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# NAVAL POSTGRADUATE SCHOOL Monterey, California 



## THESIS

OCEANIC MIXED LAYER RESPONSE TO TIDAL PERIOD INTERNAL WAVE MOTION
by
Rolf John Burger

June 1982

Thesis Advisor:<br>R. W. Garwood, Jr.

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Tidal period internal waves and mixed layer modelling.

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interaction between vertical mixing and vertical motion depended upon the wave frequency and its phase relation to the diurnal heating cycle. Second, linear and non-linear interactions of the wave induced vertical motion with the cyclical boundary conditions can generate two-dimensional ( $x-z$ ) structure in the near-surface temperature field of an initially horizontally homogeneous ocean under the influence of horizontally homogeneous surface boundary conditions. Finally, this advective interaction increases the utility of the mixed layer model in single station forecasting.

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Oceanic Mixed Layer Response to Tidal Period Internal Wave Motion

by<br>Rolf John Burger<br>Lieutenant, United States Navy<br>B.S., University of North Carolina, 1974

Submitted in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE IN METEOROLOGY AND OCEANOGRAPHY
from the
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June 1982

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## I. INTRODUCTION

In recent years much study of internal waves has focused on the linear and non-linear wave interactions. Models for wave prediction have been developed and tested to match the theory. There have been attempts to measure the internal wave fields in situ. Some of the most useful of these have been the observations of temperature and pressure field variations with depth, providing a basis for computations of the vertical motions and interactions involved. These studies and analyses of data demonstrate the importance of the internal wave field in upper ocean and near surface dynamics.

Roth et al. (1981) presented clear evidence that internal waves are not as critical in an analysis of the dynamics of mixing at depth because they have little variability. However, near the surface, where they interact with other phenomena such as cyclical heat flux, wind stress, and other boundary conditions, their effect can be considerable when dealing with the budgets of heat and momentum. Holloway (1980) pointed out that internal waves are not weak waves (in the sense of weak resonant interaction theory) and have a high impact on mixed layer dynamics down to and including turbulent scales of motion. Pinkel (1981) showed that the near-surface wave field has a profound effect on the vertical velocity field in the upper ocean.

Some mixed layer experiments such as those by Linden (1975) and Kantha et al. (1977) have shown that internal wave radiation may play an important role in controlling the rate of deepening of a turbulent
mixed layer in a stratified fluid. Displacements induced by the turbulence in the base of the mixed layer served to form internal waves which removed kinetic energy of motion from the mixed layer. Since this energy is also important in the entrainment process, these induced internal waves, of relatively high frequency, serve to retard deepening of a mixed layer. Townsend (1968) has derived an expression for the rate of generation of the internal waves and their energy by a steady disturbance field which is advected relative to a stratified layer at a constant speed. The theory was extended by Bell (1978) to cover general space-time dependence of the disturbance field. These works indicate that internal waves could be a means of retarding the growth of mixed layers, through the removal of the kinetic energy in the layers.

Such studies all indicate that the presence of internal waves should be taken into account as a possibly important factor in the mechanical energy budget which controls mixing. It is hypothesized that internal waves which have periods comparable to the dominant time scales of the atmospheric forcing (surface heat flux and wind stress) will be the most likely to result in an interaction with the entrainment mixing process in terms of a positive feedback (increased mixing) effect.

## A. BACKGROUND DATA ANALYSIS

During the period August-September 1977, a mixed layer experiment (iIILE) was conducted at Ocean Station $P\left(55^{\circ} \mathrm{N}, 145^{\circ} \mathrm{W}\right)$ in the North Pacific. The depth of the mixed layer varied between about five and thirty meters during the period of the experiment. Temperature was continuously observed at fixed depths for a period of three weeks. A spectrum analysis of this data (See Figure 10 in later section) shows some important features.

First, a major spectral peak was at about $12.79 \pm .02$ hours, with secondary peaks at $6.62 \pm .13$ and $3.94 \pm .06$ hours. Lower order spectral peaks are at 4.9 hours, and at several frequencies having periods under 3.5 hours. The peak at 12.79 hours is clearly associated with the semi-diurnal tide ( 12.45 hrs .), with the other two peaks at 6.62 and 3.94 hours probably being higher harmonics. At all three depths of the analysis it is clear that the semi-diurnal internal tidal wave, with its harmonics, is the dominant cause of vertical motion at the base of the mixed layer.

Second, the tidal peaks, including the harmonics, are very coherent vertically throughout the depths analyzed. They are present in all three depths analyzed, below the layer, and in the mixed layer, with very little frequency or phase shift.

Finally, the spectral peaks at other than tidal frequencies are not vertically coherent. Also, there is a large variation in non-tidal spectral densities for observations above and below the thirty meter depth-- the top of the seasonal thermocline at this time of year. This suggests that something has happened to the higher frequency waves near the surface making the interpretation of their effect on mixed layer dynamics hard to judge from simple theories. However, at the same time it suggests that the effect of tidal internal waves may be quite easily discerned.

These results confirm the work of Pinkel (1981) whose studies showed that of the wave fields in the upper ocean, the tide and its harmonics have the greatest vertical coherence.

Studies by Simpson and Paulson (1979) demonstrated the variability of the temperature structure in the upper ocean during the POLE experiment.


Shook (1980) showed that these variations. were a response to vertical and horizontal advective processes. In a study of Pacific Ocean ship track temperature data, Butler (1981) found definite cyclical variations in the sea surface temperature structure.

From these studies of actual data, it is obvious that (1) the tidal period internal waves and their harmonics are an important source of energy in the upper ocean, and (2) surface layer temperature patchiness exists in the ocean, and may in part be generated by interaction of mixing and internal wave motion.

## B. ADVECTION IN MODELLING

To date little has been published concerning the interaction of tidal-period internal waves and entrainment by the mixed layer. De Szoeke (1980) has discussed advection in the mixed layer as a result of horizontal variability of the surface wind stress. He showed that advection, upwelling and downwelling, was an important parameter in mixed layer dynamics, resulting from the boundary forcing of the wind stress. Cushman-Roisin (1981), Garwood et al. (1981), and Price (1981) also demonstrated that advection can play an important role in the dynamics of the mixed layer, but the interaction with internal gravity waves is not treated. De Szoeke and Rhines (1981) discussed coastal upwelling at length, but the advective motion was conceived as a necessary factor to achieve a realistic balance for surface heating, transport, and mixing, in order to produce observed coastal temperature fields. Price et al. (1978) included vertical advection in the test of a mixed layer formulation during storm conditions. They found that the phase dependence between wind stress and mixed layer deepening, and the mixed layer deepening itself were reasonably simulated.

Within this discussion of a limited amount of relevant literature on the advective process it has been shown that vertical advection must play an important role in storm response, coastal circulation, and fronts. No one has examined the role of internal wave motion, on the tidal scale, in the open ocean where the geometry and boundary conditions are rather one-dimensional.

In possibly the most relevant previous effort, Spigel (1980) looked at the coupling of internal wave motion and entrainment in a two-layer lake model, having a mixed layer which is based upon the one-dimensional turbulent kinetic energy budget. Time scales of internal wave motion, wave decay, and entrainment were widely separated. Using a seiche effect in a basin, he demonstrated that internal waves did in fact play an important role in the entrainment zone.

With this background it was believed that the time has come for an investigation of the effects of internal vertical motion in the oceanic mixed layer. The motion is there. Observations show that it dominates Eulerian measurements in the upper thermocline. The importance of this motion to mixed layer evolution needs to be established.
C. SCOPE OF THIS STUDY

The scope of this initial study shall be confined to a limited number of wave frequencies. There is insufficient time to concentrate on all the types of internal vertical motion, and all of the possible couplings and interactions in this thesis. The purpose of this initial research was to investigate the effect of dominant tidal-period internal
wave vertical motion on oceanic mixed layer dynamics, and to discern the effect upon the transient thermocline as observed during the mixed layer experiment. The effects of the vertical motion are explored using a one-dimensional bulk model of the mixed layer originally proposed by Garwood $(1976,1977)$ and expanded here to include a prescribed vertical velocity.

The first hypothesis to be examined is can alteration of the existing mixed layer model be reasonably done? Can a relatively simple mathematical transformation be applied to include a prescribed vertical motion without altering the basic physics of the entrainment process? The study will show that this in fact can be done, as revealed by a careful examination of the interactions at each step of the alteration of the original model.

A second hypothesis to be examined is that vertical motion produced by long period internal gravity waves for which the Rossby number is small, through interaction with cyclical boundary conditions (wind stress and heat flux) can produce two-dimensional ( $x-z$ ) structures in the temperature field of a simple infinite ocean having horizontally homogeneous surface boundary conditions. An objective will be to demonstrate the frequency dependence of the internal vertical motion upon the phases of the cyclical heat fields, and the wind stress. It will be shown that there is a non-linear interaction between superimposed diurnal and semi-diurnal internal tidal waves. Finally, this study will show the relative strengths of the resultant two-dimensional structures, and relate these to their importance in forming an improved thermodynamic ocean model.

## II. MIXED LAYER MODEL AND VERTICAL MOTION THEORY

A. MIXED LAYER MODEL

To study the effects of internal vertical motion on mixed layer evolution, the Garwood $(1976,1977)$ mixed layer model was used in a onedimensional form. Garwood developed an ocean mixed layer model using the Navier-Stokes equation of motion with the geostrophic component eliminated, the continuity equation for incompressible water, the heat equation from the first law of thermodynamics, the conservation of salt equation, and a linearized equation of state. The entrainment hypothesis depends upon the relative distribution of turbulent energy between horizontal and vertical components and is offered as a plausible mechanism for governing both entrainment and layer retreat.

Garwood suggested that planetary rotation influences the dissipation of turbulence for deeper mixed layers and enables a cyclical steady state to exist on an annual basis. Furthermore, the rate of entrainment for the stable regime cannot be a simple linear extrapolation of the unstable situations. Unlike the atmospheric boundary case, most of the solar radiation does not penetrate the layer. Therefore, downward turbulent heat flux in the oceanic boundary layer is as important as the upward flux during the course of both diurnal and annual cycles. The non-linearity of the interface entrainment tendency parameter, which is greatest for stable surface boundary conditions, results in a modulation of the long term trend of the mixed layer depth by the diurnal component of surface heat flux. In this model, buoyant production is somewhat more efficient than shear production as a source
of energy for vertical mixing because of its unique effect on the vertical component of the turbulent velocity.

The governing heat equation for this model assumes two forms: one for the mixed layer, the other for below the mixed layer. For the mixed layer, the equation has the form

$$
\begin{equation*}
\frac{\partial\langle T\rangle}{\partial t}=\frac{\left(\overline{\left.w^{\prime} T^{\prime}\right)_{-h}}-\overline{\left(w^{\prime} T^{\prime}\right)_{0}}\right.}{h}+\frac{q g}{\rho} \frac{C_{p} h}{C_{-h} Q d z} \tag{1}
\end{equation*}
$$

where $<>$ denotes the mixed layer average and $Q$ is the net heat flux, a product of downward solar radiation, upward turbulent heat flux to the surface, and surface back radiation. Below the mixed layer the heat equation becomes
where $K$ is the eddy viscosity, and $-\partial(w T) / \partial z$ is the term added for the vertical motion.

The buoyancy flux at the entrainment zone is modelled according to

$$
\begin{equation*}
\operatorname{ag}\left(\overline{\left(w^{\prime} T^{\prime}\right)}-h=-\frac{\left\langle\overline{w^{\prime} w^{\prime}}\right\rangle^{1 / 2}\langle\bar{E}\rangle}{h}\right. \tag{3}
\end{equation*}
$$

which is derived from the local turbulent kinetic energy (TKE) budget where

$$
\begin{equation*}
z=-h \tag{3a}
\end{equation*}
$$

In the entrainment zone, the buoyant damping attributed to ( $\left.W^{\top} T^{\top}\right)_{-h}$ is assumed to be balanced by the convergence of TKE flux. This convergence is modeled as a function of the TKE kinetic energy components and the vertical distance over which this energy must be transported. < $\left.\bar{W}^{\top} W^{\top}\right\rangle$ and $\overline{\bar{E}}>$ are the bulk values of the vertical component and the total TKE,
respectively. The prognostic equations for these mixed-layer turbulance variables are derived using the bulk second-order closure methods of Garwood (1977), and are

$$
\begin{align*}
\frac{\partial}{\partial t}\left(\frac{h\langle\bar{E}\rangle)=m u_{*}^{3}-\frac{a g h\left(\bar{w}^{\top} T^{\top}\right.}{2}-h}{2 R_{1}^{*}}+\right. & +a g h\left[\frac{\left.\left(\bar{w}^{\top}\right)^{\top}\right)_{-h}+\left(\overline{w^{\top}{ }^{\top}}\right)}{2}\right] \\
& -(\langle\bar{E}>\mathbb{1 R}+f h)\langle\bar{E}\rangle \tag{4}
\end{align*}
$$

$$
\begin{align*}
\frac{\partial}{\partial t} \frac{\left(h\left\langle\overline{w^{\prime} w^{\prime}}\right\rangle\right)}{2}= & \operatorname{agh}\left[\frac{\left.\left(\bar{w}^{\prime} T^{\prime}\right)_{-h}+\overline{\left(w^{\prime} T^{\prime}\right)_{0}}\right]+}{2}\right. \\
& \left(\langle\bar{E}\rangle-3\left\langle\overline{\left.w^{\prime} w^{\prime}\right\rangle}\right\rangle\langle\bar{E}\rangle 1 / 2-\frac{(\langle\bar{E}\rangle 12+f h)\langle\bar{E}\rangle}{3}\right. \tag{5}
\end{align*}
$$

where

$$
u_{\star}=\left(\tau / p_{0}\right)^{1 / 2}
$$

with $\tau$ being the magnitude of the surface wind stress and $\rho_{0}$ is the air density, assumed to be a constant, and

$$
R_{1}^{*}=\frac{a g h \Delta T}{\left(\Delta u^{2}+\Delta v^{2}\right)}
$$

is the bulk Richardson number. The change in the total TKE, $\langle\bar{E}\rangle$, is a function of wind stress shear production, entrainment shear production, bouyant production/damping, and dissipation. The change in the vertical component, $\left\langle\overline{\left.W^{\top} W^{\top}\right\rangle}\right.$, is a function of bouyant production/damping, pressure redistribution, and dissipation. The quantities $\Delta T, \Delta u$, and $\Delta v$ are the jump values between the mixed layer (assumed to be vertically uniform), and the underlying stable waters. These bulk TKE equations are solved algebraically assuming quasi-steady state for the TKE budgets

$$
\frac{\partial}{\partial \dagger}(h<\bar{E}>)=0
$$

and

$$
\frac{\partial}{\partial t}\left(h\left\langle w^{\prime} w^{\prime}\right\rangle\right)=0
$$

With this assumption the computed values of $\left\langle\bar{W}^{\top} W^{\top}\right\rangle$ and $\langle\bar{E}\rangle$ are in phase with the surface boundary conditions--the wind stress, $p_{0} u_{*}^{2}$, and the buoyancy flux, $a g\left(\overline{W^{\top} T}\right)_{0}$.

Once the entrainment bouyancy flux is known from (3), the downward fluxes of momentum associated with entrainment at the base of the mixed layer, $\left(\overline{w^{\top} u^{\top}}\right)_{-h}$ and $\left(\overline{w^{\top} v^{\top}}\right)_{-i l}$ are given by

$$
\begin{align*}
-\left(\overline{w^{\prime} u^{\prime}}\right)_{-h} & =w_{e} \Delta u=-\left(\overline{w^{\prime} T^{\prime}}\right)_{-h} \frac{\Delta u}{\Delta T}  \tag{6}\\
-\left(\overline{\left(w^{\prime} v^{\prime}\right)_{-h}}\right. & =w_{e} \Delta v=-\left(\overline{\left.w^{\prime} T^{\prime}\right)_{-h} \frac{\Delta v}{\Delta T}}\right. \tag{7}
\end{align*}
$$

Here the entrainment velocity is

$$
\begin{equation*}
w_{e}=-\frac{\left(w^{\prime} T^{\prime}\right)_{-h}}{\Delta T} \tag{3}
\end{equation*}
$$

and the entrainment velocity is also defined as

$$
\begin{equation*}
w_{e}=\frac{d h}{d t}+w_{-h} \tag{9}
\end{equation*}
$$

if $\mathrm{w}_{2}$ is greater than zero. If $\mathrm{w}_{\mathrm{e}}$ is not greater than zero, equations (10) and (11) are applied.

## 1. Boundary Layer Deepening by Entrainment

The entrainment heat flux ( $\left.\bar{W}^{\top} T^{T}\right)_{-h}$ is determined algebraically from (3), and the steady-state forms of (4) and (5) in terms of given values of $u_{*},\left(\overline{w^{\top} T}\right)_{0}, h$ and $R_{i}^{*}$. This heat flux is then applied to the given temperature profile. The new $h$ and new momentum and temperature

profiles are unique solutions provided that (i) the mixed layer is homogeneous, (ii) the value of $h$ is just large enough to prevent unstable density profiles ( $\partial \mathrm{T} / \partial \mathrm{z}$ greater than zero), and (iii) heat is conserved.

## 2. Boundary Layer Shallowing

Shallowing occurs when there is adequate vertical turbulent energy to transport heat down to the base of the existing mixed layer.
 the new layer boundary, and will be the base of the new mixed layer. In this model, this occurs when ( $\overline{W^{\top} W^{\top} \text { ) approaches zero. Then (3) is }}$ no longer applicable, and the steady state forms of (4) and (5) reduce to

$$
\begin{align*}
& 0=m u_{*}^{3}-\left(\langle\bar{E}\rangle^{1 / 2}+f h\right)\langle\bar{E}\rangle+\frac{a g h\left(w^{\top} T^{\top}\right)_{0}}{2} 0  \tag{10}\\
& 0=\langle\bar{E}\rangle^{3 / 2}-\frac{\left(\langle\bar{E}\rangle^{1 / 2}+f h\right)\langle\bar{E}\rangle}{3}+\frac{\operatorname{agh}\left(\bar{w}^{\top} T^{\top}\right)_{0}}{2} 0 \tag{11}
\end{align*}
$$

Without the planetary rotation term, or if the downward surface buoyancy flux is dominant, $h$ is proportional to the Obukhov length scale

$$
L=u_{*}^{3} / a g\left(w^{\prime} T^{\prime}\right)_{0}
$$

If, on the other hand, ( $\left.\bar{w}^{\top} T^{\top}\right)_{0}=0, h$ is proportional to $u_{*} / f$. In general, however, the depth of the shallowing mixed layer is a function of the two non-dimensional parameters, $h / L$ and $h f / u_{*}$.

Heat and momentum are conserved in the profile specifications and, because the mechanical energy budget requires conservation of potential energy on the time scale of the turbulence, a numerical procedure is used that also preserves potential energy. This is
intended to ensure that the deepening rate is as correct as possible when the mixed layer deepens to its former position.
B. VERTICAL MOTION THEORY

By following a water particle as it is vertically advected by a prescribed velocity field, $w(x, z, t)$, the distortion of any temperature profile, $T(z)$, after a small, but finite length of time, $(\Delta t)$, may be computed by non-linear theory.

If we assume

$$
\begin{equation*}
w=\frac{-A z}{D} \cos (\omega t+k x) \tag{12}
\end{equation*}
$$

is prescribed, then

$$
\begin{equation*}
\frac{d z}{d t}=\frac{-A z \cos (\omega t+k x)}{D} \tag{13}
\end{equation*}
$$

This may be reformed and integrated

$$
\int_{z_{1}}^{z_{2}} \frac{d_{2}}{z}=\int_{t_{1}}^{t_{2}}-\frac{A}{D} \cos (\omega t+k x) d t
$$

where a water particle starting at time $t_{1}$ at position $z_{1}$ is advected to position $z_{2}$ at time $t_{2}$. Or following any constant isotherm initially at position $z_{1}$, to $z_{2}$ at $t_{2}$ gives

$$
\begin{equation*}
z_{2}=z_{1} \exp \left\{\frac{A}{\omega D}\left[\sin \left(\omega t_{2}+k x\right)-\sin \left(\omega t_{1}+k x\right)\right]\right\} \tag{14}
\end{equation*}
$$

If $t=t_{2}-t_{1} \ll \omega^{-1}$, then

$$
\begin{equation*}
z_{2} \cong z_{1} \exp \left\{\frac{\Delta \Delta t}{D}\left[\cos \left(\omega\left(t_{1}+\frac{t}{2}\right)+k x\right)\right]\right\} \tag{15}
\end{equation*}
$$

or

$$
\begin{equation*}
z_{2} \cong z_{1} e^{R} \tag{16}
\end{equation*}
$$

电
which is the general form approximated by the progression of equations in this study of internal vertical motion. Internal vertical motion is applied to the mixed layer model through equation (10). dh/dt, due to the internal vertical advection, $W_{-h}$, is derived from equation (16).

