s=\frac{h_f}{l}$	Diameters. $\frac{3}{8}''=.03125$ feet. Velo'y Ft. per Sec.	$\frac{3}{8}''=.03125$ feet. Cubic Feet per Sec.	$\frac{1}{2}''=.04167$ ft. Velo'y Ft. per Sec.	$\frac{1}{2}''=.04167$ ft. Cubic Ft. per Sec.	$\frac{3}{4}''=.0625$ ft. Velo'y Ft. per Sec.	$\frac{3}{4}''=.0625$ ft. Cubic Ft. per Sec.	$1''=.0833$ ft. Velo'y Ft. per Sec.	$1''=.0833$ ft. Cubic Ft. per Sec.
21.12	.004	....	......	....	....	....	....	....	....
26.40	.005	....	......	....	....	....	....	....	....
31.68	.006	....	......	....	....	....	....	....	....
36.96	.007	....	......	....	....	....	....	....	....
42.24	.008	....	......	....	....	....	....	1.04	.0057
47.52	.009	....	......	....	....	....	....	1.13	.0062
52.80	.010	....	......	....	....	1.03	.0032	1.24	.0068
63.36	.012	0.698	0.00054	0.89	.0012	1.14	.0035	1.43	.0078
73.92	.014	0.763	0.00059	0.91	.0013	1.23	.0038	1.54	.0084
84.48	.016	0.826	0.00063	0.99	.0014	1.34	.0041	1.64	.0089
95.04	.018	0.888	0.00069	1.05	.0014	1.45	.0045	1.76	.0096
105.6	.02	0.947	0.00073	1.10	.0015	1.52	.0047	1.81	.0099
158.4	.03	1.18	0 00090	1.44	.0020	1.92	.0059	2.28	.0125
211.2	.04	1.38	0.0011	1.77	.0024	2.30	.0071	2.73	.0149
264.0	.05	1.58	0.0013	2.04	.0028	2.60	.0080	3.05	.0167
316.8	.06	1.79	0.0014	2.31	.0032	2.85	.0087	3.40	.0186
369.6	.07	1.98	0.0015	2.49	.0034	3.10	.0095	3.64	.0199
422.4	.08	2.13	0.0016	2.68	.0037	3.30	.0101	3.92	.0214
475.2	.09	2.36	0.0018	2.85	.0039	3.54	.0109	4.18	.0228
528.0	.10	2.49	0.0019	3.04	.0041	3.73	.0114	4.44	.0242
633.0	.12	2.76	0.0021	3.32	.0045	4.18	.0128	4.90	.0268
739.2	.14	3.04	0.0023	3.46	.0047	4.50	.0138	5.29	.0289
844.0	.16	3.30	0.0025	3.84	.0052	4.83	.0148	5.64	.0308
950.4	.18	3.50	0.0027	4.09	.0056	5.11	.0157	6.00	.0328
1050.	.20	3.71	0.0028	4.31	.0059	5.40	.0166	6.33	.0346
1320.	.25	4.15	0.0032	4.83	.0066	6.10	.0187	7.14	.0390
1584.	.30	4.58	0.0035	5.36	.0073	6.73	.0206	7.90	.0432
2112.	.40	5.32	0.0041	6.26	.0086	7.79	.0239	9.13	.0499
2640.	.50	5.99	0.0048	7.07	.0097	8.82	.0271	10.34	.0565
3168.	.60	6.62	0.0051	7.80	.0107	9.79	.0300	11.57	.0632
3696.	.70	7.20	0.0055	8.47	.0116	10.76	.0330	12.71	.0694
4224.	.80	7 75	0.0059	9.14	.0125	11.72	.0357	....	....
4752.	.90	8.22	0.0063	9.80	.0134	12.60	.0379	....	....
5280.	1.00	8.74	0.0067	10.39	.0142	....	....	....	....

TABLE 17.

Velocities and Quantities of Flow in Clean Iron Pipes due given Slopes and Diameters.

Fall per Mile. Feet.	Sine of Slope h $s=\frac{f}{l}$	Diameters. 1½″=.125 feet. Velo' Ft per Sec.	Cubic Feet per Sec.	1¾″=.1458 ft. Velo'y Ft. per Sec.	Cubic Ft. per Sec.	2″=.1667 ft. Velo'y Ft. per Sec.	Cubic Ft. per Sec.	2½″=.4167 ft. Velo'y Ft. per Sec.	Cubic Ft. per Sec.
21.12	.004	.....	....	.....	.....	1.18	.0258	1.35	.0460
26.40	.005	.....	....	1.21	.0201	1.34	.0292	1.52	.0518
31.68	.006	1.19	.0146	1.36	.0227	1.50	.0327	1.68	.0573
36.96	.007	1.29	.0158	1.45	.0243	1.60	.0349	1.81	.0617
42.24	.008	1.39	.0171	1.58	.0264	1.73	.0378	1.04	.0662
47.52	.009	1.48	.0182	1.70	.0284	1.87	.0408	2.07	.0706
52.80	.010	1.60	.0196	1.79	.0299	1.98	.0432	2.20	.0750
63.36	.012	1.73	.0212	1.95	.0326	2.22	.0484	2.44	.0832
73.92	.014	1.86	.0228	2.13	.0356	2.36	.0515	2.62	.0003
84.48	.016	2.01	.0247	2.22	.0371	2.50	.0546	2.78	.0948
95.04	.018	2.10	.0258	2.35	·0392	2.63	.0574	2.93	.0999
105.6	.02	2.28	.0279	2.53	.0422	2.80	.0611	3.15	.1074
158.4	.03	2.81	.0346	3.10	.0518	3.39	.0740	3.81	.1299
211.2	.04	3.37	.0413	3.69	.0617	4.00	.0873	4.51	.1538
264.0	.05	3.73	.0458	4.28	.0715	5.02	.1095	5.34	.1821
316.8	.06	4.11	.0504	4.69	.0783	5.50	.1200	5.85	.1995
369.6	.07	4.42	.0542	5.02	.0838	5.90	.1288	6.34	.2162
422.4	.08	4.73	.0580	5.36	.0895	6.30	.1375	6.77	.2309
475.2	.09	5.05	.0619	5.63	.0940	6.61	.1442	7.19	.2442
528.0	.10	5.48	.0672	6.01	.1003	6.98	.1523	7.61	.2595
633.0	.12	6.03	.0740	6.65	.1110	7.49	.1634	8.27	.2820
739.2	.14	6.54	.0802	7.19	.1200	8.01	.1748	8.89	.3031
840.0	.16	7.01	.0862	7.70	.1285	8.50	.1855	9.48	.3233
950.0	.18	7.50	.0923	8.22	.1372	8.96	.1955	10.04	.3424
1056.	.20	7.88	.0969	8.69	.1450	9.38	.2047	10.63	.3624
1320.	.25	8.77	.1079	9.69	.1617	10.43	.2276	11.86	.4054
1584.	.30	9.65	.1187	10.62	.1773	11.38	.2483	13.15	.4084
2112.	.40	11.23	.1380	12.28	.2050	12.98	.2833	.....	....
2640.	.50	12.60	.1550	.....	.....	.....	.....	.....	.....

TABLE 17.

Velocities and Quantities of Flow in Clean Iron Pipes, due given Slopes and Diameters.

Fall per Mile. Feet.	Sine of Slope. $s=\frac{h}{l}$	Diameters. 3″=.25 feet. Veloc'y Ft. per Sec.	Cubic Ft. per Sec.	4″=.333 ft. Velo'y Ft per Sec.	Cubic Ft per Sec.	6″=.5 feet. Velo'y Ft per Sec.	Cubic Ft per Sec.	8″=.667 Feet. Velo'y Ft per Sec.	Cubic Ft. per Sec.
8 448	.0016	......	......	.....	.....	.....	.....	1.64	.573
8.976	.0017	......	......	.....	.....	.....	.....	1.68	.586
9.504	.0018	......	......	.....	.....	1.44	.282	1.75	.611
10.032	.0019	......	......	.....	.....	1.48	.290	1.80	.628
10.560	.0020	......	......	1.25	.1091	1.52	.298	1.83	.639
11.616	.0022	......	......	1.31	.1144	1.60	.314	1.93	.659
12.672	.0024	1.10	.0540	1.36	.1187	1.68	.330	2.02	.703
13.728	.0026	1.16	.0569	1.42	.1235	1.76	.346	2.11	.737
14 784	.0028	1.22	.0599	1.48	.1298	1 83	.359	2.22	.768
15.840	.0030	1.28	.0630	1.53	.1335	1.92	.377	2.32	.808
18.480	.0035	1.41	.0692	1.68	.1465	2.10	.395	2.51	.876
21.120	.004	1 53	.0749	1.79	.1562	2.26	.444	2.69	.931
26.40	.005	1.71	.0839	2.03	.1771	2.53	.496	2.99	1.015
31.68	.006	1.86	.0915	2 20	.1923	2.79	.548	3.32	1.157
36.96	.007	2.02	.0992	2.46	.2146	2.79	.589	3.32	1.262
42.24	.008	2.16	.1060	2.67	.2339	3.00	.631	3.62	1.344
47.52	.009	2 28	.1119	2.82	.2460	3 22	.672	3.85	1.424
52.80	.010	2.43	.1190	2.96	.2582	3.43	.721	4.08	1.496
63.36	.012	2.68	.1313	3 23	.2893	3.67	.784	4.29	1.614
73.92	.014	2.88	.1413	3.48	.3036	3.99	.858	4.71	1.782
84.48	.016	3.07	.1507	3.71	.3237	4.37	.922	5.11	1.916
94.04	.018	3.24	.1590	3.91	.3412	4.70	.975	5.49	2.033
105.6	.02	3.50	.1717	4 13	.3607	4 97	1.022	5.83	2.155
158.4	.03	4.24	.2081	5.16	.4502	5 21	1.263	6.18	2.667
211.2	.04	5.03	.2469	6.11	5331	6.43	1.484	7.64	3.145
264.0	.05	5.67	.2785	6.83	.5954	7.56	1.665	9.01	3.513
316.8	.06	6 21	.3049	7.32	.6390	8.48	1.929	10.06	3.847
369.0	.07	6.79	.3331	7.99	.6967	9.83	1.976	11.02	4.196
422 0	.08	7.25	.3559	8.60	.7506	10.06	2.114	12.02	......
475 2	.09	7.78	.3816	9.13	.7960	10.92	2.274	.....	......
528.0	.10	8.24	.4043	9.70	.8467	11.58	2.399	.....	......
633.6	.12	9 05	.4440	10.63	.9270	12.21	.....	.....	......
739 2	.14	9.77	.4977	11.53	1.006	.....	.....	.....	......
844.8	.16	10.46	.5131	12.38	1.081	.....	.....	.....	......
950.4	.18	11.08	.5436	.....	.....	.....	.....	.....	......
1056.	.20	11.88	.5832	.....	.....	.....	.....	.....	......
1320.	.25	13.29	.6523	.....	.....	.....	.....	.....	......

TABLE 17.

Velocities and Quantities of Flow in Clean Iron Pipes due given Slopes and Diameters.

Fall per Mile. Feet.	Sine of Slope. $s = \frac{h}{l}$	9″=.75 feet. Veloci'y Feet per Sec.	9″=.75 feet. Cubic Feet per Sec.	10″=.833 ft. Velo'y Ft per Sec.	10″=.833 ft. Cubic Ft per Sec.	11″=.917 ft. Velo'y Ft per Sec.	11″=.917 ft. Cubic Ft per Sec.	″=1 foot. Veloci'y Feet Per Sec.	″=1 foot. Cubic Ft per Sec.
4.752	.0009	......	......	.....	.....	.....	.....	1.54	1.210
5.280	.0010	......	......	.....	.....	.....	.....	1.61	1.265
5.808	.0011	......	......	.....	.....	1.62	1.069	1.68	1.319
6.336	.0012	......	......	1.61	.878	1.70	1.122	1.79	1.402
6.864	.0013	......	......	1.69	.922	1.79	1.181	1.88	1.476
7.392	.0014	1.66	.735	1.76	.960	1.85	1.221	1.94	1.489
7.920	.0015	1.72	.761	1.84	.984	1.92	1.267	2.01	1.579
8.448	.0016	1.78	.788	1.92	1.047	2.00	1.320	2.08	1.634
8.976	.0017	1.83	.810	1.99	1.085	2.07	1.366	2.15	1.689
9.504	.0018	1.89	.836	2.04	1.110	2.12	1.399	2.20	1.728
10.032	.0019	1.96	.867	2.13	1.162	2.20	1.452	2.27	1.783
10.560	.0020	2.01	.890	2.19	1.194	2.26	1.492	2.33	1.826
11.616	.0022	2.12	.938	2.32	1.265	2.39	1.577	2.47	1.940
12.672	.0024	2.22	.982	2.43	1.325	2.50	1.650	2.58	2.026
13.728	.0026	2.32	1.025	2.53	1.377	2.61	1.723	2.70	2.117
14.784	.0028	2.41	1.065	2.61	1.423	2.71	1.789	2.81	2.207
15.840	.0030	2.50	1.105	2.70	1.470	2.81	1.855	2.93	2.297
18.480	.0035	2.71	1.198	2.91	1.587	3.02	1.993	3.14	2.466
21.120	.004	2.88	1.273	3.09	1.683	3.24	2.138	3.39	2.662
26.400	.005	3.20	1.414	3.42	1.865	3.63	2.176	3.85	3.020
31.680	.006	3.54	1.565	3.78	2.059	4.00	2.640	4.22	3.310
36.960	.007	3.84	1.697	4.08	2.222	4.33	2.858	4.58	3.601
42.240	.008	4.11	1.816	4.37	2.383	4.64	3.062	4.91	3.856
47.520	.009	4.34	1.918	4.61	2.514	4.90	3.234	5.19	4.072
52.80	.010	4.58	2.024	4.88	2.662	5.18	3.419	5.48	4.305
63.36	.012	5.04	2.228	5.38	2.932	5.70	3.762	6,02	4,728
73.92	.014	5.50	2.431	5.89	3.210	6.18	4.079	6.49	5.094
84.48	.016	5.91	2.612	6.33	3.450	6.65	4.389	6,98	5.482
95.04	.018	6.29	2.780	6.75	3.679	7.09	4.676	7.44	5.838
105.6	.02	6.62	2.926	7.07	3.856	7.45	4.917	7.84	6.100
158.4	03	8.18	3.616	8.73	4.762	9.22	6.085	9.72	7.630
211.2	.04	9.60	4.243	10.20	5.563	10.74	7.088	12.28	8.860
264.0	.05	10.81	4.778	12.29	6.704	12.49	8.243	12.69	9.967
316.8	.06	11.85	5.238	.....	.....	.....	.....	......	.....

TABLE 17.

Velocities and Quantities of Flow in Clean Iron Pipes, due Given Slopes and Diameters.

Fall per Mile. Feet.	Sine of Slope. $s=\frac{h}{l}$	14″=1.167 Feet. Veloci'y Feet per Sec.	14″=1.167 Feet. Cubic Ft. per Sec.	15″=1.25 Ft. Velo'y Ft per Sec.	15″=1.25 Ft. Cubic Ft per Sec.	16″=1.333 Ft. Velo'y Ft per Sec.	16″=1.333 Ft. Cubic Ft per Sec.	18″=1.5 Feet. Veloc'y Ft. per Sec.	18″=1.5 Feet. Cubic Ft per Sec.
2.112	.0004	......	......	.....	.....	.....	.....	1.46	2.61
2.640	.0005	......	......	.....	.....	.....	.....	1.56	2.79
3.168	.0006	......	......	.....	.....	1.50	2.04	1.66	2.97
3.696	.0007	......	......	1.56	1.91	1.61	2.25	1.76	3 10
4.224	.0008	1.61	1.71	1.67	2.05	1.74	2.43	1.85	3.27
4.752	.0009	1.71	1.83	1.78	2.19	1.86	2.59	1.98	3.49
5.280	.0010	1.80	1.91	1.87	2.30	1.95	2.72	2.07	3.66
5.808	.0011	1.90	2.02	1.98	2.43	2.07	2.88	2.20	3.88
6.336	.0012	1.98	2.11	2.07	2.54	2.16	3.02	2.30	4.06
6.864	.0013	2.04	2.18	2.16	2.65	2.28	3.18	2.40	4.23
7.392	.0014	2.13	2.27	2.24	2.75	2.35	3.28	2.49	4 40
7.920	.0015	2.20	2.35	2.31	2 84	2.43	3.39	2.61	4.61
8.448	.0016	2.29	2.44	2.39	2.94	2.50	3.49	2.69	4.75
8.976	.0017	2.38	2.54	2.48	3.08	2.59	3.62	2.78	4.90
9.504	.0018	2.43	2.59	2.53	3.11	2.64	3.69	2.85	5.03
10.032	.0019	2.50	2.67	2.61	3.21	2.73	3.81	2.93	5.17
10.560	.0020	2.55	2.72	2.68	3.29	2.81	3.92	3.00	5.30
11.616	.0022	2.70	2.88	2.82	3.47	2.95	4.12	3.19	5.63
12.672	.0024	2.83	3.02	2.96	3.63	3.10	4.32	3.32	5.87
13.728	.0026	2.95	3.15	3.09	3.79	3.23	4.51	3.50	6.18
14.784	.0028	3.08	3.29	3.22	3.95	3.36	4.68	3 61	6.38
15.840	.0030	3.20	3.42	3.34	4.11	3.49	4.87	3.76	6.64
18.48	.0035	3.47	3.62	3.63	4.46	3.80	5.31	4.06	7.17
21.12	.004	3.74	3.99	3.90	4.78	4.06	5.67	4.33	7.65
26.40	.005	4.18	4.46	4.38	5.37	4 58	6.39	4.90	8.66
31.68	.006	4.60	4.91	4.81	5.91	5.03	7.02	5.40	9.54
36 96	.007	5 03	5.37	5.26	6.45	5.49	7.66	5.84	10.33
42.24	.008	5.40	5.77	5.62	6.90	5.85	8.16	6.28	11.09
47.52	.009	5.73	6.11	5.95	7.31	6.19	8.64	6.63	11.71
52.80	.010	6.03	6 44	6.27	7.70	6.52	9.10	7.00	12.37
63.36	.012	6.56	7.00	6.84	8.39	7.12	9.95	7.73	13.65
73.92	.014	7.12	7.60	7.45	9.15	7.79	10.87	8.35	14.75
84.48	.016	7.66	8.17	7.99	9.81	8.33	11.63	8.97	15 84
95.04	.018	8.17	8.93	8.03	10.47	8.90	12.43	9.57	16 90
105.6	.02	8.67	9.26	9.04	11.09	9.41	13.14	10.10	17.85
158.4	.03	10.69	11.39	11.13	13.66	11.58	16.17	12.37	21.86
211.2	.04	12.38	13.22	12.91	15.84	13.45	18.77	......	.....

TABLE 17.

Velocities and Quantities of Flow in Clean Iron Pipes due given Slopes and Diameters.

Fall per Mile. Feet.	Sine of Slope. $s=\frac{h}{l}$	20″=1.667 feet. Veloci'y Feet per Sec	20″=1.667 feet. Cubic Feet per Sec.	22″=1.833 feet. Veloci'y Feet per Sec.	22″=1.833 feet. Cubic Feet per Sec.	24″=2 feet. Velo'y Ft per Sec.	24″=2 feet. Cubic Ft per Sec.	27″=2.25 feet Velo'y Ft per Sec.	27″=2.25 feet Cubic Ft per Sec.
2.112	.0004	......	......	......	......	.....	.....	2.00	7.95
2.640	.0005	......	......	......	......	1.78	5.59	2.08	8.27
3.168	.0006	1.66	3.61	1.80	4.61	1.94	6.10	2.10	8.51
3.696	.0007	1.86	4.07	1.99	5.25	2.12	6.64	2.29	8.91
4.224	.0008	2.00	4.35	2.13	5.62	2.27	7.13	2.39	9.30
4.752	.0009	2.15	4.68	2.28	6.01	2.41	7.56	2.58	10.26
5.280	.0010	2 26	4.92	2.39	6.32	2.53	7.95	2.70	10.74
5.808	.0011	2.36	5.15	2.51	6.62	2.66	8.34	2.88	11.45
6.336	.0012	2.48	5.40	2.63	6.94	2.79	8.75	3.00	11.93
6.864	.0013	2.58	5.62	2.74	7.24	2.91	9.14	3.16	12.54
7.392	.0014	2.68	5.82	2.85	7.51	3.02	9.47	3.26	12.96
7.920	.0015	2.78	6.05	2.95	7.78	3.12	9.80	3.40	13.49
8.448	.0016	2.88	6.27	3.05	8.05	3.23	10.13	3.52	13.98
8.976	.0017	2.97	6.48	3.17	8.36	3.37	10.57	3.63	14.41
9.504	.0018	3.05	6.65	3.24	8.55	3.43	10.77	3.73	14.81
10.032	.0019	3.17	6.92	3.35	8.85	3.54	11.10	3.83	15.21
10.560	.0020	3.23	7.05	3.43	9.07	3.64	11.43	3.93	15.63
11.616	.0022	3.40	7.42	3.62	9.55	3.84	12.05	4.14	16.44
12.672	.0024	3.57	7.79	3.79	10.01	4.02	12 61	4.34	17.23
13.728	.0026	3.73	8.14	3.97	10.48	4.21	13.23	4.53	18.01
14.784	.0028	3.89	8.48	4.14	10.91	4.39	13.79	4.72	18.75
15.84	.0030	4.02	8.77	4.28	11.29	4.54	14.25	4.91	19.50
18.48	.0035	4.35	9.49	4.64	12.25	4.94	15.50	5.32	21.13
21.12	.004	4.66	10.16	4.97	13.12	5.29	16.62	5.69	22.62
26.40	.005	5.24	11.43	5.60	14.78	5.96	18.71	6.37	25.34
31.68	.006	5.77	12.59	6.10	16.20	6.50	20.42	6.98	27.74
36.96	.007	6.26	13.66	6.64	17.53	7.02	22.05	7.52	29.96
42.24	.008	6.72	14.66	7.12	18.78	7.52	23.61	8.05	31.99
47.52	.009	7.13	15.54	7.55	19.93	7.98	25.07	8.55	33.97
52.80	.010	7.55	16.47	7.98	21.06	8.41	26.42	9.03	35.89
63.36	.012	8.25	17.99	8.74	23.07	9.24	29.03	10.00	39.76
73.92	.014	8.94	19.49	9.48	24.68	10.03	31.49	10.87	43.22
84.48	.016	9.64	21.03	10.21	26.97	10.79	33.90	11.72	46.57
95.04	.018	10.30	22.45	10.91	29.70	11.52	36.18	12.09	48.06
105.6	.02	10.80	23.56	11.52	31.15	12.24	38 45	.....	.....
158.4	.03	12.23	28.86	12.59	33.21	.....	.....	.....	.....
211.2	.04	13.50	29 58	......	......	.....	.....	.....	.....

TABLE 17.

Velocities and Quantities of Flow in Clean Iron Pipes, due Slopes and Diameters.

Fall per Mile. Feet.	Sine of Slope. $s=\frac{h_f}{l}$	Diameters. 30″=2.5 Feet. Veloci'y Feet per Sec.	Cubic Feet per Sec.	33″=2.75 Ft. Velo'y Ft per Sec.	Cubic Ft per Sec.	36″=3. Feet. Velo'y Ft per Sec.	Cubic Ft per Sec.	40″=3.333 Ft. Velo'y Ft Per Sec.	Cubic Ft. per Sec.
1.056	.0002	1.34	6.58	1.45	8.61	1.46	10.29	1.59	13.88
1.584	.0003	1.59	7.78	1.68	10.21	1.80	12.70	1.95	17.00
2.112	.0004	1.83	8.99	1.96	11.65	2 06	14.56	2.26	19.68
2.640	.0005	2.09	10.24	2.18	12.92	2.31	16.35	2.53	22.08
3.168	.0006	2 24	10.97	2.36	13.99	2 55	18.02	2 80	24.43
3.696	.0007	2.43	11.90	2.55	15.14	2.80	19.76	3.01	26.27
4.224	.0008	2.62	12.84	2.76	16.36	2.95	20.85	3.23	28.14
4.752	.0009	2.75	13.48	2.96	17.58	3.16	22.30	3.42	29.80
5.280	.0010	2.90	14.21	3.16	18.74	3.32	23.47	3.61	31.46
5.808	.0011	3.07	15.05	3.29	19.54	3.53	24.91	3.81	33.25
6.336	.0012	3.22	15 81	3.42	20.28	3.70	26.12	3.98	34.68
6.864	.0013	3.36	16.47	3.59	21 29	3.85	27.20	4.15	36.21
7.392	.0014	3.50	17.18	3.70	22.20	4.00	28.24	4.31	37.57
7.920	.0015	3.66	17.94	3.88	23.01	4.13	29.19	4.49	39.18
8.448	.0016	3.79	18.58	4.00	23.76	4.29	30.29	4.65	40.54
8.976	.0017	3.92	19.21	4.12	24.47	4.45	31.42	4.80	41.88
9.504	.0018	4.01	19.66	4.25	25.22	4.60	32.48	4.94	43.07
10.032	.0019	4.14	20.32	4.40	26.14	4.73	33.40	5.08	44.28
10.560	.0020	4,24	20.79	4.54	26.94	4.88	34.49	5,18	45.20
11.616	,0022	4.45	21.80	4.76	28.27	5.12	36.15	5.52	48.12
12 672	,0024	4.65	22.83	5.00	29.02	5.34	37.74	5.79	50.48
13.728	.0026	4.88	23.93	5.23	31.06	5.58	39.40	6.04	52.67
14.784	.0028	5.07	24.86	5.44	32.28	5.78	40.86	6.31	55,04
15.84	,0030	5.27	25.87	5.66	33.62	5.98	42.28	6.46	56.33
18.48	,0035	5.70	27.96	6.09	36 17	6.50	45.95	7.00	61.09
21.12	.004	6.08	29.84	6.49	38.57	6.91	48.83	7.50	65.41
26.40	,005	6.84	33.55	7 26	43.12	7.77	54,89	8.38	73.09
31.68	.006	7.50	36.79	7.98	47.40	8.48	59.95	9.21	80.32
36.96	,007	8.08	39.66	8.65	51.35	9.22	65.17	9.94	86.70
42.24	.008	8.64	42.39	9.25	54.91	9.88	69.80	10.61	92.58
47.52	.009	9.22	45.23	9.80	58.20	10.52	74.33	11.23	98.00
52.80	.010	9.72	47.71	10.38	61.62	11.10	78.46	11.92	104.00
63.36	.012	10.78	52.91	11.45	68.00	11.72	82.84	.....	......
73.92	.014	11.75	57.65	12.45	73.95	.....	.....	.....	......

