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PART III.

XV. *On a Class of Invariants.**

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Communicated by Professor CAYLEY, *LL.D., F.R.S.*

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I HAVE not seen it noticed by any mathematician that in the theory of Linear Differential Equations there are two important classes of functions of the coefficients which have remarkable analogies to the invariants of Algebraic Binary Quantics; consequently I venture to call attention to their existence and also to give examples of their application in the present paper.

For convenience I write the equation with binomial coefficients thus

$$\frac{d^ny}{dx^n}+nP_1\frac{d^{n-1}y}{dx^{n-1}}+\frac{n.n-1.}{\underline{|2}}P_2\frac{d^{n-2}y}{dx^{n-2}}+\ldots+P_ny=0 \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where of course P_1, P_2, &c., are functions of x only.

If now we remove the coefficient of $\frac{d^{n-1}y}{dx^{n-1}}$ by changing y to $ye^{-\int P_1dx}$ the equation, wanting the second term, may be written,

$$\frac{d^ny}{dx^n}+\frac{n.n-1.}{\underline{|2}}H\frac{d^{n-2}y}{dx^{n-2}}+\frac{n.n-1.n-2.}{\underline{|3}}G\frac{d^{n-3}y}{dx^{n-3}}$$

$$+\frac{n.n-1.n-2.n-3.}{\underline{|4}}(I+3H^2)\frac{d^{n-4}y}{dx^{n-4}}+\ldots=0 \quad . \quad . \quad . \quad (2)$$

where we have

$$H=P_2-P_1^2-\frac{dP_1}{dx}$$

$$G=2P_1^3-3P_1P_2+P_3-\frac{d^2P_1}{dx^2}$$

$$I=-6P_1^4+12P_1^2P_2-4P_1P_3-3P_2^2+P_4-\frac{d^3P_1}{dx^3}$$

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* Since the publication of the abstract of this paper the Rev. R. HARLEY has mentioned to me that the first class of functions treated of here have been already investigated by Sir JAMES COCKLE; having consulted the memoirs I was referred to by Mr. HARLEY, I think little similarity will be found between Sir JAMES COCKLE's results and mine.—J. C. M.

Now H, G, I and the remaining coefficients of equation (2) are in a certain sense invariants of the original equation (1), for they remain unaltered if in equation (1) we change y to $yf(x)$, where $f(x)$ is any function of x, and then divide by $f(x)$ so as to make the coefficient of $\frac{d^ny}{dx^n}$ unity; thus writing the equation so transformed

$$\frac{d^ny}{dx^n}+nQ_1\frac{d^{n-1}y}{dx^{n-2}}+\frac{n.n-1.}{\underline{|2}}Q_2\frac{d^{n-2}y}{dx^{n-2}}+\ldots Q_ny=0 \qquad (3)$$

and for convenience writing $f(x)$, $\frac{df}{dx}$, $\frac{d^2f}{dx^2}$, &c., f, f', f'' we find

$$Q_1=\frac{f'+P_1f}{f},\ Q_2=\frac{f''+2P_1f'+P_2f}{f}$$

$$Q_3=\frac{f'''+3P_1f''+3P_2f'+P_3f}{f}$$

Hence also

$$\frac{dQ_1}{dx}=\frac{f''f-f'^2}{f^2}+\frac{dP_1}{dx}$$

$$\frac{d^2Q_1}{dx^2}=\frac{f'''}{f}-\frac{3f'f''}{f^2}+\frac{2f'^3}{f^3}+\frac{d^2P_1}{dx^2}$$

from which we easily prove

$$Q_2-Q_1^2-\frac{dQ_1}{dx}=P_2-P_1^2-\frac{dP_1}{dx}$$

and

$$2Q_1^3-3Q_1Q_2+Q_3-\frac{d^2Q_1}{dx^2}=2P_1^3-3P_1P_2+P_3-\frac{d^2P_1}{dx^2}$$

In a similar manner we find

$$-6Q_1^4+12Q_1^2Q_2-4Q_1Q_3-3Q_2^2+Q_4-\frac{d^3Q_1}{dx^3}=I$$

The theorem proved in these cases may be easily shown to be generally true, as follows:—If from equation (3) we remove the second term by the substitution of $ye^{-\int Q_1 dx}$ for y it is evident the result must be identical with (2); but the coefficients of the resulting equation are the same functions of Q_1, Q_2, Q_3, &c., as the coefficients of (2) are of P_1, P_2, P_3, &c., hence the theorem is proved.

I proceed now to some particular applications of the general theorem.

The quadratic.

$$\frac{d^2y}{dx^2}+2P_1\frac{dy}{dx}+P_2y=0$$

Here we have the invariant H or

$$P_2-P_1^2-\frac{dP_1}{dx}$$

Let us now seek the condition that the two solutions of the equation $y=y_1$ and $y=y_2$ should be connected by the relation $y_1=y_2x$, which relation, since it depends only on the ratios of y_1 and y_2, must be expressible in terms of the invariant H.

If in the equation we change y to yy_2 the solutions of the resulting equation will be $y=1$ and $y=x$; hence if the equation is

$$\frac{d^2y}{dx^2}+2Q_1\frac{dy}{dx}+Q_2y=0$$

we have $Q_2=0$, $Q_1=0$ remembering now that

$$Q_2-Q_1^2-\frac{dQ_1}{dx}=H$$

the required condition is

$$H=0$$

To solve the equation in this case we have

$$Q_1=\frac{1}{y_2}\left(\frac{dy_2}{dx}+P_1y_2\right)=0$$

Hence $y_2=e^{-\int P_1dx}$ and the complete solution is

$$y=e^{-\int P_1dx}\{Ax+B\}$$

where A and B are arbitrary constants.

We may remark also, as is at once obvious, that the condition $H=0$ is the eliminant of the equations

$$\frac{d^2y}{dx^2}+2P_1\frac{dy}{dx}+P_2y=0$$

and

$$\frac{dy}{dx}+P_1y=0$$

If we seek the mere general condition that the solutions should be connected by the relation $y_1=y_2f(x)$, where $f(x)$ is some given function of x, transforming as before by the substitution of yy_2 for y the resulting equation

$$\frac{d^2y}{dx^2}+2Q_1\frac{dy}{dx}+Q_2y=0$$

must have for solutions $y=1$ and $y=f(x)$, hence we have

$$Q_2=0 \text{ and } f''(x)+2Q_1f'(x)=0$$

therefore

$$\frac{dQ_1}{dx}+Q_1^2+H=0$$

or substituting the value of Q_1 we get

$$2\frac{f'''(x)}{f'(x)}-3\left(\frac{f''(x)}{f'(x)}\right)^2-4H=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

the required condition.

To solve the equation in this case, remembering that $Q_1=\frac{1}{y_2}\left(\frac{dy_2}{dx}+P_1y_2\right)$, we at once see that y_2 is the solution of the equation

$$2f'(x)\frac{dy}{dx}+\{2P_1f'(x)+f''(x)\}y=0$$

from which

$$y_2=\frac{e^{-\int P_1dx}}{\sqrt{f'(x)}}$$

and the complete solution is

$$y=\frac{e^{-\int P_1dx}}{\sqrt{f'(x)}}\{Af(x)+B\}$$

where A and B are arbitrary constants.

If now in equation (4) we suppose that $f(x)$ is not known, replacing $f(x)$ by y we see the solution of the equation

$$2\frac{d^3y}{dx^3}\cdot\frac{dy}{dx}\cdot-3\left(\frac{d^2y}{dx^2}\right)^2-4H\left(\frac{dy}{dx}\right)^2=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

is

$$y=\frac{Ay_1+By_2}{Cy_1+Dy_2}$$

where A, B, C, D are arbitrary constants and y_1, y_2 the solutions of the linear equation

$$\frac{d^2y}{dx^2}+2P_1\frac{dy}{dx}+P_2y=0$$

Again it appears that if $y=\phi(x)$ satisfies equation (5) the complete solution of it is

$$y=\frac{A\phi(x)+B}{C\phi(x)+D}$$

Let us now consider the two equations

$$\frac{d^2y}{dx^2}+2P_1\frac{dy}{dx}+P_2y=0$$

and

$$\frac{d^2y}{dx^2}+2R_1\frac{dy}{dx}+R_2y=0$$

and let us seek the condition that the two solutions of the first equation, y_1 and y_2, should be connected with the two solutions of the second, y_3 and y_4, by the relation

$$\frac{y_1}{y_2}=\frac{y_3}{y_4}$$

Let $f(x)$ be the common value of these fractions, then referring to equation (4) we have at once the required condition, viz.:

$$P_2-P_1^2-\frac{dP_1}{dx}=R_2-R_1^2-\frac{dR_1}{dx}$$

It may be remarked that if H does not equal 0 but a constant, say k, the complete solution of the equation

$$\frac{d^2y}{dx^2}+2P_1\frac{dy}{dx}+P_2y=0$$

is

$$y=e^{-\int P_1dx}\{A\sin\sqrt{k}x+B\cos\sqrt{k}x\}$$

The cubic.

Let us now consider the equation of the third order

$$\frac{d^3y}{dx^3}+3P_1\frac{d^2y}{dx^2}+3P_2\frac{dy}{dx}+P_3y=0$$

we have two invariants G and H; and the equation becomes by removing the second term

$$\frac{d^3y}{dx^3}+3H\frac{dy}{dx}+Gy=0$$

Let us now consider what relation must exist between H and G in order that two solutions of the equation y_1 and y_2 should be connected by the relation $y_1=y_2x$.

Transform the cubic by substituting yy_2 for y and let the resulting equation be

$$\frac{d^3y}{dx^3}+3Q_1\frac{d^2y}{dx^2}+3Q_2\frac{dy}{dx}+Q_3y=0$$

Since this equation is satisfied by $y=1$ and also by $y=x$ we have at once

$$Q_3=0 \text{ and } Q_2=0$$

hence

$$-Q_1^2-\frac{dQ_1}{dx}=H$$

$$2Q_1^3-\frac{d^2Q_1}{dx^2}=G$$

from which we find

$$G-\frac{dH}{dx}+2HQ_1=0$$

Substituting the value of Q_1 derived from this equation in H and reducing we find for our required condition

$$4H^3+G^2-\left(\frac{dH}{dx}\right)^2+2H\left(\frac{d^2H}{dx^2}-\frac{dG}{dx}\right)=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

This condition, as is seen, is the eliminant of the equations

$$\frac{d^3y}{dx^3}+3P_1\frac{d^2y}{dx^2}+3P_2\frac{dy}{dx}+P_3y=0$$

and

$$\frac{d^2y}{dx^2}+2P_1\frac{dy}{dx}+P_2y \qquad =0$$

To solve the equation when condition (6) is true. First consider the equation with the second term removed, and we have to find y_2 the equations

$$\frac{d^3y}{dx^3}+3H\frac{dy}{dx}+Gy=0$$

$$\frac{d^2y}{dx^2}+Hy \qquad =0$$

of which equations y_2 must be a common solution, as follows at once from the conditions $Q_3=0$, $Q_2=0$. Thence we easily find

$$y_2=\sqrt{H}e^{-\frac{1}{2}\int\frac{G}{H}dx}$$

and therefore

$$y_1=x\sqrt{H}e^{-\frac{1}{2}\int\frac{G}{H}dx}$$

These values of y_1 and y_2 are to be multiplied by $e^{-\int P_1dx}$ to get the corresponding solutions of the cubic when the second term is not removed.

To get the third solution we have

$$\frac{d^3y}{dx^3}+3Q_1\frac{d^2y}{dx^2}=0$$

which gives, remembering the relation

$$G-\frac{dH}{dx}+2HQ_1=0$$

$$y=\iint H^{\frac{3}{2}}e^{-\frac{3}{2}\int\frac{G}{H}dx}dx^2$$

thence we easily find the complete solution of the original cubic

$$y=\sqrt{H}e^{-\int\left(P_1+\frac{G}{2H}\right)dx}\left\{A+Bx+C\iint H^{\frac{3}{2}}e^{-\frac{3}{2}\int\frac{G}{H}dx}dx^2\right\}$$

where A, B, and C are arbitrary constants.

If we have the two conditions $G=0$ and $H=0$, then

$$y_1=y_2x=y_3x^2$$

and the complete solution is

$$y=e^{-\int P_1dx}\{A+Bx+Cx^2\}$$

$G=0$ expresses the condition that $y=e^{-\int P_1dx}$ should be a solution of the equation.

If we wish to find the conditions necessary that the following relations should exist between the solutions of the cubic

$$y_1=y_3\phi(x), \qquad y_2=y_3\psi(x)$$

where $\phi(x)$ and $\psi(x)$ are known functions of x; change as before the cubic by substituting yy_3 for y and let it become

$$\frac{d^3y}{dx^3}+3Q_1\frac{d^2y}{dx^2}+3Q_2\frac{dy}{dx}+Q_3y=0$$

we must evidently have $Q_3=0$, hence we have

$$\phi'''+3Q_1\phi''+3Q_2\phi'=0$$

$$\psi'''+3Q_1\psi''+3Q_2\psi'=0$$

we have also

$$Q_2-Q_1{}^2-\frac{dQ_1}{dx}=H$$

$$Q_1{}^3-3Q_1Q_2-\frac{d^2Q_1}{dx^2}=G$$

Now if we let

$$\frac{\phi'''\psi'-\psi'''\phi'}{3(\phi''\psi'-\psi''\phi')}\equiv -F(x)$$

and

$$\frac{\phi'''\psi''-\psi'''\phi''}{3(\phi''\psi'-\psi''\phi')}\equiv f(x)$$

we have $Q_1=F(x)$, $Q_2=f(x)$, and the conditions sought are

$$H+F'(x)+(F(x))^2-f(x)=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

and

$$G+F''(x)+3F(x)f(x)-(F(x))^3=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

To solve the equation in this case we have

$$Q_1=\frac{1}{y_3}\left(\frac{dy_3}{dx}+P_1y_3\right)$$

thence y_3 must be the solution of the equation

$$\frac{dy}{dx}+\{P_1-F(x)\}y=0$$

from which we have

$$y=e^{\int\{F(x)-P_1\}dx}$$

or

$$y=\frac{e^{-\int P_1dx}}{(\phi''\psi'-\psi''\phi')^{\frac{1}{3}}}$$

and therefore the complete solution of the cubic is

$$y=\frac{e^{-\int P_1dx}}{(\phi''\psi'-\psi''\phi')^{\frac{1}{3}}}\{A+B\phi(x)+C\psi(x)\}$$

If we make $\phi(x)=\frac{1}{\psi(x)}$ and then eliminate $\psi(x)$ between equations (7) and (8) we should evidently obtain the condition necessary that the relation $y_1y_2=y_3^2$ should exist. I have, however, obtained the condition in a much more simple manner, and found it to be

$$2G-3\frac{dH}{dx}=0$$

which result I give further on in the present paper.

Let us now consider the more general problem; to determine the relation between the coefficients of the cubic in order that two solutions should be connected by the relation $y_1=y_2f(x)$ where $f(x)$ is a given function of x.

As before, change y to y_2y, and the equation must become of the form

$$\frac{d^3y}{dx^3}+3Q_1\frac{d^2y}{dx^2}+3Q_2\frac{dy}{dx}=0$$

Q_3 vanishing.

We have then

$$f'''+3Q_1f''+3Q_2f'=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Also

$$Q_2-Q_1^2-\frac{dQ_1}{dx}=H \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

$$2Q_1^3-3Q_1Q_2-\frac{d^2Q_1}{dx^2}=G \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

and the problem is to eliminate Q_1 and Q_2 from these equations.

If we substitute in (10) and (11) the value of Q_2 found from (9), we find they can be written in the forms

$$-\mathrm{R}^2-\frac{d\mathrm{R}}{dx}=\mathrm{S}$$

$$2\mathrm{R}^3+\mathrm{RK}-\frac{d^2\mathrm{R}}{dx^2}=\mathrm{T}$$

where

$$\mathrm{R}=\mathrm{Q}_1+\frac{f''}{2f'},\quad \mathrm{S}=\mathrm{H}-\frac{f'''}{6f'}+\frac{f''^2}{4f'^2}$$

$$\mathrm{K}=\frac{2f'f'''-3f''^2}{2f'^2},\quad \mathrm{T}=\mathrm{G}-\frac{f''^3}{f'^3}+\frac{2f''f'''}{f'^2}-\frac{f^{(\mathrm{iv})}}{2f'}$$

we have

$$\frac{d\mathrm{S}}{dx}-\mathrm{T}=(2\mathrm{S}-\mathrm{K})\mathrm{R} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (12)$$

Substituting this value of R in S and reducing we find for the required condition

$$4\mathrm{S}^3+\mathrm{T}^2-\left(\frac{d\mathrm{S}}{dx}\right)^2+2\mathrm{S}\left(\frac{d^2\mathrm{S}}{dx^2}-\frac{d\mathrm{T}}{dx}\right)+\mathrm{SK}^2-\mathrm{LS}^2\mathrm{K}+\mathrm{K}\left(\frac{d\mathrm{T}}{dx}-\frac{d^2\mathrm{S}}{dx^2}\right)-\frac{d\mathrm{K}}{dx}\left(\mathrm{T}-\frac{d\mathrm{S}}{dx}\right)=0\ . \quad (13)$$

If K vanishes or $f'^2=\mathrm{A}f''^3$, A being a constant, equation (13) is derived from (6) by changing H and G to S and T respectively.

If we regard $f(x)$ as unknown, equal u say, equation (12) is the differential equation of the fifth order, of which the complete solution is

$$u=\frac{\mathrm{A}y_1+\mathrm{B}y_2+\mathrm{C}y_3}{\mathrm{D}y_1+\mathrm{E}y_2+\mathrm{F}y_3}$$

where y_1, y_2, y_3 are solutions of the equation

$$\frac{d^3y}{dx^3}+3\mathrm{P}_1\frac{d^2y}{dx^2}+3\mathrm{P}_2\frac{dy}{dx}+\mathrm{P}_3y=0$$

and A, B, C, D, E, F arbitrary constants.

To solve the cubic when condition (13) holds. From equations (9) and 12 we find at once Q_1 and Q_2 in terms of x, let their values be respectively $\phi(x)$ and $\psi(x)$ we have then

$$\phi(x)=\frac{\frac{d\mathrm{S}}{dx}-\mathrm{T}}{2\mathrm{S}-\mathrm{K}}-\frac{f''}{2f'}$$

$$\psi(x)=\frac{f''^2}{2f'^2}-\frac{\frac{d\mathrm{S}}{dx}-\mathrm{T}}{2\mathrm{S}-\mathrm{K}}\cdot\frac{f''}{f'}-\frac{f'''}{3f'}$$

Now since

$$\mathrm{Q}_1=\frac{1}{y_2}\left(\frac{dy_2}{dx}+\mathrm{P}_1y_2\right)$$

we have

$$y_2=e^{\int(\phi(x)-\mathrm{P}_1)dx}$$

therefore

$$y_1=f(x)e^{\int(\phi(x)-\mathrm{P}_1)dx}$$

To determine y_3, let $y_3=y_2\chi(x)$ and we have to determine χ

$$\chi'''+3\mathrm{Q}_1\chi''+3\mathrm{Q}_2\chi'=0$$

but also

$$f'''+3\mathrm{Q}_1f''+3\mathrm{Q}_2f'=0$$

from which we easily find

$$\chi''f'-\chi'f''=e^{3\int\mathrm{Q}_2dx}=e^{3\int\psi(x}$$

therefore

$$\chi=\int\left\{f'\int\frac{1}{f'^2}e^{3\int\psi(x)dx}dx\right\}dx$$

and the complete solution of the cubic is in the case we are considering.

$$y=e^{\int(\phi(x)-\mathrm{P}_1)dx}\int\left\{f'\int\frac{1}{f'^2}\,e^{3\int\psi(x)dx}dx\right\}dx$$

remembering that an arbitrary constant is implied in each integration.

The quartic.

Consider now the equation of the fourth order

$$\frac{d^4y}{dx^4}+4\mathrm{P}_1\frac{d^3y}{dx^3}+6\mathrm{P}_2\frac{d^2y}{dx^2}+4\mathrm{P}_3\frac{dy}{dx}+\mathrm{P}_4y=0$$

we have three invariants H, G, and I, and the equation with its second term removed becomes

$$\frac{d^4y}{dx^4}+6\mathrm{H}\frac{d^2y}{dx^2}+4\mathrm{G}\frac{dy}{dx}+(\mathrm{I}+3\mathrm{H}^2)y=0$$

If we seek the condition that two solutions of the equation should be connected by the relation $y_1=y_2x$, transforming the equation by changing y to yy_2, the result must be of the form

$$\frac{d^4y}{dx^4}+4\mathrm{Q}_1\frac{d^3y}{dx^3}+6\mathrm{Q}_2\frac{d^2y}{dx^2}=0$$

Hence we have

$$\mathrm{H}=\mathrm{Q}_2-\mathrm{Q}_1{}^2-\frac{d\mathrm{Q}_1}{dx}$$

$$\mathrm{G}=2\mathrm{Q}_1{}^3-3\mathrm{Q}_1\mathrm{Q}_2-\frac{d^2\mathrm{Q}_1}{dx^2}$$

$$\mathrm{I}=-6\mathrm{Q}_1{}^4+12\mathrm{Q}_1{}^2\mathrm{Q}_2-3\mathrm{Q}_2{}^2-\frac{d^3\mathrm{Q}_1}{dx^3}$$

and the condition required is the result of eliminating Q_1 and Q_2 from these equations.

The required result is also the eliminant of the equations

$$\frac{d^4y}{dx^4}+6\mathrm{H}\frac{d^2y}{dx^2}+4\mathrm{G}\frac{dy}{dx}+(\mathrm{I}+3\mathrm{H}^2)y=0$$

and

$$\frac{d^3y}{dx^3}+3\mathrm{H}\frac{dy}{dx}+\mathrm{G}y=0$$

as is evident.

Having obtained the result in a very cumbersome form only I do not give it, but at once go on to cases where two or three conditions subsist between the solutions of the quartic.

Let us first consider the case where

$$y_1=y_2x=y_3x^2$$

changing y to yy_3 the resulting equation must be of the form

$$\frac{d^4y}{dx^4}+4\mathrm{Q}_1\frac{d^3y}{dx^3}=0$$

Hence we have

$$\mathrm{H}=-\mathrm{Q}_1{}^2-\frac{d\mathrm{Q}_1}{dx},\ \mathrm{G}=2\mathrm{Q}_1{}^3-\frac{d^2\mathrm{Q}_1}{dx^2}$$

$$\mathrm{I}=-6\mathrm{Q}_1{}^4-\frac{d^3\mathrm{Q}_1}{dx^3}$$

from which

$$\mathrm{G}-\frac{d\mathrm{H}}{dx}+2\mathrm{Q}_1\mathrm{H}=0,\ \mathrm{I}-\frac{d\mathrm{G}}{dx}=6\mathrm{Q}_1{}^2\mathrm{H}$$

one of the required conditions is then given by equation (6), and the second is easily found to be

$$2\mathrm{H}\left(\mathrm{I}-\frac{d\mathrm{G}}{dx}\right)=3\left(\mathrm{G}-\frac{d\mathrm{H}}{dx}\right)^2$$

To solve the equation, we find at once as in the case of the equation of the third order

$$y_3=e^{-\int\mathrm{P}_1dx}\sqrt{\mathrm{H}}e^{-\frac{1}{2}\int\frac{\mathrm{G}}{\mathrm{H}}dx}$$

and y_4 is the solution of the equation

$$\frac{d^4y}{dx^4}+4\mathrm{Q}_1\frac{d^3y}{dx^3}=0$$

multiplied by y_3.

Hence remembering the value of Q_1, we get for the complete solution of the quartic

$$y=\mathrm{A}\sqrt{\mathrm{H}}e^{-\int\left(\mathrm{P}_1+\frac{\mathrm{G}}{2\mathrm{H}}\right)dx}\left\{\iiint\mathrm{H}^2e^{-2\int\frac{\mathrm{G}}{\mathrm{H}}dx}dx^3+\mathrm{B}x^2+\mathrm{C}x+\mathrm{D}\right\}$$

If $y_1=y_2x=y_3x^2=y_4x^3$ then we have

$$H=0,\ G=0,\ I=0$$

and the complete solution is

$$y=e^{-\int P_1dx}(Ax^3+Bx^2+Cx+D)$$

It is obvious that when H, G, and I instead of being each equal to 0 are each equal to a given constant the equation can be at once solved.

If we seek the conditions for

$$y_1=y_4\phi(x),\ y_2=y_4\psi(x),\ y_3=y_4\chi(x)$$

where $\phi(x)$, $\psi(x)$, $\chi(x)$ are given functions of x; then changing y to yy_4 if the resulting equation be written

$$\frac{d^4y}{dx^4}+4Q_1\frac{d^3y}{dx^3}+6Q_2\frac{d^2y}{dx^2}+4Q_3\frac{dy}{dx}=0$$

we have

$$\phi^{(iv)}+4Q_1\phi'''+6Q_2\phi''+4Q_3\phi'=0$$
$$\psi^{(iv)}+4Q_1\psi'''+6Q_2\psi''+4Q_3\psi'=0$$
$$\chi^{(iv)}+4Q_1\chi'''+6Q_2\chi''+4Q_3\chi'=0$$

from which equations we find Q_1, Q_2, Q_3 are terms of x. Let these values be $F_1(x)$, $F_2(x)$, and $F_3(x)$, and the required conditions are

$$H=F_2(x)-(F_1(x))^2-F_1'(x)$$
$$G=2(F_1(x))^3-3F_1(x)F_2(x)+2F_3(x)-F_1''(x)$$
$$I=-6(F_1(x))^4+12(F_1(x))^2F_2(x)-4F_1(x)F_3(x)-3(F_2(x))^2-F_1'''(x)$$

To find the solution in this case, we have

$$Q_1y_4=\frac{dy_4}{dx}+P_1y_4$$

Hence

$$y_4=e^{\int(Q_1-P_1)dx}$$

Now if we let

$$\begin{vmatrix}\phi' & \psi' & \chi' \\ \phi'' & \psi'' & \chi'' \\ \phi''' & \psi''' & \chi'''\end{vmatrix}=\Delta$$

we find from the previous equations

$$Q_1=-\frac{\frac{d\Delta}{dx}}{4\Delta}$$

and therefore we have for the complete solution

$$y=\frac{e^{-\int \mathrm{P}_1 dx}}{\Delta^{\frac{1}{4}}}\{\mathrm{A}\phi(x)+\mathrm{B}\psi(x)+\mathrm{C}\chi(x)+\mathrm{D}\}$$

If

$$\phi(x)=e^{\alpha x},\quad \psi(x)=e^{\beta x},\quad \chi(x)=e^{\gamma x},$$

Q_1, Q_2, Q_3 are constants determined from the equations previously given; in fact we easily find

$$4\mathrm{Q}_1=-(\alpha+\beta+\gamma),\quad 6\mathrm{Q}_2=\alpha\beta+\alpha\gamma+\beta\gamma,\quad 4\mathrm{Q}_3=-\alpha\beta\gamma$$

H, G, and I are constants, and the solution is

$$y=e^{-\frac{\alpha+\beta+\gamma}{4}x-\int \mathrm{P}_1 dx}\{\mathrm{A}e^{\alpha x}+\mathrm{B}e^{\beta x}+\mathrm{C}e^{\gamma x}+\mathrm{D}\}$$

II.

In addition to the class of invariants of Linear Differential Equations which I have discussed in the first part of this paper, there is also another class worth noticing, namely, functions of the coefficients of the equation which remain unaltered when the *independent* variable is changed. I propose now to consider these functions.

The quadratic.

If we take the equation

$$\frac{d^2y}{dx^2}+2\mathrm{P}_1\frac{dy}{dx}+\mathrm{P}_2 y=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

and let $x=\phi(t)$, the new equation is

$$\frac{d^2y}{dt^2}+2\mathrm{Q}_1\frac{dy}{dt}+\mathrm{Q}_2 y=0$$

where

$$2\mathrm{Q}_1=2\mathrm{P}_1\phi'-\frac{\phi''}{\phi'},\quad \mathrm{Q}_2=\mathrm{P}_2\phi'^2$$

From these values of Q_1 and Q_2 we easily find the identity

$$\frac{\frac{d\mathrm{Q}_2}{dt}+4\mathrm{Q}_1\mathrm{Q}_2}{\mathrm{Q}_2^{\frac{3}{2}}}\equiv\frac{\frac{d\mathrm{P}_2}{dx}+4\mathrm{P}_1\mathrm{P}_2}{\mathrm{P}_2^{\frac{3}{2}}}$$

Hence we see that

$$\frac{\frac{d\mathrm{P}_2}{dx}+4\mathrm{P}_1\mathrm{P}_2}{\mathrm{P}_2^{\frac{3}{2}}}$$

is an invariant of this kind of the equation of the second order.

To arrive at this invariant directly, let us suppose that the second term is removed from the equation by the change of the independent variable; we have then, supposing the transformed equation to be

$$\frac{d^2y}{dt^2}+uy=0$$

$$2\mathrm{P}_1\phi'-\frac{\phi''}{\phi'}=0, \quad u=\mathrm{P}_2\phi'^2$$

Hence

$$\frac{du}{dt}=\phi'^3\frac{d\mathrm{P}_2}{dx}+2\mathrm{P}_2\phi'\phi''=\phi'^3\left(\frac{d\mathrm{P}_2}{dx}+4\mathrm{P}_1\mathrm{P}_2\right)$$

therefore

$$\frac{\frac{d\mathrm{P}_2}{dx}+4\mathrm{P}_1\mathrm{P}_2}{\mathrm{P}_2^{\frac{3}{2}}}=\frac{1}{u^{\frac{3}{2}}}\frac{du}{dt}$$

from which it is evident that

$$\frac{\frac{d\mathrm{P}_2}{dx}+4\mathrm{P}_1\mathrm{P}_2}{\mathrm{P}_2^{\frac{3}{2}}}$$

remains the same when the independent variable is changed.

I shall denote this invariant by the letter J; and I now propose to give some examples of its use.

Let us seek to determine what relation among the coefficients of the equation expresses the condition that the two solutions of equation (14) y_1 and y_2 should be connected by the equation $y_1y_2=1$.

Transform the independent variable so that e^t shall be a solution of the new equation, then e^{-t} must also be a solution. Let the new equation be

$$\frac{d^2y}{dt^2}+2\mathrm{Q}_1\frac{dy}{dt}+\mathrm{Q}_2y=0$$

and we have

$$1+2\mathrm{Q}_1+\mathrm{Q}_2=0$$

$$1-2\mathrm{Q}_1+\mathrm{Q}_2=0$$

from which we find $\mathrm{Q}_1=0$, $\mathrm{Q}_2=-1$, but

$$\mathrm{J}=\frac{\frac{d\mathrm{Q}_2}{dt}+4\mathrm{Q}_1\mathrm{Q}_2}{\mathrm{Q}_2^{\frac{3}{2}}}=0$$

hence the required condition is J=0 or

$$\frac{d\mathrm{P}_2}{dx}+4\mathrm{P}_1\mathrm{P}_2=0$$

to solve the equation in this case, we have

$$Q_2 = P_2\phi'^2 \quad \text{or} \quad 1 + P_2\frac{dx^2}{dt^2} = 0$$

from which

$$t = e^{\int\sqrt{-P_2}dx}$$

and the complete solution is

$$y = Ae^{\int\sqrt{-P_2}dx} + Be^{-\int\sqrt{-P_2}dx}$$

where A and B are arbitrary constants.

Again, let us seek the condition for $y_1y_2^2 = 1$, transforming as before so that $y_1 = e^t$, we find now that the coefficients of the transformed equation are connected by the relations

$$1 + 2Q_1 + Q_2 = 0$$
$$4 - 4Q_1 + Q_2 = 0$$

therefore

$$2Q_1 = 1, \ Q_2 = -2$$

and we have

$$J^2 + 2 = 0$$

or

$$\left(\frac{dP_2}{dx} + 4P_1P_2\right)^2 + 2P_2^3 = 0$$

for the required condition.

In this case we find as before the solution to be

$$y = Ae^{\frac{1}{\sqrt{2}}\int\sqrt{-P_2}dx} + Be^{-\sqrt{2}\int\sqrt{-P_2}dx}$$

More generally if $y_1 = y_2^m$ we have, using the same transformation as before, the following equations connecting the coefficients of the transformed equation

$$1 + 2Q_1 \ \ + Q_2 = 0$$
$$m^2 + 2mQ_1 + Q_2 = 0$$

from which

$$2Q_1 = -(m+1), \quad Q_2 = m$$

and we find as before for the required condition

$$m\left(\frac{dP_2}{dx} + 4P_1P_2\right)^2 - 4(m+1)^2P_2^3 = 0$$

the solution being

$$y = Ae^{-\frac{1}{\sqrt{m}}\int\sqrt{P_2}dx} + Be^{\sqrt{m}\int\sqrt{P_2}dx}$$

this investigation fails, as it should, when $m = 1$.

As another example let us suppose y_1 and y_2 connected by any relation; for convenience take $y_2=f(\log y_1)$ and let us seek to determine how the coefficients of the equation are related, and also its solution.

Transforming as before we have

$$1+2\mathrm{Q}_1 \quad +\mathrm{Q}_2 \quad =0$$
$$f''(t)+2f'(t)\mathrm{Q}_1+\mathrm{Q}_2 f(t)=0$$

from which

$$2\mathrm{Q}_1=\frac{f(t)-f''(t)}{f'(t)-f(t)}$$
$$\mathrm{Q}_2=\frac{f'(t)-f''(t)}{f(t)-f'(t)}$$

from this we have

$$\int\sqrt{\mathrm{P}_2}dx=\int\sqrt{\frac{f'(t)-f''(t)}{f(t)-f'(t)}}dt=\mathrm{F}(t) \text{ say}$$

hence

$$t=\mathrm{F}^{-1}\left\{\int\sqrt{\mathrm{P}_2}dx\right\}$$

and the complete solution of the equation is

$$y=\mathrm{A}e^{\mathrm{F}^{-1}\{\int\sqrt{\mathrm{P}_2}dx\}}+\mathrm{B}f\{\mathrm{F}^{-1}\{\int\sqrt{\mathrm{P}_2}dx\}\}$$

to determine the condition between P_1 and P_2 we have

$$\frac{\frac{d\mathrm{Q}_1}{dt}+4\mathrm{Q}_1\mathrm{Q}_2}{\mathrm{Q}_2^{\frac{3}{2}}}=\mathrm{J}$$

substituting $\mathrm{F}^{-1}\{\int\sqrt{\mathrm{P}_2}dx\}$ for t in the left-hand side of this equation we get the required result.

It is to be remarked that $\int\sqrt{\mathrm{P}_2}dx$ is an invariant, since if the substitution $x=\phi(t)$ removes the second term from the equation we have

$$\int\sqrt{\mathrm{P}_2}dx=\int\sqrt{u}dt$$

where u has the same meaning as before.

From the equation $\mathrm{J}=0$ which expresses the condition $y_1y_2=1$ we can derive the linear differential equation of the third order, of which the solution is

$$y=\mathrm{A}y_1^2+\mathrm{B}y_2^2+\mathrm{C}y_1y_2$$

where A, B, and C are arbitrary constants.

As I afterwards make use of this equation I shall here give a full investigation of it.

The condition $J=0$, it is easily seen may be written

$$\frac{d^2P_1}{dx^2}+6P_1\frac{dP_1}{dx}+4P_1^3+4P_1H+\frac{dH}{dx}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

where H is the invariant of the first kind previously considered.

If now in the equation

$$\frac{d^2y}{dx^2}+2P_1\frac{dy}{dx}+P_2y=0$$

we change y to $y\sqrt{z}$ we find the equation

$$\frac{d^2y}{dx^2}+2Q_1\frac{dy}{dx}+Q_2y=0$$

where

$$2Q_1=2P_1+\frac{1}{z}\frac{dz}{dx}$$

Substituting this value of Q_1 for P_1 in condition (15) and reducing we get

$$\frac{d^3z}{dx^3}+6P_1\frac{d^2z}{dx^2}+2\left(\frac{dP_1}{dx}+4P_1^2+2P_2\right)\frac{dz}{dx}+2\left(4P_1P_2+\frac{dP_2}{dx}\right)z=0 \quad . \quad . \quad . \quad . \quad (A)$$

this differential equation in z is evidently the required equation.

It is to be remarked that if we remove the second term from this equation it becomes

$$\frac{d^3z}{dx^3}+4H\frac{dz}{dx}+2\frac{dH}{dx}z=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (B)$$

The cubic.

Let us now consider the equation of the third order

$$\frac{d^3y}{dx^3}+3P_1\frac{d^2y}{dx^2}+3P_2\frac{dy}{dx}+P_3y=0$$

Substituting $\phi(t)$ for x we get

$$\frac{d^3y}{dt^3}+3Q_1\frac{d^2y}{dt^2}+3Q_2\frac{dy}{dt}+Q_3y=0$$

where

$$3Q_1=3P_1\phi'-\frac{3\phi''}{\phi'}, \quad Q_3=P_3\phi'^3$$

$$3Q_2=2P_2\phi'^2-\frac{\phi'''}{\phi'}+\frac{3\phi''^2}{\phi'^2}-3P_1\phi''$$

Now let

$$Q_1=0,\ 3Q_2=u,\ Q_3=v$$

from which

$$P_1=\frac{\phi''}{\phi'^2},\ u=3P_2\phi'^2-\frac{\phi''}{\phi'},\ v=P_3\phi'^3$$

from which equations two distinct invariants, of the second kind, of the cubic may be found, thus

$$\frac{dv}{dt}=\phi'^4\frac{dP_3}{dx}+3\phi'^2\phi''P_3=\phi'^4\left(\frac{dP_3}{dx}+3P_1P_3\right)$$

therefore

$$\frac{\frac{dP_3}{dx}+3P_1P_3}{P_3^{\frac{4}{3}}}=\frac{1}{v^{\frac{4}{3}}}\frac{dv}{dt}$$

Hence

$$\frac{\frac{dP_3}{dx}+3P_1P_3}{P_3^{\frac{4}{3}}}$$

is an invariant which I shall call I_1.

Again

$$\frac{dP_1}{dx}=-\frac{2\phi''^2}{\phi'^4}+\frac{\phi''}{\phi'^3}$$

or

$$\phi'^2\left\{\frac{dP_1}{dx}+2P_1^2-3P_2\right\}=-u$$

Hence we have

$$\frac{\frac{dP_1}{dx}+2P_1^2-3P_2}{P_3^{\frac{2}{3}}}=-\frac{u}{v^{\frac{2}{3}}}$$

and we have another invariant

$$\frac{\frac{dP_1}{dx}+2P_1^2-3P_2}{P_3^{\frac{2}{3}}}$$

which I shall call I_2.

We have also, calling $-\frac{dP_1}{dx}-2P_1^2+3P_2$, L,

$$\phi'^2L=u$$

therefore

$$\phi'^3\frac{dL}{dx}+2\phi'\phi''L=\frac{du}{dt}$$

from which

$$\frac{\frac{dL}{dx}+2P_1L}{L^{\frac{3}{2}}}=\frac{1}{u^{\frac{3}{2}}}\frac{du}{dt}$$

and we have the invariant

$$\frac{\frac{dL}{dx}+2P_1L}{L^{\frac{3}{2}}},$$

I_3 say.

I_3, however, is not a distinct invariant, since it is evident from the method of forming it that it can be expressed in terms of I_1 and I_2.

It is to be remarked that if I is any invariant of the cubic, of the kind we are considering, then

$$\frac{1}{P_3^{\frac{1}{3}}}\frac{dI}{dx} \quad \text{and} \quad \int P_3^{\frac{1}{3}} I dx$$

are also invariants of the same kind, as follows at once from the relation

$$P_3\phi'^3 = v \quad \text{or} \quad P_3^{\frac{1}{3}}dx = v^{\frac{1}{3}}dt.$$

Let us now seek the condition that two solutions of the cubic, y_1, y_2 should be connected by the relation $y_1y_2=1$.

Transform the equation so that e^t shall be a solution and let it become

$$\frac{d^3y}{dt^3}+3Q_1\frac{d^2y}{dt^2}+3Q_2\frac{dy}{dt}+Q_3y=0$$

and we have

$$1+3Q_1+3Q_2+Q_3=0$$

$$1-3Q_1+3Q_2-Q_3=0$$

from which

$$1+3Q_2=0, \quad 3Q_1+Q_3=0$$

Substituting for Q_2 and Q_1 in the equations

$$\frac{\frac{dQ_3}{dt}+3Q_1Q_3}{Q_3^{\frac{4}{3}}}=I_1$$

$$\frac{\frac{dQ_1}{dt}+2Q_1^2-3Q_2}{Q_3^{\frac{2}{3}}}=I_2$$

then letting $Q_3=z^3$ and reducing we get

$$3\frac{dz}{dt}=I_1z^2+z^4 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

$$9z^2\frac{dz}{dt}=2z^6-9I_2z^2+9$$

from which equations we are to eliminate z. Eliminating $\frac{dz}{dt}$, we get

$$z^6+3I_1z^4+9I_2z^2-9=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

Now differentiating with regard to t and then substituting for $\frac{dz}{dt}$ from (16) we find

$$2z^6+4I_1z^4+z^2\left(4I_1^2-6I_2+\frac{3}{P_3^{\frac{1}{3}}}\frac{dI_1}{dx}\right)-6I_1I_2+\frac{9}{P_3^{\frac{1}{3}}}\frac{dI_2}{dx}=0 \quad . \quad . \quad . \quad (18)$$

Hence the required result is the eliminant of the two cubic equations (17) and (18).

I do not give here the expanded result thus obtained, as I have arrived at it in a more compact form, as follows :

Remove the second term, from the cubic we are considering, by change of the independent variable, and let the result be

$$\frac{d^3y}{dt^3}+u\frac{dy}{dt}+vy=0$$

Suppose now that $y_1y_2=1$, and let

$$\frac{d^2y}{dt^2}+2Q_1\frac{dy}{dt}+Q_2y=0$$

be the linear equation of which the solutions are y_1 and y_2, it is evident then that the cubic may be written in the form

$$\frac{d}{dt}\left\{\frac{d^2y}{dt^2}+2Q_1\frac{dy}{dt}+Q_2y\right\}-2\lambda\left\{\frac{d^2y}{dt^2}+2Q_1\frac{dy}{dt}+Q_2y\right\}=0$$

Since this equation is evidently satisfied by y_1 and y_2, and we can determine λ so that any other function of t shall satisfy it.

Comparing coefficients we find

$$2Q_1-2\lambda=0, \quad 2\frac{dQ_1}{dt}-2\lambda Q_1+Q_2=u$$

$$\frac{dQ_2}{dt}-2\lambda Q_2=v$$

we have also in consequence of the given condition

$$\frac{dQ_2}{dt}+4Q_1Q_2=0$$

From these we have

$$6Q_2\lambda+v=0 \quad 3Q_2+2\int vdt=0$$

$$2\frac{d\lambda}{dt}-2\lambda^2+Q_2=u$$

Hence we easily derive

$$16\left\{\int vdt\right\}^3+24u\left\{\int vdt\right\}^2-12\frac{dv}{dt}\int vdt+15v^2=0 \quad . \quad . \quad . \quad . \quad . \quad (19)$$

which is the required result expressed in terms of the invariants u and v.

To write the result in terms of I_1 and I_2, we have $I_1=\frac{1}{v^{\frac{4}{3}}}\frac{dv}{dt}$ from which we get

$$v=e^{\int P_3^{\frac{1}{3}}I_1dx}$$

and therefore

$$\int v\,dt=\int \mathrm{P}_3^{\frac{1}{3}}e^{\frac{2}{3}\int \mathrm{P}_3^{\frac{1}{3}}\mathrm{I}_1dx}$$

$$\frac{dv}{dt}=\mathrm{I}_1e^{\frac{4}{3}\int \mathrm{I}_1\mathrm{P}_3^{\frac{1}{3}}dx} \qquad u=-\mathrm{I}_2e^{\frac{2}{3}\int \mathrm{P}_3^{\frac{1}{3}}\mathrm{I}_1dx}$$

Hence letting

$$\mathrm{S}=e^{\int \mathrm{P}_3^{\frac{1}{3}}\mathrm{I}_1dx}, \quad \mathrm{R}=\int \mathrm{P}_3^{\frac{1}{3}}e^{\frac{2}{3}\int \mathrm{I}_1\mathrm{P}_3^{\frac{1}{3}}dx}$$

Condition (19) may be written in terms of the coefficients of the cubic without its second term being removed as follows:

$$16\mathrm{R}^3-24\mathrm{I}_2\mathrm{R}^2\mathrm{S}^{\frac{2}{3}}-12\mathrm{I}_1\mathrm{R}\mathrm{S}^{\frac{4}{3}}+15\mathrm{S}^2=0$$

Having expressed Q_2 and v as functions of x as above, the cubic may in this case be solved as follows:

We have from previous results the solutions of

$$\frac{d^2y}{dt^2}+\mathrm{Q}_1\frac{dy}{dt}+\mathrm{Q}_2y=0$$

where Q_1 and Q_2 are connected by the relation

$$\frac{d\mathrm{Q}_2}{dt}+4\mathrm{Q}_1\mathrm{Q}_2=0$$

$$y_1=e^{\int \sqrt{-\mathrm{Q}_2}dt}, \quad y_2=e^{-\int \sqrt{-\mathrm{Q}_2}dt}$$

Hence two solutions of the cubic are

$$y_1=e^{\int \frac{\sqrt{-\mathrm{Q}_2}}{v^{\frac{1}{3}}}\mathrm{P}_3^{\frac{1}{3}}dx}, \quad y_2=e^{\int \frac{\sqrt{-\mathrm{Q}_2}}{v^{\frac{1}{3}}}\mathrm{P}_3^{\frac{1}{3}}dx}$$

and the complete solution may then be found.

If we seek the conditions for

$$y_1=y_2^{\,m}=y_3^{\,n}$$

transform so that y_1 may become e^t, and as before we have

$$\begin{aligned}1+3\mathrm{Q}_1+3\mathrm{Q}_2+\mathrm{Q}_3&=0\\ m^3+3\mathrm{Q}_1m^2+3\mathrm{Q}_2m+\mathrm{Q}_3&=0\\ n^3+3\mathrm{Q}_1n^2+3\mathrm{Q}_2n+\mathrm{Q}_3&=0\end{aligned}$$

from which we find Q_1, Q_2, Q_3 three constants α, β, γ, say. Hence the required conditions are

$$\frac{3\alpha}{\gamma^{\frac{4}{3}}}=\mathrm{I}_1 \quad \text{and} \quad \frac{2\alpha^2-3\beta}{\gamma^{\frac{2}{3}}}=\mathrm{I}_2$$

and the solution is easily found to be

$$y=\mathrm{A}e^{\int\left(\frac{\mathrm{P}_3}{\gamma}\right)^{\frac{1}{3}}dx}+\mathrm{B}e^{m\int\left(\frac{\mathrm{P}_3}{\gamma}\right)^{\frac{1}{3}}dx}+\mathrm{C}e^{n\int\left(\frac{\mathrm{P}_3}{\gamma}\right)^{\frac{1}{3}}dx}$$

More generally let us seek the conditions that y_2 and y_3 should be any given functions of y_1.

Write for convenience $y_2=\phi(\log y_1)$, $y_3=\psi(\log y_1)$ then transforming as before we have

$$\begin{aligned} 1\ \ +3Q_1\ \ +3Q_2\ \ +Q_3&=0\\ \phi'''+3Q_1\phi''+3Q_2\phi'+Q_3\phi&=0\\ \psi'''+3Q_1\psi''+3Q_2\psi'+Q_3\psi&=0 \end{aligned}$$

Now from these equations we find

$$Q_3=-\frac{\begin{vmatrix} 1 & 1 & 1 \\ \phi''' & \phi'' & \phi' \\ \psi''' & \psi'' & \psi' \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ \phi & \phi'' & \phi' \\ \psi & \psi'' & \psi' \end{vmatrix}}$$

Hence from the relation $\int P_3^{\frac{1}{3}}dx=\int Q_3^{\frac{1}{3}}dt$ we find $\int P_3^{\frac{1}{3}}dx=F(t)$ where F is a known function, and the complete solution of the cubic is

$$y=Ae^{F^{-1}\{\int P_3^{\frac{1}{3}}dx\}}+B\phi\left\{F^{-1}\left\{\int P_3^{\frac{1}{3}}dx\right\}\right\}+C\psi\left\{F^{-1}\left\{\int P_3^{\frac{1}{3}}dx\right\}\right\}$$

To find the required conditions we have only to substitute $F^{-1}\{\int P_3^{\frac{1}{3}}dx\}$ for t in the left-hand sides of equations

$$\frac{\frac{dQ_3}{dt}+3Q_1Q_3}{Q_3^{\frac{4}{3}}}=I_1, \quad \frac{\frac{dQ_1}{dt}+2Q_1^2-3Q_2}{Q_3^{\frac{2}{3}}}=I_2$$

Q_1, Q_2, Q_3 being found in terms of t from equations previously given.

I proceed now to consider an invariant of the cubic which is particularly worth noticing.

Referring to the values of I_2 and I_3 given before, we find

$$I_3-\frac{2}{(-I_2)^{\frac{3}{2}}}\equiv\frac{\frac{dL}{dx}+2P_1L-2P_3}{L^{\frac{3}{2}}}=K \quad \text{(say)}$$

and $K=0$ is the condition that the solutions of the equation of the third order should be connected by the relation $y_3^2=y_1y_2$; as follows.

Let $y_1=z_1^2$, $y_2=z_2^2$, and let the equation of which the solutions are z_1 and z_2 be

$$\frac{d^2z}{dx^2}+2Q_1\frac{dz}{dx}+Q_2z=0$$

Hence referring to equation (A) we see that

$$\frac{d^3y}{dx^3}+3P_1\frac{d^2y}{dx^2}+3P_2\frac{dy}{dx}+P_3y=0$$

may be expressed in the form

$$\frac{d^3y}{dx^3}+6Q_1\frac{d^2y}{dx^2}+2\left(\frac{dQ_1}{dx}+4Q_1^2+2Q_2\right)\frac{dy}{dx}+2\left(4Q_1Q_2+\frac{dQ_2}{dx}\right)y=0$$

therefore

$$P_1=2Q_1, \quad P_3=2\frac{dQ_2}{dx}+8Q_1Q_2$$

$$3P_2=2\frac{dQ_1}{dx}+8Q_1^2+4Q_2$$

From these equations eliminating Q_1, Q_2 we get

$$\frac{dL}{dx}+2P_1L-2P_3=0 \text{ or } K=0$$

as the required condition.

The relation $y_3^2=y_1y_2$ involving only the ratios of the solutions must be also expressible in terms of the invariants of the first kind considered in this paper, and in fact we find

$$\frac{dL}{dx}+2P_1L-2P_3=3\frac{dH}{dx}-2G$$

where H and G have the same meaning as before.

To arrive directly at this condition in terms of H and G, we see on referring to (B) that the cubic with the second term removed by the substitution for y of $ye^{-\int P_1dx}$, viz.,

$$\frac{d^3y}{dx^3}+3H\frac{dy}{dx}+Gy=0$$

can be written in the form

$$\frac{d^3y}{dx^3}+4H_1\frac{dy}{dx}+2\frac{dH_1}{dx}y=0$$

where

$$H_1=Q_2-Q_1^2-\frac{dQ_1}{dx}$$

Therefore

$$3H=4H_1, \quad G=2\frac{dH_1}{dx}$$

and eliminating H_1 we find

$$3\frac{dH}{dx}-2G=0$$

the required result.

Hence the condition $y_3^2=y_1y_2$ may be expressed in either of the forms

$$\mathrm{K}=0 \quad \text{or} \quad 3\frac{d\mathrm{H}}{dx}-2\mathrm{G}=0$$

we have in fact the relation

$$\mathrm{K}^2=\mathrm{L}^3\left(3\frac{d\mathrm{H}}{dx}-2\mathrm{G}\right)^2$$

We see also that in this case the solution of the cubic is reduced to that of the quadratic

$$\frac{d^2y}{dx^2}+\mathrm{P}_1\frac{dy}{dx}+\frac{\mathrm{L}}{4}y=0$$

the solutions of which are $\sqrt{y_1}$ and $\sqrt{y_2}$.

The quartic.

To find the invariants of the second kind of the equation of the fourth order

$$\frac{d^4y}{dx^4}+4\mathrm{P}_1\frac{d^3y}{dx^3}+6\mathrm{P}_2\frac{d^2y}{dx^2}+4\mathrm{P}_3\frac{dy}{dx}+\mathrm{P}_4y=0$$

let us suppose the second term removed by the substitution $x=\phi(t)$, then writing the result in the form

$$\frac{d^4y}{dt^4}+u\frac{d^2y}{dt^2}+v\frac{dy}{dt}+w=0$$

we find

$$2\mathrm{P}_1\phi'-\frac{3\phi''}{\phi'}=0 \qquad w=\mathrm{P}_4\phi'^4$$

$$u=\frac{15\phi''^2}{\phi'^2}-\frac{4\phi'''}{\phi'}-12\mathrm{P}_1\phi''+6\mathrm{P}_2\phi'^2 \quad \ldots\ldots\ldots \quad (\alpha)$$

$$v=\frac{10\phi'''\phi''}{\phi'^2}-\frac{15\phi''^3}{\phi'^3}-\frac{\phi^{(\mathrm{iv})}}{\phi'}+12\mathrm{P}_1\frac{\phi''^2}{\phi'}-4\mathrm{P}_1\phi'''-6\mathrm{P}_2\phi''\phi'+4\mathrm{P}_3\phi'^3 \quad \ldots \quad (\beta)$$

From the first of these equations we have

$$2\frac{d\mathrm{P}_1}{dx}=\frac{3\phi'''}{\phi'^3}-\frac{6\phi''^2}{\phi'^2}=\frac{3\phi'''}{\phi'^3}-\tfrac{8}{3}\mathrm{P}_1^2$$

therefore by substitution in (α) we find

$$9u=\phi'^2\left(54\mathrm{P}_2-44\mathrm{P}_1^2-24\frac{d\mathrm{P}_1}{dx}\right)$$

Hence we have

$$\frac{54\mathrm{P}_2-44\mathrm{P}_1^2-24\dfrac{d\mathrm{P}_1}{dx}}{\sqrt{\mathrm{P}_4}}=\frac{9u}{\sqrt{w}}$$

or

$$\frac{54\mathrm{P}_2-44\mathrm{P}_1^2-24\dfrac{d\mathrm{P}_1}{dx}}{\sqrt{\mathrm{P}_4}}=\mathrm{J}_1 \text{ say}$$

is an invariant of the quartic of the kind we are considering.

Again by differentiating the equation

$$\frac{\phi'''}{\phi'^3}=\tfrac{2}{3}\frac{d\mathrm{P}_1}{dx}+\tfrac{8}{9}\mathrm{P}_1^2$$

we get

$$\begin{aligned}\frac{\phi^{(\mathrm{iv})}}{\phi'^4}&=\tfrac{2}{3}\frac{d^2\mathrm{P}_1}{dx^2}+\tfrac{16}{9}\mathrm{P}_1\frac{d\mathrm{P}_1}{dx}+\frac{3\phi'''\phi''}{\phi'^5}\\ &=\tfrac{2}{3}\frac{d^2\mathrm{P}_1}{dx^2}+\tfrac{16}{9}\mathrm{P}_1\frac{d\mathrm{P}_1}{dx}+2\mathrm{P}_1\left(\tfrac{2}{3}\frac{d\mathrm{P}_1}{dx}+\tfrac{8}{9}\mathrm{P}_1^2\right)\\ &=\tfrac{2}{3}\cdot\frac{d^2\mathrm{P}_1}{dx^2}+\tfrac{28}{9}\mathrm{P}_1\frac{d\mathrm{P}_1}{dx}+\tfrac{16}{9}\mathrm{P}_1^3\end{aligned}$$

substituting in (β) we find

$$27v=\phi'^3\left\{40\mathrm{P}_1^3-36\mathrm{P}_1\frac{d\mathrm{P}_1}{dx}-18\frac{d^2\mathrm{P}_1}{dx^2}-108\mathrm{P}_1\mathrm{P}_2+108\mathrm{P}_3\right\}$$

hence we see that

$$\frac{40\mathrm{P}_1^3-36\mathrm{P}_1\dfrac{d\mathrm{P}_1}{dx}-18\dfrac{d^2\mathrm{P}_1}{dx^2}-108\mathrm{P}_1\mathrm{P}_2+108\mathrm{P}_3}{\mathrm{P}_4^{\frac{3}{4}}}$$

is an invariant, which I shall call J_2.

We easily find a third invariant from the equations

$$2\mathrm{P}_1=\frac{3\phi''}{\phi'^2} \quad \text{and} \quad w=\mathrm{P}_4\mathrm{Q}'^4,$$

namely,

$$\frac{3\dfrac{d\mathrm{P}_4}{dx}+8\mathrm{P}_1\mathrm{P}_4}{\mathrm{P}_4^{\frac{5}{4}}}$$

which may be called J_3.

By aid of the invariants J_1, J_2, J_3 we can solve problems with respect to the quartic which are analogous to those already treated of in the case of the cubic.

As an example, let the solutions be related by the equations

$$y_1 = y_2{}^m = y_3{}^n = y_4{}^p$$

then transforming so that y_1 shall be e^t we find

$$J_1 = \text{a constant, } J_2 = \text{a constant, } J_3 = \text{a constant ;}$$

and in this case the complete solution is

$$y = \mathrm{A}e^{\int\left(\frac{P_4}{\delta}\right)^{\frac{1}{4}}dx} + \mathrm{B}e^{m\int\left(\frac{P_4}{\delta}\right)^{\frac{1}{4}}dx} + \mathrm{C}e^{n\int\left(\frac{P_4}{\delta}\right)^{\frac{1}{4}}dx} + \mathrm{D}e^{p\int\left(\frac{P_4}{\delta}\right)^{\frac{1}{4}}dx}$$

where

$$\delta = mnp$$

XVI. *Description of portions of a Tusk of a Proboscidian Mammal.* (Notelephas* australis, OWEN.)

By Professor OWEN, *C.B., F.R.S., &c.*

Received March 21,—Read March 30, 1882.

[PLATE 51.]

THE subjects of the present communication were discovered by the late FRED. N. ISAAC, Esq., in a superficial drift-deposit of a ravine in a district of Darling Downs, Australia, about 60 miles to the eastward of Moreton Bay, and have been submitted to me by his nephew, E. THURSTON HOLLAND, Esq.

They are portions of a tusk and indicate a species of Mammal larger than the *Diprotodon*, consequently the largest aboriginal land Mammal of which any satisfactory evidence has, hitherto, reached me from Australia.

Besides the larger portions of the tusk figured in the accompanying drawing (Plate 51, figs. 1–8) were many smaller portions or fragments of apparently the same tusk.

At first sight they suggested evidence confirmatory of that which, in 1843, was brought from Australia by Count STEZELECKI,—a molar, viz., of a Mastodon, stated to have been obtained from a native of the interior of New South Wales, but which appeared to lack the grounds for the admission of a Proboscidian into the work 'On the Fossil Mammals of Australia,' up to the date of its publication, 1877. That molar presented the characters of the *Mastodon andium* of South America, and it is too large to be associated with the tusk under description, supposing this to have come from the upper jaw of a full-grown individual of its species. I, however, subjoin the Count's notice of the molar tooth.†

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* Νοτος south, ἐλεφας ivory.

† After alluding to some Marsupial Fossils I also had submitted to me, the author proceeds :—" To these relics may be added a molar 'bone' (tooth) of the Mastodon, similar to the *Mastodon angustidens*, and provisionally called by Professor OWEN *Mastodon australis*, and which I bought from a native of Boree, the station of Captain RYAN, through the agency of the overseer of the station. The natives in giving the bone stated that similar ones and larger still might be got further in the interior ; but that, owing to the hostility of the tribe upon whose grounds the bones are to be found, it was impossible for him to venture in that time in search for more ; as, however, he promised to exert himself at some future period in order to supply me with some better specimens, I have left a reward with the man second in command at the station, and which was to be given to the native on his redeeming his pledge. Should

The largest of the portions of the tusk (Plate 51, figs. 1–4) is from near the base, and includes part of the pulp-cavity, *p*.

It is 5 inches in length, of a full elliptical, almost circular shape transversely; measuring across at the larger end (fig. 3), 2 inches 6 lines by 2 inches 5 lines; at the smaller end (fig. 4), 2 inches by 1 inch 10 lines. The diameters of the pulp-cavity are, at the larger end, 1 inch 1 line by 1 inch; at the smaller end, 6 lines by 5 lines. The thickness of the wall of ivory, at the larger end, varies from 8 to 9 lines, and is the same at the smaller end, not decreasing in the ratio of the pulp-cavity.

In this portion of tusk the outer or cemental layer and much of the outermost layer of dentine are lost. In the smaller portions of the tusk, where the cemental layer is preserved, it has a smooth outer surface; that of the dentine which is exposed shows the shallow longitudinal linear impressions which may be seen in that part of the tusk of most Elephants recent and fossil. The transversely fractured surfaces present the decussating curvilinear lines characteristic of true or Proboscidian ivory; but they are more minute than in the tusks of the African or Indian Elephants. The dentine has, also, the compactness of ivory, and reveals the microscopic characters presently to be described. The longitudinal linear impressions on the outer layer of dentine are of varying, but nowhere of great depth, mostly alternating in this character. The deeper lines run along with interspaces of from $\frac{1}{4}$ to $\frac{1}{2}$ an inch, and such defined longitudinal tracts are commonly impressed by a fainter line running along or near their middle. A tract bounded by the deeper lines, half an inch in breadth, shows two of the shallower longitudinal impressions. The general surface is smooth and polished; the inner surface, toward the pulp-cavity, is smooth.

The present portion of tusk shows a slight departure from straightness; a degree of curvature is more evident in the second, rather narrower but longer, portion (ib., figs. 5, 6). This portion is 6½ inches in length, 2 inches 1 line by 2 inches in thickness at the largest end, 1 inch 10 lines by 1 inch 9 lines at the smaller end (ib., fig. 6). The inner layers of the dentine at the larger end have been too much and irregularly broken away to indicate satisfactorily the size and termination there of the pulp cavity; but three inches in advance the tusk is solid, and the same absence of pulp-cavity is shown at the smaller end, where the final consolidation is indicated by a dark spot. On the surface of this portion of tusk, where the cement is wanting, the longitudinal lineation of the outer layer of dentine has become fainter.

The third portion (ib., figs. 7, 8) is from near the apical extremity of the tusk; it is solid and the outer layers of dentine, coated by the smooth cement, are partially

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future enterprise lead travellers to that quarter, it will be deserving their while to push the enquiry further and add more evidence regarding the existence of the Mastodontoid animals of New Holland."—'Physical Description of New South Wales and Van Dieman's Land,' 8vo., London, 1845, p. 312.

Subsequent unintermitting correspondence in relation to discovery and acquisition of Australian fossil remains have failed to bring to my notice any evidence of a Proboscidian Mammal before the reception of the subjects of the present communication.

preserved. At the larger end (ib., fig. 8) the diameters are 1 inch 4 lines and 1 inch 1 line ; at the smaller end they are 9 lines and 6 lines ; but here a line's breadth should be added to both admeasurements, through lack of the outer layers of ivory with its thin coating of smooth cement.

Besides the three portions of tusk above described many fragments remained after the adjustment of the pieces forming the parts of the tusk figured in Plate 51. I estimate the length of the part of the tusk collected by Mr. Isaac and transmitted to me at 16 inches ; adding the portion wanted to connect the part fig. 5, with the part fig. 7, about 2 feet in length of the tusk would be represented. The size of the pulp-cavity in figs. 1–3 indicates the portion so figured to have come from near the exit of the tusk from its socket. Of some of the detached fragments microscopical sections have been made.

These sections of dentinal (Plate 51, figs. 9, 10) and cemental (ib., fig. 11) portions of the tusk demonstrate the characteristic structures of these tissues, which have been described by Retzius* and myself.†

The dentinal tubes present, at a little distance from the pulp-cavity, the characteristic minute size—$\frac{1}{5000}$th of an inch in diameter ; and soon, as they recede toward the cement, show the peculiar, strong undulatory course (fig. 9) answering to the easier "secondary curves" of those tubes in the dentine of most other kinds of Mammalian teeth. The degree of curvature in the ivory of the present extinct Proboscidian is even greater in the section of the fossil ivory here figured (fig. 9) than in the section of recent ivory of the Indian Elephant, the subject of plate 149, *d*, in the 'Odontography.'‡ In the section of dentine shown by the higher power (fig. 10), are seen the strata of extremely minute opaque cells, unusually numerous in ivory, in the interspaces of the tubes.

The cemental part of the fossil repeats the elephantine characters : the radiated cells (fig. 11) are larger, averaging $\frac{1}{2500}$th of an inch in diameter, and are more uniform in size and shape than in most other Mammalian teeth ; they also show, in transverse section, the circular figure characteristic of Proboscidian incisors.§

Fractured portions of a single tooth may seem to be a slender basis for predicating of a wider geographical distribution of the Proboscidian order than has, hitherto, been assigned to it. And, moreover, if the rest of the elephantine structures should be conformable, as I presume they would be, we here have, supposing the Dingo to be a human introduction into the Australian continent, a gyrencephalous exception to the characteristic aboriginal Mammalian organisation of that remote southern continent. I am encouraged, however, to submit the present evidence to the Society, by the successive

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* Mikroskopiska Undersökningar öfver Jädernes särdeles Tandbenets struktur. Stockholm, 1837.

† 'Odontography,' 4to., 1840–45.

‡ Ibid.

§ Ibid., p. 641.

confirmations of the previously hugest known extinct Marsupial, ultimately establishing the ordinal, generic, and specific characters of *Diprotodon australis*, of which the primal indication was but a portion of a tusk.* This tusk, however, was evidently one of a pair which had issued in an almost horizontal direction from the symphysis of the lower jaw: and both incisors had been partially coated with true enamel. Such indication bespoke a species akin to the still existing pouched quadrupeds of Australia, represented by the Wombats and Kangaroos.

The initial fossil, large as it seemed, proved afterwards to be part of an immature individual. It was obtained, as is well known, from a cavern, which had been haunted by the largest known marsupial Carnivore (*Thylacoleo*); and the relations to the locality and companion fossils recalled those of the remains of Elephants and Rhinoceroses which have been a prey to spelæan Lions haunting the caves in our own island.

And here I may remark that the cave in Wellington Valley, originally discovered and explored by Major Sir THOMAS MITCHELL, F.G.S., has since, by the enlightened liberality of the Government of New South Wales, been subjected to a more searching exploration by the accomplished naturalist and curator of the Museum of Natural History at Sydney, ED. P. RAMSAY, F.L.S.

Among the additional evidences of the *Thylacoleo*—the only carnivore to which could be referred the introduction of immature Diprotodons whose remains showed indications that they had fallen a prey—was a portion of the cranium with the articular cavity for the lower jaw. This, instead of the shallow undulatory surface of the vegetarian Marsupials, showed the deep transversely extended groove for the reception of the transversely extended fore-and-aft convexity of the mandibular condyle: the joint thus conforming, as in the Felines, with the carnassial character of the dentition of *Thylacoleo*.

As, year by year, further evidences arrived contributing to the restoration of *Diprotodon*, it may be hoped that similar materials for the reconstruction of *Notelephas australis* may reach this country.

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* Appendix to 'Three Expeditions into the Interior of Eastern Australia,' by Sir THOMAS MITCHELL, F.G.S., Surveyor-General of Australia; vol. ii., 8vo., 1838, plate 31, fig. 1.

Description of the Plate.

PLATE 51.

Notelephas australis.

Fig. 1. Side view of the basal part of the tusk.
Fig. 2. Ib. of the opposite side of ditto.
Fig. 3. Transversely fractured surface of the basal end of ditto.
Fig. 4. Ib. of the opposite end of ditto.
Fig. 5. Side view of the succeeding portion of the tusk.
Fig. 6. Transversely fractured end view of ditto.
Fig. 7. Side view of a terminal portion of the tusk.
Fig. 8. Transversely fractured surface of the basal end of ditto.
Fig. 9. Longitudinal section of dentine, magnified 50 diameters.
Fig. 10. Ib. ib., magnified 8 diameters.
Fig. 11. Transverse section of cement, magnified 90 diameters.

Figs. 1–8 are of the natural size.

XVII. *Memoir on the Theta-Functions, particularly those of two variables.*

By A. R. Forsyth, *B.A., Fellow of Trinity College, Cambridge.*

Communicated by Professor A. Cayley, *LL.D., F.R.S.*

Received December 9,—Read December 22, 1881.

The following paper is divided into four sections. Section I. deals with what may be called Rosenhain's theory; under the guidance of Professor H. J. S. Smith's paper on the single theta-functions (in vol. i. of London Math. Soc. Proc.), there is investigated a general theorem for the product of four double theta-functions with different characteristics and variables, the definition being

$$\Phi\left\{\begin{pmatrix}\lambda, & \rho\\ \mu, & \nu\end{pmatrix}x,\ y\right\}=\sum_{m=-\infty}^{m=\infty}\ \sum_{n=-\infty}^{n=\infty}(-1)^{m\lambda+n\rho}p^{\frac{1}{4}(2m+\mu)^2}q^{\frac{1}{4}(2n+\nu)^2}r^{\frac{1}{2}(2m+\mu)(2n+\nu)}e^{(2m+\mu)\frac{i\pi x}{2\mathrm{K}}+(2n+\nu)\frac{i\pi y}{2\Lambda}},$$

the product being equal to the sum of 16 similar products; and the equation is shown to include 4096 particular cases. Quadratic relations are established between the functions; and the 15 quotients of all of them but one by that one are expressed in terms of two new variables x_1, x_2, the connexion between x_1, x_2 and the original variables x, y being

$$x=\int^{x_1}\frac{\mathrm{A}+\mathrm{B}z}{\sqrt{\mathrm{Z}}}dz+\int^{x_2}\frac{\mathrm{A}+\mathrm{B}z}{\sqrt{\mathrm{Z}}}dz$$

$$y=\int^{x_1}\frac{\mathrm{A}'+\mathrm{B}'z}{\sqrt{\mathrm{Z}}}dz+\int^{x_2}\frac{\mathrm{A}'+\mathrm{B}'z}{\sqrt{\mathrm{Z}}}dz$$

where

$$\mathrm{Z}=z(1-z)(1-\kappa_1^{\,2}z)(1-\kappa_2^{\,2}z)(1-\kappa_3^{\,2}z)$$

and A, B, A′, B′, κ_1, κ_2, κ_3 are perfectly determinate constants. The quadruple periodicity of the functions is investigated at the beginning of the section, and at the end definite-integral expressions for the periods are obtained.

Section II. gives the expansions of all the functions

(i) in trigonometrical series;
(ii) in ascending powers of x and y.

To obtain the latter, use is made of a theorem there proved :—

$$\Phi\left\{\begin{pmatrix}\lambda, & \rho\\ \mu, & \nu\end{pmatrix}x,\ y\right\}=e^{-\frac{2\mathrm{K}\Lambda\log r}{\pi^2}\frac{d^2}{dxdy}}\theta_{\mu,\,\lambda}(x)\,\theta_{\nu,\,\rho}(y)$$

$\theta_{\mu,\lambda}(x)$, $\theta_{\nu,\rho}(y)$ being single theta-functions. From it are also obtained the expressions for the four periods, as well as a proof of the product theorem of Section I.; and the function Φ is shown to satisfy two differential equations of the form

$$\frac{d^2\Phi}{dx^2}-2x\left(\kappa'^2-\frac{\mathrm{E}}{\mathrm{K}}\right)\frac{d\Phi}{dx}+2\kappa\kappa'^2\frac{d\Phi}{d\kappa}=0$$

(κ, κ', E having the ordinary meaning in reference to $\theta_{\mu,\lambda}(x)$), and an equation of the form

$$r\frac{d\Phi}{dr}+\frac{2\mathrm{K}\Lambda}{\pi^2}\frac{d^2\Phi}{dxdy}=0.$$

Section III. forms the expression of the addition theorem. Although no addition theorem proper exists for theta-functions (that is to say, $\Phi(x+\xi,\ y+\eta)$ cannot be written down in terms of functions of x, y and of ξ, η), an expression is obtainable for

$$\Phi(x+\xi,\ y+\eta)\ .\ \Phi'(x-\xi,\ y-\eta)$$

Φ, Φ' being either the same or different functions. Since any one function of the sum may be combined with any function of the difference of the variables, 256 equations are necessary; and these are written down in 16 sets of 16 each.

In Section IV. many of the properties already proved for the double theta-functions are generalized for the "r" tuple functions. Among these are:—

(i.) The periodicity as in Section I.;

(ii.) The product theorem, which gives the product of four functions as the sum of 4^r products of four functions; from it several general relations are deduced;

(iii.) The analogue of the theorem in Section II., viz.:—

$$\Phi\left\{\begin{pmatrix}\lambda_1, \lambda_2, \ldots, \lambda_r\\ \mu_1, \mu_2, \ldots, \mu_r\end{pmatrix}x_1,\ x_2,\ \ldots,\ x_r\right\}=e^{-\frac{2}{\pi^2}\sum\limits_{s=1}^{s=r}\sum\limits_{t=1}^{t=r}\mathrm{K}_s\mathrm{K}_t\log p_{s,t}\frac{d^2}{dx_sdx_t}}\prod_{t=1}^{t=r}\theta_{\mu_t,\lambda_t}(x_t)$$

(iv.) The r differential equations of the form

$$\frac{d^2\Phi}{dx_r}-2x_r\left(\kappa'^2_r-\frac{\mathrm{E}_r}{\mathrm{K}_r}\right)\frac{d\Phi}{dx_r}+2\kappa_r\kappa'^2_r\frac{d\Phi}{d\kappa_r}=0$$

and the $\frac{1}{2}r(r-1)$ of the form

$$p_{s,t}\frac{d\Phi}{dp_{s,t}}+\frac{2\mathrm{K}_s\mathrm{K}_t}{\pi^2}\frac{d^2\Phi}{dx_sdx_t}=0$$

all satisfied by Φ.

INTRODUCTION.

Literature of the Subject.

1. The published investigations on the double theta-functions may be conveniently divided into four classes :—

(i.) Those concerning the algebraical integrals of the equations

$$\frac{dx}{\sqrt{X}}+\frac{dy}{\sqrt{Y}}+\frac{dz}{\sqrt{Z}}=0$$

$$\frac{xdx}{\sqrt{X}}+\frac{ydy}{\sqrt{Y}}+\frac{zdz}{\sqrt{Z}}=0$$

where X, Y, Z are of the form

$$\xi(1-\xi)(1-\kappa\xi)(1-\lambda\xi)(1-\mu\xi)$$

or

$$(a-\xi)(b-\xi)(c-\xi)(d-\xi)(e-\xi)(f-\xi);$$

(ii.) Those concerning the theta-functions, properly so-called, proceeding from the definitions and investigating the relations which hold between functions of different (and of the same) arguments ;

(iii.) The transformation theory ;

(iv.) The applications to geometry, principally in reference to Kummer's 16-nodal quartic surface.

2. The principal papers are :—

For (i.) I. Abel. His chief memoir is one occurring in the 'Mémoires des Savans Étrangers,' t. vii., 1841 (but presented to the French Academy in 1826), under the title "Mémoire sur une propriété générale d'une classe très-étendue de fonctions transcendantes," pp. 176 sqq.; the particular case of (i.) is considered p. 260. Several other papers, less important, on the transcendental functions occur in the collected edition of his works.

II. Jacobi. (α) "Considerationes generales de transcendentibus Abelianis," 'Crelle,' t. ix. (1832), p. 394 ;

(β) "De functionibus quadrupliciter periodicis quibus theoria transcendentium Abelianarum innititur," 'Crelle,' t. xiii. (1835), p. 55 ;

(γ) "Demonstratio nova theorematis Abeliani," 'Crelle,' t. xxiv. (1842), p. 28.

III. Richelot. "Ueber die Integration eines merkwürdigen systems Differentialgleichungen," 'Crelle,' t. xxiii. (1842), p. 354.

IV. CAYLEY. (α) "A Memoir on the Double θ-Functions," 'Crelle,' t. lxxxv. (1878), p. 214;

(β) "On the Double θ-Functions," 'Crelle,' t. lxxxvii. (1879), p. 74;

(γ) "On the Addition of the Double θ-Functions," 'Crelle,' t. lxxxviii. (1880), p. 74.

For (ii.) I. ROSENHAIN. "Mémoire sur les fonctions de deux variables et à quatre périodes," Mém. des Sav. Étr., t. xi., p. 361. This obtained the prize given by the Paris Academy of Sciences in 1846.

II. GÖPEL. "Theoriæ transcendentium Abelianarum primi ordinis adumbratio levis," 'Crelle,' t. xxxv. (1847), p. 277.

III. WEIERSTRASS. "Zur Theorie der Abelschen Functionen," 'Crelle,' t. xlvii. (1854), p. 289; also t. lii. (1856), p. 285.

IV. RIEMANN. "Theorie der Abelschen Functionen," 'Crelle,' t. liv.; Ges. Werke, p. 81.

V. CAYLEY. "A Memoir on the Single and Double Theta-Functions," Phil. Trans., 1881.

VI. BRIOSCHI. "La relazione di GÖPEL per funzioni iperellittiche d'ordine qualunque," Ann. di Mat., t. x. (1881).

For (iii.) I. HERMITE. "Sur la theorie de la transformation des fonctions Abéliennes," Comptes Rendus, t. xl. (1855).

II. KÖNIGSBERGER. (α) "Ueber die Transformation der Abelschen Functionen erster Ordnung," 'Crelle,' t. lxiv., p. 17 (1865). In this occurs part of the addition theorem;

(β) "Ueber die Transformation des zweiten Grades für die Abelschen Functionen erster Ordnung," 'Crelle,' t. lxvii. (1866), p. 58; a continuation of which, dealing with the modular equations, occurs in the Math. Ann., t. i. (1869), p. 163.

For (iv.) I. KUMMER. "Ueber die algebraische Strahlen-systeme," Berl. Abh. (1866).

II. CAYLEY. (α) "On the Double θ-Functions in connexion with a 16-nodal Quartic Surface," 'Crelle,' t. lxxxiii. (1877), p. 210;

(β) "On the 16-nodal Quartic Surface," 'Crelle,' t. lxxxiv. (1878), p. 238.

III. BORCHARDT. "Ueber die Darstellung der Kummerschen Fläche vierter Ordnung mit sechzehn Knotenpunkten durch die Göpelsche biquadratische Relation zwische vier Theta-functionen mit zwei Variabeln," 'Crelle,' t. lxxxiii. (1877), p. 234.

IV. WEBER. "Ueber die Kummersche Fläche vierter Ordnung," 'Crelle,' t. lxxxiv. (1878), p. 332.

SECTION I.

3. The general double theta-function is defined by the equation

$$\Phi\left\{\begin{pmatrix}\lambda, & \rho\\ \mu, & \nu\end{pmatrix}x,\ y\right\}=\sum_{m=-\infty}^{m=\infty}\ \sum_{n=-\infty}^{n=\infty}(-1)^{m\lambda+n\rho}p^{\frac{(2m+\mu)^2}{4}}q^{\frac{(2n+\nu)^2}{4}}r^{\frac{(2m+\mu)(2n+\nu)}{2}}v^{x(2m+\mu)}w^{y(2n+\nu)}.\quad . \qquad (1)$$

in which λ, μ, ρ, ν are given integers (afterwards taken to be each either zero or unity) and $\begin{pmatrix}\lambda, & \rho\\ \mu, & \nu\end{pmatrix}$ is called the characteristic; x, y are the variables; p, q, r, v, w are known constants, called the parameters; and the double summation extends to all positive and negative integral values (including zero) of m and n. To ensure the convergence of the doubly infinite series it is necessary that the real part of

$$(2m+\mu)^2\log p+(2n+\nu)^2\log q+2(2m+\mu)(2n+\nu)\log r$$

should be negative for all real values of m and n; beyond this restriction, there is no limitation to the form or the values of p, q and r.

4. It follows at once from the definition that

$$\Phi\left\{\begin{pmatrix}\lambda+2, & \rho\\ \mu & , \nu\end{pmatrix}\right\}=\Phi\left\{\begin{pmatrix}\lambda, & \rho\\ \mu, & \nu\end{pmatrix}\right\}=\Phi\left\{\begin{pmatrix}\lambda, & \rho+2\\ \mu, & \nu\end{pmatrix}\right\}\quad .\ .\ .\ .\ . \qquad (2)$$

$$(-1)^{\lambda}\Phi\left\{\begin{pmatrix}\lambda & , \rho\\ \mu+2, & \nu\end{pmatrix}\right\}=\Phi\left\{\begin{pmatrix}\lambda, & \rho\\ \mu, & \nu\end{pmatrix}\right\}=\Phi\left\{\begin{pmatrix}\lambda, & \rho\\ \mu, & \nu+2\end{pmatrix}\right\}\quad .\ .\ .\ .\ . \qquad (3)$$

the variables being the same throughout. Hence there are, in all, sixteen distinct functions, obtained by assigning to the four numbers of the characteristic the values zero and unity and taking all possible combinations.

Also from the definition

$$\begin{aligned}\Phi\left\{\begin{pmatrix}\lambda, & \rho\\ \mu, & \nu\end{pmatrix}-x,\ -y\right\}&=\sum_{m=-\infty}^{m=\infty}\ \sum_{n=-\infty}^{n=\infty}(-1)^{m\lambda+n\rho}p^{\frac{(2m+\mu)^2}{4}}q^{\frac{(2n+\nu)^2}{4}}r^{\frac{(2m+\mu)(2n+\nu)}{2}}v^{-x(2m+\mu)}w^{-y(2n+\nu)}\\ &=(-1)^{\lambda\mu+\nu\rho}\sum_{m=-\infty}^{m=\infty}\ \sum_{n=-\infty}^{n=\infty}(-1)^{(m+\mu)\lambda+(n+\nu)\rho}p^{\frac{(-2\overline{m+\mu}+\mu)^2}{4}}q^{\frac{(-2\overline{n+\nu}+\nu)^2}{4}}\\ &\qquad r^{\frac{(-2\overline{m+\mu}+\mu)(-2\overline{n+\nu}+\nu)}{2}}v^{x(-2\overline{m+\mu}+\mu)}w^{y(-2\overline{n+\nu}+\nu)}\\ &=(-1)^{\lambda\mu+\nu\rho}\sum_{m'=-\infty}^{m'=\infty}\ \sum_{n'=-\infty}^{n'=\infty}(-1)^{m'\lambda+n'\rho}p^{\frac{(2m+\mu)^2}{4}}q^{\frac{(2n'+\nu)^2}{4}}r^{\frac{(2m'+\mu)(2n'+\nu)}{2}}v^{x(2m'+\mu)}w^{y(2n+\nu)}\\ &=(-1)^{\lambda\mu+\nu\rho}\Phi\left\{\begin{pmatrix}\lambda, & \rho\\ \mu, & \nu\end{pmatrix}x,\ y\right\}.\quad .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ . \qquad (4)\end{aligned}$$

Hence there are ten even and six uneven functions, the latter being denoted by an asterisk in the following table, showing the correspondence between the notations that are used. The current-number notation adopted in this paper was chosen to coincide as closely as possible with that of Professor CAYLEY (with which it is almost identical as will be seen from the table) and with ROSENHAIN'S notation; that of WEIERSTRASS is given as quoted by KÖNIGSBERGER. In the table i denotes $\sqrt{-1}$.

Asterisk denotes odd function.	HERMITE'S characteristic.	Current-number adopted in this paper.	ROSENHAIN.	GÖPEL.	CAYLEY'S characteristic.	CAYLEY'S current-number.	WEIERSTRASS.
	$\Phi\begin{matrix}0, 0\\0, 0\end{matrix}$	ϑ_0	ϕ_{33}	P'''	$\vartheta\begin{pmatrix}0, 0\\0, 0\end{pmatrix}$	ϑ_0	ϑ_5
	$\begin{matrix}0, 0\\0, 1\end{matrix}$	ϑ_2	ϕ_{32}	Q'''	$\begin{matrix}0, 1\\0, 0\end{matrix}$	ϑ_2	ϑ_{01}
*	$\begin{matrix}0, 1\\0, 1\end{matrix}$	ϑ_{10}	ϕ_{31}	iQ''	$\frac{1}{i}\vartheta\begin{pmatrix}0, 1\\0, 1\end{pmatrix}$	$\frac{1}{i}\vartheta_{10}$	$i\vartheta_{02}$
	$\begin{matrix}0, 1\\0, 0\end{matrix}$	ϑ_8	ϕ_{30}	P''	$\begin{matrix}0, 0\\0, 1\end{matrix}$	ϑ_8	ϑ_{12}
	$\begin{matrix}0, 0\\1, 0\end{matrix}$	ϑ_1	ϕ_{23}	R'''	$\begin{matrix}1, 0\\0, 0\end{matrix}$	ϑ_1	ϑ_4
	$\begin{matrix}0, 0\\1, 1\end{matrix}$	ϑ_3	ϕ_{22}	S'''	$\begin{matrix}1, 1\\0, 0\end{matrix}$	ϑ_3	ϑ_{23}
*	$\begin{matrix}0, 1\\1, 1\end{matrix}$	ϑ_{11}	ϕ_{21}	iS''	$\frac{1}{i}\vartheta\begin{pmatrix}1, 1\\0, 1\end{pmatrix}$	$\frac{1}{i}\vartheta_{11}$	$i\vartheta_{13}$
	$\begin{matrix}0, 1\\1, 0\end{matrix}$	ϑ_9	ϕ_{20}	R''	$\begin{matrix}1, 0\\0, 1\end{matrix}$	ϑ_9	ϑ_{03}
*	$\begin{matrix}1, 0\\1, 0\end{matrix}$	ϑ_5	ϕ_{13}	iR'	$\frac{1}{i}\vartheta\begin{pmatrix}1, 0\\1, 0\end{pmatrix}$	$\frac{1}{i}\vartheta_5$	$i\vartheta_3$
*	$\begin{matrix}1, 0\\1, 1\end{matrix}$	ϑ_7	ϕ_{12}	iS'	$\frac{1}{i}\vartheta\begin{pmatrix}1, 1\\1, 0\end{pmatrix}$	$\frac{1}{i}\vartheta_7$	$i\vartheta_{24}$
	$\begin{matrix}1, 1\\1, 1\end{matrix}$	ϑ_{15}	ϕ_{11}	S	$-\vartheta\begin{pmatrix}1, 1\\1, 1\end{pmatrix}$	$-\vartheta_{15}$	$-\vartheta_{14}$
*	$\begin{matrix}1, 1\\1, 0\end{matrix}$	ϑ_{13}	ϕ_{10}	iR	$\frac{1}{i}\vartheta\begin{pmatrix}1, 0\\1, 1\end{pmatrix}$	$\frac{1}{i}\vartheta_{13}$	$i\vartheta_{04}$
	$\begin{matrix}1\ 0\\0, 0\end{matrix}$	ϑ_4	ϕ_{03}	P'	$\begin{matrix}0, 0\\1, 0\end{matrix}$	ϑ_4	ϑ_{34}
	$\begin{matrix}1, 0\\0, 1\end{matrix}$	ϑ_6	ϕ_{02}	Q'	$\begin{matrix}0, 1\\1, 0\end{matrix}$	ϑ_6	ϑ_2
*	$\begin{matrix}1, 1\\0, 1\end{matrix}$	ϑ_{14}	ϕ_{01}	iQ	$\frac{1}{i}\vartheta\begin{pmatrix}0, 1\\1, 1\end{pmatrix}$	$\frac{1}{i}\vartheta_{14}$	$i\vartheta_1$
	$\begin{matrix}1, 1\\0, 0\end{matrix}$	ϑ_{12}	ϕ_{00}	P	$\begin{matrix}0, 0\\1, 1\end{matrix}$	ϑ_{12}	ϑ_0

5. In the general definition of Φ substitute

$$v=e^{\frac{i\pi}{2\mathrm{K}}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$w=e^{\frac{i\pi}{2\Lambda}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

so that

$$\Phi\left\{\begin{pmatrix}\lambda, \rho\\ \mu, \nu\end{pmatrix}x, y\right\}=\sum_{m=-\infty}^{m=\infty}\sum_{n=-\infty}^{n=\infty}(-1)^{m\lambda+n\rho}e^{\frac{1}{4}\left\{(2m+\mu)^2\log p+(2n+\nu)^2\log q+2(2m+\mu)(2n+\nu)\log r\right\}+\frac{i\pi}{2}\left\{(2m+\mu)\frac{x}{\mathrm{K}}+(2n+\nu)\frac{y}{\Lambda}\right\}}$$

Obviously

$$\begin{aligned}\Phi\left\{\begin{pmatrix}\lambda, \rho\\ \mu, \nu\end{pmatrix}x, y\right\}&=\Phi\left\{\begin{pmatrix}\lambda, \rho\\ \mu, \nu\end{pmatrix}x+4\mathrm{K}, y\right\}\\ &=\Phi\left\{\begin{pmatrix}\lambda, \rho\\ \mu, \nu\end{pmatrix}x, y+4\Lambda\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)\end{aligned}$$

so that 4K and zero, zero and 4Λ, form two pairs of actual periods, conjugate in x and y, for Φ.

Since

$$e^{-\frac{\pi^2x^2}{4\mathrm{K}^2\log p}}\Phi\left\{\begin{pmatrix}\lambda, \rho\\ \mu, \nu\end{pmatrix}x, y\right\}=\sum_{m=-\infty}^{m=\infty}\sum_{n=-\infty}^{n=\infty}(-1)^{m\lambda+n\rho}e^{\frac{1}{\log p}\left\{i\frac{\pi x}{2\mathrm{K}}+\frac{2m+\mu}{2}\log p\right\}^2+(2n+\nu)\left\{i\frac{\pi y}{2\Lambda}+\frac{2m+\mu}{2}\log r\right\}+\frac{(2n+\nu)^2}{4}\log q}$$

and the right-hand side is unaltered by writing

and $$\left.\begin{aligned}x+\frac{4\mathrm{K}}{\pi i}\log p \text{ for } x\\ y+\frac{4\Lambda}{\pi i}\log r \text{ for } y\end{aligned}\right\},$$

$\frac{4\mathrm{K}}{\pi i}\log p$, $\frac{4\Lambda}{\pi i}\log r$ form a pair of quasi-periods for Φ, conjugate in x and y. Again

$$e^{-\frac{\pi^2y^2}{4\Lambda^2\log q}}\Phi\left\{\begin{pmatrix}\lambda, \rho\\ \mu, \nu\end{pmatrix}x, y\right\}=\sum_{m=-\infty}^{m=\infty}\sum_{n=-\infty}^{n=\infty}(-1)^{m\lambda+n\rho}e^{\frac{1}{\log q}\left\{i\frac{\pi y}{2\Lambda}+\frac{2n+\nu}{2}\log q\right\}^2+(2m+\mu)\left\{i\frac{\pi x}{2\mathrm{K}}+\frac{2n+\nu}{2}\log r\right\}+\frac{(2m+\mu)^2}{4}\log p}$$

and the right-hand side is unaltered by writing

and $$\left.\begin{aligned}x+\frac{4\mathrm{K}}{\pi i}\log r \text{ for } x\\ y+\frac{4\Lambda}{\pi i}\log q \text{ for } y\end{aligned}\right\},$$

so that $\frac{4\mathrm{K}}{\pi i}\log r$, $\frac{4\Lambda}{\pi i}\log q$ form another pair of quasi-periods for Φ, conjugate in x and y.

Actual.				Quasi.			
x.	Hermite's Notation.	y.	Hermite's Notation.	x.	Hermite's Notation.	y.	Hermite's Notation.
4K	Ω_0	0	Υ_0	$\frac{4\mathrm{K}}{\pi i}\log p$	Ω_3	$\frac{4\Lambda}{\pi i}\log r$	Υ_3
0	Ω_1	4Λ	Υ_1	$\frac{4\mathrm{K}}{\pi i}\log r$	Ω_4	$\frac{4\Lambda}{\pi i}\log q$	Υ_4

This table exhibits the four pairs of conjugate periods in x and y; one relation between them is immediately deduced, viz. :

$$\Omega_0\Upsilon_3-\Omega_3\Upsilon_0+\Omega_1\Upsilon_2-\Omega_2\Upsilon_1=0$$

an equation which Hermite makes fundamental in his transformation theory (Comptes Rendus, t. xl.) The following equations, giving the relations for quarter and half period increase of the variables, are easily obtained :—

$$\left.\begin{aligned}\Phi\left\{\begin{pmatrix}\lambda, \rho\\ \mu, \nu\end{pmatrix}x+\mathrm{K},\ y\right\}&=(-1)^{\frac{\mu}{2}}\Phi\left\{\begin{pmatrix}\lambda+1, \rho\\ \mu, \nu\end{pmatrix}x,\ y\right\}\\ \Phi\left\{\begin{pmatrix}\lambda, \rho\\ \mu, \nu\end{pmatrix}x+2\mathrm{K},\ y\right\}&=(-1)^{\mu}\Phi\left\{\begin{pmatrix}\lambda, \rho\\ \mu, \nu\end{pmatrix}x,\ y\right\}\end{aligned}\right\} \quad \ldots\ldots \quad (8)$$

$$\left.\begin{aligned}\Phi\left\{\begin{pmatrix}\lambda, \rho\\ \mu, \nu\end{pmatrix}x,\ y+\ \Lambda\right\}&=(-1)^{\frac{\nu}{2}}\,\Phi\left\{\begin{pmatrix}\lambda, \rho+1\\ \mu, \nu\end{pmatrix}x,\ y\right\}\\ \Phi\left\{\begin{pmatrix}\lambda, \rho\\ \mu, \nu\end{pmatrix}x,\ y+2\Lambda\right\}&=(-1)^{\nu}\,\Phi\left\{\begin{pmatrix}\lambda, \rho\\ \mu, \nu\end{pmatrix}x,\ y\right\}\end{aligned}\right\} \quad \ldots\ldots \quad (9)$$

$$\left.\begin{aligned}\Phi\left\{\begin{pmatrix}\lambda, \rho\\ \mu, \nu\end{pmatrix}x+\frac{\mathrm{K}}{\pi i}\log p,\ y+\frac{\Lambda}{\pi i}\log r\right\}&=p^{-\frac{1}{4}}e^{-\frac{i\pi x}{2\mathrm{K}}}\,\Phi\left\{\begin{pmatrix}\lambda\ \ \ \ , \rho\\ \mu+1, \nu\end{pmatrix}x,\ y\right\}\\ \Phi\left\{\begin{pmatrix}\lambda, \rho\\ \mu, \nu\end{pmatrix}x+\frac{2\mathrm{K}}{\pi i}\log p,\ y+\frac{2\Lambda}{\pi i}\log r\right\}&=p^{-1}e^{-\frac{i\pi x}{\mathrm{K}}}(-1)^{\lambda}\Phi\left\{\begin{pmatrix}\lambda, \rho\\ \mu, \nu\end{pmatrix}x,\ y\right\}\end{aligned}\right\} \quad \ldots \quad (10)$$

$$\left.\begin{aligned}\Phi\left\{\begin{pmatrix}\lambda, \rho\\ \mu, \nu\end{pmatrix}x+\frac{\mathrm{K}}{\pi i}\log r,\ y+\frac{\Lambda}{\pi i}\log q\right\}&=q^{-\frac{1}{4}}e^{-\frac{i\pi y}{2\Lambda}}\,\Phi\left\{\begin{pmatrix}\lambda, \rho\\ \mu, \nu+1\end{pmatrix}x,\ y\right\}\\ \Phi\left\{\begin{pmatrix}\lambda, \rho\\ \mu, \nu\end{pmatrix}x+\frac{2\mathrm{K}}{\pi i}\log r,\ y+\frac{2\Lambda}{\pi i}\log q\right\}&=q^{-1}e^{-\frac{i\pi y}{\Lambda}}(-1)^{\rho}\,\Phi\left\{\begin{pmatrix}\lambda, \rho\\ \mu, \nu\end{pmatrix}x,\ y\right\}\end{aligned}\right\} \quad \ldots \quad (11)$$

$$\left.\begin{aligned}
&\frac{\Phi\left\{\begin{pmatrix}\lambda, & \rho\\ \mu, & \nu\end{pmatrix}x+\frac{\mathrm{K}}{\pi i}\log p,\ y+\frac{\Lambda}{\pi i}\log r\right\}}{\Phi\left\{\begin{pmatrix}\lambda', & \rho'\\ \mu', & \nu'\end{pmatrix}x+\frac{\mathrm{K}}{\pi i}\log p,\ y+\frac{\Lambda}{\pi i}\log r\right\}}=\frac{\Phi\left\{\begin{pmatrix}\lambda & ,\ \rho\\ \mu,+1, & \nu\end{pmatrix}x,\ y\right\}}{\Phi\left\{\begin{pmatrix}\lambda' & ,\ \rho'\\ \mu'+1, & \nu'\end{pmatrix}x,\ y\right\}}\\
&\frac{\Phi\left\{\begin{pmatrix}\lambda, & \rho\\ \mu, & \nu\end{pmatrix}x+\frac{2\mathrm{K}}{\pi i}\log p,\ y+\frac{2\Lambda}{\pi i}\log r\right\}}{\Phi\left\{\begin{pmatrix}\lambda', & \rho'\\ \mu', & \nu'\end{pmatrix}x+\frac{2\mathrm{K}}{\pi i}\log p,\ y+\frac{2\Lambda}{\pi i}\log r\right\}}=(-1)^{\lambda+\lambda'}\frac{\Phi\left\{\begin{pmatrix}\lambda, & \rho\\ \mu, & \nu\end{pmatrix}x,\ y\right\}}{\Phi\left\{\begin{pmatrix}\lambda', & \rho'\\ \mu', & \nu'\end{pmatrix}x,\ y\right\}}
\end{aligned}\right\} \quad . \quad . \quad . \quad (12)$$

$$\left.\begin{aligned}
&\frac{\Phi\left\{\begin{pmatrix}\lambda, & \rho\\ \mu, & \nu+1\end{pmatrix}x+\frac{\mathrm{K}}{\pi i}\log r,\ y+\frac{\Lambda}{\pi i}\log q\right\}}{\Phi\left\{\begin{pmatrix}\lambda', & \rho'\\ \mu', & \nu'\end{pmatrix}x+\frac{\mathrm{K}}{\pi i}\log r,\ y+\frac{\Lambda}{\pi i}\log q\right\}}=\frac{\Phi\left\{\begin{pmatrix}\lambda, & \rho\\ \mu, & \nu+1\end{pmatrix}x,\ y\right\}}{\Phi\left\{\begin{pmatrix}\lambda', & \rho'\\ \mu', & \nu'+1\end{pmatrix}x,\ y\right\}}\\
&\frac{\Phi\left\{\begin{pmatrix}\lambda, & \rho\\ \mu, & \nu\end{pmatrix}x+\frac{2\mathrm{K}}{\pi i}\log r,\ y+\frac{2\Lambda}{\pi i}\log q\right\}}{\Phi\left\{\begin{pmatrix}\lambda', & \rho'\\ \mu', & \nu'\end{pmatrix}x+\frac{2\mathrm{K}}{\pi i}\log r,\ y+\frac{2\Lambda}{\pi i}\log q\right\}}=(-1)^{\rho+\rho'}\frac{\Phi\left\{\begin{pmatrix}\lambda, & \rho\\ \mu, & \nu\end{pmatrix}x,\ y\right\}}{\Phi\left\{\begin{pmatrix}\lambda', & \rho'\\ \mu', & \nu'\end{pmatrix}x,\ y\right\}}
\end{aligned}\right\} \quad . \quad . \quad . \quad (13)$$

$$\frac{\Phi\left\{\begin{pmatrix}\lambda, & \rho\\ \mu, & \nu\end{pmatrix}x+s\,\mathrm{K}+\frac{t\mathrm{K}}{\pi i}\log p+\frac{z\mathrm{K}}{\pi i}\log r,\ y+s_1\,\Lambda+\frac{t\Lambda}{\pi i}\log r+\frac{z\Lambda}{\pi i}\log q\right\}}{\Phi\left\{\begin{pmatrix}\lambda', & \rho'\\ \mu', & \nu'\end{pmatrix}x+s'\mathrm{K}+\frac{t\mathrm{K}}{\pi i}\log p+\frac{z\mathrm{K}}{\pi i}\log r,\ y+s'_1\Lambda+\frac{t\Lambda}{\pi i}\log r+\frac{z\Lambda}{\pi i}\log q\right\}}$$

$$=(-1)^{\frac{1}{2}\{(s+s')\mu+(s_1+s'_1)\nu\}}\frac{\Phi\left\{\begin{pmatrix}\lambda+s, & \rho+s_1\\ \mu+t, & \nu+z\end{pmatrix}x,\ y\right\}}{\Phi\left\{\begin{pmatrix}\lambda'+s', & \rho'+s'_1\\ \mu'+t, & \nu'+z\end{pmatrix}x,\ y\right\}} \quad . \quad (14)$$

where, in the last formula, s, s_1, s', s'_1, t, z are integers, and the functions on the right hand side may be reduced by formulæ (2), (3) to functions in which the numbers of the characteristics differ from λ, μ, ρ, ν, λ', μ', ρ', ν', respectively by less than 2. This combination of periods in quotients is similar to the combination of real and imaginary periods in elliptic functions.

The product theorem.

6. We multiply four theta-functions

$$\Phi\left\{\begin{pmatrix}\lambda_1, & \rho_1\\ \mu_1, & \nu_1\end{pmatrix}x_1,\ y_1\right\},\ \Phi\left\{\begin{pmatrix}\lambda_2, & \rho_2\\ \mu_2, & \nu_2\end{pmatrix}x_2,\ y_2\right\},\ \Phi\left\{\begin{pmatrix}\lambda_3, & \rho_3\\ \mu_3, & \nu_3\end{pmatrix}x_3,\ y_3\right\},\ \Phi\left\{\begin{pmatrix}\lambda_4, & \rho_4\\ \mu_4, & \nu_4\end{pmatrix}x_4,\ y_4\right\},$$

such that each sum of the four corresponding numbers in the characteristics is even. Let such a product be denoted by

$$\Pi\Phi\left\{\begin{pmatrix}\lambda, & \rho\\ \mu, & \nu\end{pmatrix}x,\ y\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

indicating that there are four functions having numbers λ, μ, ρ, ν with subscript indices 1, 2, 3, 4. Taking the general definition given in (1) for Φ, let

$$\mathrm{M}_1+2m_1=\mathrm{M}_2+2m_2=\mathrm{M}_3+2m_3=\mathrm{M}_4+2m_4=m_1+m_2+m_3+m_4$$
$$\mathrm{N}_1+2n_1=\mathrm{N}_2+2n_2=\mathrm{N}_3+2n_3=\mathrm{N}_4+2n_4=n_1+n_2+n_3+n_4$$

$$\left.\begin{aligned} 2(\Lambda_1+\lambda_1)&=2(\Lambda_2+\lambda_2)=2(\Lambda_3+\lambda_3)=2(\Lambda_4+\lambda_4)=\lambda_1+\lambda_2+\lambda_3+\lambda_4 \\ 2(\sigma_1+\mu_1)&=2(\sigma_2+\mu_2)=2(\sigma_3+\mu_3)=2(\sigma_4+\mu_4)=\mu_1+\mu_2+\mu_3+\mu_4 \\ 2(\mathrm{P}_1+\rho_1)&=2(\mathrm{P}_2+\rho_2)=2(\mathrm{P}_3+\rho_3)=2(\mathrm{P}_4+\rho_4)=\rho_1+\rho_2+\rho_3+\rho_4 \\ 2(\sigma'_1+\nu_1)&=2(\sigma'_2+\nu_2)=2(\sigma'_3+\nu_3)=2(\sigma'_4+\nu_4)=\nu_1+\nu_2+\nu_3+\nu_4 \end{aligned}\right\} \qquad (16)$$

which contain the assumption that $\Sigma\lambda$, $\Sigma\mu$, $\Sigma\nu$, $\Sigma\rho$ are all even; and

$$\left.\begin{aligned} 2(\mathrm{X}_1+x_1)&=2(\mathrm{X}_2+x_2)=2(\mathrm{X}_3+x_3)=2(\mathrm{X}_4+x_4)=x_1+x_2+x_3+x_4 \\ 2(\mathrm{Y}_1+y_1)&=2(\mathrm{Y}_2+y_2)=2(\mathrm{Y}_3+y_3)=2(\mathrm{Y}_4+y_4)=y_1+y_2+y_3+y_4 \end{aligned}\right\} \quad . \quad . \qquad (17)$$

In the course of the proof the equivalent of the algebraical identities

$$\Lambda_1^2+\Lambda_2^2+\Lambda_3^2+\Lambda_4^2=\lambda_1^2+\lambda_2^2+\lambda_3^2+\lambda_4^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (18)$$

$$\Lambda_1\mathrm{P}_1+\Lambda_2\mathrm{P}_2+\Lambda_3\mathrm{P}_3+\Lambda_4\mathrm{P}_4=\lambda_1\rho_1+\lambda_2\rho_2+\lambda_3\rho_3+\lambda_4\rho_4 \quad . \quad . \quad . \quad . \qquad (19)$$

will be used. We have

$$\begin{aligned} 4\Pi\Phi\left\{\begin{pmatrix}\lambda, & \rho \\ \mu, & \nu\end{pmatrix} x,\ y\right\} = 4\Sigma(-1)^{m_1\lambda_1+\,\ldots\,+m_4\lambda_4+n_1\rho_1+\,\ldots\,+n_4\rho_4} &\times p^{\frac{(2m_1+\mu_1)^2+\,\ldots\,+(2m_4+\mu_4)^2}{4}} \\ \times v^{(2m_1+\mu_1)x_1+\,\ldots\,+(2m_4+\mu_4)x_4} &\times q^{\frac{(2n_1+\nu_1)^2+\,\ldots\,+(2n_4+\nu_4)^2}{4}} \\ \times w^{(2n_1+\nu_1)y_1+\,\ldots\,+(2n_4+\nu_4)y_4} &\times r^{\frac{(2m_1+\mu_1)(2n_1+\nu_1)+\,\ldots\,+(2m_4+\mu_4)(2n_4+\nu_4)}{2}} \end{aligned} \qquad (20)$$

where the summation is over all integral values of the m's and n's from $-\infty$ to $+\infty$. Using (18), (19), the indices on the right-hand side of (20) can be at once transformed; and the following equations give these transformed values:—

$$\begin{aligned} m_1\lambda_1+\,\ldots\,+m_4\lambda_4&=\tfrac{1}{2}(\mathrm{M}_1\Lambda_1+\,\ldots\,+\mathrm{M}_4\Lambda_4) \\ n_1\rho_1+\,\ldots\,+n_4\rho_4&=\tfrac{1}{2}\{\mathrm{N}_1\mathrm{P}_1+\,\ldots\,+\mathrm{N}_4\mathrm{P}_4\} \\ (2m_1+\mu_1)^2+\,\ldots\,+(2m_4+\mu_4)^2&=(\mathrm{M}_1+\sigma_1)^2+\,\ldots\,+(\mathrm{M}_4+\sigma_4)^2 \\ (2n_1+\nu_1)^2+\,\ldots\,+(2n_4+\nu_4)^2&=(\mathrm{N}_1+\sigma'_1)^2+\,\ldots\,+(\mathrm{N}_4+\sigma'_4)^2 \\ (2m_1+\mu_1)x_1+\,\ldots\,+(2m_4+\mu_4)x_4&=(\mathrm{M}_1+\sigma_1)\mathrm{X}_1+\,\ldots\,+(\mathrm{M}_4+\sigma_4)\mathrm{X}_4 \\ (2n_1+\nu_1)y_1+\,\ldots\,+(2n_4+\nu_4)y_4&=(\mathrm{N}_1+\sigma'_1)\mathrm{Y}_1+\,\ldots\,+(\mathrm{N}_4+\sigma'_4)\mathrm{Y}_4 \\ (2m_1+\mu_1)(2n_1+\nu_1)+\,\ldots\,+(2m_4+\mu_4)(2n_4+\nu_4)&=(\mathrm{M}_1+\sigma_1)(\mathrm{N}_1+\sigma'_1)+\,\ldots \\ &\qquad+(\mathrm{M}_4+\sigma_4)(\mathrm{N}_4+\sigma'_4) \end{aligned}$$

The substitution of these in (20) gives

$$4\Pi\Phi\left\{\begin{pmatrix}\lambda, \rho\\ \mu, \nu\end{pmatrix}x, y\right\}=4\Sigma(-1)^{\frac{1}{2}\{(\mathrm{M}_1\Lambda_1+\ldots+\mathrm{M}_4\Lambda_4)+(\mathrm{N}_1\mathrm{P}_1+\ldots+\mathrm{N}_4\mathrm{P}_4)\}}p^{\frac{1}{4}\{(\mathrm{M}_1+\sigma_1)^2+\ldots+(\mathrm{M}_4+\sigma_4)^2\}}$$

$$q^{\frac{1}{4}\{(\mathrm{N}_1+\sigma'_1)^2+\ldots+(\mathrm{N}_4+\sigma'_4)^2\}}v^{(\mathrm{M}_1+\sigma_1)\mathrm{X}_1+\ldots+(\mathrm{M}_4+\sigma_4)\mathrm{X}_4}w^{(\mathrm{N}_1+\sigma'_1)\mathrm{Y}_1+\ldots+(\mathrm{N}_4+\sigma'_4)\mathrm{Y}_4}$$

$$r^{\frac{1}{2}\{(\mathrm{M}_1+\sigma_1)(\mathrm{N}_1+\sigma'_1)+\ldots+(\mathrm{M}_4+\sigma_4)(\mathrm{N}_4+\sigma'_4)\}} \quad . \quad . \quad (21)$$

the summation being taken for all values of the M's and N's defined by the equations, *i.e.*, for all integral values which give integral values to the *m*'s and *n*'s. Now the difference between any two of the M's is even, so that they are either all even, or all uneven. Taking the first of these cases, let

$$\mathrm{M}_1=2\mathrm{M}'_1, \quad \mathrm{M}_2=2\mathrm{M}'_2, \quad \mathrm{M}_3=2\mathrm{M}'_3, \quad \mathrm{M}_4=2\mathrm{M}'_4$$

then since

$$4m_1=-\mathrm{M}_1+\mathrm{M}_2+\mathrm{M}_3+\mathrm{M}_4$$

and similar expressions hold for $4m_2$, $4m_3$, $4m_4$ it is sufficient that

$$\mathrm{M}'_1+\mathrm{M}'_2+\mathrm{M}'_3+\mathrm{M}'_4=\text{ even.}$$

Taking the second case, let

$$\mathrm{M}_1=2\mathrm{M}'_1+1, \quad \mathrm{M}_2=2\mathrm{M}'_2+1, \quad \mathrm{M}_3=2\mathrm{M}'_3+1, \quad \mathrm{M}_4=2\mathrm{M}'_4+1$$

it is sufficient that

$$\mathrm{M}'_1+\mathrm{M}'_2+\mathrm{M}'_3+\mathrm{M}'_4=\text{ uneven.}$$

With corresponding quantities substituted for the N's in the two cases exactly similar relations hold. Separate the terms in (21) and denote by

Σ_1. those in which $\mathrm{M}'_1+\ldots+\mathrm{M}'_4=$ even, and $\mathrm{N}'_1+\ldots+\mathrm{N}'_4=$ even;
Σ_2. ,, ,, ,, $=$ even, ,, ,, $=$ odd;
Σ_3. ,, ,, ,, $=$ odd, ,, ,, $=$ even;
Σ_4. ,, ,, ,, $=$ odd, ,, ,, $=$ odd.

Also write

$$\lambda_1+\lambda_2+\lambda_3+\lambda_4=\Lambda_1+\Lambda_2+\Lambda_3+\Lambda_4=2\Lambda'$$
$$\rho_1+\rho_2+\rho_3+\rho_4=\mathrm{P}_1+\mathrm{P}_2+\mathrm{P}_3+\mathrm{P}_4=2\mathrm{P}';$$

and, for shortness, let Q_1, Q_2, Q_3, Q_4 denote the general terms in Σ_1, Σ_2, Σ_3, Σ_4 respectively, so that

$$(-1)^{\mathrm{M}'_1\Lambda_1+\ldots+\mathrm{M}'_4\Lambda_4+\mathrm{N}_1\mathrm{P}_1+\ldots+\mathrm{N}'_4\mathrm{P}_4}\mathrm{Q}_1$$
$$=p^{\frac{1}{4}\{(2\mathrm{M}_1+\sigma_1)^2+\ldots\}}q^{\frac{1}{4}\{(2\mathrm{N}'_1+\sigma'_1)^2+\ldots\}}r^{\frac{1}{2}\{(2\mathrm{M}_1+\sigma'_1)(2\mathrm{N}'_1+\sigma'_1)+\ldots\}}v^{(2\mathrm{M}'_1+\sigma_1)\mathrm{X}_1+\ldots}w^{(2\mathrm{N}'_1+\sigma'_1)\mathrm{Y}_1+\ldots}$$

$$(-1)^{\mathrm{M}_1\Lambda_1+\ldots+\mathrm{M}'_4\Lambda_4+\mathrm{N}'_1\mathrm{P}_1+\ldots+\mathrm{N}'_4\mathrm{P}_4}\mathrm{Q}_2$$
$$=p^{\frac{1}{4}\{(2\mathrm{M}'_1+\sigma_1)^2+\ldots\}}q^{\frac{1}{4}\{(2\mathrm{N}'_1+1+\sigma'_1)^2+\ldots\}}r^{\frac{1}{2}\{(2\mathrm{M}'_1+\sigma_1)(2\mathrm{N}'_1+1+\sigma_1)+\ldots\}}v^{(2\mathrm{M}'_1+\sigma_1)\mathrm{X}_1+\ldots}w^{(2\mathrm{N}'_1+1+\sigma'_1)\mathrm{Y}_1+\ldots}$$

$$(-1)^{\mathrm{M}_1\Lambda_1+\ldots+\mathrm{M}'_4\Lambda_4+\mathrm{N}_1\mathrm{P}_1+\ldots+\mathrm{N}_4\mathrm{P}_4}\mathrm{Q}_3$$
$$=p^{\frac{1}{4}\{(2\mathrm{M}_1+1+\sigma_1)^2+\ldots\}}q^{\frac{1}{4}\{(2\mathrm{N}'_1+\sigma_1)^2+\ldots\}}r^{\frac{1}{2}\{(2\mathrm{M}'_1+1+\sigma_1)(2\mathrm{N}'_1+\sigma_1)+\ldots\}}v^{(2\mathrm{M}'_1+1+\sigma_1)\mathrm{X}_1+\ldots}w^{(2\mathrm{N}_1+\sigma'_1)\mathrm{Y}_1+\ldots}$$

$$(-1)^{\mathrm{M}_1\Lambda_1+\ldots+\mathrm{M}'_4\Lambda_4+\mathrm{N}'_1\mathrm{P}_1+\ldots+\mathrm{N}'_4\mathrm{P}_4}\mathrm{Q}_4$$
$$=p^{\frac{1}{4}\{(2\mathrm{M}'_1+1+\sigma_1)^2+\ldots\}}q^{\frac{1}{4}\{(2\mathrm{N}'_1+1+\sigma'_1)^2+\ldots\}}r^{\frac{1}{2}\{(2\mathrm{M}_1+1+\sigma_1)(2\mathrm{N}_1+1+\sigma'_1)+\ldots\}}v^{(2\mathrm{M}'_1+1+\sigma_1)\mathrm{X}_1+\ldots}w^{(2\mathrm{N}_1+1+\sigma'_1)\mathrm{Y}_1+\ldots}$$

and (21) becomes

$$4\Pi\Phi\left\{\begin{pmatrix}\lambda, & \rho\\ \mu, & \nu\end{pmatrix}x,\ y\right\}=4\Sigma_1.\mathrm{Q}_1+(-1)^{\mathrm{P}'}4\Sigma_2.\mathrm{Q}_2+(-1)^{\Lambda'}4\Sigma_3.\mathrm{Q}_3+(-1)^{\Lambda+\mathrm{P}'}4\Sigma_4.\mathrm{Q}_4\,. \quad (22)$$

Consider these four sums separately; then

$$4\Sigma_1.\mathrm{Q}_1=\Sigma\Sigma\mathrm{Q}+\Sigma\Sigma(-1)^{\mathrm{N}'_1+\mathrm{N}'_2+\mathrm{N}'_3+\mathrm{N}_4}\mathrm{Q}_1+\Sigma\Sigma(-1)^{\mathrm{M}'_1+\mathrm{M}'_2+\mathrm{M}'_3+\mathrm{M}'_4}\mathrm{Q}_1$$
$$+\Sigma\Sigma(-1)^{\mathrm{N}'_1+\ldots+\mathrm{N}'_4+\mathrm{M}_1+\ldots+\mathrm{M}'_4}\mathrm{Q}_1$$

where the summations on the right hand side are now taken without restriction for all integral values of the M's and N's between $-\infty$ and $+\infty$; and with similar removal of the restrictions on the values of M and N to which the summation extends

$$4\Sigma_2.\mathrm{Q}_2=\Sigma\Sigma\mathrm{Q}_2-\Sigma\Sigma(-1)^{\mathrm{N}'_1+\mathrm{N}'_2+\mathrm{N}'_3+\mathrm{N}'_4}\mathrm{Q}_2+\Sigma\Sigma(-1)^{\mathrm{M}'_1+\mathrm{M}'_2+\mathrm{M}'_3+\mathrm{M}'_4}\mathrm{Q}_2$$
$$-\Sigma\Sigma(-1)^{\mathrm{N}_1+\ldots+\mathrm{N}'_4+\mathrm{M}'_1+\ldots+\mathrm{M}'_4}\mathrm{Q}_2$$

$$4\Sigma_3.\mathrm{Q}_3=\Sigma\Sigma\mathrm{Q}_3+\Sigma\Sigma(-1)^{\mathrm{N}'_1+\mathrm{N}'_2+\mathrm{N}'_3+\mathrm{N}'_4}\mathrm{Q}_3-\Sigma\Sigma(-1)^{\mathrm{M}'_1+\mathrm{M}'_2+\mathrm{M}'_3+\mathrm{M}'_4}\mathrm{Q}_3$$
$$-\Sigma\Sigma(-1)^{\mathrm{N}'_1+\ldots+\mathrm{N}_4+\mathrm{M}'_1+\ldots+\mathrm{M}'_4}\mathrm{Q}_3$$

$$4\Sigma_4.\mathrm{Q}_4=\Sigma\Sigma\mathrm{Q}_4-\Sigma\Sigma(-1)^{\mathrm{N}'_1+\mathrm{N}'_2+\mathrm{N}'_3+\mathrm{N}'_4}\mathrm{Q}_4-\Sigma\Sigma(-1)^{\mathrm{M}'_1+\mathrm{M}'_2+\mathrm{M}'_3+\mathrm{M}'_4}\mathrm{Q}_4$$
$$+\Sigma\Sigma(-1)^{\mathrm{N}'_1+\ldots+\mathrm{N}'_4+\mathrm{M}'_1+\ldots+\mathrm{M}'_4}\mathrm{Q}_4$$

Thus

$$4\Pi\Phi\left\{\begin{pmatrix}\lambda, & \rho\\ \mu, & \nu\end{pmatrix}x,\ y\right\}=\text{sum of sixteen double summations.}$$

But each of these double summations is the product of four double theta-functions: thus, in particular,

$Q_1 =$ general term in

$$\Phi\left\{\begin{pmatrix}\Lambda_1, & P_1\\ \sigma_1, & \sigma'_1\end{pmatrix}X_1,\ Y_1\right\}\Phi\left\{\begin{pmatrix}\Lambda_2, & P_2\\ \sigma_2, & \sigma'_2\end{pmatrix}X_2,\ Y_2\right\}\Phi\left\{\begin{pmatrix}\Lambda_3, & P_3\\ \sigma_3, & \sigma'_3\end{pmatrix}X_3,\ Y_3\right\}\Phi\left\{\begin{pmatrix}\Lambda_4, & P_4\\ \sigma_4, & \sigma'_4\end{pmatrix}X_4,\ Y_4\right\}$$

$$\textit{i.e.}\text{ in }\Pi\Phi\left\{\begin{pmatrix}\Lambda, & P\\ \sigma, & \sigma'\end{pmatrix}X,\ Y\right\};$$

$$(-1)^{N_1+N_2+N'_3+N'_4}Q_1 = \text{general term in }\Pi\Phi\left\{\begin{pmatrix}\Lambda, & P+1\\ \sigma, & \sigma'\end{pmatrix}X,\ Y\right\};$$

and so on for the others; and it follows that the expression of the product theorem is given by

$$4\Pi\Phi\left\{\begin{pmatrix}\lambda, & \rho\\ \mu, & \nu\end{pmatrix}x,\ y\right\}$$

$$\begin{array}{l|l}
 & +(-1)^{\Lambda'}\text{ into} \\
=\Pi\Phi\left\{\begin{pmatrix}\Lambda, & P\\ \sigma, & \sigma\end{pmatrix}X,\ Y\right\} & \quad\Pi\Phi\left\{\begin{pmatrix}\Lambda & , P\\ \sigma+1, & \sigma'\end{pmatrix}X,\ Y\right\} \\
+\Pi\Phi\left\{\begin{pmatrix}\Lambda, & P+1\\ \sigma, & \sigma'\end{pmatrix}X,\ Y\right\} & +\Pi\Phi\left\{\begin{pmatrix}\Lambda & , P+1\\ \sigma+1, & \sigma'\end{pmatrix}X,\ Y\right\} \\
+\Pi\Phi\left\{\begin{pmatrix}\Lambda+1, & P\\ \sigma & ,\sigma'\end{pmatrix}X,\ Y\right\} & -\Pi\Phi\left\{\begin{pmatrix}\Lambda+1, & P\\ \sigma+1 & \sigma'\end{pmatrix}X,\ Y\right\} \\
+\Pi\Phi\left\{\begin{pmatrix}\Lambda+1, & P+1\\ \sigma & ,\sigma'\end{pmatrix}X,\ Y\right\} & -\Pi\Phi\left\{\begin{pmatrix}\Lambda+1, & P+1\\ \sigma+1, & \sigma'\end{pmatrix}X,\ Y\right\}
\end{array}$$

$$\begin{array}{|l|l}
+(-1)^{P'}\text{ into} & +(-1)^{\Lambda'+P'}\text{ into} \\
\quad\Pi\Phi\left\{\begin{pmatrix}\Lambda, & P\\ \sigma & \sigma'+1\end{pmatrix}X,\ Y\right\} & \quad\Pi\Phi\left\{\begin{pmatrix}\Lambda & , P\\ \sigma+1, & \sigma'+1\end{pmatrix}X,\ Y\right\} \\
-\Pi\Phi\left\{\begin{pmatrix}\Lambda, & P+1\\ \sigma, & \sigma'+1\end{pmatrix}X,\ Y\right\} & -\Pi\Phi\left\{\begin{pmatrix}\Lambda & ,P+1\\ \sigma+1, & \sigma'+1\end{pmatrix}X,\ Y\right\} \\
+\Pi\Phi\left\{\begin{pmatrix}\Lambda+1, & P\\ \sigma & ,\sigma'+1\end{pmatrix}X,\ Y\right\} & -\Pi\Phi\left\{\begin{pmatrix}\Lambda+1, & P\\ \sigma+1, & \sigma'+1\end{pmatrix}X,\ Y\right\} \\
-\Pi\Phi\left\{\begin{pmatrix}\Lambda+1, & P+1\\ \sigma & ,\sigma'+1\end{pmatrix}X,\ Y\right\} & +\Pi\Phi\left\{\begin{pmatrix}\Lambda+1, & P+1\\ \sigma+1, & \sigma'+1\end{pmatrix}X,\ Y\right\}
\end{array} \quad . \quad . \quad . \quad . \quad (23).$$

7. This product theorem does not comprise 16^4 equations, as might be expected; in defining Λ, P, σ, σ' it was assumed that the sums of the four λ's, of the four μ's, of the four ρ's, and of the four ν's, were each even. Thus when λ_1, λ_2, λ_3, are known, λ_4, being limited to the values zero and unity, is also known; and similarly for the other numbers. Hence the number of equations comprised is 16^3, *i.e.*, is 4096.

