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THE  
NATURAL ARITHMETIC

BOOK III

BY

ISAAC O. WINSLOW, M.A.

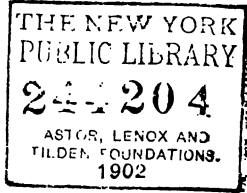
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AMERICAN BOOK COMPANY

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WIN. AB. III.

W. P. I

## PURPOSES AND DISTINCTIVE FEATURES

THE purposes of this series of Arithmetics are:—

1. *To present the subjects in a spiral order.*

Instead of presenting the general subjects of addition, subtraction, multiplication, division, fractions, etc., as complete wholes in regular succession, each subject is divided into parts with reference to the difficulty of the principles involved. The easier principles of various subjects are treated together, while the more difficult principles are reserved until the child has gained the power to apprehend them easily.

2. *To make the work easy.*

In the belief that it is better to keep mathematical work a little behind the child's mental grasp than to advance it beyond that limit, the work designed for the different grades has been made somewhat easier than that usually found in text-books. The pupil is kept busy with a varied application of the principles that he has already mastered instead of being too rapidly crowded forward into greater difficulties.

3. *To give the subject variety and interest.*

The problems are based upon facts and principles gathered from the different branches of study, as history, geography, nature study, astronomy, and physics, as well as on the customary commercial transactions, thus correlating arithmetic with other studies and adding distinctly to its vividness and interest.

4. *To develop genuine mathematical thought.*

There is a large amount of mental work interspersed with the written work. Each new subject is first developed with numbers that are not too large for mental solution. After the principle has been well established, written practice with larger numbers is introduced. Originality is also promoted by exercises requiring the pupils to make problems for themselves from given data.

5. *To give prominence to the idea of magnitude.*

The psychological fact that all mathematical knowledge is a system of relations, or ratios, has been recognized throughout the series. For the purpose of suggesting to the pupils the application of number to magnitude, the simple geometric forms have been gradually introduced.

---

In Book III the treatment of the several branches of the subject is completed, and, as a means of bringing together the parts of each subject in a general review, a summary is given of all the processes involved in common fractions, decimal fractions, and percentage.

A simple treatment of algebraic methods with a development of the algebraic equation is made a special feature of the book. In order to correlate arithmetic and algebra, problems are frequently introduced which admit of either an arithmetical or an algebraic solution.

To relieve the lack of interest which a prolonged continuance of drill in abstract processes is liable to produce, many of the problems are based upon interesting facts in nature and life.

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# BOOK III

## Notation and Numeration

*Read the following numbers :*

Use the word "and" only at the decimal point.

1.	2.	3.
9,050	.015	16.0103
40,200	27.08	.21005
205,065	5.1052	800.0803
5,060,130	250.0009	4.010203
70,010,080	6000.006	215.0010505

4. Write seven million, forty thousand, five hundred thirty.

5. Write thirty-five and two thousand sixty-one ten-thousandths.

6. Write four hundred and seven hundred six ten-thousandths.

7. Write sixty-two thousand and four hundred thousandths.

8. Write twenty thousand four hundred six millionths.

9. Write four thousand and four ten-thousandths.

10. Write three thousand sixteen hundred-thousandths.

11. Write one thousand five and three hundred nine ten-thousandths.



## Common Fractions

See Book II, pages 15, 18, 25, 43, 120, 224, 241.

*Add :*

- |  |  |
|--|--|
| 1. $\frac{3}{4} + \frac{2}{3} + \frac{5}{6}$             | 5. $9\frac{1}{2} + 7\frac{4}{5} + 8\frac{2}{3}$    |
| 2. $\frac{2}{5} + \frac{1}{3} + \frac{1\frac{1}{2}}{15}$ | 6. $5\frac{3}{8} + 9\frac{5}{12} + 5\frac{5}{8}$   |
| 3. $\frac{5}{9} + \frac{1}{4} + \frac{1\frac{1}{2}}{18}$ | 7. $10\frac{3}{8} + 9\frac{7}{15} + 12\frac{1}{6}$ |
| 4. $\frac{7}{12} + \frac{8}{9} + \frac{5}{6}$            | 8. $8\frac{3}{10} + 2\frac{1}{15} + 4\frac{1}{12}$ |

*Subtract :*

- |                                       |                                    |
|---------------------------------------|------------------------------------|
| 9. $\frac{5}{8}$ from $\frac{4}{3}$   | 13. $7\frac{1}{2} - 4\frac{3}{8}$  |
| 10. $\frac{5}{12}$ from $\frac{7}{8}$ | 14. $9\frac{5}{8} - 2\frac{3}{8}$  |
| 11. $\frac{4}{7}$ from $\frac{2}{3}$  | 15. $12\frac{3}{4} - 9\frac{3}{8}$ |
| 12. $\frac{7}{15}$ from $\frac{2}{4}$ | 16. $15\frac{5}{8} - 5\frac{5}{8}$ |

*Multiply :*

- |                                     |   |
|-------------------------------------|---|
| 17. $\frac{5}{11}$ by 6             | 21. $8\frac{4}{5} \times 10\frac{3}{4}$   |
| 18. 25 by $\frac{7}{12}$            | 22. $12\frac{3}{8} \times 9\frac{1}{2}$   |
| 19. $\frac{2}{3}$ by $\frac{9}{10}$ | 23. $3\frac{2}{5} \times 2\frac{4}{7}$    |
| 20. $\frac{7}{18}$ by $\frac{5}{9}$ | 24. $15\frac{7}{8} \times 10\frac{3}{11}$ |

*Divide :*

- |                                     |                                      |
|-------------------------------------|--------------------------------------|
| 25. $\frac{1}{8}$ by 3              | 29. $35\frac{10}{11} \div 5$         |
| 26. $\frac{1}{9}$ by $\frac{2}{19}$ | 30. $10 \div 1\frac{1}{2}$           |
| 27. $\frac{8}{9}$ by $\frac{1}{3}$  | 31. $12\frac{3}{8} \div \frac{2}{3}$ |
| 28. $1\frac{1}{2}$ by $\frac{2}{3}$ | 32. $9\frac{1}{2} \div 2\frac{1}{5}$ |

*Change to decimal fractions :*

- |                    |                    |                              |
|--------------------|--------------------|------------------------------|
| 33. $\frac{1}{2}$  | 37. $\frac{4}{5}$  | 41. $\frac{9}{50}$           |
| 34. $\frac{2}{5}$  | 38. $\frac{3}{4}$  | 42. $\frac{2\frac{1}{2}}{5}$ |
| 35. $\frac{7}{10}$ | 39. $\frac{1}{20}$ | 43. $\frac{1}{20}$           |
| 36. $\frac{1}{4}$  | 40. $\frac{1}{25}$ | 44. $\frac{87}{100}$         |

## Decimal Fractions

See Book II, pages 115, 144, 161, 224.

1.  $24.37 + 5.4304 + .0213 + 125.4$
2.  $8.801 + 542.16 + 1.0302 + .0157$
3.  $.12503 + .01615 + 19.439 + 100.9$
4.  $235.1 + 61.4378 + 5.0321 + .0013$
5.  $94.521 + 2.3846 + 804.5 - 251.63$
6.  $2.563 + .01356 + 113.154 - 48.1562$
7.  $465.02 + 10.1523 - 245.18 - .2367$
8.  $2.3681 - .458124 + 183.45 - 19.6542$

*Multiply :*

- |                   |                           |
|-------------------|---------------------------|
| 9. .0136 by 12    | 14. $24.03 \times 100$    |
| 10. .0478 by 10   | 15. $376.051 \times 1000$ |
| 11. .256 by 15    | 16. $39.42 \times 8.27$   |
| 12. .375 by .25   | 17. $164.9 \times .0015$  |
| 13. .0095 by .035 | 18. $.23489 \times 365$   |

*Divide :*

- |                    |                       |
|--------------------|-----------------------|
| 19. 27.625 by 25   | 24. $314.8 \div 1000$ |
| 20. .01372 by 12   | 25. $2520 \div .05$   |
| 21. 56.347 by 7.8  | 26. $35.7 \div .017$  |
| 22. 3.1416 by 4.13 | 27. $7564 \div .02$   |
| 23. 21.536 by 100  | 28. $5000 \div .001$  |

*Change to common fractions :*

- |          |           |             |
|----------|-----------|-------------|
| 29. .8   | 34. .025  | 39. 3.5     |
| 30. .25  | 35. .08   | 40. 15.05   |
| 31. .125 | 36. .004  | 41. 9.125   |
| 32. .225 | 37. .056  | 42. 12.075  |
| 33. .365 | 38. .1245 | 43. 73.0125 |

## Compound Quantities

See Book II, pages 26, 42, 66, 84, 160.

1. Add 3 gal. 2 qt. 1 pt. 3 gi., 4 gal. 3 qt.  $1\frac{1}{2}$  pt.
2. From 8 bu. 2 pk. 4 qt. take 4 bu. 3 pk. 2 qt. 1 pt.
3. Multiply 5 yd. 2 ft. 10 in. by 6.
4. Divide 15 lb. 6 oz. by 12.
5. How many times will 1 sq. ft. 72 sq. in. be contained in 9 sq. yd. 4 sq. ft. 7 sq. in. ?
6. The circumference of a wheel is 11 ft. 4 in. How far does the carriage move while the wheel makes 1000 revolutions ?
7. What is the exact number of days from July 20th to Oct. 12th ?
8. Find the time from Sept. 10, 1886, to Nov. 12, 1890.
9. Find the time from Mar. 15, 1874, to Jan. 18, 1879.
10. How many feet of lumber will there be in 30 boards, each 12 feet long and 15 inches wide ?
11. How many square yards are there in the walls of a room that is  $15\frac{1}{2}$  feet long,  $9\frac{3}{4}$  feet wide, and 8 feet high ?
12. How many cords are there in a pile of wood 4 feet wide, 4 feet high, and 28 feet long ?
13. How many cords are there in a pile of wood 3 feet wide, 8 feet high, and 16 feet long ?
14. How many breadths of carpet will it take to carpet a room 18 ft.  $\times$  12 ft. with carpet  $\frac{3}{4}$  of a yard wide, if the breadths run lengthwise and no allowance is made for waste in matching ?
15. How many yards of carpet will it take to carpet a room 14'  $\times$  13' with carpet 1 yard wide, if the breadths run lengthwise and each breadth must be cut 1 foot longer *than the room* for the purpose of matching ?

## Compound Quantities

*Find the cost of:*

1. 17 gal. 3 qt. of milk at  $4\frac{1}{2}$  cents a quart.
2. 3 pk. 7 qt. 1 pt. of berries at 7 cents a quart.
3. 5 bu. 2 pk. 4 qt. of potatoes at 45 cents a bushel.
4. 3 pk. 5 qt. of apples at 60 cents a bushel.
5. 14 yd. 2 ft. 6 in. of cloth at 38 cents a yard.
6. 8 yd. 1 ft. 4 in. of pipe at 7 cents a foot.
7. 1 ft. 10 in. of ribbon at 30 cents a yard.
8. 12 rd. 3 yd.  $2\frac{1}{2}$  ft. of ditch at 18 cents a foot.
9. 23 rd. 2 yd. 1 ft. of wall at \$7.50 a rod.
10. 16 sq. yd. 5 sq. ft. of concrete at 38 cents a sq. yd.
11. 49 sq. rd. 35 sq. ft. of land at  $42\frac{1}{2}$  cents a sq. ft.
12. 124 sq. rd. 50 sq. ft. of land at \$5.50 a sq. rd.
13. 115 sq. rd. of land at \$90 an acre.
14. 13 A. 45 sq. rd. of land at \$75 an acre.
15.  $4\frac{1}{2}$  sq. mi. of land at \$25 an acre.
16. 2 sq. mi. 100 sq. rd. of land at \$40 an acre.
17. A wall 20 ft.  $\times$  4 ft.  $\times$  3 ft. 6 in. at 15¢ a cu. ft.
18. A cellar 30 ft.  $\times$   $18\frac{1}{2}$  ft.  $\times$   $7\frac{1}{2}$  ft. at 48¢ a cu. yd.
19. A pile of wood 25 ft.  $\times$  12 ft.  $\times$   $6\frac{1}{2}$  ft. at \$6 a cord.
20. A pile of wood  $18\frac{1}{2}$  ft.  $\times$  14 ft.  $\times$   $7\frac{1}{2}$  ft. at \$1.15 a cord foot.
21. 3560 lb. of coal at \$6.50 a ton.
22. 5365 lb. of hay at 85¢ per 100 lb.
23. 4 gross  $6\frac{1}{2}$  doz. pencils at \$2 a gross.
24. 10 reams 7 quires of paper at 8¢ a quire.

## Percentage

See Book II, pages 168, 176, 195, 212.

1. What is 5 per cent of 100? of 200? of 500?
2. What is 8% of 100? of 50? of 25?
3. What is 1% of 3200? 4% of 3200? 10% of 3200?
4. What common fraction equals  $12\frac{1}{2}\%$ ?  $16\frac{2}{3}\%$ ? 20%?
5. What common fraction equals 25%?  $33\frac{1}{3}\%$ ?  $62\frac{1}{2}\%$ ?
6. What is  $12\frac{1}{2}\%$  of 48?  $16\frac{2}{3}\%$  of 48? 20% of 60?
7. What is 25% of 36?  $33\frac{1}{3}\%$  of 45?  $62\frac{1}{2}\%$  of 16?
8. Of what number is 12 1%? 2%? 4%?
9. Of what number is 60 20%? 30%? 50%?
10. 7 is  $12\frac{1}{2}\%$  of what number?  $16\frac{2}{3}\%$  of what number?
11. 9 is 20% of what number? 25% of what number?
12. What is 1% of \$1250? 10% of \$1250?
13. What is 1% of \$240?  $\frac{1}{2}\%$  of \$240?
14. What is 1% of 416?  $1\frac{1}{4}\%$  of 416?
15. What is the profit on goods costing \$225, and sold at a gain of 30%?
16. What is the loss on a car load of potatoes bought for \$180 and sold at a loss of  $12\frac{1}{2}\%$ ?
17. By gaining 20% on goods I gained \$35. What was the cost?
18. By losing 4% on goods I lost \$32. What was the cost?
19. An agent charges 15% for collecting bills. He has collected \$160. How much will his commission be?
20. An agent's commission amounts to \$40. His rate is 5%. Upon what sum is the commission reckoned?
21. How much will it cost to insure a house for \$2600  $1\frac{1}{2}\%$ ?

## Drill Work

*Add both vertically and horizontally the following statements of cash receipts for the successive weeks:*

	MON.	TUES.	WED.	THUR.	FRI.	SAT.	TOTAL
1st	\$ 2561.45	\$ 1562.73	\$ 4830.26	\$ 2056.42	\$ 1852.07	\$ 675.25	
2d	584.32	1082.36	873.41	456.90	748.92	1742.80	
3d	2542.76	3682.48	1206.89	872.45	1097.34	2367.45	
4th	834.05	798.30	2430.54	2108.76	2573.80	870.34	
5th	3782.60	3672.45	867.30	3210.45	897.04	1672.00	
6th	2435.00	1243.50	923.56	680.20	587.70	2140.78	
7th	685.30	834.01	1768.43	2436.80	2874.34	638.00	
8th	1345.60	2937.00	2437.89	860.43	1367.45	921.00	
9th	1743.92	873.78	624.30	328.46	924.08	1253.08	
10th	801.42	720.85	987.43	1026.87	643.28	864.23	
Total							

	MON.	TUES.	WED.	THUR.	FRI.	SAT.	TOTAL
1st	\$ 3287.40	\$ 923.75	\$ 1432.87	\$ 834.54	\$ 6378.23	\$ 925.00	
2d	832.00	1254.76	823.45	2410.34	823.78	3267.40	
3d	234.75	904.3	84.54	125.67	89.70	236.84	
4th	2873.52	973.82	823.46	925.25	775.24	1402.40	
5th	1205.34	2467.89	1345.23	1623.86	1243.80	989.75	
6th	765.34	970.75	972.04	1328.75	2234.37	1734.52	
7th	3875.43	1325.43	876.43	634.52	863.00	2007.26	
8th	990.70	750.25	1450.80	2123.88	1123.12	890.85	
9th	1450.50	865.32	784.35	988.78	878.87	1334.54	
10th	2132.45	1540.42	2110.40	1654.42	2457.67	2004.45	
Total							

## Problems from Geography

1. The earth is 25,000 miles in circumference. It is now possible to make a trip around the world in 40 days. What is the average possible rate per day for the journey?

2. When the railroad is completed from Cairo to Cape Town, connecting northern and southern Africa, the length of the road will be about 5500 miles. How many days will it take to make the trip, at the rate of 30 miles an hour, if we allow for 20 stops of 10 minutes each?

3. There are now about 170,000 miles of telegraphic cable in the world. How many times would all this cable reach continuously around the world?

4. When the canal is completed across the isthmus between North America and South America, more than 10,000 miles will be saved in the trip from New York to San Francisco. If we reckon 12 miles an hour as an average rate for a vessel, about how many days will the voyage be shortened?

5. It is estimated that this canal will cost \$125,000,000. How much would have to be collected each month from the ships that pass through to pay 5% annually on this sum?

6. If the tunnel which is now being built through the Alps will be  $12\frac{3}{4}$  miles long, and will cost \$14,000,000, what will be the average cost for each foot in length?

7. The great tunnel under the city of New York will cost about \$35,000,000. When the population of the city is 3,500,000, how much must this tunnel be worth upon the average, each year, to every inhabitant, in order that the total advantage may equal an income of 6% on the *investment*?

## Interest

**Interest** is a sum paid or allowed for the use of money. It is reckoned in per cent for each year.

The **principal** is the money for the use of which interest is paid.

The **amount** is the sum of the principal and the interest.

At 4% the interest of \$1 for one year is 4 cents, for 2 years, 8 cents, for  $2\frac{1}{2}$  years, 10 cents, etc. At 5% the interest of \$1 for 1 year is 5 cents, the interest of \$2 for 1 year is 10 cents, etc.

1. What is the interest of \$3 for 1 year at 5%?
2. What is the interest of \$3 for 3 years at 5%?
3. What part of a year are 2 months? 3 months? 4 months? 6 months? 9 months?
4. What is the interest of \$1 at 6% for 6 months? 4 months? 8 months?

Since the interest for 1 year is 6¢, for 8 months or  $\frac{2}{3}$  year, it is  $\frac{2}{3}$  of 6¢ or 4¢.

5. What is the interest of \$1 at 4% for 3 months? For 6 months? For 9 months?
6. What is the interest of \$1 for 2 years and 3 months at 8%?

*Give the interest of the following:*

7. \$1 for 2 yr. at 3%.
8. \$2 for 1 yr. at 3%.
9. \$6 for 3 yr. at 4%.
10. \$5 for 4 yr. at 5%.
11. \$8 for 6 yr. at 6%.
12. \$4 for 2 yr. 4 mo. at 3%.
13. \$10 for 3 yr. 3 mo. at 4%.
14. \$12 for 5 yr. 6 mo. at 5%.
15. \$8 for 3 yr. 3 mo. at 7%.
16. \$15 for 4 yr. 2 mo. at 6%.
17. How much interest will \$5 gain in  $3\frac{1}{2}$  yr. at 8%?
18. What will be the amount of \$10 for 3 yr. at 4%?

The interest of \$10 for 3 yr. at 4% is 12% of \$10 or \$1.20. The amount is \$10 + \$1.20 or \$11.20.

19. What will be the amount of \$15 for 2 yr. at 6%?
20. Find the amount of \$12 for 3 yr. at 3%.



## Longitude and Time

1. In which of two places is the time earlier, a place farther east or one farther west? Why?

Since the earth, in its daily revolution, turns through  $360^\circ$  in 24 hours, it turns through  $15^\circ$  in one hour, and through  $1^\circ$  in  $\frac{1}{15}$  of an hour, or 4 minutes. Since it turns through  $1^\circ$  or  $60'$  of longitude in 4 minutes of time, it turns through  $15'$  of longitude in 1 minute of time, and through  $15''$  of longitude in one second of time.

*The number of degrees, minutes, and seconds of longitude is fifteen times the number of hours, minutes, and seconds of time.*

2. At noon, what time of day is it at a place  $15^\circ$  farther west? At a place  $15^\circ$  farther east? At a place  $30^\circ$  farther west? At a place  $45^\circ$  farther east?

3. At 8 o'clock, what time is it at a place which is  $60^\circ$  farther west? At a place  $90^\circ$  farther east?

How many degrees of longitude farther west or east has a place where the time is:

- |                                  |                                  |
|----------------------------------|----------------------------------|
| 4. 2 hours earlier?              | 7. $1\frac{1}{2}$ hours later?   |
| 5. 4 hours later?                | 8. $1\frac{2}{3}$ hours earlier? |
| 6. $1\frac{1}{2}$ hours earlier? | 9. $5\frac{1}{2}$ hours later?   |

10. What is the difference in longitude between two places whose difference in time is 4 hr. 30 min.?

$15 \times 4 = 60$ , the number of degrees of longitude;  $15 \times 30 = 450$ , the number of minutes of longitude, which equal  $7^\circ 30'$ ;  $60^\circ + 7^\circ 30' = 67^\circ 30'$ .

11. What is the difference in time, if the difference in longitude is  $20^\circ$ ?

$$20 \div 15 = 1\frac{1}{3} \text{ hours or } 1 \text{ hr. } 20 \text{ min.}$$

12. What is the difference in longitude, if the difference in time is 2 hr. 40 min.?

## Longitude and Time

1. How many minutes are there in a degree of longitude?

2. How many minutes of longitude correspond to one minute of time?

3. What is the difference in time between two places whose difference in longitude is  $30'$ ?  $45'$ ?  $7\frac{1}{2}'$ ?

4. What is the difference in longitude between two places whose difference in time is 2 minutes?  $2\frac{1}{2}$  minutes? 1 minute 20 seconds? 2 minutes 40 seconds?

5. What difference in longitude corresponds to a difference of 2 hr. 3 min. 2 sec.? 5 hr. 12 min. 3 sec.? 4 hr. 15 min. 20 sec.?

$$\begin{array}{r} 2 \text{ hr.} \quad 3 \text{ min.} \quad 2 \text{ sec.} \\ \hline 15 \\ \hline 30^\circ \quad 45 \text{ min.} \quad 30'' \text{ longitude} \end{array}$$

6. What difference in time corresponds to a difference in longitude of  $45^\circ 30' 15''$ ?  $20^\circ 12' 30''$ ?  $7^\circ 19' 5''$ ?

$$\begin{array}{r} 15 \overline{)45^\circ \quad 30' \quad 15''} \\ \hline 3 \text{ hr.} \quad 2 \text{ min.} \quad 1 \text{ sec. time} \end{array}$$

7. What is the difference in longitude between two places, one of which is  $85^\circ$  west longitude and the other  $73^\circ$  west longitude?

8. What is the difference in time between two places, one of which is  $68^\circ$  west longitude and the other  $20^\circ$  east longitude?

*Find the difference in time between:*

9. Cleveland,  $81^\circ 39'$  W. and Brooklyn,  $73^\circ 58'$  W.

10. Detroit,  $83^\circ 5' 7''$  W. and Vienna,  $16^\circ 22' 22''$  E.

11. Glasgow,  $4^\circ 17' 6''$  W. and Rome,  $12^\circ 27' 58''$  E.

12. What time is it in Cleveland when it is 4 P.M. in Brooklyn?

13. Find the time at Rome when it is 5 A.M. at Glasgow.

## Life Insurance

Life insurance companies agree to pay a certain sum of money on the death of a person to his heirs, or to pay the sum to himself if he survives a certain number of years. In return for this the person insured pays annually a sum called a **premium**. The premium is usually reckoned as a certain sum per year for each \$1000 of insurance.

The amount of the premium depends upon the age of the person at the time of insurance, since the older he is the sooner he is likely to die.

The following table gives the average number of years which it has been found that people live after reaching certain ages :

AGE	EXPECTANCY	AGE	EXPECTANCY	AGE	EXPECTANCY	AGE	EXPECTANCY
40	27.61	45	24.46	50	21.11	55	17.58
41	26.97	46	23.82	51	20.39	56	16.89
42	26.34	47	23.17	52	19.68	57	16.21
43	25.71	48	22.50	53	18.97	58	15.55
44	25.09	49	21.81	54	18.28	59	14.92

1. According to the above table, if a man 45 years of age should pay \$30 a year for each \$1000 of insurance, what would be the total amount of his payments if he should live to the end of the period of expectancy?

2. A person was insured for \$10,000 at an annual premium of \$32.20 per \$1000. He died after making 18 annual payments. How much less did he pay in premiums than the company paid at his death?

3. At an annual premium of \$20.80 per \$1000, how much will be the quarterly payment on a policy of \$15,000?

## Interest

1. What is the interest of \$1 at 6% for 6 months? For 2 months? For 1 month?
2. How many mills are there in a cent? In half a cent?
3. Express in cents and mills the interest of \$1 at 6% for 3 months. For 7 months. For 11 months.
4. What is the interest of \$1 for 3 yr. 5 mo. at 6%, expressed in cents and mills?
5. What is the interest of \$10 for the same time and rate?
6. What part of the interest of a sum of money at 6% would be the interest of the same sum at 1%? At 2%? At 3%? At 4%? At 5%? At 7%? At 8%?
7. What part less is the interest of a sum at 5% than at 6%? At 4% than at 6%?
8. What part more is the interest of a sum at 8% than at 6%? At 9% than at 6%?
9. If the interest of a certain sum for a certain time at 6% is \$24, what would it be at 5%? At 2%? At 4%? At 7%?
10. If the interest of a certain sum for a certain time at 6% is \$18, what would it be at 1%? At  $4\frac{1}{2}\%$ ? At 7%? At  $5\frac{1}{2}\%$ ?
11. What is the interest of \$4 for 2 yr. 6 mo. at 6%?
12. What would be the interest of the same at 5%?
13. What is the interest of \$8 for 1 yr. 8 mo. at 6%?
14. What would it be at  $3\frac{1}{2}\%$ ?
15. What is the interest of \$20 for 3 yr. at 6%?
16. What would it be at 4%?

## Interest

1. Find the interest of \$16 for 2 yr. 5 mo. at 5%.

$$\begin{array}{r}
 \$.12 \text{ interest of } \$1 \text{ for 2 years at } 6\%. \\
 \underline{.025} \text{ interest of } \$1 \text{ for 5 months at } 6\%. \\
 \$.145 \text{ interest of } \$1 \text{ for 2 yr. 5 mo. at } 6\%. \\
 \underline{16} \\
 870 \\
 \underline{145} \\
 6) \underline{\$2.320} \text{ interest of } \$16 \text{ for 2 yr. 5 mo. at } 6\%. \\
 \underline{\$.386\frac{2}{3}} \text{ interest of } \$16 \text{ for 2 yr. 5 mo. at } 1\%. \\
 5 \\
 \underline{\$1.933\frac{1}{3}} \text{ interest of } \$16 \text{ for 2 yr. 5 mo. at } 5\%.
 \end{array}$$

Take three distinct steps in solving each of the following problems :

*First*, find the interest of \$1 for the given time at 6%.

*Second*, find the interest of the given number of dollars for the given time at 6%.

*Third*, from the interest of the whole sum at 6% obtain the interest at the required per cent.

*Find the interest of:*

2. \$12 for 2 yr. at 3%.
3. \$100 for 3 yr. 6 mo. at 5%.
4. \$200 for 5 yr. 3 mo. at 4%.
5. \$240 for 4 yr. 5 mo. at 7%.
6. \$500 for 6 yr. 9 mo. at 8%.
7. \$375 for 1 yr. 5 mo. at 6%.
8. \$248 for 3 yr. 3 mo. at 5%.
9. \$85.50 for 8 mo. at 3%.
10. \$164 for 4 yr. 7 mo. at 5%.
11. \$236.80 for 2 yr. 5 mo. at 4%.

## Problems from Physics

See Book II, pp. 86, 189, 239.

1. Sound travels in the air at the rate of about 1100 feet a second. How long does it take sound to travel 5 miles?

2. Light travels 186,000 miles a second. How many seconds does it take light to reach the earth from a star 200,000,000 miles distant?

3. A heavy body will fall in a certain number of seconds a number of feet equal to about 16 times the square of the number of seconds. How far will a stone fall in 8 seconds?

4. How long will it take a stone to fall 64 feet?

5. The specific gravity of a substance is the number of times heavier it is than pure water. A cubic foot of pure water weighs 1000 ounces. The specific gravity of mercury is 13.59. Find how many pounds a cubic foot of mercury would weigh.

6. Find how many ounces a cubic inch of mercury would weigh.

7. The pressure of the air supports the column of mercury in the tube of the barometer. The weight of the column of mercury is equal to the pressure of the air upon the same surface. If the column of mercury is 30 inches high and rests upon one square inch, find the weight of the mercury.

8. Find the pressure of the atmosphere per square inch when the column of mercury is 30.7 inches high.

9. How much will be the pressure of the atmosphere when the barometer indicates 28.3 inches?

## Review Problems

1. Through how many degrees of longitude does the sun seem to pass in an hour? Through how many minutes of longitude in a minute? Through how many seconds of longitude in a second?

2. If a man travels until his watch is 2 hours too fast, in which direction does he travel? Through how many degrees does he travel?

3. If a man travels until his watch is 2 minutes too slow, through how many minutes of longitude does he travel?

4. If the days are 2 hr. 20 min. longer than the nights, how long are the nights?

5. If the nights are 1 hr. 36 min. 10 sec. longer than the days, how long are the days?

6. If I buy cloth measured with a yardstick which is 2 inches too short, and pay 36 cents a yard, how much do I lose by the false measure in buying 20 yards?

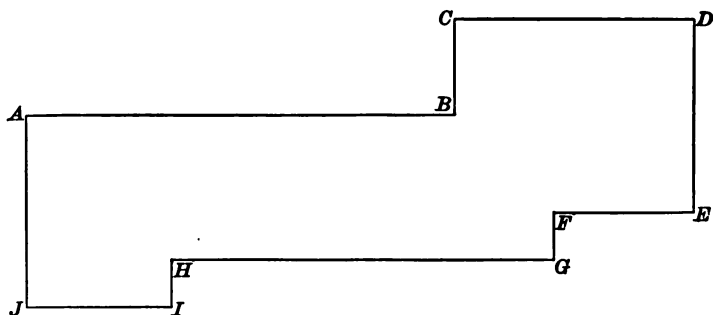
7. If sugar is sold at the rate of 20 pounds for a dollar, but the scale indicates a pound for only 14 ounces, how many pounds are really sold for a dollar?

8. How much will it cost to have insurance placed upon 10,500 lb. of wool worth 36 cents a pound, for  $\frac{3}{4}$  of its value, at  $\frac{3}{4}$ %?

9. A man bought a house for \$4000. He received \$30 a month for rent, and paid \$40 each year for repairs, and  $\frac{2}{3}$ % of  $\frac{3}{4}$  of its value for insurance. If the value of the property did not depreciate, how much had he gained for the use of his capital at the end of 4 years?

10. At the end of 5 years the house was destroyed by fire. How much did he gain or lose by the transaction?

## Measurements



The diagram represents a farm. It is drawn to the scale of  $\frac{1}{8}$  of an inch to 10 rods.

1. Find how far it is from  $A$  to  $D$  by way of  $B$  and  $C$ .
2. Which is the nearer way from  $J$  to  $D$ , to follow the line by way of  $A$  and  $C$ , or by way of  $I$  and  $E$ ?
3. If 2 acres were to be sold in the form of a rectangular strip taken from the east end of the farm, how many rods wide would the strip be?
4. If the rectangle  $CDE$  were completed by a continuation of the fences, how many acres would it contain?
5. If the rectangle  $JIH$  were completed, how many acres would it contain?
6. If the rectangle  $AJI$  were completed, how many acres would it contain?
7. If the fence  $CB$  were continued across the farm, how many acres would there be in the part east of the fence?
8. Find how many acres there are in the whole farm.
9. If the fence  $IH$  were continued across the farm so as to convert the west end of the farm into a pasture, how many cows could be pastured in it, if each cow must have at least 2 acres?





## Standard Time

In traveling from west to east or from east to west, since the true time as determined by the sun constantly changes, a watch becomes too slow or too fast. To avoid the confusion which would result from this, it is agreed in the United States to set the clocks so that there shall be only three changes of a whole hour each in passing across the country. The country is divided into four sections. All the people of any one of these sections set their clocks and watches at what would be the true time for the circle of longitude that passes through the middle of the section. The circles adopted as standards are  $15^\circ$  apart, as indicated upon the map on p. 26.

1. At 5 o'clock, what time is it at a place which is  $30^\circ$  farther east?
2. What time is it at a place  $45^\circ$  farther west?
3. According to standard time, when it is noon in New York, what time is it in California? (Consult the map.)
4. When it is 8 A.M. in Colorado, what time is it in New Jersey?
5. When it is 1 P.M. in Wisconsin, what time is it in Oregon?
6. How should a watch be changed in going across central Nebraska from the western to the eastern part? in going from Massachusetts to Montana?
7. How much difference is there between the true time and the standard time at a place whose longitude is  $80^\circ$  W.?  
The standard time for the place is the true time for longitude  $75^\circ$  W. What time corresponds to  $5^\circ$  of longitude?
8. What is the difference between the true time and the standard time, at a place whose longitude is  $85^\circ$  W.?  
 $97\frac{1}{2}^\circ$  W.?  $118^\circ 30'$  W.?  $94^\circ 45'$  W.?

## Interest

See pp. 17, 21, 22.

*Find the interest at 4% :*

1. Of \$60 for 3 yr. 6 mo.
2. Of \$35 for 1 yr. 3 mo.
3. Of \$87 for 2 yr. 8 mo.
4. Of \$48 for 5 yr. 2 mo.
5. Of \$64 for 4 yr. 10 mo.

*Find the interest at  $4\frac{1}{2}$ % :*

6. Of \$25.60 for 2 yr. 8 mo.
7. Of \$48.20 for 3 yr. 2 mo.
8. Of \$72.45 for 4 yr. 5 mo.
9. Of \$65.80 for 1 yr. 4 mo.
10. Of \$92.40 for 5 yr. 6 mo.

Since  $4\frac{1}{2}$  is  $\frac{3}{4}$  of 6, the interest at  $4\frac{1}{2}$ % is  $\frac{3}{4}$  of the interest at 6%.

*Find the interest at 9% :*

11. Of \$100 for 5 yr. 1 mo.
12. Of \$250 for 3 yr. 4 mo.
13. Of \$375 for 4 yr. 7 mo.
14. Of \$762.40 for 2 yr. 1 mo.
15. Of \$428.30 for 8 yr. 4 mo.

*Find the interest at  $7\frac{1}{2}$ % :*

16. Of \$95 for 2 yr. 10 mo.
17. Of \$79.45 for 1 yr. 9 mo.
18. Of \$37.65 for 10 yr. 6 mo.
19. Of \$81.32 for 4 yr. 11 mo.
20. Of \$132.60 for 2 yr. 7 mo.

What part of 6 is  $7\frac{1}{2}$ ? The interest at  $7\frac{1}{2}$ % is  $\frac{5}{6}$  of the interest at

## Original Problems

*Make problems from the following statements, and solve them:*

Each pupil should make problems from the statements. As one pupil reads his problem, the class should solve it.

1. A pile of wood is 28 ft. long.
2. A room is 24 ft. long and 18 ft. wide.
3. The circumference of a wheel is 9 ft. 6 in.
4. Goods which cost \$ 75 were sold at a gain.
5. Goods which cost \$150 were sold at a loss.
6. An agent charges 10% for collecting bills.
7. A house is insured at  $1\frac{1}{4}\%$ .
8. One of two places is  $35^\circ$  farther west than the other.
9. The difference in time between two places is  $3\frac{1}{2}$  hours.
10. Brooklyn is at  $73^\circ 58' W.$ , and San Francisco at  $122^\circ 26' 12'' W.$
11. A man traveled until his watch was 1 hr. 15 min. too slow.
12. Another man traveled until his watch was 2 hr. 45 min. too fast.
13. A man 45 years old had his life insured.
14. A man had his life insured at the age of 48 and died at the age of 56.
15. Sound travels in the air at the rate of about 1100 feet a second.
16. The steam from the whistle of a locomotive is seen at a distance of 4 miles.
17. A stone was 3 seconds in falling.
18. Light travels at the rate of 186,000 miles a second.

## Algebra

If the letter  $a$  represents the length of one side square, whatever this length may be, 4 times  $a$ , or will represent the perimeter of the square. If  $a$  represents 2,  $3a$  will represent 6,  $4a$  will represent 8, etc. this way it is possible to reason about quantities with using definite numbers.

*Find the value of each of the following expression*  
 $a = 1, b = 2, c = 3,$  and  $d = 4$ :

1.  $a + b + c = ?$

2.  $a + c - b = ?$

3.  $2a + b = ?$

4.  $3a + 2c = ?$

5.  $4b + c - d = ?$

6.  $c + d - b = ?$

7.  $b \times c = ?$

8.  $a \times b \times c = ?$

9.  $b \times c \times d = ?$

10.  $2a \times b \times d = ?$

11.  $\frac{a}{b} + c = ?$

12.  $\frac{d}{b} - a = ?$

13.  $\frac{2b}{d} + 2a = ?$

14.  $3c - \frac{2b}{a} = ?$

15.  $\frac{b+d}{b} = ?$

16.  $4d - \frac{b}{d} = ?$

*Find the value of the following expressions if*  $a = 3, b = 1,$  and  $d = 8$ :

17.  $a + b + c + d = ?$

18.  $a + b - d = ?$

19.  $b - a + d = ?$

20.  $c + d - a - b = ?$

21.  $b - c + d - a = ?$

22.  $a + b + c - d = ?$

23.  $\frac{a}{b} + c = ?$

24.  $\frac{d}{c} - \frac{b}{a} = ?$


25.  $\frac{b+c}{d} + \frac{a}{b} = ?$

26.  $\frac{d+2c}{b} + \frac{3a}{c} = ?$

## Algebraic Problems

The algebraic method of solving a problem consists in making use of a symbol to represent any unknown number or quantity in the problem. The letters of the alphabet are generally used as symbols.

1. James has twice as many apples as his brother John. Together they have 12 apples. How many has each?



$x = \text{John's}$   
 $2x = \text{James'}$   
 $x + 2x = 12$   
 $3x = 12$   
 $x = 4$   
 $2x = 8$

If we let the letter  $x$  represent the number that John has, since James has twice as many, two times  $x$ , or  $2x$ , will represent the number that James has. Three times  $x$ , or  $3x$ , will equal the whole number 12, and once  $x$ , or  $x$ , will be 4.

2. In my right hand I have three times as many cents as in my left hand. In both hands I have 24 cents. How many have I in each hand?

3. Divide the number 96 into two such parts that one part may be five times as large as the other.

4. A number added to ten times itself equals 352. What is the number?

5. A father is four times as old as his son. The sum of their ages is 40 years. How old is the son?

6. The sum of five times a number and three times the number is 120. What is the number?

7. Divide \$400 between two men so that one man may have 7 times as much as the other.

8. A number increased by 5 times itself equals 174. What is the number?

## Algebraic Problems

1. A farmer has a number of horses, twice as many cows, and three times as many sheep. In all he has 30 animals. How many of each kind has he?

$$\begin{array}{c} x \qquad \qquad 2x \qquad \qquad 3x \\ \hline \boxed{\phantom{x}} + \boxed{\phantom{x}} \boxed{\phantom{x}} + \boxed{\phantom{x}} \boxed{\phantom{x}} \boxed{\phantom{x}} = 30 \end{array}$$

2. In an orchard there are three times as many pear trees as peach trees and five times as many apple trees as peach trees. In all there are 72 trees. How many of each kind are there?

3. Twice a number, three times the number, and five times the number would together equal 200. What is the number?

4. In a yard there are twice as many ducks as geese and twice as many hens as ducks. In all there are 42. How many ducks are there?

5. Charles has three times as many cents as Jennie, and Henry has four times as many as Jennie. The three together have 72. How many has each?

6. If three times a certain number were taken from seven times the number, and five times the number were added to the remainder, the result would be 162. What is the number?

7. Clara is four times as old as Ruth, and Mary is half as old as Clara. The sum of their ages is 21 years. How old is each?

8. A man paid \$28 for a coat, a hat, and some shoes. He paid four times as much for the coat as for the shoes, and twice as much for the coat as for the hat. What was the cost of the shoes?

## Algebraic Problems

1. Divide 48 marbles between two children so that one child shall have three times as many as the other.

$$x + 3x = 48.$$

2. The difference between seven times a number and three times the number is 92. What is the number?

3. B has three times as much money as A, and C has twice as much as B. Together they have \$200. How much has each?

4. A boy had a certain number of papers. He bought four times as many more and then sold one half as many as he bought. He then had 21 papers. How many papers did he sell?

$$x = \text{the number he had. } x + 4x - 2x = 21.$$

5. I have a certain sum of money. If I should receive 5 times as much more and should spend  $\frac{1}{2}$  of all that I then had, I should have \$63 left. How much have I?

$$x + 5x = 6x; \frac{1}{2} \text{ of } 6x = 3x; 6x - 3x = 3x; 3x = 63.$$

6. If from 7 times a certain number twice the number is taken, the remainder will be 60. What is the number?

7. If a certain number is multiplied by 9 and 3 times the number is taken from the product, the result will be 66. Find the number.

*Make problems from the following and solve them:*

8.  $12x - 7x = 65$

13.  $9x - 4x + x = 84$

9.  $x + 3x + 12x = 48$

14.  $6x + x - 4x = 51$

10.  $7x - 5x = 18$

15.  $12x - 9x + 4x = 87\frac{1}{2}$

11.  $x + 8x - 4x = 70$

16.  $50x - 23x - 10x = 119$

12.  $8x - 3x - x = 40$

17.  $32x - 30x + 8x = 1000$



## Ratio

The **ratio** of one number, or quantity, to another is the number of times that the first will contain the second, or the number expressing the size of the first when the second is regarded as the unit of measure.

The ratio of a rectangle 2 inches wide and 6 inches long to a 2-inch square is 3, because the rectangle will contain the square 3 times, or because, if the square is regarded as the unit of measure, there are 3 units in the rectangle.

*What is the ratio :*

1. Of 35 to 5 ?
2. Of  $7\frac{1}{2}$  to  $\frac{1}{4}$  ?
3. Of  $3\frac{1}{3}$  to 20 ?
4. Of  $\frac{5}{8}$  to  $7\frac{1}{8}$  ?
5. Of  $12\frac{1}{2}$  to 75 ?
6. Of  $\frac{3}{4}$  to  $\frac{15}{16}$  ?
7. Of  $\frac{12}{8}$  to  $\frac{4}{6}$  ?
8. Of  $4\frac{1}{7}$  to  $9\frac{1}{3}$  ?
9. Of 17.9 to 5.2 ?
10. Of 8.23 to 2.08 ?

*What is the ratio :*

11. Of a 9-inch square to a 3-inch square ?
12. Of a 10-inch square to an inch square ?
13. Of a 10-inch square to a 2-inch square ?
14. Of a rectangle 25 in.  $\times$   $4\frac{1}{2}$  in. to a rectangle  $7\frac{1}{2}$  in.  $\times$  3 in. ?
15. Of a triangle whose base is  $18\frac{1}{4}$  ft. and altitude  $12\frac{1}{3}$  ft. to a triangle whose base is  $8\frac{1}{2}$  ft. and altitude  $6\frac{1}{4}$  ft. ?
16. Of the price of 45 yards of cloth to the price of  $2\frac{1}{2}$  yards of cloth ?
17. Of the time required to walk  $7\frac{1}{2}$  miles to the time required to walk 90 miles ?
18. Of 75% of \$300 to  $7\frac{1}{2}$ % of \$300 ?
19. Of the interest of \$365.50 to the interest of \$50 with the same time and rate ?

## Decimal Fractions

1. Add twenty-five and two hundred fifty-six thousandths; seven hundred ninety-six ten thousandths; two hundred fifty-eight and seventy-five hundred-thousandths; thirty-two millionths.

2. Add one hundred ninety and nine tenths; two thousand, five hundred; ninety-four and twenty-seven thousand, five hundred thirty-five hundred-thousandths; nine hundred-thousandths.

3. From six thousandths take six hundred-thousandths.

4. From twenty-one and two hundred five ten-thousandths take four and nine hundred nine thousandths.

5. Multiply five hundredths by five tenths.

6. Multiply thirty-five hundred-thousandths by two thousandths.

7. Multiply six hundred seventy-five hundred-thousandths by one tenth.

8. Multiply four hundred ninety-five ten-thousandths by one hundred.

9. Multiply nine and three hundredths by five hundred.

10. Divide twenty-four hundredths by three.

11. Divide eight and seven hundred thirty-six thousandths by twelve.

12. Divide twenty-eight hundredths by ten.

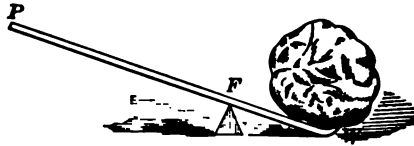
13. Divide twenty-three and forty-two hundredths by one hundred.

14. Divide six hundred forty-three thousandths by one thousand.

15. Change to common fractions: .9; .25; .08; .36; .035.

16. Change to common fractions: .15;  $.12\frac{1}{2}$ ; .005;  $.87\frac{1}{2}$ ; .055.

## Lever



When one long object, as a bar, rests upon a point of support, with a weight at one point and a force applied at another point, we have a mechanical principle called the principle or law of the lever. The bar is called the **lever**, the point of rest the pivot or **fulcrum**, the object to be moved the **weight**, and the force applied the **power**. The amount of force required to move or balance the weight depends upon the relative distances of the **power** and weight from the fulcrum. A weight of 10 lb. can be raised by a power of 5 lb. if the power is twice as far from the fulcrum as the weight is. *The power multiplied by its distance from the fulcrum is always equal to the weight multiplied by its distance from the fulcrum.*

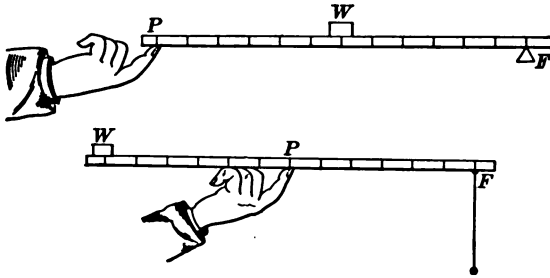
When the fulcrum is between the power and the weight, the lever is called a lever of the **first class**.

1. If 6 times a certain number equals 8 times 9, what is the number?

2. If a foot rule is rested upon a support at a point 3 in. from one end, and a weight of 6 oz. is placed upon this end, how heavy an object must be placed upon the other end to balance it?

3. If a board 15 ft. long is rested over a fence so that 10 ft. of the board is on one side of the fence, and a child weighing 80 lb. sits upon the end which is 5 ft. from the fence, how heavy must a second child be to be able to raise the first child from the ground by pulling down on the other end of the board?

## Levers



In a lever of the **second class**, the object to be moved or the weight is between the fulcrum or resting point of the lever and the point where the power is applied.

In a lever of the **third class**, the power is applied between the fulcrum and the weight.

In all cases the same law applies. See p. 36.

1. A workman wishes to raise a heavy object by pushing under it a lever 15 ft. long so that the end of the lever will rest upon a point beyond it. If the distance between the object and the fulcrum is 3 ft., and the object weighs a ton, how many pounds must the man be able to lift at the other end of the lever in order to raise the object?

2. In a lever of the third class, if the distance from the power to the fulcrum is 20 in. and the distance from the weight to the fulcrum is 50 in., how much power must be applied to raise a weight of 10 lb.?

3. If an oar rests in the rowlock at its middle point, how many pounds of force are applied to the boat by pulling 150 lb. at the end of the oar?

4. How many pounds are applied if the distance of the rowlock from the water is twice that of the hands from the rowlock?

## Levers

Let  $P$  represent the power,  $W$  the weight,  $F$  the fulcrum,  $PD$  the power distance, or distance of the power from the fulcrum, and  $WD$  the weight distance, or distance of the weight from the fulcrum.

*Find the missing term:*

1.  $P = 25$  lb.  $PD = 15$  in.  $W = 100$  lb.
2.  $P = 60$  lb.  $W = 240$  lb.  $WD = 6$  ft.
3.  $W = 500$  lb.  $WD = 4\frac{1}{2}$  ft.  $PD = 7$  ft.
4.  $PD = 27$  in.  $WD = 5$  in.  $P = 50$  lb.
5.  $P = 200$  lb.  $W = 2000$  lb.  $PD = 20$  ft.

*In a lever of the first class:*

6.  $P = 20$  lb. From  $P$  to  $F$  it is 10 in.; from  $F$  to  $W$  3 in. Find  $W$ .
7.  $W = 75$  lb. From  $P$  to  $W$  it is 24 in.; from  $P$  to  $F$  18 in. Find  $P$ .
8.  $W = 250$  lb.  $P = 40$  lb. From  $W$  to  $F$  it is 15 in. Find  $PD$ .

*In a lever of the second class:*

9.  $P = 80$  lb. From  $P$  to  $F$  it is 15 ft.; from  $W$  to  $F$  4 ft. Find  $W$ .
10.  $W = 1000$  lb.  $P = 200$  lb. From  $F$  to  $W$  it is 9 in. Find  $PD$ .
11. From  $F$  to  $P$  it is 16 in.; from  $F$  to  $W$  4 in.  $P = 35$  lb. Find  $W$ .

*In a lever of the third class:*

12.  $P = 400$  lb. From  $P$  to  $W$  it is 16 in.; from  $P$  to  $F$  16 in. Find  $W$ .
13.  $W = 36$  lb. From  $W$  to  $F$  it is 42 in.; from  $P$  to  $W$  10 in. Find  $P$ .

## Interest

For ordinary purposes in reckoning interest the months are regarded as having 30 days each. Since the interest of \$1 at 6% for 1 month, or 30 days is 5 mills, for 6 days it is 1 mill.

1. What part of a mill does a dollar gain in 3 days? In 2 days? In 1 day? In 4 days? In 5 days?

2. What is the interest of \$1 at 6% for 6 days? For 12 days? For 24 days? For 9 days? For 21 days?

*The interest of \$1 at 6% is half as many cents as there are months and one sixth as many mills as there are days.*

*At 6% give the interest of \$1 for the following periods of time :*

3. 1 yr. 4 mo. 6 da.
4. 3 yr. 3 mo. 12 da.
5. 2 yr. 5 mo. 9 da.
6. 5 yr. 8 mo. 14 da.
7. 4 yr. 7 mo. 13 da.

*Find the interest of the following at 6% :*

First find the interest of \$1.

8. \$12 for 6 mo. 15 da.
9. \$25 for 2 yr. 3 mo. 18 da.
10. \$42.18 for 3 yr. 1 mo. 7 da.
11. \$48.50 for 5 yr. 3 mo. 9 da.
12. \$100 for 8 yr. 4 mo. 15 da.

*Find the interest of the following :*

13. \$48 for 49 da. at 6%.
14. \$120 for 6 mo. 18 da. at 3%.
15. \$32.50 for 2 yr. 1 mo. 6 da. at 5%.
16. \$245 for 3 yr. 2 mo. 12 da. at 4%.
17. \$362.50 for 5 yr. 4 mo. 24 da. at 7%.

## Percentage

1. 8 is what per cent of 50 ?

8 is  $\frac{8}{50}$  of 50;  $\frac{8}{50} = \frac{16}{100}$  or 16%; hence 8 is 16 per cent of 50.

2. 9 is what per cent of 54 ?

9 is  $\frac{1}{6}$  of 54, and  $\frac{1}{6}$  is the same as  $\frac{16\frac{2}{3}}{100}$ , hence 9 is 16 $\frac{2}{3}$  per cent of 54.

The per cent may often be found in this way by inspection.

3. 83 is what per cent of 4250 ?

1 per cent of 4250 is 42.50. 83 is as many per cent of 4250 as 42.50 is contained times in 83.

*Find what per cent the one is of the other. Find it by inspection when possible :*

- |                             |                         |
|-----------------------------|-------------------------|
| 4. 17 of 100.               | 19. 75 of 375.          |
| 5. 7 of 50.                 | 20. 14 of 350.          |
| 6. 6 of 25.                 | 21. 16 of 80.           |
| 7. 12 of 200.               | 22. 112 of 700.         |
| 8. 6 of 150.                | 23. 50 of 90.           |
| 9. 13 of 20.                | 24. 12.5 of 37.5.       |
| 10. 10 of 240.              | 25. 94 of 385.          |
| 11. 19 of 300.              | 26. 38 of 96.           |
| 12. 76 of 400.              | 27. 12.35 of 4200.      |
| 13. $7\frac{1}{2}$ of 10.   | 28. 24.83 of 1356.      |
| 14. $3\frac{1}{5}$ of 20.   | 29. 8.9 of 75.42.       |
| 15. $4\frac{3}{4}$ of 25.   | 30. \$275 of \$625.     |
| 16. $6\frac{3}{8}$ of 300.  | 31. \$125.34 of \$748.  |
| 17. $15\frac{5}{8}$ of 500. | 32. \$95.75 of \$2500.  |
| 18. 200 of 1000.            | 33. \$325.60 of \$8500. |

## Problems from Geography and Astronomy

See Book II, pp. 117, 244.

How many degrees are there in a circumference?

1. The equatorial diameter of the earth is 7926 miles. Find its circumference.

2. Find the length in miles of one degree of longitude at the equator of the earth.

3. Find the number of square miles upon the earth's surface.

4. If about  $\frac{1}{4}$  of the earth's surface is land and the entire population of the earth is about 1,523,000,000, how many people are there, upon the average, to the square mile?

5. The diameter of the moon is 2160 miles. What is its circumference?

6. The same side of the moon is always turned toward the earth. About how many square miles of the moon are there which we never see?

7. How far does any point at the equator of the earth move in a day on account of the revolution of the earth upon its axis?

8. At a point between the equator and the north pole where the circle, or parallel of latitude, is  $\frac{2}{3}$  as large as the circle at the equator, how many miles an hour are points on that circle carried by the revolution of the earth?

9. The moon is about 240,000 miles from the earth. How many miles does it travel in going around the earth?

10. The moon revolves around the earth in about 27 days. About how many miles a day does it travel in its revolution?



## Interest

*Find the interest at 6%:*

1. Of \$100 for 3 yr. 4 mo.
2. Of \$200 for 6 mo. 21 da.
3. Of \$240 for 2 yr. 3 mo. 12 da.
4. Of \$350 for 3 yr. 1 mo. 21 da.

*Find the interest at  $3\frac{1}{2}\%$ :*

5. Of \$75 for 2 yr. 8 mo.
6. Of \$145 for 9 mo. 22 da.
7. Of \$82.50 for 7 mo. 9 da.
8. Of \$90 for 3 yr. 2 mo. 7 da.

*Find the interest at  $4\frac{1}{2}\%$ :*

9. Of \$125 for 5 yr. 4 mo. 6 da.
10. Of \$216 for 2 yr. 7 mo. 9 da.
11. Of \$325 for 6 yr. 11 mo. 14 da.
12. Of \$418 for 7 yr. 1 mo. 8 da.

*Find the interest:*

13. Of \$50 for 1 yr. 6 mo. at 6%.
14. Of \$85 for 2 yr. 4 mo. at 5%.
15. Of \$122 for 4 yr. 8 mo. 18 da. at 3%.
16. Of \$248 for 5 yr. 2 mo. 10 da. at 7%.

*Find the amount:*

17. Of \$100 for 1 yr. 6 mo. at 6%.
18. Of \$160 for 3 yr. 5 mo. 12 da. at 4%.
19. Of \$45.85 for 9 mo. 18 da. at  $4\frac{1}{2}\%$ .
20. Of \$253.40 for 2 yr. 3 mo. 2 da. at 5%.

## Review Problems

1. If a man travels until his watch is 1 hr. 45 min. too slow, in which direction and through how many degrees of longitude does he travel?

2. What is the difference in longitude between two places whose difference in time is 2 hr. 30 min. 20 sec.?

3. Berlin is  $13^{\circ} 23' 43''$  E. The difference in time between Berlin and St. Petersburg is 1 hr. 7 min. 27 sec. Find the longitude of St. Petersburg.

4. London is  $0^{\circ} 5' 38''$  W., and Sydney, Australia,  $151^{\circ} 11' 0''$  E. What is the difference in time between the two places?

5. What is the interest of \$40 for 3 yr. 5 mo. at 6%?

6. What is the interest of \$60 for 5 yr. 2 mo. at  $4\frac{1}{2}\%$ ?

7. What is the amount of a note of \$100, with interest added, for 3 yr. 9 mo. at 5%?

8. What is the amount of a note of \$140 for 2 yr. 1 mo. at 7%?

9. Find the amount of a note of \$200 for 7 yr. 5 mo. at  $5\frac{1}{2}\%$ .

10. In the case of a lever the power is 100 lb., the weight 500 lb., and the distance of the weight 3 feet. Find the distance of the power.

11. The power is 135 lb., the distance of the power  $12\frac{1}{2}$  feet, and the distance of the weight  $1\frac{1}{2}$  feet. Find the weight.

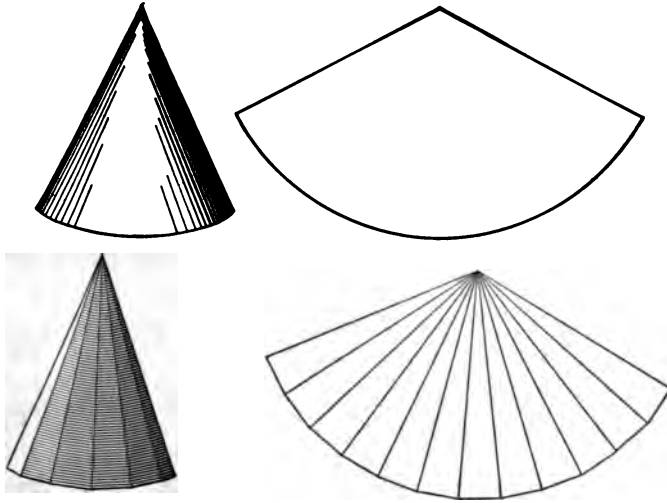
12. The power is  $32\frac{1}{4}$  lb., the weight 160 lb., and the distance of the power 11 feet. Find the distance of the weight.

## Drill Work

In adding these columns combine at a glance groups of figures whose sum is 10, and thus add by 10's.

<b>1.</b>	<b>2.</b>	<b>3.</b>	<b>4.</b>	<b>5.</b>
245	630	784	908	826
865	479	263	524	284
342	251	847	586	543
768	857	255	500	567
129	383	405	358	372
981	720	650	752	738
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>
<b>6.</b>	<b>7.</b>	<b>8.</b>	<b>9.</b>	<b>10.</b>
243	406	320	235	454
314	253	245	220	300
553	451	545	655	356
472	130	332	405	530
216	546	246	460	243
422	434	532	645	337
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>
<b>11.</b>	<b>12.</b>	<b>13.</b>	<b>14.</b>	<b>15.</b>
8052	5409	8095	\$ 25.34	\$ 76.45
2251	1383	2505	34.56	45.65
1143	4257	6545	20.55	32.35
2714	3020	2536	56.45	33.75
7630	7534	5534	63.37	54.25
2425	4643	5463	25.33	50.45
1004	4403	9242	20.28	46.45
6721	2766	1455	92.82	65.65
1337	4334	2252	17.75	25.45
2200	6273	1373	43.35	15.35
4312	5623	1211	34.64	33.35
4541	5284	6324	36.46	42.45
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>

## Cones



A **cone** is a solid that has a circle for its base, and that terminates in a point at its upper extremity. A cone may be regarded as a pyramid whose lateral surface is composed of a large number of very small triangles. If we could divide the lateral surface of a cone on one line of its slant height and spread it out upon a plane surface, what form would it assume?

1. Find the lateral surface of a cone the circumference of whose base is 25 inches, and whose slant height is 14 inches.
2. Find the diameter of the base of a cone whose circumference is 62.832 inches.
3. What is the circumference of the base of a cone, if the radius of the base is 24.15 feet?
4. Find the lateral surface of a cone, the diameter of whose base is 32 inches, and whose slant height is 6 feet.

## Statistics

1. The population of the United States from 1870 to 1900 increased from 38,500,000 to about 76,000,000. What has been the average increase for each year?

2. In 1870 there were 5871 newspapers and other periodicals published. In 1900 there were about 21,000. How many times did the number multiply during the period?

3. How many people were there for each paper published in 1870?

4. How many less people were there for each paper published in 1900 than in 1870?

5. In 1870 there were 28,492 post offices. How many people were there for each post office?

6. In 1900 there were 75,000 post offices. How many less people were there for each post office in 1900 than in 1870?

7. In 1870, 9,000,000 telegraphic messages were sent, and in 1899 about 77,000,000. Find whether the increase in the number of telegraphic messages was greater or less, proportionately, than the increase in population.

8. In 1870, 1,630,000 people in the country had deposited in the savings banks \$550,000,000. What was the average size of these deposits?

9. How much did this amount average for each person in the total population?

10. In 1900, 5,687,000 persons had \$2,230,000,000 deposited in institutions for savings. What was the average then for each depositor?

11. Find how much the average amount for each person the population increased in the 30 years.

## Review Problems

1. The longitude of Boston is  $71^{\circ} 3' 30''$  W., and that of New York  $74^{\circ} 0' 3''$  W. What is the true difference in time?

2. The longitude of San Francisco is  $122^{\circ} 26' 15''$  W. What is the true difference in time between New York and San Francisco?

3. What is the true difference in time between Boston and San Francisco?

4. Honolulu is  $157^{\circ} 52' 0''$  W. In sailing from San Francisco to Honolulu how much must a watch be set back?

5. If a circular piece of paper 20 inches in diameter is cut in halves, and one half is formed into a cone, what will be the slant height of the cone?

6. What will be the circumference of the base of the cone?

7. What will be the diameter of the base?

8. What will be the convex surface?

9. If a circular piece of paper 24 inches in diameter is cut in quarters, and one of the quarters is formed into a cone, what will be the slant height of the cone?

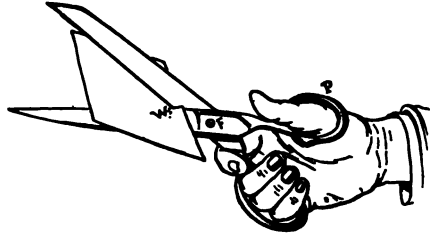
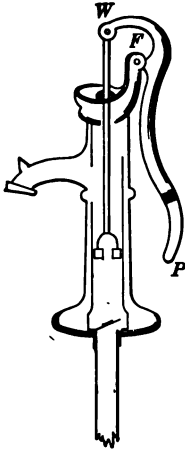
10. What will be the diameter of the base?

11. What will be the convex surface?

12. If from a circular piece of paper whose circumference is 30 inches a section is removed by cutting from the circumference to the center so as to remove 6 inches of the circumference, and the edges are brought together so as to form a cone, what will be the diameter of the base of the cone?

13. What will be the convex surface of the cone?

## Lever



See pages 36, 37.

Which of the three classes of levers are pump handles? Nut crackers? Shears?

1. If it is 25 inches from the end of the handle of a pump to the bolt by which it is fastened, and 5 inches from this bolt to the piston rod, how many pounds of pressure must be applied at the end of the handle to raise 100 lb. of water?

2. If the entire length of the pump handle is 36 inches and the shorter part is 8 inches, how many pounds of water can be raised by a child who can pull down 35 lb. upon the handle?

3. In a nut cracker, which is 6 inches long from the joint to the point where the fingers press, the nut is placed 2 inches from the joint. How much pressure can be applied to the nut by pinching 20 lb.?

4. How many pounds could be applied, if the nut were  $1\frac{1}{2}$  inches from the joint?

## Standard Time

Refer to page 27.

*Give the difference in standard time and state which is earlier:*

1. Between Boston and Chicago.
2. Between Boston and Charleston, S.C.
3. Between New York and Denver.
4. Between Minneapolis and Savannah.
5. Between Seattle and Kansas City.

*How should a watch be changed in going:*

6. From San Francisco to St. Louis?
7. From Cincinnati to Austin?
8. From Richmond, Va., to St. Paul?
9. From Salt Lake City to Indianapolis?
10. From Portland, Me., to Portland, Ore.?

*Find the difference in longitude between each place and the line of the central meridian of the section in which it is located:*

11. Omaha, whose longitude is  $95^{\circ} 56' 14''$  W.
12. New York, whose longitude is  $74^{\circ} 0' 3''$  W.
13. St. Paul, whose longitude is  $93^{\circ} 5'$  W.
14. San Francisco, whose longitude is  $122^{\circ} 23' 54''$  W.
15. Washington, whose longitude is  $77^{\circ} 0' 28''$  W.

*How much earlier or later than the true time is the standard time:*

16. At Boston, whose longitude is  $71^{\circ} 3' 48''$  W.?
17. At St. Louis, whose longitude is  $90^{\circ} 12' 23''$  W.?
18. At Louisville, Ky., whose longitude is  $85^{\circ} 30'$  W.?
19. At Pittsburg, Pa., whose longitude is  $80^{\circ} 2'$  W.?
20. At Portland, Me., whose longitude is  $70^{\circ} 15' 18''$  W.?



## Percentage

Review Book II., pages 212, 229.

1. Find 17 per cent of \$350.
2. Find  $\frac{3}{8}\%$  of 4000 lb.
3.  $17\frac{1}{2}$  is what per cent of 25?
4. \$125 is what per cent of \$500?
5. 45 is 15% of what number?
6. \$72 is  $4\frac{1}{2}\%$  of what sum of money?
7. \$1400 is 350% of a certain sum. What is the sum?
8. 560 is  $12\frac{1}{2}\%$  less than what number?
9. 791 is 13% more than what number?
10. A merchant bought a car load of bananas consisting of 850 bunches. He sold 10% of them to one man, 17% to another, and 35% to another. How many bunches remained?
11. My house is worth \$7200. If I have it insured for  $\frac{3}{4}$  of its value at  $\frac{3}{4}\%$ , what will be the amount of the premium?
12. Of 400 immigrants who arrived on a certain steamer 94 could not write. What per cent of the number could not write?
13. Fifteen per cent of the value of a certain stock of goods is \$1252.50. Find the value of the goods.
14. By selling goods for \$728 I gained 12%. What was the cost of the goods?
15. A dealer sold some flour at a loss of 20%. He lost \$90. There were 150 barrels of flour. How much did it cost per barrel?
16. A commission merchant sold 60 tubs of butter the total weight of which, including the tubs, was 3315 pounds. The average weight of the tubs was  $5\frac{1}{2}$  pounds each. He received  $21\frac{1}{2}$  cents a pound. His commission was 5%. What sum did he remit to the shipper?

## Original Problems

*Make problems and solve them :*

1. A father is three times as old as his son.
2. A farmer has a number of sheep, pigs, and hens.
3. A boy had a number of papers. He bought more and then sold some.
4. The fulcrum of a lever is between the power and the weight.
5. The weight is between the power and the fulcrum.
6. The lever is a lever of the third class.
7. The difference in time between two places is 3 hr. 15 min. 45 sec.
8. One place is  $30^{\circ}$  E. and another is  $165^{\circ}$  W.
9. The diameter of the base of a cone is 3 ft. 6 in.
10. The slant height of a cone is 4 ft. 8 in.
11. A church spire is in the form of a cone. Its slant height is 65 feet.
12. A pump handle is four feet long.
13. A board rests over a fence. There is a child upon each end.
14. A nut cracker is 9 inches long.
15. A bar 12 feet long is pushed under a stone.
16. The handles of the shears of a tinsmith are two feet long.
17. The length of an old well sweep is 18 feet. The weight upon one end is 100 lb.
18. A merchant bought a car load of oranges.
19. I had my house insured for  $\frac{1}{2}$  of its value.
20. A dealer sold some flour at a gain of 10%.

## Algebra

An expression which indicates that two numbers or quantities are equal is called an **equation**. The expression  $x + 3x = 16$  is an equation.  $x + 3x$  is called the first member of the equation, and 16 is called the second member.

*Make problems which would be represented by the following equations and solve them :*

1.  $x + 2x = 42$

4.  $x + 2x + 4x = 126$

2.  $2x + 5x = 70$

5.  $2x + 3x + 6x = 66$

3.  $\frac{1}{2}x - x = 48$

6.  $6x - \frac{1}{2}x + 5x = 140$

7. One half of a certain number is 17. What is the number?

8. A number added to one half of itself equals 18. What is the number?

$\frac{1}{2}$  of  $x$  may be written  $\frac{x}{2}$ .  $1\frac{1}{2}$  times  $x$  or  $\frac{3}{2}$  of  $x$  may be written  $\frac{3x}{2}$ .  
If three halves of  $x$  equal 18, what will one half of  $x$  equal?  $x = ?$

9. Divide the number 36 into two such parts that one part may be half as large as the other.

10. The width of an oblong is  $\frac{1}{3}$  of its length. The sum of the length and width is 28 inches. How long is it?

11. The width of a rectangle is  $\frac{1}{4}$  of its length. The distance around the rectangle is 20 feet. What is its area?

12.  $\frac{2}{3}$  of a number and  $\frac{1}{6}$  of a number equals 60. What is the number?

$$\frac{5x}{6} = 60; \quad \frac{x}{6} = 12; \quad x = 72.$$

13.  $\frac{7}{10}$  of a number less  $\frac{1}{5}$  of the number equals 42. What is the number?

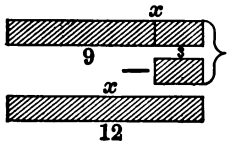
## Algebra

$$5=5 \qquad 9=9 \qquad 6=6 \qquad 8=8$$

$$5+3=5+3 \qquad 9-4=9-4 \qquad 3 \times 6=3 \times 6 \qquad 8+2=8+2$$

It is evident that we may add the same number to both members of an equation, subtract the same number from both members, multiply both members by the same number, or divide both members by the same number, and still have a true equation.