TABLE 17.

Velocities and Quantities of Flow in Clean Iron Pipes due Slopes and Diameters.

Fall per Mile. Feet.	Sine of Slope. $s=\frac{h}{l}$	DIAMETERS. 44″=3.667 feet. Veloci'y Feet per Sec.	Cubic Feet per Sec.	48″=4 feet Velo'y Ft per Sec.	Cubic Ft per Sec.	54″=4.5 feet. Velo'y Ft per Sec.	Cubic Ft per Sec.	60″=5 feet. Velo'y Ft per Sec.	Cubic Feet per Sec.
0.528	.0001	1.22	12.89	1.28	16.08	1.38	21.96	1.52	29.77
1.056	.0002	1.72	18.15	1.83	22.98	1.95	30.97	2 08	40.84
1.584	.0003	2.10	22 22	2 22	27.89	2.42	38.53	2 65	52.09
2 112	.0004	2.42	25.55	2.62	32 93	2.84	45.12	3.01	59.04
2.640	.0005	2.74	28.87	2.95	37.00	3.16	50.23	3.44	67.56
3.168	.0006	2.98	31.46	3 20	40.21	3.49	55.51	3.79	74.32
3.696	.0007	3.27	34 47	3.48	43 67	3.79	60.21	4.10	80.51
4.224	.0008	3.51	37.05	3.73	46.81	4.00	63.61	4.40	86.30
4.752	.0009	3.70	39.01	3.90	49.06	4.24	67.20	4.69	91.99
5.280	.0010	3.89	41.06	4.15	52.15	4.55	72 37	4.94	96.98
5.808	.0011	4 08	42.09	4.38	54 95	4 76	75.71	5.22	102.4
6.336	.0012	4.26	44 97	4.57	57.36	4.98	79.13	5.47	107.3
6.864	.0013	4.43	46.77	4.78	60.07	5.19	82.54	5.68	115.5
7.392	.0014	4.63	48.83	4.94	62.02	5.40	85.90	5 94	116.5
7.920	.0015	4.80	50.62	5.13	64 47	5.63	89.52	6.10	119.7
8.448	.0016	4.97	52.46	5.30	66.53	5.82	92.47	6.30	123.7
8.976	.0017	5.12	54.04	5.45	68.50	6.00	95.35	6.50	127.6
9.504	.0018	5.26	55.48	5.62	70.62	6.14	97.65	6.69	131.3
10.032	.0019	5.40	57.01	5.79	72.75	6.30	100.2	6.87	134.8
10.560	.0020	5.58	58 85	5.92	74.44	6.53	103.8	7.07	138.8
11.616	.0022	5 85	61.71	6.23	78.29	6.84	108.8	7.44	146.0
12.672	.0024	6.10	64 35	6.50	81.68	7.14	113.5	7.77	152 6
13.728	.0026	6 34	66.87	6.78	85.20	7.45	118.5	8.08	158.7
14.784	.0028	6.59	69 57	7.04	88 46	7.74	123.1	8 38	164.5
15.84	.0030	6.85	72.32	7.30	91.73	8.06	128.2	8.68	170 4
18.48	.0035	7.98	77.95	7.99	100.4	8.74	138.9	9.37	184.0
21.12	.004	7.92	83.60	8.43	105.9	9.30	147.9	10.06	197.5
26.40	.005	8.85	93.37	9.50	119.3	10.43	165.8	11.30	220.0
31.68	.006	9.78	103.3	10.42	130.9	11.47	182.4	12.44	244.3
36.96	.007	10.59	111.7	11.31	142.1	12.45	190.0	.....	......
42 24	.008	11.36	119.9	12.25	153.9	.....	.....	.....	......
47.52	.009	12.15	128.3	.....	.....	.....	.....	.....	......

TABLE 17.

Velocities and Quantities of Flow in Clean Iron Pipes, due Slopes and Diameters.

Fall per M'le. Feet.	Sine of Slope. h $s=\frac{f}{l}$	DIAMETERS. 72″=6 feet. Veloci'y Feet per Sec.	Cubic Feet per Sec.	84″=7 feet. Veloc'y Ft per Sec.	Cubic Ft per Sec.	96″=8 feet. Vel'cy Ft per Sec.	Cubic Ft per Sec	120 =10 feet. Vel'cy Ft per Sec.	Cubic Ft per Sec.
0.528	.0001	1.66	46 99	1.91	75.43	2 14	107.8	2.52	198.1
1.056	.0002	2.04	57.65	2.72	104.6	3.03	152.5	3.65	286.5
1.584	.0003	2.92	82.53	3.28	126.2	3.75	188.5	4.49	352.3
2.112	.0004	3.40	95.99	3.78	145.4	4.35	218.8	5.20	408.5
2.640	.0005	3 87	109.4	4.23	162 8	4 88	245.3	5.80	458.7
3.168	.0006	4.30	121 6	4.60	177.0	5.32	267.4	6.41	503.6
3.696	.0007	4.67	132.0	4.99	192 0	5.78	290.5	6.94	545.1
4.224	.0008	4.95	140.0	5.40	207.8	6.19	310.9	7.42	582.7
4.752	.0009	5 26	148 7	5.78	222.4	6.60	324.2	7.87	618.1
5.280	.0010	5.58	157.8	6.11	235.1	6.97	350.5	8.30	651.4
5.808	.0011	5.87	166.0	6 58	253.3	7.29	366.2	8.70	683 2
6.336	.0012	6.12	173.0	6.88	264.8	7.60	382.0	9.09	713.5
6.864	.0013	6 34	179.3	7.15	275.2	7.92	397.9	9.48	744.3
7.392	.0014	66.3	187 5	7.48	287.7	8.25	414.7	9.84	772.5
7.920	.0015	6.86	193 9	7.70	296.4	8.51	427.8	10,18	799.6
8.448	.0016	7.08	200 2	8.00	307.9	8 82	443.1	10.51	825.7
8.976	0017	7.30	206.4	8.22	316.2	9.10	457.4	10.84	850.4
9.504	.0018	7.50	212.1	8.49	326 7	9.36	470.5	11.15	875.9
10.302	.0019	7.70	217.7	8.73	335.8	9.58	481.5	11.45	899.9
10.560	.0020	7.97	225.2	9 05	348.3	9.88	496.4	11.78	925.3
11.616	.0022	8.33	235.5	9.48	364.9	10.40	522.8	12.35	971.4
12.672	.0024	8 72	246.4	9.88	389.1	10.89	547.4	12.90	1013.6
13.728	.0026	9.06	256.2	10.28	394.4	11.34	570.0	13.43	1057.4
14.784	.0028	9.45	267.2	10,61	408.4	11.78	592.1	13.94	1094.8
15.84	.0030	9.83	277.9	11.00	423.4	12.18	612.0	.....	......
18.48	.0035	10.62	299.7	12.55	483.0	.....	.....	.....	......
21.12	.004	11.34	320.7	......	......	.....	.....	.....	......
26.40	.005	12 68	358.5	......	......	.....	.....	.....	......

APPLICATION OF TABLE 17.

Ex. 70.—The diameter of a pipe being 4 feet, and the fall per mile 5.28 feet, what is the discharge in cubic feet per second?

Cal.—In "fall per mile" column, Table 17, find 5.28 feet, opposite which, in right hand column, "4 feet diameter," will be found 52.15 cubic feet, the discharge sought.

Ex. 71.—The fall per mile being 10.56 feet, a pipe of what diameter will be requisite to discharge 100 cubic feet per second?

Cal.—In "fall per mile" column, Table 17, find 10.56 feet, opposite which find 100 cubic feet, or nearest approximate thereto: 103.8 cubic feet is found in right hand column, headed 4.5 feet diameter, which, in practice, is sufficiently near the diameter sought in most cases.

Ex. 72.—The flow of 30 cubic feet of water per second through a pipe 2.5 feet diameter, 5 miles long, is employed at a hydraulic mine, whose elevation is 400 feet below the elevation of the inlet end of the pipe. What is the effective head of the water at the mine?

Cal.—In "2.5 feet" diameter column, Table 17, find 29.84 cubic feet nearest approximate to the given flow of 30 feet, opposite which, in the "fall per mile" column, is found 21.12 feet, the loss of head per mile. The loss in 5 miles, the given length of pipe, will be:

$$21.12\times5=105.60 \text{ feet.}$$

400—105.60=294.4 feet, effective head—*Ans.*

Ex. 73.—The data being the same as in Ex. 72, except that the diameter of the pipe is 3 feet instead of 2.5, what is the effective head of the water at the mine?

Cal.—Find in "3 feet," right hand diameter column, Table 17, 30.29 cubic feet, nearest approximate to 30 cubic feet, the given quantity, opposite which, in "fall per mile" column, is found 4.448 feet, the loss per mile.

Then 8.448×5=42.24 feet loss of head in 5 miles; 400—42.24=357.76 feet effective head.=*Ans.*

Comparing results with respect to the 2.5-foot pipe of Ex. 72, and the 3-foot of Ex. 73:

357.76—294.40=62.36 feet, it is seen that the loss of head in the 3-foot pipe is 63.36 feet less than in the 2.5-foot pipe, which, in matters of economy, is of no little importance.

Ex. 74.—The elevation of the Guenoc Reservoir Site of the Feather River Water Co. is 1015 feet above the city base of San Francisco. The measured distance of pipe-line between the reservoir and the city is 104.83 miles. How many gallons of water a day (24 hours) will a pipe 40 inches diameter, whose inlet is at the reservoir, and outlet at San Francisco, deliver at an elevation of 350 feet above city base?

Cal.—1015—350=665 feet, total fall; 665÷104.83

=6.34 feet fall per mile. In "fall per mile" column, Table 17, the nearest approximate fall to 6.34 feet is 6.336 feet, opposite which, in "40 inches," right hand column "diameters," is found 34.68 cubic feet, the discharge per second.

$24 \times 60 \times 60 = 86{,}400$ seconds in 24 hours; $34.68 \times 86{,}400 = 2{,}996{,}352$ cubic feet discharge in 24 hours.

In 1 cubic foot are 7.5 gallons nearly; $2{,}996{,}352 \times 7.5 = 22{,}472{,}640$ gallons.—*Ans.*

Ex. 75.—At a quartz mill, requiring 225 effective horse-power, the efficiency of the water wheel, in excess of the loss by nozzle resistance, is 60 per cent; the length of the pipe line is 3 miles, and the elevation of the reservoir, at point of water supply, is 500 feet above the point of application of the water at the mill. Required the diameter of the pipe to carry the requisite quantity of water?

Cal. 1st.—$225 \div .60 = 375$, total horse-power.

$375 \times 550 = 206{,}250$ "foot pounds;" $206{,}250 \div 62.5 = 3300$ cubic feet$\times 1$ foot, which we will term *foot volume.*

Assume the loss of head by the internal surface resistances of pipe 10.56 feet per mile; then $10.56 \times 3 = 31.68$ feet, total loss of head.

$500 - 31.68 = 468.32$ feet, effective fall; $3300 \div 468.32 = 7.05$ cubic feet flow required per second.

In "fall per mile" column, Table 17, find 10.56 feet, opposite which, in right hand column, "diameters," find 7.05 cubic feet. This is found under heading "20 inches." Hence the diameter of the required pipe is 20 inches.—*Ans.*

Cal. 2d.—By Cal. 1st, the "foot volume," corresponding to the "foot pounds," required at the mill, is = 3300 cubic feet × 1 foot.

Assume 4.224 feet, the loss of head per mile due pipe resistances; then 4.224 × 3 = 12.672, total loss of head; 500—12.672 = 487.328 feet, effective fall; 3300 ÷ 487.328 = 6.77 cubic feet required per second.

In "fall per mile" column find 4.224 feet, opposite which, in right hand column, "diameters," find 7.32 cubic feet. This is found under heading "24 inches." Hence the diameter sought is = 24 inches.—*Ans.*

Cal. 3d.—By Cal. 1st, the "foot volume," corresponding to the "foot pounds," required at the mill, is = 3300 cubic feet × 1 foot.

Assume 21.12 feet, the loss of head per mile due pipe resistances; then 21.12 × 3 = 63.36 feet, total loss of head; 500—63.36 = 436.64 feet, effective head; 3300 ÷ 436.64 = 7.55 cubic feet required per second.

In fall per mile column find 21.12 feet, opposite which, in right hand column, "diameters," is found 7.65 cubic feet—a very near approximate to 7.55 cubic feet, the required quantity. This is found under heading "18 inches." Hence the diameter sought is = 18 inches.—*Ans.*

Maximum Work.—To determine the maximum work, which water subjected to flow through a long pipe under pressure will perform, on issuing from the pipe:

Let h=the total head, exclusive of the inlet head, which, rarely in practice exceeds 1.5 feet.

$x=h_f$, the friction head, expended in overcoming the resistances within the pipe.

d=diameter of pipe.

l=length of pipe.

$r=\frac{d}{4}$, hydraulic mean radius regarded constant.

c_f=coefficient of friction of resistances within the pipe regarded constant.

g=acceleration of gravity.

v=velocity of water in the pipe per second.

ñ=ratio of circumference to diameter of pipe.

$h_{,}=h-x$, head for effective work.

W=total weight of water discharged per second.

w=weight of a cubic foot of water.

$a=\frac{\tilde{n}\, d^2}{4}$, area of cross section of the pipe.

u=maximum work performed by the water per second. Then,

$$u=\mathrm{W}\, h_{,}=a\, v\, w\, h_{,}. \qquad (142)$$

Substituting in (142), the values of

$$a=\frac{\tilde{n}\, d^2}{4},\ v=\left\{\frac{2\, g\, r\, h_f}{c_f\, l}\right\}^{\frac{1}{2}} \text{ of (Eq. 129);}$$

$$r=\frac{d}{4} \text{ and } h_{,}=h-x;$$

$$u=\mathrm{W}\, h_{,}=\frac{\tilde{n}\, d^2}{4}\left\{\frac{2\, g\, d}{4\, c_f\, l}\right\}^{\frac{1}{2}} x^{\frac{1}{2}}\, (h-x). \qquad (143)$$

Differentiating (142), omitting the constant factors ñ, d, g, c_f l, 2 and 4,

$$\frac{du}{dx}=0=\frac{hx^{-\frac{1}{2}}}{2}-\frac{3x^{\frac{1}{2}}}{2}. \qquad (144)$$

Transposing and reducing (144),

$$x=\frac{h}{3}. \qquad (145)$$

Differentiating (144),

$$\frac{d^2u}{dx^2}=-hx^{-\frac{3}{2}}-3x^{-\frac{1}{2}} \qquad (146)$$

As the second differential coefficient is negative, the function u representing the work performed is a maximum where $x=\frac{h}{3}$, as found in Eq. (145).

Thus it is shown that, in theory, the work performed by water after flowing through a pipe, is a maximum when the head expended in overcoming the resistances of the pipe is equal to one-third of the total head, exclusive of the inlet head. A modification occurs, however, with respect to this result, owing to the experimental coefficient of resistance being variable by undetermined law, instead of constant, as assumed in our solution. Another source of variation is evidently due to difference in diameters of pipes. An inspection of Table 17 shows, that in practice the ratio of the head expended in overcoming the resistances in a clean iron pipe is approximately equal to $\frac{3}{8}$, as a mean, instead of $\frac{1}{3}$, as found, of the total head. The remaining part of the total head amounting to nearly $\frac{5}{8}$ as a mean, applies to effective work.

Cal. 4th.—By Cal. 1st, the "foot volume" (substituted for "foot pounds" for convenience of calculation) required at the mill, is 3300 cubic feet $\times$ 1 foot.

Applying the ratio, $\frac{3}{8}$, proposed in the preceding article, there results: $500 \times \frac{3}{8} = 187.5$ feet, loss of head by friction; $1875 \div .3 = 62.5$ feet, loss of head per mile.

In "fall per mile" column, the nearest approximate to 62.5 feet is 63.36 feet, loss of head per mile; then $63.36 \times 3 = 190.08$ feet, total loss of head; $500 - 190.08 = 309.92$ feet effective head; $3300 \div 309.92 = 10.65$ cubic feet flow required per second.

Opposite 63.36 feet in "fall per mile column," Table 17, is found 13.65 cubic feet, nearest approximate to 10.65 cubic feet, the required quantity. This is found in "18 inches" column of contents for "diameters." Hence the pipe meeting the requirements is=18 inches diameter.—*Ans.*

Remark.—In "16-inch column," opposite 63.36 feet "fall per mile," is found 9.95 cubic feet, which is less than the required quantity. Hence a 16-inch pipe is too small. The velocity in a 17-inch pipe is $v = (\frac{64.4}{.00491} \times .012 \times \frac{17}{48})^{\frac{1}{2}} = 7.46$ feet per second for 63.36 feet fall per mile. The discharge per second in a 17-inch pipe will be $(\frac{17}{12})^2 \times .7854 \times 7.46 = 11.76$ cubic feet per second.

This amount, 11.76, exceeds the required amount of 10.65 cubic feet. Hence a 17-inch pipe will carry sufficient water to do the work. The margin of safety, $11.76 - 10.65 = 1.11$ cubic feet, however, is small.

An 18-inch pipe affording a margin of safety of $13.65 - 10.65 = 3$ cubic feet, seems by no means large. Even a 20-inch pipe, with a fall of 63.36 feet per mile, and affording a margin of 7.34 cubic feet per second, would not exceed the limits imposed by D'Aubuisson, in his advice to engineers, if, indeed, it would the limits of true economy.

Reverting to the results obtained by 1st, 2nd and 3rd calculations for Ex. 75, and to the tabulated results corresponding respectively to these, or approximately so, it will be seen that in the first case there is no margin of safety, in the second .55 cubic feet, and in the third .10 cubic feet per second. This in practice would be inadmissible. A margin of 33 per cent is none too large. So that if on portions of the lines steeper grades could not be had, it would be better to employ a "22-inch" pipe in the first case, a "27-inch" in the second, and a "20-inch" in the third case.

Inlet Head.—The inlet head is equal to the sum of the head expended in generating the velocity in a pipe, and the head expended in overcoming the resistance of entry.

Thus transposing Eq. (109), we have the inlet head,

$$h_i = h_v + h_e. \qquad (147)$$

Substituting the values of $h_v = \frac{v^2}{2g}$ of Eq. (110), and of $h_e = c_e v^2$ of Eq. (116), noting that $c_e = .505$ of Eq. (127),

$$h_i = 1.505 \frac{v^2}{2g} = .0234 v^2. \qquad (148)$$

Rule 34.—The inlet head is equal to .0234 times the square of the velocity in the pipe.

TABLE 18.

Velocities in Pipes and Corresponding Inlet Heads.

Veloci'y Feet.	Inlet Head. Feet.	Velocity. Feet.	Inlet Head. Feet.	Velocity Feet.	Inlet Head. Feet.	Velocity Feet.	Inlet Head. Feet.
.80	.015	3.68	.316	6.32	.933	10.00	2.34
.90	.019	3.76	.331	6.37	.948	10.50	2.58
1.00	.023	3.85	.346	6.42	.963	11.00	2.83
1.13	.030	3.93	.361	6.47	.978	11.50	3.10
1.27	.038	4.00	.374	6.52	.993	12.00	3.37
1.39	.045	4.09	.391	6.57	1.01	12.50	3.66
1.50	.053	4.17	.406	6.61	1.02	13.00	3.96
1.60	.060	4.25	.421	6.66	1.04	13.50	4.27
1.70	.068	4.32	.436	6.71	1.05	14.00	4.59
1.79	.075	4.39	.452	6.76	1.07	14.50	4.92
1.88	.083	4.47	.467	6.81	1.08	15.00	5.27
1.97	.090	4.54	.482	6.86	1.10	15.50	5.62
2.00	.094	4.61	.497	6.91	1.11	16.00	5.99
2.04	.098	4.68	.512	6.95	1.13	16.50	6.38
2.12	.105	4.75	.527	7.00	1.15	17.00	6.76
2.20	.113	4.81	.542	7.04	1.16	17.50	7.17
2.27	.120	4.87	.557	7.09	1.17	18.00	7.58
2.34	.128	4.94	.572	7.13	1.19	18.50	7.79
2.41	.135	5.00	.585	7.18	1.20	19.00	8.45
2.47	.143	5.07	.606	7.22	1.22	19.50	8.90
2.54	.150	5.14	.617	7.26	1.23	20.00	9.36
2.60	.158	5.20	.632	7.31	1.25	20.50	9.83
2.66	.166	5.26	.647	7.35	1.26	21.00	10.32
2.72	.173	5.32	.662	7.40	1.28	21.50	10.82
2.78	.181	5.38	.677	7.44	1.29	22.00	11.33
2.84	.188	5.44	.692	7.48	1.31	22.50	11.85
2.89	.196	5.50	.707	7.53	1.32	23.00	12.38
2.95	.202	5.56	.722	7.57	1.34	23.50	12.92
3.00	.211	5.62	.737	7.61	1.35	24.00	13.48
3.05	.218	5.67	.753	7.65	1.37	24.50	14.05
3.11	.226	5.73	.768	7.70	1.38	25.00	14.63
3.16	.232	5.79	.783	7.74	1.40	25.50	15.22
3.21	.241	5.85	.798	7.78	1.41	26.00	15.82
3.26	.248	5.90	.813	7.82	1.43	27.00	17.05
3.31	.256	5.95	.828	7.86	1.44	28.00	18.35
3.36	.263	6.00	.843	7.90	1.46	29.00	19.78
3.40	.271	6.06	.858	7.94	1.47	30.00	21.06
3.45	.278	6.11	.873	8.00	1.50	35.00	28.67
3.50	.286	6.17	.888	8.50	1.69	40.00	37.34
3.55	.293	6.22	.903	9.00	1.90	45.00	47.39
3.59	.301	6.28	.918	9.50	2.11	50.00	58.50

Ex. 76.—The velocity in a 14-inch pipe is 4.18 feet. What is the total head, the pipe being one mile long?

Cal. 1st.—Find in velocity column, Table 17, for "14 inches diameters" of pipes, the given velocity, 4.18 feet, opposite which, in "fall per mile" column, is found 26.40 feet, the head required to overcome the resistances of the pipe.

In velocity column, Table 18, find 4.17 feet, nearest approximate to the given velocity, opposite which, in "inlet head" column, is found 4.06 feet. Then

$26.40 + .406 = 26.806$ feet, total head.—*Ans.*

Cal. 2d.—Find, as by Cal. 1st, the fall 26.40 feet.

By Rule 37:

$(4.18)^2 \times .02337 = .4083$ feet, inlet head; $26.40 + .4083 = 26.8083$ feet, total head.—*Ans.*

Ex. 77.—A pipe, 33 inches diameter, being 5 miles long, and the velocity of flow in it 9.80 feet per second, what is the total head?

Cal. 1st.—In Table 17, opposite 9.80 feet, velocity for a "33-inch" pipe, find in "fall per mile column" $47.52 \times 5 = 237.6$ feet, head due resistances in pipe.

In Table 18, in velocity column, the nearest approximate to the given velocity is 9.83 feet, opposite which, in "inlet head" column, is found 2.257 feet; then $237.6 + 2.257 = 239.857$ feet, total head.—*Ans.*

Cal. 2d.—Find, as in Cal. 1st, the friction head= 237.6 feet.

By Rule 34:

$(9.8)^2 \times .02337 = 2.244$ feet, inlet head; $237.6 + 2.244 = 239.844$ feet, total head.—*Ans.*

Remark.—The inlet head, except in case of great velocity, is, in practice, usually omitted as insignificant. Thus applying it in Ex. 75, the velocity due the friction head, 63.36 feet, employed in Cal. 4th, is by Table 17, 7.73 feet for an "18-inch" pipe.

By Table 18, the inlet head, due 7.74 feet velocity nearest approximate to 7.73 feet, is 1.40 feet, which is seen to be small in comparison with the given head of 500 feet.

EQUATIONS AND RULES FOR VELOCITY, HEAD, LENGTH, DIAMETER, AND VOLUME, OF FLOW FOR CLEAN PIPES.

The general equation for the volume of flow is:

$$q = a\,v. \qquad (149)$$

Substituting in (149), the value of $a = \frac{\pi\, d^2}{4}$ (area of cross section of pipe), and the value of v of (129), noting that $r = \frac{d}{4}$,

$$q = \tfrac{1}{2} \times \frac{\pi}{4} \left\{ \frac{2g}{c_f} \right\}^{\frac{1}{2}} \left\{ \frac{h_f\, d^5}{l} \right\}^{\frac{1}{2}} \qquad (150)$$

For convenience of notation put,

$$c_{\prime} = \tfrac{1}{2} \times \frac{\pi}{4} \left\{ \frac{2g}{c_f} \right\}^{\frac{1}{2}} = 3.1514 \left\{ \frac{1}{c_f} \right\}^{\frac{1}{2}} \qquad (151)$$

Then, $$q = c_{\prime} \left\{ \frac{h_f\, d^5}{l} \right\}^{\frac{1}{2}} \qquad (152)$$

Transposing (152) successively with respect to h_f, d, l, and reducing,

$$h_f = \left\{ \frac{l\, q^2}{c_i^2\, d^5} \right\}; \qquad (153)$$

$$l = \left\{ \frac{c_i^2\, h_f\, d^5}{q^2} \right\}; \qquad (154)$$

$$d = \left\{ \frac{l\, q^2}{c_i^2\, h_f} \right\}^{\frac{1}{5}}. \qquad (155)$$

Substituting in Eq. (151), the values of ñ=3.1416, $2g=64.4$, and of $c_f=.00644$, as a mean coefficient of resistance within the pipe:

$$c_i = 39.27. \qquad (156)$$

By reference to Table 16, the coefficient of resistance .00644, employed in finding the value of c_i of Eq. (151), is due a velocity of 2.25 feet per second in a 6-inch pipe, 5 feet velocity in a 3-inch pipe, and .7 feet velocity in a 12-inch pipe. Its range thus appears too limited for general application.