8. If any uniform increase or decrease be made in a set of corresponding numbers

in the left-hand side; the same increase or decrease occurs in the right-hand side; thus if each of the λ's be increased by unity, by unity also will each of the Λ's be increased. Hence, as immediate deductions from (23), are obtained the following formulæ, including as particular cases, many of ROSENHAIN'S formulæ contained in the table at the end of his memoir.

$$\Pi\Phi\begin{pmatrix}\lambda, & \rho\\ \mu, & \nu\end{pmatrix}+\Pi\Phi\begin{pmatrix}\lambda+1, & \rho\\ \mu & , \nu\end{pmatrix}+\Pi\Phi\begin{pmatrix}\lambda, & \rho+1\\ \mu, & \nu\end{pmatrix}+\Pi\Phi\begin{pmatrix}\lambda+1, & \rho+1\\ \mu & , \nu\end{pmatrix}$$
$$=\Pi\Phi\begin{pmatrix}\Lambda, & \mathrm{P}\\ \sigma, & \sigma'\end{pmatrix}+\Pi\Phi\begin{pmatrix}\Lambda+1, & \mathrm{P}\\ \sigma & , \sigma'\end{pmatrix}+\Pi\Phi\begin{pmatrix}\Lambda, & \mathrm{P}+1\\ \sigma, & \sigma'\end{pmatrix}+\Pi\Phi\begin{pmatrix}\Lambda+1, & \mathrm{P}+1\\ \sigma & , \sigma'\end{pmatrix} \qquad (24)$$

where on the right-hand side the variables are $\mathrm{X}_1, \mathrm{X}_2, \mathrm{X}_3, \mathrm{X}_4, \mathrm{Y}_1, \mathrm{Y}_2, \mathrm{Y}_3, \mathrm{Y}_4$,
and ,, left ,, ,, ,, $x_1, x_2, x_3, x_4, y_1, y_2, y_3, y_4$.

$$\Pi\Phi\begin{pmatrix}\lambda, & \rho\\ \mu, & \nu\end{pmatrix}+\Pi\Phi\begin{pmatrix}\lambda+1, & \rho\\ \mu & , \nu\end{pmatrix}-\Pi\Phi\begin{pmatrix}\lambda, & \rho+1\\ \mu, & \nu\end{pmatrix}-\Pi\Phi\begin{pmatrix}\lambda+1, & \rho+1\\ \mu & , \nu\end{pmatrix}$$
$$=(-1)^{\mathrm{P}'}\left[\Pi\Phi\begin{pmatrix}\Lambda, & \mathrm{P}\\ \sigma, & \sigma'+1\end{pmatrix}+\Pi\Phi\begin{pmatrix}\Lambda+1, & \mathrm{P}\\ \sigma & , \sigma'+1\end{pmatrix}-\Pi\Phi\begin{pmatrix}\Lambda, & \mathrm{P}+1\\ \sigma, & \sigma'+1\end{pmatrix}-\Pi\Phi\begin{pmatrix}\Lambda+1, & \mathrm{P}+1\\ \sigma & , \sigma'+1\end{pmatrix}\right] \qquad (25)$$

$$\Pi\Phi\begin{pmatrix}\lambda, & \rho\\ \mu, & \nu\end{pmatrix}-\Pi\Phi\begin{pmatrix}\lambda+1, & \rho\\ \mu & , \nu\end{pmatrix}+\Pi\Phi\begin{pmatrix}\lambda, & \rho+1\\ \mu, & \nu\end{pmatrix}-\Pi\Phi\begin{pmatrix}\lambda+1, & \rho+1\\ \mu & , \nu\end{pmatrix}$$
$$=(-1)^{\Lambda'}\left[\Pi\Phi\begin{pmatrix}\Lambda & , \mathrm{P}\\ \sigma+1, & \sigma'\end{pmatrix}-\Pi\Phi\begin{pmatrix}\Lambda+1, & \mathrm{P}\\ \sigma+1, & \sigma'\end{pmatrix}+\Pi\Phi\begin{pmatrix}\Lambda & , \mathrm{P}+1\\ \sigma+1, & \sigma'\end{pmatrix}-\Pi\Phi\begin{pmatrix}\Lambda+1, & \mathrm{P}+1\\ \sigma+1, & \sigma'\end{pmatrix}\right] \qquad (26)$$

$$\Pi\Phi\begin{pmatrix}\lambda, & \rho\\ \mu, & \nu\end{pmatrix}-\Pi\Phi\begin{pmatrix}\lambda+1, & \rho\\ \mu, & \nu\end{pmatrix}-\Pi\Phi\begin{pmatrix}\lambda, & \rho+1\\ \mu, & \nu\end{pmatrix}+\Pi\Phi\begin{pmatrix}\lambda+1, & \rho+1\\ \mu, & \nu\end{pmatrix}$$
$$=(-1)^{\Lambda'+\mathrm{P}'}\left[\Pi\Phi\begin{pmatrix}\Lambda, & \mathrm{P}\\ \sigma+1, & \sigma'+1\end{pmatrix}-\Pi\Phi\begin{pmatrix}\Lambda+1, & \mathrm{P}\\ \sigma+1, & \sigma'+1\end{pmatrix}\right.$$
$$\left.-\Pi\Phi\begin{pmatrix}\Lambda, & \mathrm{P}+1\\ \sigma+1, & \sigma'+1\end{pmatrix}+\Pi\Phi\begin{pmatrix}\Lambda+1, & \mathrm{P}+1\\ \sigma+1, & \sigma'+1\end{pmatrix}\right] \quad . \quad . \quad . \quad . \quad (27).$$

To these must be added a set of four obtained by increasing each of the x's by $\frac{\mathrm{K}}{\pi i}\log p$ and each of the y's by $\frac{\Lambda}{\pi i}\log r$, conjugate quarter periods: by these substitutions (24) gives

$$\Pi\Phi\begin{pmatrix}\lambda, & \rho\\ \mu+1, & \nu\end{pmatrix}+\Pi\Phi\begin{pmatrix}\lambda+1, & \rho\\ \mu+1, & \nu\end{pmatrix}+\Pi\Phi\begin{pmatrix}\lambda, & \rho+1\\ \mu+1, & \nu\end{pmatrix}+\Pi\Phi\begin{pmatrix}\lambda+1, & \rho+1\\ \mu+1, & \nu\end{pmatrix}$$
$$=\Pi\Phi\begin{pmatrix}\Lambda, & \mathrm{P}\\ \sigma+1, & \sigma'\end{pmatrix}+\Pi\Phi\begin{pmatrix}\Lambda+1, & \mathrm{P}\\ \sigma+1, & \sigma'\end{pmatrix}+\Pi\Phi\begin{pmatrix}\Lambda, & \mathrm{P}+1\\ \sigma+1, & \sigma'\end{pmatrix}+\Pi\Phi\begin{pmatrix}\Lambda+1, & \mathrm{P}+1\\ \sigma+1, & \sigma'\end{pmatrix} \qquad (28).$$

Another set of four is obtained by changing x into $x+\frac{\mathrm{K}}{\pi i}\log r$

and y ,, $y+\frac{\Lambda}{\pi i}\log q$:

thus (24) gives

$$\Pi\Phi\begin{pmatrix}\lambda, & \rho \\ \mu, & \nu+1\end{pmatrix}+\Pi\Phi\begin{pmatrix}\lambda+1, & \rho \\ \mu, & \nu+1\end{pmatrix}+\Pi\Phi\begin{pmatrix}\lambda, & \rho+1 \\ \mu, & \nu+1\end{pmatrix}+\Pi\Phi\begin{pmatrix}\lambda+1, & \rho+1 \\ \mu, & \nu+1\end{pmatrix}$$
$$=\Pi\Phi\begin{pmatrix}\Lambda, & \mathrm{P} \\ \sigma, & \sigma'+1\end{pmatrix}+\Pi\Phi\begin{pmatrix}\Lambda+1, & \mathrm{P} \\ \sigma, & \sigma'+1\end{pmatrix}+\Pi\Phi\begin{pmatrix}\Lambda, & \mathrm{P}+1 \\ \sigma, & \sigma'+1\end{pmatrix}+\Pi\Phi\begin{pmatrix}\Lambda+1, & \mathrm{P}+1 \\ \sigma, & \sigma'+1\end{pmatrix} \qquad (29);$$

and a fourth set of four from the same equations by increasing each of the x's and y's by $\frac{\mathrm{K}}{\pi i}\log pr$, $\frac{\Lambda}{\pi i}\log rq$ respectively: thus (24) gives

$$\Pi\Phi\begin{pmatrix}\lambda, & \rho \\ \mu+1, & \nu+1\end{pmatrix}+\Pi\Phi\begin{pmatrix}\lambda+1, & \rho \\ \mu+1, & \nu+1\end{pmatrix}+\Pi\Phi\begin{pmatrix}\lambda, & \rho+1 \\ \mu+1, & \nu+1\end{pmatrix}+\Pi\Phi\begin{pmatrix}\lambda+1, & \rho+1 \\ \mu+1, & \nu+1\end{pmatrix}$$
$$=\Pi\Phi\begin{pmatrix}\Lambda, & \mathrm{P} \\ \sigma+1, & \sigma'+1\end{pmatrix}+\Pi\Phi\begin{pmatrix}\Lambda+1, & \mathrm{P} \\ \sigma+1, & \sigma'+1\end{pmatrix}+\Pi\Phi\begin{pmatrix}\Lambda, & \mathrm{P}+1 \\ \sigma+1, & \sigma'+1\end{pmatrix}+\Pi\Phi\begin{pmatrix}\Lambda+1, & \mathrm{P}+1 \\ \sigma+1, & \sigma'+1\end{pmatrix} \qquad (30).$$

But as might be expected from formulæ (8) and (9) with (17), these equations (28) (29) and (30) could be obtained by uniform increase of the numbers μ and ν in the characteristics of (24).

9. Let the value of a function when both the variables are zero be denoted by c, with the same subscript number as marks the function in the current-number notation; for shortness, let

$$\begin{array}{rcl} \vartheta_r(x, y) & \text{be written} & \vartheta_r \\ \vartheta_r(\xi, \eta) & ,, & \theta_r \\ \text{and } \vartheta_r(x+\xi, y+\eta) & ,, & \vartheta_r(x+\xi). \end{array}$$

Then the following equations are obtained from (23) by assigning suitable values to the numbers of the characteristics and to the variables.

$$\begin{array}{lll}
3c_0^2\,\vartheta_0^2 = & c_1^2\vartheta_1^2+c_2^2\vartheta_2^2+c_3^2\vartheta_3^2+c_4^2\vartheta_4^2+c_6^2\vartheta_6^2+c_8^2\vartheta_8^2+c_9^2\vartheta_9^2+c_{12}^2\vartheta_{12}^2+c_{15}^2\vartheta_{15}^2 \quad . \quad . & \text{(i)} \\
3c_1^2\,\vartheta_1^2 = & c_0^2\vartheta_0^2+c_2^2\vartheta_2^2+c_3^2\vartheta_3^2-c_4^2\vartheta_4^2-c_6^2\vartheta_6^2+c_8^2\vartheta_8^2+c_9^2\vartheta_9^2-c_{12}^2\vartheta_{12}^2-c_{15}^2\vartheta_{15}^2 \quad . \quad . & \text{(ii)} \\
3c_2^2\,\vartheta_2^2 = & c_0^2\vartheta_0^2+c_1^2\vartheta_1^2+c_3^2\vartheta_3^2+c_4^2\vartheta_4^2+c_6^2\vartheta_6^2-c_8^2\vartheta_8^2-c_9^2\vartheta_9^2-c_{12}^2\vartheta_{12}^2-c_{15}^2\vartheta_{15}^2 \quad . \quad . & \text{(iii)} \\
3c_3^2\,\vartheta_3^2 = & c_0^2\vartheta_0^2+c_1^2\vartheta_1^2+c_2^2\vartheta_2^2-c_4^2\vartheta_4^2-c_6^2\vartheta_6^2-c_8^2\vartheta_8^2-c_9^2\vartheta_9^2+c_{12}^2\vartheta_{12}^2+c_{15}^2\vartheta_{15}^2 \quad . \quad . & \text{(iv)} \\
3c_4^2\,\vartheta_4^2 = & c_0^2\vartheta_0^2-c_1^2\vartheta_1^2+c_2^2\vartheta_2^2-c_3^2\vartheta_3^2+c_6^2\vartheta_6^2+c_8^2\vartheta_8^2-c_9^2\vartheta_9^2+c_{12}^2\vartheta_{12}^2-c_{15}^2\vartheta_{15}^2 \quad . \quad . & \text{(v)} \\
3c_6^2\,\vartheta_6^2 = & c_0^2\vartheta_0^2-c_1^2\vartheta_1^2+c_2^2\vartheta_2^2-c_3^2\vartheta_3^2+c_4^2\vartheta_4^2-c_8^2\vartheta_8^2+c_9^2\vartheta_9^2-c_{12}^2\vartheta_{12}^2+c_{15}^2\vartheta_{15}^2 \quad . \quad . & \text{(vi)} \\
3c_8^2\,\vartheta_8^2 = & c_0^2\vartheta_0^2+c_1^2\vartheta_1^2-c_2^2\vartheta_2^2-c_3^2\vartheta_3^2+c_4^2\vartheta_4^2-c_6^2\vartheta_6^2+c_9^2\vartheta_9^2+c_{12}^2\vartheta_{12}^2-c_{15}^2\vartheta_{15}^2 \quad . \quad . & \text{(vii)} \\
3c_9^2\,\vartheta_9^2 = & c_0^2\vartheta_0^2+c_1^2\vartheta_1^2-c_2^2\vartheta_2^2-c_3^2\vartheta_3^2-c_4^2\vartheta_4^2+c_6^2\vartheta_6^2+c_8^2\vartheta_8^2-c_{12}^2\vartheta_{12}^2+c_{15}^2\vartheta_{15}^2 \quad . \quad . & \text{(viii)} \\
3c_{12}^2\vartheta_{12}^2 = & c_0^2\vartheta_0^2-c_1^2\vartheta_1^2-c_2^2\vartheta_2^2+c_3^2\vartheta_3^2+c_4^2\vartheta_4^2-c_6^2\vartheta_6^2+c_8^2\vartheta_8^2-c_9^2\vartheta_9^2+c_{15}^2\vartheta_{15}^2 \quad . \quad . & \text{(ix)} \\
3c_{15}^2\vartheta_{15}^2 = & c_0^2\vartheta_0^2-c_1^2\vartheta_1^2-c_2^2\vartheta_2^2+c_3^2\vartheta_3^2-c_4^2\vartheta_4^2+c_6^2\vartheta_6^2-c_8^2\vartheta_8^2+c_9^2\vartheta_9^2+c_{12}^2\vartheta_{12}^2 \quad . \quad . & \text{(x)}
\end{array}$$

These are not independent: thus, adding equations (ii)–(x), equation (i) is obtained; and simpler relations equivalent to these will be deduced later on.

$$3c_0^2\vartheta_5^2 = c_4^2\vartheta_1^2+c_6^2\vartheta_3^2-c_1^2\vartheta_4^2-c_3^2\vartheta_6^2-c_2^2\vartheta_7^2+c_{12}^2\vartheta_9^2-c_{15}^2\vartheta_{10}^2-c_9^2\vartheta_{12}^2+c_8^2\vartheta_{13}^2 \quad . \tag{xi}$$

$$3c_2^2\vartheta_7^2 = c_4^2\vartheta_1^2+c_6^2\vartheta_3^2-c_1^2\vartheta_4^2+c_0^2\vartheta_5^2-c_3^2\vartheta_6^2-c_{12}^2\vartheta_9^2+c_{15}^2\vartheta_{10}^2+c_9^2\vartheta_{12}^2-c_8^2\vartheta_{13}^2 \quad . \tag{xii}$$

$$3c_{15}^2\vartheta_{10}^2 = c_4^2\vartheta_1^2-c_6^2\vartheta_3^2-c_1^2\vartheta_4^2-c_0^2\vartheta_5^2+c_3^2\vartheta_6^2+c_2^2\vartheta_7^2-c_{12}^2\vartheta_9^2+c_9^2\vartheta_{12}^2+c_8^2\vartheta_{13}^2 \quad . \tag{xiii}$$

$$3c_8^2\vartheta_{13}^2 = c_4^2\vartheta_1^2-c_6^2\vartheta_3^2-c_1^2\vartheta_4^2+c_0^2\vartheta_5^2+c_3^2\vartheta_6^2-c_2^2\vartheta_7^2+c_{12}^2\vartheta_9^2+c_{15}^2\vartheta_{10}^2-c_9^2\vartheta_{12}^2 \quad . \tag{xiv}$$

$$3c_0^2\vartheta_{13}^2 = c_{12}^2\vartheta_1^2+c_{15}^2\vartheta_2^2-c_9^2\vartheta_4^2+c_8^2\vartheta_5^2+c_4^2\vartheta_9^2-c_6^2\vartheta_{11}^2-c_1^2\vartheta_{12}^2+c_3^2\vartheta_{14}^2-c_2^2\vartheta_{15}^2 \quad . \tag{xv}$$

$$3c_8^2\vartheta_5^2 = c_{12}^2\vartheta_1^2-c_{15}^2\vartheta_2^2-c_9^2\vartheta_4^2+c_4^2\vartheta_9^2+c_6^2\vartheta_{11}^2-c_1^2\vartheta_{12}^2+c_0^2\vartheta_{13}^2-c_3^2\vartheta_{14}^2+c_2^2\vartheta_{15}^2 \quad . \tag{xvi}$$

$$3c_6^2\vartheta_{11}^2 = c_{12}^2\vartheta_1^2-c_{15}^2\vartheta_2^2+c_9^2\vartheta_4^2+c_8^2\vartheta_5^2-c_4^2\vartheta_9^2-c_1^2\vartheta_{12}^2-c_0^2\vartheta_{13}^2+c_3^2\vartheta_{14}^2+c_2^2\vartheta_{15}^2 \quad . \tag{xvii}$$

$$3c_3^2\vartheta_{14}^2 = c_{12}^2\vartheta_1^2+c_{15}^2\vartheta_2^2+c_9^2\vartheta_4^2-c_8^2\vartheta_5^2-c_4^2\vartheta_9^2+c_6^2\vartheta_{11}^2-c_1^2\vartheta_{12}^2+c_0^2\vartheta_{13}^2-c_2^2\vartheta_{15}^2 \quad . \tag{xviii}$$

$$3c_1^2\vartheta_{12}^2 = c_{12}^2\vartheta_1^2+c_{15}^2\vartheta_2^2+c_9^2\vartheta_4^2-c_8^2\vartheta_5^2+c_4^2\vartheta_9^2-c_6^2\vartheta_{11}^2-c_0^2\vartheta_{13}^2-c_3^2\vartheta_{14}^2+c_2^2\vartheta_{15}^2 \quad . \tag{xix}$$

$$3c_{12}^2\vartheta_1^2 = +c_{15}^2\vartheta_2^2+c_9^2\vartheta_4^2+c_8^2\vartheta_5^2+c_4^2\vartheta_9^2+c_6^2\vartheta_{11}^2+c_1^2\vartheta_{12}^2+c_0^2\vartheta_{13}^2+c_3^2\vartheta_{14}^2+c_2^2\vartheta_{15}^2 \quad . \tag{xx}$$

$$3c_2^2\vartheta_{13}^2 = c_{15}^2\vartheta_0^2+c_{12}^2\vartheta_3^2-c_9^2\vartheta_6^2+c_8^2\vartheta_7^2+c_6^2\vartheta_9^2-c_4^2\vartheta_{11}^2-c_3^2\vartheta_{12}^2+c_1^2\vartheta_{14}^2-c_0^2\vartheta_{15}^2 \quad . \tag{xxi}$$

$$3c_3^2\vartheta_{12}^2 = c_{15}^2\vartheta_0^2+c_{12}^2\vartheta_3^2+c_9^2\vartheta_6^2-c_8^2\vartheta_7^2+c_6^2\vartheta_9^2-c_4^2\vartheta_{11}^2-c_2^2\vartheta_{13}^2-c_1^2\vartheta_{14}^2+c_0^2\vartheta_{15}^2 \quad . \tag{xxii}$$

$$3c_{15}^2\vartheta_{14}^2 = -c_1^2\vartheta_0^2+c_0^2\vartheta_1^2+c_3^2\vartheta_2^2-c_2^2\vartheta_3^2-c_4^2\vartheta_5^2+c_6^2\vartheta_7^2+c_9^2\vartheta_8^2-c_8^2\vartheta_9^2+c_{12}^2\vartheta_{13}^2 \quad . \tag{xxiii}$$

$$3c_{12}^2\vartheta_{13}^2 = -c_1^2\vartheta_0^2+c_0^2\vartheta_1^2+c_3^2\vartheta_2^2-c_2^2\vartheta_3^2+c_4^2\vartheta_5^2-c_6^2\vartheta_7^2-c_9^2\vartheta_8^2+c_8^2\vartheta_9^2-c_{15}^2\vartheta_{14}^2 \quad . \tag{xxiv}$$

$$3c_{15}^2\vartheta_{11}^2 = c_4^2\vartheta_0^2-c_6^2\vartheta_2^2-c_0^2\vartheta_4^2-c_1^2\vartheta_5^2+c_2^2\vartheta_6^2+c_3^2\vartheta_7^2-c_{12}^2\vartheta_8^2+c_8^2\vartheta_{12}^2+c_9^2\vartheta_{13}^2 \quad . \tag{xxv}$$

$$3c_9^2\vartheta_{13}^2 = c_4^2\vartheta_0^2-c_6^2\vartheta_2^2-c_0^2\vartheta_4^2+c_1^2\vartheta_5^2+c_2^2\vartheta_6^2-c_3^2\vartheta_7^2+c_{12}^2\vartheta_8^2+c_{15}^2\vartheta_{11}^2-c_8^2\vartheta_{12}^2 \quad . \tag{xxvi}$$

$$c_0 c_2 \vartheta_8 \vartheta_{10} = c_1 c_3 \vartheta_9 \vartheta_{11} + c_4 c_6 \vartheta_{12}\vartheta_{14} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \tag{xxvii}$$

$$\vartheta_0 \vartheta_5 \theta_0 \theta_5 = \vartheta_2\vartheta_7\theta_2\theta_7 + \vartheta_8\vartheta_{13}\theta_8\theta_{13} - \vartheta_{10}\vartheta_{15}\theta_{10}\theta_{15} \quad . \; . \; . \; . \; . \; . \; . \tag{xxviii}$$

$$\vartheta_{10}\vartheta_{11}\theta_{14}\theta_{15} = \vartheta_8\vartheta_9\theta_{12}\theta_{13} + \vartheta_2\vartheta_3\theta_6\theta_7 - \vartheta_0\vartheta_1\theta_4\theta_5 \quad . \; . \; . \; . \; . \; . \; . \tag{xxix}$$

10. By equations (i)–(x) we have

$$c_0^2\vartheta_0^2-c_{12}^2\vartheta_{12}^2 = c_1^2\vartheta_1^2+c_6^2\vartheta_6^2 = c_2^2\vartheta_2^2+c_9^2\vartheta_9^2 \quad . \; . \; . \; . \; . \; . \tag{31}$$

$$c_0^2\vartheta_0^2-c_3^2\vartheta_3^2 = c_6^2\vartheta_6^2+c_8^2\vartheta_8^2 = c_4^2\vartheta_4^2+c_9^2\vartheta_9^2 \quad . \; . \; . \; . \; . \; . \tag{32}$$

$$c_0^2\vartheta_0^2-c_{15}^2\vartheta_{15}^2 = c_2^2\vartheta_2^2+c_8^2\vartheta_8^2 = c_1^2\vartheta_1^2+c_4^2\vartheta_4^2 \quad . \; . \; . \; . \; . \; . \tag{33}$$

giving six distinct expressions for $c_0^2\vartheta_0^2$; and six can be obtained for each of the even functions in this form.

By (xi)–(xiv),

$$-c_8^2\vartheta_{13}^2+c_{15}^2\vartheta_{10}^2 = c_9^2\vartheta_{12}^2-c_{12}^2\vartheta_9^2 = -c_0^2\vartheta_5^2+c_2^2\vartheta_7^2 \quad . \; . \; . \; . \tag{34}$$

By (xv)–(xviii),

$$c_0^2\vartheta_{13}^2-c_8^2\vartheta_5^2 = c_{15}^2\vartheta_2^2-c_2^2\vartheta_{15}^2 = c_3^2\vartheta_{14}^2-c_6^2\vartheta_{11}^2 \quad . \; . \; . \; . \tag{35}$$

By (xv), (xvii), (xix), (xx),

$$c_1^2\vartheta_{12}^2+c_0^2\vartheta_{13}^2 = c_4^2\vartheta_9^2+c_{15}^2\vartheta_2^2 = c_{12}^2\vartheta_1^2-c_6^2\vartheta_{11}^2 \quad . \; . \; . \; . \tag{36}$$

By (xxi), (xxii), and two others,

$$c_2^2\vartheta_{13}^2+c_3^2\vartheta_{12}^2 = c_6^2\vartheta_9^2+c_{15}^2\vartheta_0^2 = c_{12}^2\vartheta_3^2-c_4^2\vartheta_{11}^2 \quad . \; . \; . \; . \tag{37}$$

By (xxiii), (xxvi), and two others,

$$c_{15}^2\vartheta_{14}^2-c_{12}^2\vartheta_{13}^2=-c_8^2\vartheta_9^2+c_9^2\vartheta_8^2=-c_4^2\vartheta_5^2+c_6^2\vartheta_7^2 \quad . \quad . \quad . \quad . \quad (38)$$

By (xxv), (xxvi), and two others,

$$c_{15}^2\vartheta_{11}^2-c_9^2\vartheta_{13}^2=-c_{12}^2\vartheta_8^2+c_8^2\vartheta_{12}^2=-c_1^2\vartheta_5^2+c_3^2\vartheta_7^2 \quad . \quad . \quad . \quad . \quad (39)$$

The first members of (34), (36), (37) are given by ROSENHAIN, in the paper already cited, in his formula (94), and of (38), (39) in his formula (99); but the second members are not noticed. The equivalents of (xxvii), (xxviii), (xxix) are given in his formulæ (98) and (102); and of the following equations (40), and (α)–($\iota\epsilon$), in (89) and (90).

11. By making both the variables zero in (31), (32), (33) there at once follow the equations

$$\left.\begin{aligned} c_0^4-c_{12}^4&=c_1^4+c_6^4=c_2^4+c_9^4 \\ c_0^4-c_3^4&=c_6^4+c_8^4=c_4^4+c_9^4 \\ c_0^4-c_{15}^4&=c_2^4+c_8^4=c_1^4+c_4^4 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (40)$$

and the following are obtained from (23),

$$\left\{\begin{aligned} c_0^2c_{12}^2&=c_4^2c_8^2+c_3^2c_{15}^2 && \ldots\ldots\ldots (\alpha) \\ c_3^2c_{12}^2&=c_6^2c_9^2+c_0^2c_{15}^2 && \ldots\ldots\ldots (\beta) \\ c_0^2c_3^2&=c_1^2c_2^2+c_{12}^2c_{15}^2 && \ldots\ldots\ldots (\gamma) \end{aligned}\right.$$

$$\left\{\begin{aligned} c_0^2c_4^2&=c_2^2c_6^2+c_8^2c_{12}^2 && \ldots\ldots\ldots (\delta) \\ c_1^2c_4^2&=c_3^2c_6^2+c_9^2c_{12}^2 && \ldots\ldots\ldots (\epsilon) \\ c_0^2c_1^2&=c_2^2c_3^2+c_8^2c_9^2 && \ldots\ldots\ldots (\varsigma) \end{aligned}\right.$$

$$\left\{\begin{aligned} c_0^2c_8^2&=c_1^2c_9^2+c_4^2c_{12}^2 && \ldots\ldots\ldots (\zeta) \\ c_2^2c_8^2&=c_3^2c_9^2+c_6^2c_{12}^2 && \ldots\ldots\ldots (\eta) \\ c_0^2c_2^2&=c_1^2c_3^2+c_4^2c_6^2 && \ldots\ldots\ldots (\theta) \end{aligned}\right.$$

$$\left\{\begin{aligned} c_2^2c_4^2&=c_0^2c_6^2+c_9^2c_{15}^2 && \ldots\ldots\ldots (\iota) \\ c_3^2c_4^2&=c_1^2c_6^2+c_8^2c_{15}^2 && \ldots\ldots\ldots (\iota\alpha) \\ c_2^2c_{12}^2&=c_6^2c_8^2+c_1^2c_{15}^2 && \ldots\ldots\ldots (\iota\beta) \end{aligned}\right.$$

$$\left\{\begin{aligned} c_1^2c_8^2&=c_0^2c_9^2+c_6^2c_{15}^2 && \ldots\ldots\ldots (\iota\gamma) \\ c_3^2c_8^2&=c_2^2c_9^2+c_4^2c_{15}^2 && \ldots\ldots\ldots (\iota\delta) \\ c_1^2c_{12}^2&=c_4^2c_9^2+c_2^2c_{15}^2 && \ldots\ldots\ldots (\iota\epsilon) \end{aligned}\right.$$

which agree with Rosenhain's set except $(\iota\gamma)$, in which his left-hand side is equivalent to $c_2{}^2c_8{}^2$, probably a misprint. It is worthy of remark that the sum or difference of the subscript numbers is the same for the same equation, which is also the case with many of the equations (31)–(39).

12. Eliminating $\vartheta_5{}^2$ between (38), (39) we have

$$\vartheta_7{}^2(c_1{}^2c_6{}^2-c_3{}^2c_4{}^2)=-c_1{}^2c_8{}^2\vartheta_9{}^2+(c_1{}^2c_9{}^2+c_4{}^2c_{12}{}^2)\vartheta_8{}^2-c_4{}^2c_8{}^2\vartheta_{12}{}^2$$

or by $(\iota\alpha)$ and (ζ)

$$\left.\begin{array}{lrl}
 & c_{15}{}^2\vartheta_7{}^2= & -c_0{}^2\vartheta_8{}^2+c_1{}^2\vartheta_9{}^2+c_4{}^2\vartheta_{12}{}^2 \\
\text{Similarly from the same equations} & & \\
 & c_{15}{}^2\vartheta_5{}^2= & -c_2{}^2\vartheta_8{}^2+c_3{}^2\vartheta_9{}^2+c_6{}^2\vartheta_{12}{}^2 \\
\text{By (35), (36)} & c_3{}^2\vartheta_{14}{}^2= & c_{12}{}^2\vartheta_1{}^2-c_4{}^2\vartheta_9{}^2-c_2{}^2\vartheta_{15}{}^2 \\
 & c_0{}^2\vartheta_{13}{}^2= & c_{15}{}^2\vartheta_2{}^2+c_4{}^2\vartheta_9{}^2-c_1{}^2\vartheta_{12}{}^2 \\
 & c_6{}^2\vartheta_{11}{}^2= & c_{12}{}^2\vartheta_1{}^2-c_{15}{}^2\vartheta_2{}^2-c_4{}^2\vartheta_9{}^2 \\
\text{and therefore by (34)} & c_0{}^2\vartheta_{10}{}^2= & c_8{}^2\vartheta_2{}^2-c_3{}^2\vartheta_9{}^2-c_6{}^2\vartheta_{12}{}^2
\end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (41)$$

which give the squares of the uneven functions in terms of squares of the even functions.

13. Following Rosenhain (91), put

$$\kappa_1{}^2=\frac{c_1{}^2c_3{}^2}{c_0{}^2c_2{}^2} \qquad \kappa'_1{}^2=\frac{c_4{}^2c_6{}^2}{c_0{}^2c_2{}^2}; \quad \text{hence by } (\theta) \qquad \kappa_1{}^2+\kappa'_1{}^2=1 \quad . \quad . \quad . \quad (42)$$

$$\kappa_2{}^2=\frac{c_3{}^2c_9{}^2}{c_2{}^2c_8{}^2} \qquad \kappa'_2{}^2=\frac{c_6{}^2c_{12}{}^2}{c_2{}^2c_8{}^2}; \quad \text{,,} \quad (\eta) \qquad \kappa_2{}^2+\kappa'_2{}^2=1 \quad . \quad . \quad . \quad (43)$$

$$\kappa_3{}^2=\frac{c_1{}^2c_9{}^2}{c_0{}^2c_8{}^2} \qquad \kappa'_3{}^2=\frac{c_4{}^2c_{12}{}^2}{c_0{}^2c_8{}^2}; \quad \text{,,} \quad (\zeta) \qquad \kappa_3{}^2+\kappa'_3{}^2=1 \quad . \quad . \quad . \quad (44).$$

Then

$$\kappa_1{}^2-\kappa_2{}^2=\frac{c_3{}^2}{c_2{}^2}\,\frac{c_1{}^2c_8{}^2-c_0{}^2c_9{}^2}{c_0{}^2c_8{}^2}=\frac{c_3{}^2c_6{}^2c_{15}{}^2}{c_0{}^2c_2{}^2c_8{}^2}=\mathrm{K}_3{}^2 \text{ say}; \quad . \quad . \quad . \quad . \quad . \quad (45)$$

$$\kappa_1{}^2-\kappa_3{}^2=\frac{c_1{}^2c_4{}^2c_{15}{}^2}{c_0{}^2c_2{}^2c_8{}^2}=\mathrm{K}_2{}^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (46)$$

$$\kappa_2{}^2-\kappa_3{}^2=\frac{c_9{}^2c_{12}{}^2c_{15}{}^2}{c_0{}^2c_2{}^2c_8{}^2}=\mathrm{K}_1{}^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (47)$$

and from these the following expressions for the ratios of the c's are easily obtained:

$$\left.\begin{array}{lll}
\dfrac{c_1^4}{c_0^4}=\dfrac{\kappa_1^2\kappa_3^2}{\kappa_2^2} & \dfrac{c_4^4}{c_0^4}=\dfrac{\kappa'^2_1\kappa'^2_3}{\kappa'^2_2} & \dfrac{c_9^4}{c_0^4}=\dfrac{\kappa'^2_1\kappa_3^2\mathrm{K}_1^2}{\kappa'^2_2\mathrm{K}_2^2} \\
\dfrac{c_2^4}{c_0^4}=\dfrac{\kappa_3^2\kappa'^2_3\mathrm{K}_3^2}{\kappa_2^2\kappa'^2_2\mathrm{K}_2^2} & \dfrac{c_6^4}{c_0^4}=\dfrac{\kappa'^2_1\kappa_3^2\mathrm{K}_3^2}{\kappa_2^2\mathrm{K}_2^2} & \dfrac{c_{12}^4}{c_0^4}=\dfrac{\kappa_1^2\kappa'^2_3\mathrm{K}_1^2}{\kappa_2^2\mathrm{K}_2^2} \\
\dfrac{c_3^4}{c_0^4}=\dfrac{\kappa_1^2\kappa'^2_3\mathrm{K}_3^2}{\kappa'^2_2\mathrm{K}_2^2} & \dfrac{c_8^4}{c_0^4}=\dfrac{\kappa_1^2\kappa'^2_1\mathrm{K}_1^2}{\kappa_2^2\kappa'^2_2\mathrm{K}_2^2} & \dfrac{c_{15}^4}{c_0^4}=\dfrac{\mathrm{K}_3^2\mathrm{K}_1^2}{\kappa_2^2\kappa'^2_2}
\end{array}\right\} \quad . \quad . \quad . \quad . \quad (48).$$

If now these be substituted in the equations of which (41) are a type, a set of algebraical identities is obtained; in fact, putting $\kappa_1^2=a$, $\kappa_2^2=b$, $\kappa_3^2=c$, a, b, c being perfectly independent

$$b(a-c)=a(1-c)(b-c)+ac(a-c)+c(1-a)(a-b)$$

$$(b-c)(a-b)=ac(1-b)+(1-a)(1-c)b-b(1-b)$$

and from these

$$a(1-c)(b-c)(1-b)+c(1-a)(1-b)(a-b)+b(1-c)(1-a)(c-a)$$
$$=(a-b)(b-c)(c-a)$$

and many others similar to these, all of which admit of immediate verification.

14. Two other equations, which will afterwards be useful, are

$$\begin{aligned}
&c_8c_9\{\vartheta_{12}(x+\xi)\vartheta_{13}(x-\xi)-\vartheta_{13}(x+\xi)\vartheta_{12}(x-\xi)\}\\
&\quad=\vartheta_{10}\vartheta_{11}\theta_{14}\theta_{15}-\vartheta_8\vartheta_9\theta_{12}\theta_{13}+\vartheta_2\vartheta_3\theta_6\theta_7-\vartheta_0\vartheta_1\theta_4\theta_5\\
&\quad=2(\vartheta_2\vartheta_3\theta_6\theta_7-\vartheta_0\vartheta_1\theta_4\theta_5) \text{ by (xxix)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (49)
\end{aligned}$$

$$\begin{aligned}
&c_9c_{12}\{\vartheta_9(x+\xi)\vartheta_{12}(x-\xi)-\vartheta_{12}(x+\xi)\vartheta_9(x-\xi)\}\\
&\quad=\vartheta_0\vartheta_5\theta_0\theta_5-\vartheta_2\vartheta_7\theta_2\theta_7+\vartheta_8\vartheta_{13}\theta_8\theta_{13}-\vartheta_{10}\vartheta_{15}\theta_{10}\theta_{15}\\
&\quad=2(\vartheta_0\vartheta_5\theta_0\theta_5-\vartheta_2\vartheta_7\theta_2\theta_7) \text{ by (xxviii)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (50).
\end{aligned}$$

Connexion with the hyperelliptic integrals.

15. Taking the fifteen ratios obtained by dividing all the functions but one by that one, it follows from the relations already established as (31), (32), (33), . . . that any thirteen of them can be expressed in terms of the remaining two, or that all these

ratios can be expressed in terms of two new variables. Re-arranging now the first parts of (34), (36), (37), and substituting for the c's in terms of the κ's, we have

$$\left.\begin{aligned}
-\frac{\kappa_1}{\kappa_2\kappa_3}\frac{\vartheta_{13}^2}{\vartheta_{12}^2}+\frac{\kappa_1\kappa'_2\kappa'_3}{\kappa'_1\kappa_2\kappa_3}\frac{\vartheta_9^2}{\vartheta_{12}^2}+\frac{\mathrm{K}_2\mathrm{K}_3}{\kappa'_1\kappa_2\kappa_3}\frac{\vartheta_{10}^2}{\vartheta_{12}^2}=1\\
-\frac{\kappa_2}{\kappa_3\kappa_1}\frac{\vartheta_{13}^2}{\vartheta_{12}^2}+\frac{\kappa'_1\kappa_2\kappa'_3}{\kappa_1\kappa'_2\kappa_3}\frac{\vartheta_9^2}{\vartheta_{12}^2}+\frac{\mathrm{K}_3\mathrm{K}_1}{\kappa_1\kappa'_2\kappa_3}\frac{\vartheta_2^2}{\vartheta_{12}^2}=1\\
-\frac{\kappa_3}{\kappa_1\kappa_2}\frac{\vartheta_{13}^2}{\vartheta_{12}^2}+\frac{\kappa'_1\kappa'_2\kappa_3}{\kappa_1\kappa_2\kappa'_3}\frac{\vartheta_9^2}{\vartheta_{12}^2}+\frac{\mathrm{K}_1\mathrm{K}_2}{\kappa_1\kappa_2\kappa'_3}\frac{\vartheta_0^2}{\vartheta_{12}^2}=1
\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (51)$$

agreeing with ROSENHAIN (95). Assuming x_1, x_2 as the new variables, put

$$\frac{\vartheta_{13}^2}{\vartheta_{12}^2}=\mathrm{A}x_1x_2$$

$$\frac{\vartheta_9^2}{\vartheta_{12}^2}=\mathrm{B}(1-x_1)(1-x_2)$$

then if

$$\frac{\vartheta_{10}^2}{\vartheta_{12}^2}=\mathrm{C}(1-\kappa_1^2x_1)(1-\kappa_1^2x_2)$$

$$\frac{\vartheta_2^2}{\vartheta_{12}^2}=\mathrm{D}(1-\kappa_2^2x_1)(1-\kappa_2^2x_2)$$

$$\frac{\vartheta_0^2}{\vartheta_{12}^2}=\mathrm{E}(1-\kappa_3^2x_1)(1-\kappa_3^2x_2).$$

the equations (51) are satisfied if

$$\mathrm{A}=-\kappa_1\kappa_2\kappa_3 \qquad \mathrm{B}=-\frac{\kappa_1\kappa_2\kappa_3}{\kappa'_1\kappa'_2\kappa'_3} \qquad \mathrm{C}=\frac{\kappa_2\kappa_3}{\kappa'_1\mathrm{K}_3\mathrm{K}_2}$$

$$\mathrm{D}=\frac{\kappa_3\kappa_1}{\kappa'_2\mathrm{K}_1\mathrm{K}_3} \qquad \mathrm{E}=\frac{\kappa_1\kappa_2}{\kappa'_3\mathrm{K}_2\mathrm{K}_1}$$

16. The other ratios involve x_1, x_2 irrationally; thus to find $\left(\frac{\vartheta_8}{\vartheta_{12}}\right)^2$, ROSENHAIN uses equations corresponding to (xxvii), (38), (39), and eliminates ϑ_{11}, ϑ_{14} between them, giving a quadratic in $\left(\frac{\vartheta_8}{\vartheta_{12}}\right)^2$. Having obtained this, the expressions for the other functions follow by substituting for the ratios already found in the equations (31)–(39); and the complete system of expressions for the fifteen ratios is as follows, the + sign being usually taken throughout (see CAYLEY, 'Crelle,' t. 88, p. 81):

$$\left.\begin{aligned}
&\frac{\vartheta_{13}^2}{\vartheta_{12}^2}=-\kappa_1\kappa_2\kappa_3 x_1x_2\\
&\frac{\vartheta_{9}^2}{\vartheta_{12}^2}=-\frac{\kappa_1\kappa_2\kappa_3}{\kappa'_1\kappa'_2\kappa'_3}(1-x_1)(1-x_2)\\
&\frac{\vartheta_{10}^2}{\vartheta_{12}^2}=\quad\frac{\kappa_2\kappa_3}{\kappa'_1\mathrm{K}_2\mathrm{K}_3}(1-\kappa_1^2x_1)(1-\kappa_1^2x_2)\\
&\frac{\vartheta_{2}^2}{\vartheta_{12}^2}=\quad\frac{\kappa_3\kappa_1}{\kappa'_2\mathrm{K}_1\mathrm{K}_3}(1-\kappa_2^2x_1)(1-\kappa_2^2x_2)\\
&\frac{\vartheta_{0}^2}{\vartheta_{12}^2}=\quad\frac{\kappa_1\kappa_2}{\kappa'_3\mathrm{K}_1\mathrm{K}_2}(1-\kappa_3^2x_1)(1-\kappa_3^2x_2)\\
&\frac{\vartheta_{1}^2}{\vartheta_{12}^2}=\quad\frac{\kappa_3}{\kappa'_3\mathrm{K}_1\mathrm{K}_2}\\
&\qquad\frac{\{\sqrt{x_2(1-x_1)(1-\kappa_1^2x_1)(1-\kappa_2^2x_1)(1-\kappa_3^2x_2)}\pm\sqrt{x_1(1-x_2)(1-\kappa_1^2x_2)(1-\kappa_2^2x_2)(1-\kappa_3^2x_1)}\}^2}{(x_1-x_2)^2}\\
&\frac{\vartheta_{3}^2}{\vartheta_{12}^2}=\quad\frac{\kappa_2}{\kappa'_2\mathrm{K}_1\mathrm{K}_3}\\
&\qquad\frac{\{\sqrt{x_2(1-x_1)(1-\kappa_1^2x_1)(1-\kappa_2^2x_2)(1-\kappa_3^2x_1)}\pm\sqrt{x_1(1-x_2)(1-\kappa_1^2x_2)(1-\kappa_2^2x_1)(1-\kappa_3^2x_2)}\}^2}{(x_1-x_2)^2}\\
&\frac{\vartheta_{4}^2}{\vartheta_{12}^2}=-\frac{\kappa_1\kappa_2}{\kappa'_1\kappa'_2\mathrm{K}_1\mathrm{K}_2}\\
&\qquad\frac{\{\sqrt{x_1(1-x_1)(1-\kappa_1^2x_2)(1-\kappa_2^2x_2)(1-\kappa_3^2x_1)}\pm\sqrt{x_2(1-x_2)(1-\kappa_1^2x_1)(1-\kappa_2^2x_1)(1-\kappa_3^2x_2)}\}^2}{(x_1-x_2)^2}\\
&\frac{\vartheta_{5}^2}{\vartheta_{12}^2}=\quad\frac{\kappa_3}{\kappa'_1\kappa'_2\mathrm{K}_1\mathrm{K}_2}\\
&\qquad\frac{\{\sqrt{x_1(1-x_2)(1-\kappa_1^2x_1)(1-\kappa_2^2x_1)(1-\kappa_3^2x_2)}\pm\sqrt{x_2(1-x_1)(1-\kappa_1^2x_2)(1-\kappa_2^2x_2)(1-\kappa_3^2x_1)}\}^2}{(x_1-x_2)^2}\\
&\frac{\vartheta_{6}^2}{\vartheta_{12}^2}=-\frac{\kappa_1\kappa_3}{\kappa'_1\kappa'_3\mathrm{K}_1\mathrm{K}_3}\\
&\qquad\frac{\{\sqrt{x_1(1-x_1)(1-\kappa_1^2x_2)(1-\kappa_2^2x_1)(1-\kappa_3^2x_2)}\pm\sqrt{x_2(1-x_2)(1-\kappa_1^2x_1)(1-\kappa_2^2x_2)(1-\kappa_3^2x_1)}\}^2}{(x_1-x_2)^2}\\
&\frac{\vartheta_{7}^2}{\vartheta_{12}^2}=\quad\frac{\kappa_2}{\kappa'_1\kappa'_3\mathrm{K}_1\mathrm{K}_3}\\
&\qquad\frac{\{\sqrt{x_1(1-x_2)(1-\kappa_1^2x_1)(1-\kappa_2^2x_2)(1-\kappa_3^2x_1)}\pm\sqrt{x_2(1-x_1)(1-\kappa_1^2x_2)(1-\kappa_2^2x_1)(1-\kappa_3^2x_2)}\}^2}{(x_1-x_2)^2}\\
&\frac{\vartheta_{8}^2}{\vartheta_{12}^2}=-\frac{1}{\kappa'_1\kappa'_2\kappa'_3}\\
&\qquad\frac{\{\sqrt{x_2(1-x_2)(1-\kappa_1^2x_1)(1-\kappa_2^2x_1)(1-\kappa_3^2x_1)}\pm\sqrt{x_1(1-x_1)(1-\kappa_1^2x_2)(1-\kappa_2^2x_2)(1-\kappa_3^2x_2)}\}^2}{(x_1-x_2)^2}
\end{aligned}\right\}\quad(52)$$

$$\left.\begin{aligned}
\frac{\vartheta_{11}^2}{\vartheta_{12}^2} &= \frac{\kappa_2}{\kappa'_2\mathrm{K}_3\mathrm{K}_1} \frac{\{\sqrt{x_2(1-x_1)(1-\kappa_1^2x_2)(1-\kappa_2^2x_1)(1-\kappa_3^2x_1)}\pm\sqrt{x_1(1-x_2)(1-\kappa_1^2x_1)(1-\kappa_2^2x_2)(1-\kappa_3^2x_2)}\}^2}{(x_1-x_2)^2} \\
\frac{\vartheta_{14}^2}{\vartheta_{12}^2} &= -\frac{\kappa_2\kappa_3}{\kappa'_2\kappa'_3\mathrm{K}_2\mathrm{K}_3} \frac{\{\sqrt{x_1(1-x_1)(1-\kappa_1^2x_1)(1-\kappa_2^2x_2)(1-\kappa_3^2x_2)}\pm\sqrt{x_2(1-x_2)(1-\kappa_1^2x_2)(1-\kappa_2^2x_1)(1-\kappa_3^2x_1)}\}^2}{(x_1-x_2)^2} \\
\frac{\vartheta_{15}^2}{\vartheta_{12}^2} &= \frac{\kappa_1}{\kappa'_2\kappa'_3\mathrm{K}_2\mathrm{K}_3} \frac{\{\sqrt{x_1(1-x_2)(1-\kappa_1^2x_2)(1-\kappa_2^2x_1)(1-\kappa_3^2x_1)}\pm\sqrt{x_2(1-x_1)(1-\kappa_1^2x_1)(1-\kappa_2^2x_2)(1-\kappa_3^2x_2)}\}^2}{(x_1-x_2)^2}
\end{aligned}\right\}$$

which correspond with ROSENHAIN's formula (97).

17. It is now necessary to find relations between x_1, x_2 and x, y. Let

$$c_5=\frac{d\vartheta_5}{dx_0} \qquad c_7=\frac{d\vartheta_7}{dx_0}$$

$$c'_5=\frac{d\vartheta_5}{dy_0} \qquad c'_7=\frac{d\vartheta_7}{dy_0}$$

where $\frac{d}{dx_0}$, $\frac{d}{dy_0}$ imply that, after the differential of the function has been taken, both the variables are to be put zero. Differentiating the equations (49), (50) with regard to ξ and then putting ξ, η zero, and noticing that

$$\frac{df(x+\xi)}{d\xi}=\frac{df(x+\xi)}{dx}$$

$$\frac{df(x-\xi)}{d\xi}=-\frac{df(x-\xi)}{dx},$$

we have from (49)

$$c_8c_9\left(\vartheta_{13}\frac{d\vartheta_{12}}{dx}-\vartheta_{12}\frac{d\vartheta_{13}}{dx}\right)=c_6c_7\vartheta_2\vartheta_3-c_4c_5\vartheta_0\vartheta_1$$

or

$$c_8c_9\frac{d}{dx}\left(\frac{\vartheta_{13}}{\vartheta_{12}}\right)=c_4c_5\frac{\vartheta_0}{\vartheta_{12}}\frac{\vartheta_1}{\vartheta_{12}}-c_6c_7\frac{\vartheta_2}{\vartheta_{12}}\frac{\vartheta_3}{\vartheta_{12}} \quad \ldots\ldots\ldots \quad (53)$$

and similarly from (50)

$$c_9c_{12}\frac{d}{dx}\left(\frac{\vartheta_9}{\vartheta_{21}}\right)=c_0c_5\frac{\vartheta_0}{\vartheta_{12}}\frac{\vartheta_5}{\vartheta_{12}}-c_2c_7\frac{\vartheta_2}{\vartheta_{12}}\frac{\vartheta_7}{\vartheta_{12}} \quad \ldots\ldots\ldots \quad (54).$$

Differentiating the same equations with regard to η and proceeding in the same manner we obtain

$$c_8 c_9 \frac{d}{dy}\left(\frac{\vartheta_{13}}{\vartheta_{12}}\right)=c_4 c'_5 \frac{\vartheta_0}{\vartheta_{12}} \frac{\vartheta_1}{\vartheta_{12}}-c_6 c'_7 \frac{\vartheta_2}{\vartheta_{12}} \frac{\vartheta_3}{\vartheta_{12}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (55)$$

$$c_9 c_{12} \frac{d}{dy}\left(\frac{\vartheta_9}{\vartheta_{12}}\right)=c_0 c'_5 \frac{\vartheta_0}{\vartheta_{12}} \frac{\vartheta_5}{\vartheta_{12}}-c_2 c'_7 \frac{\vartheta_2}{\vartheta_{12}} \frac{\vartheta_7}{\vartheta_{12}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (56)$$

which correspond to ROSENHAIN'S (104), (105). Let

$$\left.\begin{aligned} X_1&=x_1(1-x_1)(1-\kappa_1{}^2x_1)(1-\kappa_2{}^2x_1)(1-\kappa_3{}^2x_1) \\ X_2&=x_2(1-x_2)(1-\kappa_1{}^2x_2)(1-\kappa_2{}^2x_2)(1-\kappa_3{}^2x_2) \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (57)$$

Substituting in (53), (54) from (52),

$$x_1\frac{dx_2}{dx}+x_2\frac{dx_1}{dx}=\frac{\alpha\{\sqrt{X_1}x_2(1-\kappa_3{}^2x_2)-\sqrt{X_2}x_1(1-\kappa_3{}^2x_1)\}-\beta\{\sqrt{X_1}x_2(1-\kappa_2{}^2x_2)-\sqrt{X_2}x_1(1-\kappa_2{}^2x_1)\}}{x_2-x_1}$$

$$(1-x_1)\frac{dx_2}{dx}+(1-x_2)\frac{dx_1}{dx}$$
$$=\frac{\alpha\{\sqrt{X_1}(1-x_2)(1-\kappa_3{}^2x_2)-\sqrt{X_2}(1-x_1)(1-\kappa_3{}^2x_1)\}-\beta\{\sqrt{X_1}(1-x_2)(1-\kappa_2{}^2x_2)-\sqrt{X_2}(1-x_1)(1-\kappa_2{}^2x_1)\}}{x_2-x_1}$$

where α, β are functions of κ_1, κ_2, κ_3 and of c_5, c_7 which will be afterwards seen to be themselves functions of κ_1, κ_2, κ_3. From these

$$\frac{dx_2}{dx}=\frac{\beta(1-\kappa_2{}^2x_1)-\alpha(1-\kappa_3{}^2x_1)}{x_2-x_1}\sqrt{X_2}=\frac{\gamma+\delta x_1}{x_2-x_1}\sqrt{X_2}, \text{ say;}$$

$$\frac{dx_1}{dx}=\frac{\alpha(1-\kappa_3{}^2x_2)-\beta(1-\kappa_2{}^2x_2)}{x_2-x_1}\sqrt{X_1}=-\frac{\gamma+\delta x_2}{x_2-x_1}\sqrt{X_1}.$$

Similarly

$$\frac{dx_2}{dy}=-\frac{\gamma'+\delta' x_1}{x_2-x_1}\sqrt{X_2}$$

$$\frac{dx_1}{dy}=\frac{\gamma'+\delta' x_2}{x_2-x_1}\sqrt{X_1}.$$

If, then, we write

$$\left.\begin{aligned} dx&=\frac{A+Bx_1}{\sqrt{X_1}}\,dx_1+\frac{A+Bx_2}{\sqrt{X_2}}\,dx_2 \\ dy&=\frac{A'+B'x_1}{\sqrt{X_1}}dx_1+\frac{A'+B'x_2}{\sqrt{X_2}}dx_2 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (58)$$

the foregoing equations are satisfied, provided

$$\begin{aligned}
B\gamma\ -A\delta\ &=1\\
A'\delta'-B'\gamma'&=1\\
B'\gamma\ -A'\delta\ &=0\\
B\gamma'\ -A\delta'\ &=0
\end{aligned}$$

hence

$$\frac{B}{\delta'}=\frac{A}{\gamma'}=\frac{1}{\gamma\delta'-\gamma'\delta}$$

$$\frac{B'}{\delta}=\frac{A'}{\gamma}=\frac{1}{\gamma\delta'-\gamma'\delta}$$

and therefore A, B, A′, B′ are determinate functions of κ_1, κ_2, κ_3. The equations (58) are the well-known equations for the hyperelliptic integrals of the first kind.

On the expressions of the quarter-periods as definite integrals.

18. Integrating (58)

$$x=\int_0^{x_1}\frac{A+Bx}{\sqrt{X}}dx+\int_a^{x_2}\frac{A+Bx}{\sqrt{X}}dx$$

$$y=\int_0^{x_1}\frac{A'+B'x}{\sqrt{X}}dx+\int_b^{x_2}\frac{A'+B'x}{\sqrt{X}}dx$$

a, b being constants: we proceed to find some integrals giving the values of the periods.

19. (i.) Let $x=0$, $y=0$, so that all the uneven functions vanish; then, by (52), $x_1=0$, $x_2=\frac{1}{\kappa_1^2}$; and hence

$$0=\int_a^{\frac{1}{\kappa_1^2}}\frac{A+Bx}{\sqrt{X}}dx$$

$$0=\int_b^{\frac{1}{\kappa^2}}\frac{A'+B'x}{\sqrt{X}}dx.$$

(ii.) Let $x=K$, $y=0$; referring to formula (8) the functions which vanish are seen to be ϑ_1, ϑ_3, ϑ_9, ϑ_{10}, ϑ_{14}, ϑ_{15}; and to ensure this $x_1=1$, $x_2=\frac{1}{\kappa_1^2}$, so that

$$K=\int_0^1\frac{A+Bx}{\sqrt{X}}dx\ +\int_a^{\frac{1}{\kappa_1^2}}\frac{A+Bx}{\sqrt{X}}dx$$

$$0=\int_0^1\frac{A'+B'x}{\sqrt{X}}dx+\int_b^{\frac{1}{\kappa_1^2}}\frac{A'+B'x}{\sqrt{X}}dx.$$

(iii.) Let $x=0$, $y=\Lambda$: the vanishing functions are ϑ_2, ϑ_3, ϑ_5, ϑ_6, ϑ_{13}, ϑ_{15}; hence $x_1=\frac{1}{\kappa_2^2}$, $x_2=0$, and

$$0=\int_0^{\frac{1}{\kappa_2^2}}\frac{A+Bx}{\sqrt{X}}dx+\int_a^0\frac{A+Bx}{\sqrt{X}}dx$$

$$\Lambda=\int_0^{\frac{1}{\kappa_2^2}}\frac{A'+B'x}{\sqrt{X}}dx+\int_b^0\frac{A'+B'x}{\sqrt{X}}dx.$$

(iv.) Let $x=\frac{K}{\pi i}\log pr$, $y=\frac{\Lambda}{\pi i}\log rq$: the vanishing functions are ϑ_4, ϑ_6, ϑ_8, ϑ_9, ϑ_{13}, ϑ_{14}; hence $x_1=0$, $x_2=1$, and

$$\frac{K}{\pi i}\log pr=\int_a^1\frac{A+Bx}{\sqrt{X}}dx$$

$$\frac{\Lambda}{\pi i}\log rq=\int_b^1\frac{A'+B'x}{\sqrt{X}}dx.$$

(v.) Let $x=K+\frac{K}{\pi i}\log r$, $y=\Lambda+\frac{\Lambda}{\pi i}\log q$: the vanishing functions are ϑ_0, ϑ_3, ϑ_4, ϑ_5, ϑ_9, ϑ_{11}; hence $x_1=\frac{1}{\kappa_3^2}$, $x_2=1$, and

$$K+\frac{K}{\pi i}\log r=\int_0^{\frac{1}{\kappa_3^2}}\frac{A+Bx}{\sqrt{X}}dx+\int_a^1\frac{A+Bx}{\sqrt{X}}dx$$

$$\Lambda+\frac{\Lambda}{\pi i}\log q=\int_0^{\frac{1}{\kappa_3^2}}\frac{A'+B'x}{\sqrt{X}}dx+\int_b^1\frac{A'+B'x}{\sqrt{X}}dx.$$

By the elimination of a, b these ten equations reduce to the following eight:—

$$\left.\begin{aligned}0&=\int_0^1\frac{A'+B'x}{\sqrt{X}}dx\\ K&=\int_0^1\frac{A+Bx}{\sqrt{X}}dx\end{aligned}\right\}\quad\ldots\ldots\ldots\quad(59)$$

$$\left.\begin{aligned}\frac{Ki}{\pi}\log pr&=\int_1^{\frac{1}{\kappa_1^2}}\frac{A+Bx}{\sqrt{X}}dx\\ \frac{\Lambda i}{\pi}\log rq&=\int_1^{\frac{1}{\kappa_1^2}}\frac{A'+B'x}{\sqrt{X}}dx\end{aligned}\right\}\quad\ldots\ldots\ldots\quad(60)$$

$$\left.\begin{aligned}0&=\int_{\frac{1}{\kappa_1^2}}^{\frac{1}{\kappa_2^2}}\frac{A+Bx}{\sqrt{X}}dx\\ \Lambda&=\int_{\frac{1}{\kappa_1^2}}^{\frac{1}{\kappa_2^2}}\frac{A'+B'x}{\sqrt{X}}dx\end{aligned}\right\}\quad\ldots\ldots\ldots\quad(61)$$

$$\left.\begin{aligned}\frac{\mathrm{K}}{\pi i}\log r&=\int_{\frac{1}{\kappa_2^2}}^{\frac{1}{\kappa_3^2}}\frac{\mathrm{A}+\mathrm{B}x}{\sqrt{\mathrm{X}}}dx\\ \frac{\Lambda}{\pi i}\log q&=\int_{\frac{1}{\kappa_2^2}}^{\frac{1}{\kappa_3^2}}\frac{\mathrm{A}'+\mathrm{B}'x}{\sqrt{\mathrm{X}}}dx\end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (62).$$

20. Let

$$\left.\begin{aligned}\int_0^1\frac{dx}{\sqrt{\mathrm{X}}}&=\mathrm{K}_{01} & \int_0^1\frac{xdx}{\sqrt{\mathrm{X}}}&=\mathrm{K}_{11}\\ \int_0^1\frac{x^2dx}{\sqrt{\mathrm{X}}}&=\mathrm{K}_{21} & \int_0^1\frac{x^3dx}{\sqrt{\mathrm{X}}}&=\mathrm{K}_{31}\end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (63)$$

Then

$$\frac{1}{\kappa_1}\frac{d\mathrm{K}_{01}}{d\kappa_1}-\kappa_1\frac{d\mathrm{K}_{11}}{d\kappa_1}=\mathrm{K}_{11}$$

$$\frac{1}{\kappa_1}\frac{d\mathrm{K}_{11}}{d\kappa_1}-\kappa_1\frac{d\mathrm{K}_{21}}{d\kappa_1}=\mathrm{K}_{21}$$

$$\frac{1}{\kappa_1}\frac{d\mathrm{K}_{21}}{d\kappa_1}-\kappa_1\frac{d\mathrm{K}_{31}}{d\kappa_1}=\mathrm{K}_{31}$$

And

$$\kappa_1\frac{d\mathrm{K}_{01}}{d\kappa_1}+\mathrm{K}_{01}=\int_0^1\frac{dx}{\sqrt{\mathrm{X}}(1-\kappa_1^2x)}$$

and

$$\frac{d}{dx}\cdot\frac{\sqrt{\mathrm{X}}}{1-\kappa_1^2x}=\frac{\mathrm{P}_1}{\sqrt{\mathrm{X}}.(1-\kappa_1^2x)}-\frac{\mathrm{R}_1+\mathrm{S}_1x+\mathrm{T}_1x^2+\mathrm{U}_1x^2}{\sqrt{\mathrm{X}}}$$

so that

$$\kappa_1\frac{d\mathrm{K}_{01}}{d\kappa_1}+\mathrm{K}_{01}=\mathrm{PK}_{31}+\mathrm{QK}_{21}+\mathrm{RK}_{11}+\mathrm{SK}_{01}$$

P, Q, R, S being determinate functions of κ_1, κ_2, κ_3. By eliminating K_{11}, K_{21}, K_{31} from these equations it is obvious that K_{01} satisfies a differential equation of the fourth order in κ_1 as the independent variable. K_{11} will satisfy a similar equation; and hence also K, equal to

$$\mathrm{AK}_{01}+\mathrm{BK}_{11},$$

(A, B being functions of κ_1, κ_2, κ_3,) will satisfy a linear differential equation in κ_1 of the fourth order.

SECTION II.

21. If $\theta_{\mu,\lambda}$ be the general single theta-function, then

$$\theta_{\mu,\lambda}(x)=\sum_{m=-\infty}^{m=\infty}(-1)^{m\lambda}p^{\frac{(2m+\mu)^2}{4}}e^{(2m+\mu)\frac{i\pi x}{2K}}$$

[This notation, since it is already in use, is adopted in preference to $\theta\left\{\binom{\lambda}{\mu}x\right\}$ which would better agree with the definition of the double theta-function in (1) and with that of the "r" tuple function to be given later.]

As is well known, there are three even functions and uneven ; these are

$$\text{(even)}\qquad \theta_{0,0}(x)=1+2p\cos 2x\frac{\pi}{2K}+2p^4\cos 4x\frac{\pi}{2K}+2p^9\cos 6x\frac{\pi}{2K}+\ldots$$

$$\text{(even)}\qquad \theta_{0,1}(x)=1-2p\cos 2x\frac{\pi}{2K}+2p^4\cos 4x\frac{\pi}{2K}-2p^9\cos 6x\frac{\pi}{2K}+\ldots$$

$$\text{(even)}\qquad \theta_{1,0}(x)=2p^{\frac{1}{4}}\cos x\frac{\pi}{2K}+2p^{\frac{9}{4}}\cos 3x\frac{\pi}{2K}+2p^{\frac{25}{4}}\cos 5x\frac{\pi}{2K}+\ldots$$

$$\text{(uneven)}\ \frac{1}{i}\theta_{1,1}(x)=2p^{\frac{1}{4}}\sin x\frac{\pi}{2K}-2p^{\frac{9}{4}}\sin 3x\frac{\pi}{2K}+2p^{\frac{25}{4}}\sin 5x\frac{\pi}{2K}+\ldots$$

and

$$sn\,x=\frac{1}{i\sqrt{\kappa}}\frac{\theta_{1,1}(x)}{\theta_{0,1}(x)}$$

$$cn\,x=\sqrt{\frac{\kappa'}{\kappa}}\frac{\theta_{1,0}(x)}{\theta_{0,1}(x)}$$

$$dn\,x=\sqrt{\kappa'}\frac{\theta_{0,0}(x)}{\theta_{0,1}(x)}$$

22. Writing in the definition of Φ given in (1)

$$v=e^{\frac{i\pi}{2K}}\qquad\qquad w=e^{\frac{i\pi}{2\Lambda}}$$

we have the following series of expressions for the ϑ's.

$$\vartheta_0=1+2\sum_{m=1}^{m=\infty}p^{m^2}\cos\frac{m\pi x}{K}+2\sum_{m=1}^{m=\infty}q^{n^2}\cos\frac{n\pi y}{\Lambda}$$
$$+2\sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty}p^{m^2}q^{n^2}\left\{r^{2mn}\cos\pi\left(\frac{mx}{K}+\frac{ny}{\Lambda}\right)+r^{-2mn}\cos\pi\left(\frac{mx}{K}-\frac{ny}{\Lambda}\right)\right\}\quad . \quad . \quad (64)$$

and therefore

$$2\sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty}p^{m^2}q^{n^2}\left\{r^{2mn}\cos\pi\left(\frac{mx}{\mathrm{K}}+\frac{ny}{\Lambda}\right)+r^{-2mn}\cos\pi\left(\frac{mx}{\mathrm{K}}-\frac{ny}{\Lambda}\right)\right\}=\vartheta_0-\theta_{0,0}(x)-\theta_{0,0}(y)+1\quad.\quad(65)$$

the parameters of $\theta_{0,0}(x)$ being p and K, and of $\theta_{0,0}(y)$ q and Λ, the same applying to the other single theta-functions of x and of y which occur below.

$$\vartheta_1=2\sum_{m=0}^{m=\infty}p^{(m+\frac{1}{2})^2}\cos\,(m+\tfrac{1}{2})\frac{\pi x}{\mathrm{K}}+2\sum_{m=0}^{m=\infty}\sum_{n=1}^{n=\infty}p^{(m+\frac{1}{2})^2}q^{n^2}\left[r^{(2m+1)n}\cos\pi\left\{(m+\tfrac{1}{2})\frac{x}{\mathrm{K}}+\frac{ny}{\Lambda}\right\}\right.$$
$$\left.+r^{-(2m+1)n}\cos\pi\left\{(m+\tfrac{1}{2})\frac{x}{\mathrm{K}}-\frac{ny}{\Lambda}\right\}\right]\quad(66);$$

and hence

$$2\sum_{m=0}^{m=\infty}\sum_{n=1}^{n=\infty}p^{(m+\frac{1}{2})^2}q^{n^2}\left[r^{(2m+1)n}\cos\pi\left\{(m+\tfrac{1}{2})\frac{x}{\mathrm{K}}+\frac{ny}{\Lambda}\right\}+r^{-(2m+1)n}\cos\pi\left\{(m+\tfrac{1}{2})\frac{x}{\mathrm{K}}-\frac{ny}{\Lambda}\right\}\right]$$
$$=\vartheta_1-\theta_{1,0}(x)\quad(67).$$

$$\vartheta_2=2\sum_{n=0}^{n=\infty}q^{(n+\frac{1}{2})^2}\cos)\;(n+\tfrac{1}{2})\frac{\pi y}{\Lambda}+2\sum_{n=0}^{n=\infty}\sum_{m=1}^{m=\infty}p^{m^2}q^{(n+\frac{1}{2})^2}\left[r^{(2n+1)m}\cos\pi\left\{\frac{mx}{\mathrm{K}}+(n+\tfrac{1}{2})\frac{y}{\Lambda}\right\}\right.$$
$$\left.+r^{-(2n+1)m}\cos\pi\left\{\frac{mx}{\mathrm{K}}-(n+\tfrac{1}{2})\frac{y}{\Lambda}\right\}\right].\quad(68);$$

and hence

$$2\sum_{n=0}^{n=\infty}\sum_{m=1}^{m=\infty}p^{m^2}q^{(n+\frac{1}{2})^2}\left[r^{(2n+1)m}\cos\pi\left\{\frac{mx}{\mathrm{K}}+(n+\tfrac{1}{2})\frac{y}{\Lambda}\right\}+r^{-(2n+1)m}\cos\pi\left\{\frac{mx}{\mathrm{K}}-(n+\tfrac{1}{2})\frac{y}{\Lambda}\right\}\right]$$
$$=\vartheta_2-\theta_{1,0}(y)\quad(69).$$

$$\vartheta_3=2\sum_{m=0}^{m=\infty}\sum_{n=0}^{n=\infty}p^{(m+\frac{1}{2})^2}q^{(n+\frac{1}{2})^2}\left[r^{2(m+\frac{1}{2})(n+\frac{1}{2})}\cos\pi\left\{(m+\tfrac{1}{2})\frac{x}{\mathrm{K}}+(n+\tfrac{1}{2})\frac{y}{\Lambda}\right\}\right.$$
$$\left.+r^{-2(m+\frac{1}{2})(n+\frac{1}{2})}\cos\pi\left\{(m+\tfrac{1}{2})\frac{x}{\mathrm{K}}-(n+\tfrac{1}{2})\frac{y}{\Lambda}\right\}\right]\quad.\quad.\quad(70).$$

Similarly

$$2\sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty}(-1)^m p^{m^2}q^{n^2}\left[r^{2mn}\cos\pi\left(\frac{mx}{\mathrm{K}}+\frac{ny}{\Lambda}\right)+r^{-2mn}\cos\pi\left(\frac{mx}{\mathrm{K}}-\frac{ny}{\Lambda}\right)\right]$$
$$=\vartheta_4-\theta_{0,1}(x)-\theta_{0,0}(y)+1\quad.\quad.\quad.\quad(71).$$

$$2\sum_{m=0}^{m=\infty}\sum_{n=1}^{n=\infty}(-1)^{m+\frac{1}{2}}p^{(m+\frac{1}{2})^2}q^{n^2}\left[r^{(2m+1)n}\sin\pi\left\{(m+\tfrac{1}{2})\frac{x}{\mathrm{K}}+\frac{ny}{\Lambda}\right\}\right.$$
$$\left.+r^{-(2m+1)}\sin\pi\left\{(m+\tfrac{1}{2})\frac{x}{\mathrm{K}}-\frac{ny}{\Lambda}\right\}\right]=\vartheta_5-\theta_{1,1}(x)\quad.\quad.\quad.\quad(72).$$

$$2\sum_{m=1}^{m=\infty}\sum_{n=0}^{n=\infty}(-1)^m p^{m^2}q^{(n+\frac{1}{2})^2}\left[r^{(2n+1)m}\cos\pi\left\{\frac{mx}{\mathrm{K}}+(n+\tfrac{1}{2})\frac{y}{\Lambda}\right\}\right.$$
$$\left.+r^{-(2n+1)m}\cos\pi\left\{\frac{mx}{\mathrm{K}}-(n+\tfrac{1}{2})\frac{y}{\Lambda}\right\}\right]=\vartheta_6-\theta_{1,0}(y)\quad\text{. . .}\quad(73).$$

$$2\sum_{m=0}^{m=\infty}\sum_{n=0}^{n=\infty}(-1)^{m+\frac{1}{2}}p^{(m+\frac{1}{2})^2}q^{(n+\frac{1}{2})^2}\left[r^{2(m+\frac{1}{2})(n+\frac{1}{2})}\sin\pi\left\{(m+\tfrac{1}{2})\frac{x}{\mathrm{K}}+(n+\tfrac{1}{2})\frac{y}{\Lambda}\right\}\right.$$
$$\left.+r^{-(2m+\frac{1}{2})(n+\frac{1}{2})}\sin\pi\left\{(m+\tfrac{1}{2})\frac{x}{\mathrm{K}}-(n+\tfrac{1}{2})\frac{y}{\Lambda}\right\}\right]=\vartheta_7\quad\text{. . .}\quad(74).$$

$$2\sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty}(-1)^n p^{m^2}q^{n^2}\left[r^{2mn}\cos\pi\left(\frac{mx}{\mathrm{K}}+\frac{ny}{\Lambda}\right)+r^{-2mn}\cos\pi\left(\frac{mx}{\mathrm{K}}-\frac{ny}{\Lambda}\right)\right]$$
$$=\vartheta_8-\theta_{0,0}(x)-\theta_{0,1}(y)+1\quad\text{. .}\quad(75).$$

$$2\sum_{m=0}^{m=\infty}\sum_{n=1}^{n=\infty}(-1)^n p^{(m+\frac{1}{2})^2}q^{n^2}\left[r^{(2m+1)n}\cos\pi\left\{(m+\tfrac{1}{2})\frac{x}{\mathrm{K}}+\frac{ny}{\Lambda}\right\}\right.$$
$$\left.+r^{-(2m+1)n}\cos\pi\left\{(m+\tfrac{1}{2})\frac{x}{\mathrm{K}}-\frac{ny}{\Lambda}\right\}\right]=\vartheta_9-\theta_{1,0}(x)\quad\text{. .}\quad(76).$$

$$2\sum_{m=1}^{m=\infty}\sum_{n=0}^{n=\infty}(-1)^{n+\frac{1}{2}}p^{m^2}q^{(n+\frac{1}{2})^2}\left[r^{(2n+1)m}\sin\pi\left\{\frac{mx}{\mathrm{K}}+(n+\tfrac{1}{2})\frac{y}{\Lambda}\right\}\right.$$
$$\left.+r^{-(2n+1)m}\sin\pi\left\{\frac{mx}{\mathrm{K}}-(n+\tfrac{1}{2})\frac{y}{\Lambda}\right\}\right]=\vartheta_{10}-\theta_{1,1}(y)\quad\text{. . .}\quad(77).$$

$$2\sum_{m=0}^{m=\infty}\sum_{n=0}^{n=\infty}(-1)^{n+\frac{1}{2}}p^{(m+\frac{1}{2})^2}q^{(n+\frac{1}{2})^2}\left[r^{2(m+\frac{1}{2})(n+\frac{1}{2})}\sin\pi\left\{(m+\tfrac{1}{2})\frac{x}{\mathrm{K}}+(n+\tfrac{1}{2})\frac{y}{\Lambda}\right\}\right.$$
$$\left.+r^{-2(m+\frac{1}{2})(n+\frac{1}{2})}\sin\pi\left\{(m+\tfrac{1}{2})\frac{x}{\mathrm{K}}-(n+\tfrac{1}{2})\frac{y}{\Lambda}\right\}\right]=\vartheta_{11}\quad\text{. .}\quad(78).$$

$$2\sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty}(-1)^{m+n}p^{m^2}q^{n^2}\left[r^{2mn}\cos\pi\left(\frac{mx}{\mathrm{K}}+\frac{ny}{\Lambda}\right)+r^{-2mn}\cos\pi\left(\frac{mx}{\mathrm{K}}+\frac{ny}{\Lambda}\right)\right]$$
$$=\vartheta_{12}-\theta_{0,1}(x)-\theta_{0,1}(y)+1\quad\text{. . .}\quad(79).$$

$$2\sum_{m=0}^{m=\infty}\sum_{n=1}^{n=\infty}(-1)^{m+\frac{1}{2}+n}p^{(m+\frac{1}{2})^2}q^{n^2}\left[r^{(2m+1)n}\sin\pi\left\{(m+\tfrac{1}{2})\frac{x}{\mathrm{K}}+\frac{ny}{\Lambda}\right\}\right.$$
$$\left.+r^{-(2m+1)n}\sin\pi\left\{(m+\tfrac{1}{2})\frac{x}{\mathrm{K}}-\frac{ny}{\Lambda}\right\}\right]=\vartheta_{13}-\theta_{1,1}(x)\quad\text{. . .}\quad(80).$$

$$2\sum_{m=1}^{m=\infty}\sum_{n=0}^{n=\infty}(-1)^{m+n+\frac{1}{2}}p^{m^2}q^{(n+\frac{1}{2})^2}\left[r^{(2n+1)m}\sin\pi\left\{\frac{mx}{\mathrm{K}}+(n+\tfrac{1}{2})\frac{y}{\Lambda}\right\}\right.$$
$$\left.+r^{-(2n+1)m}\sin\pi\left\{\frac{mx}{\mathrm{K}}-(n+\tfrac{1}{2})\frac{y}{\Lambda}\right\}\right]=\vartheta_{14}-\theta_{1,1}(y)\quad\text{. . .}\quad(81).$$

$$2\sum_{m=0}^{m=\infty}\sum_{n=0}^{n=\infty}(-1)^{m+n}p^{(m+\frac{1}{2})^2}q^{(n+\frac{1}{2})^2}\Big[r^{2(m+\frac{1}{2})(n+\frac{1}{2})}\cos\pi\Big\{(m+\tfrac{1}{2})\frac{x}{\mathrm{K}}+(n+\tfrac{1}{2})\frac{y}{\Lambda}\Big\}$$
$$-r^{-2(m+\frac{1}{2})(n+\frac{1}{2})}\cos\pi\Big\{(m+\tfrac{1}{2})\frac{x}{\mathrm{K}}-(n+\tfrac{1}{2})\frac{y}{\Lambda}\Big\}\Big]=\vartheta_{15}\quad . \quad . \qquad (82).$$

From these formulæ many other double summations may be deduced: the following may be taken as specimens.

$$8\sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty}p^{4m^2}q^{4n^2}\Big[r^{8mn}\cos 2\pi\Big(\frac{mx}{\mathrm{K}}+\frac{ny}{\Lambda}\Big)+r^{-8mn}\cos 2\pi\Big(\frac{mx}{\mathrm{K}}-\frac{ny}{\Lambda}\Big)\Big]$$
$$=\vartheta_0+\vartheta_4+\vartheta_8+\vartheta_{12}-2\theta_{0,0}(x)-2\theta_{0,0}(y)-2\theta_{0,1}(x)-2\theta_{0,1}(y)+4\quad . \qquad (83).$$

$$8\sum_{m=0}^{m=\infty}\sum_{n=1}^{n=\infty}p^{(2m+1)^2}q^{4n^2}\Big[r^{4n(2m+1)}\cos\pi\Big\{\frac{(2m+1)x}{\mathrm{K}}+\frac{2ny}{\Lambda}\Big\}+r^{-4n(2m+1)}\cos\pi\Big\{\frac{(2m+1)x}{\mathrm{K}}-\frac{2ny}{\Lambda}\Big\}\Big]$$
$$=\vartheta_0-\vartheta_4+\vartheta_8-\vartheta_{12}-2\theta_{0,0}(x)+2\theta_{0,1}(x)\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (84).$$

$$8\sum_{m=1}^{m=\infty}\sum_{n=0}^{n=\infty}p^{4m^2}q^{(2n+1)^2}\Big[r^{4m(2n+1)}\cos\pi\Big\{\frac{2mx}{\mathrm{K}}+\frac{(2n+1)y}{\Lambda}\Big\}+r^{-4m(2n+1)}\cos\pi\Big\{\frac{2mx}{\mathrm{K}}-\frac{(2n+1)y}{\Lambda}\Big\}\Big]$$
$$=\vartheta_0+\vartheta_4-\vartheta_8-\vartheta_{12}-2\theta_{0,0}(y)+2\theta_{0,1}(y)\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (85).$$

$$8\sum_{m=0}^{m=\infty}\sum_{n=0}^{n=\infty}p^{(2m+1)^2}q^{(2n+1)^2}\Big[r^{2(2m+1)(2n+1)}\cos\pi\Big\{\frac{(2m+1)x}{\mathrm{K}}+\frac{(2n+1)y}{\Lambda}\Big\}$$
$$+r^{-2(2m+1)(2n+1)}\cos\pi\Big\{\frac{(2m+1)x}{\mathrm{K}}-\frac{(2n+1)y}{\Lambda}\Big\}\Big]=\vartheta_0-\vartheta_4-\vartheta_8+\vartheta_{12}\quad . \qquad (86).$$

$$4\sum_{m=1}^{m=\infty}\sum_{n=0}^{n=\infty}p^{4m^2}q^{(n+\frac{1}{2})^2}\Big[r^{2m(2n+1)}\cos\pi\Big\{\frac{2mx}{\mathrm{K}}+(n+\tfrac{1}{2})\frac{y}{\Lambda}\Big\}+r^{-2m(2n+1)}\cos\pi\Big\{\frac{2mx}{\mathrm{K}}-(n+\tfrac{1}{2})\frac{y}{\Lambda}\Big\}\Big]$$
$$=\vartheta_2+\vartheta_6-2\theta_{1,0}(y)\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (87).$$

$$4\sum_{m=0}^{m=\infty}\sum_{n=0}^{n=\infty}p^{(2m+1)^2}q^{(n+\frac{1}{2})^2}\Big[r^{(2m+1)(2n+1)}\cos\pi\Big\{\frac{(2m+1)x}{\mathrm{K}}+(n+\tfrac{1}{2})\frac{y}{\Lambda}\Big\}$$
$$+r^{-(2m+1)(2n+1)}\cos\pi\Big\{\frac{(2m+1)x}{\mathrm{K}}-(n+\tfrac{1}{2})\frac{y}{\Lambda}\Big\}\Big]=\vartheta_2-\vartheta_6\quad . \qquad (88).$$

$$4\sum_{m=0}^{m=\infty}\sum_{n=1}^{n=\infty}p^{(m+\frac{1}{2})^2}q^{4n^2}\Big[r^{2n(2m+1)}\cos\pi\Big\{(m+\tfrac{1}{2})\frac{x}{\mathrm{K}}+\frac{2ny}{\Lambda}\Big\}+r^{-2n(2m+1)}\cos\pi\Big\{(m+\tfrac{1}{2})\frac{x}{\mathrm{K}}-\frac{2ny}{\Lambda}\Big\}\Big]$$
$$=\vartheta_1+\vartheta_9-2\theta_{1,0}(x)\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (89).$$

$$4\sum_{m=0}^{m=\infty}\sum_{n=0}^{n=\infty}p^{(m+\frac{1}{2})^2}q^{(2n+1)^2}\Big[r^{(2m+1)(2n+1)}\cos\pi\Big\{(m+\tfrac{1}{2})\frac{x}{\mathrm{K}}+\frac{(2n+1)y}{\Lambda}\Big\}$$
$$+r^{-(2m+1)(2n+1)}\cos\pi\Big\{(m+\tfrac{1}{2})\frac{x}{\mathrm{K}}-\frac{(2n+1)y}{\Lambda}\Big\}\Big]=\vartheta_1-\vartheta_9\quad . \qquad (90).$$

24. Writing $\log r=\frac{i\pi}{2\mathrm{K}}\log\rho'$, then

$$\Phi\left\{\begin{pmatrix}\lambda, & \rho\\ \mu, & \nu\end{pmatrix}x,\, y\right\}=\sum_{m=-\infty}^{m=\infty}\sum_{m=-\infty}^{n=\infty}(-1)^{m\lambda+n\rho}p^{\frac{(2m+\mu)^2}{4}}q^{\frac{(2n+\nu)^2}{4}}e^{\frac{(2n+\nu)i\pi y}{2\Lambda}}e^{\frac{(2m+\mu)i\pi}{2\mathrm{K}}\left(x+2\frac{n+\nu}{2}\log\rho'\right)}$$

$$=\sum_{n=-\infty}^{n=\infty}(-1)^{n\rho}q^{\frac{(2n+\nu)^2}{4}}e^{\frac{(n+\nu)i\pi y}{2\Lambda}}\theta_{\mu,\lambda}\left(x+\frac{2n+\nu}{2}\log\rho'\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (91),$$

which is, in effect, ROSENHAIN's definition of the double theta-functions. Taking a particular case,

$$\vartheta_0=\sum_{n=-\infty}^{n=\infty}q^{n^2}e^{2n\frac{i\pi y}{2\Lambda}}\theta_{0,0}(x+n\log\rho')$$

$$=\theta_{0,0}(x)+q\cos 2y\frac{\pi}{2\Lambda}\{\theta_{0,0}(x+\log\rho')+\theta_{0,0}(x-\log\rho')\}$$

$$+q^4\cos 4y\frac{\pi}{2\Lambda}\{\theta_{0,0}(x+2\log\rho')+\theta_{0,0}(x-2\log\rho')\}$$

$$+q^9\cos 6y\frac{\pi}{2\Lambda}\{\theta_{0,0}(x+3\log\rho')+\theta_{0,0}(x-3\log\rho')\}$$

$$+\dots$$

$$+iq\sin 2y\frac{\pi}{2\Lambda}\{\theta_{0,0}(x+\log\rho')-\theta_{0,0}(x-\log\rho')\}$$

$$+iq^4\sin 4y\frac{\pi}{2\Lambda}\{\theta_{0,0}(x+2\log\rho')-\theta_{0,0}(x-2\log\rho')\}$$

$$+iq^9\sin 6y\frac{\pi}{2\Lambda}\{\theta_{0,0}(x+3\log\rho')-\theta_{0,0}(x-3\log\rho')\}$$

$$+\dots$$

Expanding by TAYLOR's theorem and re-arranging, this gives

$$\vartheta_0=\theta_{0,0}(x)\left[1+2q\cos 2y\frac{\pi}{2\Lambda}+2q^4\cos 4y\frac{\pi}{2\Lambda}+2q^9\cos 6y\frac{\pi}{2\Lambda}+\dots\right]$$

$$+i\log\rho'\frac{d\theta_{0,0}(x)}{dx}\left[2q\sin 2y\frac{\pi}{2\Lambda}+2.2q^4\sin 4y\frac{\pi}{2\Lambda}+3.2q^9\sin 6y\frac{\pi}{2\Lambda}+\dots\right]$$

$$+(\log\rho')^2\frac{1}{2!}\frac{d^2\theta_{0,0}(x)}{dx^2}\left[2q\cos 2y\frac{\pi}{2\Lambda}+2^2.2q^4\cos 4y\frac{\pi}{2\Lambda}+3^2.2q^9\cos 6y\frac{\pi}{2\Lambda}+\dots\right]$$

$$+i(\log\rho')^3\frac{1}{3!}\frac{d^3\theta_{0,0}(x)}{dx^3}\left[2q\sin 2y\frac{\pi}{2\Lambda}+2^3.2q^4\sin 4y\frac{\pi}{2\Lambda}+3^3.2q^9\sin 6y\frac{\pi}{2\Lambda}+\dots\right]$$

$$+\dots$$

The bracket in the first term $= \quad \theta_{0,0}(y)$

,, ,, second ,, $=-\dfrac{\Lambda}{\pi}\dfrac{d\theta_{0,0}(y)}{dy}$

,, ,, third ,, $=-\left(\dfrac{\Lambda}{\pi}\right)^2\dfrac{d^2\theta_{0,0}(y)}{dy^2}$

,, ,, fourth ,, $=\quad\left(\dfrac{\Lambda}{\pi}\right)^3\dfrac{d^3\theta_{0,0}(y)}{dy^3}$

,, ,, n^{th} ,, $=\pm\left(\dfrac{\Lambda}{\pi}\right)^n\dfrac{d^n\theta_{0,0}(y)}{dy^n}$,

the sign being + if $n=4p$ or $4p+1$, and − if $n=4p+2$ or $4p+3$.

Since

$$\log r=\frac{i\pi}{2\mathrm{K}}\log\rho'$$

$$\therefore\ \log\rho'=\frac{2\mathrm{K}}{\pi}\cdot\frac{1}{i}\cdot\log r$$

Hence

$$\vartheta_0=\theta_{0,0}(x)\,\theta_{0,0}(y)\quad\frac{2\mathrm{K}\Lambda\log r}{\pi^2}\,\frac{d\theta_{0,0}(x)}{dx}\,\frac{d\theta_{0,0}(y)}{dy}+\left(\frac{2\mathrm{K}\Lambda\log r}{\pi^2}\right)^2\frac{1}{2!}\,\frac{d^2\theta_{0,0}(x)}{dx^2}\,\frac{d^2\theta_{0,0}(y)}{dy^2}+\ldots$$

$$+(-1)^s\left(\frac{2\mathrm{K}\Lambda\log r}{\pi^2}\right)^s\frac{1}{s!}\,\frac{d^s\theta_{0,0}(x)}{dx^s}\,\frac{d^s\theta_{0,0}(y)}{dy^s}+\ldots\quad.\quad.\quad(92)$$

which may be expressed in the symbolical form

$$\vartheta_0=e^{-\frac{2\mathrm{K}\Lambda\log r}{\pi^2}\frac{d^2}{dxdy}}\theta_{0,0}(x)\theta_{0,0}(y)\,.\quad.\quad.\quad.\quad.\quad.\quad.\quad.\quad(93)\,;$$

and it may be proved by an exactly similar process to hold for all functions, so that generally

$$\Phi\left\{\begin{pmatrix}\lambda,\rho\\ \mu,\nu\end{pmatrix}x,\,y\right\}=e^{-\frac{2\mathrm{K}\Lambda\log r}{\pi^2}\frac{d^2}{dxdy}}\theta_{\mu,\lambda}(x)\theta_{\nu,\rho}(y)\quad.\quad.\quad.\quad.\quad.\quad.\quad(94)$$

the parameters of $\theta_{\mu,\lambda}(x)$ being p, K; and of $\theta_{\nu,\rho}(y)$ q, Λ.

25. The functions Φ have already been distinguished by the oddness or evenness of $\mu\lambda+\nu\rho$; this formula (94) enables us to verify the division of the even and uneven functions given in the table. The latter will be obtained by taking one of the single θ's even and the other uneven; and since there are three even and one uneven function θ for each of the variables, there will be six uneven functions Φ, obtained by taking the uneven function of each variable with the three even functions of the other variable: hence there will be ten even functions, since there are 16 $(=4^2)$ in all.

26. The periodicity also easily follows. Since $\theta(x)$ and all its differentials are periodical in 4K and $\theta(y)$ and all its differentials in 4Λ, we have

$$\begin{aligned}\Phi\{x+4m'\mathrm{K},\, y+4n'\Lambda\}&=e^{-\frac{2\mathrm{K}\Lambda\log r}{\pi^2}\frac{d^2}{dxdy}}\theta_{\mu,\lambda}(x+4m'\mathrm{K})\theta_{\nu,\rho}(y+4n'\Lambda)\\&=e^{-\frac{2\mathrm{K}\Lambda\log r}{\pi^2}\frac{d^2}{dxdy}}\theta_{\mu,\lambda}(x)\theta_{\nu,\rho}(y)\\&=\Phi\{x,y\}\quad.\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\qquad(95)\end{aligned}$$

giving the two pairs of conjugate periods 4K and 0, 0 and 4Λ.

27. If K′ be defined by the relation

$$-\pi\frac{\mathrm{K}'}{\mathrm{K}}=\log p$$

then it is known that

$$\theta_{\mu,\lambda}(x+4i\mathrm{K}')=p^{-4}e^{-\frac{2\pi ix}{\mathrm{K}}}\theta_{\mu,\lambda}(x)$$

Hence

$$\Phi\{x+4i\mathrm{K}',\,y\}=e^{-\frac{2\mathrm{K}\Lambda\log r}{\pi^2}\frac{d^2}{dxdy}}p^{-4}e^{-\frac{2\pi ix}{\mathrm{K}}}\theta_{\mu,\lambda}(x)\theta_{\nu,\rho}(y)$$

$$\begin{aligned}=p^{-4}\Big[e^{-\frac{2\pi ix}{\mathrm{K}}}\theta_{\mu,\lambda}(x)\theta_{\nu,\rho}(y)-\frac{2\mathrm{K}\Lambda\log r}{\pi^2}\,\frac{d\theta_{\nu,\rho}(y)}{dy}\,\frac{d}{dx}\Big\{e^{-\frac{2\pi ix}{\mathrm{K}}}\theta_{\mu,\lambda}(x)\Big\}\\+\Big(\frac{2\mathrm{K}\Lambda\log r}{\pi^2}\Big)^2\frac{1}{2!}\,\frac{d^2\theta_{\nu,\rho}(y)}{dy^2}\,\frac{d^2}{dx^2}\Big\{e^{-\frac{2\pi ix}{\mathrm{K}}}\theta_{\mu,\lambda}(x)\Big\}-\ .\ .\ .\Big]\end{aligned}$$

$$\begin{aligned}&=p^{-4}e^{-\frac{2\pi ix}{\mathrm{K}}}\theta_{\mu,\lambda}(x)\Big[\theta_{\nu,\rho}(y)-\frac{4\Lambda}{\pi i}\log r\frac{d\theta_{\nu,\rho}(y)}{dy}+\Big(\frac{4\Lambda}{\pi i}\log r\Big)^2\frac{1}{2!}\,\frac{d^2\theta_{\nu,\rho}(y)}{dy^2}-\ .\ .\ .\Big]\\&-p^{-4}e^{-\frac{2\pi ix}{\mathrm{K}}}\,\frac{d\theta_{\mu,\lambda}(x)}{dx}\,\frac{2\mathrm{K}\Lambda\log r}{\pi^2}\Big[\frac{d\theta_{\nu,\rho}(y)}{dy}-\frac{4\Lambda}{\pi i}\log r\frac{d^2\theta_{\nu,\rho}(y)}{dy^2}+\Big(\frac{4\Lambda}{\pi i}\log r\Big)^2\frac{1}{2!}\,\frac{d^3\theta_{\nu,\rho}(y)}{dy^3}-\ .\ .\ .\Big]\\&+p^{-4}e^{-\frac{2\pi ix}{\mathrm{K}}}\frac{d^2\theta_{\mu,\lambda}(x)}{dx^2}\,\frac{1}{2!}\Big(\frac{2\mathrm{K}\Lambda\log r}{\pi^2}\Big)^2\Big[\frac{d^2\theta_{\nu,\rho}(y)}{dy^2}-\frac{4\Lambda}{\pi i}\log r\,\frac{d^3\theta_{\nu,\rho}(y)}{dy^3}+\Big(\frac{4\Lambda}{\pi i}\log r\Big)^2\frac{1}{2!}\,\frac{d^4\theta_{\nu,\rho}(y)}{dy^2}-\ .\ .\ .\Big]\\&-\ .\ .\ .\end{aligned}$$

$$=p^{-4}e^{-\frac{2\pi ix}{\mathrm{K}}}\Big[\theta_{\mu,\lambda}(x)\theta_{\nu,\rho}\Big(y-\frac{4\Lambda}{\pi i}\log r\Big)-\frac{2\mathrm{K}\Lambda}{\pi^2}\log r\frac{d\theta_{\mu,\lambda}(x)}{dx}\,\frac{d\theta_{\nu,\rho}\Big(y-\frac{4\Lambda}{\pi i}\log r\Big)}{dy}+\ .\ .\ .\Big]$$

and therefore

$$\begin{aligned}\Phi\Big\{x+4i\mathrm{K}',\ y+\frac{4\Lambda}{\pi i}\log r\Big\}=p^{-4}e^{-\frac{2\pi ix}{\mathrm{K}}}\Big[\theta_{\mu,\lambda}(x)\theta_{\nu,\rho}(y)-\frac{2\mathrm{K}\Lambda\log r}{\pi^2}\,\frac{d\theta_{\mu,\lambda}(x)}{dx}\,\frac{d\theta_{\nu,\rho}(y)}{dy}\\+\Big(\frac{2\mathrm{K}\Lambda\log r}{\pi^2}\Big)^2\frac{1}{2!}\,\frac{d^2\theta_{\mu,\lambda}(x)}{dx^2}\,\frac{d^2\theta_{\nu,\rho}(y)}{dy^2}-\ .\ .\ .\Big]\end{aligned}$$

that is

$$\Phi\left\{x+\frac{4\mathrm{K}}{\pi i}\log p,\ y+\frac{4\Lambda}{\pi i}\log r\right\}=p^{-1}e^{-\frac{2\pi ix}{\mathrm{K}}}\Phi\{x,y\}.$$

Similarly

$$\left.\Phi\left\{x+\frac{4\mathrm{K}}{\pi i}\log r,\ y+\frac{4\Lambda}{\pi i}\log q\right\}=q^{-1}e^{-\frac{2\pi iy}{\Lambda}}\Phi\{x,y\}\right\} \quad . \quad . \quad . \quad . \quad (96).$$

which give the two pairs of conjugate quasi-periods,

$$\frac{4\mathrm{K}}{\pi i}\log p \text{ and } \frac{4\Lambda}{\pi i}\log r,\quad \frac{4\mathrm{K}}{\pi i}\log r \text{ and } \frac{4\Lambda}{\pi i}\log q.$$

28. The verification of the expansions in doubly infinite series of sines and cosines is easily effected: for substituting in (92) the expressions

$$\theta_{0,0}(x)=1+2p\cos\frac{\pi x}{\mathrm{K}}+2p^4\cos\frac{2\pi x}{\mathrm{K}}+2p^9\cos\frac{3\pi x}{\mathrm{K}}+\ldots$$

$$\theta_{0,0}(y)=1+2q\cos\frac{\pi y}{\Lambda}+2q^4\cos\frac{2\pi y}{\Lambda}+2q^9\cos\frac{3\pi y}{\Lambda}+\ldots$$

the coefficient, on the right hand side, of $\cos\pi\left(\frac{mx}{\mathrm{K}}-\frac{ny}{\Lambda}\right)$ is

$$=2\left[p^{m^2}q^{n^2}-2mn\log r.p^{m^2}q^{n^2}+\frac{(2\log r)^2}{2!}m^2n^2p^{m^2}q^{n^2}-\ldots\right]$$

$$=2p^{m^2}q^{n^2}r^{-2mn}$$

which is right.

Second proof of the product theorem (23).

29. The product theorem for single theta-functions, as given by Professor SMITH (Lond. Math. Soc. Proc., vol. i.) is, with the notation of Section I,

$$2\Pi\theta_{\mu,\lambda}(x)=\Pi\theta_{\sigma,\Lambda}(\mathrm{X})+\Pi\theta_{\sigma,\Lambda+1}(\mathrm{X})+(-1)^{\Lambda'}\{\Pi\theta_{\sigma+1,\Lambda}(\mathrm{X})-\Pi\theta_{\sigma+1,\Lambda+1}(\mathrm{X})\} \quad . \quad (97)$$

Now, using (94), we have

$$\Phi\left\{\begin{pmatrix}\lambda, \rho\\ \mu, \nu\end{pmatrix}x, y\right\}\Phi\left\{\begin{pmatrix}\lambda', \rho'\\ \mu', \nu'\end{pmatrix}x', y'\right\}=e^{-\frac{2\mathrm{K}\Lambda\log r}{\pi^2}\left(\frac{d^2}{dxdy}+\frac{d^2}{dx'dy'}\right)}\theta_{\mu,\lambda}(x)\theta_{\mu',\lambda'}(x')\theta_{\nu,\rho}(y)\theta_{\nu',\rho}(y')$$

and therefore

$$\Pi\Phi\left\{\begin{pmatrix}\lambda, \rho\\ \mu, \nu\end{pmatrix}x, y\right\}=e^{-\frac{2\mathrm{K}\Lambda\log r}{\pi^2}\left(\frac{d^2}{dx_1dy_1}+\frac{d^2}{dx_2dy_2}+\frac{d^2}{dx_3dy_3}+\frac{d^2}{dx_4dy_4}\right)}\Pi\theta_{\mu,\lambda}(x)\Pi\theta_{\nu,\rho}(y) \quad . \quad . \quad (98)$$

By the values of X, Y we have

$$\frac{d}{dx_1}=\tfrac{1}{2}\left(-\frac{d}{dX_1}+\frac{d}{dX_2}+\frac{d}{dX_3}+\frac{d}{dX_4}\right)$$

$$\frac{d}{dx_2}=\tfrac{1}{2}\left(\frac{d}{dX_1}-\frac{d}{dX_2}+\frac{d}{dX_3}+\frac{d}{dX_4}\right)$$

$$\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot$$

$$\frac{d}{dy_1}=\tfrac{1}{2}\left(-\frac{d}{dY_1}+\frac{d}{dY_2}+\frac{d}{dY_3}+\frac{d}{dY_4}\right)$$

$$\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot$$

hence

$$\frac{d^2}{dx_1dy_1}+\frac{d^2}{dx_2dy_2}+\frac{d^2}{dx_3dy_3}+\frac{d^2}{dx_4dy_4}=\frac{d^2}{dX_1dY_1}+\frac{d^2}{dX_2dY_2}+\frac{d^2}{dX_3dY_3}+\frac{d^2}{dX_4dY_4}\cdot\quad\cdot\quad(99).$$

By means of (97) and the corresponding theorem for $\Pi\theta_{\nu,\rho}(y)$, an expression is obtained for $\Pi\theta_{\mu,\lambda}(x)\Pi\theta_{\nu,\rho}(y)$, containing 16 terms; substitute this in (98) and transpose the operator by (99), and then, by (94), express each term as the product of four Φ's, and there will result the theorem already given in (23).

On the differential equations satisfied by $\Phi\left\{\binom{\lambda,\rho}{\mu,\nu}x, y\right\}$.

30. From the theory of elliptic functions it is known that if

$$K=\int_0^{\frac{\pi}{2}}\frac{d\theta}{\sqrt{1-\kappa^2\sin^2\theta}}$$

$$\Lambda=\int_0^{\frac{\pi}{2}}\frac{d\theta}{\sqrt{1-\lambda^2\sin^2\theta}}$$

K, Λ, k, λ are definite functions of p, q; viz.:

$$\sqrt{\frac{2K}{\pi}}=1+2p+2p^4+2p^9+\ldots$$

$$\sqrt{\frac{2\Lambda}{\pi}}=1+2q+2q^4+2q^9+\ldots$$

$$\sqrt{\kappa}=\frac{2p^{\frac{1}{4}}+2p^{\frac{9}{4}}+2p^{\frac{25}{4}}+\ldots}{1+2p+2p^4+2p^9+\ldots}$$

$$\sqrt{\lambda}=\frac{2q^{\frac{1}{4}}+2q^{\frac{9}{4}}+2q^{\frac{25}{4}}+\ldots}{1+2q+2q^4+2q^9+\ldots}$$

Also K, κ are given each as a function of the other by the respective differential equations

$$(1-\kappa^2)\frac{d^2\mathrm{K}}{d\kappa^2}+\frac{1-3\kappa^2}{\kappa}\frac{d\mathrm{K}}{d\kappa}-\mathrm{K}=0$$

$$(1-\kappa^2)\frac{d^2\kappa}{d\mathrm{K}^2}-\frac{1-3\kappa^2}{\kappa}\left(\frac{d\kappa}{d\mathrm{K}}\right)^2+\mathrm{K}\left(\frac{d\kappa}{d\mathrm{K}}\right)^3=0,$$

so that κ may be considered known, and likewise κ', E, given by

$$\kappa^2+\kappa'^2=1, \qquad \mathrm{E}=\int_0^{\frac{\pi}{2}}\sqrt{1-\kappa^2\sin^2\theta}\,d\theta.$$

Similarly if

$$\lambda'^2+\lambda^2=1,$$

$$\Gamma=\int_0^{\frac{\pi}{2}}\sqrt{1-\lambda^2\sin^2\theta}\,d\theta,$$

λ', Γ may be considered known.

31. It is proved in CAYLEY's 'Elliptic Functions,' § 310, that the general single theta-function satisfies the differential equation

$$\frac{d^2\theta}{dx^2}-2x\left(\kappa'^2-\frac{\mathrm{E}}{\mathrm{K}}\right)\frac{d\theta}{dx}+2\kappa\kappa'^2\frac{d\theta}{d\kappa}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (100)$$

and

$$\frac{d\mathrm{K}}{d\kappa}=\frac{\mathrm{E}}{\kappa\kappa'^2}-\frac{\mathrm{K}}{\kappa}=-\frac{\mathrm{K}}{\kappa\kappa'^2}\left(\kappa'^2-\frac{\mathrm{E}}{\mathrm{K}}\right)$$

so that (100) may be written in the form

$$\frac{d^2\theta}{dx^2}+\frac{2\kappa\kappa'^2}{\mathrm{K}}\frac{d\mathrm{K}}{d\kappa}x\frac{d\theta}{dx}+2\kappa\kappa'^2\frac{d\theta}{d\kappa}=0.$$

Differentiating s times with respect to x

$$\frac{d^2}{dx^2}\frac{d^s\theta}{dx^s}+\frac{2\kappa\kappa'^2}{\mathrm{K}}\frac{d\mathrm{K}}{d\kappa}x\frac{d}{dx}\frac{d^s\theta}{dx^s}+2s\frac{\kappa\kappa'^2}{\mathrm{K}}\frac{d\mathrm{K}}{d\kappa}\frac{d^s\theta}{dx^s}+2\kappa\kappa'^2\frac{d}{d\kappa}\frac{d^s\theta}{dx^s}=0 \quad . \quad . \quad . \quad (101)$$

Now the general term in Φ in (94) is a numerical multiple of

$$\psi'=\left(\frac{2\mathrm{K}\Lambda\log r}{\pi^2}\right)^s\frac{d^s\theta(x)}{dx^s}\frac{d^s\theta(y)}{dy^s}$$

or of

$$\psi=\mathrm{K}^s\frac{d^s\theta}{dx^s}$$

so far as x, κ are concerned. Then

$$\frac{d^2\psi}{dx^2}+\frac{2\kappa\kappa'^2}{\mathrm{K}}\frac{d\mathrm{K}}{d\kappa}x\frac{d\psi}{dx}=\mathrm{K}^s\,[\text{first two terms in (101)}]$$

and

$$\frac{d\psi}{d\kappa}=\mathrm{K}^s\left[\frac{d}{d\kappa}\frac{d^s\theta}{dx^s}+\frac{s}{\mathrm{K}}\frac{d\mathrm{K}}{d\kappa}\frac{d^s\theta}{dx^s}\right]$$

Hence

$$\frac{d^2\psi}{dx^2}+\frac{2\kappa\kappa'^2}{\mathrm{K}}\frac{d\mathrm{K}}{d\kappa}x\frac{d\psi}{dx}+2\kappa\kappa'^2\frac{d\psi}{d\kappa}=0$$

and therefore also Φ, the sum of the terms ψ, satisfies the differential equation

$$\frac{d^2\Phi}{dx^2}+\frac{2\kappa\kappa'^2}{\mathrm{K}}\frac{d\mathrm{K}}{d\kappa}x\frac{d\Phi}{dx}+2\kappa\kappa'^2\frac{d\Phi}{d\kappa}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (102)$$

or restoring the usual coefficient in the second term

$$\frac{d^2\Phi}{dx^2}-2x\left(\kappa'^2-\frac{\mathrm{E}}{\mathrm{K}}\right)\frac{d\Phi}{dx}+2\kappa\kappa'^2\frac{d\Phi}{d\kappa}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (103)$$

Similarly Φ satisfies

$$\frac{d^2\Phi}{dy^2}-2y\left(\lambda'^2-\frac{\Gamma}{\Lambda}\right)\frac{d\Phi}{dy}+2\lambda\lambda'^2\frac{d\Phi}{d\lambda}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (104),$$

or

$$\frac{d^2\Phi}{dy^2}+\frac{2\lambda\lambda'^2}{\Lambda}\frac{d\Lambda}{d\lambda}y\frac{d\Phi}{dy}+2\lambda\lambda'^2\frac{d\Phi}{d\lambda}=0 \quad . \quad . \quad . \quad . \quad . \quad . \qquad (105);$$

and from (94) it at once follows that

$$r\frac{d\Phi}{dr}+\frac{2\mathrm{K}\Lambda}{\pi^2}\frac{d^2\Phi}{dxdy}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (106).$$

32. All these equations can be deduced from the general definition of Φ, viz.:—

$$\Phi=\sum_{m=-\infty}^{m=\infty}\sum_{n=-\infty}^{n=\infty}(-1)^{m\lambda+n\rho}p^{\frac{(2m+\mu)^2}{4}}q^{\frac{(2n+\nu)^2}{4}}r^{\frac{(2m+\mu)(2n+\nu)}{2}}e^{\frac{i\pi}{2}\left\{(2m+\mu)\frac{x}{\mathrm{K}}+(2n+\nu)\frac{y}{\Lambda}\right\}}$$

The equation (106) is obviously satisfied. Consider the general term in Φ to obtain (102); it is a multiple of

$$u=p^{\frac{(2m+\mu)^2}{4}}e^{\frac{i\pi x}{2\mathrm{K}}(2m+\mu)}$$

the coefficient being independent of x, κ. Now

$$\frac{d^2u}{dx^2}=-\frac{(2m+\mu)^2\pi^2}{4\mathrm{K}^2}u$$

$$\frac{du}{dp}=\frac{(2m+\mu)^2}{4p}u-\frac{i\pi x(2m+\mu)}{2\mathrm{K}^2}\frac{d\mathrm{K}}{dp}u$$

and

$$x\frac{du}{dx}=\frac{i\pi x(2m+\mu)}{2\mathrm{K}}u$$

Hence

$$\frac{du}{dp}=-\frac{\mathrm{K}^2}{\pi^2 p}\frac{d^2u}{dx^2}-\frac{1}{\mathrm{K}}\frac{d\mathrm{K}}{dp}x\frac{du}{dx}\quad\ldots\ldots\ldots\quad(107)$$

Also

$$p=e^{-\pi\frac{\mathrm{K}'}{\mathrm{K}}}$$

$$\therefore\frac{1}{p}\frac{dp}{d\kappa}=-\pi\frac{\mathrm{K}\frac{d\mathrm{K}'}{d\kappa}-\mathrm{K}'\frac{d\mathrm{K}}{d\kappa}}{\mathrm{K}^2}$$

$$=-\frac{\pi}{\kappa\kappa'^2\mathrm{K}^2}\{-\mathrm{KE}'-\mathrm{K}'\mathrm{E}+\mathrm{KK}'\}$$

$$=\frac{\pi^2}{2\kappa\kappa'^2\mathrm{K}^2}$$

Multiplying (107) throughout by $\frac{dp}{d\kappa}$ and substituting in the first term on the right-hand side the value just found for $\frac{1}{p}\frac{dp}{d\kappa}$, we have

$$\frac{du}{dk}=-\frac{1}{2\kappa\kappa'^2}\frac{d^2u}{dx^2}-\frac{1}{\mathrm{K}}\frac{d\mathrm{K}}{d\kappa}x\frac{du}{dx}$$

and hence Φ, the sum of the terms u, satisfies

$$\frac{d^2\Phi}{dx^2}+\frac{2\kappa\kappa'^2}{\mathrm{K}}\frac{d\mathrm{K}}{d\kappa}x\frac{d\Phi}{dx}+2\kappa\kappa'^2\frac{d\Phi}{d\kappa}=0$$

which is (102). The quantity $\frac{d\mathrm{K}}{d\kappa}$ may be explicitly expressed in the terms of p as follows. We have

$$\theta_{0,0}\left(x\frac{2\mathrm{K}}{\pi}\right)=1-p^2.1-p^4.1-p^6\ldots(1+2p\cos 2x+p^2)(1+2p^3\cos 2x+p^6)\ldots$$

and

$$\theta_{0,0}(0)=\left(\frac{2\mathrm{K}}{\pi}\right)^{\frac{1}{2}}$$

Writing u for $\frac{2\mathrm{K}x}{\pi}$ and taking logarithmic differentials

$$-\tfrac{1}{4}\frac{1}{\theta_{0,0}(u)}\,\frac{2\mathrm{K}}{\pi}\,\frac{d\theta_{0,0}(u)}{du}=\frac{p\sin 2x}{1+2p\cos 2x+p^2}+\frac{p^3\cos 2x}{1+2p^3\cos 2x+p^6}+\ \ldots$$

$$=\sin 2x\left[\frac{p}{1+2p\cos 2x+p^2}+\frac{p^3}{1+2p^3\cos 2x+p^6}+\frac{p^5}{1+2p^5\cos 2x+p^{10}}+\ \ldots\right]$$

Differentiate again with regard to x, and then put x zero :

$$-\tfrac{1}{4}\left(\frac{\pi}{2\mathrm{K}}\right)^{\frac{1}{2}}\left(\frac{2\mathrm{K}}{\pi}\right)^2\frac{d^2\theta_{0,0}(u)}{du_0^{\,2}}=2\left[\frac{p}{(1+p)^2}+\frac{p^3}{(1+p^3)^2}+\frac{p^5}{(1+p^5)^2}+\ \ldots\right]$$

But by (100)

$$-\frac{d^2\theta_{0,0}(x)}{dx_0^{\,2}}=2\kappa\kappa'^2\frac{d\theta_{0,0}(0)}{d\kappa}$$

$$=2\kappa\kappa'^2\sqrt{\frac{1}{2\pi\mathrm{K}}}\,\frac{d\mathrm{K}}{d\kappa}$$

Hence

$$\frac{\kappa\kappa'^2\mathrm{K}}{2\pi^2}\,\frac{d\mathrm{K}}{d\kappa}=\frac{p}{(1+p)^2}+\frac{p^3}{(1+p^3)^2}+\frac{p^5}{(1+p^5)^2}+\frac{p^7}{(1+p^7)^2}+\ \ldots$$

so that now all the coefficients in (102) are known explicitly in terms of p.

On the constants.

33. From the definitions of ϑ_0, c_0 we have

$$\begin{aligned}c_0=1&+2p+2p^4+2p^2+\ \ldots\ +2q+2q^4+2q^9+\ \ldots\\&+2p\ \left\{q\left(r^2+\frac{1}{r^2}\right)+q^4\left(r^4+\frac{1}{r^4}\right)+q^9\left(r^6+\frac{1}{r^6}\right)+\ \ldots\right\}\\&+2p^4\left\{q\left(r^4+\frac{1}{r^4}\right)+q^4\left(r^8+\frac{1}{r^8}\right)+q^9\left(r^{12}+\frac{1}{r^{12}}\right)+\ \ldots\right\}\\&+2p^9\left\{q\left(r^6+\frac{1}{r^6}\right)+q^4\left(r^{12}+\frac{1}{r^{12}}\right)+q^9\left(r^{18}+\frac{1}{r^{12}}\right)+\ \ldots\right\}\\&+\ \ldots\end{aligned}$$

with similar series for c_1, c_2, c_3, c_4, c_6, c_8, c_9, c_{12}, c_{15}. Substituting these in the complete set of sixty equations of which (41) are the type, there will result algebraical identities in three quantities p, q, r (mutually independent), corresponding to identities in the "q-series" in elliptic functions. As an example

$$\begin{aligned}
&\Big[1+2q+2q^4+2q^9+\ldots+2p\left\{1+q\left(r^2+\frac{1}{r^2}\right)+q^4\left(r^4+\frac{1}{r^4}\right)+q^9\left(r^6+\frac{1}{r^6}\right)+\ldots\right\}\\
&\qquad+2p^4\left\{1+q\left(r^4+\frac{1}{r^4}\right)+q^4\left(r^8+\frac{1}{r^8}\right)+q^9\left(r^{12}+\frac{1}{r^{12}}\right)+\ldots\right\}\\
&\qquad+2p^9\left\{1+q\left(r^6+\frac{1}{r^6}\right)+q^4\left(r^{12}+\frac{1}{r^{12}}\right)+\ldots\right\}+\ldots\Big]^4\\
&=\Big[2p^{\frac{1}{4}}\left\{1+q\left(r+\frac{1}{r}\right)+q^4\left(r^2+\frac{1}{r^2}\right)+q^9\left(r^3+\frac{1}{r^3}\right)+\ldots\right\}\\
&\qquad+2p^{\frac{9}{4}}\left\{1+q\left(r^3+\frac{1}{r^3}\right)+q^4\left(r^6+\frac{1}{r^6}\right)+q^9\left(r^9+\frac{1}{r^9}\right)+\ldots\right\}\\
&\qquad+2p^{\frac{25}{4}}\left\{1+q\left(r^5+\frac{1}{r^5}\right)+q^4\left(r^{10}+\frac{1}{r^{10}}\right)+q^9\left(r^{15}+\frac{1}{r^{15}}\right)+\ldots\right\}+\ldots\Big]^4\\
&+\Big[2q^{\frac{1}{4}}\left\{1-p\left(r+\frac{1}{r}\right)+p^4\left(r^2+\frac{1}{r^2}\right)-p^9\left(r^3+\frac{1}{r^3}\right)+\ldots\right\}\\
&\qquad+2q^{\frac{9}{4}}\left\{1-p\left(r^3+\frac{1}{r^3}\right)+p^4\left(r^6+\frac{1}{r^6}\right)-p^9\left(r^9+\frac{1}{r^9}\right)+\ldots\right\}\\
&\qquad+2q^{\frac{25}{4}}\left\{1-p\left(r^5+\frac{1}{r^5}\right)+p^4\left(r^{10}+\frac{1}{r^{10}}\right)-p^9\left(r^{15}+\frac{1}{r^{15}}\right)+\ldots\right\}+\ldots\Big]^4\\
&+\Big[1-2q+2q^4-2q^9+\ldots-2p\left\{1-q\left(r^2+\frac{1}{r^2}\right)+q^4\left(r^4+\frac{1}{r^4}\right)-q^9\left(r^6+\frac{1}{r^6}\right)+\ldots\right\}\\
&\qquad+2p^4\left\{1-q\left(r^4+\frac{1}{r^4}\right)+q^4\left(r^8+\frac{1}{r^8}\right)-q^9\left(r^{12}+\frac{1}{r^{12}}\right)+\ldots\right\}\\
&\qquad-2p^9\left\{1-q\left(r^6+\frac{1}{r^6}\right)+q^4\left(r^{12}+\frac{1}{r^{12}}\right)-q^9\left(r^{18}+\frac{1}{r^{18}}\right)+\ldots\right\}+\ldots\Big]^4
\end{aligned}$$

Also equations (42), (43), (44) give expressions for κ_1, κ_2, κ_3, in terms of p, q, r; and other identities are obtained from the equation (α)–$(\iota\epsilon)$.

34. Let $\kappa^2=c$, $\kappa'^2=c'$; then θ being any single theta-function

$$\frac{d^2\theta}{dx^2}-2\alpha x\frac{d\theta}{dx}+\beta\frac{d\theta}{dc}=0$$

where

$$\alpha=c'-\frac{\mathrm{E}}{\mathrm{K}}=-\frac{2cc'}{\mathrm{K}}\frac{d\mathrm{K}}{dc},$$

$$\beta=4cc'.$$

First, let θ be one of the three even functions: differentiating $2n$ times with respect to x, and then putting x zero

$$\frac{d^{2n+2}\theta}{dx_0^{2n+2}}-4n\alpha\frac{d^{2n}\theta}{dx_0^{2n}}+\beta\frac{d}{dc}\frac{d^{2n}\theta}{dx_0^{2n}}=0$$

and therefore

$$\frac{1}{4cc'}\frac{d^{2n+2}\theta}{dx_0^{\,2n+2}}=-\frac{2n}{\mathrm{K}}\frac{d\mathrm{K}}{dc}\frac{d^{2n}\theta}{dx_0^{\,2n}}-\frac{d}{dc}\frac{d^{2n}\theta}{dx_0^{\,2n}}$$

$$=-\mathrm{K}^{-2n}\frac{d}{dc}\cdot\mathrm{K}^{2n}\frac{d^{2n}\theta}{dx_0^{\,2n}}$$

that is

$$\mathrm{K}^{2n+2}\frac{d^{2n+2}\theta}{dx_0^{\,2n+2}}=\left(-4cc'\mathrm{K}^2\frac{d}{dc}\right)\mathrm{K}^{2n}\frac{d^{2n}\theta}{dx_0^{\,2n}}$$

$$=\left(-4cc'\mathrm{K}^2\frac{d}{dc}\right)^{n+1}\theta(0)\quad.\quad.\quad.\quad.\quad.\quad.\quad.\quad.\quad(108).$$

Now

$$\theta_{0,0}(0)=\left(\frac{2}{\pi}\right)^{\frac{1}{2}}\mathrm{K}^{\frac{1}{2}}$$

$$\theta_{0,1}(0)=\left(\frac{}{\pi}\right)^{\frac{1}{2}}\mathrm{K}^{\frac{1}{2}}c'^{\frac{1}{4}}$$

$$\theta_{1,0}(0)=\left(\frac{2}{\pi}\right)^{\frac{1}{2}}\mathrm{K}^{\frac{1}{2}}c^{\frac{1}{4}}$$

and thus, by (108),

$$\mathrm{K}^{2n}\frac{d^{2n}\theta_{0,0}}{dx_0^{\,2n}}=(-1)^n\left(\frac{2}{\pi}\right)^{\frac{1}{2}}\left(4cc'\mathrm{K}^2\frac{d}{dc}\right)^n\mathrm{K}^{\frac{1}{2}}\quad.\quad.\quad.\quad.\quad.\quad.\quad(109).$$

$$\mathrm{K}^{2n}\frac{d^{2n}\theta_{0,1}}{dx_0^{\,2n}}=(-1)^n\left(\frac{2}{\pi}\right)^{\frac{1}{2}}\left(4cc'\mathrm{K}^2\frac{d}{dc}\right)^n\mathrm{K}^{\frac{1}{2}}c'^{\frac{1}{4}}\quad.\quad.\quad.\quad.\quad.\quad.\quad(110).$$

$$\mathrm{K}^{2n}\frac{d^{2n}\theta_{1,0}}{dx_0^{\,2n}}=(-1)^n\left(\frac{2}{\pi}\right)^{\frac{1}{2}}\left(4cc'\mathrm{K}^2\frac{d}{dc}\right)^n\mathrm{K}^{\frac{1}{2}}c^{\frac{1}{4}}\quad.\quad.\quad.\quad.\quad.\quad.\quad(111).$$

Similarly, if $\lambda^2=\gamma$, $\lambda'^2=\gamma'$,

$$\Lambda^{2n}\frac{d^{2n}\theta_{0,0}}{dy_0^{\,2n}}=(-1)^n\left(\frac{2}{\pi}\right)^{\frac{1}{2}}\left(4\gamma\gamma'\Lambda^2\frac{d}{d\gamma}\right)^n\Lambda^{\frac{1}{2}}.\quad.\quad.\quad.\quad.\quad.\quad.\quad.\quad(112)$$

$$\Lambda^{2n}\frac{d^{2n}\theta_{0,1}}{dy_0^{\,2n}}=(-1)^n\left(\frac{2}{\pi}\right)^{\frac{1}{2}}\left(4\gamma\gamma'\Lambda^2\frac{d}{d\gamma}\right)^n\Lambda^{\frac{1}{2}}\gamma'^{\frac{1}{4}}\quad.\quad.\quad.\quad.\quad.\quad.\quad(113)$$

$$\Lambda^{2n}\frac{d^{2n}\theta_{1,0}}{dy_0^{\,2n}}=(-1)^n\left(\frac{2}{\pi}\right)^{\frac{1}{2}}\left(4\gamma\gamma'\Lambda^2\frac{d}{d\gamma}\right)^n\Lambda^{\frac{1}{2}}\gamma^{\frac{1}{4}}\quad.\quad.\quad.\quad.\quad.\quad.\quad(114).$$

Next, let θ be the uneven function; differentiating the equation $(2n+1)$ times with regard to x and putting x zero

$$\frac{d^{2n+3}\theta_{1,1}}{dx_0^{\,2n+3}}-2\alpha(2n+1)\frac{d^{2n+1}\theta_{1,1}}{dx_0^{\,2n+1}}+\beta\frac{d}{dc}\,\frac{d^{2n+1}\theta_{1,1}}{dx_0^{\,2n+1}}=0$$

and therefore

$$\frac{1}{4cc'}\frac{d^{2n+3}\theta_{1,1}}{dx_0^{\,2n+3}}=-\frac{2n+1}{\mathrm{K}}\frac{d\mathrm{K}}{dc}\frac{d^{2n+1}\theta_{1,1}}{dx_0^{\,2n+1}}-\frac{d}{dc}\frac{d^{2n+1}\theta_{1,1}}{dx_0^{\,2n+1}}$$

$$\mathrm{K}^{2n+3}\frac{d^{2n+3}\theta_{1,1}}{dx_0^{\,2n+3}}=-4cc'\mathrm{K}^2\frac{d}{dc}\mathrm{K}^{2n+1}\frac{d^{2n+1}\theta_{1,1}}{dx_0^{\,2n+1}}$$

that is

$$\mathrm{K}^{2n+1}\frac{d^{2n+1}\theta_{1,1}}{dx_0^{\,2n+1}}=\left(-4cc'\mathrm{K}^2\frac{d}{dc}\right)^n\mathrm{K}\frac{d\theta_{1,1}}{dx_0}\quad .\ .\ .\ .\ .\ .\ .\quad (115).$$

Now

$$\frac{1}{i}\theta_{1,1}\left(\frac{2\mathrm{K}x}{\pi}\right)=2p^{\frac{1}{4}}\sin x-2p^{\frac{9}{4}}\sin 3x+2p^{\frac{25}{4}}\sin 5x-\ .\ .\ .$$
$$=2p^{\frac{1}{4}}(1-p^2)(1-p^2)(1-p^6)\ .\ .\ .\sin x(1-2p^2\cos 2x+p^2)(1-2p^4\cos 2x+p^8)\ .\ .\ .$$

Hence

$$\frac{\mathrm{K}}{\pi i}\frac{d\theta_{1,1}}{dx_0}=p^{\frac{1}{4}}\{(1-p^2)(1-p^4)(1-p^6\ .\ .\ .\}^3$$
$$=\left(\frac{2}{\pi}\right)^{\frac{1}{2}}\frac{1}{\pi}c^{\frac{1}{4}}c'^{\frac{1}{4}}\mathrm{K}^{\frac{3}{2}}$$

Thus (115) gives

$$\mathrm{K}^{2n+1}\frac{d^{2n+1}\theta_{1,1}}{dx_0^{\,2n+1}}=(-1)^{n+\frac{1}{2}}\left(\frac{2}{\pi}\right)^{\frac{1}{2}}\left(4cc'\mathrm{K}^2\frac{d}{dc}\right)^n c^{\frac{1}{4}}c'^{\frac{1}{4}}\mathrm{K}^{\frac{3}{2}}\quad .\ .\ .\ .\ .\quad (116).$$

Similarly

$$\Lambda^{2n+1}\frac{d^{2n+1}\theta_{1,1}}{dy_0^{\,2n+1}}=(-1)^{n+\frac{1}{2}}\left(\frac{2}{\pi}\right)^{\frac{1}{2}}\left(4\gamma\gamma'\Lambda^2\frac{d}{d\gamma}\right)^n \gamma^{\frac{1}{4}}\gamma'^{\frac{1}{4}}\Lambda^{\frac{3}{2}}\quad .\ .\ .\ .\ .\quad (117).$$

35. Another form may be given to several of these formulæ. Let

$$\left.\begin{aligned}\log p&=p'\\ \log q&=q'\\ 2\log r&=r'\end{aligned}\right\}\quad .\ .\ .\ .\ .\ .\ .\ .\ .\ .\quad (118).$$

Then

$$\frac{dp'}{d\kappa}=\frac{1}{p}\frac{dp}{d\kappa}=\frac{\pi^2}{2\kappa\kappa'^2\mathrm{K}^2}$$

and therefore

$$4cc'\mathrm{K}^2\frac{d}{dc}=\pi^2\frac{d}{dp'}\quad .\ .\ .\ .\ .\ .\ .\ .\ .\quad (119),$$

and

$$4\gamma\gamma'\Lambda^2\frac{d}{d\gamma}=\pi^2\frac{d}{dq'}.\quad .\ .\ .\ .\ .\ .\ .\ .\ .\ .\quad (120)$$

These formulæ practically contain the expansions in powers of x of the single theta-functions; restoring κ, κ', these are

$$\left.\begin{aligned}
\theta_{0,0}\left(\frac{\mathrm{K}x}{\pi}\right)&=\left(\frac{2}{\pi}\right)^{\frac{1}{2}}\left[\mathrm{K}^{\frac{1}{2}}-\frac{x^2}{2!}\frac{d.\mathrm{K}^{\frac{1}{2}}}{dp'}+\frac{x^4}{4!}\frac{d^2.\mathrm{K}^{\frac{1}{2}}}{dp'^2}-\frac{x^6}{6!}\frac{d^3.\mathrm{K}^{\frac{1}{2}}}{dp'^3}+\ldots\right]\\
\theta_{0,1}\left(\frac{\mathrm{K}x}{\pi}\right)&=\left(\frac{2}{\pi}\right)^{\frac{1}{2}}\left[(\kappa'\mathrm{K})^{\frac{1}{2}}-\frac{x^2}{2!}\frac{d.(\kappa'\mathrm{K})^{\frac{1}{2}}}{dp'}+\frac{x^4}{4!}\frac{d^2.(\kappa'\mathrm{K})^{\frac{1}{2}}}{dp'^2}-\frac{x^6}{6!}\frac{d^3.(\kappa'\mathrm{K})^{\frac{1}{2}}}{dp'^3}+\ldots\right]\\
\theta_{1,0}\left(\frac{\mathrm{K}x}{\pi}\right)&=\left(\frac{2}{\pi}\right)\left[(\kappa\mathrm{K})^{\frac{1}{2}}-\frac{x^2}{2!}\frac{d.(\kappa\mathrm{K})^{\frac{1}{2}}}{dp'}+\frac{x^4}{4!}\frac{d^2.(\kappa\mathrm{K})^{\frac{1}{2}}}{dp'^2}-\frac{x^6}{6!}\frac{d^3.(\kappa\mathrm{K})^{\frac{1}{2}}}{dp'^3}+\ldots\right]\\
\frac{1}{i}\,\theta_{1,1}\left(\frac{\mathrm{K}x}{\pi}\right)&=\left(\frac{2}{\pi^3}\right)^{\frac{1}{2}}\left[(\kappa\kappa'\mathrm{K}^3)^{\frac{1}{2}}x-\frac{x^3}{3!}\frac{d.(\kappa\kappa'\mathrm{K}^3)^{\frac{1}{2}}}{dp'}+\frac{x^5}{5!}\frac{d^2.(\kappa\kappa'\mathrm{K}^3)^{\frac{1}{2}}}{dp'^2}-\ldots\right]
\end{aligned}\right\}\;.\quad(121).$$

Expanding the cosines in the right-hand side of

$$\theta_{0,0}\left(\frac{\mathrm{K}x}{\pi}\right)=1+2p\cos x+2p^4\cos 2x+2p^9\cos 3x+\ \ldots$$

and equating coefficients of x^n, we obtain

$$\frac{d^n.\mathrm{K}^{\frac{1}{2}}}{dp'^n}=(2\pi)^{\frac{1}{2}}\{p+4^np^4+9^np^9+16^np^{16}+\ \ldots\}$$

which is easily deducible from

$$\left(\frac{2\mathrm{K}}{\pi}\right)^{\frac{1}{2}}=1+2p+2p^4+2p^9+\ \ldots$$

and so verifies the above expansions.

