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A Model for the Distribution Function for Significant Wave Height

by

Edward F. Thompson

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A model based on a three-parameter Weibull distribution function is given for the long-term distribution of significant wave height. The model, formulated in dimensionless terms, is believed to provide a more general representation than the corresponding models given in the Shore Protection Manual (SPM). A procedure for using available data from a site to estimate model parameters is described. The procedure extends the use of available data and leads to a model which more closely follows the data than the procedures in the SPM. The procedure is applied to shallow-water gage data from Nags Head, North Carolina. Example problems are given to illustrate the use of the model at the Nags Head site.

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PREFACE

Two empirical models for the distribution of significant wave height are given in Section 4.332 of the Shore Protection Manual. A similar model is presented in this report. The model is based on a three-parameter Weibull distribution function. Parameters in the model are evaluated from a large sample of shallow-water gage data at Nags Head, North Carolina. The model, which more closely represents available data than either of the previous models, is particularly useful for statistical prediction of extreme significant wave heights in shallow water at Nags Head. The technique is applicable for other gage sites. This work was carried out under the waves and coastal flooding research program of the U.S. Army Coastal Engineering Research Center (CERC).

This report was prepared by Edward F. Thompson, Hydraulic Engineer, under the supervision of Dr. C.L. Vincent, Chief, Coastal Oceanography Branch. The author gratefully acknowledges Dr. D.L. Harris, formerly CERC Senior Scientist, who provided valuable comments on this study, and J. Peworchik of CERC who processed the Nags Head data for the study.

Comments on this publication are invited.

Approved for publication in accordance with Public Law 1966, 79th Congress, approved 31 July 1945, as supplemented by Public Law 1972, 88th Congress, approved 7 November 1963.

TED E. BISHOP
Colonel, Corps of Engineers
Commander and Director

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TABLE

FIGURE

CONVERSION FACTORS, U.S. CUSTOMARY TO METRIC (SI) UNITS OF MEASUREMENT

U.S. customary units of measurement used in this report can be converted to metric (SI) units as follows:

Multiply	by	To obtain
inches	25.4	millimeters
	2.54	centimeters
square inches	6.452	square centimeters
cubic inches	16.39	cubic centimeters
feet	30.48	centimeters
	0.3048	meters
square feet	0.0929	square meters
cubic feet	0.0283	cubic meters
yards	0.9144	meters
square yards	0.836	square meters
cubic yards	0.7646	cubic meters
miles	1.6093	kilometers
square miles	259.0	hectares
knots	1.852	kilometers per hour
acres	0.4047	hectares
foot-pounds	1.3558	newton meters
millibars	1.0197×10^{-3}	kilograms per square centimeter
ounces	28.35	grams
pounds	453.6	grams
	0.4536	kilograms
ton, long	1.0160	metric tons
ton, short	0.9072	metric tons
degrees (angel)	0.01745	radians
Fahrenheit degrees	5/9	Celsius degrees or Kelvins

To obtain Celsius (C) temperature readings from Fahrenheit (F) readings, use formula: $C = (5/9)(F - 32)$.

To obtain Kelvin (K) readings, use formula: $K = (5/9)(F - 32) + 273.15$.

SYMBOLS AND DEFINITIONS

d	water depth
$F\left[H_{sc} \geq \hat{H}_{sc}\right]$	probability that the ratio $H_s/\bar{H}_s \equiv H_{sc}$ is greater than or equal to a specified ratio $\hat{H}_{sc}$
g	acceleration due to gravity
H_s	significant wave height
$\bar{H}_s$	mean significant wave height
H_{sc}	significant wave height divided by mean significant height
$\tilde{H}_{sc}$	parameter in Weibull distribution function
$\hat{H}_{sc}$	specified value of H_{sc}
$H_{sc\ min}$	minimum expected value of H_{sc}, parameter in Weibull distribution function
T	wave period
α	parameter in Weibull distribution function
σ	standard deviation of significant wave height

A MODEL FOR THE DISTRIBUTION FUNCTION FOR SIGNIFICANT WAVE HEIGHT

by
Edward F. Thompson

I. INTRODUCTION

The long-term distribution of significant wave height at a site can be estimated from empirical data or by either of two empirical models in the Shore Protection Manual (SPM), Section 4.332 (U.S. Army, Corps of Engineers, Coastal Engineering Research Center, 1977)[1]. Where sufficient data are available, direct use of empirical data is usually preferable to either SPM model. However, proper estimation of the long-term distribution of significant height requires the use of an integral number (preferably 3 or more) of reasonably complete years of data. This requirement is often difficult to meet because of intermittent failure to obtain observations and limited number of years of data collection at a site.

