OUR OWN

SCHOOL ARITHMETIC.

BY

S. LANDER, A. M.

GREENSBORO, N. C.:
PUBLISHED BY STERLING, CAMPBELL & ALBRIGHT.
RICHMOND, VA.: W. HARGRAVE WHITE.
1863.

OUR OWN

SCHOOL ARITHMETIC.

BY

S. LANDER, A. M.

GREENSBORO, N. C.:
PUBLISHED BY STERLING, CAMPBELL & ALBRIGHT.
RICHMOND, VA., W. HARGRAVE WHITE.
1863.

PREFACE.

In presenting to the public perhaps the first Arithmetic whose authorship and publication belong exclusively to the Confederate States, I call attention to the following as its leading characteristics.

1. The pupil is furnished with a model for each class of operations, by which he may know precisely what kind of explanation is required of him as he recites.

2. The distinction between abstract and concrete numbers is carefully kept up throughout the whole book.

3. The Tables of Relations of Concrete Numbers are unusually full and convenient.

4. The problems are designed to call into exercise the pupil's practical common sense, as well as to assist him in acquiring a correct knowledge of Arithmetic.

5. The results of about two-thirds of the problems are given: those of the remainder are omitted, and a few of those given contain intentional errors, to test the pupil's self-reliance.

6. Progressions, and Mensuration, and the ordinary methods of extracting roots, are excluded entirely, because they lie beyond the province of Arithmetic.

7. No space is wasted by the insertion of questions on the text. A teacher who can not instruct without the help of questions, will succeed but poorly even with them.

8. The discouraging contradictions which are so numerous in "our best Arithmetics," have been sedulously avoided.

For the neat appearance of the book I am much indebted to my publishers, who have spared neither pains nor expense to bring it out creditably; and I am under especial obligations to my friend PROF. THEO. F. WOLLE, of Edgeworth Female Seminary, without whose constant vigilance in revising the sheets, no approximation to its present accuracy could have been attained.

I invite my fellow-teachers to try the book by the only sure test, the test of the school-room; and I will thankfully receive any propositions of improvement which their examinations may suggest.

Our Own Primary Arithmetic will follow this as soon as possible.

S. LANDER.

LINCOLNTON, N. C., August 1, 1863.

CONTENTS.

ARITHMETIC.

INTRODUCTION.

§ 1. ARITHMETIC is the science of numbers.

§ 2. A *unit* is any single thing; as, one, one dollar.

§ 3. A *number* is a collection of units; as, three, two dollars, four men.

§ 4. An *abstract* number is one whose unit is not specified; as, two, forty, seventy-one, eight.

§ 5. A *concrete* number is one whose unit is specified; as, ten dollars, forty men, seventy-one bales, eight books.

§ 6. Two or more numbers are *similar* when they have the same unit; as, two, five, and seventy; three men and six men.

§ 7. Two or more numbers are *dissimilar* when they have different units; as, two, five dollars, seventy men, three books.

Note.—All abstract numbers are similar.

§ 8. A *compound* number is a concrete number expressed in two or more denominations; as, three dollars, fifty cents; ten hogsheads, forty gallons, three gills; ten miles, seven furlongs, seventeen rods.

ARABIC NOTATION.

§ 9. NOTATION is the method of expressing numbers by figures. The Arabic system, which is the one in common use, is called also the *decimal* system, partly because it employs ten figures. These figures are: 0 naught or zero, 1 one, 2 two, 3 three, 4 four, 5 five, 6 six, 7 seven, 8 eight, 9 nine.

The figure 0 is used to fill vacant places, and is omitted in reading.

§ 10. A *ten* is a collection of ten units, and is called a unit of the *second order*.

A *hundred* is ten tens, or one hundred units, and is called a unit of the *third order*.

A *thousand* is ten hundreds, or one thousand units; and is called a unit of the *fourth order*.

So, ten units of any order make one of the next higher.

§ 11. A single figure denotes units; as, 3, three units. 5, five units, 7, seven units.

§ 12. When two figures are written together, the one on the right denotes units, and the other tens; as, 23, two tens and three units, that is, twenty-three units; 34, three tens and four units, that is, thirty-four units, or, simply, thirty-four.

Read 27, 56, 73, 37, 84, 48, 99, 76, 43, 55, 79, 80, 18.

§ 13. When three figures are written together, the one on the right denotes units, the next tens, and the other hundreds; as, 123, one hundred, two tens, and three units, or, one hundred and twenty-three; 321, three hundred and twenty-one; 132, one hundred and thirty-two: 402, four hundred and two.

Read 647, 864, 420, 301, 753, 537, 357, 735, 608, 740, 047, 306, 700, 609, 069, 009, 290, 391, 001.

§ 14. When more than three figures are written together, they are separated into *periods* of three figures each, beginning at the right; and, in each period, the three figures denote respectively units, tens, and hundreds, of that period.

§ 15. The names of the periods and their order from right to left are given in the following

TABLE.

Duodecillions.	Undecillions.	Decillions.	Nonillions.	Octillions.	Septillions.	Sextillions.	Quintillions.	Quadrillions.	Trillions.	Billions.	Millions.	Thousands.	Units.
123,	004,	500,	060,	000,	000,	700,	089,	000,	897,	060,	000,	543,	210.

The above number is read,—one hundred and twenty-three duodecillions, four undecillions, five hundred decillions, sixty nonillions, seven hundred sextillions, eighty-nine quintillions, eight hundred and ninety-seven trillions, sixty billions, five hundred and forty-three thousand, two hundred and ten.

RULE FOR READING NUMBERS.—*Separate the figures into periods of three figures each, beginning at the right; then, beginning at the left, read each period as if it stood alone, and pronounce the name of the period after reading it.*

Read the following: 12, 21, 37, 86; 793, 842, 209, 319; 2346, 7907, 5432, 8642, 4001, 1861, 1775; 24608, 13579, 10724, 40047, 78009; 475213, 570903, 400101, 300003; 1230456, 2040608, 7901035, 8000005; 70083790, 245000542, 5430102046, 146070080009, 9000800706040, 6000050001.

Write the following numbers in figures:

35. Seventy-four.

36. Four hundred and forty-eight.

37. Five thousand, three hundred and ninety-seven.

38. Sixty thousand, and seventeen.

39. Seven hundred and forty thousand, eight hundred and forty-one.

40. Eight millions, seventy thousand, and seventy-nine.

41. Ninety-four millions, sixteen thousand, four hundred and fourteen.

42. Three hundred millions, and three.

43. Two billions, two millions, two thousand, and two.

44. Ten billions, ten millions, ten thousand, and ten.

45. Nine hundred and twenty-five billions, eight thousand, and sixteen.

46. Eight trillions, seven billions, sixty millions, five thousand, and four.

47. Seventy trillions, eighty-nine millions, and twenty-one.

48. Six hundred and forty-two trillions, three hundred thousand.

49. Fifty-three quadrillions, eleven billions, and seventy-three.

50. Four hundred and four quintillions, two hundred and two millions.

51. Thirty sextillions, forty quintillions, fifty trillions, six hundred and two.

52. Two octillions, four quadrillions, six hundred and eight thousand.

53. Ten decillions, twelve nonillions, fourteen millions, and ninety-nine.

54. Nine nonillions, ten millions, and twenty-seven.

ROMAN NOTATION.

§ 16. The Roman Notation employs the following seven letters: I one, V five, X ten, L fifty, C one hundred, D five hundred, and M one thousand.

All integral numbers may be denoted by combining these letters according to the following rules:

1. Any letter doubled denotes twice its simple value; tripled denotes three times, and so on. Thus, II=2, XX=20, CCC=300.

2. If a letter of less value is placed *after* one of greater value, the less is to be added to the greater. Thus, VI=6, XV=15, CCL=250.

3. If a letter of less value is placed *before* one of greater value, the less is to be subtracted from the greater. Thus, IV=4, XC=90, CD=400.

4. If a letter of less value is placed *between* two of greater value, the less is to be subtracted from the sum of the other two. Thus, XIX=19, XIV=14, XCIX=99.

5. A dash placed over a letter multiplies its value by 1000. Thus, $\overline{\text{L}}$=50000, $\overline{\text{C}}$=100000.

The above rules are sufficiently exemplified in the following

TABLE.

I=1	XI=11	XXI=21	C=100
II=2	XII=12	XXII=22	CC=200
III=3	XIII=13	XXIII=23	CD=400
IV=4	XIV=14	XXX=30	D=500
V=5	XV=15	XL=40	DC=600
VI=6	XVI=16	L=50	M=1000
VII=7	XVII=17	LX=60	MC=1100
VIII=8	XVIII=18	LXX=70	MM=2000
IX=9	XIX=19	LXXX=80	$\overline{\text{M}}$=1000000
X=10	XX=20	XC=90	MDCCCLXIII=1863

OPERATIONS.

There are four operations in Arithmetic; Addition, Subtraction, Multiplication, and Division. We will explain these operations in succession, first with reference to abstract numbers, and afterwards with reference to concrete numbers.

ADDITION OF ABSTRACT NUMBERS.

§ 17. ADDITION is the operation of finding one number equal to several other numbers put together.

§ 18. The *result* of *addition* is called the *sum* of the numbers added. Thus, 10 is the *sum* of 6 and 4.

§ 19. The *sign* of *addition*, +, is read *plus*. When placed before a number, it denotes that it is to be added to any other *additive* number with which it is connected. Thus, 6+4, 6 *plus* 4, denotes four added to six.

§ 20. The *sign* of *equality*, =, is read *is equal to*. When placed between two expressions it denotes that they are equal to each other. Thus, 6+4=10 Also, 7+4+3=8+6.

ADDITION TABLE.

2 and 0 are 2	3 and 0 are 3	4 and 0 are 4	5 and 0 are 5
2 and 1 are 3	3 and 1 are 4	4 and 1 are 5	5 and 1 are 6
2 and 2 are 4	3 and 2 are 5	4 and 2 are 6	5 and 2 are 7
2 and 3 are 5	3 and 3 are 6	4 and 3 are 7	5 and 3 are 8
2 and 4 are 6	3 and 4 are 7	4 and 4 are 8	5 and 4 are 9
2 and 5 are 7	3 and 5 are 8	4 and 5 are 9	5 and 5 are 10
2 and 6 are 8	3 and 6 are 9	4 and 6 are 10	5 and 6 are 11
2 and 7 are 9	3 and 7 are 10	4 and 7 are 11	5 and 7 are 12
2 and 8 are 10	3 and 8 are 11	4 and 8 are 12	5 and 8 are 13
2 and 9 are 11	3 and 9 are 12	4 and 9 are 13	5 and 9 are 14

6 and 0 are 6	7 and 0 are 7	8 and 0 are 8	9 and 0 are 9
6 and 1 are 7	7 and 1 are 8	8 and 1 are 9	9 and 1 are 10
6 and 2 are 8	7 and 2 are 9	8 and 2 are 10	9 and 2 are 11
6 and 3 are 9	7 and 3 are 10	8 and 3 are 11	9 and 3 are 12
6 and 4 are 10	7 and 4 are 11	8 and 4 are 12	9 and 4 are 13
6 and 5 are 11	7 and 5 are 12	8 and 5 are 13	9 and 5 are 14
6 and 6 are 12	7 and 6 are 13	8 and 6 are 14	9 and 6 are 15
6 and 7 are 13	7 and 7 are 14	8 and 7 are 15	9 and 7 are 16
6 and 8 are 14	7 and 8 are 15	8 and 8 are 16	9 and 8 are 17
6 and 9 are 15	7 and 9 are 16	8 and 9 are 17	9 and 9 are 18

Note.—Let the above table be thoroughly memorized before the pupil advances further.

Ex. 1. Add together 102741, 42102, and 3050.

102741
42102
3050
147893

§ 21. MODEL.—2 and 1 are 3; 5 and 4 are 9, 1 and 7 are 8; 3 and 2 are 5, and 2 are 7; 4; 1. The sum is 147893.

Note.—Let the teacher see to it that the pupil recites precisely according to the model both here and wherever a model is given.

EXPLANATION.—First, the numbers are arranged with units of the same order in the same column. Then, beginning at the right, the numbers in each column are added together, and the sum is placed underneath in the same column.

2. Add together 23456, 10203, and 56030. Sum, 89689.
3. Find the sum of 120242, 334124, and 224612.

Note.—Let the pupil first say, "Add the numbers together," and then proceed as in the model.

4. What is the sum of 2400, 1505, and 3074? Ans. 6979.
5. Add 270, 102, 314, and 301 together. Sum, 987.
6. Add together 94085, 16275, and 3367.

94085
16275
3367
Sum, 113727

§ 22. MODEL.—7 and 5 are 12, and 5 are 17, set down 7; 1 and 6 are 7, and 7 are 14, and 8 are 22, set down 2; 2 and 3 are 5, and 2 are 7; 3 and 6 are 9, and 4 are 13, set down 3; 1 and 1 are 2, and 9 are 11, set down 11. The sum is 113727.

EXPLANATION.—After arranging the numbers as in § 21, the sum of the column of units is found to be 17 units, that is, 1 ten and 7 units; hence, the 7 is placed under the column of units, and the 1 is afterwards added in with the column of tens. The sum of the column of tens are 22 tens, that is, 2 hundreds and 2 tens; hence, the right hand 2 is placed under the column of tens, and the other 2 is added in with the column of hundreds. The sum of the column of hundreds is 7 hundreds, and the 7 is placed underneath in that column. The sum of the column of thousands is 13 thousands, that is, 1 ten-thousand and 3 thousands; hence, the 3 is placed in the column of thousands, and the 1 is added in with the column of ten-thousands. The sum of the column of ten-thousands is 11 ten-thousands, that is, 1 hundred-thousand and 1 ten-thousand; hence, the right hand 1 is placed in the column of ten-thousands, and the other 1 in the place of hundred-thousands.

RULE.—*Arrange the numbers with units of the same order in the same column.*

Beginning at the right, find the sum of each column; if this sum is expressed by one figure, set it down under the column; but if it is expressed by more than one figure, set the right hand figure under this column, and add the remaining figure or figures in with the next column.

Set down the whole sum of the last column.

PROOF.—1. Add as before, but begin at the top of each column.

Or, 2. Find the sum of all the numbers but one, and to this sum add the number excepted.

Ex. 7. Add together 234, 156, 987, and 358. Sum, 1735.

8. Add together 1020, 304, 56, and 9. Sum, 1389.

9. Add together 2739, 9647, 271, 17, and 2950.

10. Add together 169078, 270189, and 928608.

Sum, 1367875.

11. Add together 27090, 2709, 27905, 27, 2709050, and 270. Sum, 3010051.

12. Find the sum of 369764, 275863, 10794, 273, 102469, and 1861.

13. Find the sum of 173594, 240680, 10305, 678, and 976531. Sum, 1401788.

14. Find the sum of 97347825, 89734782, 28973478, 828973478, and 98289734. Sum, 1143319297.

15. Find the sum of 1928374560, 192837456, 1928, 19283745, 1928374, 192837, and 19283.

16. 907050301 + 80604020 + 123123123 = what?

Ans. 1110777444.

17. 146 + 1375 + 13795 + 246820 + 24682 = what?

Ans. 286818.

18. 2620 + 6202 + 7593 + 3694 + 1735 = what?

19. What is the sum of 3426, 9120634, 52714, 9987, 1137, and 97579?

20. What is the sum of 26322, 50555, 37684, 898955, and 9024? Ans. 1022540.

21. What is the sum of 41084, 293347, 9139919, and 46552? Ans. 9520902.

22. What is the sum of 245301, 586642, 51407, 1752, 71283, and 42061?

23. What is the sum of 10, 105, 1057, 10572, 105723, 1057234, 10572349, 105723496, and 1057234968?

Ans. 1174705532.

24. What is the sum of 135792468, 246813579, 159483726, 372684951, 123456789, 896745321, 896453217, and 400500746? Ans. 3231930797.

SUBTRACTION OF ABSTRACT NUMBERS.

§ 23. SUBTRACTION is the operation of finding the difference between two numbers, by taking the less from the greater.

§ 24. The number *to be subtracted* is called the *subtrahend.*

§ 25. The number *to be diminished* is called the *minuend.*

§ 26. The *result* of subtraction is called the *remainder* or the *difference.*

§ 27. The *sign* of subtraction, —, is read *minus.* When placed before a number, it denotes that it is to be subtracted from the number with which it is connected. Thus, 6—4, 6 *minus* 4, denotes 4 taken from 6. Also, 7—3=4.

§ 28. The remainder is not changed by increasing the minuend and the subtrahend equally. Thus,

Min.	27	27+15=42	27+240=267	27+306=333
Sub.	16	16+15=31	16+240=256	16+306=322
Rem.	11	11	11	11

SUBTRACTION TABLE.

1 from 1 leaves 0	2 from 2 leaves 0	3 from 3 leaves 0
1 from 2 leaves 1	2 from 3 leaves 1	3 from 4 leaves 1
1 from 3 leaves 2	2 from 4 leaves 2	3 from 5 leaves 2
1 from 4 leaves 3	2 from 5 leaves 3	3 from 6 leaves 3
1 from 5 leaves 4	2 from 6 leaves 4	3 from 7 leaves 4
1 from 6 leaves 5	2 from 7 leaves 5	3 from 8 leaves 5
1 from 7 leaves 6	2 from 8 leaves 6	3 from 9 leaves 6
1 from 8 leaves 7	2 from 9 leaves 7	3 from 10 leaves 7
1 from 9 leaves 8	2 from 10 leaves 8	3 from 11 leaves 8
1 from 10 leaves 9	2 from 11 leaves 9	3 from 12 leaves 9

4 from 4 leaves 0	5 from 5 leaves 0	6 from 6 leaves 0
4 from 5 leaves 1	5 from 6 leaves 1	6 from 7 leaves 1
4 from 6 leaves 2	5 from 7 leaves 2	6 from 8 leaves 2
4 from 7 leaves 3	5 from 8 leaves 3	6 from 9 leaves 3
4 from 8 leaves 4	5 from 9 leaves 4	6 from 10 leaves 4
4 from 9 leaves 5	5 from 10 leaves 5	6 from 11 leaves 5
4 from 10 leaves 6	5 from 11 leaves 6	6 from 12 leaves 6
4 from 11 leaves 7	5 from 12 leaves 7	6 from 13 leaves 7
4 from 12 leaves 8	5 from 13 leaves 8	6 from 14 leaves 8
4 from 13 leaves 9	5 from 14 leaves 9	6 from 15 leaves 9

7 from 7 leaves 0	8 from 8 leaves 0	9 from 9 leaves 0
7 from 8 leaves 1	8 from 9 leaves 1	9 from 10 leaves 1
7 from 9 leaves 2	8 from 10 leaves 2	9 from 11 leaves 2
7 from 10 leaves 3	8 from 11 leaves 3	9 from 12 leaves 3
7 from 11 leaves 4	8 from 12 leaves 4	9 from 13 leaves 4
7 from 12 leaves 5	8 from 13 leaves 5	9 from 14 leaves 5
7 from 13 leaves 6	8 from 14 leaves 6	9 from 15 leaves 6
7 from 14 leaves 7	8 from 15 leaves 7	9 from 16 leaves 7
7 from 15 leaves 8	8 from 16 leaves 8	9 from 17 leaves 8
7 from 16 leaves 9	8 from 17 leaves 9	9 from 18 leaves 9

Ex. 1. From 976348 subtract 35127.

Min.	976348
Sub.	35127
Rem.	941221

§ 29. MODEL.—7 from 8 leaves 1; 2 from 4 leaves 2; 1 from 3 leaves 2; 5 from 6 leaves 1; 3 from 7 leaves 4; 0 from 9 leaves 9. The remainder is 941221.

EXPLANATION.—The subtrahend is placed under the minuend, with units of the same order in the same column. Then, beginning at the right, each figure of the subtrahend is taken from the corresponding figure of the minuend, and the remainder is set underneath in the same column.

2. From 127936 subtract 14312. Rem. 113624.
3. From 96898 subtract 13456.
4. Subtract 864231 from 987654. Rem. 123423.
5. Subtract 1024370 from 12357799. Rem. 11333429.

6. Subtract 327739 from 573647.

Min.	573647
Sub.	327739
Rem.	245908

§ 30. MODEL.—9 from 17 leaves 8; 4 from 4 leaves 0; 7 from 16 leaves 9; 8 from 13 leaves 5; 3 from 7 leaves 4; 3 from 5 leaves 2. The remainder is 245908.

EXPLANATION.—After arranging the numbers as in § 29, it is required to take 9 units from 7 units: this can not be done; hence, 1 ten, that is, 10 units, is added to the minuend, giving 17 units, from which 9 units taken leaves 8 units. Then, because the minuend is increased 10 units or 1 ten, the subtrahend must be increased the same amount (§ 28). This gives 4 tens to be taken from the 4 tens of the minuend, leaving 0 tens. Again, 7 hundreds can not be taken from 6 hundreds; hence, 1 thousand, that is, 10 hundreds, is added to the minuend, giving 16 hundreds, from which 7 hundreds taken leaves 9 hundreds. Then, because the minuend is increased 10 hundreds or 1 thousand, the subtrahend must be increased the same amount.

The same kind of reasoning will explain the rest of the operation.

RULE.—*Place the subtrahend under the minuend, with units of the same order in the same column.*

Beginning at the right, take each figure of the subtrahend from the corresponding figure of the minuend.

If any figure of the minuend is less than the corresponding figure of the subtrahend, add 10 *to this minuend figure, and add* 1 *to the subtrahend figure in the next column.*

PROOF.—1. Add the remainder to the subtrahend; the sum will be equal to the minuend.

Or, 2. Subtract the remainder from the minuend; the difference will be equal to the subtrahend.

Ex. 7. From 896 take 307. Rem. 589.
8. From 1842 take 961. Rem. 881.
9. From 2719 take 1827.
10. From 12791 take 9872. Rem. 2919.
11. From 24598 take 20689. Rem. 3909.
12. From 978637 take 97863.
13. From 1654278 take 755429. Rem. 898849.
14. Take 678902 from 896454. Rem. 217552.
15. Take 1724937 from 1963869.
16. Take 23468579 from 60050040. Rem. 36581461.
17. Take 9879789 from 9900000. Rem. 20211.
18. Take 7896845 from 10000000.
19. Minuend=1234567, Subtrahend=765432.
Rem. 469135.

Note.—Begin by saying, "Subtract the Subtrahend from the Minuend."

20. Min.=290178, Sub.=108405. Rem. 181765.
21. Sub.=20499, Min.=1900623.
22. Sub.=987631, Min.=8765413. Rem. 7777782.
23. 12646723—9758944=what? Ans. 2887879.
24. 2468000—970053=what?
25. What is the difference between one million, and ninety-nine? Ans. 999901.

Note.—Begin, "Subtract the less number from the greater."

26. What is the difference between thirty-seven billions, and eleven? Ans. 36999999989.
27. What is the difference between nine thousand six hundred and thirteen, and five hundred and forty-two?
28. What is the difference between eight thousand and twenty-six, and eight hundred and twenty-six? Ans. 7200.
29. What is the difference between five thousand four hundred and ninety, and seven hundred and sixty-two?

MULTIPLICATION OF ABSTRACT NUMBERS.

§ 31. MULTIPLICATION is the operation of finding a number which shall contain *one* of two given numbers as many times as there are units in *the other*.

Thus, 3 times 6 are 18: here 6 is multiplied by 3, because 18 contains 6, 3 times.

§ 32. The number *to be multiplied* is called the *multiplicand*.

§ 33. The *multiplying* number is called the *multiplier*.

§ 34. The *result* of multiplication is called the *product*.

§ 35. Either the multiplicand or the multiplier is called a *factor* of the product, and they both are called its *factors*.

In general, one number is a factor of any other number which contains it an exact number of times.

Thus, 3 is a factor of 18; 4 is a factor of 12, or of 20; 5 is a factor of 10, of 15, of 30, or of 45.

§ 36. The *sign* of multiplication, $\times$, when placed between two numbers, denotes that one of them is to be multiplied by the other. It is read *times*, when placed after the multiplier, and *multiplied by*, when placed after the multiplicand. Thus, to denote that 6 is to be multiplied by 3, we may say, 3×6, 3 *times* 6, or 6×3, 6 *multiplied by* 3. To denote the successive multiplication of more than two numbers, periods are used. Thus, $2.3.5=30$. 2 times 3 times $5=30$.

§ 37. The product of any two abstract factors is the same, no matter which is used as multiplier. Thus, $3\times6=6\times3=18$; $4\times5=5\times4=20$; $10\times8=8\times10=80$.

MULTIPLICATION TABLE.

Once 0 is 0	Twice 0 are 0	3 times 0 are 0
Once 1 is 1	Twice 1 are 2	3 times 1 are 3
Once 2 is 2	Twice 2 are 4	3 times 2 are 6
Once 3 is 3	Twice 3 are 6	3 times 3 are 9
Once 4 is 4	Twice 4 are 8	3 times 4 are 12
Once 5 is 5	Twice 5 are 10	3 times 5 are 15
Once 6 is 6	Twice 6 are 12	3 times 6 are 18
Once 7 is 7	Twice 7 are 14	3 times 7 are 21
Once 8 is 8	Twice 8 are 16	3 times 8 are 24
Once 9 is 9	Twice 9 are 18	3 times 9 are 27
Once 10 is 10	Twice 10 are 20	3 times 10 are 30
Once 11 is 11	Twice 11 are 22	3 times 11 are 33
Once 12 is 12	Twice 12 are 24	3 times 12 are 36

4 times 0 are 0	5 times 0 are 0	6 times 0 are 0
4 times 1 are 4	5 times 1 are 5	6 times 1 are 6
4 times 2 are 8	5 times 2 are 10	6 times 2 are 12
4 times 3 are 12	5 times 3 are 15	6 times 3 are 18
4 times 4 are 16	5 times 4 are 20	6 times 4 are 24
4 times 5 are 20	5 times 5 are 25	6 times 5 are 30
4 times 6 are 24	5 times 6 are 30	6 times 6 are 36
4 times 7 are 28	5 times 7 are 35	6 times 7 are 42
4 times 8 are 32	5 times 8 are 40	6 times 8 are 48
4 times 9 are 36	5 times 9 are 45	6 times 9 are 54
4 times 10 are 40	5 times 10 are 50	6 times 10 are 60
4 times 11 are 44	5 times 11 are 55	6 times 11 are 66
4 times 12 are 48	5 times 12 are 60	6 times 12 are 72

7 times 0 are 0	8 times 0 are 0	9 times 0 are 0
7 times 1 are 7	8 times 1 are 8	9 times 1 are 9
7 times 2 are 14	8 times 2 are 16	9 times 2 are 18
7 times 3 are 21	8 times 3 are 24	9 times 3 are 27
7 times 4 are 28	8 times 4 are 32	9 times 4 are 36
7 times 5 are 35	8 times 5 are 40	9 times 5 are 45
7 times 6 are 42	8 times 6 are 48	9 times 6 are 54
7 times 7 are 49	8 times 7 are 56	9 times 7 are 63
7 times 8 are 56	8 times 8 are 64	9 times 8 are 72
7 times 9 are 63	8 times 9 are 72	9 times 9 are 81
7 times 10 are 70	8 times 10 are 80	9 times 10 are 90
7 times 11 are 77	8 times 11 are 88	9 times 11 are 99
7 times 12 are 84	8 times 12 are 96	9 times 12 are 108

10 times 0 are 0	11 times 0 are 0	12 times 0 are 0
10 times 1 are 10	11 times 1 are 11	12 times 1 are 12
10 times 2 are 20	11 times 2 are 22	12 times 2 are 24
10 times 3 are 30	11 times 3 are 33	12 times 3 are 36
10 times 4 are 40	11 times 4 are 44	12 times 4 are 48
10 times 5 are 50	11 times 5 are 55	12 times 5 are 60
10 times 6 are 60	11 times 6 are 66	12 times 6 are 72
10 times 7 are 70	11 times 7 are 77	12 times 7 are 84
10 times 8 are 80	11 times 8 are 88	12 times 8 are 96
10 times 9 are 90	11 times 9 are 99	12 times 9 are 108
10 times 10 are 100	11 times 10 are 110	12 times 10 are 120
10 times 11 are 110	11 times 11 are 121	12 times 11 are 132
10 times 12 are 120	11 times 12 are 132	12 times 12 are 144

Ex. 1. Multiply 24307 by 3.

Multiplicand,	24307
Multiplier,	3
Product,	72921

§ 38. MODEL.—3 times 7 are 21, set down 1; 3 times 0 are 0, and 2 are 2; 3 times 3 are 9; 3 times 4 are 12, set down 2; 3 times 2 are 6, and 1 are 7. The product is 72921.

EXPLANATION.—The smaller factor is placed under the larger. Then, beginning at the right, each figure of the upper number is taken 3 times, the right hand figure of each product is set down, and the remaining figure, if any, is added to the next product. 3 times 7 units are 21 units, that is, 2 tens and 1 unit; hence, 1 unit is set in the units' place, and 2 tens are added to the product of the tens.

2. Multiply 24307 by 40.

Multiplicand,	24307
Multiplier,	40
Product,	972280

§ 39. MODEL.—4 times 7 are 28, set down 8; 4 times 0 are 0, and 2 are 2; 4 times 3 are 12, set down 2; 4 times 4 are 16, and 1 are 17, set down 7; 4 times 2 are 8, and 1 are 9: annex 0. The product is 972280.

EXPLANATION.—Since 10 units of any order make one of the next order on the left, any number is multiplied by 10

by merely moving each of its figures one place to the left, and putting a 0 in the place of units. Hence, to multiply by 40, each figure of the product by 4 is set one place to the left, and the units' place is filled with a 0.

3. Multiply 24307 by 43.

Multiplicand,	24307
Multiplier,	43
1st partial prod.	72921
2nd partial prod.	97228
Product,	1045201

§ 40. MODEL.—3 times 7 are 21, set down 1; 3 times 0 are 0, and 2 are 2; 3 times 3 are 9; 3 times 4 are 12, set down 2; 3 times 2 are 6, and 1 are 7:—4 times 7 are 28, set down 8 under 2; 4 times 0 are 0, and 2 are 2; 4 times 3 are 12, set down 2: 4 times 4 are 16, and 1 are 17, set down 7; 4 times 2 are 8, and 1 are 9. Add the partial products: 1; 8 and 2 are 10, set down 0; 1 and 2 are 3, and 9 are 12, set down 2; 1 and 2 are 3, and 2 are 5; 7 and 7 are 14, set down 4; 1 and 9 are 10, set down 10. The product is 1045201.

EXPLANATION.—The upper number is multiplied, first by 3, as in § 38, and then by 40, as in § 39, except that the 0 at the right is omitted, as being unnecessary, since the several figures can be placed in their proper columns without it. It must be remembered, however, that the second partial product is not 97,228, but 972,280.

4. Multiply 3047 by 246279.

5. Multiply 794378 by 4608.

Multiplier,	246279	Multiplicand,	794378
Multiplicand,	3047	Multiplier,	4608
	1723953		6355024
	985116		4766268
	738837		3177512
Product,	750412113	Product,	3660493824

RULE.—1. *When either factor contains but one valuable figure. Set the smaller factor under the larger. Beginning at the right, multiply each figure of the upper number by the lower number, set down the right hand figure of the product, and add the remaining figure, if any, to the next product; but set down the whole of the last product.*

2. *When the smaller factor contains more than one valuable figure. Set it under the larger; multiply the upper factor by each figure of the lower, setting the first figure of each partial product under the multiplying figure which produced it, and add the partial products together in that order.*

PROOF.—Multiply the lower factor by the upper.

Ex. 6. Multiply 3469 by 3.	Prod. 10407.
7. Multiply 4 by 268.	Prod. 1072.
8. Multiply 45274 by 5.	Prod. 226370.
9. Multiply 56295 by 6.	
10. Multiply 75397 by 7.	Prod. 527779.
11. Multiply 9 by 98765.	Prod. 888885.
12. Multiply 21179 by 27.	
13. Multiply 97825 by 34.	Prod. 3326050.
14. Multiply 86906 by 45.	Prod. 3910770.
15. Multiply 279362 by 52.	
16. Multiply 192837 by 67.	Prod. 12920079.
17. Multiply 293705 by 75.	Prod. 22027875.
18. Multiply 246835 by 83.	
19. $1964326 \times 98 =$ what?	Ans. 192503948.

Note.—Begin, "Multiply the first number by the second."

20. What is the product of 2975×375? Ans. 1115625.

21. What is the product of 3047×287?

22. What is the product of 40535×493? Ans. 19983755.
23. What is the product of 4027×4027? Ans. 16216729.
24. $719 \times 729 =$ what?
25. $92730465 \times 1794 =$ what? Ans. 166358454210.
26. $8162035 \times 28645 =$ what? Ans. 233801492575.

DIVISION OF ABSTRACT NUMBERS.

§ 41. Division is the operation of finding how many times one number is contained in an other. Thus, 4 in 20, 5 times: here 20 is divided by 4, since 4 is contained 5 times in 20.

§ 42. Or, Division is the operation of separating a number into some number of equal parts. Thus, if 20 is divided into 4 equal parts, each of the parts is 5.

§ 43. The number *to be divided* is called the *dividend*.

§ 44. The *dividing* number is called the *divisor*.

§ 45. The *result* of division is called the *quotient*.

§ 46. When the division is not complete, the undivided part *of the dividend* is called the *remainder*. Thus, 8 in 35, 4 times, with 3 over; here 35 is the dividend, 8 is the divisor, 4 is the quotient, and 3 is the remainder.

§ 47. The sign of division, $\div$, is read *divided by*. When placed between two numbers, it denotes that the one before it is to be divided by the one after it. Thus, $20 \div 5 = 4$.

§ 48. Division is sometimes denoted by placing the dividend over the divisor with a line between them. Thus, $\frac{20}{5} = 4$.

DIVISION TABLE.

1 in 0, no time	2 in 0, no time	3 in 0, no time
1 in 1, once	2 in 2, once	3 in 3, once
1 in 2, twice	2 in 4, twice	3 in 6, twice
1 in 3, 3 times	2 in 6, 3 times	3 in 9, 3 times
1 in 4, 4 times	2 in 8, 4 times	3 in 12, 4 times
1 in 5, 5 times	2 in 10, 5 times	3 in 15, 5 times
1 in 6, 6 times	2 in 12, 6 times	3 in 18, 6 times
1 in 7, 7 times	2 in 14, 7 times	3 in 21, 7 times
1 in 8, 8 times	2 in 16, 8 times	3 in 24, 8 times
1 in 9, 9 times	2 in 18, 9 times	3 in 27, 9 times

4 in 0, no time	5 in 0, no time	6 in 0, no time
4 in 4, once	5 in 5, once	6 in 6, once
4 in 8, twice	5 in 10, twice	6 in 12, twice
4 in 12, 3 times	5 in 15, 3 times	6 in 18, 3 times
4 in 16, 4 times	5 in 20, 4 times	6 in 24, 4 times
4 in 20, 5 times	5 in 25, 5 times	6 in 30, 5 times
4 in 24, 6 times	5 in 30, 6 times	6 in 36, 6 times
4 in 26, 7 times	5 in 35, 7 times	6 in 42, 7 times
4 in 32, 8 times	5 in 40, 8 times	6 in 48, 8 times
4 in 36, 9 times	5 in 45, 9 times	6 in 54, 9 times

7 in 0, no time	8 in 0, no time	9 in 0, no time
7 in 7, once	8 in 8, once	9 in 9, once
7 in 14, twice	8 in 16, twice	9 in 18, twice
7 in 21, 3 times	8 in 24, 3 times	9 in 27, 3 times
7 in 28, 4 times	8 in 32, 4 times	9 in 36, 4 times
7 in 35, 5 times	8 in 40, 5 times	9 in 45, 5 times
7 in 42, 6 times	8 in 48, 6 times	9 in 54, 6 times
7 in 49, 7 times	8 in 56, 7 times	9 in 63, 7 times
7 in 56, 8 times	8 in 64, 8 times	9 in 72, 8 times
7 in 63, 9 times	8 in 72, 9 times	9 in 81, 9 times

10 in 0, no time	11 in 0, no time	12 in 0, no time
10 in 10, once	11 in 11, once	12 in 12, once
10 in 20, twice	11 in 22, twice	12 in 24, twice
10 in 30, 3 times	11 in 33, 3 times	12 in 36, 3 times
10 in 40, 4 times	11 in 44, 4 times	12 in 48, 4 times
10 in 50, 5 times	11 in 55, 5 times	12 in 60, 5 times
10 in 60, 6 times	11 in 66, 6 times	12 in 72, 6 times
10 in 70, 7 times	11 in 77, 7 times	12 in 84, 7 times
10 in 80, 8 times	11 in 88, 8 times	12 in 96, 8 times
10 in 90, 9 times	11 in 99, 9 times	12 in 108, 9 times

I. SHORT DIVISION.

Ex. 1. Divide 3096 by 3.

Divisor, 3)3096, Dividend.
1032, Quotient.

§ 49. MODEL.—3 in 3, once; 3 in 0, no time; 3 in 9, 3 times; 3 in 6, twice.

The quotient is 1032.

EXPLANATION.—The divisor is placed on the left of the dividend. Then, beginning at the left, the number in each order of units is divided by 3, and each quotient figure is set in its proper column.

Ex. 2. Divide 80624 by 2. Quot. 40312.

3. Divide 8048 by 4. Quot. 2012.

4. Divide 90369 by 3. Quot. 30123.

5. Divide 17120 by 8.

8)17120
2140

§ 50. MODEL.—8 in 17, twice, with 1 over, set down 2; 8 in 11, once, with 3 over, set down 1; 8 in 32, 4 times; 8 in 0, no time.

The quotient is 2140.

EXPLANATION.—8 is not contained in 1, that is, in 1 ten-thousand, in its present form; hence, 1 ten-thousand is reduced to 10 thousands, and added to the 7 thousands, making 17 thousands. 8 is contained twice in 16; so that there is 1 thousand still undivided. This is reduced to 10 hundreds, and added to the 1 hundred, making 11 hundreds. 8 is contained once in 8; so that there are 3 hundreds still undivided. These are reduced to 30 tens, and added to the 2 tens, making 32 tens. 8 is contained in 32 just 4 times. The 0 of the dividend is retained in the quotient, to cause the several quotient figures, 2 thousands, 1 hundred, and 4 tens, to occupy their proper places

Ex. 6. Divide 36374 by 9.

9)36374
4041...5

§ 51. MODEL.—9 in 36, 4 times; 9 in 3, 0 time, with 3 over, set down 0; 9 in 37, 4 times, with 1 over, set down 4; 9 in 14, once, with 5 over, set down 1 in the quotient, and 5 as remainder. The quotient is 4041, and the remainder 5.

EXPLANATION.—The division of the 5 units might be denoted $\frac{5}{9}$, as in § 48.

RULE.—*Set the divisor on the left of the dividend, with a line between them, and one under the dividend.*

Beginning at the left, see how many times the divisor is contained in each figure of the dividend, and set the result under the dividend.

Whenever there is a remainder, prefix it to the next figure of the dividend, before dividing.

If the divisor is not contained in any figure, except the first, set 0 under such figure, and regard it as a remainder.

PROOF.—Multiply the quotient by the divisor: the product, increased by the remainder, if any, will be equal to the dividend.

Ex. 7. Divide 73052 by 2. Quot. 36526.
8. Divide 222345 by 3. Quot. 74115.
9. Divide 123456 by 4.
10. Divide 790530 by 5. Quot. 158106.
11. Divide 78920472 by 6. Quot. 13153412.
12. Divide 945 by 7.
13. Divide 1240128 by 8. Quot. 155016.
14. Divide 743200173 by 9. Quot. 82577797.
15. Divide 4703750 by 10.
16. Divide 9009 by 11. Quot. 819.
17. Divide 721428 by 12. Quot. 60119.

II. LONG DIVISION.

Ex. 18. Divide 2966232 by 925.

Dividend, 2966232	925, Divisor.
2775	3206, Quotient
1912	
1850	
6232	
5550	
682, Remainder.	

§ 52. MODEL.—9 in 29, 3 times; multiply the divisor by 3; 3 times 5 are 15, set down 5; 3 times 2 are 6, and 1 are 7; 3 times 9 are 27, set down 27: subtract the product from the dividend; 2; 5 from 6 leaves 1; 7 from 16 leaves 9; 8 from 9 leaves 1:—9 in 19, twice; multiply the divisor by 2; twice 5 are 10, set down 0; twice 2 are 4, and 1 are 5; twice 9 are 18, set down 18: subtract the product from the previous remainder; 3; 0 from 2 leaves 2; 5 from 11 leaves 6; 9 from 9 leaves 0:—9 in 6, no time; annex 2:—9 in 62, 6 times; multiply the divisor by 6; 6 times 5 are 30, set down 0; 6 times 2 are 12, and 3 are 15, set down 5; 6 times 9 are 54, and 1 are 55: subtract the product from the previous remainder; 0 from 2 leaves 2; 5 from 13 leaves 8; 6 from 12 leaves 6; 6 from 6 leaves 0. The quotient is 3206, and the remainder 682.

EXPLANATION.—The divisor is placed on the right of the dividend, for convenience in multiplying. The number 9 is used throughout as a *trial divisor*. As two figures of the real divisor are thus omitted, two figures of each partial dividend must be omitted also. Hence, in the third step, we say 9 in 6, and not 9 in 62, until we have annexed an additional figure. The first quotient figure stands for 3000; hence the first product is really 2775000, and the first remainder 191232; but, as we do not need all these

figures for the next step, we begin to subtract only one place to the right of the last valuable figure in the product. The division of the remainder might be expressed as in § 48.

Ex. 19. Divide 6593 by 19.

6593	19
57	347
89	
76	
133	
133	
0	

§ 53. MODEL.—2 in 6, 3 times; multiply the divisor by 3; 3 times 9 are 27; set down 7; 3 times 1 are 3, and 2 are 5: subtract the product from the dividend; 9; 7 from 15 leaves 8; 6 from 6 leaves 0:—2 in 8, 4 times; multiply the divisor by 4; 4 times 9 are 36, set down 6; 4 times 1 are 4, and 3 are 7: subtract this product from the previous remainder; 3; 6 from 9 leaves 3; 7 from 8 leaves 1:—2 in 13, 7 times; multiply the divisor by 7; 7 times 9 are 63, set down 3; 7 times 1 are 7, and 6 are 13, set down 13: subtract the product from the previous remainder; 0. The quotient is 347.

EXPLANATION.—If the second figure of the divisor is less than 5, the first figure is the trial divisor; but, if the second figure is greater than 5, the trial divisor is one more than the first figure. If, on multiplying, a quotient figure be found to be too large or too small, let it be diminished or increased a unit at a time until the right result is attained.

RULE.—*Set the divisor on the right of the dividend, with a line between them, and one under the divisor.*

Beginning at the left, see how often the divisor is contained in the first part of the dividend: the result will be the first figure of the quotient. Multiply the divisor by this quotient figure, and subtract the product from that part of the dividend which was used, annexing to the remainder the next figure of the dividend.

Take this remainder as a second partial dividend, and from it obtain the second quotient figure. Multiply the divisor by this figure, and subtract the product from the previous remainder, annexing to the second remainder the next figure of the dividend.

Continue this process till all the figures of the dividend have been used.

If any partial dividend will not contain the divisor, set 0 *in the quotient, annex an other figure of the dividend, and divide again.*

PROOF. 1.—The same as in § 51, for short division.

Or, 2. Subtract the remainder, if any, from the dividend; divide this remainder by the quotient, and the result will be the divisor.

Ex. 20 Divide 18950 by 25. Quot. 758.

21 Divide 17136 by 36.

22. Divide 42581 by 49 Quot. 869.

23. Dividend=16701, Divisor=57. Quot. 293.

Note.—Begin, "Divide the Dividend by the Divisor."

24. Dividend=26536, Divisor=62.

25. Dividend=15076872, Divisor=72. Quot. 209401.

26. Dividend=30744, Divisor=84. Quot. 366.

27. Divisor=97, Dividend=84002.

28. Divisor=125, Dividend=15625. Quot. 125.

29. Divisor=273, Dividend=104832. Quot. 384.

30. Divisor=354, Dividend=94164.

31. Divisor=465, Dividend=267375. Quot. 575.

32. Divisor=531, Dividend=340902. Quot. 642.

33. Divisor=685, Dividend=543205.

34. Divisor=721, Dividend=2728264. Quot. 3784.

35. Divisor=829, Dividend=5697717.

36. Divisor=937, Dividend=981976. Quot. 1048.
37. 5754375÷1125=what? Ans. 5115.
38. 4515625÷2125=what?
39. 48284964÷3094=what? Ans. 15606.
40. 24896825÷4105=what? Ans. 6065.
41. 27206656÷5216=what?
42. 45782172÷6327=what? Ans. 7236.
43. 313201258÷7153=what? Ans. 43786.
44. 293834463995÷8405=what?
45. 572473044÷9516=what? Ans. 60159.
46. 93939874943÷10471=what? Ans. 8971433.
47. 151807041÷12321=what?
48. Dividend=1274153376, Divisor=23456.
Quot. 54321.
49. Dividend=1839739176, Divisor=34056.
Quot. 54021.
50. Dividend=2642079580, Divisor=40565.
51. Dividend=2900124304, Divisor=56504.
Quot. 51326.
52. Divisor=65405, Dividend=667719645.
Quot. 10209.
53. Divisor=74316, Dividend=4734969624.
54. Divisor=81634, Dividend=7571145330.
Quot. 92745.
55. Divisor=95703, Dividend=1299551037.
Quot. 13579.
56. Divisor=97531, Dividend=2999956029.
57. Divisor=36805, Dividend=800655970.
Quot. 21754.
58. Divisor=234282, Dividend=83596737522.
Quot. 356821.
59. Divisor=5276431, Dividend=7105901538475.

CONTRACTION IN ADDITION.

Note.—The judicious teacher will omit this and most of the following contractions as his classes proceed through the book the first time.

Ex. 1. Add together the following numbers:

469375
237924
472437
853214
975318
242326
3250594

§ 54. MODEL.—26 and 10 are 36, and 8 are 44, and 10 are 54, and 4 are 58, and 30 are 88, and 7 are 95, and 20 are 115, and 4 are 119, and 70 are 189, and 5 are 194, set down 94:—1 and 23 are 24, and 50 are 74, and 3 are 77, and 30 are 107, and 2 are 109, and 20 are 129, and 4 are 133, and 70 are 203, and 9 are 212, and 90 are 302, and 3 are 305, set down 05:—3 and 24 are 27, and 90 are 117, and 7 are 124, and 80 are 204, and 5 are 209, and 40 are 249, and 7 are 256, and 20 are 276, and 3 are 279, and 40 are 319, and 6 are 325, set down 325. The sum is 3250594.

EXPLANATION.—Beginning at the right, and taking two columns at a time, we take in first the tens and then the units, as we go up the column, and set down the two right hand figures of each sum.

Ex. 2.	3.	4.	5.	6.	7.
123456	1234	235689	14250663	819349	120341
789012	5678	124578	32215941	720258	989052
345678	9012	135792	10340285	630167	878163
901234	3456	468097	92341967	541076	767274
567890	7890	531086	82395786	452985	656385
987654	1357	420987	17084657	363894	545496
321098	9246	654321	40558476	274703	432107
765432	8987	555775	91623378	185612	321098
4801454	46860	3126325	380811153		

CONTRACTION IN SUBTRACTION.

Ex. 1. From 970347 take the sum of 14375, 226899, 12534, and 369708.

970347
14375
226899
12534
369708
346831

§ 55. MODEL.—8 and 4 are 12, and 9 are 21, and 5 are 26, from 27 leaves 1; 2 and 3 are 5, and 9 are 14, and 7 are 21, from 24 leaves 3; 2 and 7 are 9, and 5 are 14, and 8 are 22, and 3 are 25, from 33 leaves 8; 3 and 9 are 12, and 2 are 14, and 6 are 20, and 4 are 24, from 30 leaves 6; 3 and 6 are 9, and 1 are 10, and 2 are 12, and 1 are 13, from 17 leaves 4; 1 and 3 are 4, and 2 are 6, from 9 leaves 3. The remainder is 346831.

EXPLANATION.—As 26, the sum of the subtrahend units, can not be taken from 7, the units of the minuend, we add 2 tens, that is, 20 units, to the minuend, and afterwards add 2 tens to the subtrahend. (§ 28.)

Note.—Let the pupil be required to use this contraction whenever it can be applied.

Ex. 2. From 1000 take 9+98+176+254+289.

Rem. 174.

3. From 9125 take 8+88+888+1297+3945.

Rem. 2899.

4. From 10275 take 1245+3735+2986+895.

Rem. 1414.

5. From 87579 take 1477+2796+8972+10896.

Rem. 63438.

6. From 120225 take 246+1357+97531+1358.
7. From 72575 take 575+2575+4575+15575.
8. From 4970 take 250-|-325-|-348-|-2211.
9. From 22907 take 3916-|-2821-|-4302-|-2309.

CONTRACTIONS IN MULTIPLICATION.

Ex. 1. Multiply 7325 by 100.

732500.

§ 56. MODEL.—Annex two naughts to the multiplicand. The product is 732500.