Similarly

$$\frac{d^n.(\kappa'\mathrm{K})^{\frac{1}{2}}}{dp'^n}=(2\pi)^{\frac{1}{2}}\{-p+4^np^4-9^np^9+16^np^{16}-\ \ldots\}$$

$$\frac{d^n.(\kappa\mathrm{K})^{\frac{1}{2}}}{dp'^n}=(2\pi)^{\frac{1}{2}}\left\{\frac{1}{4^n}p^{\frac{1}{4}}+\left(\frac{9}{4}\right)^np^{\frac{9}{4}}+\left(\frac{25}{4}\right)^np^{\frac{25}{4}}+\ \ldots\right\}$$

$$\frac{d^n.(\kappa\kappa'\mathrm{K}^3)^{\frac{1}{2}}}{dp'^n}=(2\pi^3)^{\frac{1}{2}}\left\{\frac{1}{4^n}p^{\frac{1}{4}}-\left(\frac{9}{4}\right)^np^{\frac{9}{4}}+\left(\frac{25}{4}\right)^np^{\frac{25}{4}}-\ \ldots\right\}\quad .\ .\ .\ .\ (122).$$

36. By means of the same formulæ it is possible to obtain expressions for all the constant coefficients which arise in the expansions of all the ϑ's in powers of x and y. Since by formula (4)

$$\vartheta_0(x,\,y)=\vartheta_0(-x,-y)$$

it follows that

$$\vartheta_0=c_0-\frac{1}{2!}(\mathrm{B}_{0,0},\ \mathrm{B}_{0,1},\ \mathrm{B}_{0,2}\between x,\,y)^2+\ldots$$

$$+\frac{(-1)^n}{2n!}(\mathrm{N}_{0,0},\ \mathrm{N}_{0,1},\ \mathrm{N}_{0,2},\ldots,\ \mathrm{N}_{0,r},\ldots,\ \mathrm{N}_{0,2n}\between x,\,y)^{2n}+\ldots\quad .\ .\ (123)$$

where

$$(-1)^n\mathrm{N}_{0,s}=\frac{d^{2n}\vartheta_0}{dx_0^{\ 2n-s}dy_0^{\ s}}$$

the zero subscript in the differential coefficient implying that the variables are made to vanish after differentiation. Now

$$\vartheta_0=\theta_{0,0}(x)\theta_{0,0}(y)-\frac{r'K\Lambda}{\pi^2}\frac{d\theta_{0,0}}{dx}\frac{d\theta_{0,0}}{dy}+\left(\frac{r'K\Lambda}{\pi^2}\right)^2\frac{1}{2!}\frac{d^2\theta_{0,0}}{dx^2}\frac{d^2\theta_{0,0}}{dy^2}-\dots$$

Remembering that an uneven differential of an even function is an uneven function and that an even differential is an even function, we have

$$(-1)^n N_{0,2s}=\frac{d^{2n-2s}\theta_{0,0}}{dx_0^{\,2n-2s}}\frac{d^{2s}\theta_{0,0}}{dy_0^{\,2s}}+\frac{1}{2!}\left(\frac{r'K\Lambda}{\pi^2}\right)^2\frac{d^{2n-2s+2}\theta_{0,0}}{dx_0^{\,2n-2s+2}}\frac{d^{2s+2}\theta_{0,0}}{dy_0^{\,2s+2}}+\dots$$

$$(-1)^n N_{0,2s+1}=-\frac{r'K\Lambda}{\pi^2}\frac{d^{2n-2s}\theta_{0,0}}{dx_0^{\,2n-2s}}\frac{d^{2s+2}\theta_{0,0}}{dy_0^{\,2s+2}}-\frac{1}{3!}\left(\frac{r'K\Lambda}{\pi^2}\right)^3\frac{d^{2n-2s+2}\theta_{0,0}}{dx_0^{\,2n-2s+2}}\frac{d^{2s+4}\theta_{0,0}}{dy_0^{\,2s+4}}-\dots$$

Let

$$\Delta_1=\frac{2}{\pi}\left[1+\frac{r'^2}{2!}\frac{d^2}{dp'dq'}+\frac{r'^4}{4!}\frac{d^4}{dp'^2dq'^2}+\dots\right]$$

$$=\frac{2}{\pi}\cosh\left[r'\left(\frac{d^2}{dp'dq'}\right)^{\frac{1}{2}}\right] \quad \dots\dots\dots \quad (124),$$

$$\Delta_2=\frac{2}{\pi}\left[1+\frac{r'^2}{3!}\frac{d^2}{dp'dq'}+\frac{r'^4}{5!}\frac{d^4}{dp'^2dq'^2}+\dots\right]$$

$$=\frac{2}{\pi r'\left(\frac{d^2}{dp'dq'}\right)^{\frac{1}{2}}}\sinh\left[r'\left(\frac{d^2}{dp'dq'}\right)^{\frac{1}{2}}\right] \quad \dots\dots\dots \quad (125).$$

Then we have

$$c_0=\Delta_1.K^{\frac{1}{2}}\Lambda^{\frac{1}{2}} \quad \dots\dots\dots \quad (126),$$

$$N_{0,2s}=\left(\frac{\pi}{K}\right)^{2(n-s)}\left(\frac{\pi}{\Lambda}\right)^{2s}\frac{d^n}{dp'^{\,n-s}dq'^{\,s}}.c_0 \quad \dots\dots\dots \quad (127),$$

and

$$N_{0,2s+1}=\frac{r'K\Lambda}{\pi^2}\left(\frac{\pi}{K}\right)^{2(n-s)}\left(\frac{\pi}{\Lambda}\right)^{2(s+1)}\frac{d^{n+1}}{dp'^{\,n-s}dq'^{\,s+1}}\Delta_2K^{\frac{1}{2}}\Lambda^{\frac{1}{2}} \quad \dots\dots \quad (128).$$

The remaining formulæ are obtained by an exactly similar process and are as follow.

$$\vartheta_1=c_1-\frac{1}{2!}(B_{1,0},\ B_{1,1},\ B_{1,2}\between x,\ y)^2+\dots$$

$$+\frac{(-1)^n}{2n!}(N_{1,0},\ N_{1,1},\ \dots,\ N_{1,s},\ \dots,\ N_{1,2n}\between x,\ y)^{2n}+\dots \quad \dots\dots \quad (129),$$

where

$$c_1=\Delta_1.K^{\frac{1}{2}}c^{\frac{1}{4}}\Lambda^{\frac{1}{2}} \quad \dots\dots\dots \quad (130),$$

$$N_{1,2s}=\left(\frac{\pi}{K}\right)^{2(n-s)}\left(\frac{\pi}{\Lambda}\right)^{2s}\frac{d^n}{dp'^{\,n-s}dq'^{\,s}}c_1 \quad \dots\dots\dots \quad (131),$$

and

$$N_{1,2s+1}=\frac{r'K\Lambda}{\pi^2}\left(\frac{\pi}{K}\right)^{2(n-s)}\left(\frac{\pi}{\Lambda}\right)^{2(s+1)}\frac{d^{n+1}}{dp'^{\,n-s}dq'^{\,s+1}}\Delta_2.K^{\frac{1}{2}}c^{\frac{1}{4}}\Lambda^{\frac{1}{2}} \quad \dots\dots \quad (132).$$

$$\vartheta_2 = c_2 - \frac{1}{2\,!}(\mathrm{B}_{2,0},\ \mathrm{B}_{2,1},\ \mathrm{B}_{2,2})\!(x,\ y)^2 + \ldots$$

$$+ \frac{(-1)^n}{2n\,!}(\mathrm{N}_{2,0},\ \mathrm{N}_{2,1},\ \mathrm{N}_{2,2},\ \ldots,\ \mathrm{N}_{2,s},\ \ldots,\ \mathrm{N}_{2,2n})\!(x,\ y)^{2n} + \ldots \tag{133}$$

where

$$c_2 = \Delta_1 . \mathrm{K}^{\frac{1}{2}} \Lambda^{\frac{1}{2}} \gamma^{\frac{1}{4}} \tag{134}$$

$$\mathrm{N}_{2,2s} = \left(\frac{\pi}{\mathrm{K}}\right)^{2(n-s)} \left(\frac{\pi}{\Lambda}\right)^{2s} \frac{d^n}{dp'^{n-s} dq'^s} c_2 \tag{135}$$

and

$$\mathrm{N}_{2,2s+1} = \frac{r'\mathrm{K}\Lambda}{\pi^2} \left(\frac{\pi}{\mathrm{K}}\right)^{2(n-s)} \left(\frac{\pi}{\Lambda}\right)^{2(s+1)} \frac{d^{n+1}}{dp'^{n-s} dq'^{s+1}} \Delta_2 . \mathrm{K}^{\frac{1}{2}} \Lambda^{\frac{1}{2}} \gamma^{\frac{1}{4}} \tag{136}$$

$$\vartheta_3 = c_3 - \frac{1}{2\,!}(\mathrm{B}_{3,0},\ \mathrm{B}_{3,1},\ \mathrm{B}_{3,2})\!(x,\ y)^2 + \ldots$$

$$+ \frac{(-1)^n}{2n\,!}(\mathrm{N}_{3,0},\ \mathrm{N}_{3,1},\ \mathrm{N}_{3,2},\ \ldots,\ \mathrm{N}_{3,s},\ \ldots,\ \mathrm{N}_{3,2n})\!(x,\ y)^{2n} + \ldots \tag{137}$$

where

$$c_3 = \Delta_1 . \mathrm{K}^{\frac{1}{2}} c^{\frac{1}{4}} \Lambda^{\frac{1}{2}} \gamma^{\frac{1}{4}} \tag{138}$$

$$\mathrm{N}_{3,2s} = \left(\frac{\pi}{\mathrm{K}}\right)^{2(n-s)} \left(\frac{\pi}{\Lambda}\right)^{2s} \frac{d^n}{dp'^{n-s} dq'^s} c_3 \tag{139}$$

$$\mathrm{N}_{3,2s+1} = \frac{r'\mathrm{K}\Lambda}{\pi^2} \left(\frac{\pi}{\mathrm{K}}\right)^{2(n-s)} \left(\frac{\pi}{\Lambda}\right)^{2(s+1)} \frac{d^{n+1}}{dp'^{n-s} dq'^{s+1}} \Delta_2 . \mathrm{K}^{\frac{1}{2}} c^{\frac{1}{4}} \Lambda^{\frac{1}{2}} \gamma^{\frac{1}{4}} \tag{140}$$

$$\vartheta_4 = c_4 - \frac{1}{2\,!}(\mathrm{B}_{4,0},\ \mathrm{B}_{4,1},\ \mathrm{B}_{4,2})\!(x,\ y)^2 + \ldots$$

$$+ \frac{(-1)^n}{2n\,!}(\mathrm{N}_{4,0},\ \mathrm{N}_{4,1},\ \mathrm{N}_{4,2},\ \ldots,\ \mathrm{N}_{4,s},\ \ldots,\ \mathrm{N}_{4,2n})\!(x,\ y)^{2n} + \ldots \tag{141}$$

where

$$c_4 = \Delta_1 . \mathrm{K}^{\frac{1}{2}} c'^{\frac{1}{4}} \Lambda^{\frac{1}{2}} \tag{142}$$

$$\mathrm{N}_{4,2s} = \left(\frac{\pi}{\mathrm{K}}\right)^{2(n-s)} \left(\frac{\pi}{\Lambda}\right)^{2s} \frac{d^n}{dp'^{n-s} dq'^s} c_4 \tag{143}$$

$$\mathrm{N}_{4,2s+1} = \frac{r'\mathrm{K}\Lambda}{\pi^2} \left(\frac{\pi}{\mathrm{K}}\right)^{2(n-s)} \left(\frac{\pi}{\Lambda}\right)^{2(s+1)} \frac{d^{n+1}}{dp'^{n-s} dq'^{s+1}} \Delta_2 . \mathrm{K}^{\frac{1}{2}} c'^{\frac{1}{4}} \Lambda^{\frac{1}{2}} \tag{144}$$

$$\frac{1}{i}\vartheta_5=c_5x+c'_5y-\frac{1}{3\,!}(\mathrm{C}_{5,0},\ \mathrm{C}_{5,1},\ \mathrm{C}_{5,2},\ \mathrm{C}_{5,3})\!\!\!(\,x,\ y)^3+\ \ldots$$

$$+\frac{(-1)^n}{2n+1\,!}(\mathrm{P}_{5,0},\ \mathrm{P}_{5,1},\ \mathrm{P}_{5,2},\ \ldots,\ \mathrm{P}_{5,s},\ \ldots,\ \mathrm{P}_{5,2n+1})\!\!\!(\,x,\ y)^{2n+1}+\ \ldots \quad . \quad (145),$$

where

$$c_5=\frac{1}{\mathrm{K}}\Delta_1.c^{\frac{1}{4}}c'^{\frac{1}{4}}\mathrm{K}^{\frac{3}{2}}\Lambda^{\frac{1}{2}}\ \ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ . \quad (146),$$

$$c_5{}'=\frac{r'\mathrm{K}\Lambda}{\pi}\Delta_2.c^{\frac{1}{4}}c'^{\frac{1}{4}}\mathrm{K}^{\frac{1}{2}}\left(\frac{\pi}{\Lambda}\right)^2\frac{d\Lambda^{\frac{1}{2}}}{dq'}\ \ .\ .\ .\ .\ .\ .\ . \quad (147),$$

$$\mathrm{P}_{5,2s}=\frac{1}{\mathrm{K}}\left(\frac{\pi}{\mathrm{K}}\right)^{2(n-s)}\left(\frac{\pi}{\Lambda}\right)^{2s}\frac{d^n}{dp'^{n-s}dq'^s}\mathrm{K}c_5\ .\ .\ .\ .\ .\ .\ . \quad (148),$$

$$\mathrm{P}_{5,2s+1}=\mathrm{K}\Lambda\left(\frac{\pi}{\mathrm{K}}\right)^{2(n-s)}\left(\frac{\pi}{\Lambda}\right)^{2s}\frac{d^n}{dp'^{n-s}dq'^s}\,\frac{c_5{}'}{\mathrm{K}\Lambda}\ \ .\ .\ .\ .\ .\ .\ . \quad (149).$$

$$\vartheta_6=c_6-\frac{1}{2\,!}(\mathrm{B}_{6,0},\ \mathrm{B}_{6,1},\ \mathrm{B}_{6,2})\!\!\!(\,x,\ y)^2+\ \ldots$$

$$+\frac{(-1)^n}{2n\,!}\Big(\mathrm{N}_{6,0},\ \mathrm{N}_{6,1},\ \mathrm{N}_{6,2},\ \ldots,\ \mathrm{N}_{6,s},\ \ldots,\ \mathrm{N}_{6,2n})\!\!\!(\,x,\ y)^{2n}+\ .\ .\ .\ .\ . \quad (150),$$

where

$$c_6=\Delta_1.\mathrm{K}^{\frac{1}{2}}c'^{\frac{1}{4}}\Lambda^{\frac{1}{2}}\gamma^{\frac{1}{4}}.\ \ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ . \quad (151),$$

$$\mathrm{N}_{6,2s}=\left(\frac{\pi}{\mathrm{K}}\right)^{2(n-s)}\left(\frac{\pi}{\Lambda}\right)^{2s}\frac{d^n}{dp'^{n-s}dq'^s}c_6\ \ .\ .\ .\ .\ .\ .\ .\ .\ .\ . \quad (152),$$

$$\mathrm{N}_{6,2s+1}=\frac{r'\mathrm{K}\Lambda}{\pi^2}\left(\frac{\pi}{\mathrm{K}}\right)^{2(n-s)}\left(\frac{\pi}{\Lambda}\right)^{2(s+1)}\frac{d^{n+1}}{dp'^{n-s}dq'^{s+1}}\Delta_2.\mathrm{K}^{\frac{1}{2}}c'^{\frac{1}{4}}\Lambda^{\frac{1}{2}}\gamma^{\frac{1}{4}}\ \ .\ .\ .\ . \quad (153).$$

$$\frac{1}{i}\vartheta_7=c_7x+c'_7y-\frac{1}{3\,!}(\mathrm{C}_{7,0},\ \mathrm{C}_{7,1},\ \mathrm{C}_{7,2},\ \mathrm{C}_{7,3})\!\!\!(\,x,\ y)^3+\ \ldots$$

$$+\frac{(-1)^n}{2n+1\,!}(\mathrm{P}_{7,0},\ \mathrm{P}_{7,1},\ \mathrm{P}_{7,2},\ \ldots,\ \mathrm{P}_{7,s},\ \ldots,\ \mathrm{P}_{7,2n+1})\!\!\!(\,x,\ y)^{2n+1}+\ .\ .\quad . \quad (154),$$

where

$$c_7=\frac{1}{\mathrm{K}}\Delta_1.c^{\frac{1}{4}}c'^{\frac{1}{4}}\mathrm{K}^{\frac{3}{2}}\gamma^{\frac{1}{4}}\Lambda^{\frac{1}{2}}\ \ .\ .\ .\ .\ .\ .\ .\ .\ .\ . \quad (155),$$

$$\mathrm{P}_{7,2s}=\frac{1}{\mathrm{K}}\left(\frac{\pi}{\mathrm{K}}\right)^{2(n-s)}\left(\frac{\pi}{\Lambda}\right)^{2s}\frac{d^n}{dp'^{n-s}dq'^s}\mathrm{K}c_7\ .\ .\ .\ .\ .\ .\ . \quad (156),$$

$$c'_7=\frac{r'\mathrm{K}\Lambda}{\pi^2}\Delta_2.c^{\frac{1}{4}}c'^{\frac{1}{4}}\mathrm{K}^{\frac{1}{2}}\frac{\pi^2}{\Lambda^2}\,\frac{d.\gamma^{\frac{1}{4}}\Lambda^{\frac{1}{2}}}{dq'}\ \ .\ .\ .\ .\ .\ .\ .\ . \quad (157),$$

$$\mathrm{P}_{7,2s+1}=\mathrm{K}\Lambda\left(\frac{\pi}{\mathrm{K}}\right)^{2(n-s)}\left(\frac{\pi}{\Lambda}\right)^{2s}\frac{d^n}{dp'^{n-s}dq'^s}\,\frac{c_7{}'}{\mathrm{K}\Lambda}\ \ .\ .\ .\ .\ .\ .\ . \quad (158).$$

$$\vartheta_8=c_8-\frac{1}{2!}(\mathrm{B}_{8,0},\ \mathrm{B}_{8,1},\ \mathrm{B}_{8,2})\!\!(x,\ y)^2+\ \ldots$$

$$+\frac{(-1)^n}{2n1!}(\mathrm{N}_{8,0},\ \mathrm{N}_{8,1},\ \mathrm{N}_{8,2},\ \ldots,\ \mathrm{N}_{8,s},\ \ldots,\ \mathrm{N}_{8,2n})\!\!(x,\ y)^{2n}+\ \ldots\ldots \quad (159),$$

where

$$c_8=\Delta_1.\mathrm{K}^{\frac{1}{2}}\gamma'^{\frac{1}{4}}\Lambda^{\frac{1}{2}} \quad \ldots\ldots \quad (160),$$

$$\mathrm{N}_{8,2s}=\left(\frac{\pi}{\mathrm{K}}\right)^{2(n-s)}\left(\frac{\pi}{\Lambda}\right)^{2s}\frac{d^n}{dp'^{n-s}dq'^s}c_8 \quad \ldots\ldots \quad (161),$$

$$\mathrm{N}_{8,2s+1}=\frac{r'\mathrm{K}\Lambda}{\pi^2}\left(\frac{\pi}{\mathrm{K}}\right)^{2(n-s)}\left(\frac{\pi}{\Lambda}\right)^{2(s+1)}\frac{d^{n+1}}{dp'^{n-s}dq'^{s+1}}\Delta_2.\mathrm{K}^{\frac{1}{2}}\gamma'^{\frac{1}{4}}\Lambda^{\frac{1}{2}} \quad \ldots\ldots \quad (162).$$

$$\vartheta_9=c_9-\frac{1}{2!}(\mathrm{B}_{9,0},\ \mathrm{B}_{9,1},\ \mathrm{B}_{9,2})\!\!(x,\ y)^2+\ \ldots$$

$$+\frac{(-1)^n}{2n!}(\mathrm{N}_{9,0},\ \mathrm{N}_{9,1},\ \mathrm{N}_{9,2},\ \ldots,\ \mathrm{N}_{9,s},\ \ldots,\ \mathrm{N}_{9,2n})\!\!(x,\ y)^{2n}+\ \ldots\ldots \quad (163),$$

where

$$c_9=\Delta_1.\,c^{\frac{1}{4}}\mathrm{K}^{\frac{1}{2}}\gamma'^{\frac{1}{4}}\Lambda^{\frac{1}{2}} \quad \ldots\ldots \quad (164),$$

$$\mathrm{N}_{9,2s}=\left(\frac{\pi}{\mathrm{K}}\right)^{2(n-s)}\left(\frac{\pi}{\Lambda}\right)^{2s}\frac{d^n}{dp'^{n-s}dq'^s}c_9 \quad \ldots\ldots \quad (165),$$

$$\mathrm{N}_{9,2s+1}=\frac{r'\mathrm{K}\Lambda}{\pi^2}\left(\frac{\pi}{\mathrm{K}}\right)^{2(n-s)}\left(\frac{\pi}{\Lambda}\right)^{2(s+1)}\frac{d^{n+1}}{dp'^{n-s}dq'^{s+1}}\Delta_2.\,c^{\frac{1}{4}}\mathrm{K}^{\frac{1}{2}}\gamma'^{\frac{1}{4}}\Lambda^{\frac{1}{2}} \quad \ldots\ldots \quad (166).$$

$$\frac{1}{i}\vartheta_{10}=c_{10}x+c'_{10}y-\frac{1}{3!}(\mathrm{C}_{10,0},\ \mathrm{C}_{10,1},\ \mathrm{C}_{10,2},\ \mathrm{C}_{10,3})\!\!(x,\ y)^3+\ \ldots$$

$$+\frac{(-1)^n}{2n+1!}(\mathrm{P}_{10,0},\ \mathrm{P}_{10,1},\ \mathrm{P}_{10,2},\ \ldots,\ \mathrm{P}_{10,s},\ \ldots,\ \mathrm{P}_{10,2n+1})\!\!(x,\ y)^{2n+1}+\ \ldots \quad (167),$$

where

$$c_{10}=\frac{r'\mathrm{K}\Lambda}{\pi^2}\Delta_2.\,\gamma^{\frac{1}{4}}\gamma'^{\frac{1}{4}}\Lambda^{\frac{1}{2}}\left(\frac{\pi}{\mathrm{K}}\right)^2\frac{d\mathrm{K}^{\frac{1}{2}}}{dp'} \quad \ldots\ldots \quad (168),$$

$$c'_{10}=\frac{1}{\Lambda}\Delta_1.\gamma^{\frac{1}{4}}\gamma'^{\frac{1}{4}}\mathrm{K}^{\frac{1}{2}}\Lambda^{\frac{3}{2}} \quad \ldots\ldots \quad (169),$$

$$\mathrm{P}_{10,2s}=\mathrm{K}\Lambda\left(\frac{\pi}{\mathrm{K}}\right)^{2(n-s)}\left(\frac{\pi}{\Lambda}\right)^{2s}\frac{d^n}{dp'^{n-s}dq'^s}\,\frac{c_{10}}{\mathrm{K}\Lambda} \quad \ldots\ldots \quad (170),$$

$$\mathrm{P}_{10,2s+1}=\frac{1}{\Lambda}\left(\frac{\pi}{\mathrm{K}}\right)^{2(n-s)}\left(\frac{\pi}{\Lambda}\right)^{2s}\frac{d^n}{dp'^{n-s}dq'^s}\Lambda c'_{10} \quad \ldots\ldots \quad (171).$$

$$\frac{1}{i}\vartheta_{11}=c_{11}x+c'_{11}y-\frac{1}{3\,!}(\mathrm{C}_{11,0},\ \mathrm{C}_{11,1},\ \mathrm{C}_{11,2},\ \mathrm{C}_{11,3}\between x,\ y)^3+\ldots$$

$$+\frac{(-1)^n}{2n+1\,!}(\mathrm{P}_{11,0},\ \mathrm{P}_{11,1},\ \mathrm{P}_{11,2},\ \ldots,\ \mathrm{P}_{11,s},\ \ldots,\ \mathrm{P}_{11,2n+1}\between x,\ y)^{2n+1}+\ldots \quad (172),$$

where

$$c_{11}=\frac{r'\mathrm{K}\Lambda}{\pi^2}\Delta_2.\gamma^{\frac{1}{4}}\gamma'^{\frac{1}{4}}\Lambda^{\frac{1}{2}}\left(\frac{\pi}{\mathrm{K}}\right)^2\frac{d.c^{\frac{1}{4}}\mathrm{K}^{\frac{1}{2}}}{dp'} \quad (173),$$

$$c'_{11}=\frac{1}{\Lambda}\Delta_1.\gamma^{\frac{1}{4}}\gamma'^{\frac{1}{4}}c^{\frac{1}{4}}\mathrm{K}^{\frac{1}{2}}\Lambda^{\frac{3}{2}} \quad (174),$$

$$\mathrm{P}_{11,2s}=\mathrm{K}\Lambda\left(\frac{\pi}{\mathrm{K}}\right)^{2(n-s)}\left(\frac{\pi}{\Lambda}\right)^{2s}\frac{d^n}{dp'^{n-s}dq'^s}\,\frac{c_{11}}{\mathrm{K}\Lambda} \quad (175),$$

$$\mathrm{P}_{11,2s+1}=\frac{1}{\Lambda}\left(\frac{\pi}{\Lambda}\right)^{2(n-s)}\left(\frac{\pi}{\mathrm{K}}\right)^{2s}\frac{d^n}{dp'^{n-s}dq'^s}\Lambda c'_{11} \quad (176).$$

$$\vartheta_{12}=c_{12}-\frac{1}{2\,!}(\mathrm{B}_{12,0},\ \mathrm{B}_{12,1},\ \mathrm{B}_{12,2}\between x,\ y)^2+\ldots$$

$$+\frac{(-1)^n}{2n!}(\mathrm{N}_{12,0},\ \mathrm{N}_{12,1},\ \mathrm{N}_{12,2},\ \ldots,\ \mathrm{N}_{12,s},\ \ldots,\ \mathrm{N}_{12,2n}\between x,\ y)^{2n}+\ldots \quad (177),$$

where

$$c_{12}=\Delta_1.c'^{\frac{1}{4}}\mathrm{K}^{\frac{1}{2}}\gamma'^{\frac{1}{4}}\Lambda^{\frac{1}{2}}\ , \quad (178),$$

$$\mathrm{N}_{12,2s}=\left(\frac{\pi}{\mathrm{K}}\right)^{2(n-s)}\left(\frac{\pi}{\Lambda}\right)^{2s}\frac{d^n}{dp'^{n-s}dq'^s}\,c_{12} \quad (179),$$

$$\mathrm{N}_{12,2s+1}=\frac{r'\mathrm{K}\Lambda}{\pi^2}\left(\frac{\pi}{\mathrm{K}}\right)^{2(n-s)}\left(\frac{\pi}{\Lambda}\right)^{2(s+1)}\frac{d^{n+1}}{dp'^{n-s}dq'^{s+1}}\Delta_2.c'^{\frac{1}{4}}\mathrm{K}^{\frac{1}{2}}\gamma'^{\frac{1}{4}}\Lambda^{\frac{1}{2}} \quad (180).$$

$$\frac{1}{i}\vartheta_{13}=c_{13}x+c'_{13}y-\frac{1}{3\,!}(\mathrm{C}_{13,0},\ \mathrm{C}_{13,1},\ \mathrm{C}_{13,2},\ \mathrm{C}_{13,3}\between x,\ y)^3+\ldots$$

$$+\frac{(-1)^n}{2n+1\,!}(\mathrm{P}_{13,0},\ \mathrm{P}_{13,1},\ \mathrm{P}_{13,2},\ \ldots,\ \mathrm{P}_{13,s},\ \ldots,\ \mathrm{P}_{13,2n+1}\between x,\ y)^{2n+1}+\ldots \quad (181),$$

where

$$c_{13}=\frac{1}{\mathrm{K}}\Delta_1.c^{\frac{1}{4}}c'^{\frac{1}{4}}\mathrm{K}^{\frac{3}{2}}\gamma'^{\frac{1}{4}}\Lambda^{\frac{1}{2}} \quad (182),$$

$$c'_{13}=\frac{r'\mathrm{K}\Lambda}{\pi^2}\Delta_2.c^{\frac{1}{4}}c'^{\frac{1}{4}}\mathrm{K}^{\frac{1}{2}}\left(\frac{\pi}{\Lambda}\right)^2\frac{d\gamma'^{\frac{1}{4}}\Lambda^{\frac{1}{2}}}{dq'} \quad (183),$$

$$\mathrm{P}_{13,2s}=\frac{1}{\mathrm{K}}\left(\frac{\pi}{\mathrm{K}}\right)^{2(n-s)}\left(\frac{\pi}{\Lambda}\right)^{2s}\frac{d^n}{dp'^{n-s}dq'^s}\,\mathrm{K}c_{13} \quad (184),$$

$$\mathrm{P}_{13,2s+1}=\mathrm{K}\Lambda\left(\frac{\pi}{\mathrm{K}}\right)^{2(n-s)}\left(\frac{\pi}{\Lambda}\right)^{2s}\frac{d^n}{dp'^{n-s}dq'^s}\,\frac{c'_{13}}{\mathrm{K}\Lambda}. \quad (185).$$

$$\frac{1}{i}\vartheta_{14}=c_{14}x+c'_{14}y-\frac{1}{3!}(\mathrm{C}_{14,0},\ \mathrm{C}_{14,1},\ \mathrm{C}_{14,2},\ \mathrm{C}_{14,3}\between x,\ y)^3+\dots$$
$$+\frac{(-1)^n}{2n+1!}(\mathrm{P}_{14,0},\ \mathrm{P}_{14,1},\ \mathrm{P}_{14,2},\ \dots,\ \mathrm{P}_{14,s},\ \dots,\ \mathrm{P}_{14,2n+1}\between x,\ y)^{2n+1}+\dots \quad (186),$$

where

$$c_{14}=\frac{r'\mathrm{K}\Lambda}{\pi^2}\Delta_2.\gamma^{\frac{1}{4}}\gamma'^{\frac{1}{4}}\Lambda^{\frac{1}{2}}\left(\frac{\pi}{\mathrm{K}}\right)^2\frac{d.c'^{\frac{1}{4}}\mathrm{K}^{\frac{1}{2}}}{dp'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (187),$$

$$c'_{14}=\frac{1}{\Lambda}\Delta_1.c^{\frac{1}{4}}\mathrm{K}^{\frac{1}{2}}\gamma^{\frac{1}{4}}\gamma'^{\frac{1}{4}}\Lambda^{\frac{3}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (188),$$

$$\mathrm{P}_{14,2s}=\mathrm{K}\Lambda\left(\frac{\pi}{\mathrm{K}}\right)^{2(n-s)}\left(\frac{\pi}{\Lambda}\right)^{2s}\frac{d^n}{dp'^{n-s}dq'^s}\frac{c_{14}}{\mathrm{K}\Lambda} \quad . \quad . \quad . \quad . \quad . \quad . \quad (189),$$

$$\mathrm{P}_{14,2s+1}=\frac{1}{\Lambda}\left(\frac{\pi}{\mathrm{K}}\right)^{2(n-s)}\left(\frac{\pi}{\Lambda}\right)^{2s}\frac{d^n}{dp'^{n-s}dq'^s}\Lambda c'_{14} \quad . \quad . \quad . \quad . \quad . \quad . \quad (190).$$

Lastly

$$\vartheta_{15}=c_{15}-\frac{1}{2!}(\mathrm{B}_{15,0},\ \mathrm{B}_{15,1},\ \mathrm{B}_{15,2}\between x,\ y)^2$$
$$+\frac{(-1)^n}{2n!}(\mathrm{N}_{15,0},\ \mathrm{N}_{15,1},\ \mathrm{N}_{15,2},\ \dots,\ \mathrm{N}_{15,s},\ \dots,\ \mathrm{N}_{15,2n}\between x,\ y)^{2n}+\dots \quad (191),$$

where

$$c_{15}=\frac{r'}{\pi^2}\Delta_2.c^{\frac{1}{4}}c'^{\frac{1}{4}}\gamma^{\frac{1}{4}}\gamma'^{\frac{1}{4}}\mathrm{K}^{\frac{3}{2}}\Lambda^{\frac{3}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (192),$$

$$\mathrm{N}_{15,2s}=\left(\frac{\pi}{\mathrm{K}}\right)^{2(n-s)}\left(\frac{\pi}{\Lambda}\right)^{2s}\frac{d^n}{dp'^{n-s}dq'^s}c_{15} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (193),$$

$$\mathrm{N}_{15,2s+1}=\frac{1}{\mathrm{K}\Lambda}\left(\frac{\pi}{\mathrm{K}}\right)^{2(n-s-1)}\left(\frac{\pi}{\Lambda}\right)^{2s}\frac{d^{n-1}}{dp'^{n-s-1}dq'^s}\Delta_1.c^{\frac{1}{4}}c'^{\frac{1}{4}}\gamma^{\frac{1}{4}}\gamma'^{\frac{1}{4}}\mathrm{K}^{\frac{1}{2}}\Lambda^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad (194).$$

37. The formulæ (42), (43), (44) give expressions for κ_1, κ_2, κ_3 in terms of the c's; and therefore, by the preceding, all the κ's of § 13 can be expressed in terms of K, Λ. In fact, we have

$$\left.\begin{aligned}\kappa_1&=\frac{(\Delta_1.\mathrm{K}^{\frac{1}{2}}\Lambda^{\frac{1}{2}}c^{\frac{1}{4}})\times(\Delta_1.\mathrm{K}^{\frac{1}{2}}\Lambda^{\frac{1}{2}}c^{\frac{1}{4}}\gamma^{\frac{1}{4}})}{(\Delta_1.\mathrm{K}^{\frac{1}{2}}\Lambda^{\frac{1}{2}})\times(\Delta_1.\mathrm{K}^{\frac{1}{2}}\Lambda^{\frac{1}{2}}\gamma^{\frac{1}{4}})}\\ \kappa'_1&=\frac{(\Delta_1.\mathrm{K}^{\frac{1}{2}}\Lambda^{\frac{1}{2}}c'^{\frac{1}{4}})\times(\Delta_1.\mathrm{K}^{\frac{1}{2}}\Lambda^{\frac{1}{2}}c'^{\frac{1}{4}}\gamma^{\frac{1}{4}})}{(\Delta_1.\mathrm{K}^{\frac{1}{2}}\Lambda^{\frac{1}{2}})\times(\Lambda_1.\mathrm{K}^{\frac{1}{2}}\Lambda^{\frac{1}{2}}\gamma^{\frac{1}{4}})}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (195)$$

$$\left.\begin{aligned}\kappa_2&=\frac{(\Delta_1.\mathrm{K}^{\frac{1}{2}}\Lambda^{\frac{1}{2}}c^{\frac{1}{4}}\gamma^{\frac{1}{4}})\times(\Delta_1.\mathrm{K}^{\frac{1}{2}}\Lambda^{\frac{1}{2}}c^{\frac{1}{4}}\gamma'^{\frac{1}{4}})}{(\Delta_1.\mathrm{K}^{\frac{1}{2}}\Lambda^{\frac{1}{2}}\gamma^{\frac{1}{4}})\times(\Delta_1.\mathrm{K}^{\frac{1}{2}}\Lambda^{\frac{1}{2}}\gamma'^{\frac{1}{4}}}\\ \kappa'_2&=\frac{(\Delta_1.\mathrm{K}^{\frac{1}{2}}\Lambda^{\frac{1}{2}}c'^{\frac{1}{4}}\gamma^{\frac{1}{4}})\times(\Delta_1.\mathrm{K}^{\frac{1}{2}}\Lambda^{\frac{1}{2}}c'^{\frac{1}{4}}\gamma'^{\frac{1}{4}})}{(\Delta_1.\mathrm{K}^{\frac{1}{2}}\Delta^{\frac{1}{2}}\gamma^{\frac{1}{4}})\times(\Delta_1.\mathrm{K}^{\frac{1}{2}}\Lambda^{\frac{1}{2}}\gamma'^{\frac{1}{4}})}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (196)$$

$$\left.\begin{aligned}\kappa_3&=\frac{(\Delta_1.\mathrm{K}^{\frac{1}{2}}\Lambda^{\frac{1}{2}}c^{\frac{1}{4}})\times(\Delta_1.\mathrm{K}^{\frac{1}{2}}\Lambda^{\frac{1}{2}}c^{\frac{1}{4}}\gamma'^{\frac{1}{4}})}{(\Delta_1.\mathrm{K}^{\frac{1}{2}}\Lambda^{\frac{1}{2}})\times(\Delta_1.\mathrm{K}^{\frac{1}{2}}\Lambda^{\frac{1}{2}}\gamma'^{\frac{1}{4}})}\\ \kappa'_3&=\frac{(\Delta_1.\mathrm{K}^{\frac{1}{2}}\Lambda^{\frac{1}{2}}c'^{\frac{1}{4}})\times(\Delta_1.\mathrm{K}^{\frac{1}{2}}\Lambda^{\frac{1}{2}}c'^{\frac{1}{4}}\gamma'^{\frac{1}{4}})}{(\Delta_1.\mathrm{K}^{\frac{1}{2}}\Lambda^{\frac{1}{2}})\times(\Delta_1.\mathrm{K}^{\frac{1}{2}}\Lambda^{\frac{1}{2}}\gamma'^{\frac{1}{4}})}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (197)$$

$$\mathrm{K}_3=\frac{r'}{\pi^2}\frac{(\Delta_1.\mathrm{K}^{\frac{1}{2}}\Lambda^{\frac{1}{2}}c^{\frac{1}{4}}\gamma^{\frac{1}{4}})\times(\Delta_1.\mathrm{K}^{\frac{1}{2}}\Lambda^{\frac{1}{2}}c'^{\frac{1}{4}}\gamma^{\frac{1}{4}})\times(\Delta_2.\mathrm{K}^{\frac{3}{2}}\Lambda^{\frac{3}{2}}c^{\frac{1}{4}}c'^{\frac{1}{4}}\gamma^{\frac{1}{4}}\gamma'^{\frac{1}{4}})}{(\Delta_1.\mathrm{K}^{\frac{1}{2}}\Lambda^{\frac{1}{2}})\times(\Delta_1.\mathrm{K}^{\frac{1}{2}}\Lambda^{\frac{1}{2}}\gamma^{\frac{1}{4}})\times(\Delta_1.\mathrm{K}^{\frac{1}{2}}\Lambda^{\frac{1}{2}}\gamma'^{\frac{1}{4}})} \quad . \quad . \quad . \quad . \quad (198)$$

$$\mathrm{K}_2=\frac{r'}{\pi^2}\frac{(\Delta_1.\mathrm{K}^{\frac{1}{2}}\Lambda^{\frac{1}{2}}c^{\frac{1}{4}})\times(\Delta_1.\mathrm{K}^{\frac{1}{2}}\Lambda^{\frac{1}{2}}c'^{\frac{1}{4}})\times(\Delta_2.\mathrm{K}^{\frac{3}{2}}\Lambda^{\frac{3}{2}}c^{\frac{1}{4}}c'^{\frac{1}{4}}\gamma^{\frac{1}{4}}\gamma'^{\frac{1}{4}})}{(\Delta_1.\mathrm{K}^{\frac{1}{2}}\Lambda^{\frac{1}{2}})\times(\Delta_1.\mathrm{K}^{\frac{1}{2}}\Lambda^{\frac{1}{2}}\gamma^{\frac{1}{4}})\times(\Delta_1.\mathrm{K}^{\frac{1}{2}}\Lambda^{\frac{1}{2}}\gamma'^{\frac{1}{4}})} \quad . \quad . \quad . \quad . \quad . \quad (199)$$

$$\mathrm{K}_1=\frac{r'}{\pi^2}\frac{(\Delta_1.\mathrm{K}^{\frac{1}{2}}\Lambda^{\frac{1}{2}}c^{\frac{1}{4}}\gamma'^{\frac{1}{4}})\times(\Delta_1.\mathrm{K}^{\frac{1}{2}}\Lambda^{\frac{1}{2}}c'^{\frac{1}{4}}\gamma'^{\frac{1}{4}})\times(\Delta_2.\mathrm{K}^{\frac{3}{2}}\Lambda^{\frac{3}{2}}c^{\frac{1}{4}}c'^{\frac{1}{4}}\gamma^{\frac{1}{4}}\gamma'^{\frac{1}{4}})}{(\Delta_1.\mathrm{K}^{\frac{1}{2}}\Lambda^{\frac{1}{2}})\times(\Delta_1.\mathrm{K}^{\frac{1}{2}}\Lambda^{\frac{1}{2}}\gamma^{\frac{1}{4}})\times(\Delta_1.\mathrm{K}^{\frac{1}{2}}\Lambda^{\frac{1}{2}}\gamma'^{\frac{1}{4}})} \quad . \quad . \quad . \quad . \quad (200).$$

SECTION III.

The Addition Theorem.

38. There is no addition theorem proper for the theta-functions, but the product of a theta-function of the sum of two variables by a theta-function of the difference of those variables can be expressed in terms of the functions of those variables. Thus with the previous notation for the single theta-functions

$$\theta_{0,1}{}^2(0)\theta_{0,1}(u+v)\theta_{0,1}(u-v)=\theta_{0,1}{}^2(u)\theta_{0,1}{}^2(v)-\theta_{1,1}{}^2(u)\theta_{1,1}{}^2(v), \quad . \quad . \quad . \quad . \quad . \quad (201),$$

$$\theta_{0,1}{}^2(0)\theta_{1,1}(u+v)\theta_{1,1}(u-v)=\theta_{1,1}{}^2(u)\theta_{0,1}{}^2(v)-\theta_{1,1}{}^2(v)\theta_{0,1}{}^2(u), \quad . \quad . \quad . \quad . \quad . \quad (202),$$

$$\begin{aligned}\theta_{0,0}(0)\theta_{0,1}(0)\theta_{1,1}(u+v)\theta_{0,1}(u-v)=&\,\theta_{0,1}(u)\theta_{1,1}(u)\theta_{1,0}(v)\theta_{0,0}(v)\\ &+\theta_{0,1}(v)\theta_{1,1}(v)\theta_{1,0}(u)\theta_{0,0}(u) \quad . \quad . \quad (203).\end{aligned}$$

$$\frac{\theta_{1,0}{}^2(0)}{\theta_{0,0}{}^2(0)}=\kappa, \quad \text{and} \quad \frac{\theta_{0,1}{}^2(0)}{\theta_{0,0}{}^2(0)}=\kappa'.$$

Dividing the third of these by the first and substituting for the θ's, there results the ordinary expression for $sn(u+v)$; and the division of the second by the first gives $sn(u+v)sn(u-v)$. The object of the present section is to obtain, by means of the theorem (23), the complete expression of the sum-and-difference theorem for the double theta-functions; it will be given by 256 formulæ similar to (201), (202), (203).

39. Some abbreviations in the notation are desirable; in the subsequent formulæ

$$\left.\begin{array}{lll}\Theta & \text{denotes} & \vartheta(x+\xi,\, y+\eta),\\ \Theta' & \quad ,, & \vartheta(x-\xi,\, y-\eta),\\ \vartheta & \quad ,, & \vartheta(x,\, y),\\ \theta & \quad ,, & \vartheta(\xi,\, \eta),\end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (204),$$

and, in order to simplify the first forms which are obtained from (23), some subsidiary equations are necessary. For example, writing down equations similar to (i)–(x) in Section I., but involving $\theta^2\vartheta^2$ instead of $c^2\vartheta^2$, the following simpler relations are found to be their equivalent and include (31), (32), (33) as particular cases.

$$\left.\begin{aligned}
\theta_0^2\vartheta_0^2+\theta_5^2\vartheta_5^2+\theta_{14}^2\vartheta_{14}^2&=\theta_1^2\vartheta_1^2+\theta_6^2\vartheta_6^2+\theta_{12}^2\vartheta_{12}^2\\
\theta_0^2\vartheta_0^2+\theta_{10}^2\vartheta_{10}^2+\theta_{13}^2\vartheta_{13}^2&=\theta_2^2\vartheta_2^2+\theta_9^2\vartheta_9^2+\theta_{12}^2\vartheta_{12}^2\\
\theta_0^2\vartheta_0^2+\theta_7^2\vartheta_7^2+\theta_{10}^2\vartheta_{10}^2&=\theta_3^2\vartheta_3^2+\theta_6^2\vartheta_6^2+\theta_8^2\vartheta_8^2\\
\theta_0^2\vartheta_0^2+\theta_5^2\vartheta_5^2+\theta_{11}^2\vartheta_{11}^2&=\theta_3^2\vartheta_3^2+\theta_4^2\vartheta_4^2+\theta_9^2\vartheta_9^2\\
\theta_0^2\vartheta_0^2+\theta_{11}^2\vartheta_{11}^2+\theta_{14}^2\vartheta_{14}^2&=\theta_2^2\vartheta_2^2+\theta_8^2\vartheta_8^2+\theta_{15}^2\vartheta_{15}^2\\
\theta_0^2\vartheta_0^2+\theta_7^2\vartheta_7^2+\theta_{13}^2\vartheta_{13}^2&=\theta_1^2\vartheta_1^2+\theta_4^2\vartheta_4^2+\theta_{15}^2\vartheta_{15}^2\\
\theta_1^2\vartheta_1^2+\theta_6^2\vartheta_6^2+\theta_{11}^2\vartheta_{11}^2&=\theta_2^2\vartheta_2^2+\theta_7^2\vartheta_7^2+\theta_9^2\vartheta_9^2\\
\theta_1^2\vartheta_1^2+\theta_4^2\vartheta_4^2+\theta_{10}^2\vartheta_{10}^2&=\theta_2^2\vartheta_2^2+\theta_5^2\vartheta_5^2+\theta_8^2\vartheta_8^2\\
\theta_4^2\vartheta_4^2+\theta_9^2\vartheta_9^2+\theta_{14}^2\vartheta_{14}^2&=\theta_6^2\vartheta_6^2+\theta_8^2\vartheta_8^2+\theta_{13}^2\vartheta_{13}^2\\
\theta_5^2\vartheta_5^2+\theta_{11}^2\vartheta_{11}^2+\theta_{14}^2\vartheta_{14}^2&=\theta_7^2\vartheta_7^2+\theta_{10}^2\vartheta_{10}^2+\theta_{13}^2\vartheta_{13}^2
\end{aligned}\right\}\quad . \quad . \quad . \quad (205)$$

Then by (23) we have, for instance,

$$\begin{aligned}
4c_{12}^2\Theta_{12}\Theta'_{12}=&\,\theta_0^2\vartheta_0^2+\theta_4^2\vartheta_4^2+\theta_8^2\vartheta_8^2+\theta_{12}^2\vartheta_{12}^2+\theta_3^2\vartheta_3^2+\theta_7^2\vartheta_7^2+\theta_{11}^2\vartheta_{11}^2+\theta_{15}^2\vartheta_{15}^2\\
&-\theta_1^2\vartheta_1^2-\theta_5^2\vartheta_5^2-\theta_9^2\vartheta_9^2-\theta_{13}^2\vartheta_{13}^2-\theta_2^2\vartheta_2^2-\theta_6^2\vartheta_6^2-\theta_{10}^2\vartheta_{10}^2-\theta_{14}^2\vartheta_{14}^2\\
=&\,4(\theta_{12}^2\vartheta_{12}^2+\theta_{11}^2\vartheta_{11}^2-\theta_{10}^2\vartheta_{10}^2-\theta_{13}^2\vartheta_{13}^2)
\end{aligned}$$

by equations (205) and therefore

$$c_{12}^2\Theta_{12}\Theta'_{12}=\theta_{12}^2\vartheta_{12}^2+\theta_{11}^2\vartheta_{11}^2-\theta_{10}^2\vartheta_{10}^2-\theta_{13}^2\vartheta_{13}^2$$

which reduces to an identity when ξ, η are both zero. Another similar set is

$$\left.\begin{aligned}
\theta_0^2\vartheta_5^2+\theta_2^2\vartheta_7^2+\theta_3^2\vartheta_6^2+\theta_1^2\vartheta_4^2&=\theta_5^2\vartheta_0^2+\theta_7^2\vartheta_2^2+\theta_6^2\vartheta_3^2+\theta_4^2\vartheta_1^2\\
\theta_0^2\vartheta_5^2+\theta_7^2\vartheta_2^2+\theta_9^2\vartheta_{12}^2+\theta_{14}^2\vartheta_{11}^2&=\theta_5^2\vartheta_0^2+\theta_2^2\vartheta_7^2+\theta_{12}^2\vartheta_9^2+\theta_{11}^2\vartheta_{14}^2\\
\theta_8^2\vartheta_{13}^2+\theta_{10}^2\vartheta_{15}^2+\theta_9^2\vartheta_{12}^2+\theta_{11}^2\vartheta_{14}^2&=\theta_{13}^2\vartheta_8^2+\theta_{15}^2\vartheta_{10}^2+\theta_{12}^2\vartheta_9^2+\theta_{14}^2\vartheta_{11}^2\\
\theta_8^2\vartheta_{13}^2+\theta_{15}^2\vartheta_{10}^2+\theta_1^2\vartheta_4^2+\theta_6^2\vartheta_3^2&=\theta_{13}^2\vartheta_8^2+\theta_{10}^2\vartheta_{15}^2+\theta_4^2\vartheta_1^2+\theta_3^2\vartheta_6^2\\
\theta_0^2\vartheta_5^2+\theta_{13}^2\vartheta_8^2+\theta_3^2\vartheta_6^2+\theta_{14}^2\vartheta_{11}^2&=\theta_5^2\vartheta_0^2+\theta_8^2\vartheta_{13}^2+\theta_6^2\vartheta_3^2+\theta_{11}^2\vartheta_{14}^2\\
\theta_6^2\vartheta_3^2+\theta_{14}^2\vartheta_{11}^2+\theta_7^2\vartheta_2^2+\theta_{15}^2\vartheta_{10}^2&=\theta_3^2\vartheta_6^2+\theta_{11}^2\vartheta_{14}^2+\theta_2^2\vartheta_7^2+\theta_{10}^2\vartheta_{15}^2\\
\theta_2^2\vartheta_7^2+\theta_{15}^2\vartheta_{10}^2+\theta_1^2\vartheta_4^2+\theta_{12}^2\vartheta_9^2&=\theta_7^2\vartheta_2^2+\theta_{10}^2\vartheta_{15}^2+\theta_4^2\vartheta_1^2+\theta_9^2\vartheta_{12}^2\\
\theta_{12}^2\vartheta_9^2+\theta_4^2\vartheta_1^2+\theta_{13}^2\vartheta_8^2+\theta_5^2\vartheta_0^2&=\theta_9^2\vartheta_{12}^2+\theta_1^2\vartheta_4^2+\theta_8^2\vartheta_{13}^2+\theta_0^2\vartheta_5^2\\
\theta_{12}^2\vartheta_9^2+\theta_6^2\vartheta_3^2+\theta_5^2\vartheta_0^2+\theta_{15}^2\vartheta_{10}^2&=\theta_9^2\vartheta_{12}^2+\theta_3^2\vartheta_6^2+\theta_0^2\vartheta_5^2+\theta_{10}^2\vartheta_{15}^2\\
\theta_{12}^2\vartheta_9^2+\theta_3^2\vartheta_6^2+\theta_{13}^2\vartheta_8^2+\theta_2^2\vartheta_7^2&=\theta_9^2\vartheta_{12}^2+\theta_6^2\vartheta_3^2+\theta_8^2\vartheta_{13}^2+\theta_7^2\vartheta_2^2\\
\theta_8^2\vartheta_{13}^2+\theta_2^2\vartheta_7^2+\theta_1^2\vartheta_4^2+\theta_{11}^2\vartheta_{14}^2&=\theta_{13}^2\vartheta_8^2+\theta_7^2\vartheta_2^2+\theta_4^2\vartheta_1^2+\theta_{14}^2\vartheta_{11}^2\\
\theta_0^2\vartheta_5^2+\theta_{15}^2\vartheta_{10}^2+\theta_1^2\vartheta_4^2+\theta_{14}^2\vartheta_{11}^2&=\theta_5^2\vartheta_0^2+\theta_{10}^2\vartheta_{15}^2+\theta_4^2\vartheta_1^2+\theta_{11}^2\vartheta_{14}^2
\end{aligned}\right\}\quad (206)$$

and by means of these we obtain, among others, the result

$$c_8^2\Theta_{13}\Theta'_{13}=\theta_8^2\vartheta_{13}^2-\theta_{13}^2\vartheta_8^2+\theta_{11}^2\vartheta_{14}^2-\theta_{14}^2\vartheta_{11}^2,$$

which as in the previous case reduces to an identity when ξ, η are both zero. A set similar to the last, and likewise necessary, is

$$\left.\begin{array}{l}
\theta_8^2\vartheta_2^2+\theta_{12}^2\vartheta_6^2+\theta_{14}^2\vartheta_4^2+\theta_{10}^2\vartheta_0^2=\theta_2^2\vartheta_8^2+\theta_6^2\vartheta_{12}^2+\theta_4^2\vartheta_{14}^2+\theta_0^2\vartheta_{10}^2\\
\theta_8^2\vartheta_2^2+\theta_6^2\vartheta_{12}^2+\theta_{11}^2\vartheta_1^2+\theta_5^2\vartheta_{15}^2=\theta_2^2\vartheta_8^2+\theta_{12}^2\vartheta_6^2+\theta_1^2\vartheta_{11}^2+\theta_{15}^2\vartheta_5^2\\
\theta_1^2\vartheta_{11}^2+\theta_5^2\vartheta_{15}^2+\theta_7^2\vartheta_{13}^2+\theta_3^2\vartheta_9^2=\theta_{11}^2\vartheta_1^2+\theta_{15}^2\vartheta_5^2+\theta_{13}^2\vartheta_7^2+\theta_9^2\vartheta_3^2\\
\theta_{13}^2\vartheta_7^2+\theta_3^2\vartheta_9^2+\theta_{14}^2\vartheta_4^2+\theta_0^2\vartheta_{10}^2=\theta_7^2\vartheta_{13}^2+\theta_9^2\vartheta_3^2+\theta_4^2\vartheta_{14}^2+\theta_{10}^2\vartheta_0^2\\
\theta_1^2\vartheta_{11}^2+\theta_4^2\vartheta_{14}^2+\theta_2^2\vartheta_8^2+\theta_7^2\vartheta_{13}^2=\theta_{11}^2\vartheta_1^2+\theta_{14}^2\vartheta_4^2+\theta_8^2\vartheta_2^2+\theta_{13}^2\vartheta_7^2\\
\theta_1^2\vartheta_{11}^2+\theta_{14}^2\vartheta_4^2+\theta_3^2\vartheta_9^2+\theta_{12}^2\vartheta_6^2=\theta_{11}^2\vartheta_1^2+\theta_4^2\vartheta_{14}^2+\theta_9^2\vartheta_3^2+\theta_6^2\vartheta_{12}^2\\
\theta_{12}^2\vartheta_6^2+\theta_9^2\vartheta_3^2+\theta_{10}^2\vartheta_0^2+\theta_{15}^2\vartheta_5^2=\theta_6^2\vartheta_{12}^2+\theta_3^2\vartheta_9^2+\theta_0^2\vartheta_{10}^2+\theta_5^2\vartheta_{15}^2\\
\theta_0^2\vartheta_{10}^2+\theta_{15}^2\vartheta_5^2+\theta_2^2\vartheta_8^2+\theta_{13}^2\vartheta_7^2=\theta_{10}^2\vartheta_0^2+\theta_5^2\vartheta_{15}^2+\theta_8^2\vartheta_2^2+\theta_7^2\vartheta_{13}^2\\
\theta_1^2\vartheta_{11}^2+\theta_0^2\vartheta_{10}^2+\theta_2^2\vartheta_8^2+\theta_3^2\theta_9^2=\theta_{11}^2\vartheta_1^2+\theta_{10}^2\vartheta_0^2+\theta_8^2\vartheta_2^2+\theta_9^2\vartheta_3^2\\
\theta_1^2\vartheta_{11}^2+\theta_{10}^2\vartheta_0^2+\theta_7^2\vartheta_{13}^2+\theta_{12}^2\vartheta_6^2=\theta_{11}^2\vartheta_1^2+\theta_0^2\vartheta_{10}^2+\theta_{13}^2\vartheta_7^2+\theta_6^2\vartheta_{12}^2\\
\theta_{12}^2\vartheta_6^2+\theta_{13}^2\vartheta_7^2+\theta_{14}^2\vartheta_4^2+\theta_{15}^2\vartheta_5^2=\theta_6^2\vartheta_{12}^2+\theta_7^2\vartheta_{13}^2+\theta_4^2\vartheta_{14}^2+\theta_5^2\vartheta_{15}^2\\
\theta_8^2\vartheta_2^2+\theta_3^2\vartheta_9^2+\theta_{14}^2\vartheta_4^2+\theta_5^2\vartheta_{15}^2=\theta_2^2\vartheta_8^2+\theta_9^2\vartheta_3^2+\theta_4^2\vartheta_{14}^2+\theta_{15}^2\vartheta_5^2
\end{array}\right\}.\quad(207).$$

The equations necessary to reduce the forms first given for such expressions as $c_2c_3\Theta_1\Theta'_0$ will be found at the end of the sixteen sets, each of sixteen equations.

40. First Set, with Θ'_0.

$$\begin{array}{l}
c_0c_0\,\Theta_0\,\Theta'_0=\theta_0\theta_0\,\vartheta_0\,\vartheta_0+\theta_7\,\theta_7\,\vartheta_7\,\vartheta_7+\theta_{10}\theta_{10}\vartheta_{10}\vartheta_{10}+\theta_{13}\theta_{13}\vartheta_{13}\vartheta_{13}\\
c_2c_3\,\Theta_1\,\Theta'_0=\theta_2\theta_3\,\vartheta_1\,\vartheta_0+\theta_6\,\theta_7\,\vartheta_4\,\vartheta_5-\theta_{10}\theta_{11}\vartheta_8\,\vartheta_9-\theta_{14}\theta_{15}\vartheta_{12}\vartheta_{13}\\
c_1c_3\,\Theta_2\,\Theta'_0=\theta_1\theta_3\,\vartheta_2\,\vartheta_0-\theta_5\,\theta_7\,\vartheta_4\,\vartheta_6+\theta_9\,\theta_{11}\vartheta_8\,\vartheta_{10}-\theta_{13}\theta_{15}\vartheta_{12}\vartheta_{14}\\
c_1c_2\,\Theta_3\,\Theta'_0=\theta_1\theta_2\,\vartheta_3\,\vartheta_0+\theta_4\,\theta_7\,\vartheta_5\,\vartheta_6+\theta_8\,\theta_{11}\vartheta_9\,\vartheta_{10}+\theta_{13}\theta_{14}\vartheta_{12}\vartheta_{15}\\
c_8c_{12}\Theta_4\,\Theta'_0=\theta_8\theta_{12}\vartheta_4\,\vartheta_0+\theta_{10}\theta_{14}\vartheta_2\,\vartheta_6-\theta_9\,\theta_{13}\vartheta_1\,\vartheta_5-\theta_{11}\theta_{15}\vartheta_3\,\vartheta_7\\
c_1c_4\,\Theta_5\,\Theta'_0=\theta_1\theta_4\,\vartheta_5\,\vartheta_0+\theta_0\,\theta_5\,\vartheta_1\,\vartheta_4+\theta_{10}\theta_{15}\vartheta_{11}\vartheta_{14}+\theta_{14}\theta_{11}\vartheta_{10}\vartheta_{15}\\
c_9c_{15}\Theta_6\,\Theta'_0=\theta_9\theta_{15}\vartheta_6\,\vartheta_0+\theta_1\,\theta_7\,\vartheta_8\,\vartheta_{14}-\theta_{13}\theta_{11}\vartheta_4\,\vartheta_2-\theta_3\,\theta_5\,\vartheta_{10}\vartheta_{12}\\
c_1c_6\,\Theta_7\,\Theta'_0=\theta_1\theta_6\,\vartheta_7\,\vartheta_0+\theta_0\,\theta_7\,\vartheta_1\,\vartheta_6+\theta_{10}\theta_{13}\vartheta_{12}\vartheta_{11}+\theta_{11}\theta_{12}\vartheta_{10}\vartheta_{13}\\
c_4c_{12}\Theta_8\,\Theta'_0=\theta_4\theta_{12}\vartheta_8\,\vartheta_0+\theta_5\,\theta_{13}\vartheta_9\,\vartheta_1-\theta_6\,\theta_{14}\vartheta_2\,\vartheta_{10}-\theta_7\,\theta_{15}\vartheta_3\,\vartheta_{11}\\
c_6c_{15}\Theta_9\,\Theta'_0=\theta_6\theta_{15}\vartheta_9\,\vartheta_0-\theta_{14}\theta_7\,\vartheta_8\,\vartheta_1+\theta_2\,\theta_{11}\vartheta_4\,\vartheta_{13}-\theta_3\,\theta_{10}\vartheta_{12}\vartheta_5\\
c_2c_8\,\Theta_{10}\Theta'_0=\theta_2\theta_8\,\vartheta_{10}\vartheta_0+\theta_{10}\theta_0\,\vartheta_8\,\vartheta_2+\theta_{13}\theta_7\,\vartheta_5\,\vartheta_{15}+\theta_5\,\theta_{15}\vartheta_7\,\vartheta_{13}\\
c_3c_8\,\Theta_{11}\Theta'_0=\theta_3\theta_8\,\vartheta_{11}\vartheta_0+\theta_{11}\theta_0\,\vartheta_8\,\vartheta_3+\theta_5\,\theta_{14}\vartheta_6\,\vartheta_{13}+\theta_6\,\theta_{13}\vartheta_5\,\vartheta_{14}\\
c_4c_8\,\Theta_{12}\Theta'_0=\theta_4\theta_8\,\vartheta_{12}\vartheta_0-\theta_1\,\theta_{13}\vartheta_5\,\vartheta_9+\theta_6\,\theta_{10}\vartheta_2\,\vartheta_{14}-\theta_7\,\theta_{11}\vartheta_3\,\vartheta_{15}\\
c_1c_{12}\Theta_{13}\Theta'_0=\theta_1\theta_{12}\vartheta_{13}\vartheta_0+\theta_{13}\theta_0\,\vartheta_1\,\vartheta_{12}-\theta_7\,\theta_{10}\vartheta_6\,\vartheta_{11}-\theta_6\,\theta_{11}\vartheta_7\,\vartheta_{10}\\
c_2c_{12}\Theta_{14}\Theta'_0=\theta_2\theta_{12}\vartheta_{14}\vartheta_0+\theta_{14}\theta_0\,\vartheta_2\,\vartheta_{12}-\theta_5\,\theta_{11}\vartheta_7\,\vartheta_9-\theta_7\,\theta_9\,\vartheta_5\,\vartheta_{11}\\
c_6c_9\,\Theta_{15}\Theta'_0=\theta_6\theta_9\,\vartheta_{15}\vartheta_0+\theta_5\,\theta_{10}\vartheta_{12}\vartheta_3+\theta_4\,\theta_{11}\vartheta_2\,\vartheta_{13}+\theta_7\,\theta_8\,\vartheta_1\,\vartheta_{14}\quad.
\end{array}$$

Königsberger, in his paper already referred to, gives a set of sixteen similar equations expressing the sixteen functions $\Theta_r\Theta'_0$ in terms of products to which the above are similar; but the constants on the left-hand side are for the most part different from his, with the result of making the combinations on the right-hand side different.

41. SECOND Set, with Θ'_1.

$$\begin{aligned}
c_2c_3\,\Theta_0\,\Theta'_1&=\theta_2\theta_3\,\vartheta_0\,\vartheta_1-\theta_6\,\theta_7\,\vartheta_4\vartheta_5\,-\theta_{10}\theta_{11}\vartheta_8\,\vartheta_9\,+\theta_{14}\theta_{15}\vartheta_{12}\vartheta_{13}\\
c_1c_1\,\Theta_1\,\Theta'_1&=\theta_1\theta_1\,\vartheta_1\,\vartheta_1-\theta_7\,\theta_7\,\vartheta_7\vartheta_7\,+\theta_{11}\theta_{11}\vartheta_{11}\vartheta_{11}-\theta_{13}\theta_{13}\vartheta_{13}\vartheta_{13}\\
c_0c_3\,\Theta_2\,\Theta'_1&=\theta_0\theta_3\,\vartheta_2\,\vartheta_1-\theta_5\,\theta_6\,\vartheta_4\vartheta_7\,+\theta_8\,\theta_{11}\vartheta_9\,\vartheta_{10}-\theta_{13}\theta_{14}\vartheta_{12}\vartheta_{15}\\
c_0c_2\,\Theta_3\,\Theta'_1&=\theta_0\theta_2\,\vartheta_3\,\vartheta_1+\theta_4\,\theta_6\,\vartheta_5\vartheta_7\,+\theta_8\,\theta_{10}\vartheta_9\,\vartheta_{11}+\theta_{13}\theta_{15}\vartheta_{12}\vartheta_{14}\\
c_3c_6\,\Theta_4\,\Theta'_1&=\theta_3\theta_6\,\vartheta_4\,\vartheta_1-\theta_2\,\theta_7\,\vartheta_5\vartheta_0\,-\theta_{11}\theta_{14}\vartheta_9\,\vartheta_{12}+\theta_{10}\theta_{15}\vartheta_8\,\vartheta_{13}\\
c_2c_6\,\Theta_5\,\Theta'_1&=\theta_2\theta_6\,\vartheta_5\,\vartheta_1+\theta_1\,\theta_5\,\vartheta_2\vartheta_6\,-\theta_{10}\theta_{14}\vartheta_9\,\vartheta_{13}-\theta_9\,\theta_{13}\vartheta_{10}\vartheta_{14}\\
c_4c_3\,\Theta_6\,\Theta'_1&=\theta_4\theta_3\,\vartheta_6\,\vartheta_1-\theta_2\,\theta_5\,\vartheta_7\vartheta_0\,+\theta_{11}\theta_{12}\vartheta_9\,\vartheta_{14}-\theta_{13}\theta_{10}\vartheta_8\,\vartheta_{15}\\
c_0c_6\,\Theta_7\,\Theta'_1&=\theta_0\theta_6\,\vartheta_7\,\vartheta_1+\theta_7\,\theta_1\,\vartheta_0\vartheta_6\,+\theta_{11}\theta_{13}\vartheta_{10}\vartheta_{12}+\theta_{10}\theta_{12}\vartheta_{11}\vartheta_{13}\\
c_0c_9\,\Theta_8\,\Theta'_1&=\theta_0\theta_9\,\vartheta_8\,\vartheta_1-\theta_3\,\theta_{10}\vartheta_2\vartheta_{11}-\theta_4\,\theta_{13}\vartheta_5\,\vartheta_{12}+\theta_7\,\theta_{14}\vartheta_6\,\vartheta_{15}\\
c_0c_8\,\Theta_9\,\Theta'_1&=\theta_0\theta_8\,\vartheta_9\,\vartheta_1-\theta_2\,\theta_{10}\vartheta_3\vartheta_{11}+\theta_5\,\theta_{13}\vartheta_4\,\vartheta_{12}-\theta_7\,\theta_{15}\vartheta_6\,\vartheta_{14}\\
c_3c_8\,\Theta_{10}\Theta'_1&=\theta_3\theta_8\,\vartheta_{10}\vartheta_1+\theta_{10}\theta_1\,\vartheta_3\vartheta_8\,-\theta_7\,\theta_{12}\vartheta_5\,\vartheta_{14}-\theta_5\,\theta_{14}\vartheta_7\,\vartheta_{12}\\
c_3c_9\,\Theta_{11}\Theta'_1&=\theta_3\theta_9\,\vartheta_{11}\vartheta_1+\theta_{11}\theta_1\,\vartheta_3\vartheta_9\,-\theta_5\,\theta_{15}\vartheta_7\,\vartheta_{13}-\theta_7\,\theta_{13}\vartheta_5\,\vartheta_{15}\\
c_4c_9\,\Theta_{12}\Theta'_1&=\theta_4\theta_9\,\vartheta_{12}\vartheta_1-\theta_0\,\theta_{13}\vartheta_8\vartheta_5\,-\theta_3\,\theta_{14}\vartheta_6\,\vartheta_{11}+\theta_7\,\theta_{10}\vartheta_2\,\vartheta_{15}\\
c_0c_{12}\Theta_{13}\Theta'_1&=\theta_0\theta_{12}\vartheta_{13}\vartheta_1+\theta_1\,\theta_{13}\vartheta_0\vartheta_{12}-\theta_7\,\theta_{11}\vartheta_6\,\vartheta_{10}-\theta_6\,\theta_{10}\vartheta_7\,\vartheta_{11}\\
c_0c_{15}\Theta_{14}\Theta'_1&=\theta_0\theta_{15}\vartheta_{14}\vartheta_1+\theta_{14}\theta_1\,\vartheta_0\vartheta_{15}-\theta_5\,\theta_{10}\vartheta_4\,\vartheta_{11}-\theta_4\,\theta_{11}\vartheta_5\,\vartheta_{10}\\
c_6c_8\,\Theta_{15}\Theta'_1&=\theta_6\theta_8\,\vartheta_{15}\vartheta_1+\theta_5\,\theta_{11}\vartheta_2\vartheta_{12}+\theta_0\,\theta_{14}\vartheta_9\,\vartheta_7\,+\theta_3\,\theta_{13}\vartheta_4\,\vartheta_{10}
\end{aligned}$$

42. THIRD Set, with Θ'_2.

$$\begin{aligned}
c_1c_3\,\Theta_0\,\Theta'_2&=\theta_1\theta_3\,\vartheta_0\,\vartheta_2-\theta_5\,\theta_7\,\vartheta_4\vartheta_6\,-\theta_9\,\theta_{11}\vartheta_8\,\vartheta_{10}+\theta_{13}\theta_{15}\vartheta_{12}\vartheta_{14}\\
c_0c_3\,\Theta_1\,\Theta'_2&=\theta_0\theta_3\,\vartheta_1\,\vartheta_2+\theta_5\,\theta_6\,\vartheta_4\vartheta_7\,-\theta_8\,\theta_{11}\vartheta_9\,\vartheta_{10}-\theta_{13}\theta_{14}\vartheta_{12}\vartheta_{15}\\
c_2c_2\,\Theta_2\,\Theta'_2&=\theta_2\theta_2\,\vartheta_2\,\vartheta_2+\theta_5\,\theta_5\,\vartheta_5\vartheta_5\,-\theta_{10}\theta_{10}\vartheta_{10}\vartheta_{10}-\theta_{13}\theta_{13}\vartheta_{13}\vartheta_{13}\\
c_0c_1\,\Theta_3\,\Theta'_2&=\theta_0\theta_1\,\vartheta_3\,\vartheta_2+\theta_4\,\theta_5\,\vartheta_6\vartheta_7\,+\theta_{10}\theta_{11}\vartheta_8\,\vartheta_9\,+\theta_{14}\theta_{15}\vartheta_{12}\vartheta_{13}\\
c_0c_6\,\Theta_4\,\Theta'_2&=\theta_0\theta_6\,\vartheta_4\,\vartheta_2-\theta_3\,\theta_5\,\vartheta_1\vartheta_7\,-\theta_{10}\theta_{12}\vartheta_8\,\vartheta_{14}+\theta_{11}\theta_{13}\vartheta_9\,\vartheta_{15}\\
c_1c_6\,\Theta_5\,\Theta'_2&=\theta_1\theta_6\,\vartheta_5\,\vartheta_2+\theta_2\,\theta_5\,\vartheta_1\vartheta_6\,-\theta_{10}\theta_{13}\vartheta_9\,\vartheta_{14}-\theta_9\,\theta_{14}\vartheta_{10}\vartheta_{13}\\
c_0c_4\,\Theta_6\,\Theta'_2&=\theta_0\theta_4\,\vartheta_6\,\vartheta_2-\theta_1\,\theta_5\,\vartheta_3\vartheta_7\,+\theta_{10}\theta_{14}\vartheta_8\,\vartheta_{12}-\theta_{11}\theta_{15}\vartheta_9\,\vartheta_{13}\\
c_1c_4\,\Theta_7\,\Theta'_2&=\theta_1\theta_4\,\vartheta_7\,\vartheta_2+\theta_7\,\theta_2\,\vartheta_1\vartheta_4\,+\theta_{11}\theta_{14}\vartheta_8\,\vartheta_{13}+\theta_8\,\theta_{13}\vartheta_{11}\vartheta_{14}\\
c_3c_9\,\Theta_8\,\Theta'_2&=\theta_3\theta_9\,\vartheta_8\,\vartheta_2-\theta_1\,\theta_{11}\vartheta_0\vartheta_{10}-\theta_7\,\theta_{13}\vartheta_{12}\vartheta_6\,+\theta_5\,\theta_{15}\vartheta_4\,\vartheta_{14}\\
c_3c_8\,\Theta_9\,\Theta'_2&=\theta_3\theta_8\,\vartheta_9\,\vartheta_2-\theta_0\,\theta_{11}\vartheta_1\vartheta_{10}+\theta_6\,\theta_{13}\vartheta_7\,\vartheta_{12}-\theta_5\,\theta_{14}\vartheta_4\,\vartheta_{15}\\
c_0c_8\,\Theta_{10}\Theta'_2&=\theta_0\theta_8\,\vartheta_{10}\vartheta_2+\theta_2\,\theta_{10}\vartheta_0\vartheta_8\,+\theta_7\,\theta_{15}\vartheta_5\,\vartheta_{13}+\theta_5\,\theta_{13}\vartheta_7\,\vartheta_{15}\\
c_0c_9\,\Theta_{11}\Theta'_2&=\theta_0\theta_9\,\vartheta_{11}\vartheta_2+\theta_2\,\theta_{11}\vartheta_0\vartheta_9\,+\theta_7\,\theta_{14}\vartheta_5\,\vartheta_{12}+\theta_5\,\theta_{12}\vartheta_7\,\vartheta_{14}\\
c_6c_8\,\Theta_{12}\Theta'_2&=\theta_6\theta_8\,\vartheta_{12}\vartheta_2-\theta_3\,\theta_{13}\vartheta_7\vartheta_9\,-\theta_0\,\theta_{14}\vartheta_4\,\vartheta_{10}+\theta_5\,\theta_{11}\vartheta_1\,\vartheta_{15}\\
c_3c_{12}\Theta_{13}\Theta'_2&=\theta_3\theta_{12}\vartheta_{13}\vartheta_2+\theta_2\,\theta_{13}\vartheta_3\vartheta_{12}-\theta_5\,\theta_{10}\vartheta_4\,\vartheta_{11}-\theta_4\,\theta_{11}\vartheta_5\,\vartheta_{10}\\
c_0c_{12}\Theta_{14}\Theta'_2&=\theta_0\theta_{12}\vartheta_{14}\vartheta_2+\theta_2\,\theta_{14}\vartheta_0\vartheta_{12}-\theta_{11}\theta_7\,\vartheta_5\,\vartheta_9\,-\theta_5\,\theta_9\,\vartheta_{11}\vartheta_7\\
c_1c_{12}\Theta_{15}\Theta'_2&=\theta_1\theta_{12}\vartheta_{15}\vartheta_2+\theta_{11}\theta_6\,\vartheta_5\vartheta_8\,+\theta_7\,\theta_{10}\vartheta_4\,\vartheta_9\,+\theta_0\,\theta_{13}\vartheta_3\,\vartheta_{14}
\end{aligned}$$

43. FOURTH Set—with Θ'_3.

$$c_1c_2\,\Theta_0\,\Theta'_3=\theta_1\theta_2\,\vartheta_0\,\vartheta_3-\theta_4\,\theta_7\,\vartheta_5\,\vartheta_6-\theta_8\,\theta_{11}\vartheta_9\,\vartheta_{10}+\theta_{13}\theta_{14}\vartheta_{12}\vartheta_{15}$$

$$c_0c_2\,\Theta_1\,\Theta'_3=\theta_0\theta_2\,\vartheta_1\,\vartheta_3+\theta_4\,\theta_6\,\vartheta_5\,\vartheta_7-\theta_8\,\theta_{10}\vartheta_9\,\vartheta_{11}-\theta_{13}\theta_{15}\vartheta_{12}\vartheta_{14}$$

$$c_0c_1\,\Theta_2\,\Theta'_3=\theta_0\theta_1\,\vartheta_2\,\vartheta_3-\theta_4\,\theta_5\,\vartheta_6\,\vartheta_7+\theta_{10}\theta_{11}\vartheta_8\,\vartheta_9-\theta_{14}\theta_{15}\vartheta_{12}\vartheta_{13}$$

$$c_3c_3\,\Theta_3\,\Theta'_3=\theta_3\theta_3\,\vartheta_3\,\vartheta_3-\theta_5\,\theta_5\,\vartheta_5\,\vartheta_5-\theta_{11}\theta_{11}\vartheta_{11}\vartheta_{11}+\theta_{13}\theta_{13}\vartheta_{13}\vartheta_{13}$$

$$c_1c_6\,\Theta_4\,\Theta'_3=\theta_1\theta_6\,\vartheta_4\,\vartheta_3-\theta_0\,\theta_7\,\vartheta_2\,\vartheta_5-\theta_{11}\theta_{12}\vartheta_9\,\vartheta_{14}+\theta_{10}\theta_{13}\vartheta_8\,\vartheta_{15}$$

$$c_0c_6\,\Theta_5\,\Theta'_3=\theta_0\theta_6\,\vartheta_5\,\vartheta_3+\theta_0\,\theta_3\,\vartheta_5\,\vartheta_6-\theta_{11}\theta_{13}\vartheta_8\,\vartheta_{14}-\theta_8\,\theta_{14}\vartheta_{11}\vartheta_{13}$$

$$c_1c_4\,\Theta_6\,\Theta'_3=\theta_1\theta_4\,\vartheta_6\,\vartheta_3-\theta_0\,\theta_5\,\vartheta_2\,\vartheta_7+\theta_{11}\theta_{14}\vartheta_9\,\vartheta_{12}-\theta_{10}\theta_{15}\vartheta_8\,\vartheta_{13}$$

$$c_0c_4\,\Theta_7\,\Theta'_3=\theta_0\theta_4\,\vartheta_7\,\vartheta_3+\theta_7\,\theta_3\,\vartheta_0\,\vartheta_4+\theta_{10}\theta_{14}\vartheta_9\,\vartheta_{13}+\theta_9\,\theta_{13}\vartheta_{10}\vartheta_{14}$$

$$c_2c_9\,\Theta_8\,\Theta'_3=\theta_2\theta_9\,\vartheta_8\,\vartheta_3-\theta_0\,\theta_{11}\vartheta_1\,\vartheta_{10}-\theta_7\,\theta_{12}\vartheta_6\,\vartheta_{13}+\theta_5\,\theta_{14}\vartheta_4\,\vartheta_{15}$$

$$c_2c_8\,\Theta_9\,\Theta'_3=\theta_2\theta_8\,\vartheta_9\,\vartheta_3-\theta_0\,\theta_{10}\vartheta_1\,\vartheta_{11}+\theta_7\,\theta_{13}\vartheta_6\,\vartheta_{12}-\theta_4\,\theta_{14}\,\vartheta_5\vartheta_{15}$$

$$c_1c_8\,\Theta_{10}\Theta'_3=\theta_1\theta_8\,\vartheta_{10}\vartheta_3+\theta_{10}\theta_3\,\vartheta_1\,\vartheta_8-\theta_7\,\theta_{14}\vartheta_5\,\vartheta_{12}-\theta_5\,\theta_{12}\vartheta_7\,\vartheta_{11}$$

$$c_0c_8\,\Theta_{11}\Theta'_3=\theta_0\theta_8\,\vartheta_{11}\vartheta_3+\theta_{11}\theta_3\,\vartheta_0\,\vartheta_8+\theta_5\,\theta_{13}\vartheta_6\,\vartheta_{14}+\theta_6\,\theta_{14}\vartheta_5\,\vartheta_{13}$$

$$c_0c_{15}\Theta_{12}\Theta'_3=\theta_0\theta_{15}\vartheta_{12}\vartheta_3-\theta_5\,\theta_{10}\vartheta_6\,\vartheta_9-\theta_4\,\theta_{11}\vartheta_7\,\vartheta_8+\theta_6\,\theta_{14}\vartheta_2\,\vartheta_{13}$$

$$c_2c_{12}\Theta_{13}\Theta'_3=\theta_2\theta_{12}\vartheta_{13}\vartheta_3+\theta_3\,\theta_{13}\vartheta_2\,\vartheta_{12}-\theta_5\,\theta_{11}\vartheta_{10}\vartheta_4-\theta_4\,\theta_{10}\vartheta_5\,\vartheta_{11}$$

$$c_4c_9\,\Theta_{14}\Theta'_3=\theta_4\theta_9\,\vartheta_{14}\vartheta_3+\theta_3\,\theta_{14}\vartheta_4\,\vartheta_9-\theta_0\,\theta_{13}\vartheta_7\,\vartheta_{10}-\theta_7\,\theta_{10}\vartheta_0\,\vartheta_{13}$$

$$c_0c_{12}\Theta_{15}\Theta'_3=\theta_0\theta_{12}\vartheta_{15}\vartheta_3+\theta_9\,\theta_5\,\vartheta_{10}\vartheta_6+\theta_{11}\theta_7\,\vartheta_4\,\vartheta_8+\theta_2\,\theta_{14}\vartheta_1\,\vartheta_{13}$$

44. FIFTH Set—with Θ'_4.

$$c_8c_{12}\Theta_0\,\Theta'_4=\theta_8\theta_{12}\vartheta_0\,\vartheta_4+\theta_{10}\theta_{14}\vartheta_2\,\vartheta_6+\theta_9\,\theta_{13}\vartheta_1\,\vartheta_5+\theta_{11}\theta_{15}\vartheta_3\,\vartheta_7$$

$$c_3c_6\,\Theta_1\,\Theta'_4=\theta_3\theta_6\,\vartheta_1\,\vartheta_4+\theta_2\,\theta_7\,\vartheta_0\,\vartheta_5-\theta_{11}\theta_{14}\vartheta_9\,\vartheta_{12}-\theta_{10}\theta_{15}\vartheta_8\,\vartheta_{13}$$

$$c_0c_6\,\Theta_2\,\Theta'_4=\theta_0\theta_6\,\vartheta_2\,\vartheta_4+\theta_3\,\theta_5\,\vartheta_1\,\vartheta_7+\theta_{10}\theta_{12}\vartheta_8\,\vartheta_{14}+\theta_{11}\theta_{13}\vartheta_9\,\vartheta_{15}$$

$$c_1c_6\,\Theta_3\,\Theta'_4=\theta_1\theta_6\,\vartheta_3\,\vartheta_4+\theta_0\,\theta_7\,\vartheta_2\,\vartheta_5+\theta_{11}\theta_{12}\vartheta_9\,\vartheta_{14}+\theta_{10}\theta_{13}\vartheta_8\,\vartheta_{15}$$

$$c_4c_4\,\Theta_4\,\Theta'_4=\theta_4\theta_4\,\vartheta_4\,\vartheta_4-\theta_7\,\theta_7\,\vartheta_7\,\vartheta_7-\theta_{13}\theta_{13}\vartheta_{13}\vartheta_{13}+\theta_{14}\theta_{14}\vartheta_{14}\vartheta_{14}$$

$$c_0c_1\,\Theta_5\,\Theta'_4=\theta_0\theta_1\,\vartheta_5\,\vartheta_4+\theta_5\,\theta_4\,\vartheta_0\,\vartheta_1+\theta_{14}\theta_{15}\vartheta_{10}\vartheta_{11}+\theta_{10}\theta_{11}\vartheta_{14}\vartheta_{15}$$

$$c_1c_3\,\Theta_6\,\Theta'_4=\theta_1\theta_3\,\vartheta_6\,\vartheta_4-\theta_5\,\theta_7\,\vartheta_0\,\vartheta_2-\theta_{13}\theta_{15}\vartheta_8\,\vartheta_{10}+\theta_9\,\theta_{11}\vartheta_{12}\vartheta_{14}$$

$$c_0c_3\,\Theta_7\,\Theta'_4=\theta_0\theta_3\,\vartheta_7\,\vartheta_4+\theta_7\,\theta_4\,\vartheta_0\,\vartheta_3+\theta_{13}\theta_{14}\vartheta_9\,\vartheta_{10}+\theta_9\,\theta_{10}\vartheta_{13}\vartheta_{14}$$

$$c_0c_{12}\Theta_8\,\Theta'_4=\theta_0\theta_{12}\vartheta_8\,\vartheta_4-\theta_2\,\theta_{14}\vartheta_6\,\vartheta_{10}+\theta_5\,\theta_9\,\vartheta_1\,\vartheta_{13}-\theta_7\,\theta_{11}\vartheta_3\,\vartheta_{15}$$

$$c_1c_{12}\Theta_9\,\Theta'_4=\theta_1\theta_{12}\vartheta_9\,\vartheta_4+\theta_8\,\theta_5\,\vartheta_0\,\vartheta_{13}-\theta_3\,\theta_{14}\vartheta_6\,\vartheta_{11}-\theta_7\,\theta_{10}\vartheta_2\,\vartheta_{15}$$

$$c_6c_8\,\Theta_{10}\Theta'_4=\theta_6\theta_8\,\vartheta_{10}\vartheta_4+\theta_{10}\theta_4\,\vartheta_6\,\vartheta_8+\theta_5\,\theta_{11}\vartheta_7\,\vartheta_9+\theta_7\,\theta_9\,\vartheta_5\,\vartheta_{11}$$

$$c_6c_9\,\Theta_{11}\Theta'_4=\theta_6\theta_9\,\vartheta_{11}\vartheta_4+\theta_4\,\theta_{11}\vartheta_6\,\vartheta_9+\theta_5\,\theta_{10}\vartheta_7\,\vartheta_8+\theta_7\,\theta_8\,\vartheta_5\,\vartheta_{10}$$

$$c_0c_8\,\Theta_{12}\Theta'_4=\theta_0\theta_8\,\vartheta_{12}\vartheta_4-\theta_{13}\theta_5\,\vartheta_1\,\vartheta_9-\theta_2\,\theta_{10}\vartheta_6\,\vartheta_{14}+\theta_7\,\theta_{15}\vartheta_{11}\vartheta_3$$

$$c_0c_9\,\Theta_{13}\Theta'_4=\theta_0\theta_9\,\vartheta_{13}\vartheta_4+\theta_4\,\theta_{13}\vartheta_0\,\vartheta_9-\theta_7\,\theta_{14}\vartheta_3\,\vartheta_{10}-\theta_3\,\theta_{10}\vartheta_7\,\vartheta_{14}$$

$$c_3c_9\,\Theta_{14}\Theta'_4=\theta_3\theta_9\,\vartheta_{14}\vartheta_4+\theta_4\,\theta_{14}\vartheta_3\,\vartheta_9-\theta_7\,\theta_{13}\vartheta_0\,\vartheta_{10}-\theta_0\,\theta_{10}\vartheta_7\,\vartheta_{13}$$

$$c_2c_9\,\Theta_{15}\Theta'_4=\theta_2\theta_9\,\vartheta_{15}\vartheta_4+\theta_7\,\theta_{12}\vartheta_1\,\vartheta_{10}+\theta_{14}\theta_5\,\vartheta_3\,\vartheta_8+\theta_0\,\theta_{11}\vartheta_6\,\vartheta_{13}$$

45. SIXTH Set, with Θ'_5.

$$c_1c_4\,\Theta_0\,\Theta'_5=\theta_1\theta_4\,\vartheta_0\,\vartheta_5-\theta_0\,\theta_5\vartheta_1\vartheta_4\,-\theta_{10}\theta_{15}\vartheta_{11}\vartheta_{14}+\theta_{11}\theta_{14}\vartheta_{10}\vartheta_{15}$$

$$c_2c_6\,\Theta_1\,\Theta'_5=\theta_2\theta_6\,\vartheta_1\,\vartheta_5-\theta_1\,\theta_5\vartheta_2\vartheta_6\,-\theta_{10}\theta_{14}\vartheta_9\,\vartheta_{13}+\theta_9\,\theta_{13}\vartheta_{10}\vartheta_{14}$$

$$c_1c_6\,\Theta_2\,\Theta'_5=\theta_1\theta_6\,\vartheta_2\,\vartheta_5-\theta_2\,\theta_5\vartheta_1\vartheta_6\,-\theta_{10}\theta_{13}\vartheta_9\,\vartheta_{14}+\theta_9\,\theta_{14}\vartheta_{10}\vartheta_{13}$$

$$c_0c_6\,\Theta_3\,\Theta'_5=\theta_0\theta_6\,\vartheta_3\,\vartheta_5-\theta_3\,\theta_5\vartheta_0\vartheta_6\,-\theta_{11}\theta_{13}\vartheta_8\,\vartheta_{14}+\theta_8\,\theta_{14}\vartheta_{11}\vartheta_{13}$$

$$c_0c_1\,\Theta_4\,\Theta'_5=\theta_0\theta_1\,\vartheta_4\,\vartheta_5-\theta_4\,\theta_5\vartheta_0\vartheta_1\,-\theta_{14}\theta_{15}\vartheta_{10}\vartheta_{11}+\theta_{10}\theta_{11}\vartheta_{14}\vartheta_{15}$$

$$c_0c_0\,\Theta_5\,\Theta'_5=\theta_0\theta_0\,\vartheta_5\,\vartheta_5-\theta_5\,\theta_5\vartheta_0\vartheta_0\,-\theta_{11}\theta_{11}\vartheta_{14}\vartheta_{14}+\theta_{14}\theta_{14}\vartheta_{11}\vartheta_{11}$$

$$c_1c_2\,\Theta_6\,\Theta'_5=\theta_1\theta_2\,\vartheta_6\,\vartheta_5-\theta_6\,\theta_5\vartheta_1\vartheta_2\,-\theta_{13}\theta_{14}\vartheta_9\,\vartheta_{10}+\theta_9\,\theta_{10}\vartheta_{13}\vartheta_{14}$$

$$c_0c_2\,\Theta_7\,\Theta'_5=\theta_0\theta_2\,\vartheta_7\,\vartheta_5-\theta_7\,\theta_5\vartheta_0\vartheta_2\,-\theta_{13}\theta_{15}\vartheta_8\,\vartheta_{10}+\theta_8\,\theta_{10}\vartheta_{13}\vartheta_{15}$$

$$c_1c_{12}\Theta_8\,\Theta'_5=\theta_1\theta_{12}\vartheta_8\,\vartheta_5-\theta_8\,\theta_5\vartheta_1\vartheta_{12}-\theta_3\,\theta_{14}\vartheta_7\,\vartheta_{10}+\theta_7\,\theta_{10}\vartheta_3\,\vartheta_{14}$$

$$c_0c_{12}\Theta_9\,\Theta'_5=\theta_0\theta_{12}\vartheta_9\,\vartheta_5-\theta_9\,\theta_5\vartheta_0\vartheta_{12}-\theta_2\,\theta_{14}\vartheta_7\,\vartheta_{11}+\theta_7\,\theta_{11}\vartheta_2\,\vartheta_{14}$$

$$c_6c_9\,\Theta_{10}\Theta'_5=\theta_6\theta_9\,\vartheta_{10}\vartheta_5-\theta_{10}\theta_5\vartheta_6\vartheta_9\,-\theta_2\,\theta_{13}\vartheta_1\,\vartheta_{14}+\theta_1\,\theta_{14}\vartheta_2\,\vartheta_{13}$$

$$c_6c_8\,\Theta_{11}\Theta'_5=\theta_6\theta_8\,\vartheta_{11}\vartheta_5-\theta_{11}\theta_5\vartheta_6\vartheta_8\,-\theta_7\,\theta_9\,\vartheta_4\,\vartheta_{10}+\theta_4\,\theta_{10}\vartheta_7\,\vartheta_9$$

$$c_1c_8\,\Theta_{12}\Theta'_5=\theta_1\theta_8\,\vartheta_{12}\vartheta_5-\theta_{12}\theta_5\vartheta_1\vartheta_8\,-\theta_3\,\theta_{10}\vartheta_7\,\vartheta_{14}+\theta_7\,\theta_{14}\vartheta_3\,\vartheta_{10}$$

$$c_0c_8\,\Theta_{13}\Theta'_5=\theta_0\theta_8\,\vartheta_{13}\vartheta_5-\theta_{13}\theta_5\vartheta_0\vartheta_8\,-\theta_6\,\theta_{14}\vartheta_3\,\vartheta_{11}+\theta_3\,\theta_{11}\vartheta_6\,\vartheta_{14}$$

$$c_3c_8\,\Theta_{14}\Theta'_5=\theta_3\theta_8\,\vartheta_{14}\vartheta_5-\theta_{14}\theta_5\vartheta_3\vartheta_8\,-\theta_7\,\theta_{12}\vartheta_1\,\vartheta_{10}+\theta_1\,\theta_{10}\vartheta_7\,\vartheta_{12}$$

$$c_2c_8\,\Theta_{15}\Theta'_5=\theta_2\theta_8\,\vartheta_{15}\vartheta_5-\theta_{15}\theta_5\vartheta_2\vartheta_8\,-\theta_7\,\theta_{13}\vartheta_0\,\vartheta_{10}+\theta_0\,\theta_{10}\vartheta_7\,\vartheta_{13}$$

46. SEVENTH Set, with Θ'_6.

$$c_9c_{15}\Theta_0\,\Theta'_6=\theta_9\theta_{15}\vartheta_0\,\vartheta_6-\theta_1\,\theta_7\,\vartheta_8\vartheta_{14}-\theta_{13}\theta_{11}\vartheta_4\,\vartheta_2\,+\theta_3\,\theta_5\,\vartheta_{10}\vartheta_{12}$$

$$c_4c_3\,\Theta_1\,\Theta'_6=\theta_4\theta_3\,\vartheta_1\,\vartheta_6+\theta_2\,\theta_5\,\vartheta_0\vartheta_7\,-\theta_{11}\theta_{12}\vartheta_9\,\vartheta_{14}-\theta_{13}\theta_{10}\vartheta_8\,\vartheta_{15}$$

$$c_0c_4\,\Theta_2\,\Theta'_6=\theta_0\theta_4\,\vartheta_2\,\vartheta_6+\theta_1\,\theta_5\,\vartheta_3\vartheta_7\,+\theta_{10}\theta_{14}\vartheta_8\,\vartheta_{12}+\theta_{11}\theta_{15}\vartheta_9\,\vartheta_{13}$$

$$c_1c_4\,\Theta_3\,\Theta'_6=\theta_1\theta_4\,\vartheta_3\,\vartheta_6+\theta_0\,\theta_5\,\vartheta_2\vartheta_7\,+\theta_{11}\theta_{14}\vartheta_9\,\vartheta_{12}+\theta_{10}\theta_{15}\vartheta_8\,\vartheta_{13}$$

$$c_1c_3\,\Theta_4\,\Theta'_6=\theta_1\theta_3\,\vartheta_4\,\vartheta_6-\theta_5\,\theta_7\,\vartheta_0\vartheta_2\,+\theta_{13}\theta_{15}\vartheta_8\,\vartheta_{10}-\theta_9\,\theta_{11}\vartheta_{12}\vartheta_{14}$$

$$c_1c_2\,\Theta_5\,\Theta'_6=\theta_1\theta_2\,\vartheta_5\,\vartheta_6+\theta_5\,\theta_6\,\vartheta_1\vartheta_2\,-\theta_9\theta_{10}\vartheta_{13}\,\vartheta_{14}-\theta_{13}\theta_{14}\vartheta_9\,\vartheta_{10}$$

$$c_6c_6\,\Theta_6\,\Theta'_6=\theta_6\theta_6\,\vartheta_6\,\vartheta_6-\theta_5\,\theta_5\,\vartheta_5\vartheta_5\,+\theta_{13}\theta_{13}\vartheta_{13}\vartheta_{13}-\theta_{14}\theta_{14}\vartheta_{14}\vartheta_{14}$$

$$c_0c_1\,\Theta_7\,\Theta'_6=\theta_0\theta_1\,\vartheta_7\,\vartheta_6+\theta_7\,\theta_6\,\vartheta_0\vartheta_1\,+\theta_{10}\theta_{11}\vartheta_{12}\vartheta_{13}+\theta_{12}\theta_{13}\vartheta_{10}\vartheta_{11}$$

$$c_2c_{12}\Theta_8\,\Theta'_6=\theta_2\theta_{12}\vartheta_8\,\vartheta_6+\theta_7\,\theta_9\,\vartheta_3\vartheta_{13}-\theta_0\,\theta_{14}\vartheta_4\,\vartheta_{10}-\theta_5\,\theta_{11}\vartheta_1\,\vartheta_{15}$$

$$c_0c_{15}\Theta_9\,\Theta'_6=\theta_0\theta_{15}\vartheta_9\,\vartheta_6-\theta_5\,\theta_{10}\vartheta_3\vartheta_{12}+\theta_2\,\theta_{13}\vartheta_4\,\vartheta_{11}-\theta_8\,\theta_7\,\vartheta_1\,\vartheta_{14}$$

$$c_0c_{12}\Theta_{10}\Theta'_6=\theta_0\theta_{12}\vartheta_{10}\vartheta_6+\theta_{10}\theta_6\,\vartheta_0\vartheta_{12}+\theta_1\,\theta_{13}\vartheta_7\,\vartheta_{11}+\theta_7\,\theta_{11}\vartheta_1\,\vartheta_{12}$$

$$c_9c_4\,\Theta_{11}\Theta'_6=\theta_9\theta_4\,\vartheta_{11}\vartheta_6+\theta_{11}\theta_6\,\vartheta_9\vartheta_4\,+\theta_7\,\theta_{10}\vartheta_5\,\vartheta_8\,+\theta_5\,\theta_8\,\vartheta_7\,\vartheta_{10}$$

$$c_2c_8\,\Theta_{12}\Theta'_6=\theta_2\theta_8\,\vartheta_{12}\vartheta_6-\theta_{13}\theta_7\,\vartheta_3\vartheta_9\,-\theta_0\,\theta_{10}\vartheta_4\,\vartheta_{14}+\theta_5\,\theta_{15}\vartheta_1\,\vartheta_{11}$$

$$c_3c_8\,\Theta_{13}\Theta'_6=\theta_3\theta_8\,\vartheta_{13}\vartheta_6+\theta_{13}\theta_6\,\vartheta_3\vartheta_8\,-\theta_{14}\theta_5\,\vartheta_0\,\vartheta_{11}-\theta_0\,\theta_{11}\vartheta_5\,\vartheta_{14}$$

$$c_0c_8\,\Theta_{14}\Theta'_6=\theta_0\theta_8\,\vartheta_{14}\vartheta_6+\theta_{14}\theta_6\,\vartheta_0\vartheta_8\,-\theta_5\,\theta_{13}\vartheta_3\,\vartheta_{11}-\theta_3\,\theta_{11}\vartheta_5\,\vartheta_{13}$$

$$c_0c_9\,\Theta_{15}\Theta'_6=\theta_0\theta_9\,\vartheta_{15}\vartheta_6+\theta_7\,\theta_{14}\vartheta_1\vartheta_8\,+\theta_4\,\theta_{13}\vartheta_2\,\vartheta_{11}+\theta_3\,\theta_{10}\vartheta_5\,\vartheta_{12}$$

47. Eighth Set, with Θ'_7.

$$c_1c_6\,\Theta_0\,\Theta'_7=\theta_1\theta_6\,\vartheta_0\,\vartheta_7-\theta_0\,\theta_7\vartheta_1\vartheta_6-\theta_{11}\theta_{12}\vartheta_{10}\vartheta_{13}+\theta_{10}\theta_{13}\vartheta_{11}\vartheta_{12}$$
$$c_0c_6\,\Theta_1\,\Theta'_7=\theta_0\theta_6\,\vartheta_1\,\vartheta_7-\theta_1\,\theta_7\vartheta_0\vartheta_6-\theta_{10}\theta_{12}\vartheta_{11}\vartheta_{13}+\theta_{11}\theta_{13}\vartheta_{10}\vartheta_{12}$$
$$c_1c_4\,\Theta_2\,\Theta'_7=\theta_1\theta_4\,\vartheta_2\,\vartheta_7-\theta_2\,\theta_7\vartheta_1\vartheta_4-\theta_8\,\theta_{13}\vartheta_{11}\vartheta_{14}+\theta_{11}\theta_{14}\vartheta_8\,\vartheta_{13}$$
$$c_0c_4\,\Theta_3\,\Theta'_7=\theta_0\theta_4\,\vartheta_3\,\vartheta_7-\theta_3\,\theta_7\vartheta_0\vartheta_4-\theta_9\,\theta_{13}\vartheta_{10}\vartheta_{14}+\theta_{10}\theta_{14}\vartheta_9\,\vartheta_{13}$$
$$c_0c_3\,\Theta_4\,\Theta'_7=\theta_0\theta_3\,\vartheta_4\,\vartheta_7-\theta_4\,\theta_7\vartheta_0\vartheta_3-\theta_9\,\theta_{10}\vartheta_{13}\vartheta_{14}+\theta_{13}\theta_{14}\vartheta_9\,\vartheta_{10}$$
$$c_0c_2\,\Theta_5\,\Theta'_7=\theta_0\theta_2\,\vartheta_5\,\vartheta_7-\theta_5\,\theta_7\vartheta_0\vartheta_2-\theta_8\,\theta_{10}\vartheta_{13}\vartheta_{15}+\theta_{13}\theta_{15}\vartheta_8\,\vartheta_{10}$$
$$c_0c_1\,\Theta_6\,\Theta'_7=\theta_0\theta_1\,\vartheta_6\,\vartheta_7-\theta_6\,\theta_7\vartheta_0\vartheta_1-\theta_{12}\theta_{13}\vartheta_{10}\vartheta_{11}+\theta_{10}\theta_{11}\vartheta_{12}\vartheta_{13}$$
$$c_2c_2\,\Theta_7\,\Theta'_7=\theta_2\theta_2\,\vartheta_7\,\vartheta_7-\theta_7\,\theta_7\vartheta_2\vartheta_2-\theta_{14}\theta_{14}\vartheta_{11}\vartheta_{11}+\theta_{11}\theta_{11}\vartheta_{14}\vartheta_{14}$$
$$c_3c_{12}\Theta_8\,\Theta'_7=\theta_3\theta_{12}\vartheta_8\,\vartheta_7-\theta_8\,\theta_7\vartheta_3\vartheta_{12}-\theta_1\,\theta_{14}\vartheta_{10}\vartheta_5+\theta_5\,\theta_{10}\vartheta_1\,\vartheta_{14}$$
$$c_2c_{12}\Theta_9\,\Theta'_7=\theta_2\theta_{12}\vartheta_9\,\vartheta_7-\theta_9\,\theta_7\vartheta_2\vartheta_{12}-\theta_0\,\theta_{14}\vartheta_{11}\vartheta_5+\theta_5\,\theta_{11}\vartheta_0\,\vartheta_{14}$$
$$c_4c_9\,\Theta_{10}\Theta'_7=\theta_4\theta_9\,\vartheta_{10}\vartheta_7-\theta_{10}\theta_7\vartheta_4\vartheta_9-\theta_5\,\theta_8\,\vartheta_6\,\vartheta_{11}+\theta_6\,\theta_{11}\vartheta_5\,\vartheta_8$$
$$c_0c_{12}\Theta_{11}\Theta'_7=\theta_0\theta_{12}\vartheta_{11}\vartheta_7-\theta_{11}\theta_7\vartheta_0\vartheta_{12}-\theta_1\,\theta_{13}\vartheta_6\,\vartheta_{10}+\theta_6\,\theta_{10}\vartheta_1\,\vartheta_{13}$$
$$c_2c_9\,\Theta_{12}\Theta'_7=\theta_2\theta_9\,\vartheta_{12}\vartheta_7-\theta_{12}\theta_7\vartheta_2\vartheta_9-\theta_0\,\theta_{11}\vartheta_5\,\vartheta_{14}+\theta_5\,\theta_{14}\vartheta_0\,\vartheta_{11}$$
$$c_2c_8\,\Theta_{13}\Theta'_7=\theta_2\theta_8\,\vartheta_{13}\vartheta_7-\theta_{13}\theta_7\vartheta_2\vartheta_8-\theta_5\,\theta_{15}\vartheta_0\,\vartheta_{10}+\theta_0\,\theta_{10}\vartheta_5\,\vartheta_{15}$$
$$c_0c_9\,\Theta_{14}\Theta'_7=\theta_0\theta_9\,\vartheta_{14}\vartheta_7-\theta_{14}\theta_7\vartheta_0\vartheta_9-\theta_5\,\theta_{12}\vartheta_2\,\vartheta_{11}+\theta_2\,\theta_{11}\vartheta_5\,\vartheta_{12}$$
$$c_0c_8\,\Theta_{15}\Theta'_7=\theta_0\theta_8\,\vartheta_{15}\vartheta_7-\theta_{15}\theta_7\vartheta_0\vartheta_8-\theta_5\,\theta_{13}\vartheta_2\,\vartheta_{10}+\theta_2\,\theta_{10}\vartheta_5\,\vartheta_{13}$$

48. Ninth Set, with Θ'_8.

$$c_4c_{12}\Theta_0\,\Theta'_8=\theta_4\theta_{12}\vartheta_0\,\vartheta_8+\theta_5\,\theta_{13}\vartheta_9\vartheta_1+\theta_6\,\theta_{14}\vartheta_2\vartheta_{10}+\theta_7\,\theta_{15}\vartheta_3\,\vartheta_{11}$$
$$c_0c_9\,\Theta_1\,\Theta'_8=\theta_0\theta_9\,\vartheta_1\,\vartheta_8+\theta_3\,\theta_{10}\vartheta_2\vartheta_{11}+\theta_4\,\theta_{13}\vartheta_5\vartheta_{12}+\theta_7\,\theta_{14}\vartheta_6\,\vartheta_{15}$$
$$c_3c_9\,\Theta_2\,\Theta'_8=\theta_3\theta_9\,\vartheta_2\,\vartheta_8+\theta_{11}\theta_1\,\vartheta_0\vartheta_{10}-\theta_7\,\theta_{13}\vartheta_6\vartheta_{12}-\theta_5\,\theta_{15}\vartheta_4\,\vartheta_{14}$$
$$c_2c_9\,\Theta_3\,\Theta'_8=\theta_2\theta_9\,\vartheta_3\,\vartheta_8+\theta_0\,\theta_{11}\vartheta_1\vartheta_{10}+\theta_7\,\theta_{12}\vartheta_6\vartheta_{13}+\theta_5\,\theta_{14}\vartheta_4\,\vartheta_{15}$$
$$c_0c_{12}\Theta_4\,\Theta'_8=\theta_0\theta_{12}\vartheta_4\,\vartheta_8+\theta_2\,\theta_{14}\vartheta_6\vartheta_{10}-\theta_5\,\theta_9\,\vartheta_1\vartheta_{13}-\theta_7\,\theta_{11}\vartheta_3\,\vartheta_{15}$$
$$c_1c_{12}\Theta_5\,\Theta'_8=\theta_1\theta_{12}\vartheta_5\,\vartheta_8+\theta_5\,\theta_8\,\vartheta_1\vartheta_{12}+\theta_3\,\theta_{14}\vartheta_7\vartheta_{10}+\theta_7\,\theta_{10}\vartheta_3\,\vartheta_{14}$$
$$c_2c_{12}\Theta_6\,\Theta'_8=\theta_2\theta_{12}\vartheta_6\,\vartheta_8-\theta_7\,\theta_9\,\vartheta_3\vartheta_{13}+\theta_0\,\theta_{14}\vartheta_4\vartheta_{10}-\theta_5\,\theta_{11}\vartheta_1\,\vartheta_{15}$$
$$c_3c_{12}\Theta_7\,\Theta'_8=\theta_3\theta_{12}\vartheta_7\,\vartheta_8+\theta_7\,\theta_8\,\vartheta_3\vartheta_{12}+\theta_1\,\theta_{14}\vartheta_5\vartheta_{10}+\theta_5\,\theta_{10}\vartheta_1\,\vartheta_{14}$$
$$c_8c_8\,\Theta_8\,\Theta'_8=\theta_8\theta_8\,\vartheta_8\,\vartheta_8+\theta_5\,\theta_5\,\vartheta_5\vartheta_5-\theta_7\,\theta_7\,\vartheta_7\vartheta_7-\theta_{10}\theta_{10}\vartheta_{10}\vartheta_{10}$$
$$c_0c_1\,\Theta_9\,\Theta'_8=\theta_0\theta_1\,\vartheta_9\,\vartheta_8-\theta_{10}\theta_{11}\vartheta_2\vartheta_3-\theta_{14}\theta_{15}\vartheta_6\vartheta_7+\theta_4\,\theta_5\,\vartheta_{12}\vartheta_{13}$$
$$c_0c_2\,\Theta_{10}\Theta'_8=\theta_0\theta_2\,\vartheta_{10}\vartheta_8+\theta_{10}\theta_8\,\vartheta_0\vartheta_2+\theta_{13}\theta_{15}\vartheta_5\vartheta_7+\theta_5\,\theta_7\,\vartheta_{13}\vartheta_{15}$$
$$c_0c_3\,\Theta_{11}\Theta'_8=\theta_0\theta_3\,\vartheta_{11}\vartheta_8+\theta_{11}\theta_8\,\vartheta_0\vartheta_3+\theta_{13}\theta_{14}\vartheta_5\vartheta_6+\theta_5\,\theta_6\,\vartheta_{13}\vartheta_{14}$$
$$c_0c_4\,\Theta_{12}\Theta'_8=\theta_0\theta_4\,\vartheta_{12}\vartheta_8-\theta_{10}\theta_{14}\vartheta_2\vartheta_6+\theta_{11}\theta_{15}\vartheta_3\vartheta_7-\theta_1\theta_5\,\vartheta_9\,\vartheta_{13}$$
$$c_1c_4\,\Theta_{13}\Theta'_8=\theta_1\theta_4\,\vartheta_{13}\vartheta_8+\theta_{13}\theta_8\,\vartheta_1\vartheta_4-\theta_{11}\theta_{14}\vartheta_7\vartheta_2-\theta_2\theta_7\,\vartheta_{11}\vartheta_{14}$$
$$c_0c_6\,\Theta_{14}\Theta'_8=\theta_0\theta_6\,\vartheta_{14}\vartheta_8+\theta_{14}\theta_8\,\vartheta_0\vartheta_6-\theta_{11}\theta_{13}\vartheta_3\vartheta_5-\theta_3\theta_5\,\vartheta_{11}\vartheta_{13}$$
$$c_3c_4\,\Theta_{15}\Theta'_8=\theta_3\theta_4\,\vartheta_{15}\vartheta_8+\theta_{10}\theta_{13}\vartheta_1\vartheta_6+\theta_9\,\theta_{14}\vartheta_2\vartheta_5+\theta_0\theta_7\,\vartheta_{11}\vartheta_{12}$$

49. TENTH Set—with Θ'_9.

$$c_6c_{15}\Theta_0\,\Theta'_9=\theta_6\theta_{15}\vartheta_0\,\vartheta_9-\theta_{14}\theta_7\,\vartheta_8\,\vartheta_1-\theta_2\,\theta_{11}\vartheta_4\,\vartheta_{13}+\theta_3\,\theta_{10}\vartheta_{12}\vartheta_5$$

$$c_0c_8\,\Theta_1\,\Theta'_9=\theta_0\theta_8\,\vartheta_1\,\vartheta_9+\theta_2\,\theta_{10}\vartheta_3\,\vartheta_{11}+\theta_5\,\theta_{13}\vartheta_4\,\vartheta_{12}+\theta_7\,\theta_{15}\vartheta_6\,\vartheta_{14}$$

$$c_3c_8\,\Theta_2\,\Theta'_9=\theta_3\theta_8\,\vartheta_2\,\vartheta_9+\theta_0\,\theta_{11}\vartheta_1\,\vartheta_{10}-\theta_6\,\theta_{13}\vartheta_7\,\vartheta_{12}-\theta_5\,\theta_{14}\vartheta_4\,\vartheta_{15}$$

$$c_2c_8\,\Theta_3\,\Theta'_9=\theta_2\theta_8\,\vartheta_3\,\vartheta_9+\theta_0\,\theta_{10}\vartheta_1\,\vartheta_{11}+\theta_7\,\theta_{13}\vartheta_6\,\vartheta_{12}+\theta_4\,\theta_{14}\vartheta_5\,\vartheta_{15}$$

$$c_1c_{12}\Theta_4\,\Theta'_9=\theta_1\theta_{12}\vartheta_4\,\vartheta_9-\theta_8\,\theta_5\,\vartheta_0\,\vartheta_{13}+\theta_3\,\theta_{14}\vartheta_6\,\vartheta_{11}-\theta_7\,\theta_{10}\vartheta_2\,\vartheta_{15}$$

$$c_0c_{12}\Theta_5\,\Theta'_9=\theta_0\theta_{12}\vartheta_5\,\vartheta_9+\theta_5\,\theta_9\,\vartheta_0\,\vartheta_{12}+\theta_2\,\theta_{14}\vartheta_7\,\vartheta_{11}+\theta_7\,\theta_{11}\vartheta_2\,\vartheta_{14}$$

$$c_0c_{15}\Theta_6\,\Theta'_9=\theta_0\theta_{15}\vartheta_6\,\vartheta_9-\theta_5\,\theta_{10}\vartheta_3\,\vartheta_{12}-\theta_2\,\theta_{13}\vartheta_4\,\vartheta_{11}+\theta_7\,\theta_8\,\vartheta_1\,\vartheta_{14}$$

$$c_2c_{12}\Theta_7\,\Theta'_9=\theta_2\theta_{12}\vartheta_7\,\vartheta_9+\theta_7\,\theta_9\,\vartheta_2\,\vartheta_{12}+\theta_0\,\theta_{14}\vartheta_5\,\vartheta_{11}+\theta_5\,\theta_{11}\vartheta_0\,\vartheta_{14}$$

$$c_0c_1\,\Theta_8\,\Theta'_9=\theta_0\theta_1\,\vartheta_8\,\vartheta_9-\theta_{10}\theta_{11}\vartheta_2\,\vartheta_3+\theta_{14}\theta_{15}\vartheta_6\,\vartheta_7-\theta_4\,\theta_5\,\vartheta_{12}\vartheta_{13}$$

$$c_9c_9\,\Theta_9\,\Theta'_9=\theta_9\theta_9\,\vartheta_9\,\vartheta_9-\theta_{10}\theta_{10}\vartheta_{10}\vartheta_{10}-\theta_{13}\theta_{13}\vartheta_{13}\vartheta_{13}+\theta_{14}\theta_{14}\vartheta_{14}\vartheta_{14}$$

$$c_0c_3\,\Theta_{10}\Theta'_9=\theta_0\theta_3\,\vartheta_{10}\vartheta_9+\theta_{10}\theta_9\,\vartheta_0\,\vartheta_3-\theta_{13}\theta_{14}\vartheta_4\,\vartheta_7-\theta_4\,\theta_7\,\vartheta_{13}\vartheta_{14}$$

$$c_0c_2\,\Theta_{11}\Theta'_9=\theta_0\theta_2\,\vartheta_{11}\vartheta_9+\theta_{11}\theta_9\,\vartheta_0\,\vartheta_2+\theta_{12}\theta_{14}\vartheta_5\,\vartheta_7+\theta_5\,\theta_7\,\vartheta_{12}\vartheta_{14}$$

$$c_9c_{12}\Theta_{12}\Theta'_9=\theta_9\theta_{12}\vartheta_{12}\vartheta_9+\theta_0\,\theta_5\,\vartheta_0\,\vartheta_5-\theta_2\,\theta_7\,\vartheta_2\,\vartheta_7-\theta_{11}\theta_{14}\vartheta_{11}\vartheta_{14}$$

$$c_0c_4\,\Theta_{13}\Theta'_9=\theta_0\theta_4\,\vartheta_{13}\vartheta_9+\theta_{13}\theta_9\,\vartheta_0\,\vartheta_4-\theta_{11}\theta_{15}\vartheta_6\,\vartheta_2-\theta_2\,\theta_6\,\vartheta_{11}\vartheta_{15}$$

$$c_3c_4\,\Theta_{14}\Theta'_9=\theta_3\theta_4\,\vartheta_{14}\vartheta_9+\theta_{14}\theta_9\,\vartheta_3\,\vartheta_4-\theta_{10}\theta_{13}\vartheta_0\,\vartheta_7-\theta_0\,\theta_7\,\vartheta_{10}\vartheta_{13}$$

$$c_0c_6\,\Theta_{15}\Theta'_9=\theta_0\theta_6\,\vartheta_{15}\vartheta_9+\theta_{11}\theta_{13}\vartheta_2\,\vartheta_4+\theta_8\,\theta_{14}\vartheta_1\,\vartheta_7+\theta_3\,\theta_5\,\vartheta_{10}\vartheta_{12}$$

50. ELEVENTH Set—with Θ'_{10}.

$$c_2\,c_8\,\Theta_0\,\Theta'_{10}=\theta_2\,\theta_8\,\vartheta_0\,\vartheta_{10}-\theta_0\,\theta_{10}\vartheta_2\,\vartheta_8-\theta_5\,\theta_{15}\vartheta_7\,\vartheta_{13}+\theta_7\,\theta_{13}\vartheta_5\,\vartheta_{15}$$

$$c_3\,c_8\,\Theta_1\,\Theta'_{10}=\theta_3\,\theta_8\,\vartheta_1\,\vartheta_{10}-\theta_1\,\theta_{10}\vartheta_3\,\vartheta_8-\theta_5\,\theta_{14}\vartheta_7\,\vartheta_{12}+\theta_7\,\theta_{12}\vartheta_5\,\vartheta_{14}$$

$$c_0\,c_8\,\Theta_2\,\Theta'_{10}=\theta_0\,\theta_8\,\vartheta_2\,\vartheta_{10}-\theta_2\,\theta_{10}\vartheta_0\,\vartheta_8-\theta_7\,\theta_{15}\vartheta_5\,\vartheta_{13}+\theta_5\,\theta_{13}\vartheta_7\,\vartheta_{15}$$

$$c_1\,c_8\,\Theta_3\,\Theta'_{10}=\theta_1\,\theta_8\,\vartheta_3\,\vartheta_{10}-\theta_3\,\theta_{10}\vartheta_1\,\vartheta_8-\theta_7\,\theta_{14}\vartheta_5\,\vartheta_{12}+\theta_5\,\theta_{12}\vartheta_7\,\vartheta_{14}$$

$$c_6\,c_8\,\Theta_4\,\Theta'_{10}=\theta_6\,\theta_8\,\vartheta_4\,\vartheta_{10}-\theta_4\,\theta_{10}\vartheta_6\,\vartheta_8-\theta_7\,\theta_9\,\vartheta_5\,\vartheta_{11}+\theta_5\,\theta_{11}\vartheta_7\,\vartheta_9$$

$$c_6\,c_9\,\Theta_5\,\Theta'_{10}=\theta_6\,\theta_9\,\vartheta_5\,\vartheta_{10}-\theta_5\,\theta_{10}\vartheta_6\,\vartheta_9-\theta_4\,\theta_{14}\vartheta_7\,\vartheta_{13}+\theta_2\,\theta_{13}\vartheta_1\,\vartheta_{14}$$

$$c_0\,c_{12}\Theta_6\,\Theta'_{10}=\theta_0\,\theta_{12}\vartheta_6\,\vartheta_{10}-\theta_6\,\theta_{10}\vartheta_0\,\vartheta_{12}-\theta_1\,\theta_{13}\vartheta_7\,\vartheta_{11}+\theta_7\,\theta_{11}\vartheta_1\,\vartheta_{13}$$

$$c_4\,c_9\,\Theta_7\,\Theta'_{10}=\theta_4\,\theta_9\,\vartheta_7\,\vartheta_{10}-\theta_7\,\theta_{10}\vartheta_4\,\vartheta_9-\theta_6\,\theta_{11}\vartheta_5\,\vartheta_8+\theta_5\,\theta_8\,\vartheta_6\,\vartheta_{11}$$

$$c_0\,c_2\,\Theta_8\,\Theta'_{10}=\theta_0\,\theta_2\,\vartheta_8\,\vartheta_{10}-\theta_8\,\theta_{10}\vartheta_0\,\vartheta_2-\theta_{13}\theta_{15}\vartheta_5\,\vartheta_7+\theta_5\,\theta_7\,\vartheta_{13}\vartheta_{15}$$

$$c_0\,c_3\,\Theta_9\,\Theta'_{10}=\theta_0\,\theta_3\,\vartheta_9\,\vartheta_{10}-\theta_9\,\theta_{10}\vartheta_0\,\vartheta_3-\theta_{13}\theta_{14}\vartheta_4\,\vartheta_7+\theta_4\,\theta_7\,\vartheta_{13}\vartheta_{14}$$

$$c_{15}c_{15}\Theta_{10}\Theta'_{10}=\theta_{15}\theta_{15}\vartheta_{10}\vartheta_{10}-\theta_{10}\theta_{10}\vartheta_{15}\vartheta_{15}-\theta_{11}\theta_{11}\vartheta_{14}\vartheta_{14}+\theta_{14}\theta_{14}\vartheta_{11}\vartheta_{11}$$

$$c_0\,c_1\,\Theta_{11}\Theta'_{10}=\theta_0\,\theta_1\,\vartheta_{11}\vartheta_{10}-\theta_{11}\theta_{10}\vartheta_0\,\vartheta_1-\theta_6\,\theta_7\,\vartheta_{12}\vartheta_{13}+\theta_{12}\theta_{13}\vartheta_6\,\vartheta_7$$

$$c_9\,c_{15}\Theta_{12}\Theta'_{10}=\theta_9\,\theta_{15}\vartheta_{12}\vartheta_{10}-\theta_{12}\theta_{10}\vartheta_9\,\vartheta_{15}-\theta_{11}\theta_{13}\vartheta_8\,\vartheta_{14}+\theta_8\,\theta_{14}\vartheta_{11}\vartheta_{13}$$

$$c_3\,c_4\,\Theta_{13}\Theta'_{10}=\theta_3\,\theta_4\,\vartheta_{13}\vartheta_{10}-\theta_{13}\theta_{10}\vartheta_3\,\vartheta_4-\theta_{11}\theta_{12}\vartheta_2\,\vartheta_5+\theta_2\,\theta_5\,\vartheta_{11}\vartheta_{12}$$

$$c_0\,c_4\,\Theta_{14}\Theta'_{10}=\theta_0\,\theta_4\,\vartheta_{14}\vartheta_{10}-\theta_{14}\theta_{10}\vartheta_0\,\vartheta_4-\theta_9\,\theta_{13}\vartheta_3\,\vartheta_7+\theta_3\,\theta_7\,\vartheta_9\,\vartheta_{13}$$

$$c_1\,c_4\,\Theta_{15}\Theta'_{10}=\theta_1\,\theta_4\,\vartheta_{15}\vartheta_{10}-\theta_{15}\theta_{10}\vartheta_1\,\vartheta_4-\theta_{11}\theta_{14}\vartheta_0\,\vartheta_5+\theta_0\,\theta_5\,\vartheta_{11}\vartheta_{14}$$

