AN OBJECTIVE STUDY OF THE INFLUENCE OF MOISTURE DISTRIBUTION AND LAPSE RATE UPON VERTICAL STABILITY

BY

ELSTON WYATT

AN OBJECTIVE STUDY OF THE INFLUENCE OF MOISTURE DISTRIBUTION AND LAPSE RATE UPON VERTICAL STABILITY

by

Elston Wyatt
Lieutenant, United States Navy

Submitted in partial fulfillment
of the requirements
for the degree of
MASTER OF SCIENCE
IN AEROLOGY

United States Naval Postgraduate School
Monterey, California
1950

This work is accepted as fulfilling
the thesis requirements for the degree of
Master of Science in Aerology

from the
United States Naval Postgraduate School

PREFACE

"An Objective Study of the Influence of Moisture Distribution and Lapse Rate Upon Vertical Stability" is a paper designed for use in aerology as a stability analysis of the atmosphere. The work was done at the U. S. Naval Postgraduate School, Monterey, California, as a partial requirement for the degree of Master of Science in Aerology.

The author wishes to acknowledge the help and guidance of Professor W. D. Duthie of the Postgraduate School.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

TABLE OF SYMBOLS AND ABBREVIATIONS

T	temperature
f	relative humidity
$\overline{RH}$	mean relative humidity
mb	millibars
☈	thunderstorm
☁	cumulo nimbus cloud
ϟ	lightning
z	height
No. ☈	no thunderstorm
ρ	density
M	mass
t	time
p	pressure
α	specific volume
v	vertical velocity
R	gas constant
$\dot{C}$	circulation acceleration
γ	lapse rate
γ_d	dry adiabatic lapse rate
γ_m	saturated adiabatic lapse rate
V	velocity
A	area

INTRODUCTION

The analysis of the vertical stability of the atmosphere by means of the upper air sounding is a standard procedure employed by aerologists as a forecasting aid. The two methods of analysis presently employed most frequently are the slice method and the parcel method.

Evidence has recently been accumulated, particularly by the U. S. government thunderstorm projects of 1946 and 1947, to suggest that lateral mixing between ascending cloud air and environment influences the vertical development of cumulus clouds. Investigations of Austin [1] [2] and Stommel [11] indicate graphical methods for determining the effect of entrainment (lateral mixing) upon lapse rate. Byers and Braham [6], in their report of the results of the thunderstorm projects, give estimated values of entrainment rates.

The slice method as established by Bjerknes [4] [5] and Petterssen [10] and modified by Beers [3] is noteworthy in that it provides a numerical index of vertical stability. As presently employed, however, this method assumes that there is no mixing between ascending cloud air and the environment. Furthermore, Cressman [8], by a modification of the slice method, has shown the qualitative influence of the field of horizontal divergence upon convective activity. To modify the Beers slice method of stability analysis so as to include both the effects of entrainment and of divergence would provide an improved tool for studying the vertical stability of an air mass and for forecasting the development of cumulus activity. To this end the efforts of the

research were initially directed. The approach taken was theoretical. The effect of mass divergence was introduced into the slice method equations, as was a factor of entrainment rate. It was anticipated that the inclusion into the slice method equations of mathematic expressions for divergence and entrainment would introduce complexities; however it was thought that some clear estimation of the effect of these factors in combination could be obtained. Such was not the case. The primary difficulty encountered was that of analytical representation of the mixing process.

As a different approach, a statistical study of vertical stability was undertaken for a single station, with especial emphasis upon the effect of vertical distribution of moisture. By a comparison of the upper air variates of relative humidity and temperature with the observed weather, an objective technique for stability analysis was developed. This technique indicates the influence of vertical moisture distribution upon cumulus activity and is applicable to stations other than the one investigated.

CHAPTER I

PROCEDURES

The investigation of the slice method of stability analysis as modified by the introduction of expressions for divergence and entrainment was purely theoretical and mathematical. This development is set forth in the appendix to the thesis.

The variates employed in establishing an objective technique for forecasting the stability of an air mass are temperature and relative humidity. The lapse rate of temperature within an air mass is an obvious measure of the vertical stability of the air. The vertical distribution of moisture influences convective stability in such a way that a given layer, if dry above and moist below, will be convectively unstable, and if moist above and dry below will be convectively stable. Hence, an objective technique based upon vertical distribution of temperature and relative humidity should provide a measure of the vertical stability of an air mass.

For the purposes of this study, the upper air soundings for the Weather Bureau station at Dodge City, Kansas were used. Data were taken from the Upper Air Bulletins for the months of August and September, 1949, and surface observations were obtained from the daily surface weather maps of the Postgraduate School.

Soundings were plotted from the upper air data for 0300Z and 1500Z of each day of August and September 1949. From these, relative humidities at standard and significant levels were computed. The relative humidities and temperatures at 850, 700, and 500 millibars were the principal

variates employed. These were recorded for each sounding and differences were computed as indicated in Table I.

As an aid in determining suitable combinations of variates for use in an objective technique, graphs were drawn plotting variates against time and weather conditions; and scatter diagrams were prepared.

From the graphs of Δf_3 against time, it is apparent that while vertical distribution of moisture influences stability, this influence is not independent of other variates. In particular, the influence of moisture distribution upon vertical stability varies with different lapse rates of temperature. ΔT_3, the difference between the 850 and 500 mb. temperatures, is a measure of lapse rate; it was observed that whenever ΔT_3 was extreme (large or small), lapse rate and mean relative humidity appeared to be in themselves adequate measures of stability and the effect upon stability of moisture distribution in the vertical was negligible. On the other hand, for intermediate values of ΔT_3 the effect of moisture distribution upon stability was marked.

Accordingly, as a preliminary step in obtaining objective criteria for use in evaluation of the soundings, limits for four different ΔT_3 groups were selected. For each ΔT_3 group, moisture criteria were discovered (on the basis of the two months' data) for forecasting the occurrence or non-occurrence of thunderstorm at Dodge City during the $21\frac{1}{2}$ hours subsequent to each sounding. The ΔT_3 classifications, together with their individual objective moisture criteria are given in Table 2.

TABLE 1

T_{850} temperature at 850 mb. level in ^{o}F.

f_{850} relative humidity at 850 mb. level in %.

$\Delta T_1 = T_{850} - T_{700}$

$\Delta T_2 = T_{700} - T_{500}$

$\Delta T_3 = T_{850} - T_{500}$

$\Delta f_1 = f_{850} - f_{700}$

$\Delta f_2 = f_{700} - f_{500}$

$\Delta f_3 = f_{850} - f_{500}$

Δf max the most positive of the three variates Δf_1, Δf_2, Δf_3.

$\overline{RH} = \frac{f_{850} + f_{700} + f_{500}}{3}$ = mean relative humidity for 850 - 500 mb. layer.

TABLE 2

Classification of Soundings According to ΔT_3 value:

Case I: $\Delta T_3 < 20$

Case II: ΔT_3 20 - 25

Case IIIA: ΔT_3 26 - 30, $\overline{RH} < 53\%$

Case IIIB: ΔT_3 26 - 30, $\overline{RH} > 52\%$

Case IV: $\Delta T_3 > 30$

Subdivision of actual thunderstorm occurrences into "frontal" and "air mass" types was not made. Only three cases of frontal thunderstorms were identified from the surface data for the period under study. It should be pointed out, however, that most of the thunderstorm occurrences observed were during periods when Dodge City was situated on the western side of a high pressure cell. Dodge City was, therefore, located within a field of convergence which resulted from southerly flow and which was fully as capable of releasing convective instability as a frontal passage would have been. Accordingly, a distinction between "frontal" and "air mass" thunderstorm types was not considered essential.

From examination of the individual soundings after classification into ΔT_3 groups, moisture criteria were established for predicting the occurrence or non-occurrence of thunderstorm or cumulo-nimbus development. These moisture criteria vary among the different ΔT_3 groups. For Case I ($\Delta T_3 < 20$), which occurred only seven times, it appears that absolute stability is great enough so that neither moisture content nor distribution of moisture is likely to be sufficiently influential to produce thunderstorm activity. At the other extreme with Case IV ($\Delta T_3 > 30$), the investigation indicates that thunderstorm activity is probable whenever the mean relative humidity is not extremely low and that vertical distribution of moisture is not of great significance. Cases II and III indicate the importance of distribution of moisture in a consideration of stability. In both

these cases many different combinations of variates were investigated with regard to their influence upon stability. It was discovered that mean relative humidity and the distribution of relative humidity among the standard levels were the most satisfactory variates for use in forecasting the possibility of thunderstorm development.

In order to indicate clearly the results of this investigation, forecasting "rules" for the different ΔT_3 cases were established and applied to each sounding. The "rules", based solely upon the upper air data and surface observations at Dodge City for two months, are presented in Table 3.

TABLE 3

Forecast Rules

Case I: $\Delta T_3 < 20$

Rule: forecast "no thunderstorm" during next period ($21\frac{1}{2}$ hrs.).

Case II: $19 < \Delta T_3 < 26$

Rule: forecast "thunderstorm" if $\overline{RH} > 37$ and $\Delta f_{max} > 24$, and both are larger than at sounding 12 hours previous. Otherwise forecast "no thunderstorm".

Case IIIA: $25 < \Delta T_3 < 31$, $\overline{RH} < 53\%$

Rule: forecast "thunderstorm" if $\overline{RH} > 36$ and $\Delta f_1 > \Delta f_2 > \Delta f_3$ with both Δf_1 and $\Delta f_2 < 21$. Otherwise forecast "no thunderstorm."

Case IIIB: $25 < \Delta T_3 < 31$, $\overline{RH} > 52\%$

Rule: forecast "thunderstorm".

Case IV: $\Delta T_3 > 30$

Rule: forecast "thunderstorm" if $\overline{RH} > 30$; otherwise forecast "no thunderstorm".

CHAPTER II

RESULTS

Since the "forecast rules" were established as a result of comprehensive examination of the upper air soundings at Dodge City, it is to be expected that, when applied to these same soundings, they should give accurate results. Such is the case. For the "verification" of a forecast, the weather as plotted on the 0030 Z and 1230 Z surface maps on file at the Postgraduate School was used. A forecast of "thunderstorm" was considered "verified" whenever either the 0030 Z or the 1230 Z surface maps subsequent to the upper air sounding (upon which the forecast was based) indicated thunderstorm or cumulo nimbus development at Dodge City or at stations 450, 463, and 465 which are nearby Dodge City. Lightning and showers when accompanied by cumulus activity were also considered to verify a "thunderstorm" forecast. During the period under investigation there occurred at these surface stations seventeen thunderstorms, five cases of cumulo nimbus development without thunder, one case of lightning and one case of showers accompanied by cumulus development.

It is unfortunate that hourly surface data were not available. The actual weather conditions were known only at 12 hour periods, obviating the possibility of investigating with any accuracy the precise time of thunderstorm occurrence. Furthermore, since past weather is reported only for a period of six hours prior to each surface map, there is a gap of six hours between surface maps during which thunderstorm might have occur-

red without being noted in this investigation. Accordingly, it is possible that, had complete surface observations been available, a higher percentage of correct forecasts would have resulted. The results obtained when the forecast rules were applied to Dodge City for August and September 1949 are given in Table 4. Percentage correct forecasts and skill scores are indicated for the five different cases. It should be noted that a perfect "fit" between observed thunderstorm occurrence (or non-occurrence) and the developed moisture and lapse rate criteria would result in a percentage correct forecasts of 100 and a skill score of one.

Since sufficient surface data for Dodge City was not available for use as test data, the accuracy of the forecast rules could not be adequately evaluated. It is not imagined that an objective technique based upon a mere two months' data and upon such an arbitrarily limited number of variates as the scope of this investigation permitted would meet a strict test of accuracy when applied to independent data. In particular, it is almost certain that, because of the small size of the sample tested, limits of the humidity and moisture distribution criteria were fixed much too rigidly in order to "forecast" successfully the highest possible number of occurrences and non-occurrences of thunderstorm. It is highly possible that several "freak" cases within the small two months' sample which was tested cast a disproportionate weight in the establishment of criteria and limits.

Table 4 indicates the results of a test made on upper air data at Oklahoma City for the month of August 1949. This test indicates that

TABLE 4

Dodge City		Number of R> Occurrences	Number of No-R> Occurrences	Percentage Correct Forecasts	Skill Score
Case I	Forecast R>	0	0		
	Forecast No R>	0	7	100	
Case II	Forecast R>	6	3	89	0.68
	Forecast No R>	1	26		
Case IIIA	Forecast R>	4	0	100	1.00
	Forecast No R>	0	25		
Case IIIB	Forecast R>	15	3	83	
	Forecast No R>	0	0		
Case IV	Forecast R>	11	1	92	
	Forecast No R>	0	0		
Total	Forecast R>	36	7	92	0.84
	Forecast No R>	1	58		

Oklahoma City		Number of R> Occurrences	Number of No-R> Occurrences	Percentage Correct Forecasts	Skill Score
Total	Forecast R>	14	8	75	0.49
	Forecast No R>	4	22		

the "forecast rules" established for Dodge City are not sufficiently embracing to be used without modification at other stations. The effects of divergence and of advection and the factors of lapse rate and of moisture distribution at other than standard levels should certainly be included in a comprehensive application of an objective stability analysis. These effects and factors were of necessity omitted in this study.

Nevertheless, the investigation indicates a method for obtaining rapidly and objectively a measure of the stability of an air mass and for assisting the forecaster in solving the difficult thunderstorm case.

