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U. S. DEPARTMENT OF AGRICULTURE,

DIVISION OF FORESTRY.

MEASURING THE FOREST CROP.

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LETTER OF TRANSMITTAL.

U. S. DEPARTMENT OF AGRICULTURE,
DIVISION OF FORESTRY,
Washington, D. C., June 24, 1898.

SIR: I have the honor to transmit herewith for publication a brief presentation of the methods by which measurements of felled and standing trees and of whole forest growths are best performed, together with a discussion of a method employed in ascertaining the rate of growth of trees and forest crops, developed in the Division of Forestry, by Mr. A. K. Mlodziansky.

The publication of these (with exception of the last) more or less elementary methods of procedure, selected from a large number of methods that have been developed, seems justified at the present time, as with the waning of forest supplies more accurate methods of measuring the forest crop are indicated. Moreover, the entire business arrangements of a well-conducted forest management are based upon a knowledge of the amount of product which may be had or expected from a given area. This knowledge can not be satisfactorily ascertained by mere estimates; hence mathematical methods must be employed.

Especially is this true with regard to the rate of growth at which the forest crop develops, for all financial calculations of the profitableness of forest management presuppose this knowledge. The method developed by Mr. Mlodziansky in the work of the Division, with regard to collecting and collating the data in ascertaining the rate of growth of white pine and other species, the results of which are presently to be published, will recommend itself for the rapidity with which a large number of measurements can be coordinated and summarized.

Respectfully,

B. E. FERNOW, *Chief.*

Hon. JAMES WILSON,
Secretary of Agriculture.

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MEASURING THE FOREST CROP.

INTRODUCTION.

The methods of measuring wood when cut are well known. For firewood and billets, for pulp wood, spokes, staves, etc., the cord of 128 cubic feet is employed; for telegraph poles, posts, etc., the linear foot, with diameter limits, furnishes the measure; for saw logs various standard log rules are used, which pretend to give the amount of timber that can be sawed from logs of given lengths and smallest measured diameter. We say "pretend," for in fact the amounts given in these log rules, or scalers' books, do not in most cases coincide with the amount obtained by the miller. That amount depends upon the care with which the miller handles the log and the character of the saw he employs.

It is not, however, the measuring of the cut wood that we propose to discuss here, but the measuring of the standing crop as it is found in the forest. This knowledge, not only of what amount of wood is standing on an acre at a given time, but what amount grows in a year or has grown in a given period, is of great importance with a crop which requires many years to mature, and does not, like a field crop, have a definite period when it is ripe, but with which the harvest depends on the question when it is profitable to cut the crop.

The amount which grows each year varies at different periods of the life of the crop, hence if we want to determine when it is most profitable to cut the crop we must be able to measure its growth and to determine whether the yearly or periodic increment is such as to make it desirable to let the crop stand because it increases in value in due proportion to the cost of its standing, or to cut it because the wood made per year ceases to pay interest on the cost.

In order to measure the amount of timber standing and the amount of wood growing we must know the methods of measuring (1) the contents of a single tree; (2) the contents of a stand of trees or growing stock; (3) the rate at which single trees and whole stands grow under varying conditions and at various ages.

While full knowledge of the subject may be acquired only by special study and application, familiarity with the simplest method is within the easy reach of everyone interested or engaged in lumbering or forestry operations, and only the simplest methods are to be discussed here.

MEASUREMENT OF STANDING TREES.

HEIGHT MEASURING.

There are various methods employed in determining the height of a standing tree; of these the geometrical method may be recommended for its simplicity and sufficient accuracy. At some distance from the tree (fig. 1), where both top and base are readily visible, place a pole from 4 to 5 feet long (SF) perpendicularly in the ground; put in the ground another and longer pole (DE) at some distance from the first one, so that the poles and tree are situated in the same vertical plane.

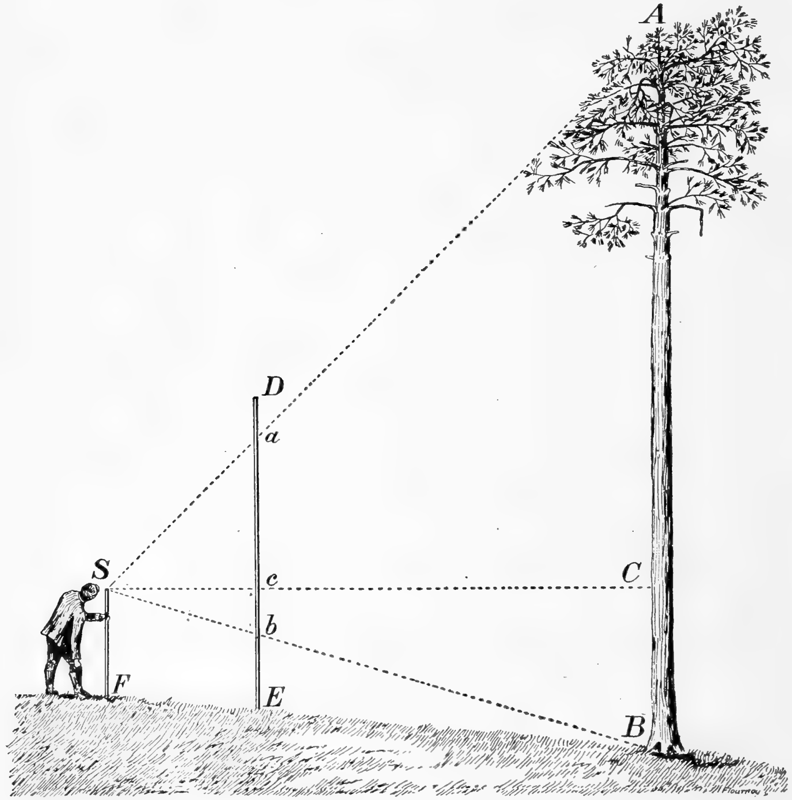

Fig. 1.—Measuring the height of a tree by means of two poles.

Sight from the top of the smaller pole the base and the top of the tree and note the points where your lines of vision intersect the longer pole; measure the distance between them; measure also the horizontal distance between the small pole and the tree and that between the two poles. Multiply the first distance by the second and divide by the third, the result being the height of the tree $\left(\frac{ab \times SC}{Sc}\right)$.

Example: Let the distance between the points where the lines of vision intersect be 6 feet, the distance between the pole and tree 30 feet, the distance between the poles 2 feet; then the height of the tree equals $\frac{6\times30}{2}=90$ feet.

Another simple method, where possible, is to measure the shadow of the tree and of a pole or man, when the unknown height (h) of the tree is in the same ratio to the known length of its shadow (s) as the length of the pole (p) to that of its shadow (ps), both of which are also known;

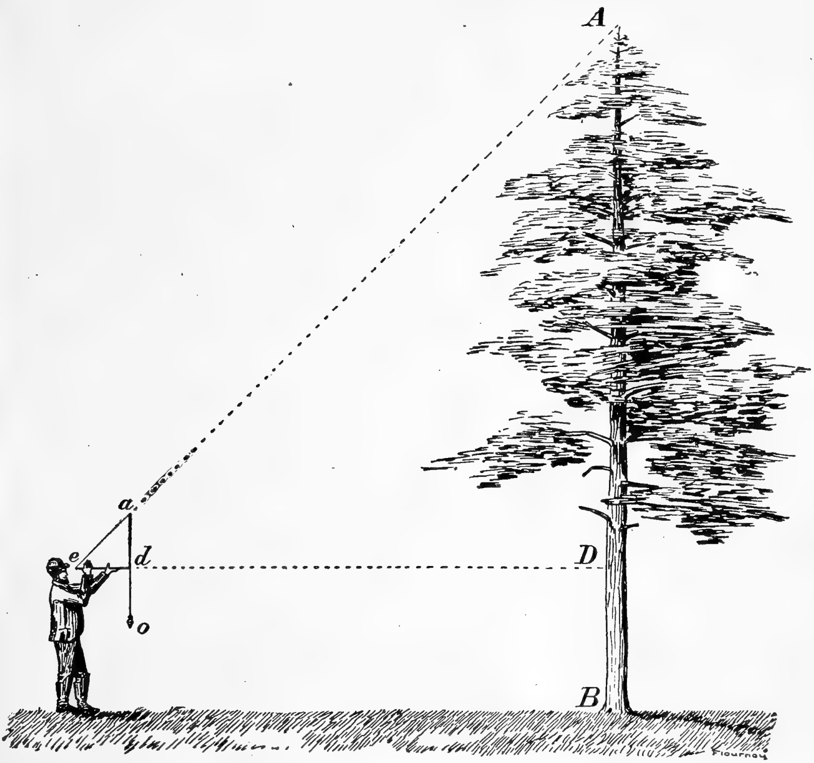

FIG. 2.—Measuring the height of a tree by means of a right-angled isosceles triangle.

that is to say, the height is equal to the product of the tree's shadow and the pole's length divided by the length of the pole's shadow $\left(h=\frac{s\times p}{ps}\right)$.

There are various instruments for measuring the height of a standing tree, based on the same principles as the first-mentioned simple method. The calculations are usually placed on the scale of the instrument and the height can be read off at once. The simplest one is a right-angled isosceles triangle, which may easily be made of pasteboard or wood. In using this triangle the observer should select a spot on the same level with the base of the tree at a distance approximately equal to the height of the tree (fig. 2).

Place the triangle to the eye so as to sight along the longer side, while holding the shorter sides (with the aid of a plumb line) so that the one is strictly vertical, the other horizontal; then shift your position forward or backward until you can just sight the top of the tree; measure your distance from the tree and add the height of your eye above the ground; the sum gives the height of the tree. After some practice with either of these two methods on trees standing in the open, one may become sufficiently expert in estimating the heights of trees to meet most requirements.

The most convenient instrument which may be recommended for measuring the height of trees is the so-called "mirror hypsometer" of Faustman.

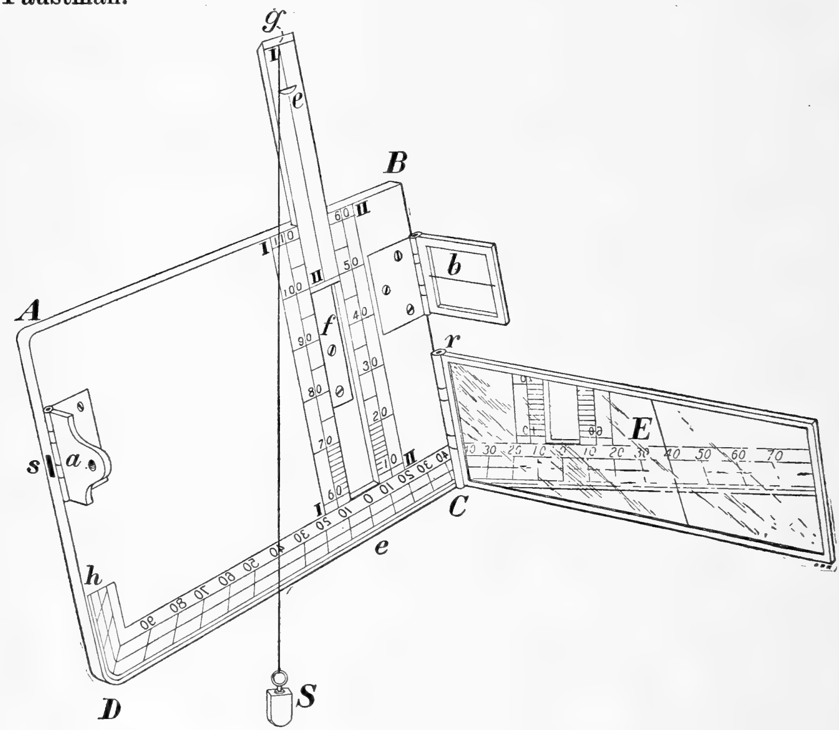

FIG. 3.—Faustman's mirror hypsometer.

The instrument (fig. 3) consists of the following parts:

ABCD—Rectangular wooden board (or brass frame), 7.3 inches long and 3.1 inches wide.

a—Eyepiece made of brass.

b—Frame with hair line.

ge—Sliding scale for registering the distance from the observer to the tree. It consists of two parts, the shifting part with the attachment of the plumb line *g*S, and the graduated part with a spring attachment (*f*) to keep the shifting part in position.

CD*h*—Height scale from which the height is read off.

E—Mirror, of similar length with the board and 1 inch wide, in which the height scale is reflected.

In using the instrument the observer should select a spot from which the top of the tree is distinctly seen; then measuring off the distance from the tree in feet, shift the sliding scale until its lower end stands opposite this distance. Thereupon sight from the eyepiece past the hair line and the top of the tree and read off in the mirror the figure which the plumb line strikes on the scale. In the same manner sight the base of the tree and find the corresponding figure. The sum of the two figures represents the height of the tree when the observer is situated above the level of the base of the tree. When the observer is situated below the level of the base of the tree the difference between those figures should be taken in order to obtain the height of the tree. The figure represents the position of the instrument when in use, the observer being supposed as on the same level with the base of the tree and as shown on the sliding scale at 100 feet distant; the height of the tree, as indicated by the position of the plumb line reflected in the mirror, is 40 feet. When the instrument is not in use all its parts are easily folded and put into a case, which can be conveniently kept in the pocket.

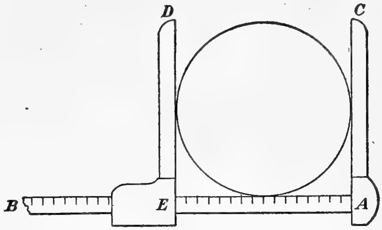

FIG. 4.—Calipers for measuring the diameter of trees.

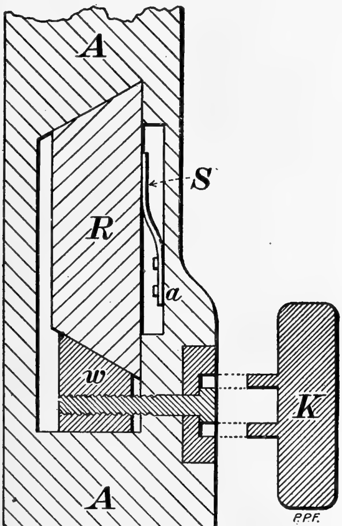

FIG. 5.—Section of the movable arm of Heyer's calipers.

MEASUREMENT OF DIAMETER.

The diameter of a standing tree is usually taken breast-high or above the swelling of the base and measured by a pair of calipers, the essential parts of which are a graduated rule, divided in inches and subdivisions, with two arms (fig. 4), one of which (AC) is fixed at right angles to the graduated rule (AB), while the other may be shifted along the rule, remaining parallel to the immovable arm (AC).

In measuring, the calipers are usually placed breast-high horizontally against the trunk, so that both the rule and the immovable arm touch it; then the movable arm is shifted along the rule until it is brought in touch with the trunk, when the diameter can be read off on the rule. The length of the rule depends upon the size of the trees to be measured, and the length of the arms should not be less than half of that of the graduated rule. Calipers from 4 to 5 feet long will answer in most cases.*

Since trees are rarely cylindrical, being often larger in one direction than in the other, it is advisable to make two measurements and take the average, or else take care to measure the estimated average diameter. Instead of measuring the diameter, the circumference may be measured by a tape and the diameter determined by dividing it by 3.14, which is the ratio of the circumference to the diameter.

MEASUREMENT OF VOLUME.

In determining the volume of a standing tree the stem or bole only is considered; the cubic contents of the branches may be estimated by themselves. It is rather difficult to determine the volume of a standing tree because geometrical forms which exactly correspond to the shape of a stem are not known. Moreover, the shapes of trunks differ with age, with species, and with the soil and forest conditions under which they grow; hence we can obtain the volume only approximately by comparing it to the mathematical form which it resembles most nearly. The form of a stem of a tree is neither a cone nor a cylinder, but resembles most closely the form known as a paraboloid. The volume of a paraboloid equals the product of its base by one-half of its height. The base of the tree is taken at a distance from the ground, usually breast-high, where the irregularities of the trunk caused by the root swellings terminate. Here the tree is calipered, and the area for the corresponding diameter (found in the area table, p. 37) is multiplied by one-half of the height of the tree.

Example: Let the height of the tree be 90 feet, the diameter, breast-high, 21 inches. The area corresponding to a circle of 21 inches diameter is 2.40 square feet. The volume of the tree then equals $\frac{2.40 \times 90}{2} = 108$ cubic feet.

Another method, devised by a German forester, Mr. Pressler, may be recommended for determining the volume of a standing tree: Find a place along the stem (fig. 6) where its diameter (d) is exactly one-

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*The calipers should be so constructed that the arms work strictly parallel to each other and at right angles to the rule; it should, therefore, be made of wood which is not easily affected by moisture. Air-dry pear wood may be recommended as a material least subject to shrinkage. Swelling and shrinking of the wood makes the shifting of the arm either difficult or too easy, often throwing the arm out of the perpendicular, thus destroying the required parallelism between the arms. To avoid this various constructions of calipers have been adopted. The calipers of Gustav Heyer, a section of which is given in fig. 5, may be recommended. A represents the section of the movable arm; R is the cross section of the rule; S a spring fastened at A pressing on the rule and pushing it down; w is the cross section of a wedge made of brass and fastened to a screw which can be moved by the key K. By moving the wedge backward and forward the rule can be tightened or released, thus enabling the observer to regulate the shifting of the movable arm without throwing it out of the perpendicular.

half of that at breast height (D); this point is called the guide point. This point can be determined by estimate after some practice or else by use of a simple instrument (fig. 7) consisting of three hollow cylinders (A, B, and C), which fit one into the other. The instrument then can be lengthened and shortened in the same way as an ordinary telescope. The cylinders may be made of stiff manila or other similar paper. Into the outer cylinder (A) two pins (*k* and *l*) are thrust 1 inch from the end; they can be moved in and out, permitting a change of distance between their heads. Cylinders A and B are of the same length, 13 inches each, while that of C is 2 inches long. The end of cylinder C is closed by a paper cover, in the center of which a hole (*y*), of one-fourth inch in diameter, is made as an eyepiece. Looking through the eyepiece (*y*), arrange the heads of the pins so that the distance between them coincides exactly with the diameter of the tree at breast height. Without changing the distance between the heads of the pins, the observer draws out the cylinders so as to double the former length, allowing for that purpose the two inside cylinders to project into each other 1 inch; then range the telescope up the trunk until a point is found where the diameter of the tree again corresponds with the distance between the heads of the pins. At this point the diameter of the tree is one-half of that at breast height. To obtain the volume of the tree, estimate or measure the height of the guide point, add 2 feet, and multiply this sum by two-thirds of the area corresponding to the diameter (D) measured at breast height.

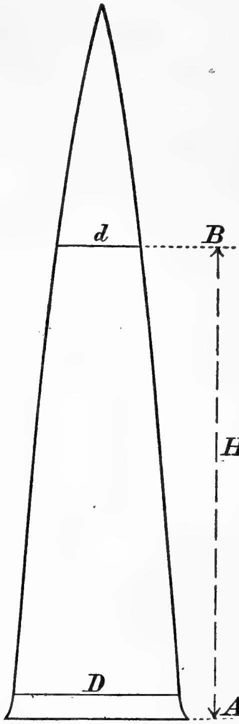

Fig. 6.—Pressler's method of determining the volume of a standing tree.

Example: A tree of 26 inches in diameter at breast height is 13 inches in diameter at a height of 60 feet from the ground—that is to say, the height of the tree to the guide point equals 60 feet. Adding 2 to 60 and multiplying by two-thirds of 3.69 (3.69 square feet represents

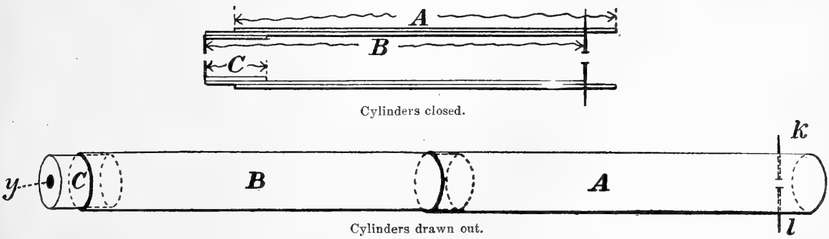

Fig. 7.—Instrument used for determining the guide point.

the area of a circle with a diameter of 26 inches), we find the volume of the tree to be 152.5 cubic feet. The merit of this method lies in its being equally applicable to trees of various geometrical forms; it is correct for trees of parabolic and conical forms; for trees representing the form of a cone with a concave surface the difference is only 1.4 per cent.

MEASUREMENT OF VOLUME OF A STANDING TREE BY EMPLOYING THE FACTOR OF SHAPE.

The trunks of trees, as has been mentioned, differ in shape. The shape of the trunk of a cypress, a spruce, or a fir is totally different from that of a pine, hemlock, or oak. The cypress, spruce, and fir, tapering rapidly toward the top of the tree, form stems resembling either a cone, as in the spruce and fir, or a neloid or conical shape with a concave surface as in the cypress. The pine, the hemlock, and most of the hardwood trees, tapering more gradually toward the top, form stems of a conical shape with a convex surface. An oak or a tulip tree, on the other hand, may nearly approach the shape of a cylinder. As we have stated before, trees never attain a mathematical form, but only approximate more or less closely one or the other form.

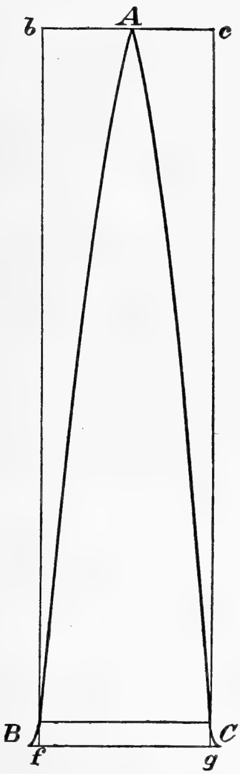

FIG. 8.—Determining the factor of shape.

The European foresters noticed long ago that there exists a relation between the actual volume of a tree and that of a regular geometrical body of corresponding dimensions. From actual calculation they learned further that this relation, varying with the kinds of trees, their dimensions, and conditions of growth, seems to be strikingly uniform. In Germany, for instance, there were measured more than forty thousand individual trees of various species and, all of them being felled, the forester was able to determine their volume in an accurate way. The actual volume of each individual tree thus obtained was compared with that of a cylinder of the same height and of the diameter at breast height. This comparison proved that the actual volume of the tree when divided by that of the cylinder of the corresponding dimensions gives a quotient which is constant for trees of the same species, approximately the same dimensions, and grown under the same forest conditions.

This quotient showing the taper of the tree, or the relation between the volume of a tree and of a cylinder of the same height and diameter breast high, is called the *factor of shape* or *form factor;* it is usually expressed in decimals and represents arithmetically the form of the stems.

For instance, if we take a tree of 22 inches diameter and 82 feet in height (fig. 8), whose volume by careful measurement we have found to be 93.1 cubic feet, we determine its form arithmetically or its factor of shape by dividing the volume of the tree by the volume of a cylinder of the same dimensions, which is 216.5 cubic feet. The factor of shape is, therefore, $\frac{93.1}{216.5}=0.43$. That means that the volume of the tree is forty-three hundredths of the volume of a cylinder of the same diameter and height. Applying this method when factors of shape have been determined by a number of previous measurements, the diameter and height of the tree are measured, the volume of the corresponding cylinder found, and that volume multiplied by the factor of shape in order to obtain the cubic contents of the tree. This method gives more accurate results than those obtained from calculations of geometrical forms which the stems of the trees are supposed to represent. The factors of shape of a species may be determined from a number of accurate measurements of the volume of felled trees.