For future study, the salinity budget can be handled in a like manner.
促

## III. EXPERIMENTS

## A. PRODUCTS

The products used to identify features in the model runs and used for comparison are:

1. Table (Sample Figure la) showing the hourly change in the mixed layer primary variables:
$H$ is the mixed layer depth in meters.
$T$ is the mixed layer temperature in degrees centigrade.
$S$ is the mixed layer salinity in parts per thousand.
TAU is the surface wind stress in dynes per square centimeter.
QNET is the net heat flux out of the mixed layer in watts per square meter.

WDD is the value of the vertical motion parameter affecting the model in centimeters per second.

DH is the change in mixed layer depth due to the vertical motion for that time period.
2. Contours of the temperature variation, $T(z, t)$, (Sample Figure 1) between $z=0$ and $z=-60$ meters over the initial 24 hours of the model run. Varying resolution is used to better resolve the evaluation of $T(z, t)$. Temperature contours are in degrees centigrade.
3. Contours of temperature variation similar to 2 , in a quasi twodimensional mode. This is accomplished by running the model 24 times, each time passing the prescribed vertical motion through the 24 hour cycle of the model at a different, progressive phase. The 24 runs are then sorted by hour giving 24 data sets of 24 independent points with a spatial separation of $k x$ from hour one through hour twenty-four.

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It is assumed that there is a two-dimensional internal wave in $x$ and $z$ passing through this ocean system. Heat flux and wind stress are taken to be the same at all points, with no interaction between any of the points (advection is limited to the vertical).
4. Progressive temperature profiles, $T(z)$, to a depth of 60 meters (Sample Figure lc). Only the first profile matches the horizontal temperature scale which is degrees centigrade. The remaining profiles have been consecutively offset. The accompanying table has the mixed layer temperatures in degrees centigrade, and the mixed layer depth in meters.
5. Progressive temperature profiles as in 4 , but in the twodimensional mode of 3 . Profiles are offset from the first, again, to provide an insight to the two-dimensional nature to the data.

## B. STANDARDS

Prior to the examination of the addition of vertical motion to the mixed layer model (MLM), it is beneficial, here, to examine the results of running the mixed layer model in the two modes which will be used as the standards for comparison with the vertical motion modelling experiments. Both standard cases are runs of the mixed layer model from which the effects of evaporation and precipitation have been removed. This has been done to aid in the interpretation of later results by limiting the number of interaction variables to a manageable quantity. The first standard is a run of the MLiM in the primarily winddriven deepening mode. The second standard is for the MLM in a cyclical shallowing/deepening response to a strong downward heat flux.

1. Standard 1 (SI)

This run was initialized with a temperature profile having a surface and mixed layer temperature of $15.0^{\circ} \mathrm{C}$. At 5.0 m the initial temperature decreases linearly with the form

$$
\begin{equation*}
T(z)=T_{1}-5.0 \times 10^{-4}(Z-H) \tag{17}
\end{equation*}
$$

Where $Z$ is the absolute value of $z$ in centimeters, and $H$ is the mixed layer depth in centimeters. This profile extends to 200 m . The resulting profile is shown in Figure 2.

The surface wind stress (TAU) is a constant 0.50 dynes per square centimeter for the entire run. Net heating reaches a maximum of 302.5 watts per square meter at hour six (local noon). Net heat loss after sunset is a constant 100.8 watts per square meter.

With no vertical motion the layer deepens continuously over the two-day period of the model run (Figure 1), as expected for primarily wind-driven mixing. In examining the first twenty-four hour period it is seen that the mixed layer deepens from an initial depth of 4.5 m to a depth of 14.7 m . The layer cools from $15^{\circ} \mathrm{C}$ to $14.767^{\circ} \mathrm{C}$ over the period, with a short period of mixed layer temperature increase of six hours related to that time when the net heat flux is sufficient to overcome the cooling attributed to the entrainment of cooler water into the layer.

The salinity in the layer remains constant because there is no precipitation or evaporation allowed which could change the surface salinity, and the below layer salinity is assumed to be a constant 34.00 part per thousand. Therefore, there are no sources of water with a salinity other than 34.00 parts per thousand.


Figure 2. Initial temperature and salinity profiles for Case S1.

The downward solar radiation, $Q_{s}$, increases from sunrise (hour 1) to noon (hour 6), and then decreases to zero at sunset (hour 12.) A constant is assumed for the net upward turbulent heat flux to the atmosphere, plus back radiation of 100.8 watts per square meter for the entire day.

The one-day response to this cycle can be seen in the contours (Figure 1b). The mixed layer deepens continuously over the first 24 hours. Below the layer temperature, at any given depth not penetrated as the layer deepens, remains constant over time. In the layer the temperature is seen to cool slightly initially, then warm to a maximum value at about hour nine ( 1500 local time), and then cool again for the remainder of the period.

In the $T(z)$ graph this sequence is seen again (Figure lc). Here it is seen that all of the slopes of the profiles below the layer are the same from hour one to twenty-four, indicating that with no vertical motion, the characteristics below the layer will not change with time. The included table shows the relationship of the temperature, in degrees centigrade, and the layer depth, in meters, and indicates the rate and magnitude of the deepening.

The rate of deepening (Table l) reflects the surface boundary conditions at any given time as well as layer depth and stratification below. As the run begins, the layer deepens quickly in response to the surface wind stress and the short distance over which it must act, as seen in equations (3) and (4). At hour one the layer deepens beginning at a shallow value where the wind flux has a high impact from equation (4), during a period where the heat flux into the layer
is a minimum. As the heat flux increases and the layer deepens the wind stress is relatively less effective in deepening, and the entrainment now effected by equation (5) slows until hour nine when the solar radiation goes to zero, and the deepening rate increases. This increase is relatively small because of the relatively large temperature jump $(\Delta T)$ in the entrainment zone by this time. After hour eleven ( 1800 local time) the minor heat loss, small wind stress and increased depth involved all act to slowly decrease the deepening rate.

Table 1. Deepening Rate in Meters Per Hour for Case S1

| Hour | Deepening <br> Rate | Hour | Deepening <br> Rate |
| :---: | :---: | :---: | :---: |
| 1 | 1.7613 | 13 | 0.3296 |
| 2 | 1.0227 | 14 | 0.3376 |
| 3 | 0.7271 | 15 | 0.3287 |
| 4 | 0.4877 | 17 | 0.3017 |
| 5 | 0.4242 | 18 | 0.3083 |
| 6 | 0.3221 | 19 | 0.3115 |
| 7 | 0.2941 | 20 | 0.2759 |
| 8 | 0.2964 | 21 | 0.2872 |
| 9 | 0.2904 | 22 | 0.2703 |
| 10 | 0.2917 | 23 | 0.2560 |
| 11 | 0.3291 | 0.2607 |  |
| 12 | 0.3580 |  |  |

2. Standard 2 (S2)

This run is begun with the same initial layer temperature as for standard 1. Here, however, the initial mixed layer depth is taken to be 49.5 meters. At 50.0 meters, the profile again assumes the linear variability of equation (18), extending to 200.0 meters (Figure 3).

All other parameters are the same as those of Sl except for the prescribed surface heat flux. The net heat flux has increased to


Figure 3. Initial temperature and salinity profiles for Case S2.
电
a maximum of 504.2 watts per square meter by assuming an extremely strong solar heating maximum $\left(Q_{S}\right)$ of 600 watts per square meter at local noon.

In this case the layer depth remains at 49.5 meters for the first hour (Figure 21a) because the wind stress is not strong enough, coupled with the available heat energy, to cause further entrainment at this depth, and the solar heating is not yet strong enough to cause ( $\overline{W^{\top} W^{\top}}$ ) to go to zero so that equations (10) and (11) are applied. By the second hour the net downward surface heat flux and the surface wind stress are sufficient to cause ( $\overline{W^{\top} W^{\top}}$ ) to go to zero, and a new $h$ is formulated through the solution of equations (10) and (11), forming a new surface mixed layer with a depth of about 18 m . The layer then continues to shallow until hour 6 (local noon) in response to the increased downward heat flux. Stirring by the wind stress is relatively small as its effect is overcome by the buoyancy flux associated with net surface heat flux. The layer temperature increases as long as there is heating sufficient to overcome the minor cooling due to the entrainment of cooler below layer water which resumes at local noon, but is not strong until hour 10 (1600 local time). At that time the layer cools due to the combined effects of the surface heat loss, and the entrainment of the cooler below layer waters.

The salinity, again, remains constant because there is no source of water of a salinity other than 34.00 parts per thousand. As shown by the contours (Figure 11b) the original layer of 50 meters shallows to hour 6 , not forming a distinct layer, since equations (10) and (11) are applied at each hourly step, until hour 7. The transient thermocline prior to hour 7 is weak. This is seen more
clearly, perhaps, on the $T(z)$ profiles (Figure $11 c$ ). The originally sharp interface at 50 meters becomes diffuse with time due to small $\left(K_{T}=0.2 \mathrm{~cm}^{2} / \mathrm{s}\right)$ below-layer diffusion. This is clearly seen in the $T(z)$ profiles, and it is the reason for the non-horizontal contours near the 50 m depth. The variability of the temperature structure below the newly formed surface mixed layer is a function of the shallowing adjustment by equations (10) and (11) as applied from hour two through six.

Again it is evident in the $T(z)$ profiles that the profiles keep the same slope below the layer except for the originally sharp interface at 50 meters where diffusion of the layer occurs with time. This diffusion area is also seen to hold at a relatively constant depth with time.

The $T(z, t)$ evolution reflects the turbulent kinetic energy budget for this problem (Table 2). The initial decrease in $h$ from a deep layer to a shallow layer is clearly the result of the surface heating and the surface wind stress acting to form a new, warmer, shallower surface layer. Energy is increased in the surface mixed layer due to excessive solar insolation during the day. The Obukhov length is reduced from hour two through six, and the newly forming mixed layer adjusts accordingly. By hour seven the solar insolation begins to reduce. As this happens, the Obukhov length increases, allowing the newly formed layer to deepen. Deepening does not keep up with the Obukhov length because of the damping due to the energy transport which must take place during the mixing process. By hour 11 the net heat flux is negligible decreasing to a constant surface
loss after sunset. The relative magnitude of the depth increase ( $\partial \mathrm{h} / \partial \mathrm{t}$ ) with time when compared with that of Sl is due to the strength of the temperature jump ( $\Delta T$ ) at the layer, and the available energy in the layer. In SI a $\Delta T$ of the order of $.4^{\circ} \mathrm{C}$ was encountered, while in S2, due to the slow formation of the new surface layer, a $\Delta T$ of the order $.08^{\circ} \mathrm{C}$ is the maximum encountered. This allows for more rapid deepening. The rate of deepening increases after hour 16 (2200 local time) because, as seen in the $T(z)$ profiles (Figure llc) the $\Delta T$ at the entrainment zone is decreasing with time due to the net upward surface heat flux.

Table 2. Deepening Rate in Meters Per Hour for Case S2.

| Hour | Deepening <br> Rate | Hour | Deepening <br> Rate |
| ---: | ---: | ---: | :---: |
| 1 | 0.0152 | 13 | 0.8464 |
| 2 | -31.4079 | 14 | 0.8317 |
| 3 | -1.9247 | 15 | 0.3252 |
| 4 | -3.1646 | 16 | 0.8235 |
| 5 | -0.9575 | 17 | 0.8278 |
| 6 | -0.2935 | 18 | 0.8404 |
| 7 | 0.3766 | 19 | 0.8596 |
| 8 | 0.3445 | 20 | 0.8879 |
| 9 | 0.4944 | 21 | 0.9186 |
| 10 | 0.5383 | 22 | 0.9516 |
| 11 | 0.6793 | 23 | 0.9870 |
| 12 | 0.8371 | 24 | 1.0256 |

## C. EXPERIMENTS

The addition here to the existing theory of Garwood (1976-77) is approached in a step-by-step procedure, gradually increasing the complexity of the problem. A quick reference for this progression can be seen in Table 3. A basic assumption that was made in all of the

Table 3. Initial conditions, boundary conditions, and progression of vertical motion inputs for all experiments.

## For all Cases:

1. Net surface heat loss is constant at 100.8 watts/m ${ }^{2}$.
2. Surface wind stress is constant at 0.5 dynes $/ \mathrm{cm}^{2}$.
3. The initial layer temperature is $15.0^{\circ} \mathrm{C}$.
4. The salinity is constant to 200 m at $34.00 \%$.
5. No precipitation or evaporation is allowed during the run.
6. The vertical motion is exponentially decreasing to the surface. Maximum motion is held at $36.0 \mathrm{~cm} / \mathrm{hr}$ at 10 m , except for Case IVb , where it is $18.0 \mathrm{~cm} / \mathrm{hr}$ at 10 m.

| Case | $\begin{gathered} \text { B. C. } \\ Q_{s} \max . \\ \text { (watts/m} \left.)^{2}\right) \end{gathered}$ | $\begin{aligned} & \text { I. } \\ & \text { Temp. } \\ & \text { Profile } \end{aligned}$ | $\begin{gathered} \text { MLD } \\ \text { (III) } \end{gathered}$ | Vertical Motion |
| :---: | :---: | :---: | :---: | :---: |
| S1 | 400.00 | Fig. 2 | 4.5 | None |
| S2 | 600.00 | Fig. 3 | 49.5 | - None |
| Ia | S1 | S1 | S1 |  |
| Ib | S2 | S2 | S2 | Constant downwelling during model run. |
| Ic | S1 | S1 | Sl |  |
| Id | S2 | S2 | S2 | Constant upwelling during model run. |
| IIa | S1 | S1 | S1 | Pulsed downwelling, 12 hour cycle. Phase shifted to peak at hours 5,18 , and 24. |
| IIb | S2 | S2 | S2 |  |
| IIc | S1 | S1 | S1 | Pulsed upwelling, 12 hour cycle. Phase shifted to peak at hours 5,18 , and 24. |
| IId | S2 | S2 | S2 |  |
| IIIa | S1 | S1 | S1 | Single diurnal wave passage. Phase shifted to give maximum downward motion at all hours. |
| IIIb | S2 | S2 | S2 | Hours 1, 7, 13, and 19 discussed. |
| IVa | S2 | S2 | S2 | Continuious diurnal wave system passage. Same phase shifts as Case III. Hours 6, 12, 18, and 24 discussed. |
| IVb | S2 | S2 | S2 | Contimuous semi-diurnal wave system passage. Phase shifted as in Case III. Runs with peak motions at hours 6 and 18 , and 12 and 24 are discussed. |
| Va | S2 | S2 | S2 | Superimposed diurnal and semi-diurnal continuous wave system passage. Both waves in phase. Input phase shifted as in Case III. |
| Vb | S2 | S2 | S2 | As in Case Va, but phases of diurnal and semidiurnal waves varied relative to one another. |


work was that the vertical motion moved the water up and down, but did not directly induce turbulence (i.e., mixing). This is a valid assumption if no shear is associated with this motion.

The cases of progressively complicated vertical motion examined were:

1. Linearly (in $z$ ) varying vertical motion, constant over the entire period.
a. Upwelling
b. Downwelling
2. Linearly varying vertical motion pulsed in time.
a. Short daily upwelling/downwelling events.
3. Linear internal wave motion.
a. Passage of a single wave pulse through the experimental point over the 24 hour daily cycle.
4. Hon-linear wave variations coupled with diurnal and semidiurnal boundary conditions (heating, wind).
5. Non-linear response to superimposed waves of different periods. Diurnal and semi-diurnal tidal waves are combined and phaseshifted to determine non-linear interactions.
6. Case I

3oth upwelling and downwelling are realistically induced with the realization that in a mixed homogeneous layer the vertical displacement due to a linear $w(z)$ can be shown to be exponential, going to zero at the surface. Hathematically we know that

$$
\begin{equation*}
\frac{d z}{d \dagger}=w(z) \tag{18}
\end{equation*}
$$

if

$$
w(z)=\frac{A z}{D}
$$

where $D$ is a scaling depth, it follows that

$$
\frac{d z}{d t}=\frac{A z}{D}
$$

or

$$
\frac{d z}{z}=\frac{A d t}{D}
$$

Integration from $t_{1}$ to $t_{2}$ yields

$$
\begin{equation*}
z_{2}=z_{1} e^{\frac{A \Delta t}{D}} \tag{19}
\end{equation*}
$$

which is the equation upon which the following work is based, and which agrees with equation (16) derived earlier.

The temperature and salinity profiles must reflect "correct" variation if equations (16) and (19) are correct. Upwelling and/or downwelling will not cause isolines to break the surface or each other. Mixing must be done to induce this. Therefore, above the mixed layer depth $T_{\text {new }}$ and $S_{\text {new }}$ must equal $T_{\text {old }}$ and $S_{\text {old }}$ when there is vertical motion without mixing or surface exchange. Below the entrainment depth the profiles must be altered to reflect the type of vertical motion (upward or downward) in accordance with equation (16). Figure 4 shows the expected result from upwelling on a temperature profile. The mixed layer depth has decreased from time one to time two (after upwelling), and the slope of the profile below the mixed layer has been decreased. In all the cases the $\Delta T$ at the entrainment zone has been preserved.
a. Constant Downwelling (Case Ia) (Figure 12)

The initial conditions used are the same as those for Sl
(See Table 3). A constant downward vertical motion is introduced giving the vertical motion parameter, WDD, (WDD $=A t / D$ ) a value of .03600 , which equates to a value of vertical motion of $.01 \mathrm{~cm} / \mathrm{s}$ or $36 \mathrm{~cm} / \mathrm{hr}$ at

ten meters. This is a rather large value, but it demonstrates the interactions well. The vertical motion is applied beginning the first hour, and is not applied as the initial condition on the temperature and salinity profiles.

The result of this downward motion is, as expected, a more rapid deepening of the surface mixed layer (Figure 12a) when compared to S1. Comparing the depth change due to the mixing in this case (Table 4) to the S 1 mixing, the same sequence of events as described for S 1 is seen, and the values are comparable, indicating that the physics of the model has not been altered by the addition of the vertical motion parameter in the downwelling mode. It is noted that the effect of the vertical motion quickly becomes greater than the effect of mixing due to the large value of vertical mixing assigned in this run of the model.

Table 4. Deepening Rate in Meters Per Hour for the Mixing and Vertical Motion of Case Ia.

| Hour | Tixing <br> Deepening <br> Rate | Vertical Motion <br> Deepening <br> Rate | Hour | Mixing <br> Deepening <br> Rate | Verticle itotion <br> Deepening <br> Rate |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1.7613 | 0.2295 | 13 | 0.3201 | 0.5729 |
| 2 | 0.9705 | 0.2735 | 14 | 0.3169 | 0.6055 |
| 3 | 0.6606 | 0.3077 | 15 | 0.3134 | 0.6392 |
| 4 | 0.4759 | 0.3365 | 16 | 0.3102 | 0.6740 |
| 5 | 0.3535 | 0.3618 | 17 | 0.3068 | 0.7099 |
| 6 | 0.2945 | 0.3858 | 18 | 0.3034 | 0.7471 |
| 7 | 0.2444 | 0.4089 | 19 | 0.3017 | 0.7855 |
| 8 | 0.2282 | 0.4323 | 20 | 0.2983 | 0.8252 |
| 9 | 0.2394 | 0.4569 | 21 | 0.2949 | 0.8663 |
| 10 | 0.2513 | 0.4828 | 22 | 0.2914 | 0.9087 |
| 11 | 0.2793 | 0.5108 | 23 | 0.2881 | 0.9526 |
| 12 | 0.3219 | 0.5413 | 24 | 0.2094 | 0.9952 |

The sequence is vividly displayed in the contours (Figure 12b)
where it is seen that the temperature contours deepen as time progresses
as one would expect in a downelling case. The surface heating due to the incoming solar radiation is noted again, followed by cooling as the sun sets and heat is then lost at the surface and colder water is entrained at the bottom of the mixed layer. It is also noted that in only twenty-four hours, the vertical motion added here has deepened the layer to almost twice the value of $S 1$.