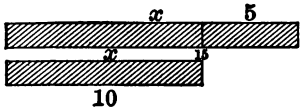
1. If 3 is taken from a certain number the remainder is 9. What is the number?



$$\begin{array}{l}
 x \\
 \hline
 9 \qquad 3 \\
 \hline
 x - 3 = 9 \qquad x - 3 + 3 = 9 + 3 \\
 x = 9 + 3 \qquad \qquad x = 12
 \end{array}$$

We wish to find the whole value of  $x$ . In other words, we wish to have  $x$  stand alone as the first member of the equation. Since the first member as it stands is 3 less than  $x$ , by adding 3 we get  $x$  for the first member. Since it is necessary also to add 3 to the second member to keep the equation true, we have  $x = 12$ . This work may be done in one step by transferring the 3 from the first to the second member and *changing its sign*.

2. If 5 is added to a certain number the result is 15. Find the number.



$$\begin{array}{l}
 x \qquad 5 \\
 \hline
 = 15 \qquad x + 5 = 15 \qquad x + 5 - 5 = 15 - 5 \\
 \hline
 x \\
 \hline
 = 10 \qquad x = 15 - 5 \qquad x = 10
 \end{array}$$

To get  $x$  alone as the first member of the equation we must subtract 5 from each member. This may be done in one step by transferring the 5 from the first to the second member and *changing its sign*.

*Find the value of  $x$  in the following equations:*

$$\begin{array}{lll}
 3. \ x - 7 = 13 & 5. \ 2x - 4 = 20 & 7. \ x + 3x - 6 = 18 \\
 4. \ x + 10 = 35 & 6. \ 3x + 5 = 17 & 8. \ 4x + 3 - x = 22
 \end{array}$$



## Algebra

1.  $6x - 7 = 3x + 17$

First change the equation so as to remove the  $3x$  from the second member, and then again so as to remove the  $-7$  from the first member. Or transfer both in one step and *change the signs* of the terms transferred.

$$6x - 3x = 17 + 7$$

2.  $10x - 12 = 4x + 24$

4.  $30 + x = 84 - 2x$

3.  $8x + 10 = 5x + 25$

5.  $60 - x = 100 - 5x$

6.  $15 + x = 10 + 2x$

This, when changed, becomes  $x - 2x = 10 - 15$ . To add 10 and then subtract 15 is the same as to subtract 5. So to add  $x$  and subtract  $2x$  is the same as to subtract  $x$ . The equation becomes  $-x = -5$ . If to subtract  $x$  is the same as to subtract 5, then  $x = 5$ .

7.  $x + 35 = 3x + 15$

10.  $6x - 8 = 24 - 8$

8.  $70 + 2x = 10 + 6x$

11.  $x - 8 = 10 - 2x$

9.  $10x + 4 = 20 + 14$

12.  $10x - 40 = 50 - 8x$

13. If 3 is taken from six times a certain number, the remainder will be 9 more than twice the number. What is the number?

14. If 4 is added to twice a number, the result will be 10 less than nine times the number. What is the number?

15. Eight times the money which I have and \$9 more would be \$65. How much have I?

16. Five times as much money as I have, less \$20, would be three times as much as I have. How much have I?

17. In two schoolrooms there are 65 children. In one there are 13 more than in the other. How many are there in each room?

18. A father's age is twice his son's age. The father's age decreased by 13 years would equal the son's age increased by 9 years. How old is each?

## Problems for Arithmetic or Algebra

1. A binder bound for me a certain number of histories, 3 times as many novels, and 4 times as many books on science. In all he bound up 24 volumes. How many of each kind did he bind?

2. A farmer has a certain number of geese, twice as many turkeys, and three times as many chickens. In all he has 72. How many of each has he?

3. I have 3 times as many 5-cent pieces as dimes, and 4 times as many cents as dimes. In all I have 80 coins. How many have I of each?

4. Divide \$10 between 2 men, giving to the first \$1 more than twice as much as to the second.

Let  $x$  = amount given to the second. Then  $2x + 1$  = amount given to the first. Usually it is most convenient to represent the smallest quantity by  $x$ .

5. Divide \$8 between 2 men, giving the first \$1 less than twice as much as the second.

6. Place 11 silver coins in 2 piles, putting 3 more than 3 times as many in the first as in the second.

7. Place 24 silver coins in 2 piles, putting 4 less than 3 times as many in the first pile as in the second.

8. Divide 13 apples among 3 children, giving to the second 1 apple more than to the first, and to the third twice as many as to the first.

9. My money, increased by 4 times as much and \$5 more, would equal \$70. How much have I?

10. In the garden there are 4 less than 3 times as many pinks as roses. In all there are 28 flowers. How many are there of each?

## Problems from Astronomy .

First estimate the answers.

1. Light travels at the rate of about 186,000 miles a second. If it could travel in a curved line, how many times would it go around the earth in a second ?

2. The distance to the moon is 240,000 miles. How long does it take the light which is reflected from the moon to come from the moon to the earth ?

3. It is about 93,000,000 miles from the earth to the sun. How long does it take light to reach us from the sun ?

4. The planet Neptune is 2,800,000,000 miles from the sun. At a time when it is at the same distance from the earth how long does it take light to reach us from Neptune ?

5. Venus is 67,200,000 miles from the sun. When Venus is on the side of the sun toward the earth, about how far is it from the earth ?

6. When at this nearest position to the earth, how long does it take light to come from Venus to the earth ?

7. The distance to the nearest fixed star is about 25,000,000,000,000 miles. How long has the light which we see in the direction of the star been traveling toward us ?

8. How many times as far is it from the earth to the nearest fixed star as from the earth to the sun ?

9. Sound travels in the air at the rate of about 1100 feet per second. How many times faster does light travel than sound ?

10. At the rate of 1100 ft. per second, how long would it take for sound to travel from the earth to the moon ?



## Review Problems

\$800

NEW YORK, Jan. 12, 1900.

Six months after date, for value received, I promise to pay to Charles H. Sawyer, or order, Eight Hundred dollars with interest at 5 per cent per annum.

HENRY P. CLARKE.

1. Find the amount due upon the above note at the time of maturity.

The amount due is \$800 plus the interest for 6 months.

2. Find what would have been due, if it had not been paid until 20 days after it became due?

3. Find the interest of a note of \$400, dated Nov. 10, 1898, and paid May 5, 1900, with interest at  $4\frac{1}{2}\%$ .

Find the time between the dates by the second method indicated in Book II., page 84.

4. Find the amount of a note of \$225, dated July 12, 1899, and paid Oct. 1, 1900, with interest at  $5\frac{1}{2}\%$ .

5. Find the amount of a note of \$316, dated Nov. 17, 1898, and paid May 16, 1900, with interest at 7%.

6. What is the interest upon a note of \$120.50 from Sept. 5, 1899, to Jan. 1, 1901, at 5%.

7. What sum must be invested at 5% to yield an annual income of \$750?

\$750 is 5% of what number?

8. When the fulcrum of a lever is between the power and the weight, and the distance of the power from the fulcrum is 4 times that of the weight from the fulcrum, how much power is necessary to raise a weight of 35 lb. 12 oz.?

9. When the power is between the weight and the fulcrum, and the ratio of the distances from the fulcrum is as 1 to 5, how heavy must the weight be to correspond to a power of 60 pounds?

## Apothecaries' Weight

See page 282.

Apothecaries' weight is used by druggists in mixing medicines.

1. How many drams are there in 120 gr. ? 180 gr. ? 30 gr. ?
2. How many ounces in 72  $\text{ʒ}$ ? 36  $\text{ʒ}$ ? 8  $\text{ʒ}$ ?
3. How many pounds in 96  $\text{ʒ}$ ? 144  $\text{ʒ}$ ? 24  $\text{ʒ}$ ?
4. How many grains in 1  $\text{ʒ}$ ?  $\frac{1}{2}$   $\text{ʒ}$ ? 3  $\text{ʒ}$ ?
5. How many grains in  $\frac{1}{2}$  lb?  $\frac{1}{4}$  lb? 2 lb?
6. How many pills of 2 grains each can be made from 33 2  $\text{ʒ}$  of quinine?
7. A pound Avoirdupois weighs 7000 grains. Which is heavier, an Apothecaries' pound or an Avoirdupois pound, and how much?
8. What part of an Avoirdupois pound is an Apothecaries' pound?
9. How many grains in an Avoirdupois ounce?
10. How many grains in an Apothecaries' ounce?
11. What is the ratio of an Apothecaries' ounce to an Avoirdupois ounce?
12. Change 10 pounds Avoirdupois weight to Apothecaries' weight.
13. How many 5 grain powders can an apothecary put up from 2 Avoirdupois pounds?
14. How many pounds Avoirdupois are there in 20 pounds Troy?
15. Change to grains: 2 lb 8  $\text{ʒ}$ , 3 lb 4  $\text{ʒ}$  43.
16. Change to scruples: 6 lb 3  $\text{ʒ}$ , 2  $\text{ʒ}$  13 2  $\text{ʒ}$ .
17. Change to higher denominations: 42 gr.; 18  $\text{ʒ}$ ; 48  $\text{ʒ}$ .



## Problems from Geography

The map on page 60 is drawn to the scale of 500 miles to the inch.

1. Estimate the shortest distance from Boston to San Francisco.
2. About how long would it take a train to go in nearly a straight line from Boston to San Francisco at an average rate of 40 miles an hour?
3. About what part of the average length of the country is its average width?
4. Estimate the number of square miles in the country.
5. Estimate what part of the whole country is included in New England.
6. About what is the average size of the states?
7. Estimate the size in square miles of Pennsylvania, Alabama, Iowa, Montana, Colorado, California.
8. About how many states of the size of Texas could be formed from the United States?
9. Which states are of about an average size?
10. How much larger is California than the average of the states?
11. About what is the ratio of the area of Massachusetts to that of New York?
12. What is the ratio of the area of New York to that of Texas?
13. What is the ratio of the area of Massachusetts to that of Texas?
14. About what part of the country is west of the Rocky Mountains?
15. What part of the country is drained by the *Mississippi River and its tributaries*?

### Miscellaneous Problems

1. If I buy a roll of wall paper 24 feet long and 18 inches wide to cover a box 3 feet 6 inches long, 2 feet wide, and 1 foot 6 inches high, how many square feet of paper will be left if there is no waste?

2. What is the volume of a cube whose entire surface is 54 square inches?

3. The length of a rectangle is 5 times its width. It contains 125 sq. ft. What is its length?

4. How many square rods in a piece of land 99 ft. long and 66 ft. wide?

5. If one of the acute angles of a right triangle measures  $25^{\circ} 12'$ , what is the size of the other acute angle?

See p. 78.

6. How large is each of the angles of an equilateral triangle?

7. If one of the angles at the base of an isosceles triangle is  $36^{\circ} 30'$ , what is the size of each of the other angles?

8. An excavation is made for a cellar 60 ft. long, 40 ft. wide, and 7 ft. deep. A cellar wall  $2\frac{1}{2}$  ft. wide is built around the four sides. How many cubic feet are there in the wall?

9. What is the distance around the cellar after the wall is built?

10. How many square feet are there on the inside of the walls of the cellar?

11. How many square feet are there in the bottom of the cellar?

12. How much will it cost to cement the bottom of the cellar at 25 cents a square yard?

## Easy Methods

*Give the results orally:*

1. How many times is  $12\frac{1}{2}$  contained in 100? In 500?
2. How many times is  $16\frac{2}{3}$  contained in 100? In 800?
3. How many times is 20 contained in 100? In 700?
4. How many times is 25 contained in 100? In 900?
5. How many times is  $33\frac{1}{3}$  contained in 100? In 1500?
6. How many times is 50 contained in 200? In 2400?

These fractional parts of 100 are used so frequently in business calculations that it is important to become familiar with them.

7. Multiply  $12\frac{1}{2}$  by 48.

8 times  $12\frac{1}{2}$  are 100. Since 48 is 6 times 8, 48 times  $12\frac{1}{2}$  is 6 times 100, or 600.

8. Divide 500 by  $16\frac{2}{3}$ .

$16\frac{2}{3}$  is contained in 100 6 times, and in 500 5 times 6 times or 30 times.

In multiplying or dividing think how many times the number it takes to make 100.

- |                                   |  |
|-----------------------------------|--|
| 9. $300 \div 12\frac{1}{2} = ?$   | 21. $950 \div 50 = ?$                      |
| 10. $250 \div 12\frac{1}{2} = ?$  | 22. $20 \times 45 = ?$                     |
| 11. $8 \times 12\frac{1}{2} = ?$  | 23. $70 \times 20 = ?$                     |
| 12. $12\frac{1}{2} \times 32 = ?$ | 24. $700 \div 25 = ?$                      |
| 13. $600 \div 16\frac{2}{3} = ?$  | 25. $475 \div 25 = ?$                      |
| 14. $150 \div 16\frac{2}{3} = ?$  | 26. $25 \times 13 = ?$                     |
| 15. $12 \times 16\frac{2}{3} = ?$ | 27. $19 \times 25 = ?$                     |
| 16. $16\frac{2}{3} \times 30 = ?$ | 28. $600 \div 33\frac{1}{3} = ?$           |
| 17. $500 \div 20 = ?$             | 29. $17 \times 50 = ?$                     |
| 18. $1000 \div 20 = ?$            | 30. $87\frac{1}{2} \div 12\frac{1}{2} = ?$ |
| 19. $18 \times 33\frac{1}{3} = ?$ | 31. $66\frac{2}{3} \div 16\frac{2}{3} = ?$ |
| 20. $33\frac{1}{3} \times 22 = ?$ | 32. $11 \times 12\frac{1}{2} = ?$          |

## Troy Weight

See p. 281.

Troy weight is used in weighing gold and silver.

1. Change to grains: 15 pwt.; 2 pwt. 8 gr.; 3 pwt. 12 gr.
2. Change to grains: 1 oz.; 2 oz. 5 pwt.; 2 oz. 10 gr.
3. Change to larger weights: 30 oz.; 350 pwt.; 5000 gr.
4. What part of an ounce is 5 pwt.? 12 gr.? 4 gr.?
5. What part of a pound is 10 pwt.? 12 gr.?
6. What part of a pound is 6 oz. 10 pwt.?
7. What is the weight of 12 spoons, each of which weighs 2 oz. 12 pwt. 5 gr.?
8. How many gold rings each weighing 3 pwt. 4 gr. can be made from half a pound of gold?

A Troy pound and an Apothecaries' pound are equal, each weighing 5760 grains. An Avoirdupois pound weighs 7000 grains.

9. Which is heavier, and how much, a pound of sugar or a pound of gold?

10. Change 48 Troy pounds to Avoirdupois pounds.

11. How many Troy pounds in  $\frac{1}{4}$  of a ton Avoirdupois?

12. By the "Bland Silver Bill" in 1878 silver dollars weighing  $412\frac{1}{2}$  gr. each were ordered to be coined and were made legal for the payment of debts. How many dollars could be coined from 100 Troy pounds of silver?

13. The silver in these dollars was really worth only 92 cents. How many silver dollars could be made from \$1000 worth of silver bullion?

14. Between the years 1849 and 1856 over \$400,000,000 worth of gold was found in California. A pound of pure gold is worth \$248.04. What was the total weight of *this gold*?

## Percentage

*Find :*

- |                             |                               |
|-----------------------------|-------------------------------|
| 1. 1% of 500.               | 5. 1% of \$2400.              |
| 2. 12% of 500.              | 6. $\frac{1}{2}$ % of \$2400. |
| 3. $4\frac{1}{2}$ % of 640. | 7. $\frac{2}{5}$ % of \$750.  |
| 4. 17% of 325.              | 8. $1\frac{3}{8}$ % of \$656. |

*Find the number of which :*

- |                             |                             |
|-----------------------------|-----------------------------|
| 9. 18 is 3%.                | 13. 78 is $\frac{1}{2}$ %.  |
| 10. 75 is 5%.               | 14. 60 is $1\frac{1}{2}$ %. |
| 11. 90 is $4\frac{1}{2}$ %. | 15. 350 is 7%.              |
| 12. 56 is 16%.              | 16. 648 is 108%.            |

*The first is what per cent of the second :*

- |                 |                     |
|-----------------|---------------------|
| 17. 17 of 50 ?  | 21. 10 of 500 ?     |
| 18. 13 of 20 ?  | 22. \$60 of \$750 ? |
| 19. 69 of 300 ? | 23. 35 of 243 ?     |
| 20. 75 of 250 ? | 24. 46.8 of 954 ?   |

*The first is what per cent more than the second :*

- |                    |                      |
|--------------------|----------------------|
| 25. 112 than 100 ? | 29. 33 than 25 ?     |
| 26. 56 than 50 ?   | 30. 156 than 150 ?   |
| 27. 27 than 20 ?   | 31. 2675 than 2500 ? |
| 28. 318 than 300 ? | 32. 2460 than 2000 ? |

*The first is what per cent less than the second :*

- |                  |                     |
|------------------|---------------------|
| 33. 4 than 5 ?   | 37. 166 than 200 ?  |
| 34. 17 than 20 ? | 38. 228 than 240 ?  |
| 35. 12 than 60 ? | 39. 504 than 560 ?  |
| 36. 15 than 75 ? | 40. 720 than 1200 ? |



## Miscellaneous Problems

1. An auctioneer sells goods on a commission of 2%. How much does he receive for selling goods to the value of \$1200?

2. A broker sold \$1600 of stock on a commission of  $\frac{1}{8}\%$ . What was his commission?

3. A commission merchant received a commission of 5% for selling goods. If his commission amounted to \$30, for what sum were the goods sold?

4. An agent remitted \$242.50 from a sale of goods, after deducting his commission of 3%. What price did he receive for the goods?

5. After a man had spent  $\frac{2}{3}$  and  $\frac{1}{6}$  of his money he had \$26 left. How much had he at first?

6. A man had a certain sum of money. After he had earned  $\frac{1}{2}$  as much more he had \$63. How much had he at first?

7. How many pieces 2 ft. 3 in. long can be cut from a piece of cloth measuring  $44\frac{3}{4}$  yd.?

8. What is the total amount of milk in 16 cans, each containing 2 gal. 3 qt. 1 pt.?

9. How many cords of wood are there in a pile of cord wood 4 ft. high and 67 ft. long?

10. Add two hundred seventy-five ten-thousandths, twenty-three and seven hundredths, forty-two millionths.

11. From twenty-five take twenty-five ten-thousandths.

12. Multiply two hundred sixty-three and fifty-one thousandths by three and eighty-five thousandths.

13. Divide two hundred twenty-five and fifteen hundredths by fifteen hundred-thousandths.

## Thermometers

On a Fahrenheit thermometer the freezing point is marked  $32^{\circ}$  and the boiling point  $212^{\circ}$ . On a Centigrade thermometer the freezing point is  $0^{\circ}$  and the boiling point  $100^{\circ}$ . Therefore  $212^{\circ} - 32^{\circ}$  or  $180^{\circ}$  Fahr. correspond to  $100^{\circ}$  Cent. The Fahrenheit thermometer is used most commonly in the United States and England, and the Centigrade in France. The minus sign is used in the following examples to indicate degrees below zero :

1. How many degrees F. correspond to  $1^{\circ}$  C. ? What part of a degree C. to  $1^{\circ}$  F. ?

2. When the reading of a Fahrenheit thermometer is  $54^{\circ}$ , what will be the reading of a Centigrade thermometer ?

The degrees F. above freezing point are  $54^{\circ} - 32^{\circ} = 22^{\circ}$ ;  $\frac{5}{9}$  of  $22^{\circ} = 12\frac{2}{9}^{\circ}$  C. above freezing =  $12\frac{2}{9}^{\circ}$  C. above zero.

3. When the Fahrenheit thermometer is at zero, what will be the reading of a Centigrade thermometer ?

How much is the thermometer below the freezing point?  $32^{\circ}$  F. = how many degrees C. below zero ?



*Change from F. to C:*

*Change from C. to F:*

4.  $60^{\circ}$    6.  $150^{\circ}$    8.  $-20^{\circ}$    10.  $15^{\circ}$    12.  $-18^{\circ}$    14.  $70^{\circ}$   
 5.  $12^{\circ}$    7.  $100^{\circ}$    9.  $62^{\circ}$    11.  $42^{\circ}$    13.  $12^{\circ}$    15.  $-24^{\circ}$
16. Find the average of the Fahrenheit readings  $12^{\circ}$ ,  $5^{\circ}$ ,  $8^{\circ}$ ,  $2^{\circ}$ ,  $6^{\circ}$ .
17. Find the average of the Centigrade readings  $72^{\circ}$ ,  $65^{\circ}$ ,  $48^{\circ}$ ,  $79^{\circ}$ .

## Division of Fractions

1. Divide  $\frac{5}{7}$  by  $\frac{2}{3}$ .

First method:  $\frac{5}{7} \div \frac{2}{3} = \frac{15}{14} + \frac{14}{14} = \frac{29}{14}$  or  $1\frac{15}{14}$ .

Second method:  $\frac{5}{7} \div \frac{2}{3} = \frac{5}{7} \times \frac{3}{2} = \frac{15}{14}$  or  $1\frac{15}{14}$ .

We may first divide  $\frac{5}{7}$  by 2, which will give  $\frac{5}{14}$ ; but since we wish to divide by only  $\frac{1}{2}$  of 2, the answer must be 3 times  $\frac{5}{14}$ , or  $\frac{15}{14}$ . Notice that we have divided by the numerator and multiplied by the denominator of the divisor; or we have multiplied by the divisor with its terms inverted.

To divide a fraction by a fraction we may *invert the divisor and then multiply the numerators and denominators together*. For this purpose a whole number may be written in the form of a fraction with 1 for the denominator. 12 is the same as  $1\frac{12}{1}$ .

*Divide:*

- |                                      |                                      |                                       |
|--------------------------------------|--------------------------------------|---------------------------------------|
| 2. 2 by $\frac{1}{2}$                | 8. $\frac{2}{3}$ by $\frac{1}{2}$    | 14. $\frac{3}{4}$ by $\frac{2}{3}$    |
| 3. 2 by $\frac{2}{3}$                | 9. $\frac{2}{3}$ by $\frac{1}{4}$    | 15. $\frac{4}{5}$ by $\frac{5}{8}$    |
| 4. 6 by $\frac{1}{3}$                | 10. $\frac{5}{9}$ by $\frac{1}{5}$   | 16. $\frac{7}{12}$ by $\frac{2}{7}$   |
| 5. 6 by $\frac{2}{3}$                | 11. $\frac{7}{10}$ by $\frac{1}{8}$  | 17. $\frac{11}{20}$ by $\frac{2}{3}$  |
| 6. 8 by $\frac{3}{4}$                | 12. $\frac{11}{2}$ by $\frac{1}{6}$  | 18. $\frac{17}{35}$ by $\frac{2}{7}$  |
| 7. 9 by $\frac{1}{5}$                | 13. $\frac{12}{5}$ by $\frac{2}{5}$  | 19. $\frac{25}{6}$ by $\frac{2}{3}$   |
| 20. $3\frac{1}{2}$ by $\frac{1}{2}$  | 26. $4\frac{1}{2}$ by $1\frac{1}{2}$ | 32. 25 by $12\frac{1}{2}$             |
| 21. $5\frac{1}{4}$ by $\frac{3}{4}$  | 27. $4\frac{1}{2}$ by $1\frac{1}{3}$ | 33. 25 by $12\frac{1}{3}$             |
| 22. $6\frac{2}{5}$ by $\frac{4}{5}$  | 28. $5\frac{1}{3}$ by $\frac{2}{3}$  | 34. $62\frac{1}{2}$ by $6\frac{1}{4}$ |
| 23. $8\frac{1}{3}$ by $\frac{5}{6}$  | 29. $5\frac{1}{3}$ by $2\frac{2}{3}$ | 35. $62\frac{1}{2}$ by $6\frac{1}{5}$ |
| 24. $9\frac{3}{4}$ by $\frac{5}{8}$  | 30. $6\frac{1}{4}$ by $3\frac{1}{6}$ | 36. $87\frac{1}{3}$ by $9\frac{2}{3}$ |
| 25. $10\frac{2}{5}$ by $\frac{1}{4}$ | 31. $9\frac{2}{7}$ by $4\frac{1}{2}$ | 37. $96\frac{3}{4}$ by 15             |

## Review Problems

1. A commission merchant sold 200 barrels of flour at \$4.50 a barrel on a commission of 3%. What was the amount of his commission?

2. An agent received \$20 for selling potatoes at 50 cents a bushel on a commission of 4%. How many bushels did he sell?

3. If I buy a gross of pencils for \$2.50, and sell  $\frac{1}{2}$  of them at the rate of 2 for 5 cents, and the remainder for 3 cents each, what will be the profit?

*What is the interest of:*

4. \$1 for 4 yr. at 6%?

6. \$5 for  $3\frac{1}{2}$  yr. at 6%?

5. \$3 for 2 yr. at 6%?

7. \$12 for 2 yr. at 4%?

8. The distance from the earth to the moon is 240,000 miles, and to the sun 93,000,000 miles. How many times farther is it to the sun than to the moon?

9. Find about how far the earth travels in its revolution around the sun.

Find the length of its path, which is nearly the circumference of a circle.

10. How many times the distance which the moon traverses in its revolution around the earth is the distance which the earth traverses in its revolution around the sun?

11. How many grains are there in 2 lb. 6 oz. 8 pwt.?

12. How many Avoirdupois pounds will it take to make 175 Troy pounds?

13. If a Fahrenheit thermometer should fall  $15^{\circ}$  in a night, how much would a Centigrade thermometer fall?

14. When the Fahrenheit thermometer registers  $8^{\circ}$  below zero, what does the Centigrade thermometer register?

## Measurements

Review p. 45, and Book II, pp. 214, 227, 244.

*Find the measurements of the regular prism :*

1. The ends are 8 in. square. Surface of ends = ?
2. 9 in. square ; 16 in. long. Find the lateral surface.
3. 2 ft. 4 in. square ; 4 ft. long. Entire surface = ?
4. 3 ft. 2 in. square ; 6 ft. long. Entire surface = ?

*Find the measurements of the cylinder  $3\frac{1}{2}$  ft. long :*

5. The diameter is 15 in. Find the circumference.
6. The radius is 10 in. Find the lateral surface.
7. The circumference is 42 in. Find the diameter.
8. The circumference is 38 in. Find the radius.

*Find the lateral surface of the regular pyramid :*

9. The base is 4 ft. square. The slant height is 6 ft.
10. The base is  $6\frac{1}{2}$  ft. square. The slant height is 10 ft.
11. Base a triangle. Each side 18 in. Slant height 3 ft.
12. Base a triangle. Each side 3 ft. Slant height 5 ft. 2 in.

*Find the measurements of the cone. The slant height is 40 inches :*

13. Diameter of base is 20 in. Find its circumference.
14. Circumference of base is 18 in. Find its radius.
15. Radius of base is 8 in. Find the lateral surface.
16. Diameter of base is 2 ft. Find the lateral surface.

*Find the measurements of the sphere :*

17. The diameter is 25 in. Find the circumference.
18. The circumference is 6 ft. Find the diameter.
19. The diameter is 5 ft. Find the surface.
20. The radius is 18 in. Find the surface.

## Original Problems

*Make problems and solve them :*

1. Light travels at the rate of about 186,000 miles a second.
2. Sound travels in the air at the rate of about 1100 feet per second.
3. The sound of thunder is heard 10 seconds after the flash of the lightning is seen.
4. The planet Jupiter is 475,700,000 miles from the sun.
5. A note of \$300 gained \$3.75 at 6%.
6. I receive an annual income of \$400 from a certain investment.
7. A manufacturing druggist buys quinine enough to make 10,000 2-grain pills.
8. A can do  $\frac{1}{3}$  of a piece of work in an hour, and B  $\frac{1}{5}$  of it in an hour.
9. A silver manufacturer buys silver enough to make 100 doz. silver spoons, each spoon weighing 2 oz. 12 pwt.
10. An apothecary buys 10 lb. of drugs by Avoirdupois weight, and sells them in prescriptions by Apothecaries' weight.
11. An auctioneer sells goods to the amount of \$850 on a commission of 3%.
12. A commission merchant received \$60 as his commission for selling goods on a commission of 5%.
13. An agent remitted \$114 after deducting his commission of 5%.
14. A Fahrenheit and a Centigrade thermometer hang near each other. The Fahrenheit thermometer indicates  $12^{\circ}$ .
15. A field in the form of a trapezoid is 60 rods long, 12 rods wide on one of the parallel sides, and  $15\frac{1}{2}$  rods on the other.

## Algebra

In the equation  $\frac{8}{4} = \frac{4}{2}$  the second member may be changed so that the equation will be  $\frac{8}{4} = \frac{8}{4}$ . Then  $8 = 8$ . When all the terms of an equation are similar fractions, the numerators of all the terms will form a true equation.

1. Solve the equation  $\frac{x}{4} - \frac{1}{2} = \frac{3}{2}$ .

$$\frac{x}{4} - \frac{1}{2} = \frac{3}{2}; \quad \frac{x}{4} - \frac{2}{4} = \frac{6}{4}; \quad x - 2 = 6; \quad x = 8$$

$$\frac{x}{4} \times 4 = x; \quad \frac{1}{2} \times 4 = 2; \quad \frac{3}{2} \times 4 = 6; \quad x - 2 = 6; \quad x = 8.$$

The denominators may also be removed by multiplying both members of the equation by 4, the largest number containing all the denominators. This has the same effect as changing all the members to similar fractions.

2. Solve  $\frac{4x}{5} + 6 = \frac{2x}{3} + \frac{x}{4} + \frac{5}{2}$ .

Multiply each term by 60.

$$48x + 360 = 40x + 15x + 150$$

$$48x - 40x - 15x = 150 - 360$$

$$-7x = -210 \text{ and } x = 30.$$

3.  $\frac{x}{2} - 4 = \frac{x}{4}$

6.  $\frac{2x}{3} - 8 = \frac{x}{2} - 4$

4.  $\frac{4x}{3} + 2 = 18$

7.  $\frac{x}{2} + \frac{x}{3} + \frac{x}{4} = \frac{26}{2}$

5.  $\frac{2x}{4} - 2 = \frac{x}{3}$

8.  $x - \frac{x}{3} + 2 = 10$

9. Two thirds of a certain number, increased by 6, would equal 30. Find the number.

10. If to a certain number one half of itself and one fourth of itself are added, the sum will be 35. Find the number.

11. The sum of two numbers is 27. The second number is four fifths of the first. What are the numbers?

## Algebra

A parenthesis inclosing two or more quantities indicates that the quantities inclosed are to be regarded as a single quantity. The combined value of the quantities is to be added or subtracted according to the sign before the parenthesis. When the parenthesis is removed, care must be taken that the value of the whole expression is not changed.

$$10 + (6 - 2) = 14$$

$$10 + 6 - 2 = 14$$

When there is a plus sign before the parenthesis the value of the expression is not changed by removing the parenthesis. To add 6 and subtract 2 produces the same effect as to add the difference between 6 and 2.

$$10 - (6 - 2) = 6$$

$$10 - (6 + 2) = 2$$

$$10 - 6 + 2 = 6$$

$$10 - 6 - 2 = 2$$

To subtract the difference between 6 and 2 is not the same as to subtract both 6 and 2. If 6 is first subtracted, 2 must then be added to correct the error. If both 6 and 2 are to be subtracted, the minus sign must be placed before both when the parenthesis is removed.

For these reasons, in removing a parenthesis which is preceded by the minus sign, *the signs of all the terms within the parenthesis must be changed.*

If the first term within the parenthesis has no sign, plus is understood.

*Remove the parentheses and find value of  $x$ :*

1.  $x + (x - 4) = 20$

5.  $18 + x = (30 - x)$

2.  $2x - (x + 5) = 3$

6.  $10 + (x - 5) = 18 - (x - 5)$

3.  $12 - (7 - x) = 25$

7.  $\frac{x}{2} + (x + 3) = 18$

4.  $(x + 4) + (x - 4) = 14$

8.  $\frac{x}{3} - (11 - x) = 1$



## Algebra

1. Solve the equation  $2x - \frac{x-4}{2} = 14$ .

$2x - \frac{x-4}{2} = 14$ <p>(Multiply by 2)</p> $4x - (x-4) = 28$ $4x - x + 4 = 28$ $3x = 24$ $x = 8$	<p>When the numerator of a fraction contains two or more terms, these terms are regarded as a single quantity, as if they were inclosed by a parenthesis. If the denominator is caused to disappear, care must be taken that the signs give the true value of the numerator, as in the case of the removal of a parenthesis.</p>
--	--

2.  $2x + \frac{x-2}{3} = 18$

5.  $\frac{6x}{3} + \frac{x-2}{3} = \frac{61}{3}$

3.  $4x - \frac{x+3}{2} = 16$

6.  $\frac{8x}{2} - \frac{x+3}{2} = \frac{32}{2}$

4.  $6x + x - 2 = 61$

7.  $8x - x - 3 = 32$

In the second example, since the fraction is preceded by a plus sign, no change takes place in the signs when the denominators are removed. In the third example the signs after the minus sign must be changed so that the whole quantity  $x + 3$  may be subtracted.

8.  $12 + \frac{x+2}{4} = 15$

14.  $\frac{x}{5} - \frac{x-2}{3} = 18 - x$

9.  $\frac{2x+1}{3} = \frac{3x-1}{4}$

15.  $\frac{x-2}{3} - \frac{x-3}{4} = 2$

10.  $5x - \frac{x-3}{3} = 29$

16.  $\frac{7+23x}{4} = 18 + \frac{x-1}{2}$

11.  $\frac{x-1}{2} = \frac{x-2}{3} + 2$

17.  $\frac{x}{3} - (x-9) = \frac{x-1}{11}$

12.  $\frac{2x-3}{5} = 4 - \frac{x+1}{10}$

18.  $\frac{x-7}{10} - \frac{x+1}{6} = \frac{x-7}{5}$

13.  $\frac{3x+6}{9} + 5 = \frac{10x+2}{6}$

19.  $\frac{2x-1}{3} = \frac{8x+2}{6} - \frac{8x-4}{9}$

## Algebraic Problems

1. If 4 is added to a certain number,  $\frac{1}{2}$  of this sum will equal 5 more than  $\frac{1}{4}$  of the number. What is the number?

2. If 4 is added to  $\frac{1}{2}$  of a number, the sum equals  $\frac{3}{4}$  of the number plus 2. What is the number?

3. If 5 is subtracted from a certain number,  $\frac{1}{4}$  of the remainder will equal  $\frac{1}{5}$  of the number. Find the number.

4. Fred has 3 more than  $\frac{1}{2}$  as many marbles as Herbert. Together they have 33 marbles. How many has each?

5. After William had spent 7 cents,  $\frac{1}{3}$  of what remained were one more than  $\frac{1}{4}$  of what he had at first. How many had he at first?

6. The sum of two numbers is 28, and  $\frac{1}{4}$  of the greater number is equal to  $\frac{1}{3}$  of the less. What are the numbers?

If we let  $x$  equal the smaller number, since their sum is 28, the greater number will be  $28 - x$ ;  $\frac{28 - x}{4} = \frac{x}{3}$ .

7. The sum of two numbers is 63, and  $\frac{1}{4}$  of one of the numbers is equal to  $\frac{1}{5}$  of the other. Find the numbers.

8. Divide the number 50 into two such parts that  $\frac{1}{2}$  of one part plus  $\frac{1}{6}$  of the other part shall equal 10.

9. Two men have equal sums of money. After one man gains \$300 and the other loses \$100, the former has twice as much as the latter. How much had each at first?

10. Divide \$195 among three men so that the second shall have \$10 more than the first, and the third shall have  $\frac{1}{2}$  as much as the first and second together.

11. Three men had equal sums of money. The first lost \$25, the second gained \$50, and the third gained \$75. They then had in all \$460. How much had each at first?

### Problems for Arithmetic or Algebra

1. The sum of twice a number and  $\frac{1}{3}$  of the number is 28. What is the number?

2. The sum of the ages of three sisters is 47 years. The youngest is 12 years old, and the next 3 years older. What is the age of the oldest?

3. If to a number  $\frac{1}{2}$  of itself and 3 more is added, the result will be 27. What is the number?

4. If  $\frac{1}{3}$  of a number is subtracted from twice the number, the result will be 21. What is the number?

5. I spent  $\frac{3}{4}$  of my money for a barrel of flour, and with the remainder bought 13 pounds of sugar at 5 cents a pound. What was the cost of the flour?

6.  $\frac{2}{5}$  of a certain number is equal to the number decreased by 14. What is the number?

7. A man has sheep in 3 separate flocks. In the first flock there are 5 more than in the second, and in the third flock there are 10 more than in the second. In all he has 45 sheep. How many are there in each flock?

8. Two numbers are in the ratio of 2 and 3, and their sum is 20. What are the numbers?

Let  $x$  = one number. Then  $\frac{2x}{3}$  = the other number.

9. Two numbers are in the same ratio as 3 and 4, and their sum is 21. What are the numbers?

10. A has \$84, and B \$60. How many dollars must A give to B so that the two may have equal sums?

$$84 - x = 60 + x.$$

11. A has \$25 and B \$95. How much must B give A so that the two may have equal sums?

## Review Problems

1. What part of a pound is 3 oz. ?
2. What part of a bushel is 5 qt. ?
3. What part of a mile is 16 rd. ?
4. What part of a square mile is 25 A. ?
5. 7 is what per cent of 25?
6. 11 is what per cent of  $33\frac{1}{3}$  ?
7. 15 is what per cent of 16 ?
8. 42 is what per cent of 300 ?
9. 250 is what per cent of 5000 ?
10. A man owned  $\frac{3}{5}$  of a lot of land, and sold  $\frac{2}{5}$  of what he owned. What part of the whole lot did he sell ?
11. What part of the lot had he remaining ?
12. A man invested  $\frac{5}{8}$  of his money in business, and then invested  $\frac{1}{3}$  of what he had left in a house. What part of the whole remained ?
13. He had \$3000 remaining. How much had he at first ?
14. A dealer expended  $\frac{1}{3}$  of his money for dry goods, and  $\frac{1}{4}$  of the remainder for groceries. The groceries cost him \$600. How much money had he at first ?
15. If the temperature during the day from 8 A.M. to 2 P.M. rises from  $-10^{\circ}$  to  $26^{\circ}$ , what is the average rise per hour ?
16. What is the average between  $43^{\circ}$  and  $-15^{\circ}$  ?
17. What is the average of  $5^{\circ}$ ,  $13^{\circ}$ , and  $18^{\circ}$  ?
18. What is the average between  $12^{\circ}$  and  $-16^{\circ}$  ?
19. What Centigrade reading corresponds to  $27^{\circ}$  F. ?
20. What Fahrenheit reading corresponds to  $27^{\circ}$  C. ?

## Problems from Geography

First estimate the results.

1. In descending below the surface of the earth, as in deep mines, the temperature is found to be about one degree warmer for each 51 feet of descent. At this rate of increase, if the temperature of the earth near its surface is about 40° F., find at what depth the temperature is 212° F., the boiling point of water.

2. At a depth of how many miles would the temperature reach 3000°, and be sufficient to melt wrought iron?

3. The coast of the Netherlands is sinking at the rate of about  $\frac{1}{8}$  of an inch a year. If this continues, how much higher will the dikes have to be built in the next thousand years?

4. Great Salt Lake, in Utah, is elevated 4200 feet above the level of the sea. What should be the height of the barometer at that altitude to compare with 30 inches at sea level?

The barometer falls about 1 inch for each 1000 feet of ascent.

5. The Niagara River once flowed over the bluff directly into Lake Ontario. The water has worn away the bluff so as to move the falls back about 7 miles. It has been estimated that the river bed has been worn back in this way about 14 inches a year. At this rate how long has it been since the river began to flow over the bluff into the lake?

6. The surface of the Dead Sea is about  $\frac{1}{4}$  of a mile below the level of the ocean. If the barometer stands at 30 inches at sea level, how high should the column of mercury be at the level of the Dead Sea?

### Miscellaneous Problems

1. What is the area of a rectangle 4 yd.  $2\frac{1}{2}$  ft. long and 3 yd.  $2\frac{1}{4}$  ft. wide?
2. A rectangular lot is 12 rd. long and 9 rd. wide. How many yards is it around it?
3. 3 boys together purchase a ball. The first pays for  $\frac{1}{3}$  of it, the second for  $\frac{1}{5}$  of it, and the third pays 21 cents. What is the price of the ball?
4. What part of an Avoirdupois pound is  $\frac{1}{2}$  of a Troy pound?
5. How many silver spoons, each weighing 2 oz. 4 pwt., can be made from a piece of silver weighing 12 pounds Avoirdupois?
6. If the temperature of water is  $65^{\circ}$  F., what would a Centigrade thermometer indicate?
7. When the temperature is  $-10^{\circ}$  C., what would be the reading on a Fahrenheit thermometer?
8. If a barometer should be carried high enough above the earth to fall to 23 inches, what would be the weight of the atmosphere above it?
9. Multiply  $4\frac{2}{3}$  by  $5\frac{3}{4}$ ;  $8\frac{2}{7}$  by  $4\frac{1}{5}$ ;  $12\frac{1}{5}$  by  $9\frac{3}{4}$ .
10. Divide  $7\frac{1}{2}$  by  $2\frac{1}{4}$ ;  $7\frac{1}{5}$  by  $13\frac{3}{4}$ ;  $11\frac{7}{9}$  by  $4\frac{2}{3}$ .
11. At  $\$1\frac{3}{8}$  a yard, how many yards of cloth can be bought for  $\$12\frac{4}{5}$ ?
12. At  $5\frac{3}{8}$  cents a pound, what will be the cost of  $24\frac{3}{4}$  lb. of sugar?
13. How many pieces of ribbon, each  $6\frac{3}{4}$  inches long, can be cut from  $8\frac{5}{9}$  yards?
14. If  $5\frac{1}{2}$  tons of coal cost  $\$35\frac{1}{4}$ , how much will  $3\frac{1}{2}$  tons cost at the same rate?

### Miscellaneous Problems

1. The difference between two numbers is 7, and the smaller number is 8. What is the larger number?
2. The difference between two numbers is 10, and the larger number is 22. What is the smaller number?
3. Put 17 lb. of sugar into 2 bags, putting  $3\frac{1}{2}$  lb. more into one than into the other.
4. The specific gravity of granite is about 2.7. What is the weight of a cubic foot of granite?
5. A cubic foot of silver weighs 655 pounds. What is the specific gravity of silver?
6. If the specific gravity of sea water is 1.025, what is the pressure per square foot at a depth of 200 feet below the surface of the ocean due to the weight of the water above?
7. A pump handle is 3 feet long. The bolt is 28 inches from the end of the handle. If the column of water to be raised weighs 120 pounds, how much pressure must be applied at the end of the handle?
8. What are the smallest factors of 60?
9. If 35 is multiplied by the product of 2 and 3, what will be the factors of the product?
10. If 60 is divided by the product of 2 and 3, what will be the factors of the quotient?
11. If 540 is divided by 30, what will be the factors of the quotient?
12. If the factors of the multiplier are 2, 3, and 7, and the factors of the multiplicand are 3, 5, and 11, what is the product? What are the factors of the product?
13. If the factors of the dividend are 2, 5, 11, and 17, and those of the quotient are 5 and 17, what is the divisor? What are the factors of the divisor?

## Squares—Rectangles

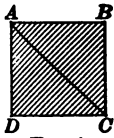


FIG. 1.

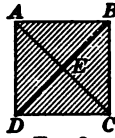


FIG. 2.



FIG. 3.

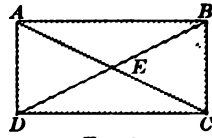


FIG. 4.

See Book II, pp. 78, 85, 123, 231.

1. In the square (Fig. 1) what kind of a triangle is  $ABC$ ?
2. Can it be shown that  $ABC$  is equal to  $ADC$ ?  
Show that one triangle will exactly fit upon the other.
3. How large is the angle at  $B$ ? The angle  $BAC$ ?  $ACD$ ?
4. In Fig. 2 how large is the angle  $BAE$ ?  $ABE$ ?  $AEB$ ?  $DCE$ ?
5. Can it be shown that the triangle  $AEB$  is equal to the triangle  $DEC$ ?
6. In Fig. 3 which angle is equal to the angle  $BAC$ ?
7. Which angle is equal to the angle  $BCA$ ?
8. Show that the triangle  $ABC$  is equal to the triangle  $ADC$ .
9. If the angle  $BAC$  is an angle of  $30^\circ$ , how large is the angle  $BCA$ ?  $CAD$ ?  $ACD$ ?
10. In Fig. 4 how many corresponding pairs of triangles are there?
11. Which triangle is equal to  $ABD$ ?  $ADC$ ?  $AEB$ ?  $AED$ ?
12. If the angle  $ABE$  is an angle of  $30^\circ$ , how large is the angle  $EBC$ ?  $BAE$ ?  $EAD$ ?  $AEB$ ?  $DEC$ ?  $ADE$ ?
13. In Fig. 1 if  $AD$  is 10 feet, how many square feet are there in the triangle  $ADC$ ?



## Common Fractions and Decimal Fractions

1. Change .8 to a common fraction. Change .275 to a common fraction.

$$.8 = 8/10 = 4/5. \quad .275 = 275/1000 = 11/40.$$

*Write the decimal fraction as a common fraction and reduce this fraction to its lowest terms.*

2. Change  $\frac{1}{2}$  to a decimal fraction. Change  $\frac{5}{8}$  to a decimal fraction.

$$1/2 = 5/10 = .5 \quad 5/8 = 5 \div 8 \quad \begin{array}{r} 8 \overline{) 5.000} \\ \underline{.625} \end{array}$$

Since  $\frac{5}{8}$  equals 5 divided by 8, we divide 5, with two decimal ciphers annexed, by 8.

From this we may obtain the formal rule: *Annex ciphers to the numerator and divide by the denominator, pointing off as many decimal figures as there are ciphers annexed.*

*Change to common fractions in smallest terms:*

3. .5	9. .08	15. .225	21. .0013
4. .8	10. .015	16. .250	22. .0125
5. .25	11. .025	17. .0225	23. .0005
6. .45	12. .035	18. 5.125	24. .0008
7. .65	13. .045	19. 10.625	25. .00012
8. .95	14. .125	20. 24.0625	26. 35.15625

*Change to decimal fractions:*

27. $\frac{9}{10}$	33. $\frac{7}{12}$	39. $2\frac{1}{2}$	45. $8\frac{1}{4}$
28. $\frac{5}{8}$	34. $\frac{9}{13}$	40. $3\frac{1}{4}$	46. $10\frac{1}{8}$
29. $\frac{11}{12}$	35. $\frac{18}{19}$	41. $5\frac{7}{8}$	47. $18\frac{1}{20}$
30. $\frac{7}{15}$	36. $\frac{1}{3}$	42. $8\frac{5}{7}$	48. $22\frac{3}{5}$
31. $\frac{13}{25}$	37. $\frac{2}{3}$	43. $12\frac{8}{9}$	49. $25\frac{3}{50}$
32. $\frac{2}{10}$	38. $\frac{7}{15}$	44. $15\frac{7}{11}$	50. $43\frac{1}{50}$

## Percentage

1. What per cent is 27 of 300 ? 45 of 375 ? 122 of 300 ?
2. What per cent is 24 cents of \$48 ? \$34.16 of \$683.20 ?
3. 15 is what part of 90 ? What per cent of 90 ?
4. 15,000,000 is what per cent of 90,000,000 ?
5. The annual rainfall of New England is about 35 inches in the northern part and about 50 inches along the southern coast. What per cent is the rainfall in the northern part of that in the southern part ?
6. What per cent less of rain and snow has the northern part than the southern part ?
7. What per cent more has the southern part than the northern part ?
8. In central New England the annual precipitation is about 42 inches. What per cent is this of the precipitation along the coast ?
9. This is what per cent less than the precipitation along the coast ?
10. What per cent more of precipitation has central New England than northern New England ?
11. There are about 1,500,000,000 people in the world. If the population of the United States is 76,000,000, what per cent of all the people in the world are the people of the United States ?
12. There are about 700,000,000 people of the Caucasian race. What per cent of the people of the world are Caucasians ?
13. The total area of the globe is about 197,000,000 sq. mi. The land occupies somewhat more than 52,000,000 sq. mi. About what per cent of the earth's area is land ?

## Heat

A unit of heat is the heat required to raise a pound of water one degree Fahrenheit. To raise 10 pounds of water one degree would require 10 units of heat. To raise 10 pounds five degrees would require 50 units.

1. About 142 units of heat are required to melt a pound of ice. How many units will it take to melt  $3\frac{1}{2}$  pounds of ice, if the ice is at  $32^{\circ}$  F.?

2. If a pound of ice at  $32^{\circ}$  is melted in a pound of water at  $200^{\circ}$ , to what point will the temperature of the water be reduced?

How many units of heat will be withdrawn from the water?

3. If a pound of ice is melted in 2 pounds of water at  $150^{\circ}$ , to what point will the temperature of the water be reduced?

4. What must be the temperature of a pound of water in order that a pound of ice may be entirely melted in it?

How many degrees above the freezing point must the temperature of the water be?

5. What must be the temperature of 8 pounds of water in order that 2 pounds of ice may be entirely melted in it?

6. If a block of ice is placed in a tank containing 20 pounds of water at  $70^{\circ}$ , how many pounds of ice will be melted?

How many single units of heat are there in the water above the freezing point?

7. How much ice could be melted with a pound of steam at  $212^{\circ}$ ?

8. If 20 pounds of water should be converted into steam, and the steam should be injected into a tightly closed barrel containing ice, how many pounds of the ice would be melted, if no allowance is made for the escape of the heat?

## The Parenthesis

Review p. 73.

1. Take 3 from 7 and subtract the remainder from 12.
2. Subtract 7 from 12 and add 3 to the remainder.
3.  $12 - (7 - 3) = ?$
4.  $12 - 7 + 3 = ?$   $12 - (7 - 3) = 12 - 4$  or  $12 - 7 + 3 = ?$

When a parenthesis is preceded by the minus sign, the signs of all the terms within it must be changed if the parenthesis is removed.

*Give the results of the following :*

5.  $18 - (8 - 3) + 7$
6.  $18 - 8 + 3 + 7$
7.  $25 - (16 - 5) - 3$
8.  $25 - 16 + 5 - 3$
9.  $28 - (12 + 4) + 6$
10.  $28 - 12 - 4 + 6$

*Write the following as they should be without the parentheses :*

11.  $32 - (8 + 5) + (9 - 4)$
12.  $35 + (9 - 4) - (12 - 5)$
13.  $40 - (10 + 3) + (8 - 6)$
14.  $42 + (20 + 6) - (25 - 5)$
15.  $(31 + 10) - 12 + (31 - 10)$
16.  $(36 - 15) + 15 - (36 - 15)$

*Give the results of the following :*

17.  $16 - (8 + 3) + (10 - 5)$
18.  $20 + (10 - 3) - (15 - 7)$
19.  $32 - (12 + 4) + (16 - 4)$
20.  $43 + (18 + 4) - (28 - 8)$

## Review Problems

1. When the power is applied to a lever between the fulcrum and the weight, if the power is 10 feet from the fulcrum, and the weight 20 feet from the power, how much power must be applied to raise a weight of 160 pounds?

2. If the power and weight are on opposite sides of the fulcrum of a lever, and the weight of 50 pounds is 10 inches from the fulcrum, how much power must be applied at a distance of 25 inches from the fulcrum to balance it?

3. What common fraction is equivalent to .025? .0125? .375?

4. What decimal fraction is equivalent to  $\frac{7}{12}$ ?  $\frac{1}{75}$ ?  $\frac{3}{25}$ ?

5. If one of the angles formed by the diagonal of a rectangle is  $32^{\circ} 15'$ , how large is each of the other three angles formed by the diagonal?

6. If one of the angles formed at the corner of a rectangle by a diagonal is  $35^{\circ}$ , what must be the sizes of the angles formed at the center by the two diagonals?

7. Through how many degrees of longitude does the earth turn in  $2\frac{1}{2}$  hours?

8. What is the true difference in time between two places one of which is  $27^{\circ} 20' 15''$  east longitude, and the other  $78^{\circ} 18' 12''$  west longitude?

9. Find the difference in longitude between two places whose true difference in time is 2 hr. 20 min. 30 sec.

10. The longitude of London is  $0^{\circ} 5' 48''$  W., and that of Paris  $2^{\circ} 20' 22''$  E. What is the true difference in time between the two places?

11. What is the longitude of a place where the true time is 2 hr. 30 min. earlier than the time at London?

## Construction

See pp. 81, 287, and Book II, pp. 92, 222, 231.

1. Construct a triangle with one side 3 inches long and one 2 inches long and an angle of  $115^\circ$  included between them. What should be the sum of the other two angles? Measure them. Regarding this triangle as one half of a parallelogram, complete the parallelogram. Draw a perpendicular dotted line for the altitude. Find the area of the parallelogram.

2. Construct an isosceles triangle with an angle of  $70^\circ$  as the third, or unequal, angle. What must be the size of each of the remaining angles? Measure them. Draw a line from the vertex of the angle of  $70^\circ$  to the middle point of the opposite side. Measure the parts of the triangles thus formed and find which parts of the two triangles are correspondingly equal to each other. Show that the two triangles are equal.

3. Construct a triangle with one side 3 inches long and with two angles of  $60^\circ$  each. Measure the other two sides. What kind of a triangle is it? Draw a line from an angle perpendicular to the opposite side. Which of the angles are equal to each other?

4. Construct a 3-inch square. Draw the diagonals. Measure the sides and angles of all the triangles formed. Which lines are equal? Which angles are equal? Which triangles are equal?

5. Construct a rectangle 4 inches long, 2 inches wide. Draw the diagonals. Which lines are equal? Which angles are equal? Which triangles are equal?

6. Construct a parallelogram with sides 4 inches and 3 inches long and an angle of  $60^\circ$ . Draw the diagonals. Which lines are equal? Which angles are equal? Which triangles are equal?

## Heat

See p. 84.

*How many units of heat are necessary:*

1. To raise 10 lb. of water  $30^{\circ}$  F.?
2. To raise 15 lb. of water  $45^{\circ}$  ?
3. To raise 12 lb. of water from  $40^{\circ}$  F. to  $75^{\circ}$  F.?
4. To raise 65 lb. of water from  $32^{\circ}$  to  $90^{\circ}$  ?
5. To raise 100 lb. of water from  $60^{\circ}$  to  $212^{\circ}$  ?
6. To raise 80 lb. from the freezing point to  $100^{\circ}$  ?
7. To raise 250 lb. from the freezing point to  $212^{\circ}$  ?

*How many pounds are necessary:*

8. Water at  $60^{\circ}$  F. to melt 5 lb of ice at  $32^{\circ}$  ?
9. Water at  $100^{\circ}$  to melt 20 lb. of ice ?
10. Water at  $150^{\circ}$  to melt 50 lb. of ice ?
11. Water at the boiling point to melt 100 lb. of ice ?
12. Steam at  $212^{\circ}$  to melt 40 lb. of ice ?
13. Steam at  $212^{\circ}$  to raise 100 lb. of water from  $50^{\circ}$  to  $100^{\circ}$  ?
14. Steam at  $212^{\circ}$  to raise 60 lb. of water from  $100^{\circ}$  to  $140^{\circ}$  ?

*What will be the temperature of the combination :*

15. 20 lb. of water at  $40^{\circ}$  F. with 30 lb. at  $50^{\circ}$  F.?
16. 45 lb. water at  $60^{\circ}$  with 75 lb. at  $90^{\circ}$  ?
17. 100 lb. of water at  $42^{\circ}$  with 180 lb. at  $88^{\circ}$  ?
18. 10 lb. of steam at  $212^{\circ}$  with 20 lb. of water at  $100^{\circ}$  ?
19. 30 lb. of steam at  $212^{\circ}$  with 50 lb. of water at  $75^{\circ}$  ?
20. 25 lb. of steam at  $212^{\circ}$  with 5 lb. of ice at  $32^{\circ}$  ?
21. 8 lb. of ice at  $32^{\circ}$  melted in 20 lb. of water at  $150^{\circ}$  ?

## Original Problems

*Make problems and solve them :*

1. The temperature of the earth increases one degree for each 51 feet of descent.

2. The Centigrade thermometer indicates  $24^{\circ}$ .

3. The difference between two numbers is 12, and the larger is 17.

4. There are 35 quarts of milk in 2 cans, and 11 more quarts in one than in the other.

5. A pump handle is  $3\frac{1}{2}$  feet long.

6. A cubic foot of lead weighs 11,350 ounces.

7. 80 is divided by the product of the factors 2 and 5.

8. The factors of the dividend are 2, 5, 7, and 11.

9. A rectangle is divided into four triangles by its diagonals.

10. Edith is twice as old as Margaret, and Louise is  $\frac{1}{2}$  as old as Edith.

11. An acre of land is divided into 15 house lots.

12. One of the angles at the corner of a rectangle, formed by the diagonals, measures  $40^{\circ}$ .

13. A quantity of water weighing 20 pounds is heated enough to raise the temperature  $10^{\circ}$  F.

14. A pound of ice at  $32^{\circ}$  F. is melted in 10 pounds of water at  $100^{\circ}$  F.

15. A block of ice is placed in a barrel containing 100 pounds of water at  $52^{\circ}$  F.

16. 5 pounds of water are converted into steam, and the steam is used to melt ice.

17. A man owned  $\frac{6}{7}$  of a farm and sold  $\frac{5}{8}$  of his share.

18. A man owned  $\frac{3}{6}$  of a farm and afterwards bought  $\frac{1}{4}$  of the remainder.



## Algebra

If  $a = 1$ ,  $b = 3$ ,  $c = 5$ ,  $d = 2$ ,  $e = 4$ , find the values of the following expressions:

1.  $a + b + c + d$

2.  $a + c - d + b$

3.  $\frac{e}{d} + b + (c - d) - d$

4.  $c - \frac{b - a}{d} + \frac{b + d}{a}$

5.  $\frac{a + b}{d} + \frac{b + c}{e} - \frac{c - d}{b}$

$$5 + 5 + 5 + 5 + 5 + 5 = 6 \times 5 = 30$$

$$a + a + a + a + a + a = 6 \times a = 6a$$

Several equal algebraic quantities may be combined by placing a figure before one of the equal quantities indicating how many have been combined. The figure is called the **coefficient** of the term. 6 is the coefficient of the term  $6a$ , and indicates that 6 of the quantities represented by  $a$  have been combined.

When terms differ in their coefficients only, they are called **similar terms**.

*Combine the similar terms in the following expressions:*

6.  $a + a + a + b + b =$

7.  $a + 2a + 4a + a + 3a =$

8.  $3a + 2b + a + 3b + 2a =$

9.  $a + 2b + 3c + 3a + b + 4c =$

10.  $4a + 2a - 3a + a - 2a =$

11.  $3b + 4c - d - 2c + 3d =$

12.  $7a + 5b - (4a - 2a) + 3b - c =$

13.  $6a - a + 4b - 3b + 2b - 3a =$

## Algebra

*Add or combine the following :*

1.	2.	3.	4.	5.	6.
$a$	$5x$	$2b$	$m$	$-3n$	$12c$
$5a$	$-2x$	$b$	$-3m$	$10n$	$2c$
$3a$	$3x$	$7b$	$6m$	$n$	$-8c$
$a$	$x$	$-3b$	$-2m$	$-2n$	$15c$
<u><math>6a</math></u>	<u><math>-4x</math></u>	<u><math>4b</math></u>	<u><math>5m</math></u>	<u><math>4n</math></u>	<u><math>-c</math></u>

7.	8.	9.
$2a + 4b - c$	$6a - 5b + 2d$	$8m + 5n + 12$
<u><math>5a - 2b + 3c</math></u>	<u><math>4a - 2b + 3d</math></u>	<u><math>-3m + 7n - 15</math></u>

10.	11.	12.
$x + 2y - z$	$3x - 5y + 8$	$12x + 10y + 3z$
<u><math>2x - 3y + z</math></u>	<u><math>2x + 5y - 4</math></u>	<u><math>-12x - 5y + 2z</math></u>

13.	14.	15.
$5a - b + 3c$	$4m + 4n + 4$	$15x - 12y + z$
<u><math>-10a + 2b - 8c</math></u>	<u><math>-4m + 3n + 12</math></u>	<u><math>4y - 8x - z</math></u>

16.	17.
$3a - 4c + d - 8b + 5$	$4a + 3b - 5c + 6d + e$
<u><math>5b - 2a + 3d + c - 5</math></u>	<u><math>a - e + d - 2b + 6</math></u>

18.	19.	20.	21.
$2a + b$	$b + 9$	$7m - n$	$x - y$
$-c + 2b$	$-c + 4d$	$2n + 6$	$7 + x$
$4d - a$	$3e - 8$	$4m - 5$	$3y - 10$
<u><math>2c + 5</math></u>	<u><math>4b - d</math></u>	<u><math>10 - 2n</math></u>	<u><math>4x - 2y</math></u>

## Algebra

See pp. 22, 73.

1.  $5x + (3x - 5) - (2x + 2) = 47$

2.  $\frac{4x}{5} + x - (12 + x) = (15 - x)$

3.  $3x + \frac{x}{7} - 8 = \frac{x}{7} + 13$

4.  $9 - \left(\frac{x}{3} - \frac{x}{4}\right) = x - \frac{x}{3}$

Notice that equal quantities may be removed from both sides of an equation at any time.

5.  $7x - \frac{x-4}{4} = \frac{x+16}{8} + (7x-4)$

6.  $\frac{3x-17}{13} + 2x = x + 9 + \frac{3x-17}{13}$

7.  $x + \frac{5x}{4} - (29 + 3x) = \frac{10x}{8} + 25 - (3x + 29)$

8.  $x + 2x + 3x - 4x + \frac{x}{2} + \frac{x}{3} - \frac{x}{4} = 93$

9. If to a certain number  $\frac{1}{2}$  of itself,  $\frac{1}{3}$  of itself, and  $\frac{1}{4}$  of itself are added, the result will be 4 more than twice the number. What is the number?

10. If 4 is added to a certain number, and  $\frac{1}{4}$  of this sum is taken from twice the number, the result will be 8 less than twice the number. What is the number?

11. The sum of three sides of a triangle is 32 inches. The second side is 4 inches more than the first side, and the third side  $\frac{1}{3}$  of the sum of the other two sides. What is the length of each side?

12. In a right triangle one of the acute angles is  $9^\circ$  smaller than  $\frac{1}{2}$  of the other acute angle. How large is *each angle of the triangle*?

## Algebra

1.  $x + \frac{2x}{3} + \frac{x}{6} = 22$

6.  $5x + (x - 12) = 12$

2.  $\frac{x}{7} + \frac{x}{2} - 1 = 8$

7.  $35 - (2x + 6) = 15$

3.  $\frac{2x}{5} - \frac{3x}{10} = 3$

8.  $7x - (15 - x) = (x + 6)$

4.  $18 - \frac{x}{6} = 21 - \frac{2x}{3}$

9.  $\frac{3x}{4} - \left(12 - \frac{x}{2}\right) = x - 9$

5.  $\frac{5x}{12} - \frac{x}{6} = \frac{1}{8}$

10.  $16 + \left(\frac{x}{2} - \frac{x}{3}\right) = 20$

11. A certain town between Boston and Albany is three fifths as far from Boston as from Albany. The distance from Boston to Albany is about 200 miles. How far is the town from Boston?

12. The width of a block of ice is two thirds of its length. Its thickness is one fourth of its length. The sum of the length, width, and thickness is 46 inches. What are the dimensions?

13. If 5 is taken from a certain number and the difference is added to twice the number, the sum will be 31. What is the number?

14. If 7 is added to a certain number and the sum is taken from five times the number, the remainder will be 49. What is the number?

*Make problems for these equations and solve them:*

15.  $\frac{3x}{4} - \frac{x}{2} = 9$

18.  $7x + (x - 4) = 36$

16.  $15 - \frac{x}{3} = 18 - \frac{5x}{6}$

19.  $3x - (3x - 8) = x - 8$

17.  $x + \frac{x}{2} + \frac{x}{5} = 34$

20.  $12 - \left(\frac{x}{4} + \frac{x}{3}\right) = 5$

## Problems in Interest

1. What principal will gain \$4.80 interest in 4 years at 6%?

\$1 in 4 years at 6% will gain \$.24. It will take as many dollars to gain \$4.80 as \$.24 is contained times in \$4.80. *Ans.* \$20.

Or, let  $x$  = the principal. Since 1% or  $\frac{1}{100}$  of  $x$  would be represented by  $\frac{x}{100}$ , 24% of  $x = \frac{24x}{100}$ . Then  $\frac{24x}{100} = \$4.80$ ;  $24x = \$480$ ; and  $x = \$20$ .

2. In how many years will \$50 gain \$12 at 4%?

In 1 year \$50 at 4% will gain \$2. It will take as many years to gain \$12 as \$2 is contained times in \$12. *Ans.* 6 yr.

Or, let  $x$  = the time. Then  $\frac{4x}{100} \times 50 = 12$ ;  $200x = 1200$ ; and  $x = 6$  yr.

3. At what rate will \$200 gain \$25 in 2 years?

At 1% \$200 would gain \$2 in 2 years. It will take as many per cent to gain \$25 as \$2 is contained times in \$25. *Ans.* 6¼%.

Or, let  $x$  = the rate. Then  $\frac{2x}{100} \times 200 = 25$ ;  $400x = 2500$ ; and  $x = 6\frac{1}{4}\%$ .

*Find by arithmetic or by algebra the missing term in the following:*

	Principal	Time	Rate	Interest
4.	\$100	3 yr.	6%	?
5.	?	2 yr. 4 mo.	7%	\$40
6.	\$300	?	5%	\$60
7.	\$125	8 mo.	?	\$ 3.75
8.	\$ 45.50	9 mo. 12 da.	7½%	?
9.	\$700	?	4%	\$63
10.	\$ 90	4 yr. 2 mo.	?	\$11.25
11.	?	10 mo. 15 da.	6%	\$10.50

## Drill Table

A great number of examples for practice may be obtained from this page by adding the columns as far as any given point, by dividing a column, and from the sum of one part taking the sum of the other part, by proceeding both vertically and horizontally, etc.

	1.	2.	3.	4.	5.
<i>a.</i>	56,432	78,425	47,221	67,658	49,503
<i>b.</i>	70,569	48,672	85,825	59,476	96,351
<i>c.</i>	40,606	83,536	96,843	22,823	78,056
<i>d.</i>	89,504	25,237	77,343	45,295	37,564
<i>e.</i>	73,302	86,473	67,454	84,642	42,126
<i>f.</i>	41,509	75,225	67,532	64,978	80,795
<i>g.</i>	87,400	47,231	72,835	58,674	57,868
<i>h.</i>	94,375	45,437	97,653	92,432	43,686
<i>i.</i>	62,792	48,693	37,409	67,538	92,465
<i>j.</i>	86,947	79,428	88,453	75,445	84,564
<i>k.</i>	52,943	54,667	65,895	36,542	74,568
<i>l.</i>	25,759	93,477	69,875	85,436	95,456
<i>m.</i>	47,479	46,235	79,891	73,549	79,654
<i>n.</i>	56,773	84,506	35,947	68,556	65,989
<i>o.</i>	79,086	65,382	67,536	45,465	56,458
<i>p.</i>	68,574	95,734	80,509	64,013	80,475
<i>q.</i>	47,379	90,480	54,384	34,536	58,079
<i>r.</i>	76,470	67,283	76,208	98,435	46,575
<i>s.</i>	69,593	66,743	68,579	80,306	68,542
<i>t.</i>	56,964	85,613	82,732	58,968	75,842
<i>u.</i>	46,702	92,574	57,642	74,216	68,545
<i>v.</i>	61,452	80,987	61,939	92,871	86,456
<i>w.</i>	43,286	69,443	72,074	38,195	76,390
<i>x.</i>	89,374	52,761	41,763	85,862	45,178
<i>y.</i>	63,296	27,759	77,312	48,027	75,235
<i>z.</i>	43,199	53,547	33,206	74,529	39,414

## Review Problems

1. What part of the interest of a sum of money at 6% would be the interest of the same sum at 3%? At 2%? At 4%?
2. If the interest upon a certain sum of money for a certain time at 6% is \$40, what would it be at 3%? At 4%? At 12%? At 9%?
3. What part of 6 is  $4\frac{1}{2}$ ?  $1\frac{1}{2}$ ?  $7\frac{1}{2}$ ?
4. If the interest upon a certain sum for a certain time at 6% is \$60, what would it be at  $1\frac{1}{2}$ %? At  $4\frac{1}{2}$ %? At  $7\frac{1}{2}$ %?
5. Find the interest of \$250 for 2 yr. 6 mo. 12 da. at 3%. At  $4\frac{1}{2}$ %. At  $7\frac{1}{2}$ %.
6. Find the interest of \$148.50 from July 12, 1897 to May 24, 1899, at 8%.
7. Find the interest of \$342.80 from Feb. 18, 1895 to June 2, 1898, at  $5\frac{1}{2}$ %.
8. What is the area of a circle whose diameter is 10 feet?
9. Find the number of feet in the circumference of a circle whose radius is 40 inches.
10. Find the area of this circle.
11. Find the area of a triangle whose base is 4 ft. 5 in. and whose altitude is 6 ft. 9 in.
12. One of the parallel sides of a parallelogram is  $3\frac{1}{2}$  yd. long. The perpendicular distance between the parallel sides is  $5\frac{1}{2}$  ft. Find the area.
13. The parallel sides of a trapezoid are  $18\frac{1}{2}$  in. and  $26\frac{1}{2}$  in. The perpendicular distance between them is 14 inches. Find the area.

## Taxes

When money is needed for public purposes, a tax is assessed upon the people. The amount to be raised by taxation is generally divided among the people who own property according to the amount of property which they own.

If the sum of \$5000 is to be raised upon property valued at \$250,000, the tax upon each dollar will be  $\frac{1}{250000}$  of \$5000.

Property is of two kinds, real estate and personal property. Real estate is immovable property, as land and houses. Personal property is movable property, as money, goods, live stock, etc.

A poll tax is usually assessed upon the male citizens. The amount of the poll taxes is then taken from the whole amount to be raised, and the balance is raised upon the property.

1. If \$2000 is to be raised by taxation upon the property, and all the property in the town subject to taxation is \$150,000, what will be the tax on each dollar of property?

2. In a certain town the valuation of the real estate is \$1,650,000, and the valuation of the personal property is \$1,150,000. The town raises by taxation on this property the sum of \$33,600. What is the tax on each dollar?