EXPLANATION.—We annex to the multiplicand as many ciphers as there are annexed to the 1 of the multiplier. (§ 39.)

Ex. 2. Multiply 1358 by 10. Prod. 13580.
3. Multiply 2468 by 100. Prod. 246800.
4. Multiply 4579 by 1000. Prod. 4579000.
5. Multiply 86725 by 10000.
6. Multiply 1020 by 100.
7. Multiply 32500 by 1000.

8. Multiply 32500 by 25000.

32500
25000
1625
650
812500000

§ 57. MODEL.—5 times 5 are 25, set down 5; 5 times 2 are 10, and 2 are 12, set down 2; 5 times 3 are 15, and 1 are 16, set down 16:—twice 5 are 10, set down 0 under 2; twice 2 are 4, and 1 are 5; twice 3 are 6:—add the partial products: 5; 2; 5 and 6 are 11, set down 1; 1 and 6 are 7, and 1 are 8:—annex 5 naughts. The product is 812500000.

EXPLANATION.—After finding the product of the valuable figures, we annex to it as many naughts as there are in the right of both the factors.

Ex. 9. Multiply 27500 by 350. Prod. 9625000.
10. Multiply 1250 by 1500. Prod. 1875000.
11. Multiply 747000 by 250. Prod. 186750000.
12. Multiply 19500 by 1400. Prod. 27300000.
13. Multiply 124750 by 3000.
14. Multiply 2795000 by 2700.

15. Multiply 3759 by 104.

3759×104
15036
390936

§ 58. MODEL.—4 times 9 are 36, set down 6, two places to the right of 9; 4 times 5 are 20, and 3 are 23, set down 3; 4 times 7 are 28, and 2 are 30, set down 0; 4 times 3 are 12, and 3 are 15, set down 15:—add the partial products:—6; 3; 9; 5 and 5 are 10, set down 0; 1 and 1 are 2, and 7 are 9; 3. The product is 390936.

EXPLANATION.—If the multiplier has only two valuable figures, the first of which is 1, we multiply by the other valuable figure, and set the first figure of the product as far to the right of the units figure of the multiplicand as this figure is to the right of the 1.

Ex. 16. Multiply 2376 by 12. Prod. 28512.
17. Multiply 47475 by 107. Prod. 5079825.
18. Multiply 57875 by 10080. Prod. 583380000.
19. Multiply 275 by 1009. Prod. 277475.
20. Multiply 4479 by 10006.
21. Multiply 795310 by 10500.

22. Multiply 1025 by 7001.

1025×7001
7175
7176025

§ 59. MODEL.—7 times 5 are 35, set down 5, three places to the left of 5; 7 times 2 are 14, and 3 are 17, set down 7; 7 times 0 are 0, and 1 is 1; 7 times 1 are 7:—add the partial products:—5; 2; 0; 5 and 1 are 6; 7; 1; 7. The product is 7176025.

EXPLANATION.—If the multiplier has only two valuable figures, the last of which is 1, we multiply by the other valuable figure, and set the first figure of the product as far to the left of the units figure of the multiplicand as this figure is to the left of the 1.

Ex. 23. Multiply 7893 by 51. Prod. 402543.

24. Multiply 4685 by 601. Prod. 2815685.
25. Multiply 23795 by 7010. Prod. 166802950.
26. Multiply 1375 by 8001.
27. Multiply 20478 by 90010.

28. Multiply 27346 by 99.

2734600
27346
2707254

§ 60. MODEL.—Annex 2 naughts to the multiplicand:—subtract the multiplicand from the result; 6 from 10 leaves 4; 5 from 10 leaves 5; 4 from 6 leaves 2; 7 from 14 leaves 7; 3 from 3 leaves 0; 0 from 7 leaves 7; 0 from 2 leaves 2. The product is 2707254.

EXPLANATION.—Since 9 is 1 less than 10, we may multiply any number by 9, by subtracting the number from 10 times itself. If therefore the multiplier consists of 9's alone, we annex to the multiplicand as many naughts as there are nines in the multiplier, and subtract the multiplicand from the result.

Ex. 29. Multiply 124795 by 9. Prod. 1123155.
30. Multiply 24735 by 99. Prod. 2448765.
31. Multiply 1469 by 999. Prod. 1467531.
32. Multiply 70095 by 99. Prod. 6939405.
33. Multiply 9999 by 256. (§ 37.) Prod. 2559744.
34. Multiply 1276538 by 999.
35. Multiply 8365712 by 99.

36. Multiply 2754 by 54.

27540
2754
24786
6
148716

§ 61. MODEL.—54=9 times 6. First, multiply by 9:— (§ 60.) 4 from 10 leaves 6; 6 from 14 leaves 8; 8 from 15 leaves 7; 3 from 7 leaves 4; 0 from 2 leaves 2. The product is 24786. Multiply this product by 6:— 6 times 6 are 36, set down 6; 6 times 8 are 48, and 3 are 51, set down 1; 6 times 7 are 42, and 5 are 47,

set down 7; 6 times 4 are 24, and 4 are 28, set down 8; 6 times 2 are 12, and 2 are 14, set down 14. The product is 148716.

EXPLANATION.—If the multiplier is the product of two or more numbers, we may multiply the multiplicand by either of those numbers, and this product by an other, and so on.

Ex. 37. Multiply 3725 by 35. Prod. 130375.
38. Multiply 17575 by 48. Prod. 843600.
39. Multiply 473729 by 49. Prod. 23212721.
40. Multiply 27936 by 56. Prod. 1564416.
41. Multiply 124684 by 64.
42. Multiply 247372 by 72.

43. Multiply 21357 by 714.

21357
714
149499
298998
15248898

§ 62. MODEL.—14 is twice 7. First, multiply by 7:— 7 times 7 are 49, set down 9 under 7 of the multiplier; 7 times 5 are 35, and 4 are 39, set down 9; 7 times 3 are 21, and 3 are 24, set down 4; 7 times 1 are 7, and 2 are 9; 7 times 2 are 14, set down 14. The product is 149499. Multiply this product by 2:— twice 9 are 18, set down 8 under 4 of the multiplier; twice 9 are 18, and 1 are 19, set down 9; twice 4 are 8, and 1 are 9; twice 9 are 18, set down 8; twice 4 are 8, and 1 are 9; twice 1 are 2. Add the partial products: 8; 9; 9 and 9 are 18, set down 8; 1 and 8 are 9, and 9 are 18, set down 8; 1 and 9 are 10, and 4 are 14, set down 4; 1 and 2 are 3, and 9 are 12, set down 2; 1 and 4 are 5; 1. The product is 15248898.

EXPLANATION.—If one part of the multiplier is a factor of an other, the work may be contracted as in the model, placing the first figure of each product immediately under

the right hand figure of the corresponding part of the multiplier.

Ex. 44. Multiply 12479 by 654. Prod. 8161266.
45. Multiply 24793 by 56248. Prod. 1394556664.
46. Multiply 97635 by 53545. Prod. 5227866075.
47. Multiply 86436 by 497. Prod. 42958692.
48. Multiply 23047 by 488.
49. Multiply 902756 by 366108.

---

50. Multiply 225 by 25.

4)22500
5625

§63. MODEL.—Annex 2 naughts to the multiplicand:—divide the result by 4:—4 in 22, 5 times, with 2 over, set down 5; 4 in 25, 6 times, with 1 over, set down 6; 4 in 10, twice, with 2 over, set down 2; 4 in 20, 5 times. The product is 5625.

EXPLANATION.—Annexing 2 naughts multiplies by 100, (§ 56): hence, since $100=4\times25$, we divide the product by 4, to get the true product.

Ex. 51. Multiply 10275 by 25. Prod. 256875.
52. Multiply 28832 by 25. Prod. 720800.
53. Multiply 72725 by 25. Prod. 1818125.
54. Multiply 84287 by 25. Prod. 2107175.
55. Multiply 96248 by 25.
56. Multiply 8324728 by 25.

---

57. Multiply 274 by 125.

8)274000
34250

§ 64. MODEL.—Annex 3 naughts to the multiplicand:—divide the result by 8:—8 in 27, 3 times, with 3 over, set down 3; 8 in 34, 4 times with 2 over, set down 4; 8 in 20, twice, with 4 over, set down 2; 8 in 40, 5 times; 8 in 0, no time. The product is 34250.

EXPLANATION.—Annexing 3 naughts multiplies by 1000, (§ 56): hence, since $1000=8\times125$, we divide the product by 8, to get the true product.

Ex. 58. Multiply 125 by 125. Prod. 15625.
59. Multiply 625 by 125. Prod. 78125.
60. Multiply 1776 by 125. Prod. 222000.
61. Multiply 34079 by 125. Prod. 4259875.
62. Multiply 934478 by 125.
63. Multiply 7840349 by 125.

CONTRACTIONS IN DIVISION.

Ex. 1. Divide 12564 by 100.

125,64 — § 65. MODEL.—Cut off two figures at the right. The quotient is 125, and the remainder 64.

EXPLANATION.—We cut off at the right, for remainder, as many figures as there are naughts at the right of the 1 of the divisor. The remaining figures on the left constitute the quotient.

2. Divide 34000 by 10. Quot. 3400.
3. Divide 74500 by 100. Quot. 745.
4. Divide 19740 by 100. Quot. 197; Rem. 40.
5. Divide 246000 by 1000. Quot. 246.
6. Divide 147375 by 1000.
7. Divide 24680 by 100.

8. Divide 98630 by 800.

8,00)986,30
123—230 — § 66. MODEL.—Cut off the 2 naughts at the right of the divisor, and 2 figures at the right of the dividend:—then, 8 in 9, once, with 1 over, set down 1; 8 in 18, twice, with 2 over, set down 2; 8 in 26, 3 times, with 2 over. The quotient is 123, and the remainder 230.

EXPLANATION.—The remainder after dividing is prefixed to the dividend figures cut off, to constitute the true remainder.

Ex. 9. Divide 127569 by 270. Quot. 4724; Rem. 189.
10. Divide 56000 by 700. Quot. 80.
11. Divide 3230000 by 1700. Quot. 1900.
12. Divide 24600 by 2400. Quot. 10 : Rem. 600.
13. Divide 7346790 by 72900.
14. Divide 135073 by 21800.

---

15. Divide 275 by 5.

275
2
55,0

§ 67. MODEL.—Multiply the dividend by 2: twice 5 are 10, set down 0; twice 7 are 14, and 1 are 15, set down 5; twice 2 are 4, and 1 are 5:—divide this product by 10. (§ 65.) The quotient is 55.

EXPLANATION.—Since the dividend is already 5 times the required quotient, multiplying it by 2 gives (2×5) 10 times the quotient. The part cut off at the right, by this plan, is twice the true remainder.

Ex. 16. Divide 10024 by 5. Quot. 2004; Rem. 4.
17. Divide 2725 by 5. Quot. 545.
18. Divide 49720 by 5. Quot. 9944.
19. Divide 598405 by 5. Quot. 119681.
20. Divide 479324 by 5.
21. Divide 2379456 by 5.

---

22. Divide 329 by 25.

329
4
13,16

§ 68. MODEL.—Multiply the dividend by 4: 4 times 9=36, set down 6; 4 times 2=8, and 3 =11, set down 1; 4 times 3=12, and 1=13, set down 13:—divide this product by 100 (§ 65.) The quotient is 13, and the remainder 4.

EXPLANATION.—Since the dividend is already 25 times the required quotient, multiplying it by 4 gives (4×25) 100 times the quotient. The part cut off at the right, by this plan, is 4 times the true remainder.

Ex. 23. Divide 293235 by 25. Quot. 11729; Rem. 10.

24. Divide 148532 by 25. Quot. 5941; Rem. 7.

25. Divide 2475 by 25. Quot. 99.

26. Divide 193450 by 25. Quot. 7738.

27. Divide 34795 by 25.

28. Divide 107059 by 25.

---

29. Divide 23725 by 125.

23725
8
189,800

§ 69. MODEL.—Multiply the dividend by 8: 8 times 5 are 40, set down 0; 8 times 2 are 16, and 4 are 20, set down 0; 8 times 7 are 56, and 2 are 58, set down 8; 8 times 3 are 24, and 5 are 29, set down 9; 8 times 2 are 16, and 2 are 18, set down 18:—divide this product by 1000. (§ 65.) The quotient is 189, and the remainder 100.

EXPLANATION.—Since the dividend is already 125 times the required quotient, multiplying it by 8 gives (8×125) 1000 times the quotient. The part cut off at the right, by this plan, is 8 times the true remainder.

Ex. 30. Divide 724350 by 125. Quot. 5794; Rem. 100.

31. Divide 111000 by 125. Quot. 888.

32. Divide 246625 by 125. Quot. 1973.

33. Divide 57935 by 125. Quot. 463; Rem. 60.

34. Divide 793575 by 125.

35. Divide 125364 by 125.

---

36. Divide 10202 by 42.

2)10202
3)5101
7)1700—1
242—6

§ 70. MODEL.—42=2 times 3 times 7. First, divide by 2:—2 in 10, 5 times; 2 in 2, once; 2 in 0, no time; 2 in 2, once:—divide this quotient by 3:—3 in 5, once, with 2 over, set down 1; 3 in 21, 7 times; 3 in 0, no time; 3 in 1, no time, with 1 over, set down 0 in the quotient, and 1 as remainder:—divide this quotient by 7:—7 in 17, twice, with 3 over, set down 2; 7 in 30, 4 times, with 2 over, set down 4; 7 in 20, twice, with 6 over, set down 2 in the quotient, and 6 as remainder. The quotient is 242, and the remainder 38.

EXPLANATION.—If the divisor is the product of two or more numbers, we may divide the dividend by either of those numbers, and the quotient by an other, and so on. The true remainder is found by multiplying each remainder by all the divisors previous to the one which produced it, and adding together the several products.

Ex. 37. Divide 7346 by 56. Quot. 131; Rem. 10.
38. Divide 347934 by 35. Quot. 9940; Rem. 34.
39. Divide 92384 by 64. Quot. 1443; Rem. 32.
40. Divide 83495 by 45. Quot. 1855; Rem. 20.
41. Divide 745106 by 72.
42. Divide 656215 by 96.

43. Divide 34635 by 285.

34635 | 285
613 | 121
435
150

§ 71. MODEL.—3 in 3, once:—once 5 is 5, from 6 leaves 1; once 8 is 8, from 14 leaves 6; once 2 is 2, and 1 are 3, from 3 leaves 0: annex 3:—3 in 6, twice:—twice 5 are 10, from 13 leaves 3; twice 8 are 16, and 1 are 17, from 21 leaves 4; twice 2 are 4, and 2 are 6, from 6 leaves 0: annex 5:—3 in 4, once:—once 5 is 5, from 5 leaves 0; once 8 is 8, from 13 leaves 5; once 2 is 2, and 1 are 3, from 4 leaves 1. The quotient is 121, and the remainder 150.

EXPLANATION.—The products are not written, but are immediately substracted as in § 55.

Note.—Let all the exercises in Long Division hereafter be performed by this plan.

Ex. 44. Divide 136895 by 725. Quot. 188; Rem. 595.
45. Divide 247986 by 836. Quot. 296; Rem. 230.
46. Divide 358097 by 749. Quot. 478; Rem. 75.
47. Divide 469108 by 5275. Quot. 88; Rem. 4908.
48. Divide 5702195 by 4386.
49. Divide 68132050 by 5295.

GENERAL PRINCIPLES OF DIVISION.

§ 72. If the divisor remain unchanged, and the dividend be multiplied by any number, the quotient will be multiplied by the same number. Thus, $32 \div 8 = 4$: then, $64 \div 8 = 8$.

§ 73. If the divisor remain unchanged, and the dividend be divided by any number, the quotient will be divided by the same number. Thus, $32 \div 8 = 4$: then, $16 \div 8 = 2$.

§ 74. If the dividend remain unchanged, and the divisor be multiplied by any number, the quotient will be divided by the same number. Thus, $32 \div 8 = 4$: then, $32 \div 16 = 2$.

§ 75. If the dividend remain unchanged, and the divisor be divided by any number, the quotient will be multiplied by the same number. Thus, $32 \div 8 = 4$: then, $32 \div 4 = 8$.

§ 76. If the dividend and the divisor be both multiplied by the same number, the quotient will remain unchanged. Thus, $32 \div 8 = 4$: then, $64 \div 16 = 4$.

§ 77. If the dividend and the divisor be both divided by the same number, the quotient will remain unchanged.—Thus, $32 \div 8 = 4$: then, $16 \div 4 = 4$.

PROMISCUOUS PROBLEMS.

1. The subtrahend is thirty thousand and forty-five; the remainder is forty-six thousand eight hundred and ninety: what is the minuend? Ans. 76935.

§ 78. Minuend—Subtrahend=Remainder.
Minuend—Remainder=Subtrahend.
Subtrahend+Remainder=Minuend.

2. The minuend is three hundred thousand; the subtrahend is ninety-nine thousand three hundred and seventy-four: what is the remainder? Ans. 200624.

3. The minuend is seventy thousand and twenty-nine; the remainder is sixty-five thousand and forty-six: what is the subtrahend?

4. The multiplicand is twenty-seven thousand and four; the product is seven hundred and twenty-nine millions, two hundred and sixteen thousand, and sixteen: what is the multiplier? Ans. 27004.

§ 79. Multiplicand×Multiplier=Product.
Product÷Multiplier=Multiplicand.
Product÷Multiplicand=Multiplier.

5. The multiplicand is four thousand and seventy-two; the multiplier is one thousand one hundred and six: what is the product? Ans. 4503632.

6. The product is ninety-three thousand three hundred and sixty-one; the multiplier is eighty-nine: what is the multiplicand?

7. The divisor is one thousand and nine; the quotient is nine hundred and ten: what is the dividend? Ans. 918190.

§ 80. Dividend÷Divisor=Quotient.
Divisor×Quotient=Dividend.
(Dividend—Remainder)÷Quotient=Divisor.
Quotient×Divisor+Remainder=Dividend.

8. The dividend is nine hundred and forty-five thousand, eight hundred and eighty-eight; the divisor is two thousand and four: what is the quotient? Ans. 472.

9. The dividend is one hundred and forty-eight thousand; the quotient is three hundred and forty-two; the remainder is two hundred and fifty-six: what is the divisor?

10. The quotient is one thousand and three; the divisor is one thousand and two: the remainder is one thousand and one: what is the dividend? Ans. 1006007.

11. Find the sum of two hundred and forty-five thousand, nine hundred and seven, seventy-four thousand and seventy-four, one hundred and nine thousand and nine, and three hundred and ninety-seven. Sum, 429387.

12. Find the difference between two hundred thousand, and one hundred and eighty-seven thousand six hundred and fifty-four.

13. Find the product of one million three hundred and seventy-five, and one thousand three hundred and seventy-five. Prod. 1375515625.

14. Find the quotient of three millions divided by six thousand two hundred and seventy-nine.

Quot. 477; Rem. 4917.

15. What number is that from which if 2407, 4072, 724, and 7240 be subtracted, the remainder will be 7042?

16. What number is that to which if 2407, 4072, 724, and 7240 be added, the sum will be 15000? Ans. 557.

17. What number is that by which if 2047 be multiplied, the product will be 15151894? Ans. 7402.

18. What number is that by which if 2025042 be divided, the quotient will be 2021?

19. $247+1023-934+3720-4142+245=$what?

20. $(247-154)\div 3+(247+154)\times 3=$what? Ans. 1234.

§ 81. A parenthesis enclosing two or more numbers shows that their united value is to be subjected to the operation indicated immediately before or after the parenthesis. For example, in the preceding problem, the difference of 247 and 154 is to be divided by 3, and the sum of 247 and 154 is to be multiplied by 3, and the product and the quotient are to be added together.

Two numbers thus connected are called a *binomial;* three numbers are called a *trinomial;* four, a *tetranomial;* five, a *pentanomial;* six, a *hexanomial, &c.*

The 20th problem is read, "Binomial 247 minus 154 divided by 3 plus binomial 247 plus 154 multiplied by 3 is equal to what?"

21. $3247+247-47+7-(247-47+7)=$what?
22. $(987-876+333)\div(765-543)+210-95=$what?
Ans. 117.
23. $27-30\div10+(475-399)\div4=$what? Ans. 43.
24. $(204-60)\div6-(90-|-10)\div5-|-(76-|-12)\div4=$what?
25. $204-60\div6-90-|-10\div5-|-(76-|-12)\div4=$what?
Ans. 128.
26. $(204-60)\div6-|-90-|-10\div5-|-(76-|-12)\div4=$what?
27. $(204-60)\div6-(90-|-10)\div5-|-76-12\div4=$what?
28. $204-60\div6-(90-|-10\div5-|-76)-12\div4=$what?
Ans. 23.
29. $123-|-41-(123-41)-|-123\times41-123\div41=$what?
Ans. 5122.
30. $123-|-41-(123-41)-|-(123\times41-123)\div41=$what?
31. $\{[(742\div2)\div53]\times27-1\}\div53=$what? Ans. 1.
32. $[(199-78)\div11-(199-43)\div78]\times(12-3)=$what?
Ans. 81.
33. $[(117-43)\times2]\div37-|-(138-128)\times37=$what?

MEASURES AND MULTIPLES.

§ 82. An *even* number is one which can be exactly divided by 2. Thus, 12, 4, 36, 58, and 70, are *even* numbers.

Note.—All even numbers end in either 2, 4, 6, 8, or 0.

§ 83. An *odd* number is one which can not be exactly divided by 2. Thus, 9, 17, 25, 33, and 41, are *odd* numbers.

Note.—All odd numbers end in either 1, 3, 5, 7, or 9.

§ 84. A *prime* number is one which is not the product of two other numbers. Thus, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, and 97, are all the *prime* numbers less than 100.

TABLE of PRIME NUMBERS up to 1000.

2	43	103	173	241	317	401	479	571	647	739	827	919
3	47	107	179	251	331	409	487	577	653	743	829	929
5	53	109	181	257	337	419	491	587	659	751	839	937
7	59	113	191	263	347	421	499	593	661	757	853	941
11	61	127	193	269	349	431	503	599	673	761	857	947
13	67	131	197	271	353	433	509	601	677	769	859	953
17	71	137	199	277	359	439	521	607	683	773	863	967
19	73	139	211	281	367	443	523	613	691	787	877	971
23	79	149	223	283	373	449	541	617	701	797	881	977
29	83	151	227	293	379	457	547	619	709	809	883	983
31	89	157	229	307	383	461	557	631	719	811	887	991
37	97	163	233	311	389	463	563	641	727	821	907	997
41	101	167	239	313	397	467	569	643	733	823	911	

§ 85. A *composite* number is one which is the product of two other numbers. Thus, 4, 6, 9, 15, 21, and 30, are *composite* numbers, because $2\times2=4$, $2\times3=6$, $3\times3=9$, $3\times5=15$, $3\times7=21$, and $5\times6=30$.

Is 20 prime, or composite? 25? 28? 31? 34? 37? 40? 43? 50? 57? 64? 71? 78? 85? 92? 99? 106? 217? 328? 439?

§ 86. Powers.—The *first power* of a number is the number itself. Thus, 5 is the *first power* of 5; 7, of 7; 10, of 10.

The *second power* of a number is the product of the number multiplied by itself. Thus, 36 is the *second power* of 6, because $6\times6=36$: 81, of 9, because $9\times9=81$: 100, of 10, because $10\times10=100$.

The second power of a number is usually called its *square*.

The *third power* of a number is the product of the number multiplied by its *square*. Thus, 8 is the *third power* of 2, because $2\times4=8$: 64, of 4, because $4\times16=64$: 216, of 6, because $6\times36=216$: 512, of 8, because $8\times64=512$: 1000, of 10, because $10\times100=1000$.

The third power of a number is usually called its *cube*.

In like manner, what is the *fourth power* of a number? What is the *sixth* power? The *ninth* power? &c.

§ 87. Roots.—The *first root* of a number is the number itself.

The *second root*, or the *square root*, of a number is one of the *two equal* factors which produce it. Thus, 5 is the square root of 25, because $5\times5=25$.

15 has no square root, because its two factors, 3 and 5, are not equal.

The *third root*, or the *cube root*, of a number is one of the *three equal* factors which produce it. Thus, 3 is the cube root of 27, because $3.3.3=27$.

30 has no cube root, because its three factors, 2, 3, and 5, are not equal. 25 has none, because it has only two equal factors, 5 and 5. 16 has none, because it has four equal factors, 2, 2, 2, and 2.

In like manner, what is the *fourth root* of a number? What is the *seventh* root? The *sixteenth* root? &c.

§ 88. The *prime factors* of a composite number are the prime numbers by whose continued multiplication the number is produced. Thus, the prime factors of 9 are 3 and 3, because $3\times3=9$: the prime factors of 60 are 2, 2, 3, and 5, because $2.2.3.5=60$.

§ 89. A *measure* of a number is a number which is contained in it a number of times *without a remainder*. Thus, 3 is a measure of 12, because 3 is contained *exactly* 4 times in 12: 4 is a measure of 36, because 4 is contained *exactly* 9 times in 36.

Is 5 a measure of 10? 25? 37? 40? 53? 65? 80?

Is 6 a measure of 7? 12? 20? 30? 39? 48? 54?

Is 7 a measure of 14? 19? 28? 36? 42? 48? 63?

§ 90. A *multiple* of a number is a number which contains it a number of times *without a remainder*. Thus, 12 is a multiple of 3, because 12 contains 3 *exactly* 4 times: 36 is a multiple of 4, because 36 contains 4 *exactly* 9 times.

Is 40 a multiple of 2? 3? 4? 5? 6? 7? 8? 9? 10? 20?

Is 56 a multiple of 2? 4? 7? 8? 9? 10? 14? 20? 24?

Is 60 a multiple of 2? 3? 4? 5? 6? 7? 8? 10? 12? 15?

§ 91. *Common* means belonging equally to two or more numbers.

§ 92. One number is a *common* measure of *two* or more numbers, if it is a measure of each of them. Thus, 3 is a measure of 9, also of 12, also of 18; hence, 3 is a *common* measure of 9, 12, and 18. Also, 4 is a *common* measure of 8, 24, 32, and 48.

Is 2 a common measure of 4, 6, and 10?

Is 3 a common measure of 6, 10, and 15?

Is 4 a common measure of 12, 16, and 20?

Two or more numbers may have several common meas-

ures. Thus, 24 and 36 have as common measures 2, 3, 4, 6, and 12. In this case, 12 is, of course, the *greatest* common measure of 24 and 36.

§ 93. One number is a *common* multiple of two or more numbers, if it is a multiple of each of them. Thus, 40 is a multiple of 5, also of 8, also of 10; hence, 40 is a *common* multiple of 5, 8, and 10. Also, 45 is a *common* multiple of 3, 5, and 9.

Is 10 a common multiple of 2 and 5?

Is 15 a common multiple of 3 and 6?

Is 30 a common multiple of 2, 5, and 10?

Two or more numbers always have several common multiples. Thus, 4, 3, and 6, have as common multiples 12, 24, 36, 48, 60, &c. In this case, 12 is, of course, the *least* common multiple of 4, 3, and 6.

§ 94. Two or more numbers are *prime to each other*, if they have no common measure. Thus, 81 and 64 are prime to each other. Also, 20, 27, and 77 are prime to each other.

§ 95. 2 is a measure of every number which ends in either 2, 4, 6, 8, or 0. (§ 82. Note.)

3 is a measure of a number, if it is a measure of the sum of the figures which denote the number. Thus, 3 is a measure of 246, or 462, or 624, or 642, or 426, or 264, or 2064, or 4602, &c., because 3 is a measure of 6+4+2, that is, of 12.

4 is a measure of a number, if it is a measure of the number denoted by its two right hand figures. Thus, 4 is a measure of 768, or 1860, or 95372, or 1112316, because 4 is a measure of 68, or 60, or 72, or 16.

5 is a measure of every number which ends in either 5 or 0. Thus, 5 is a measure of 20, or 55, or 100, or 275.

6 is a measure of every even number of which 3 is a measure. Thus, 6 is a measure of 462, or 4512, or 1236: but not of 471, or 6321.

8 is a measure of a number, if it is a measure of the number denoted by its three right hand figures. Thus, 8 is a measure of 34800, or 753064, because 8 is a measure of 800, or 64.

9 is a measure of a number, if it is a measure of the sum of the figures which denote the number. Thus, 9 is a measure of 891, or 1728, or 253269, because 9 is a measure of 18, or 18, or 27.

10 is a measure of every number which ends in 0.

100 is a measure of every number which ends in 2 naughts.

Is 2 a measure of 3040? 4047? 28? 1112? 10124?

Is 3 a measure of 258? 369? 12345678? 5169? 2571?

Is 4 a measure of 125784? 24680? 57932? 14760? 1112?

Is 5 a measure of 245? 12450? 7824? 12570? 3457?

Is 6 a measure of 570? 378? 45342? 123456? 12324?

Is 8 a measure of 5070120? 247080? 1479008? 1234?

Is 9 a measure of 1234566? 68472? 1357? 1476?

Is 10 a measure of 240? 245? 3795? 7630? 1460?

§ 96. A measure of a number is a measure of any one of its multiples. Thus, 6 is a measure of 18: then it is a measure of 36, or 54, or 72, or 90.

§ 97. A common measure of two or more numbers is a measure of their sum. Thus, 8 is a common measure of 16, 24, and 40: then it is a measure of 80.

§ 98. A common measure of two numbers is a measure of their difference. Thus, 9 is a common measure of 18 and 54: then it is a measure of 36.

PRIME FACTORS.

Ex. 1. Resolve 7800 into its prime factors.

2)7800
2)3900
2)1950
5)975
5)195
3)39
13

§ 99. MODEL.—Divide the number by 2. (§ 50). Divide the quotient by 2. Divide this quotient by 2. Divide this quotient by 5. Divide this quotient by 5. Divide this quotient by 3. This quotient is a prime number. The prime factors of 7800 are 2, 2, 2, 5, 5, 3, and 13.

EXPLANATION.—It is better to divide first by 2 as often as possible, then by 5, and then by the other prime numbers in succession. The several divisors and the last quotient are evidently the prime factors of the number.

RULE.—*Divide the given number by one of its prime measures; divide the quotient by one of* its *prime measures; continue thus dividing until a prime number is obtained fo a quotient: the several divisors and the last quotient will be the prime factors of the given number.*

PROOF.—The continued product of the prime factors will be equal to the given number.

Ex. 2. Resolve 524 into its prime factors.

P. F. 2, 2, and 131.

3. Resolve 460 into its prime factors.

4. Resolve 770 into its prime factors.

P. F. 2, 5, 7, and 11.

5. Resolve 880 into its prime factors.

P. F. 2, 2, 2, 2, 5, and 11.

6. Resolve 999 into its prime factors.

7. Find the prime factors of 1040.

P. F. 2, 2, 2, 2, 5, and 13.

8. Find the prime factors of 1160.

P. F. 2, 2, 5, and 29.

9. Find the prime factors of 1275.

10. What are the prime factors of 1300?

Ans. 2, 2, 5, 5, and 13.

11. What are the prime factors of 1590?

Ans. 2, 5, 3, and 53.

12. What are the prime factors of 1738?

13. What are the prime factors of 19500?

Ans. 2, 2, 5, 5, 5, 3, and 13.

14. What are the prime factors of 966000?

Ans 2, 2, 2, 2, 5, 5, 5, 3, 7, and 23.

15 What are the prime factors of 825000?

16. What are the prime factors of 1357200?

Ans. 2, 2, 2, 2, 5, 5, 3, 3, 13, and 29.

INVOLUTION.

§ 100. INVOLUTION is the process of finding a *power* of a number. From the definitions of the several powers in § 86, it is evident that any power of a number is found by taking the number as a factor in multiplication as many times as there are units in the number of the power.

Ex. 1. What is the square of 7? Ans 49.

2. What is the cube of 3? Ans. 27.

3. What is the fourth power of 2?

4. What is the fifth power of 2? Ans. 32.

5. What is the fourth power of 5? Ans 625.

6. What is the cube of 9?

7. What is the square of 19? Ans. 361.

8. What is the cube of 15? Ans. 3375.

9. What is the fourth power of 20?

EVOLUTION.

§ 101 EVOLUTION is the process of finding a *root* of a given power. The method here explained is applicable only to such numbers as have precise roots. The method of extracting approximate roots of imperfect powers can not be explained without the use of algebraic formulas, and consequently is not given in this treatise.

Ex 1. What is the cube root of 216?

$$\left\{\begin{array}{l} 2)216 \\ 2)108 \\ 2)54 \end{array}\right. \qquad \left\{\begin{array}{l} 3)27 \\ 3)9 \\ 3 \end{array}\right.$$

§ 102. MODEL.—Resolve the given number into its prime factors. (§ 99.) It contains three twos and three threes. Hence, its cube root is $2\times3=6$.

EXPLANATION.—Since the cube root of a number is one of the three equal factors which produce it, we separate the prime factors into sets of three equal prime factors each, and selecting one from each set, the product of those selected is evidently the cube root of the given number. For any other root, we separate into sets of as many prime factors each as there are units in the number of the root.

Note.—If the prime factors can not be separated as above, the required root can not be *exactly* found, either by this, or by any other method.

RULE.—*Resolve the given power into its prime factors: separate the factors into groups of as many equal factors each as there are units in the number of the root; select one factor from each group, and multiply together those selected: their product will be the root required.*

PROOF.—Raise the root to the corresponding power. The result will be equal to the given number.

Ex. 2. What is the square root of 100? Ans. 10.
3. What is the cube root of 125?
4. What is the fourth root of 1296? Ans. 6.
5. What is the fifth root of 243? Ans. 3.
6. What is the sixth root of 64?
7. What is the fourth root of 10000? Ans. 10.
8. What is the fifth root of 1024? Ans. 4.
9. What is the cube root of 3375?
10. What is the square root of 12321? Ans. 111.
11. What is the square root of 65536? Ans. 256.
12. What is the fourth root of 65536?
13. What is the eighth root of 65536? Ans. 4.
14. What is the sixteenth root of 65536? Ans. 2.
15. What is the square root of 390625?
16. What is the fourth root of 390625? Ans. 25.
17. What is the eighth root of 390625? Ans. 5.
18. What is the cube root of 10077696?
19. What is the ninth root of 10077696? Ans. 6.
20. What is the cube root of 42875? Ans. 35.
21. What is the square root of 122500?
22. What is the square root of 7569? Ans. 87.

GREATEST COMMON MEASURE.

Ex. 1. Find the greatest common measure of 60, 150, and 480.

2)60, 150, 480
5)30, 75, 240
3) 6, 15, 48
2, 5, 16
2.5.3=30

§ 103. MODEL.—Divide each of the given numbers by 2. (§ 50). Divide each of these quotients by 5. Divide each of these quotients by 3. These quotients are prime to each other. 2.5.3=30. 30 is the greatest common measure of the given numbers.

EXPLANATION.—In this operation it is not necessary for the divisors to be prime numbers. We might have divided by 10 and by 3, or by 5 and by 6.

RULE.—*Divide each of the given numbers by any one of their common measures; divide each of these quotients by one of* their *common measures; continue thus dividing until the quotients become prime to each other: the continued product of the divisors will be the greatest common measure of the given numbers.*

Ex. 2. Find the greatest common measure of 36, 126, 216, and 234.

$36=2.2.3.3$
$126=2.\ 3.3.7$
$216=2.2.3.3.\ 2.3$
$234=2.\ 3.3.\ \ 13$
$2.3.3.=18.$

§ 104. MODEL.—Resolve 36 into its prime factors. (§ 99). $36=2.2.3.3$. Resolve 126 into its prime factors. $126=2.3.3.7$. Resolve 216 into its prime factors. $216=2.2.3.3.2.3$. Resolve 234 into its prime factors. $234=2.3.3.13$. $2.3.3=18$. 18 is the greatest common measure of the given numbers.

EXPLANATION.—The prime factors are arranged with equal factors in the same column, as far as possible. The full columns contain the factors that are common to all the numbers. The product of these factors is the greatest common measure of the numbers.

RULE.—*Resolve each of the given numbers into its prime factors; select those factors which are common to all the numbers: the continued product of these factors will be the greatest common measure of the given numbers.*

Ex. 3. Find the greatest common measure of 108 and 261.

```
261|108
216|2
---
108|45
 90|2
---
 45|18
 36|2
---
 18|9
 18|2
---
  0
```

§ 105. Model.—Divide 261 by 108. (§ 71). Divide 108 by 45. Divide 45 by 18. Divide 18 by 9. There is no remainder. 9 is the greatest common measure of the given numbers.

Explanation.—9 is a measure of 18, (§ 89); hence it is a measure of 2×18, or 36, (§ 96); hence, of $36+9$, or 45, (§ 97); hence, of 2×45, or 90, (§ 96); hence, of 108, (§ 97); hence, of 216, (§ 96); hence, of 261, (§ 97); hence it is a common measure of 108 and 261.

Rule.—*Divide the larger number by the smaller; then divide the smaller number by the remainder, and continue dividing the last divisor by the last remainder, until there is no remainder: the last divisor will be the greatest common measure of the given numbers.*

To find the greatest common measure of more than two numbers, find the greatest common measure of two of them, then find the greatest common measure of this measure and an other of the numbers, and so on: the last common measure will be the greatest common measure of all the numbers.

Either of the above methods may be used in the following exercises.

Ex. 4. Find the greatest common measure of 48, 64, and 112. G. C. M 16.

5. Find the greatest common measure of 68, 119, and 357.

6. Find the greatest common measure of 60, 90, and 165.

7. Find the greatest common measure of 39, 52, 91, and 143. G. C. M. 13.
8. Find the greatest common measure of 40, 60, and 200. G. C. M. 20.
9. Find the greatest common measure of 96, 128, and 320.
10. Find the greatest common measure of 164, 246, 287, and 451. G. C. M. 41.
11. Find the greatest common measure of 63, 126, 315, and 441. G. C. M. 63.
12. Find the greatest common measure of 150, 375, and 675.
13. Find the greatest common measure of 40, 60, 68, and 204. G. C. M. 4.
14. Find the greatest common measure of 214, 642, and 856. G. C. M. 214.
15. Find the greatest common measure of 63, 189, 315, and 693.
16. Find the greatest common measure of 152, 380, and 532. G. C. M. 76.
17. Find the greatest common measure of 170, 187, and 340. G. C. M. 17.
18. Find the greatest common measure of 100, 120, 240, and 480.
19. Find the greatest common measure of 114, 190, and 1140. G. C. M. 38.
20. Find the greatest common measure of 54, 108, 324, and 378. G. C. M. 54.
21. Find the greatest common measure of 56, 84, 140, and 196. G. C. M. 28.
22. Find the greatest common measure of 75, 225, 375, and 675. G. C. M. 75.
23. Find the greatest common measure of 46, 115, and 161.

LEAST COMMON MULTIPLE.

Ex. 1. Find the least common multiple of 40, 60, and 150.

2)40, 60, 150	§106 MODEL.—Divide each of the numbers by 2. (§ 50). Divide some of the quotients by 2. Divide each of these quotients by 5. Divide some of these quotients by 3. These quotients are prime to each other. 2.2.5.3.2.5=600. 600 is the least common multiple of the given numbers.
2)20, 30, 75	
5)10, 15, 75	
3)2, 3, 15	
2. 1. 5	
2.2.5.3.2.5=600	

EXPLANATION.—We divide two or more of the given numbers by any *prime* number that will divide them without a remainder; and two or more of the resulting numbers by any prime number that will divide them without a remainder; and so on, till the quotients are prime to each other:—remembering to repeat in the line below, such numbers as cannot be divided. By this means, every factor of each number is used, and hence the result is a *common multiple* of the numbers; but no factor of either number is used more than once, and hence the result is their *least* common multiple.

RULE.—*Divide two or more of the given numbers by any prime common measure; take the quotients and the undivided numbers for a new set; divide two or more of* them *by any prime common measure; and so on, until the resulting numbers are prime to each other; the continued product of the resulting numbers and all the divisors will be the least common multiple of the given numbers.*

Ex. 2. Find the least common multiple of 36, 126, and 216.

36=2.2.3.3
126=2. 3.3.7
216=2.2.3.3.2.3
2.2.3.3.7.2.3=1512

§ 107. MODEL.—Resolve 36 into its prime factors. (§ 99.) 36=2.2.3.3. Resolve 126 into its prime factors. 126=2.3.3.7. Resolve 216 into its prime factors. 216=2.2.3.3.2.3.—2.2.3.3.7.2.3=1512. 1512 is the least common multiple of the given numbers.

EXPLANATION.—The prime factors are arranged as in § 104, and one factor is taken from each column, whether full or not.

RULE.—*Resolve each of the given numbers into its prime factors; multiply together all the factors of the largest number, and all the factors of the other numbers that are not found in the largest number; the product will be the least common multiple of the given numbers.*

Either of the above methods may be used in the following exercises.

Ex. 3. Find the least common multiple of 5, 6, and 7.

4. Find the least common multiple of 2, 4, 6, 8, 12, and 16. L. C. M. 48.

5. Find the least common multiple of 3, 6, 9, 12, and 18. L. C. M. 36.

6. Find the least common multiple of 5, 10, 12, and 15.

7. Find the least common multiple of 6, 12, 24, and 48. L. C. M. 48.

8. Find the least common multiple of 8, 24, and 72. L. C. M. 72.

9. Find the least common multiple of 3, 9, 18, and 72.

10. Find the least common multiple of 2, 3, 4, 5, 6, 10, 12, 15, and 20. L. C. M. 60.

11. Find the least common multiple of 3, 5, 7, and 11. L. C. M. 1155.

12. Find the least common multiple of 2, 3, 4, 6, 8, 12, and 24.

13. Find the least common multiple of 3, 7, and 13.
L. C. M. 273.

14. Find the least common multiple of 2, 4, 7, and 14.
L. C. M. 28.

15. Find the least common multiple of 3, 5, 15, and 30.

16. Find the least common multiple of 2, 4, 8, 16, and 32.
L. C. M. 32.

17. Find the least common multiple of 3, 4, 6, 8, and 9.
L. C. M. 72.

18. Find the least common multiple of 2, 3, 6, and 9.

19. Find the least common multiple of 4, 6, 8, 12, 16, and 32. L. C. M. 96.

20. Find the least common multiple of 2, 4, 5, 10, and 20.
L. C. M. 20.

PROMISCUOUS PROBLEMS.

1. Read 279301682038040.

2. Read 1207300804000975O.

3. Write twenty-seven billions, three hundred and three millions, four hundred and seventy-five thousand, and eighty-nine.

4. Write five hundred and five billions, and fifty-five.

5. Add 3 millions 24 thousand and 17, 4 hundred thousand 7 hundred and 98, 4 millions 247 thousand and 56, and 724 thousand 8 hundred and 29. Sum, 8396700.

6. Add twenty, 2 hundred and 2, 2 thousand and 27, 20 thousand 278, 202 thousand 7 hundred and 89, and 2 millions 27 thousand 8 hundred and 90.

7. From 9 millions and 9, subtract 5 millions 789 thousand 6 hundred and 54. Rem. 3210355.

8. From 80 millions 85 thousand and 8, subtract 65 millions 764 thousand 3 hundred and 75. Rem. 14320633.

9. Multiply 4 hundred and 70 thousand 8 hundred and 7, by 4 thousand 8 hundred and 7.

10. Multiply 90 thousand 7 hundred and 5, by 80 thousand 6 hundred and 4. Prod. 7311185820.

11. Divide 2 billions 59 millions 191 thousand and 72, by 50 thousand 7 hundred and 9. Quot. 40608.

12. Divide 8 billions 777 millions 887 thousand 5 hundred and 31, by 97 thousand 5 hundred and 31.

13. The minuend is 4 hundred thousand 4 hundred; the subtrahend is 364 thousand 7 hundred and 26: what is the remainder? Ans. 35674.

14 The minuend is 57 thousand and 57; the subtrahend is 27 thousand 5 hundred and 79: what is the remainder? Ans. 19478.

15. The minuend is 75 thousand and 63; the remainder is 36 thousand and 57: what is the subtrahend?

16. The subtrahend is 3 millions and 75; the remainder is 5 hundred thousand 7 hundred and 5: what is the minuend? Ans. 3500780.

17. The remainder is 777 thousand 7 hundred and 7; the subtrahend is 654 thousand 3 hundred and 25: what is the minuend? Ans. 1432032.

18. The multiplicand is 3 millions and 75; the multiplier is 5 hundred thousand 7 hundred and 5: what is the product?

19. The multiplier is 3 thousand 3 hundred and 3; the multiplicand is 75 thousand 4 hundred and 25: what is the product? Ans. 249128775.

20. The product is 670 millions 592 thousand 745; the multiplier is 12 thousand 345: what is the multiplicand?
Ans. 54321.

21. The multiplicand is 40 thousand 5 hundred and 6; the product is 413 millions 282 thousand 7 hundred and 18: what is the multiplier?

22. The dividend is 1 billion 546 millions 263 thousand 5 hundred and 4; the divisor is 71 thousand 2 hundred and 17: what is the quotient? Ans. 21712.

23. The dividend is 2 billions 162 millions 6 hundred thousand; the remainder is 19 thousand 4 hundred and 90; the quotient is 24 thousand and 6: what is the divisor?
Ans. 90085.

24. The divisor is 14 thousand and 20; the quotient is 2 thousand 3 hundred and 45: what is the dividend?

25. The divisor is 7 thousand and 2; the quotient is 2 thousand and 7; the remainder is 2 thousand and 7: what is the dividend? Ans. 14055021.

26. Resolve 3 thousand and 80 into its prime factors.
P. F. 2, 2, 2, 5, 7, 11.

27. Resolve 5 thousand 4 hundred and 60 into its prime factors.

28. Resolve 4 thousand and 4 into its prime factors.
P. F. 2, 2, 7, 11, 13.

29. Find the greatest common measure of 58, 87, and 2610. G. C. M. 29.

30. Find the greatest common measure of 118, 177, and 295.

31. Find the greatest common measure of 48, 80, 128, and 176. G. C. M. 16.

32. Find the least common multiple of 3, 7, 9, 12, and 18. L. C. M. 252.

33. Find the least common multiple of 2, 5, 8, 11, and 14.

34. Find the least common multiple of 2, 4, 7, 11, 16, and 22. L. C. M. 1232.

35. What number is that to which if 1234, 8912, 5678, 4567, and 9123 be added, the sum will be 47275? Ans. 17761.

36. What number is that from which if 1234, 8912, 5678, 4567, and 9123 be subtracted, the remainder will be 47275?

37. What number is that by which if 9876 be multiplied, the product will be 121919220? Ans. 12345.

38. What number is that by which if 5483896 be divided, the quotient will be 2468? Ans. 2222.

FRACTIONS.

§ 108. A FRACTION is a part of a unit. Thus, one half, three fourths, two fifths, five sixths, four sevenths, three eighths, five ninths, and seven tenths, are fractions.

Fractions are of two kinds, COMMON and DECIMAL.

COMMON FRACTIONS.

§ 109. A common fraction, or, simply, a fraction, is *denoted* by two *terms*, one above, and the other below, a horizontal line. The term above is called the *numerator;* the term below is called the *denominator.* Thus, the above fractions are denoted, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{2}{5}$, $\frac{5}{6}$, $\frac{4}{7}$, $\frac{3}{8}$, $\frac{5}{9}$, $\frac{7}{10}$. The numerators are 1, 3, 2, 5, 4, 3, 5, and 7: the denominators are 2, 4, 5, 6, 7, 8, 9, and 10.

Point out the numerator and the denominator of each of the following fractions: $\frac{1}{4}$, $\frac{2}{3}$, $\frac{3}{5}$, $\frac{1}{6}$, $\frac{2}{7}$, $\frac{5}{8}$, $\frac{4}{9}$, $\frac{3}{10}$, $\frac{7}{11}$, $\frac{5}{12}$, $\frac{4}{13}$, $\frac{5}{14}$, $\frac{7}{15}$, $\frac{9}{16}$, $\frac{3}{17}$, $\frac{5}{18}$, $\frac{2}{19}$, $\frac{3}{20}$, $\frac{4}{21}$, $\frac{5}{22}$, $\frac{6}{23}$, $\frac{7}{24}$, $\frac{8}{25}$, $\frac{9}{26}$, $\frac{10}{27}$, $\frac{11}{28}$, $\frac{12}{29}$, $\frac{13}{30}$, $\frac{14}{31}$, $\frac{15}{32}$, $\frac{16}{33}$, $\frac{9}{34}$, $\frac{18}{35}$, $\frac{250}{1200}$, $\frac{37}{2000}$, $\frac{45}{376}$, $\frac{279}{972}$.

§ 110. A fraction is *read* by pronouncing after the numerator the ordinal of the denominator in the singular or the plural number according as the numerator is one or more than one. Thus, $\frac{1}{5}$ is read, one fifth; $\frac{2}{5}$, two fifths; $\frac{3}{21}$, three twenty-firsts; $\frac{4}{32}$, four thirty-seconds; $\frac{5}{209}$, five two hundred and-ninths: $\frac{6}{2002}$, six two thousand-and-seconds; $\frac{7}{3106}$, seven three thousand-one-hundred-and sixths.