Below we give the factors of shape for white pine when situated in a moderately dense forest. They are based upon 722 individual trees, which, being felled, were measured and the results collated in the Division of Forestry, with a view of determining the rate of growth of the species:

Diameter at breast height.	Corresponding factors of shape.	Diameter at breast height.	Corresponding factors of shape.	Diameter at breast height.	Corresponding factors of shape.	Diameter at breast height.	Corresponding factors of shape.
Inches.		*Inches.*		*Inches.*		*Inches.*	
6	0.51	17	0.46	28	0.42	39	
7	0.50	18	0.45	29		40	
8	0.50	19	0.44	30		41	
9	0.49	20	0.44	31		42	0.40
10	0.49	21	0.43	32		43	
11	0.48	22	0.43	33	0.41	44	
12	0.48	23	0.42	34		45	
13	0.48	24	0.42	35		46	0.39
14	0.47	25	0.42	36			
15	0.47	26	0.42	37			
16	0.46	27	0.42	38	0.40		

It is seen that for a pine from 29 to 36 inches in diameter the factor of shape is 0.41. Suppose we are to determine the volume of a standing white pine of 31 inches in diameter, breast high, and 130 feet in height. The volume of a cylinder of 31 inches in diameter and 130 feet high is equal to 681.4 cubic feet. Multiplying 681.4 by the factor of shape (0.41) we determine the volume of the tree to be 279.4 cubic feet.

MEASUREMENT OF FELLED TREES.

HEIGHT AND DIAMETER MEASURING.

The height of a felled tree is measured either by a tape (a steel tape measure being most accurate) or by a measuring pole from 4 to 8 feet long. The diameter of a felled tree at any given place is measured by calipers described above. It is always advisable to note the average diameter of two measurements taken at a right angle.

MEASUREMENT OF VOLUME.

The volume of a felled tree may be determined with more accuracy than that of a standing tree, for the tree on the ground may be measured in parts and the volume of each part determined separately. While the tree, taken as a whole, does not closely resemble any of the known geometrical forms, the portions of the tree, especially when they are small, may be compared to some of the known forms with less hesitation. Of the various methods which may be employed in determining the volume of a felled tree, the following are recommended:

(1) *When great accuracy is required and the volume of the stump is included.*—Divide the tree into sections, each 4 feet in length, and caliper at right angles at each section, noting the average of the two diameters, including that of the butt. Find in the table the areas corresponding to the diameters noted and add all together. Multiply the sum of the areas by 4 feet and the product will be the total volume of the tree, if the last measurement with the calipers was taken at 2 eet from the top. If the last measurement with the calipers was taken

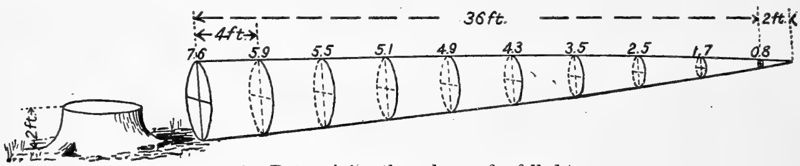

FIG. 9.—Determining the volume of a felled tree.

at a greater distance from the top, the volume of the top part must yet be added. The volume of the leader, or top, equals one-third the product of the basal area by its length. The base of the leader is taken at 2 feet from the last point of measurement with the calipers. The diameter of the leader is measured and the corresponding basal area found in the area table. Example: Let the average diameter of the butt be 7.6 inches (fig. 9), and the average diameters calipered from butt for every 4 feet be, consecutively, 5.9, 5.5, 5.1, 4.9, 4.3, 3.5, 2.5, 1.7, and 0.8 inches. The last measurement with the calipers was taken at 2 feet from the top. In the area tables we find:

Location.	Diameters.	Areas.
	Inches.	*Sq. ft.*
Butt	7.6	0.315
4 feet from butt	5.9	.190
8 feet from butt	5.5	.165
12 feet from butt	5.1	.142
16 feet from butt	4.9	.131
20 feet from butt	4.3	.101
24 feet from butt	3.5	.067 1.111
28 feet from butt	2.5	.034
32 feet from butt	1.7	.016
36 feet from butt	0.8	.003
		1.164

Multiplying the sum of the areas 1.164 by 4, we find the total volume of the tree to be 4.6 cubic feet. In case the last measurement with the calipers was 3.5 inches, i. e., it was taken at 14 feet from the top, then the volume of the leader must be added to the product of 1.111×4, which is 4.4. The base of the leader begins 2 feet above the last measurement with the calipers; the diameter measured here is 3 inches and the corresponding basal area equals 0.049 square feet; one-third the product of 0.049 by 12 (length of leader) is 0.02, which, added to 4.4, makes 4.6, the total volume of the tree, the same as obtained from previous calculations.

(2) *When less accuracy is required and the volume of the stump is excluded.*—The tree is calipered at the butt and in a few other places where it is most convenient; each log length, for instance. The volume of each portion between two measurements with the calipers equals one-half the product of its length multiplied by the sum of the areas corresponding to the diameters thus measured. The total volume of the tree is determined by summing up the volumes of all the parts thus separately calculated, including also the leader. The last measurement of the tree with the calipers is taken as the base of the leader, the volume of which is calculated in the same way as given above.

Example: Let us determine the volume of the tree taken above by calipering it at the butt, at 12 and at 20 feet from the butt:

The average diameter at the butt equals 7.6 inches; the corresponding area, 0.315 square foot.

The average diameter 12 feet from the butt equals 5.1 inches; the corresponding area, 0.142 square foot.

The average diameter 20 feet from the butt equals 4.3 inches; the corresponding area, 0.101 square foot.

The sum of the areas of butt and top of first length is 0.457; the length of first log is 12 feet; the volume then equals one-half of 12 multiplied by 0.457, or 2.74 cubic feet. The distance between the second and third measurements with the calipers is 20 feet minus 12 feet, equals 8 feet, and the volume of this portion of tree equals one-half of 8 multiplied by 0.243 equals 0.97 cubic feet. The volume of the leader equals one-third of 18 multiplied by 0.101 equals 0.61 cubic feet. The total volume of tree (stump excluded) is determined by taking the sum of 2.74, 0.97, and 0.61, which equals 4.3 cubic feet.

MEASUREMENT OF A STAND OF TREES OR GROWING STOCK OF A FOREST.*

On first thought it appears to be a very simple problem to measure the contents of a stand of trees or a forest, since a forest is an aggregate of single trees whose volume we already know how to determine. It appears as though we should need only to measure each tree and add the results. But this would be an expensive operation, and since absolute accuracy is neither necessary nor attainable, a method of averaging is employed in which the trees composing the forest are grouped into classes and only sample trees of each class are measured. The measurements are extended either over the whole forest or over only small typical areas, usually called "sample areas."

DETERMINATION OF THE GROWING STOCK BY EXTENDING THE MEASUREMENTS OVER THE WHOLE FOREST.

When the forest is not large its growing stock is usually determined by extending the measurements over the whole forest, i.e., the diameters, breast-high, of all the trees constituting the forest are measured with calipers. Of course the diameter measurements of different species are kept separate. If there exists an interdependence between the height and diameter growth, i. e., if the species grows at a uniform rate and trees of larger or smaller diameters are correspondingly taller or shorter in height, there is no necessity of measuring the heights; the average height is then determined by a sample tree, the selection of which will be discussed later. But if the height is not proportional to the diameter, i. e., if trees of the same species and equal diameters differ considerably in height, then classification by height becomes also necessary, and the scoring of trees is done not only by diameter but also by height classes. Differences in height development usually occur when the same species are found in the same forest under different soil conditions. A shallow compact clay soil for instance would produce relatively different proportions from a deep, loose, loamy sand.

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* In this country it has so far been customary only to estimate the growing stock. The result necessarily is mostly far from the truth even with the most expert estimator, and as the estimator is usually employed by a purchaser, the estimate usually comes out from 10 to 30 per cent and more below the actual volume.

There are various ways in which estimators proceed. One of the most frequently used is to establish by either measurement or estimate for the district to be estimated, the average number of superficial feet per tree, then the trees are counted and their number multiplied by the figure obtained for the average tree, making allowance at the same time for breakage, defects, etc.

This method is especially in use where one species uniformly developed is to be estimated. More detailed estimates are made when several species of economic value are to be taken into account.

When the formation of height classes is not necessary, the following method of scoring may be recommended:

Form No. 1.—*Diameter measurements.*

Diameter, breast high, in inches.	NAME OF THE SPECIES.									
	OAK	Total in each diameter class.	HARD MAPLE	Total in each diameter class.	HICKORY	Total in each diameter class.	ASH	Total in each diameter class.	BLACK WALNUT	Total in each diam. class.
8	[illegible]	23								
9	[illegible]	76								
10	[illegible]	87	[illegible]	26						
11	[illegible]	172	[illegible]	75						
12	[illegible]	43			[illegible]	63				
13	[illegible]	97								
14	[illegible]	115			[illegible]	16				
15	[illegible]	84								
16	[illegible]	164	[illegible]	91			[illegible]	56		
17	[illegible]	181					[illegible]	29		
18	[illegible]	46								
19	[illegible]	67	[illegible]	21						
20	[illegible]	88								
21	[illegible]	118							[illegible]	5
22	[illegible]	78							[illegible]	68
23	[illegible]	32								
24	[illegible]	64							[illegible]	11
25	[illegible]	85							[illegible]	12
	Total	1620	Total	213	Total	79	Total	85	Total	96

When the formation of height classes is necessary, the form No. 1 may be modified as follows:

FORM No. 2.—*Diameter measurements.*

Diameter breast high.	Oak.				Hard maple.				Hickory.				Ash.		Black walnut.	
	Height, Class I.		Height, Class II.		Height, Class I.		Height, Class II.		Height, Class I.		Height, Class II.					
		Number of trees of each diameter.		Number of trees of each diameter.		Number of trees of each diameter.		Number of trees of each diameter.		Number of trees of each diameter.		Number of trees of each diameter.		Number of trees of each diameter.		Number of trees of each diameter.
In.																
8																
9																
10																

Form No. 2 would be applicable should the oak, the hard maple, and the hickory of our hard-wood grove differ in height so as to necessitate the formation of two height classes for each of these species. Each height class then will have to be treated like a separate species. Of course only the species of economic value are measured. In the example represented by Form No. 1, five species were supposed to form the stand. In measuring, fractions of less than one-half of an inch are disregarded while those over half an inch are counted as full inches. Each tree calipered is scored in the appropriate species column on its proper diameter line by a mark, each fifth score crossing the four preceding, so that groups of five scores are made for more convenient addition. The measuring can be done more expeditiously if two or more persons divide the labor of scoring, calipering, and marking the calipered trees so as to avoid repetition in measuring; one scorer following two measurers who call out species and measured diameter, and mark the measured trees, or else one or two assistants blaze the trees to keep the work in line, preventing repetition as well as omission of trees.

When all the trees have been scored, the volume of each species may be determined either (1) by felling and measuring in detail a sample tree representing the average of all the trees of the species, or (2) by felling and measuring a number of sample trees, each representing the average of a diameter class, or (3) for the greatest accuracy, by felling and measuring a proportionate number of sample trees for each diameter, the proportion felled being a fixed percentage of the number of trees of each diameter.

DETERMINATION OF VOLUME BY MEANS OF AN AVERAGE SAMPLE TREE.

This method requires for each species (*a*) a calculation to determine the diameter of the average sample tree; (*b*) the selection of the sample tree in the forest and its measurement; and, finally (*c*) the calculation of the total volume of the species.

CALCULATION FOR AVERAGE SAMPLE TREE.

To find the diameter of the average sample tree it is first necessary to find the basal area of its cross section, breast-high. To do this, the basal areas of the cross sections, breast-high, of all the calipered trees must first be found. This is done by finding in the area table (p. 37) the areas corresponding to each diameter represented in the calipered trees, multiplying these figures by the number of trees of that diameter and adding the results.* The addition represents the total basal area of the cross sections of all the trees. If, now, we divide this by the total number of trees of the species, we get the basal area of the average sample tree, and from the area table obtain the diameter corresponding to that basal area.

Applying this method, for instance, to determine the diameter of the average sample tree for the oak, in our area table on page 37, we find that the basal area of—

	Square feet.
23 oaks of 8 inches in diameter equals	8.03
76 oaks of 9 inches in diameter equals	33.58
87 oaks of 10 inches in diameter equals	47.45
172 oaks of 11 inches in diameter equals	113.52
43 oaks of 12 inches in diameter equals	33.77
97 oaks of 13 inches in diameter equals	89.41
115 oaks of 14 inches in diameter equals	122.94
84 oaks of 15 inches in diameter equals	103.08
164 oaks of 16 inches in diameter equals	228.99
181 oaks of 17 inches in diameter equals	285.31
46 oaks of 18 inches in diameter equals	81.29
67 oaks of 19 inches in diameter equals	131.92
88 oaks of 20 inches in diameter equals	191.99
118 oaks of 21 inches in diameter equals	283.83
78 oaks of 22 inches in diameter equals	205.91
32 oaks of 23 inches in diameter equals	92.33
4 oaks of 24 inches in diameter equals	201.06
85 oaks of 25 inches in diameter equals	289.75
	2,534.16

The total basal area of the 1,620 oaks equals 2,534.16 square feet. Dividing this area by the number of trees we find that 1.56 square feet is the basal area of the average sample tree which corresponds to a diameter of 16.9 inches.

SELECTION AND MEASUREMENT OF SAMPLE TREES.

When the diameter of the sample tree has been determined, a thrifty tree of the species with such a diameter should be selected in the forest. Care should be taken that the sample tree is not situated in an opening nor on a road nor in a crowded growth; also that it have an average well-developed crown, and that it be sound, straight, and free from wind shakes. The sample tree so selected is felled, measured, and its

* The product may be obtained directly from the tables of volume, as explained.

volume determined in the way explained on page 16, where the measuring of a felled tree is discussed.

CALCULATION OF THE TOTAL VOLUME OF THE STAND.

The volume of the sample tree thus obtained represents in the average all the trees of the species. The total volume, then, of the species may be determined by multiplying the volume of the sample tree by the number of trees of the species contained in the forest. When a species is represented by a large number of trees it is always advisable to select more than one sample tree and determine separately for each its volume in cubic and superficial feet. There will be noticed a difference between the volumes of the sample trees notwithstanding their diameters and heights are the same; this is due to the difference in the tapering of the sample trees or, in other words, to the difference of the factor of shape, which though small is invariably noticed even among trees of the same dimensions. The average volume of the sample trees, whether in cubic or superficial feet, is then multiplied by the number of trees of the species in the forest, in order to obtain the total volume of the species. For the oak measured and recorded in the above example five sample trees of 16.9 inches in diameter at breast height and of the same height were selected in the grove. All of them were felled and sawed into logs. The following are their actual volumes given in cubic and superficial feet:

Sample tree.	Cubic feet.	B. M.
No. 1	46.2	180
No. 2	48.3	204
No. 3	44.8	156
No. 4	47.5	192
No. 5	42.1	148

These five sample trees give in the average 45.8 cubic feet and 176 feet B. M. Multiplying these two averages by 1,620 (number of oaks) the volume of the oak equals: (1) $45.8 \times 1{,}620 = 74{,}196$ cubic feet; (2) $176 \times 1{,}620 = 285{,}120$ B. M. The same operations and calculations are made for each species of the stand.

The following form (No. 3) shows how the measurements are finally collated for computing the growing stock of the grove:

FORM No. 3.—*Showing the computing of growing stock by means of average sample trees.*

Name of species. (Average height, 60 feet.)	Diameter at breast height.	Number of trees of each diameter.	Basal area of each diameter.	Sample tree.					Volume of growing stock.	
				Average dimensions.		Sample tree No.	Volume.			
				Basal area.	Corresponding diameter.		Cubic feet.	B. M.	Cubic feet.	B. M.
1	2	3	4	5	6	7	8	9	10	11
	Inches.		*Sq. feet.*	*Sq. feet.*						
Oak	8	23	8.03							
	9	76	33.58	......	......	1	46.2	180		
	10	87	47.45							
	11	172	113.52							
	12	43	33.77	......	......	2	48.3	204		
	13	97	89.41							
	14	115	122.94							
	15	84	103.08	......	......	3	44.8	156		
	16	164	228.99							
	17	181	285.31							
	18	46	81.29	......	......	4	47.5	192		
	19	67	131.92							
	20	88	191.99							
	21	118	283.83	......	......	5	42.1	148		
	22	78	205.91							
	23	32	92.33							
	24	64	201.06							
	25	85	289.75	1.56	16.9					
Total	......	1,620	2,534.16	......	......	......	......	......	74,196	285,120
Average	......	......	......	......	......	......	45.8	176		
Hard maple	10	26	14.18							
	11	75	49.50							
	16	91	127.06							
	19	21	41.35	1.09	14.1	1	32.7	120		
Total	......	213	232.09	......	......	......	......	......	6,965	25,560
Hickory	12	63	49.48							
	14	16	17.10	0.84	12.4	1	25.5	90		
Total	......	79	66.58	......	......	......	......	......	2,014	7,110
Ash	16	56	78.19							
	17	29	45.71	1.46	16.4	1	43.8	168		
Total	......	85	123.90	......	......	......	......	......	3,723	14,280
Black walnut	21	5	12.03							
	22	68	179.51							
	24	11	34.56							
	25	12	40.91	2.78	22.6	1	83.4	336		
Total	......	96	267.01	......	......	......	......	......	8,006	32,256

Total contents, 94,904 cubic feet; 364,326 B. M.

Having recorded in columns 1 to 9 the measurements and calculations of all the trees and of the respective sample trees for each species, the volumes in columns 10 and 11 are found by multiplying the volume of the sample tree by the number of trees of each species, and to obtain the volume of the growing stock of the whole grove the last two columns are added up, which addition shows the grove to contain 94,904 cubic feet of wood, from which 364,326 feet B. M. might be obtained, the balance to be turned into slabs, sawdust, firewood, etc.

DETERMINING VOLUME BY SAMPLE TREES OF DIAMETER CLASSES.

More accurate results in ascertaining the growing stock of a forest may be obtained by arranging the trees of each species in diameter classes and then finding and measuring a sample tree of each class. The calculation to determine the basal area and, hence, the diameter of the sample tree of a diameter class, the selection of the sample tree in the forest and its measuring and the calculation of the volume of the diameter class are performed in the same way as described above. Each diameter class is to comprise trees differing not more than 4 inches in diameter at breast height. The oak of our hard-wood grove, then, would be divided into five diameter classes. The first diameter class would contain trees from 8 to 11 inches in diameter, inclusive; the second, trees from 12 to 15 inches, inclusive; the third, trees from 16 to 19 inches, inclusive; the fourth, trees from 20 to 23 inches, inclusive; and the fifth, trees from 24 to 25 inches, inclusive.

The first diameter class would then contain:

	No. of trees.	Basal area.
		Square feet.
8 inches in diameter	23	8.03
9 inches in diameter	76	33.58
10 inches in diameter	87	47.45
11 inches in diameter	172	113.52
Total	358	202.58

The basal area of this diameter class, 202.58 square feet divided by 358, the number of trees it contains, gives the basal area of its sample tree as 0.56 square foot, which corresponds to a diameter of 10.1 inches. Two sample trees of 10.1 inches selected accordingly in the forest among the oaks had in the average a volume of 16.8 cubic feet and scaled 60 feet B. M. Multiplying 16.8 and 60 each by 358 (number of trees in the class), we obtain 6,014.4 cubic feet and 21,480 feet B. M., which is the volume of the first diameter class in cubic and superficial feet respectively. The same process is repeated for the other diameter classes and species, selecting and measuring a smaller or larger number of sample trees as the diameter class contains a smaller or larger total number of trees. The final addition gives the volume of the stand. The accompanying table (Form No. 4) illustrates in detail the manner of recording and computing the growing stock of our hard-wood grove by arranging each species in diameter classes.

FORM No 4.—*Showing the computing of growing stock by arranging the species in diameter classes.*

Name of species.	Diameter at breast height.	Number of trees of each diameter.	Basal area of each diameter.	Diameter classes. Class number.	Diameter classes. Number of trees.	Diameter classes. Basal area.	Sample tree. Average dimension. Basal area.	Sample tree. Average dimension. Corresponding diameter.	Sample tree. Average actual volume. Cubic feet.	Sample tree. Average actual volume. B. M.	Number of sample tree of each size class.	Volume of each diameter class. Cubic feet.	Volume of each diameter class. B. M.
	In.		*Sq. ft.*			*Sq. ft.*	*Sq. ft.*	*Inches.*					
Oak	8	23	8.03	1	358	202.58	0.56	10.1	16.8	60	2	6,014.4	21,480
	9	76	33.58										
	10	87	47.45										
	11	172	113.52										
	12	43	33.77	2	339	349.20	1.03	13.7	30.9	120	2	10,475.1	40,680
	13	97	89.41										
	14	115	122.94										
	15	84	103.08										
	16	164	228.99	3	458	727.51	1.59	17.1	47.7	192	3	21,846.6	87,936
	17	181	285.31										
	18	46	81.29										
	19	67	131.92										
	20	88	191.99	4	316	747.06	2.36	20.8	70.8	336	2	22,372.8	106,176
	21	118	283.83										
	22	78	205.91										
	23	32	92.33										
	24	64	201.06	5	149	480.81	3.23	24.3	96.9	480	1	14,438.1	71,520
	25	85	289.75										
Hard maple	10	26	14.18	1	101	63.68	0.63	10.7	18.9	70	1	1,908.9	7,070
	11	75	49.50										
	16	91	127.06	2	112	168.41	1.50	16.6	45.0	180	1	5,040	20,160
	19	21	41.35										
Hickory	12	63	49.48	1	79	66.58	0.84	12.4	25.5	90	1	2,014	7,110
	14	16	17.10										
Ash	16	56	78.19	1	85	123.90	1.46	16.4	43.8	168	1	3,723	14,280
	17	29	45.71										
Black walnut	21	5	12.03	1	73	191.54	2.62	21.9	78.6	384	2	5,737.8	28,032
	22	68	179.51										
	24	11	34.56	2	23	75.47	3.28	24.5	98.4	480	1	2,263.2	11,040
	25	12	40.91										

Total contents, 95,834 cubic feet; 415,484 B. M.

The black walnut in the blank is divided into two diameter classes to maintain the uniformity of the diameter classification adopted for our hard-wood grove; otherwise all the trees of the black walnut could have been included in one diameter class.

DETERMINING VOLUME BY FELLING AND MEASURING A PROPORTIONAL NUMBER OF SAMPLE TREES FOR EACH DIAMETER.

Still greater accuracy of result can be obtained if instead of choosing at haphazard a number of sample trees of each diameter class, a definite proportion of the trees of each class or of each diameter is used for the computation. For instance, we may decide to measure 1 per cent of the trees of each diameter. All sizes of timber are then represented by sample trees in the proportion in which they occur in the forest; we have in the sample trees, then, an exact counterpart of the entire growth reduced in proportion. The relation between the volumes of the whole forest and the proportionately reduced forest of sample trees is exactly the same as that which exists between their corresponding basal areas; hence, dividing the basal area of the whole forest by the basal area of all the sample trees, and multiplying the quotient thus obtained by the volume of all the sample trees, the growing stock of the forest is determined. This method requires neither calculations to determine the dimensions of the sample trees nor the separate measuring of each to determine the volume. All the sample trees of the corresponding diameters are directly selected in the forest, felled and sawed into logs of desired length, which logs are piled together and the volume of the pile determined as a whole in cords or in cubic feet. Or else the number of superficial feet of all the sample trees can be accurately and directly obtained by sawing the logs into boards and other kinds of lumber.

