## TECHNICAL REPORT

# A METHOD FOR ESTIMATING THE FLUSHING <br> TIME OF ESTUARIES AND EMBAYMENTS 

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## ABSTRACT

A relationship is derived from the circulation pattern of a piedmont-type estuary which expresses the time required for a contaminated estuarine volume containing dissolved or suspended matter to be replaced by tidal action and river flow. The volume of such an estuary consists of an upper layer of mixed water with a net seaward movement, overlying a layer of sea water with a net landward movement. These volumes are separated by a surface of no net horizontal motion through which there is a net vertical movement of sea water into the mixed layer. Hence, the lower layer is renewed by tidal action alone while the mixed layer is renewed by tidal action and river runoff.

Since the net seaward flow of mixed water is equal in volume to the net landward flow of sea water plus the river flow, the balance should be indicated by tidal current data. Thus, it is possible to compute the flushing time without first computing the river flow.

In embayments and/or estuaries during a dry season, a relae flushing time the tidal prism the segments. incentration of 1 the exchange

## FOREWORD

The equations and formulas developed through a purely theoretical approach to oceanography sometimes are difficult to apply directly to a specific naval operation or problem. At times slight modifications in the theoretical approach make these formulas more usable for practical problems; at other times a totally new or different approach is required. In the flushing problem, a combination of theoretical considerations has provided better methods of calculation for the "general" estuary. This report describes such a method currently being used for determining the flushing characteristics of estuaries.


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## PART I

## Estuaries

The river water entering an estuary mixes with salt water and continues its movement to the sea. When the temperature of the river water does not fall below that of the sea water, a net seaward movement of mixed water occurs in the upper layers of the estuary. When the volume of river water received by the estuary in a tidal cycle is constant over a period of time, the volume of river water reaching the sea becomes equal to that received from the land in a tidal cycle.

Over a period of time the salinity at a point in the estuary will remain practically constant. Since salt is constantly being lost from the upper levels of the estuary, because of the net seaward volume transport from these levels, there must be a net volume transport of salt into the lower levels of the estuary to balance this loss. The situation just described is illustrated in Figure 1.

From the standpoint of the net volume transport pattern of Figure 1 , an estuary can be divided into two volume sections, $A$ and $B$, as in Figure 1. Here the net flow in $A$ is toward the sea, while the net flow


FIGURE I CROSS SECTION OF PIEDMONT-TYPE ESTUARY SHOWING NET CIRCULATION PATTERN
in $B$ is toward the land. Because of the opposing directions of the net flow velocities, a surface cd must exist where the net horizontal transport of salt is equal to zero. Moreover, there must exist through the surface cd a net volume transport of salt per tidal cycle from $B$ into $A$, equal to that entering the seaward boundary of $B$ in a tidal cycle. If, now, the volume of river water received from the land in a tidal cycle is designated by $R$, and $Q_{B}$ represents the net volume transport of sea water in $B$, the net volume transport of mixed water in $A$ will be represented by:

$$
\begin{equation*}
Q_{A}=R+Q_{B} . \tag{1}
\end{equation*}
$$

The hatched zone in Figure 1 is assumed to contain water which has very little motion, and hence this zone does not relate to the problem at hand. Also, the water in the shaded zone of Figure 1 will be considered to be dead, because the sea water arriving at point $g$ is expected to pass from $f$ rather than uphill from some point $e$.

When an estuary is shallow, some river water will flow toward the land in $B$, but when the depth is relatively great, the flow in $B$ will consist chiefly of sea water.

It is now assumed that a contaminant will be dispersed uniformly

through the entire estuary of Figure 1 at mean high water. Consider now the motion of a water particle from position 1 through position 18 in Figure 2. Obviously, it is only the net movement of the particle which contributes to its passage through the estuary. It is, therefore, required to find this net movement in order to solve the problem at hand.

One method for accomplishing this would be to determine the average flood and ebb velocities through the seaward boundaries of $A$ and $B$. Then

$$
\begin{equation*}
Q_{A}=h d\left(U_{E} t_{E}-U_{F} t_{F}\right) \tag{2}
\end{equation*}
$$

where, hd is the mean area at the seaward boundary of $A, U_{E}$ and UF refer to mean ebb and flood, respectively, and $t$ is the duration of flood and ebb flow. Again, if the river flow $R$ is computed independently,

$$
\begin{equation*}
Q_{B}=e d\left(U_{F} t_{F}-U_{E} t_{E}\right) \tag{3}
\end{equation*}
$$

from which (2) can be determined since $Q_{A}=R+Q_{B}$.
The case for which only river flow data is available for the solution of the problem will be discussed later on.