The $T(z)$ plot (Figure 12c) demonstrates the deepening involved and the effects on the temperature profile below the mixed layer. Note the increased slope ( $d T / d z$ ) with time, as warmer nearsurface temperatures (water) are carried downward by the downwelling. Also, one can see the effect of the preservation of the $\Delta T$ profile at the entrainment zone from the mixing step through the vertical motion progression in the model where at times there is an apparent discontinuity in the profile. This does not appear to cause a problem in the execution of the model, and will be seen in many of the following $T(z)$ plots. One can further see that the vertical motion addition has caused a slight decrease in the jump strength of $\Delta T$ at the layer over that in S1. This can be attributed to the reduced mixing for the larger mixed layer depths. Where, with no downwelling a $\Delta T$ of $1.0^{\circ} \mathrm{C}$ may have occurred in 18 meters, it now occurs at a greater depth. This effect is transmitted into the entrainment zone as this below layer water is mixed in, causing the $\Delta T$ to be reduced.
b. Constant Downwelling (Case Ib) (Figure 13)

The initial conditions are those of S2 (See Table 3). A constant downward motion is introduced equal to that used in the previous case.

The result of the downward motion is again, as expected, a more rapid deepening of the layer after the new surface mixed layer is formed through the surface wind stress and heat flux as applied by equations (10) and (11) (Figure 13a). Comparing the depth change due to the mixing here (Table 5) to the $S 2$ mixing shows that the values are of the same order, which shows that the energy relations of the original model are not impaired by the addition of vertical motion in this mode of the model either. Here it is noted that mixing drives the production of a new surface mixed layer beginning at hour 2 (0800 local) (Figure 13b). The shallowing due to the surface heating is dominant until hour seven (1300 local) when the downwelling becomes the dominant factor until hour nine. From hour nine to hour fifteen the effects of mixing and downwelling are of the same order. After hour

Table 5. Deepening rate in meters per hour for the Mixing and Vertical Motion in Case Ib.

| Hour | Sixing <br> Deepening <br> Rate | Vertical iotion <br> Deepening <br> Rate | Hour | Mixing <br> Deepening <br> Rate | Vertical Motion <br> Deepening <br> Rate |
| ---: | ---: | :---: | :---: | :---: | :---: |
| 1 | 0.0152 | 1.8150 | 13 | 0.8093 | 0.6986 |
| 2 | -33.4052 | 0.6571 | 14 | 0.7343 | 0.7511 |
| 3 | -2.5282 | 0.5885 | 15 | 0.7445 | 0.8059 |
| 4 | -3.7245 | 0.4735 | 16 | 0.7131 | 0.8616 |
| 5 | -1.4432 | 0.4392 | 17 | 0.7029 | 0.9190 |
| 6 | -.7286 | 0.4286 | 18 | 0.6768 | 0.9775 |
| 7 | 0.1233 | 0.4489 | 19 | 0.6716 | 1.0379 |
| 8 | 0.3275 | 0.4773 | 20 | 0.6628 | 1.1002 |
| 9 | 0.4266 | 0.5104 | 21 | 0.6420 | 1.1641 |
| 10 | 0.5787 | 0.5504 | 22 | 0.6316 | 1.2299 |
| 11 | 0.6446 | 0.5942 | 23 | 0.6224 | 1.2978 |
| 12 | 0.8003 | 0.6986 | 24 | 0.6120 | 1.3678 |

fifteen the downwelling again, slowly, becomes the dominant factor for layer deepening.


It is seen that the layer temperature, again, immediately begins to warm (Figure 13c), and in fact, while the dominant factor is the large amount of surface heating, the layer warms at exactly the same rate as in S2. After hour ten the layer again begins to cool, but because of the deepening of the mixed layer due to downwelling, the mixing of the surface wind stress is reduced. This, along with the fact that a deeper layer has available more mass of water to dilute the effect of the entrained water, results in less cooling over the remaining twelve hours.

Because of the downwelling effect on diffusing the temperature gradient it is also seen (Figure 13b) that the entrainment zone (layer boundary) remains rather diffuse until hour eight, while in S2 it was sharpened by hour six. This helps explain the ragnitude of the deepening effect in the period from hour nine to fifteen, thoughi the depth of the entrainment zone is increasing faster than that in S2 and reaches a difference of five meters from that in $S 2$ by hour fifteen. The $\Delta T$ across the bottom of the layer for the downwelling case is slightly less than that in $S 2$, allowing the mixing to be a more important factor for deepening in this case. The following decrease in the mixing is then due to the ever increasing depth through which the mixing must take place. The wind stress must act over distances that are twice as large as those in $S 2$, and although the $\Delta T$ at the layer depth is not as great, the effect of the depth is quite noticeable.

The original deep layer is seen to begin to diffuse, but is quickly carried below the 60 meter level of the graphs. As stated above, the new surface mixed layer begins to show a significant $\Delta T$
at hour eight. Extremely close examination of the $T(z)$ profiles for S 2 and the profiles for this run does show a smaller $\Delta T$ at the entrainment zone for this case during the period from hour seven to hour sixteen. After hour 16 there is no discernable difference in the $\Delta T$ in the profiles.
c. Constant Upwelling (Case Ic) (Figure 14)

The initial conditions are those of S1. A constant upward vertical motion is introduced beginning at hour one giving the vertical motion parameter, WDD, a value of . 03600 , or about $36 \mathrm{~cm} / \mathrm{hr}$ at ten meters. Again, this is high, but it is done with the realization that later work will be with wave motion where this will not be an unrealistic value, and using it here provides a good benchmark for the effects produced by this motion.

Table 5. Deepening Rate in Meters Per Hour for the Mixing and Vertical motion of Case Ic.

| Hour | Mixing <br> Deepening <br> Rate | Yertical Motion <br> Deepening <br> Rate | Hour | Mixing <br> Depening <br> Rate | Vertical Motion <br> Deepening <br> Rate |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1.7613 | -.2214 | 13 | 0.4239 | -.3026 |
| 2 | 1.1031 | -.2526 | 14 | 0.4528 | -.3079 |
| 3 | 0.6746 | -.2675 | 15 | 0.4845 | -.3142 |
| 4 | 0.6103 | -.2796 | 16 | 0.4774 | -.3199 |
| 5 | 0.4954 | -.2873 | 17 | 0.2909 | -.3189 |
| 6 | 0.3417 | -.2892 | 18 | 0.2871 | -.3178 |
| 7 | 0.3184 | -.2902 | 19 | 0.3046 | -.3173 |
| 8 | 0.3097 | -.2909 | 20 | 0.3223 | -.3175 |
| 9 | 0.3133 | -.2917 | 21 | 0.3407 | -.3183 |
| 10 | 0.3254 | -.2929 | 22 | 0.2596 | -.363 |
| 11 | 0.3435 | -.2947 | 23 | 0.2739 | -.3148 |
| 12 | 0.3938 | -.2982 | 24 | 0.2891 | -.3138 |

The result of the addition of this upward motion is, as
anticipated, a slowing of the deepening rate of the mixed layer (Figure 14a) when compared to S 1 . In fact, due to the size of the vertical
motion parameter, an effect is produced which almost holds the mixed layer at a constant depth. This is displayed more dramatically after two model days have been executed, and can be seen to reach a near equilibrium condition after four to five days. Comparing the deepening rates due to the effect of the mixing in the model and the vertical motion in this case (Table 5) to the 51 mixing it can be seen quite clearly that the effects described for the mixed layer model standard, S1, are still present. From equations (1) and (3) the slightly increased magnitude of the cooling of the mixed layer by entrainment and the subsequent heating are attributed to the reduced depth over which the wind stress and heat flux must be distributed. Again, it can be seen, that the model TKE balance, equations (3-5), is not violated by the addition of this upwelling motion.

The layer temperature initially falls (Figure 14a), then rises until 1400 local time. It then falls again in response to the mixing and the surface heating cjcle, as in S1. The heating of the layer is not as great here as in the downwelling case (Figure 13), because the entrainment is sufficiently enlarged due to the smaller $h$, and because cooler water becomes available for entrainment more quickly due to the upwelling. This is more clearly seen in comparing the contoured fields, $T(z, t)$, (Figure 14b) of this case with those of the downwelling case (Figure 13b). By the end of the twenty-four hour period this has resulted in a very strong temperature jump, $\Delta T$, at the entrainment zone, as is seen best in the $T(z)$ profiles (Figure 14c). It is seen that $\Delta T$ grows with time.

The strengthening of the jump, and the lifting of the cooler water is demonstrated clearly by the contours, and it is
clearly evident the contours not only lift, but also merge in time, indicating the $\Delta T$ grows with time. This effect is seen in the $T(z)$ profiles by the increased gradient of the profiles with time.

Equilibrium is reached in this case as the jump gets too large for the mixing to have an effect greater than the effect of the vertical motion. Equation (8) shows that dh/dt vanishes as $\mathrm{w}_{\mathrm{e}}$ approaches $w_{-h}$.
d. Constant Upwelling (Case Id) (Figure 15)

The initial conditions applied are identical to those of S2. A constant upward vertical motion is introduced at hour one of about $36 \mathrm{~cm} / \mathrm{hr}$ at ten meters, as before.

The net result of the vertical motion after 24 hours is a reduced layer depth in comparison to the case having no vertical motion $(S 2)$. The total reduction in depth of six meters, however, is not as great as the previous case (Ic). After two days it is seen that the mixed layer is approaching an equilibrium depth of near eleven meters (Figure 15a).

The deepening rates (Table 6), when compared to those of S2, demonstrate that the basic model is still preserved in this run. From hours two through five the model is in the shallowing mode, and the new layer depth is established by equations (10) and (11), while there is a massive influx of solar radiation to the sea surface. The increased deepening rate due to the mixing seen from hour twelve through twenty-three is attributed to the effect of equations (1), (4) and (5). The mixed layer is maintained at a shallow level by the upward vertical motion, therefore, the mixing takes place over a much smaller distance than in 52.

Table 6. Deepening Rate in Meters Per Hour for the Mixing and Vertical Motion of Case Id.

| Hour | Mixing <br> Deepening <br> Rate | Vertical iotion <br> Deepening <br> Rate | Hour | Mixing <br> Depening <br> Rate | Vertical Motion <br> Deepening <br> Rate |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.0152 | -1.7509 | 13 | 0.9465 | -.4751 |
| 2 | -29.4675 | -0.6470 | 14 | 0.9470 | -.4917 |
| 3 | -1.3404 | -0.5767 | 15 | 1.0159 | -.5103 |
| 4 | -2.6147 | -0.4639 | 16 | 1.0611 | -.5298 |
| 5 | -0.5166 | -0.4292 | 17 | 1.1449 | -.515 |
| 6 | 0.1312 | -0.4186 | 18 | 1.2301 | -.5755 |
| 7 | 0.3085 | -0.4148 | 19 | 1.3043 | -.6013 |
| 8 | 0.4578 | -0.4163 | 20 | 1.4087 | -.6298 |
| 9 | 0.5885 | -0.4224 | 21 | 1.5577 | -.6626 |
| 10 | 0.6775 | -0.4314 | 22 | 1.6638 | -.6980 |
| 11 | 0.7113 | -0.4413 | 23 | 1.3743 | -.7219 |
| 12 | 0.9078 | -0.4578 | 24 | 0.6909 | -.7208 |

Further, there is a concentration of energy in this shallower layer. The great variance in the last hour is a direct result of the magnitude of the upwelling, which, by hour four, has set up a large gradient in the temperature (Figure 15b). By hour twenty-four a gradient has been set up with such a large slope that mixing is becoming very difficult. Continued upward motion of this magnitude, without shallowing as is seen on the second day, would quickly make it extremely difficult to entrain much water into the mixed layer.

The progression in the change of the mixed layer temperature (Figure 15a) shows that as the model is shallowing, governed by equation (10) and (11), the same adjustment takes place as is seen in S2. The temperatures produced are exactly those seen in S2. This is another indication that the model is performing as before. Since $w_{e}$ is zero, the upwelling has no effect on the setup of the new layer. If the $H$ values had been listed immediately after the mixing phase of the computation, this could be seen more clearly. After the layer again begins

to mix downward, however, the upwelling begins to have some effect on $T$ and $h$ due to the upward transport of the entrainment zone. Though the layer is one to two meters shallower than in $S 2$, the temperature of the layer is the same as that in the deeper standard to hour ten. At hour ten the heat flux of equation (1) has been reduced sufficiently so that wind mixing again becomes the dominant deepening force. At this point the upwelling has moved water cooler than that in S2 at the same time to the entrainment zone. The change in entrainment rate due to the upwelling, the entrainment of cooler waters, and the net surface heat loss after hour eleven combine to produce a cooling of $.147^{\circ} \mathrm{C}$ from the maximum warm temperature, as compared to $.109^{\circ} \mathrm{C}$ for $S 2$. For the mass of water involved, and the relatively short time involved, this is a significant difference.

The effect of the upward vertical motion is perhaps best shown in the $T(z)$ diagram (Figure 15 c ), where it is seen that the model initially adjusts, through equations (10) and (11), to a new surface layer which slowly strengthens, in terms of the jump, $\Delta T$, as it entrains. As the process continues the profiles below the layer rise by upwelling and the initial deep interface is smoothed by diffusion until it disappears. By hour twenty-four a sufficiently stable profile has been set up to make deepening of the layer more difficult. This indeed is the case in the second day of the run (Figure 15a) as the layer reaches a temporary equilibrium. It is temporary, in that once the profile has been reached which makes deepening sufficiently difficult through equations (4) and (5), the dominant force becomes the upwelling, $w_{-h}$ of equation (9).


The contours (Figure 15b) further demonstrate the effect of the upward lifting of the cooler water, as the contours tend to converge with time. The initial interface is quickly lost, as the model smooths this into its uplifted profile.

## 2. Case II

The next step of this progressive series of tests is to explore the effects of a simple pulsed motion varying in time. This is accomplished by varying the input motion in a sinusoidal pattern. By using the equation

$$
\begin{equation*}
w_{0}(t)=A A \sin \left(\frac{2 \pi t}{24}+k x\right) \tag{20}
\end{equation*}
$$

as the particular form of equation (12) applied for this case, where $A A$ is the velocity scale factor such that $A A=A z / D$ is scaled for an appropriate motion at 100 meters, and limiting the value of $w_{0}(t)$ to only values in one direction (+ or -), and zero, a pattern of motion can be produced such as that seen in Figure 5. A pulse of vertical motion of twelve hour duration is produced, followed by a twelve hour period of no motion. To determine the relationship of this motion with the cyclical boundary conditions one need only shift the phase of the pulse relation. This will cause the motion to peak at varying selected times during any twenty-four hour period run.
a. Pulsed Downwelling (Case IIa) (Figure 16, 17 and 18)

The initial conditions are the same as those of S1. Three different phase relationships of the downwelling to the cyclical boundary conditions are presented. In the first (Figure 16) the maximum of the downwelling occurs $180^{\circ}$ out of phase with the heating
cycle (that period of the model defined as night, or the cooling cycle). In the second (Figure 17), the downwelling occurs during the last seven hours of night, and during the first five hours of daylight. The maximum downwelling occurs six hours before the heating maximum. Finally, in the third case (Figure 18), the downwelling occurs in phase with the maximum heating cycle. In all three cases, the downwelling event is a twelve hour pulse with a maximum value of $36 \mathrm{~cm} / \mathrm{hr}$ at ten meters.
(1) $180^{\circ}$ Out of Phase (Figure 16). For this case it is seen that the model evolves exactly as does Sl until hour thirteen when downwelling begins (Figure 16a). All the fields produced are exactly the same as in Sl until hour thirteen for which the evolution has been discussed.

After hour thirteen the downwelling begins to transport the surface water downward, and causes a decrease in the jump, $\Delta T$, strength of a few hundredths of a degree. This combined effect causes the deepening of the mixed layer to increase here over that of Sl by about four meters over the twenty-four hour period. Additionally, it decreases the change of temperature of the mixed layer because the layer is initially deeper, thereby, reducing mixing, and increasing the dilution of the entrained waters, and by causing entrainment of water which is not as cool as that of $S 1$, over the same period. These effects are apparent in the contours (Figure 16b), and in the $T(z)$ profile (Figure 16c). The downwelling effect is seen most dramatically in the contours, where the downward pulse is evident, along with the described temperature variations.
(2) Haximum Downwelling Before Heating (Figure 17). Here, the downwelling is applied during the last six hours of the mixing,

and during the first five hours of the heating cycle. The six hours of downwelling during the last six hours of night serve only to deepen the layer in the manner previously described. The first five hours of downwelling coupled with the heating and wind stress through equation (9), produces a noted change in the temperature fields (Figure 17c) from that of S1. This coupling leads to an increase of the layer temperature at the peak heating period of $.008^{\circ} \mathrm{C}$ for this run over that of the previous ( 8 T increase), and over that of the standard, S 1. After twenty-four hours this difference has decreased to $.005^{\circ} \mathrm{C}$ as compared to the previous run, and by two days the difference in temperatures is negligible, though the layer depth is slightly shallower after two days for this run. By only partially coupling the effects of downwelling and heating, a shallower mixed layer has been produced with somewhat warmer temperatures.

The surface temperature evolution and the below layer evolution are best revealed by the contoured fields (Figure 17b). It is apparent here that another phenomenon is present. As the downwelling commences the near layer contours do in fact diverge as expected. The deeper contours, however, initially narrow as the pulse forms, then widen to the below layer temperature spreading effect described previously. This convergence during the peak motion period is a direct result of the mathematical treatment of the below layer temperature profiles through equation (16).
(3) Coupled Heating and Downwelling (Figure 18). Here the downwelling peaks one hour before the maximum surface heat flux. The combination of downwelling and wind stress, equation (9) through equations (16) and (8), are sufficiently strong to cause the effect of
the heat flux, equation (1), to be negligible, and the layer deepens to almost three meters more than that of Sl by hour eleven. Also, the layer temperature has been increased significantly over SI due to the downwelling effect on the layer which causes the entrainment of less water than previously during the period of maximum surface heating. By combining the downwelling with the heating, a maximum temperature change of $.211^{\circ} \mathrm{C}$ for SI has been reduced to $.162^{\circ} \mathrm{C}$ for this run over the first twenty-four hours. This has also produced a mixed layer which is $.06^{\circ} \mathrm{C}$ warmer and three meters deeper than SI after only twenty-four hours. This reflects a substantial change in the energetics of the system, and may be a significant mechanism for the generation of horizontal temperature structure. After the pulse, the layer is mixed by the action of the wind stress in the presence of a smaller $\Delta t$--reduced by the downwelling accompanied by a small downward surface heat flux. This explains why the deepening is not as great after the downwelling pulse as would be expected if one considered only the reduced jump strength.

In combination these three phased runs demonstrate the effect of the interaction of the downwelling and the diurnal heating cycle. In all cases a relative heating of the mixed layer is seen because of the depth increase. The deeper layer reduces the effect of the mixing, and reduces the entrainment. As the downwelling event couples with the heating, the final mixed layer depth is reduced, over that of the uncoupled cases, and greater surface temperatures (layer temperatures) are achieved. The rudiments of a pattern in surface features which could be induced over a large ocean by simply varying the coupling of these two basic effects is now seen.
b. Pulsed Downwelling (Case IIb) (Figures 19, 20 and 21)

The initial conditions are those of $S 2$. The magnitudes of the downwelling, and the phase relations are the same as the previous case.
(1) $180^{\circ}$ Out of Phase with Heating (Figure 19). Because the downwelling event occurs after the surface heating has stopped (at night), the model has gone througn the shallowing mode, equations (10) and (11), and produced a new surface mixed layer exactly as in S2 during the daylight period. The effect of the downwelling is seen at hour fourteen when mixed layer deepening is first observed. The cumulative effect of the downwelling is to deepen the layer by about five meters more than in $S 2$ in the twenty-four hour period (Figure 19a). The layer temperature, due to the downwelling effect, remains $.011^{\circ} \mathrm{C}$ warmer after the period. Destruction of the original layer at 50 m is caused by diffusion, and the model's mathematical smoothing in the production of new temperature contours due to the downwelling in accordance with equation (16).
(2) Maximum Downwelling Before Heating (Figure 20). The shallowing of the model through equations (10) and (11) during the surface heating period overrides the downwelling, so that by hour five this run is essentially the same as $S 2$, with only a slight temperature difference. Comparing the contours (Figure 11 b and Figure 20b) shows that this run is the same as $\$ 2$ until the downwelling pulse begins at hour nineteen, except the early downwelling has deepened the layer and moved the cooler water away from the entrainment zone, relative to the temperatures in S2. The layer then deepens due to wind stress mixing, and downwelling. The net result of this coupled action is a

layer about four meters deeper than $\$ 2$ after twenty-four hours, but with a temperature essentially equal to that of S2.
(3) Coupled Heating and Downwelling (Figure 21). Here the shallowing mode is still the dominant factor, with the downwelling playing only a minor role in that it deepens the newly forming layer, and moves the relatively cooler water away from the entrainment zone. Because the model is in the shallowing mode, as $\mathrm{w}_{\mathrm{e}}$ approaches zero, the new value of $h$ is dependent on equations (10) and (11), as is the temperature of the new mixed layer through equation (1). Since the downwelling adjustment is done after the evaluation of the shallowing $h$, the downwelling does deepen the newly formed layer, but no temperature adjustment is made. This results in a layer that appears to be deeper than in S2, and yet maintains the same temperature. The net result of these interactions after twenty-four hours is a layer that is one meter deeper than in $S 2$, and. $.012^{\circ} \mathrm{C}$ warmer. Here the coupling effect was negligible.