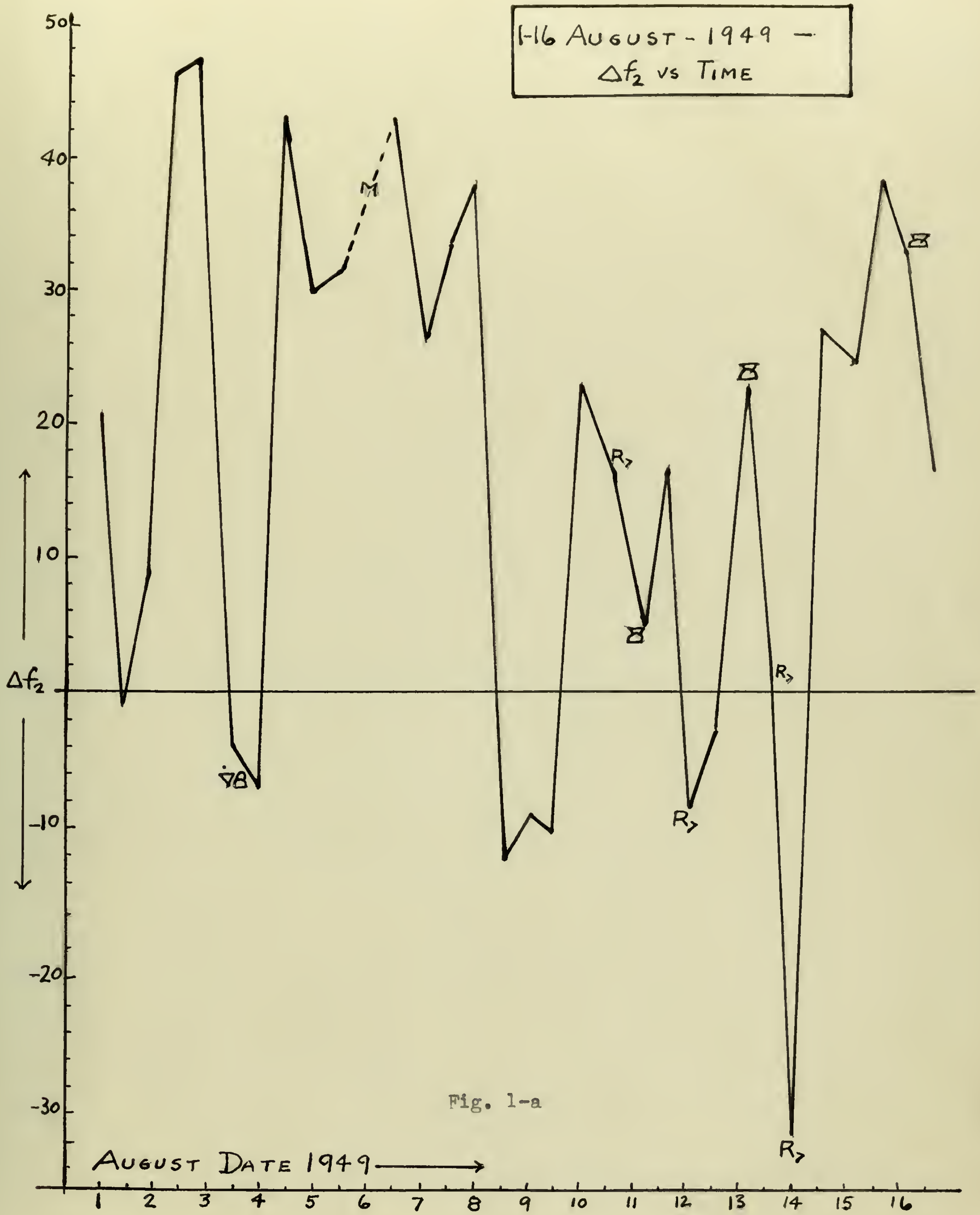
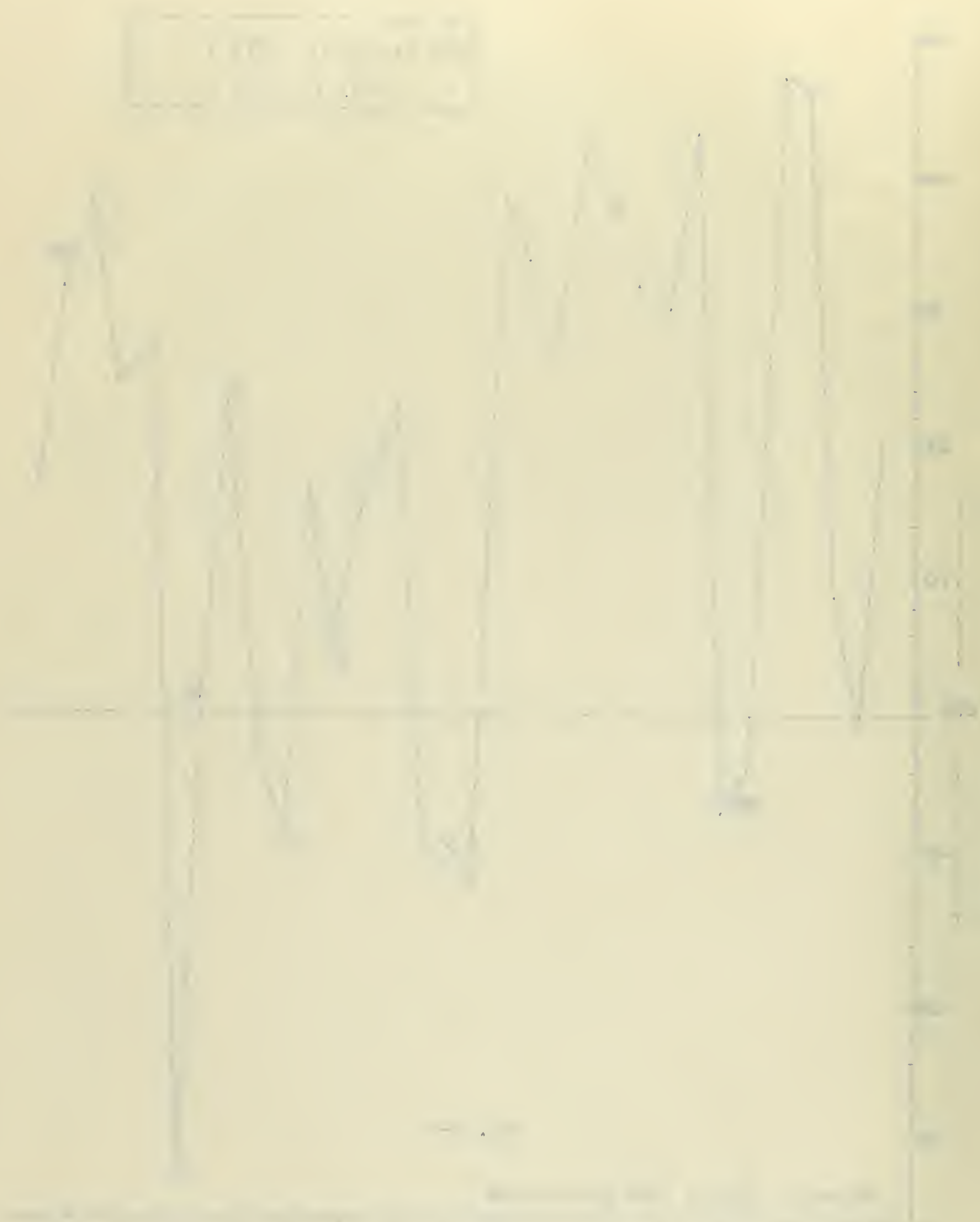

1-16 August - 1949 —
Δf2 vs Time
50
40
30
20
10
Δf2
-10
-20
-30
M
R7
R7
R7
R7
August Date 1949
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16

Fig. 1-a



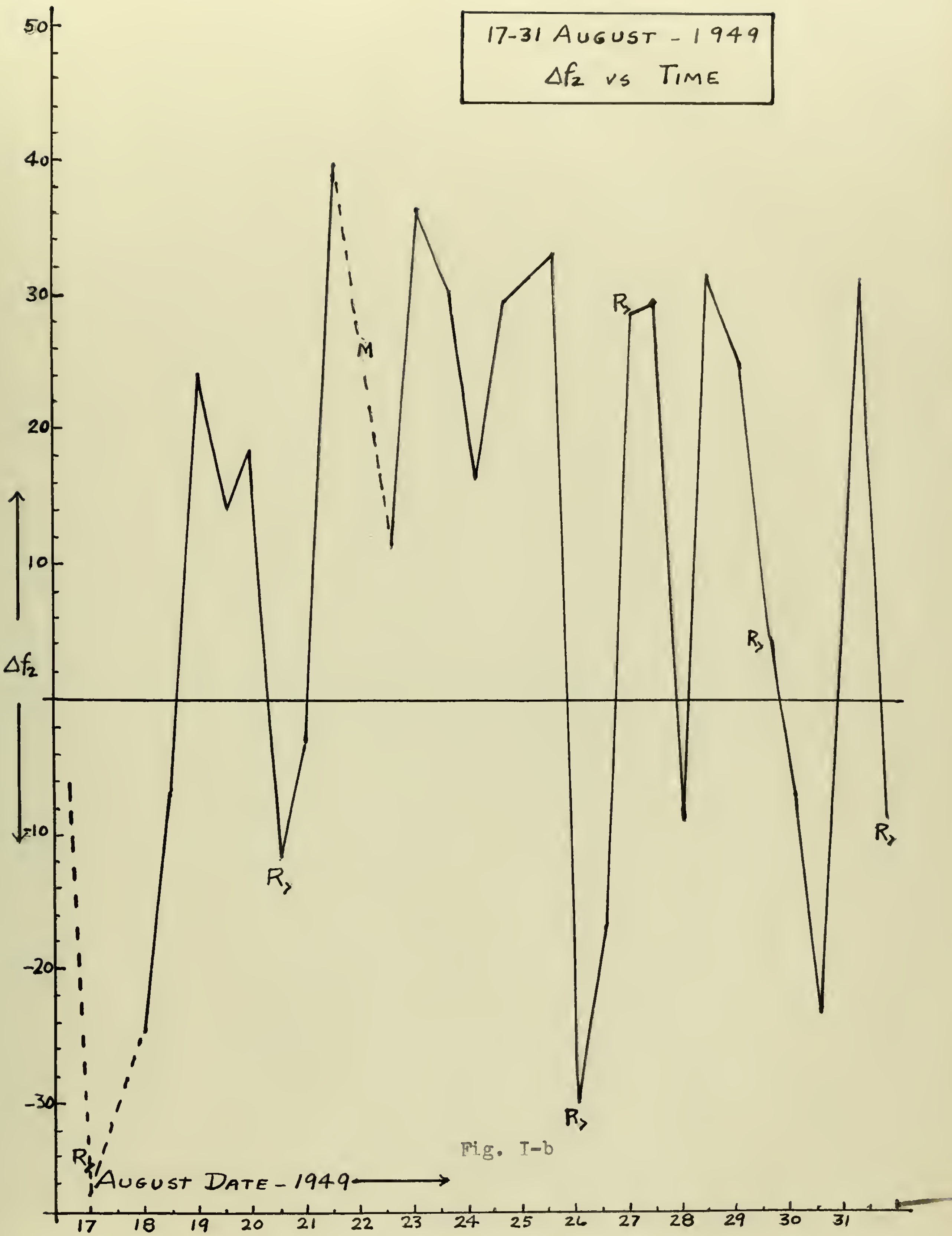

17-31 AUGUST - 1949
Δf2 VS TIME
50
40
30
20
10
-10
-20
-30
Δf2
M
R>
AUGUST DATE - 1949
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31