Substituting the value of c_i of (156) in Eqs. (152), (153), (154) and (155), there results:

$$q = 39.27 \left\{ \frac{h_f\, d^5}{l} \right\}^{\frac{1}{2}}; \qquad (157)$$

$$h_f = .000648 \left\{ \frac{l\, q^2}{d^5} \right\}; \qquad (158)$$

$$l = 1542.13 \left\{ \frac{h_f\, d^5}{q^2} \right\}; \qquad (159)$$

$$d = .23034 \left\{ \frac{l\, q^2}{h_f} \right\}^{\frac{1}{5}}. \qquad (160)$$

Equations (157), (158), (159) and (160), expressed as written rules are as follows:

Rule 35.—The quantity of flow in cubic feet per second, in a clean pipe, is equal to 39.27 times the square root of the quotient arising from dividing the product of the head, and the 5th power of the diameter, both in feet measure, by the length of the pipe in feet.

Rule 35 corresponds to Eq. (157).

Rule 36.—The head is equal to .000648 times the quotient arising from dividing the product of the length of pipe in feet, and the square of the discharge in cubic feet per second, by the 5th power of the diameter of the pipe.

Rule 36 corresponds to Eq. (158).

Rule 37.—The length of a pipe is equal 1542.13 times the quotient arising from dividing the product of the head and the 5th power of the diameter by the square of the discharge in cubic feet per second.

Rule 37 corresponds to Eq. (157).

Rule 38.—The diameter is equal to .23034 times the 5th root of the quotient arising from dividing the product of the length of the pipe in feet, and the square of the discharge in cubic feet by the head in feet.

Rule 38 corresponds to Eq. (160).

Rule 39.—The values of quantity of flow, the head, length of pipe, and diameter of pipe are found by Table 17.

Case 1st.—Quantity of flow being required, divide the given head by the given length, both in feet. Find in "sine of slope" column, the sine equal to this quotient, opposite which, in discharge column for the given diameter, will be found the quantity of flow sought.

Case 2d.—The head being sought, find the given discharge, or nearest approximate thereto, in the discharge column for the given diameter, opposite which, in "sine of slope" column, will be found the proper sine, which, multiply by the given length of pipe. The product will be the head sought.

Case 3d.—The length of pipe being required, find in the discharge column for the given diameter, the given quantity of flow, opposite which, in "sine of slope" column, will be found the proper sine. Divide the given head by this sine, the quotient will be the length of pipe required.

Case 4th.—The diameter being required, divide the given head by the given length of pipe. Find in "sine of slope" column the sine equal to this quotient, opposite which, in discharge column, find the given quantity of flow, or nearest approximate thereto. The diameter for this discharge will be the diameter sought.

Ex. 78.—The head being 40 feet, the pipe 6 inches diameter, 10,000 feet long, what is the discharge in cubic feet per second?

Cal. 1st.—Substituting the given values $d=6''=.5$ feet, $h_f=40$ feet, and $l=10{,}000$ feet in Eq. (157);

$$\frac{h_f d^5}{l} = (.5)^5 \times 40 \div 10{,}000 = .000125;$$

$39.27 \times (.000125)^{\frac{1}{2}} = .440$ cubic feet.—*Ans.*

Employ Rule 39, case 1st.

Cal. 2d.—Dividing the given head by the given length of pipe, $s = 40 \div 10{,}000 = .004$, sine of slope.

In Table 17, find the sine of slope, opposite which, in discharge column, for 6-inch pipe, is found the quantity sought=.444 cubic feet.—*Ans.*

Ex. 79.—A pipe 3 inches diameter, 10,000 feet long, discharges .247 cubic feet per second, what is the head?

Cal. 1st.—Substituting the given values, $d=$ 3 inches, $l=10{,}000$ feet, and $q=247$ cubic feet in Eq. (158):

$$\frac{l\,q^2}{d^5} = (.247)^2 \times 10{,}000 \div (.25)^5 = 61.7;$$

$.000648 \times 61.74 = 400.05$ feet head.—*Ans.*

Employ Rule 39, case 2.

Cal. 2d.—In Table 17, find in 3-in. discharge column, .2469 cubic feet, nearest approximate to the given quantity .247, opposite which, in sine of slope column, is found .04. Then, $h = l\,s$.

$.04 \times 10{,}000 = 400$ feet head.—*Ans.*

Ex. 80.—If, under a head of 42 feet, a pipe 4 inches diameter discharges .3 of a cubic foot of water per second, what is the pipe's length?

Cal. 1st.—Substituting the given values of $d = 4'' = \frac{1}{3}$ foot, $h = 42$, and $q = .3$ cubic feet in Eq. (159),

$l=1542.13\times 42\times(\frac{1}{3})^5\div(.3)^2=2961.6$ feet.—*Ans.*

Employ Rule 39, case 3.

Cal. 2d.—In Table 17, find in discharge column for 4-inch pipe, .3036 cubic feet, nearest approximate to the given quantity .3 cubic feet, opposite which, in sine of slope column, is found .014; then $l=\frac{h}{s}$; $l=\frac{42}{.014}$ $=3000$ feet.—*Ans.*

Ex. 81.—The head of water being 280 feet, required the diameter of a pipe 4000 feet long, that will discharge 80,000 gallons in 24 hours?

Cal. 1st.—$q=80,000\div(24\times 60\times 60\times 7.5)=.124$ cubic feet, discharge per second.

Substituting the given values, $q=.124$ cubic feet, $h_f=280$ feet, and $l=4000$ feet in Eq. (160);

$d=.23034\left(4000\times(.124)^2\div 280\right)^{\frac{1}{5}}=.17$ feet$=2.04$ inches diameter.—*Ans.*

Employ Rule 39, case 4.

Cal. 2d.—$s=\frac{h}{l}=\frac{280}{4000}=.07$, sine of slope.

In Table 17, find sine of slope .07, opposite which, in discharge column, find .1286 cubic feet per second, nearest approximate to the given volume .124. This is found under heading "2 inches" diameter. The diameter required then is=2 inches.—*Ans.*

Coefficient of Flow.

When the coefficient of flow in a long pipe is $c_{\prime}=$

39.27, the coefficient of velocity deduced from Eq. (130) or (129), is

$$c=\left\{\frac{2g}{c_f}\right\}^{\frac{1}{2}}=\frac{64.4}{.00644}=100. \qquad (161)$$

In which case Eq. (130) becomes

$$v=100\ (r\ s)^{\frac{1}{2}}, \qquad (162)$$

and Eq. (129), by observing that $r=\frac{d}{4}$,

$$\text{becomes } v=50\left\{\frac{h_f\,d}{l}\right\}^{\frac{1}{2}}. \qquad (163)$$

Several standard authors on hydraulics find as follows:

Chezy finds $c=100$. (164)

Eytelwein finds $c=100$. (165)

Leslie finds $c=100$. (166)

D'Aubuisson finds $c=95.6$. (167)

Blackwell finds $c=95.83$. (168)

Hawksley finds $c=96.09$. (169)

Bartlett finds $c=95.88$. (170)

Jackson finds (small pipes) $c=100$. (171)

Fanning finds (mean for small pipes), $c=100$. (172)

D'Arcy finds (for larger pipes) $c=113.8$. (173)

Substituting the value of $c=113.8$, as found by

D'Arcy, the value of $\pi = 3.1416$, in Eq. (150), and reducing,

$$q = 44.69 \left\{ \frac{h_f d^5}{l} \right\}^{\frac{1}{2}}. \qquad (174)$$

COMPARISON OF RESULTS OBTAINED BY EQUATION (174) AND BY TABLE 17.

If the head of water be 100 feet, the pipe 1 foot diameter, and 10,000 feet long, the quantity of flow by Eq. (174), will be $q = 4.469$ cubic feet per second, and by Table 17, $q = 4.305$ cubic feet per second.

The head being 16 feet, the pipe 4 feet diameter, and 10,000 feet long, the quantity of flow by Eq. (174) will be $q = 57.20$ cubic feet per second, and by Table 17, $q = 68.50$ cubic feet per second.

When the head is 100 feet, the pipe 6 inches diameter, and 10,000 feet long, the discharge by Eq. (174) will be $q = .79$ cubic feet per second, and by Table 17 $q = .72$ cubic feet per second.

The head being 20 feet, the pipe 8 feet diameter, and 10,000 feet long, the quantity of flow by Eq. (174) will be $q = 361.8$ cubic feet per second, and by Table 17 $q = 496.4$ cubic feet per second.

These comparisons show that the coefficient, 113.8, proposed as a mean, is too large for small pipes and too small for large pipes; that it is adapted to a pipe 1 foot diameter, with a limited range for either smaller or larger diameters.

They illustrate, also, that the engineer, in the practice of his profession, cannot safely venture far from an established fact in hydraulics, without experiment as a guide.

Bent Pipes.

Bends occurring in pipes resist the flow of water through them.

To determine the additional head requisite to overcome this resistance, Weisbach (partly on the experiments of Du Buat, but chiefly on his own experiments), gives substantially the following formulas and tables compiled from them:

Angular Bends.

Let $h_{\prime}$=the additional head required to overcome one angular bend.

m=one-half the angle of deflection of the bend.

z=the coefficient of bend or knee resistance.

v=velocity of flow. Then,

$$h_{\prime}=z\left\{\frac{v^2}{2g}\right\}; \qquad (175)$$

$$z=0.9457 \sin.^2 m+2.047 \sin.^4 m. \qquad (176)$$

Table 19 is calculated from Eq. (176).

TABLE 19.

Coefficients for Bend Resistances in Pipe.

$m°$	10°	20°	30°	40°	45°	50°	55°	60°	65°	70°
z	.046	.139	.364	.740	.984	1.260	1.556	1.861	2.158	2.43

Rule 40.—The additional head required to overcome one angular bend is, in case the head generating the velocity be given, equal to the product of the given head, and the coefficient of bend or knee resistance, due the given angle of deflection found in Table 19; and is, in case the velocity be given, equal to the product of said coefficient and the square of the given velocity, divided by 64.4.

Ex. 82.—The velocity of water in a pipe, in which occurs one rectangular bend, is 10 feet per second. What additional head will be requisite to overcome the resistance of the head?

Cal.—By Rule 40, square of velocity divided by $64.4 = 10 \times 10 \div 64.4 = 1.553$ feet; $m = \frac{90^\circ}{2} = 45^\circ$, one-half the angle of deflection.

By Table 19, the value of z, corresponding to 45°, is .984; then $1.553 \times .984 = 1.528$ feet, additional head.—*Ans.*

CURVED BENDS.

Let $h_{\prime}$=additional head required to overcome the resistance of curvature,

b=angle of curvature of the pipe,

R=radius of curvature of the bend,

r=radius of the pipe,

$z_{\prime}$=coefficient of resistance;]

$$h_{\prime} = \frac{z_{\prime} b^\circ v^2}{180^\circ} = \frac{z_{\prime} b v^2}{\tilde{n}(2g)}. \qquad (177)$$

$$z_{,}=0.131+1.847\left\{\frac{r}{R}\right\}^{\frac{7}{2}}. \qquad (178)$$

Eq. (178), from which Table 20 is computed, is for pipes with circular cross sections.

TABLE 20.

Coefficients of Resistance of Curvature with Circular Transverse Sections.

$\frac{r}{R}$....	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$z_{,}$....	.131	.138	.158	.206	.294	.440	.661	.977	1.408	1.978

Rule 41.—The additional head required to overcome the resistance of curvature of a bent pipe, is equal to the product of the head, $h=\left\{\frac{v^2}{2g}\right\}$, generating the velocity, the ratio $\left\{\frac{b°}{180°}=\frac{b}{\tilde{n}}\right\}$ of 180° or ñ, to the angle of deflection or length of bend, and the coefficient of resistance, corresponding to the ratio of the radius of curvature to the radius of the pipe.

Ex. 83.—The velocity of water in a pipe 2 inches diameter, is 16 feet per second: what additional head will be requisite to overcome the resistances of a bend in the pipe, whose radius of curvature is 2 inches, and angle of deflection 90°?

Cal.—Head generating velocity, equal to the square of the velocity, divided by 64.4, as $h_f = 16 \times 16 \div 64.4 = 3.975$

$\frac{r}{R} = \frac{1}{2} = .5$, ratio of given radii; $b = \frac{90°}{180°} = \frac{1}{2}$, ratio of 180°, to given angle of deflection.

By Table 20, the coefficient corresponding to .5, the ratio of radii is .294. Then by Rule 41,

$$3.975 \times \tfrac{1}{2} \times .294 = .584 \text{ feet.—} Ans.$$

$$z_{\prime\prime} = 0.124 + 3.104 \left\{ \frac{r}{R} \right\}^{\frac{7}{2}}. \qquad (179)$$

In Eq. (179), from which Table 21 is computed, r represents half the width of a rectangular pipe; and R, the radius of curvature of the axis.

TABLE 21.

Coefficients of Resistance of Curvature in Pipes with Rectangular Transverse Sections.

$\frac{r}{R}$ . . .	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$z_{\prime\prime}$	1.124	.135	.180	.250	.398	.643	1.015	1.546	2.271	3.228

Remark.—Rule 41 applies to finding the additional head required to overcome the resistance of curvature of a bent pipe, whose cross section is rectangular, by employing Table 21 instead of Table 20.

Total Head.—Let in general:

$$h_x = n\, z_x \left\{ \frac{v^2}{2g} \right\}. \qquad (180)$$

represent the additional head requisite to overcome the resistances of n, given bends — angular or curved—in a pipe, the form of whose cross section is given—circular or rectangular.

Combining (180) and (122), observing that $c=.505$ of (127), $\frac{p}{a}=\frac{1}{r}=\frac{4}{d}$, and $2g=64.4$,

$$h+h_x=\left\{1.505+\frac{4\,c_f\,l}{d}+n\,z_x\right\}\frac{v^2}{64.4}; \qquad (181)$$

Ex. 84. A pipe 1 foot diameter, 500 feet long, has five quadrant or right angled bends: the radius of curvature of each bend is 1 foot; the velocity of water through the pipe is 8.025 feet per second. What is the total head ?

Cal. By Table 16, the coefficient corresponding to a velocity of 8 feet per second in a pipe whose diameter is $d=1$ ft., is $c_f=.0052$.

Square of velocity divided by 64.4; $8.025\times8.025\div64.4=1$. In the present case $n=5\frac{b}{180°}=5\times\frac{90°}{180°}=2.5$. Ratio of the given radii, $\frac{r}{R}=\frac{.5}{1}=.5$.

By Table 20, when $\frac{r}{R}=.5$, $z=\frac{z}{x}=.294$.

Substituting in (181) the values of $c_f=.0052$, $d=1$ foot, $l=500$ feet, $n=2.5$, and $z_x=z_1=.294$.

$h+h_x=(1.505+4\times.0052\times500+2.5\times.294)=12.64$ feet.—*Ans.*

FLOW OF WATER THROUGH NOZZLES.

The resistance to the flow of water in conically convergent tubes, estimated for the smaller orifice, is less than the resistance in a cylindrical tube with equal orifice. Thus, Table 13 shows that the coefficient of velocity from a tube converging with an angle 3° 10′ is .894; and that the coefficient for a cylindrical tube of equal diameter is .829. Let t represent this ratio:

$$t = \frac{.894}{.829}. \qquad (182)$$

Let v=the experimental velocity of water in a cylindrical pipe, the ratio of whose diameter to length is as 1:10; v_1=the theoretical velocity in same pipe; c_f=coefficient of resistance, varying from .006 to .004; c_n=mean coefficient of velocity with respect to the smaller orifice of the nozzles:

$$c_n = t\left\{\frac{v}{v_1}\right\}. \qquad (183)$$

Substituting the values of c_f=.004, d=1, and l=10 in (128),

$$v = (2\,g\,h)^{\frac{1}{2}}\left\{\frac{1}{1.505 + \frac{.004 \times 10 \times 4}{1}}\right\}^{\frac{1}{2}} = .775(2gh)^{\frac{1}{2}}; \qquad (184)$$

$$v_1 = (2\,g\,h)^{\frac{1}{2}}, \text{ as per Eq. (8).} \qquad (185)$$

Substituting the values of v, v_1, and t, in (183), c_n=.836, nozzle coefficient (186).

TABLE 22.

Flow of Water Through Nozzles.

Smaller diameter : to length : : 1 : 10 ; angle of convergence, 3° 10.

Head. Feet.	Diameters of Nozzles.							
	1 inch. Cu. ft.	1.5″ Cu. ft.	2.″ Cu. ft.	2.5″ Cu. ft.	3.″ Cu. ft.	3.5″ Cu. ft.	4.″ Cu. ft.	4.5″ Cu. ft.
10.	.115	.258	.458	.715	1.03	1.39	1.92	2.32
12.5	.128	.288	.510	.797	1.16	1.56	2.05	2.60
15.	.131	.315	.562	.875	1.26	1.72	2.25	2.84
17.5	.151	.340	.605	.94	1.36	1.85	2.42	3.06
20.	.162	.364	.647	1.02	1.46	1.99	2.59	3.28
22.5	.171	.386	.686	1.08	1.54	2.10	2.75	3.48
25.	.182	.407	.725	1.13	1.63	2.26	2.90	3.67
27.5	.190	.426	.768	1.19	1.71	2.32	3.03	3.84
30.	.204	.446	.811	1.26	1.81	2.48	3.24	4.10
32.5	.208	.464	.825	1.30	1.87	2.53	3.30	4.18
35.	.214	.482	.857	1.35	1.93	2.63	3.43	4.34
40.	.230	.520	.92	1.43	2.07	2.81	3.67	4.64
45.	.242	.553	.97	1.52	2.18	2.97	3.88	4.93
50.	.256	.583	1.04	1.60	2.30	3.13	4.09	5.25
60.	.273	.638	1.13	1.77	2.56	3.43	4.49	5.75
70.	.307	.690	1.22	1.92	2.75	3.75	4.88	6.16
80.	.328	.737	1.31	2.04	2.95	4.01	5.26	6.64
90.	.347	.778	1.39	2.20	3.06	5.54	5.54	7.00
100.	.366	.824	1.47	2.29	3.30	5.87	5.87	7.41
125.	.410	.92	1.64	2.56	3.67	6.75	6.55	8.26
150.	.449	1.01	1.80	2.80	4.03	7.20	7.20	9.07
175.	.485	1.09	1.94	3.02	4.36	7.78	7.78	9.80
200.	.518	1.16	2.07	3.23	4.59	8.28	8.28	10.45
250.	.584	1.31	2.32	3.62	5.22	9.29	9.29	11.75
300.	.635	1.43	2.54	3.96	5.67	10.15	10.15	12.88
350.	.686	1.54	2.74	4.28	6.16	11.98	10.98	13.85
400.	.733	1.65	2.93	4.58	6.59	11.74	11.74	14.82
450.	.778	1.75	3.11	4.86	6.98	12.46	12.46	15.71
500.	.820	1.84	3.28	5.12	7.38	13.10	13.10	16.60
550.	.859	1.89	3.44	5.36	7.56	13.75	13.75	17.01
600.	.899	2.01	3.59	5.61	8.13	14.36	14.36	18.06
700.	.95	2.21	3.92	6.11	8.86	15.70	15.70	19.93
800.	1.04	2.32	4.14	6.47	9.29	16.56	16.56	20.90
900.	1.10	2.48	4.39	6.87	9.90	17.57	17.57	22.27
1000.	1.16	2.60	4.64	7.24	10.40	18.58	18.58	23.40

TABLE 22.

Flow of Water Through Nozzles.

Smaller diameter : to length : : 1: 10 ; angle of convergence 3° 10.

Head Feet.	Diameters of Nozzle.							
	5 inch. Cu. Ft.	5.5″ Cu. ft.	6.″ Cu. ft.	7.″ Cu. ft.	8.″ Cu. ft.	9.″ Cu. ft.	10.″ Cu. ft.	12.″ Cu. ft.
10.	2.86	3.46	4.13	5.60	7.69	9.29	11.44	16.48
12.5	3.19	3.86	4.62	6.26	1.18	10.40	12.79	18.44
15.	3.51	4.23	5.05	6.86	9.00	11.36	14.10	20.20
17.5	3.78	4.57	5.44	7.42	9.67	12.24	15.14	21.75
20.	4.04	4.89	5 83	7.93	10.35	13.12	16.18	23.32
22.5	4.31	5.17	6.18	8.41	10.98	13.92	17.17	24.74
25.	4.53	5.47	6.51	9.13	11.59	14 64	18.08	25.95
27.5	4.75	5.74	6.82	9.30	12.13	15.35	18.97	27.30
30.	5.07	6.14	7.29	9.94	12.98	16.39	20.27	29.14
32.5	5.15	6.33	7.38	10 10	13.20	16.72	20.62	29.72
35.0	5.35	6.47	7.72	10.49	13.73	17.36	21.40	30.86
40.	5.73	6.91	8.25	11.24	14.65	18.66	22.88	33.00
45.	6.06	7.34	8.75	11.89	15.59	19.79	24.26	34.98
50.	6.39	7.83	9.11	12 63	16.35	20.82	25.58	36.83
60.	7.00	8.48	10.11	13.73	17.92	22.71	28.03	40.39
70.	7.66	9.27	11.02	14.48	19.51	24.47	30.27	42.56
80.	8.19	9.91	11.81	16.06	21.02	26.57	32.36	46.65
90.	8.68	10.42	12.46	17.03	22.18	28.03	34.75	49.82
100.	9.15	11.08	13.18	17.95	23.47	29.65	36.63	52.70
125.	10.24	12.38	14.69	20.07	26.21	33.05	40.96	58.75
150.	11.21	13.57	16.13	21.98	28.80	36.29	44.86	64.51
175.	12.11	13.76	17.42	23.75	31.10	39.20	48.47	69.70
200.	12.91	15.76	18.58	25.38	33.12	41.80	51.80	74.30
250.	15.48	17.52	20.88	28.39	37.15	46.98	57.92	83.52
300.	15.86	19.20	22.90	31.09	40.61	51.52	63.45	90.58
350.	17.14	19.84	24.57	33.59	43.92	55 40	68.54	97.5
400.	18.31	22.16	26.35	35.90	46.94	59.29	73.27	105.4
450.	19.43	23.51	27.94	38.08	49.82	62.86	77.72	111.8
500.	20.47	24.79	29.52	39.60	52.42	66.42	81.92	118.1
550.	21.47	25.99	30 24	42.10	55.01	68.04	85.91	121.0
600.	22.44	27.14	32.11	43.97	57.46	72.25	89.74	128.5
700.	24.46	29.59	35.42	47.93	62.78	79.70	9.79	141.7
800.	25.89	31.43	37.15	50.76	66.24	83.52	103 6	148.6
900.	26.47	33.25	39.60	53.89	70.27	89.10	109.8	158.4
1000.	28 95	35.04	41.62	56.75	74.30	93.6	115 9	166.5

Ex. 85.—A nozzle being 6 inches diameter at its discharge end, 5 feet long, will discharge under a head of 150 feet how many cubic feet of water per second?

Cal.—In "6-inch" diameter column, opposite 150 feet in "head" column, Table 22, the quantity sought is found=16.13 cubic feet.

RELATIVE CARRYING CAPACITIES OF CLEAN, FOUL AND VERY FOUL WATER PIPES.

In the preceding discussion with respect to the carrying capacities of water pipes, they have been considered clean.

Assuming the coefficient of resistance for a clean pipe, .00644; for a rough or foul pipe, .0082; and for a very foul pipe, .012, then will the relative volumes of flow be:

For clean pipes,	1.0000;
For foul pipes,	.8863;
For very foul pipes,	.7325.

Rule 42.—In case a water pipe is rough and foul, multiply the flow for a clean pipe in Table 17 (with same diameter, length and head), by 9, 8, 7, etc., according to the degree of foulness, as judgment shall dictate.

Ex. 89.—A long pipe, 6 inches diameter, has a fall of 18.48 feet per mile. What will be the flow in cubic feet per second, if its coefficient of discharge with respect to a clean pipe be .8?

Cal.—In Table 17, find opposite the given fall 18.48 feet, in discharge column, for a 6-inch pipe, .395 cubic feet. This is for a clean pipe.

Then by Rule 42, $.395 \times .8 = .316$ cubic feet.—*Ans.*

Pressure Ordinates.—A mean pressure ordinate is the vertical distance between the axis of the pipe and the hydraulic gradient. Thus in Fig. 19, C E is the hydraulic gradient, and $(a\,b+r)$, $(c\,d+r)$, or $(e\,f+r)$, is a mean pressure ordinate. If the axis of the pipe be depressed below the base, as represented in Fig. 19, by the dotted line, D a, c, e, E; $a\,b$, $c\,d$ or $e\,f$, will be the mean pressure ordinate for the given point, a, c or e. To find the value of the ordinate in pounds pressure:

Let h_o=the pressure ordinate, or hight of water column in feet, as $a\,b$; p=pounds pressure per square inch of the ordinate or column; whence

$$p=\frac{62.5}{144},\ h_o=.434\ h_o \text{ pounds nearly.} \qquad (187)$$

Rule 43.—The pressure in pounds of a vertical column of water, whose cross section is one square inch, is equal to .434 times the hight of the column or ordinate in feet. Rule 43 corresponds to Eq. (187).

Thickness of thin pipes requisite to withstand a given pressure in pounds per square inch.

Let p=pounds pressure per square inch; t=thickness of pipe in inches; r=radius of pipe; k=modulus of strength, working load, or safety, as shall be given.

By Bartlett's Mechanics, page 507,

$$t=\frac{p\,r}{k}. \qquad (188)$$

Rule 44.—The thickness in inches of a thin pipe, requisite to withstand a given pressure per square inch is equal to the quotient arising from dividing the product of the pounds pressure per square inch, and the radius of the pipe in inches by the given modulus of the material in the pipe. Rule 44 corresponds to Eq. (188).

Ex. 90.—The water pipe, 34.19 inches in diameter, at the hydraulic works of the Spring Valley Company, Butte County, California, laid in 1870, now in perfect condition, as reported by the engineer in charge, is subjected to a working strain of 17,549 pounds per square inch. The greatest pressure ordinate is 887 feet in hight. What is the thickness of the iron in the pipe?

Cal.—By Rule 43, $p=887\times.434=385$ pounds pressure per square inch, nearly; $r=34.19\div2=17.095$, radius of pipe.