But, if the denominator is 2, the fraction is read half or halves, and not second or seconds. Thus, $\frac{1}{2}$, one half; $\frac{3}{2}$, three halves.

Read the following: $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{6}{7}$, $\frac{7}{8}$, $\frac{2}{9}$, $\frac{3}{10}$, $\frac{4}{11}$, $\frac{5}{12}$, $\frac{6}{13}$, $\frac{3}{14}$, $\frac{4}{15}$, $\frac{5}{16}$, $\frac{6}{17}$, $\frac{7}{18}$, $\frac{8}{19}$, $\frac{9}{20}$, $\frac{10}{21}$, $\frac{3}{22}$, $\frac{4}{23}$, $\frac{5}{24}$, $\frac{6}{25}$, $\frac{7}{26}$, $\frac{8}{27}$, $\frac{9}{28}$, $\frac{10}{29}$, $\frac{11}{30}$, $\frac{12}{31}$, $\frac{13}{32}$, $\frac{14}{33}$, $\frac{15}{34}$, $\frac{16}{35}$, $\frac{17}{36}$, $\frac{18}{37}$, $\frac{15}{38}$.

§ 111. A fraction is *produced* by dividing a unit or a number into some number of equal parts. Thus, $\frac{1}{4}$ is produced by dividing the unit into four equal parts; $\frac{2}{5}$, by dividing 2 into 5 equal parts; $\frac{5}{9}$, by dividing 5 into 9 equal parts.

The numerator is the dividend, the denominator is the divisor, and the value of the fraction is the quotient. See § 48.

How is $\frac{3}{4}$ produced? $\frac{4}{5}$? $\frac{5}{6}$? $\frac{7}{8}$? $\frac{9}{10}$? $\frac{9}{11}$? $\frac{5}{12}$? $\frac{10}{13}$? $\frac{17}{21}$? $\frac{19}{23}$? $\frac{17}{24}$? $\frac{27}{35}$?

§ 112. Otherwise, a fraction may be *produced* by dividing a *unit* into some number of equal parts and considering either one or several of these parts. Each of these parts is called a *fractional unit;* and a fraction is either one or several fractional units. The denominator shows

into how many parts the unit is divided, and the numerator shows how many parts there are in the fraction. Thus, in $\frac{3}{5}$, one fifth is the fractional unit, and the fraction contains three of these units; in $\frac{7}{9}$, the fractional unit is one ninth, and the fraction contains seven of them.

In this view, how is $\frac{8}{11}$ produced? $\frac{3}{13}$? $\frac{5}{8}$? $\frac{2}{7}$? $\frac{1}{10}$? $\frac{6}{25}$? $\frac{7}{24}$? $\frac{8}{25}$? $\frac{3}{7}$? $\frac{2}{9}$? $\frac{5}{11}$? $\frac{6}{13}$?

§ 113. The *value* of a fraction is the quotient of its numerator divided by its denominator. This value depends on the value of the fractional units, as well as on the number of them. If the fractional units of several fractions are equal, of course the greatest fraction is the one which has the most fractional units. That is, if the denominators are equal, the greatest fraction is the one which has the greatest numerator. Again, if the number of fractional units in several fractions is the same, of course the greatest fraction is the one which has the largest fractional unit. But, the larger the number of parts into which a unit is divided, the smaller each part must be. Therefore, if the numerators of several fractions are equal, the greatest fraction is the one which has the smallest denominator.

How do $\frac{2}{5}$ and $\frac{4}{5}$ compare in value? $\frac{1}{6}$ and $\frac{5}{6}$? $\frac{2}{7}$ and $\frac{5}{7}$? $\frac{1}{9}$ and $\frac{8}{9}$? $\frac{3}{11}$ and $\frac{4}{11}$? $\frac{8}{13}$ and $\frac{6}{13}$? $\frac{7}{15}$ and $\frac{8}{15}$? $\frac{11}{19}$ and $\frac{4}{19}$? $\frac{5}{6}$ and $\frac{5}{7}$? $\frac{7}{13}$ and $\frac{7}{12}$? $\frac{9}{10}$ and $\frac{9}{11}$? $\frac{3}{7}$ and $\frac{3}{10}$? $\frac{4}{9}$ and $\frac{4}{21}$? $\frac{16}{21}$ and $\frac{16}{23}$? $\frac{25}{29}$ and $\frac{25}{31}$? $\frac{27}{29}$ and $\frac{27}{28}$? $\frac{33}{35}$ and $\frac{33}{34}$? $\frac{7}{11}$ and $\frac{7}{9}$?

From the definition in § 108, the number of fractional units in a fraction must be less than the number of parts into which the unit is divided; that is, the numerator must be less than the denominator. Larger numbers, however, may be expressed in a fractional form; and such expressions are improperly called fractions also. Hence the following distinctions:—

§ 114. A *proper* fraction is one whose numerator is *less* than its denominator, and whose value is less than a unit. Thus, $\frac{2}{3}$, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{4}{6}$, $\frac{1}{6}$, $\frac{2}{7}$, $\frac{3}{8}$, $\frac{4}{9}$, $\frac{75}{100}$, $\frac{4}{25}$, $\frac{7}{10}$, and $\frac{11}{12}$, are proper fractions.

§ 115. An *improper* fraction is one whose numerator is *not less* than its denominator, and whose value is not less than a unit. Thus, $\frac{3}{3}$, $\frac{4}{3}$, $\frac{5}{5}$, $\frac{7}{2}$, $\frac{8}{3}$, $\frac{9}{4}$, $\frac{10}{7}$, $\frac{25}{3}$, $\frac{75}{5}$, $\frac{100}{20}$, and $\frac{27}{10}$, are improper fractions.

Is $\frac{1}{4}$ a proper or an improper fraction? $\frac{2}{3}$? $\frac{6}{5}$? $\frac{7}{3}$? $\frac{3}{18}$? $\frac{4}{11}$? $\frac{5}{1.3}$? $\frac{11}{4}$? $\frac{12}{5}$? $\frac{17}{20}$? $\frac{20}{17}$? $\frac{20}{25}$? $\frac{17}{15}$? $\frac{9}{10}$? $\frac{8}{9}$? $\frac{8}{8}$? $\frac{9}{7}$? $\frac{7}{9}$?

§ 116. A unit is often called, for distinction, an *integral* unit; and a number of integral units is called an integer, or an integral number.

§ 117. A *mixed* number is one composed of an integer and a fraction. Thus, $5\frac{1}{2}$, read five and one half, is a mixed number. So also, $6\frac{1}{4}$, $3\frac{1}{8}$, $18\frac{3}{4}$, $31\frac{1}{4}$, $66\frac{2}{3}$.

A fractional unit or a fraction may be divided into any number of equal parts. Thus, if $\frac{1}{8}$ be divided into 3 equal parts, each of the parts is $\frac{1}{3}$ of $\frac{1}{8}$. So, if $\frac{1}{3}$ of $\frac{1}{8}$ be divided into 4 equal parts, 3 of these parts are $\frac{3}{4}$ of $\frac{1}{3}$ of $\frac{1}{8}$. Such expressions are called compound fractions. Hence,

§ 118. A *compound* fraction is a fraction of a fraction. Thus, $\frac{2}{3}$ of $\frac{3}{4}$, $\frac{1}{9}$ of $\frac{3}{17}$, $\frac{2}{3}$ of $\frac{1}{2}$, $\frac{27}{50}$ of $12\frac{1}{2}$, and $\frac{5}{6}$ of $\frac{3}{8}$ of $33\frac{1}{3}$ are compound fractions.

§ 119. A *complex* fraction is one which has a fraction for its numerator or its denominator or each. Thus, $\frac{2}{5\frac{1}{2}}$, read two divided by five and one half, is a complex fraction. So also, $\frac{6\frac{1}{4}}{\frac{2}{5}}$, $\frac{12\frac{1}{2}}{37\frac{1}{2}}$, $\frac{\frac{1}{2}\text{ of }\frac{2}{3}}{33\frac{1}{3}}$, $\frac{\frac{3}{4}\text{ of }18\frac{3}{4}}{\frac{1}{2}\text{ of }12\frac{1}{2}}$.

REDUCTION OF COMMON FRACTIONS.

§ 120. To *reduce* any number, either fractional or integral, is to change its *form* of expression *without changing its value.* Thus, a unit may be reduced to $\frac{4}{4}$, or to $\frac{7}{7}$, or to $\frac{10}{10}$:—$\frac{3}{4}$ may be reduced to $\frac{6}{8}$, or to $\frac{12}{16}$, or to $\frac{9}{12}$:—$6\frac{1}{4}$ may be reduced to $\frac{25}{4}$, or to $\frac{50}{8}$:—and $\frac{25}{2}$ may be reduced to $12\frac{1}{2}$.

§ 121. A fraction is in its *lowest terms* when its terms are prime to each other. (§ 94). Thus, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{2}{5}$, $\frac{31}{7}$, $\frac{5}{8}$, $\frac{7}{9}$, and $\frac{10}{9}$, are each in its lowest terms.

Ex. 1. Reduce $\frac{144}{576}$ to its lowest terms.

$4\,|\,\frac{144}{576}\,|\,4\,|\,\frac{36}{144}\,|\,9\,|\,\frac{9}{36}\,|\,\frac{1}{4}$

§ 122. Model.—Divide both terms by 4: 4 in 144, 36 times; 4 in 576, 144 times: divide both these quotients by 4: 4 in 36, 9 times; 4 in 144, 36 times: divide both these quotients by 9: 9 in 9, once; 9 in 36, 4 times. These quotients are prime to each other: hence, $\frac{1}{4}$ is the given fraction in its lowest terms.

Explanation.—By comparing §§ 77 and 111, it is evident that the value of a fraction is not changed by dividing both its terms by the same number: and by successive divisions, its terms may always be made prime to each other.

Ex. 2. Reduce $\frac{567}{675}$ to its lowest terms.

$27\,\frac{567}{675}\,|\,\frac{21}{25}$

§ 123. Model.—Find the greatest common measure of the terms of the fraction. (§ 103). Their greatest common measure is 27. Divide both terms by 27: 27 in 567, 21 times; 27 in 675, 25 times. 21 twenty-fifths is the given fraction in its lowest terms.

Rule for reducing a fraction to its lowest terms.

1. *Divide both terms by any common measure; divide both these quotients by any common measure; and so on, until the quotients are prime to each other; the last quotients will be the lowest terms of the given fraction.*

Or, 2. *Divide both terms by their greatest common measure: the quotients will be the lowest terms.*

Ex. 3. Reduce $\frac{75}{225}$ to its lowest terms.

4. Reduce $\frac{210}{280}$ to its lowest terms. Value, $\frac{3}{4}$.

5. Reduce $\frac{340}{425}$ to its lowest terms. Val. $\frac{4}{5}$.

6. Reduce $\frac{460}{552}$ to its lowest terms.

7. Reduce $\frac{2323}{2717}$ to its lowest terms. Val. $\frac{9}{11}$.

8. Reduce $\frac{1661}{1983}$ to its lowest terms. Val. $\frac{11}{13}$.

9. Reduce $\frac{997}{1994}$ to its lowest terms.

10. Reduce $\frac{4207}{5409}$ to its lowest terms. Val. $\frac{7}{9}$.

11. Reduce $\frac{2721}{9977}$ to its lowest terms. Val. $\frac{3}{11}$.

12. Reduce $\frac{2863}{3272}$ to its lowest terms.

13. Reduce $\frac{708}{720}$ to its lowest terms. Val. $\frac{59}{60}$.

14. Reduce $\frac{264}{520}$ to its lowest terms. Val. $\frac{33}{65}$.

15. Reduce $\frac{100335}{232665}$ to its lowest terms.

16. Reduce $\frac{3502}{7576}$ to its lowest terms. Val. $\frac{449}{947}$.

17. Reduce $\frac{679}{6979}$ to its lowest terms. Val. $\frac{97}{997}$.

18. Reduce $\frac{2448}{8712}$ to its lowest terms.

19. Reduce $\frac{4081}{8173}$ to its lowest terms. Val. $\frac{371}{743}$.

20. Reduce $\frac{306900}{967225}$ to its lowest terms. Val. $\frac{401}{977}$.

---

21. In $\frac{868}{16}$ how many units?

868'16
68 $54\frac{4}{16}=54\frac{1}{4}$
4,

§ 124. Model.—Divide the numerator by the denominator. (§ 71). The quotient is 54, and the remainder 4. Reduce $\frac{4}{16}$ to its lowest terms. The given fraction is equal to $54\frac{1}{4}$.

Explanation.—Since 16 sixteenths make a unit, the number of units in 868 sixteenths is equal to the number of times 16 is contained in 868. See § 113.

Rule for reducing an improper fraction to a whole or a mixed number.

Divide the numerator by the denominator; the quotient will be the integral part. Place the denominator under the remainder for the fractional part.

Ex. 22. In $\frac{75}{3}$ how many units? Ans. 25.

23. In $\frac{97}{4}$ how many units? Ans. $24\frac{1}{4}$.

24. In $\frac{103}{5}$ how many units?

25. In $\frac{27}{7}$ how many units? Ans. $3\frac{6}{7}$.

26. Reduce $\frac{180}{9}$ to units. Val. 20.

27. Reduce $\frac{180}{12}$ to units.

28. Reduce $\frac{220}{15}$ to units. Val. $14\frac{2}{3}$.

29. Reduce $\frac{130}{16}$ to units. Val. $8\frac{1}{8}$.

30. Reduce $\frac{275}{19}$ to units.

31. Reduce $\frac{280}{21}$ to a mixed number. Val. $13\frac{8}{7}$.

32. Reduce $\frac{299}{24}$ to a mixed number. Val. $12\frac{11}{24}$.

33. Reduce $\frac{320}{27}$ to a mixed number.

34. Reduce $\frac{344}{31}$ to a mixed number. Val. $11\frac{3}{31}$.

35. Reduce $\frac{371}{36}$ to a mixed number. Val. $10\frac{11}{36}$.

36. Reduce $\frac{402}{43}$ to a mixed number.

37. Reduce $\frac{438}{52}$ to a mixed number. Val. $8\frac{7}{13}$.

38. Reduce $\frac{481}{64}$ to a mixed number. Val. $7\frac{33}{64}$.

39. Reduce $\frac{513}{79}$ to a mixed number.

40. Reduce $\frac{577}{95}$ to a mixed number. Val. $6\frac{7}{95}$.

41. Reduce 10 units to fourths.

§ 125. Model.—Multiply 4 fourths by 10. $10=\frac{40}{4}$ The product is 40 fourths: hence 10 units = 40 fourths.

Explanation.—Since 4 fourths make a unit, 10 units = 10 times 4 fourths; that is, 40 fourths.

Rule for reducing a whole number to any fractional denomination.

Multiply the number of fractional units in a unit by the number of units.

Ex. 42. In 3 units how many fifths?

43. In 5 units how many sevenths? Ans. $\frac{35}{7}$.

44. In 6 units how many ninths? Ans. $\frac{54}{9}$.

45. In 7 units how many elevenths?

46. Reduce 8 to thirteenths. Val. $\frac{104}{13}$.

47. Reduce 9 to fifteenths. Val. $\frac{135}{15}$.

48. Reduce 11 to seventeenths.

49. Reduce 12 to twentieths. Val. $\frac{240}{20}$.

50. Reduce 13 to twenty-fourths. Val. $\frac{312}{24}$.

51. Reduce 14 to twenty-ninths.

52. Reduce 15 to thirty-fifths. Val. $\frac{525}{35}$.

53. Reduce 16 to a fraction with denominator 40.
Val. $\frac{640}{40}$.

54. Reduce 17 to a fraction with denominator 46.

55. Reduce 18 to a fraction with denominator 53.
Val. $\frac{954}{53}$.

56. Reduce 19 to a fraction with denominator 59.
Val. $\frac{1121}{59}$.

57. Reduce 20 to a fraction with denominator 65.

58. Reduce 21 to a fraction with denominator 71.
Val. $\frac{1491}{71}$.

59. Reduce 22 to a fraction with denominator 77.

60. Reduce 23 to a fraction with denominator 85.

---

61. Reduce $16\frac{1}{2}$ to an improper fraction.

$16\frac{1}{2}=\frac{32}{2}+\frac{1}{2}=\frac{33}{2}$ § 126. Model.—Reduce 16 units to halves. Add 32 halves and 1 half.—The sum is 33 halves: hence $16\frac{1}{2}$ is equal to 33 halves.

Rule for reducing a mixed number to an improper fraction.

Reduce the integer to the denomination of the fraction; add the two numerators together, and under their sum set the common denominator.

Ex. 62. Reduce $3\frac{2}{3}$ to thirds. Val. $\frac{11}{3}$.

63. Reduce $4\frac{1}{4}$ to fourths.

64. Reduce $6\frac{2}{5}$ to fifths. Val. $\frac{32}{5}$.

65. Reduce $8\frac{5}{6}$ to sixths. Val. $\frac{53}{6}$.

66. Reduce $10\frac{3}{7}$ to sevenths.

67. Reduce $12\frac{5}{8}$ to eighths. Val. $\frac{101}{8}$.

68. Reduce $14\frac{1}{9}$ to ninths. Val. $\frac{127}{9}$.

69. Reduce $16\frac{3}{10}$ to tenths.

70. In $17\frac{2}{11}$ how many elevenths? Ans. $\frac{189}{11}$.

71. In $18\frac{7}{12}$ how many twelfths? Ans. $\frac{223}{12}$.

72. In $19\frac{3}{14}$ how many fourteenths?

73. In $20\frac{5}{16}$ how many sixteenths? Ans. $\frac{325}{16}$.

74. In $21\frac{5}{18}$ how many eighteenths? Ans. $\frac{385}{18}$.

75. In $22\frac{8}{21}$ how many twenty firsts?

76. Reduce $23\frac{7}{24}$ to an improper fraction. Val. $\frac{559}{24}$.

77. Reduce $24\frac{10}{27}$ to an improper fraction. Val. $\frac{678}{27}$.

78. Reduce $25\frac{9}{29}$ to an improper fraction.

79. Reduce $52\frac{2}{33}$ to an improper fraction. Val. $\frac{1718}{33}$.

80. Reduce $65\frac{56}{65}$ to an improper fraction. Val. $\frac{4281}{65}$.

---

81. Reduce $\frac{3}{4}$ to twentieths.

$\frac{3}{4}=\frac{15}{20}$ § 127. Model.—5 twentieths make one fourth. Multiply both terms by 5: 5 times 3 are 15; 5 times 4 are 20. $\frac{3}{4}$ is equal to 15 twentieths.

Explanation.—By comparing §§ 76 and 111, it is evident that the value of a fraction is not changed by multiplying both its terms by the same number. We divide the required denominator by the given one, and multiply both terms of the fraction by the quotient.

Rule for reducing a fraction to a larger denominator.

Multiply both terms by the quotient of the required denominator divided by the given one.

Ex. 82. Reduce $\frac{2}{5}$ to tenths. Val. $\frac{4}{10}$.
83. Reduce $\frac{5}{6}$ to eighteenths. Val. $\frac{15}{18}$.
84. Reduce $\frac{5}{7}$ to thirty-fifths.
85. Reduce $\frac{3}{8}$ to fortieths. Val. $\frac{15}{40}$.
86. Reduce $\frac{7}{9}$ to sixty-thirds. Val. $\frac{49}{63}$.
87. Reduce $\frac{9}{10}$ to ninetieths.
88. Reduce $\frac{7}{11}$ to ninety-ninths. Val. $\frac{63}{99}$.
89. Reduce $\frac{5}{12}$ to sixtieths. Val. $\frac{25}{60}$.
90. Reduce $\frac{6}{13}$ to sixty-fifths.

---

91. Reduce $\frac{25}{100}$ to twentieths.

$5\left|\frac{25}{100}\right|\frac{5}{20}$

§ 128. Model.—Divide both terms by 5: 5 in 25, 5 times; 5 in 100, 20 times. 25 one hundredths=5 twentieths.

For explanation, see § 122.

Rule for reducing a fraction to a lower denominator.

Divide both terms by the quotient of the given denominator divided by the required one.

Ex. 92. Reduce $\frac{9}{42}$ to fourteenths. Val. $\frac{3}{14}$.
93. Reduce $\frac{16}{60}$ to fifteenths.
94. Reduce $\frac{35}{80}$ to sixteenths. Val. $\frac{7}{16}$.
95. Reduce $\frac{30}{108}$ to eighteenths. Val. $\frac{5}{18}$.
96. In $\frac{12}{120}$ how many twentieths?
97. In $\frac{21}{63}$ how many twenty firsts? Ans. $\frac{7}{21}$.
98. In $\frac{121}{253}$ how many twenty-thirds? Ans. $\frac{11}{23}$.
99. In $\frac{56}{168}$ how many twenty-fourths?
100. In $\frac{390}{900}$ how many thirtieths? Ans. $\frac{13}{30}$.

---

101. Reduce $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{8}$, to a common denominator. (§91).

$\frac{1}{2}$ $\frac{3}{4}$ $\frac{5}{8}$
$\frac{32}{64}$ $\frac{48}{64}$ $\frac{40}{64}$

§ 129. Model.—Multiply both terms of the first fraction by 32; 32 times 1 are 32; 32 times 2 are 64: multiply both terms of the second fraction by 16; 16 times 3 are 48; 16 times 4 are 64: multiply both terms

of the third fraction by 8; 8 times 5 are 40; 8 times 8 are 64. The given fractions are respectively equal to 32, 48, and 40 sixty-fourths.

EXPLANATION.—The values of the fractions are not changed, because both terms of each fraction are multiplied by the same number: and the denominators are alike, because each one is produced by multiplying together all the given denominators. The multiplier 32 for the first fraction is 4×8, the product of the other two denominators. And so for the others.

RULE for reducing fractions to a common denominator.

Multiply both terms of each fraction by the product of the other *denominators.*

Ex. 102. Reduce $\frac{2}{3}$, $\frac{1}{4}$, and $\frac{5}{6}$, to a common denominator.

103. Reduce $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{5}$, to a common denominator.

Val. $\frac{15}{30}$, $\frac{10}{30}$, $\frac{6}{30}$.

104. Reduce $\frac{1}{3}$, $\frac{3}{4}$, and $\frac{2}{5}$, to a common denominator.

Val. $\frac{20}{60}$, $\frac{45}{60}$, $\frac{24}{60}$.

105. Reduce $\frac{1}{4}$, $\frac{2}{5}$, and $\frac{5}{6}$, to a common denominator.

106. Reduce $\frac{1}{5}$, $\frac{1}{6}$, and $\frac{2}{7}$, to a common denominator.

Val. $\frac{42}{210}$, $\frac{35}{210}$, $\frac{60}{210}$.

107. Reduce $\frac{1}{6}$, $\frac{2}{7}$, and $\frac{3}{8}$, to a common denominator.

Val. $\frac{56}{336}$, $\frac{96}{336}$, $\frac{126}{336}$.

108. Reduce $\frac{1}{7}$, $\frac{3}{8}$, and $\frac{5}{9}$, to a common denominator.

109. Reduce $\frac{1}{8}$, $\frac{2}{9}$, and $\frac{3}{10}$, to a common denominator.

Val. $\frac{90}{720}$, $\frac{160}{720}$, $\frac{216}{720}$.

110. Reduce $\frac{1}{9}$, $\frac{3}{10}$, $\frac{5}{12}$, and $\frac{7}{20}$, to a common denominator.

Val. $\frac{2400}{21600}$, $\frac{6480}{21600}$, $\frac{9000}{21600}$, $\frac{7560}{21600}$.

111. Reduce $\frac{1}{8}$, $\frac{3}{10}$, and $\frac{5}{13}$, to a common denominator.

112. Reduce $\frac{1}{7}$, $\frac{2}{9}$, and $\frac{3}{11}$, to a common denominator.

Val. $\frac{99}{693}$, $\frac{154}{693}$, $\frac{189}{693}$.

113. Reduce $\frac{1}{6}$, $\frac{3}{8}$, and $\frac{5}{10}$, to a common denominator.

114. Reduce $\frac{1}{6}$, $\frac{2}{7}$, and $\frac{4}{9}$, to a common denominator.

115. Reduce $\frac{1}{4}$, $\frac{1}{6}$, and $\frac{2}{9}$, to a common denominator.

Val. $\frac{54}{216}$, $\frac{36}{216}$, $\frac{48}{216}$.

116. Reduce $\frac{1}{4}$, $\frac{2}{5}$, and $\frac{5}{7}$, to a common denominator.

Val. $\frac{35}{140}$, $\frac{56}{140}$, $\frac{100}{140}$.

117. Reduce $\frac{1}{3}$, $\frac{3}{7}$, and $\frac{2}{9}$, to a common denominator.

118. Reduce $\frac{1}{4}$, $\frac{2}{5}$, and $\frac{2}{9}$, to a common denominator.

Val. $\frac{45}{180}$, $\frac{72}{180}$, $\frac{40}{180}$.

119. Reduce $\frac{1}{4}$, $\frac{2}{7}$, and $\frac{5}{10}$, to a common denominator.

Val. $\frac{70}{280}$, $\frac{80}{280}$, $\frac{140}{280}$.

120. Reduce $\frac{1}{5}$, $\frac{3}{8}$, and $\frac{3}{11}$, to a common denominator.

---

121. Reduce $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{8}$, to their least common denominator.

$\frac{1}{2}$ $\frac{3}{4}$ $\frac{5}{8}$

$\frac{4}{8}$ $\frac{6}{8}$ $\frac{5}{8}$

§ 130. Model.—Find the least common multiple of the denominators. (§ 106). 8 is their least common multiple. Multiply both terms of the first fraction by 4: 4 times 1 are 4; 4 times 2 are 8: multiply both terms of the second fraction by 2: twice 3 are 6; twice 4 are 8; the third fraction is already of the required denomination.—The given fractions are respectively equal to 4, 6, and 5 eighths.

Explanation.—To find the proper multiplier for the terms of either fraction, we divide the least common multiple by its denominator. See § 127.

Rule for reducing fractions to their least common denominator.

Find the least common multiple of the denominators, and reduce each fraction to the denomination expressed by this multiple. Each fraction must first be in its lowest terms.

Ex. 122. Reduce $\frac{2}{3}$, $\frac{1}{4}$, and $\frac{5}{6}$, to their least common denominator. Val. $\frac{8}{12}$, $\frac{3}{12}$, $\frac{10}{12}$.

123. Reduce $\frac{1}{2}$, $\frac{2}{5}$, and $\frac{5}{6}$, to their least common denominator.

124. Reduce $\frac{1}{6}$, $\frac{2}{7}$, and $\frac{3}{8}$, to their least common denominator. Val. $\frac{28}{168}$, $\frac{48}{168}$, $\frac{63}{168}$.

125. Reduce $\frac{1}{6}$, $\frac{3}{8}$, and $\frac{5}{10}$, to their least common denominator. Val. $\frac{4}{24}$, $\frac{9}{24}$, $\frac{12}{24}$.

126. Reduce $\frac{1}{4}$, $\frac{1}{6}$, and $\frac{2}{9}$, to their least common denominator.

127. Reduce $\frac{1}{4}$, $\frac{2}{7}$, and $\frac{5}{10}$, to their least common denominator. Val. $\frac{7}{28}$, $\frac{8}{28}$, $\frac{14}{28}$.

128. Reduce $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{6}$, to their least common denominator. Val. $\frac{6}{12}$, $\frac{4}{12}$, $\frac{3}{12}$, $\frac{2}{12}$.

129. Reduce $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, and $\frac{1}{12}$, to their least common denominator.

130. Reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{11}{12}$, to their least common denominator. Val. $\frac{16}{24}$, $\frac{18}{24}$, $\frac{20}{24}$, $\frac{21}{24}$, $\frac{22}{24}$.

131. Reduce $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{3}{16}$, and $\frac{19}{32}$, to their least common denominator. Val. $\frac{16}{32}$, $\frac{24}{32}$, $\frac{20}{32}$, $\frac{6}{32}$, $\frac{19}{32}$.

132. Reduce $\frac{1}{3}$, $\frac{5}{6}$, $\frac{11}{12}$, and $\frac{17}{24}$, to their least common denominator.

133. Reduce $\frac{5}{8}$, $\frac{8}{16}$, $\frac{7}{28}$, and $\frac{3}{24}$, to their least common denominator. Val. $\frac{5}{8}$, $\frac{4}{8}$, $\frac{2}{8}$, $\frac{1}{8}$.

134. Reduce $\frac{1}{2}$, $\frac{3}{6}$, $\frac{8}{12}$, and $\frac{12}{16}$, to their least common denominator. Val. $\frac{6}{12}$, $\frac{4}{12}$, $\frac{8}{12}$, $\frac{9}{12}$.

135. Reduce $\frac{9}{12}$, $\frac{6}{12}$, $\frac{2}{16}$, and $\frac{8}{16}$, to their least common denominator.

136. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{6}$, to their least common denominator. Val. $\frac{30}{60}$, $\frac{40}{60}$, $\frac{45}{60}$, $\frac{48}{60}$, $\frac{50}{60}$.

137. Reduce $\frac{1}{5}$, $\frac{2}{7}$, $\frac{6}{15}$, $\frac{18}{21}$, and $\frac{19}{35}$, to their least common denominator. Val. $\frac{7}{35}$, $\frac{10}{35}$, $\frac{14}{35}$, $\frac{30}{35}$, $\frac{19}{35}$.

138. Reduce $\frac{1}{2}$, $\frac{3}{4}$, $\frac{4}{6}$, $\frac{3}{8}$, $\frac{7}{10}$, $\frac{3}{20}$, and $\frac{27}{40}$, to their least common denominator.

139. Reduce $\frac{1}{3}$, $\frac{3}{5}$, $\frac{4}{9}$, $\frac{7}{15}$, and $\frac{11}{45}$, to their least common denominator. Val. $\frac{15}{45}$, $\frac{27}{45}$, $\frac{20}{45}$, $\frac{21}{45}$, $\frac{11}{45}$.

140. Reduce $\frac{1}{2}$, $\frac{2}{5}$, $\frac{3}{10}$, $\frac{4}{25}$, and $\frac{7}{50}$, to their least common denominator. Val. $\frac{25}{50}$, $\frac{20}{50}$, $\frac{15}{50}$, $\frac{8}{50}$, $\frac{7}{50}$.

ADDITION OF COMMON FRACTIONS.

Ex. 1. Add $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, and $\frac{7}{8}$.

$\frac{1}{8}+\frac{3}{8}+\frac{5}{8}+\frac{7}{8}=\frac{16}{8}=2$

§ 131. MODEL.—1 and 3 are 4, and 5 are 9, and 7 are 16.—16 eighths is equal to 2. The sum is 2.

EXPLANATION.—Since all the fractions have the same fractional unit, their numerators are added for the numerator of the sum, and the common denominator is taken as its denominator.

Ex. 2. Add $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{7}{8}$.

$\frac{1}{2}+\frac{3}{4}+\frac{7}{8}$

$\frac{4}{8}+\frac{6}{8}+\frac{7}{8}=\frac{17}{8}=2\frac{1}{8}$

§ 132. MODEL.—Reduce the given fractions to their least common denominator. (§ 130). 4 and 6 are 10, and 7 are 17. 17 eighths is equal to $2\frac{1}{8}$. The sum is $2\frac{1}{8}$.

EXPLANATION.—It is evidently impossible to add the given fractions without reduction. 3 fourths and 7 eighths make neither 10 fourths nor 10 eighths. It is not essential to reduce to the *least* common denominator; but this generally requires less labor than to reduce simply to a common denominator.

Ex. 3. Add $24\frac{1}{4}$, $351\frac{1}{2}$, $179\frac{7}{8}$, and 187.

$24\frac{1}{4}$
$351\frac{1}{2}$
$179\frac{7}{8}$
187
$742\frac{5}{8}$

$\frac{1}{4}+\frac{1}{2}+\frac{7}{8}$

$\frac{2}{8}+\frac{4}{8}+\frac{7}{8}=\frac{13}{8}=1\frac{5}{8}$

§ 133. MODEL.—Reduce the fractions to their least common denominator. (§ 130).—2 and 4 are 6, and 7 are 13. 13 eighths is equal to 1 and 5 eighths, set down $\frac{5}{8}$; 1 and 7 are 8, and 9 are 17, and 1 are 18, and 4 are 22, set down 2; 2 and 8 are 10, and 7 are 17, and 5 are 22, and 2 are 24, set down 4; 2 and 1 are 3, and 1 are 4, and 3 are 7. The sum is $742\frac{5}{8}$.

RULE.—*Reduce the fractions to their least common denominator: add the numerators, and under their sum set the common denominator. Reduce the result to its lowest terms or to a mixed number, as the case may be.*

Ex. 4. Add $\frac{1}{4}$, $\frac{2}{4}$, and $\frac{3}{4}$. Sum, $1\frac{1}{2}$.

5. Add $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, and $\frac{4}{5}$. Sum, 2.

6. Add $\frac{1}{6}$, $\frac{2}{6}$, $\frac{3}{6}$, $\frac{4}{6}$, and $\frac{5}{6}$.

7. Add $\frac{2}{7}$, $\frac{4}{7}$, $\frac{5}{7}$, and $\frac{6}{7}$. Sum, $2\frac{3}{7}$.

8. Add $\frac{1}{8}$, $\frac{2}{8}$, $\frac{3}{8}$, and $\frac{7}{8}$. Sum, $1\frac{5}{8}$.

9. Add $\frac{2}{9}$, $\frac{4}{9}$, $\frac{5}{9}$, $\frac{7}{9}$, and $\frac{8}{9}$.

10. Add $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$. Sum, $2\frac{43}{60}$.

11. Add $\frac{3}{4}$, $\frac{3}{5}$, $\frac{3}{6}$, and $\frac{3}{7}$. Sum, $2\frac{39}{140}$.

12. Add $\frac{1}{2}$, $\frac{3}{4}$, $\frac{2}{5}$, $\frac{3}{10}$, and $\frac{7}{20}$.

13. Add $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{7}$, and $\frac{9}{14}$. Sum, $1\frac{25}{28}$.

14. Add $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{8}$, $\frac{7}{16}$, and $\frac{19}{32}$. Sum, $2\frac{5}{32}$.

15. Add $\frac{1}{3}$, $\frac{5}{6}$, $\frac{8}{9}$, $\frac{11}{12}$, and $\frac{17}{18}$.

16. Add $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{11}{12}$, and $\frac{15}{16}$. Sum, $4\frac{39}{48}$.

17. Add $\frac{3}{4}$, $\frac{5}{6}$, $\frac{3}{8}$, $\frac{5}{12}$, $\frac{3}{16}$, and $\frac{5}{32}$. Sum, $2\frac{23}{32}$.

18. Add $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{8}{9}$.

19. Add $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$, and $\frac{1}{11}$. Sum, $\frac{886}{1155}$.

20. Add $2\frac{1}{2}$, $3\frac{2}{3}$, $4\frac{3}{4}$, and $5\frac{4}{5}$. Sum, $16\frac{43}{60}$.

21. Add 4, $8\frac{1}{3}$, $9\frac{1}{2}$, and $11\frac{3}{5}$.

22. Find the sum of $10\frac{1}{2}$, $21\frac{3}{4}$, $32\frac{2}{5}$, $43\frac{3}{10}$, and $54\frac{7}{20}$.
Sum, $162\frac{3}{10}$.

23. Find the sum of 19, $20\frac{1}{2}$, $16\frac{1}{4}$, and $27\frac{3}{8}$. Sum, $83\frac{1}{8}$.

24. Find the sum of $\frac{17}{18}$, $2\frac{11}{12}$, $3\frac{8}{9}$, $25\frac{5}{6}$, and $33\frac{1}{3}$.

25. Find the sum of $12\frac{1}{2}$, $18\frac{3}{4}$, $33\frac{1}{3}$, $87\frac{1}{2}$, and $93\frac{3}{4}$.
Sum, $245\frac{5}{6}$.

26. What is the sum of $\frac{8}{9}$, $6\frac{5}{6}$, $3\frac{2}{3}$, $2\frac{1}{3}$, and 98?
Ans. $111\frac{5}{9}$.

27. What is the sum of 1, $2\frac{1}{2}$, $3\frac{1}{3}$, $6\frac{1}{6}$, and $9\frac{1}{9}$?

28. What is the sum of $4\frac{3}{4}$, $5\frac{6}{7}$, $17\frac{3}{4}$, and $18\frac{5}{12}$?

Ans. $46\frac{5}{84}$.

29. What is the sum of $2\frac{1}{7}$, $25\frac{1}{4}$, $125\frac{4}{7}$, and $325\frac{5}{12}$?

Ans. $478\frac{8}{21}$.

30. What is the sum of $1\frac{2}{3}$, $4\frac{5}{6}$, $7\frac{8}{9}$, $10\frac{11}{12}$, and $13\frac{14}{15}$?

SUBTRACTION OF COMMON FRACTIONS.

Ex. 1. From $\frac{7}{8}$ take $\frac{3}{8}$.

$\frac{7}{8}-\frac{3}{8}=\frac{4}{8}=\frac{1}{2}$

§ 134. MODEL.—3 from 7 leaves 4. 4 eighths is equal to $\frac{1}{2}$. The remainder is $\frac{1}{2}$.

Ex. 2. From $\frac{1}{2}$ take $\frac{1}{3}$.

$\frac{1}{2}-\frac{1}{3}$

$\frac{3}{6}-\frac{2}{6}=\frac{1}{6}$

§ 135. MODEL.—Reduce the fractions to their least common denominator. (§ 130). 2 from 3 leaves 1, that is, 1 sixth. The remainder is $\frac{1}{6}$.

Ex. 3. From $32\frac{7}{8}$ take $18\frac{3}{4}$.

$32\frac{7}{8}$
$18\frac{3}{4}$
$14\frac{1}{8}$

$\frac{7}{8}-\frac{3}{4}$

$\frac{7}{8}-\frac{6}{8}=\frac{1}{8}$

§ 136. MODEL.—Reduce the fractions to their least common denominator. (§ 130). 6 from 7 leaves 1, set down $\frac{1}{8}$; 8 from 12 leaves 4; 2 from 3 leaves 1. The remainder is $14\frac{1}{8}$.

EXPLANATION.—Any number of fractional units may evidently be subtracted from a larger number of fractional units of the same denomination, just as one number of simple units is subtracted from an other. If the given fractions have different denominators, they must first be reduced to a common denominator: $\frac{7}{8}-\frac{3}{4}$=neither $\frac{4}{8}$ nor $\frac{4}{4}$, just as 7 dollars—3 cents=neither 4 dollars nor 4 cents.

Ex. 4. From 27 take $19\frac{1}{8}$.

27
$19\frac{1}{8}$
$7\frac{7}{8}$

§ 137. MODEL.—1 from 8 leaves 7, set down $\frac{7}{8}$; 10 from 17 leaves 7; 2 from 2 leaves 0. The remainder is $7\frac{7}{8}$.

Ex. 5. From $9\frac{1}{8}$ take $6\frac{1}{2}$.

$9\frac{1}{8}$ $\quad \frac{1}{8}-\frac{1}{2}$
$6\frac{1}{2}$
$2\frac{5}{8}$ $\quad \frac{1}{8}-\frac{4}{8}$

§ 138. MODEL.—Reduce the fractions to their least common denominator.—(§ 130). 4 from 9 leaves 5, set down $\frac{5}{8}$; 7 from 9 leaves 2. The remainder is $2\frac{5}{8}$.

EXPLANATION.—When the fraction in the minuend is less than that in the subtrahend, we add an integral unit to the minuend fraction, subtract the subtrahend fraction from this sum, and then add 1 to the units of the subtrahend before subtracting from the units of the minuend.

RULE.—*Reduce the fractions to their least common denominator; subtract the numerator of the subtrahend from the numerator of the minuend; and under the remainder set the common denominator.*

If, in subtracting one mixed number from an other, the subtrahend fraction should be larger than the one in the minuend, reduce an integral unit to the common denomination of the fractions, add it to the minuend fraction, subtract the subtrahend fraction from this sum, and add one to the subtrahend in the column of units.

Ex. 6. Subtract $\frac{3}{4}$ from $\frac{9}{10}$.

7. Subtract $\frac{1}{3}$ from $\frac{3}{5}$. Rem. $\frac{4}{15}$.
8. Subtract $\frac{3}{10}$ from $\frac{7}{15}$. Rem. $\frac{1}{6}$.
9. Subtract $\frac{2}{9}$ from $\frac{11}{12}$.
10. Subtract $\frac{2}{7}$ from $\frac{5}{9}$. Rem. $\frac{17}{63}$.
11. From $\frac{17}{18}$ take $\frac{3}{4}$. Rem. $\frac{7}{36}$.
12. From $\frac{17}{20}$ take $\frac{3}{5}$.
13. From $5\frac{2}{3}$ take $\frac{4}{7}$. Rem. $5\frac{2}{21}$.
14. From $7\frac{1}{9}$ take $4\frac{1}{12}$. Rem. $3\frac{1}{36}$.
15. From $8\frac{3}{7}$ take $7\frac{5}{6}$.
16. Minuend$=17\frac{1}{17}$; Subtrahend$=6\frac{1}{3}$. Rem. $10\frac{37}{51}$.

17. Minuend=$200\frac{5}{12}$; Subtrahend=$105\frac{3}{4}$. Rem. $94\frac{2}{3}$.
18. Minuend=$42\frac{1}{2}$; Subtrahend=$27\frac{9}{10}$.
19. Minuend=$72\frac{1}{3}$; Subtrahend=$24\frac{5}{12}$. Rem. $47\frac{3}{4}$.
20. Minuend=175; Subtrahend=$83\frac{3}{4}$. Rem. $91\frac{1}{4}$.
21. Subtrahend=$66\frac{2}{3}$; Minuend=$106\frac{1}{4}$.
22. Subtrahend=$17\frac{1}{7}$; Minuend=$27\frac{1}{3}$. Rem. $10\frac{4}{21}$.
23. Subtrahend=$1\frac{2}{3}$; Minuend=$4\frac{5}{6}$. Rem. $3\frac{1}{6}$.
24. Subtrahend=$7\frac{8}{9}$; Minuend=$8\frac{9}{10}$.
25. Subtrahend=$11\frac{13}{16}$; Minuend=$20\frac{25}{31}$. Rem. $8\frac{493}{496}$.
26. What is the difference between $12\frac{11}{12}$ and $21\frac{11}{21}$?

Ans. $8\frac{17}{252}$.

27. What is the difference between $16\frac{17}{18}$ and $10\frac{11}{12}$?
28. What is the difference between 100 and $33\frac{1}{3}$?

Ans. $66\frac{2}{3}$.

29. What is the difference between $19\frac{1}{9}$ and $20\frac{1}{10}$?

Ans. $\frac{89}{90}$.

30. What is the difference between 75 and $68\frac{3}{4}$?

MULTIPLICATION OF COMMON FRACTIONS.

Ex. 1. Multiply $\frac{3}{8}$ by 7.

$\frac{3}{8}\times 7=\frac{21}{8}=2\frac{5}{8}$

§ 139. MODEL.—7 times 3 are 21: 21 eighths is equal to $2\frac{5}{8}$. The product is $2\frac{5}{8}$.

EXPLANATION.—Comparing §§ 72 and 111, we see that the value of a fraction is multiplied by a whole number by multiplying its numerator by the number.

Ex. 2. Multiply $\frac{5}{9}$ by 3.

$\frac{5}{9}\times 3=\frac{5}{3}=1\frac{2}{3}$

§ 140. MODEL.—3 in 9, 3 times: 5 thirds is equal to $1\frac{2}{3}$. The product is $1\frac{2}{3}$.

EXPLANATION.—Comparing §§ 75 and 111, we see that the value of a fraction is multiplied by a whole number by dividing its denominator by the number. When the multiplier is a measure of the denominator, this method is preferable to the other.

Ex. 3. Multiply $47\frac{3}{4}$ by 9.

$$\begin{array}{r} 47\frac{3}{4} \\ 9 \\ \hline 429\frac{3}{4} \end{array}$$

§ 141. MODEL.—9 times 3 are 27 : 27 fourths is equal to $6\frac{3}{4}$, set down $\frac{3}{4}$; 9 times 7 are 63, and 6 are 69, set down 9; 9 times 4 are 36, and 6 are 42. The product is $429\frac{3}{4}$.

EXPLANATION.—As in whole numbers, we begin with the lowest denomination, and reduce each partial product to the next higher denomination, setting down the remaining units of the denomination in question, and reserving the units of the next denomination to be added to the next product.

Ex. 4. Multiply $\frac{3}{4}$ by $\frac{7}{8}$.

$\frac{3}{4} \times \frac{7}{8} = \frac{21}{32}$

§ 142. MODEL.—7 times 3 are 21 : 8 times 4 are 32. The product is $\frac{21}{32}$.

EXPLANATION.—To multiply by $\frac{7}{8}$ is the same as to multiply by 7 and divide the product by 8. 7 times 3 fourths $=21$ fourths, and 21 fourths $\div 8 = 21$ thirty-seconds : since a fraction (or a quotient) is divided by multiplying the denominator (or the divisor). (§§ 74, 111).

Ex. 5. Multiply $30\frac{1}{4}$ by $\frac{1}{4}$.

$30\frac{1}{4} \times \frac{1}{4}$

$\frac{121}{4} \times \frac{1}{4} = \frac{121}{16} = 7\frac{9}{16}$

§ 143. MODEL.—Reduce $30\frac{1}{4}$ to fourths. (§126). It is equal to 121 fourths. Once 121 is 121: 4 times 4 are 16. 121 sixteenths is equal to $7\frac{9}{16}$. The product is $7\frac{9}{16}$.

Ex. 6. Multiply $30\frac{1}{4}$ by $5\frac{1}{2}$.

$30\frac{1}{4} \times 5\frac{1}{2}$

$\frac{121}{4} \times \frac{11}{2} = \frac{1331}{8} = 166\frac{3}{8}$

§ 144. MODEL.—Reduce the mixed numbers to improper fractions. (§ 126). 11 times 121 are 1331: twice 4 are 8. 1331 eighths is equal to $166\frac{3}{8}$. The product is $166\frac{3}{8}$.

EXPLANATION.—It is often easier to reduce a mixed number to an improper fraction before multiplying, if the other factor is not a whole number.

RULE.—To multiply a simple fraction by a whole number.

Divide the denominator of the fraction, or else multiply its numerator, by the whole number.

To multiply a fraction by a fraction.

Multiply each term of the one fraction by the corresponding term of the other.

A *mixed number* may be reduced to an improper fraction, or its parts may be multiplied separately.

Ex. 7. Reduce $\frac{2}{3}$ of $\frac{4}{5}$ to a simple fraction.

$\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$

§ 145. MODEL.—Twice 4 are 8: 3 times 5 are 15. The given fraction is equal to $\frac{8}{15}$.

EXPLANATION.—One third of 1 fifth is evidently 1 fifteenth; 1 third of 4 fifths is 4 times 1 fifteenth, that is, 4 fifteenths; and 2 thirds of 4 fifths is twice 4 fifteenths, that is, 8 fifteenths.

RULE for reducing a compound fraction to a simple one.

Multiply together the several fractions which compose it.

Ex. 8. Multiply $\frac{4}{9}$ by 4. Prod. $1\frac{7}{9}$.

9. Multiply $\frac{6}{7}$ by 7.

10. Multiply $\frac{7}{8}$ by 8. Prod. 7.

11. Multiply $\frac{5}{6}$ by 12. Prod. 10.

12. Multiply $\frac{5}{11}$ by 15.

13. Multiply $\frac{7}{10}$ by 5. Prod. $3\frac{1}{2}$.
14. Multiply $2\frac{1}{2}$ by 7. Prod. $17\frac{1}{2}$.
15. Multiply $8\frac{2}{3}$ by 8.
16. Multiply $16\frac{2}{3}$ by 15. Prod. 250.
17. Multiply $19\frac{7}{8}$ by 20. Prod. $397\frac{1}{2}$.
18. Multiply $207\frac{5}{6}$ by 13.
19. What is the product of $315\frac{5}{9}$ and 19? Prod. $5995\frac{5}{9}$.
20. What is the product of $\frac{5}{7}$ and $\frac{3}{5}$? Prod. $\frac{3}{7}$.
21. What is the product of $\frac{2}{9}$ and $\frac{3}{2}$?
22. What is the product of $\frac{15}{16}$ and $\frac{8}{15}$? Prod. $\frac{1}{2}$.
23. Reduce $\frac{1}{2}$ of $\frac{2}{3}$ to a simple fraction. Val. $\frac{1}{3}$.
24. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ to a simple fraction.
25. Reduce $\frac{4}{5}$ of $\frac{6}{7}$ to a simple fraction. Val. $\frac{24}{35}$.
26. Reduce $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{5}{6}$ to a simple fraction. Val. $\frac{5}{16}$.
27. Reduce $\frac{1}{3}$ of $\frac{5}{7}$ of $\frac{9}{10}$ to a simple fraction.
28. Reduce $\frac{1}{3}$ of $7\frac{1}{2}$ to a simple fraction. Val. $2\frac{1}{2}$.
29. Reduce $\frac{1}{2}$ of $\frac{1}{5}$ of $7\frac{1}{2}$ to a simple fraction. Val. $1\frac{1}{4}$.
30. Reduce $\frac{2}{5}$ of $\frac{3}{7}$ of $8\frac{1}{3}$ to a simple fraction.
31. Find the product of $\frac{2}{3}$ of $\frac{3}{4}$ and $\frac{1}{2}$ of $12\frac{1}{2}$. Prod. $1\frac{9}{16}$.
32. Find the product of $\frac{5}{6}$ of $\frac{7}{8}$ and $33\frac{1}{3}$. Prod. $24\frac{11}{36}$.
33. Find the product of $\frac{1}{2}$ of $66\frac{2}{3}$ and $\frac{1}{3}$ of 100.
34. Find the product of $\frac{2}{5}$ of 250 and $\frac{3}{7}$ of 21. Prod. 900.
35. Find the product of $\frac{1}{2}$ of $\frac{1}{3}$ of 210 and $\frac{1}{4}$ of $83\frac{3}{4}$.
Prod. $670\frac{5}{16}$.
36. What is the product of $16\frac{1}{2}$ and $16\frac{1}{2}$?
37. What is the product of $30\frac{1}{4}$ and $60\frac{1}{2}$? Ans. $1830\frac{1}{8}$.
38. What is the product of $111\frac{1}{10}$ and $20\frac{1}{2}$? Ans. $2277\frac{11}{20}$.
39. What is the product of 275 and $\frac{1}{2}$ of $\frac{3}{4}$ of 36?
40. What is the product of 303 and $\frac{2}{7}$ of 20? Ans. $1731\frac{3}{7}$.
41. What is the product of $3\frac{2}{7}$ and $4\frac{14}{33}$?
42. What is the product of $\frac{2}{9}$ of $\frac{3}{5}$ and $\frac{5}{8}$ of $3\frac{3}{7}$?