3. In a certain town the valuation of the real estate and personal property amounts to \$860,000. The town raises \$14,390 by taxation. There are 420 persons upon whom a poll tax of \$1.50 each is assessed. What will be the rate of taxation on the property?

4. What will be the tax of one of the taxpayers who has property to the amount of \$2460 and who pays a poll tax?



## Taxes

*Find the rate of taxation :*

1. The property is \$450,000; the sum raised \$9000.
2. The property is \$842,000; the sum raised \$10,525.
3. The property is \$627,800; the sum raised \$14,439.40.
4. The property is \$785,000; the sum raised \$13,737.50.
5. The property is \$2,500,000; the sum raised \$42,500.
6. The property is \$1,640,000; the sum raised \$27,880.
7. The property is \$384,200; the sum raised \$8068.20.
8. The property is \$569,300; the sum raised \$9108.80.

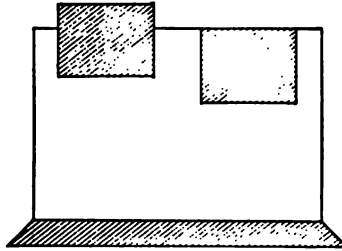
*Find the amount of the tax :*

9. The property is \$3560; the rate is 15 mills.
10. The property is \$4500; the rate is 14 mills.
11. The property is \$25,000; the rate is 17 mills.
12. The property is \$18,420; the rate is  $15\frac{1}{2}$  mills.
13. Property \$9450; rate 13 mills; poll tax \$1.00.
14. Property \$2300; rate  $16\frac{1}{2}$  mills; poll tax \$1.50.
15. Property \$1500; rate 12 mills; poll tax \$2.00.
16. Property \$8350; rate  $13\frac{3}{4}$  mills; poll tax \$1.00.

*Find the rate and the amount of the tax :*

17. Property \$350,000; raised \$5250. Find tax on \$2000.
18. Property \$638,000; raised \$9570. Find tax on \$3200.
19. Property \$550,000; raised \$6875. Find tax on \$8500.
20. Property \$975,000; raised \$13,650. Find tax on \$6750.
21. Property \$365,800; raised \$5852.80. Find tax on \$250.

## Buoyancy of Water



The upward pressure of water at any point is always just enough to support the weight of the water which is above that point. If a section of water at the surface could be removed, and some object of the same weight as water put in its place, the object would remain in the same position without rising or sinking. If the object were heavier than water, then the upward pressure of the water underneath would not be sufficient to support it, and it would sink. If it were lighter than water, it would rise until its weight would just equal the weight of a quantity of water sufficient to fill the space which it still occupies beneath the surface.

1. A cubic foot of water weighs 1000 ounces. If a cubic foot of some kind of wood should weigh exactly 1000 ounces, would this wood sink or rise when placed in water?

2. If a cubic foot of wood should weigh 500 ounces, what part of it would rise above the water in floating?

3. What would be the specific gravity of this wood?

See Book II, p. 189.

4. If the specific gravity of a substance is  $\frac{2}{3}$ , what part of it will float above water?

5. The specific gravity of cork is about  $\frac{1}{4}$ . What part of a piece of cork would sink beneath the surface in floating?

## Buoyancy of Water

1. If a block of wood is  $\frac{3}{4}$  as heavy as water, what part of the block will rise above the surface when floating?

2. If the specific gravity of a certain kind of wood is .6, what part of the wood will rise above the surface of the water when it is floating?

3. If a block of this wood rises 4 inches above the surface of the water, how thick is the block?

4. A cubical block of wood measuring 12 inches each way is floating in water. Its specific gravity is  $\frac{5}{8}$ . How many cubic inches of water does the block displace?

5. How many more cubic inches would it displace, if it should sink to the level of the water?

6. How many pounds of weight would it be necessary to place upon it to cause it to sink to the level of the water?

7. How many cubic inches of water must be displaced in order that the pressure of the water may support a man weighing 150 pounds?

8. The specific gravity of ice is .92. To what depth will a block of ice a foot thick sink in fresh water?

9. Find how many inches a block of ice  $5\frac{1}{4}$  feet thick would rise above the surface of fresh water.

10. The specific gravity of poplar wood is about .4. How many inches of a block of poplar wood 20 inches thick will float above water?

11. The specific gravity of ash wood is about .8. If an ash block rises 2 inches above the water in floating, how thick is the block?

12. A piece of ash timber 8 ft. long, 18 in. wide, and 18 in. thick is floating in water. How many cubic feet are below the surface of the water?

### Miscellaneous Problems

1. I bought a house for \$4000, and spent a sum equal to  $12\frac{1}{2}\%$  of the cost to repair it. The taxes and other expenses amount to \$40 a year. What must be the annual rent in order that I may have a clear income of 5% of the total cost?

2. The income from an investment which pays  $3\frac{1}{2}\%$  is \$140. How much is invested?

3. My property is insured at  $\frac{1}{2}\%$ . The premium is \$14. For what sum is the property insured?

4. What would be the commission at  $2\frac{1}{2}\%$  for selling 250 barrels of flour at \$4.20 a barrel?

5. In 1900 there were about 100,000 Chinese in the United States. The total population of the main body of the country was 76,131,000. What was the ratio of the number of Chinamen to the total population?

6. Wind having a velocity of 50 miles an hour exerts a pressure of about 12 pounds to the square foot. What would be the total pressure exerted upon the side of a building 40 ft. long and 30 ft. wide?

7. A man wishing to move a stone places the end of a bar upon the ground at a point 8 inches from the point where the bar touches the stone. The total length of the bar is 5 feet. It will require a pressure of 900 pounds to move the stone. How many pounds will it be necessary to lift upon the end of the bar?

8. What is the exact number of days from Apr. 5th to Aug. 10th?

9. What is the exact interest of \$120, reckoned according to the exact number of days, from June 5th to Oct. 8th, at 4%?

## Pentagons

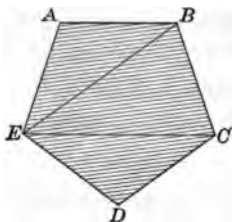


FIG. 1.

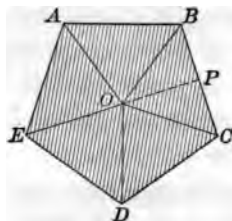


FIG. 2.

A plane figure bounded by five straight lines is called a **pentagon**. In a regular pentagon the sides are all equal and the angles are all equal.

The sum of all the angles of the triangles into which the pentagon is divided equals the sum of all the angles of the pentagon. *The sum of all the angles of any triangle is  $180^\circ$ .*

1. How large is each angle of the regular pentagon?
2. What kind of a triangle is  $ABE$ ?  $EDC$ ?
3. What kind of a triangle is  $EBC$ ?
4. Show that the triangle  $ABE$  is equal to the triangle  $DCE$ .

Show that the one would exactly cover the other, if placed upon it.

5. How large is the angle at  $A$ ? The angle  $ABC$ ? The angle  $AEB$ ?
6. How large is the angle  $EBC$ ?  $ECD$ ?  $ECB$ ?  
Take the angle  $ABE$  from the whole angle  $ABC$ .
7. In Fig. 2, if the point  $O$  is at the center of the regular pentagon, how do the five triangles formed compare with each other?
8. How large is the angle  $ABO$ ?  $BAO$ ?  $AOB$ ?
9. If the line  $OP$  is perpendicular to the line  $BC$ , how large is the angle  $OPB$ ?  $POB$ ?

## Review Problems

1. A merchant sold a bill of goods amounting to \$1200. He deducted  $\frac{1}{3}$  of the bill, and then deducted  $\frac{1}{10}$  of the remainder for cash. What was the selling price?

2. A merchant deducted  $\frac{1}{4}$  of the list price of a bill of goods, and sold them for \$150. What was the list price?

3. In 1864 it took nearly \$3 in "greenbacks" to buy what a gold dollar would buy. How much was a dollar bill really worth?

4. What was the real cost of a barrel of flour at \$18 in "greenbacks"?

5. If one of the angles of a parallelogram is  $42^{\circ} 40'$ , how large is each of the other angles?

See Book II, pp. 78, 92.

6. If one of the angles of a parallelogram is  $36^{\circ}$ , and one of the angles formed by a diagonal dividing this angle is  $16^{\circ}$ , what must be the size of each of the other angles in the figure?

7. Find the interest at 5% upon a note of \$168.75, dated Jan. 1, 1899, and paid Apr. 20, 1900.

8. Find the exact interest reckoned by days at  $4\frac{1}{2}\%$  on a note of \$50, dated Sept. 30, 1899, and paid Feb. 4, 1900.

9. If the tax rate is 17 mills on a dollar, and my taxable property amounts to \$3250, what will be my tax including a poll tax of \$2?

10. What will be the total amount of tax raised in a town in which the taxable real estate is assessed at \$2,345,000 and the personal property at \$1,462,000, if the rate is 13 mills on a dollar, and there are 1642 poll taxes at \$1.50?

11. If the rate of taxation is 15 mills on a dollar, and the tax on my property is \$648.75, for how much property am I taxed?

## Duties

Import duties are sums charged by the government for goods imported from foreign countries. The duty is sometimes reckoned at a certain rate per pound, per yard, etc., called **specific** duty, and sometimes at a certain per cent of the value of the goods, called **ad valorem** duty. Sometimes goods are subject to duties of both of these forms.

1. A merchant imported 5000 lb. of hemp. By the tariff adopted in 1897 the duty on hemp is 2 cents per pound. What was the amount of the duty?

2. The duty on molasses of a certain grade is 3 cents per gallon. What would be the duty on 10 hogsheads of molasses containing 58 gallons each?

3. The duty on Brussels carpet is 22 cents per square yard and 40% *ad valorem*. How many square yards are there in 400 yards  $\frac{3}{4}$  of a yard wide? What would be the specific duty upon 400 yards of Brussels carpet?

4. If the carpet costs 90 cents per yard, what is the amount of the *ad valorem* duty?

5. The duty on woolen cloth is 44 cents per pound and 60% *ad valorem*. Find the entire import duty on 1200 yards of woolens, if three yards weigh a pound, and the goods are worth \$1.25 a yard.

6. The duty on flannels is 33 cents per pound and 35% *ad valorem*. A merchant imports 2000 yards of flannel, 4 yards of which weigh a pound. The cost price is 25 cents a yard. Find the entire cost per yard, including the duty.

7. What would be the amount of the import duty on 2000 lb. of hemp and 180 yd. of Brussels carpet worth 75¢ a yard?

## Miscellaneous Problems

1. How many square inches of glass are there on the front of a mirror which is  $3\frac{1}{2}$  feet long and 20 inches wide, measured outside the frame, if the width of the frame is 5 inches?

2. What is the distance around the glass of the mirror inside the frame?

3. A garden which is surrounded by a concrete walk 3 ft. wide, is 4 rods long and 2 rods wide, measured outside the walk. How far is it around the garden inside the walk?

4. How many square feet are there in the walk?

5. How many square yards does the garden contain inside the walk?

6. A note of \$200, dated July 15, 1898, was due 2 years after date with interest at 5%. What was the amount of both principal and interest to be paid when the note became due?

7. Find the amount of the following note when due :

\$360

BUFFALO, N.Y., July 1, 1900.

Nine months after date, I promise to pay to the order of Charles Davidson, Three Hundred Sixty Dollars, with interest at 7%. Value received. F. H. LANGLEY.

8. The taxable real estate in a certain town is \$640,000 and the personal property \$300,000. The sum of \$11,750 is to be raised upon the property. What will be the rate?

9. What will be the tax of a man living in this town who owns taxable property to the amount of \$3120 and who pays a poll tax of \$2?

10. What would be the tax in the same town on property valued at \$9350?



## Easy Methods

To multiply by a number which is a fractional part of 100, it is often easier first to multiply by 100 and then divide the product by the number indicating the required fractional part.

To divide by such a number it is often easier first to divide by 100 and then multiply the quotient.

1. Multiply 16 by  $12\frac{1}{2}$ .

$12\frac{1}{2}$  is  $\frac{1}{4}$  of 100. 100 times 16 is 1600.  $12\frac{1}{2}$  times 16 is one eighth of 1600, or 200.

2. Divide 340 by 25.

25 is  $\frac{1}{4}$  of 100. 100 is contained in 340, 3.4 times. 25 is contained in 340, four times 3.4 times, or 13.6 times.

*Give the results orally as far as possible :*

3.  $12\frac{1}{2} \times 48 = ?$

4.  $12\frac{1}{2} \times 264 = ?$

5.  $12\frac{1}{2} \times 75.2 = ?$

6.  $16\frac{2}{3} \times 66 = ?$

7.  $10.8 \times 16\frac{2}{3} = ?$

8.  $20 \times 35 = ?$

9.  $20 \times 255 = ?$

10.  $20 \times 4.85 = ?$

11.  $25 \times 248 = ?$

12.  $27 \times 33\frac{1}{3} = ?$

13.  $500 \div 12\frac{1}{2} = ?$

14.  $420 \div 12\frac{1}{2} = ?$

15.  $2500 \div 16\frac{2}{3} = ?$

16.  $1150 \div 16\frac{2}{3} = ?$

17.  $1200 \div 20 = ?$

18.  $3450 \div 25 = ?$

19.  $12\frac{1}{2} \times 3160 = ?$

20.  $16\frac{2}{3} \times 3.1416 = ?$

21.  $20 \times 24.985 = ?$

22.  $25 \times 74804 = ?$

23.  $33\frac{1}{3} \times .0354 = ?$

24.  $50 \times 9.8764 = ?$

25.  $1742 \div 12\frac{1}{2} = ?$

26.  $243.56 \div 16\frac{2}{3} = ?$

27.  $.0385 \div 20 = ?$

28.  $86320 \div 25 = ?$

29.  $417.8 \div 33\frac{1}{3} = ?$

30.  $408576 \div 50 = ?$

31.  $79.36 \times 12\frac{1}{2} = ?$

32.  $8.34 \times 16\frac{2}{3} = ?$

33.  $38160 \div 25 = ?$

34.  $435.17 \div 33\frac{1}{3} = ?$

## Bank Discount

Banks serve as places for depositing money, provide currency for circulation, exchange one kind of money for another, borrow and lend money, and collect money on drafts and notes.

Banks usually lend money for short periods of time and reckon the exact number of days. When money is loaned at a bank, the interest is reckoned beforehand, at the time of the loan, and is deducted from the face of the note. The amount received by the borrower is called the proceeds. This is the difference between the face of the note and the discount, or interest.

1. Find the bank discount, and proceeds of a note for \$500 due in 60 days at 6%.

The bank discount is the interest of \$500 for 60 da. at 6% or \$5.  
The proceeds is the difference between \$500 and \$5 or \$495.

*Find the bank discount of the following notes :*

- |                                       |                                       |
|---------------------------------------|---------------------------------------|
| 2. \$200, 60 da. at 6%.               | 8. \$85, 42 da. at 4%.                |
| 3. \$300, 30 da. at 5%.               | 9. \$65, 48 da. at 6%.                |
| 4. \$760, 90 da. at 7%.               | 10. \$40, 30 da. at 7%.               |
| 5. \$125, 60 da. at $4\frac{1}{2}$ %. | 11. \$75, 80 da. at 5%.               |
| 6. \$240, 30 da. at 6%.               | 12. \$50, 56 da. at $4\frac{1}{2}$ %. |
| 7. \$500, 90 da. at $5\frac{1}{2}$ %. | 13. \$96, 64 da. at $5\frac{1}{2}$ %. |

*Find the proceeds of the following notes :*

- |  |  |
|--|--|
| 14. \$360, 60 da. at 5%.               | 20. \$125.50, 4 mo. at 6%.               |
| 15. \$400, 30 da. at 6%.               | 21. \$348.30, 6 mo. at 7%.               |
| 16. \$700, 90 da. at $4\frac{1}{2}$ %. | 22. \$118.20, 3 mo. at 5%.               |
| 17. \$580, 36 da. at $5\frac{1}{2}$ %. | 23. \$675.50, 5 mo. at $5\frac{1}{2}$ %. |
| 18. \$290, 45 da. at 7%.               | 24. \$416.30, 4 mo. at $6\frac{1}{2}$ %. |
| 19. \$285, 74 da. at 8%.               | 25. \$548.95, 6 mo. at 4%.               |

## Bank Discount

*Find the bank discount :*

1. Note of \$400 for 60 days at 6%.
2. Note of \$650 for 90 days at 5%.
3. Note of \$240 for 30 days at 4%.
4. Note of \$580 for 60 days at  $4\frac{1}{2}\%$ .
5. Note of \$350 for 45 days at  $3\frac{1}{2}\%$ .
6. Note of \$830 for 32 days at  $5\frac{1}{2}\%$ .

*Find the proceeds :*

7. Note of \$365 for 30 days at 7%.
8. Note of \$160 for 60 days at  $5\frac{1}{2}\%$ .
9. Note of \$250 for 90 days at  $4\frac{1}{2}\%$ .
10. Note of \$640 for 60 days at  $6\frac{1}{2}\%$ .
11. Note of \$280 for 35 days at 4%.
12. Note of \$980 for 56 days at 5%.

*Find the time and the discount :*

13. \$100 from Mar. 1 to Apr. 30 at 6%.
14. \$200 from Apr. 10 to June 5 at 5%.
15. \$260 from Oct. 20 to Nov. 30 at  $4\frac{1}{2}\%$ .
16. \$400 from Sept. 4 to Oct. 11 at  $5\frac{1}{2}\%$ .
17. \$525 from Aug. 2 to Nov. 20 at 7%.
18. \$600 from July 31 to Sept. 10 at 6%.

*Find the time and the proceeds :*

19. \$256.20 from Mar. 1 to Apr. 21 at 6%.
20. \$135.40 from Mar. 20 to May 5 at  $4\frac{1}{2}\%$ .
21. \$316.85 from May 9 to Aug. 5 at 5%.
22. \$560.30 from Apr. 2 to July 22 at 7%.
23. \$754.80 from June 15 to Aug. 31 at  $7\frac{1}{2}\%$ .
24. \$952.45 from May 5 to Sept. 1 at 6%.

## Construction

See p. 102 and Book II, pp. 151, 231.

1. Construct a right triangle containing an angle of  $35^\circ$ .

2. Construct an isosceles triangle containing an angle of  $52^\circ$ .

3. Construct an equilateral triangle containing a side  $3\frac{1}{4}$  inches long.

4. Make a circle with a radius  $1\frac{1}{2}$  inches long. Draw a diameter. At the center make a right angle and draw a second diameter perpendicular to the first. At the extremities of the diameters make right angles and prolong the sides in both directions until they meet each other. What figure do these lines form? Find the circumference of the circle. Find the area of the circle.

5. Make a line  $1\frac{1}{2}$  inches long. At one end make an angle of  $108^\circ$ , and make the second side  $1\frac{1}{2}$  inches long. At the end of this line, and on the same side as before, make an angle of  $108^\circ$ , and make the side  $1\frac{1}{2}$  inches long. Proceed as before, till the starting point is reached. What is the size of the last angle? What figure has been formed?

6. Construct a regular pentagon with a side  $1\frac{3}{4}$  inches long. From the vertex of one angle draw two lines dividing the pentagon into three triangles. Measure the angles of these triangles. Which angles are equal? Which triangles are equal? What kind of triangles are the three triangles?

7. Construct a regular pentagon with a side 2 inches long. By testing find some point within that is equally distant from the vertices of all the angles. Draw lines from this point to the vertices of the angles. In the triangles thus formed which parts are equal? Which triangles are equal?

## Review Problems

1. A block of wood is 4 ft. long, 8 in. wide, and 6 in. thick. What must be its weight in order that it may sink to the level of the surface of the water ?

2. A log is 14 inches square and 12 feet long. How many cubic inches does it contain ?

3. The specific gravity of the log is .6. When floating in water how many cubic inches of water will it displace ?

4. How heavy a person would the log support without sinking ?

5. The specific gravity of cork is about  $\frac{1}{4}$ . How many pounds will a life preserver, containing one cubic foot of cork, hold up above water ?

6. A man holds one end of a bar 8 ft. long and the other end rests upon a support. If an object weighing 100 pounds is placed upon the bar at a point 2 feet from the point of support, how many pounds will the man have to lift ?

7. If the safety valve of a steam boiler is 10 inches from the point of attachment of the lever and a weight of 20 pounds is required to keep it down when placed at a distance of 30 inches from the point of attachment, what is the total pressure of the steam in the valve ?

8. If the diameter of the valve is 3 inches, against how many square inches does the steam press in the valve ?

Find the area of the circle.

9. What is the pressure of the steam in the boiler per square inch ?

10. When the power is applied to a lever between the fulcrum and the weight, how much power is necessary to raise a weight of 600 lb. if the distance of the weight from the fulcrum is three times the distance of the power from the fulcrum ?

## Original Problems

*Make problems and solve them :*

1. If 16 is added to three times a number, the result will be five times the number.
2. The sum of two numbers is 26, and the smaller is 2 more than half the larger.
3. A certain number is diminished by 5 and then increased by 12.
4.  $\frac{3}{8}$  of a number is taken from  $\frac{5}{8}$  of the number.
5. The radius of a circle is 20 inches.
6. A circle is inscribed in a square. The area of the square is 225 inches.
7. The valuation of the property in a certain town is \$980,000. The amount to be raised on the property is \$11,760.
8. The rate of taxation in a certain town is 15 mills on a dollar. The tax of a resident of the town is \$45, besides his poll tax.
9. The income from an investment which pays  $4\frac{1}{2}\%$  is \$225.
10. Wind with a velocity of 50 miles an hour exerts a pressure of about 12 pounds per square foot.
11. A merchant deducted  $\frac{1}{5}$  of the list price of goods and sold them for \$40.
12. The duty on Brussels carpet is 22 cents per square yard and 40% *ad valorem*.
13. A note of \$500 is discounted at a bank for 90 days.
14. A note of \$300, dated Apr. 20, 1900, and due in 60 days, was discounted at a bank.
15. The specific gravity of a certain kind of wood is .7.

## Algebra

In algebraic addition or subtraction similar terms should be placed in the same column.

When there is a term in the minuend with no similar term in the subtrahend it is written in the remainder without change.

When there is a term in the subtrahend having a plus sign and there is no similar term in the minuend from which it is to be taken or with which it is to be combined, it is written in the remainder with its sign changed to minus because it is to be subtracted.

*Subtract :*

1.	2.	3.	4.	5.
$7a$	$16x$	$4a + 2b$	$12m + 5n$	$4a + 9b + 10$
$\underline{3a}$	$\underline{3x}$	$\underline{2a + b}$	$\underline{4n + 6m}$	$\underline{8 + 2a + 7b}$
$4a$				

6.	7.	8.
$9a + 3b$	$8a - 3b + 10$	$11x + 15$
$\underline{4a}$	$\underline{4a + 6}$	$\underline{3x + y + 7}$

9.	10.	11.
$5a - b + 7c - 12$	$9x + 7y - 4z$	$20x - y + 6$
$\underline{2a + 3c + 2d + 3}$	$\underline{2x + y + 2z}$	$\underline{2y + z + 13}$

*Subtract :*

12.  $4x + 3z$  from  $10x + 5z$ .
13.  $6a + 2b$  from  $8a + b$ .
14.  $18 + (x + y)$  from  $42 - (x - y)$ .
15.  $a + 4c + 3d + 5$  from  $3a - 2b - c - 6$ .
16.  $5a + (2b + 3c) + 4d$  from  $16a + (c - d)$ .
17.  $7a + 2b + c + d$  from  $10a - b + c - d$ .

## Algebra

1. Subtract  $8 - 5$  from 10. Subtract  $8x - 5y$  from  $10x$ . Subtract  $-8x - 5y$  from  $10x + 7y$ .

$$\frac{10}{8 - 5}$$

$$\frac{2 + 5}{2 + 5}$$

$$\frac{10x}{8x - 5y}$$

$$\frac{2x + 5y}{2x + 5y}$$

$$\frac{10x + 7y}{-8x - 5y}$$

$$\frac{18x + 12y}{18x + 12y}$$

When there is a term with the minus sign in the subtrahend it serves to decrease the subtrahend and so to increase the remainder. If the other portion of the subtrahend is taken from the minuend before deducting this term, the minus term must be added to the remainder to correct the error. *In subtracting, those terms of the subtrahend which have the plus sign are subtracted and those which have the minus sign are added.* In other words, the subtrahend, with all its signs changed, is added to the minuend.

*Subtract :*

2.

$$\begin{array}{r} 12a \\ 7a - 3b \\ \hline \end{array}$$

3.

$$\begin{array}{r} 9a - 4b + 7c \\ a + 3b - 2c \\ \hline \end{array}$$

4.

$$\begin{array}{r} 7m + 3n - 6 \\ 4m - 2n + 8 \\ \hline \end{array}$$

5.

$$\begin{array}{r} 4x - 5y + 6z - 3 \\ x - y + 7z - 5 \\ \hline \end{array}$$

6.

$$\begin{array}{r} 13 - 5a + 3d \\ 2a - d + 3c \\ \hline \end{array}$$

7.

$$\begin{array}{r} 3a - 4b + 3c - 4d \\ 3a - 4b - 3c + 4d \\ \hline \end{array}$$

8. Add  $3a - 2b$  and  $6a + 5b$ .
9. Add  $4x - 3y$ ,  $5x - 2y$ , and  $4y - 7x$ .
10. Add  $7x + 3y + z$ ,  $y - 5x$ , and  $4x - 2y + z$ .
11. Add  $7m - 5n$ ,  $3m + 4n$ , and  $3n - 4m$ .
12. Add  $9a + 7b - 8c + 12$  and  $-5a + b + 5c - 3$ .
13. From  $10a$  take  $5a - 4$ .
14. From  $8a + 5b$  take  $6a - 2b - c$ .
15. From  $2x - y + z$  take  $x - y - z$ .
16. From  $9x + 3y - 7z$  take  $2x - 4y + 8z - 3$ .



## Algebra

*Find the value of  $x$ :*

1.  $5x + 7 = 3x + 19$

2.  $8x - 15 = 25 - 2x$

3.  $3x - (12 - x) = 2x + 8$

4.  $\frac{x}{2} + \frac{3x}{8} + x = 15$

5.  $25 + \frac{x+8}{4} = 29 - \frac{x-8}{4}$

6. Two of three brothers have equal sums of money. The third brother has \$10 more than  $\frac{1}{3}$  as much as the other two together. They all together have \$50. How much has each?

7. Thirteen more than  $\frac{1}{2}$  of a certain number is seventeen less than  $\frac{7}{8}$  of the number. What is the number?

8. If I should subtract 12 from a certain number and should add the remainder to four times the number, this sum would equal 3 more than four times the number. Find the number.

9. A father and son worked 18 days. The son received  $\frac{1}{4}$  as much as the father. The wages of both amounted to \$45. How much did each earn per day?

*Make problems for these equations:*

10.  $2x + \frac{2x}{3} - 6 = 26$

11.  $4x + 10 = 35 - (15 - x)$

12.  $15x + \frac{15x}{6} = 52\frac{1}{2}$

13.  $x + \frac{x+1}{2} = 3x - \frac{7x+2}{5}$

## Problems for Arithmetic or Algebra

1. If the multiplicand is 356 and the product is 2848, what is the multiplier?

Let  $x =$  multiplier. Then  $356x = 2848$ .

2. If the product is 2526 and the multiplier is 6, what is the multiplicand?

3. If the dividend is 12,513 and the quotient is 23, what is the divisor?

Let  $x =$  divisor. Then  $23x = 12513$ .

4. If the quotient is 8126 and the divisor is 18, what is the dividend?

5. The area of a rectangle is 504 sq. ft. Its length is 42 ft. What is its width?

6. The area of a triangle is 736 sq. in. Its base is 3 ft. 10 in. What is its altitude?

7. The area of a trapezoid is 136 sq. ft. Its altitude is 8 ft. One of the parallel sides is 21 ft. long. How long is the other side?

8. The length of a room is to its width as 4 to 3. The distance around it is 112 feet. What are its length and width?

9. One angle of a triangle is  $27\frac{1}{2}^\circ$ . A second angle measures  $84\frac{1}{4}^\circ$ . How large is the third angle?

10. The angle at the vertex of an isosceles triangle is an angle of  $50^\circ$ . What is the size of the angles at the base?

11. One of the angles of a parallelogram measures  $42^\circ$ . What is the size of each of the other angles?

12. One angle in a right triangle measures  $50\frac{1}{2}^\circ$ . What is the size of the other acute angle?

13. The circumference of a circle is 52 inches. Find the diameter.

### Miscellaneous Problems

1. If I buy a dollar's worth of sugar at the rate of 20 pounds for a dollar, but find that I receive only 15 ounces for a pound, what price per pound do I really pay?
2. This is at the rate of how many pounds for a dollar?
3. A piece of cloth measured with a yardstick 2 inches too short seemed to be 9 yd.  $1\frac{1}{2}$  ft. long. How long was it?
4. A piece of cloth measured with a yardstick 1 inch too long seemed to contain 12 yards. How many yards did it really contain?
5. The diameter of a cylinder is 22 inches, and its length 42 inches. Find the number of square inches on the two ends.
6. Find the number of square feet in the entire surface of the cylinder.
7. A square prism is 5 ft. long and 15 in. square. What is the total length of all the edges?
8. What is the total area of the prism?
9. What is the sum of all the angles of a pentagon?
10. How large is one of the angles at the center of a pentagon formed by lines from the center to the vertices?
11. About 21% of the bulk of pure air is oxygen. If a child inhales 28 cu. in. of air at each breath, and about  $\frac{1}{4}$  of the oxygen taken into the lungs remains in the system, how much oxygen is taken into the system at each breath?
12. How many cubic feet of oxygen will be consumed by the child in an hour, if he breathes 20 times a minute?
13. How many cubic feet of oxygen would be consumed *per class* of 30 children in an hour?

## The Tide

1. The interval between successive high tides is about 12 hr. 26 min. If it is high tide precisely at noon to-day, at what time will high tide occur to-morrow?
2. There are two tidal waves opposite each other on the earth, following each other around. If there were no continents to obstruct its way, how long would it take for one of these waves to move entirely round the earth?
3. When it is high tide at noon on a certain day, at what time in the day will high tide occur 5 days later?
4. How long will it be before high tide will occur at about noon again?
5. Find how many times high tide occurs each year.
6. How long is the period from low tide to high tide?
7. Find how many miles an hour the tidal waves move on a great circle of the earth.
8. At a place where the difference between low tide and high tide is 50 inches, what is the average amount of change per hour?
9. In the Bay of Fundy the tide rises and falls 70 feet. How many inches is the average rise or fall per hour?
10. If it is low tide at 9.30 A.M. on April 5th, find at what time low tide will occur on April 8th.
11. If it is low tide at 5.20 A.M. on May 24th, at what times will it be high tide on June 1st?
12. On a gently sloping beach where the tide rises 6 feet, and in doing so moves up the beach for a distance of 12 rods, how many inches per minute would one have to move back to keep out of the water?

## Powers and Roots

The **square** of a number is the product obtained by multiplying the number by an equal number.  $7 \times 7 = 49$ . 49 is the square of 7.  $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$ .  $\frac{4}{9}$  is the square of  $\frac{2}{3}$ .

The **square root** of a number is one of the two equal factors of the number. 7 is the square root of 49,  $\frac{2}{3}$  is the square root of  $\frac{4}{9}$ .

The **cube** of a number is the result obtained by multiplying together three equal numbers.  $3 \times 3 \times 3 = 27$ .  $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$ . 27 is the cube of 3.  $\frac{1}{8}$  is the cube of  $\frac{1}{2}$ .

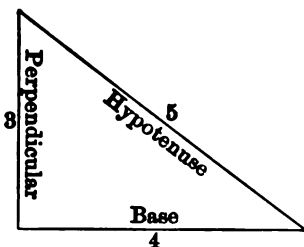
The **cube root** of a number is one of the three equal factors of the number. 3 is the cube root of 27.  $\frac{1}{2}$  is the cube root of  $\frac{1}{8}$ .

The square root of a number, indicating the number of square units in a square, gives the length of the square.

The cube root of a number, which indicates the number of cubic units in a cube, gives the length of the cube.

1. What is the square of 5? The square root of 36?
2. What is the cube of 3? The cube root of 125?
3. A square field contains 81 sq. rd. How long is it?
4. A cubical block contains 64 cubic inches. How long is it?
5. The product of two equal numbers is 144. What are the numbers?
6. The product of three equal numbers is 216. What are the numbers?
7. The product of four equal numbers is 625. What are the numbers?
8. What is the square of  $\frac{2}{3}$ ? The square root of  $\frac{4}{9}$ ?
9. What is the cube of  $\frac{1}{2}$ ? The cube root of  $\frac{1}{8}$ ?
10. What is the cube of  $\frac{2}{3}$ ? The cube root of  $\frac{8}{27}$ ?

## Right Triangles



See Book II, p. 127.

In any right triangle *the square of the hypotenuse is equal to the sum of the squares of the base and the perpendicular.*

$$H^2 = B^2 + P^2; \quad B^2 = H^2 - P^2; \quad P^2 = H^2 - B^2.$$

$$25 = 16 + 9; \quad 16 = 25 - 9; \quad 9 = 25 - 16.$$

1. In a right triangle the base is 8 feet and the perpendicular 6 feet. How long is the hypotenuse?
2. If the hypotenuse is 15 feet and the perpendicular 12 feet, how long is the base?
3. A rectangle is 16 feet long and 12 feet wide. Find the length of its diagonal.
4. A ladder 13 feet long reaches a window which is 12 feet above the street. How far from the house is the foot of the ladder placed?
5. The distance between the opposite corners of a room is 17 feet. The room is 15 feet long. How wide is it?
6. A line 25 feet long is stretched from the top of a tree to a point on the ground 15 feet from the tree. How tall is the tree?
7. The distance between the opposite corners of a rectangular field is 26 rods. The length of the field is 24 rods. Find its width.
8. If it takes a rope 13 feet long to reach from the top of the mast of a boat to a point 5 feet from the foot of the mast, how tall is the mast?

### Miscellaneous Problems

1. A collector was paid 10% commission for collecting over-due accounts. He earned \$18.25. How much did he collect?
2. A collector received \$65 for collecting bills at 20% commission. How much did his employer receive?
3. How much will it cost to insure furniture for \$800 at  $1\frac{1}{2}\%$ ?
4. How much will it cost to insure a house worth \$3600 for  $\frac{2}{3}$  of its value at  $1\frac{3}{4}\%$ ?
5. What is the number of feet of lumber in a plank 16 ft. long, 18 in. wide, and 3 in. thick?
6. Find the number of feet of lumber required to make the floor of a barn 42 ft. long,  $18\frac{1}{4}$  ft. wide, if the planks are 2 inches thick.
7. Find the convex surface of a cone, the circumference of whose base is 14 ft. 6 in., and whose slant height is 12 ft. 8 in.
8. Find the lateral surface of a square pyramid whose base is 11 ft. square, and whose slant height is 12 ft. 4 in.
9. What is the bank discount of a note of \$460 discounted for 60 days at 5%?
10. What are the proceeds of a note of \$132 discounted at a bank for 90 days at  $4\frac{1}{2}\%$ ?
11. I sold 500 bushels of wheat at 60 cents a bushel and received a note due in 60 days. I had the note discounted at a bank at 5%. To how much in cash was the note equivalent?
12. A merchant bought 300 barrels of flour at \$4.50 a barrel, payable in 6 months. He sold it immediately for \$4.45 a barrel, and put the money at interest at 6%. At the end of the 6 months he paid for the flour. How much *did he gain or lose* upon the transaction?

## Ratio

See p. 34.

*What is the ratio :*

1. Of a rectangle 40 ft.  $\times$   $3\frac{1}{2}$  ft. to a 5-foot square ?
2. Of a triangle whose base is 18 in. and altitude 18 in. to an 18-inch square ?
3. Of a trapezoid whose parallel sides are 15 in. and 12 in. and altitude 10 in. to a triangle whose base is 15 in. and altitude 10 in. ?
4. Of a right triangle whose base and perpendicular are 3 ft. 8 in. and 4 ft. 3 in. to a rectangle 10 ft. 9 in. long and 4 ft. 2 in. wide ?
5. Of a 14-inch square to the circle inscribed in the square ?
6. Of the surface of an 8-inch cube to the surface of a square prism 8 inches square and 2 ft. long ?
7. Of the surface of a foot cube to the surface of the largest sphere that can be contained in the cube ?
8. In a township 6 miles square, which has a population of 7200, how many people are there per square mile ?
9. What is the ratio of the population to the number of square miles ?
10. When a house worth \$5400 is rented for \$30 a month, what is the ratio of the annual rent to the value of the house ?
11. If \$200 is put at interest for 5 years, at 6 %, what is the ratio of the interest for that time to the principal ?
12. What is the ratio of the principal to the amount ?
13. What is the ratio of the interest to the amount ?



### Miscellaneous Problems

1. What fraction of a foot is  $3\frac{1}{2}$  in.?  $6\frac{2}{3}$  in.?  $10\frac{3}{8}$  in.?
2. What fraction of a yard is 1 ft. 4 in.? 2 ft. 3 in.?
3. What part of 100 is  $9\frac{1}{11}$ ?  $14\frac{2}{3}$ ?  $44\frac{4}{5}$ ?
4. What is the value of a pile of wood 20 ft. long, 4 ft. wide, and  $5\frac{1}{2}$  ft. high, at \$6.50 a cord?
5. A rectangular cistern is 8 ft. long, 7 ft. 3 in. wide, and 5 ft. 6 in. deep. How many square feet are there in the bottom and sides?
6. What is the area of a trapezoid whose parallel sides are 120 ft. and 44 ft., and whose altitude is 60 ft.?
7. Find the lateral surface of a triangular prism each side of whose base is  $4\frac{1}{3}$  ft. and whose length is 10 ft.
8. The circumference of a circular flower bed is 32 ft. 3 in. Find its diameter.
9. Find the lateral surface of a cylinder whose diameter is 15 in. and whose length is 20 in.
10. Find the exact number of days in the period of discount of a note, dated Jan. 20, 1900, written for 90 days, and discounted Feb. 15.
11. Find the bank discount of a note of \$275, dated Aug. 5, due in 60 days, and discounted Aug. 17 at 5%.
12. Find the proceeds of a note of \$324.50 dated March 16, 1900, due in 90 days from date, discounted at a bank May 2, 1900 at 5%.
13. I sold a lot of land, receiving \$400 in cash and a note of \$300 due in 6 months. The note was discounted at a bank at 5%. How much did I really get for the land?
14. How much better is \$500 in cash than a note of \$508, due in 4 months, which can be discounted at a bank at 5%?

## Hexagons

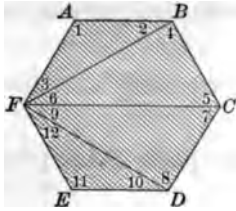


FIG. 1.

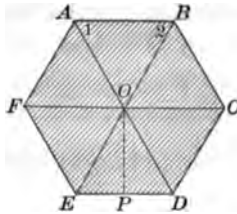


FIG. 2.

A **regular hexagon** is a polygon having 6 equal sides and 6 equal angles.

It is evident that the sum of all the interior angles of a polygon equals the sum of all the angles of the triangles into which it may be divided.

1. In the regular hexagon (Fig. 1) what is the sum of all the angles into which the hexagon is divided?

2. How large is each of the angles of the hexagon?

3. Since  $FA = AB$ , what kind of a triangle is  $FAB$ ? How does it compare with  $FED$ ?

4. How large is the angle 1? The angle 2?

5. How large is the angle 12? 10? 4? 8?

6. How much is the sum of the angles 6 and 9?

Take the sum of the angles 3 and 12 from the whole angle at  $F$ .

7. How large is the angle 6? The angle 9? The angle 5? The angle 7?

8. In the regular hexagon (Fig. 2) if lines are drawn from the center  $O$  to the vertices, what is the sum of all the angles of each of these triangles?

9. What is the sum of all the angles around the point  $O$ ?

10. How large is one of the angles formed at  $O$ ?

11. How large is the angle 1? The angle 2?

12. What kind of a triangle is each of the triangles?

## Review

1. Divide the number 39 into two parts in the proportion of 5 to 8.

$5 + 8 = 13$ . If we take  $\frac{5}{13}$  and  $\frac{8}{13}$  respectively of 39, the results will be in the proportion of 5 to 8.

2. Divide \$142.66 into two parts in the proportion of 3 to 4.

3. Two men purchase a quantity of hay for \$351. In dividing it one gets 12 tons and the other 15 tons. How much should each pay?

4. The length of a room is  $1\frac{1}{3}$  times its width. The sum of the length and width is 35 feet. How long is the room?

5. The length of a room is  $\frac{5}{4}$  of its width. The sum of the length and width is 27 feet. How long is it?

6. If the distance around a rectangle is 96 feet, and it is  $\frac{5}{3}$  as long as it is wide, how many square feet does it contain?

7. A man paid  $\frac{2}{5}$  of his money for rent, and  $\frac{1}{3}$  of the remainder for groceries, and had \$30 left. How much had he at first?

8. A man paid  $\frac{1}{6}$  of his money for a hat, and  $\frac{1}{4}$  of his money for a pair of shoes. The shoes cost \$2 more than the hat. How much had he at first?

9. Find the interest of \$235.63 for 3 yr. 7 mo. 15 da. at 4%.

10. Find the bank discount of a note of \$365 for 90 days at  $5\frac{1}{2}$ %.

11. Find the proceeds of a note of \$450, dated July 12, due in 4 months after date, discounted at a bank Sept. 24, at 4%.

## Building

A house is to be built 50 ft. long and 34 ft. wide. The walls are to be 24 ft. high. The roof is to extend 12 feet higher than the walls. The rafters are 22 ft. long, and the roof extends 2 feet over the sides and ends.

1. Find the number of square feet in the walls including the gables.

2. Find the number of square feet in the roof.

3. In boarding the house the open spaces left for windows and doors amount to 800 sq. ft. If we regard one half of these open spaces as equivalent to the waste in fitting the boards, what will be the cost of the boards at \$18 a thousand feet?

4. 4 bunches of shingles are reckoned as 1000. A thousand shingles will cover about 100 sq. ft., when laid with 4 inches exposed to the weather. How many bunches of shingles will it take for the house?

5. How much will it cost for the shingles at \$3.75 per M., if the shingles are laid 4 inches to the weather?

6. Clapboards are usually 4 ft. long and 6 in. wide, and are tied in bundles of 25 in a bundle. When clapboards are laid with  $3\frac{1}{2}$  in. to the weather, how much surface does one clapboard cover?

7. Reckoning one clapboard for each square foot, how many bundles of clapboards will it take to cover the walls and the gables of the house?

8. How much will the clapboards cost at \$45 per M.?

9. How much will it cost to paint the clapboards with two coats of paint at the rate of 80 cents per 100 sq. ft. for each coat?

10. How much will it cost to lay floors in the two stories two boards thick, if the boards cost \$24 per M.?

### Miscellaneous Problems

1. A boy earns  $\frac{1}{2}$  as much as his older brother, and the older brother earns  $\frac{1}{2}$  as much as his father. The three together earn \$56 a week. How much does each earn?

2. A man left  $\frac{1}{3}$  of his property to his wife,  $\frac{1}{3}$  of the remainder to his daughter, and what then remained to his son. What part of the property did each receive?

3. If the wife received \$1000 more than the daughter, how much did each receive?

4. Find the surface of a circle whose diameter is 14 inches.

5. Find the entire surface of a cylinder whose diameter is 16 inches and length 40 inches.

6. What is the convex surface of a cone, the diameter of whose base is 12 inches, and whose slant height is 15 inches?

7. A barn is 60 feet long and 30 feet wide, and the roof extends 8 feet above the eaves. What is the length of the rafters, if they extend 1 foot beyond the eaves?

8. The roof extends 1 foot over each end. How much will it cost to shingle it, if the price of the shingles is \$3.50 per M., and they are laid 4 inches to the weather?

9. How much less would it cost, if the shingles were laid 5 inches to the weather, than if laid 4 inches to the weather?

10. If we reckon one clapboard for each square foot, and clapboards cost \$40 per M., find how much it will cost for the clapboards to cover the walls and gables of a barn 40 feet long, 24 feet wide, and 16 feet high to the eaves, if the roof extends 9 feet above the eaves.

## Bank Discount

When a note is discounted at a bank at some time after it was written, the discount is reckoned for the time remaining before it will become due. A note dated May 15, and due 60 days after date, will be due July 14. If this note is discounted June 10, the term of discount will be the time from June 10 to July 14, or 34 days.

By the law in some states, 3 days, called **days of grace**, are added to the time written upon the note, so that a note written to become due in 60 days will become due in 63 days after date.

*Find the number of days between the following dates :*

1. Jan. 1 and Mar. 15.
2. Mar. 2 and Apr. 12.
3. May 18 and June 28.
4. June 30 and Aug. 5.
5. July 6 and Oct. 10.
6. July 17 and Aug. 31.
7. Sept. 2 and Nov. 1.
8. Oct. 18 and Dec. 30.

*Find the term of discount of the following notes :*

9. Date May 1 ; due in 60 days ; discounted May 10.
10. Date June 12 ; due in 30 days ; discounted June 17.
11. Date July 15 ; due in 90 days ; discounted Aug. 12.
12. Date Aug. 30 ; due in 4 months ; discounted Nov. 1.

*Find the term of discount, adding three days of grace :*

13. Date Mar. 5 ; due in 60 days ; discounted Apr. 1.
14. Date May 10 ; due in 6 months ; discounted July 15.
15. Date Oct. 12 ; due in 90 days ; discounted Dec. 4.
16. Date Nov. 20 ; due in 3 months ; discounted Dec. 15.

## Problems from Physics

See pp. 36, 37, 99.

1. The water of the Dead Sea is about 10 times as salt as that of the ocean. The specific gravity of the water of the ocean is about 1.02, and that of the water of the Dead Sea about 1.2. What is the weight of a cubic foot of the water of the Dead Sea?

2. What is the pressure of the water of the Dead Sea, upon a square foot, at a depth of 10 feet?

3. If the body of a man displaces 2 cubic feet of water, how many pounds of support does he receive from the water while swimming in fresh water?

4. How many pounds of support would he receive while swimming in ordinary sea water?

5. How many more pounds of support would he receive from the water of the Dead Sea than from ordinary sea water?

6. If a branch will support a boy weighing 100 lb., at a point 10 feet from the trunk, how large a weight would it support at a point 2 feet from the trunk?

7. A tree is to be pulled over by a team of horses. The horses can pull 2000 pounds. If the chain should be attached to the tree one foot from the ground, it would require 24,000 pounds of force to uproot it. At how high a point must the chain be attached in order that the horses may succeed?

8. A certain tree is 40 feet tall. Another tree near it is 120 feet tall. If the former has foliage enough to receive from the wind a force of 1500 pounds, and the latter enough to receive a force of 2500 pounds, about how many times stronger must the latter be at the root than *the former* in order to be able to stand?

## Original Problems

*Make problems and solve them :*

1. It is high tide at 25 minutes past 1 o'clock.
2. It was high tide at 7.45, Dec. 20th.
3. An agent received \$35 for collecting bills at a commission of 25 %.
4. The base of a square pyramid is 6 ft. 4 in. square, and the slant height is 8 ft. 6 in.
5. A merchant sold a car load of corn, consisting of 400 bushels, at 55 cents a bushel, and received in payment a note due in 60 days.
6. Jan. 1, 1898, I deposited \$200 in the savings bank.
7. A note was written Mar. 20, 1899, due in 90 days, and discounted at a bank Apr. 26th.
8. The side of a regular hexagon is 10 inches.
9. Two men together purchase a car load of oats. In dividing, one receives 350 bushels and the other 250 bushels.
10. A man expended  $\frac{1}{3}$  of his money for a suit of clothes, and  $\frac{1}{5}$  of it for an overcoat.
11. A note is written June 16th, payable in 90 days.
12. A man left  $\frac{1}{2}$  of his property to his wife, and  $\frac{2}{5}$  of the remainder to his daughter.
13. A barn is 28 feet long and 14 feet wide, and the roof extends 10 feet above the eaves.
14. The rafters project 18 inches over the eaves.
15. A thousand shingles will cover 100 square feet.
16. The clapboards cost \$45 a thousand.



## Algebra

To indicate the product of algebraic quantities which are expressed by symbols it is not necessary to write the sign of multiplication between them. It is understood that if they are written in succession with no sign between them, they are to be multiplied together.  $abc$  indicates the product of  $a$ ,  $b$ , and  $c$ .

When there are two or more equal factors the symbol is written but once, and the number of such equal factors is indicated by a small figure called the **exponent** written above, at the right.  $a^2$  indicates the product of  $a$  and  $a$ ;  $b^3$  indicates the product of  $b$ ,  $b$ , and  $b$ , or of  $b^2$  and  $b$ ;  $3ab^2c$  the product of 3,  $a$ ,  $b$ ,  $b$ , and  $c$ .

*Multiply :*

1. $ab$ $cd$ <hr/> $abcd$	2. $ab$ $ab$ <hr/> $a^2b^2$	3. $abc$ $b^2$ <hr/> $ab^3c$	4. $x^2y$ $xy$ <hr/>	5. $xy^2z^3$ $x^2y$ <hr/>
6. $3ab$ $7$ <hr/> $21ab$	7. $5a^2b$ $abc^2$ <hr/>	8. $4bc^2$ $5cd^2$ <hr/>	9. $8x^2y$ $8x^2y$ <hr/>	10. $24m$ $3n^2$ <hr/>

In dividing one quantity by another the factors of the divisor are removed from the factors of the dividend.  $2 \times 3 \times 4 \times 5$  divided by  $2 \times 4$  will give  $3 \times 5$ .

*Divide :*

11. $ab)abcd$ $cd$	12. $bc)ab^2c$ $ab$	13. $xy)x^2y^2$ $xy$	14. $xyz)xyz^2$ $z$	15. $xyz^2)x^2y^4z^3$ $xy^3z$
16. $ab)5ab^2$	17. $7)7a^2b^3$	18. $2)10ab$	19. $4a^2)8a^2b^4$	20. $6mn)12m^2n^2$

## Algebra

An expression containing a single term is called a **monomial**. An expression containing more than one term is called a **polynomial**. To multiply or divide a polynomial, each of the terms must be multiplied or divided.

$$\frac{ab^2 - 2b}{3ab}$$

$$\frac{3a^2b^3 - 6ab^2}{3a^2b^3 - 6ab^2}$$

$$\frac{2m^2n \mid 4m^3n^2 + 2m^2n^3}{2mn + n^2}$$

1. Add  $4m + 7n - 3y$ ,  $5m - 3n + 4y$ .
2. Add  $4x^2y - 2xy^2$ ,  $2x^2y + xy$ ,  $6xy - 3xy^2$ .
3. Add  $8a^2b^2 - 4ab + ab^3$ ,  $a^2b^2 + ab + ab^3$ .
4. Add  $10m^2 - 5mn + 3n^2$ ,  $6mn - 5m^2 - 2n^2$ .
5. Add  $x^2 + 4ax^2 - 8ax^3$ ,  $4x^2 - 4ax^2 + 8ax^3$ .
6. From  $6x^2 + 4y^2$  subtract  $3x^2 - 2y^2$ .
7. From  $12a^2 - 6ab + a$  subtract  $10a^2 - 5ab - b$ .
8. From  $6x^5 - 10x^3y^3 + 4xy$  subtract  $3x^3y^3 - xy + 2x^5$ .
9. From  $8ab + 5b^2 - 4cd$  subtract  $9ab - 4b^2 + 3cd$ .
10. From  $7m^2n - 5mn^2 - 2mn + 4$  subtract  $3m^2n + mn + 10$ .
11. Multiply  $a^2 + 2a$  by 4.
12. Multiply  $m + 2mn - n^2$  by  $2m$ .
13. Multiply  $4x - 7xy + xy^2$  by  $3xy$ .
14. Multiply  $x^3 - 2x^2 + 5x - 3$  by  $8x^2y$ .
15. Divide  $a^3 + a^2b$  by  $a^2$ .
16. Divide  $a^2bc - 4ab^2c^2$  by  $ac$ .
17. Divide  $21a^4b^3c + 28a^3b^2c$  by  $7abc$ .
18. Divide  $6x^3 - 9x^5 + 12x^6 - 15x^8$  by  $3x^3$ .
19. Divide  $4m^2n^4x^2 - 6mn^3x^3 - 8amn^2x^4 + 6m^2n^3x^2$  by  $2mn^2x^2$ .

## Problems for Arithmetic or Algebra

1. 450 is 3% of a certain number. What is the number?
2. 325 is  $\frac{5}{8}$  of what number?
3. 675 is 15% of what number?
4. \$14.40 is a certain per cent of \$240. What is the per cent?
5. \$119.70 is what per cent of \$1260?
6. A commission merchant received \$38.27 as commission for selling goods to the amount of \$765.40. What was his rate?
7. It cost \$40 to insure a house for \$3200. What was the rate?
8. By selling goods at a loss of 12% I lost \$26.94. What was the cost of the goods?
9. The interest of \$80 for 2 yr. 6 mo., at a certain rate is \$10. What is the rate?
10. At what rate of interest will \$250 gain \$70 in 4 yr. 8 mo.?
11. What principal will gain \$47 in 1 yr. 6 mo. 24 da., at 4%?
12. My tax is \$51.70. The rate is 14 mills. My poll tax is \$1.50. For what sum am I taxed?
13. A man's tax including the poll tax is \$35.75. He is taxed for \$2250. The rate is 15 mills. How much is the poll tax?
14. The income from a certain investment is \$720. The rate of interest is 4%. How much is the investment?
15. The sum of \$25,000 invested in stock yields a yearly income of \$1125. What rate does the investment pay?

## Problems for Arithmetic or Algebra

1. The area of a parallelogram is 420 sq. in. The perpendicular distance between two parallel sides is 1 ft. 7 in. What is the length of these sides?
2. One half of the base of an isosceles triangle is 16 inches. The area of the triangle is 2 square feet. Find the altitude of the triangle.
3. A farm 120 rods wide contains 300 acres. How long is it?
4. The area of a trapezoid is 108 sq. in. One of the parallel sides is 16 in. and the other 20 in. Find the altitude.
5. A square prism is 13 inches square. It contains 3380 cubic inches. What is its length?
6. The diameter of a cylinder is 10 inches. Its lateral surface is 1570.8 square inches. What is its length?
7. A side of the base of a square pyramid is 1 ft. 8 in. Its lateral surface is 1200 square inches. Find the slant height.
8. The radius of the base of a cone is 4 inches. The lateral surface is 125.664 square inches. Find the slant height.
9. The lateral surface of a regular triangular pyramid is 120 sq. ft. The slant height is 10 ft. Find the length of one side of the base.
10. The lateral surface of a square pyramid is 216 sq. ft. The slant height is 12 ft. Find the length of the base.
11. The slant height of a regular triangular pyramid is 16 ft. The lateral surface is 168 sq. ft. Find the length of one side of the base.

## Least Common Multiple

A **multiple** of a number is a number in which it is contained some number of times without a remainder. 15 is a multiple of 5, because it will contain 5 without a remainder.

1. Give some numbers which are multiples of 3; 7; 12; 17.

2. Of what number is 10 a multiple? 16? 24? 96?

A **common multiple** of several numbers is a number that will contain each of them without a remainder.

3. Give some number which is a common multiple of 2 and 3. Of 2, 3, and 4. Of 3 and 5. Of 2, 4, and 5.

The **least common multiple** of two or more numbers is the least number that will contain each of them without a remainder.

4. What is the least common multiple of 2 and 4? Of 3, 4, and 6? Of 2, 3, and 5?

A number that contains all the factors that occur in several numbers will contain each of the numbers without a remainder.

5. Find the least common multiple of 8, 10, and 20.

$8 = 2 \times 2 \times 2$  A number that is to contain either 8, 12, or  
 $12 = 2 \times 2 \times 3$  20 without a remainder must contain among its  
 $20 = 2 \times 2 \times 5$  factors three 2's in order to contain the number 8.  
It must contain in addition a factor 3 in order to  
contain the number 12, and it must contain a fac-  
tor 5 in order to contain the number 20. Hence the least common  
multiple of the three numbers is  $2 \times 2 \times 2 \times 3 \times 5$  or 120.

*Find the least common multiple of:*

6. 8, 16, 24

10. 27, 30, 42

7. 28, 40, 56

11. 12, 36, 60, 84

8. 15, 20, 60

12. 10, 35, 20, 49

9. 24, 40, 72

13. 16, 24, 80, 112

## The Air

A cubic foot of ordinary air weighs about 1.2 ounces. In ordinary air there are about 4 parts of carbonic acid gas in every 10,000 parts of air. Carbonic acid gas is about one and a half times as heavy as common air. If a certain volume of air weighs 1000 ounces, the same volume of carbonic acid gas would weigh 1525 ounces. From  $\frac{1}{80}$  to  $\frac{1}{200}$  part of the atmosphere is water vapor.

1. Find the weight of the air in a room 30 ft.  $\times$  30 ft.  $\times$  13 ft.
2. If this room were entirely filled with carbonic acid gas, what would be the weight of the gas?
3. If the air of the room is of ordinary purity, find the number of cubic feet of carbonic acid gas contained in it.
4. Find the weight of this amount of the gas.
5. If a man should inhale 35 cubic inches of air at each breath, how many breaths would the air in the room furnish, if none of the air should be inhaled a second time?
6. If he should breathe 18 times a minute, how long would the air last him, if he should inhale none of it a second time?
7. In the "Black Hole" at Calcutta, 146 British prisoners were confined in a dungeon measuring 18 feet each way, and the next morning only 23 of them remained alive. How many cubic feet of air did the dungeon contain?
8. How many single breaths of 35 cubic inches each would this air furnish without being inhaled a second time?
9. If a man breathes 18 times a minute, how many times does he breathe in 24 hours?

## Review Problems

1. A merchant sold 20 yards of cloth for \$15, and by so doing lost  $\frac{1}{4}$  of the cost. What was the cost per yard?

2. I sold a horse for \$45, and lost  $\frac{1}{4}$  of the cost. For what sum should I have sold him to gain  $\frac{1}{5}$  of the cost?

3. 3 boxes are filled with apples. The first contains  $\frac{3}{4}$  as many as the second, and the second  $\frac{1}{2}$  as many as the third. The third contains 24 bushels. How many bushels are there in the first box?

4. If a block of wood floats with one half out of water, how much does it weigh per cubic foot?

See p. 99.

5. If a block of wood floats with one fourth out of water, what is its specific gravity?

6. If a block of wood containing 4 cubic feet floats with one fifth above water, how many pounds of weight will it be necessary to place upon it to sink it?

7. A floating log containing 20 cubic feet floats with one fourth above water. How many pounds will it support without sinking?

8. The specific gravity of a log of wood is .65. How many pounds of extra weight will each cubic foot of the wood support?

9. Find the least common multiple of 27, 35, 42, and 70.

10. Find the l. c. m. of 28, 40, 56, and 60.

Before adding the following fractions, find the least common multiple of the denominators, which is called the least common denominator.

11. Add  $\frac{7}{8}$ ,  $\frac{11}{12}$ ,  $\frac{8}{15}$ .

13. Add  $\frac{1}{5}$ ,  $\frac{19}{40}$ ,  $\frac{31}{80}$ .

12. Add  $\frac{5}{9}$ ,  $\frac{3}{18}$ ,  $\frac{16}{45}$ .

14. Add  $\frac{1}{8}$ ,  $\frac{5}{24}$ ,  $\frac{2}{3}$ .

## Greatest Common Divisor

The **greatest common divisor** of two or more numbers is the largest number that will be contained in all of the numbers without a remainder. 8 is the greatest common divisor of 24 and 32, because it is the largest number that is contained in both 24 and 32 without a remainder.

1. What is the greatest common divisor of 2 and 4? Of 4 and 6? Of 10 and 15?

2. What is the greatest common divisor of 42 and 60?

$$42 = 2 \times 3 \times 7$$

$$60 = 2 \times 2 \times 3 \times 5$$

A number that is to be contained in either of the numbers 42 and 60 without a remainder may contain the factor 2, since this is a factor of each of the numbers. For the same reason it may contain the factor 3. It cannot contain a second 2, a 7, or a 5, since neither of these occurs in both of the numbers. Hence the greatest common divisor of the two numbers is  $2 \times 3$  or 6.

3. Find the smallest factors of 30 and 36.

4. What is the product of all the factors that are common to both of these numbers?

5. Find the greatest common divisor of 48 and 84. Of 66 and 132. Of 147 and 366.

6. Find the greatest common divisor of 60, 84, and 96. Of 28, 56, and 98. Of 140, 168, and 224.

7. What is the longest measure that will exactly measure two lines 56 and 64 inches long?

8. Find the largest number by which both the numerator and the denominator of  $\frac{288}{420}$  may be divided.

*Reduce to smallest terms :*

9.  $\frac{204}{228}$  ;  $\frac{144}{180}$  ;  $\frac{115}{253}$

10.  $\frac{240}{312}$  ;  $\frac{288}{480}$  ;  $\frac{252}{396}$

11.  $\frac{195}{210}$  ;  $\frac{363}{605}$  ;  $\frac{264}{480}$

12.  $\frac{175}{210}$  ;  $\frac{125}{625}$  ;  $\frac{182}{196}$

13.  $\frac{465}{470}$  ;  $\frac{187}{340}$  ;  $\frac{196}{322}$

14.  $\frac{112}{480}$  ;  $\frac{200}{540}$  ;  $\frac{725}{1000}$



### Miscellaneous Problems

1. A man earned \$3 a day and in 60 days earned \$150. How many days was he idle?

2. A man earned \$2 a day, but when idle paid 50 cents a day for his board. In 3 days he saved \$3.50. How many days did he work?

3. A man earned \$4 a day, but when idle paid 50 cents a day for his board. In 6 days he saved \$10.50. How many days did he work?

4. A man owned  $\frac{5}{8}$  of a flock of sheep. He sold  $\frac{3}{8}$  of his share and had 220 sheep left. How many were there in the whole flock?

5. A man owned  $\frac{5}{8}$  of a farm. He sold  $\frac{3}{8}$  of his share for \$1500. What was the value of the farm?

6. What number increased by 10 and diminished by 15 equals 12?

7. What number diminished by 6 and increased by 11 equals 21?

8. What number, increased by 12, diminished by 15, and then increased by 17, equals 75?

9. The two parallel sides of a field in the form of a trapezoid are 80 rd. and 120 rd., and the distance between two sides 30 rd. What is the value of the field at \$54 an acre?

10. One of the two equal sides of a field in the form of a parallelogram is 68 rd., and the perpendicular distance between these sides is 24 rd. How many acres does the field contain?

11. A field in the form of a regular hexagon measures 10 rd. 7 ft. on each side. What is the distance around it?

## Percentage

Review p. 40.

1. What is 1% of 35?
2. If .35 is 1% of 35, what per cent of 35 is 7?
3. What per cent of 275 is 22?
4. \$1.50 is what per cent of \$50?
5. 17 is what per cent of 340?
6. If I buy goods for \$2, and sell them for \$1.80, what per cent of the cost price is the selling price?
7. What per cent of the cost do I lose?
8. If I sell goods for \$62, which cost me \$50, what is the gain per cent?
9. If the whole number of pupils in a school is 542, and 325 are girls, what per cent are girls?
10. What per cent more of girls are there than boys?
11. What per cent less of boys are there than girls?
12. A man paid a premium of \$30 for insuring a house for \$3000. What was the rate?
13. An agent's commission for selling goods to the amount of \$455 was \$22.75. What was his rate?
14. My tax on property valued at \$8300 was \$124.50. What was the rate of taxation?
15. At what rate will \$256.80 gain \$11.556 of interest in a year?
16. In a certain town containing a population of 42,324 people 22,143 are native born. What per cent of the population are native born?
17. If the sum of \$937.50 is gained upon an investment of \$7500, what per cent is gained?

## Bank Discount

*Find the time when the notes become due, the term of discount, the discount, and the proceeds. Do not add days of grace:*

1. \$500. BOSTON, MASS., May 12, 1900.

Three months after date, for value received, I promise to pay Charles Taylor, or order, Five Hundred Dollars, at the Mechanics' National Bank.

PHILIP N. HENDERSON.

Discounted May 25, 1900, at 6%.

2. \$640.50. ST. LOUIS, MO., Sept. 10, 1900.

Sixty days after date, I promise to pay William Bailey, or order, Six Hundred Forty  $\frac{50}{100}$  Dollars, at the First National Bank. Value received.

HENRY R. PARKER.

Discounted Oct. 1, 1900, at 5%.

3. \$262.48. NEW ORLEANS, LA., Jan. 1, 1901.

Four months after date, for value received, I promise to pay F. R. Stoddard & Co. Two Hundred Sixty-two  $\frac{48}{100}$  Dollars, at the Citizens' Bank, with interest at 6%.

ALFRED T. SPENCER.

Discounted Feb. 16, at 5%.

Since the note bears interest, find the discount upon the sum to which it will amount when due.

4. \$750. CHICAGO, ILL., March 10, 1901.

Six months after date, for value received, I promise to pay Charles H. Billings Seven Hundred Fifty Dollars, at the First National Bank, with interest at 5%.

Discounted May 25, at  $5\frac{1}{2}$ %.

## Prisms



See Book II, p. 227.

The edges of a right prism are perpendicular to the surfaces of the ends, or the bases. *The volume of a right prism is equal to the area of one end multiplied by the altitude or height.*

1. What is the volume of a square prism, if the area of one end is 12 square feet and the length is 12 feet ?
2. What is the volume of any right prism whose base contains 15 square inches and whose height is 8 inches ?
3. If the ends of a prism are right triangles, whose two sides near the right angles are 20 inches and 16 inches, what is the area of each triangle ?
4. If the length of the prism is 4 feet, what is the volume of the prism ?
5. Find the volume of a square prism 7 inches square and 20 inches long.
6. The volume of a hexagonal prism is 972 cubic inches. Its length is 1 foot. Find the area of the end.
7. The volume of a pentagonal prism is 2880 cubic inches. The area of the end is 1 square foot. Find the length.
8. The triangle at the end of a triangular prism has a base  $16\frac{1}{2}$  inches long and an altitude of 10 inches. The length of the prism is 3 ft. 4 in. Find its volume.

## Review

1. A merchant insured his goods for \$1200, paying \$40 as premium. What was the rate of the insurance?
2. My property is assessed for \$4200. What tax must I pay, if the rate of taxation is  $1\frac{1}{4}$  per cent?
3. How many tiles 6 in.  $\times$  4 in. will exactly cover a floor 12 ft. 6 in. wide and 16 ft. 4 in. long?
4. How many bricks 8 in.  $\times$  4 in. will exactly cover a yard 45 ft.  $\times$  20 ft.?
5. Find the interest of \$375 from Nov. 12, 1898, to Dec. 30, 1899, at  $5\frac{1}{2}\%$ .
6. Find the amount of a note of \$642.30 dated May 5, 1897, and paid Oct. 12, 1899, with interest at 4%.

*Find the least common multiple of:*

7. 24, 36, 50, 75
8. 60, 90, 100, 270
9. 80, 240, 360, 420

*Add the following:*

10.  $\frac{2}{3}$ ,  $\frac{3}{4}$ ,  $\frac{5}{8}$
11.  $\frac{4}{15}$ ,  $\frac{17}{30}$ ,  $\frac{22}{45}$
12.  $\frac{19}{40}$ ,  $\frac{27}{35}$ ,  $\frac{39}{60}$

*Find the greatest common divisor of:*

13. 36, 60, 96
14. 48, 60, 64

*Reduce to smallest terms:*

15.  $\frac{36}{72}$ ;  $\frac{44}{78}$ ;  $\frac{78}{91}$ ;  $\frac{66}{99}$
16.  $\frac{48}{84}$ ;  $\frac{144}{256}$ ;  $\frac{115}{200}$ ;  $\frac{639}{1737}$

## Easy Methods

Review pp. 63, 106.

*Learn the following table:*

$\frac{1}{2}$ of 100 = 50	$\frac{1}{2} = .50$	$\frac{1}{6}$ of 100 = $16\frac{2}{3}$	$\frac{1}{6} = .16\frac{2}{3}$
$\frac{1}{3}$ of 100 = $33\frac{1}{3}$	$\frac{1}{3} = .33\frac{1}{3}$	$\frac{5}{8}$ of 100 = $83\frac{1}{8}$	$\frac{5}{8} = .83\frac{1}{8}$
$\frac{2}{3}$ of 100 = $66\frac{2}{3}$	$\frac{2}{3} = .66\frac{2}{3}$	$\frac{1}{7}$ of 100 = $14\frac{2}{7}$	$\frac{1}{7} = .14\frac{2}{7}$
$\frac{1}{4}$ of 100 = 25	$\frac{1}{4} = .25$	$\frac{1}{8}$ of 100 = $12\frac{1}{2}$	$\frac{1}{8} = .12\frac{1}{2}$
$\frac{3}{4}$ of 100 = 75	$\frac{3}{4} = .75$	$\frac{3}{8}$ of 100 = $37\frac{1}{2}$	$\frac{3}{8} = .37\frac{1}{2}$
$\frac{1}{5}$ of 100 = 20	$\frac{1}{5} = .20$	$\frac{5}{8}$ of 100 = $62\frac{1}{2}$	$\frac{5}{8} = .62\frac{1}{2}$
$\frac{3}{5}$ of 100 = 60	$\frac{3}{5} = .60$	$\frac{7}{8}$ of 100 = $87\frac{1}{2}$	$\frac{7}{8} = .87\frac{1}{2}$
$\frac{4}{5}$ of 100 = 80	$\frac{4}{5} = .80$	$\frac{1}{9}$ of 100 = $11\frac{1}{9}$	$\frac{1}{9} = .11\frac{1}{9}$

*Work the following examples mentally as far as possible:*

- |                                |                                  |
|--------------------------------|----------------------------------|
| 1. $500 \times 12\frac{1}{2}$  | 17. $128 \times 62\frac{1}{2}$   |
| 2. $384 \times 16\frac{2}{3}$  | 18. $640 \times 87\frac{1}{2}$   |
| 3. $840 \times 25$             | 19. $720 \times 83\frac{1}{3}$   |
| 4. $969 \times 33\frac{1}{3}$  | 20. $400 \times 75$              |
| 5. $786 \times 50$             | 21. $8000 \times .12\frac{1}{2}$ |
| 6. $480 \div 12\frac{1}{2}$    | 22. $6300 \times .14\frac{2}{7}$ |
| 7. $650 \div 16\frac{2}{3}$    | 23. $2646 \times .16\frac{2}{3}$ |
| 8. $800 \div 25$               | 24. $8280 \times .11\frac{1}{9}$ |
| 9. $750 \div 33\frac{1}{3}$    | 25. $3755 \times .20$            |
| 10. $765 \div 50$              | 26. $5872 \times .25$            |
| 11. $450 \times 11\frac{1}{9}$ | 27. $7545 \times .33\frac{1}{3}$ |
| 12. $560 \times 14\frac{2}{7}$ | 28. $2482 \times .50$            |
| 13. $720 \div 11\frac{1}{9}$   | 29. $4800 \times 62\frac{1}{2}$  |
| 14. $870 \div 14\frac{2}{7}$   | 30. $7500 \times 66\frac{2}{3}$  |
| 15. $360 \times 66\frac{2}{3}$ | 31. $3648 \times .75$            |
| 16. $400 \times 37\frac{1}{2}$ | 32. $4200 \times 83\frac{1}{3}$  |

## Review Problems

1. If a barrel of apples containing  $2\frac{1}{2}$  bushels is bought for \$2.25, and sold at 40 cents a peck, what is the gain?

