Se. Co = Profit / Capital ?

Tax on [illegible] ?

FOREST VALUATION

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BY

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I. INTRODUCTION

A. LITERATURE.

Endres, Dr. Max. Lehrbuch der Waldwertrechnung und Forststatik, 2nd ed. 1911, published by Julius Springer, Berlin; 308 pp.

Dr. Endres is Professor of Forestry at the University of München, and author of the great "Forst Politik," one of the most instructive books in forestry literature. On pages 1-4 is given a full list of the literature of the subject. In addition the literature and historic facts of interest are added to all important points in the treatise. An excellent set of tables of compound interest, etc., forms the appendix. As Dr. Endres says (p. 2), "Scientific forest valuation is based on the labors of Faustman, Pressler and Gustav Heyer," and so it is but natural that this excellent and most complete treatise should follow the works of these three great men and represent what is accepted today as correct and scientific in forest valuation.

Stoetzer, Dr. H. Waldwertrechnung und Forstliche Statik, Frankfurt, 1894; 4th ed. 1908.

Dr. Stoetzer's work as author, teacher and administrator, and also as editor of the great Handbuch, is well known. In Valuation and Statics he has stood and stands today one of the foremost authorities.

Martin, Dr. H. Die Forstliche Statik, Berlin, 1905, Julius Springer, 361 pp.

Dr. Martin is Professor of Forestry at Tharandt, formerly at Eberswalde, author of "Forsteinrichtung," a prolific writer, well known teacher and recognized authority. This book is entirely devoted to Statics of Forestry; its position is defined in the sentence (p. 22), "As to the system and contents of this book, the author closely follows Hundeshagen." In his introduction, one of the most interesting and instructive chapters ever published on Forest Valuation and Statics, he discusses the historic development and points out the fact that the useful study of Statics was opposed in practice chiefly because the writers clung too much to mathematics and formulæ. The chapter, "Choice between agriculture and forestry in the use of land," to which thirty-five per cent of the book is devoted is one of the best discussions of this subject and particularly interesting at the present time to students of forestry and economics in the United States.

Schlich, Dr. W. Manual of Forestry, vol. III, 1895, 4th ed. 1911.

A brief chapter, fifty-nine pages in the first edition, following closely the treatment by German authors.

Schenck, Dr. C. A. Forest Finance, published by Dr. Schenck, Biltmore, N. C., 1909.

Chapman, H. H. Forest Valuation, Wiley & Sons, New York, 1915; 310 pp.

Professor Chapman treats the subject from the American standpoint, and adds a number of topics not ordinarily treated in this connection, particularly, Appraisal of damages; Forest taxation; Stumpage values; Future values of forest products, and Risks. Convenient tables of the various formulæ, compound interest values and logarithms form the appendix.

Fernow, Dr. B. E. Forest Economics and Forest History.

Dr. Fernow, now Professor of Forestry at Toronto University, himself a student of Gustav Heyer, points out the historic development and the application of Forest Valuation and Statics.

B. SCOPE AND APPLICATION OF FOREST VALUATION AND STATICS.

1. General.

In Forestry, just as in farming and other lines of business, nearly every transaction involves expense and income and demands judgment of values. If a piece of land, the timber on the land, or a piece of forest (land and timber together) is bought, sold, rented, leased, or in any way contracted for, the first question concerns the value of the property under consideration. If a forester plants an acre of land to forest trees the owner wants to know what such work should cost and if under the particular conditions it may be expected to pay. Since it may take the crop fifty or more years to grow to useful size, the question as to whether it will pay or not involves the future and the owner must of necessity have faith in certain premises. He must believe that the trees will grow and thrive on the particular site, that a certain amount of expense will suffice to care for them, that they will reach a useful size in a certain time, and finally, that they will have a definite value when cut.

If he has no faith in any of these premises it is an inexcusable "gamble" for him to plant the trees. But to have faith in business premises is not peculiar to forestry; a man buying a farm believes that the land will produce certain crops, produce them in ten or twenty years just as now, and that the crops have a certain value, justifying his labor and expense. Some years a dry season, frost, hail or insects destroy most of his crops, but in the long run he maintains a successful average.

2. Valuation and Statics.

When the buyer estimates and examines a stand of timber which he intends to cut over at once, he merely determines the value for immediate use and the work is one of **simple forest valuation.**

When a lumberman buys a body of timber with a view of supplying his business for the next twenty years, the case is less simple. He has to reckon with the future and with many uncertainties; the timber may be old and become defective before he is ready to cut; fire and insects may destroy; certain railway developments which he expects may not take place; the prices of timber may not advance; taxes, interest, or other expense may eat up all the expected profits. It is a case of nice judgment in which the different factors and conditions are estimated, if possible, in money value and balanced to arrive at a final result. It is a case of weighing various conditions; weighing effort and expense against expected results. It is no longer simple valuation, but **complex valuation** with a heavy proportion of **statics,** of the science and art of weighing cause and effect in our case, costs and results.

When the owner of a piece of land decides to change from farm crop to forest, he does so because he has weighed the advantages and disadvantages of both lines and found that the forest will pay better. This weighing is Forest Statics. The "pay better" need not be in money at all.

When a forester decides to give up natural reproduction and plant five year transplants of spruce, he does, or should do this only after careful weighing or calculation. His calculation should show clearly that he may expect a material gain by going to this greater expense of planting. In making his calculation he starts with the cost of planting. This he knows from experience. He figures on a more even stand, more rapid growth, earlier thinnings, more income from thinning and on an earlier final cut, saving him several years waiting, use of the land and interest on the value of the stand. Aside from the planting, all seems guess work. But this is not the case; he has the experience of centuries, of thousands of men, of many thousand acres of timber to guide him. He has faith in this experience and decides to plant. His process of weighing possibilities is Forest Statics. It computes extra costs and forecasts extra results.

It is clear that Forest Valuation and Forest Statics are not sharply separated, that in most cases of valuation more or less statics is involved and for this reason the two are generally treated together in forestry, just as in other forms of valuation.

In the above cases the lumberman or the forester might prefer not to go through any tedious process of reasoning, analysis or calculation, and simply "jump at" the conclusion that he could pay ten dollars per acre for the timber, or in the case of the forester that he could better afford to plant than to go on with natural reproduction. No doubt much of this is done. Some gifted or lucky men succeed, most men lose. The gifted man has his own short method and his results are correct in proportion as he is clever. The average man can not and does not jump at conclusions or guess results if he can avoid it. And it is to help the average man analyze a given case, weigh the various factors and combine them in a scientific and effective manner that modern forest valuation was developed. The importance of this development in forestry is emphasized by the fact that the forester commonly deals with large properties, not his own, where it becomes necessary to place the case before the owner or his agent in a perfectly clear and convincing form. At such a time every item must be clearly stated in a form capable of detailed discussion, the methods of analysis and calculation must be explained and stand the test of examination and trial.

3. Application of Valuation and Statics.

As in every other business, so in forestry, the work of valuation or the determination of cost or value of property (land, timber, etc.,) or of operations (planting, thinning, protection, logging, etc.,) follows every line of work and is always in evidence. For example its application in a few of the most important branches of forest business:

1. In **purchase, sale and exchange of property,** such as land timber, or forest (land and timber together) the necessity of ascer-

taining the value of the property bought or sold is evident. This kind of work in the United States at the present time is very important and will be more important in the future than it has been in the past. The mere cruise for merchantable timber will no longer satisfy men buying and selling forest properties. To sell a forest for the value of the merchantable stuff alone is equivalent to giving away over half the value in many localities and this condition is rapidly extending.

2. **Holding of timber** for future use is a common practice among lumbermen. In buying for ten or twenty years it is necessary to compute expenses, taxes, interest, protection, probable losses, and it requires an estimate or forecast of the price which may be obtained at the end of the period or at the time of cutting. Taxes and interest together with other current expenses come every year and accumulate with compound interest; the final value must be discounted to the present time. There is a right way of doing this and many wrong ways and some of our lumbermen in the Lake Region and elsewhere are finding today that it is not always easy to make a state tax commission see the right way of computing the true value of a lot of stumpage which can not possibly be cut before ten or twenty years.

3. **Timber contracts, leases,** rights of way, etc., are common and necessary. A long time contract for hemlock in a hemlock and hardwood forest may become a very difficult and troublesome affair. To draw up such a contract requires a forester's judgment; to settle it equitably involves forest valuation and statics.

4. **Damage in timber,** usually trespass or fire, is very common. In the past it was impossible for the courts to administer justice; these cases were among the most unsatisfactory; the evidence was neither clear nor convincing, the methods of calculation could not be explained, they were never agreed upon; it was a battle of unproved assertion backed by a pretence of experience and authority. As a result the courts granted nothing beyond a simple provable loss of material actually saleable at the time of destruction. Since modern forest valuation has found its way into the woods business this situation has been relieved very materially and the courts are glad to be able to grant real justice based upon an orderly, intelligent method of determining the values. In fact the application of modern forest valuation in the United States had its first tests in damage cases.

5. **Taxation** requires assessment of the true value of the property. As long as it was a fight with local assessors the old

methods alone were effective. Since the state tax commission takes a hand in the assessment it becomes necessary to apply forest valuation to show that if hemlock is worth five dollars per M. feet on the stump for immediate use that the stuff to be cut in ten years from now is not worth five dollars unless it doubles in value during the ten years which it has not done in the last ten years and will not in the next. Similarly in our efforts at tax reform. Several states, notably Pennsylvania, New York, Massachusetts and Connecticut have initiated change of laws regarding taxation of forest property. In all cases a "yield tax" was introduced and the taxation of the land was modified. But what is a reasonable tax on stumpage? Is it ten per cent as in Pennsylvania, or six per cent as in Massachusetts and what basis has either state for its estimate? Guessing and juggling a few figures will not give any permanent, satisfactory relief; it will require modern forest valuation to supply a basis.

6. The **right use of land** is one of the most important economic questions in two-thirds of the United States. Is it wise to try to farm all lands in Michigan, New York and Pennsylvania; should part be devoted to forest and if so what part, in what amount; what will be the economic result if these lands are used to raise timber; how may we determine which of the two, forest or field should have preference in a given case? Booster oratory or decision by the man in the office may deceive some people for a time but it will lead to great losses in the end. Pennsylvania, the Lake Region and other districts today have millions of acres of idle, unused lands entailing a loss of millions of dollars. The chief obstacle to any improvement in this situation is the land boomer and his friends who continue to convince the legislatures that all land is farm land. The loss is great and has accumulated for years. Nothing but the application of sound methods of valuation and the recognition of this valuation by the legislatures will prepare for the remedy and change great money loss to income for the people.

7. The **Management of the forest** is and must always be the most important subject of forest valuation and statics. As stated before, every time an acre of land is planted to trees the question arises—does it pay—should a different kind of tree, cheaper plant stock, cheaper methods of planting be used. Later on—does it pay to thin, should the thinning be light or heavy, and how often can we afford to thin in this stand. All along there is expense for protection—should it be cheap, less than five cents per acre, will it pay to go as high as ten cents, is it better to use many men or depend on outfit, etc. In the end comes the question—is it more profitable to

cut the stuff at the age of fifty, eighty or a hundred years. These and many other questions must be studied and answered for each stand, for each neighborhood, not merely once but repeatedly because the answer varies with the premises, with success of silviculture, market, transportation, cost of logging, labor in general, and many other conditions. It is evident that in management the work of valuation is largely a matter of weighing or statics, though a great deal of simple valuation also enters. Every year dozens of stands must be estimated and millions of feet of timber appraised as to their proper stumpage value.

C. HISTORY.

Forest valuation is old. Timber was an object of traffic with the ancients, and traffic is unthinkable without valuation or the setting of a price on the object bought or sold. When timber was shipped or rafted, forests sold or exchanged, or mortgaged as is recorded as early as the days of Charlemagne, an important part of the transaction was the determination of the value of the forest or timber.

Even the more modern conception of forest valuation the comparison of the growing stock to a capital increasing at a certain per cent, the influence of the rotation or long interval between planting and harvest, and finally the application of an orderly mathematical analysis to discover the proper rotation and to compare different kinds of forest, beech, spruce, etc., date back one hundred and fifty years and more.

According to Endres (p. 183) the Forest Order of the principality of Neuburg, Bavaria, of the year 1577 makes it clear that the yield from the forest is in the nature of interest on a capital and that the person in possession has no right to cut more than the growth or proper yield, being entitled only to the interest.

As early as 1623 the English economist Culpepper explained that the forest could not produce a large interest rate. In 1721 Reaumur pointed out that too long a rotation meant real money loss, and in 1742 Buffon in his "Memoirs sur la culture des forêts" discussed the same subject and pointed out the accumulation of expenses in growing a forest crop.

In 1764 the great von Zanthier, first to establish a "Meisterschule" for foresters, used compound interest calculations in forest valuation; worked out a comparison for different rotations in beech, a comparison of beech, spruce and oak in which all expenses are prolonged to the end of the rotation. He also prepared the first money yield tables. Endres says of these calculations that they were satisfactory (zweckentsprechend) so that we have here a perfectly up-to-date piece of valuation and statics one hundred and fifty years old.

In 1765 a writer in Stahl's Forstmagazin discusses a proper interest rate to use in forest valuation and brings up the same arguments used to this day to show that the ordinary commercial rate, then four per cent, was too high.

With the rapid development of forest regulation and literature about the end of the eighteenth century came considerable discussion of forest valuation, for it now became necessary to promulgate regular instructions to guide the practicing forest officials. G. L. Hartig, Cotta, König and Pfeil all busied themselves with this subject. König in 1813 first expressed clearly the value of using the net rental on the land which might be secured in forestry as proper measure not only of the value of the land but of the success of the operation in silviculture, protection and utilization.

In 1823 Hundeshagen first uses the term forest statics and defines it as the "art of measuring the productive forces and results in forestry."

About the same time König in his "Forstmathematik" discusses the profitableness of forestry and introduces the idea of the profit of undertaking (unternehmergewinn) and that of the per cent at which a given forest works. In 1849 Faustman, more or less independently, worked out the expectation or income value of the soil, first expressed by König, and gave to forest valuation this analysis in a formula—Se—which has never been changed or even modified materially since, and which is recognized as the most important analysis in forest statics. He showed that this analysis is applicable alike to forests cut over at intervals (intermittent) and to regulated properties with yearly cut.

In 1858 Pressler began publishing his works "Rationelle Waldwirt" and "Waldau des höchsten Ertrags" which really form the beginning of a new epoch in forest valuation.

In 1865 Gustav Heyer published the first complete treatise on valuation and followed this up in 1871 by the first treatise on forest statics. Lehr worked over the subject for the Handbuch in 1887, and Kraft, in 1882, distinguished himself in his efforts to determine correct methods of measuring the value-growth in the forest, in his "Zuwachsrechnungen."

The practice, as usual, was reluctant to accept everything. Yield tables and every kindred means of helping the forester to know the volume and value of the stand and the growth in time found no difficulty in adoption by the practice. But when it came to formulae with compound interest and especially when these formulae seemed to be wrong as soon as five per cent, the current interest rate, was used and when they seemed to lead to wholesale reduction of rotations, the practicing forester ceased to have faith and refused to accept.

Common sense and experience showed the forester that it required at least eighty years (in a given case) to grow marketable sizes of spruce, that smaller stuff was drug on the market. When Pressler's and Heyer's calculations and formulae demanded a "financial" rotation of fifty years it was clear to the forester that something was radically wrong and even good men like Borggreve, Urich, Bose and Baur denounced the whole method.

Some discussion is still going on, but the truth is gradually becoming evident to all, that all these calculations have nothing to do with the raising of timber, that this is left to silviculture and regulation as guided by growth on one hand and market on the other, and that these calculations merely try to supply a correct measure, a reasonable, orderly, acceptable analysis to show exactly what forestry is doing and what it can do in any given case. If then the growth of timber is so slow that the crop increases only at the rate of two per cent compound, there is no amount of calculation and no juggling of methods and figures which will change the plain fact and only one question remains—will we keep on raising timber or quit? If the timber is needed and the land will not grow any other crops or do better it is quite certain that forestry will continue regardless of the rate per cent which it makes on the capital.

At the same time it is becoming evident to all that valuation is necessary. As pointed out before, the owner wants to know what is actually made, what the land is worth if used to raise timber, whether expensive planting of transplants is actually better than cheap natural reproduction, etc. And he is not willing to take the forester's mere word for it, he wants to see by what method of calculation and upon what premises the recommendation is based.

In the efforts of bringing correct forest valuation and statics into the forest business the labors of Judeich, master of forest regulation, stand out conspicuously. In his long and successful career at the head of the Saxon forests, he not only recommended but introduced everything of value into the practice and he may well be regarded as the most powerful exponent of modern forest statics and especially of the general usefulness of Se.

II. ARITHMETIC OF FOREST VALUATION

a. General.

A stand of timber fifty years old after planting may be worth two hundred dollars per acre. If this stand is to remain and continue to grow until it is eighty years old, it is right for the owner to ask—at what rate is the stand, now fifty years old, growing, what per cent is this growth on the two hundred dollars which the stand is now worth, etc.

Since the stand does not pay or give up any interest, the two hundred dollars or the value of the fifty year old stand is evidently like a capital out at interest, where the interest remains unpaid, but is added to the capital every year. In other words, the stand of timber grows like capital out at compound interest.

If the owner wishes to know what this stand has cost to produce or grow to the age of fifty years he adds up the various items, cost of planting, care and taxes during the fifty years, and the rent on the land. The ten dollars per acre spent in planting have been out for fifty years without drawing any interest, they have grown, therefore, at compound interest for fifty years. The expense of taxes and care comes every year, the sum paid out the first year has been out for forty-nine years, the sum paid the second year has been out forty-eight years, etc., for none of this interest has been paid.

If the owner wishes to forecast or estimate what this stand is worth to him today if it is to continue to grow until it is eighty years old, he must first decide that when eighty years old the stand will bring, say, five hundred dollars per acre and then discount the five hundred dollars to the present day, i. e., for thirty years.

From this it is evident that forest valuation works a good deal with the arithmetic of interest, with discount and with the summation of series, usually geometric series, the geometrical progressions of many writers.

The following are the more common forms of interest here.[1]

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[1] The elementary treatment of these simple problems in literal arithmetic has been fully justified by fifteen years experience in University teaching. The student will do well not only to go over this ground very thoroughly, work out all formulae on paper and repeat often, but also to speak aloud the true meaning of each formula and learn to state, as in geometry, exactly what each formula means.

b. Discount, Prolongation and Rate.

If $75 is put out at 4% compound interest for eight years, the capital and interest to be paid at the end of the period, the $75 capital grows as follows:

at the start, $C_0 = \$75$; at the end of
first year, $C_1 = 75 +$ interest for one year $= 75 + 75\ (.04)$
$= 75\ (1 + .04) = 75\ (1.04)$;
second year, $C_2 = 75\ (1.04) +$ interest $= 75\ (1.04)\ (1.04)$
$= 75\ (1.04^2)$;
third year, $C_3 = 75\ (1.04^2)\ (1.04) = 75\ (1.04^3)$;
eighth year, $C_8 = 75\ (1.04^7)\ (1.04) = 75\ (1.04^8)$;
or, in general, $C_n = C_0\ (1.0p^n)$;
that is: the final capital equals the initial capital multiplied by $1.0p^n$.

From the above: $\frac{C_n}{1.0p^n} = C_0$ or, the initial capital equals the final capital divided by $1.0p^n$, or it is equal to the final capital **discounted** for **n** years at **p** per cent.

Also: $\frac{C_n}{C_0} = 1.0p^n$, or $1.0p^n$ equals the final capital divided by the initial capital.

Since the numerical value of $1.0p^n$ may be found in the table, the value of **p**, or the interest rate is readily determined.

$C_0\ (1.0p^n)$ signifies that the initial capital C_0 is **prolonged** at **p** per cent for **n** years. $\frac{Cn}{1.0p^n}$ signifies that the final capital **Cn** is discounted at **p** per cent, **n** years.[2]

c. Summation of Geometrical Series.

If the yearly expenses on a property are $500, and these continue for fifty years and money is worth 3%, what will these expenses amount to? Evidently the first $500 is out at compound interest for 49 years, the second for 48 years, etc., and it is desirable to find a short way of computing these various amounts.

[2] The word capital is used here in a rather loose way, perhaps, but not more so than in ordinary conversation, etc., and it is a very helpful term to employ in these explanations.

1. Fundamentally this series is:

$$\text{sum, } S = a + ar + ar^2 + ar^3 + ar^4$$

and

$$Sr = ar + ar^2 + ar^3 + ar^4 + ar^5$$

subtracting the upper from the lower,

$$Sr - S = ar^5 - a$$

or

$$S(r-1) = a(r^5-1)$$

when

$$S = \frac{a(r^5-1)}{(r-1)}$$

since 5 is the number of terms in the series, or **n**, and this form is perfectly general, it may be written:

$$S = \frac{a(r^n-1)}{(r-1)}$$

In this series **a** is the regular payment, and **r** is the ratio between any two consecutive terms, as: $\frac{ar^2}{ar} = r$.

2. Applying this to the above case of a yearly or current expense of $500 at 3%.

$$\text{sum, } S = 500(1.03^{49}) + 500(1.03^{48}) + \ldots\ldots\ldots\ldots + 500$$

$$S(1.03) = 500(1.03^{50}) + 500(1.03^{49}) + \text{etc.}\ldots\ldots\ldots + 500(1.03)$$

here 1.03 is the ratio, i. e.,

$$\frac{500(1.03^{49})}{500(1.03^{48})} = 1.03$$

subtracting:

$$S(1.03-1) = 500(1.03^{50}-1)$$

$$S = \frac{500(1.03^{50}-1)}{(1.03-1)}$$

By looking up 1.03^{50} in the tables (see Appendix), the computation becomes perfectly simple and requires little time.

3. Since this same process applies to any similar case it may be written as a general formula:

$$S = \frac{a(1.0p^n-1)}{(1.0p-1)}$$

which may be expressed:

The sum of a series of payments **a** coming every year, continued for **n** years and compounded at **p** per cent.

Since this sum represents the value of all these payments at the end of the period, it may be termed the end value of the series.[3]

Query: Is the sum $\frac{500\ (1.03^{50}-1)}{(1.03-1)}$ the same whether it begins in 1902 or in 1925, whether it begins in ten years from now, or has run for fifteen years?

d. Present Value of the Series.

A farmer or forester may be able to get $500 rent per year from a certain property. He may wish to know the value of twenty years rent of this property with a view to raising money or selling the twenty years' rent.

1. The end value or sum of the twenty years' rent is found as above and is expressed:

$$\text{sum or end value} = \frac{500\ (1.03^{20}-1)}{1.03-1}$$

But what is the value of this sum today?

Keeping in mind that

$$C_0 = \frac{C_n}{1.0p^n}$$

and that **Cn** is the above sum or end value of the series:

$$\text{present value, } C_0 = \frac{C_{20}}{1.03^{20}} = \frac{500\ (1.03^{20}-1)}{(1.03-1)\ 1.03^{20}}$$

Expressed in words: The present value of a series of payments of $500 coming every year, continued twenty years, compounded at 3% and discounted at the same per cent, the first payment coming a year from now, in general,

$$\text{present value of series} = \frac{a\ (1.0p^n-1)}{(1.0p-1)\ (1.0p^n)}$$

2. Assuming that the above series runs for twenty years and suppose that it has already run for seven years, what is its present value? Evidently the owner is seven years to the good, he does not

[3] The student will do well to write these formulae exactly as he has learned to develop them. The temptation is to write, for instance, $S = \frac{a\ (1.0p^n-1)}{.0p}$. This is correct, but it loses the connection, the student forgets how he came by it. In solving ordinary problems write out the general formula and keep this before you. This will avoid many mistakes.

need to discount it for twenty years but only for twenty less seven or thirteen, and the case may be written:

$$\text{present value} = \frac{500\,(1.03^{20} - 1)}{(1.03 - 1)\; 1.03^{20-7}}$$

$$\text{or} = \frac{500\,(1.03^{20} - 1)\; 1.03^{7}}{(1.03 - 1)\; 1.03^{20}}$$

this latter having the advantage that it keeps the whole story clearly before the student.

If in the above case the series does not begin until fifteen years from today, evidently the discount to present time must be longer by these fifteen years, and the case may be written:

$$\text{present value} = \frac{500\,(1.03^{20} - 1)}{(1.03 - 1)\; 1.03^{20+15}}$$

or to make it more clear,

$$= \frac{500\,(1.03^{20} - 1)}{(1.03 - 1)\;(1.03^{20})\;(1.03^{15})}$$

e. Series of Periodic Payments.

If a forest property can be cut over every fifteen years and at each return the cut nets \$500 it may become of interest to determine what the value of ten cuts of the forest is. Assuming that the property has just been cut over so that the next cut comes in fifteen years and the last cut in one hundred and fifty years, the end value or the value of the sum of these cuts is:

1. End value or sum:

$$S = 500\,(1.03^{150-15}) + 500\,(1.03^{150-30}) \ldots\ldots 500$$

$$S\,(1.03^{15}) = 500\,(1.03^{150}) + 500\,(1.03^{150-15}) \ldots\ldots\ldots 500\,(1.03^{15})$$

(Note that the ratio here is 1.03^{15}.)
Subtracting:

$$S\,(1.03^{15} - 1) = 500\,(1.03^{150} - 1)$$

$$S = \frac{500\,(1.03^{150} - 1)}{(1.03^{15} - 1)}$$

If in the above case we let fifteen years $=$ **t**, and 10, the number of cuts $=$ **n**, then $150 =$ **nt**, and we may write the general form:

$$S = \frac{a\,(1.0p^{nt} - 1)}{(1.0p^{t} - 1)}$$

Note: The student will recognize these periodic cases by the exponent **t** in the denominator.

Expressed in words: The end value or sum of a series of payments of \$500 each coming every fifteen years continued for ten payments compounded at 3% and the first payment to be made in fifteen years.

Query: Will the above sum or end value be greater or smaller if the first cut comes in five years instead of fifteen years?

2. Present value of the sum of a series of periodic payments. The case is the same as under d—1 and 2.

Present value is:

$$\frac{a\ (1.0p^{nt} - 1)}{(1.0p^{t} - 1)\ 1.0p^{nt}}$$

and is modified according to the time when the series begins, or, in the above case, when the first cut is made by writing it:

$$\frac{a\ (1.0p^{nt} - 1)\ 1.0p^{t-x}}{(1.0p^{t} - 1)\ 1.0p^{nt}}$$

where x is the number of years before the first cut or payment.

f. Sum of Infinite Series.

When a farmer buys a farm he really buys the yearly income or rent of the farm for all time, at least there is no set limit. In practice about fifty years' rent is worth as much as the farm, but this fact does not alter the nature of the bargain.

1. Using the formula as developed under c—2 and 3, and taking its present value as under d—1, present value:

$$\frac{a\ (1.0p^{n} - 1)}{(1.0p - 1)\ 1.0p^{n}}$$

and letting **n**=infinity or ∞ the formula becomes:

$$\text{present value} = \frac{a\ (1.0p^{\infty} - 1)}{(1.0p - 1)\ 1.0p^{\infty}}$$

and since $1.0p^{\infty} - 1 = 1.0p^{\infty}$: present value

$$\frac{a(1.0p^{\infty})}{(1.0p - 1)\quad 1.0p^{\infty}} = \frac{a}{1.0p - 1} = \frac{a}{.0p}$$

If **a** is the rental per year or \$500, and p = 5, then the present value of all the rentals from this farm forever are worth $\frac{500}{.05}$ or \$10,000, which simply means that the farm is worth \$10,000 if it brings a net income of \$500 and the buyer is willing to take 5% on his money. The $10,000 is the **income value of the farm** at 5%, and is equal to the net income **capitalized** at 5%.

2. If the income is periodic, comes every fifteen years, as in the case of the forest cut over every fifteen years, the foregoing is modified as follows:

$$\text{present value} = \frac{500\ (1.03^{\infty\ 15} - 1)}{(1.03^{15} - 1)\ \ (1.03^{\infty\ 15})}$$

$$= \frac{500\ (1.03^{\infty\ 15})}{1.03^{15} - 1\ \ (1.03^{\infty\ 15})}$$

$$= \frac{500}{(1.03 - 1)}$$

or in general:

$$\frac{A}{(1.0p^{t} - 1)}$$

where A is the periodic rent or money income at each cut in the above case. If in the above case the first cut comes in six years instead of fifteen the owner is fifteen less six or nine years to the good and this may be expressed as follows: value of property:

$$= \frac{500\ \ 1.03^{9}}{(1.03^{15} - 1)}$$

That is, the sum is prolonged at 3% for nine years, or in general.

$$= \frac{A\ \ 1.0p^{t-x}}{(1.0p^{t} - 1)}$$

where x is the number of years before the first payment is due.

Note: Care must be taken to see that the sum A is really the sum in every case. To illustrate: A forest of 100 acres can be cut over every 20 years for $50 worth of timber (stumpage value) per acre. The yearly expenses for the 20 years sum up to $15 per acre so that the net income A is 50-15 or $35. If now the first cut may be made at once the value of this property is:

$$\text{income value} = \$50 + \frac{35}{1.03^{20} - 1},\ \text{and not}\ \frac{35\ \ 1.03^{20-0}}{1.03^{20} - 1}$$

because the first income is larger by 15 than the established future incomes.

g. Change of Periodic to Yearly Payment.

Where a forest property furnishes an income only every twenty years it may be desirable to convert this periodic income into a yearly one. If this property, on account of the periodic income is worth, or has a safe income value of:

$$\frac{\$48000}{1.03^{20}-1}=\frac{48000}{1.80-1}=\$60{,}000$$

and the bank or trust company is willing to accept this as fact and pay the owner a yearly interest on the full $60,000 at 3%; the owner of the forest would receive $0.03 \times 60{,}000 = \$1{,}800$ per year.

This may also be stated as follows:

$$\frac{\$48{,}000}{1.03^{20}-1}=\frac{1800}{.03}=60{,}000,$$

or the yearly income,

$$1800=\frac{\$48{,}000\ (0.03)}{1.03^{20}-1}$$

In general then:

$$\frac{A}{(1.op^{t}-1)}=\frac{a}{(.op)},\ \text{or}\ a=\frac{A\ (.op)}{1.op^{t}-1}$$

Table of Formulae.

No.	Formula.	Meaning of Formula.
1.	$C_n = C_0(1.op^n)$	Prolongation of initial capital C_0 for **n** years at **p** per cent.
2.	$C_0 = \frac{C_n}{1.op^n}$	Discount of final capital C_n for **n** years at **p** per cent.
3.	$\frac{C_n}{C_0} = 1.op^n$	Final capital C_n divided by initial capital C_0 to find interest rate **p**.
4.	$C_n = S = \frac{a(1.op^n - 1)}{(1.op - 1)}$	Sum, or end value of a series of payments **a** coming every year continued for **n** years compounded at **p** per cent.
5.	$C_0 = \frac{a(1.op^n - 1)}{(1.op - 1)\ 1.op^n}$	Present value of a series of payments **a** coming every year continued for **n** years compounded at **p** per cent and discounted at the same per cent, first payment to come in one year from now.

6. $$C_n = S = \frac{a(1.0p^{nt}-1)}{(1.0p^t-1)}$$

End value or sum of a series of payments **a** coming every **t** years continued for **n** payments compounded at **p** per cent.

7. $$C_0 = \frac{a(1.0p^{nt}-1)}{(1.0p^t-1)\ 1.0p^{nt}}$$

Present value of a series of payments **a** coming every **t** years continued for **n** payments compounded at **p** per cent, discounted at same per cent, the first payment to come in **t** years.

8. $$C_0 = \frac{a(1.0p^{nt}-1)\ 1.0p^{t-x}}{(1.0p^t-1)\ 10p^{nt}}$$

The same as No. 7 but with the first payment coming in **x** years so that the sum must be prolonged for t—**x** years. Where **x** is greater than **t** this becomes a discount.

9. $$C_0 = \frac{a}{0.0p}$$

Income value of a property producing a net income **a** every year computed by capitalizing the income **a** at **p** per cent; or the present value of a series of payments **a** coming every year continued forever compounded and discounted at **p** per cent, the first payment to come in one year from now.

10. $$C_0 = \frac{A}{(1.0p^t-1)}$$

Present value of a series of payments **A** coming every **t** years continuing forever compounded and discounted at **p** per cent, the first payment to come in **t** years from now. This is also the income value of a property furnishing a net income **A** every **t** years computed at **p** per cent.

11. $$C_0 = \frac{A\ \ 1.0p^{t-x}}{(1.0p^t-1)}$$

Same as No. 10 but with the first payment coming in **x** years from now.

12. $$a = \frac{A\ (0.0p)}{1.0p^t-1}$$

Changing a periodic rental **A** coming every **t** years into a yearly rental **a** where the value of the property is computed by capitalizing the income at **p** per cent.

III. APPLICATION OF VALUATION

A. TIMBER CROP, ITS NATURE AND VALUE.

Timber was not only important to man in the past, it was a necessity, a basic condition to every reasonable development. With the great progress in manufacture, extended use of iron and steel, especially with the introduction of cement the importance of wood seemed to decrease materially and it was commonly predicted that wood had really ceased to be a necessity. But the actual facts, the consumption per head, the great traffic, export and import of timber, the intimate relation even dependence of many of our industries on a liberal use of wood and the unexpected rise in price of timber, the world over, all flatly contradict the common assertions and prophecy of the past years. (See Fernow, Economics.)

The people of Europe use more wood today than formerly and pay higher prices.

The timber import into Great Britain more than doubled between 1850 and 1890 and is about seventy per cent greater now than in 1890, having exceeded one hundred and twenty millions.

Germany, an exporter until 1860, is the second greatest timber importer in the world and this in spite of intensive forestry. France is a growing importer and is exceeded in its import by Italy.

In the United States the consumption per head has increased from about three hundred and fifty feet b. m. of lumber in 1880 to over four hundred feet in 1910, in spite of the fact that stumpage and lumber have increased in price by about one hundred per cent.

Our railways have not been able to free themselves from the wooden tie, the telegraph and telephone lines use wooden poles, as much lumber goes into ship building now as previously, charcoal iron is still a preferred product, pulp and paper industries demand more wood every year and nearly every new invention, even to the propeller of the aeroplane calls for wood.

In keeping with this increased demand for wood prices have advanced for a century. While the price of rye, the stable breadstuff, in Prussia decreased by fifteen per cent, the price of timber increased by fifty-five per cent, between 1860 and 1903, and while the income from the field remained nearly stationary the income from forest, state forest, during the same period increased by one hundred and eighty per cent in Bavaria, one hundred and forty-seven per cent in Prussia and by over one hundred per cent for all Germany.

Generally the demand is for lumber or larger sizes, and not merely cordwood or quickly grown small stuff. The demand also is more for conifers than for hardwoods; in the United States the consumption of conifers to hardwoods being about three to one.

Larger sizes require longer time to grow so that a forester of central Europe does not expect to cut his timber before it is eighty years old, the average rotations for conifers staying close to one hundred years.

The forests of the world are limited, the really useful ones amount to about one acre for each human being. In the United States we now have about five acres of woods per head of population. The population increases and the forests decrease. Even under present conditions the people of the United States are dependent for the future on the growth of five acres per head. This growth at present is practically nothing. Even if cared for, it would probably just suffice for the needs of the population as it is at present.

This means that the future will demand reduction in consumption regardless of all human efforts. With this reduction will come further advance in price. In many localities this reduction has already come, the supply of raw material for hundreds of mills is gone and from the state of Michigan alone industries with over sixty million dollars investment have disappeared in the past twenty years. The whole Lake Region, New York, Pennsylvania, parts of the South and parts of Canada share in this change. In many localities over large areas the change has been from industrial prosperity to utter desolation, so that towns have been deserted, railway lines taken up or abandoned.

The forest crop has its peculiarities. Categorically stated:

1. The forest takes many years to grow.

2. It can use sand lands, cold, steep, rocky, even poorly drained lands, and so utilize large areas not useful otherwise.

3. The forest improves the land, and protects it against erosion.

4. It is a much more certain crop than the field crops.

5. It is more independent of man, reproduces and grows very well without human effort wherever climate, chiefly moisture, is at all congenial to tree growth.

6. It requires not merely land but involves large investment in the growing stock.

A forester with ten thousand acres of woods properly regulated so that he can cut every year, would cut about one hundred acres of ripe timber. The other ninety-nine hundred acres have timber from one to ninety-nine years old and this body of growing stock is worth about four or five times as much as the land on which it stands.

In farming there is very little of this kind of investment, the orchard trees, bushes of smaller fruit and meadow grass resemble it.

7. The crop is not ripened in a week or even a year but may be cut when it is thirty, fifty or a hundred years old.

8. In case of accident, fire, insects, etc., a large part of the damaged timber can be used, providing there are means of getting it out, and a ready market.

9. Timber after it is cut is not a perishable material like potatoes, fruits, etc., but is improved by seasoning. A body of sawed timber may make from three to five per cent on its value in this way. This gives independence in holding.

10. Timber crop requires further manufacture, sawmill, planing mill, pulp, etc. It stimulates industries.

11. The forest furnishes more material for transportation per acre than the farm. The latter about two hundred and fifty pounds per acre, the forest about one thousand pounds.

12. The forest requires little help, therefore making larger net income.

13. From all ordinary and inferior lands the forest makes as large a gross income as does ordinary farming at present prices.

14. To the rural people the forest is a necessity to preserve sufficient independence with regard to fuel and building material.

15. The forest prevents erosion, agriculture leads to erosion. The forest is the only large agency at the disposal of man to influence the flow of water in streams, safely, effectively and permanently. In mountainous countries where the forest, because of the topography, covers seventy-five per cent of the land it is the greatest factor in water distribution.

16. Forest influences evaporation and rainfall, protects, locally, against drying wind. It encourages bird and insect life and tends to maintain a biological equilibrium, helpful to man, especially in agriculture.

17. The forest is the greatest and most important feature of the beauty of the earth. Thinking man will never wish to do without the forest wherever it can be made to grow.

B. RISK IN FORESTRY.

The following statement is made with an apology; it is not made because there is any special risk in forestry, or that forestry is less safe a business than farming or other industries. It is made because there is a very strong prejudice in the United States and because this prejudice is constantly being fostered and its arguments repeated in legislatures, among timber owners and others, as an excuse for not doing their simple duty to the country and the forests they control. The people of central Europe do not discuss the risks in forestry. They have practiced forestry for more than five hundred years. But many of the people of the United States who have never practiced forestry at all, who still prefer to let the native forests burn up rather than make any kind of adequate effort at protection, are fully convinced that forestry is not practicable because forest fires can not be stopped.

In speaking of risks, it is usually the fire danger which is in the minds of the people. As is shown in detail in the chapter on fire insurance in forestry, this danger is universally overrated. For forty years the matter of forest fire insurance has been agitated abroad, but in spite of the fact that some good companies are ready to take up this insurance at $1.80 per $1,000 property, the forester generally has not felt the need of insurance and at every meeting where this is discussed it is pointed out that the actual losses from fire, even in the pinery districts of North Germany do not amount to more than one-tenth the amount of this premium so there is no reason why large owners should spend their money in this direction. The statistical reports of Baden and Württemberg do not find it necessary even to mention losses from fire, and Bavaria for years staid below four cents on the thousand dollars worth of woods.

In the United States without any effort to prevent the fires and with many people eager to get rid of the forest, burning the forest intentionally, the losses from fire have been very great. The report of the United States Commission of Conservation in 1909 gives the estimated losses per year, since 1870, at about fifty million dollars for saleable material. This is about ten cents per acre per year and therefore not far from the premium rate now demanded by the German insurance companies, so that it was as cheap to let the woods burn as it would have been to insure them in up to date companies, even if this could have been done. It is evident then that even in the United States in spite of all neglect, lack of law and law enforcement, in spite of land clearing, etc., which made burning necessary, the fire losses in the forest are relatively small. Had they been spread over all forests evenly instead of being concentrated at certain points, there would not be the general fear of fire risk in forestry.

As a real menace to forestry insects and wood-destroying fungi are much more serious. Even in the best cared for forests of Germany, fungi, producing defects and decay in the timber, and even killing young trees, are a constant trouble. But this is true of the wild woods as it is of the cultivated forest and it is true of the farm crop more than of the woods. One of the greatest tasks of the United States Department of Agriculture and of all state experiment stations is to find remedies for the many plant enemies of our crops and fruits. Millions are spent every year by the farmer in spraying, etc., and millions more are lost for lack of effective treatment. The same is true with insects; scale, louse, moth, weevil, chinch bug, wheat fly, and potato beetle have, and do cost many millions every year. If the fly gets into wheat, the chinch bug into the corn or the boll weevil into cotton the loss is often complete. An attack of insects in timber, even if severe enough to kill, is only a partial loss, all large stuff can be used, wherever forestry has become a real business with means of utilization and a market. While the German forester is much more concerned about insects and fungi than about forest fires, it does not occur to anyone to think of these difficulties as a risk sufficient to discourage forestry as a business. Weber in the *Handbuch* and Endres in his *Forst Politik* made no effort to estimate these losses. Like those from fire in the United States they are great both here and abroad but they even up as they do in farming. To the small owner they are serious, in large holdings they form a regular trouble to figure with. Endres (p. 95) mentions the losses from wind, snow, ice and insects, (leaves out fire) and says that in all computation of averages these losses are inconspicuous—"Kommen nur wenig zum Ausdruck."