DIVISION OF COMMON FRACTIONS.

Ex. 1. Divide $\frac{15}{16}$ by 3.

$\frac{15}{16} \div 3 = \frac{5}{16}$

§ 146. MODEL.—3 in 15, 5 times. The quotient is $\frac{5}{16}$.

EXPLANATION.—Comparing §§ 73 and 111, we see that the value of a fraction is divided by a whole number by dividing its numerator by the number.

Ex. 2. Divide $\frac{3}{4}$ by 5.

$\frac{3}{4} \div 5 = \frac{3}{20}$

§ 147. MODEL.—5 times 4 are 20. The quotient is 3 twentieths.

EXPLANATION.—Comparing §§ 74 and 111, we see that the value of a fraction is divided by a whole number by multiplying its denominator by the number.

Ex. 3. Divide $\frac{15}{16}$ by $\frac{3}{4}$.

$\frac{15}{16} \div \frac{3}{4} = \frac{5}{4} = 1\frac{1}{4}$

§ 148. MODEL.—3 in 15, 5 times: 4 in 16, 4 times. 5 fourths is equal to $1\frac{1}{4}$. The quotient is $1\frac{1}{4}$.

EXPLANATION.—15 sixteenths $\div 3 = 5$ sixteenths (§ 146): but the divisor 3 *fourths* is only one *fourth* of 3; hence the quotient is 4 times 5 sixteenths, that is, 5 fourths. (§ 140).

Again, since division is the reverse of multiplication, the process for division should be the reverse of that for multiplication: and since $\frac{3}{4} \times \frac{5}{7} = \frac{15}{28}$, it is evident that $\frac{15}{28} \div \frac{3}{4} = \frac{5}{7}$.

Ex. 4. Divide $\frac{3}{4}$ by $\frac{7}{8}$.

$\frac{3}{4} \div \frac{7}{8} = \frac{24}{28} = \frac{6}{7}$

§ 149. MODEL.—8 times 3 are 24: 7 times 4 are 28. 24 twenty-eighths is equal to $\frac{6}{7}$. The quotient is $\frac{6}{7}$.

EXPLANATION.—3 fourths $\div 7 = 3$ twenty-eighths (§147): but the divisor 7 eighths is only one *eighth* of 7; hence, by § 75, the quotient is 8 times 3 twenty-eighths, that is, 24 twenty-eighths. (§ 139).

Again, Multiplying both terms of the dividend by 56, we have $\frac{16}{24}\frac{8}{4} \div \frac{7}{8} = \frac{24}{28} = \frac{6}{7}$. Or, Multiplying both terms by 14, we have $\frac{42}{56} \div \frac{7}{8} = \frac{6}{7}$, the same result as before.

Ex. 5. Divide $273\frac{1}{3}$ by 5.

$5)273\frac{1}{3}$

$54\frac{2}{3}$

§ 150. MODEL.—5 in 27, 5 times with 2 over, set down 5; 5 in 23, 4 times with 3 over, set down 4; 5 in 10, twice, set down $\frac{2}{3}$. The quotient is $54\frac{2}{3}$.

EXPLANATION.—We divide the integer as usual, and reduce the 3 remaining units to thirds, making 9 thirds, which added to the given 1 third makes 10 thirds, and this divided by 5 gives 2 thirds. If the numerator of $\frac{10}{3}$ had not been divisible by 5, we would have multiplied its denominator by the divisor, as in § 147.

Ex. 6. Divide $3\frac{1}{3}$ by $12\frac{2}{3}$.

$3\frac{1}{3} \div 12\frac{2}{3}$

$\frac{10}{3} \div \frac{38}{3} = \frac{30}{114} = \frac{5}{19}$

§ 151. MODEL.—Reduce the given mixed numbers to improper fractions. (§ 126). 3 times 10 are 30: 38 times 3 are 114. 30 one-hundred-and-fourteenths is equal to 5 nineteenths. The quotient is 5 nineteenths.

RULE.—To divide a simple fraction by a whole number.

Divide the numerator of the fraction, or else multiply its denominator, by the whole number.

To divide a fraction by a fraction.

Divide each term of the dividend by the corresponding term of the divisor. Or, Multiply each term of the dividend by the other term of the divisor.

To divide a whole number by a fraction.

Divide the dividend by the denominator of the divisor, and multiply the quotient by the numerator.

A *mixed number* will mostly better be reduced to an improper fraction.

Ex. 7. Reduce $\frac{2\frac{1}{2}}{\frac{1}{2}\text{ of }\frac{3}{4}}$ to a simple fraction.

$2\frac{1}{2} \div \frac{1}{2}$ of $\frac{3}{4}$

$\frac{5}{2} \div \frac{3}{8} = \frac{40}{6} = 6\frac{2}{3}$

§ 152. MODEL.—Reduce the terms to simple fractions. Divide $\frac{5}{2}$ by $\frac{3}{8}$. 8 times 5 are 40: 3 times 2 are 6.—40 sixths is equal to $6\frac{2}{3}$. The given fraction is equal to $6\frac{2}{3}$.

RULE for reducing a complex fraction to a simple one. *Divide its numerator by its denominator.*

Ex. 8. Divide $\frac{3}{4}$ by 5. Quot. $\frac{3}{20}$.

9. Divide $\frac{6}{7}$ by 8.

10. Divide $\frac{15}{21}$ by 3. Quot. $\frac{5}{21}$.

11. Divide $\frac{18}{25}$ by 6. Quot. $\frac{3}{25}$.

12. Divide 40 by $\frac{4}{5}$.

13. Divide 200 by $\frac{3}{7}$. Quot. $466\frac{2}{3}$.

14. Divide 175 by $\frac{7}{25}$. Quot. 49.

15. Divide $\frac{15}{16}$ by $\frac{3}{8}$.

16. Divide $\frac{20}{33}$ by $\frac{2}{3}$. Quot. $\frac{10}{11}$.

17. Divide $\frac{12}{25}$ by $\frac{3}{5}$. Quot. $\frac{4}{5}$.

18. Dividend $=\frac{8}{9}$: divisor $=\frac{2}{3}$.

19. Dividend $=\frac{3}{5}$: divisor $=\frac{2}{7}$. Quot. $2\frac{1}{10}$.

20. Dividend $=\frac{2}{9}$: divisor $=\frac{3}{5}$. Quot. $\frac{10}{27}$.

21. Dividend $=\frac{1}{10}$: divisor $=\frac{1}{3}$.

22. Divisor $=\frac{3}{4}$: dividend $=\frac{5}{6}$. Quot. $1\frac{1}{9}$.

23. Divisor $=\frac{5}{6}$: dividend $=\frac{3}{4}$. Quot. $\frac{9}{10}$.

24. Divisor $=\frac{7}{8}$: dividend $=\frac{5}{8}$.

25. Divisor $=\frac{7}{8}$: dividend $=\frac{7}{9}$. Quot. $\frac{8}{9}$.

26. Divisor $=\frac{1}{2}$ of $\frac{3}{5}$: dividend $=\frac{4}{9}$. Quot. $1\frac{13}{27}$.

27. Divisor $=\frac{2}{3}$ of $\frac{4}{5}$: dividend $=\frac{6}{7}$.

28. Divisor $=\frac{5}{6}$ of $\frac{3}{8}$: dividend $=\frac{1}{2}$ of $\frac{3}{5}$. Quot. $\frac{24}{25}$.

29. Divisor $=\frac{3}{7}$ of $\frac{2}{9}$: dividend $=\frac{2}{5}$ of $\frac{3}{10}$. Quot: $1\frac{13}{50}$.

30. Divisor $=\frac{1}{2}$ of $12\frac{1}{2}$: dividend $=\frac{2}{3}$ of $\frac{3}{11}$.

31. Dividend $=12\frac{1}{2}$: divisor $=4$. Quot. $3\frac{1}{8}$.
32. Dividend $=207\frac{1}{5}$: divisor $=6$. Quot. $34\frac{8}{15}$.
33. Dividend $=45\frac{3}{4}$: divisor $=18\frac{3}{4}$.
34. Dividend $=70\frac{1}{2}$: divisor $=68\frac{3}{4}$. Quot. $1\frac{7}{275}$.
35. Dividend $=27\frac{1}{3}$: divisor $=55\frac{1}{7}$. Quot. $\frac{287}{579}$.
36. Dividend $=\frac{1}{3}$ of $28\frac{1}{4}$: divisor $=\frac{2}{3}$ of $43\frac{3}{4}$.
37. Dividend $=\frac{2}{7}$ of $\frac{3}{5}$: divisor $=\frac{1}{3}$ of 275. Quot. $\frac{18}{9625}$.
38. Dividend $=\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{7}{8}$: divisor $=\frac{2}{3}$ of $17\frac{1}{2}$. Quot. $\frac{5}{64}$.
39. Dividend $=\frac{2}{7}$ of $27\frac{1}{2}$: divisor $=\frac{3}{8}$ of $38\frac{1}{3}$.
40. Reduce $\dfrac{2\frac{1}{2}}{\frac{3}{4}}$ to a simple fraction. Val. $3\frac{1}{3}$.
41. Reduce $\dfrac{4\frac{1}{4}}{5\frac{2}{5}}$ to a simple fraction. Val. $\frac{85}{108}$.
42. Reduce $\dfrac{6\frac{5}{6}}{4\frac{1}{5}}$ to a simple fraction.
43. Reduce $\dfrac{\frac{1}{2}\text{ of }\frac{3}{5}}{7\frac{1}{2}}$ to a simple fraction. Val. $\frac{1}{25}$.
44. Reduce $\dfrac{\frac{2}{3}\text{ of }4\frac{1}{2}}{\frac{1}{5}\text{ of }19}$ to a simple fraction. Val. $\frac{15}{19}$.
45. Reduce $\dfrac{\frac{1}{2}\text{ of }\frac{3}{5}}{\frac{2}{3}\text{ of }7\frac{1}{2}}$ to a simple fraction.
46. Reduce $\dfrac{27}{\frac{1}{3}\text{ of }30}$ to a simple fraction. Val. $2\frac{7}{10}$.
47. Reduce $\dfrac{\frac{1}{5}\text{ of }20}{\frac{2}{3}\text{ of }17\frac{1}{3}}$ to a simple fraction. Val. $\frac{9}{26}$.
48. Reduce $\dfrac{\frac{2}{7}\text{ of }2\frac{2}{7}}{346\frac{1}{2}}$ to a simple fraction.
49. Reduce $\dfrac{2\frac{1}{2}}{7\frac{1}{2}}$ to a simple fraction. Val. $\frac{1}{3}$.
50. Reduce $\dfrac{\frac{1}{9}\text{ of }27\frac{1}{3}}{\frac{2}{7}\text{ of }72\frac{2}{7}}$ to a simple fraction. Val. $\frac{2009}{12669}$.

CANCELLATION.

§ 153. In multiplication of fractions, and in some other similar operations, the labor may be often diminished by canceling all the factors common to the numerators and the denominators, and afterwards multiplying together the remaining factors of each. This is simply reducing the result to its lowest terms in advance.

It is customary to draw a line through a number that has been canceled.

Ex. 1. Multiply $\frac{45}{49}$ by $\frac{7}{90}$.

$$\frac{\cancel{45}}{\cancel{49}} \times \frac{7}{\cancel{90}} = \frac{1}{14}$$

7 2

MODEL.— 45 in 45, once; 45 in 90, twice: 7 in 7, once; 7 in 49, 7 times: the numerator is 1; the denominator is $7 \times 2 = 14$. The product is $\frac{1}{14}$.

Ex. 2. Divide $\frac{1}{2}$ of $\frac{3}{7}$ of $\frac{5}{9}$ by $\frac{6}{9}$ of $\frac{10}{21}$ of $\frac{1}{27}$.

$\frac{1}{2}$ of $\frac{3}{7}$ of $\frac{5}{9} \div \frac{6}{9}$ of $\frac{10}{21}$ of $\frac{1}{27}$

3

$$\frac{1}{2} \text{ of } \frac{\cancel{3}}{7} \text{ of } \frac{\cancel{5}}{\cancel{9}} \times \frac{\cancel{9}}{\cancel{6}} \text{ of } \frac{\cancel{21}}{\cancel{10}} \text{ of } \frac{27}{1} = \frac{81}{8} = 10\frac{1}{8}$$

2 2

MODEL.—3 in 3, once; 3 in 6, twice: 7 in 7, once; 7 in 21, 3 times: 9 in 9, once; 9 in 9, once: 5 in 5, once; 5 in 10, twice: the numerator is $3 \times 27 = 81$; the denominator is $2.2.2 = 8$. The quotient is $\frac{81}{8} = 10\frac{1}{8}$.

Ex. 3. Divide the product of 77 and 96 by the product of 22 and 24.

2

7 $\cancel{48}$

$$\frac{\cancel{77} \times \cancel{96}}{\cancel{22} \times \cancel{24}} = 14$$

$\cancel{2}$

MODEL.— 11 in 77, 7 times; 11 in 22, twice: 2 in 2, once; 2 in 96, 48 times: 24 in 24, once; 24 in 48, twice. The quotient is $7 \times 2 = 14$.

Ex. 4. Divide $11 \times 21 \times 26$ by $3 \times 13 \times 14$.

$$\frac{\overset{7}{\cancel{11}} \times \overset{2}{\cancel{21}} \times \overset{2}{\cancel{26}}}{\underset{1}{\cancel{3}} \times \underset{1}{\cancel{13}} \times \underset{2}{\cancel{14}}}$$

MODEL.— 3 in 3, once; 3 in 21, 7 times: 7 in 7, once; 7 in 14, twice: 13 in 26, twice; 13 in 13, once: 2 in 2, once; 2 in 2, once. The quotient is 11.

Ex. 5. Multiply $\frac{3}{7}$ of $\frac{14}{27}$ by $\frac{9}{10}$ of $\frac{20}{38}$ of $\frac{19}{25}$. Prod. $\frac{2}{25}$.

6. Multiply $\frac{1}{4}$ of $\frac{2}{5}$ of $\frac{10}{15}$ by $\frac{3}{5}$ of $\frac{25}{29}$.
7. Multiply $\frac{2}{7}$ of $\frac{14}{9}$ by $\frac{3}{4}$ of $\frac{12}{15}$. Prod. $\frac{4}{15}$.
8. Divide $\frac{17}{21}$ of $\frac{13}{20}$ by $\frac{34}{7}$ of $\frac{26}{5}$ of $\frac{1}{12}$. Quot. $\frac{1}{4}$.
9. Divide $\frac{27}{25}$ of $\frac{15}{18}$ by $\frac{9}{15}$ of $\frac{30}{36}$.
10. Divide $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{7}{8}$ by $\frac{3}{8}$ of $\frac{15}{24}$. Quot. $2\frac{1}{3}$.
11. Divide the product of 22 and 56 by the product of 44, 28, and 16. Quot. $\frac{1}{16}$.
12. Divide the product of 72 and 96 by the product of 60 and 64.
13. Divide the product of 27, 28, and 29 by the product of 35, 36, and 37. Quot. $\frac{87}{185}$.
14. Divide $10 \times 11 \times 12$ by $22 \times 24 \times 30$. Quot. $\frac{1}{12}$.
15. Divide $25 \times 27 \times 32 \times 36$ by $15 \times 18 \times 24 \times 28$.

PROMISCUOUS PROBLEMS.

1. What is the sum of 275, 386, 497, and 608?
2. What is the difference between 275386 and 497608?

Ans. 222222.

3. What is the product of 275386497 and 608?
4. What is the quotient of 275386497 by 608?

Ans. $452938\frac{193}{608}$.

5. Add the difference between 395 and 422 to the sum of 39, 54, and 202. Sum, 3

6. Subtract the sum of 25 and 19 from their product.
7. Multiply the difference of 25 and 19 by their sum.
Prod. 264.
8. Divide the product of 36 and 45 by their difference.
Quot. 180.
9. Resolve 7050 into its prime factors.
10. What is the greatest common measure of 25, 250, and 375? Ans. 25.
11. What is the least common multiple of 5, 6, 10, and 12?
Ans. 60.
12. Reduce $\frac{252}{1260}$ to its lowest terms.
13. In $\frac{565}{55}$ how many units? Ans. $10\frac{5}{11}$.
14. In 19 units how many nineteenths? Ans. $\frac{361}{19}$.
15. In $15\frac{3}{5}$ how many fifths?
16. In $\frac{5}{9}$ how many forty-fifths? Ans. $\frac{25}{45}$.
17. In $\frac{28}{100}$ how many twenty-fifths? Ans. $\frac{7}{25}$.
18. Reduce $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{9}$ to a common denominator.
19. Reduce $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{9}$ to their least common denominator.
Val. $\frac{27}{36}$, $\frac{30}{36}$, $\frac{28}{36}$.
20. Add $\frac{3}{14}$, $\frac{1}{28}$, and $\frac{3}{4}$. Sum, 1.
21. What is the sum of $\frac{1}{2}$ of $\frac{4}{9}$ and $\frac{4}{9}$ of $\frac{1}{2}$?
22. What is the sum of $\frac{2}{3}$ of $10\frac{1}{3}$ and $\frac{2\frac{1}{2}}{\frac{1}{4}\text{ of }17}$? Ans. $7\frac{73}{156}$.
23. What is the difference between $19\frac{7}{9}$ and $26\frac{1}{2}$?
Ans. $6\frac{13}{18}$.
24. What is the difference between $\frac{1}{3}$ of 27 and $\frac{2}{9}$ of 24?
25. What is the product of $27\frac{1}{2}$ and $\frac{1}{3}$ of 77? Ans. $705\frac{5}{6}$.
26. What is the product of $\frac{\frac{1}{2}\text{ of }5\frac{1}{2}}{22\frac{1}{2}}$ and $\frac{45}{\frac{1}{2}\text{ of }11}$? Ans. 1.
27. What is the quotient of $\frac{2}{9}$ of 47 by $25\frac{1}{6}$?
28. What is the product of $\frac{2}{5}$ of 47 and $\frac{1}{6}$ of 25?
Ans. $43\frac{14}{27}$.

29. What is the quotient of $\frac{\frac{1}{2} \text{ of } 27\frac{1}{4}}{25\frac{1}{3}}$ by $\frac{\frac{3}{4} \text{ of } 19}{83\frac{3}{4}}$?

Ans. $3\frac{163}{1162}$.

30. Add the product of $\frac{1}{2}$ of 27 and $\frac{3}{4}$ of $\frac{5}{9}$ to their difference.

31. Subtract the quotient of $\frac{2}{7}$ of 45 by $\frac{1}{3}$ of 24 from their sum. Rem. $19\frac{1}{4}$.

32. Multiply the sum of $\frac{\frac{1}{2} \text{ of } \frac{5}{9}}{\frac{1}{3} \text{ of } \frac{5}{11}}$ and $\frac{5}{9}$ of $73\frac{1}{3}$ by their difference. Prod. $262\frac{607}{2916}$.

33. Divide the product of $25\frac{1}{5}$ and $17\frac{1}{7}$ by their sum.

34. What number is that to which if $3\frac{2}{3}$, $5\frac{3}{5}$, $6\frac{5}{6}$, and $10\frac{9}{10}$, be added, the sum will be $30\frac{1}{3}$? Ans. $3\frac{1}{3}$.

35. What number is that from which if $3\frac{2}{3}$, $5\frac{3}{5}$, $6\frac{5}{6}$, and $10\frac{9}{10}$, be subtracted, the remainder will be $30\frac{1}{3}$? Ans. $57\frac{1}{3}$.

36. What number is that by which if the sum of $3\frac{2}{3}$, $5\frac{3}{5}$, $6\frac{5}{6}$, and $10\frac{9}{10}$, be multiplied, the product will be $30\frac{1}{3}$?

37. What number is that by which if the sum of $3\frac{2}{3}$, $5\frac{3}{5}$, $6\frac{5}{6}$, and $10\frac{9}{10}$, be divided, the quotient will be $30\frac{1}{3}$?

Ans. $\frac{81}{91}$.

38. What is the sum of $\frac{3}{7}$, $\frac{3}{9}$, 13, and $18\frac{3}{15}$?

39. What is the difference between $\frac{3}{5}$ and $\frac{9}{16}$? Ans. $\frac{3}{80}$.

40. What is the product of $5\frac{1}{4}$ and $\frac{1}{6}$?

41. What is the quotient of $\frac{2}{3}\frac{7}{7}\frac{5}{5}\frac{5}{8}$ by 19?

42. $3+5-7\div5+16\div7-15\div6=$what? Ans. $6\frac{27}{70}$.

43. $(3+5-7)\div5+16\div7-15\div6=$what?

44. $3+(5+7)\div5+16\div7-15\div6=$what? Ans. $5\frac{138}{70}$.

45. $3+5-7\div5+16+7-15\div6=$what? Ans. $27\frac{1}{10}$.

46. $(3+5-7)\div5+(16+15-7)\div6=$what?

47. $3+(5+7)\div5+16+(15-7)\div6=$what? Ans. $22\frac{11}{15}$.

48. $3+(7-5)\div(5+16)+(15-7)\div6=$what? Ans. $4\frac{3}{7}$.

49. $3+7-5\div(5+16+15)-7\div6=$what?

DECIMAL FRACTIONS.

§ 154. A *decimal* fraction is one whose denominator is some *power of ten* and is not expressed in writing.

§ 155. In the Arabic or decimal system of notation (§10), we observed that, in passing from the units' place to the left, a unit of any order is *ten times* a unit of the preceding order; or that, in passing from left to right, a unit of any order is *one tenth* of a unit of the preceding order. If this law be extended to the right of units, the next order will be tenths, the next hundredths, the next thousandths, &c., as in the following

TABLE.

Thousands.	Hundreds.	Tens.	Units.	*Units' Point.*	Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hundred-thousandths.	Millionths.	Ten-millionths.	Hundred-millionths.	Billionths.	Ten-billionths.	Hundred-billionths.	Trillionths.
2	3	4	5	.	2	3	4	5	7	6	5	9	5	8	7	5

As 100 is $\frac{1}{10}$ of 1000, 10 is $\frac{1}{10}$ of 100, and 1 is $\frac{1}{10}$ of 10, so one tenth is $\frac{1}{10}$ of 1, one hundredth is $\frac{1}{10}$ of 1 tenth, one thousandth is $\frac{1}{10}$ of one hundredth, &c.

§ 156. To write any number of tenths, then, we simply put the proper figure one place to the right of units; for hundredths, we put the figure two places to the right, &c. To determine the position of units and the relative positions of the fractional orders, we place a period, called the

units' point, between units and tenths; or to the left of tenths, if the expression is entirely fractional. Thus, 2.3, two and three tenths; 3.02, three and two hundredths; 5.32, five and three tenths and two hundredths; .005, five thousandths; .0006, six ten-thousandths; .00004, four hundred-thousandths; .000008, eight millionths.

In integral numbers, this point, being unnecessary, is never written: but in fractional or mixed expressions, it must never be omitted.

§ 157. It will be observed that the number of places occupied by the numerator of a decimal fraction is equal to the number of naughts in its denominator. If the ordinary expression of the numerator does not require so many places, each place intervening between the units' point and the left hand figure of the numerator must be filled with a naught. Thus, .002, 2 thousandths; .023, 23 thousandths; .0203, 203 ten-thousandths; .0023, 23 ten-thousandths; .0004, 4 ten-thousandths; .002034, 2034 millionths.

§ 158. A decimal fraction is *read*, like a common fraction, by pronouncing after the numerator the ordinal of the denominator. Sometimes, in reading a mixed number, to prevent ambiguity, it is necessary to pronounce the word "units" after the integer. Thus, three hundred and fifteen thousandths is written .315; but 300.015 is three hundred units and fifteen thousandths: so, 7000.0275 is read seven thousand units and two hundred and seventy-five ten-thousandths.

Read the following decimal fractions:— .1, .3, .5, .7, .8; .01, .05, .09, .11, .25, .34, .47, .51, .63, .75, .87, .99; .001, .005, .015, .025, .075, .125, .219, .375, .487, .567, .605, .777, .808, .999; .0001, .0012, .0125, .1275, .3525, .6225, .7203,

.8007, .9883, .9999; .00001, .00014, .00225, .03275, .33125, .42075, .53003, .70007, .87078, .99999; .000001, .000017, .000175, .003175, .063175, .475327, .796305, .634008, .320075, .200017, .200325; .0000001, .0000025, .0000275, .0020705, .0357675, .7500786; .00027625, .02700625, .23450275, .00073513, .23570025, .125346798, .000000125, .0007600025, .27340709025, .70030005345, .000257025702.

Read the following mixed numbers:— 3.3, 70.5, 35.7, 2.02, 3.25, 75.75, 24.05, 7.07, 30.003, 400.025, 25.125, 375.375, 1.001, 2.325, 2.0275, 300.0025, 17.0017, 1.0005, 2000.0002, 21.2125, 325.03725, 9180.20025, 1000.02207, 7025.00025, 6278.374375, 2000.0002325, 3375.00000765, 27.0000027, 3200.000000075, 2500.0000036975.

Write the following in figures:

110. Seventy-four hundredths.
111. Four hundred and forty-eight thousandths.
112. Five hundred units and three hundredths.
113. Seventy-five thousandths.
114. Five hundred and three thousandths.
115. Five hundred units and three thousandths.
116. Three hundred and twenty-seven ten-thousandths.
117. Three hundred units and twenty-seven ten-thousandths.
118. Seventeen and seventeen hundred-thousandths.
119. One thousand units and two thousand two hundred and seven hundred-thousandths.
120. Three thousand two hundred units and seventy-five millionths.
121. Six hundred and three ten-thousandths.
122. Two thousand four hundred and sixty-one and three hundred and nineteen millionths.

ADDITION OF DECIMAL FRACTIONS.

Ex. 1. Add .3, .23, .175, and .025.

.3
.23
.175
.025
.73

§ 159. MODEL.—5 and 5 are 10; 1 and 2 are 3, and 7 are 10, and 3 are 13, set down 3; 1 and 1 are 2, and 2 are 4, and 3 are 7.—Point before 7. The sum is .73.

EXPLANATION.—Beginning at the right, we find the sum of the first column to be 10 thousandths, equal to 1 hundredth exactly. We do not set down the naught here, because a naught at the right of a decimal fraction does not assist in determining the orders of the other figures. The 1 hundredth is added in with the column of hundredths, which amounts to 13 hundredths, equal to 1 tenth and 3 hundredths. Setting 3 under the column of hundredths, we add the 1 tenth in with the column of tenths. We then place the units' point at the left of the tenths. (§ 156).

Ex. 2. Add .3, 3.5, 3.15, 35.25, and 171.275.

.3
3.5
3.15
35.25
171.275
213.475

§ 160. MODEL.—5; 7 and 5 are 12, and 5 are 17, set down 7; 1 and 2 are 3, and 2 are 5, and 1 are 6, and 5 are 11, and 3 are 14, set down 4; 1 and 1 are 2, and 5 are 7, and 3 are 10, and 3 are 13, set down 3; 1 and 7 are 8, and 3 are 11, set down 1; 1 and 1 are 2. Point before 4. The sum is 213.475.

EXPLANATION.—The sum of the column of tenths being 14, that is, 1 unit and 4 tenths, we set 4 under the column of tenths, and add 1 to the column of units. We place the units' point between the units and the tenths. (§ 156).

RULE.—*Arrange the numbers with units of the same order in the same column; and add as in whole numbers.* (§ 22).

Place the units' point on the left of the tenths figure in the sum.

PROOF.—The same as in whole numbers. (§ 22).

Ex. 3. Add 1.2, 3.56, 45.67, and 56.789.

4. Add 1.3, 5.79, 24.68, and 90.275. Sum, 122.045.
5. Add 27.72, 365.9, 125.008, and 236.115. Sum, 754.743.
6. Add 135.709, 246.008, 145.008, and 236.709.
7. Add 1.35795, 135.795, and 13579.5.
8. Find the sum of 2.465, 25.009, 160.206, and 146.27. Sum, 333.950.
9. Find the sum of 100.0001, [illegible]1.4012, 412.5124, and 421.5214.
10. Find the sum of 1234.56, 78.9012, 3456.789, 10.234567, and 890.13575. Sum, 5670.620517.
11. Find the sum of 907.0503, 890.7054, 785.4321, and 25.457. Sum, 2608.6448.
12. Find the sum of 12.012575, 120.125725, 1201.257725, and .276825.
13. Find the sum of .760027, .000176, .012012, and .027945. Sum, .800160.
14. What is the sum of .230495, .341507, .452618, and .563729? Ans. 1.588349.
15. What is the sum of 2.30495, 34.1507, 452.618, and 5637.029?
16. What is the sum of 12.000012, 250.0025, 75.075, and 175.0175? Ans. 512.095012.
17. 175 + 6.115 + 123.1341 + 172.21275 + 5637.175 = what? Ans. 6113.63685.

18. 52.8672+549.72+927.365+57.10715+13.575=what?
19. 79.105+131.187+19.4201+2643.13+34.8364=what?
Ans. 2907.6785.
20. 3844.04 + .444584 + 6.14644 + 6847.34 + 77.9899= what?
Ans. 10775.960924.

SUBTRACTION OF DECIMAL FRACTIONS.

Ex. 1. From 275.075 take 87.1275.

275.075
87.1275
187.9475

§ 161. MODEL.—5 from 10 leaves 5; 8 from 15 leaves 7; 3 from 7 leaves 4; 1 from 10 leaves 9; 8 from 15 leaves 7; 9 from 17 leaves 8; 1 from 2 leaves 1. Point before 9. The remainder is 187.9475.

EXPLANATION.—After placing the subtrahend under the minuend with units of the same order in the same column, we find 5 ten-thousandths in the subtrahend and no ten-thousandths in the minuend. Adding 1 thousandth, that is, 10 ten-thousandths, to the minuend, we subtract from this the 5 ten-thousandths of the subtrahend. Then, because the minuend is increased 10 ten-thousandths or 1 thousandth, the subtrahend must be increased the same amount. (§ 28).. The same kind of reasoning will explain the rest of the operation. We place the units' point between the units and the tenths. (§ 156).

RULE.—*Place the subtrahend under the minuend, with units of the same order in the same column, and subtract as in whole numbers.* (§ 30).

Place the units' point on the left of the tenths figure in the remainder. (§ 156).

PROOF.—The same as in whole numbers. (§ 30).

Ex. 2. From 8.96 take 3.07. Rem. 5.89.

3. From 2.719 take 1.827.

4. From 97.8637 take 9.7863. Rem. 88.0774.

5. Take 67.8902 from 896.454. Rem. 828.5638.

6. Take 17.24937 from 1963.869.

7. Take 234.68579 from 6005.004. Rem. 5770.31821.

8. Take 98.79789 from 99.000099. Rem. .202209.

9. Minuend is 1234.567; Subtrahend is .76542.

10. Minuend is 29017.05; Subtrahend is 10.8405.
Rem. 29006.2095.

11. Minuend is 2098.76; Subtrahend is 454.698.
Rem. 1644.062.

12. Minuend is 1201.257725; Subtrahend is 120.125575.

13. Subtrahend is .012095; Minuend is .027945.
Rem. .01585.

14. Subtrahend is 2.30495; Minuend is 34.1507.
Rem. 31.84575.

15. Subtrahend is 12.000012; Minuend is 250.0025.

16. Subtrahend is 75.075; Minuend is 175.0175.
Rem. 99.9425.

17. 5637.175 − 172.21275 = what? Ans. 5464.96225.

18. 927.305 − 57.190715 = what?

19. What is the difference between one millionth, and ninety-nine thousandths? Ans. .098999.

20. What is the difference between thirty-seven billionths, and one hundred and eleven thousandths?
Ans. .110999963.

21. What is the difference between six billionths, and nine hundred and ninety-nine thousandths?

22. What is the difference between three millionths, and three hundred and six thousandths? Ans. .305997.

MULTIPLICATION OF DECIMAL FRACTIONS.

Ex. 1. Multiply 5.3 by 6.25.

6.25
5.3
1875
3125
33.125

§ 162. MODEL.—3 times 5 are 15, set down 5; 3 times 2 are 6, and 1 are 7; 3 times 6 are 18:—5 times 5 are 25, set down 5 under 7; 5 times 2 are 10, and 2 are 12, set down 2; 5 times 6 are 30, and 1 are 31:—add the partial products: 5; 5 and 7 are 12, set down 2; 1 and 2 are 3, and 8 are 11, set down 1; 1 and 1 are 2, and 1 are 3; 3. Point before 1. The product is 33.125.

EXPLANATION.—Reducing both factors to improper fractions, and multiplying as in § 142, we have $\frac{625}{100} \times \frac{53}{10} = \frac{33125}{1000}$; and this product reduced to a mixed number becomes 33.125, as in the model. If any decimal mixed number be reduced to an improper fraction, the numerator will consist of the same figures as the given mixed number. Hence we multiply as in whole numbers. The location of the units' point in the product is found by observing that the number of naughts in the denominator of either factor is the same as the number of figures in the numerator, and that the product of any two powers of ten is obtained by annexing to 1 as many naughts as there are in both factors together. There are, therefore, as many fractional figures in the product as in both factors together.

Ex. 2. Multiply .15 by .3.

.15
.3
.045

§ 163. MODEL.—3 times 5 are 15, set down 5; 3 times 1 are 3, and 1 are 4. Prefix one naught. Point before 0. The product is .045.

EXPLANATION.—When the product does not contain enough figures to express its proper denomination, we prefix one or more naughts to supply this deficiency.

RULE.—*Multiply as in whole numbers, and point off as many fractional figures in the product as there are in both the factors, prefixing naughts when necessary to make up the number.*

PROOF.—The same as in whole numbers. (§ 40).

Ex. 3. Multiply 12.42 by 3.2.

4. Multiply 25.25 by 2.5. Prod. 63.125.
5. Multiply .25 by .25. Prod. .0625.
6. Multiply 5.5 by 5.5.
7. Multiply 211.79 by 2.7. Prod. 571.833.
8. Multiply 97.825 by .34. Prod. 33.2605.
9. Multiply 275.005 by 5.005.
10. Multiply 869.06 by .045. Prod. 39.1077.
11. Multiply 27.9362 by .0052. Prod. .14526824.
12. $192.837 \times 6.7 =$ what?
13. $293.705 \times .075 =$ what? Ans. 22.027875.
14. $3.047 \times 2.87 =$ what? Ans. 8.74489.
15. $2.975 \times .375 =$ what?
16. $4.027 \times 402.7 =$ what? Ans. 1621.6729.
17. What is the product of 247.742 and 10.035?
18. What is the product of 307.0005 and .000375?
19. What is the product of 175.025 and 25.0175?

Ans. 4378.6879375.

20. What is the product of 1200.375 and 162.625?

DIVISION OF DECIMAL FRACTIONS.

Ex. 1. Divide 2.25 by .3.

$$.3)2.25 \quad 7.5$$

§ 164. MODEL.—3 in 22, 7 times, with 1 over, set down 7; 3 in 15, 5 times. Point before 5. The quotient is 7.5.

EXPLANATION.—Since the divisor and the quotient are factors of the dividend, there must be as many fractional figures in the dividend as there are in both the factors. (§ 162). Hence, to find the number of fractional figures in the quotient, we subtract the number in the divisor from the number in the dividend.

Ex. 2. Divide 26.4 by 8.25.

26.40	8.25
2475	3.2
1650	
1650	
0	

§ 165. MODEL.—Annex one naught to the dividend: 8 in 26, 3 times; multiply the divisor by 3; 15, 7, 24; subtract the product from the dividend; 5, 6, 1; annex 0: 8 in 16, twice; multiply the divisor by 2; 10, 5, 16; subtract the product from the previous remainder; 0. Point before 2. The quotient is 3.2.

EXPLANATION.—As the number of fractional figures in the divisor exceeds the number in the dividend, we annex a naught to the dividend to make them equal. We afterwards find it necessary to annex an other naught to complete the division. This makes 3 fractional figures in the dividend; and, as there are 2 in the divisor, there must be one in the quotient.

Ex. 3. Divide 4 by 15.

4.00	15
30	.266+
100	
90	
100	
90	
10	

§ 166. MODEL.—Annex 2 naughts to the dividend; 15 in 40, twice; multiply the divisor by 2; 10, 3; subtract the product from the dividend; 0, 0, 1; 15 in 100, 6 times; multiply the divisor by 6; 30, 9; subtract the product from the previous remainder; 0, 1; annex 1: 15 in 100, 6 times; &c. Point before 2.—The quotient is .266+.

EXPLANATION.—Since the dividend can be extended only by annexing naughts, it is evident that, if the same remainder should occur twice in succession, the same quotient figure will occur and will give rise to the same remainder again ; so that the same circuit of operations will occur perpetually. In such cases the quotient can not be obtained exactly, but we can always make an approximation sufficiently near for any practical purpose.

RULE.—*Divide as in whole numbers, and point off as many fractional figures in the quotient as the number in the dividend exceeds the number in the divisor, prefixing naughts when necessary to make up the number.*

If the number of fractional figures in the divisor exceeds the number in the dividend, annex to the dividend as many naughts as may be necessary to make the number in the dividend at least equal to the number in the divisor.

Note.—When the division can not be exactly performed, we put the sign + at the right of the quotient.

PROOF.—The same as in whole numbers. (§ 53).

Ex. 4. Divide 1728 by .12.

5. Divide 1728 by 1.2.
6. Divide 172.8 by 12.
7. Divide 17.28 by 12.
8. Divide 13 by 245.
9. Divide 2.7 by 900.
10. Divide 189.75 by .759. Quot. 250.
11. Divide 84.099 by .097. Quot. 867.
12. Dividend is 4435.2, divisor is .84.
13. Dividend is .8928, divisor is 1.24. Quot. .72.
14. Dividend is 7049.754, divisor is 8.7034. Quot. 810.
15. Dividend is 2.4416, divisor is 43.6.

16. Divisor is 47, dividend is 22.09. Quot. .47.
17. Divisor is 18.07, dividend is .12649. Quot. .007.
18. Divisor is 180.7, dividend is .012649.
19. Divisor is .125, dividend is 2.25. Quot. 18.
20. Divisor is 18, dividend is 19.

CONTRACTION IN MULTIPLICATION.

Ex. 1. Multiply 23.25 by 10.

$23.25 \times 10 = 232.5$

§ 167. MODEL.—Remove the point one place to the right. The product is 232.5.

EXPLANATION.—To multiply by any power of ten, we simply remove the units' point as many places to the right as there are naughts in the multiplier, annexing naughts when necessary. See § 155.

Ex. 2. Multiply 232.5 by 100. Prod. 23250.
3. Multiply 10.25 by 1000.
4. Multiply 246.25 by 100. Prod. 24625.
5. Multiply 875.275 by 10. Prod. 8752.75.
6. Multiply 96.0025 by 10000.
7. Multiply .0025 by 1000. Prod. 2.5.
8. Multiply .0007 by 100000. Prod. 70.
9. Multiply .05 by 1000000.
10. Multiply .0065 by 10000. Prod. 65.

CONTRACTION IN DIVISION.

Ex. 1. Divide 23.25 by 10.

$23.25 \div 10 = 2.325$

§ 168. MODEL.—Remove the point one place to the left. The quotient is 2.325.

EXPLANATION.—To divide by any power of ten, we simply remove the units' point as many places to the left as there are naughts in the divisor, prefixing naughts when necessary. See §155.

Ex. 2. Divide 2.325 by 100. Quot. .02325.
3. Divide 10.25 by 1000.
4. Divide 246.25 by 100. Quot. 2.4625.
5. Divide 875.275 by 10. Quot. 87.5275.
6. Divide 96.0025 by 10000.
7. Divide 2500 by 1000. Quot. 2.5.
8. Divide 7000 by 100000. Quot. .07.
9. Divide .05 by 1000000.
10. Divide .0065 by 10000. Quot. .00000065.

RELATIONS OF COMMON AND DECIMAL FRACTIONS.

§ 169. Every decimal fraction may be expressed in the form of a common fraction by simply *removing the units' point, writing the denominator under the numerator, and reducing, if necessary, to its lowest terms.* Thus, $.5=\frac{5}{10}=\frac{1}{2}$. Also, $.25=\frac{25}{100}=\frac{1}{4}$.

Ex. 1. Reduce .375 to a common fraction. Val. $\frac{3}{8}$.
2. Reduce .625 to a common fraction. Val. $\frac{5}{8}$.
3. Reduce .1875 to a common fraction.
4. Reduce .3125 to a common fraction. Val. $\frac{5}{16}$.
5. Reduce .05 to a common fraction. Val. $\frac{1}{02}$.
6. Reduce .0015 to a common fraction.
7. Reduce 00025 to a common fraction. Val. $\frac{1}{4000}$.

8. Reduce .004375 to a common fraction. Val. $\frac{7}{1600}$.
9. Reduce .08125 to a common fraction.
10. Reduce .0175 to a common fraction. Val. $\frac{7}{400}$.

§ 170. If the denominator of a common fraction has no other prime factor than 2 or 5, it may be reduced to a decimal form by multiplying both its terms by such a number as will make the denominator a power of ten, removing the denominator, and putting the units' point at its proper place in the numerator. Thus, multiplying both terms of $\frac{1}{4}$ by 25, we have $\frac{25}{100}$, which may be written, .25.

Ex. 11. Reduce $\frac{2}{5}$ to a decimal fraction. Val. .4.
12. Reduce $\frac{3}{4}$ to a decimal fraction.
13. Reduce $\frac{5}{8}$ to a decimal fraction. Val. .625.
14. Reduce $\frac{7}{20}$ to a decimal fraction. Val. .35.
15. Reduce $\frac{14}{5}$ to a decimal fraction.
16. Reduce $\frac{19}{40}$ to a decimal fraction. Val. .475.
17. Reduce $\frac{3}{80}$ to a decimal fraction. Val. .0375.
18. Reduce $\frac{27}{50}$ to a decimal fraction.
19. Reduce $\frac{9}{16}$ to a decimal fraction. Val. .5625.
20. Reduce $\frac{3}{32}$ to a decimal fraction. Val. .09375.

§ 171. If the denominator of a common fraction has neither 2 nor 5 as a prime factor, it cannot be reduced to a decimal form. We can make an approximation, however, sufficiently near for all practical purposes, by the following plan. Taking the example of last section, if we multiply both terms of $\frac{1}{4}$ by 100, we have $\frac{100}{400}$, and then dividing both terms by 4, we have $\frac{25}{100}$, that is, .25. In other words, *A common fraction is reduced to a decimal form by dividing its numerator by its denominator.* (§ 166). This is the general rule, and is but a repetition of what we learned in § 113. But let us attempt to apply this rule to the frac-

tion $\frac{2}{3}$. Dividing, we have 3 in 20, 6 times, with 2 over; again, annexing an other naught, we have 3 in 20, 6 times, with 2 over; and so on, evidently forever. Again, reduce $\frac{2}{11}$ to a decimal form. Dividing, we have, 11 in 20, once, with 9 over; 11 in 90, 8 times, with 2 over; 11 in 20, once, with 9 over, again; and 11 in 90, 8 times, with 2 over, again; and so, evidently, these two quotient figures might be repeated to the end of time.

11)2.0000
.1818+

§ 172. Such expressions as these are called *pure repetends*, and they are denoted by placing a dot over the repeating figure when there is but one, or by placing dots over the first and last repeating figures when there are several.

Thus, $\frac{2}{3}=.\dot{6}$; $\frac{2}{11}=.\dot{1}\dot{8}$; $\frac{2}{9}\frac{7}{9}\frac{5}{9}=.\dot{2}75$.

Ex. 21. Reduce $\frac{1}{3}$ to a repetend.

22. Reduce $\frac{2}{7}$ to a repetend. Val. .285714.

23. Reduce $\frac{3}{11}$ to a repetend. Val. .27.

24. Reduce $\frac{4}{13}$ to a repetend.

25. Reduce $\frac{5}{17}$ to a repetend. Val. .2941176470588235.

§ 173. If the denominator of a common fraction has either 2 or 5 or both, and other prime factors, the quotient of its numerator by its denominator will be partly a decimal fraction and partly a repetend.

Thus, $\frac{5}{6}=.8333+$, or .83. Also, $\frac{5}{12}=.41666+$, or .416. Also, $\frac{5}{24}=.2083$; and $\frac{5}{48}=.10416$.

These expressions are called *mixed repetends*.

Ex. 26. Reduce $\frac{7}{12}$ to a mixed repetend. Val. .583.

27. Reduce $\frac{1}{6}$ to a mixed repetend.

28. Reduce $\frac{3}{14}$ to a mixed repetend. Val. .2142857.

29. Reduce $\frac{7}{15}$ to a mixed repetend. Val. .46.

30. Reduce $\frac{11}{18}$ to a mixed repetend.

§ 174. *To reduce a pure repetend to a common fraction, we remove the units' point, write for denominator as many nines as there are repeating figures, and reduce the result to its lowest terms.*

For, $\frac{1}{9}=.\dot{1}$, $\frac{2}{9}=.\dot{2}$, $\frac{3}{9}$ or $\frac{1}{3}=.\dot{3}$, $\frac{7}{9}=.\dot{7}$, $\frac{8}{9}=.\dot{8}$:

Again, $\frac{1}{99}=.0\dot{1}$, $\frac{5}{99}=.0\dot{5}$, $\frac{10}{99}=.\dot{1}0$, $\frac{25}{99}=.\dot{2}5$, $\frac{50}{99}=.\dot{5}0$:

Also, $\frac{1}{999}=.00\dot{1}$, $\frac{10}{999}=.0\dot{1}0$, $\frac{75}{999}=.0\dot{7}5$, $\frac{275}{999}=.\dot{2}75$, &c.

§ 175. From these facts we learn that a pure repetend is read by pronouncing after its numerator the ordinal of the number formed of as many nines as there are figures in the repetend. Thus, $.\dot{7}=\frac{7}{9}$, $.8\dot{7}=\frac{87}{99}$, &c.

Ex. 31. Reduce $.2\dot{7}$ to a common fraction. $\frac{3}{11}$.

32. Reduce $.7\dot{2}$ to a common fraction. $\frac{8}{11}$.

33. Reduce $.3\dot{6}$ to a common fraction.

34. Reduce $.\dot{1}35$ to a common fraction. $\frac{5}{37}$.

35. Reduce $.2\dot{7}9$ to a common fraction. $\frac{31}{111}$.

36. Reduce $.7\dot{9}2$ to a common fraction.

37. Reduce $.8\dot{0}\dot{1}$ to a common fraction. $\frac{89}{111}$.

38. Reduce $.9001$ to a common fraction. $\frac{9001}{9999}$.

39. Reduce $.8\dot{8}7\dot{7}$ to a common fraction.

40. Reduce $.\dot{9}765$ to a common fraction. $\frac{1085}{1111}$.

§ 176. A mixed repetend is a complex fraction, having for its denominator some power of ten, and for its numerator a mixed number: the fractional part of the mixed number having for its denominator a series of nines.

Thus, .83 is $\frac{8\frac{3}{9}}{10}$; .416 is $\frac{41\frac{6}{9}}{100}$; .2083 is $\frac{208\frac{3}{9}}{1000}$.