Fig. I-b

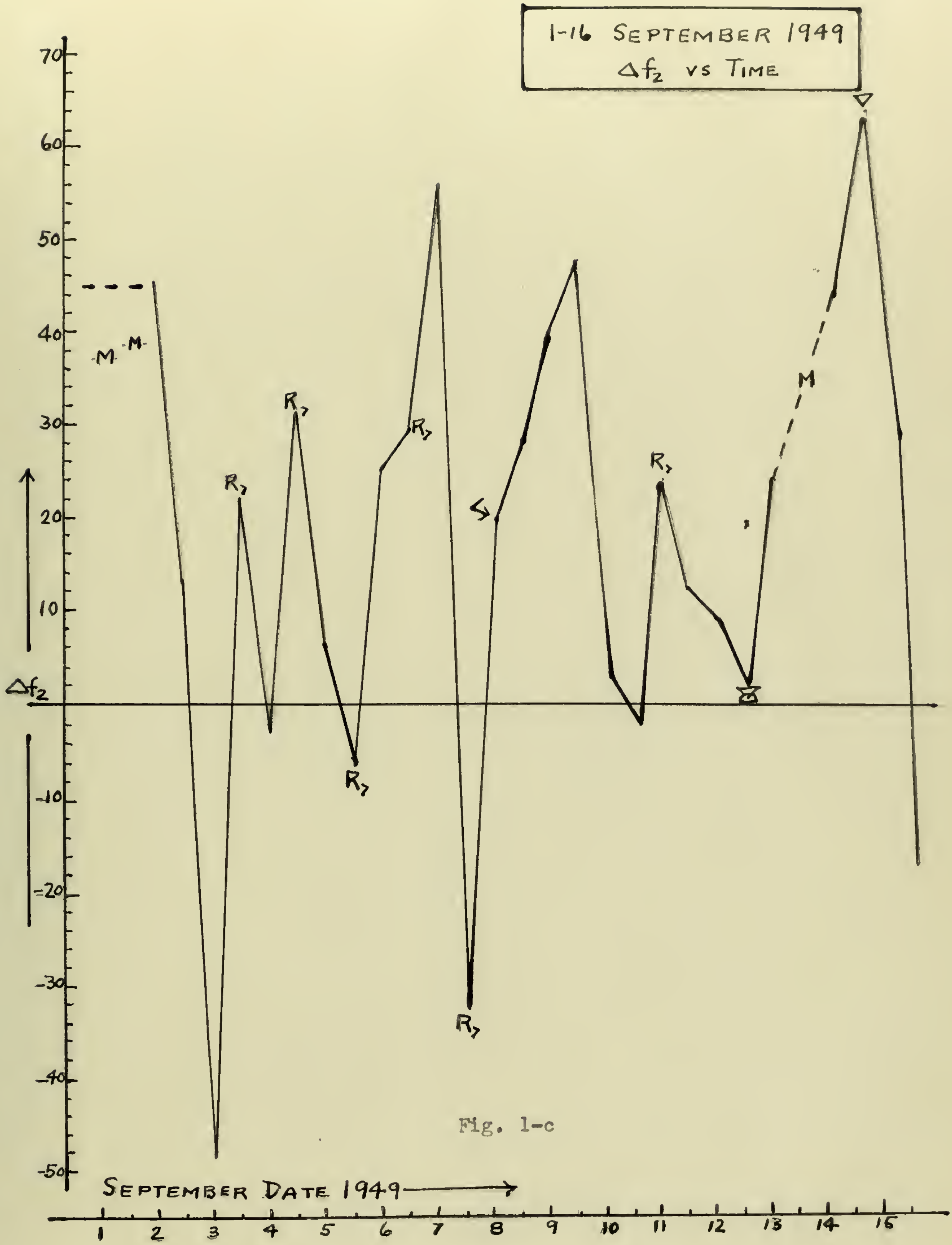
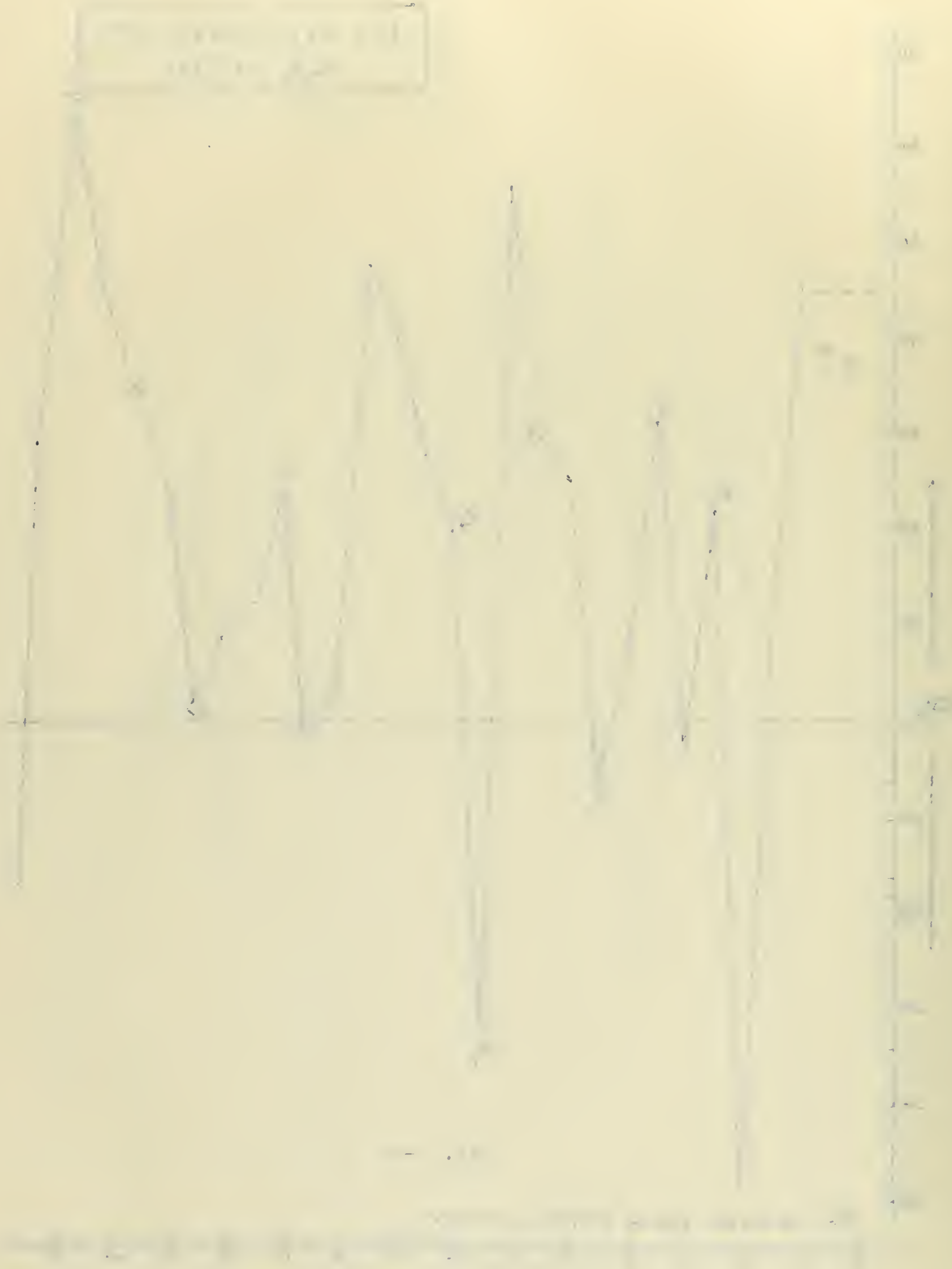

1-16 SEPTEMBER 1949
Δf2 VS TIME
70
60
50
40
30
20
10
Δf2
-10
-20
-30
-40
-50
M
M
R7
M
SEPTEMBER DATE 1949
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

Fig. 1-c



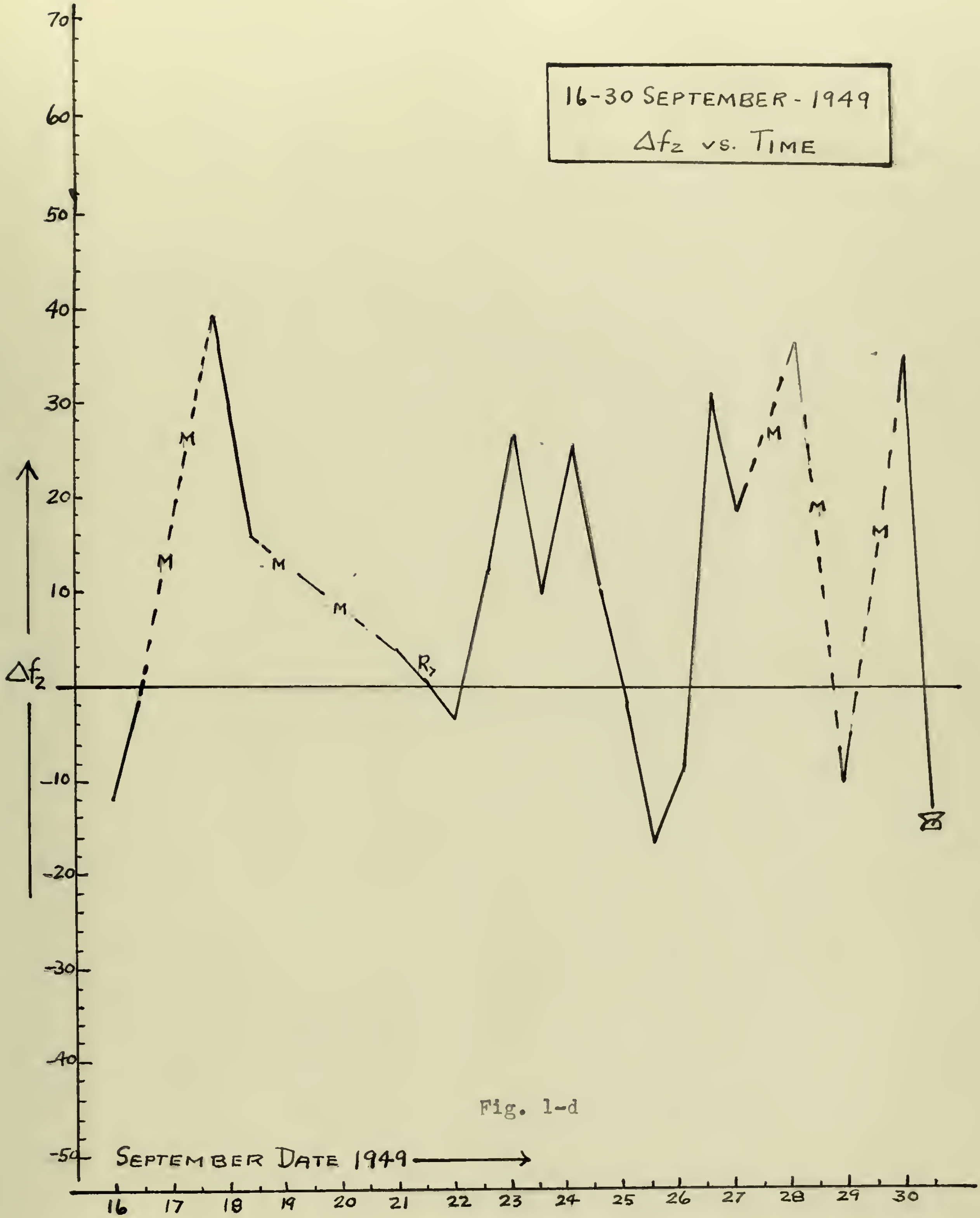
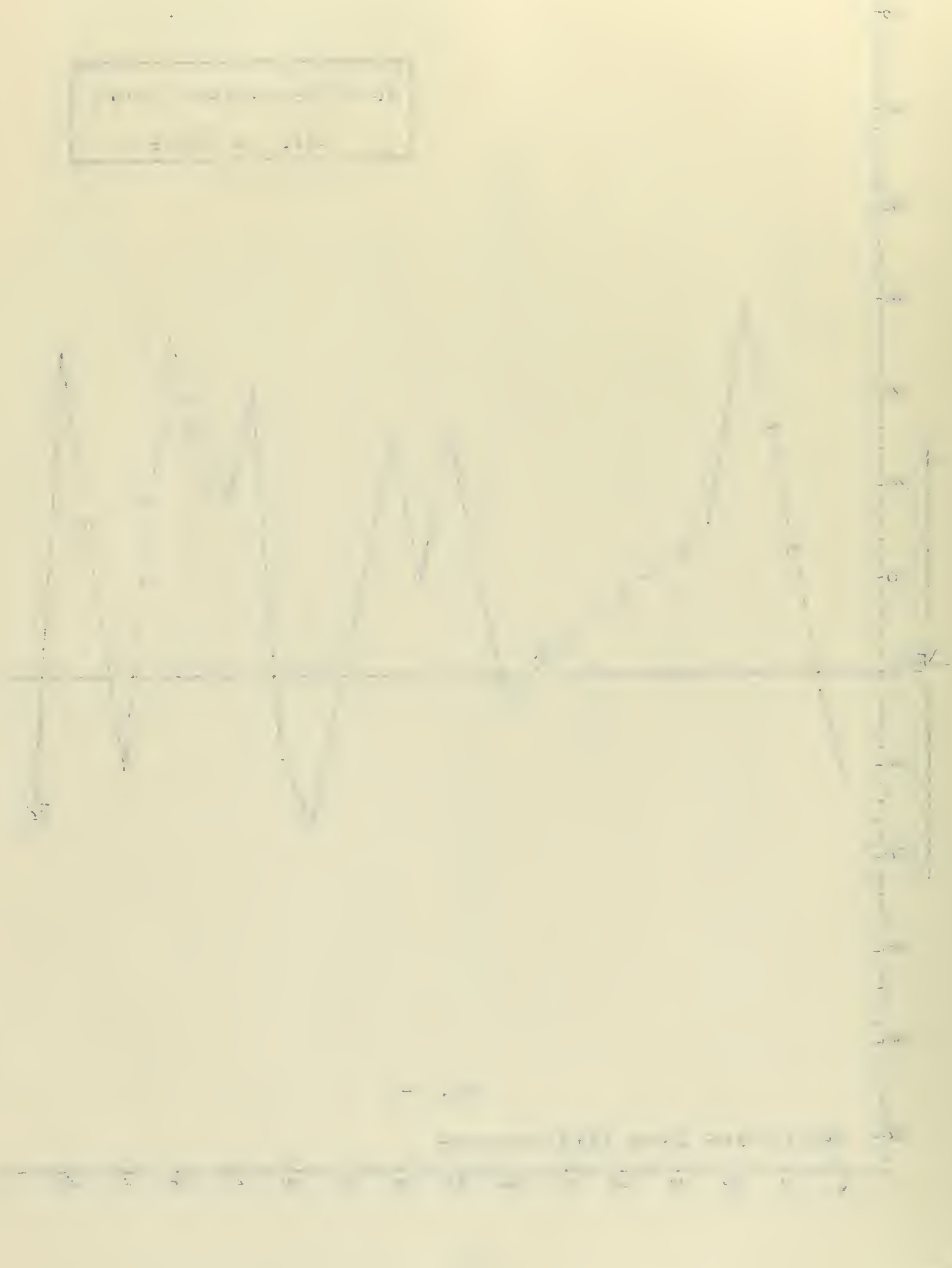
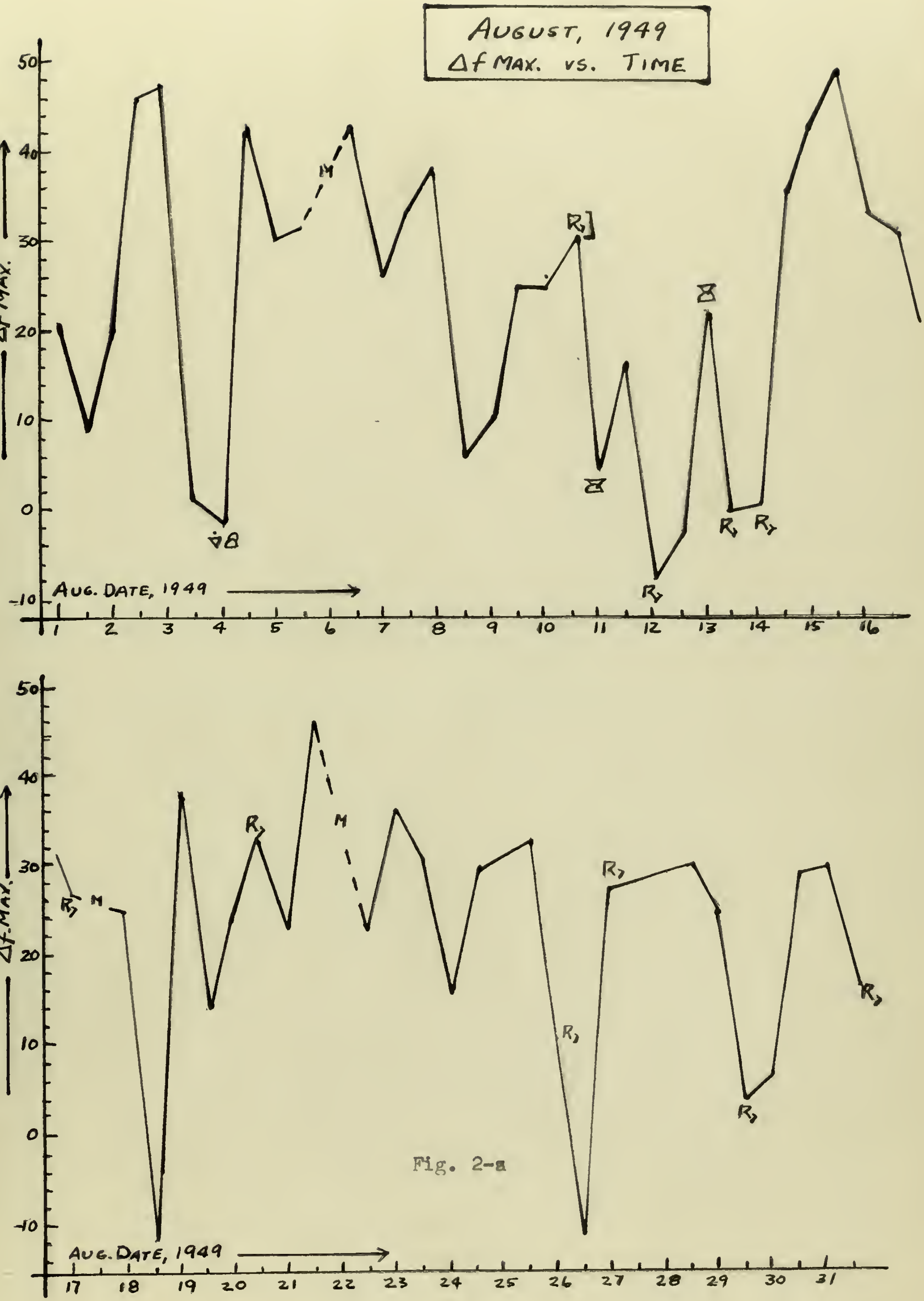