2. What is the profit upon 100 bushels of potatoes bought at 60 cents a bushel, and sold at \$1 a bushel, if they shrink 10% in measure before they are sold?

3. What would be the profit upon 50 boxes of oranges bought at \$2.00 each, and sold at \$2.75 each, if 20% are found spoiled?

4. A steamer, having engines of 1000 horse power, is to go on a 10 days' voyage. How many cubic feet of coal must it take, if the rate of consumption of coal is 7 pounds per horse power per hour, and 40 cubic feet of coal weigh a ton?

5. If granite weighs 160 pounds per cubic foot, how many tons of paving stones 12 in. long, 4 in. wide, and 9 in. deep, will be required to pave a street 50 ft. wide and 400 ft. long, if  $\frac{1}{10}$  of the area is deducted for the space between the blocks?

6. What is the entire surface of a square prism, the perimeter of whose base is 36 in., and whose altitude is 22 in.?

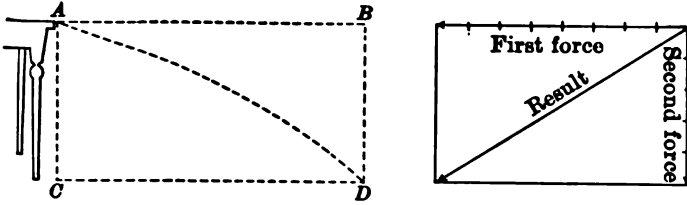
7. What is the volume of a square prism, the perimeter of whose base is 28 ft., and whose altitude is 43 ft.?

8. What is the lateral surface of a regular hexagonal prism, one side of whose base is 10 in., and whose altitude is  $12\frac{1}{2}$  in.?

9. Find the lateral surface of a pentagonal pyramid, one side of whose base is  $8\frac{1}{2}$  inches, and whose slant height is  $17\frac{1}{4}$  inches.

10. Find the lateral surface of a hexagonal pyramid, one side of whose base is 1 ft. 8 in., and whose slant height is 5 ft. 4 in.

## Forces



When two or more forces act upon an object at the same time, each force produces the same effect as if it acted alone. If a rolling ball falls from a table, it will move as far from the table in falling as it would have rolled in the same time. The momentum of the ball and the force of gravitation act independently. If two forces act upon an object in different directions, the object will move in the direction of the diagonal of a parallelogram formed with the lengths of the sides representing the forces.

See p. 119.

1. How far will a body fall in 1 second? In 2 seconds? In 3 seconds?
2. A ball is thrown in a horizontal direction from a point 16 feet above the ground. How long will it be before it will reach the ground?
3. If it first strikes the ground at a point 60 feet from the starting point, how swiftly was it thrown?
4. A shot is discharged from a fort in a horizontal direction at a point 64 feet above the water. In how many seconds will it strike the water?
5. A ball is struck with a force sufficient to send it north 12 yards in a second, and at the same instant with another force sufficient to send it east 9 yards in a second. How far will it go the first second?

Find the length of the hypotenuse of the right triangle.



## Review

1. There are 231 cubic inches in a gallon. A cubic foot of water weighs 1000 ounces. Find the weight of a gallon of water.

2. If the specific gravity of milk is 1.03, find the weight of a gallon of milk.

3. If a bottle holds 32 ounces of water, how many ounces of oil will it hold, if the specific gravity of the oil is .954?

4. The specific gravity of mercury is 13.59. How many ounces of mercury would the same bottle hold?

5. How many cubic feet of water will be contained in a tank in the form a square prism, whose end is 40 inches square and whose length is 12 feet?

6. Find how many pounds of water the tank would hold.

7. A cubic foot of ice weighs 58 pounds. How many pounds of solid ice would the tank hold?

8. If the tank full of water should be all frozen, how many cubic feet of ice would be raised above the level of the tank?

How many cubic feet of ice would the water produce?

*Find the greatest common measure of:*

9. 24, 30, 36

11. 36, 81, 135

10. 42, 84, 126

12. 51, 105, 243

13. Reduce to smallest terms  $\frac{60}{150}$ ;  $\frac{36}{132}$ ;  $\frac{75}{450}$ ;  $\frac{144}{720}$ .

14. Find the least common multiple of 12, 15, and 20.

15. Find the least number of acres in a farm that can be divided exactly into fields of either 12, 18, or 24 acres.

16. What is the shortest distance that can be exactly measured by either a 3 ft. stick, a 5 ft. stick, or a 10 ft. stick?

## Construction

See pp. 102, 123, and Book II, p. 199.

1. Construct a trapezoid with one parallel side  $3\frac{1}{4}$  inches long, the other  $2\frac{3}{4}$  inches long, and containing an angle of  $75^\circ$ .

2. Construct a right triangle with the sides near the right angle 4 inches and  $2\frac{1}{4}$  inches. Regarding this triangle as one half of a rectangle, complete the rectangle.

3. Construct a triangle with two sides  $3\frac{1}{2}$  inches and  $2\frac{3}{4}$  inches, and an angle of  $118^\circ$ . Draw a dotted line which measures the altitude of the triangle. Find the area of the triangle. Regarding the triangle as a part of a parallelogram, complete the parallelogram.

4. Construct a regular pentagon with a side  $2\frac{1}{2}$  inches long. Find the center, and draw lines from the center to the vertices of the angles. Measure the parts of the triangles. Which triangles are equal to each other? What kind of triangles are formed? From the center draw a dotted line to the middle point of one side. Find the area of the triangle. Find the area of the pentagon.

5. Draw a line 2 inches long. At one end make an angle of  $120^\circ$ , and make the second side 2 inches long. Make another angle of  $120^\circ$  on the same side as before, and continue till the starting point is reached. What figure has been formed? From the vertex of one angle draw lines dividing the hexagon into triangles. Measure the parts, and find which triangles are equal to each other.

6. Construct a regular hexagon with a side  $1\frac{3}{4}$  inches long. Draw lines connecting the vertices of angles which are directly opposite to each other. What kind of triangles have been formed? Measure the altitude of one triangle, and find the area of all the triangles.

## Original Problems

*Make problems and solve them :*

1. A square contains 144 square feet.
2. The length of an oblong is four times its width. It contains 144 square feet.
3. The sum of \$2000 is loaned at 4%.
4. A block of wood floats with  $\frac{1}{5}$  out of water.
5. A man owned  $\frac{4}{5}$  of a flock of sheep. He bought  $\frac{1}{2}$  of the other part, and then had 90 sheep.
6. The two parallel sides of a field in the form of a trapezoid are 60 rods and 90 rods.
7. The whole number of pupils in a certain school is 445, and 210 are boys.
8. A square prism is 4 ft. 6 in. square and 8 ft. long.
9. A regular triangular prism is 35 inches long, and each lateral face is 10 inches wide.
10. A hexagonal prism is 4 feet long, and each side of the base is 6 inches.
11. I insured a house at  $1\frac{1}{2}\%$  on  $\frac{2}{3}$  of its value, and the premium amounted to \$30.
12. A pile of bricks is 10 ft. long, 4 ft. wide, and 6 ft. high.
13. The engines of a certain steamer have 600 horse power. The rate of consumption of coal is 8 pounds per horse power per hour.
14. A tank 2 feet long and 8 feet wide holds 9 gallons of water.
15. A cubic foot of ice weighs 58 pounds.
16. An ice cart contains 40 blocks of ice, each  $30\text{ in.} \times 24\text{ in.} \times 12\text{ in.}$

## Algebra

Quantities preceded by the plus sign are called **positive quantities**. Those preceded by the minus sign are called **negative quantities**.

*Multiply :*

1.

$$\begin{array}{r} 7 - 3 \\ \underline{\quad} \\ 14 - 6 \end{array}$$

2.

$$\begin{array}{r} 6 - 2 \\ \underline{5 - 3} \\ 30 - 10 \\ \quad - 18 + 6 \\ \hline 30 - 28 + 6 \end{array}$$

3.

$$\begin{array}{r} x - y \\ \underline{a - b} \\ ax - ay \\ \quad - bx + by \\ \hline ax - ay - bx + by \end{array}$$

In No. 1 we first have  $2 \times 7$ , but since we wish only 2 times 3 less than 7, or  $2 \times 4$ , we must subtract  $2 \times 3$ . In No. 2 we first multiply  $6 - 2$  by 5 as in No. 1 without regard to the term  $- 3$ . Now since we do not wish to find 5 times the quantity  $6 - 2$ , but twice that quantity, we must subtract 3 times the quantity. We first subtract  $3 \times 6$  or 18, but, since the multiplicand is 2 less than 6, we have subtracted too much and must now add  $3 \times 2$  to correct this.

Notice that in multiplication correct results will be obtained by observing the rule: "*Like signs produce plus and unlike signs produce minus.*"

4.

$$\begin{array}{r} a + b \\ \underline{\quad} \\ a \end{array}$$

5.

$$\begin{array}{r} a - b \\ \underline{\quad} \\ b \end{array}$$

6.

$$\begin{array}{r} a - b \\ \underline{\quad} \\ a + b \end{array}$$

7.

$$\begin{array}{r} a - b \\ \underline{\quad} \\ a - b \end{array}$$

8.

$$\begin{array}{r} 2ab - 5ab^2c \\ \underline{\quad} \\ b + c \end{array}$$

9.

$$\begin{array}{r} 2m^2 - 5mn - 8 \\ \underline{\quad} \\ 3m - 4 \end{array}$$

10.

$$\begin{array}{r} x^2 - 2xy + y^2 \\ \underline{\quad} \\ x - y \end{array}$$

11. Multiply  $6a + 5$  by  $5a - 8$ .
12. Multiply  $a + b - 1$  by  $a - b + 1$ .
13. Multiply  $4x + 6y - 7$  by  $x + 4y$ .

## Algebra

See p. 118.

1. Express the product of 2, 3, 5,  $a$ ,  $b$ ,  $b$ ,  $b$ , and  $c$ .
2. What are the smallest factors of  $36 a^2 b^3 c$ ?
3. What are the smallest factors of  $75 xy^2 z^5$ ?
4. Find the smallest factors of  $4 a^4 b^2$  and divide these factors into two equal groups.
5. Find the product of one of the two equal groups of factors of  $16 m^6 n^4$ .
6. What two equal quantities will produce  $100 x^2 y^4$  when multiplied together?
7. What is the square of  $5 a^2 b$ ?
8. What is the square root of  $81 a^4 b^8 c^2$ ?
9. Find the square root of  $324 x^4 y^6$ .
10. Find the square root of  $5625 m^6 n^{10}$ .
11. Find the smallest factors of  $a^3 b^6$ , and divide these into three equal groups.
12. Find one of the three equal factors of  $27 x^6 y^9$ .
13. What is the cube of  $3 a^2 b^3$ ?
14. Find the cube root of  $64 a^6 b^9$ .
15. Find the cube root of  $216 m^9 n^{12}$ .
16. If  $x^2 = 16$ , what is the value of  $x$ ?

*Find the value of  $x$  in these equations :*

17.  $x^2 = 30 - 5$

18.  $x^2 + 6 = 61 - 6$

19.  $2x^2 - 5 = 28 - 15$

First find the value of  $2x^2$ ; then of  $x^2$ ; then of  $x$ .

20.  $5x^2 = 48 + 2x^2$

### Problems for Arithmetic or Algebra

1. Annie is twice as old as Margaret, and the sum of their ages is 21 years. How old is each?
2. The sum of two numbers is 20, and the greater is 3 times the less. What are the numbers?
3. The difference of two numbers is 4, and the greater is twice the less. What are the numbers?
4. If 15 is added to twice a number, the result will be  $4\frac{1}{2}$  times the number. What is the number?
5. If 24 is taken from 5 times a number, the remainder will be twice the number. What is the number?
6. The sum of two numbers is 29, and the larger is 5 more than 3 times the smaller. What are the numbers?
7. The sum of two numbers is 18, and the smaller is 3 more than half the larger. What are the numbers?
8.  $\frac{1}{2}$  and  $\frac{1}{3}$  of a sum of money is \$12 less than the whole sum. What is the sum?
9. When  $\frac{3}{4}$  of a number is taken from  $\frac{5}{8}$  of the number, the remainder is 2. Find the number.
10.  $\frac{2}{3}$  of a number added to  $\frac{5}{8}$  of it is 8 more than the number. Find the number.
11. If a certain number is increased by 9, and then diminished by 15, the result will be 10. Find the number.
12. If 12 is subtracted from a certain number and 8 is added to the remainder, the result will be 16. Find the number.
13. A man having a certain sum of money spent \$5 and earned \$8. He then had \$23. How much had he at first?
14. If  $3\frac{2}{3}$  times a number is taken from  $5\frac{5}{8}$  times the number, the remainder will be 39. What is the number?

## Problems for Arithmetic or Algebra

See p. 94.

1. What principal will gain \$31.50 interest in 4 yr. 8 mo. at  $4\frac{1}{2}\%$ ?
2. In what time will \$220 gain \$2.42 at 6%?
3. At what rate will \$560 gain \$56 in 2 yr. 6 mo.?
4. What principal will amount to \$69.60 in 3 yr. 2 mo. 12 da., with interest at 5%?

Let  $x$  = principal. Amount of \$1 = \$1.16. Amount of  $x$  dollars =  $1.16x$ ;  $1.16x = \$69.60$ . Find  $x$ .

5. What principal will amount to \$244.80 in 3 yr. 2 mo. 12 da. at 7%?
6. What principal will amount to \$140.75 in 2 yr. 1 mo. 6 da. at 6%?
7. What principal will gain \$180 interest in 1 yr. 3 mo. at 6%?
8. In what time will \$2000 gain \$180 at 6%?
9. What principal will amount to \$21,600 in 1 yr. 4 mo. at 4%?
10. In what time will \$5610 gain \$420.75 at 5%?
11. At what rate will \$5610 gain \$420.75 in 1 yr. 6 mo.?
12. What principal will amount to \$3886.89 in 3 yr. 6 mo. at 7%?
13. At what rate will a principal of \$3122 gain \$764.89 in 3 yr. 6 mo.?
14. A square contains 169 square inches. What is its length?
15. The length of a rectangle is 4 times its width. It contains 196 square inches. How wide is it?

## SUMMARY OF PROCESSES WITH FRACTIONS

### Reduction of Fractions

*To change a mixed number to an improper fraction :*

1. Change  $12\frac{7}{8}$  to an improper fraction.

$12\frac{7}{8} = 1\frac{1}{2}$ . In 1 whole there are 8 eighths. In 12 wholes there are 12 times 8 eighths, or 96 eighths. 96 eighths and 7 eighths are 103 eighths.

Change to improper fractions:  $17\frac{2}{3}$ ;  $25\frac{5}{8}$ ;  $48\frac{4}{5}$ ;  $87\frac{1}{2}$ ;  $65\frac{1}{13}$ ;  $92\frac{2}{15}$ ;  $107\frac{2}{16}$ ;  $123\frac{7}{18}$ .

*To change an improper fraction to a mixed number :*

2. Change  $\frac{77}{9}$  to a mixed number.

$\frac{77}{9} = 8\frac{5}{9}$ . There are 9 ninths in 1 whole. In 77 ninths there are as many wholes as 9 is contained times in 77, or 8 wholes, with 5 ninths remaining.

Change to mixed numbers:  $\frac{27}{6}$ ;  $\frac{25}{13}$ ;  $\frac{79}{16}$ ;  $\frac{120}{17}$ ;  $\frac{135}{24}$ ;  $\frac{187}{31}$ ;  $\frac{249}{42}$ ;  $\frac{587}{53}$ .

*To change a fraction to lower or higher terms :*

3. Change  $\frac{2}{3}$  to lower terms. To higher terms.

$\frac{2}{3} \div \frac{1}{3} = \frac{2}{1}$ ;  $\frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$ . In either dividing or multiplying both the numerator and the denominator by the same number the value of the fraction is not changed.

Change to lower terms:  $\frac{12}{18}$ ;  $\frac{15}{25}$ ;  $\frac{45}{90}$ ;  $\frac{36}{126}$ ;  $\frac{50}{85}$ ;  $\frac{26}{120}$ ;  $\frac{100}{150}$ ;  $\frac{160}{320}$ .

*To change a fraction to its lowest terms :*

4. Change  $\frac{24}{30}$  to its lowest terms.

$$\frac{24}{30} \div \frac{6}{6} = \frac{4}{5}; \frac{24}{30} \div \frac{3}{3} = \frac{8}{10}; \text{ or } \frac{24}{30} \div \frac{2}{2} = \frac{12}{15}.$$

Either divide both the numerator and the denominator by some number that is contained in both without a remainder, and then again by some other number, and so on; or divide both the numerator and the denominator by their greatest common divisor.

Change to their lowest terms:  $\frac{75}{150}$ ;  $\frac{36}{108}$ ;  $\frac{128}{144}$ ;  $\frac{216}{252}$ ;  $\frac{240}{336}$ ;  $\frac{260}{480}$ ;  $\frac{144}{432}$ ;  $\frac{482}{880}$ .



## Reduction of Fractions

*To change fractions to the same denominator :*

1. Change  $\frac{1}{2}$ ,  $\frac{2}{3}$ , and  $\frac{5}{6}$  to 18ths.

$$\frac{1}{2} \times \frac{9}{9} = \frac{9}{18}, \quad \frac{2}{3} \times \frac{6}{6} = \frac{12}{18}, \quad \frac{5}{6} \times \frac{3}{3} = \frac{15}{18}.$$

To change  $\frac{1}{2}$  to 18ths we must multiply both numerator and denominator by 9, since 2 is contained in 18 9 times. To change  $\frac{2}{3}$  to 18ths we must multiply both numerator and denominator by 6, and to change  $\frac{5}{6}$  to 18ths we must multiply both terms by 3.

$\frac{1}{2}$ ,  $\frac{2}{3}$ , and  $\frac{5}{6}$  are the same as  $\frac{9}{18}$ ,  $\frac{12}{18}$ , and  $\frac{15}{18}$ .

When several fractions have the same denominator they are called **similar fractions**.

*Change to the same denominator :*

- |  |  |
|--|--|
| 2. $\frac{3}{4}, \frac{5}{6}, \frac{1}{2}, \frac{2}{3}$    | 7. $\frac{1}{3}, \frac{2}{5}, \frac{11}{15}, \frac{3}{8}$      |
| 3. $\frac{2}{5}, \frac{1}{6}, \frac{7}{15}, \frac{1}{2}$   | 8. $\frac{7}{8}, \frac{1}{3}, \frac{7}{16}, \frac{19}{24}$     |
| 4. $\frac{5}{9}, \frac{5}{6}, \frac{1}{4}, \frac{11}{12}$  | 9. $\frac{1}{20}, \frac{7}{30}, \frac{1}{2}, \frac{3}{4}$      |
| 5. $\frac{1}{10}, \frac{2}{5}, \frac{5}{8}, \frac{19}{20}$ | 10. $\frac{5}{9}, \frac{7}{8}, \frac{5}{12}, \frac{1}{36}$     |
| 6. $\frac{5}{7}, \frac{1}{6}, \frac{2}{3}, \frac{4}{12}$   | 11. $\frac{19}{25}, \frac{7}{20}, \frac{13}{50}, \frac{9}{10}$ |

*To change fractions to the least common denominator :*

12. Change  $\frac{3}{4}$  and  $\frac{5}{6}$  to the least common denominator.

$\frac{3}{4}$  and  $\frac{5}{6}$  are the same as  $\frac{3}{4}$  and  $\frac{5}{6}$ , but they are also the same as  $\frac{9}{12}$  and  $\frac{10}{12}$ . In the former case the fractions have a common denominator, but in the latter case they have the least denominator that they can have in common. This least denominator is the least common multiple of the denominators.

*Find the least common multiple of the denominators and make the fractions similar :*

- |  |   |
|--|---|
| 13. $\frac{3}{8}, \frac{7}{48}, \frac{9}{16}, \frac{13}{24}$   | 17. $\frac{15}{6}, \frac{6}{7}, \frac{5}{8}, \frac{3}{16}$    |
| 14. $\frac{3}{5}, \frac{5}{9}, \frac{11}{18}, \frac{7}{15}$    | 18. $\frac{5}{26}, \frac{3}{20}, \frac{1}{13}, \frac{1}{7}$   |
| 15. $\frac{5}{14}, \frac{1}{3}, \frac{17}{21}, \frac{11}{12}$  | 19. $\frac{9}{10}, \frac{3}{40}, \frac{7}{25}, \frac{29}{30}$ |
| 16. $\frac{7}{24}, \frac{5}{12}, \frac{25}{28}, \frac{17}{18}$ | 20. $\frac{3}{16}, \frac{9}{32}, \frac{10}{24}, \frac{7}{36}$ |

## Addition and Subtraction of Fractions

Before fractions can be added or subtracted they must be changed to the same denomination.

1. Add  $\frac{5}{12}$ ,  $\frac{7}{15}$ , and  $\frac{8}{20}$ .

$$\frac{5}{12} + \frac{7}{15} + \frac{8}{20} = \frac{25}{60} + \frac{28}{60} + \frac{24}{60} = \frac{77}{60}, \text{ or } 1\frac{17}{60}.$$

Make the fractions similar and add the numerators, writing the sum over the common denominator.

2. Subtract  $\frac{2}{3}$  from  $\frac{16}{17}$ .

$$\frac{16}{17} - \frac{2}{3} = \frac{48}{51} - \frac{34}{51} = \frac{14}{51}.$$

After making the fractions similar, subtract the numerator of the subtrahend from that of the minuend and write the result over the common denominator.

- |  |   |
|--|---|
| <p>3. <math>\frac{5}{8} + \frac{7}{15} + \frac{11}{20} = ?</math></p> <p>4. <math>\frac{3}{8} + \frac{7}{9} + \frac{13}{24} = ?</math></p> <p>5. <math>\frac{6}{7} + \frac{1}{14} + \frac{5}{16} = ?</math></p> <p>6. <math>\frac{3}{7} + \frac{5}{28} + \frac{4}{21} = ?</math></p> <p>7. <math>\frac{5}{9} + \frac{3}{4} + \frac{5}{18} = ?</math></p> <p>8. <math>\frac{13}{24} + \frac{7}{16} - \frac{5}{12} = ?</math></p> <p>9. <math>\frac{11}{15} + \frac{13}{20} - \frac{5}{28} = ?</math></p> <p>10. <math>\frac{3}{8} - \frac{1}{14} + \frac{7}{24} = ?</math></p> <p>11. <math>\frac{7}{9} + \frac{17}{36} - \frac{10}{24} = ?</math></p> <p>12. <math>\frac{14}{15} + \frac{1}{18} + \frac{7}{30} = ?</math></p> <p>13. <math>\frac{5}{21} + \frac{13}{38} + \frac{10}{49} = ?</math></p> <p>14. <math>\frac{17}{30} + \frac{23}{35} + \frac{37}{48} = ?</math></p> <p>15. <math>\frac{23}{42} + \frac{33}{56} + \frac{51}{72} = ?</math></p> <p>16. <math>\frac{10}{36} + \frac{23}{42} + \frac{25}{48} = ?</math></p> | <p>17. <math>7\frac{2}{13} + 12\frac{8}{11} = ?</math></p> <p>18. <math>5\frac{3}{17} - 2\frac{5}{16} = ?</math></p> <p>19. <math>16\frac{3}{15} + 15\frac{3}{4} = ?</math></p> <p>20. <math>14\frac{1}{4} + 12\frac{3}{8} - 3\frac{1}{2} = ?</math></p> <p>21. <math>25\frac{17}{42} - 1\frac{1}{15} = ?</math></p> <p>22. <math>16\frac{1}{8} + 9\frac{1}{5} = ?</math></p> <p>23. <math>42\frac{1}{3} - 19\frac{0}{10} = ?</math></p> <p>24. <math>82\frac{1}{4} + 63\frac{11}{15} = ?</math></p> <p>25. <math>58\frac{1}{3} - 26\frac{7}{5} = ?</math></p> <p>26. <math>\frac{1}{10} + \frac{1}{5} + \frac{3}{8} = ?</math></p> <p>27. <math>\frac{1}{17} + \frac{3}{28} = ?</math></p> <p>28. <math>\frac{2}{7} - \frac{1}{4} = ?</math></p> <p>29. <math>\frac{1}{9} + \frac{2}{7} = ?</math></p> <p>30. <math>\frac{2}{4} - \frac{2}{6} = ?</math></p> |
|--|---|

## Multiplication of Fractions

*To multiply a fraction by a whole number:*

1. Multiply  $\frac{3}{8}$  by 4.

$\frac{3}{8} \times 4$  or  $4 \times \frac{3}{8} = 1\frac{3}{2}$  or  $1\frac{1}{2}$ . Since the numerator indicates the number of parts, to multiply the fraction is to multiply the numerator.

*To multiply a mixed number by a whole number:*

2. Multiply  $21\frac{3}{8}$  by 4.

$$21\frac{3}{8}$$

$$\frac{4}{4}$$

$$\frac{84}{1}$$

$$\frac{12}{8}$$

$$\frac{85\frac{1}{2}}{1}$$

Multiply the two parts separately, and add the products.

*To multiply a fraction by a fraction:*

3. Multiply  $\frac{7}{10}$  by  $\frac{2}{3}$ .

If we multiply  $\frac{7}{10}$  by 2 we have  $\frac{14}{10}$ ; but we wish to multiply by only one third of 2, hence we find one third of this by multiplying the denominator by 3. Therefore we multiply the denominators together and the numerators together.

4.  $\frac{2}{8} \times 7 = ?$

5.  $9 \times \frac{5}{6} = ?$

6.  $\frac{7}{8} \times 12 = ?$

7.  $15 \times \frac{3}{4} = ?$

8.  $\frac{3}{8}$  of 18 = ?

9.  $5\frac{1}{2} \times 6 = ?$

10.  $23\frac{2}{5} \times 7 = ?$

11.  $9 \times 34\frac{7}{8} = ?$

12.  $15 \times 17\frac{1}{5} = ?$

13.  $25 \times 40\frac{2}{10} = ?$

14.  $\frac{3}{8} \times \frac{5}{8} = ?$

15.  $\frac{9}{10} \times \frac{4}{7} = ?$

16.  $\frac{15}{16} \times \frac{5}{9} = ?$

17.  $\frac{7}{12} \times \frac{35}{41} = ?$

18.  $\frac{7}{18} \times 16 = ?$

19.  $\frac{12}{20}$  of 300 = ?

20.  $45\frac{6}{7} \times 14 = ?$

21.  $32 \times 71\frac{1}{2} = ?$

22.  $\frac{17}{10} \times \frac{40}{9} = ?$

23.  $\frac{37}{80} \times \frac{30}{31} = ?$

## Multiplication of Fractions

*To multiply a mixed number by a fraction or a mixed number:*

1. Multiply  $24\frac{1}{2}$  by  $\frac{3}{4}$ ;  $12\frac{1}{2}$  by  $8\frac{2}{3}$ ;  $5\frac{1}{2}$  by  $4\frac{1}{5}$ .

(a)  $24\frac{1}{2}$

$$\frac{\frac{3}{4}}{18}$$

$$\frac{\frac{3}{4}}{18\frac{1}{2}}$$

(b)  $12\frac{1}{2}$

$$\frac{8\frac{2}{3}}{96}$$

$$\frac{4}{8} \\ \frac{1}{2} \\ \hline 108\frac{1}{2}$$

(c)  $5\frac{1}{2} \times 4\frac{1}{5} = \frac{11}{2} \times \frac{21}{5} = \frac{231}{10}$  or  $23\frac{1}{10}$

When the numbers are of such a nature that there will not be many fractions in the parts of the result, it is better to adopt the direct method, (a) and (b). In other cases the mixed numbers may be changed to the form of fractions (c).

- |   |  |
|---|--|
| 2. $10\frac{1}{2} \times 1\frac{1}{2} = ?$    | 19. $7\frac{1}{2} \times 9\frac{2}{3} = ?$   |
| 3. $\frac{2}{3} \times 20\frac{5}{8} = ?$     | 20. $12\frac{3}{5} \times \frac{1}{9} = ?$   |
| 4. $15\frac{3}{5} \times \frac{1}{8} = ?$     | 21. $\frac{6}{7} \times 1\frac{1}{2} = ?$    |
| 5. $12\frac{3}{5} \times \frac{2}{4} = ?$     | 22. $2\frac{1}{4} \times 3\frac{1}{5} = ?$   |
| 6. $\frac{6}{7} \times 28\frac{7}{8} = ?$     | 23. $\frac{2}{3} \times 80\frac{1}{2} = ?$   |
| 7. $24\frac{2}{3} \times 12\frac{1}{2} = ?$   | 24. $21\frac{6}{7} \times \frac{2}{3} = ?$   |
| 8. $16\frac{4}{7} \times 14\frac{2}{3} = ?$   | 25. $30\frac{5}{8} \times 12\frac{2}{3} = ?$ |
| 9. $27\frac{9}{10} \times 20\frac{8}{9} = ?$  | 26. $36\frac{1}{4} \times 16\frac{3}{8} = ?$ |
| 10. $35\frac{5}{8} \times 40\frac{3}{8} = ?$  | 27. $9\frac{2}{5} \times 4\frac{1}{7} = ?$   |
| 11. $\frac{1}{5} \times 17\frac{4}{11} = ?$   | 28. $\frac{1}{6}$ of $32\frac{1}{3} = ?$     |
| 12. $46\frac{1}{2} \times \frac{1}{2} = ?$    | 29. $\frac{7}{20}$ of $60\frac{1}{7} = ?$    |
| 13. $50\frac{4}{11} \times 4\frac{9}{10} = ?$ | 30. $42\frac{5}{8} \times 36\frac{2}{3} = ?$ |
| 14. $\frac{1}{8} \times 33\frac{4}{7} = ?$    | 31. $17\frac{3}{4} \times 19\frac{2}{7} = ?$ |
| 15. $\frac{1}{12} \times 48\frac{10}{11} = ?$ | 32. $\frac{9}{10} \times 84\frac{1}{6} = ?$  |
| 16. $69\frac{1}{5} \times \frac{1}{5} = ?$    | 33. $\frac{2}{7}$ of $74\frac{1}{9} = ?$     |
| 17. $3\frac{7}{8} \times 5\frac{2}{15} = ?$   | 34. $\frac{5}{8}$ of $80\frac{1}{2} = ?$     |
| 18. $\frac{4}{5} \times 38\frac{5}{8} = ?$    | 35. $98\frac{1}{3} \times 92\frac{5}{8} = ?$ |

## Division of Fractions

*To divide a fraction or a mixed number by a whole number :*

1. Divide  $\frac{8}{9}$  by 4.  $15\frac{6}{7}$  by 3.  $\frac{2}{3}$  by 5.

When the numerator of the fraction will contain the divisor an even number of times, simply divide the numerator, as in (a).

(a)

$$\frac{8}{9} \div 4 = \frac{2}{9}, \text{ or}$$

$$\frac{1}{4} \text{ of } \frac{8}{9} = \frac{2}{9}$$

When each part of the mixed number will contain the divisor an even number of times, divide each separately, as in (b).

(b)

$$15\frac{6}{7} \div 3 = 5\frac{2}{7}, \text{ or}$$

$$\frac{1}{3} \text{ of } 15\frac{6}{7} = 5\frac{2}{7}$$

When the numerator will not contain the divisor an even number of times, multiply the denominator by the divisor, as in (c).  $\frac{1}{3}$  of  $\frac{2}{3}$  is  $\frac{2}{15}$ . In this case we have the same number of parts in the quotient as in the dividend, but each part is only one fifth as large. When necessary, change mixed numbers to improper fractions.

(c)

$$\frac{2}{3} \div 5 = \frac{2}{15}, \text{ or}$$

$$\frac{1}{5} \text{ of } \frac{2}{3} = \frac{2}{15}$$

2.  $\frac{6}{7} \div 3 = ?$

3.  $\frac{8}{9} \div 2 = ?$

4.  $1\frac{5}{6} \div 5 = ?$

5.  $2\frac{8}{9} \div 2 = ?$

6.  $12\frac{2}{3} \div 2 = ?$

7.  $24\frac{1}{5} \div 4 = ?$

8.  $10\frac{5}{8} \div 5 = ?$

9.  $181\frac{2}{3} \div 6 = ?$

10.  $\frac{3}{4} \div 5 = ?$

11.  $\frac{1}{7} \div 4 = ?$

12.  $1\frac{1}{2} \div 5 = ?$

13.  $1\frac{1}{2} \div 6 = ?$

14.  $3\frac{1}{7} \div 12 = ?$

15.  $42\frac{6}{7} \div 6 = ?$

16.  $\frac{4}{5} \div 7 = ?$

17.  $\frac{9}{10} \div 2 = ?$

18.  $2\frac{1}{2} \div 7 = ?$

19.  $35\frac{5}{8} \div 5 = ?$

20.  $12\frac{1}{2} \div 3 = ?$

21.  $\frac{3}{4} \div 12 = ?$

22.  $\frac{4}{7} \div 14 = ?$

23.  $\frac{9}{16} \div 10 = ?$

24.  $36\frac{8}{9} \div 4 = ?$

25.  $1\frac{8}{3} \div 7 = ?$

26.  $95\frac{10}{11} \div 5 = ?$

27.  $1\frac{7}{10} \div 4 = ?$

28.  $\frac{6}{7} \div 16 = ?$

29.  $804\frac{3}{4} \div 40 = ?$

## Division of Fractions

*To divide a whole number, a fraction, or a mixed number, by a fraction or a mixed number :*

1. Divide 4 by  $\frac{1}{2}$ ; 6 by  $1\frac{1}{2}$ ;  $\frac{4}{5}$  by  $\frac{2}{5}$ ;  $6\frac{1}{3}$  by  $\frac{1}{3}$ ;  $17\frac{1}{2}$  by  $2\frac{1}{2}$ .

### FIRST METHOD

- (a)  $4 \div \frac{1}{2} = 8$       When it may be readily seen by inspection how many times the divisor is contained in the dividend, no further process is necessary.  $\frac{1}{2}$  is contained in 4 twice and in  $4$  eight times (a).  $1\frac{1}{2}$  is contained in 6 four times (b).  
 (b)  $6 \div 1\frac{1}{2} = 4$   
 (c)  $\frac{4}{5} \div \frac{2}{5} = 2$       2 fifths is contained in 4 fifths twice (c).  $\frac{1}{3}$  is contained in  $6\frac{1}{3}$  nineteen times (d).  $2\frac{1}{2}$  is contained in 5 twice, in 15 three times, and in  $17\frac{1}{2}$  seven times (e).  
 (d)  $6\frac{1}{3} \div \frac{1}{3} = 19$   
 (e)  $17\frac{1}{2} \div 2\frac{1}{2} = 7$

2.  $5 \div \frac{1}{3} = ?$

3.  $7 \div \frac{1}{5} = ?$

4.  $6 \div \frac{2}{3} = ?$

5.  $\frac{1}{2} \div \frac{1}{4} = ?$

6.  $\frac{2}{3} \div \frac{1}{6} = ?$

7.  $\frac{1}{15} \div \frac{7}{15} = ?$

8.  $\frac{9}{10} \div \frac{1}{5} = ?$

9.  $4\frac{1}{2} \div \frac{1}{4} = ?$

10.  $5\frac{2}{3} \div \frac{2}{3} = ?$

11.  $10 \div 3\frac{1}{3} = ?$

12.  $12 \div \frac{1}{2} = ?$

13.  $13 \div \frac{1}{5} = ?$

14.  $9\frac{1}{7} \div \frac{1}{7} = ?$

15.  $9\frac{1}{7} \div \frac{1}{14} = ?$

16.  $10 \div 2\frac{1}{2} = ?$

17.  $20 \div 3\frac{1}{3} = ?$

18.  $23\frac{1}{3} \div 3\frac{1}{3} = ?$

19.  $12\frac{1}{2} \div 2\frac{1}{2} = ?$

20.  $\frac{7}{8} \div \frac{1}{16} = ?$

21.  $\frac{1}{16} \div \frac{3}{32} = ?$

22.  $15 \div \frac{3}{5} = ?$

23.  $2\frac{6}{7} \div \frac{2}{7} = ?$

24.  $\frac{5}{9} \div \frac{1}{18} = ?$

25.  $4\frac{3}{4} \div 2\frac{3}{8} = ?$

26.  $25\frac{5}{8} \div 5\frac{1}{8} = ?$

27.  $42\frac{7}{8} \div 6\frac{1}{8} = ?$

28.  $125\frac{3}{8} \div 25\frac{1}{8} = ?$

29.  $100 \div \frac{1}{2} = ?$

30.  $100 \div \frac{1}{8} = ?$

31.  $100 \div 12\frac{1}{2} = ?$

32.  $100 \div 6\frac{1}{4} = ?$

33.  $200 \div 6\frac{1}{4} = ?$

34.  $40 \div 2\frac{1}{2} = ?$

35.  $60 \div 7\frac{1}{2} = ?$

## Division of Fractions

1. Divide  $\frac{4}{5}$  by  $\frac{2}{3}$ ;  $5\frac{3}{8}$  by  $1\frac{1}{2}$ .

### SECOND METHOD

$$\frac{4}{5} \div \frac{2}{3} = \frac{4}{5} \div \frac{1}{\frac{3}{2}} = \frac{4}{5} \times \frac{3}{2} \text{ or } \frac{6}{5} \text{ or } 1\frac{1}{5}.$$

$$5\frac{3}{8} \div 1\frac{1}{2} = \frac{43}{8} \div \frac{3}{2} = \frac{43}{8} \times \frac{2}{3} = \frac{43}{12} \text{ or } 3\frac{7}{12}.$$

In all cases in which the quotient cannot be easily found by inspection, the mixed numbers may be changed to the form of fractions and the fractions made similar. Then one numerator may be divided by the other.

10 fifteenths is contained in 12 fifteenths as many times as 10 are contained in 12.

- |  |   |
|--|---|
| 2. $\frac{5}{6} \div \frac{2}{3} = ?$      | 19. $\frac{18}{41} \div \frac{2}{3} = ?$    |
| 3. $\frac{9}{10} \div \frac{4}{5} = ?$     | 20. $\frac{15}{37} \div \frac{1}{2} = ?$    |
| 4. $\frac{7}{18} \div \frac{3}{4} = ?$     | 21. $\frac{11}{30} \div \frac{8}{9} = ?$    |
| 5. $\frac{2}{4} \div \frac{5}{7} = ?$      | 22. $4\frac{1}{2} \div 2\frac{1}{3} = ?$    |
| 6. $\frac{3}{10} \div \frac{5}{6} = ?$     | 23. $5\frac{1}{5} \div 1\frac{1}{4} = ?$    |
| 7. $2\frac{1}{4} \div \frac{3}{7} = ?$     | 24. $12\frac{2}{3} \div 2\frac{1}{5} = ?$   |
| 8. $4\frac{1}{5} \div \frac{5}{9} = ?$     | 25. $18\frac{6}{7} \div 1\frac{1}{3} = ?$   |
| 9. $7\frac{8}{9} \div \frac{5}{8} = ?$     | 26. $15 \div 3\frac{2}{3} = ?$              |
| 10. $8\frac{5}{6} \div 1\frac{1}{12} = ?$  | 27. $27 \div 5\frac{1}{2} = ?$              |
| 11. $7\frac{1}{3} \div \frac{6}{5} = ?$    | 28. $32\frac{1}{3} \div 7\frac{2}{5} = ?$   |
| 12. $\frac{9}{10} \div 1\frac{1}{3} = ?$   | 29. $29 \div 1\frac{1}{9} = ?$              |
| 13. $\frac{17}{21} \div \frac{3}{7} = ?$   | 30. $36\frac{5}{16} \div \frac{1}{2} = ?$   |
| 14. $3\frac{1}{5} \div 17\frac{1}{2} = ?$  | 31. $75 \div 2\frac{3}{8} = ?$              |
| 15. $12 \div 3\frac{2}{5} = ?$             | 32. $100 \div \frac{3}{13} = ?$             |
| 16. $19 \div 5\frac{5}{8} = ?$             | 33. $150 \div 1\frac{7}{19} = ?$            |
| 17. $14\frac{2}{7} \div 1\frac{3}{14} = ?$ | 34. $116\frac{2}{3} \div 2\frac{2}{3} = ?$  |
| 18. $16\frac{8}{9} \div \frac{5}{6} = ?$   | 35. $79\frac{5}{6} \div 48\frac{3}{17} = ?$ |

## Division of Fractions

### THIRD METHOD

1. Divide  $\frac{6}{7}$  by  $\frac{5}{9}$ .

$\frac{6}{7}$  divided by 5 would be  $\frac{6}{35}$ , but since we wish to divide only by one ninth of 5, the quotient is nine times  $\frac{6}{35}$ , or  $\frac{54}{35}$ .

$$\frac{6}{7} \div \frac{5}{9} = \frac{6}{7} \times \frac{9}{5} = \frac{54}{35} \text{ or } 1\frac{19}{35}.$$

Therefore, instead of making the fractions similar we may divide by the numerator of the divisor and then multiply by its denominator, or *invert the divisor and proceed as in multiplication of fractions.*

- |  |  |
|--|--|
| <p>2. <math>\frac{2}{3} \div \frac{1}{4} = ?</math></p> <p>3. <math>\frac{6}{7} \div \frac{1}{8} = ?</math></p> <p>4. <math>\frac{15}{16} \div \frac{15}{19} = ?</math></p> <p>5. <math>4 \div \frac{3}{7} = ?</math></p> <p>6. <math>18 \div \frac{5}{9} = ?</math></p> <p>7. <math>2\frac{1}{2} \div \frac{1}{3} = ?</math></p> <p>8. <math>4\frac{6}{7} \div \frac{2}{5} = ?</math></p> <p>9. <math>8\frac{1}{3} \div 1\frac{2}{5} = ?</math></p> <p>10. <math>4\frac{1}{2} \div 2\frac{1}{3} = ?</math></p> <p>11. <math>7 \div 3\frac{1}{3} = ?</math></p> <p>12. <math>9 \div 1\frac{2}{5} = ?</math></p> <p>13. <math>16 \div 4\frac{5}{8} = ?</math></p> <p>14. <math>\frac{9}{10} \div \frac{6}{17} = ?</math></p> <p>15. <math>\frac{11}{13} \div \frac{13}{22} = ?</math></p> <p>16. <math>3\frac{1}{11} \div 1\frac{4}{5} = ?</math></p> <p>17. <math>10\frac{3}{17} \div \frac{7}{8} = ?</math></p> <p>18. <math>12 \div \frac{3}{16} = ?</math></p> <p>19. <math>\frac{17}{18} \div 1\frac{1}{15} = ?</math></p> <p>20. <math>22\frac{8}{9} \div \frac{7}{18} = ?</math></p> | <p>21. <math>\frac{16}{17} \div \frac{8}{10} = ?</math></p> <p>22. <math>\frac{15}{27} \div \frac{5}{8} = ?</math></p> <p>23. <math>\frac{21}{17} \div \frac{21}{5} = ?</math></p> <p>24. <math>\frac{11}{45} \div 1\frac{1}{9} = ?</math></p> <p>25. <math>5\frac{2}{3} \div \frac{2}{7} = ?</math></p> <p>26. <math>81\frac{1}{2} \div 1\frac{7}{18} = ?</math></p> <p>27. <math>12\frac{1}{2} \div 25\frac{1}{3} = ?</math></p> <p>28. <math>42\frac{4}{5} \div 9\frac{5}{8} = ?</math></p> <p>29. <math>100 \div 17\frac{1}{5} = ?</math></p> <p>30. <math>250 \div 27\frac{5}{6} = ?</math></p> <p>31. <math>52\frac{1}{3} \div 52\frac{1}{4} = ?</math></p> <p>32. <math>68\frac{5}{8} \div 34\frac{5}{8} = ?</math></p> <p>33. <math>86 \div 17\frac{5}{9} = ?</math></p> <p>34. <math>93\frac{1}{18} \div \frac{6}{5} = ?</math></p> <p>35. <math>\frac{49}{60} \div \frac{12}{25} = ?</math></p> <p>36. <math>\frac{87}{89} \div 3\frac{1}{4} = ?</math></p> <p>37. <math>180 \div \frac{37}{8} = ?</math></p> <p>38. <math>681\frac{1}{2} \div 1\frac{1}{3} = ?</math></p> <p>39. <math>1181\frac{1}{17} \div 231\frac{5}{21} = ?</math></p> |
|--|--|



## Complex Fractions

*Reduce the complex fractions to their simplest forms:*

1. Reduce  $\frac{\frac{5}{9}}{\frac{6}{9}}$  to its simplest form;  $\frac{8\frac{6}{7}}{9\frac{4}{5}}$ .

$$\frac{\frac{5}{9}}{\frac{6}{9}} = \frac{5}{6} \div 9 = \frac{5}{54}, \quad \frac{8\frac{6}{7}}{9\frac{4}{5}} = \frac{42}{49} = \frac{62}{7} \div \frac{49}{5} = \frac{62}{7} \times \frac{5}{49} = \frac{310}{343}$$

- |   |  |   |
|---|--|---|
| 2. $\frac{\frac{3}{4}}{9}$  | 7. $\frac{6\frac{1}{2}}{8\frac{3}{5}}$   | 12. $\frac{3\frac{2}{3}}{4\frac{2}{5}}$   |
| 3. $\frac{\frac{8}{9}}{14}$   | 8. $\frac{3\frac{1}{2}}{6\frac{3}{8}}$   | 13. $\frac{24\frac{4}{5}}{41\frac{3}{8}}$   |
| 4. $\frac{6\frac{1}{4}}{10}$  | 9. $\frac{7}{\frac{11}{\frac{5}{6}}}$  | 14. $\frac{25}{5\frac{7}{8}}$   |
| 5. $\frac{6\frac{1}{4}}{16\frac{3}{8}}$   | 10. $\frac{6\frac{3}{4}}{100}$   | 15. $\frac{1\frac{1}{5}}{4\frac{3}{8}}$   |
| 6. $\frac{8\frac{6}{7}}{9\frac{4}{5}}$  | 11. $\frac{7\frac{5}{6}}{5\frac{3}{8}}$  | 16. $\frac{87\frac{1}{2}}{100}$   |
| 17. $\frac{5\frac{1}{2}}{\frac{1}{2} \text{ of } \frac{3}{4}}$                                | 24. $\frac{12\frac{6}{7}}{\frac{3}{7} \times 5 \times \frac{4}{5}}$                            | 31. $\frac{15\frac{1}{2} + 9\frac{1}{4}}{24\frac{4}{5} \div 4}$   |
| 18. $\frac{2 \times 5\frac{1}{2}}{3\frac{1}{2} \times 4\frac{1}{2}}$                          | 25. $\frac{\frac{5}{8} \times \frac{3}{4}}{4\frac{3}{8} - 2\frac{1}{4}}$                       | 32. $\frac{\frac{3}{4} + \frac{3}{4} \text{ of } \frac{8}{9}}{\frac{3}{4} - \frac{3}{4} \text{ of } \frac{8}{9}}$                       |
| 19. $\frac{1\frac{5}{6} + 2\frac{1}{2}}{2\frac{1}{4} - 1\frac{1}{8}}$                         | 26. $\frac{6\frac{1}{2} \times 3 - 1\frac{1}{4}}{12\frac{1}{6} + 2 \times 2\frac{1}{2}}$       | 33. $\frac{\frac{1}{2} \text{ of } \frac{2}{3} \text{ of } \frac{3}{4}}{\frac{2}{3} \text{ of } \frac{4}{5} \text{ of } \frac{10}{11}}$ |
| 20. $\frac{2\frac{1}{9} \times \frac{4}{5}}{\frac{7}{12} \text{ of } \frac{1}{2}\frac{5}{8}}$ | 27. $\frac{\frac{2}{3} + \frac{1}{4} + 2}{\frac{1}{12} + \frac{2}{3} + \frac{5}{6}}$           | 34. $\frac{3\frac{4}{5} + 7\frac{3}{8}}{3\frac{4}{5} \times 7\frac{3}{8}}$  |
| 21. $\frac{6\frac{5}{8} \times 2\frac{1}{4}}{9\frac{1}{5} \times 6\frac{3}{8}}$               | 28. $\frac{12 \div 3\frac{1}{4}}{18 \div 3\frac{1}{2}}$  | 35. $\frac{7\frac{1}{2} \times 6 - 2\frac{1}{2}}{8\frac{1}{4} - 4 \times \frac{3}{8}}$  |
| 22. $\frac{2\frac{3}{4} - 1\frac{1}{2}}{4\frac{1}{4} - 2\frac{5}{8}}$                         | 29. $\frac{7 + 3\frac{1}{4} \times 6}{15 + 7\frac{1}{8} \times 5}$                             | 36. $\frac{25\frac{1}{2} \times 12\frac{1}{5}}{\frac{9}{10} + \frac{3}{8} + \frac{1}{5}}$   |
| 23. $\frac{\frac{4}{9} \times \frac{3}{8}}{\frac{3}{4} \times \frac{8}{9}}$                   | 30. $\frac{\frac{3}{8} + \frac{4}{9} + \frac{1}{2}}{\frac{1}{16} + \frac{5}{4} + \frac{7}{8}}$ | 37. $\frac{28 + 4\frac{1}{8}}{37 \div 5\frac{1}{8}}$  |

### Miscellaneous Problems

1. Upon an ordinary leaf of an apple tree there are as many as 100,000 openings for the admission of air from which carbon is obtained for the growth of the tree. How many such openings are there upon all the leaves of a tree which has 10,000 leaves?

2. There are about 2 miles of silk fiber in the cocoon of a silkworm of the size of a robin's egg. How many of these cocoons would furnish silk enough to form a continuous line reaching twice around the earth?

3. How many times would the silk from one of these cocoons reach across a web of cloth which is  $\frac{3}{4}$  of a yard wide?

4. If a ball is struck with two blows at the same instant, one of which would send it south 12 feet in a second, and the other west 16 feet in a second, in which direction and how far will it roll in the second?

5. A ball is struck with two blows at the same instant in directions at right angles to each other. Each blow is sufficient to send it 20 feet in a second. About how far does it roll in the second?

*Simplify:*

$$6. \frac{6\frac{1}{2}}{3\frac{1}{18}}$$

$$10. \frac{\frac{3}{8} \times 5\frac{2}{3}}{2\frac{1}{8} + 1\frac{1}{8}}$$

$$14. \frac{34\frac{3}{8}}{12\frac{3}{8} - 8\frac{9}{10}}$$

$$7. \frac{37\frac{1}{2}}{112\frac{1}{2}}$$

$$11. \frac{9.37 - .12}{24 + .08}$$

$$15. \frac{3\frac{1}{2} \times 6 - 4\frac{1}{4}}{12\frac{1}{4} - 3 \times 2\frac{1}{8}}$$

$$8. \frac{66\frac{2}{3}}{200}$$

$$12. \frac{55\frac{2}{3} - (36 - .9)}{12\frac{1}{10} - .7\frac{2}{3}}$$

$$16. \frac{\frac{2}{3} \text{ of } 7\frac{5}{8}}{\frac{4}{5} \text{ of } 12\frac{2}{3}}$$

$$9. \frac{\frac{5}{8} + \frac{1}{8}}{\frac{3}{4} \text{ of } 67}$$

$$13. \frac{15\frac{1}{2} - .6\frac{1}{4}}{8\frac{1}{3} - 5.2}$$

$$17. \frac{16\frac{1}{2} + 5 \times 3\frac{5}{8}}{22\frac{1}{4} \times 7 - 4\frac{1}{8}}$$

## Stocks

The capital of any public company is called **stock**. The stock is usually divided into shares. The original value of one of the shares is called its **par value**. The par value is usually \$100, \$50, or \$25.

In the examples in this book the par value of a share is \$100 unless otherwise stated.

The sum for which a share can be sold at any particular time is called the **market value** of the stock.

The dividend is a percentage of the *par value* paid yearly on the stock. As the dividend is always reckoned on the par value, it does not change with the fluctuating market value.

A bond is a note, bearing interest, issued by a corporation or by the government.

An agent who buys and sells stock for others is called a **broker**. His commission is called **brokerage**.

1. What is the cost of 40 shares of railroad stock at 95, if no brokerage is paid?

$$40 \times \$95 = \$3800.$$

2. What is the cost of 35 shares of stock at  $106\frac{1}{2}$ , without brokerage?

3. Find the cost of 20 shares of railroad stock at  $87\frac{1}{2}$ , including brokerage at  $\frac{1}{8}\%$ .

The brokerage is reckoned upon the par value.

$$20 \times (87\frac{1}{2} + \frac{1}{8}) = \$1752.50.$$

4. Find the cost of 75 shares of stock at  $112\frac{5}{8}$ , including the usual brokerage of  $\frac{1}{8}\%$ .

5. Find the cost of a \$1000 United States bond at  $104\frac{1}{4}$ , including brokerage.

6. What would be the cost of eight \$1000 bonds at  $107\frac{3}{8}$ , *uding* brokerage?

## Stocks

The following is the market quotation of stock of April 2, 1900 :

### NOON QUOTATIONS

#### *Government Bonds*

U.S. 4's, 134. <sup>1</sup>	U.S. reg. 5's, 114½.
U.S. 2's, 100½.	U.S. war 3's, 109½.

#### *General List*

Am. Sugar, 104½.	Northern Pacific, 61½.
Gen. Electric, 131½.	People's Gas, 109½.
Missouri Pac., 49½. <sup>2</sup>	So. Pacific, 42.

1. At above quotations, how much would 20 shares of Gen. Electric cost, including the brokerage of  $\frac{1}{8}\%$  ?

*Find the cost, including brokerage :*

2. Of 12 shares of American Sugar.
3. Of 18 shares of Missouri Pacific.
4. Of 50 shares of Southern Pacific.
5. If I buy U.S. 3's at 150, what per cent of income shall I receive on the investment ?

I receive \$3 on every \$150.  $\frac{3}{150} = 2\%$

*Find the per cent of income, disregarding brokerage :*

6. On U.S. 4's bought at 120.
7. On U.S. 5's bought at 110.
8. On U.S. 3's bought at 109½.
9. For \$617.50, how many shares of Northern Pacific stock could I buy at the quotations, brokerage  $\frac{1}{8}\%$  ?

$\$61\frac{1}{2} + \$\frac{1}{8} = \$61\frac{1}{8}$ , cost of one share;  $\$617.50 \div \$61\frac{1}{8} = 10$ , the number of shares.

10. How many shares of Gen. Electric could I buy for \$1584 at quotations, with brokerage at  $\frac{1}{8}\%$  ?

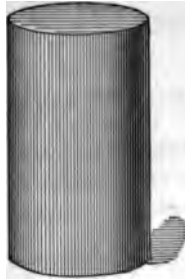
<sup>1</sup> This means that one \$100 United States bond yielding 4% interest, or dividend, can be bought in the market for \$134.

<sup>2</sup> This means that one \$100 share of Missouri Pacific stock can be bought for \$49½.

## Review

1. Add  $22\frac{5}{8}$ ,  $75\frac{5}{8}$ ,  $27\frac{11}{11}$ ,  $82\frac{7}{8}$ .
2. Add  $17\frac{5}{8}$ ,  $32\frac{7}{8}$ ,  $45\frac{11}{4}$ ,  $62\frac{7}{8}$ .
3. From  $22\frac{3}{4}$  take  $13\frac{1}{10}$ .
4. From  $\frac{3}{2}\frac{1}{7}$  take  $\frac{4}{7}\frac{2}{8}$ .
5. Multiply  $125\frac{3}{4}$  by  $46\frac{5}{8}$ .
6. Multiply  $322\frac{6}{7}$  by  $56\frac{5}{8}$ .
7. Divide  $5245\frac{1}{2}$  by 15.
8. Divide  $463\frac{1}{2}$  by  $32\frac{1}{4}$ .
9. Find  $\frac{1}{2}$  of  $\frac{3}{8}$  of  $\frac{3}{4}$  of  $\frac{5}{8}$ .
10. Find  $\frac{1}{8}$  of  $\frac{7}{15}$  of  $12\frac{1}{2}$ .
11. How many strips of wall paper, 18 inches wide, will be required to paper the walls of a room 18 ft. long,  $13\frac{1}{2}$  ft. wide?
12. How much will it cost to surround a lot 80 ft. long and 45 ft. wide, with a tight board fence 4 ft. high, if the boards cost \$14.75 per M. ?
13. What will be the cost of a strip of land for a road 66 ft. wide and 2 miles long, at \$30 an acre ?
14. A rifle bullet is discharged in a horizontal direction from a cliff 256 feet above the level of the sea. It reaches the water at a point 4800 feet from the cliff. What is the average velocity of the bullet?  
See p. 23. How long does it take the bullet to fall 256 feet?
15. If the water of a river is moving at the rate of 6 miles an hour, how many feet is it moving a second ?
16. If this water falls over a bluff 144 feet high, how many feet from the foot of the bluff will it strike the river bed again ?

## Cylinders



*The volume of a cylinder, like the volume of a prism, is found by multiplying the area of one end by the altitude.*

1. The area of the end or base of a cylinder is 27 sq. in. and its length 10 in. What is its volume?
2. Find the circumference of a cylinder whose diameter is 25 inches.
3. Find the area of one end of a cylinder whose diameter is 20 in.
4. Find the volume of a cylinder whose diameter is  $36\frac{1}{2}$  in. and whose height is 4 ft.
5. How many cubic feet of water will be contained in a cylindrical standpipe  $15\frac{1}{2}$  ft. in diameter and 30 ft. high?
6. How many cubic feet are there in a cylindrical watering cart 6 ft. in diameter and 12 ft. 6 in. long?
7. How long will it take to fill this cart, if the water runs through a hose at the rate of 200 cubic inches a second?
8. When the cart is filled, what will be the weight of the load of water?
9. How many cubic inches are there in a cylindrical bar of iron 6 in. in diameter and 10 ft. 4 in. long?
10. If the specific gravity of iron is 7.2, how much will the bar of iron weigh?

## Partnership

1. Divide the number 54 into 3 parts in the proportion of 2, 3, and 4.

We are to take three fractional parts of 54, which shall be in the proportion of 2, 3, and 4. If we regard 54 as divided into  $2+3+4$ , or 9 equal parts, the first of the three parts required will be 2 of these 9 parts, or  $\frac{2}{9}$  of 54, or 12. The second part will be  $\frac{3}{9}$  of 54, or 18; and the third part  $\frac{4}{9}$  of 54, or 24.

2. If \$25 is divided between two persons, so that one receives \$2 as often as the other receives \$3, how much does each receive?

3. Divide the number 84 into two parts in the proportion of 5 to 7.

4. Divide the number 100 into three parts in the proportion of 2, 3, and 5.

5. Divide the number 24 into two parts in the proportion of 30 to 50.

6. Divide \$1000 into three parts in the proportion of 1, 3, and 6.

7. Divide \$180 among three men in the proportion of 2, 3, and 4.

8. Divide \$4000 among three persons in the proportion of 7, 13, and 20.

9. Two men engage in business, with an agreement that the profits shall be divided in the proportion of the amounts of capital invested. The first invests \$3000 and the second \$7000. The profits amount to \$2000. What is each man's share?

10. Three persons, A, B, and C, engage in business. A invests \$2000, B \$3000, and C \$4000. The gain is \$3600. *What is each man's share?*

## Review Problems

1. What is the number of days from July 25th to Oct. 12th ?
2. What is the number of days from Jan. 10, 1896, to April 15th of the same year ?
3. If a watch gains 5 seconds in 8 hours, how long will it take it to gain 10 minutes ?
4. Two clocks are together at 12 o'clock. One loses 4 minutes a day, and the other gains 8 minutes a day. In how many days will they be an hour apart ?
5. What is the cost of 30 shares of bank stock of \$100 each, bought at 15% premium, with brokerage of  $\frac{1}{8}\%$  ?
6. How much is lost in buying 100 shares of telegraph stock of \$100 each, at 5% premium, and selling at 12% discount, without brokerage ?
7. What will be the gain in buying 60 shares of stock of \$50 each, at  $47\frac{1}{2}$ , and selling at  $52\frac{1}{2}$ , including brokerage of  $\frac{1}{8}\%$  in both cases ?
8. What is the area of the two ends of a cylinder which is 12' 4" in diameter ?<sup>1</sup>
9. Find the convex surface of a cylinder whose diameter is 6' 2" and the length 11' 8".
10. The convex surface of a cylinder is 63 sq. ft., and its circumference is 4' 4". How long is it ?
11. How many cubic feet are there in a log in the form of a cylinder whose length is 15' 3" and diameter 10" ?
12. If the specific gravity of this log is .85, how many such logs will it take to weigh a ton ?

<sup>1</sup> The sign ' is used for feet and '' for inches.



## Review Problems

1. A man paid \$12.60 for insuring his barn at  $1\frac{1}{2}$  per cent. For what sum was the barn insured?
2. If I have to pay \$36 for insuring my house for  $\frac{3}{4}$  of its value, at  $1\frac{1}{5}$  per cent, what is the value of the house?
3. My goods were insured in one company for \$800, and in another company for \$1200. They were damaged to the extent of \$500. What sum had each company to pay?
4. If I buy at 120, stock which yields 6% of its par value, what per cent shall I receive on my investment?
5. If 8% stock is bought at 75, what per cent will the investment pay?
6. I own stock which yields quarterly dividends of  $1\frac{1}{4}$  per cent. I bought the stock at  $62\frac{1}{2}$ . Its par value is \$100 per share. What rate of interest am I receiving on my investment?
7. How many bushels will be contained in a cylindrical tank whose diameter is 52 inches, and whose depth is 4 feet 9 inches?
8. What must be the depth of a cylindrical tank 16 inches in diameter, to contain  $8\frac{1}{2}$  gallons of water?
9. A and B entered into partnership. A furnished \$12,000 of the capital and B \$8000. Their profits were \$5200. What was the share of each?
10. A and B purchased land for \$18,000. A invested three times as much as B. They sold the land at a profit of \$3600. What was each one's share of the profit?
11. A, B, and C hire a pasture for \$105. A puts in 3 cows, B 5 cows, and C 7 cows. What part should each pay?

## Measurements

*Find the measurements of the regular prism 3 ft. long :*

1. Square prism ; end 6 in. square. Lateral surface = ?
2. Square prism ; end 5 in. square. Volume = ?
3. Triangular prism ; face 8 in. wide. Lateral surface = ?
4. Triangular prism ; area of end 65 sq. in. Volume = ?
5. Pentagonal prism ; face 10 in. wide. Lateral surface = ?
6. Pentagonal prism ; area of end 70 sq. in. Volume = ?
7. Hexagonal prism ; face 7 in. wide. Lateral surface = ?

*Find the measurements of the cylinder :*

8. Diameter 2 ft. 4 in. Find the circumference.
9. Radius 15 in. Find the surface of one end.
10. Radius 10 in. ; length 20 in. Volume = ?
11. Circumference 25 in. Area of one end = ?
12. Circumference 24 in. ; length 16 in. Volume = ?
13. Diameter 3 ft. ; length 4 ft. Lateral surface = ?
14. Radius 2 ft. ; length  $3\frac{1}{2}$  ft. Volume = ?

*Find the measurements of the regular pyramid :*

15. Square pyramid ; base 2 ft. square ; slant height 5 ft. Lateral surface = ?
16. Square pyramid ; base 9 in. square. Area of base = ?
17. Triangular pyramid ; side 8 in. Perimeter of base = ?
18. The slant height is 16 in. Lateral surface = ?
19. Hexagonal pyramid ; side 5 in. Perimeter of base = ?

## Original Problems

*Make problems and solve them :*

1. A train runs 75 rods a minute.
2. A field is 48 rods long and 36 rods wide.
3. A shot is discharged from a gun at the rate of 1200 feet per second.
4. One number is  $\frac{2}{5}$  of another, and the difference between them is 16.
5. William's age is  $\frac{2}{5}$  of his father's age, and the sum of their ages is 56.
6.  $\frac{11}{8}$  of a number is 18 more than the number itself.
7. A ball is struck with two blows at the same instant. One of the blows tends to send the ball east and the other to send it north.
8. A speculator bought 100 shares of Southern Pacific railroad stock at 42, with brokerage of  $\frac{1}{8}\%$ .
9. A public road 4 rods wide is built across a farm which is 80 rods wide.
10. The diameter of a cylinder is 3 ft. 4 in., and its height 9 ft.
11. A bar of iron is 3 inches in diameter and 10 feet long.
12. 10 shares of bank stock having a par value of \$100 each, are sold at 92 below par.
13. \$36 is divided between two persons so that the first receives \$5 as often as the second receives \$4.
14. Three men engage in partnership.
15. Two clocks are together at 12 o'clock. One loses 3 minutes a day and the other gains 2 minutes a day.
16. A cylindrical tank, 14 inches wide and 4 feet deep, filled with water.

## Algebra

When a quantity in a parenthesis is immediately preceded or followed by another quantity, with no sign between, the two quantities are to be multiplied together.  $5a(4x^2 - 3y)$  means  $5a$  times the whole quantity  $4x^2 - 3y$ .

If  $a = 2$ ,  $b = 4$ ,  $c = 3$ , and  $d = 10$ , find the values of the following expressions:

1.  $5a + 4(b + c) - (d + 2c)$

See pp. 73, 85.

2.  $8a^2 - 2b(b - c) + (d - b)$

3.  $b^2 - 2bc + c^2$

4.  $\frac{a+d}{b} + \frac{b-a}{a} - \frac{d-b}{c}$

See p. 74.

5.  $a^4 + 2ab^2c - \frac{3(b-a)}{c}$

Find the value of  $x$ :

6.  $x + \frac{x-1}{2} - \frac{x-2}{3} = 4 + \frac{x-1}{2}$

7.  $4(x+2) + 6 = 10(x+2) - 10(x-2)$

8.  $\frac{3(x+1)}{4} = 12$

9.  $45 - \frac{5(1+x)}{2} = 39 - \frac{10(5-x)}{5}$

10. Find the sum of  $3a^2 + 4ab$ ,  $c^2 - a^2$ ,  $2c^2 - 2ab$ .

11. From  $4x^4y^2 - 3x^2y + 2y^3$  take  $y^3 - 2x^4y^2 + x^2y$ .

12. Multiply  $a^2 - 2b^3 + 2c - 4d$  by  $2a - 3c$ .

13. Divide  $m^4n^2 + m^3n^3 - m^2n^4$  by  $m^2n^2$ .

### Problems for Arithmetic or Algebra

1. John has a sum of money and Mary has one half as much. Together they have 36 cents. How much has each?
2. One number is three times another, and the difference between them is 40. What are the numbers?
3. In one pocket I have 6 cents more than 4 times as much money as in the other. In both pockets I have 66 cents. How many have I in each?
4. The difference between two numbers is 17, and the larger number is 25. What is the smaller number?
5.  $\frac{1}{2}$  of a number added to  $\frac{1}{6}$  of the number equals 20. What is the number?
6. A horse and carriage are together worth \$320, and the horse is worth three times as much as the carriage. What is the value of each?
7. 8 times the number of marbles that a boy has, less 3 marbles, would be 37 marbles. How many has he?
8. The sum of two numbers is 100, and the greater is 4 times the less. What are the numbers?
9. William's age is  $\frac{3}{4}$  that of his sister. The sum of their ages is 30 years. How old is each?
10. If to a certain number one half of itself is added, the sum will be 18. What is the number?
11.  $\frac{5}{6}$  of a number is 9 less than the number itself. What is the number?
12.  $\frac{3}{8}$  of a number is 15 more than the number. What is the number?
13. If 5 is taken from  $\frac{1}{2}$  of a number the remainder will be  $\frac{1}{4}$  of the number. What is the number?
14. If 6 is added to  $\frac{2}{3}$  of a number, the sum will be  $\frac{5}{6}$  of the number. What is the number?

## Proportion

When two fractions, as  $\frac{3}{4}$  and  $\frac{6}{8}$ , are equal to each other, the product of the numerator of the first and the denominator of the second is always equal to the product of the denominator of the first and the numerator of the second.

When two ratios are equal, their corresponding terms are proportional to each other. A statement of equality between two ratios is called a **proportion**, as  $2:3 = 6:9$ .

In a proportion the two middle terms are called the **means**, and the terms at the ends the **extremes**. In the above proportion 3 and 6 are the means and 2 and 9 the extremes.

*The product of the means is always equal to the product of the extremes.*

1. The product of two numbers is 24, and one of the numbers is 3. What is the other number?

2. The product of the two means in a proportion is 36, and one of the extremes is 4. What is the other extreme?

*Find the unknown term in each of the following proportions:*

3.  $6:8 = 24:x$

7.  $10:17 = 25:x$

4.  $5:35 = x:70$

8.  $36:x = 4\frac{1}{2}:9\frac{1}{2}$

5.  $14:x = 44:11$

9.  $112:74 = x:30$

6.  $x:21 = 9:63$

10.  $x:56 = 35:100$

11. If 4 oranges cost 12 cents, how much will 7 oranges cost at the same rate?

The ratio of the oranges is equal to the ratio of the cents corresponding to the oranges.  $4:7 = 12:x$ .  $4x = 84$ ;  $x = 21$ .