It is often convenient to model an observed distribution of significant wave height. A model provides a simple parameterization of the observed distribution as well as a systematic method for extrapolating to probabilities beyond the data (although extrapolations are always much more uncertain than the part of the distribution well supported by data because of long-term variability in storms producing extreme wave conditions). Since there is no compelling physical basis for favoring any particular model, models are chosen to fit observed distributions of significant height. The models in the SPM were proposed as a tool for representing the distribution of the highest 50 to 80 percent of observed significant heights. The model presented in equation 4-6 of the SPM is a two-parameter modified exponential distribution which is further simplified in equation 4-9 of the SPM to a one-parameter distribution.

The model presented in this report is based on a three-parameter Weibull distribution function. The three-parameter model can better fit observations than either the one- or two-parameter models in the SPM. Parameters are evaluated to optimize the fit to empirical data from a gage at Nags Head, North Carolina. The model is formulated in dimensionless terms so that the effect of mean significant wave height level is removed. The advantages of using dimensionless terms are that more complete use is made of available data and that general characteristics of the distribution of significant height in addition to the mean can be readily examined. The dimensionless distribution function may be relatively invariant compared to mean significant height variations along a short section of coast. Hence, the model presented in this report is believed to provide a more general representation than the models in the SPM.

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[1]U.S. ARMY, CORPS OF ENGINEERS, COASTAL ENGINEERING RESEARCH CENTER, *Shore Protection Manual*, 3d ed., Vols. I, II, and III, Stock No. 008-022-00113-1, U.S. Government Printing Office, Washington, D.C., 1977, 1,262 pp.

II. WEIBULL DISTRIBUTION FUNCTION

A general model which can be used to approximate the empirical distribution of significant wave height is

$$F\left[H_{sc} \geq \hat{H}_{sc}\right] = e^{-\left(\frac{\hat{H}_{sc} - H_{sc\ min}}{\tilde{H}_{sc}}\right)^{\alpha}} \tag{1}$$

where

H_{sc} = significant wave height divided by mean significant height

$\hat{H}_{sc}$ = specified value of H_{sc}

$F\left[H_{sc} \geq \hat{H}_{sc}\right]$ = probability that H_{sc} is greater than or equal to a specified ratio $\hat{H}_{sc}$,

$H_{sc\ min}$ = minimum expected value of H_{sc}

$\tilde{H}_{sc}$, α = other parameters in distribution function.

Equation (1) is a form of the Weibull distribution function with three parameters ($H_{sc\ min}$, $\tilde{H}_{sc}$, and α).

III. COMPILATION OF EMPIRICAL DISTRIBUTION

The parameters in equation (1) must be evaluated for each site by optimizing the agreement between equation (1) and the empirical, dimensionless distribution of significant height at the site using the following procedure:

(a) Assemble all significant heights obtained by reliable, consistent analysis methods at a particular site;

(b) delete significant heights from months in which more than 50 percent of the possible observations are missing;

(c) compute mean significant height for each remaining month;

(d) divide each significant height by the appropriate monthly mean significant height; and

(e) combine the dimensionless significant heights in step (d) from all months into one distribution.

The implicit assumption in step (e) is that monthly variations in wave conditions can be completely represented by variations in monthly *mean* significant height but that variations relative to the mean are consistent from month to month. The assumption may not be valid at sites strongly affected by hurricane waves unless hurricane waves are treated separately.

IV. APPLICATION TO SHALLOW-WATER GAGE SITE AT NAGS HEAD, NORTH CAROLINA

Wave data used to test the model were obtained from a pier-mounted staff gage in a 16-foot water depth at Nags Head, North Carolina (see Thompson, 1977)[2]. Digital records were collected and analyzed at 6-hour intervals, with numerous interruptions, from December 1968 to March 1978. Significant wave heights from each of 54 relatively complete months were processed, as discussed previously, to form one dimensionless distribution containing 5,220 observations (see Fig.). The empirical distribution extends to an exceedance percentage of 0.02. Many cases at low exceedance percentages may be affected by the limited water depth at the Nags Head site, as indicated by the hatched area in the Figure.