16-30 SEPTEMBER - 1949
Δf2 vs. TIME
70
60
50
40
30
20
10
Δf2
-10
-20
-30
-40
-50
M
M
M
M
R7
M
M
M
M
SEPTEMBER DATE 1949
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30

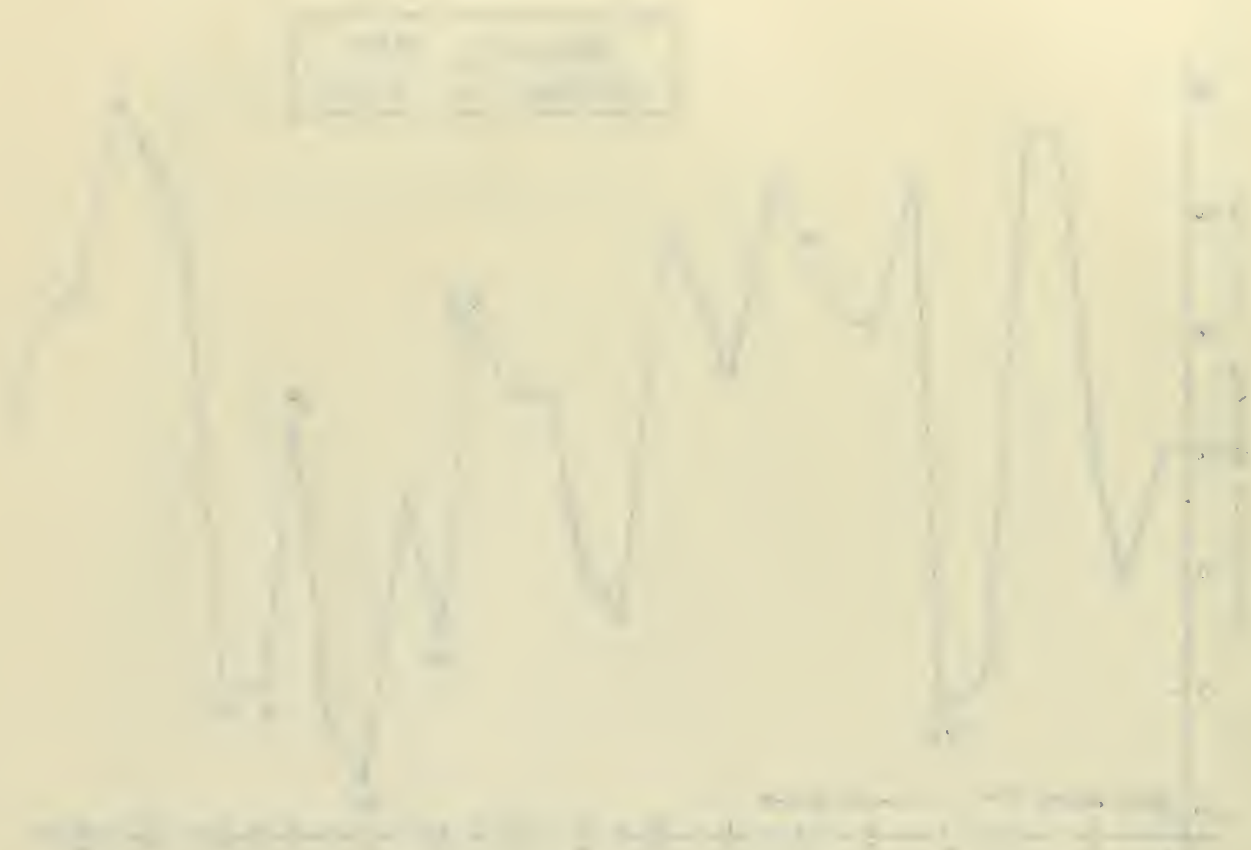
Fig. 1-d

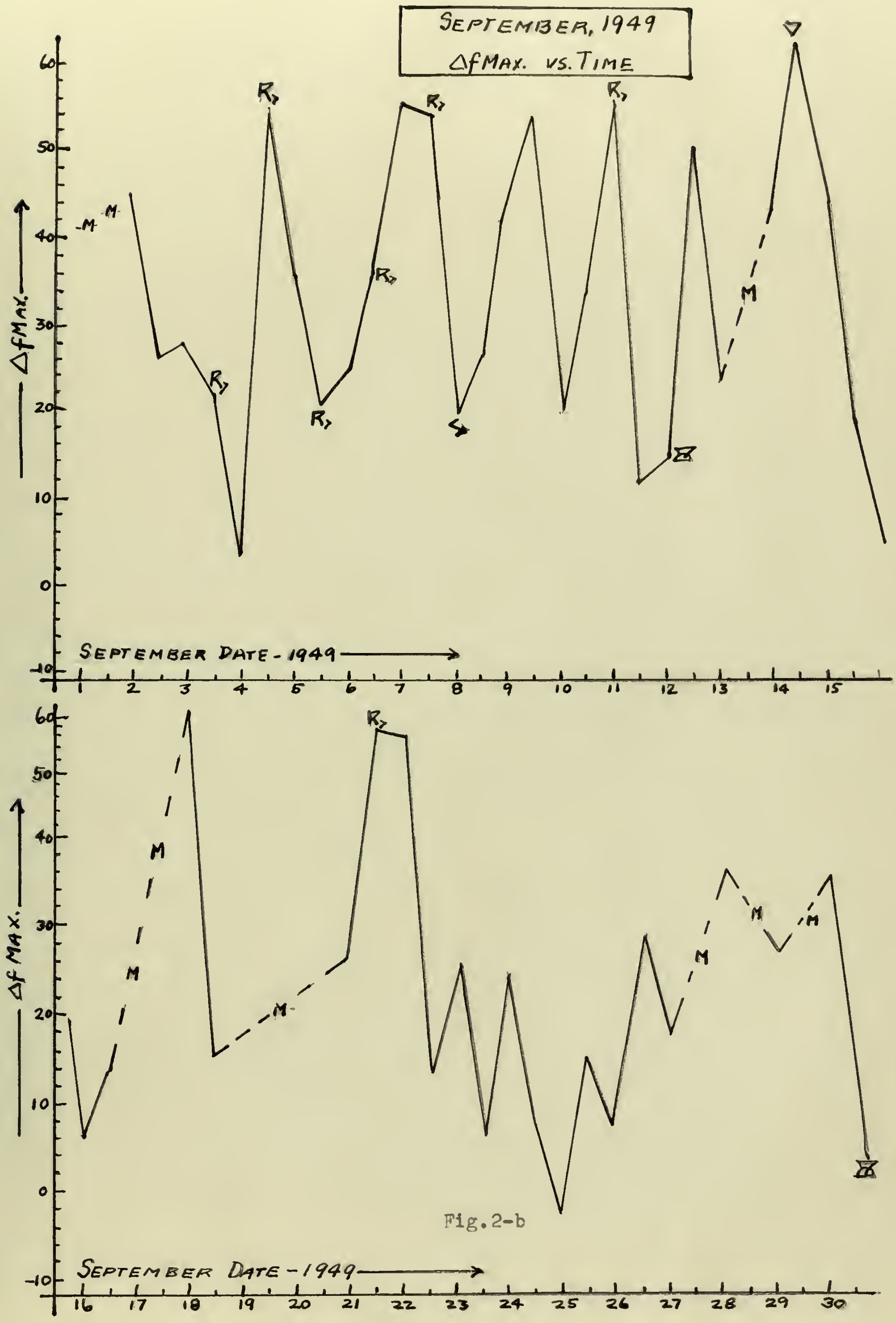




AUGUST, 1949
Δf MAX. vs. TIME
Δf MAX.
AUG. DATE, 1949
Δf.MAX.
AUG. DATE, 1949

Fig. 2-a





SEPTEMBER, 1949
ΔfMAX. VS. TIME