By Rule 44, $t=385\times17.095\div17{,}549=.3754$ inches thickness, or $\frac{3}{8}$ of an inch.—*Ans.*

Remark.—The modulus k=17,549 pounds, working load, seems rather too high for ordinary plate iron.

TABLE 23.

Moduli of Strength, Working Load and Safety—Weisbach.

Names of Substances.	Modulus of Strength. Pounds.	Modulus of Working load. Pounds.	Modulus of Safety. Pounds.
Iron in bars	59.805	20,622	10,311
Iron in plates	56,712	....	9,280
Cast iron....................	19 592	14,436	3,094
Steel.......................	123 700	37,120	20,622
Copper.....................	38,151	....	6,187
Lead........................	....	....	329
Box, oak, fir, firm Scotch fir, pine (American)—Bartlett..	12,373	3,094	1,237

INVERTED SIPHON.

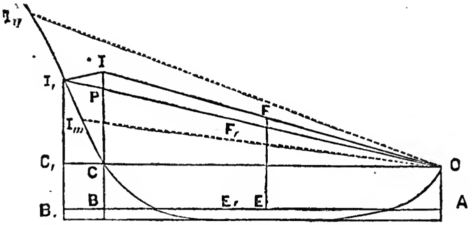


FIG. 20.

A pipe, as represented by $I_{\prime\prime}$ C G O, Fig. 20, employed in conveying water across a valley or other depression between two more elevated points, as $I_{\prime}$ and O, is termed an *inverted siphon.* It is a matter of no little importance in practice to determine the relation of the hights of inlet and outlet of an inverted siphon. Thus the elevation, O, being given, it is demonstrable that if $I_{\prime\prime}$ be the point of inlet, the weight of the pipe will be greater than if the inlet be taken at a lower elevation. It is also demonstrable if $I_{\prime\prime\prime}$ be the point of inlet, the weight of the pipe will be greater than if the inlet were taken at a higher elevation.

For example, having given the elevations of the several stations, and distances between them, of an inverted siphon, whose length is l feet, let it be required to find the hight of the inlet point, above the outlet point, so that the weight, W, pounds of the material employed in the construction of the siphon, carrying q, cubic feet of water per second, shall be a minimum?

Let l=length of siphon in feet;

W=weight of siphon in pounds;

q=cubic feet flow of water per second;

$d_{\prime}=\frac{2r}{12}$, diameter of siphon in feet;

t=thickness of shell of siphon in inches;

w=weight of a cubic foot of material in siphon;

h=working modulus of strength of material;

p=mean pressure in pounds per square inch;

x=hydraulic head, such that the weight, W, shall be a minimum.

Let, in Fig. 20, O represent the point of outlet; O C (approximately equal to O $C_{\prime}$), a horizontal line, intersecting the pipe line in C; O A=a, determined from the given levelling data, the ordinate of mean hydrostatic pressure, in the part, O G C, of the siphon filled to the level, O C; then

$$O\,A = \frac{\text{area } O\,G\,C}{O\,C} = a. \qquad (189)$$

Let the perpendicular, $C\,I = x$, so that the area of the triangle, O C I, shall equal the area of $O\,C\,I_{,}$.

Then will the mean pressure ordinate in feet for the entire pipe, be:

$$E\,F = \frac{2a + x}{2}. \qquad (190)$$

In most cases, without considerable error, C I may be taken equal to $C_{,}\,I_{,}$, thereby assuming that the area, $C\,P\,I_{,}$, is equal to the area, O P I.

If greater accuracy be required, make O A of such a hight that the trapezoid, $O\,A\,B_{,}\,I_{,}$, shall equal the area embraced by the pipe line, $I_{,}\,C\,G\,O$, and the hydraulic gradient, $O\,I_{,}$. In case the exact length of the pipe is not given, but the approximate horizontal distance, $O\,C_{,} = A\,B_{,}$, is known approximately, put

$A\,B_{,} + \frac{B_{,}\,I_{,} + O\,A}{2} = l_{,}$, length of pipe.

Then

$$E_{,}\,F_{,} = \frac{2a_{,} + x_{,}}{2}. \qquad (190)$$

Substituting the value of $E\,F = h_o$ in (187),

$$p = .434\left\{\frac{2a + x}{2}\right\}. \qquad (191)$$

Substituting the values of p of (191), and of $r=\frac{12d_{,}}{2}$, in (188),

$$t=\frac{1.303d_{,}\,(2a+x)}{k}. \qquad (192)$$

The obvious equation for the weight of the siphon is:

$$\mathrm{W}=\frac{\tilde{n}d_{,}\,t\,w\,l}{12}. \qquad (193)$$

Substituting the value of t of (192) in (193),

$$\mathrm{W}=\frac{.1085\tilde{n}\,d_{,}^{2}\,w\,l\,(2a+x)}{k}. \qquad (194)$$

Squaring both members of Eq. (155), observing that $x=h_{f}$,

$$d_{,}^{2}=\frac{1}{c_{,}^{\frac{4}{5}}}\left\{\frac{l\,q^{2}}{x}\right\}^{\frac{2}{5}}. \qquad (195)$$

Substituting the value of $d_{,}^{2}$ of (195) in (194),

$$\mathrm{W}=\frac{.1085\,\tilde{n}\,w\,l^{\frac{7}{5}}\,q^{\frac{4}{5}}}{c^{\frac{4}{5}}\,k}\left\{\frac{2a+x}{x^{\frac{2}{5}}}\right\}. \qquad (196)$$

Differentiating (196), omitting the constant factor outside the parenthesis,

$$\frac{d\,\mathrm{W}}{dx}=-\frac{4a}{5}\,x^{-\frac{7}{5}}+\tfrac{3}{5}\,x^{-\frac{2}{5}}=0. \qquad (197)$$

Reducing (197), $$x=\frac{4a}{3}. \qquad (198)$$

Decomposing (197) into factors, and differentiating that factor which reduces to 0 (zero), the second differential becomes:

$$\frac{d^2 W}{dx^2} = \frac{4a^{-1\frac{2}{5}}}{5}, \qquad (199)$$

which, being positive, determines that W is a minimum, when $x = \frac{4a}{3}$, as found in Eq. (198).

SPECIAL VERIFICATION.

To verify, by special example, the correctness of the result, showing that the weight, W, of an inverted siphon, carrying a given quantity, q, of water, is a minimum, when the head, $h_f = x = \frac{4a}{3}$: substitute the value of $x = \frac{4a}{3}$, also different values, one greater and one less than $\frac{4a}{3}$, as $x_{\prime} = \frac{5a}{3}$, and $x_{\prime\prime} = \frac{3a}{3}$ in Eq. (196).

For convenience of notation, let $f_{\prime}$ = the constant factor outside of the parenthesis. The substitutions being made:

When $x = \frac{4a}{3}$,

$$W = 2.971 f_{\prime} a^{\frac{3}{5}}; \qquad (200)$$

When $x_{\prime} = \frac{5a}{3}$,

$$W_{\prime} = 2.989 f_{\prime} a^{\frac{3}{5}}; \qquad (201)$$

And when $x_{\prime\prime}=\frac{3a}{3}=a$,

$$W_{\prime\prime}=3f_{\prime}\,a^{\frac{3}{5}}. \qquad (202)$$

Since the literal factors in Eqs. (200), (201) and (202) are identical, the numerical factors in these equations express the relative weights of W, $W_{\prime}$ and $W_{\prime\prime}$ in the given order.

These factors conclusively show that the greatest economy is attained in the weight of the siphon, when the value of $x=\frac{4a}{3}$, as determined in Eq. (198).

Minimum with respect to pressure; diameter, thickkess, and weight of an inverted siphon—Minimum mean pressure per square inch.

Substituting the value of $x=\frac{4a}{3}$ in Eq. (191),

$$p=.7234a. \qquad (203)$$

Minimum Diameter.

Substituting in (155), the values of $x=\frac{4a}{3}$ of (198), $c_{\prime}=3.1514\left\{\frac{1}{c_f}\right\}^{\frac{1}{2}}$ of (151), and recollecting that ñ$=h_f$,

$$d_{\prime}=.5965\left\{\frac{c_f\,l\,q^2}{a}\right\}^{\frac{1}{5}}. \qquad (204)$$

Minimum Mean Thickness.

Substituting the values of $\text{ñ}=\frac{4a}{3}$, and of d, of (204) in (192),

$$t=\frac{2.5908}{k}\left(c_f\, l\, q^2\, a^4\right)^{\frac{1}{5}}. \qquad (205)$$

Minimum Weight.

Substituting the values of $d_{\prime}$ of (204), and of t of (205) in (193),

$$W=\frac{.4046w}{k}\left(c_f^2\, l^7\, q^4\, a^3\right)^{\frac{1}{5}}. \qquad (206)$$

In obtaining the value of (206), the inner diameter of the pipe has been employed—the same as used with respect to discharge—whereas the shell of the pipe being exterior to this diameter, its weight is somewhat greater than represented by the value of W (206), as will readily appear by the following:

Let A=area of cross section of pipe with respect to internal diameter.

$A_{\prime}$=area of cross section with respect to external diameter.

$$\text{Then } A=\frac{\text{ñ}}{4}d_{\prime}^2; \qquad (207)$$

And $A_{,}=\frac{ñ}{4}\left(d_{,}+2t\right)^{2}=\frac{ñ}{4}\left(d_{,}^{2}+4d_{,}t+4t^{2}\right)$. (208)

Deducting (207) from (208), and decomposing the difference into factors,

$$A_{,}-A=ñd_{,}t\left\{1+\frac{t}{d_{,}}\right\}. \quad (209)$$

Farther, the pipe has been regarded seamless, whereas, in most cases, water pipes—especially the larger class—are constructed with rivets and laps, or bands, whose weight must be added to that of W of (206).

Let n=this weight, expressed in form of percentage; thus:

$$W_{,}=W\ (1+n). \quad (210)$$

Let $W_{,,}$=the entire weight of pipe, including corrections shown by (209) and (210).

Then as $ñd_{,}t$ only, as respects area of cross section of shell, is involved in the value of W of (206), will

$$W_{,,}=4046w\left(\frac{c_{f}^{2}\,l^{7}\,q^{4}\,a^{3}}{k}\right)^{\frac{1}{5}}\left\{1+\frac{t}{d}\right\}\left(1+n\right). \quad (211)$$

If the pipe is very thin, the factor $\left\{1+\frac{t}{d}\right\}$ may be omitted; if seamless, $(1+n)$, will be omitted.

To find the mean hydraulic pressure below the level of the outlet.

Rule 45.—Find in square feet, from the levelling data, the area embraced, as represented in Fig. 20, within the boundary of the line of pipe, O G C (axis of pipe), and the horizontal line, O C, drawn from the outlet, O, and intersecting the pipe line in C. Divide this area by the length of the line, O C, in feet, the quotient will be the mean hydrostatic ordinate in feet, represented by O A. The length of this ordinate, multiplied by the decimal, .434, will be the mean hydrostatic pressure in pounds sought.

Rule 45 corresponds to Eq. (189).

To find the hydraulic head, such that the weight of the material employed in the construction of the pipe carrying a given quantity of water, shall be a minimum.

Rule 46.—Divide four times the length of the hydrostatic ordinate, as found according to Rule 45, by 3; the quotient will be the hydraulic head, C I (Fig. 20), required.

Rule 46 corresponds to Eq. (198).

Remark.—In Fig. 20, I represents the point at which the pressure in the pipe begins. This point determined, that of I, will readily be found, from the levelling data, and will require to have an additional head sufficient only to discharge the given quantity of water at I.

TO FIND THE MEAN ORDINATE, E F, FIG. 20, INVOLVING BOTH THE HYDRAULIC HEAD, C I, AND THE HYDROSTATIC ORDINATE, A O.

Rule 47.—Divide the sum of the hydraulic head, C I, and twice the hydrostatic ordinate by 2.

Rule 47 corresponds to Eq. (190).

TO FIND THE MEAN PRESSURE PER SQUARE INCH IN POUNDS FOR THE ENTIRE PIPE.

Rule 48.—Multiply the mean hydrostatic ordinate, O A, Fig. 20, in feet by the decimal .7234.

Rule 48 corresponds to Eq. (203).

TO FIND THE MINIMUM DIAMETER.

Rule 49.—The minimum diameter is equal to .5965 times the fifth root of the quotient arising from dividing the product of the coefficient of resistance, the length of the pipe and the square of the discharge per second, by the hight of the hydrostatic ordinate, O A, Fig. 20, in feet.

Rule 49 corresponds to Eq. (204).

To find the minimum mean thickness.

Rule 50.—Multiply the fifth root of the product of the coefficient of resistance, the length of the pipe, the square of the discharge per second, and the fourth power of the mean hydrostatic ordinate, O A, Fig. 20, by the quotient arising from dividing 2.5908 by the modulus of the working load or of safety, as shall be required.

Rule 50 corresponds to Eq. (205).

To find the minimum weight.

Rule 51.—Case 1.—The pipe being very thin, multiply the fifth root of the product of the square of the coefficient of resistance, the seventh power of the length of the pipe, the fourth power of the discharge per second, and the cube of the mean hydrostatic ordinate, O A, Fig. 20, by the quotient arising from dividing .4046 times the weight of a cubic foot of the material in the pipe by the modulus of the material.

Case 2.—The pipe being thick as $\frac{3}{16}$ of an inch or more, and seamless, multiply the result obtained, according to Case 1, by 1 (unit), increased by the quotient arising from dividing the thickness of the shell by the inner diameter.

Case 3.—The pipe being thick, as $\frac{3}{16}$ of an inch or more, and constructed with rivets and laps or bands, multiply the result obtained, according to Case 2, by 1 (unit), increased by their relative weight to that of the pipe.

Rule 51 corresponds to Eq. (211).

The moduli with respect to the strength, working load, and safety are given in Table 23.

The modulus of working load, as shown in Ex. 90, is $k=17,549$ pounds.

Unless the iron is extra in quality, the modulus ought to be less, as $k=14,000$.

The weight of a cubic foot of iron is usually estimated at 480 pounds.

TABLE 24.

Number, Thickness and Weight of One Square Foot of Sheet Iron.

Birmingham Gauge.						American Gauge.—Haswell					
No.	Thi'k in.	Lbs.	No	Thi'k in.	Lbs.	No.	Thi'k in.	Lbs.	No.	Thick in.	Lbs.
0000	.454	18.35	17	.058	2.34	0000	46	18.63	19	.036	1.45
000	.425	17.18	18	.049	1.98	000	.41	16.58	20	.032	1.29
00	.38	15.36	19	.042	1.70	00	.365	14.77	21	.028	1.15
0	.34	13.74	20	.035	1.42	0	.325	13.15	22	.025	1.03
1	.3	12.13	21	.032	1.29	1	.289	11.70	23	.023	.913
2	.284	11.48	22	.028	1.13	2	.258	10.43	24	.020	.814
3	.259	10.47	23	.025	1.01	3	.229	9.29	25	.018	.724
4	.238	9.62	24	.022	.889	4	.204	8.27	26	.016	.644
5	.22	8.89	25	.02	.808	5	.182	7.37	27	.014	.574
6	.203	8.21	26	.018	.723	6	.162	6.56	28	.013	.511
7	.18	7.28	27	.016	.647	7	.144	5.84	29	.011	.455
8	.165	6.67	28	.014	.566	8	.128	5.20	30	.010	.405
9	.148	5.98	29	.013	.525	9	.114	4.63	31	.009	.360
10	.134	5.42	30	.012	.485	10	.102	4.13	32	.008	.321
11	.12	4.85	31	.010	.404	11	.091	3.67	33	.007	.286
12	.109	4.41	32	.009	.364	12	.081	3.27	34	.0063	.254
13	.095	3.84	33	.008	.323	13	.072	2.92	35	.0056	.226
14	.083	3.36	34	.007	.283	14	.064	2.59	36	.005	.202
15	.072	2.91	35	.005	.202	15	.057	2.31	37	.0045	.180
16	.065	2.63	36	.004	.162	16	.051	2.05	38	.004	.159
						17	.045	1.83	39	.0035	.142
						18	.040	1.63	40	.0031	.127

Ex. 91.—The following data from a sheet-iron inverted siphon being given, viz:

Length of pipe, 128.5 miles.

Elevations with respect to sea level:
- Point of inlet, 1300 feet.
- Point of outlet, 350 feet.

Mean hydrostatic ordinate, as O A, Fig. 20, 305.5 feet; discharge of water per second, 37.57 cubic feet; modulus of safety of the iron, 14,000 pounds; weight of iron per cubic foot, 485 pounds; allowance for bands, laps and rivets, 15 per cent; cost of laid pipe per pound, 10 cents. Required, the minimum diameter, thickness of shell, weight and cost of the siphon? Required, also, the diameter, thickness of shell, weight and cost of the siphon, if 950 feet, the full hydraulic head, be employed?

Cal. 1st.—The given hydrostatic ordinate is 305.5 feet.

By Rule 46, corresponding to Eq. (198), $305.5\times4\div3=407.34$ hydraulic head; $407.34\div128.5=3.17$ feet fall per mile.

By Table 17, it is seen that for 3.17 feet fall per mile, the pipe carrying 37.57 cubic feet per second will approximate 48 inches in diameter, and that the corresponding velocity is 3.20 feet per second.

By Table 16, for a velocity of 3 feet in a 48-inch pipe, the coefficient of resistance is $=.0038$.

By Rule 49, corresponding to Eq. (204),

$.5965\left(\frac{.0038\times128.5\times5280\times(37.57)^2}{305.5}\right)^{\frac{1}{5}}=3.898$ feet $=$ 46.776 inches, minimum diameter (internal).—*Ans.*

By Rule 50, corresponding to Eq. (205),

$\left\{\frac{2.5908}{14000}.(.0038\times128.5\times5280\times(305.5)^4\right\}^{\frac{1}{5}}=.3694$ inches in thickness.—*Ans.*

By Rule 51, corresponding to Eq. (211),

$\frac{.4046\times485}{14000}\left\{(.0038)^2(128.5\times5280)^7(37.57)^4(305.5)^3\right\}^{\frac{1}{5}}$

$(1+\frac{.3694}{46.776})\times 1.15=143790750$ pounds, minimum weight.—*Ans.*

Whence at 10c. per pound: Cost=$14,379,075.00.—*Ans.*

Cal. 2d.—1300—350=950 feet total head; $950\div 128.5=7.392$ feet fall per mile.

By Table 17, the diameter of pipe having 7.392 feet fall per mile, and discharge 37.57 cubic feet of water per second is=40 inches.

By Rule 47, corresponding to Eq. (190), $(305.5\times 2+950)\div 2=780.5$ feet mean ordinate for the entire pipe.

By Rule 43, corresponding to Eq. (187), $780.5\times .434=338.737$ mean ordinate in pounds for the entire pipe.

By Rule 44, corresponding to Eq. (188), $338.737\times 20\div 14000=.4839$ inches thickness of pipe; $40\times 3.1416\div 12=10.472$ feet circumference of pipe; $10.472\times 128.5\times 5280\times 485\times .4839\div 12=138954840$ pounds weight of pipe, assumed seamless, and estimated for the internal diameter.

By Rule 51, corresponding to Eq. (211), cases 2 and 3, $138954840(1+\frac{.4839}{40})\times 1.15=161{,}747{,}880$ pounds weight of pipe, employing the full head of 950 feet.—*Ans.*

Whence, at 10c. per pound: Cost=$16,174,788.00.—*Ans.*

The difference in these results, viz., $16,174,788.00—$14,379,075.00=$1,795,713.00, which amounts to a saving of over 11 per cent by the application of the principle hereinbefore demonstrated with respect to the minimum weight and minimum cost of an inverted siphon.

There will, in fact, be a greater saving in practice, arising from a less length of pipe under pressure, in case of employing the smaller head.

FLOW OF WATER IN OPEN CHANNELS AND NATURAL STREAMS.

The flow of water in open channels and natural streams is subject to the same laws which govern its flow in pipes. The force producing motion in the water, and overcoming the resistances of the water way, is that of gravity applied to an inclined plane. A greater variety of forms, with respect to cross section of streams, is presented in open channels and natural streams than in pipes, thereby changing to a greater extent the relations between the perimeters and the areas of the cross sections of the former, than of the latter. Thus, in the case of pipes, the "hydraulic mean radius" has been shown, uniformly, equal to one-fourth of the diameter, while in open channels their mean depths vary indefinitely.

The "mean depth" of an open channel or natural stream is the ratio of the perimeter to the area of the cross section of the stream. For the most part in hydraulic computations, that portion of the perimeter which bounds the bottom and sides of this area, termed the "wet perimeter," is employed.

The "air perimeter," whose value does not often exceed one-tenth of an equal length of the wet perimeter, unless strong winds or other disturbing influences obtain, is considered when great accuracy is required:

Let a=area of cross section of stream;

p=wet perimeter;

$m\,p$=air perimeter;

m=coefficient of air perimeter;

r=hydraulic mean depth.

$$\text{Then } r=\frac{a}{p}. \qquad (212)$$

$$r_{,}=\frac{a}{p+m\,p}. \qquad (213)$$

Other things being equal, the greater the ratio of the perimeter to the area of the cross section of a stream of water, the less will be the resistance to flow.

The forms of cross sections, generally applied to water ways, are rectangular and trapezoidal.

Form of rectangle of maximum carrying capacity.

Let p=perimeter (omitting air perimeter);

x=hight of rectangle;

Then $p-2x$=width of rectangle.

$$\frac{px-2x^2}{p}=\frac{p}{4}=r, \text{ maximum.} \qquad (214)$$

Differentiating (214), $x=\frac{p}{4}$, hight; (215)

$$p-2x=\frac{2p}{4}, \text{ width.} \qquad (216)$$

FORM OF TRAPEZOID OF REGULAR FIGURE OF MAXIMUM CARRYING CAPACITY.

In Fig. 21, let t=B A E, angle of slope of bank; $\frac{p}{3}$=a side; then hight$=\frac{p}{3}$ sin t. Mean width$=\frac{p}{3}+\frac{p}{3}$ cos t:

$$\frac{p^2}{9p}(\sin t+\sin t \cos t)=r \text{ maximum.} \qquad (217)$$

Differentiating (217), observing that $\sin^2 t=1-\cos^2 t$,

$$\cos^2 t+\frac{\cos t}{2}=\tfrac{1}{2}. \qquad (218)$$

Reducing (218), $\cos t=\frac{1}{2}=.5$. (219)

By table natural sines, $t=60°$. (220)

Of the regular figures, the semi-circle consisting of an infinite number of sides, so that at any point cos t $=1=r$, offers the least resistance to flow.

By equations (215) and (216), it is seen that the form of a rectangle, offering the least resistance to flow, has its base or width equal to twice its hight; and by equations (219) and (220), it is seen that of the regular figures, the trapezoid whose angle of slope is 60°, in other words the semi-hexagon, offers the least resistance to flow.

THE ANGLE OF SLOPE AND THE AREA BEING GIVEN TO DETERMINE THE MOST APPROPRIATE FORM OF A CANAL.

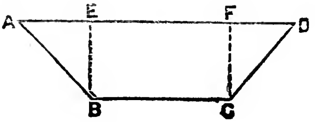


FIG. 21.

Let in Fig. 21, p=AB+BC+CD=perimeter; b=BC=bottom; t=angle A; n=cot t; a=area; x=d=BE, depth of canal. Then

$$AE=nx; \qquad (221)$$

$$AB=x(1+n^2)^{\frac{1}{2}}; \qquad (222)$$

$$p=b+2x(1+n^2)\ ; \qquad (223)$$

$$a=(b+nx)x; \qquad (224)$$

$$\text{whence } b=\frac{a-nx^2}{x}. \qquad (225)$$

Substituting value of b of (225) in (223) and dividing then both members by a,

$$\frac{p}{a}=\frac{1}{x}+\frac{x}{a}\left\{(1+n^2)^{\frac{1}{2}}-n\right\}\text{minimum.} \qquad (226)$$

Differentiating (226) and reducing,

$$x=\left\{\frac{a}{2(1+n^2)^{\frac{1}{2}}-n}\right\}^{\frac{1}{2}}. \qquad (227)$$

Substituting the values of $n = \cot t = \frac{\cos t}{\sin t}$ and $(1 + n^2)^{\frac{1}{2}} = \frac{1}{\sin t}$ in (227),

$$x = \left\{ \frac{a \sin t}{2 - \cos t} \right\}^{\frac{1}{2}}. \qquad (228)$$

$$\text{From (225) } b = \frac{a}{x} - x \cot t. \qquad (229)$$

TABLE 25.

Dimensions of the most suitable forms of Canals, corresponding to different angles of slopes, and to a given area of cross section.

Angle of Slope $= t$.	Ratio of Perp. to Base.	Relative Slope. n.	Depth. $\frac{d}{\sqrt{a}}$	Bottom Width. $\frac{b}{\sqrt{a}}$	$\frac{n\,d}{\sqrt{a}}$	Top Width. $\frac{b + 2n\,d}{\sqrt{a}}$	Perimeter. $\frac{p}{\sqrt{a}}$
90° 00′	1 on 0	.0	0.707	1.414	.0	1.414	2.828
78° 41′	5 on 1	0.200	0.734	1.217	0.147	1.510	2.713
75° 58′	4 on 1	0.250	0.734	1.161	0.186	1.533	2.692
71° 34′	3 on 1	0.333	0.752	1.079	0.251	1.580	2.656
63° 26′	2 on 1	0.500	0.759	0.938	0.379	1.697	2.635
60° 00′	26 on 15	0.577	0.760	0.877	0.439	1.755	2.632
56° 19′	3 on 2	0.667	0.759	0.812	0.506	1.824	2 635
53° 8′	4 on 3	0.750	0.757	0.753	0.568	1.892	2.645
51° 20′	5 on 4	0.800	0.753	0.724	0.603	1.960	2.654
45° 00′	1 on 1	1.000	0.740	0.613	0.740	2.092	2.704
40° 00′	21 on 25	1.192	0.722	0.525	0.860	2.246	2.771
36° 52′	3 on 4	1.333	0.707	0.471	0.943	2.557	2.828
35° 00′	7 on 10	1.402	0.697	0.439	0.995	2.430	2.870
33° 41′	2 on 3	1.500	0 689	0.418	1.034	2.465	2.989
30° 00′	23 on 40	1.732	0.664	0.356	1.150	2.656	3 012
26° 34′	1 on 2	2.000	0.636	0.300	1.272	2.844	3.144
21° 48′	2 on 5	2.500	0.589	0.228	1.471	3.170	3.397
18° 26′	1 on 3	3.000	0.548	0.188	1.645	3.478	3.646
14° 2′	1 on 4	4.000	0.485	0.119	1.941	4.001	4.121
11° 19′	1 on 5	5.000	0.441	0.062	2.205	4.472	4.519
semi-cir.	....	....	0.798	....	....	1.596	2.507

To find the dimensions of the most suitable form of canal, when the angle of slope of the banks and the area of cross section are given.