12. If 3 men can dig a ditch 5 rods long in a day, how long a ditch can 8 men dig in the same time?

13. If \$240 will gain \$36 of interest in a certain time, how much interest will \$600 gain in the same time?

## Proportion

1. If 16 yards of cloth cost \$48, how much will 65 yards cost?
2. If 24 men earn \$108 in 3 days, how much will they earn in 6 weeks, at the same rate?
3. How many oranges can be bought for 80¢, at the rate of 3 for 10¢?
4. If a flagstaff 96 ft. high casts a shadow 64 ft. long, what is the height of a steeple which casts a shadow 82 ft. long, at the same time?
5. How many bushels of corn can be bought for \$200, if 65 bushels can be bought for \$27.30?
6. At the rate of 16¢ a peck, what would be the cost of 15 bu. 3 pk. of apples?
7. If the cost of plowing  $7\frac{1}{2}$  acres of land is \$16.50, what will be the cost, at the same rate, of plowing 200 acres?
8. If a wheel revolves 80 times in going 1000 feet, how many times will it revolve in going a mile and a half?
9. A rectangle 8 inches wide and  $12\frac{1}{2}$  inches long contains 100 square inches. What must be the length of a rectangle of the same width to contain 4 square feet?
10. If a man can walk 95 miles in 9 days, how far would he walk, at the same rate, in  $23\frac{1}{2}$  days?
11. If 122 yards of cloth are sufficient for 10 suits of clothes, how many yards are sufficient for 7 suits?
12. If 50 men can make 800 pair of shoes in a week by working 10 hours a day, how many pairs could they make *in working* 8 hours a day?

### Miscellaneous Problems

1. How many feet of lumber are there in 10 planks 12 ft. long,  $1\frac{1}{2}$  ft. wide, and 2 in. thick?

2. How many feet of lumber are there in a plank 10 ft. long, 10 in. wide at one end, 8 in. wide at the other end, and 3 in. thick?

3. The base of a cylindrical column of water is 5 sq. ft. and its height 10 ft. Find how many cubic feet it contains.

4. What is the weight of the water?

5. A pipe 3 in. in diameter and 12 ft. long is filled with water. Find how many cubic inches of water there are.

6. If the pipe stands perpendicularly, what is the total pressure at the bottom?

7. A, B, and C form a partnership. A invests \$4200, B \$3500, and C \$4300. If they gain \$4200, what will be each man's share?

8. Two men, A and B, engage in business with an agreement that the profits shall be divided according to the equivalents of their investments for 1 month. A has \$4000 in the business for 3 months, and B \$2000 for 4 months. They make \$1600. What is each man's share?

$3 \times \$4000 = \$12000$ ;  $4 \times \$2000 = \$8000$ ;  $\$12000 + \$8000 = \$20000$ .

A's share is  $\frac{12000}{20000}$  or  $\frac{3}{5}$  of \$1600 = \$960;

B's share is  $\frac{8000}{20000}$  or  $\frac{2}{5}$  of \$1600 = \$640.

9. A, B, and C engage in business together. A invests \$8000 for 1 month, B \$4000 for 2 months, and C \$2250 for 4 months. The profit is \$6000. What is each partner's share?



## Problems from Astronomy

1. The moon revolves around the earth in 27 da. 7 hr. 43 min. 11 sec. How many revolutions does it make in a year?
2. The planet Mars revolves around the sun in 687 days. How many times does the earth go round the sun while Mars is making its revolution?
3. The great comet of 1811 has a period of revolution of 3065 years. When will it next be visible from the earth?
4. It takes the planet Jupiter 4332.5 days to revolve round the sun. How many of our years make one year for Jupiter?
5. The planet Neptune makes its revolution in 60,126.7 days. How many of our years make a complete year for Neptune?
6. The planets revolve around the sun in paths which are nearly circles. The mean distance of Mercury from the sun is 36,000,000 miles. Find the approximate length of the path of its revolution.
7. Mercury makes its revolution in 87.9 days. Find how many miles it moves per day.
8. The mean distance of Saturn from the sun is 886,000,000 miles. Find the length of its path of revolution.
9. Saturn revolves around the sun in 10,759.2 days. How does its daily rate of travel through the heavens compare with that of Mercury?
10. The mean distance of the earth from the sun is about 93,000,000 miles. Find the length of its annual orbit.
11. How many miles more or less per day does the earth travel than Saturn?

### Miscellaneous Problems

*Find the value of the unknown quantities :*

1.  $\frac{8}{9} : x = 5\frac{1}{2} : 6\frac{1}{4}$

2. 6 yd. : 7 ft. = \$1.50 :  $x$

3.  $x : 16\frac{1}{2} = 120 : 184$

4.  $16\frac{2}{3} : 5\frac{1}{6} = x : 3\frac{1}{4}$

5. If  $\frac{5}{8}$  of an acre of land is worth \$37.50, how much are  $3\frac{5}{8}$  acres worth at the same rate ?

6. Three men, A, B, and C, engage in partnership. A has \$2000 in the business 5 months, B \$3000 4 months, and C \$4000 2 months. The profits of the business amount to \$3600. What is each man's share ?

7. The estimated increase of the real and personal property of the United States from 1880 to 1890 was nearly 50%. At the same rate of increase the value of the total property of the country at the end of the century would be about \$100,000,000,000. What was the total amount of the property in 1890 ?

8. What was it in 1880 ?

9. The population of the country at the end of the century was about 76,000,000. How much property was there per capita ?

10. In the first years of Washington's administration it cost about \$7,000,000 a year to carry on the government. The population was about 4,000,000. What was the ratio of expense to the population ?

11. The annual expense of the government is now about \$685,000,000. If we reckon the population at 76,000,000, what is the present ratio of expense ?

## Review

1. Divide .000024 by .082.
2. Multiply \$568 by  $.15\frac{1}{2}$ .
3. Change 1.45 sq. yd. to the fraction of a square rod.
4. Change  $\frac{3}{4}$  of a pint to the decimal of a gallon.
5. What part of a mile is  $\frac{3}{4}$  of a rod?
6. What part of a ton is  $62\frac{1}{2}$  lb.?
7. At \$ $4\frac{3}{4}$  a barrel, how many barrels of flour can be bought for \$80 $\frac{3}{4}$ ?
8. At \$ $\frac{4}{5}$  a yard, what will be the cost of  $15\frac{3}{8}$  yards of cloth?
9. Which is greater,  $\frac{7}{15}$  or  $\frac{5}{13}$ ?
10. If 4 is added to both terms of the fraction  $\frac{7}{8}$ , how much will the fraction be increased or diminished?
11. If 5 is subtracted from both terms of the fraction  $\frac{11}{12}$ , how much will the fraction be increased or diminished?
12. What will be the cost of 20 shares of bank stock, whose par value is \$100 each, at 80?
13. What will be the cost of 10 shares of railroad stock, whose par value is \$100 each, at  $112\frac{1}{2}$ , including brokerage at  $\frac{1}{8}\%$ ?
14. A man sold \$10,000 of 4% railroad stock at  $73\frac{3}{4}$ , paying brokerage of  $\frac{1}{8}\%$ . How much did he receive?
15. If I receive an annual dividend of \$400 on 5% bank stock, how many shares of \$50 each do I own?
16. If I have sufficient 4% government bonds of \$500 each to yield quarterly interest to the amount of \$260, how many bonds have I?
17. How many 3% bonds of \$100 each will it take to yield a semi-annual income of \$450?

## Proportion

In some cases in the comparison of quantities, the corresponding terms are not directly proportional, but are inversely proportional.

1. If 2 men can do a piece of work in 7 days, how long would it take 4 men to do the same work ?

In this case the greater the number of men the less the number of days. The days are not directly proportional to the men, but inversely proportional. The proportion is not  $2 : 4 = 7 : x$  but  $4 : 2 = 7 : x$ .

2. If 8 men can do a piece of work in 10 days, how long will it take 3 men to do an equal piece of work ?

3. If a certain number of men can build 20 rods of fence in 3 days, how many rods can they build in 7 days ?

Are the rods and the days directly proportional or inversely proportional ?

4. How many men will it take to do a piece of work in 12 days that 4 men can do in 9 days ?

5. If 13 horses eat 42 bu. 3 pk. 3 qt. of oats in 18 days, how many oats will they eat in the month of May ?

6. If a quantity of oats will last 8 horses  $5\frac{1}{2}$  days, how long would the same quantity last 12 horses ?

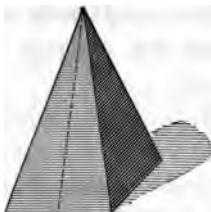
7. If one rectangle is 8 ft. long and 5 ft. wide, and another containing the same number of square feet is 4 feet wide, how long is it ?

8. How many yards of cloth  $\frac{3}{4}$  of a yard wide will it take to line 12 yards of cloth  $\frac{1}{4}$  of a yard wide ?

9. If  $12\frac{1}{2}$  acres are sufficient pasture for 4 cows, how many acres will be sufficient for 25 cows ?

10. The area of one end of a square prism is 36 square feet, and its length is 12 feet. The length of a second prism which contains the same volume is 10 feet. What is the area of one end of the second ?

## Pyramids



The base of a regular pyramid is a regular polygon, and the vertex is directly over the center of the base. Pyramids are triangular, square, pentagonal, hexagonal, etc., according to the forms of their bases. *The volume of a pyramid is  $\frac{1}{3}$  of the volume of a prism having an equal base and altitude, or  $\frac{1}{3}$  of the product of the area of the base and altitude.*

1. What is the volume of a prism whose base has an area of 24 sq. ft. and whose altitude is 6 ft. ?

2. What is the volume of a pyramid whose base has an area of 24 sq. ft. and whose altitude is 6 feet ?

$$24 \times 6 \times \frac{1}{3} = 48 \text{ cu. ft.}$$

3. What is the volume of a pyramid whose base is 9 feet square and whose altitude is 12 ft. 4 in. ?

4. Find the difference between the volume of this pyramid and that of a prism having the same dimensions.

5. In a triangular pyramid one side of the triangle forming the base is 4 ft. 6 in. The perpendicular distance from this side to the opposite vertex is 3 ft. 4 in. Find the area of the base.

6. The altitude of the pyramid is 5 ft. 3 in. Find the volume.

7. What is the volume of a square pyramid one side of whose base is 18 in. and whose altitude is  $24\frac{1}{4}$  in. ?

## Problems for Arithmetic or Algebra

See pp. 94, 152.

1. What interest will \$1 gain in 2 years at 5%?
2. How many dollars will gain 50 cents in 2 years at 5%?
3. What is the amount of \$1 for 1 yr. 6 mo. at 6%?
4. How many dollars will it take to amount to \$3.27 in the same time and at the same rate?
5. How much will \$5 gain in 4 yr. at 1%?
6. At what per cent would \$5 gain 80 cents in the same time?
7. What interest will \$100 gain in 1 yr. at 4%?
8. In how many years will \$100 gain \$20 at 4%?
9. What principal will gain \$12 in 6 months at 6%?
10. What principal will amount to \$275 in 1 yr. 3 mo. at 8%?
11. At what rate will \$75 gain \$9.45 in 4 yr. 2 mo. 12 da.?
12. In what time will \$140 gain \$18.20 at 4%?
13. What sum at 6% will amount to \$718.21 in 2 yr.?
14. What sum invested at 6% will produce a semi-annual income of \$3150?
15. At what rate will \$3750 produce \$225 of interest in 1 yr. 6 mo.?
16. In what time will \$1500 yield \$116.92 of interest at 3%?
17. If I pay \$31.75 for the use of \$1500 for 4 mo. 7 da., what is the rate?
18. Find how long it will take for a sum of money to double itself at simple interest at  $3\frac{1}{2}\%$ .

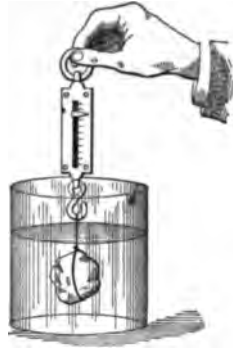
## Review Problems

1. Cloth which cost \$1.20 a yard was sold at an advance of  $33\frac{1}{3}\%$ . What was the selling price?
2. A lot of 400 barrels of flour worth \$5 a barrel was insured for  $\frac{3}{4}$  of its value at  $\frac{1}{3}\%$ . What was the premium?
3. A collector collected \$460 on a commission of 5%. What sum did he pay to his employer?
4. How many men will it take to do a piece of work in 10 days which 12 men can do in 15 days?
5. What must be the length of a piece of land 64 rd. wide in order that it may contain the same number of acres as a piece of land 200 rd. long and 80 rd. wide?
6. How many yards of carpet  $\frac{3}{4}$  of a yard wide will cover the same space as a carpet  $\frac{1}{2}$  of a yard wide and 30 yards long?
7. What is the entire surface of a square pyramid the perimeter of whose base is 20 ft. and whose slant height is  $50\frac{1}{2}$  ft.?
8. What is the volume of a square pyramid the perimeter of whose base is 36 ft. and whose altitude is 32 ft. 8 in.?
9. An ancient historian reported that the side of the base of the Great Pyramid in Egypt was 883 pedes, or Roman feet. Two pedes make a cubit, and a cubit is equal to 20.7 inches. What was the length of the side of the pyramid?
10. The Great Pyramid is a square pyramid, and is 450 feet high. If we take 760 feet as the length of its base, what is the distance around it?
11. How many miles of wall 10 ft. high and 1 ft. thick *could be made* with the stone contained in it?

## Buoyancy of Water — Specific Gravity

See pp. 23, 99.

Water, by its buoyant force, presses upward against an object as much as the weight of the water displaced by the object, so that the object will weigh so much the less in water than in the air. If an object weighs 500 ounces less in water than in the air, the water which it has displaced would weigh 500 ounces; hence one half of a cubic foot of water must have been displaced.



1. What is the weight of a cubical block a foot long which will just float in water without rising above the surface?
2. If the block were twice as heavy as water, how much would it weigh if attached to the scale by a string and suspended in water?
3. If an object weighs  $\frac{3}{4}$  as much in the water as in the air, what is its specific gravity?
4. If an object weighs  $\frac{1}{4}$  more in the air than in water, what is its specific gravity?
5. If a glass is exactly full of water and an object is lowered into it, how much water will overflow?
6. If the water which overflows weighs 20 ounces, and the object weighs 50 ounces when suspended in water, what is the entire weight of the object?
7. What is the specific gravity of the object?
8. If an object, when lowered into a glass full of water, causes  $\frac{1}{5}$  of a cubic foot of water to overflow, and weighs 10 pounds when suspended in the water, what is its specific gravity?



## Review Problems

1. If I insure my life for \$4000, and the premium is  $\frac{3}{4}\%$  quarterly, how much will the premium amount to in 3 years?

2. A man whose property is assessed for \$6000 is taxed 12 mills on a dollar. How much is his tax?

\$500.

SPRINGFIELD, ILL., March 15, 1899.

For value received, I promise to pay to the order of James Haskell, Five Hundred Dollars, on demand, with interest at 5%.

JOSEPH EVERETT.

3. Find how much was due on the above note Jan. 1, 1900.

4. Find how much would have been due if it had not been paid until Aug. 23, 1900.

5. What is the interest of \$560 from Sept. 19, 1895, to Apr. 1, 1898, at 4%?

6. What is the amount of \$275 with interest for 1 yr. 7 mo. 13 da., at  $5\frac{1}{2}\%$ ?

7. What sum of money will produce \$106.08 of interest in 6 yr. 3 mo. 15 da., at  $4\frac{1}{2}\%$ ?

8. What is the rate when the amount of \$255 for 2 yr. 7 mo. 6 da. is \$288.15?

9. In what time will \$420 gain \$37.80 at 4%?

10. What is the volume of a square prism one end of which is 14 in. square, and whose altitude is 27 in.?

11. What is the volume of a square pyramid whose base is 17 in. square, and whose altitude is 39 in.?

12. What is the lateral surface of a triangular pyramid each side of whose base is 8 ft. 9 in. and whose slant height is 12 ft. 9 in.?

## Construction

See pp. 102, 123.

1. Construct a triangle with an angle of  $48^\circ$ , a side 3 inches long, and an altitude of  $4\frac{1}{2}$  inches.
2. Construct a right triangle with a base 2 inches long and an hypotenuse 4 inches long.
3. Construct a parallelogram with a side  $3\frac{1}{4}$  inches long and an angle of  $52^\circ$ . Upon the same base construct a rectangle which will have the same area as the parallelogram. Find the area.
4. Make a triangle with a base  $3\frac{1}{2}$  inches long and an angle of  $65^\circ$ . Regarding this triangle as a part of a parallelogram, complete the parallelogram. Upon the same base make a rectangle whose area will be equal to that of the parallelogram. Find the area of the parallelogram and of the triangle.
5. Make a trapezoid with a base 4 inches long. Make a rectangle upon the trapezoid which will have the same area as the trapezoid. Find the area.
6. Make a triangle with a base 4 inches long and an altitude  $3\frac{1}{2}$  inches. Make another triangle with a base  $3\frac{1}{2}$  inches long which shall be equal in area to the first triangle.
7. Make a regular pentagon with a side  $1\frac{1}{2}$  inches long. Find the center. Divide the pentagon into triangles. Find the area.
8. Make a regular hexagon with a side  $1\frac{1}{2}$  inches long. Find the center. Divide the hexagon into triangles. Find the area.
9. Make a square  $2\frac{3}{4}$  inches long. Inscribe a circle. Find the area of the circle, and of the part of the square outside the circle.

## Original Problems

*Make problems and give answers :*

1. One of the two equal means in a proportion is 12. One of the extremes is 6.
2. One of the extremes in a proportion is 8 and the other is 18.
3. 4 men can dig a ditch 7 rods long in a day.
4. A pipe 4 inches in diameter and 12 feet long is filled with water.
5. The moon revolves around the earth in 27 da. 7 hr. 43 min. 11 sec.
6. The planet Uranus revolves around the sun in 30,686.8 days.
7. The distance of Uranus from the sun is 1,753,869,000 miles.
8. 5 men can build a piece of fence in 3 days.
9. One rectangle is 12 feet long and 10 feet wide. Another is 8 feet wide.
10.  $10\frac{1}{2}$  acres of grass are sufficient for three cows.
11. In carrying a barometer up a mountain to a certain height it falls 3 inches.
12. The base of a prism contains 12 square feet, and its altitude is 6 feet.
13. The diameter of a cylinder is 26 inches.
14. The base of a square pyramid is 8 feet square, and the altitude is 15 feet.
15. A cylindrical standpipe 20 feet in diameter is filled with water to a depth of 24 feet.
16. A cubic foot of marble weighs 2800 ounces.
17. *The specific gravity of sea water is about 1.025.*

## Algebra

1. Add  $5x^3 - 3xy$ ,  $5xy - 3y^2$ ,  $8y^2 - 6x^2$ .
2. Add  $m^2 + mn + n^2$ ,  $m^2 - 3n^2$ ,  $2m^2 - 3mn - 6n^2$ .
3. From  $a^2 + 2ab + b^2$  take  $a^2 - 2ab + b^2$ .
4. From  $7a^2b^2 - ab^2 + 8ab^3c$  take  $2ab^2 + 4a^2b^2 - ab^3c$ .
5. Multiply  $3x^2y^2 - 5xy^3 + y^4$  by  $4xy^2$ .
6. Multiply  $a^2 - 2ab + b^2$  by  $a - 2b$ .
7. Divide  $8m^3 - 4m^2 + 6m$  by  $2m$ .
8. Divide  $15m^4n^5 - 12m^3n^5 + 21m^2n^7$  by  $3m^2n^4$ .
9.  $x + 6 - \frac{2x - 7}{3} = 11$ .
10.  $\frac{x + 7}{3} = 7 - \frac{x - 4}{7}$ .
11.  $\frac{x - 2}{3} - \frac{x - 6}{4} = \frac{x - 2}{6}$ .
12.  $\frac{3x + 5}{2} - 3(x + 1) = 32 - 5(x - 1)$ .
13.  $8x - \frac{2(x - 4)}{4} = 10x - \frac{10(x + 2)}{5}$ .

14. A number of coins amount to \$9. The number of quarter dollars is one half of the number of dimes. How many of each are there?

Let  $x$  = number of dimes and  $\frac{x}{2}$  number of quarters. Then  $10x$  = value of dimes in cents and  $\frac{25x}{2}$  = value of quarters in cents.  
 $10x + \frac{25x}{2} = 900$ .

15. After a man had sold  $\frac{1}{2}$  of his chickens and  $\frac{1}{5}$  of them, he had remaining 12 more than  $\frac{1}{4}$  of them. How many had he at first?

16. A man bought a horse and a cow for \$95. The cow cost \$25 more than  $\frac{1}{3}$  as much as the horse. What was the cost of each?

## Problems for Arithmetic or Algebra

1. 12 is what per cent of 50?
2. 56 is 30% less than a certain number. What is the number?
3. 60 is 20% more than some number. What is the number?
4. An agent in Chicago purchased for me a car load of flour at \$3.80 a barrel. His commission was  $2\frac{1}{2}\%$ . The car contained 120 barrels. The freight bill was \$45. How much did the agent pay for the flour?
5. What was the amount of his commission?
6. How much was the freight per barrel?
7. What was the entire cost of the car load of flour?
8. For what price per barrel must I sell the flour to gain 15% of the cost?
9. By the reduction of the rate of taxation in a certain town from 17 mills to 14 mills on a dollar a man's tax was made \$6 less. For how much was his property assessed?
10. If an agent charges \$105 for purchasing goods to the value of \$4200, what is his rate of commission?
11. An agent sold 500 bushels of potatoes at 60 cents a bushel, and charged \$15. What was his rate?
12. If the sum of the principal and interest of a note for a year, at 5%, is \$367.50, what is the principal?
13. If I have a note of \$450 due in 2 years without interest, and the money would gain 6% if I had it now to invest, what is the real value of the note?

What sum, if invested now, would amount to \$450 in 2 years?  
 $\$450 \div 1.12 = ?$

14. What is the present value of a note of \$600 due in 6 mo. without interest, if the use of money is worth 4%?

## SUMMARY OF PROCESSES WITH DECIMAL FRACTIONS

### Addition and Subtraction of Decimal Fractions

*To add or subtract decimal fractions or mixed numbers :*

1. Add .065, 16.18 and 5.0103.

$$\begin{array}{r} .065 \\ 16.18 \\ \underline{5.0103} \\ 21.2553 \end{array}$$
 Since decimal numbers decrease toward the right in the same way as the whole numbers, care must be taken to add or subtract in each column only numbers of the same order.

2. Subtract 9.1004 from 36.05.

$$\begin{array}{r} 36.05 \\ \underline{9.1004} \\ 26.9496 \end{array}$$
 Since ciphers written at the right of decimals, as at the left of whole numbers, are of no value, the vacant places in the column may be imagined to be filled with ciphers.

3. Add 2.5, 18.52, 375, .248.
4. Add 16.34, 9.005, 100.5, .092.
5. Add 137.04, 95, .17, 8.0675.
6. Add 1840.3, 375, .0032, 12.0102.
7. From 35.82 take 7.4.
8. From 4.203 take .0507.
9. From 23.0303 take .30303.
10. From .0161 take .005151.
11. Find the sum of eight thousandths, seven hundredths, and fourteen ten-thousandths.
12. Find the sum of twelve and four tenths, seven hundred fifty, and three hundred seventy-two hundred-thousandths.
13. From eight and fifteen thousandths take two and nine ten-thousandths.
14. From twenty-three thousandths take twelve millionths.
15. Find the sum of eighty-five and forty-six ten-thousandths, two hundred seven hundred-thousandths, and eight thousand twenty-four millionths.

## Multiplication of Decimal Fractions

*To multiply decimal fractions or mixed numbers :*

1. Multiply .027 by .08. 4.35 by 2.4.

(a)  $.027 \times .08 = \frac{27}{1000} \times \frac{8}{100} = \frac{216}{100000}$  or .00216

$\begin{array}{r} .027 \\ .08 \\ \hline .00216 \end{array}$  In decimal fractions the figures given are simply the numerators of the fractions. The denominators are indicated by the positions of the decimal points. Since thousandths multiplied by hundredths will produce hundred-thousandths, we must have 5 decimal places in the product of (a).

(b)  $\begin{array}{r} 4.35 \\ 2.4 \\ \hline 1740 \\ 870 \\ \hline 10.440 \end{array}$   $4.35 \times 2.4 = \frac{435}{100} \times \frac{24}{10} = \frac{10440}{1000}$  or 10.440

Since hundredths multiplied by tenths produce thousandths, we must have 3 decimal places in the product of (b).

*The decimal places in the product must equal the sum of those in the multiplicand and the multiplier.*

*Multiply :*

- |                   |                      |
|-------------------|----------------------|
| 2. 16 by .16      | 17. 2.756 by 13.59   |
| 3. 8.5 by .245    | 18. .1250 by 2.50    |
| 4. 625 by .025    | 19. 26 by 3.1416     |
| 5. 5.013 by .001  | 20. 19.35 by 3.1416  |
| 6. 1.728 by 120   | 21. 5100 by .0051    |
| 7. 2050 by .205   | 22. 1000 by .001     |
| 8. 60.01 by .2002 | 23. 20.050 by 36     |
| 9. .0056 by 36.04 | 24. 178.92 by 6.300  |
| 10. 352 by 1.056  | 25. 1.025 by .018    |
| 11. 75.9 by .001  | 26. 42 by 1.7036     |
| 12. 1200 by .003  | 27. 200 by .002      |
| 13. .0015 by 15   | 28. 1000 by .022     |
| 14. .0101 by 99   | 29. 1500 by .15      |
| 15. .1213 by 18   | 30. 9.0102 by .005   |
| 16. 19.19 by .019 | 31. 35.107 by 24.209 |

## Division of Decimal Fractions

*To divide decimal fractions or mixed numbers :*

1. Divide 48.2 by .024.

FIRST EXPLANATION

$$\begin{aligned} 48.2 \div .024 &= \frac{482}{100} \div \frac{24}{1000} = \\ &= \frac{48200}{1000} \div \frac{24}{1000} = 1800 \end{aligned}$$

We make the numbers similar by changing the dividend to thousandths. The division is then a division of the numerators and the answer is a whole number.

When the number of decimal figures in the dividend is not equal to the number in the divisor, we make it equal by annexing ciphers, and then divide as in whole numbers.

2. Divide 1.404 by .26. 79.82 by .325.

(a)	SECOND EXPLANATION	(b)
$\frac{5.4}{.26)1.404}$		$\frac{245.6}{.325)79.8200}$

Dividing (a) as in whole numbers, the result is 5.4. Since the number of decimal places in a product equals the *sum* of the number in multiplicand and multiplier, the number of places in a quotient must equal the *difference* between the number in dividend and divisor. This difference is 1. We therefore point off one decimal place and our answer is 5.4.

When as in (b) the number of decimal places in the divisor exceeds those in the dividend, ciphers should be annexed to the dividend until its number of places equals or exceeds those in the divisor. As the annexing of decimal ciphers does not alter the value of a decimal, the division may be extended as far as desirable by annexing such ciphers to the dividend.

*We always point off as many decimal places in the quotient as the number of decimal places in the dividend exceeds the number in the divisor.*

- |                        |                         |                            |
|------------------------|-------------------------|----------------------------|
| 3. $.375 \div 5 = ?$   | 8. $15.73 \div .13 = ?$ | 13. $76 \div 10.87 = ?$    |
| 4. $.375 \div .5 = ?$  | 9. $1.573 \div 1.3 = ?$ | 14. $25 \div 3.1416 = ?$   |
| 5. $.375 \div .05 = ?$ | 10. $1573 \div .13 = ?$ | 15. $17.2 \div 3.1416 = ?$ |
| 6. $15.73 \div 13 = ?$ | 11. $6.39 \div 60 = ?$  | 16. $.0036 \div 125 = ?$   |
| 7. $.1573 \div 13 = ?$ | 12. $50 \div .005 = ?$  | 17. $6.4 \div .0021 = ?$   |



## Decimal Fractions and Common Fractions

*To change a decimal fraction to a common fraction:*

1. Change .125 to a common fraction in its lowest terms.

$$.125 = \frac{125}{1000} = \frac{1}{8}$$

Since in a decimal fraction the denominator is understood, we need simply change the decimal fraction to the form of a common fraction, and then reduce it to lower terms if possible.

*Change to common fractions in their lowest terms:*

2. .8	7. .08	12. .175
3. .16	8. .025	13. .0125
4. .35	9. .006	14. .0065
5. .04	10. .015	15. .0625
6. .45	11. .625	16. .1875

*To change a common fraction to a decimal fraction:*

17. Change  $\frac{5}{8}$  to a decimal fraction.

$$\frac{5}{8} = 5 \div 8. \quad \begin{array}{r} .625 \\ 8 \overline{)5.000} \end{array}$$

Annex decimal ciphers to the numerator and divide by the denominator. If there still is a remainder, continue to annex ciphers as far as desired.

In some cases the change may be made at sight by changing the fraction so that its denominator will be some multiple of 10.

$$\frac{1}{5} = \frac{2}{10} \text{ or } .2$$

$$\frac{3}{8} = \frac{375}{1000} \text{ or } .375$$

*Change to decimal fractions:*

18. $\frac{1}{2}$	22. $\frac{7}{8}$	26. $\frac{7}{11}$	30. $\frac{7}{115}$
19. $\frac{2}{5}$	23. $\frac{1}{6}$	27. $\frac{15}{16}$	31. $\frac{25}{1284}$
20. $\frac{5}{8}$	24. $\frac{2}{3}$	28. $\frac{13}{18}$	32. $\frac{9}{750}$
21. $\frac{7}{10}$	25. $\frac{5}{9}$	29. $\frac{19}{21}$	33. $\frac{47}{2856}$

## French and German Money

In French money 100 centimes make a franc. A franc equals \$0.193 in United States money.

In German money 100 pfennigs equal a mark. A mark equals \$0.2385 in United States money.

1. About how many centimes are equivalent to a cent in United States money?

2. What is the exact equivalent of a centime in United States money?

3. Find the value in United States money of 20 francs, 60 centimes.

4. Find the exact value in French money of 25¢. Of \$1. Of \$5.

5. About how many German pfennigs are equivalent to a cent?

6. What is the exact equivalent of a pfennig in United States money?

7. Find the value in United States money of 10 marks, 50 pfennigs.

8. Find the value in German money of 40¢. Of \$30.

*Find the exact value of the following in United States money:*

- |                 |                              |
|-----------------|------------------------------|
| 9. 35 centimes  | 14. 25 francs, 30 centimes   |
| 10. 75 centimes | 15. 150 francs, 12 centimes  |
| 11. 200 francs  | 16. 1250 francs, 40 centimes |
| 12. 64 pfennigs | 17. 15 marks, 30 pfennigs    |
| 13. 800 marks   | 18. 2520 marks, 60 pfennigs  |

*Find the value of the following in French money:*

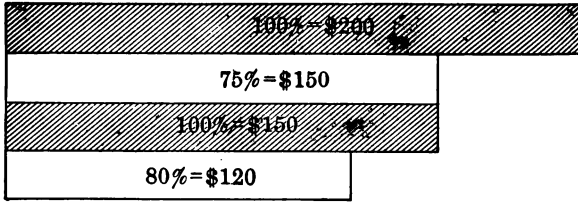
- |         |            |           |              |
|---------|------------|-----------|--------------|
| 19. 12¢ | 21. 85¢    | 23. \$100 | 25. \$24.50  |
| 20. 50¢ | 22. \$1.00 | 24. \$165 | 26. \$148.20 |

*Find the value of the following in German money:*

- |         |            |           |              |
|---------|------------|-----------|--------------|
| 27. 42¢ | 29. 95¢    | 31. \$150 | 33. \$36.30  |
| 28. 63¢ | 30. \$1.00 | 32. \$700 | 34. \$173.50 |

## Percentage

Discounts of 25% and 20% on \$200



Goods are sometimes subject to two or three discounts below the list price when sold. Such discount is called **commercial discount** or **trade discount**.

1. If shoes listed at \$4.00 a pair are discounted 20%, what is the discount price ?

$$80\% \text{ of } \$4 = ?$$

2. If the shoes are again discounted at 10% below the first discount price, what is the second discount price ?

$$80\% \text{ of } \$4 \times 90\% = ?$$

3. What would have been the price if the shoes had been discounted but once at 30% ?

4. What is the selling price of goods listed at \$1.00 and subject to discounts of 25% and 20% ?

$$75\% \text{ of } \$1.00 \times 80\% = ?$$

5. A wholesale merchant sold a bill of goods. The list price was \$480 and the discounts  $16\frac{2}{3}\%$  and 10%. What was the net amount of the bill ?

6. Find the difference between the net amounts of a bill of goods for \$400 if sold at trade discounts of 25% and 15%, or at a single discount of 40%.

7. If I should buy goods which are listed at \$500, at 20% below the list price, and sell them at 10% below the list price, how much should I gain ?

8. What per cent should I gain on what they cost me ?

## Miscellaneous Problems

1. If 12 gas burners, each of which consumes 5 cu. ft. of gas an hour, are burned 3 hours each night, what will be the cost of the gas for a year of 365 days, at \$1.25 a thousand feet?

2. What is the circumference of a wheel that revolves 3820 times in going 10 miles?

3. How many feet of lumber are there in 12 scantlings, each 4 inches square at the end, and 15 feet long?

4. What principal will gain \$51.20 in 3 yr. 2 mo. 12 da. at 5%?

5. What principal will amount to \$500 in 2 years at 6%?

6. What is the present worth of a note of \$200 which is payable in one year without interest, if the use of money is worth 6%?

The present worth is the *principal* which will amount to \$200 in a year at 6%. Therefore it is  $\$200 \div 1.06$ .

7. What is the present worth of a note which promises to pay \$360 in six months, if money is worth 4%?

8. There are 231 cubic inches in a gallon. Find the weight of a gallon of water.

9. The specific gravity of a certain ore is 6.4. What would be the weight of a piece of the ore large enough to displace a gallon of water?

10. What would be the weight of the same piece of ore in water?

11. If the specific gravity of sea water is 1.026, what would be the weight in sea water of a cubic foot of metal whose specific gravity is 5.138?

## Square Root—Cube Root

The **square root** of a number is equal to the product of one half of the factors in each of its groups of equal factors. The **cube root** of a number is equal to the product of one third of its factors. The **fourth root** is equal to the product of one fourth of the factors, etc.

$324 = 2 \times 2 \times 3 \times 3 \times 3 \times 3$ . The square root of 324 is  $2 \times 3 \times 3$ , or 18.

$1000 = 2 \times 2 \times 2 \times 5 \times 5 \times 5$ . The cube root of 1000 is  $2 \times 5$ , or 10. In this way it is easy to find a root of a number which is a perfect power, that is, a number whose root may be found without a remainder.

1. What are the smallest factors of 36?
2. What is the square root of 36?
3. What are the smallest factors of 216?
4. What is the cube root of 216?
5. Find the factors and the square root of 225.
6. Find the factors and the cube root of 1000.
7. Find the cube root of 1728.
8. Find the cube root of 5832. Of 27,000.
9. What is the fourth root of 810,000?
10. What is the length of one side of a square which contains 1764 sq. in.?
11. What is the length of a square lot of land which contains 2025 sq. ft.?
12. Find the length of the edge of a cube which contains 13,824 cu. in.
13. Find the length of the hypotenuse of a right triangle, whose sides are 72 ft. and 54 ft.
14. Find the length of the edge of a cube, whose volume is equal to the volume of a rectangular solid which  $9' \times 4' \times 2'$ .

### Miscellaneous Problems

1. What is the bank discount of a note of \$450, due in 6 months, discounted at a bank at 6%?

2. What are the proceeds of a note of \$240, due in 4 months, discounted at a bank at  $4\frac{1}{2}\%$ ?

3. A note due in 3 months was discounted at a bank at 8% and yielded \$196 proceeds. What was the face of the note?

The discount is 2% of the face, and the proceeds 98% of the face. Therefore \$196 = 98% of the face, and the face = \$196  $\div$  .98.

4. A note due in 9 months was discounted at a bank at 4% and yielded \$3055.50 proceeds. Find the face of the note.

5. A note due in 2 months was discounted at a bank at 6% and yielded \$561.33 proceeds. Find the face of the note.

6. What is the gain per cent in buying a horse for \$150 and selling him for \$180?

7. What is the loss per cent in buying a house for \$2000 and selling it for \$1950?

8. On a bill of goods with a list price of \$200, is it better to accept trade discounts of 25 and 5, or of 20 and 10, and how much is the difference?

9. I bought goods, the price of which was \$360, at discounts of 20%, and  $16\frac{2}{3}\%$ . I sold the goods for \$300. What was the gain per cent?

10. What is the amount of a bill of goods, the list price of which is \$425.75, subject to trade discounts of 15% and 12%?

11. I sold goods for \$280, at an advance of 12% above the cost. What was the cost?

## Review

- |  |  |
|--|--|
| <p>1. <math>\frac{7}{8} + \frac{4}{5} + \frac{9}{10}</math></p> <p>2. <math>1\frac{1}{2} + \frac{2}{3} + 1\frac{7}{8}</math></p> <p>3. <math>\frac{5}{9} + 1\frac{4}{5} + \frac{1}{12}</math></p> <p>4. <math>\frac{3}{7} + \frac{4}{5} + \frac{1}{6}</math></p> <p>5. <math>1\frac{3}{5} + 1\frac{1}{30} + \frac{7}{48}</math></p> <p>6. <math>\frac{9}{10} + \frac{4}{35} + \frac{27}{50}</math></p> | <p>7. <math>\frac{7}{12} - 1\frac{9}{12}</math></p> <p>8. <math>\frac{37}{56} - 1\frac{1}{84}</math></p> <p>9. <math>\frac{49}{50} - \frac{49}{80}</math></p> <p>10. <math>21\frac{1}{2} - 21\frac{3}{8}</math></p> <p>11. <math>3\frac{9}{16} - 1\frac{17}{24}</math></p> <p>12. <math>6\frac{5}{48} - \frac{7}{60}</math></p>  |
| <p>13. <math>\frac{7}{18} \times 5</math></p> <p>14. <math>9 \times \frac{5}{27}</math></p> <p>15. <math>\frac{7}{36} \times 12</math></p> <p>16. <math>\frac{9}{57} \times 18</math></p> <p>17. <math>17\frac{2}{3} \times 12</math></p> <p>18. <math>48 \times 23\frac{5}{8}</math></p>  | <p>19. <math>\frac{6}{7} \times \frac{1}{2}</math></p> <p>20. <math>\frac{3}{4} \times 1\frac{2}{7}</math></p> <p>21. <math>\frac{3}{8}</math> of <math>48\frac{8}{11}</math></p> <p>22. <math>\frac{5}{8}</math> of <math>1\frac{2}{3}</math></p> <p>23. <math>25\frac{5}{8} \times 36\frac{4}{5}</math></p> <p>24. <math>3\frac{5}{8} \times 2\frac{9}{8}</math></p> |
| <p>25. <math>1\frac{5}{8} \div 3</math></p> <p>26. <math>1\frac{7}{8} \div 7</math></p> <p>27. <math>12 \div \frac{3}{8}</math></p> <p>28. <math>1\frac{6}{7} \div 1\frac{4}{7}</math></p> <p>29. <math>\frac{9}{10} \div \frac{1}{8}</math></p> <p>30. <math>1\frac{3}{4} \div \frac{4}{36}</math></p>  | <p>31. <math>\frac{7}{8} \div 3\frac{1}{2}</math></p> <p>32. <math>9\frac{7}{12} \div \frac{3}{4}</math></p> <p>33. <math>35\frac{1}{2} \div 7\frac{1}{4}</math></p> <p>34. <math>42\frac{2}{3} \div 4\frac{1}{5}</math></p> <p>35. <math>58\frac{7}{9} \div 9\frac{2}{7}</math></p> <p>36. <math>135\frac{4}{5} \div 12\frac{3}{7}</math></p>                         |
| <p>37. <math>1\frac{1}{2} + 1\frac{4}{5}</math></p> <p>38. <math>1\frac{5}{7} + \frac{8}{9}</math></p> <p style="padding-left: 2em;"><math>\frac{25}{27} - \frac{7}{18}</math></p> <p style="padding-left: 2em;"><math>\frac{49}{15} - 1\frac{1}{7}</math></p>   | <p>41. <math>\frac{37}{8} \times 1\frac{8}{7}</math></p> <p>42. <math>24\frac{1}{3} \times 16\frac{1}{5}</math></p> <p>43. <math>1\frac{5}{8} \div \frac{30}{31}</math></p> <p>44. <math>75\frac{7}{8} \div 13\frac{2}{7}</math></p>   |

### Problems from Physics

1. The specific gravity of iron is 7.2. What will a cubic foot of it weigh in the air?
2. What will a cubic foot of iron weigh in water?
3. The specific gravity of marble is 2.8. What will 144 cu. in. of it weigh in the air?
4. What will 144 cu. in. of marble weigh in water?
5. If the specific gravity of sea water is 1.026, what is the weight of a cubic foot of sea water?
6. How much less will a cubic foot of any substance weigh in sea water than in air?
7. How much less will a cubic foot of any substance weigh in sea water than in pure water?
8. If  $\frac{1}{2}$  of a cubic foot of silver weighs 4735 ounces when weighed in pure water, what is its specific gravity?
9. If  $\frac{1}{3}$  of a cubic foot of lead weighs 2587 ounces in water, what is its specific gravity?
10. The specific gravity of milk is 1.03. What is the weight of a cubic foot of milk?
11. How much less will an object which contains  $\frac{1}{4}$  of a cubic foot weigh in milk than in the air?
12. How much less will it weigh in milk than in pure water?
13.  $\frac{1}{5}$  of a cubic foot of gold would weigh 3850 ounces in the air. How much would it weigh in water?
14. What is the specific gravity of gold?
15. Find how much a cubic inch of gold would weigh in the air.



## Cones

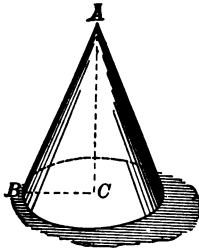


FIG. 1.

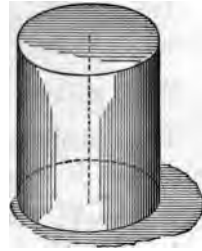


FIG. 2.

See pp. 45, 167.

*The volume of a cone is  $\frac{1}{3}$  of the volume of a cylinder having an equal base and altitude, or  $\frac{1}{3}$  of the product of the area of its base and its altitude.*

1. What is the volume of a cone, if the area of the base is 13 feet and the altitude 9 feet?

2. What is the area of the base of a cone whose diameter is  $16\frac{1}{2}$  inches?

3. What is the volume of a cone, if the diameter of the base is  $16\frac{1}{2}$  in. and the altitude 30 in.?

4. In Fig. 1, what kind of a triangle is formed by the slant height  $AB$ , the altitude  $AC$ , and the radius  $BC$ ?

5. If  $AB$  is 25 inches, and  $BC$  15 inches, what is the altitude  $AC$ ?

6. What is the altitude of a cone the diameter of whose base is 24 ft. and whose slant height is 20 ft.?

7. Find the volume of a cone the circumference of whose base is 31.416 ft. and whose altitude is 12 ft.

Find the diameter and area of the base.

8. How many cubic feet are there in a cone whose altitude is 12 ft. and the diameter of whose base is 4 ft.?

9. Find the convex surface of a cone the circumference whose base is 37.6992 ft. and whose slant height is 10 ft.

## Powers and Roots

Review p. 118.

$\sqrt{4}$  indicates the square root of 4;  $\sqrt[3]{8}$  the cube root of 8;  $\sqrt[4]{16}$  the fourth root of 16, etc.

$3^2$  indicates the square of 3 or the product of 3 and 3.  $4^3$  indicates the third power of 4, or the product of 4, 4, and 4. The small figure indicating the power is called the exponent.

1.  $5^2 = ?$

2.  $7^3 = ?$

3.  $10^4 = ?$

4.  $9^5 = ?$

5.  $18^3 = ?$

11.  $\sqrt{81} = ?$

12.  $\sqrt{100} = ?$

13.  $\sqrt[3]{64} = ?$

14.  $\sqrt[3]{125} = ?$

15.  $\sqrt[3]{216} = ?$

6.  $(\frac{1}{4})^2 = ?$

7.  $(\frac{3}{5})^3 = ?$

8.  $(\frac{7}{8})^4 = ?$

9.  $.9^3 = ?$

10.  $.15^4 = ?$

16.  $\sqrt{\frac{9}{25}} = ?$

17.  $\sqrt{.64} = ?$

18.  $\sqrt[3]{.216} = ?$

19.  $\sqrt[3]{\frac{27}{64}} = ?$

20.  $\sqrt[4]{\frac{16}{81}} = ?$

*Find the square root of the following:*

21. 169

22. 256

23. 324

24. 400

25. 484

26. 784

27. 1024

28. 1225

*Find the cube root of the following:*

29. 216

30. 343

31. 729

32. 5832

33. 8000

34. 13,824

35. 19,683

36. 32,768

### Miscellaneous Problems

1. What is the square root of 225?
2. What is the square root of 289?
3. What is the cube root of 729?
4. What is the cube root of 1728?
5. Find the fifth power of 8.
6. Find the fourth power of .25.
7. What is the volume of a cube whose edge is 2.03 inches?
8. What is the volume of a cube, one of whose sides is 121 square inches?
9. A can do a piece of work in 10 days, and B can do the same amount in 12 days. How long would it take both together to do the work?

What part of the work can each do in one day? What part can both together do in one day? How long, then, will it take both together to do the whole work?

10. A can build a piece of fence in 15 days. B could build it in 18 days, and C in 30 days. In how many days could they build the fence if all should work together?
11. If A can do a piece of work in 8 days, and B in 12 days, what part of the work can both together do in one day?
12. What part of the work can both together do in 3 days?
13. What part would remain to be done after they had worked together upon it for 3 days?
14. If I should sell goods at an advance of 15% above the cost, and by so doing should gain \$6.60, what is the cost?

## Stocks

1. The capital of a certain stock company is \$40,000. Profits amounting to \$3600 are to be distributed among the stockholders. How much profit is there for each \$100 of the stock?

2. What is the per cent of dividend to be declared?

3. How much will a stockholder receive who holds 25 shares of \$100 each?

4. What per cent will a share of stock pay which costs \$80 and which yields 4% a year reckoned upon the par value of \$100?

The income on \$100 is \$4; \$4 is what per cent of \$80?

5. What is the per cent of income upon a share of 5% stock whose par value is 100, purchased at 80?

6. What is the per cent of income upon a share of 5% stock whose par value is 50, purchased at 60?

7. When stock costs  $64\frac{7}{8}$ , what will be the cost of 40 shares, including the brokerage of  $\frac{1}{8}\%$ ?

$$40 \times (\$64\frac{7}{8} + \$\frac{1}{8}) = ?$$

8. What will be the cost of 40 shares of stock whose par value is \$50 each, purchased at  $43\frac{5}{8}$  without brokerage?

9. What will be the profit in buying 10 shares of stock whose par value is \$100 each, at 88, and selling at  $95\frac{1}{2}$ , if brokerage of  $\frac{1}{8}\%$  is paid both in buying and selling?

$$\text{Cost} = (\$88 + \$\frac{1}{8}) \times 10; \text{selling price} = (\$95\frac{1}{2} - \$\frac{1}{8}) \times 10.$$

10. How much is gained in buying 30 shares of stock whose par value is 100, at 116, and selling it at  $119\frac{1}{2}$ , paying  $\frac{1}{8}\%$  brokerage in both cases?

11. How much would be lost in buying 50 shares of stock at 117 and selling at  $110\frac{1}{2}$ , paying brokerage in both cases?

### Miscellaneous Problems

1. A man began business with a capital of \$5000. The first year he gained 20%, and added it to his capital. The second year he gained  $16\frac{2}{3}\%$ , and added it to his capital. How much capital had he then?

$$\$5000 \times \frac{3}{4} \times \frac{4}{3} = ? \text{ (Cancel.)}$$

2. A man engaging in business gained 25% the first year, and added it to his capital. The second year he gained 20%. He then had \$6000. How much had he at first?

$$x \times \frac{5}{4} \times \frac{4}{5} = \$6000.$$

3. I loaned \$2000 at 5% interest. At the end of the year I received both the interest and the principal, and loaned the entire amount again at 6%. How much money had I at the end of the second year?

4. I loaned a sum of money at 10%. At the end of each year I collected the interest and added it to the principal. At the end of the second year I had \$1210. How much did I loan at first?

5. What is the simple interest of \$1250 for 3 yr. 5 mo. 17 da. at  $4\frac{1}{2}\%$ ?

6. What amount is due Dec. 15, 1899, on a note of \$500, dated March 5, 1896, and bearing simple interest at 5%?

7. What is the volume of a cylinder whose diameter is 4 ft. 6 in. and length 12 ft.?

8. Find the volume of a cone the diameter of whose base is 8 feet, and whose altitude is 14 feet.

9. Find the convex surface of a cone the diameter of whose base is  $3\frac{1}{4}$  ft., and whose slant height is 12 ft.

10. What is the volume of a pyramid whose base is 7 ft. square, and whose altitude is 24 ft.?

## Work — Energy — Momentum

The amount of work or energy expended in raising one pound one foot against the force of gravity is called a **foot pound**. A foot pound is the unit by which mechanical force is reckoned.

The **momentum** of a moving body depends both upon its weight and the speed with which it is moving. The momentum is found by multiplying the weight of the body in pounds by the number of feet through which it is moving per second.

1. How many foot pounds of force are necessary to raise an object weighing 25 pounds one foot? 25 pounds 3 feet?

2. How much energy is expended in raising a stone weighing 5000 lb. 20 feet?

3. What is the momentum of a body weighing 10 pounds and moving at the rate of 25 feet a second?

4. A mass of 500 pounds moves with a velocity of 20 feet per second. How many units of work are stored up in it?

5. Find the number of units of work in a 24-pound shot leaving a gun with a velocity of 1200 feet per second.

6. A ball weighing 5 pounds is rolling at the rate of 10 feet per second. How many units of momentum has it?

7. If this ball meets another coming in the opposite direction, and the second ball weighs 5 pounds, and is rolling at the rate of 5 feet per second, what will be the result?

8. A rifle bullet weighing 2 ounces leaves a gun with a velocity of 1000 feet per second. What is its momentum?

9. How fast would a cannon ball weighing 50 pounds have to move to have the same momentum as this bullet?

1. What is the bank discount of a note of \$2000, due in 90 days, discounted at 4% ?

2. What will be the proceeds of a note of \$640 when discounted at a bank for 60 days at  $5\frac{1}{2}\%$  ?

3. Reduce to smallest terms:  $\frac{78}{286}$  ;  $\frac{204}{808}$  ;  $\frac{528}{924}$ .

4. What fractional part of a year of 365 days is the time from July 1st to Oct. 14th ?

5. 300 is 20% more than what number ?

6. 300 is 20% less than what number ?

7. If goods are sold for \$4.90, at a loss of  $12\frac{1}{2}\%$ , what was the cost ?

8. If goods which cost \$5.50 are sold for \$6.00, what is the gain per cent ?

9. How many cubic inches of capacity has a dipper in the form of an inverted cone, whose diameter, measured across the top, is 10 inches, and whose depth is 12 inches ?

10. How many gallons does this dipper hold ?

*Find the square root of the following :*

11. 2401

15. 2025

12. 2916

16. 5184

13. 3025

17. 8100

14. 5625

18. 10,000

19.  $\sqrt{.09} = ?$

23.  $\sqrt{\frac{25}{86}} = ?$

20.  $\sqrt{.0625} = ?$

24.  $\sqrt{\frac{86}{625}} = ?$

21.  $\sqrt[3]{.008} = ?$

25.  $\sqrt[3]{\frac{8}{27}} = ?$

22.  $\sqrt[3]{.125} = ?$

26.  $\sqrt[3]{\frac{125}{216}} = ?$

## Review Problems

1. The base of a right triangle is 28 ft., and the perpendicular 21 ft. Find the hypotenuse.
2. What is the length of the longest line that can be drawn on the floor of a room  $20' \times 15'$ ?
3. A lot of land in the form of a square contains 8100 sq. ft. What is the length of its side?
4. A house is 48 ft. wide and its roof extends 32 ft. above the eaves. The rafters project 2 ft. over the eaves. How long are the rafters?
5. The floor of a hall contains 1152 sq. ft. Its width is  $\frac{1}{2}$  of its length. How long is it?
6. A piece of land contains 3072 sq. rd. Its width is  $\frac{1}{3}$  of its length. How long is it?
7. What is the length of one edge of a cube which contains 13,824 cubic inches?
8. The height of a regular square prism is twice the width of its base. It contains 11,664 cubic inches. What is the width of its base?
9. What fraction, when squared, will become  $\frac{4}{9}$ ?
10. What fraction, when cubed, will become  $\frac{27}{125}$ ?
11. What is the length of one side of a square which contains  $\frac{2}{5}$  of a square rod?
12. What is the length of one edge of a cube which contains  $\frac{8}{27}$  of a cubic yard?
13. Find the area of a circle inscribed in a square whose length is 15 inches.
14. Find the length of a cube which contains 8000 cubic inches.



## Percentage

Advance of 20% and reduction of 25%.

100		
120		
90		

1. A merchant marked goods to sell at a profit of 20%, but was afterwards obliged to reduce his price 25%. What per cent did he lose upon the goods?

$$120\% \times 75\% \text{ or } \frac{6}{5} \times \frac{3}{4} = 90\%. \quad \text{Loss} = 10\%$$

2. A merchant sold goods to a customer at a profit of 20%, but was afterwards able to collect only 50% of the bill. What per cent did the merchant lose on the sale?

3. A merchant bought cloth, the list price of which was 75 cents, at a discount of 20%, and sold it at an advance of 25% on the cost. What was the selling price?

4. If 25% is gained by selling cloth at \$1 a yard, find what per cent will be gained or lost by selling it at 70 cents a yard.

5. A real estate dealer sold a lot of land for \$840, gaining 20% on the cost. What was the cost?

6. He sold another lot for \$840, losing 20% of the cost. What was the cost of this lot?

7. How much did he gain or lose by the two sales?

8. The selling price of goods is at a loss of 20%. What per cent must the selling price be advanced in order that there may be no loss?

80% is how many per cent less than 100%? What per cent of 80% is 20%?

9. What per cent would it be necessary to advance the selling price in order to gain 10%? 30%? 50%?

## Surveyors' Measure

In measuring land surveyors use what is called surveyors' measure. The chain used is  $\frac{1}{80}$  of a mile long. Each chain contains 100 links.

1. How many rods long is a chain? How many feet?
2. Find how many inches there are in a link.
3. How long is the side of a field which measures 10 chains and 20 links?
4. How many acres are there in a rectangular field 35 chains long and 16 chains 25 links wide?
5. A square piece of land measures 40 chains on a side. How many miles is it around it?
6. How many chains are there in 1320 feet?
7. How many chains in 1584 inches?
8. How many feet in 3 mi. 20 ch.?
9. What part of a mile is 16 chains?
10. How many chains is it around a township 6 miles square?
11. How many square chains are there in a quarter-section of the township?
12. How many square chains in a square mile?
13. What part of a square mile is 10 sq. ch.?
14. What part of a square mile is a lot of land 10 chains square?
15. How many square chains in an acre?
16. How many times will a chain have to be placed in order to measure the distance around 10 acres in the form of a square?
17. How many acres in 6750 sq. ch.?
18. What would be the cost of 22 acres, 20 square rods of land, at \$10.50 a square chain?

## Original Problems

*Make problems and solve them :*

1. The product of three equal numbers is 125.
2. A house worth \$3000 is rented for \$400.
3. By selling goods for \$28 I lose  $12\frac{1}{2}\%$ .
4. The specific gravity of a certain ore is 6.5.
5. A piece of this ore, when lowered into water, causes 500 ounces to overflow.
6. Shoes listed at \$3.00 a pair are subject to discounts of 20% and  $12\frac{1}{2}\%$ .
7. On a bill of \$500 there are three discounts of 20%, 15%, and 5%.
8. Each of four gas burners consumes 4 cubic feet of gas an hour.
9. I have a note of \$250 due in 6 months without interest.
10. A square field contains 676 square rods.
11. The two extremes of a proportion are 2 and 32.
12. A rectangle is 20 inches long and 12 inches wide.
13. A note, due in 60 days, was discounted at a bank at 5%.
14. A bill of goods amounting to \$324.50 is subject to trade discounts of 25% and 8%.
15. The diameter of the base of a cone is 16 inches, and its altitude is 3 feet.
16. A can do a piece of work in 6 days and B in 10 days.
17. A stone weighing 800 lb. was elevated 12 feet by a hoisting crane.
18. An engine is capable of raising 6 tons 10 feet in *one minute*.

## Algebra

Review pp. 118, 203.

1. Find the smallest factors of  $30 a^{21} b^5 x^3$ .
2. Divide the factors of  $64 m^4 x^8$  into two equal groups.
3. Find the square root of  $81 a^4 b^6 c^2$ .
4. Find the cube root of  $216 x^3 y^9 z^6$ .
5. If  $16 x^4 y^2$  represents the area of a certain square field, what will represent its length?
6. If  $125 m^3 n^{12}$  represents the volume of a cubical block of marble, what will represent its length?

*Simplify these expressions :*

- |                              |                        |
|------------------------------|------------------------|
| 7. $\sqrt{a^2 x^2}$          | 12. $(a^2)^2$          |
| 8. $\sqrt{4 m^4}$            | 13. $(2xy)^3$          |
| 9. $\sqrt{25 m^2 n^6}$       | 14. $\sqrt{a^4}$       |
| 10. $\sqrt[3]{a^6 b^3}$      | 15. $\sqrt{25 b^2}$    |
| 11. $\sqrt[3]{8 a^9 b^{12}}$ | 16. $\sqrt[4]{16 a^4}$ |

17. If the square root of a number is 7, what is the number?
  18. If  $\sqrt{x} = 12$ , what is the value of  $x$ ?
  19. If  $3\sqrt{x} = 24$ , what is the value of  $x$ ?
- $\sqrt{x} = 24 \div 3$ , or 8. Find  $x$ .

*Simplify :*

- |                             |                                 |
|-----------------------------|---------------------------------|
| 20. $2\sqrt{a^2}$           | 23. $2a\sqrt{b^2}$              |
| 21. $5\sqrt{16 x^4}$        | 24. $4a^2\sqrt{4 a^2}$          |
| 22. $10\sqrt[3]{8 a^3 b^6}$ | 25. $3ay\sqrt[3]{8 a^3 y^{12}}$ |

*Find the value of  $x$  :*

- |                       |                          |
|-----------------------|--------------------------|
| 26. $\sqrt{x} = 40$   | 29. $\sqrt[3]{x} = 12$   |
| 27. $2\sqrt{x} = 30$  | 30. $4\sqrt[3]{x} = 60$  |
| 28. $5\sqrt{x} = 100$ | 31. $6\sqrt[3]{x} = 300$ |

## Algebraic Problems

*Make equations and solve them:*

1. The surface of a triangle is  $126\frac{1}{2}$  feet. Its base is 23 feet. Find the altitude.

2. One of the sides of a right triangle is twice the other side. The square of the hypotenuse is 80 in. Find the length of the shorter side.

3. The width of a rectangle is three fourths of its length. The diagonal is 20 feet. Find the width.

4. The surface of a parallelogram is 240 sq. ft. The distance between two parallel sides is  $12\frac{3}{4}$  feet. Find the length of one of the parallel sides.

5. The surface of an isosceles triangle is 144 sq. ft. Its altitude is twice its base. What is the altitude?

6. One of the two parallel sides of a trapezoid is  $2\frac{1}{2}$  feet and the other 5 feet. The area is 450 sq. in. Find the perpendicular distance between the parallel sides.

7. The lateral surface of a cylinder is 75.3984 sq. ft. Its length is 6 feet. Find its diameter.

First find the circumference.

8. The length of a square prism is 27 inches. Its volume is 2187 cu. in. Find its width.

9. Find the diameter of the base of a cone whose slant height is 15 inches and whose lateral surface is 235.62 sq. in.

10. The area of a circle is 19.635 sq. in. Find its diameter.

11. The surface of a sphere is 201.0624 sq. in. Find its diameter.

12. The volume of a sphere is 523.6 cu. in. Find its diameter.

## Problems for Arithmetic or Algebra

1. 17 is 25 per cent of what number ?
2. A commission merchant received \$16.80 as his commission for selling goods whose value was \$560. What was his rate ?
3. I paid \$36 as a premium for insuring my house at  $1\frac{1}{2}\%$ . For what sum was the house insured ?
4. The property of a certain man is assessed for \$8500. His tax on the property is \$119. What is the rate of taxation ?
5. What principal will gain \$12.10 in a year at  $5\%$ ?
6. What principal will gain \$11.40 in 3 years at  $4\%$ ?
7. What principal will amount to \$144.20 in a year at  $3\%$ ?
8. What principal will amount to \$382.32 in  $4\frac{1}{2}$  years at  $4\%$ ?
9. In how many years will \$200 gain \$44 interest at  $5\frac{1}{2}\%$ ?
10. How many years will be required for \$1000 at interest at  $7\%$  to amount to \$1245 ?
11. What is the face of a note which, when discounted at a bank for 60 days at  $6\%$ , will yield \$376.20 ?
12. If a note of \$500 gains \$30 interest in 2 yr. 4 mo., what is the rate ?
13. I have a note of \$164.80 due in one year without interest. If I had the cash, the use of it would be worth  $7\%$ . What is the real value of the note at the present time ?
14. Find the present value of a note of \$375, due in 1 yr. 6 mo. without interest, if the use of money is worth  $4\%$ .

## SUMMARY OF PROCESSES IN PERCENTAGE

*To find any per cent of a number.*

1. Find 8% of 340.

$$\begin{array}{r} 3.40 \\ 8 \\ \hline 27.20 \end{array}$$

1% or  $\frac{1}{100}$  of 340 is 3.40, and 8% is 8 times 3.40 or 27.20.

$$\begin{array}{r} 340 \\ .08 \\ \hline 27.20 \end{array}$$

It is equally correct to find eight hundredths in the decimal form.

2. What is 5% of 240 ?
3. What is 15% of 180 ?
4. What is 23% of 500 ?
5. What is  $6\frac{1}{2}\%$  of \$ 315 ?
6. What is 42% of \$ 1850 ?
7. What is  $7\frac{1}{4}\%$  of \$ 86.40 ?
8. What is  $\frac{3}{8}\%$  of \$ 2000 ?
9. What is  $11\frac{1}{2}\%$  of \$ 465 ?

In many cases it is easier to give the per cent the form of a simple fraction and reduce it to smaller terms.

10. Find  $12\frac{1}{2}\%$  of 168.

$$12\frac{1}{2}\% = \frac{12\frac{1}{2}}{100} = \frac{1}{8}. \quad \frac{1}{8} \text{ of } 168 = 21.$$

11. What is 20% of 25 ?
12. What is  $16\frac{2}{3}\%$  of 834 ?
13. What is 25% of 1648 ?
14. What is  $33\frac{1}{3}\%$  of 7500 ?
15. What is 80% of 255 ?
16. What is  $62\frac{1}{2}\%$  of \$ 448 ?
17. What is  $37\frac{1}{2}\%$  of \$ 6.30 ?
18. What is 125% of \$ 932 ?

## Percentage

*To find what per cent one number is of another.*

1. 7 is what per cent of 20.

**FIRST EXPLANATION**

7 is  $\frac{7}{20}$  of 20.  $\frac{7}{20} = \frac{35}{100}$ ; hence 7 is 35% of 20.

This is the easier method when the ratio of the numbers is such that it can be easily changed to hundredths by inspection.

2. 385 is what per cent of 2500 ?

**SECOND EXPLANATION**

385 is  $\frac{385}{2500}$  of 2500;  $385 \div 2500 = 2500 \overline{)385.00} \begin{array}{r} .154 \\ \end{array} = .154 = 15\frac{1}{2}\%$ .

3. 17 is what per cent of 50 ?
4. 13 is what per cent of 20 ?
5. 15 is what per cent of 60 ?
6. 30 is what per cent of 90 ?
7. 160 is what per cent of 640 ?
8. \$25 is what per cent of \$400 ?
9. \$120 is what per cent of \$720 ?
10. 210 men are what per cent of 840 men ?
11. 72 lb. are what per cent of 360 lb. ?
12. \$60 of yearly interest is what per cent of \$1000 ?
13. \$22.50 of commission is what per cent of \$750 collected ?
14. If an investment of \$8000 yields an income of \$420, what is the rate ?
15. If the premium for insurance to the amount of \$2400 is \$30, what rate is charged ?
16. If the tax upon \$5320 of property is \$69.16, how much is the tax rate ?



## Percentage

*To find the number of which the given number is a certain per cent.*

1. 40 is 8% of what number ?

Since 40 is 8% of the required number, 1% of the number is  $\frac{1}{8}$  of 40, or 5, and 100% of the number is 100 times 5, or 500.

2. 25 is  $33\frac{1}{3}\%$  of what number ?

$33\frac{1}{3}\% = \frac{33\frac{1}{3}}{100} = \frac{1}{3}$ . If 25 is  $\frac{1}{3}$  of a certain number, the number must be 3 times 25, or 75.

3. 27 is 3% of what number ?

4. 25 is 5% of what number ?

5. 80 is  $66\frac{2}{3}\%$  of what number ?

6. 25.2 is 4% of what number ?

7. \$2400 is 120% of what sum ?

8. \$3000 is 200% of what sum ?

9. 560 lb. are  $37\frac{1}{2}\%$  of how many pounds ?

10. 480 bu. are 40% of how many bushels ?

11. The yearly interest upon a sum of money at 4% amounts to \$324.16. What is the sum ?

12. The discount of a note discounted at a bank for 6 mo. at 6% was \$25.50. What was the face of the note ?

13. The commission upon the sale of a car load of potatoes at 5% was \$12.35. For how much was the car load sold ?

14. The premium for insuring property for 2 years at  $2\frac{1}{4}\%$  a year was \$72. For how much was the property insured ?

15. In a town where the tax rate is 14 mills a man pays a tax of \$105. Upon how much property does he pay tax ?

## Percentage

*To find a certain per cent more or less than a given number.*

1. What is 12% more than 250 ?

$$\begin{array}{r} 250 \\ 1.12 \\ \hline 500 \\ 250 \\ \hline 250 \\ \hline 280.00 \end{array} \quad \begin{array}{l} 12\% \text{ more than } 250 \text{ is } 112\% \text{ of } 250, \text{ which is } 280. \end{array}$$

2. What is 15% less than \$ 756 ?

$$\begin{array}{r} \$ 756 \\ .85 \\ \hline 3780 \\ 6048 \\ \hline \$ 642.60 \end{array} \quad \begin{array}{l} 15\% \text{ less than } \$ 756 \text{ is } 85\% \text{ of } \$ 756, \text{ which is } \$ 642.60. \end{array}$$

3. Find 25% more than 72.
4. Find 12% more than 250.
5. Find 60% less than 125.
6. Find 40% less than 475.
7. Find 15% more than 3540.
8. Find  $4\frac{1}{2}\%$  more than \$ 215.60.
9. Find  $5\frac{1}{4}\%$  less than \$ 164.20.
10. Find 80% less than \$ 2000.
11. Find 100% more than \$ 200.
12. Find 35% less than 500 tons.
13. Find  $22\frac{1}{2}\%$  more than 800 barrels.
14. Find  $16\frac{2}{3}\%$  less than 450 children.
15. Goods which cost \$ 275 are advanced 15%. What is the advanced price ?
16. A bill of goods amounting to \$ 324.50 is discounted 20%. What is the amount of the bill as discounted ?

## Percentage

*To find a number which will equal the given number when increased or diminished by the given per cent.*

1. What number when increased by 20% of itself equals 240 ?

240 must be 100% + 20%, or 120% of the required number.

1% of the number is  $\frac{1}{120}$  of 240, which is 2, and 100% = 100 times 2, or 200. Or,  $240 \div 1.20 = 200$ .

2. What number decreased by 30% of itself equals 35 ?

35 must be  $\frac{7}{10}$  of the number.  $\frac{1}{10}$  of the number = 5 and  $\frac{7}{10} = 50$ . Or,  $35 \div .70 = 50$ .

*What number increased by :*

- |   |  |
|---|--|
| 3. 25% of itself = 600 ?                | 11. 50% of itself = 21 ?               |
| 4. 12% of itself = 336 ?                | 12. 100% of itself = 4 ?               |
| 5. 30% of itself = 520 ?                | 13. 20% of itself = 12 ?               |
| 6. $12\frac{1}{2}$ % of itself = 108 ?  | 14. 1% of itself = 202 ?               |
| 7. 40% of itself = 560 ?                | 15. $2\frac{1}{2}$ % of itself = 82 ?  |
| 8. 10% of itself = 2200 ?               | 16. $16\frac{2}{3}$ % of itself = 42 ? |
| 9. 15% of itself = 690 ?                | 17. $37\frac{1}{2}$ % of itself = 77 ? |
| 10. $4\frac{1}{2}$ % of itself = 2110 ? | 18. 200% of itself = 15 ?              |

*What number decreased by :*

- |                            |   |
|----------------------------|---|
| 19. 50% of itself = 326 ?  | 27. $12\frac{1}{2}$ % of itself = 7 ?               |
| 20. 25% of itself = 429 ?  | 28. $16\frac{2}{3}$ % of itself = 10 ?              |
| 21. 20% of itself = 1264 ? | 29. $33\frac{1}{3}$ % of itself = $18\frac{2}{3}$ ? |
| 22. 18% of itself = 410 ?  | 30. $37\frac{1}{2}$ % of itself = $12\frac{1}{2}$ ? |
| 23. 6% of itself = 752 ?   | 31. $62\frac{1}{2}$ % of itself = $21\frac{1}{4}$ ? |
| 24. 4% of itself = 720 ?   | 32. 90% of itself = $\frac{1}{2}$ ?                 |
| 25. 35% of itself = 208 ?  | 33. 10% of itself = $\frac{9}{10}$ ?                |
| 26. 60% of itself = 2840 ? | 34. 100% of itself = 0 ?                            |

## Review Problems

1. What is the length of a square which contains 169 sq. ft. ?

2. The length of an oblong is three times its width. It contains 432 sq. ft. How wide is it ?

Make a diagram for the oblong.