51. TWELFTH Set, with Θ'_{11}.

$$c_3c_8\,\Theta_0\,\Theta'_{11}=\theta_3\theta_8\,\vartheta_0\,\vartheta_{11}-\theta_0\,\theta_{11}\vartheta_3\vartheta_8-\theta_6\,\theta_{13}\vartheta_5\,\vartheta_{14}+\theta_5\,\theta_{14}\vartheta_6\,\vartheta_{13}$$

$$c_3c_9\,\Theta_1\,\Theta'_{11}=\theta_3\theta_9\,\vartheta_1\,\vartheta_{11}-\theta_1\,\theta_{11}\vartheta_3\vartheta_9-\theta_7\,\theta_{13}\vartheta_5\,\vartheta_{15}+\theta_5\,\theta_{15}\vartheta_7\,\vartheta_{13}$$

$$c_0c_9\,\Theta_2\,\Theta'_{11}=\theta_0\theta_9\,\vartheta_2\,\vartheta_{11}-\theta_2\,\theta_{11}\vartheta_0\vartheta_9-\theta_5\,\theta_{12}\vartheta_7\,\vartheta_{14}+\theta_7\,\theta_{14}\vartheta_5\,\vartheta_{12}$$

$$c_0c_8\,\Theta_3\,\Theta'_{11}=\theta_0\theta_8\,\vartheta_3\,\vartheta_{11}-\theta_3\,\theta_{11}\vartheta_0\vartheta_8-\theta_6\,\theta_{14}\vartheta_5\,\vartheta_{13}+\theta_5\,\theta_{13}\vartheta_6\,\vartheta_{14}$$

$$c_6c_9\,\Theta_4\,\Theta'_{11}=\theta_6\theta_9\,\vartheta_4\,\vartheta_{11}-\theta_4\,\theta_{11}\vartheta_6\vartheta_9-\theta_7\,\theta_8\,\vartheta_5\,\vartheta_{10}+\theta_5\,\theta_{10}\vartheta_7\,\vartheta_8$$

$$c_6c_8\,\Theta_5\,\Theta'_{11}=\theta_6\theta_8\,\vartheta_5\,\vartheta_{11}-\theta_5\,\theta_{11}\vartheta_6\vartheta_8-\theta_4\,\theta_{10}\vartheta_7\,\vartheta_9+\theta_7\,\theta_9\,\vartheta_4\,\vartheta_{10}$$

$$c_4c_9\,\Theta_6\,\Theta'_{11}=\theta_4\theta_9\,\vartheta_6\,\vartheta_{11}-\theta_6\,\theta_{11}\vartheta_4\vartheta_9-\theta_5\,\theta_8\,\vartheta_7\,\vartheta_{10}+\theta_7\,\theta_{10}\vartheta_5\,\vartheta_8$$

$$c_0c_{12}\Theta_7\,\Theta'_{11}=\theta_0\theta_{12}\vartheta_7\,\vartheta_{11}-\theta_7\,\theta_{11}\vartheta_0\vartheta_{12}-\theta_6\,\theta_{10}\vartheta_1\,\vartheta_{13}+\theta_1\,\theta_{13}\vartheta_6\,\vartheta_{10}$$

$$c_0c_3\,\Theta_8\,\Theta'_{11}=\theta_0\theta_3\,\vartheta_8\,\vartheta_{11}-\theta_8\,\theta_{11}\vartheta_0\vartheta_3-\theta_5\,\theta_6\,\vartheta_{13}\vartheta_{14}+\theta_{13}\theta_{14}\vartheta_5\,\vartheta_6$$

$$c_0c_2\,\Theta_9\,\Theta'_{11}=\theta_0\theta_2\,\vartheta_9\,\vartheta_{11}-\theta_9\,\theta_{11}\vartheta_0\vartheta_2-\theta_{12}\theta_{14}\vartheta_5\,\vartheta_7+\theta_5\,\theta_7\,\vartheta_{12}\vartheta_{14}$$

$$c_0c_1\,\Theta_{10}\Theta'_{11}=\theta_0\theta_1\,\vartheta_{10}\vartheta_{11}-\theta_{10}\theta_{11}\vartheta_0\vartheta_1-\theta_{12}\theta_{13}\vartheta_6\,\vartheta_7+\theta_6\,\theta_7\,\vartheta_{12}\vartheta_{13}$$

$$c_1c_1\,\Theta_{11}\Theta'_{11}=\theta_1\theta_1\,\vartheta_{11}\vartheta_{11}-\theta_{11}\theta_{11}\vartheta_1\vartheta_1-\theta_{13}\theta_{13}\vartheta_7\,\vartheta_7+\theta_7\,\theta_7\,\vartheta_{13}\vartheta_{13}$$

$$c_1c_6\,\Theta_{12}\Theta'_{11}=\theta_1\theta_6\,\vartheta_{12}\vartheta_{11}-\theta_{12}\theta_{11}\vartheta_1\vartheta_6-\theta_0\,\theta_2\,\vartheta_{10}\vartheta_{13}+\theta_{10}\theta_{13}\vartheta_0\,\vartheta_7$$

$$c_0c_6\,\Theta_{13}\Theta'_{11}=\theta_0\theta_6\,\vartheta_{13}\vartheta_{11}-\theta_{13}\theta_{11}\vartheta_0\vartheta_6-\theta_{10}\theta_{12}\vartheta_1\,\vartheta_7+\theta_1\,\theta_7\,\vartheta_{10}\vartheta_{12}$$

$$c_1c_4\,\Theta_{14}\Theta'_{11}=\theta_1\theta_4\,\vartheta_{14}\vartheta_{11}-\theta_{14}\theta_{11}\vartheta_1\vartheta_4-\theta_0\,\theta_5\,\vartheta_{10}\vartheta_{15}+\theta_{10}\theta_{15}\vartheta_0\,\vartheta_5$$

$$c_0c_4\,\Theta_{15}\Theta'_{11}=\theta_0\theta_4\,\vartheta_{15}\vartheta_{11}-\theta_{15}\theta_{11}\vartheta_0\vartheta_4-\theta_{10}\theta_{14}\vartheta_1\,\vartheta_5+\theta_1\,\theta_5\,\vartheta_{10}\vartheta_{14}$$

52. THIRTEENTH Set, with Θ'_{12}.

$$c_4\,c_8\,\Theta_0\,\Theta'_{12}=\theta_4\,\theta_8\,\vartheta_0\,\vartheta_{12}+\theta_1\,\theta_{13}\vartheta_5\,\vartheta_9-\theta_6\,\theta_{10}\vartheta_2\,\vartheta_{11}-\theta_7\,\theta_{11}\vartheta_3\,\vartheta_{15}$$

$$c_4\,c_9\,\Theta_1\,\Theta'_{12}=\theta_4\,\theta_9\,\vartheta_1\,\vartheta_{12}+\theta_0\,\theta_{13}\vartheta_5\,\vartheta_8+\theta_3\,\theta_{14}\vartheta_6\,\vartheta_{11}+\theta_7\,\theta_{10}\vartheta_2\,\vartheta_{15}$$

$$c_6\,c_8\,\Theta_2\,\Theta'_{12}=\theta_6\,\theta_8\,\vartheta_2\,\vartheta_{12}+\theta_3\,\theta_{13}\vartheta_7\,\vartheta_9+\theta_0\,\theta_{14}\vartheta_4\,\vartheta_{10}+\theta_5\,\theta_{11}\vartheta_1\,\vartheta_{15}$$

$$c_0\,c_{15}\Theta_3\,\Theta'_{12}=\theta_0\,\theta_{15}\vartheta_3\,\vartheta_{12}-\theta_5\,\theta_{10}\vartheta_6\,\vartheta_9+\theta_4\,\theta_{11}\vartheta_7\,\vartheta_8-\theta_6\,\theta_{14}\vartheta_2\,\vartheta_{13}$$

$$c_0\,c_8\,\Theta_4\,\Theta'_{12}=\theta_0\,\theta_8\,\vartheta_4\,\vartheta_{12}-\theta_{13}\theta_5\,\vartheta_1\,\vartheta_9-\theta_7\,\theta_{15}\vartheta_3\,\vartheta_{11}+\theta_2\,\theta_{10}\vartheta_6\,\vartheta_{14}$$

$$c_1\,c_8\,\Theta_5\,\Theta'_{12}=\theta_1\,\theta_8\,\vartheta_5\,\vartheta_{12}+\theta_5\,\theta_{12}\vartheta_1\,\vartheta_8+\theta_{14}\theta_7\,\vartheta_3\,\vartheta_{10}+\theta_3\,\theta_{10}\vartheta_7\,\vartheta_{14}$$

$$c_2\,c_8\,\Theta_6\,\Theta'_{12}=\theta_2\,\theta_8\,\vartheta_6\,\vartheta_{12}-\theta_{13}\theta_7\,\vartheta_3\,\vartheta_9-\theta_5\,\theta_{15}\vartheta_1\,\vartheta_{11}+\theta_0\,\theta_{10}\vartheta_4\,\vartheta_{14}$$

$$c_2\,c_9\,\Theta_7\,\Theta'_{12}=\theta_2\,\theta_9\,\vartheta_7\,\vartheta_{12}+\theta_7\,\theta_{12}\vartheta_2\,\vartheta_9+\theta_5\,\theta_{14}\vartheta_0\,\vartheta_{11}+\theta_0\,\theta_{11}\vartheta_5\,\vartheta_{14}$$

$$c_0\,c_4\,\Theta_8\,\Theta'_{12}=\theta_0\,\theta_4\,\vartheta_8\,\vartheta_{12}+\theta_1\,\theta_5\,\vartheta_9\,\vartheta_{13}-\theta_{10}\theta_{14}\vartheta_2\,\vartheta_6-\theta_{11}\theta_{15}\vartheta_3\,\vartheta_7$$

$$c_9\,c_{12}\Theta_9\,\Theta'_{12}=\theta_9\,\theta_{12}\vartheta_9\,\vartheta_{12}+\theta_0\,\theta_5\,\vartheta_0\,\vartheta_5-\theta_2\,\theta_7\,\vartheta_2\,\vartheta_7-\theta_{11}\theta_{14}\vartheta_{11}\vartheta_{14}$$

$$c_9\,c_{15}\Theta_{10}\Theta'_{12}=\theta_9\,\theta_{15}\vartheta_{10}\vartheta_{12}+\theta_{10}\theta_{12}\vartheta_9\,\vartheta_{15}-\theta_8\,\theta_{14}\vartheta_{11}\vartheta_{13}-\theta_{11}\theta_{13}\vartheta_8\,\vartheta_{14}$$

$$c_1\,c_6\,\Theta_{11}\Theta'_{12}=\theta_1\,\theta_6\,\vartheta_{11}\vartheta_{12}+\theta_{11}\theta_{12}\vartheta_1\,\vartheta_6+\theta_{10}\theta_{13}\vartheta_0\,\vartheta_7+\theta_0\,\theta_7\,\vartheta_{10}\vartheta_{13}$$

$$c_{12}c_{12}\Theta_{12}\Theta'_{12}=\theta_{12}\theta_{12}\vartheta_{12}\vartheta_{12}-\theta_{10}\theta_{10}\vartheta_{10}\vartheta_{10}+\theta_{11}\theta_{11}\vartheta_{11}\vartheta_{11}-\theta_{13}\theta_{13}\vartheta_{13}\vartheta_{13}$$

$$c_8\,c_9\,\Theta_{13}\Theta'_{12}=\theta_8\,\theta_9\,\vartheta_{13}\vartheta_{12}+\theta_{13}\theta_{12}\vartheta_8\,\vartheta_9-\theta_{14}\theta_{15}\vartheta_{10}\vartheta_{11}-\theta_{10}\theta_{11}\vartheta_{14}\vartheta_{15}$$

$$c_0\,c_2\,\Theta_{14}\Theta'_{12}=\theta_0\,\theta_2\,\vartheta_{14}\vartheta_{12}+\theta_{14}\theta_{12}\vartheta_0\,\vartheta_2-\theta_7\,\theta_5\,\vartheta_9\,\vartheta_{11}-\theta_9\,\theta_{11}\vartheta_5\,\vartheta_7$$

$$c_0\,c_3\,\Theta_{15}\Theta'_{12}=\theta_0\,\theta_3\,\vartheta_{15}\vartheta_{12}+\theta_{13}\theta_{14}\vartheta_1\,\vartheta_2+\theta_8\,\theta_{11}\vartheta_7\,\vartheta_4+\theta_5\,\theta_6\,\vartheta_9\,\vartheta_{10}$$

53. Fourteenth Set, with Θ'_{13}.

$$c_1c_{12}\Theta_0\,\Theta'_{13}=\theta_1\theta_{12}\vartheta_0\,\vartheta_{13}-\theta_0\,\theta_{13}\vartheta_1\vartheta_{12}-\theta_7\,\theta_{10}\vartheta_6\,\vartheta_{11}+\theta_6\,\theta_{11}\vartheta_7\,\vartheta_{10}$$
$$c_0c_{12}\Theta_1\,\Theta'_{13}=\theta_0\theta_{12}\vartheta_1\,\vartheta_{13}-\theta_1\,\theta_{13}\vartheta_0\vartheta_{12}-\theta_7\,\theta_{11}\vartheta_6\,\vartheta_{10}+\theta_6\,\theta_{10}\vartheta_7\,\vartheta_{11}$$
$$c_3c_{12}\Theta_2\,\Theta'_{13}=\theta_3\theta_{12}\vartheta_2\,\vartheta_{13}-\theta_2\,\theta_{13}\vartheta_3\vartheta_{12}-\theta_5\,\theta_{10}\vartheta_4\,\vartheta_{11}+\theta_4\,\theta_{11}\vartheta_5\,\vartheta_{10}$$
$$c_2c_{12}\Theta_3\,\Theta'_{13}=\theta_2\theta_{12}\vartheta_3\,\vartheta_{13}-\theta_3\,\theta_{13}\vartheta_2\vartheta_{12}-\theta_5\,\theta_{11}\vartheta_4\,\vartheta_{10}+\theta_4\,\theta_{10}\vartheta_5\,\vartheta_{11}$$
$$c_0c_9\,\Theta_4\,\Theta'_{13}=\theta_0\theta_9\,\vartheta_4\,\vartheta_{13}-\theta_4\,\theta_{13}\vartheta_0\,\vartheta_9-\theta_7\,\theta_{14}\vartheta_3\,\vartheta_{10}+\theta_3\,\theta_{10}\vartheta_7\,\vartheta_{14}$$
$$c_0c_8\,\Theta_5\,\Theta'_{13}=\theta_0\theta_8\,\vartheta_5\,\vartheta_{13}-\theta_5\,\theta_{13}\vartheta_0\,\vartheta_8-\theta_3\,\theta_{11}\vartheta_6\,\vartheta_{14}+\theta_6\,\theta_{14}\vartheta_3\,\vartheta_{11}$$
$$c_3c_8\,\Theta_6\,\Theta'_{13}=\theta_3\theta_8\,\vartheta_6\,\vartheta_{13}-\theta_6\,\theta_{13}\vartheta_3\,\vartheta_8-\theta_5\,\theta_{14}\vartheta_0\,\vartheta_{11}+\theta_0\,\theta_{11}\vartheta_5\,\vartheta_{14}$$
$$c_2c_8\,\Theta_7\,\Theta'_{13}=\theta_2\theta_8\,\vartheta_7\,\vartheta_{13}-\theta_7\,\theta_{13}\vartheta_2\,\vartheta_8-\theta_0\,\theta_{10}\vartheta_5\,\vartheta_{15}+\theta_5\,\theta_{15}\vartheta_0\,\vartheta_{10}$$
$$c_1c_4\,\Theta_8\,\Theta'_{13}=\theta_1\theta_4\,\vartheta_8\,\vartheta_{13}-\theta_8\,\theta_{13}\vartheta_1\,\vartheta_4-\theta_{11}\theta_{14}\vartheta_2\,\vartheta_7\,+\theta_2\,\theta_7\vartheta_{11}\,\vartheta_{14}$$
$$c_0c_4\,\Theta_9\,\Theta'_{13}=\theta_0\theta_4\,\vartheta_9\,\vartheta_{13}-\theta_9\,\theta_{13}\vartheta_0\,\vartheta_4-\theta_2\,\theta_6\,\vartheta_{11}\vartheta_{15}+\theta_{11}\theta_{15}\vartheta_2\,\vartheta_6$$
$$c_3c_4\,\Theta_{10}\Theta'_{13}=\theta_3\theta_4\,\vartheta_{10}\vartheta_{13}-\theta_{10}\theta_{13}\vartheta_3\,\vartheta_4-\theta_2\,\theta_5\,\vartheta_{11}\vartheta_{12}+\theta_{11}\theta_{12}\vartheta_2\,\vartheta_5$$
$$c_0c_6\,\Theta_{11}\Theta'_{13}=\theta_0\theta_6\,\vartheta_{11}\vartheta_{13}-\theta_{11}\theta_{13}\vartheta_0\,\vartheta_6-\theta_1\,\theta_7\,\vartheta_{10}\vartheta_{12}+\theta_{10}\theta_{12}\vartheta_1\,\vartheta_7$$
$$c_8c_9\,\Theta_{12}\Theta'_{13}=\theta_8\theta_9\,\vartheta_{12}\vartheta_{13}-\theta_{12}\theta_{13}\vartheta_8\,\vartheta_9-\theta_{10}\theta_{11}\vartheta_{14}\vartheta_{15}+\theta_{14}\theta_{15}\vartheta_{10}\vartheta_{11}$$
$$c_8c_8\,\Theta_{13}\Theta'_{13}=\theta_8\theta_8\,\vartheta_{13}\vartheta_{13}-\theta_{13}\theta_{13}\vartheta_8\,\vartheta_8-\theta_{14}\theta_{14}\vartheta_{11}\vartheta_{11}+\theta_{11}\theta_{11}\vartheta_{14}\vartheta_{14}$$
$$c_0c_3\,\Theta_{14}\Theta'_{13}=\theta_0\theta_3\,\vartheta_{14}\vartheta_{13}-\theta_{14}\theta_{13}\vartheta_0\,\vartheta_3-\theta_4\,\theta_7\,\vartheta_9\,\vartheta_{10}+\theta_9\,\theta_{10}\vartheta_4\,\vartheta_7$$
$$c_0c_2\,\Theta_{15}\Theta'_{13}=\theta_0\theta_2\,\vartheta_{15}\vartheta_{13}-\theta_{15}\theta_{13}\vartheta_0\,\vartheta_2-\theta_5\,\theta_7\,\vartheta_8\,\vartheta_{10}+\theta_8\,\theta_{10}\vartheta_5\,\vartheta_7$$

54. Fifteenth Set, with Θ'_{14}.

$$c_2c_{12}\Theta_0\,\Theta'_{14}=\theta_2\theta_{12}\vartheta_0\,\vartheta_{14}-\theta_0\,\theta_{14}\vartheta_2\vartheta_{12}-\theta_5\,\theta_{11}\vartheta_7\vartheta_9\,+\theta_7\theta_9\,\vartheta_5\,\vartheta_{11}$$
$$c_0c_{15}\Theta_1\,\Theta'_{14}=\theta_0\theta_{15}\vartheta_1\,\vartheta_{14}-\theta_1\,\theta_{14}\vartheta_0\vartheta_{15}-\theta_5\,\theta_{10}\vartheta_4\vartheta_{11}+\theta_4\theta_{11}\vartheta_5\,\vartheta_{10}$$
$$c_0c_{12}\Theta_2\,\Theta'_{14}=\theta_0\theta_{12}\vartheta_2\,\vartheta_{14}-\theta_2\,\theta_{14}\vartheta_0\vartheta_{12}-\theta_{11}\theta_7\,\vartheta_5\vartheta_9\,+\theta_5\theta_9\,\vartheta_7\,\vartheta_{11}$$
$$c_4c_9\,\Theta_3\,\Theta'_{14}=\theta_4\theta_9\,\vartheta_3\,\vartheta_{14}-\theta_3\,\theta_{14}\vartheta_4\vartheta_9\,-\theta_7\,\theta_{10}\vartheta_0\vartheta_{13}+\theta_0\theta_{13}\vartheta_7\,\vartheta_{10}$$
$$c_3c_9\,\Theta_4\,\Theta'_{14}=\theta_3\theta_9\,\vartheta_4\,\vartheta_{14}-\theta_4\,\theta_{14}\vartheta_3\vartheta_9\,-\theta_7\,\theta_{13}\vartheta_0\vartheta_{10}+\theta_0\theta_{10}\vartheta_7\,\vartheta_{13}$$
$$c_3c_8\,\Theta_5\,\Theta'_{14}=\theta_3\theta_8\,\vartheta_5\,\vartheta_{14}-\theta_5\,\theta_{14}\vartheta_3\vartheta_8\,-\theta_1\,\theta_{10}\vartheta_7\vartheta_{12}+\theta_7\theta_{12}\vartheta_1\,\vartheta_{10}$$
$$c_0c_8\,\Theta_6\,\Theta'_{14}=\theta_0\theta_8\,\vartheta_6\,\vartheta_{14}-\theta_6\,\theta_{14}\vartheta_0\vartheta_8\,-\theta_5\,\theta_{13}\vartheta_3\vartheta_{11}+\theta_3\theta_{11}\vartheta_5\,\vartheta_{13}$$
$$c_0c_9\,\Theta_7\,\Theta'_{14}=\theta_0\theta_9\,\vartheta_7\,\vartheta_{14}-\theta_7\,\theta_{14}\vartheta_0\vartheta_9\,-\theta_2\,\theta_{11}\vartheta_5\vartheta_{12}+\theta_5\theta_{12}\vartheta_2\,\vartheta_{11}$$
$$c_0c_6\,\Theta_8\,\Theta'_{14}=\theta_0\theta_6\,\vartheta_8\,\vartheta_{14}-\theta_8\,\theta_{14}\vartheta_0\vartheta_6\,-\theta_{11}\theta_{13}\vartheta_3\vartheta_5\,+\theta_3\theta_5\,\vartheta_{11}\vartheta_{13}$$
$$c_3c_4\,\Theta_9\,\Theta'_{14}=\theta_3\theta_4\,\vartheta_9\,\vartheta_{14}-\theta_9\,\theta_{14}\vartheta_3\vartheta_4\,-\theta_{10}\theta_{13}\vartheta_0\vartheta_7\,+\theta_0\theta_7\,\vartheta_{10}\vartheta_{13}$$
$$c_0c_4\,\Theta_{10}\Theta'_{14}=\theta_0\theta_4\,\vartheta_{10}\vartheta_{14}-\theta_{10}\theta_{14}\vartheta_0\vartheta_4\,-\theta_3\,\theta_7\,\vartheta_9\vartheta_{13}+\theta_9\theta_{13}\vartheta_3\,\vartheta_7$$
$$c_1c_4\,\Theta_{11}\Theta'_{14}=\theta_1\theta_4\,\vartheta_{11}\vartheta_{14}-\theta_{11}\theta_{14}\vartheta_1\vartheta_4\,-\theta_{10}\theta_{15}\vartheta_0\vartheta_5\,+\theta_0\theta_5\,\vartheta_{10}\vartheta_{15}$$
$$c_0c_2\,\Theta_{12}\Theta'_{14}=\theta_0\theta_2\,\vartheta_{12}\vartheta_{14}-\theta_{12}\theta_{14}\vartheta_0\vartheta_2\,-\theta_5\,\theta_7\,\vartheta_9\vartheta_{11}+\theta_9\theta_{11}\vartheta_5\,\vartheta_7$$
$$c_0c_3\,\Theta_{13}\Theta'_{14}=\theta_0\theta_3\,\vartheta_{13}\vartheta_{14}-\theta_{13}\theta_{14}\vartheta_0\vartheta_3\,-\theta_9\,\theta_{10}\vartheta_4\vartheta_7\,+\theta_4\theta_7\,\vartheta_9\,\vartheta_{10}$$
$$c_4c_4\,\Theta_{14}\Theta'_{14}=\theta_4\theta_4\,\vartheta_{14}\vartheta_{14}-\theta_{14}\theta_{14}\vartheta_4\vartheta_4\,-\theta_{13}\theta_{13}\vartheta_7\vartheta_7\,+\theta_7\theta_7\,\vartheta_{13}\vartheta_{13}$$
$$c_0c_1\,\Theta_{15}\Theta'_{14}=\theta_0\theta_1\,\vartheta_{15}\vartheta_{14}-\theta_{14}\theta_{15}\vartheta_0\vartheta_1\,-\theta_{10}\theta_{11}\vartheta_4\vartheta_5\,+\theta_4\theta_5\,\vartheta_{10}\vartheta_{11}$$

55. SIXTEENTH Set, with Θ'_{15}.

$$
\begin{aligned}
c_6 c_9 \Theta_0 \Theta'_{15} &= \theta_6 \theta_9 \vartheta_0 \vartheta_{15} + \theta_5 \theta_{10} \vartheta_{12} \vartheta_3 - \theta_4 \theta_{11} \vartheta_2 \vartheta_{13} - \theta_7 \theta_8 \vartheta_1 \vartheta_{14} \\
c_6 c_8 \Theta_1 \Theta'_{15} &= \theta_6 \theta_8 \vartheta_1 \vartheta_{15} + \theta_5 \theta_{11} \vartheta_2 \vartheta_{12} - \theta_0 \theta_{14} \vartheta_7 \vartheta_9 - \theta_3 \theta_{13} \vartheta_4 \vartheta_{10} \\
c_1 c_{12} \Theta_2 \Theta'_{15} &= \theta_1 \theta_{12} \vartheta_2 \vartheta_{15} + \theta_7 \theta_{10} \vartheta_4 \vartheta_9 - \theta_6 \theta_{11} \vartheta_5 \vartheta_8 - \theta_0 \theta_{13} \vartheta_3 \vartheta_{14} \\
c_0 c_{12} \Theta_3 \Theta'_{15} &= \theta_0 \theta_{12} \vartheta_3 \vartheta_{15} + \theta_7 \theta_{11} \vartheta_4 \vartheta_8 - \theta_5 \theta_9 \vartheta_{10} \vartheta_6 - \theta_2 \theta_{14} \vartheta_1 \vartheta_{13} \\
c_2 c_9 \Theta_4 \Theta'_{15} &= \theta_2 \theta_9 \vartheta_4 \vartheta_{15} + \theta_5 \theta_{14} \vartheta_3 \vartheta_8 - \theta_7 \theta_{12} \vartheta_1 \vartheta_{10} - \theta_0 \theta_{11} \vartheta_6 \vartheta_{13} \\
c_2 c_8 \Theta_5 \Theta'_{15} &= \theta_2 \theta_8 \vartheta_5 \vartheta_{15} + \theta_5 \theta_{15} \vartheta_2 \vartheta_8 - \theta_7 \theta_{13} \vartheta_0 \vartheta_{10} - \theta_0 \theta_{10} \vartheta_7 \vartheta_{13} \\
c_0 c_9 \Theta_6 \Theta'_{15} &= \theta_0 \theta_9 \vartheta_6 \vartheta_{15} + \theta_7 \theta_{14} \vartheta_1 \vartheta_8 - \theta_4 \theta_{13} \vartheta_2 \vartheta_{11} - \theta_3 \theta_{10} \vartheta_5 \vartheta_{12} \\
c_0 c_8 \Theta_7 \Theta'_{15} &= \theta_0 \theta_8 \vartheta_7 \vartheta_{15} + \theta_7 \theta_{15} \vartheta_0 \vartheta_8 - \theta_5 \theta_{13} \vartheta_2 \vartheta_{10} - \theta_2 \theta_{10} \vartheta_5 \vartheta_{13} \\
c_3 c_4 \Theta_8 \Theta'_{15} &= \theta_3 \theta_4 \vartheta_8 \vartheta_{15} + \theta_{10} \theta_{13} \vartheta_1 \vartheta_6 - \theta_9 \theta_{14} \vartheta_2 \vartheta_5 - \theta_0 \theta_7 \vartheta_{11} \vartheta_{12} \\
c_0 c_6 \Theta_9 \Theta'_{15} &= \theta_0 \theta_6 \vartheta_9 \vartheta_{15} + \theta_{11} \theta_{13} \vartheta_2 \vartheta_4 - \theta_8 \theta_{14} \vartheta_1 \vartheta_7 - \theta_3 \theta_5 \vartheta_{10} \vartheta_{12} \\
c_1 c_4 \Theta_{10} \Theta'_{15} &= \theta_1 \theta_4 \vartheta_{10} \vartheta_{15} + \theta_{10} \theta_{15} \vartheta_1 \vartheta_4 - \theta_{11} \theta_{14} \vartheta_0 \vartheta_5 - \theta_0 \theta_5 \vartheta_{11} \vartheta_{14} \\
c_0 c_4 \Theta_{11} \Theta'_{15} &= \theta_0 \theta_4 \vartheta_{11} \vartheta_{15} + \theta_{11} \theta_{15} \vartheta_0 \vartheta_4 - \theta_{10} \theta_{14} \vartheta_1 \vartheta_5 - \theta_1 \theta_5 \vartheta_{10} \vartheta_{14} \\
c_0 c_3 \Theta_{12} \Theta'_{15} &= \theta_0 \theta_3 \vartheta_{12} \vartheta_{15} + \theta_{13} \theta_{14} \vartheta_1 \vartheta_2 - \theta_8 \theta_{11} \vartheta_7 \vartheta_4 - \theta_5 \theta_6 \vartheta_9 \vartheta_{10} \\
c_0 c_2 \Theta_{13} \Theta'_{15} &= \theta_0 \theta_2 \vartheta_{13} \vartheta_{15} + \theta_{13} \theta_{15} \vartheta_0 \vartheta_2 - \theta_5 \theta_7 \vartheta_8 \vartheta_{10} - \theta_8 \theta_{10} \vartheta_5 \vartheta_7 \\
c_0 c_1 \Theta_{14} \Theta'_{15} &= \theta_0 \theta_1 \vartheta_{14} \vartheta_{15} + \theta_{14} \theta_{15} \vartheta_0 \vartheta_0 - \theta_{10} \theta_{11} \vartheta_4 \vartheta_5 - \theta_4 \theta_5 \vartheta_{11} \vartheta_{10} \\
c_{15} c_{15} \Theta_{15} \Theta'_{15} &= \theta_{15} \theta_{15} \vartheta_{15} \vartheta_{15} + \theta_{10} \theta_{10} \vartheta_{10} \vartheta_{10} - \theta_{11} \theta_{11} \vartheta_{11} \vartheta_{11} - \theta_{14} \theta_{14} \vartheta_{14} \vartheta_{14}
\end{aligned}
$$

56. Any one equation giving $cc\Theta_r\Theta'_n$ where r, n are different, also gives $cc\Theta_n\Theta'_r$ by changing the sign of ξ, η: thus from

$$c_2 c_3 \Theta_1 \Theta'_0 = \theta_2 \theta_3 \vartheta_1 \vartheta_0 + \theta_6 \theta_7 \vartheta_4 \vartheta_5 - \theta_{10} \theta_{11} \vartheta_8 \vartheta_9 - \theta_{14} \theta_{15} \vartheta_{12} \vartheta_{13}$$

there follows in this manner

$$c_2 c_3 \Theta_0 \Theta'_1 = \theta_2 \theta_3 \vartheta_0 \vartheta_1 - \theta_6 \theta_7 \vartheta_4 \vartheta_5 - \theta_{10} \theta_{11} \vartheta_8 \vartheta_9 + \theta_{14} \theta_{15} \vartheta_{12} \vartheta_{13}.$$

In addition to the equations given in formulæ (205), (206), (207), the following include most of those used in order to obtain the above sixteen sets in the form in which they are presented.

$$\left.\begin{aligned}
\theta_2\theta_3\vartheta_1\vartheta_0+\theta_8\theta_9\vartheta_{10}\vartheta_{11}&=\theta_0\theta_1\vartheta_2\vartheta_3+\theta_{10}\theta_{11}\vartheta_8\vartheta_9\\
\theta_4\theta_5\vartheta_6\vartheta_7+\theta_{14}\theta_{15}\vartheta_{12}\vartheta_{13}&=\theta_6\theta_7\vartheta_4\vartheta_5+\theta_{12}\theta_{13}\vartheta_{14}\vartheta_{15}\\
\theta_1\theta_3\vartheta_2\vartheta_0+\theta_4\theta_6\vartheta_5\vartheta_7&=\theta_5\theta_7\vartheta_4\vartheta_6+\theta_0\theta_2\vartheta_1\vartheta_3\\
\theta_{13}\theta_{15}\vartheta_{12}\vartheta_{14}+\theta_8\theta_{10}\vartheta_9\vartheta_{11}&=\theta_{12}\theta_{14}\vartheta_{13}\vartheta_{15}+\theta_9\theta_{11}\vartheta_8\vartheta_{10}\\
\theta_1\theta_2\vartheta_3\vartheta_0+\theta_{13}\theta_{14}\vartheta_{12}\vartheta_{15}&=\theta_0\theta_3\vartheta_1\vartheta_2+\theta_{12}\theta_{15}\vartheta_{13}\vartheta_{14}\\
\theta_4\theta_7\vartheta_5\vartheta_6+\theta_8\theta_{11}\vartheta_9\vartheta_{10}&=\theta_5\theta_6\vartheta_4\vartheta_7+\theta_9\theta_{10}\vartheta_8\vartheta_{11}
\end{aligned}\right\}\quad . \quad . \quad . \quad (208)$$

$$\left.\begin{aligned}
\theta_8\theta_{12}\vartheta_4\vartheta_0+\theta_{10}\theta_{14}\vartheta_2\vartheta_6&=\theta_0\theta_4\vartheta_8\vartheta_{12}+\theta_2\theta_6\vartheta_{10}\vartheta_{14}\\
\theta_9\theta_{13}\vartheta_1\vartheta_5+\theta_{11}\theta_{15}\vartheta_7\vartheta_3&=\theta_1\theta_5\vartheta_9\vartheta_{13}+\theta_7\theta_3\vartheta_{11}\vartheta_{15}\\
\theta_4\theta_{12}\vartheta_8\vartheta_0+\theta_5\theta_{13}\vartheta_1\vartheta_9&=\theta_0\theta_8\vartheta_4\vartheta_{12}+\theta_1\theta_9\vartheta_5\vartheta_{13}\\
\theta_2\theta_{10}\vartheta_6\vartheta_{14}+\theta_{11}\theta_3\vartheta_7\vartheta_{15}&=\theta_6\theta_{14}\vartheta_2\vartheta_{10}+\theta_7\theta_{15}\vartheta_3\vartheta_{11}\\
\theta_4\theta_8\vartheta_{12}\vartheta_0+\theta_{15}\theta_3\vartheta_7\vartheta_{11}&=\theta_0\theta_{12}\vartheta_4\vartheta_8+\theta_7\theta_{11}\vartheta_3\vartheta_{15}\\
\theta_{10}\theta_6\vartheta_2\vartheta_{14}+\theta_5\theta_9\vartheta_1\vartheta_{13}&=\theta_1\theta_{13}\vartheta_5\vartheta_9+\theta_2\theta_{14}\vartheta_6\vartheta_{10}
\end{aligned}\right\}\quad . \quad . \quad . \quad (209)$$

$$\left.\begin{aligned}
\theta_9\theta_{15}\vartheta_6\vartheta_0+\theta_4\theta_2\vartheta_{11}\vartheta_{13}&=\theta_0\theta_6\vartheta_9\vartheta_{15}+\theta_{11}\theta_{13}\vartheta_2\vartheta_4\\
\theta_3\theta_5\vartheta_{10}\vartheta_{12}+\theta_8\theta_{14}\vartheta_1\vartheta_7&=\theta_1\theta_7\vartheta_8\vartheta_{14}+\theta_{10}\theta_{12}\vartheta_3\vartheta_5\\
\theta_6\theta_{15}\vartheta_9\vartheta_0+\theta_8\theta_1\vartheta_{14}\vartheta_7&=\theta_{14}\theta_7\vartheta_1\vartheta_8+\theta_9\theta_0\vartheta_6\vartheta_{15}\\
\theta_2\theta_{11}\vartheta_4\vartheta_{13}+\theta_5\theta_{12}\vartheta_3\vartheta_{10}&=\theta_3\theta_{10}\vartheta_5\vartheta_{12}+\theta_4\theta_{13}\vartheta_2\vartheta_{11}\\
\theta_6\theta_9\vartheta_{15}\vartheta_0+\theta_5\theta_{10}\vartheta_3\vartheta_{12}&=\theta_0\theta_{15}\vartheta_6\vartheta_9+\theta_3\theta_{12}\vartheta_5\vartheta_{10}\\
\theta_8\theta_7\vartheta_1\vartheta_{14}+\theta_4\theta_{11}\vartheta_2\vartheta_{13}&=\theta_2\theta_{13}\vartheta_4\vartheta_{11}+\theta_1\theta_{14}\vartheta_7\vartheta_8
\end{aligned}\right\}\quad . \quad . \quad . \quad (210)$$

$$\left.\begin{aligned}
\theta_1\theta_9\vartheta_0\vartheta_8+\theta_4\theta_{12}\vartheta_5\vartheta_{13}&=\theta_0\theta_8\vartheta_1\vartheta_9+\theta_5\theta_{13}\vartheta_4\vartheta_{12}\\
\theta_{11}\theta_3\vartheta_2\vartheta_{10}+\theta_6\theta_{14}\vartheta_7\vartheta_{15}&=\theta_2\theta_{10}\vartheta_3\vartheta_{11}+\theta_7\theta_{15}\vartheta_6\vartheta_{14}\\
\theta_1\theta_8\vartheta_0\vartheta_9+\theta_6\theta_{15}\vartheta_7\vartheta_{14}&=\theta_0\theta_9\vartheta_1\vartheta_8+\theta_7\theta_{14}\vartheta_6\vartheta_{15}\\
\theta_{11}\theta_2\vartheta_3\vartheta_{10}+\theta_5\theta_{12}\vartheta_4\vartheta_{13}&=\theta_3\theta_{10}\vartheta_2\vartheta_{11}+\theta_4\theta_{13}\vartheta_5\vartheta_{12}\\
\theta_8\theta_9\vartheta_0\vartheta_1+\theta_{10}\theta_{11}\vartheta_2\vartheta_3&=\theta_0\theta_1\vartheta_8\vartheta_9+\theta_2\theta_3\vartheta_{10}\vartheta_{11}\\
\theta_{12}\theta_{13}\vartheta_4\vartheta_5+\theta_{14}\theta_{15}\vartheta_6\vartheta_7&=\theta_4\theta_5\vartheta_{12}\vartheta_{13}+\theta_6\theta_7\vartheta_{14}\vartheta_{15}
\end{aligned}\right\}\quad . \quad . \quad . \quad (211)$$

$$\left.\begin{aligned}
\theta_0\theta_{12}\vartheta_{14}\vartheta_2+\theta_3\theta_{15}\vartheta_{13}\vartheta_1&=\theta_4\theta_8\vartheta_6\vartheta_{10}+\theta_{11}\theta_7\vartheta_5\vartheta_9\\
\theta_0\theta_2\vartheta_{14}\vartheta_{12}+\theta_3\theta_1\vartheta_{13}\vartheta_{15}&=\theta_4\theta_6\vartheta_8\vartheta_{10}+\theta_5\theta_7\vartheta_9\vartheta_{11}\\
\theta_2\theta_{12}\vartheta_{14}\vartheta_0+\theta_{15}\theta_1\vartheta_3\vartheta_{13}&=\theta_6\theta_8\vartheta_4\vartheta_{10}+\theta_5\theta_{11}\vartheta_7\vartheta_9
\end{aligned}\right\}\quad . \quad . \quad . \quad (212)$$

$$\left.\begin{aligned}
\theta_0\theta_6\vartheta_7\vartheta_1+\theta_{11}\theta_{13}\vartheta_{10}\vartheta_{12}&=\theta_9\theta_{15}\vartheta_8\vartheta_{14}+\theta_2\theta_4\vartheta_3\vartheta_5\\
\theta_0\theta_1\vartheta_7\vartheta_6+\theta_{10}\theta_{11}\vartheta_{12}\vartheta_{13}&=\theta_8\theta_9\vartheta_{14}\vartheta_{15}+\theta_2\theta_3\vartheta_4\vartheta_5\\
\theta_1\theta_6\vartheta_7\vartheta_0+\theta_{10}\theta_{13}\vartheta_{12}\vartheta_{11}&=\theta_8\theta_{15}\vartheta_9\vartheta_{14}+\theta_4\theta_3\vartheta_2\vartheta_5
\end{aligned}\right\}\quad . \quad . \quad . \quad (213)$$

$$\left.\begin{aligned}
\theta_0\theta_3\vartheta_{11}\vartheta_8+\theta_{13}\theta_{14}\vartheta_5\vartheta_6&=\theta_{12}\theta_{15}\vartheta_4\vartheta_7+\theta_1\theta_2\vartheta_9\vartheta_{10}\\
\theta_3\theta_8\vartheta_{11}\vartheta_0+\theta_5\theta_{14}\vartheta_{13}\vartheta_6&=\theta_4\theta_{15}\vartheta_{12}\vartheta_7+\theta_2\theta_9\vartheta_1\vartheta_{10}\\
\theta_0\theta_8\vartheta_{11}\vartheta_3+\theta_5\theta_{13}\vartheta_6\vartheta_{14}&=\theta_4\theta_{12}\vartheta_7\vartheta_{15}+\theta_1\theta_9\vartheta_2\vartheta_{10}
\end{aligned}\right\}\quad . \quad . \quad . \quad (214)$$

$$\left.\begin{aligned}
\theta_0\theta_8\vartheta_{10}\vartheta_2+\theta_5\theta_{13}\vartheta_7\vartheta_{15}&=\theta_4\theta_{12}\vartheta_6\vartheta_{14}+\theta_1\theta_9\vartheta_3\vartheta_{11}\\
\theta_2\theta_8\vartheta_{10}\vartheta_0+\theta_7\theta_{13}\vartheta_5\vartheta_{15}&=\theta_6\theta_{12}\vartheta_4\vartheta_{14}+\theta_3\theta_9\vartheta_1\vartheta_{11}\\
\theta_0\theta_2\vartheta_{10}\vartheta_8+\theta_5\theta_7\vartheta_{13}\vartheta_{15}&=\theta_4\theta_6\vartheta_{12}\vartheta_{14}+\theta_1\theta_3\vartheta_9\vartheta_{11}
\end{aligned}\right\}\quad . \quad . \quad . \quad (215)$$

$$\left.\begin{aligned}
\theta_2\theta_6\vartheta_5\vartheta_1+\theta_8\theta_{12}\vartheta_{11}\vartheta_{15}&=\theta_{10}\theta_{14}\vartheta_9\vartheta_{13}+\theta_0\theta_4\vartheta_3\vartheta_7\\
\theta_1\theta_6\vartheta_5\vartheta_2+\theta_8\theta_{15}\vartheta_{11}\vartheta_{12}&=\theta_{10}\theta_{13}\vartheta_9\vartheta_{14}+\theta_3\theta_4\vartheta_0\vartheta_7\\
\theta_1\theta_2\vartheta_5\vartheta_6+\theta_{12}\theta_{15}\vartheta_{11}\vartheta_8&=\theta_{13}\theta_{14}\vartheta_9\vartheta_{10}+\theta_0\theta_3\vartheta_4\vartheta_7
\end{aligned}\right\}\quad . \quad . \quad . \quad (216)$$

$$\left.\begin{aligned}
\theta_4\theta_6\vartheta_1\vartheta_3+\theta_5\theta_7\vartheta_0\vartheta_2&=\theta_1\theta_3\vartheta_4\vartheta_6+\theta_0\theta_2\vartheta_5\vartheta_7\\
\theta_9\theta_{11}\vartheta_{12}\vartheta_{14}+\theta_8\theta_{10}\vartheta_{13}\vartheta_{15}&=\theta_{12}\theta_{14}\vartheta_9\vartheta_{11}+\theta_{13}\theta_{15}\vartheta_8\vartheta_{10}\\
\theta_3\theta_4\vartheta_1\vartheta_6+\theta_8\theta_{15}\vartheta_{10}\vartheta_{13}&=\theta_1\theta_6\vartheta_4\vartheta_3+\theta_{10}\theta_{13}\vartheta_8\vartheta_{15}\\
\theta_2\theta_5\vartheta_0\vartheta_7+\theta_9\theta_{14}\vartheta_{11}\vartheta_{12}&=\theta_0\theta_7\vartheta_2\vartheta_5+\theta_{11}\theta_{12}\vartheta_9\vartheta_{14}\\
\theta_3\theta_6\vartheta_1\vartheta_4+\theta_9\theta_{12}\vartheta_{11}\vartheta_{14}&=\theta_1\theta_4\vartheta_3\vartheta_6+\theta_{11}\theta_{14}\vartheta_9\vartheta_{12}\\
\theta_2\theta_7\vartheta_0\vartheta_5+\theta_8\theta_{13}\vartheta_{15}\vartheta_{10}&=\theta_0\theta_5\vartheta_2\vartheta_7+\theta_{10}\theta_{15}\vartheta_8\vartheta_{13}
\end{aligned}\right\}\quad . \quad . \quad . \quad (217)$$

$$\left.\begin{aligned}
\theta_0\theta_3\vartheta_{15}\vartheta_{12}+\theta_{13}\theta_{14}\vartheta_1\vartheta_2&=\theta_{15}\theta_{12}\vartheta_0\vartheta_3+\theta_1\theta_2\vartheta_{13}\vartheta_{14}\\
\theta_3\theta_{12}\vartheta_{15}\vartheta_0+\theta_5\theta_{10}\vartheta_9\vartheta_6&=\theta_{15}\theta_0\vartheta_3\vartheta_{12}+\theta_9\theta_6\vartheta_5\vartheta_{10}\\
\theta_0\theta_{12}\vartheta_{15}\vartheta_3+\theta_{11}\theta_7\vartheta_4\vartheta_8&=\theta_4\theta_8\vartheta_{11}\vartheta_7+\theta_{15}\theta_3\vartheta_0\vartheta_{12}\\
\theta_2\theta_{14}\vartheta_1\vartheta_{13}+\theta_9\theta_5\vartheta_{10}\vartheta_6&=\theta_1\theta_{13}\vartheta_2\vartheta_{14}+\theta_6\theta_{10}\vartheta_9\vartheta_5\\
\theta_{11}\theta_4\vartheta_7\vartheta_8+\theta_{13}\theta_2\vartheta_1\vartheta_{14}&=\theta_7\theta_8\vartheta_4\vartheta_{11}+\theta_1\theta_{14}\vartheta_2\vartheta_{13}\\
\theta_8\theta_{11}\vartheta_4\vartheta_7+\theta_5\theta_6\vartheta_9\vartheta_{10}&=\theta_4\theta_7\vartheta_8\vartheta_{11}+\theta_9\theta_{10}\vartheta_5\vartheta_6
\end{aligned}\right\}\quad . \quad . \quad . \quad (218)$$

$$\left.\begin{aligned}
\theta_9\theta_4\vartheta_{14}\vartheta_3+\theta_2\theta_{15}\vartheta_5\vartheta_8&=\theta_1\theta_{12}\vartheta_6\vartheta_{11}+\theta_7\theta_{10}\vartheta_0\vartheta_{13}\\
\theta_3\theta_9\vartheta_{14}\vartheta_4+\theta_2\theta_8\vartheta_5\vartheta_{15}&=\theta_6\theta_{12}\vartheta_1\vartheta_{11}+\theta_7\theta_{13}\vartheta_0\vartheta_{10}\\
\theta_3\theta_4\vartheta_{14}\vartheta_9+\theta_8\theta_{15}\vartheta_5\vartheta_2&=\theta_1\theta_6\vartheta_{12}\vartheta_{11}+\theta_{10}\theta_{13}\vartheta_0\vartheta_7
\end{aligned}\right\}\quad . \quad . \quad . \quad (219)$$

and others, as follow, which do not admit of being arranged symmetrically, the equations necessary to complete systems such as (218) or (219) not having been used :

$$\begin{aligned}
&\theta_1\theta_4\vartheta_0\vartheta_5+\theta_{14}\theta_{11}\vartheta_{10}\vartheta_{15}=\theta_3\theta_6\vartheta_2\vartheta_7+\theta_9\theta_{12}\vartheta_8\vartheta_{13}\\
&\theta_1\theta_0\vartheta_4\vartheta_5+\theta_{10}\theta_{11}\vartheta_{14}\vartheta_{15}=\theta_3\theta_2\vartheta_6\vartheta_7+\theta_8\theta_9\vartheta_{12}\vartheta_{13}\\
&\theta_1\theta_{12}\vartheta_{13}\vartheta_0+\theta_2\theta_{15}\vartheta_{14}\vartheta_3=\theta_4\theta_9\vartheta_8\vartheta_5+\theta_{10}\theta_7\vartheta_6\vartheta_{11}\\
&\theta_0\theta_{12}\vartheta_{13}\vartheta_1+\theta_3\theta_{15}\vartheta_{14}\vartheta_2=\theta_4\theta_8\vartheta_5\vartheta_9+\theta_{11}\theta_7\vartheta_6\vartheta_{10}\\
&\theta_3\theta_8\vartheta_{10}\vartheta_1+\theta_4\theta_{15}\vartheta_{13}\vartheta_6=\theta_2\theta_9\vartheta_0\vartheta_{11}+\theta_5\theta_{14}\vartheta_{12}\vartheta_7\\
&\theta_1\theta_8\vartheta_{10}\vartheta_3+\theta_6\theta_{15}\vartheta_{13}\vartheta_4=\theta_0\theta_9\vartheta_2\vartheta_{11}+\theta_7\theta_{14}\vartheta_{12}\vartheta_5\\
&\theta_0\theta_{15}\vartheta_{14}\vartheta_1+\theta_3\theta_{12}\vartheta_2\vartheta_{13}=\theta_6\theta_9\vartheta_8\vartheta_7+\theta_5\theta_{10}\vartheta_4\vartheta_{11}\\
&\theta_0\theta_1\vartheta_{14}\vartheta_{15}+\theta_2\theta_3\vartheta_{12}\vartheta_{13}=\theta_8\theta_9\vartheta_6\vartheta_7+\theta_{10}\theta_{11}\vartheta_4\vartheta_5\\
&\theta_0\theta_9\vartheta_{11}\vartheta_2+\theta_7\theta_{14}\vartheta_{12}\vartheta_5=\theta_1\theta_8\vartheta_3\vartheta_{10}+\theta_6\theta_{15}\vartheta_4\vartheta_{13}\\
&\theta_0\theta_2\vartheta_{11}\vartheta_9+\theta_7\theta_5\vartheta_{12}\vartheta_{14}=\theta_1\theta_3\vartheta_8\vartheta_{10}+\theta_6\theta_4\vartheta_{15}\vartheta_{13}\\
&\theta_0\theta_4\vartheta_3\vartheta_7+\theta_{10}\theta_{14}\vartheta_9\vartheta_{13}=\theta_8\theta_{12}\vartheta_{11}\vartheta_{15}+\theta_2\theta_6\vartheta_1\vartheta_5\\
&\theta_0\theta_3\vartheta_4\vartheta_7+\theta_{13}\theta_{14}\vartheta_9\vartheta_{10}=\theta_{15}\theta_{12}\vartheta_{11}\vartheta_8+\theta_2\theta_1\vartheta_6\vartheta_5
\end{aligned}$$

and many of a similar form.

57. If there be four pairs of arguments x_1, y_1, x_2, y_2, x_3, y_3, x_4, y_4, such that

$$x_1+x_2+x_3+x_4=0=y_1+y_2+y_3+y_4$$

then with the notation of the first section we have

$$\begin{aligned}
&X_1+x_1=X_2+x_2=X_3+x_3=X_4+x_4=0\\
&Y_1+y_1=Y_2+y_2=Y_3+y_3=Y_4+y_4=0
\end{aligned}$$

and the product theorem (23) will give results similar to (205), (206), (207). As examples, if we put

$$\Pi=\vartheta(x_1, y_1)\vartheta(x_2, y_2)\vartheta(x_3, y_3)\vartheta(x_4, y_4)$$

the equations corresponding to (205) will be

$$\begin{aligned}
&\Pi_0+\Pi_5+\Pi_{14}=\Pi_1+\Pi_6+\Pi_{12}\\
&\Pi_0+\Pi_{10}+\Pi_{13}=\Pi_2+\Pi_9+\Pi_{12}
\end{aligned}$$

and so on; and if

$$\Pi_s^r=\vartheta_r(x_1, y_1)\vartheta_r(x_2, y_2)\vartheta_s(x_3, y_3)\vartheta_s(x_4, y_4)$$

the first equations of the sets corresponding to (206), (207) will be respectively

$$\begin{aligned}
&\Pi_5^0+\Pi_7^2+\Pi_6^3+\Pi_4^1=\Pi_0^5+\Pi_2^7+\Pi_3^6+\Pi_1^4\\
&\Pi_2^8+\Pi_6^{12}+\Pi_4^{14}+\Pi_0^{10}=\Pi_8^2+\Pi_{12}^6+\Pi_{14}^4+\Pi_{10}^0
\end{aligned}$$

58. Again, if

$$a+b+c+d=0$$
$$a'+b'+c'+d'=0$$

and in the general theorem we put

$$x_1=u+a \qquad x_2=u+b \qquad x_3=u+c \qquad x_4=u+d$$
$$y_1=v+a' \qquad y_2=v+b' \qquad y_3=v+c' \qquad y_4=v+d'$$

then $X_1=u-a$, $Y_1=v-a'$, and so for the others: and it is not difficult to prove that

$$\begin{aligned}&\vartheta_0(u+a)\vartheta_1(u+b)\vartheta_2(u+c)\vartheta_3(u+d)+\vartheta_2(u+a)\vartheta_3(u+b)\vartheta_0(u+c)\vartheta_1(u+d)\\&\qquad+\vartheta_0(u-a)\vartheta_1(u-b)\vartheta_2(u-c)\vartheta_3(u-d)+\vartheta_2(u-a)\vartheta_3(u-b)\vartheta_0(a-c)\vartheta_1(u-d)\\&=\vartheta_3(u+a)\vartheta_2(u+b)\vartheta_1(u+c)\vartheta_0(u+d)+\vartheta_1(u+a)\vartheta_0(u+b)\vartheta_3(u+c)\vartheta_2(u+d)\\&\qquad+\vartheta_3(u-a)\vartheta_2(u-b)\vartheta_1(u-c)\vartheta_0(u-d)+\vartheta_1(u-a)\vartheta_0(u-b)\vartheta_3(u-c)\vartheta_2(u+d)\end{aligned}$$

$\vartheta(u+a), \ldots$ denoting $\vartheta(u+a,\ v+a'), \ldots$, with other relations of the same kind between the theta-functions.

Section IV.

The "r" tuple theta-functions.

59. The general "r" tuple theta-functions is defined by the equation

$$\begin{aligned}\Phi\left\{\begin{pmatrix}\lambda_1, \lambda_2, \ldots, \lambda_r\\ \nu_1, \nu_2, \ldots, \nu_r\end{pmatrix} x_1, x_2, \ldots, x_r\right\}&=\Sigma\Sigma\ldots(-1)^{m_1\lambda_1+\ldots+m_r\lambda_r}p_1^{\left(m_1+\frac{\nu_1}{2}\right)^2}p_2^{\left(m_2+\frac{\nu_2}{2}\right)^2}\ldots p_r^{\left(m_r+\frac{\nu_r}{2}\right)^2}\\&\quad p_{1,2}^{2\left(m_1+\frac{\nu_1}{2}\right)\left(m_2+\frac{\nu_2}{2}\right)}\ldots p_{s,t}^{2\left(m_s+\frac{\nu_s}{2}\right)\left(m_t+\frac{\nu_t}{2}\right)}\ldots v_1^{(2m_1+\nu_1)r_1}v_2^{(2m_2+\nu_2)x_2}\ldots v_r^{(2m_r+\nu_r)x_r}\quad . \quad . \qquad (220)\end{aligned}$$

in which $\lambda_1, \lambda_2, \ldots, \lambda_r, \nu_1, \nu_2, \ldots, \nu_r$ are given integers (afterwards taken to be either zero or unity) and $\begin{pmatrix}\lambda_1, \lambda_2, \ldots, \lambda_r\\ \nu_1, \nu_2, \ldots, \nu_r\end{pmatrix}$ is called the characteristic; $x_1, x_2, \ldots, x_r$ are the variables; $p_1, p_2, \ldots, p_r, p_{1,2}, \ldots, p_{s,t}, \ldots, v_1, v_2, \ldots, v_r$ are $\frac{r\,.\,r+3}{2}$ constants and are called the parameters; and the "r" tuple summation extends to all positive and negative integral values from $-\infty$ to $+\infty$ (including zero) of $m_1, m_2, \ldots, m_r$. To ensure the convergence of the series it is necessary that the real part of

$$(2m_1+\nu_1)^2\log p_1+\ldots+2(2m_1+\nu_1)(2m_2+\nu^2)\log p_{1,2}+\ldots$$

should be negative for all real values of the m's; beyond this restriction the $\frac{r\,.\,r+1}{2}$ quantities p are of any form or value whatever.

60. From the definition it at once follows that

$$\Phi\begin{Bmatrix}\lambda_1, \lambda_2, \ldots, \lambda_s+2, \ldots, \lambda_r\\ \nu_1, \nu_2, \ldots, \nu_s, \quad \ldots, \nu_r\end{Bmatrix}=\Phi\begin{Bmatrix}\lambda_1, \lambda_2, \ldots, \lambda_s, \ldots, \lambda_r\\ \nu_1, \nu_2, \ldots, \nu_s, \ldots, \nu_r\end{Bmatrix}$$

$$=(-1)^{\lambda_s}\Phi\begin{Bmatrix}\lambda_1, \lambda_2, \ldots, \lambda_s, \quad \ldots, \lambda_r\\ \nu_1, \nu_2, \ldots, \nu_s+2, \ldots, \nu_r\end{Bmatrix} \quad . \quad . \quad . \quad . \quad (221)$$

the variables being the same throughout. Hence it is obvious that the number of distinct functions is $2^{2r}=4^r$. Also

$$\Phi\left\{\begin{pmatrix}\lambda_1, \lambda_2, \ldots, \lambda_r\\ \nu_1, \nu_2, \ldots, \nu_r\end{pmatrix}-x_1, \ -x_2, \ldots, \ -x_r\right\}$$

$$=(-1)^{\lambda_1\nu_1+\ldots+\lambda_r\nu_r}\Phi\left\{\begin{pmatrix}\lambda_1, \lambda_2, \ldots, \lambda_r\\ \nu_1, \nu_2, \ldots, \nu_r\end{pmatrix}x_1, x_2, \ldots, x_r\right\} \quad . \quad . \quad . \quad (222)$$

a formula which enables us to distinguish between even and uneven functions. For each of the expressions, as $\lambda_s\nu_s$, zero value may arise in three ways, viz.: $\lambda_s=0$, $\nu_s=0$; $\lambda_s=1$, $\nu_s=0$; $\lambda_s=0$, $\nu_s=1$; and a value unity arises in one way, $\lambda_s=1$, $\nu_s=1$; and an uneven function will occur when the number of units in the index is odd. Thus if P, Q denote the numbers of even and uneven functions respectively

$$P=3^r+\frac{r.r-1}{1.2}3^{r-2}+\frac{r.r-1.r-2.r-3}{1.2.3.4}3^{r-4}+\ldots$$

$$Q=r.3^{r-1}+\frac{r.r-1.r-2}{1.2.3}3^{r-3}+\ldots$$

and therefore

$$P+Q=(3+1)^r=2^{2r}$$

$$P-Q=(3-1)^r=2^r$$

so that

$$P=2^{2r-1}+2^{r-1}$$

$$Q=2^{2r-1}-2^{r-1}.$$

Periodicity.

61. Putting

$$v_s = e^{\frac{i\pi}{2K_s}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (223)$$

where s has in turn every value from 1 to r, in the definition of Φ there follow at once the r distinct sets of actual periods for Φ :—

x_1	x_2	x_3				x_s	x_r
$4K_1$	0	0				0	0
0	$4K_2$	0				0	0
0	0	$4K_3$				0	0
0	0	0				$4K_s$	0
0	0	0				0	$4K_r$

and

$$\left.\begin{aligned}
&\Phi\left\{\begin{pmatrix}\lambda_1, \lambda_2, \ldots, \lambda_s, \ldots, \lambda_r \\ \nu_1, \nu_2, \ldots, \nu_s, \ldots, \nu_r\end{pmatrix} x_1, x_2, \ldots, x_s + K_s, \ldots, x_r\right\} \\
&\qquad = (-1)^{\frac{1}{2}\nu_s} \Phi\left\{\begin{pmatrix}\lambda_1, \lambda_2, \ldots, \lambda_s + 1, \ldots, \lambda_r \\ \nu_1, \nu_2, \ldots, \nu_s \quad, \ldots, \nu_r\end{pmatrix} x_1, x_2, \ldots, x_s, \ldots x_r\right\} \\
&\Phi\left\{\begin{pmatrix}\lambda_1, \lambda_2, \ldots, \lambda_s, \ldots, \lambda_r \\ \nu_1, \nu_2, \ldots, \nu_s, \ldots, \nu_r\end{pmatrix} x_1, x_2, \ldots, x_s + 2K_s, \ldots, x_r\right\} \\
&\qquad = (-1)^{\nu_s} \Phi\left\{\begin{pmatrix}\lambda_1, \lambda_2, \ldots, \lambda_s, \ldots, \lambda_r \\ \nu_1, \nu_2, \ldots, \nu_s, \ldots, \nu_r\end{pmatrix} x_1, x_2, \ldots, x_s, \ldots, x_r\right\}
\end{aligned}\right\} . \quad (224).$$

By a method similar to that before used can be obtained the following set of quasi-periods :—

x_1	x_2			x_s	x_r
$\frac{4K_1}{\pi i}\log p_1$	$\frac{4K_2}{\pi i}\log p_{1,2}$			$\frac{4K_s}{\pi i}\log p_{1,s}$	$\frac{4K_r}{\pi i}\log p_{1,r}$
$\frac{4K_1}{\pi i}\log p_{1,2}$	$\frac{4K_2}{\pi i}\log p_2$			$\frac{4K_s}{\pi i}\log p_{2,s}$	$\frac{4K_r}{\pi i}\log p_{2,r}$
$\frac{4K_1}{\pi i}\log p_{1,s}$	$\frac{4K_2}{\pi i}\log p_{2,s}$			$\frac{4K_s}{\pi i}\log p_s$	$\frac{4K_r}{\pi i}\log p_{s,r}$
$\frac{4K_1}{\pi i}\log p_{1,r}$	$\frac{4K_2}{\pi i}\log p_{2,r}$			$\frac{4K_s}{\pi i}\log p_{s,r}$	$\frac{4K_r}{\pi i}\log p_r$

and

$$\left.\begin{aligned}
&\Phi\left\{\begin{pmatrix}\lambda_1, \lambda_2, \ldots, \lambda_s, \ldots, \lambda_r\\ \nu_1, \nu_2, \ldots, \nu_s, \ldots, \nu_r\end{pmatrix} x_1+\frac{K_1}{\pi i}\log p_{1,s},\ x_2+\frac{K_2}{\pi i}\log p_{2,s}, \ldots, x_s\right.\\
&\qquad\qquad\qquad\left. +\frac{K_s}{\pi i}\log p_s, \ldots, x_r+\frac{K_r}{\pi i}\log p_{r,s}\right\}\\
&\quad =p_s^{-\frac{1}{4}}e^{-\frac{i\pi x_s}{2K_s}}\Phi\left\{\begin{pmatrix}\lambda_1, \lambda_2, \ldots, \lambda_s \quad, \ldots, \lambda_r\\ \nu_1, \nu_2, \ldots, \nu_s+1, \ldots, \nu_r\end{pmatrix} x_1, x_2, \ldots, x_s, \ldots, x_r\right\}\\
&\Phi\left\{\begin{pmatrix}\lambda_1, \lambda_2, \ldots, \lambda_s, \ldots, \lambda_r\\ \nu_1, \nu_2, \ldots, \nu_s, \ldots, \nu_r\end{pmatrix} x_1+\frac{2K_1}{\pi i}\log p_{1,s},\ x_2+\frac{2K_2}{\pi i}\log p_{2,s}, \ldots, x_s\right.\\
&\qquad\qquad\qquad\left. +\frac{2K_s}{\pi i}\log p_s, \ldots, x_r+\frac{2K_r}{\pi i}\log p_{s,r}\right\}\\
&\quad =p_s^{-1}e^{-\frac{i\pi x_s}{K_s}}(-1)^{\lambda_s}\Phi\left\{\begin{pmatrix}\lambda_1, \lambda_2, \ldots, \lambda_s, \ldots, \lambda_r\\ \nu_1, \nu_2, \ldots, \nu_s, \ldots, \nu_r\end{pmatrix} x_1, x_2, \ldots, x_s, \ldots, x_r\right\}
\end{aligned}\right\} \quad (225).$$

Product theorem.

62. We multiply four functions $\Phi\ \Phi'\ \Phi''\ \Phi'''$ (in which the variables are $x_1, \ldots, x'_1, \ldots, x''_1, \ldots, x'''_1, \ldots$, and the characteristics are $\begin{pmatrix}\lambda_1, \ldots\\ \nu_1, \ldots\end{pmatrix}$, $\begin{pmatrix}\lambda'_1, \ldots\\ \nu'_1, \ldots\end{pmatrix}$, $\begin{pmatrix}\lambda''_1, \ldots\\ \nu''_1, \ldots\end{pmatrix}$, and $\begin{pmatrix}\lambda'''_1, \ldots\\ \nu'''_1, \ldots\end{pmatrix}$ respectively, the sums of the four corresponding numbers being all even); and we find that the product is the sum of 4^r products. Denote such a product by

$$\Pi\Phi\left\{\begin{pmatrix}\lambda_1, \lambda_2, \ldots, \lambda_r \\ \nu_1, \nu_2, \ldots, \nu_r\end{pmatrix} x_1, x_2, \ldots, x_r\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (226)$$

Let

$$\begin{aligned}
&\mathrm{M}_t+2m_t=\mathrm{M}'_t+2m'_t=\mathrm{M}''_t+2m''_t=\mathrm{M}'''_t+2m'''_t=m'_t+m'_t+m''_t+m''_t \\
&2(\mathrm{X}_t+x_t)=2(\mathrm{X}'_t+x'_t)=2(\mathrm{X}''_t+x''_t)=2(\mathrm{X}'''_t+x'''_t)=x_t+x'_t+x''_t+x'''_t \\
&2(\Lambda_t+\lambda_t)=2(\Lambda'_t+\lambda'_t)=2(\Lambda''_t+\lambda''_t)=2(\Lambda'''_t+\lambda'''_t)=\lambda_t+\lambda'_t+\lambda''_t+\lambda'''_t=2\mathrm{L}_t \\
&2(\mathrm{N}_t+\nu_t)=2(\mathrm{N}'_t+\nu'_t)=2(\mathrm{N}''_t+\nu''_t)=2(\mathrm{N}'''_t+\nu'''_t)=\nu_t+\nu'_t+\nu''_t+\nu'''_t
\end{aligned}$$

in which for t are to be substituted, in succession, the values 1, 2, 3, . . . , r. Then

$$m_t\lambda_t+m'_t\lambda'_t+m''_t\lambda''_t+m'''_t\lambda'''_t=\tfrac{1}{2}(\mathrm{M}_t\Lambda_t+\mathrm{M}'_t\Lambda'_t+\mathrm{M}''_t\Lambda''_t+\mathrm{M}'''_t\Lambda'''_t)$$

$$\begin{aligned}
&(2m_t+\nu_t)^2+(2m'_t+\nu'_t)^2+(2m''_t+\nu''_t)^2+(2m'''_t+\nu'''_t) \\
&\qquad=(\mathrm{M}_t+\mathrm{N}_t)^2+(\mathrm{M}'_t+\mathrm{N}'_t)^2+(\mathrm{M}''_t+\mathrm{N}''_t)^2+(\mathrm{M}'''_t+\mathrm{N}'''_t)
\end{aligned}$$

$$\begin{aligned}
&(2m_t+\nu_t)(2m_s+\nu_s)+\ldots+(2m'''_t+\nu'''_t)(2m'''_s+\nu'''_s) \\
&\qquad=(\mathrm{M}_t+\mathrm{N}_t)(\mathrm{M}_s+\mathrm{N}_s)+\ldots+(\mathrm{M}'''_t+\mathrm{N}'''_t)(\mathrm{M}'''_s+\mathrm{N}'''_s)
\end{aligned}$$

$$(m_t+\nu_t)x_t+\ldots+(2m'''_t+\nu'''_t)x'''_t=(\mathrm{M}_t+\mathrm{N}_t)\mathrm{X}_t+\ldots+(\mathrm{M}'''_t+\mathrm{N}'''_t)\mathrm{X}'''_t.$$

These, substituted in $\Phi\Phi'\Phi''\Phi'''$, give

$$\begin{aligned}
\Phi\Phi'\Phi''\Phi'''=\Sigma\Sigma\ldots(-1)^{\sum\limits_{t=1}^{t=r}\frac{\mathrm{M}_t\Lambda_t+\mathrm{M}'_t\Lambda'_t+\mathrm{M}''_t\Lambda''_t+\mathrm{M}'''_t\Lambda'''_t}{2}}\, & p_1^{\frac{1}{4}\{(\mathrm{M}_1+\mathrm{N}_1)^2+\ldots+(\mathrm{M}'''_1+\mathrm{N}'''_1)^2\}}\ldots \\
& p_r^{\frac{1}{4}\{(\mathrm{M}_r+\mathrm{N}_r)^2+\ldots+(\mathrm{M}'''_r+\mathrm{N}'''_r)^2\}}\, p_{1,2}^{\frac{1}{2}\{(\mathrm{M}_1+\mathrm{N}_1)(\mathrm{M}_2+\mathrm{N}_2)+\ldots+(\mathrm{M}'''_1+\mathrm{N}'''_1)(\mathrm{M}'''_2+\mathrm{N}'''_2)\}}\ldots \\
& p_{s,r}^{\frac{1}{2}\{(\mathrm{M}_s+\mathrm{N}_s)(\mathrm{M}_r+\mathrm{N}_r)+\ldots+(\mathrm{M}'''_s+\mathrm{N}'''_s)(\mathrm{M}'''_r+\mathrm{N}'''_r)\}}\, v_1^{(\mathrm{M}_1+\mathrm{N}_1)\mathrm{X}_1+\ldots+(\mathrm{M}'''_1+\mathrm{N}'''_1)\mathrm{X}'''_1}\ldots \\
& v_r^{(\mathrm{M}_r+\mathrm{N}_r)\mathrm{X}_r+\ldots+(\mathrm{M}'''_r+\mathrm{N}'''_r)\mathrm{X}'''_r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (227)
\end{aligned}$$

the summation being taken for all values of the M's defined by the preceding equations. Now the difference between any two of the M's with the same suffix is even, so that all the M's with the same suffix are either even or uneven. In the former case let

$$\mathrm{M}_t=2\mu_t \qquad \mathrm{M}'_t=2\mu'_t \qquad \mathrm{M}''_t=2\mu''_t \qquad \mathrm{M}'''_t=2\mu'''_t$$

and it will be found that if the equations are satisfied

$$\mu_t+\mu'_t+\mu''_t+\mu'''_t=\text{even}.$$

In the latter case, let

$$\mathrm{M}_t=2\mu_t+1 \qquad \mathrm{M}'_t=2\mu'_t+1 \qquad \mathrm{M}''_t=2\mu''_t+1 \qquad \mathrm{M}'''_t=2\mu'''_t+1$$

and then it will be necessary that

$$\mu_t+\mu'_t+\mu''_t+\mu'''_t=\text{uneven}.$$

Separate now the general term in (227) into parts corresponding to the particular cases of the values of the M's (*i.e.*, whether they are even or uneven), and denote them as follows :—

Σ_0 when all the M's are even, and general term P_0:
Σ_t when all the M's except the M_t's are even, and general term P_t:
$\Sigma_{s,t}$ when all the M's except the M_s, M_t are even, and general term $P_{s,t}$:

and so on ; also let

$\overset{t=r}{\underset{t=1}{S}}\Sigma_t=$ sum of all the terms which have one set of M's uneven and all the rest even,

$\overset{s=r}{\underset{s=1}{S}}\ \overset{t=r}{\underset{t=1}{S}}\Sigma_{s,t}=$sum of all the terms which have two sets of M's uneven and all the rest even,

and so on ; making the number of distinct terms on the right-hand side

$$
\begin{array}{ll}
1 & \text{in which no sets of M are uneven}\\
+r & \text{in which one set is uneven}\\
+\dfrac{r.r-1}{2\,!} & \text{in which two sets are uneven}\\
+\dfrac{r.r-1.r-2}{3\,!} & \text{in which three sets are uneven}\\
+\ldots & \\
+1 & \text{in which all the sets are uneven}
\end{array}
$$

viz. : $=2^r$ in all ; and then

$$\Phi\Phi'\Phi''\Phi'''=\Sigma_0P_0+\overset{t=r}{\underset{t=1}{S}}(-1)^{L_t}\Sigma_tP_t+\overset{s=r}{\underset{s=1}{S}}\ \overset{t=r}{\underset{t=1}{S}}(-1)^{L_s+L_t}\Sigma_{s,t}P_{s,t}+\ldots\quad.\quad.\quad(228).$$

In this

$$
\begin{aligned}
P_0=(-1)^{\sum\limits_{t=1}^{t=r}(\mu_t\Lambda_t+\mu'_t\Lambda'_t+\mu''_t\Lambda''_t+\mu'''_t\Lambda'''_t)}&\,p_1^{\frac{1}{4}\{(2\mu_1+N_1)^2+\ldots+(2\mu'''_1+N'''_1)^2\}}\ldots p_r^{\frac{1}{4}\{(2\mu_r+N_r)+\ldots+(2\mu'''_r+N'''_r)^2\}}\\
&p_{1,2}^{\frac{1}{2}\{(2\mu_1+N_1)(2\mu_2+N_2)+\ldots+(2\mu'''_1+N'''_1)(2\mu'''_2+N'''_2)\}}\ldots\\
&v_1^{(2\mu_1+N_1)X_1+\ldots+(2\mu'''_1+N'''_1)X'''_1}\ldots v_r^{(2\mu_r+N_r)X_r+\ldots+(2\mu'''_r+N'''_r)X'''_r}
\end{aligned}
$$

$$
\begin{aligned}
P_{s,t}=(-1)^{\sum\limits_{t=1}^{t=r}(\mu_t\Lambda_t+\mu'_t\Lambda'_t+\mu''_t\Lambda''_t+\mu'''_t\Lambda'''_t)}&\,p_1^{\frac{1}{4}\{(2\mu_1+N_1)^2+\ldots+(2\mu''_1+N'''_1)^2\}}\ldots p_t^{\frac{1}{4}\{(2\mu_t+1+N_t)^2+\ldots+(2\mu'''_t+1+N'''_t)^2\}}\\
&p_s^{\frac{1}{4}\{(2\mu_s+1+N_s)^2+\ldots+(2\mu''_s+1+N'''_s)^2\}}\ldots p_r^{\frac{1}{4}\{(2\mu_r+N_r)^2+\ldots+(2\mu'''_r+N'''_r)^2\}}\ldots\\
&p_{1,t}^{\frac{1}{2}\{(2\mu_1+N_1)(2\mu_t+1+N_t)+\ldots+(2\mu'''_1+N''_1)(2\mu'''_t+1+N'''_t)\}}p_{s,t}^{\frac{1}{2}\{(2\mu_s+1+N_s)(2\mu_t+1+N_t)+\ldots+(2\mu'''_s+1+N'''_s)(2\mu'''_t+1+N'''_t)\}}\\
&v_1^{(2\mu_1+N_1)X_1+\ldots+(2\mu'''_1+N''_1)X'''_1}\ldots v_s^{(2\mu_s+1+N_s)X_s+\ldots+(2\mu'''_s+1+N'''_s)X'''_s}\ldots\\
&v_t^{(2\mu_t+1+N_t)X_t+\ldots+(2\mu'''_t+1+N'''_t)X'''_t}\ldots v_r^{(2\mu_r+N_r)X_r+\ldots+(2\mu'''_r+N''_r)X'''_r}
\end{aligned}
$$

and similarly for the others. Taking the terms in (228) separately we have

$$
\begin{aligned}
2^r\Sigma_0\mathrm{P}_0 = \Sigma_0\mathrm{P}_0 + \overset{t=r}{\underset{t=1}{\mathrm{S}}}\,\Sigma_0(-1)^{\Sigma\mu_t}\mathrm{P}_0 + \overset{t=r}{\underset{t=1}{\mathrm{S}}}\;\overset{s=r}{\underset{s=1}{\mathrm{S}}}\,\Sigma_0(-1)^{\Sigma\mu_t+\Sigma\mu_s}\mathrm{P}_0 \\
+\overset{t=r}{\underset{t=1}{\mathrm{S}}}\;\overset{s=r}{\underset{s=1}{\mathrm{S}}}\;\overset{u=r}{\underset{u=1}{\mathrm{S}}}\,\Sigma_0(-1)^{\Sigma\mu_t+\Sigma\mu_s+\Sigma\mu_u}\mathrm{P}_0 + \dots \quad (229)
\end{aligned}
$$

in which

$$\Sigma\mu_t = \mu_t + \mu'_t + \mu''_t + \mu'''_t$$

and the summation on the right-hand side is taken for all values of the μ's, without restriction, from $-\infty$ to $+\infty$; the factor 2^r is prefixed to the left-hand side because there would remain 2^r terms of the initial value of $\Sigma_0\mathrm{P}_0$ were the right-hand side written out at full length. Now

$$\mathrm{P}_0 = \text{general term in } \Pi\Phi\left\{\begin{pmatrix}\Lambda_1, & \Lambda_2, & \dots, & \Lambda_s, & \dots, & \Lambda_r\\ \mathrm{N}_1, & \mathrm{N}_2, & \dots, & \mathrm{N}_s, & \dots, & \mathrm{N}_r\end{pmatrix}\mathrm{X}_1,\ \mathrm{X}_2,\ \dots,\ \mathrm{X}_s\ \dots,\ \mathrm{X}_r\right\}$$

$$(-1)^{\Sigma\mu_1}\mathrm{P}_0 = \quad ,, \qquad ,, \qquad \Pi\Phi\left\{\begin{pmatrix}\Lambda_1+1, & \Lambda_2, & \dots, & \Lambda_s, & \dots, & \Lambda_r\\ \mathrm{N}_1 & , \mathrm{N}_2, & \dots, & \mathrm{N}_s, & \dots, & \mathrm{N}_r\end{pmatrix}\mathrm{X}_1,\ \mathrm{X}_2,\ \dots,\ \mathrm{X}_s,\ \dots,\ \mathrm{X}_r\right\}$$

. . . . . .

and therefore

$$
\begin{aligned}
2^r\Sigma_0\mathrm{P}_0 = {} & \Pi\Phi\left\{\begin{pmatrix}\Lambda_1, & \Lambda_2, & \dots, & \Lambda_s, & \dots, & \Lambda_r\\ \mathrm{N}_1, & \mathrm{N}_2, & \dots, & \mathrm{N}_s, & \dots, & \mathrm{N}_r\end{pmatrix}\mathrm{X}_1,\ \mathrm{X}_2,\ \dots,\ \mathrm{X}_r\right. \\
& + \sum_{s=1}^{s=r}\Pi\Phi\left\{\begin{pmatrix}\Lambda_1, & \Lambda_2, & \dots, & \Lambda_s+1, & \dots, & \Lambda_r\\ \mathrm{N}_1, & \mathrm{N}_2, & \dots, & \mathrm{N}_s & , \dots, & \mathrm{N}_r\end{pmatrix}\mathrm{X}_1,\ \mathrm{X}_2,\ \dots,\ \mathrm{X}_r\right\} \\
& + \sum_{s=1}^{s=r}\ \sum_{t=1}^{t=r}\Pi\Phi\left\{\begin{pmatrix}\Lambda_1, & \Lambda_2, & \dots, & \Lambda_s+1, & \Lambda_t+1, & \dots, & \Lambda_r\\ \mathrm{N}_1, & \mathrm{N}_2, & \dots, & \mathrm{N}_s & , \mathrm{N}_t & , \dots, & \mathrm{N}_r\end{pmatrix}\mathrm{X}_1,\ \mathrm{X}_2,\ \dots,\ \mathrm{X}_r\right\} \\
& + \sum_{s=1}^{s=r}\ \sum_{t=1}^{t=r}\ \sum_{u=1}^{u=r}\Pi\Phi\left\{\begin{pmatrix}\Lambda_1, & \Lambda_2, & \dots, & \Lambda_s+1, & \Lambda_t+1, & \Lambda_u+1, & \dots, & \Lambda_r\\ \mathrm{N}_1, & \mathrm{N}_2, & \dots, & \mathrm{N}_s & , \mathrm{N}_t & , \mathrm{N}_u & , \dots, & \mathrm{N}_r\end{pmatrix}\mathrm{X}_1,\ \mathrm{X}_2,\ \dots,\ \mathrm{X}_r\right\} \\
& + \dots \quad (230)
\end{aligned}
$$

where in that expression on the right-hand side which has κ of the upper row of numbers in its characteristic of the form Λ_s+1 there will be $\frac{r!}{\kappa!\,\overline{r-\kappa}!}$ independent terms, and these of course are the only ones to be included in the "κ" tuple summation. Thus $2^r\Sigma_0\mathrm{P}_0$ is equal to the sum of 2^r products of four functions, to each product being prefixed a positive sign.

Again

$$
\begin{aligned}
2^r\Sigma_t\mathrm{P}_t = \Sigma\mathrm{P}_t - \Sigma(-1)^{\Sigma\mu_t}\mathrm{P}_t + \overset{s=r}{\underset{s=1}{\mathrm{S}}}\,\Sigma(-1)^{\Sigma\mu_s}\mathrm{P}_t - \overset{s=r}{\underset{s=1}{\mathrm{S}}}\,\Sigma(-1)^{\Sigma\mu_t+\Sigma\mu_s}\mathrm{P}_t \\
+\overset{s=r}{\underset{s=1}{\mathrm{S}}}\;\overset{u=r}{\underset{u=1}{\mathrm{S}}}\,\Sigma(-1)^{\Sigma\mu_s+\Sigma\mu_u}\mathrm{P}_t - \dots \quad (231)
\end{aligned}
$$

where in the third and fourth terms s is not to take the value t; in the fifth term neither s nor u may take the value t, and so on; and all restrictions on the values of the μ's have been removed. In the expression the number of negative terms will be

$$1+r-1+\frac{r-1.r-2}{1.2}+\ldots+r-1+1=2^{r-1};$$

i.e., half the terms will be affected with a negative sign. Now

$$\mathrm{P}_t=\text{general term in } \Pi\Phi\left\{\begin{pmatrix}\Lambda_1, & \Lambda_2, & \ldots, & \Lambda_t & , \ldots, & \Lambda_r\\ \mathrm{N}_1, & \mathrm{N}_2, & \ldots, & \mathrm{N}_t+1, & \ldots, & \mathrm{N}_r\end{pmatrix}\mathrm{X}_1, \mathrm{X}_2, \ldots, \mathrm{X}_r\right\}$$

$$(-1)^{\Sigma\mu_s}\mathrm{P}_t=\quad\text{,,}\quad\text{,,}\quad \Pi\Phi\left\{\begin{pmatrix}\Lambda_1, & \Lambda_2, & \ldots, & \Lambda_s+1, & \ldots, & \Lambda_t & , \ldots, & \Lambda_r\\ \mathrm{N}_1, & \mathrm{N}_2, & \ldots, & \mathrm{N}_s & , \ldots, & \mathrm{N}_t+1, & \ldots, & \mathrm{N}_r\end{pmatrix}\mathrm{X}_1, \mathrm{X}_2, \ldots, \mathrm{X}_r\right\}$$

and so for the others: thus

$$\begin{aligned}2\Sigma_t\mathrm{P}_t=&\,\Pi\Phi\left\{\begin{pmatrix}\Lambda_1, & \Lambda_2, & \ldots, & \Lambda_t & , \ldots, & \Lambda_r\\ \mathrm{N}_1, & \mathrm{N}_2, & \ldots, & \mathrm{N}_t+1, & \ldots, & \mathrm{N}_r\end{pmatrix}\mathrm{X}_1, \mathrm{X}_2, \ldots, \mathrm{X}_r\right\}\\ &+\sum_{s=1}^{s=r}\pm\Pi\Phi\left\{\begin{pmatrix}\Lambda_1, & \Lambda_2, & \ldots, & \Lambda_s+1, & \ldots, & \Lambda_t & , \ldots, & \Lambda_r\\ \mathrm{N}_1, & \mathrm{N}_2, & \ldots, & \mathrm{N}_s & , \ldots, & \mathrm{N}_t+1, & \ldots, & \mathrm{N}_r\end{pmatrix}\mathrm{X}_1, \mathrm{X}_2, \ldots; \mathrm{X}_r\right\}\\ &+\sum_{s=1}^{s=r}\sum_{u=1}^{u=r}\pm\Pi\Phi\left\{\begin{pmatrix}\Lambda_1, & \Lambda_2, & \ldots, & \Lambda_s+1, & \ldots, & \Lambda_t & , \ldots, & \Lambda_u+1, & \ldots, & \Lambda_r\\ \mathrm{N}_1, & \mathrm{N}_2, & \ldots, & \mathrm{N}_s & , \ldots, & \mathrm{N}_t+1, & \ldots. & \mathrm{N}_u & , \ldots, & \mathrm{N}_r\end{pmatrix}\mathrm{X}_1, \mathrm{X}_2, \ldots, \mathrm{X}_r\right\}\\ &+\ldots \qquad (232)\end{aligned}$$

an equal number (2^{r-1}) of terms being affected with a positive and with a negative sign; the rule by which it is applied will shortly be pointed out. Similarly

$$\begin{aligned}2^r\Sigma_{s,t}\mathrm{P}_{s,t}=&\,\Pi\Phi\left\{\begin{pmatrix}\Lambda_1, & \Lambda_2, & \ldots, & \Lambda_s & , \Lambda_t & , \ldots, & \Lambda_r\\ \mathrm{N}_1, & \mathrm{N}_2, & \ldots, & \mathrm{N}_s+1, & \mathrm{N}_t+1, & \ldots, & \mathrm{N}_r\end{pmatrix}\mathrm{X}_1, \mathrm{X}_2, \ldots, \mathrm{X}_r\right\}\\ &+\sum_{\kappa=1}^{\kappa=r}\pm\Pi\Phi\left\{\begin{pmatrix}\Lambda_1, & \Lambda_2, & \ldots, & \Lambda_\kappa+1, & \ldots, & \Lambda_s & , \ldots, & \Lambda_t & , \ldots, & \Lambda_r\\ \mathrm{N}_1, & \mathrm{N}_2, & \ldots, & \mathrm{N}_\kappa & , \ldots, & \mathrm{N}_s+1, & \ldots, & \mathrm{N}_t+1, & \ldots, & \mathrm{N}_r\end{pmatrix}\mathrm{X}_1, \mathrm{X}_2, \ldots, \mathrm{X}_r\right\}\\ &+\sum_{\kappa=1}^{\kappa=r}\sum_{l=1}^{l=r}\pm\Pi\Phi\left\{\begin{pmatrix}\Lambda_1, & \Lambda_2, & \ldots, & \Lambda_\kappa+1, & \ldots, & \Lambda_l+1, & \ldots, & \Lambda_s & , \ldots, & \Lambda_t & , \ldots, & \Lambda_r\\ \mathrm{N}_1, & \mathrm{N}_2, & \ldots, & \mathrm{N}_\kappa & , \ldots, & \mathrm{N}_l & , \ldots, & \mathrm{N}_s+1, & \ldots, & \mathrm{N}_t+1, & \ldots, & \mathrm{N}_r\end{pmatrix}\right.\\ &\qquad\qquad\left.\mathrm{X}_1, \mathrm{X}_2, \ldots, \mathrm{X}_r\right\}+\ldots \qquad (233)\end{aligned}$$

half the terms, as in (232), being affected with a negative sign.

Proceeding in this manner, (228) gives

$$2^r\Phi\Phi'\Phi''\Phi'''=\text{ sum of } 4^r \text{ products of four functions } \Phi \quad . \quad . \quad . \quad (234).$$

63. Each of these products is affected with a positive or negative sign, determined by the following rule (Rule I.); and it has this sign modified by being multiplied, according to another rule (Rule II.), by a definite power of negative unity. Taking the first term, viz.:—

$$\Pi\Phi\left\{\begin{pmatrix}\Lambda_1, \Lambda_2, \ldots, \Lambda_r \\ \mathrm{N}_1, \mathrm{N}_2, \ldots, \mathrm{N}_r\end{pmatrix}\mathrm{X}_1, \mathrm{X}_2, \ldots, \mathrm{X}_r\right\}$$

the characteristics for the remaining 4^r-1 similar products may be written down as follows: for the upper rows take all possible combinations of

$$\begin{cases}\Lambda_1, \\ \Lambda_1+1,\end{cases}\begin{cases}\Lambda_2, \\ \Lambda_2+1,\end{cases}\begin{cases}\Lambda_3, \\ \Lambda_3+1,\end{cases}\ldots,\begin{cases}\Lambda_s, \\ \Lambda_s+1,\end{cases}\ldots,\begin{cases}\Lambda_r, \\ \Lambda_r+1,\end{cases}$$

by selecting one from each bracketed pair; and similarly, for the lower rows, from

$$\begin{cases}\mathrm{N}_1, \\ \mathrm{N}_1+1,\end{cases}\begin{cases}\mathrm{N}_2, \\ \mathrm{N}_2+1,\end{cases}\begin{cases}\mathrm{N}_3, \\ \mathrm{N}_3+1,\end{cases}\ldots,\begin{cases}\mathrm{N}_s, \\ \mathrm{N}_s+1,\end{cases}\ldots,\begin{cases}\mathrm{N}_r, \\ \mathrm{N}_r+1.\end{cases}$$

Defining that number in the lower row of the characteristic as "corresponding" with another in the upper row (and vice versâ) when the two have the same suffix, the following are the rules above referred to:—

Rule I. If, in the typical characteristic of any product, there be an odd number of pairs of corresponding numbers such that each member of a pair differs by unity from the member holding the same position in the first term, viz.: in

$$\Pi\Phi\left\{\begin{pmatrix}\Lambda_1, \Lambda_2, \ldots, \Lambda_r \\ \mathrm{N}_1, \mathrm{N}_2, \ldots, \mathrm{N}_r\end{pmatrix}\mathrm{X}_1, \mathrm{X}_2, \ldots, \mathrm{X}_r\right\},$$

then to that product is prefixed a negative sign; but if there be an even number of such pairs, a positive sign must be prefixed.

(As in the algebraical expression of the theorem the numbers will be of the form N_s+1, N_t+1 and not of the form $\mathrm{N}_s-1=\mathrm{N}_s+1-2$, $\mathrm{N}_t-1=\mathrm{N}_t+1-2$, which might by formula (220) cause a difference of sign, the above rule is perfectly determinate).

Rule II. To find the index of $(-1)^{\frac{1}{2}}$ in order to prefix the proper power of (-1) to a product, there must be taken the sum of the numbers in the upper rows of the characteristics of the four functions in the first term, viz.: in

$$\Pi\Phi\left\{\begin{pmatrix}\Lambda_1, \Lambda_2, \ldots, \Lambda_r \\ \mathrm{N}_1, \mathrm{N}_2, \ldots, \mathrm{N}_r\end{pmatrix}\mathrm{X}_1, \mathrm{X}_2, \ldots, \mathrm{X}_r\right\},$$

corresponding to those in the lower row which hold the same position as the numbers, differing from them by unity, in the lower row of the typical characteristic of the product.

(Since it has been assumed that

$$\Lambda_t+\Lambda'_t+\Lambda''_t+\Lambda'''_t=\text{even}$$

for all values of t which occur, no imaginary quantities are introduced).

Thus, as a term on the right-hand side, there will be

$$-(-1)^{\frac{1}{2}[\Sigma\Lambda_l+\Sigma\Lambda_m+\Sigma\Lambda_n+\Sigma\Lambda_p]}$$

$$\Pi\Phi\left\{\begin{pmatrix}\Lambda_1, & \Lambda_2, & \Lambda_3+1, & \ldots, & \Lambda_\kappa+1, & \Lambda_l+1, & \Lambda_m+1, & \Lambda_n+1, & \Lambda_p & , \Lambda_q, & \ldots, & \Lambda_r \\ N_1, & N_2, & N_3 & ,\ldots, & N_\kappa & , N_l+1, & N_m+1, & N_n+1, & N_p+1, & N_q, & \ldots, & N_r\end{pmatrix}X_1,\ X_2,\ \ldots,\ X_r\right\}$$

where

$$\Sigma\Lambda_l=\Lambda_l+\Lambda'_l+\Lambda''_l+\Lambda'''_l.$$

The coefficient of $\frac{1}{2}$ in the index of -1 is $\Sigma\Lambda_l+\Sigma\Lambda_m+\Sigma\Lambda_n+\Sigma\Lambda_p$, by Rule II., since the numbers N_l+1, N_m+1, N_n+1, N_p+1 are all that differ by unity from those in the first term, and the sum of the numbers which correspond to $N_l, \ldots, N_m, \ldots, N_n, \ldots, N_p, \ldots$, in the four functions in that term is $\Sigma\Lambda_l+\Sigma\Lambda_m+\Sigma\Lambda_n+\Sigma\Lambda_p$; and a $-$ sign is prefixed, by Rule I., because there is an odd number of pairs of corresponding numbers—$\left\{\begin{matrix}\Lambda_l+1,\\ N_l+1,\end{matrix}\right.\left\{\begin{matrix}\Lambda_m+1,\\ N_m+1,\end{matrix}\right.\left\{\begin{matrix}\Lambda_n+1,\\ N_n+1,\end{matrix}\right.$ each member of which differs by unity from the members of the similarly situated pairs in the first term. So another term will be

$$(-1)^{\frac{1}{2}\{\Sigma\Lambda_l+\Sigma\Lambda_m+\Sigma\Lambda_n+\Sigma\Lambda_q\}}\Pi\Phi\left\{\left(\begin{matrix}\Lambda_1, & \Lambda_2, & \Lambda_3+1, & \ldots, & \Lambda_k+1, & \Lambda_l+1, \\ N_1, & N_2, & N_3 & ,\ldots, & N_k & , N_l+1,\end{matrix}\right.\right.$$

$$\left.\left.\begin{matrix}\Lambda_m+1, & \Lambda_n & ,\ldots, & \Lambda_p+1, & \Lambda_q & ,\ldots, & \Lambda_r \\ N_m+1, & N_n+1, & \ldots, & N_p & , N_q+1, & \ldots, & N_r\end{matrix}\right)X_1,\ X_2,\ \ldots,\ X_r\right\};$$

and the sign and coefficient of any term may be written down from an inspection of its characteristic.

64. As it has been proved that every number in the characteristic is either zero or unity, and the assumption has been made that the sum of any four similarly situated numbers in the characteristics of the four functions is even, the general product theorem comprises $(4^r)^3$, *i.e.*, 2^{6r}, particular cases, the variables being still left perfectly general.

65. In a manner similar to that adopted in Section I. the following formulæ are obtainable :—

$$\left.\begin{aligned}
&\Pi\Phi\begin{pmatrix}\lambda_1, \lambda_2, \ldots, \lambda_r \\ \nu_1, \nu_2, \ldots, \nu_r\end{pmatrix}+\mathop{\mathrm{S}}\limits_{t=1}^{t=r}\Pi\Phi\begin{pmatrix}\lambda_1, \lambda_2, \ldots, \lambda_t+1, \ldots, \lambda_r \\ \nu_1, \nu_2, \ldots, \nu_t\quad, \ldots, \nu_r\end{pmatrix}\\
&\qquad+\mathop{\mathrm{S}}\limits_{s=1}^{s=r}\mathop{\mathrm{S}}\limits_{t=1}^{t=r}\Pi\Phi\begin{pmatrix}\lambda_1, \lambda_2, \ldots, \lambda_s+1, \ldots, \lambda_t+1, \ldots, \lambda_r \\ \nu_1, \nu_2, \ldots, \nu_s\quad, \ldots, \nu_t\quad, \ldots, \nu_r\end{pmatrix}+\ldots\\
&\quad\ldots+\mathop{\mathrm{S}}\limits_{s=1}^{s=r}\mathop{\mathrm{S}}\limits_{t=1}^{t=r}\Pi\Phi\begin{pmatrix}\lambda_1+1, \lambda_2+1, \lambda_3+1, \ldots, \lambda_s, \ldots, \lambda_t, \ldots, \lambda_r+1 \\ \nu_1\quad, \nu_2\quad, \nu_3\quad, \ldots, \nu_s, \ldots, \nu_t, \ldots, \nu_r\end{pmatrix}\\
&\qquad+\mathop{\mathrm{S}}\limits_{s=1}^{s=r}\Pi\Phi\begin{pmatrix}\lambda_1+1, \lambda_2+1, \ldots, \lambda_s, \ldots, \lambda_r+1 \\ \nu_1\quad, \nu_2\quad, \ldots, \nu_s, \ldots, \nu_r\end{pmatrix}+\Pi\Phi\begin{pmatrix}\lambda_1+1, \lambda_2+1, \ldots, \lambda_r+1 \\ \nu_1\quad, \nu_2\quad, \ldots, \nu_r\end{pmatrix}\\
&=\text{same expression with }\left.\begin{matrix}\Lambda\text{ written for }\lambda\\ \mathrm{N}\qquad,,\qquad\nu\end{matrix}\right\}\text{throughout}
\end{aligned}\right\}\quad . \qquad (235)$$

the variables on the left-hand side being

$$x_1, x_2, \ldots x_r, x'_1, x'_2, \ldots x'_r, x''_1, x''_2, \ldots x''_r, x'''_1, x'''_2, \ldots x'''_r,$$

and the variables on the right-hand side being

$$\mathrm{X}_1, \mathrm{X}_2, \ldots, \mathrm{X}_r, \mathrm{X}'_1, \mathrm{X}'_2, \ldots, \mathrm{X}'_r, \mathrm{X}''_1, \mathrm{X}''_2, \ldots, \mathrm{X}''_r, \mathrm{X}'''_1, \mathrm{X}'''_2, \ldots, \mathrm{X}'''_r.$$

Again

$$\left.\begin{aligned}
&\Pi\Phi\begin{pmatrix}\lambda_1, \lambda_2, \ldots, \lambda_t, \ldots, \lambda_r \\ \nu_1, \nu_2, \ldots, \nu_t, \ldots, \nu_r\end{pmatrix}-\Pi\Phi\begin{pmatrix}\lambda_1, \lambda_2, \ldots, \lambda_t+1, \ldots, \lambda_r \\ \nu_1, \nu_2, \ldots, \nu_t\quad, \ldots, \nu_r\end{pmatrix}\\
&\qquad+\mathop{\mathrm{S}}\limits_{s=1}^{s=r}\Pi\Phi\begin{pmatrix}\lambda_1, \lambda_2, \ldots, \lambda_s+1, \ldots, \lambda_r \\ \nu_1, \nu_2, \ldots, \nu_t\quad, \ldots, \nu_r\end{pmatrix}\\
&\qquad-\mathop{\mathrm{S}}\limits_{s=1}^{s=r}\Pi\Phi\begin{pmatrix}\lambda_1, \lambda_2, \ldots, \lambda_s+1, \ldots, \lambda_t+1, \ldots, \lambda_r \\ \nu_1, \nu_2, \ldots, \nu_s\quad, \ldots, \nu_t\quad, \ldots, \nu_r\end{pmatrix}\\
&\qquad+\mathop{\mathrm{S}}\limits_{s=1}^{s=r}\mathop{\mathrm{S}}\limits_{u=1}^{u=r}\Pi\Phi\begin{pmatrix}\lambda_1, \lambda_2, \ldots, \lambda_s+1, \ldots, \lambda_u+1, \ldots, \lambda_r \\ \nu_1, \nu_2, \ldots, \nu_s\quad, \ldots, \nu_u\quad, \ldots, \nu_r\end{pmatrix}-\ldots\\
&\quad\ldots-\mathop{\mathrm{S}}\limits_{s=1}^{s=r}\mathop{\mathrm{S}}\limits_{u=1}^{u=r}\Pi\Phi\begin{pmatrix}\lambda_1+1, \lambda_2+1, \ldots, \lambda_s, \ldots, \lambda_u, \ldots, \lambda_r+1 \\ \nu_1\quad, \nu_2\quad, \ldots, \nu_s, \ldots, \nu_u, \ldots, \nu_r\end{pmatrix}\\
&\qquad+\mathop{\mathrm{S}}\limits_{s=1}^{s=r}\Pi\Phi\begin{pmatrix}\lambda_1+1, \lambda_2+1, \ldots, \lambda_s, \ldots, \lambda_t, \ldots, \lambda_r+1 \\ \nu_1\quad, \nu_2\quad, \ldots, \nu_s, \ldots, \nu_t, \ldots, \nu_r\end{pmatrix}\\
&\qquad-\mathop{\mathrm{S}}\limits_{s=1}^{s=r}\Pi\Phi\begin{pmatrix}\lambda_1+1, \lambda_2+1, \ldots, \lambda_s, \ldots, \lambda_r+1 \\ \nu_1\quad, \nu_2\quad, \ldots, \nu_s, \ldots; \nu_r\end{pmatrix}\\
&\qquad+\Pi\Phi\begin{pmatrix}\lambda_1+1, \lambda_2+1, \ldots, \lambda_t, \ldots, \lambda_r+1 \\ \nu_1\quad, \nu_2\quad, \ldots, \nu_t, \ldots, \nu_r\end{pmatrix}-\Pi\Phi\begin{pmatrix}\lambda_1+1, \lambda_2+1, \ldots, \lambda_r+1 \\ \nu_1\quad, \nu_2\quad, \ldots, \nu_r\end{pmatrix}\\
&=(-1)^{\frac{1}{2}\{\lambda_t+\lambda'_t+\lambda''_t+\lambda'''_t\}}[\text{same expression with }\Lambda\text{ written for }\lambda\text{ and N for }\nu\\
&\qquad\qquad\text{throughout except for }\nu_t\text{, for which write }\mathrm{N}_t+1\text{;}\\
&\qquad\qquad\text{the variables being as in (235)}]
\end{aligned}\right\}\quad . \qquad (236)$$

This equation (236) is equivalent to r equations by substituting in succession $t=1, 2, 3, \ldots, r$; and a summation indicated by $\mathop{\mathrm{S}}\limits_{s=1}^{s=r}$ implies that s is to have every integer value 1, 2, 3, . . . , r, except t. Again

$$
\left.\begin{aligned}
&\Pi\Phi\begin{pmatrix}\lambda_1, \lambda_2, \ldots, \lambda_r \\ \nu_1, \nu_2, \ldots, \nu_r\end{pmatrix} - \Pi\Phi\begin{pmatrix}\lambda_1, \lambda_2, \ldots, \lambda_s+1, \ldots, \lambda_t, \ldots, \lambda_r \\ \nu_1, \nu_2, \ldots, \nu_s \quad, \ldots, \nu_t, \ldots, \nu_r\end{pmatrix} \\
&\qquad -\Pi\Phi\begin{pmatrix}\lambda_1, \lambda_2, \ldots, \lambda_s, \ldots, \lambda_t+1, \ldots, \lambda_r \\ \nu_1, \nu_2, \ldots, \nu_s, \ldots, \nu_t \quad, \ldots, \nu_r\end{pmatrix} \\
&\qquad +\mathop{\mathrm{S}}\limits_{l=1}^{l=r}\Pi\Phi\begin{pmatrix}\lambda_1, \lambda_2, \ldots, \lambda_l+1, \ldots, \lambda_s, \lambda_t, \ldots, \lambda_r \\ \nu_1, \nu_2, \ldots, \nu_l \quad, \ldots, \nu_s, \nu_t, \ldots, \nu_r\end{pmatrix} \\
&\qquad +\Pi\Phi\begin{pmatrix}\lambda_1, \lambda_2, \ldots, \lambda_s+1, \lambda_t+1, \ldots, \lambda_r \\ \nu_1, \nu_2, \ldots, \nu_s \quad, \nu_t \quad, \ldots, \nu_r\end{pmatrix} \\
&\qquad -\mathop{\mathrm{S}}\limits_{l=1}^{l=r}\Pi\Phi\begin{pmatrix}\lambda_1, \lambda_2, \ldots, \lambda_l+1, \ldots, \lambda_s, \ldots, \lambda_t+1, \ldots, \lambda_r \\ \nu_1, \nu_2, \ldots, \nu_l \quad, \ldots, \nu_s, \ldots, \nu_t \quad, \ldots. \nu_r\end{pmatrix} \\
&\qquad -\mathop{\mathrm{S}}\limits_{l=1}^{l=r}\Pi\Phi\begin{pmatrix}\lambda_1, \lambda_2, \ldots, \lambda_l+1, \ldots, \lambda_s+1, \ldots, \lambda_t, \ldots, \lambda_r \\ \nu_1, \nu_2, \ldots, \nu_l \quad, \ldots, \nu_s \quad, \ldots, \nu_t, \ldots. \nu_r\end{pmatrix} \\
&\qquad +\mathop{\mathrm{S}}\limits_{l=1}^{l=r}\mathop{\mathrm{S}}\limits_{m=1}^{m=r}\Pi\Phi\begin{pmatrix}\lambda_1, \lambda_2, \ldots, \lambda_l+1, \ldots, \lambda_m+1, \ldots, \lambda_s, \ldots, \lambda_t, \ldots, \lambda_r \\ \nu_1, \nu_2, \ldots, \nu_l \quad, \ldots, \nu_m \quad, \ldots, \nu_s, \ldots, \nu_t, \ldots, \nu_r\end{pmatrix} - \ldots \\
&\quad \ldots +\mathop{\mathrm{S}}\limits_{l=1}^{l=r}\mathop{\mathrm{S}}\limits_{m=1}^{m=r}\Pi\Phi\begin{pmatrix}\lambda_1+1, \lambda_2+1, \ldots, \lambda_l, \ldots, \lambda_m, \ldots, \lambda_s+1, \ldots, \lambda_t+1, \ldots, \lambda_r+1 \\ \nu_1 \quad, \nu_2 \quad, \ldots, \nu_l, \ldots, \nu_m, \ldots, \nu_s \quad, \ldots, \nu_t \quad, \ldots, \nu_r\end{pmatrix} \\
&\qquad -\mathop{\mathrm{S}}\limits_{l=r}^{l=r}\Pi\Phi\begin{pmatrix}\lambda_1+1, \lambda_2+1, \ldots, \lambda_l, \ldots, \lambda_s, \lambda_t+1, \ldots, \lambda_r+1 \\ \nu_1 \quad, \nu_2 \quad, \ldots, \nu_l, \ldots, \nu_s, \nu_t \quad, \ldots, \nu_r\end{pmatrix} \\
&\qquad -\mathop{\mathrm{S}}\limits_{l=1}^{l=r}\Pi\Phi\begin{pmatrix}\lambda_1+1, \lambda_2+1, \ldots, \lambda_l, \ldots, \lambda_s+1, \ldots, \lambda_t, \ldots, \lambda_r+1 \\ \nu_1 \quad, \nu_2 \quad, \ldots, \nu_l, \ldots, \nu_s \quad, \ldots, \nu_t, \ldots, \nu_r\end{pmatrix} \\
&\qquad +\Pi\Phi\begin{pmatrix}\lambda_1+1, \lambda_2+1, \ldots, \lambda_s, \ldots, \lambda_t, \ldots, \lambda_r+1 \\ \nu_1 \quad, \nu_2 \quad, \ldots, \nu_s, \ldots, \nu_t, \ldots. \nu_r\end{pmatrix} \\
&\qquad +\mathop{\mathrm{S}}\limits_{l=1}^{l=r}\Pi\Phi\begin{pmatrix}\lambda_1+1, \lambda_2+1, \ldots, \lambda_l, \ldots, \lambda_s+1, \ldots, \lambda_t+1, \ldots, \lambda_r+1 \\ \nu_1 \quad, \nu_2 \quad, \ldots, \nu_l, \ldots, \nu_s \quad, \ldots, \nu_t \quad, \ldots, \nu_r\end{pmatrix} \\
&\qquad -\Pi\Phi\begin{pmatrix}\lambda_1+1, \lambda_2+1, \ldots, \lambda_s, \ldots, \lambda_t+1, \ldots, \lambda_r+1 \\ \nu_1 \quad, \nu_2 \quad, \ldots, \nu_s, \ldots, \nu_t \quad, \ldots, \nu_r\end{pmatrix} \\
&\qquad -\Pi\Phi\begin{pmatrix}\lambda_1+1, \lambda_2+1. \ldots, \lambda_s+1, \ldots, \lambda_t, \ldots, \lambda_r+1 \\ \nu_1 \quad, \nu_2 \quad, \ldots, \nu_s \quad, \ldots, \nu_t, \ldots, \nu_r\end{pmatrix} \\
&\qquad +\Pi\Phi\begin{pmatrix}\lambda_1+1, \lambda_2+1, \ldots, \lambda_s+1, \ldots, \lambda_t+1, \ldots, \lambda_r+1 \\ \nu_1 \quad. \nu_2 \quad, \ldots, \nu_s \quad, \ldots, \nu_t \quad, \ldots, \nu_r\end{pmatrix} \\
&=(-1)^{\frac{1}{2}(\lambda_t+\lambda'_t+\lambda''_t+\lambda'''_t+\lambda_s+\lambda'_s+\lambda''_s+\lambda'''_s)} \Big[\text{same expression with } \Lambda \text{ written for } \lambda \text{ and N} \\
&\qquad\qquad \text{for } \nu \text{ throughout, except for } \nu_s,\ \nu_t \text{ for} \\
&\qquad\qquad \text{which write respectively } \mathrm{N}_s+1,\ \mathrm{N}_t+1\,; \\
&\qquad\qquad \text{the variables being as in (235)}\Big]
\end{aligned}\right\} \quad (237)
$$

in which s, t are to have in succession the values 1, 2, 3, . . . , r, (but never the same value together); and $\mathop{\mathrm{S}}\limits_{l=1}^{l=r}$ implies summation for every value of l in 1, 2, 3, . . . , r, except $l=s$, and $l=t$. Thus the equation comprises $\frac{1}{2}r(r-1)$ cases.

66. From an inspection of these equations, it is seen that the lower row in the characteristic of each term is the same throughout the same side of the same equation; and this holds throughout the system of 2^r equations of which the above are examples. To write down the equation in which κ of the numbers in the lower row on the right-hand side differ by unity from $\mathrm{N}_1, \mathrm{N}_2, \ldots, \mathrm{N}_r$, (and which is therefore an equation comprising $\frac{r!}{\kappa! \, r-\kappa!}$ cases), take that group of 2^r terms in the general product theorem having this lower row for the common lower row of the characteristic and multiply the group by

$$(-1)^{\frac{1}{2}\overset{t}{\mathrm{S}}\Sigma\Lambda_t}$$

where $\Sigma\Lambda_t$ has already been defined and $\overset{t}{\mathrm{S}}$ implies that summation is to be taken for those κ values of t which have their numbers in the lower row of the characteristic of the form N_t+1; this is the right-hand side of the equation. To obtain the left-hand side the coefficient $(-1)^{\frac{1}{2}\overset{t}{\mathrm{S}}\Sigma\Lambda_t}$ is dropped, as well as all the units in the numbers $\mathrm{N}_t+1, \ldots$; and λ, ν, x are substituted for Λ, N, X respectively. Thus to a term of the form

$$(-1)^{\frac{1}{2}(\Sigma\Lambda_l+\Sigma\Lambda_m+\Sigma\Lambda_n+\Sigma\Lambda_q)}\Pi\Phi\left\{\begin{pmatrix}\Lambda_1, \Lambda_2, \Lambda_3+1, \ldots, \Lambda_\kappa+1, \ldots, \Lambda_l+1, \ldots, \\ \mathrm{N}_1, \mathrm{N}_2, \mathrm{N}_3 \quad, \ldots, \mathrm{N}_\kappa \quad, \ldots, \mathrm{N}_l+1, \ldots,\end{pmatrix}\right.$$

$$\left.\begin{pmatrix}\Lambda_m+1, \ldots, \Lambda_n \quad, \ldots, \Lambda_p+1, \ldots, \Lambda_q \quad, \ldots, \Lambda_r \\ \mathrm{N}_m+1, \ldots, \mathrm{N}_n+1, \ldots, \mathrm{N}_p \quad, \ldots, \mathrm{N}_q+1, \ldots, \mathrm{N}_r\end{pmatrix}\mathrm{X}_1, \mathrm{X}_2, \ldots, \mathrm{X}_r\right\}$$

on the right-hand side, there will on the left-hand side be a term of the form

$$\Pi\Phi\left\{\begin{pmatrix}\lambda_1, \lambda_2, \lambda_3+1, \ldots, \lambda_\kappa+1, \ldots, \lambda_l+1, \ldots, \lambda_m+1, \ldots, \lambda_n, \ldots, \lambda_p+1, \ldots, \lambda_q, \ldots, \lambda_r \\ \nu_1, \nu_2, \nu_3 \quad, \ldots, \nu_\kappa \quad, \ldots, \nu_l \quad, \ldots, \nu_m \quad, \ldots, \nu_n, \ldots, \nu_p \quad, \ldots, \nu_q, \ldots, \nu_r\end{pmatrix}\right.$$

$$\left. x_1, x_2, \ldots, x_r\right\}$$

67. By increasing each variable x (and therefore also X, from its definition) by the quarter quasi-period in any set of conjugate quasi-periods, and taking all combinations of these (amounting in number to 2^r), each equation of the above system of 2^r equations gives rise to 2^r further equations. The reason of this is that each function is periodic with the exception of a factor

$$p_s^{-\frac{1}{4}}e^{-\frac{i\pi x_s}{2\mathrm{K}_s}}$$

and therefore a product on the left-hand side is periodic with the exception of a factor

$$p_s^{-1}e^{-\frac{i\pi}{2\mathrm{K}_s}(x_s+x'_s+x''_s+x'''_s)}$$

while the corresponding factor on the right-hand side is

$$p_s^{-1}e^{-\frac{i\pi}{2K_s}(X_s+X'_s+X''_s+X'''_s)}$$

and these are equal in virtue of the relation

$$x_s+x'_s+x''_s+x'''_s=X_s+X'_s+X''_s+X'''_s.$$

As an example, by the substitutions $x_1+\frac{K_1}{\pi i}\log p_{1,s}$ for $x_1, \dots, x_s+\frac{K_s}{\pi i}\log p_s$ for $x_s, \dots,$ (235) gives

$$\left.\begin{aligned}
&\Pi\Phi\begin{pmatrix}\lambda_1, \lambda_2, \dots, \lambda_s & , \dots, \lambda_r\\ \nu_1, \nu_2, \dots, \nu_s+1, & \dots, \nu_r\end{pmatrix}+\overset{t=r}{\underset{t=1}{\mathrm{S}}}\Pi\Phi\begin{pmatrix}\lambda_1, \lambda_2, \dots, \lambda_s & , \dots, \lambda_t+1, \dots, \lambda_r\\ \nu_1, \nu_2, \dots, \nu_s+1, & \dots, \nu_t \quad , \dots, \nu_r\end{pmatrix}\\
&\qquad+\overset{t=r}{\underset{t=1}{\mathrm{S}}}\;\overset{u=r}{\underset{u=1}{\mathrm{S}}}\Pi\Phi\begin{pmatrix}\lambda_1, \lambda_2, \dots, \lambda_s & , \dots, \lambda_t+1, \dots, \lambda_u+1, \dots, \lambda_r\\ \nu_1, \nu_2, \dots, \nu_s+1, & \dots, \nu_t \quad , \dots, \nu_u \quad , \dots, \nu_r\end{pmatrix}+\dots\\
&\qquad+\overset{t=r}{\underset{t=1}{\mathrm{S}}}\;\overset{u=r}{\underset{u=1}{\mathrm{S}}}\Pi\Phi\begin{pmatrix}\lambda_1+1, \lambda_2+1, \dots, \lambda_s+1, \lambda_t, \lambda_u, \dots, \lambda_r+1\\ \nu_1 \quad , \nu_2 \quad \dots, \nu_s+1, \nu_t, \nu_u, \dots, \nu_r\end{pmatrix}\\
&\qquad+\overset{t=r}{\underset{t=1}{\mathrm{S}}}\Pi\Phi\begin{pmatrix}\lambda_1+1, \lambda_2+1, \dots, \lambda_s+1, \dots, \lambda_t, \dots, \lambda_r+1\\ \nu_1 \quad , \nu_2 \quad , \dots, \nu_s+1, \dots, \nu_t, \dots, \nu_r\end{pmatrix}\\
&\qquad+\Pi\Phi\begin{pmatrix}\lambda_1+1, \lambda_2+1, \dots, \lambda_s+1, \dots, \lambda_r+1\\ \nu_1 \quad , \nu_2 \quad , \dots, \nu_s+1, \dots, \nu_r\end{pmatrix}\\
&=\text{same expression with } \Lambda, \mathrm{N}, \mathrm{X} \text{ written throughout for } \lambda, \nu, x \text{ respectively}
\end{aligned}\right\} \quad (238).$$

The same remark with regard to (238) may be made here as at the end of § 8.

68. We obviously have from the definition

$$\begin{aligned}
&\Phi_r\left[\begin{pmatrix}\lambda_1, \lambda_2, \dots, \lambda_r\\ \nu_1, \nu_2, \dots, \nu_r\end{pmatrix}x_1, x_2, \dots, x_r\right]\\
&\qquad=\sum_{m_1=-\infty}^{m_1=\infty}(-1)^{m_1\lambda_1}p_1^{\left(m_1+\frac{\nu_1}{2}\right)^2}e^{(2m_1+\nu_1)\frac{i\pi x_1}{2K_1}}\Phi_{r-1}\left[\begin{pmatrix}\lambda_2, \lambda_3, \dots, \lambda_r\\ \nu_2, \nu_3, \dots, \nu_r\end{pmatrix}x'_2, x'_3, \dots, x'_r\right]. \quad (239)
\end{aligned}$$

where Φ_r, Φ_{r-1} are functions of the orders r, $r-1$ respectively, and

$$\left.\begin{aligned}
x'_s&=x_s+\frac{2m_1+\nu_1}{2}\log p'_{1,s}\\
\log p_{1,s}&=\frac{i\pi}{2K_s}\log p'_{1,s}
\end{aligned}\right\} \quad \dots\dots\dots \quad (240).$$

Putting all the numbers in the characteristic of Φ_r zero, we have

$$\underset{0,r}{\Phi}(x_1, x_2, \ldots, x_r)$$

$$= \sum_{m_1=-\infty}^{m_1=\infty} p_1^{m_1^2} e^{m_1 \frac{i\pi x_1}{K_1}} \underset{0,r-1}{\Phi}(x_2+m_1 \log p'_{1,2}, x_3+m_1 \log p'_{1,3}, \ldots, x_r+m_1 \log p'_{1,r})$$

$$\begin{aligned}= \underset{0,r-1}{\Phi}(x_2, x_3, \ldots, x_r)&+p_1 \cos \frac{\pi x_1}{K_1}\Big[\underset{0,r-1}{\Phi}(x_2+ \log p'_{1,2}, \ldots)+\underset{0,r-1}{\Phi}(x_2- \log p'_{1,2}, \ldots)\Big]\\ &+ip_1 \sin \frac{\pi x_1}{K_1}\Big[\underset{0,r-1}{\Phi}(x_2+ \log p'_{1,2}, \ldots)-\underset{0,r-1}{\Phi}(x_2- \log p'_{1,2}, \ldots)\Big]\\ &+p_1^4 \cos \frac{2\pi x_1}{K_1}\Big[\underset{0,r-1}{\Phi}(x_2+2 \log p'_{1,2}, \ldots)+\underset{0,r-1}{\Phi}(x_2-2 \log p'_{1,2}, \ldots)\Big]\\ &+ip_1^4 \sin \frac{2\pi x_1}{K_1}\Big[\underset{0,r-1}{\Phi}(x_2+2 \log p'_{1,2}, \ldots)-\underset{0,r-1}{\Phi}(x_2-2 \log p'_{1,2}, \ldots)\Big]\\ &+\ldots\end{aligned}$$

Expanding and arranging, this gives

$$\begin{aligned}\underset{0,r}{\Phi}(x_1, x_2, \ldots, x_r)=&\underset{0,r-1}{\Phi}(x_2, x_3, \ldots, x_r)\Big[1+2p_1 \cos \frac{\pi x_1}{K_1}+2p_1^4 \cos \frac{2\pi x_1}{K_1}+\ldots\Big]\\ &+2i\Big\{p_1 \sin \frac{\pi x_1}{K_1}+2p_1^4 \sin \frac{2\pi x_1}{K_1}+3p_1^9 \sin \frac{3\pi x_1}{K_1}+\ldots\Big\}\Big(\log p'_{1,2}\frac{d}{dx_2}+\log p'_{1,3}\frac{d}{dx_3}+\ldots\Big)\underset{0,r-1}{\Phi}\\ &+\frac{1}{2!}\,2\Big\{p_1 \cos \frac{\pi x_1}{K_1}+2^2.p_1^4 \cos \frac{2\pi x_1}{K_1}+\ldots\Big\}\Big(\log p'_{1,2}\frac{d}{dx_2}+\log p'_{1,3}\frac{d}{dx_3}+\ldots\Big)^2\underset{0,r-1}{\Phi}\\ &+\ldots\end{aligned}$$

$$\begin{aligned}=\underset{0,r-1}{\Phi}\,\theta_{0,0}(x_1)&-\frac{K_1}{\pi}\frac{d\theta_{0,0}(x_1)}{dx_1}\Big[\frac{2K_2}{\pi}\log p_{1,2}\frac{d}{dx_2}+\frac{2K_3}{\pi}\log p_{1,3}\frac{d}{dx_3}+\ldots\Big]\underset{0,r-1}{\Phi}\\ &+\frac{1}{2!}\Big(\frac{K_1}{\pi}\Big)^2\frac{d^2\theta_{0,0}(x_1)}{dx_1^2}\Big[\frac{2K_2}{\pi}\log p_{1,2}\frac{d}{dx_2}+\frac{2K_3}{\pi}\log p_{1,3}\frac{d}{dx_3}+\ldots\Big]_{0,r-1}\Phi+\ldots \quad (241)\end{aligned}$$

$$=e^{-\frac{2K_1}{\pi^2}\sum\limits_{s=2}^{s=r}(\log p_{1,s})K_s\frac{d^2}{dx_1 dx_s}}\theta_{0,0}(x_1)\underset{0,r-1}{\Phi}(x_2, x_3, \ldots, x_r) \quad \ldots\ldots \quad (242)$$

$$=e^{-\frac{2}{\pi^2}\sum\limits_{s=1}^{s=r}\sum\limits_{t=1}^{t=r}K_sK_t\log p_{s,t}\frac{d^2}{dx_s dx_t}}\prod_{t=1}^{t=r}\theta_{0,0}(x_t) \quad \ldots\ldots\ldots\ldots \quad (243),$$

where the double summation in the index implies that *s*, *t* are to have every value 1, 2, 3. . . . , *r* (but never the same value together); and

$$\prod_{t=1}^{t=r}\theta_{0,0}(x_t)=\theta_{0,0}(x_1)\theta_{0,0}(x_2)\ldots\theta_{0,0}(x_r).$$

The theorem for the general function is

$$\Phi\left\{\begin{pmatrix}\lambda_1, & \lambda_2, & \ldots, & \lambda_r\\ \nu_1, & \nu_2, & \ldots, & \nu_r\end{pmatrix}x_1, x_2, \ldots, x_r\right\}=e^{-\frac{2}{\pi^2}\sum\limits_{s=1}^{s=r}\sum\limits_{t=1}^{t=r}K_sK_t\log p_{s,t}\frac{d^2}{dx_s dx_t}}\prod_{u=1}^{u=r}\theta_{\nu_u,\lambda_u}(x_u) \quad . \quad (244).$$

69. Since $\theta(x_u)$ satisfies the general differential equation

$$\frac{d^2\theta}{dx_u^2}-2x_u\left(\kappa'^2_u-\frac{\mathrm{E}_u}{\mathrm{K}_u}\right)\frac{d\theta}{dx_u}+2\kappa_u\kappa'^2_u\frac{d\theta}{d\kappa_u}=0$$

and the general term in Φ, so far as concerns x_u, is a numerical multiple of

$$\mathrm{K}_u^m\frac{d^m\theta(x_u)}{dx_u^m}$$

it follows, exactly as in Section II., that Φ satisfies the r equations of the form

$$\frac{d^2\Phi}{dx_u^2}-2x_u\left(\kappa'^2_u-\frac{\mathrm{E}_u}{\mathrm{K}_u}\right)\frac{d\Phi}{dx_u}+2\kappa_u\kappa'^2_u\frac{d\Phi}{d\kappa_u}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (245).$$

That this is satisfied can be verified by means of the definition of Φ; and the same is true of the $\frac{1}{2}r(r-1)$ equations of the type

$$p_{s,t}\frac{d\Phi}{dp_{s,t}}+\frac{2\mathrm{K}_s\mathrm{K}_t}{\pi^2}\frac{d^2\Phi}{dx_s dx_t}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (246)$$

all satisfied by Φ.