ΔfMAX.
SEPTEMBER DATE - 1949
ΔfMAX.
SEPTEMBER DATE - 1949

Fig.2-b

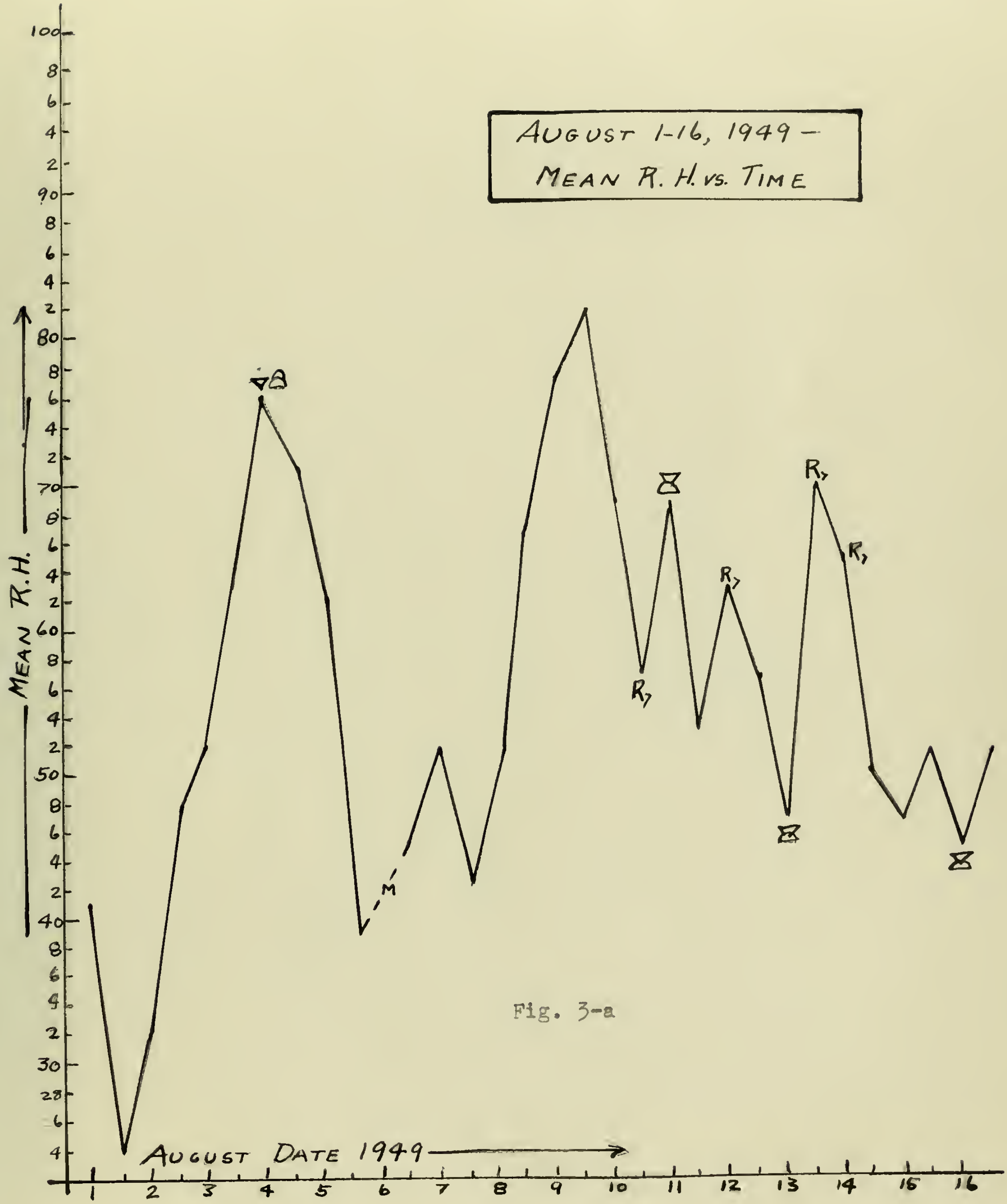
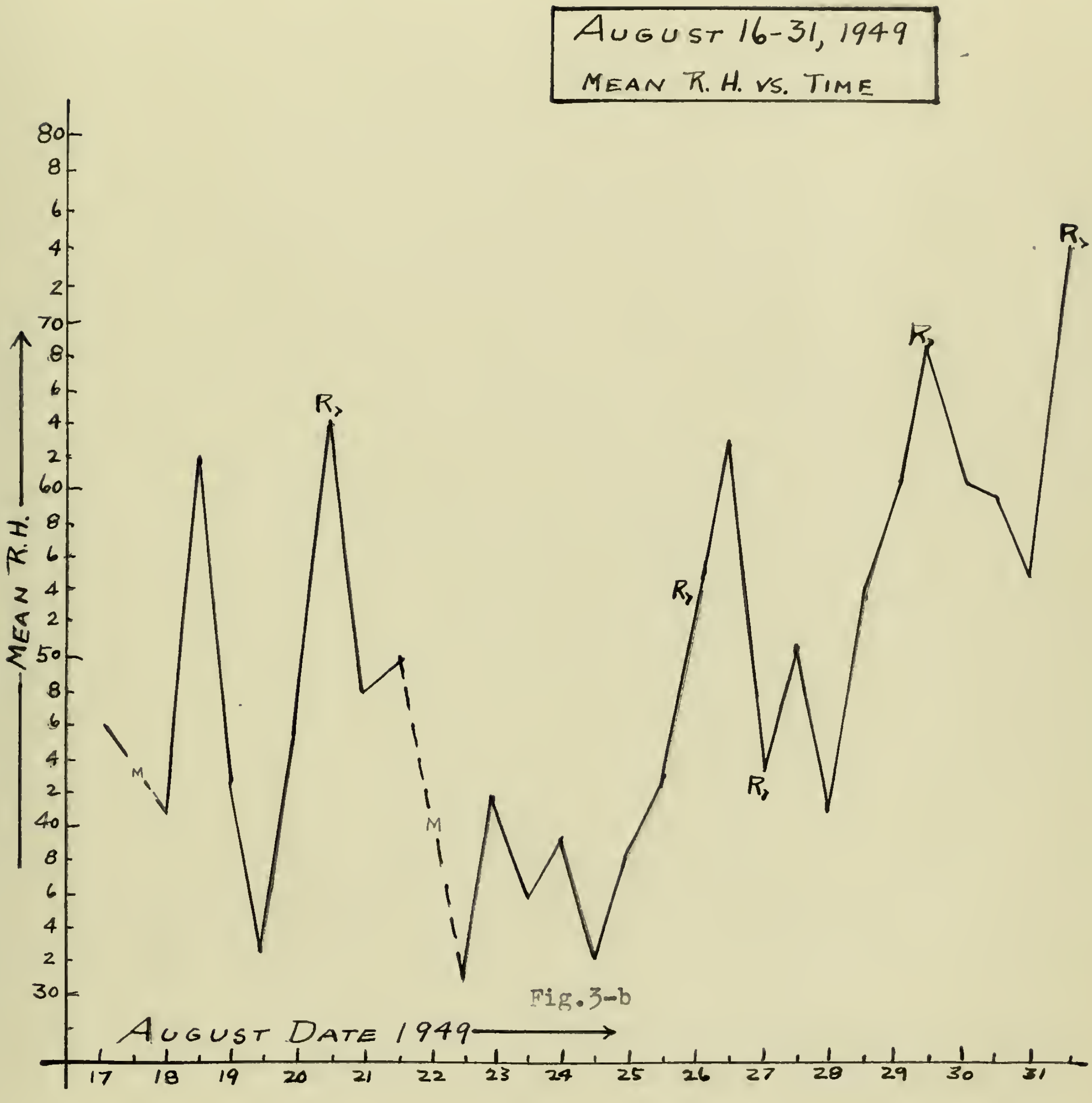
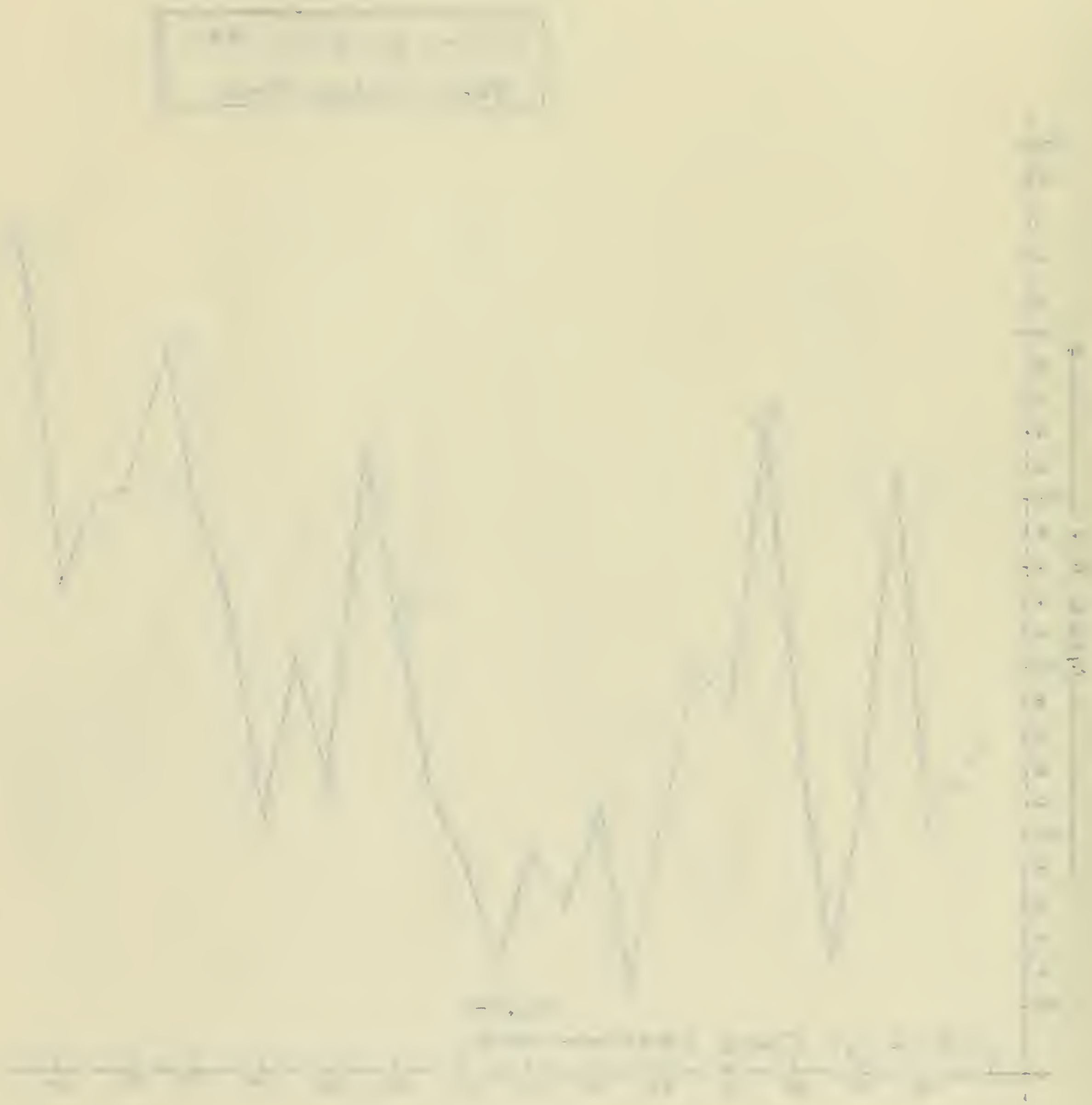


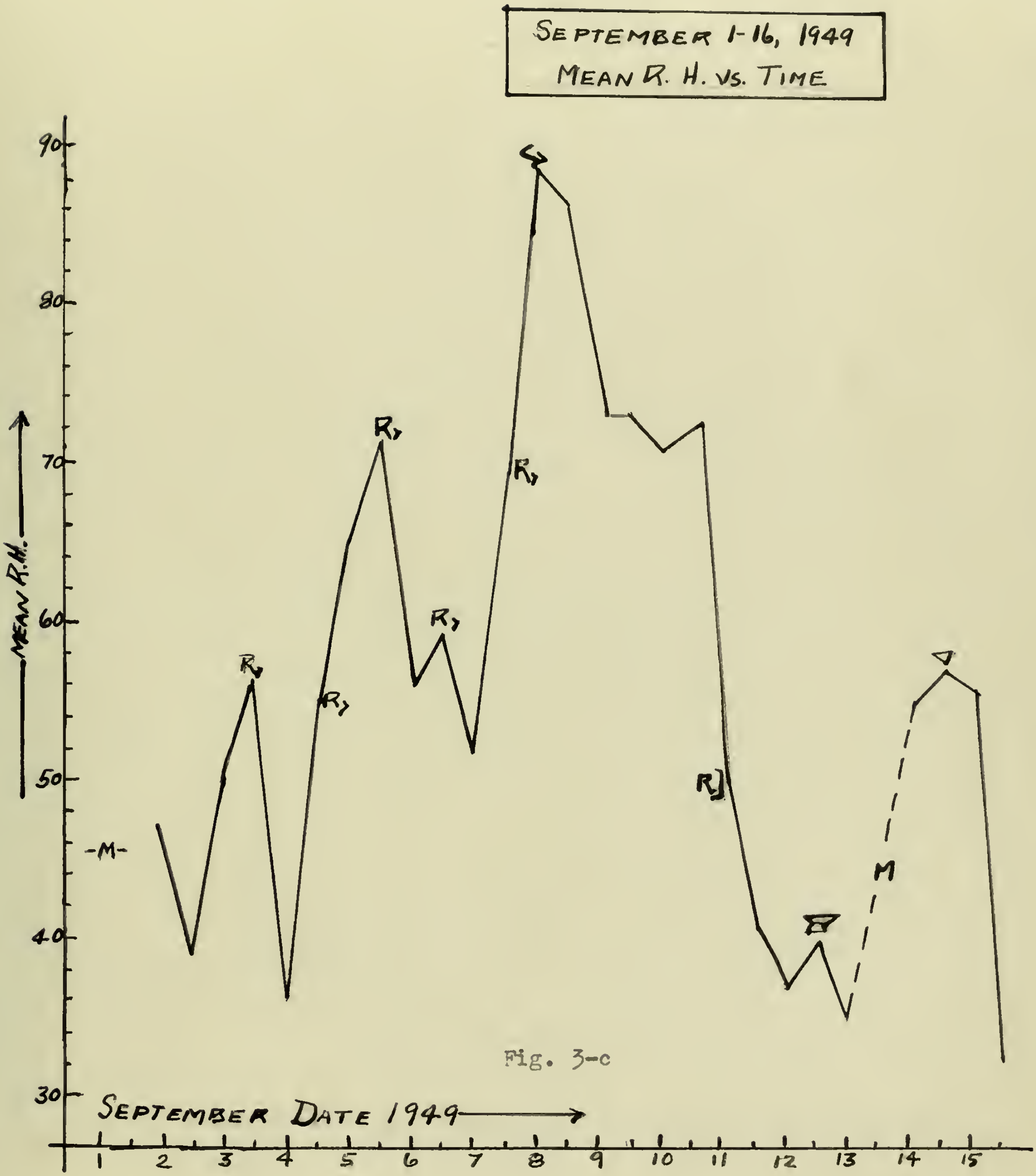
Fig. 3-a



August 16-31, 1949
Mean R. H. vs. Time
Mean R.H.
August Date 1949
80
70
60
50
40
30
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31