3. The length of a rectangle is 50% greater than its width. It contains 96 sq. ft. How long is it ?

4. What is the length of the edge of a cube whose entire surface is 150 sq. in. ?

What is the surface of one face of the cube ?

5. What is the length of the edge of a cube which contains 216 cu. in. ?

6. The length of a square prism is twice its width. It contains 250 cu. in. How wide is it ?

Think of the prism as divided into two cubes.

7. The length of a square prism is 3 times its width. It contains 192 cu. in. How long is it ?

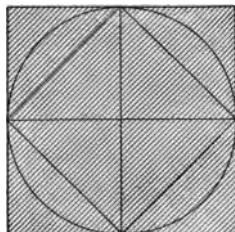
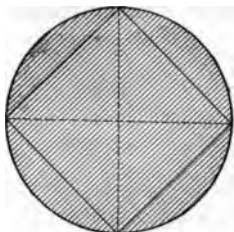
8. How many yards of carpet, one yard wide, will it take to exactly cover a floor 16 ft. 4 in. long and 15 ft. wide ?

9. How many yards of carpet,  $\frac{3}{4}$  of a yard wide, will it take to carpet a floor 22 ft. 6 in. long and 16 ft. wide, if the strips run lengthwise, and no allowance is made for waste in matching ?

10. When the fulcrum is between the power and the weight, if the distance from the power to the fulcrum is  $6\frac{1}{2}$  ft. and the distance from the weight to the fulcrum is  $2\frac{1}{2}$  ft., how many pounds of power will be necessary to balance a weight of 520 pounds ?

11. The weight distance is 28 in. The weight is 1600 lb. The power is 250 lb. Find the power distance.

## Squares and Circles



Notice that the diagonals of a square divide it into right triangles. See Book II, p. 151.

1. When a square is inscribed in a circle whose radius is 5 inches, what is the length of the diagonal of the square?

2. Find the approximate length of one side of the square.

3. Find the circumference of the circle.

4. What is the area of the square?

Observe that the square is composed of four parts which may be so combined as to make two smaller squares.

5. Find the area of that part of the circle which is outside the square.

Find the area of the whole circle.

6. Find the diameter of a circle whose circumference is 62.832 inches.

7. Find the area of a square inscribed in this circle.

8. Find the difference between the surface of the circle and the surface of the inscribed square.

9. What is the area of the largest square that can be inscribed in a circle whose diameter is 28 inches?

10. What is the area of the largest square that can be inscribed in a circle whose circumference is 314.16?

## Review Problems

See p. 207.

1. If the heart beats, on an average, at the rate of 72 strokes a minute, how many times does it beat in a day?

2. At each beat the heart of an average person pumps about 6 ounces of blood. If it beats 72 times a minute, how many tons of blood are moved every day?

3. How many tons of blood are moved in this way every year?

4. The total amount of work done by the heart in a full-sized man, per hour, is about equivalent to lifting a ton to the height of 8 feet. How many tons would be lifted one foot by a force equivalent to the work done by the heart in a year of 365 days?

5. A cannon ball weighing 10 lb. is moving at the rate of 700 ft. per second. Another cannon ball weighing 16 lb. is moving at the rate of 500 ft. per second. What is the ratio of the momentum of the first to that of the second?

See pp. 119, 198, 203.

6. Find the square root of 900.

7. Find the square root of .0625.

8. What is the length of the diagonal of a rectangle 24 ft long, 18 ft. wide?

9. Find approximately the diagonal of a square which is 7 ft. square.

10. The rafters of a house are 30 ft. long and the width of the house is 48 ft. What is the height of the gable?

11. A rectangular farm is 80 rods wide and contains 200 acres. What is its length?

## **Problems from Geography and History**

- 1.** In 1845 the United States offered Spain \$100,000,000 for Cuba. The area of Cuba is 45,884 square miles. How much per acre was the price offered?
- 2.** The population of Cuba is about 1,600,000. How many acres are there for each inhabitant?
- 3.** Porto Rico has an area of 3531 square miles. Its population in 1900 was about 1,000,000. How many more people per square mile has Porto Rico than Cuba?
- 4.** In the Hawaiian Islands there are about 14,000 Americans and English, 40,000 Japanese and Chinese, 9000 Portuguese, and 75,000 natives. What is the per cent of each class of the population?
- 5.** The population of the Philippine Islands is about 7,000,000. The total area of the islands is 114,361 sq. mi. About what is the population per square mile?
- 6.** The annual exports from the port of Manila include \$8,000,000 of hemp, \$6,000,000 of sugar, \$2,000,000 of tobacco, \$2,500,000 of gold, and \$1,250,000 of coffee. Find the average total value of these exports for each inhabitant of the islands.
- 7.** The direct cost of the recent war with Spain was about \$130,000,000. If to this sum \$70,000,000 more is added for the indirect cost of the war, how much would this make the islands acquired by the war cost per square mile?
- 8.** The total continental territory of the United States is 3,681,661 square miles. The total territory of the groups of islands recently acquired is 124,601 square miles. About what part of the total territory belonging to the country is that of these islands?

## Review Problems

See p. 207.

1. A ball weighing 15 pounds is rolling at the rate of 20 feet a second and meets another ball weighing 12 pounds, coming in the opposite direction at the rate of 10 feet a second. How fast will the larger ball continue to roll after the collision?

Find the difference of momentum.

2. Which has the greater amount of momentum, a rifle bullet weighing  $\frac{1}{2}$  of an ounce with a velocity of 2000 feet a second, or a 5-pound ball rolling at the rate of 10 feet a second?

3. What is the amount of momentum of a locomotive weighing 60 tons, moving at the rate of 25 miles an hour?

4. How many pounds of continuous force will it take to stop such a locomotive in 2 minutes?

Find how much momentum must be overcome each second.

5. How many feet are there in 3 chains and 20 links?

6. How many square rods are there in a field that is 10 ch. 50 li. long and 6 ch. 25 li. wide?

7. Find the number of square feet in a lot that measures 3 ch. 10 li. in width and 5 ch. 30 li. in length?

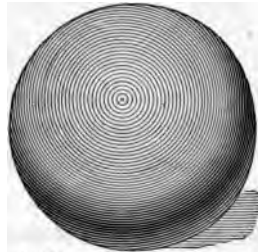
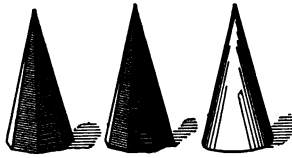
8. The surveyors in measuring a rectangular lot of land counted  $24\frac{1}{2}$  chains on one side and  $32\frac{1}{4}$  chains on another. Find how many acres there are in the lot.

9. A rectangular field containing 60 acres measures 24 chains on one side. What is the other dimension?

10. One side of a triangular field measures 25 chains. The perpendicular distance from this side to the opposite angle is  $18\frac{1}{2}$  chains. How many acres are there in the field?



## Spheres



See pp. 182, 202, and Book II, p. 244.

A sphere may be regarded as composed of a great number of pyramids or cones, having their bases in the surface of the sphere and their vertices at the center. *The volume of a sphere is equal to  $\frac{1}{3}$  of the product of its surface and its radius.*

1. Find the combined volume of 10 equal pyramids, each of whose bases contain 2 square inches, and whose common altitude is 9 inches.
2. What is the diameter of a sphere whose circumference is 314.16 in. ?
3. How many square inches are there in the surface of this sphere ?
4. Find the volume of the sphere.
5. What is the surface of a sphere whose diameter is 25 feet ?
6. What is its volume ?
7. Find the number of cubic inches in a hemisphere 3 feet in diameter.
8. What is the weight of a sphere of iron 10 inches in diameter, if the specific gravity of the iron is 7.2 ?
9. If a 10-inch cubical block of wood is turned in a lathe so as to take the form of a sphere, how many cubic inches of the wood are taken off ?

### Miscellaneous Problems

1. Find one of the two equal factors of 1296.
2. Find the length of a square field that contains 2304 sq. rd.
3. Find two equal numbers, the product of which is equal to the product of 12 and 27.

When the two means of a proportion are the same, the mean is called a **mean proportional** between the two extremes.

$$4 : 8 = 8 : 16.$$

4. Find the value of  $x$  in the proportion  $5 : x = x : 45$ .  
 $x^2 = 225$ .  $x = \sqrt{225} = 15$ .

5. Find a mean proportional between 8 and 200.  
 $8 : x = x : 200$ . Find  $x$ .

Find a mean proportional:

6. Between 2 and 8.
7. Between 8 and 18.
8. Between 3 and 27.
9. Between 6 and 150.
10. Between 5 and 125.
11. Between 5 and 500.
12. Two squares are 18 inches and 24 inches long. How long is a square which is as large as both together?
13. What is the hypotenuse of a right triangle whose sides are 28 ft. and 21 ft.?
14. What is the altitude of a right triangle whose hypotenuse is 17 ft. and base 8 ft.?
15. Two squares are 82 ft. and 18 ft. long. What is the length of a square equal to the difference between them?
16. Find the length of the diagonal of a rectangle which is 80 ft. long and 60 ft. wide.
17. What is the length of a rectangle whose diagonal is 205 ft. and width 45 ft.?

## Percentage

*Find the selling price :*

1. List price \$100. Discounts 20% and 10%.
2. List price \$350. Discounts 15% and 10%.
3. List price \$465. Discounts 25% and 5%.
4. List price \$116. Discounts 15% and 5%.
5. List price \$25.60. Discounts 30% and 10%.
6. List price \$86.40. Discounts 20%, 10%, and 5%.
7. List price \$375. Discounts 30%, 10%, and 5%.
8. List price \$750. Discounts 25%, 15%, and 5%.

*Find the gain or loss :*

9. Cost \$200. Marked 10% above cost, then reduced 10%.
10. Cost \$300. Marked 15% above cost, then reduced 10%.
11. Cost \$60. Marked 10% below cost, then advanced 10%.
12. Cost \$80. Marked 15% below cost, then advanced 10%.
13. Cost \$160. Marked 15% above cost, then reduced 10%.
14. Cost \$40. Marked 25% above cost, then reduced 15%.
15. Cost \$75. Marked  $33\frac{1}{3}\%$  above cost, then reduced 20%.
16. Cost \$500. Marked 12% below cost, then advanced 20%.

*Find the gain or loss per cent :*

17. Cost \$300. Sold for \$345.
18. Cost \$450. Sold for \$405.

## Review Problems ✓

1. How many chains are there in 15 miles ?
2. How many square chains are there in  $16\frac{1}{4}$  acres ?
3. The specific gravity of cork is .24. How many ounces of water will be displaced by a cubic foot of floating cork ?
4. How many pounds of support does a person receive in the water by wearing a life preserver containing a cubic foot of cork ?
5. A piece of oak wood having a specific gravity of .73 and a volume of 46 cubic inches floats in water. How much water does it displace ?
6. If a piece of wood weighing 180 pounds floats in water with  $\frac{5}{8}$  of its volume below the surface, what is the volume of the part below the surface ?  
How many pounds of water does it displace ?
7. What is the volume of the whole piece ?
8. A wholesale dealer paid \$2 for a barrel of potatoes. He sold them to a retail dealer at a gain of 20%, and the retail dealer sold them to his customers at a gain of 30%. How much did the retail dealer get for them ?
9. A tailor marked a coat which cost him \$8 so as to gain 40%. He afterward discounted the price 20%. What was the selling price ?
10. I sold 2 houses which cost me \$2300 each. I sold one at a gain of 25%, and the other at a loss of 18%. How much did I gain by the whole transaction ?
11. I sold goods at a loss of 16%, and received \$42. How much would it have been necessary to obtain for them in order to gain 12% ?

## Masonry

In stone masonry the contents of walls are reckoned in perches or in cords. A perch is  $24\frac{1}{4}$  cubic feet.

Bricks are usually 8 in.  $\times$  4 in.  $\times$  2 in. It takes about 7 bricks, together with the mortar, to cover 1 square foot of the surface of a wall. 22 bricks, together with the mortar, are reckoned for a cubic foot of wall.

For the length of the walls of a building masons take the measurement around the outside. The extra length reckoned in this way compensates for the extra cost of building the corners. For the same reason no allowance is usually made for the openings.

1. What is the actual number of cubic feet of stone masonry in the wall of a cellar which is 42 feet long, 24 feet wide, and 7 feet deep, if the wall is built 30 inches thick?

2. How many cubic feet would a mason actually reckon in estimating the cost of this wall?

3. What would be the cost of building the wall at \$4.25 a cord?

4. How many square feet are there in the outer surface of a rectangular cistern built 15 ft. long, 8 ft. wide, and 6 ft. high?

5. How many bricks will it take to build the cistern with bricks and mortar, if the walls are to be made one brick thick?

Take the outside measurement for the length of the walls.

6. How many bricks would be reckoned if the walls were to be 2 bricks thick?

7. A house is to be built 36 ft. long, 28 ft. wide, and 20 ft. high to the eaves. How many bricks will it take to build the walls one brick thick?

## Partial Payments

When only a part of the amount due on a note is paid at one time, or payments are made in part from time to time, there are different methods of computing the amounts due. According to the usual method in the United States the interest is reckoned upon the note to the time when the payment is made, the payment is deducted, and the balance is regarded as a new principal.

1. \$ 650.

BOSTON, MASS., June 1, 1897.

On demand, for value received, I promise to pay John H. Hawley, or order, Six Hundred Fifty Dollars, with interest at 6 per cent.

S. M. BARROWS.

On this note a payment of \$ 100 was made Jan. 19, 1898, and a payment of \$ 200 Mar. 1, 1900.

Find how much was due upon the note when the balance was paid, Oct. 25, 1900.

Principal . . . . .	\$650.00
Int. from June 1, 1897, to Jan. 19, 1898 . . . . .	24.70
Amount, Jan. 19, 1898 . . . . .	<u>674.70</u>
Payment, Jan. 19, 1898 . . . . .	100.00
New principal . . . . .	574.70
Int. from Jan. 19, 1898, to Mar. 1, 1900 . . . . .	72.99
Amount, Mar. 1, 1900 . . . . .	<u>647.69</u>
Payment, Mar. 1, 1900 . . . . .	200.00
New principal . . . . .	447.69
Int. from Mar. 1, 1900, to Oct. 25, 1900 . . . . .	17.46
Amount due, Oct. 25, 1900 . . . . .	<u>\$465.15</u>

2. Upon a note of \$ 600, dated June 1, 1898, bearing interest at 5%, with an indorsement of \$100, Apr. 16, 1899, and another of \$ 200, Aug. 13, 1900, find how much was due Jan. 16, 1901.

## Partial Payments

When any payment made is not large enough to pay the interest which has accrued at the time of the payment, the payment is simply credited and held till sufficient payments have been made to cover the interest, and then the interest is computed and the amount of the payments is subtracted.

1. \$1200.

CHICAGO, ILL., April 15, 1896.

On demand, for value received, I promise to pay Ernest Southard, or order, One Thousand Two Hundred Dollars, with interest at 5 per cent.

CHARLES H. BAILEY.

Payments: Jan. 15, 1898, \$50; June, 3, 1898, \$200.

Find how much was due on this note March 5, 1900.

Upon this note the interest from Apr. 15, 1896, to Jan. 15, 1898, \$105, is more than the payment \$50. Therefore the date Jan. 15, 1898, is disregarded, and the payment of \$50 is added to the payment of \$200 made later.

Principal . . . . .	\$1200.00
Int. from Apr. 15, 1896, to June 3, 1898 . . . . .	128.00
Amount, June 3, 1898 . . . . .	1328.00
The sum of the two payments . . . . .	250.00
New principal . . . . .	1078.00
Int. from June 3, 1898, to Mar. 21, 1900 . . . . .	97.02
Amount due, Mar. 21, 1900 . . . . .	1175.02

2. Upon a note of \$350, dated Nov. 15, 1894, bearing interest at 6%, a payment of \$100 was made Apr. 10, 1896, and another of \$25, July 21, 1898. How much was due Jan. 25, 1899?

3. Find how much was due Apr. 1, 1901, upon a note of \$248.50, dated Sept. 5, 1897, bearing interest at 5%, if one payment of \$20 was made Jan. 1, 1901.

## Partial Payments

*Apply the United States rule :*

1. A note of \$400, dated May 24, 1896, was indorsed as follows : Mar. 1, 1897, \$100 ; Dec. 25, 1897, \$150. The rate of interest was 6%. How much was due July 1, 1898 ?

2. A note of \$1000, dated Jan. 15, 1897, was indorsed as follows : Jan. 1, 1898, \$50 ; Jan. 1, 1899, \$300. The rate of interest was 6%. How much was due July 1, 1899 ?

3. A note of \$800, dated July 1, 1895, had the following indorsements : Jan. 1, 1896, \$250 ; Sept. 1, 1896, \$100. The rate of interest was 5%. How much was due July 1, 1897 ?

4. A note of \$300 was given Apr. 12, 1898, bearing interest at 7%. On this note the following sums were paid : Jan. 1, 1899, \$75 ; July 1, 1900, \$20. Find how much was due Jan. 1, 1901.

5. A note of \$1500, dated Sept. 20, 1894, had the following indorsements : Jan. 1, 1895, \$20 ; Jan. 1, 1896, \$300 ; Aug. 1, 1897, \$500. The rate of interest was 6%. Find how much was due Apr. 16, 1898.

6. A note of \$2000, dated Aug. 2, 1895, was indorsed as follows : July 15, 1896, \$500 ; Mar. 21, 1897, \$100. The rate of interest was 5%. How much was due Jan. 1, 1898 ?

7. A note of \$1600, dated May 10, 1897, had the following indorsements : Dec. 15, 1898, \$200 ; July 1, 1899, \$150 ; Nov. 1, 1900, \$300. The rate of interest was 5%. How much was due Feb. 1, 1901 ?

8. A note of \$745.20, dated July 15, 1898, had an indorsement of \$75 Jan. 5, 1901. The rate of interest was  $5\frac{1}{2}\%$ . How much was due Apr. 15, 1901 ?



## Construction

See pp. 102, 123, 222.

1. Draw a right triangle with the hypotenuse 4 inches long and an angle of  $65^\circ$ .
2. Draw an isosceles triangle with one of the equal sides  $3\frac{1}{2}$  inches long and with an angle of  $95^\circ$ .
3. Draw a line  $3\frac{1}{2}$  inches long. Regarding this line as a diagonal of a square, construct the square.
4. Draw a line 4 inches long. Regarding this line as a diagonal of a parallelogram, construct the parallelogram.
5. Draw a line  $4\frac{1}{2}$  inches long. Regarding this line as a diagonal of a trapezoid, construct the trapezoid.
6. Construct a regular pentagon with sides  $1\frac{1}{8}$  inches long. Find the center of the figure and find the area of the pentagon.
7. Construct a regular hexagon with sides  $2\frac{1}{2}$  inches long. Find the center and the area of the hexagon.
8. Make a square 3 inches long. Draw the diagonals. With a radius equal to the distance from the center to the middle point of one side, inscribe a circle. With a radius equal to one half of the diagonal of the square, circumscribe a circle. Find the area of the inscribed circle. Find the area of the circumscribed circle. Find the area of the space between the inscribed circle and the circumscribed circle.
9. Make a circle with a radius  $2\frac{1}{2}$  inches long. Draw the diameters perpendicular to each other. Draw straight lines connecting the extremities of the diameters. What figure is formed by the lines? Inscribe a circle within the square. Find the area of the circumscribed circle. Find the area of the inscribed circle. Find the area of the *square*.

## Review

1. Add twenty-seven and thirty-five hundredths; one hundred five and fifteen thousandths; two hundred five hundred-thousandths.

2. Add one thousand twenty and seventeen thousandths; twenty-five thousand and twenty-five thousandths; seven and two hundred-eight ten-thousandths.

3. From one thousand nine hundred and thirty-eight thousandths take eight hundred-forty and four hundredths.

4. From twelve thousand and twenty-four ten-thousandths take seven thousand four and nine thousandths.

*Perform the operations indicated:*

- |                           |                           |
|---------------------------|---------------------------|
| 5. $25.832 \times 150$    | 20. $100.03 \times .0011$ |
| 6. $.8536 \times .35$     | 21. $.1513 \times .112$   |
| 7. $.7641 \times 2.031$   | 22. $1614 \times .0015$   |
| 8. $.0517 \times 1.023$   | 23. $21.318 \times 1000$  |
| 9. $35.017 \times 100$    | 24. $13.416 \times .001$  |
| 10. $.3518 \times 1000$   | 25. $31.542 \div 1000$    |
| 11. $2.1316 \times 400$   | 26. $42.637 \div .001$    |
| 12. $1500 \times .0015$   | 27. $1.0213 \times 4000$  |
| 13. $.625 \div 25$        | 28. $2.0016 \times .004$  |
| 14. $1250 \div .25$       | 29. $.00581 \div 2000$    |
| 15. $82.32 \div 1.2$      | 30. $.00362 \div .002$    |
| 16. $243.81 \times 3.18$  | 31. $42000 \div 5000$     |
| 17. $30.516 \div .004$    | 32. $57000 \div .005$     |
| 18. $180.76 \times .1035$ | 33. $67020 \times .0105$  |
| 19. $218.09 \times .0104$ | 34. $2603.9 \div .0013$   |

## Original Problems

*Make problems and solve them:*

1. A square is inscribed in a circle whose radius is 8 inches.
2. A circle is inscribed in a square that is 4 feet square.
3. The heart of an average person pumps 6 ounces of blood at each beat.
4. A cannon ball weighing 20 pounds is moving at the rate of 600 feet a second.
5. A square lot of land contains 9216 square feet.
6. A cubical block contains 15,625 cubic inches.
7. A ball weighing 20 pounds rolls at the rate of 10 feet a second. Another weighing 8 pounds meets it, coming in the opposite direction, at the rate of 15 feet a second.
8. A locomotive weighing 50 tons is moving at the rate of 20 miles an hour.
9. A merchant's price for goods was 30% above cost, but he discounted this price 10%.
10. 20% is gained by selling cloth at 30 cents a yard.
11. A fraction, when squared, becomes  $\frac{49}{4}$ .
12. A fraction, when cubed, becomes  $\frac{216}{4}$ .
13. The diameter of a sphere is 18 inches.
14. The circumference of a sphere is 44 inches.
15. A 12-inch cubical block is turned in a lathe so as to form a sphere.
16. A certain farm is 25 chains long, and 22 chains 2 rods wide.
17. A cellar is dug 36 feet long, 22 feet wide, and 6 feet deep.

## Proportion

Review p. 227.

*Find the value of the unknown term:*

1.  $6 : x = 3 : 15$
2.  $8 : 5 = x : 20$
3.  $100 : 24 = 10 : x$
4.  $x : 12\frac{1}{2} = 18 : \frac{1}{2}$
5.  $2\frac{1}{2} : 16\frac{3}{4} = 24 : \frac{x}{2}$
6.  $x : 8 = 8 : 2$
7.  $7 : 5 = x : 35$
8.  $10 : 2x = 3 : 15$
9.  $x : 9 = 24 : 3$
10.  $2 : x = 4 : 14$
11.  $15 : 37 = 43 : x$
12.  $42 : 62\frac{1}{2} = x : 100$
13.  $x : 58 = 33\frac{1}{3} : 250$
14.  $84 : x = 75 : 16\frac{2}{3}$
15.  $150 : 275 = x : 360$
16.  $96 : 135 = 42\frac{1}{2} : x$
17.  $\frac{x}{5} : \frac{7}{10} = \frac{3}{4} : \frac{5}{6}$
18.  $27 : \frac{x}{3} = 60 : \frac{40}{3}$
19.  $12 : 15 = 15 : x$
20.  $4 : 8 = 8 : x$
21.  $4 : x = x : 16$
22.  $3 : x = x : 27$
23.  $4 : x = x : 36$
24.  $8 : x = x : 32$
25. Find a mean proportional between 5 and 20.
26. Find a mean proportional between 16 and 36.
27. If 7 pencils cost 12 cents, how much will 25 pencils cost at the same rate?
28. If 5000 yards of cloth are manufactured at a certain mill each day, when the mill runs 9 hours a day, how many yards will be manufactured when the mill runs  $10\frac{1}{2}$  hours a day.
29. If 9 men can do a piece of work in 5 days, in how many days can 4 men do the same work?
30. If 8 men can paint 5 houses in a week, how many can 40 men paint in the same time?

### Problems for Arithmetic or Algebra

1. The difference between  $\frac{1}{3}$  and  $\frac{1}{2}$  of a certain number is 15. What is the number?

2.  $\frac{1}{5}$  and  $\frac{1}{6}$  of a certain number equals 22. What is the number?

3. If from  $\frac{1}{3}$  of my age  $\frac{1}{5}$  of my age is subtracted, the remainder will be 6 years. How old am I?

4. Divide \$15 between 2 men, giving the first \$3 more than  $\frac{1}{2}$  of what the second receives.

5. In a school there are 420 pupils. There are twice as many girls as boys. How many are there of each?

6. The difference between two numbers is 6, and their sum is 20. What are the numbers?

7. A boy having a certain number of tops lost 5 and bought 8, and then had 12. How many had he at first?

8. A boy having a certain number of cents earned 12 more and spent 16, and then had 25. How many had he at first?

9. A man spent  $\frac{3}{8}$  of his money, and then earned  $\frac{1}{4}$  as much as he had left. He then had \$10 more than at first. How much had he at first?

10. In an orchard there are 7 more than twice as many apple trees as pear trees. In all there are 133 trees. How many are there of each kind?

11. The difference of two numbers is 24, and the smaller number equals  $\frac{5}{8}$  of the larger number. What are the numbers?

12. The difference of two numbers is 12, and the numbers are to each other as 7 to 10. What are the numbers?

*Let  $7x =$  one number and  $10x$  the other.  $10x - 7x = 12.$*

### Problems for Arithmetic or Algebra

1. When a boy had walked five times around a pond and 75 rods more, he had walked 5 miles. How far is it around the pond?

2. Robert's age is  $\frac{2}{5}$  of his father's age. Three times his age added to his father's age would make 88 years. What is the age of each?

3. If 5 is taken from three times a certain number,  $\frac{1}{5}$  of the remainder will be 15 less than twice the number. Find the number.

4. If a certain number is taken from 30, and the remainder is subtracted from three times the number, this remainder will be 2 more than twice the number. What is the number?

5. A farmer bought an adjoining farm  $\frac{1}{3}$  as large as his original farm, and another which was  $\frac{1}{4}$  as large as the original farm. He then had 380 acres. How large was his original farm?

6. The length of a rectangular lot is 7 times its width. It contains 9 acres 135 square rods. What is the width?

7. Find the dimensions of a block of granite in the form of a square prism whose length is 5 times the width, or thickness, and which contains 1080 cubic inches.

8. Find a mean proportional between 8 and 32.

9. Find the length of a square field which is equal in area to two fields, one of which is 50 rods long and 20 rods wide, and the other 40 rods long and 15 rods wide.

10. If \$40 will gain \$2.30, how much will \$67 gain in the same time and at the same rate?

11. It takes  $7\frac{1}{2}$  yards of lining for 12 yards of cloth which is 30 inches wide. How wide is the lining?

## Review

1. What is the average of the numbers 9, 15, and 18?
2. What is the average of the numbers 85, 93, 76,  $64\frac{1}{2}$ , 82,  $89\frac{1}{4}$ , and  $99\frac{3}{4}$ ?
3. The average of five numbers is 10. To what is the average reduced by combining with these five numbers the numbers 6, 7, and 9?
4. A school contains 20 boys and 15 girls. The average age of the boys is 16 years, and that of the girls  $13\frac{2}{3}$  years. What is the average age of all together?
5. If I mark goods, which cost \$ 1.20, so as to gain  $33\frac{1}{3}\%$ , and afterward sell them at 50% less than the marked price, what per cent of the cost do I lose?
6. If I mark goods, which cost \$ 2, so as to gain 20%, and afterwards mark them again 20% above the first marked price, what per cent do I gain on the cost by selling at the second marked price?
7. I bought a bill of goods at 10% below the marked price, and paid \$67.50. What was the marked price?
8. I bought a bill of goods at 10% below the marked price, and had to sell them at 10% below the cost. I received \$81. How much did they cost me?
9. What was the marked price?
10. Find the surface of a sphere whose diameter is 8 inches.
11. Find the volume of a sphere whose diameter is 15 inches.
12. Find the surface of a sphere whose circumference is 31.416 feet.
13. Find the volume of a sphere whose circumference is 314.16 feet.

## Powers and Roots

Review pp. 198, 203.

*Find the square root of the following :*

- |          |                      |
|----------|----------------------|
| 1. 729   | 6. $\frac{49}{81}$   |
| 2. 576   | 7. $\frac{9}{16}$    |
| 3. .49   | 8. $\frac{100}{144}$ |
| 4. .0144 | 9. $\frac{25}{169}$  |
| 5. .0009 | 10. $\frac{1}{100}$  |

*Find the cube root of the following :*

- |             |                       |
|-------------|-----------------------|
| 11. 216     | 16. $\frac{8}{27}$    |
| 12. .343    | 17. $\frac{64}{125}$  |
| 13. 1728    | 18. $\frac{1}{216}$   |
| 14. .064    | 19. $\frac{008}{027}$ |
| 15. .000125 | 20. $\frac{004}{343}$ |

*Perform the operations indicated :*

- |                  |                       |
|------------------|-----------------------|
| 21. $\sqrt{16}$  | 26. $\sqrt[3]{27000}$ |
| 22. $\sqrt{400}$ | 27. $\sqrt[3]{.512}$  |
| 23. $\sqrt{169}$ | 28. $\sqrt{6.25}$     |
| 24. $\sqrt{900}$ | 29. $\sqrt[4]{81}$    |
| 25. $\sqrt{625}$ | 30. $\sqrt[5]{32}$    |

*Find the indicated powers :*

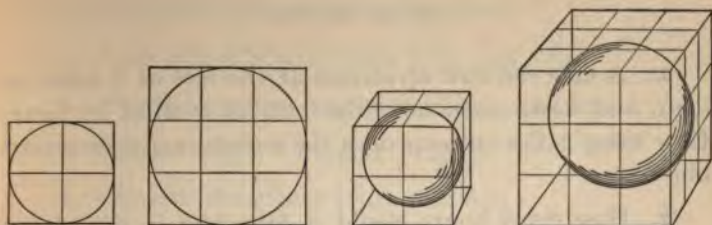
- |            |                       |
|------------|-----------------------|
| 31. $5^2$  | 36. $(\frac{1}{2})^3$ |
| 32. $4^3$  | 37. $(\frac{2}{3})^4$ |
| 33. $2^5$  | 38. $(\frac{5}{8})^2$ |
| 34. $25^3$ | 39. $.12^2$           |
| 35. $16^4$ | 40. $.005^3$          |



## Review

1. Find  $12\frac{1}{2}\%$  of 9240.
2. Find  $16\frac{3}{4}\%$  of \$175.44.
3. Find  $9\frac{1}{2}\%$  of \$375.62.
4. Find  $\frac{1}{4}\%$  of \$123.40.
5. Find 115% of 2351.
6. 15 is what per cent of 90?
7. 36 is what per cent of 300?
8. 85 is what per cent of 1000?
9. \$24.75 is what per cent of \$450?
10. 92 is what per cent of 378?
11. 45 is 5% of what number?
12. 63 is  $3\frac{1}{2}\%$  of what number?
13. \$126 is  $4\frac{1}{2}\%$  of what sum?
14. \$315 is  $1\frac{1}{2}\%$  of what sum?
15. \$168 is 112% of what sum?
16. What is 15% more than 50?
17. What is 12% less than 350?
18. What is  $4\frac{1}{2}\%$  less than \$600?
19. What is 85% more than \$27.30?
20. What is 42% less than \$63.70?
21. What number increased 50% equals 30?
22. What number decreased 10% equals 81?
23. What number increased  $12\frac{1}{2}\%$  equals 225?
24. What sum increased 6% equals \$42.40?
25. What sum decreased 7% equals \$418.50?

## Similar Forms



The **areas** of similar plane figures are to each other as the **squares** of their corresponding dimensions. The **volumes** of similar solid bodies are to each other as the **cubes** of their corresponding dimensions.

The ratio of the area of a 3-inch square to that of a 6-inch square is not the same as the ratio of 3 to 6, but as the ratio of the square of 3 to the square of 6, or as 9 to 36.

The volume of a sphere whose diameter is 2 feet is to the volume of another whose diameter is 5 feet as the cube of 2 is to the cube of 5, or as 8 is to 125.

1. The lengths of 2 squares are 1 foot and 2 feet. How do the areas of the squares compare with each other?

2. The diameters of 2 circles are 3 feet and 5 feet. What is the ratio of their areas?

3. How many 2-inch cubes are equal in volume to a 6-inch cube?

4. What is the ratio of the entire surface of a 3-inch cube to the entire surface of a 4-inch cube?

5. The end of a square prism is 5 inches square, and its length is 12 inches. What is its entire surface?

6. What is the entire surface of a square prism similarly shaped, which is 20 inches long?

## Review Problems

See pp. 226, 243.

1. A man can row up stream at the rate of 4 miles an hour, and down stream at the rate of 8 miles an hour. How many miles an hour does the current assist or retard him?
2. How many hours would it take him to row down stream 16 miles and back to the point of starting?
3. How many miles could he go down stream and get back in 15 hours?
4. Find the difference between the surface of a foot cube and the surface of the largest sphere that can be inclosed in it.
5. If a cubical block of wood one foot long weighs 50 ounces, find the weight of the largest sphere that can be formed from it.
6. The specific gravity of cast iron is 7.2. Find the weight of a 6-inch cannon ball.
7. By the law of the relation between similar solids find from this the weight of a 4-inch cannon ball.
8. If the area of a certain regular pentagon is 16 sq. ft., what is the area of a second regular pentagon, one of whose sides is 4 times as long as a side of the first?
9. Find the area of a circle inscribed in a 10-inch square.
10. What is the area of a circle inscribed in a 5-inch square?
11. If a man 5 ft. 6 in. tall weighs 140 lb., what would *be the weight* of a man of similar form 6 ft. tall?

*Find the ratio between the similar forms :*

1. Two triangles with sides 6 in. and 3 in.
2. Parallelograms: altitudes 10 in. and 4 in.
3. Cubes: edges 7 in. and 2 in.
4. Circles: diameters 10 in. and 3 in.
5. Pentagons : sides 18 in. and 8 in.
6. Cylinders : lengths 5 ft. and 9 in.
7. Pyramids : slant heights 21 in. and 4 in.
8. Hexagons : diagonals 13 in. and 11 in.
9. Spheres : diameters 28 in. and 10 in.
10. Trapezoids : altitudes 2 ft. 4 in. and 1 ft. 2 in.
11. Prisms : sides 15 in. and 9 in.
12. Cones : slant heights 3 ft. 6 in. and 18 in.

*Find the missing quantity in comparing the similar objects :*

13. Two rectangles are 22 ft. and 6 ft. long. The area of the first is 220 sq. ft.
14. The diameters of the bases of two cones are 18 in. and 11 in. The lateral surface of the first is 5 sq. ft.
15. Two oxen are 4 ft. 8 in. and 5 ft. tall. The weight of the first is 1000 lb.
16. A trout 13 in. long weighs 14 oz. Another trout is 17 in. long.
17. Two elm trees are 50 ft. tall and 30 ft. tall. The shadow of the first covers 1000 sq. ft.
18. A man 5 ft. 9 in. tall weighs 150 lb. Another man is 6 ft. 2 in. tall.

## Partial Payments—Merchants' Method

When notes that have been paid in part are settled within a year from the time when they were given, merchants are accustomed to find the amount of the principal for the whole time and then to subtract from this the amount of each payment with the interest from the time the payment was made to the time of settlement.

1. \$860.

CINCINNATI, O., May 10, 1898.

On demand, for value received, I promise to pay William Jackson, or order, Eight Hundred Sixty Dollars, with interest at 6 per cent.

HENRY GOULD.

Indorsements: July 22, 1898, \$200; Nov. 12, 1898, \$300.

Find by the merchants' method how much was due Jan. 1, 1899.

Find the amounts of \$200 and \$300 from the times when these payments were made to Jan. 1, 1899. Find the amount of \$860 for the time from May 10, 1898, to Jan. 1, 1899, and subtract from this amount the amounts of the payments.

*Find by the merchants' method the amount due upon the following notes at the time of settlement:*

2. A note of \$1000, dated Jan. 15, 1900, bearing interest at 5%; settled Jan. 1, 1901. Indorsements: May 12, 1900, \$300; Aug. 6, \$250.

3. A note of \$700, dated Feb. 12, 1900, bearing interest at 4%; settled Dec. 1, 1900. Indorsements: May 12, \$150; July 25, \$400; Nov. 10, \$50.

4. A note of \$300, dated June 18, 1900, bearing interest at  $5\frac{1}{2}\%$ ; settled Feb. 1, 1901. Indorsements: Sept. 10, 1900, \$75; Dec. 20, 1900, \$125.

## Review Problems

See pp. 207, 226, 248.

1. If I buy goods at 40% below the list price and sell them at 20% below the list price, what per cent do I gain?

2. If the list price of goods is \$2.00 and I buy them at 25% below the list price, and sell them at 15% below the list price, how much do I gain? What per cent do I gain?

3. A merchant paid \$50 premium for insuring flour at 2%. He had it insured for  $\frac{1}{4}$  of its cost. There were 600 barrels. How much did the flour cost per barrel?

4. If the area of a triangle whose altitude is 8 inches is 42 sq. in., what is the area of a similar triangle whose altitude is 11 inches?

5. The entire surface of a 10-inch cube is 600 sq. in. What is the entire surface of a 32-inch cube?

6. If a cup  $4\frac{1}{2}$  inches in diameter will contain 52 cubic inches of water, how many cubic inches will an exactly similar cup contain which is  $7\frac{1}{2}$  inches in diameter?

7. If a man 5 ft. 8 in. tall weighs 150 lb., what should be the weight of a similarly formed man who is 6 ft. 2 in. tall?

8. How much work is done by a man weighing 160 lb. in ascending stairs to the height of 65 feet?

9. What is the momentum of a locomotive weighing 90 tons and moving at the rate of a mile in 2 minutes?

10. Find the number of cubic inches of capacity of an hemispherical bowl which measures 8 inches across the top.

11. Find how many quarts this bowl would contain.

## Measurements

*Find the measurements of the regular pyramid :*

1. Base 6 in. square ; slant height 32 in. Lateral surface = ?
2. Base 4 ft. square ; altitude  $5\frac{1}{2}$  ft. Volume = ?
3. Base a triangle ; side 15 in. Perimeter = ?
4. The slant height is 34 in. Lateral surface = ?
5. Area of base  $2\frac{1}{2}$  sq. ft. ; altitude  $4\frac{1}{2}$  ft. Volume = ?
6. Base a pentagon ; side 7 in. Perimeter of base = ?
7. The slant height is 30 in. Lateral surface = ?
8. Base 112 sq. in. ; altitude 14 in. Volume = ?

*Find the measurements of the cone :*

9. Circumference of base 18 in. Diameter = ?
10. Radius of base 7 in. Area of base = ?
11. Diameter of base 14 in. Circumference = ?
12. The slant height is 22 in. Lateral surface = ?
13. Radius of base  $4\frac{1}{2}$  ft. Area of base = ?
14. The altitude is  $6\frac{1}{2}$  ft. Volume = ?
15. Diameter of base 7 ft. ; altitude 10 ft. Volume = ?
16. Radius of base 20 in. ; altitude 45 in. Volume = ?

*Find the measurements of the sphere :*

17. Circumference  $4\frac{1}{2}$  ft. Diameter = ?
18. Diameter 28 in. Circumference = ?
19. Diameter 32 in. Surface = ?
20. Diameter 32 in. Volume = ?
21. Radius 1 ft. Volume = ?
22. Circumference 4 ft. 6 in. Volume = ?
23. Diameter 35 in. Surface = ?
24. Radius 15 in. Volume = ?

## Problems from Astronomy

1. The distance from the center of the earth to either pole is 20,855,121 feet. How many miles is it?
2. The distance from the center of the earth to the equator is 20,926,062 feet. What is the true equatorial diameter of the earth?
3. Find the difference in miles between the shortest and the longest diameter of the earth.
4. Regarding the average diameter as 7913 miles, find the average circumference.
5. Find the true circumference at the equator.
6. Find the volume of the earth in cubic miles.
7. The diameter of the moon is 2160 miles. Find how many times greater the surface of the earth is than the surface of the moon.
8. Find how many times greater the volume of the earth is than the volume of the moon.
9. The diameter of the sun is about 866,000 miles. About how many times longer is the diameter of the sun than the diameter of the earth?
10. Since the two are similar bodies, about how many times the surface of the earth is the surface of the sun?
11. How many times the surface of the moon is the surface of the sun?
12. The distance of the moon from the earth is about 240,000 miles. How many times the diameter of the moon is its distance from the earth?
13. The distance of the sun from the earth is about 93,000,000 miles. How many times the diameter of the sun is its distance from the earth?



## Review Problems

1. If the list price of goods is \$4.00, which make the lower rate and how much, discounts of 15% and 10%, or of 10% and 15%?
2. How much will a creditor lose on a debt of \$2530, if the debtor can pay only 45 cents on a dollar?
3. The estate of a bankrupt amounted to \$46,000. His liabilities were \$75,950. The expense of the settlement was \$430. How much could he pay on a dollar?
4. If I gain \$90 by selling goods at a gain of 18%, what was the cost of the goods?
5. What is the cost of insuring property worth \$15,000 for  $\frac{3}{4}$  of its value, at  $\frac{3}{4}$ %?
6. If the premium for insuring property at  $1\frac{1}{4}$ % is \$15, for what amount is the property insured?
7. A commission merchant remits to the shipper \$237.50 for the sale of goods, after having deducted his commission of 5%. For what sum were the goods sold?
8. A broker's commission for selling stock at  $\frac{1}{8}$ % brokerage was \$12.50. What was the par value of the stock sold?
9. A clothier bought suits at \$12.00 each, and marked them to sell at 25% profit. He afterwards reduced the marked price 10%. What per cent did he finally gain upon the cost?
10. A clothier bought coats at \$8.00 each, and marked them to be sold at a gain of 25%. He afterward advanced his price 20%. What per cent did he finally gain upon the cost?
11. A dealer marked his goods to be sold at a gain of 20%. His price for a certain kind of cloth was 48 cents a yard. He finally sold it, however, for 45 cents a yard. *What per cent did he gain?*

## Ratio

See p. 34.

1. What is the ratio of the volume of a 3-inch cube to that of a 15-inch cube?

2. What is the ratio of a square prism 10 inches square and 2 ft. 4 in. long to a 5-inch cube?

3. What is the ratio of a square prism a foot square and 3 ft. long to the largest cylinder that can be contained within it?

4. Find the ratio of a cone whose altitude is 15 in. and the diameter of whose base is 16 in. to a cylinder having the same base and altitude.

5. How many cones, the diameter of whose bases is 9 inches and whose altitude is 12 inches, will be equal to a sphere whose radius is 12 inches?

6. How many times the volume of a sphere whose diameter is 8 inches is that of a cylinder just large enough to contain the sphere?

7. If an agent charges  $2\frac{1}{2}\%$  for selling goods, what is the ratio of the price of the goods to the amount of the commission?

8. What is the ratio of the commission to the sum remitted to the owner?

9. At the rate of  $3\frac{1}{2}\%$  simple interest, in how many years will the interest upon a sum of money equal the principal?

10. At 5% interest, what is the ratio of the principal to the interest for 2 yr. 6 mo.?

11. A merchant bought some goods for \$6000 and sold them at a gain of \$750. What was the ratio of the cost to the gain? Of the gain to the selling price? Of the selling price to the cost?

### Miscellaneous Problems

1. Water in freezing expands about  $7\frac{1}{2}\%$ . Find the weight of a cubic foot of ice.

2. What is the specific gravity of ice ?

3. If a block of ice is 20 inches thick, how many inches will the top of the block rise above the surface of the water in floating ?

4. Water expands about  $\frac{1}{24}$  of its volume when heated from the freezing point to the boiling point. Find the weight of a cubic foot of boiling water.

5. If a cubic foot of ice should be melted, how many cubic inches of water would there be ?

6. If this water should be heated to the boiling point, how many cubic inches of boiling water would there be ?

7. If the water should be heated enough to convert it into steam, and the steam should be injected into a barrel of ice, how many pounds of ice would be melted, if none of the heat of the steam were wasted ?

See p. 84.

8. If a pound of meat is equivalent in food value to  $1\frac{1}{2}$  pounds of flour, how many pounds of meat are equivalent in food value to a barrel of flour ?

9. When flour costs \$6 a barrel, what would be the corresponding food value of a pound of meat ?

10. What should be the price per barrel of flour to correspond in food value to meat at 10 cents per pound ?

11. The average advance in wages per hour during the past 40 years has been about 75%. If the wages for a certain kind of work now is \$3.50 a day, what was the *e paid* for the same kind of work 40 years ago ?

## Original Problems

*Make problems and solve them:*

1. The average age of 10 boys in school is 15 years, 3 months, and that of 12 girls is 14 years, 4 months.

2. I bought a bill of goods at a discount of 20%, and sold them at a gain of 10%. I sold them for \$44.

3. Two similar prisms are 15 inches and 10 inches long.

4. The diameters of two spheres are to each other as 13 to 29. The surface of the smaller sphere is 38 feet.

5. A block of wood floats with  $\frac{1}{8}$  of its volume above the surface of the water.

6. The specific gravity of a certain kind of wood is .75.

7. By a reduction of the rate of taxation from 14 mills to 11 mills on a dollar, the tax of a certain taxpayer was reduced \$24.30.

8. A man can row down stream at the rate of 8 miles an hour, and up stream at the rate of 2 miles an hour.

9. The heights of two men of similar form are 5 ft. 6 in. and 5 ft. 10 in. The smaller man weighs 130 lb.

10. The entire surface of a 9-inch cube is 486 inches.

11. The surface of one side of a cube is 81 square inches.

12. If  $\frac{1}{4}$  of a number is subtracted from 3 times the number, the result is 55.

13. When water freezes it expands about  $7\frac{1}{2}\%$ .

14. A cylindrical bucket 9 inches in diameter is full of water, and the water entirely freezes.

15. A block of ice 3 feet long,  $2\frac{1}{2}$  feet wide, and 20 inches thick, is melted.

## Fractions

- |  |  |
|--|--|
| <p>1. Add <math>\frac{2}{5}</math>, <math>\frac{11}{20}</math>, and <math>\frac{23}{40}</math>.</p> <p>2. Add <math>\frac{11}{8}</math>, <math>\frac{3}{7}</math>, and <math>\frac{17}{42}</math>.</p> <p>3. Add <math>\frac{2}{9}</math>, <math>\frac{4}{57}</math>, and <math>\frac{13}{78}</math>.</p> <p>4. Add <math>\frac{5}{17}</math>, <math>\frac{9}{18}</math>, and <math>\frac{5}{12}</math>.</p> <p>5. Add <math>\frac{7}{48}</math>, <math>\frac{5}{69}</math>, and <math>\frac{7}{16}</math>.</p> <p>6. Add <math>\frac{61}{70}</math>, <math>\frac{1}{18}</math>, and <math>\frac{14}{15}</math>.</p> | <p>7. Add <math>\frac{124}{238}</math> and <math>\frac{131}{182}</math>.</p> <p>8. Add <math>\frac{75}{251}</math> and <math>\frac{83}{128}</math>.</p> <p>9. Add <math>\frac{231}{230}</math> and <math>\frac{24}{318}</math>.</p> <p>10. Add <math>\frac{115}{104}</math> and <math>\frac{120}{178}</math>.</p> <p>11. Add <math>\frac{224}{25}</math> and <math>\frac{200}{67}</math>.</p> <p>12. Add <math>\frac{188}{50}</math> and <math>\frac{217}{481}</math>.</p> |
| <p>13. From <math>\frac{17}{20}</math> take <math>\frac{9}{18}</math>.</p> <p>14. From <math>\frac{25}{5}</math> take <math>\frac{3}{10}</math>.</p> <p>15. From <math>\frac{27}{87}</math> take <math>\frac{5}{14}</math>.</p> <p>16. From <math>\frac{28}{3}</math> take <math>\frac{2}{9}</math>.</p> <p>17. From <math>\frac{72}{108}</math> take <math>\frac{17}{64}</math>.</p> <p>18. From <math>\frac{83}{223}</math> take <math>\frac{19}{187}</math>.</p>  | <p>19. From 56 take <math>2\frac{18}{9}</math>.</p> <p>20. From 65 take <math>8\frac{7}{7}</math>.</p> <p>21. From <math>43\frac{5}{11}</math> take <math>24\frac{2}{7}</math>.</p> <p>22. From <math>69\frac{5}{2}</math> take <math>48\frac{2}{16}</math>.</p> <p>23. From <math>85\frac{7}{13}</math> take <math>52\frac{25}{98}</math>.</p> <p>24. From <math>96\frac{15}{99}</math> take <math>38\frac{17}{30}</math>.</p>  |
| <p>25. Multiply <math>\frac{17}{21}</math> by <math>\frac{11}{17}</math>.</p> <p>26. Multiply <math>\frac{28}{49}</math> by <math>\frac{5}{19}</math>.</p> <p>27. Multiply <math>\frac{13}{21}</math> by <math>23\frac{5}{7}</math>.</p> <p>28. Multiply <math>17\frac{5}{11}</math> by <math>21\frac{5}{8}</math>.</p> <p>29. Multiply <math>113\frac{8}{9}</math> by <math>49\frac{11}{12}</math>.</p>   | <p>30. Multiply <math>\frac{9}{17}</math> by <math>5\frac{2}{2}</math>.</p> <p>31. Multiply <math>2\frac{5}{3}</math> by <math>4\frac{5}{6}</math>.</p> <p>32. Multiply <math>\frac{125}{225}</math> by <math>\frac{112}{13}</math>.</p> <p>33. Multiply <math>\frac{93}{350}</math> by <math>2\frac{47}{10}</math>.</p> <p>34. Multiply <math>\frac{161}{500}</math> by <math>\frac{247}{657}</math>.</p>   |
| <p>35. Divide 38 by <math>12\frac{1}{2}</math>.</p> <p>36. Divide 77 by <math>23\frac{7}{8}</math>.</p> <p>37. Divide <math>8\frac{2}{4}</math> by <math>2\frac{7}{4}</math>.</p> <p>38. Divide <math>8\frac{2}{7}</math> by <math>7\frac{6}{8}</math>.</p> <p style="padding-left: 2em;">Divide <math>4\frac{2}{9}</math> by <math>6\frac{2}{9}</math>.</p>   | <p>40. Divide <math>9\frac{5}{24}</math> by <math>12\frac{1}{8}</math>.</p> <p>41. Divide <math>14\frac{5}{8}</math> by <math>8\frac{5}{13}</math>.</p> <p>42. Divide <math>47\frac{2}{17}</math> by 56.</p> <p>43. Divide <math>\frac{110}{27}</math> by <math>\frac{134}{185}</math>.</p> <p>44. Divide <math>\frac{216}{354}</math> by <math>\frac{158}{249}</math>.</p>  |

## Problems for Arithmetic or Algebra

1. A farmer has three times as many acres of corn as of potatoes, and five times as many acres of grass as of corn. In all he has 95 acres. How many acres has he of each?

2. A has seven times as much money invested in business as B has. A has \$15,000 more invested than B has. How much has each invested?

3. A certain number plus  $\frac{3}{4}$  of itself plus  $\frac{1}{2}$  of itself equals 148. Find the number.

4. Find two numbers whose sum is 52 and one of which is three times the other.

5. A man bought a horse and sold him so as to gain  $\frac{1}{3}$  as much as the cost. The selling price was \$126. What was the cost?

6. There are baskets of peaches of two sizes at a fruit store. In each of the four smaller baskets there is a certain number, and in each of the six larger baskets there are three times as many. In all there are 440 peaches. How many are there in the different baskets?

7. A boy had a certain number of papers. He bought as many more, and then sold  $\frac{3}{4}$  of what he had. He had 10 papers left. How many had he at first?

8. A man and his two sons worked 12 days. The older son received the same wages as the father, and the younger son  $\frac{2}{3}$  as much as the father. In all they received \$48. How much did each receive per day?

9. The width of a certain field is  $\frac{1}{3}$  of its length. The distance around the field is 56 rods. Find its area.

10. A triangular field contains 25 acres. The length of one side is 52 rods. What is the perpendicular distance from this side to the opposite angle?

## Square Root

A perfect square is the product of two equal factors or of two equal groups of factors.

The square root of any number which is a perfect square may be found by first finding its factors. To find the approximate square root of a number which is not a perfect square, another process is necessary.

$1 \times 1 = 1$	$10 \times 10 = 100$
$2 \times 2 = 4$	$99 \times 99 = 9,801$
$3 \times 3 = 9$	$100 \times 100 = 10,000$
$5 \times 5 = 25$	$999 \times 999 = 998,001$
$9 \times 9 = 81$	$1000 \times 1000 = 1,000,000$

Note that the square of a number consisting of one figure is composed of not more than two figures; the square of a number consisting of two figures is composed of not less than three and not more than four figures; the square of a number consisting of three figures is composed of not less than five and not more than six figures, etc. Hence when a number is squared the square of the units' figure is contained within the first group of two figures, reckoning from the right, the square of the tens' figure within the second group of two figures, and so on.

The square of 234 is 5'47'56. The square of 4 is contained within the first group, or section, 56. The square of the 3, or 30, is contained within the second section, 47, or 4700. The square of the 2, or 200, is contained within the third section, 5, or 50,000.

The excess in each of these sections over the squares of the separate parts of 234 is due to the fact that the square of the whole combined is more than the sum of the squares of the separate parts.

## Square Root — Algebraic Explanation

1. Find the square of  $a + b$ .

$$\begin{array}{r} a + b \\ a + b \\ \hline a^2 + ab \\ \quad ab + b^2 \\ \hline a^2 + 2ab + b^2 \\ = a^2 + b(2a + b) \end{array}$$

Since  $a$  and  $b$  may represent any numbers, we see from the multiplication that *the square of the sum of two numbers is always equal to the square of the first number, plus twice the product of the first and the second, plus the square of the second; or the square of the first, plus the second, times the sum of twice the first and the second.*

2. Square 43.

$$\begin{array}{r} 40 + 3 \\ 40 + 3 \\ \hline 40^2 + 3 \times 40 \\ \quad 3 \times 40 + 3^2 \\ \hline 40^2 + 2(3 \times 40) + 3^2 \\ = 40^2 + 3(2 \times 40 + 3) \end{array}$$

In the number 43, since the 4 is tens, its real value is 40, and the number may be written in the form  $40 + 3$ . The square of the number in this form gives a result in the same form as in the case of the square of  $a + b$ .

3. Find the square root of 1849.

$$\begin{array}{r} 40^2 = \begin{array}{l} 1849 \\ \underline{1600} \\ 249 \end{array} \begin{array}{l} 40 + 3 \\ \quad 43 \end{array} \\ 2 \times 40 = 80 \quad \begin{array}{l} 249 \\ \underline{249} \end{array} \\ 2 \times 40 + 3 = 83 \quad \begin{array}{l} 249 \\ \underline{249} \end{array} \end{array}$$

By dividing the figures of the number into sections we know that the square root of the number will consist of two figures. The greatest number of tens whose square is contained in 1849 is 4 tens, or 40,

and the square is 1600. After subtracting 1600 from the number, the 249 remaining must contain twice the product of the tens and the units, and the square of the units. Since the square of the units must be a comparatively small number, we may regard 249 as consisting approximately of twice the product of the tens and the units. We know that twice the product of the tens is 80; hence the units must be approximately the number of times that 80 is contained in 249, or 3. Then  $3(2 \times 40 + 3)$  or  $3 \times 83$ , or 249, will complete the parts of the square of  $40 + 3$ , leaving no remainder.

*Find the square root of the following numbers and explain the process:*

- |         |         |         |          |
|---------|---------|---------|----------|
| 4. 784  | 6. 2025 | 8. 2704 | 10. 3844 |
| 5. 1024 | 7. 1444 | 9. 3136 | 11. 4489 |



## Percentage

1. If goods are bought for \$1250 and sold for \$1500, what per cent is gained?

2. I sold a car load of corn for \$242 and gained 10%. What per cent should I have gained if I had received \$253?

3. If a tax of \$60 is paid on a building worth \$4000, what is the value of a building upon which a tax of \$13.50 is paid, at the same rate?

4. Find the amount of difference between buying goods at discounts of 25% and 10% and at discounts of 10% and 25%, if the list price in each case is \$300.

5. I bought corn for \$860 and wheat for \$735. On the corn I gained 18% and on the wheat I lost 14%. How much did I gain by the whole transaction?

6. Which is the better investment, 5% stock at 80 or 7% stock at 120?

7. I bought flour at \$4.20 a barrel, on 6 months' credit. I sold it immediately for cash, at an advance of 10%. I put the proceeds at interest at 6%. When I paid for the flour, at the end of 6 months, how much had I gained per barrel?

8. Find the bank discount of a note of \$525 for 60 days, at  $5\frac{1}{2}$  per cent.

9. \$400.

PITTSBURG, PA., Jan. 5, 1901.

Three months after date, for value received, I promise to pay C. M. Henry, or order, Four Hundred Dollars, with interest at 5%.

This note was discounted at a bank at 6%, Feb. 24, 1901. Find the proceeds.

## Algebraic Problems

1. A man paid \$468 for an equal number of cows and sheep. He paid \$35 each for the cows and \$4 each for the sheep. How many of each did he buy?

2. I have \$1.68 in dimes, 5-cent pieces, and cents. I have twice as many dimes as 5-cent pieces, and three times as many cents as 5-cent pieces. How many of each have I?

3. A gentleman, after giving away  $\frac{3}{8}$  and  $\frac{1}{8}$  of his money, had \$10,000 left. How much had he at first?

4. A piece of land containing 180 acres is divided into 3 fields. The first field contains 10 acres more than the second, and the second 20 acres less than the third. How many acres are there in each of the 3 fields?

5. I bought 70 stamps. I bought 10 more 2-cent stamps than 1-cent stamps and 25 more 2-cent stamps than 5-cent stamps. How many of each did I buy?

6. The length of a rectangular field is 12 rods more than its width. The distance around it is 84 rods. Find its length and width.

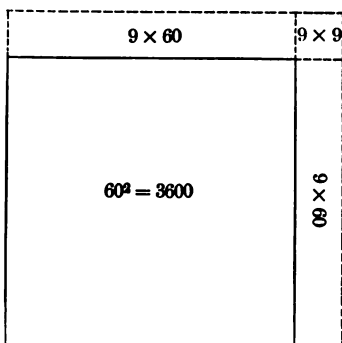
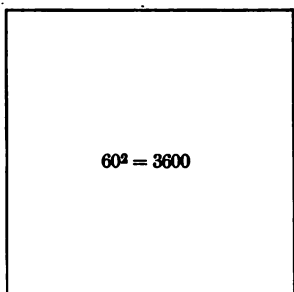
7. The length of a rectangular field is five times its width. It contains 1125 square rods. Find its length and width.

8. In 6 days 16 men and 16 boys earn \$259.20. The boys earn  $\frac{1}{3}$  as much as the men. Find the daily wages of each.

9. The time past 12 o'clock equals  $\frac{1}{3}$  of the time before 1 o'clock. What time is it?

10. If the time past 4 o'clock is  $\frac{4}{5}$  of the time remaining before 6 o'clock, what time is it?

## Square Root — Geometric Explanation



Any number may represent the number of square units in a square. The square root will be the length of one side of the square.

1. Find the square root of 4761.

$$\begin{array}{r}
 47\overline{)61} \mid 60+9 \\
 \underline{3600} \phantom{00} \\
 2 \times 60 = 120 \phantom{00} \\
 2 \times 60 + 9 = 129 \phantom{00} \\
 \underline{1161} \\
 1161
 \end{array}$$

The problem is to construct the largest possible square from 4761 square units. We first construct the largest possible perfect square from 4000 square units. This square is 60 units long and contains 3600 square units. We have 1161 square units remaining to be added to the square to increase its size.

The additions will consist of two rectangles, each 60 units long, and a small square whose length will be equal to the width of the rectangles. These additions may be imagined to form a continuous rectangle whose approximate length is  $2 \times 60$ , or 120. Since 1161 is the number of square units in the additions, and 120 is their approximate length, their width will be approximately the number of times that 120 is contained in 1161, or 9. If 9 is the true width of the additions, it must also be the length of the small square, and  $120 + 9$ , or 129, must be the true length of all the additions. Multiplying this number by 9, which is the width of the additions, we have 1161 square units for the entire surface of the additions. In making the additions we have thus used all the square units that remained.

*Find the square root and explain the process geometrically:*

2. 1225

3. 2209

4. 2809

5. 3364

## Square Root — Formal Explanation

1. Find the square root of 54,756.

$$\begin{array}{r}
 5'47'56(234 \\
 \underline{4} \\
 43)147 \\
 \underline{129} \\
 464)1856 \\
 \underline{1856}
 \end{array}$$

Beginning at the right and separating the figures into sections of two figures each, we have three sections, hence there will be three figures in the root. The greatest perfect square in the first section is 4, and its square root is 2. Placing the root figure at the right and subtracting its square, we have 1 remaining. Annexing the next section of figures, we have 147. We double the root figure, 2, making 4, which we place at the left of 147. This trial divisor, 4, which is to be regarded as 40, is contained in 147 three times. We write the 3 in the root and also annex it to the 4, making 43 for the complete divisor. Multiplying the complete divisor by the root figure, we have 129. Subtracting this and annexing the next section of figures, we have 1856. We next double the root figures, 23, for a new trial divisor, writing it at the left of 1856 and regarding it as 460. This is contained in 1856 four times. We write the 4 in the root and annex it to 46, making 464 for the complete divisor. Multiplying the complete divisor by the last root figure, we have no remainder, hence the square root of 47,560 is 234.

2. Find the square root of 162,409.

In this example the trial divisor 8, which is to be considered as 80, will not be contained in 24. We write zero both in the root and at the right of the 8, making the new trial divisor 80, which is to be regarded as 800. We now bring down the next section of figures. 800 is contained in 2409 three times. We write the 3 in the root and annex it to the trial divisor to complete the divisor. Multiplying the complete divisor by 3, we have no remainder.

$$\begin{array}{r}
 16'24'09(403 \\
 \underline{16} \\
 803)2409 \\
 \underline{2409}
 \end{array}$$

*Find the square root of the following:*

- |            |             |             |
|------------|-------------|-------------|
| 3. 98,596  | 8. 390,625  | 13. 574,564 |
| 4. 99,225  | 9. 484,416  | 14. 662,596 |
| 5. 205,209 | 10. 522,729 | 15. 678,976 |
| 6. 256,036 | 11. 577,600 | 16. 810,000 |
| 7. 316,969 | 12. 651,249 | 17. 829,921 |

### Miscellaneous Problems

1. To make 18% profit, at what price must goods be marked that cost \$7.50?
2. An agent collected bills for me on a commission of 10%. He paid me \$630. How much did he collect?
3. A board 16 ft. long and 9 in. wide contains the same number of square feet as another board that is 8 ft. long. How wide is the latter board?
4. Find the number of square yards of plastering in a room  $16' \times 14' \times 9'$ , making no allowance for doors, windows, etc.
5. A field 80 rods long contains 72 acres. How many feet of wire will it take to build a fence 4 wires high around it?
6. A room 36 ft.  $\times$  22 ft.  $\times$  14 ft. contains 40 persons. How many cubic feet of air space are there for each person?
7. When a gold dollar was worth \$1.12 in paper money, what was the value in gold of a \$20 bill?
8. Find the value of half a dozen silver spoons, each weighing 3 oz. 2 pwt., at \$1.15 per ounce.
9. If silver coin is  $\frac{9}{10}$  pure silver, how many ounces of silver are there in 500 silver dollars, each weighing  $412\frac{1}{2}$  grains?
10. If gold coin contains 90% gold, 9% silver, and 1% copper, find the amount of pure gold in a \$10 gold piece weighing 258 grains.
11. If the specific gravity of gold is 19.258, how many pounds Avoirdupois would a gold brick weigh of the size of a common brick?
12. If the specific gravity of silver is 10.474, what would be the weight of a silver brick?

## Measurements

1. A tank  $12\frac{1}{2}$  ft. long,  $2\frac{1}{4}$  ft. wide, and 16 in. deep is filled with water. How many gallons does it contain?
2. How many bushels of oats can be put into a bin that is 6 ft. long,  $2\frac{1}{2}$  ft. wide, and 3 ft. deep?
3. In a rainfall of 1.2 inches, how many pounds of water fall upon a lot 100 ft. long and 60 ft. wide?
4. How much will it cost to pave a street 18 rd. long and 32 ft. wide, at 55¢ a square yard?
5. How much will it cost to build a road 3 miles, 90 rods long, at \$2000 a mile?
6. How many square rods are there in a lot that is 60 ft. wide at one end, 40 ft. wide at the other end, and 10 rd. long?
7. What will be the cost of a piece of land 4200 ft. long, 3600 ft. wide, at \$200 an acre?
8. A lot of land is 24 rd. wide. How long a piece of it will make an acre?
9. Find the cost of 15 boards, each 17 ft. long, 8 in. wide, and 1 in. thick, at  $4\frac{1}{2}$ ¢ a foot, board measure.
10. Find the cost of a pile of wood 42 ft. long, 4 ft. wide, and 7 ft. high, at \$5.60 a cord.
11. A pile of wood 60 ft. long, 4 ft. wide, contains 12 cords. How high is it?
12. How much will 5 lb. 10 oz. of butter cost, at 23¢ a pound?
13. Find the cost of 5250 lb. of coal, at \$5.25 a ton.
14. If 7 lb. 4 oz. of cheese costs \$1.16, how much will 3 lb. 2 oz. cost?

## Square Root — Decimal Fractions

1. Find the square root of .0676.

To find the square root of a decimal fraction, we begin at the decimal point and proceed toward the right, dividing the figures into sections of two figures each. Since each figure of the root corresponds to one section in the number, the first figure of the root will be tenths, the second hundredths, etc.

$$\begin{array}{r} .06'76'(.26 \\ \quad 4 \\ \hline 46)276 \\ \quad 276 \\ \hline \end{array}$$

2. Find the square root of .469.

$$\begin{array}{r} .46'90'(.6848+ \\ \quad 36 \\ \hline 128)1090 \\ \quad 1024 \\ \hline 1364) \quad 6600 \\ \quad 5456 \\ \hline 13688)114400 \\ \quad 109504 \\ \hline \quad \quad 4896 \end{array}$$

When a decimal number contains an odd number of figures, a cipher may be annexed so as to complete the last section of figures. Since any number of ciphers annexed to a decimal number does not change its value, we may continue to add to the remainder sections of two ciphers each and prolong the process indefinitely. The longer it is continued, the more exact will the root become.

In the case of a whole number with a decimal fraction, we begin at the decimal point and divide the figures into sections of two figures each, thus: 3'05.61'04'3.

When any whole number is not a perfect square, ciphers may be annexed, and the root carried to decimal figures.

A common fraction whose numerator and denominator are not perfect squares may be changed to a decimal fraction and then the square root of the decimal fraction may be found.  $\sqrt{\frac{9}{8}} = \sqrt{.625} = .79 +$ .

In the case of a mixed number the fraction may be first changed to the decimal form.  $\sqrt{5\frac{1}{8}} = \sqrt{5.1666+} = 2.27 +$ .

*Find the square root of:*

- |          |           |             |                       |                      |
|----------|-----------|-------------|-----------------------|----------------------|
| 3. .0625 | 6. .937   | 9. 43.712   | 12. 210.006           | 15. $\frac{9}{10}$   |
| 4. .0841 | 7. .03246 | 10. 1249.5  | 13. $\frac{121}{961}$ | 16. $\frac{3}{8}$    |
| 5. .1369 | 8. 156.25 | 11. 821.015 | 14. $\frac{15}{64}$   | 17. $7\frac{11}{14}$ |

### Problems involving Square Root

1. Find the length of one side of a square floor that contains 6889 square feet.

2. Three times the square of a certain number is 17,328. Find the number.

3. One fourth of the square of a certain number is 3844. Find the number.

4. The length of a rectangle is twice its width. It contains 4418 square inches. What is its width?

5. How many rods long is a square field that contains 640 acres?

6. How large a square floor can be made with 25 boards, each 16 ft. long and 15 in. wide?

7. Find the length of a square which would be equal in area to the sum of three rectangles, the first of which is 10 ft. long,  $5\frac{1}{2}$  ft. wide; the second  $15\frac{3}{8}$  ft. long, 12 ft. wide; and the third 25 ft. long,  $7\frac{1}{8}$  ft. wide.

8. Find the length of a square that would be equal in area to a triangle whose base is 8 ft. 10 in. and whose altitude is 6 ft. 4 in.

9. Find the length of a square that is equal in area to a circle whose diameter is 12 feet.

10. A circle 20 ft. in diameter is inscribed in a square. How much longer is this square than a square that would be equal in area to the circle?

11. Find the length of a square that would be equal in area to a trapezoid whose parallel sides are 16 ft. 8 in. and 20 ft. 4 in. and whose altitude is 8 ft.

12. If as many as possible of 3000 men are arranged in a square with as many lines as there are men in each line, how many men will be left over?



## Geometric Forms

1. How much farther is it around a square that is 20 ft. square than around a circle that is 20 ft. in diameter?
2. Find the difference in area between a 15-inch square and a circle 15 inches in diameter.
3. One of the angles at the base of an isosceles triangle is  $27\frac{1}{2}^{\circ}$ . How large is the angle at the vertex?
4. One of the angles of a right triangle is  $43^{\circ} 20'$ . How large are the other angles?
5. When lines are drawn connecting the center of a regular pentagon with each of the angles, how large is each of the angles at the center?
6. What must be the length of a cylinder that is 2 feet in diameter to contain 50 cubic feet?
7. At 9¢ per square yard, how much will it cost to paint the outer surface of a cylindrical standpipe whose diameter is 15 ft. and whose height is 30 ft.?
8. How many cubic inches are there in a stone in the form of a square pyramid 18 inches high, whose base is 14 inches square?
9. The diameter of the base of a cone is 16 inches. Its slant height is 10 inches. Find its true altitude.
10. Find the number of cubic feet in a cone whose height is 15 feet and the diameter of whose base is 8 feet.
11. A pile of grain in the form of a cone is 7 feet high and covers a space 16 feet in diameter. Find how many bushels there are in the pile.
12. How many square inches are there in the surface of a globe whose circumference is 25.1328 inches?
13. How many square feet are there in the surface of a hemispherical dome whose diameter is 18 feet?