The choice of the per cent or the proportion of trees to be taken as sample trees is influenced by the accuracy to be attained and the size of the area to be measured. If a tolerably satisfactory representation is to be had, not less than 10 to 15 trees, or at least 1 per cent, should be used.

Suppose that in order to determine the volume of the 1,620 oaks of our hard-wood grove, recorded in Form No. 1 (p. 19), it was decided to take 1 per cent, or in all 17 sample trees. If the fraction is less than one-half it is disregarded; if more than one-half it is considered as one. Then the number of sample trees for the oak would be determined as follows:

23 trees of 8 inches diameter require … $\frac{23}{100}$—no sample tree.
76 trees of 9 inches diameter require … $\frac{76}{100}$—one sample tree.
87 trees of 10 inches diameter require … $\frac{87}{100}$—one sample tree.
172 trees of 11 inches diameter require … $\frac{172}{100}$—two sample trees.
43 trees of 12 inches diameter require … $\frac{43}{100}$—no sample tree.
97 trees of 13 inches diameter require … $\frac{97}{100}$—one sample tree.
115 trees of 14 inches diameter require … $\frac{115}{100}$—one sample tree.
84 trees of 15 inches diameter require … $\frac{84}{100}$—one sample tree.
164 trees of 16 inches diameter require … $\frac{164}{100}$—two sample trees.
181 trees of 17 inches diameter require … $\frac{181}{100}$—two sample trees.
46 trees of 18 inches diameter require … $\frac{46}{100}$—no sample tree.
67 trees of 19 inches diameter require … $\frac{67}{100}$—one sample tree.
88 trees of 20 inches diameter require … $\frac{88}{100}$—one sample tree.
118 trees of 21 inches diameter require … $\frac{118}{100}$—one sample tree.
78 trees of 22 inches diameter require … $\frac{78}{100}$—one sample tree.
32 trees of 23 inches diameter require … $\frac{32}{100}$—no sample tree.
64 trees of 24 inches diameter require … $\frac{64}{100}$—one sample tree.
85 trees of 25 inches diameter require … $\frac{85}{100}$—one sample tree.

The 17 sample trees of the corresponding diameters are then selected in the forest, felled, and sawed up into logs, which are piled together with the tops of all the sample trees. Let the pile be equal to 6½ cords, or 832 cubic feet. Let us suppose that the 6½ cords were sawed into lumber and furnished 3,360 feet B. M. From the measurements with the calipers, recorded in Form No. 1, we know the basal area of the oak to be equal to 2,534.16 square feet; the basal area of 17 sample trees we find in the area table to be equal to 27.22 square feet. Dividing 2,534.16 by 27.22 we obtain a quotient equal to 93.1. Multiplying 93.1 by 832, the volume of the sample trees in cubic feet, we determine the volume of the oak to be 77.459 cubic feet; or, multiplying the quotient, 93.1, by 3,360, the number of superficial feet furnished by the sample trees after they passed through the mill, we obtain 312,816 feet B. M., which is the total amount of merchantable lumber contained in the oak of our hardwood grove.

The volume of the other species may be determined in the same manner, and then the growing stock of the grove is obtained by adding together the volume of the trees of all its species.

DETERMINATION OF THE GROWING STOCK BY MEANS OF SAMPLE AREAS.

It is always possible to find in a forest a small area the contents of which represent an average proportion of either the whole forest or of at least a considerable portion of it. The volume of this small area may be easily and rapidly determined by one of the methods above described. Such an area may be called a sample area, and the contents found on it per acre may be called an acre yield. If the small area represents an average condition of the whole forest, then, in order to obtain the growing stock of the whole forest, the acre yield of the sample area need only be multiplied by the number of acres in the forest; when the sample area represents only the conditions of a portion of the forest, then the acre yield multiplied by the number of acres involved in that portion gives only the growing stock of that portion, and for other portions of different conditions corresponding acre yields must be found.

Example: Let a forest containing 100 acres have three distinct forest conditions represented, each by 40, 35, or 25 acres, respectively. Let the 40 acres be represented by a sample area of one-half an acre; the 35 acres by a sample area of 1½ acres, and the 25 acres by a sample area of one-fourth an acre. Let the volumes of the sample areas determined by one of the methods given above be—

(1) The volume of the one-half acre equals 3,000 cubic feet and 12,000 B. M.
(2) The volume of the 1½ acre equals 12,000 cubic feet and 48,000 B. M.
(3) The volume of the one-fourth acre equals 2,500 cubic feet and 10,000 B. M.

The acre yields of the corresponding portion of the forest equal then—

(1) 3,000 cubic feet and 12,000 feet B. M. multiplied each by 2 equals 6,000 cubic feet and 24,000 feet B. M.

(2) 12,000 cubic feet and 48,000 feet B. M. divided each by 1½ equals 8,000 cubic feet and 32,000 feet B. M.

(3) 2,500 cubic feet and 10,000 feet B. M. multiplied each by 4 equals 10,000 cubic feet and 40,000 feet B. M.

The volume of an acre of the forest condition represented by the 40 acres equals 6,000 cubic feet and 24,000 feet B. M.; multiplied each by 40 equals 240,000 cubic feet and 960,000 feet B. M.

The volume of an acre of the forest condition represented by the 35 acres equals 8,000 cubic feet and 32,000 feet B. M.; multiplied each by 35 equals 280,000 cubic feet and 1,120,000 feet B. M.

The volume of an acre of the forest condition represented by 25 acres equals 10,000 cubic feet and 40,000 feet B. M.; multiplying each by 25 gives 250,000 cubic feet and 1,000,000 feet B. M. Adding together the volume of these three portions of the forest, we find the growing stock of the forest equals 770,000 cubic feet or 3,080,000 feet B. M.

In selecting the sample area care should be taken that—

(1) The species found in the forest be proportionally represented on the sample area.

(2) The density of crown cover of the sample area and the percentage of openings be the same as in the forest.

(3) All the sizes of timber and the corresponding number of trees of each size be found on the sample area in the same proportion as found among the trees in the whole forest or the portion to which the sample area refers. The selection of the sample area is a delicate operation and therefore requires considerable skill on the part of the estimator. The selected sample area should be staked off in the form of a square, which may contain from one-fourth to 2 acres.

HOW TO DETERMINE THE RATE OF GROWTH.

As the knowledge of the contents of a forest growth is necessary in order to determine its present value for purposes of sale or purchase, so the knowledge of the rate at which its contents are changing, increasing or decreasing is of the highest importance in determining the profitableness of wood growth. The questions whether the annual or periodic increase is sufficient to pay interest on the investment, and whether it is proper to cut and utilize the wood crop or to allow it to grow and accumulate longer, are answered by measuring its rate of growth.

Just as the contents of a forest or acre or stand is ascertained by means of measuring one or more sample trees, the rate of growth of the stand, acre, or forest for any period may be ascertained from these sample trees. This calculation may concern itself either with the rate at which the height accretion takes place, or the diameter accretion, or the volume or mass accretion. It may also be made with reference to a longer or shorter period. If the period is taken as one year we may call it annual or yearly accretion; if for a number of years, for instance a decade, we may call it periodic accretion. Again, the annual accretion may be that of the one year for which we measure, the current annual accretion; or else it may be the average of a number of years, the average annual accretion, which is found by dividing the height, diameter, or volume by the number of years it has taken to grow. For instance, a tree 120 years old, containing 87 cubic feet, would show an average annual accretion of $\frac{120}{87}=0.72$ cubic feet, while its current annual accretion for the one hundred and twentieth year may be 1.4 cubic feet; and if we had ascertained the volume which it formed in the last ten years, as 15 cubic feet, this would be the periodic accretion for that decade.

The measurements by which the accretion, annual or periodic, is ascertained, rely upon the fact that, in all temperate zones at least, trees form annually one layer of wood, which appears on a cross section of a tree as a ring, more or less clearly defined, and on its longitudinal section made through the pith as a section of an enveloping cone (fig. 10). Hence by counting and measuring the rings appearing on cross sections taken at various heights from the ground, or by counting and measuring the enveloping cones appearing on the corresponding longitudinal sections made through the pith, not only the age, the progress in diameter, and area increase of the sections, but its height and volume development can be easily and accurately ascertained. Let us, for example, analyze the tree represented in fig. 10: A represents the longitudinal section of the tree made through its pith; B represents the tree in cross sections, made (1) at the surface of the ground; (2) at 13 feet; (3) at 25 feet; (4) at 37 feet, and (5) at 49 feet from the ground; the total height of the tree is 54 feet. Each ring of a cross section corresponds to an enveloping cone, and the number of concentric rings counted on a cross section, as seen from fig. 10, corresponds with the number of enveloping cones counted above each section.

Just as the width of the concentric rings on both sides of the center on a cross section determines the annual increase of the diameter, so the distance between the apexes of two enveloping cones determines the annual increase of the height. It is clear that the difference between the number of rings counted at the bottom and top sections of a log gives the number of years which it has taken to produce the length of the log. Or, if we take the lowest section of the tree, cut so that all the years of its growth are contained in the section (as in fig. 10), and deduct from the number of rings found on this section the number of rings found on any higher section, the difference then equals the number of years during which the tree had grown to attain the height of the higher section. Or, again, the number of rings counted on a cross section gives also the period of time during which the portion of the tree situated above has developed its height. Thus we find that during the period of forty-four years, the age of the tree, the trunk has reached 54 feet in height. The average annual growth in height is therefore equal to 54 feet divided by 44, equals 14.7 inches.

From the second cross section we find that the tree had grown 40 feet in the last thirty-one years (number of rings on that section), which means 15.8 inches annually during that period; or subtracting from the total age of the tree (44) the age of the second cross section (31) we find that the tree during the first thirteen years of its life has reached the height of the second section, i. e. 13 feet, which means that the tree

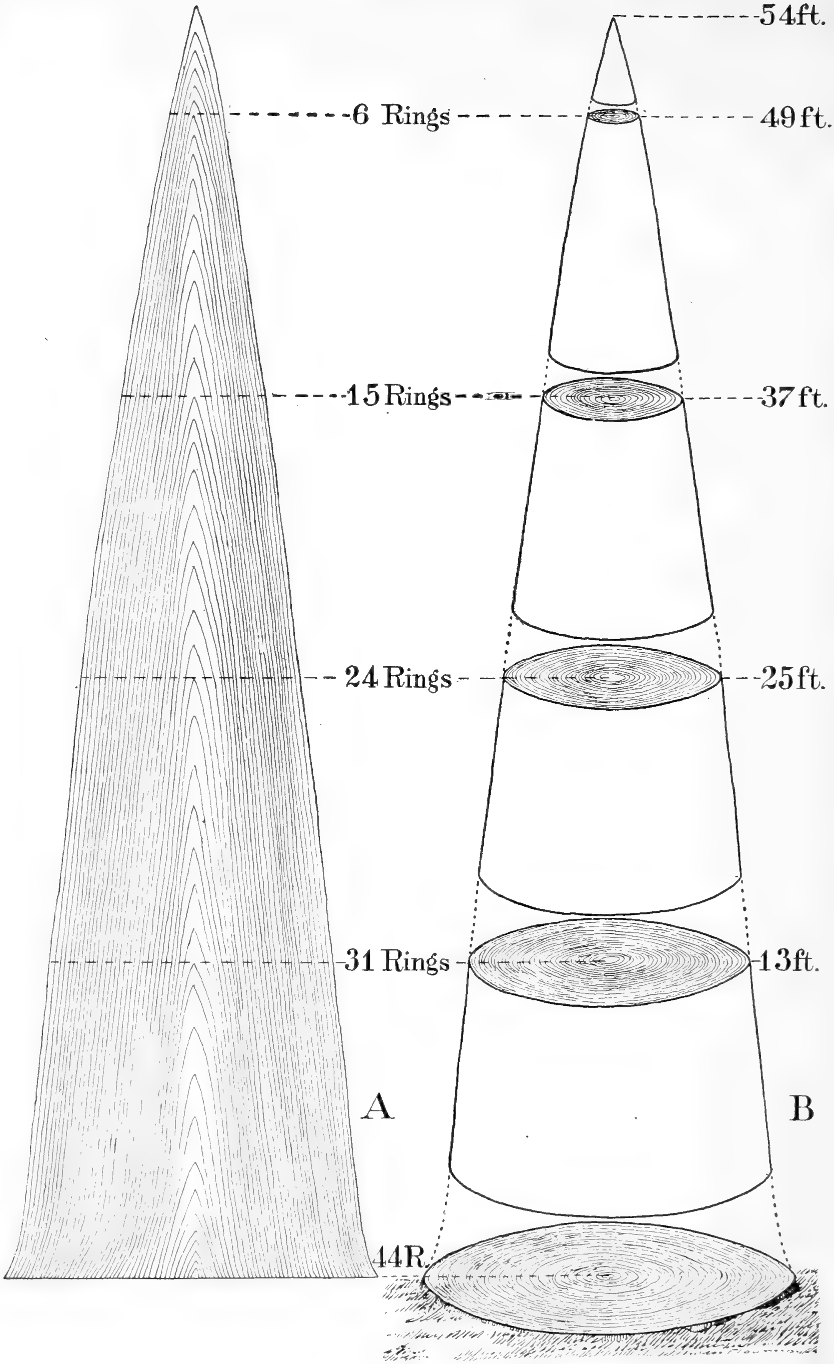


FIG. 10.—Showing the progressive development of a tree. A. Longitudinal section made through the pith and showing the sections of the enveloping cones. B. Cross-sections showing number of rings at various heights from the ground.

during this period grew annually 1 foot in height. By similar reasoning we find from the third cross section that the tree had grown 29 feet for the last twenty-four years, or 14.5 inches annually, and 25 feet for the first twenty years, or 15 inches annually; from the fourth cross section that the tree had grown 17 feet for the last fifteen years, or 13.6 inches annually, and 37 feet for the first twenty-nine years, or 15.2 inches annually; from the fifth cross section that the tree had grown 5 feet for the last six years, or 10 inches annually, and 49 feet for the first thirty-eight years, or 15.6 inches annually.

The rate at which the diameter and the area of any cross section of the tree increases can be easily ascertained by measuring the width of the rings on the various cross sections and finding in the table (p. 37) their corresponding areas. Thus we find, for instance, that on the second cross section—

The first 10 rings measure 3.9 inches.
The first 20 rings measure 6.2 inches.
The first 30 rings measure 7.9 inches.

The corresponding areas found in the table are, respectively, 0.8, 0.21, and 0.33 square feet. Subtracting either the diameters from each other or the corresponding areas, we can ascertain accurately the growth in diameter or area for the respective periods of time.

The rate at which the volume of the tree increases may be easily determined for any period of time by calculating the volume of as many enveloping cones as there are years in the period. Various methods may be employed to ascertain, for instance, the volume of the last period of years. The simplest one is: (1) To determine the volume of the upper portion of the tree, which has for its base a cross section containing as many rings as there are years in the period by considering that portion as a paraboloid; (2) determine the volume of each of the logs into which the lower portion of the tree is sawed, with and without the width of the last number of rings (number of years in the period), as explained on page 16, and deduct the sum of the last from the first volume; (3) adding to the difference thus obtained the volume of the upper portion, the growth for the desired period of time is ascertained. For the tree represented in fig. 10, the mass-accretion for the last 6, 15, 24, and 31 years could be conveniently ascertained, and thus the current annual accretion for those respective periods accurately calculated. Generally trees are not analyzed with such completeness, and simple methods have been devised for determining the accretion of a single tree or a forest.

DETERMINING THE ACCRETION OF A STANDING TREE.

In determining the average annual accretion the age and volume of the tree must be first ascertained. The age of a standing tree can be obtained only by observation, which is based on actual counting of the rings on stumps of felled trees of the same size, same species, and grown on the same site, or at least in the same locality and under the same conditions.

In determining the current accretion it is better to establish the increase of volume for the last five or ten years and assume that the current accretions were the same annually during that period; it is safer to make this assumption than to deal with a single year's increase, which is an unstable quantity changing with the season.

The current accretion of a standing tree may be conveniently expressed in per cent of volume of the tree. If the increase of actual volume is to be expressed, then it ought to be calculated with simple interest; but if the mass of a tree is looked upon as a capital, then it is proper to consider the accretions as returns on the capital represented by the amount of wood and to calculate it with compound interest in order to establish the expediency and profitableness of the investment.

MASS ACCRETION WITH SIMPLE INTEREST.

If the present volume of a standing tree is 115 cubic feet, and that of the same tree five years ago 109 cubic feet, then $115 - 109 = 6$ gives the increase of volume for the last five years; the accretion for one year is $\frac{6}{5} = 1.2$ cubic feet. Dividing 1.2 by 109 and multiplying the quotient by 100 we find the current annual accretion equals 1 per cent expressed in per cent of volume. But while the present volume of the tree can be easily determined by employing one of the described methods, the volume which the standing tree had five years ago is difficult to establish. It is necessary, therefore, in order to determine the current accretion of standing trees to devise a method which should not require the determination of the present and past volumes of the tree. Suppose a standing tree, the accretion of which we are to determine, has a basal area at breast height, which we will designate for convenience sake by a letter A; let the basal area which the tree had five years ago be a, then the present and past volumes of the tree may be represented by the following products:

(1) Present volume: Base A multiplied by one-half of the height of the tree.

(2) Volume five years ago: Base a multiplied by one-half of the height which the tree had five years ago.

Suppose also that the tree is considered after it has reached its full-height growth (a number of species reach it before 100 years of age), then the height accretion for five years is comparatively small. Disregarding this small difference, the proportion between the present and past volumes of the standing tree equals the proportion which exists between their basal areas A and a; in other words, the per cent of volume accretion is the same as that of the area accretion. The per cent of the area accretion may be easily determined when the diameter which the tree had five years ago is established. This can be ascertained by cutting out a chip or else by using an instrument—Pressler's increment borer,* by means of which a cylinder of wood can be extracted from the stem and the width of the rings measured. Taking twice the width of the last five rings and subtracting it from the present diameter (at breast height), the diameter the tree had five years ago is nearly enough determined.

Example: Let the present diameter of a standing tree be 22½ inches at breast height; let the width of the first five rings from the periphery, measured on the cylinder extracted by the Pressler borer, or on the chip of wood cut out, be two-eighths of an inch. Multiplying the two-eighths by 2 gives one-half inch as the diameter increment for the last five years, and subtracting the half inch from 22½, we find that the diameter the tree had five years ago equals 22 inches. In the tables for areas (page 37) we find that the area corresponding to 22½ is 2.76 square feet; that corresponding to 22 inches, 2.64; the difference (2.76—2.64=0.12)

* Pressler's apparatus (fig. 11) consists of a hollow borer, slightly tapering from the handle toward the point, inserted into a handle; a flat-toothed wedge, which for convenience of measurement is graduated into centimeters; and a cradle, being of a small semicylindrical piece of tin, used to hold the chip when measuring in order to avoid its breaking; the handle also is hollow, so as to receive the borer, wedge, and cradle when the instrument is not in use. The borer is screwed in a radial direction into the tree, at right angles to its axis, to the desired depth, whereby a cylindrical column or chip of wood enters the hollow borer; then the wedge is inserted through the hollow borer between the chip and the inner wall of the borer, with its toothed side toward the wood and firmly pressed in. The borer is now screwed backward one or two turns, whereby the chip is severed at its base from the tree; a few more forward turns of the borer cause the chip to be pushed back until it can easily be withdrawn by the use of the wedge and placed into the cradle. In this way a chip of wood is obtained from 2 to 5 inches long, according to the length of the borer. The width of the concentric rings is then measured. If the rings are not distinct, a smooth surface may be prepared with a sharp knife.

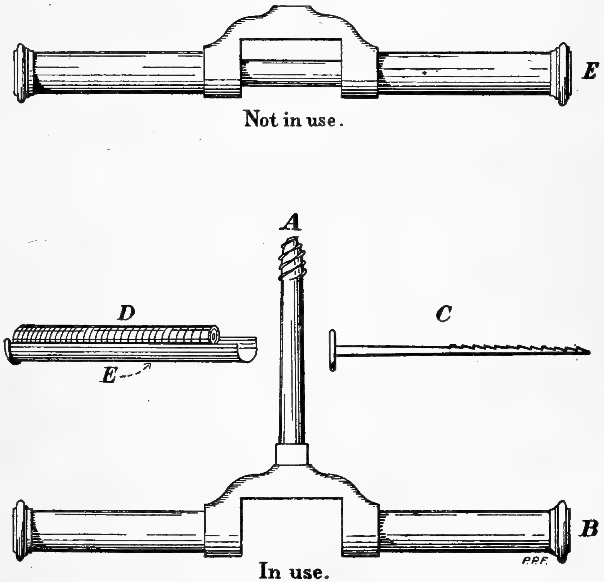


FIG. 11.—Pressler's accretion borer.

divided by 5 gives 0.024 as the current area accretion for one year; dividing this one-year growth by 2.64 and multiplying the result by 100, we find that 6 is the per cent of area, and thus of volume accretion of that particular tree of 22½ inches in diameter at breast height. The per cent of volume accretion, as has been seen, may be expressed by a fraction, the numerator of which is the volume increase for one year, and the denominator is the volume of the tree previous to that year.

The difference between the successive current accretions, though increasing with age, are small in proportion to the increase of the respective volumes, which are always enlarged by one year's growth. In other words, the fraction or the per cent of accretion it represents decreases steadily with age.

Pressler gives a simple formula ($\frac{100}{A}$) which expresses the per cent of accretion of the tree when it has reached its maximum stage of growth. "A" is the age of the tree when it reaches the stage of maximum growth, i. e., when the current accretion becomes equal to the average annual accretion. If the per cent of accretion of the tree obtained from calculations is larger than $\frac{100}{A}$, it shows that the average annual accretion still inc eases; when it is less than $\frac{100}{A}$, the average annual accretion is on the decrease.

MASS ACCRETION WITH COMPOUND INTEREST.

In determining the mass accretion with compound interest the general formula of compound interest could be applied. To avoid calculations by logarithms Pressler gives a formula of his own, and a table of figures based on it, the practical application of which is very simple: Measure the diameter of the standing tree at breast height; extract by Pressler's borer a cylinder of wood and measure off the width of the last n years (n designates the number of years in the period for which the calculation is to be made, generally five or ten years); then divide the diameter by double the width of the last n rings, and the so called relative diameter is established. Finding then the relative diameter, thus obtained, in the column of the relative diameters (Pressler's table, p. 40); the corresponding number, given in the same line with the relative diameter, should be divided by n, and the quotient will be the per cent of accretion with compound interest.

Example: Let us take the same tree for which the per cent of accretion was determined with simple interest; its present diameter at breast height is 22½ inches; the width for the last five years is two-eighths of an inch. Dividing 22½ by double the width of the last 5 rings, we find the relative diameter equals 45: ($22\frac{1}{2} \div \frac{1}{2} = 45$). In Pressler's table we find that 6.7 corresponds to the relative diameter of 45 when the tree is of a very thrifty growth. Dividing 6.7 by 5 we find the current annual growth equals 1.3 per cent with compound interest.

In Pressler's table on page 40, for each relative diameter two figures are given—one for an average thrifty growing tree, the other for a very thrifty growing tree. The general appearance and the crown development of the tree will indicate which one of the figures should be taken for calculations.