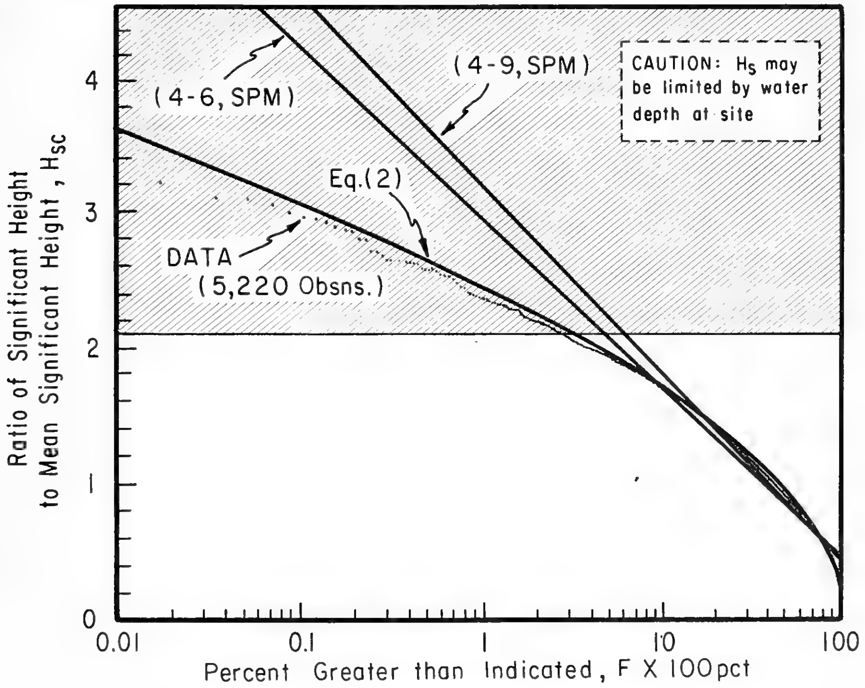

Figure. Distribution of significant wave height, Nags Head, North Carolina.

[2]THOMPSON, E.F., "Wave Climate at Selected Locations Along U.S. Coasts," TR 77-1, U.S. Army, Corps of Engineers, Coastal Engineering Research Center, Fort Belvoir, Va., Jan. 1977.

Values for the parameters in equation (1) were estimated from the empirical distribution of significant heights at Nags Head by the method described in the Appendix to give

$$F\left[H_{sc} \geq \hat{H}_{sc}\right] = e^{-\left(\frac{\hat{H}_{sc} - 0.198}{0.885}\right)^{1.65}} \qquad (2)$$

Equation (2) is shown in the Figure. This distribution function fits the empirical distribution at Nags Head better than the comparable models in the SPM. Equation (2) can be rearranged to give

$$\hat{H}_{sc} = e^{\left\{0.606 \, \ell n\left(- \ell n \, F\left[H_{sc} \geq \hat{H}_{sc}\right]\right) - 0.122\right\}} + 0.198 \qquad (3)$$

The assumption that variations relative to monthly mean significant height are consistent from month to month was tested at Nags Head by comparing empirical distributions of significant height by season. Distributions for fall (September to November), winter (December to February), and spring (March to May) are comparable to the empirical distribution in the Figure. However, the distribution for summer (June to August) indicates higher values of H_{sc} than the empirical distribution in the Figure at probabilities below about 10 percent. This discrepancy is due to exceptionally low monthly mean significant heights in summer and, in many cases, to nearshore hurricanes. Equation (2) seems to be satisfactory for estimating the annual or nonsummer distribution of significant height at Nags Head, but the summer distribution must include special consideration of hurricane-generated waves at this site.

V. EXAMPLE PROBLEMS

* * * * * * * * * * * * * * * EXAMPLE PROBLEM 1 * * * * * * * * * * * * * * *

GIVEN: Mean annual significant height of approximately $\bar{H}_s$ = 3.0 feet (0.91 meter) at Nags Head, North Carolina (see Table).

FIND:

(a) The significant height which is equaled or exceeded during 6 hours every year.