### Miscellaneous Problems

1. In what time can 30 men do a piece of work that 16 men can do in 25 days?

2. A can do a piece of work in 5 days and B could do it in 9 days. In how many days could they both together do it?

3. If a man can earn  $\frac{7}{10}$  of a dollar in  $\frac{2}{3}$  of a day, how much can he earn in  $\frac{2}{10}$  of a day?

4. I spent  $\frac{4}{5}$  of my money, and then spent  $\frac{1}{3}$  of the remainder, and had \$16 left. How much had I at first?

5. What principal will amount to \$357 in 3 yr. 2 mo. with interest at 6%?

6. What is the present value of a note of \$500, due in 4 mo. without interest, if the use of money is worth 5%?

7. In what time will \$100 gain \$100 at 4% simple interest?

8. Find the length of the base of a triangular field containing 20 acres, if the altitude is 48 rods.

9. What must be the length of a rectangular field 90 rods wide to contain 220 acres?

10. If I sell  $\frac{5}{8}$  of an article for as much as  $\frac{7}{8}$  of its cost, what per cent is gained?

11. I bought goods at 25% below the retail price and sold them at the retail price. What per cent did I gain?

12. \$2500.

CHARLESTON, S.C., Apr. 1, 1898.

Three years after date, for value received, I promise to pay Arthur Mason, or order, Two Thousand Five Hundred Dollars, with interest at 6%.

W. H. JACKSON.

On this note were the following indorsements: Aug. 1, 1898, \$500; Dec. 23, 1899, \$100. Find how much was due Apr. 1, 1901.

## Problems involving Square Root

Review p. 119.

1. Find the hypotenuse of a right triangle whose base is 52 inches and whose altitude is 39 inches.

2. Find the altitude of a right triangle whose hypotenuse is 8 ft. 8 in. and whose base is 5 ft. 4 in.

3. Find the altitude of an isosceles triangle whose base is 20 ft. and one of whose equal sides is 30 ft.

4. Find the altitude of an equilateral triangle, one of whose sides is 4 ft. 8 in.

5. Find the diagonal of a square that is 15 inches long.

6. Find the diagonal of a square whose area is 324 sq. in.

7. A room is 8 yards long and  $5\frac{3}{4}$  yards wide. Find the distance between two opposite corners.

8. When a pole 75 feet tall casts a shadow 60 feet long, how far is it in a straight line from the top of the pole to the end of the shadow?

9. A rectangular field is 75 rods long and 25 rods wide. How much distance is saved in going from one corner to the opposite corner by going in a direct line instead of following the boundary?

10. A and B start from the same point. A travels north 4 miles an hour, and B travels east 5 miles an hour. How far apart will they be when they have traveled 6 hours?

11. A boy flying a kite upon a level plain lets out 250 feet of string. A second boy, standing directly under the kite, is 190 feet from the first boy. If we allow 20 feet for the curvature of the string, how high is the kite?

## Compound Interest

When the interest of money is added to the principal so as to form a new principal every year, every 6 months, or at other regular intervals, the interest reckoned in this way is called compound interest.

1. Find the compound interest of \$200 for 3 years at 6%.

\$200	Principal.
<u>12</u>	Interest for first year.
\$212	Amount, and new principal.
<u>12.72</u>	Interest for second year.
\$224.72	Amount and new principal.
<u>13.48</u>	Interest for third year.
\$238.20	Amount.
<u>200</u>	
\$38.20	Compound Interest.

When there is a part of a regular period of time remaining, find the interest for the short period and add it as before.

*Find the compound interest of:*

2. \$100, 2 yr. with 6% interest added annually.
3. \$60, 3 yr. with 4% interest added annually.
4. \$90, 1 yr. 6 mo. with 6% interest added semi-annually.
5. \$400, 2 yr. 3 mo. with 5% interest added annually.

*Find the amount at compound interest of:*

6. \$300, 4 yr. at 4%, interest added annually.
7. \$250, 2 yr. at 6%, interest added semi-annually.
8. \$500, 1 yr. 3 mo. at 4%, interest added quarterly.
9. \$160, 3 yr. 5 mo. at 5%, interest added annually.
10. Find the difference between the simple interest and the compound interest of \$100 for 5 years at 6%, if the compound interest is added annually.

### Miscellaneous Problems

1. Find the number of square feet in a garden that is 2 rods square.
2. Find the length of one side of a square field containing 10 acres.
3. A rectangular field that is three times as long as it is wide contains 48 acres. How many feet wide is it?
4. The distance between the opposite corners of a rectangular field is 75 rods. The width of the field is 40 rods. Find its length.
5. The width of a house is 32 feet. The length of the rafters is  $22\frac{1}{2}$  feet. The rafters project  $2\frac{1}{2}$  feet over the side of the building. Find the height of the gable.
6. A pole 16 feet high casts a shadow 28 feet long. How far is it from the top of the pole to the end of the shadow?
7. How far from the base of a building must a ladder 40 feet long be placed, to reach a window 32 feet from the ground?
8. How many balls of iron 2 inches in diameter will it take to weigh as much as a ball of the same kind of iron 8 inches in diameter?
9. If a regular hexagon contains 256 square feet, how many square feet are there in a second regular hexagon whose side is 5 times that of the first?
10. If a ball 4 inches in diameter weighs 20 ounces, what is the weight of a ball of the same kind of wood 15 inches in diameter?
11. If a man 5 ft. 9 in. tall weighs 145 lb., about what would be the weight of a man of exactly similar proportions who is 6 ft. tall?

### Problems for Arithmetic or Algebra

1. If 7 is subtracted from four times a number, and the remainder is multiplied by 5, the result is 85. What is the number?

2. Eight men agreed to pay equal sums of money, but as three of them failed to pay, each of the others was obliged to pay \$3 more. How much did each agree to pay?

3. Twice the difference between a father's age and his son's age is 46 years. The sum of their ages is 67 years. How old is each?

4. A grocer has two kinds of tea worth 40 cents a pound and 60 cents a pound. He mixes enough to make 50 pounds. The mixture is worth 48 cents a pound. How many pounds of each kind are there in the mixture?

5. At a certain time in the month of June  $\frac{2}{3}$  of the number of days of the month which are past is equal to the number remaining. How many days remain?

6. A son's age is  $\frac{1}{2}$  of his father's age. Ten years ago his age was  $\frac{1}{3}$  of his father's age. How old is he now?

7. The greater of two numbers is 15 less than five times the less. The less is  $\frac{2}{5}$  of the greater. What are the numbers?

8. A boy bought some peaches at the rate of 4 for 3 cents, and sold them at the rate of 8 for 7 cents. He gained 10 cents. How many peaches did he buy?

9. One third, one fourth, and one fifth of a certain number, added together, will equal 26 less than the number itself. What is the number?

10. Two adjoining lots contain in all 326 square rods. The larger lot is square, and the smaller is 10 rods long and 7 rods wide. What is the length of the larger lot?

## Metric System — Long Measure

The French system, or metric system, of weights and measures is in common use in many countries. It is generally used in all countries for scientific purposes, as in chemistry, physics, etc.

The unit for measuring is the **meter**, which is 39.37 inches long. Measures larger than a meter are equal to 10 meters, 100 meters, or 1000 meters. Measures smaller than a meter are equal to  $\frac{1}{10}$  of a meter,  $\frac{1}{100}$  of a meter, or  $\frac{1}{1000}$  of a meter.

In the measures of the metric system the prefix *milli* means *one thousandth* of the principal unit; *centi*, *one hundredth*; *deci*, *one tenth*. The prefix *deka* means *ten times* the principal unit; *hekto*, *one hundred times*; *kilo*, *one thousand times*.

TABLE

10 millimeters ( <sup>mm</sup> )	= 1 centimeter ( <sup>cm</sup> ).
10 centimeters	= 1 decimeter ( <sup>dm</sup> ).
10 decimeters	= 1 <b>meter</b> ( <sup>m</sup> ).
10 meters	= 1 dekameter ( <sup>Dm</sup> ).
10 dekameters	= 1 hektometer ( <sup>Hm</sup> ).
10 hektometers	= 1 kilometer ( <sup>Km</sup> ).



1. How many decimeters are there in a meter?
2. How many millimeters in a meter?
3. How many meters in a kilometer?
4. Change 5 meters to centimeters.
5. Change 5 kilometers to millimeters.
6. How many dekameters are there in 500 decimeters?
7. How many decimeters are there in 20 kilometers?

## Metric System

Since the metric system is a decimal system, the change from one denomination to another is made by changing the position of the decimal point.

247.5 centimeters = 2.475 meters. We divide the number of centimeters by 100 by moving the decimal point two places to the left.

25.84 kilometers = 25,840 meters. We multiply by 1000 by moving the decimal point three places to the right.

1. Change  $2325^m$  to centimeters; to kilometers.
2. Change  $8.25^{km}$  to meters; to millimeters.
3. Change  $.83^m$  to centimeters; to millimeters.
4. Find the sum in meters of  $25^m$ ,  $150^{cm}$ , and  $85^{dm}$ .

First change them all to meters.

5. Find the sum in decimeters of  $32.5^{dm}$ ,  $500^{mm}$ ,  $12^m$ , and  $7^{km}$ .

6. From  $175^m$  take  $175^{dm}$ .
7. From  $355^m$  take  $2565^{cm}$ .

8. If cloth costs \$1.25 a meter, what will be the cost of  $8.3^m$  of cloth?

9. If a man walks  $50^{km}$  in 10 hours, how long will it take him to walk  $1000^m$ ?

10. How many meters apart are two places which are 17.25 kilometers apart?

11. Find the combined length of four boards which measure separately:  $7.45^m$ ,  $45.5^{dm}$ ,  $5.3^{dm}$ , and  $3.8^m$ .

12. How many pieces of rope  $1.2^m$  long can be made by cutting a piece  $16.8^m$  long?



## Metric System

### EQUIVALENTS

1 centimeter = .3937079 in.	1 dekameter = 32.80899 ft.
1 decimeter = 3.937079 in.	1 hektometer = 19.92781 rd.
1 meter = 39.37079 in.	1 kilometer = .621382 mi.


1. How many decimeters are there in a foot ?
2. How much longer is a meter than a yard ?
3. How many meters are equal to 70 yards ?
4. How many feet are there in a hektometer ?
5. How many feet are there in a kilometer ?
6. How many more feet are there in a mile than in a kilometer ?
7. How many more yards than meters are there in a mile ?
8. How many hektometers are there in 10 rods ?
9. How many kilometers are there in 100 miles ?
10. How many inches are equal to 20 decimeters ?
11. How many miles are equal to 50 kilometers ?
12. Sound travels at the rate of about 1100 feet per second. How many meters does it travel in a second ?
13. How many meters distant is a locomotive if the steam of the whistle is seen 7 seconds before the sound is heard ?
14. If the mercury in a barometer stands 30 inches high, how many millimeters high will it stand in a French barometer ?
15. If a man should step exactly a meter each time, how many steps would he take in walking a mile and a *half* ?

## Metric System

The metric unit of surface is a **square meter**. In surface measure 100 units of each denomination make one of the next higher.

The denominations of surface measure correspond to those of linear measure as follows :

TABLE

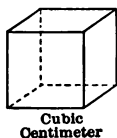
100 square millimeters ( $q^{mm}$ )	= 1 square centimeter ( $q^{cm}$ ).	 <small>Square Centimeter</small>
100 square centimeters	= 1 square decimeter ( $q^{dm}$ ).	
100 square decimeters	= 1 square meter ( $q^m$ ).	
100 square meters	= 1 square dekameter ( $q^{Dm}$ ).	
100 square dekameters	= 1 square hektometer ( $q^{Hm}$ ).	
100 square hektometers	= 1 square kilometer ( $q^{Km}$ ).	

In measuring land the square meter is called a **centare** ( $^a$ ), the square dekameter an **are** ( $^*$ ), and the square hektometer a **hektare** ( $^{Ha}$ ).

1. How many square decimeters are there in a square meter ?
2. How many square centimeters in a square meter ?
3. How many square meters in a square kilometer ?
4. Change  $5^{qHm}$  to square meters.
5. Change  $7^{qKm}$  to square meters.
6. Change  $35^{qm} 4^{qcm}$  to square centimeters.
7. How many square meters are there in a rectangular surface  $35.6^m$  long and  $22.4^m$  wide ?
8. What is the area of a triangle whose base is  $85^{cm}$  and altitude  $1.2^m$  ?
9. Find the area of a circle whose diameter is  $2.5^m$ .
10. How many rugs, each  $1^m$  long,  $30^{cm}$  wide, would it take to entirely cover a floor which is  $12.5^m$  long and  $6^m$  wide ?

## Metric System

A metric unit of volume is a **cubic meter**. The different denominations correspond to those of linear measure. 1000 units of each denomination in cubic measure make one unit of the next higher denomination.



TABLE

1000 cubic millimeters ( <sup>cu mm</sup> )	= 1 cubic centimeter ( <sup>cu cm</sup> ).
1000 cubic centimeters	= 1 cubic decimeter ( <sup>cu dm</sup> ).
1000 cubic decimeters	= 1 cubic meter ( <sup>cu m</sup> ).

For measuring wood the cubic meter is generally called a **stere** (<sup>st</sup>), the cubic decimeter a **decistere** (<sup>dst</sup>), and the cubic dekameter a **dekastere** (<sup>Dst</sup>).

1. How many cubic decimeters are there in a cubic meter?
2. How many cubic centimeters in a cubic meter?
3. How many cubic meters are there in  $5000^{\text{cu cm}}$ ?
4. How many cubic meters in  $60,000^{\text{cu cm}}$ ?
5. Change  $9^{\text{cu m}}$  to cubic centimeters.
6. Change  $7^{\text{cu dm}}$  to cubic millimeters.
7. How many cubic centimeters are there in a block  $12^{\text{cm}}$  long,  $8^{\text{cm}}$  wide, and  $5^{\text{cm}}$  high?
8. How many centimeters long is the edge of a cube which contains  $64,000^{\text{cu dm}}$ ?
9. How many square centimeters are there in one face of a cube which contains  $729^{\text{cu cm}}$ ?
10. How many cubic meters are there in a wall which is  $25^{\text{m}}$  long,  $84^{\text{dm}}$  high, and  $76^{\text{cm}}$  wide?
11. A pile of wood contains 1800 cubic meters. It is  $15^{\text{m}}$  long, and  $12^{\text{m}}$  wide. How high is it?

## Metric System

### EQUIVALENTS

1 sq. centimeter = .155 + sq. in.    1 sq. dekameter = 119.6034 sq. yd.

1 sq. decimeter = 15.5 + sq. in.    1 sq. hektometer = 2.47114 A.

1 sq. meter = 1.196 + sq. yd.    1 sq. kilometer = 247.114 A.

1 cu. centimeter = .061 + cu. in.

1 cu. decimeter = 61.026 + cu. in.

1 cu. meter = 35.316 + cu. ft.

1 decistere = 3.531 + cu. ft.

1 stere = 35.316 + cu. ft.



1 dekastere = 13.079 + cu. yd.

1. How many square feet are there in a square meter?
2. How many square meters are there in 500 square feet?
3. How many square yards are equal to  $325^{\text{a m}}$ ?
4. How many square meters are there in a room that is 15 feet long and 10 feet wide?
5. How many steres are equal to a cord?
6. How many steres are there in a pile of wood that is 20 ft. long, 4 ft. wide, and 6 ft. high?
7. How many cubic feet are equal to  $18^{\text{cu cm}}$ ?
8. How many cubic meters are there in 100 cubic inches?
9. How many cubic meters are there in a wall that is 20 ft. long, 6 ft. high, and 2 ft. thick?
10. How many cubic feet of capacity has a tank that is  $4.5^{\text{m}}$  long,  $2.4^{\text{m}}$  wide, and  $1.7^{\text{m}}$  deep?
11. What would be the cost of a pile of wood  $9^{\text{m}}$  long,  $1^{\text{m}}$  wide, and  $2^{\text{m}}$  high, at \$4.25 a cord?
12. How many ares are there in 40 acres?
13. How many hektares are there in 60 acres of land?

## Metric System

The metric unit of weight is called a **gram**. It is the weight of a cubic centimeter of water.

TABLE

10 milligrams ( <sup>mg</sup> )	= 1 centigram ( <sup>cg</sup> ).	
10 centigrams	= 1 decigram ( <sup>dg</sup> ).	
10 decigrams	= 1 gram ( <sup>g</sup> ).	
10 grams	= 1 dekagram ( <sup>Dg</sup> ).	
10 dekagrams	= 1 hektogram ( <sup>Hg</sup> ).	
10 hektograms	= 1 kilogram ( <sup>Kg</sup> ).	
1000 kilograms	= 1 metric ton ( <sup>T</sup> ) or <b>tonneau</b> .	<b>Cubic Centimeter</b> <b>Gram Weight</b>

1. How many milligrams are there in a gram?
2. How many grams in a kilogram?
3. How many centigrams in a kilogram?
4. How many centigrams in 5<sup>Kg</sup>?
5. How many grams in 15,000<sup>mg</sup>?
6. Change 5<sup>Kg</sup>, 5<sup>Hg</sup> to grams.
7. Change 2<sup>Kg</sup>, 3<sup>g</sup> to centigrams.
8. A gram equals 15.43235 grains. There are 7000 grains in an Avoirdupois pound. Find how many grams are equal to a pound Avoirdupois.
9. A cubic foot of water weighs 1000 ounces. Find how many grams a cubic inch of water weighs.
10. How many cubic centimeters are there in a cubic inch?
11. A tank is 3 feet long, 2 feet wide, and 18 inches deep. Find how many hektograms of water it will contain.

## Metric System

The metric unit of capacity is called a liter. It is the same as a cubic decimeter.

TABLE

10 milliliters ( <sup>ml</sup> )	= 1 centiliter ( <sup>cl</sup> ).
10 centiliters	= 1 deciliter ( <sup>dl</sup> ).
10 deciliters	= 1 liter ( <sup>l</sup> ).
10 liters	= 1 dekaliter ( <sup>Dl</sup> ).
10 dekaliters	= 1 hektoliter ( <sup>Hl</sup> ).
10 hektoliters	= 1 kiloliter ( <sup>Kl</sup> ).

1. How many liters are equivalent to a cubic meter of capacity?
2. How many centiliters are equivalent to a cubic meter?
3. How many cubic meters are equivalent to 4<sup>Hl</sup>?
4. How many hektoliters will a bin hold which is 12<sup>m</sup> long, 5½<sup>m</sup> wide, and 4<sup>m</sup> high?
5. At the rate of \$ 1.50 per hektoliter, how much will 5 dekaliters of wheat cost?
6. How many liters of capacity has a tank which contains 2.5<sup>cu m</sup>?
7. A liter equals 1.05671 liquid quarts. How many liters of milk are there in a can which contains 4½ gallons?
8. Find how many quarts are equivalent to 25 liters.
9. How many liters are equivalent to 25 quarts?
10. A liter contains 61.026 cubic inches. How many liters are there in a bushel, which contains 2150.42 cubic inches?
11. Find how many bushels there are in 500 liters.
12. Find what part of a quart of dry measure equals a liter.

## Tables

### LIQUID MEASURE

4 gills (gi.)	= 1 pint (pt.).
2 pints	= 1 quart (qt.).
4 quarts	= 1 gallon (gal.).
1 gal.	= 231 cubic inches.

### DRY MEASURE

2 pints (pt.)	= 1 quart (qt.).
8 quarts	= 1 peck (pk.).
4 pecks	= 1 bushel (bu.).
1 bushel	= 2150.42 cubic inches.

### LINEAR MEASURE

12 inches (in.)	= 1 foot (ft.).
3 feet	= 1 yard (yd.).
5½ yards	= 1 rod (rd.).
16½ feet	= 1 rod.
320 rods	= 1 mile (mi.).
1760 yards	= 1 mile.
5280 feet	= 1 mile.

### SURFACE MEASURE

144 square inches (sq. in.)	= 1 square foot (sq. ft.).
9 square feet	= 1 square yard (sq. yd.).
30¼ square yards	= 1 square rod (sq. rd.).
272¼ square feet	= 1 square rod.
160 square rods	= 1 acre (A.).
4840 square yards	= 1 acre.
43560 square feet	= 1 acre.
640 acres	= 1 square mile (sq. mi.).

### SURVEYORS' MEASURE

<i>Long Measure</i>		<i>Square Measure</i>
7.92 in.	= 1 link (l.).	16 sq. rd. = 1 sq. ch.
100 links	= 1 chain (ch.).	10 sq. ch. = 1 A.
80 chains	= 1 mile (mi.).	640 A. = 1 sq. mi.
		1 sq. mi. = 1 section (sec.).
		36 sec. = 1 township.

## Tables

### ANGULAR MEASURE

60 seconds (")	= 1 minute (')
60 minutes	= 1 degree (°).
360 degrees	= 1 circle.

### SOLID OR CUBIC MEASURE

1728 cubic inches (cu. in.)	= 1 cubic foot (cu. ft.).
27 cubic feet	= 1 cubic yard (cu. yd.).

### WOOD MEASURE

16 cubic feet	= 1 cord foot (cd. ft.).
8 cord feet, or 128 cubic feet	} = 1 cord (cd.).

### AVOIRDUPOIS WEIGHT

Avoirdupois Weight is used in weighing all articles except gold, silver, and precious stones.

16 ounces (oz.)	= 1 pound (lb.).
100 pounds	= 1 hundredweight (cwt.).
2000 pounds	= 1 ton (T.).

The long ton is used in the United States customhouses and in wholesale transactions in iron and coal.

112 pounds Avoirdupois	= 1 long hundredweight.
2240 pounds Avoirdupois	= 1 long ton.

The pound Avoirdupois contains 7000 grains.

### TROY WEIGHT

Troy Weight is used in weighing gold, silver, and jewels.

24 grains (gr.)	= 1 pennyweight (pwt.).
20 pennyweights	= 1 ounce (oz.).
12 ounces	= 1 pound (lb.).



## Tables

### APOTHECARIES' WEIGHT

Apothecaries' Weight is used in mixing medicines.

20 grains (gr.) = 1 scruple (℞).                      8 drams = 1 ounce (℥).  
 3 scruples = 1 dram (ʒ).                              12 ounces = 1 pound (lb).

The pound Troy and the Apothecaries' pound are equal, each weighing 5760 grains. The pound Avoirdupois weighs 7000 Troy or Apothecary grains.

The ounce Troy and the Apothecaries' ounce are each 480 grains; the ounce Avoirdupois is 437½ grains.

### TIME MEASURE

60 seconds (sec.) = 1 minute (min.).                      365 days = 1 common year (yr.).  
 60 minutes = 1 hour (hr.).                                      366 days = 1 leap year.  
 24 hours = 1 day (da.).    100 years = 1 century.  
 7 days = 1 week (wk.).

The names of the months (mo.), called calendar months, and the number of days in each are:

	da.		da.
1. January (Jan.) . . . . .	31	7. July . . . . .	31
2. February (Feb.) . . . . .	28 or 29	8. August (Aug.) . . . . .	31
3. March (Mar.) . . . . .	31	9. September (Sept.) . . . . .	30
4. April (Apr.) . . . . .	30	10. October (Oct.) . . . . .	31
5. May . . . . .	31	11. November (Nov.) . . . . .	30
6. June . . . . .	30	12. December (Dec.) . . . . .	31

### UNITED STATES MONEY

10 mills (m.) = 1 cent (¢).  
 10 cents = 1 dime (d.).  
 10 dimes = 1 dollar (\$).  
 10 dollars = 1 Eagle (E.).

### ENGLISH MONEY

4 farthings (far.) = 1 penny (d.).  
 12 pence = 1 shilling (sh.).  
 20 shillings = 1 pound or sovereign (£).

### FRENCH MONEY

100 centimes (ct.) = 1 franc (fr.)

### GERMAN MONEY

100 pfennigs (pf.) = 1 mark (mk.)

### MISCELLANEOUS TABLE

12 units = 1 dozen.		20 units = 1 score.
12 dozen = 1 gross.		24 sheets = 1 quire.
12 gross = 1 great gross.		20 quires = 1 ream.

## For Reference

**Acute angle.** An acute angle is smaller than a right angle.

**Altitude.** The altitude of a figure is the perpendicular distance from the base to the highest point, or the point farthest from the base.

**Angle.** An angle is the difference in direction of two lines.

**Area.** The area of a square, a rectangle, or a parallelogram, is equal to the product of its base and its altitude.

The area of a triangle is equal to one half of the product of its base and altitude.

The area of a trapezoid is equal to the product of the average between its two parallel sides and the perpendicular distance between them.

The area of a circle is equal to one half of the product of its radius and circumference.

**Atmospheric pressure.** The pressure of the atmosphere upon each square inch at the level of the sea is about 14.7 pounds.

**Barometer.** The barometer falls about an inch in each 1000 feet of ascent.

**Base.** The base of a figure is the line or plane upon which it is supposed to rest.

**Bricks.** A brick is 8 in. long, 4 in. wide, and 2 in. thick. On the side of a wall 7 bricks with the mortar cover about 1 sq. ft. About 22 bricks with the mortar will fill 1 cu. ft.

**Buoyancy.** The buoyancy, or supporting force, of a liquid is sufficient to raise to the surface of the liquid an object whose weight is equal to that of the liquid displaced.

**Bushels.** A bushel contains 2150.42 cu. in. There are about  $\frac{1}{4}$  as many bushels in a bin as there are cubic feet. Of fruits and vegetables there are about  $\frac{1}{4}$  as many bushels as cubic feet.

**Circle.** A circle is a plane figure bounded by a curved line, all points of which are at equal distances from the center.

**Circumference.** The circumference of a circle is its boundary line. The circumference of a circle is 3.1416 times its diameter.

**Clapboards.** A clapboard is usually 4 ft. long and 6 in. wide. There are 25 in a bunch. A clapboard will cover about one square foot of surface.

## For Reference

**Cone.** A cone may be regarded as a pyramid whose base contains an infinitely large number of sides, so as to become a circle.

**Cylinder.** A cylinder is a solid whose ends or bases are circles.

**Diagonal.** A diagonal is a straight line connecting two angles that are not adjacent to each other.

**Diameter.** The diameter is the distance across the circle through the center.

**Equilateral triangle.** An equilateral triangle is a triangle all of whose sides are equal.

**Falling bodies.** A heavy body will fall about 16 feet in one second,  $2 \times 2 \times 16$  feet in two seconds,  $3 \times 3 \times 16$  feet in three seconds, etc. The distance through which a body falls is proportional to the square of the number representing the time of the fall.

**Foot-pound.** A foot-pound is the amount of work expended in raising one pound one foot high.

**Forces.** When two or more forces act upon an object at the same time, the effect is the same as if each force acted alone.

**Gallons.** A gallon contains 231 cu. in. A gallon of water weighs about  $8\frac{1}{4}$  lb. A cubic foot of water weighs  $62\frac{1}{2}$  lb.

**Hexagon.** A hexagon is a polygon having six sides.

**Horse power.** A horse power is sufficient power to raise 550 pounds one foot per second.

**Isosceles triangle.** An isosceles triangle is a triangle two of whose sides are equal.

**Lateral surface.** The lateral surface of a solid is the surface of its sides exclusive of its ends or bases.

The lateral surface of a cylinder is equal to the circumference of its base multiplied by its altitude.

The lateral surface of a regular pyramid or cone is equal to one half of the product of the perimeter or circumference of the base and the slant height.

**Laths.** A lath is 4 ft. long and  $1\frac{1}{2}$  in. wide. Laths are usually left  $\frac{3}{8}$  of an inch apart when nailed. There are 50 laths in a bunch. A bunch will cover 3 square yards of surface.

**Lever.** In any application of the lever the number of units of the power times the number of units of its distance from the fulcrum is

## For Reference

equal to the number of units of the weight times the number of units of its distance from the fulcrum.

**Light.** Light travels at the rate of about 186,000 miles a second.

**Momentum.** The momentum of a body is its weight in pounds multiplied by the number of feet through which it is moving per second.

**Oblong.** An oblong is a rectangle whose length exceeds its width.

**Obtuse angle.** An obtuse angle is larger than a right angle.

**Paper.** In America wall paper is usually 18 in. wide. A single roll is 24 ft. long, a double roll 48 ft. long.

**Parallelogram.** A parallelogram is a quadrilateral whose opposite sides are parallel to each other.

**Pentagon.** A pentagon is a polygon having five sides.

**Perimeter.** The perimeter of a plane figure is its boundary line.

**Plane.** A plane is a surface such that a straight line joining any two points in it lies wholly within that surface.

**Polygon.** A polygon is a plane figure bounded by straight lines.

In a regular polygon the sides are all equal and the angles are all equal.

**Pressure.** The pressure of liquids at any point is equal in all directions.

**Prism.** A prism is a solid whose ends are polygons and whose sides are parallelograms.

**Pyramid.** A pyramid is a solid whose base is a polygon and whose lateral faces are triangles.

**Quadrilateral.** A quadrilateral is a polygon having four sides.

**Radius.** The radius is the distance from the center of a circle to the circumference.

**Rectangle.** A rectangle is a quadrilateral all of whose angles are right angles.

**Right angle.** A right angle is an angle of  $90^\circ$ .

**Right triangle.** A right triangle is a triangle which has one right angle.

**Shingles.** Shingles are packed and bound in bunches. Four bunches of shingles make 1000. A thousand shingles will cover 100 square feet, when laid with 4 inches exposed to the weather.

## For Reference

**Slant height.** The slant height of a pyramid is the perpendicular distance from its vertex to one of the sides of its base.

**Sound.** Sound travels in the air at the rate of about 1100 feet a second, and in the water at the rate of about 4700 feet a second.

**Specific gravity.** The specific gravity of a substance is the ratio of its weight to the weight of water.

**Sphere.** A sphere is a solid, all points of whose surface are equally distant from the center.

**Square.** A square is a quadrilateral, all of whose angles are right angles and all of whose sides are equal.

**Stone masonry.** Stone is reckoned in cubic feet, in cords, or in perches. A perch is  $24\frac{1}{4}$  cu. ft.

**Surface.** The surface of a sphere is equal to the product of its diameter and its circumference.

See also lateral surface.

**Trapezoid.** A trapezoid is a quadrilateral, two of whose sides are parallel to each other.

**Triangle.** A triangle is a polygon having three sides.  
The sum of the angles of any triangle equals  $180^\circ$ .

**Unit of heat.** A unit of heat is the amount of heat required to raise a pound of water one degree Fahrenheit.

**Vertex.** The vertex of a figure is its highest point, or the point of the angle opposite the base.

**Volume.** The volume of a solid is the number of cubic units which it contains.

The volume of a prism or of a cylinder is equal to the area of one end multiplied by the length.

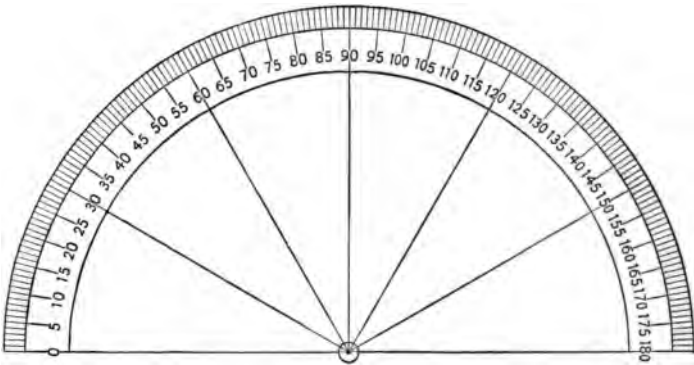
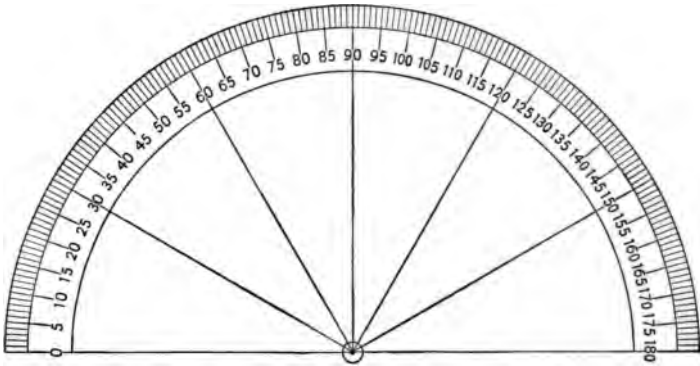
The volume of a pyramid or of a cone is equal to one third of the product of the area of the base and the altitude.

The volume of a sphere is equal to its surface multiplied by one third of its radius.

**Water.** A cubic foot of water weighs 1000 ounces.

Water in freezing expands about  $7\frac{1}{2}$  per cent.

## Protractors



Paste one of these upon a piece of stiff paper or cardboard and cut it out to use with pp. 87, 109, etc.



## ANSWERS

**Page 10.** — 1.  $2\frac{1}{2}$ . 2.  $1\frac{3}{8}$ . 3.  $1\frac{1}{2}$ . 4.  $2\frac{11}{16}$ . 5.  $26\frac{1}{2}$ . 6.  $20\frac{1}{2}$ . 7.  $32\frac{1}{2}$ .  
 8.  $15\frac{1}{2}$ . 9.  $\frac{7}{10}$ . 10.  $\frac{11}{4}$ . 11.  $\frac{2}{1}$ . 12.  $\frac{1}{10}$ . 13.  $3\frac{1}{2}$ . 14.  $7\frac{1}{2}$ . 15.  $3\frac{2}{3}$ .  
 16.  $9\frac{3}{4}$ . 17.  $2\frac{8}{11}$ . 18.  $14\frac{7}{12}$ . 19.  $\frac{2}{3}$ . 20.  $\frac{35}{144}$ . 21.  $94\frac{1}{2}$ . 22.  $120\frac{1}{2}$ .  
 23.  $8\frac{2}{3}$ . 24.  $162\frac{2}{9}$ . 25.  $\frac{1}{10}$ . 26. 6. 27.  $2\frac{2}{3}$ . 28.  $2\frac{2}{24}$ . 29.  $7\frac{1}{11}$ . 30.  $6\frac{1}{2}$ .  
 31. 10. 32.  $4\frac{7}{22}$ . 33. .5. 34. .6. 35. .7. 36. .25. 37. .8. 38. .75.  
 39. .05. 40. .04. 41. .18. 42. .96. 43. .95. 44. .87.

**Page 11.** — 1. 155.2217. 2. 551.5069. 3. 120.48018. 4. 301.5712.  
 5. 649.7756. 6. 67.57436. 7. 229.7556. 8. 165.705776. 9. .1632.  
 10. .478. 11. 3.84. 12. .09375. 13. .0003325. 14. 2403. 15. 376,051.  
 16. 326.0034. 17. .24735. 18. 85.73485. 19. 1.105. 20. .00114+.  
 21. 7.223+. 22. .7606+. 23. .21536. 24. .3148. 25. 50,400. 26. 2100.  
 27. 378,200. 28. 5,000,000. 29.  $\frac{1}{2}$ . 30.  $\frac{1}{4}$ . 31.  $\frac{1}{8}$ . 32.  $\frac{9}{40}$ . 33.  $\frac{13}{200}$ .  
 34.  $\frac{1}{10}$ . 35.  $\frac{2}{3}$ . 36.  $\frac{1}{30}$ . 37.  $1\frac{1}{2}$ . 38.  $\frac{249}{2000}$ . 39.  $3\frac{1}{2}$ . 40.  $15\frac{1}{10}$ .  
 41.  $9\frac{1}{2}$ . 42.  $12\frac{3}{4}$ . 43.  $73\frac{1}{10}$ .

**Page 12.** — 1. 8 gal. 2 qt. 1 pt. 1 gi. 2. 3 bu. 3 pk. 1 qt. 1 pt.  
 3. 35 yd. 2 ft. 4. 1 lb.  $4\frac{1}{2}$  oz. 5.  $56\frac{1}{2}$ . 6. 2 mi.  $773\frac{1}{2}$  ft. 7. 84.  
 8. 4 yr. 2 mo. 2 da. 9. 4 yr. 10 mo. 3 da. 10. 450. 11.  $44\frac{2}{3}$ . 12.  $3\frac{1}{2}$ .  
 13. 3. 14. 36. 15. 25.

**Page 13.** — 1. \$3.20. 2. \$2.21. 3. \$2.53. 4. 54¢. 5. \$5.64.  
 6. \$1.77. 7. 18¢. 8. \$37.71. 9. \$175.68. 10. \$6.29. 11. \$5684.48.  
 12. \$683.01. 13. \$64.69. 14. \$996.09. 15. \$72,000. 16. \$51,225.  
 17. \$42. 18. \$74. 19. \$91.41. 20. \$139.62. 21. \$11.57. 22. \$45.60.  
 23. \$9.08. 24. \$16.56.

**Page 14.** — 15. \$67.50. 16. \$22.50. 17. \$175. 18. \$800. 19. \$24.  
 20. \$800. 21. \$39.

**Page 15.** — 1. Mon. \$17,316.42; Tues. \$17,407.46; Wed. \$16,950.01;  
 Thur. \$14,037.74; Fri. \$13,566.02; Sat. \$13,144.93. 1st, \$13,538.18; 2d,  
 \$5488.71; 3d, \$11,769.37; 4th, \$9615.79; 5th, \$14,101.84; 6th, \$8010.74;  
 7th, \$9236.88; 8th, \$9869.37; 9th, \$5747.62; 10th, \$5044.08; total,  
 \$92,422.58. 2. Mon. \$17,647.43; Tues. \$11,168.82; Wed. \$10,703.57;  
 Thur. \$12,650.01; Fri. \$16,867.78; Sat. \$14,793.01. 1st, \$13,781.79; 2d,  
 \$9411.73; 3d, \$867.93; 4th, \$7773.69; 5th, \$8875.87; 6th, \$8005.77; 7th,  
 \$9582.07; 8th, \$7329.60; 9th, \$6302.36; 10th, \$11,899.81; total, \$83,830.62.

**Page 16.** — 1. 625 mi. 2. 7 da.  $18\frac{1}{2}$  hr. 3.  $6\frac{1}{2}$ . 4. About 35.  
 5. \$520,833.33. 6. \$213.83. 7. About 60¢.

**Page 17.** — 12. 28¢. 13. \$1.30. 14. \$3.30. 15. \$1.82. 16. \$3.75.  
 17. \$1.40. 18. \$11.20. 19. \$16.80. 20. \$13.08.

**Page 19.** — 8. 5 hr. 52 min. 9. 30 min. 44 sec. 10. 6 hr. 37 min.  
 49  $\frac{1}{2}$  sec. 11. 1 hr. 7 min.  $\frac{1}{3}$  sec. 12. 29 min. 16 sec. past 3. 13. 7 min.  
 $\frac{1}{3}$  sec. past 6.

**Page 20.** — 1. \$750 for each \$1000. 2. \$4204. 3. \$78.

**Page 22.** — 2. 72¢. 3. \$17.50. 4. \$42. 5. \$74.20. 6. \$270.  
 7. \$31.88. 8. \$40.30. 9. \$1.71. 10. \$37.58. 11. \$22.89.



## Answers

- Page 23.**—1. About 24 sec. 2. About 1075. 3. 1024 ft. 4. 2 sec.  
5. 849½. 6. 7.80+. 7. 14.74+ lb. 8. 15.08+ lb. 9. 13.9+ lb.
- Page 24.**—4. 10 hr. 50 min. 5. 11 hr. 11 min. 55 sec. 6. 40¢.  
7. 17½. 8. \$18.90. 9. \$1200. 10. Lost the interest, less \$500.
- Page 25.**—1. 310 rd. 2. No difference. 3. 4. 4. 50. 5. 7½. 6. 80.  
7. 55. 8. 126½. 9. 15.
- Page 27.**—3. 9 A.M. 4. 10 A.M. 5. 11 A.M. 7. 20 min. 8. 20 min.;  
30 min.; 6 min.; 19 min.
- Page 28.**—1. \$8.40. 2. \$1.75. 3. \$9.28. 4. \$9.92. 5. \$12.37.  
6. \$3.07. 7. \$6.87. 8. \$14.40. 9. \$3.95. 10. \$22.87. 11. \$45.75.  
12. \$75. 13. \$154.89. 14. \$142.95. 15. \$321.23. 16. \$20.19.  
17. \$10.43. 18. \$29.65. 19. \$29.99. 20. \$25.69.
- Page 31.**—2. 18 and 6. 3. 80 and 16. 4. 32. 5. 8 years. 6. 15.  
7. \$350 and \$50. 8. 29.
- Page 32.**—1. 5 horses, 10 cows, and 15 sheep. 2. 8 peach; 24 pear;  
40 apple. 3. 20. 4. 6 geese; 12 ducks; 24 hens. 5. Jennie, 9; Charles,  
27; Henry, 36. 6. 18. 7. Ruth, 3; Clara, 12; Mary, 6. 8. \$4.
- Page 33.**—1. 36 and 12. 2. 23. 3. A, \$20; B, \$60; C, \$120.  
4. 14. 5. \$21. 6. 12. 7. 11. 8. 13. 9. 3. 10. 9. 11. 14. 12. 10.  
13. 14. 14. 17. 15. 12½. 16. 7. 17. 100.
- Page 34.**—1. 7. 2. 30. 3. ½. 4. ⅓. 5. ¼. 6. ⅕. 7. 1⅓. 8. 1⅔.  
9. 3⅓. 10. 3⅓. 11. 9. 12. 100. 13. 25. 14. 5. 15. 4⅓. 16. 18.  
17. ⅓. 18. 10. 19. 7.31.
- Page 35.**—1. 283.336382. 2. 2785.17544. 3. .00594. 4. 16.1115.  
5. .025. 6. .0000007. 7. .000675. 8. 4.95. 9. 4615. 10. .08.  
11. 7.28. 12. .028. 13. .2342. 14. .000643. 15. ⅓; ¼; ⅕; ⅖; ⅗;  
16. ⅘; ⅙; ⅚; ⅛; ⅞.
- Page 36.**—1. 12. 2. 2 oz. 3. 40 lb.
- Page 37.**—1. 400. 2. 25 lb. 3. 300. 4. 225.
- Page 38.**—1. WD = 3½ in. 2. PD = 24 ft. 3. P = 321½ lb.  
4. W = 270 lb. 5. WD = 2 ft. 6. W = 66⅔ lb. 7. P = 25 lb.  
8. PD = 93½ in. 9. W = 300 lb. 10. PD = 45 in. 11. W = 140 lb.  
12. W = 200 lb. 13. P = 47½ lb.
- Page 39.**—8. 39¢. 9. \$3.45. 10. \$7.85. 11. \$15.35. 12. \$50.25.  
13. 39¢. 14. \$1.98. 15. \$3.41. 16. \$31.36. 17. \$137.03.
- Page 40.**—4. 17%. 5. 14%. 6. 24%. 7. 6%. 8. 4%. 9. 65%.  
10. 4⅓%. 11. 6⅓%. 12. 19%. 13. 75%. 14. 16%. 15. 19%. 16. 2⅓%.  
17. 3⅓%. 18. 20%. 19. 20%. 20. 4%. 21. 20%. 22. 16%. 23. 55⅓%.  
24. 35⅓%. 25. 24⅓%. 26. 39⅓%. 27. 47%. 28. 111⅓%.  
29. 113⅓%. 30. 44%. 31. 163⅓%. 32. 3⅓%. 33. 311⅓%.
- Page 41.**—1. 24,900.3216 mi. 2. About 69½ mi. 3. About  
197,359,949. 4. About 31. 5. 6785.856 mi. 6. About 7,328,724.  
7. About 24,900 mi. 8. About 692. 9. 1,507,968. 10. About 55,850.
- Page 42.**—1. \$20. 2. \$6.70. 3. \$32.88. 4. \$65.98. 5. \$7.  
6. \$4.12. 7. \$1.76. 8. \$10.04. 9. \$30.09. 10. \$25.35. 11. \$101.73.  
12. \$133.66. 13. \$4.50. 14. \$9.92. 15. \$17.26. 16. \$90.18.  
17. \$109. 18. \$182.08. 19. \$47.50. 20. \$281.98.
- Page 43.**—1. East, 26¼°. 2. 37° 35'. 3. 30° 15' 28" E. 4. 10 hr.  
5 min. 6⅓ sec. 5. \$8.20. 6. \$13.95. 7. \$118.75. 8. \$160.42.  
9. \$281.68. 10. 15 ft. 11. 1125 lb. 12. 2 ft. 2⅓ in.
- Page 44.**—1. 3330. 2. 3320. 3. 3204. 4. 3623. 5. 3330. 6. 2220.

## Answers

7. 2220. 8. 2220. 9. 2620. 10. 2220. 11. 44,330. 12. 54,929.  
 13. 52,635. 14. \$470.90. 15. \$521.60.  
 Page 45.—1. 175 sq. in. 2. 20 in. 3. 151.73928 ft. 4. 25.1328 sq. ft.  
 Page 46.—1. 1,250,000. 2. About  $3\frac{1}{2}$ . 3. 6558. 4. 2930. 5. 1351. 6. 338. 7. Greater. 8. \$337.42. 9. \$14.29. 10. \$302.12. 11. \$15.05.  
 Page 47.—1. 11 min.  $46\frac{1}{2}$  sec. 2. 3 hr. 13 min.  $44\frac{1}{2}$  sec. 3. 3 hr. 25 min. 31 sec. 4. 2 hr. 21 min. 43 sec. 5. 10 in. 6. 31.416 in. 7. 10 in. 8. 157.08 sq. in. 9. 12 in. 10. 6 in. 11. 113.0976 sq. in. 12. 7.6394 + in. 13. 57.29 + sq. in.  
 Page 48.—1. 20. 2.  $122\frac{1}{2}$ . 3. 60 lb. 4. 80 lb.  
 Page 49.—11.  $5^{\circ} 56' 14''$ . 12.  $59' 57''$ . 13.  $3^{\circ} 5'$ . 14.  $2^{\circ} 23' 54''$ . 15.  $2^{\circ} 0' 28''$ . 16. 15 min.  $44\frac{1}{2}$  sec. later. 17.  $49\frac{1}{2}$  sec. earlier. 18. 18 min. later. 19. 20 min. 8 sec. earlier. 20. 18 min.  $58\frac{1}{2}$  sec. later.  
 Page 50.—1. \$59.50. 2. 15 lb. 3. 70%<sub>o</sub>. 4. 25%<sub>o</sub>. 5. 300. 6. \$1600. 7. \$400. 8. 640. 9. 700. 10. 323. 11. \$36. 12.  $23\frac{1}{2}\%$ . 13. \$8350. 14. \$650. 15. \$8. 16. \$609.69.  
 Page 52.—1. 14. 2. 10. 3. 16. 4. 18. 5. 6. 6. 20. 7. 34. 8. 12. 9. 12 and 24. 10. 21 in. 11. 16 sq. ft. 12. 84.  
 Page 53.—3. 20. 4. 25. 5. 12. 6. 4. 7. 6. 8. 6.  
 Page 54.—2. 4. 3. 2. 4. 4. 5. 7. 6. 10. 7. 5. 8. 3. 9. 2. 10. 6. 11. 5. 12. \$4. 13. 14. 14. 15. 15. 12. 16. 40 ft. 17. 14 ft.  
 Page 55.—2. 6. 3. 5. 4. 18. 5. 10. 6. 5. 7. 10. 8. 15. 9. 3. 10. 4. 11. 6. 12. 5. 13. 3. 14. 2. 15. \$7. 16. \$10. 17. 39 and 26. 18. 44 yr. and 22 yr.  
 Page 56.—1. 3, 9, and 12. 2. 12, 24, and 36. 3. 10, 30, and 40. 4. \$7 and \$3. 5. \$5 and \$3. 6. 9 and 2. 7. 17 and 7. 8. 3, 4, and 6. 9. \$13. 10. 20 and 8.  
 Page 57.—1. About  $7\frac{1}{2}$  times. 2.  $1\frac{1}{2}$  sec. 3.  $8\frac{1}{2}$  min. 4. 4 hr. 10 min.  $53\frac{1}{2}$  sec. 5. 25,800,000 mi. 6. 2 min.  $18\frac{1}{2}$  sec. 7. About  $4\frac{1}{2}$  yr. 8. About 268,817. 9. 892,800. 10.  $13\frac{1}{2}$  da.  
 Page 58.—1. \$820. 2. \$822.22. 3. \$26.75. 4. \$240.09. 5. \$349.12. 6. \$7.97. 7. \$15,000. 8. 8 lb. 15 oz. 9. 12 lb.  
 Page 59.—1. 2; 3;  $\frac{1}{2}$ . 2. 3;  $1\frac{1}{2}$ ;  $\frac{1}{2}$ . 3. 1;  $1\frac{1}{2}$ ;  $\frac{1}{2}$ . 4. 480; 240; 1440. 5. 2880; 1440; 11,520. 6. 110. 7. Avoirdupois 1240 gr. 8.  $1\frac{1}{2}$ . 9.  $437\frac{1}{2}$ . 10. 480. 11.  $1\frac{1}{2}$ . 12. 12 lb  $1\frac{3}{4}$  6  $\frac{3}{4}$  2  $\frac{1}{2}$ . 13. 2800. 14.  $16\frac{1}{2}$ . 15. 15,360 gr.; 19,440 gr. 16. 1800  $\oslash$ ; 53  $\oslash$ . 17. 2  $\oslash$  2 gr.; 6  $\oslash$ ; 3 lb. 4  $\oslash$ .  
 Page 62.—1.  $5\frac{1}{2}$ . 2. 27 cu. in. 3. 25 ft. 4. 24. 5.  $64^{\circ} 48'$ . 6.  $60^{\circ}$ . 7.  $36^{\circ} 30'$ ;  $107^{\circ}$ . 8. 3325. 9. 180 ft. 10. 1260. 11. 1925. 12. \$53.47.  
 Page 64.—1. 360 gr.; 56 gr.; 84 gr. 2. 480 gr.; 1080 gr.; 970 gr. 3. 2 lb. 6 oz.; 1 lb. 5 oz. 10 pwt.; 10 oz. 8 pwt. 8 gr. 4.  $\frac{1}{4}$ ;  $\frac{1}{10}$ ;  $1\frac{1}{2}$ . 5.  $\frac{1}{2}$ ;  $\frac{1}{10}$ . 6.  $\frac{1}{4}$ . 7. 2 lb. 7 oz. 6 pwt. 12 gr. 8. 37, with 68 gr. left. 9. Sugar, 1240 gr. 10.  $39\frac{1}{4}$  lb. 11.  $607\frac{1}{2}$ . 12. 1396, with 150 gr. over. 13. Nearly 1087. 14. 1,612,643.12 + lb.  
 Page 65.—1. 5. 2. 60. 3. 28.8. 4. 55.25. 5. 24. 6. 12. 7. 3. 8. 9.02. 9. 600. 10. 1500. 11. 2000. 12. 350. 13. 15,600. 14. 4000. 15. 5000. 16. 600. 17. 34%. 18. 65%. 19. 23%. 20. 30%. 21. 2%. 22. 8%. 23.  $14\frac{1}{4}\%$ . 24.  $4\frac{1}{4}\%$ . 25. 12%. 26. 12%. 27. 35%.

## Answers

- 28.**  $6\%$    **29.**  $32\%$    **30.**  $4\%$    **31.**  $7\%$    **32.**  $23\%$    **33.**  $20\%$    **34.**  $15\%$   
**35.**  $80\%$    **36.**  $80\%$    **37.**  $17\%$    **38.**  $5\%$    **39.**  $10\%$    **40.**  $40\%$
- Page 66.**—1. \$24.   2. \$2.   3. \$600.   4. \$250.   5. \$60.   6. \$35.  
**7.**  $59\frac{1}{2}$ .   **8.** 46 gal.   **9.**  $8\frac{1}{2}$ .   **10.** 23,097,642.   **11.** 24,9975.   **12.** 811,512,335.  
**13.** 1,501,000.
- Page 67.**—1. 1.8;  $\frac{1}{2}$ .   2.  $12\frac{1}{2}^\circ$ .   3.  $-17\frac{1}{2}^\circ$ .   4.  $15\frac{1}{2}^\circ$ .   5.  $-11\frac{1}{2}^\circ$ .  
**6.**  $65\frac{1}{2}^\circ$ .   **7.**  $37\frac{1}{2}^\circ$ .   **8.**  $-28\frac{1}{2}^\circ$ .   **9.**  $16\frac{1}{2}^\circ$ .   **10.**  $59^\circ$ .   **11.**  $107\frac{1}{2}^\circ$ .   **12.**  $-1^\circ$ .  
**13.**  $53\frac{1}{2}^\circ$ .   **14.**  $158^\circ$ .   **15.**  $-11\frac{1}{2}^\circ$ .   **16.**  $6\frac{1}{2}^\circ$ .   **17.**  $66^\circ$ .
- Page 68.**—2. 4.   3.  $1\frac{1}{2}$ .   4. 18.   5. 9.   6.  $10\frac{1}{2}$ .   7. 45.   8. 3.   9.  $1\frac{1}{2}$ .  
**10.**  $2\frac{1}{2}$ .   **11.**  $5\frac{1}{2}$ .   **12.**  $5\frac{1}{2}$ .   **13.**  $2\frac{1}{2}$ .   **14.**  $1\frac{1}{2}$ .   **15.**  $1\frac{1}{2}$ .   **16.**  $1\frac{1}{2}$ .  
**17.**  $\frac{1}{2}$ .   **18.**  $1\frac{1}{2}$ .   **19.**  $1\frac{1}{2}$ .   **20.** 7.   **21.** 7.   **22.** 8.   **23.** 10.   **24.**  $15\frac{1}{2}$ .  
**25.**  $42\frac{1}{2}$ .   **26.** 3.   **27.**  $3\frac{1}{2}$ .   **28.** 8.   **29.** 2.   **30.**  $1\frac{1}{2}$ .   **31.**  $2\frac{1}{2}$ .   **32.** 2.  
**33.**  $2\frac{1}{2}$ .   **34.** 10.   **35.**  $10\frac{1}{2}$ .   **36.**  $9\frac{1}{2}$ .   **37.**  $6\frac{1}{2}$ .
- Page 69.**—1. \$27.   2. 1000.   3. \$1.46.   4. 24¢.   5. 36¢.   6. \$1.05.  
**7.** 96¢.   8.  $387\frac{1}{2}$ .   9. 292,168,800 mi.   10. About  $387\frac{1}{2}$ .   11. 14,592.  
**12.** 144.   **13.**  $8\frac{1}{2}^\circ$ .   **14.**  $-22\frac{1}{2}^\circ$ .
- Page 70.**—1. 128 sq. in.   2. 576 sq. in.   3.  $48\frac{1}{2}$  sq. ft.   4.  $96\frac{1}{2}$  sq. ft.  
**5.** 47,124 in.   **6.** 18,326 sq. ft.   **7.** 13,368 + in.   **8.** 6,047 + in.   **9.** 48 sq. ft.  
**10.** 130 sq. ft.   **11.**  $6\frac{1}{2}$  sq. ft.   **12.**  $23\frac{1}{2}$  sq. ft.   **13.** 62,832 in.   **14.** 2,864 + in.  
**15.** 6,981 + sq. ft.   **16.** 10,472 sq. ft.   **17.** 78.54 in.   **18.** 1,909 + ft.  
**19.** 78.54 sq. ft.   **20.** 28,274 + sq. ft.
- Page 72.**—3. 16.   4. 12.   5. 12.   6. 24.   7. 12.   8. 12.   9. 36.  
**10.** 20.   **11.** 15 and 12.
- Page 73.**—1. 12.   2. 8.   3. 20.   4. 7.   5. 6.   6. 9.   7. 10.   8. 9.
- Page 74.**—2. 8.   3. 5.   4. 9.   5. 9.   6. 5.   7. 5.   8. 10.   9. 7.   10. 6.  
**11.** 11.   **12.** 9.   **13.** 4.   **14.** 20.   **15.** 23.   **16.** 3.   **17.** 12.   **18.** 2.   **19.** 5.
- Page 75.**—1. 12.   2. 8.   3. 25.   4. 20 and 13.   5. 40.   6. 16 and 12.  
**7.** 28 and 35.   8. 5 and 45.   9. \$500.   10. \$60; \$70; \$65.   11. \$120.
- Page 76.**—1. 12.   2. 20 yr.   3. 16.   4. 14.   5. \$1.95.   6. 35.   7. 10,  
 15, and 20.   8. 8 and 12.   9. 9 and 12.   10. \$12.   11. \$35.
- Page 77.**—1.  $\frac{1}{3}$ .   2.  $\frac{1}{2}$ .   3.  $\frac{1}{20}$ .   4.  $\frac{1}{2}$ .   5. 28%.   6. 33%.   7.  $93\frac{1}{2}\%$ .  
**8.**  $14\%$ .   **9.**  $5\%$ .   **10.**  $\frac{1}{5}$ .   **11.**  $\frac{1}{2}$ .   **12.**  $\frac{1}{2}$ .   **13.** \$12,000.   **14.** \$1800.  
**15.**  $6^\circ$ .   **16.**  $14^\circ$ .   **17.**  $12^\circ$ .   **18.**  $-2^\circ$ .   **19.**  $-2\frac{1}{2}^\circ$ .   **20.**  $80.6^\circ$ .
- Page 78.**—1. 8772 ft.   2.  $28\frac{1}{2}$ .   3. 10 ft. 5 in.   4. About 26 in.  
**5.** 31,680 yr.   6. About 31.3 in.
- Page 79.**—1.  $163\frac{1}{2}$  sq. ft.   2. 231.   3. 45¢.   4.  $\frac{1}{2}$ .   5. 79, with 576  
 gr. left.   6.  $18\frac{1}{2}^\circ$ .   7.  $14^\circ$ .   8. 11.3 lb. per sq. in.   9.  $26\frac{1}{2}$ ;  $34\frac{1}{2}$ ; 118¢.  
**10.**  $3\frac{1}{2}$ ;  $4\frac{1}{2}$ ;  $2\frac{1}{2}$ .   **11.**  $9\frac{1}{2}$ .   **12.** \$1.33.   **13.** 45, with  $4\frac{1}{2}$  in. left.  
**14.** \$22.43.
- Page 80.**—1. 15.   2. 12.   3.  $6\frac{1}{2}$  lb. and  $10\frac{1}{2}$  lb.   4. 2700 oz.   5. 10.48.  
**6.** 12,812 $\frac{1}{2}$  lb.   7.  $34\frac{1}{2}$  lb.   8. 2, 2, 3, 5.   9. 2, 3, 5, 7.   10. 2, 5.   11. 2, 3, 3.  
**12.** 6930; 2, 3, 3, 5, 7, 11.   **13.** 22; 2 and 11.
- Page 81.**—3.  $90^\circ$ ;  $45^\circ$ ;  $45^\circ$ .   4.  $45^\circ$ ;  $45^\circ$ ;  $90^\circ$ ;  $45^\circ$ .   6. ACD.  
**7.** DAC.   9.  $60^\circ$ ;  $60^\circ$ ;  $30^\circ$ .   10. 6.   11. BDC; ABC; DEC; BEC.  
**12.**  $60^\circ$ ;  $30^\circ$ ;  $60^\circ$ ;  $120^\circ$ ;  $120^\circ$ ;  $60^\circ$ .   **13.** 50.
- Page 82.**—3.  $\frac{1}{2}$ .   4.  $\frac{1}{2}$ .   5.  $\frac{1}{2}$ .   6.  $\frac{1}{2}$ .   7.  $\frac{1}{2}$ .   8.  $\frac{1}{2}$ .   9.  $\frac{1}{2}$ .   10.  $\frac{1}{2}$ .  
**11.**  $\frac{1}{2}$ .   **12.**  $\frac{1}{2}$ .   **13.**  $\frac{1}{2}$ .   **14.**  $\frac{1}{2}$ .   **15.**  $\frac{1}{2}$ .   **16.**  $\frac{1}{2}$ .   **17.**  $\frac{1}{2}$ .   **18.**  $\frac{1}{2}$ .  
**19.**  $10\frac{1}{2}$ .   **20.**  $24\frac{1}{2}$ .   **21.**  $100\frac{1}{2}$ .   **22.**  $1\frac{1}{2}$ .   **23.**  $200\frac{1}{2}$ .   **24.**  $125\frac{1}{2}$ .   **25.**  $150\frac{1}{2}$ .  
**26.**  $35\frac{1}{2}$ .   **27.** 9.   **28.** 625.   **29.** 9166+.   **30.** 4666+.   **31.** 52.   **32.** 525.  
**33.** 5833+.   **34.** 6923+.   **35.** 9473+.   **36.** 3333+.   **37.** 6666+.  
**38.** 4666+.   **39.** 2.5.   **40.** 3.25.   **41.** 5.875.   **42.** 8.7142+.   **43.** 12.8888+.

## Answers

44. 15.6363+. 45. 8.25. 46. 10.125. 47. 18.05. 48. 22.9714+.  
 49. 25.06. 50. 43.0125.
- Page 83.—1.  $9\%$ ;  $12\%$ ;  $40\frac{1}{2}\%$ . 2.  $\frac{1}{2}\%$ ;  $5\%$ . 3.  $\frac{1}{2}$ ;  $10\frac{1}{2}\%$ . 4.  $16\frac{1}{2}\%$ .  
 5.  $70\%$ . 6.  $30\%$ . 7.  $42\frac{3}{4}\%$ . 8.  $84\%$ . 9.  $10\%$ . 10.  $20\%$ . 11. About  
 $5\%$ . 12.  $46\frac{1}{2}\%$ . 13. About  $20\frac{1}{2}\%$ .
- Page 84.—1. 497. 2.  $58^\circ$ . 3.  $79^\circ$ . 4.  $174^\circ$ . 5.  $67\frac{1}{2}^\circ$ . 6.  $5\frac{1}{2}\frac{1}{2}$ .  
 7.  $1\frac{1}{2}$  lb. 8.  $25\frac{1}{2}\frac{1}{2}$ .
- Page 85.—5. 20. 6. 20. 7. 11. 8. 11. 9. 18. 10. 18. 11. 24.  
 12. 33. 13. 29. 14. 48. 15. 50. 16. 15. 17. 10. 18. 19. 19. 28.  
 20. 45.
- Page 86.—1. 480 lb. 2. 20 lb. 3.  $\frac{1}{16}$ ;  $\frac{1}{16}$ ;  $\frac{1}{16}$ . 4. .583+; .0133+; 12.  
 5.  $57^\circ 45'$ ;  $32^\circ 15'$ ;  $57^\circ 45'$ . 6.  $110^\circ$ ;  $70^\circ$ . 7.  $37\frac{1}{2}$ . 8. 7 hr. 2 min.  $33\frac{1}{2}$   
 sec. 9.  $35^\circ 7' 30''$ . 10. 9 min.  $44\frac{1}{2}$  sec. 11.  $37^\circ 35' 48''$  W.
- Page 88.—1. 300. 2. 675. 3. 420. 4. 3770. 5. 15,200. 6. 5440.  
 7. 45,000. 8.  $25\frac{1}{2}$ . 9.  $41\frac{1}{2}$ . 10.  $60\frac{1}{2}$ . 11.  $78\frac{1}{2}$ . 12.  $31\frac{1}{2}$ . 13.  $27\frac{1}{2}$ .  
 14.  $13\frac{1}{2}$ . 15.  $46^\circ$ . 16.  $78\frac{1}{2}^\circ$ . 17.  $71\frac{1}{2}^\circ$ . 18.  $137\frac{1}{2}^\circ$ . 19.  $126\frac{1}{2}^\circ$ . 20.  $158\frac{1}{2}^\circ$ .  
 21.  $75\frac{1}{2}^\circ$ .
- Page 91.—1. 16 a. 2. 3 x. 3. 11 b. 4. 7 m. 5. 10 n. 6. 20 c.  
 7.  $7a + 2b + 2c$ . 8.  $10a - 7b + 5d$ . 9.  $5m + 12n - 3$ . 10.  $3x - y$ .  
 11.  $5x + 4$ . 12.  $5y + 5z$ . 13.  $-5a + b - 5c$ . 14.  $7n + 16$ .  
 15.  $7x - 8y$ . 16.  $a - 3b - 3c + 4d$ . 17.  $5a + b - 5c + 7d + 6$ .  
 18.  $a + 3b + c + 4d + 5$ . 19.  $5b - c + 3d + 3e + 1$ . 20.  $11m - n + 11$ .  
 21.  $6x - 3$ .
- Page 92.—1. 9. 2. 15. 3. 7. 4. 12. 5. 8. 6. 9. 7. 25. 8. 36.  
 9. 48. 10. 28. 11. 10 in.; 14 in.; 8 in. 12.  $35^\circ$ ;  $55^\circ$ ;  $90^\circ$ .
- Page 93.—1. 12. 2. 14. 3. 30. 4. 6. 5.  $\frac{1}{2}$ . 6. 4. 7. 7. 8. 3.  
 9. 12. 10. 24. 11. 75 mi. 12. 24 in.; 16 in.; 6 in. 13. 12. 14. 14.  
 15. 36. 16. 6. 17. 20. 18. 5. 19. 16. 20. 12.
- Page 94.—4. \$18. 5. \$250. 6. 4 yr. 7.  $4\frac{1}{2}\%$ . 8. \$2.67.  
 9. 2 yr. 3 mo. 10.  $3\%$ . 11. \$200.
- Page 96.—5. \$19; \$28.50; \$47.50. 6. \$22.18. 7. \$62.01.  
 8. 78.54 sq. ft. 9. 20.944. 10. 34.906 + sq. ft. 11.  $14\frac{1}{2}$  sq. ft.  
 12.  $57\frac{1}{2}$  sq. ft. 13. 315 sq. in.
- Page 97.—1.  $13\frac{1}{2}$  mills. 2. 12 mills. 3. 16 mills. 4. \$40.86.
- Page 98.—1. 20 mills. 2.  $12\frac{1}{2}$  mills. 3. 23 mills. 4.  $17\frac{1}{2}$  mills.  
 5. 17 mills. 6. 17 mills. 7. 21 mills. 8. 16 mills. 9. \$53.40.  
 10. \$63. 11. \$425. 12. \$285.51. 13. \$123.85. 14. \$39.45. 15. \$20.  
 16. \$115.81. 17. \$30. 18. \$48. 19. \$106.25. 20. \$94.50.  
 21. \$148.
- Page 99.—2.  $\frac{1}{2}$ . 3.  $\frac{1}{2}$ . 4.  $\frac{1}{2}$ . 5.  $\frac{1}{2}$ .
- Page 100.—1.  $\frac{1}{2}$ . 2. 4. 3. 10 in. 4. 1080. 5. 648. 6.  $23\frac{1}{2}$ .  
 7.  $414\frac{1}{2}$ . 8. 11.04 in. 9. 5.52. 10. 12. 11. 10 in. 12.  $14\frac{1}{2}$ .
- Page 101.—1. \$265. 2. \$4000. 3. \$2800. 4. \$26.25. 5. About  
 $\frac{1}{10}$ . 6. 14,400 lb. 7. 120 lb. 8. 127. 9. \$1.67.
- Page 102.—1.  $108^\circ$ . 2.  $108^\circ$ ;  $108^\circ$ ;  $36^\circ$ . 3.  $72^\circ$ ;  $36^\circ$ ;  $72^\circ$ .  
 4.  $54^\circ$ ;  $54^\circ$ ;  $72^\circ$ . 5.  $90^\circ$ ;  $36^\circ$ .
- Page 103.—1. \$864. 2. \$200. 3.  $33\frac{1}{2}\%$ . 4. \$6. 5.  $137^\circ 20'$ ;  
 $42^\circ 40'$ ;  $137^\circ 20'$ . 6.  $20^\circ$ ;  $144^\circ$ ;  $16^\circ$ ;  $20^\circ$ ;  $144^\circ$ . 7. \$10.99. 8.  $79^\circ$ .  
 9. \$57.25. 10. \$51,954. 11. \$43,250.
- Page 104.—1. \$100. 2. \$17.40. 3. \$66. 4. \$144. 5. \$1076.  
 6. \$340. 7. \$123.70.

## Answers

**Page 105.**—1. 320. 2. 84 in. 3. 174 ft. 4. 558. 5. 180.  
6. \$220. 7. \$378.90. 8.  $12\frac{1}{2}$  mills. 9. \$41. 10. \$116.88.

**Page 107.**—2. \$2. 3. \$1.25. 4. \$13.30. 5. 94¢. 6. \$1.20.  
7. \$6.88. 8. 40¢. 9. 52¢. 10. 23¢. 11. 83¢. 12. 35¢. 13. 94¢.  
14. \$357. 15. \$398. 16. \$692.12. 17. \$576.81. 18. \$287.46.  
19. \$280.32. 20. \$122.99. 21. \$336.11. 22. \$116.43. 23. \$660.02.  
24. \$407.28. 25. \$537.97.

**Page 108.**—1. \$4. 2. \$8.13. 3. 80¢. 4. \$4.35. 5. \$1.53.  
6. \$4.06. 7. \$362.87. 8. \$158.53. 9. \$247.19. 10. \$633.07.  
11. \$278.91. 12. \$972.38. 13. 60 da.; \$1. 14. 56 da.; \$1.56.  
15. 41 da.; \$1.33. 16. 37 da.; \$2.26. 17. 110 da.; \$11.23. 18. 41 da.;  
\$4.10. 19. 51 da.; \$254.02. 20. 46 da.; \$134.62. 21. 88 da.; \$312.98.  
22. 111 da.; \$548.21. 23. 77 da.; \$742.69. 24. 119 da.; \$933.56.

**Page 110.**—1.  $83\frac{1}{2}$  lb. 2. 28,224. 3. 16,934.4. 4. 408.3 + lb.  
5. About  $46\frac{1}{2}$ . 6. 25. 7. 60 lb. 8. 7,0686. 9. About  $8\frac{1}{2}$  lb.  
10. 1800 lb.