To reduce a mixed repetend to a simple common fraction, we must first reduce the numerator to an improper fraction. This makes it necessary to multiply the integral part by 9 or by a series of nines; and this multiplication can be most

readily accomplished by § 60. Take the second of the above examples, for instance. Annexing one naught to 41, and subtracting 41 from the result, we have 369 as the product of the integral part by the denominator. To this product adding the numerator 6, we have 375 as the numerator of the improper fraction. Dividing $\frac{375}{9}$ by 100, we find $\frac{375}{900}$, which should then be reduced to its lowest terms.

$$\begin{array}{r} 410 \\ 41 \\ \hline 369 \end{array} \qquad \begin{array}{r} 369 \\ 6 \\ \hline 375 \\ \hline 9 \end{array} \qquad \frac{375}{9} \div 100 = \frac{375}{900}$$

This result could be more easily obtained by *subtracting the decimal part from the whole repetend for the numerator, and by taking for the denominator as many nines as there are repeating figures, followed by as many naughts as there are decimal figures.*

Thus, $.83 = \frac{75}{90} = \frac{5}{6}$; $.2083 = (2083 - 208) = \frac{1875}{9000} = \frac{5}{24}$.

Ex. 41. Reduce .123 to a common fraction. $\frac{37}{300}$.

42. Reduce .5075 to a common fraction.

43. Reduce .7797 to a common fraction. $\frac{386}{495}$.

44. Reduce .176 to a common fraction. $\frac{35}{198}$.

45. Reduce .4554 to a common fraction.

PROMISCUOUS PROBLEMS.

1. What is the sum of 247 millionths, 26 ten-thousandths, 163 hundred-thousandths, 3 thousandths, and 19 hundredths? Ans. .197477.

2. What is the difference between 19 units and 19 millionths? Ans. 18.999981.

3. What is the product of 273 thousandths and 117 ten-thousandths?
4. What is the quotient of 17 ten-thousandths by 16 hundredths? Ans. .10625.
5. What is the sum of the product of 5 tenths and 5 hundredths, and the quotient of 5 tenths by 5 hundredths? Ans. 10.025.
6. What is the difference between the sum of 6 hundredths and 6 units, and the product of 6 hundredths and 6 units?
7. What is the product of the sum of 12 thousandths and 34 hundredths, and their difference? Ans. .115456.
8. What is the quotient of the product of 506 thousandths and 78 hundredths by their sum? Ans. .306905+.
9. Add 27 hundredths, 538 thousandths, 64 ten-thousandths, and 9768 millionths.
10. Subtract the product of 39 hundredths and 54 thousandths from their sum. Rem. .42294.
11. Multiply the quotient of 36 hundredths by 45 ten-thousandths by their difference. Prod. 28.44.
12. Divide the sum of 497 thousandths and 608 ten-thousandths by their difference.
13. What number is that to which if 13 hundredths, 13 thousandths, 13 ten-thousandths, and 13 millionths be added, the sum will be 13 units? Ans. 12.855687.
14. What number is that from which if 11 hundredths, 12 thousandths, 13 ten-thousandths, and 14 hundred-thousandths be subtracted, the remainder will be 15 millionths? Ans. 123455.
15. What number is that by which if 79 thousandths be multiplied, the product will be 54115 billionths?

16. What number is that by which if 6375 millionths be divided, the quotient will be 5 thousandths?

Ans. 1.275.

17. The subtrahend is 25 ten-thousandths, the minuend is 2 tenths; what is the remainder? Ans. .1975.

18. The subtrahend is 25 thousandths, the remainder is 2 hundredths; what is the minuend?

19. The remainder is 13 millionths, the minuend is 13 thousandths; what is the subtrahend? Ans. .012987.

20. The multiplicand is 75 thousandths, the multiplier is 25 ten-thousandths; what is the product?

Ans. .0001875.

21. The multiplier is 18 thousandths, the product is 369 millionths; what is the multiplicand?

22. The product is 1482 ten-millionths, the multiplicand is 95 hundredths; what is the multiplier?

Ans. .000156.

23. The divisor is 19 hundredths, the quotient is 21 thousandths; what is the dividend? Ans. .00399.

24. The dividend is 65 and 12 hundredths, the divisor is 17 and 6 tenths; what is the quotient?

25. The quotient is 14 hundredths, the dividend is 322 thousandths; what is the divisor? Ans. 2.3.

26. What are the prime factors of 3500?

Ans. 2, 2, 5, 5, 5, and 7.

27. What are the prime factors of 756?

28. What different prime numbers will exactly divide 700?

Ans. 2, 5, and 7.

29. What different prime numbers will exactly divide 850?

Ans. 2, 5, and 17.

30. What is the least common multiple of 7, 8, 10, and 14?

31. What is the smallest number that may be exactly divided by either 9, 10, 12, or 15? Ans. 180.
32. What is the smallest number that may be exactly divided by either 24, 36, 48, or 72? Ans. 144.
33. What is the greatest common measure of 45, 54, and 108?
34. What is the largest number that will exactly divide either 75, 100, or 150? Ans. 25.
35. What is the largest number that will exactly divide either 96, 192, or 240? Ans. 48.
36. What is the sum of $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{9}{10}$?
37. What is the difference between $\frac{3}{7}$ and $\frac{3}{8}$? Ans. $\frac{3}{56}$.
38. What is the product of $\frac{7}{9}$ and $\frac{10}{11}$? Ans. $\frac{70}{99}$.
39. What is the quotient of $\frac{7}{10}$ divided by $\frac{9}{14}$?
40. What is the value of $\frac{7\frac{1}{2}}{15\frac{1}{4}}$? Ans. $\frac{30}{61}$.
41. What is the value of $\frac{2}{5}$ of $\frac{3}{7}$ of $7\frac{1}{4}$? Ans. $1\frac{17}{70}$.
42. What is the value of $.25+.025+.715+.225$?
43. What is the value of $.0237-.002375$? Ans. .021325.
44. What is the value of $.027\times.0027$? Ans. .0000729.
45. What is the value of $.0144\div3.6$?
46. What is the sum of $\frac{3}{10}$, $\frac{4}{12}$, $\frac{7}{15}$, and $\frac{11}{20}$? Ans. $1\frac{13}{20}$.
47. What is the difference between $\frac{9}{21}$ and $\frac{7}{30}$?
48. What is the sum of 216 thousandths, 37 hundredths, 15 ten-thousandths, and 10 units? Ans. 10.5875.
49. What is the difference between 206 ten-thousandths, and 27 millionths?
50. What is the value of $.211+3.07+29.6+.0735$? Ans. 32.9545.
51. What is the value of $.6501\times.736089$?
52. What is the value of $.4396\div9.3$?

CONCRETE NUMBERS.

§ 177. The relations of the concrete numbers in most common use are set forth in the following

TABLES.

I. United States Money.

		1 mill,	(m.)	=	$\frac{1}{10}$ of a cent;
10 mills	=	1 cent,	(ct.)	=	$\frac{1}{10}$ " " dime;
10 cents	=	1 dime,	(d.)	=	$\frac{1}{10}$ " " dollar;
10 dimes	=	1 dollar,	($)	=	$\frac{1}{10}$ " " eagle;
10 dollars	=	1 eagle,	(E.)		

m.		ct.		d.		$		E.
1	=	$\frac{1}{10}$	=	$\frac{1}{100}$	=	$\frac{1}{1000}$	=	$\frac{1}{10000}$
10	=	1	=	$\frac{1}{10}$	=	$\frac{1}{100}$	=	$\frac{1}{1000}$
100	=	10	=	1	=	$\frac{1}{10}$	=	$\frac{1}{100}$
1000	=	100	=	10	=	1	=	$\frac{1}{10}$
10000	=	1000	=	100	=	10	=	1.

The denominations *dime* and *eagle* are very little used in calculation. In stead of 14E. 5$, 7d, 5ct., we usually write $145.75.

II. English Currency; or, Sterling Money.

		1 farthing,	(qr.)	=	$\frac{1}{4}$ of a penny;
4 farthings	=	1 penny,	(d.)	=	$\frac{1}{12}$ " " shilling;
12 pence	=	1 shilling,	(s.)	=	$\frac{1}{20}$ " " pound;
20 shillings	=	1 pound,	(£).		

qr.		d.		s.		£
1	=	$\frac{1}{4}$	=	$\frac{1}{48}$	=	$\frac{1}{960}$
4	=	1	=	$\frac{1}{12}$	=	$\frac{1}{240}$
48	=	12	=	1	=	$\frac{1}{20}$
960	=	240	=	20	=	1

The pound sterling is represented by a gold coin, called a *sovereign*, valued at $4.84, U. S. currency.

Farthings are usually written as fractions of a penny.

III. French Currency.

1 centime, (cent.) = $\frac{1}{10}$ of a decime;
10 centimes = 1 decime, (dec.) = $\frac{1}{10}$ " " franc;
10 decimes = 1 franc, (fr.).

cent.		dec.		fr.
1	=	$\frac{1}{10}$	=	$\frac{1}{100}$
10	=	1	=	$\frac{1}{10}$
100	=	10	=	1

Accounts are kept in francs and centimes.

The franc is valued at 18ct. 6m., U. S. currency.

IV. Troy Weight.

USED FOR WEIGHING GOLD, SILVER, JEWELS, &c.

1 grain, (gr.) = $\frac{1}{24}$ of a pennyweight;
24 grains = 1 pennyweight, (dwt.) = $\frac{1}{20}$ of an ounce;
20 pennyweights = 1 ounce, (oz.) = $\frac{1}{12}$ of a pound;
12 ounces = 1 pound, (lb.)

gr.		dwt.		oz.		lb.
1	=	$\frac{1}{24}$	=	$\frac{1}{480}$	=	$\frac{1}{5760}$
24	=	1	=	$\frac{1}{20}$	=	$\frac{1}{240}$
480	=	20	=	1	=	$\frac{1}{12}$
5760	=	240	=	12	=	1

V. Apothecaries' Weight.

USED IN *MIXING* MEDICINES.

1 grain, (gr.) = $\frac{1}{20}$ of a scruple;
20 grains = 1 scruple, (sc.) or ℈ = $\frac{1}{3}$ " " dram;
3 scruples = 1 dram, (dr.) or ʒ = $\frac{1}{8}$ " " ounce;
8 drams = 1 ounce, (oz.) or ℥ = $\frac{1}{12}$ " " pound;
12 ounces = 1 pound, (lb.) or lb

gr.		sc.		dr.		oz.		lb.
1	=	$\frac{1}{20}$	=	$\frac{1}{60}$	=	$\frac{1}{480}$	=	$\frac{1}{5760}$
20	=	1	=	$\frac{1}{3}$	=	$\frac{1}{24}$	=	$\frac{1}{288}$
60	=	3	=	1	=	$\frac{1}{8}$	=	$\frac{1}{96}$
480	=	24	=	8	=	1	=	$\frac{1}{12}$
5760	=	288	=	96	=	12	=	1

The pound Apothecaries' is the same as the pound Troy.

VI. Avoirdupois Weight.

USED FOR WEIGHING ALL ARTICLES EXCEPT THOSE MENTIONED ABOVE.

1 dram, (dr.) = $\frac{1}{16}$ of an ounce;
16 drams = 1 ounce, (oz.) = $\frac{1}{16}$ of a pound;
16 ounces = 1 pound, (lb.) = $\frac{1}{25}$ of a quarter;
25 pounds = 1 quarter, (qr.) = $\frac{1}{4}$ of a hundredweight;
4 quarters = 1 hundredweight, (cwt.) = $\frac{1}{20}$ of a ton;
20 hundredweight = 1 ton, (T.).

dr.		oz.		lb.		qr.		cwt.		T.
1	=	$\frac{1}{16}$	=	$\frac{1}{256}$	=	$\frac{1}{6400}$	=	$\frac{1}{25600}$	=	$\frac{1}{512000}$
16	=	1	=	$\frac{1}{16}$	=	$\frac{1}{400}$	=	$\frac{1}{1600}$	=	$\frac{1}{32000}$
256	=	16	=	1	=	$\frac{1}{25}$	=	$\frac{1}{100}$	=	$\frac{1}{2000}$
6400	=	400	=	25	=	1	=	$\frac{1}{4}$	=	$\frac{1}{80}$
25600	=	1600	=	100	=	4	=	1	=	$\frac{1}{20}$
512000	=	32000	=	2000	=	80	=	20	=	1

144 pounds Avoirdupois = 175 pounds Troy or Apothecaries'.

1 lb. Avoir. = 7000 gr. Troy; 1 oz. Avoir. = 437.5 gr. Troy.

The following denominations also belong here:

28	pounds	=	1 long quarter;
112	"	=	1 long hundredweight;
2240	"	=	1 long ton;
14	"	=	1 stone;
21½	stone	=	1 pig;
8	pigs	=	1 fother.

50	pounds	of	salt	=	1 bushel.
56	"	"	corn	=	1 bushel.
60	"	"	wheat	=	1 bushel.
56	"	"	butter	=	1 firkin.
100	"	"	salt fish	=	1 quintal.
196	"	"	flour	=	1 barrel.
200	"	"	beef, pork, or fish	=	1 barrel.

VII. French Weights.

	1 milligramme	=	$\frac{1}{10}$ of a centigramme;
10 milligrammes	= 1 centigramme	=	$\frac{1}{10}$ " " decigramme;
10 centigrammes	= 1 decigramme	=	$\frac{1}{10}$ " " gramme;
10 decigrammes	= 1 gramme	=	$\frac{1}{10}$ " " decagramme;
10 grammes	= 1 decagramme	=	$\frac{1}{10}$ " " hectogramme;
10 decagrammes	= 1 hectogramme	=	$\frac{1}{10}$ " " kilogramme;
10 hectogrammes	= 1 kilogramme	=	$\frac{1}{100}$ " " quintal;
100 kilogrammes	= 1 quintal	=	$\frac{1}{10}$ " " millier;
10 quintals	= 1 millier, or 1 ton of sea water.		

1 gramme = 15.433 grains Troy.

VIII. Long Measure; or, Linear Measure.

USED IN MEASURING LINES, OR DISTANCES.

	1 inch,	(in.)	$= \frac{1}{12}$ of a foot;
12 inches	= 1 foot,	(ft.)	$= \frac{1}{3}$ " " yard;
3 feet	= 1 yard,	(yd.)	$= \frac{2}{11}$ " " rod;
$5\frac{1}{2}$ yards	= 1 rod,	(rd.)	$= \frac{1}{40}$ " " furlong;
40 rods	= 1 furlong,	(fur.)	$= \frac{1}{8}$ " " mile;
8 furlongs	= 1 mile,	(mi.)	

in.	ft.	yd.	rd.	fur.	mi.
1 =	$\frac{1}{12}$ =	$\frac{1}{36}$ =	$\frac{1}{198}$ =	$\frac{1}{7920}$ =	$\frac{1}{63360}$
12 =	1 =	$\frac{1}{3}$ =	$\frac{2}{33}$ =	$\frac{1}{660}$ =	$\frac{1}{5280}$
36 =	3 =	1 =	$\frac{2}{11}$ =	$\frac{1}{220}$ =	$\frac{1}{1760}$
198 =	$16\frac{1}{2}$ =	$5\frac{1}{2}$ =	1 =	$\frac{1}{40}$ =	$\frac{1}{320}$
7920 =	660 =	220 =	40 =	1 =	$\frac{1}{8}$
63360 =	5280 =	1760 =	320 =	8 =	1

The following denominations are sometimes used:

3 barley corns	= 1 inch;
6 points	= 1 line;
12 lines	= 1 inch;
4 inches	= 1 hand;
9 "	= 1 span;
18 "	= 1 cubit;
21.9 "	= 1 sacred cubit;
3 feet	= 1 pace;
6 feet	= 1 fathom;
$69\frac{1}{6}$ miles	= 1 degree of latitude.

IX. Surveyor's Long Measure.

7.92 inches	= 1 link,	(l.)	$= \frac{1}{25}$ of a rod;
25 links	= 1 rod,	(rd.)	$= \frac{1}{4}$ " " chain;
4 rods	= 1 chain,	(ch.)	$= \frac{1}{10}$ " " furlong;
10 chains	= 1 furlong,	(fur.)	$= \frac{1}{8}$ " " mile;
8 furlongs	= 1 mile,	(mi.)	

in.	l.	rd.	ch.	fur.	mi.
7.92 =	1 =	$\frac{1}{25}$ =	$\frac{1}{100}$ =	$\frac{1}{1000}$ =	$\frac{1}{8000}$
198 =	25 =	1 =	$\frac{1}{4}$ =	$\frac{1}{40}$ =	$\frac{1}{320}$
792 =	100 =	4 =	1 =	$\frac{1}{10}$ =	$\frac{1}{80}$
7920 =	1000 =	40 =	10 =	1 =	$\frac{1}{8}$
63360 =	8000 =	320 =	80 =	8 =	1

X. Square Measure.

USED FOR MEASURING SURFACES OF LAND, PAINTING, PLASTERING, PAVING, &c.

1 square inch, (sq.in.) = $\frac{1}{144}$ of a square foot;
144 square inches = 1 " foot, (sq.ft.) = $\frac{1}{9}$ " " " yard;
9 " feet = 1 " yard, (sq.yd.) = $\frac{4}{121}$ " " perch;
$30\frac{1}{4}$ " yards = 1 perch, (P.) = $\frac{1}{40}$ " " rood;
40 perches = 1 rood, (R.) = $\frac{1}{4}$ " " acre;
4 roods = 1 acre, (A.) = $\frac{1}{640}$ " " square mile
640 acres = 1 square mile, (sq.mi.).

sq.in.	sq.ft.	sq.yd.	P.	R.	A.	sq. mi.
1 =	$\frac{1}{144}$ =	$\frac{1}{1296}$ =	$\frac{1}{39204}$ =	$\frac{1}{1568160}$ =	$\frac{1}{6272640}$ =	$\frac{1}{4014489600}$
144 =	1 =	$\frac{1}{9}$ =	$\frac{4}{1089}$ =	$\frac{1}{10890}$ =	$\frac{1}{43560}$ =	$\frac{1}{27878400}$
1296 =	9 =	1 =	$\frac{4}{121}$ =	$\frac{1}{1210}$ =	$\frac{1}{4840}$ =	$\frac{1}{3097600}$
39204 =	$272\frac{1}{4}$ =	$30\frac{1}{4}$ =	1 =	$\frac{1}{40}$ =	$\frac{1}{160}$ =	$\frac{1}{102400}$
1568160 =	10890 =	1210 =	40 =	1 =	$\frac{1}{4}$ =	$\frac{1}{2560}$
6272640 =	43560 =	4840 =	160 =	4 =	1 =	$\frac{1}{640}$
4014489600 =	27878400 =	3097600 =	102400 =	2560 =	640 =	1

XI. Cubic Measure.

USED FOR MEASURING THE CONTENTS OF SOLIDS.

1 cubic inch, (cu.in.) = $\frac{1}{1728}$ of a cubic ft.;
1728 cubic inches = 1 " foot, (cu.ft.) = $\frac{1}{27}$ " " " yd.;
27 " feet = 1 " yard, (cu.yd.).

cu. in.		cu. ft.		cu. yd.
1	=	$\frac{1}{1728}$	=	$\frac{1}{46656}$
1728	=	1	=	$\frac{1}{27}$
46656	=	27	=	1

Also, 40 cubic feet of round timber = 1 ton;
50 " " " hewn timber = 1 ton;
42 " " " shipping = 1 ton;
16 " " " wood = 1 cord foot;
128 " " " wood = 1 cord.

Also, 231 cubic inches = 1 gallon Liquid or Wine Measure;
$268\frac{4}{5}$ " " = 1 " Dry Measure;
282 " " = 1 " Ale Measure, (out of use.)
$537\frac{3}{5}$ " " = 1 peck;
$2150\frac{2}{5}$ " " = 1 bushel.

XII. Liquid Measure; or, Wine Measure.

USED IN MEASURING LIQUIDS; AS, MOLASSES, SPIRITS. WINE, WATER, &c.

1 gill, (gi.) = $\frac{1}{4}$ of a pint;
4 gills = 1 pint, (pt.) = $\frac{1}{2}$ " " quart;
2 pints = 1 quart, (qt.) = $\frac{1}{4}$ " " gallon;
4 quarts = 1 gallon, (gal.) = $\frac{2}{63}$ " " barrel;
$31\frac{1}{2}$ gallons = 1 barrel, (bbl.) = $\frac{1}{2}$ " " hogshead;
2 barrels = 1 hogshead (hhd.) = $\frac{1}{2}$ " " pipe;
2 hogsheads = 1 pipe, (pi.) = $\frac{1}{2}$ " " tun;
2 pipes = 1 tun, (tun).

gi.	pt.	qt.	gal.	bbl.	hhd.	pi.	tun.
1 =	$\frac{1}{4}$ =	$\frac{1}{8}$ =	$\frac{1}{32}$ =	$\frac{1}{1008}$ =	$\frac{1}{2016}$ =	$\frac{1}{4032}$ =	$\frac{1}{8064}$
4 =	1 =	$\frac{1}{2}$ =	$\frac{1}{8}$ =	$\frac{1}{252}$ =	$\frac{1}{504}$ =	$\frac{1}{1008}$ =	$\frac{1}{2016}$
8 =	2 =	1 =	$\frac{1}{4}$ =	$\frac{1}{126}$ =	$\frac{1}{252}$ =	$\frac{1}{504}$ =	$\frac{1}{1008}$
32 =	8 =	4 =	1 =	$\frac{2}{63}$ =	$\frac{1}{63}$ =	$\frac{1}{126}$ =	$\frac{1}{252}$
1008 =	252 =	126 =	$31\frac{1}{2}$ =	1 =	$\frac{1}{2}$ =	$\frac{1}{4}$ =	$\frac{1}{8}$
2016 =	504 =	252 =	63 =	2 =	1 =	$\frac{1}{2}$ =	$\frac{1}{4}$
4032 =	1008 =	504 =	126 =	4 =	2 =	1 =	$\frac{1}{2}$
8064 =	2016 =	1008 =	252 =	8 =	4 =	2 =	1

Also, 42 gallons = 1 tierce;
2 tierces = 1 puncheon.

XIII. Ale Measure.

FORMERLY USED FOR MEASURING MALT LIQUORS AND MILK, WHICH NOW, HOWEVER, ARE GENERALLY MEASURED BY LIQUID MEASURE.

1 pint, (pt.) = $\frac{1}{2}$ of a quart;
2 pints = 1 quart, (qt.) = $\frac{1}{4}$ " " gallon;
4 quarts = 1 gallon, (gal.) = $\frac{1}{36}$ " " barrel;
36 gallons = 1 barrel, (bbl.) = $\frac{2}{3}$ " " hogshead;
1½ barrels = 1 hogshead, (hhd.)

Also, 9 gallons = 1 firkin,
2 firkins = 1 kilderkin.

XIV. Dry Measure.

USED FOR MEASURING GRAIN, FRUITS, VEGETABLES, SALT, &c.

1 pint, (pt.) = $\frac{1}{2}$ of a quart;
2 pints = 1 quart, (qt.) = $\frac{1}{4}$ " " gallon;
4 quarts = 1 gallon, (gal.) = $\frac{1}{2}$ " " peck;
2 gallons = 1 peck, (pk.) = $\frac{1}{4}$ " " bushel;
4 pecks = 1 bushel, (bu.)

pt.		qt.		gal.		pk.		bu.
1	=	$\frac{1}{2}$	=	$\frac{1}{8}$	=	$\frac{1}{16}$	=	$\frac{1}{64}$
2	=	1	=	$\frac{1}{4}$	=	$\frac{1}{8}$	=	$\frac{1}{32}$
8	=	4	=	1	=	$\frac{1}{2}$	=	$\frac{1}{8}$
16	=	8	=	2	=	1	=	$\frac{1}{4}$
64	=	32	=	8	=	4	=	1

Also, 5 bushels = 1 barrel, of corn;
8 bushels = 1 quarter;
36 bushels = 1 chaldron.

In the Confederate States, corn is usually bought and sold by the barrel. A barrel of corn should contain 280 pounds.

XV. Time.

	1 second,	(sec.)	$\frac{1}{60}$	of a minute;
60 seconds	= 1 minute,	(min.)	$\frac{1}{60}$	" an hour;
60 minutes	= 1 hour,	(hr.)	$\frac{1}{24}$	" a day;
24 hours	= 1 day,	(da.)	$\frac{4}{1461}$	" " year;
$365\frac{1}{4}$ days	= 1 year,	(yr.)	$\frac{1}{10}$	" " decade;
10 years	= 1 decade,	(dec.)	$\frac{1}{10}$	" " century;
10 decades	= 1 century,	(cent.)		

sec. min. hr. da. yr. dec. cent.

$$1=\frac{1}{60}=\frac{1}{3600}=\frac{1}{86400}=\frac{1}{31557600}=\frac{1}{315576000}=\frac{1}{3155760000}$$

$$60=1=\frac{1}{60}=\frac{1}{1440}=\frac{1}{525960}=\frac{1}{5259600}=\frac{1}{52596000}$$

$$3600=60=1=\frac{1}{24}=\frac{1}{8766}=\frac{1}{87660}=\frac{1}{876600}$$

$$86400=1440=24=1=\frac{4}{1461}=\frac{2}{7305}=\frac{1}{36525}$$

$$31557600=525960=8766=365\tfrac{1}{4}=1=\frac{1}{10}=\frac{1}{100}$$

$$315576000=5259660=87660=3652\tfrac{1}{2}=10=1=\frac{1}{10}$$

$$3155760000=52596000=876600=36525=100=10=1$$

Also, 7 days = 1 week, (wk.);
30 or 31 days = 1 month, (mo.);
12 months = 1 year.

According to the table, $365\frac{1}{4}$ days make a year. To obviate the difficulty arising from the fraction, we reckon three years of 365 days each, and one of 366 days. This long year is called *leap year*. The leap years are those whose numbers are exactly divisible by 4; except that the centennial years are not leap years unless their numbers are exactly divisible by 400. Thus, 1860 and 1848 were leap years; but 1900 will not be leap year, because it is not divisible by 400.

The year is also divided into four seasons; Spring, Summer, Autumn, and Winter. These consist of the following months:

Spring,	3. March,	(Mar.)	has	31	days.
	4. April,	(Apr.)	"	30	"
	5. May,	(May)	"	31	"
Summer,	6. June,	(Jun.)	"	30	"
	7. July,	(Jul.)	"	31	"
	8. August,	(Aug.)	"	31	"
Autumn,	9. September,	(Sept.)	"	30	"
	10. October,	(Oct.)	"	31	"
	11. November,	(Nov.)	"	30	"
Winter,	12. December,	(Dec.)	"	31	"
	1. January,	(Jan.)	"	31	"
	2. February,	(Feb.)	"	28	" leap year, 29.

In most business transactions 30 days are considered a month.

XVI. Circular Measure.

USED IN SURVEYING, GEOGRAPHY, AND ASTRONOMY.

1 second, (″ or sec.) $=\frac{1}{60}$ of a minute;
60 seconds = 1 minute, (′ or min.) $=\frac{1}{60}$ " " degree;
60 minutes = 1 degree, (° or deg.) $=\frac{1}{30}$ " " sign;
30 degrees = 1 sign, (S.) $=\frac{1}{12}$ " " circumference;
12 signs = 1 circumference, (C.)

″		′		°		S.		C.
1	=	$\frac{1}{60}$	=	$\frac{1}{3600}$	=	$\frac{1}{108000}$	=	$\frac{1}{1296000}$
60	=	1	=	$\frac{1}{60}$	=	$\frac{1}{1800}$	=	$\frac{1}{21600}$
3600	=	60	=	1	=	$\frac{1}{30}$	=	$\frac{1}{360}$
108000	=	1800	=	30	=	1	=	$\frac{1}{12}$
1296000	=	21600	=	360	=	12	=	1

Also, 60 degrees = 1 sextant $=\frac{1}{6}$ of a circumference;
And, 90 " = 1 quadrant $=\frac{1}{4}$ " " "

XVII. Paper.

	1 sheet,	(sh.)	= $\frac{1}{24}$	of a quire;
24 sheets	= 1 quire,	(qr.)	= $\frac{1}{20}$	" " ream;
20 quires	= 1 ream,	(rm.)	= $\frac{1}{2}$	" " bundle;
2 reams	= 1 bundle,	(bdle.)	= $\frac{1}{5}$	" " bale;
5 bundles	= 1 bale.			

XVIII. Duodecimals.

	1 unit,		= $\frac{1}{12}$	of a dozen;
12 units	= 1 dozen,	(doz.)	= $\frac{1}{12}$	" " gross;
12 dozen	= 1 gross,	(gr.)	= $\frac{1}{12}$	" " great gross;
12 gross	= 1 great gross.			

Also, 20 units = 1 score.

OPERATIONS ON CONCRETE NUMBERS.

The numerical processes are the same for concrete numbers as for abstract. In this place, therefore, we are to discuss only the denominations of the several results.

ADDITION OF CONCRETE NUMBERS.

§ 178. Dissimilar numbers can not be added together. Thus, 3 dollars and 5 cents make neither 8 dollars nor 8 cents.

§ 179. The *sum* of several similar numbers is similar to the numbers added. Thus, 3 dollars and 5 dollars make 8 *dollars;* 3 cents and 5 cents make 8 *cents.*

Ex. 1. Add $1075, $2157, $3779, and $4209.

2. Add £47, £53, £29, and £63. Sum, £192.
3. Add 27fr., 36fr., 297fr., and 365fr.
4. Add 29lb., 37lb., 49lb., and 58lb. Sum, 173lb.
5. Add 45sc., 28sc., 143sc., and 287sc. Sum, 503sc.
6. Add 100cwt., 205cwt., 177cwt., and 329cwt.
7. Add 2479 grammes, 147 grammes, and 986 grammes. Sum, 3612gram.
8. Add 276yd., 299yd., 469yd., and 357yd. Sum, 1401yd.
9. Add 79mi., 227mi., 37mi., and 475mi.
10. Add 306cu. ft., 279cu. ft., and 520cu. ft. Sum, 1105cu. ft.
11. Add 575A., 209A., 105A., and 258A. Sum, 1147A.
12. Add 27gal., 72gal., 298gal., and 143gal.
13. Add 15bbl., 28bbl., 19bbl., 247bbl., and 86bbl. Sum, 395bbl.
14. Add 47bu., 475bu., 407bu., and 4750bu. Sum, 5679bu.
15. Add 27da., 38da., 52da., and 93da.
16. Add 12°, 26°, 37°, and 45°. Sum, 120°.
17. Add 10rm., 14rm., 7rm., and 22rm. Sum, 53rm.
18. Add 6doz., 27doz., 14doz., and 97doz.
19. Add $12\frac{1}{2}$lb., $33\frac{1}{3}$lb., $37\frac{1}{2}$lb., and $83\frac{1}{3}$lb. Sum, $166\frac{2}{3}$lb.
20. Add $3\frac{2}{3}$mi., $16\frac{2}{3}$mi., $18\frac{3}{4}$mi., $62\frac{1}{2}$mi., and $42\frac{1}{8}$mi. Sum, $143\frac{17}{24}$mi.
21. Add $19\frac{1}{5}$qt., $20\frac{1}{2}$qt., $7\frac{3}{4}$qt., and $28\frac{2}{3}$qt.
22. Add 3.25hr., 6.5hr., .275hr., and 700.075hr. Sum, 710.1hr.
23. Add 47.5pt., 57.75pt., .375pt., and .0625pt. Sum, 105.6875pt.
24. Add 9.73pk., 10.01pk., 17.75pk., and .1775pk.
25. Add 10.11pk., 7.369pk., and 1.002pk.

SUBTRACTION OF CONCRETE NUMBERS.

§ 180. Subtraction can not be performed upon dissimilar numbers. Thus, 3 cents from 5 dollars leaves neither 2 cents nor 2 dollars.

§ 181. The *difference* of two similar numbers is similar to those numbers. Thus, 3 dollars from 5 dollars leaves 2 *dollars ;* 3 cents from 5 cents leaves 2 *cents.*

Ex. 1. From £245 take £196. Rem. £49.
2. From 25cwt. take 6cwt. Rem. 19cwt.
3. From 793rd. take 546rd.
4. From 17246sq. ft. take 8472sq. ft. Rem. 8774sq. ft.
5. From 635cu. yd. take 473cu. yd. Rem. 162cu. yd.
6. From 47 decigrammes take 29 decigrammes.
7. From 479hhd. take 398hhd. Rem. 81hhd.
8. From 272pt. take 199pt. Rem. 73pt.
9. From 365da. take 175da.
10. From 360° take 275° Rem. 85°.
11. From $27\frac{3}{4}$s. take $19\frac{1}{2}$s. Rem. $8\frac{1}{4}$s.
12. From \$75$\frac{1}{2}$ take \$59$\frac{1}{4}$.
13. From \$19.75 take \$ .99. Rem. \$18.76.
14. From $270\frac{1}{4}$fr. take $197\frac{3}{4}$fr. Rem. $72\frac{1}{2}$fr.
15. From 77in. take 17.75in.
16. From 3706sq. yd. take $897\frac{1}{8}$sq. yd. Rem. $2808\frac{7}{8}$sq. yd.
17. From $\frac{1}{2}$ of 246cu. ft. take $\frac{1}{4}$ of 317cu. ft.
18. From $525\frac{1}{8}$qt. take $252\frac{7}{8}$qt.
19. From 27bu. take 17.25bu. Rem. 9.75bu.
20. From $83\frac{1}{3}$cu. in. take $31\frac{1}{4}$cu. in. Rem. $2\frac{1}{12}$cu. in.
21. From 725dwt. take 339.17dwt.
22. From .2468d. take .08642d. Rem. .16038d.
23. From .175lb. take .017lb. Rem. .158lb.

MULTIPLICATION OF CONCRETE NUMBERS.

§ 182. Every *multiplier* must be an abstract number.—Thus, if we wish to find the cost of 3 yards at 25 cents a yard, it is evidently absurd to say, "3 yards times 25 cents," or, "25 cents multiplied by 3 yards." We multiply 25 cents by 3, because 3 yards cost 3 times the price of 1 yard, that is, 3 times 25 cents.

§ 183. The *product* is always similar to the *multiplicand.* Thus, 3 times 25 *cents* are evidently 75 *cents;* 6×7 abstract units$=$42 abstract units; $4 \times \$10 = \$40$; 5×6-yards $=$30 yards.

Ex. 1. Multiply \$3179 by 27.	Prod. \$85833.
2. Multiply 2764bu. by 46.	Prod. 127144bu.
3. Multiply 365da. by 19.	
4. Multiply 347oz. by 83.	Prod. 28801oz.
5. Multiply 2047cwt. by 109.	Prod. 223123cwt.
6. Multiply 347fr. by 201.	
7. Multiply 467A. by 5297.	Prod. 2473699A.
8. Multiply 6386pi. by 578.	Prod. 3691108pi.
9. Multiply 7475pk. by 689.	
10. Multiply £69 by 4234.	Prod. £292146.
11. Multiply 224 by 4759.	Prod. 1066016.
12. Multiply 8564wk. by 790.	
13. Multiply 9563 by 801.	Prod. 7659963.
14. Multiply 10742doz. by 912.	Prod. 9769704doz.
15. Multiply 20s. by 16750.	
16. Multiply $5\frac{1}{2}$yd. by 746.	Prod. 4103yd.
17. Multiply 16.5ft. by 165.	Prod. 2722.5ft.
18. Multiply 30.25sq. yd. by 3.025.	
19. Multiply 7.92in. by 198.	Prod. 1568.16in.

20. Multiply 31½gal. by 1008. Prod. 31752gal.
21. Multiply 365¼da. by 365¼.
22. Multiply $29.75 by 29.75. Prod. $885.0625.
23. Multiply $100.375 by 37.5. Prod. $3764.0625.
24. Multiply 279.5 by 27.95.

DIVISION OF CONCRETE NUMBERS.

§184. Division is the reverse of multiplication. In multiplication, the two factors are given, to find the product; in division, the product and one of its factors are given, to find the other factor. The dividend corresponds to the product; the divisor may correspond to either the multiplicand or the multiplier, and the quotient corresponds to the other.

Thus, $6 \times 25\text{gal.} = 150\text{gal.}$

Conversely, $150\text{gal.} \div 6 = 25\text{gal.}$

Or, $150\text{gal.} \div 25\text{gal.} = 6.$

§185. Either the divisor or the quotient must be abstract, and the other must be similar to the dividend.

In other words, if the dividend and the divisor are similar, the quotient is abstract: if the divisor is abstract, the quotient is similar to the dividend.

The remainder is always similar to the dividend. (§46).

Ex. 1. Dividend=45ct., divisor=3. Quot. 15ct.
2. Dividend=$750, divisor=$25. Quot. 30.
3. Dividend=1000bu., divisor=40.
4. Dividend=245lb., divisor=5. Quot. 49lb.
5. Dividend=3003, divisor=11. Quot. 273.
6. Dividend=1728cu. in., divisor=48cu. in.
7. Dividend=7007yd., divisor=13yd. Quot. 539.

8. Divisor=17mi., dividend=289mi. Quot. 17.
9. Divisor=25, dividend=1175gi.
10. Divisor=27, dividend=297hr. Quot. 11hr.
11. Divisor=109, dividend=2398qt. Quot. 22qt.
12. Divisor=245cu. ft., dividend=5880cu. ft.
13. Divide 642780 dozen by 36 dozen. Quot. 17855.
14. Divide 79008oz. by 96. Quot. 823oz.
15. Divide 847665qr. by 345qr.
16. Divide 3475cwt. by 296. Quot. 11.739+cwt.
17. Divide 1001s. by 27s. Quot. 37.074.
18. Divide $35\frac{1}{2}$ft. by $17\frac{3}{4}$ft.
19. Divide 372.25sq. yd. by 250sq. yd. Quot. 1.489.
20. Divide $\frac{1}{2}$A. by $13\frac{1}{2}$. Quot. .037A.
21. Divide $243\frac{3}{4}$lb. by $19\frac{4}{5}$.
22. Divide 799.6T. by 87.5T. Quot. 9.104+.
23. Divide 34.15 grammes by 19.25 grammes.
24. Divide 177pt. by 771. Quot. .2295+pt.

REDUCTION OF CONCRETE NUMBERS.

§ 186. A compound number may be reduced to a simple one, or a simple concrete number to a compound one by the application of the following rule according to the circumstances of the case.

§ 187. RULE.—*Find from the proper table the value of one of the given units in terms of the required denomination; and multiply this value by the number of the given units.*

Ex. 1. Reduce 6 gallons to pints.

§ 188. MODEL. 6gal.=6×4qt.=24qt.

24qt.=24×2pt.=48pt.

Hence, 6gal.=48pt.

EXPLANATION.—Since 4qt.=1gal., 6gal., that is, 6 times 1gal.=6 times 4qt. And since 2pt.=1qt., 24qt., or 24 times 1qt.=24 times 2pt.

Observe that in each instance the product is similar to the multiplicand. (§ 183.)

Otherwise, 6gal.=6×8pt.=48pt.

Ex. 2. Reduce 6gal. 3qt. 1pt. to pt.

§ 189. MODEL. 6gal.=6×4qt.=24qt.

24+3=27 27qt.=27×2pt.=54pt.

54+1=55 Hence, 6gal. 3qt. 1pt.=55pt.

EXPLANATION.—After reducing the 6gal. to qt., the given 3qt. may be added to the result. (§ 179.) And after reducing the 27qt. to pt., the given 1pt. may be added to this result.

Otherwise, 6gal.=6×4qt.=24qt.=24×2pt.=48pt.

3 " = 3×2 " = 6 "

1 " = 1 "

Hence, 6gal. 3qt. 1pt.=55pt.

Otherwise, 6gal.=6×8pt.=48pt.

3qt. =3×2 " = 6 "

1pt. = 1 "

Hence, 6gal. 3qt. 1pt.=55pt.

Evidently the final result is not affected by the order in which the several reductions are performed.

Ex. 3. Reduce $\frac{1}{10}$lb. to oz., dwt., &c.

§ 190. MODEL. $\frac{1}{10}$lb.=$\frac{1}{10}$ of 12oz.=$1\frac{1}{5}$oz.

$\frac{1}{5}$oz.=$\frac{1}{4}$ of 20dwt.=4dwt.

Hence, $\frac{1}{10}$lb.=1oz. 4dwt.

EXPLANATION.—This example differs from the first only in the fact that here each multiplier is a fraction.

Ex. 4. Reduce 3795P. to A., R., &c.

§ 191. MODEL. $3795P. = 3795 \times \frac{1}{40} R. = \frac{3795}{40} R. = 94R.\ 35P.$

$94R. = 94 \times \frac{1}{4} A. = \frac{94}{4} A. = 23A.\ 2R.$

Hence, $3795P. = 23A.\ 2R.\ 35P.$

EXPLANATION.—This example differs from the preceding only in the fact that here each multiplicand is a fraction.

Ex. 5. Reduce 6gal. 3qt. 1pt. 3gi. to hhd.

§ 192. MODEL. $3gi. = 3 \times \frac{1}{4} pt. = \frac{3}{4} pt.$

$1\frac{3}{4} pt. = 1\frac{3}{4} \times \frac{1}{2} qt. = \frac{7}{8} qt.$

$3\frac{7}{8} qt. = 3\frac{7}{8} \times \frac{1}{4} gal. = \frac{31}{32} gal.$

$6\frac{31}{32} gal. = 6\frac{31}{32} \times \frac{1}{63} hhd. = \frac{223}{2016} hhd.$

EXPLANATION.—Here both factors are fractional.

Ex. 6. Reduce 30bu. 1pk. 3qt. 1pt. to pk.

§ 193. MODEL. $30bu. = 30 \times 4pk. = 120pk.$

$1pk. = 1$ "

$3qt. = 3 \times \frac{1}{8} pk. = .375$

$1pt. = \frac{1}{16} pk. = .0625$

Hence, $30bu.\ 1pk.\ 3qt.\ 1pt. = 121.4375pk.$

EXPLANATION.—This example is but a combination of two of the preceding ones, and seems to require no additional explanation.

Ex. 7. In $14, how many mills? Ans. 14000m.

8. In £15, how many pence? Ans. 3600d.

9. In 19fr., how many centimes?

10. In 22lb., Troy, how many dwt.? Ans. 5280dwt.

11. In 25lb., Apothecaries', how many scruples?

Ans. 7200sc.

12. In 26lb., Avoirdupois, how many drams?

13. In 31 hectogrammes, how many decigrammes?

Ans. 31000dec.

14. In 45 miles, how many feet? Ans. 237600ft.

15. In 49fur., how many chains?
16. In 50A., how many square yards? Ans. 242000sq. yd.
17. In 55cu. yd., how many cu. in.? Ans. 2566080cu. in.
18. In 72gal., how many gi.?
19. In 64bu., how many qt.? Ans. 2048qt.
20. In 20da., how many sec.? Ans. 1728000sec.
21. In 29°, how many seconds?
22. In 17rm., how many sheets? Ans. 8160sh.
23. In 5gr. gross, how many doz.? Ans. 720doz.
24. In £4, 3s. 2d., how many qr.?
25. In 5fr. 7dec. 8cent., how many centimes?
Ans. 578cent.
26. In 6lb. 5oz. 3dwt., how many gr.? Ans. 37032gr.
27. In 3lb. 6oz. 5dr. 2sc., how many sc.?
28. In 28T. 10cwt. 3qr., how many lb.? Ans. 57075lb.
29. In 1 millier, 5 quintals, how many grammes?
Ans. 1500000gr.
30. In 2rd. 3yd. 2ft., how many in.?
31. In 10 chains, 1 rod, how many links? Ans. 1025lk.
32. In 4sq. yd. 6sq. ft., how many sq. in.? Ans. 6048sq.in.
33. In 10cu. ft. 400cu. in., how many cu. in.?
34. In 2 tuns, 1pi. 1hhd., how many gal.? Ans. 693gal.
35. In 5bu. 2pk. 1gal. 3qt., how many pt.? Ans. 366pt.
36. In 1cent. 6dec. 5yr., how many yr.?
37. In 2S. 25° 45′, how many seconds? Ans. 308700″.
38. In 2rm. 10qr. 12sh., how many sh.? Ans. 1212sh.
39. In 3gr. 4doz., how many units?
40. Reduce $\frac{1}{3}$£ to s. and d. Val. 6s. 8d.
41. Reduce $\frac{2}{3}$fr. to decimes. Val. 6$\frac{2}{3}$dec.
42. Reduce $\frac{2}{5}$lb. Troy, to oz., dwt., &c.
43. Reduce $\frac{1}{5}$ lb. Apothecaries', to oz., &c.
Val. 2oz. 3dr. 12gr.

44. Reduce $\frac{1}{6}$T. to cwt., qr., &c. Val. 3cwt. 1qr. 8$\frac{1}{2}$lb.
45. Reduce $\frac{1}{3}$mi. to fur., &c.
46. Reduce $\frac{1}{7}$A. to R., P., &c. Val. 22$\frac{5}{7}$P.
47. Reduce $\frac{1}{6}$cu. yd. to cu. ft., &c. Val. 4cu. ft. 864cu. in.
48. Reduce $\frac{3}{7}$gal. to qt., pt., and gi.
49. Reduce $\frac{7}{10}$bu. to pk., gal., &c. Val. 2pk. 1gal. 2$\frac{2}{5}$qt.
50. Reduce $\frac{1}{5}$da. to hr., min., &c. Val. 4hr. 48min.
51. Reduce .375° to min. and sec.
52. Reduce .13rm. to qr. and sh. Val. 2qr. 14.4sh.
53. Reduce .15gr. to doz. and units. Val. 1doz. 9.6un.
54. Reduce 975qr. to £.
55. Reduce 5000gr. to lb. Val. 10oz. 8dwt. 8gr.
56. Reduce 300sc. to lb. Apothecaries'. Val. 11b. 4dr.
57. Reduce 600000dr. to T.
58. Reduce 11000in. to mi. Val. 1fur. 15rd. 3yd. 2in.
59. Reduce 60000lk. to mi. Val. 7mi. 40ch.
60. Reduce 4000000sq. yd. to sq. mi.
61. Reduce 60000cu. in. to cu. yd.
Val. 1cu. yd. 7cu. ft. 1248cu. in.
62. Reduce 10000gi. to tuns. Val. 1 tun, 60gal. 2qt.
63. Reduce 1000pt. to hhd., Ale Measure.
64. Reduce 250pt. to bu. Val. 3bu. 3pk. 5qt.
65. Reduce 600000min. to yr. Val. 1yr. 51da. 16hr.
66. Reduce 2000000″ to circumferences.
67. Reduce 27000sh. to rm. Val. 56rm. 5qr.
68. Reduce 19000 units to gr. gross.
Val. 10gr. gr. 11gr. 11doz. 4 units.
69. Reduce 11d. 3qr. to £.
70. Reduce 9oz. 9dwt. 9gr. to lb. Val. .789+lb.
71. Reduce 6dr. 2sc. 15gr. to lb. Val. .07204+lb.
72. Reduce 1qr. 15lb. to T.

73. Reduce 20rd. 5yd. to fur. Val. .5227fur.
74. Reduce 2rd. 20lk. to ch. Val. .7ch.
75. Reduce 1R. 10P. to A.
76. Reduce 1cu. ft. 10cu. in. to cu. yd.
Val. .03725+cu. yd.
77. Reduce 1pt. 1gi. to gal. Val. .15625gal.
78. Reduce 1hhd. 1bbl. to tuns.
79. Reduce 3pk. 1gal. 3qt. to bu. Val. .96875bu.
80. Reduce 10hr. 15min. 30sec. to da. Val. .4274+da.
81. Reduce 1° 10′ 30″ to S.
82. Reduce 2qr. 12sh. to rm. Val. .125rm.
83. Reduce 1gr. 10doz. 10 units, to gr. gross.
Val. .1585648igr. gr.
84. Reduce £2, 10s. 6d. 3qr. to s.
85. Reduce 10lb. 9oz. 9dwt. 9gr. to oz. Val. 129.46875oz.
86. Reduce 3lb. 5oz. 5dr. 1sc. 10gr. to dr. Val. 333.5dr.
87. Reduce 1T. 10cwt. 1qr. 20lb. to cwt.
88. Reduce 1mi. 7fur. 20rd. 3yd. to rd. Val. 620.54rd.
89. Reduce 3ch. 2rd. 10lk. to rd. Val. 14.4rd.
90. Reduce 10A. 3R. 20P. to R.
91. Reduce 2cu. yd. 6cu. ft. 75cu. in. to cu. ft.
Val. 60.0434+cu. ft.
92. Reduce 10gal. 1qt. 1pt. 3gi. to pt. Val. 83.75pt.
93. Reduce 2bu. 1pk. 3qt. to pk.
94. Reduce 1da. 1hr. 1min. 1sec. to min.
Val. 1501.016min.
95. Reduce 1° 10′ 30″ to minutes. Val. 70½′.
96. Reduce 2rm. 3qr. 5sh. to qr.
97. Reduce 1T. 1cwt. 1qr. 1lb. 1oz. to lb.
Val. 2126.0625lb.
98. Reduce 1sq. yd. 1sq. ft. 1sq. in. to sq. ft.
Val. 10.00694sq. ft.

PROMISCUOUS PROBLEMS.

1. Bought a dress for $12, a cloak for $15, a bonnet for $7, and a pair of gloves for $1: what did they all cost?

$12
15
7
1
$35

§ 194. MODEL.—The whole cost is the sum of the several prices: hence, add $12, $15, $7, and $1. (§ 179). The sum is $35: hence, they all cost $35.