Rule 52.—Employing Table 25, multiply the square root of the given area of cross section by the number in the table, which is opposite the given angle of slope of bank, and in the column of the denomination of the dimension sought, as "depth," "bottom width," "top width," "perimeter" (wet).

Ex. 92.—What dimensions must be given to the cross section of a canal—trapezoid of regular form—whose discharge of 64 cubic feet, with a velocity of 4 feet per second, is a maximum?

Cal.—By Eq. (220), it is shown that the angle of slope$=60°$, when the carrying capacity of a regular trapezoidal canal is a maximum.

$64\div4=16$ square feet, area of cross section.

$(16)^{\frac{1}{2}}=4$ square root of area of cross section.

By Table 25, Rule 52, opposite 60°, the angle of slope in "depth" column, find .760; that is, $\frac{d_{,}}{\sqrt{a}}=.760$; or $d=.760\sqrt{a}$.

But, as shown above, the square root of the area of cross section is$=4$ feet; hence, $d_{,}=.760\times4=3.04$ feet, depth of canal.—*Ans.*

Farther, opposite 60°, in "bottom width" column, find .877; that is, $\frac{b}{\sqrt{a}}=.877$; or $b=.877\sqrt{a}$, but as shown above $\sqrt{a}=4$; hence, $b=.877\times4=3.508$ feet, bottom width.—*Ans.*

Farther, opposite 60° find in "top width column" 1.755 feet; that is, $\frac{b+2nd}{\sqrt{a}}=1.755$ feet; or $b+2nd=1.755\times4=7.020$ feet, top with.—*Ans.*

Farther, opposite 60° in "perimeter column," find 2.632; that is, $p=2.632\times4=10.528$ feet perimeter.—*Ans.*

Ex. 93.—The following data for a canal in loose earth being given, viz: Discharge of water, 50 cubic feet per second; velocity of flow, 2 feet per second; angle of slope, 26° 34′ equivalent to 1∶2; it is required to determine the dimensions of the most appropriate form of cross section of the canal ?

Cal.—$50\div2=25$ square feet, area of cross section; $\sqrt{a}=\sqrt{25}=5$ square root of area of cross section.

By Table 25, opposite 26° 34′, the given angle of slope (1∶2), in depth column, find .636; that is $\frac{d_{\prime}}{\sqrt{a}}=.636$; or $d_{\prime}=.636\sqrt{a}$.

Substituting value of $\sqrt{a}=5$; in this, $d_{\prime}=.636\times5=3.18$ feet, depth.—*Ans.*

Farther, in column of "bottom width," find $b=.300\sqrt{a}$, or by substituting 5 for $\sqrt{a}$, $b=.300\times5=1.5$ feet, bottom width.—*Ans.*

Farther, in column of "top width," find $b\times2nd_{\prime}=$

$2.844\sqrt{a}$, or by substituting 5 for $\sqrt{a}$, $b+2nd=2.844\times5=14.22$ feet, top width.—*Ans.*

Farther, in "perimeter" column, find $\frac{p}{\sqrt{a}}=3.144$, or $p=3.144\times5=15.72$ feet, perimeter.—*Ans.*

FORMULAS FOR THE FLOW OF WATER IN OPEN STREAMS.

Equation (130) is an expression for uniform velocity of water in open streams, as well as in pipes. In case the air perimeter of an open stream be considered, the value of $r_{,}=\frac{a}{p+mp}$ of (213) must be substituted for r in (130); but in case the air perimeter be omitted, equation (130) for the velocity of water in pipes remains unchanged for the velocity of water in open streams, that is:

$$v=\left\{\frac{2g}{c_f}\right\}^{\frac{1}{2}}\left\{rs\right\}^{\frac{1}{2}}. \qquad (230)$$

In the factor $\left\{\frac{2g}{c_f}\right\}^{\frac{1}{2}}$, c_f is a variable coefficient, whose value depends upon experiment. This simple equation covers all cases in practice, providing the value of c_f be known.

Of the numerous formulas in use for finding the velocity of water in open streams, the following from Kutter, taken in connection with his table of coefficients for roughness of stream bed, seems entitled to the foremost position, on account of its directness and wide range of application.

Kutter's Formula.

$$v=\left\{\frac{\frac{1.811}{n}+41.6+\frac{0.00281}{s}}{1+\left(41.6+\frac{0.00281}{s}\right)\frac{n}{r^{\frac{1}{2}}}}\right\}(r\,s)^{\frac{1}{2}}. \qquad (231)$$

In (231), v, r and s, respectively, denote velocity, hydraulic mean depth, and sine of slope, and n represents the coefficient of roughness of the stream bed.

Eq. (231) may be somewhat simplified by putting

$$x=n\left\{41.6+\frac{0.00281}{s}\right\}; \qquad (232)$$

When

$$v=\left\{\left(\frac{x+1.811}{x+r^{\frac{1}{2}}}\right)\frac{r}{n}\right\}(s)^{\frac{1}{2}}. \qquad (233)$$

The first factor of the second member of Eq. (231) is equal to the first factor of the second member of Eq. (230), and being deduced from a wide range of experiments with great care and masterly skill, furnishes the best means known to the present writer of determining, in the absence of actual experiment in a special case, the value of $\left\{\frac{2g}{c_f}\right\}^{\frac{1}{2}}$. Let c_k =this expression from Kutter's formula; then

$$v=c_k\,(r\,s)^{\frac{1}{2}}. \qquad (234)$$

TABLE 26.

Coefficients (n) for Roughness of Stream Beds. Compiled from Kutter, Jackson and Fanning.

n=.009 well planed timber in perfect order.

n=.010 cement, glazed, coated material.

n=.012 unplaned timber in flumes.

n=.013 brickwork, cast and wrought-iron.

n=.015 canvas lining, rectangular flumes with battens .5 inch apart.

n=.017 rubble, also, earth in highly regular cases.

n=.020 coarse rubble, set dry, in bad condition, very firm regular gravel, and flumes with battens 2 inches apart.

n=.0225, dry, coarse rubble in bad order; earth canals and channels above average.

n=.0250 earth, canals and channels in good order.

n=.0275 earth, canals and channels below average.

n=.030 earth, canals and channels in bad order.

n=.035 rivers and canals in bad order, overgrown with vegetation, and strewn with stones.

n=.070 rivers in earth, with stones and weeds in great quantities.

The principal elements, besides the force of gravity involved in the determination of the velocity of an open stream of water, are the hydraulic mean depth, the sine of slope, and the degree of roughness of the stream bed. Now, if these relations be not changed, it is evident that the velocity of the water will neither be increased nor diminished by varying the form and magnitude of the sectional area of the stream.

Illustrative of this proposition:

1st.—The hydraulic mean radius or mean depth of a circular pipe, in which d=diameter, is:

$$r=\frac{a}{p}=\frac{\tilde{n}\, d^2}{4\, \tilde{n}\, d}=\frac{d}{4}. \qquad (235)$$

2d.—The hydraulic mean depth of a V flume ("vee flume"), right angled at the bottom, in which d=the slant hight or width of side,

$$r=\frac{a}{p}=\frac{d^2}{2\times 2d}=\frac{d}{4} \qquad (236)$$

3d. The hydraulic mean depth of a rectangular flume or canal, in which d=the width, and $\frac{d}{2}$=the depth is,

$$r=\frac{a}{p}=\frac{d^2}{2\times 2d}=\frac{d}{4}. \qquad (237)$$

4th. To determine a trapezoidal flume or canal, in which the mean width and the angle of slope of the sides are given, such that the hydraulic mean depth shall be equal to one fourth of the mean width.

Let $d_{,}$=mean width.

x=width of side or slant hight.

y=bottom width.

t=angle of slope of sides.

Then

$$x \sin t = \text{depth}. \qquad (338)$$

$$x \cos t + y = d. \qquad (239)$$

$$\frac{dx \sin t}{2x + y} = \frac{d}{4}. \qquad (240)$$

$$\text{Whence: } x = \frac{d}{\cos t + 4 \sin t - 2}; \qquad (241)$$

$$\text{And } y = d \left\{ \frac{4 \sin t - 2}{\cos t + 4 \sin t - 2.} \right\}. \qquad (242)$$

Case 1st.—Put $t = 45°$.

By Table of natural sines and cosines, sin 45°= .70711; cos 45°=.70711.

Substituting values of sin t and cos t in (241) and (242),

$$x = .6512\, d. \qquad (243)$$

$$y = .5395\, d. \qquad (244)$$

$$\text{Depth} = x \sin t = .4605\, d. \qquad (245)$$

$$\text{And } r = \frac{a}{p} = \frac{.4605\, d^2}{1.842\, d} = \frac{d}{4}. \qquad (246)$$

Case 2nd.—Put $t = 60°$.

By Table of natural sines and cosines, sin 60°= .86603; cos 60°=.50000.

Substituting values of sin 60° and cos 60° in (241) and (242),

$$x = .5091\, d. \qquad (247)$$

$$y = .7454\, d. \qquad (248)$$

$$\text{Depth } x \sin t = .4409\, d. \qquad (249)$$

$$\text{And } r = \frac{p}{a} = \frac{.4409 d^2}{1.7636 d} = \frac{d}{4}. \qquad (250)$$

By giving different values to t in (241) and (242), the form of sectional area of a trapezoidal flume or canal may be varied indefinitely without change of the hydraulic mean depth, $r=\frac{d}{4}$.

The forms given in which $t=45°$ and $t=60°$, seem to occur most frequently in practice.

Now, as under like conditions, excepting the forms of stream beds, the velocity of water in flumes or canals is equal to its velocity in pipes of equal hydraulic mean radii or depths. Table 17 giving the velocity of flow in pipes, applies equally well to this class of flumes or canals.

To find the velocity of water in a flume or canal whose hydraulic mean depth is equal to that of a pipe of given diameter.

Rule 53.—Case 1st—The flume being V shaped, that is, quadrant in form (usually termed "vee flume)," find opposite the given sine of slope or fall per mile, in Table 17, the velocity of flow due a pipe whose diameter is equal to the side, width or slant hight of the flume. The quantity so found will be the velocity sought.

Case 2d.—The flume or canal being either rectangular or trapezoidal, find, opposite the given sine of slope or fall per mile in Table 17, the velocity of flow due a pipe whose diameter is equal to the mean width of the flume or canal. The quantity so found will be the velocity sought.

TO FIND THE FLOW OF WATER IN CUBIC FEET IN A FLUME OR CANAL WHOSE HYDRAULIC MEAN DEPTH IS EQUAL TO THAT OF A PIPE OF GIVEN DIAMETER.

Rule 54.—Case 1st.—The flume being V shaped, quadrant in form, a "vee flume," find the velocity as directed in Rule 53, and multiply the velocity so found by one half the square of the side width or slant-depth of the ratio.

Case 2nd.—The flume or canal being rectangular, multiply the velocity found as directed in Rule 53, by one half the square of the width of the water way.

Case 3d.—The flume or canal being trapezoidal, if the angle of the slope of the sides is equal to 45°, multiply the velocity found as directed in Rule 53, by .4605 times the square of the mean width; if the angle of slope of the sides is equal to 60°, multiply the velocity by .4409 times the square of the mean width of the water way, and, in general, if the angle of slope of the sides be equal to *t*, multiply the velocity found as directed in Rule 53, by the product of the square of the mean width, and the ratio of the mean width to the vertical depth of the water in the flume or canal.

Rule 55.—Another method of finding the flow of a given flume is to multiply the discharge of a circular pipe, Table 17, of equal fall per mile, and equal hydraulic mean depth, by the ratio of the sectional area of the pipe to the sectional area of the given flume; that is, if the flume is either quadrant in form, a "V flume," or if it is rectangular, having its width equal to twice its depth, multiply by .637; if it is trapezoidal, multiply by .586 when the angle of slope of the side is 45°, and by .561 when it is 60°.

Coefficient of roughness involved in Table 17.

The measure of the degree of roughness of the inner surfaces of the pipes for which the flow of water under pressure has been computed in Table 17, closely approximates .011 when referred to Kutter's scale of coefficients (Table 26 of the present work): that is $n = .011$.

Ex. 94.—The fall per mile being 15.84 feet, what will be the velocity and discharge per second of water in a "V flume," whose side width or slant depth of water is 2.5 feet, and the coefficient of roughness of inner surfaces equal to .011 ?

Cal.—With respect to velocity by Rule 53, in Table 17, opposite the given fall 15.84, in velocity column of 2.5 feet diameter, find 5.27 feet.—*Ans.*

Calculation with respect to discharge: One-half the square of side, $2.5 \times 2.5 \div 2 = 3.125$ square feet.

By Rule 51, $5.27 \times 3.125 = 16.47$ cubic feet.—*Ans.*

Or, by Rule 55, find, by Table 17, the discharge of a pipe 2.5 diameter, for the given fall, 25.87 cubic feet.

$$25.87 \times .637 = 16.48 \text{ cubic feet.—}Ans.$$

Ex. 95.—The fall being 5.28 feet per mile, what will be the discharge of water flowing in a flume whose mean width is 8 feet, angle of side slope 45°, and coefficient of roughness, $n = .011$.

Cal.—By Table 17, for a fall of 5.28 feet per mile, and 8-foot pipe, the discharge per second is 350.5 cubic feet. Then by Rule 55,

$$350.5 \times .586 = 205.39 \text{ cubic feet.—}Ans.$$

Remark.—With respect to the flow of water in pipes not differing largely in size, it may be assumed without material error in practice, that the velocities are proportionate to the respective diameters.

If greater accuracy be required, recourse to (133) will need be had.

Ex. 96.—The fall being 2.64 feet per mile, what is the discharge per second in a flume 9 feet wide, 4.5 feet deep, and the coefficient of roughness, $n = .011$?

Cal.—By Table 17, the velocities for a fall of 2.64 feet per mile, due pipes 8 and 10 feet diameter—

mean $=\frac{8+10}{2}=9$ feet—are 4.88, and 5.84 feet—mean $=\frac{4.88+5.84}{2}=5.36$ feet, velocity per second.

Whence, $5.36\times9\times9\div2=217.08$ cubic feet.—*Ans.*

Kutter's Formula for the Flow of Water in in Open Streams.

Among the most eminent experimentists in hydraulics, during the present half century, have been D'Arcy and Bazin, Humphreys and Abbot, and Kutter. Prior to this time, the science of hydraulics was largely speculative and incoherent. The older hydraulicians had determined on meager data certain laws which they erroneously held susceptible of general application.

D'Arcy and Bazin having collected a large amount of experimental data, deduced therefrom and published in 1835 and 1865, a formula better adapted than any preceding for finding the flow of water in open streams and pipes of medium size. The report of Humphreys and Abbot on the "Physics and Hydraulics" of the Mississippi river, published in 1861, is very justly esteemed by engineers, first in importance, as to the extent, accuracy, and value of its contributions to experimental hydraulics. Their formulas, however, deduced from their experiments, besides being quite complex and tedious of application, give results too low for the flow of water in small and medium sized streams. Thus, between the formulas of D'Arcy and Bazin, and those of Humphreys and Abbot, there existed a wide hiatus, till it was effectually closed up in 1870 by the introduction of Kutter's formula.

The mode in brief of Herr Kutter, in accomplishing this difficult and laborious task, is substantially as follows:

1st.—To divide the great mass of the observed and trustworthy results at his command, appertaining to the flow of water in open streams, into twelve classes, arranged as shown in Table 26, with reference to the degree of roughness of the stream-beds.

2d.—To adopt, on careful comparison of various formulas for the velocity of water under the imposed conditions, the following formula of Chezy as the basis:

$$v = c\,(r\,s)^{\frac{1}{2}}, \qquad (251)$$

noting that c is variable.

3d.—For the determination of the values of c, that shall, when substituted with the given values of r, the hydraulic mean depth, and of s, the sine of slope in the Chezy formula (251), yield results respectively corresponding to those determined by observation, he makes extensive experiments with several trial formulas, among which is that of D'Arcy and Bazin, hitherto considered in our discussion of the flow of water in pipes.

Of these trial formulas, the following finally adopted is one devised by himself, which is not only more simple in form than that of D'Arcy and Bazin, but yields results nearer in accord with those observed:

$$c = \frac{a'}{1 + \frac{b'}{r^{\frac{1}{2}}}}. \qquad (252)$$

This formula, however, is faulty, in that it is limited in application.

Thus by inversion, it is shown to be an equation to a straight line whose abscissa $= \frac{1}{r^{\frac{1}{2}}}$, whereas the plotted results of observation indicate a curve, and further show that a' is dependent upon the value of b'—in a word, that a' and b' are variables, and not constants, as at first assumed.

4th.—To generalize Eq. (252) the variable terms z for a' and x for b' are substituted in it; whence,

$$c = \frac{z}{1 + \frac{x}{r^{\frac{1}{2}}}}. \qquad (253)$$

5th.—"After much examination and further comparison," (Kutter's words) he puts,

$$z = a + \frac{l}{n}; \qquad (254)$$

$$x = n\,z - l = a\,n. \qquad (255)$$

Substituting these values of z and x in (253), we have

$$c = \frac{a + \frac{l}{n}}{1 + \frac{a\,n}{r^{\frac{1}{2}}}}. \qquad (256)$$

Equation (256) is found, however, not suited to the extremes of inclination of water surface, nor to the extreme limits of sectional area.

6th.—To meet these requirements, in other words to render the formula applicable to all cases whatever, Kutter noting that "when $r=$ infinity, c will$=z$, and the coefficients z will have their values represented by an hyperbolic curve," makes in Eq. (253),

$$z = \mathrm{A} + \frac{m}{s}, \qquad (257)$$

in which A denotes the semi-axis of an hyperbola, m the tangent of the inclination of its asymtotes with the axis of abscissa, and $\frac{1}{s}$ (s representing the sine of slope) the abscissæ; whence,

$$z = a + \frac{l}{n} + \frac{m}{s}; \qquad (258)$$

$$x = n\,z - l = \left\{ a + \frac{m}{s} \right\} n. \qquad (259)$$

Substituting the values of z and x of Eqs. (258) and (259) in (253), there results the equation in its general form for the value of the coefficient c, viz:

$$c=\frac{a+\frac{l}{n}+\frac{m}{s}}{1+\left\{a+\frac{m}{s}\right\}\frac{n}{r^{\frac{1}{2}}}} \tag{260}$$

Substituting the value of c of Eq. (260) in (251), there results the general formula of Kutter for the velocity of water in open streams, viz:

$$v=\left\{\frac{a+\frac{l}{n}+\frac{m}{s}}{1+\left\{a+\frac{m}{s}\right\}\frac{n}{r^{\frac{1}{2}}}}\right\}(r\ s)^{\frac{1}{2}}. \tag{261}$$

Combining (257) and (254),

$$A+\frac{m}{s}=a+\frac{l}{n}. \tag{262}$$

Making s infinite in Eq. (262),

$$A=a+\frac{l}{n}. \tag{263}$$

To determine the relation of c to n, in its simplest form, let

$$\frac{1}{r^{\frac{1}{2}}}=l, \tag{264}$$

in which $l=1$, as found by trial.

Substituting this value of l, and of $\frac{1}{r^{\frac{1}{2}}}$ in Eq. (256), and reducing, we obtain the relation sought,

$$\frac{1}{c}=n. \tag{265}$$

which, as determined for the Mississippi river, is

$$n=\frac{1}{c}=.027. \qquad (266)$$

The value of A in an hyperbola, coinciding with the curve formed by plotting observed results, is found to be:

$$A=60. \qquad (267)$$

Substituting the values of $A=60$ of (267), $l=1$ of (264), and $n=.027$ of (266) in (263), transposing and reducing,

$$a=A-\frac{n}{l}=60-37=23. \qquad (268)$$

To find the value of tangent m, let an extreme case be taken, in which the values, as determined from the plotted curve, are:

$$s=0.00000363; \qquad (269)$$

$$z=487. \qquad (270)$$

Substituting these values, together with that of $A=60$ of (267) in (257), transposing and reducing,

$$m=(z-A)s=(487-60)\times 0.00000363=0.00155. \quad (271)$$

Subsituting these constant values of $a=23$ of (268), $l=1$ of (264), and $m=0.00155$ in (261), there results for metrical measures:

$$v=\left\{\frac{23+\frac{1}{n}+\frac{0.00155}{s}}{1+\left(23+\frac{0.00155}{s}\right)\frac{m}{r^{\frac{1}{2}}}}\right\}(r\,s)^{\frac{1}{2}}. \qquad (272)$$

To reduce metrical measures employed in Eq. (272) to those of a different system, let e denote the ratio of the former to the latter, noting that n and s, representing ratios, are not affected by the reduction.

$$\text{Substitute } z=23+\frac{1}{n}+\frac{0.00155}{s}; \qquad (273)$$

$$x=\left\{23+\frac{0.00155}{s}\right\}n; \qquad (274)$$

$$v=\left\{\frac{z}{1+\frac{x}{r^{\frac{1}{2}}}}\right\}(r\,s)^{\frac{1}{2}}=\left\{\frac{r\,z}{r^{\frac{1}{2}}+x}\right\}(r\,s)^{\frac{1}{2}}. \qquad (275)$$

Let z', x', r' and v' represent respectively the terms to which z, x, r and v, of the metrical system are to be reduced, then will $z=\frac{z'}{e}$, $x=\frac{x'}{e}$, $r=\frac{r'}{e}$ and $v=\frac{v'}{e}$.

Substituting these values of z, x, r and v in (275)

$$\frac{v'}{e}=v=\frac{r'^{\frac{1}{2}}}{e^{\frac{1}{2}}}\left\{\frac{\frac{z'}{e}}{\frac{r'^{\frac{1}{2}}}{e^{\frac{1}{2}}}+\frac{x'}{e}}\right\}\frac{r'^{\frac{1}{2}}}{e^{\frac{1}{2}}}s^{\frac{1}{2}}. \qquad (276)$$

Multiplying in (276), both numerator and denominator within the parenthesis by $e^{\frac{1}{2}}$; also, multiplying both sides of the equation by e,

$$v'=v\,e=r'^{\frac{1}{2}}\left\{\frac{\frac{z'}{e^{\frac{1}{2}}}}{r^{\frac{1}{2}}+\frac{x'}{e^{\frac{1}{2}}}}\right\}r'^{\frac{1}{2}}\,s^{\frac{1}{2}}=\left\{\frac{\frac{z'}{e^{\frac{1}{2}}}}{1+\frac{x'}{e\,r'^{\frac{1}{2}}}}\right\}(r'\,s)^{\frac{1}{2}}. \qquad (277)$$

Substituting the values of $\frac{z'}{e^{\frac{1}{2}}}=e^{\frac{1}{2}}z$, and $\frac{x'}{e^{\frac{1}{2}}}=e^{\frac{1}{2}}x$, in (277).

$$v'=\left\{\frac{e^{\frac{1}{2}}z}{1+\frac{e\,x}{r'^{\frac{1}{2}}}}\right\}(r'\,s)^{\frac{1}{2}}. \qquad (278)$$

With respect to the Kutter formula, equation (278) is general for the reduction of the measures employed in it to those of another system. An inspection shows that by multiplying z and x, by $e^{\frac{1}{2}}$ (the square root of the ratio of the different measures), each term of the equation [1 (unit) being common,] will be the denomination sought.

To render the equation in terms of English feet,

Let e=3.281, the number of feet in a meter. (279)

Substituting the value of $e^{\frac{1}{2}}$=1.811 in (278) after restoring the values of z and x of (273) and (274), and omitting the accents with respect to v' and r',

$$v=\left\{\frac{41.6+\frac{1.811}{n}+\frac{0.00281}{s}}{1+\left\{41.6+\frac{0.00281}{s}\right\}\frac{n}{r^{\frac{1}{2}}}}\right\}(r\,s)^{\frac{1}{2}}. \qquad (280)$$

Equation (280) for the purposes of application may be somewhat simplified in the following manner.

Putting the numerator inclosed in brackets under the following form:

$$\text{Numerator}=\frac{1}{n}\left\{\left(41.6+\frac{0.00281}{s}\right)n+1.811\right\}, \qquad (281)$$

and putting $x=\left(41.6+\dfrac{0.00281}{s}\right)n.$ (282)

Substituting x for its value in (280), and then multiplying both numerator and denominator by $r^{\frac{1}{2}}$, there results:

$$v=\frac{r}{n}\left(\frac{x+1.811}{x+r^{\frac{1}{2}}}\right)s^{\frac{1}{2}}. \qquad (283)$$

APPLICATION OF THE KUTTER FORMULA AS RENDERED BY EQS. (282) AND (283).

Ex. 97.—In a rectangular flume or canal four feet wide, two feet deep, the fall per mile is 4.752 feet (equivalent to a sine of slope, $s=.0009$), and the coefficient of roughness of whose bed is $n=.025$. What is the velocity of flow per second?

Cal.—Area of cross section, $4\times2=8$; wet perimeter, $4+2+2=8$.

Hydraulic mean depth, $8\div8=1$.

Square root hydraulic mean depth, $r^{\frac{1}{2}}=\sqrt{1}=1$.

Square root of sine of slope, $s^{\frac{1}{2}}=\sqrt{.0009}=.03$.

Substituting the given values of $s=.0009$, and $n=.025$ in Eq. (282),

$$x=\left(41.6+\frac{0.00281}{.0009}\right)\times.025=1.118.$$

Substituting the value of $n=.025$, and the values as found of $x=1.118$, $s^{\frac{1}{2}}=.03$, and $r^{\frac{1}{2}}=1$ in Eq. (283),

$$v=\frac{1}{.025}\left(\frac{1.118+1.811}{1.118+1}\right)\times .03.$$

Reducing, $v=40\left(\frac{2.928}{2.118}\right)\times .03=1.658$ feet.—*Ans.*

Ex. 98.—In a rectangular flume or canal four feet wide, two feet deep, the fall per mile is 4.752 (equivalent to a sine of slope $s=.0009$), and the coefficient of roughness, of whose bed is $n=.012$, what is the velocity of flow per second?