In combination these three runs of the model in the downwelling mode with S 2 initial conditions showed the opposite progression of those runs using the si initial conditions. The maximum variations of temperature and mixed layer depth occurs in the case of $180^{\circ}$ out of phase downwelling with the heating cycle, where a deeper, warmer mixed layer is formed during the twenty-four hour period. In the case of the $S 1$ initial conditions the maximum deviations occurred during the coupled run.
C. Pulsed Upwelling (Case IIC) (Figure 22, 23 and 24) The initial conditions are those of ST. The phase
relationships used are the same as in the pulsed downwelling case, as are the magnitudes of the effects.
(1) $180^{\circ}$ Out of Phase with Heating (Figure 22). The effect here is negligible until hour thirteen, and this run is exactly the same as Si until then. At hour thirteen the upwelling begins to increase the magnitude of $\Delta T$. Colder water is moved up toward the entrainment zone below the layer, and the layer itself is moved upward, as the wind stress acts to deepen the layer by mixing. At hour twentyone it is seen that the combination of the upwelling and the magnitude of $\Delta T$ are great enough to actually shallow the layer for that hour.

The net effect of the upwelling in this case is to create a mixed layer that is about 2.6 meters shallower than in 51 over the twenty-four hour period, and $.03^{\circ} \mathrm{C}$ cooler (Figure 22a). Also, there is a maximum 8 T in the mixed layer of $.241^{\circ} \mathrm{C}$, where mixing only produced a 8 T of $.211^{\circ} \mathrm{C}$ in S 1 . This has the potential of producing an observable surface temperature effect.

The effect of contour convergence during the pulse mentioned in the downwelling case is seen here again (Figure 22b). The overall effect of the upwelling, to converge the contours from before to after upwelling, is seen as expected.
(2) Maximum Upwelling Before Heating (Figure 23). In
this case the upwelling immediately begins to lift the entrainment zone against the effect of the wind stress, causing an increase in the effect of the mixing, and reducing the water available for dilution of the entrained, cold water. In equation (9), $w_{-h}$ is of opposite sign to that of $w_{e}$, reducing, or changing the sign of $d h / d t$.
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This results in cooler layer temperatures over the same period as described for S1, and shallower mixed layer depths. Numerically, the net result of this effect over the twenty-four hour period is to produce a layer that is 2.5 m shallower than that of S 1 , and $.04^{\circ} \mathrm{C}$ cooler (Figure 23c). The maximum $8 T$ of the layer over the period is $.240^{\circ} \mathrm{C}$, or about the same as in the previous upwelling case.
(3) Coupled Heating and Upwelling (Figure 24). The coupling of the upwelling and the heating cycle produce a mixed layer that is only 1.9 meters shallower than that of S 1 in the twenty-four hour period, but a layer that is much cooler. The reduction of the layer depth during the heating period causes the maximum temperature to be $.013^{\circ} \mathrm{C}$ colder, and causes the layer temperature at the end of the twenty-four hour period to be $.071^{\circ} \mathrm{C}$ colder than SI , a significant change. The maximum 8 T of the layer for this period is $.269^{\circ} \mathrm{C}$.

As in the downwelling case with the shallow initial layer conditions, the maximum surface variations in temperature is in the coupled upwelling case.
d. Pulsed Upwelling (Case IId) (Figure 25 and 26)

The initial conditions are those of S 2 . The phase relationships used are the same as those in the previous three model runs. Magnitudes of the upwelling are the same as in. the previous run.
(1) $180^{\circ}$ Out of Phase with Heating (Figure 25). The shallowing proceeds as in S2 for the first six hours, and a new layer is formed (Figure 25b). At hour thirteen the upwelling begins and several effects are noted. First, the deepening immediately slows, relative to that in 52 , as the upwelling lifts the layer, and
increases the $\Delta T$ at the entrainment zone. Second, the layer temperature decreases faster relative to that in 52 , due to the shallowing of the layer, and entrainment of relatively cooler water into the layer. Finally, there is an effect not previously observed. The initial layer, which is being diffused in the previous runs is now reinforced during the upwelling (Figure 25c), and a two-layer system is formed over the first twenty-four hours.

The net result of this case is a surface layer that is 2.4 meters shallower and $.022^{\circ} \mathrm{C}$ cooler than S 2 over the period. This leads to a slightly increased surface signature over the non-upwelling case. Because of the reinforcement of the initial layer there is also a potential for a substantially recognizable $T(z)$ signature for this case.
(2) Maximum Upwelling Before Heating (Figure 26). The shal-
lowing and upwelling combine to make this a relatively uninteresting case. Though the large upwelling has some initial effect, the fact that $w_{e}$ goes to zero forcing the model to be driven by equations (10) and (11) soon overcomes the upwelling, and by hour four this case is identical to S 2 and the previous run with the exception that the upwelling has advected cooler water closer to the entrainment zone. The difference in water temperature and mixed layer depth between this case and S 2 are negligible by hour eighteen. At hour nineteen upwelling begins again, increasing the relative effect of the wind stress mixing, causing increased entrainment of cool water. The net effect of the upwelling and the wind stress mixing over the depth involved is to produce a layer that is 2.2 meters shallower than in S2, but at only $.004^{\circ} \mathrm{C}$ cooler after twenty-four hours. This would
not produce a noticeable effect at the surface. The two-layer type system seen in the previous case could, again, provide a signature to this sort of upwelling event.
(3) Coupled Heating and Upwelling (Figure 26). The shallowing prescribed by equations (10) and (11) due to the excessive heating tends, initially, to override any effect of the upwelling. By hour six there is only a . 4 meter depth difference over that of S2, and essentially no temperature difference in the mixed layer. After that point, the upwelling begins to combine with the mixing to augment the cooling of the layer, and slowing its deepening.

The net result after twenty-four hours is to produce a layer that is about as deep as that of $S 2$, but that is as cool as the case of the evening upwelling. This represents a substantial energy difference in the runs. Again, a slight surface temperature signature could be seen with this run in comparison with the standard.
3. Case III

The next test is that of the interaction of mixing witin a single cycle internal wave. By stepping the phase of an input sinusoid through the twenty-four hour progression of the model, a quasi twodimensional motion can be prescribed.

For a wave of the form

$$
\text { wave amplitude } \propto A \sin (\omega t+k x)
$$

the vertical motion would be of the form

$$
\begin{equation*}
w(x, z, t) \propto A \omega \cos (\omega t+k x) \tag{21}
\end{equation*}
$$

as seen in Figure 6. By increasing the value of $k x$, the wave is made to travel in the $-x$ direction.

Figure 6. Vertical motion-wave form relationship: Vertical motion (dashed line) relative to wave (solid line) passing through a given point ( $x$ ) at time $t(h r)$. Amplitudes are not absolute (See equation 21).
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The initial conditions used were those of cases S1 and S2. The vertical motion was introduced at hour one, and the amplitude was varied as $t$ varied in equation (21), to give a wave induced vertical motion. This was then run twenty-four times, each time starting at a different phase of the wave, to test the interrelation of the wave input with the cyclical heat flux. For simplicity, only four major phase relationships are discussed here, as they will demonstrate the primary effects. Also, here, the two-dimensional field will be discussed for the first time.

The wave form of the desired vertical motion does appear in all four of the displayed runs. The difference in each run is the phase of the vertical motion in relation to the heat flux (daily solar heating cycle).
a. Shallow Initial Layer (Case IIIa) (Figures 27-33)

In all four phase relationships presented, maximum downward motion at hours $1,7,13$, and 19, the initial conditions are those of S1. The magnitude of the peak upward and downward motions is $36 \mathrm{~cm} / \mathrm{hr}$ at ten feet, as before. The wave is introduced at hour one, and progresses through the system through hour twenty-four.
(1) Peak Upward at Hour 7 (Figure 27). In this case the initial upward motion, hours one through twelve, the heating and the surface wind stress combine to give a slowly deepening layer, which, due to the upward advection of the layer and of cooler, below layer water, is almost $.1^{\circ} \mathrm{C}$ cooler at its maximum temperature than SI . At the end of the upward cycle the layer has deepened about two meters less than $S l$ at the same time, and has set up a slightly larger $\Delta T$ at
the entrainment zone. As a result, the following downward cycle produces a smaller effect in combination with the wind stress in deepening than that seen previously. Because of the initial upward motion effects on the temperature profile, the combined effect of wind stress mixing, surface cooling, and the downward motion produce a layer that, after twenty-four hours, is only two meters deeper than SI. Additionally, the downward effect of the motion on the temperature gradient is sufficient only to make up about 60 percent of the deviation in temperatures caused by the upwelling motion, as to that of S1.

The maximum 8 T for this case is $.245^{\circ} \mathrm{C}$. It, also,
produced the deepest layer of the four runs discussed here, over the twenty-four hour period.
(2) Peak Upward at Hour 1 (Figure 28). In this case the upward motion was present during the first six hours of the heating cycle only. The downward motion cycle tben progressed for twelve hours, followed by six hours of upward motion to complete the twenty-four hour sequence.

During the first six hours the layer deepened to within less than one meter of the depth reached by hour six in S1, but the layer temperature is seen to be about $.01^{\circ} \mathrm{C}$ cooler. The reversal of the motion was too late in the heating cycle to overcome that trend, and by hour nine, the time of the maximum surface temperature for Sl, this experiment produced a surface temperature that is $.013^{\circ} \mathrm{C}$ cooler. The layer depth reached here is within half a meter of the $S 1$ depth for the same time. The difference is attributed to the initial lifting of the entrainment zone.

As the downward motion continues, along with the surface cooling, the layer deepens more quickly while the cooling of the layer slows as the effect of mixing is reduced. A maximum depth is reached two hours after the downward motion has stopped, when the upward motion is of sufficient magnitude to overcome the wind stress mixing. The layer becomes shallower from its extreme depth of 15.56 meters to 14.94 meters by hour twenty-four, with associated layer cooling due to surface heat loss, and increased entrainment due to the increased effect of the wind stress mixing.

The maximum 8 T for this run is $.189^{\circ} \mathrm{C}$. The final layer depth is within .3 meters of 51 for hour twenty-four. The variations of depth and temperature during the run are obviously much greater than for ST.
(3) Peak Upward Motion at Hour 19 (Figure 29). The fact that the downwelling and the heating cycle are in phase, as in Case IIa, produces, by hour nine, a layer that is two meters deeper and $.006^{\circ} \mathrm{C}$ warmer than Sl for this same time. The following upward motion is sufficient to halt layer deepening at hour sixteen in opposition to the mixing, but is not sufficient to cool the layer as much as for S1. Tine final layer temperature is $.035^{\circ} \mathrm{C}$ warmer than that in S1, while the final layer depth is about one meter less, thougn at hour sixteen a depth of 14.2 meters was reached. The maximum $8 T$ for the mixed layer in this case was $.181^{\circ} \mathrm{C}$.
(4) Peak Upward Motion at Hour 13 (Figure 30). This case produced the warmest layer temperature at hour nine, though the downward motion was only present during the first six hours of the
surface heating cycle. The initial downward motion deepened the layer, and thereby reduced mixing in this period, over the Sl case, allowing the surface layer to be heated to $14.988^{\circ} \mathrm{C}$, an increase of $.01^{\circ} \mathrm{C}$ over that in S1. The following upward motion was sufficient, hours eleven through fourteen, to hold the mixed layer at a relatively constant depth. This allowed a relatively constant, and strong wind stress entrainment to combine with surface cooling to rapidly cool the layer. The result after twenty-four hours is to produce a layer of about the same depth as that of S1, but one that is sligntly cooler.

The 8 T here was $.237^{\circ} \mathrm{C}$, a very noticeable difference from S1, though the final product did not vary much in layer temperature or layer depth.
(5) Two Dimensional Data (Figure 31, 32 and 33). The two dimensional oceanic field was produced as previously discussed. The data covered here is that of the evolution of the ocean system at hours $6,12,18$, and 24.

By nour six the wave has progressed into the oceanic field, and its effect below the layer is dramatic, (Figure 3la). It has produced a mixed layer that has a mean depth of about 9 meters (initial $h$ was 4.5 meters) with a mean temperature of $14.957^{\circ} \mathrm{C}$. A temperature structure, as could be measured at the surface in $x$, is being formed. The present maximum difference in surface temperatures (in $x$ ) is $.016^{\circ} \mathrm{C}$.

By nour twelve (Figure 31b) the effect of the wave is at its maximum, as seen in the amplitude of the contours below the layer. The layer is generally deeper than at hour six with a mean depth of about 11.5 ineters. The deviation from the mean is much
greater for this time, however. The mean of the layer temperature has gone to $14.950^{\circ} \mathrm{C}$, a slight decrease from hour six, but more significant is the deviation of the surface temperatures in the $x$ direction. There is now a very definite variation in $x$ of $.051^{\circ} \mathrm{C}$.

The layer has deepened to a mean depth of about 13 meters by hour eighteen (Figure 31c), with the effect of the wave beginning to be reduced in the system. The mean layer temperature has dropped to $14.846^{\circ} \mathrm{C}$, a cooling of almost $.05^{\circ} \mathrm{C}$ in the six hours between illustrations. The surface temperature deviation has reached $.091^{\circ} \mathrm{C}$. Narm and cold temperature bands are becoming defined in x .

The single wave has passed through the ocean by hour twenty-four, and only the remnant of its passage is seen (Figure 31a). In an ocean that is defined by Sl conditions everywhere, the twentyfour hour product would be an ocean with a mixed layer depth constant at 14.65 meters, and mixed layer temperature of $14.767^{\circ} \mathrm{C}$. With the introduction of a single diurnal wave cycle, a variable condition ocean has been produced. The mean depth of this ocean is 15.012 meters, with local variations of up to +1.73 meters and -1.32 meters, with depths at small x being somewhat greater than depths at large x . The mean layer temperature is $14.764^{\circ} \mathrm{C}$, but more importantly, the surface temperature variation is now $.095^{\circ} \mathrm{C}$, in a clearly defined pattern. The pattern that exists is that of a narrow zone of relatively cool water centered near $k x=5$, with a wider zone of relatively warmer water centered near $k x=18$. Below the layer the isotherms are found to be parallel after the passage of this single wave cycle.
b. Deep Initial Layer (Case IIIb) (Figures 34-40) The initial conditions for these runs were the same as
for S2. The wave cycle passed through the system as in the previous runs (IIIa). The wave form is harder to discern in these runs, but looking at the contours below the original layer (Figures 34-37) the same forms as the previous run are apparent.
(1) Peak Upward Motion at Hour 7 (Figure 34). As in the previous deep initial layer cases, the shallowing due to the surface heating drives the model in the first six hours. The coincident upward motion during this time combines with the shallowing and mixing to produce a layer that is about 1.5 meters shallower, but at the same temperature as in S2, by hour ten. Two noticeable effects during this period of upward motion are (i) a slight shallowing of the newly formed surface layer, and (ii) the upward transport of cooler below layer water, setting up a larger $\Delta T$ at the entrainment zone for this case over that of the standard. The downward motion then contributes to the deepening of the layer as does the mixing which must initially act over a relatively shallow depth. This entrains cool water, forming a quickly deepening and cooling surface mixed layer. As the downward motion decreases the cooling slows, and the increase in the depth of the layer is seen to be caused mostly by the vertical motion. Of all the phases examined this produced the deepest layer ( 30.34 m ) and has the second coolest layer at the end of the period $\left(15.059^{\circ} \mathrm{C}\right)$ in relation to the other three cases discussed.

The 8 T for this case is $.115^{\circ} \mathrm{C}$. This is about half that for the shallow layer case discussed previously. The original deep interface can be followed on the contours (Figure 34b) over the twenty-four hour period. During the downward motion it, again, is

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diffusing, while during the upward case there is a slight reinforcement of the interface.
(2) Peak Upward Motion at Hour 1 (Figure 35). Again it is seen that the shallowing mode of the mixed layer model, equations (10) and (11), controls the layer development to hour ten. The layer depth reached through the shallowing, upward, and downward motion combination by hour ten has reached a depth that is about .5 meters deeper than that in S2 for the same time. The temperature is the same as S2. Also, the downward motion makes entrainment of water into the mixed layer due to mixing more difficult, such that cooling after hour ten progresses much more slowly than in that of the previous case, and that of S2. The net result in the layer depth of this run is a layer that is about as deep as that of $S 2$ after the twenty-four hour period. The model did not deepen more than $S 2$ due to the dominant upward motion during the last six hours. The mixed layer temperature was sligntly warmer than that of $S 2$ at the end of the run $\left(.071^{\circ} \mathrm{C}\right)$. Also, it can been seen that the below layer water is somewhat warmer than in S 2 for the end of the period, due to the combined effects of the heating and downward vertical motion timing.

The 87 for this case is $.092^{\circ} \mathrm{C}$. This is less significant than in the previous run.
(3) Peak Upward Notion at Hour 19 (Figure 36). Again, the shallowing mode of the model controls the initial ten hour layer temperature adjustment. The downward vertical motion, however, drives the layer two meters deeper during this time causing reduced mixing due to the surface wind stress. The upward motion keeps the layer
from deepening as quickly as before after hour twelve. Because of the below layer temperature adjustment by the downward motion and the initial deepening of the layer, the layer, also, does not cool as quickly. This case produced the shallowest final layer depth, and the second warmest layer at the end of the period, relative to the other three runs.

The 8 T for this case is $.103^{\circ} \mathrm{C}$, which is about the same as the first case which was $180^{\circ}$ out of phase with this run.
(4) Peak Upward Motion at Hour 13 (Figure 37). The coincident shallowing, wind stress mixing, heat flux, and cycled vertical motion over the first ten hours produce a surface mixed layer that has the same temperature as $S 2$, and almost the same layer depth (to within less than .5 meters). Below the layer, however, the upward motion has made relatively cooler water available by hour ten (in comparison with S 2 ), and the layer, though not rapidly deepening due to the continued upward motion, cools quickly as large quantities of water are entrained over a relatively shallow depth, a trend which continues to the end of the period. The net result is a mixed layer that is 1.5 meters deeper than that of $S 2$, but is also the coolest layer of the four runs. Because of the below layer temperature profiles which are set up by the upward motion for the midtwelve hours, the continued run of this model over a second day gives continued rapid cooling of the layer due to increased entrainment of cooler, upward advected water, and to increased effects of mixing. A noticeable deepening does take place at the end of the period due to the downward motion, which accounts for the fact that this run ended deeper than $S 2$.

The $8 T$ for this run is $.125^{\circ} \mathrm{C}$. This is the largest of the four runs, but it is still only half of the maximum 8 seen for the shallow initial layer case.
(5) Two-Dimensional Data (Figure 38, 39 and 40). The twodimensional oceanic field is produced, here, as previously discussed, for the single wave cycle. The data covered is for hours $6,12,18$, and 24. A common feature is the varying below layer temperature structure set up because of the introduction of the initial deep layer. Because of this, the changes in the thermocline below the initial deep interface best show the effect of the wave progression through the system.

As in the shallow case, by hour six, the wave has progressed into the oceanic system producing some horizontal sea surface temperature structure. However, the effect on the surface temperature is almost negligible, in that the variability of the surface temperature in $x$ is only $.001^{\circ} \mathrm{C}$. This is due to the shallowing mode response of the model over the first six hours. Even with this, however, the depth structure has varied (Figure 38a) so that there has been a change from the constant structure that would be seen in S2 of 11.767 meters for the layer depth, to a layer of mean depth of 11.769 meters + . 35 meters, indicating that an adjustment due to the wave is taking place.