70. Expressions for the constant terms in the even functions and for all coefficients in the expansions of all the functions in powers of the x's may be obtained as before. Noticing that

$$\theta_{\nu,\lambda}(0)=\left(\frac{2}{\pi}\right)^{\frac{1}{2}}\mathrm{K}^{\frac{1}{2}}c^{\frac{\bar{\nu}}{4}}c'^{\frac{\lambda}{4}}$$

(ν, λ being either zero or unity, but not both unity at the same time) we have

$$\mathrm{C}\begin{pmatrix}\lambda_1, & \lambda_2, & \ldots, & \lambda_r \\ \nu_1, & \nu_2, & \ldots, & \nu_r\end{pmatrix}=\left(\frac{2}{\pi}\right)^{\frac{r}{2}}\nabla_0 . \prod_{t=1}^{t=r}\mathrm{K}_t^{\frac{1}{2}}c_t^{\frac{\nu_t}{4}}c'^{\frac{\lambda_t}{4}}_t \quad . \quad . \quad . \quad . \quad . \quad . \quad (247)$$

where

$$\nabla_0=\cosh\left\{2\sum_{s=1}^{s=r}\sum_{t=1}^{t=r}\log p_{s,t}\left(\frac{d^2}{dp_s dp_t}\right)^{\frac{1}{2}}\right\} . \quad . \quad . \quad . \quad . \quad . \quad . \quad (248)$$

and in the summation s, t are not to have the same value together. This gives the constant term in all those 3^r even functions in the characteristics of which no two corresponding numbers are unity at the same time. Similarly if we put

$$\begin{aligned}\pi^2\nabla_{l,m} &= 2\log p_{l,m}+\frac{2^3}{3!}\left\{(\log p_{l,m})^3\frac{d^2}{dp_l dp_m}+3\log p_{l,m}\sum_{s=1}^{s=r}\sum_{t=1}^{t=r}(\log p_{s,t})^2\frac{d^2}{dp_s dp_t}\right\}+\dots\\ &= \frac{1}{2\left(\dfrac{d^2}{dp_l dp_m}\right)^{\frac{1}{2}}}\left[\sinh 2\left\{\log p_{l,m}\left(\frac{d^2}{dp_l dp_m}\right)^{\frac{1}{2}}+\sum_{s=1}^{s=r}\sum_{t=1}^{t=r}\log p_{s,t}\left(\frac{d^2}{dp_s dp_t}\right)^{\frac{1}{2}}\right\}\right.\\ &\qquad\left.+\sinh 2\left\{\log p_{l,m}\left(\frac{d^2}{dp_l dp_m}\right)-\sum_{s=1}^{s=r}\sum_{t=1}^{t=r}\log p_{s,t}\left(\frac{d^2}{dp_s dp_t}\right)^{\frac{1}{2}}\right\}\right] \quad . \quad . \quad . \quad (249)\end{aligned}$$

where in the summation s, t take all the values $1, 2, 3, \dots, r$, except l and m together, then

$$C\begin{pmatrix}\lambda_1, \lambda_2, \dots, 1, \dots, 1, \dots, \lambda_r\\ \nu_1, \nu_2, \dots, 1, \dots, 1, \dots, \nu_r\end{pmatrix}=\left(\frac{2}{\pi}\right)^{\frac{r}{2}}\nabla_{l,m}.c_l^{\frac{1}{4}}c'^{\frac{1}{4}}_l c_m^{\frac{1}{4}}c'^{\frac{1}{4}}_m K_l^{\frac{3}{2}}K_m^{\frac{3}{2}}\prod_{t=1}^{t=r}c_t^{\frac{\nu_t}{4}}c'^{\frac{\lambda_t}{4}}_t K_t^{\frac{1}{2}} \quad . \quad (250)$$

in which t has all values except l, m. This formula gives the constant terms for all those 3^{r-2} even functions in the characteristics of which $\lambda_l=\lambda_m=1=\nu_l=\nu_m$, but no other corresponding numbers are unity at the same time; and since l, m may be any whatever of the suffixes, this formula comprises $\frac{1}{2}r(r-1)3^{r-2}$ constants.

The above will suffice to indicate how all the constants may be obtained.

NOTE.—Since the above memoir was written I have seen a paper by Professor H. J. S. SMITH (in vol. x. of Lond. Math. Soc. Proc., 1879) in which the results of §§ 6, 62, and 63 are given in an equivalent but somewhat different and more concise form. [Sept. 29, 1882.]

XVIII. *On Seismic Experiments.*

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[Plate 52.]

The following paper is an account of a series of experiments made at the Akabane Engineering Works, Tokio, for the purpose of investigating a number of phenomena connected with earthquake motion.

In 1851 Mr. Robert Mallet, by means of a number of carefully-performed experiments, determined the velocity with which vibrations were transmitted through various media. These vibrations were produced by exploding charges of gunpowder. (See Report of British Association, 1857, also Philosophical Transactions, 1861, 1862, Appendix). As maximum and minimum velocities obtained in feet per second, Mr. Mallet found

	Feet per second.
In sand	824·915
In solid granite	1664·576

In 1876, at the time of the Hellgate explosions at the entrance to the New York Harbour, experiments were arranged by General Abbott for like determinations. These experiments gave velocities varying between 3,000 and 8,300 feet per second.

By a different series of experiments on the "compressibility of solid cubes" of rock, Mr. Mallet determined "the mean modulus of elasticity of the material," and inferred the fact that "owing to discontinuity of rocky masses as found in nature, nearly $\frac{7}{8}$ of the full velocity is lost." (See 'The Eruption of Vesuvius in 1872,' by Professor Luigi Palmieri, with notes by Robert Mallet, p. 18.)

These determinations of velocity of wave transit are, so far as we are aware, all that has yet been done in the experimental investigation of phenomena connected with the transmission of vibratory motion through the earth's crust.

In the experiments described in this paper the disturbance was produced by allowing a heavy weight to fall from a height which could be varied from zero up to 35 feet.

The following are the principal points towards which we have directed our attention with regard to the resulting vibrations.

1. The difference in the magnitude and character of the motions produced at stations variously situated with regard to the point at which the blow was struck. These stations were sometimes taken on a level plain at different distances from the origin of the motion. At other times they were taken at various points on the side and summit of a hill. They were also taken so that a deep cutting in the form of a pond intervened between the point where the blow was struck and the observing station.
2. The relation between normal and transverse vibration as simultaneously exhibited at various stations.
3. The velocity of transmission of normal and of transverse vibrations.

The Akabane Engineering Works where these experiments were carried on are situated on the southern side of Tokio, upon a flat alluvial plain bordering a low spur running out from one of the table-land-like elevations which characterise certain portions of the city lying at a short distance from the sea board. The soil, with the exception of that which is immediately upon the surface, may be described as a hardened mud, very similar in character to that which is now being deposited in the delta-like formations at the mouths of the various rivers which enter the bay.

The plan which is attached (Plate 52) shows the general arrangement of the works. The buildings which are shaded are built of brick; the others are light structures built of wood. The ball which gave the shock was always dropped at the point A, it weighed 1,710 lbs., and could be conveniently raised up to a height of 35 feet.

The stations where the effects of the blows were observed are on the lines A B, A C, A D, &c.

The stations on these lines are distinguished from each other by numbers indicating their distance in feet from the point A where the ball fell. The only exception to these are the stations on the line J K, which are marked according to their distances in feet from the corner of the pond J.

The sections on the lines A F and A H will give the contours of the hill. The pond has perpendicular walled sides, and is about 10 feet deep. The lines A B, A C, A D are along ground which is practically level. At the commencement of the experiments the ground was somewhat soft, and the ball at the first few blows sank to a considerable depth.

We may here call attention to the fact that the vibrations which we have investigated, as in the case of vibrations produced in Mr. MALLET's experiments by an explosion of gunpowder, are the result of a disturbance produced on the surface of the ground, whereas in actual earthquake motion it is quite possible that the disturbances we feel may have an origin which is deeply seated beneath the superficial crust. The only advantage which we can claim for our method of producing vibrations, as compared with the production of them by the ignition of some explosive material, is that we are able to make a more definite estimate of the force of the blow.

In giving the following results, which have been classified, so far as it was convenient, according to their object, we call attention to the fact that the fall of the ball was, when we could obtain observers and instruments, used to determine several distinct results.

Thus a blow which was used to *measure* the *velocity* along the line A B was used to determine the relative amount of vibration transmitted along various other lines. This remark explains why the same fall of the ball is referred to in different groups of experiments. These experiments were first projected in November, 1880. During the month of December instruments, &c., were being prepared and the necessary arrangements entered into for the carrying of them out. Of days actually spent in making experiments there were nine. As in some cases, after a set of experiments, new base lines had to be measured, old instruments altered, or new ones designed for the improvements of the work already done, or for the carrying out of new work, and as at the same time much college work, &c., had to be attended to, these nine days were not consecutive.

In consequence of having on each day of experiment to relay our telegraph, reset the various instruments, obtain coolies to work at the windlass in winding up the ball, &c., it was but seldom that more than four or five falls could be obtained in one day. The following table shows the numbers of the falls which took place upon each of the days.

EXPERIMENTS at the Akabane Works, Tokio, January and February, 1881.

	Number of falls.
First day	1, 2.
Second day	3, 4, 5, 6, 7, 8.
Third day	9, 10, 11, 12.
Fourth day	13, 14, 15, 16.
Fifth day	17, 18, 19, 20.
Sixth day	21, 22, 23, 24.
Seventh day	25, 26, 27, 28, 29, 30.
Eighth day	31, 32, 33, 34, 35, 36.
Ninth day.	37, 38, 39, 40.

The object in giving the above table is that, having numbered our experiments according to the fall of the ball, it can be seen on which day the experiment took place. This is necessary, because after every fall on any particular day the ground on which the ball fell became harder, and this hardening has apparently had its effect in the nature of the records. Also on different days the ground generally varied slightly in its character, according as it had or had not been subjected to severe frost during the previous night.

First Set of Experiments.

These were preliminary experiments to determine the relative amounts of energy received at stations variously situated with regard to the point at which the ball fell.

The general method which was followed in making these experiments was by means of a winch to wind the ball up to a certain height, pull a catch, and allow it to drop.

To observe the amount of vibration which was produced in consequence of this blow, a number of similar bottles containing similar amounts of mercury were employed. One or two of these were placed at each station with an observer, who noted, by means of the seconds' hand of a watch, the length of time that the oscillation produced by the blow continued. These bottles had wide necks, an internal diameter of about 40 millims., and were filled with mercury to a depth equal to their diameter. In order to see the vibration of the mercury distinctly, in some of the experiments a small float consisting of a thin circular piece of sheet iron about 6 millims. in diameter, from the centre of which there was a thin piece of wire projecting up through the neck of the bottle, was placed upon the surface of the mercury.

In consequence of a slight motion in the mercury producing a considerable motion in the ship-like mast of the float, the length of time that the mercury oscillated could be easily noted. These floats were made as similar as possible. After the bottle had been firmly planted on the ground, and the mast of the float remained steady on a position passing through the centre of the neck of the bottle, the whole was covered with a large beaker to act as a shade.

This method of experiment is evidently at its best only capable of giving a relative estimate of the amount of motion at the different observing stations.

I.—Fall of ball on the line A B.

All the bottles, which were numbered from 1 to 6 consecutively, were placed in a row at the 30 feet station. The ball fell from a height of 30·5 feet. Owing to the ground being very soft it sank into the soil for fully 1 foot. All the pointers oscillated for about 20 seconds with the exception of that in bottle number 4, which oscillated for nearly 30 seconds.

II.—Fall of ball on the line A B. Ball fell 30·5 feet.

Number of bottle.	Station.	Time of motion of the mercury.	Remarks.
	feet.	seconds.	
6	30	14 (?)	These observations were not carefully made.
4 5	100	About 28	
2 3	200	About 15	

III., IV., and V.—FALLS of ball on the line A D. Ball fell 31 feet.

Number of bottle.	Observer.	Station.	Number of experiment.		
			III. Time of motion.	IV. Time of motion.	V. Time of motion.
		feet.	seconds.	seconds.	seconds.
6 and 3	MILNE . .	100	20	21	22
4 and 5	KINCH . .	200	20	22	. .
4 and 5	KINCH . .	270	Not observed	Not observed	20

VI.—FALL of ball on the line A B. Ball fell 31 feet.

Number of bottle.	Observer.	Station.	Time of motion.
		feet.	seconds.
6 and 3	MILNE . .	100	30
4 and 5	KINCH . .	400	13 and 15

VII. and VIII.—FALL of ball on the line A B. Ball fell 31 feet.

In these experiments the floats were taken out of the bottles and the movement of the surface of the mercury observed directly.

Number of bottle.	Observer.	Station.	Number of experiments.	
			VII. Time of motion.	VIII. Time of motion.
		feet.	seconds.	seconds.
3	MILNE . .	100	13	16
6	NEMBRINI .	300	11	13
4 and 5	KINCH . .	400	8	11

During these last experiments it may be remarked that the observers at the 400 and the 560 feet stations distinctly heard the sound of the ball striking the ground before seeing any motion in the mercury.

IX., X., XI., and XII.—Falls of ball on the line A B. Ball fell 35 feet upon a large block of iron which had been placed in the hole made by the ball.

In these experiments a number of similar saucers containing equal quantities of mercury were substituted for the bottles.

Station.	Number of the experiment.							
	IX.		X.		XI.		XII.	
	Time of motion.	Observer.	Time of motion.	Observer.	Time of motion.	Observer.	Time of motion.	Observer.
feet.	seconds.		seconds.		seconds.		seconds.	
100	9	Milne	14	Milne	15	Gray	20	Milne
300	10	Gray	8 or 9	Angas	7	Angas	..	..
450	..	. . . .	5	Nembrini	8	Nembrini	8	Nembrini

During the XI. Experiment a saucer of mercury was observed at the 75 feet station on the line J K by Brindley, but no motion was detected.

XIII., XIV., XV., and XVI.—Falls of ball on the line J K to show the effect of the pond in cutting off the transmission of vibrations. Ball fell 35 feet upon the block of iron, which had been almost driven out of sight.

Station.	Number of the experiment.							
	XIII.		XIV.		XV.		XVI.	
	Time of motion.	Observer.	Time of motion.	Observer.	Time of motion.	Observer.	Time of motion.	Observer.
feet.	seconds.		seconds.		seconds.		seconds.	
0	9	Milne	10	Nembrini	..	. . . .	..	..
30	8	Tamaki	10	Tamaki	..	. . . .	..	..
60	..	. . . .	5	Milne	9	Nembrini	9	Nembrini
90	..	. . . .	..	. . . .	1	Tamaki	0	Tamaki
120	..	. . . .	..	. . . .	0	Milne	0	Milne

XVII.—Fall of ball on the line A H (see section). Ball fell 35 feet.

Station 140 feet. When the ball fell the observer felt a strong shake and the mercury in the saucer was in consequence put in motion for 12 seconds.

Station 100 feet. Here also a strong shake was felt, but as the ground was in a continuous state of vibration due to the working of a neighbouring engine it was difficult to say how much of the motion produced in the mercury was alone due to the blow.

XVIII. and XIX.—Fall of ball. Ball fell 35 feet.
Line A F and line A E (see sections).

	XVIII.		XIX.		Mean.
	Time of motion.	Observer.	Time of motion.	Observer.	
	seconds.		seconds.		
Line A F, station 180 .	24	Nembrini .	10	Milne . .	17
Line A E, station 130 .	10	Milne . .	13	Nembrini .	11

These experiments apparently show that neither a small hill nor a cutting have the effect of preventing the propagation of vibrations.

This brings us to the end of all the experiments where the movement in a vessel of mercury was used as an indicator of the relative amounts of motion which were transmitted to the various stations. The only value of these observations, which were necessarily somewhat rough, is the indication they give as to where the motion was relatively strong, slight, hardly perceptible, &c., and of points where it had died out or owing to the configuration of the ground had been unable to reach.

Second Set of Experiments.

In the following three experiments, in consequence of having employed a seismometer, we obtained a close approximation to the maximum amplitude of the motion of an earth particle. These indications also showed us whether the movements recorded were due to normal or transverse vibrations.

The seismometer may be briefly described as follows :—

A frame having a spherical base supports on a pivot at a point a little above the centre of curvature of this base a heavy lead ring, of such a weight that when placed at the centre of oscillation of the frame it produces approximately neutral equilibrium.

To the edge of this ring, at points distant from each other by one quarter the circumference, two threads were attached. These threads were carried outwards in the direction of the radii through their points of attachment and passed over light pulleys attached to a second frame which was fixed to the earth. When this second frame was moved, in consequence of the motion of the earth, the ring, by its inertia, caused the threads to turn the pulleys; these pulleys were furnished with indices giving a multiplication of 40 for the motion on one side of the normal position of the earth.

EXPERIMENT XX.—Ball fell 35 feet.

Line A E. Station 130 feet.

The seismometer was so placed that the thread of one pointer bore about E. and W., or was parallel to the edge of the pond, whilst the thread of the second pointer was at right angles to the first thread.

The second pointer moved about 1 millim. as if by a transverse wave.

This shows a total motion of ·05 millim. in the ground.

EXPERIMENT XXI.—Ball fell 35 feet.

Line A E. Station 130 feet.

The seismometer was so placed that one thread pointed directly to the point where the ball fell. At the time of the shock the pointer of this thread did not move, but the pointer at right angles moved about 1 millim., again indicating that the instrument had only been affected by transversal vibrations.

EXPERIMENT XXII.—Ball fell 35 feet.

Line A H. 100 feet station.

The seismometer was so placed that the thread of one pointer was in the direction of the weight, the thread of the other pointer being at right angles to this direction.

Each of the pointers moved about 2 millims.

THIRD SET OF EXPERIMENTS.

In the following experiments a clear graphical distinction between normal and transverse vibrations was obtained at a number of different stations, together with the maximum amplitudes of each of these two distinct movements.

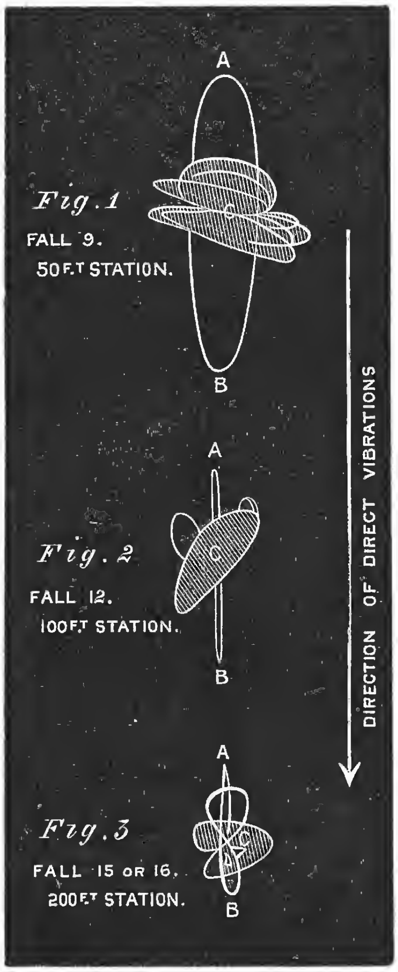
The instrument employed in obtaining these records was a small rolling sphere seismograph, writing its movements directly upon a smoked glass plate by means of a pointer.

The instrument may be described as follows:—At the centre of curvature of a small hemisphere a heavy lead ring rested on a point. To the hemisphere a light pointer was attached, of such a length that its end moved ten times as far as the bottom of the hemisphere. At the end of this pointer a sliding needle was arranged so that its point rested against a smoked glass plate, and when moved by the motion of the earth under the sphere, wrote that motion magnified 10 times on the plate.

When the ball fell, the pointer could be seen to make one or more back and forth quick movements in the direction of the line joining the instrument and the point where the ball fell, and then immediately afterwards to change suddenly into a motion which was inclined to the first movements. The records drawn upon the smoked glass plates corresponded with the observed movements.

The only exceptions, which were two in number, where the above description of the motion does not apply, was when the ball only fell a short distance (8 and 11 feet), and the observing point was at the 50 feet station. In these two instances simply a series of long narrow ellipses were drawn, their longer axes being in the direction of the line joining the position of the falling weight and the instrument.

Of the other figures which were drawn we give three examples, each of which show the motion of the ground magnified 85 times. It will be seen that these figures are so arranged that their greatest lengths are parallel. The direction of greatest motion was that of a line joining the instrument and the point where the ball fell.



At the commencement the needle of the pointer was resting at or very near to the centre of the figure C. In all cases when a clear diagram was obtained it moved to A, then down to B, and back again towards C. At or about this point its motion was suddenly deflected. In many cases this deflection took place nearly at right angles to the first motion, as is shown in fig. 1. Here a great number of flattened ellipse-like figures were drawn one on the top of another so as to render this central portion of the diagram confused.

In fig. 2 these central motions were performed in a direction inclined at about 45° to the first motions.

In fig. 3, about which it may be remarked that other diagrams taken at the same station were very similar, a lemniscate-like motion is exceedingly well pronounced.

The shaded portions of these three figures indicate the parts where the lines were so numerous that they could not be distinctly separated from each other.

The stations at which these figures were drawn and the fall of the ball to which they correspond are indicated beneath each.

In the first column of the following table a number is given showing on which fall of the ball the experiment was made. The second column indicates at which station, or the distance in feet from the point where the ball fell, at which the experiment was made—with the exception of the fall No. 23 all these stations are on the line A D.

The third column shows the maximum amplitude of the normal, and the fourth column that of the transverse motion—excepting the falls 27, 28, and 29 (see Remarks), the weight fell 35 feet.

Number of fall.	Station.	Maximum amplitudes.		Remarks.
		Normal vibrations.	Transverse vibrations.	
	feet.	millims.	millims.	
9	50	7	4·5	The ball fell on a block of iron.
10	50	10	2·5	Transverse motion not very distinct.
11	50	10	2·25	,, ,, ,,
12	100	4·5	2	Transverse motion at an angle of about 45° with the normal vibrations. As the ground hardened under the weight, the transverse vibration apparently became less.
13	100	3·5	2·75	The character of the records drawn for these two experiments are very similar, but slightly different from that obtained from Fall No. 12.
14	100	3·5	3	
15	200	2	1·5	These two records appear to be exactly the same, the transverse motion is now more than at right angles to the normal motion.
16	200	2	1·5	
17	100	4·5	2	
18	100	4·25	2·5	
20	150	5	3·5	
21	250	2	1·5	Not completely written.
22	250	1·75	1·75	Somewhat complicated.
23	130	1	1·25	This record was made at the 130 feet station on A E.
26	50	9 (?)	5	The record is somewhat confused.
27	50	8·5	4·5	Ball only fell 25 feet. The record is not good.
29	50	9	2	Ball only fell 11 feet } Transverse measurements represent the breadth of the normal ellipses.
30	50	8	2·5	,, ,, 8 feet

FOURTH SET OF EXPERIMENTS.

In these experiments a pair of horizontal lever seismographs were used to register the motion of the ground. They indicated their motion on smoked glass plates.

From the following brief description of these instruments it will be seen that if one of them was so placed that its pointer was in the direction of the line joining the centre of disturbance and the observing station, it could only fully register transverse vibrations, whereas normal vibrations would not be recorded.

The instruments were therefore placed with their pointers at right angles to each other, one of them pointing directly towards the weight. In this manner one of them was caused to give a full record of normal vibrations and the other of transverse vibrations.

The seismograph here used was almost identical in form with EWING'S "astatic horizontal lever seismograph."

The principle is that of a mass supported on a horizontal arm which can turn freely round a vertical axis. The mass consisted in this case of an iron ring about 5 kilos. in weight, and was pivoted so that it could turn round a vertical axis through its centre at right angles to the plane of the ring. The distance between the axis of the lever and of the mass was about 2 centims., and the length of the index such as to give a multiplication of 12.

The various columns in the following records of the experiments with these instruments have the same meaning as those in the last set of records.

Number of fall.	Station.	Maximum amplitudes.		Remarks.
		Direct	Transverse.	
	feet.	millims.	millims.	
19	100	7·5 (?)	1·25	
24	50	9	2·5	
25	30	1	. .	On the line J K.

FIFTH SET OF EXPERIMENTS.

These experiments, which were made on the line A D, chiefly differ from the third set in the fact that the smoked glass plates, instead of being at rest whilst the pointers of the seismographs were indicating the back and forth motions of the ground, were drawn along horizontally by means of clockwork. These plates were strips of glass about 2 feet in length, and from 3 to 6 inches in breadth, carried on a small three-wheeled carriage. This carriage was attached to a strip of paper which was pulled along by means of a MORSE telegraph instrument. An advantage in using this instrument was, that an observer was enabled to tick off very conveniently intervals of time upon the moving strip of paper. When the signal was given for the catch to be pulled to allow the ball to fall, an observer started the clockwork, so that before the ball reached the ground the carriage was fairly in motion. When the observer saw that the pointers of the seismograph had ceased to write, the clock was stopped, and the whole arrangement allowed to stand still until the ball could be again wound up and a second record obtained. In this way several successive records were obtained without the introduction of possible errors due to the resetting of the instruments.

The seismograph employed was either the small rolling sphere seismograph, or a pair of horizontal lever instruments.

When the horizontal lever seismographs were used—they were so placed that one of them could only fully record normal vibrations and the other transversal vibrations—it could be seen that the writing of the normal vibrations commenced slightly before that of the transversal ones. During these experiments, in order to determine the interval of time it had taken for these two sets of vibrations to travel from the falling weight to the station at which the instrument was placed, a third pointer was allowed to rest upon the smoked glass plate close to the pointers of the horizontal lever seismographs. As the plate travelled along, this third pointer, when not interfered with, described a straight line. By means of the shock communicated by the ball to the ground, a specially contrived instrument, which was placed about 10 feet from the point where the ball struck, closed an electric circuit. This closing of a circuit caused an electromagnet to suddenly deflect the third pointer and produce a sudden break or deviation in the line being drawn by it. The instant when the vibrations reached a point 10 feet away from the place where the ball struck the ground was thus very clearly marked upon the smoked glass plates. The time at which they reached the station where the plate was situated, was indicated by the pointers of the seismograph ceasing to draw a straight line and commencing to write the vibrations affecting the instrument to which they belonged. The velocities deduced in this manner for the transmission of normal and transverse vibrations are discussed under the sixth set of experiments.

Records obtained with horizontal-lever seismograph. Ball fell 35 feet, unless differently specified.

Number of fall.	Station.	Maximum amplitudes.		Number of visible vibrations.		Remarks.
		Normal vibrations.	Transverse vibrations.	Normal.	Transverse.	
	feet.	millims.	millims.	millims.	millims.	
17	100	3	2	6	13	Normal vibrations drawn in 1 second, transverse in 2 seconds.
18	100	3	1·5	7	12	Transverse drawn in about 3 seconds (?). In both the above records of the transverse vibrations small irregular ripples apparently due to the direct wave or to a looseness in the joints of the instruments are shown.
21	200	1	1	4	12	
22	200	1	1·5	6	12	
23	200	1	1	6	10	
26	100	2	1·25	16	18	Produced in 4 seconds.
27	100	2	1·25	15	18	,, 3½ seconds. Ball fell 25 feet.
28	100	1	1·5	8	9	,, 3½ ,, ,, 18 ,,
30	100	0·75	1	11	16	,, 2½ ,, ,, 8 ,,

Records obtained from the rolling sphere seismograph. Ball fell 35 feet.

Number of fall.	Station.	Maximum amplitudes.		Remarks.
		Normal vibrations.	Transverse vibrations.	
	feet.	millims.	millims.	
19	100	4	1·25	18 distinct waves in 3 seconds.
24	250	1	(?)	25* waves. Direction of motion of plate was parallel to the direction of the transverse waves, which therefore cannot be measured.
25	250	2	(?)	34* waves in 10 seconds. The transverse motions are too much compounded with the direct vibrations to be measured.

Sixth Set of Experiments.

On the line A D.

The only difference between these experiments and the preceding ones is that the smoked glass plate was drawn along parallel to one of the pointers in order to avoid the possibility of any rotational movement taking place. This precaution, although apparently necessary when making experiments for the determination of velocity, does not, in the result, show that the former method gave any errors.

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* It is probable that either from bad adjustment of the instrument or from an independent motion of the ground the instrument has exaggerated the duration of the motion in these two cases.

Transverse vibrations.

Fall 32. 250 feet station.

The transverse motion commenced slowly. After 1·5 second it rose to an amplitude of 3 millims., and it continued, but gradually falling, for more than 6 seconds.

Normal vibrations.

Fall 33. 250 feet station.

There appears to have been about 6·5 normal vibrations per second.

The first distinct motion is apparently one of suction or drawing in of the ground towards the point where the blow was struck. The maximum motion is very near to the commencement of the record, which can only be seen for a length equivalent to a period of 3 seconds.

Normal vibrations.

Fall 34. 250 feet station.

About 11 normal vibrations are visible. There appears to have been 5·5 vibrations per second. The amplitude at the commencement is 1 millim. At the end of 1 second it is ·5 millim. This amplitude decreases regularly to zero, which is reached at the end of 4 seconds.

At first there is a *very slight* motion of compression. The first distinct motion is one of suction or drawing inwards towards the weight. The motion reaches a maximum ·25 second from the commencement.

Transverse vibrations.

Fall 35. 250 feet station.

Here there are 6 waves per second. The movement commenced faintly, rose to a maximum of 1 millim. after about 1 second, and then died out as it commenced.

Motion was observed for 4 seconds.

Normal vibrations.

Fall 36. 100 feet station.

The first distinct motion is a rarefaction or drawing in towards the weight. The greatest amplitude is 2·25 millims. Simultaneously with this record, a complete record of the *vertical* motion was drawn. (See seventh set of experiments.)

SEVENTH SET OF EXPERIMENTS.

These experiments were made for the purpose of determining vertical motion. The instrument employed was a cylindrical tin can half-filled with water, with a flexible sheet indiarubber bottom. As this can is raised or lowered, the flexible bottom synchronously palpitates. These palpitations are recorded by means of a pointer or lever axled in the centre of the flexible bottom and again to a point in contact with the framework of the can; with this contrivance the actual up and down motion of the bottom of the can is multiplied 21 times.

VERTICAL motion on the line A D.

Fall of ball.	Station.	Rise of ground.	Fall of ground.	Total motion.
32	50	6	3	9
33	100	2·5	2	4·5
35	150	. .	. .	1
34	200	. .	. .	0

From this table it will be observed that the vertical motion, which may be a component of the transverse vibrations, dies out very rapidly. The rapid dying out of the vertical movement is possibly due to the free surface. It is remarkable that this wave of distortion should die out so much more quickly than that at right angles to it.

Fall 36. 100 feet station.

Here the vertical motion was registered on a moving glass plate attached at right angles to a plate on which normal vibrations were being written. The first distinct movement appears as if the earth had gone downwards. The double amplitude of the first large wave is 6·5 millims., the downward half of the movement being larger than the upward half. The largest motion to one side 3·5 millims. It diminishes very rapidly at first but afterwards more slowly.

In the following four experiments also made on the line A D, two similar and similarly placed horizontal lever seismographs were allowed to write the movements of the ground simultaneously at two stations. For the object of these experiments, see eighth set of experiments.

Normal vibrations.

Fall 37. 50 feet station.

There were about 8 vibrations per second. The first movement appeared to be one of compression. The vibrations were sensible for 1·5 seconds. The first 2 vibrations are very distinct.

250 feet station.

There were about 8 vibrations per second. The first movement is one of compression. The greatest double amplitude is 1·5 millims. The vibrations are more regular than they are at the 50 feet station. The first 4 vibrations can be easily compared with the first 4 at the 50 feet station.

Normal vibrations.

Fall 38.

These records at the 50 feet and 250 feet stations were very similar to those of the previous experiments.

Transverse vibrations.

Fall 39. 50 feet station.

There were at the commencement 16 vibrations per second. After 1 second there were 8 vibrations per second. This change in the rate of vibration may possibly be due to a disturbance caused by the direct wave, and therefore only apparent.

Vibrations continued for 2 seconds.

250 feet station.

There were about 8 vibrations per second. The greatest double amplitude is ·8 millim. There is no appearance of the rapid vibrations which were observed at the 50 feet station.

The characteristic waves of the 50 feet station can be here recognised.

Transverse wave.

Fall 40.

The character of the vibrations is similar to that of the previous experiment.

EIGHTH SET (VELOCITY).

The method first adopted for the measurement of the velocity of transit was to arrange a very sensitive circuit-closer in a position only a few feet distant from the point where the weight fell. This circuit-closer being connected with a pair of wires leading to a chronograph, situated at a point 560 feet distant from the circuit-closer, served to bring this chronograph in action. An observer stationed beside the chronograph, with his hand on a contact key, broke the circuit at the instant the surface of mercury in a vessel placed on the ground beside him was set into vibration. The corrections, personal and instrumental, in this method were evidently somewhat difficult, and it was abandoned before any very good results were obtained by it. The lowest result obtained by this method was about 380 feet per second and the highest about 930 feet, the average being 630 feet. The explanation of this very high result was no doubt to be found partly in the circuit-closer, which was at that time in various ways imperfect, and partly in the fact that the fall of the weight could be readily heard through the air. This may have caused the observer to anticipate the very small vibration which was produced in the mercury. The circuit-closer was caused sometimes by the rapid up and down movement of the earth to leap out of contact immediately after having closed the circuit; in such cases the chronograph did not act until the circuit became permanently closed. The result of this was evidently to reduce the apparent time of transit and hence increase the velocity.

The second cause of error above indicated was rendered very likely by the fact that the motion of the machinery in the works kept up a continual vibration in the ground.

An automatic circuit-breaker was next introduced into this arrangement and the distance reduced to 200 feet. A correction for instrumental error was in this case determined from observations at 100 feet. With this arrangement a mean velocity of transit equal to 563 feet per second was obtained.

The state of the ground was in this case affected to some extent by frost, the temperature the previous night being 9° Fahr. below freezing.

The circuit-closer was next altered and three more determinations made, the mean result of which gave 379 feet per second. In this case, however, the previously determined correction had to be used owing to the fact that a new correction could not be made that day, and was not afterwards made.

This result is somewhat uncertain, because a new circuit-breaker was here used and more improvements introduced in the closer.

Neither of these methods having given results with which we felt at all satisfied, we turned our attention to the registration of the motion of the ground in conjunction with the time. We were led to this method of experiment through some trials of a seismograph, designed by one of us, which we had taken advantage of these experiments to make. This instrument (Gray's rolling sphere seismograph) proved so sensitive that the motion of the ground could be plainly written at a distance of about 400 feet. The first trial of this method was made as follows:—A glass plate mounted on three wheels was arranged in such a way that it could be pulled forward uniformly by clockwork under the writing point of the seismograph. A separate arrangement consisting of an electromagnet and writing levers was fitted to write on the same plate. This electromagnet was placed in circuit with the circuit-closer previously used, and hence when the circuit was closed a mark was made on the plate; a short time after this the seismograph began to write, and the interval between gave the time of transit subject to the error of the circuit-closer. The error of the circuit-closer was again determined by bringing the recording apparatus 150 nearer to it. This, however, also proved inconvenient and on the whole unsatisfactory, as it caused a great number of somewhat laborious experiments to be made which could be obviously avoided by doubling our apparatus and taking the difference of time between two stations. The propriety of using an instrument which would only record a component of the earth's motion soon suggested itself when we came to consider the question of the relative velocities of the direct and transverse vibrations. A very approximate estimate of this could be got by the instrument just mentioned, but as it wrote the resultant motion there was a little difficulty in determining the exact point at which the transverse wave became felt. To get over the difficulty we had recourse to a pair of horizontal lever seismographs in our possession. These instruments have already been referred to, and for a full account of them see Transactions of the Seismological Society of Japan, vol. ii.

The measurements made by the first method here mentioned gave a velocity of transit equal to 446 feet per second for direct and 353 for transverse wave.

The individual experiments in this case give for direct wave 454, 446, 436, 449, for transverse 360, 345.

By the second method, which evidently requires no correction of any kind except for clock rate, we obtained a mean velocity of 396·5 feet for direct wave.

The individual experiments being 399, 394, the mean velocity for transverse wave was at the same time found to be 360 from two experiments giving respectively 367 and 353, the rate at which the plate moved was determined by causing a small pendulum to act as a periodic circuit-closer, and by so doing to make a series of marks simultaneously on the two plates at distances apart which represented in the actual experiments $\frac{10}{20}$ths of a second of time.

The variations in velocity, although partly due to different methods of experimenting, were no doubt to a certain extent due to different conditions of the soil, there being a variable amount of frost during the period (somewhat extended) which we found necessary for the whole series of experiments.

The difference between the results of the last two methods are probably due to difference in the ground. An interval of more than a week intervened between these two set of determinations, and in the meantime the weather had changed from a minimum temperature a few degrees below zero to a minimum temperature a little above zero.

Giving double value to the last two sets of experiments and leaving out altogether the first set we obtain a mean velocity for the direct wave of 438 feet. Again, from the last two sets we get a mean of 357 for velocity of transverse wave. These results are probably near the truth.

DESCRIPTION OF FIGURES.

PLATE 52.

Fig. 4. (See fifth set of experiments, fall 25). This shows a representation, magnified two and a-half times, of the diagram drawn at the 250 feet station by the rolling seismograph.

The arrow crossing the diagram shows the direction of the direct wave. The first two vibrations are apparently normal ones, but the succeeding waves show the interference of these vibrations with transverse motions.

The most interesting point about the diagram is, perhaps, the evidence of a cycle which is passed through in about five vibrations. Corresponding vibrations are similarly numbered.

Fig. 5. (See fifth set of experiments, fall 17.) Fig. 5 is a representation, magnified two and a-half times, of the record taken by two horizontal lever seismographs at the 100 feet station for the 17th fall of the ball.

The line A represents the record of transverse vibrations, and B that of the direct. These two sets of vibrations were recorded simultaneously on the same plate. The direct vibrations are apparently more rapid than the transverse ones.

The chief point to be observed about this diagram is that the normal vibrations commence a short time before the transverse ones.

It will be observed that although the normal vibrations are at first greater in amplitude than the transversal ones, they die out more quickly than the latter.

Corresponding points in time are joined by cross lines.

This record may be taken as characteristic of other records taken in the same manner.

Fig. 6. (See seventh set of experiments, fall 36.) This diagram shows a magnified record of the normal vibrations on the line A, and vertical vibrations on the line B, as taken simultaneously at the 100 feet station. It will be observed that the vertical motion like the transverse motion is a little behind the direct motion. This we should anticipate, the vertical being like the transverse, a wave of distortion.

Fig. 7. (See sixth set of experiments, fall 33.) This diagram illustrates one of the methods adapted for the determination of the velocity of propagation. In this method a circuit-closer and seismograph were used (see eighth set of experiments).

The diagram was drawn at the 250 feet station, and as shown on this paper has been magnified two and a-half times.

The upper line was drawn by a point on the end of the lever which could be deflected by an electromagnet, which magnet was in electric connexion with the circuit-closer placed near the falling ball.

The lower line shows the diagram drawn by a horizontal lever seismograph, arranged to record normal vibrations. The portions *a b* were drawn before the circuit was closed by the fall of the ball; when the circuit was closed near to the ball the pointer of the electromagnet was deflected to *c* and held deflected.

The plate travelled the interval represented by *b′ d′* before vibrations commenced to be recorded at the 250 feet station.

Fig 8. (See seventh set of experiments, fall 37 or 38). This diagram is a magnified representation of two records of normal vibrations taken simultaneously, the upper one at a 50 feet station and the lower one at a 250 feet station on the same line. Corresponding points *in time* on these two diagrams are marked with similar figures.

The chief point illustrated is the last and probably the best method of obtaining velocity.

By comparing the two records we see that vibrations took place at the 50 feet station considerably before reaching the 250 feet station.

By measuring the interval between similar points, as for instance the beginning of the motion on the two diagrams, the time taken for the vibration to pass from one station to the other can be calculated, and from this the velocity of propagation between these two stations deduced. This method of experiment has the great advantage that no instrumental error can enter into the result. It is of course possible that a vibration which reached the 50 foot station, and was there registered as the beginning of the motion, might not reach the 250 foot station with sufficient amplitude to be registered. An error from this cause can, however, be easily avoided by an examination of the records.

General Conclusions.

The first set of experiments, which was an attempt to obtain an estimate of the rate at which a disturbance produced at a point on the earth's surface is absorbed, were not very satisfactory. The times which the mercury continued to vibrate at the various stations, as given by various observers, did not agree well with each other, and no doubt depended to a great extent on the mode of observing. When, for instance, the reflection of the sun from the surface of the mercury could be seen, the motion could be detected much longer than when a diffused light was used.

The observations all show a dying out of the vibration, as a matter of course, but the only interesting observation is the total cutting off of the vibration by the pond when the point of observation was at a sufficient distance from the corner. This shows that the motion observed near the corner must have been due to the creeping round of the vibrations transmitted along the side. More definite information is given by the second set of experiments. These show that the vibrations which pass up the hill and round the pond are for the most part transverse.

In the third set of experiments we have evidence from the written records that the amplitude of motion is nearly inversely as the distance. The change in the nature of the ground under the falling weight interfered considerably, however, in these experiments. The character of these static records is also very interesting, as this no doubt gives a good indication of the nature of the actual movement in the earth. It would appear also from these, and from subsequent records, that the direct vibrations, although the largest at first, die out more rapidly than the transverse motions. This may have an interesting bearing on the direction observations in earthquakes.

The apparently quicker rate of vibrations of the normal motions, as compared with the transversal movements, is a point worthy of attention. Also the quicker rate of either of these sets of vibrations as compared with the average rate of vibration as experienced in any of the recent earthquakes in Tokio may be noticed. It may also be remarked that the actual earthquake records show a motion which is usually exceedingly irregular.

The actual displacement of a particle from its normal position in the artificial disturbances seems never to have been above 0·5 millim. at a distance of 50 feet, and at a distance of 250 feet it seems to have been from 0·1 to 0·05 millim. Perhaps the most remarkable point made out from these experiments is the very slow rate at which the disturbance is propagated. The experimental verification of the slower rate of propagation in transverse vibrations is also interesting.

This slow rate of propagation accounts for the observation, so often made when earthquakes take place in Tokio, that a rumbling sound precedes them. This rumbling sound is no doubt in many cases the cracking and creaking of buildings reaching the ear through the air sooner than the disturbance in the ground becomes sensible. It is possible that very reliable observations of direction and locality from which the disturbance emanates may be founded on this.

In conclusion, we beg to tender our best thanks to the directors of the Akabane Works for the facilities given for the carrying out of our experiments; to the Telegraph Department for the loan of wire; to Messrs. TAMAKI, KITAKA, and other gentlemen who from time to time afforded us assistance in taking observations.

XIX. *Report of an Examination of the Meteorites of Cranbourne, in Australia; of Rowton, in Shropshire; and of Middlesbrough, in Yorkshire.*

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Communicated by H. Debus, *Ph.D., F.R.S.*

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[Plate 53.]

I. The Siderites of Cranbourne, near Melbourne, Australia.

Already, in 1854, it was known that masses of iron lay near Western Port, south-east of Melbourne. Mr. E. Fitzgibbons, the Secretary of the Municipality of Melbourne, was the first to direct attention to their meteoric characters, and he succeeded in removing enough of the larger mass to have the pieces forged into a horseshoe.

Two masses of meteoric iron were discovered in Victoria, and they were first reported upon by the late W. Haidinger in the 'Sitzungsberichte Akad. Wien'* in 1861. The smaller block became the property of Mr. Abel, the engineer; the larger one was purchased for a sovereign by Mr. A. Bruce, now of Chislehurst. It appears that Mr. Bruce had seen a piece of iron, which had the appearance of being meteoric iron, in the fireplace of a squatter there, and he asked the man if any more of that kind was to be met with in that neighbourhood. He was conducted to a spot in the adjoining parish of Sherwood, where an irregular spur of iron projected from the surface, and he there and then purchased it with the intention of presenting it to the British Museum. Later on, when they proceeded to dig round it and uncover its sides, they were astonished at its large size; various sums of money were offered Mr. Bruce for the splendid block, but his one answer to all such offers was: "No! I have bought it for a sovereign; and I am going to give it to the British Museum." As has been stated, a point only of the iron was above the surface. Its position in the ground is well shown in a photograph taken on the spot by my late friend, Mr. R. Daintree, the Agent-General for Queensland, after the tertiary sandstone enclosing it had been removed. It is the same sandstone which crops out at Broughton; with basalt from 12 to 15 feet below, as on the coast at Western Port. Bruce states that the lower bed is silurian, and that the block of iron penetrated a foot or more into it.

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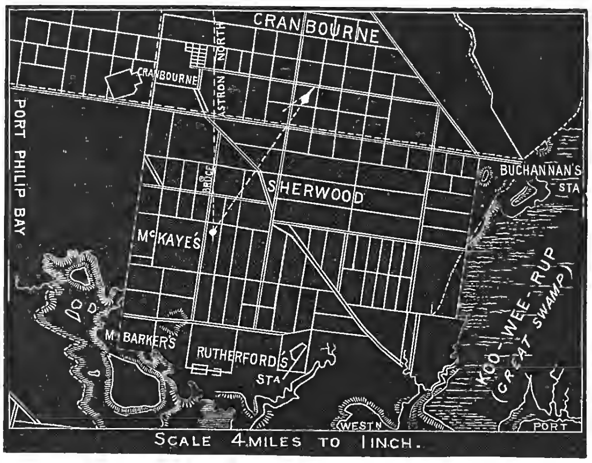
* W. Haidinger, 'Sitzungsberichte Akad. Wien,' xliv., 18th April, 6th June, and 17th October, 1861; xlv., 65, 9th January, 1862.

Early in 1861 the spot was visited by Dr. Neumayer and Mr. Abel; one mass was found to weigh several hundredweight, the other from three to four tons. Their relative position is shown in the accompanying small sketch map of the district. They were found to be beyond all question native, or rather meteoric, iron covered with a crust of the usual characters, in which the customary hollows were not wanting. This statement is, however, somewhat misleading; no crust corresponding to that of magnetite, such as is presented by the Rowton siderite (see *infra*), is met with, but in place a layer of considerable thickness of hydrated oxides and magnetite, indicating a long period during which the blocks had lain in the earth. The relative positions of the two masses was S. 34° W. and N. 34° E. (magnetic declination), and they were 3·6 miles (60 miles to a degree at the equator) apart. Both lay close to the surface, and were only so deeply imbedded that a point protruded from the soil. The latitude of the smaller block, which lies north of the other, is 38° 8′ S. and the longitude 145° 22′ E.; those of the larger being latitude 38° 11′ S. and longitude 145° 20′ E. of Greenwich. The height above sea level of the former was 107 feet, and of the latter 127 feet.



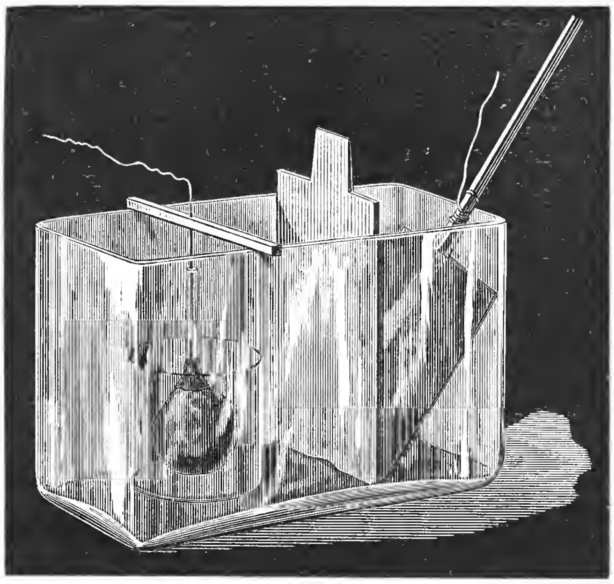
They showed no polarity beyond that due to the action of the earth. The under side of each mass was strongly south magnetic, and on the upper side north magnetic. The longer axis of the Bruce meteorite, the larger mass, is about 5 English feet, and it lay exactly in the magnetic meridian of the place.

Neumayer made a number of determinations of the specific gravity of the nickel-iron of the smaller mass, in the possession of Mr. Abel, ranging from 7·12 to 7·6, the crust being 3·66. This block was sent to the International Exhibition in London in 1862. The larger was brought down to Melbourne and placed in the University Grounds there, near the shore, and unfortunately exposed to the action of the sea-water. Efforts were made to delay the shipment of the Bruce meteorite to England, but eventually the smaller block was bought by the Trustees of the British Museum for £300, and it was presented to the Colonial Museum; the Bruce meteorite was then sent to this country.

When it reached the British Museum some holes were drilled into its under surface, and it was fixed on a turntable in the first room of the Mineral Gallery. It was found to decay to a considerable extent; fragments oxidised and crumbled off, and drops of iron chloride exuded here and there. This, however, was stopped to a very great extent by injecting it with clear shellac varnish, and keeping it in a glass case provided with trays containing caustic lime. By this means the destruction has been reduced to a minimum. It was noticed that the part of the meteorite which was so rapidly decaying presented a very marked crystalline character; that the tetrahedral structure broke up into plates, and between them were very thin plates of another constituent, which less readily underwent change. The action of moisture on these series of plates was like that of the exciting liquid of a galvanic cell, and caused the oxidation to proceed very rapidly. Many of the fragments which came off at this time were selected and reduced again to the firm solid original condition and present beautiful structure. Of this I shall have more to say later on.

II. *Proportion of nickel and other constituents present in the nickel-iron.*

It was at once noticed that the meteorite consisted entirely of metallic minerals, that it contained no rocky matter whatever. One of the first experiments which suggested itself was to determine whether the iron was only alloyed with nickel, cobalt, copper, &c., or whether it contained combined carbon. A weighed portion was suspended by a platinum wire, carefully covered up with glass and caoutchouc, in a solution of recrystallised salt, and connected with a Bunsen cell, in the apparatus shown in the accompanying woodcut. The positive cell was kept slightly acid from time to time as it grew alkaline. Nickel-iron, weighing 5·9989 grms., was dissolved in this way,



and the greater part of the insoluble ingredients was found to consist of very minute bright apparently square prisms, which pervade all the nickel-iron, and apparently constitute nearly 1 per cent. of its mass. These prisms are acted upon very slowly and with considerable difficulty by hydrogen chloride, but disappear readily in hydrogen nitrate. But I shall return to the consideration of the characters and composition of the prisms later on. The absence of all combined carbon was fully established. The nickel-iron thus dissolved was found to contain of:

	Per cent.
Prisms	0·932
Nickel	7·651
Cobalt	0·501
Copper	0·0156
Silicium	0·172

Some of the largest nickel-iron crystals, and cleavages of them, were examined for other constituents than iron with the following results:—I. was a tetrahedron of iron with cleavages parallel to the faces of the tetrahedron; II. was similar to I. but thinner; III. were several examples of cleavage plates, firm not pliant, thicker than the paper-like plates which will be described later on; IV. were thinner plates, but not pliant ones; V. were thick cleavage plates; and VI. some borings. The following ingredients were met with:—

	I.	II.	III.	IV.	V.	VI.
Insoluble part . .	1·405	0·072	0·103, 0·106, 0·724	none	none	0·137
Nickel.	..	7·837, 7·712, 7·529, 7·504	9·764, 6·476	..	..	..
Nickel and cobalt .	8·057	..	..	9·801	9·046	..
Cobalt.	..	0·601	0·756	..	..	..
Phosphorus . . .	..	0·187	0·018	0·059	..	..
Sulphur	..	..	0·023	..	..	..

The rusted fragments of the meteorite, which were very carefully picked over, yielded many very good crystals of nickel-iron. These were reduced in porcelain tubes in hydrogen, a large quantity of hydrogen chloride was extracted from them, and dozens of perfectly complete tetrahedra of nickel-iron as well as many cleavage pieces with sharp edges were safely preserved.

III. *Edmondsonite, a well defined mineral species, occurring in the Cranbourne meteorite.*

In one of the early notes on the Bruce meteorite published by W. Haidinger, in 1862, he wrote: "Vielleicht finden sich in der That innerhalb der Meteoreisenmassen . . . selbst manche Sättigungspunkte, welche wirklich verschiedene Mineralspecies darstellen." Such an instance presents itself in the thin paper-like pliant plates which lie on the faces of the tetrahedra of nickel-iron and between the large plates of the crystals of nickel-iron; they are in the form of equilateral triangles or are lozenge-shaped, have the thickness of stout writing paper and, unlike the plates of nickel-iron, are quite pliant. They are strongly magnetic, are of a pure white colour, and have evidently been extruded from the nickel-iron at the time of formation. They are soluble in hydrogen chloride and nitrate. As the examination of them was made in the case of some which had been reduced in hydrogen, a further portion picked direct from the fragments which had come off the meteorite was taken; both kinds were found to be equally pliant. The fresh plates taken direct from the meteorite contained 0·688 per cent. of phosphorus. Analysis of the plates showed them to consist of:

Iron . . .	70·138 ÷ 28 = 2·504	: 5
Nickel . . .	29·744 ÷ 29·5 = 1·008	: 2
	99·882	

This is evidently an alloy of very well defined composition, which has been extruded from the nickel-iron under special conditions when the latter was saturated with it and ready to expel it. It is the constituent of nickel-iron which forms the fine lines constituting the Wiedmannstättian figures, and not schreibersite, as usually stated in writings on the etched figures of meteoric iron. Tänite is the name which Professor GUSTAV ROSE gave to leaves containing 13·2 per cent. of nickel, and which he stated to form the figures on an etched surface. Dr. K. G. ZIMMERMANN, in a letter to one of the editors of the 'Jahrbuch für Mineralogie,' 1861, p. 557, proposed the name "meteorine" for a new metal occurring in the Cranbourne meteorite which he found to contain no copper, nickel, or cobalt. The substance referred to in both cases was evidently the little plates above described. As the composition of this mineral has now for the first time been definitely made out, I propose to call it Edmondsonite, in memory of the late GEORGE EDMONDSON, the Head Master of Queenwood College, Hampshire, a great lover of science; a man with whom I had the honour to be long and intimately connected.

A curious accident should here be described which established the fact that the alloy is a definite chemical compound. A number of pieces of nickel-iron from this meteorite which had become rusty were heated in a porcelain tube in a current of hydrogen. During the progress of the experiment, which was conducted out of doors, it came on to rain, and some drops touched the hot tube and cracked it. Air slowly entered the crack and oxidised the iron till it acquired a bright blue colour; while the little plate of Edmondsonite remained colourless (Plate 53, fig. 1). This result accords with the conclusion arrived at by STODART and FARADAY some sixty years ago,* on the oxidation of alloys of iron and nickel. An alloy of iron, or rather of the best Bombay wootz,

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* FARADAY'S 'Experimental Researches in Chemistry and Physics,' p. 63. TAYLOR and FRANCIS, 1859.

with 10 per cent. of nickel made by them in 1820, in imitation of the Siberian meteoric iron of Krasnojarsk, in which CHILDREN found as a mean of three analysis 8·96 per cent.* of nickel, was compared, as regards its powers of undergoing oxidation, with pure iron. And the authors say: "The colour, when polished, had a yellow tinge. A piece of the alloy has been exposed to moist air for a considerable time together with a piece of pure iron; they are both a little rusty, not, however, to the same extent, that with the nickel being but slightly acted upon comparatively to the action on the pure iron; it thus appears that nickel, when combined with iron, has some effect in preventing oxidation, though certainly not to the extent that has at times been attributed to it. It is a curious fact that the same quantity of the nickel alloyed with steel instead of preventing its rusting appeared to accelerate it very rapidly."

IV. *Troilite of the Cranbourne siderite.*

The BRUCE meteorite contains many nodules of troilite lying here and there amongst the plates and crystals of nickel-iron, always in rounded masses, only very occasionally an ill-defined cleavage plane being met with. They vary in size from half an inch to more than two inches in length, are usually covered with a thin layer of graphite, sometimes with some daubréelite surrounding them; and one nodule, consisting of graphite, was found to enclose troilite which had aggregated inside the graphite in a curious way, so that the section of the nodule suggested the outline of a holly leaf. Plate 53, fig. 2, represents a section of the nodule of graphite, the shaded enclosed part representing the sulphide. Excepting daubréelite, troilite is the only sulphide found in this meteorite and, it need hardly be said, was not in the slightest degree magnetic. A specimen of pounded and dried mineral was digested with a quantity of carbon disulphide, which had been twice distilled, for a day and a-half, and sulphur amounting to 0·0207 per cent. was dissolved. A portion chosen for analysis was found to possess the following composition:—

	I.	II.	III.	IV.
Insoluble part . . .	0·215	2·297	. .	. .
Iron	. .	62·150	63·613	. .
Sulphur	36·543	. .	36·207	36·250
Nickel	. .	0·446	. .	. .
Copper	. .	0·079	. .	. .
Chlorine	. .	0·130	. .	. .

or, as the mean of these determinations:

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* BERZELIUS found nickel 10·73 per cent. and cobalt 0·46 per cent. in the Krasnojarsk nickel-iron.

		FeS requires
Iron	=63·613	63·64
Sulphur	=36·333	36·36
Copper	= 0·079	. .
Chlorine	= 0·130	. .
	100·155	100·00

V. *Square, strongly magnetic, prisms of iron-nickel phosphide.*

The next mineral, the composition of which we have to consider, is that forming the prisms which, as we have already seen, are scattered throughout the mass of the nickel-iron and form nearly 1 per cent. of its mass. They resist the action of hydrogen chloride and are only dissolved after long treatment with very strong acid; they dissolve, on the other hand, easily in hydrogen nitrate. They exhibit strong magnetic characters. They seem to be identical with the mineral to which GUSTAV ROSE gave the name of rhabdite. They appear to form square prisms, and the terminal faces of the prism could rarely be met with.

The prisms were exceedingly brittle and were rarely, if ever, of their normal length.

It was a difficult matter to obtain the prisms quite free from organic matter (dried varnish, &c.), but the following very pure material was at last obtained:—

	I.	II.	III.	Mean.	$(Fe_4Ni_3)P$.
Nickel . . .	49·715	. .	48·955	49·335	48·38
Iron	36·666	39·519	38·540	38·242	38·23
Phosphorus . .	[13·619]	12·586	12·645	12·950	13·39
					100·00

The specific gravity of several specimens of the prisms gave numbers varying from 6·326 to 6·78.

A few years ago Professor DAUBRÉE* pointed out the great resemblance which he had traced between the artificial phosphide of iron, Fe_8P, which M. SIDOT had succeeded in preparing, and the rhabdite of meteoric iron. I have to offer my hearty thanks to Professor DAUBRÉE for permitting me to inspect some of M. SIDOT's crystals, which bore the closest resemblance to the above crystals. More recently, in the spring of last year, M. E. MALLARD† communicated a note to the 'Comptes Rendus,' on phosphide of iron found among the products of the spontaneous fires in the coal mines at Commentry. The crystals are square prisms, terminated by a pyramid, are strongly

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[*] G. A. DAUBRÉE, 'Comptes Rendus,' lxxiv., 1427; and M. SIDOT, 'Comptes Rendus,' lxxiv., 1425.

[†] M. E. MALLARD, "Sur la production d'un phosphure de fer cristallisé et du feldspath anorthite, dans les incendies des houillères de Commentry." 'Comptes Rendus,' 1881, xcii., 933.

magnetic, have a specific gravity of 6·71 and the composition indicated by the formula Fe_7P. They, of course, contain no trace of nickel; in all other respects, however, they bear the closest resemblance to the above body.

VI. *Schreibersite in the Cranbourne siderite.*

When the crude nickel-iron of the meteorite was treated with hydrogen chloride till action ceased, coarse insoluble particles, mixed with a black powder, and the needles remained: they could both be removed by decantation and repeated washings. It was then subjected to a thorough cleansing with hydrogen chloride, with dilute nitric acid, with water, with a mixture of ether, alcohol, benzol, and chloroform, and finally, when dried, with the magnet. In this way the coarse powder was obtained in a pure state; it consisted of a very brittle, very magnetic, coarse powder, which dissolved easily in strong hydrogen nitrate. Analysis gave the following results:—

	I.	II.		
Iron	56·245	55·990	56·117 ÷ 28 = 2·004	2·993
Nickel. . . .	29·176	..	29·176 ÷ 29·5 = 0·989	
Phosphorus . .	13·505	..	13·505 ÷ 31 = 0·435 × 7 = 3·045	
	98·798			

This is, doubtless, the mineral *Schreibersite* which appears to have the composition indicated by the formula $(Fe_2Ni)_7P$. The material, as already stated, consisted of a coarse powder, of faceless irregular fragments of a very brittle constituent of the meteorite. Search was accordingly made for crystals, and occasionally, but very rarely, larger bodies which might when broken up have formed this powder were hit upon. One was met with, a large brass-coloured oblique crystal which readily cleaved across the base; it was but slightly acted upon by hydrogen chloride or nitrate, both of which, however, on long continued boiling dissolve it slowly; in *aqua regia*, on the other hand, it quickly disappears. When heated a fragment of one of these crystals quickly became of a dark brown colour. Analyses of these crystals gave the following results:—

	I.	II.		
Iron	69·251	69·843	69·547 ÷ 28 = 2·484 =	2·972
Nickel* . . .	..	..	14·410 ÷ 29·5 = 0·488 =	
Phosphorus . .	15·420	16·666	16·043 ÷ 31 = 0·517 = 0·517	
			100·000	

which results point to $(Fe_9Ni_2)P_2$ as the true representative of its composition. It

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* Both determinations were lost.

does not accord very well with the analysis of the powder, and the relation of one body to the other must be left till fresh material comes to hand.

VII. *Curious crystals with dark centres, occurring in the Cranbourne siderite.*

Mention should here be made of a curious crystal which on two or three occasions was met with while searching through the *débris* of the meteorite. It consisted apparently of a square prism, which, while the sides were quite bright and metallic, had a square centre of a dull almost black colour; it very readily broke across the prism. On Plate 53, fig. 3, is represented such a prism broken across, showing the dark centre. An analysis of this compound gave the following results:—

Iron	67·480 ÷ 28	2·410
Nickel	20·318 ÷ 29·5	0·688
Phosphorus	12·317 ÷ 31	0·397
	100·115	

which numbers agree with the formula $(Fe_7Ni_2)_8P$.

VIII. *Graphite.*

Graphite occurs occasionally, but rarely, as nodules; sometimes as nodules, enclosing troilite, like the one already referred to; sometimes in large sheet-like masses, in one case about four inches in length and two inches wide. A specimen was carefully dried and pounded and burnt in a current of oxygen and gave numbers which show it to have the composition:

Carbon	89·661
Hydrogen	0·257
Residue (iron, &c.)	10·412
	100·330

IX. *Gases occluded by the nickel-iron.*

The nickel-iron was further examined for occluded gases. A portion of the nickel-iron borings removed from the under surface was selected and was heated in a porcelain tube connected with a Sprengel pump. Gas amounting in bulk to 3·59 times the volume of the iron was extracted and was found on analysis to have the following composition:—

Carbonic acid	0·12
Carbonic oxide	31·88
Hydrogen	45·79
Marsh gas	4·55
Nitrogen	17·66
	100·00

X. The Rowton Siderite.

The metallic mass which I shall next proceed to describe is one of unusual interest in more than one respect: in the first place, before it fell only one iron meteorite was known to have fallen in Great Britain, while eight stony meteorites that have fallen in the British islands are in the national collection; and, secondly, of the more than 300 meteorites which are contained in the collection in the Natural History Museum, more than 100 are unquestionably iron meteorites, and of these the fall of seven only has been witnessed.

The circumstances attending the fall of the Rowton iron are as follows. At about 20 minutes to 4 o'clock on the afternoon of the 20th of April, 1876, a strange rumbling noise was heard in the atmosphere, followed almost instantaneously by a startling explosion resembling a discharge of heavy artillery. There was neither lightning nor thunder, but rain was falling heavily, the sky being obscured with dark clouds for some time both before and after the incident related. About an hour after the explosion Mr. George Brooks had occasion to go to a turf field in his occupation adjoining the Wellington and Market Drayton Railway, about a mile north of the Wrekin, when his attention was attracted to a hole cut in the ground. The land where it fell, it should be stated, is part of the property of the Duke of Cleveland, at Rowton, near Wellington, in Shropshire; and Mr. Ashdown, the agent of the Duke, exerted himself in the matter, and obtained his Grace's assent to the meteorite being presented to the trustees of the British Museum.

As regards the hole which was found in the field, Mr. Brooks probed the opening with a stick and discovered a lump of metal of irregular shape, which proved to be a meteorite, weighing $7\frac{3}{4}$ lbs. It had penetrated to a depth of 18 inches, passing through 4 inches of soil and 14 inches of solid clay down to the gravel. The hole is nearly perpendicular, but the stone appears to have fallen in a south-easterly direction. Some men were at work at the time within a short distance, and they, together with many other people in the neighbourhood, heard the noise of explosion. According to other observers, the sound was heard as of something falling during a heavy shower of rain, accompanied by a hissing and then a rumbling noise. It is, moreover, stated that when Mr. Brooks found the mass "it was quite warm." Mr. Wills described it as being black on the surface and apparently covered with a scale of metallic oxides; but at the point where it impinged on the earth the oxides had been removed, and the metallic character of the mass had been revealed.

When the meteorite reached the British Museum it was at once seen that it was wholly metallic in structure and was covered with a very thin pellicle of the jet-black magnetic oxide of iron, and only where this had been removed by abrasion with the soil is the bright metallic surface of the nickel-iron revealed. The depth to which the meteorite penetrated the soil is proof of how much momentum still remained to it, partly due, no doubt, to the approximately vertical direction with which it entered the atmosphere, and in some degree to the higher density of an iron mass as compared with one of stone, the rocky meteorites rarely penetrating to so considerable a depth. The meteorite closely resembles the siderite of Nedagolla, in India, as Professor STORY-MASKELYNE, M.P., F.R.S., has pointed out.

XI. *The nickel-iron of the Rowton siderite.*

Some fragments which had been removed by the lapidary's wheel were submitted to analysis, with the following results:—

	I.	II.
Iron	91·250	91·046
Nickel	8·582	9·077
Cobalt	0·371	
Copper	trace.	trace.
	100·203	100·123

This nickel-iron has the composition closely approaching that of what may be called a normal nickel-iron—in short, the metals are in the ratio in which they are met with in their oxides when precipitated from an iron solution, containing an excess of nickel oxide, by ammonia, both when a large excess of ammonium chloride is present and when it is absent. As a result of several analytical determinations it was found to be:

Iron	=0·1231	=91·12
Nickel	=0·0120	= 8·88
	0·1351	100·00

XII. *The troilite of the Rowton siderite.*

One of the fragments of nickel-iron devoted to the analytical examination was found to contain a section of a nodule of troilite; this easily dropped out of the iron; where it was in close contact with the alloy it was covered with a very thin layer of graphite. No cleavage planes were noticed on the specimen; it was examined with a magnetic needle and found not to be in the slightest degree magnetic. It was shown on analysis to have the composition:

		The protosulphide requires
Sulphur	= 36·073	36·36
Iron	=[63·927]	63·64
	100·000	100·00

Some fragments of the iron were sawn into very thin plates, and were kept quite cool all the time by a current of methylated spirit; they were carefully dried and weighed, and the gas drawn from them when at a bright red-heat with a SPRENGEL pump. The plates of iron taken measured 1·198 cub. centim., and the gas collected after many hours' heating was 6·38 times the bulk of the metal. This is about double the quantity met with by GRAHAM and MALLET in other meteoric irons which had lain a long time in the ground.

XIII. *The gases occluded by the Rowton nickel-iron.*

After subtracting a little oxygen and the corresponding amount of nitrogen, due probably to the entrance of a little air into the apparatus, the gas was found to have the following percentage composition:—

Carbonic acid	5·155
Hydrogen.	77·778
Carbonic oxide	7·345
Nitrogen	9·722
	100·000

Plate 53 shows a drawing of the mass (fig. 4, actual size) as well as a sketch of the figures developed by etching the surface with bromine (fig. 5); they show larger figures than are usual, with less of the bright extruded ingredient, doubtless a compound rich in phosphorus. The small darker-coloured rounded mass near the bottom of the section is a nodule of troilite.

XIV. THE METEORITE OF MIDDLESBROUGH, YORKSHIRE.

During the past year a very beautiful specimen of a meteorite fell near Middlesbrough, in Yorkshire. It struck the earth at a spot called Pennyman's Siding, on the North-Eastern Railway Company's branch line from Middlesbrough to Guisbrough, about one mile and three-quarters from the former town. Its descent was witnessed by W. ELLINOR and three platelayers, who heard a whizzing or rushing noise in the air followed in a second or two by a sudden blow of a body striking the ground not far from them: the spot was found to be 48 yards from where they stood.

The fall took place on the 14th March, 1881, at 3·35 P.M. The wind was from the north-east, and it was a clear and bright but rather cold afternoon. At more distant places, as Northallerton and four miles to the eastward, the sound resembled the boom of a gun; no luminous or cloud-forming phenomena are reported. The character of the hole, according to Professor ALEXANDER HERSCHEL, who at once visited the spot, points to the fall having been vertical or nearly so. The stone was "new milk warm" when found, and weighed 3 lbs. 8¼ oz.; the dark surface is entirely fused and crusted, and has scarcely suffered by the fall. The stone forms a low pyramid, slightly scolloped, 6¼ inches in length, 5 inches wide, and 3 inches in height. The rounded summit and sloping sides are scored and grooved deeply with a polish like black lead, in waving furrows running to the base, showing that this side came foremost during the fusing action of the atmosphere which the meteorite underwent in its flight. The rear or base is equally fused or branded by heat, but is rough, dull brown in colour, and not scored or furrowed.

The meteorite penetrated the soil to a depth of 11 inches, and the penetration line apparently slopes about 10° from the vertical from the S.S.E.; it passed through 7 or 8 inches of coke-ballast, and thereafter brick-earth or coarse clay to the remaining depth. From experiments made by Professor HERSCHEL on the power of penetration of a cast-iron model of the meteorite, it is calculated that the actual velocity of fall with which the stone struck the ground must have been 412 feet per second. As it would acquire this velocity by falling freely through half-a-mile, it is clear how little of the original planetary speed with which it entered the atmosphere can have remained to affect its fall.

The interior of the stone has a greyish-white appearance, and is evidently for the most part composed of silicates: frequent bright metallic granules are to be seen, and they appear to be entirely or almost entirely granules of nickel-iron. The rocky portion varies from grey to pure white, of which there are patches, and while the greater part appears to be homogeneous in structure, there are many enclosed chondra of large size and of a darker grey than the body of the stone.

In the well-developed markings of the exterior of the stone it bears a close resemblance, as Professor HERSCHEL points out, to the meteorite of KARAKOL (Kirgis Steppe, May 9th, 1840), of which Professor GOEBEL gives a figure in his paper of 1866 in the 'Mélanges physiques et chimiques de l'Académie Impériale de St. Pétersbourg,' vii., 318–324.

The railway company, who at the time this notice was written retained possession of the stone, kindly permitted a few fragments to be removed for examination; and I shall now proceed to describe the results of the chemical analysis of them. It has since been presented to the Yorkshire Philosophical Society, and is now preserved in the museum at York.

XV. *The nickel-iron enclosed in the rocky constituents of the meteorite.*

A quantity was dried and weighed and treated with mercury chloride, and it was found that 9·379 per cent. of constituents were removed. As already stated, when examined under a microscope the metallic particles appeared to consist entirely, or almost entirely, of nickel-iron. The nickel-iron was found to have the following composition :—

Iron	76·990
Nickel	21·320
Cobalt	1·690
	100·000

The remaining constituents, consisting of purely rocky matter, amounting to 90·621 per cent., are thus composed :—

A. Soluble silicate	54·315
B. Insoluble silicate	36·306
	90·621

XVI. *The gelatinisable constituents of the rocky portion of the aerolite.*

The soluble portion and the silicic acid belonging to that portion was found to have the following composition :—

		Oxygen.
Silicic acid	41·100	21·92
Iron protoxide	27·960	6·213
Magnesia	30·940	12·380
	100·000	18·593

These numbers indicate the presence of one olivine of the form $2(\frac{1}{3}Fe, \frac{2}{3}Mg)O, SiO_2$ or one closely resembling that which occurs in the Lancé stone, which fell July 13th, 1872, and was examined by DAUBRÉE. No lime and no alumina were found in the soluble part, though carefully sought for.

XVII. *The insoluble silicates in the rocky portion of the Middlesbrough aerolite.*

The constituents of the insoluble part were as follow :—

		Oxygen.
Silicic acid	55·389	29·541
Alumina, and a little chromium oxide	4·770	2·223
Iron protoxide	23·580	5·241
Lime	4·373	1·249
Magnesia	11·043	4·417
		10·907
Alkalies	not determined	
	99·155	

If the chief silicate in the above portion be regarded as bronzite it most closely resembles that met with in the meteorites of Iowa Co., Iowa, east of Marengo, which fell 12th February, 1875 ;[*] if, on the other hand, as is more probable, it be regarded as a lime-magnesia-iron augite it is closely allied to the augite of the stones of STANNERN and JUVINAS. The aluminous constituent is doubtless labradorite, and is probably present as some of the occasional chondra which are seen in a microscopic section of the meteorite.

In Plate 53 the front furrowed side is shown in figs. 6 and 7 and the back view in fig. 8. They require no further explanation.

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[*] J. L. SMITH, Amer. Jour. Sc. [3], vol. x., 1875, p. 363.

XX. *On the Development of the Ossicula auditus in the Higher Mammalia.*

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Communicated by Dr. ALLEN THOMSON, *F.R.S.*

Received March 16,—Read March 30, 1882.

[PLATES 54–58.]

CONTENTS.

INTRODUCTION.

SINCE the discovery by MECKEL of the cartilage which bears his name and the subsequent discovery by RATHKE of the cartilaginous rods immediately succeeding it, anatomists and embryologists have not failed to be intensely interested in the elucidation of their history. HUSCHKE was the first to limit the origin of the ossicula to the cartilages of the first two post-oral arches, but it is to REICHERT that the first full and masterly description of these two cartilages is due. Looking at the wide-spread existence of these two rods in all classes of the Vertebrata, and the differentiation taking place at their proximal extremities, which may result either in the formation of a suspensory apparatus, or in one which, changing its function, is subordinated to the organ of hearing, it can scarcely be wondered at that the opinions concerning the homology of the differentiated parts should be various and conflicting. Nor can it be otherwise, seeing that as yet there is no general agreement as to what parts are or are not derived from or connected with the upper extremities of these two cartilages in any one class of the Vertebrata. This portion of my work refers only to the Mammalia, but I hope soon to complete my observations upon the remaining classes of the Vertebrata.

Before passing to the description of my own work I shall give a brief summary of the various views that have been advanced respecting the origin of the ossicula auditus.

HISTORICAL.

MECKEL (1) described the cartilage of the first post-oral arch as a mere process of the malleus, but did not further refer to the origin of the ossicula. RATHKE and VALENTIN (2) describe the ossicula as having their origin in two nodules of cartilage which made their appearance in the tympanic cavity; one, the earlier and larger of the two, formed the elements of the incus, malleus, and MECKEL'S process; while the other, which appeared later, formed the stapes.

HUSCHKE (3) describes the malleus as arising from the cartilage of the first post-oral arch, the incus from the cartilage of the second, while the two cartilages united at their proximal extremities to form the stapes.

BURDACH (4), while accepting with some doubts the last-mentioned origin for the malleus and incus, thought the stapes ought to be considered as a bud from the wall of the labyrinth.

REICHERT (5) divides the first post-oral cartilage into three portions, the posterior of which connected the cartilage to the cranial vertebræ, and did not undergo any further change, but finally disappeared; the median division formed the incus with its processes; the anterior, the malleus, and MECKEL'S process.

The second post-oral cartilage was at first connected to the base of the cranium in the region of the post-sphenoid; this part, however, soon disappeared, and the remainder of the cartilage formed in succession the stapes, stapedius muscle, eminentia pyramidalis, styloid process, stylo-hyoid ligament (generally), and lesser cornu of the hyoid bone.

GÜNTHER (6) while in the main accepting REICHERT'S views respecting the cartilage of the first arch, considered his description of the second, so far as it referred to the stapes, to be untenable. He described the origin of the stapes as follows: from the median of the three divisions of the first post-oral cartilage a nodule arose which applied itself to the wall of the labyrinth; formed a depression there—the future fenestra ovalis; this nodule, by the growth of the parts, chiefly the cochlear part of the labyrinth, became more remote from its place of origin, and formed a process, which process becoming bent on itself, formed a horizontal part—the stapes, and a vertical part—the long crus of the incus; this division formed also the incus with its processes. Malleus, incus, and stapes were thus derivatives of the first post-oral cartilage.

MAGITOT et ROBIN (7) describe the malleus as arising from the intra-tympanic portion of the Meckelian cartilage. The incus and stapes they consider to arise independently, but in what manner they do not make clear; they mention that the stapes is a perforated and not a solid ball of cartilage at its first appearance, as had been described by REICHERT. BRUCH (8) describes the mandibular cartilage, with the malleus and the incus, as forming one continuous strip of cartilage; but he also mentions that he has seen the malleus and incus take their origin in a special cartilage which soon united with the mandibular one.

The stapes appears at the upper end of the hyoidean cartilage as a somewhat square-shaped cartilage, whether continuous with it or not he does not state; it is the latest of the ossicula to appear.

HUXLEY (9) in his earlier writings accepted the views advocated by REICHERT, but at a later period he has advanced views of his own based upon comparative anatomical work, and more especially on the condition of parts found in "sphenodon punctatum." He describes the malleus as forming the proximal extremity of the mandibular cartilage. He divides the hyoidean into a part above and a part below the stapes; the former becomes the incus or supra stapedial, which is connected by a distinct ossification—the orbicular bone—to the stapes; the latter becomes the infra-stapedial or stylo-hyal of the Mammalia, which may be connected to the supra-stapedial either by means of the stapedius muscle, which passes to the orbicular bone, or by means of a ligament which passes to the short crus of the incus.

He mentions also that in many Fishes and Amphibia the proximal ends of the two cartilages are united into a single plate.

PARKER (10) describes the malleus also as the proximal end of the mandibular cartilage, the apex of which grows downwards over the first visceral cleft and forms the manubrium mallei, the shoulder of the bend forming the head with its articular surface; the hyoidean cartilage undergoes segmentation; the upper part becomes the incus, the apex of the bar growing downwards forms an attachment by its capitulum (which becomes the orbicular bone) with the stapes; the shoulder of the bend becomes the body and articular surface of the incus, which unites with the corresponding part of the mandibular cartilage. From the shoulder or bend a little boss grows backwards which becomes the short process of the incus. The rest of the cartilage is carried backwards and downwards, but is connected to the previously described part by a ligament, the inter-hyal, in which a nodule of cartilage appears, one end of which becomes attached to the head of the stapes, the other is buried in the substance of the stapedius muscle. The head of that part of the cartilage carried downwards is bifurcated; the outer division becomes attached to the tegmen tympani, the inner to the ear sac immediately in front of the exit of the cranial nerve. The stapes is derived from the median of three projections, which appear on the external surface of the wall of the labyrinth; it frees itself from the wall, leaving a gap, the future fenestra ovalis, which is closed by the separated nodule; externally, the nodule is covered by delicate indifferent tissue ready to become cartilage, and union takes place between it and the orbicular apex of the hyoidean rod.

SEMMER (11) corroborates REICHERT'S conclusions respecting the origin of the malleus and incus. He describes the stapes as appearing in the form of an oval group of cells situated immediately behind and above the proximal end of the stylo-hyal and only indistinctly separate from it.

It is, however, clearly separated from the long crus of the incus by the upper wall of the tympanic sulcus, and by a thick layer of indifferent tissue. It is in all probability developed out of the second visceral arch; but it is impossible to prove that, owing to the second visceral cleft having already disappeared.

HUNT (12) considers them to arise in embryonic connective tissue elements. He does not mention whether they are continuous with any cartilage or not; he cannot accept MR. PARKER'S views respecting the stapes, but describes it as arising, like the other ossicles, in embryonic connective tissue elements.

GRUBER (13) agrees with PARKER as to the stapes and fenestra ovalis. He does not accept the view that the cartilages give origin to the other ossicles, but considers them to be developed, like the stapes, from the original substance of the head vertebræ.

LÖWE (14) describes all three ossicles as having been at one time continuous; also, as having their origin in a visceral cartilage, but from which cartilage he does not make clear.

KÖLLIKER (15) accepts REICHERT'S conclusions respecting the malleus and incus, but differs from him and from more recent workers respecting the origin of the stapes. He describes the hyoidean cartilage as being at its commencement a thin, elongated rod, stretching from, and continuous with, the cartilaginous petrous bone to the middle line in front, the cartilages of opposite sides failing, however, to meet there. The changes that take place at a later period are that the upper and the lower ends ossify to form the styloid process and the lesser cornu of the hyoid bone, while the connexion between these may be fibrous, cartilaginous, or bony. The origin of the stapes he leaves doubtful, but he denies altogether the contentions of PARKER and GRUBER that it springs from the labyrinth in the cartilaginous condition, as well as those of REICHERT, that it forms a part of the hyoidean cartilage, although the cartilage and the stapes are very close to one another.

SALENSKY'S (16) papers form the last contribution to the working out of this subject in the Mammalia (Pig and Sheep embryoes to which his observations were restricted), and I shall therefore state them in detail. In 2 centim. embryoes the mandibular and hyoidean cartilages are cylindrical in shape and limit the hyomandibular cleft, the one above, the other below. They attach themselves to the ear capsule and their extremities are connected by embryonic tissue. The changes that rapidly take place at the proximal extremity of the mandibular cartilage are, that it thickens and bends downwards, two furrows making their appearance, which divide the extremity into three parts, the posterior of which forms the elements of the body and long crus of the incus, the latter being attached by embryonic tissue to the upper extremity of the hyoidean cartilage. The short crus, later in its appearance, is described as growing backwards from the body. The median thickening forms the malleus, while its downgrowing process forms the neck and manubrium. The anterior thickening soon disappears and has no significance. The separation between the posterior and median thickenings is complete in embryoes measuring 2·7 centims. The union between the long crus of the incus and the upper end of the hyoidean cartilage is described as being secondary. The part of the mandibular cartilage which remains after the separation of the incus forms the malleus and MECKEL's process.

The stapes is described as making its appearance first in embryoes measuring 2·7 centims. At first it forms an accumulation of cells, in the middle of which is seen the cross-section of a vessel which he names the arteria mandibularis. He describes this as a branch of a vessel which is situated dorsally to the facial nerve, and which he calls carotis interna. This I shall show further on to be the primitive jugular vein, and I shall further demonstrate that the arterial branch does not go to the mandible. The stapes is also described as being independent of the cartilages of the two first post-oral arches and of the periotic capsule.

These various and conflicting opinions may be reduced to order as follows :—All observers are agreed that the malleus has its origin in the mandibular cartilage ; that the incus is either the proximal extremity of the mandibular cartilage or the proximal extremity of the hyoidean cartilage ; that the stapes has its origin in the mandibular or the hyoidean cartilage, or is a bud from the periotic capsule, or built independently round a small artery.

PERSONAL WORK.

METHODS OF PREPARATION, AND GENERAL STATEMENTS.

The embryoes were prepared after the manner of KLEINENBERG ; that is, they were placed in his solution of picric acid for periods varying from five to forty-eight hours according to size. They were then placed in 50 per cent. alcohol for twenty-four hours, then in 75 per cent. for a similar period, and finally in absolute alcohol ; stained in his solution of hæmotoxylin, placed again in absolute alcohol, then in bergamot oil ; embedded in a mixture of spermaceti and castor oil, cut, cleared in a mixture of creosote and turpentine, and finally mounted in Canada balsam. The embryoes were measured by placing their most projecting parts (mid brain and curve of tail) between the limbs of a compass, and the length read off on a centimetre scale.

The work was intended primarily to settle the relation of the incus to the cartilages of the mandibular and hyoidean arches, and the relation of the stapes to the hyoidean cartilage and the periotic capsule.

It has been done upon embryoes of the following Mammals.

Rat embryoes.—The earliest under 8 millims ; then at 8 millims., 1 centim., 1·3 centim., 2 centims., 2·2 centims., 2·5 centims., 3·5 centims., 4 centims., and at the last stage just before birth.

Pig embryoes.—1 centim., 1·5 centim., 2 centims., 2·3 centims., and at 2·6 centims.

Dog embryoes.—1 centim., 1·3 centim., and at 2·5 centims.

Sheep embryoes.—For a series of these I have to thank Dr. ALLEN THOMSON. 1 centim., 1·4 centim., 1·6 centim., 2 centims., and at 4 centims.

Rabbit embryoes.—1 centim., and at 1·5 centim.

Human embryoes.—1 centim., fully, and at 4 centims. The head and body of the last embryo measured about the length stated.

In addition to these I have made observations upon the embryoes of the Mouse and Calf, but these were not complete.

In the embryoes, and at the stages mentioned above, I have made complete series of sections (in many cases several complete series), in the long vertical (the plane being either parallel to the middle line of the head or along the obliquity of the post-oral arches, in order to cut their cartilages along their whole length), transverse vertical (with reference to the ossicula), and obliquely transverse approaching the horizontal planes. These were numbered so that I had corresponding sections at different stages for study and comparison. During the course of the work I soon learned that the incus was quite as distinct from both cartilages when they could properly be called so, as it was at birth or at adult age, so that I had to work upon embryoes at a stage preceding the true cartilaginous one, that is, at a stage between that in which there was not the slightest trace of cartilage to be detected, and that in which the cartilages of the arches were sharply and clearly defined, and in which the cartilage cells had acquired the characteristic hyaline appearance. In order that I should have abundance of material for the study of this particular stage, I cut in the long vertical direction the heads of seven embryo Pigs, five Dogs, five Rabbits, four Sheep, seven Rats, and one Human embryo. But here again the difficulty arose that although the cartilages could be roughly distinguished, yet they were not limited by any sharp line of demarcation, but faded gradually away into the adjacent mesoblastic or embryonic tissue, from which they differed only in greater aggregation of round cells; hence it is that there is room for difference of opinion among rival homologists, especially those who, resting their claims solely upon the condition of the embryonic cartilages at this immature stage, fail to recognise the value of a knowledge of their form and relations throughout their entire distribution in the vertebrate series, and of the great importance of the relation of the nerves in determining the true homology of the parts in dispute. As I shall show further on, the value of the mandibular branch of the seventh nerve (chorda tympani) in the solution of this particular question is considerable. In the endeavour to make the embryonic history of these parts clear, I shall first of all describe the parts coming into more or less close connexion with the ossicula auditus, but more in a morphological than in a histological sense; then, secondly, I shall give a description of the proximal extremities of the first two post-oral cartilages, which is in effect a description of the malleus and incus, and compare the embryonic condition of these ossicles with their form in the adult. I shall also give under this heading a separate description of the embryonic history and adult condition of the stapes and the artery with which it is connected, and finally, briefly summarise the results which may be drawn from the work.

Description of the Work.

First.—The parts entering into more or less close connexion with the ossicula are the gasserian ganglion and maxillary and mandibular branches of the fifth nerve, the ganglion, trunk, and mandibular (chorda tympani) branch of the facial nerve, the primitive jugular vein, the hyomandibular cleft, the meatus auditorius externus, the auditory vesicle and its capsule, the dorsal aorta, and its branch passing towards the stapes, and lastly the tympanic annulus.

The gasserian ganglion is situated dorsad of the cleft of the mouth, over which its maxillary and mandibular branches are placed. In the earliest embryoes figured (Plate 54, fig. 1; and Plate 55, fig. 11) the nerves and ganglia are of large relative size, in the older ones they are less conspicuous.

The mandibular branch of the fifth divides into two parts which come into close relation with the cartilage of the arch a little distance in front of its proximal extremity, one part runs internal to the cartilage and is joined by the mandibular branch of the seventh, the other passes obliquely external to the cartilage (Plate 54, figs. 1, 4, 5; Plate 55, fig. 11; and Plate 58, fig. 39). The ganglion of the seventh is situated posterior to and near the ventral border of the gasserian ganglion, compared with which it is very small (Plate 54, fig. 6; Plate 55, fig. 12; Plate 56, figs. 22, 24, 26; and Plate 58, fig. 39). The trunk of the nerve at first runs in an antero-posterior direction, lying with the primitive jugular vein, between the canal portion of the labyrinth and the dorsal and external portion of the hyomandibular cleft (Plate 54, fig. 6; Plate 55, figs. 11, 12; and Plate 56, fig. 24). At the level of the lower border of the periotic capsule it turns almost at right angles to its former direction, turning round and passing external to the hyoïdean cartilage on its way to the face.

Where the above change in the direction of the axis of the trunk takes place the mandibular or chorda tympani branch is given off; in older embryoes this branch passes off the trunk of the nerve after it emerges from under the cover of the periotic process (a process given off from the periotic capsule to join the hyoidean cartilage); it is of large size in very young embryoes (Plate 54, fig. 1; and Plate 55, fig. 11), and passes between the hypoblast and epiblast forming the closed dorsal and external portion of the hyomandibular cleft, then runs in a ventral direction and joins the internal branch of the mandibular division of the fifth nerve. In older embryoes it comes into relation with the cartilages of the first two post-oral arches passing external to the hyoidean, internal to the proximal extremity of the mandibular (Plate 54, figs. 4, 5, 8; Plate 55, figs. 10, 16; Plate 56, figs. 19, 21, 25; and Plate 58, figs. 39, 40).

This relation to the cartilages is common to the embryoes of the human subject and the lower animals; but while in the embryoes of the lower animals the nerve never comes into relation with the incus; in the human embryo it has the same relation to the long crus (Plate 55, fig. 16) as to the hyoidean cartilage. This relation, if not secondary or acquired, points suggestively to the origin of the incus. The mandibular branch and the trunk of the seventh have been described as being homologous to the maxillary and mandibular branches of the fifth, but the mandibular branch of the seventh differs from the maxillary division of the fifth in not being ganglionic at its origin and in joining the mandibular division of that nerve.

In sections of embryoes before birth the trunk of the seventh has acquired its characteristic adult course (Plate 58, fig. 36); that is to say, the anterior portion of the trunk is pushed in a ventral direction by the growth chiefly of the cochlear part of the labyrinth.

The primitive jugular vein in the earliest embryoes is of very large size, being the most conspicuous feature in the sections. It returns the blood from the entire head. It lies dorsad of the gasserian ganglion and seventh nerve, ventrad of the periotic capsule, and then passes backwards towards the heart (Plate 54, figs. 3–6; Plate 55, figs. 17, 18; Plate 56, figs. 20, 22, 24; and Plate 58, fig. 39). This is the vein that SALENSKY names arteria carotis interna. It has the same relations as the cardinal vein in the Chick's head. When the ossicula begin to develope it commences to disappear, and is soon replaced by the internal and external jugular veins.

In the earliest condition of the hyomandibular cleft in the dorsal region, the hypoblast lining approaches and touches the external epiblast. I feel convinced that this cleft never actually forms a perforation in this region (region of membrana tympani), although it is perforated ventrad of this. Should the above supposition be correct, the formation of the membrana tympani would be very easy to understand; the hypoblast lining forming the mucous lining of the adult membrane, the epiblast the cuticular covering, while the fibrous layer is derived from the ingrowth of mesoblast, which along with the proximal extremity of the mandibular cartilage separates these two layers from each other (Plate 55, fig. 18; Plate 56, figs. 23, 26; and Plate 57, fig. 29). In the oldest embryoes the manubrium mallei occupies the entire space between the dorsal part of the cleft and the bottom of the meatus auditorius externus (Plate 55, fig. 13; and Plate 57, fig. 31).

HIS (17) has recently raised doubts as to the correctness of the usually accepted statements first advanced by RATHKE and HUSCHKE regarding the open condition of the clefts. These authors passed bristles from the external surface through the clefts, and as they appeared in the cavity of the pharynx, they concluded that the clefts were open. HIS, on the contrary, states that the bristles were passed through the delicate epiblast and hypoblast, which come into close contact with each other. He does not press his observations, but leaves them as matter for future determination.

The dorsal and external part of the cleft, following MOLDENHAÜER (18), may be called the sulcus tympanicus, while the ventral and internal portion opening into the pharynx may be termed the tubal portion of the sulcus. This sulcus does not expand until after birth, when the fœtus begins to breathe air; the walls of it, even in the oldest embryoes, being close together, although in some of the figures, owing to the obliquity of the sections, they are a little distance apart (Plate 57, figs. 29, 30, 31, 32, and 34).

There is at first no external auditory meatus; it is formed later on by the growth outwards of the side walls of the head (Plate 55, figs. 17, 18; Plate 56, fig. 23; Plate 57, figs. 29 and 31), leaving a gap leading towards the dorsal part of the hyomandibular cleft (region of membrana tympani). The growth outwards of the wall of the head incloses a median thickening, which contains the ventrally curved extremity of the mandibular cartilage (Plate 55, fig. 18); the dorsal region of this growth becomes bent upon itself, forming a fold of the integument, which unfolding after birth becomes the long dorsally directed pinna of the Rodents (Plate 57, fig. 31).

The auditory vesicle, after it has been shut off from the external epiblast, has an oval form (Plate 54, figs. 2 and 3), and is situated between the hind brain and the external epiblast. Ventrad of the vesicles runs the seventh nerve, the primitive jugular vein and dorsal aorta. The ganglion acusticum and auditory epithelium are very intimately connected, a fact already noticed by BALFOUR and MARSHALL (Plate 54, fig. 2; and Plate 55, fig. 17). The ganglion has two well marked divisions, corresponding to the cochlear and vestibular portions of the nerve (Plate 54, fig. 9; Plate 55, fig. 18; and Plate 56, fig. 23), although in embryoes near the period of birth it is single (Plate 57, fig. 32).

In slightly older embryoes the vesicle has undergone the ordinary Mammalian complications (Plate 55, fig. 17; and Plate 56, figs. 20 and 23); the ventral end of the vesicle passes inwards towards the base of the skull, forming the canalis cochlearis and sacculus hemisphericus; in the hollow of the canal lies the ganglion acusticum; from the dorsal end of the vesicle, on the side nearest the brain cavity, there passes off an elongated diverticulum, the recessus labyrinthi, external to which, and running for some distance parallel with it, lies the superior semicircular canal.

The external semicircular canal passes from the vestibular cavity outwards, dorsad of the seventh nerve and primitive jugular vein towards the epiblast (Plate 55, fig. 17; Plate 56, figs. 20 and 23). The complicated vesicle becomes surrounded by a thickened layer of densely packed cells, which follows the outlines of the several parts of the vesicle, with the exception of the recessus vestibuli. This layer of cells is laid down at the same time as the elementary cartilages in the arches, and the changes by which it is converted into hyaline cartilage go on simultaneously with those occurring in the cells of the cartilages.

The points of interest in connexion with the periotic capsule are that it is deficient on its internal surface, for the ganglion acusticum and its vestibular and cochlear branches; it is also deficient externally, for the fenestra ovalis and fenestra rotunda.

These fenestræ differ in their development in the following manner: in sections from rat embryoes 8 millims. in length, in which the wall surrounding the labyrinth can first be said to be roughly differentiated from the adjacent mesoblast, a gap for neither fenestra exists; but in sections from embryoes 13 millims. in length, while the wall is continuous at the position of the future fenestra ovalis (Plate 57, fig. 27), a well marked deficiency exists for the fenestra rotunda (Plate 57, fig. 28).

The primitive jugular vein and seventh nerve divide the auditory region on an external view into two parts. Ventrad of these structures are the proximal extremities of the first two post-oral cartilages; dorsad of them lies the semicircular canal portion of the labyrinth; this portion of the periotic capsule is roughly circular in outline, but flattened ventrally, and more or less pointed dorsally. Along the anterior and posterior borders run in a dorsal direction the superior and posterior semicircular canals; these turning ventrally unite together, and open by a common aperture into the vestibular portion of the labyrinth; the external semicircular canal lies between the ventral extremities of the last two, and has a direction parallel to the vessel and nerve (Plate 54, fig. 6; Plate 56, figs. 19, 21, and 22; Plate 57, figs. 33 and 34; and Plate 58, fig. 39).

I shall describe the dorsal aorta and the branch passing from it in connexion with the stapes.

The tympanic annulus can first be detected in Rat embryoes 2 centims. in length; it lies within the proximal extremities of the mandibular and hyoidean cartilages, ventrad of the external wall of the sulcus tympanicus, and internal to the bottom of the meatus auditorius externus; it is deficient dorsally, the proximal extremity of the mandibular cartilage appears to bend round and be supported by it (Plate 55, fig. 15). In the oldest Rat embryoes and especially in the young of carnivorous animals at birth (Cat, Dog, Leopard), a membranous splint intrevenes between it and the mandibular cartilage (Plate 57, fig. 34); this, however, soon unites with the periotic, or the annulus.

In the oldest embryoes examined there is no trace of the papery bulla or bony meatus auditorius externus, which are continuous with and lie internal and external to the annulus in the adult (Plate 55, figs. 13, 14, and 16; Plate 57, fig. 31; Plate 58, figs. 35, 37, and 40).

Second.—Embryonic history of the proximal extremities of the first two post-oral cartilages, being in effect a description of the malleus and incus, and a comparison between the embryonic and adult form of these ossicles.

I have never been able to detect, notwithstanding the large number of embryo heads which I cut for the study of the earliest condition of the cartilages, a stage in which these could be described as being straight.

There was always a slight bending of the proximal extremities of both cartilages, that of the mandibular depressing the dorsal wall of the meatus auditorius externus towards the ventral, and forming the elements of the neck and manubrium of the malleus, while that of the hyoidean had its concavity directed towards the mandibular, to which it closely applied itself. The proximal extremity of this cartilage forms the head of the incus, the long crus in the majority of embryoes, those of the Pig especially, being continuous with the rest of the cartilage (Plate 54, fig. 5), but in some Dog and Rabbit embryoes there was a slight gap between the long crus and the remainder of the cartilage (Plate 54, fig. 4); passing dorsally from the head of the incus is a short process (slightly exaggerated in Plate 54, fig. 5) which eventually becomes the short crus of the incus.

The long crus has at this stage no bend inwards towards the stapedial ring (Plate 56, fig. 24). The distal end of the hyoidean cartilage is closely connected to the cartilage of the first branchial arch (thyro-hyal), so much so that if microscopic sectional appearances were alone to be considered they could be described as being continuous.

The drawings do not do justice to the immature condition of these cartilages, there being no sharp limiting lines in nature. In older embryoes there is a striking similarity in the appearance of the cartilages in the different embryoes (at the same developmental stage), Plate 54, figs. 8 and 9; and Plate 55, fig. 10, for the Pig; Plate 55, fig. 18; and Plate 56, fig. 19, for the Sheep; Plate 56, fig. 21, for the Dog; and Plate 56, figs. 25 and 26; and Plate 57, fig. 27, for the Rat. In the entire series the several parts of the malleus and incus can be readily distinguished; the apex of the long crus of the incus has but a slight turn inwards towards the stapedial ring (Plate 54, figs. 7 and 9; Plate 55, fig. 18; and Plate 57, fig. 27), and it is separated from the remainder of the cartilage (which joins the periotic process of the periotic capsule) by a slight interval, but it agrees with the cartilage both in its form and general direction in a very marked manner. It also, and this is equally important, agrees with the hyoidean cartilage in every histological particular; it takes up the colouring matter in the same way, and its cells assume the hyaline character at the same period of time—those of the mandibular cartilage having undergone that change at an earlier date.

The seventh nerve and its chorda tympani, or mandibular branch, have the relations to the cartilages already described. In still older embryoes (Plate 55, figs. 13, 14, 15, and 16, for Human embryo; Plate 57, figs. 29, 30, 31, 32, 33, 34; and Plate 58, figs. 35, 36, 37, 38, for the Rat) the individual parts of the adult ossicula are fully developed. The joint between the malleus and incus is now for the first time clearly evident, and its V-shape is worthy of notice (Plate 58, fig. 36). The manubrium is large, and of the same shape as in the adult; it occupies the entire space between the sulcus tympanicus and the meatus auditorius externus (Plate 55, fig. 13; and Plate 57, fig. 31); at its junction with the neck is a well marked process, which I have called posterior (DORAN'S (19) orbicular process or apophysis—a bad name, because of the orbicular apophysis of the incus). The processus muscularis of HYRTL and the tensor tympani muscle are well developed.

The cartilages of the Human embryo are very massive (Plate 55, figs. 15 and 16), but have otherwise much the same shape as those of other animals.

On comparing these embryonic parts with those of the adult malleus and incus (Plate 58, fig. 40), the derivation of these can be understood at a glance. The head of the malleus in the adult Rat articulates with the incus, and from it pass two bony processes of considerable size—one has a direction forwards, and soon tapers to a point; this may be called the mandibular process, because it is ossified from the cartilage (DORAN (19) calls it process head of malleus); the other, with a direction downwards, is the neck. From the lower part of this passes in a forward, downward, and inward direction, a long sabre-shaped process, with its edges looking inwards and outwards, the manubrium mallei. At the lower part of the neck on its internal surface, is the processus muscularis of HYRTL; while at the junction of the neck with the manubrium are three other processes, one of which looking backwards is very constant and called posterior, another looks outwards, like the processus brevis of the Human malleus, and the third looks forwards. Passing from this last to the tapering point of the mandibular process, is the free edge of a papery lamina of bone, the extent of which depends chiefly on the length of the neck; it bears the same relation to the mandibular cartilage that the papery bulla does to the tympanic annulus, and it developes mainly after the fœtus has begun to breathe air, when the tympanic cavity expands. The processus gracilis of the adult Human malleus must be formed by the atrophy of the mandibular cartilage along the dotted line in Plate 55, fig. 15, the head and short neck of the Human malleus corresponding to the entire thickness of the cartilage, the lower margin of which persists as the processus gracilis. A study of the plates illustrating DORAN'S (19) paper, which show the various forms of mallei throughout the Mammalian orders, will make clear at once the morphological significance of the mandibular process, which is the true remnant in the adult of the mandibular cartilage of the embryo; the whole thickness of the cartilage may ossify, as in the Rat (Plate 58, fig. 40), or its lower margin only, as in the Human embryo, while there may be all grades of persistence in the thickness of the ossified cartilage between these two extremes. The adult incus (Plate 58, fig. 40) corresponds in every particular save one with its appearance in the embryo. When looked at from the outside, the body and the two crura are at once recognised; the short crus is but little, if at all, inferior in size to the long crus; the latter stands over the upper end of the ossified hyoidean cartilage (tympano-hyal of FLOWER), seen better after the removal of the bulla. The seventh nerve lies behind the tympano-hyal and the bulla; its mandibular branch passes external to the first-named, and is found with the greatest difficulty in the adult, as also in fine sections of the older embryoes, the nerves being less conspicuous because of the development of the surrounding parts. It is on looking at the incus from the anterior view that the only difference is seen to exist between its embryonic and adult form. In the adult (Plate 58, fig. 40, B) the orbicular apophysis has a pedicular attachment to the apex of the long crus on its internal surface. In the older embryoes the distal extremity of the long crus of the incus is flexed inwards, but has not any trace of the characteristic adult constriction. The orbicular apophysis cannot therefore be longer spoken of as a bone distinct from the long crus of the incus before or after birth, but is simply a small portion of the long crus separated by constriction from, and assuming a direction at right angles to it, accommodating itself in this way to the stapes.

Embryonic history and adult condition of the stapes, together with the artery in connexion with it.

This ossiculum appears contemporaneously with the cartilages in the arches and the cartilage surrounding the labyrinth; it lies dorsad of the sulcus tympanicus, ventrad of the seventh nerve and primitive jugular vein, external to the unperforated wall of the labyrinth with which at first it is not even in contact, and internal to the long crus of the incus (Plate 54, figs. 6, 7; Plate 55, figs. 12, 18; and Plate 56, fig. 24), with which it is much more closely connected, although from the different direction of its cells it cannot be described as being actually continuous with the hyoidean cartilage; it has the form of a circular ring of cells of uniform thickness. Through the centre of the ring passes a vessel (Plate 54, figs. 2, 6, 7, 9; Plate 55, figs. 12, 18; Plate 56, figs. 22, 23, 24; Plate 57, fig. 27; and Plate 58, figs. 35, 36), which has never yet been described in the embryonic condition nor its value recognised in the formation of this ossiculum. True, SALENSKY has seen the vessel, but he describes it as being a branch of a vessel which he calls carotis interna, but which I have shown to be the primitive jugular or anterior cardinal vein. He also mentions that it ends in the mandibular arch. OTTO (20) was the first to describe and figure this artery in the adult. He found it in the hybernating animals and imagined that some connexion existed between the course pursued by this vessel and the peculiar function of hybernation. HYRTL (21) described it (at first unaware of OTTO's previous observations) in the Cheiroptera, Insectivora, the genus Lemur, and many Rodents. He named it the "Steigbügelarterie"; he found that it supplied part of the brain, the orbit and its contents, and the whole of the superior maxilla; he described also the homologue of this vessel in Man, which might have one of three sources—(*a*) the accessory middle meningeal, pursuing an unusual course; (*b*) the stylo-mastoid artery; or (*c*) a small branch from the anastomosis formed usually between the stylo-mastoid and vidian arteries. It has also been described by MECKEL (22) in the Hedgehog and Dormouse, but called by him carotis interna, although doubtless it was this same vessel. This artery may be either free between the crura so that in the macerated skull no trace of it is left, or it may be surrounded by a bony canal which may have osseous union to one of the crura, so that a vertical bolt may occupy the space between the crura. This has been figured and named the pessulus by CARLISLE (23) in the Marmot and Guinea Pig. He considered that the function of the pessulus was to prevent the stapes from falling into the vestibule.

In the Mammalian embryoes from which my sections have been made, there are two forms of this artery, one which disappears very early represented by the embryoes of the Pig, Dog, Sheep, Calf, and Human subject; the other which persists throughout life as in the embryoes of the Rat—the former may be called arteria stapedialis, the latter arteria stapedio-maxillaris, a name already applied to it by HYRTL in the adult. In embryoes of the Rat the artery has the following course (Plate 56, figs. 22 and 24):—The carotis communis, when it approaches the ventral part of the cartilaginous wall of the cochlea, divides into two branches, one considerably in front of the other, passes internal to the cochlear wall on its way to the brain cavity, and corresponds to carotis interna; the other passes external to the wall of the cochlea, dorsad of and slightly internal to the sulcus tympanicus, through the embryonic ring of the stapes, threading it, as it were, then passes ventrad of the seventh nerve, external to its ganglion, internal to the mandibular division of the fifth nerve, then comes into relation with its maxillary division, and runs underneath this nerve in the whole of its extent until it terminates in branches to the face. During this course it gives off several small branches, which pass in various directions, one of which, however, passes into the mandibular arch. The artery in the embryoes of the Pig, which may be taken to represent those in which it early disappears, has the same course from the carotis communis through the ring of the stapes upwards as far as the ganglion of the seventh nerve, where it divides into two small branches (Plate 54, fig. 6), which pass external to the gasserian ganglion, and beyond which I have failed to trace them. In the adult Rat it is a large and important artery and has the following course:—The carotis communis is a long straight artery, and runs from the aorta to the upper part of the air tube; it there divides into two branches, the external of which passes up over the outer surface of the mandible, forming the arteria facialis; the internal is a short vessel, which soon divides into two branches, the internal of which passes between the anterior Eustachian thickened portion of the bulla and the basisphenoid on its way to the brain cavity, and corresponds to carotis interna; the external and larger of the two branches passes through a foramen formed in the line of articulation between the hinder part of the bulla and the petrosal into the tympanic cavity, then passes up in front of the fenestra rotunda, grooving the cochlear wall, between the crura of the stapes, over the cerebral surface of the bulla, through a foramen in the alisphenoid, then runs under the maxillary division of the fifth nerve in the whole of its length until it terminates in branches to the face, corresponding, in the latter part of its course, to the infra-orbital artery in the human subject. This artery, then, plays a very important part in the formation of this perforated ossiculum; in truth, it bears the same relation to the stapes that the embryonic vertebral artery does to the cartilaginous transverse processes of the cervical vertebræ, or that any artery bears to a cartilaginous mass through which it has to pass, that is, it leaves a foramen. Hence it is a fair deduction that in all Mammalia possessed of a perforated stapes this artery exists always in embryonic and occasionally in adult life. It has some particular relation to RATHKE's arches, which I have not as yet thoroughly worked out. I had thought at first that it was the artery to the hyoidean arch, in which case it would have afforded strong presumptive evidence in favour of the stapes being a part of the cartilage of this arch; but the careful tracing of the artery in the embryo Rat and its distribution to the maxillary arch made me forego the idea.

In older embryoes the stapes approaches and appears to indent the periotic wall (Plate 54, fig. 9; and Plate 57, fig. 27), the cellular ring being as yet uniform in its dimensions. In Rat embryoes 2 centims. in length the ring has lost its uniform character, the basal portion being much larger than that which comes into relation with the long crus of the incus; it is also in a line with the cartilaginous wall of the labyrinth, with which it appears to be almost continuous (Plate 57, fig. 30); it is separated from the vestibular portion of the membranous labyrinth by the remains of its primitive wall. The stapes in embryoes before birth has assumed somewhat of the appearance and proportions of the adult stapes—it has a well-marked head, two crura and a base, which is still separated from the membranous labyrinth (vestibular portion) by connective tissue (Plate 58, fig. 37); there is also a well-marked articulation between its head and the long crus of the incus, similar to what exists between the malleus and incus; the wall of the cochlea ventrad of the stapes is circular in outline and bulges externally, so that the stapes appears to be buried in the wall (Plate 57, fig. 32; and Plate 58, fig. 35).

On examining the stapes of the adult Rat with the lens it is found to bear a very close resemblance to the form of the human stapes, differing only in point of size (Plate 58, fig. 40, A).

It has a well-marked head, at the junction of which with the posterior crus is a slight tubercle for the attachment of the stapedius muscle. Of the two crura the anterior descends from the head to the base almost vertically, the posterior being the more curved; the base projects beyond the crura and it is wider, the head, crura, and base are fluted internally so that the adult stapes is the merest outside form or semblance of bone, a change which takes place after birth.

The stapedius muscle agrees in its development with the tensor tympani muscle, or with any other in the region of the head. The tubercle on the posterior crus has the same relation to this muscle that the processus muscularis of HYRTL has to the tensor tympani (the muscle is seen in Plate 57, fig. 34; and Plate 58, figs. 37, 38); it lies some distance internal to the hyoidean cartilage, from which it is separated by the seventh nerve. The nucleus in its tendon, named inter-hyal by Professor PARKER, is not present in any of the embryoes from which my sections have been made; moreover, it cannot have any relation to the hyoidean cartilage, or it would agree with it in its developmental history. When the nucleus in the tendon of this muscle is present it may be looked upon as comparable to the nuclei found in the tendons of other muscles.

Summary and Results.

Summarizing this work the following conclusions can with safety be drawn.

First.—That the malleus is the proximal extremity of the mandibular cartilage, and that its manubrium is the last part to be developed.

Second.—That the incus is the proximal extremity of the hyoidean cartilage. That this is determined not only by continuity of embryonic cartilage (which is not always present) and its morphological and histological agreement with the remainder of the cartilage, but also by the relation which the chorda tympani nerve (mandibular branch of the seventh) bears to its long crus. That its orbicular apophysis is the last to develope; its pedicular form of attachment appearing after birth.

Third.—That the stapes is developed as a circular ring of cells round the artery which has been described. That its connexion with the hyoidean cartilage is at first more close than with the periotic capsule; that of its individual parts the head and margins of the base are the last to appear, and that the fluted condition of the head, crura, and base is developed after birth.

In conclusion, I have to express my warmest acknowledgments to my former teacher, Dr. Allen Thomson, but for whom this paper could not possibly have been in its present form. I am also much indebted to Professor Parker for information concerning the form and modifications of the two cartilages in the lower Vertebrates.

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EXPLANATION OF THE FIGURES.

All the figures were drawn (with the exception of the last two) with a Zeiss' camera (Oberhäuser) and Hartnack objective. The numbers attached indicate in diameters the magnifying power employed. Some of the drawings are from single sections, but the majority have been made by combining consecutive sections.

The outlines in such cases were generally drawn from a single section, and are strictly accurate. The details were filled in from the appearances seen with a Zeiss Obj. A, Ocul. 2 × 55, or with Obj. D, Ocul. 2 × 235.

They have been restricted to those essential to the work in hand.

Alphabetical List of Reference Letters.

au.v.	Auditory vesicle.
a.st.	Arteria stapedialis.
ao.	Aorta.
a.st.m.	Arteria stapedio-maxillaris.
a.c.c.	Arteria carotis communis.
a.c.i.	Arteria carotis interna.
a.v.	Arteria vertebralis.
art.i.st.	Articulation between incus and stapes.
art.b.p.	Articulation between bulla and petrosal.
B.	Bulla.
Br^1.	First branchial arch.
Br^2.	Second branchial arch.
b.cr.	Basis cranii.
ch.t.	Chorda tympani.
ch.	Chorda dorsalis.
c.co.	Canalis cochlearis.
c.he.	Cerebral hemispheres.
ce.	Cerebellum.
e.s.ca.	External semicircular canal.
f.br.	Fore brain.
f.l.	Fore limb.
fe.r.	Fenestra rotunda.
G.ga.	Ganglion gasserii.
g.v.ac.	Vestibular portion of ganglion acusticum.
g.c.ac.	Cochlear portion of ganglion acusticum.
h.br.	Hind brain.
hy.c.	Hyoidean cartilage.
Hy.	Hyoid arch.
H.	Heart.
in.	Incus.
in.m.	Investing mass.
la.	Lamina of papery bone between mandibular process of malleus and tubercle at the junction of the neck with the manubrium (the free edge of which is called by Doran processus gracilis).
m.au.e.	Meatus auditorius externus.
mn.c.	Mandibular cartilage.
mn.	Mandible.
m.	Malleus.
ma.h.	Head of malleus.
ma.n.	Neck of malleus.
m.ma.	Manubrium mallei.
m.obl.	Medulla oblongata.

m.br.	Mid brain.
Mx.	Maxillary arch.
Mn.	Mandibular arch.
mn.pr.	Mandibular process.
op.	Optic.
olf.p.	Olfactory pit.
oc.c.	Occipital cartilage.
or.ap.	Orbicular apophysis of the incus.
olf.n.	Olfactory nerve.
pe.c.	Periotic capsule.
p.s.ca.	Posterior semicircular canal.
pe.pr.	Periotic process.
p.v.au.	Vestibular portion of auditory nerve.
p.c.au.	Cochlear portion of auditory nerve.
p.pr.	Posterior process.
pe.	Periotic.
p.eust.	Eustachian portion of bulla.
pt.i.	Pterygoideus internus.
pt.c.	Pterygoid cartilage.
r.v.	Recessus vestibuli.
r.ce.	Recessus cerebelli.
s.ty.	Sulcus tympanicus.
s.s.ca.	Superior semicircular canal.
st.	Stapes.
s.hem.	Sacculus hemisphericus.
st.a.c.	Anterior crus of stapes.
st.m.	Stapedius muscle.
s.p.s.ca.	United portion of superior and posterior semicircular canals.
th.hy.c.	Thyro-hyoid cartilage (cartilage of first branchial arch).
t.ty.	Tensor tympani.
ty.a.	Tympanic annulus.
th.c.	Thyroid cartilage.
t.p.	Tubal portion of sulcus tubo-tympanicus.
tub.	Tubercle on posterior crus of stapes for stapedius muscle.
th.	Vesicle of the third ventricle.
v.j.pv.	Vena jugularis primitiva.
v.j.	Vena jugularis.
v.c.	Vertebra cervicalis.
ve.	Vestibule.
v.j.ex.	Vena jugularis externa.
V^1.	Ophthalmic branch of fifth nerve.
V^2.	Maxillary branch of the fifth.
V^3.*ib.*	Mandibular branch of fifth (internal division).
V^3.*e.b.*	Mandibular branch of fifth (external division).
VII.	Seventh (nervus facialis).
VIII.	Auditory.
IX.	Glossopharyngeal.
X.	Pneumogastric.
XI.	Spinal accessory.
XII.	Hypoglossal.

Forty Figures in all: Of these there are 11 in Pig embryoes, 6 in the Human embryo, 3 in the Sheep, 2 in the Dog, and 18 in the Rat.

PLATE 54.

Figs. 1–3. Sections through Pig embryoes 1 centim. in length × 25.

Fig. 1. Longitudinal vertical section, showing gasserian ganglion, maxillary and mandibular branches of the fifth, the latter dividing into two; the seventh nerve, with its mandibular (chorda tympani) branch joining the inner of the two branches of the mandibular division of the fifth.

Fig. 2. Longitudinal vertical section, showing auditory vesicle, ganglion acusticum, dorsal aorta with the origin of the arteria stapedialis. Both figures have been cut along the obliquity of the arches.

Fig. 3. Transverse vertical section, showing the auditory vesicle, primitive jugular vein, facial nerve, and dorsal aortæ; the section is very oblique, shows on one side the open cleft.

Figs. 4–7. Sections through Pig embryoes 1·4 centim. in length × 25.

In the first three figures the sections are made along the obliquity of the arches. In all the figures a large number of consecutive sections have been used to fill in the outlines of the cartilages and periotic capsule.

Fig. 4. Longitudinal vertical section, showing eye, nose, mouth, cartilages of mandibular and hyoidean arches, mandibular branch of the fifth, with its outer and inner divisions, facial nerve with its mandibular (chorda tympani) branch, and meatus auditorius externus.

Fig. 5. Longitudinal vertical section much the same as preceding, but hyoidean cartilage continuous throughout its entire length, short crus of incus commencing to be formed, although too strongly indicated in the diagram.

Fig. 6. Longitudinal vertical section, showing gasserian ganglion, ganglion of olfactory nerve giving off two sets of branches, standing over nose cleft, ganglion and trunk of facial; stapes and stapedial artery, primitive jugular vein, periotic capsule and semi-circular canals, and junction of hyoidean cartilage with cartilage of first branchial arch (thyro-hyoid).

Fig. 7. Transverse vertical section, showing incus, hyoidean cartilage, stapes, and stapedial artery, periotic capsule, basis cranii,

gasserian ganglion, primitive jugular vein, facial nerve, with its mandibular (chorda tympani) branch, mouth cavity, and sulcus tympanicus.

Figs. 8–9. Sections of Pig embryoes 2 centims. in length × 25.

Fig. 8. A stage later than fig. 5.

Fig. 9. A stage later than fig. 7. Shows in addition the cochlear and vestibular divisions of the ganglion acusticum.

PLATE 55.

Fig. 10. Longitudinal vertical section of Pig embryo 2·6 centims. in length × 25, showing the cartilages of the first two post-oral arches, the periotic capsule with the semicircular canals, and the facial nerve with its mandibular (chorda tympani) branch.

Figs. 11–12. Sections of Human embryo fully 1 centim. in length × 25.

Fig. 11. Longitudinal vertical section, parallel to the middle line of the head, showing maxillary and mandibular branches of the fifth nerve, facial nerve with its mandibular (chorda tympani) branch; outlines of the ganglia of the fifth and seventh; and mandibular and hyoidean cartilages.

Fig. 12. Transverse vertical section showing basis cranii, carotis interna, arteria stapedialis, periotic capsule, stapes, hyoidean cartilage, ganglion and trunk of facial, and ganglion gasserii.

Figs. 13–16. Sections of Human embryo 4 centims. in length × 25.

Fig. 13. Transverse vertical section showing periotic capsule, occipital and hyoidean cartilages, tympanic annulus, sulcus tympanicus, meatus auditorius externus, facial nerve, head, neck (with processus muscularis of HYRTL), and manubrium of the malleus, and tensor tympani muscle.

Fig. 14. Transverse vertical section, through a plane posterior to that of fig. 13, showing head and long crus of the incus, and stapes, no artery (which has disappeared).

Fig. 15. Longitudinal vertical section, showing mandibular and hyoidean cartilages, head, neck, and manubrium of the malleus, incus, tympanic annulus and sulcus tympanicus.

Fig. 16. Longitudinal vertical section, showing part of mandibular cartilage, incus (body and crura), periotic capsule, hyoidean cartilage undergoing segmentation, facial nerve with chorda tympani (mandibular) branch, manubrium mallei, with processus muscularis and tensor tympani muscle, sulcus tympanicus and tympanic annulus.

Figs. 17–19. Sections of Sheep embryoes × 25.

Fig. 17. Transverse vertical section of embryo 1 centim. in length (a little oblique), showing canalis cochlearis, cochlear division of the ganglion acusticum, recessus vestibuli, superior and external semicircular canals, primitive jugular vein, facial nerve, sulcus tubo-tympanicus and arteria stapedialis.

Fig. 18. Obliquely transverse vertical section of embryo 1·4 centim. in length, showing cochlear and vestibular divisions of ganglion acusticum, periotic capsule, primitive jugular vein, facial nerve, stapes, arteria stapedialis, long crus of incus, sulcus tubo-tympanicus, neck of malleus, chorda tympani (mandibular branch of seventh) and hyoidean cartilage.

PLATE 56.

Fig. 19. Longitudinal vertical section of embryo 2 centims. in length, showing cartilages of mandibular and hyoidean arches. Periotic capsule and semicircular canals, facial nerve with its chorda tympani branch and meatus auditorius externus.

Figs. 20–21. Sections of Dog embryoes × 25.

Fig. 20. Obliquely transverse vertical section of embryo 1·3 centim. in length, showing hind brain, canalis cochlearis, sacculus hemisphericus, recessus vestibuli, superior and external semicircular canals, primitive jugular vein, facial nerve, arteria stapedialis, and sulcus tubo-tympanicus. (The cartilage is not represented in this section although it was present.)

Fig. 21. Longitudinal vertical section of embryo 2·5 centims. in length, showing cartilages of mandibular and hyoidean arches, periotic capsule, periotic process, facial nerve with mandibular (chorda tympani) branch, and meatus auditorius externus.

Figs. 22–38. Sections of Rat embryoes × 25.

Figs. 22–24. Sections from embryoes about 9 millims. in length.

Fig. 22. Longitudinal vertical section parallel to middle line of head, showing periotic capsule with semicircular canals, ganglion acusticum, primitive jugular vein, facial nerve with its ganglion, mandibular and hyoidean cartilages, stapes, arteria stapedio-maxillaris, sulcus tympanicus, gasserian ganglion with its maxillary and mandibular branches, eye and olfactory pit.

Fig. 23. Obliquely vertical section showing canalis cochlearis, cochlear and vestibular divisions of ganglion acusticum, recessus vestibuli, superior and external semicircular canals, periotic capsule,

primitive jugular vein, facial nerve, hyoidean cartilage in the whole of its length (partly in outline), mandibular branch of seventh, anterior crus of stapes, part of neck of malleus, basis cranii, sulcus tubo-tympanicus, arteria stapedio-maxillaris, thyro-hyoid cartilage (cartilage of first branchial arch), and thyroid cartilage.