DETERMINING THE ACCRETION OF A FELLED TREE.

The average annual accretion of a felled tree may be determined with greater accuracy, because the volume and the age of the tree can be obtained with more exactness. The age of the tree is established by counting the rings on the stump section and adding to the number counted five or six years, which were required by the tree to reach the height of the stump.

The current accretion for any given number of years of a felled tree may be determined as follows:

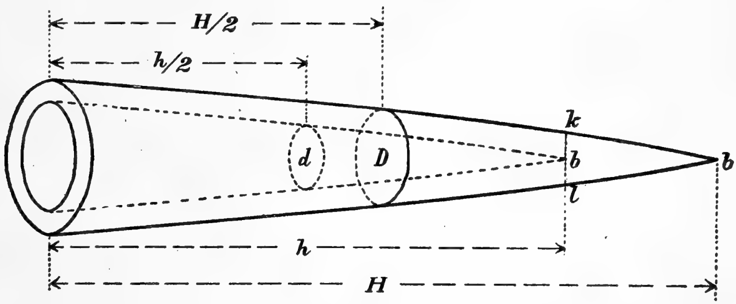


FIG. 12.—Determining the current accretion of a felled tree.

Find the volume of the felled tree by multiplying its height (H) by the basal area which corresponds to the diameter measured at the middle of the tree (D), bark excluded; then, by some trial, find a place near the top (see fig. 12) where the section (*kl*) contains as many rings as there are years in the period (*n*) for which the accretion is to be determined. Then from the middle of the topless portion (*d*) of the tree extract, with Pressler's accretion borer, a cylinder of wood; measure off on that cylinder the width of the last *n* rings and subtract twice that width from the outside diameter measured here without bark. The difference gives the diameter the tree had *n* years ago. Multiplying the basal area corresponding to that diameter by the length (*h*) of the topless portion of the tree the volume of the tree as it was *n* years ago is ascertained. The difference between the present and past volumes gives the periodic accretion for the last *n* years.

Pressler simplifies this method by measuring at the middle of the topless stem $\left(\frac{h}{2}\right)$ both the present diameter and that of the tree as it was *n* years ago and calculates the respective volumes by employing the length of the topless stem. The excess in volume which results for the present tree due to the fact that the diameter which should have been taken halfway of the full length tree $\left(\text{at } \frac{H}{2}\right)$ was measured too low, is counterbalanced by neglecting the volume of the top for the n years.

DETERMINING THE ACCRETION OF A FOREST.

There are several methods employed in determining the mass accretion of a stand of trees for a short period (from ten to twenty years), during which it may be safely assumed that the number of trees of the stand will not diminish owing to natural thinning (death). Of these the following may be recommended:

When the growing stock of a forest has been ascertained by means of an average sample tree, then the accretion of that tree for a given number of years should be multiplied by the number of trees the forest contains, in order to determine the rate of growth of the forest for that period of time. In case the growing stock of the forest was established by arranging its trees in diameter classes, the accretion of each diameter class is determined separately by multiplying the accretion of the sample tree by the number of trees involved in the class. Adding together the accretion of all the diameter classes we obtain a sum which represents the rate of growth of the forest for the period of time in question.

When the growing stock of a forest is determined by means of a proportional number of sample trees representing each diameter, the accretions of the sample trees, calculated for each of them separately, are added together and the sum is multiplied by the quotient obtained by dividing the basal area of the forest by that of all the sample trees. The product gives the rate of growth of the forest during the period for which the accretion of the sample trees was determined.

Table of areas of circles for diameters of 1 inch to 60 inches.

Diameter.	Area of circle.	Diameter.	Area of circle.	Diameter.	Area of circle.	Diameter.	Area of circle.	Diameter.	Area of circle.	Diameter.	Area of circle.
In.	*Sq. ft.*	*In.*	*Sq. ft.*	*In.*	*Sq. ft.*	*In.*	*Sq. ft.*	*In.*	*Sq. ft.*	*In.*	*Sq. ft.*
1.0	0.0055	3.0	0.0491	5.0	0.1364	7.0	0.2673	9.0	0.4418	11.0	0.6600
.1	.0067	.1	.0524	.1	.1418	.1	.2750	.1	.4517	.1	.6721
.2	.0079	.2	.0559	.2	.1474	.2	.2828	.2	.4617	.2	.6842
.3	.0092	.3	.0594	.3	.1532	.3	.2907	.3	.4718	.3	.6965
.4	.0107	.4	.0631	.4	.1590	.4	.2987	.4	.4820	.4	.7089
.5	.0123	.5	.0669	.5	.1650	.5	.3068	.5	.4923	.5	.7214
.6	.0140	.6	.0707	.6	.1710	.6	.3151	.6	.5027	.6	.7340
.7	.0158	.7	.0747	.7	.1772	.7	.3234	.7	.5132	.7	.7467
.8	.0177	.8	.0788	.8	.1835	.8	.3319	.8	.5238	.8	.7595
.9	.0197	.9	.0830	.9	.1899	.9	.3404	.9	.5345	.9	.7724
2.0	.0218	4.0	.0873	6.0	.1963	8.0	.3491	10.0	.5454	12.0	.7854
.1	.0240	.1	.0917	.1	.2029	.1	.3579	.1	.5564	.1	.7986
.2	.0264	.2	.0963	.2	.2096	.2	.3668	.2	.5675	.2	.8118
.3	.0289	.3	.1009	.3	.2164	.3	.3758	.3	.5787	.3	.8252
.4	.0314	.4	.1056	.4	.2234	.4	.3849	.4	.5900	.4	.8387
.5	.0341	.5	.1105	.5	.2304	.5	.3941	.5	.6014	.5	.8523
.6	.0369	.6	.1154	.6	.2376	.6	.4034	.6	.6129	.6	.8660
.7	.0398	.7	.1205	.7	.2448	.7	.4129	.7	.6245	.7	.8798
.8	.0428	.8	.1257	.8	.2522	.8	.4224	.8	.6362	.8	.8937
.9	.0459	.9	.1310	.9	.2597	.9	.4321	.9	.6481	.9	.9077
13.0	0.9218	16.4	1.4670	19.8	2.1382	23.2	2.9356	26.6	3.8591	30	4.9087
.1	.9360	.5	1.4849	.9	2.1599	.3	2.9610	.7	3.8882	31	5.2414
.2	.9504	.6	1.5030	20.0	2.1817	.4	2.9864	.8	3.9174	32	5.5851
.3	.9648	.7	1.5212	.1	2.2036	.5	3.0120	.9	3.9467	33	5.9396
.4	.9794	.8	1.5394	.2	2.2256	.6	3.0377	27.0	3.9761	34	6.3050
.5	.9941	.9	1.5578	.3	2.2477	.7	3.0635	.1	4.0056	35	6.6813
.6	1.0089	17.0	1.5763	.4	2.2699	.8	3.0894	.2	4.0353	36	7.0686
.7	1.0237	.1	1.5949	.5	2.2922	.9	3.1154	.3	4.0650	37	7.4667
.8	1.0387	.2	1.6136	.6	2.3146	24.0	3.1416	.4	4.0948	38	7.8758
.9	1.0538	.3	1.6324	.7	2.3371	.1	3.1679	.5	4.1248	39	8.2958
14.0	1.0690	.4	1.6513	.8	2.3597	.2	3.1942	.6	4.1548	40	8.7266
.1	1.0843	.5	1.6703	.9	2.3825	.3	3.2207	.7	4.1850	41	9.1684
.2	1.0997	.6	1.6894	21.0	2.4053	.4	3.2471	.8	4.2152	42	9.6211
.3	1.1153	.7	1.7087	.1	2.4283	.5	3.2748	.9	4.2456	43	10.0847
.4	1.1309	.8	1.7280	.2	2.4514	.6	3.3006	28.0	4.2761	44	10.5592
.5	1.1467	.9	1.7475	.3	2.4745	.7	3.3275	.1	4.3067	45	11.0447
.6	1.1626	18.0	1.7671	.4	2.4978	.8	3.3545	.2	4.3374	46	11.5410
.7	1.1785	.1	1.7868	.5	2.5212	.9	3.3816	.3	4.3681	47	12.0482
.8	1.1946	.2	1.8066	.6	2.5447	25.0	3.4088	.4	4.3991	48	12.5664
.9	1.2108	.3	1.8265	.7	2.5684	.1	3.4361	.5	4.4301	49	13.0954
15.0	1.2272	.4	1.8465	.8	2.5921	.2	3.4636	.6	4.4612	50	13.6354
.1	1.2437	.5	1.8666	.9	2.6159	.3	3.4911	.7	4.4925	51	14.1863
.2	1.2602	.6	1.8869	22.0	2.6398	.4	3.5188	.8	4.5238	52	14.7480
.3	1.2768	.7	1.9072	.1	2.6638	.5	3.5465	.9	4.5553	53	15.3207
.4	1.2936	.8	1.9277	.2	2.6880	.6	3.5744	29.0	4.5869	54	15.9043
.5	1.3104	.9	1.9482	.3	2.7122	.7	3.6024	.1	4.6186	55	16.4988
.6	1.3274	19.0	1.9689	.4	2.7366	.8	3.6305	.2	4.6504	56	17.1042
.7	1.3444	.1	1.9897	.5	2.7611	.9	3.6587	.3	4.6823	57	17.7206
.8	1.3616	.2	2.0206	.6	2.7857	26.0	3.6870	.4	4.7143	58	18.3478
.9	1.3789	.3	2.0316	.7	2.8104	.1	3.7154	.5	4.7464	59	18.9859
16.0	1.3963	.4	2.0527	.8	2.8352	.2	3.7439	.6	4.7787	60	19.6350
.1	1.4138	.5	2.0739	.9	2.8602	.3	3.7725	.7	4.8110		
.2	1.4314	.6	2.0952	23.0	2.8852	.4	3.8013	.8	4.8435		
.3	1.4492	.7	2.1167	.1	2.9103	.5	3.8301	.9	4.8760		

Table of the volumes of cylinders and the sum

Length of cylinder or number of circles.	Diameter in inches.							
	1.	2.	3.	4.	5.	6.	7.	8.
1	0.0055	0.0218	0.0491	0.0873	0.1364	0.1963	0.2673	0.3491
2	.0110	.0436	.0982	.1746	.2728	.3926	.5346	.6982
3	.0165	.0654	.1473	.2619	.4092	.5889	.8019	1.0473
4	.0220	.0872	.1964	.3492	.5456	.7852	1.0692	1.3964
5	.0275	.1090	.2455	.4365	.6820	.9815	1.3365	1.7455
6	.0330	.1308	.2946	.5238	.8184	1.1778	1.6038	2.0946
7	.0385	.1526	.3437	.6111	.9548	1.3741	1.8711	2.4437
8	.0440	.1744	.3928	.6984	1.0912	1.5704	2.1384	2.7928
9	.0495	.1962	.4419	.7857	1.2276	1.7667	2.4057	3.1419
	17.	18.	19.	20.	21.	22.	23.	24.
1	1.5763	1.7671	1.9689	2.1817	2.4053	2.6398	2.8852	3.1416
2	3.1526	3.5342	3.9378	4.3634	4.8106	5.2796	5.7704	6.2832
3	4.7289	5.3013	5.9067	6.5451	7.2159	7.9194	8.6556	9.4248
4	6.3052	7.0684	7.8756	8.7268	9.6212	10.5592	11.5408	12.5664
5	7.8815	8.8355	9.8445	10.9085	12.0265	13.1990	14.4260	15.7080
6	9.4578	10.6026	11.8134	13.0902	14.4318	15.8388	17.3112	18.8496
7	11.0341	12.3697	13.7823	15.2719	16.8371	18.4786	20.1964	21.9912
8	12.6104	14.1368	15.7512	17.4536	19.2424	21.1184	23.0816	25.1328
9	14.1867	15.9039	17.7201	19.6853	21.6477	23.7582	25.9668	28.2744
	33.	34.	35.	36.	37.	38.	39.	40.
1	5.9396	6.3050	6.6813	7.0686	7.4667	7.8758	8.2958	8.7266
2	11.8792	12.6100	13.3626	14.1372	14.9334	15.7516	16.5916	17.4532
3	17.8188	18.9150	20.0439	21.2058	22.4001	23.6274	24.8874	26.1798
4	23.7584	25.2200	26.7252	28.2744	29.8668	31.5032	33.1832	34.9064
5	29.6980	31.5250	33.4065	35.3430	37.3335	39.3790	41.4790	43.6330
6	35.6376	37.8300	40.0878	42.4116	44.8002	47.2548	49.7748	52.3596
7	41.5772	44.1350	46.7691	49.4802	52.2669	55.1306	58.0706	61.0862
8	47.5168	50.4400	53.4504	56.5488	59.7336	63.0064	66.3664	69.8128
9	53.4564	56.7450	60.1317	63.6174	67.2003	70.8822	74.6622	78.5394

of circles, for diameters of 1 inch to 48 inches.

Diameter in inches.								Length of cylinder or number of circles.
9.	10.	11.	12.	13.	14.	15.	16.	
0.4418	0.5454	0.6600	0.7854	0.9218	1.0690	1.2272	1.3963	1
.8836	1.0908	1.3200	1.5708	1.8436	2.1380	2.4544	2.7926	2
1.3254	1.6362	1.9800	2.3562	2.7654	3.2070	3.6816	4.1889	3
1.7672	2.1816	2.6400	3.1416	3.6872	4.2760	4.9088	5.5852	4
2.2090	2.7270	3.3000	3.9270	4.6090	5.3450	6.1360	6.9815	5
2.6508	3.2724	3.9600	4.7124	5.5308	6.4140	7.3632	8.3778	6
3.0926	3.8178	4.6200	5.4978	6.4526	7.4830	8.5904	9.7741	7
3.5344	4.3632	5.2800	6.2832	7.3744	8.5520	9.8176	11.1704	8
3.9762	4.9086	5.9400	7.0686	8.2962	9.6210	11.0448	12.5667	9
25.	26.	27.	28.	29.	30.	31.	32.	
3.4088	3.6870	3.9761	4.2761	4.5869	4.9087	5.2414	5.5851	1
6.8176	7.3740	7.9522	8.5522	9.1738	9.8174	10.4828	11.1702	2
10.2264	11.0610	11.9283	12.8283	13.7607	14.7261	15.7242	16.7553	3
13.6352	14.7480	15.9044	17.1044	18.3476	19.6348	20.9656	22.3404	4
17.0440	18.4350	19.8805	21.3805	22.9345	24.5435	26.2070	27.9255	5
20.4528	22.1220	23.8566	25.6566	27.5214	29.4522	31.4484	33.5106	6
23.8616	25.8090	27.8327	29.9327	32.1083	34.3609	36.6898	39 0957	7
27.2704	29.4960	31.8088	34.2088	36.6952	39.2696	41.9312	44.6808	8
30.6792	33.1830	35.7849	38.4849	41.2821	44.1783	47.1726	50.2659	9
41.	42.	43.	44.	45.	46.	47.	48.	
9.1684	9.6211	10.0847	10.5592	11.0447	11.5410	12.0482	12.5664	1
18.3368	19.2422	20.1694	21.1184	22.0894	23 0820	24.0964	25.1328	2
27.5052	28.8633	30.2541	31.6776	33.1341	34.6230	36.1446	37.6992	3
36.6736	38.4844	40.3388	42.2368	44.1788	46.1640	48.1928	50.2656	4
45.8420	48.1055	50.4235	52.7960	55.2235	57.7050	60.2410	62.8320	5
55.0104	57.7266	60.5082	63.3552	66.2682	69.2460	72.2892	75.3984	6
64.1788	67.3477	70.5929	73.9144	77.3129	80.7870	84.3374	87.9648	7
73.3472	76.9688	80.6776	84.4736	88.3576	92.3280	96.3856	100.5312	8
82.5156	86.5899	90.7623	95.0328	99.4023	103.8690	108.4338	113.0976	9

Pressler's table.

Relative diameter.	Average thrifty tree.	Very thrifty tree.	Relative diameter.	Average thrifty tree.	Very thrifty tree.	Relative diameter.	Average thrifty tree.	Very thrifty tree.	Relative diameter.	Average thrifty tree.	Very thrifty tree.
2.0	144	156	6.8	42	47	13.2	21.0	24.0	33.0	8.2	9.2
2.1	138	150	6.9	41	46	13.4	21.0	23.0	33.5	8.1	9.1
2.2	132	144	7.0	40	45	13.6	20.0	23.0	34.0	7.9	8.9
2.3	127	139	7.1	40	45	13.8	20.0	23.0	34.5	7.8	8.8
2.4	122	134	7.2	39	44	14.0	20.0	22.0	35.0	7.7	8.6
2.5	117	129	7.3	39	44	14.2	19.0	22.0	35.5	7.6	8.5
2.6	113	124	7.4	38	43	14.4	19.0	22.0	36.0	7.5	8.4
2.7	109	120	7.5	38	42	14.6	19.0	21.0	37.0	7.3	8.2
2.8	105	116	7.6	37	42	14.8	19.0	21.0	38.0	7.1	8.0
2.9	101	112	7.7	37	41	15.0	18.0	21.0	39.0	6.9	7.8
3.0	98	109	7.8	36	41	15.2	18.0	20.0	40.0	6.8	7.6
3.1	95	105	7.9	36	40	15.4	18.0	20.0	41.0	6.6	7.4
3.2	92	102	8.0	35	40	15.6	18.0	20.0	42.0	6.4	7.2
3.3	89	99	8.1	35	39	15.8	17.0	20.0	43.0	6.3	7.1
3.4	86	96	8.2	34	39	16.0	17.0	19.0	44.0	6.1	6.9
3.5	84	93	8.3	34	38	16.5	17.0	19.0	45.0	6.0	6.7
3.6	81	91	8.4	34	38	17.0	16.0	18.0	46.0	5.9	6.6
3.7	79	88	8.5	33	37	17.5	16.0	18.0	47.0	5.8	6.5
3.8	77	86	8.6	33	37	18.0	15.0	17.0	48.0	5.6	6.3
3.9	75	84	8.7	32	36	18.5	15.0	17.0	50.0	5.4	6.1
4.0	73	81	8.8	32	36	19.0	14.0	16.0	52.0	5.2	5.9
4.1	71	79	8.9	32	35	19.5	14.0	16.0	54.0	5.1	5.7
4.2	69	77	9.0	31	35	20.0	14.0	15.0	56.0	4.9	5.5
4.3	68	76	9.1	31	35	20.5	13.0	15.0	58.0	4.7	5.3
4.4	66	74	9.2	31	34	21.0	13.0	15.0	60.0	4.5	5.1
4.5	65	72	9.3	30	34	21.5	13.0	14.0	62.0	4.4	4.9
4.6	63	70	9.4	30	34	22.0	12.0	14.0	64.0	4.2	4.7
4.7	62	69	9.5	29	33	22.5	12.0	14.0	66.0	4.1	4.6
4.8	60	67	9.6	29	33	23.0	12.0	13.0	68.0	3.9	4.4
4.9	59	66	9.7	29	32	23.5	12.0	13.0	70.0	3.8	4.3
5.0	58	65	9.8	29	32	24.0	11.0	13.0	72.0	3.7	4.2
5.1	56	63	9.9	28	32	24.5	11.0	12.0	74.0	3.6	4.1
5.2	55	62	10.0	28	31	25.0	11.0	12.0	76.0	3.6	4.0
5.3	54	61	10.2	27	31	25.5	11.0	12.0	78.0	3.5	3.9
5.4	53	60	10.4	27	30	26.0	10.0	12.0	80.0	3.4	3.8
5.5	52	59	10.6	26	30	26.5	10.0	12.0	85.0	3.2	3.6
5.6	51	57	10.8	26	29	27.0	10.0	11.0	90.0	3.0	3.4
5.7	50	56	11.0	25	28	27.5	9.9	11.0	100.0	2.7	3.0
5.8	49	55	11.2	25	28	28.0	9.7	11.0	110.0	2.4	2.7
5.9	49	54	11.4	24	27	28.5	9.5	11.0	120.0	2.2	2.5
6.0	48	53	11.6	24	27	29.0	9.3	11.0	130.0	2.1	2.3
6.1	47	53	11.8	23	26	29.5	9.2	10.5	140.0	1.9	2.2
6.2	46	52	12.0	23	26	30.0	9.0	10.0	150.0	1.8	2.0
6.3	45	51	12.2	23	26	30.5	8.9	10.0	170.0	1.6	1.8
6.4	45	50	12.4	22	25	31.0	8.7	9.8	200.0	1.3	1.5
6.5	44	49	12.6	22	25	31.5	8.6	9.7	250.0	1.1	1.2
6.6	43	48	12.8	22	24	32.0	8.5	9.5	300.0	0.9	1.0
6.7	42	48	13.0	21	24	32.5	8.4	9.4			

A METHOD OF INVESTIGATING TIMBER GROWTH.

When a forest is to be bought, sold, or assessed, or when its timber is to be cut, the questions which are to be solved relate either to its growing stock alone—if the forest is to be disposed of immediately—or to the progress of its growth during the period for which its standing timber is intended to be kept. The mastering of the methods discussed on the preceding pages enables one to solve these questions with ample accuracy. With additional knowledge relating to local prices on cleared land, market condition of lumber and timber, cost of labor, transportation, etc., the present value of the forest, or that which it may have at the expiration of a short period, is easily ascertained. The determination of the growing stock in such cases terminates the inquiries. But when a forest is intended for rational management, with the expectation of making it yield continual returns, the knowledge of its growing stock serves only as a guiding point from which the way toward obtaining other information of equal importance becomes clearer. The success of conducting the other inquiries relating to the forest, with a view to working out an adequate plan for its management, depends upon knowledge of the sylvicultural possibilities of the species comprising it. The question, for instance, of how far the actual annual growth of the forest differs from the annual growth possible for the locality, under the given conditions and species or the age at which the cutting should be made in order to insure that possible annual growth, can be properly answered when the requirements of the species composing the forest and the rate at which those species grow at various ages and under various situations is known. In Europe the sylvicultural requirements of forest trees have been ascertained in an experimental, or rather historical, way. The forest districts into which the state forests were divided at the beginning of the epoch of forest regulation kept records registering the results attained by their forest trees under the various situations and treatment. These records, together with the casual and experimental observations organized later on by the European foresters, have accumulated a good deal of valuable and sound data, the systematic teaching of which has attained the rank of a science known under the name of sylviculture. It has taken Europe almost two centuries to work out its sylviculture. It would take the United States, with ten times as many species of economic value as are found in Europe, considerably more time to work out its sylviculture should it ignore the progress made in forestry science, and in spite of it follow the slow historical method of investigating timber growth.

It has been shown in the preceding pages that the progressive development of a single tree may be determined in an analytical way. The European foresters apply that analytical method for the examination of the average sample trees. The analysis of a few individual trees is sufficient to enable the forester, with the general knowledge he possesses of the rate of growth of the particular species, to determine the factor of the locality, i. e., to determine in what way the locality affects the general law of growth of the species. But when the sylvicultural requirements of a species are totally unknown, an analysis of a few individual trees is scarcely sufficient even to indicate the rate of growth of the species. To be sure, by analyzing a large number of trees taken on various sites and under various situations the rate of growth of the species could be determined, but if each of the number of individual trees were to be analyzed separately, as is done usually by European foresters, the work would be too cumbersome. It was thought necessary, therefore, in the work for the Division of Forestry of ascertaining the rate of growth of our species, to modify the European analytical method so as to make it more applicable for the thorough investigation of timber growth.