By hour twelve much more temperature and layer $(h(x))$ variation is beginning to take shape. At hour twelve the case without the vertical motion has a layer depth of 15.037 meters, and a layer temperature of $15.159^{\circ} \mathrm{C}$ everywhere. In this case, where the maximum effect of the wave has been reached, the layer depth has a mean of

15.231 meters with a deviation of $\pm 2.5$ meters. The variability in the surface temperature in $x$ has reached $.01^{\circ} \mathrm{C}$. Though the variability in the surface temperature is not very large, a pattern is again beginning to emerge in the $x$ direction. At small $x$ cooler water is forming, while at large $x$ a band of warmer water is emerging.

The layer has continued to deepen to hour eighteen. The layer depth has increased to a mean of $19.46 \mathrm{~m}, \pm 3.4 \mathrm{~m}$, and the variation in the surface temperature structure in $x$ has reached $.03^{\circ} \mathrm{C}$. The warm-cold pattern seen at hour six is repeated and strengthened. Though the temperature difference between warm and cool water is not as great as in the shallow case, the banding of the patches is much more distinct here (Figure 38, Figure 31).

At the end of the period the wave has, again, been removed from the system, and only the remnant of its passage is seen. The final layer depth is deeper than in the previous hours, and much deeper than in the shallow case. The variability of $h$ in $x$ is also much greater here than in the shallow case. The surface temperature patchiness is well-defined, but the variability of the temperature in $x$ is only $.035^{\circ} \mathrm{C}$.

Comparing the two cases, shallow and deep initial layers, it is seen that for the same amplitude wave the model produces two distinct ocean types. For the shallow case an ocean is formed that has a relatively shallow layer with a large temperature variation in the $x$ direction, but with that temperature variation primarily in wide bands. For the deep case a deep surface layer is formed with smaller variability in the surface temperature. However, the temp-. erature structure in $x$ is much better defined. Also in the deep case,
the patches of warm and cold water are much closer to each other (in $x$ ). This could suggest a mechanism for generating or sharpening SST fronts through surface heating, and internal solitary wave passage.
4. Case IV

It was noted in Case III that after twenty-four hours the wave form had passed through the ocean system, and had left behind only the residue of its effect on the system, over and above the standard effects. This can be compared to a solitary wave passing through an ocean system. While this has theoretical advantages, it is not a very realistic problem, in that wave forms seldom take on this sort of motion. One possible exception may be solitons generated at extreme tidal stages (Apel, 1982).

The next progression, therefore is to look at a continuous wave system that passes through the ocean with constant form, until the run is terminated. This is done by simply modifying the mathematics of the previous case to include initialization of the temperature and salinity profiles with the wave form according to the following.

We know that

$$
w(x, z, t)=\frac{A A_{2} \cos (\omega t+k x)}{D}
$$

and

$$
w=\frac{-\partial 8 h}{\partial t}
$$

or

$$
8 h=-\int w \partial t
$$

therefore

$$
8 h=-\frac{A A z}{\omega D} \sin (\omega t+k x)
$$

For initial displacement

$$
8 h(x, 2,0)=8 h_{0}
$$

therefore

$$
\begin{equation*}
8 h_{0}=\frac{-A A_{2}}{\omega D} \sin (k x) \tag{22}
\end{equation*}
$$

Applying equation (22) to form the initialized layer depth, and combining this with equation (16) to initialize the temperature profile, a continuous wave field can now be passed through the model ocean (Figure 7).

From this point on the discussion will be confined to the deep initial layer case, S2 standard, and variations thereof. This is done because all the primary effects discussed to this point appear in both the shallow and deep cases, as do the effects to be discussed in varying degrees, and it is not necessary to continuously reiterate this discussion.
a. Diurnal Wave (Case IVa) (Figures 41, 42 and 43)

Because of the overriding effect of the shallowing mode of the model in the first six hours of the runs, the results produced here were exactly the same as those discussed in Case IIIb, for the phase relationships. This was of course expected from the theory.
(1) Two-Dimensional Data (Figure 44, 45 and 46). Again, because of the dominance of the shallowing mode, the hour $6,12,18$, and 24 two-dimensional fields produced here are the same as those of Case IIIb for the area of the entrainment zone, and the mixed layer. Below the layer, however, the wave form is now seen to progress through the system in a continuous mode. This is best seen in the contours

Figure 7. Diurnal wave relationship (of continuous diurnal waveform to model location ( $x$ ). Amplitude variable.)
in which the amplitude of the system remains a constant, but the wave form is shifted with time.

It can be seen that the patch of warmer waters forms (Figure 44) and takes a phase that barely leads the peak of the wave system. The entrainment zone ( $h(x)$ ) now takes the form of the passing wave system, as would be expected, and yet was not obvious in the previous case. The divergence and convergence of below layer fields is also obvious here, where previously it was not particularly obvious.
b. Semi-diurnal Wave (Case IVb) (Figures 47-50)

A dominant internal wave system of the temperate oceans is the semi-diurnal tidal cycle. South of $30^{\circ} \mathrm{N}$ there is also a strong diurnal tidal cycle. A semi-diurnal tidal field can be produced for testing in this model by simply adjusting the frequency of the prescribed $w$ for equation (21), and setting the initial profiles with the appropriate frequency of equation (22) and (17).

The amplitude of the motion remains the same as the diurnal cycle case. Because the cycle of wave mixing interactions repeats itself after a $2 \pi$ phase shift, which occurs after 12 hours in this case, only the cases of six and twelve hour phase shifts will be discussed. The 18 and 24 hour phase shifts are merely repeats.

## (1) Six Hour Shift (Figure 47a and b). Figure 8 shows

 the relationship of the heating cycle to the wave motion in this case. The vertical motion is seen to immediately combine with the shallowing mode of the model through the initialization to give a somewhat shallower initial layer. The shallowing mode of the model holds the layer temperature the same as in S2 to hour six, while the verticalFigure 8. Phase relationship of Case IVb (six hour shift) of semi-diurnal waveform to diurnal
motion varies the layer depth, and alters the below layer temperature structure. By hour six a half-cycle of the wave has passed through the model point, and a layer has been produced that is almost one meter deeper than S2 at this time, but which has the same layer temperature. Continued but decreasing downward motion combines with the downward heat flux until hour nine to slowly deepen the layer, but the layer continues to warm. This process continues through hour ten, though upward motion has begun because the heating and entrainment of relatively warm water are greater than the effect of the resistance to deepening due to the surface heat flux, and the small upward motion. By hour eleven the upward motion dominates, and the layer shallows while it cools slightly due to the increased entrainment. At hour twelve the layer is as shallow as it will ever get, and the first wave has passed. Now decreased upward motion followed by downward motion, combined with surface cooling and mixing through a progressively cooler surface mixed layer quickly increases the layer depth to a maximum of 26.7 meters at hour twenty-one. This is deeper than $S 2$ gets in the entire twenty-four hour period. Yet, the layer is not as cool as in the S2 case. By hour twenty-one the upward motion is again dominant and the layer shallows.
(2) Twelve Hour Shift (Figure 47 c and d). Figure 9 shows the relationship of the heating cycle with the wave in this case. The initialization combines with the vertical motion and shallowing mode to produce an initial layer that is deeper than that in $S 2$, but soon becomes shallower, as the new surface layer is established. By hour six a layer of the same temperature as $S 2$, but .72 meters shallower


Figure 9. Phase relationship of Case IVb (twelve hour shift).
has been formed. The net result of the cyclical interactions in this case is a layer that is 1.5 meters deeper than in S2, but which is at about the same temperature. The difference in the layer temperature between this run, and the previous is $.004^{\circ} \mathrm{C}$. However, the layer depth is 2.2 m different.
(3) Two-Dimensional Data (Figure 48, 49 and 50). At hour six a very discernable depth structure has been set up at the bottom of the mixed layer which reflects the structure of the passing wave. Above the entrainment zone the temperature structure, as produced by the shallowing mode, is homogeneous. Below the layer the structure is highly varied in $x$ reflecting the combined effects of the vertical motion to this time. The mean layer depth is 11.75 meters, while that of $S 2$ is 11.77 m , showing the domination of the shallowing to this point. A variation of $h$ in $x$ of $\pm .75$ meters is already exhibited.

By hour twelve the mixed layer depth structure in $x$, and the layer temperature variability in x have evolved considerably over that of hour six. At the entrainment zone the structure is approximately sinusoidal, as expected. Over the last six hours a large amount of water from below the layer has been entrained while surface heating has occurred, and the mixed layer has heated and cooled accordingly to begin to set up the pattern of warm and cold bands of temperature in $x$. The mean depth of the layer is now 15.21 meters $\pm 2.50$ meters, while that for S 2 is 15.04 meters. The temperature variation in $x$ as seen at the surface is now $.012^{\circ} \mathrm{C}$.

The structure of temperature and layer depth profile has been further accented by hour eighteen. The layer has continued

to deepen, and the fluctuation in $x$ increased. The banding of the in layer temperature pattern in x has become more defined.

After twenty-four hours a layer has been produced that has a depth of $26.14 \mathrm{~m}, \pm 3.3 \mathrm{~m}$. The surface temperature pattern in $x$ is quite clearly defined, though the variation in temperature is only. $.015^{\circ} \mathrm{C}$.
5. Case V (Superimposed Waves) (Figure 51, 52 and 53)

Since oceanic internal waves are almost never simple sinusoids, one more progression is considered. Spectral analysis of the MILE data (Figure 10 ) at station $P\left(55^{\circ} \mathrm{N}\right)$ shows the dominant wave field to be the semi-diurnal tide superimposed by, at least, the 6-7 hour harmonic of the tidal wave. Internal wave theory indicates that the period of the internal waves must be less than or equal to the inertial period. North of approximately $30^{\circ} \mathrm{N}$, no diurnal internal waves may propagate, and therefore, they are not seen in the MILE data. South of this, however, diurnal and semi-diurnal tidal period waves may propagate. Therefore, since previous discussions have been concerned with diurnal and semi-diurnal waves, and there is justification for their presence, these waves will be superimposed to form a slightly more complicated wave system.

The combined wave is directed through the two-dimensional ocean which has the initial conditions of $S 2$, from right to left (large $k x$ to small $k x$ ). The amplitude of the motion is such that both waves peak at the same amplitude at their peak motions, though one has half the period of the other. Both waves start with an amplitude of vertical motion in phase at time one.

Figure 10. HILE spectral analysis (of temperature variation at $50 \mathrm{~m}(\square), 47 \mathrm{~m}(+)$, and $32 \mathrm{~m}(\mathcal{D})$.

The superimposed waves reflect all of the interactions discussed in the previous sections. Further, this simple addition of two waveforms has developed several new features. The new features can be traced in their development from hour one through hour twenty-four, where they become most evident. Since this is the case, the discussion will be limited to the case of the two-dimensional ocean at hour twentyfour.

By this time a complex structure has emerged for the two-dimensional ocean, both in temperature and in profile of the layer depth. An ocean has been formed that has a mean layer depth of 26.78 m , but this depth has a variation that is not symmetrical from small to large x . At small $x$ the variation from the mean depth is seen to be a maximum of -5.45 m , while at large x the layer depth varies to a depth deviation from the mean of +7.83 m . An ocean has been formed that has a deep layer at one end, and a shallow layer at the other. Further, a deviation of the surface temperature in the $x$ direction of $.042^{\circ} \mathrm{C}$ is seen. The pattern of the warm and cold bands again is seen, but this also is no longer symmetrical. The temperature pattern, as seen at the surface, is a cold band beginning at small kx, progressing through a weak gradient to a warm band centered near $k x=15$. A strong gradient is then encountered leading to a cold band centered about $k x=21$. Through the introduction of a simple two wave system, an extremely diverse ocean has been produced.

Shifting the phase relationships of the two internal waves, and running the model again, resulted in similar patterns. Positions of the temperature maxima and minima in the layer, and the position of the depth variations shift, but their relative positions are maintained.
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The temperature variation in the $x$ direction as seen at the surface does not change with the phase changes.

## IV. CONCLUSIONS AND RECOMMENDATIONS

The analysis of the mixing response to the prescribed vertical motion shows that vertical advection can be successfully introduced into this one-dimensional mixed layer model without numerical difficulties. This is important in that it indicates that a single station forecasting model can be altered to include the variability of long internal wave motion. This step is the first of a progressive development of a single station forecasting model that may in the future include other local advective effects.

Using an ensemble of one-dimensional stations, a two-dimensional mixing response to prescribed internal wave motion was simulated. Although the initial temperature field (without wave) and the surface boundary conditions were both horizontally homogeneous, a significant wave-induced horizontal structure was created in the $T(x, z)$ field. Both linear and non-linear effects have been demonstrated in the simple single wave and superimposed wave cases. Horizontal variability has been shown, both in mixed layer depth and in mixed layer temperature structure. The mixed layer temperature develops a corrollary structure of alternating warm and cold patches. In the case of the superimposed waves of different frequencies, this patchiness pattern can be complex, and can give rise to horizontal gradients which could be frontogenetic.

There is an important relationship between the phase of internal motion and the cyclical diurnal heat flux boundary condition. This is clearly demonstrated in the numerical experiments in which the phase of the input pulse or wave system has been varied.


In the case of superimposed diurnal and semi-diurnal waves, the horizontal variation in temperature in the mixed layer was about $.04^{\circ} \mathrm{C}$ after only a single diurnal cycle. In the same period, the mixed layer depth varied by about 13 m . The wave dispersion relation for a two layer ocean at $25^{\circ} \mathrm{N}$ gives a horizontal wave length scale of from 10 to 100 Km , depending upon stratification. This wave-mixing interaction may, therefore, be an important mechanism for the generation of sea surface patchiness as found by Butler (1981).

Further study must be done in this area. Clearly, more complicated waveforms will produce more complex responses. The model should be tested with salinity changes, including evaporation and precipitation. Tests should be conducted on data sets such as those of iIILE. Comparison of the mixed layer model results using liILE boundary conditions, with and without vertical advection, as observed in spectrum analysis, should further demonstrate the importance of addition of vertical motion to mixed layer models.

Finally, the wave systems should be tested in an interactive mode with the mixing of the model. Energy exchanges, as prescribed by wave theory and the studies of Linden (1975), and Kantha et al. (1977) should be allowed, and the relative significance of the different mechanism should be evaluated. Internal vertical motion is an important dynamic variable, and should be treated as such in the future of mixed layer modelling.

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| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 19.3 | 15.001 | 34.000 | 0.50 | - 55.7 | 0.0 | 0.0 |
| 1 | 2 | 18.1 | 15.009 | 34.000 | 0.50 | -201.5 | 0.0 | 0.0 |
| 1 | 3 | 16.2 | 15.024 | 34.000 | 0.50 | -326.8 | 0.0 | 0.0 |
| 1 | 4 | 13.0 | 15.047 | 34.000 | 0.50 | -423.0 | 0.0 | 0.0 |
| 1 | 5 | 12.1 | 15.075 | 34.000 | 0.50 | -483.5 | 0.0 | 0.0 |
| 1 | 6 | 11.8 | 15.105 | 34.000 | 0.50 | -504.2 | 0.0 | 0.0 |
| 1 | 7 | 12.1 | 15.132 | 34.000 | 0.50 | -483.7 | 0.0 | 0.0 |
| 1 | 3 | 12.5 | 15.154 | 34.000 | 0.50 | -423.5 | 0.0 | 0.0 |
| 1 | 9 | 13.0 | 15.168 | 34.000 | 0.50 | -327.5 | 0.0 | 0.0 |
| 1 | 10 | 13.5 | 15.175 | 34.000 | 0.50 | -202.4 | 0.0 | 0.0 |
| 1 | 11 | 14.2 | 15.172 | 34.000 | 0.50 | - 56.6 | 0.0 | 0.0 |
| 1 | 12 | 15.0 | 15.160 | 34.000 | 0.50 | 99.9 | 0.0 | 0.0 |
| 1 | 13 | 15.9 | 15.149 | 34.000 | 0.50 | 100.8 | 0.0 | 0.0 |
| 1 | 14 | 18.7 | 15.139 | 34.000 | 0.50 | 100.8 | 0.0 | 0.0 |
| 1 | 15 | 17.5 | 15.129 | 34.000 | 0.50 | 100.8 | 0.0 | 0.0 |
| 1 | 16 | 18.4 | 15.120 | 34.000 | 3.50 | 100.8 | 0.0 | 0.0 |
| 1 | 17 | 19.2 | 15.112 | 34.000 | 0.50 | 100.8 | 0.0 | 0.0 |
| 1 | 18 | 20.0 | 15.104 | 34.000 | 0.50 | 100.8 | 0.0 | 0.0 |
| 1 | 19 | 20.9 | 15.097 | 34.000 | 0.50 | 100.8 | 0.0 | 0.0 |
| 1 | 20 | 21.8 | 15.090 | 34.000 | 0.50 | 100.8 | 0.0 | 0.0 |
| 1 | 21 | 22.7 | 15.083 | 34.000 | 0.50 | 100.8 | 0.0 | 0.0 |
| 1 | 22 | 23.6 | 15.077 | 34.000 | 0.50 | 100.8 | 0.0 | 0.0 |
| 1 | 23 | 24.6 | 15.071 | 34.000 | 0.50 | 100.8 | 0.0 | 0.0 |
| 1 | 24 | 25.7 | \$5.066 | 34.000 | 0.50 | 100.8 | 0.0 | 0.0 |
| 2 | 1 | 26.2 | 15.066 | 34.000 | 0.50 | - 55.7 | 0.0 | 0.0 |
| 2 | 2 | 22.4 | 15.073 | 34.000 | 0.50 | -201.5 | 0.0 | 0.0 |
| 2 | 3 | 15.1 | 15.089 | 34.000 | 0.50 | -325.8 | 0.0 | 0.0 |
| 2 | 4 | 13.2 | 15.112 | 34.000 | 0.50 | -423.0 | 0.0 | 0.0 |
| 2 | 5 | 12.0 | 15.140 | 34.000 | 0.50 | -483.5 | 0.0 | 0.0 |
| 2 | 6 | 11.7 | 15.169 | 34.000 | 0.50 | -504.2 | 0.0 | 0.0 |
| 2 | ? | 12.0 | 15.197 | 34.000 | 0.50 | -483.7 | 0.0 | 0.0 |
| 2 | B | 12.4 | 15.219 | 34.000 | 0.50 | -423.5 | 0.0 | 0.0 |
| 2 | 9 | 12.9 | 15.234 | 34.000 | 0.50 | -327.5 | 0.0 | 0.0 |
| 2 | 10 | 13.4 | 15.240 | 34.000 | 0.50 | -202.4 | 0.0 | 0.0 |
| 2 | 11 | 14.1 | 15.237 | 34.000 | 0.50 | - 56.6 | 0.0 | 0.0 |
| 2 | 12 | 14.9 | 15.225 | 34.000 | 0.50 | 99.9 | 0.0 | 0.0 |
| 2 | 13 | 15.7 | 15.214 | 34.000 | 0.50 | 100.8 | 0.0 | 0.0 |
| 2 | 14 | 16.5 | 15.203 | 34.000 | 0.50 | 100.3 | 0.0 | 0.0 |
| 2 | 15 | 17.4 | 15.194 | 34.000 | 0.50 | 100.8 | 0.0 | 0.0 |
| 2 | 16 | 18.2 | 15.185 | 34.000 | 0.50 | 100.8 | 0.0 | 0.0 |
| 2 | 17 | 19.1 | 15.177 | 34.000 | 0.50 | 100.8 | 0.0 | 0.0 |
| 2 | 18 | 20.0 | 15.169 | 34.000 | 0.50 | 100.8 | 3.0 | 0.0 |
| 2 | 19 | 20.9 | 15.161 | 34.000 | 0.50 | 100.8 | 0.0 | 0.0 |
| 2 | 20 | $21.9{ }^{\circ}$ | 15.155 | 34.000 | 0.50 | 100.8 | 0.0 | 0.0 |
| 2 | 21 | 22.8 | 15.143 | 34.000 | 0.30 | 100.8 | 0.0 | 0.0 |
| 2 | 22 | 23.8 | 15.142 | 34.000 | 0.50 | 100.3 | 0.0 | 0.0 |
| 2 | 23 | 24.7 | 15.136 | 34.000 | 0.50 | 100.8 | 0.0 | 0.0 |
| 2 | 24 | 25.6 | 15.130 | 34.000 | 0.50 | 100.8 | 0.0 | 0.0 |