As regards the growth and reproduction of the crop itself which expresses more properly the certainty or success in crop production, the forest far excels the farm crop. From Michigan to the Gulf, the forest was practically unbroken, there were no large "burns" or other gaps. Nature had reproduced one crop after another, the crops had grown to old age, died and given way to new crops. Windstorms, insects and fungi had collected their toll but the forest was there and asked no help. In the most intensive forestry in Germany less than twenty per cent of the expenses are devoted to planting, etc., to protection and care of the crop, about fifty per cent is spent in harvesting. Where the forest has not been abused and where the climate is good the new crop comes in without any help or delay, and even where planting is justified by better returns, this planting makes rarely more than ten per cent of the total expenses on a forestry property. The growth of the forest is quite secure, a dry season may produce less wood, but the next season makes up for it and the harvest of the year does not show the effects at all. Frost, hail, extremes of cold and heat, etc., all affect the forest, but only in a small degree and the cut of the year remains unchanged. Insects may partly defoliate a stand of oak and hurt it quite severely; but if the stand recovers, as is usually the case, the man who harvests this stand will never know the difference and the effects of the injury are hardly felt or known by the business.

In farming the time of ripening is a critical period; a few days of dry weather, dry hot winds, a hail storm, rain, etc., may largely destroy the season's yield. Nothing of this kind exists in forestry. If a sixty-year-old stand is not growing well, if fungi have started their work and the stand begins to be defective it is cut and used and another put in its place.

In a twenty-year average farming over large areas must be content with about sixty per cent full or normal crop, even in the great staples, wheat, corn, etc., while forestry, where properly practiced at all, produces over seventy per cent of a full crop. In fruit farming it is doubtful if the average crop is twenty per cent of a regular full crop.

From this sketch it is evident that the risk in forestry is small, smaller than in farming and much smaller than in most city business. It is this very security which has distinguished the state forests and forests of other large owners and which has made the forest the valuable and desirable property it is. Where the forest is neglected, stocked with poor species and handled by poor methods, coppice in France, for instance, and where mismanagement has left unmade suitable improvements, it is only natural that the value of the property is small, in keeping with low income. But even in these cases it is not risk but low income, due to neglect, which is the fault.

C. THE INTEREST RATE IN FORESTRY.

a. General.

When a farmer who paid $6,000 for a 100-acre farm finds that for 10 years back he averaged $300 net income per year for his crops he naturally asks—what per cent is this on my investment? If he is satisfied with the 5% which it made he will be satisfied with farm business, if he feels that he ought to make 10% on his money he will want to sell out.

In this case it is a matter of experience and fact, with $300 income 5% is what the farm made on $6,000 cost price. If now a man wishes to buy this farm and is shown that the farm made $300 net income per year he may say: the farm is good for $300 per year, is a safe property, I may as well put my money into this farm as keep it in the bank at three per cent. To him the income value of the farm is: $\frac{\$300}{0.03}$ or $10,000 and he may be satisfied to pay this price.

To another man who values his money at 6% the farm is worth only half this amount. In this case the three per cent or six per cent are set by the man who wishes to buy the farm and judges its value by the income it makes and a per cent which he determines himself.

In forestry, too, these two fundamental cases of determining interest rate occur and very commonly are both present in the same calculation.

If a planted stand of pine sixty years old is ready to cut and worth $300 per acre the owner may want to know what per cent was made on the investment. He knows that it cost $10 to plant, that the land cost $20 per acre and that it cost 60 cents per acre each year to take care of the forest and pay taxes. The ten dollars for planting have been out at compound interest for fifty-nine years, and the question arises—at what per cent? Evidently it is for the man to choose this per cent. When the calculation is finished and he finds that all expenses for the sixty years amount to $180 per acre he can find the net income and real value of his business.

The calculation of what interest rate a stand of timber actually makes or has made is comparatively simple and there is little disagreement as to method. But what per cent to assume or set in calculations of forest valuation is quite a different question, and the opinions often differ and have differed since valuation began.

To choose a high rate, such as the current rate of interest paid by business men and farmers gives the calculation a business-like air and seems practical. But even the reasonable rate of five per cent will seem to prove most forestry a failure or lead to a demand for absurdly low rotations, etc. To choose a low rate, say two or three per cent, is unpopular, seems to discredit forestry, sounds theoretic and unpractical.

But the choice must be made, forest valuation is impossible without it. The man who puts money into land and timber for forestry must have some reasonable calculation to show what he expects of the business. Tax commissions and legislatures will not change their methods unless shown convincing facts as argument. It is self-evident that the assumed interest rate and all calculations based on this rate have nothing at all to do with the growth of the timber and all these calculations can possibly do is to give a reasonable measure of what actually takes place and compare forestry as a business with other kinds of business. The question is—is there a reasonable and acceptable basis for interest rate to set in forestry?

b. Interest rates paid and made.

1. In the United States today the ordinary loan pays about 5%, timberland owners and many industrials pay 6% and over, the farmer pays about 6 % in the east and north, larger per cents in the south and west, so that it was reported in 1912 that the farmer of the United States borrows about 6,000 million dollars at an average rate of 8%. Large railroad companies, etc., pay four or five per cent. Cities and states pay about four per cent on their bonds, the United States Government can borrow at from two to two and a half per cent, and the savings banks with their enormous deposits pay generally three per cent, while millions of dollars are deposited in banks and draw no interest.

In Germany mortgages pay about four or four and a half per cent, bonds about 3.4% to 3.8%.

Interest rates are higher in new and undeveloped countries, the West pays higher than the East.

In the fourteenth century the common rate in Germany was ten per cent, it sank to five per cent by the sixteenth century. After the Napoleonic wars it went up to eight per cent, then it sank to three per cent by 1870, rose again to five and then declined to present rates.

2. The fact that a man pays six per cent on his mortgage is no indication that the man makes six per cent in his business, whatever that may be. This seems self-evident, and yet it is the most common fallacy in connection with this discussion to assume that because a certain per cent is paid by the men of a certain industry when in need of money this rate is also made or approximated by the industry.

a. Since agriculture in the broad sense really includes forestry, it is fair to use farm business as a criterion and see what farm business pays. The data for the following analysis are taken from circular 132-A, 1913, of the United States Department of Agriculture, and based on the Census of 1910.

The average farm of the United States has 138 acres and is valued at $6,443, including stock and implements.

The total or gross income is $980, leaving off cents, of which $860 is the value of the crop, including stuff fed.

In the summary on page four of circular 132, the data are arranged as follows:

"Total gross income	$980
Total expenses	340
Net farm income	640
Interest on investment 5%	322
Labor income, including unpaid family labor and all the farm furnishes toward the family living except milk and cream. Does not include income from outside sources, and the amount paid for live stock bought must be deducted from this sum"	318
"Interest on mortgage, $1,715 at 6%	$102
Available for live stock and for family living	$537"

These same data may be arranged as follows, where it is assumed that the farmer and his family should receive for their labor at least the wages of one farm laborer without board, since they board themselves, and that it takes $50 per year to replace teams and other live stock.

Gross income per year		$980
Expenses:		
Labor	$102	
Fertilizer	18	
Feed	47	
To maintain buildings	49	
To maintain implements	39	
Taxes, at 0.6%	38	
Miscellaneous	44	
Salary of farmer and family	360	
Replacement of live stock	50	$750
Net income		$230

This $230 is **three and a half per cent** on the $6,443 capital. If the $102 interest on the mortgage is deducted there remains only two per cent interest actually made by the average farm of the United States in 1910.

Since the average farmer with his family can not be engaged at less than $500 per year and usually demand part or all the groceries for the household paid by the employer it is evident that the above three and a half per cent is too high and it is doubtful if two per cent is actually made.

This fully agrees with the findings of Cornell Agricultural College and with bulletin 41, United States Department of Agriculture, 1914, which shows that the interest made on selected farms in Indiana, Illinois and Iowa in 1911 was about three and a half per cent and in cash rent system less than three per cent. It is evident that many farmers make no interest at all and do not even make the common farm hand wages.

Whether this fact is explained by saying that farm products are sold too cheaply or that cost of production is too high is immaterial, the fact remains that the rate actually made by the most important industry of the country and the industry most closely related to forestry is less than three per cent. In Germany farming paid about two per cent, in some provinces only one and a half, in 1899.

b. What other kinds of business actually make is little known and much disguised. If the losses and gains of our stores, factories, railways, etc., could be known, and if the natural wealth, timber, coal, etc., could be deducted and also the entrepreneur's risk and effort, if these could be ascertained, it is doubtful if the business of the country really makes one and a half per cent, even in good times like 1900-1910.

Generally it is a fact that the current rate paid by the farmer is not made by the farm, it is equally certain that as an average it is not made by other business. To employ or set the current rate as measure, then, would discourage most of our ordinary industries.

c. What rates a good forest business makes is well illustrated by the state forests of Saxony where the rate actually made has been 2.3 to 2.6% for over forty years. Since a great deal of forest property is assessed on a basis of income and an assumed rate of interest this interest rate in Germany frequently is an argument in circle.

d. Germany today uses an interest rate of 3% or less, in Baden 2½, in all calculations of values of land, timber or forest.

e. In adopting an interest rate in forest valuation we have the choice and say:

1. Adopt a rate which the ordinary forest business can make, which encourages the improvements and methods necessary for a secure paying enterprise. The best measure and basis is the farm business.

2. Adopt a rate which the business can not make, which tends to abandonment of land and great loss, which even under good conditions discourages all improvements and all better but more expensive methods.

Germany, Switzerland and Denmark have, unconsciously chosen the first, France and England the second, England has no forests, France little and mostly poor forests. See also Professor Kirkland's article on the influence of the interest rate on timber production. Washington University Forestry Club Annual, 1915.

D. VALUATION OF LAND OR SOIL.

Generally, land is valuable in proportion as it produces rent. A city lot is valuable owing to its location, a field or forest land is valuable owing to the crop it can produce. The value of the crop is the measure of the value of the land. But even in the simplest case the matter is more or less complicated. The value of the crop does not depend only on the kind and amount of the crop, but also on the market and transportation and so on the location of field or forest.

Again the same field may be used to raise wheat, corn, hay or timber and for each crop the same field may have a very different value. In farming, nature demands a change of crop and it is impossible or at least unprofitable to raise wheat on the same area continuously. Generally the crop making the largest income is most trying to the land and succeeds only on good land and at reasonable intervals. Some crops, like tobacco, are very exhausting, while others like clover and timber tend to improve the land.

a. Cost value, sale—and income—or expectation—value.

1. If a man bought a tract of land ten years ago for five thousand dollars and if the income from this land has paid expenses and interest it is evident that this tract today costs him the original five thousand. Generally however, the case is complicated, the income has been irregular, greater or less than expenses and interest, and so the **cost value** of a piece of land is not as clear or easily computed as it would seem.

2. The **sale or market value** of land seems to require no explanation. It is determined by sales actually made in the district, by supply and demand. It is interesting to see what conditions are most important in determining this sale value. The conditions may be divided as follows:

a. Conditions affecting the Income:

1. Conditions which are part of the property itself; area and shape of the property, soil, topography, water supplies and drainage, improvements.

2. Conditions which are not part of the property, or which extend beyond the property: climate, market, facilities for transportation, labor, taxation, character and business of the surrounding people, and demand for properties.

b. Conditions not affecting the income but important from the standpoint of sentiment:

Beauty of the property and surroundings, social conditions, church, school, etc., family ties, habits, love of sport, "land hunger," etc.

With farmlands it is often more the second group of conditions, (b) which set the price, so that generally, the world over, farm lands are rated and paid for at a price higher than is warranted by income. But even in forestry it is frequently this second set of conditions which are decisive so that the majority of large estates held as forest parks today in the United States are held more for love of scenery or sport than as properties for income.

The sale value fluctuates greatly with demand for land and is easily affected by "booms." It is an old experience in the United States to see a few land dealers, by means of shrewd advertising, succeed in a few sales at high prices. Whether always bona fide or not, such sales tend to raise the price of land for the entire district, in spite of the fact that there is no justification for this advance in larger crops or better prices.

2. The **income value** of the land, or **expectation value** depends on the crop, rent or income which the land produces and the interest rate which is assumed or is set by the individual making the valuation. As indicated before:

$$\frac{\text{net income}}{0.0p} = \text{income value},$$

where **p** is the interest rate assumed and where the income is a yearly one as in farming.

The income itself depends on the conditions enumerated under sale value and the ability of the owner.

The interest rate usually varies with:

a. Outside conditions:

Money market, location of property (old country as against newly settled districts), good and bad times, etc.

b. Conditions of property and business itself:

Size of property, term of investment, regularity of income, safety of property and income.

b. Cost-value and sale-value of land or soil in forest properties in the United States.

Aside from the sales of small woodlots, etc., it is generally true that the land itself is not considered in sales of timber. In the past and even at the present time it is the common practice in buying and selling timberlands to estimate the merchantable timber or the stuff now ready for ax and market and to set a price only on this marketable material leaving out of consideration both the land and the growing stock smaller than merchantable. In many cases even part of the large timber is considered non-merchantable and is not paid for. For instance, balsam, cedar and oak in California, hemlock and balsam in the northwest, Douglas fir and balsam in the northern Rockies, hardwoods in the northeast, gum and other hardwoods in the south.

Generally, then, the cost value and the sale value of forest lands in the real forest districts of the United States is practically nothing. In the Great Lakes Region millions of acres were simply abandoned by the lumbermen after the timber had been cut and allowed to revert for non-payment of taxes. Of late these lands are turned over to some land company, often the same men, to sell to settlers. In the south millions of acres of cutover lands can be bought at less than three dollars per acre though there is a remnant of tree growth usually worth the price paid for the land. Michigan and Wisconsin and Ontario have sold millions of acres in the last twenty years at prices generally not over one dollar per acre and in these cases, too, the land was bought more for the remnants of timber than the land itself.

New York bought over one and one-half million acres of land, over seventy-five per cent covered with timber, at about three dollars and sixty cents; Pennsylvania bought about a million acres at two dollars and twenty-seven cents; Pisgah forest of eighty thousand acres was bought in 1914 at five dollars for land and timber and over thirty thousand acres in the White Mountains were bought at about four and a half dollars an acre, though in both cases numerous old abandoned farms were included, and the remaining timber is worth the price paid. In 1915 a tract of over thirty-six thousand acres in North Carolina was bought at one dollar and ninety cents an acre for land and timber.

c. INCOME VALUE OF LAND OR SOIL IN FORESTRY.

Expectation value of soil or Se.

When a man buys land to raise timber and does this as a matter of business and not merely sentiment, for sport or summer home, etc., the case is one of investment and he values the land in proportion as it produces an income or crop. In business the income, here net income, which can be made from the land by raising a particular crop, timber, is the only reliable measure of value.

To the man who is in the business of raising the crop, whether farm crop or timber, this net income per acre is not only the correct measure of the value of the land but it is also a measure of the work or operation of raising the crop. A good farmer or forester who selects the right kinds of crop, best suited to the soil and climate, cares for the land, sows, tends and harvests well, gets a larger net income than a less competent man. Any mistake or neglect reduces the income and with this the income value of the soil or land.

In this way the income value of the land becomes a measure of the value of land and the effectiveness of its management.

1. General method of calculation for Se.

In the case of farm property where the income from each acre is a yearly one, the income value of a given acre of land is the net income capitalized. If an acre of land on a ten-year average can produce $10 net income and money is rated at 3% the income value of this acre of land is $\frac{10}{0.03}$ or $33 330

Where an acre of land is planted to pine and the pine requires eighty years to grow to acceptable size, the income from this acre comes not yearly but every eighty years. It is a periodic income or rental which is assumed as in the case of a farm to come forever.

The present value of these net incomes, coming every eighty years is the value of the acre of land. If the timber is worth $400 and it has cost $175 to raise it, the net income is $225 and the present value of all these net incomes, coming every eighty years,

$$Se = \frac{A}{(1.0p^r - 1)} = \frac{225}{(1.03^{80} - 1)} = \frac{225}{(10.64 - 1)} = \$23.40.$$

which means that this particular acre of land is worth $23.40 when used to raise pine where this is allowed to grow to the age of eighty years, where the expenses are $175 during the eighty years, and where the final crop is worth $400 and money is rated at 3%. That the value of this acre may change if a different kind of timber is raised or if the pine is cut at sixty years, or expenses are lower or higher, or if a different rate of interest is set, is evident. It is clear too, that if the man pays $23.40 per acre for this land and uses it as outlined above, he makes three per cent on his investment and no more. This **Se** then, is rather the maximum price which he can actually pay.

It is evident, also, that these same considerations hold in case of a piece of farmland. The same acre of land produces a larger income with sugar beets or potatoes than with wheat and more with wheat than hay, so that the same acre has different income value in different years owing to change in crops, yield, expenses, etc. Yet this income value is considered the safest measure of value and forms the basis of all farm valuation and assessment.

2. The usual case of Se.

a. Premises:

Area, 40 acres; method, clear cut, and plant.

Rotation **r**, 80 years; interest rate **p**, 3%.

Cost of planting **c**, $6 per acre.

Yearly expenses, taxes and care, **e**, twenty-five cents per acre.

Yield of timber eighty years old, or **Yr**, 30 M ft. at $10, $300 per acre.

Thinning when stand is 20 years old, $\mathbf{T}_{20}$, just pays expenses, hence no income.

Thinning when stand is 40 years old, $\mathbf{T}_{40}$, yields 5 cords at $3 or $15 per acre.

Thinning when stand is 60 years old, $\mathbf{T}_{60}$, yields 10 cords at $4.50 or $45 per acre.

b. Gross income per acre at end of rotation:

Final yield or cut 80 years old, **Yr**	$300
Thinning when 20 years old, **T20**	0
Thinning when 40 years old, **T40**, $15, with interest for 80-40 years or $\$15(1.03^{40})$	50.40
Thinning when 60 years old, **T60**, $45 with interest for 80-60 years or $\$45(1.03^{20})$	81
Total	$431.40

c. Total expenses per acre at end of rotation.

Cost of planting, **c**, \$6 with interest for eighty years or 6 (1.03^{80})......\$ 63.84

Care, protection, and taxes, **e**, 0.25, coming every year with interest.

$$\frac{0.25(1.03^{80}-1)}{(1.03-1)} \quad \ldots\ldots\ldots\ldots \$\ 80.25$$

Total expense \$144.09

d. The net income per acre at end of rotation:

\$431.40
144.09
\$287.31 per acre.

This net income of \$287.31 per acre may be and is expected to come every 80 or **r** years and the value of all these net incomes or the value of the acre of land producing them is:

value of land per acre, **Se** $\dfrac{287.31}{(1.03^{80}-1)} = \$29.80$

value of 40 acres: \$1,192.

In the above simple case it is assumed that the plantation succeeds and that it does not require any filling in or cultivation, etc. If such work has to be done it is charged exactly like the plantation itself. Similarly, if there are more thinnings as there would be where intensive work is justified and also where there are other incomes, such as grazing, turpentine, etc., these incomes are credited and prolonged exactly like the thinnings in the above simple case.

It is evident from the above that this analysis is clear and logical that it resembles computations made in other lines of business, requires only ordinary interest arithmetic and that there is no necessity for putting it into a formula which often seems to hinder rather than help.

Nevertheless it is convenient and makes the matter much clearer and easier to discuss, and to understand the relation and influence of the various factors if it is arranged in a concise formula, and the usual arrangement follows closely the above sample case:

e. Formula of **Se**:

$$Se = \frac{Yr + Ta(1.0p^{r-a}) + Tb(1.0p^{r-b}) + Tc(1.0p^{r-c}) - c(1.0p^{r}) - \dfrac{e(1.0p^{r}-1)}{(1.0p-1)}}{(1.0p^{r}-1)}$$

where **Yr** is the stumpage value of one acre of timber **r** years old;

Ta the stumpage value of the thinnings from one acre **a** years old, etc.;

c the cost of planting, or otherwise reproducing, one acre;

e the regular yearly expenses of tax and care for one acre of land;

r the rotation;

p the interest rate employed or assumed;

Se the expectation or income value of one acre of land or soil.*

In the above formula the term $\mathbf{e}\,\dfrac{(1.0p^{r} - 1)}{(1.0p - 1)}$ may also be written $\dfrac{e}{(1.0p - 1)}(1.0p^{r} - 1) = E(1.0p^{r} - 1)$ where $E = \dfrac{e}{.0p} = e$ capitalized so that **E** is a sum of money which will bear a yearly interest **e** at **p** per cent and therefore, if put out at interest will "take care" of the regular yearly expenses **e**.

The formula then may also be written:

$$Se = \frac{Yr + Ta(1.0p^{r-a})) + \ldots\ldots etc. - c(1.0p^{r}) - E(1.0p^{r} - 1)}{(1.0p^{r} - 1)}$$

which may be and usually is simplified into:

$$Se = \frac{Yr + Ta(1.0p^{r-a}) + \ldots\ldots etc. - c(10p^{r})}{(1.0p^{r} - 1)} - E\,{**}$$

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*It is of great help to the student to use these terms or symbols consistently and use no more terms than are actually necessary. To use **Y** and **Yr** interchangeably or use **Yr** for one acre and for a given area of many acres, etc., soon confuses even able students. Where new cases come up, especially cases which are not really standard, but exceptional, it is much better to write out fully what each part means and use the full analysis, as in the above sample case, rather than try to force it into a formula.

** This formula is written by Endres as follows:

$$Se = \frac{Yr + Ta(1.0p^{r-a}) + \ldots\ldots\ldots etc. - c}{(1.0p^{r} - 1)} - (c + E)$$

which means exactly the same but changes the order for **c** so that it confuses the beginner and the general student. As stated before, the average student will find it helpful to stay closely with the simple forms and write them out in full so that the formula shows clearly just how it was derived.

E. VALUE OF SOIL AND STATICS.

Just as in farming, so in forestry, the net income is larger on good land and with good management than on poor land or with less efficient work. Since the income value of the land is the net income capitalized at an arbitrary, accepted per cent this income value serves as a measure of land and management. With the expenses constant, or what may be assumed the same, with the same kind and quality of management, the income and with this the income value increases only with the quality of the soil; the better the land the greater its income value. Similarly, on the same acre of land the income and with this the income value of the land increases with the efficiency or quality and correctness of the work and management.

It is interesting to see how the income value of the soil differs with change in the various factors, what these several factors depend upon, and how they vary under ordinary conditions.

a. Relation of Se and its Factors.

1. Increase of the **final cut, Yr,** increases the income and therefore the value of the land. Since **Yr** is the value of one acre of timber **r** years old it is evident that everything which increases this **Yr,** such as good planting, right thinning, good prices, cheap logging, etc. also increases the net income and income value.

2. **Thinnings** not only increase the total income but as they come long before the final cut can be made, twenty to sixty years in ordinary cases, they help to bear expenses which are piling up at compound interest.

A good thinning practice raises the quality of the final cut and so increases **Yr**; a very severe thinning practice may cut down the **Yr** in volume and if too early also in quality.

3. Cheap **planting** or small **c** means small expenses, for every dollar spent, [redacted] at three per cent, becomes nineteen dollars in a hundred year rotation. But if cheap planting means poor planting it may lead to a poor stand, reduce the income from thinnings and cut down the final cut or **Yr.** A large **c,** therefore may mean larger income. From the formula it is clear that whenever $c\ (1.0p^{r})$ is larger than $Yr + Ta\ (1.0p^{r-a})$, etc., the term becomes negative, a condition which applies to practically all young stands.

4. The **current yearly expenses** for taxes, protection, etc., accumulate with compound interest and reduce the income. They are

generally small in forestry and their importance in the business is easily overrated.

Where cheap protection means unreliable protection, the kind that fails in danger season, the saving in current expenses is the proverbial penny wisdom. As appears from the formula, the capitalized expenses **E** affect the income or the income value of the soil uniformly; they are constant regardless of rotation and so do not cause the maximum of income value to come either earlier or later.

A glance at the formula shows that whenever **E** is greater than

$$\frac{Yr + Ta(1.0p^{r-a}) \text{ etc.} - c(1.0p^{r})}{(1.0p^{r} - 1)}$$

the business is carried on at a loss and the income value of the soil becomes negative.

5. The **rotation, r,** affects the value of the crop and in this way the net income and income value. In spruce, pine, maple, etc., a five or ten year rotation would mean a final cut of mere brush of practically no value. Even a twenty year rotation would in most cases not even pay for the plantation. For these short rotations then, **Se** is negative regardless of quality of land or management. In the above cases the business would just about pay expenses at a thirty year rotation; the income value would continue to increase and reach a maximum at about eighty years. After that the expenses, planting and current expenses, together with the discount would grow faster than the timber and **Se** would decrease, in spite of the fact that the final cut would be larger at one hundred years than at eighty years.

The following figures for spruce site III (Endres, p. 275) illustrate this: At three per cent **Se** is:

Age, or rotation years	30	40	50	60	70	80	90	100	110	120
Income value of land per acre, Se.....$	—14	13	38	53	59	59	56	51	46	41

which shows that even in the German forest and at three per cent a rotation of thirty years leads to loss and negative value of **Se** of $14 per acre; that **Se** increases with increase in rotation up to seventy years, declines after eighty years and reaches a value of $41 with a 120 year rotation.

6. The **interest rate, p,** which is assumed in these calculations greatly affects the results. If in the sample case outlined above the

interest rate of five per cent is set instead of three, the **Se** of $29.80 changes to a negative quantity; i. e., the same land and management which gives to the land an income value of $29.80 per acre becomes a losing business to the man who values his money at five per cent. The same is illustrated by the following figures for spruce, site II (Endres, p. 274). The income value of the land, **Se**, with an 80 year rotation is at

3 %........................	$105 per acre
2½%........................	173 per acre
2 %........................	285 per acre

b. Values of the Factors of Se, and Variation in these Factors under different conditions.

1. **The final cut, Yr,** depends on the growth to produce the crop and on market to pay for it.

a. Both site and species affect the final cut. These possible variations are laid down in the yield tables for different species at least so far as volume and size or quality are concerned. In the more modern tables and for the settled conditions of the Old World they are commonly worked out for money values or made into money yield tables. The yield tables giving size and volume of timber per acre have been a matter of experience and growth. Even for the different districts of central Europe they are being improved continually and will be modified further with better methods of silviculture, added experience and more accurate and extended statistics. Nevertheless, for central Europe they are good now, quite the best we have; they are more reliable than farm yield tables and far surpass in safety the forecasting of ordinary business enterprises. In all new forest districts including the United States, data for yield tables are gradually accumulating. As a beginning the practice takes what it finds in the wild woods and the assumption is that what nature has produced unaided will be produced again on the same land and by the same species. Since the wild woods generally do not produce as much timber nor as good a quality, for ordinary rotations, the figures of yield and growth are conservative. This is doubly true of money yield tables. In fact money yield tables for most parts of the United States where they are based on present stumpage prices are conservative to the point of uselessness, as is clearly shown by comparing present stumpage prices with those of twenty-five years ago.

b. The influence of site or quality of land, including climate, etc., is very great. Endres, (p. 94) using Lorey, Weise and Baur's tables gives the following figures for the one hundred-year-old stand of timber fully stocked:

	Yield per acre. On site I Cubic feet.	Relative figures or per cent for different sites.				
		I	II	III	IV	V
Pine	9200	100	78	61	48	38
Spruce	16000	100	81	64	49	35
White fir ..	17200	100	81	64	49	—
Beech	10400	100	84	65	49	33

Since this classification itself is really based on arbitrary but generally accepted figures for the volume of timber per acre at a given age it may be said that the above figures are an argument in a circle. But the important fact is that in actual practice the forester deals with lands on which the yield at one hundred years differs as above. The figures clearly show therefore, the great influence of the site on the same kind of timber. That the influence of site is just as great in the United States as in central Europe is certain; the western yellow pine in the Black Hills makes a two or three log tree and yields about five or ten thousand feet per acre, while in parts of California and Oregon it is a six to eight log tree yielding forty thousand feet and over.

The influence of site on the money value of the final cut, the **Yr** of the calculation, is even greater than on the volume, since the quality and price are largely matters of size and good land produces larger timber in the same time. Schwappach in his tables of 1902 gives the following for spruce, one hundred years old: Volume of site I per acre is 13800 cubic feet, its value $2,085; putting the values for site I equal to 100, the relative figures for the different sites are:

Sites...................	I	II	III	IV
Volume	100	85	65	50
Value per acre.........	100	75	54	36

so that site IV produces half as much timber in volume, but only a little over one-third in value. In pine the average price per 100 cubic feet of timber in the stand 120 years old, in Prussia is given by Schwappach (pine, 1908, p. 144) as follows:

Site I	$12.00 per 100 cubic feet
Site II	11.20 per 100 cubic feet
Site III	9.10 per 100 cubic feet
Site IV	8.25 per 100 cubic feet
Site V	6.60 per 100 cubic feet

These figures well illustrate the effect of site on size, quality, and price of the material.

The final cut in practice and for large areas involving a variety of sites is best illustrated by the cut in the German state forests. According to Endres' *Forst Politik*, p. 96, in the year 1900 the cut for the state forests was set as follows: cubic feet of stuff three inches and over, per acre of forest area:

Baden	73 cubic feet
Wüttemberg	71 cubic feet
Saxony	70 cubic feet
Bavaria	60 cubic feet
Prussia	43 cubic feet

Which means about sixty or sixty-five per cent of what the yield table calls for, on a basis of site III and the usual rotations. Since this condition is rapidly changing owing to the conservative cutting in the past, the later figures for Württemberg are interesting.

In the three years 1906-1908, the cut of the state forests averaged one hundred and three cubic feet per acre of woods. Of this about one-third is hardwoods, mostly beech. Using beech with one hundred and twenty year rotation and spruce with ninety year rotation, and assuming on an average forty per cent of area site II, and sixty per cent site III (very nearly the actual condition as reported by Graner) the cut according to Schwappach's tables should be one hundred and forty cubic feet to be full or normal. Being one hundred and three cubic feet it is seventy-three per cent of normal, or in other words, the **practice today in Wurttemberg is seventy-three per cent efficient**, or successful. Since the cut is increasing in all states it is clear why the state forest authorities of Bavaria in their late instructions consider seventy-five or eighty per cent a possible goal.

c. The value of the final cut, **Yr** is greater for spruce than for pine, greater for pine than beech, etc. The following German figures for site II, age one hundred years, ilustrate this:

Spruce	$1563 per acre
Pine	627 per acre
Oak	377 per acre
Beech	347 per acre

While this comparison is not quite fair, since oak does not really develop quality until after one hundred years, yet it bears out the general experience that the hardwoods do not make the values like conifers, and that tolerant conifers produce more value per acre than intolerants. When it is remembered that the land producing pine is generally poor sand, while oak land is good loam or clay, commonly fair agricultural land, the above comparison is even more impressive.

d. The final cut depends on the methods of silviculture. A slow, long drawn out, natural restocking of the land wastes time and can not possibly produce the same volume and value of **Yr** which a good prompt reproduction does. Defective or imperfect cover at time of reproduction wastes space and leads to irregular stands where some trees have too much, others too little room and light. Very dense natural reproduction leads to dense stands, great competition, and if no thinning is practicable, leads to great loss of material by death and decay, loss of growth, and, besides producing less healthy and safe conditions, it leads to smaller timber, small volume and value of **Yr.**

A good planting practice assures prompt reproduction, proper spacing, healthy growth, safe conditions, and with these, larger and better timber and larger **Yr.** A saving of five years in the rotation or what amounts to the same, a **Yr** five years better in size, volume and value may be secured by good planting.

Mixed stands have often been claimed to produce a larger cut per acre. The proof has never been furnished; the practice abroad does not believe it, the pure stand is gaining and not losing ground. Mayr was right in pointing out the poorer development of crown and waste of space on the line of contact between different species such as beech and spruce, etc.

Clear cutting methods save in logging and so increase **Yr.** On good ground with proper improvements, roads, etc., the skidding is saved and buyers haul directly from the area. In parts of Germany this item alone offsets the cost of planting. Methods like the coppice produce small, cheap stuff, often difficult to market. The general experience of France with its non-paying coppice woods on relatively valuable sites clearly proves this.

The regular timber forest with rotations over sixty years alone makes a satisfactory **Yr.**

e. Up to reasonable limits the final cut varies almost directly with the length of the rotation. German experience indicates that for pine, spruce, fir and beech a one hundred and forty year rotation produces as high a **Yr** as is likely to be produced by any higher rotation. The difficulty of keeping the stand intact and growing is very great and generally demands the cut before one hundred and forty.

While the effects of the methods of treatment, silviculture, on **Yr** are quite well understood and fully recognized, their estimate in dollars and cents is not easy. This is especially true of the effects of thinning, and it is one of the important tasks of the forest experiment stations today, to gather reliable data based on careful experiment. Schwappach has made an effort to estimate the effects of thinnings in beech and spruce, and to a less extent in pine, but these efforts are still tentative. That a full and clear appreciation of these facts really constitutes the most important part of a forester's judgment regarding his business, is self-evident.

f. The market with its prices and grading determines the value of the crop when once it is produced. In the United States the market is irregular, uncertain and localized; it is excellent in New York, Boston, Chicago, etc., but poor in the forest districts. Most of the timber has to be transported over long distances and the price at the mill and in the forest is low as a result. Cypress goes from the Gulf to the northern states and east; red fir, sugar pine and redwood are railroaded clear across the continent.

A good telephone pole or a 1,000 feet of good pine retail for as much money in the towns of southern Michigan, Ohio or Indiana as they do in Germany. But an enormous supply of merchantable or mature stuff ready for the ax encourages destructive competition among manufacturers so that while good quality pine is retailed in Michigan as high as one hundred dollars per thousand feet it may be sold by the manufacturer in California or Oregon for less than cost of production, leaving no **Yr** at all for the stand in the forest.

These conditions are rapidly changing and the value of the stand in the United States as in Germany will be determined largely by the cost of growing the timber.

As it is, the stumpage price or unit price of **Yr** has increased rapidly being now more than one hundred per cent greater than twenty-five years ago.

In Europe timber prices have increased for one hundred years past, they have practically doubled between 1830 and 1890, but have remained rather constant since that time.

The following figures from Endre's *Forst Politik,* p. 120, will illustrate changes in price of timber sold from state forests in Germany.

Average price in dollars per 100 cubic feet solid, paid in the state forests of:

Average for the years	Prussia All wood $ per 100 c. ft.	Bavaria All wood $ per 100 c. ft.	Baden All wood $ per 100 c. ft	Saxony Only wood 3″ and over $ per 100 c. ft.
1830–39	2.50	....	....	4.60
1840–49	3.00	....	....	5.70
1850–59	3.20	3.50	4.10	6.40
1860–69	3.80	4.10	6.50	7.90
1870–79	4.40	5.20	7.90	9.20
1880–89	4.40	5.00	6.40	9.30
1890–99	4.90	5.80	7.60	10.20
1900	6.60	7.00	8.70	11.80

These prices were paid for the timber cut or cut and piled, in part at least, skidded to the road, i. e., in the form in which the timber is usually sold by the authorities. Log timber is usually sold full length, cordwood is piled in the usual way.

Prices for pine in eastern Prussia in sound clean sticks containing thirty-five cubic feet or more according to Schwappach, 1908, p. 143, are $13 to $15.40 per 100 cubic feet solid, or about $16 to $19 per thousand feet b.m. actual stumpage. For spruce, Schwappach gives the folowing prices:

Logs class I ....$20.40 per M. feet bm. stumpage
Logs class II .... 19.20 per M. feet bm. stumpage
Logs class III .... 18.80 per M. feet bm. stumpage

For oak in Prussian state forests the same author gives prices as follow for logs cut and skidded:

Logs over	24 inch diameter	$42 per 100 cubic feet	or	$53 per M. feet bm.
Logs	20–24 inch diameter	35 per 100 cubic feet	or	44 per M. feet bm.
Logs	16–20 inch diameter	28 per 100 cubic feet	or	35 per M. feet bm.
Logs	12–16 inch diameter	17 per 100 cubic feet	or	22 per M. feet bm.

For beech saw timber Schwappach uses the average price of $7 per 100 cubic feet or $8.90 per M feet stumpage for north Germany and $2.10 to $4.90 per 100 cubic feet solid for cordwood and small timber, also on the stump.

Average prices for log timber cut and usually peeled and skidded, i. e., ready to haul from the woods, in the state forests of Württemberg were as follows:

	Oak		Conifers	
	$ per 100 c.ft.	$ per M. ft. bm.	$ per 100 c.ft.	$ per M. ft. bm.
1880–84	17.70	22.25	9.00	11.50
1885–89	17.40	23.00	9.50	11.60
1890–94	24.00	30.00	10.50	13.20
1895–99	24.60	30.75	12.80	16.00

g. The cost of exploitation and transportation are intimately associated with market and greatly affect the value of the final cut. In the United States the cost of getting the timber cut, skidded and hauled to the railway, landing, etc., has not changed very much in spite of the great variety of conditions under which the work must be done. A cost of from four to six dollars per M feet of logs would probably include seventy-five per cent of all operations. Nor has this cost changed materially in the last twenty years. Generally the employment of machinery and the utter disregard for the safety and condition of the forest have enabled forest utilization to keep down the expenses of logging or immediate exploitation.

Timber exploitation in Europe works with cheaper labor, less equipment and less efficiency. In districts with good and ample road systems it is cheaper than the work in the United States, in all difficult situations and whenever it works over long distances as is the rule in the United States, it is not cheaper and often eats up the larger part of the value of the cut.

The cost of exploitation, (Werbung's *Kosten*), for all timber in the state forests was as follows in 1900:

Prussia ....$0.98 per 100 cubic feet, or about $1.25 per M. feet bm.
Saxony .... 1.26 per 100 cubic feet, or about 1.55 per M. feet bm.
Württemberg ...... 1.33 per 100 cubic feet, or about 1.65 per M. feet bm.

But these figures are not readily comparable to logging costs in the United States. The stuff includes cordwood, poles, ties, mine-props, etc. Moreover the material is commonly not skidded, and even if skidded, it is so only for a very short distance, generally less than two hundred yards.

The logging costs in the German states have increased in keeping with better wages; this increase is over fifty per cent since 1870 and the increase continues. Usually the costs vary but little from year to year and so may be estimated very closely. Transportation of logs to mill and of timber and lumber to markets affects **Yr** very seriously everywhere. In the United States the transportation of logs to mill formerly meant long drives, costly improvements, much loss by "sinkers" and a great deal of risk. Today it involves costly railway construction where the one cut must pay for all improvement in this way subtracting heavily from the value of the standing material.

Transportation to market in the United States is normally long distance haul. The average haul in the United States for cypress and southern pine may be assumed to be over six hundred miles between mill and customer; for western timber over fifteen hundred miles, and even for Lake states' stuff over three hundred miles, and this practically all by rail.

In Europe too the influence of transportation costs on the price of timber is evident in spite of shorter railway haul and more extensive use of waterways. The same kind of timber in east Prussia and Poland sells for fifty to one hundred per cent less than it does in the industrial Rhine districts.

The influence of a good system of roads is well illustrated in the Black Forest, while the utter lack of roads in parts of the Alps prevents many villages from having any material income from the forest properties.

h. The volume, quality and value of the final cut depends on the care and protection of the stand and the presence or absence of injurious agencies like storm, fire, insects, fungi, drought, snow, sleet, etc.

On every large property some of these enemies are at work at different times and it is the rare exception that a stand of timber passes through its entire life of one hundred years or more without suffering more, or less injury. Fungi and insects are always present and need only suitable conditions to multiply into regular epidemics. Often these enemies are assisted through the mistakes of the forester in selecting the wrong species for the site in question, by keeping too dense a stand, etc. It is due to these enemies and injurious conditions that most stands of timber, especially intolerants, "break," or open up long before the end of the rotation and often compel earlier cutting or underplanting, and on poor sites, adoption of shorter rotation.

Much of the trouble, particularly from fire, can be averted by proper protection and much can be done by prompt and frequent thinning and underplanting of older stands on poor, sandy sites. For this reason a saving in current expenses or in caring for the stand may prove false economy by reducing the final cut.

2. **Thinnings.** The incomes from thinnings are set down in most modern yield tables for the important species and the several sites and have of late been expressed as money tables. These tables for thinnings are not as fully developed as the tables for the final yield because of the great difference in the practice of thinnings. Different practicing foresters still disagree within rather wide limits in their views regarding the proper degree of thinning. In addition the market, income, labor, etc., may enable one forester to thin early and often and compel another to postpone thinnings, and to come only at long intervals. Modern practice in good intensive work expects a thinning every ten years and in young stands of oak on good sites five years is considered a long interval.

Like the final cut, **Yr**, the thinnings vary in volume and value income with species, site and practice.

a. For a stand one hundred years old, on site I, as per Schwappach, the sum of all thinnings taken out during the life of the stand and the volume and value of the main stand one hundred years old are as follows for wood of all sizes per acre:

	Main stand 100 years old		Thinnings taken out during life of stand	
	Volume cu. ft.	$ per acre	Volume cu. ft.	$ per acre
Pine	6550	627	6100	363
Spruce ...	11500	2028	10800	450
Oak	6150	798	7800	535
Beech ...	9200	431	5720	193

Arranging these values of thinnings as per cents of the volume and value of the final cut gives the following interesting comparison:

Thinnings expressed as per cent of main stand or final cut of stand one hundred years old, site I

	In volume	In value
Oak	126%	67%
Pine	94%	57%
Spruce	94%	22%
Beech	62%	45%

In oak and pine thinning out of a large volume is necessary, in spruce and beech it is not necessary but it is possible and desirable. The smaller value of thinnings is due to inferior quality, is most conspicuous in spruce and least in beech where even the final cut is largely firewood. The tendency at present is for heavier thinnings and a larger return from this source. In pine (Schwappach, p. 5) the thinnings in 1889 made about thirty-five per cent of the total growth while in 1908 they made about fifty per cent and the same change is apparent in the treatment of other species.