Fig. 24. Nearly horizontal section, showing chorda dorsalis, investing mass, one of the cervical vertebræ, with vertebral artery, arteria carotis communis, arteria carotis interna, arteria stapedio-maxillaris, periotic capsule, stapes, incus, hyoidean cartilage, facial nerve, glosso-pharyngeal, pneumogastric, spinal accessory and hypo-glossal nerves, and gasserian ganglion.

Figs. 25–28. Sections from embryoes 1·3 centim. in length.

Fig. 25. Longitudinal vertical section, showing cartilages of mandibular and hyoidean arches, the various parts of malleus and incus, facial nerve with its mandibular (chorda tympani) branch and external auditory meatus.

Fig. 26. Transverse vertical section, showing malleus, sulcus tympanicus, throat cavity, meatus auditorius externus, periotic capsule, arteria carotis interna, gasserian ganglion, ganglion of facial, hyoidean cartilage, and basis cranii.

PLATE 57.

Fig. 27. Transverse vertical section, from the same series as preceding figure, but through a plane more posterior, showing incus and stapes, arteria stapedio-maxillaris, arteria carotis communis, arteria carotis interna, periotic capsule, basis cranii, hyoidean cartilage, and mandibular branch of seventh nerve.

Fig. 28. Transverse vertical section through a plane posterior to fig. 27, showing periotic capsule with primitive gap for fenestra rotunda, facial, glosso-pharyngeal, and pneumogastric nerves, basis cranii, cervical vertebræ, and primitive jugular vein.

Figs. 29–30. Sections from embryoes 2 centims. in length.

Fig. 29. Transverse vertical section, showing head, neck, and manubrium of the malleus, tensor tympani muscle, basis cranii, periotic capsule, gasserian ganglion, ganglion of the facial, meatus auditorius externus, and sulcus tympanicus.

Fig. 30. Transverse vertical section through a plane posterior to fig. 29, showing incus and stapes, with arteria stapedio-maxillaris.

Figs. 31–38. Sections from embryoes nearly 4 centims. in length.

Fig. 31. Transverse vertical section, showing malleus and its relation to sulcus tympanicus and meatus auditorius externus, tympanic annulus, basis cranii, and thyroid cartilage.

Fig. 32. Transverse vertical section through a plane posterior to fig. 31, showing ganglion acusticum, and the cochlear and vestibular portions of the nerve, basis cranii, thyroid cartilage, tympanic annulus, periotic capsule, incus, stapes and arteria stapedio maxillaris, articulation between incus and stapes; and vestibule of labyrinth.

Fig. 33. Longitudinal vertical section made parallel to middle line of head, showing malleus, incus, periotic process, hyoidean cartilage, facial nerve with its mandibular division, meatus auditorius externus, periotic capsule with the canals partly in dotted outline, recessus cerebelli for flocculus, and exoccipital cartilage.

Fig. 34. Longitudinal vertical section from same series as fig. 33, in a plane more internal, showing mandibular cartilage, and membranous splint of bone immediately underneath it, tympanic annulus, meatus auditorius externus, stapedius muscle in which there is no cartilage, facial nerve, periotic capsule and canals, and hyoidean cartilage.

PLATE 58.

Fig. 35. Longitudinal vertical section in a plane more internal than fig. 34, showing meatus auditorius externus, sulcus tympanicus, manubrium and posterior process of the malleus, hyoidean cartilage, arteria stapedio-maxillaris, and stapes, facial nerve, periotic capsule, and exoccipital cartilage.

Fig. 36. Nearly horizontal section above the level of tympanic sulcus, showing periotic capsule and process, ganglion and trunk of facial, incus and malleus with articulation between them, mandible, pterygoideus internus muscle, gasserian ganglion, and its maxillary and mandibular divisions.

Fig. 37. Nearly horizontal section from same series as preceding figure but cutting tympanic sulcus, showing stapes and its relation to periotic capsule, and long crus of incus, facial nerve, stapedius muscle, neck of malleus, mandibular cartilage, tympanic annulus, tensor tympani muscle, and pterygoid cartilage.

Fig. 38. Nearly horizontal section in a plane ventrad to that of fig. 37, showing meatus auditorius externus in part, sulcus tympanicus,

tensor tympani muscle and neck of malleus, hyoidean cartilage, facial nerve, and stapedius muscle, periotic capsule, mandibular cartilage and mandible.

Fig. 39. Head of Pig embryo 1·4 centim. in length × 14, somewhat diagrammatic, designed to show the cartilages of the first two post-oral arches, the periotic capsule and semicircular canals, ganglion and trunk of facial with its mandibular branch, ganglion of ninth nerve, gasserian ganglion with its ophthalmic, maxillary and mandibular branches, primitive jugular vein, cerebral hemisphere, vesicle of third ventricle, midbrain, cerebellum, medulla oblongata, ventricles, auricle and aortic bulb of heart, fore-limb, optic cup, olfactory pit, and maxillary, mandibular, hyoid, and indications of first and second branchial arches.

Fig. 40. Right bulla and periotic bone of adult Rat × 10 (parts of the bony meatus auditorius and periotic have been removed to expose the external ossicles *in situ*), showing mandibular process (what DORAN calls process head) of malleus, lamina of papery bone between this process and tubercle at junction of neck and manubrium (the free edge of which is called processus gracilis by DORAN), posterior process (DORAN'S orbicular apophysis), manubrium mallei, body and crura of incus, ossified hyoidean cartilage (tympano-hyal of FLOWER), facial nerve with its mandibular branch, bulla, and Eustachian portion of bulla, articulation between bulla and petrosal.

Fig. 40, A. Stapes of adult Rat × 10, showing head, crura, and base, with an attempt to show the fluted condition of these, and tubercle on posterior crus for stapedius muscle.

Fig. 40, B. Long crus and orbicular apophysis of incus × 10, showing pedicular attachment of orbicular apophysis to long crus.

XXI. *Contributions to the Anatomy of the Central Nervous System in Vertebrate Animals.*

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[PLATES 59–63.]

Part I.—ICHTHYOPSIDA. Section I.—PISCES. Subsection I.—TELEOSTEI.

Appendix.

ON THE BRAIN OF THE MORMYRIDÆ.

MARCUSEN[*] has given a very good *résumé* of the zoological history of this family from the time of their first discovery by HASSELQUIST, a pupil of LINNEUS, up to the date of the publication of his paper by the St. Petersburg Academy, and as his treatise is easily accessible there is no occasion for my going into that part of the subject here. The external appearance of these Fishes gives no indication at all of anything extraordinary in the structure of the brain, and as the zoologist above referred to, did not pay much attention to anatomy, more than half a century elapsed before any steps were taken in that direction.

One of the species at least was very well known to the ancient Egyptians, to whom it was a sacred animal, and its portrait is even now quite easily distinguishable on some of the monuments.

A figure of one species is to be found in vol. 19, p. 250, of CUVIER and VALENCIENNES' 'Histoires Naturelles des Poissons;' RÜPPEL,[†] also, has given plates of three species. The position in nature of this family is defined in Dr. GÜNTHER'S catalogue of fishes.

ERDL[‡] was the first anatomical writer who mentioned any peculiarity about the

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[*] Die Familie der Mormyren. Mém. de l'Acad. Impériale des Sciences de St. Pétersbourg, viie serie, tome vii., 1864.

[†] Beschreibung u. Abbildungen mehrerer neuer Fische im Nil entdeckt, 1829 to 1835.

[‡] Ueber d. Gehirn der Fischgattung Mormyrus. Gelehrte Anzeigen herausgegeben von Mitgliedern der K. Bayer. Akad. der Wiss., Bd. 23, 1846.

brain in these Fishes. He supposed that they had a Mammalian cerebrum; the pink colour and the minute striations of the surface, which superficially resemble convolutions, led him into this error.

MARCUSEN went more thoroughly into the subject; but even he, in his first* paper, was of the same opinion.

About this time ECKER† wrote a description of a brain of a very small species, probably a young *Mormyrus Cyprinoides;* the conclusion he came to was, that the highly-developed organ which is the subject of this paper, was the corpus quadrigeminum.

Eleven years after the first slight sketch referred to above, MARCUSEN ‡ took a view of the brain of this Fish more nearly approaching the truth, inasmuch as he supposed that the "Eigenthümliches Organ," as he termed it, was part of the cerebellum.

OEFFINGER,§ whose paper, so far as I know, was the last that has been written on this subject, added nothing to the information possessed by his predecessors.

These Fishes are by no means easily procured by ordinary tourists on the Nile.

V. MIKLUCHO-MACLAY ‖ has placed it on record that he was unable to see one during his stay in Egypt. He appears to have been there twice, but his residence each time must have been short; else his great powers as a traveller would surely have come to his assistance in this, as in other matters.

I was more fortunate, and by enquiring of native fishermen, through the medium of my dragoman, succeeded in obtaining a sufficient supply of living specimens.

The two species which I obtained most abundantly were, *Hyperopisus dorsalis* (GÜNTHER), which was described by MARCUSEN under the name of *Phagrus dorsalis,* and *Mormyrus oxyrhynchus,* but this latter was much more rare than the former. Of a species of *Mormyrops* I obtained one or two specimens.

The mode of procedure which I adopted in the treatment of the brains of these Fishes was the following. All sensation, or at least all sensation of pain, having been eliminated by section of the spinal cord at, or nearly opposite the region of the pectoral fins, parts of the top or sides of the skull together with the facial bones and all superfluous tissue were removed, and the skull containing the brain was placed in MÜLLER's fluid and spirits of wine in the proportion of one-third of the latter to two-thirds of the former. This solution was changed the next day, and again in three or four days, one more change was made at about the tenth day, and after being in this fluid for three weeks, the brain was removed into a 2 per cent. solution of potass-bichromate. The potass-bichromate was changed once a fortnight. It is not absolutely necessary to change the solution so often; the more frequently it is renewed,

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* Gaz. Méd. de Paris. Sur quelques particularités des Mormyrus, 1853, p. 136. Also Bull. de la classe Phys. Math. de l'Acad. Imp. des Sciences de St. Pétersbourg, tom. xii., 1854.

† Anatomische Beschreibung des Gehirns vom Karpfenartigen Nilhecht. Leipzig, 1854.

‡ *Loc. cit.*

§ Neue Untersuchungen ü. d. Bau d. Gehirns vom Nilhecht. Arch. f. Anat., 1867.

‖ Beiträge z. vergleichenden Neurologie d. Wirbelthiere, p. 69, note.

the quicker the hardening process goes on. It appears that it is not desirable that the specimens should be kept in the same solution more than one month.

This method is a modification of that proposed by HAMILTON,* and is much more convenient than the process which I formerly used, which was to place the brain first in absolute alcohol and iodine and then in chromic acid.

The MÜLLER's fluid and spirits of wine has very great penetrating power, so that it is not necessary to remove the brain entirely from the skull, but two or three holes made in the latter are quite sufficient. So far both methods are equal; but the latter has a great advantage inasmuch as it does not cause any contraction in the nervous tissue; on the contrary, the brain swells and becomes enlarged so as partially to project through the holes in the skull. The hardening process takes a long time to be effected. The brain requires at least six months to become sufficiently firm to be submitted to the section knife; but it remains in that state for twelve months longer, more or less.

Here a curious thing is to be noticed, indicating, probably, different chemical composition in brains of various species; for instance, the brain of *Mormyrus* is still in a good condition for making thin sections after having remained eighteen months in the hardening fluid, while those of some other Teleostei, Gurnard or Ballan Wrasse, for instance, are too friable to make satisfactory sections after twelve months.

In consequence of this slow method of hardening the nervous tissue, the histological elements are not contracted or distorted, but show much better the form which they possessed during life; so that we see none of those angular nuclei or cells which MEYNERT† reports that he has seen in sections of the brain of Fishes.

Another advantage especially in travelling is, that there is no necessity, as there is with absolute alcohol and chromic acid, to make sections soon after they have been put into the hardening fluid on account of their becoming too friable; but they can be safely left until a convenient opportunity occurs of attending to them.

When the brains have become sufficiently hard, sections of any degree of fineness can be made in the microtome; and after having been stained with rosaniline or carmine, they can be cleared with oil of cloves and preserved in Canada balsam. RANVIER‡ and other writers on histology recommend that before the application of the oil of cloves, the sections should be immersed in alcohol; this is not at all necessary for the nervous system, since the oil of cloves renders the sections quite transparent even when they are taken direct from the watery fluid; the only precaution required being to absorb all superfluous moisture with blotting paper. This makes one operation less to perform, and it is obvious that when a hundred sections or more have to be placed in consecutive order, the fewer processes they have to go through the better.

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* Journal of Anatomy and Physiology, vol. xii., 1878.

† STRICKER's Handbook, Sydenham Society, vol. ii.

‡ Traité Technique d'Histologie, p. 112. Paris, 1875–78.

In order to obtain an idea of the brain of a *Mormyrus,* we may take that organ of any ordinary Teleostean, and imagine a fungoid growth taking place from the region in front of the cerebellum; we may then imagine this growth to burst through the tecta lobi optici, forcing them asunder, repressing them to the basal part of the brain, and then to spread out in all directions, covering over and concealing every one of the remaining portions of the brain.

We have thus a stalk or peduncle on each side of the region referred to above, and expansions which take the form of wings growing in every direction—forward, upward, outward, and backward—until the obstacle of the walls of the skull is encountered, when, growth still continuing, foldings in various directions occur: thus the anteriorly directed wing turns backward on itself on reaching the front extremity of the cranium, the dorsal wing turns inward on attaining the roof of the skull, the outer wing turns upward under the same condition, while the posterior wing ends in a free edge directed backward. This is the state of affairs in the young animal, but as age creeps on the complication in the folding of these wings increases. Another lobe becomes developed between the outer and the ascending wings; this also projects forward between the anterior wings of each side; the posterior wing now becomes folded into a number of small transverse convolutions, and the dorsal wing develops an additional fold at its upper extremity.

MARCUSEN[*] tried to found genera partly on the greater or less extent of these wings, but this arrangement, so far as it is based on the nervous system, will not hold good; because even in species with the most highly developed brain, the more simple arrangement prevails in young specimens.

Each wing is provided on the outer side with numerous excessively fine parallel ridges, whose structure will be described presently; these ridges follow every curve and inequality of the surface, so that their sections present various appearances.

The consequence of this arrangement is that those parts of the wings which are turned back or folded show only the layer of medullary fibres, and are therefore white, while the parts which are not reflected, such as the posterior wing and the upper part of the dorsal wing, show the minute ridges which give these portions a pinkish colour in the fresh state, and cause the minute striations which gave occasion to the comparison with the convolutions of the cerebrum of Mammalia; we shall see presently that they have nothing to do with these convolutions, but that in fact, they belong to quite another part of the brain.

I have not given figures of the external aspect of the brain of the *Mormyridæ,* because MARCUSEN'S[†] illustrations are very good, although his drawings of the sections and of the microscopic anatomy leave much to be desired.

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[*] *Loc. cit.*

[†] *Loc. cit.*

Microscopic Anatomy of the Brain.

Lobi Olfactorii.

With the exception of the wings and of the tuberculum impar, the other parts of the brain have very nearly the same structure as in the ordinary Teleostean Fishes.

The *Mormyridæ* belong to that division of the Teleostei in which the olfactory lobes are situated at the extremity of a long peduncle far in advance of the anterior end of the brain and very near to the olfactory organ.

They are fusiform in shape and are placed near the anterior extremity of the olfactory peduncle like a bead on a string, and the olfactory nerve pursues a short course before plunging into the sense organ.

The structure of the olfactory lobes resembles that of the corresponding lobe in *Mugil cephalus;* on the outside is seen a layer of the fibres which go to form the olfactory nerve. Many of these fibres are broad and bandlike, resembling the fibres of REMAK in the sympathetic nerves; they surround the whole lobe with the exception of the broader end which is directed towards the brain and from which the peduncle emerges. Towards the inner side of this layer of fibres and mingled with them is a stratum of larger sized cells which occur sometimes in groups of eight or ten and occasionally singly. They resemble those figured in my paper on the brain of Teleostei.[*]

The central part of this lobe is occupied by numerous nerve cells of the smallest category, which some writers consider to be nuclei, but which, in Fishes at least, are certainly cells if MAX SCHULTZE's[†] definition of a cell is to be accepted. They are rounded or oval, sometimes fusiform or pear-shaped. The more circular ones measure from 0·004 millim. to 0·005 millim. in diameter; the fusiform ones with a process at each end measure about 0·005 millim. by 0·0067 millim. The rounded cells are occasionally seen to give off processes. These small nerve cells occupy a space the basis of which is a granular material in which numerous fibrillæ are seen to ramify. The fibres of the olfactory commissure, on their entrance into the lobe, go directly into this central part; each fibre appears to be composed of an axis cylinder with a very small medullary sheath; they are smaller in diameter than the fibres of the olfactory nerve.

The layers of the olfactory lobe are arranged here as in *M. cephalus.* The fibres of the olfactory nerve are outside and in front, the larger cells come next, and the smaller cells with the fibres of the olfactory peduncle occupy the centre and the posterior part, thus confirming the comparison of the structure of this lobe in the Mammalia with that of the corresponding lobe in Teleostei, which I made in the case of *M. cephalus.*[‡]

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[*] Phil. Trans., 1878.

[†] Ü. Muskelkörper und das was man eine Zelle zu nennen habe. Archiv f. Anatomie, 1861.

[‡] *Loc. cit.*, p. 750.

Cerebral Lobes.

These lobes resemble in structure the corresponding parts in *M. cephalus.* Each lobe is partially divided into two by a transverse horizontal depression, which separates a rounded dorsal knob from the ventral portion. The olfactory peduncle is attached to a minute oval eminence situated at the inferior edge of the anterior part of the ventral lobe. Externally there is a single layer of epithelial cells, each sending a long process into the substance of the lobe perpendicular to the surface; within this there is a layer of small cells, sharply defined at parts from the central portions of the lobe. These cells are packed close together with a comparatively small quantity of neuroglia between them; many of them measure from 0·006 millim. to 0·0083 millim. in diameter; others are more oval in shape; the majority show a nucleus and a nucleolus; in some places they are separated from the central portion by a layer of fine transversely directed fibrillæ. This layer is shown in my paper on the brain of Teleostei.*

Fritsch† is of opinion that these cells belong to the connective tissue elements, and compares them to the layer of granular neuroglia which bounds the grey matter of the cerebrum externally in Mammalia; but to me they seem to present the characteristics of nerve cells, although this is a point which is often difficult to determine; in *M. cephalus* they certainly do not join the processes from the epithelial cells, which they ought to do if they were connective tissue elements. If they are nerve cells the layer of neuroglia referred to by Fritsch is not present in the brain of Teleostei, although a corresponding one is found in the brain of the Elasmobranchii.

The central portion of these lobes (*br.br.*) is occupied principally by neuroglia of a granular appearance in which are dispersed at intervals cells of a larger size than those of the outer layer. They measure generally about 0·016 millim. by 0·012 millim., with a nucleus 0·008 millim. by 0·006 millim.; they mostly have a smooth outline with about three processes. A few were occasionally met with which possessed an irregular outline with five or six processes. Round each cell there was ordinarily to be seen a clear space; concerning the nature of this space anatomists are by no means clear; Boll‡ considers that it is an artificial production; Obersteiner§ mentioned several reasons in favour of the view that it is natural; Henle‖ and Forel¶ are entirely of this opinion; for myself I am undecided at present. In the *Mormyridæ* these spaces certainly do not present so natural an appearance as they had in *M. cephalus.* Perhaps sections of frozen brain tissue might decide the point, though this is by no means

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* *Loc. cit.*, Plate 63, fig. 17.

† Untersuchungen ü. den feineren Bau des Fischgehirns. Page 48.

‡ Die Histiologie u. Histiogenese d. Nervösen central organe. Archiv f. Psychiatrie, Bd. iv.

§ Beit. z. Kentniss vom feineren Bau der Kleinhirn. Sitzb. d. Math-Nat. Classe d. k. Akad. der Wissenschaft, Wien. Bd. lx.

‖ Nervenlehre.

¶ Untersuch. ü. d. Hauberegion, &c. Archiv. f. Psychiatrie, Bd. vii., 1877, p. 449.

certain, for even if they are natural and not artificial they might so collapse in the soft fresh state as to become obliterated.

The Hypoaria.

These bodies are not so well developed in the *Mormyridæ* as in *M. cephalus*—they scarcely project at all below the ventral surface of the brain, and do not form separate lobes; the ventricle is also very small; the trigonum fissum, however, is rather prominent, and forms a separate tubercle on the inferior surface. The hypophysis or pituitary body is attached to the apex of the trigonum, and the infundibulum passes through it. With regard to the minute structure of these parts it resembles that of the corresponding lobes in the *M. cephalus* in respect to the arrangement of the cells, which are dispersed separately throughout the neuroglia; but many of the cells themselves differ in shape in being multipolar and in having as many as six processes, while some are pear-shaped. They usually measure about 0·0093 millim. by 0·007 millim. The nearly obliterated ventricle and the infundibulum are surrounded by a layer of small cells as in other Teleostei. These bodies may be considered to be in a rudimentary condition, and this is shown also by the absence of the nucleus rotundus and the transverse commissure connected with it.

Tecta Lobi Optici.

The position of these bodies in the *Mormyridæ* has become quite reversed. Instead of arching over the tori semicirculares on the dorsal side of the brain as in the ordinary Teleostei, they become relegated to the inferior surface and occupy a position below and to the outside of those bodies; the roof in fact has been transferred to the foundation.

The tecta appear to have attained this position in consequence of the enormous development of the wings alluded to in the first part of this paper.

Accompanying this change of position there is a considerable degradation of structure. The seven layers which are distinguishable in *M. cephalus* are here reduced to two or at the most three; the ependyma has disappeared; there are only slight indications of radial striation. The outer layer in some places is separable into two subordinate divisions. The *Mormyridæ* are not good subjects for deciding any histological points in the structure of these lobes which as I have before remarked are decidedly undeveloped; I was therefore not surprised to find that I am unable either to confirm or refute the statements of BELLONCI* as to the presence of a minute network of fibrillæ in the external layer of the tecta lobi optici, and I have not yet had

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* Ricerche intorno all' intima tessitura del cervello dei Teleostei. Reale Accademia dei Lincei, Anno cclxxvi., 1878 and 1879. Also by the same author, Ü. den Ursprung des Nervus Opticus. Zeitschr. f. wiss. Zoologie, Bd. xxxv., 1880.

an opportunity of testing the method which he followed in obtaining his results, a description of which he was kind enough to send me. These results are, however, very interesting, showing as they do the mode of origin of the optic nerve primarily from a net-work of the finest possible fibrillæ, a statement which, however probable, had not before been demonstrated.

The two outer layers occupy only a small part of the tectum, the remainder extending to the inner surface corresponds to the second, third, fourth, fifth, and sixth layers in *M. cephalus;* it consists principally of neuroglia in which radial striation is indistinctly visible. The fusiform cells which ordinarily form a conspicuous feature in this lobe are few and far between; but they certainly are present and occur close to the outer edge of this inner layer. They are placed more obliquely than in *M. cephalus,* and as in that species, their outwardly directed process, which is probably the protoplasmic process, goes into the external layer. FRITSCH,[*] in his large work on the structure of the brain in Fishes, denies the existence of these cells, and supposes that STIEDA mistook a crossing of two capillaries for them; this seems to show that the former writer did not make his sections sufficiently thin; if they are properly made, so that only one layer of cells occupies their thickness, such a mistake would be impossible.

The cells which correspond to the sixth layer do not form here a distinctly continuous layer, but occur in scattered groups along the inner surface of the tectum; they are slightly larger than the cells in *M. cephalus;* they measure about 0·005 millim. by 0·004 millim. The interrupted layer thus formed is interspersed with longitudinally-directed fibres, which correspond to the fifth layer of the tecta in Teleostei; these fibres are medullated, each of them appearing to be a single fibrilla clothed with a very narrow medullary sheath. The reason why longitudinally directed fibres in one species correspond to transversely directed fibres in another depends on the position of the tori longitudinales, which in *Mormyrus* are widely separated from each other by the development of the wings of the valvula cerebelli. The tori longitudinales are only in contact in front, and posteriorly they are placed along the outer edge of the tecta and end in a free point. The consequence of this arrangement is that the commissure of these lobes only exists at the anterior end, and the fibres destined to form it must therefore necessarily run longitudinally. The structure of the tori longitudinales in *Mormyrus* resembles that of the corresponding parts in *M. cephalus* so far as they are made up of the smallest-sized cells; but the arrangement of these cells is different; they do not occur in rows between bundles of radiating fibrillæ, but quite irregularly, in a net-work of the smallest-sized fibres; they vary very much in form; some are rounded, some pear-shaped, some oval, and others irregular in outline; they give off numerous processes.

I termed these bodies "fornix" in my paper on the brain of Teleostei, following

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[*] *Loc. cit.*, p. 51.

Gottsche,* who gave them that appellation without any reference to the part of the same name in the Mammalian brain; but it seems that the other term, tori longitudinales, is more appropriate; the former term seems to indicate a theory; but Fritsch, although he uses the latter name, really considers that the bodies in question homologise with the fornix of the human subject; this I hope to prove in the end is not the case, and thus the term fornix is a misnomer, as having been applied to a different structure. These objections do not apply to the name tori longitudinales, which involves no sort of consideration as to theory, but simply states a fact of form.

Tori Semicirculares.

The tecta lobi optici having changed their position, and having been relegated to the inferior surface of the brain, while the tori semicirculares have retained theirs without displacement, the relative position of these two parts comes to be that of the egg to the egg-cup, or, as Ecker puts it, that of an oyster to its shell. The torus semicircularis forms a tuberosity of rounded form, bounded below and on the outside by the tectum. It may be divided into two parts, easily distinguishable from each other, the upper part consisting principally of nerve cells and fibrillæ dispersed in a granular neuroglia. The lower part on which it rests contains more nerve fibres, which belong to the system of the commissura ansulata.

The upper part differs in structure from the corresponding part in *M. cephalus* and *Labrax lupus* in the same diffused way as the tectum. The small cells which do not show such elongated processes as in the species just cited, are scattered irregularly among the fibrillæ which traverse this body in all directions through the upper section of the tuberosity. They measure from about 0·005 millim. to 0·0035 millim. in diameter, and each shows a well defined nucleus and nucleolus, the latter being a mere speck.

The larger cells occur sparingly; they have a clear space round them, and are of moderate size, being about 0·019 millim. by 0·0093 millim.; the nucleus generally measured 0·006 millim. in diameter, and the nucleolus 0·0017 millim.; with these which were smooth and rounded or oval in outline, occurred others which were of irregular shape, with three processes; these latter are rather smaller, measuring 0·012 millim. by 0·008 millim., with a nucleus 0·003 millim. in diameter; some other cells which are still smaller, measuring 0·006 millim. by 0·007 millim., seem to form a transition to those of the smallest category mentioned above.

Cerebellum.

The cerebellum (fig. 1) has the usual structure of this lobe in Teleostei; here it is a tongue-shaped process directed forward, in which respect it differs from that of most Fishes, where as a rule it is turned backward, unless it is not sufficiently developed to

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* Arch. f. Anatomie, 1835.

form more than a small tuberosity bridging over the fourth ventricle. It resembles, however, the form of the cerebellum in Fishes of the family Siluridæ, of which *Schilbe Hasselquistii*, found also in the Nile, may be taken as an example, but in comparison to the size of the brain it is much smaller, since in *S. Hasselquistii* it predominates over the other lobes in size, while here it is overshadowed and compressed by the enormous size of the wings of the valvula cerebelli. In structure it agrees with that of the cerebellum in all Vertebrata.

DENISSENKO[*], in his interesting paper on the anatomy of this lobe, divides it into three layers, viz.: the molecular, which is external; the granular coming next, and the fibrous occupying the centre of the lobule; but MEYNERT[†] makes a separate layer for the Purkinje cells, which is more in accordance with the state of affairs in the Teleostei; I have never found these cells so irregularly dispersed through the molecular layer in those Fishes which I have examined as is described by DENISSENKO. The nerve fibres also do not form a distinct layer, except quite at the root of the organ.

Adopting MEYNERT's idea, I shall therefore speak of the Purkinje cells as forming a distinct layer, and term it the intermediate layer.

Thus we have in the cerebellum of the *Mormyrus* three layers, counting from the outside towards the centre, viz.: the molecular, the intermediate, and the granular layers; the fibrous layer may be looked on as part of, and diffused through, the last mentioned one. These correspond to the first three layers in *M. cephalus*, and the fibrous stratum in the parts where it forms a distinct layer would in like manner correspond to the fourth.

The radial striation which characterises the molecular layer is very strongly marked in this species. This appearance is caused by prolongations from the single layer of epethelial cells which cover the whole surface of the cerebellum; each cell sends a process for a considerable distance into this layer. STIEDA[‡] has published a good description of these cells, to which he has given the appropriate name "stiftzelle;" BOLL[§] showed how they were developed from the so-called "Deiters," or connective tissue cells, so as to form a membrane beneath the pia mater. These peculiar cells appear to exist on and form a covering for most parts of the brain, *e.g.*, the cerebrum, and the tecta lobi optici, but they do not form such well marked and distinct striations in other organs as they do in the cerebellum. This layer is not formed entirely of connective tissue, although these processes do occupy a considerable part of it, and cause the striated appearance which gives it such a characteristic aspect; there is a large quantity of molecular matter as well as small cells in it. These small cells are extremely scarce, although occasionally met with; they resemble the cells of the granular layer, and have a nucleus and a nucleolus, which latter is a mere

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[*] Zur Frage ü. den Bau der Kleinhirnrinde, &c., Arch. f. Mikroskopisce Anatomie, Bd. xiv., 1877.

[†] STRICKER's Handbook, Sydenham Society, vol. ii.

[‡] Studien ü. d. Centralnervensystem der Wirbelthiere, p. 154.

[§] *Loc. cit.*

speck; they are usually pear-shaped, and have a process at each end; they measure about 0·006 millim. long by 0·0045 millim. broad. There is probably also a network of fine fibres in the molecular layer of the cerebellum, such as is found in the corresponding layers of the wings; but I have not been able to add anything to what I remarked on this subject in my paper on the brain of the Teleostei.*

The intermediate layer consists of the Purkinje cells and a bundle of longitudinal medullated fibres of small size, which appear to join eventually the crus cerebelli. The Purkinje cells resemble those of the Teleostei in general, in being arranged parallel to the long axis of the lobe: they are shorter and thicker than the corresponding cells in *M. cephalus;* their usual size is 0·018 millim. long by 0·016 millim. broad; some are as long as 0·022 millim.; the nucleus is about 0·009 millim. in diameter, and the nucleolus 0·002 millim.; there is a circular space round the nucleolus, which is clearer and smoother than the remainder of the contents of the nucleus: these cells then resemble those of the motor horn of the grey matter of the spinal cord, in the disk-like nucleolus and the clear space which surrounds it. Each Purkinje cell has two processes, one thicker, corresponding to the protoplasmic processes of the cells of the spinal cord; the other finer, corresponding to the axis-cylinder process of the same. I never could perceive any prolongation of this into the nucleus, but on the other hand, the axis-cylinder process was in one case distinctly traceable into one of the medullated fibres of the longitudinal bundles belonging to the intermediate layer. This fact confirms the statement of KOSCHENIKOFF,† and also accords well with the theory which I suggested in the description of the cerebellum in *M. cephalus*, and which only required this arrangement to complete the statement.

It now remains to consider the granular layer of the cerebellum; DENISSENKO described a method by which he thought he had demonstrated the existence of two different species of cells in this layer. During the process of double staining with eosin and hæmatoxylin, he maintained that one kind of cell takes up one colour and another kind absorbs the other colour; he thus had the so-called eosin cells and the hæmatoxylin cells. Whatever may be the case in higher animals, in Fishes there exists only one species of cell in this layer; I tried the experiment several times, carefully following DENISSENKO's directions; but I never could find that the cells of this layer exercised any selection as to the material they took up, but the result was that all the cells assumed a more or less light purple uniform tint according as the eosin or hæmatoxylin solutions were the stronger: from this one must infer that there is no difference of composition in the cells forming this layer in Fishes. In shape they resemble those of the third layer of the cerebellum in *M. cephalus;* they belong to the smallest category, generally measuring about 0·004 millim. in diameter; they mostly possess a nucleus and nucleolus; some are even still smaller, being 0·0034 millim. long by 0·003 millim., but the nucleus is of the same size as that of the larger specimens.

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* *Loc. cit.*

† Axencylinder Vortsatz der Nervenzellen im Kleingehirn, &c., Arch. f. Mik. Anatomie, Bd. v., 1869.

They have a tendency to be arranged in distinct rows, the spaces between which are occupied by nervous fibrillæ.

There has been a good deal of dispute as to the nature of these cells. Some histologists think that they are connective tissue cells, others that they are nervous elements ; with these latter I must throw in my lot ; there seems to be no manner of doubt but that they are nervous ; and it is difficult to imagine that a whole lobe of a brain should be almost entirely made up of connective tissue, or what object could be fulfilled by such an arrangement ; it would not be required to support the few remaining elements which would then consist of little else than the Purkinje cells which one would suppose might be better packed up in a smaller compass.

Valvula Cerebelli.

I hope to prove eventually that the wings described at the commencement of this paper are nothing more than the lateral parts of the valvula cerebelli enormously developed ; but for the present I will confine myself to a description of their structure. They consist of two parts : a central portion, which is simply a continuation of the cerebellum, containing precisely the same elements arranged in precisely the same manner ; and a pair of wings.

In many Fishes this part forms nearly the whole of the valvula cerebelli, the wings not being much developed ; but here, although it is much more extensive than in many Fishes in which it forms the greater part of the valvula, it occupies but a small portion of the entire structure.

It consists of three tubercles (fig. 1) arranged one in front of the other, diminishing in size from behind forward ; of these the most posterior appears actually to be part of the cerebellum. They each present the molecular, intermediate and granular layers. The first of these three layers can be traced continuously from the cerebellum over these tubercles and ends in front of the foremost with an undulating free edge as if an indefinite development of these bodies might take place. The wings arise from each side of this central portion and cover more or less the remaining lobes of the brain according to the age of the animal. The greater part of their mass is formed by an accumulation of minute cells which closely resemble those of the granular layer of the cerebellum ; with these are found numerous bundles of fibrillæ. This accumulation of cells takes the form of thick plates which are bent and folded as formerly explained. On one side of these plates a series of closely arranged ridges are placed (fig. 3). Each of these ridges is a cerebellum in miniature and consists of the regular number of layers, the molecular, the granular, the intermediate, and the fibrous. They are so placed that the molecular layers of a pair of ridges are arranged in close contact, with only a process of the pia mater between them, then comes the granular and the intermediate, and finally the fibrous layer is interposed between a pair of ridges on the other side, one fibrous layer serving for two contiguous ridges ; the fibrillæ belonging to this layer are of extreme fineness and are unprovided with a medullary sheath, being at these parts merely axis cylinders.

Each ridge is inserted into the main body of the wing, like a nail into a plank, with a conical insertion consisting only of the molecular layer. The granular-basal part of the wings forms a conical projection between each pair of molecular layers, and the pia mater passes down between these, forming a double membrane which at the summit of this conical process of the basal part divides into two, one accompanying each molecular layer to the extremity of its insertion where it disappears.

We have thus a series of ridges in alternately reversed positions, occurring in fact in pairs back to back. Where the wings are comparatively straight, the only contact that the ridges have with their basal portion is by the insertion at one edge; but where curves and foldings occur the ridges become more or less distorted, and may be in contact with those parts by the sides as well as by the edges. With regard to the minute structure of these ridges and the remainder of the wings we will begin with a consideration of the molecular layers. These are generally separate from each other at the summit of the ridge, but in some parts this layer of one ridge is continuous with that of the next; but in those parts where the wings end in a free edge, and the ridges are on the external surface, the end of each ridge where it abuts on this free edge, goes round to join the next, forming a loop. The striated arrangement of the molecular layer is very apparent and the palisade cells (as one may translate STIEDA's term "stiftzelle") are very well developed. The pia mater generally penetrates between each pair of molecular layers, and where this is so the striations are directed at right angles to the width of the ridge as seen in section, or in other words parallel to its thickness; but in the ridges along some parts of the dorsal surface of the wing, where they are external, the pia mater does not penetrate between them but simply covers their free edge; in this case the striations are not directed at right angles but parallel to the width of the ridge; this seems to show that the striation is due to the palisade cells and not to the ramifications of the protoplasmic processes of the Purkinje cells, to which I was formerly inclined to attribute it. Besides this, the prolongations of these cells may very often be seen actually sending processes into the molecular layer. I can also confirm BOLL's statement that the palisade cells forms a membrane distinct from the pia mater; for this latter membrane can be torn off leaving the layer of palisade cells intact.

As previously mentioned, the arrangement of the layers in these ridges is not quite the same as in the cerebellum; the granular layer becomes interposed between the molecular and the intermediate layers, instead of the reverse being the case; this granular layer is continuous with the basal portion of the wings, but differs somewhat in this, that in the basal portions of the wings the cells are more uniform in size, whereas in the ridges the cells vary more in dimension; the smaller ones (fig. 17, *b.*) are oval or rounded, with nucleus and nucleolus; generally they are provided with two or three processes; they measure usually about 0·004 millim. or less; one of those figured is

0·004 millim. long by 0·003 millim. wide, and the nucleus is 0·002 millim. in diameter; the other is 0·0034 millim. in diameter with a nucleus of the same size as that of the one first mentioned. The cells of the larger size seem to be transitional forms to the Purkinje cells. The one figured (fig. 17, *c.*) measures 0·006 millim. long by 0·0054 wide, the nucleus is 0·003 in diameter, and the nucleolus, which in this case is rather larger than is usual in this species of cell, measures nearly 0·001 millim. in diameter. These two kinds of cells are irregularly mingled with each other, but the smaller sort are more often found nearer the molecular layer and extend some distance into it. The basis of this granular layer contains a fine network of fibrillæ (fig. 17, *h.*) which connects the cells together and extends into the molecular layer.

The ridges are bounded on the other side by a single row of cells which appear to correspond to the Purkinje cells of the cerebellum; their stratum may therefore have the same denomination of intermediate layer (fig. 17), and the term is still appropriate, for although they are not intermediate between the granular layer and the molecular—the former having encroached more on the latter than in the cerebellum—they are still intermediate between the granular and the fibrous layer, which latter is common to two contiguous ridges. As just mentioned, these cells are arranged in a single row in ordinarily formed ridges, *i.e.*, in ridges which occupy the straight parts of the wings, and are bounded on one side by the granular layer, and on the other by a space which separates one ridge from the other, and in which the fibrils from the fibrous layer are found, which presumably join the axis cylinder processes of these cells, although I have not actually been able to see the junction.

These cells are generally pear-shaped or oval, and usually have two processes, one at each end; one of these is the protoplasmic process, which can often be traced through the granular into the molecular layer. This process usually runs straight; but in the cell which formed the subject of the drawing (fig. 17, *a.*) it had accidentally become bent; the other process is probably the axis cylinder process, and it is not difficult to understand that it may join the fibrillæ from the fibrous layer. Occasionally cells occur which are more or less triangular in shape, in which case the apex is turned towards the granular layer, and two processes are given off from the broad end, which probably join the fibrils. All these Purkinje cells are of larger size than the largest of those belonging to the granular layer, although smaller than the Purkinje cells of the cerebellum. Their measurements are as follows (fig. 17, *a.*):—length, 0·01 millim.; width, 0·0068 millim.; nucleus, 0·0048 millim. in diameter; and nucleolus 0·0015 millim. Another cell is rather longer and larger, being 0·016 millim. long and 0·012 millim. wide, with a nucleus 0·006 millim. in diameter; but the nucleolus is smaller, being only 0·0005 millim. in diameter. The arrangement of these cells in those ridges which are placed on the straight part (either horizontal or perpendicular) of the wings is shown in the figure (fig. 17); but where there are curves and foldings, as at the angles of junction of horizontal with perpendicular wings, other cells are added to the single row of Purkinje cells so as to form groups filling up the depressions caused by these foldings. These cells are of about the same size as the others, but differ in shape, being multipolar instead of bipolar. In other respects they belong to the same class, having a large oval or rounded nucleus and an easily distinguishable nucleolus. The specimen shown in the figure (fig. 17, *f.*) is a representative cell, having six processes, none of which can be singled out from the others as an axis cylinder process. This cell measures 0·01 millim. long by 0·008 millim. wide; the nucleus measures 0·006 millim. by 0·004 millim., and the nucleolus about 0·001 millim. in diameter.

These cells are situated among a network of fibrillæ, which fills up the space between them; but I could not trace any of the processes of the cells into this network.

In other respects the ridges at these curves and foldings have the same structure as elsewhere, the only difference being that at these places there is a larger space than ordinary between two contiguous ridges, which is consequently filled in with the cells resembling those just described.

The basal portions of the wings, which form the greater part of the whole mass to which the ridges appear merely as appendages, are entirely composed of extremely small cells which correspond in every respect with those of the granular layer of the cerebellum in structure, together with very fine nerve fibres, which are accumulated in greater numbers towards the outer side. The fibres run in a direction from the roots of the wings towards the apex, and give a longitudinally striated appearance to the part especially near the outer surface. In places where a sufficiently thin section has been obtained to see their arrangement, these cells are found to be placed in long, narrow groups, one or two deep, following the direction of the fibres, which form a network of elongated meshes. The cells are attached to them by short stalks in the same way as is represented in the figure of the torus longitudinalis (fornix) in my paper on the brain of the Teleostei.* This arrangement becomes less apparent, and the cells are more irregularly scattered towards the inner side of the wing, where the fibrillæ take a less decidedly longitudinal direction. The cells themselves are round, oval, or fusiform, sometimes with three, sometimes with two processes, one from each end, and occasionally with only one (fig. 17, *g.*); they measure about 0·003 millim. long by 0·0026 millim. broad, one larger than those figured was 0·0034 long, with a nucleus of the same size as that of the other.

MARCUSEN was of opinion that the anterior lobes of the wings had a different structure from the remainder, but such is not the case. The anatomical arrangements are precisely the same throughout the whole extent of this very extensive organ, which might be considered extremely complicated were it not that it is entirely made up of repetitions of the same structures.

The difference to be remarked between the anatomy of the ridges on the wings of the valvula cerebelli and that of the cerebellum is reduced to three points: first, the arrangement of the layers comes in a different order; secondly, the Purkinje cells are

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* Fig. 18, *e*. Phil. Trans., 1878.

directed with their long axis towards the molecular layer, instead of parallel with it; and thirdly, the granular layer itself contains cells of various sizes instead of all being uniform in dimensions.

Medulla Oblongata.

We now come to the consideration of the medulla oblongata. This contains an extraordinarily developed tuberosity, which MARCUSEN[*] took to be the cerebellum, and which he was puzzled to find had not the histological character of that organ. That it is not the cerebellum is certain; what its real homology is will appear presently. In the meantime we can turn our attention to its minute structure. A reference to the figure (fig. 1) shows that it is an immensely developed tuberosity of a rounded shape placed immediately above the medulla oblongata and behind the cerebellum. It appears as a new development superadded to the usual elements of the Teleostean brain. It may be denominated for the present "Tuberculum Impar."

A section, either longitudinal or transverse, shows that six layers from outside inwards may be distinguished by the different degrees in which they absorb the colour of the staining fluid.

The first layer consists of small cells which become deeply coloured by the staining fluid.

The second layer contains sections of obliquely directed bundles of nerve fibres.

The third is a smoothly-granular layer which does not become so highly coloured as the outside, but which shows faint indications of radial striation.

The fourth layer is a narrow stratum of variously-sized cells which become intensely coloured by the staining fluid.

The fifth layer is occupied by a complex of medullated fibres, which form a complicated felt-like mass of intricate texture.

The sixth layer exists only at the anterior end of the tuberculum impar, and consists of finely granulated matter. This is replaced at the posterior end by a circular hollow space which seems a mere gap in the tissue, and is not a ventricle properly speaking, since it is not lined by epithelium.

Further inspection shows that the first or outer layer is not continuous throughout. Behind, it is deficient on the dorsal surface, the layers of each side ending within a short distance from each other in a club-shaped termination (fig. 8), but towards the anterior end of the tuberculum impar they meet in the mid line (fig. 7), while quite at the front of that tubercle they are continuous with a part (fig. 1) which is intercalated between the cerebellum and the tubercle in question. Both in the club-shaped dorsal termination at the posterior end of this layer, and at the anterior end, where the layers of the two sides meet in the mid line, the thickness is increased by the introduction of a new material consisting of a granular basis through which numerous cells of a very large size are interspersed, and the portion which is intercalated between

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[*] *Loc. cit.*

the tubercle and the cerebellum is entirely formed by this material together with fibres running from the base to the apex.

The remainder of the first or external layer is found to be composed of a very fine network of fibrillæ, together with cells in considerable numbers, which are enclosed in a mesh-work of connective tissue. These cells are of two different sizes (fig. 18, *d, c.*), and are either round, oval, or pear-shaped. The smaller kind measured about 0·0045 millim. long by 0·004 millim. wide, with a nucleus 0·003 millim. in diameter, and a nucleolus which is a mere speck. Some of these cells are seen to give off several very slender processes which join the fine network of fibrillæ forming the basis of this layer. This class of cell is the most numerous, but others of a larger size occur more sparingly, these are often pear-shaped, with two processes, one from each side; they measure about 0·01 millim. long by 0·006 millim. wide, the nucleus being 0·004 millim. long by 0·003 millim. wide, and with a disk-like nucleolus measuring about 0·001 millim. Besides these cells and the net-work of fine fibrillæ, bundles also of these latter are seen penetrating this layer principally running in a longitudinal direction; some also are transverse and others oblique. The smaller class of cells are found throughout, but predominate in the posterior part of this layer, while the larger ones are found mostly in the anterior portion.

The granular basis of the club-shaped dorsal termination of this layer and the anterior end of the same, might be looked upon as a separate layer, but it does not extend over the whole tuberculum impar, but is confined to the dorsal and anterior portions only, where it forms a sort of crest. This granular portion of the outer layer contains, in addition to the elements already mentioned in describing that layer, viz.: small-sized cells and fibrillæ, cells of a very large kind, equal to, if not surpassing, those of the spinal cord in magnitude; these occur principally in the dorsal accumulation, where they form a special ganglion, but they are also found in the anterior process. One of the largest measures 0·052 millim. in length and 0·02 millim. in width; the nucleus, oval in shape, is 0·013 millim. long by 0·01 millim. broad (fig. 18, *f.*); it contains a distinct disk-like nucleolus, 0·003 in diameter, which has a circular clear space round it; this cell passes off gradually at one end into a broad protoplasmic process, and at the other more sharply into a narrower, probably axis-cylinder, process. This cell seems to be of an exceptional size; most of them are smaller, not much exceeding 0·03 millim. in diameter, but all show the same characteristics—broad, disk-like nucleolus, and usually two processes. Some of the cells also show distinct traces of striation of the cell-contents.

If these cells may be looked upon as motor corpuscles, they seem to form an exception to the rule that motor elements occur only on the ventral surface—a rule which is perhaps not of universal application, as there are exceptions to it, even in the spinal cord in Fishes.

The second layer requires no comment, as it consists simply of bundles of nerve-fibres, belonging, according to their position, either to the trifacial or the vagus. They are cut off obliquely as they descend from above downwards; they extend over the dorsal edge of the third layer, even where the first layer is deficient; and here they form what may be looked upon as a commissure.

The third layer (figs. 7 and 8) is the widest of all of them, and consists of a finely granular material, showing distinct traces of radial striation, which is more apparent in some places than in others. It seems to have the same sort of composition as the molecular layer of the cerebellum; the absence of palisade cells would account for the striation being very slightly indicated, and would seem to show how much of these markings might be attributed to the presence of the palisade cells, and how much to the nature of the material itself and to the direction of the nervous fibrillæ.

The fourth layer (figs. 7 and 8) consists of cells of two sizes arranged in an arch equally curved in both directions. The smaller cells form a continuous line which is always conspicuous, in whatever direction the sections are made, on account of the intense colour which they take up from the staining fluid. They are generally bipolar and are so placed that their processes run in a radial direction, in apparent continuation of the striæ of the third layer; they thus seem to be a medium of communication between the fifth layer and those layers situated more externally.

These cells have all well-marked nuclei and nucleoli, and are of three sizes; the largest, generally spherical, measure 0·0143 millim. long (fig. 18, *a.*) by 0·013 millim. wide, nucleus 0·008 millim., and nucleolus 0·002 millim. Of the middle-sized cells the length of one representative cell (fig. 18, *b.*) is 0·009 millim. long by 0·006 millim. broad, the nucleus is oval, 0·0045 millim. long by 0·003 millim. wide; there is here also a disk-like nucleolus measuring 0·0015 millim. in diameter. A process is seen to be directed from the small end of this cell towards the third layer; the comparative breadth and appearance of this prolongation characterise it as a protoplasmic process. A fine fibre coming from the other side of this cell, if it existed, is not visible. This kind of cell seems to form a transition between the larger kind resembling motor cells of the spinal cord and the smaller cells which some writers consider to be connected with sensation, but they have more affinity to the former, as is shown by their large circular nucleoli.

The smallest species of cells (fig. 18, *c.*) in this layer range from 0·006 millim. to 0·004 millim. in diameter; they are generally nearly round or oval; they have a nucleus and nucleolus; their processes are directed radially; the thicker is turned towards the third layer, and the other, which is finer, towards the fifth layer. The smaller cells are arranged in groups, irregularly scattered through the granular basis of the layer; the larger ones either occur singly or in groups, or else in some places occupy a curved row: these are placed at the edge of the fourth layer between it and the third.

The fifth layer consists almost entirely of a complicated interlacement of medullated fibres; those at the anterior end belong to the trifacial, and those at the posterior end to the vagus. Anteriorly they enclose the sixth layer, while posteriorly they surround a central vacant space. In addition to these fibres there are also a few cells scattered throughout the layer; they resemble the largest sized cells of the fourth layer, but are on an average of slightly larger dimensions; sometimes they have three processes, one of which can be traced for some distance into the complex of fibres forming the exit of the nerve. The sixth layer is formed of finely granular material resembling that of the third layer; it occupies the anterior end of the tuberculum, and forms a sort of kernel round which the other layers are placed.

We have now described those parts of the brain of *Mormyrus* which differ from, and are in fact additions to, the nervous centres of an ordinary Teleostean; the remaining portions show very slight variations from the general plan.

The Central Cavities.

In tracing the central canal of the spinal cord and its prolongations forward, it is to be remarked that the cavities show a tendency to contraction and almost obliteration. At a very short distance in front of the point where the central canal of the spinal cord merges into the fourth ventricle, this latter cavity (fig. 10) becomes plugged with a mass of finely granular material, which in the central line shows indications of its original separation into two halves; this contains part of the ganglion of the vagus. Farther forward this ventricle exhibits an extremely deep narrow fissure (fig. 9), which owes its depth and narrowness to the great development of the vagal tuberosities. At a point beneath the posterior end of the tuberculum impar the ventricle (fig. 8) becomes again plugged by a development of what appears to be connective tissue, corresponding to that material which surrounds the central canal of the spinal cord. There are, however, two spaces left one above the other, the lower one lined by an endothelial layer of cells. They both lie beneath the space which occupies the centre of this part of the tuberculum impar. Towards the anterior end of the tuberculum impar the fourth ventricle (fig. 7) becomes reduced to an extremely small canal, triangular in outline, with the apex directed downward; in some specimens the lumen is scarcely perceptible. This corresponds to the narrower portion of the fourth ventricle in *M. cephalus*; after this it again enlarges (fig. 6) beneath the cerebellum, and shows a triangular lumen which seems to correspond to the anterior end of the fourth ventricle in other Teleostei; beyond this it again becomes contracted as the aqueduct of Sylvius (fig. 4) into a very small perpendicular slit, which soon extends through the hypoarium nearly to the inferior surface of the brain (fig. 3). This slit is occupied in its centre by a small coil of an epithelial tubular structure, which appears to correspond to the saccus vasculosus of the ordinary Teleostei. After this the aqueduct opens into what would be the ventricle of the optic lobe, were it not that it is quite open to the external surface, and by the peculiar position of the tecta, has come to be above instead of below them. The growth of the wings in fact has quite forced open this ventricle and separated the valvula cerebelli and themselves from the tori semicirculares and the tecta, so that when a transverse section is made there is nothing to prevent their falling apart.

Spinal Cord.

A section of the spinal cord shows the same features as in *M. cephalus*. The grey matter here forms a figure like a St. Andrew's cross (figs. 13 and 14). The dorsal horns are not very much developed, while the ventral horns have about the usual appearance. These latter show the ventral ganglion with from six to ten cells in one section, together with the central ganglion with seldom more than two, but these of a larger size.

Three pairs of columns may be distinguished—the dorsal columns, situated between the two dorsal horns of grey matter; the lateral columns placed between the dorsal and ventral horns: these are characterised by the nerve fibres having a tendency to fall into separate bundles, thus differing from the corresponding columns in *M. cephalus*, where they form a more undivided mass; and finally the ventral columns, situated in the space between the two ventral horns; these have the same sort of large fibres as in ordinary Teleostei, and contain also the characteristic Mauthner's fibres. RANVIER[*] considers that there is no physiological significance in the various sizes of the nerve fibres, but still it is a curious circumstance that the ventral columns should always contain fibres of so much larger calibre than the others.

The dorsal columns are traceable forward through the tuberosities of the vagus and the outer and inferior angle of the tuberculum impar into the posterior and outer part of the cerebellum; they, together with the tuberosity of the vagus, probably homologise with the restiform columns or bodies and contain finer fibres than either the lateral or ventral column.

The lateral columns are also composed of fine fibres, finer than those of the ventral columns, except at the part where they border on the dorsal horns. Here some of the fibres are of a larger size. When they arrive at the medulla oblongata, these lateral columns are arranged as just mentioned in separate bundles slightly resembling the formatio reticularis of that part in the Elasmobranchii. They can be traced forward as far as the large transverse commissure of the tuberculum impar, which some of them appear to join, others go forward as far as the inner side of the torus semicircularis, being joined by some fibres from the central portion of the above-mentioned commissure, but many of them are dispersed in the grey matter of the medulla.

The ventral longitudinal columns are divided into two bundles by the ventral transverse commissure, as in *M. cephalus*. In the medulla oblongata further forward, they are separated by the immensely developed transverse decussating commissure (fig. 7) of the tuberculum impar. The Mauthner's fibres accompany the upper or central longitudinal column and decussate immediately under the angle formed by the posterior root of the trifacial as it turns outward towards its place of exit from the medulla

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[*] Leçons sur l'Histologie du Systeme Nerveux. Paris, vol. i, p. 100.

oblongata. At the point of their crossing, the Mauthner fibres present the appearance as if the axis cylinder had broken up into fibrillæ. The medullary sheaths are entire, and in them are seen several blue dots* like sections of fibrillæ; in the next section forward both medullary sheaths and blue points have completely disappeared, while in the few sections behind this part, the axis cylinder is seen gradually breaking up into these fibrillæ, the lower one becoming decomposed at a point behind the upper. This confirms what I have long suspected: that the axis cylinders of the Mauthner's fibres are really composed of fibrillæ, and as the Mauthner are merely gigantic nerve fibres it might confirm the views of MAX SCHULTZE,† who was of the opinion that ordinary nerve axis cylinders are made up of fibrillæ, united together into a single fibre.

The central longitudinal columns disappear at a very short distance in front of the decussation above mentioned, and the ventral longitudinal columns take their place beneath the floor of the fourth ventricle, at the anterior end of the commissure of the tuberculum impar. At about the position of the ganglion of the oculomotorius these disappear as compact bundles, but can be traced forward as isolated fibres and small fasciculi as far as the region of the posterior commissure to which they appear to contribute fibres.

Transverse Commissures of the Central Nervous System.

In the spinal cord two transverse commissures are present, corresponding to those of *M. cephalus;* they connect the dorsal and ventral horns of grey matter of each side; the former is placed close to the dorsal edge of the central canal and the latter occupies its usual place between the ventral and central columns (figs. 13, 14, and 15).

More anteriorly an immense decussating commissure (fig. 7) is met with which is connected with the tuberculum impar, and is developed in like proportions. This is evidently the continuation forward of the ventral commissure of the cord; it occupies the same level and separates the two ventral columns in the same manner. It exists only at the anterior part of the tuberculum where it forms a communication between one side of that lobe and the opposite side of the medulla oblongata. Its bundles are seen to arise from the complex of fibres of origin of the trifacial in the central portion of the tuberculum, then to wind round the central nucleus of finely granular matter or sixth layer, to pass beneath what there is of the fourth ventricle and finally to pass beneath that ventricle to the other side, where they are lost quite close to the lateral and inferior edge of the medulla oblongata. This commissure is traceable in the myelencephalon as far forward as the posterior side of the cerebellum.

The commissura ansulata (fig. 6) occurs further back than in *Mugil*, being placed beneath the cerebellum instead of beneath the middle of the valvula cerebelli. That it is the commissure which is displaced backward, rather than that the cerebellum is displaced forward, is I think indicated by the fact that the root of the oculomotorius

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* The staining fluid was rosaniline.

† STRICKER's Handbook, Sydenham Society.

is directed obliquely backward from its ganglion to its exit, whereas in ordinary Teleostei it passes directly downward; I speak here of course in reference to the position which these animals habitually assume. The commissura ansulata consists of the same two divisions as in *M. cephalus*, but the parts which it connects are slightly different. It derives its lateral fibres from the base of the wings to which it acts as a commissure; the upper part of its decussating bundles, instead of going to the torus semicircularis as in *M. cephalus*, goes into and is lost in the granular layer of the anterior end of the cerebellum. Upon further consideration I find that the homology which I gave for this commissure in the brain of *M. cephalus* was most probably wrong, and the fact that some of its fibres in this animal are derived from the cerebellum seems to confirm its determination, as the pons varolii, which was also adopted by Fritsch.*

The posterior commissure (fig. 3) occupies its usual position behind the third ventricle and the infundibulum, but here it is more isolated, for the posterior wall of the infundibulum appears to be wanting, so that the fissure in the floor of the aqueduct of Sylvius communicates with it without any partition; it forms a communication between regions corresponding to those united by this commissure in *M. cephalus*.

The anterior end of the dorsal commissure of the tecta (figs. 2 and 3) is all that remains of that commissure which in Teleostei generally extends for the whole length of those bodies. This is placed immediately above the posterior commissure and even extends in front of it; this relation contributes material for forming an opinion as to the homology of the tecta, which will be touched upon presently.

The deep transverse commissure, and with it the nucleus rotundus, have completely disappeared in this family, and have left not a trace behind.

Deep Origins of the Cranial Nerves.

In the *Mormyridæ* all the nerves can be easily found except the trochleares and the abducentes; these, probably from the great degradation of the muscles of the eyeball, have become reduced to such extreme minuteness as readily to escape detection.

The optic nerves, although of small size, present, at their entrance into the optic tract, a true chiasma nervorum opticorum, formed by bundles which decussate, and at the same time form a felt-like structure. The optic tract, at its entrance, forms a thick layer of fibres surrounding the trigonum fissum and the hypoarium on three sides (figs. 2 and 3). I am sorry to have to differ from Bellonci † as to the origin of the optic nerve. The family of the *Mormyridæ* present rather good subjects for investigating the derivations of this nerve, as the tecta have been pressed forward and downward, so as to be placed immediately above the exit of the nerve in question, and consequently the optic tract passes directly upward into the tecta. I find that I cannot subscribe to the opinion that these latter bodies form the only origin of the optic nerves; on the contrary, some fibres are distinctly seen to emerge from the inferior part of the trigonum fissum and from the region containing the small cells which line the infun-

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* *Loc. cit.*, p. 74, fig. 44, *p.v.*

† *Loc. cit.*

dibulum, and to join the optic tract. As to these fibres being derived from the posterior commissure, this is very unlikely, since this commissure is situated at a considerable distance away from this particular part.* The optic tracts are seen going upward and entering the roots of the tecta in two separate portions; the inferior portion curves over the angle formed by their junction with the hypoarium and streams into the outer layer of those bodies, while the upper portion enters them by numerous small radiating bundles, which cross the bundles of the commissure of the tectum at a large angle and are lost in the internal layer; that is to say, the layer which corresponds to the third and fourth layers in the tectum of *M. cephalus.* The inferior portion (fig. 2) of the optic tract presents a peculiar arrangement of fibres which form a mass of decussations projecting above the dorsal edge of the optic tract in the central line, from which two or three bundles emerge, going obliquely upward and outward, crossing the crus cerebri to join the main body of the optic tract at the point where a complex of fibres is about to be formed, previous to their entrance into the tecta as above described. I could not make out with certainty that any of the fibres were derived from the tori semicirculares. These bodies are displaced further back in comparison to the tecta than in *M. cephalus,* or rather have retained to a greater extent their proper position, the latter having, as it were, moved forward, so that no part of the optic tract passes beneath their base. But there is a bundle of fibres (fig. 3) which passes downward along their inner edge, and appears to join the complex of fibres of the optic tract.

The oculomotorius (figs. 4 and 5) is derived from a ganglion situated on the floor of the ventricle of the optic lobe corresponding to its origin in *M. cephalus.* The only peculiarity about this nerve is that the fibres run obliquely backward towards their point of exit near the commissura ansulata instead of perpendicularly downward.

The trifacial (fig. 7) presents the same number and derivation of its roots as in *M. cephalus,* and in addition derives a large number of its fibres from the anterior end of the tuberculum impar.

The acusticus (fig. 7) also shows a complete concordance with the arrangement in *M. cephalus.* It passes forward in close contact with the central longitudinal column† and turns outward with the upper column of the trifacial, both forming an angle. FRITSCH has suggested that the latter presents the marks of the facialis root in the brain of Mammalia, to which I would add that the acusticus root passing in close contact with it tends to increase the resemblance.

The vagus (figs. 8, 9, 10) also presents roots of origin corresponding to those in *M. cephalus,* with the exception, perhaps, of the one from the cerebellum, which does not seem to be present. Some addition to its fibres is derived from the posterior part of

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* There does not appear to be anything extraordinary about this fact, since the trigonum fissum homologises with the tuber cinereum, and this part in the human subject sends fibres into the optic tract. ('Human Anatomy,' QUAIN, latest edition.)

† In the figure 7 this does not show well, the parts being too small, but in the original proparation three distinct bundles of fibres are apparent here.

the tuberculum impar. The only peculiarity about it is that the ganglion of origin of the posterior root is situated higher than the corresponding part in *M. cephalus*, being placed above, instead of below, the floor of the fourth ventricle.

The spinal nerves (figs. 13 and 14) take their origin very much in the same way as in the case of *M. cephalus.* The dorsal roots, after running a short distance dorsad along the edge of the section, pass downward and inward to enter the apex of the dorsal horn of grey matter by two bundles, and there they are soon quite lost to view. The ventral roots are derived from two sources—one from the transverse commissure, which separates the two ventral longitudinal bundles; the other comes from the ventral horn of grey matter of the same side. These two bundles join at the ventral edge of the cord and emerge together as a single root. The fasciculus from the transverse commissure is probably derived from the ventral horn of the opposite side.

Homologies of the Brain in the Mormyridæ.

We have now to consider what relation the brain of *Mormyrus* bears to that of the ordinary Teleostei. What are the homologies of those highly developed wings which MARCUSEN, in despair of finding their signification, termed the "peculiar organ" (Eigenthümliches Organ)? What the homology of the tuberculum impar? If we examine a longitudinal section (fig. 1) we see at once that they are new formations. We have only to remove them and to allow the displaced organs to resume their proper position, and we have a brain like that of any other Teleostean. There is a slight difference in the direction of the cerebellum, but that occurs in other families, such as the Siluridæ. In order to solve this problem we have examined the characteristics of this "peculiar organ," and we have found, in the first place, that it contains all the elements of the cerebellum arranged in a slightly different manner; secondly, we have found that it is not the cerebellum, since that organ exists, presenting the ordinary form and arrangement of tissue; thirdly, that its roots are derived from that part of the brain which lies immediately in front of that organ; the tecta lobi optici occupy this position in ordinary Teleostei, but here they are pressed towards the base of the brain without being otherwise deformed. The only organ then occupying this position that remains for comparison is the valvula cerebelli. This part is highly developed in the Ballan Wrasse (*Labrus maculatus*), and if we examine a transverse section through it (fig. 15), we find in the centre a double layer of the molecular layer indicating the fold of cerebellum by which it is formed; on each side we see that the organ stretches out into a wing, which is composed of a folded stratum of the molecular layer, below which the remainder of its bulk is made up of the granular layer. Even here it looks as if an attempt had been made to pack a large amount of tissue into a small space.

We now turn to a section (fig. 3) through the brain of a *Mormyrus*, taken perhaps a little farther in advance. We find here the same arrangement; in the centre there is a double molecular layer below and a single layer above; this is accounted for by the greater number of foldings undergone by the molecular layer, and would explain itself by a reference to the longitudinal section (fig. 1), and obviously this is a difference of degree and not of kind; next we have on each side a wing which goes out into a much more complicated arrangement than in *L. maculatus.*

We are now in a position to explain the morphology of the "peculiar organ" in *Mormyrus.* It is easy enough to imagine that the ancestors of the *Mormyridæ* had a brain resembling that of the Ballan Wrasse, possessing a valvula cerebelli of no larger size; but there is nothing to prevent this valvula being endowed with a capacity for indefinite growth. It may be seen that the external extremity of the molecular layer on the right side in the figure (fig. 15) has begun to turn down; it might go on growing, and might form another fold like the one already present, and after that another, and so on until it breaks through the tecta lobi optici, and this going on to an indefinite extent in every direction would eventually produce such a brain as is possessed by the *Mormyridæ* of the present day.

To solve the problem of the tuberculum impar we must turn to an entirely different genus of Fishes; the well-known Gold Carp, such favorites with some people as ornaments for ponds and fountains, present the key to this difficulty.

These Fishes have a tubercle placed in the fourth ventricle behind the cerebellum; in the species that I investigated there were two tubercles in this position, one placed in front of the other. On examining the figure (fig. 16), which is a section across the medulla oblongata of a Gold Fish (*Cyprinus* sp. ?), it is found that there is a tubercle in the centre, and on each side a sharp ridge; the composition of these beginning on the outside of the ridges, which are the vagal tuberosities, is as follows:—

First, there is a layer of small cells.
Secondly, a layer of finely granular material.
Thirdly, a layer of nerve fibres.
Fourthly, a granular layer with large cells dispersed in it.

All these are in the vagal tuberosities; then comes the tuberculum, which occupies the space between them; it consists of—

Fifthly, a layer of small cells; and
Sixthly, a layer of granular material, which occupies the greater part of the tubercle in the centre.

At the junction of this tubercle with the vagal tuberosities there are sections of a few bundles of nerve fibres.

We have now to consider how the tuberculum impar of the *Mormyrus* may be explained by a consideration of these facts.

The organ in question is, as we have seen, placed immediately behind the cerebellum, and at first sight this seems to be the only circumstance that it has in common with that of the Carp, since there are in the *Mormyrus* no sharply projecting ridges, and the succession of tissues does not apparently correspond; but a little consideration will show that with a slight alteration all these layers may be made to do so. In the Carp we find that the fourth layer is composed of a granular material in which large cells are dispersed; we have only to imagine these cells to be collected at the summit instead of at the sides, when they would homologise with the club-shaped dorsal end of the outer layer and the anterior side of the tuberculum impar of the *Mormyrus*. After this our course is plain, and making allowance for its enormous development, we have layers in tolerably exact correspondence with each other. The outer layer in the vagal tuberosities of *Cyprinus* is evidently the homologue of the outer layer of the *Mormyrus*, then comes the finely granular layer; the third and fourth layers in *Cyprinus* being by some means displaced, we immediately come upon the outer granular layer of the tubercle, which in the places where it is free, extends round the whole circumference: this would correspond to the fourth layer of the tuberculum impar. It may be supposed that the inferior fibres have become developed at the expense of the upper ones, and so would come to be derived from the inner side of this granular layer, and thus correspond to the fifth layer in *Mormyrus*. We have now only the finely granular interior of the tubercle remaining to be accounted for: this would answer to the sixth layer, which is the finely granular material forming the central portion of the anterior part of the tuberculum impar, and which presents the same relation to the remains of the fourth ventricle, and to the central longitudinal columns of the medulla oblongata in both Fishes. From these considerations, the conclusion is obvious, that the tuberculum impar of the *Mormyridæ* corresponds to the tuberculum impar situated in the fourth ventricle of *Cyprinus* together with the tuberosities of the vagus; the three bodies which are nearly separate in *Cyprinus* having become indissolubly united in *Mormyrus*.

I have thus, as far as I am able, explained the special formations in the brain of the *Mormyrus;* but while it is comparatively easy to account morphologically for these peculiarities, it becomes very much more difficult, if not impossible, with our present amount of knowledge, to account for them physiologically. What can possibly be the function of this enormously developed organ and of the innumerable repetitions of the same structure? As far as I could learn there was nothing extraordinary about the habits of these Fishes that could throw any light on it. It is true that their eyes are very small, while their organs of hearing are comparatively highly developed, as was shown by FISCHER,[*] but there does not seem to be any connexion between these two series of facts. Other Fishes have complicated organs of hearing, and, as far as I know, neither these nor the blind or semi-blind Fishes inhabiting caves have any such peculiarity in the structure of their brain. But the most curious fact about these Fishes is, that notwithstanding the enormous development of the brain, the nerves emerging therefrom have nothing like a corresponding amount of development; most of them are quite small, some escape detection, and the only nerves of any size, the trifacial and the vagus, do not present anything out of the common, either in extent or dis-

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[*] Ueber d. Gehörorgan der Fischgattung Mormyrus. Freiburg, 1854.

tribution. These Fishes certainly have organs placed along the side near the tail, which by some writers have been looked upon as electric lobes, and which, although presenting some of the structure of those bodies, yet show that they are pseudo-electric organs, since they give no shocks when the Fish is handled. This fact, however, seems to have no connexion with the development of the brain, for the lateral line and dorsal nerves which supply those structures are by no means of enormous size. This subject must therefore remain in obscurity until some competent person may be in a position to throw light on it by means of physiological experiments, to which the formation of the brain lends itself with great facility, because it is placed close beneath the skull, which is very thin, and the wings form plates of nervous matter easily removable without injury to other parts of the organ.

Conclusion.

Although the brain of the *Mormyridæ,* being squeezed almost out of all recognition, does not throw much light on the problem of the homologies of the various parts of the central nervous system in Fishes, yet I must take this opportunity of making some remarks on the interpretations put forward by FRITSCH[*] in his great work, which I had not time to study before my paper on the brain of the Teleostei was finished. As far as I can make out from his rather involved style, he considers that the tecta lobi optici[†] taken together represent the persistent cortex of the primitive anterior vesicle of the embryo ;[‡] nevertheless, he recognises in the commissure of the tecta the homologue of the corpus callosum[§] and in the torus longitudinalis that of the fornix of the Mammalian brain. Both these structures are developed from what FRITSCH terms the secondary vesicles,[||] from which the cerebral hemispheres arise. How they get transferred to the primary vesicle this author does not explain. Further, he homologises the tori semicirculares with the thalami optici, and certain undefined regions lying to the inside of them with the corpora quadrigemina.

It seems to me that there are two fixed points in the brain of Fishes from which it would be possible to deduce the homologies of all the remainder. The first of these is the pineal gland or epiphysis, together with the infundibulum, which opens beneath it, and the pituitary body or hypophysis which is placed at the inferior extremity of the latter.

EHLERS[¶] has shown that the epiphysis is a structure universally present in the Vertebrata, and that it invariably occupies a corresponding position throughout. The probability is that the infundibulum is a channel older even than the Vertebrate stock itself ; and in reference to this subject perhaps I might be permitted to express my gratification

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[*] *Loc. cit.*

[†] *Loc. cit.*, pp. 22 and 54.

[‡] Primäres Vorderhirn.

[§] *Loc. cit.*, p. 51, fig. 42.

[||] "Secundäres Vorderhirn."

[¶] Die Epiphyse am Gehirn d. Plagiostomen. Zeitsch. f. wiss. Zoologie, Bd. 30 (supplement), 1878.

that the suggestion thrown out by me in 1878, that the œsophagus of the primitive ancestors of the Vertebrata passed through the infundibulum, should also have occurred to Professor OWEN, who, in a paper recently read before the Linnean Society, entitled "Homology of the Conario-Hypophyseal Tract, or the so-called Pineal and Pituitary Glands," showed that in the embryo there are indications of there having been a communication between the primitive intestine (before the permanent mouth appeared) and the canal going through the infundibulum and the third ventricle to the present dorsal surface of the brain, so that the lateral walls of the third ventricle and the crura cerebri in Sharks and (consequently in Teleostei) homologise with the commissures between the supra and infra-œsophageal ganglia of insects.

The second fixed point would be the ganglion of origin and the place of exit of the oculomotorius. If the foramen of exit of a cranial nerve be sufficient to fix the homology of the bone of the skull through which it passes, then it must be conceded that the ganglion of origin of a nerve ought to fix the homologies of the parts of the brain lying near it.

We have then these two fixed points, and applying them to the central nervous system, we find that the first indicates the third ventricle or thalamencephalon, and consequently the posterior commissure, which two parts have always a fixed relation to each other. The determination of the posterior commissure incidentally involves that of the corpus quadrigeminum; for FOREL[*] shows that this commissure is in juxtaposition to the transverse commissure of the anterior corpus quadrigeminum; so in Fishes the posterior commissure is in close relation to another commissure which FRITSCH marks *c. anterior*, but which is nothing more than the continuation forward of the transverse commissure of the tecta lobi optici. This line of argument is supported by the consideration of the position of the ganglion of the oculomotorius, which in the Mammalia is placed immediately beneath the aqueduct of Sylvius and the anterior corpus quadrigeminum. In Fishes the corresponding ganglion is placed beneath the ventricle of the optic lobe and the tectum lobi optici.

In one of his figures[†] FRITSCH has made it appear as if this ganglion were at the side of the aqueduct of Sylvius instead of beneath it, as it ought to be.

This converging line of argument is supported by another mode of reasoning based upon structure, which although taking a subordinate position is yet of use, as supporting considerations derived from other sources. Thus MEYNERT[‡] has shown that in the human subject the anterior corpus quadrigeminum contains radial fibres on which fusiform cells are developed. The tecta lobi optici in Fishes have radial fibres with fusiform cells developed thereon.

The united force of these arguments compels me to homologise the tecta lobi optici

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[*] Beiträge z. Kentniss des Thalamus Opticus, &c. Sitzber. d. k. Akad. der Wiss., Wien, Bd. lxiii. Math. Nat. Classe, fig. 4; also MEYNERT, fig. 249, *l.c.*

[†] *Loc. cit.*, fig. 32.

[‡] *Loc. cit.*

of Fishes not only with one of the pairs of the corpora quadrigemina but especially with the anterior pair, as STIEDA long ago pointed out. The ventricle of the optic lobe would be simply the remains of the ventricle which is always found in the mesencephalon of the embryo in Vertebrata. It does not seem to me that FRITSCH's homologies have any force as against these arguments.

The torus semicircularis could not homologise with the thalami optici, but would seem more likely to correspond to the corpus geniculatum externum,* a ganglion in the course of the optic tract, which according to FOREL† becomes larger in the lower Mammalia.

The region round about the posterior commissure in Fishes homologises better with the thalami optici; the part which extends behind that commissure in Mammalia becomes much less developed, as FOREL‡ has also pointed out, in descending the scale.

The torus longitudinalis, if this line of argument carries any force, could not homologise with the fornix, as the former is a nervous centre, closely resembling the granular layer of the cerebellum in structure, and the latter is simply a set of longitudinal commissural fibres. With these principal points, the minor interpretations of FRITSCH of course fall to the ground. Finally it seems to me that as Fishes retain so many marks of being in an embryological position, they would possess a brain in accordance with these conditions, and to find all the parts of a Mammalian cerebrum even in a rudimentary state, much more in the comparatively developed form insisted upon by FRITSCH, would be, so to speak, an anachronism. The brain would not be in accordance with the rest of the organisation.

EXPLANATION OF PLATES.

The following letters have the same signification throughout :—

a.	Arteries.
aq.Sy.	Aqueduct of Sylvius.
a.v.c.	Ala of the valvula cerebelli.
a.w.	Anterior wings of the valvula cerebelli.
c.a.	Commissura ansulata.
c.c.	Crura cerebri.
c.ca.	Central canal of the spinal cord.

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* I have only just received the interesting article by P. MAYSER on the brain of Teleostei (Zeitsch. f. Wiss. Zool., Bd., xxxvi., 1881.) I am glad to find that in most cases he is in accord with me. Curiously enough, he compares the tori semicirculares with the corpus geniculatum internum.

† *Loc. cit.*, pp. 29 and 30.

‡ *Loc. cit.*, p. 30.

cbl.	Cerebellum.
c.cbl.	Crura cerebelli.
ce.	Cerebral lobes.
c.g.	Central ganglion of the spinal cord.
ch.o.	Chiasma of the optic nerves.
c.l.c.	Central longitudinal column.
com.w.	Commissure of the wings of the valvula cerebelli.
c.w.	Central wing of the valvula cerebelli.
d.c.	Dorsal columns of the spinal cord.
d.h.	Dorsal horn of grey matter in the spinal cord.
d.r.	Dorsal roots of the spinal nerve.
d.t.i.	Decussating fibres from the tuberculum impar.
d.w.	Dorsal wing of the valvula cerebelli.
f.l.	Fibrous layer of the cerebellum and of the valvula of the same.
f.r.	Formatio reticularis of the medulla oblongata.
g.l.	Granular layer of the cerebellum and of the valvula of the same.
gn.mo.	Ganglion of the oculomotorius.
gn.tf.	Ganglion of the trifacial.
gn.v.	Ganglion of the vagus.
gn.vh.	Ganglion of the ventral horn of the spinal cord.
hy.	Hypoarium.
i.l.	Intermediate layer of the cerebellum and of the valvula cerebelli.
in.	Infundibulum.
i.n.t.i.	Internal nucleus of the tuberculum impar.
l.c.	Lateral columns of the spinal cord.
l.c. 5.	Lower column of the trifacial.
l.w.	Lateral wing of the valvula cerebelli.
m.	Medulla oblongata.
m.f.	Mauthner's fibres.
m.l.	Molecular layer of cerebellum and valvula cerebelli.
n. 3.	Oculomotorius.
n. 5.	Trifacial.
n. 7.	Facial.
n. 8.	Acusticus.
n. 9.	Glossopharyngeal.
n. 10.	Vagus.
op.tr.	Optic tract.
p.c.	Posterior commissure.
p.m.	Pia mater.
p.w.	Posterior wing of the valvula cerebelli.
r.	Raphé.

r.c.	Restiform column.
s.g.c.	Substantia gelatinosa centralis.
s.r.	Fourth ventricle or sinus rhomboidalis.
s.v.	Saccus vasculosus.
t.	Tectum lobi optici.
t.c.	Transverse commissure of the cerebrum.
t.c.t.	Transverse commissure of the tectum.
t.c.v.	Transverse commissure at the root of the vagus.
t.f.	Trigonum fissum.
t.i.	Tuberculum impar.
t.l.	Torus longitudinalis.
t.s.	Torus semicircularis.
t.v.	Tuberosity of the vagus.
u.c. 5.	Upper column of the trifacial.
v.c.	Valvula cerebelli.
v.g.	These appear to correspond to the vincula gelatinosa centralia of FRITSCH, *l.c.*, p. 44.
v.h.	Ventricle of the hypoarium.
v.h.g.	Ventral horn of grey matter in the spinal cord.
v.l.c.	Ventral longitudinal column.
v.op.l.	Ventricle of the optic lobe.
v.r.	Ventral root of the spinal nerves.
v.t.c.	Ventral transverse commissure in the medulla oblongata and the spinal cord.
v.th.	Third ventricle or thalamencephalon.

PLATE 59.

Fig. 1. Longitudinal and vertical (sagittal of authors) section through the brain of a young *Hyperopisus dorsalis,* close to the central line. × $8\frac{1}{2}$ diameters about. The dorsal part of the lateral wing is left in outline. The anterior wing has become slightly bent upward; it ought to be parallel to the long axis of the brain.

** Show the line of section taken in fig. 3.

Fig. 2. Transverse section through the anterior part of the optic tract and the tecta lobi optici of another specimen of the same species. The lateral and dorsal wings of the valvula cerebelli have been left in outline. × $15\frac{1}{2}$ diameters nearly.

Fig. 3. Transverse section through the centre of the tectum lobi optici at a point slightly behind the last, taken from the same specimen. × $15\frac{1}{2}$ diameters nearly.

PLATE 60.

Fig. 4. Transverse section through parts slightly behind the last. Taken from an older specimen of *H. dorsalis*. × $15\frac{1}{2}$ diameters nearly.

Fig. 5. Transverse section through the aqueduct of Sylvius and the trigonum fissum of a younger specimen. × $8\frac{1}{2}$ diameters nearly.

Fig. 6. Transverse section taken farther behind, and falling through the main part of the commissura ansulata and the posterior part of the cerebellum. Taken from the same specimen as fig. 4. × $15\frac{1}{2}$ diameters nearly.

PLATE 61.

Fig. 7. Transverse section through the anterior part of the tuberculum impar. Taken from the same specimen as the last. × $15\frac{1}{2}$ diameters nearly.

Fig. 8. Transverse section through the posterior part of the tuberculum impar. Taken from the same specimen. × $15\frac{1}{2}$ diameters nearly.

PLATE 62.

Fig. 9. Transverse section through the posterior part of the fourth ventricle. The extreme posterior end of the tuberculum impar, which projects over it, is not represented. This is taken from another specimen of the same species. × 37 diameters.

Fig. 10. Transverse section through the posterior end of the fourth ventricle. Taken from the same specimen as fig. 8. × 37 diameters.

Fig. 11. Transverse section taken close to the point of transition of the spinal cord into the medulla oblongata. Taken from the same specimen as the last. × 37 diameters.

Fig. 12. Transverse section taken from the same specimen at a point a short distance behind the last. × 37 diameters.

Fig. 13. Transverse section through the spinal cord of a younger specimen of the same species, to show the origin of the ventral roots of the spinal nerves × 37 diameters.

PLATE 63.

Fig. 14. Transverse section through the spinal cord of an older specimen of the same species, to show the origin of the dorsal roots of the spinal nerves. × 37 diameters.

Fig. 15. Transverse section through the valvula cerebelli of the Ballan Wrasse (*Labrus maculatus*). × 23 diameters.

The tecta lobi optici and the upper part of the tori semicirculares are represented in outline.

Fig. 16. Transverse section through the posterior part of the fourth ventricle of a species of Gold Fish (*Cyprinus* sp. ?), to show the origin of the vagus and the tuberculum impar. × 23 diameters.

Fig. 17. A section through one of the ridges of the wing of the valvula cerebelli from a *Mormyrus oxyrhynchus*. × 450 diameters. The molecular layer of the twin ridge is seen in close contact, the only separation being by a process from the pia mater. The fibrous layer on the side next the Purkinje cells is omitted, having dropped out of the section.

p.c. Purkinje cells.

Fig. 17. *a.* A Purkinje cell from the single layer. × 1140 diameters; the axis cylinder process had been broken off, and is therefore not represented.

e.f. Other Purkinje cells. × 650 diameters.

e. Taken from a part of the intermediate layer where only one row of cells is to be found. Both the axis cylinder and the protoplasmic processes are to be seen.

f. Taken from a part where the cells occur in groups.

b.c.d. Cells taken from the intermediate layer. × 1140 diameters.

g. Cells from the granular layer forming the basal portions of the wings. × 1140 diameters.

h. Section through the junction of the molecular with the intermediate layer, showing the network of fibres. × 430 diameters. *m.* indicates the side towards the molecular layer.

Fig. 18. Transverse section through the posterior end of the tuberculum impar. × 50 diameters.

a. One of larger cells of the fourth layer. × 450 diameters.

b. Another larger cell of the fourth layer. × 1140 diameters.

c. A small cell of the same layer. × 1140 diameters.

d. Two cells of the first layer taken from the posterior end of the tuberculum impar. × 1140 diameters.

e. Cells from the anterior part of the same layer. × 1140 diameters.

f. Largest sized cells from the club-shaped process. × 450 diameters.

PLATE 62.

Fig. 19. Ventral surface of the brain of *H. dorsalis* twice the natural size.

Fig. 20. Side view of the brain of *Mormyops* (sp. ?) part of the skull being removed.

XXII. *On the Influence of the Galvanic Current on the Excitability of the Motor Nerves of Man.*[*]

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[PLATES 64, 65.]

INTRODUCTION.

THE verification for undissected nerve of the established laws of electrotonus for dissected nerve has been the object of several previous researches. Of some the results have been negative, of others contradictory, of but few fairly confirmatory. PFLÜGER'S chief conclusions have received an occasional illustration from experiments made on Man, but these can hardly be said to carry support, still less independent proof.

The first definite experiments on the subject were made by FICK (Medic. Physik., 1866, p. 377). He tested his own ulnar nerve but failed to obtain any evidence of electrotonic change of excitability. EULENBURG (Arch. f. Klin. Med., 1867, p. 117) and ERB (ibid., pp. 238, 513) simultaneously came to opposite conclusions. According to EULENBURG, evidence of descending anelectrotonus and katelectrotonus was uniformly obtained by testing the nerve below the ascending and descending galvanic currents respectively. ERB, on the contrary, found the excitability diminished below the kathode, increased below the anode—an apparently anomalous result which HELMHOLTZ (Nat. Med. Verein., Heidelberg, 1867) attributed to rapid current-diffusion, and consequent establishment in the vicinity of the electrode, of opposite "odic" points in the nerve, wherever the current enters or leaves it. And, in effect, acting on this suggestion by testing the nerve with a small electrode, introduced through a large perforated electrode (polarising), ERB found the excitability diminished during anodic, increased during kathodic influence. SAMT ('Der Electrotonus am Menschen,' Berlin, 1868), who also sought for descending electrotonus, came to similar conclusions as EULENBURG, though his experiments were far from yielding uniform results; he attributes the inconstancy of the results to the "inconstancy of the

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[*] Towards the expenses of this research a grant was made by the British Medical Association on the recommendation of the Scientific Grants Committee of the Association.

nervous matter," a view in which he was followed by CYON ('Principes d'Electrothérapie,' Paris, 1872, p. 123), whose results were equally inconstant. BRÜCKNER (Deutsches Klinik, 1868, No. 43) and RUNGE (Arch. f. Klin. Med., 1870, p. 536) had recourse to the method of uniting the testing induction with the polarising galvanic currents in one circuit. They both obtained, in the main, similar results,—when the two currents flowed together, there was an increased effect both at the anode and at the kathode; when the two currents were opposed, the effect was diminished at both poles of the galvanic current, especially at the anode. In their interpretation, however, these two observers took opposite views. RUNGE explained everything by an assumed "summation" of electromotive forces; BRÜCKNER attributed the effects to physiological causes, though his arguments were not such as to carry conviction. None of these experimenters, with the exception of CYON, used the graphic method; they estimated the changes by watching the contractions.

METHODS AND RATIONALE.

We employed three modes of excitation: (1.) Induction currents; (2.) Makes and breaks of a continuous current; (3.) Mechanical stimulation. The electrodes* were applied as follows:—One electrode of large area—the "indifferent" electrode—was applied to any convenient part of the body remote from the part explored; the other electrode of small area—the "exploring" or "testing" electrode—was applied to selected points along the course of favourably situated nerves, and the effects at this movable electrode were alone considered.

We shall describe these effects under the polar terms "*anodic*" and "*kathodic*," without reference to any assumed direction of current in the nerve; for not only is the localisation of an efficient current in a given direction problematical for the imbedded nerve, but a simple experiment suffices to show that the position of the indifferent, whether central or peripheral to the exploring electrode, does not in any way influence the results obtained at either pole.

The condition which we thought necessary to fulfil throughout our experiments was the coextension of the points of excitation and of polarisation; our reason being that owing to current diffusion and consequent establishment of opposite electrodes in the nerve in the immediate neighbourhood of the electrode, the electrotonic state is variable in kind, degree, and distribution. This condition is fulfilled by conjoining the testing and polarising currents in one circuit, and by applying one electrode only to the nerve.† The other method, by which the two circuits are separate, gave in our hands discordant results, similar to those of previous observers, who found that the

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* We did not use unpolarisable electrodes in these experiments, but plates of metal covered with chamois leather.

† This method was first used by ECKHARDT (Beiträge zur Anat. und Phys., Giessen, 1858-4, p. 28), afterwards by PFLÜGER ('Elektrotonus,' p. 394) and by von BEZOLD ('Erregung der Nerven und Muskeln,' p. 212); on Man it has been employed by BRÜCKNER (*loc. cit.*) and by RUNGE (*loc. cit.*).

effects varied in kind and degree.* HELMHOLTZ' hypothesis undoubtedly accounts for some of these apparent contradictions, which may be due to the fact that when a proximal region, or series of points, in a nerve is submitted to the polar action of a given electrode, there exists a distal region, or series of points, in that nerve which is submitted to polar influence of the opposite sign to that of the electrode (using the terms "distal" and "proximal" with reference to the electrode as centre).

But if this hypothesis is true in the case of the polarising current, it must hold also for the testing current, whether galvanic or induced, since the laws of diffusion are the same for both, and one cannot ensure the coincidence of excitation and polarisation by separate electrodes. Another contingency capable of vitiating the results obtained by this same method is, that derived currents may be established through the testing electrode of sufficient strength to alter the irritability of the nerve. We adopted the theory of a mixed polar action for both polarising and testing currents, as a working hypothesis at the beginning of our experiments, relying upon the following considerations to justify its assumption:—

(1.) The presence of an anodic make contraction speaks in its favour, since, given the axiom of PFLÜGER and CHAUVEAU, that make contraction is kathodic, break contraction anodic, it follows that with anodic make there coexists an associated kathodic make.

(2.) The elementary fact that, over certain points of nerve, the muscles that contract at anodic make are not the same as those that contract at anodic break also speaks in its favour, since it indicates that at the two events the points of excitation are not identical.

To denote the nerve-regions that are subjected to the two kinds of influence, we use the terms "*polar*" and "*peripolar.*" The area of electrical influence within which the density is sufficient to produce physiological effect is schematically represented as consisting of a polar region immediately beneath the electrode, in which the electrical relation is of the same sign as that of the electrode, and of a surrounding peripolar region in which the electrical relation is of the opposite sign (Diagram I.). We shall use this representation for the most convenient expression of our facts; it must

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* Our experiments with this method were few, and for the reasons given in the text we do not lay any stress upon the results obtained. These were as follows:—

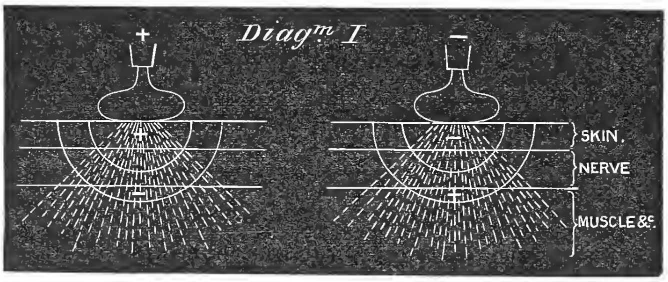
With the galvanic kathode above or below the negative polarising electrode the make contraction (K.C.C.) was diminished. On breaking the current a contraction appeared (K O.C.?).

With the galvanic anode above or below the negative polarising electrode the break contraction (A.O.C.) was diminished; the make contraction (A.C.C.) variable.

On testing with the make and break induction currents the effect of the kathode of either was diminished in the neighbourhood of the polarising kathode, increased in that of the polarising anode; the effect of the anode was increased in the neighbourhood of the polarising kathode; variable in that of the polarising anode.

With reference to the after-effects, they appeared to show increase after diminution of effect; diminution after increase.

however be understood that we are dealing with physiological signs alone, and that we have not determined the actual physical conditions that underlie them. With the method we employ, the actual limits of the provinces of the two effects (anodic and kathodic) are a secondary matter, for, admitting the existence of these effects, it is sufficient to know that they are coextensive for the two electrical components (testing and polarising) at the common electrode, that the principle of opposite polar and peripolar electrotonic states applies to the testing as well as to the polarising current, and that as the current density is greater in the polar than in the peripolar province, the electrotonic states are proportionately more marked in the former than in the latter.



To secure the coincidence of excitation with polar modification in the case of mechanical stimuli, we transmitted the testing blow through the movable electrode to a superficial nerve resting on bone.

The strength of the induction current was altered by altering the distance of the secondary from the primary coil of a DU BOIS apparatus. With the ordinary arrangement (one or two volts for the primary circuit and a resistance of about 1 ohm) the make induction shock is hardly effective on the human body, and examination is restricted to the effects of the break induction current. In order, therefore, to examine also the effects of the make induction current, we adopted the modification of HENRY,* viz., introduction of resistance into a primary circuit supplied by a battery of many cells, in order to obtain make and break induction shocks of about equal strength. The interruptions were made by hand.†

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* See DU BOIS-REYMOND, 'Ges. Abhandlungen,' vol. i., p. 230; HENRY, Trans. Phil. Soc., Philadelphia, vol. viii., pp. 7, 8 (1840); and Phil. Magazine, vol. xviii., p. 488 (1841).

† This method gives sufficiently regular stimuli, as may be perceived from the figures, care being of course taken to make and break contact as uniformly as possible. To this purpose we employed a simple spring key in the battery circuit, removing the finger rapidly, so that the break was effected by the recovery of the spring. We may remark that whereas by breaking rapidly a more energetic current is obtained than by breaking slowly, the contrary obtains in the case of the make, where a more gradual contact is more effectual than a sudden one. We have, however, also used a capillary contact with a mechanical interruption as a relay without improving upon the regularity of interruption. We prefer the manual method because it requires less apparatus, and because the capillary contact is well suited for make only. Used for adding to or subtracting from a galvanic current, the latter method would also necessitate an undesirable period of short-circuiting, and we used a HELMHOLTZ' key as will be described.

The strength of the polarising current was varied by altering the number of cells (freshly charged LECLANCHÉ'S with conglomerate plates), and to avoid fallacies arising from changes of resistance occurring during experiment, controlled by a galvanometer constructed to measure from 1 to 50 thousandths of a B.A. unit (milliwebers). In order further to eliminate the error caused by changes of resistance (vide *infra*) we introduced an additional resistance into circuit, maintaining the current at its original strength by employing greater electromotive force (*e.g.*, by 4000 ohms + galvanometer resistance about 750 ohms, *i.e.*, three times that of the average resistance of the body in the conditions of our experiments, the possible effects of changing resistance are divided by four). By this device we ascertained that our results were practically independent of alteration of current strength by alteration of resistance. It occasioned, however, the observation that addition of resistance in the secondary circuit diminishes the effect of the anode of the induction current (make or break) far more markedly than that of the kathode, a point which is illustrated in Tracing 32.

We observed a fact of the same kind with the HENRY modification of the current, viz.: that on gradually increasing the strength of current by bringing the secondary nearer to the primary coil, a break positive effect, which appeared with lower strength, was gradually overtaken by the corresponding make negative effect, which under certain circumstances might even surpass the break positive effect with higher strength (Tracing 33). We found the former phenomenon to be independent of the nature of the resistance apparatus used, and of its position in the circuit. We did not find anything analogous for the anode and kathode of the continuous current, the diminution of the contraction at the anode being parallel to that at the kathode when the resistance in circuit is augmented; on the other hand, we noticed that when wire coils are used, with the same current strength, the make effect at both poles is greater without than with additional resistance in circuit, the break effect remaining unaffected. This diminution of the make effect is doubtless due to the generation of an extra current in the resistance coil.

To record the muscular contractions which give the measure of nerve-excitability, we employed a small thick indiarubber bag, strapped to the part chosen for exploration, and connected by a flexible tube with a MAREY'S tympanum. The excursions of the lever were recorded on a smoked cylinder travelling at a convenient speed (about 2 centims. per minute). We preferred this mode of estimating excitability by strength of contraction, to that of estimating it by minimum stimulus, and we only determined the minimum stimulus for the sake of comparison. Whether during polarisation, increase seen on the figure was due to increased contraction of a given amount of muscle, or to the added contraction of previously unexcited muscle, or to both factors, does not affect the evidence for our present purpose. We did not, strictly speaking, measure the nerve excitability, but only determined whether it was increased or diminished, for the amplitude of the record does not increase proportionally with the strength of contraction.

The greater number of our experiments were made on the peroneal nerve close to the tendon of the biceps, or at the head of the fibula, and all the accompanying figures were taken from the anterior tibial group of muscles. We, however, obtained similar results on other nerves, and with other groups of muscles, viz.: by stimulation of the supra-clavicular nerves, and of the median and ulnar nerves, using the muscles of the fore-arm as the indicator of effects. The experiments were made on ourselves.

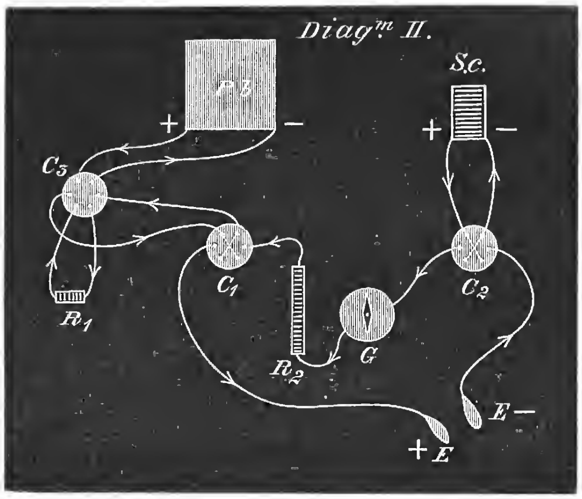
I. POLAR ALTERATIONS OF EXCITABILITY TESTED BY INDUCTION CURRENTS.

Preliminary.—Before using induction currents to judge of alterations effected by the galvanic current, we examined the effects of long series of induction breaks and makes. Our experiments gave the following results:—

1. The height of successive contractions by make or break induction currents approaches more and more gradually to a maximum. The figures show a marked and progressive increase, similar to the "stair-case" increase obtained with repeated excitations of the ventricle apex (Tracing 12).
2. The stronger the excitations the more rapid is the initial increase.

We do not propose to discuss these points here, but only record them in evidence of an alteration effected by induction currents, which must not be attributed to the galvanic current when it coexists. Anticipating upon some remarks we shall have to make on ascending and descending series by makes and breaks of the galvanic current, we interpret the progressive ascent to signify progressively increasing excitability as an after-effect of each successive excitation. We also observe that the ascent is more marked with the kathode than with the anode.

Electrical connexions were established as shown in Diagram II.



The poles of a galvanic battery (*P.b.*) are connected with the commutator (C_1), provided with its cross wires; a commutator (C_3), without its cross wires being inserted between the battery and its commutator (C_1), and serving to direct the current from the secondary coil (*S.c.*) through the battery or through the resistance (R_1) not less than that of the battery.* The secondary coil (*S.c.*) is connected with the commutator (C_2). To the outer holes of the commutators (C_1, C_2) are connected the two electrodes (*E*, *E*). The inner holes of (C_1, C_2) are connected through a galvanometer (G) and a resistance (R_2).

Either current can thus be reversed independently of the other by the commutators C_1, C_2, and the movable electrode can be made at will kathode or anode of the make or break induction current, with or without kathode or anode of the galvanic current.

The combinations possible are :—

Anode	of the break	induction	current with	Anode	of the galvanic	current.
Kathode	,,	,,	,,	Kathode	,,	,,
Anode	,,	,,	,,	Kathode	,,	,,
Kathode	,,	,,	,,	Anode	,,	,,
Anode	of the make	induction	current with	Anode	of the galvanic	current.
Kathode	,,	,,	,,	Kathode	,,	,,
Anode	,,	,,	,,	Kathode	,,	,,
Kathode	,,	,,	,,	Anode	,,	,,

These are schematically represented in the following diagram,—*III. A.* representing the conditions of make excitation, *III. B.* those of break excitation.

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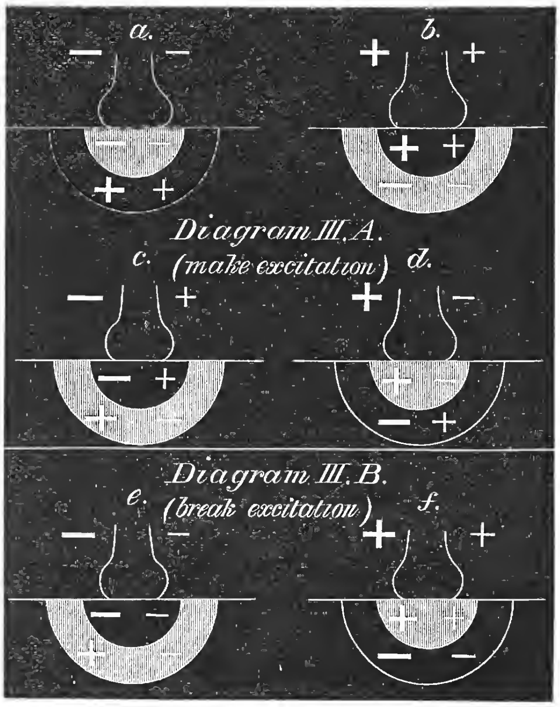
* We found, however, that this precaution may be safely dispensed with, the battery used being of low internal resistance.



The accompanying diagram represents the four polar combinations of the testing and polarising currents, and the coincidence of their respective kathodic and anodic regions. The signs in large type are intended to denote the kathode and anode of the polarising current, and the corresponding electrotonic states of the polar and peripolar regions. The signs in small type are intended to apply similarly to the testing current; "make," and "break" here refer to the commencement and cessation of a galvanic current, not to make and break induced currents, both of which give rise to make excitation. Figs. *a*, *b*, *c*, *d*, apply to the electrotonic modifications of the make excitation; figs. *e*, *f* to those of the break excitation; the region of excitation is in each case shaded in the diagram. The combinations are as follows:—

a. Polar	excitation	coincides	with polar	katelectrotonus.
b. Peripolar	,,	,,	peripolar	,,
c. Peripolar	,,	,,	peripolar	anelectrotonus.
d. Polar	,,	,,	polar	,,
e. Peripolar	,,	,,	peripolar	,,
f. Polar	,,	,,	polar	,,

A. *Polar alterations of excitability tested by the break induction current.*

Our first series of experiments were made with the ordinary arrangement of the coil (an electromotive force of 2 volts and a resistance of 1 ohm for the primary circuit), and gave the following results:—

1. *The effect of the kathode of the break induction current is greater than that of the anode.*

Our interpretation of this fact is as follows:—The current consists in an almost simultaneous make and break, and the stimulus depends on the former alone. With the kathode the polar region is kathodic, with the anode the peripolar region is kathodic; since the excitation proceeds in each case from the kathodic region, and since the current density is greater in the polar than in the peripolar region, the effect of the kathode is greater than that of the anode.

2. *The effect of the kathode of the break induction current is increased when that kathode is also kathode of the galvanic current.*

With the kathode of the induction current the polar region is kathodic, and excitation proceeds therefrom. With the kathode of the galvanic current the polar region is kathodic, and excitation proceeds from a region in which the excitability is increased. The effect is therefore increased.

3. *The effect of the anode of the break induction current is increased when that anode is also anode of the galvanic current.*

With the anode of the induction current excitation proceeds from the peripolar region. With the anode of the galvanic current that region is kathodic, and excitation proceeds from a region in which the excitability is increased. The effect is therefore increased.

4. *The effect of the kathode of the break induction shock is diminished when that kathode is also anode of the galvanic current.*

With the kathode of the induction current the polar region is kathodic, and excitation proceeds therefrom. With the anode of the galvanic current the polar region is anodic, and excitation proceeds from a region in which the excitability is diminished. The effect is therefore diminished.

5. *The effect of the anode of the break induction shock is diminished when that anode is also kathode of the galvanic current.*

With the anode of the induction current excitation proceeds from the peripolar region. With the kathode of the galvanic current that region is anodic, and excitation proceeds from a region in which the excitability is diminished. The effect is therefore diminished.

6. *The increase in the effect of the kathode of the break induction current when that kathode is also kathode of the galvanic current, is greater than the increase in the effect of the anode of the break induction current when that anode is also anode of the galvanic current.*

With the double kathode it is the polar region whence excitation proceeds and wherein excitability is increased; with the double anode it is the peripolar region whence excitation proceeds and wherein excitability is increased. The augmentation in the effect of a given stimulus is greater in the former case than in the latter because the density is greater.

7. *The diminution in the effect of the kathode of the break induction shock when that kathode is also anode of the galvanic current, is greater than the diminution in the effect*

of the anode of the break induction current when that anode is also kathode of the galvanic current.

With the combined induction kathode and galvanic anode the excitation and the diminished excitability are of the polar region with the combined induction anode and galvanic kathode, the excitation and the diminished excitability are of the peripolar region.

The diminution of a given effect is greater in the former case than in the latter. That diminution is such that the effect of the combined *faradic* anode and *galvanic* kathode is greater than that of the combined faradic kathode and galvanic anode. This fact was observed by BRÜCKNER, who saw in it a proof of electrotonus inasmuch as the previously more effective faradic kathode is rendered the less effective during the passage of an opposed galvanic current.

To the statements 1, 2, 3, 4, 6, 7 we have found no exception in all our experiments; statement 5, though expressing the usual event with currents of medium strength, requires qualification by this additional statement.

8. *With increasing strength of the galvanic current, the effect of the anode of the break induction current, when that anode is also kathode of the galvanic current diminishes to a minimum, and with further increase in the strength of the galvanic current increases up to and beyond the original normal.*

It is a constant phenomenon which is shown best with weak and moderate strengths of the induction current. We conjecture that it signifies extension of katelectrotonic influence in the previously anelectrotonic region (physiological) with increasing strength of the galvanic current. The increased excitability is in the polar region, the excitation is in the peripolar region in which excitability is diminished. But apart from its theoretical import, we lay stress on the fact as showing augmentation of excitability in the presence of opposed electromotive forces. We may observe further that with strong induction currents it does not appear, that with subminimal induction currents the increase is alone seen and only with greater strength of the galvanic current, and that exceptionally with induction currents of moderate strength the increase brought about by the galvanic current appears without previous decrease.

In seeking confirmation of the hypothesis applied to the fact expressed in the preceding statement, we were led to look for an analogous effect with the combined induction kathode and galvanic anode, and found ourselves entitled to qualify statement 4 by the following statement:—

9. *With increasing strength of the galvanic current, the effect of the kathode of the break induction current, when that kathode is also anode of the galvanic current, diminishes to a minimum, and with further increase in the strength of the galvanic current increases, but not up to the original normal within endurable strength of the galvanic current.*

After several failures we verified this statement with moderate strength of the induction current. It is the converse of the preceding observation, and our interpretation is likewise a supposition that the phenomenon signifies invasion of katelectrotonic by anelectrotonic region (physiological) with increasing strength of the galvanic current. The increased excitability is in the peripolar region, the excitation is in the polar region in which excitability is diminished. The difficulty with which this phenomenon is brought into evidence, in comparison with its converse, is accounted for by the supposition that in the converse case we have to deal with the invasion of peripolar anelectrotonic by polar katelectrotonic region, and in the present case with invasion of polar anelectrotonic by peripolar katelectrotonic region. The difficulty is due to the pain caused by the strong galvanic current required, and to the tetanus which such a current evokes, so that the increase appears only when the conditions are exceptionally favourable to electrical density in the peripolar region as compared with density in the polar region; we noticed in effect that when this increase was brought about, kathodic polarization of increasing strength effected increase in the effect of the induction anode without preceding diminution. The difference between the augmentation of the faradic anode under the influence of the galvanic kathode and that of the faradic kathode under the influence of the galvanic anode may be formally stated as follows.

10. *The increasing effect of the combined faradic anode and galvanic kathode takes place with a weaker galvanic current than that of the combined induction kathode and galvanic anode; the increase is greater and more rapid in the former case than in the latter.*

We record finally, under these statements relating to the break induction current, an observation relating to the make induction current, because it was a salient feature in our experiments with the ordinary arrangements of the coil, and because its mention emphasises the superiority of the kathode of the make induction current combined with the kathode of the galvanic current, over all other combinations of the two poles.

11. *With the ordinary arrangement of the coil used there was no contraction to the make induction current with all combinations and all strengths of the induction and galvanic currents, except the combined anode of the break induction current* (i.e., *kathode of the make induction current*) *and kathode of the galvanic current.*

The interpretation of this fact is identical with that for the kathode of the break induction current when also kathode of the galvanic current. Excitation thus coincides with increased excitability in the polar region.

B. *Polar alterations of excitability tested by the make induction current.*

In order to use make induction currents as the test of excitability, we modified the coil after the method of HENRY, using 30 LECLANCHÉ cells and a resistance of 100 ohms in the primary circuit; the results differed in no respect from the results previously obtained with break induction currents, and it is therefore only necessary to add that we have verified all the above statements for the make as well as for the break induction current (with the exception of statement 9, which we did not attempt to verify).

Using the following abbreviations,

F− for the effect of the kathode of the induction current (make or break).
F+ " " anode " " "
G− for polarisation by the galvanic kathode.
G+ " " " anode.

the above results may be summed up thus :—

			Tracings:
(1)	F−	> F+	—
(2)	F− with G−	> F−	14, 20
(3)	F+ with G+	> F+	19
(4)	F− with G+	< F−	17, 18, 13*b*
(5)	F+ with G−	$\overset{<}{\underset{>}{=}}$ F+	13*a*, 16
(6)	Increase of F− with G−	> Increase of F+ with G+	19 to 22
(7)	Diminution of F− with G+	> Diminution of F+ with G−	23 and 24
(8)	F+ with increasing G− is at first diminished, then increased to above normal		13*a*
(9)	F− with increasing G+ is at first diminished, then begins to increase		13*b*
(10)	The increase from minimum F+ with G− occurs with weaker G than that from minimum F− with G+. With further increase of G it is greater in the case of F+ with G− than in that of F− with G+		13*a*, *b*

For convenience of reference we have given with each of the above statements a number referring to an illustrative figure.

C. *Minimum exciting strength of an induction current with increasing strength of a polarising current.*

The following tables contain numbers expressing in millimetres the distance of the secondary coil at which contractions were first marked on the cylinder in the absence of polarisation, and in the presence of polarisation by 10 and 20 cells respectively.

Induction pole.	Distance of coil.		Number of cells of polarising current.	
F+	113	113	0	G+
,,	117	120	10	,,
,,	135	140	20	,,
F−	117	115	0	G−
,,	131	122	10	,,
,,	151	155	20	,,
F+	113	113	0	G−
,,	111	116	10	,,
,,	112	112	20	,,
F−	118	118	0	G+
,,	108	110	10	,,
,,	103	104	20	,,

These numbers agree with the conclusions already obtained, but this method is less satisfactory than the graphic method; the determination of the minimum F+G− is especially variable, as indeed might have been expected from our previous results.

D. *Proof of the physiological nature of the interference when faradic and galvanic currents in one circuit are opposed in direction.*

It has been shown by one of us* that the latent time of the break contraction on Man far exceeds that of the make contraction. That the length of this period depends on a persistence of anelectrotonus after its galvanic provocative has ceased was proved by showing that the recovery of a faradic tetanus, interfered with by anodal polarisation for a given time, does not occur within a period equivalent to a make latency, but is delayed for a period equivalent to a break latency. The existence of this period of delay is proof of physiological versus physical interference. From this datum it can be shown that the diminution of effect when the two currents are opposed is always anelectrotonic, whether the combined exploring pole be faradic kathode with galvanic anode or faradic anode with galvanic kathode, for in either case the period of persistence of anelectrotonus is observed.

The following experiment constitutes the easiest demonstration of electrotonus on Man; it is an old experiment, but has not hitherto received a physiological interpretation:—

Holding in both hands the poles of a secondary coil with vibrating interrupter, so that both arms are tetanised, it may be observed with suitable distance of coil and

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* A. Waller, in an oral communication to the Physiological Society of London, November, 1881. (Arch. de Physiologie, Avril, 1882.)

strength of galvanic current that a given tetanus is greatly increased, or that previously ineffectual stimuli tetanise when a galvanic current traverses the coil in the same direction as that of the break induction current; and, conversely, that a tetanus is greatly weakened or abolished when a galvanic current traverses the coil and body in the opposite direction to that of the break induction current. With the two currents in the same direction, all induction kathodes in the body coincide with galvanic kathodes, and the increased effect is katelectrotonic; with the two currents in opposite directions, all induction kathodes in the body coincide with galvanic anodes, and the diminished effect is anelectrotonic. The latter statement is proved by showing that the interference is physiological, inasmuch as the time which elapses between the removal of the galvanic current and the contraction produced by the released *faradic* excitation is greater than a make latency, and may be as long as a break latency. From this it is a legitimate inference that the former statement is also true (Tracing 25).

II. POLAR ALTERATIONS OF EXCITABILITY DURING THE PASSAGE OF A GALVANIC CURRENT TESTED BY MAKES AND BREAKS OF A GALVANIC CURRENT.

Preliminary.—Before using the make and break of a galvanic current as the test of alterations during and after the passage of an uninterrupted galvanic current in the same circuit, we examined the series of effects of the former alone. As was to be expected, this test showed more marked variations than had been found with induction currents. The full discussion of the alterations with time and current strength of the excitability during and after galvanisation demands however prolonged investigation, and will form the subject of a separate paper. We shall only anticipate upon that part of the question sufficiently to justify certain statements.

We observe that a horizontal series of make contractions, with equal duration and interval of current, can be converted into an ascending series by using a stronger current, or by altering the excitation rhythm to one of short duration and long intervals; that, conversely, a horizontal series can be converted into a descending series by using a weaker current, or by altering the excitation rhythm to one of long duration and short interval.

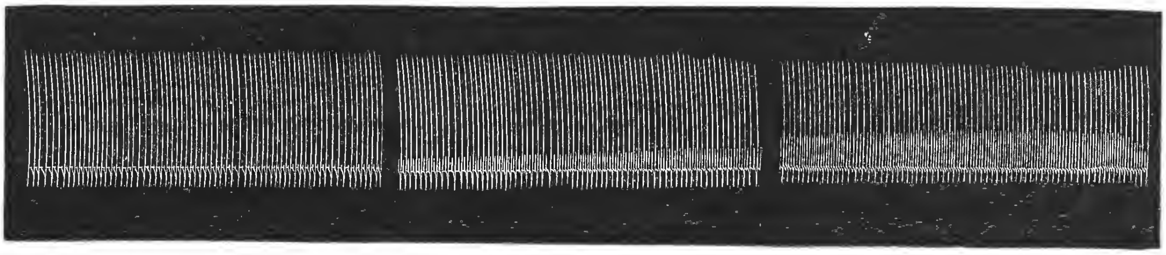
We shall show, when we come to treat of the after-effects, that excitability after kathodal polarisation is at first diminished, subsequently increased (Tracings 8, 34). We also borrow an observation from a future paper on which we base the statement that this diminution of excitability increases in degree and in duration up to a certain limit of strength and duration of polarisation, and that beyond that limit it diminishes and sooner gives place to a more marked increase of excitability. Upon these data we base the statements that descending series indicate that each make excitation occurs during the period of diminished excitability consequent on previous make, and that ascending series indicate that each make excitation occurs during increased excitability consequent on previous make.

We also observe that at the termination of an ascending series a weaker current than the original current will cause a greater contraction than at first; that at the termination of a descending series a stronger current than the original current will cause a smaller contraction than at first. These facts indicate in the former case diminution of increased excitability, in the latter case a summation of diminished excitability which however very rapidly gives way to increase.

We notice that, *cæteris paribus*, ascending series are most frequently observed at the beginning of experiments (independently of any increase of current strength), that descending are most frequently observed after the nerve has been for some time subjected to experiment. This probably signifies that increased excitability is more apparent before the general excitability has been raised, diminished excitability while the general excitability is raised above normal, and we shall give an observation to show that the excitability is raised above the normal, and so remains for a considerable period after polarisation (Tracing 32).

We observe finally that anodic opening contractions at first increase, then gradually diminish, and that it requires far stronger or more prolonged currents to produce the opening contraction at the end than at the beginning of the experiment.

Tracing A.

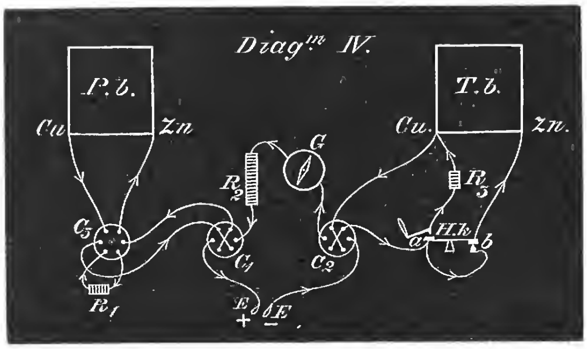


Series of anodic closure and opening contractions by 150 cells (17 milliwebers), 9000 ohms additional resistance in circuit. The interruptions in the tracing represent periods of about $2\frac{1}{2}$ minutes each during which the opening contractions gradually diminished.

Amidst complex conditions of time, current-strength, and excitability, various results are naturally to be expected, and it is to be borne in mind that gradual alterations of current-strength, by alteration of resistance and by polarisation currents, will adulterate the developing changes of excitability (vide pp. 980, 981). On this account we had recourse to the introduction of a considerable additional resistance in order to dilute the effects of physical change, and to disengage, as far as possible, physiological changes from the physical adulteration. All things considered, we concluded that *a descending series, with an excitation rhythm, which previously gave a horizontal series, signifies decreasing excitability; that, conversely, an ascending series indicates increasing excitability; declining excitability is proved* à fortiori *when, with constant current-strength and excitation rhythm, a previously ascending series is con-*

verted into a descending series; increasing excitability when a previously descending series is converted into an ascending series.

Tracing 26 shows that it is not indifferent whether make and break of a galvanic current be effected in the principal circuit, or in a deriving metallic circuit.



Electrical connexions were established as in Diagram IV.; they are substantially the same as in Diagram II. with the substitution of a battery *T.b.* for the coil *S.c.*, and with the addition of a HELMHOLTZ' key (*H.k.*) so arranged that the current of *T.b.* can be added to or subtracted from the current of *P.b.* without breaking circuit and without short circuiting *T.b.* for any appreciable period. By reference to the diagram it will be understood that the make and break of *T.b.* occur at the end *a* of the key; by depressing the handle *T.b.* is bridged at *a*, and its current cut off, further short circuiting being prevented by the instantly succeeding break at *b*. By raising the handle the bridge is re-established at *b*, and instantly after broken at *a*, so that the current is made. The two inferior surfaces at *a* and *b* are connected; a resistance R_3, not less than that of the battery *T.b.* being introduced between the battery and the upper surface of the side *a* of the key, so that the current of *P.b.* be not transferred to a path of less resistance when the testing battery is cut out of circuit. This resistance was only used unfavourably to the results anticipated, viz.: in examining for increased excitability during polarisation; but its use was found to be a superfluous precaution as had been found for the resistance R_1.

We always used the two galvanic currents in the same direction, hence we obtained by the above arrangement a comparison of the effects of a given current with the effects of the same current added to or subtracted from a pre-established current, viz.:

Anodic make alone or added to an anodic current.
Kathodic ,, ,, a kathodic ,,
Anodic break alone or subtracted from an anodic current.
Kathodic ,, ,, a kathodic ,,

The experiments on which the following statements are based, were made with 11,000 ohms additional resistance in circuit:—

1. *The effect of kathodic make is greater than that of anodic make.*

With the kathode, the polar region is kathodic; with the anode, the peripolar region is kathodic. The make excitation proceeds from the kathodic region, and the current density is greater in the polar than in the peripolar region; the effect at make of the kathodic is therefore greater than that at make of the anode.

2. *The effect of anodic break is greater than that of kathodic break.*

With the anode, the polar region is anodic; with the kathode, the peripolar region is anodic. The break excitation proceeds from the anodic region, and is greater when it is polar than when it is peripolar.

3. *The effect of kathodic make is increased during the flow of a kathodic current.*

Kathodic excitation and kathodic increased excitability are both of the polar region, and the effect is therefore greater with than without the kathodic current.

4. *The effect of anodic make is increased during the flow of an anodic current.*

Kathodic excitation and kathodic increased excitability are both of the peripolar region.

5. *The effect of anodic break is diminished during the flow of an anodic current.*

With the anode of the testing current the break excitation proceeds from the polar region which is anodic, and in which anodic influence is maintained by the remaining current. The break excitation thus falls on a region in which the excitability is depressed, and its effect is diminished or abolished.

6. *The effect of kathodic break is diminished during the flow of a kathodic current.*

The break excitation falls on the peripolar region in which the excitability is depressed by the anodic influence of the remaining current.

7. *The increase in the effect of kathodic make during the flow of a kathodic current is greater than the increase in the effect of an anodic make during the flow of an anodic current.*

The kathodic increase is greater when it is polar than when it is peripolar, *i.e.*, greater with the double kathode than with the double anode.

8. *The diminution in the effect of anodic break during the flow of an anodic current is greater than the diminution in the effect of kathodic break during the flow of a kathodic current.*

The anodic diminution of excitability is greater when it is polar than when it is peripolar, *i.e.*, the diminution is greater with the double anode than with the double kathode.

Using the following abbreviations:—

K.C.C. for the kathodic closure contraction.
A.C.C. ,, anodic ,, ,,
K.O.C. ,, kathodic opening ,,
A.O.C. ,, anodic ,, ,,
K.D. ,, kathodic current duration.
A.D. ,, anodic ,, ,,

the above results may be thus summed up :—

1. K.C.C. > A.C.C.
2. A.O.C. > K.O.C.
3. K.C.C. during K.D. > K.C.C.
4. A.C.C. during A.D. > A.C.C.
5. A.O.C. ,, A.D. < A.O.C.
6. K.O.C. ,, K.D. < K.O.C.
7. Increase of K.C.C. with K.D. > Increase of A.C.C. with A.D.
8. Diminution of A.O.C. with A.D. > Diminution of K.O.C. with K.D.

These statements are illustrated in Tracings 35 and 36, with the exception of statements 6 and 8, which we have verified, but of which it is difficult to obtain a good record.

Minimum Exciting Strength of a Galvanic Current Added to or Subtracted from a Pre-existing Galvanic Current.

We determined the strength of current necessary to produce minimum contraction with polarising currents of increasing strength. The experiments were made on the peroneal nerve, with and without additional resistance in circuit, and are embodied in the following table :—

First appearance on tracing of	Number of cells of testing current. (without additional resistance.)			Number of cells of polarising current.
	a.	*b.*	*c.*	
K.C.C. . . .	9	9	7	0
	8	8	..	2
	7	7	5	4
	6	6	..	6
	5	5	3	8
	4	4	..	10
	3	3	2	12
	3	3	..	14
	2	2	..	16
A.C.C. . . .	10	10	9	0
	8	9	..	2
	8	8	7	4
	7	7	..	6
	6	6	5	8
	5	5	..	10
	4	4	4	12
	3	3	..	14

First appearance on tracing of	Number of cells of testing current. (with 1700 ohms additional resistance.)			Number of cells of polarising current.
	a.	*b.*	*c.*	
A.O.C. . . .	14	12	12	0
	16	14	14	2
	18	16	20	4
	22	20	24	6
	28	24	28	8
	30	26	..	10
K.C.C. . . .	20	19	23	0
	19	18	..	2
	18	18	..	4
	18	17	..	6
	17	16	20	8
	16	15	..	10
	14	14	..	12
	13	13	..	14
	13	12	15	16
	12	11	..	18
	11	10	..	20
	10	10	12	22
	10	9	..	24
	9	9	..	26
	9	8	..	28
	..	..	9	32
A.C.C. . . .	17	17	15	0
	15	16	..	2
	15	15	..	4
	13	13	..	6
	12	12	12	8
	10	10	..	10
	10	10	..	12
	9	9	..	14
	9	8	8	16
	8	8	..	18
	..	..	5	24
	..	..	3	32

N.B.—The figures in columns *a* and *b* were taken on the same day, those in column *c* on another occasion. The minimum stimulus of the A.C.C. is the most uncertain.

This table shows that alterations of excitability increase with the strength of the polarising or pre-existing current, and justifies the statement that :—*The greater the density of pre-existing current, the smaller the increase of density, and the greater the decrease of density necessary to effect stimulus.*

Tracings 35 and 36 illustrate the statement that :—*The greater the density of pre-existing current, the greater the excitatory effect of a given increase of density, the less the excitatory effect of a given diminution of density.*

III. Polar Alterations of Excitability tested by Mechanical Excitation.

In order to ensure the coincidence of excitation with polar alteration of excitability, we used the electrode itself as the medium of mechanical excitation. We have found only one point of the body which lends itself to the mechanical excitation of nerve, viz., the elbow, at the place where the ulnar nerve runs immediately under the skin upon a resisting bony groove. Here it is possible, though not always easy, to elicit regular contractions by striking a small electrode held steadily over the nerve, with gentle and regular blows of an ordinary hammer. The test may appear a rough one, but the results are sufficiently uniform and evident to leave no doubt of the reality of the alterations.