2. A owns $10475 in real estate, $3850 in slaves, $4095 in good notes, and $1415 in cash; what is the value of his whole estate? A. $19835.

3. Three men form a partnership: A invests $2445; B, $2890; C, $1950: what is the whole investment?

4. A miller bought from one man 147 bushels of wheat, from an other 98 bushels, and from a third 273 bushels; how much wheat did he buy from the three?

5. A farmer raised on one farm 415 bushels of wheat, 548 bushels of corn, 327 hundred weight of hay; on the other, 293 bushels of wheat, 487 bushels of corn, 286 hundred weight of hay: how much did he raise on both farms?

A. 708bu. wheat, 1035bu. corn, 613cwt. hay.

6. Bought a farm for $2875, and sold it for $3225; what did I gain?

$3225
2875
$350

§ 195. MODEL.—The gain is the difference between what I gave and what I received: hence, subtract $2875 from $3225. (§181.) The difference is $350: hence, I gained $350.

7. A farmer owning 725 acres, sells 375 acres; how much land has he remaining? A. 350A.

8. A man divides $3000 among three sons, giving A $985, and B $1235: how much does he give C? A. $780.
9. Burnt a kiln of 100000 bricks; sold at different times 3475, 2800, 40150, and 35000; how many are still unsold?
10. The distance from Charlotte to Goldsboro', via High Point, is 223mi., from Charlotte to High Point is 79mi.; how far is High Point from Goldsboro'? A. 144mi.
11. What cost 247lb. of bacon at 19ct. per lb.?

247 × 19ct.
2223
247
4693ct.

§196. MODEL.—247lb. cost 247 times the cost of 1lb.: hence, multiply 19ct. by 247. (§183.) The product is 4693ct.: hence, the bacon cost 4693ct.

12. How many cents are in 25 dollars?
13. How many gallons in 14hhd.? A. 882gal.
14. What will 94bbl. of flour cost at $8 per bbl.? $752.
15. How many pages in 475 volumes of 296 pages each?
16. A father divides $5460 equally among his 4 sons; what does each son receive?

4)$5460
$1365

§197. MODEL.—Each son's share is one fourth of the whole: hence, divide $5460 by 4. (§185.) The quotient is $1365: hence, each son's share is $1365.

17. If 755A. of land cost $12835, what will one acre cost? A. $17.
18. If 125 slaves sell for $75125, what is their average value?
19. If 85 bales of cotton weigh 38675lb., what does each bale weigh? A. 455lb.
20. On 475A. of land I raised 15675bu. of wheat; how much per acre? A. 33bu.
21. In 47824lb. flour, how many bbl.

47824lb.	196lb.
862	244
784	
000	

§ 198. MODEL.—As 196lb. make a bbl., the number of bbl. is equal to the number of times 196lb. are contained in 47824lb.: hence, divide 47824lb. by 196lb. (§ 185.) The quotient is 244: hence, there are 244bbl.

22. How many cu. yd. in 13122cu. ft.? A. 486cu. yd.
23. In 11823s., how many G.? A. 563G.
24. How many Acres can be bought for $5658 at $23 per Acre?
25. If a vessel make 376mi. per day, how long will she be in making 7144mi.? A. 19da.
26. Find the sum of two thousand and forty-seven, three thousand six hundred and fifty, sixty-three thousand and five, ten thousand four hundred and three, and four hundred and seven. Sum, 79512.
27. Find the difference between ten thousand and forty-two, and eight thousand seven hundred and ninety-nine.
28. What is the product of seven thousand three hundred and seventy-five, and one hundred and twenty-five? A. 921875.
29. What is the quotient of eight thousand six hundred and twenty-five, by one hundred and twenty-five? A. 69.
30. How many days in 4wk.?
31. How many hours in 28da.? A. 672hr.
32. How many minutes in 672hr.? A. 40320min.
33. How many seconds in 40320min.?
34. The minuend is 91 thousand 8 hundred and 75, the subtrahend 8 thousand 9 hundred and 69; what is the remainder? A. 82906.
35. The subtrahend is 4 thousand 2 hundred and 96, the

remainder 6 thousand 2 hundred and 84; what is the minuend? A. 10580.

36. The remainder is 7 hundred thousand and 94, the minuend 2 millions 3 thousand; what is the subtrahend?
37. How many hours in 40320min.? A. 672hr
38. How many days in 672hr.? A. 28da.
39. How many weeks in 28da.?
40. How many min. in 2419200sec.? A. 40320min.
41. The multiplicand is 37 millions 43 thousand and 25, the multiplier 8 thousand and 64; what is the product? A. 298714953600.
42. The multiplicand is 7 hundred and 25, the product 593 thousand 7 hundred and 75; what is the multiplier?
43. The multiplier is 4 thousand 9 hundred and 7, the product 42 millions 813 thousand 575; what is the multiplicand? A. 8725.
44. What cost 347yd. of rope at 9ct. per foot? A. $93.69.
45. How many qt. in 7gal. 2qt.?
46. How many qt. in 8gal. 1qt.?
47. How many pt. in 8gal. 1qt. 1pt.?
48. How many gi. in 7gal. 3qt. 1pt. 3gi.?
49. The dividend is 11 millions 210 thousand 202, the divisor 7 thousand and 2; what is the quotient? A. 161.
50. The divisor is 8 thousand and 4, the quotient 5 thousand and 90; what is the dividend? A. 40740360.
51. The quotient is 1 million 2 thousand and 3, the dividend 1 trillion 4 billions 10 millions 12 thousand and 9; what is the divisor?
52. How many sq. mi. in 228888P.? A. 2sq. mi. 150A. 2R. 8P.
53. How many R. in 1728P.? A. 43R. 8P.

54. How many sq. mi. in 1895A.?
55. How many A. in 1806P.? A. 11A. 1R. 6P.
56. What is the sum of 7 thousand, 83 thousand and 40, 9 hundred and 70, and 17 times 5 hundred and 79? A. 100853.
57. What is the difference between the product of 85 and 307, and the quotient of 999875 by 125?
58. How many lb. in 7qr.? A. 175lb.
59. How many oz. in 25lb. Avoir.? A. 400oz.
60. The distance from High Point to Greensboro' is 15mi., from Greensboro' to Shops 22mi., from Shops to Raleigh 53mi.; how far is it from High Point to Raleigh, via Greensboro' and Shops?
61. The distance from Charlotte to High Point is 79mi., from High Point to Raleigh 95mi., from Raleigh to Goldsboro' 49mi.; how far is it from Charlotte to Goldsboro', via High Point and Raleigh? A. 223mi.
62. Bought a pair of horses for $375, a set of harness for $55, and a buggy for $187; what did the whole cost? A. $617.
63. Paid $789 for a lot of tobacco, and sold it for $910; gained how much?
64. How many units in 14doz. and 7? A. 175.
65. How many units in 3 score and 10? A. 70.
66. How many doz. in 12 gross?
67. How many units in 10 great gross? A. 17280.
68. Bought 3 stone of potatoes at 2ct. per lb.; what did they cost? A. 84ct.
69. Bought 1000lb. of fish at $9 per quintal; what did I pay?
70. What cost 616lb. of butter at $15 a firkin? A. $175.
71. What cost 247bbl. of flour at $5 per bbl.? A. $1235.

72. How far will a train of cars go in 3 days, at 16 miles per hour?

73. Bought 16yd. of calico at 15ct., 7yd. of gingham at 25ct., 9yd. of flannel at 68ct., and 25yd. of domestics at 10ct.; paid 16bu. of corn at 68ct.; how much is still due? A. $1.89.

74. If a book of 155 pages has 29 lines on each page, and 39 letters in each line, how many letters are in the book? A. 175305 letters.

75. I deposited in bank $10050: having drawn out $15, $175, $237, $375, $4165, $394, and $3968, how much have I still on deposit?

76. The Bible contains 31173 verses: how many verses must I read each day, to finish it in one year?
A. 85 verses a day, and 148 verses over.

77. How many sheets of paper in 20 quires? A. 480sh.

78. How many sheets in 14 reams?

79. How many reams in 180 quires? A. 9rm.

80. How many quires in 19 reams? A. 380qr.

81. A stock-dealer bought 47 cows at $19, 29 horses at $135, 53 mules at $97, and 155 sheep at $3: he received for them 347 acres of land at $26, and $4125 in money; how much did he gain?

82. What will 574bbl. of pork cost at $13 per bbl.?
A. $7462.

83. How far will a man travel in 6da. at 29mi. per da.?
A. 174mi.

84. A planter who worked 57 hands, raised 399 bales of cotton: how many bales did he raise to the hand?

85. In $45, how many ct.? A. 4500ct.

86. In £4, 5s. 6d., how many d.? A. 1026d.

87. In 240dwt., how many oz.?
88. In 39sc., how many dr.? A. 13dr.
89. In 3T. 3qr. 20lb. 12oz., how many oz.? A. 97532oz.
90. In 7920in., how many yd.?
91. In 4mi., how many ch.? A. 320ch.
92. In 1568160sq. in., how many sq. yd.? A. 1210sq. yd.
93. In 4sq. mi., how many A.?
94. In 4cu.yd. 12cu.ft., how many cu.in.? A. 207360cu.in.
95. In 3025gi., how many hhd.? A. 1hhd. 31gal. 2qt. 1gi.
96. In 5bu., how many pt.?
97. In 3da. 10hr. 15min., how many sec.? A. 296100sec.
98. In 3S. 3° 3′ 3″, how many seconds? A. 334983″.
99. In 2gr. gr, 3gr. 4doz. and 5, how many units?
100. In 6rm. 7qr. 8sh., how many sh.? A. 3056sh.
101. In 3gal. 3qt. 3gi., how many qt.? A. 15.375qt.
102. In 10bu. 1pk. 1gal. 1pt., how many pk.?
103. In £6, 6s. 6d. 3qr., how many s.? A. 126.5625s.
104. Add $\frac{1}{4}$lb., $\frac{1}{2}$oz., $\frac{1}{3}$dwt., and $\frac{1}{5}$gr., in gr.
Sum, 1688.2gr.
105. Add 3.5hr., 7.75min., and .15sec., in min.
106. Add $\frac{1}{4}$A., $\frac{1}{5}$R., and $\frac{1}{10}$P., in P. Sum, 48.1P.
107. Add .25cu. yd., .375cu. ft., and .625cu. in., in cu. ft.
Sum, 7.12536+cu.ft.
108. From .9cwt., take .25lb., in oz.
109. From .75lb., take .5dwt., in oz. Rem. 8.975oz.
110. From 10.875s., take 9.15d., in qr. Rem. 485.4qr.
111. From 5.5da., take 5.5min., in min.
112. Multiply .75gal. by 7.5, in pt. Prod. 45pt.
113. Multiply 2.25A. by .125, in P. Prod. 45P.
114. Divide 4.5mi. by 5.4, in rd.
115. Divide 1.55s. by 2.3, in d. Quot. 8.08695d.

OPERATIONS ON COMPOUND NUMBERS.

§ 199. The operations on compound numbers are analogous to the corresponding ones on abstract numbers.

ADDITION OF COMPOUND NUMBERS.

Ex. 1. Add together 4hhd. 25gal. 3qt., 5hhd. 20gal. 2qt., 7hhd. 17gal. 2qt.

4hhd.	25gal.	3qt.
5 "	20 "	2 "
7 "	17 "	2 "
17 "	0 "	3 "

§ 200. MODEL.—2 and 2 are 4, and 3 are 7, 7qt., equal to 1gal. 3qt., set down 3 : 1 and 17 are 18, and 20 are 38, and 25 are 63, 63 gal., equal to 1hhd., set down 0 : 1 and 7 are 8, and 5 are 13, and 4 are 17, 17hhd. The sum is 17hhd. 3qt.

EXPLANATION.—In simple numbers ten units of any denomination make one of the next higher. In compound numbers this uniformity of relation does not exist. Thus, in the example above, 4qt. make 1gal., but 63gal. make 1 hhd. With this exception, the explanation in § 22 will suffice for this case.

Ex. 2. Add £10, 14s. 9d. 3qr., £5, 16s. 6d. 2qr., £7, 10d. 1qr., £12, 9s. 9d. 3qr.

3. Add £4, 10s. 11d., £7, 8s. 9d. 3qr., £8, 10d., and 16s. 3qr. Sum, £20, 16s. 7d. 2qr.

4. Add 10lb. 10oz. 10dwt. 10gr., 12lb. 9oz. 6dwt. 3gr., 9lb. 11oz. 13dwt. 15gr., and 24lb. 8oz. 15dwt. 20gr. Sum, 58lb. 4oz. 6dwt.

5. Add 3lb. 6oz. 9dwt. 12gr., 6lb. 8oz. 10dwt. 12gr., 8lb. 11oz. 14dwt. 17gr., and 14lb. 11oz. 8dwt. 5gr.

6. Add 10lb. 9oz. 7dr. 2sc. 15gr., 10oz. 6dr. 1sc. 10gr., 15 lb. 11oz. 7dr. 2sc. 19gr., and 3lb. 4oz. 5dr. 6gr.
Sum, 31lb. 1oz. 3dr. 1sc. 10gr.
7. Add 10T. 10cwt. 10lb. 10oz. 10dr., 14T. 15cwt. 3qr. 15lb. 13oz. 15dr., and 25T. 7cwt. 1qr. 20lb. 8oz. 8dr.
Sum, 50T. 13cwt. 1qr. 22lb. 1oz. 1dr.
8. Add 3sq.mi. 300A. 2R. 25P., 7sq.mi. 525A. 3R. 10P., 19sq.mi. 285A. 3R. 19P., and 250A. 25P.
Sum, 31sq.mi. 82A. 1R. 39P.
9. Add 19cu.yd. 19cu.ft. 19cu.in., 25cu.yd. 25cu.ft. 250cu. in., and 100cu.yd. 15cu.ft. 1555cu.in.
10. Add 4hhd. 40gal. 2qt. 1pt. 3gi., 10hhd. 10gal. 1qt. 1pt. 1gi., and 20hhd. 43gal. 3qt. 1pt. 3gi.
Sum, 35hhd. 32gal. 3gi.
11. Add 10bu. 3pk. 7qt. 1pt., 9bu. 2pk. 6qt. 1pt., 16bu. 3pk. 6qt., and 15bu. 1pk. 5qt. 1pt. Sum, 53bu. 1qt. 1pt.
12. Add 30da. 10hr. 30min. 30sec., 15da. 15hr. 15min. 15 sec., and 10da. 20hr. 45min. 15sec.
13. Add 25° 15′ 25″, 75° 24′ 50″, and 15° 50′ 45″.
Sum, 116° 31′.
14. Add 2rm. 10qr. 12sh., 4rm. 15qr. 18sh., and 3rm. 9qr. 10sh.
15. Add 2gr. gross, 10 gross, 7doz. 5 units, 4gr. gross, 8 gross, 6doz. 7 units, and 5 gross, 8doz. 6 units.

SUBTRACTION OF COMPOUND NUMBERS.

Ex. 1. From £17, 5s. 6d. 3qr., take £8, 10s. 9d. 2qr.

£17,	5s.	6d.	3qr.
8,	10“	9“	2“
8,	14“	9“	1“

§ 201. Model.—2 from 3 leaves 1; 9 from 18 leaves 9; 11 from 25 leaves 14; 9 from 17 leaves 8. The remainder is £8, 14s. 9d. 1qr.

EXPLANATION.—As 9d. can not be taken from 6d., we add 1s., that is 12d., to the minuend, and subtract 9d. from 18d. We then add 1s. to the subtrahend, and proceed. See §§ 28, 30.

Ex. 2. From 50lb. 6oz. 15dwt. 19gr., take 10lb. 17dwt.
Rem. 40lb. 5oz. 18dwt. 19gr.

3. From 15lb. 15gr., take 12lb. 9oz. 10dwt. 12gr.
4. From 10T. 10cwt. 10oz., take 5T. 15cwt. 20lb. 12oz. 10dr. Rem. 4T. 14cwt. 3qr. 4lb. 13oz. 6dr.
5. From 6sq.mi. 2R., take 375A. 25P.
Rem. 5sq.mi. 265A. 1R. 15P.
6. From 250cu.yd. 20cu.ft. 875cu.in., take 79cu.yd. 25cu. ft. 695cu.in.
7. From 15T. 15cwt. 3qr. 15lb., take 10T. 19cwt. 3qr. 19lb. Rem. 4T. 15cwt. 3qr. 21lb.
8. From 10hhd. 10gal. 1qt. 1pt., take 9hhd. 33gal. 3qt. 3gi. Rem. 39gal. 2qt. 1gi.
9. From 4 tuns, 1pi. 1hhd. 5gal. 2qt. 3gi., take 2 tuns, 60 gal. 3qt. 3gi.
10. From 175bu. 1pk. 3qt. 1pt., take 54bu. 3pk. 2qt.
Rem. 120bu. 2pk. 1qt. 1pt.
11. From 27bu. 2pk. 1pt., take 13bu. 5qt.
Rem. 14bu. 1pk. 3qt. 1pt.
12. From 30da. 10hr. 15min., take 17da. 15hr. 15sec.
13. From 180°, take 74° 14′ 45″. Rem. 105° 45′ 15″.
14. From 90°, take 35° 41′ 15″. Rem. 55° 18′ 45″.
15. From 100° 17′ 30″, take 90° 25′ 45″.
16. From 22T. 8cwt. 2qr. 20lb., take 12T. 18cwt. 22lb.
Rem. 9T. 10cwt. 1qr. 23lb.
17. From 16hhd. 24gal. 3qt. 2pt., take 14hhd. 37gal. 3qt.
18. From 236bu. 2pk. 5qt. 1pt., take 17bu. 2pk. 7qt. 2pt.

MULTIPLICATION OF COMPOUND NUMBERS.

Ex. 1. Multiply £17, 5s. 6d. 3qr. by 15.

£17, 5s. 6d. 3qr.
15
259, 3 " 5 " 1 "

§202. MODEL.—$15 \times 3 = 45$. 45qr. = 11d. 1qr.; set down 1: $15 \times 6 = 90$, and 11 = 101. 101d. = 8s. 5d.; set down 5: $15 \times 5 = 75$, and 8 = 83. 83s. = £4, 3s.; set down 3: $15 \times 17 = 255$, and 4 = 259. The product is £259, 3s. 5d. 1qr.

EXPLANATION.—See §§ 38, 200.

Ex. 2. Multiply £53, 10s. 9d. 2qr. by 4.

Prod. £214, 3s. 2d.

3. Multiply 13° 15′ 45″ by 7.
4. Multiply 25° 30′ 45″ by 10. Prod. 255° 7′ 30″.
5. Multiply 50lb. 6oz. 15dwt. 19gr. by 13.

Prod. 657lb. 4oz. 5dwt. 7gr.

6. Multiply 12lb. 9oz. 10dwt. 12gr. by 16.
7. Multiply 5T. 15cwt. 20lb. 12oz. 10dr. by 19.

Prod. 109T. 8cwt. 3qr. 17lb. 9oz. 14dr.

8. Multiply 2sq. mi. 200A. 2R. 20P. by 22.

Prod. 50sq. mi. 573A. 3R.

9. Multiply 3cu. yd. 25cu. ft. 750cu. in. by 25.

Prod. 98cu. yd. 14cu. ft. 1470cu. in.

10. Multiply 9hhd. 33gal. 3qt. 3gi. by 21.
11. Multiply 2 tuns, 60gal. 3qt. 3gi. by 24.

Prod. 53tuns, 1pi. 1hhd. 11gal. 1qt.

12. Multiply 25bu. 3pk. 1qt. 1pt. by 29.
13. Multiply 10bu. 1pk. 4qt. by 35. Prod. 363bu. 4qt.
14. Multiply 10da. 10hr. 10min. 10sec. by 41.

Prod. 1yr. 62da. 8hr. 56min. 50sec.

15. Multiply 17da. 15min. 15sec. by 50.

DIVISION OF COMPOUND NUMBERS.

Ex. 1. Divide 15° 15′ 50″ by 10.

10)15° 15′ 50″
1° 31′ 35″

§ 203. MODEL.—10 in 15, once, with 5 over, set down 1; 10 in 315, 31 times, with 5 over, set down 31; 10 in 350, 35 times. The quotient is 1° 31′ 35″.

EXPLANATION.—10 is contained once in 10; so that there are 5° undivided. These 5° are reduced to 300′, and added to the 15′, making 315′. In like manner, the 5′ undivided are reduced to 300″, and added to the 50″, making 350″.

Ex. 2. Divide £30, 16s. 2d. 1qr. by 3.

3. Divide £60, 1s. 5d. by 4.

4. Divide 29lb. 2oz. 2dwt. 2gr. by 5.
Quot. 5lb. 10oz. 10gr.

5. Divide 242lb. 5oz. 11dwt. 16gr. by 8.
Quot. 30lb. 3oz. 13dwt. 23gr.

6. Divide 448lb. 10oz. 14dr. by 11.

7. Divide 32T. 2qr. by 15. Quot. 2T. 2cwt. 2qr. 20lb.

8. Divide 52sq. yd. 5sq. ft. 128sq. in. by 20.
Quot. 2sq. yd. 5sq. ft. 100sq. in.

9. Divide 97cu. yd. 22cu. ft. 80cu. in. by 26.

10. Divide 91gal. 1qt. 1pt. by 34.
Quot. 2gal. 2qt. 1pt. 2gi.

11. Divide 79 tuns, 1qt. 1gi. by 45.
Quot. 1tun, 1pi. 1hhd. 1gal. 1qt. 1pt. 1gi.

12. Divide 600bu. 3pk. 6qt. by 60.

13. Divide 8wk. 3da. 7hr. 43min. 20sec. by 70.
Quot. 20hr. 20min. 20sec.

14. Divide 1150° 31′ 15″ by 75. Quot. 15° 20′ 25″.

15. Divide 1° 41′ 40″ by 100.

16. Divide 57T. 19cwt. 1qr. 17lb. 14oz. by 9cwt. 1qr. 17lb. 10oz.

57T. 19cwt. 1qr. 17lb. 14oz.=1855086oz.
9cwt. 1qr. 17lb. 10oz.=15082oz.
1855086oz.÷15082oz.=123.

§ 204. MODEL.—Reduce the dividend to oz. (§ 189). Reduce the divisor to oz. (§ 189). Divide the dividend by the divisor. (§ 185). The quotient is 123.

Ex. 17. Divide 2941bu. by 45bu. 3pk. 6qt. 1pt.

Quot. 64.

18. Divide 97T. 11cwt. 3qr. 11lb. 10oz. by 1T. 6cwt. 2qr. 26lb. 10oz.
19. Divide 17bu. 1pk. 6qt. by 2bu. 3pk. 5qt. Quot. 6.
20. Divide 51A. 1R. 11P. by 1A. 1P. Quot. 51.
21. Divide 10 tuns, 2hhd. 17gal. 2pt. by 39gal. 6pt.
22. Divide £27, 2s. 6d. by 15s. 6d. Quot. 35.

PROMISCUOUS PROBLEMS.

1. What is the least common multiple of 15, 24, and 27?
2. What is the least common multiple of 9, 25, and 45?

L. C. M. 225.

3. What is the greatest common measure of 505, 1111, and 3434?
4. What is the greatest common measure of 1015, 1260, and 1330? G. C. M. 35.
5. What are the prime factors of 6105?
6. What are the prime factors of 4060?
7. Divide £113, 13s. 4d. by 31. Quot. £3, 13s. 4d.
8. Divide 10 tuns, 1pi. 17gal. 2pt. by 67.
9. Divide 50T. 4cwt. 2qr. 14lb. by 23cwt. 3qr. 17lb.

10. Divide 1572yd. by 32yd. 3qr. Quot. 48.
11. Multiply 25oz. 8dwt. 17gr. by 100.
Prod. 211lb. 11oz. 10dwt. 20gr.
12. Multiply 21da. 18hr. 42min. by 75.
13. Subtract 40A. 3R. 25P. from 79A. 15P.
Rem. 38A. 30P.
14. Subtract 4 tuns, 1pi. 1hhd. 25gal. 3qt. from 5 tuns, 1qt.
15. Add 60mi., 40mi. 7fur. 39rd., and 19mi. 1rd.
16. Add 13° 14′ 15″, 16° 17′ 25″, 25° 19′ 47″, and 3° 15″.
17. Divide $1521808938 by 234. Quot. $6503457.
18. Divide 14265hhd. by 45hhd.
19. Multiply 4327bu. by 102. Prod. 441354bu.
20. Multiply 47935gal. by 275. Prod. 13182125gal.
21. Subtract 2598328fur. from 3002575fur.
22. Subtract 187564329gi. from 923465781gi.
Rem. 735901452gi.
23. Add 2479A., 3580A., 1358A., and 9745A.
24. Add £13575, £23495, £9475, and £31525.
25. Divide 82960332 by 84. Quot. 987265; Rem. 72.
26. Divide 82071 by 99. Quot. 829.
27. Multiply 24068 by 13579.
28. Multiply 1020908 by 8979091. Prod. 9166825834628.
29. Subtract 3987456002 from 4567398745.
30. Subtract 246+357+1298+982 from 3120.
31. Add 20030405, 910285, 5821090, and 9706845.
Sum, 36468625.
32. Add 123, 1234, 12345, 123456, 1234567, 12345678, and 123456789. Sum, 137174192.
33. A. raised 125 bales of cotton, 517bu. corn, 629bu. wheat, and 119bu. rye; B., 217 bales of cotton, 865bu. corn, 798bu. wheat, and 143bu. rye; C., 94 bales of cotton, 424bu. corn, 517bu. wheat, and 77bu. rye; and D.,

111 bales of cotton, 512bu. corn, 558bu. wheat, and 98bu. rye. How much of each article did they all raise?

34. A farmer went to town with $100, and spent $9 for molasses, $13 for sugar, $11 for coffee, $8 for rice, $17 for dry goods, and $25 for leather. How much money had he left? Ans. $17.

35. Bought 47 acres of land at $19, 5 horses at $125, 10 head of cattle at $21, 14 sheep at $3, and a two-horse wagon for $65; what did they all cost? Ans. $1835.

36. Sold 75 firkins of butter for $1350; how much was that a firkin?

37. What number is that, to which if 245, 379, 124, 212, and 399 be added, the sum will be 1525? Ans. 166.

38. What number is that, from which if the sum of 245, 379, 124, 212, and 399 be subtracted, the remainder will be 1525? Ans. 2884.

39. What number is that, by which if twice 19 be multiplied, the product will be the difference between 4127 and 2759?

40. What number is that, by which if 4235 be divided, the quotient will be 77? Ans. 55.

41. A miller has 5 bins, one of which holds 43bu. 3pk. 5qt.; the second, 39bu. 1pk. 3qt.; the third, 45bu. 1qt. 1pt.; the fourth, 53bu. 2pk.; the fifth, 34bu. 3pk. 1pt.; what is their united capacity? Ans. 216bu. 2pk. 2qt.

42. How much time elapsed between Jan. 20th, 1833, and May 25th, 1861?

43. Bought 4 lots of land, containing 3R. 27P. each; how many A. did I buy? Ans. 3A. 2R. 28P.

44. A wine merchant has 269gal. 2gi. of wine in 30 equal vessels; how much wine is there in each vessel?

45. Reduce £40, 19s. 11d. 3qr. to qr.
46. Reduce 11lb. 11oz. 19dwt. 23gr. to gr. Val. 69119gr.
47. Reduce 3℔. 11℥. 7ʒ. 2℈. 19gr. to gr. Val. 23039gr.
48. Reduce 2T. 19cwt. 3qr. 24lb. 15oz. 15dr. to dr.
49. Reduce 4L. 2mi. 7fur. 35rd. to rd. Val. 4795rd.
50. Reduce 12yd. 2ft. 11in. to in. Val. 467in.
51. Reduce 2sq. mi. 600A. 3R. 35P. to P.
52. Reduce 25sq. yd. 8sq. ft. 100sq. in. to sq. in.
53. Reduce 21sq. mi. 250A. 2R. to R. Val. 54762R.
54. Reduce 5cu. in. 20cu. ft. 1600cu. in. to cu. in.
55. Reduce 3 tuns, 1pi. 1hhd. to hhd. Val. 15hhd.
56. Reduce 3hhd. 60gal. 3qt. 1pt. to pt. Val. 1999pt.
57. Reduce 2gal. 1qt. 1pt. 3gi. to gi.
58. Reduce 5bu. 3pk. 7qt. 1pt. to pt. Val. 383pt.
59. Reduce 3C. 75yr. 300da. to da. Val. 137175da.
60. Reduce 4da. 10hr. 25min. to sec.
61. Reduce 25° 10′ 15″ to seconds. Val. 90615″.
62. Reduce 2rm. 15qr. 12sh. to sh. Val. 1332sh.
63. Reduce 4 score and 5 to units.
64. Reduce 1000qr. to £. Val. £1, 10d.
65. Reduce 6000gr. to lb. Troy. Val. 1lb. 10dwt.
66. Reduce 600000dr. to T.
67. Reduce 1000rd. to mi. Val. 3m. 1fur.
68. Reduce 2000sq. in. to sq. yd.
69. Reduce 200000sq. rd. to sq. mi.
70. Reduce 60000cu. in. to cu. yd.
71. Reduce 100gi. to gal. Val. 3gal. 1pt.
72. Reduce 500qt. to hhd.
73. Reduce 100pt. to bu. Val. 1bu. 2pk. 2qt.
74. Reduce 4000sec. to hr. Val. 1hr. 6min. 40sec.
75. Reduce 200hr. to wk.
76. Reduce 4000″ to degrees. Val. 1° 6′ 40″.

77. Bought 3 firkins of butter at 20ct. per lb., 20qt. molasses at $1 per gal., 3 stone of potatoes at 3ct. per lb., and 980lb. flour at $8 per bbl.; what did they all cost?

78. Bought 5doz. Arithmetics at 30ct. apiece; sold them all for $27: how much apiece did I gain or lose?

79. A owes B for 500lb. of salt fish at $8 a quintal; B owes A for 3bbl. flour at $7 a bbl., 10bu. corn at 70ct. per bu., and 5bu. rye at 80ct. per bu.: how does their account stand? A owes B $18.

80. From 500 subtract the sum of 225, 120, and 75; divide the remainder by the difference between 1000 and 960; multiply the quotient by 17; and add 16 to the product. Sum, 50.

81. Find the sum of the product of 88 and 11, and the quotient of 88 by 8.

82. A man has 1184bu. of wheat and 468bu. of corn, which he wishes to pack in equal bags as large as possible. How many bushels will each bag hold; and how many bags will be required? Ans. 4bu., and 413 bags.

83. What is the value of $85+77-64+6\times19-132\div4$?

84. What is the value of $15498\div54+41\times63-27\times55$?

85. Find the least common multiple of 4, 44, 132, and 792.

86. Find the greatest common measure of 4, 44, 132, and 792. G. C. M. 4.

87. Two men travel in the same direction from the same place, one 40mi. a day, the other 33mi. a day; how far apart are they in 7 days?

88. Two men travel in contrary directions from the same place, one 40mi. per day, the other 33mi. per day; how far apart are they in 7 days? Ans. 511mi.

89. If 10 persons use a barrel of flour in 57 days, how long will a barrel last one person? Ans. 570da.

90. What is the sum of 3 numbers, of which the first is 28, the second 8 times the first, and the third one seventh of the second?

91. The difference is one hundred thousand four hundred and seventy-six, the minuend is one million; what is the subtrahend? Ans. 899524.

92. The minuend is one hundred thousand, the subtrahend is sixty-seven thousand seven hundred and forty-four; what is the remainder? Ans. 32256.

93. The subtrahend is seven hundred thousand and forty-nine, the remainder is ninety-nine thousand two hundred and seventy-eight; what is the minuend?

94. The multiplicand is thirty-six thousand seven hundred and seven, the multiplier is eighty thousand and one; what is the product? Ans. 2936596707.

95. The multiplier is eight hundred and four, the product is sixty-one thousand nine hundred and eight; what is the multiplicand? Ans. 77.

96. The product is eighteen billions two hundred and twenty thousand, the multiplicand is two thousand two hundred; what is the multiplier?

97. The divisor is one hundred and twenty-five, the dividend is nine hundred and eighty-seven thousand six hundred and twenty-five; what is the quotient? Ans. 7901.

98. The dividend is thirty-four thousand eight hundred and forty-eight, the quotient is one hundred and thirty-two; what is the divisor? Ans. 264.

99. The quotient is thirty thousand and seventy, the divisor is seven hundred and eight; what is the dividend?

100. What cost 18rm. of paper at $4, 3doz. Arithmetics at $6 a dozen, and 24 Algebras at $12 a dozen?

ALIQUOT PARTS.

§ 205. An aliquot fraction is a simple fraction whose numerator is 1. Thus, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{10}$, $\frac{1}{20}$, are aliquot fractions.

An aliquot part of a number is a part denoted by an aliquot fraction. Thus, 3 is an aliquot part of 12, 10s. is an aliquot part of £1, 20da. is an aliquot part of 2mo.

Ex. 1. What is the cost of 5A. 3R. 25P. of land at $45.50 per A.?

2) $45.50=1A.
5.

$227.50=5A.
2) 22.75 =2R. } 3R.
2) 11.375=1R. }
4) 5.687=20P. } 25P.
1.421= 5P. }

$268.733

§ 206. Model.—5A. cost 5 times as much as 1A.;—hence, multiply the cost of 1A. by 5. (§ 183). 2R. is one half of 1A.; hence, divide the cost of 1A. by 2. (§ 185). 1R. is one half of 2R.; hence, divide the cost of 2R. by 2. 20P. is one half of 1R.; hence, divide the cost of 1R. by 2. 5P. is one fourth of 20P.; hence, divide the cost of 20P. by 4. Add the several costs together. The sum is $268.733: hence, 5A. 3R. 25P. cost $268.733.

Ex. 2. What is the cost of 17T. 15cwt. 3qr. 10lb. of iron at $36 per T.? Ans. $640.53.

3. What is the cost of 10bu. 3pk. 4qt. 1pt. of grass seed at $8 per bu.?

4. What is the yield of 45A. 3R. 15P. of wheat land at 20bu. per A.? Ans. 916.875bu.

5. What is the value of 2lb. 9oz. 10dwt. 6gr. of plate at $15 per lb.? Ans. $41.88.

6. What is the value of 5T. 10cwt. 1qr. 5lb. of hay at $25 per T.?

7. What is the cost of 15bu. 1pk. 5qt. of dried peaches at $6 per bu.? Ans. $92.4375.
8. What is the cost of 4gal. 1qt. 1pt. of wine at $4 per gal.? Ans. $17.50.
9. What is the cost of 37rd. 2¾yd. of fencing at $3.50 per rd.?
10. What is the cost of 7yd. 2ft. 6in. of cloth at $7.25 per yd.? Ans. $56.791+.
11. What is the cost of 30sq.yd. 4sq.ft. 72sq.in. of painting at $.75 per sq.yd.? Ans. $22.875.
12. What is the cost of 10cd. 80cu.ft. of wood at $2.50 per cd.?
13. What will a man earn in 9mo. 10da. at $25 per mo.? Ans. $233.33⅓.
14. What will 10cd. of wood cost at $2.62½ per cd.?

2)$10	= cost of 10cd.	at	$1.00	per	cd.	
2						
$20	= "	"	"	2.00	"	"
4) 5	= "	"	"	.50	"	"
1.25	= "	"	"	.12½	"	"
$26.25	= "	"	"	2.62½	"	"

§ 207. Model.—10 cords at $1 would cost $10. The cost at $2 is twice the cost at $1; hence, multiply the cost at $1 by 2. The cost at 50ct. is one half of the cost at $1; hence, divide the cost at $1 by 2. The cost at 12½ct. is one fourth of the cost at 50ct.; hence, divide the cost at 50ct. by 4. Add the several costs together. The sum is $26.25: hence, 10 cords at $2.62½ cost $26.25.

15. What is the cost of 360bu. of wheat at $1.37½ per bu.?
16. What is the cost of 15yd. of cloth at £1, 4s. 9d. per yd.? Ans. £18, 11s. 3d.

CONTRACTION IN MULTIPLICATION.

Ex. 1. Multiply 279 by $33\frac{1}{3}$.

3)27900
9300

§ 208. MODEL.—Annex 2 naughts to the multiplicand:—divide the result by 3. The product is 9300.

EXPLANATION.—See § 63. When the multiplier is an aliquot part of any power of ten, we may abridge the work by multiplying by this power of ten (§ 56) and dividing the result by the denominator of the aliquot fraction.

Ex. 2. Multiply 72 by $12\frac{1}{2}$. Prod. 900.
3. Multiply 72 by $16\frac{2}{3}$.
4. Multiply 77 by $14\frac{2}{7}$. Prod. 1100.
5. Multiply 63 by $1\frac{1}{9}$. Prod. 70.
6. Multiply 684 by $166\frac{2}{3}$.
7. Multiply 273 by $333\frac{1}{3}$. Prod. 91000.

CONTRACTION IN DIVISION.

Ex. 1. Divide 2000 by $142\frac{6}{7}$.

2000
7
14,000

§ 209. MODEL.—Multiply the dividend by 7:—divide the product by 1000. (§ 65.) The quotient is 14.

EXPLANATION.—See § 68. When the divisor is an aliquot part of any power of ten, we may abridge the work by multiplying the dividend by the denominator of the aliquot fraction and dividing the product by the power of ten. (§65.)

Ex. 2. Divide 150 by $33\frac{1}{3}$. Quot. 4.5.
3. Divide 250 by $14\frac{2}{7}$.
4. Divide 1500 by $166\frac{2}{3}$. Quot. 9.

5. Divide 245 by $12\frac{1}{2}$. Quot. 19.6.
6. Divide 1375 by $11\frac{1}{9}$.
7. Divide 2468 by $333\frac{1}{3}$. Quot. 7.404.

PROMISCUOUS PROBLEMS.

1. Find 246+2468+24680+20468+24068+24608.
Sum, 96538.
2. Find the sum of one half, three fourths, two fifths, and one sixth. Sum, $1\frac{4}{60}$.
3. Add 315, 31.57, 3157, 3.157, and .3157.
4. A farmer raised $357 worth of corn, $475.50 worth of wheat, $123.75 worth of rye, and 446.37\frac{1}{2}$ worth of other products; what was the total value of the products of his farm? Ans. 1402.62\frac{1}{2}$.
5. A's gold mine yielded $\frac{1}{2}$lb. in one week, B's yielded 7 oz., and C's 125.5dwt.; how many ounces did the three mines yield? Ans. 19.275oz.
6. A tobacco planter raised from one field 7cwt. 1qr. 20 lb., from an other 10cwt. 3qr. 10lb., and from an other 15cwt. 15lb.; what amount did he raise in all?
7. A man sold 14 horses at $125.75 a head, and 25 head of cattle at 19.87\frac{1}{2}$ a head; what did he receive for them all? Ans. $2257.375.
8. Find the value of 135−1357+13570−3571−3157.
Val. 5620.
9. What is the difference between thirteen fourteenths and fourteen fifteenths?
10. From 500.05 take 65.556. Rem. 434.494.
11. A dealer sold a lot of bacon for 2425.16\frac{2}{3}$, which cost him 2177.43\frac{3}{4}$; what did he gain by the trade?

12. Find the difference in R. between $\frac{1}{6}$A. and 27.5P.
13. Washington was born Feb. 22, 1732, and died Dec. 14, 1799; what was his age? Ans. 67yr. 9mo. 22da.
14. A gave B 250lb. of beef at 8ct. for 33lb. of leather at 60ct.; how does their account stand?
Ans. B owes A 20ct.
15. What is the product of 7903×3907?
16. Multiply three eighths by two and two thirds.
Prod. 1.
17. Find the value of 19.275×21.125. Val. 407.184375.
18. A planter who works 47 hands raises to each hand ten bales of cotton averaging 445lb.; how much does his cotton yield him at 9ct. per lb.?
19. A farmer owns 4 farms containing each 47A. 25P.; what do the four contain? Ans. 188A. 2R. 20P.
20. A jeweller has 25 gold rings weighing $3\frac{1}{2}$dwt. each; how many oz. do they all weigh? Ans. $4\frac{3}{8}$oz.
21. Bought 22gal. of molasses at 75ct., 247lb. of sugar at $16\frac{2}{3}$ct., 175lb. of rice at $6\frac{1}{4}$ct., and $57\frac{1}{2}$lb. of coffee at $18\frac{3}{4}$ct.; what was my bill?
22. Find the value of $25000 \div 125 + 1475 \div 25$. Val. 259.
23. What is the quotient of three fourths by seventeen thirty-thirds? Ans. $1\frac{31}{68}$.
24. What is the value of $17.375 \div 2.5 - 9.63 \div 3.3$?
25. In 392981b. of flour, how many bbl.? Ans. $200\frac{1}{2}$bbl.
26. A father divided 778A. 3R. 21P. equally among his seven children; what was each child's share?
Ans. 111A. 1R. 3P.
27. What is the cost of 273bu. of wheat at $\$1.66\frac{2}{3}$ per bu.?
28. What is the cost of 29rm. of paper at \$2.75 per rm.?
Ans. \$79.75.
29. What part of 10da. is 7hr. 15min.? Ans. $\frac{29}{960}$.

30. What part of 5gal. 1qt. is 3qt. 3gi.?
31. Bought $\frac{4}{5}$ of 17T. 3qr. of iron at 5ct. per lb.; what did I pay? Ans. $1363.
32. In 3lb. Avoirdupois, how many oz. Troy?
Ans. 43.75oz.
33. What part of 1lb. Avoir. is 1lb. Troy?
34. How many cubic inches in 40qt. Wine Measure?
Ans. 2310cu.in.
35. What cost 3doz. Arithmetics at $2.75 per doz.,17 slates at 14ct., 5 gross of steel pens at 93$\frac{3}{4}$ct. per gross, and 300 slate pencils at 31$\frac{1}{4}$ct. per hundred?
36. Bought 156bbl. of flour for $936, and sold the same at $8.45 per bbl.; what did I gain?
37. In 1bu., how many qt. Wine Measure?
Ans. 37.236qt.
38. How much heavier is a pound of feathers than a pound of gold? Ans. 1240gr.
39. Virginia contains 61352sq.mi.; North Carolina, 55500; South Carolina, 28000; Georgia, 58000; Florida, 59268; Alabama, 50722; Mississippi, 47151; Louisiana, 41346; Texas, 325520; Arkansas, 52198; Missouri, 65037; Tennessee, 44000; and Kentucky, 37680. What is the area of the Confederate States of America?
40. In 1lb. Troy, how many oz. Avoir.?
Ans. 13.0281+oz.
41. What will 33$\frac{1}{2}$yd. of cloth cost at $4.75 per yd.?
Ans. $159.125.
42. What will 17bbl. of flour cost at $7.875 per bbl.?
43. What will 66bu. of wheat cost at $1.125 per bu.?
44. A man borrowed $189.75, and paid at one time $37.375, at an other $23.625, and at an other $19.4375; how much does he still owe? Ans. $109.3125.

45. A lady bought a silk dress for $21.875, a lace mantle for $15.50, a pair of cloth gaiters for $3.25, and a bonnet for $9.625; what did they all cost?

46. If 25yd. of cloth cost $85.50, what does 1yd. cost?
Ans. $3.42.

47. What will 365lb. of flour cost at $4 per hundred?
Ans. $14.60.

48. How many working days are there in a common year?

49. If a man receives $2000 a year, how much is that a day?
Ans. $5.479.

50. Bought 5bu. at $1.37½ per bu., and sold them at 5ct. per qt. Wine Measure; how much did I gain?
Ans. $2.434+.

51. How many steps of 28in. each, does a soldier take in marching 5 miles?

52. How many bottles containing 1qt. ½gi. each, can be filled from a hogshead of French Brandy?
Ans. 237.295 bottles.

53. If a family use 15bbl. of flour in a year, how much is that a day? Ans. 8.05lb.

54. If a man travel 29mi. 7fur. 15rd. per day, how far will he travel in 5wk. if he rest on Sunday?

55. A lady went shopping with £$\frac{5}{9}$, and spent $\frac{1}{3}$ of 14s.; how much had she left? Ans. 6$\frac{4}{9}$s.

56. How many days are there from Jan. 17 to April 6?
Ans. 79da.

57. Sold one load of hay weighing 1.1T., an other weighing 1$\frac{5}{9}$T., and a third weighing 17.3cwt.; what did the three loads weigh?

58. Bought $\frac{1}{2}$ of $\frac{1}{3}$ of an acre in one lot, 49P. in an other, and $\frac{1}{3}$ of 10R. in an other; what did the three lots cost at $69.6875 per A.? Ans. $91.029+.

59. What will 2550lb. of corn cost at 40ct. per bu.?
Ans. $18.21.

60. What will 1000lb. of wheat cost at $1.37½ per bu.?

61. If 1bu. of wheat will make 45lb. of flour, how many bbl. will 1500lb. make? Ans. 5.74bbl.

62. How many seconds were there in the winter of 1859–60?
Ans. 7862400sec.

63. How many minutes were there in the summer of 1860?

64. How many acres of land at $1 per sq.yd. can be bought for $15000? Ans. 3.099A.

65. What will 200mi. of Telegraph wire cost at 10ct. per yd.? Ans. $35200.

66. How many pounds of flour in 75bbl.?

67. What is the difference in height between a man 5ft. 11in. high, and a horse 16 hands high? Ans. 7in.

68. Bought 10lb. of rhubarb at $6.50 per lb. Avoir., and sold it at 50ct. per oz. Troy; what did I gain?
Ans. $7.916.

69. What cost 2127ft. of lumber at $ .8375 per hundred?

70. What cost 37560 bricks at $7.75 per thousand?
Ans. $291.09.

71. What cost 17 firkins of butter at 18¾ct. per lb.?
Ans. $178.50.

72. What cost 5.5 stone of potatoes at 1.5ct. per lb.?

73. What decimal fraction is equal to 62½÷129?
Ans. .484496+.

74. A man dying left $27000 to be divided so that his widow should have one third of it, each one of 4 sons one seventh of the remainder, and each one of 5 daughters one fifth of what was left; what was each daughter's share? Ans. $1542.857.

75. A druggist having bought 60gal. of oil for $97.50, lost 6.25gal. by leakage, and sold the remainder at $2.125 per gal.: what did he gain?

76. A merchant bought two bales of domestic, containing each 20 bolts of 33yd. at 13.25ct. per yd.: what did he pay? Ans. $174.90.

77. Bought 300bbl. flour at $6.75 per bbl.; sold one third of it at $7.375 per bbl., one half of the remainder at $7.9375 per bbl., and the rest at $8.50 per bbl.: how much did I gain? Ans. $356.25.

78. What will 2250bu. corn cost at $\frac{5}{8}$ of a dollar per bu.?

79. A gentleman's house cost him four times as much as his furniture, and both together cost $4435; what did his furniture cost? Ans. $887.

80. A grocer had 7cwt. 3qr. of sugar, and sold at different times $3\frac{1}{3}$cwt., $3\frac{1}{3}$qr., and 127lb.: how many lb. has he remaining? Ans. $231\frac{1}{3}$lb.

81. A planter sold 15 bales of cotton averaging 445.5lb. at 9ct. per lb., and with the proceeds bought land at $21.25 per A.: how much did he buy?

82. If $4\frac{3}{4}$yd. of cloth cost $12\frac{5}{9}$, what will 1yd. cost?

83. What cost 29A. 1R. 18P. of land at $45.625 per A.? Ans. $1339.664+.

84. What cost 9T. 16cwt. 15lb. of iron at $37.75 per T.?

85. A man left $\frac{1}{2}$ of his estate to his wife, $\frac{1}{3}$ of the remainder to his son, and the remaining $2500 to his daughter: what was his estate? Ans. $7500.

86. If $16\frac{1}{2}$ days' work cost $19.75, what will $3\frac{1}{4}$ days' work cost? Ans. $3.89+.

87. What must I pay for $6\frac{3}{4}$lb. of butter at 35ct. per lb., $12\frac{1}{2}$doz. eggs at 15ct. per doz., 10 chickens at $18\frac{3}{4}$ct. apiece, and 30 cucumbers at 10ct. per doz.?

88. If 1bbl. of tar cost $3.875, what will 17bbl. cost?
Ans. $65.875.

89. What cost 247lb. dried blackberries, at 15ct. per lb.?
Ans. $37.05.

90. What will 100lb. of coffee cost at 6lb. to the dollar?

91. How many dollars will pay for 15 pieces of French calico, each containing 27yd., at 1.2fr. per yd.?
Ans. $90.396.

92. How many dollars will pay for 75 gross of Gillott's pens at 3s. 6d. per gross? Ans. $63.525.

93. What will 45bu. corn cost at 5½ dimes per bu.?

94. What will 727lb. salt cost at $1.25 per bu.?
Ans. $18.175.

95. What will 3bbl. flour cost at $.05 per lb.? Ans. $29.40.

96. How many bu. of corn can be hauled by a team which can haul just 50bu. of wheat?

97. John's height is 3 cubits and a span; his pony is 14 hands high; what is the difference of their heights?
Ans. 7in.

98. How many ft. of water is drawn by a vessel which can not sail in less than 3 fathoms 2 feet? Ans. 20ft.

99. What will 1280cu.ft. of wood cost at $1.75 per cord?

100. What should be paid for 570bu. of corn, at $2.50 per bbl.? Ans. $285.

101. A merchant bought 21 pieces of cloth, each containing 41 yards, for which he paid $1260; he sold the cloth at $1.75 per yd.: did he gain or lose by the bargain?
Ans. He gained $246.75.