The influence of site on thinnings is ilustrated in the following figures for the stand of pine one hundred and twenty years old:

	Main or final cut, 120 years old.		Sum of thinnings during life of stand.	
	Volume cu. ft.	$ per acre	Volume cu. ft.	$ per acre
Site I	6860	774	7540	505
Site II	5640	600	6650	365
Site III	4550	368	5340	284
Site IV	3620	268	3940	203
Site V	2760	153	2800	136

From the foregoing figures it is apparent that site greatly affects the volume and value of thinnings and also that the difference for different sites corresponds closely to that of the final stand.

b. The influence of the method of treatment or silviculture extends beyond the mere practice of thinning itself. A dense reproduction like good stands of natural reproduction, or dense seeding and planting, call for early thinning. In many localities even in central Europe such a thinning is made at a loss. Thinnings of oak ten years old from usual seeding, also thinnings of beech and even pine and spruce, come under this description. Here the method, if artificial, by wider spacing may save money in seeds and plants and again save the expense of early thinnings, besides producing larger material in the same period of time.

The mixed forest, especially mixture of hardwoods and conifers, introduces disadvantages as well as advantages, with the former still prevailing. Thinnings of mixed stands require more judgment and care and generally increase the difficulty of selling the material. In hardwoods like ash, hickory, maple, where small second growth stuff is valuable the mixture may prove an advantage. As stated before, the greatest differences in the income from thinnings rest with the thinning practice itself. Fifty and more years ago many foresters still believed in the dense stand, removed only dead and dying trees. This did practically nothing to stimulate the growth of the stand, either in volume or quality, and did not relieve the stand of the struggle between individual trees. This method furnished very little of value in thinnings and tended to produce smaller timber for the final cut.

As indicated by Schwappach's tables the modern practice is now taking out as much volume in thinnings as is expected in the final cut and the tendency is for further increase. With a closer utilization and better prices for small stuff it seems a possibility to increase the income from thinnings and in pine, oak and many hardwoods, make it approach closely the money value of the final cut.

3. The **cost of reproduction, c** of the formula, varies with site species and methods and is greatly affected by enemies, fungi, insects, rodents, also by season, storm, frost, or conditions which may not be classed, ordinarily, with site. From a business point of view, as expressed in the income value of the land, it is not enough that reproduction may be accomplished but it is necessary that a new stand be established with certainty and safety, and in reasonable time. Generally natural reproduction is considered cheaper than planting. In many cases this is true. But if a stand of beech is opened in the shelterwood plan and has perhaps a growth of less than one per cent in value with only about one-half cover it is poor economy to wait ten years and more, as has actually been done even of late years, for natural re-seeding. In some cases where this was observed in 1910 the land readily produces from four to seven dollars worth of timber per acre and year so that the expense of ten years waiting would plant up the area several times.

a. Artificial reproduction.

Schwappach in his tables for pine uses ten dollars per acre as cost of planting on site I and V, the best and poorest lands. For sites II to IV he uses eight dollars per acre. The higher price for site I is due to the troublesome weeds and brush which come into clear cut areas on good land and usually require extra work to keep down.

Endres, "Valuation," uses twelve dollars per acre and Schwappach ten for all sites in spruce, for white fir Endres uses ten dollars but evidently he has in mind that much of this is reproduced naturally. For pine Endres follows Schwappach and uses ten dollars, but does so for site III as well as I. For beech, where again artificial and natural reproduction are both employed and figures for costs are mixed, he uses five dollars per acre. Reproduction of oak in Germany is rather costly since a great deal of protective work, fencing, etc., must be done to prevent damage from game. Yet in oak, as in beech the cost varies from almost nothing in good natural reproduction to expensive planting of large transplants. The greater part of oak is started artificially by very dense seeding. Schwappach therefore puts reproduction costs in oak at fifteen dollars per acre.

For the state forests of Württemberg, nearly five hundred thousand acres, with sixty-nine per cent conifers and forty per cent occupied by beech and white fir where natural reproduction prevails, the official costs of reproduction include: drainage, seeding, whether in the open or to assist natural reproduction, planting of all kinds as well as maintenance of nurseries and purchase of seeds and plants. But it does not include the extra expenses in logging due to methods of natural reproduction, an item which would require estimate from one piece of work to the next and so far is never introduced in forest statistics. The following is taken from Graner, for 1908:

Total expense for reproduction $129,000, or 26 cents per acre of forest, of this total there was for (round numbers):

1.	Ditching	$ 1100
2.	Seeding, new, 150 acres, corrections 40 acres, cost per acre $10.30, total	2000
3.	Planting, new plantations 4430 acres, corrections 1130, cost per acre $10.03, cost per 1000 plants set out, $3.30, total	55800
4.	Maintenance of nurseries, 500 acres	49100

Cost of restocking lands or reproduction forms only seven per cent of all expenses as against 12 per cent spent for roads in 1908.

The high cost of seeding is due to the large proportion of oak. In the pineries of Prussia reproduction by artificial seeding has been revived by Splettstösser and the cost in some cases is less than a dollar and a half an acre. But in dry years this method fails and the cost of successful reproduction by seeding is still uncertain and greater than is usually assumed.

For the state forests of Prussia reproduction costs in 1900 were about twenty-seven cents per acre of forest. These expenses have increased from about nine cents in 1870.

In the Saxony state forests the average costs for restocking were about twenty cents per acre of forest, including ditching work, having approximately doubled since 1870.

In the United States the cost of planting and seeding is very variable. In the eastern states for conifers and most native hardwoods it may be figured approximately:

Cost of raising plants: three year seedlings—$1 per 1,000; four year transplants, or 2-2 plants, $3 per 1,000; to set out either conifers or hardwoods about $4 per 1,000. Since 1,000 per acre is quite sufficient, the cost per acre is $5 to $7. To this must be added thirty to fifty per cent for mishaps, bringing the cost per acre to $7—$10, or about the same as the cost in Europe, where, however, much denser planting is practiced. For oak and beech about two to five bushels of seed per acre is used. With acorns at a dollar and a half, a common price, the seeding costs about $10 or $12 per acre.

These figures vary with site, labor, cost of seed, etc., and will be high wherever conditions are adverse to forest growth and call for extra effort. The experience of the United States Forest Service though the most extensive in this country can not well be used here, since the conditions under which most of this work has been done so far are very difficult, particularly as regards site, accessibility, cost of labor, and, in many localities, the work of seeding suffered excessively through rodents, which make this method almost useless. Experience in district 1, Montana, etc., indicates that even here the cost of "effective" or successful planting is now close to $11 per acre, and that much planting is done at about $7 per acre, and seeding at about $2.50 per acre in conifers. But it should be added here that seeding has generally been a failure in these western forests and is largely abandoned.

b. Natural reproduction.

The cost of natural reproduction is assumed to be zero. This is not true in most cases. If natural reproduction is slow it wastes valuable time, years of rent are lost. If it produces a stand in which a portion of the trees are of poor species and not wanted and this condition requires the cutting out of material at a loss, this expense may very well be charged to the form of reproduction. And even where the natural reproduction is perfect as to species and time, but produces dense thickets of stuff which cause extra expense by early thinning or involve years of struggle and consequent loss of growth, the method is certainly not without cost to the owner of the forest. Unfortunately statistics on these points are still too imperfect to base general statements on. The universal adoption of clear cutting and planting in pine and spruce in German and other forests is the best wholesale evidence that natural reproduction involves expense, or, what is the same thing, that artificial reproduction saves money. Similarly, when the best Danish foresters find it to their advantage to spend fifteen or twenty dollars in tilling, liming and artificial seeding to assist so-called natural reproduction, we have here artificial seeding and the old stand is useful only as shelter.

In good coppice on good sites the expense of cleaning out useless sprouts and useless species is small and if need be can be neglected. But such saving is poor business on excellent site which should produce from ten to twelve dollars worth of good material per acre and year.

In selection forest of our ordinary hardwoods natural reproduction is quite certain, fairly prompt, but often made up of the less desirable species and either means a loss in the yield or an expense for cutting out this material.

In situations where a dry climate, lack of seed, etc., prevent prompt reproduction, the waste by having a part of the land unused for years may be a very considerable expense, as pointed out above.

In nearly all cases, except coppice, natural reproduction increases the cost of logging. In many cases in central Europe today this amounts to more than twice the cost of planting and should be charged to reproduction and not deducted from the final yield as is always done.

In remote mountainous districts where the cold climate retards growth and the site at best produces but a small yield, lodge pole, Engelman spruce, tamarack in swamp, jack pine, etc., the cost of artificial reproduction may not be warranted except perhaps as a starter on burned over districts, and here the natural reproduction in spite of its loss in time and rent may still be the only economical method. And even on better sites in our unsettled districts, western yellow pine, etc., with enormous masses of excellent but unsalable material, natural reproduction may be indicated. But to leave large areas of these better sites entirely bare for many years would be a mistake. This is recognized so that even now planting is being started in many localities.

4. **Current expenses, e,** usually consist of administration and protection, improvements and taxes.

a. Taxes are beyond the control of the owner; they are known and remain fairly constant for large rural properties for many years. In the United States under ordinary settled conditions rural property pays about six or eight dollars on the thousand dollars worth of property or about twelve dollars on a two-thirds valuation.

This will no doubt remain so whether forest taxation continues on the present plan or changes to a yield tax or single tax, for in the end the income or the income value of the forest determines the ability of the owner to pay.

b. Administration and protection are usually given together in recent statistics. Endres, *"Valuation,"* p. 38, quotes the following for the state forests of Germany: Prussia, Bavaria and Alsace-Lorain about seventy cents per acre; Baden eighty-six cents, Württemberg one dollar and five cents and Saxony one dollar and thirty-five cents per acre. Usually about sixty per cent of this goes for administration proper and the rest for protection. Since so much of the under-forester's or ranger's time is spent in work where protection and supervision is combined it is impracticable to separate the two. It is doubtful, however, if protection proper is done for less than twenty cents per acre in any of these state forests. According to Schwappach, the larger private estates in south Germany spend about forty-five or sixty-five cents per acre and year for administration and protection.

In the United States there are no satisfactory, comparable figures on this point. The total appropriation for the care of approximately one hundred and sixty million acres of national forests is only about five million dollars or three cents per acre, a figure which can not be considered here for it does not even indicate a reasonable interest in the property. Of this three cents it is assumed that about two cents an acre are devoted to administration and protection. But everything is new; a large amount of labor is absolutely necessary to survey, locate, map and describe or record things, to enable any administration to be carried on at all, and in addition much work is necessary to attend to the orderly disposition of timber, grazing, etc., so that it is doubtful if much more than one cent an acre is actually devoted to protection and supervision.

It is difficult, as yet, to estimate what will constitute a satisfacfactory figure for administration and protection in our extensive enterprises. Recent experience would indicate that protection of the kind that will really protect can not be furnished under ten cents an acre; about half this sum should be allowed for administration if the property is to be properly regulated, inspected and its business recorded. Undue economy in either direction is costly.

c. Improvements in forest properties consist chiefly in roads and besides these in buildings for foresters and rangers, and in telephone lines. Occasionally special improvements like chutes, flumes, etc., are provided, but they may well be charged to exploitation and deducted from the gross yield.

For the most part improvements are developed gradually and the expense for improvement is fitted to the income, and may therefore remain fairly constant for a long term of years. In the state forests of Europe the construction and maintenance of roads is usually the only item quoted in the reports. In the state forests of Württemberg road work in the five years ending 1908, averaged about forty cents per acre of forest and of this twenty-four cents was for maintenance. Ever since 1875 the roads have received over twenty-five cents per acre and year, and while new construction will become less in time the item of roads will always amount to over thirty cents per acre and year.

In the United States there is as yet little experience in this matter outside the United States Forest Service and as stated, the Service has had too little support to do more than make a beginning.

On a good forest property with satisfactory market, etc., a suitable road system is a necessity and it is reasonable to suppose that this will cost here what it does abroad.

Combining the factors of current expenses for our conditions the following estimate will suit a large part of our forests:

Taxes	15 cents per acre
Protection	10 cents per acre
Administration	5 cents per acre
Improvements	10 cents per acre
Total	40 cents per acre

This should be regarded rather as minimum than average for good, intensive work.

That these expenses must be suited to the income or the value of the crop which any tract can produce is evident. From the formula it is apparent that: $Yr + Ta\ (1.0p^{r-a})$ must be greater than $c\ (1.0p^{r}) + E\ (1.0p^{r} - 1)$.

If we take the value of the thinnings, prolonged, to be equal to one half **Yr** or the final cut, and use ten dollars for planting and forty cents for current expenses, eighty years as rotation and figure with three per cent, then the final cut must be worth more than one hundred and seventy-five dollars to produce any rent on the land. In jack pine, tamarack, or high mountain timber where the final cut can not be expected to be worth over fifty dollars and in the mountain forest requires perhaps one hundred and fifty years to grow, it is evident that planting should be avoided and the current expenses cut to one half or less of the above estimate.

5. **The rotation** is largely a matter of species, site and market and also of treatment. The first condition is to produce material which can be sold at fair price and in large quantities. Generally, then, the market determines the size and quality while site, species and method determine the time required to produce the demanded size and quality.

In central Europe oak requires over one hundred and fifty years, pine saw timber and beech over one hundred and twenty, while spruce and balsam are now commonly cut at about one hundred years.

The following figures of age, class, conditions for the timber forests of all Germany are instructive in this connection:

Area in 1,000 acres of different age classes in 1900.

	Over 100 years.	80-100	60-80	40-60	20-40	to 20	bare
Oak	244	147	181	208	250	272	25
Beech	792	823	937	816	657	511	29
Pine	975	1133	1801	2859	3319	3383	53
Spruce	403	548	875	1191	1517	1517	17
White fir	150	118	120	108	122	116	15
Larch	0.3	1.6	5	8	7	9	—

From a financial standpoint the temptation is to set a rotation producing the largest **Se** or largest interest rate on the business.

For spruce in Saxony state forests according to Schulze, the following is true with present prices:

Rotation 50–60 years, rate made	5 %
60–70	3.7%
70–80	3.2%
80–90	2.6%
90–100	2.2%

But if a rotation of fifty to sixty years was attempted on all Saxon state forest lands the price of smaller sizes would drop and the result would be a failure. Again, for large areas of poor sites, IV and V, such a rotation would not produce marketable stuff at all.

In fact, according to the same author, a rotation of fifty-five or sixty-five years is most profitable, even with present prices, on only eleven per cent of all forests, while the rest require, owing to slower growth, rotations of over sixty-five and as high as one hundred.

With better machinery, manufacture of pulp and extensive use of timber in mining, etc., the use of smaller sizes has been increased. In conifers timber ten to fourteen inches d.b.h. with bark can be sold at good price and in largest quantities. In addition the modern practice of thinning gives more space to each tree, stimulates growth and so produces these sizes in shorter time. Accordingly, there is a tendency toward shorter rotations in all timber except oak, where satisfactory quality requires trees over eighteen inches d.b.h. In the United States the timber industries are rapidly adapting themselves to the use of smaller sizes so that, locally, at least, as small stuff is sawed and worked up as in Europe. In the handling of defective stuff the United States methods far excell those of the old world. It is reasonable, therefore, to assume that the conditions determining the rotation will be the same here as abroad. With warmer climate and better growth, shorter rotations such as fifty years even for saw timber are anticipated, established or accepted in legislation, in Massachusetts and Connecticut. In loblolly pine, hardwoods in river bottoms, etc., no doubt this will apply, but in all cases the quality of "grain," fine or broad rings, will also be considered by the market and may call for denser stands and longer time.

6. The interest rate, p, has been discussed before. Its effect on calculation of the income value of the land and of the success of the forest business is very great. The interest rate actually made in the forest business in the state forests of Germany varies from two to three per cent on the sale value of the property. An interest rate of five per cent will probably prove ninety per cent of European forestry a failure. In the United States at present and for some time to come the unsettled conditions of values in timber and timber lands will make it possible to acquire lands at prices which will assure higher rates. In time the real value of land and timber will approach the true income value and the interest rate made in the business will be normal just as it is in agriculture and other lines of business. To make these calculations reasonable, comparable and useful it would be well if a uniform rate could be adopted and since three per cent has been used extensively in European works it commends itself for adoption here.

c. Actual values of Se.

The following figures are taken from Endres' *Valuation,* and Schwappach's *Tables of Growth.* The interest rate is three per cent.

Income or expectation value of land per acre for different species and rotations.

Rotation years	Spruce on sites I	II	III	White fir on sites I	III	Pine on sites I	III	Beech on sites I	III
	dollars per acre								
30	51	10	—14	—10	—30	43	—3	4	—19
40	100	47	13	67	—5	59	7	25	—4
50	139	77	38	134	25	66	13	32	6
60	159	98	53	156	38	67	13	35	7
70	166	105	59	156	44	64	13	35	7
80	161	105	59	145	44	59	10	34	4
90	151	99	56	130	41	53	8	31	2
100	138	91	51	114	36	48	5	27	2
110	126	83	46	99	30	43	4	24	—2
120	117	75	41	86	25	40	2	21	—4

For white oak in Germany two per cent, the values are as follows for **Se**:

Rotation, years	Site I	Site II	Site III
100	$262	$131	$56
120	272	133	63
140	260	134	60
160	236	124	53
180	215	113	49
200	201	101	44

The above figures are based on normal yield tables of Schwappach for spruce, pine and oak, Eichorn, 1902, for white fir, and Grundner, 1904, for beech. They asume cost of reproduction twelve dollars per acre for spruce, ten for fir and pine, five for beech and fifteen for oak. The current expenses are taken at ninety cents per acre and year for all but oak where sixty cents is used. Since the above values are based on normal yield tables, assuming full and satisfactory stands, the values require reduction in practice by at least the amount by which the practice now fails to keep the forest in fully stocked good growing condition. This for the state forests of Württemberg would be less than thirty per cent, and for most state forests would vary from thirty to forty per cent.

An examination of values for **Se** at three and two per cent reveals the interesting fact that the maximum for **Se** comes too early at three per cent, and of course still earlier at higher rates, for a safe technical rotation which works for the sizes most demanded by the present market. On a basis of two per cent the maximum **Se** and the best technical rotation fairly coincide for all species except oak where even two per cent brings maximum **Se** too early by at least forty years. This seems to indicate clearly that the true value growth of the forest is about two per cent for pine and spruce and below two per cent or about one and one-half for white oak in the German forest and present conditions of market.

On the other hand a two per cent rate makes forest lands generally more valuable than farm lands of far better quality for all species except oak. In oak even a two per cent rate gives to the land no higher value than the same land would have for farming.

d. Appreciation of the calculation of the income value of the land.

According to Endres, von Zanthier's articles in Stahl's *Forstmagazin,* 1764, may be considered the earliest satisfactory efforts to develop money yield tables and a calculation by which different methods may be compared.

The first calculation of the income value of the soil was published by König in his *Anleitung zur Holztaxation,* 1813. His formula agrees with the present except that he used **c** in place of

$$\frac{c\,(1.0p^r)}{(1.0p^r - 1)}.$$

The formula as now used was first developed by Faustman in 1849 in the *Allgemeine Forst und Jagdzeitung,* under the title Waldbodenrentenformel, formula of soil rent in forest.

The formula has been and is still being attacked in spite of the fact that the basis for the calculation, final cut and thinnings, cost of planting and current expenses are well known, the values, yield tables, etc., constantly used, and universally accepted as the best there is, and as being as reliable as any similar compilations in other lines of business. The calculation itself as a simple problem in arithmetic is no longer discussed.

Usually the criticism is one of the three following:

a. Forest valuation, in Germany, does not ordinarily start with bare land but with a forest business fairly regulated for a long time. There is no need of any compound interest calculation. This argument seems convincing but is not true. In modern forest practice, excepting selection forest, every acre of forest is cut once in each rotation. Whether this is done in clear cutting in a single year or in shelterwood in several years, the land is cleared of the old timber and a new stand is started. To start this stand and care for it during eighty or a hundred years costs money and when the stand is ripe, the question is: what has it cost to produce? Since the money spent in planting, protection, administration, taxes, etc., has not paid any interest it is proper that this interest be compounded yearly and charged to the cost of producing the stand exactly as is done in the above calculation for **Se.**

b. The second criticism is that the values for final yield and thinnings are useless, since timber prices advance so rapidly that they may double and more during one rotation. This argument is valid but it does not affect the case at all. If a man examines two farms with a view to buying, he is apt to judge on a basis of present prices and if one farm produces thirty bushels of corn per acre and the other fifty bushels he will consider that the latter is worth more than the former. And he will be right regardless of any change in the price of corn. Exactly the same thing is true in forestry. If Faustman applied his calculation to two tracts of forestland sixty years ago and found one of them worth twice as much as the other the important point in his calculation is still valid in spite of the fact that timber has doubled in value during this time. And even the absolute value was, for his purpose, entirely right, for no one is willing to buy a farm or forest on the assumption that in one hundred years the crops will be worth double the price. Occasionally there are cases where an advance in price is practically certain and may well be introduced even in calculation of the business. For instance a plantation of pine set out today in the eastern United States may take eighty years to mature and will then certainly not be sold at present stumpage prices. But even in such a case it is not necessary or desirable to assume uncertain advances in prices and most men would prefer to estimate the value of their enterprise on the basis of present prices, using, no doubt, maximum rather than average figures.

c. The third criticism is that the calculation is a failure as soon as "reasonable" or "current" rates of interest are employed. This point has been discussed under "Rates of interest." As pointed out, current rates of interest are normally far above what large industries and business actually make. On the basis of six per cent farming in the United States is a losing business, though the farmer pays this as a current rate. The forests and their growth depend on natural laws and have nothing to do with interest rates. The only use of an interest rate in the study of these forests and their growth is to serve as a measure. We can use five per cent and have all values of **Se** negative and still secure a very good comparison between different tracts of land, between different species and methods of treatment. But these negative values while making possible this comparison will give a distorted picture of things; they will tell the truth and yet mislead. The absolute values will be correct so far as the five per cent basis goes but they will show a good safe business bringing in millions and giving paying employment to thousands as being a losing enterprise.

This same criticism was amplified by pointing out that this form of calculation on a basis of ordinary interest rates would lead to a ridiculous and disastrous shortening of rotation. In part this was provoked by enthusiasts in mathematics who actually argued for such shortening of rotation. That shorter rotations were needed most authorities agreed but rotations of forty or fifty years in spruce, etc., were evidently wrong and the practitioner saw this. The mistake was one of judgment in establishing the value of timber. As stated under rotation, the maximum **Se** for Saxon spruce with present prices would come at about fifty years. At present the great body of timber offered to the market is larger than is produced in fifty years and the prices of the smaller, less useful stuff are purposely kept up by not offering too much of this kind of material. If the Saxon state forest should go to a fifty year rotation this small stuff would at once become a drug on the market and what seemed profitable would prove a loss.

But this merely proves that in deciding on the proper rotation and consequent final yield, it is necessary to use some judgment and consider the demands of the market.

Good material, clean stuff, fourteen inches in spruce, etc., need not fear depression in prices. Its market is practically unlimited in central Europe and therefore the rotation is set to produce this material.

The calculation of the income value of the land devoted to forest is not mere formula, in fact the formula may well be dispensed with, it is an analysis of value such as is used in other lines of business. The estimate of value or the assessment of every farm involves this same analysis and the only difference lies in the time element, a peculiarity of the forest business.

Just as the forester uses yield tables, expenses of starting the crop, caring for this crop, taxes, improvements, etc., so the farmer has accepted standard figures for costs of plowing, seeding, cultivating, his "stock figures" or yield tables, and finally his average prices which guide him in setting a value on the land, on different crops and methods.

e. The Se in coppice woods.

Since in ordinary coppice the crop is cut clean and the ground cleared there is a temptation to use the calculation for **Se** without any allowance and put cost of reproduction equal to zero. This is not correct for a piece of coppice woods recently cut is not bare land, but bare land plus living stumps. These stumps are the equivalent of a good plantation safely established and so have value.

f. The Se in the selection forest.

Here the crop is never entirely removed. Where the forest has been in use for a long time and satisfactory records of growth have been established the stumpage value of the material removed during one rotation and properly prolonged may be substituted for $Yr + \Sigma Ta\ (1.0p^{r-a})$ in the usual calculation.

In most cases it is simpler to estimate the **Yr**, etc., from the size, quality and growth of the timber found upon the land at any time and make up a new case.

F. VALUE OF STAND OF TIMBER OR THE GROWING STOCK IN THE EVEN-AGED STAND.

A stand of white pine now fifty years old may be sold without the land to a man who intends cutting the timber at once. In a neighborhood in New Hampshire where many such stands are cut every year, this kind of a stand will have a fairly definite **market- or sale value.**

Stands of white pine in this same vicinity but less than thirty years old would probably have no ready market and there would be no established or accepted sale value, while there would be ready sale value for any stand over fifty years old. Under ordinary conditions stands not yet ready for use, non-merchantable or immature, have no regular sale value and this is true even of stands abroad, for the sale of such stands is not sufficiently common, even there.

The same stand may be sold to a man who wants to keep it until it is seventy years old and of better size and quality. He would have considerable expense holding the stand; interest on the money he has to pay for the stand, rent to the owner of the land , care and protection for twenty years and taxes for this period. To determine what he could pay for the stand he would assume a certain value which the stand is expected to have at seventy years, deduct all expenses and find what he may expect to get at the end of twenty years when the stand is cut. He would discount this to present date and get the present value of the stand on the basis of his expectations. This would be the **expectation value** of the stand.

The owner who wishes to sell the stand would set the price first of all by the amount which it has cost to produce the stand, by the **cost value** of the stand. This cost value of an even-aged, planted stand he can determine exactly as the farmer finds the cost of producing an acre of corn. He charges the expense of planting, the rent on the land, taxes and care. But unlike the farmer he has to charge and compound these sums for the whole fifty years or the age of the stand.

Any stand of timber regardless of age or size has the usual three values, cost, sale and expectation, with the sale value rather indefinite for non-merchantable stands.

a. Sale value of growing stock.

Since this is largely a matter of stumpage of marketable stuff either for immediate use or to hold for future use, and since in the United States this is important chiefly in connection with our wild woods, it is discussed under stumpage value in chapter VI.

b. Cost value of growing stock.

The usual case for an even-aged stand of timber is as follows:

1. Area of stand, 80 acres; age, **m**, 45 years, interest rate, **p**, 2%.

Premises per acre:

Cost value of land, **Sc**, $5, therefore rent **Sc** (.op) is ten cents.

Cost of planting, **c**, $8.

Current expenses yearly, **e**, 50 cents.

Income from thinnings:

Thinning at age of 20 years,	**Ta**,	costs $3, negative income,
30	**Tb**,	just paid for cutting,
40	**Tc**,	5 cords of wood worth $10.

2. Expenses incurred to produce the stand or grow it to the age of 45 years:

Cost of planting prolonged, **c** $(1.0p^m)$,
or 8 (1.02^{45}) .... $ 19.40
Current yearly expenses **e** with interest.

$$\frac{e(1.0p^m - 1)}{1.0p - 1} = E(1.0p^m - 1) \text{ where } E = \frac{e}{.0p}$$

and yearly rent of land with interest,

$$\text{rent } \frac{(1.0p^m - 1)}{1.0p - 1} = Sc(1.0p^m - 1),$$

simplifying $(E + Sc)\ (1.0p^m - 1)$ or
$(25 + 5)\ (1.02^{45} - 1)$ .... $ 43.13

Thinning at age of 20 years, cost with interest.

$Ta(1.0p^{m-a}) = 3(1.02^{25})$ 4.92

Total expenses per acre .... $ 67.45

3. Income per acre obtained from the stand during the 45 years,

Thinning at 30 (just paid expense) .... $ 0
Thinning at 40, $10 with interest for five years....... 11.04

Total income $11.04

4. Net cost per acre:

$$\$67.45 - \$11.04 = \$56.41$$

and cost value of the stand of 80 acres, $4,512.

5. This calculation is usually expressed in a formula as follows:

$$^mGc = c(1.0p^m) + (Sc + E)\ (1.0p^m - 1) - (Ta(1.0p^{m-a}) + Tb \text{ etc.})$$

when **Ta** is made at a loss or is a negative quantity, **—Ta,** is deducted from costs as **—(—Ta)** as in the sample case above.

In the formula ^{m}**Gc** means: the **cost value of one acre of the growing stock, m years old.**

6. The above case fully illustrates the usual calculation. If natural reproduction replaces planting, **c** may be zero. Similarly, if there are expenses of clearing the land, as on a burn, of cultivating the plantation, of filling fail places, etc., these expenses are charged and prolonged exactly as planting, **c.**

7. In the above case the cost value **Sc** of the land was used. In most cases it is but natural that this should be so. But where the cost value is practically zero, quite common where the land and timber were bought together and only the merchantable timber considered in the price, the cost value is misleading, the land may readily be worth five or ten dollars an acre and so have an accepted sale value.

In such cases the sale value, **Ss**, is used. Occasionally the land has neither cost nor sale value which could be agreed upon, for instance, state and national lands, remote mountain or non-agricultural land. Here the temptation is to ignore the value of the land entirely. This is wrong since any land which can and does, or actually has produced a crop of timber having value, has value itself. Inconsistency here is liable to lead to trouble for sooner or later the value of these very lands may come in question. In such cases, the **Se** properly based and discounted should be substituted as **S.**

To employ **Se** without deductions is hardly advisable since **Se** is rather the maximum value of the land for the particular purpose and to employ a maximum price in these calculations is hardly safe or satisfactory.

c. Expectation value of growing stock.

Ordinary case: Spruce, clear cut, and plant; rotation, ninety years; area, eighty acres; age of stand, forty-five years, p, two per cent.

1. Premises per acre:

Land value **Sc**	$ 20
Yearly current expense **e**	0.50
Final yield, **Yr**, stumpage value, 40 M. feet at $10 or	400
Thinning at 50 years, stumpage	30
Thinning at 70 years, stumpage	60

2. Gross income per acre at end of rotation:

Final yield	$400
Thinning at 50 and interest for 40 years	66
Thinning at 70 and interest for 20 years	89
Total	$555

3. Expenses per acre to end of rotation:

Rent of land and current expenses treated as in case of cost value,

$$(Sc + E)\ (1.0p^{r-m} - 1) = (20 + 25)\ (1.02^{45} - 1) \ldots \$64.$$

4. Net income per acre at end of rotation or net value of the stand then, \$555 — \$64 or \$491.

5. Present value of the stand on basis of expected income: Final value discounted to date, or

$$\frac{491}{1.02^{45}} = \$202$$

Value of the stand on 80 acres = \$16,160.

6. This calculation may be put in a general formula:

$$^{m}Ge = \frac{Yr + Ta(1.0p^{r-a}) + Tb(1.0p^{r-b}) \text{ etc.} - (Sc + E)\ (1.0p^{r-m} - 1)}{1.0p^{r-m}}$$

where ^{m}Ge means **the expectation value of one acre of growing stock, m years old.**

7. It is evident that proper allowances must be made in estimating future incomes, i. e., that these values must be reasonably conservative. It also appears that if, in the above case, \$202 per acre is paid and the values are secured as per estimate, the buyer does not make anything over and above his two per cent on all investment. Evidently then, the full expectation value is the maximum which can be paid for the stand.

Where risks of injury from fire, etc., are great, such risks are allowed for in the calculation by proper deduction from the values of the final yield and thinnings.

d. Which value to use.

In selling and buying a house the owner first considers the cost value or what the property cost him. The buyer cares little or nothing about this cost value but considers the market or sale value or else the value which he may make on the property. So in the valuation of the stand of timber. The buyer does not care whether the cost of planting was high or low, and how much expense the owner had in care, protection, taxes, etc., he merely cares about the values which he can make out of the timber, the sale or expectation value.

This situation is complicated by the uncertainties of a living crop. Commonly the age and condition of the stand decide. If a stand of pine ten years old is injured or destroyed the question almost naturally is—what did it cost to reproduce or grow it, what will it cost to replace the stand, i. e., what is the cost value?

In older stands it is the sale value which determines. But in many cases, especially in new districts, as in the far west, it often happens that even a fine, mature stand of timber has no sale value at all, but is certain to have a value as soon as a certain railway is finished. Similarly a stand of lodge pole may be nearly mature and yet be a few inches below the marketable size; here evidently the stand has a future value or expectation value, though neither sale value nor an accepted cost value exist.

In the courts the sale value has always been accepted. Of late, since these matters have been more carefully considered and the evidence has been sufficient and clear, both cost and expectation value have been admitted. For young stands, less than half rotation, the cost value or cost of replacing is the more satisfactory.

e. Value of land and growing stock, together, i, e., value of forest with even age stands.

This is merely a matter of addition of the values of land and timber. In adding, the question arises as to the use of cost, sale or expectation value of both land and timber. As stated before, this depends on the viewpoint of the person making the valuation. The owner of a tract of timber is apt to use cost values for both land and timber, the buyer will use sale values as far as possible. If the stand is young, he too is apt to employ cost value for the stand, but is quite certain to use the sale value of the land, for it would be useless to employ cost values where this is zero or at least uncertain. Both buyer and seller should work out the expectation value to get a fair measure and check on all sale value estimates. Since the growing stock is expected to pay rent on the land, the value of the land is not changed by the fact that the stand must remain for twenty or more years on the land before removal and so prevents use of the land for other purposes.

G. VALUE OF THE GROWING STOCK OF A REGULATED OR NORMAL FOREST.

When the State of Michigan starts out to establish a million acres of state forest it must determine: how much capital is to be tied up in this property and how much of it in land and how much in perishable growing stock or trees of all ages from the seedling to mature timber. The same is true of any owner or buyer who considers selling, buying or developing a real forest business. The importance of the growing stock in such a business is great since it usually makes from sixty to eighty per cent of the entire value of the property.

a. The regulated or normal forest.

1. Premises: Area, 16,000 acres.

Method, clear cut, and plant, even age stands.

Rotation, 80 years.

Thinnings every 10 years, beginning when the stand is ten years old, the last at seventy years.

2. Condition of forest in spring just after planting:

Two hundred acres just planted, growing stock 0 years old.

Two hundred acres growing stock 1 year old, etc.

Two hundred acres 79 years old, or r-1 years old.

3. Yearly work, expense and income on this forest:

Two hundred acres are cut, income 200 x Yr.

Fourteen hundred acres thinned, income T_{10} (200), T_{20} (200), T_{30} (200), etc.

Two hundred acres are planted, 200 x c.

Sixteen thousand acres protected and cared for in road building and other improvements, taxes paid, cost 16,000 x e.

Sixteen thousand acres growing stock pay rent of land 16,000 x Sc (.op).

4. A proper sample of this regulated forest would contain 80 acres, one acre just planted, one acre with one year old stuff, one acre two years old, etc. to one acre 79 years old, taken from the oldest stands. Whatever is true as to the average value of this sample of eighty acres applies to the whole forest of 16,000 acres so that if the growing stock on the sample eighty acres is worth fifty dolars an acre on the average, the growing stock on the 16,000 is worth fifty dollars an acre average.

b. Expectation value of the growing stock of the regulated or normal forest.

For sake of simplicity the rotation is assumed to be only ten years so that the sample of the regulated forest contains ten acres. Diagram, Fig. 1, illustrates this 10-acre sample and indicates the present value of yields, **Yr**, and thinnings, which are to be obtained.

The yearly work and expense and income on this 10-acre sample are:

One acre cut, income **Yr.**

One acre thinned, when stand is 4 years old, income $\mathbf{T_4}$.

One acre planted, costs **c.**

Ten acres protected, cared for, taxes paid, expenses 10 x **e.** Ten acres of growing stock pay rent on the 10 acres of land, cost 10 x Sc (.op).

2. Adding the values of the growing stock of the 10 acres gives the value of the growing stock for the entire sample of 10 acres and dividing this sum gives the average value per acre for the sample, and also of the regulated forest.

Age 9	8 y.	7 y.	6 y.	5 y.	4 y.	3 y.	2 y.	1 y.	0 y.
r-1	r-2	r-3	r-4	r-5	r-6	r-7	r-8	r-9	r-10
Value of Yr:									
$\frac{Yr}{1.0p^1}$	$\frac{Yr}{1.0p^2}$	$\frac{Yr}{1.0p^3}$	$\frac{Yr}{1.0p^4}$	$\frac{Yr}{1.0p^5}$	$\frac{Yr}{1.0p^6}$	$\frac{Yr}{1.0p^7}$	$\frac{Yr}{1.0p^8}$	$\frac{Yr}{1.0p^9}$	$\frac{Yr}{1.0p^{10}}$
Value of Thinnings expected:									
one expected	0.	0.	0	0	just removed	$\frac{T_4(1.0p^{r-4})}{1.0p^7} = \frac{T_4}{1.0p^1}$	$\frac{T_4(1.0p^{r-4})}{1.0p^8}$ $\frac{T_4}{1.0p^2}$	$\frac{T_4(1.0p^{r-4})}{1.0p^9}$ $\frac{T_4}{1.0p^3}$	$\frac{T_4(1.0p^{r-4})}{1.0p^{10}}$ $\frac{T_4}{1.0p^4}$

Fig. 1. Diagram representing 10 acres (*r* acres), a sample of a forest regulated on a year rotation, indicating the age of the growing stock on each acre; the present value of e final yield (*Yr*) from each, and the present value of the thinnings expected from each re. Since *Yr* on the acre of 7 year old stuff is to be cut in 3 years, its final value, or umpage value is discounted for 3 years. On the several acres 4, or more years old, no rther thinning is to be expected, but the stand 3 years old will furnish a thinning next ar, and hence the value of this is discounted for one year, etc.

Since the general formula for expectation value of growing stock reads:

$$^{m}Ge = \frac{Yr + Ta(1.0p^{r-a}) - (Sc + E)\ (1.0p^{r-m} - 1)}{1.0p^{r-m}}$$

the value of the several acres beginning with the oldest stand may be written:

$$^{9}Ge = \frac{Yr + 0 - (Sc + E)\ (1.0p^{r-9} - 1)}{1.0p^{r-9}}$$

NOTE:—The thinning from this acre has been taken and no further thinning is expected.

$$^{8}Ge = \frac{Yr + 0 - (Sc + E)\ (1.0p^{r-8} - 1)}{1.0p^{r-8}} \text{ etc.}$$

$$^{8}Ge = \frac{Yr + T_4(1.0p^{r-4}) - (Sc + E)\ (1.0p^{r-8} - 1)}{1.0p^{r-8}}$$

$$\text{and } {}^{0}Ge = \frac{Yr + T_4(1.0p^{r-4}) - (Sc + E)\ (1.0p^{r-0} - 1)}{1.0p^{r-0}}*$$

To get a common denominator for Σ Ta, treat in this manner:

$$\frac{Ta(1.0p^a - 1)}{1.0p^a(1.0p - 1} \times \frac{1.0p^{r-a}}{1.0p^{r-a}} = \frac{Ta(1.0p^{r-a})\ (1.0p^a - 1)}{1.0p^r(1.0p - 1}$$

adding we have:

$$^{0-(r-1)}Ge = \frac{(Yr + E + Sc)\ (1.0p^r - 1) + Ta(1.0p^{r-a})\ (1.0p^a - 1)}{(1.0p^r)\ (1.0p - 1)} - r(Sc + E)$$

as the expectation value of the growing stock on the ten-acre, or **r** acre sample. Dividing this by **r** the average value of the growing stock per acre is obtained and the formula reads:

$$^{0-(r-1)}Ge \text{ average per acre} =$$

$$\frac{(Yr + E + Sc)\ (1.0p^r - 1) + Ta(1.0p^{r-a})\ (1.0p^a - 1)}{r(1.0p^r)\ (1.0p - 1)} - (Sc + E)$$

3. To illustrate: Assume an area 16,000; clear cut, and plant, **p**, 2%.

Rotation, 80 years.

Premises per acre: final yield or **Yr**, stumpage value per acre, $400.

Thinning at 20 years, no income.

Thinning at 40 years, $10 stumpage.

Thinning at 60 years, $20 stumpage.

Current expenses $1.50, hence $\mathbf{E} = \frac{1.50}{.02}$, or $75.

Cost value of land, **Sc**, $10.

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* For the class it is helpful to write these out as follows:

$$^{m}Ge = \frac{Yr}{1.0p^{r-m}} + \frac{Ta(1.0p^{r-a})}{1.0p^{r-m}} - \frac{(Sc + E)\,1.0p^{r-m}}{1.0p^{r-m}} + \frac{(Sc + E)}{1.0p^{r-m}}$$

this makes addition of ΣYr, ΣTa, etc., much easier and clearer.

The average value per acre of the growing stock on this forest is:

$$\frac{(400+75+10)\ (1.02^{80}-1)+10(1.02^{40})\ (1.02^{40}-1)+20(1.02^{20})\ (1.02^{60}-1)}{80(1.02^{80})\ (1.02-1)}$$

$$-(75+10)$$

$= 253 - 85 = \$168$ per acre, or about \$2,688,000 for the property.

This case closely resembles a fair case in spruce in Germany, site II, except that the cost value of the land is taken rather low. The calculation indicates that the growth produced by this growing stock will pay two per cent on the capital invested in the property and composed of \$160,000 for land and \$2,688,000 in growing stock.

4. Simplifying the above formula by substituting **Se**, or its value for **Sc**, results as follows and gives a very useful form:

$$\text{since}\,Se = \frac{Yr + Ta(1.0p^{r-a}) - c(1.0p^{r}) - E(1.0p^{r}-1)}{(1.0p^{r}-1)}$$

$Se\,(1.0p^{r}-1)$ may replace $Sc\,(1.0p^{r}-1)$ in the above formula as follows:

$$^{0-(r-1)}Ge \text{ per acre} =$$

$$\frac{Yr(1.0p^{r}-1)+E(1.0p^{r}-1)+Yr+Ta(1.0p^{r-a})-c(1.0p^{r})-E(1.0p^{r}-1)+}{r(1.0p^{r})\ (1.0p-1)}$$

$$\frac{Ta(1.0p^{r-a})\ (1.0p^{a}-1)}{r(1.0p^{r})\ (1.0p-1)}-(E+Se)$$

which simplifies into:

$$\frac{Yr+Ta-c}{r(1.0p-1)}-(E+Se)=\frac{Yr+Ta-c-re}{r(1.0p-1)}-Se$$

In this last form the formula clearly shows the work, expense and income of the **r** acre sample, of the regulated forest. The yearly income is **Yr** and **Ta**; the yearly expenses are **c** and **re**, the difference is the net income from the r acres. This net income divided by **r** is the net income per acre, and this capitalized gives the income value of the forest, i. e., land and timber together, so that the value of the growing stock is this income value of property minus the value of the land.