When the kathode rests on the nerve, the polar region being therefore kathodic, the effect of mechanical excitation is increased; when the anode rests on the nerve, the polar region being therefore anodic, the effect of mechanical excitation is diminished or abolished.

On breaking the current the contractions appear in both cases greater than before.

A possible objection must be forestalled. It might be said that the increased effect during the kathodic current, is partly due to the change of *density* in the current, arising from the sudden compression of the tissues lying between the electrode and the nerve. This is not consistent with the fact that such a change of *density* would increase the contractions when a strong polarising anode is applied, whereas the contrary is observed. We may also remark that the effects are equally well observed with a large additional resistance in circuit.

IV. Polar Alterations of Excitability Subsequent to the Passage of the Galvanic Current.*

Preliminary.—Before drawing conclusions as to the state of excitability after the passage of the galvanic current, it is necessary to realise the possible influence of polarisation currents and alterations of resistance upon the current-strength.

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* The after-effects have been alluded to by some of the authors quoted above (Eulenburg, Samt). They form the subject of a laborious inquiry by Remak (" Ueber Modificirende Wirkungen Galvanischer

Changes of current-strength depend upon various conditions which we have not been able to determine satisfactorily for the want of a sufficiently delicate galvanometric record. Pending such a determination, which will be necessary when we come to the investigation of the time factor in electrotonic and post-electrotonic processes, we record the alterations which we have hitherto observed. When the electrodes are first applied the current grows gradually to a maximum, owing to the permeation of the skin with moisture, and, perhaps, to the accompanying vascular turgidity. On applying the electrodes to the skin previously prepared by soaking, the current at once reaches a maximum from which it declines at first rapidly, then more and more slowly. On renewed make of the same current instantly after interruption the galvanometer shows that the current has not recovered its strength. On renewed make after another interruption of 30–60 seconds the galvanometer indicates that the current has recovered. If a current that has been allowed to flow until the galvanometer needle has ceased to fall is suddenly reversed, the current-strength is indicated to be greater in the new direction. These effects are such as would be produced, at least partially, by polarisation of the electrodes, and that this is so can be shown in the usual way by leaving the electrodes only connected with the galvanometer, either in direct contact with one another, or with the body intervening. Current is indicated in either case, greater of course in the former. We observe another fact indicating a fallacy to which the estimation of after-effects is especially liable, viz.: that after a given duration of current, with a given deflection of the needle, a given interval is followed by a greater deflection which rapidly subsides to the original deflection—a phenomenon which may repeat itself several times in succession, and obtains whether the electrodes are kept in broken or in closed circuit during the interval. Without committing ourselves to any positive statement, we may remark that our observations led us to suppose that during the passage of the galvanic current, an opposed electromotive force is developed within the body which rapidly subsides. But however this may be, the importance of these alterations of current-strength as a source of fallacy are obvious, and they especially concern the question of after-effects. We have not been able as yet to assign to them their exact share for and against the manifestation of the after-effects. But we have, we think, succeeded in demonstrating the rough facts in spite of the fallacy, and it will be one of the objects of a future paper to find numerical expression for this vague quantity.

Finally we may mention a further possible source of error, which however cannot be eliminated should it exist, viz.: the unequal alteration by the current of the conductivity of the nerve and of the surrounding tissues. It is conceivable that the conductivity of the surroundings of nerve should be increased in a greater proportion than

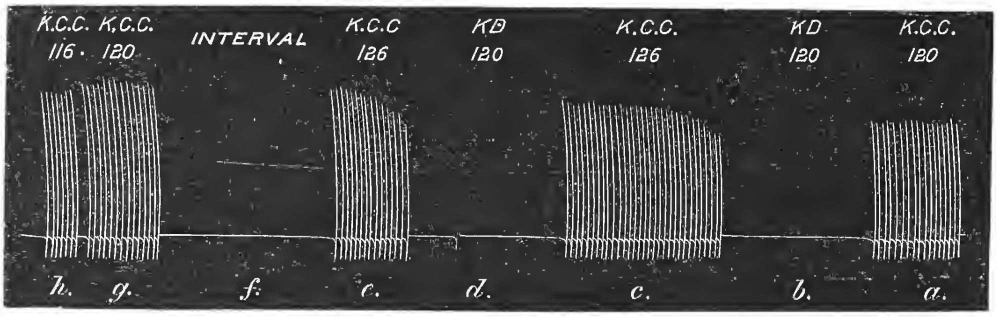
Ströme," &c. Deutsches Archiv f. Klin. Med., 1876, p. 264). He lays stress upon the fallacies arising from alterations in the current-strength, and used the galvanic test only. He states that K.C.C. is increased after K.D., and variable after A.D.; that A.C.C. is increased after K.D., and variable after A.D. He does not seem to have determined any alteration of the opening contraction.

that of the latter, being more vascular and susceptible, therefore of greater increase of vascularity under the influence of the current.

We made the conditions of experiment unfavourable to the results anticipated by using a weaker current than the original current when it was desired to prove augmentation of excitability, a stronger current than the original current when it was desired to prove diminution of excitability. This was done for the galvanic test by altering the number of cells so that the galvanometer deflection was evidently smaller or greater than before. The instruments at our disposal did not allow us to apply this *à fortiori* device to all the cases of the question, but we established our principal results by its means, and we think that the proof of these may be taken to cover the remaining cases of which the results are congruent.

Tracing B, with the accompanying description, are given as formal examples of the application of this proof as regards the polar region. Tracings C and D illustrate a simpler and more direct proof of the physiological nature of the after-kathodic change.

Tracing B.

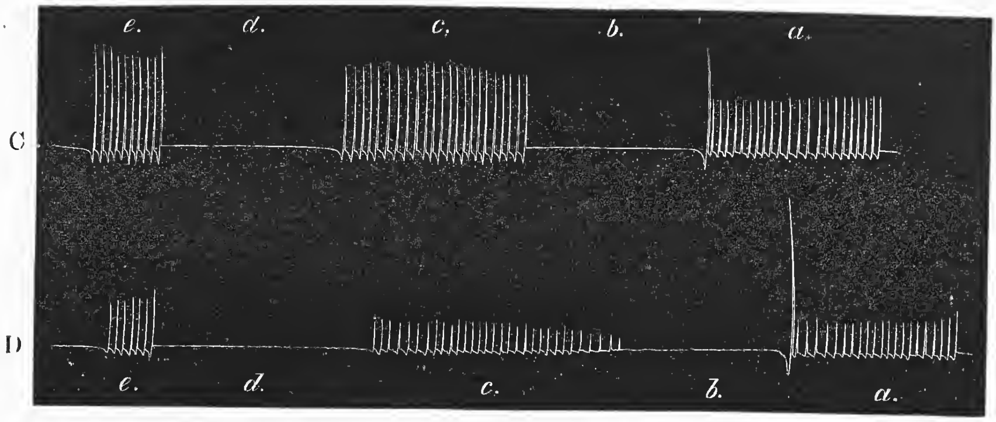


Tracing B. 11,000 ohms additional resistance in circuit.

	Cells.
a. K.C.C.	120
b. K.D.	120
c. K.C.C.	126
d. K.D.	120
e. K.C.C.	126
f. Interval.	
g. K.C.C.	120
h. K.C.C.	116

The contractions at (*e*) with stronger current are smaller than those at (*a*); therefore the excitability is diminished. The contractions at (*h*) with weaker current are greater than those at (*a*); therefore the excitability is increased.

Tracings C and D.



Tracing C. 750 ohms additional resistance.

	Cells.	m.w.
a. K.C.C.	12	5
b. K.D.	28	13
c. K.C.C.	12	5·5
d. Interval.		
e. K.C.C.	12	5·5

Tracing D. 5000 ohms additional resistance.

	Cells.	m.w.
a. K.C.C.	30	5
b. K.D.	70	13
c. K.C.C.	30	5
d. Interval.		
e. K.C.C.	30	5

The comparison of series (*c*) in the two preceding figures shows that unless a considerable resistance be added in circuit, the after-kathodic diminution may be masked owing to an increase of current-strength.

The following statements (the letters have the same meaning as at page 972) summarise the results of our experiments on the peroneal nerve, with the precautions taken of an approximate galvanometric control and of an additional resistance of from 10,000 to 20,000 ohms in circuit. This being about six to twelve times the resistance of the body, the possible effects of physical change were divided by so much, and the effects of altered excitability practically disengaged from the fallacy. The cases in which we have proved *à fortiori* the physiological nature of the change are marked with an asterisk (*) in table of after-effects by the galvanic test.

After-effects tested by the Induction Current.

1. F+ after G+ is less than before.
2. F− after G− is less ,,
3. F+ after G− is greater ,,
4. F− after G+ is greater ,,

That is to say: *kathodic excitation of a given region produces less effect after than before kathodic polarisation, greater effect after than before anodic polarisation.*

5. The diminution of F− after G− is greater than the diminution of F+ after G+
6. The increase of F− after G+ is greater than the increase of F+ after G−

That is to say: *the after-kathodic diminution and the after-anodic increase are more marked in the polar than in the peripolar region.*

These statements are illustrated in Tracings 15–20. They refer to the immediate after-effects, *i.e.*, those manifest within the first few seconds after polarisation has ceased, and we must add the statement that *the after-anodic increase is of long duration, the after-kathodic diminution soon gives way to an increase of long duration* (Tracing 34).

After-effects tested by the Galvanic Current.

	Immediately.	Subsequently.	
1. K.C.C. after K.D. is	less*	greater*	than before polarisation.
2. K.O.C. ,, K.D. ,,	greater	?	,, ,,
3. A.C.C. ,, A.D. ,,	less*	greater*	,, ,,
4. A.O.C. ,, A.D. ,,	greater*	less*	,, ,,
5. K.C.C. ,, A.D. ,,	greater	greater	,, ,,
6. K.O.C. ,, A.D. ,,	less	?	,, ,,
7. A.C.C. ,, K.D. ,,	greater	greater	,, ,,
8. A.O.C. ,, K.D. ,,	less	less	,, ,,

That is to say: *kathodic excitation produces less effect after than before kathodic polarisation, greater effect after than before anodic polarisation.*

9. The diminution of K.C.C. after K.D. is greater than the diminution of A.C.C. after A.D.
10. The diminution of A.O.C. after K.D. is greater than the diminution of K.O.C. after A.D.
11. The increase of A.O.C. after A.D. is greater than the increase of K.O.C. after K.D.
12. The increase of K.C.C. after A.D. is greater than the increase of A.C.C. after K.D.

That is to say: *the after-kathodic diminution and the after-anodic increase are more marked in the polar than in the peripolar region.*

These statements, with the exception of 2 and 6 referring to the K.O.C., and consequently also of 10 and 11, are illustrated in Tracings 28–31. They refer to the immediate after-effects.

We are not at present in a position to discuss the subsequent after-effects, which we reserve until we shall have investigated the *course* of electrotonic and post-electrotonic changes. We cannot, however, omit drawing attention to the peculiarity affecting the opening excitation, which constitutes an exception to the rule that the subsequent after-effect of polarisation of either sign is increased excitability. The subsequent after-effect of anodic polarization as tested by anodic break is apparently diminished excitability, yet by testing in the polar region by kathodic make, evidence of increased excitability is obtained when the effect of the original anodic break may be almost lost. To what specific difference in the excitatory process at make and at break this may point we cannot conjecture, and we have only to remark that the fact is of the same nature as the gradual diminution of successive anodic break contractions.

DESCRIPTION OF TRACINGS.

(N.B.—All Tracings, except Tracing 25, are to be read from right to left. The cylinder speed is about 2 centims. per minute.)

Tracings 1–7. Series of kathodic closure contractions with different strength and rhythm of current. 13,000 ohms in circuit.

	Strength.			Duration of flow.	Duration of interval.
	Cells.	m.w.			
Tracing 1.	75	4	1st part	3″	1″
			2nd part	1″	3″
Tracing 2.	150	9	1st part	1″	3″
			2nd part	3″	1″
Tracing 3.	150	9		3″	1″
Tracing 4.	75	4		1″	3″
Tracing 5.	75	4		2″	2″
Tracing 6.	150	9		2″	2″
Tracing 7.	75	4		2″	2″

In support of the argument that descending series indicate that make excitation occurs during the diminished excitability consequent on previous make, ascending series that it occurs during the increased excitability consequent on previous make, we may refer to Tracings 1 and 2, in which the rhythm was reversed in the course of each series. The comparison of Tracings 6 and 7 shows, we believe, that the period of diminution is longer with the weaker than with the stronger current. Tracing 4 as compared with Tracing 5 shows diminution of the descent by prolonging the interval and shortening the duration of flow; we suppose that the make excitation in the former case does not, as in the latter, coincide in time with a state of diminished excitability.

Tracing 3 as compared with Tracing 6 shows diminution of the ascent by shortening the interval and prolonging the duration of current; we suppose that in the former case, as compared with the latter, the make excitation falls within a period of diminished excitability.

Tracings 8–14 show the effect of the induction current modified by a galvanic current in the same circuit; without additional resistance.

Tracing 8. Break induction kathodic contractions before, during, and after passage of a galvanic current in the same direction. Distance of coil 10 centims. Galvanic current of 12 cells.

Tracing 9. Break induction anodic contractions before, during, and after passage of a galvanic current in the same direction. Distance of coil 9 centims. Galvanic current of 12 cells.

Tracing 10. Break induction kathodic contractions before, during, and after passage of a galvanic current in the opposite direction. Distance of coil 9·5 centims. Galvanic current of 4 cells.

Tracing 11. Break induction anodic contractions before, during, and after the passage of a galvanic current in the opposite direction. Distance of coil 7 centims. Galvanic current of 12 cells.

Tracing 12. Series of contractions by the break induction kathode, with increasing strengths of current. The figures accompanying each group of contractions denote the distance in centims. of secondary from primary coil.

Tracing 13. Effect of break induction anode with increasing strength of an opposed galvanic current. The first group of five contractions is taken in the absence of any galvanic current, the 2nd group with a galvanic current of 2 cells, the 3rd with 4 cells . . . , the 21st with 40 cells (19 milliwebers). Distance of coil 6 centims. 1000 ohms additional resistance in circuit.

Tracing 13*b*. Effect of break induction kathode with increasing strength of an opposed galvanic current. The first group of three contractions is taken in the absence of galvanic current, the 2nd group with 2 cells, the third with 4 cells . . . , the 15th with 28 cells. Distance of coil 8 centims. No additional resistance in circuit.

Tracing 14. Series of make and break induction contractions, with and without galvanic current in circuit. HENRY's modification (E. M. F. of 30 volts; 100 ohms in primary circuit). Reading from right to left the first of each pair of contractions is by the make, the second by the break current. 1000 ohms added in circuit.

a.	Make induction anode		and	break induction kathode.	
b.	Make induction kathode		,,	break induction anode.	
c.	Make induction anode with galvanic kathode		,,	break induction kathode with galvanic kathode.	
d.	Make induction kathode with galvanic anode	abolished	,,	break induction anode with galvanic anode.	
e.	Make induction anode with galvanic anode		,,	break induction kathode with galvanic anode.	abolished
f.	Make induction kathode with galvanic kathode		,,	break induction anode with galvanic kathode.	

Tracings 15–20 : Illustrate the same facts as Tracings 8–11, and show more clearly the after-effects. 11,000 ohms additional resistance in circuit.

	Induction current.	Distance of coil. centims.	During.	After.	Galvanic current, 50 cells (3·5 m.w.).
Tracing 15.	—	4	increased	diminished	—
Tracing 16.	+	0	diminished	increased	—
Tracing 17.	—	5	diminished	increased	+
Tracing 18.	—	0	diminished	increased	+
Tracing 19.	+	0	increased	diminished	+
Tracing 20.	—	5	increased	diminished	—

The greater increase of F — by G — than of F + by G + is seen in the comparison of Tracing 19 with Tracing 20; the greater diminution of F — by G + than of F + by G — in the comparison of Tracing 10 with Tracing 11.

The diminution of F — after G — (Tracings 8, 15, 20); the diminution of F + after G + (Tracing 9); and the superiority of the former over the latter are also to be recognised in Tracings 15–20 especially. Likewise the increase of F — after G + (Tracing 17), the increase of F + after G — (Tracing 18), and the superiority of the former over the latter. The make induction contraction is seen in Tracing 11.

Tracings 21–24 show the effect of the break induction current with increasing strength of a galvanic current in the same circuit. No additional resistance in circuit.

Tracing 21. Induction kathodic contractions with increasing galvanic current in the same direction. The first group of five contractions is taken in the absence of galvanic current, the second with a current of 2 cells . . ., the last with a current of 20 cells. (In the next three tracings the galvanic current is increased as in this by 2 cells with each successive group.) Distance of coil 10·5 centims.

Tracing 22. Induction anodic contractions with increasing galvanic current in the same direction. Distance of coil 9 centims.

Tracing 23. Induction kathodic contractions with increasing galvanic current in the opposite direction. The contractions are much reduced with 4 cells, abolished with 6 cells. Distance of coil 9·5 centims.

Tracing 24. Induction anodic contractions with increasing galvanic current in the opposite direction. The contractions decrease as the current increases to 12 cells, increase as the current increases from 14 cells. Distance of coil 8 centims.

Tracing 25. The upper line shows the latency of three coinciding contractions by closure of the galvanic current through both arms. The small vertical line marks the instant of excitation.

The lower line shows the interval between the instant of removal of the galvanic current and contraction by excitation by the (physiologically) released faradic current, the poles being for the arm explored *anode galvanic* and *kathode break faradic*. Three contractions coincide.

The middle line shows the same latency with the poles reversed, these being for the arm explored *kathode galvanic* and *anode break faradic*.

The closure latency (upper line) is determined in the usual way by a catch fixed to the cylinder. The liberation interval (two lower lines) is determined by causing a metal surface fixed to the cylinder to short-circuit the galvanic current for a given period, thus subtracting it from the combined faradic and galvanic circuit. The contractions registered are taken from the right forearm. The interruptions of the inducing current are made by a reed vibrating 200 per second. A chronograph records 100 vibrations per second below the tracing.

Tracing 26. The two groups are kathodic closure contractions by 10 cells (6 milliwebers). 1000 ohms' resistance in circuit. For the group of contractions to the right the closure excitations were made by bridging a break in the principal circuit; for the group to the left by breaking a bridge in a deriving metallic circuit.

Tracing 27. The two groups are anodic closure and opening contractions by 30 cells (23 milliwebers), 1000 ohms in circuit. For the group to the right the closure excitations were made by bridging a break in the principal circuit, the opening excitations by breaking a bridge in that circuit; for the group to the left the closure excitations were made by breaking a bridge in a deriving metallic circuit, the opening excitations by bridging such a break.

The tracings show that a stronger make excitation is obtained by closing a key in the principal circuit than by opening a key in a deriving metallic circuit, and that a stronger break excitation is obtained by closing a key in a deriving metallic circuit than by opening a key in the principal circuit. The effects of excitation made in these two ways must not therefore be compared.

Tracings 28–31 show series of contractions by make and break of a galvanic current before and after the passage of a galvanic current in the same and in the opposite direction for 1 minute. Each series lasts 1 minute. Duration of testing current, 1 second; interval, 2 seconds.

Tracing.	Testing current.	Resistance in circuit.	Number of cells.	Milli-webers.	Polarising current.	Resistance in circuit.	Number of cells.	Milli-webers.	After effect.
28	K.C.C.	13,000	70	3	An.	13,000	150	9	Increase
29	K.C.C.	,,	,,	,,	Kat.	,,	,,	,,	Diminution
30	A.C.C.	8000	110	11	,,	8000	110	11	Increase
	A.O.C.	,,	,,	,,	,,	,,	,,	,,	Diminution
31	A.C.C.	,,	,,	,,	An.	,,	,,	,,	Diminution
	A.O.C.	,,	,,	,,	,,	,,	,,	,,	Increase

The greater diminution of K.C.C. after K.D. than of A.C.C. after A.D. may be recognised by comparing Tracings 29 and 31.

The greater increase of K.C.C. after A.D. than of A.C.C. after A.D. may be recognised by comparing Tracings 28 and 30.

Tracing 31 shows the diminution of A.O.C.

Tracing 32 shows the effect of the addition of resistance in the secondary circuit on the response to the break induction kathode and anode respectively. The first group (reading from right to left) is a series of kathodic contractions, the second group is a series of anodic contractions without added resistance. The third group is a series of kathodic, the fourth a series of anodic contractions with 1000 ohms added resistance. The fifth and sixth groups are taken with 2000 ohms added resistance, the anode group being only just visible. On further additional resistance the anode ceased to produce any effect. The seventh group is a series of kathodic contractions with 3000 ohms added. The eighth group (barely visible) is a series with 4000 ohms. With 5000 ohms the kathode ceased to produce any effects. Distance of coil 9 centims.

Tracing 33 shows the gradual appearance of contraction with the make (kathodic) and break (anodic) induction currents of gradually increasing strength. HENRY'S modification.

The first four groups of three contractions each are by the anode of the break current with the coil at 9·5, 9, 8·5, 8 centims., at which distances the kathode of the make produced no effect. The last four groups of six contractions each are by the kathode of the make and by the anode of the break at 7·5, 7, 6·5, 6 centims. The first of each pair of contractions is by the make current, the second by the break current. The former (kathodic) gradually overtakes the latter (anodic).

Tracing 34. To show kathodic after-effect. Distance of secondary from primary coil 9·5 centims. No additional resistance in circuit. The first group to the right of the tracing is of induction kathodic contractions before polarisation; the second group is of contractions by the same induction current during the 8th minute of kathodic polarisation by 20 cells (12 m.w.). The following series shows the absence and subsequent rise of contractions after polarisation. The small groups to the left of tracing, marked 1, 2, 3, 4, 5, 6, 7, are of contractions taken at the end of 1, 2, 3, 4, 5, 6, 7 intervals of 15 minutes each, and shows the gradual decline of excitability, the strength of the induction current remaining the same (9·5 centims.) throughout the experiment.

Tracing 35. Series of kathodic closure contractions alternately without and with an uninterrupted galvanic current in the same circuit, 1000 ohms in circuit. Each series last about 1 minute, the duration of current made is about 1 second, that of the interval is about 2 seconds. The strength of current giving the K.C. contractions is the same throughout, viz.: by 10 cells (6 milliwebers). In the three successive series taken during the passage of the uninterrupted current, the strength of the latter is increased, being by 2, 4, and 6 cells in each successive series. The figure shows that the augmentation of K.C.C. is greater with greater strength of the polarising current.

Tracing 36. Series of anodic closure and opening contractions by 30 cells (23 milliwebers) taken on the same plan as the preceding series, with and without an uninterrupted galvanic current in the same direction. It shows augmentation of A.C.C. and diminution of A.O.C. increasing with increasing strength of the polarising current. On comparison with the preceding series it may be recognised that the augmentation of A.C.C. during polarisation is less marked than that of K.C.C.

XXIII. THE CROONIAN LECTURE.—*On the Rhythm of the Heart of the Frog, and on the Nature of the Action of the Vagus Nerve.*

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[PLATES 66–70.]

IN all investigations upon the causation of the beat of the heart, one question stands forward prominently before all others, viz. : What is the relative share taken by the ganglion cells and the muscular tissue respectively in the production of its spontaneous rhythmical beats ? And in any discussion upon the action of the cardiac nerves, the most important question always is, How far do they act on the ganglion cells, how far on the muscular tissue directly?

In recent times a variety of investigations have pointed unmistakably to the conclusion that rhythmical action can occur in muscular tissue under the influence of a constant stimulus without the intervention of ganglion cells, or at all events of any well-defined recognisable nerve-cells ; in other words, certain kinds of muscular tissue possess the faculty of transforming the effects of a continuous stimulus into a discontinuous result.

Again, there can be no doubt whatever that in the heart of the Frog the rhythmical beat is markedly associated with the presence of certain ganglion cells, especially the cells of REMAK's ganglia, so that the first question which it is absolutely necessary to answer is, Does the normal rhythm of the heart as a whole depend upon separate rhythmical impulses passing from certain motor ganglia to the muscular tissue, each of which impulses causes a contraction, so that the rhythm is due to the nerve cells, and the muscle simply gives expression to it, or do the motor cells send to the muscles a continuous series of subminimal impulses, the effects of which the muscle sums up, so as to produce from time to time a single beat ; in other words, Is the rhythm due to the muscle, while the nerve cells supply the constant stimulus ?

Further, before it is possible to consider the action of the cardiac nerves, it is necessary to come to some conclusion respecting the different attributes of the cardiac muscle apart from the question of rhythm, and it seems to me that the three which especially demand attention are the following :—

1. Contraction-power, as measured by the extent of the contraction.
2. Excitability, as measured by the response or non-response of the muscle to the stimulus.
3. Tonicity, as measured by the extent of the relaxation of the tissue at the commencement of each contraction.

I propose, then, to discuss in the first part of this paper not only the nature of the normal rhythm of the heart, but also certain variations in the three above-mentioned attributes of the cardiac muscle, together with their relations to each other. Afterwards I will proceed to the second part and endeavour to determine the respective actions of the vagus nerve upon the ganglion cells and the cardiac muscle.

In order to investigate the different problems suggested above, it is clearly necessary to employ the graphic method, and at the same time it is very advisable to obtain a method which shall register simultaneously as many of the different factors involved as possible.

As far as the question of tonicity is concerned, I have chiefly made use of the arrangement described in my paper* "On the Tonicity of the Heart and Blood-vessels," and those experiments I propose to consider later on; there are, however, considerable difficulties of interpretation in using this method for the determination of the force of contraction of different parts of the heart. I therefore resolved to investigate the nature of the cardiac contractions in the bloodless heart after the same plan as has been used with such marked success in the case of ordinary striped muscle and its nerve.

The vagus nerve, usually on the left side, is dissected out so as to be as long as possible, and a ligature attached to one end; the two aortic trunks are cut close to the bulbus, and a thread is tied to the extremity of the loose auricular flap, at the junction of the two auricles, which is exposed by the removal of the aortic trunks; another thread is attached to the extreme apex of the ventricle, and the heart with a piece of the œsophagus and trachea behind it, and with the vagus nerve intact, is removed from the body. In taking the heart out the lungs are cut away, and the venous sinus carefully left intact. One of KRONECKER's forceps is now made to clip the free end of the œsophagus and trachea, and held tight in a suitable holder (Plate 66, F). The thread attached to the auricles is fixed by means of a small hook to a lever A, which is capable of movement in a vertical plane, while that attached to the ventricle is fixed in a similar way to another exactly similar lever B, which can slide up and down on the same support below the lever A. The upper lever to which the auricles are attached is drawn up above the horizontal position by means of an elastic thread fixed to it near the fulcrum, and also fixed to the support above it, as is seen at C in the figure. In this way the heart is suspended between the two levers A and B, the lower one of which is drawn upwards by its contraction, the upper one downwards. Between the two a clamp D is now placed so as just to hold the tissue tight in the auriculo-ventricular groove, and the two levers are adjusted so that they are parallel and horizontal. The clamp is made of two limbs of strong but thin metal with carefully rounded edges, which are capable of being approximated to each other by means of a fine micrometer screw. The levers are provided with aluminium-foil points, and are adjusted so that the two writing-points are exactly in the same vertical line when the levers are horizontal. By

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* Journal of Physiol., vol. iii., p. 48.

means of a drum the two levers write at the same moment upon the blackened paper, and the upper lever moving downwards records the contractions of the auricle, while the lower lever moving upwards traces those of the ventricle.

The vagus nerve is now laid upon a pair of platinum electrodes E, and kept moist by laying over it a piece of filter paper dipped into normal saline solution. The electrodes are in connexion with an ordinary induction coil driven by a single DANIELL cell, the interruptor of which is a tuning-fork instead of the usual hammer. In addition to the two tracings described above, the usual time and stimulation markers write on the blackened paper.

In the description just given I have spoken of the clamp as being placed in the auriculo-ventricular groove, so as to register the auricular and ventricular contractions simultaneously; it can, however, be placed across any portion of the heart, for example between the sinus and auricles, so as to register the sinus and auricular contractions; midway between the base and the apex of the ventricle, so as to register the contractions of the base and apex simultaneously, &c.

Besides the advantage obtained by the power of registering at will the contractions of any two contiguous parts of the heart, the clamp enables us to study the effects of compression at different points much more delicately than the old plan of ligaturing, for it is possible either just to hold the tissue so as not to injure it physiologically or to compress it up to any required amount by the simple movement of the micrometer screw.

Further, it is possible to cut the ventricle open from base to apex, and to apply poisons or fluids of any kind to that part of the heart which is situated below the clamp without any chance of their reaching the parts above the clamp (the latter being provided with a flange on each side to prevent the fluids passing over it). Also the parts on the one side of the clamp can be heated or cooled independently of the parts on the other side, and under all these different conditions the effect of stimulation of the vagus nerve can be studied. Throughout the course of each experiment the heart is kept moist by the application of salt solution.

PART I.

ON THE RHYTHM OF THE HEART.

If we imagine a portion of muscle made to contract rhythmically by the action of certain cells situated at a distance from it, then it is clear that the rate of rhythm will be independent of any increase in the excitability of the muscle, if the rhythm is due to separate impulses passing from the cells to the muscles, each of which is able to cause a contraction; on the other hand, an increase in the muscular excitability must cause a quicker rate of rhythm, if the rhythm is dependent upon some process of summation of stimuli taking place in the muscle itself.

Now, by the method described above it is possible to influence in different ways either the ventricle alone on the one side of the clamp without affecting the sinus and auricles on the other side, or only the auricles and other parts on that side of the clamp without touching the ventricle; so that it is possible to increase or diminish the excitability of the ventricular muscle without affecting the discharges from the motor ganglia, or to act upon the motor ganglia without altering the excitability of the ventricle. Of the various means for effecting this object, it will be sufficient to give examples of the action of heat, atropin and muscarin.

When the whole heart is heated the rate of rhythm is always greatly increased, and the same, as is well known, is true in the case of either the ventricle or apex when isolated and beating spontaneously. In this latter case the greater rapidity of rhythm is partly, at all events, to be ascribed to the increase of excitability in the muscle due to the heating.

Suppose, now, in the heart suspended as described above the ventricle alone be heated, then its excitability will be increased* while that of the auricles will remain the same as before, and therefore if the rhythm be dependent upon the muscular tissue, as in the case of the isolated apex, the ventricle ought to beat at a quicker rate than the auricles whose excitability has not been altered. On the other hand, if the rhythm is due to discrete impulses passing from the motor ganglia to the ventricle, then no increase in the excitability of the latter ought to make the smallest alteration in the rate of the beats, because upon that view the ventricle does not contract except when an impulse reaches it, and the motor ganglia remaining outside the range of the heating the rate at which their discharges take place remains unaltered and is unaffected by any alteration in the excitability of the ventricular muscle. Of these two views experiment proves that the latter is the true one.

If the clamp be placed in the auriculo-ventricular groove and be tightened so as just to hold the tissue firmly, then both auricles and ventricle continue to beat with perfect regularity for hours, each ventricular beat following in orderly sequence upon each auricular. In order to heat the parts of the heart on one side of the clamp without heating those on the other side I have used a spiral of thin platinum or copper wire which is placed round either the ventricle or the auricles and sinus and is in connexion by means of a key with a battery of two or three GROVE cells. When the ventricle alone is to be heated it is placed in connexion with the upper lever and the auricles with the lower; if the heating is intended to affect only the sinus and auricles then the arrangement is reversed. That this method works well is shown by the fact that whereas a thermometer suspended within the spiral may show a rise up to 50° or 60° C. under the heating influence of three GROVE cells, yet a thermometer placed against the tissue just below the clamp does not show any rise at all, when the spiral is in position round that part of the heart which is situated above the clamp.

Heating the sinus and auricles alone in this way causes a most marked increase in the rate of rhythm both of the auricles and ventricle, with other phenomena which will be mentioned presently (Plate 67, fig. 1), showing that the method of heating is effective. On the other hand, when the ventricle alone is heated no alteration *in the*

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* KRONECKER, "Das charact. Merkmal der Herzmuskel-Beweg." LUDWIG's Festgabe, 1874.

rate of rhythm either of ventricle or auricles is produced; the rate of the auricular beats continues the same; the ventricular beat follows upon each auricular with the same regularity as before, and, in fact, as far as the rhythm is concerned the ventricle might not have been heated (Plate 67, fig. 2). That the heating was effective, however, the figure clearly shows, for the force of the ventricular contractions is markedly diminished during the time that the muscle was heated: a fact of considerable importance, as proving that heat acts upon the muscular tissue of the heart in such a way as to diminish the force of its contractions apart altogether from the rapidity of the rate of rhythm of those contractions.

This experiment seems to me positive proof that in the whole heart the rhythm is due to discrete impulses proceeding from certain motor ganglia to the muscular tissue, each of which impulses causes a contraction of that tissue. Further evidence to the same effect is given by a series of results which may be generalized as follows:—

Any influence which, applied to the auricles and sinus alone, causes an alteration in the rhythm of the auricles, affects the rate of the ventricular beats synchronously, while the same influence, applied to the ventricle alone, causes no initial alteration in the rhythm either of ventricle or auricles.

Thus cold slows both the auricular and ventricular beats when applied to the auricles and sinus, while the same amount of cold produces no effect on the rhythm when applied to the ventricle alone. Atropin sulphate (1 p.c. solution) dropped on to the auricles and sinus slows the rate of beating of the whole heart most markedly (Plate 70, fig. 26 B) while the same solution applied to the cut-open ventricle produces no initial effect upon the rhythm. So, too, with muscarin sulphate, a marked slowing when applied to the auricles and sinus; no alteration of rhythm, for some time at all events, when confined to the ventricle. (See Plate 70, figs. 26 A, 27.)

Many more instances of the same kind might be given, but enough has been said already to permit us to formulate the following proposition:—

The rhythmical contractions of the heart as a whole are caused by discrete impulses which proceed rhythmically from certain motor ganglia to the auricles and ventricle.

The apparent discrepancy between this assertion and the rhythmical phenomena exhibited by the isolated ventricle and apex will be discussed at the end of this part of my subject.

On the want of sequence between the ventricular and auricular contractions.

If, however, we conclude that separate impulses pass from the motor ganglia to the ventricle, the further question arises, What conditions are necessary so that each one of these impulses should cause a contraction of the ventricular muscle?

In the ventricle or apex alone we know, from the experiments of KRONECKER,[*] that

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[*] *Op. cit.*

a series of single induction shocks of sufficient strength, sent through the muscle at a definite and invariable rate, will cause a synchronous series of contractions; and that if then the muscle be cooled down, the same series of single stimuli will cause a series of contractions synchronous not with every stimulus, but with every second stimulus. Again, v. BASCH* has shown that a series of single stimuli of the same strength may cause a series of contractions synchronous with every second stimulus when each stimulus is so weak as to be unable to cause a contraction by itself.

From these two observations we may draw the conclusion that a definite relation between the strength of the stimulus and the excitability of the ventricular muscle is necessary, in order that the muscle should contract synchronously with the series of artificial stimuli; and also that the muscle may be made to respond synchronously to every second stimulus, instead of to every stimulus, either by lowering the excitability of the muscle, the strength of the stimulus remaining unaltered, or by lessening the stimulus, while the excitability remains the same.

In his latest paper,† v. BASCH suggests as an explanation of this half-rhythm that a stimulus which is insufficient to cause a contraction increases the excitability of the muscle, so that a subsequent stimulus of the same strength is thereby enabled to produce a contraction.

These observations of KRONECKER and v. BASCH may be taken as evidence that the ventricular muscle is able to sum up the effects of two or even more artificial stimuli; but so far we have no proof that it is able to act in the same way towards the normal impulses coming from the motor ganglia.

This evidence I have been able to supply by my method of experimentation, and have come to the conclusion that a definite relation between the strength of the impulses from the motor ganglia, and the excitability of the ventricular muscle, is requisite to ensure the occurrence of a series of ventricular beats synchronous with the impulses coming from the motor ganglia. Here, too, just as in the case of the artificial stimuli, a certain number of the impulses prove inefficient to cause a contraction when the excitability of the muscle is lowered sufficiently; here, too, a diminution in the strength of the impulses below a certain limit causes the ventricle to beat synchronously with every two, three, or more impulses, instead of with every impulse, although the excitability of the muscle remains the same.

In this case, then, as well as in the case of artificial stimulation, the probable explanation of the half-rhythm observed is to be found in the following extension of the suggestion made by v. BASCH, viz.: *when the relation between the strength of the impulses coming from the motor ganglia and the excitability of the ventricular muscle is such that each separate impulse is not able to cause a beat of the ventricle, then the ventricle does not necessarily remain quiescent, but beats synchronously with every second*

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[*] Sitzber. d. k. Akad. d. Wiss. (Wien.) Bd. lxxix., III. Abth.

[†] Arch. f. Anat. u. Physiol. (Physiol. Abth.). 1880, S. 283.

impulse, because each impulse, though unable to cause a contraction of the muscle, may increase its excitability to the height necessary for the production of a beat by the time the next impulse reaches the tissue.

The proof of these assertions depends upon the fact that, by the method of clamping, it is possible to alter the relation between the strength of the impulses and the excitability of the ventricular muscle without affecting the relation between the motor ganglia and the auricular muscle. The rate, therefore, at which the impulses are sent out from the motor ganglia is indicated by the rate of the auricular beats, and the extent of synchronism between them and the beats of the ventricle determines the number of impulses that have been made inefficient, as far as the causation of a ventricular beat is concerned, by the action of each special operation.

This want of sequence between the auricular and ventricular contractions can be obtained by three different methods.

1. By tightening the clamp in the auriculo-ventricular groove.
2. By heating the auricles and sinus without heating the ventricle.
3. By the application of various poisons to the ventricle alone.

By the use of the micrometer screw attached to the clamp it is possible to compress the tissue so slightly, and yet to hold it with sufficient firmness, that the sequence of ventricular upon auricular beats continues without alteration or interruption; it is possible also to compress it so strongly that the motor impulses can no longer reach the ventricle, and therefore the ventricle remains quiescent, while the auricles continue beating with unaltered rhythm; and it is possible to compress it to any extent intermediate between these two extremes, and to observe the alteration in the ventricular rhythm so produced. This latter case is that with which we are specially concerned at present, and in every instance I have found that with gradual tightening of the clamp the ventricle does not pass abruptly into a state of quiescence, but that the increased compression always causes the ventricle to beat slower than the auricles. This slower rate of beating is never independent of the auricular rate, but is connected with it in such a way that the ventricle, instead of beating synchronously with each beat of the auricles, beats synchronously with every second, third, fourth, or more auricular beat. The commonest effect by far, and the one which is most permanent, is that in which the ventricle beats synchronously with every second beat of the auricles; and, indeed, by careful manipulation of the micrometer screw, it is always possible to bring about this particular kind of rhythm, and in many cases to make the ventricle beat permanently with this rhythm for the rest of the experiment. In other cases this rhythm may last for a considerable length of time, and then the ventricle return to a rate synchronous with that of the auricles. So, too, when the clamp is first tightened the half-rhythm may be caused immediately, or the ventricle may cease to beat for a short time, and then commence to beat with intervals between the beats, corresponding to a large number of auricular beats, such as 1 ventricular beat to 6 auricular; quickly passing through such intermediate stages as 1 ventricular to 5 auricular, 1 to 4, 1 to 3, it settles down to the rhythm 1 to 2, at which it remains permanently, or ultimately returns to a rate synchronous with the auricular (Plate 67, fig. 3).

In some cases the rhythm of the ventricular beats has remained for a long time synchronous with every third beat of the auricles, instead of every second, as in Plate 68, fig. 14, and in other cases, for a shorter time, with every fourth beat. This latter rate of rhythm occurs frequently in the apex of the heart when the clamp is placed across the middle of the ventricle and the beats of the apex and base of the ventricle are compared (see Plate 69, fig. 16); here, too, however, the more permanent rate caused by the compression of the clamp is that in which the apex beats synchronously with every second beat of the base.

These facts show that direct compression of the tissue between the motor ganglia and the ventricle causes a certain number of the impulses to be ineffective as far as the causation of a ventricular beat is concerned, although, as shown by the auricular beats, that compression has not interfered with the rate of discharge from the ganglia. Either, then, the compression of the clamp in the auriculo-ventricular groove has caused a diminution in the excitability of the ventricular muscle without altering the strength of the impulses coming to it, or it has diminished the strength of the impulses to the muscle and so caused it to respond to every second instead of to every impulse. Which of these two solutions is the most probable will be better discussed after the consideration of the effects upon the ventricular rhythm of heating the auricles and sinus only, and of the application of poisons to the ventricle alone.

When by the method described above the auricles and sinus alone are heated, any variations which are noticed in the rhythm of the ventricle must be due to the action of the heat upon the motor impulses before they have reached the ventricular muscle and not to any alteration in the excitability of the muscle itself, for the latter is out of the range of the heating.

In many respects there is a decided resemblance between the action of the clamp and the effect of heating the auricles and sinus upon the rhythm of the ventricle. Thus, if the heating has been sufficient only slightly to accelerate and diminish the auricular contractions, then it is found that the ventricle is able to beat synchronously with the accelerated auricular beats, just as a slight compression of the clamp does not interfere with the regular sequence of ventricular upon auricular beats. If the heating continues the auricular beats become weaker and more rapid, and the curves then show that the ventricle is unable to respond with the same rapidity, and ceasing suddenly to beat synchronously with the auricles, it answers with a single contraction to every two contractions of the auricles. When the heating is removed and the auricles begin to beat less frequently, the ventricle again alters its rate so as to beat synchronously with the auricles with the same suddenness as in its original alteration of rhythm. So, too, an increase in the tightness of the clamp makes the ventricle beat synchronously with every second beat of the auricles, and upon loosening the clamp the original sequence of beats returns.

Finally, in some cases heating the auricles and sinus causes the ventricle more or less suddenly to cease beating and remain quiescent in the relaxed condition, although, as will be shown hereafter, we have reason to think that the motor impulses are still passing to the muscle; similarly, when the clamp in the auriculo-ventricular groove is tightened sufficiently the ventricle remains quiescent, even although there is proof that the impulses have not been entirely prevented from reaching the muscle (see Plate 69, figs. 17 and 18).

Again, it is to be noticed that when the ventricle is made to beat synchronously with every second beat of the auricles, either by heating the auricles and sinus as in Plate 67, fig. 1), or by tightening the clamp as in Plate 67, fig. 3, the slower contractions are always larger than the previous ones, and this greater force of the contraction is only partly, at all events, due to the fact that with a slower beat the muscle contracts from a position of greater relaxation than with a quicker one. The figures, it is true, show that the lever does fall lower between the contractions when the ventricle is beating slowly than when it is beating quickly—a fact which proves that, with the ordinary rate of beating, the ventricle has not time to relax to its full extent before another contraction commences—but they also show that the lever rises above the level of the previous quicker beats; in other words, the slower beats are accompanied by an increase in the force of the contraction as well as by an increase in the relaxation of the muscle between the beats.

The similarity between the effects of clamping and those of heating the auricles and sinus upon the rhythm of the ventricle is so marked and consistent, that we may fairly conclude that any explanation which is sufficient for the one phenomenon must also resolve the other.

What factors, then, can be conceived as common to these two methods by which such similar results may be produced?

In the first place, it may be argued that the excitability of the ventricular muscle has been diminished by the clamping, and therefore the ventricle can only respond once to every two impulses. The possibility of such a causation for the production of the half-rhythm is shown by the fact that curare or muscarin, when applied to the ventricle alone, does ultimately cause that muscle to beat synchronously with every second auricular beat, without any alteration of the rate of the auricular contractions.

It is, however, to be noticed that these poisons never produce this effect until they have acted so powerfully upon the muscular tissue of the ventricle as markedly to lower the strength of its contractions; that, in fact, as one would expect, any method by which the excitability of the muscle is lowered to such an extent that it is only able to respond to every second impulse, must at the same time greatly diminish the strength of the contractions of that muscle.

This fact that a diminution of the contraction force accompanies the production of the half-rhythm when it is clearly due to a diminution in the excitability of the ventricular muscle, points unmistakably to the conclusion that the half-rhythm caused by clamping, in which, as has been shown, the ventricle contracts with greater force than before the clamp was tightened, is not due to any diminution of muscular excitability caused by the action of the clamp. This is further shown by the following facts :—

1. The effect of tightening the clamp upon the rhythm of the ventricle is the same in kind, whether the clamp be near the ventricle, as in the auriculo-ventricular groove, or far from it, as at the junction of the sinus and auricles ; *i.e.*, whether the clamp be in a position where it might possibly lower the excitability of the muscle by direct injury, or so far removed as to make it impossible to affect the muscle directly.

2. When the clamp is placed across the middle of the ventricle the base may continue to beat with the same rhythm as the auricles, while the apex beats synchronously with every second, third, fourth, or more beats of the base ; and it seems impossible to imagine that a direct mechanical injury across the middle of a muscle should so markedly lower the excitability of one side without perceptibly diminishing that of the other.

3. It frequently happens, as already mentioned, that a standstill of the ventricle occurs immediately upon the clamp being tightened, but that after an interval the ventricle commences to beat again ; if, therefore, the clamp acts by diminishing the excitability of the ventricular muscle, this recommencement of the beats must be due to a gradual recovery of that excitability from the first depressing effect of the increased compression ; this recovery of the excitability ought therefore to be capable of measurement by the method of sending single induction shocks through the muscle at regular intervals, after it has been rendered quiescent by tightening the clamp in the auriculo-ventricular groove. I have made this experiment, and with single shocks sent through every 3 or 5 seconds with the secondary coil carefully regulated so as just to give a contraction at every shock, I could find no evidence whatever that a weaker stimulus was sufficient to produce contractions synchronous with the stimuli, even after a length of time much greater than is ever seen between the time of tightening the clamp and the recommencement of spontaneous beats.

For these different reasons, then, it is most probable that the compression of the clamp does not alter the rhythm of the ventricle by lowering the excitability of its muscular tissue, and we may infer that the same conclusion holds good in reference to the effect of heating the auricles and sinus. Another explanation must therefore be found.

Since the experiments of MAREY and others it is recognised as a fact that every contraction of the ventricle is accompanied by a marked diminution of the excitability of the muscle, so that if minimal stimuli be applied to the ventricle at too rapid a rate the muscle will not be able to contract to each stimulus, because it has not had time to recover sufficiently from the loss of excitability caused by the previous contraction. For this reason, then, we can conceive that any influence like heating the auricles and sinus only, which causes the motor ganglia to send out impulses at a much quicker rate without increasing the excitability of the ventricle, may cause the ventricle to respond synchronously to every second of these quicker impulses, instead of to every impulse.

Now, although this explanation may be sufficient in the case of the half-rhythm produced by heating the sinus and auricles only, it is clearly inefficient to explain the action of tightening the clamp, for in this latter case no alteration in the rate of the auricular contractions occurs.

Since, therefore, the compression by means of the clamp in the sulcus does not produce its effects upon the ventricle by lowering its excitability, or by altering the rate of the impulses from the motor ganglia, it is most probable that it does act by weakening the strength of the impulses in the same way as suggested by v. BASCH, in order to explain the half-rhythm caused by the use of sufficiently weak electrical stimuli.

Further, the similarity between the effects of tightening the clamp and of heating the sinus and auricles alone, is sufficiently great to suggest that in both cases the effects are produced in the same way, and therefore the main reason why heating the sinus and auricles caused the observed alteration in the rhythm of the ventricle is to be found in the fact that, when the motor ganglia are heated, not only is the rate of discharge of the impulses quickened, but also the quicker impulses are of necessity weaker than before.

To sum up the conclusions arrived at so far, we have the following propositions :—

1. The rhythm of the heart is caused by discrete motor impulses passing to the muscular tissue from certain motor ganglia.

2. Each one of these impulses produces a contraction of the ventricle only when a due relation exists between the strength of the impulses and the excitability of the ventricular muscle.

3. When each impulse is inefficient to cause a contraction of the ventricle, the ventricular muscle has the power of summing up the effects of two or more of these inefficient impulses, and so continues to beat rhythmically, though no longer synchronously with every impulse.

4. The easiest explanation of this summation process is as follows : Every impulse which is inefficient to produce a muscular contraction increases the excitability of the muscle, and therefore makes it easier for a second similar impulse to cause a contraction.

5. The impulses can be made inefficient to produce contractions synchronous with them by lowering sufficiently the excitability of the ventricle, as is seen in the action of poisons, even although the rate and strength of the impulses remain unaltered.

6. The impulses can also be made inefficient when the excitability of the muscle is unchanged by diminishing the strength of the impulse, as is seen in the effects of compressing the tissue between the ventricle and the motor ganglia or of heating the auricles and sinus without heating the ventricle.

7. There is a limit to the extent to which a series of inefficient impulses can raise the excitability of the muscle so that the ventricle can remain absolutely quiescent, even although the impulses still pass to it, when those impulses are sufficiently weakened, as will be shown in the second part of this paper.

The assertion that the rhythm of the heart is normally due to separate impulses discharged from the motor ganglia in the venous sinus, each of which causes a contraction of the cardiac muscle, is not intended to imply that these impulses necessarily travel along simple nerve fibres without passing through one or more interpolated ganglion cells, but only that whatever the nature of the path of the conduction of the nervous impulse may be, separate discharges from the motor ganglia at one end of the path reach the muscle as separate impulses at the other end.

So, too, I have spoken of the motor ganglia in the venous sinus without meaning thereby to assert that these ganglia may not extend slightly beyond the sinus into the auricular septum as asserted by LÖWIT* (for as a matter of fact experiments which I have made expressly for this purpose have convinced me of the truth of his assertion), or that the other groups of ganglion cells found in different parts of the heart may not upon occasion perform motor functions.

The whole question of the action of these different groups of nerve cells deserves separate discussion. In this paper I have purposely abstained from that discussion because my object is to describe certain definite well established facts rather than to enlarge upon all the various topics connected with the theory of the heart's action.

In this first part of the paper, therefore, I have confined myself to the question of the relation of the muscular tissue to the impulses coming to it, and have attempted to prove not only that the normal beat of the heart is dependent upon separate impulses coming to the cardiac muscle, but that, in addition, the muscular tissue is of such a character that if from any cause the impulses discharged from the motor ganglia should become too weak to cause a contraction, the rhythmical action of the heart can still continue though at a slower rate, because each of these impulses, abortive though it is to produce a contraction, increases the excitability of the muscle, and therefore the latter responds rhythmically to every two or three of the impulses coming to it.

This relation between the stimulus and the excitability of the muscle, which in the whole heart is to be regarded as a special safeguard for the maintenance of its rhythmical action and not the prime cause of that action, is on the other hand the chief factor in the causation of the rhythm of the isolated apex of the ventricle.

The large number of investigations on this subject which have been made of recent years may, as far as the rhythm is concerned, be summed up by saying that the muscular tissue of the apex contracts rhythmically under the influence of a sufficient continually acting stimulus, whether that stimulus be electrical, chemical or mechanical. Thus spontaneous rhythmical contractions occur when the constant current or a weak tetanizing current is sent through the tissue, and the rapidity of the rhythm varies

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* PFLÜGER's Archiv, Bd. xxiii., S. 313.

directly with the strength of the current; also fluids containing different chemical substances, especially alkaline substances like sodium hydrate and sodium carbonate, which according to BIEDERMANN[*] increase most markedly the excitability of the striated muscular tissue, produce when supplied to the apex more or less rhythmical contractions. Further, nutritive fluids alone, such as the blood of the animal itself, cause the same effect, provided that in addition a certain pressure exists within the apex cavity; and the effect of this distension of the walls by pressure, which is the *essential point* in this method of producing rhythmical apex contractions, may fairly be classed as a continuous mechanical stimulation. Here, also, according to LUDWIG and LUCHSINGER,[†] the rapidity of the rhythm varies directly with the extent of this pressure.

The facts which have just been urged show that the ventricular muscle has the power of summing up the effects of two, three, or more impulses so as to produce a contraction though each of those impulses is inefficient of itself to cause the muscle to contract. In the same way it is possible to conceive that the muscle should be able to sum up the effects of a series of stimuli which follow one another so closely as to deserve the title of a continuous rather than a discontinuous stimulation. If, then, we have reason to suppose that in the first case the discontinuous stimuli ultimately produce a contraction by increasing the excitability of the muscle, so in the second case we may suppose that a continuous stimulation causes a rhythmical series of contractions by acting in the same way. The difference between the two cases would consist in the extent to which the excitability was increased; in the one, the muscle is not rendered sufficiently excitable to contract without the assistance of a distinct separate stimulus; in the other, the excitability is increased so much that no separate stimulus is required: the muscle, in fact, can be spoken of as capable of spontaneous contraction.

PART II.

ON THE ACTION OF THE VAGUS NERVE.

Ever since the brothers WEBER discovered that stimulation of the vagus nerve caused the heart to stand still, physiologists have been incessantly endeavouring to discover the reason of this standstill, with the result that an enormous mass of facts in connexion with the action of the nerve has been accumulated, and a variety of hypotheses have been suggested to explain these facts.

Thus it was found that the vagus at one time causes complete stoppage, at another only slowing of the heart's rhythm; this fact has led to the view that the complete cessation of beats is simply a prolonged slowing, and therefore that the nerve by its direct action upon the motor ganglia causes the discharges from the ganglia to take place at a slower rate.

tzungsber. d. k. Akad. d. Wiss. (Wien). Bd. lxxxii., 1880.

[†] PFLÜGER'S Archiv, Bd. xxv., S. 211.

Another view, which largely depends for its support upon the action of various poisons, is, that the nerve does not produce standstill by direct interference with the motor ganglia, but because it excites some inhibitory mechanism which is interpolated between these ganglia and the muscular tissue. Both these views have one hypothesis in common, viz.: that the inhibitory action of the nerve is due to the prevention of the discharges from the motor ganglia from reaching the muscular tissue, and not to the action of the nerve upon the muscle itself. In pharmacological literature especially this hypothesis has become so predominant that again and again the statement is made that such and such a poison paralyses or stimulates some part or other of the vagus nerve, when the facts only show in the first case that the vagus is no longer able to produce standstill, and in the second that the poison has by its action caused the heart to beat slower.

Again, a number of observers, notably SCHIFF,[*] have asserted that stimulation of the vagus nerve of the Frog under certain circumstances always causes acceleration of the heart's rhythm, and not slowing: a result which, conjoined with the action of atropia, has led to the supposition that the Frog's vagus contains accelerator as well as inhibitory fibres, and a fresh set of hypotheses has been made with respect to the relative vitality of these two sets of fibres.

Finally, it has been long known that the contractions which immediately follow the standstill caused by vagus stimulation are much weaker than the normal contractions, and NUËL[†] has shown that stimulation of the nerve weakens, at all events the auricular contractions, even without a preliminary standstill.

We have, therefore, according to present views at least, three sets of fibres in the Frog's vagus, viz.: fibres which are capable of slowing or entirely preventing the discharges from the motor ganglia; fibres which accelerate those discharges; and fibres which *in some way or other* diminish the force of the cardiac contractions. In addition, we have various statements that sometimes only one vagus is active, that at certain times of the year the vagus loses all power, &c.

The literature of the subject then affords ample evidence of considerable discrepancy not only as to the explanation of the action of the vagus, but also as to observed facts connected with that action. The reason of this confusion will appear in the following pages, and I will say at present that it is based upon a view of the nature of the action of the vagus, which appears to me to be too much one-sided. Too great prominence has hitherto been assigned to the action of the nerve upon the motor ganglia—too little to its action upon the muscular tissue.

The most striking feature of vagus stimulation is a more or less prolonged standstill of the heart. There is no reason *à priori* to assign this standstill to the action of the nerve upon the rate of the discharges from the motor ganglia rather than to its action upon the muscular tissue of the heart. Either conception is perfectly possible, and its truth can be tested directly by experiment.

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[*] PFLÜGER'S Archiv, Bd. xviii., S. 172.

[†] Ibid., Bd. ix., S. 83.

Thus if standstill is invariably due to a slowing of the rate of the discharges from the motor ganglia, experiment ought naturally to show that the beats which first occur after the standstill are more infrequent than before the nerve was stimulated, and that the subsequent recovery of the rhythm up to or beyond the original frequency takes place gradually and not abruptly. Also a stimulation which is unable to cause complete standstill ought to produce the effect nearest to standstill, viz.: a greater or less slowing of the heart independently of any alteration in the force of the contractions.

If, on the other hand, standstill is invariably caused by the action of the nerve upon the muscular tissue, so that the latter does not respond to the impulses coming to it, then we should expect to find (in accordance with the law of BOWDITCH* that the force of the contractions of the ventricle is not dependent upon the strength of the stimulus); that the first beats which occur after the standstill are very small, that the force of the contractions gradually increases up to or beyond the orignal force, and that the rate of rhythm after the standstill is not necessarily slower than before the stimulation of the nerve. Also a stimulation which is unable to cause complete standstill ought to produce the effect nearest to standstill, viz.: a more or less marked diminution in the force of the contractions independently of any alteration of rhythm.

The majority of physiologists have hitherto accepted the first of these views as the *invariable* explanation of the standstill of the heart caused by the action of the vagus nerve, and in accordance with this view always speak of the vagus as causing *inhibition* of the heart's action.

In this present paper I propose to give evidence that, at all events in the heart removed from the body, the second view is the true one, and in accordance therefore with this conception I have throughout spoken of the vagus as causing *quiescence* of the heart.

It is of course possible and indeed probable that both these views may be true, and that therefore the vagus may cause standstill sometimes by the one action and sometimes by the other. Whether this is so or not I am not at present able to judge; I can only say that in the suspended heart I have never as yet seen any reason to suppose that the vagus is capable of producing such a prolonged slowing of the rate of the discharges from the motor ganglia as to entitle that slowing to the name of standstill of the heart; while on the other hand the quiescence of the heart produced by the action of the nerve upon the force of the muscular contractions may last for a considerable time.

Again, physiologists and pharmacologists have hitherto considered that the inhibitory action of the nerve is abolished when its stimulation no longer causes standstill or slowing of the heart. This view will clearly require modification if the quiescence of the heart is due to the action of the nerve upon the force of the contractions rather than upon the rate of the discharges from the motor ganglia. Indeed, it will be no

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* LUDWIG'S Arbeiten, 1871, S. 139.

longer possible to speak of the inhibitory action of the nerve apart from its whole action. In every case, then, it will be necessary, before we can assert that its action is abolished, to determine that the vagus is no longer able to cause any alteration, not only in the rate of rhythm, but also in the strength of the cardiac contractions.

I use the term *alteration*, rather than *diminution*, of the force of the contractions advisedly, for it will not be sufficient to show that the vagus is no longer able to cause any diminution; it must also be shown that it is unable to cause any *increase* of the force of the beats, because, as will be seen, the most permanent and invariable function of the vagus (in the heart removed from the body, at all events) is to *increase the strength* of the muscular contractions.

Before describing the results of the experiments upon which the assertions made above are based, it is desirable to say a few words about the conditions under which they were conducted. In every case the vagus nerve was carefully dissected out nearly or quite up to its ganglion and placed on the electrodes, so that there was about a quarter of an inch of free space between the electrodes and the nearest tissue. The experiments were made in the months of February, March, April, May, June, and July of this year, and the animals used were Toads or large, in most cases freshly caught, specimens of *Rana temporaria*. The left vagus was invariably used, and I can safely assert that in no single instance has its stimulation failed to produce some effect upon the heart, although, as will be seen, that effect is not always the same.

In my first experiments the heart was suspended by means of KRONECKER's forceps attached to the bulbus aortæ, the ventricle was slit open from base to apex, and the contractions of the ventricle alone were registered by means of a lever which was attached by a thread to the very apex. The typical effect of vagus stimulation under these circumstances is seen in Plate 67, fig. 4, which shows that by this method complete inhibition can be obtained; that when the ventricle begins to beat again the first beats are very small, but that they soon begin to increase in size and shortly after the end of the stimulation reach a maximum which is very muchgreater than be for the stimulation. From this maximum the force of the contractions very slowly diminishes to the original size.

I give this curve simply to show that the nature of the phenomena seen upon vagus stimulation when the clamp is used cannot be ascribed to the use of the clamp, for, as the subsequent curves will show, there is no essential difference between the curves produced whether the clamp is absent or present. As, however, the results are much more interesting when the auricular and ventricular beats are simultaneously recorded, I will proceed at once to describe the effects of stimulating the nerve when the clamp is in the auriculo-ventricular groove and is tightened so as just to hold the tissue; the ventricle is therefore beating synchronously with every beat of the auricles.

As the large majority of my experiments have been made with the clamp in this position, I am enabled to draw conclusions from a very great number of curves, each one of which represents a separate stimulation of the vagus nerve. Upon examination of these curves taken as a whole it is seen that although stimulation of the vagus produces different effects at different times, these effects can all be reduced to the three following types :—

1. Complete quiescence of both ventricle and auricles, followed by contractions which at first are scarcely visible, but which rapidly increase in size until at the maximum they are much greater than before the stimulation of the nerve ; from this maximum they very gradually decrease until the original force of contraction is again reached (Plate 67, fig. 6, curve B).

2. During the stimulation no quiescence of either ventricle or auricles, but simply a diminution of the size of the contractions, followed by a rapid and marked augmentation of the contraction curve beyond the original height, and then a slow gradual diminution to the size before the nerve was stimulated (Plate 67, fig. 6, curve A ; Plate 68, figs. 8, 9, &c.).

3. No primary diminution, but from the commencement of the stimulation the beats increase in force, and after a time gradually return again to the original size (Plate 68, fig. 10). Between these three types every conceivable variation may occur, so that a series of curves may be selected in which no line of demarcation can be drawn between complete primary quiescence, or to use the usual term, inhibition, on the one hand, and a simple primary augmentation of the contraction force on the other. Such a series is to a certain extent represented by Plates 67, 68, figs. 6, B, 6, A, 8, 9, 10.

Further, the curves show that not only does the height of the contractions vary in a graduated series from the null point on the one side up to a height much greater than before the stimulation on the other, but also that this variation is independent of the rate of rhythm. In the majority of experiments the rhythm is seen to be decidedly accelerated by the stimulation of the nerve, in others no alteration of rhythm can be perceived. Thus in Plate 67, fig. 6, B, as soon as beats appear after the primary quiescence they show a slight acceleration of rhythm; so, too, in such figures as Plate 68, figs. 9, 10, where the nerve stimulation produced a primary diminution and a primary augmentation of the force of the contractions respectively, the rhythm is seen to be accelerated or unaltered from the commencement of the stimulation.

In all cases where acceleration occurs, the return to the original rate is slow and gradual, in the same way as the return to the original contraction force.

Another proof that this quiescence, which is caused by stimulation of the vagus, is not due to any interference with the rate of discharge from the motor ganglia, is most markedly shown by the fact that stimulation of the nerve often reduces the ventricle to a state of complete quiescence, while the auricles not only continue beating but beat with greater rapidity than before. Thus we may have complete quiescence of the ventricle with the auricles so nearly quiescent that their accelerated contractions are barely visible on the curve (Plate 67, fig. 7), in other cases with the auricular contractions more and more plainly visible (Plate 67, fig. 5), until finally we may obtain curves in which the ventricular beats are diminished down to complete quiescence, while the auricular are, from the commencement of the stimulation, augmented in force (Plate 67, fig. 11). The reverse, however, never occurs—I have never seen quiescence of the auricles with simultaneous primary augmentation in the force of the ventricular contractions.

A careful examination of the large number of curves which I possess will, I venture to think, afford clear evidence of the truth of the following statements :—

1. The effect of stimulation of the vagus nerve, which is most constant, is an augmentation of the force of the contractions, both of the auricles and ventricle; an augmentation which cannot be regarded as a mere rebound, so to speak, from a previous diminution of the force of the contractions.

2. In the case of both auricles and ventricle a series of curves can be obtained which show that stimulation of the vagus is able to produce a perfect gradation of effects upon the strength of the contractions irrespectively of the rate of rhythm : a gradation which extends from a simple augmentation of the force of the contractions as the result of the nerve stimulation, down to so great a primary diminution of that force that the contractions are no longer visible, and therefore quiescence results from the stimulation of the nerve.

3. The auricles and ventricle may be in different phases of this gradation at the same moment, with the qualification that, if quiescence be called the lowest and primary augmentation of the force of the contractions, the highest term of the series, the ventricle, is always in a lower phase than the auricles.

The auricular curves are different from the ventricular in one or two other details; thus the maximum effect on the force of the contractions is reached more quickly after the end of the stimulation in the former than in the latter, so that the auricular beats have already reached their maximum, while those of the ventricle are still increasing. (Plate 68, figs. 9, 10). The auricles also return to their original force of contraction much more quickly than does the ventricle ; as is seen in Plate 68, figs. 9, 10. The same figures show another peculiarity of the auricular curves, which is especially characteristic of the Toad's heart, viz.: after the maximum force of contraction has been reached, the contractions gradually diminish to a height less than before the stimulation, and then again increase up to or slightly beyond their original height : a peculiarity which I have never as yet observed in the ventricular curves.

I have already said sufficient to justify the assertion which I made above, that the vagus is never absolutely inefficient ; it always produces some effect or other upon stimulation, and I have described a variety of the effects which it does produce. Hitherto I have considered the curves as a whole, and have shown that the different effects seen when the nerve is stimulated—as far at least as the action of the nerve upon the force of the contractions is concerned—can all be arranged in a graduated series in which no line of demarcation can be drawn separating one especial effect from another. It remains to be seen whether a similar gradation can be produced by a series of stimulations in each separate experiment, and, if so, what conditions are necessary for its production.

In the first place, stimulation of the vagus is much more likely to reduce the force of the contractions down to complete quiescence immediately after removal of the heart from the body, than after it has been suspended for some time; and, indeed, a variety of facts can be brought together which all point in one direction, viz.: that the production of quiescence by stimulation of the nerve stands in intimate relation with the existence of a suitable nutrition of the heart. Thus it frequently happens that typical inhibition can be obtained before the heart is removed from the body, although after it has been suspended it is impossible by any strength of stimulation to reduce the strength of the contractions to such an extent as to cause standstill.

Again, experiments made in June for this express purpose show that if the heart be suspended as quickly after removal as possible, and the circulation be kept intact till the last moment, then, with a certain strength of current the first stimulation caused absolute quiescence; the next, although of the same strength and length, only a primary diminution of the force of the contractions, and with succeeding stimulations of the same strength and the same length and following upon one another at as nearly as possible the same intervals of time, there was a decided tendency for the primary diminution of the contraction force not only to diminish in extent, but also to affect a smaller number of beats, until at last the stimulation caused an increase in the force of the contractions with a previous diminution of only one or two or even no beats at all (see Plate 68, fig. 9).

Indeed, it can be said that in the course of each separate experiment the successive stimulations of the nerve tend to show the same gradation of effects from complete quiescence followed by augmentation on the one hand, up to a simple primary augmentation of the force of the contractions on the other, as has already been pointed out from the consideration of the whole number of curves.

In the second place, the examination of my curves indicates that the nature of the special nutritive condition of the heart, upon which the phenomenon of vagus standstill depends, may possibly be given by the chemical examination of a large number of hearts at different times of the year, for as far as I can judge from this first series of experiments, standstill is more likely to occur upon vagus stimulation, after the heart has been cut out and suspended, in the early spring months of the year than in the summer months. Indeed, in some cases in March, stimulation was still able to cause quiescence even after the heart had remained suspended for nearly two hours.

In Plate 68, fig. 12, curve A, I give an example of quiescence of the auricular contractions caused by vagus stimulation, which was obtained about an hour and a half after removal of the heart from the body, during which time the vagus had been stimulated nine times. In curve B of the same figure, which is a continuation of curve A (except that between A and B the vagus was stimulated so that the beats at the commencement of the curve B represent the maximum contractions due to that stimulation) the vagus was stimulated with the same strength of current at frequent intervals, so that the auricle was not allowed to reach the maximum of its contractions. The curve shows well not only that quiescence was at first produced, but also the resolution of that quiescence into barely visible rhythmical contractions.

From these experiments the conclusion can be drawn that the impairment of the nutrition of the heart caused by the removal of the circulation is not of itself sufficient to take away from the vagus the power of reducing the force of the contractions to the null point; but rather that, in addition, some unknown factor, upon which this diminution of the beats depends, is more powerful at one time than at another.

In all that has been said hitherto, no evidence has been given that the vagus is able to cause any slowing of the rate at which impulses are discharged from the motor ganglia, but, on the contrary, clear evidence that it is able to accelerate that rate, and although the large majority of my curves point directly to that conclusion, yet occasionally a curve is obtained in which an undoubted slowing has taken place. In some cases it is possible that the slowing may be due to the failure of certain impulses to produce a contraction as is undoubtedly the case in the slower ventricular beats seen in Plate 69, fig. 15, where the stimulation of the vagus caused the ventricle to beat for a time synchronously with every second auricular beat. On the other hand, such slowing as is seen in Plate 67, fig. 13, where the recovery of the rate of rhythm after the end of the stimulation was gradual both in auricles and ventricle, cannot be explained, except upon the supposition that the nerve stimulation had slowed the rate of the discharges from the motor ganglia.

Indeed, the fact that the vagus is able to cause a slowing of the heart's rhythm is so well established as not to admit of doubt; all I contend for is, that that slowing is not always a *sufficient* explanation of the phenomenon known as vagus inhibition.

Further, I venture to suggest that the hypothesis that the vagus contains two sets of fibres going to the motor ganglia, of which the one causes upon stimulation acceleration, the other a slowing of the rate of discharge from the motor ganglia, is no more necessary than the similar supposition that the nerve contains two sets of fibres supplying the muscular tissue, of which the one causes a diminution, the other an increase of the force of the contractions.

The experiment from which Plate 67, fig. 13, is taken shows clearly the rapid diminution of the power of the vagus to produce a slowing of the heart when the heart is removed from the body, and suggests that that removal affects the action of the nerve upon the rate of rhythm in the same direction as it affects its action upon the force of the muscular contractions. Thus, in Plate 67, fig. 13, the effect of the first stimulation of the nerve immediately after the heart was suspended, is given. The second stimulation, about a minute after the first, still caused a decided slowing, but very much less marked in extent than before. The third stimulation showed that the nerve was no longer able to produce any slowing, although the effect upon the force of the contractions was as great as in the previous stimulations.

On the influence of the vagus nerve upon the excitability of the ventricular muscle.

Seeing that the vagus acts so powerfully upon the force of the muscular contractions, it is natural to expect that it also has an influence upon the excitability of the muscular tissue. The nature of this influence can be determined by the following considerations.

In the first part of this paper I have attempted to prove that the ventricle beats synchronously with the auricles when a due relation subsists between the strength of the impulses from the motor ganglia and the excitability of the ventricle. When, however, the strength of the impulses is diminished either by clamping or by the action of heat upon the motor ganglia, this due relation is destroyed and the ventricle either beats synchronously with every second, third, or more contractions of the auricles or else does not beat at all. If under these circumstances the excitability of the ventricular muscle be increased, then the relation between the strength of the impulses and the muscular excitability, which is necessary for synchronous contractions, will be restored, either fully or partially, according to the extent to which the excitability is raised. On the other hand, if the excitability be lowered then that due relation will be still further destroyed, and each beat of the ventricle will follow upon a still larger number of auricular beats than before.

In accordance with this method of testing the excitability, experiment proves that the vagus may cause a diminution of the excitability of the ventricle during the first part of its action, during, therefore, the period when it is known to diminish the force of the contractions; and may cause an increase of excitability when it augments the force of the contractions.

I will now proceed to give the facts upon which the truth of this proposition depends, and will first examine the action of the nerve when the clamp in the auriculo-ventricular groove is tightened so as to make the ventricle beat synchronously with every second or third beat of the auricles.

In this case the most striking effect of stimulation of the nerve is the alteration which is produced in the rate of the ventricular rhythm rather than the variations in the force of the ventricular contractions. It is true that the nerve stimulation may still cause the force of the ventricular contractions to be diminished and augmented in the same way as already described, but neither the diminution nor the augmentation are so manifest as when the ventricle is beating synchronously with every auricular beat; this is doubtless largely owing to the greater size of the ventricular contractions in consequence of their slower rate of rhythm. This is, however, not the main distinction between the two cases.

When the ventricle is beating synchronously with every beat of the auricles we have seen that this synchronism continues during the period of diminution as well as during the period of augmentation of the force of the contractions; when, however, the ventricle is beating synchronously with every second beat of the auricles the same law does not necessarily hold; in many cases stimulation of the nerve causes a sudden alteration in the relation of the ventricular to the auricular rhythm.

In the first place the rhythm of the ventricle becomes synchronous with that of the auricles for a longer or shorter period. This period of synchronism corresponds to the time when the contractions would have been greatest if the ventricle had been beating in due sequence with the auricles; it may therefore occur sometimes during the stimulation, at other times some considerable time after the end of the stimulation. Thus in Plate 68, fig. 14, the ventricle was beating synchronously with every third beat of the auricles ; for a definite period after the end of the stimulation of the vagus its rhythm was changed so that it beat synchronously with every second instead of with every third auricular beat. Similar figures might be given to show how the vagus causes the ventricle to beat synchronously with every beat of the auricles instead of with every second beat.

These curves show that stimulation of the vagus increased the excitability of the ventricle sufficiently to enable it either wholly or partially to recover for a time the normal sequence of its beats upon those of the auricles, and also that increase of excitability took place at the same time as the maximum increase in the force of the contractions.

In the second place, stimulation of the vagus may cause a primary slowing of such a character that a variable number of auricular beats are dropped out from the ventricular curve, and then afterwards the ventricle may regain its original rhythm or even become synchronous with every beat of the auricles (Plate 69, fig 15).

Further, this slowing of the ventricle may not only be coincident with an acceleration of the auricles, but in fact is more likely to occur when stimulation of the nerve causes a marked auricular acceleration. This, at first sight paradoxical, assertion follows necessarily upon the arguments put forward in the first part of this paper, if it be assumed that the vagus diminishes the excitability of the muscle at the same time that it depresses the strength of the contractions ; for the greater rapidity of the auricular contractions denotes not only quicker but also weaker impulses from the motor ganglia ; and it is clear that, if the impulses be weakened at the same time that the excitability is lessened, a larger number of impulses will require to be summed up in order to produce a contraction, than if the excitability only is diminished.

Again, if the clamp be placed across the middle of the ventricle, and the contractions of the base and apex of the ventricle be registered, the same diminution of excitability can be observed. Thus, when the apex is beating synchronously with every second or, as in Plate 69, fig. 16, with every fourth beat of the base, then coincident with the diminution and acceleration of the contractions of the base, which occur during the stimulation of the nerve, the beats of the apex take place at a slower rate than before, because each apex contraction corresponds to a larger number, and therefore probably to a longer series of base-contractions than before the stimulation.

Thus in the instance given (Plate 69, fig. 16) I have marked the corresponding contractions on the two sides with the same numbers, and it is seen that before the stimulation of the vagus the relation between the rhythm of the apex and the base was such that each apex-contraction corresponded to four base-contractions. During the stimulation the beats of the base became weaker and more frequent, while those of the apex were so much slower that only two beats took place, of which the first corresponded to twelve beats of the base and the second to eight beats. After the stimulation the original relation between the rhythm of the apex and base was restored, so that each apex-contraction again corresponded to four base-contractions. Curves similar to this are those most frequently obtained with the clamp in this position.

These facts alone make it probable that the vagus lowers the excitability of the muscle at one time and increases it at another. Now although this diminution of excitability occurs at that period of the vagus action when the stimulation of the nerve is known to cause a diminution of the force of the contractions, yet it is clear that the connexion between the two is relative and not absolute. As has been seen, when the contractions are synchronous with the impulses, stimulation of the nerve is able to reduce the force of the contractions to the vanishing point, and yet up to the last moment of visible contractions every impulse is followed by a contraction—*i.e.*, the excitability does not in this case fall sufficiently, in relation to the strength of the impulses, to prevent the muscle from responding to every impulse, even although the contractions become so small as to be almost invisible. Since, therefore, we have reason to suppose that the vagus stimulation does diminish the excitability at the first period of its action—*i.e.*, at the time when it diminishes the force of the contractions—and yet leaves the cardiac muscle still able to respond synchronously to the impulses, provided that the latter have not been diminished in strength, the inevitable conclusion is, that normally the excitability of the muscular tissue is greater than is necessary to enable it just to respond to every impulse. In other words, a range is allowed within which the excitability may fluctuate without thereby preventing the synchronous response of the muscle to the impulses, as long as the strength of the impulses does not diminish. As a rule the lower limit of this range is not passed, in consequence of vagus stimulation, when the strength of the impulses is not interfered with; and therefore in these cases the diminution of excitability caused by the vagus is not rendered visible. This diminution can, however, be made manifest by first reducing the excitability of the muscle to some extent. Thus if muscarin be applied to the ventricle alone, when it is beating synchronously with every beat of the auricles, and the vagus be stimulated when the muscarin has, without altering the synchronism, greatly reduced the force of the ventricular contractions, then the stimulation may cause the ventricle to beat synchronously with every second auricular beat for a short period, showing that the stimulation of the nerve, added to the effect of the muscarin, has reduced the excitability of the ventricle to a greater extent than the muscarin alone had done.

Hitherto I have chiefly spoken about the *diminution* of excitability caused by the vagus, and will therefore now proceed to give further proofs that the vagus increases the excitability of the muscle at the same time that it augments the force of its contractions. The most striking proof of this fact, in addition to what has been already said, is given by the effect upon the ventricle of stimulation of the nerve when the ventricle is rendered quiescent either by heating the sinus and auricles or by tightening the clamp in the auriculo-ventricular groove.

In both cases the effect is the same, and is well exemplified in the series of Plate 69, fig. 17, curves A, B, C, D, which represent the effects of vagus stimulation in a single heating experiment. Plate 69, fig. 18, shows the corresponding effect when standstill is produced by clamping.

A consideration of these and similar curves shows that when, whether from the effect of clamping alone or from the combined effects of heating the motor ganglia and clamping, the ventricle is not beating, stimulation of the vagus causes a series of ventricular beats which commences as a rule some little time after the end of the stimulation. These contractions last for a certain time, and then the ventricle again becomes quiescent, until, with a new stimulation of the nerve, a new group of contractions is produced. Again, the contractions in each of these series are synchronous with the auricular contractions, and are strong vigorous contractions such as would have been produced had the ventricle been beating when the nerve was stimulated. This shows that the beats in each of these series, as far as their mere occurrence is concerned, are due to motor impulses which pass to the ventricle at the same rate as to the auricles, and therefore that during the previous quiescence the motor impulses were still passing to the ventricle, but were not able to produce any contractions. Again, these series of contractions do not as a rule take place during the stimulation of the nerve, but some time after the stimulation, and the position of each series corresponds closely to the position of the maximum contractions seen after the end of the stimulation of the nerve in the ordinary curve of vagus stimulation. An explanation, therefore, of this action of the vagus is clearly given by the supposition that by the action of the nerve the excitability of the ventricular muscle is increased at the same time that the force of the contractions is increased. Hence as soon as the excitability is sufficiently raised the motor impulses which have been rendered inefficient by the action of the clamp, or by the action of heat upon the motor ganglia, are able to cause contractions: and these contractions last until by the passing off of the vagus influence the excitability has again sunk too low for the impulses to be able to cause a contraction.

The beats in each series are not necessarily synchronous with every impulse from the beginning to the end of the series. As far as I have seen they are usually synchronous in the middle beats of each group, but the series may begin and end with one or two beats which are synchronous with every second auricular contraction instead of with every one. This latter form of curve simply denotes that the rise and fall of the excitability has not been so abrupt as in the first case, as can easily be seen by drawing a diagram to represent the curve of excitability in relation to the strength of the impulses.

Further, it was noticed in the course of the experiment from which Plate 69, fig. 17, is taken, that after a while the same strength of stimulation caused a series of ventricular contractions when the auricles and sinus were not heated, and no effect whatever when the nerve was stimulated during the time of heating; in other words, when the strength of the impulses was made still weaker by the heating, the rise of excitability caused by the vagus stimulation was not sufficient to make those impulses efficient to cause ventricular contractions.

Also (as is seen in Plate 69, fig. 17, B, C, D) during the course of the experiment each series of ventricular contractions lasted on the whole a shorter and shorter time, and commenced later and later after the end of the stimulation—a fact which is to be explained by the progressive exhaustion of the whole heart.

On the action of the vagus upon the ventricular muscle when that muscle is beating with alternately strong and weak beats.

When the heart is suspended in the manner already described and the ventricle is beating in due sequence with the auricles, it frequently happens that, although the auricular contractions are all equal in force, the ventricle is seen to be beating with alternately weak and strong contractions, as is shown in Plates 67, 69, figs. 7, 19, 21. If, now, the vagus be stimulated, this alternation in the size of the ventricular contractions disappears simultaneously with the increase in the force of the contractions caused by the nerve stimulation, and reappears again as the contractions return to their original size; and if the nerve be stimulated a second time, while the contractions are still equal in force, then with the primary diminution in force caused by the stimulation the alternately weak and strong beats again appear, and again give way to beats of equal force with the subsequent augmentation of the force of the contractions. This is well shown in Plate 69, fig. 19.

It is difficult to come to any final conclusion upon the causation of this alternation in the strength of the ventricular contractions, though it is possible to somewhat limit the area of discussion. In the first place, they are clearly local in origin, for by clamping across the middle of the ventricle the same alternation in strength is sometimes seen in the beats of the apex, while those of the base of the ventricle remain of the same strength throughout (Plate 69, fig. 20). Here, too, the vagus removes this alternation in the same way as in the case of the whole ventricle. Also, as far as I have seen, this phenomenon is confined to the ventricle. I have never seen any appearance of this alternation of contraction-force in the auricular beats. In the second place, they are not necessarily due to the clamp, for the same alternation may occur when the ventricle alone is suspended and no clamp is used.

Further, the sizes of these alternately weak and strong contractions are clearly dependent upon each other, so that, other things being equal, the more marked the diminution in the weak contractions the more marked also is the increase in the strong ones which alternate with them; therefore if, as sometimes happens, the alternation disappears, temporarily though it may be, when the ventricle is moistened with normal saline solution, then the size of the equal contractions is intermediate between the size of the alternately weak and strong ones. This relation is beautifully seen when, as is apt to occur, a secondary rhythm appears in these contractions, as is shown in Plate 69, fig. 21. Here it is seen that the weak contractions increase to a maximum while the strong contractions diminish simultaneously to a minimum, and *vice versâ*, the minima of the weak correspond to the maxima of the strong.

Now we know from the experiments of BOWDITCH[*] that the force of the ventricular contractions is independent of the strength of the stimulus. The explanation, therefore, of this alternation in the force of the contractions must be sought for in the

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[*] *Op. cit.*

muscular tissue itself, and it seems to me that the most probable explanation is that a larger amount of tissue contracts when the beats are large than when they are small, and that, therefore, in all probability, certain portions of the ventricle respond only to every second impulse, while other portions respond to every impulse. The observations of AUBERT* show that by the direct action of a blow a circumscribed area of the ventricular muscle can be made to remain quiescent, while the rest of the ventricle is contracting rhythmically. I am inclined, therefore, to suggest that, owing to some cause in the manipulation, such as cutting open the ventricle, or some other cause which affects the ventricle unequally, the excitability of the ventricular muscle is at the time not absolutely the same throughout, so that, although the impulses remain the same in strength, yet certain parts which possess a lower excitability are able to respond only to every second impulse, while the rest of the tissue responds to every impulse. In this way, if the strength of the contractions depends upon the amount of tissue contracting, we see not only that every second beat must be larger, but also that the size of each strong contraction must vary inversely as the size of each corresponding weaker contraction.

Upon this assumption the action of the vagus admits of easy explanation, for it is evident that the alternation disappears during the time when we have reason to suppose that the maximum rise of excitability caused by vagus stimulation is produced, and reappears when the normal excitability is regained, or when the excitability is lowered below the normal by the action of the vagus. If, therefore, we look upon this alternation in the strength of the contractions as due to a local lowering of excitability, so that certain portions contract once only to every two contractions of the rest of the ventricle, then it is clear that as soon as the excitability of these parts is raised sufficiently for them to respond to every impulse, every contraction must become as strong as every other contraction, for every part of the ventricle will contract to every impulse. On the other hand, if the general excitability is lowered there must be a greater tendency for this alternation to occur; thus, it sometimes happens that when the ventricle is beating with contractions of equal height, stimulation of the vagus causes not only a lowering of the height of the contractions, but also a temporary appearance of alternately strong and weak beats.

Also, when there is a great difference in size between the consecutive contractions, it is possible for the stimulation of the vagus to fail in increasing the excitability of all portions of the ventricle to an extent sufficient to cause them to respond synchronously with every impulse, so that the alternation does not entirely disappear in any part of the curve, although the difference in height between consecutive beats is not so great during the period of augmentation of the force of the contractions as before the stimulation; in other words, the nerve stimulation has sufficiently increased the excitability of some portions of the affected area, though not of the whole of it.

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* PFLÜGER's Archiv, Bd. xxiv., S. 357.

On the action of the vagus nerve upon the tonicity of the heart.

The question whether any particular influence upon the heart causes the relaxation of the tissue to become greater or less than the normal, is always attended with the difficulty that it is impossible to say what the normal amount of relaxation really is. Thus with the ordinary rate of rhythm the amount of relaxation is always greater with a slower beat than with a quicker one, a fact which is to be explained by the simple supposition that the condition of contraction has not absolutely passed away when the next contraction takes place unless the contractions follow at a slower rate than the normal; so that for this reason alone a greater relaxation must occur if the vagus causes a slowing of the beats and a less relaxation if the beats are made more frequent. On the other hand, if the tissue is relaxed when the rhythm of the beats is accelerated or unaltered, we may then fairly speak of an increased relaxation of the tissue, and if the relaxation is less, when the rate is the same or slower, then we may speak of a diminished relaxation of the tissue. Thus, for instance, every experiment shows that muscarin and lactic acid truly increase, while sodium hydrate and digitalin truly diminish the normal relaxation of the tissue.

Now in endeavouring to estimate the extent of relaxation in a hollow muscle like the ventricle, it is clear that the measurement of its capacity will afford a better chance of obtaining a visible amount of relaxation than any mere linear measurement such as is given by the method described above. I have therefore endeavoured to find out whether the vagus really causes any relaxation of the ventricle by the following method:—

The instrument which I have used is a modification of Roy's tonometer* and is represented in Plate 70, figs. 22, 23. The glass chamber A is in connexion by means of a metal tube with the recording chamber B, to the lower end of which the membrane used by Roy is tied and within which the piston attached to this membrane is able to move up and down and so to cause corresponding movements of the lever. The funnel and tap on the top of the metal chamber B is for the purpose of filling this chamber and the connecting tube C with oil. The tap in the tube C enables us to free this tube from air; and somewhat below the middle of the chamber B is a tap which allows oil to be drawn out of the apparatus by means of the indiarubber tube D, and so to exhaust the pressure within the whole apparatus and raise the lever to the horizontal position as described by Roy in the original description of his tonometer. The glass chamber A is provided with a vulcanite stopper which fits absolutely tight, so that any variation in the contents of the chamber A must be transmitted to the chamber B through the tube C, and cause a corresponding variation in the position of the lever. There is no necessity that the chamber A should be filled with oil; it may be filled with normal salt solution or any other fluid without preventing the working of the membrane, because the lower extremity of the chamber B is on the same level with the top of the chamber A, and therefore the oil in the chamber B will not be contaminated by the salt solution in A.

The vulcanite stopper, of which a separate drawing is given on Plate 70, fig. 23, contains five holes of exactly the same diameter; small portions of a piece of metal tubing which has been ground accurately to fit these holes can be inserted into one or more of them, according to the requirements of each experiment, and those

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* Journal of Physiol., vol. i., p. 452.

holes which are not required can be closed absolutely by vulcanite plugs, of which one is represented in the drawing.

Different instruments can be fixed into any of these pieces of metal tubing, and so the apparatus be made available for a variety of experiments, either upon the whole heart or on the ventricle or apex alone.

In the figure the vulcanite stopper is represented when fitted up for the purpose of investigating the action of the vagus nerve upon the whole heart when an artificial solution of blood is flowing into it.

The metal cannula E, which fits tightly into any one of the pieces of metal tubing, is tied into the commencement of the venous sinus—the small glass cannula in the middle is fixed into one of the aortic trunks; the plate F is a cork plate fixed on a piece of thin metal, which is soldered to its corresponding piece of metal tubing. This piece of tubing can either be sealed hermetically or be left open and attached to a manometer, in order to observe the pressure within the apparatus. The glass tube G, which is hermetically sealed at both ends, carries within it a pair of platinum electrodes, bent at their extremities into the form of a loop. Two very small holes are blown exactly opposite to each other in the middle of this tube, and the electrodes are fixed within it so that the line joining the two holes is just below the lowest point of the electrodes. Through these holes the vagus nerve can be drawn by a thread attached to its extremity, and by pinning the end of the thread on to the cork slab F, the nerve will remain always in contact with the electrodes. Since the glass tube which contains the electrodes is closed both above and below, the air within it does not escape, and therefore the part of the nerve which lies on the electrodes is during the whole time of the experiment surrounded by air, although the heart itself is plunged in normal salt solution. In this way all escape of current is avoided.

While the heart is still within the body the arterial cannula is tied into the right aortic trunk, and the metal venous cannula into the commencement of the venous sinus (this cannula has an internal diameter of fully 3 millims.). The two superior venæ cavæ are tied without injuring the vagus nerve on the left side, the roots of both lungs and the left aortic trunk are ligatured, the lungs are cut away, and the heart with the left vagus intact is removed from the body. The cannulæ are then filled with blood solution and slipped on to their respective pieces of metal tubing; the heart is fixed in its place by means of pins stuck through the œsophagus and surrounding tissue into the cork slab F. The vagus nerve is then gently drawn through the glass tube over the electrodes and fixed by means of a pin through the thread attached to its extremity.

The vulcanite stopper is now placed in position in the chamber A, and the superfluous salt solution allowed to escape through that hole, which is afterwards closed by the vulcanite plug, as shown in the figure. The oil is now allowed to run out of the indiarubber tube D until the lever is horizontal, and the tracing can then begin.

The arrangement for keeping constant the pressure of the fluid sent into the heart is the same as already described in my former paper.* When the lever is made horizontal by the running out of the oil, it is found that the negative pressure in the chamber A amounts to 5 centims. of water. Therefore the pressure of the fluid in the venous side of the heart is 5 centims. of water, when the level of the blood solution is the same as that of the extremity of the cannula in the venous sinus.

The curves obtained by this method (Plate 70, fig. 24) are sufficiently striking, but unfortunately not easy to interpret. From the nature of the arrangement it is clear that the beats in the curves represent the changes in volume of the whole heart when beating with a blood supply at a constant pressure. Therefore when the auricle contracts that contraction ought not to be registered at all, for it causes simply a transference of fluid from one part of the heart to another, unless by the contraction a certain amount of fluid is thrown backwards along the inflow tube. Again, when the ventricle

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* *Op. cit.*

contracts the curve registers not the amount of fluid thrown out by that contraction, but the difference between that amount and the amount which has flowed into the auricles during the time of that contraction; and this latter amount will depend upon the extent to which the auricles have completed their relaxation when the ventricle begins to contract. It seemed to me, therefore, impossible to come to any satisfactory conclusions when auricles and ventricle were both beating. In consequence, I have attempted to prevent the auricles from beating by raising the pressure of the blood solution flowing into them sufficiently high, and as Roy[*] has asserted that a pressure of 15 centims. is almost sufficient to prevent their contracting, I have always used a higher pressure than this, and have always carefully observed that there were no movements of the auricles visible to the naked eye or registered on the tracing before I proceeded to stimulate the vagus nerve. In this way I have seen most distinct relaxation of the ventricle upon vagus stimulation without any slowing of the rate of rhythm. Thus Plate 70, figs. 25 A and B, represents two cases taken from the same heart as fig. 24; the pressure within the auricles was 28 and 29 centims. of water respectively, and as is seen by the gradual slope of the curve in fig. 25 B as a whole, there was a slight amount of leakage from some part in consequence of the high pressure; still the curves show clearly enough that the contractions which occur when the nerve is stimulated start from a lower level of the lever; and as no variations in the condition of the auricles was to be noticed, it seems to me that the only explanation of such curves as these is that the vagus causes a true relaxation of the muscular tissue of the ventricle, and in all probability of the auricles as well.

If, now, we turn to the consideration of the curves obtained by the method of suspending the heart, we see decided indications of the same fact; in many of the curves it can be noticed that with the diminution of the beats caused by vagus stimulation, the diastolic line formed by joining the points of greatest relaxation falls very slightly nearer to the abscissa than before the stimulation (Plate 69, figs. 15, 17 A, &c.).

Another peculiarity can be seen in many of these curves, which is sometimes much more pronounced than at other times, viz.: that with the increase in the force of the contractions, the diastolic line is raised higher above the abscissa; this is well shown in Plates 68, 69, figs. 10 and 17 A. This diminution of relaxation between the beats may be due to an increased tonicity of the ventricle, coincident with the increase in the force of the contractions, or it may be caused mechanically by the arrangement of the apparatus. Thus, when the clamp is not very tight the movements of both auricle and ventricle are transmitted mechanically through the clamp to the other side, so that the contraction of the ventricle not only pulls the lower lever upwards but also slightly moves the upper lever downwards, and the reverse with the contractions of the auricles; therefore, if the ventricular contraction comes before the auricular lever has reached its lowest position of relaxation, then the pull of the ventricle will not only prevent the auricular lever falling any lower but will also increase the size of the next auricular contraction. This mechanical influence of the ventricular contractions is seen in such cases as Plate 68, fig. 14, where the ventricular beats have produced the appearance of strong and weak beats in the auricular curve. It is possible then to explain the rise in the diastolic line, both of auricles and ventricle, which often occurs after stimulation of the vagus nerve by the fact that owing to the increase in the force of the contractions, a stronger mechanical effect is produced upon the contractions of that part of the heart

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[*] *Op. cit.*, p. 469.

which is situated on the opposite side of the clamp. That this will help to explain many of the cases of diminished relaxation after vagus stimulation there can be no doubt. Still, I have seen cases in which such an explanation does not seem sufficient, especially when the ventricle is beating with alternately strong and weak beats.

Further, such an explanation will not account for the peculiar shape of the diastolic line in the series of ventricular beats which occur as a consequence of vagus stimulation when the ventricle has been reduced to standstill either by tightening the clamp or heating the auricles and sinus. The series of curves given in Plate 69, figs. 17 and 18, show, firstly, that when the ventricle is made to beat with the same rhythm as the auricles the relaxation between the beats is very much less than that of the quiescent ventricle; and, secondly, that the muscle relaxes less between consecutive beats near the commencement of each group of beats than later on in that group, although as is seen in curve C, fig. 17, the rate of rhythm has remained the same throughout the series of beats. This diminution of relaxation at the commencement of each group is clearly not due to any mechanical action of the auricular contractions; for, as is seen in all the figures, and especially in fig. 18, where the maximum contractions of the auricles occurred before the ventricle began to beat, those contractions caused no appreciable effect upon the position of the ventricular lever.

I am inclined, therefore, to think that it is possible that the increase of the contraction force caused by the action of the vagus nerve may be accompanied by an increase of tonicity, and that the diminution in the contraction force is certainly accompanied by a diminution of tonicity. I am the more emboldened to consider this a possibility by the fact that AUBERT[*] has already, from observations on the action of the Dog's heart, suggested that cardio-tonic nerves for the heart exist, and by their action cause the greater relaxation which exists during the stimulation of the vagus and the diminution of relaxation which he noticed after the end of the stimulation of the nerve.

The effect of atropin, muscarin, and curare upon the action of the vagus.

Before mentioning the effect of these poisons upon the vagus action, I will endeavour to sum up briefly their actions upon the motor ganglia and the muscular tissue respectively, as far, at least, as can be judged from these present experiments.

Atropin applied to the ventricle alone does not alter its rate of rhythm or the force of its contraction except after repeated applications. From this we can say that atropin does not prevent the muscle from contracting to its full extent or diminish its excitability except in extreme doses.

Muscarin applied to the ventricle alone rapidily diminishes the force of its contractions without altering the rate of rhythm, and may ultimately make the ventricle beat synchronously with every second auricular beat. Muscarin, therefore, prevents the full contraction of the muscle and diminishes its excitability.

Curare applied to the ventricle alone produces at first no alteration in the force of its contractions or in its rhythm; after repeated doses it diminishes the contraction force and often causes the ventricle to respond once only to every two auricle contractions. Curare, therefore, ultimately prevents the full contraction of the muscle and lowers its excitability.

Atropin applied to the sinus and auricles only, slows the rhythm both of auricles and ventricle. With this slowing it causes the ventricle to beat synchronously with

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[*] PLÜGER'S Archiv, Bd. v., S. 621.

the auricles, if it has previously been beating with half-rhythm, and increases the strength of the contractions both of auricles and ventricle. Also as long as the auricles are beating, the ventricle beats synchronously with them. From these facts I conclude that atropin slows the rate of the discharges from the motor ganglia without making them weaker; with the slower rate the impulses are therefore stronger than before, and the contractions must also be stronger, up to a certain extent of slowing, because with a certain slow rate of rhythm the maximum contractions of the heart are obtained. Whether the strength of the impulses is absolutely increased apart from the presumably beneficial effects of the slower rate I cannot say positively, though I am inclined to think they may be, as in some cases it has seemed to me that after atropin has been given it is necessary to screw the clamp up tighter than usual in order to cause the ventricle to remain quiescent.

Muscarin applied to the auricles and sinus only, slows the rhythm both of auricles and ventricle, weakens the auricular contractions, and may cause the ventricle to beat with half-rhythm or to remain quiescent, before the auricular contractions have ceased. This shows that muscarin not only slows the rate of the discharges from the motor ganglia but also weakens the strength of the impulses.

Curare applied to the auricles and sinus only, produces at first hardly any effect; it soon, however, slows the rhythm and may ultimately cause the ventricle to beat with half-rhythm. It therefore slows the rate of discharges from the motor ganglia and ultimately weakens the strength of the impulses.

If the heart be suspended with the cut-open ventricle downwards and muscarin be applied to the ventricle alone, then the ventricular beats are weakened without altering the rhythm, as in Plate 70, fig. 26, curve A. If the preparation be now turned over without loosening the clamp, so that the auricles are downwards, atropin can be applied to the auricles and sinus alone without any chance of its reaching the ventricle. Then, as is seen in Plate 70, fig. 26, curve B, where the atropin was applied to the auricles and sinus at the point denoted by the arrow, the rhythm both of auricles and ventricle is slowed, and in consequence the ventricular as well as the auricular contractions are slightly strengthened in force. If, now, atropin be applied to the ventricle, the ventricular contractions immediately increase very greatly in strength, as is shown in Plate 70, fig. 26, curve C, which is the direct continuation of the curve B. This shows that atropin removes the weakening effect of muscarin on the muscular tissue itself.

Again, if the heart be beating slowly in consequence of the application of muscarin to the sinus and auricles alone, then atropin applied to the tissue on the same side of the clamp does not quicken the rate of rhythm, although it may cause the auricular beats to become stronger. Atropin, however, may undoubtedly cause the heart to beat again when it has ceased beating from the application of muscarin.

We are now in a position to consider the effects of these three poisons upon the action of the vagus nerve, and I will take first the case of atropin upon the whole heart.

Atropin when applied to the whole heart ultimately removes all the different effects of vagus stimulation, it prevents any diminution of the force of the contractions, and *à fortiori* any standstill, it prevents any increase of the contraction force and it prevents any acceleration. Examination of the various atropin curves which I possess shows that the first effect of atropin is to lessen the extent of the diminution of the force of the contractions caused by the stimulation of the nerve, so that in this stage the vagus finally produces acceleration* and augmentation of the beats only. In the further stage with the slow vigorous contractions characteristic of the atropinized heart, less and less acceleration and less and less augmentation of the force of the beats occurs, until at last stimulation of the nerve, even with the strongest current, is absolutely ineffective.

This experiment not only proves that atropin prevents the action of the vagus, but it also does away with an objection that might by some possibility be made against such results of vagus stimulation as I have recorded, viz.: that they are due, in part at all events, to escape of current, an objection which I believe has already been urged against SCHELSKE'S† assertion that in certain cases where the heart was not beating, he was able to make it beat again by stimulation of the vagus. I have, however, satisfied myself, apart from the action of atropin, that the effects seen are due to the vagus stimulation alone, and not to any escape of current. Thus the space between the electrodes and the tissue is considerable, and great care was always taken that there should be no communication between the electrodes and the heart except by means of the bare nerve. Again, if the thread to which the nerve is attached be first well moistened and then placed on the electrodes instead of the nerve, no effect whatever is produced upon sending the current through, although upon gently drawing the nerve on the electrodes which have throughout remained in the same position, the characteristic effect is immediately produced on stimulation. Also if the electrodes be placed directly upon the tissue between the heart and the forceps which hold the œsophagus, no effect whatever is produced on the heart with a current of the same strength as is sufficient to cause all the effects which are seen when the nerve is stimulated.

Since the vagus acts both upon the motor ganglion and the muscular tissue, and atropin is able to remove the whole effect of the action of the nerve, it naturally follows that by the local action of poisons it may be possible to abolish the vagus action in one part of the heart while leaving it intact in another. With this view I have applied the poisons either to the ventricle alone or to the sinus and auricles alone, and have obtained certain somewhat striking results; at the same time the number of these experiments is hardly sufficient as yet to allow me to say that these results are in every case so constant as to be entirely due to the action of the poison upon the sinus and auricles in one case or the ventricle in the other.

Thus, for instance, in Plate 70, fig. 27, muscarin had been applied to the sinus and auricles only, until, as is seen, the auricular beats had become slow, irregular, and somewhat weaker. The vagus was stimulated between the two vertical lines, and, as the figure shows, no effect was produced either on the rhythm or on the force of the auricular contractions (although previous to application of the muscarin the auricular contractions were augmented in force by the nerve stimulation); while on the other hand the ventricular contractions were increased in force to as great an extent as at the beginning of the experiment. Also the

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* SCHIFF, Molesch. Unters., 1865, S. 58.

† Ueber die Veränderungen der Erregbarkeit durch d. Wärme, S. 20. Heidelberg, 1860.

fact that the ventricular rhythm was throughout the same as the auricular shows that the muscarin had caused the slowness and irregularity of the rhythm by its action upon the motor ganglia, and not because it had rendered the auricular muscle unable to respond to every impulse. The curve affords, too, a good proof that separate and not continuous impulses pass from the ganglia to the ventricle, otherwise it is impossible to conceive that with the increased excitability of the muscle caused by the vagus action it would be obliged to contract in the same slow irregular manner as the auricles.

Very soon after this curve was taken the muscarin caused the action of the motor ganglia to cease altogether, and then it was found that stimulation of the vagus was unable to make either ventricle or auricle beat.

This shows that the vagus is *not a motor* nerve to the muscle, for if it were so it must have caused contractions of the ventricle, seeing how powerfully it had been able to affect the ventricle a very short time before.

Again, cases occur where the curves show that muscarin applied to the auricular side only had very greatly reduced the size of the auricular beats, somewhat slowed the rhythm, and by its weakening action on the strength of the impulses caused the ventricle (which had previously been beating synchronously with the auricles) to remain quiescent. Stimulation of the nerve produced only a slight quickening of the auricular beats, and after the end of the stimulation a series of ventricular beats similar to what has already been described when the ventricles were reduced to standstill either by clamping or heating the auricles and sinus.

Muscarin, then, is apparently able to prevent the action of the vagus upon the auricles when applied to the auricles and sinus alone, without thereby preventing the action of the nerve upon the ventricle.

When atropin is applied to the sinus and auricles alone, then, simultaneously with the appearance of the slow vigorous beats thereby caused, the action of the vagus both on the rate of rhythm and upon the force of the ventricular as well as the auricular contractions entirely disappears. In fact, atropin applied to the sinus and auricles alone is as effective as when applied to the whole heart.

Curare applied to the auricles and sinus alone is able to prevent the action of the vagus nerve upon the auricular muscle, as far, at all events, as the diminution of the force of the contractions is concerned, without, as in the case of muscarin, greatly diminishing the force of the auricular contractions, while at the same time it leaves intact the action of the nerve upon the force of the ventricular contractions.

When any one of these three poisons is applied to the ventricle alone, the action of the vagus upon the force of the ventricular contractions is gradually removed without affecting its action upon the rhythm of the heart or upon the force of the auricular contractions. The usual sequence of events is as follows :—Soon after the application of the poison the nerve stimulation is no longer able to cause any diminution of the force of the contractions of the ventricle, so that in this stage it causes simply an increase of force with acceleration, if the auricular beats are accelerated. The maximum of this increase is less, and takes place later after the stimulation than before the application of the poison. As the action of the poison continues, the ventricular beats become smaller and smaller, and the augmentation of their contractions due to the vagus stimulation becomes less and less marked, until at last the vagus produces no alteration in the size of the contractions, but only an alteration of rhythm in accordance with the alteration of rate of the auricular contractions.*

CONCLUSION.

The curves and arguments given in this paper show clearly that the vagus nerve exercises a powerful influence upon the muscular tissue of the heart as well as upon the motor ganglia. That it acts upon the muscular tissue is proved by the fact that its stimulation causes a profound alteration of the force of the muscular contractions, together with corresponding variations in the excitability and tonicity of the muscle, without necessarily altering the rate of rhythm.

Its action upon the force of the contractions may be in the direction of diminution or of augmentation, so that the nerve sometimes causes a diminution of the force of the contractions followed by augmentation, sometimes augmentation alone without any previous diminution.

This diminution may vary greatly both in extent and duration ; it may be very slight or so great that the contractions disappear altogether, so that in this way complete quiescence may be caused ; it may be that the first few beats only after the beginning of the stimulation are affected or the effect may continue until some little time after the end of the stimulation.

The extent of this diminution is in close connexion with the nutritive condition of the heart, so that quiescence followed by augmentation is more likely to occur when the heart is first suspended ; and augmentation alone, without previous diminution, after the heart has been removed from the circulation for some time.

Augmentation, therefore, without a previous diminution, is that effect of the stimulation of the vagus upon the muscular tissue which is the most enduring ; and in accordance with this, poisons which gradually prevent the whole action of the vagus upon the muscular tissue always remove first the power of the nerve to cause a diminution of the force of the contractions, and *à fortiori* quiescence, and afterwards its power to cause augmentation of that force.

The vagus, also, in all probability acts upon the motor ganglia in the same direction as upon the muscular tissue. It is able at one time to cause slowing, at another acceleration without a previous slowing. The slowing is more likely to occur before removal of the heart, or immediately after the heart has been suspended ; acceleration alone or no effect at all after the removal of the circulation or after the heart has been suspended some time.

So, too, poisons which entirely remove the action of the vagus first prevent the nerve from causing slowing, and only finally prevent the causation of acceleration.

If, then, we compare slowing of the rate of the discharges from the motor ganglia to

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* N.B.—The hearts of Toads are apparently less susceptible to the action of all three of these poisons than those of Frogs. For instance, it takes much longer time to remove the whole effect of the action of the vagus by means of atropin (1 p. c. solution) in the case of the former than in the case of the latter.

diminution of the force of the contractions of the cardiac muscle, and acceleration of that rate to augmentation of that force, we can say that the action of the vagus upon all parts of the heart is similar; and that the difference of its action at different times depends not upon a multiplicity of nerve fibres, each possessing different functions, but rather upon the different effects produced by the same nerve in consequence of variations in the condition of nutrition of the heart as a whole—variations of nutrition to which naturally the more delicately-organised ganglion cells would prove more sensitive than the comparatively coarser muscular fibres.

How, then, can we imagine to ourselves the action of the nerve upon the heart? In the first place, it is clear that the nerve cannot be considered as the motor nerve of the muscle in the sense in which the words motor nerve are ordinarily used. The nerve itself cannot upon stimulation cause a contraction of the cardiac muscle; but causes such a modification in the chemical processes going on in the muscle as to modify the force of the contractions which take place when the impulses from the motor ganglia reach the muscle. In fact, two distinct processes are going on—the one, *motor*, represented by the impulses from the motor ganglia, and corresponding to the blow by which such an explosive substance as nitro-glycerine, for instance, is exploded; the other, *trophic*, represented by the action of the vagus upon the muscular substance, and corresponding to something affecting the nature or properties of the explosive substance itself.

The problem, then, to be explained is, what conception can be formed of the nature of the changes caused in the muscle by the action of the nerve, which shall satisfy the conditions that the same nerve can produce in different cases a graduated series of different effects extending from a simple increase in the force of the contractions on the one side down to diminution of force even to complete standstill on the other?

In trying to form a theory to explain new facts, it is always advisable to see if any theory which has been devised to explain somewhat similar facts will apply to this case also. With this consideration I cannot help thinking that the recent views which have been put forward to explain the changes going on in gland cells in a condition of rest and activity may help in the conception of the changes going on in the cardiac muscle cells under the influence of such a nerve as the vagus. In all secretory glands it appears to be the fact that the formation of the ultimate products of secretion is a gradual process, three stages of which can be recognised, viz.: the growth of the protoplasm, the formation of zymogen from the protoplasm, and the formation of the ultimate products from the zymogen; and according to LANGLEY[*] these three processes not only go on at the same time, but also there may be, under different circumstances, a difference in the relative rates at which they proceed.

If, then, we imagine in the cardiac muscle an initial or ground substance, or, in correspondence with the gland cell, a muscle protoplasm and a final or explosive substance which is not self-explosible, but requires a stimulus to fire it off, then we may conceive that the protoplasm does not form the explosive substance directly, but that there are intermediate stages corresponding to the zymogenic stage in the gland. According to this view there would be in the cardiac muscle during its normal activity three processes going on: the growth of the protoplasm with the aid of the raw material from the nutritive fluid surrounding the muscle fibres; the transformation of the protoplasm into intermediate products, which are non-explosible, or at least explosible with difficulty; and the formation from these of easily explosible material; in fact, a gradual succession from stable to more and more unstable combinations. If, then, the formative activity of the muscle be distributed over these three stages in the right proportion, it follows that impulses passing to the muscle at equal times must all cause contractions of equal force. If, however, that activity be exclusively directed towards the formation of the muscle protoplasm or the intermediate non-explosible substances it is conceivable that no contractions should be able to take place, because no suitable material was ready for the impulses to act upon, though as a natural consequence of this greater activity in the formation of the intermediate substances the ultimate effect would be that the subsequent contractions would be very much greater than before. On this view, then, the force of the contraction at any time depends upon the relative rates at which these three processes proceed.

In this way I can imagine standstill to take place, and it is also quite conceivable that in certain conditions of nutrition of the heart it may occur more easily than in others; thus, as we have seen in the heart cut out and suspended, complete quiescence may in some cases be easily obtained, in others not at all; so, also, in the heart through which salt solution is flowing I have often obtained standstill upon stimulation of the vagus, thus confirming LUDWIG and LUCHSINGER,[*] while in other cases I have only obtained acceleration, thus confirming SCHIFF.[†]

Again, if the whole heart or the ventricle alone be beating in the apparatus described on page 1019, while salt solution is flowing through it, it will beat regularly and well for a long time with the salt solution, and if while it is still beating, whether strongly or weakly, blood solution be sent into it under the same conditions of pressure, &c., as the salt solution, then it often happens that as soon as the blood is seen to reach the heart or the ventricle the heart stops still in the relaxed condition, sometimes for a considerable length of time, and subsequently begins again to beat with the improved beats due to the blood solution. In the same heart this experiment may be repeated again and again; in other hearts the sudden supply of nutrient material contained in the blood does not cause any stoppage whatever, but a simple gradual improvement in the contractions. Upon the hypothesis suggested above this means that in consequence of the sudden supply of blood the whole energy available is in the one case engaged in the growth of the protoplasm from the raw material, so that no contractions can take place, while in the other it is distributed more equally over the different processes going on in the muscle.

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[*] *Op. cit.*

[†] *Op. cit.*

As a conceivable hypothesis, then, it seems to me we may say—the vagus increases the activity of the formative processes going on in the muscle, and it will produce therefore augmentation of the force of the contractions alone, or diminution followed by augmentation, or even standstill followed by augmentation, according as that activity is more or less equally distributed over the different stages of the process, the ultimate end of which is the formation of the final contractile substance.

Again, although I am not prepared at present to offer any definite theory to explain the action of the nerve upon the motor ganglia, yet still it is impossible to help being struck with the apparent resemblances between its action on these cells and on the muscle cells; thus just as in the case of the muscle the nerve may at one time cause diminution followed by augmentation of the contractions, at another time augmentation alone without any previous diminution, so in the case of the motor nerve cells, the vagus may under certain conditions cause a slowing of the rate of discharge of the impulses followed by acceleration, under other conditions acceleration alone without any previous slowing.

I venture, therefore, to suggest that—

> The vagus is the trophic nerve of both the muscular tissue and the motor ganglia, meaning thereby that it increases the activity of the various formative processes going on in both these kinds of tissue, and it produces all its effects by virtue of this quality.

In all that I have hitherto said I have been speaking entirely of the hearts of Frogs and Toads, and so far as these animals are concerned I venture to think that my views are in harmony with an experiment of FOSTER and DEW-SMITH,[*] who showed that although the constant current was able to cause rhythmical contractions in the isolated and quiescent apex or ventricle, or in the whole heart when rendered quiescent by the STANNIUS ligature, yet it was unable to do so when the heart was reduced to standstill by the stimulation of the vagus. This remarkable experiment, which, as far as I know, has never been disputed or explained, seems to me to follow naturally from the experiments and conclusions put forward in this paper.

As yet I have not made any experiments upon the hearts of warm-blooded animals, and cannot therefore assert from my own observations that the vagus acts in a similar way upon them; there are, however, certain experiments recorded which render this conclusion more or less probable. Thus PANUM and GIANUZZI[†] have both observed that in Rabbits with weakly-beating hearts, stimulation of the vagus causes the beats to become much stronger. TRAUBE[‡] has observed that in a curarised animal the heart beats for a longer time after the cessation of artificial respiration with the vagi intact than when these nerves have been previously cut; and BROWN-SEQUARD[§] has quite

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[*] Journ. Anat. and Physiol., vol. x.

[†] HERMANN'S Handbuch der Phyiol., Bd. iv., S. 384.

[‡] Allgem. Med. Centralzeitung, 1864, N. 42.

[§] Gaz. Med. de Paris, 1880, p. 391.

recently noticed that if two Rabbits are killed at the same moment by cutting open the aorta, and if in the one the vagus has been strongly stimulated just before the aorta is opened and not in the other, then that heart which has been affected by the nerve stimulation will beat stronger and continue to beat a longer time than the heart of the Rabbit in which the vagus had not been stimulated.

These experiments render it probable that the action of the vagus on the warm-blooded is similar to its action on the cold-blooded heart.

Evidence of another kind is given by the observations of EICHORST[*] and ZANDER,[†] who find that the death which results from section of both vagi is not entirely due to inanition, but partly to degenerative changes in the heart itself; they therefore conclude that the vagi nerves exercise a trophic influence on the heart.

Hitherto I have tacitly assumed that the vagus acts directly upon the muscular tissue and the motor ganglia respectively, and not through the aid of some intermediate apparatus. Now although the experiments with muscarin and curare which I have just mentioned tend to confirm this view, for in each case the local application produced only local results whether upon the muscle or on the motor ganglia, yet the action of atropin is difficult to explain on this hypothesis. If atropin acts directly upon the muscular tissue and the motor ganglia, so as in each case to slow the rate at which the formative processes are going on, without at the same time diminishing the extent of those processes, then it is possible to conceive it as acting in direct opposition to such an influence as the vagus nerve which I have supposed to act by expediting those same formative processes; were this the whole of its action, however, it ought to prove as local in its effects as muscarin or curare. Since, however, atropin removes the whole action of the vagus nerve upon the ventricle when it has been applied to the sinus and auricles only, the question naturally arises, how is this to be explained? Is there a trophic centre for the ventricle which is situated either in the auricles or venous sinus, upon which both atropin and the vagus act? At present I do not see any absolute necessity to draw such a conclusion from this action of atropin, and therefore prefer to leave this question entirely undecided in the hope that further researches may enable me to come to some definite conclusion upon the matter.

In conclusion, I desire again to call attention to the fact that the action of the vagus upon the muscular tissue of the heart is characterised by two different sets of effects.

1. Diminution of contraction force associated with a diminution in the excitability and a diminution of the tonicity of the cardiac muscle.

2. Increase of contraction force associated with an increase in the excitability and possibly also with an increase in the tonicity of the cardiac muscle.

Upon the view put forth above that a series of substances are formed between the muscle protoplasm and the final explosive substance, it follows that the chemical combinations which are formed in the lower part of this series possess less explosive

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[*] 'Die trophischen Beziehungen der Nervi Vagi zum Herzmuskel.' Berlin, 1878.

[†] PFLÜGER'S Archiv, Bd. xix., S. 263.

energy, are less excitable, and less compact than the combinations which occur in the higher part of the series. Finally, I cannot help being struck with the fact that alkaline substances such as sodium hydrate or sodium carbonate increase the excitability, increase the tonicity and increase the force of the contractions in weakly-beating hearts; while, on the other hand, such substances as lactic acid diminish the excitability, diminish the tonicity and diminish the force of the contractions of the cardiac muscle. Whether the resemblance in these two cases is purely fictitious, or whether the trophic action of the vagus nerve is connected with some such action of acids and alkalies I cannot say, and only put forward the resemblance in the hope that a possible clue may thus be found to the chemical action of the vagus upon the muscular substance of the heart.

POSTSCRIPT.

(Added July 3rd, 1882.)

Throughout this paper I have spoken of impulses proceeding from the sinus to the ventricle without attempting to define the nature of the stimulus which causes the ventricle to contract, or the path along which the impulse travels. Subsequent investigations which I have made upon the hearts of Tortoises have thrown new light upon the cause of the sequence of the ventricular upon the auricular beat, and point to a possible explanation of the want of sequence observed under different circumstances which is somewhat different to that suggested above.

The heart of the Tortoise is removed from the body and suspended between two levers in the manner above described; no clamp is used, but the aortic trunk is held firmly in a suitable holder. By means of section of the auricle in different directions and to different extents I find that—

1. The sequence of the ventricular upon the auricular contractions is associated with the passage of a wave of contraction along the auricular muscular fibres to the auriculo-ventricular groove, and is not dependent upon the integrity of the large nerve trunks between the sinus and the ventricle.
2. By means of section of the auricle the passage of this wave can be *blocked* and the ventricle in consequence remains quiescent.
3. If the section be severe the block will be complete; no contractions will pass. If less severe, the block will be partial and then *every second* contraction will pass, and as soon as it reaches the auriculo-ventricular groove will cause a contraction of the ventricle.

In this way by hindering the passage of a contraction wave to the ventricle, the ventricle can be made to beat with every second contraction of the sinus and that part of the auricle in connexion with the sinus, in precisely the same way as by tightening the clamp between the sinus and ventricle of the Frog. The probability therefore is strong that the clamp produces the half-rhythm observed because it blocks the passage of a contraction wave to the ventricle, and not because it weakens a nervous impulse from the sinus to the ventricle.

Again, when by section of the auricle of the Tortoise only every second contraction passes the block and induces a contraction of the ventricle, stimulation of the vagus is able to cause every contraction to pass, in a manner similar to that observed in the case of the Frog when the clamp was tightened. This, then, would seem to show that in this latter case too the vagus produced this effect, not so much by its influence upon the excitability of the *ventricular* muscle as by its power to remove the blocking of the *auricular* contractions caused by the pressure of the clamp upon the auricular fibres at their termination in the auriculo-ventricular groove.

Lastly, I find that with a section of the auricular muscle, such that every contraction is able to pass at a certain rhythm, only every second contraction passes at a quicker rhythm; *i.e.*, the time required for the auricular tissue to recover itself after a contraction is much longer at the blocking point than at other parts of the auricle. In this way many cases of half-rhythm which are coincident with an increased rapidity of rhythm are to be explained.

DESCRIPTION OF FIGURES.

In all the figures the divisions on the line traced by the time marker correspond to intervals of two seconds. The duration of the vagus stimulation is in all cases marked by two thin vertical lines. All the curves read from left to right.

PLATES 66–70.

Fig. 1. March 17, 1881. Frog. Auricles and sinus alone heated between the two vertical lines.

Fig. 2. March 28, 1881. Toad. Ventricle alone heated between the two vertical lines.

Fig. 3. June 17, 1881. Frog. Clamp in auriculo-ventricular groove tightened at the place denoted by the arrow.

Fig. 4. Feb. 16, 1881. Frog. Stimulation of vagus. Sec. coil at 13 centims. No clamp. Ventricular beats only registered.

Fig. 5. March 17, 1881. Frog. Stimulation of vagus. Sec. coil at 10 centims. Auricles and sinus were heated. In consequence, auricles are beating very rapidly, and ventricle does not respond to every auricular beat.

Fig. 6. March 15, 1881. Frog. Stimulation of vagus. Sec. coil at 8 centims. The stimulation was probably weaker for some reason in curve A than in curve B.

Fig. 7. March 8, 1881. Frog. Stimulation of vagus. Sec. coil at 10 centims. The auricular contractions are just visible during the ventricular standstill.

Fig. 8. June 23, 1881. Toad. Stimulation of vagus. Sec. coil at 10 centims.

Fig. 9. June 20, 1881. Toad. Stimulation of vagus. Sec. coil at 10 centims.

Fig. 10. April 11, 1881. Toad. Stimulation of vagus. Sec. coil at 5 centims. The ventricle was unable to register the maximum of its contractions, as the upper and lower levers came in contact.

Fig. 11. March 1, 1881. Frog. Curve B. Stimulation of vagus. Sec. coil gradually pushed from 15 to 8 centims.

Fig. 12. March 15, 1881. Frog. Stimulation of vagus. Sec. coil at 5 centims. Auricular beats only shown. Curve B is the continuation of curve A, except that the vagus was stimulated between the two curves.

Fig. 13. March 10, 1881. Frog. Stimulation of the vagus. Sec. coil at 10 centims.

Fig. 14. May 12, 1881. Toad. Stimulation of vagus. Sec. coil at 8 centims. Ventricle beating once to every 3 auricular beats.

Fig. 15. April 11, 1881. Toad. Stimulation of vagus. Sec. coil at 10 centims.

Fig. 16. Feb. 25, 1881. Frog. Stimulation of vagus. Sec. coil at 8 centims. Contractions of base of ventricle, upper curve; of apex, lower curve.

Fig. 17. March 17, 1881. Frog. Curves A, B, C, D. Series of curves to show effect of vagus stimulation, when the ventricle is rendered quiescent, by heating the auricles and sinus.

Fig. 18. June 6, 1881. Toad. Stimulation of vagus. Sec. coil at 7 centims. Clamp tightened so as to render ventricle quiescent.

Fig. 19. May 9, 1881. Frog. Stimulation of vagus. Sec. coil at 9 and 8 centims. Ventricle beating with alternately strong and weak contractions. Intermediate part of the curves indicated by the dotted lines.

Fig. 20. Feb. 23, 1881. Apex and base of ventricle. Apex beating with alternately weak and strong contractions. Base upper, apex lower curve.

Fig. 21. June 16, 1881. Frog. Secondary rhythm in the alternately strong and weak contractions of the ventricle.

Figs. 22 and 23. Apparatus as described on page 1019.

Fig. 24. Nov. 4, 1880. Toad. Stimulation of the vagus. Sec. coil at 6 centims. Heart in modified Roy's tonometer. Pressure of blood solution 5 centims.

Fig. 25 A and B. Nov. 4, 1880. Stimulation of vagus. Sec. coil at 4 and 3 centims. Pressure of blood solution 28 centims. in A, 29 centims. in B.

Fig. 26. May 19, 1881. Frog. Curve A. Muscarin sulphate, 5 per cent. solution applied to the ventricle at the point marked by an arrow. Upper curve auricle, lower curve ventricle. Curve B. Atropin sulphate, 1 per cent. solution applied at arrow to the auricles and sinus only. Upper curve ventricle, lower curve auricle. Curve C. Atropin sulphate applied to ventricle at the arrow. Upper curve ventricle, lower curve auricle.

Fig. 27. July 16, 1881. Toad. Stimulation of vagus. Sec. coil at 10 centims. Muscarin sulphate, 5 per cent. solution had been applied to the auricles and sinus only.

XXIV. *An Attempt at a complete Osteology of* Hypsilophodon Foxii; *a British Wealden Dinosaur.*

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[PLATES 71–82.]

THE dinosauria are peculiarly interesting and instructive on account of the combinations in their skeletons of structures which now only occur separately in those of extant Sauropsida; and also on account of their forming a link between more specialised Reptiles and Birds.* The need of such an osteology of a Wealden dinosaur as might serve for a type and aid to those who are working out our fossil reptiles, long felt, has lately become increasingly urgent through the discoveries in the United States of large numbers of remains in such preservation and abundance as to make their reconstruction a relatively light task. Some of the members indicated by these remains resemble certain of our Wealden fauna, of which our knowledge is very imperfect and scanty, so that a strict comparison of the American and British Wealden forms appears likely to throw much light upon the latter.† Unfortunately our own material does not yet exist in a form available for this purpose; for although a very large number of memoirs have been written on the dinosauria of our Cretaceous and Wealden formations, nothing approaching to a complete osteology of any one of them based on the study of remains recovered from British Cretaceous and Wealden formations has yet been published. The reason is not far to seek. In our Wealden beds their remains are usually so scattered, disconnected, and not seldom mutilated, that their identification and reconstruction are exceptionally difficult. As regards *Iguanodon Mantelli* a complete osteology may be expected from Belgian workers whenever the magnificent remains in the Musée d'Histoire Naturelle at Brussels, obtained in 1878 from an extension of the Wealden formation at Bernissart, shall have been wrought out—a task which I am authoritatively informed is not likely to be accomplished for several years. But with respect to *Hypsilophodon* something

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* HUXLEY, "Dinosauria and Birds." Quart. Journ. Geol. Soc, 1870, p. 12.