According to the analytical method as employed by the European foresters, each of the trees measured for analytical purposes is analyzed separately, and for each individual tree a table of growth is prepared. Then all the tables of growth are classified according to forest conditions, ages, and degrees of dominance. Thereupon the tables assigned to a group are averaged, and a table representing the rate of growth of the group is thus obtained. Suppose, for instance, that 50 trees of a given species were measured on a site under the same forest conditions and then analyzed. Suppose further that the corresponding 50 tables of growth, each of course representing the progressive development of a single tree, have been divided into two distinct groups, according to the accepted classification, one group containing 29 and the other 21. Finding the average of the 29 tables and that of the 21 tables, the 50 analyzed trees would have been finally represented by 2 tables, each representing the rate of growth of the corresponding group.

These operations can be simplified by starting with the classification of the trees when their measurements are taken in the forest, then proceeding with the averaging of those measurements for each group separately, and finally analyzing only the average tree of each group.

The classification of trees can be performed in a more efficient and accurate way in the forest than in the office. The measurements for each group of trees can be taken separately and so arranged as to permit the entering of sets of corresponding measurements of a homogeneous nature on separate sheets, thus facilitating their averaging. For instance, all the measurements of cross sections taken at uniform heights from ground would be entered on one sheet for all the trees of the same group. By averaging, then, these homogeneous sets of measurements, figures would be obtained representing the measurements of an average tree of the group. The analysis of that average tree would determine the progressive development of the group. Thus by reversing the process of analysis the rate of growth of the 50 trees taken in our example could be determined by the analysis of only 2 trees, each being an average for one of the two corresponding groups.

While the work in the field and the averaging either of the tables of growth or the sets of homogeneous measurements will consume the same amount of time, the time required for the analysis itself of the 50 trees would differ in the two cases in the proportion of 2 to 50, i. e., the modified method would have required only one twenty-fifth of the time that would have been consumed by the analytical method as practiced by European foresters. The saving of time will be more appreciated when it is known that the complete analysis of a single tree of seven to nine cross sections, including the preparation of the table of growth, takes a day's work. Determination of the rate of growth of the 50 trees would consume 50 working days, while under the present arrangement 2 days are sufficient to arrive at the same results.

The detailed discussion of the method, accompanied by an actual example, will enable the reader to understand its working more clearly.

FIELD WORK.

Under the modified method here presented the field work constitutes the most delicate part of the tree analysis. The reliability of the tables of growth calculated in the office for various groups of trees and the deductions made from them depend not only upon the accuracy with which the measurements of individual trees are taken, but also upon the knowledge and skill employed in classifying the trees and describing the conditions under which they grow.

The field work begins with a general description of the station. Blank No. 1, given in the appendix, may be recommended for that purpose. The geographical climate of a station is determined by its latitude. Special attention must be given to ascertaining the physical or local climate, for it has a direct effect upon the rate of growth and quality of timber. The local climate depends upon the general configuration, elevation, general trend of the valleys or mountains, nature of soil, and proximity of sea. All these local features must be carefully noted. If the locality is provided with a meteorological station the record of that station for average monthly temperature, for average monthly precipitation, and monthly means of relative humidity should be procured for the greatest possible length of time.

In most cases such climatic data can not be obtained, and that is why it is advisable to carefully examine every local feature that may exert an influence upon the temperature and humidity of the atmosphere.

FOREST CONDITIONS OF STATION.

In describing the forest conditions of the station typical forms should be indicated, and, if possible, each typical form represented by one or more sample areas. Such sample areas, besides having a statistical value, furnish valuable information on which to outline the general forest conditions surrounding the species under investigation.

It is exceedingly desirable to procure trees for analysis from each typical form of forest conditions of the station, especially so when the typical forms differ considerably from each other in soil and drainage conditions or in the composition and density of the forest. The climate being thus eliminated, it becomes more easy to determine the effect of each of the various factors upon the rate of growth of the species.

In many instances the timber investigator has little choice in selecting conditions for measuring trees, because the detailed measurement necessary for analytical purposes requires the felling of the trees and their being sawed up into logs, which operation is regulated by the lumber camps. The operators of lumber camps usually confine themselves for each winter to limited forest areas, which seldom offer a wide range of forest and other conditions. A station may comprise several camps, each of which, of course, may represent distinct forest conditions. But in most cases the timber investigator will have to connect his work with the operations of the lumber camp, and direct his attention at least to such spots of the lumber area where the species is found in primeval forest conditions and not affected by natural dangers. The spots selected must be carefully and minutely described, accompanying the description by a sample area staked off exactly in the place where the trees will be afterwards felled and measured for analysis. Blank No. 2 in the appendix may be conveniently used both for description and measuring purposes of the sample area, which should be, if possible, an acre in extent. The method of taking the acre-yield measurements has been discussed in one of the preceding chapters. As regards the description of the sample area, the example given in the blank shows exactly how it should be done.

ASPECT.

In a mountainous country the aspect or exposure must be noticed, for it exerts an influence on the climate and hence upon the growth of trees. The northern aspect has diffused light, comparatively little heat, and the soil, due to low temperature, remains moist, thus favoring rapid growth. The eastern aspect is the most favorable for forest vegetation, because the sun shines obliquely and during the coolest hours of the day; the temperature and the light are moderate, permitting the soil to retain its moisture, which again favors active growth.

The southern exposure has the sun almost all day, causing intense light, heat, and high temperature, which dries the soil rapidly and, consequently, retards tree growth. On the western exposure the sun shines during the hottest hours; again the high temperature makes it difficult to retain the moisture of the soil. Of course, the nature of the prevailing winds will modify the influence exercised by the exposure upon tree growth, and it is desirable, therefore, to note the directions of the wind, its velocity, and the amount of moisture and heat with which it is charged. It should be also mentioned that on northern and eastern exposures, especially on the northern, vegetation is retarded and the trees usually escape spring frost, but are apt to suffer from early autumn frost, owing to the incomplete lignification of their shoots. On southern and western exposures vegetation begins early and young forests often fall the victim of spring frosts.

SOIL AND DRAINAGE CONDITIONS.

In describing the soil it must be borne in mind that the fertility of a soil for sylvicultural purposes is determined by its physical properties rather than by its chemical composition. Forest vegetation requires little inorganic matter. The amount of inorganic matter barely exceeds one-half of that required by agricultural products, and then a great portion of it is returned to the soil by the fall and decomposition of the leaves and branches. The mineral constituents absolutely necessary for the growth and development of trees are: Potash, calcium, iron, magnesia, phosphorus, and sulphur. Most soils contain these mineral substances in a sufficient quantity to meet the requirements of forest vegetation. The important thing to know about the soils devoted to sylviculture is the relative quantities of the principal components of the soil, i. e., of sand, clay, limestone, and organic matter which the soil contains; for the proportion in which these are mixed determines the texture of the soil, hence its physical properties. A chemical analysis of the soil can not supply this information, nor does the nature of the rocks give a clue, for the same rocks do not always form similar soils, and the products of decomposition do not always remain together; besides the soil formed from a rock varies in its properties with the stages of disintegration. The principal components of the soil can best be determined by mechanical analysis, which is recommended whenever possible. But for purposes at hand the timber investigator can attain good results by examining the soil in the field in the following manner: Take a certain quantity of soil, say a pound, and dry it thoroughly at approximately the temperature of boiling water. The difference in weight before and after it was dried gives the amount of moisture. Crumble the dry soil into powder, take a certain quantity of it and mix it with water while stirring; let the mixture stand for a while and decant carefully the turbid liquid. Repeat this process several times until the last water to be poured off becomes altogether clear. What is left at the bottom of the vessel constitutes sand; dry and weigh it. The turbid liquid contains clay, limestone, and organic matter. Add gradually to the turbid liquid hydrochloric acid until it turns litmus paper red; filter it well and dry the residue, which contains clay and organic matter. Ignite the residue in order to burn the organic matter; the difference in weight before and after it was ignited gives the amount of organic matter, the rest constitutes clay. Adding together the weight thus obtained for sand, clay, and organic matter and subtracting the sum from the original weight of the part of the dry soil taken for analysis, the amount of limestone is roughly determined. According to the proportion in which the principal components are mixed, soils are classified as sand, loam, clay, and lime.

Sandy soils contain 75 per cent or more sand; the remainder is clay. When clay constitutes from 15 to 25 per cent, the soil is called loamy sand; when the clay is found to be 10 per cent or less, the soil is considered as a sand. Loamy soils contain from 60 to 70 per cent of sand, about 5 per cent of lime, 5 per cent of iron oxides, while the rest is made up of clay. When the clay constitutes 40 per cent, the soil is considered as loam; when the clay constitutes 30 per cent, it is considered as a sandy loam.

Clayey soils are those which contain 50 per cent of clay and more. When clay and loam are mixed half-and-half, the soil is called clayey loam; but when clay constitutes more than 60 per cent, the soil is considered as a clay.

Limy soils are those which contain over 10 per cent of carbonate of lime. According to the proportion of lime found in the soils, they are subdivided into:

Marl soils, containing from 10 to 20 per cent of carbonate of lime.
Loamy lime, containing about 30 per cent of carbonate of lime.
Clayey lime, containing about 40 per cent of carbonate of lime.
Lime, containing about 50 per cent of carbonate of lime.

These are the principal classes of soil; besides, there may be distinguished two other classes of soil, namely, humus and ferruginous soil. Humus soil contains 20 per cent and more of vegetable mold. Ferruginous soil contains from 10 to 25 per cent of iron oxide. It can be easily recognized by its brown red color.

The above classification, which is commonly used in European forestry practice, is useful because at once suggestive of the physical properties of the soil.

PHYSICAL PROPERTIES OF SOIL.

The physical properties of a soil of importance to sylviculture are those which determine its moisture conditions. Upon the amount of moisture in the soil depends the chemical activity of the soil, its temperature, and the supply of water for the growth of the plant. With regard to moisture, soils are classified by European foresters as follows:

Wet, where water flows from the clod without pressure being applied.

Moist, where water drops from the clod on pressure being applied.

Fresh, where traces of moisture are felt by pressing a handful of soil.

Dry, where traces of moisture are not felt, but when rubbed the soil does not resolve into dust.

Arid, where on rubbing the soil crumbles into dust.

The chief physical properties of soil are:

(1) *Hygroscopicity.*—Hygroscopicity of soil, or the capacity with which it absorbs and retains water, depends upon the fineness of the soil particles and is, therefore, in direct proportion to the compactness of the soil. The power of absorbing rain water is the greatest in lime, then comes clay, loam, and sand. The aqueous vapors of the atmosphere are best absorbed by clay, next by lime, loam, and sand.

(2) *Tenacity.*—Tenacity, or the degree of cohesiveness between the particles of the soil, depends upon the size of the particles. Clay and sand represent in this respect two extremes. The first one, consisting of very fine grain, represents the most tenacious, while the latter, consisting of granular and coarser grain, represents the least tenacious soil. With regard to tenacity the soils may be classified as heavy, mild, light, loose, and shifting. To the class of heavy, stiff, or tenacious soils belong clays, clayey loams, limes, and marls. Heavy soils are characterized by the deep cracks they form when suddenly dried. To the class of mild soils belong loams, sandy loams, and loamy limes. Mild soils crack when suddenly dried, but are able to retain

the form of clods. Loamy sand and sandy marl are considered as light soils, which are characterized by being capable of forming clods when moist. Sand is considered as a loose soil, which is incapable of forming clods even when moist. To the class of shifting soils belong the sand drifts and dunes. Tenacious soils are unfavorable for tree growth, because, firstly, they offer considerable resistance to the penetration of roots and their ramification throughout the soil; secondly, the circulation of air and moisture in such soils is greatly impeded. Consequently, tenacious soils are either excessively moist or excessively dry. They absorb water slowly, but in large quantities. The power of soils to retain moisture is in direct proportion to their tenacity. So the power of retaining moisture is the greatest in clay; next comes lime, loam, and, finally, sand.

(3) *Permeability.*—Permeability, or the capacity of the soil to diffuse its moisture, is proportional to the size of the soil particles. It is the greatest in sand, next in loam, lime, and least in clay.

(4) *Warmth.*—The warmth of soil, or facility with which it absorbs heat, depends, aside from the atmospheric temperature, upon its color and the quantity of moisture it contains. Clayey soils do not easily raise or lower their temperature with the corresponding increase or decrease of the atmospheric temperature—they are cold soils. Sandy soils are very active and respond to even slight changes in the atmospheric temperature—they are warm soils.

(5) *Depth.*—The upper layer, which is penetrated by the roots, is spoken of as the soil; from it the trees draw the mineral nutriments. What is below the soil is considered as the subsoil, which may be of the same nature with the soil or may differ from it.

Depth of soil in sylviculture is rather a relative conception. Soils which are shallow for one kind of trees may be considered deep when applied to other kinds of trees. It depends altogether on the nature and development of the root systems of the species. Each species, then, could have its own classification of soils as regards their depth; but for practical purposes a general and uniform classification may be adopted. The classification adopted by the experiment stations in Germany, given below, may be recommended: Very shallow, up to 6 inches; shallow, from 6 inches to 1 foot; medium, from 1 foot to 2 feet; deep, from 2 feet to 4 feet; very deep, over 4 feet.

Deep soils are very favorable, even for species with shallow root systems, for they can retain the moisture longer, while the moisture conditions of shallow soils depends upon the nature of the subsoil. Usually shallow soils suffer either from excess of moisture or from drought.

Vegetable mold tends to modify extreme differences of the soils. It makes stiff soils less tenacious and binds loose soils; it warms cold and cools warmer soils; it increases the depth of soil; it is capable of holding large masses of waters, which it gives gradually to the lower layer;

it condenses aqueous vapors, carbonic-acid gas, and ammonia from the atmosphere, which, together with the carbonic acid-gas it develops, assists the decomposition of the mineral substances in the soil.

SUBSOIL.

The subsoil, if different in nature from the soil, should be described in the same manner as the soil.

SOIL COVER.

Under soil cover is meant the weedy or herbaceous forest plants that grow in the soil. It should be noted, because frequently such weeds indicate the quality of the soil.

FOREST CONDITIONS OF SAMPLE AREA.

In describing the forest conditions of the sample area it is advisable to be as concise as possible. The blanks left for the composition of forest should be filled out after the measurements of the acre yield have been taken, because then the proportion in which the species found in the forest area mixed may be accurately determined. Very small trees, such as those under 3 inches in diameter (breast high) and under 20 feet high, should be counted separately and be considered as undergrowth. All shrub forms should also be mentioned as a part of the undergrowth.

The description of the sample area is usually concluded with general remarks relating to the appearance of the stand of trees, development of crowns, quality of timber, average ages of the species composing the stand, and such other items as may for any reason be found necessary.

DENSITY OF FOREST.

The density of a forest is usually judged by it's canopy or degree of contact of the crowns of the trees. When the crowns are in touch with each other, forming a close canopy, the density is considered normal and the forest fully stocked. This condition is designated by a unit. The degrees of opening of the crown cover are expressed in decimals, thus permitting the making of 10 degrees of density. The density factor is simply a short expression of the light conditions of the forest, and the 10 degrees are established not with the expectation of getting the exact mark of density, but to enable the forester to indicate it with more facility. It is not worth while to puzzle the brains over the solution as to whether the density is 0.5 or 0.6, especially when it is remembered that the method of designating the density is in itself imperfect and mostly based upon the general impression of the forester.

ACRE-YIELD MEASUREMENT.

In measuring the trees on the sample area special attention should be given to the classification of trees resulting from various stages of development which they have attained, the basis of such classification being height and crown development. For the purposes at hand it is sufficient to consider only three classes, namely, dominant, codominant, and oppressed. Dominant trees are those which overtop their neighbors and possess fully developed crowns. Codominant trees are those which, although being of the same height as the dominant trees, possess poorly developed crowns, usually compressed on all sides by neighboring trees. Oppressed trees are those whose crowns are still less developed than those of the codominant trees; they are not only compressed, but also somewhat overtopped by the neighboring trees. It is advisable to adopt a conventional system of marking the trees of each class of dominance by blazing the bark of trees below the height of the stump, when the class of dominance assigned to a tree while standing may be either verified or rejected afterwards when the tree is felled and measured in detail.

DEDUCED RESULTS.

The sample area leaves a general impression upon the mind of the timber investigator. This impression should be utilized by converting it into figures of forest economic value, which can be easily remembered. It is exceedingly desirable that the timber investigator should deduce all the results relating to the acre-yield in the manner shown in Blank No. 2 immediately after the sample area is described and measured. Thereafter, when, while the details are fresh in mind, the timber investigator meets with similar forest conditions, he is in a position to estimate the standing timber at a glance, using previous experiences as a basis of judgment.

THE MEASURING OF FELLED TREES FOR ANALYSIS.

When the trees on the sample area are felled and sawed into logs the timber investigator should begin the detailed measurements. The sawyers are followed, and as each tree is sawed into logs the measurements are made before the logs are removed from the place where the tree fell.

The tree is calipered first at breast height, and then at intervals of 8 feet from the ground, until a point on the trunk is reached where the diameter measures 5 inches or less. In keeping a record of the measurements, the entry for each tree includes its serial number; height of tree; height to base of crown; character of growth, i. e., whether dominant, codominant, or oppressed; condition of timber, i. e., whether sound, defective, crooked, wind shaken, clear, or knotty; amount of merchantable timber (determined by scaling the logs of the tree right on the spot where it is felled); the position of the tree and surrounding species, and other remarks. Blank No. 3 of the appendix gives the measurements of the trees felled on sample area described in Blank No. 2. When the calipering is completed and recorded, the measurements of growth at the stump and at the top of each log is made by counting and measuring the rings on the average radius of each cross section, as indicated in Blank No. 4 of the appendix. The most systematic results are secured if all the trees are cut into logs of equal length, say 16 feet. Each blank of set No. 4 is intended for recording the measurements of cross sections from all the trees at approximately the same height from the ground separately for each group; thus one of Blank No. 4 records the results of stump measurements for the trees of each of the three groups (dominant, codominant, and oppressed), while the other records the results of cross section at the top of first log (18 feet from ground), the third. that taken at the top of the second log (34 feet from ground), etc. In the appendix are given two blanks of set No. 4; one records the stump section, the other the cross section taken at the top of the third log (50 feet from the ground). The groups of trees in this particular instance had nine cross sections, which were recorded in nine blanks of set No. 4. The measuring of the cross sections should begin with the stump section, because the number of rings counted on this section determines the age of the tree (of course, the allowance made for the height of the stump must be added), which must be known for establishing age classes before the measurements of sections are registered. Trees either of the same age or differing not more than twenty years for old trees and ten years for younger trees, usually constitute one age class. The rings of a section are carefully counted on the average radius, and the distances for 10, 20, 30, 40, etc., rings from the center to the periphery are noted in millimeters. The entire radius is also noted. Besides, for each cross section should be noted—

(1) Number of tree to which it belongs.
(2) Exact height from ground.
(3) Thickness of bark.
(4) Number and width of rings in the sapwood.
(5) Number of rings on the cross sections.

OFFICE WORK.

TABULATION OF MEASURED TREES.

The office work begins with a concise description of the forest conditions of the trees measured for analysis, accompanied by a tabulation for each group of such measurements and calculations, as are illustrated in the following form:

FORM NO. 5.

(Site: *f*. Age-class: 240–260 years. Species: White pine.)

Location.	Description of site.	Tree number.	Age.	Diameter (breast high).	Total height.	Height to base of crown.	Rings per inch on stump.	Volume. Tree.	Volume. Merchantable timber.	Factor of shape.	Ratio of length of crown to total height of tree.	Lumber product under present practice; per cent used of total volume of stem.	Remarks.
								Cu. ft.	*B. M.*				
Du Bois, Clearfield County, Pa. Latitude, 41° 3′. Longitude, 78° 45′. Altitude, 1,200 to 1,400 feet.	Hemlock, mixed with white pine, with scattering maple, beech, and birch, on a hill sloping toward southwest, where it is bordered by the left-hand branch of Narrow Creek. The moderately dense undergrowth consists of very young beech, hemlock, and occasional birch and cucumber.	1	260	35⅓	158	90	7.6	435.4	3,030	0.40	0.43	58	Dominant.
		2	260	36	157	90	7.0	481.3	3,401	.43	.42	59	
		3	259	32	152	84	7.8	396.0	2,637	.46	.44	55	
		4	241	32	150	62	6.6	347.7	2,079	.41	.59	50	
		10	244	33	146	96	6.8	365.9	2,384	.42	.34	54	
		12	262	28	156	88	9.0	285.8	1,648	.43	.43	47	
		18	265	39	153	88	6.0	511.1	3,318	.40	.42	54	
		19	250	34	150	78	6.3	402.4	2,397	.42	.48	49	
		20	266	44	144	100	5.7	638.4	4,388	.42	.30	57	
		21	245	34	146	92	7.1	366.7	2,248	.40	.37	51	
		23	248	34	242	90	7.2	373.4	2,318	.42	.37	51	
		33	259	33	133	91	8.0	304.5	1,770	.40	.31	48	
		34	262	33	146	90	7.4	369.2	2,220	.42	.38	50	
		35	263	31	144	82	8.5	275.2	1,458	.36	.43	44	
		36	241	31½	134	88	7.1	307.7	1,853	.42	.34	50	
		37	261	37	146	106	6.7	482.9	2,970	.44	.27	50	
	Average..	...	255	34	147	88	7.0	390.0	2,507	.41	.39	52	
	Soil: Yellow clay loam of a medium grain (fine shales in it), deep, fresh, well drained, with 2 to 3 inch mold on top and with a surface cover of scanty leaves, fern, tea berries, and scattering dogwood (laurel in northeast corner and on north side). Subsoil: Laminated shale of an indefinite depth.	28	261	28½	138	75	9.8	264.3	1,551	.43	.45	49	Codominant.
		25	244	28½	138	107	7.7	298.1	1,954	.49	.22	54	
		24	245	25	130	84	9.3	192.1	1,102	.43	.35	48	
		22	246	31	130	82	7.3	310.3	1,731	.45	.37	46	
		5	264	29	140	100	8.4	300.4	1,905	.47	.28	52	
		6	264	29	140	110	8.5	291.4	1,631	.45	21	47	
		7	262	29	152	112	9.5	302.8	1,854	.46	.26	51	
		8	235	29	142	86	....	248.6	1,318	.38	.39	44	
		9	236	32	142	84	....	287.7	1,648	.36	.41	48	
		11	244	30	141	81	7.5	305.3	1,947	.44	.42	53	
		13	258	23	147	93	9.6	206.0	1,048	.48	.37	42	
		14	242	25	139	98	....	217.1	1,233	.46	.30	47	
		15	262	26	136	98	....	257.2	1,389	.51	.28	45	
		16	235	24½	124	93	....	163.8	815	.40	.25	41	
		17	262	25	128	108	....	214.4	1,183	.49	.16	46	
		26	245	26	136	98	9.3	199.2	1,021	.40	.28	47	
		30	259	26½	134	90	9.2	228.6	1,336	.44	.32	48	
		29	264	28	141	84	9.2	276.5	1,577	.46	.40	47	
		31	262	25½	132	88	10.0	191.8	863	.41	.33	37	
		32	261	26	142	99	9.1	239.9	1,322	.46	.30	46	
	Average..	...	252	27	138	93	9.0	250.0	1,421	.44	.32	47	
		27	259	19	132	94	11.6	138.8	683	.53	.29	41	Oppressed.
		38	260	23	137	96	11.1	189.6	987	.48	.30	43	
		39	258	20½	123	109	13.0	130.9	558	.46	.11	35	
		40	261	16½	120	82	13.7	89.6	339	.50	.31	31	
	Average..	...	259	20	128	95	12.3	137.0	642	.49	.25	37	

ANALYSIS.