(b) The significant height which is equaled or exceeded during 1 hour every year.

SOLUTION:

(a) The exceedance percentage ($F[H_{sc} \geq \hat{H}_{sc}] \times 100$ percent) is

$$\frac{1}{365 \times \frac{24}{6}} \times 100 = 0.0685 \text{ percent}$$

Table. Monthly and annual significant wave height statistics, Nags Head, North Carolina

Month	Year	No. of Obsns.	$\bar{H}_s$ (ft)	σ (ft)	Month	Year	No. of Obsns.	$\bar{H}_s$ (ft)	σ (ft)
12	1968	96	2.62	1.34	04	1974	73	2.50	1.19
01	1969	112	3.04	1.59	09	1974	75	2.55	0.98
02	1969	103	4.12	2.01	12	1974	118	3.09	1.59
03	1969	106	3.66	1.85	01	1975	93	3.38	1.58
04	1969	90	2.43	1.27	03	1975	63	2.97	1.72
05	1969	85	2.32	1.08	08	1975	63	1.86	0.86
07	1969	112	2.01	1.02	09	1975	86	2.99	1.10
08	1969	94	2.29	1.07	10	1975	92	3.35	1.40
09	1969	105	3.36	1.74	11	1975	90	3.02	1.49
Annual	Dec. 1968–Oct. 1969	1,006	3.00	1.72	02	1976	87	2.30	1.09
09	1971	117	3.44	1.78	03	1976	99	2.71	1.13
10	1971	117	3.34	2.02	04	1976	72	2.58	1.33
11	1971	78	3.88	1.71	06	1976	65	2.06	1.01
12	1971	120	3.39	1.94	10	1976	113	2.85	1.14
01	1972	82	2.38	0.90	11	1976	113	2.21	1.19
02	1972	116	3.81	2.03	12	1976	109	2.63	1.26
03	1972	123	2.93	1.21	01	1977	89	2.90	1.18
04	1972	110	3.10	1.67	02	1977	98	2.21	1.18
05	1972	120	3.19	1.53	03	1977	118	2.34	0.93
06	1972	101	2.15	0.98	04	1977	93	1.97	0.78
08	1972	88	2.24	1.33	05	1977	82	1.60	0.72
Annual	Sept. 1971–Aug. 1972	1,173	3.10	1.71	06	1977	91	1.68	0.82
09	1972	106	3.06	2.03	08	1977	88	1.40	0.59
10	1972	109	3.77	1.84	12	1977	93	3.20	1.40
11	1972	96	4.58	1.40	01	1978	100	3.26	1.21
12	1972	97	2.96	1.61	03	1978	82	3.48	1.23
01	1973	97	3.36	1.61					
02	1973	92	4.43	2.09					
03	1973	114	4.33	2.28					
05	1973	89	2.38	1.00					
Annual	Sept. 1972–May 1973	854	3.55	1.93					

From the Figure or equation (3),

$$\hat{H}_{sc} = \frac{H_s}{\bar{H}_s} = 3.13,$$

$$H_s = 9.4 \text{ feet (2.9 meters).}$$

(b) The exceedance percentage is

$$\frac{1}{365 \times 24} \times 100 = 0.0114 \text{ percent.}$$

From the Figure or equation (3),

$$\hat{H}_{sc} = \frac{H_s}{\bar{H}_s} = 3.57,$$

$$H_s = 10.7 \text{ feet (3.3 meters).}$$

Check to see if H_s exceeds depth-limited height.

$$\frac{d}{H_s} = \frac{16}{10.7} = 1.5$$

From Figure 2-66 in the SPM, depth-limited breaking may be possible if

$$\frac{H_s}{gT^2} > 0.0172$$

This condition corresponds to

$$T < \sqrt{\frac{H_s}{0.0172\ g}} = \sqrt{\frac{10.7}{0.0172 \times 32.2}} = 4.4 \text{ seconds.}$$

Since a period of 4.4 seconds or less is unreasonably short for a 10.7-foot-high wave at this site (see Thompson, 1977)[3], depth-limited breaking is not expected to be a consideration in this example.

* * * * * * * * * * * * * * * EXAMPLE PROBLEM 2 * * * * * * * * * * * * * * *

GIVEN: Mean significant height of approximately $\bar{H}_s$ = 3.4 feet (1.0 meter) in February at Nags Head, North Carolina (see Table).