Cal.—It will be noted that Ex. 98 differs from Ex. 97, as relates to coefficient of roughness of the bed only.

Substituting the given value of $n=.012$, the values as found of $x=.536$, $s^{\frac{1}{2}}=.03$, and $r^{\frac{1}{2}}=1$ in Eq. (283),

$$v=\frac{1}{.012}\left(\frac{.536+1.811}{.536+1}\right)\times .03.$$

Reducing, $v=83.33\left(\frac{2.347}{1.536}\right)\times .03=3.82$ feet.—*Ans.*

Ex. 99.—It is required to construct a rectangular canal, whose fall per mile shall be 2.112 feet (equivalent to sine $s=.0004$), and coefficient of roughness of bed $n=.025$, what must be its depth and width, for it to discharge 108.2 cubic feet per second?

Cal.—Let $q=a\,v=108.2$ cubic feet, the discharge per second. (*a*)

Substitute the value of v of (283) in Eq. (a),

$$q=\frac{ar}{n}\left(\frac{x+1.811}{x+r^{\frac{1}{2}}}\right)s^{\frac{1}{2}}. \qquad (b)$$

Arranging terms of Eq. (b) with respect to r,

$$a\,r-\frac{n\,q\,r^{\frac{1}{2}}}{(x+1.811)s^{\frac{1}{2}}}=\frac{n\,q\,x}{(x+1.811)s^{\frac{1}{2}}}. \qquad (c)$$

Substituting the values of s and n in (282),

$$x=\left(41.6+\frac{0.00281}{.0004}\right)\times .025=1.216. \qquad (d)$$

In a rectangular canal, whose depth is d, and width $2\,d$, the hydraulic mean depth is:

$$r=\frac{d}{2}; \qquad (e)$$

$$\text{area, } a=2\,d^2. \qquad (f)$$

Substituting the values of $x=1.216$, $s^{\frac{1}{2}}=.02$, $n=$ 025, $r=\frac{d}{2}$, $a=2\,d^2$, and $q=108.2$ in Eq. (c), and ob-erving that $d^3=d^{\frac{6}{2}}$,

$$d^{\frac{6}{2}}-31.59\,d^{\frac{1}{2}}=54.33. \qquad (g)$$

Solving Eq. (g) by Hutton's method for the resolution of equations of the higher order:

				−31.59	54.33	2.236
2	4	8	16	32.	.82	
2	8	24	64			
				.41	53.51	
4	12	32	80	160.	43.06	
2	12	48	160	54.89		
					10.45	
6	24	80	240	215.3	8.65	
2	16	80	34.5	62.3		
					1.80	
8	40	160	274.5	277.6	1.79	
2	20	12.5	37.1	10.7		
10	60	172.5	311.6	288.3		
2	2.4	13.	39.8	10.8		
12	62.4	185.5	351.4	299.1		
	2.5	13.5	6.4			
	64.9	199.0	357.8			
	2.5	14.				
	67.4	213.				
	2.6	2.				
	70.0	215.				
	2.6					
	72.6					

Thus $d^{\frac{1}{2}}=2.236$. (h)

Squaring, $d=5$ feet depth; whence $2d=10$ feet width.—*Ans.*

Ex. 100.—In a rectangular canal two feet deep, four feet wide, the fall per mile is 4.752 feet, equivalent to sine of slope $s=.0009$, and the observed velocity per second 2.578 feet, what is the value of the coefficient n for the roughness of the bed?

$$Cal.\text{—Let } m = 41.6 + \frac{.00281}{s}. \qquad (a)$$

Substituting m of (a) for its value in (282),

$$x = m\,n. \qquad (b)$$

Substituting $m\,n$ for x in (283), and arranging terms with respect to n,

$$n^2 + \left(\frac{v\,r^{\frac{1}{2}} - r\,s^{\frac{1}{2}}m}{v\,m}\right)n = 1.811\,r\,s^{\frac{1}{2}}. \qquad (c)$$

Substituting the value of $s = .0009$ in Eq. (a),

$$m = 41.6 + \frac{.00281}{.0009} = 44.7. \qquad (d)$$

From the given data, the hydraulic mean depth is found to be:

$$r = 4 \times 2 \div (4 + 2 + 2) = 1. \qquad (e)$$

Substituting in Eq. (c), the values of $m = 44.7$ of Eq. (d), $r = 1$, $r^{\frac{1}{2}} = 1$, $s^{\frac{1}{2}} = .03$, and the given value of $v = 2.578$ feet.

$$n^2 + .01072\,n = .000471. \qquad (f)$$

$$\text{Completing, square } n^2 + \int + \left(.00536\right)^2 = .0004997. \qquad (g)$$

Extracting root and transposing, $n = .017$.—*Ans.*

Ex. 101.—In a rectangular canal two feet deep, four feet wide, the coefficient of roughness of the bed is $n = .012$, the observed velocity of flow 4.946 feet per second, what is the sine of slope?

Cal.—Arranging terms of Eq. (280), with respect to s,

$$s^{\frac{3}{2}}-\left(\frac{41.6\,n^2\,v+nv\,r^{\frac{1}{2}}}{41.6n\,r+1.811r}\right)s^{\frac{2}{2}}+\left(\frac{.00281\,r}{41.6\,n\,r+1.811\,r}\right)s^{\frac{1}{2}}=\frac{.00281\,n^2\,v}{41.6\,n\,r+1.811\,r}. \qquad (a)$$

Substituting the given values of $v=4.946$, $n=.012$, and the value of $r=1$, $r^{\frac{1}{2}}=1$, as found in the preceding example, in Eq. (a):

$$s^{\frac{3}{2}}-.03852\,s^{\frac{2}{2}}+.00001459\,s^{\frac{1}{2}}=.0000008663. \qquad (b)$$

Solving Eq. (b) by Hutton's method for the resolution of cubic equations:

−.03852	+.0000146	+.0000008663	.03872
.03	−.0002556	−.000007230	
−.00852	.0002410	.0000080963	
.03	.0006444	.0000070336	
.02148	.0004034	.0000010647	
.03	.0004758	.0000010306	
.05148	.0008792	.0000000341	
.008	.0005398	.0000000308	
.05948	.0014190	33	
.008	.0000533		
.06748	.0014723		
.008	.0000538		
.07548	.0015261		
.0007	.0000015		
.07618	.0015276		
.0007			
.07688			
.0007			
.07758			

Thus $s^{\frac{1}{2}}=.03872.$ (c)

Squaring, $s=.0015$, sine of slope.—*Ans.*

Remark.—It will be observed that Eq. (*c*) obtained in the solution of Ex. 98, Eq. (*c*) in that of Ex. 99, and Eq. (*a*) in that of Ex. (100), are general and applicable for the solution respectively of all similar problems.

Table 27 has been computed by the author of the present work direct from Eqs. (282) and (283), for the velocity of water in open streams differing in regime and slope, and varying from the size of a small ditch to that of the Mississippi river. The computation has been made for four different values of n, to-wit: $n=.012$, $n=.017$, $n=.025$, and $n=.035$.

(For explanation of the values of n, see Table 26.)

Thus, the hydraulic mean depth being $r=\frac{a}{p}=1$, and the fall per mile F=5.28 feet, or sine of slope $s=.001$, the velocity per second for the respective values of n, as shown by Table 27, are:

When $n=.012$, the velocity is $v=4.028$ feet;
$n=.017$, the velocity is $v=2.720$ feet;
$n=.025$, the velocity is $v=1.755$ feet;
$n=.035$, the velocity is $v=1.190$ feet.

In the several headings of the table, F represents in feet the fall per mile, and s the equivalent sine of slope.

Table 28 has been prepared to be used in connection with Table 27. In the trapezoidal forms of canal beds, considered in this table, the bottom and sides are equal each to each in the same cross section.

Table 29 has been computed to facilitate in finding the wet perimeter of the bed of a trapezoidal canal.

Let b=bottom width;

d=depth of water=depth of bank;

m=ratio of depth to base of bank;

$m\,d$=base of bank;

y=slope of bank;

p=wet perimeter.

$$\text{Then } y=d\,(1+m^2)^{\frac{1}{2}}; \qquad (284)$$

$$p=2\,d\,(1+m^2)^{\frac{1}{2}}+b. \qquad (285)$$

The computed values of the base and bank slope for a unit depth are arranged under their respective headings.

TABLE 27.

Flow of Water per Second in Open Streams, the Coefficient of Roughness of whose beds is n=.012.

Hydraulic Mean Depth, $r=\frac{a}{p}$	F=.0528. s=.0001. Velocity. Feet.	F=.264. s=.0005. Velocity. Feet.	F=.528. s=.0001. Velocity. Feet.	F=1.056. s=.0002. Velocity. Feet.	F=1.584. s=.0003. Velocity. Feet.	F=2.112. s=.0004. Velocity. Feet.	F=2.64. s=.0005. Velocity. Feet.	F=3.168. s=.0006. Velocity. Feet.
.25	.0856	.2627	.4126	.6250	.7864	.9206	1.038	1.144
.3	.1016	.3064	.4780	.7204	.9046	1.058	1.192	1.313
.4	.1329	.3894	.6004	.8989	1.134	1.312	1.477	1.612
.5	.1635	.4676	.7146	1.062	1.325	1.545	1.739	1.912
.6	.1925	.5416	.8213	1.214	1.513	1.762	1.981	2.178
.7	.2226	.6123	.9227	1.359	1.690	1.966	2.209	2.428
.8	.2508	.6804	1.020	1.495	1.856	2.159	2.425	2.664
.9	.2796	.7457	1.111	1.625	2.016	2.343	2.631	2.889
1.	.3073	.8090	1.201	1.751	2.170	2.520	2.828	3.034
1.25	.3752	.9493	1.411	2.044	2.527	2.932	3.287	3.608
1.5	.4408	1.100	1.606	2.316	2.858	3.312	3.711	4.071
2.	.5665	1.359	1.960	2.806	3.453	3.995	4.47[illegible]	4.905
2.5	.6866	1.596	2.281	3.247	3.987	4.606	5.158	5.652
3.	.8017	1.815	2.576	3.651	4.477	5.171	5.783	6.335
3.5	.9127	2.021	2.852	4.121	4.930	5.691	6.362	6.967
4.	.9968	2.216	3.111	4.380	5.356	6.189	6.905	7.561
4.5	1.124	2.402	3.357	4.714	5.759	6.640	7.419	8.122
5.	1.226	2.579	3.590	5.030	6.141	7.077	7.822	8.653
5.5	1.324	2.741	3.814	5.333	6.506	7.495	8.370	9.160
6.	1.421	2.977	4.029	5.624	6.856	7.896	8.615	9.647
6.5	1.516	3.135	4.236	5.903	7.192	8.281	9.244	10.11
7.	1.608	3.290	4.435	6.171	7.515	8.651	9.655	10.56
7.5	1.699	3.443	4.629	6.433	7.831	9.012	10.06	11.00
8.	1.788	3.590	4.817	6.686	8.134	9.359	10.44	11.42
8.5	1.875	3.656	4.909	6.931	8.430	9.695	10.81	11.83
9.	1.961	3.792	5.176	7.169	8.715	10.02	11.18	12.26
9.5	2.046	3.926	5.349	7.402	8.996	10.34	11.54	12.61
10.	2.128	4.056	5.518	7.632	9.268	10.66	11.88	12.99
11.	2.292	4.309	5.846	8.067	9.796	11.25	12.55	13.73
12.	2.450	4.551	6.157	8.485	10.29	11.83	13.19	14.42
13.	2.604	4.785	6.458	8.889	10.78	12.39	13.81	15.09
14.	2.754	5.010	6.748	9.278	11.25	12.92	14.40	15.74
15.	2.900	5.228	7.029	9.652	11.70	13.43	14.97	16.36
16.	3.044	5.439	7.299	10.01	12.13	13.93	15.52	16.97
17.	3.184	5.645	7.563	10.37	12.56	14.42	16.06	17.55
18.	3.322	5.846	7.820	10.71	12.97	14.89	16.58	18.13
19.	3.457	6.166	8.069	11.04	13.37	15.35	17.09	18.68
20.	3.590	6.361	8.313	11.37	13.76	15.79	17.59	19.22
21.	3.720	6.551	8.551	11.68	14.14	16.23	18.07	19.75
22.	3.848	6.737	8.783	12.00	14.51	16.65	18.54	20.26
23.	3.974	6.918	9.010	12.30	14.87	17.06	19.01	....
24.	4.098	7.096	9.233	12.60	15.23	17.47	19.46	....
25.	4.220	7.271	9.451	12.88	15.58	17.87	19.95	....
50.	6.843	10.89	13.95	....	....	....	....	....
100.	10.79	16.07	20.36	....	....	....	....	....

TABLE 27.

Flow of Water per Second in Open Streams, the Coefficient of Roughness of whose beds is n=.012.

Hydraulic Mean Depth, $r=\frac{a}{p}$	F=3.696. s=.0007. Velocity. Feet.	F=4.224. s=.0008. Velocity. Feet.	F=4.752. s=.0009. Velocity. Feet.	F=5.28. s=.001. Velocity. Feet.	F=7.92. s=.0015. Velocity. Feet.	F=10.56. s=.002 Velocity. Feet.	F=26.40. s=.005. Velocity. Feet.	F=52.80. s=.01. Velocity. Feet.
.25	1.241	1.330	1.415	1.494	1.841	2.132	3.391	4.919
.3	1.423	1.526	1.623	1.713	2.110	2.443	3.884	5.504
.4	1.761	1.898	2.007	2.118	2.606	3.017	4.762	6.790
.5	2.072	2.220	2.359	2.489	3.134	3.544	5.625	7.968
.6	2.358	2.526	2.685	2.833	3.403	4.029	6.388	9.053
.7	2.629	2.815	2.991	3.154	3.877	4.485	7.280	10.07
.8	2.883	3.088	3.257	3.459	4.253	4.919	7.794	11.03
.9	3.127	3.348	3.556	3.750	4.605	5.326	8.385	11.95
1.0	3.360	3.596	3.820	4.028	4.946	5.719	9.064	12.83
1.25	3.902	4.176	4.435	4.676	5.738	6.633	10.50	14.87
1.5	4.403	4.710	5.001	5.272	6.468	7.475	11.81	16.74
2.	5.302	5.671	6.018	6.344	7.778	8.986	14.22	20.11
2.5	6.107	6.531	6.851	7.304	8.951	10.33	16.35	23.13
3.	6.843	7.317	7.674	8.180	10.02	11.84	18.30	25.88
3.5	7.525	8.044	8.436	8.992	11.01	12.71	20.10	28.43
4.	8.165	8.727	9.151	9.153	11.94	13.78	21.79	30.81
4.5	8.769	9.372	9.826	10.47	12.82	14.80	23.39	33.07
5.	9.341	9.983	10.46	11.15	13.65	15.75	24.90	35.20
5.5	9.888	10.56	11.20	11.80	14.44	16.67	26.34	37.24
6.	10.41	11.13	11.79	12.44	15.21	17.55	27.73	39.19
6.5	10.91	11.66	12.36	13.02	15.94	18.39	29.05	41.07
7.	11.40	12.17	12.91	13.60	16.64	19.20	30.32	42.86
7.5	11.87	12.68	13.44	14.16	17.32	19.99	31.57	44.62
8.	12.32	13.16	13.96	14.70	17.98	20.75	32.77	46.31
8.5	12.76	13.63	14.45	15.23	18.62	21.43	33.93	......
9.	13.19	14.09	14.94	15.73	19.24	22.20	35.05	......
9.5	13.61	14.54	15.41	16.23	19.85	22.90	......	......
10.	14.02	14.97	15.87	16.71	20.44	23.58	......	......
11.	14.81	15.81	16.76	17.64	21.58	......	......	......
12.	15.56	16.61	17.61	18.54	22.67	......	......	......
13.	16.28	17.38	18.42	19.41	......	......	......	......
14.	16.97	18.13	19.21	20.28	......	......	......	......
15.	17.64	18.84	19.97	......	......	......	......	......
16.	18.29	19.53	20.71	......	......	......	......	......
17.	18.93	28.21	......	......	......	......	......	......
18.	19.54	......	......	......	......	......	......	......
19.	20.14	......	......	......	......	......	......	......

TABLE 27.

Flow of Water per Second in Open Streams, the Coefficient of Roughness of whose beds is n=.017.

Hydraulic Mean Depth, $r=\frac{a}{p}$	F=.0528. s=.00001. Velocity. Feet.	F=.264. s=.00005. Velocity. Feet.	F=.528. s=.0001. Velocity. Feet.	F=1.056. s=.0002. Velocity. Feet.	F=1.584. s=.0003. Velocity. Feet.	F=2.112. s=.0004 Velocity. Feet.	F=2.64. s=.0005. Velocity. Feet.	F=3.168. s=.0006. Velocity. Feet
.25	.0567	.1704	.2615	.3956	.5004	.5844	.6597	.7288
.3	.0675	.2000	.3051	.4595	.5800	.6769	.7635	.8412
.4	.0887	.2568	.3877	.5796	.7296	.8499	.9577	1.054
.5	.1095	.3110	.4657	.6922	.8691	1.011	1.138	1.256
.6	.1301	.3628	.5272	.7979	.9997	1.162	1.307	1.438
.7	.1503	.4127	.6101	.8985	1.123	1.305	1.467	1.612
.8	.1702	.4612	.6778	.9951	1.239	1.441	1.619	1.781
.9	.1892	.5079	.7433	1.086	1.355	1.572	1.766	1.986
1.	.2093	.5535	.8066	1.176	1.465	1.698	1.907	2.095
1.25	.2569	.6625	.9565	1.386	1.722	1.989	2.237	2.456
1.5	.3034	.7659	1.097	1.581	1.957	2.269	2.544	2.792
2.0	.3934	.9581	1.356	1.939	2.398	2.768	3.109	3.402
2.5	.4802	1.136	1.592	2.338	2.793	3.221	3.605	3.952
3.	.5629	1.302	1.812	2.563	3.157	3.808	4.069	4.458
3.5	.6457	1.460	2.018	2.843	3.496	4.025	4.506	4.929
4.	.7253	1.609	2.213	3.100	3.816	4.390	4.906	5.372
4.5	.8030	1.753	2.455	3.358	4.119	4.735	5.291	5.792
5.	.8590	1.890	2.576	3.594	4.407	5.064	5.657	6.191
5.5	.9533	2.023	2.745	3.824	4.683	5.379	6.007	6.574
6.	1.026	2.101	2.910	4.044	4.948	5.682	6.344	6.941
6.5	1.098	2.275	3.068	4.256	5.204	5.973	6.667	7.134
7.	1.168	2.394	3.220	4.460	5.450	6.253	6.978	7.633
7.5	1.238	2.512	3.361	4.659	5.690	6.526	7.282	7.964
8.	1.307	2.582	3.433	4.851	5.921	6.790	7.575	8.280
8.5	1.373	2.740	3.570	5.038	6.147	7.047	7.861	8.595
9.	1.439	2.845	3.790	5.220	6.365	7.296	8.137	8.896
9.5	1.505	2.951	3.923	5.397	6.579	7.540	8.407	9.191
10.	1.569	3.055	4.054	5.444	6.788	7.777	8.673	9.479
11.	1.696	3.256	4.307	6.044	7.192	8.256	9.182	10.03
12.	1.820	3.519	4.539	6.226	7.572	8.878	9.669	10.56
13.	1.939	3.638	4.783	6.537	7.950	9.100	10.14	11.08
14.	2.059	3.819	5.009	6.680	8.309	9.487	10.59	11.57
15.	2.175	3.994	5.257	7.125	8.654	9.901	11.03	12.05
16.	2.289	4.165	5.438	7.404	8.990	10.28	11.45	12.51
17.	2.401	4.331	5.644	7.675	9.317	10.65	11.86	12.96
18.	2.511	4.493	5.846	7.940	9.635	11.01	12.26	13.39
19.	2.619	4.651	6.181	8.196	9.943	11.36	12.65	13.82
20.	2.726	4.806	6.231	8.448	10.24	11.71	13.03	14.23
21.	2.836	4.957	6.417	8.694	10.54	12.04	13.40	14.64
22.	2.935	5.096	6.598	8.933	10.82	12.37	13.77	15.03
23.	3.036	5.249	6.777	9.273	11.11	12.69	14.12	15.42
24.	3.137	5.391	6.952	9.397	11.38	13.00	14.47	15.80
25.	3.237	5.531	7.123	9.622	11.65	13.31	14.81	16.17
50.	5.402	8.439	10.67	14.27	17.22	19.64	21.83	....
100.	8.766	12.64	15.75	20.90	....	....	....	....

TABLE 27.

Flow of Water per Second in Open Streams, the Coefficient of Roughness of whose beds is n=.017.

Hydraulic Mean Depth, $r=\frac{a}{p}$	F=3.696. s=.0007. Velocity. Feet.	F=4.224. s=.0008. Velocity. Feet.	F=4.752. s=.0009. Velocity. Feet.	F=5.28. s=.001. Velocity. Feet.	F=7.92. s=.0015. Velocity. Feet.	F=10.56. s=.002. Velocity. Feet.	F=26.4. s=.005. Velocity. Feet.	F=52.8. s=.01. Velocity. Feet.
.25	.7893	.8463	.9002	.9508	1.172	1.358	2.160	3.061
.3	.9128	.9785	1.039	1.074	1.354	1.568	2.494	3.534
.4	1.143	1.200	1.303	1.375	1.693	1.961	3.116	4.414
.5	1.358	1.455	1.546	1.632	2.009	2.325	3.692	5.230
.6	1.558	1.708	1.773	1.872	2.302	2.664	4.228	5.988
.7	1.748	1.871	1.988	2.098	2.580	2.985	4.736	6.707
.8	1.928	2.064	2.193	2.314	2.844	3.290	5.219	7.474
.9	2.102	2.249	2.445	2.521	3.098	3.568	5.680	8.042
1.	2.268	2.427	2.578	2.720	3.344	3.863	6.124	8.669
1.25	2.658	2.654	3.020	3.185	3.911	4.521	7.163	10.14
1.5	3.020	3.231	3.430	3.618	4.543	5.130	8.143	11.44
2.	3.678	3.931	4.174	4.401	5.347	6.540	9.869	13.96
2.5	4.271	4.567	4.844	5.108	6.260	7.231	11.44	16.18
3.	4.817	5.149	5.462	5.758	7.053	8.145	12.88	18.22
3.5	5.324	5.823	6.036	6.362	7.790	8.995	14.22	20.11
4.	5.802	6.200	6.526	6.931	8.484	9.794	15.48	21.89
4.5	6.255	6.683	7.086	7.469	9.141	10.55	16.67	23.57
5.	6.684	7.141	7.572	7.979	9.764	11.26	17.81	25.17
5.5	7.096	7.583	8.036	8.469	10.36	11.95	18.88	26.70
6.	7.491	8.003	8.483	8.775	10.93	12.61	19.93	28.17
6.5	7.871	8.408	8.912	9.390	11.48	13.25	20.93	29.48
7.	8.212	8.797	9.324	9.824	12.01	13.86	21.88	30.94
7.5	8.594	9.178	9.683	10.25	12.53	14.45	22.82	32.26
8.	8.938	9.545	10.12	10.66	13.01	15.03	23.73	33.54
8.5	9.166	9.902	10.49	11.05	13.51	15.23	24.61	....
9.	9.597	10.25	10.86	11.44	13.98	16.13	25.46	....
9.5	9.914	10.59	11.22	11.82	14.44	16.66	....	....
10.	10.22	10.91	11.57	12.18	14.89	17.18	....	....
11.	10.82	11.55	12.24	12.93	15.76	18.18	....	....
12.	11.39	12.16	12.89	13.57	16.58	19.12	....	....
13.	11.95	12.75	13.51	14.23	17.38	20.00	....	....
14.	12.48	13.32	14.11	14.86	18.10	....	....	....
15.	13.99	13.86	14.69	15.47	18.89	....	....	....
16.	13.49	14.36	15.25	16.06	19.61	....	....	....
17.	13.97	14.91	15.79	16.64	20.31	....	....	....
18.	14.44	15.41	16.32	17.19	....	....	....	....
19.	14.89	15.90	16.80	17.73	....	....	....	....
20.	15.34	16.33	17.34	18.26	....	....	....	....
21.	15.78	16.82	17.83	18.78	....	....	....	....
22.	16.20	17.29	18.31	19.28	....	....	....	....
23.	16.62	17.73	18.78	19.77	....	....	....	....
24.	17.03	18.17	19.24	....	....	....	....	....
25.	17.43	18.59	19.69	....	....	....	....	....

TABLE 27.

Flow of Water per Second in Open Streams, the Coefficient of Roughness of whose beds is n=.025.