The cross sections of each group are averaged in order to obtain the measurements of the cross sections of the average tree of the group, which is then constructed and analyzed in the manner discussed under caption "Rate of growth" in preceding pages.

As an example, the analysis of the dominant group is given below. It begins with averaging the widths for the groups of rings from center to periphery for each of the cross sections from which the area and accretion are calculated.

Cross section of the stump of the average tree of the group.

SITE *f*.—Dominant group of 16 trees.

CROSS SECTION: Stump.

Height from ground, 2 feet.
Width of bark, 43 millimeters.
Number of rings in sap, 35.
Width of sap, 32 millimeters.
Total number of rings, 249.

Number of rings (center to periphery).	Radius.	Diameter.		Area.	Accretion for decades	
					Area.	Diameter.
	Mm.	*Mm.*	*Inches.*	*Sq. ft.*	*Sq. ft.*	*Inches.*
10	29	58	2.3	0.03	0.03	2.3
20	64	128	5.1	.14	.11	2.8
30	93	186	7.4	.30	.16	2.3
40	119	238	9.5	.59	.19	2.1
50	143	286	11.4	.71	.22	1.9
60	166	332	13.3	.96	.25	1.9
70	193	386	15.4	1.29	.33	2.1
80	215	430	17.2	1.61	.32	1.8
90	231	462	18.5	1.87	.26	1.3
100	246	492	19.7	2.12	.25	1.2
110	269	538	21.5	2.52	.40	1.8
120	287	574	23.0	2.88	.36	1.5
130	304	608	24.3	3.22	.34	1.3
140	320	640	25.6	3.57	.35	1.3
150	334	668	26.7	3.89	.32	1.1
160	345	690	27.6	4.15	.26	.9
170	357	714	28.6	4.46	.31	1.0
180	369	738	29.5	4.75	.29	.9
190	379	758	30.3	5.01	.26	.8
200	390	780	31.2	5.31	.30	.9
210	401	802	32.1	5.62	.31	.9
220	411	822	32.9	5.90	.28	.8
230	422	844	33.8	6.23	.33	.9
240	432	864	34.6	6.52	.29	.8
249	437	874	35.0	....	....	....

Cross section at 18 feet from the ground of the average tree of the group.

SITE *f*.—Dominant group of 16 trees.

CROSS SECTION: NO. 1.

Height from ground, 18 feet.
Width of bark, 22 millimeters.
Number of rings in sap, 44.
Width of sap, 32 millimeters.
Number of rings, 238.

Number of rings (center to periphery).	Radius.	Diameter.		Area.	Accretion for decades.	
					Area.	Diameter.
	Mm.	*Mm.*	*Inches.*	*Sq. ft.*	*Sq. ft.*	*Inches.*
10	43	86	3.4	0.06	0.06	3.4
20	79	158	6.3	.22	.16	2.9
30	104	208	8.3	.37	.15	2.0
40	124	248	9.9	.53	.16	1.6
50	144	288	11.5	.72	.19	1.6
60	159	318	12.7	.88	.16	1.2
70	174	348	13.9	1.05	.17	1.2
80	186	372	14.9	1.21	.16	1.0
90	199	398	15.9	1.38	.17	1.0
100	214	428	17.1	1.59	.21	1.2
110	227	454	18.2	1.81	.22	1.1
120	240	480	19.2	2.01	.20	1.0
130	252	504	20.2	2.22	.21	1.0
140	262	524	21.0	2.40	.18	.8
150	270	540	21.6	2.54	.14	.6
160	279	558	22.3	2.71	.17	.7
170	288	576	23.0	2.88	.17	.7
180	296	592	23.7	3.06	.18	.7
190	304	608	24.3	3.22	.16	.6
200	312	624	25.0	3.41	.19	.7
210	320	640	25.6	3.57	.16	.6
220	328	656	26.2	3.74	.17	.6
230	335	670	26.8	3.92	.18	.6
238	341	682	27.3	....	....	....

Cross section at 34 feet from the ground of the average tree of the group.

SITE *f*.—Dominant group of 16 trees.

CROSS SECTION: NO. 2.

Height from ground, 34 feet.
Width of bark, 18 millimeters.
Number of rings in sap, 48.
Width of sap, 32 millimeters.
Number of rings, 228.

Number of rings (center to periphery).	Radius.	Diameter.		Area.	Accretion for decades.	
					Area.	Diameter.
	Mm.	*Mm.*	*Inches.*	*Sq. ft.*	*Sq. ft.*	*Inches.*
10	39	78	3.1	0.05	0.05	3.1
20	81	162	6.5	.23	.18	3.4
30	106	212	8.5	.39	.16	2.0
40	128	256	10.2	.57	.18	1.7
50	145	290	11.6	.73	.16	1.4
60	160	320	12.8	.89	.16	1.2
70	172	344	13.8	1.04	.15	1.0
80	186	372	14.9	1.21	.17	1.1
90	199	398	15.9	1.38	.17	1.0
100	213	426	17.0	1.58	.20	1.1
110	225	450	18.0	1.77	.19	1.0
120	235	470	18.8	1.93	.16	.8
130	245	490	19.6	2.09	.16	.8
140	253	506	20.2	2.22	.13	.6
150	262	524	21.0	2.40	.18	.8
160	270	540	21.6	2.54	.14	.6
170	277	554	22.2	2.69	.15	.6
180	285	570	22.8	2.83	.14	.6
190	293	586	23.4	2.99	.16	.6
200	301	602	24.1	3.17	.18	.7
210	310	620	24.8	3.35	.18	.7
220	320	640	25.6	3.57	.22	.8
228	325	650	26.0	....	....	....

Cross section at 50 feet from the ground of the average tree of the group.

SITE *f*.—Dominant group of 16 trees.

CROSS SECTION: NO. 3.

Height from ground, 50 feet.
Width of bark, 15 millimeters.
Number of rings in sap, 46.
Width of sap, 33 millimeters.
Number of rings, 216.

Number of rings (center to periphery).	Radius.	Diameter.		Area.	Accretion for decades.	
					Area.	Diameter.
	Mm.	*Mm.*	*Inches.*	*Sq. ft.*	*Sq. ft.*	*Inches.*
10	40	80	3.2	0.05	0.05	3.2
20	76	152	6.1	.20	.15	2.9
30	102	204	8.2	.37	.17	2.1
40	122	244	9.8	.52	.15	1.6
50	138	276	11.0	.66	.14	1.2
60	151	302	12.1	.80	.14	1.1
70	165	330	13.2	.95	.15	1.1
80	180	360	14.4	1.13	.18	1.2
90	193	386	15.4	1.29	.16	1.0
100	204	408	16.3	1.45	.16	.9
110	214	428	17.1	1.59	.14	.8
120	223	446	17.8	1.73	.14	.7
130	233	466	18.6	1.89	.16	.8
140	241	482	19.3	2.03	.14	.7
150	250	500	20.0	2.18	.15	.7
160	257	514	20.6	2.34	.16	.6
170	265	530	21.2	2.45	.11	.6
180	273	546	21.8	2.59	.14	.6
190	281	562	22.5	2.76	.17	.7
200	289	578	23.1	2.91	.15	.6
216	294	588	23.5	....	....	....

Cross section at 66 feet from the ground of the average tree of the group

SITE *f*.—Dominant group of 16 trees.

CROSS SECTION: NO. 4.

Height from ground, 66 feet.
Width of bark, 15 millimeters.
Number of rings in sap, 46.
Width of sap, 34 millimeters.
Number of rings, 203.

Number of rings (center to periphery).	Radius.	Diameter.		Area.	Accretion for decades.	
					Area.	Diameter.
	Mm.	*Mm.*	*Inches*	*Sq. ft.*	*Sq. ft.*	*Inches.*
10	34	68	2.7	0.04	0.04	2.7
20	64	128	5.1	.14	.10	2.4
30	93	186	7.4	.30	.16	2.3
40	112	224	9.0	44	.14	1.6
50	128	256	10.2	.57	.13	1.2
60	143	286	11.4	.71	.14	1.2
70	158	316	12.6	.86	.15	1.2
80	171	342	13.7	1.02	.16	1.1
90	182	364	14.6	1.16	.14	.9
100	193	486	15.4	1.29	.13	.8
110	203	406	16.2	1.43	.14	.8
120	212	424	17.0	1.58	.15	.8
130	221	442	17.7	1.71	.13	.7
140	230	460	18.4	1.85	.14	.7
150	240	480	19.2	2.01	.16	.8
160	247	494	19.8	2.14	.13	.6
170	255	510	20.4	2.27	.13	.6
180	263	526	21.0	2.40	.13	.6
190	270	540	21.6	2.54	.14	.6
200	276	552	22.1	2.66	.12	.5
203	272	544	21.8	....	....	....

Cross section at 82 feet from the ground of the average tree of the group.

SITE *f*.—Dominant group of 16 trees.

CROSS SECTION: NO. 5.

Height from ground, 82 feet.
Width of bark, 13 millimeters.
Number of rings in sap, 43.
Width of sap, 35 millimeters.
Number of rings, 185.

Number of rings (center to periphery).	Radius.	Diameter.		Area.	Accretion for decades.	
					Area.	Diameter.
	Mm.	*Mm.*	*Inches.*	*Sq. ft.*	*Sq. ft.*	*Inches.*
10	30	60	2.4	0.03	0.03	2.4
20	58	116	4.6	.11	.08	2.2
30	79	158	6.3	.22	.11	1.7
40	97	194	7.8	.33	.11	1.5
50	114	228	9.1	.45	.12	1.3
60	129	258	10.3	.58	.13	1.2
70	143	286	11.4	.71	.13	1.1
80	156	312	12.5	.85	.14	1.1
90	169	338	13.5	.99	.14	1.0
100	180	360	14.4	1.13	.14	.9
110	191	382	15.3	1.28	.15	.9
120	201	402	16.1	1.41	.13	.8
130	211	422	16.9	1.56	.15	.8
140	221	442	17.7	1.71	.15	.8
150	230	460	18.4	1.85	.14	.7
160	239	478	19.1	1.99	.14	.7
170	247	494	19.8	2.14	.15	.7
180	255	510	20.4	2.27	.13	.6
185	251	502	20.1	....	....	....

Cross section at 99 feet from the ground of the average tree of the group.

SITE *f*.—Dominant group of 16 trees.

CROSS SECTION: NO. 6.

Height from ground, 99 feet.
Width of bark, 11 millimeters.
Number of rings in sap, 37.
Width of sap, 33 millimeters.
Number of rings, 161.

Number of rings (center to periphery).	Radius.	Diameter.		Area.	Accretion for decades.	
					Area.	Diameter.
	Mm.	*Mm.*	*Inches.*	*Sq. ft.*	*Sq. ft.*	*Inches.*
10	21	42	1.7	0.01	0.01	1.7
20	42	84	3.4	.06	.05	1.7
30	62	124	5.0	.14	.08	1.6
40	79	158	6.3	.22	.08	1.3
50	96	192	7.7	.32	.10	1.4
60	111	222	8.9	.43	.11	1.2
70	124	248	9.9	.53	.10	1.0
80	137	274	11.0	.66	.13	1.1
90	148	296	11.8	.76	.10	.8
100	160	320	12.8	.89	.13	1.0
110	170	340	13.6	1.01	.12	.8
120	181	362	14.5	1.15	.14	.9
130	194	388	15.5	1.31	.16	1.0
140	204	408	16.3	1.45	.14	.8
150	215	430	17.2	1.61	.16	.9
160	225	450	18.0	1.77	.16	.8
161	226	452	18.1	....	....	....

Cross section at 114 feet from the ground of the average tree of group.

SITE *f*.—Dominant group of 16 trees.

CROSS SECTION: NO. 7.

Height from ground, 114 feet.
Width of bark, 8 millimeters.
Number of rings in sap, 34.
Width of sap, 33 millimeters.
Number of rings. 122.

Number of rings (center to periphery.)	Radius.	Diameter.		Area.	Accretion for decades.	
					Area.	Diameter.
	Mm.	*Mm.*	*Inches.*	*Sq. ft.*	*Sq. ft.*	*Inches.*
10	18	36	1.4	0.01	0.01	1.4
20	36	72	2.9	.04	.03	1.5
30	52	104	4.2	.10	.06	1.3
40	67	134	5.4	.16	.06	1.2
50	81	162	6.5	.23	.07	1.1
60	94	188	7.5	.31	.08	1.0
70	108	216	8.6	.40	.09	1.1
80	119	238	9.5	.49	.09	.9
90	130	260	10.4	.59	.10	.9
100	140	280	11.2	.68	.09	.8
110	151	302	12.1	.80	.12	.9
120	161	322	12.9	.91	.11	.8
122	162	324	13.0	....	....	....

Cross section at 129 feet from the ground of the average tree of group.

SITE *f*.—Dominant group of 16 trees.

CROSS SECTION: NO. 8.

Height from ground, 129 feet.
Width of bark, 6 millimeters.
Number of rings in sap, 31.
Width of sap, 30 millimeters.
Number of rings, 91.

Number of rings (center to periphery).	Radius.	Diameter.		Area.	Accretion for decades.	
					Area.	Diameter.
	Mm	*Mm.*	*Inches.*	*Sq.ft.*	*Sq.ft.*	*Inches.*
10	14	28	1.1	0.01	0.01	1.1
20	29	58	2.3	.03	.02	1.2
30	43	86	3.4	.06	.03	1.1
40	56	112	4.5	.11	.05	1.1
50	70	140	5.6	.17	.06	1.1
60	81	162	6.5	.23	.06	.9
70	92	184	7.4	.30	.07	.9
80	104	208	8.3	.38	.08	.9
90	116	232	9.3	.47	.09	1.0
91	117	234	9.4	....	....	....

The preceding tabulations are used for constructing the average tree of the group, as shown in fig. 13.

HEIGHT GROWTH.

The progressive development of the height growth is determined by means of graphical interpolation, taking as a basis the heights and ages of the cross sections as explained under caption "Rate of growth." The tree under analysis reached the height of the first cross section (18 feet) in the course of 17 years (age of tree minus age of section);

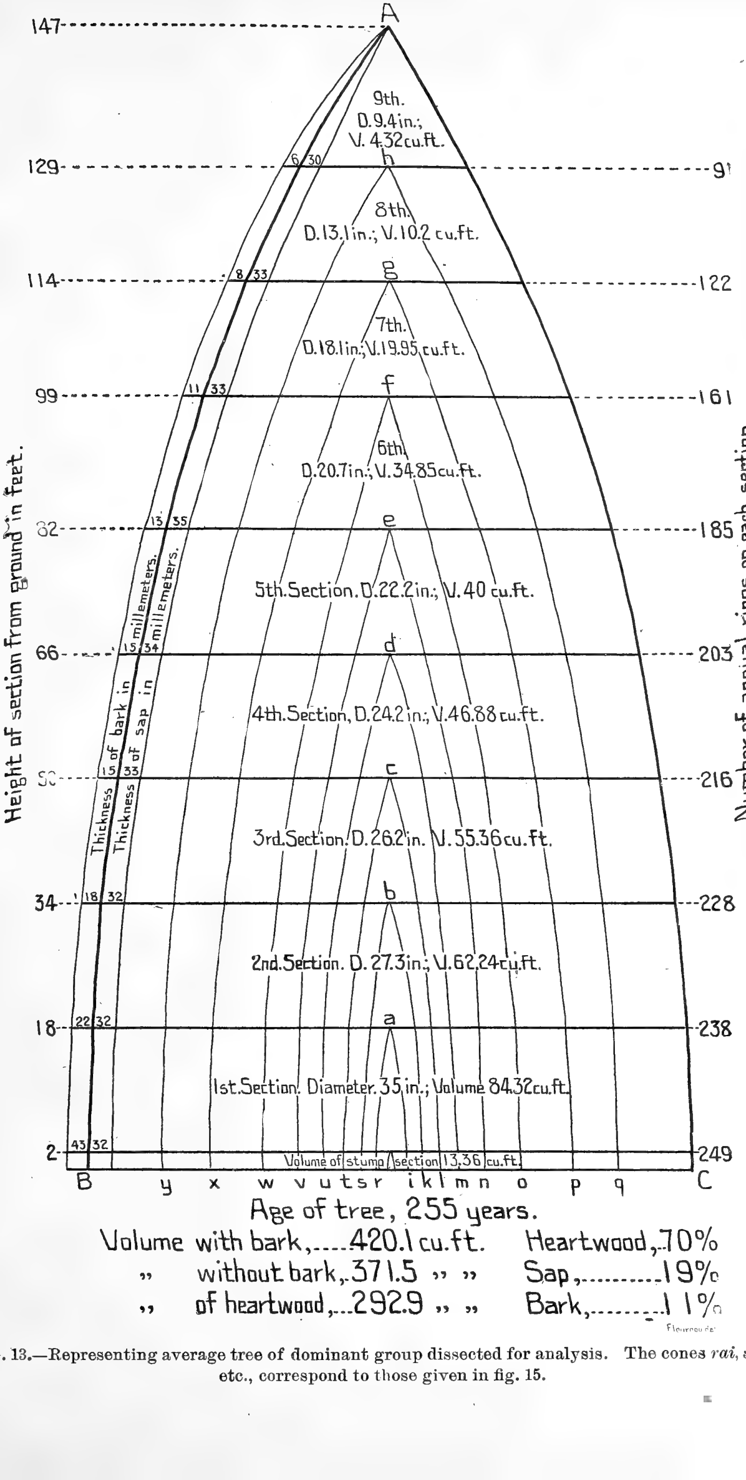


FIG. 13.—Representing average tree of dominant group dissected for analysis. The cones *rai*, *sbk*, *tcl*, etc., correspond to those given in fig. 15.

it reached the height of the second section (34 feet) in the course of 27 years (age of tree minus age of second cross section), etc. Making an allowance of six years which was required by the tree to attain the height of the stump (2 feet), the following were the heights reached by the tree at corresponding ages:

A height of 2 feet in 6 years.
A height of 18 feet in 17 years.
A height of 34 feet in 27 years.
A height of 50 feet in 39 years.
A height of 66 feet in 52 years.
A height of 82 feet in 70 years.
A height of 99 feet in 94 years.
A height of 114 feet in 133 years.
A height of 129 feet in 164 years.
A height of 147 (total height) in 255 years (total age).

These figures are used for constructing a curve of height growth as follows: Take cross-section paper (see fig. 14) and let the horizontal line AB represent the age of the tree and the vertical line AC its corresponding height. Locate each of the above 10 points on the cross-section paper with reference to age and height lines and connect them. The curve thus obtained will represent graphically the height growth of the average tree of the group.

DIAMETER GROWTH.

The progressive development of the diameter on each of the cross sections can be determined in the same graphical way by plotting the age of the section on the horizontal line and the corresponding distance from the center on the vertical line. Connecting all the points thus located, a curve is obtained representing graphically the diameter growth on the particular cross section of the tree. For the tree under analysis nine curves should be constructed in order to study the diameter growth of the tree.

VOLUME GROWTH.

The detail measurements of the cross sections of the average tree of the group enables one to determine the volume the tree had when—

17 years old (age of tree minus age of first section).
27 years old (age of tree minus age of second section).
39 years old (age of tree minus age of third section).
52 years old (age of tree minus age of fourth section).
70 years old (age of tree minus age of fifth section).
94 years old (age of tree minus age of sixth section).
133 years old (age of tree minus age of seventh section).
164 years old (age of tree minus age of eighth section).

The volume the tree had when 17 years old is determined as follows (see fig. 15; cone *rai*): Calculate the diameter of the central 11 rings on the stump section and find in the tables (page 37) the area of its corresponding circle. Multiply this area by the height of the stump (2 feet); multiply also this area by one-half the length of the first section.

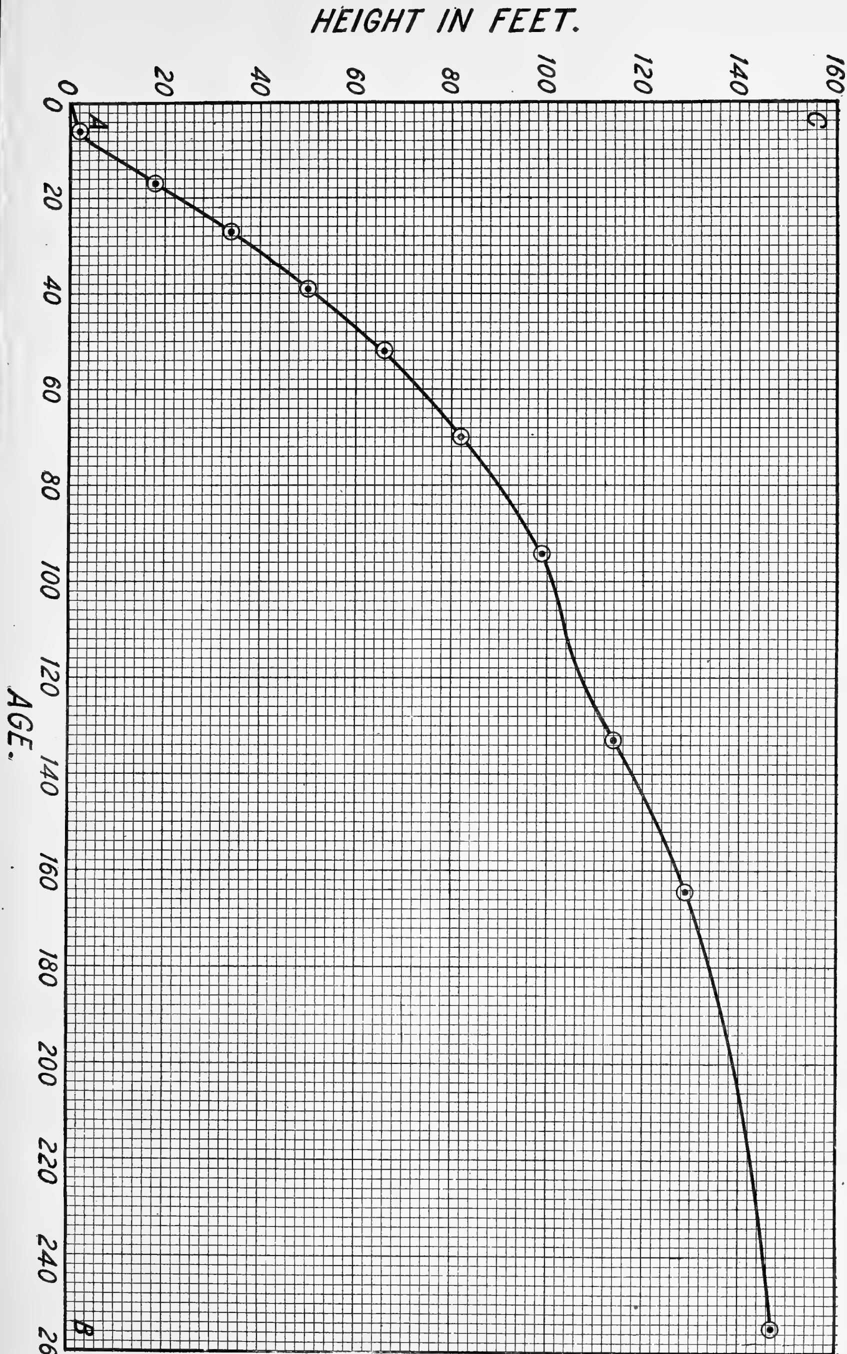


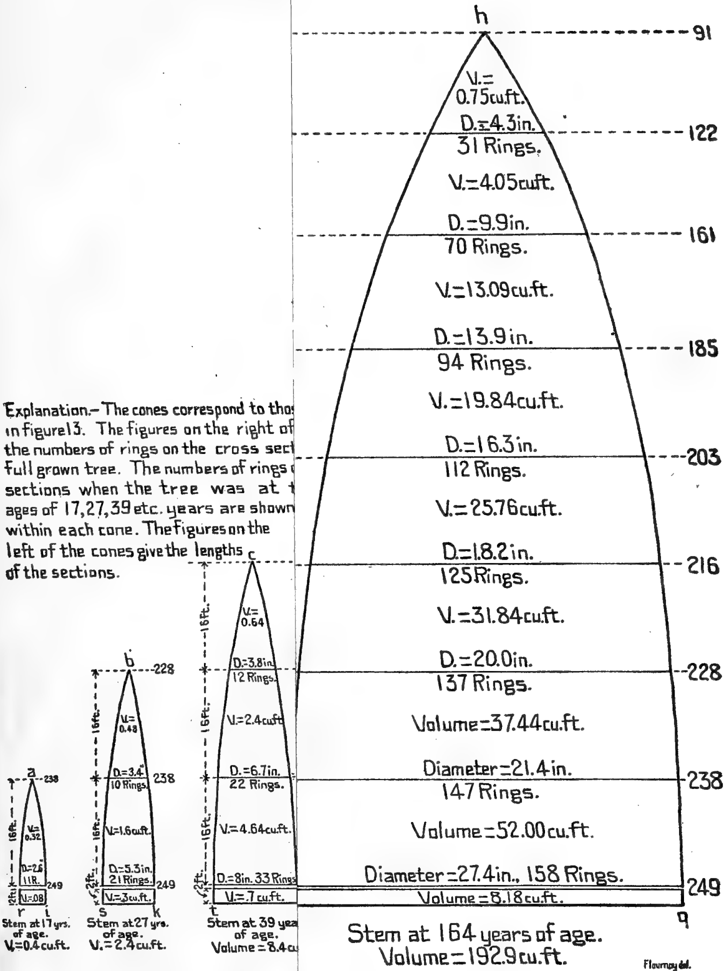
FIG. 14.—Graphic representation of height growth.