Using the premises set before and putting $c = \$10$ and replacing Sc with Se at \$28 we have:

$$\text{Value of growing stock per acre}\ \frac{400+10+20-10-120}{80(.02)}-28=\$159.$$

In this particular case the result, \$159, compares closely enough with \$168 as obtained by the ordinary formula. In any case it is necessary to keep in mind that this simple formula depends on substituting **Se** for other values of soil, **Sc** or **Ss.**

c. **Cost value of the growing stock of a regulated forest.**

In this case the value of the growing stock of the ten-acre sample is obtained by adding the values of the ten acres, writing the value of each by the general formula as developed before:

$$^{m}Gc = c(1.0p^{m}) + (E + Sc)(1.0p^{m} - 1) - Ta(1.0p^{m-a})$$

where again **m** is the age of any one stand.

1. This leads to the formula:

$$^{0-(r-1)}Gc \text{ per acre} = \frac{(c + E + Sc)(1.0p^{r} - 1) - Ta(1.0p^{r-a} - 1)}{r(1.0p - 1)} - (Sc + E)$$

2. To illustrate: forest of 16,000 acres, clear cut, and plant; rotation, 80 years; p, 2%; premises per acre:

Cost value of land, $10.
Cost of planting, $10.
Current expense, $1.50, hence E, $75.
Thinning at 20 years, no income.
Thinning at 40 years, $10.
Thinning at 60 years, $20.

Then the average cost value of growing stock per acre:

$$\frac{(10 + 75 + 10)(1.02^{80} - 1) - 10(1.02^{40} - 1) - 20(1.02^{20} - 1)}{80(1.02 - 1)} - (75 + 10) = \$131$$

and for 16,000 acres about $2,096,000.

3. Simplification of this formula by substituting $Se(1.0p^{r} - 1)$ for $Sc(1.0p^{r} - 1)$ results in the same form found for $^{0-(r-1)}$**Ge** so that:

$$^{0-(r-1)}Gc = {}^{0-(r-1)}Ge$$

when **Se** is employed as the value of the land in place of **Sc,** or **Ss.**

H. VALUE OF THE GROWING STOCK IN THE ALL-AGED OR MANY-AGED FOREST.

Two cases arise: the regulated and the irregular forest.

a. The **stand is truly all-aged and regular** as in the case of a well managed selection forest. Here old and young are in proper proportion, all age classes are represented and occupy equal areas. The difference between the growing stock of this forest and that of a forest of even aged stands is merely in the distribution of old and young on the ground. Theoretically at least, the following holds: if the rotation is 120 years, the area 3,600 acres, the period of return 20 years, so that each year 180 acres are cut over, each one of the 180 acre lots contains six age classes, $\frac{\text{rotation}}{\text{return}}$, and on the 3,600 acres all age classes, from 1—120 year old stuff, are represented, provided reproduction is secured promptly, as is here assumed. On each 180-acre lot the trees of the six age classes would occupy the same amount of ground, 30 acres, not in separated areas, but mixed, so that every acre would have old, young and middle aged stuff, i. e., trees of all six age classes. If this forest were cut over every ten years, each lot would have twelve age classes, and if cut over every year every lot should contain every age class from one to one hundred and twenty years old.

The work, expenses and income from this regulated forest of many-aged stands would be similar to that on the regulated forest of even aged stands. Every year there is:

a. A cut of 180 acres, (but not clear cut), which takes:

1. The mature, 120-year-old stuff, theoretically occupying 30 acres, but scattered all over the 180 acres.

2. Thinnings, and stuff cut to improve the woods on the 180 acres, really from 150 acres.

b. The entire 3,600 acres are protected and cared for in improvements, etc., and have taxes paid on them.

c. The growing stock must pay rent on the 3,600 acres of land.

To illustrate the computation: The yearly income is the stuff cut from the 180 acres, or 30 Yr + 150 acres thinnings, say, $10,000; the expenses $5,500 per year; net income $4,500 per year; capitalizing this at 3% gives the forest an income value or expectation value of $\frac{4500}{.03}$ or $150,000 for land and timber together.

To find the value of the growing stock on this forest the value of the land is deducted. This value of the land may be cost value or sale value or expectation value, according to the object of the calculation or viewpoint of the person making this valuation. If the land cost $10 per acre and this value appears satisfactory to use, the growing stock on this forest is worth $114,000, or about $31.60 per acre, on the basis of the yearly income it can produce and is expected to produce.

b. The **irregular forest of many-aged stands** requires a valuation from stand to stand and no general calculation can apply. Such a forest can not possibly yield a uniform income, steadily until it is changed by systematic regulation. In our wild woods the following three forms or conditions are common:

1. The stand is generally mature, the merchantable timber makes up seventy-five per cent and more of all growing stock. Such stands have little value beyond that of the merchantable timber, for usually the rest is stunted and is injured during the removal of the old stuff. This, then, is a case of stumpage values.

2. The stand is largely middle age stuff, which may be expected to be merchantable in a certain number of years, as seen by growth studies. Here it is simply a case of ^{m}**Ge,** as developed before.

3. The stand is mostly young stuff, such as reproduction on burns, windfall, etc. In these cases the cost value, ^{m}**Gc,** should prevail. Where such reproduction comes in with open, mature, or over mature stands, the area of the young stand is estimated; its cost value and the stumpage value of the old stuff are determined separately.

IV. RELATION OF CAPITAL AND INCOME IN FORESTRY

A. NATURE OF INCOME AND PROFIT IN FORESTRY.

1. General notion of profit.

a. If an article for a store is bought at one dollar and sold at a dollar and a quarter there is twenty-five cents profit, or the profit is 25%.

b. If a house, bought at $1,000, is sold within two months at $1,200, the profit is $200, or 20%.

c. An office building in a small town cost $150,000, there is little demand for this and the income after paying taxes, running expenses and upkeep is about $3,000 per year. There is this net income but no one considers the building a profitable one. To the contrary it is well known as a "losing proposition." Savings banks and the best commercial papers would pay twice this income. In figuring the loss it is asumed that the capital of $150,000 should have been invested at 4%, the income therfore, $6,000, that the yearly loss is $3,000, or 2% on the money; and if the building had to be sold this would mean a total loss of $75,000. We have income here but certainly no profit.

d. A house is bought at $1,000 and current rate of interest is 5%, the true net income from this house is $80 per year. Evidently the new owner makes 8% on his money and the house nets him 3% more than the current interest rate. In this case there is a profit, even though the owner does not sell, and this profit is 3% and not 8%. In this way the net income from the house divides itself into two parts, a reasonable regular income on the capital invested, i. e., the 5% on the $1,000, and a 3% profit.

Since on this point authors vary, some putting the net income as synonymous or at least interchangeable with profit, the position taken in these discussions is here stated.

e. A farm is bought at $5,000, the new owner borrows $2,000 additional at 5% as a working capital to buy teams, tools, etc. He produces a net income of $700 or 10% on all the money put into the enterprise. He pays his creditor $100 yearly interest and retains $600. This $600 makes 12% on the farmer's own money so that he has his regular 5% income and 7% profit in this business. But even here there is a doubt as to whether part of this seven per cent profit is not a personal matter.

These few cases suffice to show that the matter of profit is not as simple as it seems; a great variety of combinations necessarily arise, there may be profit even though there is no sale of property, and also there are various viewpoints possible.

f. A man buys a house for $5,000, he borrows the entire sum, giving the house as security. In one month he sells the house at $7,500, and returns teh cash with a few dollars interest. The profit is practically $2,500 and is not made on any real investment on his part. It is usually stated that his credit is used, etc., but in many cases this is not true at all; the capitalist does not trust the man but considers the house good for the money.

It is neither ordinary profit nor wages but a special form of profit paying him for his enterprise, shrewdness or more often his good luck, exactly as in gambling.

2. Profit from income producing properties.

a. In case under 1, c, a house is bought at $1,000, the interest rate is 5%, the net income $80 or 8% on the money invested, the yearly profit is $30 or 3%. The cost value of the house, **Hc** is $1,000, income value **He** is $\frac{80}{.05}$ or $1,600, the total profit is $1,600 — $1,000 or $600 and may be stated as **He** — **Hc**.

b. A forest property is bought at $250,000, it produces $10,000 per year or 4% on the investment, net income. Whether this would be regarded a profitable investment or not depends on the purchaser, chances for investment, the current rate of interest, etc. In central Europe it would be considered an excellent property, in the United States at present, it would be considered poor, in the west, barely paying in the middle states, fair in the east. Any one who could easily get five per cent on his money would not buy this property, and would certainly not speak of a profit in this case.

Assume now that the property yields $20,000 per year or 8%, then the case is similar to that of the house, i. e.,

Cost value of forest, $250,000.

Income value $\frac{20,000}{.05}$ or 400,000.

The yearly profit is \$20,000 — \$12,500 or \$7,500, or expressed in per cent it is 8% — 5% or 3%; and expressed as a total it is \$400,000 — \$250,000 or \$150,000 which may be stated as:

profit = **Fe** — **Fc.**

That the same reasoning applies to land which is to be used for forest is evident and leads to the statement: Profit = **Se** — **Sc.**

3. **Profit from a forest not producing regular yearly income.** Usual case: A tract of 40 acres of timber is bought at \$5,000, the owner is not ready to cut the timber, it is not to be sold, or else no offer is made; in many cases of this kind in the more remote districts, it might be impossible at times to get a buyer even at half the real value of the timber; there is neither loss nor profit here actually established until the timber is cut or some sale is made. As a forecast, however, it may be possible and worth while to work out an expectation value in the manner outlined before, and then the probable or expected profit is equal to the expectation value of this 40 acres minus the cost value, in this case \$5,000.

From the foregoing illustration, it is evident that the term profit is clear and that it is consistently employed in the ordinary store, and in ordinary sale of property. But when employed in connection with the regular income from a house, farm or forest, factory, etc., it is not clear, being confused, commonly, with gross, or net incomes and largely loses its usefulness.

B. INTEREST MADE BY THE FOREST.

In forestry the question frequently comes up—what rate of interest is made by the growth of timber on a given tract. Three cases may be distinguished:

a. The growth of a single stand such as a 40-acre stand of even aged timber.

b. The growth of a regulated forest, or more precisely, of a working section with perfect age class conditions.

c. The growth of timber which may be produced on an area of land which is bare to start with.

1. **The interest made by a single even aged stand of timber.**

Here the usual case considers the growth for only a short period, say 10 years, but frequently, however, it demands the interest made by the growth for an entire rotation.

a. Interest made by the growth of a stand during a short period or part of a rotation: If the cost value of the land is $10 per acre and the cost value of the stand, now 30 years old, is $80, then the amount invested in this enterprise is $90 per acre. Evidently the only increase in value of this piece of forest is due to the growth of the timber. If the growth in volume, quality and market price amounts to $20 in 5 years then this $20 represents the interest for 5 years on the $90 invested, and approximately the rate of interest is: $\frac{20 \times 100}{5 \times 90}$ or 4.4%. While this calculation is a fair approximation it requires modification for larger enterprises, and better analysis. The following illustrates the usual modification.

Area, 40 acres; stand, even aged 50 years old at beginning of period, and 55 at end; **p**, 2%; land, $10 per acre; yearly expenses, **e**, $1 per acre; therfore $\frac{e}{.02}$ or **E** is $50; the value of the stand at 50 years is $250 and at 55 is $300. The reasoning then is: $1.0p^n = \frac{Cn}{Co}$.

The initial capital, **Co**, is made up of stand at 50 years, land, and **E**, the final capital, **Cn**, of stand at 55, plus land and **E**, and we may set:

$$1.0p^5 = \frac{Y^{55} + S + E}{Y^{50} + S + E} = \frac{\$300 + 10 + 50}{\$250 + 10 + 50} = 1.16 \text{ and hence } \mathbf{p} = 3\%.$$

This per cent made by the current growth of the stand may be termed "forest per cent," or P_f, and is the "Weiser" per cent of the German authors.

The formula as written by Endres is:

$$W = 100 \left(\sqrt[n]{\frac{A_{x+n} + B + V}{A_x + B + V}} - 1 \right)$$

or in our terms,

$$p_f = 100 \left(\sqrt[n]{\frac{Y_{x+n} + S + E}{Y_x + S + E}} - 1 \right)$$

which readily changes into the above simple form.

This form of analysis was used by Pressler and Heyer over fifty years ago. It is useful in determinig whether a given stand should be cut or left standing, the usual assumption is that if the per cent drops below 3% or whatever per cent may reasonably be demanded by the owner, the stand is no longer worth holding.

While the calculation is quite simple the actual determination in the woods is not easy and is never more than an estimate reinforced by measuring the outer rings of a few trees.

From these measurements the **Pv** or growth in volume in per cent is determined by Schneider's formula; the per cent growth in quality is estimated on a basis of diameter at beginning and end of the period, and the per cent increase in market price is a figure worked out for several years in any given district. Having this, the growth of the stand is expressed in per cent for the three lines and Pressler conceived the idea of simply using these as follows: assuming the actual growth in value per year is: $(Pv + Pq + Ps)\,\frac{Ym}{100}$ and that this growth is made on a capital Ym + Se.

$$\text{Then } p_f = (p_v + p_q + p_s)\,\frac{Y_m\,(100)}{100\,(Y_m + Se)} = (p_v + p_q + p_s)\,\frac{Y_m}{Y_m + Se}$$

To illustrate: If the stand is worth \$250 and land \$10 per acre and

$$p_v = 1.3\%$$
$$p_q = 1$$
$$p_s = 1$$

$$\text{then } p_f = (1.3 + 1 + 1)\,\frac{250}{250 + 10} \text{ or } 3.1\%.$$

Since this method or inquiry is usually applied only to older stands where the value of the timber is very large as compared with the value of the land, the value of $\frac{Y_m}{Y_m + Se}$ may be put = 1 and for ordinary purposes, therfore, it is quite sufficient to state: $p_f = p_v + p_q + p_s$; in the above case $p_f = 1.3 + 1 + 1 = 3.3\%$, a very convenient form.

In the United States where at present the value of timber is usually by M. ft. b.m. on rather uniform price per M. feet, the case of working out the data for $\mathbf{p_f}$ may take the following form:

Assume stand of pine, pure, even aged, 80 years old; volume table exists for species and district; from borings or felling the diameter growth of the timber is ascertained, the stand is calipered and volume known; stumpage \$10 per M. feet; assume that the stand now, 80 years old, contains 250 M. feet. Value of this $250 \times 10 =$ \$2,500.

From growth study it appears that the stand 10 years ago contained 200 M. feet, valued at \$2,000. Then the growth in value, beside increase in market price, is \$500 in 10 years. Expressed as per cent:

$$1.0p_x^{10} = \frac{2500}{2000} = 1.25 \qquad p_x = 2.3\%.$$

This $\mathbf{p}_x$ or 2.3% includes both the growth in volume and the growth in quality as it finds expression in our present day practice of scaling timber so that we may state: $2.3\% = p_x = p_v + p_q$, and this requires merely the addition of $\mathbf{p}_s$ or the growth in market price to state the total growth in value.

b. Rate of interest made by the growth of an even aged stand during the entire rotation, i. e., from the time of planting to time of cutting.

Here again it is convenient to apply the fundamental form of reasoning that

$$1.0p_t^r = \frac{Cr}{Co} = \frac{\text{Capital or values at end of rotation}}{\text{Capital or values at beginning}}$$

The initial capital **Co** is made up of: land, **Sc**; cost of planting, **c**, or the value of young growth; capital **E**, to take care of the stand, pay taxes, protection, etc.

While this capital, **E**, is not really on hand, the results are the same, the owner must provide from some source, an amount equal to the interest on **E** at **p** per cent. At the end, or time of cutting the final capital, **Cr**, into which **Co** has grown, is made up of: land, **Sc**; capital for expenses, **E**, stand of ripe timber, **Yr**, and the statement becomes:

$$1.0p_t^r = \frac{\text{stand} + \text{land} + \text{expense capital}}{\text{land} + \text{expense capital} + \text{cost of planting}} = \frac{Yr + Sc + E}{c + Sc + E}$$

Concrete case: Land \$15 per acre, $\mathbf{p} = 2\%$, $\mathbf{e} = \$1$, hence **E** $\frac{1.00}{.02} = \$50$; **Yr**, \$400; **c**, \$12; **r**, 80 years; then:

$$1.0p_t^{80} = \frac{400 + 15 + 50}{12 + 15 + 50} = \frac{465}{77} = 6.04$$

and from tables: $p_t = 2.3\%$.

The above formula may also be written:

$$p = 100 \left(\sqrt[r]{\frac{Y + S + E}{S + C + E}} - 1 \right)$$

which is nothing more than the old form of the Weiser per cent, where the period includes the entire rotation, **c** the cost of planting represents the value of the stand just planted or at the beginning, and **Yr** is the value of the stand at the end of the period.

Whether the value of the land is taken at cost, sale or expectation value is optional. Certainly the introduction of **Se** means the use of a maximum which is not to be recommended. Since the values of land are apt to change very decidedly during a period of eighty years it is not only permissible but even proper to set a higher value for **S** in the numerator, i. e., in the final capital, than in the denominator or initial capital.

In some recent publications the formula is written in a modified form as:

$$1.0p^r = \frac{Yr + S - E_1}{S + C} \text{ and } E_1 \text{ is equivalent to } e\frac{1.0p^r - 1}{1.0p - 1}$$

or merely the end value of all expenses during this one rotation compounded at ordinary interest rate, the assumption being that money is borrowed (actually or not) at ordinary rates for this purpose of paying taxes, protection, etc. It is evident that this, as well as many other forms, is possible and that some uniformity in the use of the letters and writing of these formulæ will greatly help all concerned.

Where thinnings are made, and this must be assumed as the normal condition in forestry, the above formula is incomplete and must be modified to take care of the receipts from thinnings. This may be done as follows:

$$1.0pr^r = \frac{Yr + \Sigma T_q(1.0p^{r-q}) + Sc + E}{c + Sc + E}$$

2. Rate of interest made by a regulated forest, or one producing a regular yearly income.

Since the yearly net income is in the nature of interest on the capital invested in the forest and since this income is known, it is necessary only to determine the value of the property, stand, and land. But as this may be cost, sale, or expectation value of either land or stand, a variety of conditions are possible.

Where the forest has been purchased in recent time, where the income so far received has paid all expenses including an acceptable rate of interest, and where the management has prevented any deterioration of the forest, the cost price is evidently the capital invested, and the rate of interest made by this forest is:

$$pr = \frac{\text{yearly net income}}{\text{cost value of forest}} \times 100$$

If the man bought the forest at $250,000 and receives $10,000 per year net income, he evidently makes 4%.

If this same man paid $250,000 ten years ago and has "put back" $50,000 to develop a better road system then evidently the forest costs him more nearly $300,000 and the $10,000 income is about 3% on the cost value of the property. Similarly, if instead of getting $10,000 net income he has saved with a view to improving the growing stock on the property and has secured only $5,000 per year net income, the $5,000 reduction of cut is an investment and increases the cost value.

Again, the forest may be part of an old estate or it may have been bought twenty years ago at a "bargain" so that the present cost value is not clear and certainly not equal to the present sale value of the property. In this case the income is not referred to the cost value but to the sale value.

With properties which are not common objects of sale like state forests, national forests, etc., the value of the property itself is based on the very income made from it together with an assumed interest rate. The forest producing $10,000 per year net income would be valued by capitalizing the $10,000 at, say, 3%, and the capital value would be $\frac{10{,}000}{.03}$ or $333,000.

If this capitalization is now accepted it may serve as a measure to find the forest per cent for future years. For the present, however, this value of the forest must be regarded as arbitrary, for it would have been just as fair to use two or four as the per cent of capitalization, and for the present, therefore, the per cent or interest rate which this forest makes can not be determined for lack of acceptable basis, the value of the property on which the $10,000 were made.

In determining the rate of interest made by any property producing regular yearly income there enters the notion or custom of "watering stock." In the above case, for instance, the owner would at once set a much higher sale value in place of the 250,000 cost value and in this way bring down not the income but the apparent rate of interest. Practically all state forests of Germany never cost the states any money, they were simply taken charge of by the governments, paid their way and all improvements to the present day. And yet their value is set so high that the splendid net income barely makes 3%, in most cases not that. The strange part is that this high value is not a fictitious one but a very real thing and that these properties could readily be sold at these high prices.

We have the same situation in the case of farm properties in Europe as well as in our country.

3. Rate of interest made in a forestry enterprise starting with bare land.

Under this head come the ordinary plantations, where no forest exists, where the bare land is considered as the real capital, where the money for planting, etc., is or may be borrowed at the interest rate used in the calculation. In this case then:

$$p_x = \frac{\text{yearly income} \times 100}{\text{capital}} = \frac{\text{yearly income} \times 100}{\text{cost value of land}} = \frac{a \times 100}{Sc}$$

Since the income here is not a yearly one but a periodic one, every **r** years, the value of the equivalent yearly income may be found by the formula:

$$a = \frac{A\ .0p}{1.0p^r - 1}$$

The periodic income is:

$$A = Yr + Ta(1.0p^{r-a}) - C(1.0p^r) - E(1.0p^r - 1)$$

and the above calculation for yearly income is then:

$$a = \frac{(Yr + Ta(1.0p^{r-a}) - C(1.0p^r) - E(1.0p^r - 1)\ .0p}{1.0p^r - 1} = Se(.0p), \text{ and:}$$

$$p_x = \frac{a \times 100}{Sc} = \frac{Se\ .0p \times 100}{Sc} = \frac{Se}{Sc})\ .p$$

To illustrate:

If the land costs $10 per acre, the accepted rate, **p**, is 3%, and the forest enterprise is such that it pays 3% on $30, i. e., makes the land worth $30, then evidently the man makes on his cost value of land on the $10, not 3% but 9%, or, as per formula:

$$p_x = \frac{30}{10} \times 3 \text{ or } 9.$$

This is an interesting statement and has value especially in the United States where cost values of forest lands are still very low.

To illustrate: Cut over lands in the southern states may be bought at $2 per acre. The **Se** even on a very conservative basis is $20 per acre. To a company financially able to hold and develop such lands as forest properties the enterprise would make:

$$p_x = \frac{20}{2} 3 = 30\ \%\ \text{on the cost value of the land.}$$

It should be noted that **p** in this case should be the **p** employed in calculation of **Se**, and also that the p_x, the 30% in the above case, is the interest made only on the cost value of the land, on **Sc**, and that the other parts of the investment, costs of planting, taxes, protection, etc., produce simply **p** the accepted rate of interest, usually 3% in these studies.

C. COMPARISON OF PROPERTIES WITH YEARLY AND WITH INTERMITTENT INCOME.

The great advantage of having any kind of business in a condition of producing approximately equal and yearly incomes is quite evident. Ordinarily an enterprise is not considered a business at all until it does produce a yearly income at least balancing expenses. In most cases it is difficult to interest and enlist financial support or purchasers unless there is an assured yearly return at least equal to a conservative rate of interest, except in cases where a body of natural wealth is in view, as in standing timber, coal veins, etc., or else in cases of great possibilities or probabilities where the element of lottery or gamble enters. Financial concerns under public control are generally forbidden to lend on securities of this kind. The regular yearly income has been, therefore, one of the aims in forest business and forms one of the principal tasks of forest regulation. Merely to emphasize and illustrate this fact the following concrete case is here considered.

Assume two properties of 1,000 acres each; rotation 80 years; **p**, 3%; **Yr**, $400; **Sc**, $10; average growth of ripe stuff per acre, one cord per year, value per cord, $4; cost of planting, **c**, $10; current expenses, **e**, $1 per acre. Thinnings left out for sake of simplicity.

Comparison of yearly and intermittent working.

	Property with yearly cut and yearly income.	Property with a cut and income every 80 years.
1. Growing stock at beginning.	40,000 cords.	None.
2. Growth of ripe stuff during the 80 years.	80,000 cords.	80,000 cords.
3. Growing stock at end of 80 years.	40,000 cords.	80,000 cords.
4. Value of property at beginning.	$10,000 for land and about $60,000 for growing stock.	$10,000 for land nothing for growing stock; might enter plantation.
5. Income during the 80 years.	$320,000 plus interest.	None, thinnings not considered.
6. Income at 80, i. e., in last year.	$4,000.	$320,000.
7. Condition at end of first twenty years:	Has Earned: Per year: Income: 4000. Expenses: Rent 335. Tax and care 1000. Planting 125. Total expense 1460. Net income per year, $2540. Net income for 20 years $2540 \frac{(1.03^{20}-1)}{1.01-1}$ or about $67,700.	In Debt: Expenses per year: Rent 335. Tax and care 1000. Total 1335. For 20 years' expenses: $1335 \frac{(1.03^{20}-1)}{1.03-1}$ or $35,600. For planting 10,000. Plus interest or $10,000 (1.03^{20}) = 18,000$. Total expenses for 20 years: $53,600.

This simple case of a comparatively small property brings out the great disadvantages of the intermittent working of a forest property. The man who owns 1,000 acres of land and sees before him the piling up of over $50,000 expenses in only one-fourth of the rotation is not very much encouraged to go into the forest business. It also illustrates why in common sense and fairness the owner of such a property is not able to pay taxes like the owner of a regulated forest who has a net income every year. It also illustrates the fairness, in fact the necessity, of deferring the taxes to the time when the timber is cut and an income secured, such taxes to stand as a lien against the property.*

* Read: "The Need of Working Plans on National Forests and the Policies Which Should be Embodied in Them," by Professor Burt P. Kirkland, in Proceedings of the Society of American Foresters, Oct. 1915. This article shows clearly the necessity of working for a sustained yearly cut in the forests, and the principles laid down in this article apply in all real forestry work the world over.

V. ROTATION

In planning a forest business it becomes necessary to decide upon a rotation. To do this is one of the most important tasks of forest regulation and requires a most careful consideration of all conditions and factors concerned. Among these the most important are the market, site, species and treatment.

The market sets its requirements as to species, size and quality which must be raised, and it is useless to discuss rotations which fail to raise timber of the required size. Site, species, and treatment, particularly practice of planting and thinning decide the length of time in which the desired size and quality of timber can be raised. Being largely a weighing of conditions and factors, the setting of the rotation is primarily a problem in statics and perhaps the most important task of statics in forestry.

In so far as the conditions and results require expression in dollars and cents, the basis for the study of a proper rotation is supplied by forest valuation. In any ordinary forest business the aim is to keep the forest and land in best possible condition and at the same time make the largest income.

Assuming the silvicultural conditions cared for, the best rotation is the one furnishing the best income for the longest time.

There are two distinct ways of judging this income:

1. We may demand the largest net income per acre of established forest;

2. We may wish to secure the largest per cent on the money invested in the forest or the largest net rental on the soil, the maximum **Se.**

These two viewpoints have for many years divided the foresters of central Europe into two schools, the division still exists and promises to continue for a long time.

a. Rotation for largest net income per acre of forest.

The assumption here is that we start with a forest and not merely bare land; that this forest is more or less regulated and able to produce a yearly income. All of these conditions actually apply to the majority of forest properties of central Europe.

The following case illustrates the calculation:

Area of forest, 16,000 acres; rotation, 80 years; age classes fairly regular. From this forest we can select an 80 acres sample with stands of all ages from 1-80 years. From this sample we would get each year:

One acre of ripe stuff to cut or **Yr,**

Several acres of thinnings, or **ΣTq,**

And we should have expenses of one acre to plant, or **c,** and 80 acres to pay taxes, protection, etc., or **re,** which may be put into the following form:

average net income per acre =

$$\frac{Yr + \Sigma T_q - c - re}{r}$$

or according to the following table for spruce site II, for a rotation of

$$60 \text{ years} \quad \frac{583 + 71 - 12 - 54}{60} = \$\,9.80$$

$$80 \text{ years} \quad \frac{980 + 232 - 12 - 72}{80} = \$14.10$$

Spruce site II, Germany, Endres, after Schwappach, gross income, expenses, net income and **Se** for different rotations from 30 to 120 years for use in formula. Average net income per acre

$$\frac{Yr + \Sigma T_q - C - re}{r}$$

Rotation Years.	Gross Income.			Expenses.			Net Income.		
	Yr or final cut per acre r years old.	Σ Tq sum of thin-nings from r acres.	Total income from r acres of normal forest.	c plant-ing of one acre.	re current expense tax, pro-tection etc., for r acres e $0.90	Total expense for r acres of normal forest.	Total from r acres of normal forest.	Average per acre.	Se on basis of 3% rental value one acre bare land
	$	$	$	$	$	$	$	$	$
30	87	..	87	12	27	39	48	1.60	10
40	203	7	210	12	36	48	163	4.00	47
50	371	31	402	12	45	57	354	6.90	77
60	583	71	654	12	54	66	588	9.80	98
70	786	137	923	12	63	75	848	12.10	105
80	980	232	1212	12	72	84	1129	14.10	105
90	1123	351	1474	12	81	93	1382	15.30	99
100	1222	488	1710	12	90	102	1609	16.00	91
110	1281	683	1919	12	99	111	1808	16.40	83
120	1296	791	2087	12	108	120	1967	16.30	75

From the foregoing table it appears that the average net income with spruce site II and present prices, is but $1.60 per acre for a rotation of thirty years, rises rapidly and steadily to $15.30 for a ninety year rotation and passes a maximum of $16.40 at a rotation of about 110 years.

b. Rotation for largest net rental on land, maximum Se, and with this of highest per cent interest on the investment. See also chapter on **Se.** The assumption is that the bare land is the real capital and that by using it to raise timber this land is made to pay a rental and the object is to decide upon a rotation which will secure the highest net rental per acre, i. e., the highest **Se.**

That a forest producing the largest **Se** also pays the largest rate of interest or **p** is evident from a previous study into **p** where it is shown that:

$$p_x = \frac{Se}{Sc}\, p, \text{ the rate of } \mathbf{p} \text{ on } \mathbf{Sc}.$$

Since **Sc** or the cost value of the land is fixed and **p** is the accepted rate of interest which is also fixed, it is evident that $\mathbf{p}_x$ increases directly with **Se.** This calculation applies to every stand or every acre in the forest and therfore to the entire property.

Generally longer rotation means larger and better timber, larger growing stock, larger capital, larger net income per acre of property. As seen from the table for spruce, site II, and from discussion of **Se** it means a larger **Se** and better rate of interest. But this reaches a maximum after which the **Se** and with it the rate of interest made by the forest decreases. Formerly unduly long rotations and large timber were the rule and in most forest districts there was no regard to the relation of this rotation with the capital invested and the income secured. It was against this kind of luxury in forest management that Pressler and Heyer raised their voices and used the mathematics of forest valuation and statics. And it was the fundamental formula for **Se** first developed by Faustman which served to make clear the mistakes of very long rotations. At first the current rates of interest, four and five per cent, were employed. But as seen from discussion of **Se** these higher rates not only reduce **Se** but bring the maximum **Se** very early and so suggest or demand too short rotations. That there must be a limit to shortening of rotation and that good marketable timber must be raised regardless of any formulæ was evident to every good practitioner. Unnecessary stress on mathematical formulæ on the one side and common sense with an aversion for formulæ, especially for bringing in compound interest discussions into the business of raising timber, on the other, have prevented the two schools from coming together.

Of late there is a better understanding, it is evident to all that the forest must produce good marketable stuff, and that if the forest can not make more than 3% in doing this, there is little use of introducing 5% in the formulæ. On the other hand it is clear that the planting of small saplings at $50 per acre and holding old stands which have not earned 1% for a quarter of a century are mistakes to be avoided.

With an interest rate of 2%, the rotation of highest **Se,** the financial rotation coincides closely with the technical rotation or that producing the timber most in demand and now in actual use.

A few figures may serve to illustrate this:

Ordinary rotations for regular timber forest in central Europe run about as follows:

Pine and spruce	80 to 100 years
Balsam and beech	100 to 120 years
Oak	150 to 200 years

For spruce in Germany as seen from the tables under discussion of **Se,** the maximum **Se,** and therfore p_f, for site I-IV, on a basis of 2% comes at 70-80 years, at a 3% basis in 60-70 years.

For pine in Germany on sites I-IV, on a basis of 2% as well as 3% the maximum **Se** comes at 60-70 years.

For balsam in Germany according to Lorey, maximum **Se** for sites II-III on a basis of 2½% comes at 100-110 years, which according to Lorey is also the preferable age from the standpoint of silviculture.

For oak in Germany, according to Schwappach, a rate of 2% puts the maximum **Se** at 120-130 years, but here the modern technical requirements demand a rotation of 160 or over.

For beech in Germany the maximum **Se** on 2% basis for sites I-IV comes at about 80 years, a rotation which does not produce satisfactory saw timber and similar goods.

Poor sites such as poor sands in the pinery of north Germany are unable with any rotation to make a 3% income under the ordinary conditions of German management, and barely work out 2% for land and costs. That the rotation can be shortened by planting good transplants and by an energetic practice of thinning is evident. In addition, the modern means of manufacture have made it possible to use smaller sizes and so work toward shorter rotations in the forest.

VI. VALUE OF STUMPAGE

Under present conditions in the United States timber is generally bought and sold as standing timber or on the stump, and usually goes with the land, so that in most cases it is really a sale of the forest, i. e., land together with the entire growing stock and not merely a sale of the timber.

In these ordinary sales of timber or forest, the price paid is determined merely by the value of the merchantable timber and the rest of the growing stock as well as the land is left out of consideration, thrown into the bargain. Of late, especially on the national forests larger sales have been made where the value of the merchantable timber alone was concerned.

In some cases, timber, whether forest or timber alone, is bought for immediate use or cutting, in other cases the timber is bought years ahead with a view of supplying a regular lumber or timber business, or else just for speculation, when it has to be held for years before any return is secured from the money invested. In this way a great variety of cases arise, each having its own peculiar conditions as to time, location, market, cost of exploitation, etc.

But all of these cases have a few fundamental conditions in common and these only can here be considered.

Timber, here only merchantable timber, like other property has three forms of value: cost, sale, and expectation value.

A. Cost value of stumpage.

1. Where timber is raised the cost value of the merchantable timber can be determined in the ways before described. But generally the time between planting and harvest is so long, that this cost value is rarely considered, and mature timber even in central Europe is estimated by sale, rather than cost value. But it is well to keep in mind that with present prices of timber from twelve to twenty dollars per M. feet b.m. and cut into logs for spruce and pine, the per cent made in the forest business is generally three per cent or less, so that present prices may be regarded as cost prices with three per cent and no profit, in spite of the fact that forestry pays a larger net rent per acre than farming.

2. Where timber is bought for immediate use, as is much of the timber purchased on national forests, etc., the price paid is the cost value.

But where timber was bought years ago, in advance of cutting, the present cost value involves: price paid; interest, compound, usually 5-6% in the United States; taxes with compound interest; care of property, of late some money for fire protection, with interest; losses by fire, theft, insects, windfall, etc.

Part of these losses is made up by growth, but in most cases it is not safe to figure on any material growth. On large areas the growth is usually balanced by decay, on smaller areas, the decay or loss may far outrun growth, as in timber infested by insects, etc. In these cases only a detailed examination determines the true status. Leaving out this very uncertain element of losses by fire, etc., the cost value results in a case approximately like the following:

Area, 2,000 acres; price paid 10 years ago, $40,000;

Present value of this 40,000 $(1.05^{10}) = \$65,160$;

Taxes at 1½% on half cost price, $300 per year;

Care of property by some local agent, lawyer, etc., together with expenses of fire control and occasional visit by cruiser, etc., $200 per year. These two items with interest:

$$(300 + 200)\frac{(1.05^{10} - 1)}{1.05 - 1} = \$6,200,$$

Total cost, $71,360, or nearly double the original cost price.

This explains a common phrase among timber owners in the United States—that the value of a property must double every ten years if there is to be no loss.

B. Sale value of stumpage.

This may be considered under two heads:

The ordinary market price of stumpage as determined by actual sales.

The stumpage value for immediate use as determined by an analysis of the various factors entering into, or affecting this value.

1. **The ordinary market price of stumpage** should approximate: value of lumber at mill—cost of logging and milling, where a proper profit, etc., forms part of costs.

But so far the market price of stumpage has not been determined in this way. Probably more than ninety per cent of all stumpage bought during the past twenty-five years was paid at prices quite independent of the prices of lumber and cost of logging, and determined chiefly by the activity of large buyers. A number of causes contributed to this peculiar condition. Large quantities, thousands of acres of fine stumpage were secured without pay from the United

States government under the homestead act, large areas were alienated at the nominal price of two dollars and a half per acre under the Timber and Stone act and by Commutation of homesteads, large areas were given as land grants to railways, etc. Several of our states, Florida, Oregon, Idaho, New Mexico and others sold lands, given them by the government under various acts, at prices in no way graded by the price of lumber. Even our lake states sold timber at give-away prices. In this way Michigan sold 1½ million acres of land in the ten years ending 1910, at prices usually below two dollars per acre and most of it below a dollar and a quarter, and most of this land was bought for the timber. Large areas of timberlands have been held by small owners, or owners financially unable to exploit and market the material and in many cases, a lack of market, transportation facilities, etc., prevented the owners from doing anything with the timber. Aside from these personal factors there has always been the great balance of demand and supply. Even today with more than fifty years cut in sight, of timber already matured, it is useless to estimate the value on a basis of immediate exploitation and present prices.

To illustrate: assuming prices of lumber, mill run, to stay at about $15 per M. feet, and cost of logging and milling to average $10 per M. feet. Then the true value of stumpage should be $5 per M. feet average.

But this is not true. The first year's cut is worth the $5, but the second, third, etc. year's cut is not worth it. The stumpage cut in the tenth, twentieth or thirtieth year, at five per cent discount is worth only $\frac{5.00}{1.62}$, $\frac{5.00}{2.65}$, $\frac{5.00}{4.32}$, or $3.10, $1.89 and $1.15 per M. feet.

In keeping with these peculiar and unsettled conditions of stumpage prices stumpage of pine in Minnesota brought $12 while in the same year millions of better stumpage was bought in Oregon for less than $1 per M. feet.

The man in Minnesota could use it at once and make money, the man in Oregon bought it to hold and he had to hold, and may lose money at it.

Stumpage prices today vary not only for species but vary with locality, with market, railway facilities, habit of the people of the district, etc. Average figures for stumpage have little value, they vary chiefly between $3 and $7, go as high as $20 and as low as $1, and generally make from 10 to 30% of the value of the lumber at the mill.

In central Europe most of the timber is cut, or cut and skidded to the road and sold not really as stumpage but as logs ready for hauling. Here the prices have gradually settled to a fairly exact condition, the value of the logs depending on mill or market value, and cost of transportation.

The prices vary little from year to year, have risen steadily for many years, about doubled, in Germany in the last sixty years. The timber is bought at auction, the bids usually naming a per cent of the price set by the forest office.

In pine and spruce the timber is classified by the cubic contents of the stem, i. e., the number of cubic feet per stem. The following figures illustrate:

Spruce, logs cut full length, ready to haul:

Stem Class:	Contents:	Price per c. ft.	Price M. ft. bm.
Class I	100 c. ft. or over	14 cts. per c. ft.	$20
Class II	70-100 c. ft.	13 cts. per c. ft.	$19
Class III	35- 70 c. ft.	11 cts. per c. ft.	$17
Class IV	18- 35 c. ft.	10 cts. per c. ft.	$15

Pine, stems cut full length, ready to haul:

Stem Class:	Contents:	Price per c. ft.	Price M. ft. bm.
Class I	70 c. ft. or over	15 cts.	$22
Class II	35-70 c. ft.	13 cts.	$19
Class III	18-35 c. ft.	9 cts	$13
Class IV	below 18 c. ft.	7 cts.	$10

Cordwood: 4 cts. solid, or $3.60 per cord.

White oak, classification by middle diameter of stem:

Stem Class:	Middle Diameter:	Price per c. ft.	Price M. ft. bm.
Class I	24 inches and over	42 cts.	$60
Class II	20-24	35 cts.	$50
Class III	16-20	28 cts.	$40
Class IV	12-16	17 cts.	$20
Class V	under 12	10 cts.	$10

Beech usually sells at 4-7 cents per cubic foot and most of it, even clear logs 12 inches diameter and better go as cordwood. This cordwood is commonly sold from $3-$4 per cord.

Table of size and value of average tree, all for site I:

Age	Pine.		Spruce		Oak		Beech	
	Diam. "	Value	Dbh. "	Value	Dbh. "	Value	Dbh. "	Value
60	9.5	$1.10	9.0	2.35	8	0.85	8	.55
80	12	2.90	12.6	4.50	12	3	10.8	1.30
100	14	4.50	15.8	8.30	16	9.80	13	2.25
120	16	6.80	18.4	11.40	18	22.50	15	3.25
140	17.2	9.60			21.5	28	16.8	4.20
160					23	34		
180					24.5	48		
200					26	52		

The above prices are for straight, well-cleaned, sound timber. They clearly indicate that prices in parts of the United States are rapidly approaching and in many cases have fully reached those paid in the forests of south Germany and other European countries. In fact it is doubtful if the same grade of large stems could be bought for the same prices in Ohio or Indiana and several other states.

2. **Sale value of stumpage,** determined by consideration of the various factors. Here the calculation attempts to show:

price of lumber at mill;

cost of logging and milling;

the assumption being that the stumpage price should be the difference between these two, i. e., if the lumber is worth $15 and it costs $10 to log and mill then the stumpage should be $5.

a. Price of lumber at mill.

Where the lumber is sold "mill run" this value is easily obtained providing the sale of the lumber is bona fide and under normal conditions.

Where lumber is graded at mill and sold by grades to various dealers, attempts are made to get at the true average price by learning the per cent each grade makes of the cut and the price of each grade.