When $Q_{A}$ and $Q_{B}$ can be determined, as above, the following statements can be made:
(a) Except for the first several tidal cycles following the initial contamination of the estuary, the contaminant in $B$ must leave the estuary via cd and the opening hd (Fig. 3).
(b) Any contaminant leaving the estuary in a tidal cycle will be contained in a volume $Q_{A}$.

The following assumptions are made to supplement (a) and (b): The contaminant remaining in the estuary at any time becomes uniformly distributed at high tide; and if the removal of the contaminant is excessive or deficient during a time interval, the rate of removal will compensate during a following time interval.

We can now write exchange ratios for sections $A$ and $B$, as

$$
\begin{equation*}
r_{A}=\frac{Q_{A}}{A} \tag{4}
\end{equation*}
$$

and

$$
\begin{equation*}
r_{B}=\frac{Q_{B}}{B}, \tag{5}
\end{equation*}
$$

respectively. These ratios express the fraction of volume $A$ which is lost to the sea in a tidal cycle, and the fraction of volume $B$ which passes into section $A$ in a tidal cycle. The actual volume of mixed water containing contaminant, which is removed from the estuary with each ebb of the tide, is:

$$
\begin{equation*}
r_{A} A=Q_{A} \tag{6}
\end{equation*}
$$

Consider now the high water situation in the estuary at the end of the first tidal cycle following initial contamination. The volume of contaminated water removed with the first ebb tide is given by (6); the contaminant remaining in section A would, therefore, be given by

$$
\begin{equation*}
A\left(1-r_{A}\right) \tag{7}
\end{equation*}
$$

had not the quantity

$$
\begin{equation*}
r_{B} B \tag{8}
\end{equation*}
$$

entered section A from section $B$. The remaining volume of contaminant in $B$, is:

$$
\begin{equation*}
B\left(1-r_{B}\right), \tag{9}
\end{equation*}
$$

at the end of the first tidal cycle.
The contaminant lost from section A during the second tidal cycle is, from (7) and (8),

$$
\begin{equation*}
\left.r_{A} C_{1}=r_{A}\left[A\left(1-r_{A}\right)+r_{B} B\right)\right], \tag{10}
\end{equation*}
$$

and that remaining in section $A$ at the time of the second high water is

$$
\begin{equation*}
C_{2}=\left[A\left(1-r_{A}\right)+r_{B} B\right]\left(1-r_{A}\right)+r_{B} B\left(1-r_{B}\right), \tag{11}
\end{equation*}
$$

where, as before, the quantity $r_{B} B\left(1-r_{B}\right)$
entered A along with the salt necessary to replenish that lost to the sea with volume $Q_{A}$.

Similarly, for the third tidal cycle,

$$
\begin{equation*}
C_{3}=\left\{\left[A\left(1-r_{A}\right)+r_{B} B\right]\left(1-r_{A}\right)+r_{B} B\left(1-r_{B}\right)\right\}\left(1-r_{A}\right)+r_{B} B\left(1-r_{B}\right)^{2} \tag{12}
\end{equation*}
$$

represents the remaining contaminant in section $A$.
The terms in (12) can be rearranged to give the equivalent expression

$$
\begin{equation*}
C_{3}=A\left(1-r_{A}\right)^{3}+r_{B} B\left[\left(1-r_{A}\right)^{2}+\left(1-r_{A}\right)\left(1-r_{B}\right)+\left(1-r_{B}\right)^{2}\right] . \tag{13}
\end{equation*}
$$