The sum of these products gives the volume of the first 17 enveloping cones, or, what is the same, the volume the tree had when 17 years old.

The volume of the first 27 enveloping cones—i. e., the volume the tree had when 27 years old—is obtained as follows (see fig. 15; cone *sbk*): Determine the diameter of the first 21 central rings on the stump section (difference between the ages of stump and second sections); determine the diameter of the first 10 central rings on the first cross section (difference between the ages of first and second sections); calculate the volume of the portion between stump and first sections, considering it as frustum of cone or as a paraboloid; calculate the volume of the portion above the first section, considering it as a paraboloid. The volume of stump the tree had when 27 years old is calculated by considering it as a cylinder, with a diameter equal to that of the first 21 central rings taken on the stump of the tree. By adding together the volumes thus calculated the volume the tree had when 27 years old is obtained. The volumes the tree had when 39, 52, 70, 94, 133, and 164 years old are determined in the same manner (see fig. 15; cones *tcl*, *udm*, *ven*, *wfo*, and *xgp*)—i. e., the volume of portions of tree between two consecutive sections are calculated, considering them either as frustums of cones or as frustums of paraboloids, while the volume of the last portion of tree is calculated, considering it as a paraboloid; the volume of stump is calculated, considering it as a cylinder with the diameter taken on stump section.

The total volume of the tree is also calculated as explained above. The progressive development of volume growth may be then determined by means of graphical interpolation, as shown by fig. 16, which represents the volume growth of the average tree of the group under analysis.

The analysis is generally concluded by collating the figures relating to height, diameter, area, and volume growth in the form of the following table:



Explanation.—The cones correspond to thos in figure13. The figures on the right of the numbers of rings on the cross sect full grown tree. The numbers of rings sections when the tree was at t ages of 17, 27, 39 etc. years are shown within each cone. The figures on the left of the cones give the lengths of the sections.

5107——Face

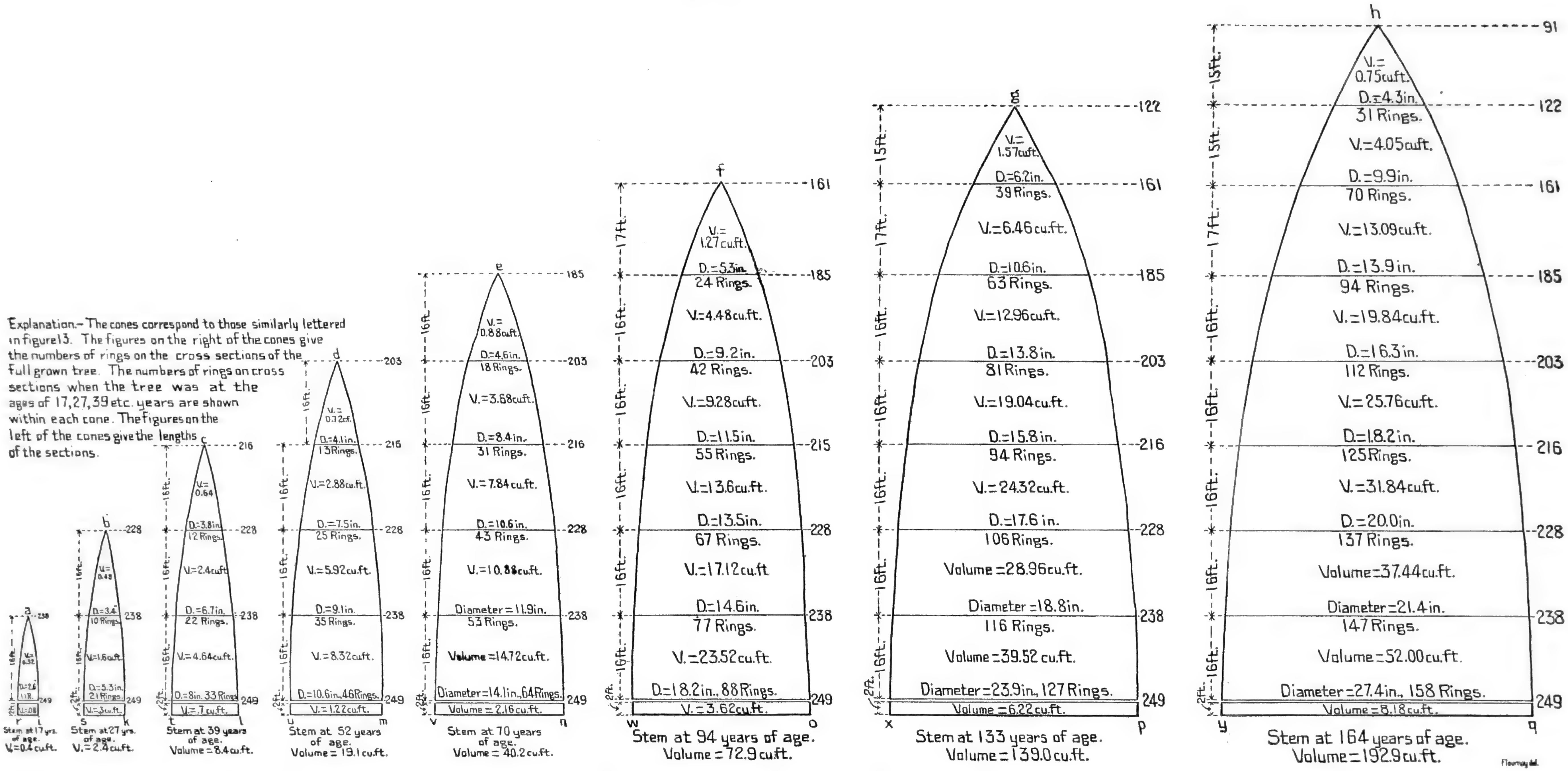
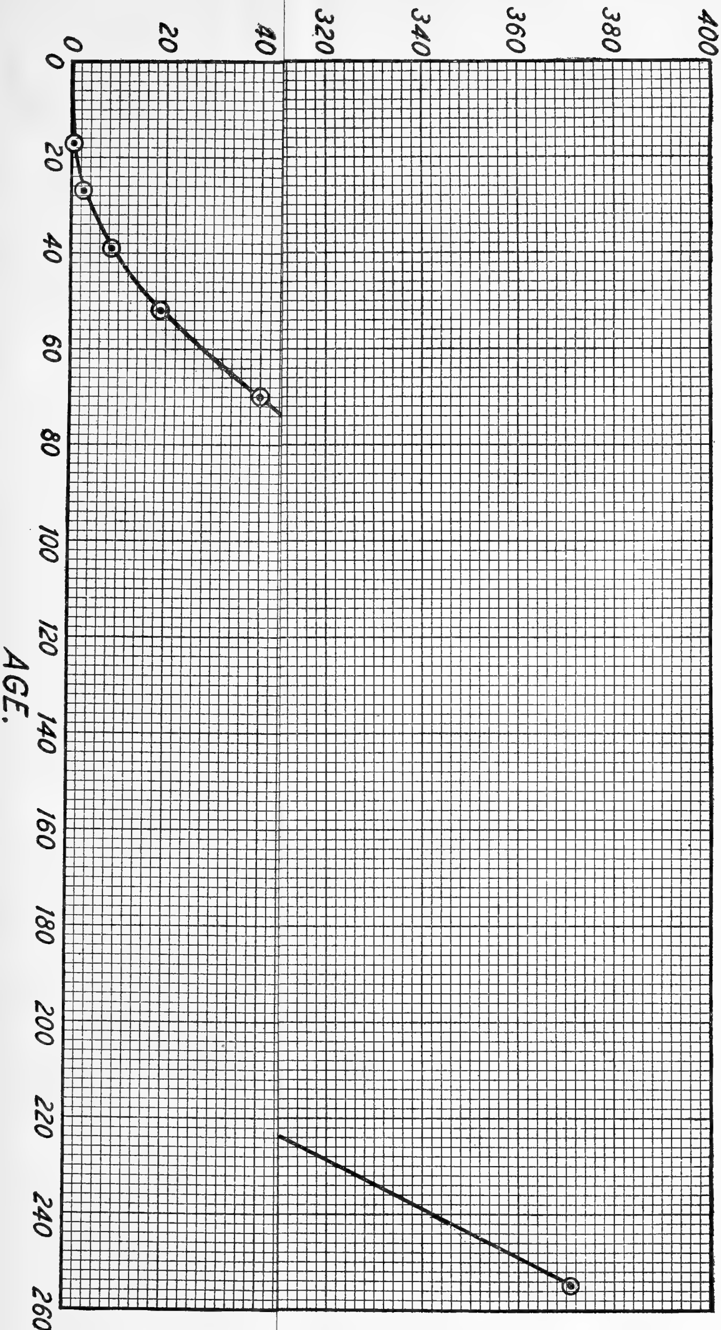


FIG. 15.—Cones representing size of stem at successive ages.


AGE.
0
20
40
60
80
100
120
140
160
180
200
220
240
260
0
20
40
320
340
360
380
400

5107——Face

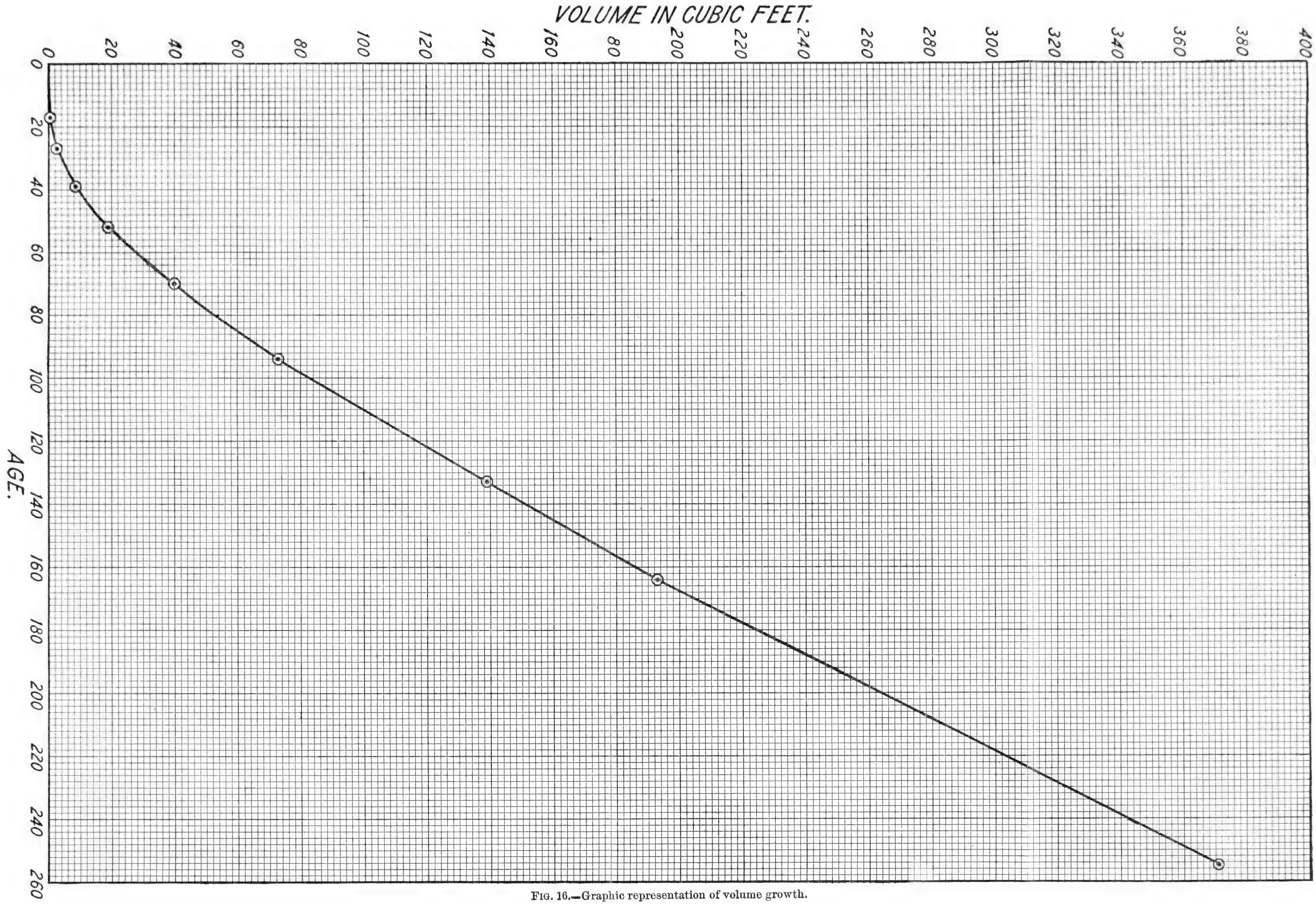


FIG. 16.—Graphic representation of volume growth.

FORM NO. 6.—*Table showing the rate of growth of the dominant group.*

SITE: *f.* SPECIES: White pine.

Dominant growth (16 trees).

Age.	Diameter at height of 2½ feet (without bark).	Total height of tree.	Volume of stem (without bark).	Relative per cent of total volume: Heartwood.	Relative per cent of total volume: Sapwood.	Relative per cent of total volume: Bark.	Periodical accretion: Decade.	Height.	Cross-section 1 at height of 2½ feet: Diameter.	Cross-section 1: Area.	Cross-section 2 at height of 18 feet: Diameter	Cross-section 2: Area	Cross-section 3 at height of 34 feet: Diameter.	Cross-section 3: Area.	Cross-section 4 at height of 50 feet: Diameter.	Cross-section 4: Area.	Cross-section 5 at height of 66 feet: Diameter.	Cross-section 5: Area.	Cross-section 6 at height of 82 feet: Diameter.	Cross-section 6: Area.	Cross-section 7 at height of 99 feet: Diameter.	Cross-section 7: Area.	Cross-section 8 at height of 114 feet: Diameter.	Cross-section 8: Area.	Cross-section 9 at height of 129 feet: Diameter.	Cross-section 9: Area.	Volume.	Average annual accretion.	Current periodic accretion.
	In.	*Feet.*	*Cubic feet.*	*Per ct.*	*Per ct.*	*Per ct.*		*Ft.*	*In.*	*Sq. feet.*	*In.*	*Sq. feet.*	*In.*	*Sq. feet.*	*In.*	*Sq. feet.*	*In.*	*Sq. feet.*	*In.*	*Sq. feet.*	*In.*	*Sq. feet.*	*In.*	*Sq. feet.*	*In.*	*Sq. feet.*	*Cubic feet.*	*Cubic feet.*	*Cu. feet.*
10	1.2	8	(?)				First	8	2.3	0.03	3.4	0.06	3.1	0.05	3.2	0.05	2.7	0.04	2.4	0.03	1.7	0.01	1.4	0.01	1.1	0.01	(?)	(?)	(?)
20	3.4	23	0.8				Second	15	2.8	.11	2.9	.16	3.4	.18	2.9	.15	2.4	.10	2.2	.08	1.7	.05	1.5	.03	1.2	.02	(?)	0.04	(?)
30	6.1	38	3.6				Third	15	2.3	.16	2.0	.15	2.0	.16	2.1	.17	2.3	.16	1.7	.11	1.6	.08	1.3	.06	1.1	.03	2.8	.12	0.28
40	8.2	51	9.0				Fourth	13	2.1	.19	1.6	.16	1.7	.18	1.6	.15	1.6	.14	1.5	.11	1.2	.08	1.2	.06	1.1	.05	5.4	.22	.54
50	10.2	64	17.0				Fifth	13	1.9	.22	1.6	.19	1.4	.16	1.2	.14	1.2	.13	1.3	.12	1.4	.10	1.1	.07	1.1	.06	8.0	.34	.80
60	12.2	73½	27.0				Sixth	9½	1.9	.25	1.2	.16	1.2	.16	1.1	.14	1.2	.14	1.2	.13	1.2	.11	1.0	.08	.9	.06	10.0	.45	1.00
70	14.2	82	40.0				Seventh	8½	2.1	.33	1.2	.17	1.0	.15	1.1	.15	1.2	.15	1.1	.13	1.0	.10	1.1	.09	.9	.07	13.0	.57	1.30
80	16.1	90	52.0				Eighth	8	1.8	.32	1.0	.16	1.1	.17	1.2	.18	1.1	.16	1.1	.14	1.1	.13	.9	.09	.9	.08	12.0	.65	1.20
90	17.8	96	64.0				Ninth	6	1.3	.26	1.0	.17	1.0	.17	1.0	.16	.9	.14	1.0	.14	.8	.10	.9	.10	1.0	.09	12.0	.71	1.20
100	19.0	102	80.0				Tenth	6	1.2	.25	1.2	.21	1.1	.20	.9	.16	.8	.13	.9	.14	1.0	.13	.8	.09			16.0	.80	1.60
110	20.4	107	98.0				Eleventh	5	1.8	.40	1.1	.22	1.0	.19	.8	.14	.8	.14	.9	.15	.8	.12	.9	.12			18.0	.89	1.80
120	22.0	110½	116.0				Twelfth	3½	1.5	.36	1.0	.20	.8	.16	.7	.14	.8	.15	.8	.13	.9	.14	.8	.11			18.0	.98	1.80
130	23.5	113½	133.0				Thirteenth	3	1.3	.34	1.0	.21	.8	.16	.8	.16	.7	.13	.8	.15	1.0	.16					17.0	1.02	1.70
140	24.8	117	150.5				Fourteenth	3½	1.3	.35	.8	.18	.6	.13	.7	.14	.7	.14	.8	.15	.8	.14					17.5	1.07	1.75
150	26.0	122	167.5				Fifteenth	5	1.1	.32	.6	.14	.8	.18	.7	.15	.8	.16	.7	.14	.9	.16					17.0	1.12	1.70
160	27.0	127	184.5				Sixteenth	5	.9	.26	.7	.17	.6	.14	.6	.16	.6	.13	.7	.14	.8	.16					17.0	1.15	1.70
170	28.0	130	202.0				Seventeenth	3	1.0	.31	.7	.17	.6	.15	.6	.11	.6	.13	.7	.15							17.5	1.19	1.75
180	29.0	132	221.0				Eighteenth	2	.9	.29	.7	.18	.6	.14	.6	.14	.6	.13	.6	.13							19.0	1.23	1.90
190	29.8	134	240.0				Nineteenth	2	.8	.26	.6	.16	.6	.16	.7	.17	.6	.14									19.0	1.26	1.90
200	30.6	136	259.0				Twentieth	2	.9	.30	.7	.19	.7	.18	.6	.15	.5	.12									19.0	1.29	1.90
210	31.6	138	277.0				Twenty-first	2	.9	.31	.6	.16	.7	.18	.7	.18											18.0	1.32	1.80
220	32.4	140	298.5				Twenty-second	2	.8	.28	.6	.17	.8	.22													21.5	1.35	2.15
230	33.2	142	320.0				Twenty-third	2	.9	.33	.6	.18															21.5	1.39	2.15
240	34.1	144	340.0				Twenty-fourth	2	.8	.29																	20.0	1.41	2.00
250	35.0	146	361.0	70	19	11	Twenty-fifth	2																			21.0	1.44	2.10

APPENDIX.

BLANK NO. 1.

UNITED STATES DEPARTMENT OF AGRICULTURE,
DIVISION OF FORESTRY.

RECORDS OF TREE MEASUREMENTS.

Name of collector: N. N.
Species: White pine.
Year: 1897.

GENERAL DESCRIPTION OF STATION: A.
(Denoted by capital letter.)

State: Pennsylvania. County: Clearfield. Town: Dubois.
Longitude: 78° 45′. Latitude: 41° 3′. Altitude: 1,200 to 1,500 feet.
General configuration: Plains hills plateau *mountainous.*
General trend of valleys or hills: (Not noted.)
Climatic features: (Meteorological tables furnished.)

General forest conditions of the region: This forest area, in 1876, extended over 20,000 acres. The lumber operations carried on for twenty years by Mr. Du Bois have left only from 1,500 to 2,000 acres of standing timber in a primeval condition.

Three typical forms of forest conditions are suggested to the observer:

(1) Hemlock and white-pine forest, with an admixture of mature hardwoods and a number of young hardwoods and young hemlock, which form the undergrowth.

(2) Hemlock mixed with white pine, with scattering hardwoods. The undergrowth, usually moderately dense, consists mainly of young hemlock with the admixture of young hardwoods.

(3) Hardwoods intermixed with white pine and scattering hemlock. The undergrowth here consists mainly of young hardwoods.

Among the hardwoods the oak, birch, and the maple form the staple of the hardwood forest, while the beech, the chestnut, the hickory, the cucumber, the ash, the cherry, and the basswood are comparatively few in number.

The region has a uniform soil and subsoil, as may be judged by the sample areas NN 5, 6, and 7, and is well provided with moisture by the many streams crossing it all over in different directions.

BLANK NO. 2.

DESCRIPTION OF SITE: *f*.

[Denoted by small letter *f*.]

Sample area, No. 5: (One acre.)

Conformation of surface: Hill sloping toward southwest, where it is bordered by the Irish Narrow Creek.