In some cases this is quite simple, in most it is not. Any change in the quality of the timber in the woods affects the per cent of grades. Where several species are logged together such as hemlock, maple and beech, etc., the proportion as well as the quality vary from one tract to another.

To use the general prices of the district and agree on some proportion of kinds and grades is usually as far as the analysis can go.

b. Cost of milling.

This also involves various considerations differing radically for different cases. In one case the logs go to an established mill, the particular body of timber is only one of several or many which are cut at this mill. The large mills along, or at the mouths of the rivers in Michigan and Wisconsin are of this order. In this case the cost of milling is quite simple, there is a regular market value of this milling toll which is fairly well established.

In other cases the mill is established to cut the particular body of timber and no more, it must be torn down and moved away, is largely a loss and with it also a number of other buildings, quarters for men, boarding houses, office, store, shops, etc. In such a case the cost of milling involves:

1. Labor, material and repairs;
2. Depreciation of all buildings and machinery;
3. Interest, taxes, insurance, supervision and profits.

Generally the direct cost of labor and materials is fairly well known, varies within narrow limits, mostly between \$1.50 and \$3 per M. feet b.m.

The depreciation of mill and machinery and buildings may be treated as follows:

Mill and buildings may be assumed to last throughout the job, the different machines may and may not need separate treatment. Suppose the amount of stumpage sufficient to furnish 10 years cutting. Let the mill and other buildings, including those machines which are fairly certain to last the ten years be worth \$150,000 and let the interest rate be 5%. Then this outfit together with interest costs at the end of the ten years:

$150{,}000\ (1.05^{10})$—the wrecking value of the outfit.

If the latter is \$10,000, then the end value is 240,000 — 10,000 or \$230,000.

Usually we desire to know what yearly sum will take care of this \$230,000 and from this derive the cost of the depreciation per M. feet of lumber. To get this we may set for the above case:

yearly cost of depreciation = X and $X \dfrac{(1.05^{10} - 1)}{1.05 - 1} = \$230{,}000$, and

from this X = \$18,400.

If the yearly cut is 20 million the depreciation is 92 cents per M. feet. It should be noted that this 92 cents pays for this part of the outfit including all interests.

c. The cost of logging.

Logging is a very simple kind of business but the conditions under which it is done are variable and many, and tend to complicate matters. Size and quantity of timber per acre, mixture of species, mixture of large and small stuff, distances, topography, surface, rocks, brush, swamp, weather, snow, etc., all exert their influence. The men are scattered over a large area and not easily supervised. Where the timber is large as in the Pacific coast country or located in swamps as cypress so that heavy machinery must be employed the matter is made still more difficult.

Nevertheless the cost of logging like milling varies within narrow limits, the good parts of the job making up for the bad. Usually it may be said that the cost of logging varies from 4 to 6 dollars per M. feet b.m., rarely going below 4, occasionally going up to 8 in mountain country. For eastern United States it runs per M. feet logs about as follows:

Felling and cutting into logs	$ .75
Skidding and swamping	1.25
Hauling to landing or railway	1.50
Railway haul or drive	1.00
Overhead charges of supervision replacement of equipment, teams, camps, roads	1.50
	$6.00

In these items the profit is included with each item.

For large timber on the Pacific coast:

Felling and cutting into logs	$ .60
Yarding and loading	1.00
Hauling to mill	1.50
Booming, scaling, etc.	.40
Direct costs	$3.50
Depreciation of machinery and wire cables on basis of the time they last and how many millions they handle, also railway track and rolling stock, etc.	$2.00
Taxes, insurance, interest on money invested, supervision and profit	1.50
	$7.00

Usually logging is cheaper in the Great Lakes country and South than in East and West. But these general average figures are little more than guide marks, each job has its own peculiarities and it is not easy to standardize where considerations are so variable and where the human element is so very important.

Logging as done in the United States is the work of a transition period, many of the methods will disappear, simpler methods under more uniform conditions will take their place. When that time comes the cost of logging in the United States as is now the case in many of the forest districts of Europe, may be predicted within very close limits.

Where the sale of stumpage involves large amounts of timber, requiring ten or more years to remove, though cutting begins at once, the case of sale value grades into the expectation value, and demands special consideration. On the national forests large bodies of timber are sold, the timber is paid for as it is cut, the buyer does not really own the stumpage, he takes no risks in case of fire or other injury, except so far as the injury affects his camps, equipment and the use of his outfit or investments. In this case he not only can pay full stumpage value as calculated above but can pay higher prices for stumpage as the prices of lumber increase. For this reason provision is made in these contracts to review or re-arrange the price scale at fixed intervals or on a particular scale.

C. Expectation value of stumpage.

Leaving out the question of timberland speculation there still arise many cases where this value is involved. In buying a large lumber business, mill, railway equipment and body of timber, the first question is: what is the timber worth? Suppose the concern has four hundred million feet or twenty years' supply of stumpage, the present value of stumpage for immediate use being five dollars per M. feet. Evidently it would be a mistake to pay for the four hundred million at the rate of five dollars per M. feet unless the buyer is very certain that the increase in value of lumber warrants this. How much increase would this mean?

If he pays \$5 per M. feet, or 2 million dollars for the 400 million feet, and if money is worth 5%, then this sum grows to 2 million $\times$ (1.05^{20}) or \$5,300,000 in round numbers. The lumber values do not rise year by year but rise by groups of years usually, and with this rises the value of stumpage. Assume that the stumpage stays at \$5 for the first 10 years and then rises to a constant figure for the second 10 years, how much must this be? About \$13 per M. feet.

If he pays $5 and stumpage does not rise he loses upwards of two million dollars in the twenty years. What can he pay if he expects stumpage to stay at $5? About $3.10 per M. feet.

In these calculations no attention is paid to expenses, taxes, protection, etc., which the owner has to incur in holding these forest lands. The way of computing these was indicated under cost value of stumpage and in other connections.

From what has been said about stumpage valuation it is evident that in most cases it is a matter of business judgment and intimate knowledge of conditions which guides the investor. This knowledge is largely one of forest utilization. The methods of calculation or arrangement of data present nothing new but follow the general lines of forest valuation.

The traffic in stumpage as it has existed in the United States up to the present time will be modified in time and take more and more the form of timber sale now in use on the national forests and in the more settled districts change to the form of timber disposal now in use in central Europe. Good silviculture will press steadily for full control of the timber cutting.

VII. DAMAGES IN TIMBER

Here belong a great variety of cases, from gas injury of shade trees in the town to the burning of large bodies of timber by fires started by a railway locomotive. And it is not merely the cases where someone is liable or supposed to be so, but the numberless additional cases where it is part of the forest business to ascertain damages done by fire, storm, insects, etc.

Normally, the damage is only to the live, growing trees, but at times there is added damage, since it may cost considerable sums to put the land in shape for a new crop. Commonly these cases are complicated. A fire running through a stand of hardwoods may kill half the trees and leave the rest in an injured condition so that a few survive staying alive for twenty years, while most of them die during the first five years. In such cases it is anything but easy to determine the exact facts, or condition.

Such cases are usually complicated still further by the fact that parts or all of the timber could be used if cut at once, but that various circumstances prevent the owner from cutting, so it may be several years before he gets to all of the lands burned over and in this way loses nearly all of the timber. How far conditions compel him and how far he is at fault himself is usually impossible to ascertain accurately.

Only a few sample cases may be considered here:

a. The shade tree in town.

A tree forty years old, satisfactory in every way, is destroyed by a gas leak. What is the true value destroyed? The owner guided by his feelings would probably place the value at several hundred, the person responsible for the leak would value it as fire wood.

The following calculation may be helpful and approach a just settlement as nearly as possible:

Cost of establishing a good tree, \$15; interest at 5%; 25 years before the tree is really serving its purpose, after that the tree pays its way, pays by its service as shade tree interest on the cost of establishing it.

The value of the tree is then $15\ (1.05^{25}) = \$51.80$ and this value remains as long as the tree is in good condition. Cost of caring for the tree, etc., might also be added.

b. Young plantation of forest trees.

Spruce twenty-five years old destroyed by fire; evidently the stuff is of little use save as inferior fire wood. Assuming the cost value of the stand to be forty-five dollars and the value of the damaged wood to be ten dollars per acre the real damage is clearly thirty-five dollars.

If the wood is of no value and the man spends ten dollars per acre to get the land cleared sufficiently to plant another crop, the loss to him is fifty-five dollars. In the past the courts in the United States judged the value of the timber solely as sale value. Since the timber on this plantation has no sale value and since the man actually incurred these losses it is only fair that he should receive pay accordingly.

To ask damages in proportion to an expectation value of this stand, on assumption that at the age of eighty years, or fifty-five years hence, the stand may be worth five hundred dollars an acre, while perfectly reasonable, would not appeal to any court of justice.

c. Older plantation of pine.

Fifty-year-old stand completely destroyed, sale value is only one hundred and twenty dollars an acre, but the expectation value on an eighty year rotation is two hundred dollars. Which should be allowed? Evidently the latter, or at least a compromise between sale and expectation value. In this case the costs of establishing the stand are remote; it may have been natural reproduction on non-agricultural land, etc., and might be less than fifty dollars. So it would not be fair to restrict the price to this cost value. On the other hand, expensive planting on costly land and much extra care, cultivating, etc., might bring the cost value to two hundred and fifty dollars, which in all probability would not be allowed.

d. Ordinary Stand of wild woods.

Timber on forty acres of hardwoods and hemlock is largely destroyed by fire. History and conditions of the forty acres of forest: bought 12 years ago at $800; interest rate 5%; ready to cut 7 years from now; timber 400 M. feet, present stumpage price $5 per M. feet; land now worth $10 per acre when cut over; expect stumpage to go to $6 in the next 7 years. Value of timber after fire, $500; tax and care for the 40, $30 per year.

Estimate of the damage may be set in one of three ways:

1. By cost value:

Value before fire:	$\$800\,(1.05^{12})$	$1434
Tax and care	$\dfrac{30\,(1.05^{12}-1)}{(1.05-1)}$	480
Total		$1914

Value after fire:

Land	$400
Timber	500
Total	$900

damage, $1,014.

2. By sale value:

Value before fire: land	$400
Timber 400 × 5	2000
Total	$2400

damage, $1,500.

In this case it must be shown that the sale value could actually be obtained without delay. The fact that timber is worth $5 stumpage in the particular district is not sufficient evidence.

3. By expectation value:

Expectation value of the 40 before fire:

Value of timber 400 × 6	$2400
Value of land	400
Total gross value in 7 years	$2800

Expenses during 7 years:

$\dfrac{30(1.05^{7}-1)}{1.05-1}$	$ 244
Present value $\dfrac{2556}{1.05^{7}}$	1817

Value after fire, $900; damage, $917.

In cases of this kind the value of the young stuff, i. e., the non-merchantable part of the growing stock is usually left out of the calculation, though frequently it has a greater value than the merchantable stuff.

e. Young Stands in Wild Woods.

In the more remote districts of the United States the value of the land, and the value of young, or non-merchantable timber is debatable. The determination of the cost of production of such a stand, with calculations of compound interest, rent, charge of protection, etc., is not well understood, and not well received, and the same is even more true with regard to the expectation value.

For such cases it has been suggested to use a method strongly advocated for years by Frey in Germany (See Zeitschirft für F. and J., 1915, p. 284), and employ the sale value exclusively as the basis of estimate. Ordinarily this would work out about as follows:

The ripe stand, 120 years old, is worth $300. A stand 40 years old is worth $\frac{300 \times 40}{120}$. Generally, then, the value of any stand is based on the average yearly growth in value, in the above case: $\frac{300}{120} = \$2.50$ per acre and year. Due allowance is made for degree of stocking or condition of the stand.

The advantage of this method lies in its simplicity; it expects the trespasser to pay only for the wood actually on the ground and destroyed. For stands older than one-half the rotation this method is satisfactory; for young stands it does not quite do justice to the owner. Thus if a plantation is destroyed 4 years after being established the $2.50 per acre and year (in above case) would hardly clean the ground and replant, and the owner would have nothing for the four years' growth and expenses. To charge the cost of planting and protection, etc., to the $10.00 actual value per acre, in this case, is really to ask pay for the stand twice, once, its actual value, and once on basis of cost of production, or at least a large part of this.

Where natural reproduction is assumed to require 10-20 years time, this period of reproduction is made part of the rotation; i. e., the rotation includes the entire time from one harvest to the next. For the above case the crop worth $300 is produced in 120 years. If an additional 16 years is required for natural reproduction the average yearly growth is: $\frac{300}{120 + 16} = \$2.20$ per acre and year. In the usual case of natural reproduction a stand of trees with an average of 20 years may contain trees from about 12 to 28 years old;

the land has been occupied and used for 28 years, and the question arises: should the trespasser pay for a stand 20 years old or one 28 years old? Evidently the latter, at least in all cases of clear cutting. In the shelter-woods where an old stand shares in the growth during the 16 years in which the new stand is established, it may suffice to charge for 20 years' growth.

It is evident that this method of computing the value of the stand lacks analysis and must always be defective especially with young stands, where the computation of the regular cost value of stand is much more satisfactory.

f. The lumberman's case.

Two years after the fire, claimedly caused by the locomotive of a railway company, the lumber company sues the railway company and claims: Lumber company bought, merely as "permit" or "license," from the Canadian province, 460 million feet stumpage, it had cut three years and still owned 400 million feet, it has the right to continue lumbering on the "timber limit" as long as there is any timber; it pays 50 cents per M. feet "royalty," or stumpage; it invested $200,000 in mill, railway and equipment, and aimed to cut the timber in next 20 years; was making a profit of $2 per M. feet in the business. The fire destroyed 200 million and so reduced the cut by nearly 10 years, since most of the timber destroyed is not yet accessible.

The particular claim set forth:

1. Lump sum to cover loss in manufacture of lumber and for increased cost of logging due to reduction of total stand available for the plant.

2. Value of stumpage actually destroyed.

3. Value of equipment, camps, railway bridges, etc., destroyed.

4. Logs and ties burned up.

5. Value of young timber, not yet merchantable, destroyed.

Among the questions which came up were the following:

1. Is all dead timber on the area killed by the particular fire?

2. Is all dead timber actually a loss, and if not how much has been saved, how much could be saved, and at what gain?

3. What is the company's basis for claiming that it can make $2 per M. feet, and how is this affected by future logging from more remote portions of the tract?

4. What effort did the company make to protect its logs and equipment?

5. What right has the company to claim pay for immature stuff when the forest belongs to the province and the company pays stumpage as it cuts?

6. Is not the province the real owner of the stumpage and the one entitled to pay for stumpage destroyed?

7. What rate of interest should be used in calculation of damages?

Other complications came up but need no mention here. Evidently claim numbers five and two for young stuff and stumpage requires a court decision as to rights of a "limit holder." If favorable, then the expectation value or **Ge** should be established on basis of growth study, and cruise.

Claims three and four are clearly right, if properly substantiated.

Claims one and two are not easily separated since one depends on two. The simplest calculation is offered by establishing the income or expectation value of the whole business before the fire and the same value after the fire and the difference should be the loss.

Expectation value before the fire: the cut is 20 million feet per year with profit, (evidently net income and profit), \$2 per M. feet, or a total of \$40,000 per year; interest at 3%; since this would have continued for 20 years the present value of these 20 years income is:

$$40{,}000\,\frac{(1.03^{20}-1)}{(1.03-1)\quad 1.03^{20}}=\$591{,}000$$

plus the provable wrecking value of the outfit at end of 20 years, discounted to present time; let this be 10,000, then the total is \$601,000.

The expectation value after the fire on basis of only \$1.50 profit, due to extra work of logging.

$$30000\,\frac{(1.03^{10}-1)}{1.03-1\quad 1.03^{10}}=\$257{,}000$$

plus the wrecking value at end of ten years, discounted to present time; if this is \$50,000, then the present value is \$307,000, and the loss due to fire is about \$294,000.

Another way of attacking this problem is to ascertain the value of the stumpage on a liberal basis. Good stumpage is sold at \$2 in this district. Assume that 200 million feet are destroyed, that this timber would have formed a uniform part of the cut during the 20 years. Then its present value is as follows:

Every year 10 million would have been cut, valued at \$20,000;

total present value: $20{,}000 \frac{(1.03^{20} - 1)}{(1.03 - 1)\ 103^{20}} = \$296{,}000.$

In questions of this kind there is still some uncertainty as to how far a company or person should be held responsible in cases of fire. In the city the owner of a burning building is generally not held liable for harm which comes to other properties from the burning of his. In any case the public and both parties to the suit are more or less closely bound together and should suffer together.

This case sufficiently illustrates efforts at determining damage. It is evident that it is chiefly a matter of accurate examination into all the facts and the establishment of a reasonable basis which is needed. Detail knowledge of the business and honest dealing should solve these problems outside the courts.

VIII. TAXATION OF FORESTS

The subject of forest taxation is primarily one of public policy; the attitude of government toward the forest business. But since the basis of equitable taxation of forests must be supplied by forest valuation a brief treatment of this subject is necessary.

a. Taxation is for public good, to collect money for public expenses. There are three fundamental principles apparent in all taxation except punitive forms which have no place here:

1. All people are asked to share in proportion as they are able to pay.

2. Taxation should be no more irksome or offensive than is necessary, indirect taxation, revenue, tariff, etc.

3. Taxation should not interfere or injure useful or necessary business. While rarely expressed in just this way, the many modifications of the methods of taxation to suit different kinds of business were based on this principle fully as much as on the difficulty of finding a satisfactory basis.

Even in the United States we have taxes of various kinds: ordinary property tax for house, farm, valuables, etc.; income tax, national, and also for railways by states, poll tax or head tax, "road tax" to be worked out or paid for, tariff duties, internal revenues, licenses to do business, special taxes, like present war tax, and other forms.

Taxes are gathered by United States authorities for the national government and by local authorities for town, county and state.

b. In ordinary taxation of real estate in the United States we have to distinguish between **assessment and rate.**

1. **The assessment** sets a value on the property. It usually takes the ordinary sale or market value, it may take the entire value or only a certain, more or less uncertain fraction. This latter condition is not always fixed by law and even where it is, it is commonly modified in practice and gives the assessor a chance to favor certain persons or properties. It is clear that this local agent, an ordinary elective officer, without special training, experience or other qualities, can modify the amount of taxes paid on any particular property and in this way he becomes the most important part in the entire tax system.

That this is not easily overcome by law is well illustrated in the report of the State Tax Commission of Michigan where it is shown that about one half of the state, in 1914, was assessed at nearly full value, (ninety-seven per cent), and the other half at only sixty-nine per cent, in spite of the law which requires assessment at full cash value, and in spite of the efforts of past tax commissioners. As late as 1911 the assessment for the whole state was about sixty-one per cent of the full value.

Assessment is not always simple. A 160-acre farm with ordinary buildings may make a good net income and yet sell for only $50 an acre while a similar farm with fine brick buildings, etc., may not pay expenses and yet sell for $100 an acre. Fundamentally the owner of the first farm is better able to pay. In practice the second farmer pays double the taxes.

A store or factory doing poor business may not be worth the land and buildings, or visible property, one doing a good business may be worth five times the sale value of the visible property. Of two railways from Detroit to Chicago the shorter one is the best paying and most valuable but the longer route cost more money to build and maintain, etc.

2. **The rate of taxation** is set to meet the expenses necessary or supposed to be necessary by state, county and town. The total taxes, state, county, town, including road, school, etc., for Michigan have been about fifteen to eighteen per thousand dollars worth of property. Of $16.55 total taxes in 1902, $12.26 were for local taxes, town, school, roads, etc., $2.41 for county and only $1.88 for state. Naturally the local taxes vary most, and vary from one school district to another, even in the same township. The great range for different counties in the same state is apparent from the following: for 1901 the average tax rate for the different counties in Michigan was as follows:

Over $ 50 per $1000 property in	1 county
40-49	5
30-39	13
20-29	30
15-19	18
10-14	15
Below 10 dollars	1

It is apparent that in any county where timber is assessed at full value and taxed at forty dollars per thousand of value, the interest and taxes eat up the property every six or ten years. Since this rate is an arbitrary affair without limits and left to local politicians, it is quite common that the tax rate and not the tax assessment is the more dreaded phase of taxation. Good, old settled, rural districts in southern Michigan commonly fall below ten dollars per thousand of property while poorly settled, new, districts are normally high in their tax rates, in fact one of the most sparsely settled counties in 1901 had an average tax rate of over sixty-three dollars per thousand of property. It is doubtful if rural taxes should be allowed to go much above ten dollars per thousand.

In this connection the following table is interesting:

Tax Rate Per Thousand Dollars of Assessed Valuation, according to bulletin 109, Department of Commerce and Labor, 1910, p. 849:

	1902	1890	1880	1870	1860
Continental United States	$20.50	$18.50	$18.30	$19.80	$ 7.80
North Atlantic, Maine-Penna.	18.30	16.40	17.70	18.70	9.70
South Atlantic, Delaware-Florida	16.00	14.40	12.70	14.10	3.80
North Central, Ohio-Wisconsin-Kansas-Minnesota	24.70	23.70	20.90	24.00	11.70
South Central, Kentucky-Arkansas-Alabama-Texas	17.30	14.30	16.30	18.00	4.10
Western, including Pacific Coast	25.50	19.90	22.30	25.50	17.70
Ohio	23.80	19.10	16.80	20.10	10.00
Indiana	19.70	18.20	16.40	16.30	9.00
Illinois	51.50	40.90	31.30	45.20	15.70
Michigan	16.60	16.10	16.70	19.90	10.80
New York	22.20	19.80	21.30	24.70	11.00
Pennsylvania	14.90	14.00	17.10	18.70	12.10
Massachusetts	15.80	14.60	15.40	15.70	9.60
Wisconsin	13.60	22.90	18.70	16.20	12.50

The above table indicates that Michigan, in this, as many other matters is very nearly average and also that for the entire Union fifteen to twenty dollars per thousand is considered a reasonable rate. The tendency to increase in rate may be expected to give way to a tendency to decrease with about twelve dollars per thousand as the proper minimum for the state and about six dollars for rural property. In circular 132, A, United States Department of Agriculture, 1913, this rate of six dollars per thousand, of farm property, is accepted as average for farm property in the United States. As pointed out, property is not assessed at full value and even in states where the law demands this it is not done consistently. In most states the assessment is at about 2/3 value for rural property and varies from twenty-five to one hundred per cent in city property with large amounts escaping taxation altogether.

c. When farm **taxes were gathered as "tithes,"** or one-tenth of the crop, not in money but in grain, etc., the **tax was** clearly a **personal tax** and of the nature of income tax, on gross income.

When this changed to a fixed money tax, based on the value of the farm and this latter based on the fertility of the farm, it lost the character of a personal tax though still of the nature of a tax on an implied or estimated income.

Today in the United States the farm or forest is assessed at its sale value; the tax books are based on the property, it is the property which is taxed regardless of ownership. If the owner neglects to pay, it is not the owner who is looked up and addressed, but the property itself which is at once taken charge of and sold for taxes, etc.

The property tax as it applies to real estate, farm and forest, then, is **no longer a personal tax,** it ignores the owner and condition of ownership, is not concerned whether the owner is able financially, or poor and in debt, whether or not he makes any income from this property.

d. **In Europe forests are taxed** in various ways. Usually the forest property is so regulated that a yearly income is secured and the ordinary forest property resembles the farm and the ability of the owner to pay is much more easily determined.

The three forms of forest taxation most generally applied in central Europe are: ground tax; income tax; property tax.

1. The **ground Tax,** "Grundsteuer," or "Ertragsteuer," of the Germans is a tax on the soil, based on its estimated income or else on an official estimate of the productivity, "Kataster," and is used exactly like the ordinary tax on farm property or real estate in the United States; where the farm is largely assessed according to the crop it produces.

2. The **property tax** of European states is a regulator in cases where the property does not produce an income in keeping with its value. An empty lot in the city is only an expense to the owner but it has a sale value. Similarly a farm in the outskirts of a city may be worth a thousand dollars an acre and yet as a farm may not make as large a net income as another farm three miles away and valued

at only one hundred an acre. Private parks, summer homes, etc., belong in this class. In these cases the sale value of the property is taken in the assessment.

3. **Income tax is a personal tax** and takes a certain part, say one-tenth, of the net income. The income is taken as an average of three or five years and all expenses are deducted. In these expenses are included also the interest which the owner of the property is paying on debt. The following case illustrates:

Area of property, 10,000 acres;
Forest of spruce, all one site;
Clear cut and plant, rotation, 100 years;
Yr, $500, yearly cut, 100 acres, worth $50,000;
Yearly expenses, $1.50 an acre, or $15,000 total;
Tax rate, one-tenth of net income. The taxes are:

$$0.10 \times (50{,}000 - 15{,}000) \text{ or } \$3{,}500.$$

This amount is seven per cent of the stumpage value of the ripe timber cut during the year so that a yield tax of seven per cent in this typical case is equivalent to ten per cent on net income.

If in the above case there was a mortgage of $200,000 at 4% on the property the $8,000 of interest would be added to the expenses and the taxes reduced to $2,700.

4. Where the forest is not regulated it does not produce a yearly income but produces an income only at long intervals, (intermittent working of Schlich), as for instance a single plantation, all one age and kind where, if we neglect thinnings, no income is secured from time of planting till the stand is ripe. In such cases the method of taxation by income tax is not readily applicable and various modifications are employed. The simplest of these and most nearly in keeping with the principle of income tax is the postponing of payment until the timber is ripe and then paying a sufficient sum in keeping with the large income at that time.

In other cases the value of the intermittent income is calculated and converted into a yearly income by the formula:

$$\frac{\text{periodic income}}{1.0p^{r}-1} = \frac{\text{yearly income}}{1.0p-1}$$

$$\therefore \text{yearly income} = \frac{\text{periodic income}}{1.0p^{r}-1}\,.0p.$$

But this means practically a return to the ground tax system which in these cases no doubt deserves preference.

5. The state and local taxes in Germany are not always computed by the same methods, in fact it is common for state taxes to be levied as income tax and local taxes as ground or property tax.

Since the state forests pay local taxes, corresponding to county, town, highway, etc., taxes with us, and since these local taxes represent probably not less than seventy-five per cent of all taxes paid, the following figures are of interest, doubly so because they affect a large proportion of all forests of the several states.

Local taxes paid on the state forests of Württemberg on a total area of about 490,000 acres:

Years.	Value of forest or net income capitalized at 10 %. $ per acre.	Yearly net income per acre.	Local taxes paid by state.				
			Total $1000.	Per acre. $	Tax on 1000 property. $	Tax as % of net income. %	Tax as % of stumpage. %
1880–89	26	$2.69	80.9	.16	6.10	6	4.2
1890–99	38	3.86	105	.21	5.50	5.4	4.3
1900	50	5.07	117	.23	4.60	4.5	3.7
1901	54	5.41	119	.24	4.45	4.4	3.4
1902	47	4.73	126	.25	5.30	5.2	4.1
1903	51	5.10	128	.26	5.10	5.1	3.9
1904	57	5.79	105	.21	3.60	3.6	2.9
1905	61	6.10	167	.34	5.55	5.6	4.2
1906	67	6.78	178	.36	5.35	5.3	4.1
1907	74	7.47	188	.38	5.10	5.1	4.1
1908	65	6.50	200	.41	6.30	6.3	4.9

Assuming these local taxes to be seventy-five per cent of total taxes the forests of Württemberg were taxed in 1908 on a basis of about $8.40 per $1,000; or 8.4 of net income, or 6.5 stumpage tax, or 54 cents an acre. The following figures represent the same condition for the state forests of Baden for the year 1902, arranged by the political districts:

Local taxes paid by the state forests of Baden in 1902:

District.	Area of forests, 1000 acres.	Net yearly income per acre.	Value of forest on basis of net income and 10 %.	Local Taxes.		Tax on $1000 property.	Tax as % of net income.	Tax as % of stumpage.
				Total 1000.	Per acre.			
		$	$	$	$	$	%	%
1	13	8.48	84	3.5	.27	3.20	3.2	2.7
2	4.5	3.52	35	.8	.18	5.10	5.1	4.
3	110	5.98	59	32.5	.29	4.90	4.9	3.5
4	50	5.92	59	23.7	.47	8.00	8.	6.1
5	46	6.19	61	4.5	.09	1.48	1.4	2.1
6	8	4.13	41	3.2	.40	9.80	9.8	7.
7	14	2.04	20	3.2	.22	11.00	11.	7.
Entire state	248	5.78	57	73	.29	5.10	5.1	4.4

The great variation in the local taxation as it appears in this table is, no doubt, due to the fact that so many villages and towns in parts of Baden have large communal properties which relieve local taxation very materially.

On previous assumption of local taxes representing seventy-five per cent of total taxes the Baden forests were taxed in 1902 about thirty-eight cents an acre; six dollars and eighty cents per thousand dollars worth of property; six and eight-tenths per cent of net income; or five and eight-tenths per cent of stumpage cut per year.

In Prussia with about 6,540,000 acres of state forest the local taxes for many years varied between three and a half and four and a half per cent of total expenses. Taking the year 1892-93 as basis of income and expenses, the following results:

Total income per acre	$ 2.66
Total expenses per acre	1.38
Net income per acre	1.27
Value of forest per acre on 10% basis	12.70
Local taxes at 4% of expenses per acre: 5.2 cents,	
Local taxes per $1000 property	4.20
As per cent of net income	4.2%
As per cent of stumpage	2.3%

Assuming again the local taxes to be seventy-five per cent of total taxes these forests appear taxed at about seven cents per acre; or five dollars and sixty cents per thousand; five and six-tenths of net income, or three per cent of value of stumpage.

6. The **general tendency in forest taxation in Europe** is toward some form of income tax but it is doubtful if ground tax and property tax will ever be entirely dispensed with. An important and interesting fact is that forestry as a business is not only possible but thrives under a variety of methods of taxation and that in the best forest districts, Württemberg, Saxony and Baden, the oldest form of ground tax is still employed.

This clearly proves that it is not so much a matter of method of taxation as it is an understanding of forestry as a business and a sense of justice which is needed, here, as in many other lines of taxation. The application of some one simple method by one authority, as is done today in the United States for farm property, would certainly be preferable to the irregular variety of taxation, in method, rate and authority of most European countries.

e. Forest taxation in the United States.

1. Forests are taxed in all states of the Union as real estate, the timber and the land, or crop and land being taxed together. The taxation is not uniform; in one county the forest may be assessed low as wild land, in another the timber is carefully estimated and its value added. Non-resident owners and therefore a large part of the forest owners usually fare worse than residents of the township or county. A common mistake is to assess stumpage which can not possibly come into use for years, as if it were ready to cut at once. For instance—a lumber company has twenty years' supply of stumpage; its mill is the most important enterprise in the county, furnishes opportunity for labor, market for produce and a supply of cheap building material. Stumpage is worth six dollars per M. feet. At a recent revision an effort was made to assess the twenty years supply at this rate. It was shown that by doing so with the prevailing tax rate of three per cent, the taxes and interest would eat up the value of a large part of this stumpage long before the timber could be used at the mill.

The complaint usually is that the present methods of taxation force the owners to cut and thereby discourage the holding of forests.

2. In actual forestry practice, the present method of assessing the stand at full sale value and taxing it at a rate of three per cent would prove prohibitory. To illustrate: a plantation of white pine

costs ten dollars an acre and is located on land worth ten dollars more. It is assessed at twenty dollars and pays taxes at three per cent for thirty years. So far the owner has had no income but has paid sixty cents per year for taxes alone. This together with interest at three per cent makes thirty years holding of the plantation cost $28.54 for taxes. If now the value of the stand

At 30 years is	$100
At 50 years is	250
At 70 years is	400

and if the assessment is raised, as it should be according to law in some states, to these values at thirty and at fifty years the case is as follows:

Taxes and interest between 30-50 years	$ 80
Taxes and interest between 50-70 years	200

and the growth in value of the stand between fifty and seventy years fails by as much as fifty dollars to make good the expense of taxes.

In addition to taxes there is a charge of $39.14 between 30 and 50 and a charge of $70.68 between 50 and 70 on the original investment of land and plantation.

The above case presents no special peculiarities, the rotation of seventy years is below the ordinary and the value of the stand at seventy is liberal. It is evident therefore that with present methods of assessment and rates, forestry, even at such favorable conditions as in the above case, must be carried on at a large loss.

The fundamental fallacy of this method lies in the fact that the same year's growth is taxed over and over again, for seventy years, in the above case. This is never done in taxation of farm property.

3. From the standpoint of the state or people the following is true: To the state the taxes on standing timber are only of moderate importance; in the Lake states and east of very little importance. On the other hand the secondary benefit through manufacture of timber is of great value to the state. With the counties this varies. In well settled farm counties the taxes from woodlots are unimportant and there is no reason why woodlots should not be treated like the rest of the farm land, i. e., the value of the timber left out of consideration. In sparsely settled forested counties the taxes from timber are important and there is usually clamor for large sums and also for a steady income, to enable development.

In some localities this is justified, in most places it is not, for usually the sawmill, etc., are worth more to the counties than any direct taxes. Lack of state control and the unreasonable insistence on high local taxes have repeated the story of the goose of the golden egg a great many times in all our forest districts.

The claims of the local people are usually of this order: the forests were not raised by their present owners but given away by the people at nominal prices to encourage the timber business and general development. The owners are not in the forestry business and have no intention of perpetuating the forest. There is no reason why a man should not pay taxes on a hundred thousand dollars which he invests in timber exactly as if he had invested it in other property. The holding of large areas of forest is a form of land monopoly and generally inimical to settlement and development.

f. Reform in forest taxation in the United States.

1. For about forty years efforts have been made to encourage forestry either by modifying taxation or by actual bonus. Among the earliest of these efforts was the Timber Culture Act of 1873, giving United States lands on condition of establishment of a stand of forest trees. Since that time different states have enacted a variety of laws. Usually the aim was to encourage planting of woods in small tracts.

In some cases the plantation was exempt entirely or partially, in others only the land was taxed and assessed at nominal value, etc. In most cases these laws, excepting the United States Timber Culture Act, produced no results whatever. In the last five years, Connecticut, Massachusetts, New York and Pennsylvania have enacted well planned laws.

In recent legislation the following three essential points in our transition stages of forestry in the United States are considered:

The requirement of the local people of a continuous income is met by a yearly tax on the land, either on its sale value, or a nominal value.

Lack of income from plantations and immature stands is considered by deferring the tax on the timber to the time of cutting, i. e., making it a **harvest** or **yield tax.**

The variable and peculiar condition of merchantable timber is taken care of by varying the amount of the yield tax.

In the Connecticut law of 1913 the following provisions are made:

1. The application of the new tax law is optional with the owner.

2. Land worth twenty-five dollars or more is not considered forest land and does not come under the provisions of the act.

3. An existing forest placed under the new law pays not over ten per mill on actual value of the forest as a yearly tax, and when the timber is cut it pays a yield tax which is varied as follows:

If cut in first ten years, yield tax....	2% of stumpage value
2nd	3% of stumpage value
3rd	4% of stumpage value
4th	5% of stumpage value
5th	6% of stumpage value
after 5th	7% of stumpage value

4. New plantations are not exempt, but pay:

a. Yearly tax on the assessed value of the land alone at not over ten per mill;

b. Yield tax of ten per cent of the stumpage value of the material removed, to be paid when the timber is cut.

In the Pennsylvania law of 1913 the harvest or yield tax is also set at ten per cent of the stumpage value, the land is assessed at one dollar per acre and the state pays the yearly land tax and the owner of forests is free to avail himself of the law.

g. Basis of new tax laws.

1. The various efforts at some reform in forest taxation clearly show a lack of uniformity in the basis for such tax law. Even with regard to the land we have:

Lands worth over twenty-five dollars excluded;

Lands assessed at sale value;

Lands assessed at one dollar, in Pennsylvania, and assessment at five dollars recommended by Massachusetts.

A reasonable sale value should be employed, and there is no reason why even fifty dollar land should be excluded. Efforts at tax dodging would be taken care of by the sale value assessment.

2. The greatest amount of uncertainty, however, was involved in finding a proper rate for the yield tax. Should it be five or ten per cent, should it be ten per cent of the stumpage, of logs at landing, at mill, etc.?

Pennsylvania and Connecticut employed ten per cent without apparently attempting to develop a reasonable basis or giving an explanation.

The Massachusetts Commission, see Senate document number 426, 1914, made an effort which proves instructive. Its premises are:

Rotation, 50 years, also assumed by Connecticut and Vermont in recent acts (setting a general rotation is of very doubtful value and certainly 50 years will not raise proper sizes for general market);

Tax rate, \$17.39 per \$1,000 worth of property:

Yield at 50 years, \$240; value of land, \$5 per acre; interest rate for total tax, 8%; for land tax, 5%.

The plan is to pay a yearly tax on the land and pay a yield tax at the time of cutting the timber. The reasoning is this: the income from this property consists in the timber cut. The tax on this income or timber is the tax for the whole property, timber and land.

The land tax is paid to furnish a yearly income for the local government but should be deducted from the yield tax which is the principal tax which the property should pay.

Computation of yield tax or principal tax:

The present value of **Yr** pays taxes every year at rate of \$17.39 per \$1,000, and this is to be compounded at 8%, so that for \$1 of **Yr** we have:

$$\frac{1}{1.08^{50}} \times \frac{17.39}{1000}\left(\frac{1.08^{50}-1}{1.08-1}\right) = \text{yield tax for each \$1 Yr, or \$0.212.}$$

Of this 21 cents only 2/3 is taken, since it is customary in Massachusetts to tax rural property on a 2/3 valuation. The yield tax then is 14 cents for each dollar of final yield or thirty-three dollars and sixty cents an acre, if \$240 worth of timber is cut per acre.

The land tax is computed as follows:

$$5 \times \frac{17.39}{1000}\left(\frac{1.05^{50}-1}{1.05-1}\right)$$

or \$19.11 an acre, which makes about 12 cents per \$1 of final yield. Again taking only 2/3 of this, or 8 cents and subtracting this from the 14 cents yield tax leaves a **final yield tax of 6 cents per dollar of stumpage,** to be paid at the end of the rotation or when the timber is cut. In this way the commission decided that a yield tax of six per cent is a proper tax to charge.

An attempt to use the Massachusetts plan on other rotations gives, even with a uniform land price of five dollars an acre, the following yield tax:

Rotation	50 years	6%
	75	3%
	100	—10%

so that the state would be rebating as soon as reasonable rotations are employed.

3. In seeking a proper basis for any tax rate the following may be assumed:

The most important point of all is that the tax rate on forests should be no higher than that on other rural property. If farm property generally pays less than twelve per thousand in practically all parts of the United States then forest property should pay the same, but certainly not more. This principle is recognized in the Massachusetts plan.

Again if the value of farm property universally is judged by the income which it can produce the same measure is fair for forest property.

A yield tax should be fair not only for one rotation but for all reasonable rotations.

In judging the value of a farm or forest for taxation by capitalizing the net income the interest rate should be fairly high, since the owner takes all risks and the state takes none. This also seems recognized by the Massachusetts commission, and it is common practice in valuation of real estate.

If a yield tax is fair for one stand of timber it is fair for any other stand regardless of methods and regulation, and the proper rate may, therefore, be found by a study of the regulated forest.

That a regulated forest property would pay more taxes, even at the same yield tax rate than a property worked on the intermittent plan, is self-evident.

The following case brings out the relations of property tax at 12 per 1,000 and the yield tax for the stands of a regulated forest at different rotations.

Premises: **Yr** is taken at about half the normal yield in spruce, site II, as per Schwappach.

Yr at 50 years	$200
Yr at 75 years	400
Yr at 100 years	600

land, $5 per acre, **p**—10% to capitalize net income and find income value of the property, Current yearly expenses $1 per acre; cost of planting, $10 per acre.

Tax rate, 12 per 1,000 as being about average for rural property in the United States.

Relation of property tax at 12 per 1,000 and yield tax in regulated forest.

Rotation, years.	50	75	100
Minimum number of acres in regulated forest, i. e., r acres	50	75	100
Yearly gross income from r acres, neglect thinnings	$ 200	$ 400	$ 600
Yearly expenses on r acres, current and planting	$ 60	$ 85	$ 110
Yearly net income from r acres	$ 140	$ 315	$ 490
Value of the r acres regulated forest, i. e., net income capitalized at ten per cent	$1400	$3150	$4900
Taxes, as property tax at rate of 12 per 1000 on r acres	$ 16.80	$ 37.80	$ 58.80
Property tax, 12 per 1000, per acre	$ 0.33	$ 0.50	$ 0.58
Property tax of 12 per 1000 as a per cent of stumpage cut, or **Yr**	8.4%	9.4%	9.8%
Property tax of 12 per 1000 as a per cent of net income	12%	12%	12%

It is evident from the above that a twelve per mill tax on the true value of a regulated forest corresponds closely to a yield tax of nine or ten per cent of the stumpage value of the timber.

The introduction of the land tax disturbs the simplicity and even the justice of the yield tax method since land values differ and this difference is not always made up by the difference in final yield. On lands where the timber grows slowly and requires about one hundred years to reach marketable size the yearly land tax on five dollar land even at twelve per mill and compounded at as low as three per cent reaches the formidable sum of over thirty-six dollars an acre, a sum which would seriously discourage the planting of bare lands and frequently exceed the ten per cent yield tax. For some time to come and only as an expedient to bring about some tolerable reform the land tax may be necessary, but a tax rate not to exceed ten per mill should then be employed. Cases like parks and costly estates in the vicinity of cities should receive special treatment and be taxed by the ordinary tax method.