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| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 6.0 | 14.956 | 34.000 | 0.50 | 3.5 | -. 03600 | 22.140 |
| 1 | 2 | 6.9 | 14.937 | 34.000 | 0.50 | -100.8 | -. 03600 | 25.258 |
| 1 | 3 | 7.3 | 14.931 | 34.000 | 0.50 | -184.3 | -. 03600 | 26.750 |
| 1 | 4 | 7.6 | 14.934 | 34.000 | 0.50 | $-248.4$ | -. 03600 | 27.962 |
| 1 | 5 | 7.8 | 14.941 | 34.000 | 0.50 | -288.7 | -. 03600 | 28.725 |
| 1 |  | 7.9 | 14.949 | 34.000 | 0.50 | -302.5 | -. 03600 | 28.917 |
| 1 | 7 | 7.9 | 14.957 | 34.000 | 0.50 | -288.9 | .. 03600 | 29.021 |
| 1 | 8 | 7.9 | 14.961 | 34.000 | 0.50 | -248.7 | -. 03600 | 29.089 |
| 1 | 9 | 8.0 | 14.959 | 34.000 | 0.50 | $-184.7$ | -. 03600 | - 29.169 |
| 1 | 10 | 8.0 | 14.949 | 34.000 | 0.50 | -101.3 | -. 03600 | - 29.288 |
| 1 | 11 | 8.0 | 14.930 | 34.000 | 0.50 | 4. 1 | -. 03600 | - 29.467 |
| 1 | 12 | 3.1 | 14.898 | 34.000 | 0.50 | 100.2 | -. 03600 | 29.817 |
| 1 | 13 | 3.3 | 14.869 | 34.800 | 0.50 | 100.8 | -. 03600 | 30.262 |
| 1 | 14 | 8.4 | 14.342 | 34.000 | 0.50 | 100.8 | -. 03600 | 30.793 |
| 1 | 15 | 9.6 | 14.815 | 34.000 | 0.50 | 100.8 | -. 03600 | 31.417 |
| 1 | 16 | 3.7 | 14.789 | 34.000 | 0.50 | 100.8 | -. 03600 | 31.994 |
| 1 | 17 | 3.7 | 14.763 | 34.000 | 0.50 | 100.8 | -. 03600 | 31.892 |
| 1 | 18 | 3.7 | 14.735 | 34.000 | 0.50 | 100.8 | -. 03600 | 31.779 |
| 1 | 19 | 9.7 | 14.709 | 34.000 | 0.50 | 100.8 | -. 03600 | - 31.733 |
| 1 | 20 | 3.7 | 14.683 | 34.000 | 0.50 | 100.8 | -. 03600 | - 31.750 |
| 1 | 21 | 3.7 | 14.657 | 34.000 | 0.50 | 100.8 | -. 03600 | - 31.832 |
| 1 | 22 | 8.6 | 14.631 | 34.000 | 0.50 | 100.3 | -. 03600 | - 31.625 |
| 1 | 23 | 3.6 | 14.605 | 34.000 | 0.50 | 100.8 | -. 03600 | - 31.475 |
| 1 | 24 | 3.6 | 14.573 | 34.000 | 0.50 | 100.8 | -. 03600 | 31.384 |
| : | 1 | a. 5 | 14.556 | 34.200 | 0.50 | - 3.5 | -. 03600 | - 31.297 |
| 2 | 2 | 3.5 | 14.547 | 34.000 | 0.50 | -100.8 | -. 03600 | - 31.155 |
| 2 | 3 | 3. 3 | 14.546 | 34.000 | 0.50 | -184.3 | -. 03600 | - 30.956 |
| 2 | + | 9.3 | 14.551 | 34.000 | 0.50 | -248.4 | -. 03600 | - 30.709 |
| 2 | 5 | a. 3 | 14.559 | 34.000 | 0.30 | -288.7 | -. 03600 | - 30.435 |
| 2 | 6 | 9.2 | 14.554 | 34.000 | 0.50 | -302.5 | -. 03600 | - 30.162 |
| 2 | 7 | 9. 2 | 14.562 | 34.000 | 0.50 | -288.9 | -. 03600 | - 29.951 |
| 2 | * | 8.1 | 14.567 | 34.000 | 0.50 | -248.7 | -. 03600 | - 29.802 |
| 2 | 9 | 9.1 | 14.565 | 34.000 | 0.50 | -184.7 | -. 03600 | - 29.745 |
| 2 | 10 | 8.1 | 14.555 | 34.000 | 0.50 | -101.3 | -. 03600 | - 29.308 |
| 2 | 11 | 9.2 | 14.535 | 34.000 | 0.50 | - 4.1 | -. 03600 | - 30.018 |
| 2 | 12 | 8.3 | 14.417 | 34.000 | 0.50 | 100.2 | -. 03600 | - 30.407 |
| 2 | 13 | 8.5 | 14.449 | 34.000 | 0.50 | 100.8 | -. 03600 | - 31.019 |
| 2 | 14 | 9.7 | 14.422 | 34.000 | 0.50 | 100.8 | -. 03600 | - 31.729 |
| 2 | 15 | 8.7 | 14.396 | 34.000 | 0.50 | 10C. 8 | -. 03600 | - 31.972 |
| 2 | 16 | a. 5 | 14. 369 | 34.000 | 0.50 | 100.8 | -. 03600 | - 31.256 |
| 2 | 17 | 8. 3 | 14.341 | 34.000 | 0.50 | 100.8 | . . 03600 | - 30.583 |
| 2 | 19 | 8.2 | 14.304 | 34.000 | 0.50 | 100.8 | . .03600 | - 29.953 |
| 2 | 19 | 8.0 | 14.274 | 34.000 | 0.50 | 100.8 | -. 03600 | - 29.370 |
| 2 | 20 | 7.9 | 14.243 | 34.000 | 0.50 | 100.8 | -. 03600 | - 28.828 |
| 2 | 21 | 7.7 | 14.212 | 34.000 | 0.50 | 100.8 | -. 03600 | - 28.323 |
| 2 | 22 | 7.6 | 14.179 | 34.000 | 0.50 | 100.8 | -. 03600 | - 27.792 |
| 2 | 23 | 1.4 | 14.146 | 34.000 | 0.50 | 100.8 | -. 03600 | - 27.302 |
| 2 | 24 | 7. | 14.095 | 34.000 | 0.50 | 100 | 0 | - 26.35 |

(a)









| Qnet | W0 |
| :---: | :---: |
| - 3.5 | 0.03477 |
| -100.8 | 0.03118 |
| -184.3 | 0.02546 |
| -248.4 | 0.01800 |
| -288.7 | 0.00932 |
| -302.5 | 0.0 |
| -288.7 | 0.0 |
| -248.7 | 0.0 |
| -184.7 | 0.0 |
| -101.3 | 0.0 |
| - 4.1 | 0.0 |
| 100.2 | 0.0 |
| 100.8 | 0.0 |
| 100.8 | 0.0 |
| 100.8 | 0.0 |
| 100.8 | 0.0 |
| 100.8 | 0.0 |
| 100.8 | 0.0 |
| 100.8 | 0.00932 |
| 100.8 | 0.01800 |
| 1.00 .8 | 0.02546 |
| 100.8 | 0.03118 |
| 100.8 | 0.03477 |
| 100.8 | 0.03600 |
| - 3.5 | 0.03477 |
| -100.8 | 0.03118 |
| -184.3 | 0.02546 |
| -248.4 | 0.01800 |
| -288.7 | 0.00932 |
| -302.5 | 0.0 |
| -288.7 | 0.0 |
| -248.7 | 0.0 |
| -184.7 | 0.0 |
| -101.3 | 0.0 |
| - 4.1 | 0.0 |
| 100.2 | 0.0 |
| 100.8 | 0.0 |
| 100.8 | 0.0 |
| 100.8 | 0.0 |
| 100.8 | 0.0 |
| 100.8 | 0.0 |
| 100.8 | 0.0 |
| 100.8 | 0.00932 |
| 100.8 | 0.01500 |
| 100.8 | 0.02546 |
| 100.8 | 0.03118 |
| 100.8 | 0.03477 |

DH

### 22.156 <br> 23.156 23.607 <br> 21.546 16.469 <br> 16.469 8.979 0.0 <br> 0.0 0.0 0.0 0.0 0.0 0.0 <br> 0.0 0.0 <br> 0.0 0.0 0.0 <br> 0.0 0.0 0.0 <br> 0.0 0.0 0.0 0.0 0.0 13.167 26.290 6.290 9.712 5.5827 63.132 <br> 63.342 <br> 59.542 59.561 50.219 <br> 50.219 36.313 <br> 36.313 16.670 <br> 0.0 0.0 <br> 0.0 0.0 <br> 0.0 0.0 0.0 <br> 0.0 0.0 0.0 <br> 0.0 0.0 0.0 <br> 0.0 0.0 0.3 <br> 0.3 0.0 <br> 0.0 0.0 <br> 1.0 0.0 0.0 20.705 <br> 0.0 20.705 <br> 10.866 59.500 <br> 59.500 5.477 <br> 94.494

(a)

Figure 17. Same as figure (1), except for Case Ila wath peak at hour 24 .

电

| oay | HR | 8 | 5 | 5 | tau | QNET | woo | OH |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 49.5 | 15.001 | 34.000 | 0.50 | - 55.7 | 0.0 | 0.c |
| 1 | 2 | 18.1 | 15.009 | 34.000 | 0.50 | -201.5 | 0.0 | 0.0 |
| 1 | 3 | 16.2 | 15.024 | 34.000 | 0.50 | -326.8 | 0.0 | 0.0 |
| 1 | 4 | 13.0 | 15.047 | 34.000 | 0.50 | -423.0 | 0.0 | 0.0 |
| 1 | 5 | 12.1 | 15.075 | 34.000 | 0.50 | -483.5 | 0.0 | 0.0 |
| 1 | 6 | 11.8 | 15.105 | 34.000 | 0.50 | -504.2 | 0.0 | 0.0 |
| 1 | 7 | 12.1 | 15.132 | 34.000 | 0.50 | -483.7 | 0.0 | 0.0 |
| 2 | - | 12.5 | 15.154 | 34.000 | 0.50 | -423.5 | 0.0 | 0.0 |
| 1 | * | 13.0 | 15.168 | 34.000 | 0.50 | -327.5 | 0.0 | 0.0 |
| 1 | 10 | 13.5 | 15.175 | 34.000 | 0.50 | -202.4 | 0.0 | 0.0 |
| 1 | 11 | 14.2 | 15.172 | 34.000 | 0.50 | - 56.6 | 6.0 | 0.0 |
| 1 | 12 | 15.0 | 15.160 | 34.000 | 0.50 | 99.9 | 0.0 | 0.0 |
| 1 | 13 | 16.0 | 15.149 | 34.000 | 0.50 | 100.8 | 0.00932 | 14.869 |
| 1 | 14 | 17.2 | 15.139 | 34.000 | 0.50 | 100.8 | 0.01800 | 30.739 |
| 1 | 15 | 18.6 | 15.130 | 34.1000 | 0.50 | 100.8 | 0.02546 | 46.756 |
| 1 | 16 | 20.0 | 15.122 | 34.000 | 0.50 | 100.8 | 0.03118 | 61.334 |
| 1 | 17 | 21.5 | 15.314 | 34.000 | 0.50 | 100.8 | 0.03477 | 73.399 |
| 1 | 18 | 23.1 | 15.107 | 34.000 | 0. 50 | 100.8 | 0.03600 | 81.528 |
| 1 | 1.3 | 24.7 | 15.101 | 34.000 | 0.50 | 100.8 | 0.03477 | 84.247 |
| 1 | 20 | 26.2 | 15.096 | 34.000 | 0.50 | 100.8 | 0.03118 | 80.442 |
| 1 | 21 | 27.7 | 15.090 | 34.300 | 0.50 | 160.8 | 0.02546 | 69.511 |
| 1 | 22 | 23.9 | 15.086 | 34.000 | C. 50 | 100.8 | 0.01800 | 51.613 |
| 1 | 23 | 30.0 | 15.081 | $3 \pm .000$ | 0.50 | 100.8 | 0.00932 | 27.799 |
| - | 24 | 30.8 | 15.077 | 34.000 | 0.50 | 100.8 | 0.0 | 0.0 |
| : | 1 | 31.1 | 15.077 | 34.000 | 0.50 | - 55.7 | 0.0 | 0.0 |
| 2 |  | 21.0 | 15.085 | 34.000 | 0.50 | -201.5 | 0.0 | 0.0 |
| 2 |  | 15.4 | 15.101 | 34.000 | 0.50 | -326.8 | 0.0 | 0.0 |
| 2 | 4 | 13.1 | 15.12: | 34.000 | G. 50 | - 423.0 | 0.0 | 0.0 |
| 2 | 5 | 12.0 | 15.152 | 34.600 | 0.50 | -483.5 | c. 0 | 0.0 |
| 2 | 6 | 11.7 | 15.181 | 34.000 | 0.50 | -504.2 | 0.0 | 0.0 |
| 2 |  | 12.0 | 15.208 | 34.000 | 0.50 | -483.7 | 0.0 | 0.0 |
| 2 | 9 | 12.4 | 15.231 | 34.000 | 0.50 | -423.5 | 0.0 | 0.0 |
| 2 | , | 12.9 | 15.245 | 34.000 | 0.50 | -327.5 | 0.0 | 0.0 |
| 2 | 10 | 13.4 | 15.251 | 34.060 | 0.50 | -202.4 | 0.0 | 0.0 |
| 2 | 11 | 14.1 | 15.248 | 34.000 | 0.50 | - 56.6 | 0.0 | 0.0 |
| 2 | 12 | 14.9 | 15.236 | 34.000 | 0.50 | 79.9 | 0.0 | 0.0 |
| 2 | 13 | 15.9 | 15.225 | 34.000 | 0.50 | 100.8 | 0.00932 | 14.75 |
| 2 | 14 | 17.0 | 15.215 | 34.000 | c. 50 | 100.8 | 0.01800 | 30.35 |
| 2 | 15 | 19.3 | 15.206 | 34.000 | c. 50 | 100.8 | 0.02546 | 46.03 |
| 2 | 16 | 19.8 | 15.198 | 34.000 | 3.50 | 100.6 | 0.03118 | 60.69 |
| 2 | 17 | 21.3 | 15.190 | 34.000 | 0.50 | 100.8 | 0.03477 | 72.79 |
| 2 | 16 | 22.9 | 15.183 | 34.000 | 0.50 | 100.8 | 0.03600 | 81.05 |
| 2 | 19 | 24.6 | 15.177 | 34.000 | 0.50 | 100.8 | 0.03477 | 83.92 |
| 2 | 20 | 26.2 | 15.172 | 34.000 | c. 50 | 100.9 | 0.03118 | 80.28 |
| 2 | 21 | 27.6 | 15.166 | 34.000 | 0.50 | 100.8 | 0.02546 | 69.477 |
| 2 | 22 | 28.9 | 15.162 | 34.000 | 0.50 | 100.8 | 0.01800 | 51.644 |
| 2 | 23 | 30.0 | 15.157 | 34.600 | 0.50 | 106. ${ }^{\text {a }}$ | 0.00932 | 27.833 |
| 2 | 24 | 30.8 | 15.153 | 34.000 | c. 50 | 100.8 | 0.0 | 3.0 |

(a)



(a)

(b)

$$
\begin{gathered}
\text { UfP111 } \\
32.00
\end{gathered}
$$

$$
\begin{aligned}
& \\
& \vdots \\
& \dot{3} \\
& \dot{9} \\
& \hline \\
& \dot{3} \\
& \dot{9}-1
\end{aligned}
$$

Figure 20. Same as Eigure (1), except for Case Itb with paak at hour 24 . OtP[H (M)
(a)


| OAY | HR | 4 | T | 3 | tad | Qset | W00 | OH |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 6.0 | 14.956 | 34.000 | 0.50 | - 3.5 | -. 03477 | -21.398 |
| 1 | 2 | 6.9 | 14.937 | 34.000 | 0.50 | -100.8 | -. 03118 | -21.939 |
| 1 | 3 | 7.4 | 14.932 | 34.000 | 0.50 | -184.3 | -. 02546 | -19.111 |
| 1 | 4 | 7.9 | 14.934 | 34.000 | 0.50 | -248.4 | -. 01800 | -14.295 |
| 1 | 5 | 8.2 | 14.943 | 14.000 | 0.50 | -288.7 | -. 00932 | - 7.661 |
| 1 | 6 | 8.5 | 14.950 | 34.000 | 0.50 | -302.5 | 0.0 | 0.0 |
| 1 | 7 | 8.9 | 14.960 | 34.000 | 0.50 | -288.9 | 0.0 | 0.0 |
| 1 | 8 | 9.2 | 14.966 | 34.000 | 0.50 | -248.7 | 0.0 | 0.0 |
| 1 | 9 | 9.5 | 14.967 | 34.000 | 0.50 | -184.7 | 0.0 | 0.0 |
| 1 | 10 | 9.8 | 14.962 | 34.000 | 0.50 | -101.3 | 0.0 | 0.0 |
| 1 | 11 | 10.1 | 14.949 | 34.000 | 0.50 | - 4.1 | 0.0 | 0.0 |
| 1 | 12 | 10.4 | 14.927 | 34.000 | 0.50 | 100.2 | 0.0 | 0.0 |
| 1 | 13 | 10.8 | 14.708 | 34.000 | 0.50 | 100.8 | 0.0 | 0.0 |
| 1 | 14 | 11.1 | 14.889 | 34.000 | 0.50 | 100.8 | 0.0 | 0.0 |
| 1 | 15 | 11.4 | 14.871 | 34.000 | 0.50 | 100.8 | 0.0 | 0.0 |
| 1 | 16 | 11.7 | 14.854 | 34.000 | 0.50 | 100.8 | 0.0 | 0.0 |
| 1 | 17 | 12.0 | 14.838 | 34.000 | 0.50 | 100.8 | 0.0 | 0.0 |
| 1 | 18 | 12.3 | 14.821 | 34.000 | 0.50 | 100.8 | 0.0 | 0.0 |
| 1 | 19 | 12.5 | 14.806 | 34.000 | 0.50 | 100.8 | -. 00932 | -11.687 |
| 1 | 20 | 12.5 | 14.791 | 34.000 | 0.50 | 100.8 | -. 01800 | -22.791 |
| 1 | 21 | 12.5 | 14.776 | 34.000 | 0.50 | 100.8 | -. 02546 | -32.299 |
| 1 | 22 | 12.4 | 14.761 | 34.000 | 0.50 | 100.8 | -. 03118 | -39.426 |
| 1 | 23 | 12.3 | 14.746 | 34.000 | 0.50 | 100.8 | -. 03477 | -43.687 |
| 1 | 24 | 12.2 | 14.727 | 34.300 | 0.50 | 100.8 | -. 03600 | -44.903 |
| 2 | 1 | 12.2 | 14.719 | 34.000 | 0.50 | - 3.5 | -. 03477 | -43.006 |
| 2 | 2 | 12.1 | 14.717 | 34.000 | 0.50 | -100.8 | -. 03118 | -38.216 |
| 2 | 3 | 12.0 | 14.721 | 34.000 | 0.50 | -184.3 | -. 02546 | -30.974 |
| 2 | + | 12.0 | 14.730 | 34.000 | 0.50 | -248.1 | -. 01800 | -21.820 |
| 2 | 5 | 12.1 | 14.742 | 34.000 | 0.50 | -288.7 | -. 00932 | -11.317 |
| 2 | 6 | 12.3 | 14.749 | 34.000 | 0.50 | -302.5 | 0.0 | 0.0 |
|  | 7 | 12.4 | 14.761 | 34.000 | 0.50 | -288.9 | 0.0 | 0.0 |
| 2 | 8 | 12.6 | 14.770 | 34.000 | 0.50 | -248.7 | 0.0 | 0.0 |
| : | $\rightarrow$ | 12.3 | 14.774 | 34.000 | 0.50 | -184.7 | 0.0 | 0.0 |
| 2 | 10 | 13.0 | 14.774 | 34.000 | 0.50 | -101.3 | 0.0 | 0.0 |
| 2 | 11 | 13.2 | 14.766 | 34.000 | 0.50 | - 4.1 | 0.0 | 0.0 |
| 2 | 12 | 13.4 | 14.750 | 34.300 | 0.50 | 100.2 | 0.0 | 0.0 |
| 2 | 13 | 13.6 | 14.736 | 34.000 | 0.50 | 100.8 | 0.0 | 0.0 |
| 2 | 14 | 13.7 | 14.723 | 34.000 | 0.50 | 100.8 | 0.0 | 0.0 |
| 2 | 15 | 13.9 | 14.710 | 34.000 | 0.50 | 100.8 | 0.0 | 0.0 |
| 2 | 16 | 14.1 | 14.697 | 34.000 | 0.50 | 100.8 | 0.0 | 0.0 |
| 2 | 17 | 14.2 | 14.684 | 34.000 | 0.50 | 100.8 | 0.0 | 0.0 |
| 2 | 19 | 14.4 | 14.670 | 34.000 | 0.50 | 100.8 | 0.0 | 0.0 |
|  | 19 | 14.4 | 14.658 | 34.000 | 0.50 | 100.8 | -. 00932 | -13.497 |
| 2 | 20 | 14.3 | 14.645 | 34.000 | 0.50 | 200.8 | -. 01800 | -26.005 |
| 2 | 21 | 14.1 | 14.633 | 34.000 | 0.50 | 100.8 | -. 02546 | -36.398 |
|  | 22 | 13.8 | 14.620 | 34.000 | 0.50 | 100.8 | -. 031118 | -43.855 |
| 2 | 23 | 13.5 | 14.607 | 34.000 | 0.50 | 100.8 | -. 03477 | -47.921 |
| 2 | 24 | 13.2 | 14.592 | 34.000 | 0.50 | 100.8 | -. 03600 | -48.452 |