FIND: The significant height which is equaled or exceeded during 6 hours every February.

SOLUTION: The exceedance percentage is

$$\frac{1}{28 \times 4} \times 100 = 0.89 \text{ percent}$$

[3]THOMPSON, E.F., op. cit., p. 9.

From the Figure or equation (3),

$$\hat{H}_{sc} = \frac{H_s}{\bar{\bar{H}}_s} = 2.47,$$

$$H_s = 8.4 \text{ feet (2.6 meters).}$$

VI. SUMMARY

A three-parameter model for the distribution of significant wave height is given. A procedure for using available data from a site to compile a dimensionless distribution of significant height and to estimate parameters in the model is presented. The procedure extends the use of available data and leads to a model which more closely follows the data than procedures given in the SPM. The procedure is applied to shallow-water gage data from Nags Head, North Carolina.

APPENDIX

METHOD FOR ESTIMATING PARAMETERS IN THE WEIBULL DISTRIBUTION FUNCTION

The Weibull distribution function, equation (1), can be transformed into a form suitable for linear regression analysis. First, the natural logarithm of equation (1) is taken

$$\ell n\ F = -\left(\frac{\hat{H}_{sc} - H_{sc\ min}}{\tilde{H}_{sc}}\right)^{\alpha} \qquad \text{(A-1)}$$

Both sides of equation (A-1) are multiplied by -1, and again natural logarithms are taken

$$\ell n\ (-\ \ell n\ F) = \ell n\left[\left(\frac{\hat{H}_{sc} - H_{sc\ min}}{\tilde{H}_{sc}}\right)^{\alpha}\right] \qquad \text{(A-2)}$$

Equation (A-2) can be rewritten as

$$\ell n\ \left(\hat{H}_{sc} - H_{sc\ min}\right) = \ell n\ \tilde{H}_{sc} + \frac{1}{\alpha}\ \ell n\ (-\ \ell n\ F) \qquad \text{(A-3)}$$

Equation (A-3) is in the form

$$Y = a + b\ X, \qquad \text{(A-4)}$$

where

$Y = \ell n\left(\hat{H}_{sc} - H_{sc\ min}\right)$

$a = \ell n\ \tilde{H}_{sc}$

$b = \frac{1}{\alpha}$

$X = \ell n\ (-\ \ell n\ F)$.

An initial value of the parameter $H_{sc\ min}$ was obtained from Table 1 of Thompson and Harris (1972)[4] as the "minimum significant height" divided by the observed mean significant height. The value for Nags Head was 0.31. Alternatively, $H_{sc\ min}$ could be estimated initially as 0.38 from equation 4-8 in the SPM. The estimated value of $H_{sc\ min}$ and empirical tabulation of F as a function of H_{sc} are used to compute a table of X and Y values. Linear regression analysis is then used to estimate optimum values of a and b in

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[4]THOMPSON, E.F., and HARRIS, D.L., "A Wave Climatology for U.S. Coastal Waters," *Proceedings of Offshore Technology Conference,* May 1972, pp. 675-688 (also Reprint 1-72, U.S. Army, Corps of Engineers, Coastal Engineering Research Center, Fort Belvoir, Va., NTIS AD 746 365).

equation (A-4). Values for the parameters $\tilde{H}_{sc}$ and α in equation (1) are easily calculated from a and b by using the relationships defined for equation (A-4).

The distribution function given by equation (1) with the estimated parameters $H_{sc\ min}$, $\tilde{H}_{sc}$, and α is compared with the empirical distribution. A new value of $H_{sc\ min}$ is estimated to attempt a better fit to the empirical distribution. The new $H_{sc\ min}$ is used to compute a new table of X and Y values which is then used to estimate new values of $\tilde{H}_{sc}$ and α as before. The process is continued until an $H_{sc\ min}$ has been found which leads to a satisfactory model. The parameters for Nags Head in equation (2) are considered satisfactory because they specify a distribution function which fits the lower 99 percent of the empirical distribution reasonably well but is conservatively high in comparison to the highest 1 percent of the empirical distribution. Since the highest part of the empirical distribution is based on the smallest number of observations, it is the least well-established part of the empirical distribution, and conservatism is desirable in a model for engineering use.

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