Of late the application of the single tax has strongly been advocated for forests as well as other real property. The single tax is well suited to forestry; it would stimulate the accumulation of a good growing stock and the making of improvements, roads, proper division, etc., and so put a premium on good forestry. The action of the single tax in forestry is exactly the same as in farming, where it stimulates building and keeping of plenty live stock, while the present form, the property tax, in a way penalizes the good farmer with fine buildings and plenty of stock and machinery. But in no case should the crop, or timber and land be taxed together, as has at times been advocated by single taxers. For the existing virgin forest where the owner merely holds a stored mass of merchantable timber and is in no way practicing forestry and where the public is determined to make the owner divide the goods, practically given away by the people, there is nothing for it but to devise some compromise. Here the simple property tax as now applied, or a combination as is represented in the Connecticut law is probably as good as any other.

h. Future in forest taxation.

1. A yield tax is inconvenient. The owner of a small forest cutting a few poles or a few cords of wood finds it bothersome to record and report. This leads to exemption for domestic use, as is done in the Connecticut law. But it is hard to set limits in these exemptions, they lead to confusion and bad practice. With large owners it becomes necessary to take the word of the owner, it pries into his affairs and has all the objections commonly claimed for income taxes. The local tax official has added a great deal to his labor and where the matter is left optional it involves inspection of state foresters who lack help, money and experience.

For a regulated forest property there is no occasion to use a yield tax; it can be assessed as easily as a farm and taxed in exactly the same way.

For isolated tracts, not really managed as forest it may bridge over, but is not satisfactory enough to recommend for permanent practice. To receive a yield tax once in eighty years introduces too many chances for cheating the buyer of such property and the community.

For a beginning the yield tax is to be recommended to get away from the present methods, but the tax collecting practice in rural properties will work for a return to the property tax, properly regulated by computations based on actual income.

2. The income tax has been recommended for forest taxation, it is used abroad, it usually takes about ten per cent of net income, it is simple in practice, tends to a form resembling closely the ordinary property tax now in use. The ten per cent basis is arbitrary, whether a remnant of the old "tithe" or a gradual adaptation of public expense to personal income, yet it is generally agreed upon as common and fair.

How a ten per cent income tax compares to our ordinary twelve per mill property tax is shown by the following figures, the basis taken from circular 132, A, 1913, United States Department of Agriculture.

For the average farm of the United States the total value is $6,343, the total income $980 and the net income is given at $538. An income tax of ten per cent on $538 or $53 is equivalent to $8.30 per $1,000 of property and shows that ten per cent of net income is only about 2/3 of the ordinary 12 per 1,000 rate of taxation of rural property, where this is assessed at full value. Apparently this was felt by rural tax officials and the assessment was adjusted to make the rate reasonable, i. e., the custom of taking rural property at 2/3 value brings the common, nominal twelve per mill to an eight per mill on full value, or close to a ten per cent on net income as above computed.

3. From the above it follows that a ten per cent yield tax which is really ten per cent on the gross income is somewhat too high; that about eight per cent of the stumpage is more nearly right and that the addition of a land tax is entirely unfair. From this standpoint the Pennsylvania law requiring only the ten per cent yield tax is fairest; the Massachusetts law charging six per cent on yield and a land tax on flat rate assessment comes next, but does not work well for rotations above fifty years, and that the Connecticut law still demands an amount of tax much greater than is paid by the average farmer.

4. Taxation and protection more than all other factors combined decide whether forestry can and will be practiced by private owners. The present methods lead to confiscation; the recent laws are a good beginning but they still demand more than is fair and also lack in convenience and simplicity of application.

From the standpoint of the state it would be of great value in all forested districts to pass some simple laws, applicable to all forests and framed somewhat along the lines of the Connecticut law, to provide:

a. Taxation of all woodlots as parts of occupied farms, on a basis of simple assessment of the land, leaving out of consideration the growing stock or crop, exactly as is done in case of wheatlands, meadows, etc.

b. Exemption of forest plantations for twenty-five years and then taxation on a land and yield tax, but limiting the tax rate to five per mill and the yield tax to six per cent.

c. Taxation of existing forests, not parts of farms, on a land tax at five per mill and a progressive yield tax with a limit of six per cent.

d. Taxation of regulated forests on the property tax plan, exactly like the farm, but the value of the forest to be the income value on a basis of net income capitalized at ten per cent.

Or else: taxation of these forest properties by levy of income tax taking ten per cent of the net income; or as a third alternative, a simple yield tax of eight per cent.

e. Providing a state control which makes it impossible for local politicians to interpret and execute tax laws to suit their interests.

IX. FIRE INSURANCE IN FORESTY

The timber or growing stock in the forest is liable to destruction by fire at any time of its existence. The danger is greater for conifers than hardwoods, greater for stands from ten to thirty years old than for plantations or older timber; varies with the climate, soil, topography, and may be reduced greatly by proper improvements and protection. Since the timber or growing stock commonly makes up seventy-five per cent or more of the value of the forest property, fire danger is serious.

For the owner of a small forest, from forty to one hundred acres, who may lose in one fire the greater part of all he owns, this is no doubt true. Generally the danger is overrated. The burning of a few plantations on a large forest property is not a serious matter; they must be replaced and this adds to the current expense but it does not affect the regular income for years to come. In stands over fifty years old, the fire-injured or fire-killed timber is not a total loss, in fact, it may be utilized often up to eighty or ninety per cent of its full value. Here the fire may disturb the plans and orderly business of the forest but does not cause a serious loss. In wild woods the case has been different. In the United States fires have done enormous damage and even today are the greatest factor preventing action in forestry. According to the report of the National Conservation Commission of 1909, the fire losses in the United States may be estimated at about fifty million dollars per year for merchantable timber alone. To this the report adds a much larger sum for destruction of stuff below merchantable size and usually a total loss. Leaving out the latter, yearly fire losses of merchantable stuff for the last fifty years in the United States have been at least ten cents per acre of forest area, and if the average value of forest is twenty dollars an acre, the loss amounts to five dollars per thousand dollars worth of forest, or about seventy per cent of what good forests should pay in taxes.

The difficulty here has been a lack of protection and lack of market and roads to enable immediate use of the stuff injured or killed. It has been the rule rather than the exception that timber killed by fires in the United States has been a total loss.

This is all changing now and will change a great deal more even in the near future. It may be expected that in fifty years conditions in the United States except perhaps in the high mountain country of the west, will resemble those of Europe.

Here the fire danger is no longer considered a serious difficulty in forestry and all large owners, state and private, do not even feel the necessity of fire insurance. The following figures will illustrate:

DAMAGE FROM FOREST FIRES:*

	Prussia.		Bavaria state forests. 1877-1902	Austria not Hungary. 1881-1895
	All forests. 1881-1894	State forests. 1881-1912		
Total area of forest, million acres...	20.4	6.2	2.07	24.27
Total gross income, million dollars..	50‡	15.5	6	30‡
Net income per acre, approximate $.	1.25‡	1.25	2	0.30‡
Value of forests on basis of 3% and net income, million dollars........	810	250	120	240
Damage from forest fires $1000.....	95	——	4	35
For each 100,000 acres of forest the fires burned over, acres..........	25	21	10	14
For every $1000 gross income fire damage is in dollars.............	1.90	——	.66	1.16
For every million dollars worth of forest damage is $..............	117	——	33	137
Average number of fires per year...	388	26	91	——
Average area burned over each year, acres	4955	1500	200	3420

In the statistical works for Württemberg and Baden the damage by forest fires is not even mentioned. While it is evident from the foregoing that large forest owners have little reason to seek insurance, the case is different for the great number of small owners. For this reason forest fire insurance was discussed and planned as early as 1877 by Burckhardt and the matter agitated from that time to this.

Insurance by the state, by cooperation of forest owners and by commercial concerns, all have been discussed.

* See Endres Forst Politik, also Forstliche Verhältnisse Preussens Württemburg, etc.

‡ May be a little high since it is based on income in state forests and averaged for a number of years.

So far only commercial organizations have worked extensively in this field. Of these a department of the Gladbacher fire insurance company, starting in 1895 is the oldest and most important. In 1903 this company had more than 300,000 acres of forest covered by insurance, mostly in the densely settled Rhine province where the forest is in small tracts. The average rate has been about $1.80 per $1,000 of property.

Since then the Bayerische Hypoteken und Wechselbank, the Provincial Fire Insurance Institute of the Rheinprovinz and others have taken up this work. The rates usually run from one to five dollars per thousand dollars worth of property.

Usually it is demanded that the insurance continue for at least ten years, that the owner insure all his holdings in one company, and that he give to his property such protection as is customary in the district, this point being stipulated rather in detail. The rate of premium varies with the kind of timber and age, formerly the rates increased with age up to thirty years, this appears to have been given up and the rate starts with a maximum and decreases.

Regular published tables of yield and methods of assessing damage are provided, the cost and sale value of the stand prevail, the expectation value is permissible only under certain exceptional conditions, the payment is to cover only actual damage, any income from material salvaged is deducted, the interest rate for any calculation of values is three per cent.

The following figures from Vorster, director of the Provincial Fire Insurance Institute of the Rheinprovinz, published in Zeitschrift für Forst und Jagdwesen, 1908, p. 797, etc., will illustrate what is today good forest fire insurance as to rates, and change in these.

Kind of timber insured.	Premium per $1000 of property.
1. Pure hardwoods, coppice or timber	$0.25— .80
2. Mixed hardwoods and conifers in timber forest	.60—1.50
3. Coppice mixed with conifers	.90—3.00
4. Pure conifers:	
Up to 8 years old	3.00—4.00
8-15	2.50—4.00
15-40	1.50—2.00
over 40	.50—1.50

The following illustrates how a small forest of pine, site IV, of Schwappach, properly regulated, with present prices for timber, fares in this insurance. Figures of the Rhenish Institute are compared with those of the Gladbach Fire Insurance company.

For simplicity, the lower rates only are here considered for both companies.

Cost of insurance for a regulated forest of pine, site IV, 60 acres, rotation 60 years.

Area. Acres.	Age of Stand. Years.	Value of Stand per Acre $.	Rhein Province Institute. Premium.		Gladbach Insurance Co. Premium.	
			Rate per $1000.	Cost for 10 Acres $.	Rate per $1000.	Cost for 10 Acres $.
10	10	28	2.50	.70	4	1.12
10	20	52	1.50	.78	3.40	1.76
10	30	91	1.50	1.37	2.80	2.56
10	40	133	1.50	2.00	2.20	2.93
10	50	165	.50	.82	1.50	2.48
10	60	197	.50	.98	1.00	1.97
				Total 6.67		Total 12.84
			Per acre	.11		.21
			Per $1000 worth of woods	1.00		1.94

Assuming the yearly gross income from this sixty acres to be $225, **Yr + T$_q$**, and the total expense **c + e**, $75, the insurance would take 4.4% in one, or 8.5% in the other, of the net income of this forest. From each $1,000 of gross income this insurance takes $30 in one, $57 in the other company. Comparing this to the actual yearly losses in Prussia, $1.90, and Bavaria $0.66, as seen in the foregoing table, it is easy to see why the state forests are not insured.

So far the forest fire insurance has not been a success, the companies have not made any money in this line of insurance and the proportion of forest insured is still insignificant. Compulsory state insurance would no doubt readily solve this problem and reduce the rate to nearly the actual average loss and so reduce it to less than twenty per cent of the present rates.

In this connection the comparison of cost of insurance and cost of good fire protection is interesting. For a regulated forest of pine with a rotation of only sixty years the lowest premium is eleven cents per acre for a hundred year rotation of the same kind of woods, pine, it amounts to twenty-seven cents. But with any large property and any kind of system of protection ten cents an acre goes a long way toward preventing fire entirely and certainly can well reduce the damage to the insignificant minimum now secured in

Germany. In the national forests an expenditure of only about two cents an acre has reduced fire losses from an estimated ten cents per acre to less than one cent per acre. By an expenditure of ten cents an acre it is to be expected that this loss will be reduced still further, and what is more important, this protection will protect with a certainty regardless of conditions, and so prevent any of the great forest fires which from time to time have completely devastated many thousands of acres in a single season.

So far forest fire insurance has not really been tried in the United States. All financial concerns, including insurance companies have been afraid of forest properties; some of the reasons for this attitude are found in the following conditions:

The values are badly distributed and scattered in a forest; there is a large area for a relatively small amount of money. This is less true in the well-cared for forest with good market than with the wild woods; and it is less true of the heavy stands of timber in the Pacific coast states, but generally the fact remains. In the city, property on one acre is worth many thousands, in the forests of the eastern half of the United States it is worth about twenty dollars.

The value of wild woods is hard to ascertain with any degree of accuracy. The insurance company's agent can not simply inspect and verify; it takes a regular cruise and involves expense.

The amount of damage by a forest fire is very hard to ascertain. A lot of hardwood timber run over by fire may all leaf out and appear practically uninjured and yet half the stuff may die and start to decay before five years are past. Two estimators would often fail to agree with regard to the same tree before them.

Much of our timber is owned in separated and often widely scattered bodies.

The owners of timber are only holding stumpage; it is a speculative affair, and does not inspire confidence. The owners do not practice forestry, the forest is left without improvements, roads, etc., and there is no real care and protection.

The community, state and county do nothing, as yet, to make forest property reasonably secure.

Public opinion and habit has no regard for forest property and is quite generally inimical to the owners of forests. This attitude finds expression in the behavior of young and old in the forest, in legislation and in the enforcement of law.

What the future will bring is doubtful. For large owners of forest, who practice real forestry, there is no object in insuring, as is evident from the experience on the national forests today. For the small holding, especially the woodlot on the farm, insurance would be beneficial and may even become necessary if these valuable woods are to be preserved. For commercial companies these woodlots are not inviting as objects to insure, the difficulties in appraisal of damage alone being sufficient to discourage any company. It will require mutual insurance as now exists in many counties, or state insurance will have to supply security.

X. THE RIGHT USE OF LAND

FORESTRY VS. AGRICULTURE AND RANGE.

a. **Nature of land.** Since the area of land can not be increased and since land can not be moved from place to place, the occupancy of land, as by a farmer, is monopolistic, and the right use of the land is important, in fact, fundamental to the welfare of any people. That a farmer should have a duty, not only to himself, but to the people or state, and that the state authorities have a duty in seeing that the land of the state is put to its best use for present and future, are modern conceptions, rapidly developing and becoming influential in public economy. Even in the past, the state felt justified in taxing the shiftless owner of a good piece of land, farm or city lot, not in keeping with what it produced but according to what it should produce. Today all civilized states are spending money to devise ways and means of making the land more productive. In this effort to maintain and better the land it is of as much importance to select the right crop as it is to give the proper care, and it is here where the choice between forest and field crop asserts itself.

b. **Lands may roughly be divided** into: agricultural, forest, range and waste lands; but the lines between these four classes are rarely very sharp.

On the agricultural lands, including the garden and truck lands, the crop is to furnish food for people or animals. The forest crop, on the other hand, serves two very distinct purposes; it may merely maintain a necessary cover on mountain lands and be useful as a protective forest, or it may serve solely to produce timber and other products. The importance of the forest as protective cover is so great and has of late been so well recognized by the governments of civilized countries that Martin in his Statik classifies lands into those used to raise products and those requiring a protective cover.

In the following paragraphs the protective forest is considered as occupying absolute forest soil.

c. **The factors which determine or limit the use of land** are chiefly: climate, (temperature and moisture,) soil, topography and population. In mountain countries small patches of valley or bench land may be excellent farm land and yet not desirable for this use because of their location and size, a condition which applies, however, only to a very small per cent of our lands.

1. **Climate** is invariably the principal factor. More than half of N. America and Asia is too cold for agriculture; more than 35% of North America, more than sixty per cent of Asia and Africa, and more than eighty per cent of Australia are too dry for farming.

About forty degrees Fahrenheit average yearly temperature limits good farming; about 30°F. ends the useful forest, except protective woods. In the United States about twenty-five inches of rainfall for the year limits the natural forest; about fifteen inches sets a limit to dry farming or farming without irrigation; at ten inches of rainfall the prairie changes to desert.

In countries with an average yearly temperature less than fifty degrees Fahrenheit and with dry summers and low relative humidity all lean sandy lands have failed in maintaining satisfactory agriculture.

Topography affects agriculture; generally a five per cent slope washes as soon as plowed; a ten per cent slope gullies and is ruined. Usually steep lands become stony and lean and millions of acres of farm lands have been abandoned in Europe because of topography.

Dense population commonly stimulates the use of land for farming. In the United States millions of acres of good land are not used or poorly used for lack of labor and demand. But while it is generally true that dense population leads to the use of lands for agriculture, this is no longer true to the same extent that it was fifty or more years ago. Formerly the majority of the people lived on the farm, today over sixty per cent of the people of the United States live in town. The people do not flock to the country, they leave the country and move to town. It is the town which furnishes employment and a safe and comfortable living. This is as true of Europe as it is of the United States. The result is that millions of acres of land even in the better populated parts of the United States and the most densely populated states of Europe are not tilled, and large areas have been abandoned to deteriorate into waste lands. Dense population today, means cities, manufacture and commerce, good wages and living in the city, higher wages and less farm labor for the farm; and intensive use of good farm lands which justify labor and machinery; and abandonment of poor farm lands. In Europe this situation has shifted the use of lands from agriculture to forestry for large areas; it has led to regular appropriations by the states and the development in this direction is more rapid today than ever before. Lands worth up to fifty dollars an acre are today being converted from field into forest in southern Germany.

England has had a special commission to work on this problem. In its report this body cites the case of a tract of land of twelve thousand acres, bought at ten dollars an acre, which for years had brought only about thirty cents per acre rental, and it recommends extensive purchase of lands and reforestation in densely populated England.

d. **The countries of the Old World** are interesting in this connection: of Scandinavia less than 8% is tilled, about 50% is waste land; Spain and Italy are about 33% tilled and more than 25% is goat and sheep range which will be regarded waste land as soon as industrial conditions are such as to give the people a 50 cent per day wage.

Russia in Europe is claimed to be about 26% tilled and 37% forest, leaving 37% as waste land, with an uncertain portion of range lands.

Turkey in Asia tills about 3% of its 580 million acres, the rest being mostly waste land with various combinations of rock waste, desert and range lands.

India, China and Japan, with about half the human race as population, till probably less than 20% of their land.

Use of land in highly developed states:

	Fields.	Meadow and Pasture.	Forest.	Cities Highways and waste including some range.
Germany	48	19	26	6
France	55	14	18	13
Great Britain	30	31	3	36*

Both in France and Germany the tilled land is not on the increase. Prices of land are not advancing and the rents or income from farms is not materially better than fifty years ago.

* Includes considerable heather land.

e. Land conditions in the United States.

Region by States.	Area million acres.	Per Cent.		Rest is
		Improved land.	Forest.	
Atlantic, Maine to Georgia....	225	35%.	45%	cut over land
Gulf, Florida to Louisiana....	126	18	65	cut over land
Texas	167	12	25	prairie
Lake States, including Ohio, Indiana and Illinois........	207	51	31	cut over land
East Central, West Virginia to Arkansas	146	41	40	cut over land
West Central, Oklahoma to Dakota	250	40	4	prairie
Rocky Mountain	549	1.5	20	prairie, desert, brush and waste
Pacific, California to Washington	203	9	40	prairie, desert, brush and waste
Total United States..........	1900	25	26	

Apparently over 900 million acres are neither forest nor field, largely too dry and therefore prairie or desert. About 300 million acres are wild woods on absolute forest soils, the lands being unsuited to farming by reason of climate, cold mountain country, topography, all large mountain ranges, and to a lesser extent, poor soil, South and Lake states and the East.

What these conditions may be even in an old settled state is well seen from the following figures for Michigan:

	State.	S. Half.	N. Half.
Total area million acres.......	36.8	17	19.8
Per cent improved............	35%	64%	6%

f. Agricultural use.

Generally the agricultural possibilities of any district finally decide its use. Any good tract of agricultural land as large as an ordinary county will become farm settlement in our country. This fact, based on large experience has helped to fix in the minds of most people, and especially the so-called representative men who are in duty bound to "boost" and to "believe in the country," the assumption that all lands are agricultural lands. That this is practically true of Iowa and Illinois no one doubts, but that it is true of very few other states every one who cares to can see for himself.

Formerly the policy of the states was to get people and settle the country. Usually this policy was forcefully emphasized by the land boomer and other people who had, or believed they had a monetary interest in rapidly populating the state. Of late the authorities agree that the state wants:

1. Not merely many people but rather good and prosperous people.

2. To the state, any kind of "pauper" industry or industry in which the people can not make fair wages and a fair living is an injury. Of these pauper industries the farming of non-agricultural lands is one of the worst.

3. The state needs manufacture as well as farming, and that even the best farming districts may lose in population and business because of the one-industry condition. Consider Missouri and Iowa, and some of the best farm counties of several of our states.

4. The state wants to have its people supplied with raw material and manufactured goods by the state itself and so be as independent of import as possible. This applies as much to timber as to grain.

5. Every acre of land should raise the best possible crop and make the largest return in permanent use.

6. Labor conditions should be carefully balanced, so that labor is steadily employed and all work is well done. The great wheat districts, though excellent farm country, waste grain for lack of help in the harvest season, and fail to give employment all the rest of the year.

7. Regulation of stream flow, protection from erosion and winds and the beauty of the landscape are important and in many cases decide the choice between field and forest.

In our country public policy regarding the proper use of land is largely controlled by the views and wishes of the farmer and it is interesting to see how he is affected by the choice.

1. The farmer wants not merely good crops but good prices. A good home market is the only means of liberating the farmer

from the speculator and the manipulator of the world markets. Two good sawmills where twenty per cent of the land is tilled in a county of the Great Lakes district, are worth more to these farmers of the county than a condition of sixty per cent of the land cleared, competition trebled and the local market spoiled for lack of manufactures. This competition is especially painful in one-crop districts, potato, fruit, etc., and it is doubly bad where the average farmer is poor and compelled to sell for what he can get.

2. The farmer needs labor, cheap and convenient, usually for short periods. Without other business a district can not keep itself supplied.

3. The farmer needs railways. The farm furnishes about two hundred and fifty pounds of freight per acre and year, the forest about one thousand and the manufacturer exceeds them both.

4. The farmer wants good roads and schools and yet low taxes. The state taxes, and these in the long run determine school and road, are not paid by the farmer. In the United States the farmer owns less than one-third of the property, is assessed at about sixty per cent of its value and usually pays about sixty or seventy-five per cent of the average tax rate for the state, so he contributes less than twenty per cent to the state tax burden.

It is evident that to the farmer there is very little inducement to urge agricultural use of lands wherever this brings competition and reduces manufacture and local market. In the past this phase of the question was clouded because most farmers were as much speculators as they were farmers; they wanted to feel that their farm was increasing in value and that they would make money by selling out if they chose to do so. The utter fallacy of this reasoning to the real farmer is apparent; all he gained by the boost was a larger tax assessent, for his crop, and his income was not increased by it. A few boost sales in a county frequently upset farm assessment though they did not add a cent to the income of ninety-nine per cent of the farmers.

g. **The income from the land** has always been regarded as an important criterion of its use. A century ago before railway and modern manufacture, wood was cheap, over large areas it had no value at all, while food was a necessity and measured by a day's wage was high in price everywhere. Accordingly, the incomes from forests were very small and even foresters like Hundeshagen and Cotta felt that only the state could afford forestry on anything like fair land. Today these conditions have changed, and agriculture is called on to explain its smaller net income from better lands. In this discussion it is well to keep in mind the following:

1. Timber is a necessity to modern civilization. All substitutes have not been able to reduce the per capita consumption and in Europe any state with fair industrial development becomes a timber importer as soon as its forest area drops below thirty per cent.

2. The forest crop is safe and certain wherever forest is native and even on prairies the forest once established modifies its own local climate. On all lean and difficult, stony, rough, etc., lands the forest is far more certain than any field crop. The forest crop takes care of the land, the field crop drains it.

3. The forest crop is durable, unlike potatoes, fruit, etc. A few years seasoning increases the value of timber.

4. Prices of forest products have gone up steadily for a century and have more than doubled in half a century; prices of field crops in Europe and the United States have not changed materially for the last half century. See Endres, *Forst Politik*, p. 119, etc. Rye, the great bread stuff, was cheaper in Prussia in 1893-1903 than it was in 1860-1890. For the United States, see United States Dept. Agri. Farmer's bulletin, 645, 1914, p. 23, the value per acre of the ten most important farm crops was $15.74 in 1871, then declined rapidly and did not regain this value until 1908.

5. Forest statistics of necessity are averages since it requires many seasons to grow a crop of timber. To be comparable at all the figures for farm income therefore must be taken as averages. This is commonly overlooked and accidental maximum figures are constantly quoted and repeated. "Great money in clover-seed," "$400 per acre in cherries," etc., are repeated in journals and by booster orators until the average person is completely misled. It is one of the most useful works of the United States Department of Agriculture and the Experiment Stations to gather truthful statistics of the farm. So long as the booster succeeds in making the people believe that there is big money in sand farms, so long the public is reluctant to pay even a decent cost price for farm products.

6. Forest statistics as used here describe large averages of incomes not from the good lands but from inferior, largely non-agricultural lands. It is doubtful if more than thirty per cent of all German forest lands could be farmed continuously and successfully. If all lands of Germany were arranged in five classes of equal area,

the forest lands would take practically all of number five and about half of number four with very little of class I-III. For this reason it is quite safe to say that for the same use, the lands now in fields, etc., should by virtue of their fertility produce a much larger return than the forest lands. This as will be seen, they do only for the gross income in which a large per cent is labor, but they fail to do this for net income, clearly proving the superiority, at present prices, of the forest uses when judged by net income.

1. Income from field and farm land in the United States.

For over thirty years the United States Census authorities have gathered farm statistics. Of late the United States Department of Agriculture and also some of our Agricultural Experiment Stations have checked, verified and amplified this work and have generally proven the accuracy of the work of the Census.*

In discussing the income from land used in farming it is necessary to keep separate the income from the crop and that from the animals, etc., which represent a separate investment.

The total value of the farm crops in the United States, including the value of the material fed to live stock, was reported in circular 132, as taken from the Census of 1910, at 5,487 million dollars, or $860 per farm. The average farm has 138 acres, of which 75 acres are improved land. Allowing only 50% for cost of production, exclusive of rent of land, the 138 acres of farm land produce a gross crop income of $6.24 per acre, or a net crop income of $3.12. Even if referred to the 75 acres of improved land only the gross crop income is $11.40 and net $5.70 an acre.

United States Bulletin 645, 1914, p. 23, gives the "Yearly value per acre of ten crops combined." The crops are corn, wheat, oats,

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* See United States Crop Reporter for 1911 for cost of producing wheat, oats and potatoes in the United States.

Ohio Agricultural Experiment Station bulletin 266 cost of producing corn in Ohio, 1913.

United States Farmers' Bulletin 641, 1914, cost of producing cotton.

United States Farmers' Bulletin 635, what the farm contributes directly to the farmer's living, 1914.

United States Farmers' Bulletin 645, Agricultural Outlook, 1914, p. 23, yearly value per acre of the ten most important crops combined for 1866-1914.

Bulletin 41, United States Department of Agriculture, 1914, farm management survey of three representative areas in Indiana, Illinois and Iowa.

Farmers' Bulletin 665, 1915, Agricultural Outlook for farm wages in the United States; Farmers' Bulletin 570, 1913, and circular 132-A, 1913, which discuss the farmers' income, using Census of 1910.

barley, rye, cotton, potatoes, hay, tobacco and buckwheat. The data are presented in the following diagram.

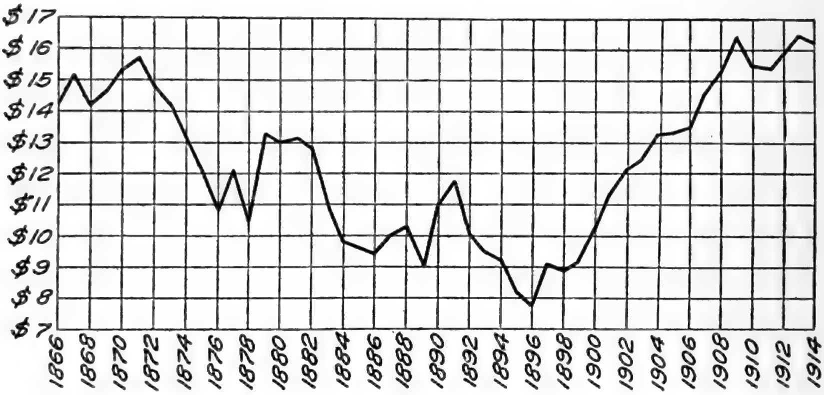

Fig. 2. Chart showing the value per acre of 10 crops combined (corn, wheat, oats, barley, rye, buckwheat, potatoes, hay, tobacco, and cotton), representing about 90% of the total cultivated area of the United States.

These figures present the gross income per acre of fields and meadows, actually tilled that year, and show that even at present prices this gross income is only about $16 an acre and that for more than 25 years prior to 1904 it was usually below $12 an acre of fields and meadows. These fields contained 47 acres per farm, or made only 34% of the farm itself. The following table indicates the relative importance of the principal crops, and presents some figures on the cost of production of farm crops in the United States, 1909.

	Total area million acres.	Total Value million dollars.	Yield per acre.	Value per acre. $	Cost of production per acre exclusive of rent. $
Corn	105	1520	32 bu.	19	8.52
Wheat	49	555	17 bu.	17	7.85
Oats	37	452	35 bu.	14	7.13
Cotton	37	825	240 lbs.	30	16.70
Potatoes	3.6	212	118 bu.	35	25
Hay	43	856	1.2 tons	14	——
Barley and rye	9.7	135	22 bu.	11	about like oats.

Bulletin 645, 1914, states that the thirteen principal crops occupied three hundred million acres, and had a value at the farm of 4,919 million dollars, or about $16.38 an acre.

Another reliable measure of farm-crop income is the rent actually paid to owners of farms. While in a few localities the rent has been boosted up to as high as $8, the general average of cash rent seems to be close to $3, closely agreeing with the average net crop income. Even this $3 rent per acre and year is not a net income to the owner since taxes and repair of buildings, etc., must still be deducted. It is also quite generally conceded that the farm is rarely well kept up in the cash rent system.

The following figures describe the crop income conditions in Germany. They are taken from Helferich *"Deutschland's Wohlstand,"* and represent average yields for 1909-1913, valued at current prices.

Rye	$14.56 per acre
Wheat	21.50
Barley	13
Potatoes	19.20
Oats	11.80
Hay	12

Which would indicate a general average close to $16 or the same as the present average for the United States.

Since less machinery is used in Germany and the fields are generally too small for economical farming, the cost of production is higher in spite of cheaper labor. The "Schlesische Landschaft," see Martin, *Statik*, p. 259, estimates the cost of producing the crop as follows: on lands of

Class 1 cost is	55-65% of value of crop
2	57-67
3	60-70
4	65-75
5	72-82

Taking land of class 3 as average and using the above $16 average for crop, the net income even from these fields and meadows in Germany is only $5.60 or about the same as from the improved land in the United States.

As regards the regularity of income the figures of the United States Department of Agriculture, see Farmer's bulletins 629, 641, 645, indicate that in hay even, the average is below seventy per cent of a fair stand. As stated in part one, the state forests of Württemberg today cut about seventy-three per cent of normal, are still improving and may be expected to be eighty per cent normal in their yield before another twenty years.

2. Income from the forest.

Since there are no forests in the United States in a regulated business condition, it is necessary to consider European results. Even in Europe it is only the state forests and the holdings of other large owners, cities, etc., which can be considered as properly taken care of in a way at all comparable to the farmer's fields.

The following table represents large averages and the figures are reliable. The great difference between Prussia on one hand and Württemberg and Saxony on the other is due to soil, sand, and climate, and in part, market; the smaller income in Bavaria is largely due to topography, mountain woods, and to a lack of development. In Bavaria and Prussia a further development, road building, etc., will continue to absorb much of the yield. Later on, in twenty-five or fifty years, this will cease and then the net income will be a much larger part of the total income.

Incomes from state forests in Germany:*

Area. Acres.	Württemberg 1908. 490,000		Saxony 1911. 431,000		Bavaria 1912. 2,330,000		Prussia 1912. 6,796,000	
Total yearly income.	$4,906,000		$4,015,000		$16,156,000		$38,490,000	
Years.	Income per acre.		Income per acre.		Income per acre.		Income per acre.	
	Gross.	Net.	Gross.	Net.	Gross.	Net.	Gross.	Net.
1850–59	$ 3.60	$ 2.11	$ 3.03	$ 1.94	———	———	$ .97	$ .54
1860–69	4.73	2.89	4.46	3.21	$ 2.32	$ 1.35	1.43	.83
1870–79	5.77	3.45	5.76	3.92	2.95	1.66	1.98	.97
1880–89	4.99	2.70	6.47	4.33	2.64	1.30	2.20	1.01
1890–99	6.35	3.87	7.13	4.50	3.73	1.81	2.68	1.29
1905	9.20	6.10	8.44	5.39	4.55	2.20	4.26	2.42
1906	10.05	6.78	8.28	5.08	4.95	2.55	4.27	2.27
1907	10.90	7.47	9.60	6.17	4.90	2.55	4.54	2.51
1908	10.00	6.50	9.22	5.58	5.50	2.95	4.45	2.21
1909	9.90	6.35	8.96	5.24	6.30	3.30	4.53	2.02
1910	———	———	9.15	5.36	———	———	4.22	1.93
1911	———	———	9.20	6.03	———	———	5.19	2.93

* See Endres, Forst Politik; Verhältnisse Württembergs; Tharander Forstliches Jahrbuch und Zeitschrift für Forst und Jagdwesen. Figures round.

From the preceding it appears:

1. The gross income per acre in the well kept forests of Württemberg and Saxony with present prices of timber is larger than the gross income from the field and meadow when the quality of soil is considered; and that it is about sixty per cent of crop income even if no allowance is made for the difference of soil.

In Prussia the gross income from forest is about thirty per cent of the gross income from the fields, a difference fully accounted for by the poor quality of the pinery sands devoted to forestry.

2. The net income from forests in Württemberg, Baden, Saxony, etc., state forests, is as large and often larger than the net income from farm lands in the same districts. This, of course, is chiefly due to the smaller cost of production since this cost is normally below fifty per cent of the value of the crop in forestry and above sixty-five per cent in farming.

3. The gross crop income and net crop income from farmland in the United States is smaller than that obtained from the state forests of Württemberg, Baden, Saxony, etc., and is but little higher than that from the poor pinery forests of Prussia.

Even if only the improved lands of our farms are considered, the gross income is little higher than that of the state forests of Württemberg and Saxony and the net income is even lower.

To some extent this condition is due, no doubt, to the fact that farm products in the United States and abroad are not paid for at a proper price in keeping with the value of soil and the cost of production. If correctly adjusted the field with its better soil and large amount of labor should produce at least twice the gross income obtained from the forest. But the fact remains that the growth produced in the forest is very large and also that the forest, owing to the large crop-capital in the form of growing stock is entitled to a correspondingly large net income.

Summing up:

1. The forest can utilize colder, frostier sites.

2. It can use rough, steep, stony and poorly drained land.

3. It can use poor soils, especially sands.

4. It maintains the fertility of the soil and therefore is a much surer crop than any farm crop.

5. Bad seasons are averaged up in forestry, an excellent stand harvested in 1914 may have been through many poor seasons but their effect is no longer seen.

6. Its products are enduring and generally gain in value by storage.

7. The crop is not ripened in a few days or even months, but can be cut at any time from the age of fifty to one hundred and fifty years.

8. It requires less labor than the farm crop.

Agricultural use on the other hand:

1. Is possible or practicable on probably less than fifty per cent of the land area of the United States.
2. It is the most important use of land.
3. It uses more unskilled labor than forestry.
4. It furnishes greater values only from better lands.
5. It furnishes less material per acre for transportation and further manufacture.
6. It encourages erosion, rapid run off of water and so injures land and disturbs water distribution.

3. Use of land for range.

Real range lands occur chiefly in dry districts, prairie and desert, and mountain countries where the forest can not hold its own on account of conditions of temperature, or moisture, or both. On better prairies of the West the range has given way to the farm. On poor soils in forest districts like the southern pinery and the sands of the Great Lake region the forage plants are readily crowded out by more frugal but useless plants, so that grazing on these lands is of very little value, and can not be compared with the use of these same lands for forest. The poor jack pine lands will grow fifty cents worth of jack pine and scrub oak per acre and year, but they will never be worth ten cents per acre and year continuously as range.

The following general averages describe the range conditions in western United States:

Number of head of range live stock and their equivalent in sheep.

United States.		West. West of plains.	Equivalent in sheep in the West.
Cattle	61 million	9 million, or 15%	50 million
Horses	24 million	2.5 million, or 10%	20 million
Sheep	52 million	28 million, or 54%	28 million

Assuming that the feed of a sheep is worth sixty cents per year and that the four million acres of tilled agricultural land in the west furnish one-third of all the feed, the range lands, approximately four hundred million acres furnish about forty million dollars worth of feed or ten cents per acre and year.

Where range lands are leased as by the state of Texas and others, or where stock is grazed at fixed rate per head as by the United States Indian Office and the United States Department of Agriculture, the price is generally below five cents per acre and year, or season. From these few figures it is clear that where land can be used for forestry or for farming the use as range is an inexcusable waste.

APPENDIX.

A. Normal Yield Tables.

These tables will prove useful in giving actual reliable data concerning forestry as a business. They will be helpful in preparing problems for class use.

American yield tables are still deficient; practically all data come from wild woods; the growth of the individual tree is usually determined on dominant trees only, the yields per acre on stands far beyond a justifiable rotation; the effects of man's care in thinning are yet to be learned. None of these data are as yet accepted by a representative body of men.

Under these conditions it seemed best to use the German tables as worked out in connection with the forest experiment stations and compiled by Professor Doctor Adam Schwappach, the foremost authority in this line of study.

The division into five sites was agreed to by the Experiment Stations in 1888, and bases the division on the total volume of the main stand and provides for the stand one hundred years old the following relations:

Site.	Pine.	Spruce and fir.	Beech.
I	100	100	100
II	78	82	80
III	60	66	64
IV	43	50	49
V	29	36	35

In Schwappach's tables this relation is fairly adhered to as is apparent from the following: total volume of the pure fully stocked normal stand one hundred years old is:

Site.	Pine.	Spruce.	Oak.	Beech.
I	6600 c. ft.	11600	6200	10100
I	100	100	100	100
II	85	83	79	80
III	69	66	58	64
IV	55	51	—	49
V	43	36	—	35

Since these German tables can not be used in the United States without modification, it seemed unnecessary to tabulate for all five sites, and only the figures for site II as nearly a good average, are here reproduced. Most of the figures are rounded off. The graphs, figures 3-8, follow Schwappach's as given in his admirable "Ertragstafeln d. wichtigeren Holzarten," 1912.

How far these tables may serve to guide and check in our work in the United States is yet to be learned. It seems reasonable, however, that for trees of similar habits, tolerance and soil requirements, etc., the yield per acre is well indicated by height growth, and that as a provisional check we may assume that if spruce in a certain district in the United States has a height growth equal to that of spruce site III in Germany, the yield per acre will also approximate that of site III, etc.

MONEY YIELD TABLE FOR SPRUCE, SITE II.

Middle and N. Germany; Schwappach; 1902.

Only wood 3" and over (Derbholz) considered.

Age.	Main Stand.			Thinnings added.	Total value. Main stand and thinnings	Average Tree of Main Stand.			
	Trees per acre.	Price per 100 c. ft.	Value total per acre.	Value per acre.	per acre.	Height	Diam b. h. o. b.	Volume	Value
		$	$	$	$	ft.	inch.	c. ft.	$
30	1800	4.80	56	—	56	27	3.2	0.5	0.03
40	1020	6.70	172	10	182	41	4.8	2.5	0.17
50	680	8.60	366	41	407	56	6.4	6.	0.54
60	485	9.30	541	104	645	67	8.	12.8	1.11
70	372	11.60	803	207	1010	76	9.5	18.8	2.15
80	295	12.60	972	336	1308	83	10.8	26.5	3.28
90	239	13.25	1092	483	1575	90	12.1	34.5	4.50
100	198	13.80	1183	648	1831	96	13.3	44.	5.90
110	167	14.40	1260	814	2074	101	14.3	53.	7.50
120	140	14.80	1305	969	2274	105	15.6	64.	9.25

MONEY YIELD TABLE FOR PINE, SITE II.

In N. Germany, Schwappach, Kiefer, 1908.

Only for wood 3″ and over (Derbholz).

Age.	Main Stand. Trees per acre.	Main Stand. Price per 100 c. ft. $	Main Stand. Value total per acre. $	Thinnings added. Value per acre. $	Total value. Main stand and thinnings per acre. $	Average Tree of Main Stand. Height ft.	Average Tree of Main Stand. Diam. inch.	Average Tree of Main Stand. Volume c. ft.	Average Tree of Main Stand. Value $
30	1560	5.80	133	17	150	33	3.7	0.9	0.08
40	840	6.10	201	55	256	43	5.3	3.	0.24
50	550	6.40	267	98	365	52	6.7	6.	0.48
60	400	6.70	333	143	476	59	8.0	9.5	0.83
70	290	7.10	423	190	613	65	9.2	14.	1.45
80	240	7.70	499	242	741	71	10.5	19.	2.05
90	200	8.40	567	299	866	75	11.5	24.5	2.83
100	160	9.20	623	360	983	79	12.5	32.	3.90
110	140	10.20	700	428	1128	83	13.5	37.	5.00
120	120	11.20	770	501	1271	85	14.5	44.	6.40
130	105	12.00	838	574	1412	87	15.2	50.	8.00
140	96	12.80	859	639	1498	89	15.6	54.	8.90

MONEY YIELD TABLE FOR OAK, SITE II.

In N. Germany, Schwappach, 1905.

Only the Yield of Stuff 3″ and over (Derbholz).