**Page 112.**—2.  $13x$ . 3.  $2a + b$ . 4.  $6m + n$ . 5.  $2a + 2b + 2$ .  
6.  $5a + 3b$ . 7.  $4a - 3b + 4$ . 8.  $8x - y + 8$ . 9.  $3a - b + 4c - 2d$   
— 15. 10.  $7x + 6y - 6z$ . 11.  $20x - 3y - z - 7$ . 12.  $6x + 2z$ .  
13.  $2a - b$ . 14.  $24 - 2x$ . 15.  $2a - 2b - 5c - 3d - 11$ . 16.  $11a -$   
 $2b - 2c - 5d$ . 17.  $3a - 3b - 2d$ .

**Page 113.**—2.  $5a + 3b$ . 3.  $8a - 7b + 9c$ . 4.  $3m + 5n - 14$ .  
5.  $3x - 4y - z + 2$ . 6.  $13 - 7a - 3c + 4d$ . 7.  $6c - 8d$ . 8.  $9a + 3b$ .  
9.  $2x - y$ . 10.  $6x + 2y + 2z$ . 11.  $6m + 2n$ . 12.  $4a + 8b - 3c + 9$ .  
13.  $5a + 4$ . 14.  $2a + 7b + c$ . 15.  $x + 2z$ . 16.  $7x + 7y - 15z + 3$ .

**Page 114.**—1. 6. 2. 4. 3. 10. 4. 8. 5. 8. 6. \$15, \$15, and \$20.  
7. 80. 8. 15. 9. \$2 and 50¢. 10. 12. 11.  $3\frac{1}{2}$ . 12. 3. 13. 9.

**Page 115.**—1. 8. 2. 421. 3.  $544\frac{2}{3}$ . 4. 56,268. 5. 12 ft.  
6. 2 ft. 8 in. 7. 13 ft. 8. 64 ft. and 48 ft. 9.  $68\frac{1}{2}^\circ$ . 10.  $65^\circ$ . 11.  $138^\circ$ ,  
 $42^\circ$ , and  $138^\circ$ . 12.  $30\frac{1}{2}^\circ$ . 13. 16.55 + in.

**Page 116.**—1.  $5\frac{1}{2}^\circ$ . 2.  $18\frac{1}{2}$ . 3. 8 yd.  $2\frac{1}{2}$  ft. 4.  $12\frac{1}{2}$ . 5. 760,2672.  
6. 25,4382 +. 7. 30 ft. 8.  $28\frac{1}{2}$  sq. ft. 9.  $540^\circ$ . 10.  $72^\circ$ . 11. 1.47 cu. in.  
12.  $1\frac{1}{4}$ . 13.  $30\frac{1}{2}$ .

**Page 117.**—1. At 12.52 P.M. 2. 24 hr. 52 min. 3. At 4.20 P.M.  
4. About 28 da. 5. 704 or 705. 6. 6 hr. 13 min. 7. About 1000.  
8. About 8 in. 9. About 134. 10. At 12.06 P.M. 11. At 6.03 A.M.  
and 6.20 p.m. 12. About 6.

**Page 119.**—1. 10 ft. 2. 9 ft. 3. 20 ft. 4. 5 ft. 5. 8 ft. 6. 20 ft.  
7. 10 rd. 8. 12 ft.

**Page 120.**—1. \$182.50. 2. \$260. 3. \$12. 4. \$42. 5. 72.  
6. 1633. 7.  $91\frac{1}{2}$  sq. ft. 8.  $271\frac{1}{2}$  sq. ft. 9. \$3.83. 10. \$130.51.  
11. \$207.50. 12. Gained \$25.05.

**Page 121.**—1.  $5\frac{1}{2}$ . 2.  $\frac{1}{2}$ . 3.  $1\frac{1}{2}$ . 4.  $10\frac{1}{75}$ . 5.  $1.27 +$ . 6.  $\frac{3}{4}$ .  
7. 1,000. 8. 200. 9. 200. 10.  $\frac{1}{2}$ . 11.  $\frac{3}{10}$ . 12.  $1\frac{1}{2}$ . 13.  $\frac{3}{4}$ .

**Page 122.**—1.  $\frac{7}{8}$ ;  $\frac{1}{10}$ ;  $\frac{1}{10}$ . 2.  $\frac{1}{2}$ ;  $\frac{1}{2}$ . 3.  $\frac{1}{10}$ ;  $\frac{1}{2}$ ;  $\frac{1}{2}$ . 4. \$22.34.  
5.  $226\frac{1}{2}$ . 6. 4920 sq. ft. 7. 130 sq. ft. 8.  $10,265\frac{1}{2}$  + ft. 9. 942.48 sq. in.  
10. 64. 11. \$1.83. 12. \$322.56. 13. \$692.50. 14. 47¢.

**Page 123.**—1.  $720^\circ$ . 2.  $120^\circ$ . 4.  $120^\circ$ ;  $30^\circ$ . 5.  $30^\circ$ ;  $30^\circ$ ;  $90^\circ$ ;  $90^\circ$ .  
6.  $00^\circ$ . 7.  $30^\circ$ ;  $30^\circ$ ;  $60^\circ$ ;  $60^\circ$ . 8.  $180^\circ$ . 9.  $360^\circ$ . 10.  $60^\circ$ .  
11.  $00^\circ$ ;  $00^\circ$ .

**Page 124.**—1. 15 and 24. 2. \$61.14 and \$81.52. 3. \$156 and \$195.

## Answers

4. 20 ft. 5. 15 ft. 6. 540. 7. \$75. 8. \$24. 9. \$34.17. 10. \$5.02.  
11. \$447.55.

Page 125.—1. 4440. 2. 2592. 3. \$119.38. 4. About 104.  
5. \$97.50. 6.  $1\frac{1}{2}$  sq. ft. 7. About 178. 8. \$200.25. 9. \$71.04.  
10. \$163.20.

Page 126.—1. \$8; \$16; \$32. 2. Wife,  $\frac{1}{3}$ ; daughter,  $\frac{2}{3}$ ; son,  $\frac{1}{3}$ .  
3. Wife, \$3000; daughter, \$2000; son \$4000. 4. 153,9384 sq. in.  
5. 2412.7488 sq. in. 6. 282.744 sq. in. 7. 18 ft. 8. \$78.12. 9. \$15.62.  
10. \$90.56.

Page 127.—1. 73. 2. 41. 3. 41. 4. 36. 5. 96. 6. 45. 7. 60.  
8. 73. 9. 51 da. 10. 25 da. 11. 62 da. 12. 59 da. 13. 36 da.  
14. 121 da. 15. 40 da. 16. 70 da.

Page 128.—1. About 75 lb. 2. 750 lb. 3. 125. 4.  $127\frac{1}{2}$ . 5.  $22\frac{1}{2}$ .  
6. 500 lb. 7. 12 ft. 8. 5.

Page 131.—1.  $9m + 4n + y$ . 2.  $6x^2y + 7xy - 5xy^2$ . 3.  $9a^2b^2 - 3ab + ab^2 + ab^3$ . 4.  $5m^2 + mn + n^2$ . 5.  $5x^2$ . 6.  $3x^2 + 6y^2$ .  
7.  $2a^2 - ab + a + b$ . 8.  $4x^6 + 5xy - 13x^3y^3$ . 9.  $9b^2 - ab - 7cd$ .  
10.  $4m^2n - 5mn^2 - 3mn - 6$ . 11.  $4a^2 + 8a$ . 12.  $2m^2 + 4m^2n - 2mn^2$ .  
13.  $12x^2y - 21x^2y^2 + 3x^2y^3$ . 14.  $8x^5y - 16x^4y + 40x^3y - 24x^2y$ . 15.  $a + b$ .  
16.  $ab - 4b^2c$ . 17.  $3a^3b^2 + 4a^2b$ . 18.  $2 - 3x^2 + 4x^3 - 5x^6$ .  
19.  $2mn^2 - 3nx - 4ax^2 + 3mn$ .

Page 132.—1. 15,000. 2. 520. 3. 4500. 4.  $6\frac{1}{2}\%$ . 5.  $9\frac{1}{2}\%$ . 6.  $5\frac{1}{2}\%$ .  
7.  $1\frac{1}{2}\%$ . 8. \$224.50. 9.  $5\frac{1}{2}\%$ . 10.  $6\%$ . 11. \$750. 12. \$3585.71.  
13. \$2. 14. \$18,000. 15.  $4\frac{1}{2}\%$ .

Page 133.—1.  $22\frac{2}{3}$  in. 2. 18 in. 3. 400 rd. 4. 6 in. 5. 20 in.  
6. 50 in. 7. 30 in. 8. 10 in. 9. 8 ft. 10. 9 ft. 11. 7 ft.

Page 134.—6. 48. 7. 280. 8. 60. 9. 360. 10. 1890. 11. 1260.  
12. 980. 13. 1680.

Page 135.—1.  $877\frac{1}{2}$  lb. 2. About 1338 lb. 3. 4.68. 4. 8.56 + oz.  
5. 577,645 $\frac{1}{2}$ . 6. 534 $\frac{1}{2}$  hr. 7. 5832. 8. 287,934 $\frac{5}{8}$ . 9. 25,920.

Page 136.—1. \$1. 2. \$72. 3. 9. 4.  $31\frac{1}{2}$  lb. 5.  $\frac{3}{4}$ . 6. 50. 7.  $312\frac{1}{2}$ .  
8.  $21\frac{1}{2}$ . 9. 1890. 10. 840. 11.  $21\frac{1}{2}$ . 12.  $1\frac{7}{10}$ . 13.  $1\frac{11}{20}$ .  
14.  $2\frac{3}{7}$ .

Page 137.—5. 12; 66; 3. 6. 12; 14; 28. 7. 8 in. 8. 12. 9.  $\frac{1}{5}$ ;  
 $\frac{4}{5}$ ;  $\frac{3}{11}$ . 10.  $\frac{10}{13}$ ;  $\frac{8}{5}$ ;  $\frac{7}{11}$ . 11.  $\frac{1}{2}$ ;  $\frac{2}{5}$ ;  $\frac{1}{20}$ . 12.  $\frac{2}{5}$ ;  $\frac{1}{5}$ ;  $\frac{1}{12}$ . 13.  $\frac{3}{4}$ ;  $\frac{1}{10}$ ;  $\frac{1}{12}$ .  
14.  $\frac{1}{30}$ ;  $\frac{10}{37}$ ;  $\frac{20}{40}$ .

Page 138.—1. 10. 2. 2. 3. 3. 4. 880. 5. \$4500. 6. 17. 7. 16.  
8. 61. 9. \$1012.50. 10.  $10\frac{1}{2}$ . 11. 62 rd. 9 ft.

Page 139.—3.  $8\frac{1}{2}\%$ . 4.  $3\frac{1}{2}\%$ . 5.  $5\frac{1}{2}\%$ . 6. 90%. 7. 10%. 8. 24%.  
9.  $50\frac{3}{4}\%$ . 10.  $49\frac{15}{17}\%$ . 11.  $33\frac{3}{13}\%$ . 12. 1%. 13. 5%. 14. 15 mills.  
15.  $4\frac{1}{2}\%$ . 16. 52.31+. 17.  $12\frac{1}{2}\%$ .

Page 140.—1. \$493.42. 2. \$637.03. 3. \$264.98. 4. \$756.07.

Page 141.—1. 144 cu. ft. 2. 120 cu. in. 3. 160 sq. in. 4.  $4\frac{1}{2}$  cu. ft.  
5. 980 cu. in. 6. 81 sq. in. 7. 20 in. 8. 3300 cu. in.

Page 142.—1.  $3\frac{1}{3}\%$ . 2. \$52.50. 3. 1225. 4. 4050. 5. \$23.38.  
6. \$704.89. 7. 1800. 8. 2700. 9. 5040. 10.  $2\frac{1}{2}$ . 11.  $1\frac{2}{5}$ . 12.  $1\frac{1}{3}$ .

13. 12. 14. 4. 15.  $\frac{1}{2}$ ;  $\frac{1}{3}$ ;  $\frac{2}{3}$ . 16.  $\frac{3}{5}$ ;  $\frac{5}{13}$ ;  $\frac{2}{5}$ ;  $\frac{7}{13}$ .  
Page 144.—1. \$1.75. 2. \$30. 3. \$10. 4. 33,600. 5. 1080.

6. 954 sq. in. 7. 2107 cu. ft. 8. 768 sq. in. 9.  $366\frac{2}{15}$  sq. in. 10.  $26\frac{1}{2}$  sq. ft.  
Page 145.—2. 1 sec. 3. 60 ft. per sec. 4. 2. 5. Northeast 15 yd.  
Page 146.—1. 133.68 + oz. 2. 137.69 + oz. 3. 30.528. 4. 434.88.

## Answers

5.  $13\frac{1}{2}$ . 6.  $8333\frac{1}{2}$ . 7.  $7733\frac{1}{2}$ . 8.  $10\frac{1}{2}$ . 9. 6. 10. 42. 11. 9. 12. 3.  
13.  $\frac{1}{2}$ ;  $\frac{1}{3}$ ;  $\frac{1}{4}$ ;  $\frac{1}{5}$ . 14. 60. 15. 72. 16. 30.

**Page 149.**—4.  $a^2 + ab$ . 5.  $ab - b^2$ . 6.  $a^2 - b^2$ . 7.  $a^2 - 2ab + b^2$ .  
8.  $2ab^2 - 5ab^2c + 2abc - 5ab^2c^2$ . 9.  $6m^3 - 8m^2 - 15m^2n + 20mn - 24m + 32$ .  
10.  $x^3 - 3x^2y + 3xy^2 - y^3$ . 11.  $30a^2 - 23a - 40$ .  
12.  $a^2 + 2b - b^2 - 1$ . 13.  $4x^2 - 7x + 22xy + 24y^2 - 28y$ .

**Page 150.**—1.  $30ab^2c$ . 2. 2, 2, 3, 3, a, a, b, b, b, c. 3. 3, 5, 5, x, y, y, z, z, z, z. 4. 2, a, a, b, and 2, a, a, b. 5.  $4m^2n^2$ . 6.  $10xy^2$ .  
7.  $25a^2b^2$ . 8.  $9a^2b^4c$ . 9.  $18x^2y^2$ . 10.  $75m^3n^5$ . 11. a, b, b; a, b, b;  
a, b, b. 12.  $3x^2y^2$ . 13.  $27a^2b^2$ . 14.  $4a^2b^2$ . 15.  $6m^2n^4$ . 16. 4. 17. 5.  
18. 7. 19. 3. 20. 4.

**Page 151.**—1. 14 and 7. 2. 15 and 5. 3. 8 and 4. 4. 6. 5. 8.  
6. 23 and 6. 7. 8 and 10. 8. \$32. 9. 24. 10. 16. 11. 16. 12. 20.  
13. \$20. 14. 18.

**Page 152.**—1. \$150. 2. 2 mo. 6 da. 3. 4%. 4. \$60. 5. \$200.  
6. \$125. 7. \$2400. 8. 1 yr. 6 mo. 9. \$20,506.33. 10. 1 yr. 6 mo.  
11. 5%. 12. \$3122. 13. 7%. 14. 13 in. 15. 7 in.

**Page 154.**—2.  $\frac{2}{3}$ ;  $\frac{1}{2}$ ;  $\frac{1}{3}$ ;  $\frac{1}{4}$ ;  $\frac{1}{5}$ . 3.  $\frac{1}{10}$ ;  $\frac{5}{70}$ ;  $\frac{1}{14}$ ;  $\frac{1}{15}$ . 4.  $\frac{60}{108}$ ;  $\frac{20}{108}$ ;  
 $\frac{17}{108}$ ;  $\frac{10}{108}$ . 5.  $\frac{1}{6}$ ;  $\frac{1}{12}$ ;  $\frac{1}{18}$ ;  $\frac{1}{24}$ . 6.  $\frac{1}{12}$ ;  $\frac{1}{18}$ ;  $\frac{1}{24}$ ;  $\frac{1}{36}$ . 7.  $\frac{1}{18}$ ;  $\frac{1}{24}$ ;  $\frac{1}{36}$ ;  
 $\frac{1}{48}$ . 8.  $\frac{1}{12}$ ;  $\frac{1}{18}$ ;  $\frac{1}{24}$ ;  $\frac{1}{36}$ . 9.  $\frac{1}{10}$ ;  $\frac{1}{15}$ ;  $\frac{1}{20}$ ;  $\frac{1}{30}$ . 10.  $\frac{1}{12}$ ;  $\frac{1}{18}$ ;  $\frac{1}{24}$ ;  
 $\frac{1}{36}$ . 11.  $\frac{7}{10}$ ;  $\frac{1}{10}$ ;  $\frac{1}{10}$ ;  $\frac{1}{10}$ . 12.  $\frac{1}{10}$ ;  $\frac{1}{10}$ ;  $\frac{1}{10}$ ;  $\frac{1}{10}$ . 13.  $\frac{1}{10}$ ;  $\frac{1}{10}$ ;  $\frac{1}{10}$ ;  
 $\frac{1}{10}$ . 14.  $\frac{1}{10}$ ;  $\frac{1}{10}$ ;  $\frac{1}{10}$ ;  $\frac{1}{10}$ . 15.  $\frac{1}{10}$ ;  $\frac{1}{10}$ ;  $\frac{1}{10}$ ;  $\frac{1}{10}$ . 16.  $\frac{1}{10}$ ;  $\frac{1}{10}$ ;  
 $\frac{1}{10}$ ;  $\frac{1}{10}$ . 17.  $\frac{1}{10}$ ;  $\frac{1}{10}$ ;  $\frac{1}{10}$ ;  $\frac{1}{10}$ . 18.  $\frac{1}{10}$ ;  $\frac{1}{10}$ ;  $\frac{1}{10}$ ;  $\frac{1}{10}$ . 19.  $\frac{1}{10}$ ;  
 $\frac{1}{10}$ ;  $\frac{1}{10}$ ;  $\frac{1}{10}$ . 20.  $\frac{1}{10}$ ;  $\frac{1}{10}$ ;  $\frac{1}{10}$ ;  $\frac{1}{10}$ .

**Page 155.**—3.  $1\frac{1}{2}$ . 4.  $1\frac{1}{2}$ . 5.  $1\frac{1}{2}$ . 6.  $\frac{1}{2}$ . 7.  $1\frac{1}{2}$ . 8.  $\frac{1}{2}$ .  
9.  $1\frac{1}{2}$ . 10.  $1\frac{1}{2}$ . 11.  $1\frac{1}{2}$ . 12.  $1\frac{1}{2}$ . 13.  $1\frac{1}{2}$ . 14.  $2\frac{1}{2}$ . 15.  $1\frac{1}{2}$ .  
16.  $1\frac{1}{2}$ . 17.  $2\frac{1}{2}$ . 18.  $2\frac{1}{2}$ . 19.  $3\frac{1}{2}$ . 20.  $2\frac{1}{2}$ . 21.  $2\frac{1}{2}$ .  
22.  $2\frac{1}{2}$ . 23.  $2\frac{1}{2}$ . 24.  $1\frac{1}{2}$ . 25.  $3\frac{1}{2}$ . 26.  $1\frac{1}{2}$ . 27.  $1\frac{1}{2}$ .  
28.  $1\frac{1}{2}$ . 29.  $1\frac{1}{2}$ . 30.  $1\frac{1}{2}$ .

**Page 156.**—4.  $4\frac{1}{2}$ . 5.  $7\frac{1}{2}$ . 6.  $10\frac{1}{2}$ . 7.  $11\frac{1}{2}$ . 8.  $10\frac{1}{2}$ . 9. 33.  
10.  $103\frac{1}{2}$ . 11.  $313\frac{1}{2}$ . 12. 258. 13.  $1022\frac{1}{2}$ . 14.  $\frac{1}{10}$ . 15.  $\frac{1}{10}$ . 16.  $\frac{1}{10}$ .  
17.  $\frac{1}{10}$ . 18.  $6\frac{1}{2}$ . 19. 285. 20. 642. 21.  $253\frac{1}{2}$ . 22.  $\frac{1}{10}$ . 23.  $\frac{1}{10}$ .  
**Page 157.**—2.  $15\frac{1}{2}$ . 3.  $12\frac{1}{2}$ . 4.  $5\frac{1}{2}$ . 5.  $9\frac{1}{2}$ . 6.  $24\frac{1}{2}$ . 7.  $308\frac{1}{2}$ .  
8.  $244\frac{1}{2}$ . 9.  $582\frac{1}{2}$ . 10.  $1446\frac{1}{2}$ . 11.  $3\frac{1}{2}$ . 12.  $23\frac{1}{2}$ . 13.  $249\frac{1}{2}$ . 14.  $41\frac{1}{2}$ .  
15.  $44\frac{1}{2}$ . 16.  $13\frac{1}{2}$ . 17.  $17\frac{1}{2}$ . 18.  $30\frac{1}{2}$ . 19.  $72\frac{1}{2}$ . 20.  $1\frac{1}{2}$ . 21.  $1\frac{1}{2}$ .  
22.  $7\frac{1}{2}$ . 23.  $30\frac{1}{2}$ . 24.  $14\frac{1}{2}$ . 25.  $379\frac{1}{2}$ . 26.  $612\frac{1}{2}$ . 27.  $38\frac{1}{2}$ . 28.  $30\frac{1}{2}$ .  
29.  $21\frac{1}{2}$ . 30.  $1551\frac{1}{2}$ . 31.  $342\frac{1}{2}$ . 32.  $75\frac{1}{2}$ . 33.  $71\frac{1}{2}$ . 34.  $79\frac{1}{2}$ .  
35.  $9108\frac{1}{2}$ .

**Page 158.**—2.  $\frac{1}{2}$ . 3.  $\frac{1}{3}$ . 4.  $\frac{1}{4}$ . 5.  $\frac{1}{5}$ . 6.  $6\frac{1}{2}$ . 7.  $6\frac{1}{2}$ . 8.  $2\frac{1}{2}$ . 9.  $3\frac{1}{2}$ .  
10.  $\frac{1}{10}$ . 11.  $\frac{1}{10}$ . 12.  $\frac{1}{10}$ . 13.  $\frac{1}{10}$ . 14.  $\frac{1}{10}$ . 15.  $7\frac{1}{2}$ . 16.  $\frac{1}{10}$ . 17.  $\frac{1}{10}$ .  
18.  $\frac{1}{10}$ . 19.  $7\frac{1}{2}$ . 20.  $4\frac{1}{2}$ . 21.  $\frac{1}{10}$ . 22.  $\frac{1}{10}$ . 23.  $18\frac{1}{2}$ . 24.  $9\frac{1}{2}$ . 25.  $18\frac{1}{2}$ .  
26.  $19\frac{1}{2}$ . 27.  $\frac{1}{10}$ . 28.  $6\frac{1}{2}$ . 29.  $21\frac{1}{2}$ .

**Page 159.**—2. 15. 3. 35. 4. 9. 5. 2. 6. 4. 7. 2. 8.  $4\frac{1}{2}$ . 9. 18.  
10. 8. 11. 3. 12. 24. 13. 65. 14. 64. 15. 128. 16. 4. 17. 6. 18. 7.  
19. 5. 20. 14. 21. 10. 22. 25. 23. 10. 24. 10. 25. 2. 26. 5. 27. 7.  
28. 5. 29. 200. 30. 800. 31. 8. 32. 16. 33. 32. 34. 16. 35. 8.

**Page 160.**—2.  $1\frac{1}{2}$ . 3.  $1\frac{1}{2}$ . 4.  $1\frac{1}{2}$ . 5.  $\frac{1}{2}$ . 6.  $\frac{1}{2}$ . 7.  $5\frac{1}{2}$ . 8.  $7\frac{1}{2}$ .  
9.  $4\frac{1}{2}$ . 10.  $9\frac{1}{2}$ . 11.  $6\frac{1}{2}$ . 12.  $1\frac{1}{2}$ . 13.  $\frac{1}{2}$ . 14.  $\frac{1}{2}$ . 15.  $3\frac{1}{2}$ .  
16.  $3\frac{1}{2}$ . 17.  $60\frac{1}{2}$ . 18.  $20\frac{1}{2}$ . 19.  $\frac{1}{2}$ . 20.  $\frac{1}{2}$ . 21.  $\frac{1}{2}$ . 22.  $1\frac{1}{2}$ .  
23.  $\frac{1}{2}$ . 24.  $5\frac{1}{2}$ . 25.  $14\frac{1}{2}$ . 26.  $4\frac{1}{2}$ . 27.  $41\frac{1}{2}$ . 28.  $4\frac{1}{2}$ . 29.  $26\frac{1}{2}$ .  
30.  $10\frac{1}{2}$ . 31.  $35\frac{1}{2}$ . 32.  $433\frac{1}{2}$ . 33.  $109\frac{1}{2}$ . 34.  $49\frac{1}{2}$ . 35.  $1\frac{1}{2}$ .

## Answers

**Page 161.**—2.  $2\frac{1}{2}$ . 3.  $6\frac{1}{2}$ . 4.  $1\frac{1}{2}$ . 5.  $9\frac{1}{2}$ . 6.  $32\frac{1}{2}$ . 7.  $7\frac{1}{2}$ . 8.  $12\frac{1}{2}$ .  
 9.  $5\frac{1}{2}$ . 10.  $1\frac{1}{2}$ . 11.  $2\frac{1}{2}$ . 12.  $6\frac{1}{2}$ . 13.  $3\frac{1}{2}$ . 14.  $2\frac{1}{2}$ . 15.  $1\frac{1}{2}$ .  
 16.  $1\frac{1}{2}$ . 17.  $11\frac{1}{2}$ . 18.  $6\frac{1}{2}$ . 19.  $3\frac{1}{2}$ . 20.  $58\frac{1}{2}$ . 21.  $1\frac{1}{2}$ . 22.  $3\frac{1}{2}$ .  
 23.  $1\frac{1}{2}$ . 24.  $1\frac{1}{2}$ . 25.  $19\frac{1}{2}$ . 26.  $9\frac{1}{2}$ . 27.  $1\frac{1}{2}$ . 28.  $4\frac{1}{2}$ . 29.  $5\frac{1}{2}$ .  
 30.  $8\frac{1}{2}$ . 31.  $1\frac{1}{2}$ . 32.  $1\frac{1}{2}$ . 33.  $4\frac{1}{2}$ . 34.  $77\frac{1}{2}$ . 35.  $2\frac{1}{2}$ .  
 36.  $1\frac{1}{2}$ . 37.  $18\frac{1}{2}$ . 38.  $18\frac{1}{2}$ . 39.  $1\frac{1}{2}$ .

**Page 162.**—2.  $1\frac{1}{2}$ . 3.  $1\frac{1}{2}$ . 4.  $1\frac{1}{2}$ . 5.  $1\frac{1}{2}$ . 6.  $1\frac{1}{2}$ . 7.  $1\frac{1}{2}$ . 8.  $1\frac{1}{2}$ .  
 9.  $1\frac{1}{2}$ . 10.  $1\frac{1}{2}$ . 11.  $1\frac{1}{2}$ . 12.  $1\frac{1}{2}$ . 13.  $1\frac{1}{2}$ . 14.  $1\frac{1}{2}$ . 15.  $1\frac{1}{2}$ . 16.  $1\frac{1}{2}$ .  
 17.  $14\frac{1}{2}$ . 18.  $1\frac{1}{2}$ . 19.  $4\frac{1}{2}$ . 20.  $4\frac{1}{2}$ . 21.  $1\frac{1}{2}$ . 22.  $1\frac{1}{2}$ . 23.  $1\frac{1}{2}$ .  
 24.  $7\frac{1}{2}$ . 25.  $1\frac{1}{2}$ . 26.  $1\frac{1}{2}$ . 27.  $1\frac{1}{2}$ . 28.  $1\frac{1}{2}$ . 29.  $1\frac{1}{2}$ . 30.  $1\frac{1}{2}$ .  
 31.  $3\frac{1}{2}$ . 32.  $17\frac{1}{2}$ . 33.  $1\frac{1}{2}$ . 34.  $1\frac{1}{2}$ . 35.  $6\frac{1}{2}$ . 36.  $210\frac{1}{2}$ . 37.  $1\frac{1}{2}$ .

**Page 163.**—1. 1,000,000,000. 2. About 25,000. 3. 4693. 4. South-west, 20 ft. 5. About 28 ft. 6. 2. 7.  $\frac{1}{2}$ . 8.  $\frac{1}{2}$ . 9.  $1\frac{1}{2}$ . 10.  $1\frac{1}{2}$ .  
 11. .0308+. 12.  $1\frac{1}{2}$ . 13.  $4\frac{1}{2}$ . 14.  $9\frac{1}{2}$ . 15.  $2\frac{1}{2}$ . 16.  $1\frac{1}{2}$ .  
 17.  $1\frac{1}{2}$ .

**Page 164.**—2. \$3727.50. 4. \$8456.25. 5. \$1043.75. 6. \$8600.

**Page 165.**—2. \$1255.50. 3. \$897.75. 4. \$2106.25. 6.  $3\frac{1}{2}\%$ .  
 7.  $4\frac{1}{2}\%$ . 8.  $2\frac{1}{2}\%$ . 9.  $2\frac{1}{2}\%$ . 10. 12.

**Page 166.**—1.  $207\frac{1}{2}$ . 2.  $157\frac{1}{2}$ . 3.  $8\frac{1}{2}$ . 4.  $1\frac{1}{2}$ . 5.  $5863\frac{1}{2}$ .  
 6.  $18259\frac{1}{2}$ . 7.  $349\frac{1}{2}$ . 8.  $14\frac{1}{2}$ . 9.  $1\frac{1}{2}$ . 10.  $4\frac{1}{2}$ . 11. 42.  
 12. \$14.75. 13. \$480. 14. 1200 ft. per second. 15.  $8\frac{1}{2}$ . 16.  $26\frac{1}{2}$ .

**Page 167.**—1. 270 cu. in. 2. 78.54 in. 3. 314.16 sq. in.  
 4. 29,0652+ cu. ft. 5. 5660.7705. 6. 353.43. 7. 50.89+ min.  
 8. 22,089 $\frac{1}{2}$  lb. 9. 3506.0256. 10. 913.0275 lb.

**Page 168.**—2. \$10 and \$15. 3. 35 and 49. 4. 20, 30, and 50.  
 5. 9 and 15. 6. \$100; \$300; \$600. 7. \$40; \$60; \$80. 8. \$700;  
 \$1300; \$2000. 9. \$600 and \$1400. 10. \$800, \$1200, and \$1600.

**Page 169.**—1. 79. 2. 96. 3. 40 da. 4. 5. 5. \$3453.75.  
 6. \$1700. 7. \$285. 8. 238.936 sq. ft. 9. 226.0206 sq. ft. 10.  $14\frac{1}{2}$  ft.  
 11. 8.3176. 12. 4.52+.

**Page 170.**—1. \$840. 2. \$4500. 3. \$200 and \$300. 4.  $5\%$ .  
 5.  $10\frac{1}{2}\%$ . 6.  $8\%$ . 7. 56.29+. 8. 9.76+ in. 9. \$3120; \$2080.  
 10. \$2700 and \$900. 11. \$21, \$35, and \$49.

**Page 171.**—1. 6 sq. ft. 2. 900 cu. in. 3. 6 sq. ft. 4. 2340 cu. in.  
 5.  $12\frac{1}{2}$  sq. ft. 6. 2520 cu. in. 7.  $10\frac{1}{2}$  sq. ft. 8. 7.3304 ft. 9. 706.86  
 sq. in. 10. 6238.2 cu. in. 11. 49.73+ sq. in. 12. 733.3824 cu. in.  
 13. 37.6992 sq. ft. 14. 43.9824 cu. ft. 15. 20 sq. ft. 16. 81 sq. in.  
 17. 24 in. 18.  $1\frac{1}{2}$  sq. ft. 19. 30 in.

**Page 173.**—1. 22. 2. 30. 3. 1. 4. 2. 5. 206. 6. 5.  
 7.  $6\frac{1}{2}$ . 8. 15. 9. 3. 10.  $2a^2 + 2ab + 3c^2$ . 11.  $6x^4y^2 - 4x^2y + y^3$ .  
 12.  $2a^2 - 4ab^2 + 4ac - 8ad - 3a^2c + 6b^2c - 6c^2 + 12cd$ . 13.  $m^2 + mn - n^2$ .

**Page 174.**—1. 24¢ and 12¢. 2. 60 and 20. 3. 54¢ and 12¢.  
 4. 8. 5. 30. 6. Horse, \$240; carriage, \$80. 7. 5. 8. 80 and 20.  
 9. 12 yr. and 18 yr. 10. 12. 11. 54. 12. 25. 13. 20. 14. 36.

**Page 175.**—3. 32. 4. 10. 5.  $3\frac{1}{2}$ . 6. 3. 7.  $42\frac{1}{2}$ . 8. 76.  
 9.  $45\frac{1}{2}$ . 10.  $19\frac{1}{2}$ . 12.  $13\frac{1}{2}$  rd. 13. \$90.

**Page 176.**—1. \$195. 2. \$1296. 3. 24. 4. 123 ft. 5.  $476\frac{1}{2}$ .  
 6. \$10.08. 7. \$425.81. 8. 633 $\frac{1}{2}$ . 9. 72 in. 10.  $248\frac{1}{2}$  mi.  
 11. 85 $\frac{1}{2}$ . 12. 640.

**Page 177.**—1. 360. 2.  $22\frac{1}{2}$ . 3. 50. 4. 3125 lb. 5. 1017.8784.  
 6. 589.05 oz. 7. \$1470, \$1225, and \$1505. 9. \$1920, \$1920, and \$2160.



## Answers

- Page 178.** — 1. 13,359+. 2. 1.88+. 3. In the year 4876. 4. 11.8+.  
 5. 164.61+. 6. 226,195,200 mi. 7. 2,573,324.23+. 8. 5,566,915,200 mi.  
 9. 2,055,914.4+ mi. less. 10. 584,387,600 mi. 11. 1,082,419.38+ more.
- Page 179.** — 1.  $1\frac{1}{5}$ . 2.  $58\frac{1}{2}\%$ . 3.  $10\frac{1}{4}$ . 4.  $10\frac{1}{4}$ . 5. \$160.  
 6. \$1200, \$1440, and \$960. 7. About \$66,000,000,000. 8. About  
 \$44,000,000,000. 9. About \$1300. 10. About  $1\frac{1}{2}$ . 11. About 9.
- Page 180.** — 1. .00075. 2. \$88.04. 3.  $\frac{1}{10}$ . 4. .09375. 5.  $\frac{1}{1000}$ .  
 6.  $\frac{1}{10}$ . 7. 17. 8. \$12.30. 9.  $\frac{1}{10}$ . 10. Increased  $\frac{1}{10}$ . 11. Diminished  $\frac{1}{10}$ .  
 12. \$1600. 13. \$1126.25. 14. \$7362.50. 15. 160. 16. 52. 17. 300.
- Page 181.** — 2.  $26\frac{1}{2}$  da. 3.  $46\frac{1}{2}$ . 4. 3. 5. 73 bu. 3 pk.  $1\frac{1}{2}$  qt. 6.  $3\frac{1}{2}$  da.  
 7. 10 ft. 8.  $13\frac{1}{2}$ . 9.  $78\frac{1}{2}$ . 10.  $43\frac{1}{2}$  sq. ft.
- Page 182.** — 1. 144 cu. ft. 2. 48 cu. ft. 3. 393 cu. ft. 4. 666 cu. ft.  
 5.  $7\frac{1}{2}$  sq. ft. 6.  $13\frac{1}{2}$  cu. ft. 7. 2673 cu. in.
- Page 183.** — 10. \$250. 11. 3%. 12. 3 yr. 3 mo. 13. \$641.26.  
 14. \$105,000. 15. 4%. 16. 2 yr. 7 mo. 5 da. 17. 6%. 18. 28 yr. 6 mo.  
 26 da.
- Page 184.** — 1. \$160. 2. \$5. 3. \$437. 4. 18. 5. 250 rd. 6.  $33\frac{1}{2}$ .  
 7. 530 sq. ft. 8. 882 cu. ft. 9. 761.58+ ft. 10. 3040 ft. 11. 1640.9+.
- Page 185.** — 1. 1000 oz. 2. 1000 oz. 3. 4. 4. 5. 6. 70 oz. 7.  $3\frac{1}{2}$ .  
 8. 5.
- Page 186.** — 1. \$360. 2. \$72. 3. \$519.86. 4. \$535.97. 5. \$56.75.  
 6. \$299.49. 7. \$374.68. 8. 5%. 9. 2 yr. 3 mo. 10. 5292 cu. in.  
 11. 3757 cu. in. 12.  $167\frac{1}{2}$  sq. ft.
- Page 189.** — 1.  $5x^2 - 6xy + 2xy + 5y^2$ . 2.  $4m^2 - 2mn - 8n^2$ . 3.  $4ab$ .  
 4.  $3a^2b^2 - 3ab^2 + 9ab^2c$ . 5.  $12x^2y^4 - 20x^2y^6 + 4xy^6$ . 6.  $a^3 - 4a^2b + 5ab^2 - 2b^3$ .  
 7.  $4m^2n - 2m + 3$ . 8.  $5m^2n - 4mn + 7n^3$ . 9. 8. 10. 11.  
 11. 14. 12.  $10\frac{1}{2}$ . 13. 12. 14. 20 and 40. 15. 240. 16. Horse, \$52.50;  
 cow, \$42.50.
- Page 190.** — 1. 24%. 2. 80. 3. 50. 4. \$456. 5. \$1140. 6.  $37\frac{1}{2}\%$ .  
 7. \$512.40. 8. \$4.91. 9. \$2000. 10.  $2\frac{1}{4}\%$ . 11. 5%. 12. \$350.  
 13. \$401.79. 14. \$588.24.
- Page 191.** — 3. 396.268. 4. 125.937. 5. 240.2775. 6. 2227.3134.  
 7. 28.42. 8. 4.1523. 9. 22.72727. 10. .010949. 11. .0794. 12. 762.40372.  
 13. 6.0141. 14. .022988. 15. 85.014694.
- Page 192.** — 2. 2.56. 3. 2.0825. 4. 15.625. 5. .005013. 6. 207.36.  
 7. 420.25. 8. 12.014002. 9. .201824. 10. 371.712. 11. .0759. 12. 3.6.  
 13. .0225. 14. .9999. 15. 2.1834. 16. .36461. 17. 37.45404. 18. .3125.  
 19. 81.6816. 20. 60.78996. 21. 26.01. 22. 1. 23. 721.8. 24. 1127.196.  
 25. .01845. 26. 71.5512. 27. .4. 28. 22. 29. 225. 30. .045051.  
 31. 849.905363.
- Page 193.** — 3. .075. 4. .75. 5. 7.5. 6. 1.21. 7. .0121. 8. 121.  
 9. 1.21. 10. 12,100. 11. .1065. 12. 10,000. 13. 6.9917+. 14. 7.9577+.  
 15. 5.4749+. 16. .0000288. 17. 3047.619+.
- Page 194.** — 2.  $\frac{1}{5}$ . 3.  $\frac{1}{10}$ . 4.  $\frac{1}{10}$ . 5.  $\frac{1}{10}$ . 6.  $\frac{1}{10}$ . 7.  $\frac{1}{10}$ . 8.  $\frac{1}{10}$ .  
 9.  $\frac{1}{100}$ . 10.  $\frac{1}{100}$ . 11.  $\frac{1}{10}$ . 12.  $\frac{1}{10}$ . 13.  $\frac{1}{10}$ . 14.  $\frac{1}{1000}$ . 15.  $\frac{1}{10}$ . 16.  $\frac{1}{10}$ .  
 18. .25. 19. .6. 20. .625. 21. .7. 22. .875. 23. .166+. 24. .666+.  
 25. .555+. 26. .6363+. 27. .9375. 28. .7222+. 29. .9047+.  
 30. .06086+. 31. .0197+. 32. .012. 33. .0199+.
- Page 195.** — 1. 5. 2. \$0.00193. 3. \$3.9758. 4. 1 fr. 29.5+ ct.; 5 fr.  
 18.1+ ct.; 25 fr. 90.6+ ct. 5. 4. 6. \$0.02385. 7. \$2.50425. 8. 1 mk.  
 67.7+ pf.; 125 mk. 78.6+ pf. 9. \$0.96755. 10. \$0.14475. 11. \$38.80.

## Answers

12. \$0.15264.      13. \$190.80.      14. \$4.8829.      15. \$28.97316.  
 16. \$241.3272.      17. \$3.64905.      18. \$601.1631.      19. 62 + ct.      20. 2 fr.  
 59 + ct.      21. 4 fr. 40 + ct.      22. 5 fr. 18 + ct.      23. 518 fr. 13 + ct.      24. 854 fr.  
 92 + ct.      25. 126 fr. 94 + ct.      26. 787 fr. 87 + ct.      27. 1 mk. 76 + pf.  
 28. 2 mk. 64 + pf.      29. 3 mk. 98 + pf.      30. 4 mk. 19 + pf.      31. 628 mk.  
 98 + pf.      32. 2935 mk. 1 + pf.      33. 152 mk. 20 + pf.      34. 727 mk. 46 + pf.  
 Page 196. — 1. \$3.20.      2. \$2.88.      3. \$2.80.      4. 60¢.      5. \$360.  
 6. \$15.      7. \$50.      8. 12½%.  
 Page 197. — 1. \$82.13.      2. 13.82 + ft.      3. 240.      4. \$320.      5. \$446.43.  
 6. \$188.68.      7. \$352.94.      8. 133.68 oz.      9. 855.55 oz.      10. 721.87 + oz.  
 11. 4112 oz.  
 Page 198. — 7. 12.      8. 18; 30.      9. 30.      10. 42 in.      11. 45 ft.  
 12. 24 in.      13. 90 ft.      14. 4 ft.  
 Page 199. — 1. \$13.50.      2. \$236.40.      3. \$200.      4. \$3150.      5. \$567.  
 6. 20%.      7. 2½%.      8. The former \$1.50 better.      9. 25%.      10. \$318.46.  
 11. \$250.  
 Page 200. — 1. 2½.      2. 2½.      3. 1½.      4. 1½.      5. 1½.      6. 1½.  
 7. 1½.      8. 1½.      9. 1½.      10. 1½.      11. 1½.      12. 5½.      13. 2½.      14. 1½.  
 15. 2½.      16. 2½.      17. 212.      18. 1134.      19. 7.      20. 7.      21. 18.      22. 1½.  
 23. 950½.      24. 11½.      25. 1½.      26. 1½.      27. 32.      28. 4.      29. 5½.      30. 4½.  
 31. 1.      32. 12½.      33. 4½.      34. 10½.      35. 6½.      36. 10½.      37. 1½.  
 38. 1½.      39. 1½.      40. 2½.      41. 1½.      42. 30½.      43. 1½.      44. 5½.  
 Page 201. — 1. 7200 oz.      2. 6200 oz.      3. 233½ oz.      4. 150 oz.  
 5. 1026 oz.      6. 1026 oz.      7. 26 oz.      8. 10.47.      9. 8.761.      10. 1030 oz.  
 11. 267½ oz.      12. 7½ oz.      13. 3650 oz.      14. 10.25.      15. 11.14 + oz.  
 Page 202. — 1. 39 cu. ft.      2. 213.82515 sq. in.      3. 2138.2515 cu. in.  
 5. 20 in.      6. 16 ft.      7. 314.16 cu. ft.      8. 50.2666.      9. 188.496 sq. ft.  
 Page 203. — 1. 25.      2. 343.      3. 10,000.      4. 59,049.      5. 5832.      6. 1½.  
 7. 1½.      8. 1½.      9. 729.      10. .00050625.      11. 9.      12. 10.      13. 4.      14. 5.  
 15. 6.      16. 1½.      17. 8.      18. 6.      19. 1.      20. 1.      21. 13.      22. 16.      23. 18.  
 24. 20.      25. 22.      26. 28.      27. 32.      28. 35.      29. 6.      30. 7.      31. 9.      32. 18.  
 33. 20.      34. 24.      35. 27.      36. 32.  
 Page 204. — 1. 15.      2. 17.      3. 9.      4. 12.      5. 32,768.      6. .00390625.  
 7. 8.365427 cu. in.      8. 1331 cu. in.      9. 5½ da.      10. 6½.      11. 1½.      12. 1½.  
 13. 1.      14. \$44.  
 Page 205. — 1. \$9.      2. 9%.      3. \$225.      4. 5%.      5. 6½%.      6. 4½%.  
 7. \$2800.      8. \$1745.      9. \$72.50.      10. \$97.50.      11. \$337.50.  
 Page 206. — 1. \$7000.      2. \$4000.      3. \$2226.      4. \$1000.      5. \$194.74.  
 6. \$594.44.      7. 190.8522 cu. ft.      8. 234.5728 cu. ft.      9. 61.2612 sq. ft.  
 10. 392 cu. ft.  
 Page 207. — 1. 25; 75.      2. 100,000 foot-pounds.      3. 250.      4. 10,000.  
 5. 28,800.      6. 50.      8. 125.      9. 2½ ft. per second.  
 Page 208. — 1. \$20.      2. \$634.13.      3. 1½; 1½; 1½.      4. 1½.      5. 250.  
 6. 375.      7. \$5.60.      8. 9½%.      9. 314.16.      10. 1.36.      11. 49.      12. 54.  
 13. 55.      14. 75.      15. 45.      16. 72.      17. 90.      18. 100.      19. 3.      20. 25.  
 21. 2.      22. 5.      23. 1.      24. 1½.      25. 1.      26. 1.  
 Page 209. — 1. 35 ft.      2. 25 ft.      3. 90 ft.      4. 42 ft.      5. 48 ft.  
 6. 96 rd.      7. 2 ft.      8. 18 in.      9. 1.      10. 1.      11. 1 rd.      12. 1 yd.  
 13. 176.715 sq. in.      14. 20 in.  
 Page 210. — 1. 10%.      2. 40%.      3. 75%.      4. 12½% lost.      5. \$700.  
 6. \$1050.      7. Lost \$70.      8. 25%.      9. 37½%; 62½%; 87½%.

## Answers

**Page 211.**—1. 4; 66. 2. 7.92. 3. 673½ ft. 4. 567. 5. 2. 6. 20. 7. 2. 8. 17,160. 9. ½. 10. 1920. 11. 1600. 12. 6400. 13. ½. 14. ½. 15. 10. 16. 40. 17. 675. 18. \$2323.13.

**Page 213.**—3.  $9a^2b^3c$ . 4.  $6xy^3z^2$ . 5.  $4x^2y$ . 6.  $5mn^4$ . 7.  $az$ . 8.  $2m^2$ . 9.  $5mn^3$ . 10.  $a^2b$ . 11.  $2a^3b^4$ . 12.  $a^4$ . 13.  $8x^3y^3$ . 14.  $a^2$ . 15.  $5b$ . 16.  $2a$ . 17. 49. 18. 144. 19. 64. 20.  $2a$ . 21.  $20x^2$ . 22.  $20ab^2$ . 23.  $2ab$ . 24.  $8a^2$ . 25.  $6a^2y^6$ . 26. 1600. 27. 225. 28. 400. 29. 1728. 30. 3375. 31. 125,000.

**Page 214.**—1. 11 ft. 2. 4 in. 3. 12 ft. 4.  $181\frac{1}{4}$  ft. 5. 24 ft. 6. 10 in. 7. 4 ft. 8. 9 in. 9. 10 in. 10. 5 in. 11. 8 in. 12. 10 in.

**Page 215.**—1. 68. 2. 3%. 3. \$2400. 4. 14 mills. 5. \$242. 6. \$95. 7. \$140. 8. \$324. 9. 4. 10. 3½. 11. \$380. 12. 2½%. 13. \$154.02. 14. \$353.77.

**Page 216.**—2. 12. 3. 27. 4. 115. 5. \$20,47½. 6. \$777. 7. \$6,264. 8. \$7.50. 9. \$6,975. 11. 5. 12. 139. 13. 412. 14. 2500. 15. 204. 16. \$280. 17. \$2,3625. 18. \$1165.

**Page 217.**—3. 34%. 4. 65%. 5. 25%. 6.  $33\frac{1}{3}$ %. 7. 25%. 8.  $61\frac{1}{2}$ %. 9.  $16\frac{1}{2}$ %. 10. 25%. 11. 20%. 12. 6%. 13. 3%. 14.  $51\frac{1}{2}$ %. 15.  $11\frac{1}{2}$ %. 16. 13 mills.

**Page 218.**—3. 900. 4. 500. 5. 120. 6. 630. 7. \$2000. 8. \$1500. 9. 1493½. 10. 1200. 11. \$8104. 12. \$850. 13. \$247. 14. \$1600. 15. \$7600.

**Page 219.**—3. 90. 4. 280. 5. 50. 6. 285. 7. 4071. 8. \$225,302. 9. \$155,5795. 10. \$400. 11. \$400. 12. 325 tons. 13. 980 barrels. 14. 375 children. 15. \$316.25. 16. \$259.60.

**Page 220.**—3. 480. 4. 300. 5. 400. 6. 96. 7. 400. 8. 2000. 9. 600. 10. 2019,138+. 11. 14. 12. 2. 13. 10. 14. 200. 15. 80. 16. 36. 17. 56. 18. 5. 19. 652. 20. 572. 21. 1580. 22. 500. 23. 800. 24. 750. 25. 320. 26. 7100. 27. 8. 28. 12. 29. 28. 30. 20. 31. 58. 32. 5. 33. 1. 34. Any number.

**Page 221.**—1. 13 ft. 2. 12 ft. 3. 12 ft. 4. 5 in. 5. 6 in. 6. 5 in. 7. 12 in. 8.  $27\frac{1}{2}$ . 9. 60. 10. 200. 11.  $179\frac{1}{2}$  in.

**Page 222.**—1. 10 in. 2. 7 in. 3. 31,416. 4. 50 sq. in. 5. 28.54 sq. in. 6. 20 in. 7. 200 sq. in. 8. 114.16 sq. in. 9. 392 sq. in. 10. 5000 sq. in.

**Page 223.**—1. 103,680. 2.  $19\frac{1}{2}$ . 3. 7095½. 4. 70,080. 5. ¾. 6. 30. 7. 25. 8. 30 ft. 9. 10 ft. 10. 18 ft. 11. 400 rd.

**Page 224.**—1. \$341. 2. About 18. 3. 248. 4.  $10.144\frac{1}{2}$ %; 28.985+%; 6.52+%; 64.347+%. 5. 61. 6. \$2.82. 7. \$1748.85. 8. ½.

**Page 225.**—1. 12 ft. a sec. 2. The rifle bullet. 3. 4,400,000. 4. 36,666½. 5.  $211\frac{1}{2}$ . 6. 1050. 7. 71,569.08. 8.  $79\frac{1}{16}$ . 9. 25 ch. 10. 23½.

**Page 226.**—1. 60 cu. in. 2. 100 in. 3. 31,416. 4. 523,600 cu. in. 5. 1963.5 sq. ft. 6. 8181.25 cu. ft. 7. 24,429.0816. 8. 136.35 + lb. 9. 476.4.

**Page 227.**—1. 36. 2. 48 rd. 3. 18 and 18. 5. 40. 6. 4. 7. 12. 8. 9. 9. 30. 10. 25. 11. 50. 12. 30 in. 13. 35 ft. 14. 15 ft. 15. 80 ft. 16. 100 ft. 17. 200 ft.

**Page 228.**—1. \$72. 2. \$267.75. 3. \$331.31. 4. \$93.67. 5. \$16.13. 6. \$59.10. 7. \$224.44. 8. \$454.22. 9. Loss, \$2. 10. Gain, \$10.50.

11. Loss, 60¢. 12. Loss, \$5.20. 13. Gain, \$5.80. 14. Gain, \$2.50. 15. Gain, \$5. 16. Gain, \$28. 17. Gain, 15%. 18. Loss, 10%.

## Answers

- Page 229.** — 1. 1200. 2.  $162\frac{1}{2}$ . 3. 240. 4.  $47\frac{1}{2}$ . 5. 33.58 cu. in.  
 6.  $2\frac{1}{2}$  cu. ft. 7.  $2\frac{1}{2}$  cu. ft. 8. \$3.12. 9. \$8.96. 10. \$161. 11. \$56.  
**Page 230.** — 1. 2135. 2. 2310. 3. \$76.70. 4. 276. 5. 1932. 6. 3864.  
 7. 17,920.  
**Page 231.** — 2. \$368.79.  
**Page 232.** — 2. \$301.27. 3. \$272.88.  
**Page 233.** — 1. \$189.71. 2. \$790.70. 3. \$509.38. 4. \$253.72.  
 5. \$957.33. 6. \$1610.05. 7. \$1220.17. 8. \$782.91.  
**Page 235.** — 1. 132,36705. 2. 26,027,0628. 3. 1059,998. 4. 4995,9934.  
 5. 3874.8. 6. 29876. 7. 1,551,8871. 8. .0528891. 9. 3501.7. 10. 351.8.  
 11. 852.64. 12. 2.25. 13. .025. 14. 5000. 15. 68.6. 16. 775,3168.  
 17. 7629. 18. 18,70866. 19. 2,268136. 20. .110033. 21. .0169456.  
 22. 2,421. 23. 21,318. 24. .013416. 25. .031542. 26. 42,637.  
 27. 4055.2. 28. .0080064. 29. .000002905. 30. 1.81. 31. 8.4.  
 32. 11,400,000. 33. 703.71. 34. 2,003,000.  
**Page 237.** — 1. 30. 2. 32. 3.  $2\frac{1}{2}$ . 4. 450. 5.  $321\frac{1}{2}$ . 6. 32.  
 7. 49. 8. 25. 9. 72. 10. 7. 11.  $106\frac{1}{2}$ . 12.  $67\frac{1}{2}$ . 13.  $71\frac{1}{2}$ .  
 14.  $18\frac{1}{2}$ . 15.  $196\frac{1}{2}$ . 16.  $59\frac{1}{2}$ . 17.  $3\frac{1}{5}$ . 18. 18. 19.  $15\frac{1}{2}$ .  
 20. 16. 21. 8. 22. 9. 23. 12. 24. 16. 25. 10. 26. 24.  
 27.  $42\frac{1}{2}$ . 28.  $583\frac{1}{2}$ . 29.  $11\frac{1}{2}$ . 30. 25.  
**Page 238.** — 1. 90. 2. 60. 3. 45 yr. 4. \$7 and \$8.  
 5. 280 and 140. 6. 13 and 7. 7. 9. 8. 29. 9. \$80. 10. 91 and 42.  
 11. 40 and 64. 12. 28 and 40.  
**Page 239.** — 1. 5265 ft. 2. 16 yr. and 40 yr. 3. 10. 4. 16.  
 5. 240 A. 6. 15 rd. 7. 30 in. long, 6 in. wide. 8. 16. 9. 40 rd.  
 10. \$3.85. 11. 48 in.  
**Page 240.** — 1. 14. 2.  $84\frac{1}{2}$ . 3. 9. 4. 15 yr. 5.  $33\frac{1}{2}\%$ .  
 6.  $44\%$ . 7. \$75. 8. \$90. 9. \$100. 10. 201,0624 sq. in.  
 11. 1767.15 cu. in. 12. 314.16 sq. ft. 13. 523,600 cu. ft.  
**Page 241.** — 1. 27. 2. 24. 3. 7. 4. .12. 5. .03. 6.  $\frac{7}{8}$ .  
 7.  $\frac{1}{2}$ . 8.  $\frac{1}{3}$ . 9.  $\frac{1}{3}$ . 10.  $\frac{1}{10}$ . 11. 6. 12. 7. 13. 12. 14. 4.  
 15. .05. 16.  $\frac{1}{3}$ . 17.  $\frac{1}{3}$ . 18.  $\frac{1}{3}$ . 19.  $\frac{2}{3}$ . 20.  $\frac{4}{7}$ . 21. 4. 22. 20.  
 23. 13. 24. 30. 25. 25. 26. 30. 27. .8. 28. 2.5. 29. 3. 30. 2.  
 31. 25. 32. 64. 33. 32. 34. 15,625. 35. 65,536. 36.  $\frac{1}{2}$ . 37.  $\frac{1}{4}$ .  
 38.  $\frac{1}{2}$ . 39. .0144. 40. .000000125.  
**Page 242.** — 1. 1155. 2. \$29.24. 3. \$35.68. 4. \$31. 5. 2703.65.  
 6.  $16\frac{1}{2}\%$ . 7.  $12\%$ . 8.  $8\frac{1}{2}\%$ . 9.  $5\frac{1}{2}\%$ . 10.  $24\frac{1}{5}\%$ . 11. 900.  
 12. 1800. 13. \$2800. 14. \$21,000. 15. \$150. 16.  $57\frac{1}{2}$ . 17. 308.  
 18. \$573. 19. \$50.51. 20. \$36.95. 21. 20. 22. 90. 23. 200.  
 24. \$40. 25. \$450.  
**Page 243.** — 2.  $\frac{2}{5}$ . 3. 27. 4.  $\frac{2}{5}$ . 5. 290 sq. in. 6.  $805\frac{1}{2}$  sq. in.  
**Page 244.** — 1. 2. 2. 6. 3. 40. 4. 2,8584 sq. ft. 5. 26.18 oz.  
 6. 29,4525 lb. 7. 8,7266 lb. 8. 256 sq. ft. 9. 78.54 sq. in.  
 10. 19,635 sq. in. 11. About 181 lb.  
**Page 245.** — 1. 4. 2.  $6\frac{1}{2}$ . 3.  $42\frac{1}{2}$ . 4.  $11\frac{1}{2}$ . 5.  $5\frac{1}{5}$ . 6.  $296\frac{1}{2}$ .  
 7.  $144\frac{1}{2}$ . 8.  $1\frac{1}{5}$ . 9.  $21\frac{1}{2}$ . 10. 4. 11.  $4\frac{1}{2}$ . 12.  $12\frac{1}{2}$ .  
 13.  $16\frac{1}{2}$  sq. ft. 14.  $1\frac{1}{2}$ . 15. About 1230 lb. 16. About 31 oz.  
 17. 360 sq. ft. 18. About 185 lb.  
**Page 246.** — 1. \$385.36. 2. \$483.42. 3. \$113.44. 4. \$107.82.  
**Page 247.** — 1.  $33\frac{1}{2}\%$ . 2. 20¢;  $13\frac{1}{2}\%$ . 3. \$5. 4.  $79\frac{1}{2}$  sq. in.

## Answers

5. 6144 sq. in. 6. 240 $\frac{1}{2}$ . 7. 198.3 + lb. 8. 10,400 units. 9. 7,920,000.  
 10. 134.0416. 11. 2.32 +.
- Page 248.**—1. 2 $\frac{1}{2}$  sq. ft. 2. 20 $\frac{1}{2}$  cu. ft. 3. 45 in. 4. 765 sq. in.  
 5. 3 $\frac{1}{2}$  cu. ft. 6. 85 in. 7. 525 sq. in. 8. 522 $\frac{1}{2}$  cu. in. 9. 5.729 + in.  
 10. 163.9384 sq. in. 11. 43.9824 in. 12. 483.8064 sq. in.  
 13. 63.6174 sq. ft. 14. 137.8377 cu. ft. 15. 128.282 cu. ft.  
 16. 18,849.6 cu. in. 17. 17.18 + in. 18. 7.3304 ft. 19. 3216.9984 sq. in.  
 20. 17,157.3248 cu. in. 21. 4.1888 cu. ft. 22. 2659.03 + cu. in.  
 23. 3848.46 sq. in. 24. 14,137.2 cu. in.
- Page 249.**—1. 3949.83 +. 2. 7926.53 + mi. 3. 26.87 + mi.  
 4. 24,859.4808 mi. 5. 24,902.01 + mi. 6. About 259,431,755,889 cu. mi.  
 7. About 13.4 +. 8. 49.16 +. 9. About 110. 10. 12,100. 11. About  
 160,741. 12. 111.11 +. 13. 107.39 +.
- Page 250.**—1. No difference. 2. \$1391.50. 3. 60¢. 4. \$500.  
 5. \$75. 6. \$1200. 7. \$250. 8. \$10,000. 9. 12 $\frac{1}{2}$ %. 10. 50%.  
 11. 12 $\frac{1}{2}$ %.
- Page 251.**—1.  $\frac{1}{15}$ . 2. 22 $\frac{1}{2}$ . 3. 1.27 +. 4.  $\frac{1}{2}$ . 5. 28 $\frac{1}{2}$ . 6. 1 $\frac{1}{2}$ .  
 7. 40. 8.  $\frac{1}{15}$ . 9. 28 $\frac{1}{2}$ . 10. 8. 11. 8;  $\frac{1}{2}$ ; 1 $\frac{1}{2}$ .
- Page 252.**—1. 925 oz. 2. .925. 3. 1 $\frac{1}{2}$ . 4. 958 $\frac{1}{2}$  oz. 5. 1598.4.  
 6. 1665. 7. 73.28 +. 8. 130 $\frac{1}{2}$ . 9. 4 $\frac{1}{2}$ ¢. 10. \$13.07. 11. \$2.
- Page 254.**—1. 1 $\frac{1}{2}$ ¢. 2. 1 $\frac{1}{2}$ ¢. 3.  $\frac{1}{15}$ . 4. 1 $\frac{1}{2}$ ¢. 5. 1 $\frac{1}{2}$ ¢. 6. 1 $\frac{1}{2}$ ¢.  
 7. 11 $\frac{1}{2}$ ¢. 8. 11 $\frac{1}{2}$ ¢. 9. 11 $\frac{1}{2}$ ¢. 10. 11 $\frac{1}{2}$ ¢. 11. 11 $\frac{1}{2}$ ¢. 12. 11 $\frac{1}{2}$ ¢.  
 13. 11 $\frac{1}{2}$ ¢. 14. 11 $\frac{1}{2}$ ¢. 15. 11 $\frac{1}{2}$ ¢. 16. 11 $\frac{1}{2}$ ¢. 17. 11 $\frac{1}{2}$ ¢. 18. 11 $\frac{1}{2}$ ¢. 19. 11 $\frac{1}{2}$ ¢.  
 20. 64¢. 21. 19¢. 22. 20 $\frac{1}{2}$ ¢. 23. 32¢. 24. 57 $\frac{1}{2}$ ¢. 25. 1¢.  
 26. 1¢. 27. 16¢. 28. 37 $\frac{1}{2}$ ¢. 29. 5684 $\frac{1}{2}$ ¢. 30. 1 $\frac{1}{2}$ ¢. 31. 1 $\frac{1}{2}$ ¢. 32. 1 $\frac{1}{2}$ ¢.  
 33. 1 $\frac{1}{2}$ ¢. 34. 1 $\frac{1}{2}$ ¢. 35. 3 $\frac{1}{2}$ ¢. 36. 3 $\frac{1}{2}$ ¢. 37. 1 $\frac{1}{2}$ ¢. 38. 1 $\frac{1}{2}$ ¢.  
 39. 1 $\frac{1}{2}$ ¢. 40. 1 $\frac{1}{2}$ ¢. 41. 1 $\frac{1}{2}$ ¢. 42. 1 $\frac{1}{2}$ ¢. 43. 1 $\frac{1}{2}$ ¢. 44. 1 $\frac{1}{2}$ ¢.
- Page 255.**—1. Potatoes, 5 A.; corn, 15 A.; grass, 75 A. 2. A, \$17,500;  
 B, \$2500. 3. 60. 4. 39 and 13. 5. \$112. 6. 20 and 60. 7. 20.  
 8. \$1.50; \$1.50; \$1. 9. 147 sq. rd. 10. 153 $\frac{1}{2}$  rd.
- Page 257.**—4. 28. 5. 32. 6. 45. 7. 38. 8. 52. 9. 56. 10. 62.  
 11. 67.
- Page 258.**—1. 20%. 2. 15%. 3. \$900. 4. No difference.  
 5. \$51.90. 6. 5% stock at 80. 7. 56¢. 8. \$4.81. 9. \$402.30.
- Page 259.**—1. 12. 2. 6 5-cent pieces; 12 10-cent pieces; 18 cents.  
 3. \$75,000. 4. 60, 50, and 70. 5. 35 2-cent; 25 1-cent; 10 5-cent.  
 6. 27 rd. and 15 rd. 7. 75 rd. and 15 rd. 8. \$1.80 and 90¢. 9. 12.15.  
 10. 4.53 $\frac{1}{2}$ .
- Page 260.**—2. 35. 3. 47. 4. 53. 5. 58.
- Page 261.**—3. 314. 4. 315. 5. 453. 6. 506. 7. 563. 8. 625.  
 9. 696. 10. 723. 11. 760. 12. 807. 13. 758. 14. 814. 15. 824.  
 16. 900. 17. 911.
- Page 262.**—1. \$8.85. 2. \$700. 3. 18 in. 4. 84 $\frac{1}{2}$ . 5. 29,568.  
 6. 277 $\frac{1}{2}$ . 7. \$17.86. 8. \$21.39. 9. 386 $\frac{1}{2}$ . 10. 232.2 gr. 11. 441 $\frac{1}{2}$ .  
 12. 24 $\frac{1}{2}$  lb.
- Page 263.**—1. 280 $\frac{1}{2}$ . 2. 36.16 +. 3. 37,500. 4. \$580.80.  
 5. \$6562.50. 6. 30 $\frac{1}{2}$ . 7. \$69,421.49. 8. 6 $\frac{1}{2}$  rd. 9. \$7.65. 10. \$51.45.  
 11. 0 $\frac{1}{2}$  ft. 12. \$1.29. 13. \$13.78. 14. 50¢.
- Page 264.**—3. 25. 4. 29. 5. 37. 6. .96 +. 7. .18 +. 8. 12.5.  
 9. 6.61 +. 10. 35.34 +. 11. 28.85 +. 12. 14.49 +. 13.  $\frac{1}{2}$ . 14. .16 +.  
 15. .94 +. 16. .61 +. 17. 2.81 +.

## Answers

**Page 265.** — 1. 83 ft.    2. 76.    3. 124.    4. 47 in.    5. 320.  
 6. 22.36 + ft. square.    7. 20.92 + ft.    8. 5.28 + ft.    9. 10.63 + ft.    10. 2.27 + ft.  
 11. 12.16 + ft.    12. 84.

**Page 266.** — 1. 17.188 ft.    2. 48.285 sq. in.    3. 125°.    4. 90° and 46° 40'.  
 5. 72°.    6. 15.9154 + ft.    7. \$141.37.    8. 1176.    9. 6 in.    10. 261.328.  
 11. 376 +.    12. 201.0624.    13. 508.9392.

**Page 267.** — 1. 13½ da.    2. 3½.    3. \$1.57½.    4. \$120.    5. \$300.  
 6. \$491.80.    7. 25 yr.    8. 133½ rd.    9. 391½ rd.    10. 40%.    11. 33½%.  
 12. \$2278.

**Page 268.** — 1. 65 in.    2. 6.83 + ft.    3. 28.28 + ft.    4. 4.04 + ft.  
 5. 21.21 + in.    6. 25.45 + in.    7. 9.79 + yd.    8. 96.04 + ft.    9. 20.94 + rd.  
 10. 38.41 + mi.    11. 129.61 + ft.

**Page 269.** — 1. \$12.36.    2. \$7.40.    3. \$8.35.    4. \$46.51.    5. \$350.96.  
 6. \$281.38.    7. \$525.51.    8. \$180.08.    9. \$3.82.

**Page 270.** — 1. 1089.    2. 40 rd.    3. 834.84 +.    4. 63.44 + rd.    5. 12 ft.  
 6. 32.24 + ft.    7. 24 ft.    8. 64.    9. 6400 sq. ft.    10. 1054½ oz.  
 11. 164 + lb.

**Page 271.** — 1. 6.    2. \$5.    3. 45 yr. and 22 yr.    4. 30 and 20.  
 5. 12.    6. 20 yr.    7. 15 and 6.    8. 80.    9. 120.    10. 16 rd.

**Page 272.** — 4. 500<sup>cm</sup>.    5. 5,000,000<sup>mm</sup>.    6. 5.    7. 200,000.  
**Page 273.** — 1. 232,500<sup>cm</sup>; 2.325<sup>Km</sup>.    3. 8250<sup>m</sup>; 8,250,000<sup>mm</sup>.  
 4. 83<sup>cm</sup>; 830<sup>mm</sup>.    5. 35<sup>m</sup>.    6. 70,157.5<sup>dm</sup>.    7. 157.5<sup>m</sup>.    8. 329.35<sup>m</sup>.  
 9. 12 min.    10. 17,250.    11. 68.8<sup>m</sup>.    12. 14.

**Page 274.** — 1. 3.047 +.    2. 3.37 + in.    3. 64.006 +.    4. 328.808 +.  
 5. 3288.08 +.    6. 1969.1 +.    7. 150.68 +.    8. 5.601 +.    9. 160.93 +.  
 10. 78.74 +.    11. 31.069 +.    12. 335.27 +.    13. 2340.8 +.    14. 761.9 +.  
 15. 2413.97 +.

**Page 275.** — 3. 1,000,000.    4. 50,000<sup>mm</sup>.    5. 7,000,000<sup>mm</sup>.  
 6. 350,004<sup>mm</sup>.    7. 797.44.    8. 5.1<sup>dm</sup>.    9. 4.90875<sup>mm</sup>.    10. 250.

**Page 276.** — 3. 5.    4. .06.    5. 9,000,000<sup>cm</sup>.    6. 7,000,000<sup>mm</sup>.  
 7. 480.    8. 400.    9. 81.    10. 159.6.    11. 10<sup>m</sup>.

**Page 277.** — 1. 10.76 +.    2. 46.4 +.    3. 388.7 +.    4. 13.94 +.    5. 3.62 +.  
 6. 13.69 +.    7. .00063 +.    8. 0016 +.    9. 6.79 +.    10. 648.4 +.    11. \$21.11.  
 12. 1618.68 +.    13. 24.28 +.

**Page 278.** — 6. 5500<sup>¢</sup>.    7. 200,300<sup>¢</sup>.    8. 453.592 +.    9. 16.4 +<sup>¢</sup>.  
 10. 16.39 +.    11. 2549.5 +.

**Page 279.** — 1. 1000.    2. 100,000.    3. 4.    4. 2640.    5. 75¢.    6. 2500.  
 7. 17.08 +.    8. 26.417 +.    9. 23.65 +.    10. 35.23 +.    11. 14.18 +.    12. .908 +.







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