(a)

$\begin{array}{lll} & \\ 14.7560 & 5.0473 & \\ 10.9370 & 5.3279 & \\ 14.9310 & 7.124 & \\ 14.9340 & 7.9100 & \\ 14.9420 & 9.1815 & \text { (C }\end{array}$
Figure 22 Same as figure (1), except
for Case ilc with peak at hour 24 .


| oay | ER | $\pi$ | $\tau$ | $s$ | rao | QNET | WDD | Dr |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 47.8 | 15.001 | 34.000 | 0.50 | - 55.7 | -. 03477 | -169.223 |
| 1 | 2 | 17.7 | 15.009 | 34.000 | 0.50 | -201.5 | -. 03118 | - 56.143 |
| 1 | 3 | 15.9 | 15.324 | 34.000 | 0.50 | -326.8 | -. 02546 | - 40.938 |
| 1 | 4 | 12.9 | 15.047 | 34.000 | 0.50 | -423.0 | -. 01800 | - 23.344 |
| 1 | 5 | 12.0 | 15.075 | 34.000 | 0.50 | -483.5 | -. 00932 | - 11.218 |
| 1 | 6 | 11.3 | 15.104 | 34.000 | 0.50 | -504.2 | -. 00000 | - 0.000 |
| 1 | 7 | 12.1 | 15.131 | 34.000 | 0.50 | -483.7 | 0.0 | 0.0 |
| 1 | 8 | 12.5 | 15.153 | 34.000 | 0.50 | -423.5 | 0.0 | 0.0 |
| 1 | 9 | 12.9 | 15.168 | 34.000 | 0.50 | -327.5 | 0.0 | 0.0 |
| 1 | 10 | 13.5 | 15.174 | 34.000 | 0.50 | -202.4 | 0.0 | 0.0 |
| 1 | 11 | 14.1 | 15.171 | 34.000 | 0.50 | - 56.6 | 0.0 | 0.0 |
| 1 | 12 | 15.0 | 15.159 | 34.300 | 0.50 | 99.9 | 0.0 | 0.0 |
| 1 | 13 | 15.8 | 15.148 | 34.000 | 0.50 | 100.8 | 0.0 | 0.0 |
| 1 | 14 | 16.6 | 15.138 | 34.000 | 0.50 | 100.8 | 0.0 | 0.0 |
| 1 | 15 | 17.4 | 15.128 | 34.000 | 0.50 | 100.8 | 0.0 | 0.0 |
| 1 | 16 | 18.2 | 15.119 | 34.000 | 0.50 | 100.8 | 0.0 | 0.0 |
| 1 | 17 | 19.0 | 15.111 | 34.000 | 0.50 | 100.8 | 0.0 | 0.0 |
| 1 | :9 | 19.9 | 15.103 | 34.000 | 0.50 | 100.8 | 0.0 | 0.0 |
| 1 | 19 | 20.6 | 15.095 | 34.000 | 0.50 | 100.8 | -. 00932 | - 19.256 |
| 1 | 20 | 21.1 | 15.088 | 34.000 | 0.50 | 100.8 | -. 01800 | - 38.331 |
| 1 | 21 | 21.5 | 15.082 | 34.000 | 0.50 | 100.8 | -. 02546 | - 55.477 |
| 1 | 22 | 21.8 | 15.075 | 34.000 | 0.50 | 100.8 | -. 03118 | - 69.136 |
| 1 | 23 | 22.1 | 15.068 | 34.000 | 0.50 | 100.8 | -. 03477 | - 78.296 |
| 1 | 24 | 22.5 | 15.062 | 34.000 | 0.50 | 100.8 | -. 03600 | - 82.402 |
| 2 | 1 | 22.4 | 15.062 | 34.000 | 0.50 | - 55.7 | -.03477 | - 99.400 |
| : | 2 | 21.9 | 15.069 | 34.000 | 0.50 | -201.5 | -. 03118 | - 69.231 |
| 2 | 3 | 14.3 | 15.085 | 34.000 | 0.50 | -326.8 | -. 02546 | - 38.163 |
| 2 | 4 | 13.0 | 15.108 | 34.000 | 0.50 | -423.0 | -. 01800 | - 23.687 |
| 2 | 5 | 11.9 | 15.136 | 34.000 | 0.50 | -493.5 | -. 00932 | - 11.136 |
| 2 | 6 | 11.3 | 15.165 | 34.000 | 0.50 | -504.2 | -. 00000 | - 0.000 |
| 2 | , | 12.1 | 15.192 | 34.000 | 0.50 | -483.7 | 0.0 | 0.0 |
| 2 | 3 | 12.4 | 15.214 | 34.000 | 0.50 | -423.7 | 0.0 | 0.0 |
| 2 | 9 | 12.9 | 15.229 | 34.000 | 0.50 | -327.5 | 0.0 | 0.0 |
| 2 | 10 | 13.4 | 15.235 | 34.000 | 0.50 | -202.4 | 0.0 | 0.0 |
| 2 | 11 | 14.1 | 15.232 | 34.200 | 0.50 | - 56.6 | 0.0 | 0.0 |
| 2 | 12 | 14.9 | 15.220 | 34.000 | 0.50 | 99.9 | 0.0 | 0.0 |
| 2 | 13 | 15.7 | 15.209 | 34.000 | 0.50 | 100.8 | 0.0 | 0.0 |
| 2 | 14 | 16.5 | 15.199 | 34.000 | 0.50 | 100.8 | 0.0 | 0.0 |
| 2 | 15 | 17.3 | 15.189 | 34.000 | 0.50 | 100.8 | 0.0 | 0.0 |
| 2 | 16 | 18.2 | 15.180 | 34.000 | 0.50 | 100.3 | 0.0 | 0.0 |
| 2 | 17 | 19.0 | 15.172 | 34.200 | 0.50 | 100.8 | 0.0 | 0.0 |
| 2 | 19 | 19.3 | 15.163 | 34.200 | 0.50 | 100.8 | 0.0 | 0.0 |
| 2 | 19 | 20.4 | 15.156 | 34.000 | 0.50 | 100.8 | -. 00932 | - 19.065 |
| 2 | 20 | 20.7 | 15.149 | 34.000 | 0.50 | 100.8 | -. 01800 | - 37.614 |
| 2 | 21 | 20.8 | 15.142 | 34.000 | 0.50 | 100.8 | -. 02546 | - 53.606 |
| 2 | 22 | 20.8 | 15.134 | 34.020 | 0.50 | 100.8 | -. 03118 | -63.729 |
| 2 | 23 | 20.7 | 15.127 | 34.000 | 0.50 | 100.8 | -. 03477 | - 73.101 |
| 2 | 24 | 20.6 | 15.120 | 34.000 | 0.50 | 100.8 | -. 03600 | - 75.372 |

(a)



|  <br>  <br>  <br>  |
| :---: |
|  |  |
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|  |  |
|  |  |

(c)

(a)






4.956 34.000 $\begin{array}{ll}14.956 & 34.000 \\ 14.939 & 14.000\end{array}$


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|  ingino wiogooooooóooooo <br>  |  <br> OHFMOONOGOOOOOOOOOOOOO - riobringosooooo oooooo <br>  |

(a)

| OX ${ }^{\text {a }}$ | 48 | H | T | 5 | tau | QNET | woo | OH |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 49.5 | 15.001 | 34.000 | 0.50 | - 55.7 | 0.0 | 0.0 |
| 1 | 2 | 19.1 | 15.009 | 34.000 | 0.50 | -201.5 | 0.0 | 0.0 |
| 1 | 3 | 16.2 | 15.024 | 34.000 | 0.50 | -326.8 | 0.0 | 0.0 |
| 1 | 4 | 13.0 | 15.047 | 14.000 | 0.50 | -423.0 | 0.0 | 0.0 |
| 1 | 5 | 12.1 | 15.075 | 34.000 | 0.50 | -483.5 | 0.0 | 0.0 |
| 1 | 6 | 11.8 | 15.105 | 34.000 | 0.50 | -504.2 | 0.0 | 0.0 |
| 1 | 7 | 12.1 | 15.132 | 34.000 | 0.50 | -483.7 | 0.0 | 0.0 |
| 1 | 8 | 12.5 | 15.154 | 34.000 | 0.50 | -423.5 | 0.0 | 0.0 |
| 1 | 9 | 13.0 | 15.168 | 34.000 | 0.50 | -327.5 | 0.0 | 0.0 |
| 1 | 10 | 13.5 | 15.175 | 34.000 | 0.50 | -202.4 | 0.0 | 0.0 |
| 1 | 11 | 14.2 | 15.172 | 34.000 | 0.50 | - 56.6 | 0.0 | 0.0 |
| 1 | 12 | 15.0 | 15.160 | 34.000 | 0.50 | 99.9 | 0.0 | 0.0 |
| 1 | 13 | 15.7 | 15.149 | 34.000 | 0.50 | 100.8 | -. 00932 | -14.731 |
| 1 | 14 | 16.3 | 15.138 | 34.000 | 0.50 | 100.8 | -. 01800 | -29.577 |
| 1 | 15 | 16.7 | 15.129 | 34.000 | 0.50 | 100.9 | -. 02546 | -43.104 |
| 1 | 16 | 17.0 | 15.119 | 34.000 | 0.50 | 100.8 | -. 03118 | -53.910 |
| 1 | 17 | 17.3 | 15.110 | 34.000 | 0.50 | 100.8 | -. 03477 | -61.301 |
| 1 | 18 | 17.6 | 15.101 | 34.000 | 0.50 | 100.8 | -. 03600 | -64.690 |
| 1 | 19 | 18.0 | 15.092 | 34.000 | 0.50 | 100.8 | -. 03477 | -63.643 |
| 1 | 20 | 19.4 | 15.084 | 34.000 | 0.50 | 100.8 | -. 03118 | -59.407 |
| 1 | 21 | 19.1 | 15.076 | 34.000 | 0.50 | 100.8 | -. 02546 | -49.130 |
| 1 | 22 | 19.9 | 15.068 | 34.000 | 0.50 | 100.8 | -. 01800 | -36.125 |
| 1 | 23 | 20.9 | 15.061 | 34.000 | 0.50 | 100.8 | -. 00932 | -19.604 |
| 1 | 24 | 22.3 | 15.054 | 34.000 | 0.50 | 100.8 | -. 00000 | -00.001 |
| 2 | 1 | 23.1 | 15.054 | 34.000 | 0.50 | - 55.7 | 0.0 | 0.0 |
| 2 | 2 | 23.1 | 15.061 | 34.000 | 0.50 | -201.5 | 0.0 | 0.0 |
| 2 | , | 14.9 | 15.077 | 34.000 | 0.50 | -326.8 | 0.0 | 0.0 |
| 2 | 4 | 13.3 | 15.100 | 34.000 | 0.50 | -423.0 | 0.0 | 0.0 |
| 2 | 5 | 12.0 | 15.128 | 34.000 | 0.50 | -483.5 | 0.0 | 0.0 |
| 2 | 6 | 11.7 | 15.157 | 34.000 | 0.50 | -504.2 | 0.0 | 0.0 |
| 2 | 7 | 12.1 | 15.1 a4 | 34.000 | 0.50 | -483.5 | 0.0 | 0.0 |
| 2 | 8 | 12.4 | 15.206 | 34.000 | 0.50 | -423.5 | 3.0 | 0.0 |
| 2 | 9 | 12.9 | 15.221 | 34.000 | 0.50 | -327.5 | 0.0 | 0.0 |
| 2 | 10 | 13.5 | 15.227 | 14.000 | 0.50 | -202.4 | 0.0 | 0.0 |
| 2 | 11 | 14.2 | 15.224 | 34.000 | 0.50 | - 56.\% | 0.0 | 0.0 |
| 2 | 12 | 15.0 | 15.212 | 34.000 | 0.50 | 99.9 | 0.0 | 0.0 |
| 2 | 13 | 15.6 | 15.201 | 34.000 | 0.50 | 100.9 | -. 00932 | -14.641 |
| 2 | 14 | !6. 2 | 15.191 | 34.000 | 0.50 | 100.8 | -. 01800 | -29.359 |
| 2 | 15 | 16.6 | 15.181 | 34.000 | 0.50 | 100.8 | -. 02546 | -42.771 |
| 2 | 16 | 16.3 | 15.172 | 34.000 | 0.50 | 100.8 | -. 03118 | -53.606 |
| 2 | 17 | 17.2 | 15.162 | 34.000 | 0.50 | 100.8 | -. 03477 | -61.038 |
| 2 | 18 | 17.6 | 25.153 | 34.000 | 0.50 | 100.8 | -. 03600 | -64.530 |
| 2 | 19 | 17.3 | 15.164 | 34.000 | 0.50 | 100.8 | -. 03477 | -63.417 |
| 2 | 20 | 13.2 | 15.136 | 34.000 | 0.50 | 100.8 | -. 03118 | -57.717 |
| 2 | 21 | 19.7 | 15.128 | 34.000 | 0.50 | 100.8 | -. 02546 | -48.092 |
| 2 | 22 | 19.1 | 15.119 | 34.000 | 0.50 | 100.8 | -. 01800 | -34.610 |
| 2 | 23 | 19.7 | 15.112 | 34.000 | 0.50 | 100.8 | -. 00932 | -18.398 |
| 2 | 24 | 20.4 | 15.104 | 34.000 | 0.50 | 100.9 | -. 00000 | -00.001 |

(c)

[^0]
(b)


TIME (HR)


| OAT | HR | R | T | $s$ | таט | QEET | WDD | OH |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 6.3 | 14.956 | 34.000 | 0.50 | - 3.5 | 0.0 | 0.0 |
| 1 | 2 | 7.2 | 14.939 | 14.000 | 0.50 | -100.8 | -. 00932 | 6.758 |
| 1 | 3 | 7.8 | 14.935 | 34.000 | 0.50 | -184.3 | . .01800 | -14.212 |
| 1 | 4 | 8.1 | 14.939 | 34.000 | 0.50 | -248.4 | -. 02546 | -20.986 |
| 1 | 5 | 8.3 | 14.947 | 14.000 | 0.50 | -288.7 | -. 03118 | -26.310 |
| 1 | 6 | 8.4 | 14.954 | 14.000 | 0.50 | -302.5 | -. 03477 | -29.781 |
| 1 | 7 | 8.5 | 14.963 | 34.000 | 0.50 | -288.9 | -. 03600 | -31.198 |
| 1 | B | 3.6 | 14.969 | 34.000 | 0.50 | -248.7 | -. 03477 | -30.526 |
| 1 | 9 | 8.8 | 14.969 | 34.000 | 0.50 | -184.7 | -. 03118 | -27.771 |
| 1 | 10 | 9.8 | 14.962 | 34.000 | 0.50 | -101.3 | -. 02546 | -22.787 |
| 1 | 11 | 9.0 | 14.946 | 34.000 | 0.50 | - 4.1 | -. 01800 | -16.328 |
| 1 | 12 | 9.2 | 14.920 | 34.300 | 0.50 | -100.2 | -. 00932 | -8.652 |
| 1 | 13 | 9.6 | 14.896 | 34.000 | 0.50 | 100.8 | 0.0 | 0.0 |
| 1 | 14 | 10.0 | 14.874 | 34.000 | 0.50 | 100.8 | 0.00932 | 9.319 |
| 1 | 15 | 10.6 | 14.853 | 34.000 | 0.50 | 100.8 | 0.01800 | 18.919 |
| 1 | 16 | 11.3 | 14.835 | 34.000 | 0.50 | 100.8 | 0.02546 | 28.302 |
| 1 | 17 | 12.0 | 14.817 | 34.000 | 0.50 | 100.8 | 0.03118 | 36.838 |
| 1 | 18 | 12.8 | 14.797 | 34.000 | 0.50 | 100.8 | 0.03477 | 43.781 |
| 1 | 19 | 13.6 | 14.783 | 34.000 | 0.50 | 100.8 | 0.03600 | 48.172 |
| 1 | 20 | 14.4 | 14.770 | 34.000 | 0.50 | 100.8 | 0.03477 | 49.208 |
| 1 | 21 | 15.1 | 14.758 | 34.000 | 0.50 | 100.8 | 0.03118 | 46.482 |
| 1 | 22 | 15.3 | 14.747 | 34.000 | 0.50 | 100.8 | 0.02546 | 39.755 |
| 1 | 23 | 16.4 | 14.736 | 34.000 | 0.50 | 100.8 | 0.01800 | 29.195 |
| 1 | 24 | 16.7 | 14.725 | 34.000 | 0.50 | 100.8 | 0.00932 | 15.526 |
| 2 | 1 | 16.9 | 14.720 | 34.000 | 0.50 | - 3.5 | 0.0 | 0.0 |
| 2 | 2 | 16.9 | 14.722 | 34.000 | 0.50 | -100.3 | -. 00932 | -15.818 |
| 2 | 3 | 16.7 | 14.728 | 34.500 | 0.50 | -184.3 | -. 01800 | -30.300 |
| 2 | 4 | 16.3 | 14.737 | 34.000 | 0.50 | -248.4 | -. 02546 | -42.078 |
| 2 | 5 | 15.9 | 14.749 | 34.000 | 0.50 | -288.7 | -. 03118 | -50.223 |
| 2 | \% | 15.4 | 14.761 | 34.000 | 0.50 | -302.5 | -.03477 | -54.338 |
| 2 | 7 | 14.9 | 14.773 | 34.000 | 0.50 | -288.9 | -. 03600 | -54.494 |
| 2 | 8 | 14.4 | 14.783 | 34.000 | 0.50 | -248.7 | -. 03477 | -51.081 |
| 2 | 9 | 14.1 | 14.788 | 34.000 | 0.50 | -184.7 | -. 03118 | -44.649 |
| 2 | 10 | 13.9 | 14.788 | 34.000 | 0. 50 | -101.3 | -. 02546 | -35.798 |
| 2 | 11 | 13.8 | 14.782 | 34.000 | 0.50 | - 4.1 | -. 01800 | -25.074 |
| 2 | 12 | 13.9 | 14.768 | 34.300 | 0.50 | 100.2 | -. 00932 | -12.981 |
| 2 | 13 | 14.1 | 14.755 | 34.000 | 0.50 | 100.8 | 0.0 | 0.0 |
| 2 | 14 | 14.4 | 14.742 | 34.000 | 0.50 | 100.8 | 0.00932 | 13.323 |
| 2 | 15 | 14.8 | 14.730 | 34.000 | 0.50 | 100.8 | 0.01800 | 26.414 |
| 2 | 16 | 15.4 | 14.718 | 34.000 | 0.50 | 100.8 | 0.02546 | 38.631 |
| 2 | 17 | 16.0 | 14.707 | 34.000 | 0.50 | 100.8 | 0.03118 | 49.224 |
| 2 | 18 | 16.8 | 14.695 | 34.000 | 0.50 | 100.8 | 0.03477 | 57.351 |
| 2 | 19 | 17.6 | 14.685 | 34.000 | 0.50 | 100.8 | 0.03600 | 62.131 |
| 2 | 20 | 13.4 | 14.675 | 34.000 | 0.50 | 100.8 | 0.03477 | 62.760 |
| 2 | 21 | 19.0 | 14.667 | 34.200 | 0.50 | 100.8 | 0.03118 | 58.667 |
|  | 22 | 19.8 | 14.659 | 34.000 | 0.50 | 100.8 | 0.02546 | 49.687 |
| 2 | 23 | 20.3 | 14.651 | 34.000 | 0.50 | 100.8 | 0.01800 | 36.190 |
| , | 24 | 20.6 | 14.642 | 34.000 | 0.50 | 100.8 | 0.00932 | 19.137 |