In (13) the sum involving the terms ( $1-r_{A}$ ) and ( $1-r_{B}$ ) is, except for other constant terms, similar to the expanded form of

$$
\left[\left(1-r_{A}\right)+\left(1-r_{B}\right)\right]^{(n-1)} .
$$

We can, therefore, write for the contaminant remaining in $A$ at the end of $n$ tidal cycles

$$
\begin{align*}
C_{n}= & A\left(1-r_{A}\right)^{n}+r_{B} B\left[\left(1-r_{A}\right)^{(n-1)}+\left(1-r_{A}\right)^{(n-2)}\left(1-r_{B}\right)+\ldots \cdots\right.  \tag{14}\\
& \left.+\left(1-r_{A}\right)\left(1-r_{B}\right)^{(n-2)}+\left(1-r_{B}\right)^{(n-1)}\right] .
\end{align*}
$$

This last expression is too complicated to be of practical use. If, in the second member on the right of (14), $r_{A}$ and $r_{B}$ are replaced by

$$
\begin{equation*}
r \neq r_{A} \cong r_{B} \neq r=\frac{r_{A}+r_{B}}{2}, \tag{15}
\end{equation*}
$$

we get

$$
\begin{equation*}
C_{n}=A\left(1-r_{A}\right)^{n}+n r B(1-r)^{(n-1)} . \tag{16}
\end{equation*}
$$

On dividing and multiplying the second member on the right of (16) by (l-r), we get

$$
\begin{equation*}
C_{n}=A\left(1-r_{A}\right)^{n}+\frac{n r}{(1-r)} B(1-r)^{n}, \tag{17}
\end{equation*}
$$

n being the number of tidal cycles.
Now, the numerical values of $r_{A}$ and $r_{B}$ in practice are expected to lie between $1 / 10$ and $1 / 50$. When substituted into a relation such as

$$
(1-r)^{\frac{1}{r}} *
$$

these values give $\quad=\left(1-\frac{1}{10}\right)^{10}=0.3487$,

* $\underset{r=0}{L(1-r)^{\frac{T}{r}}}=\frac{1}{e}=\frac{1}{2.718}=0.367$, where $e$ is the base of the natural logarithms.

$$
\text { and } \quad=\left(1-\frac{1}{50}\right)^{50}=0.3643
$$

For all practical purposes, therefore, we can write

$$
\begin{equation*}
C_{\frac{1}{r_{A}}}, \frac{1}{r}=A\left(1-r_{A}\right)^{\frac{1}{r_{A}}}+\frac{B}{(1-r)}(1-r)^{\frac{1}{r}} \tag{18}
\end{equation*}
$$

or its equivalent

$$
\begin{equation*}
C_{\frac{1}{r_{A}}, \frac{1}{r}}=0.35 A+\frac{0.35 B}{(1-r)} \tag{19}
\end{equation*}
$$

where 0.35 A is the portion of the original contaminated volume $A$ which remains in section $A$ at the end of

$$
t_{1}=\frac{1}{r_{A}}
$$

tidal cycles, and

$$
\frac{0.35 B}{(1-r)}
$$

is that portion of the total contaminant arriving from $B$ in

$$
t_{2}=\frac{I}{r}
$$

tidal cycles which would remain in $A$ at the end of this latter time.

If the curves

$$
\begin{equation*}
y_{1}=(0.35)^{m} A, \quad m=1,2, \text { etc. } \tag{20}
\end{equation*}
$$

and

$$
\begin{equation*}
y_{2}=(0.35)^{n} \frac{B}{(1-r)}, n=1,2, \text { etc. } \tag{21}
\end{equation*}
$$

are plotted for the same coordinates, but with unit lengths on the abscissa corresponding to the reciprocals of their r-values, the sum of (20) and (21) will be a curve expressing the remaining contaminant in section $A$ after any desired number of tidal cycles have occurred as illustrated in Figure 3.

For example, let
A equal 10 units
$B$ equal 14 units
$Q_{A}$ equal 0.4 unit
$\mathrm{Q}_{\mathrm{B}}$ equal 0.3 unit
The exchange ratios are $r_{A}$ equal $0.04, r_{B}$ equal 0.0214 , and $r$ equal
0.0307 , and unit lengths for the curves $y_{1}$ and $y_{2}$ are

$$
t_{1}=25
$$

and

$$
\mathrm{t}_{2}=32.5,
$$

respectively. In Figure 3 the curves $y_{1}, y_{2}$ and $y_{1}+y_{2}$ are shown for values of $m$ and $n$ equal to $1,2,3$, etc.

In practice, sometimes very little information is available upon which to base the preceding computations. We may, therefore, be required to adopt the following procedure in order to obtain a first order approximation of the flushing time.