102. A man receives ⅗ of his income, and finds it equal to $3724.16: how much is his whole income?

103. If 322 books cost $371.91, what will 248 books cost at the same rate? Ans. $286.44

RATIO.

§ 210. The *ratio* of one number to an other of the same denomination is the quotient of the second divided by the first. Thus, the ratio of 3 to 12 is 4; the ratio of 5ft. to 15ft. is 3; the ratio of \$17 to \$8 is $\frac{8}{17}$.

§ 211. Since the two numbers compared are necessarily of the same denomination, every ratio is an abstract number. (§ 185.)

§ 212. Of two numbers compared, the first is called the *antecedent*, the second is called the *consequent*, and both together are called the *terms* of the ratio. Thus, in the first ratio above, 3 is the antecedent, 12 is the consequent, and 3 and 12 are the terms.

§ 213. A ratio is usually denoted by a colon placed between the two terms. Thus, 3 : 12 is the ratio of 3 to 12; so also, 5ft. : 15ft. = 3; \$17 : \$8 = $\frac{8}{17}$.

§ 214. The ratio of two numbers of the same nature but of different denominations may be found by first reducing them to the same denomination. Thus, 3ft. : 5yd. = 5; 5ct. : \$1 = 20.

§ 215. The ratio of two numbers of different natures can not be found. Thus, 3ft. has no ratio to 3gal.

Ex. 1. What is the ratio of 3 to 6?	Ans. 2.
2. What is the ratio of 10 to 75?	Ans. 7.5.
3. What is the ratio of 27 to 9?	
4. What is the ratio of 446 to 1338?	Ans. 3.
5. What is the ratio of \$97 to \$485?	Ans. 5.
6. What is the ratio of 27qt. to 9qt.?	
7. What is the ratio of 3qt. to 5gal.?	Ans. 6.6.

8. What is the ratio of 7fur. to 11mi.? Ans. 12.57142.
9. What is the ratio of £.5 to 15s.?
10. What is the ratio of $\frac{1}{2}$ to $\frac{7}{5}$? Ans. 1.75.
11. What is the ratio of 3.75 to 11.25? Ans. 3.
12. What is the ratio of $5\frac{1}{2}$ to $17\frac{3}{4}$?
13. What is the ratio of $3\frac{1}{4}$oz. to $1\frac{1}{4}$lb. Avoir.? Ans. $6\frac{2}{13}$.
14. What is the ratio of 45min. to $\frac{1}{2}$hr.? Ans. $\frac{2}{3}$.
15. What is the ratio of 1.25cu.ft. to 2.5cu.in.?
16. What is the ratio of $\frac{3}{4}$A. to 15P.? Ans. .125.
17. What is the ratio of 1hhd. to 25gal.? Ans. .3668+.
18. What is the ratio of 1.5 cubits to 65 inches?

SIMPLE PROPORTION.

§216. A *proportion* is an equality of two ratios. Thus, the two ratios, 5 : 10, and 3in. : 6in., form a proportion.

§ 217. A proportion is denoted by a double colon between the two ratios, or by a sign of equality between them.—Thus, 5 : 10 :: 3in. : 6in., read, 5 is to 10 as 3in. is to 6in. Or, 5 : 10=3in. : 6in., read, the ratio of 5 to 10 is equal to the ratio of 3in. to 6in.

§ 218. The first two terms of a proportion are called the *first couplet;* the last two are called the *second couplet:*—the first and third terms are called the *antecedents;* the second and fourth are called the *consequents:*—the first and fourth terms are called the *extremes;* the second and third are called the *means.*

Also, the fourth term is called a *fourth proportional* to the other three: and, when the means are equal to each other, either mean is called a *mean proportional* between the two extremes.

§ 219. In every proportion, the *product* of the *extremes* is equal to the *product* of the *means*.

For, if $3 : 6 :: 7 : 14$, $6 \div 3 = 14 \div 7$, or $\frac{6}{3} = \frac{14}{7}$, multiplying both terms of the first fraction by 7, and both terms of the second by 3, we have $\frac{6 \times 7}{21} = \frac{3 \times 14}{21}$, or $6 \times 7 = 3 \times 14$.

§ 220. This property enables us to find any term of a proportion when the other terms are given.

If the two means and either extreme are given, to find the other extreme, we divide the product of the means by the given extreme.

If the two extremes and either mean are given, to find the other mean, we divide the product of the extremes by the given mean.

Ex. 1. 1st Term : 6 :: 5 : 15, what is the first term?

Ans. 2.

2. 7 : 2nd Term :: 14 : 70, what is the second term?

Ans. 35.

3. $5\frac{1}{2} : 22 ::$ 3rd Term : 40, what is the third term?

4. 8 : 1.6 :: 50 : 4th Term, what is the fourth term?

Ans. 10.

5. 15 : 1.875 :: 3rd Term : 5, what is the third term?

Ans. 40.

6. $2\frac{1}{2}$: 2nd Term :: $7\frac{1}{2} : 12\frac{1}{2}$, what is the second term?

7. 1st Term : $1\frac{1}{4} :: \frac{2}{3}$ of $\frac{4}{5} : 17$, what is the first term?

Ans. $\frac{2}{5}\frac{1}{1}$.

8. 29 : 2nd Term :: 17 : 49, what is the second term?

Ans. $83\frac{10}{17}$.

9. 15 : 18 :: 3rd Term : 24, what is the third term?

10. $2\frac{1}{2} : 3\frac{1}{2} :: 4\frac{1}{2}$: 4th Term, what is the fourth term?

Ans. 6.3.

§ 221. Whichever term is required, however, the given terms may always be arranged in the first, second, and third places, so that the work shall consist in finding a fourth proportional to the three given terms. Thus,

The first question would become, 15 : 5 :: 6 : 4th Term;

The second, 14 : 70 :: 7 : 4th Term;

The third, 12 : $5\frac{1}{2}$ :: 40 : 4th Term.

In finding a fourth proportional to three concrete numbers, the first two terms must be of the same denomination, and this common denomination must be canceled before the operation is performed. In speaking, hereafter, of the first or the second term, we will always mean the number of abstract units in such term.

Ex. 11. Find a fourth proportional to 3in., 7in., and $12.

3 : 7 :: $12 : $28

7

3)$84

$28

§222. MODEL.— Multiply the third term by the second. (§183). Divide the product by the first term. (§185). The fourth term is $28.

EXPLANATION.—The necessity of considering the first two terms abstract, is evident from the fact that $12 can not be multiplied by 7in., neither can $84 be divided by 3in.

RULE.—*Multiply the third term by the second, and divide the product by the first.*

Or, *Multiply the third term by the ratio of the first to the second.*

Ex. 12. Find a fourth proportional to 15yd., 25yd., and 10da.

13. Find a fourth proportional to $5, $75, and $40.

14. Find a fourth proportional to 7da., 15da., and £2, 6s. 9d. 4th Term, £5, $2\frac{1}{7}$d.

15. Find a fourth proportional to $5\frac{1}{2}$qt., 14qt., and \$17.75.
16. Find a fourth proportional to 3pk., 2bu., and \$7.50.
4th Term, \$20.
17. Find a fourth proportional to 7.7, 4.3, and 14s.
4th Term, 7s. 9d. $3\frac{3}{11}$qr.
18. Find a fourth proportional to $3\frac{3}{4}$, $7\frac{3}{4}$, and 10gal. 3qt.
19. Find a fourth proportional to 10gal., 3qt., and 5.5da.
4th Term, .4125da.
20. Find a fourth proportional to 3s., 10da., and 5yd.
4th Term, 1.38yd.
21. If 15bu. of corn cost \$10, what will 27bu. cost?

15 : 27 :: \$10 : \$18

27

\$270 | 15

120 | \$18

0

§ 223. MODEL.— \$10 is the third term. Since the cost of 27bu. is greater than that of 15bu., 15 is the first term, and 27 the second. Multiply the third term by the second. (§ 183.) Divide the product by the first term. (§ 185.) Hence, 27bu. cost \$18.

EXPLANATION.—The ratio of 15bu. to 27bu. is evidently the same as the ratio of the value of 15bu. to the value of 27bu., that is, the ratio of \$10 to the required amount. Hence, the propriety of the proportion. The first and second terms are considered abstract from the necessity of the case.

RULE.—*Take for the third term the given number which is of the same nature with the required term: and, if the required term is evidently greater than this third term, take the greater of the remaining terms for the second and the less for the first; but, if the required term is less than the third term, take the less of the remaining terms for the second and the greater for the first.*

Find a fourth proportional to the three terms thus arranged.

Ex. 22. If 10lb. of sugar cost $1.25, what will 17.5lb. cost? Ans. $2.1875.

23. If 27bbl. of flour cost $150.75, what will 94.5bbl. cost? Ans. $527.625.

24. What cost 11gal. of molasses, if 49gal. cost $34.47?

25. What cost 3.75A. of land, if 16.375A. cost $400? Ans. $91.60+.

26. If 12 horses eat a load of hay in 10 days, how long would a load last 5 horses? Ans. 24da.

27. If 12 horses eat a load in 10 days, how many horses would eat it in 5 days?

28. If 9lb. of tea cost $10, what will 11lb. cost?

29. If a 6 penny loaf of bread weigh 5.5oz. when flour is $4.50 per bbl., what should it weigh when flour is $7 per bbl.? Ans. 3.535oz.

30. If $\frac{1}{2}$ of a yard of cloth cost $2.75, what will $\frac{7}{16}$ of a yard cost?

31. What cost 175lb. of coffee, at $5\frac{1}{2}$ lb. for a dollar? Ans. $31.81+.

32. What cost 45 pr. of shoes, if 14 pr. cost $35.50? Ans. $114.107+.

33. If $100 gain $6 in 1yr., in how many years will it gain $100?

34. If 12 men build 20rd. of masonry in a week, how many men could build 75rd. in the same time? Ans. 45 men.

35. If 10bbl. of flour will last a company of soldiers 15da., how long will 1000lb. last them? Ans. 7.653da.

36. If 14lb. of butter cost $4.25, what will 1.75 firkins cost?

37. If a boat travels 75mi. in 6hr., how far does it go in 25min.? Ans. 5mi. 1fur. 26.6rd.

38. If 87.5lb. of coffee cost £4, 12s. 6d., what will 7.5lb. cost? Ans. 7s. 11d.

39. If a man can walk 10mi. in 3hr., how far can he walk in 5da. of 8hr. each?
40. If 5.5lb. of sugar cost \$1.00, what will 1lb. cost?
Ans. \$ .18.
41. If $5\frac{1}{2}$bu. of wheat make 1bbl. of flour, how much flour will 2500lb. of wheat make? Ans. 7.57bbl.
42. If $1\frac{5}{6}$gal. of molasses cost \$1.29, what will $1\frac{1}{2}$hhd. cost?
43. If $10\frac{1}{2}$yd. of cloth cost \$11.625, what will $16\frac{2}{3}$yd. cost?
Ans. \$18.452+.
44. If $\frac{3}{5}$ of a ship cost £500, 7s. 3d., what will $\frac{5}{16}$ of her cost? Ans. £260, 12s. 1.3125d.
45. If 3 reams of paper cost \$7.75, how many reams can be bought for \$17.75?
46. How many lb. Avoir. are equal to 500 lb. Troy?
47. A grocer was detected in using as a gallon measure a vessel containing 3qt. 1pt. $2\frac{1}{2}$gi.: how many true gallons were in 47.5 of his false gallons? A. 45.273+gal.
48. The same man used for his purchases a vessel containing 4qt. 2gi.: how many true gallons were in 47.5 of these false gallons?
49. How many of his selling gallons were in 47.5 of his buying gallons? Ans. 52.95+.
50. If 100lb. of gunpowder require 75lb. of saltpetre, how much saltpetre will 22.25lb. of gunpowder require?

COMPOUND PROPORTION.

§ 224. A compound ratio is the product of two or more simple ratios.

Thus, $\left.\begin{matrix}3 : 5\\4 : 7\end{matrix}\right\}$ :: 12 : 35, is a compound ratio.

§ 225. A compound proportion is an expression of equality between a compound and a simple ratio.

§ 226. A compound ratio is reduced to a simple one, by multiplying together its corresponding terms. Thus, in the above instance, the antecedent 12 is the product of the antecedents 3 and 4, and the consequent 35 is the product of the consequents 5 and 7.

§ 227. If the first three terms of a compound proportion are given, the fourth term may be found by multiplying the third term by the product of the second terms, and dividing this product by the product of the first terms: the first and second terms being reduced to the same denomination in each simple ratio, and then considered abstract.

Ex. 1. If 5 hands can hoe 24A. of cotton in 4da., how many Acres can 17 hands hoe in 11da.?

$$\left.\begin{matrix} 5 : 17 \\ 4 : 11 \end{matrix}\right\} :: 24\text{A.} : 224\tfrac{2}{5}\text{A.}$$

$$\begin{array}{r} 20 \quad 187 \\ \hline 24\text{A.} \\ \hline 748 \\ 374 \\ \hline 2,0)448,8\text{A.} \\ \hline 224\tfrac{2}{5}\text{A.} \end{array}$$

§ 228. Model.—24A. is the third term. Since 17 hands can hoe more than 5 hands, 5 is a first term, and 17 a second. Since more land can be hoed in 11da. than in 4 da., 4 is a first term and 11 a second. 4 times 5 are 20. 11 times 17 are 187. Multiply the third term by 187. (§ 183.) Divide the product by 20. (§ 185.) Hence the required number is $224\frac{2}{5}$A.

Explanation.—As in simple proportion, we take for the third term that which is of the same nature with the required term. The remaining numbers are in pairs of similar terms, and each pair is arranged as if the question depended upon it alone.

RULE.—*Take for the third term the given number which is of the same nature with the required term. Take the remaining numbers in pairs of the same nature, and arrange each pair as in simple proportion.*

Multiply the third term by the product of the second terms, and divide this product by the product of the first terms.

Ex. 2. If 60 men can do a piece of work in 40 days of 8 hours each, how many men can do three times the work in 90 days of 10 hours each? Ans. 64 men.

3. If 10 horses eat 88bu. of corn in 45 days, how many horses will eat 120bu. in 50 days?
4. If I can travel 75 miles in $2\frac{1}{2}$da. of $7\frac{1}{2}$hr. each, how many da. of 10 hr. each would it take me to travel 225mi.? Ans. $5\frac{5}{9}$da.
5. If 5 white men can do as much work as 7 negroes, how many days of 10hr. each will be required for 25 negroes to do a piece of work which 30 white men can do in 10 days of 9hr. each? Ans. 15.12da.
6. If by traveling 7hr. per da. at 4mi. per hr. I go 280mi. in 10da., how far will I go in 12da. by traveling 8hr. per da. at $4\frac{1}{2}$mi. per hr.?
7. If 12 men build 9 rods of wall in 10 days, how many men can build 27 rods in 5 days? Ans. 72 men.
8. If \$100 gain \$6 in 12mo., what will \$375 gain in 9mo.? Ans. \$16.875.
9. If 100lb. be carried 100mi. for 35ct., what will be the freight on 10000lb. carried 75mi.?
10. If 9 men, in 10 days, can build a wall 25rd. long, 3yd. high, and 5ft. thick, in what time can 20 men build a wall 30rd. long, 4yd. high, and 4ft. thick? A. 5.76da.
11. If 100 men, in 5da of 10hr. each, can dig a ditch 150 yd. long, 3yd. wide, and 5ft. deep, how many men, in

8da. of 9hr. each, can dig a ditch 200yd. long, $3\frac{1}{3}$yd. wide, and 2yd. deep? A. omitting fraction, 123 men.

12. If \$100 gain \$7 in 12mo., in how many months will \$700 gain \$100?

13. If \$100 gain \$8 in 12mo., what sum of money will gain \$160 in 18mo.? Ans. 1333.33$\frac{1}{3}$.

14. If \$900 gain \$135 in 18mo., what will \$100 gain in 12mo.? Ans. \$10.

15. If 6 school-girls spend \$72 pocket-money in 4wk., what will 10 girls spend in 6wk.?

16. If 900 soldiers eat 70bbl. of flour in 20da., how many days will 200bbl. last 3000 soldiers at half rations? Ans. $34\frac{2}{7}$da.

PARTITIVE PROPORTION; OR, FELLOWSHIP.

§ 229. Partitive Proportion is the division of a number into two or more parts which shall have to each other a given ratio.

The terms of the ratio are called the proportional terms, and in the operation they must be regarded abstract.

Ex. 1. Divide 450 into three parts, which shall be to each other as 2, 3, and 4.

9 : 2 :: 450 : 100
9 : 3 :: 450 : 150
9 : 4 :: 450 : 200

§ 230. Model.—2 and 3 are 5, and 4 are 9. 9 is to 2 as 450 is to the first part. Multiply 450 by 2. Divide the product by 9. The first part is 100. 9 is to 3 as 450 is to the second part. Multiply 450 by 3. Divide the product by 9. The second part is 150. 9 is to 4 as 450 is to the third part. Multiply 450 by 4. Divide the product by 9. The third part is 200. Hence, 100, 150, and 200, are the three parts required.

EXPLANATION.—One half of the first part is evidently equal to one third of the second or one fourth of the third: so that if the whole number be divided into 9 equal parts, the first will contain 2, the second 3, and the third 4, of those parts. Hence the truth of the proportions.

RULE.—*As the sum of the proportional terms is to either term, so is the whole number to be divided to the part corresponding to that term.*

PROOF.—Add the several parts together: their sum is equal to the whole number divided.

Ex. 2. Divide \$1000 into three parts which shall be to each other as, 6, 1, and 3. \$600, \$100, and \$300.

3. Divide 144A. into three parts in the proportion of $1\frac{1}{4}$, $3\frac{1}{4}$, and $4\frac{1}{2}$.
4. Divide 226gal. into four parts in the proportion of $1\frac{3}{4}$, $2\frac{1}{3}$, $2\frac{1}{2}$, and $2\frac{5}{6}$. 42gal., 56gal., 60gal., and 68gal.
5. Divide 992.95 into four parts in the proportion of 1.25, 3.2, 4.73, and 5.005. 87.5, 224, 331.1, 350.35.
6. Divide \$43.20 into four parts in the proportion of 1, 3, 5, and 7.
7. Two men, A and B, engage in business with a joint capital of \$6000, of which A furnishes \$2500 and B the remainder. What is each one's share of a gain of \$1200? Ans. \$500, \$700.
8. Two men, C and D, gain \$1275 on a capital of which C's share is double D's; what is each one's share of this gain? Ans. \$850, \$425.
9. If E invests \$2375, and F \$3225, and the firm loses \$700, how much must each partner lose?
10. A invests \$3000, and B \$3500, in a certain business, in which the first year they lose \$325. After paying

this loss from the funds of the firm, they take in C as a partner with a capital of $4000, and the second year the new firm gains $2035. How much of this gain is due to each partner? Ans. A, $570; B, $665; C, $800.

11. D and E form a partnership for two years, D contributing $5000, and E $1750. The first year they gain $1350. D spends his share of the gain, but E leaves his share among the funds of the firm. The second year they gain $2130. What is each partner's share of this last gain? Ans. D, $1500; E, $630.

12. Messrs. Jones, Smith, and Brown gained $5000; what is each man's share of this gain, if Smith owns twice as much of the capital as Brown, and Jones as much as Smith and Brown together?

13. In a certain firm, A owns $1\frac{1}{4}$ times as much stock as B, C owns $1\frac{1}{5}$ times as much as A, and D owns $1\frac{1}{6}$ times as much as C: A's gain on a year's transactions is $500; what is the gain of each of the other partners? Ans. B, $400; C, $600; D, $700.

14. A merchant owes one creditor $2000, and an other $3500: having failed, he can pay them both only $4015: how much should each creditor receive? Ans. $2555, and $1460.

15. A man dying wills to one son $2000, to an other son $1500, and to his daughter $1250; but after paying his debts his executor has in his hands only $3000. How much should he pay to each legatee?

---

16. Three partners, A, B, and C, invest as follows:—A invests $500 for 2 months; B, $400 for 3 months; and C, $300 for 5 months. They gain $740. What ought each partner to receive?

$500 \times 2 = 1000$ 3700 : 1000 :: \$740 : \$200
$400 \times 3 = 1200$ 3700 : 1200 :: \$740 : \$240
$300 \times 5 = 1500$ 3700 : 1500 :: \$740 : \$300
3700

§ 231. MODEL.—Twice 500 are 1000. 3 times 400 are 1200. 5 times 300 are 1500. The sum of these proportional parts is 3700. [Proceed as in § 230.] Hence A ought to receive \$200, B, \$240, and C, \$300.

EXPLANATION.—A's investment of \$500 for 2 months is equal to an investment of twice \$500, or \$1000, for 1 month; so B's \$400 for 3 months is equal to 3 times \$400, or \$1200, for 1 month; and C's \$300 for 5 months is equal to 5 times \$300, or \$1500, for 1 month. The several investments, being thus referred to the same unit of time, evidently furnish equitable proportional terms.

This is an example of what is called COMPOUND FELLOWSHIP.

Ex. 17. A firm of two partners gained \$1750: what was each partner's share of the gain, if A contributed \$3000 for 10 months, and B \$2500 for 1 year?

Ans. A's, \$875; B's, \$875.

18. A, B, C, and D, rented a pasture for \$100. A kept 20 head of cattle in it 6 months; B kept 25 head 5 months; C kept 30 head $5\frac{1}{2}$ months; and D kept 50 head 3 months. What part of the rent ought each man to pay?

19. In a certain partnership A contributed \$3000 Jan. 1st; B contributed \$2500 Feb. 1st; and C contributed \$4000 May 1st. On the 1st of August, they lost by fire \$4000. What part of the loss did each partner sustain? Ans. A, \$1750; B, \$1250; C, \$1000.

20. Three partners in trade gained \$3008 after 15 months' business. A put in \$1000 at first, and \$2000 3 months

afterwards; B put in at first $4000, but took out $2000 6 months afterwards; and C put in $2000 at the end of 5 months, and $2000 5 months afterwards. What was each partner's share of the gain?

Ans. A's, $1092; B's, $1176; C's, $740.

PROMISCUOUS PROBLEMS.

1. If 8bu. of wheat are worth as much as 15bu. of corn, and 5bu. of corn as much as 2cwt. of hay, and 4cwt. of hay are worth $6; how many bu. of wheat can be bought for $45?

8bu.	15	45.4.5.8bu.=7200bu.
5	2	6.2.15=180
4	6	7200bu.÷180=40bu.
45		

§ 232. MODEL.—Set 8bu. on the left, and 15 on the right; 5 on the left, and 2 on the right; 4 on the left, and 6 on the right, and 45 on the left. Multiply together the terms on the left. Multiply together the terms on the right. Divide 7200bu. by 180. The quotient is 40bu. Hence $45 will pay for 40bu. of wheat.

EXPLANATION.—This question, commonly referred to a distinct head, called CONJOINED PROPORTION, or the Chain Rule, is merely a complicated case of simple proportion, as will be seen by stating it thus:—

1. If 4cwt. hay cost $6, how many cwt. will cost $45?
   6 : 45 :: 4cwt. : 30cwt. Hence, 30cwt. hay=$45.
2. If 2cwt. hay=5bu. corn, how many bu. corn=30cwt. hay?
   2 : 30 :: 5bu. : 75bu. Hence, 75bu. corn=$45.
3. If 8bu. wheat=15bu. corn, how many bu. wheat=75bu. corn?
   15 : 75 :: 8bu. : 40bu. Hence, 40bu. wheat=$45.

Comparing this work with the model, we see that the means in the proportions are 45, 4, (30,) 5, (75,) and 8; and the extremes, except the last, are 6, (30,) 2, (75,) and 15; and that, omitting the two terms, 30 and 75, common to both, we have left in the one case the terms on the left in the model, and in the other the terms on the right. And since the product of the means is equal to the product of the extremes, the product of the terms on the left divided by the product of the terms on the right will give the required term.

The term similar to the required term is called the *odd* term, and the one equivalent to the required term is called the term of *demand*. Both of these must be placed on the left of the vertical line, and the other terms must be arranged so that equivalents shall be opposite each other, and no two similar terms on the same side. In the operation, all but the odd term must be regarded abstract.

2. If 2bbl. of flour are worth as much as $26\frac{2}{3}$bu. of corn, and 3bu. of corn as much as $7\frac{1}{2}$lb. of bacon, how many lb. of bacon are equivalent to 3bbl. of flour?

Ans. 100lb.

3. If £93 are equal to 2420fr., and $166\frac{2}{3}$fr. are equal to $31, and $7 are equal to 4bu. of wheat, how many bu. of wheat are equal to £15?

4. If A can do as much work in 5 days as B can do in 6, B can do as much in 7 days as C can do in 8, and C can do as much in 9 days as D can do in 10; in how many days can A do as much as D can do in 15?

Ans. 9.84da.

5. If 10 Ells Flemish are equal to 6 Ells English, and 4 Ells English to 5 yards, and 12 yards to 8 Ells French;

how many Ells French are equal to 16 Ells Flemish?
Ans. 8 E. Fr.

6. If 19lb. of butter are worth 30lb. of cheese, and 19lb. of cheese are worth $3; how many lb. of butter are worth $7.50?

7. If a train of cars travel a mile in 2.5min., how long will it be in going 45 miles? Ans. 1hr. 52.5min.

8. If 8 men can mow a meadow in 10 days of 13hr. each, in how many days of 11hr. each can 12 men mow it?
Ans. 17.72da.

9. A, B, and C formed a partnership for two years: the first year, they lost $500; the second year, they gained $750; how much is each partner entitled to at the end of the second year, if A contributed $4000, and B and C $3000 each to the funds of the firm?

10. A and B formed a partnership for two years from Jan. 1, 1860. On that day, A contributed $1000, and B $500: July 1, 1860, A added $500 to his investment, and Oct. 1, 1860, B added $500 to his: Jan. 1, 1861, A withdrew $250 from the funds of the firm, and Mar. 1, 1861, B contributed $500 more. They gain $1090. How much of this ought each partner to receive?
Ans. A, $600; B, $490.

11. If 5bbl. of cider are worth 8bu. of wheat, and 11bu. of wheat are worth 2T. of coal, and 3T. of coal are worth 50lb. of tea, and 4lb. of tea are worth 5oz. of quinine, and 7oz. of quinine are worth $6.50; how many dollars are 10bbl. of cider worth? Ans. $56.277.

12. If the transportation of 100lb. 100mi. cost $2.15, what will it cost to transport 2500lb. 25mi.?

13. What is the smallest number that can be exactly divided by either 12, 13, or 14? Ans. 1092.

14. What is the largest number that will exactly divide either 240, 720, or 840? Ans. 120.
15. What is the total cost of 4yd. of silk at $1.875 per yd., 8yd. of berege at $.625 per yd., 3doz. buttons at $.75 per doz., and 7.5yd. of calico at $.25 per yd.?
16. What is the total cost of 5lb. of tea at 5s. 6d., 7bu. of corn at 4s. 4d., 8bu. of wheat at 11s. 9d., and 11gal. of molasses at 7s. 3d.? Ans. £11, 11s. 7d.
17. What interval elapsed between Dec. 5, 1813, and Mar. 17, 1842? Ans. 28yr. 3mo. 12da.
18. What interval elapsed between Jan. 30, 1833, and Sept. 3, 1862?
19. John Jones was born Mar. 9, 1827, and was married when he was 22yr. 6mo. 19da. old; when was he married? Ans. Sept. 28, 1849.
20. If 5 men can plough $47\frac{1}{2}\frac{9}{4}$ acres in $7\frac{2}{5}$ days, in how many days can 6 men plough 31 acres? Ans. 4 days.
21. If $5\frac{1}{2}$cwt. of hemp cost $27.50, how much hemp will cost 33.33\frac{1}{3}$?
22. A merchant bought 3000bu. of salt: after having sold to A 100.5bu., to B 477.75bu., to C 329.8375bu., and to D 1200.25bu., how much has he left?
Ans. 891.6625bu.
23. What is the produce of 15.375A. of corn, at 8bbl. 3bu. 3pk. to the acre? Ans. 134bbl. 2bu. 2pk. 5qt.
24. A field of 25 acres produced 637.5bu. of wheat; how much was that per acre?
25. A sum of money divided equally among 17 men gives to each $17.765; if divided equally among 11 men, how much would each get? Ans. $27.455.
26. If 20 men in 35da. earn $320, how many men will earn $480 in 70da.? Ans. 15 men.

27. If 18 horses eat 10bu. of oats in 20da., how many horses will eat 60bu. in 36da.?
28. What is the greatest common measure of 75, 825, and 1575? Ans. 75.
29. What is the least common multiple of 46, 230, and 115? Ans. 230.
30. What are the different prime factors of 24400?

PERCENTAGE.

§ 233. PERCENTAGE includes all cases of proportion in which the first term is *one hundred*.

The phrase, *per centum*, that is, per hundred, is usually written, and often pronounced, *per cent.* Thus, instead of "six dollars per hundred," we usually say "6 per cent."

Ex. 1. A lawyer collected $3725; what is his commission at 3 per cent.?

100 : 3725 :: $3 : $112.75 This proportion is evidently correct: and all similar problems may be solved in the same manner. But, inasmuch as the three given terms have always the same unit, the same result will be obtained by regarding the second term concrete and the first and third abstract, by dividing the third by the first and multiplying the second by this quotient. This method, being a little less troublesome, is the one usually adopted. To explain it more fully, we must give the following definitions.

§ 234. The price or amount per hundred is called the *rate per cent.* Thus, in the above example, 3 is the rate per cent.

§ 235. If the rate per cent. be divided by 100, the quotient is called the *rate per unit.* In all operations, this is regarded as an abstract number. Thus, .03 is the rate per unit in the example above.

What is the rate per unit for 6 per cent.? for 10 per cent.? for 50 per cent.? for 75 per cent.? for $1\frac{1}{2}$ per cent.? for $\frac{1}{4}$ per cent.? for 100 per cent.? for $33\frac{1}{3}$ per cent.? for $12\frac{1}{2}$ per cent.? for $18\frac{3}{4}$ per cent.? for $\frac{1}{2}$ per cent.? for $\frac{1}{2\,5}$ per cent.?

§ 236. The number on which percentage is calculated, is called the *basis of percentage.* Thus, above, $3725 is the basis.

§ 237. The result of the operation is called the *percentage.* Thus, above, $112.75 is the percentage.

Ex. 2. Find 5 per cent. of $5750.

$5750
.05
$287.50

§ 238. MODEL.—Multiply the basis by the rate per unit. (§ 183.) The product is $287.50. Hence the percentage is $287.50.

EXPLANATION.—5 per cent. of any number is evidently 5 hundredths of that number, and this is found by multiplying the number by .05. Observe that the rate per unit is simply one hundredth of the rate per cent., and is most conveniently expressed as a decimal fraction.

RULE.—*Multiply the basis by the rate per unit. The product will be the percentage.*

Ex. 3. What is 1 per cent. of 7500?

4. What is 2 per cent. of 250? Ans. 5.
5. What is 3 per cent. of 275? Ans. 8.25.
6. What is 4 per cent. of 775?
7. What is 6 per cent. of $325? Ans. $19.50.
8. What is 7 per cent. of $9250? Ans. $647.50.
9. What is 8 per cent. of 725 men?

10. What is 9 per cent. of 1700 men? Ans. 153 men.
11. What is $\frac{1}{10}$ per cent. of \$1000? Ans. \$1.
12. What is $1\frac{1}{8}$ per cent. of \$175?
13. What is $2\frac{5}{8}$ per cent. of \$27.75? Ans. \$.7284375.
14. What is $3\frac{1}{3}$ per cent. of \$630? Ans. \$21.
15. What is $4\frac{3}{5}$ per cent. of 795?
16. What is $7\frac{1}{2}$ per cent. of 2775.25? Ans. 208.14375.
17. What is $9\frac{3}{4}$ per cent. of 473.75? Ans. 46.190625.
18. What is $10\frac{1}{10}$ per cent. of 275?
19. What is $16\frac{2}{3}$ per cent. of 1500? Ans. 250.
20. What is $66\frac{2}{3}$ per cent. of \$750? Ans. \$500.
21. What per cent. of 690 is 115?

$$\begin{array}{r|l} 115.00 & 690 \\ 690 & .16\frac{2}{3} \\ \hline 4600 & 100 \\ 4140 & 16\frac{2}{3} \\ \hline 460 & 2 \\ 230 & \overline{690} \end{array}$$

§ 239. MODEL.—Divide the percentage by the basis. (§ 52.) The quotient is $.16\frac{2}{3}$. Multiply this quotient by 100. The product is $16\frac{2}{3}$. Hence, 115 is $16\frac{2}{3}$ per cent. of 690.

EXPLANATION.—Since the percentage is equal to the basis multiplied by the rate per unit, conversely the rate per unit is equal to the percentage divided by the basis. And, since the rate per unit is one hundredth of the rate per cent., conversely the rate per cent. is found by multiplying the rate per unit by 100.

RULE.—*Divide the percentage by the basis. The quotient will be the rate per unit. Multiply the rate per unit by* 100. *The product will be the rate per cent.*

Ex. 22. What per cent. of 700 is 70? Ans. 10 per cent.
23. What per cent. of 375 is 125? Ans. $33\frac{1}{3}$ per cent.
24. What per cent. of 1000 is 125?
25. What per cent. of 550 is 110? Ans. 20 per cent.

26. What per cent. of $675 is $337.50? Ans. 50 per cent.
27. What per cent. of $1000 is $875?
28. What per cent. of $5000 is $250? Ans. 5 per cent.
29. What per cent. of $10000 is $50? Ans. $\frac{1}{2}$ per cent.
30. What per cent. of $150 is $300?
31. What per cent. of 3000 is 4000? Ans. $133\frac{1}{3}$ per cent.
32. What per cent. of 275 is 302.5? Ans. 110 per cent.
33. What per cent. of 245 is 735?
34. What per cent. of 200 is 500? Ans. 250 per cent.
35. What per cent. of 325 is 2925? Ans. 900 per cent.
36. What per cent. of $1.25 is $1.50?
37. What per cent. of $1.00 is $.375? Ans. $37\frac{1}{2}$ per cent.
38. What per cent. of $.875 is $.50? Ans. $57\frac{1}{7}$ per cent.
39. What per cent. of $.66\frac{2}{3}$ is $.22\frac{2}{9}$?
40. What per cent. of $.125 is $.0625? Ans. 50 per cent.
41. A commission merchant purchases articles amounting to $247.75; what is his commission, at $2\frac{1}{2}$ per cent.?
42. What is the commission on $312, at 12 per cent.?
43. A merchant insured a vessel and cargo, valued at $75000, at $7\frac{3}{4}$ per cent.; what did he pay?
44. What premium must I pay for the insurance of my life, the policy being $5000, and the rate 2.35 per cent.? Ans. $117.50.
45. What is the premium for insuring $9450, at $\frac{3}{5}$ per cent.?
46. What is the insurance on a dwelling and furniture valued at $25550, at $1\frac{1}{4}$ per cent.? Ans. $319.375.
47. What is the duty, at 40 per cent., on French broadcloths valued at $15375? Ans. $6150.
48. What is the duty, at 20 per cent., on $6250 worth of Italian silk?
49. At $7\frac{1}{2}$ per cent., what is the duty on an invoice of Geneva watches, valued at $7475? Ans. $560.625.

50. At 50 per cent., what is the duty on a case of Leghorn hats, worth $1500? Ans. $750.
51. What tax should be paid on $17725 worth of real estate, at $\frac{1}{2}$ per cent.?
52. What is the tax on $261000, at .15 per cent.? Ans. $391.50.
53. What is the tax on $17150, at 60ct. on $100? Ans. $102.90.
54. What is the amount of a dividend of 3 per cent., on $4200 of bank stock?
55. The North Carolina Railroad company declared a dividend of $2\frac{1}{2}$ per cent.: what did I receive on 14 shares of $100 each? Ans. $35.
56. A merchant bought broadcloth at $3.50 per yard; at what price must he sell it, to gain 40 per cent.? Ans. $4.90.

The percentage must be added to the basis.

57. A grocer bought candles at 25ct. per lb.: how must he sell them, to gain 30 per cent.?
58. If broadcloth cost $4.00 per yd., how much will it bring at a loss of 35 per cent.? Ans. $2.60.

The percentage must be subtracted from the basis.

59. A dealer bought 50bbl. of flour at $12 per bbl., but was forced to sell it at a decline of 20 per cent.: what did he get for it all? Ans. $480.
60. A speculator bought $35000 worth of cotton, and sold it at a loss of 15 per cent.: what did he receive for it?
61. A man pays $406.25 for the insurance of his dwelling, valued at $32500: what is the rate per cent.? Ans. $1\frac{1}{4}$ per cent.
62. A vessel worth $15400 was insured for $539: what was the rate per cent.? Ans. $3\frac{1}{2}$ per cent.

63. At what rate per cent. will the insurance on $11500 cost $172.50?

64. A man had his life insured for $277.50: what was the rate per cent., the policy being $10000?

Ans. 2.775 per cent.

65. If the duty on $3457 worth of goods is $1037.10, what is the rate per cent.? Ans. 30 per cent.

66. What is the rate of duty, when $12657 worth of clothing pays $6328.50?

67. If $45000 worth of property pays a tax of $229.50, what is the tax on $100? Ans. $.51.

68. I paid a broker $21.125 for investing $8450 in Government stocks; what was his rate of brokerage?

Ans. $\frac{1}{4}$ per cent.

69. My attorney charged me 260.73\frac{3}{4}$ for collecting $3476.50: what was his rate of commission?

70. I bought a farm for $4000, and sold it for $5000; what did I gain per cent.? Ans. 25 per cent.

The first cost subtracted from the selling price leaves the total gain.

71. I bought a farm for $5000, and sold it for $4000; what did I lose per cent.? Ans. 20 per cent.

The selling price subtracted from the first cost leaves the total loss. Observe that in each case the first cost *is the basis. Hence the difference in the answers of the last two questions.*

72. If I buy calico at 10ct., and sell it at 12$\frac{1}{2}$ct., what do I gain per cent.?

73. If I buy calico at 12$\frac{1}{2}$ct., and sell it at 15ct., what do I gain per cent.? Ans. 20 per cent.

74. A man bought a house for $7625, and sold it for $8387.50; what did he gain per cent.? Ans. 10 per cent.

75. A man having paid $7625 for his house, was compelled to sell it for $6862.50: how much per cent. did he lose?

---

76. By selling an article for $1300, I gain 30 per cent. on it: what did it cost me?

$1300.00 | 130	1.30 | $1000
0000	

§240. MODEL.—Divide the selling price by 1+the rate per unit. (§165.) The quotient $1000 is the first cost.

EXPLANATION.—It is evident that 1+the gain per unit : 1 :: the first cost+the whole gain : the first cost. But the selling price, $1300, is evidently the first cost+the whole gain. Then since the second term of the proportion is always 1, it is easy to see the truth of the

RULE.—*To find the first cost, when the selling price and the rate per cent. of* gain *are given. Divide the selling price by* 1 + *the rate per unit. The quotient will be the first cost.*

Ex. 77. By selling a piece of muslin for $50, I gain 100 per cent.; what did I give for it? Ans. $25.

78. What did I pay for eggs, if I gain 33⅓ per cent. by selling them for 16ct. per doz.?

79. A grocer sold a lot of sugar for $1058, gaining thereby 15 per cent.: what did the sugar cost him? A. $920.

80. A merchant sells some flour for $924, and gains 12 per cent. on it: what did he pay for it? Ans. $825.

81. The selling price is $1800, the gain 20 per cent.: what is the first cost?

---

82. A merchant sold a quantity of cloth for $1410, and thus sustained a loss of 6 per cent.: what did the cloth cost him?

$$\begin{array}{r|l} \$1410.00 & .94 \\ 94 & \$1500 \\ \hline 470 & \\ 470 & \\ \hline 000 & \end{array}$$

§ 241. MODEL.—Divide the selling price by 1—the rate per unit. (§165.) The quotient $1500 is the first cost.

EXPLANATION.—Evidently, 1 — the loss per unit : 1 :: the first cost—the whole loss : the first cost. But the first cost—the whole loss is evidently the selling price, $1410. The second term of this proportion is always 1, and hence the following

RULE.—*To find the first cost, when the selling price and the rate per cent. of* loss *are given. Divide the selling price by* 1—*the rate per unit. The quotient will be the first cost.*

Ex. 83. The selling price is $8000; the loss 20 per cent.: what is the first cost? Ans. $10000.

84. By selling flour at $12.25 per bbl., I lost 12½ per cent. of what it cost me; what did it cost?

85. I remitted $3150 to my commission merchants to lay out in groceries after retaining 5 per cent. of what he spent: how much did he spend for me? Ans. $3000.

This is precisely similar in principle to the foregoing.

86. How much sugar at 10ct. per lb. can I get by remitting $364.87½ to a merchant who charges 5 per cent. commission? Ans. 3475 lb.

87. How much stock at 15 per cent. advance can I buy for $1265?

88. How much stock at 15 per cent. below par can I buy for $935? Ans. $1100.

89. A father settled his son with property worth $10000: the first year he lost 20 per cent. of it, and the second year he gained 25 per cent. of what he had left; how much had he then? Ans. $10000.

90. A merchant sold some sugar for $1402.50, and lost thereby 15 per cent. What did it cost him?

91. How much stock at a discount of $3\frac{1}{3}$ per cent. can be bought for $5790? Ans. $6000.

SIMPLE INTEREST.

§ 242. INTEREST is the price paid by the borrower for the use of money loaned.

§ 243. The sum of money on which interest is calculated is called the *principal.*

§ 244. The sum of the principal and interest is called the *amount.*

§ 245. The price paid for the use of one hundred dollars one year is called the *rate per cent. per annum.*

Ex. 1. What is the interest of $375, for 2yr. 10mo. 20da., at 7 per cent. per annum?

$375 2yr. 10mo. 20da. 7p. c.
.07

2 | $26.25 = 1yr.
2

$52.50 = 2yr.
2 | 13.125 = 6mo. }
3 | 6.562 = 3mo. } 10mo.
2 | 2.187 = 1mo. }
3 | 1.093 = 15da. } 20da.
.364 = 5da. }

$75.831 = 2yr. 10mo. 20da.

§ 246. MODEL.—Multiply the principal by the rate per unit. (§ 183.) The product, $26.25, is the interest for 1 year. Multiply the interest for 1yr. by 2. The product is $52.50, the interest for 2yr. 6mo. is one half of 1yr. Divide the interest for 1yr. by 2. The quotient is $13.125, the interest for 6mo. 3mo. is one half of 6mo. Divide the interest for 6mo. by 2. The quotient is $6.562, the interest

for 3mo. 1mo. is one third of 3mo. Divide the interest for 3mo. by 3. The quotient is $2.187, the interest for 1 mo. 15da. is one half of 1mo. Divide the interest for 1 mo. by 2. The quotient is $1.093, the interest for 15da. 5da. is one third of 15da. Divide the interest for 15da. by 3. The quotient is $ .364, the interest for 5da. Add the partial interests together. The sum is $75.831, the interest for the whole time.

EXPLANATION.—Since the rate is 7 per cent. per annum, the interest of the given principal for 1 year is found by multiplying the principal by the rate per unit. Thus far the work is simple percentage. For longer or shorter periods of time the interest is proportional to the time: hence we take such aliquot parts of the interest for 1 year, &c., as the periods in question severally require.

In the calculation of interest, a month is considered equal to 30 days, and a year to 360 days.

RULE.—*Multiply the principal by the rate per unit. The product will be the interest for* 1 *year.*

Multiply the interest for 1 *year by the number of years, and take aliquot parts for periods of time less than a year.*

To find the amount, add the interest to the principal.

Ex. 2. What is the interest of $50 for 2yr. at 6 per cent. per annum? Ans. $6.

3. What is the interest of $75 for 6mo. at 7 per cent. per annum?

4. What is the amount of $100 for 9mo. at 8 per cent. per annum? Ans. $106.

5. What is the interest of $125 for 3yr. at 9 per cent. per annum? Ans. $33.75.

6. What is the interest of $225 for 2yr. 6mo. at 10 per cent. per annum?

7. What is the interest of $150.75 for 1yr. 3mo. at 5 per cent. per annum? Ans. $10.67.
8. What is the amount of $175.50 for 2yr. 9mo. at 6 per cent. per annum? Ans. $204.45.
9. What is the interest of $305.50 for 3yr. 5mo. 15da. at 6 per cent. per annum?
10. What is the interest of $574.95 for 4yr. 7mo. 25da. at 5 per cent. per annum? Ans. $133.75.
11. What is the interest of $615.49 for 5yr. 11mo. 22da. at 7 per cent. per annum? Ans. $257.47.
12. Find the amount of $777.75 for 3yr. 2mo. 20da. at 8 per cent. per annum.
13. Find the interest of $1225 for 5yr. 5mo. 5da. at 5 per cent. per annum. Ans. $332.62.
14. Find the interest of $1525.25 for 1yr. 2mo. at 8 per cent. per annum. Ans. $142.356.
15. Find the interest of $2790 for 2yr. 7mo. at 9 per cent. per annum.
16. Find the amount of $1724.25 for 12yr. 6mo. at 8 per cent. per annum. Ans. $3448.50.
17. Find the interest of $3500 for 7yr. 3mo. 10da. at 7 per cent. per annum. Ans. $1783.05.
18. Find the interest of $4275 for 16yr. 8mo. at 6 per cent. per annum.
19. Find the interest of $5550 for 15yr. 11mo. 27da. at 9 per cent. per annum. Ans. $7987.83¾.
20. Find the amount of $2995 for 8yr. 4mo. at 12 per cent. per annum. Ans. $5990.
21. Find the interest of $3827 for 17yr. 3mo. 15da. at 10 per cent. per annum. Ans. $6617.52.

CONCISE METHOD FOR 6 PER CENT. PER ANNUM.

Ex. 22. What is the interest of $247.50 for 2yr. 6mo. 18 da. at 6 per cent. per annum?

2yr. 6mo. 18da. = 30.6mo.

200)30.6
.153

$247.50
.153
74250
123750
24750
$37.86750

§ 247. MODEL.—Reduce the given time to months. (§ 193.) Divide the number of months by 200. (§ 165.) Multiply the principal by this quotient. (§ 162.) The product is 37.86\frac{3}{4}$, the interest required.

EXPLANATION.—Since the rate is 6 per cent. per annum, or for 12 months, one half of the number of months is the rate per cent. for any length of time: and this rate per cent. divided by 100, gives the corresponding rate per unit, by which the principal must be multiplied, to find the interest.

For any other rate, we may find the interest at 6 per cent., and increase or diminish it, as the case may require. For instance, for 7 per cent., add to the interest found by this method, one sixth of itself: for 5 per cent., from the interest thus found subtract one sixth of itself; &c. Or, generally, find the interest at 6 per cent., divide it by 6, and multiply the quotient by the given rate.

RULE.—*Divide the number of months in the given time by* 200, *and multiply the principal by the quotient. The product will be the interest.*

Or, Multiply the number of years by 6, *and divide the product by* 100: *Divide the number of months by* 2, *and divide the quotient by* 100: *Divide the number of days by*

6, *and divide the quotient by* 1000 : *Add these three results together, and multiply the principal by their sum.*

After finding the interest at 6 *per cent., as above, to find the interest at any other rate ; Divide the interest at* 6 *per cent. by* 6, *and multiply the quotient by the required rate.*

SECOND METHOD FOR 6 PER CENT.

Ex. 23. Find the interest of $275.75 for 3yr. 10mo. 21 da. at 6 per cent. per annum.

3yr. 10mo.=46mo.

3)21 $275.75
7 .467

193025
165450
110300

2)$128.77525

$64.3876

§ 248. MODEL.—Reduce the years and months to months.—Divide the number of days by 3. Annex the quotient to the number of months. Divide this result by 1000. Multiply the principal by this quotient. Divide this product by 2. The quotient is $64.38¾, the interest required.

EXPLANATION.—This method is evidently the same in principle as the preceding, and is preferable to the other only on account of its greater freedom from liability to fractions. Of course, the multiplier in each of these methods must be considered abstract.

RULE.—*To the number of months annex one third of the number of days. Divide the number thus produced by* 1000. *Multiply one half of the principal by this quotient.*

Or, Multiply the whole principal by this quotient, and divide the product by 2.

The interest for any other rate may be found as in § 247.

CONCISE METHOD FOR ANY RATE PER CENT.

Ex. 24. Find the interest of $360.60 for 6mo. 15da. at 8 per cent. per annum.

12,00)$3,60.60
$.3005
6.5
15025
18030
$1.95325
8
$15.62600

§ 249. MODEL.—Divide the principal by 1200. Multiply this quotient by 6.5. Multiply this product by 8. The product is $15.62½, the required interest.