Fig.3-b





SEPTEMBER 1-16, 1949
MEAN R. H. VS. TIME
MEAN R.H.
90
80
70
60
50
40
30
-M-
R
M
SEPTEMBER DATE 1949
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

Fig. 3-c

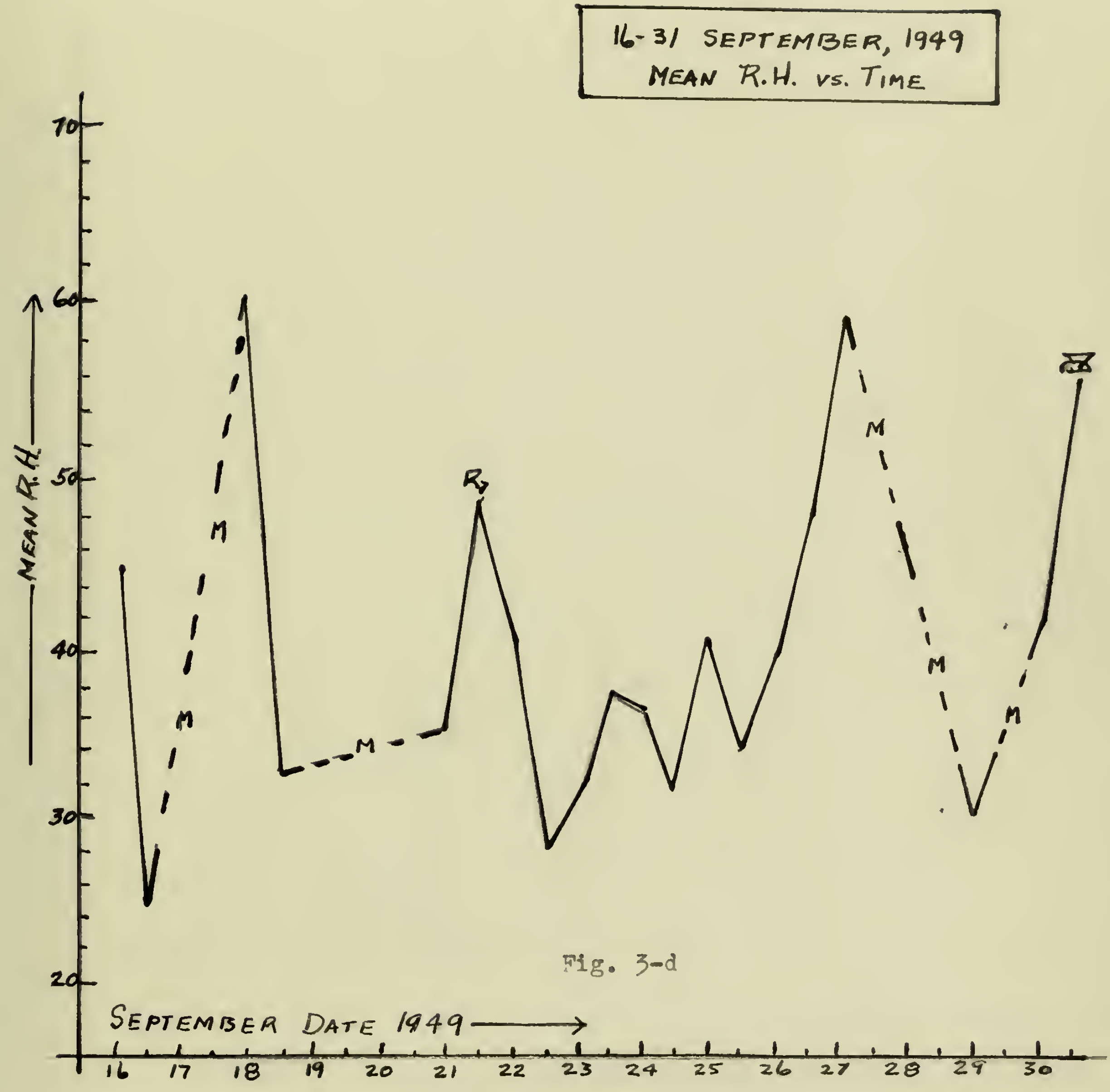
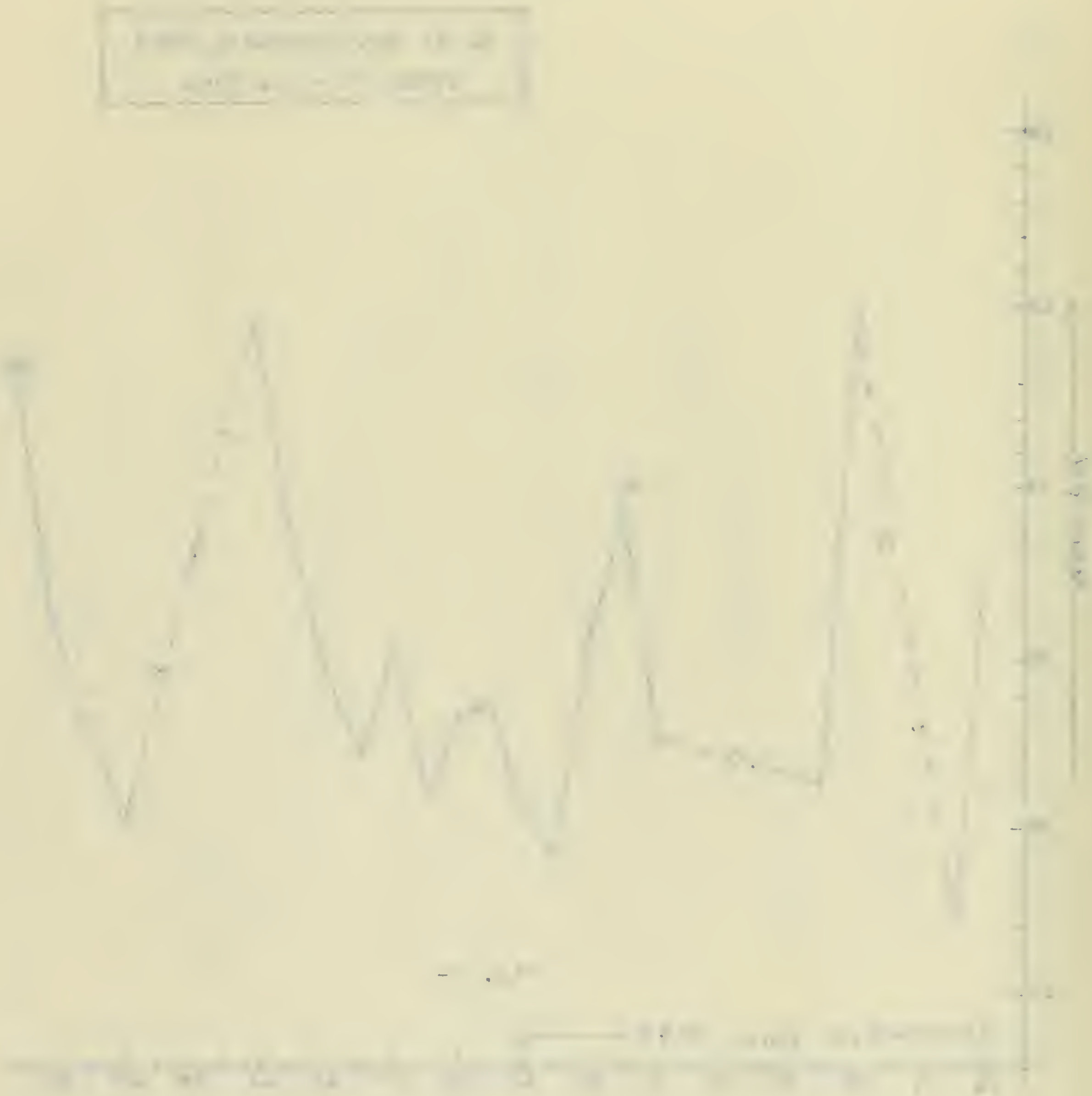
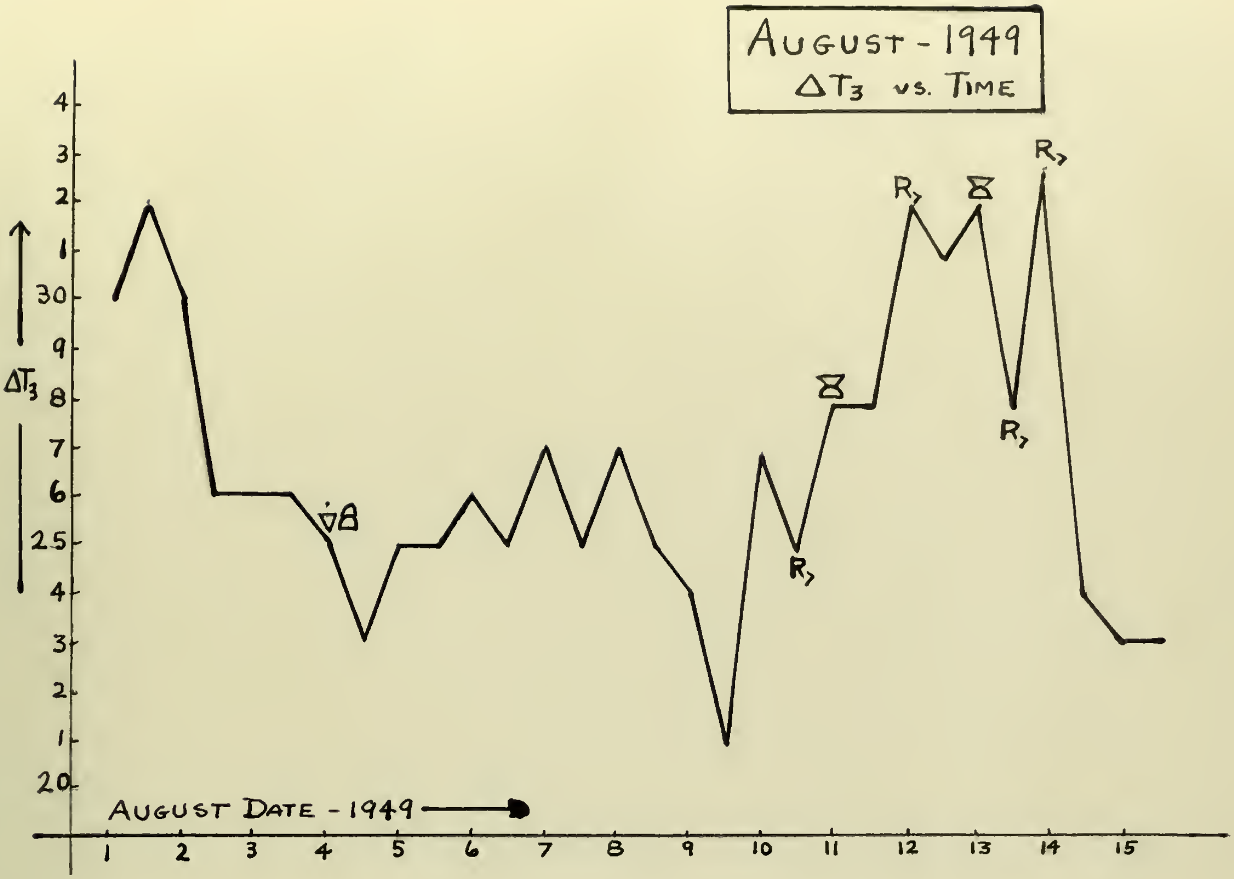

16-31 SEPTEMBER, 1949
MEAN R.H. vs. TIME
MEAN R.H.
70
60
50
40
30
20
M
M
M
R7
M
M
M
SEPTEMBER DATE 1949
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30

Fig. 3-d




AUGUST - 1949
ΔT3 vs. TIME
ΔT3
AUGUST DATE - 1949
4
3
2
1
30
9
8
7
6
25
4
3
2
1
20
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
R7
R7
R7
R7