The mean high tide volume is computed and divided into two parts, giving $A=B$. Also, we can put $r_{A}=r_{B}=r^{*}$. When substituted into (16) these give
from which

$$
\begin{align*}
C_{n} & =A(1-r)^{n}+n r A(1-r)^{(n-1)} \\
& =A(1-r)^{n}+\frac{n r A}{(1-r)}(1-r)^{n}  \tag{22}\\
& =A(1-r)^{n}\left[1+\frac{n r}{1-r}\right]
\end{align*}
$$

$$
\begin{aligned}
C_{\left(\frac{1}{r}=t_{1}\right)} & =A(1-r)^{\frac{1}{r}}\left[1+\frac{1}{1-r}\right]^{* *} \\
& =2 A(1-r)^{\frac{1}{r}} . \\
C_{\frac{1}{r}}^{r} & =(A+B)(0.35) .
\end{aligned}
$$

approximately, or
The only curve to be plotted in this situation is, therefore,

$$
\begin{equation*}
y=(A+B)(0.35)^{n}, n=1,2,3, \text { etc. } \tag{23}
\end{equation*}
$$

The value $r$ to be employed in order to determine the number of tidal cycles corresponding to $n_{1}=1, n_{2}=2$, etc. still remains to be found. (For an example see Figure 3.) Now the total volume of sea water

* Also, the value $r_{A}$ can be retained in the first member on the right of $(20)$, and $\quad r=\frac{1}{2} \quad\left(r+r_{B}\right)$ substituted in the second member on the right of (20). This procedure is probably to be preferred to the above.
**Here for simplicity ( $1-r$ ) is assumed to be close to unity. For example, when

$$
r=\frac{Q_{A}}{A}=\frac{1}{30},(1-r)=0.967
$$

which enters the estuary on the flooding tide is, except for the river water contributed in a half tidal cycle, equal to the tidal prism volume $P$. The average excursion $E$ of the sea water near the seaward boundary a of the estuary is, therefore, approximately equal to:

$$
\begin{equation*}
E=\frac{P}{a} \tag{24}
\end{equation*}
$$

where $a=$ hd + ed (see Fig. 2).
Now, for the James River estuary, a relation existed between the maximum current velocity and the net salt transport, which will be assumed to hold in general. This is to the effect that the net volume transport velocity is approximately equal to one-fifth the maximum current velocity. Multiplying (24) by $\bar{\Pi} / 2$ gives the maximum excursion

$$
\begin{equation*}
\mathrm{E}_{\max }=\frac{\pi}{2} \mathrm{E}=\frac{\pi \mathrm{P}^{*}}{2 \mathrm{a}} \tag{25}
\end{equation*}
$$

and multiplying (25) by one-fifth gives

$$
\begin{equation*}
\mathrm{E}_{\text {net }}=\frac{\pi \mathrm{P}}{10 \mathrm{a}} \tag{26}
\end{equation*}
$$

The net volume of sea water entering the estuary is

$$
\begin{equation*}
U_{\text {net }} \mathrm{a}=\frac{1}{5} \mathrm{U}_{\max } \mathrm{a}=\frac{1}{5} \frac{\pi}{2} \mathrm{Ua}=\frac{\pi \mathrm{P}}{10} \approx 2 \mathrm{Q}_{\mathrm{B}}, \quad \text { or } \tag{27}
\end{equation*}
$$

Therefore,

$$
\mathrm{Q}_{\mathrm{B}} \approx \frac{\pi \mathrm{P}^{* *}}{20}
$$

$$
\begin{equation*}
r=\frac{Q_{B}}{B}=\frac{Q_{B}}{1 / 2(A+B} \tag{28}
\end{equation*}
$$

and

$$
\begin{equation*}
r_{A}=R+Q_{B} \tag{29}
\end{equation*}
$$

$* U_{F}=U_{\text {max }} \sin \theta$, where $\theta$ varies between $o$ and $\pi$ during the flooding tide interval $t_{F}$. The area under the velocity profile is $\int_{0}^{\pi} U_{\max } \sin \theta d \theta=$ $-U_{\max }[-1-1]=2 U_{\text {max }}$. If $U$ is the average flood velocity, $U \pi=2 U_{\max }$ or $U_{\max }=\frac{h^{2}}{2} U$. Also $U_{\max } t_{F}=E_{\text {max }}$, etc.
** It is assumed that actual current velocities are not known.