(a)


Figure 28. Same as figure (1), except for Case Illa with peaks at hours 1 and

(a)


| oay | 38 | 4 | $=$ | $s$ | tad | gnet | woo | OH |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 0.0 | 14.956 | 34.000 | 0.50 | - 3.5 | -. 03600 | -22.140 |
| 1 | 2 | 6.9 | 14.937 | 34.000 | 0.50 | $-100.8$ | -. 03477 | -24.417 |
| 1 | 3 | 7.3 | 14.932 | 34.000 | 0.50 | -184.3 | -. 03118 | -23.270 |
| ! | 4 | 7.8 | 14.934 | 34.000 | 0.50 | -248.4 | -. 02546 | -20.008 |
| 1 | 5 | 8.1 | 14.942 | 34.000 | 0.50 | -288.7 | -. 01800 | -14.652 |
| : | 6 | 3.3 | 14.949 | 34.000 | 0.50 | -302.5 | -. 00932 | - 7.808 |
| 1 | 7 | 8.7 | 14.959 | 34.000 | 0.50 | -288.9 | 0.0 | 0.0 |
| 1 | 8 | 9.1 | 14.965 | 34.000 | 0.50 | -248.7 | 0.00932 | 8.440 |
| 1 | 9 | 9.5 | 14.966 | 34.000 | 0.50 | -184.7 | 0.01800 | 17.029 |
| 1 | 10 | 10.1 | 14.960 | 34.000 | 0.50 | -101.3 | 0.02546 | 25.370 |
| 1 | 11 | 10.7 | 14.948 | 34.300 | 0.50 | -4.1 | 0.03118 | 32.978 |
| 1 | 12 | 11.5 | 14.728 | 34.000 | 0.50 | 100.2 | 0.03477 | 39.220 |
| 1 | 13 | 12.2 | 14.911 | 34.000 | 0.50 | 100.8 | 0.03600 | 43.180 |
| 1 | 14 | 13.0 | 14.896 | 34.000 | 0.50 | 100.8 | 0.03477 | 44.276 |
| 1 | 15 | 13.7 | 14.382 | 34.000 | 0.50 | 100.8 | 0.03118 | 41.902 |
| 1 | 16 | 14.3 | 14.369 | 34.000 | 0.50 | 100.8 | 0.02546 | 35.893 |
| 1 | 17 | 14.3 | 14.657 | 34.000 | 0.50 | 100.8 | 0.01800 | 26.426 |
| 1 | 18 | 15.2 | 14.844 | 34.000 | 0.50 | 100.8 | 0.00932 | 14.111 |
| 1 | 19 | 15.5 | 14.833 | 34.000 | 0.50 | 100.8 | 0.0 | 0.0 |
| 1 | 20 | 15.6 | 14.822 | 34.000 | 0.50 | 100.8 | -. 00932 | -14.562 |
| 1 | 21 | 15.3 | 14.811 | 34.000 | 0.50 | 100.8 | -. 01800 | -28.200 |
| 1 | 22 | 15.4 | 14.800 | 34.000 | 0.50 | 100.8 | -. 02546 | -39.680 |
| : | 23 | 15.2 | 14.789 | 34.000 | 0.50 | 100.8 | -. 03118 | -48.077 |
| 1 | 24 | 14.9 | 14.776 | 34.000 | 0.50 | 100.8 | -. 03477 | -52.860 |
| 2 | 1 | 14.6 | 14.771 | 34.000 | 0.50 | - 3.3 | -. 01600 | -53.604 |
| 2 | 2 | 14.3 | :4.771 | 34.000 | 0.50 | -100.9 | -. 03477 | -50.584 |
| 2 | 3 | 14.0 | 14.776 | 34.000 | 0.50 | -184.3 | -. 03118 | -44.329 |
| 2 | 4 | 13.7 | 14.786 | 34.000 | 0.50 | -248.4 | -. 02546 | -35.447 |
| 2 | 5 | 13.6 | 14.798 | 34.000 | 0.50 | -288.7 | -. 01800 | -24.685 |
|  | 6 | 13.5 | 14.810 | 34.000 | 0.50 | -302.5 | -. 00932 | -12.683 |
| 2 | ? | 13.6 | 14.822 | 34.000 | 0. 50 | -288.9 | 0.0 | 0.0 |
| 2 | a | 13.9 | 14.831 | 34.000 | 0.50 | -248.7 | 0.00932 | 12.864 |
| 2 | 9 | 14.2 | 14.837 | 34.000 | 0.50 | -104.7 | 0.01800 | 25.415 |
| 2 | 10 | 14.8 | 14.837 | 34.000 | 0.50 | -101.3 | 0.02546 | 37.112 |
| 2 | 11 | 15.4 | 14.831 | 34.000 | 0.50 | - 4.1 | 0.03118 | 47.309 |
| 2 | 12 | 16.2 | 14.318 | 34.000 | 0.50 | 100.2 | 0.03477 | 55.256 |
| 2 | 13 | 17.0 | 14.808 | 34.000 | 0.50 | 100.8 | 0.03600 | 60.002 |
| 2 | 14 | 17.8 | 14.798 | 14.000 | 0.50 | 100.3 | 0.03477 | 60.692 |
| 2 | 15 | 18.5 | 14.799 | 34.000 | 0.50 | 100.8 | 0.03118 | 56.796 |
| 2 | 16 | 19.2 | 14.781 | 34.000 | 0.50 | 100.8 | 0.02546 | 48.153 |
| 2 | 17 | 19.7 | 14.773 | 34.000 | 0.50 | 100.3 | 0.01800 | 35.109 |
| 2 | 13 | 20.0 | 14.764 | 34.000 | 0.50 | 100.8 | 0.00932 | 18.583 |
|  | 13 | 20.2 | 14.756 | 34.000 | 0.50 | 100.3 | 0.0 | 0.0 |
| 2 | 20 | 20.2 | 14.749 | 34.000 | 0.50 | 100.8 | -. 00932 | -18.903 |
| 2 | 21 | 20.0 | 14.741 | 34.000 | 0.50 | 100.8 | -. 01800 | -36.339 |
| 2 | 22 | 19.7 | 14.733 | 34.000 | 0.50 | 100.8 | -. 02546 | -50.746 |
| 2 | 23 | 19.3 | 14.725 | 34.000 | 0.30 | 100.8 | -. 03118 | -60.996 |
|  |  |  | 14 | 14.000 | - | 100.3 | -. 03477 | -66.499 |








Figure 31. Contours as in figure (1), except for Case llla in two dimensional mode.
(a), (b), (c), and (d) are hour $6,12,18$ and 24 respectively.




| 3ay | 88 | d | $\pm$ | $s$ | tau | 2NET | woo | OH |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 19.5 | 15.001 | 34.000 | 0.50 | - 55.7 | 0.0 | 0.0 |
| 1 | 2 | 17.9 | 15.009 | 34.000 | 0.50 | -201.5 | -.00932 | $-16.793$ |
| 1 | 3 | 15.9 | 15.024 | 34.000 | 0.50 | -326.8 | . 01800 | -28.950 |
| 1 | 4 | 12.7 | 15.247 | 34.000 | 0.50 | -423.0 | 02546 | -32.85 |
| 1 | 5 | 11.7 | 15.075 | 34.000 | 0.50 | -483.5 | 03118 | -37.299 |
| 1 | 6 | 11.4 | 15.104 | 34.000 | 0.50 | -504.2 | -. 03477 | -40.436 |
| 1 | 7 | 11.3 | 15.132 | 34.000 | 0.50 | -483.7 | -. 03600 | -41.530 |
| 1 | 3 | 12.4 | 15.154 | 34.000 | 0.50 | -423.5 | -. 03477 | -40.316 |
| 1 | 9 | 11.6 | 15.168 | 34.000 | 0.50 | -327.5 | -. 03118 | -36.803 |
| 1 | 10 | 12.0 | 15.174 | 34.000 | 0.50 | -202.4 | -. 02546 | -30.875 |
| 1 | 11 | 12.5 | 15.169 | 34.000 | 0.50 | - 56.6 | -. 01800 | -22.641 |
| 1 | 12 | 13.2 | 15.155 | 34.000 | 0.50 | 79.9 | -. 00932 | -12.376 |
| 1 | 13 | 14.1 | 25.142 | 34.000 | 0.50 | 100.8 | 0.0 | 0.0 |
| 2 | 14 | 15.1 | 15.230 | 34.000 | 0.50 | 100.8 | 0.00932 | 14.028 |
| : | 15 | 16.3 | 15.119 | 34.000 | 0.50 | 100.8 | 0.01800 | 29.256 |
| 1 | 16 | 17.7 | 15.110 | 34.000 | 0.50 | 100.8 | 0.02546 | 44.423 |
| 1 | 17 | 19.2 | 15.102 | 34.000 | 0.50 | 100.8 | 0.03118 | 58.808 |
| 1 | 18 | 20.8 | 15.093 | 34.000 | 0.50 | 100.8 | 0.03477 | 71.045 |
| 1 | 19 | 22.5 | 15.086 | 34.000 | 0.50 | 100.8 | 0.03600 | 79.525 |
| 1 | 20 | 24.2 | 15.080 | 34.000 | 0.50 | 100.8 | 0.03477 | 82.842 |
| 1 | $\pm 1$ | 26.0 | 15.078 | 34.000 | 0.50 | 100.8 | 0.03118 | 79.704 |
| 1 | 22 | 27.6 | 15.069 | 34.000 | 0.50 | 100.8 | 0.02546 | 69.357 |
| 1 | 23 | 29.1 | 15.364 | 34.000 | 0.50 | 100.8 | 0.01800 | 51.856 |
| 1 | 24 | 30.3 | 15.060 | 34.000 | 0.50 | 100.8 | 0.00932 | 28.137 |
| 2 | 1 | 30.8 | 15.060 | 34.800 | 0.50 | - 55.7 | 0.0 | 0.0 |
| , | 2 | 20.9 | 25.069 | 34.000 | 0.50 | -201.5 | -. 00932 | -19.593 |
| 2 | 3 | 15.1 | 15.083 | 34.000 | 0.50 | -326.8 | . .01800 | -27.473 |
| 2 | 4 | 12.9 | 15.106 | 34.000 | 0.50 | -423.0 | -. 02546 | -33.187 |
| 2 | 5 | 12.7 | 15.134 | 34.000 | 0.50 | -483.5 | -. 03118 | -36. 369 |
| 2 | 6 | 11.3 | 15.164 | 34.000 | 0.50 | -504.2 | -. 03477 | -39.896 |
| 2 | ? | 11.2 | 15.191 | 34.000 | 0.50 | -483.5 | -. 03600 | -41.192 |
| 2 | 3 | 11.3 | 15.213 | 34.000 | 0.50 | -423.5 | -. 03477 | -40.094 |
| 2 | 9 | 12.6 | 15.228 | 34.000 | 0.50 | -327.5 | -. 03118 | -36.673 |
| 2 | 10 | 12.9 | 25.234 | 34.000 | 0.50 | -202.4 | -. 02546 | -30.800 |
| 2 | 11 | 12.4 | 15.229 | 34.000 | 0.50 | - 56.6 | -. 01800 | -22.569 |
| 2 | 12 | 13.2 | 15.215 | 34.000 | 0.50 | 99.9 | -. 00932 | -12.336 |
| 2 | 13 | 14.2 | 15.202 | 34.000 | 0.50 | 100.8 | 0.0 | 0.0 |
| 2 | 14 | 15.1 | 15.190 | 34.000 | 0.50 | 100.8 | 0.00932 | 14.013 |
| 2 | 15 | 16.3 | 15.179 | 34.000 | 0.50 | 100.8 | 0.01800 | 29.163 |
| 2 | 16 | 27.7 | 15.269 | 34.000 | 0.50 | 100.8 | 0.02546 | 44.461 |
| 2 | 17 | 19.2 | 15.160 | 34.000 | 0.50 | 100.8 | 0.03118 | 58.869 |
| 2 | 18 | 20.8 | 15.153 | 34.300 | 0.50 | 100.8 | 0.03477 | 71.122 |
| 2 | 19 | 22.5 | 15.146 | 34.000 | 0.50 | 100.8 | 0.03600 | 79.577 |
| 2 | 20 | 24.2 | 25.139 | 34.000 | 0.50 | 100.8 | 0.03477 | 82.821 |
| 2 | 21 | 25.9 | 15.134 | 34.000 | 0.50 | 100.8 | 0.03118 | 79.552 |
| 2 | 22 | 27.4 | 15.128 | 34.000 | 0.50 | 100.8 | 0.02546 | 68.985 |
| 2 | 23 | 28.3 | 15.124 | 34.000 | 0.50 | 100.8 | 0.01800 | 51.351 |
| 2 | 24 | 29.8 | 15.119 | 34.000 | 0.50 | 100.8 | 0.00932 | 27.646 |

(a)


| Ar | 8R | H | T | s | tad | QNET | WOD | OH |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 47.8 | 15.001 | 34.000 | 0.50 | - 55.7 | -. 03600 | -175.086 |
| 1 | 2 | 17.7 | 15.009 | 34.000 | 0.50 | -201.5 | -. 03477 | 62.530 |
| 1 | 3 | 15.8 | 15.024 | 34.000 | 0.50 | -326.8 | -. 03118 | 50.044 |
| 1 | 4 | 12.3 | 15.047 | 34.000 | 0.50 | -423.0 | -. 02546 | 32.932 |
| 1 | 5 | 11.9 | 15.075 | 34.000 | 0.50 | -483.5 | -. 01800 | 21.607 |
| 1 | 6 | 11.7 | 15.104 | 34.000 | 0.50 | -504.2 | -. 00932 | 10.945 |
| 1 | , | 12.0 | 15.131 | 34.000 | 0.50 | -433.7 | 0.0 | 0.0 |
| 1 | 8 | 12.4 | 15.154 | 34.000 | 0.50 | -423.5 | 0.00932 | 11.542 |
| 1 | * | 13.1 | 15.16 a | 34.000 | 0.50 | -327.5 | 0.01300 | 23.408 |
| 1 | 10 | 14.0 | 15.175 | 34.000 | 0.50 | -202.4 | 0.02546 | 35.288 |
| 1 | 11 | 15.2 | 15.172 | 34.000 | 0.50 | - 56.6 | 0.03118 | 46.708 |
| 1 | 12 | 16.7 | 15.161 | 34.000 | 0.50 | 99.9 | 0.03477 | 56.954 |
| 1 | 13 | 18.0 | 15.152 | 34.000 | 0.50 | 100.8 | 0.03600 | 63.705 |
| 1 | 14 | 19.4 | 15.143 | 34.000 | 0.50 | 100.8 | 0.03477 | 66.297 |
| 1 | 15 | 20.8 | 15.135 | 34.000 | 0.50 | 100.8 | 0.03118 | 63.716 |
| 1 | 16 | 22.0 | 15.12 s | 34.000 | 0.50 | 100.8 | 0.02546 | 55.248 |
| 1 | 17 | 23.1 | 15.122 | 34.000 | 0.50 | 100.8 | 0.01800 | 41.146 |
| 1 | 18 | 24.0 | 15.116 | 34.000 | 0.50 | 100.8 | 0.00932 | 22.239 |
| 1 | 19 | 24.7 | 15.110 | 34.000 | 0.50 | 100.8 | 0.0 | 0.0 |
| 1 | 20 | 25.1 | 15.204 | 34.000 | 0.50 | 100.8 | -. 00932 | - 23.531 |
| 1 | 21 | 25.1 | 15.099 | 34.000 | 0.50 | 100.8 | -. 01800 | - 46.145 |
| 1 | 22 | 25.5 | 15.093 | 34.000 | 0.50 | 100.8 | -. 02546 | - 65.790 |
| 1 | 23 | 25.5 | 15.388 | 34.000 | 0.50 | 100.8 | -. 03118 | - 80.740 |
| 1 | 24 | 25.4 | 15.083 | 34.000 | 0.50 | 100.8 | -.03477 | - 89.980 |
| 2 | 1 | 25.0 | 15.083 | 34.000 | 0.50 | - 35.7 | -. 03600 | - 91.646 |
| 2 | 2 | 22.1 | 15.090 | 34.000 | 0.50 | -201.5 | -. 03477 | - 73.064 |
| 2 | 3 | 14.7 | 15.106 | 14.000 | 0.50 | -326.8 | -. 03118 | - 46.456 |
| 2 | 4 | 13.0 | 15.128 | 34.000 | 0.50 | - $\$ 23.0$ | -. 02546 | - 33.442 |
| 2 | 5 | 11.8 | 15.157 | 34.000 | 0.50 | -483.5 | -. 01800 | - 21.441 |
| 2 | 6 | 11.7 | 15.186 | 34.000 | 0.50 | -504.2 | -. 00932 | - 10.915 |
| 2 | 7 | 11.9 | 15.214 | 34.000 | 0.50 | -483.7 | 0.0 | 0.0 |
| 2 | 8 | 12.4 | 15.236 | 34.000 | 0.50 | -423.5 | 0.00932 | 11.456 |
| 2 | 9 | 12.9 | 15.251 | 34.000 | 0.50 | -327.5 | 0.01800 | 23.239 |
| 2 | 10 | 13.9 | 15.257 | 34.000 | 0.50 | -202.4 | 0.02546 | 35.035 |
| 2 | 11 | 15.0 | 15.254 | 34.000 | 0.50 | - 56.6 | 0.03118 | 46.159 |
| 2 | 12 | 16.4 | 15.243 | 34.000 | 0.50 | 99.9 | 0.03477 | 56.054 |
| 2 | 13 | 17.3 | 15.233 | 34.000 | 0.50 | 100.8 | 0.03600 | 63.043 |
| 2 | 14 | 19.2 | 15.225 | 34.000 | 0.50 | 100.8 | 0.03477 | 65.623 |
| 2 | 15 | $\geq 0.6$ | 15.217 | 34.000 | 0.50 | 100.8 | 0.03118 | 63.110 |
| 2 | 16 | 21.8 | 15.210 | 34.000 | 0.50 | 100.8 | 0.02546 | 54.870 |
| 2 | 17 | 22.9 | 15.203 | 34.000 | 0.50 | 100.8 | 0.01800 | 40.923 |
| 2 | 18 | 23.9 | 15.197 | 34.000 | 0.50 | 100.8 | 0.00932 | 22.134 |
| 2 | 19 | 24.6 | 15.191 | 34.000 | 0.50 | 100.8 | 0.0 | 0.0 |
| 2 | 20 | 25.1 | 15.185 | 34.000 | 0.50 | 100.3 | -. 00932 | - 23.462 |
| 2 | 21 | 25.3 | 15.180 | 34.000 | 0.50 | 100.8 | -. 01800 | - 46.025 |
| 2 | 22 | 25.4 | 15.175 | 34.000 | 0.50 | 100.8 | -. 022546 | - 65.615 |
| 2 | 23 | 25.3 | 15.169 | 34.000 | 0.50 | 100.8 | -. 03118 | - 80.218 |
|  |  | 25.1 | 15.164 | 34.000 | 0. | 100 | -. 03477 | 88 |

(a)
\%



8
3
3
3
3
0
1
1
0
0
3
1
1
0

8

| $\begin{array}{r}\text { UEPIH } \\ 3 \\ 32.00 \\ \hline\end{array}$ | (M) <br> -23.00 | 18.00 | -11.00 |
| :---: | :---: | :---: | :---: |
|  <br>  |  |  |  |
|  |  |  |  |
|  |  |  |  |

(a)




Figure 38. Same as figure (31), except for Case IIIb.

(a)

(b)

(d)

TIME (HR)



$$
\frac{11}{11}
$$

(a)

(c)

DEPTH
(b)

(d)


Figure 44. Same as figure (31), except for Case IVa











$$
-
$$





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[^0]:    Figure 26. (a) and (b) sane as figure (1), except for and (b), except for Case Ild.