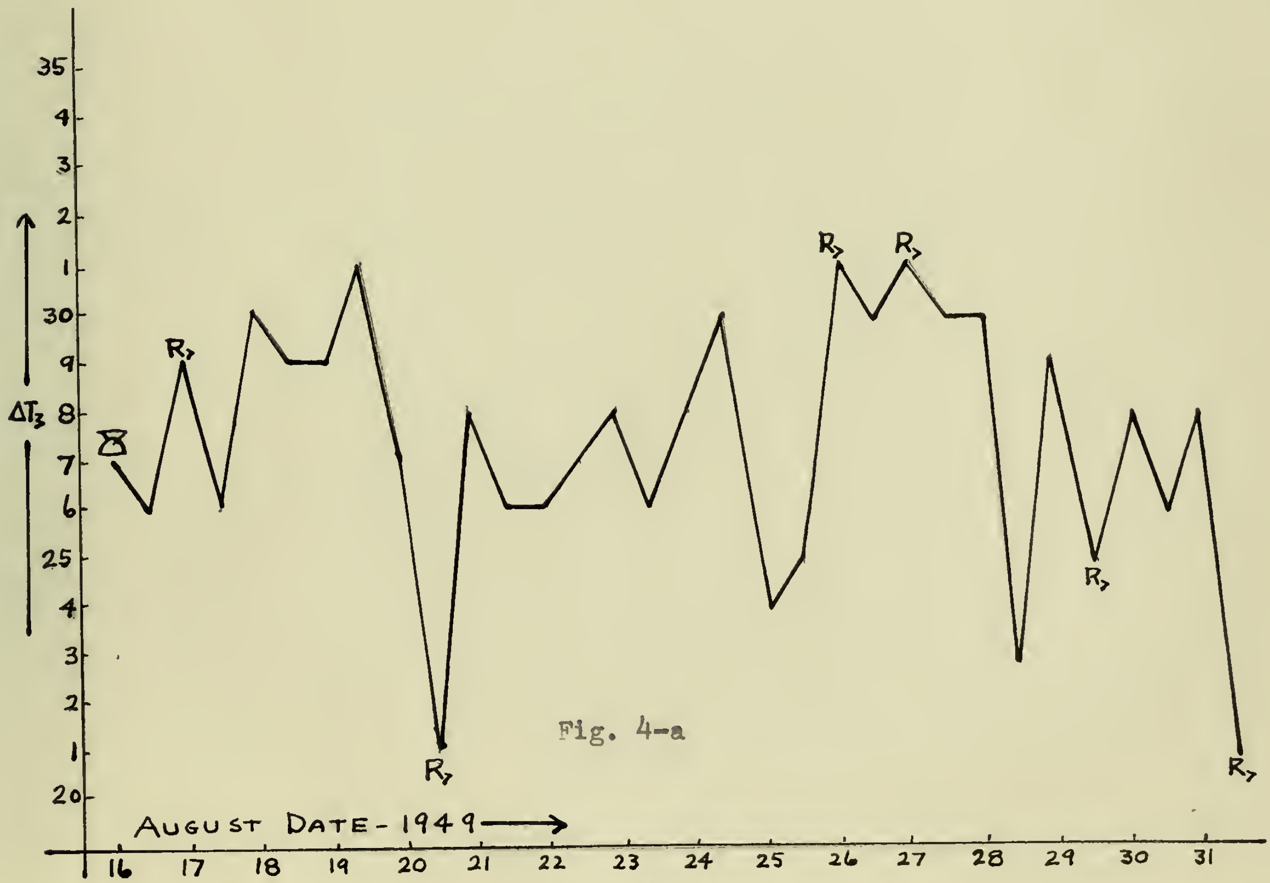

ΔT3
AUGUST DATE - 1949
35
4
3
2
1
30
9
8
7
6
25
4
3
2
1
20
16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31
R7
R7
R7
R7
R7
R7

Fig. 4-a

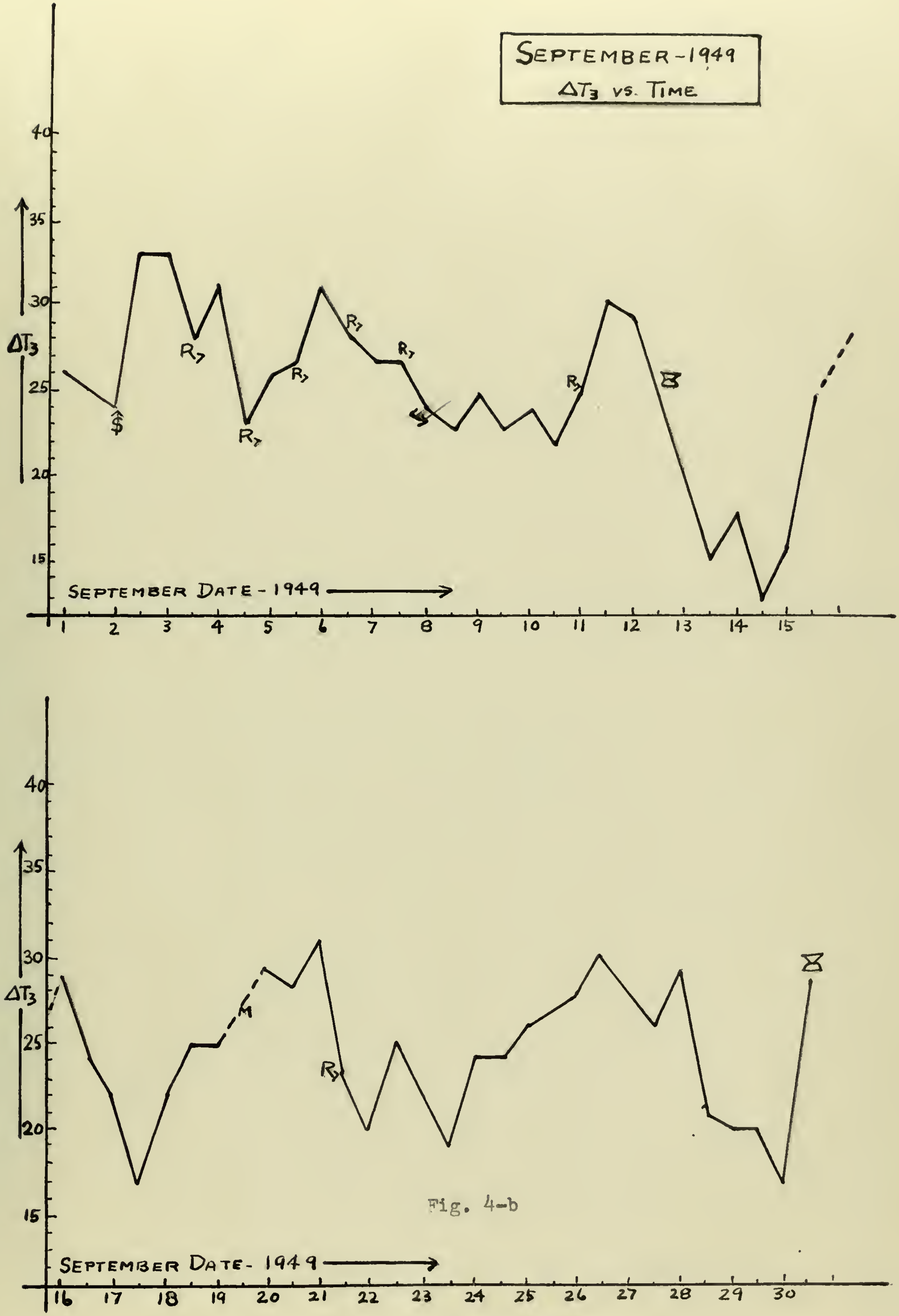
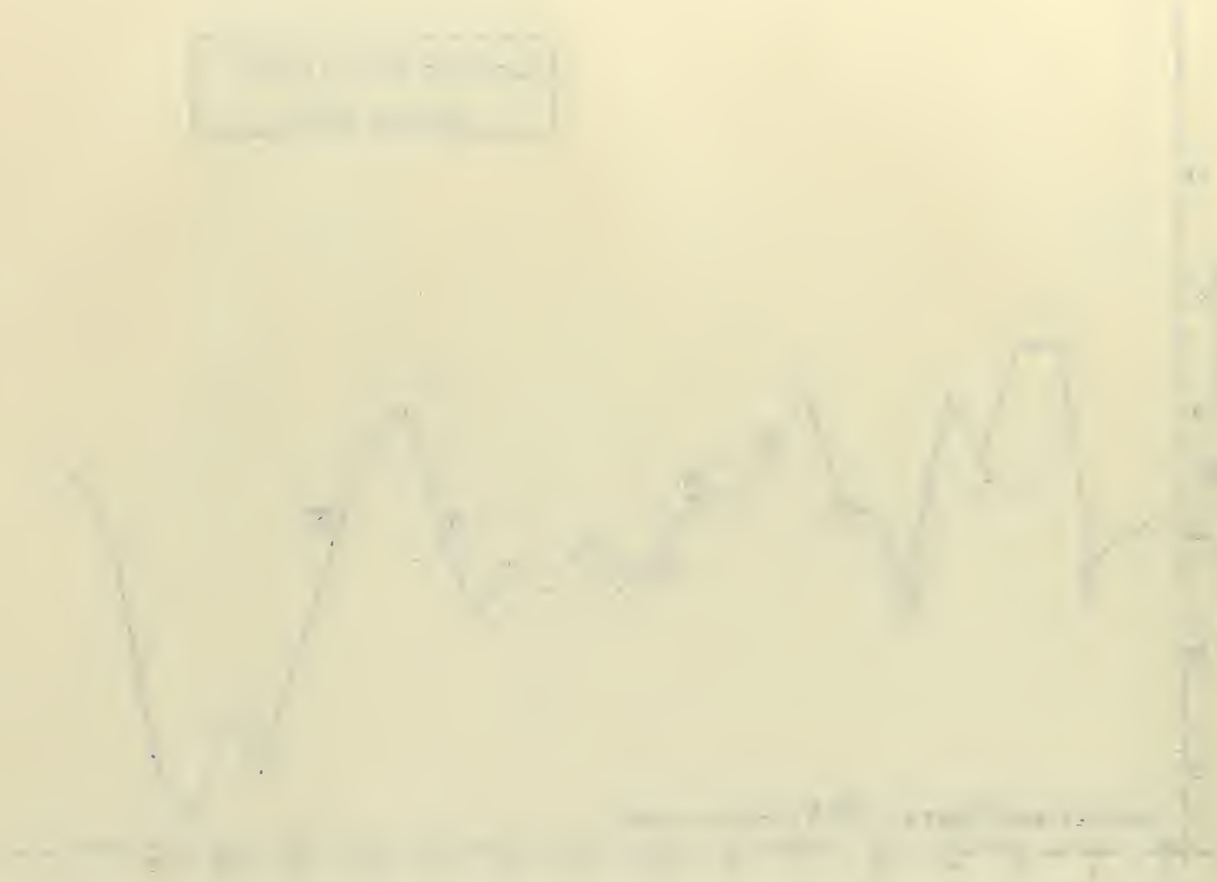

SEPTEMBER-1949
ΔT3 vs. TIME
40
35
30
25
20
15
ΔT3
R7
SEPTEMBER DATE-1949
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
40
35
30
25
20
15
ΔT3
M
R7
SEPTEMBER DATE-1949
16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