The gross method just employed should not cause as great an error in the flushing time as might at first be expected. For, when $A$ is taken too large or too small, a compensating effect results from $B$ being too small or too large. A serious deviation from a reasonable estimate of the flushing time would occur when the net salt transport velocity differed radically from one-fifth the value of the maximum current velocity.

Example: Determine the flushing time of the James River estuary.
The navigation chart used for this example was U.S. Coast and Geodetic Survey Chart No. 78. A generalized chart showing the locations of the observing stations is shown in Figure 4. The area flushed extended from a line connecting Pike Point and Newport News to Shirley near the head of the estuary. Velocity-depth curves for current station J-17 were used in place of river flow data and are shown in Figure 5. The depth of the surface of no net motion at station J-17 is about 9.5 feet and roughly corresponds to the inflection points of the salinity-depth curves of Figure 6. The surface of no net motion rises gradually from the head towards the mouth of the estuary, and is estimated to be about 9 feet below the mean water level near the boundary shown in Figure 4. The surface of no net motion will be considered to be horizontal in this example.

In Figure 4, the net landward transport of sea water through the boundary $a b$ into volume $B$ is estimated by multiplying the average velocity found at station $J-17$ by the sectional area of $B$.

The dead water volumes involved in these considerations are neglected when computing volume $B$ (see Fig. l). The average net velocity for sections $A$ and $B$ at station $J-17$ can be obtained from Figure 5 by means of a planimeter (i.e., by finding the centroid of the area between the mean flood and ebb velocity curves). The velocities thus obtained were:

Average net ebb velocity in A. . . 0.30 knot
Average net flood velocity in B . . 0.27 knot
The effective areas corresponding to the volume sections $A$ and $B$ at the boundary ab were computed to be:

$$
\begin{aligned}
& \text { Cross section for A . . . . . . . 181,065 sq. ft. } \\
& \text { Cross section for B . . . . . . } 180,087 \text { sq. ft., }
\end{aligned}
$$

and the corresponding volumes, $A$ and, $B$, were:

> Volume of section A . . . . . . $2,389 \times 10^{7} \mathrm{cu} . \mathrm{ft}$.
> Volume of section B . . . . . $2,254 \times 10^{7} \mathrm{cu} . \mathrm{ft}$.

These and the above net flow velocities give:*

$$
\begin{aligned}
& Q_{A}=198 \times 10^{7} \mathrm{cu} . \mathrm{ft} . \\
& \mathrm{Q}_{\mathrm{B}}=177 \times 10^{7} \mathrm{cu} . \mathrm{ft} .
\end{aligned}
$$

The exchange ratios are: $r=\frac{Q_{A}}{A}$. $=0.08$

$$
r_{B}=\frac{Q_{B}}{B}=0.078,
$$

and

$$
\begin{aligned}
& y_{1}=2389 \times 10^{7}(0.35)^{m} \\
& y_{2}=2254 \times 10^{7}(0.35)^{n} .
\end{aligned}
$$

The plot of these curves and their sum ( $y_{1}+y_{2}$ ) are shown in Figure 7.

## PART II

$$
\text { Embayments ( } R=0 \text { ) }
$$

When the river water entering a bay is nil or insufficient to cause the salinity of the bay to differ from that of the adjacent sea water, then the circulation pattern of Figure 1 does not exist. In Figure 8A the salinity of a bay at low water is supposed to be equal to that of the adjacent sea volume, $P$.

Let the volume $P$ be equal to the tidal prism volume of the bay. Suppose that the volume of the bay is contaminated,at high water. Then Figure 8A represents the situation at the following low water. It is assumed that the contaminant which is ejected into the sea during any ebb tide is carried away from the bay entrance before the following flood tide begins.

Now let volume $P$ push bodily into the bay displacing the volumes $P_{1}=P_{2}=P_{3}$, etc., all of which are numerically equal to $P$. We now have a second high tide situation, Figure 8B. In Figure 8A the small

[^0]numbered rectangles within $P_{1}, P_{2}$, etc. represent uniform distribution of the contaminant in the bay at low tide.