Hydraulic Mean Depth, $r=\frac{a}{p}$	F=.0528. s=.00001. Velocity. Feet.	F=.264. s=.00005. Velocity. Feet.	F=.528. s=.0001. Velocity. Feet.	F=1.056. s=.0002. Velocity. Feet.	F=1.584. s=.0003. Velocity. Feet.	F=2.112. s=.0004. Velocity. Feet.	F=2.64. s=.1005. Velocity. Feet.	F=3.168. s=.0006. Velocity. Feet.
.25	.0365	.1022	.1584	.2394	.3031	.3528	.3978	.4387
.3	.0435	.1206	.1862	.2802	.3519	.4142	.4642	.5117
.4	.0624	.1564	.2394	.3580	.4483	.5238	.5899	.6498
.5	.0712	.1909	.2901	.4316	.5393	.6296	.7086	.7800
.6	.0848	.2243	.3388	.5017	.6258	.7296	.8208	.9027
.7	.0982	.2567	.3858	.5691	.7089	.8256	.9282	1.020
.8	.1115	.2884	.4311	.6341	.7887	.9180	1.031	1.134
.9	.1247	.3192	.4761	.6966	.8653	1.006	1.130	1.243
1.	.1378	.3494	.5183	.7576	.9400	1.093	1.226	1.348
1.25	.1662	.4223	.6211	.9025	1.117	1.297	1.457	1.598
1.5	.2017	.4921	.7186	1.039	1.283	1.488	1.669	1.832
2.	.2636	.6239	.9006	1.291	1.590	1.841	2.062	2.264
2.5	.3235	.7134	1.079	1.514	1.872	2.164	2.422	2.655
3.	.3826	.8641	1.227	1.739	2.132	2.464	2.755	3.020
3.5	.4399	.9758	1.377	1.943	2.303	2.746	3.068	3.361
4.	.4965	1.083	1.519	2.136	2.610	3.010	3.371	3.684
4.5	.5518	1.214	1.656	2.321	2.832	3.264	3.645	3.993
5.	.6062	1.290	1.786	2.496	3.045	3.506	3.921	4.279
5.5	.6600	1.382	1.913	2.666	3.247	3.738	4.161	4.566
6.	.7128	1.476	2.034	2.831	3.443	3.962	4.420	4.838
6.5	.7650	1.567	2.153	2.988	3.634	4.180	4.662	5.100
7.	.8165	1.655	2.267	3.141	3.815	4.388	4.893	5.352
7.5	.8671	1.741	2.379	3.291	3.996	4.592	5.121	5.599
8.	.9202	1.826	2.488	3.433	4.169	4.788	5.340	5.840
8.5	.9607	1.909	2.594	3.575	4.337	4.982	5.552	6.070
9.	1.016	1.990	2.697	3.712	4.500	5.168	5.760	6.295
9.5	1.065	2.069	2.790	3.846	4.661	5.352	5.961	6.518
10.	1.113	2.146	2.898	3.978	4.818	5.530	6.160	6.734
11.	1.208	2.353	3.091	4.357	5.101	6.056	6.568	7.131
12.	1.299	2.444	3.276	4.491	5.413	6.208	6.911	7.552
13.	1.392	2.583	3.455	4.714	5.695	6.530	7.267	7.939
14.	1.482	2.724	3.629	4.941	5.967	6.838	7.612	8.311
15.	1.569	2.793	3.797	5.162	6.228	7.136	7.942	8.671
16.	1.658	2.988	3.960	5.375	6.484	7.426	8.262	9.021
17.	1.742	3.115	4.127	5.585	6.732	7.708	8.575	9.362
18.	1.828	3.241	4.278	5.788	6.975	7.986	8.879	9.695
19.	1.915	3.362	4.419	5.985	7.209	8.272	9.186	10.02
20.	1.992	3.480	4.514	6.178	7.439	8.532	9.488	10.33
21.	2.074	3.598	4.719	6.368	7.647	8.770	9.749	10.68
22.	2.155	3.712	4.862	6.553	7.884	9.020	10.02	10.95
23.	2.234	3.824	5.000	6.734	8.018	9.264	10.30	11.24
24.	2.312	3.934	5.150	6.911	8.310	9.502	10.56	11.52
25.	2.390	4.052	5.270	7.085	8.516	9.738	10.82	11.81
50.	4.127	6.325	8.064	10.70	12.80	14.61	16.21	17.67
100.	6.916	9.673	12.00	15.89	18.94	21.58	....	....

TABLE 27.

Flow of Water per Second in Open Streams, the Coefficient of Roughness of whose beds is n=.025.

Hydraulic Mean Depth, $r = \frac{a}{p}$	F=3.696. s=.0007. Velocity. Feet.	F=4.224. s=.0008. Velocity. Feet.	F=4.752. s=.0009. Velocity. Feet.	F=5.28. s=.001. Velocity. Feet.	F=7.92. s=.0015. Velocity. Feet.	F=10.56. s=.002. Velocity. Feet.	F=26.4. s=.005. Velocity. Feet.	F=52.8. s=.01. Velocity. Feet.
.25	.4759	.5116	.5457	.5765	.7072	.8197	1.301	1.847
.3	.5551	.5951	.6330	.6847	.8238	.9544	1.515	2.150
.4	.7045	.7548	.8028	.8491	1.044	1.209	1.918	2.722
.5	.8318	.8986	.9630	1.019	1.254	1.450	2.299	3.259
.6	.9787	1.048	1.114	1.177	1.446	1.674	2.655	3.765
.7	1.106	1.184	1.259	1.330	1.633	1.890	2.996	4.248
.8	1.228	1.315	1.398	1.477	1.813	2.097	3.324	4.711
.9	1.345	1.440	1.530	1.616	1.984	2.295	3.637	5.184
1.	1.459	1.564	1.664	1.755	2.151	2.488	3.941	5.585
1.25	1.729	1.855	1.980	2.092	2.545	2.943	4.660	6.600
1.5	1.981	2.127	2.265	2.383	2.914	3.368	5.333	7.551
2.	2.446	2.621	2.787	2.934	3.590	4.143	6.418	9.290
2.5	2.804	3.035	3.261	3.434	4.206	4.861	8.693	10.90
3.	3.262	3.487	3.702	3.899	4.775	5.514	8.719	12.34
3.5	3.630	3.880	4.113	4.237	5.310	6.131	9.694	13.71
4.	3.979	4.248	4.503	4.746	5.817	6.712	10.61	15.01
4.5	4.310	4.599	4.872	5.139	6.297	7.272	11.49	16.24
5.	4.625	4.936	5.223	5.509	6.754	7.795	12.32	17.41
5.5	4.929	5.255	5.559	5.866	7.192	8.300	13.11	18.53
6.	5.220	5.566	5.886	6.211	7.614	8.788	13.89	19.62
6.5	5.503	5.864	6.198	6.543	8.025	9.257	14.62	20.66
7.	5.773	6.152	6.501	6.862	8.416	9.709	15.33	21.67
7.5	6.640	6.432	6.795	7.178	8.803	10.15	16.03	22.65
8.	6.297	6.703	7.080	7.485	9.194	10.59	16.71	23.60
8.5	6.545	6.966	7.356	7.773	9.527	10.91	17.16	24.13
9.	6.789	7.224	7.626	8.054	9.884	11.40	17.96	25.42
9.5	7.001	7.461	7.890	8.332	10.23	11.79	18.62	....
10.	7.260	7.721	8.148	8.601	10.53	12.18	19.24	....
11.	7.948	8.454	8.919	9.306	11.21	12.89	20.39	....
12.	8.141	8.655	9.126	9.642	11.83	13.64	21.49	....
13.	8.556	9.090	9.585	10.13	12.43	14.33	22.62	....
14.	8.937	9.503	10.03	10.59	13.02	15.01	....	....
15.	9.318	9.982	10.46	11.05	13.57	15.64	....	....
16.	9.723	10.32	10.87	11.49	14.11	16.27	....	....
17.	10.09	10.71	11.28	11.92	14.64	16.88	....	....
18.	10.44	11.08	11.67	12.33	15.16	17.47	....	....
19.	10.79	11.45	12.05	12.75	15.66	18.05	....	....
20.	11.10	11.81	12.43	13.14	16.15	18.61	....	....
21.	11.46	12.16	12.80	13.53	16.61	19.16	....	....
22.	11.78	12.48	13.15	13.91	17.09	19.70	....	....
23.	12.10	12.82	13.50	14.27	17.55	20.22	....	....
24.	12.41	13.15	13.84	14.53	17.99	....	....	....
25.	12.72	13.48	14.18	15.00	18.43	....	....	....
50.	19.01	20.13	....	....	....	....	....	....

TABLE 27.

Flow of Water per Second in Open Streams, the Coefficient of Roughness of whose beds is n=.035.

Hydraulic Mean Depth, $r=\frac{a}{p}$	F=.0528. s=.0001. Velocity. Feet.	F=.264. s=.0005. Velocity. Feet.	F=.528. s=.0001. Velocity. Feet.	F=1.056. s=.0002. Velocity. Feet.	F=1.584. s=.0003. Velocity. Feet.	F=2.112. s=.0004. Velocity. Feet.	F=2.64. s=.0005. Velocity. Feet.	F=3.168. s=.0006. Velocity. Feet.
.25	.0251	.0674	.1033	.1551	.1947	.2279	.2565	.2832
.3	.0300	.0799	.1219	.1825	.2288	.2676	.3011	.3322
.4	.0397	.1040	.1545	.2353	.2944	.3439	.3867	.4215
.5	.0493	.1280	.1929	.2860	.3571	.4166	.4682	.5160
.6	.0588	.1546	.2266	.3347	.4171	.4862	.5462	.6015
.7	.0683	.1737	.2594	.3817	.4752	.5547	.6215	.6841
.8	.0771	.1959	.2944	.4274	.5313	.6184	.6941	.7628
.9	.0870	.2177	.3225	.4718	.5858	.6815	.7648	.8412
1.	.0963	.2391	.3555	.5152	.6389	.7429	.8334	.9165
1.25	.1192	.2911	.4267	.6212	.7663	.8898	.9976	1.096
1.5	.1418	.3413	.4972	.7521	.8872	1.029	1.153	1.266
2.	.2160	.4372	.6303	.9035	1.112	1.288	1.442	1.583
2.5	.2298	.5283	.7542	1.076	1.322	1.528	1.711	1.875
3.	.2727	.6297	.8734	1.296	1.518	1.753	1.961	2.100
3.5	.3147	.6991	.9861	1.392	1.704	1.966	2.198	2.407
4.	.3562	.7800	1.094	1.539	1.881	2.169	2.423	2.653
4.5	.3971	.8583	1.198	1.680	2.050	2.363	2.638	2.888
5.	.4374	.9343	1.296	1.815	2.213	2.543	2.846	3.113
5.5	.4774	1.008	1.396	1.946	2.370	2.728	3.045	3.331
6.	.5168	1.081	1.490	2.075	2.464	2.902	3.238	3.540
6.5	.5558	1.151	1.583	2.195	2.669	3.069	3.425	3.744
7.	.5944	1.219	1.671	2.314	2.811	3.232	3.605	3.940
7.5	.6327	1.287	1.759	2.431	2.971	3.391	3.782	4.132
8.	.6706	1.353	1.802	2.544	3.086	3.545	3.953	4.319
8.5	.7080	1.418	1.927	2.655	3.218	3.695	4.120	4.501
9.	.7454	1.482	2.009	2.763	3.347	3.843	4.284	4.678
9.5	.7823	1.544	2.089	2.868	3.473	3.986	4.444	4.851
10.	.8189	1.605	2.168	2.972	3.597	4.127	4.600	5.022
11.	.8911	1.726	2.315	3.173	3.837	4.401	4.904	5.351
12.	.9625	1.842	2.463	3.368	4.068	4.663	5.195	5.668
13.	1.033	1.958	2.612	3.556	4.204	4.917	5.477	5.974
14.	1.102	2.063	2.751	3.738	4.508	5.163	5.750	6.271
15.	1.171	2.174	2.886	3.914	4.717	5.401	6.014	6.557
16.	1.238	2.279	3.017	4.086	4.921	5.633	6.271	6.836
17.	1.305	2.382	3.146	4.253	5.120	5.858	6.500	7.108
18.	1.372	2.483	3.271	4.489	5.314	6.079	6.765	7.373
19.	1.437	2.582	3.394	4.576	5.502	6.293	7.003	7.631
20.	1.501	2.679	3.514	4.732	5.688	6.503	7.235	7.883
21.	1.569	2.774	3.632	4.885	5.869	6.709	7.463	8.131
22.	1.629	2.867	3.747	5.033	6.046	6.909	7.682	8.372
23.	1.692	2.951	3.861	5.180	6.228	7.105	7.903	8.607
24.	1.754	3.049	3.972	5.324	6.398	7.299	8.118	8.840
25.	1.816	3.139	4.081	5.465	6.556	7.488	8.327	9.068
50.	3.223	5.038	6.385	8.420	10.04	11.44	12.70	13.81
100.	5.825	7.878	9.991	12.71	1 .10	17.15	19.03	20.68

TABLE 27.

Flow of Water per Second in Open Streams, the Coefficient of Roughness of whose beds is n=.035.

Hydraulic Mean Depth, $r=\frac{a}{p}$	F=3.696. s=.0007. Velocity. Feet.	F=4.224. s=.0008. Velocity. Feet.	F=4.752. s=.0009. Velocity. Feet.	F=5.28. s=.001. Velocity. Feet.	F=7.92. s=.0015. Velocity. Feet.	F=10.56. s=.002. Velocity. Feet.	F=26.4. s=.005. Velocity. Feet.	F=52.8. s=.01. Velocity. Feet.
.25	.3071	.3294	.3503	.3700	.4562	.5163	.8401	1.190
.3	.3603	.3365	.4108	.4339	.5345	.6191	.9843	1.395
.4	.4621	.4954	.5266	.5561	.6846	.7927	1.259	1.784
.5	.5590	.5992	.6368	.6723	.8273	.9577	1.521	2.155
.6	.6517	.6983	.7419	.7832	.9634	1.115	1.770	2.507
.7	.7498	.7937	.8241	.8901	1.094	1.266	2.010	2.846
.8	.8270	.8858	.9411	.9932	1.221	1.412	2.240	3.173
.9	.9107	.9753	1.036	1.097	1.343	1.555	2.464	3.488
1.	.9920	1.062	1.128	1.190	1.462	1.691	2.682	3.797
1.25	1.186	1.270	1.348	1.422	1.734	2.019	3.200	4.529
1.5	1.370	1.466	1.556	1.641	2.015	2.329	3.689	5.221
2.	1.672	1.830	1.943	2.049	2.512	2.903	4.596	6.502
2.5	2.026	2.167	2.299	2.424	2.972	3.433	5.430	7.682
3.	2.393	2.482	2.633	2.650	3.400	3.927	6.201	8.783
3.5	2.600	2.779	2.947	3.107	3.804	4.393	6.945	9.820
4.	2.865	3.061	3.247	3.422	4.188	4.836	7.642	10.80
4.5	3.118	3.331	3.543	3.723	4.556	5.270	8.307	11.74
5.	3.361	3.590	3.807	4.011	4.907	5.663	8.945	12.65
5.5	3.594	3.840	4.070	4.289	5.246	5.928	9.559	13.57
6.	3.820	4.081	4.325	4.557	5.573	6.429	10.15	14.35
6.5	4.039	4.314	4.572	4.817	5.889	6.793	10.72	15.16
7.	4.250	4.539	4.810	5.067	6.194	7.145	11.28	15.94
7.5	4.459	4.759	5.044	5.313	6.493	7.490	11.81	16.70
8.	4.657	4.973	5.269	5.551	6.783	7.823	12.34	17.44
8.5	4.853	5.181	5.490	5.783	7.065	8.148	12.85	18.16
9.	5.044	5.372	5.705	6.008	7.340	8.475	13.05	18.87
9.5	5.231	5.584	5.916	6.230	7.610	8.775	13.84	19.56
10.	5.413	5.778	6.121	6.447	7.855	9.079	14.32	20.23
11.	5.781	6.155	6.521	6.867	8.385	9.668	15.24	....
12.	6.108	6.504	6.904	7.269	8.674	10.47	16.13	....
13.	6.438	6.740	7.227	7.661	9.538	10.78	17.03	....
14.	6.757	7.209	7.635	8.038	9.811	11.30	17.80	....
15.	7.064	7.537	7.982	8.403	10.25	11.82	18.62	....
16.	7.365	7.856	8.320	8.759	10.69	12.31	19.40	....
17.	7.657	8.167	8.648	9.104	11.11	12.80	20.16	....
18.	7.942	8.471	8.970	9.442	11.52	13.27	....	....
19.	8.031	8.766	9.281	9.769	11.92	13.71	....	....
20.	8.489	8.849	9.586	10.09	12.30	14.18	....	....
21.	8.755	9 125	9.886	10.43	12.69	14.61	....	....
22.	9.015	9.614	10.18	10.71	13.06	15.05	....	....
23.	9.263	9.884	10.46	11.01	13.43	15.47	....	....
24.	9.518	10.15	10.74	11.31	13.78	15.88	....	....
25.	9.763	10.41	11.02	11.60	14.14	16.28	....	....
50.	14.86	15.84	16.75	17.62	21.43	....	....	....

TABLE 28.

Dimensions of Water Ways Corresponding to Their Given Hydraulic Mean Depths.

Hydraulic Mean D'pth $r=\frac{a}{p}$	Square. 1:1		Rectangle. 1:2			Triangle. 90°		Semi-circle.	
	Side. Feet.	Area. Sq. Ft.	D'pth Ft.	Width Ft.	Area. Sq. Ft.	Side. Feet.	Area. Sq. Ft.	Diam'tr Feet.	Area. Sq. Ft.
.25	.75	.563	.5	1.0	.5	1.	.5	1.	.393
.3	.9	.81	.6	1.2	.72	1.2	.72	1.2	.565
.4	1.2	1.44	.8	1.6	1.28	1.6	1.28	1.6	1.005
.5	1.5	2.25	1.0	2.0	2.0	2.0	2.00	2.	1.571
.6	1.8	3.24	1.2	2.4	2.88	2.4	2.88	2.4	2.262
.7	2.1	4.41	1.4	2.8	3.92	2.8	3.92	2.8	3.079
.8	2.4	5.76	1.6	3.2	5.12	3.2	5.12	3.2	4.021
.9	2.7	7.29	1.8	3.6	6.48	3.6	6.48	3.6	5.089
1.	3.	9.00	2.0	4.0	8.00	4.	8.00	4.	6.283
1.25	3.75	14.06	2.5	5.0	12.5	5.	12.5	5.	9.818
1.5	4.5	20.25	3.0	6.0	18.0	6.	18.0	6.	14.137
2.	6.	36.	4.0	8.0	32.0	8.	32.0	8.	25.133
2.5	7.5	56.25	5.0	10.0	50.0	10.	50.0	10.	39.27
3.	9.	81.	6.0	12.0	72.0	12.	72.0	12.	56.549
3.5	10.5	110.25	7.0	14.0	98.0	14.	98.	14.	76.696
4.	12.	144.00	8.	16.0	128.	16.	128.	16.	100.53
4.5	13.5	182.25	9.	18.0	162.	18.	162.	18.	127.23
5.	15.	225.00	10.	20.0	200.	20.	200.	20.	157.08
5.5	16.5	272.25	11.	22.0	242.	22.	242.	22.	190.07
6.	18.	324.00	12.	24.0	288.	24.	288.	24.	226.2
6.5	19.5	380.25	13.	26.0	338.	26.	338.	26.	265.5
7.	21.	441.00	14.	28.0	392.	28.	392.	28.	307.0
7.5	22.5	506.25	15.	30.0	450.	30.	450.	30.	353.4
8.	24.	576.00	16.	32.0	512.	32.	512.	32.	402.1
9.	27.	729.00	18.	36.0	648.	36.	648.	36.	508.9
10.	30.	900.00	20.	40.0	800.	40.	800.	40.	628.3

TABLE 28.

Dimensions of Water Ways Corresponding to Their Given Hydraulic Depths.

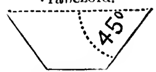
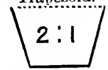
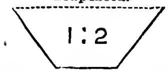
Hydraulic Mean Depth, $r=\frac{a}{p}$	Semi-Hexagon. 			Trapezoid. 			Trapezoid. 			Trapezoid. 		
	Side. Feet.	Depth. Feet.	Area. Sq. Ft.	Side. Feet.	Depth. Feet.	Area. Sq. Ft.	Side. Feet.	Depth. Feet.	Area. Sq. Ft.	Side. Feet.	Depth. Feet.	Area. Sq. Ft.
.25	.578	.5	.433	.621	.439	.466	.579	.518	.435	.885	.396	.664
.3	.693	.6	.624	.746	.527	.671	.695	.622	.626	1.067	.475	.955
.4	.924	.8	1.109	.994	.703	1.193	.927	.829	1.112	1.416	.633	1.699
.5	1.155	1.	1.732	1.243	.879	1.864	1.159	1.036	1.738	1.775	.792	2.654
.6	1.386	1.2	2.494	1.491	1.054	2.685	1.391	1.243	2.503	2.125	.950	3.822
.7	1.617	1.4	3.395	1.740	1.230	3.133	1.622	1.451	3.407	2.479	1.109	5.202
.8	1.848	1.6	4.434	1.988	1.406	4.772	1.854	1.658	4.450	2.833	1.267	7.194
.9	2.078	1.8	5.612	2.237	1.582	6.040	2.086	1.866	5.632	3.187	1.425	8.599
1.0	2.309	2.0	6.928	2.485	1.757	7.457	2.318	2.073	6.953	3.541	1.584	10.62
1.25	2.887	2.5	10.825	3.107	2.197	11.651	2.897	2.591	10.86	4.426	1.980	16.59
1.5	3.464	3.	15.588	3.728	2.636	16.777	3.476	3.109	15.64	5.312	2.375	23.88
2.0	4.619	4.	27.713	4.971	3.515	29.828	4.635	4.146	27.81	7.082	3.167	42.46
2.5	5.775	5.	43.301	6.213	4.594	46.606	5.794	5.182	43.46	9.853	3.959	66.35
3.0	6.928	6.	62.354	7.456	5.272	67.113	6.952	6.219	62.58	10.62	4.751	95.54
3.5	8.083	7.	84.870	8.698	6.152	91.348	8.110	7.255	85.17	12.39	5.543	130.04
4.	9.238	8.	110.85	9.941	7.031	119.31	9.268	8.292	111.25	14.16	6.334	169.86
4.5	10.30	9.	140.40	11.184	7.909	151.00	10.43	9.328	140.8	15.93	7.126	214.97
5.	11.55	10.	173.20	12.427	8.787	186.5	11.59	10.36	173.8	17.71	7.918	265.4
5.5	12.70	11.	209.58	13.670	9.666	225.6	12.75	11.40	210.3	19.48	8.709	321.13
6.	13.86	12.	249.42	14.913	10.[illegible]44	263.5	13.91	12.44	250.3	21.25	9.501	382.18
6.5	15.01	13.	297.72	16.155	11.423	315.1	15.06	13.47	293.8	23.02	10.29	448.53
7.	16.17	14.	339.48	17.398	12.302	365.4	16.22	14.51	340.7	24.79	11.09	520.18
7.5	17.32	15.	389.71	18.641	13.181	419.5	17.38	15.55	391.1	26.56	11.89	597.15
8.	18.48	16.	443.40	19.883	14.059	477.2	18.54	16.58	445.0	28.33	12.68	679.42
9.	20.78	18.	561.18	22.368	16.817	604.0	20.86	18.66	563.2	31.87	14.26	859.9
10.	23.09	20.	692.82	24.853	17.574	745.7	23.18	20.73	695.3	55.41	15.84	1061.6

TABLE. 29.

Relations of Depth, Base and Slope of a Bank of a Trapezoidal Canal.

Ratio of Depth to Base.	1 Depth.	m Base.	$(1 \times m^2)^{\frac{1}{2}}$ Slope.	Ratio of Depth to Base.	1 Depth.	m Base.	$(1+m^2)^{\frac{1}{2}}$ Slope.
5:1	1	.2000	1.0198	19:4	1	.2105	1.0219
24:5	1	.2083	1.0215	9:2	1	.2222	1.0244
23:5	1	.2174	1.0234	17:4	1	.2353	1.0272
22:5	1	.2273	1.0255	15:4	1	.2667	1.0349
21:5	1	.2381	1.0280	7:2	1	.2856	1.0400
4:1	1	.2500	1.0306	13:4	1	.3077	1.0463
19:5	1	.2632	1.0364	11:4	1	.3636	1.0641
18:5	1	.2778	1.0379	5:2	1	.4000	1.0770
17:5	1	.2941	1.0423	9:4	1	.4444	1.0940
16:5	1	.3125	1.0477	7:4	1	.5714	1.1517
3:1	1	.3333	1.0541	3:2	1	.6667	1.2046
14:5	1	.3571	1.0619	5:4	1	.8000	1.2806
13:5	1	.3846	1.0714	3:4	1	1.3333	1.6667
12:5	1	.4167	1.0833	1:2	1	2.0000	2.2361
11:5	1	.4545	1.0985	1:4	1	4.0000	4.1231
2:1	1	.5000	1.1180	14:3	1	.2143	1.0227
9:5	1	.5556	1.1439	13:3	1	.2308	1.0263
8:5	1	.6250	1.1793	11:3	1	.2727	1.0365
7:5	1	.7143	1.2575	10:3	1	.3000	1.0440
6:5	1	.8333	1.3017	8:3	1	.3750	1.0680
1:1	1	1.000	1.4142	7:3	1	.4286	1.0880
4:5	1	1.250	1.6008	5:3	1	.6000	1.1662
3:5	1	1.667	1.9436	4:3	1	.7500	1.2500
2:5	1	2.500	2.6926	2:3	1	1.5000	1.8028
1:5	1	5.000	5.0990	1:3	1	3.0000	3.1622
5:24	1	4.8000	4.9031	4:19	1	4.7500	4.8541
5:23	1	4.6000	4.7074	4:18	1	4.5000	4.6098
5:22	1	4.4000	4.5122	4:17	1	4.2500	4.3660
5:21	1	4.2000	4.3174	4:15	1	3.7500	3.8810
5:19	1	3.8000	3.9295	4:14	1	3.5000	3.6401
5:18	1	3.6000	3.7363	4:13	1	3.2500	3.4004
5:17	1	3.4000	3.5440	4:11	1	2.7500	2.9262
5:16	1	3.2000	3.3526	4:9	1	2.2500	2.4622
5:14	1	2.8000	2.9732	4:7	1	1.7500	2.0156
5:13	1	2.6000	2.7851	4:6	1	1.5000	1.8028
5:12	1	2.4000	2.6000	3:14	1	4.6667	4.7726
5:11	1	2.2000	2.4161	3:13	1	4.3333	4.4472
5:9	1	1.8000	2.0591	3:11	1	3.6666	3.8006
5:8	1	1.6000	1.8868	3:10	1	3.3333	3.4801
5:7	1	1.4000	1.7205	3:8	1	2.6667	2.8480
5:6	1	1.2000	1.5621	3:7	1	2.3333	2.5386

PRACTICAL APPLICATION OF TABLES 27, 28 AND 29, TO DETERMINE THE VELOCITY AND DISCHARGE OF AN OPEN STREAM OF WATER.

Rule 56.—From the given dimensions of the stream find by Table 28 or Table 29, according to the conditions, the hydraulic mean depth. Turn then to Table 27 for the given coefficient n, for roughness of bed. In this table, opposite the "hydraulic mean depth" as determined, in the column headed by the given fall per mile, will be found the velocity sought. Multiply the velocity so found by the area of the cross section of the stream for the discharge.

Ex. 102.—The side of a square flume of unplaned plank is 3 feet, the fall per mile 4.752 feet (s=.0009), and the coefficient of roughness of its bed, n=.012. (See Table 26.) What, per second, is the velocity, and what the discharge?