Age.	Main Stand. Trees per acre.	Main Stand. Price per 100 c. ft. $*	Main Stand. Value total per acre. $	Thinnings added. Value per acre. $	Total value. Main stand and thinnings per acre. $	Average Tree of Main Stand. Height ft.	Average Tree of Main Stand. Diam. inch.	Average Tree of Main Stand. Volume c. ft.	Average Tree of Main Stand. Value $
30	2650	7.85	39	—	39	26	2.	—	0.01
40	1240	7.85	82	27	109	35	3.	—	0.06
50	610	7.85	110	86	196	43	4.9	2.5	0.18
60	370	7.85	150	155	305	51	7.	6.	0.40
70	260	7.85	202	222	424	59	8.5	10.5	0.77
80	195	7.85	244	281	525	64	10.	17.5	1.25
90	145	7.85	295	332	627	69	11.5	27.	2.06
100	125	8.60	373	374	747	73	13.	36.	3.00
110	110	10.00	476	410	886	76	15.	44.	4.34
120	96	11.50	584	442	1026	79	16.	54.	6.05
130	86	13.60	735	472	1207	82	17.	64.	8.50
140	78	15.80	895	503	1398	85	18.	74.	11.40
150	71	17.20	1020	535	1555	88	19.	86.	14.30
160	65	18.00	1100	564	1664	90	19.7	97.	16.90
170	60	18.60	1177	645	1822	92	20.5	108.	19.60
180	55	19.40	1242	690	1932	94	21.2	120.	22.50
190	51	19.40	1277	736	2013	96	22.	130.	25.00
200	48	19.40	1296	786	2082	97	22.6	140.	27.00

* For the Stand below 100 y the value of 11 Mark pro. F. m. was used as the nearest average.

MONEY YIELD TABLE FOR BEECH, SITE II.

N. Germany; Schwappach; 1911.

Only wood 3″ and over (Derbholz).

Heavy Thinning (Lockerer Schluss).

Age.	Main Stand. Trees per acre.	Main Stand. Price per 100 c. ft. $	Main Stand. Value total per acre. $	Thinnings added. Value per acre. $	Total value. Main stand and thinnings per acre. $	Average Tree of Main Stand. Height ft.	Average Tree of Main Stand. Diam. b. h. o. b. inch.	Average Tree of Main Stand. Volume c. ft.	Average Tree of Main Stand. Value $
30	2900	—	—	—	—	28	2.	—	—
40	2100	3.10	41	1	42	41	2.8	0.6	0.02
50	1360	3.80	88	19	107	52	3.8	1.7	0.06
60	780	4.15	127	57	184	62	5.1	4.0	0.16
70	430	4.50	166	103	269	72	6.8	8.5	0.38
80	260	4.90	205	154	359	78	8.5	15.6	0.79
90	187	5.30	248	208	456	85	10.4	23.	1.30
100	153	5.60	292	267	559	91	11.2	31.	1.90
110	131	5.90	341	328	669	96	12.2	39.	2.60
120	112	6.30	388	393	781	101	13.2	47.	3.45
130	93	6.60	428	460	888	105	14.3	59.	4.60
140	76	7.00	458	529	987	109	15.8	70.	6.00

NORMAL YIELD TABLE FOR SPRUCE, SITE II. In Middle and North Germany; Schwappach, 1902.
Only wood 3″ and over (Derbholz) considered.

Age.	Main Stand.						Thinnings.			Total Production in Vol.	Thinnings make of total	Average Yearly Growth per acre.		Current Yearly Growth per acre.	
	No. of trees per acre.	Area Cross Section	Av. Height	Av. Diam. b. h. o. b.	Vol. per acre	Form factor	No. of trees.	Volume per acre. Taken in period	Volume per acre. Total 100			of Main Std.	total	in Vol-	in
		sq. ft.	ft.	inch.	100 c. ft.			c. ft.	c. ft.	100 c. ft.	%	c. ft.	c. ft.	c. ft.	%
25	2540	82	20	2.5	3	202	—	—	—	3	—	14	14	—	—
30	1800	98	27	3.2	10	379	740	40	—	10	4	35	36	143	41.
35	1340	112	34	3.6	17	442	460	85	1	18	6	53	53	160	15.
40	1020	125	41	4.8	25	475	320	158	3	28	10	63	66	186	10.
45	812	136	49	5.6	34	497	208	240	5	39	13	75	86	220	8.7
50	680	148	55	6.4	42	510	132	315	8	50	16	85	101	240	7,1
55	568	158	62	7.2	51	517	112	390	12	63	19	92	114	250	5.9
60	485	168	67	8.0	58	515	83	460	17	75	22	96	124	240	4.7
65	423	174	72	8.8	64	512	62	520	22	86	25	99	133	230	3.9
70	372	178	76	9.5	70	508	51	560	28	98	28	100	138	220	3.4
75	330	182	80	10.2	74	503	42	590	34	108	31	99	143	200	2.9
80	295	185	83	10.8	78	498	35	600	40	118	33	97	147	194	2.6
85	265	187	87	11.5	81	493	30	615	46	127	36	95	149	183	2.3
90	239	188	90	12.1	83	488	26	630	52	135	38	93	150	175	2.1
95	217	189	93	12.7	85	482	22	645	59	144	40	89	152	166	2.1
100	198	189	96	13.3	87	477	19	645	65	152	42	87	152	158	1.8
105	182	186	99	13.6	88	473	16	630	71	159	44	83	152	149	1.7
110	167	185	101	14.3	89	470	15	620	77	166	46	80	150	140	1.6
115	153	184	103	15.0	89	467	14	590	83	172	48	77	150	129	1.4
120	140	182	105	15.6	90	465	13	560	89	179	49.1	75	149	120	1.3

NORMAL YIELD TABLES FOR PINE IN N. GERMANY; SITE II. After Schwappach, Kiefer, 1908.
Only wood 3" and over, i. e., only "Derbholz."

Age	Main Stand. No. of trees per acre.	Main Stand. Area of Cross Section sq. ft.	Main Stand. Av. Height ft.	Main Stand. Diam. d.b.h. o. b. inch.	Main Stand. Vol. per acre 100 c. ft.	Main Stand. Form factor.	Secondary Stand or Thinnings. No. of trees per acre.	Secondary Stand or Thinnings. Area of Cross Section sq. ft.	Secondary Stand or Thinnings. Volume. Taken per period c. ft.	Secondary Stand or Thinnings. Volume. Total Vol. 100 c. ft.	Total Production in Vol., Main and Thin. 100 c. ft.	Thinnings form of Total %	Average Yearly Growth. Main Stand c. ft.	Average Yearly Growth. Total c. ft.	Current Yearly Growth. In Vol. c. ft.	Current Yearly Growth. In %
25	2200	106	28	3.0	9	310	—	8.7	43	—	9	4	37	39	—	—
30	1560	114	33	3.7	15	390	640	10.0	115	1	16	9	50	55	147	15.
35	1080	120	39	4.5	20	440	440	15.6	190	3	23	14	59	69	149	10.
40	840	125	43	5.3	25	463	240	15.2	230	5	30	18	63	77	146	7.
45	640	126	47	6.1	29	468	200	14.8	260	7	36	22	64	83	135	5.4
50	550	131	52	6.7	33	476	90	13.4	270	11	44	25	66	87	126	4.3
55	460	134	56	7.3	36	474	90	12.6	290	14	50	28	64	90	116	3.6
60	400	135	59	8.0	38	470	60	11.3	290	17	55	30	63	92	112	3.1
65	340	137	62	8.6	40	465	60	10.4	290	20	60	32	62	92	106	2.8
70	300	139	65	9.2	42	462	40	10.0	290	22	64	34	60	93	103	2.6
75	260	140	68	9.8	44	460	30	9.6	300	25	69	36	59	93	97	2.3
80	240	140	71	10.5	46	460	20	9.6	300	28	74	38	58	93	93	2.1
85	220	141	73	11.0	48	460	20	9.0	300	32	80	39	56	93	89	1.9
90	200	141	75	11.5	49	459	20	9.0	300	35	84	41	55	93	85	1.8
95	180	140	77	12.0	50	459	20	9.0	300	38	88	42	53	93	82	1.7
100	160	140	79	12.5	51	458	20	8.7	300	41	92	44	52	92	77	1.5
105	150	139	81	13.0	52	458	10	8.7	300	44	96	45	49	90	73	1.4
110	140	138	83	13.5	52	457	10	8.7	290	47	99	47	47	90	69	1.3
115	130	137	84	14.0	53	457	10	8.7	290	49	102	48	46	89	64	1.2
120	120	136	85	14.5	53	456	10	8.2	280	52	105	49	43	87	60	1.1
125	110	134	86	15.0	53	456	10	7.8	260	55	108	50	41	86	55	1.0
130	105	131	87	15.2	53	455	5	7.4	240	57	110	52	40	84	49	0.9
135	100	129	88	15.4	52	455	5	7.0	230	59	111	53	39	83	40	0.8
140	96	127	89	15.6	52	454	4	5.7	220	61	113	54	37	80	32	0.6

NORMAL YIELD TABLE FOR OAK, SITE II. In N. Germany, after Schwappach, 1905.
Only wood 3″ and over, i. e., "Derbholz."

Age.	Main Stand. No. of Trees per acre.	Main Stand. Area of Cross Section sq. ft.	Main Stand. Ave. Height ft.	Main Stand. Diam. b. h. o. b. Ave. inches.	Main Stand. Vol. per acre 100 c. ft.	Main Stand. Form factor.	Secondary Stand or Thinnings. No. of trees.	Secondary Stand or Thinnings. Volume per acre. Taken per period c. ft.	Secondary Stand or Thinnings. Volume per acre. Total 100 c. ft.	Total Production in Vol. 100 c. ft.	Thinnings make of total %	Average Yearly Growth per acre. Main Std. c. ft.	Average Yearly Growth per acre. Total c. ft.	Current Yearly Growth per acre. In Vol. c. ft.	Current Yearly Growth per acre. In %
30	2650	57	26	2.	5	346	—	—	—	5	12	17	19	52	16.
40	1240	71	36	3.2	9.8	403	1410	400	4	14	29	25	35	91	12.
50	610	80	43	4.9	15	450	630	600	10	25	39	32	52	129	10.
60	370	90	51	7.	22	483	240	700	17	39	44	37	66	146	7.8
70	260	98	59	8.5	28	501	110	900	26	54	47	42	77	143	5.6
80	195	104	64	10.	34	513	65	700	33	67	49	43	84	132	4.2
90	145	112	69	11.5	40	517	50	700	40	80	50	44	88	114	3.1
100	125	117	73	13.	45	520	20	500	45	90	50	44	90	97	2.3
110	110	122	76	15.	49	520	15	500	50	99	50	44	90	87	1.9
120	96	126	79	16.	52	522	14	400	54	106	50	43	88	74	1.5
130	86	128	82	17.	55	523	10	400	58	113	51	43	87	69	1.3
140	78	130	85	18.	58	525	8	400	62	120	51	42	86	66	1.2
150	71	131	88	19.	61	526	7	300	65	126	51	40	84	60	1.0
160	65	132	90	20.	63	527	6	400	69	132	52	40	83	54	0.9
170	60	132	92	20.5	65	529	5	300	72	137	52	39	79	52	0.8
180	55	132	94	21.	66	532	5	300	75	141	53	38	78	49	0.8
190	51	131	96	22.	67	535	4	400	79	146	53	36	78	43	0.7
200	48	130	97	22.5	68	537	3	300	82	150	54	34	76	40	0.6

NORMAL YIELD TABLE FOR BEECH, SITE II. For N. Germany; Schwappach, 1911.

Only wood 3″ and over ("Derbholz") considered.
Heavy Thinning ("Lockerer Schluss" of author).

Age.	Main Stand. No. of trees per acre.	Area of Cross Section sq. ft.	Av. Height ft.	Diam. b. h. o. b. Av. inch.	Vol. per acre 100 c. ft.	Form factor.	Secondary Stand or Thinnings. No. of trees.	Volume per acre. Taken in period c. ft.	Total 100 c. ft.	Total Growth in Volume 100 c. ft.	Thinnings make of total %	Average Yearly Growth per acre. Main Std. c. ft.	Total c. ft.	Current Yearly Growth per acre. In Volume c ft.	In %
30	2900	60	28	2.	—	—	—	—	—	—	—	—	—	—	—
35	2500	76	35	2.4	8	306	400	—	—	8	—	22	22	—	—
40	2100	88	41	2.8	14	383	400	70	—	14	4	34	37	132	9.4
45	1710	98	47	3.3	19	471	390	210	2	21	12	43	49	149	7.8
50	1360	102	52	3.8	24	441	350	350	6	30	21	47	60	160	6.8
55	1040	106	57	4.3	28	452	320	460	11	39	28	50	70	169	6.2
60	780	108	62	5.1	31	465	220	530	16	47	34	52	79	172	5.6
65	570	106	67	5.9	34	476	210	560	22	56	39	52	86	169	5.0
70	430	106	72	6.8	37	480	140	570	28	65	43	52	92	163	4.6
75	320	105	75	7.7	39	484	110	570	33	72	46	52	96	158	4.2
80	260	105	78	8.5	41	488	60	570	39	80	49	50	99	155	3.8
85	210	105	81	9.4	42	492	50	570	45	87	51	50	192	152	3.6
90	187	105	85	10.4	44	494	23	570	51	95	43	49	105	152	3.4
95	167	104	88	10.8	46	495	20	570	56	102	55	49	107	149	3.2
100	153	104	91	11.2	48	496	14	570	61	109	56	47	109	146	3.0
105	142	104	94	11.7	49	499	11	570	67	116	57	47	110	143	2.9
110	131	104	96	12.2	51	500	11	570	73	124	59	46	112	143	2.8
115	121	104	98	12.7	52	503	10	570	79	131	60	46	113	143	2.7
120	112	104	101	13.2	53	504	9	570	84	137	61	45	114	137	2.6
125	102	104	103	13.7	54	504	10	570	90	144	62	43	116	132	2.4
130	93	104	105	14.3	55	505	9	560	96	151	63	43	116	129	2.3
135	84	102	107	15.0	56	505	9	550	101	156	64	42	116	126	2.2
140	76	102	109	15.8	57	506	8	550	106	163	65	40	116	123	2.1

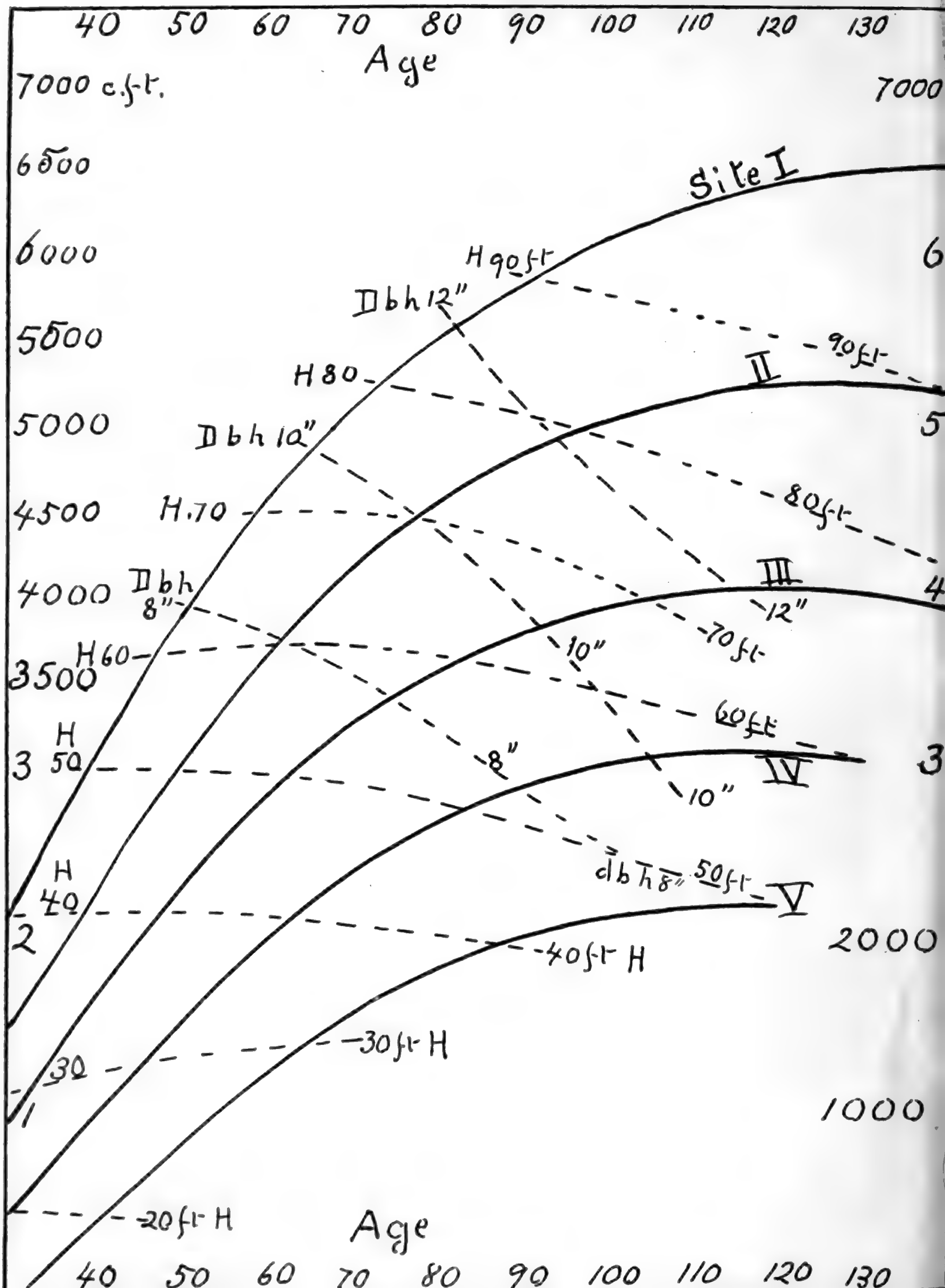

Fig. 3. **Pine**; North Germany Sites I-V; Normal Yield of main stand in cubic feet per acre. Also the points indicating the age at which the average tree attains a diameter of 8", 10", 12" and a height of 30 ft., 40 ft., etc. Only wood 3" and over is here considered.

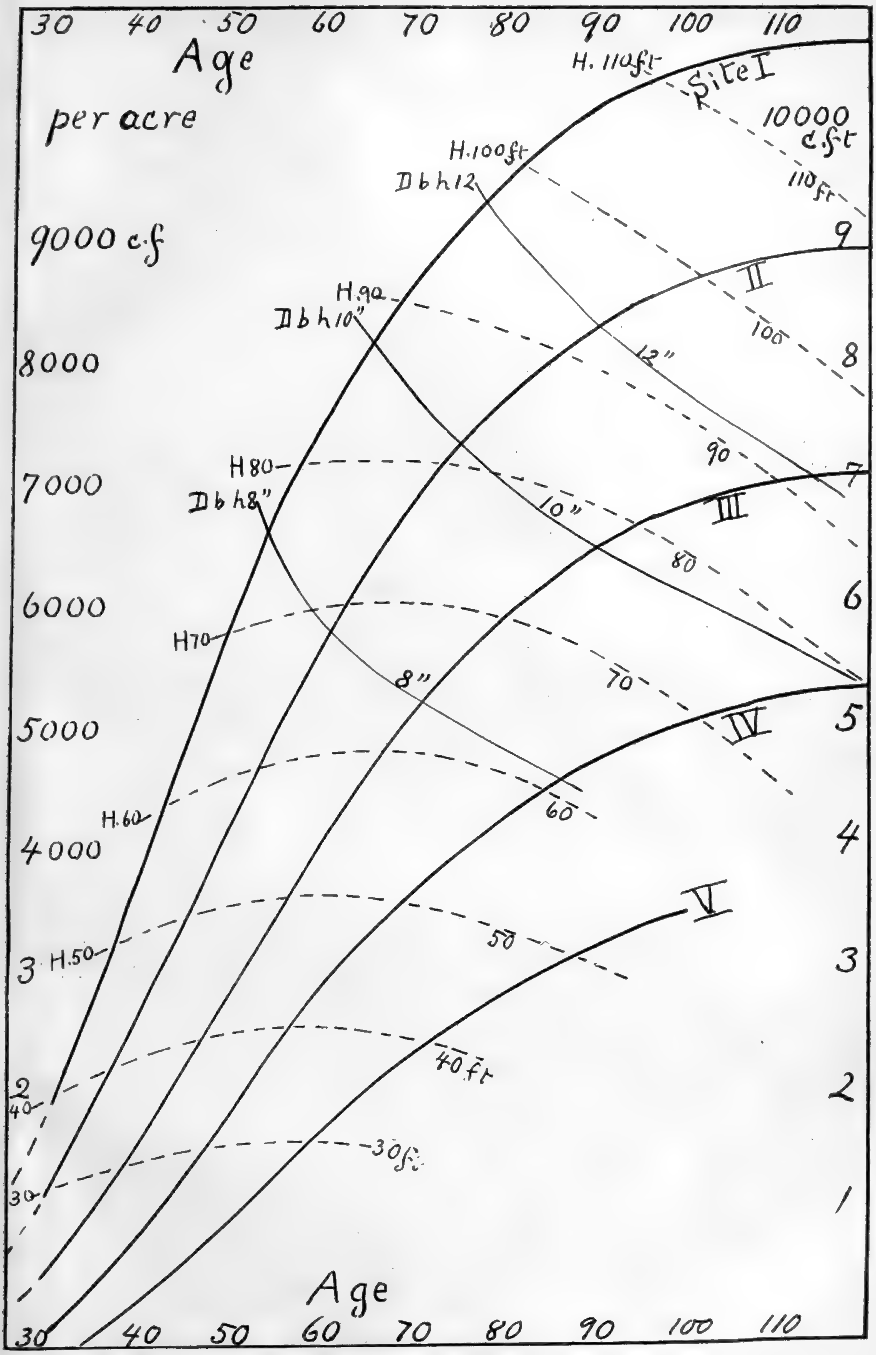

Fig. 4. **Spruce**; Middle and North Germany; Sites I-V; Normal Yield of Main Stand in cubic ft. per acre. Also the points indicating at what age the average tree attains a diameter of 8", 10" and 12", and a height of 40 ft., 50 ft., etc. Only wood 3" and over is here considered.

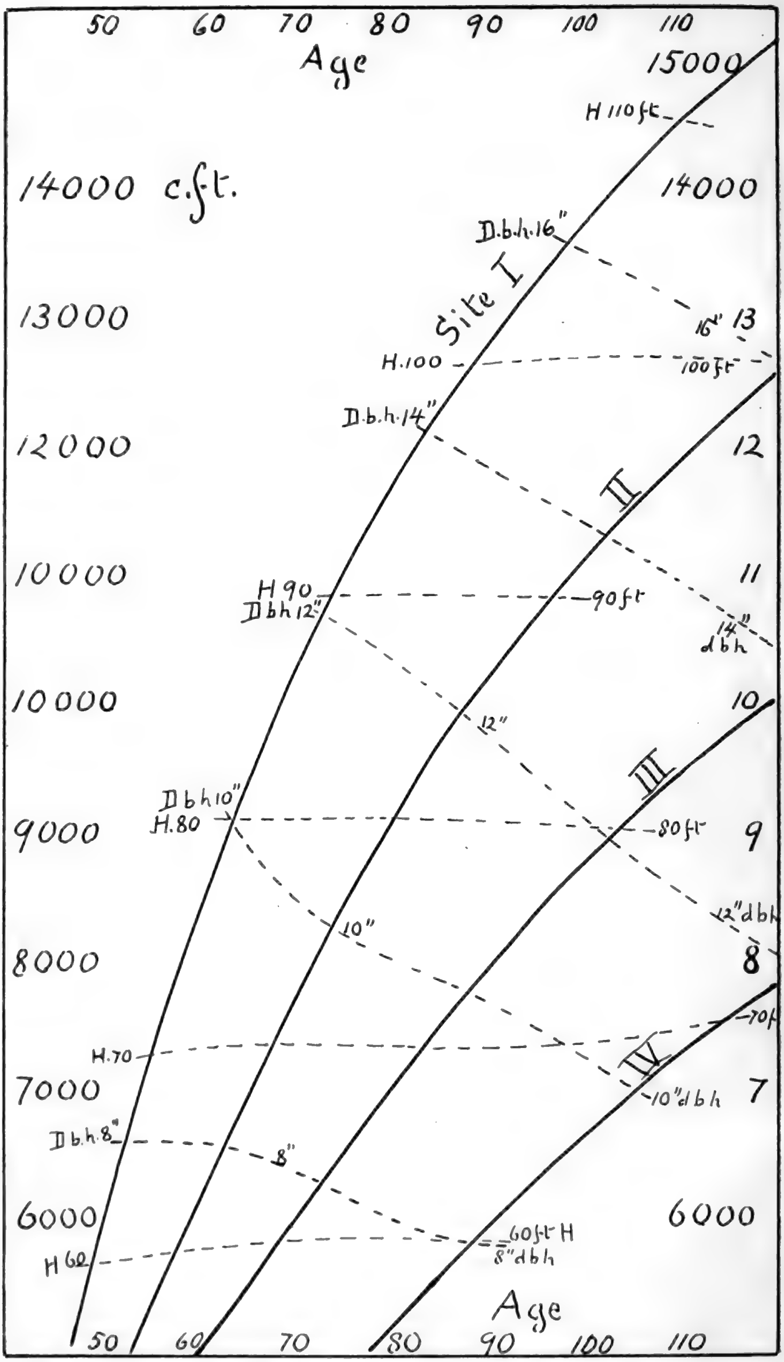

Fig. 5. **White Fir**; South Germany; Sites I-IV; Normal Yield of Main Stand in cubic feet per acre. Also the points indicating at what age the average tree attains a diameter of 8", 10", etc., and a height of 60 ft., 70 ft., etc. Only wood 3" and over is here considered.

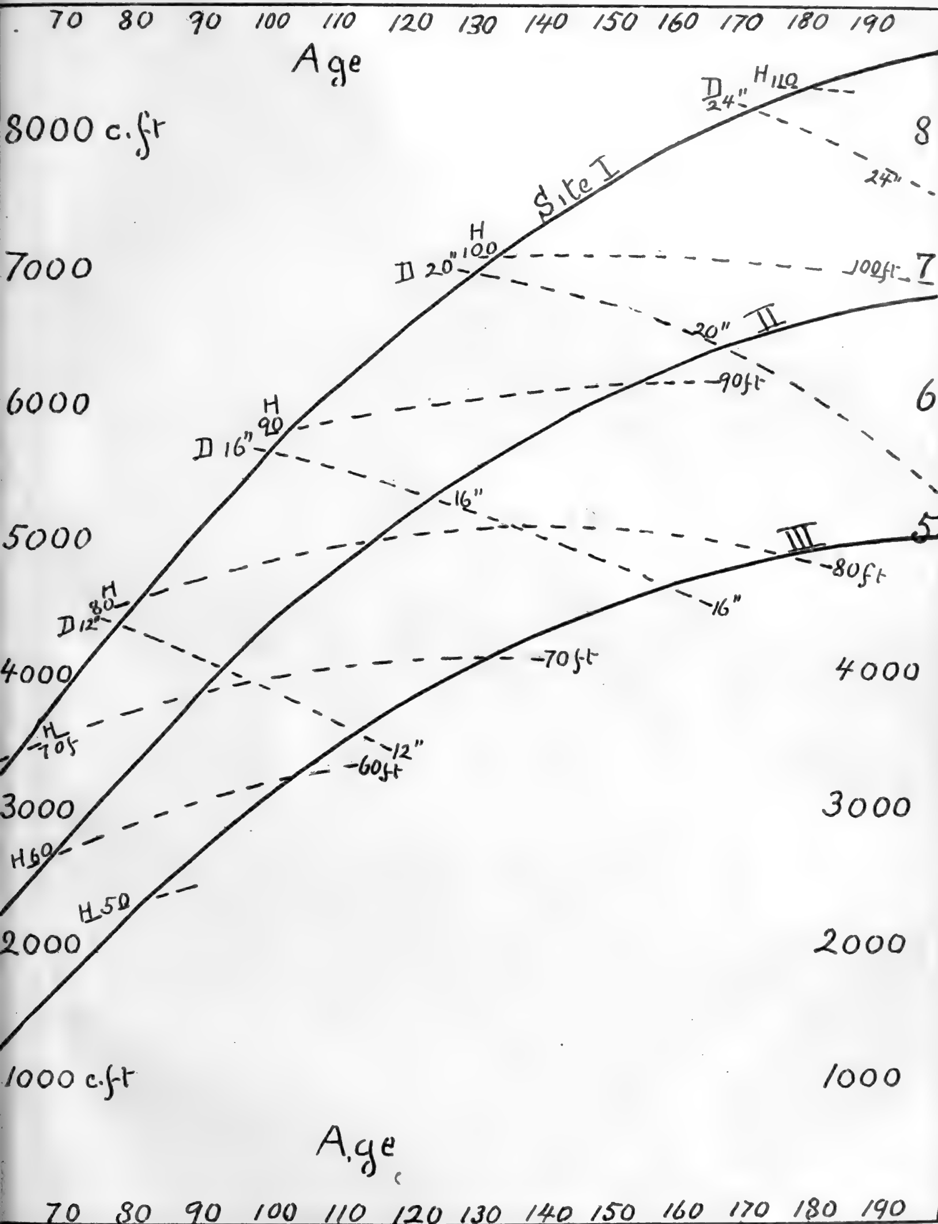

Fig. 6. **White Oak**; N. Germany; Sites I-III; Normal Yield of Main Stand in cubic feet per acre. Also points indicating at what age the average tree attains a diameter of 12″, 16″, 24″, and a height of 60 ft., 70 ft., etc. Only wood 3″ and over is here considered.

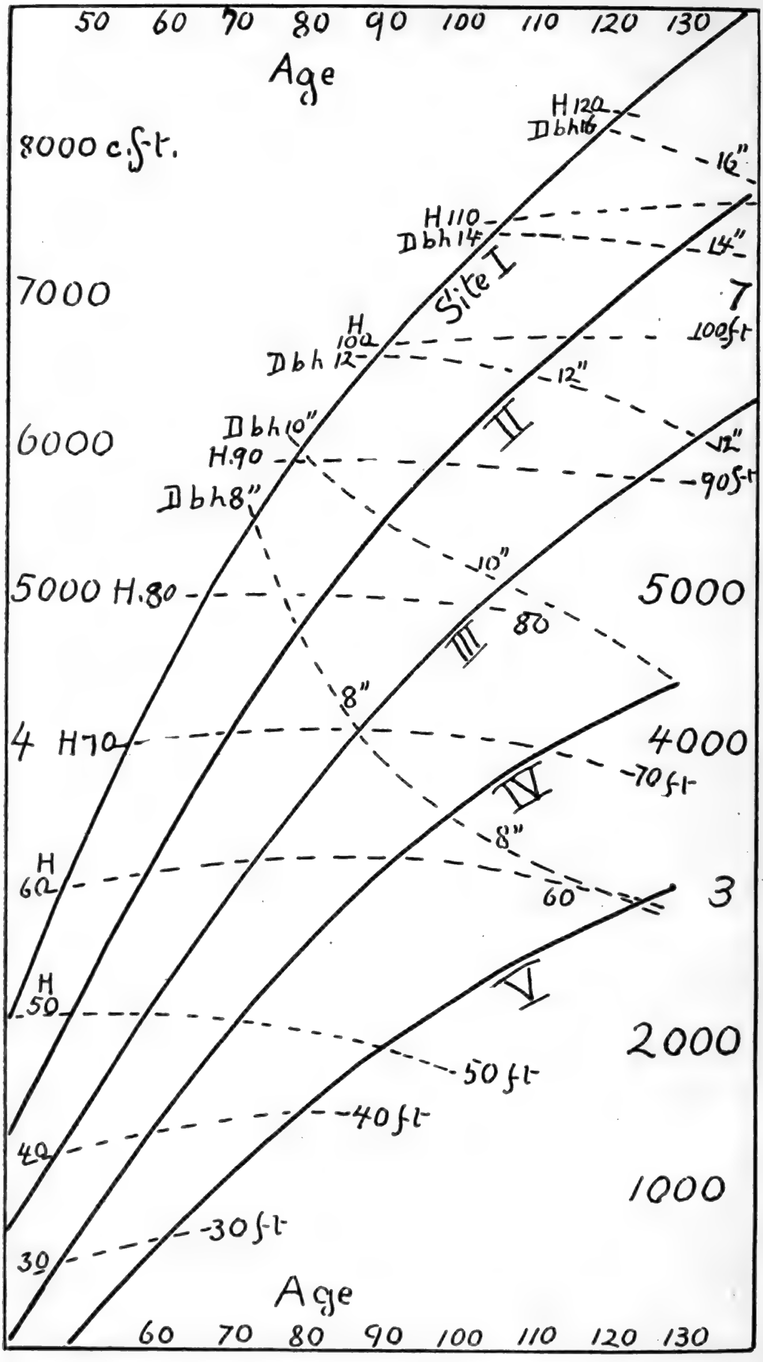

Fig. 7. **Beech**; N. Germany; Sites I-V; Normal Yield of Main Stand in cubic ft. per acre. Also points indicating at what age the average tree attains a diameter of 8″, 10″, etc., and a height of 50 ft., 60 ft., etc. Only wood 3″ and over is here considered.

B. Tables of prolongation and discount.

The following tables follow Kraft's in his "Zuwachsrechnungen." Their use is best illustrated by a few examples:

1. To find capital, Cn.

A stand of timber has now 5,500 cubic feet per acre; a growth study determines that it is growing at 2.5%. What will this stand contain in 15 years, growth being assumed to continue at present rate?

Under 2.5% find figure 1.448 opposite 15 years; multiply 5,500 by 1.448, result is the volume per acre in 15 years. The figure 1.448 is 1.025^{15}.

2. To find the initial capital Co. Same stand during the last 10 years grew at rate of 3%. What was the volume 10 years ago? Find in column of 3% the figure 1.343 opposite 10 years. Divide 5,500 by 1.343, result is the volume 10 years ago.

3. To find the growth in per cent. A stand now 50 years old can be sold at $6.10 per 100 cubic feet. From a growth study it is evident that if kept until 70 years old the stand would bring $10 per 100 cubic feet. What is the per cent growth in quality these 20 years? Keeping in mind that $\frac{Cn}{Co}$ is $1.0p^n$ we have $1.0p^{20} = \frac{10}{6.10} =$ 1.638 and p the rate of growth, 2.%. In teresting and convenient to use in this connection is the fact that the capital growing at compound interest doubles, approximately, every $\frac{72}{p}$ years. To illustrate, one dollar, or one cubic foot grows into two dollars or two cubic feet at 3% in $\frac{72}{3} = 24$ years.

Short Table of

Years	Values of $1.0p^n$, where p —						
n	2	2,5	3	3,5	4	4,5	5
10	1,21	1,28	1,34	1,41	1,48	1,55	1,62
20	1,48	1,63	1,80	1,98	2,19	2,41	2,65
30	1,81	2,09	2,42	2,80	3,24	3,74	4,32
40	2,20	2,68	3,26	3,95	4,80	5,81	7,04
50	2,69	3,43	4,38	5,58	7,10	9,03	11,46
60	3,28	4,39	5,89	7,87	10,51	14,02	18,67
70	3,99	5,63	7,91	11,11	15,57	21,78	30,42
80	4,87	7,20	10,64	15,67	23,04	33,93	49,56
90	5,94	9,22	14,30	22,11	34,11	52,53	80,73
100	7,24	11,81	19,21	31,19	50,50	81,58	131,5
110	8,83	15,12	25,82	43,99	74,75	126,7	214,2
120	10,76	19,35	34,71	62,06	110,6	196,7	348,9
130	13,12	24,78	46,64	87,54	163,8	305,5	568,3
140	15,99	31,72	62,69	123,4	242,4	474,5	925,7
150	19,49	40,60	84,25	174,2	358,9	736,9	1507
160	23,76	51,97	113,2	245,7	531,2	—	—
170	28,97	66,53	152,1	346,6	786,4	—	—
180	35,32	85,17	204,5	488,9	1164	—	—
190	43,05	109,0	274,8	689,7	1723	—	—
200	52,48	139,5	369,3	972,9	2550	—	—

Years n	Values of $1.0p^n$, where p = 0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0
1	1,001	1,002	1,003	1,004	1,005	1,006	1,007	1,008	1,009	1,010
2	1,002	1,004	1,006	1,008	1,010	1,012	1,014	1,016	1,018	1,020
3	1,003	1,006	1,009	1,012	1,015	1,018	1,021	1,024	1,027	1,030
4	1,004	1,008	1,012	1,016	1,020	1,024	1,028	1,032	1,036	1,041
5	1,005	1,010	1,015	1,020	1,025	1,030	1,035	1,041	1,046	1,051
6	1,006	1,012	1,018	1,024	1,030	1,037	1,043	1,049	1,055	1,062
7	1,007	1,014	1,021	1,028	1,036	1,043	1,050	1,057	1,065	1,072
8	1,008	1,016	1,024	1,032	1,041	1,049	1,057	1,066	1,074	1,083
9	1,009	1,018	1,027	1,037	1,046	1,055	1,065	1,074	1,084	1,094
10	1,010	1,020	1,030	1,041	1,051	1,062	1,072	1,083	1,094	1,105
11	1,011	1,022	1,033	1,045	1,056	1,068	1,080	1,092	1,104	1,116
12	1,012	1,024	1,037	1,049	1,062	1,074	1,087	1,100	1,114	1,127
13	1,013	1,026	1,040	1,053	1,067	1,081	1,095	1,109	1,124	1,138
14	1,014	1,028	1,043	1,057	1,072	1,087	1,103	1,118	1,134	1,149
15	1,015	1,030	1,046	1,062	1,078	1,094	1,110	1,127	1,144	1,161
16	1,016	1,032	1,049	1,066	1,083	1,100	1,118	1,136	1,154	1,173
17	1,017	1,035	1,052	1,070	1,088	1,107	1,126	1,145	1,165	1,184
18	1,018	1,037	1,055	1,075	1,094	1,113	1,134	1,154	1,175	1,196
19	1,019	1,039	1,059	1,079	1,099	1,120	1,142	1,163	1,186	1,208
20	1,020	1,041	1,062	1,083	1,105	1,127	1,150	1,173	1,196	1,220
21	1,021	1,043	1,065	1,087	1,110	1,134	1,158	1,182	1,207	1,232
22	1,022	1,045	1,068	1,092	1,116	1,141	1,166	1,192	1,218	1,245
23	1,023	1,047	1,071	1,096	1,122	1,148	1,174	1,201	1,229	1,257
24	1,024	1,049	1,075	1,101	1,127	1,154	1,182	1,211	1,240	1,270
25	1,025	1,051	1,078	1,105	1,133	1,161	1,190	1,220	1,251	1,282
26	1,026	1,053	1,081	1,109	1,138	1,168	1,199	1,230	1,262	1,295
27	1,027	1,055	1,084	1,114	1,144	1,175	1,207	1,240	1,274	1,308
28	1,028	1,058	1,087	1,118	1,150	1,182	1,216	1,250	1,285	1,321
29	1,029	1,060	1,091	1,123	1,156	1,189	1,224	1,260	1,297	1,334
30	1,030	1,062	1,094	1,127	1,161	1,197	1,233	1,270	1,308	1,348
35	1,036	1,072	1,111	1,150	1,191	1,233	1,277	1,322	1,368	1,417
40	1,041	1,083	1,127	1,173	1,221	1,270	1,322	1,375	1,431	1,489
45	1,046	1,094	1,144	1,197	1,252	1,309	1,369	1,431	1,497	1,565
50	1,051	1,105	1,162	1,221	1,283	1,349	1,417	1,490	1,565	1,645
55	1,057	1,116	1,179	1,246	1,316	1,390	1,468	1,550	1,637	1,729
60	1,062	1,127	1,197	1,271	1,349	1,432	1,520	1,613	1,712	1,817
65	1,067	1,139	1,215	1,296	1,383	1,475	1,574	1,679	1,790	1,909
70	1,072	1,150	1,233	1,322	1,418	1,520	1,630	1,747	1,872	2,007
75	1,078	1,162	1,252	1,349	1,454	1,566	1,687	1,818	1,958	2,109
80	1,083	1,173	1,271	1,376	1,490	1,614	1,747	1,892	2,048	2,217
85	1,089	1,185	1,290	1,404	1,528	1,663	1,809	1,969	2,142	2,330
90	1,094	1,197	1,309	1,432	1,567	1,713	1,874	2,049	2,240	2,449
95	1,100	1,209	1,329	1,461	1,606	1,765	1,940	2,132	2,342	2,574
100	1,105	1,221	1,349	1,491	1,647	1,819	2,009	2,219	2,450	2,705
110	1,116	1,246	1,390	1,551	1,731	1,931	2,154	2,402	2,680	2,988
120	1,127	1,271	1,433	1,615	1,819	2,050	2,310	2,602	2,931	3,300