At high water, Figure 8 B , some of the particles 1 , which were in volume $P_{1}$ at the previous low water, would still be expected to be found near the entrance boundary of the bay. Similarly at high water some of the particles 4 in Figure 8 A would be expected to be found at an excursion length farther into the bay as in Figure 8B. The maximum span of the contaminant on the flooding tide is thus twice that of the particle excursion distance. Thus, the contaminant in $P_{1}$ becomes distributed in the volume $P_{1}+P=2 P$. The volume of water containing contaminant which is lost during the next ebb tide is equal to $P$, and therefore the exchange ratio for volume $P_{1}$ is $\mathrm{r}_{1}=1 / 2$.

Figure 8 C shows what the situation would be at low tide if an adjustment of the remaining contaminant did not now occur because of diffusion. Since the volumes being considered are small, we assume that the process of diffusion is complete before the next flood tide begins. The effect of diffusion is to disperse the remaining contaminant uniformly (Fig. 8D). Now the total contaminant remaining in $P_{1}$ at the end of the first ebb tide is $P_{1}\left(l-r_{1}\right)$. If $P$ is taken equal to unity, then, since $r_{1}=1 / 2$, the contaminant remaining in $P_{1}$ is $r_{l}$. On this basis the contaminant in $P_{2}$, Figure $8 C$, is $2 r_{1}$, giving for the adjusted value in each of the volumes $P_{1}$ and $P_{2}$ the amount $3 \frac{r_{1}}{2}=3 / 4$. The contaminant lost from $P_{2}$ is thus $1 / 4=r_{1}{ }^{2}$. This is the exchange ratio for volume $P_{2}$. Similarly, the contaminant in $P_{3}$ and $P_{2}$ is now $2 r_{1}$ $+3 \frac{r_{1}}{2}$, and for each of the volumes $P_{2}$ and $P_{3}$ this becomes $1 / \frac{1}{2}$ $\left(2 r_{1}+3 \frac{r_{1}}{2}\right)=7 / 8$. Hence, the exchange ratio for $P_{3}$ is $r_{1}{ }^{3}$. In general we can write for the exchange ratio of the nth segment $P_{n}$,

$$
r_{n}
$$

Now suppose that the bay volume at high water is equal to

$$
n P=P_{1}+P_{2}+\ldots P_{n}
$$

The remaining contaminant near the head of the bay after

$$
t=\left(\frac{1}{r_{1}}\right)^{n}
$$

tidal cycles is approximately

$$
{ }_{\left(1-r_{1}^{n}\right)^{\frac{1}{r_{1}^{n}}}}=\frac{1}{e} P_{n}^{*}
$$

[^1]Obviously, if $n$ is large, $t$ will be too great to warrant consideration of the flushing problem, when the region near the head of the bay is considered alone. We will, therefore, consider the average situation in the bay as a whole.

Let $S$ be the sum of all the exchange ratios. Then

$$
n r_{a v}=S=r+r^{2}+\ldots \ldots+r^{n} .
$$

Multiplying by r gives,

$$
r S=r^{2}+r^{3}+\ldots \ldots+r^{(n+1)}
$$

from which $S(1-r)=r-r(n+1)$ or $S=r /(1-r)$, since $r^{(n+1)}$ is very small. Since $n$ segments were involved in the sum $S$ the average exchange ratio for the bay is

$$
r_{a v}=\frac{1}{n}=\frac{P}{V}
$$

where $P$ is the tidal prism volume and $V$, the high tide volume.


FIGURE 3 GENERAL FLUSHING CURVES


Figure 4 LOcAtion of observing stations in the james river ESTUARY


FIGURE 5 MEAN VELOCITY-DEPTH CURVES FOR CURRENT STATION J-I7


FIGURE 6 MEAN SALINITY-DEPTH CURVE AT 3 STATIONS IN THE JAMES RIVER ESTUARY


FIGURE 7 FLUSHING TIME CURVES FOR THE JAMES RIVER ESTUARY

D. LOW WATER AFTER ADJUSTMENT OF CONTAMINANT

FIGURE 8 DIAGRAM SHOWING REDISTRIBUTION OF CONTAMINANT WITH TIDAL ACTION

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[^0]:    *Flood and ebb tide intervals were taken as 6 hours each.

[^1]:    * Where $e=i s$ base of natural logarithm.

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