EXPLANATION.—The principal ÷ 100 = the interest for 1 year at 1 per cent. This interest ÷ 12 = the interest for 1 month at 1 per cent. This last × 6.5 = the interest for 6.5 months at 1 per cent. And this × 8 = the interest for 6.5 months at 8 per cent.

RULE.—*Divide the principal by* 1200. *Multiply the quotient by the number of months in the given time, and this product by the rate per cent. This last product will be the interest.*

Either of the above methods may be used in any case.

Ex. 25. Find the interest of $1349.50 for 9 days, at 6 per cent. per annum.

26. Find the interest of $3658.75 for 17 days at 6 per cent. per annum. Int. $10.366.

27. Find the interest of $5739.25 for 2mo. 24da. at 6 per cent. per annum. Int. $80.349.

28. Find the amount of $37682.375 for 2mo. 6da. at 6 per cent. per annum. Amt. $38096.88.

29. Find the amount of $1665.25 for 1yr. 11mo. 9da. at 6 per cent. per annum. Amt. $1859.25.

30. Find the interest of $4336.30 for 4yr. 8mo. 13da. at 6 per cent. per annum.

31. Find the interest of $2758.50 from July 3, 1846, to May 19, 1855, at 6 per cent. per annum.

Int. $1469.36.

To find the interval of time, the earliest date must be subtracted from the latest. In this subtraction, the number of each month in the calendar is used, and each month is taken as equal to 30 days.

1855yr.	5mo.	19da.
1846 "	7 "	3 "
8 "	10 "	16 "

Ex. 32. Find the amount of $8140.75 from Dec. 9, 1847, to Apr. 27, 1855, at 6 per cent. per annum.

Amt. $11747.10.

33. Find the interest of $34219.15 from Apr. 8, 1850, to June 15, 1855, at 7 per cent. per annum.

34. Find the interest of $6813.45 from Mar. 5, 1855, to Oct. 8, 1862, at 8 per cent. per annum.

Int. $4138.035.

35. Find the interest of $856.85 for 6yr. 8mo. 9da. at 8 per cent. per annum. Int. $458.699.

36. Find the amount of $742.40 from June 24, 1854, to Mar. 13, 1860, at 7 per cent. per annum.

37. Find the interest of $171.80 from July 29, 1857, to Sept. 1, 1861, at 10 per cent. per annum. Int. $70.24.

38. Find the interest of $670.70 from Apr. 7, 1859, to Oct. 13, 1862, at 9 per cent. per annum.

Int. $212.276.

39. Find the interest of $976.18 from Mar. 1, 1861, to Feb. 10, 1862, at $8\frac{1}{2}$ per cent. per annum.

40. Find the interest of $375.85 from Jan. 19, 1860, to Jan. 1, 1862, at 11 per cent. per annum.
Int. $80.619.

41. Find the amount of $6.89 from June 11, 1860, to June 1, 1862, at 9 per cent. per annum. Amt. $8.11.

42. What is the interest of $89.96 for 2yr. 3mo. 16da. at 8 per cent. per annum?

43. What is the interest of $325 for 6yr. 7mo. 27da. at 7¼ per cent. per annum? Ans. $156.88.

44. What is the amount of $1728 from Dec. 29, 1859, to Oct. 9, 1852, at 10 per cent. per annum? Ans. $2208.

45. What is the interest of $160.08 from May 1, 1851, to Sept. 9, 1854, at 7 per cent. per annum?

46. What is the interest of $18.62 for 3yr. 18da. at 5 per cent. per annum? Ans. $2.839.

47. What is the interest of £17, 6s. 9d. for 18mo. at 6 per cent. per annum?

£17, 6s. 9d. = £17.3375
.09
£1.560375
£1.56 = £1, 11s. 2½d.

The principal must first be reduced to pounds, and then the interest may be found by any one of the preceding methods.

Ex. 48. What is the interest of £427, 18s. 9d. for 2 years at 5¾ per cent. per annum?

49. What is the amount of £1096, 15s. 6d. for 4 years at 6½ per cent. per annum? Ans. £1381, 18s. 8d.

50. What is the amount of £120, 10s. for 2yr. 6mo. at 4¾ per cent. per annum? Ans. £134, 16s. 1¾d.

51. What is the interest of £270, 10s. 9d. for 1yr. 4mo. 20 da. at 7 per cent. per annum?

52. What is the interest of 1775fr. 75cent. for 3yr. 6mo. at 6 per cent. per annum? Ans. 372fr. 90cent.

53. What is the interest of 2070fr. 65cent. for 2yr. 8mo. 20da. at 7 per cent. per annum? Ans. 394fr. 57cent.
54. What is the amount of 3297fr. 15cent. for 3yr. 15da. at 8 per cent. per annum?
55. What is the interest of 10720fr. 25cent. for 5yr. 7mo. 10da. at 5 per cent. per annum? Ans. 3007.62fr.
56. What is the amount of 20625fr. 30cent. for 6yr. 6mo. 6da. at 6 per cent. per annum? Ans. 28689.79fr.

PARTIAL PAYMENTS.

The method here given is the one enjoined by the Supreme Court of North Carolina, and used in most, if not all, the States of the Confederacy.

§ 250. RULE.—*Find the amount of the given principal to the time of the first payment, and if this payment is greater than the interest then due, subtract the payment from the amount. Consider the remainder as a second principal, and find the amount of it from the time of the first payment to the time of the second, and if the second payment is greater than the interest last found, subtract the second payment from the second amount, and consider the remainder as a third principal: and so on.*

But if any payment is less than its corresponding interest, find the amount of the same principal to the time of the next payment, and if the sum of these two payments is greater than the interest then due, subtract their sum from the amount: but if the sum of the two payments is less than the interest then due, extend the time until the sum of the payments made shall exceed the interest due at the time of the last payment.

The principle of the rule is that the payment of a part of the debt shall not increase the debt.

Ex. 57. $725.50. RICHMOND, VA., Jan. 1, 1858.

One day after date, I promise to pay J. Jones, or order, seven hundred and twenty-five dollars and fifty cents, for value received. *L. Smith.* SEAL

On this note were the following endorsements:

Mar. 16, 1858,	$100.00
May 16, 1859,	25.50
July 1, 1861,	300.00

How much was due Oct. 8, 1862?

SOLUTION.

Original Principal,	$725.50
Interest to Mar. 16, 1858,—2m. 15da.,	9.068
Amount then due,	$734.568
Amount then paid,	100.
Second Principal,	$634.568
Interest from Mar. 16, 1858, to May 16, 1859,	$44.419
Amount then paid (less than interest)	25.50
Interest from Mar. 16, 1858, to July 1, 1861,—3y. 3m. 15d.	125.327
Amount then due,	$759.895
Sum of the two payments,	325.50
Third Principal,	$434.395
Interest from July 1, 1861, to Oct. 8, 1862,—1y. 3m. 7d.	33.086
Amount due Oct. 8, 1862,	$467.481

58. $3256.37. LINCOLNTON, N. C., Mar. 12, 1853.

On demand I promise to pay to the order of J. Reinhardt, three thousand two hundred and fifty-six dollars and thirty-seven cents, for value received.

L. Payson. SEAL

On this note were the following endorsements:

July 12, 1855,	received	$654.33
Sept. 20, 1857,	"	$246.50
Jan. 5, 1859,	"	$945.87

What was the balance due Sept. 7, 1860? Ans. $2755.41.

---

59. $108.43. COLUMBIA, S. C., Dec. 9, 1857.

With interest from date, for value received, I promise to pay J. Townsend or order one hundred and eight dollars and forty-three cents. *D. North.* SEAL

Endorsements. Mar. 3, 1858, received $50.04; Dec. 10, 1858, $13.19; May 1, 1860, $50.11. How much was due Oct. 9, 1862? Ans. $5.844.

60. A note was given at Savannah, Geo., Apr. 16, 1856, for $450. On it the following endorsements were made:—Jan. 1, 1857, received $20; Apr. 1, 1857, $14; July 16, 1857, $31; Dec. 25, 1857, $10; July 4, 1858, $18. What balance was due June 1, 1859?

Note.—When no rate of interest is mentioned in a note, the legal rate at the place where it is given is to be used. In Louisiana the legal rate is 5 per cent.: in Arkansas, Kentucky, Maryland, Missouri, North Carolina, Tennessee, and Virginia, it is 6 per cent.: in South Carolina it is 7 per cent.; and in Alabama, Florida, Georgia, Mississippi, and Texas, it is 8 per cent.

COMPOUND INTEREST.

§ 251. *Compound Interest* is the interest on both principal and interest when the interest is not paid as it falls due. In ordinary business transactions it is not allowed by law; but in a few classes of debts it is required that the interest shall be compounded annually. In such cases, the interest for one year is added to the principal; this amount becomes the principal for the second year; its amount for the third year, and so on to the last year or part of a year. The original principal subtracted from the final amount gives the compound interest.

Ex. 61. What is the compound interest of $525.75 for 3yr. 6mo. at 6 per cent., interest due annually?

SOLUTION.

Original Principal,	$525.75
Interest for the first year,	31.545
Amount,—Second Principal,	$557.295
Interest on $557.295 for the second year,	33.437
Amount,—Third Principal,	$590.732
Interest on $590.732 for the third year,	35.443
Amount,—Fourth Principal,	$626.175
Interest on $626.175 for the remaining 6mo.,	18.785
Total Amount at Compound Interest,	$644.960
Original Principal,	525.75
Compound Interest,	$119.21

Ex. 62. What is the amount at compound interest of $500 at 6 per cent. for 4yr. 3mo., interest due annually?

63. What is the amount of $1000 for 7 years at 7 per cent., compounded annually?

64. What is the amount of $1000 for 6 years at 6 per cent., compounded semi-annually? Ans. $1425.76.

65. What is the interest of $1000 for 4 years at 6 per cent., compounded quarterly? Ans. $268.98.

DISCOUNT.

§ 252. DISCOUNT is a deduction made for the payment of money before it is due.

§ 253. The *present worth* of a future debt is that sum which, at ordinary interest, will amount to the debt at the time it becomes due. The present worth bears the same relation to the debt, that the *principal* bears to the *amount.*

The problem to be solved, then, is, having given the amount, the time, and the rate, to find the principal and the interest.

§ 254. RULE.—*Find the amount of* $1 *for the given time at the given rate. Then, as the amount of* $1 *is to* $1, *so is the amount of the debt to its present worth.*

To find the discount, subtract the present worth from the amount of the debt. Or say, as the amount of $1 *is to its interest, so is the amount of the debt to the discount.*

Ex. 66. What is the present worth, and what is the discount, of a note due 6 months hence for $550 at 6 per cent.?

SOLUTION.

Amount of $1 for 6 months at 6 per cent., $1.03.

$1.03 : $1 :: $550 : $533.98, present worth.

$550 − $533.98 = $16.02 = the discount.

Or, $1.03 : $ .03 :: $550 : $16.02, the discount.

Ex. 67. What is the present worth of a note for $245, due 1 year hence when the rate of interest is 6 per cent.?

68. What discount should be allowed on a note for $525, if paid 3mo. before it is due, interest being at 7 per cent.?

69. What is the present worth of a debt of $375.50, due in 7mo. 15da., if interest is at 8 per cent.?

70. What is the discount of a note for $725, due in 10mo. 10da., interest being 7 per cent.?

71. In Mobile, Ala., one man gave another his note for $247.50, due twelve months after date. What was the present worth of the note?

72. What discount would be allowed at New Orleans on a debt of $650, due 9 months hence?

73. What is the present worth, at Little Rock, Ark., of a note for $769.35, due 5mo. 18da. hence?

74. What is the proper discount on a debt of $75.75, due 7mo. hence at Memphis, Tenn.?

75. What is the present worth of $1250, due 12 months hence at Galveston, Texas?

76. What is the discount of $250, due 8mo. hence at Lexington, Ky.?

77. What is the present worth of $55.55, due 7mo. hence at St. Louis, Mo.?

BANK DISCOUNT.

§ 255. The *present worth* or *proceeds* of a note payable in bank is the remainder obtained by subtracting from its face its interest for the time it has to run, including three additional days—called days of grace.

Thus, if I deposit with the Cashier of the Bank of Cape

Fear my note for $1000 due in 60 days, he will pay me on it only $1000—the interest of $1000 for 63 days, that is, $989.50.

§ 256. The *bank discount* of a note not yet due is the interest of the face of the note for three days more than the time it has to run.

Ex. 78. What is the present worth in bank of a note for $500 due in 30 days, at 6 per cent.?

SOLUTION.

Face of the note,	$500.
nterest of $500 for 33da.,—bank discount,	2.75
Present Worth or proceeds,	

Ex. 79. What is the proceeds of a note due in bank 60 da. hence for $250 at 6 per cent.?

80. What is the bank discount on a note for $750 due in bank in 90 days, at 6 per cent.?

81. What discount would a bank require on a note for $550.75, due 90 days hence at 8 per cent.?

82. What is the present worth of a note due in bank 90da. hence for $333.33 at 6 per cent.?

83. What is the face of a note due 60da. hence, if its present worth in bank is $500, interest being at 6 per cent.?

§ 257. The present worth of $1 : $1 :: present worth of the note : face of the note. In this case, $.9895 : $1 :: $500 : the answer.

Ex. 84. What sum, payable in 90 days, will produce $750, if discounted at a bank at 6 per cent.?

85. What sum, payable in 60 days, will produce $3000, if discounted at bank at 7 per cent.?

86. For what amount must a note be drawn, payable in 30 days, so that, if discounted in bank at 5 per cent., the proceeds will be $250?

87. What must be the face of a note payable in bank in 90 days, so that, if discounted at 6 per cent., its present worth may be $75.75?

TIME TABLE,

Showing the number of days from any day of one month to the same day of any other month next following.

From any day of	To the same day of the next											
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Jan.	365	31	59	90	120	151	181	212	243	273	304	334
Feb.	334	365	28	59	89	120	150	181	212	242	273	303
Mar.	306	337	365	31	61	92	122	153	184	214	245	275
Apr.	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June	214	245	273	304	334	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
Aug.	153	184	212	243	273	304	334	365	31	61	92	122
Sept.	122	153	181	212	242	273	303	334	365	30	61	91
Oct.	92	123	151	182	212	243	273	304	335	365	31	61
Nov.	61	92	120	151	181	212	242	273	304	334	365	30
Dec.	31	62	90	121	151	182	212	243	274	304	335	365

To find the interval of time between Sept. 3, 1862, and May 19, 1863. Find Sept. in the left hand column and May in the upper line: then at the right of Sept. and under May, is 242, the number of days from Sept. 3 to May 3. To this add 16, the number of days from May 3 to May 19. The sum 258 is the number of days required.

Again, from Jan. 25, to Sept. 9, is (243—16) 227 days.

PROMISCUOUS PROBLEMS.

1. In what time will $100 amount to $200 at 6 per cent. simple interest?

§ 258. As the interest of the given principal for 1 year : the given interest :: 1 year : the number of years.

In this case, as $6 : $100 :: 1yr. : 16yr. 8mo., the answer.

2. In what time will $200 gain $50 interest at 6 per cent. per annum?
3. In what time will $500 gain $49 interest at 7 per cent.?
4. In what time will $1000 gain $10 simple interest at 5 per cent. per annum?
5. At what rate will $100 gain $15 interest in 2yr. 6mo.?

§ 259. As the interest of the given principal at 1 per cent. : the given interest :: 1 : the rate per cent.

In this case, as $2.50 : $15 :: 1 : 6, the answer.

6. At what rate will $250 gain $250 interest in 10yr.?
7. At what rate will $427.25 gain $143.60 in 3yr. 4mo. 10da.? Ans. 10 per cent.
8. At what rate will $746 gain $83.92 in 2yr. 3mo.?
9. What principal will gain $174.56 in 1yr. 7mo. at 7 per cent.?

§ 260. As the interest of $1 for the given time at the given rate : the given interest :: $1 : the principal.

In this case, $ .1108$\frac{1}{3}$: $174.56 :: $1 : $1575, the answer.

10. What principal will gain $42 in 3yr. 6mo. at 6 per cent.? Ans. $200.
11. What principal will gain $210 in 5yr. at 6 per cent.?

12. What principal will gain $400 in 4 years at 8 per cent.?
13. What is the fourth root of 810000? Ans. 30.
14. What is the value of 2.3.5.7.11?
15. What are the prime factors of 1800?
16. A commission merchant sold goods worth $9072; what was his commission at $2\frac{1}{2}$ per cent.? Ans. $226.80.
17. A capitalist sent his broker $15400 to lay out in stocks, after retaining $\frac{1}{4}$ per cent. of the amount purchased. How much stock did he purchase? Ans. $15361.60.
18. A gentleman laid out $3025 in stocks which were 10 per cent. below par. What was the nominal value of the stock purchased?
19. If I buy coffee at 30ct. per lb., and sell it for 36ct. per lb., what per cent. do I gain? Ans. 20 per cent.
20. A merchant bought 125 bushels of wheat at $1.60 per bu., and sold it at a profit of 20 per cent.; what did he get for it? Ans. $240.
21. If I pay $12000 for a house and lot and sell them at an advance of 25 per cent., what do I gain by the transaction?
22. A merchant gave $3.51 for an article which he is willing to sell at a profit of $33\frac{1}{3}$ per cent., how must he mark it?
23. By selling a tract of land for $4704 I gain 12 per cent. on it; how much did it cost me? Ans. $4200.
24. If 3cwt. of sugar cost $23.40, what will 16cwt. 3qr. cost?
25. A merchant, failing, pays only 60ct. on the dollar of his indebtedness; how much will a man receive to whom he owes $1800? Ans. $1080.
26. What cost 462yd. of cloth at $\$1.06\frac{1}{4}$ per yd.? Ans. $\$490.87\frac{1}{2}$.
27. What cost 83bu. 3pk. 2qt. of clover seed at $8 per bu.?

28. What per cent. of $50 is $6? Ans. 12 per cent.
29. What is 115 per cent. of $287.50? Ans. $330.625.
30. At 5 per cent. commission, what would I receive for selling $240 worth of property?
31. A commission merchant sells property amounting to $550. Retaining his commission of 5 per cent., he lays out the balance after deducting a commission of $2\frac{1}{2}$ per cent. on the amount purchased. How much did he lay out?
32. What amount can I retain for commission at 8 per cent. on the amount invested, if I have received $2647.08?

AVERAGE.

§ 261. The *average* of *two* numbers is one half of their sum. Thus, the average of 7 and 13 is $(7+13)\div 2=10$.

The average of *three* numbers is one *third* of their sum.

The average of *four* numbers is one *fourth* of their sum. And so on.

§ 262. The average of two *dates* is a date lying half way between them. Thus, in any year June 23 is the average between June 1 and July 15.

Ex. 1. Find the average of 2, 4.5, 5.75, 7, and 9.25.

2.
4.5
5.75
7.
9.25
5)28.50
5.7

§ 263. MODEL.—Find the sum of the five given numbers. (§ 159.) Divide this sum by 5. (§ 164.) The quotient 5.7 is their average.

This needs no explanation.

Ex. 2. What is the average of 2, 3, 5, and 6?
3. What is the average of 2, 5, 7, and 10?
4. What is the average of 25 and 32?
5. What is the average of 34 and 19?
6. What is the average of 25, 32, and 41?
7. What is the average of 17, 29, and 63?
8. What is the average of 25, 170, and 195?
9. What is the average of 2, 102, 111, and 115?
10. What is the average of 0, 5, 7.5, 25, and 40?
11. What is the average of 1, 7, 15, 25.25, and 37.5?
12. What is the average of 3, 7.5, 5.75, 11.75, and .625?
13. What is the average of 20, 47, 35, 94.5, 79.5, and 10.01?
14. What is the average of 13, 15, 17, 29.5, 37.5, and 63.75?
15. What is the average of 0, 1, 7, 9, 25, 37, and 39?

ALLIGATION MEDIAL.

§ 264. This name is given to the process of finding the mean value of a mixture, when the values of the substances composing it are known.

Ex. 16. If 4lb. of sugar worth 10ct. per lb. are mixed with 10lb. worth 12ct. per lb., what is a pound of the mixture worth?

$4 \times 10\text{ct.} = 40\text{ct.}$
$10 \times 12\text{ “} = 120\text{ “}$
$14 \quad 160\text{ct.} \mid 14$
$14 \quad 11\frac{3}{7}\text{ct.}$
20
14
6

§ 265. Model.—Multiply 10ct. by 4. (§ 183.) Multiply 12ct. by 10. (§ 183.)—Add the products together. (§ 179.) Divide the sum by 14. (§ 185.) The quotient, $11\frac{3}{7}$ct., is the average price per lb.

EXPLANATION.—The whole mixture weighs 14lb., which evidently cost 160ct. : and 1 fourteenth of this amount is the average price per lb.

RULE.—*Divide the whole cost by the number of articles; the quotient will be the average cost per unit.*

This rule applies to several things not embraced in the definition.

Ex. 17. During 24 hours the thermometer stood for 2hr. at 55°, for 3hr. at 60°, for 4hr. at 65°, for 5hr. at 70°, for 6hr. at 75°, and for 4hr. at 80°. What was the mean temperature of the day?

18. A goldsmith mixes 10oz. of gold 16 carats fine with 6oz. 17 carats fine and 8oz. 19 carats fine; what is the fineness of the mixture?

19. A grocer mixed 4gal. of wine worth $1 a gallon, 5gal. worth $1.25 a gallon, and 10gal. worth $1.50 a gallon; what was the mixture worth per gal.?

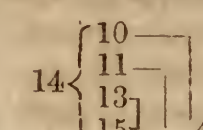
20. If 30gal. of molasses at 40ct., 40gal. at 50ct., 70gal. at 60ct., and 80gal. at 80ct., be mixed together, what is a gallon of the mixture worth? Ans. $62\frac{8}{11}$ct.

21. A farmer has 10 sheep worth $4 each, 12 worth $5 each, and 8 worth $10 each; what is their average value?

ALLIGATION ALTERNATE.

§266. This consists in finding the proportional quantities of several simple substances which shall make a compound of a given mean value. It is, therefore, the converse of the preceding.

Ex. 22. In what proportions must sugars worth 10ct., 11ct., 13ct., and 15ct., be mixed, that the compound may be worth 14ct.?



§ 267. MODEL.—Connect 10 with 15, 11 with 15, and 13 with 15. 10 from 14 leaves 4; set 4 opposite 15: 11 from 14 leaves 3; set 3 opposite 15: 13 from 14 leaves 1; set 1 opposite 15:—14 from 15 leaves 1; set 1 opposite 10, 11, and 13. Hence there must be 1lb. at 10ct., 1lb. at 11ct., and 1lb. at 13ct., to 8lb. at 15ct.

EXPLANATION.—After arranging the several prices as in the model, and placing the mean price on the left, we connect each price below the mean with one above it, and each price above the mean with one below it. Then taking the difference between each price and the mean, we set this difference opposite the price with which this price is connected; observing during the operation to consider all the prices as abstract numbers. The reason for all this is evident when we consider that each pound at 10ct. falls 4ct. below the mean, while each pound at 15ct. is only 1ct. above it. To average these two values, therefore, we must have 4lb. of the sugar at 15ct. to every one at 10ct. For a similar reason, it requires 3lb. at 15ct. to counterbalance 1lb. at 11ct. And as the mean price is equidistant between 13ct. and 15ct., these two qualities must be taken in equal quantities. So that to bring the three inferior qualities up to the required average, it is necessary to take 4+3+1, *i. e.*, 8lb. of the superior quality to 1lb. of each of the inferior qualities.

RULE. I.—*Arrange the several prices in a vertical column, and place the mean price on the left.*

Connect each price below the mean with one above it, and each price above the mean with one below it.

Find the difference between each price and the mean, and set it opposite the price with which it is connected. If only one difference stands opposite any price, it denotes the proportion of that value; but if several differences stand opposite any price, their sum denotes the proportion of that value.

II. If it is required to have a specified quantity of any value.—*Find the proportions as above. Then say, As the proportion found for this value : the quantity required for it :: the proportion for any other value : the quantity required for it.*

III. If the whole quantity of the mixture is specified.—*Find the proportions as above. Then take the sum of the proportional numbers, and say, As the sum of the proportional numbers : the required quantity of the mixture :: the proportion for any value : the quantity required for that value.*

Proof.—By Alligation Medial.

Ex. 23. In what proportions may gold of 10, 13, 14, and 22 carats fine, be mixed so that the compound may be 17 carats fine?

24. A grocer having brandy worth $1 a gallon, wishes to mix it with water so that he can sell the mixture at 80ct. a gallon. In what proportions must he mix them?

25. In what proportions may liquors worth respectively $1, $1.20, $1.40, and $1.50 be mixed, that the mixture may be worth $1.25?

26. A farmer wishes to mix 14bu. of wheat worth $1 per bu. with such a quantity worth $1.24 as will make the mixture worth $1.03; how much must he take?

27. How much tea at 80ct., 70ct., and 60ct., respectively, should be mixed with 90lb. at 90ct., so that the mixture may be worth 75ct. per lb.?

28. A merchant having 100lb. of sugar worth 10ct. per lb., mixed it with other sugar worth respectively 5, 8, and 9ct., and sold the mixture at 8½ct. How much of each quality was there in the mixture?

29. How much sugar at 10ct., and how much at 15ct. per lb., must be taken to make 60lb. worth $7.20?

Ans. 36lb. at 10ct., and 24lb. at 15ct.

30. A grocer mixes 144lb. of sugars worth respectively 12, 10, 6, and 4ct. per lb., and sells the mixture at 8ct. per lb.; how much of each quality does he take?

31. A man paid $165 to 55 persons—men, women, and boys; to each man he paid $5, to each woman $1, to each boy 50ct.; how many were there of each?

Ans. 30 men, 5 women, 20 boys.

EQUATION OF PAYMENTS.

§ 268. This consists in finding the average date at which several amounts due at different times may all be paid, so that no interest shall be either gained or lost.

Ex. 32. A owes B $25 due in 4mo., $50 due in 6mo., and $75 due in 8mo.; what is the mean time of payment?

$25 \times 4\text{mo.} = 100\text{mo.}$
50×6 " $= 300$ "
75×8 " $= 600$ "

150 15,0)100,0mo.

$6\frac{2}{3}$mo.

§ 269. MODEL.—Multiply 4mo. by 25. (§ 183.) Multiply 6mo. by 50. Multiply 8mo. by 75.—Add the products together. Add the multipliers together. Divide 1000mo. by 150. The quotient $6\frac{2}{3}$mo. is the mean time of payment.

EXPLANATION.—The interest of $25 dollars for 4 months is equal to the interest of 1 dollar for 100 months: the interest of $50 for 6mo. = the interest of $1 for 300mo.: the interest of $75 for 8mo. = the interest of $1 for 600mo. Hence the interest of the several amounts for their respective times is equal to the interest of $1 for 1000mo., and this is equal to the interest of $150 for $6\frac{2}{3}$mo. Hence it is fair that the whole amount should be paid in $6\frac{2}{3}$mo.

RULE.—*Multiply each term of credit by the number of units in the corresponding payment, and divide the sum of the products by the sum of the multipliers: the quotient will be the mean time of payment.*

Ex. 33. A man owes an other $500 due in 3mo., $400 due in 6mo., and $600 due in 9mo.; what is the average term of credit for the three debts?

34. Bought goods as follows: $400 on a credit of 6mo., $200 on $3\frac{1}{2}$mo.; and $560 on 6mo.; what average credit should be allowed me on the whole?

35. Bought $1000 worth of goods to be paid for as follows: $200 on the day of purchase, $400 in 5mo., and $400 in 15mo. What average credit should be allowed me on the whole?

36. In what time should the following amounts be paid all at once: $1600 due in 5mo., $1200 in 6mo., and $1200 in 8mo.?

37. I owe $100 to be paid Jan. 15, $200 due Feb. 15, and $300 due Mar. 9; on what day may the whole debt be paid at once?

Note.—Select some day from which the periods of credit may be supposed to commence. In this instance, Jan 15 is the most convenient. Find the interval elapsing between this date and each of the others, and then proceed according to the rule. Consider each month 30 days.

38. A man owes his neighbor $1250 due in 8mo.: but at the end of 3mo. he pays $250, and in 2mo. more he pays $150; what extension of credit should be allowed on the remainder?

$250 \times 5\text{mo.} = 1\ 50\text{mo.}$
$150 \times 3\ \text{“} = 450\ \text{“}$
$850)1700\text{mo.}$
2mo.

270. Model.—Multiply 5 mo. by 250. (§ 183) Multiply 3mo. by 150. Add the products together. Divide 1700mo. by 850. The quotient 2mo. is the extension of credit.

Explanation.—The debtor, having paid $250 (8—3) 5mo before it was due, is entitled to a credit of 1250mo. on $1: and, having paid $150, 3mo. before it was due, is therefore entitled to a credit of 450mo. on $1. For both prepayments he is entitled to a credit equivalent to $1 for 1700mo. The remainder unpaid is $1250—($250+$150) =$850: and a credit of 1700mo. on $1 is equal to a credit of 2mo. on $850.

Ex. 39. I owe $1000 due in 12mo. If I pay $100 at the end of 3mo., and $100 at the end of 4mo., how long beyond the 12mo. should my creditor wait for the payment of the balance?

40. I owe $2000 due in 6mo. If I pay $500 down, $300 in 2mo., and $200 in 3mo., in how many months from the contraction of the debt should I pay the balance?

41. A merchant owes $1200, of which $200 is to be paid in 4 months, $400 in 10 months, and the remainder in 16 months: if he pays the whole at once, at what time must he make the payment?

42. A merchant owes $1800 to be paid in 12 months, $2400 to be paid in 6 months, and $2700 to be paid in 9 months: what is the average time of payment?

PROMISCUOUS PROBLEMS.

1. Reduce £19, $8\frac{3}{8}$s. to pence.
2. Reduce 9oz. $16\frac{3}{7}$dwt. to grains.
3. Reduce $\frac{\frac{2}{3}}{\frac{7}{8}}$ of $\frac{\frac{3}{4}}{12}$ of $16\frac{1}{2}$ to its simplest form.
4. Reduce $\frac{1}{1\frac{1}{2}}$, $\frac{6\frac{7}{8}}{20\frac{5}{8}}$, $\frac{6\frac{1}{2}}{26}$, and $\frac{2}{3}$ of $\frac{4\frac{1}{2}}{9}$ to their least common denominator.
5. Add 900.01, 450.037, and 696.9 together.
6. Add $2\frac{8}{11}$, $6\frac{1}{5}$, and $12\frac{19}{22}$ together.
7. Add $\frac{7}{10}$ of $\frac{4}{5}$ of 20, $\frac{3}{7}$ of $\frac{6}{7}$ of $24\frac{1}{2}$, and $\frac{6}{11}$ of $2\frac{1}{4}$ together.
8. From $\frac{6\frac{3}{4}}{20}$ take $\frac{2\frac{1}{2}}{12}$.
9. From $\frac{5}{7}$ of $\frac{2}{3}$ of $3\frac{1}{2}$ take $\frac{5}{9}$ of $\frac{7}{8}$.
10. From $\$49\frac{3}{4}$ take $\$4.75+\$5\frac{5}{10}+\$9.30$.
11. Multiply $\frac{8\frac{1}{2}}{12}$ by $\frac{17}{20\frac{2}{5}}$.
12. What is the product of $\frac{3}{5}\frac{1}{0}$ by $\frac{17}{20\frac{2}{5}}$?
13. Multiply $\frac{9.1}{18.2}$ by $\frac{7}{11}$ of $\frac{1}{1}\frac{8}{9}$ of $5\frac{3}{7}$.
14. Divide $\frac{2}{3}$ of $3\frac{3}{4}$ by $\frac{3}{7}$ of $6\frac{1}{9}$.
15. Divide $1301\frac{2}{5}$ by 161.3.
16. Divide $\$1843\frac{1}{4}$ by $368\frac{3}{4}$.
17. What is the insurance on \$3125 at $6\frac{1}{4}$ per cent.?
18. A commission merchant sold 19 firkins at 45ct. per lb., and retained 5 per cent. commission; how much did he return to the owner?
19. What is the par value of two certificates of stock; one

for $350 at $2\frac{1}{2}$ per cent. discount, the other for $527.50 at 5 per cent. advance?

20. What is the amount of £1054, 10s. 9d. for 2yr. 9mo. at 4 per cent. per annum?
21. If the interest of a certain amount of money at 6 per cent. is $241.80, what is the interest of the same sum for the same time at $7\frac{1}{2}$ per cent.?
22. At what rate per cent. per annum will £1829, 10s. amount to £1898, 2s. $1\frac{1}{2}$d. in 9 months?
23. What is the greatest common measure of 560, 880, 1028, and 1296?
24. What is the least common multiple of 36, 18, 33, 11, and 6?
25. What is the greatest common measure of 56, 154, and 182?
26. What is the least common multiple of 2, 4, 10, 7, 14, 15, and 21?
27. Resolve 528 into its prime factors.
28. What prime factors are common to 360, 420, and 840?
29. I sold 125A. 2R. 20P. of land for $2050; how much did I gain or lose, if I gave $15.50 per A. for the land?
30. I bought a lot of English paper for £698, 10s. 6d., and sold it at a profit of 75 per cent.; how much did I receive for it in Federal currency?
31. What is the amount of £300, 10s. for 2yr. 3mo. at interest compounded semi-annually, at 8 per cent. per annum?
32. What is the square root of 509796?
33. What is the cube root of 16003008?
34. Find the greatest common measure of 1538, 2307, and 3845.

35. I sold $\frac{1}{3}$ of my land to A, $\frac{1}{4}$ of it to B, and retained 200A. for myself; how much had I at first?
36. A, B, and C trade in partnership. A invests $1000 for 12 months; B, $1500 for 10 months; and C, $2000 for 9 months. How shall their profit of $1000 be divided?
37. How many barrels of potatoes at $2.50 per bbl. should be exchanged for a hogshead of sugar weighing 1375lb. gross, worth $15.00 a hundred pounds net, tare being 8 per cent.?
38. How many firkins of butter, at 25ct. per lb., can be bought for 9mo. interest of $800 at 7 per cent. per annum?
39. Having been engaged in merchandise with a capital of $19500, I realized a profit of $33\frac{1}{3}$ per cent., which I immediately invested in land at $16.50 per A.; how many acres did I buy?
40. If I owe three notes, one for $600 due 3mo. hence, another for $800 due 6mo. hence, and the other for $1000 due 15mo. hence, in what time might I fairly pay the three notes together?
41. If 12lb. of tea @$1.20, 15lb. @$1.44, and 18lb. @$1.80, be mixed together, what is the value of 11lb. of the mixture?
42. What is the 4th power of $7\frac{1}{2}$?
43. What is the cube of 3.5?
44. What is the cube root of 19.683?
45. What is the square root of $76\frac{9}{16}$?
46. Required to fill a hogshead with two kinds of wine worth $1.20 and $1.05 per gal. respectively, so that the mixture will be worth $1.15 per gal.; how many gallons of each kind will be required?

47. The total stock in a Railroad is $1000500 the net income for a year is $50000; what dividend will I receive for $10000 worth of stock?
48. I exchanged a house and lot worth £500, 15s. for land worth $10.50 per A.; how much land did I receive?
49. A pedlar exchanged a piece of calico, rated at 21ct. per yd., for a firkin of butter worth 22½ct. per lb; how many yards of calico were there?
50. I imported 95T. of iron worth $82 per T.; what was the duty on it at 33⅓ per cent.?
51. What is the net weight of 275 bags of coffee, weighing each 73lb. gross, tare being 4 per cent.?
52. What cost 70A. 3R. 25P. of land at $25.75 per A.?
53. What cost 5T. 16cwt. 3qr. 0 iron at $4.125 per cwt.?
54. If, by selling a tract of land for $6450, I lose 4 per cent. of what it cost me, for what would I have had to sell it, to gain 6¼ per cent.?
55. Bought 25T. 16cwt. of iron at £14, 16s. per T.; and sold the whole for $2000, what did I gain or lose per T?
56. What is the present worth of $2000, due in 2yr. 3mo. 15da., interest being at 6 per cent. per annum?
57. What is the discount on £180, due in 3yr. 6mo., interest being at 8 per cent. per annum?
58. I wish to borrow $1150 in bank: interest being at 6 per cent. per annum, what must be the face of the proper note at 90 days?
59. In what time will £432, 15s., at 6 per cent. per annum, amount to £562, 11s. 6d.?
60. If the insurance of $25000 is $400, what is the rate per cent.?
61. What per cent. of 60 is 1.25?
62. What per cent. of 75 is 125?

63. What is $3\frac{1}{16}$ per cent. of $11755?
64. If $5\frac{1}{2}$A. of land cost $144.50, what will 17A. 3R. 19.375P. cost?
65. If 1.37gal. of sorghum molasses cost $1.4375, what will 13.7gal. cost?
66. Divide 17mi. 5fur. 25rd. by 1.5.
67. Multiply 3deg. 17min. 45sec. by 2.03.
68. Dividend is £1, 18s. 9.5d., divisor is 4.9, what is the quotient?
69. Dividend is 1bu. 3pk. 4.5qt., quotient is 75bu. 2pk. 4qt., what is the divisor?
70. From 1.475T. take 17cwt. 1qr. 19.29lb.
71. Add together 4.75gal., 3.07qt., 7.45pt., and 6.19gi.
72. What cost 17bbl. flour at $10 per 100lb., 3bu. salt at $1.25 per bu., and 677.5lb. pork at $0.065 per lb.?
73. If 75 persons eat 800bu. corn in 1 year, how long will 600bu. last 90 persons?
74. If 150 copies of a book of 200 pages require 6rm. 4qr. of paper, how many reams will 15000 copies of a book of 224 pages require?
75. If 1rm. of paper weigh 30lb. and cost 30ct. per lb., what will the paper cost for an edition of 1000 copies of a book which requires 5rm. 10qr. for 96 copies?
76. If $83\frac{1}{2}$T. of coal cost $405.50, what will 17T. 3cwt. 1qr. bring at $16\frac{2}{3}$ per cent. advance?
77. The second, third, and fourth terms of a proportion are $\frac{2}{5}$, $\frac{7}{9}$, and 2.5, respectively; what is the first term?
78. If the first, third, and fourth terms of a proportion are $64.96, 7cwt. 1qr., and 4cwt. 2qr., respectively, what is the second term?
79. Multiplicand is 94; product is .66; what is the multiplier?

80. I bought 625lb. of cheese for $62.50, and sold it at 12½ct. per lb.; how much per cent. did I gain?
81. I own ⅘ of a ship worth $20000, and have insured it at 2.375 per cent.; what insurance do I pay?
82. What is the amount of $2169.845 for 1yr. 10mo. 17da. at 7 per cent. per annum?
83. What are the prime factors of 7825?
84. What are the common prime factors of 875 and 1750?
85. How many hours will there be in the year 1900?
86. The Mecklenburg Declaration of Independence was made May 20, 1775; North Carolina unanimously seceded from the United States May 20, 1861; how many days elapsed between these two great events?
87. What cost 3000lb. of corn at $3.00 per bbl.?
88. What cost 5.25bbl. of flour at $ .04 per lb.?
89. An officer, in pursuit of a criminal, goes 10mi. per hr.; the criminal, who has 36mi. the start, goes 7mi. per hr.; how far must the officer go, to catch the criminal?
90. Bought 40gal. wine at $2.50 per gal.; lost 5gal. by leakage: how must I sell the remainder per gal. so as to gain 25 per cent. on the whole?
91. A vessel laden with 3000bu. wheat, found it necessary to throw 25 per cent. of her cargo overboard; what was her loss at $1.25 per bushel?
92. What is the value in Avoirdupois weight of 16lb. 5oz. 10dwt. 12gr. Troy?
93. How many sheets in 7 reams of paper?
94. If 7 silver spoons weigh 1lb. 2oz. 3dwt., what will each spoon weigh?
95. If 2A. produce 45bu. 3pk. 6qt. 1pt. of corn, how much will 32A. produce?

96. Add together $\frac{1}{3}$ of $\frac{1}{4}$ of an Acre, 75$\frac{1}{2}$P. $\frac{5}{7}$R., and $\frac{3}{8}$A.?
97. What part of a fathom is 3$\frac{1}{2}$ft.?
98. What is the amount of $3000 for 6mo. 24da. at 7$\frac{1}{2}$ per cent. per annum?
99. A and B purchased a house for $3000, of which A paid $1800, how shall they divide a rent of $350?
100. What is the square root of 576?
101. What is the 4th root of 6561?
102. What is the cube root of $\frac{9261}{1000000}$?
103. How much stock at 7 per cent. advance may be bought for $5350?
104. Bought 10rm. of paper at $3.50 per rm., and sold it at $.25 per quire, how much did I gain or lose on it all?
105. Bought 300bbl. of flour for $2250, sold $\frac{1}{4}$ of it at $6 per bbl., and the remainder at $8 per bbl., how much did I receive for the whole?
106. Reduce 26$\frac{7}{8}$ to a decimal form.
107. Multiply four thousandths by five hundredths.
108. Multiply four hundred and fifty by two hundredths.
109. Divide seven tenths by one hundredth.
110. What is the difference between thirty-five hundredths, and thirty-five thousandths?
111. What is the 2nd term of a proportion whose 1st, 3rd, and 4th terms are 7, 1.3, and 19, respectively?
112. If one acre of land costs £2, 15s. 4d., what will be the cost of 173A. 2R. 14P. at the same rate?
113. A gentleman's estate is worth £4215, 4s. a year: what may he spend per day and yet save £1000 per annum?
114. A father left his son a fortune, $\frac{1}{4}$ of which he ran through in 8 months, $\frac{2}{7}$ of the remainder lasted him 12 months longer, when he had barely £820 left: what sum did his father leave him?

115. There are 1000 men besieged in a town with provisions for 5 weeks, allowing each man 16 ounces a day. If they are reinforced by 500 more and no relief can be afforded till the end of 8 weeks, how many ounces must be given daily to each man?

116. A father gave $\frac{7}{18}$ of his estate to one son, and $\frac{7}{18}$ of the remainder to another, leaving the rest to his widow. The difference of the children's legacies was £514, 6s. 8d.: what was the widow's portion?

117. If 14cwt. 2qr. of sugar cost $129.92, what will be the price of 9cwt.?

118. If the freight of 80 tierces of sugar, each weighing $3\frac{1}{3}$cwt., 150 miles, cost $84, what must be paid for the freight of 30hhd of sugar, each weighing 12cwt., 50 miles?

119. If one pound of tea be equal in value to 50 oranges, and 70 oranges be worth 84 lemons, what is the value of a pound of tea when a lemon is worth 2 cents?

120. If 60 bushels of oats will serve 24 horses for 40 days, how long will 30 bushels serve 48 horses at the same rate?

121. What will be the cost of 2hhd. 5gal. 3qt. 2gi. of molasses, at $12\frac{1}{2}$ cents per quart?

122. What is the interest of $3153.82 for 9 years, at $4\frac{1}{4}$ per cent. per annum?

123. What is the interest of $31573.25 for 10 months at 6 per cent. per annum?

124. What will be the amount of $9537.15 for 11 years, 2 months, and 18 days at 7 per cent. per annum?

125. What will be the amount of $3758.56 for 3 years at 7 per cent., the interest being compounded semi-annually?

126. If I buy 895 gallons of molasses and lose 17 per cent. by leakage, how much have I left?
127. Bought a piece of cloth containing 150 yards for $650: what must it be sold for per yard, in order to gain $300?
128. What is the bank discount on a note of $556.27 payable in 60 days, discounted at 6 per cent. per annum?
129. The sum of two numbers is 5330, their difference is 1999: what are the two numbers?
130. How many scholars are there in a class, to which if 11 be added the number will be augmented one-sixteenth?
132. Sound travels about 1142 feet in a second. If then the flash of a cannon be seen at the moment it is fired, and the report heard 45 seconds after, what distance would the observer be from the gun?
133. What number is that which being augmented by 85, and this sum divided by 9, will give 25 for the quotient?
134. One-fifth of an army was killed in battle, $\frac{1}{6}$ part was taken prisoners, and $\frac{1}{10}$ died by sickness: if 4000 men were left, how many men did the army at first consist of?
135. The greatest of two numbers is 15 and the sum of their squares is 346: what are the two numbers?
136. At what rate per cent. will $1720.75 amount to $2325.86 in 7 years?
137. In what time will $2377.50 amount to $2852.42 at 4 per cent. per annum?
138. What principal put at interest for 7 years, at 5 per cent. per annum, will amount to $2327.89?
139. What is the greatest common measure of 945, 1560, and 22683?
140. What is the greatest common measure of 204, 1190, 1445, and 2006?
141. Find the least common multiple of 6, 9, 4, 14, and 16.

142. What is the least common multiple of 11, 17, 19, 21, and 7?

143. What is the least common multiple of 7, 15, 21, 28, 35, 100, 125?

144. Reduce $\frac{37594}{999}$ to a mixed number.

145. Reduce $149\frac{5}{9}$ to an improper fraction.

146. Reduce $375\frac{24}{99}$ to an improper fraction.

147. Reduce $17494\frac{543}{999}$ to an improper fraction.

148. Reduce $\frac{410}{510}$ to its lowest terms.

149. Reduce $\frac{3455}{17455}$ to its lowest terms.

150. Reduce $\frac{8343}{97437}$ to its lowest terms.

151. Reduce $\frac{4}{5}$, $\frac{8}{9}$, and $\frac{3}{15}$ to their least common denominator.

152. Reduce $\frac{3}{15}$, $\frac{4}{24}$, and $\frac{8}{9}$ to their least common denominator.

153. Find the least common denominator and add the fractions, $\frac{1}{16}$, $\frac{3}{7}$, $\frac{2}{8}$, and $\frac{4}{9}$.

154. Find the least common denominator and add $\frac{6}{15}$, $\frac{3}{5}$, $\frac{4}{8}$, and $\frac{5}{30}$.

155. Multiply $5\frac{1}{4}$ by $\frac{1}{6}$.

156. Multiply $\frac{12}{10}$ by $\frac{3}{4}$ of 9.

157. If 80 yards of cloth cost $340, what will 650 yards cost?

158. If 120 sheep yield 330 pounds of wool, how many pounds will be obtained from 1200 sheep?

159. If 6 gallons of molasses cost $1.95, what will 6 hogsheads cost?

160. If $\frac{4}{5}$ of a yard of cloth cost $1\frac{5}{9}$, what will $7\frac{1}{2}$ yards cost?

161. What is the cost of $28\frac{1}{2}$ yards of cloth, at $4\frac{3}{4}$ per yard?

162. What is the interest of $1914.16 for 18 years at 3½ per cent. per annum?

163. What is the amount of $7953.70 for 9 months at 6 per cent per annum?

164. A merchant has 1200 barrels of flour; he shipped 64 per cent. of it and sold the remainder: how much did he sell?

165. Two men had each $240. One of them spends 14 per cent., and the other 18½ per cent.: how many dollars more did one spend than the other?

166. What is the difference between 5½ per cent. of $800 and 6½ per cent of $1050?

167. What is the square root of 15193592?

168. What is the square root of 36372961?

169. What is the cube root of 48 28544?

170. What is the cube root of 27054036008?

171. If a person receives $1 for ⅐ of a day's work, how much is that a day?

172. What number is that of which ½, ⅓, and ¼ added together, will make 65?

Our Own Series of School Books.

PREPARED BY

Richard Sterling, A. M., & J. D. Campbell, A. M.

CONSISTING OF

I. Our Own Primer. For the use of children, pp. 34, 12mo

II. Our Own Spelling Book. For schools. pp. 112.

III. Our Own First Reader. Arranged to follow the Primer and containing easy lessons for those beginning to read, with spelling lessons accompanying each, pp. 72, 12mo.

IV. Our Own Second Reader. The lessons in this book have been carefully selected to suit the progress of those who have finished the First Reader, and at the same time to furnish both entertainment and instruction, pp. 168, 12mo.

V. Our Own Third Reader. In this book the most difficult words of each lesson are defined, to aid the pupil in understanding what he reads. Each lesson is also followed by an Exercise to be written on the black-board or slate, for the purpose of teaching the pupil to spell and punctuate. pp. 200.

VI. Our Own Fourth Reader. The lessons in this book will be found to possess high literary merit, without being above the capacity of the young reader. Definitions of the more difficult words are continued.

VII. Our Own Fifth Reader. A Rhetorical Reader for the more advanced classes in Academies and High Schools.

OUR OWN SCHOOL GRAMMARS.

BY C. W SMYTHE, A. M.

Our Own Primary Grammar. Designed for the use of beginners, and containing only the simple facts of the language. pp. 72, 12mo.

Our Own Elementary Grammar, Designed for our Schools and Academies, as a sequel to the Primary Grammar, and embracing a complete elementary statement of the subject.

OUR OWN SCHOOL ARITHMETICS.

BY S. LANDER, A. M.

Our Own Primary Arithmetic. For the use of beginners. pp. 96, 12 mo.

Our Own Mental Arithmetic. In course of preparation.

Our Own School Arithmetic. pp. 224, 12mo.

BINGHAM'S LATIN GRAMMAR.

For the use of Schools, with Exercises and Vocabularies.

Bingham's Cæsar, with Notes and Vocabulary.

Sterling, Campbell & Albright, *Publishers,*
GREENSBORO, N. C.