Soil and drainage conditions: Yellow clay loam of a medium grain (fine shale in it), deep, fresh, well drained, with 2 to 3 inch mold on top.

Subsoil: Laminated shale of an indefinite depth.

Soil cover: Scanty leaves, fern, and tea berries.

Origin of stand: Natural regeneration.

Form: Uniform; storied. White pine forms first and hemlock the second.

Composition: A stand of hemlock mixed with white pine, intermixed with scattering maple, beech, and birch.

Undergrowth: Absent; dense; moderately dense; scanty; consists of very young beech, hemlock, and occasional birch, cucumber, and dogwood (laurel in northeast corner).

Density of stand: 0.7 (in places 0.8).

REMARKS.—Crowns of white pine, generally well developed; clear and straight stems. Age of white pine 230 to 260 years. Age of hemlock almost the same as that of white pine.

ACRE-YIELD MEASUREMENT OF SITE: *f*.

Diameter at breast height (in inches).	Name of species. White pine. Dominant, height from 140 to 160 feet.	White pine. Codominant, height from — to —.	White pine. Oppressed, height from 130 to 150.	Hemlock. Dominant, height from 90 to 120.	Hemlock. Codominant, height from 80 to 120.	Hemlock. Oppressed, height from 60 to 80.	Maple. Dominant, height from 40 to 60.	Maple. Codominant, height from — to —.	Maple. Oppressed, height from — to —.	Beech. Dominant, height from 40 to 60.	Beech. Codominant, height from — to —.	Beech. Oppressed, height from — to —.	Birch. Dominant, height from 40 to 60.	Birch. Codominant, height from — to —.	Birch. Oppressed, height from — to —.	Undergrowth. Name of species.	Undergrowth. Diameter (b.h.). 3–6 inches.	Undergrowth. Diameter (b.h.). Under 3 inches.
6																		
7						///// ///// //	/											
8													/					29
9						///	/											36
10						//										Hemlock.		43
11													/					
12						////							/					
13										//								37
14						///												34
15						////	//											
16			/			/	/											
17					///	/////												
18				/	/////											Maple.		
19			/	////	//					/								11
20			/	//	////	/												
21				///	//													
22					////													
23		/	/	///														47
24																		38
25		/////		///												Beech.		31
26		////		/														42
27				////														38
28	/	///		/														
29		////		/														
30		/		/														
31	//																	
32	//															Birch.		10
33	//																	
34	///																	
35																		
36	//																	
37	/																	

38 39 40 41 42 43 44 45	 / / 															Cucumber.		4

DEDUCED RESULTS.

Total number of trees on the acre: 132, of which there were—

White pine, 37. Dominant, 41 per cent; codominant, 48 per cent; oppressed, 11 per cent.

Hemlock, 84. Dominant, 32 per cent; codominant, 26 per cent; oppressed, 42 per cent.

Maple, 5.

Beech, 3.

Birch, 3.

Total yield of the acre: Volume of stems, 15,686 cubic feet; merchantable timber, 90,103 feet, B. M., of which there was a total yield of 58 per cent of white pine and 42 per cent of hemlock.

Average annual accretion: In cubic feet, 65; merchantable timber in feet, B. M., 375.

BLANK NO. 3.

MEASUREMENTS OF STEMS.

SITE: *f*. SPECIES: WHITE PINE.

Tree No.	Diameter (in inches) at a height from ground of— Breast height.	8 feet.	16 feet.	24 feet.	32 feet.	40 feet.	48 feet.	56 feet.	64 feet.	72 feet.	80 feet.	88 feet.	96 feet.	104 feet.	112 feet.	120 feet.	128 feet.	Height to base of crown.	Total height of tree.	Character of growth. Dominant.	Codominant.	Oppressed.	Merchantable timber (B. M.).	Condition of timber. Sound.	Defective.	Crooked.	Clear.	Knotty.	Windshaken.	Surrounding species.	Remarks.
1	35½	34	29	28	28	27	26½	26	25	24	23	22	20	19	16½	13	11	90	158	×	...	...	3,030	×	...	...	...	...	...	White pine, hemlock, and small beech.	
2	36	33	31	30	29	28	28	27	26	26	25	23	21½	19½	18	15	13	90	157	×	...	...	3,401	×	...	...	...	...	...	Hemlock, pine, birch, maple, and ash.	
3	32	30	28	28	27	25½	25	24½	24½	23½	23½	21½	19½	18	15½	12½	...	84	152	×	...	...	2,637	...	...	...	...	...	...	Hemlock and pine.	
4	32	30	27½	26	26	25	24	23½	22½	21½	20½	18½	17	15	12½	10½	8	62	150	×	...	...	2,079	...	...	...	...	...	...	do	
5	29	27½	26	25	25	24	23	22	21½	21	18½	17	15½	14	11½	8	6	100	140	...	×	...	1,905	×	...	...	...	...	...	do	Crowded.
6	29	26½	25½	24½	23½	23	22½	21½	20	20	19	17½	16	15	13	9	7	110	140	...	×	...	1,631	×	...	...	...	...	...	do	
7	29	27½	25½	24½	23½	23	21½	21	21	19½	19	18	16½	15½	14	12½	9½	112	152	...	×	...	1,854	×	...	...	...	...	...	do	
8	29	26	23½	22½	22	21	20	19	18½	18½	17	16½	14½	13	11	8	6	86	142	...	×	...	1,313	Core at butt.	...	...	...	...	...	do	
9	32	28½	25½	24½	23½	23	22	21	21	18½	18	16½	15	14	11	8	7	84	142	...	×	...	1,648	do	...	...	...	...	...	do	
10	33	30½	29	27½	26½	25½	25	24	23	21½	21	19	18	15½	13½	12½	8	96	146	×	...	...	2,384	×	...	...	...	...	...	do	Crown free.
11	30	28½	27	25	24½	24	23	22½	20½	19½	18½	17½	16½	13½	11½	9	6	81	141	...	×	...	1,947	...	...	...	...	...	...	do	Somewhat crowded.
12	28	27	24½	24	23	23	22	21	20½	19½	19	17	16	13½	12	9½	8	88	156	×	...	...	1,648	...	...	...	...	...	...	do	
13	23	20½	20½	20½	20½	19½	19	18½	17	16½	15	15	14	12½	11½	9½	7½	93	147	...	×	...	1,048	...	...	...	...	...	...	do	Crowded.
14	25	23½	22½	22	21	20½	19½	18½	18	17½	16	14½	13	11½	9	6½	...	98	139	...	×	...	1,233	Core at butt.	...	...	...	...	...	do	Do.
15	26	24½	23½	22½	21½	21	21	20½	20½	18½	17½	17	16	14	12½	10	7	98	136	...	×	...	1,389	do	...	...	...	...	...	do	Top killed, somewhat crowded.
16	24½	21½	19½	19	18½	17½	17	16½	16	14½	14	12	10½	8	4½	...	...	93	124	...	×	...	815	do	...	...	...	...	...	do	Crowded.
17	25	23½	22	21	20	20	19	19	18	17½	16½	15	14	12½	8½	...	...	108	128	...	×	...	1,183	do	...	...	...	...	...	do	Do.
18	39	36	33	32½	30	29½	28½	28	26	26	23	22½	21½	18½	18	14½	11½	88	153	×	...	...	3,318	...	...	...	...	...	...	do	
19	34	32	30	29	28	27½	26½	25	24½	23	22	19½	18	16½	13	10½	8	78	134	×	...	...	2,397	...	...	...	...	...	...	do	At 134 feet top lost. Free crown (about 16 feet).

20	44	39	38	36½	35½	34	32½	31½	30½	28	27	26	23	21½	20	14½	10½	100	144	×	...	...	4,388	×	...	...	...	...	...	do	Crown free.
21	34	31½	28	27	26½	25½	24½	23½	22½	21½	21	19½	18	16½	14½	11½	8	92	146	×	...	...	2,248	×	...	...	...	...	...	do	Crown free—sound.
22	31	29	27	27	25	24	23	23	21	19½	19½	17	14½	12	9½	...	...	82	130	...	×	...	1,731	×	...	...	...	...	...	do	Crown somewhat crowded.
23	34	31½	29	28	27	25½	25	24	23½	22	20½	20½	18	16½	13½	9½	6	90	142	×	...	...	2,318	×	...	...	...	...	...	do	Crown free.
24	25	23½	22	21	20	19	18½	17½	16½	15	14	13	12	10	7	...	...	84	130	...	×	...	1,102	×	...	...	...	...	...	do	Somewhat crowded.
25	28½	26½	25½	24½	24½	24	22½	21½	21	20½	19	18	16	15½	13	11	5	107	138	...	×	...	1,954	×	...	...	...	...	...	do	Do.
26	26	24	21½	21	20½	19	18	17	16½	16	15	14	12½	11½	9	6	...	98	136	...	×	...	1,021	×	...	...	...	...	...	do	
27	19	18	17½	17½	16½	16	15½	15	14	14	13	12	11½	9½	8½	6½	...	94	132	...	...	×	683	×	...	...	...	...	...	do	Crown very crowded
28	28½	26	24	23	22½	22	21½	21	20	19	18	17	15	13½	10½	7	...	75	138	...	×	...	1,551	×	...	...	...	...	...	do	
30	26½	24½	23	22½	21	20½	20	19½	19	18	16½	15	13	11	9	6	...	90	134	...	×	...	1,336	Core at butt	...	...	...	...	...	do	Somewhat crowded.
33	33	30	27½	25½	25½	24½	23	22	21	20	18	16½	13½	12½	9	5½	...	91	133	×	...	...	1,770	×	...	...	...	...	...	do	Free exposure.
34	33	30	27½	27	26	25	25	24½	23½	22½	21	20	19	15½	15½	12	9	90	146	×	...	...	2,220	×	...	...	...	...	...	do	Do.
35	31	27	24½	23½	23	22½	21½	20½	20	19½	19	16½	14½	13	11	8	5½	82	136	×	...	...	1,458	×	...	...	...	...	...	do	Top about 18 feet, not found.
36	31½	28½	26½	26	25	24	24	23½	21½	21	19	16½	15	12	10	7½	...	88	134	×	...	...	1,855	×	...	...	...	...	...	do	Do.
37	37	35	32½	32	31	30	28½	28	26½	26	23½	22½	21½	17	14	11½	8	106	146	×	...	...	2,970	×	...	...	...	...	...	do	Do.
38	23	22	21½	20	19½	19	18	17½	17	16	15	14	12½	11	9	5	...	96	137	...	...	×	987	×	...	...	...	...	...	do	Crown very crowded
39	20½	18½	18	17½	17	16	15	14½	13½	12½	12	11½	10½	9½	5	...	...	109	123	...	...	×	558	...	...	...	...	...	...	do	Do.
40	16½	16	15½	15	14	13½	13	12½	12	11	10	8½	7½	4½	...	...	...	82	120	...	...	×	339	...	...	...	...	...	...	do	Crown very small.

BLANK NO. 4.

SECTION, Stump.

SITE: *f*. AGE CLASS: 240 to 260 years. SPECIES: White pine.

Tree No.	Height from ground.	Thickness of bark.	Sap-wood. No. of rings.	Sap-wood. Width.	No. of rings.	Radius at age of— 10	20	30	40	50	60	70	80	90	100	110	120	130	140	150
		mm		*mm*																
1	2	48	36	38	254	38	80	105	125	139	157	179	198	218	244	273	292	306	320	329
2	2	35	33	37	254	24	54	71	85	96	109	139	163	180	213	235	259	281	308	325
3	2	55	33	35	253	34	72	98	119	135	156	177	192	209	228	250	267	283	300	312
4	2	55	20	25	235	25	59	107	133	178	200	221	256	276	304	313	330	346	357	369
10	2	40	47	30	238	38	78	116	145	180	205	244	248	257	276	295	309	322	333	350
12	2	30	45	28	256	32	61	80	99	121	138	158	178	186	200	214	229	240	253	268
18	2	50	28	38	259	24	46	74	102	121	142	175	201	223	246	268	295	321	349	375
19	2	50	35	50	244	27	53	75	103	129	154	182	199	212	232	250	263	282	303	311
20	2	50	30	18	260	37	89	133	165	200	251	292	345	372	392	409	426	444	461	477
21	2	40	37	30	239	25	58	82	102	121	139	158	175	186	203	225	245	270	294	315
23	2	49	24	22	242	19	53	77	99	120	144	167	190	210	244	257	274	288	302	311
36	2	25	35	45	235	26	56	77	92	109	120	130	138	150	180	200	222	246	265	278
34	2	40	46	35	256	23	60	97	133	160	187	222	261	275	290	302	316	328	336	348
33	2	48	33	25	253	27	57	85	116	141	168	191	220	229	241	251	265	282	293	307
35	2	50	41	28	257	34	74	110	138	161	182	209	222	229	237	246	255	265	276	289
37	2	30	46	27	255	28	76	106	152	178	203	234	267	283	303	323	341	356	370	386
28	2	39	48	28	256	36	72	104	135	160	178	195	207	211	218	225	237	251	262	270
25	2	40	38	35	238	15	41	58	71	89	104	114	133	153	173	198	220	237	249	261
24	2	37	25	25	239	15	45	73	94	110	123	131	136	140	146	155	165	173	181	189
22	2	40	43	30	240	32	71	111	137	156	195	218	241	254	263	275	293	306	316	324
5	2	45	35	25	258	26	53	70	86	104	120	140	156	168	178	191	204	219	238	251
6	2	40	28	25	258	23	71	102	122	146	166	183	202	218	227	239	253	264	276	286
7	2	30	35	24	256	25	53	92	114	128	141	155	167	177	189	204	225	236	244	255
8	2	....	....	....	229 }							Core for 3 inches.								
9	2	....	....	....	230 }															
11	2	32	29	37	238	31	55	69	87	105	121	132	147	166	183	199	221	247	258	270

Tree No.	Radius at age of— 160	170	180	190	200	210	220	230	240	250	260	270	280	290	300	Entire radius.	Remarks.
1	336	341	348	359	367	376	385	395	406	410	....	....		....	....	415	
2	336	346	364	386	397	408	418	429	444	452	....	....		....	....	454	
3	319	326	335	343	351	363	375	385	389	400	....	....		....	....	405	
4	377	388	398	407	417	426	435	442	....	....	....	....		....	....	447	
10	362	370	389	398	408	415	422	430	....	....	....	....		....	....	434	
12	275	284	294	303	310	319	329	334	339	345	....	....		....	....	350	
18	391	409	426	441	457	464	481	498	513	528	....	....		....	....	541	
19	322	336	349	364	392	417	432	453	472	....	....	....		....	....	477	
20	488	500	509	516	525	537	547	544	561	567	571	....	Dominant.	....	....	....	
21	334	347	359	369	378	385	395	406	....	....	....	....		....	....	416	
23	324	342	353	359	366	374	383	413	420	....	....	....		....	....	421	
36	292	305	318	331	350	378	388	406	....	....	....	....		....	....	415	
34	357	367	376	382	390	398	408	415	423	429	....	....		....	....	432	
33	319	330	342	349	358	369	378	385	390	395	....	....		....	....	397	
35	293	304	318	329	338	345	352	358	365	373	....	....		....	....	378	
37	398	411	422	432	439	445	452	458	466	473	....	....		....	....	476	
28	274	279	285	289	293	298	306	312	320	325	....	....		....	....	328	
25	276	289	303	319	335	350	365	375	....	....	....	....		....	....	381	
24	197	208	218	237	255	277	298	314	....	....	....	....		....	....	320	
22	332	343	356	365	375	385	390	407	412	....	....	....		....	....	....	
5	263	277	294	309	324	337	347	356	363	373	....	....	Codominant.	....	....	382	
6	293	299	306	318	327	338	349	358	365	372	....	....		....	....	377	
7	263	269	278	287	295	302	312	319	323	334	....	....		....	....	337	
8																	
9																	
11	284	300	314	327	342	357	371	384	....	....	....	....		....	....	396	

13	2	42	32	20	252	20	61	93	114	131	144	158	172	185	199	214	226	236	246	255	264	272	282	290	295	300	309	316	321	326	....	....		....	....	327
14	2	....	....	....	236							Core for $4\frac{1}{2}$ inches.																								
15	2	....	....	....	258							Core for 5 inches.																								
16	2	....	....	....	229							Core for 3 inches.																								
17	2	....	....	....	256							Core for 6 inches.																								
26	2	27	29	40	239	25	60	81	91	112	121	125	132	145	150	164	176	184	193	200	211	222	234	247	266	281	295	310	....	....	....	....		....	....	321
30	2	27	40	42	253	28	58	80	102	121	143	162	175	182	193	205	217	229	240	252	262	272	281	289	297	303	309	312	320	326	....	....		....	....	329
27	2	20	45	24	253	30	66	89	106	125	141	158	173	178	183	187	193	203	209	217	224	231	236	239	242	246	252	259	264	270	....	....	Op-pressed.	....	....	$271\frac{1}{2}$
38	2	38	47	29	254	35	73	97	119	137	152	165	176	185	192	201	210	221	231	237	244	250	256	261	264	269	273	277	281	284	....	....		....	....	285
39	2	32	54	24	252	37	81	102	122	127	148	156	163	167	170	176	183	191	196	200	203	205	208	214	220	226	230	235	238	241	....	....		....	....	$241\frac{1}{2}$
40	2	22	35	16	255	29	64	81	101	117	131	141	151	155	158	162	170	177	183	186	191	195	200	205	208	211	215	221	225	229	....	....		....	....	232

BLANK NO. 4.

SECTION NO. 3.

SITE: *f*. AGE CLASS: 240 to 260 years. SPECIES: White pine.

Tree No.	Height from ground.	Thickness of bark.	Sap-wood. No. of rings.	Sap-wood. Width.	No. of rings.	Radius at age of— 10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200	210	220	230	240	250	260	270	280	290	300	Entire radius.	Remarks.
		mm		*mm*																																	
1	50	15	49	32	221	50	76	98	115	130	143	156	180	193	205	214	224	230	235	242	249	256	262	269	275	282	287						Dominant.			288	
2	50	17	45	35	214	35	64	84	102	118	135	153	169	195	201	219	231	242	253	265	277	288	297	307	317	324	....									327	
3	50	17	47	42	227	33	72	97	112	127	138	150	164	176	185	196	204	210	217	226	234	243	253	263	272	280	288									293	
4	50	11	50	30	208	40	74	107	129	148	162	175	188	200	210	219	228	237	245	253	258	266	272	278	283	....	....									286	
10	50	14	50	30	210	42	80	110	130	144	158	172	185	196	205	217	225	234	245	253	259	265	272	278	283	293	....									....	
12	50	12	55	32	228	26	76	95	116	130	142	155	169	178	189	198	205	209	214	219	225	230	235	241	247	252	258									264	
18	50	19	41	40	223	35	64	86	106	125	139	157	176	195	213	228	243	257	263	275	288	292	306	318	328	337	346									349	
19	50	20	39	38	209	40	67	90	106	120	132	146	165	180	194	207	217	230	241	253	265	279	290	302	311	....	....									317	
20	50	20	50	30	219	68	123	164	193	212	231	246	262	278	288	297	306	313	320	327	334	340	347	355	362	368	....									371	
21	50	16	40	34	202	33	69	91	108	120	132	147	163	176	189	201	211	221	231	241	249	258	268	276	284	....	....									286	
23	50	15	40	40	204	20	61	91	109	122	134	148	163	177	190	200	212	223	233	242	250	259	270	280	295	....	....									300	
36	50	15	45	38	202	25	37	53	67	78	96	115	134	150	167	178	190	201	211	221	228	236	245	253	261	....	....									263	
34	50	15	41	30	223	45	82	110	135	153	164	177	190	200	210	216	225	234	242	248	256	265	276	283	290	298	304									307	
33	50	15	48	30	223	48	78	103	124	142	149	158	167	174	183	187	196	204	213	218	223	231	238	243	248	254	258									260	
35	50	15	47	20	226	47	84	114	135	148	155	161	168	176	183	190	198	204	210	217	222	228	233	239	242	247	251									254	
37	50	12	44	30	217	51	103	145	173	195	214	229	232	244	253	261	269	277	289	295	302	309	314	321	328	331	....									340	
28	50	12	63	29	228	44	81	109	127	142	153	160	167	175	184	195	204	208	214	220	225	230	235	240	243	248	253						Codominant.			257	
25	50	15	50	40	188	38	69	92	117	133	156	172	184	194	204	215	226	238	245	253	261	268	278	....	....	....	....									284	
24	50	12	26	25	193	13	18	25	33	53	75	89	101	111	118	126	137	147	160	173	188	202	214	220	....	....	....									221	
22	50	15	43	27	203	34	74	107	129	148	161	174	187	197	208	216	224	230	235	241	247	254	260	266	271	....	....									273	
17	50	15	50	30	226	27	64	88	104	117	128	137	146	155	164	169	173	175	178	182	186	192	198	205	212	217	223									226	
5	50	15	43	33	220	34	66	83	97	107	116	125	136	148	159	170	182	193	203	212	225	233	241	250	258	265	272									....	
6	50	17	45	30	222	40	72	93	108	119	128	137	148	159	170	180	190	200	207	215	221	229	236	243	250	255	261									262	
7	48	13	50	30	222	40	73	93	107	120	130	140	150	160	170	180	187	197	205	214	222	229	237	244	250	255	260									263	
8	50	16	45	35	188	22	46	52	75	87	102	116	129	140	151	164	172	180	189	198	207	214	221	....	....	....	....									226	
9	50	15	42	35	192	35	58	78	96	113	131	150	162	177	189	198	208	214	230	241	250	257	264	273	....	....	....									275	
11	50	20	44	44	200	37	62	78	89	102	113	127	141	156	166	176	187	200	214	226	239	249	259	262	270	....	....									....	
13	50	12	53	33	223	31	57	74	90	99	110	119	128	135	142	149	155	162	168	174	181	188	195	201	208	215	221									223	

14	50	12	34	35	204	30	50	60	69	76	84	93	102	112	123	130	137	145	156	167	176	186	196	205	216	....	....	....	....	....	....	....		....	....	220
15	50	15	39	39	219	35	62	78	88	100	107	115	123	134	142	151	158	164	172	180	189	198	209	217	228	239	....	....	....	....	....	....		....	....	248
16	50	15	45	37	185	27	55	72	85	100	112	122	132	140	148	153	161	169	179	186	195	204	211	....	....	....	....	....	....	....	....	....		....	....	216
26	50	10	35	35	194	21	46	65	83	89	97	107	117	123	129	137	146	155	168	179	189	199	210	220	....	....	....	....	....	....	....	....		....	....	223
30	48	10	51	30	220	38	71	98	114	125	134	141	150	161	171	179	187	192	198	203	210	215	220	225	233	239	244	....	....	....	....	....		....	....	....
27	50	14	40	19	223	43	71	89	103	113	117	120	122	125	129	133	138	143	148	150	153	156	160	163	167	171	174	....	....	....	....	....	Op-pressed.	....	....	175½
38	50	10	53	19	223	46	69	98	111	121	129	136	145	153	160	166	172	178	183	188	192	196	201	204	207	210	212	....	....	....	....	....		....	....	213
39	50	12	52	19	226	30	66	86	99	108	113	116	120	127	134	139	143	146	148	151	155	160	165	169	173	176	179	....	....	....	....	....		....	....	181
40	50	10	52	19	222	30	55	70	79	84	89	94	99	105	109	113	116	120	123	126	130	135	137	140	145	149	153	....	....	....	....	....		....	....	155

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