Cal.—In Table 28, find in "side" column for a square flume or canal, 3 feet; opposite which, in "area" column, is found 9 square feet, and opposite which, in hydraulic mean depth column, is found 1. Turning now to Table 27, computed for coefficient of roughness of bed, n=.012, find in hydraulic mean depth column 1; opposite which, in velocity column, for the given fall, F=4.752, (s=.0009) will be found the velocity sought, viz.:

$$v=3.82 \text{ feet.}$$

Then $q = 3.82 \times 9 = 34.38$ cubic feet.—*Ans.*

Ex. 103.—The depth of a rectangular canal being 5 feet, the width 10 feet, the fall per mile 2.64 feet, $F = 2.64$, $s = .0005$, and the coefficient of roughness of the bed $n = .017$ (see Table 26), what will be the velocity of the water, and what the discharge in cubic feet?

Cal.—Find in Table 28, under "rectangle" in "depth" and "width" columns, the given depth and width 5 and 10 feet; opposite which, in "area" column, will be found 50 square feet; and in the "hydraulic mean depth column," will be found 2.5.

Turning now to Table 27, for $n = .017$, find in "hydraulic mean depth column" 2.5; opposite which, in velocity column for the given fall, $F = 2.64$, will be found the velocity sought, viz.:

$$v = 3.605 \text{ feet.}$$

Then $q = 3.605 \times 50 = 180.25$ cubic feet.—*Ans.*

Ex. 104.—The side (wetted) of a V (vee) flume of unplaned plank, being 2.4 feet, the fall per mile 26.4 feet, and the coefficient for roughness of the bed, $n = .012$ (see Table 26), what is the velocity of the water per second, and what the discharge in cubic feet?

Cal.—Find in Table 28, under triangle in side column, the given side 2.4 feet; opposite which, in area column, is found 2.88 square feet, and in "hydraulic mean depth column," .6.

Turning now to Table 27, for $n = .012$, find in "hydraulic mean depth" column, .6; opposite which, in velocity column for the given fall, F=26.4, will be found the velocity sought, viz.:

$$v=6.338 \text{ feet.}$$

Then $q=6.338\times 2.88=18.25$ cubic feet.—*Ans.*

Ex. 105.—The depth of a regular semi-hexagonal canal in earth, being 3 feet, the fall per mile F=5.28 feet ($s=.001$), and the coefficient for roughness of bed $n=.025$ (see Table 26), what is the velocity of the water and what the discharge?

Cal.—Find in Table 28, under semi-hexagon in depth column, 3 feet, opposite which, in area column, is found 15.588 square feet, and in hydraulic mean depth column 1.5.

Turning now to Table 27, for $n=.025$, find in hydraulic mean depth column 1.5, opposite which, in velocity column for the given fall, F=5.28 feet, will be found the velocity sought, viz.:

$$v=2.383 \text{ feet.}$$

Then $q=15.588\times 2.383=37.15$ cubic feet.—*Ans.*

Ex. 106.—In a trapezoidal canal (bottom-slope of bank), the angle of slope of bank is 45°, the depth 4.394 feet, the fall per mile F=7.92 feet, and the coefficient for roughness of bed $n=.017$ (see Table 26), what is the velocity and what the discharge of water per second?

Cal.—Find in Table 28, under trapezoid, with bank slope of 45°, in depth column, 4.394 feet, the given depth; opposite which, in area column, is found 46.606 square feet, and in hydraulic mean depth column, 2.5.

Turning to Table 27, find in hydraulic mean depth column 2.5, opposite which, in velocity column for the given fall, F=7.92 feet, is found the velocity sought, viz.:

$$v=6.26 \text{ feet.}$$

Then $q=46.606\times6.26=291.75$ cubic feet.—*Ans.*

Ex. 107.—In a trapezoidal canal, in which the bottom is equal a side, and the ratio of the depth to the base of bank is as 2:1, the depth of water is 2.591 feet, the fall per mile F=10.56 feet ($s=.002$) and the coefficient for roughness of bed, $n=.025$ (see Table 26), what, per second, is the velocity and what the discharge?

Cal.—Find in Table 28, under trapezoid 2:1, in depth column, the given depth 2.591 feet; opposite which in area column is found 10.86 square feet, and in hydraulic mean depth column 1.25.

Turning now to Table 27 for $n=.025$ find in hydraulic mean depth column 1.25; opposite which in velocity column for the given fall F=10.56 is found the velocity sought, viz.:

$$v=2.943 \text{ feet.}$$

Then $q=10.86\times2.943=31.96$ cubic feet.—*Ans.*

Ex. 108.—In a trapezoid canal, in which the bottom is equal to a side, and the ratio of depth to the base of the bank is as 1:2, the depth of water is

5.543 feet, the fall per mile 1.056 feet, and the coefficient for the roughness of the bed, $n=.035$ (see Table 26), what is the velocity, and what the discharge per second ?

Cal.—Find in Table 28, under trapezoid 1:2, in depth column the given depth 5.543 feet; opposite which in area column is found 130.04 square feet, and hydraulic mean depth column 3.5. Turning now to Table 27, find in hydraulic mean depth column 3.5; opposite which in velocity column for the given fall, $F=1.056$, will be found the velocity sought, viz.:

$$v=1.392 \text{ feet.}$$

Then $q=130.04\times1.392=181.02$ cubic feet.—*Ans.*

Ex. 109.—The diameter of a semi-circular canal being 8 feet, the fall per mile 26.4 feet, and the coefficient for roughness of bed $n=.012$, (see Table 26), what is the velocity per second and what the discharge ?

Cal.—Find in Table 28, under semi-circle, in diameter column 8 feet, the given diameter; opposite which in area column is found 25.133 square feet, and in hydraulic mean depth column, 2.

Turning now to Table 27, find in hydraulic mean depth column 2; opposite which in velocity column for the given fall, $F=26.4$ feet, is found the velocity sought, viz.:

$$v=14.22 \text{ feet.}$$

Then $q=25.133\times14.22=357.39$ cubic feet.—*Ans.*

Remark.—It will be observed that Table 27, is equally well adapted to finding the flow of water in circular pipes as in semi-circular canals. For the hydraulic mean depth is the same in each. Thus in Example 108, were it required to determine the velocity of water in a circular pipe running full, the only change required in the calculation would be to double the area, whence would occur a corresponding change in the result, as follows:

$$357.39 \times 2 = 714.78 \text{ cubic feet.}$$

In case of foul pipes it will be better to employ Table 27, rather than Table 17, computed for clean iron pipes, the coefficient of roughness for whose walls as shown is $n = .011$.

Ex. 110.—The observed data of a canal are as follows:

Width of bottom, 15 feet.

Depth of water, 4.5 feet.

Ratio of depth to base, 2 : 5.

Fall per mile, 3.168 feet.

Coefficient of roughness of bed .017.

What is the velocity of flow per second, and what the discharge?

Cal.—Find in Table 29 the given ratio of depth to base, viz.: 2 : 5; opposite which in columns of depth, base and slope, computed for a depth of unity are found 1, 2.5 and 2.6926. Multiplying each by the given depth, there results 4.5, 11.25 and 12.1167, the depth, base and slope of bank. The wet perimeter is equal to the sum of the bottom and twice the slope of bank:

$$p = 15 + 12.1167 \times 2 = 39.2334 \text{ feet.}$$

The mean width of the stream is equal to the sum of the bottom and of the base of the bank:

$$w = 15 + 11.25 = 26.25.$$

The area of cross-section of stream is equal to the product of the mean width and depth:

$$a = 26.25 \times 4.5 = 118.125 \text{ square feet.}$$

The hydraulic mean depth is equal to the quotient arising from dividing the area of cross-section by the wet perimeter:

$$r = \frac{a}{p} = 118.125 \div 39.2334 = 3.01.$$

Turning now to Table 27 for the given coefficient for roughness of bed, $n = .017$, find in hydraulic mean depth column 3, the nearest approximate to that determined, viz.: 3.01; opposite which in velocity column for the given fall per mile, $F = 3.168$, is found the velocity sought, viz.:

$$v = 4.458 \text{ feet.}$$

Then $q = 118.125 \times 4.458 = 526.6$ cubic feet.—*Ans.*

Ex. 111.—The data for the Sacramento river being as follows, viz.:

Fall per mile .528 feet.

Depth, 25 feet.

Width of bottom, 453.33.

Ratio of depth to base of bank, 5 : 12.

Coefficient of roughness of bed, .025.

What will be the velocity per second, and what the discharge ?

Cal.—Find in Table 29 the given ratio, 5 : 12, opposite which are found 1, 2.4 and 2.6, multiplying each by the given depth, 25 feet, there results 25, 60 and 65, the depth, base and slope.

Then wet perimeter $=453.33+65\times 2=583.33$ feet.

Mean width $=453.33+60=513.33$.

Sectional area $=513.33+25=12{,}833.25$ square feet.

Hydraulic mean depth $=12{,}833.25\div 583.33=22$.

By Table 27, the velocity for the hydraulic mean depth, 22 in velocity column for the given fall of .528 feet is:

$$v=4.862 \text{ feet.}$$

Whence, $q=12{,}833.25\times 4.862=62395.26$ cubic feet.—*Ans.*

INTERPOLATION.

For the purposes of interpolation where the extremes are not far apart, in Table 27, the velocities, without any considerable error in practice may be assumed proportionate, either to the sines of slope on one hand, or to the hydraulic mean depths on the other.

Ex. 112.—The coefficient of roughness of bed being $n=.012$, the hydraulic mean depth .5, the extreme sines of slope $s=.0015$, and $s=.005$, what, in a regu lar arithmetical series will be the six interpolated velocities ?

Cal.—By Table 27, for $n=.012$, and for the given hydraulic mean depths the velocities due the given slopes are 3.134 and 5.625 feet per second. Then

$(5.625-3.134)\div 7=.356$ common difference.

Whence, the interpolated velocities will be 3.490, 3.846, 4.202, 4.558, 4.914 and 5.270 feet.—*Ans.*

Ex. 113.—The coefficient of roughness of bed being $n=.012$, the given sine of slope $s=.0007$, the extremes of hydraulic mean depths 1 and 2, what, in a regular arithmetical series, will be the three interpolated velocities?

Cal.—By Table 27, for $n=.012$, the velocities due the given hydraulic mean depths 1 and 2, are 3.360 and 5.302 feet. Then $(5.302-3.360)\div 4=.4855$ common difference; whence the interpolated velocities will be 3.845, 4.330, and 4.816 feet.—*Ans.*

Mean Velocity.

To find the mean velocity of an open stream of water, various devices, as tight tin tubes, loaded each at one end, so as to float vertically in still water, and as nearly so in streams as the current will permit—the Pitot tube, patent logs, etc., are employed. Wooden floats, of nearly the specific gravity of water, are, however, mostly used in common practice. By these the surface velocity is taken; thence the mean velocity is determined by calculation based on experimental data.

Having chosen a straight section of a stream free as possible, to be found from eddies, roughness and foulness of bottom, and having divided the stream into sections parallel with its course, cast into it wooden floats, note the time taken by a float in each section to pass through a given distance, as 100 feet, and thence determine the surface sectional velocity per second. Divide the sum of the several velocities of the floats by their numbers; the result will be the mean surface velocity of the stream per second; whence, the mean velocity for the mean depth of the entire cross section can be calculated as above stated.

Central Surface and Corresponding Mean Velocity.

Having made several trials with floats, as above described, let the greatest velocity, well established by any one of them, be taken as the central surface velocity. For the central surface velocities, from five-tenths (.5) of a foot to six (6) feet, the corresponding mean velocities have, by the aid of experimental data, and the following empirical formula of Prony, viz.:

$$v = V\left\{\frac{V+7.782}{V+10.345}\right\}, \qquad (286)$$

been computed, and the results arranged in Table 30.

In Eq. (286), v denotes the mean velocity, and V the central surface velocity of a stream of water.

TABLE 30.

Central Surface and Corresponding Mean Velocities of Streams.

Central Surface Velocity. Feet.	Mean Velocity. Feet.	Central Surface Velocity. Feet.	Mean Velocity. Feet.
.5	.382	3.5	2.852
1.0	.774	4.0	3.284
1.5	1.174	4.5	3.721
2.0	1.584	5.0	4.165
2.5	2.000	5.5	4.609
3.0	2.424	6.0	5.058

Ex. 114.—What is the quantity of flow in a stream in which the cross section is 50 square feet and the central surface velocity 3 feet per second?

Cal.—In Table 30, opposite the given central surface velocity 3 feet, find in mean velocity column 2.424 feet. Then

$$2.424 \times 50 = 121.2 \text{ cubic feet per second.—}\textit{Ans.}$$

Rough Approximate.—A rough approximate is readily found by taking one-half the product of the surface width, central depth, and central velocity of a stream.

This rough approximate rule is based on the assumption that the cross section of stream is parabolic, and the mean velocity equal to three-fourths (.75) of the central surface velocity.

Ex. 115.—The surface width of a stream is 25 feet, the central depth 4 feet, and the central surface velocity 1.5 feet, what is the flow per second?

Cal.—$25 \times 4 \times 1.5 \div 2 = 75$ cubic feet.—*Ans.*

Quantity of Water Required for Various Mining Purposes.

Hydraulic Mining.—Hydraulic mining, properly, comprises all classes of mining in which the metallic substance sought is separated from its earthy mass or matrix by means of water. The term, however, as employed in California, is, for the most part, restricted to that class of mining in which a stream of water is projected under great pressure from a nozzle against a deep gravel deposit or earthy formation for the purposes of disintegrating the mass, thence freeing the gold and carrying off the debris. The relation of the quantity of water employed to that of material removed varies in different mines and in different parts of the same mine.

Duty of an Inch of Water.—This phrase involves in its meaning the work of disintegration; but as the projecting head is variable from 50 to 350 feet and upward, the phrase seems to refer chiefly to that portion of the work performed by the water in carrying off the debris in a sluice, the grade of which is usually

6 inches per 12 feet. Experience shows that the duty of a 24-hour miner's inch, under a 7-inch head, equivalent, as shown by Table 8, to 2,230 cubic feet flow in twenty-four hours, is in the lower portions of certain mines as follows:

	Cubic Yards.
North Bloomfield Mine	3.5
Milton Mine	2.4
Excelsior Mine	2.0
Gold Run Mine	3.5

In the upper portions of the same mines the duty of the miner's inch was much greater, say 5 to 10 cubic yards.

Thus, at the Gold Run mine, for six years to November 1, 1881, 4,389,791 cubic yards were worked with 1,124,367 miner's inches of water; whence the duty of per inch was 3.9 cubic yards.

In the State Engineer's report to the legislature of the State of California, 1880, the estimated inch duty is as follows, viz.:

	Cubic Yards.
Yuba River Mines	3.5
Bear River Mines	3.0
American River Mines	4.5

One instance is brought to the attention of the writer showing the inch duty to have been 19 cubic yards.

Drift Mining.—Drift mining consists in excavating the lower material of a gravel mine by hand, raising it through a shaft to the surface, or carrying it by wheelbarrows and cars through a tunnel to a dump, whence it is shoveled or piped into a sluice to be freed of its gold, and thereupon carried off as debris. A the larger cobble, bowlders and barren blocks of rock are usually left in a drift mine, and the material broken smaller than in hydraulic mining, a correspondingly less quantity of water is required for working a cubic yard.

The duty of a 24-hour inch (2,230 cubic feet) in this class of mining varies according to the character of the gravel, whether hard and cemented, clayey or sandy, from 3 to 20 cubic yards.

Quartz Mining.—The contents of one ton of quartz, in its normal condition in the lode, is estimated at 13 cubic feet, and at 20 cubic feet when the quartz is broken, as it usually comes from the mine. Adopting the lode measurement it is seen that a cubic yard of quartz is $27 \div 13 = 2.08$ tons nearly.

Experience shows that the duty of a miner's inch is as follows:

Duty of a miner's inch (under 4-inch pressure) in the reduction and amalgamation of silver ores in a "stamp silver mill," Nevada, 3.25 cubic yards or 6.76 tons; in the reduction and amalgamation by riffles, or copper plate, in "stamp gold mill," California, 5.78 cubic yards or 12 tons.

Duty of miner's inch (under 7-inch pressure) in the former case (silver) 4.3 cubic yards, or 8.93 tons; in the latter case (gold) 6.65 cubic yards, or 15.88 tons.

The volume of water to that of ore is, in working silver ores, Nevada, 19.5 to 1; in working gold ores, California, 11.1 to 1; in working copper ores, Lake Superior, 20 to 1.

Quantity of Water Required for Purposes of Irrigation.

As the area of land is usually expressed in denomination of acres, a convenient unit of measure for irrigating purposes is that quantity of water which will cover one acre one inch deep. This quantity is 3,630 cubic feet.

The total depth of irrigation, as practiced in California, varies for different soils and products from two to five feet.

Ex: 116.—It is proposed to irrigate 1000 acres of land, 50 inches in depth, in 100 days, by means of a canal whose fall per mile is to be 1.056 feet (s=.0002), coefficient of roughness of bed n=.017, bottom width equal to slant width of side, and ratio of depth to base of bank as 1:2, what will be the dimensions of the canal?

Cal.—$3630\times50\div100=1815$ cubic feet; $1815\times1000=1815000$ cubic feet per day; $1815000\div86400=21.007$ cubic feet per second.

Assume, by way of trial, the hydraulic mean depth to be 1.25. Then, in Table 27, for n=.017, the velocity for hydraulic mean depth 1.25 is 1.386 feet per second; whence, $21.007\div1.386=15.16$ square feet, area of cross section of canal.

By Table 28, for hydraulic mean depth 1.25, the sectional area is 16.59 square feet, under trapezoid 1:2. This approximate is sufficiently near to meet the requirements of practice. The dimensions of the canal then, as per Table 28, are: side=bottom=4.426 feet; depth=1.980 feet.—*Ans.*

If greater accuracy be required, proceed as in the solution of Ex. 99.

Ex. 117.—How many acres can be irrigated 40 inches in depth in 75 days, by means of a semi-hexagonal canal five feet deep, the fall per mile being 1.584 feet ($s=.0003$), and the coefficient for roughness of bed being $n=.025$?

Cal.—By Table 28, it is seen that the hydraulic mean depth and the area of a semi-hexagon five feet deep, are respectively: 2.5 feet and 43.301 square feet.

By Table 27, for $n=.025$, fall per mile 1.584 feet, the velocity corresponding to hydraulic mean depth 2.5 is 1.872 feet per second.

Then $43.301\times1.872=81.059472$ cubic feet per second; $81.059472\times86400\times75=525265378.5$ cubic feet; $3,630\times40=145200$ cubic feet per acre; $525265378.5\div145200=3617.5$ acres.—*Ans.*

Measurement of the Power of Water as a Motor.

The unit in the measurement of power is a foot-pound—that is, the amount of energy necessary to raise one pound weight vertically through a distance of one foot. On the other hand, one pound falling by the force of gravity through a distance of one foot, generates a foot-pound.

The amount of energy required to raise one pound vertically 550 feet, is equal to the amount of energy necessary to raise 550 pounds vertically one foot in hight.

This amount of energy rendered in one second is termed a horse-power—that is, 550 foot-pounds rendered in one second, is the value of a horse power in mechanics.

The weight of a cubic foot of fresh water is estimated in practice at 62.5 pounds.

Ex. 118.—How many horse-power will 10 cubic feet of water, applied to an overshot water wheel, 40 feet diameter, render, the efficiency of the wheel being 75 per cent, and one foot being allowed for clearance?

Cal.—40—1=39 feet, effective head; $62.5 \times 10 \times 39 \times .75 \div 550 = 33.24$ horse-power.—*Ans.*

TABLE 30.

Limiting Velocities in Open Streams.—Jackson's Hydraulic Manual.

	Feet per Second.
For the worst or most sandy soil	2.5
For sandy soil generally	2.75
For ordinary loam	3.
For firm gravel and hard soil	4.
For brick work, ashlar, or rubble in cement	5.5 to 7.5
For hard, sound, stratified rock	10.
For very hard homogeneous rock	14. or 15.
Limits usual for canals	1. to 4.
Limits for irrigating channels	1. to 3.
Limits for sewers and brick conduits	1. to 4.5
Limits for self-cleansing sewers and drainage pipes	2.5 to 4.5

Remark.—The importance of the data, given in Table 30, will be seen at a glance.

Thus, if a velocity exceeding 2.5 feet per second be given a stream in very sandy soil, destruction by erosion of the bed of the canal will ensue; while on the other hand, if the velocity shall not exceed one foot per second at first, the canal will be liable to become choked up by the growth of vegetation.

Flood-Flow of Streams.

Various formulæ have been devised for the flow of streams in times of floods. These empirical formulæ are, at best, but rough approximates to the true flow.

The following formula, given by Fanning, is an expression for the "recorded flood measurement of American streams" in New England and the Middle States:

$$Q=200\ M^{\frac{5}{6}}, \qquad (287)$$

in which Q denotes the number of cubic feet discharge per second, and M the area of water shed in square miles.

As California is more mountainous than New England and the Middle States, and fully as subject to heavy downfall of rain in the mountainous regions, it is not improbable that the flood discharge here will exceed that indicated by formula (287). Let it hence be accepted until otherwise determined.

Ex. 119.—The water-shed of the main Sacramento river contains twenty-four thousand seven hundred and eight square miles. What will be its flood discharge per second?

Cal.—$(24{,}708)^{\frac{5}{6}}\times 200=915647$ cubic feet.—*Ans.*

TABLE 32.

Miscellanies.

1 cubic foot of distilled water (U. S. standard), barometer 30 inches, 39.83° Fahr.=62.3793 lbs.

1 cubic foot of distilled water (British standard), barometer 30 inches, 62° Fahr.=62.321 lbs.

1 cubic foot of distilled water (U. S. standard)= 7.48052 gallons.

1 cubic inch of distilled water (U. S. standard)= 0.0361 lbs.

1 gallon (U. S. standard)=231 cubic inches= 0.133681 cubic feet=8.3389 pounds water.

1 gallon, imperial (British standard)=277.123 cubic inches=0.160372 cubic feet=10 lbs. water.

1 gallon (N. Y. statute measure), barometer 30 inches, 39.83° Fahr.=221.184 cubic inches=8 lbs. water.

1 pound avoirdupois=16 ounces=7000 grains (U. S. standard)=27.7015 cubic inches.

1 pound Troy=1 pound apothecary=12 ounces= 5760 grains.

1 ounce avoirdupois=437.5 grains.

1 ounce Troy=1 ounce apothecary=480 grains.

1 chain=100 links=4 rods=66 feet=792 inches.

80 chains=1 statute mile=320 rods=1760 yards= 5280 feet=63,360 inches.

1 geographical, nautical or sea-mile=6,086.5 feet in longitude; and 6,076.5 feet in latitude.

1 league (English)=3 nautical miles.

1 metre=3.2808992=3.281 in practice.

1 square metre=1 centiare=10.7643 square feet.

1 are=100 square metres=1076.43 square feet.

1 cubic metre=1 stare=35.3166 cubic feet.

1 vara=2.75 feet.

1 legua (Mexican)=5000 varas linear=13,750 feet =2.60417 miles.

100 vara lot=100 varas square=75625 square feet=1.73611 acres.

1 legua, Mexican (of land)=6.7817 square miles= 4340.27778 acres.

1 acre=4 roods=10 square chains=160 square rods=43560 square feet.

1 section=1 square mile=640 acres.

1 township=36 sections=6 miles square=36 square miles.

1 cubic yard=27 cubic feet=16,656 cubic inches.

1 hundredweight (British)=8 stone=112 pounds.

1 ton (long ton), commercial=20 hundredweight= 2240 pounds.

1 ton (short ton), U. S.=2000 pounds.

1 quintal=100 pounds.

1 fathom=6 feet; 1 cable length=120 fathoms.

1 point=$\frac{1}{72}$ of an inch.

1 line=6 points=$\frac{1}{12}$ of an inch.

12 inches=1 foot; 3 feet=1 yard.

$5\frac{1}{2}$ yards=1 rod.

1 foot board measure=1 foot square and 1 inch thick.

12 feet board measure=1 cubic foot.

1 foot-pound=work required to raise one pound vertically one foot.

1 second foot-pound=work required to raise one pound vertically one foot in one second of time.

1 minute foot-pound=work required to raise one pound vertically one foot in one minute.

1° (one degree), centigrade=1.8° (degrees), Fahrenheit.

1 barometric inch=column of mercury, with one square inch base and one inch high.

Atmospheric pressure per square inch=14.7 pounds =30 barometric inches nearly, at 39.83° Fahr.

1 ounce Troy, gold, 1000 fine=$20.6718.

1 ounce Troy, gold coin, U. S., 900 fine=$18.6046.

1 pound avoirdupois, gold coin, U. S., 900 fine= $271.375.

1 ounce Troy, silver, 1000 fine=$1.29293.

1 ounce Troy, silver, U. S., 900 fine=$1.163636.

1 pound avoirdupois, silver coin, U. S., 900 fine= $16.96969.

1 dollar, U. S. gold coin=23.22 grains gold+2.58 grains copper=25.8 grains.

1 dollar, U. S. silver coin=371.25 grains silver+ 41.25 grains copper=412.5 grains.

1 pound sterling=1 sovereign=113.001 grains gold +10.273 grains copper=123.274 grains weight, fineness 22 carats=916.6667.

1 grain gold, 1000 fine=$.0430663 mint value.

1 grain silver, 1000 fine=$.0026936 mint value.

1 gramme gold, 1000 fine=$.6646142 mint value.

1 gramme silver, 1000 fine=$.0415686 mint value.

1 cubic foot air=.0806726 pounds=564.7082 gr's.

1 pound of air at 39.83°=12.387 cubic feet by vol.

1 cubic foot hydrogen=.005042 pounds=35.2743 grains.

25 cubic feet of sand=1 ton.

18 cubic feet of earth=1 ton.

17 cubic feet of clay=1 ton.

13 cubic feet of quartz, unbroken in lode=1 ton.

18 cubic feet of gravel or earth, before digging= 27 cubic feet when dug.

20 cubic feet of quartz, broken (of ordinary fineness coming from the lode)=1 ton, contract measurement.

1 horse-power (H. P.)=550 second foot-pounds= 33000 minute foot-pounds.