Years n	Values of $1.0p^n$, where p = 1,1	1,2	1,3	1,4	1,5	1,6	1,7	1,8	1,9	2,0
1	1,011	1,012	1,013	1,014	1,015	1,016	1,017	1,018	1,019	1,020
2	1,022	1,024	1,026	1,028	1,030	1,032	1,034	1,036	1,038	1,040
3	1,033	1,036	1,040	1,043	1,046	1,049	1,052	1,055	1,058	1,061
4	1,045	1,049	1,053	1,057	1,061	1,066	1,070	1,074	1,078	1,082
5	1,056	1,061	1,067	1,072	1,077	1,083	1,088	1,093	1,099	1,104
6	1,068	1,074	1,081	1,087	1,093	1,100	1,107	1,113	1,120	1,126
7	1,080	1,087	1,095	1,102	1,110	1,118	1,125	1,133	1,141	1,149
8	1,091	1,100	1,109	1,118	1,126	1,135	1,144	1,153	1,163	1,172
9	1,103	1,113	1,123	1,133	1,143	1,154	1,164	1,174	1,185	1,195
10	1,116	1,127	1,138	1,149	1,161	1,172	1,184	1,195	1,207	1,219
11	1,128	1,140	1,153	1,165	1,178	1,191	1,204	1,217	1,230	1,243
12	1,140	1,154	1,168	1,182	1,196	1,210	1,224	1,239	1,254	1,268
13	1,153	1,168	1,183	1,198	1,214	1,229	1,245	1,261	1,277	1,294
14	1,166	1,182	1,198	1,215	1,232	1,249	1,266	1,284	1,302	1,319
15	1,178	1,196	1,214	1,232	1,250	1,269	1,288	1,307	1,326	1,346
16	1,191	1,210	1,230	1,249	1,269	1,289	1,310	1,330	1,352	1,373
17	1,204	1,225	1,246	1,267	1,288	1,310	1,332	1,354	1,377	1,400
18	1,218	1,240	1,262	1,284	1,307	1,331	1,355	1,379	1,403	1,428
19	1,231	1,254	1,278	1,302	1,327	1,352	1,378	1,404	1,430	1,457
20	1,245	1,269	1,295	1,321	1,347	1,374	1,401	1,429	1,457	1,486
21	1,258	1,285	1,312	1,339	1,367	1,396	1,425	1,455	1,485	1,516
22	1,272	1,300	1,329	1,358	1,388	1,418	1,449	1,481	1,513	1,546
23	1,286	1,316	1,346	1,377	1,408	1,441	1,474	1,507	1,542	1,577
24	1,300	1,331	1,363	1,396	1,429	1,464	1,499	1,535	1,571	1,608
25	1,315	1,347	1,381	1,416	1,451	1,487	1,524	1,562	1,601	1,641
26	1,329	1,364	1,399	1,435	1,473	1,511	1,550	1,590	1,631	1,673
27	1,343	1,380	1,417	1,456	1,495	1,535	1,576	1,619	1,662	1,707
28	1,358	1,397	1,436	1,476	1,517	1,560	1,603	1,648	1,694	1,741
29	1,373	1,413	1,454	1,497	1,540	1,585	1,631	1,678	1,726	1,776
30	1,388	1,430	1,473	1,518	1,563	1,610	1,658	1,708	1,759	1,811
35	1,467	1,518	1,572	1,627	1,684	1,743	1,804	1,867	1,932	2,000
40	1,549	1,611	1,676	1,744	1,814	1,887	1,963	2,041	2,123	2,208
45	1,636	1,710	1,788	1,869	1,954	2,043	2,135	2,232	2,333	2,438
50	1,728	1,816	1,908	2,004	2,105	2,211	2,323	2,440	2,563	2,692
55	1,825	1,927	2,035	2,148	2,268	2,394	2,527	2,668	2,816	2,972
60	1,928	2,046	2,171	2,303	2,443	2,592	2,750	2,917	3,094	3,281
65	2,036	2,171	2,315	2,469	2,632	2,806	2,991	3,189	3,399	3,623
70	2,151	2,305	2,470	2,646	2,835	3,038	3,254	3,486	3,734	4,000
75	2,272	2,446	2,635	2,837	3,055	3,289	3,541	3,811	4,103	4,416
80	2,399	2,597	2,810	3,041	3,291	3,560	3,852	4,167	4,508	4,875
85	2,534	2,756	2,998	3,260	3,545	3,854	4,191	4,556	4,952	5,383
90	2,677	2,926	3,198	3,495	3,819	4,173	4,559	4,981	5,441	5,943
95	2,827	3,106	3,411	3,746	4,114	4,518	4,960	5,446	5,978	6,562
100	2,986	3,296	3,639	4,016	4,432	4,891	5,396	5,954	6,568	7,245
110	3,331	3,714	4,140	4,615	5,144	5,732	6,387	7,116	7,928	8,831
120	3,717	4,185	4,711	5,303	5,969	6,718	7,560	8,506	9,570	10,77

Years	Values of 1.0pⁿ, where p = 2,1	2,2	2,3	2,4	2,5	2,6	2,7	2,8	2,9	3,0
1	1,021	1,022	1,023	1,024	1.025	1,026	1,027	1,028	1,029	1,030
2	1,043	1,045	1,047	1,049	1.051	1,053	1,055	1,057	1,059	1,061
3	1,064	1,068	1,071	1,074	1,077	1,080	1,083	1,086	1,090	1,093
4	1,087	1,091	1,095	1.100	1,104	1,108	1,113	1,117	1,121	1,126
5	1,110	1,115	1,120	1,126	1,131	1,137	1,140	1,148	1,154	1,159
6	1,133	1,140	1,146	1,153	1,160	1,167	1,173	1,180	1,187	1,194
7	1,157	1,165	1,173	1,181	1,189	1,197	1,205	1,213	1,222	1,230
8	1,181	1,190	1,200	1,209	1,218	1,228	1,238	1,247	1,257	1,267
9	1,206	1,216	1,227	1,238	1,249	1,260	1,271	1,282	1,293	1,305
10	1,231	1,243	1,255	1,268	1.280	1,293	1,305	1,318	1,331	1,344
11	1,257	1,271	1,284	1,297	1,312	1,326	1,341	1,355	1,370	1,384
12	1,283	1,299	1,314	1.329	1,345	1,361	1,377	1,393	1,409	1,426
13	1,310	1,327	1,344	1,361	1,379	1,396	1,414	1,432	1,450	1,468
14	1,338	1.356	1,375	1,394	1,413	1,433	1,453	1,472	1,492	1,513
15	1,366	1,386	1,407	1,427	1,448	1,470	1,492	1,513	1,536	1,558
16	1,395	1,417	1,439	1,462	1,485	1,508	1.532	1,556	1,580	1,605
17	1,424	1,448	1,472	1.497	1,522	1,547	1,573	1,599	1,626	1,653
18	1,454	1,480	1,506	1,533	1.560	1,587	1,615	1,644	1,673	1,702
19	1,484	1,512	1,541	1,569	1,599	1,629	1,659	1,690	1,721	1,753
20	1,515	1,545	1,576	1,607	1,639	1,671	1,704	1,737	1,771	1,806
21	1,547	1,579	1,612	1,646	1,680	1,714	1,750	1,786	1,823	1,860
22	1,580	1,614	1,649	1,685	1.722	1,759	1,797	1,836	1,876	1,916
23	1,613	1,650	1,687	1,726	1,765	1,805	1,846	1,887	1,930	1,974
24	1,647	1,686	1,726	1,767	1,809	1,852	1,895	1,940	1,986	2,033
25	1,681	1,723	1,766	1,809	1,854	1,900	1,947	1,995	2,044	2,094
26	1,717	1,761	1,806	1,853	1,900	1,949	1,999	2,050	2,103	2,157
27	1,753	1,800	1,848	1,897	1,948	2,000	2,053	2,108	2,164	2,221
28	1,790	1,839	1,890	1,943	1,997	2.052	2,109	2,167	2,227	2,288
29	1,827	1,880	1,934	1,989	2,046	2,105	2,165	2,227	2,291	2,357
30	1,865	1,921	1,978	2,037	2,098	2,160	2,224	2,290	2,358	2,427
35	2,070	2,142	2,216	2,294	2,373	2,456	2,541	2,629	2,720	2,811
40	2,296	2,388	2,483	2,581	2,685	2.792	2,903	3,017	3,138	3,262 3.262
45	2,548	2,663	2,782	2,907	3,038	3,174	3,316	3,465	3,620	3,782
50	2,827	2,969	3.117	3,274	3,437	3,609	3,789	3,978	4.176	4,384
55	3,136	3,310	3,493	3,686	3,889	4,103	4,329	4,567	4,818	5,082
60	3,480	3,690	3,913	4,150	4,400	4,665	4.946	5,243	5,558	5,892
65	3,861	4,115	4,385	4,672	4,978	5,304	5,650	6,019	6,412	6,830
70	4,284	4.587	4,912	5,260	5,632	6,030	6,455	6,911	7.397	7,918
75	4,753	5,115	5,504	5,923	6,372	6,856	7,375	7,934	8.534	9,179
80	5,273	5,703	6,167	6,668	7,210	7,795	8,426	9,109	9,845	10,64
85	5,850	6,358	6,909	7,508	8,157	8,862	9,627	10,46	11.35	12,34
90	6,491	7,089	7,741	8,452	9,229	10,06	11,00	12,01	13,10	14.30
95	7,702	7,904	8,673	9,517	10,44	11,46	12,57	13,78	15.12	16,58
100	7,991	8,812	9,718	10,72	11,81	13,02	14.36	15,82	17,44	19,22
110	9,836	10,88	12,20	13,58	15,12	16,60	18,74	20,86	23.21	25,83
120	12,11	13,62	15,31	17,21	19,36	21,76	24,46	27,49	30,89	34,71

Years	Values of $1.0p^n$, where p =									
n	3,1	3,2	3,3	3,4	3,5	3,6	3,7	3,8	3,9	4,0
1	1,031	1,032	1,033	1,034	1,035	1,036	1,037	1,038	1,039	1,040
2	1,063	1,065	1,067	1,070	1,071	1,073	1,075	1,077	1,080	1,082
3	1,096	1,100	1,102	1,106	1,109	1,112	1,115	1,118	1,122	1,125
4	1,130	1,134	1,139	1,143	1,148	1,152	1,157	1,161	1,165	1,170
5	1,165	1,171	1,176	1,182	1,188	1,194	1,199	1,205	1,211	1,217
6	1,201	1,208	1,215	1,222	1,229	1,236	1,244	1,251	1,258	1,265
7	1,238	1,247	1,255	1,264	1,272	1,281	1,290	1,298	1,307	1,316
8	1,277	1,287	1,297	1,307	1,317	1,327	1,337	1,348	1,358	1,369
9	1,316	1,328	1,339	1,351	1,363	1,375	1,387	1,399	1,411	1,423
10	1,357	1,370	1,384	1,397	1,411	1,424	1,438	1,452	1,466	1,480
11	1,399	1,414	1,429	1,445	1,460	1,476	1,491	1,507	1,523	1,539
12	1,443	1,459	1,476	1,494	1,511	1,529	1,547	1,565	1,583	1,601
13	1,487	1,506	1,525	1,545	1,564	1,584	1,604	1,624	1,644	1,665
14	1,533	1,554	1,576	1,597	1,619	1,641	1,663	1,686	1,709	1,732
15	1,581	1,604	1,628	1,651	1,675	1,700	1,725	1,750	1,775	1,801
16	1,630	1,655	1,681	1,707	1,734	1,761	1,788	1,816	1,844	1,873
17	1,680	1,708	1,737	1,766	1,795	1,824	1,855	1,885	1,916	1,948
18	1,733	1,763	1,794	1,826	1,857	1,890	1,924	1,957	1,991	2,026
19	1,786	1,819	1,853	1,888	1,923	1,958	1,994	2,031	2,069	2,107
20	1,842	1,878	1,915	1,952	1,990	2,029	2,068	2,108	2,149	2,191
21	1,899	1,938	1,978	2,018	2,059	2,102	2,145	2,189	2,233	2,279
22	1,958	2,000	2,043	2,087	2,132	2,177	2,224	2,272	2,320	2,370
23	2,018	2,064	2,110	2,158	2,206	2,256	2,306	2,358	2,411	2,465
24	2,081	2,130	2,180	2,231	2,283	2,337	2,392	2,448	2,505	2,563
25	2,145	2,198	2,252	2,307	2,363	2,421	2,480	2,541	2,603	2,666
26	2,212	2,268	2,326	2,385	2,446	2,508	2,572	2,637	2,704	2,772
27	2,280	2,341	2,403	2,467	2,532	2,599	2,667	2,737	2,810	2,883
28	2,351	2,416	2,482	2,550	2,620	2,692	2,766	2,841	2,919	2,999
29	2,424	2,493	2,564	2,637	2,712	2,789	2,868	2,949	3,033	3,119
30	2,499	2,573	2,649	2,727	2,807	2,889	2,974	3,061	3,151	3,243
35	2,911	3,012	3,115	3,223	3,334	3,448	3,567	3,639	3,815	3,946
40	3,391	3,525	3,664	3,809	3,959	4,115	4,277	4,445	4,620	4,801
45	3,951	4,127	4,310	4,502	4,702	4,911	5,129	5,357	5,594	5,841
50	4,602	4,830	5,070	5,321	5,585	5,861	6,151	6,455	6,776	7,107
55	5,361	5,654	5,964	6,290	6,633	6,995	7,376	7,778	8,201	8,646
60	6,245	6,619	7,015	7,434	7,878	8,348	8,846	9,373	9,930	10,52
65	7,275	7,748	8,251	8,784	9,357	9,963	10,61	11,29	12,02	12,80
70	8,473	9,069	9,706	10,39	11,11	11,89	12,78	13,59	14,56	15,57
75	9,872	10,62	11,42	12,28	13,20	14,19	15,26	16,40	17,63	18,95
80	11,50	12,43	13,43	14,51	15,68	16,94	18,29	19,76	21,34	23,05
85	13,40	14,56	15,80	17,15	18,62	20,21	21,94	23,81	25,84	28,04
90	15,61	17,03	18,58	20,27	22,11	24,12	26,31	28,69	31,29	34,12
95	18,18	19,93	21,85	23,96	26,26	28,79	31,55	34,58	37,89	41,51
100	21,16	23,32	25,71	28,32	31,19	34,36	37,82	41,66	45,88	50,50
110	28,74	31,97	35,57	39,56	44,00	48,93	54,41	[illegible]	67,28	74,76
120	39,00	43,81	49,21	55,27	62,06	69,69	78,25	87,84	98,59	110,7

Years n	Values of $1.0p^n$, where p = 4,1	4,2	4,3	4,4	4,5	4,6	[illegible]	4,8	4,9	5,0
1	1,041	1,042	1,043	1,044	1,045	1,046	1,047	1,048	1,049	1,050
2	1,083	1,086	1,088	1,090	1,092	1,094	1,096	1,098	1,100	1,103
3	1,128	1,131	1,135	1,138	1,141	[illegible]	1,148	1,151	1,154	1,158
4	1,174	1,179	1,184	1,188	1,193	[illegible]	1,202	1,206	1,211	1,216
5	1,223	1,228	1,234	1,240	1,246	1,252	1,258	1,264	1,270	1,276
6	1,273	1,280	1,287	1,295	[illegible]	1,310	1,317	1,325	1,333	1,340
7	1,325	1,334	1,343	1,352	1,361	1,370	1,379	1,389	1,398	1,407
8	1,379	1,390	1,401	1,411	1,422	1,433	1,444	1,455	1,466	1,478
9	1,436	1,448	1,461	1,473	1,486	1,499	1,512	1,532	1,539	1,551
10	1,495	1,509	1,524	1,539	1,553	1,568	1,583	1,598	1,614	1,629
11	1,556	1,573	1,589	1,610	1,623	1,640	1,657	1,675	1,693	1,710
12	1,620	1,638	1,657	1,677	1,696	1,716	1,735	1,755	1,776	1,796
13	1,686	1,707	1,729	1,750	1,772	1,794	1,817	1,840	1,863	1,886
14	1,755	1,779	1,803	1,827	1,852	1,877	1,902	1,928	1,954	1,980
15	1,827	1,854	1,881	1,908	1,935	1,963	1,992	2,020	2,050	2,079
16	1,902	1,932	1,961	1,992	2,022	2,054	2,085	2,117	2,150	2,183
17	1,980	2,013	2,046	2,079	2,113	2,148	2,183	2,219	[illegible]	[illegible]
18	2,061	2,097	2,134	2,171	2,208	2,247	2,286	2,326	[illegible]	2,407
19	2,146	2,185	2,225	2,266	2,308	2,350	2,393	2,437	2,482	2,527
20	2,234	2,277	2,321	2,366	2,412	2,458	2,506	2,554	2,603	2,653
21	2,325	2,373	2,421	2,459	2,520	2,571	2,624	2,677	2,731	2,786
22	2,421	2,472	2,525	2,579	2,634	2,690	2,747	2,805	2,865	2,925
23	2,520	2,576	2,634	2,692	2,752	2,813	2,876	2,940	3,005	3,072
24	2,623	2,684	2,747	2,811	2,876	2,943	3,011	3,081	3,152	3,225
25	2,731	2,797	2,865	2,934	3,005	3,078	3,153	3,229	3,307	3,386
26	2,843	2,915	2,988	3,064	3,141	3,220	3,301	3,384	3,469	3,556
27	2,959	3,037	3,117	3,198	3,282	3,368	3,456	3,546	3,639	[illegible]
28	3,081	3,164	3,251	3,339	3,430	3,523	3,618	3,716	3,817	3,920
29	3,207	3,297	3,390	3,486	3,584	3,685	3,788	3,895	4,004	4,110
30	3,338	3,436	3,536	3,640	3,745	3,854	3,967	4,082	4,200	4,322
35	4,081	4,221	4,365	4,514	4,667	4,826	4,990	5,160	5,335	5,516
40	4,989	5,185	5,387	5,598	5,816	6,043	6,279	6,523	6,777	7,040
45	6,099	6,369	6,650	6,943	7,248	7,567	7,900	8,246	8,608	8,985
50	7,257	7,823	8,208	8,611	9,033	9,475	9,939	10,43	10,93	11,47
55	9,116	9,610	10,13	10,68	11,26	11,87	12,51	13,18	13,89	14,64
60	11,15	11,81	12,51	13,25	14,03	14,86	15,73	16,66	17,64	18,68
65	13,62	14,50	15,44	16,43	17,48	18,60	19,79	21,06	22,41	23,84
70	16,66	17,81	19,05	20,37	21,78	23,29	24,90	26,63	28,46	30,43
75	20,36	21,88	23,52	25,27	27,15	29,17	31,33	33,66	36,16	38,83
80	24,84	26,88	29,03	31,34	33,93	36,52	39,42	42,55	45,93	49,56
85	30,43	33,02	35,82	38,86	42,16	45,73	49,60	53,79	58,33	63,25
90	37,20	40,56	44,22	48,20	52,54	57,26	62,41	68,00	74,10	80,73
95	45,48	49,82	54,58	59,78	65,47	71,70	[illegible]	85,97	94,12	103,0
100	55,60	61,20	67,37	[illegible]	81,59	89,78	[illegible]	108,7	119,6	131,5
110	83,10	92,35	102,6	[illegible]	126,7	140,8	[illegible]	173,7	192,9	214,2
120	124,2	139,4	156,4	[illegible]	196,8	220,7	[illegible]	277,5	311,2	348,9

Years n	Values of $1.0p^n$, where p = 5,1	5,2	5,3	5,4	5,5	5,6	5,7	5,8	5,9	6,0
1	1,051	1,052	1,053	1,054	1,055	1,056	1.057	1,058	1,059	1,060
2	1,105	1,107	1.109	1,111	1,113	1,115	1,117	1,119	1,122	1,124
3	1,161	1.164	1,168	1,171	1,174	1,178	1,181	1,184	1,188	1,191
4	1,220	1,226	1,230	1,234	1,239	1,244	1.248	1,253	1,258	1,262
5	1,282	1,289	1,295	1,301	1,307	1,313	1,319	1,326	1,332	1,338
6	1,348	1.356	1,363	1,371	1,379	1,387	1,395	1,403	1,411	1,419
7	1,417	1.423	1,436	1,445	1,455	1,464	1,474	1,484	1,494	1,504
8	1,489	1,500	1,512	1,523	1,535	1,547	1.558	1,570	1,582	1,594
9	1,565	1,578	1,592	1,604	1,619	1,633	1,647	1,661	1,675	1,689
10	1,645	1.660	1,676	1,692	1,708	1,724	1,741	1,757	1,774	1,791
11	1.729	1,747	1,765	1,783	1,802	1,821	1,840	1,859	1,879	1,898
12	1,817	1,837	1,859	1,880	1,901	1,923	1.945	1,967	1,990	2,012
13	1,909	1,933	1,957	1,981	2,006	2,031	2.056	2,081	2,107	2,133
14	2,007	2.033	2,061	2,088	2,116	2,144	2,173	2,202	2,231	2,261
15	2,109	2,139	2,170	2,201	2,233	2,265	2,297	2,330	2,363	2,397
16	2.216	2,250	[illegible]	2,320	2,355	2,391	2.428	2,465	2,502	2,540
17	2,329	2,367	[illegible]	2,445	2,485	2,525	2,566	2,608	2,650	2,693
18	2,448	2,491	[illegible]	2.577	2,621	2,667	2,712	2,759	2,806	2,854
19	2,573	2.620	2.668	2,716	2,766	2,815	2,867	2,919	2,972	3,026
20	2,704	2,756	2,809	2,863	2,918	2.974	3.030	3,088	3,147	3,207
21	2,842	2,900	2,958	3,018	3,078	3,140	3.203	3,267	3,383	3,400
22	2,987	3.050	3,115	3,181	3,248	3,316	3,386	3,457	3,530	3,604
23	3,136	3.209	3,280	3,352	3,426	3,502	3,579	3,657	3,738	3,829
24	3,300	3,376	3,454	3,533	3,615	3,698	3.783	3,870	3,958	4,049
25	3,468	3.551	3,637	3,724	3,813	3,905	3,998	4,094	4,192	4,292
26	3,645	3.736	3,830	3,925	4,023	4,124	4,226	4.332	4,439	4,550
27	3,831	3,930	4.033	4,137	4,245	4,354	4,467	4,583	4,701	4,822
28	4,026	4,135	4,246	4,361	4,478	4,598	4,722	4.850	4,978	5,112
29	4,231	4.350	4.471	4,596	4,724	4,856	4,993	5,130	5,272	5,418
30	4.447	4.576	4,708	4,844	4,984	5,128	5,275	5,427	5,583	5,744
35	5.703	5,896	6,095	6,301	6,514	6.734	6.960	7,195	7,436	7,686
40	7,313	7,596	7,891	8,197	8,513	8,842	9,184	9,537	9,905	10,29
45	9,378	9.788	10,22	10,66	11,13	11,61	12,12	12,64	13,19	13,77
50	12.03	12,61	13,23	13,87	14,54	15,25	15,99	16,76	17,57	18,42
55	15,42	16,25	17,02	18,04	19,01	20,02	21,09	22,23	23,40	24,65
60	19,78	20,94	22,17	23,47	24,84	26,29	27,83	29.45	31,17	32,99
65	25,36	26.98	28,70	30,52	32,46	34,53	36,72	39,05	41,52	44,14
70	32.52	34,76	37,15	39,71	42,43	45,34	48,15	51,76	55,30	59,08
75	41.71	44,79	48,10	51,65	55,46	59,54	63,92	68,62	73,66	79,06
80	53,48	57,71	62,27	67,18	72,48	78,18	84,33	90.96	98,10	105,8
85	68.59	74 36	80,62	87,38	94,73	102,7	111,3	120,6	130,6	141,6
90	87.95	95,81	104,4	113,7	123,8	134,8	146,8	159,9	174,0	189,5
95	112,8	123 4	135,1	147,9	161,8	177,0	193.7	211,9	231,8	253,5
100	144,6	159,1	174,9	192,3	211,5	232,5	249,8	280,9	305,8	339,3
110	237,9	264.1	[illegible]	325,4	361.2	400,9	444,9	493,7	547,7	607,6
120	391,1	438,4	[illegible]	550,7	617,0	691,3	488,7	867,5	[illegible]	1088

Years n	Values of $1.0p^n$, where p = 6,2	6,4	6,6	6,8	7,0	7,2	7,4	7,6	7,8	8,0
1	1,062	1,064	1,066	1,068	1,070	1,072	1,074	1.076	1,078	1,080
2	1,127	1,132	1,136	1,141	1,145	1,149	1,154	1,158	1,162	1,166
3	1,198	1,205	1,211	1,218	1,225	1,232	1,239	1,246	1,253	1,260
4	1,272	1,282	1,291	1,301	1,311	1,321	1,331	1,341	1,351	1,361
5	1,351	1,364	1,377	1,390	1,403	1,416	1,429	1,442	1,456	1,469
6	1,435	1.451	1,467	1,484	1,501	1,518	1,535	1,552	1,569	1,587
7	1.524	1,544	1,564	1,585	1,606	1,627	1,648	1,670	1,692	1,714
8	1,618	1,643	1,668	1,693	1,718	1,744	1,770	1,797	1,824	1,851
9	1,718	1,748	1,778	1,808	1,839	1,870	1,901	1,933	1,966	1,999
10	1,825	1.860	1,895	1,931	1,967	2,004	2,042	2,080	2,119	2,159
11	1,938	1,979	2,020	2,062	2,105	2,149	2,193	2,238	2,285	2,326
12	2,058	2,105	2,153	2,202	2,252	2,303	2.355	2,409	2,463	2,518
13	2,186	2,240	2,295	2,352	2,407	2,469	2,530	2,592	2,655	2,720
14	2,321	2.383	2,447	2,512	2,579	2,647	2,716	2,789	2,862	2,938
15	2,465	2,535	2,608	2,683	2,759	2,838	2,918	3,001	3,085	3,172
16	2,618	2,698	2,784	2,865	2,952	3,042	3.133	3,229	3,326	3,426
17	2,781	2,870	2,964	3,060	3,159	3,261	3,366	3,474	3,585	3,700
18	2,953	3,055	3,160	3,268	3,380	3,496	3,615	3,738	3,865	3,996
19	3.136	3,250	3,368	3,490	3,617	3,747	3,882	4,022	4,166	4,316
20	3,330	3,458	3,590	3,728	3,870	4,017	4,170	4,328	4,491	4,661
21	3,537	3,679	3,827	3,981	4,141	4,306	4,478	4,657	4,842	5,034
22	3,756	3.915	4,080	4,252	4,431	4,616	4,810	5,010	5,219	5,437
23	3,989	4,165	4,349	4,541	4,741	4,949	5,165	5,391	5,627	5,872
24	4,236	4,432	4,636	4,850	5,072	5,305	5.548	5,801	6,065	6,341
25	4,499	4,716	4,942	5,180	5,428	5,687	5,958	6,213	6,538	6,849
26	4,778	5.018	5,269	5,532	5,807	6,096	6,399	6,716	7,048	7,396
27	5.074	5,339	5,616	5,908	6,214	6,535	6,873	7,227	7,598	7,988
28	5,389	5,680	5,987	6,310	6,649	7,005	7.381	7,776	8,191	8,627
29	5,723	6,044	6,382	6,739	6,794	7,510	7,927	8,367	8,830	9,317
30	6,078	6,431	6,803	7,197	7,612	8,051	8,514	9,003	9,518	10,06
35	8.210	8,769	9,365	10,00	10,68	11,40	12,17	12,99	13,86	14,79
40	11,09	11,96	12,89	13,90	14,98	16,14	17.39	18,73	20,17	21,73
45	14,98	16,31	17,75	19,31	21,00	22,85	24,84	27,01	29,37	31,92
50	20,24	22,24	24,43	26,82	29,46	32,34	35,50	38,96	42,75	46,90
55	27,34	30,33	33,62	37,28	41,32	45,79	50,73	56,19	62,24	68,92
60	36,94	41,35	46,29	51,79	57,95	64,82	72,49	81,05	90,60	101,3
65	49,90	56,39	63,71	71,97	81,27	91,76	103,5	116,4	131,9	149,1
70	67,41	76,90	87,70	100,0	114,0	129,9	148,0	168,6	192,0	218,6
75	91,06	104,9	120,7	139,0	159,9	183,9	211,5	243.2	279,5	321,2
80	123,0	143,0	166,2	193,1	224,2	260,4	302,2	350,7	406,9	472,0
85	166,2	195,0	228,8	268,3	314,5	368,6	431,9	505,8	592,4	693,5
90	224,5	265,9	314,9	372,8	441,1	521,8	617,1	729,6	862,4	1019
95	303,3	362,6	[illegible]	517,9	618,7	738,8	881,9	1052	1256	1497
100	409,7	494,5	[illegible]	719,7	867,7	1046	1260	1518	1828	2200

Years n	Values of $1.0p^n$, where p = 8,2	8,4	8,6	8,8	9,0	9,2	9,4	9,6	9,8	10
1	1,082	1,084	1,086	1,088	1,090	1,092	1,094	1,096	1,098	1,100
2	1,171	1,175	1,179	1,184	1,188	1,193	1,197	1,201	1,206	1,210
3	1,267	1,274	1,281	1,288	1,295	1,302	1,309	1,317	1,324	1,331
4	1,371	1,381	1,391	1,401	1,412	1,422	1,432	1,443	1,454	1,464
5	1,483	1,497	1,510	1,525	1,539	1,552	1,567	1,582	1,596	1,611
6	1,605	1,623	1,641	1,659	1,677	1,696	1,714	1,733	1,752	1,772
7	1,736	1,759	1,782	1,805	1,828	1,852	1,876	1,900	1,924	1,949
8	1,878	1,907	1,935	1,964	1,993	2,022	2,052	2,082	2,113	2,144
9	2,033	2,067	2,101	2,136	2,172	2,208	2,245	2,282	2,320	2,358
10	2,199	2,240	2,282	2,324	2,367	2,411	2,456	2,501	2,547	2,594
11	2,380	2,429	2,478	2,529	2,581	2,633	2,687	2,741	2,797	2,853
12	2,575	2,632	2,691	2,751	2,813	2,875	2,939	3,004	3,071	3,139
13	2,786	2,854	2,923	2,994	3,066	3,133	3,215	3,293	3,372	3,452
14	3,014	3,093	3,174	3,257	3,337	3,429	3,518	3,609	3,702	3,798
15	3,262	3,353	3,447	3,544	3,643	3,744	3,848	3,955	4,065	4,177
16	3,529	3,635	3,744	3,855	3,970	4,089	4,210	4,334	4,463	4,595
17	3,818	3,940	4,066	4,195	4,328	4,465	4,606	4,751	4,901	5,055
18	4,131	4,271	4,415	4,564	4,417	4,875	5,038	5,207	5,381	5,560
19	4,470	4,630	4,795	4,965	5,142	5,324	5,512	5,707	5,908	6,116
20	4,837	5,019	5,207	5,402	5,605	5,814	6,030	6,254	6,487	6,728
21	5,233	5,440	5,655	5,878	6,109	6,349	6,597	6,855	7,123	7,400
22	5,662	5,897	6,141	6,395	6,659	6,933	7,217	7,513	7,821	8,140
23	6,127	6,393	6,669	6,958	7,258	7,571	7,896	8,235	8,587	8,954
24	6,629	6,930	7,243	7,570	7,911	8,267	8,638	9,025	9,429	9,850
25	7,173	7,512	7,866	8,236	8,623	9,028	9,450	9,892	10,35	10,84
26	7,761	8,143	8,542	8,961	9,399	9,858	10,34	10,84	11,37	11,92
27	8,397	8,827	9,277	9,750	10,25	10,77	11,31	11,88	12,48	13,11
28	9,086	9,568	10,08	10,61	11,17	11,76	12,37	13,02	13,71	14,42
29	9,831	10,37	10,94	11,54	12,17	12,84	13,54	14,27	15,05	15,86
30	10,64	11,24	11,88	12,56	13,27	14,03	14,81	15,64	16,52	17,45
35	15,78	16,83	17,95	19,14	20,41	21,77	23,21	24,73	26,37	28,10
40	23,39	25,19	27,11	29,19	31,41	33,80	36,37	39,12	42,08	45,26
45	34,69	37,70	40,96	44,49	48,33	52,48	56,99	61,87	67,16	72,89
50	51,45	56,43	61,87	67,83	74,36	81,50	89,30	97,84	107,2	117,4
55	76,30	84,45	93,46	103,4	114,4	126,6	140,0	154,7	171,1	189,1
60	113,2	126,4	141,2	157,7	176,0	196,5	219,3	244,7	273,0	304,5
65	167,8	189,2	213,3	240,4	270,8	305,1	343,7	387,0	435,7	490,4
70	248,8	283,2	322,2	366,5	416,7	473,8	538,5	612,0	695,3	789,8
75	369,0	423,8	486,7	558,7	641,2	735,7	843,9	967,8	1110	1272
80	547,3	634,4	735,2	851,8	986,6	1142	1323	1531	1771	2048

C. Suggestions for problems for use in teaching.

The following problems are offered here merely as suggestions. Some practice in the use of the analyses and formulae has been found necessary to fix these matters in the minds of the students.

1. Capital Co = \$50, p = 3%; what does this capital grow into in 7, 19, 37, and 83 years?

2. Capital now is \$500; has been out at 2½% compound interest; what was it worth 17, 43, and 120 years ago?

3. \$140 grew into \$190 in 17 years; what is $1.0p^{17}$ and what is p?

4. A tree contains 430 cubic feet; it is growing at 1.2%; what is the growth for 5 years in cubic feet?

5. A tree now has 430 cubic feet; 23 years ago it had 375; at what p did it grow, what is $1.0p^n$ in this case?

6. Schneider's formula says: $p_v = \frac{400}{nD}$ for older timber (D being dbh and n the number of rings in outer one inch). Suppose stand of timber has 40,000 cubic feet; average D 21 inches, and n 12; at what rate is this stand growing, what is the value of five years growth at 7 cents per cubic foot?

7. A locomotive sets fire to a stand of pine planted 25 years ago; it cost 10 dollars an acre to plant; what is the loss if only planting expense and interest at 3% is counted?

8. A stand is now 35 years old. We expect it to be worth \$350 when 80 years old; what is its present value; p = 3%, and no expenses figured.

9. A farm rents at 100 dollars, clear, over and above expenses. Rate per cent 3% and also 2%. What is the value of the farm and what is the value of 25 years use of it?

10. A forest property is worth \$10,000, based on income, but it can not be sold until 15 years from now; the property is used and no change is expected. What is its value today at 2% and at 4%?

11. A note for \$500 at 4% is due in 7 years; it has run 3 years; all interest and capital to be paid in at end of 7 years. What is the note worth now to a man who is willing to take 3% on his money?

12. A stand of timber contains 550 cords; it contained 470, 10 years ago; what was p_v for this time? If this timber now brings \$4.50 per 100 cubic feet but brought only \$3.75 per 100 cubic feet, 10 years ago, what is p_q, quality growth, in per cent? If there is no price growth, what is p_f, i. e., the growth in value of this stand?

13. What is the growth, in quality in per cent, for each decade which will keep the growth in value constant between the ages 40 and 100 years in spruce, site II, as in Schwappach tables?

14. Eight hundred acres of timber is cut over every 10 years and nets $12 per acre at each cut. Expenses per acre: taxes 15 cents, protection 10 cents. What is the value of the forest at 2% and at 3%? What is 100 years lease worth if the last cut came 6 years ago?

15. One hundred acres of land may be farmed. At farming it brings 75 cents per acre net. If planted to pine and $r = 80$ years; $Yr = 60$ cords at $3; $C = \$10$; $e = 25$ cents; $p = 4$, 3, and 2%—which use is more profitable?

16. Six thousand acres of North Carolina pinery cut over lands bought at $5 per acre; expenses per acre: taxes 10 cents, protection 5 cents, $p = 3\%$; what is Fc in 25 years? If timber is $6 per M. feet, how much must one get to make interest and expenses, and leave the forest as good as when bought?

17. Mountain lands in Pennsylvania; can return every 15 years for 6 cords of hardwood stuff, mostly coppice, worth $2 per cord on stump. Taxes 5 cents per acre; no other charges; what is the value of the woods at 3%?

18. A farmer has 60 acres inferior lands; he paid $18 an acre. He has rented this at $1 per acre for pasture. Expenses per acre: taxes 15 cents, fences 10 cents, $p = 3\%$. He plants up 30 acres to hardwoods. $C = \$10$, first cut when 15 years old. How much must he cut if income is to be as good as at pasture use, if he values stuff at $2 per cord?

19. One hundred acres coppice; non-agricultural land, can be cut every 20 years for 15 cords of stuff, last cut 9 years ago, stumpage now $1 per cord. Expected rise in market price 20 cents per cord for each of the next 5 rotations; after this constant. What is this woods worth at 3%?

20. Four thousand acres of pine woods in North Carolina can be cut over every 15 years for 5 M. feet. Last cut 7 years ago. What yearly payment or annuity can a trust company pay if they rate money at 3%?

21. One thousand acres of coppice woods in Pennsylvania have cut 15 cords every 25 years for iron furnace; stumpage $1.50. Plan is to convert into spruce for pulp on 50 year rotation. If $Yr = 30$ cords at $5; $C = \$10$; thin. $= 0$; taxes in first case 15 cents, in new case 10% of stumpage as yield tax; protection and care $= 10$ cents; $p = 3\%$. How do the two compare?

22. Spruce forest.

a. Present practice: Natural reproduction, takes 10 years, requires no artificial help; rotation 90 years, i. e., 90 from end of one reproduction to end of next; thinning needed early, 15 years after reproduction is started, costs $5 an acre.

Second thinning costs $2 per acre at age of 25.

Third thinning at 40 years brings $5 net.

Fourth thinning at 60 years brings $20 net.

Fifth thinning at 80 years brings $40 net.

Yr at 90 years would be $600 but owing to extra cost of logging is only $560.

e is $1 per acre and year, including taxes; p = 3%. What is Se here?

b. Plan is now to change this as follows:

Plant 5 year transplants space 6 x 6; plants cost $3 per M, setting costs $5 per M.

First thinning at 20 years pays expense.

Second thinning at 40 years pays $20.

Third thinning at 60 years pays $40.

Fourth thinning at 80 years pays $100.

Yr should be $600 but heavy thinning reduces volume but raises quality so that we still have $540, saving $40 on cost of logging.

e and p same as before. What is the new Se, and how does this compare to the old plan?

23. Beech forest, site II, is not satisfactory as to income. Make concrete case from Schwappach to show how this would work out if changed to spruce and to pine. Put p = 3%, e = $1, C = $10.

24. State of Michigan has been selling land inclusive of some growing stock at $1 per acre. If put in pine, C = $10, e = 50 cents, p = 2% and r = 80 years, how much pine must the state cut per acre to beat the $1, with stumpage at $10 per M?

25. Work out Se at 2, 4 and 5% for spruce, pine and oak, all for site II; C = $10, e = $1, and r = 40, 60, 80, 100 and 120 years.

Yr and thinnings as by Schwappach. Tabulate.

26. What is Se in coppice, rotation 25 years; Yr 20 cords at $2; thinnings cover expenses and taxes, planting costs $2 in each rotation, i. e., at each cut; p = 3%. Is this really Se?

27. Fe, pine in North Carolina ready to cut over now, for 10 M. feet at $6, can return every 20 years; e = 50 cents, soil = $5, p = 3%. Find Fe.

28. What is ${}^{o\text{-}(r\text{-}1)}Gc$ and ${}^{o\text{-}(r\text{-}1)}Ge$ in Spruce, Yr and thinnings as by Schwappach, site II, r = 80 years and 100 years, e = \$1, p = 3%, Sc = \$20.

29. What is Fc and Fe in the above case if Sc is replaced by Se in the calculation?

30. If the state of Michigan wanted to go into forestry and establish one million acres of forest in regulated condition what investment, i. e., what is Fc, if: soil = \$5 per acre; C = \$10 per acre; e = 50 cents; p = 2%; r = 80 y; Yr = \$200; thinnings neglected. What is Fc if Se is substituted and what is Fe then?

31. What rate of interest is made in the following case: stand at 80 years worth \$350; at 60 worth \$200, with land worth \$10 an acre, e = 50 cents and rate of interest 3%. Explain principle involved, and write out formula.

32. If C = \$10, e = 50 cents, p = 3%, soil = \$10, r = 80 years, and Yr = \$300, what p_f is made here in raising the crop?

33. A lumber company bought 10 sections of timber 10 years ago at \$15 per acre, land and timber; p = 5%; will cut 12 M. feet per acre; expenses per acre, assessment at ½ cost value; tax rate = 12 per 1,000; protection, etc., = 5 cents; no net growth, no fires, etc. losses so far. What is Fc now?

What is Fe if the cut expected is 12 M. feet at \$6 and the company expects to get at this timber in 8 years? Expenses to continue uniform. How does the Fc and Fe compare did they pay too much?

34. If average stumpage costs \$5 per M. feet what must the lumber be worth which will be cut in 40 years from now if: milling = \$4; logging = \$6; taxes 5 cents per M. feet; protection = 1 cent per M. feet; p = 5%; and land in 40 years to be worth \$10; to justify buying this now at \$5 per M. feet.

35. A saw mill costs: for buildings, \$10,000, life of this 30 years, wreckage value \$1,000; for steam engines, etc., \$15,000, life 15 years, wreckage value \$1,000; saw mill machinery \$25,000, life 10 years, wreckage value \$3,000. What is depreciation cost in this mill per year, if p = 5%?

36. A lumber company has 25 years timber on hand. It cuts 20 million feet; lands cut 10 M. feet per acre. They paid \$6 per acre. As fast as lands are cut over they sell for \$1 to a realty company. They pay 20 cents tax an acre, and also 5 cents for protection. Interest rate = 5%. What does the stumpage cost them on their 20 million feet per year?

37. A lumber company bought 30 thousand acres of timber lands; which cut 12 M. per acre; no net growth; they paid $10 an acre, build a saw mill, planing mill and other buildings at a cost of $150,000, and buy logging equipment at $50,000 more. They log for $6 and mill for $4 which includes all expenses. They get $18 per M. feet on board car at mill; cut 20 million per year. Taxes on ½ value at 10 per 1,000, interest at 5%; depreciation and replacement as follows:

None for buildings, the repairs are paid by milling costs.

All machinery is replaced once during the entire cut. Buildings cost new $60,000 and are worth at end $10,000. Machinery at start $90,000, at end $10,000. The logging outfit is kept up by logging charge of $6.

a. What is the yearly depreciation on buildings? What on machinery?

b. What does the 20 million feet lumber cost them and what is their net income per year?

c. What is the expectation value of the whole enterprise at start and after 7 years cut?

d. If a fire reduces their forest by 5 years cutting what is the damage to the company?

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Wm. C. Wells construction Co.
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