Fig. 4-b



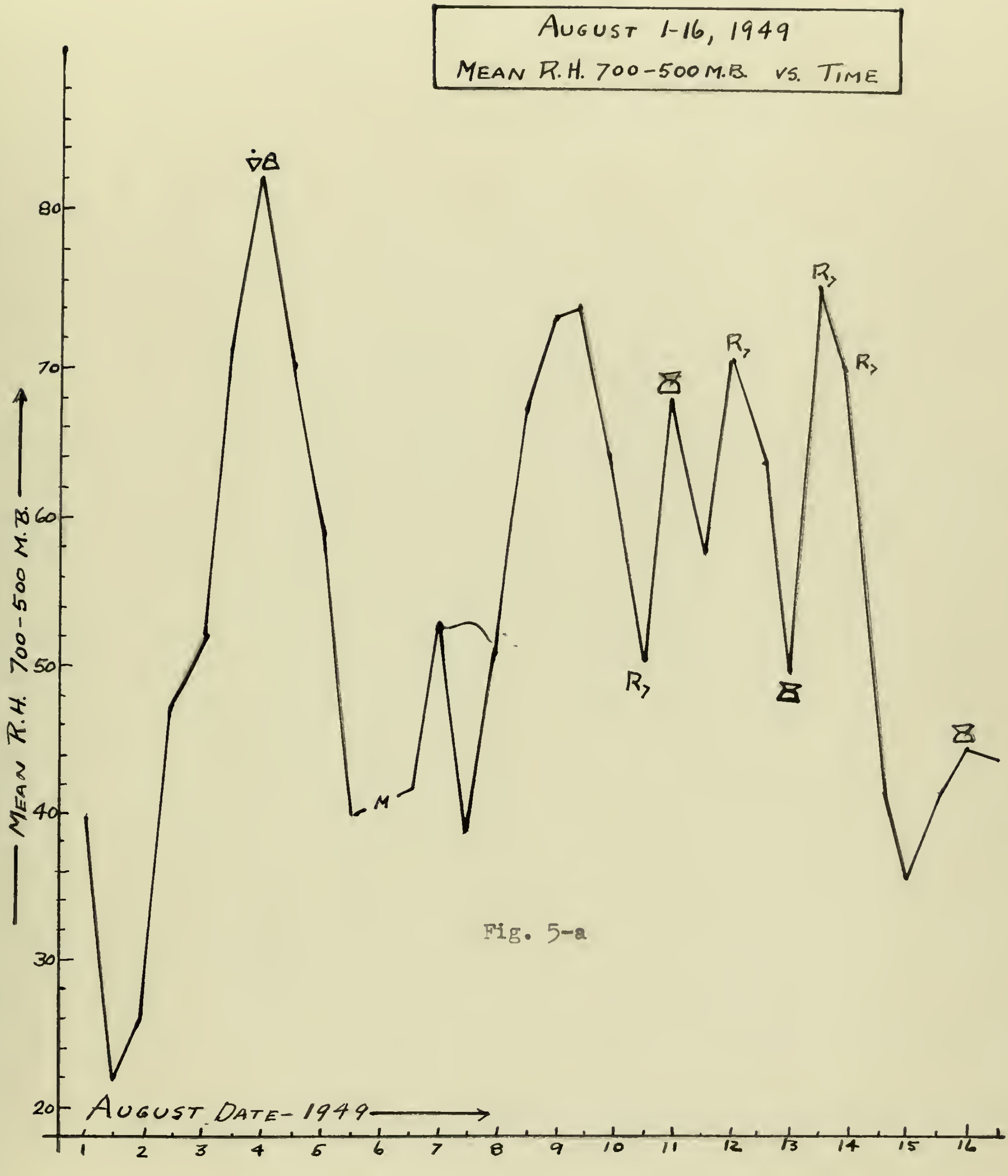
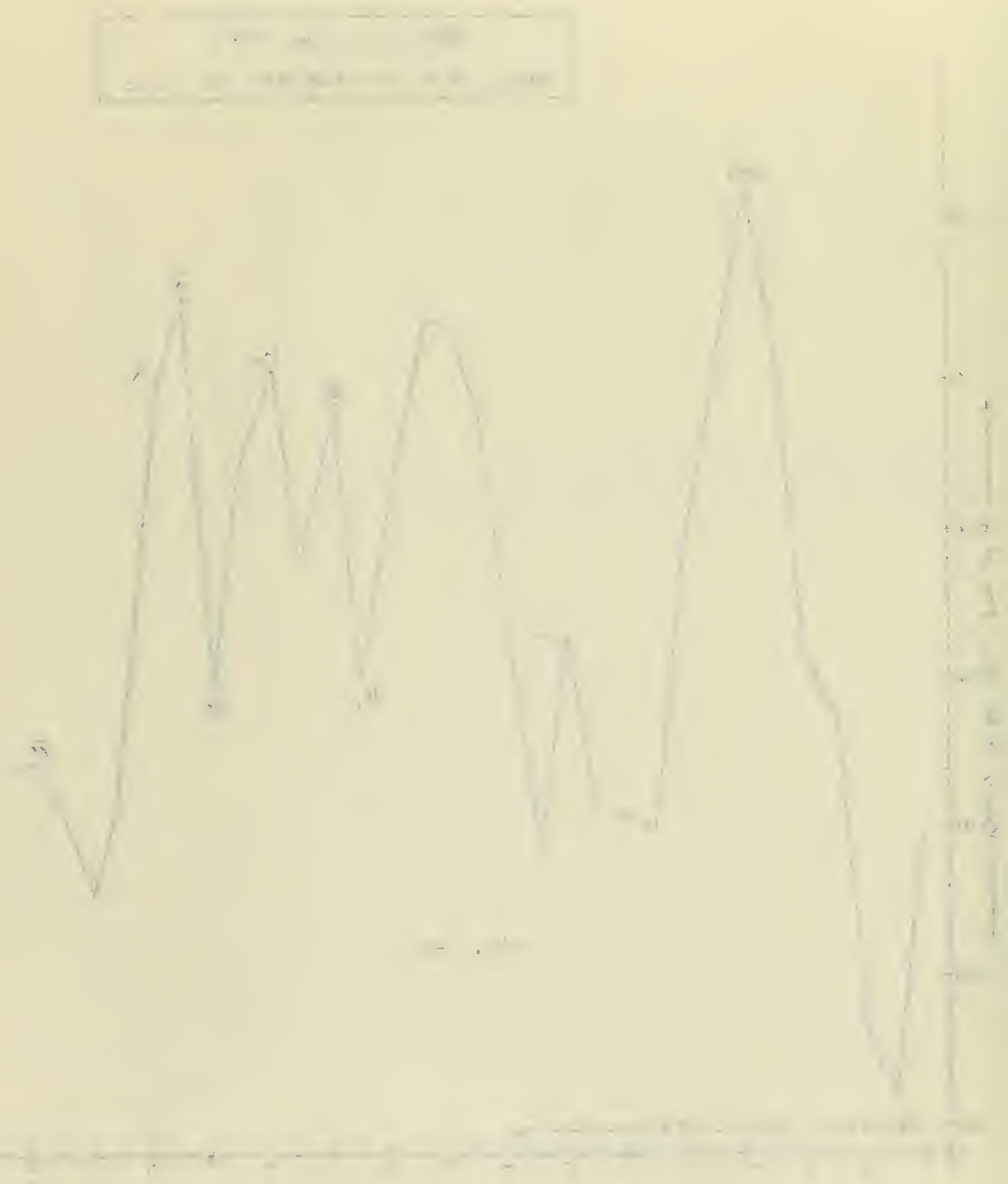

August 1-16, 1949
Mean R.H. 700-500 M.B. vs. Time
Mean R.H. 700-500 M.B.
August Date-1949
80
70
60
50
40
30
20
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
M
R7

Fig. 5-a



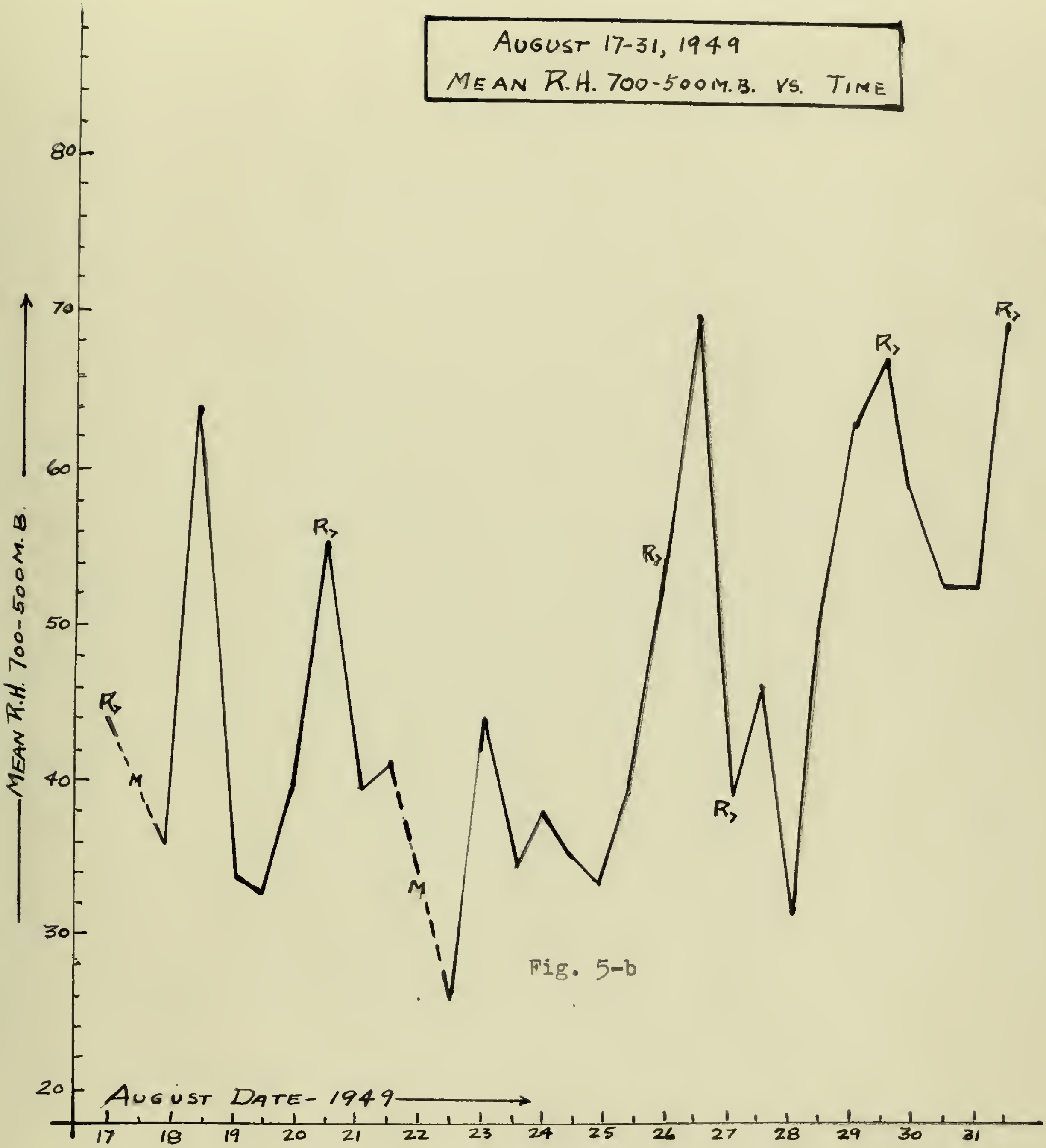

August 17-31, 1949
Mean R.H. 700-500M.B. vs. Time
Mean R.H. 700-500M.B.
80
70
60
50
40
30
20
August Date- 1949
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
R7
M

Fig. 5-b

BIBLIOGRAPHY

1. Austin, James M. A note on cumulus growth in a nonsaturated environment. Journal of Meteorology, 5:103-107, June 1948.

2. Austin, James M. and Fleisher, Aaron. A thermodynamic analysis of cumulus convection. Journal of Meteorology, 5: 240-243, October, 1948.

3. Beers, Norman R. Atmospheric stability and instability. Handbook of Meteorology. Section V:402-409; section X: 693-725. McGraw Hill, New York, 1945.

4. Bjerknes, J. Saturated ascent of air through a dry-adiabatically descending environment. Quarterly J. Roy. Meteorological Society. Vol. 65, 1938.

5. Bjerknes, V., Bjerknes, J., Solberg, H., and Bergeron, T., Physikalische hydrodynamik, Paragraph 715. Springer, Berlin, 1933.

6. Byers, Horace R. and Braham, Roscoe R., Jr. Thunderstorm structure and circulation. Journal of Meteorology. 5:71-86, June 1948.

7. Byers, Horace R. Principal results of a comprehensive investigation of the structure and dynamics of the thunderstorm. Tellus. I:6-17, November, 1949.

8. Cressman, George P. The influence of the field of horizontal divergence on convective cloudiness. Journal of Meteorology, 3:85-88, September, 1946.

9. Jones, D. R. Stability analysis by objective techniques. MS thesis, U. S. Naval Postgraduate School, Annapolis, Md. 1948.

10. Petterssen, S. Weather analysis and forecasting. McGraw Hill, New York, 1940.

11. Stommel, Henry. Entrainment of air into a cumulus cloud. Journal of Meteorology. 4:91-94, June, 1947.

APPENDIX

I

Introduction into the slice method equation of an expression for mass divergence:

The basic assumptions of the slice method are:

(a) Horizontal motion does not maintain any net inflow to or outflow from any stratum determined by the air between significant levels.

(b) Conditions are barotropic initially.

(c) All motions are adiabatic above the surface layers.

In this study, assumption (a), above, was replaced by the assumption that horizontal motion maintains a net inflow of air into the ascending column of air with a corresponding outflow of air from the descending column. Cressman [8] expresses this condition by

(1) $$M v + M' v' = M' \Delta v'$$

where: M = mass of descending air in the unit slice.
M¹ = mass of ascending air in the unit slice.
v and v¹ are corresponding upward components of velocity.
v¹ is the speed which, when multiplied by the mass of ascending air in the unit slice, gives the net rate of upward mass transport.

Beers [3] obtains as the fundamental result of the slice method

(2) $$\dot{c} = R \ln \frac{P_0}{P_1} \left(\frac{\partial T'}{\partial t} - \frac{\partial T}{\partial t} \right) \Delta t$$

Considering the case in which the ascending current is saturated, while the descending current is dry

$$\frac{\partial T'}{\partial t} = v' (\gamma - \gamma_m)$$ in the ascending saturated air.

$$\frac{\partial T}{\partial t} = v\,(\gamma - \gamma_d)$$ in the descending non-saturated air.

Making these substitutions in (2)

(3) $$\dot{c} = R \ln \frac{P_o}{P_1}\left[v'(\gamma - \gamma_m) - v\,(\gamma - \gamma_d)\right]\Delta t$$

but $M v + M' v' = M' \Delta v'$, so that

(4) $$\dot{c} = R \ln \frac{P_o}{P_1}\left[(\gamma - \gamma_m) - \frac{M'}{M}\left(1 - \frac{\Delta v'}{v'}\right)(\gamma - \gamma_d)\right]\Delta t\, v'$$

letting $$K = R \ln \left(\frac{P_o}{P_1}\right) \gamma_d\, v' \Delta t$$

(5) $$\frac{\dot{c}}{K} = \frac{\gamma}{\gamma_d}\left[1 + \frac{M'}{M}\left(1 - \frac{\Delta v'}{v'}\right)\right] - \left[\frac{\gamma_m}{\gamma_d} + \frac{M'}{M}\left(1 - \frac{\Delta v'}{v'}\right)\right]$$

which may be compared with Beers' equation III

$$\frac{C}{K} = \frac{\gamma}{\gamma_d}\left[\left(1 + \frac{M'}{M}\right) - \left(\frac{\gamma_m}{\gamma_d} + \frac{M'}{M}\right)\right]$$

to indicate that the net effect of including in the slice method equations considerations of horizontal divergence is to multiply $\frac{M'}{M}$ by the factor $\left(1 - \frac{\Delta v'}{v'}\right)$.

The entrainment rate of 100% per 500 mb. suggested by Byers [6] may be expressed as $$\frac{M'}{M} = \frac{M_o\,(500 + dp)}{500\,M_o - M_o'\,dp}$$

where: M^1 mass of ascending air at level $p = p_o - d_p$
M mass of descending air at level $p = p_o - d_p$
M_o^1 mass of ascending air at level $p = p_o$
M_o mass of descending air at level $p = p_o$

so that with assumed values of $\frac{M_o'}{M_o}$ a corresponding value of $\frac{M'}{M}$ at any level may be obtained.

was made. This proved of little use because the area term A' does not lend itself to ready evaluation.

In summation, it is apparent that the entrainment process, which can be adequately described graphically, is not subject to a simple, continuous, non-graphical mathematical method capable of rapid evaluation.

Thesis 13131
W94 Wyatt
An objective study of the influence of moisture distribution and lapse rate upon vertical stability.