† In illustration of this may be cited the resemblance between *Hadrosaurus*, LEIDY., *Agathaumus*, COPE, and *Iguanodon Mantelli;* between *Lælaps* and *Megalosaurus Bucklandi;* between *Titanosaurus*, *Atlantosaurus*, MARSH, *Camarasaurus*, COPE, and *Ornithopsis*, SEELEY (Syn. *Eucamerotus*, HULKE, *Bothriospondylus*, OWEN, *Chondrosteosaurus*, OWEN); between *Laosaurus*, MARSH, and *Hypsilophodon*, HUXLEY.

may fairly be expected of English palæontologists, for its remains occur in a manner quite exceptional—large parts of skeletons of this dinosaur, the bones of which are usually well preserved, and often maintain their normal connexions, have been obtained by the Rev. WILLIAM FOX (lately deceased) and by myself.* A study of these, prosecuted in leisure intervals during several years, enables me at length to offer a connected account of the skeleton, which I venture to hope leaves but few omissions to be supplied when additional materials shall have been acquired.

The following short list contains, it is believed, the titles of all the papers which have been written on this dinosaur :—

BIBLIOGRAPHY.

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2. HUXLEY, T. "On *Hypsilophodon Foxii* : a New Dinosaurian from the Wealden of the Isle of Wight." Quart. Journ. Geol. Soc., vol. xxvi., p. 3, plate i. 1870.
3. HULKE, J. W. "Contribution to the Anatomy of *Hypsilophodon Foxii.*" Quart. Journ. Geol. Soc., vol. xxix., p. 522, plate xxviii. 1873.
4. HULKE, J. W. "Supplemental Note on the Anatomy of *Hypsilophodon Foxii.*" Quart. Journ. Geol. Soc., vol. xxx., p. 18, plate iii. 1873.
5. OWEN, R. "Skull and Teeth of *Iguanodon Foxii* : Fossil Reptilia of the Wealden Formation." Supplement No. 5. Pal. Soc., vol. for 1873, p. 1, plates i., ii.
6. OWEN, R. Appendix to the above Supplement : "Monograph on *Iguanodon Foxii.*" P. 17.

Skull. (Plates 71, 72.)

In shape Lizard-like; the external nostrils are subterminal and divided; the orbital openings are large, and laterally directed ; the eyeball has a ring of bony plates ; and the temporal fossæ are bounded externally by an upper and a lower bar.

The present length of the best-preserved skull in Mr. Fox's collection† (Plate 71, figs. 2–4) is 9·6 centims. To this may be added 1 centim. for shortening due to displacement of the snout upon the braincase, and to slight mutilation of the occiput

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[*] GISEMENT. Remains of *Hypsilophodon Foxii* have as yet only been found in a bed which crops out a short distance west of Barnes High Cliff, and passes under the shore a few yards west of Cowleaze Chine, on the south coast of the Isle of Wight. The rock varies much often within the space of a few yards. Generally the upper 3½ feet of it consist of a cap of grey sandstone resting on sandy clay ; this is succeeded by about the same depth of mottled-red and blue clay lying on the bands of sandstone. The *Hypsilophodon* remains are almost restricted to the lower half of the bed. The only other bones ever taken out of it by Mr. Fox and myself represent a small scuted Crocodilian (*Goniopholis ?*) and a Chelonian (*Trionyx ?*) No bones referable to *Iguanodon Mantelli* have ever been found by us in this bed.

[†] Described in Memoirs 2, 5. I shall refer to this as Mr. Fox's "type skull."

and of the end of the snout, making its total length 10·6 centims. The width at the occipito-parietal crest is 22 millims.; at the middle of the parietal region it decreases several millims.; at the fronto-parietal suture it increases to 33 millims.; between the orbits it slightly contracts; in front of these cavities it again slightly expands; and thence lessens to the end of the snout.

This is evidently the skull of an immature individual. A skull in my own collection (Plate 71, fig. 1) (No. 110 Catal., Coll. HULKE) was probably 5 centims. longer, and I have seen fragments which indicate that this greater length was sometimes exceeded.

For convenience, first the occiput, then the upper surface, next the under surface of the skull, and lastly the dentition will be described.

Occiput.—The occipital plane in Mr. Fox's type skull (Plate 71, fig. 3) includes an acute angle with the upper surface, but it is probable that the inclination of these surfaces is exaggerated by the crushing to which the skull has been subjected. The occipital condyle (*oc.*), 9 millims. in its horizontal diameter, is more wide than deep. The margin of the foramen magnum is slightly swollen. Above the foramen the surface is divided by a median ridge. A large par-occipital process, 23 millims. long (*popr.*), stands out from the sides of the occiput in the level of the upper part of the foramen magnum. Owing to the damage this part of the skull has undergone, effacing the sutures, the respective shares of the several bones composing the occiput cannot be now fixed with precision, but it may, I think, be safely stated that the supraoccipital enters largely into the foramen magnum, as in *Iguanodon Prestwichii.**

Upper surface of skull.—The *parietal* bone (*pa.*) appears to me single, as in *Iguanodon Mantelli*† and *I. Prestwichii.* I find no unequivocal evidence of an interparietal suture. Its two halves meet in an angular crest, which behind is prolonged beyond a line drawn transversely to the long axis of the skull through the meeting of the upper and the posterior surfaces, and overhangs the supraoccipital bone.

From this median supraoccipital process the posterior border of the parietal bends outwards, joining the inner branch of the squamosal, with which it bounds posteriorly the upper opening of the temporal fossa (Plate 71, fig. 2). Anteriorly the parietal crest declines, and ends at the interfrontal suture. From a point slightly in advance of the middle of the crest a slight ridge-like elevation of the surface curves outwards to the post-frontal bone. It marks an expansion of the front half of the parietal bone, as occurs in *Iguanodon Mantelli*‡ and *I. Prestwichii.*§

The *frontal bones* (*fr.*) are remarkably large (Plate 71, figs. 1, 2). The suture joining their median borders is slightly raised. Their orbital border also is prominent, which makes the upper surface between the orbits slightly concave in a direction transverse

* HULKE, J. W., Quart. Journ. Geol. Soc., vol. 36, plate xviii., fig. 3.

† Ibid., vol. 27, plate xi., fig. 3.

‡ Ibid., vol. 36, plate xviii., figs. 1, 2.

§ Ibid., vol. 27, plate xi., fig. 3.

to the long axis of the skull. In a direction coincident with this axis the surface is gently convex.

The *post-frontal* (*psf.*) (Plate 71, fig. 2) is a large three-rayed bone. One ray, short and stout, articulates with the postero-external angle of the frontal, and forms the posterior and upper part of the orbital opening. A descending ray, longer than the former, joins the jugal and completes the posterior border of the orbit. The third ray, directed backwards, is applied to the outer edge of the anterior branch of the squamosal, and forms the greater part of the upper temporal bar.

The *squamosal* (*sq.*) (Plate 71, fig. 2), by an anterior branch, forms the minor part of the bar just named, and by an inner process which joins the *parietal* helps to bound posteriorly the upper temporal opening. It caps the upper end of the *quadrate,* which is fitted into a recess in its under surface (Plate 72, fig. 1). Behind, it descends some distance on the back of the *quadrate,* an arrangement which must have given to the squamosal-quadrate joint great stability. A similar arrangement obtains in *Iguanodon Mantelli,* and in the Liassic *Scelidosaurus.**

A *præfrontal* and a *lacrymal* are present (Plate 71, fig. 2) in the anterior border of the orbit. The examples I have yet met with have been too mutilated for description.

A *jugal* (Plate 71, fig. 2), a rather wide bar, applied to the posterior third of the upper border of the maxilla, and connected with the descending ray of the *post-frontal,* completes the ring of the orbit.

The presence of a *quadrato-jugal* is indicated by a thin plate abutting against the lower part of the quadrate in Mr. Fox's type skull (Plate 71, figs. 3, 4). Intercalated between the *jugal* and *quadrate,* constituting with the former a lower temporal bar, its presence in conjunction with the form of the squamoso-quadrate joint would have rendered the quadrate immovable.

The *nasals* are very large and relatively wide bones (Plate 71, fig. 2). They form a considerable portion of the upper surface of the snout. Their notched lower end exclusively forms the upper or posterior boundary of the external nares. From each of its angles it sends downwards an extremely slender process. Of these processes the inner one, closely applied to the outer side of the anterior or septal process of the præmaxilla, constitutes with this the septal margin of the nostril; whilst the outer nasal process descends upon the narial border of the external ascending præmaxillary process, with which it forms the outer margin of the nostril, as described by HUXLEY.†

Præmaxillæ (Plate 71, figs. 1, 2).—There are two præmaxillary bones. That part of the body of the præmaxilla visible on the outer surface is an oblong vertical plate, the upper border of which is notched by the nostril. The lower dentigerous border is nearly straight. In a skull in my collection (No. 110 Catal., Coll. HULKE) the vertical measurement from the narial to the dentigerous border is 11 millims., and the length of the dentigerous border is 20 millims. The external surface is smooth except quite

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* OWEN, R., "Fossil Reptilia of the Oolitic Formation," part i., plate v.

† HUXLEY, Quart. Journ. Geol. Soc., vol. xxvi., p. 4, plate i., fig. 1.

in front, where, as mentioned by HUXLEY in a description of Mr. FOX's type skull, it is wrinkled.*

In both this and my skull, No. 110, the extreme end of the snout is missing, so that the actual anterior termination of the præmaxillæ is not certainly known. From each end of the body of the præmaxilla a strong process ascends in the snout. Of these the median or anterior process, flattened, three-sided, tapering upwards to a point, ascends, closely applied to its fellow, between the mesial margins of the nasals for about one-third of the length of the snout (Plate 71, fig. 2*), forming with the descending narial process of the nasal lately described the internarial septum. The external or posterior præmaxillary process († in same fig.), wider, flatter, and longer than the anterior, intercalated between the *maxilla* and the *nasal*, curves upwards and backwards, resting in a groove in the anterior border of the maxilla, but not united to this by serrated suture. The dentigerous border in a space of 20 millims. in my skull, No. 110, contains in separate sockets five teeth, the same number as in Mr. FOX's type skull. Since in both the extremity of the snout is abraded, it is just possible that one additional tooth may have been present, although I do not think this probable; or the ends of the præmaxillæ may have been bent downwards in the form of a beak as Professor HUXLEY suggests.†

Maxilla (*mx.*) (Plate 71, figs. 1, 2).—A large bone of rudely subtriangular outline. The anterior border, convexly curved, is grooved on its outer surface for the reception of the external ascending process of the præmaxilla, as was mentioned in connexion with the description of this bone. In my skull, No. 110, this border rises to a maximum height of 30 millims. above the dentigerous margin at the second molar tooth, making here an obtuse angle with the upper border which from this point curves gently downwards to a distance of 15 millims. above the last tooth. The nearly straight dentigerous border contains in a space of 43 millims. an unbroken series of eleven teeth, one more than in Mr. FOX's type skull.

In advance of the dentigerous portion of the maxilla a thin tongue-like plate formed by the convergence of the upper and the lower border is prolonged upon the deep surface of the body of the præmaxilla, nearly if not quite reaching the anterior border of this latter. The upper margin of this tongue-like process of the maxilla must be in very close proximity to the lower and outer margin of the external nostril. Behind the hindmost tooth the maxilla narrows abruptly, and it sends backwards a stout three-sided process to which the *jugal* is united (Plate 71, fig. 1). Throughout nearly its whole length the outer surface of the maxilla, nearly in the level of the tooth-roots, is pierced by a series of conspicuous foramina as in *Iguanodon Prestwichii*, *Megalosaurus*, and *Teratosaurus*.

The *quadrate* (Plate 71, figs. 3, 4; Plate 72, fig. 1), a large, stout bone, is antero-posteriorly compressed near its articular ends and laterally flattened intermediately.

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[*] HUXLEY, ibid.

[†] HUXLEY, *loc. supra cit.*

In the fragment of a skull in Mr. Fox's collection, in which it is but little damaged and has retained its natural relations almost undisturbed, the length of the quadrate is 53 millims.; its antero-posterior dimension just below the squamosal end is 6 millims.; at the distance of 21 millims. from this end it is 10 millims; and just above the mandibular end it is 5 millims. (the length of the portion of mandible preserved which comprises the entire dentigerous and part of the edentulous spout, given for comparison, is 96 millims.). The width or transverse diameter of the quadrate near its mandibular end in a smaller mandible is 10 millims. The squamosal articulation of the quadrate was described in connexion with the former bone.

Mandible (Plate 71, figs. 3, 4; Plate 72, figs. 1, 2; and Plate 73).—This repeats in miniature that of *Iguanodon Mantelli.* It has a spout-like symphysial end (Plate 72, fig. 2); great depth of the dentigerous part; a high coronoid process from which the upper border falls abruptly to the quadrate joint, and slopes gradually towards the junction of the tooth-bearing and edentulous parts. The lower border is almost straight. The articular element is large, and the joint-surface very capacious relatively to the size of the lower end of the *quadrate,* an arrangement which would have permitted a very free movement of the *mandible* upon the quadrate. The *angular* element appears to be small, it reaches to the posterior extremity of the mandible. A splenial plate covered the large subdentinal groove.

In a slab in my collection immediately in front of a much-crushed mandible is a thin triangular bone symmetrically bent into a trough-like form appearing not unlike a continuation of the symphysial mandibular spout, one edge of the triangular bone having the same slant as the inclined border of this.

I do not identify this bone with the mandible. The close proximity of a præmaxillary tooth to it suggests that it may be connected with the præmaxilla. I mention it in order to call attention to it.

Under surface of skull.—This is described entirely from my skull No. 110, as this region in Mr. Fox's type skull appears to me too damaged to afford unequivocal evidence of the nature of the pieces of bone displayed and of their relations.

The palate is closed in front by the junction in the middle line of the triangular, præmaxillary, palatal processes (Plate 72, fig. 1). A small anterior palatine foramen is situated in the front of the palatal inter-præmaxillary suture. The posterior borders of these processes curve inwards, and, meeting in the middle line of the palate, form a backward projecting angle. This curved border is smooth, non-articular, and it apparently forms the anterior boundary of the palato-nares. To the angle are attached the two *vomers* (*v.*), which are followed by the two pterygo-palatine bars separated by a median fissure. The *palatine* bone (*pl.*), a thin bar 6 millims. wide, is deeply grooved near its union with the *pterygoid* (*pt.*). This last bone has a remarkably stout central part or body, which is traversed obliquely from without and behind inwards and forwards by a prominent ridge, ending in an angular projection at the inner border. The position and the direction of this ridge approximately coincide with those of the row of denticles in the palate of extant Iguanas. In front, the body of the pterygoid joins the palatine; behind, it has a stout boss, which doubtless articulated with a basisphenoidal swelling; and externally it sends off two processes separated by an interspace, of these one (*q.pr.*) passes backwards and outwards towards the quadrate, and the other (*ect.pr.*) outwards towards an ecto-pterygoid which connected it with the maxilla. Thus, in *Hypsilophodon* the pterygo-palatine bars are constructed upon a plan not very unlike that of the existing Iguana.

Dentition.—The dentinal formula, so far as this is shown by Mr. Fox's type skull and my No. 110, is—

Præmaxillary (incisor) teeth	$\frac{5-5}{0-0}$
Maxillary and mandibular (molars)	$\frac{11-11}{10-10}$?

But these numbers must be accepted only as an approximation, subject to correction or confirmation by better preserved remains.

The præmaxillary teeth are cylindric, and the maxillary and mandibular teeth compressed.

Præmaxillary teeth (Plate 72, figs. 3, 4).—A perfect tooth, selected for description, is 10 millims. long; of this, nearly 4 millims. belong to the crown, which is separated from the root by a slight constriction or neck. The root contracts slightly towards the crown and towards its opposite end, and is slightly dilated intermediately. Its cross-section is nearly circular. Its surface is smooth. Two teeth which I slit longitudinally had a large pulp-cavity filled with spar extending the whole length of the root into the crown. The crown is slightly and unequally compressed, the inner contour of its cross-section being more convex than the outer. Its apex is acuminate and slightly inflected, which makes the outer longitudinal contour convex, and renders the inner contour sinuous, concave near the point and convex near the root. The outer and inner surfaces meet angularly, forming a low wing, within which, and parallel with it, upon the inner surface is a minute shallow groove. In very perfect unworn crowns the marginal wing bears a row of minute tubercles just visible in a strong light to the unaided eye. Both surfaces are highly polished and smooth. Upon the outer surface a few very minute striæ are discernible, and towards the neck both surfaces are beset with exceedingly minute tubercles (not recognizable as such without a magnifyer), the collective effect of which to the unaided eye is an extremely fine wrinkling.

A transverse section through the root of a præmaxillary tooth in position showed it to be contained in a distinct separate socket. The successional teeth, as usual, descend at the inner side of those in use.

Maxillary and mandibular (molar) teeth (Plate 72, figs. 5–9).—The crowns of these are compressed, their contour is sub-rhomboidal, both surfaces are convex longitudinally and transversely. The root is long, cylindroid, tapering. One surface of the crown—

that towards the mouth in mandibular teeth, and that towards the cheek in maxillary teeth—is sculptured by longitudinal ridges passing from a raised cingulum at the junction of crown and root to the free border of the former.

A smaller and a larger variety of compressed sculptured tooth are distinguishable, the former occurring in the front of the series. In a nearly perfect tooth of the smaller variety (Plate 72, fig. 6) the cingulum makes an angle open towards the summit of the crown. From the nearly axial angle a principal ridge passes to the apex of the cutting border, having on each side of it a secondary ridge, one of which does not quite reach the cingulum. Between the free ends of the secondary ridges, which give this part of the crown a coarse serration, and the lateral terminations of the cingulum, the sides of the crown are very finely serrated, repeating in miniature the lamelliform serration of the crown in *Iguanodon Mantelli*.

In the larger variety of the compressed tooth (Plate 72, figs. 7–9), the ornamented surface of the crown is sculptured by a greater number of ridges, which are less unequal in size. Some of them divide near the cutting border of the crown, rendering this, before being worn by use, finely crenated. The sides of the crowns of these larger teeth are finely serrated, as are those of the smaller variety. The contour of the crown of the larger is rounder and less angular than that of the smaller teeth. The unridged surface of the crown of both varieties has a few very minute inconspicuous striæ. All crowns which project fully above the level of the outer border of the alveolar process bear marks of wear. They are obliquely ground. The sculptured surface, having a thick enamel, lasts longer, and forms a cutting edge which, at first, is serrated by the cross-sections of the longitudinal ridges, and later becomes simply sinuous as these ridges decline in height in the level of the lateral angles of the crown. The worn surface of a large crown is usually marked by slight elevations not deserving the name of ridges passing from the inner to the outer surface, and the fine attritional striæ discernible in all worn teeth have this direction, suggestive of grinding lateral movements of the mandible in addition to gliding and hinge-movement in one plane.

By the time the crown is worn to the level of the alveolar border of the jaw, the tapering cylindroid root has been nearly absorbed, so that very slight force would suffice to detach the remnant of a tooth in this condition.

The compressed ridged teeth are not so separately enclosed in distinct sockets as are the cylindric præmaxillary teeth, but as in *Iguanodon Mantelli* and *I. Prestwichii*, the outer wall of the dentary groove sends inwards partitions which separate the roots, and nearly if not always quite reaching the inner wall of the groove must have afforded the teeth very firm support. The successional teeth are evolved in cavities at the inner side and intermediately of those in wear (Plate 72, fig. 2).

The structure of the skull shows a combination of Lacertilian and Crocodilian characters with a great preponderance of the former. The *supra-occipital* bone enters into the ring of the *foramen* magnum as it does in Lizards and in Birds, but not in Crocodiles, in which it is excluded from this opening by the union of the *exocci-*

pitals. The divided *frontal* is another Lacertilian trait: in Crocodilians the primitive division of the *frontal* disappears before the young leaves the shell.* The form and proportions of the nasals, and the prolongation of their anterior angles as an external and an internal narial process are imitated in some extant Lizards, but not, I think, in any Crocodilians. The division of the anterior nares and its method, as in *Megalosaurus,* are also Lacertilian features; the septum nasi mainly consists of the ascending median præmaxillary processes (closely imitated in *Hatteria,* in which the præmaxilla is paired; confluent in other Lizards where the primitive separateness of the præmaxillæ is early lost) which scarcely exist in Crocodilians in which the bony septum of the external nostril, when present, consists almost exclusively of the intruded tapering anterior ends of the nasal bones. The exclusion of the maxilla from the outer boundary of the external nostril, cited by R. OWEN as a Crocodilian character,† is perhaps apparent and not real, because the maxilla does not cease at the posterior margin of the external ascending præmaxillary process, but it is prolonged forwards beneath this and would become visible in very close proximity to the outer and lower part of the nostril if this process were removed. The lower temporal bar, a Crocodilian feature, is present in *Iguanodon Mantelli,* in the Liassic *Scelidosaurus,* and one extant Lizard—*Hatteria.* The fixity of the *quadrate,* another Crocodilian trait, is not attained by its being wedged in between the skull-bones, as in Crocodiles, but is due to the form of the squamosal articulation and the presence of the lower temporal bar. The anterior position of the palato-nares; the form, proportions, and connexions of the pterygo-palatine bars; and the median cleft in the palate are all Lacertilian characters not present in Crocodiles. The occurrence of teeth in the præmaxilla of simpler form than those in the maxilla and mandible, and the smaller size and minor complexity of the crown of a small number of the foremost teeth of the maxillary and mandibular series are highly interesting as foreshadowing the divisions of the teeth in higher Vertebrates. In form, in attachment, and in their mode of succession, the maxillary and mandibular teeth resemble those of Lizards, and not those of Crocodiles.

Vertebral column.—No remains have yet been recovered which demonstrate the exact number of vertebræ in the præ- and post-sacral segments of the column. The sacrum certainly comprises five vertebræ.

Professor R. OWEN, in his account of "Part of the Skeleton of a Young *Iguanodon* (*I. Mantelli*),"‡ preserved in the British Museum, Cat. No. 39,460, suggests that the most anterior of a continuous chain of seventeen præsacral vertebræ corresponds to the fourth cervical vertebra of an Alligator.

Professor HUXLEY, referring to the same vertebra, finding its capitular process in the level of the neuro-central suture, as in the eighth cervical vertebra of a Crocodile,

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* MIALL, 'Skull of the Crocodile,' p. 32.

† OWEN, R., "Fossil Reptilia of the Wealden Dinosaurus, *Iguanodon,*" Supplement 5, p. 6.

‡ OWEN, R., "Monograph on the Fossil Reptilia of the Wealden Formation," Pal. Soc., vol. for 1855, p. 2, t. 1.

suggests that there may have been seven or eight vertebræ between the most anterior preserved in No. 39,460 and the head.*

The correctness of this inference is demonstrated by the part of a skeleton in Mr. Fox's collection represented in the accompanying sketch (Plate 73). In close proximity to the mandible and shoulder-girdle is a continuous chain of nine vertebræ, proved cervical by the position of the capitular process (*parapophysis*). In the eighth vertebra in this chain this process is in the level of the neuro-central suture, and therefore in the same position as in the most anterior of the seventeen vertebræ displayed in No. 39,460, and in the eighth cervical of Crocodiles. If to the seventeen vertebræ in this latter fossil seven are added for those missing from its neck, and we allow two or three for those in the loins hidden by the foot, the number of præsacral vertebræ will amount to twenty-seven or twenty-eight at fewest, exclusive of the atlas, which is still unknown. Of this number, reckoning as cervical all in which the capitular process is wholly or partly on the centrum, nine, exclusive of the atlas, belong to the neck. Of the others, if we reckon as lumbar all in which a short vertebral riblet, unconnected with the sternum, is attached by a single articulation to the end of a transverse process, six at fewest should be referred to the loins. The remaining ten or eleven belong to the dorsal region. The number of caudal vertebræ was considerable, probably not less than fifty.

Cervical vertebræ (Plate 73, *cv.cv′.*; Plate 74, figs. 1–8†).—These are opisthocœlous. The contour of the anterior articular end is roughly shield-shaped; it is a rhomboid figure with the upper acute angle cut off and indented by the neural canal. The vertical and horizontal diameters of this end in the most anterior vertebra of No. 39,460 Brit. Mus. Cat. (Plate 74, fig. 4) are respectively 13 millims. and 10 millims. In a corresponding vertebra in my own collection (Plate 74, fig. 7) they are 14 millims. and 11 millims. The sides of the centrum are deeply pinched in below the neuro-central suture; an expansion of the centrum towards the articular ends makes them concave in the longitudinal direction; below, they converge to a somewhat acutely angular keel. All these vertebræ have a capitular process on the side of the centrum near its front. The position of this process ascends on the side of the centrum in passing from the front to the root of the neck. All have also a tubercular process (diapophysis) on the neural arch, placed just external to the præzygapophysis. The articular surfaces of the præzygapophyses are directed upwards and inwards. The spinous process is quite dwarfed in the anterior cervical vertebræ, but at the root of the neck well developed. The postzygapophyses are a pair of long branches diverging from the back of the neural arch having the articular surface on the under side of their free end.

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* HUXLEY, TH., Quart. Journ. Geol. Soc., vol. xxvi., p. 3, plate i. 1870.

† These vertebræ are better illustrated by the MANTEL-BOWERBANK fossil (No. 39,460 Brit. Mus. Catal.) than by any others I have seen. They are figured by R. OWEN in "British Fossil Reptilia," Pal. Soc., vol. for 1855, plate i., figs. 2, 3, 4.

Dorsal and lumbar vertebræ[*] (Plate 74, figs. 11, 12; Plate 76, fig. 2).—In passing from the front to the root of the neck, the convexity of the anterior and the concavity of the posterior articular end of the centrum decrease. Centra in the anterior dorsal region have their anterior articular end plane or slightly concave, and their posterior end somewhat more concave, but in much less degree than in the neck. With diminished angularity of the inferior keel, and the removal of the parapophysis from off the centrum, the contour of the articular ends of this becomes less shield-shaped and more rounded, until in the posterior dorsal region and in the loins it is approximately circular. The depression so marked in the sides of the cervical vertebræ below the neuro-central suture decreases in approaching the trunk, and from the mid-dorsal region to the loins the sides of the centrum are approximately flat in the vertical direction. In the longitudinal direction they are rendered slightly concave by an expansion of the centrum towards its ends. The transverse processes, in the anterior dorsal region stout and relatively long, have a capitular costal facet on their front edge where they spring from the neural arch, and a tubercular facet at their free end. Towards the loins the capitular facet approaches the tubercular, and in the lumbar vertebræ the relatively short stout transverse process has only a terminal facet. The average length of the dorsal and lumbar centra in No. 39,460 Brit. Mus. Coll. is 17 millims.; that of the vertical diameter of the articular ends is 14 millims.; and that of the middle of the centrum is 12 millims. The three last lumbar vertebræ in a piece in my collection (No. 110) are each 20 millims. long. The terminal lumbar vertebra is distinguished from the others, by its greater bulk, and particularly by the enlargement of the posterior end of the centrum in adaptation to the corresponding surface of the first sacral with which it is often found anchylosed (Plates 75 and 76, fig. 2).

Sacrum (Plate 74, fig. 9; Plates 75 and 76).—This comprises, as already stated, five vertebræ. These and the terminal lumbar in mature individuals are usually anchylosed together, with sometimes, but less frequently, I think, the first post-sacral vertebræ. All the five sacral vertebræ are smaller than the terminal lumbar. The first sacral is distinguished from the others by the great lateral expansion of the front end of its centrum (Plate 75, 1 *s.*). The shape of the second sacral centrum is cylindroid; it is constricted at the middle and expanded at its ends, which makes the lower outline of the sacrum sinuous. The swelling that marks the junction of the anchylosed centra is not a uniformly tumid ring, but is greater where the inferior and lateral surfaces meet, forming here a pair of small knots, as in *Iguanodon Mantelli.* Inferiorly, instead of being angulated through the inclination of the sides, as in *I. Mantelli*, the sacral centra in *Hypsilophodon* have a shallow median groove. The lower transverse processes are attached laterally at the junctions of the centra, they pass directly outwards, and their outer ends expanding antero-posteriorly coalesce and form a series of loops

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[*] These are well illustrated by the Plate referred to in the preceding page.

or sacral foramina. The root of each lower tranverse process is therefore connected with two centra, that of the first transverse process, stouter than the others, being attached to the terminal lumbar and the first sacral centrum. The construction is that which is usual in the dinosauria. The length of the sacrum represented in Plate 75 of one in Mr. Fox's collection is 8·3 centims., and the average length of the centrum is 17 millims. The length of the last lumbar vertebra in another skeleton in the same collection is 18 millims., the diameter at the constricted middle is 14 millims., and that of the expanded posterior end is 18 millims. The superior bulk of the last lumbar centrum is well shown in two sacras in my own collection.

DIMENSIONS of centra in two sacra.

	Length.	Horizontal diameter at front.	Horizontal diameter at middle.	Horizontal diameter at posterior end.
1. In Mr. Fox's Collection.—1st sacral .	15	21	10	15
„ „ 2nd „ .	15	15	10	15
2. In No. 110 Coll. HULKE.—1st sacral .	17	19	13	16
„ „ 2nd „ .	17	16	14	..

In the sacra to which reference has been made, the last centrum is too mutilated for description. One from a larger individual is 22 millims. long. Its anterior articular end, marked with the diverging striæ present before anchylosis, has a vertical diameter of 14·5 millims., and a horizontal diameter of 19·5 millims. The shape of the centrum is cylindroid. A mutilated transverse process, preserved on the left side, is attached to the upper part of the side of the centrum, at its front, for a space of 13 millims., and since the sutural surface of the process projects 3 millims. in advance of the anterior articular end of the centrum, it must to this extent have rested on the centrum in advance. The fifth sacral nerve escaped behind the transverse process through the intervertebral foramen between this and the first caudal vertebræ. The præzygapophyses look upwards, and slightly outwards. The spinous process, 21 millims. long and 14 millims. wide at its free end, has a slight backward slant.

Post-sacral or caudal vertebræ (Plate 74, figs. 9–13; Plate 75).—The first is known by the absence of chevron-facets, and the second by the presence of a single facet, the first chevron bone being articulated with the second and third caudal vertebræ. In shape the two foremost caudal centra resemble the last sacral, but the lateral surfaces below the transverse process are more flattened, which makes the third and succeeding centra less cylindroid. Transverse processes are present in about the first fourteen vertebræ, counted from the sacrum. Those of the first caudal are small and short, they project from the side of the centrum immediately below the neuro-central suture. Those of the second caudal vertebræ are longer and flatter. The length of the transverse process increases until about the eighth vertebra, behind which it rapidly shortens and becomes an inconspicuous tubercle which soon disappears. Where this occurs the length of the centrum is slightly increased. Its shape also becomes cylindroid, with a contracted middle and swollen ends, flattened slightly, laterally, and inferiorly, where also an oblique facet at each end marks the attachment of the chevrons (Plate 74, fig. 13). The spinous processes and chevrons are reduced much more slowly than are the transverse processes. In a rudimentary form they persist to the end of the tail; where most developed, in the front half of the tail, the spinous processes have the shape of flat oblong blades. The chevrons are longer than the spinous processes; their articular end is stout, and when well preserved is wedge-shaped, the anterior facet of the wedge being slightly the larger; their free end is expanded and flattened; and intermediately their shaft is contracted and slender.

In the part of the tail shown in Plate 74, fig. 13, the average length of the centrum is 25 millims., that of the spinous process is 35 millims., and that of the chevron bones 51 millims. The longitudinal streaks in this figure are ossified tendons.

The changes of shape of its articular ends and of the length of the vertebral centrum, in passing from the cranial to the caudal end of the centrum, add another to the already numerous refutations of the dictum which for many years was a great hindrance to the reconstruction of the dinosaurian skeleton, viz.: that the shape of its ends and the length of the centrum are constant throughout the column. The double costal articulation is repeated in Crocodilians, but not the opisthocœlous form of the centrum in the neck and front of the back. The great depth of the tail is probably in adaptation to swimming.

Ribs.—Ribs (pleurapophyses) are borne by all the præsacral vertebræ. (This statement does not apply to the two first vertebræ, respecting which information is still wanted.)

In the neck the riblets are short, their vertebral end is forked, the branches lie in a nearly vertical plane, and they articulate with corresponding upper and lower vertebral transverse processes. Their ventral or free end is extended antero-posteriorly. In the anterior dorsal region the capitular branch is long. Near the middle of the back the tubercular branch is reduced to a mere tubercle placed where the long, slender capitular branch or neck and the shaft of the rib join. Ribs from the posterior dorsal region show a reduction of the length of the neck with an approximation of the head to the tubercle, until in the loins both blend in a single terminal articular facet attached to the end of the transverse process. The form and the arrangement of the ribs in the neck and back are closely repeated in extant Crocodilia, but in the loins there is a small peculiarity to which allusion has been already made—the anchylosis of the rib (pleurapophysis) with the end of the transverse process, their junction being marked by a nodal swelling. I have seen this in three skeletons of adult *Hypsilophodon*, but not in those of Crocodilia.

Shoulder girdle and forelimb.

Sternum.—The breast-bone is broad and shield-shaped (Plate 73, *st.*). Its two halves are so inclined that they make a blunt median angle or ridge along the inferior surface, which starts from the bottom of a deep notch that indents the anterior border. The lateral margins bear in front the articular surfaces for the coracoids comprising a large segment of a circle. The chord of this in the fully-grown skeleton shown in the accompanying sketch is now 30 millims., but originally it measured somewhat more as it is somewhat mutilated posteriorly. Behind the coracoid surface are the marks of attachments of ribs. Of the number of these we have as yet no certain information.

Coracoid (Plate 73, *c.*; and Plate 79, fig. 1).—This is a flat bone of a simple, rudely crescentic shape. Its curved border in the articulated skeleton, mesial, is adapted to the corresponding surface of the sternum. Its outer border, much stouter than the mesial, consists of an articular part in front, and of a deeply incurved non-articular part behind. The articular part is subdivided into a thinner anterior segment firmly articulating with the scapula, and a stout expanded posterior part—the coracoid component of the glenoid fossa. In well-preserved examples a small chink passes a short way into the body of the bone from the point where these two sub-divisions of the outer border meet, and just in front of this fissure is a perforation as in *Iguandon Mantelli.* The width of the fully-grown coracoid represented in Plate 73, *c.*, measured from the middle of the outer border to the corresponding point in the inner border, is 45 millims.; and the length of the glenoid surface is 27 millims.

Scapula (Plate 73; and Plate 79, fig. 1).—This is a long, thin, narrow, slightly recurved bone, having a general likeness to that of *Iguanodon Mantelli.* Its dorsal end, in uninjured specimens, is expanded antero-posteriorly, its shape is not symmetrical, the backward extension being greater. The anterior border is sinuous, convex in its dorsal, and concave in its ventral half; and near the ventral end is an acromial projection, a repetition on a small scale of that shown in the scapula referred to *I. Mantelli* in the collection of J. B. Holmes, Esq., of Horsham, figured by R. Owen in his "Fossil Reptilia of the Wealden Formation, Monograph *Iguanodon,*" t. xiv. From this projection a ridge curves upwards and backwards across the outer surface ending at the upper or posterior lip of the glenoid fossa. The outer surface below this ridge is depressed, whilst that above it is slightly convex. The posterior border is slightly concave. At about two-thirds of the distance between the dorsal end and the glenoid fossa a slight projection breaks the otherwise regular curve of the posterior border (Plate 73). The ventral end of the scapula presents in front a relatively thin part suturally joined to the coracoid, and behind this a smooth articular part which with the corresponding part of the coracoid composes the glenoid fossa. The length of the scapula of an immature individual in my collection (No. 98 Cat., Coll. Hulke) (Plate 79, fig. 1) is 86 millims., being nearly that of the humerus; the width of its dorsal end is 26 millims., that of its ventral end is 32 millims., of which 10 millims. belong to the glenoid, and 22 to the coracoid segment; but these numbers have only an approximate value as the bone has been injured by pressure. Some of the dimensions of the scapula represented in Plate 73, are as follows:—Length from dorsal end to acromion 14·3 centims., length of glenoid surface 2·8 centims., and that of coracoid border about 3 centims.

Forelimb.—The structure of this is still incompletely known. I have as yet only recovered very mutilated remains of it.*

Humerus (Plate 73; and Plate 79, figs. 1 and 2).—This is nearly as long as the scapula. Its proximal end is broadly expanded, convex tranversely on its dorsal aspect, and concave in the same direction in the ventral surface. The proximal end bears, nearly at its middle, a smooth sub-spherical articular surface from which a ridge-like swelling passes some distance down the dorsal surface of the bone. The anterior, radial, border swells into a stout crest which renders its outline convex in the upper third of the bone. The contour of the posterior border is a hollow curve. The shaft seems to be slightly twisted owing to a small change in the direction of its surfaces. The distal end is condylarly divided. Behind, the condyles are separated by a wide shallow groove which ascends some distance on the shaft. The length of the humerus figured in Plate 79, fig. 1, is 13·6 centims.

Ulna (Plate 79, fig. 3).—Its proximal end is larger than that of the radius and it seems to have well-formed olecranon. The shaft and the distal end are slender. Its length slightly exceeds that of the radius.

Radius (Plate 79, fig. 3).—The length of this bone is about 12·5 centims. in a skeleton in which the humerus is about 13·6 centims. long. The breadth of its carpal end is 21 millims. The expanded carpal end is the chief support of the manus.

Manus (Plate 79, fig. 3).—A proximal row of carpalia appears to consist of a large *os radiale* and a smaller *ulnare.* The metacarpals are much smaller than the metatarsals. The ungual phalanges resemble in form those of the pes, but are smaller than these.

In its extreme simplicity the shoulder girdle of *Hypsilophodon* differs from that of most extinct Lizards. Its *coracoid,* in respect of its simple crescentic form, devoid of bony procoracoid, agrees with that of every other dinosaur yet known. It is imitated in *Hatteria* and in *Chameleo,* and is very unlike the long, narrower coracoid of Crocodilians. The sternum, in respect of its shape, resembles the (cartilaginous) sternum of Crocodiles more than that of Lizards, except *Chameleo.* The scapula, as regards its length and narrowness, closely agrees with that of *Iguandon Mantelli, I. Prestwichii,* and *Megalosaurus,* and differs from that of *Hylæosaurus,* of *Scelidosaurus,* of Crocodilia and extant Lizards except *Chameleo.* The humerus in the greater expansion of its ends and the slenderness of its shafts resembles that of a Lizard more than that of a Crocodile. The inner tuberosity of the proximal end resembles that in

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* See postscript.

Iguanodon Mantelli, and in Birds. The ungual phalanges in shape resemble those of the pes, as it is now known they really do in *Iguanodon Mantelli,* the restoration of the manus of this dinosaur in the "British fossil Reptilia" being based on mistaken identifications.

Hip-girdle and hind limb.

Ilium (Plate 77).—This has the elongated form which characterises this bone in the Iguanodontidæ. Its præacetabular process, extremely long and slender, extends forwards upon the loins overhanging the posterior lumbar ribs. The acetabular arc ends anteriorly in a relatively slender pubic process directed downwards and forwards, and behind it is terminated by a low swelling which marks the attachment of the ischium. The lower border of the broad post-acetabular part, nearly straight, is slightly inflected, and is indented by a longitudinal groove.

Ischium (Plate 75; Plate 76; Plate 77).—This bone was first identified by Professor HUXLEY in the British Museum fossil No. 39,460.* It had before been considered to be a tibia.† It is the same bone which in *Iguanodon Mantelli* passed for so many years as clavicle.‡

It has the form of a long slender bar directed backwards parallel with the lower border of the post-acetabular part of the Ilium. That this is its true direction may be seen by its impression in the stone below the left Ilium in the annexed sketch made several years ago of a fossil in Mr. Fox's collection (Plate 77).§ The proximal end of the ischium is expanded. It has above a stout process, which united to a corresponding process on the Ilium behind the acetabulum, completed below and posterioly the circle of this articular hollow. This acetabular process is borne on a narrow part or neck, the axis of which makes an angle of about 90° with the long axis of the bone. Below the acetabular process and the neck is a wide rudely quadrilateral expansion directed downwards, the lower curved border of which contributes the lower margin of the acetabulum. Behind this expansion the bone rapidly contracts and continues narrow through about half its length when it again widens. Nearly midway between its acetabular and its ventral or lower end is a lip-shaped out-turned obturator process (Plate 75, *ob.p.*), against which rests the rod-like part of the *os pubis.* The lower or ventral ends of the *Ischia* seem to be symphysially united.

Pubis.—The form of this bone repeats in miniature that of *I. Mantelli* (Plate 75; Plate 76, fig. 1; and Plate 77). It has a stout short part, or body, with a smooth articular surface, which in front is attached to the pubic process of the *Ilium,* and behind meeting the *Ischium* completes below and in front the circle of the acetabulum. From

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* HUXLEY, Quart. Journ. Geol Soc., vol. xxvi., plate i., fig. 3. 1870.

† OWEN, R., "Fossil Reptilia of the Wealden Formation," Pal. Soc., vol. for 1855, p. 2, t. 1.

‡ OWEN, R., 'Fossil Reptilia of the Cretaceous Formation,' p. 105, plate xxxiv.

§ This figure shows the right *Ischium,* which has been uncovered since the above was written.

this part a broad bar-like plate extends forwards and downwards, and an extremely slender long rod passes backwards parallel with and supported by the corresponding ischial bar, which it nearly equals in length. I cannot certainly say that the ventral ends of the pubis were symphysially joined, but the appearances make this probable.

The bottom of the bony acetabulum was defective.

Femur (Plate 75; Plate 78; Plate 80).—This is a much stouter bone than the humerus. Its proximal end has a sub-globular head borne on a stout short neck, the axis of which makes nearly a right angle with that of the shaft; and a prominent outer trochanter between the upper part of which and the shaft is a deep narrow fissure. Behind the head is a small pit, and in front between the neck and the outer trochanter is a larger depression. The distal end has the usual condylar shape. The outer condyle is longer and stouter than the inner. Both project strongly backwards, and are here separated by a wide deep intercondylar groove, the outer border of which is formed by a narrow ridge which divides the intercondylar groove from a deep but much narrower groove in which the upper end of the fibula moved, the ridge itself being received in the interval between the fibula and the outer tibial condyle in flexion of the leg on the thigh. The anterior intercondylar groove is wide and shallow, contrasting strongly in these respects with the deep narrow, almost tunnel-shaped, anterior intercondylar groove in *Iguanodon*, as known in *I. Mantelli*, *I. Prestwichii*, and *I. Seelyi*. The shaft of the femur appears to be twisted owing to the alteration in the aspect of its surfaces, that which at the proximal end is external becoming at the distal end anterior. It is also bent, its upper longitudinal outline being a convex curve. The cross-section at the middle is rudely prismatic. Nearer to the upper than the lower end of the bone at the inner and posterior aspect of the shaft is a compressed triangular, in the best-preserved specimens, remarkably long-pointed inner trochanter, the apex of which is directed towards the distal end of the bone. At the inner side of this trochanter is a pit.

Tibia (Plate 80, fig. 2; Plate 81, fig. 1).—This bone in *Hypsilophodon* is longer than the femur, the opposite of that which obtains in *Iguanodon*. A nearly perfect tibia of *Hypsilophodon* measures 23·25 centims. long, the length of the femur of the same skeleton does not exceed 18 centims. The proximal end of the tibia shows a division of the articular surface into two condyles, which project posteriorly and are here separated by an intercondylar groove. A large præcnemial crest projects from the upper part of the shaft in front of the external condyle. The cross-section of the shaft is prismatic. The distal end is expanded and shaped into two malleoli, of which the outer and posterior is longer and thinner, and the inner and anterior is shorter and stouter. These are separated in front by a wide shallow groove, which ascends a slight distance on the anterior external surface, and below ends behind at a salient angle where the inner and posterior surfaces meet. The outer border of the distal half of the tibia is impressed in its anterior aspect by the fibula which here lies upon it. The axis of the shaft is twisted in such a manner that a plane laid through the long diameter of the proximal end cuts at a large angle another plane laid through the long axis of the distal end (Plate 80, fig. 8).

The following are some of the dimensions of a fully grown tibia :—Extreme length, 22·25 centims. ; breadth across the proximal condyle, 3·35 centims. ; from the back of the inner condyle to the most prominent point of the præcnemial crest, 5·7 centims. ; breadth across the malleoli, about 4·6 centims.

Fibula (Plate 77 ; Plate 78 ; Plate 81).—The upper end is flattened. It rests against the outer surface of the outer proximal condyle of the tibia behind the great præcnemial crest. In flexion of the knee it is received in a groove on the outer femoral condyle, as already mentioned. The shaft, sub-prismatic, rests in its lower half on the anterior surface of the tibia, just within its outer border. Its distal end is stout. It articulates with the anterior of the two divisions in the upper surface of the *os calcis.*

Pes.—The tarsus comprises two distinct bones corresponding to those of the proximal row of *ossa tarsalia* of other Vertebrates—an *os tibiale* or *astragalus,* and an *os fibulare* or *calcis.* Evidence of the presence and the composition of a distal row of distinct *tarsalia* is yet incomplete.

Astragalus (Plate 80, figs. 3–7).—The upper surface of this, the largest tarsal bone, is the counterpart of that of the tibia ; it is concave from front to back, and in this direction divided by a ridge that marks out two portions corresponding to the inner and part of the outer tibial malleolus. The under surface is pulley-shaped, convex from front to back, and sinuous transversely, being in this direction convex towards the ends and concave intermediately. The anterior margin is a thin lip, the posterior margin is stout. The inner end is non-articular and sub-cutaneous, the outer end (very thin by the approximation of the upper and under surface) articulates with the *os calcis.*

Os calcis (Plate 80, figs. 3–7).—The outer, non-articular, sub-cutaneous surface is crescentic. The upper border, nearly straight, is interrupted by a slight elevation, the outer end of a ridge which divides the upper surface into an anterior moiety that receives the end of the fibula, and a posterior in which the outer part of the external tibial malleolus rests. The under surface is convex from front to back. The inner surface, or rather border, very narrow, articulates with the astragalus.

The *astragalus* and *os calcis* conjointly form a pulley-shaped articular surface on which the front part of the pes moves. The interlocking of the upper surface of the conjoined bones with the ends of the tibia and fibula renders impossible any movement of them upon the leg-bones.

In a right foot of a nearly fully-grown *Hypsilophodon* I found a wedge-shaped bone inserted between the proximal end of the IVth. metatarsal and the *os calcis.* Its position and shape correspond to a bone in the pes of *Scelidosaurus Harrisonii*[*] and

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[*] OWEN, R., "British Fossil Reptilia of the Oolitic Formation," part ii., plate x., fig. 1.

certain American Dinosauria[*] regarded as the homologue of the *cuboid.* In another foot between the astragalus and the base of IVth. metatarsal I found a small bone which may be the homologue of the *external cuneiform.* These identifications must be accepted with reservation; they must be confirmed or corrected by new and less disturbed materials.

Metatarsus (Plate 77; Plate 79, fig. 4; Plate 81, figs. 2, 3).—The metatarsus of *Hypsilophodon* contains five bones, of which four are large and support functional toes, and one is rudimentary. The proximal ends of the four large metatarsals are in closest mutual apposition. Those of the two inner ones with the two distal *tarsalia* (if the identification of these latter be correct) form the distal half of the mid-tarsal joint where movement of the foot on the leg takes place.

The distal ends of these *metatarsalia* are stout, their articular surface is pulley-shaped, the pits for the attachment of lateral ligaments are large and deep. Their long, slender, prismatic shafts have a slight forward and inward curve. Counted from the inner to the outer border of the foot, the IIIrd. is the longest and largest metatarsal; the IV.th. is slightly longer than the IInd.; and this latter slightly exceeds the Ist. The following table gives some dimensions of the metatarsals in a fully-grown *Hypsilophodon*:—

	I.	II. millims.	III. millims.	IV. millims.	V. millims.
Length	†	93·	105·	87·5	32·
Breadth of proximal end. . .	. .	12·	12·5	. .	8·5
Breadth of distal end . . .	. .	. .	20·	15·	. .

The Vth. metatarsal is a small tapering styliform bone, its distal end bluntly pointed does not support any phalanges. I found it first in 1876 in close relation to the IVth. metatarsal and *os calcis*, and I have since observed it in five other hind feet.

Phalanges (Plate 79, fig. 4).—The four functional toes have respectively two, three, four and five phalanges, and therefore correspond to the Ist., IInd., IIIrd., and IVth. toes in the foot of existing Lizards and Birds. The second, third, and fourth phalanges of the IVth. (outer toe) are shorter than the other phalanges, but this shortening is less than in the same phalanges in the foot of *Iguanodon Mantelli* and *I. Prestwichii.*

The ungual phalanges are long, pointed, slightly curved (Plate 79, fig. 4); their proportions to the metatarsals and other phalanges can be seen from the annexed table of measurements of the foot of an immature individual.

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[*] Marsh, O. C., "Principal Characters of American Jurassic Dinosaurs," from American Journal of Arts and Sciences, vol. xvi., Nov., 1878, plate ix., fig. 3. *Laosaurus.*

[†] Too mutilated for measurement.

Lengths	Ist. toe. millims.	IInd. toe. millims.	IIIrd. toe. millims.	IVth. toe. millims.	Vth. toe. millims.
Metatarsals	..	63·	69·	56·	..
1st phalange	..	23·	25·	17·	..
2rd phalange	17·	14·	17·	12·	..
3rd phalange	..	21·	13·5	8·	..
4th phalange	..	..	23·5	8·	..
5th phalange	..	..	..	17·	..

The *sacrum,* as in all true dinosauria, differs in the greater number of vertebræ composing it from that of *Saurii* and *Crocodilini.* The *Ilium* closely imitates that of Aves in the great extension of its præacetabular part—a part which in *Saurii* and *Crocodilini* exists only as a mere rudiment.

The *Ischium* in respect of its length and slenderness and its direction parallel to the lower border of the post-acetabular part of the Ilium differs entirely from that of Lizards and Crocodilians and closely resembles that of Aves, as noticed first by Professor HUXLEY. The slender rod-like part of the *pubis* (*post-pubis,* O. MARSH) directed backwards parallel to the Ischium is obviously the homologue of the pubis in Aves and in *Saurii.* Its broad *præpubic* or præacetabular plate (*præpubis,* O. MARSH) has no homologue in *Saurii;* in Aves it exists as a mere rudimentary tubercle. Its homology with the bone called pubis in Crocodilians deserves consideration.

The *Ilium, Ischium,* and *Pubis* all contribute to the formation of the acetabulum, as I believe occurs in all other dinosauria of which the pelvic bones are known, *Omosaurus* being no exception to this, for the suggestion that in this dinosaur the pubis articulates only with the Ischium and is thus "seemingly" excluded from the acetabulum* obviously arises from a misconception of the homologies of the several parts of the pubic bone. A comparison of its pubis with those of *Iguanodon Mantelli, Hypsilophodon Foxii,* and certain *American dinosaurs* must make this apparent to every unbiassed mind.

The different proportions of the *femur* and *tibia* in *Hypsilophodon* and *Iguanodon Mantelli* have been noticed. In the former it is longer than the tibia, in the latter it is shorter than the tibia. In *Hypsilophodon* the inner trochanter is nearer the proximal end than it is in *Iguanodon Mantelli,* and it is also more pointed than in this. The differences in the anterior intercondylar groves in these two dinosauria are striking. The tibia is more slender than in *Iguanodon Mantelli.* The Avian resemblances of femur, tibia, and proximal row of tarsalia are very striking. The pes differs from that of *Iguanodon Mantelli* notably in the presence of a fourth functional toe. This alone would, I submit, suffice to exclude it from the genus *Iguanodon* in which the pes has but three functional toes. In having four functional toes and the rudi-

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* OWEN, R., 'Mezozoic Reptilia,' part ii., p. 64.

ment of a fifth, probably also in the presence of two distinct tarsalia answering to those of the outer side in the distal row of tarsalia in higher Vertebrates, the hind foot of *Hypsilophodon* closely agrees with that of the Liassic *Scelidosaurus Harrisonii.* The sharp pointed curved ungual phalanges, of very different form to the blunt depressed unguals of *Iguanodon Mantelli,* are obviously related to a different habit of life. *Hypsilophodon* was adapted to climbing upon rocks and trees.

Postscript.

(Added October 9, 1882.)

Since the above was written, further work by the skilful mason of the Palæontological Department, Mr. Barlow, upon a block of sandstone in the Fox Collection in the British Museum, has very recently brought into view several additional parts of the skeleton, of which the pelvis is represented by Plate 75. These afford much information respecting the structure of the fore limb, and an opportunity of comparing the fore and the hind limbs in the same individual. In the left fore limb the coracoid and scapula remain naturally articulated, and the natural relations of the humerus, ulna and radius are only slightly disturbed, but the manus had broken up and its bones were scattered before the consolidation of the rock. The right ulna and radius with the manus attached, the dorsal surface towards the spectator, are well preserved.

The shapes and proportions of the coracoid, scapula and humerus agree so nearly with those already given from other specimens as to make any detailed description of them superfluous—the chief apparent difference is the absence of the slight projection from the posterior border of the scapula shown in Plate 73. A textural difference in the fossil marks off the expanded vertebral end of the bone as a supra-scapula. The length of the scapula including its supra-scapular part is nearly 9·7 centims., and that of the humerus is 9·5 centims.

The fore-arm (Plate 79, fig. 3) is shorter than the arm. The length of the left ulna is 8·7 centims. (that of the right ulna 8·5 centims., the slight difference is explained by the different exposure of the proximal end); the radius is shorter, its length being 7·8 centims. (the right radius 7·7 centims.).

The ulna is stout and massive at its articular ends; and its shaft is laterally so compressed as to greatly narrow the posterior surface in this part. The distal end of this bone, 1·3 centims. wide, appears to me to afford—relatively to the radius—a larger support to the carpus than the same bone in *Iguanodon Mantelli.* The radius is a more slender bone than the ulna and its figure is more cylindroid. Its proximal end is slightly expanded. The capitellum is followed by a slight contraction or neck which merges into a relatively slender shaft, widening and flattening distally towards the carpal extremity; the breadth of which appears to be somewhat less than that of the corresponding part of the ulna.

In the carpus (Plate 79, fig. 3) (through which unfortunately passes a crack filled with carbonaceous matter) a proximal row of *ossa Carpalia* is clearly recognisable. This appears to me to comprise a wedge-shaped *os ulnare* proximally articulating with the ulna and radially with a polygonal *os intermedium,* whilst its distal border corresponds to the base of the IVth., and slightly to that of the IIIrd. metacarpal bone. The *intermedium* is proximally in relation chiefly with the ulna, but its radial border touches the lower end of the radius here coming into contact with an ossicle, which from its situation with respect to the radius must be regarded as an *os radiale.* This is unfortunately damaged by the crack through the rock already mentioned. Distally the *os intermedium* corresponds principally to the base of the IIIrd., and in a less extent to that of the IInd. metacarpal bone. The *os radiale* is opposed distally to the base of the Ist. and slightly to that of the IInd. metacarpal bone. The condition of this part of the fossil does not permit me to speak confidently of the presence of a distal row of *Carpalia.*

Five digits are recognisable: numbering these from the radial to the ulnar border of the paw, the Ist. metacarpal is 1·7 centim. long, and so is slightly shorter than the IInd., which is 1·85 centim. long. This, again, is slightly exceeded by the IIIrd., which is 1·95 centim. or 2 centims. long, and is the longest one, the IVth. attaining only 1·2 centim.; whilst the length of the bone which I regarded as the Vth. metacarpal is only 8 millims. In its shape and its proportions this bone has more resemblance to a digital phalanx than the other metacarpals, but in these very points, as also in its different direction to that of the other metacarpals, it corresponds so well to the Vth. metacarpal in *Iguanodon Mantelli* that I may not hesitate to regard it as such. Proximally it articulates with the *os ulnare.* Its distal end is pulley shaped, distinctly articular. The phalangeal continuations of the toe are missing. On a small scale the shape and proportions of the other metacarpals repeat the metatarsals and do not require particular description.

In the Ist. toe two digits are discernible. In the IInd. toe three phalanges are apparent, of which the proximal is nearly hidden by that of the IIIrd. toe. This latter (the IIIrd.), the line of which is slightly displaced from that of its metacarpal, certainly has four phalanges, the lengths of which are 9 millims., 5·5 millims., 4 millims., and 8 millims. (the lengths of the proximal and 2nd phalanges of the IIIrd. hind toe of the same skeleton are 2·5 centims. and 2·1 centims.).

It is fortunate that the number of phalanges, four, in this toe (IIIrd. of manus) is beyond question, because this shows an essential structural difference between the fore foot of *Hypsilophodon* and that of *Iguanodon Mantelli,* in which, upon the evidence of undisturbed specimens in the Brussels Museum, no digit has more than three phalanges. In the IVth. toe of our fossil the proximal phalanx is succeeded by a second, of which the distal half is missing, as is also the continuation of the toe, so that the number of phalanges it had before mutilation is unknown. The ungual phalanges of the manus resemble on a small scale those of the pes, as already stated. Evidently the manus of *Hypsilophodon* conforms more nearly to that of existing Lizards, its type is more generalised than that of the manus of *Iguanodon Mantelli*, which is highly specialised.

The relative smallness of the fore limb in *Hypsilophodon*, shown for the manus by the lengths of the metapodia and phalanges already given, is evident also upon a comparison of the other corresponding segments of the fore and hind limb, as may be seen by the following numbers :—

TABLE of Lengths.

	centims.
Femur*	15·0
Tibia*	15·5
IIIrd. metatarsal	7·6
Humerus	9·5
Ulna	8·7
Radius	7·8
IIIrd. metacarpal	1·9

EXPLANATION OF PLATES.

** *Denote fossils formerly in the collection of the late Rev.* WM. FOX, *recently acquired by the British Museum.* *** *Denote fossils in my own collection. Unless otherwise stated the figures represent the natural size of the fossils.*

PLATE 71.

The following lettering applies to all the figures in this plate.

pa.	The parietal bone.
fr.	The principal frontal bone.
prf.	The præfrontal bone.
psf.	The post-frontal bone.
nas.	The nasal bone.
nar.	The anterior nares.
pmx.	The præmaxilla. * Its median ascending process. † Its lateral ascending process.
mx.	The maxilla.

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* These bones are slightly mutilated and their real lengths are somewhat more.

orb.	The orbit.
utb.	The upper temporal bar.
ltb.	The lower temporal bar.
sq.	The squamosal bone.
popr.	The parotic process.
fm.	The foramen magnum.
oc.	The occipital condyle.
qu.	The quadrate bone.
mn.	The mandible.

Fig. 1. An oblique view of a skull of an adult *Hypsilophodon*. It represents the upper surface and the right side of the brain-case, and the palatal aspect of the snout which, wanting the mandible, has separated from the hind part of the cranium in front of the orbits, and is twisted on its long axis and laterally displaced.***

Fig. 2. Oblique view of the upper surface and the right side of a skull of a smaller individual.**

Fig. 3. A posterior view of the same skull.** (From a sketch by the author before the occipital condyle was mutilated.)

Fig. 4. Oblique postero-inferior view of the left mandible with the quadrate, and a piece of the lower temporal bar attached to the latter.

PLATE 72.

Fig. 1. Inner view of lateral parts of a skull (much abraded) in which the quadrate bone preserves its normal relations.**

qu.	The quadrate bone.
sq.	The squamosal bone.
mn.	The mandibular.

Fig. 2. Inner view of the anterior part of a left mandibular ramus.**

ed.sp.	Marks the edentulous symphysial spout.
sy.	The symphysis.

Fig. 3. Outer view of a præmaxillary tooth.***

Fig. 4. Side view of the same.***

Fig. 5. Two worn teeth from near the front of the maxilla with the crown of a successional tooth.***

Fig. 6. An unworn tooth from near front of the maxilla.***

Fig. 7. A crown of a larger maxillary tooth, posterior in position to figs. 5 and 6.***

Fig. 8. Side view of the same.***

Fig. 9. A large nearly perfect maxillary tooth.***

(The mark I indicates the actual length of the teeth.)

PLATE 73.

Part of a skeleton of a fully-grown *Hypsilophodon*.**

cv′, cv.	On the left hand of the figure is a continuous chain of cervical vertebræ. In front of *cv′* is the reversed mandible *Mn.*
sc.	The right scapula.
h.	The right humerus, its proximal end still resting in the glenoid fossa.
c.	The right coracoid.
st.	The sternum.

PLATE 74.

In this plate *sp* marks the spinous process. *dp.* The diapophysis. *pp.* The parapophysis. *tr.* The transverse process. *ch.* Chevron bones. *prz.* The præzygapophysis. *psz.* The postzygapophysis. *k.* The inferior keel. *nap.* The neurapophysis.

Figs. 1–4. Three foremost cervical vertebræ in the MANTELL-BOWERBANK fossil. No. 39,460, Brit. Mus. Cat. (Figured by R. O. in "Brit. Foss. Rept.," Pal. Soc., vol. 1855, tab. i.)**
Fig. 1. Left lateral view.
Fig. 2. Upper view.
Fig. 3. Under view.
Fig. 4. Anterior view of *a* in fig. 1.
Fig. 5. Lateral view of a cervical vertebræ of a younger individual.***
a. The anterior surface.
Fig. 6. Inferior view of the same.***
Fig. 7. Anterior view of the same.***
Fig. 8. Posterior view of the same.***
Fig. 9. Lateral view of the 5th sacral, and two foremost caudal vertebræ.**
Fig. 10. Front view of 5th sacral in the preceding figure.**
Fig. 11. Lateral view of a dorsal vertebra.**
Fig. 12. Lateral view of a lumbar vertebra. Its spinous process postzygapophyses, and transverse process (foreshortened) are broken off.**
Fig. 13. Lateral view of three caudal vertebræ with their chevron bones.**

PLATE 75.

Inferior view of the sacrum and pelvic bones of an immature *Hypsilophodon*.**

Ll.	The last lumbar vertebra.

1–5 *s.*	The sacral vertebræ.
1–5 *f.*	The sacral foramina.
P.	The right os pubis.
P′.	The left os pubis.
Is.	The right ischium.
Is′.	The left ischium.
ob.p.	Obturator process.
Sy.	The symphysis.
Fm.	The femora.
cv, cv′.	Caudal vertebræ.
ch.	Chevron bones.

PLATE 76.

Fig. 1. Inferior view of sacral and lumbar vertebræ with ischia and pubis of an immature *Hypsilophodon.***

Is.	Ischium.
P.	Os pubis.

Fig. 2. Inferior view of lumbar and sacral vertebræ of a fully-grown *Hypsilophodon.****

s.	Sacral vertebræ.
Ll.	Last lumbar vertebra.
a.	Acetabulum.

PLATE 77.

The pelvic bones and right hind limb of a fully-grown *Hypsilophodon.***

For their better individual representation the artist has removed these bones slightly from their relative positions in the fossil.

Il.	The ilium.
Is.	The ischium.
Pra.	The præacetabular portion of the os pubis.
Psa.	The postacetabular part of the same.
f.	A foramen corresponding to that which in the Bird transmits the tendon of the Obturator internus muscle.
a.	The acetabulum.
Fm.	The femur.
Fb.	The fibula.
T.	The tibia.
C.	The os calcis.
1–5 *m.*	The metatarsal bones.

PLATE 78.

Figs. 1–5. Views of the left femur of the skeleton of which the shoulder-girdle, &c., are shown in Plate 73.***

Fig. 1. Outer surface.
Fig. 2. Inner surface.
Fig. 3. Inferior surface.
Fig. 4. Superior surface.

ot. Outer trochanter.
i.tr. Inner trochanter.
cap. Capitulum.

Fig. 6. Side view of a femur with the tibia and fibula in almost undisturbed natural relation.***

T. Tibia.
F. Fibula.

Fig. 7. Front view of the lower end of the same femur as fig. 6.***

PLATE 79.

Fig. 1. The scapula, *Sc;* humerus, *H;* and coracoid, *C,* of an immature individual.***

g. The glenoid fossa.

Fig. 2. The right humerus of the individual whose pelvic bones are represented in Plate 75.*** Its proximal end is mutilated.

Fig. 3. The forearm and manus of the same individual.***

R. The radius.
U. The ulna.
1–5. The digits.

Fig. 4. A right hind foot.

T. The tibia.
1–4. The digits.
M. The metatarsals.

PLATE 80.

Fig. 1. Femur of an almost fully-grown individual.***

cap. The capitulum (mutilated).
ot. The outer trochanter (mutilated).
it. The inner trochanter.

Fig. 2. Outer view of a tibia (the left) of an almost fully-grown individual.***

ic. The inner condyle.
pr.c. The præcnemial crest.

Fig. 3. Front view of the lower end of the bones of the leg with the proximal tarsalia.
F. The fibula.
T. The tibia.
As. The astragalus.
Ca. The os calcis.
Fig. 4. Outer view of the same.
Fig. 5. Posterior view of the same.
Fig. 6. Inner view of the same.
Fig. 7. Inferior view of the same.
Fig. 8. View of the proximal end of the tibia (fig. 2), projected on a dotted outline of the distal end to illustrate the directions of their long axes.

PLATE 81.

Fig. 1. Front view of the same tibia as shown in Plate 80, fig. 2.
pcr. The præcnemial crest.
Fig. 2. Lateral view of bones of right hind leg and foot.
T. The tibia.
F. The fibula.
Ca. The os calcis.
1–5 *m.* The metatarsals.
Fig. 3. Oblique outer view of a left hind foot. The lettering as in fig. 2.

PLATE 82.

Restoration of the skeleton of *Hypsilophodon.*

XXV. *The Minute Anatomy of the Thymus.*

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[PLATES 83–95.]

TABLE OF CONTENTS.

The following research was begun in the year 1876, and has been carried on at intervals since that time.

We will first give the result of this research in a brief preliminary statement; secondly, the history of the subject; and finally, a detailed account.

PRELIMINARY STATEMENT.

I.—*The cortex is the more important and essential part of the follicle of the thymus. In the cortex there are two kinds of retiform tissue—one (with small meshes), which we call a reticulum, formed of fine fibres with thickened nodal points; and the other, which we call a network, composed of branched connective tissue corpuscles.*

II.—*The formation of connective tissue and of new vessels occurs chiefly by the agency of granular cells; these granular cells arise from connective tissue corpuscles, and there is great similarity between the formation of the connective tissue in the thymus, and in pathological new formations.*

III.—*A concentric corpuscle usually arises, partially from the granular cells, and partially from the epithelioid connective tissue corpuscles; and the giant cells from either one or other of these sources. The concentric corpuscles are transformed into fibrous tissue.*

IV.—*The connective tissue corpuscles in the thymus of the Dog can undergo certain changes, which finally transform them into ciliated epithelial cells.*

V.—*In involution, the gland, diminished in size, though with enlarged blood-vessels, is transformed into connective tissue, and in Mammals is buried in fat.*

VI.—*The thymus arises, in the embryo, in connective tissue.*

VII.—*There are present in the lymph issuing from the thymus, cells containing coloured blood corpuscles and hæmoglobin granules; and in the lymphatics of the thymus there are more colourless cells than in the lymphatics of the neck.*

HISTORY.

An account of the knowledge and speculations of the old writers is given very fully by HAUGSTED (1),* SIMON (2), and FRIEDLEBEN (3); consequently this part of the history is dealt with very shortly in this paper.

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* See the references, p. 1119.

Rufus of Ephesus (4) first mentioned the thymus, and noticed that it cannot be found in all bodies.

Galen (5) said that the organ has attained its greatest size in newly born animals, and diminishes after birth.

This view, that it decreases very rapidly after birth, seems to have been held from this time until the latter half of the seventeenth century; although Möbius (6) noticed a thymus in the post-mortem examination of a man thirty years old.

Glisson (7), 1671, found that it is large in children.

Mangetus (8), 1717, said that the gland is very large in newly born children, but decreases during childhood.

Verheyen (9), 1726, was the first who definitely presented the fact that the thymus grows after birth. He said that it is larger in children some years old, than in those newly born.

Hewson (10), 1777, said that the gland enlarges to the end of the first year; and since his time it has been generally accepted, that the growth of the gland is not terminated by the duration of fœtal life.

It was in the end of the seventeenth century that two anatomical statements were put forward, both of which delayed for a long time a more extended knowledge of this organ. One statement was as fruitful a source of physiological error as the other was of anatomical misconception.

In 1670, Velsch (11), in the Alpine Marmot, confounded the fat glands and the thymus. This confusion between fat glands and thymus, which was a very natural one, was continued by Harder (12), Scheuchzer (13), Pallas (14), Meckel (15), Prunelle (16), and Tiedemann (17). It was in consequence of this error that Pallas (14) in 1778 compared hibernation to fœtal life. Cuvier (18) noticed that the thymus and other glands diminish the capacity of the chest in hibernating animals. In 1817, however, Jacobson (19) showed that the glands in the thorax, which enlarge in the winter sleep, are not thymus; and that the thymus behaves in animals which hibernate, as in other animals; and maintained that the thymus has nothing in common with the peculiar glands lying in its neighbourhood, either in its development, appearance, or secretion. He was followed by Rudolphi (20) and Haugsted (1), the latter giving drawings of the two organs at various periods of life; and the facts were further confirmed by Barkow (21) and others. Simon (2), however, accepted the older doctrine, holding that the thymus is very large in hibernating animals, and he partially founded upon this supposed fact his view of the function of the organ.

The second error arose, as has been stated, about the same time. In 1673, Bartholinus (22) asserted that the thymus contains a cavity; and a great part of many subsequent anatomical researches were taken up with the various accounts of this cavity, the openings into it, and other supposed facts about it. The existence of a cavity was accepted by Hugo (23), who worked with Haller, by Lucae (24), by

ASTLEY COOPER (25), and indeed by almost all anatomists, with the exception of BICHAT (26), BERRES (27), BISCHOFF (28), and GOODSIR (29), until we come to quite recent times, when the methods of hardening the tissue had so improved, that JENDRASSIK (30), and FRIEDLEBEN (3) had grave doubts of its existence; and BERLIN (31), KLEIN (32), and AFANASSIEW (33), asserted that the thymus contains no cavity.

WHARTON (34), 1659, noticed that in young Oxen which were put to the plough, the thymus disappears much sooner than in those at rest.

HALLER (35), 1766, thought that the thymus is not entirely changed into fat, but lost in fat. "*In adipe circumfuso sepelitur.*"

HEWSON (10), 1777, first described the thymic corpuscles, and found that in size and shape they exactly resemble the central particles in the vesicles of the blood; he attributed to them the destiny of becoming nuclei for the vesicles of the blood. He tied the lymphatics of the thymus of the Calf and the Dog, and found that they contain a great number of small colourless solid particles, such as are found in the fluid of the lymphatic glands. He suspected that the lymphatic vessels were possibly the excretory ducts of the thymus, and that the structure and uses of this organ are similar to those of the lymphatic glands, to which it may be considered an appendage.

BICHAT (26), 1803, said that the thymus is composed of a mass of little vesicles.

LUCAE (24), 1811, described the gland as divided into lobes, lobules, and alveoli (follicles). He noticed that the alveoli, when seen from the surface, have a polygonal area.

Sir A. COOPER (25), 1832, gave drawings of the lobes of the Calf's thymus, with the communicating vessel. (See his fig. I.) He found that the thymus consists of ropes, which can be unravelled, and which are disposed in a spiral manner; he thought he recognised two large absorbent vessels, which run down the cervical portions and terminate in the innominate vein.

HAUGSTED (1), 1832, confirmed JACOBSON'S (19) observation as to the difference between the fat glands and the thymus, and demonstrated most clearly the increase of size in the thymus after birth; giving drawings proving both facts. He noticed that if thin layers of the thymus of infants are viewed by transmitted light, the central part of the follicles is more pellucid. He found the supposed cavity in the thymus to be hardly present in health.

GULLIVER (36), 1842, noticed a somewhat similar fact to that which had been related by WHARTON (34), as he saw that, in overdriven Lambs, the thymus shrinks; but that if the animals are afterwards well fed, the gland fills out again. GULLIVER followed HEWSON (10) in considering that this organ provides germs for the tissues of the blood.

BISCHOFF (28), 1842, also considered that the function of the thymus is to form blood corpuscles.

OESTERLIN (37), 1843, first noticed that there are cells containing pigment in the thymus.

SIMON (2), 1845, wrote an elaborate essay, which obtained the ASTLEY COOPER prize; he dissected many rare animals, having access to the collection of the Royal College of Surgeons. He gave careful and accurate accounts of the position of the thymus in many Mammals, and described its position for the first time in Birds, Reptiles, and some Amphibia, incorrectly, however, in the Frog. SIMON rejected the theory of HEWSON (10), and considered that "the presence of the gland is co-extensive with pulmonary respiration;" "that it secretes into a closed cavity;" that in most animals the secreted matter "presents itself in a fluid form;" in some animals in "the solid form of fat;" and that the gland "fulfils its use as a sinking fund of nourishment in the service of respiration."

BARKOW (21), 1846, in a monograph on hibernation, pointed out in his sixteenth chapter that it was known to ARISTOTLE and to MARTIAL, that hibernating animals are very fat. BARKOW maintained the difference between the thymus and fat glands, and gave a very accurate description of the position of the latter.

GOODSIR (29), 1846, found no cavities or reservoirs in the substance of the thymus. He asserted that the thymus consists of lobes, grouped around germinal spots, which derive material for the formation of the cells from arteries passing through them.

ROBIN (38), 1847, first noticed the thymus in the Fish.

HANDFIELD JONES (39), 1849–1852, noticed that the absorbent vessels of the thymus travel to glands in the anterior mediastinum. He found that the thymic corpuscles are not similar in appearance and structure to the colourless granular corpuscles of the blood.

HASSALL (40), 1849, said that the gland consists of a straight tube, with the follicles arranged around it in a spiral manner. He described the concentric corpuscles (which have sometimes been called the concentric corpuscles of HASSALL) as parent cells, containing several granular nuclei, each of which is surrounded by one or more concentric lamellæ.

After this time nearly every writer dwelt on the structure of the concentric corpuscles, and their opinions on that subject do not differ so very materially from those of later writers; but their views of the origin and functions of these bodies differ very widely.

We may distinguish three opinions as to the origin of the concentric corpuscles. (1) That they arise from the cells of the thymus, (*a*) by endogenous formation, HASSALL (40), WEDL (41), GÜNSBERG (42), BERLIN (31): (*b*) as a retrograde process, BRUCH (43): (*c*) in a similar manner to cancroids, VIRCHOW (44), VERNEIUL (45), and HIS (46). 1859: (*d*) arising by a fatty metamorphosis of the thymic corpuscles, ECHER (47): (*e*) from the gland cells, with a super-imposition of amorphous substance, KÖLLIKER (48), 1854, and JENDRASSIK (30): (*f*) from the gland cells, after passing through a stage of epithelial cells, PAULIZKY (49). (2) That they arise from the blood vessels, (*a*) in the same manner as the angiolithic sarcoma, CORNIL and RANVIER (50), or (*b*) from the endothelium of the vessels, AFANASSIEW (51 and 33). (3) From the remains of the epithelium, from which the thymus is originally developed, HIS (52), 1880, and STIEDA (53). KÖLLIKER (54), in 1863, said that they do not arise from gland cells. Their attachment to vessels was noticed by BERLIN (31), HIS (46), and PAULIZKY (49).

BRUCH (43), 1850, noticed that the outer cells of the concentric corpuscles are epithelioid in character. He did not believe that the gland cells are changed into fat cells, as SIMON (2) and many other authors had done; but found fat cells in the connective tissue surrounding the follicles.

ECKER (47), 1853, in a long paper, propounded the most definite and elaborate statements concerning the supposed cavity, comparing the thymus to an acinus gland. He thought that the gland consists of a central canal, closed at either end, into which, during its spiral course, many cavities open. He said that the vessels are outside the gland membrane. In describing involution, he found that points or streaks of fat pierce the gland tissue, while in the surrounding connective tissue the fat increases; that certain cells contain fat granules, and that there are intermediate forms between these cells and the concentric corpuscles, which he described as arising by fatty metamorphosis of thymic corpuscles. He noticed the occurrence of pigment in the thymus of the Tortoise. He proved that the organ discovered in the Fish by ROBIN (38) is the thymus. At nearly the same time as LEYDIG (55) he described the position of the thymus in the Frog. He failed to find any difference between the fluid in the lymphatic vessels of the thymus and in those of other parts of the body.

GERLACH (56), 1854, adhered to the view that cavities exist in the interior of the gland. He noticed that the vessel network is not so rich as had been imagined, and also that extravasations often occur in the central part of the follicle.

WEDL (41), 1854, thought that the concentric corpuscles are endogenous formations, and gave good drawings of the large granular cells.

KÖLLIKER (48), 1854, described the thymus as being formed of lobes, connected by a canal which runs spirally in the interior of the gland. On the inner side of this canal he found openings, each of which leads into a lobule. He stated that the acini are solid (the acini in his drawings are, however, the cortical parts of the follicles). He noticed vessels penetrating the alveoli. He did not think it probable that there is a direct formation of blood corpuscles from thymic corpuscles, as HEWSON (10) thought.

REMAK (57), 1855, described and figured ciliated cysts in intimate connexion with the upper part of the thymus of the young Cat. He compared them to the ciliated bladders of the mesogastrium of the Frog and the mesometrium of the Rabbit. He noticed a distinction between the cortical and medullary parts of the follicle. (See his plate 8, figs. 10*a*, 10*b*.)

LEYDIG (55), 1857, also described the position of the thymus in the Fish; he classed the thymus with the lymphatic glands, and said that it contains a network like that of the PEYER's patches.

JENDRASSIK (30), 1857, considered that the lobules are solid, for though he noticed in some cases a hollow running through the gland, he also found portions removed from the rest, in which there was no canal. He thought that the hollows arise by a softening process, and agreed with KÖLLIKER (48) that the vessels penetrate the follicile, as he noticed large veins in their centre. He concluded that the concentric corpuscles compress and obliterate the vessels, and that they are thus concerned in involution. He observed extravasations of blood, of which some were recent and some degenerated; and that degenerated blood corpuscles are found in the concentric corpuscles.

BERLIN (31), 1857, doubted if the concentric corpuscles are concerned in involution; he noticed that they are situated in the neighbourhood of the vessels, and thought the central cavity an artificial product, and that there is no change of gland cells to fat cells; he said that chromatin crystals are often found in the gland.

GÜNSBERG (42), 1857, found crystals of hæmatoidin in the thymus, and cells containing blood corpuscles identical with those that are met with in the spleen.

FUNKE (58), 1858, came to the same conclusion as HEWSON (11) and BISCHOFF (29) in regard to the function of the thymus, considering it an organ for the restoration of the morphological elements of the blood. He concluded that some of the gland cells are transformed into fat cells.

FRIEDLEBEN (3), 1858, who is the anthor of a valuable monograph, the labour of many years, found, in the thymus of the Calf, a central string in each cervical portion. He thought that there is no duct, and that the organ consists of closed follicles. In a postscript, however, he described hollows. He noticed that the richness of the vessels had been greatly over-estimated. He gave weights of the thymus during various periods of life, to prove that it increases up to the end of the second year. He noticed concentric corpuscles in the thymus of the fœtus, but did not consider them important factors in involution; he concluded that involution partly arises from the connective tissue pressing on the follicular wall, and partly from a fatty change of the contained tissue. In involution he noticed an increase in the size of the coats of the vessels; and imagined that this is due to a change in the nerves. FRIEDLEBEN quoted and confirmed RESTELLI's (59) observation that the blood of the thymic vein, in Calves three to four months old, contains numerous elements of the thymus; and that the blood coming from the thymus has a marble-like appearance. He failed to notice any difference between the contents of the lymphatic vessels, and of the lymph vessels of other parts; and stated that the number of lymphatics is much exceeded by that of the veins. He removed the thymus in young animals as RESTELLI (59) had previously done, and found that the animals lived and increased in weight very rapidly, and that in the blood of these animals there were more colourless blood corpuscles than usual.

HIS (46), 1859, described, and gave very good figures of, the connective-tissue-corpuscles, and the long threads forming the network. He considered that the thymus has a central thread, to which the lobules are attached; this thread consisting of artery, vein, lymphatic vessels, and central canal; that each acinus has a small hollow, and that all the hollows are in communication with a central canal. He gave figures showing the difference, in the follicle, between the cortical and medullary portions, drawing the latter as a dark round or oval space in the centre of the follicle; he concluded that the dark spaces are the central hollows. (See his figs. 20 and 17.) He said that all the lymphoid corpuscles show a ring of protoplasm around the nucleus, if they are treated with five per cent. phosphate of soda. HIS noticed that there are cells with pigment granules and with great red spheres like blood corpuscles, and that the concentric corpuscles are connected with the small vessels, often surround them completely, or rest on their points of division. He noted that the lymphatic vessels are in close approximation to the wall of the follicle. He tied these vessels as they leave the thymus and concluded that the corpuscles travel by these paths, although he had never seen any communication between the central cavity and the lymphatic vessels. He considers that the thymic corpuscles are changed into blood corpuscles; in describing the circulation he gave drawings of the thymus after it had been injected from the arteries and the veins, and found that the arteries run into the centre of the follicles, and ramify outwards, and that the larger veins are on the outside.

MELCHIOR (60), 1859, in his dissertation dealt chiefly with the pathology of the thymus.

TURNER (61), 1860, noticed a thymus in an adult Porpoise and in an adult Antelope.

HIRZEL and FREY (62), 1863, wrote on the hibernating gland. They noticed the fact that if these glands are exposed to the air they assume a dark red-brown colour.

KÖLLIKER (54), 1863, adhered in great measure to his former view. He described the star-shaped cell network in the follicles, and noticed large cells with multiple nuclei. He confirmed the fact observed by HEWSON (10) and HIS (46), that the lymphatic vessels contain very many corpscules, similar to those of the thymus, and described the simple concentric corpuscles as having a granular mass in their interior; he did not think that they arise from the gland cells.

PAULIZKY (49), 1863, compared the concentric corpuscles to the corpora amylacea. He adhered to the view that the elements undergo fatty degeneration. PAULIZKY'S drawings of the concentric corpuscles and of the granular cells are very good. He noticed, as REMAK (57) and JENDRASSIK (30) had previously done, that the concentric corpuscles are found in the central parts of the follicles; he thought that they are pathological formations, arising from epithelial cells; these epithelial cells taking their origin from small round cells. He described and figured the vacuolation of the granular cells.

TOLDT (63), 1868, examined the thymus of Amphibia; he gave drawings of perivascular spaces around the vessels in the thymus of the Salamander.

KLEIN (32), 1870, noticed cellular elements in the capsule. He stated that there are wide lymphatic spaces which invest the follicles; these communicate, by means of finer vessels, with the central part of the follicle. He gave the last blow to the idea that there are cavities in the thymus, and he found the individual follicles either entirely encapsulated or at times united. He described three kinds of networks in the gland: one, composed of anastomosing cells; a second, a narrow network with greater breadth of trabeculæ; and a third, formed of strong elongated fibres stretched between adjoining vessels. The cells enclosed in the network are small cells, similar to lymphatic corpuscles, large coarsely granular spheroidal bodies with one or more nuclei, and concentric corpuscles.

FLEISCH (64), 1870, in writing about the thymus of Amphibia, described and figured so-called ganglion cells; these were the granular cells already described by KÖLLIKER (54), and PAULIZKY (49), and figured by the latter.

HENLE (65), 1873, described the thymus, and appeared rather to lean to the older view, that the follicles contain cavities.

CORNIL and RANVIER (50), 1873, said that concentric bodies, calcified or otherwise, are often found in the thymus; these bodies hang from the vascular walls, and have the same origin as the angiolithic sarcoma—*i.e.*, they arise as hollow buds, which communicate with the lumen of the vessel, and growing, become pedunculated. The cellular elements are flattened, and arranged in concentric layers upon the wall of the vessel.

RAINEY (66), 1875, put forward some peculiar views as to the physiology of the thymus.

KRAUSE (67), 1876, described long spaces as existing in the gland, connected with one another: they are not canals, but connective-tissue spaces, filled by arteries, veins, lymphatic vessels, and nerves. The follicles are completely penetrated by blood vessels. The external vessels are radiating.

AFANASSIEW (51), 1877, said that the concentric corpuscles arise from the endothelium of the veins and capillaries; the endothelial cells increase by division, and fill the lumen. He found vessels in which the lumen was stopped up by the increase, and others where the endothelium was concentrically arranged. He noticed blood corpuscles in the interior of the concentric corpuscles; and also said that the walls of the vessels are often thickened, and can beget the capsule-like formations. He quoted the monograph on the development of cancer by KÖSTER, who said that concentric corpuscles in carcinoma are derived from the endothelium of the lymphatic vessels.

In a *second paper*, AFANASSIEW (33) found that the periphery of the follicles is clearly bordered, and that the central parts run into one another. In describing the network, he said that it is not formed of cells, except in the embryo; but that flat connective-tissue cells lie on the nodal points of a reticulum. There are lymphatic vessels in the interfollicular tissue, and they spread out into the follicle, in the direction of the blood vessels. The lymphatic vessels, leaving the thymus, run to lymphatic glands in the mediastinum and in the neck. He found that coloured and colourless blood corpuscles escape from the vessels, and thus an increase of connective-tissue is set up; the new tissue presses on the concentric corpuscles, which gradually fade away, and thus few of them remain in the later stages. He injected the concentric corpuscles in five cases, and said that the smallest are in size and form similar to sympathetic nerve ganglia. Involution is produced by the decay of the vessels; yet the presence of the concentric corpuscles is an expression of regressive metamorphosis. He found much pigment in the thymus of the Hedgehog; and in certain Mammals, round cells containing one or more coloured blood corpuscles. There is a fatty granular formation in the glandular elements; and fat arises in the pigmented cells, which are divided from the rest of the thymus by the bundles of connective-tissue running into the follicle; the pigmented cells are very like those of the hibernating gland.

DAHMS (68), 1877, noticed a difference, in stained sections, between the outer and inner parts of the follicles, in animals of all ages; but thought the difference due to condensation of the glandular elements at the periphery of the follicle. She considered that the centres of the concentric corpuscles are filled by two or three spherical cells, but that they are formed by lymphoid cells; and she found that there are more concentric corpuscles during involution than at other times.

WATNEY (69), 1878, in a short note, showed that the follicles consist of two parts, which differ in the character of their network, and of the cells contained in their meshes, and in the arrangement of the blood vessels. He described granular cells, which help to form, partly by a process of vacuolation, the concentric corpuscles. These granular cells are attached to the blood vessels, and to the trabeculæ. He found giant cells in the medulla, and stated that the concentric corpuscles are concerned in the formation of vessels and trabeculæ; and that in fresh preparations colourless cells are seen, which contain granules and spherules of hæmoglobin.

KLEIN (70), 1879, confirmed the fact of the existence of a cortical and medullary portion in the follicle. He said that in the medulla there are large endothelial cells, each of which fills up a mesh of reticulum; and there are transition forms between these and the granular cells, and between these last and giant cells.

KLEIN (71), 1881, found that the thymus does not much vary in the young and in the adult Guinea Pig.

WATNEY (72), 1881, in a second short note, stated that there is an increase of connective tissue during involution, inside and outside the follicle; that the cortical part of the follicle attains its greatest size at the period of the greatest activity of the gland; and that the granular cells are actively concerned in the formation of connective tissue. A network of connective tissue corpuscles and a reticulum were described as both existing in the cortical portion.

WATNEY (73), 1881, in a third short note, stated that the lymphatic vessels issuing from the thymus contain more colourless corpuscles than the lymphatic vessels of the neck; and that in the thymus of the Dog ciliated epithelium can be found, if the animal be over a certain age.

History of the views concerning the development of the thymus.

This part of the subject has been given separately.

HAUGSTED (1), 1832, found that the thymus appears in the ninth or tenth week in the human embryo, as a double organ.

ARNOLD (74), 1831 and 1837, thought that the thymus is developed from the mucous membrane of the respiratory tract, and that it arises nearly in common with the larynx and thyroid.

BISCHOFF (28), 1842, noticed the gland as early as the eighth week in the Mammalian embryo ; he considered that it arises in connexion with the thyroid.

SIMON (2), 1845, described and figured the thymus as commencing as a simple hollow tube ; the tube bulges at certain points, and gives origin to diverticula or follicles, and by the extension of this process of follicular growth, the thymus attains its bulk and complexity of structure.

ECKER, 1851–1859, in the 'Icones Physiologicæ,' gave drawings of the position of the thymus in the Chick on the nineteenth day of incubation, and in the embryo of *Squatina vulgaris.*

REMAK (57), 1855, said that the thymus is developed from part of the pharyngeal wall, and arises between the arches of the aorta ; that in the front of the neck of the Chick there is a gland which disappears very rapidly. He found that it has near relations to the thymus, although the one is not developed from the other ; he accepted ECKER's view of the position of the thymus in the neck of the Chick.

FRIEDLEBEN (3), 1858, found that the gland is not formed originally as a tube, but that it appears as a connective-tissue stroma, in which round nuclei are imbedded. This stroma multiplies on all sides ; the capsule then shows traces of blood vessels.

ROBIN (75), 1874–1875, said that by the successive invaginations of the internal folds (? of the hypoblast) the epithelium of the pharynx, of the œsophagus, of the trachæa, and of the lungs is formed ; and that the thymus and the thyroid are produced in the same manner.

AFANASSIEW (33), 1877, found that the thymus is developed in the Chick at the end of the fifth or sixth day of incubation in the reticular connective-tissue, which lies behind and outside the vessels, as masses of indifferent cells. In a fœtal Rabbit, one-fifth of an inch in length, he saw the commencing thymus in the connective-tissue in front of the carotid artery, as a heaping up of indifferent cells.

DAHMS (68), 1877, thought that the thymus arises as a mass of agglomerate little rounded bodies around a clearer part ; that in some cases there is a canal in the very early stage ; and that there is an invagination of the mucous membrane of the respiratory tract.

KÖLLIKER (76), 1879, said that it arises, as an epithelial organ, from the branchial clefts; he gave a drawing of a cross-section of the gland, in its earliest stage of development, showing a central canal. The canal is lined by many layers of epithelial cells.

The cells become gradually smaller and less distinct, until at length they look like round nuclei, and have lost their epithelial character; at the same time there is an ingrowth of vessels and connective-tissue in the thick wall of the organ. As regards the further development of the gland, KÖLLIKER agreed with the views of SIMON (2).

Besides the thymus, he found in embryoes of fifteen, sixteen, and seventeen days, an accessory thymus as a double organ between the œsophagus and the trachæa; this organ showed hollow spaces and vessels in the embryo of seventeen days old. In an embryo of fourteen days he found another organ like a thymus before the first arch of the aorta. In an embryo of sixteen days, he found the lateral part of the thyroid consisted of two parts—one was in its usual place, the other on the side of the carotid, and behind the upper end of the thymus.

HIS (52), 1880, gave drawings of closed rings of epithelium, which he said probably form the thymus. In a footnote he said that the acinous form of the gland is thus explained; that the adenoid tissue arises from the surrounding connective-tissue, and that the concentric corpuscles are the remains of the epithelium.

STIEDA (53), 1881, examined the thymus of many young Mammalian embryoes; he demonstrated and figured the thymus as arising from one of the branchial clefts (probably the third) as a hollow epithelial tube; around this tube there is a connective-tissue envelope, and between these two adenoid tissue is developed. He found it impossible to trace the epithelial elements; but he thought that the concentric corpuscles and the large cells are the remains of the original epithelial layer.

Opinions of various authors concerning the so-called plasma cells.

WALDEYER (77), in 1875, described cells with coarse granules, which are often found near the blood vessels; they are easily transformed into fat cells; he called them plasma cells.

FLEMMING (78), in 1876, found that plasma cells are more numerous in some animals than in others; he did not deny that they form fat cells, but he thought that they much resemble atrophied fat cells.

EHRLICH (79), 1877, used an aniline dye, *dahlia*, to trace out the plasma cells; he found that they do not contain fat; he noticed them, among other places, in the thymus of the Calf, where they have the form of lymphoid cells.

LÖWE (80), 1878, considered that the granules in the plasma cells are not fat granules, but the precursors of fat.

EHRLICH (81), 1879, showed that the cells, all classed by WALDEYER as plasma cells, are probably not all of a similar nature; he considered that certain of these cells should be classed separately; they are characterised by their containing granules which are very readily stained by aniline dyes.

There have been two other papers on these subjects by WESTPHAL (82) and SCHWARZE (83), neither of which I have been able to obtain. The last author said that certain granular cells occur in the blood and in the organs which form the blood; and that the granules in these cells are neither fat nor hæmoglobin.

The Methods used.

(*a*). To tease up the tissue in saline solution, either in the fresh state, or after leaving small pieces some hours in $\frac{1}{10}$ per cent. osmic acid; another method is to immerse small portions in a dilute solution of bichromate of potash, to which a considerable quantity of watery solution of eosin has been added; the pieces were usually left in the solution from twenty-four to forty-eight hours, at times from one to three weeks; and were then teased in saline solution and mounted.

(*b*). To make sections of the thymus. For this purpose the tissues were hardened in one of the following ways: (1) for ten days in $\frac{1}{6}$ to $\frac{1}{4}$ per cent. chromic acid, and then in 50 per cent. alcohol for two days, and afterwards placed in strong methylated alcohol; (2) in 2 per cent. bichromate of potash for weeks or months, and then in alcohol; (3) in chloride of gold for some hours, and then in chromic acid, and subsequently in alcohol; (4) in equal parts of $\frac{1}{2}$ per cent. osmic acid, and $\frac{1}{5}$ per cent. chromic acid, from ten to fifteen days, and then transferred to alcohol; (5) in methylated alcohol.

The sections were, except in a few instances, stained in hæmatoxylin, and mounted in Canada balsam; some of them were shaken in a test tube, from twenty minutes to four or five hours, before being mounted.

It was found that to shake out the smaller cells, a two rigid hardening of the tissue was prejudicial.

Staining by means of hæmatoxylin extract.

It has long been known to those who have used a watery solution of extract of hæmatoxylin and alum (one part of the extract to three of alum, as recommended by Professor Arnold (84) and Dr. Klein (85)) that in certain cases the resulting fluid is purple, at other times blue, and occasionally red. The red solution gives a disagreeable colour to the sections, and also stains very slowly.

In working with various microscopical specimens of the thymus, it was found, however, that certain tissues were well stained by the red solutions, and others readily by the blue. It has been my custom in many cases to double stain the specimens, using first a red solution of the colour of Plate 86, fig. 22, *c*, and deepening the staining of the specimen, and making parts of it purple-blue by a solution of the colour of Plate 86, fig. 22, *e*.

The method adopted is to leave them in a rather strong solution of *c*, from sixteen to eighteen hours. They are then washed and placed in a weak blue solution, from four to twenty-four hours. It is necessary to use the red fluid of such a strength that it will not change to blue, even during three or four days' exposure; and to insure this it is better to use it rather strong, as all weak solutions are more liable to turn blue. It is also better to keep the red solution very strong, and to filter a few drops of it into a watch-glass of water to the required strength each time it is used, as all filtering tends to turn the solution blue, especially if a clean dry filter-paper is used; and the filtering of weak solutions is prejudicial to the red colour. The blue solution is used with more water, because the weaker the colour the bluer the specimen.

Specimens of thymus treated in this way will, in most cases, show double staining; the difference of colour, it is true, is not much marked, but still is quite sufficient to clearly distinguish the cortical from the medullary part of the follicle (see Plate 83, figs. 1, 2; and Plate 84, fig. 3), or to stain the granular cells of the thymus, and the surrounding protoplasm, of two distinct colours (see Plate 85, fig. 8).

The red hæmatoxylin stains the connective-tissue, and the protoplasm of the connective-tissue-corpuscles and of the granular cells, and the walls of the vessels; while the blue deeply stains mucus, almost all nuclei, the reticulum, and the lymphoid corpuscles.*

As a rule, if only a single staining is used, the solutions which are of a purple colour are much the best; the red hæmatoxylin solutions do not stain deeply enough, though specimens stained by them bear high powers well. The blue solutions leave the protoplasm of the cells almost uncoloured; and the dark colour of the nuclei, and of the lymphoid cells, prevents the other features of the specimen from being well seen, in fact such specimens are very unsatisfactory.

The difference in colour of the hæmatoxylin solutions depends almost entirely on the alum which is used in making them. It is true that the different woods give a slightly different extract, but the variation between the extracts, whether they are made from Campeachy or Jamaica, Honduras or St. Domingo woods, is so slight as not to be of any real importance. The alum by keeping becomes more acid, and acid, as is well known to chemists, turns the blue colour of hæmatoxylin, red. The most intensely blue colour is obtained by using freshly-prepared dried alum, while old powdered crystals of alum (potash alum) give a most decided red or even a yellow colour. The proportion of alum to the hæmatoxylin extract is also somewhat important. If the proportion sinks below three of alum to one of extract, the red colour will probably be seen; but, in that case, the staining properties will not be so good. Any amount of alum, above three of alum to one of extract, appears to make little difference to the colour. To show the difference between the solutions when different alums are used, see Plate 86, fig. 22. The solutions whose colours are represented by *a*, *b*, *d*, *e*, *f*, were all made with the same extract of hæmatoxylin, on the same day, and with the same proportion of alum to extract, one of the latter to three of the former.

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* The words "lymphoid corpuscles" are used in this paper in reference only to the smaller thymic corpuscles, such as are figured in Plate 92, figs. 72, 73, 74, 75, and 76, as *Thc.*

Staining by means of extract of Brazil wood.

This extract is treated with alum, in the same manner as that from hæmatoxylin woods. The colour is very beautiful, and varies from dull red to carmine, according to the character of the alum used in making the solution. The extract stains in much the same manner as hæmatoxylin, and can be used with it as a double staining. There are three kinds of wood in the English market called Brazil wood, *i.e.*, Lima chips, Sappan wood, and Brazil wood. They all contain a staining substance: Brasilin, which changes from yellow to red, as hæmatoxylin changes from yellow to blue.[*][†]

GENERAL OBSERVATIONS.

The thymus was examined in the human fœtus and the Child, and in the following animals: the Hedgehog, the Cat, the Dog, the Lamb, the Calf, the Ox, the Alpine-Marmot, the Rat, the Guinea Pig, the Rabbit, the Pelican, the Turkey, the Pigeon, the Chick, the Alligator, the Snake, the Tortoise, the Frog, the Axolotl, the Cod-fish, and the Ray-fish.

The thymus first appears in early embryonic life. It increases rapidly in size during the life of the embryo, and during the early part of the growth of the animal. The time at which it ceases to enlarge, and the duration of its involution, differ considerably in the different classes of Vertebrates, and in the different species. In Mammals it decreases slowly, does not get much smaller until adult life, and does not finally disappear until some period of adult life has passed.[‡] In Birds it has in most instances disappeared before adult life, and the same is true of the thymus of the Frog. In the Tortoise and the Ray-fish the thymus is very large in large specimens, as in a Ray three feet five inches wide; however, in the large Rays the thymus has been in a great measure transformed into connective tissue.

The involution has been traced in the Ox, Cat, Dog, Guinea Pig,[§] Bird, Tortoise, and Ray-fish.

The position of the thymus varies much in different animals, and has been carefully described by SIMON (2), LEYDIG (55), AFANASSIEW (33), and others; so that there is no need to say any more, than that in Mammals it is found either in the thorax on the pericardium, as in the Dog; or in the neck, as in the Guinea Pig where the follicles are contiguous to the salivary gland; or extending from the pericardium to the thyoid, as in the Child; or even nearly to the angle of the jaw, as in the Calf. In Birds, it

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[*] See FLÜCKIGER and HANBURY 'Pharmacographia,' p. 216, 2nd edit., 1879.

[†] I have to thank Messrs. HEARON and SQUIRE, and Messrs. LESCHER and EVANS, for taking considerable trouble to procure me the various hæmatoxylin and Brazil woods.

[‡] It is in consequence of the want of this knowledge that it has been described by many observers as being permanent. I have seen a thymus in a man fifty years old.

[§] There is some difficulty in obtaining old Guinea Pigs, as it is difficult to ascertain the ages of the animals; and the breeders usually have no knowledge of the age of any given animal.

is found on each side of the neck, at the back of the jugular vein (see Plate 87, fig. 29); in the Reptile, lying on the vessels near the base of the heart, as in the Alligator or the Snake; or situated near the carotid artery, as in the Tortoise; in Amphibia, at the back of the angle of the jaw; in the Ray-fish, lying behind the spout cavity, and between the gills and the muscles of the back (see Plate 84, fig. 5, *Th.*).

Its appearance in Mammals is always the same—*i.e.*, a grey white mass divided up into lobules and follicles. In the Bird, it appears as an elongated mass, which to the naked eye contains a central band, on which the follicles are arranged; in the Reptile, as a small oval pale yellow mass, a little whiter than the fat of the animal; in Amphibia, as a white oval body; in the Ray-fish, as a grey-white triangular mass, very soft in texture, but in the adult animal showing the individual follicles to the naked eye.

In the following pages, the thymus, as it is found in the Mammal, will be described; and afterwards the various differences and points of agreement between the thymus of the Mammal and that of other Vertebrates will be pointed out.

The thymus is surrounded by a connective-tissue envelope; this covering sends in membranous projections, which penetrate between the separate lobes, and to a very great extent between the lobules; and penetrating into the lobules, mark out the division into follicles (see Plate 83, fig. 1). In the connective-tissue run the blood and lymphatic vessels, and a few nerves.

If we attempt with the scalpel and forceps to divide up the thymus, and select the thymus of the Calf, as being from its great size the easiest to deal with, it will be found that we can easily separate the gland into lobes, and these again into smaller lobes. In the lobes, the lobules are marked out to the naked eye, especially in portions of the gland which have been allowed to lie for some hours in 2 per cent. bichromate of potash. They are generally four-sided, and somewhat pyramidal in form (see Plate 87, fig. 24). The attempt to separate the lobes into lobules is not however always successful, as though some lobules are unconnected, in most cases they are attached to one another or to a central string which runs up the cervical portion of the thymus, as was known and depicted by various authors including Sir A. COOPER (25). This central string consists of connective-tissue and of blood vessels. The lobules are seen by the naked eye to be composed of follicles, but it is not possible to separate the individual follicles, as they are almost always attached to one another (see Plate 83, figs. 1, 2; and Plate 84, fig. 3).

ANATOMY OF THE FOLLICLE AS A WHOLE.

The follicles are, in the fully-developed thymus, chiefly of a polygonal form (see Plate 83, fig. 1). As, however, they are usually attached to one another, their forms are somewhat various. In some cases, compound branched follicles are met with (see Plate 83, figs. 1, 2; and Plate 84, fig. 3).

The follicles, except in earliest embryonic life, are composed of two parts, a cortex and medulla, which, as we shall see, differ very materially in structure. The difference between the two parts can be seen by the naked eye in a thymus which has been left for two or three weeks in bichromate of potash or monochromate of ammonia. In sections made from hardened tissues and stained in hæmatoxylin, this difference is easily seen (see Plate 83, figs. 1, 2; and Plate 84, fig. 3), and depends on two main facts: the first, that the concentric corpuscles, the giant cells, the granular cells, the numerous endothelial connective-tissue-corpuscles (together with the large vessels and fibrous-tissue met with in certain states of involution) are found, with few exceptions, in the medulla, and that therefore there are necessarily much fewer lymphoid corpuscles in that part of the follicle; the second fact is, that the reticulum is but rarely met with in the medulla, while it surrounds nearly every lymphoid corpuscle in the cortex; and the preponderating presence of these two deeply staining bodies, the lymphoid corpuscles and the nodal points of the reticulum, causes the cortex to assume a much darker and bluer colour in preparations stained by logwood. In some follicles there are seen to be two or even three medullary portions (see Plate 83, fig. 1, *c*).

The follicles are sometimes united by cortical-tissue (see Plate 83, figs. 1 and 2, *a*), and at other times by medullary-tissue (see Plate 83, figs. 1, 2; and Plate 84, fig. 3, *b*). These follicles, when united by medullary-tissue, may sometimes be spoken of as compound follicles (see Plate 84, fig. 3, *b*).

It will be noticed that processes of the connective-tissue penetrate the follicle in a radiating manner (see Plate 83, figs. 1 and 2, *c′t′*). These processes often contain somewhat large vessels (see Plate 85, fig. 6, V′), and thus the medulla is in contact with the surrounding connective-tissue and vessels; and it is not necessary for the vessels to pass through the cortical lymphoid tissue in order to reach the medulla.

In the embryonic state, the upper part of each half of the thymus may be said to consist of one follicle (see Plate 87, fig. 23). This is not at first differentiated into cortex and medulla, and the differentiation does not occur until the follicle is divided up by connective-tissue, as will now be described. The blood vessels penetrating into the follicle from the outside, pass in, surrounded by connective-tissue; and by the ingrowth of this connective-tissue, the original follicle is divided into a considerable number of follicles (see Plate 86, fig. 11).* The follicles increase in size, and are each divided, and thus the thymus is formed of a large number of follicles, partially or wholly separated from one another; these follicles forming lobules, which are often united. During the ingrowth of this connective-tissue, the blood vessels increase in size. This process goes on during the whole growth of the thymus, and the follicles increase in size and in number. Although the connective-tissue is constantly ingrowing, the bands of connective-tissue, in the fully-developed thymus, are very thin (see Plate 83, fig. 1).

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* This description applies to the development of the thymus, except in its very earliest stage; this will be described under the head of "Development."

The very same process by which the thymus is separated into follicles, and by which the follicles are multiplied in number, is also at work during involution; only then, although the connective-tissue penetrates further into the follicles, and the blood vessels enlarge, the follicles do not increase in size; on the contrary, they diminish, and thus the follicles are split up and surrounded by connective-tissue (see Plate 86, fig. 10); and the connective-tissue increasing, the follicles are finally left as islands imbedded in it (see Plate 84, fig. 4).

On examination of Plate 83, figs. 1 and 2, and Plate 84, figs. 3 and 4 (representing tissues treated exactly in the same manner and magnified to the same number of diameters) which are made specially to point out the appearances of the follicles at various stages of development and involution, it will be seen that the foregoing description is quite correct; for while in Plate 83, fig. 1, the ingrowing processes of connective tissue are quite fine, in Plate 83, fig. 2, the cortex is divided by broader bands. These bands in Plate 84, fig. 3, are still larger, and the cortical portions are still further separated from one another; and it will be also seen that the follicles have, during the process of involution, become considerably smaller; and this in two ways, one of which is only indicated in the drawings, namely, by the invasion of the medulla by connective-tissue, while the other is clearly shown, by the great shrinking both in breadth and thickness of the cortex. It will be seen in Plate 84, fig. 4, that finally the medulla becomes converted almost entirely into connective-tissue, and the cortex shrinks to thin lines of tissue.

In Mammalia this connective-tissue surrounding the follicles, and even that penetrating them, becomes, during involution, a bed for the deposit of fat cells, as will be described further on (see Plate 83, fig. 2; and Plate 84, figs. 3 and 4); but this deposit of fat does not occur in the other classes of Vertebrates, and therefore should not be insisted on as a main factor in the involution of the follicle, as has been done by so many authors; indeed, in the Bird, where the thymus disappears so early, there is no formation of fat.

The cortical part of the follicle is, as has been already stated, not differentiated from the rest of the follicle at an early period of the existence of the organ (see Plate 87, fig. 23); and when first differentiated in the fœtal thymus is not of very great extent (see Plate 86, fig. 11). In the fully-developed thymus (as in Plate 83, fig. 1) the cortex is seen to be at least twice the size of the medulla. On the other hand, as the follicle undergoes involution, the cortex shrinks more rapidly than the medulla (see Plate 84, fig. 3). The cortical part of the follicle consists in a very large measure of lymphoid cells, and it will be shown afterwards that these cells leave the thymus by the lymphatics. Although the medulla contains some lymphoid cells, it also contains other elements not peculiar to the thymus (or the lymphoid tissues), as the granular cells, giant cells, and fibrous tissue.

Briefly stated, the cortex is much larger in the fully-developed organ, and undergoes involution more rapidly than the medulla. It is composed almost entirely of lymphoid cells, such as are found in the lymphatics. These considerations point out that *the cortex of the follicle is the more important and essential part of the thymus.*

THE LYMPHATICS OF THE THYMUS.

The capsule of the follicle surrounds the cortical portion, separating the cortical-tissue from the projections of interfollicular-tissue, such as are shown at *c′t′*, in Plate 83, figs. 1 and 2 and Plate 86, fig. 10. In sections of the thymus, the capsule is seen to be composed of endothelial cells; blood vessels penetrate it obliquely, and, supported by it, enter the follicle (see Plate 87, fig. 25). The capsule in certain stages of involution contains much fibrous tissue and some cells; but in the majority of cases it appears as a double membrane, whose outer wall is also the wall of a lymphatic space. These lymphatic spaces contain many thymic corpuscles. It is however possible that some of these cells have passed into the lymphatic spaces in making the specimen.

It was not found possible to discover any lymphatic vessels in the follicles, and none can be shown to exist in the cortex of the follicle during the period of the full development of the gland; as no lymphatic vessels can be seen in sections, and there are no perivascular sheaths to the vessels (see Plate 87, figs. 25 and 26). In the medulla many of the vessels, even in the early stages of the growth of the thymus, have perivascular sheaths; and during involution almost all the vessels of the medullary portion (see Plate 94, fig. 96) and many even of the vessels of the cortex, are surrounded by perivascular spaces. These spaces may act as lymphatic vessels. It is seen from the foregoing description that the cortical part of the follicles—the more important part—has nothing which can serve as a lymphatic vessel, during its greatest development, except the lymphatic space on the outer side of the capsule.

The course of the lymphatics issuing from the thymus was traced in the Calf. The young animals were killed by chloroform, and, as soon as dead, a small quantity of a saturated solution of indigo-carmine was injected by puncture into the thymus, the small wound in the skin was closed up, and the animals were left for half-an-hour. It was found that the indigo-carmine solution passed into the lymphatic vessels, travelled along them, and marked them out, so that they could be easily followed by careful dissection.

In the neck part of the thymus the blue fluid is seen to leave the organ by a number of small vessels running by the side of the minute veins. These small lymphatic vessels form the afferent vessels of small glands lying in the neck on each side of the thymus; from these glands the efferent lymph-vessel runs down the neck, not far from the large lymphatic trunk, but never joins it, and can be traced to two other lymphatic glands lying on the trachea, close to its point of entrance into the thorax. It will be noticed that the lymphatic vessels do not run longitudinally in the gland, and that there is no large lymphatic vessel running at the back of the isthmus of the thymus, as described by HIS (46).

The indigo-carmine solution, if injected into the thoracic portion of the thymus, can be traced to lymphatic glands lying in the thorax, generally to one gland lying on the arch of the aorta, and from thence to the large veins on the left side. There seems to be no intercommunication between the lymphatics of the thorax and of the neck.

THE BLOOD VESSELS.

The most noticeable fact about the blood vessels of the thymus, in an injected specimen (see Plate 85, fig. 6), is the disposition of the large vessels in two rings:—(1) They, to a great measure, surround the follicle, more so, usually, than in Plate 85, fig. 6; and (2) form nearly a complete circle on the margin of the medullary portion of the follicle. This latter circle is not, however, outside the medulla, but situated within it, or just on its borders. This disposition of the vessels is true of all cases where the follicle is polygonal, and contains one medullary centre. Where there are two medullary portions there are two central rings of vessels, or if the medulla be in the form of long processes, the surrounding vessels run longitudinally. The ring of vessels surrounding the follicle is composed of arteries and veins, which give off or receive branches from two neighbouring follicles. The inner circle, or circles, is also composed of arteries and veins, but chiefly of the latter. The cortex is thus surrounded by large vessels running parallel to its inner and outer surfaces, and is chiefly supplied with arterial blood from the outer circle of vessels, while the inner circle collects the venous blood from the cortex and from the medulla. In the cortical portion fine capillary vessels are met with, the majority of which run in a radiating manner from the exterior inwards. There are, however, a few vessels penetrating the cortex from the arteries in the medulla, and sometimes vessels from outside run in loops, and join the veins on the outside.

The medulla, in injected specimens of the thymus, is more vascular than the cortex, on account of the ring of large vessels which it contains. The blood supply of the interior of the medullary portion varies as follows: When the thymus is in a state of growth, or during full development, the vessels in the interior of the medulla are few and fine, there being often one or more small central arteries; those parts of the medulla which contain the granular cells and the concentric corpuscles have very few and only small vessels (see Plate 85, fig. 6). In the thymus during involution, on the contrary, the vessels of the medulla are numerous and very large, often surrounded by connective tissue, and have perivascular sheaths. The large veins of the medulla, as has been described, pass out of the follicle in the bands of connective tissue which invade the cortex (see Plate 83, figs. 1 and 2; and Plate 85, fig. 6, V′), and help to form the large veins running between the follicles.

Extravasations take place in injected specimens if the pressure used is at all high. These extravasations occur only in the medullary portion; and the fluid, if it be carmine and gelatine, is found, as a coloured reticulum, between the cells.

In the early stage of development of the thymus, the blood vessels are quite small, and are found in the follicle and on its border (see Plate 87, fig. 23).

The relation of the blood vessels to the granular cells, concentric corpuscles, and network will be spoken of in treating of those tissues.

THE TISSUES COMPOSING THE CORTICAL PORTION OF THE FOLLICLE.

In thin sections of the cortex, the first feature which strikes the eye is the very great number of lymphoid corpuscles, which lie closely packed from the capsule to the edge of the medulla. They are seen to be supported by a delicate reticulum, that is, by a closely-meshed web, which is broadened at the nodal points. The meshes are very close, so that as a rule they are only large enough to enclose one lymphoid corpuscle (see Plate 85, fig. 9; and Plate 87, figs. 27 and 28). This reticulum encloses all the other tissues met with in the cortex, and thus resembles that which I have described in the mucous membrane of the intestine (see reference No. 86).

In specimens which are stained by hæmatoxylin, the difference in colour between the irregularly-shaped nodal points of the reticulum and the lymphoid corpuscles is not very great (see Plate 87, fig. 27), although the refractive powers of the two are very different. A great difference between the lymphoid cells and the nodal points is, however, noticed in specimens stained in indigo-carmine and carmine (see reference No. 87); for in this case the nodal points are stained of a blue-green colour, and the contained lymphoid corpuscles, red (see Plate 85, fig. 9).

In sections of the cortex which have been shaken, and in which the lymphoid cells and the reticulum have been washed away, we have, stretching from the capsule of endothelial cells, a network (1) of connective-tissue threads (see Plate 87, figs. 25 and 26, *th*), (2) of capillary blood vessels, and between these vessels and threads, but attached to both, a delicate but widely-meshed network, composed (3) of branching cells with long fine delicate processes (see Plate 87, fig. 25). This cell network forms an adventitia to the vessels (see Plate 87, figs. 25 and 26), and the connective-tissue threads with the network of branching cells form only one network.

This network of cells differs in two respects from that found in any other lymphoid tissues of the body. In the first place, it is a permanent network of cells—*i.e.*, it is not like that found in the follicles of the lymphatic gland (only a network of cells during very early life, to be replaced afterwards by a non-nucleated reticulum), for it exists as long as the thymus, and can be found in shaken specimens of the thymus of adult animals, as for instance, in an Ox four years old. Secondly, the network is unlike any other network, the cells with their very fine branching processes being peculiar to, and characteristic of, the cortex of the thymus gland.

It will be seen that this network is not a reticulum of threads on which epithelioid cells lie, but that the processes of the connective-tissue-corpuscles, with the cell bodies, form the greater part of the network of the cortex. If the specimen be roughly shaken, most of the cells and their processes will be shaken away (as in Plate 87, fig. 26), only those remaining which are attached to the blood vessels, and thus have a firm support.

The nuclei of the connective-tissue-corpuscles, forming the network, and of the endothelial cells of the blood vessels (see Plate 87, figs. 28 and 25) and the nuclei of the cells forming the capsule, are all similar in size and appearance; and in specimens stained in indigo-carmine and carmine, the branched connective-tissue-corpuscles, the blood vessels, and the connective-tissue threads, all stain of a red colour, and not like the reticulum, of a blue-green.

It will be seen from the foregoing description, and from Plate 87, figs. 25 and 27, *that in the cortex there are two kinds of retiform-tissue; one (with small meshes) which we have called a reticulum, formed of fine fibres with thickened nodal points; and the other, which we have called a network, composed of branched connective-tissue-corpuscles.* These are not to be considered as identical, nor as arising the one from the other, but are two distinct meshes lying one within the other.

THE TISSUES COMPOSING THE MEDULLARY PORTION OF THE FOLLICLE.

The network of branched connective-tissue-corpuscles of the cortex is continuous with a network of branched cells in the medulla, but there is a considerable difference in the two networks (compare Plate 87, fig. 25, with Plate 86, fig. 18, and Plate 93, fig. 78). In Plate 86, fig. 18, the cells are larger than in Plate 87, fig. 25; they are joined together, not by very delicate, but by coarse, sharply-defined cell processes; and the body of the cell is also much thicker. Again, the spaces enclosed by the cell processes are not of an indefinite polygonal area, but are sharply defined, and nearly spherical (see Plate 93, fig. 78).

The connective-tissue-corpuscles forming the network are not, however, always met with as branching cells, but at times are close together, and then they have no processes, but lie side by side like an endothelium (see Plate 88, fig. 30, *c′t′*, and Plate 89, fig. 45 A, B, C). These latter will be called epithelioid cells. Further, in some places, the cells are so closely approximated that no trace of cell-division can be seen; and here we find masses of protoplasm, containing three, four, or numerous nuclei, not differentiated into cells (see Plate 88, fig. 30; and Plate 90, fig. 54).

The reticulum, which is so marked a feature of the cortex, is also found to a slight extent in the medulla (see Plate 88, fig. 31; and Plate 93, fig. 83), but in this part of the follicle it does not enclose all the other tissues in its meshes. It is only seen here and there, and gives the impression which would be made if the cells which it originally enclosed had multiplied, the reticulum itself not having increased; so that the reticulum is only seen at intervals, among the connective-tissue-corpuscles or the epithelioid cells.

The medulla, therefore, contains a network, formed either (1) of branched connective-tissue-corpuscles, or (2) of epithelioid cells, or (3) of protoplasmic masses not differentiated into cells: and also contains a reticulum, which only occurs at rare intervals.

Besides these, there are many granular cells, giant cells, and concentric corpuscles, found in the medulla, each of which will now be considered.

Granular cells.

During the latter part of fœtal life, the medulla is studded in places with granular cells (see Plate 90, fig. 49), and during the remaining period of growth and involution these cells are present (see Plate 88, fig. 31). However, there are many more during the first period of involution than at any other time. They are more numerous in the Calf than in other Mammals, but they are found in all classes of Vertebrates, and in much greater number in the Mammalia and in the Frog, than in the Bird, Reptile, or Fish. They are easily recognised, in sections made from tissue hardened in bichromate, and stained in hæmatoxylin, as yellow masses with somewhat coarse granules; they are often found lying close together, arranged in bands, or in masses lying over one another in great numbers. They are not readily stained by the ordinary hæmatoxylin solutions, or by carmine, if the tissue has been hardened by chromic acid and alcohol. They are, however, stained very readily if the tissue has been hardened in osmic acid (see Plate 90, fig. 49), and are deeply stained by red hæmatoxylin solutions (see Plate 85, fig. 8).

These granular cells vary considerably in size (compare Plate 93, fig. 82; and Plate 88, fig. 36). In fresh preparations, their protoplasm closely resembles in appearance that of the giant cells of medulla of bone. In sections, some cells having very coarse granules are met with, close to others with much finer granules (see Plate 88, fig. 33, and compare figs. 33 and 34).

The granular cells may be divided, for the sake of convenience, into four varieties, although it should be clearly understood that there are intermediate forms.

They are found (1) as polygonal or rounded epithelioid cells, often lying in close approximation; the central part of the cell only is granular (see Plate 88, figs. 31 and 33); (2) as vacuolated cells, in this case the mass lying in the vacuole is granular (see Plate 88, fig. 32; and Plate 89, figs. 45 B, *va*, and 45 C, *va*); or (3) as spheroidal masses lying in cavities between the branching processes of the connective-tissue-corpuscles, as in Plate 88, fig. 30, and Plate 89, fig. 46 E; or (4) as club-shaped masses, attached to blood vessels and to newly-formed connective-tissue, as in Plate 88, figs. 35, 37, and 36, 38.

We will first consider the epithelioid cells which contain granular masses. They are well seen in Plate 88, figs. 33 and 31. The granular mass in the cells varies in size from quite small spheres (as in Plate 89, figs. 44 A, 45 A, *g*, and 41) to masses which nearly fill the cell (see Plate 89, fig. 44 F). When the contained granular mass is quite small the nucleus of the cell is nearly central (see Plate 89, figs. 44 A and 41), but as it increases in size the nucleus is forced to one side (see Plate 88, fig. 31), and the nucleus occasionally divides, as some of these cells contain several nuclei (see Plate 88, fig. 31, *g*). Sometimes, but not often, the contained granular mass is nucleated (see Plate 89, figs. 44 D and E, and 45 A, *gr*). This form of granular cell might with equal reason be described as epithelioid connective-tissue-corpuscles containing granular masses; and presently, when we come to consider the origin of the granular cells, we must lay great stress on this variety. These cells are generally situated near to one or more concentric corpuscles or to giant cells (see Plate 90, fig. 49). Traces of a reticulum can be seen among them (see Plate 88, fig. 31, *r*).

The second kind of granular cells are those which are vacuolated, and contain granular masses in their interior. They are generally polygonal or spheroidal (see Plate 88, fig. 32; and Plate 89, figs. 45 B, *va*, and 45 C, *va*), and are found in most cases close to those of the first kind, and appear to arise from them, the clear protoplasm of the cell having shrunk away from the granular mass. These vacuolated cells are found in some animals, as in the Bird (see Plate 95, fig. 98), in greater numbers and of much larger size than in Mammalia. This vacuolation of the cell, as we shall see afterwards, is a process of great importance in the formation of concentric corpuscles in some animals, and in the formation of the ciliated cysts in the Dog.

If the vacuolation of the cell proceeds further the granular mass is set free, and can assume an independent existence.

We then have the third form of granular cell, as met with in Plate 88, fig. 30, and Plate 89, figs. 46 D and E. In the two latter figures the granular mass is also vacuolated. In Plate 88, fig. 30, we find large granular masses, enclosed in a network of connective-tissue; the granular masses may be considered as separate cells. In preparations of the thymus which have been shaken, we often meet with branched networks of connective-tissue-corpuscles, from which most of the granular cells have been shaken out (see Plate 93, fig. 78).

The fourth variety of granular cells are club-shaped or rounded, and are attached to the blood vessels and the connective-tissue trabeculæ (see Plate 85, fig. 8; Plate 88, figs. 37, 38; and Plate 92, fig. 69). They are evidently not merely lying in apposition to these structures, but attached to them; as shaking for a long time (three or four hours) does not separate this connexion. Those which are attached to the vessels sometimes appear to be entirely formed of a granular mass (see Plate 92, fig. 68, and some of the cells in Plate 88, fig. 37), at other times of a clear protoplasm, nucleus, and granular mass (see Plate 88, fig. 35; and Plate 89, fig. 43); the latter stains of a different colour to the nucleus (see Plate 85, fig. 8). The lumen of the vessels appears to be quite unchanged (see Plate 85, fig. 8; and Plate 89, fig. 43), and there are no extravasations in the neighbourhood, nor any other feature to indicate that these cells have passed out from the vessel. The vessels have at times, however, a considerable thickening of the adventitia (see Plate 88, figs. 35 and 37).

The granular cells attached to the connective-tissue trabeculæ are well shown in Plate 88, figs. 36 and 37, and Plate 92, fig. 69, *gr*. In Plate 92, fig. 69, there are indications of the cells having a surrounding zone of clear protoplasm, for the nuclei are seen outside the granular mass; but in Plate 88, figs. 36 and 38, the granular masses are nucleated and constitute the whole cell. There is some resemblance between these granular cells attached to newly-formed connective-tissue, and osteoblasts attached to the edge of newly-forming bone; and this resemblance is not a deceptive one, for the granular cells are concerned in forming fibrous tissue, their processes are often seen to be fibrillated (see Plate 88, fig. 36), and the fibrillæ are continuous with those of the connective-tissue trabeculæ (see Plate 93, fig. 82). Sometimes the whole cell is directly transformed into fibrillated connective-tissue.*

The granular cells form blood vessels. They stretch across from vessel to vessel, forming bands of granular tissue. In most cases these slender bands of tissue become fibrous-tissue. To demonstrate the formation of new vessels, it is therefore necessary to select those granular bands stretching between the vessels, where one part of the band, or newly-forming vessel, is hollow, and the other part is still formed of a granular cell (see Plate 94, fig. 94) or of granular cells (see Plate 94, fig. 93). In the former case the vessel is intra-epithelial, and in the latter it is inter-epithelial.

From what do the granular cells arise? Their position near the blood vessels, their being massed in the medulla of the follicle, where at times extravasations take place, might suggest the idea that they arise from extravasated colourless blood corpuscles. However, that is not the case; and the figures given in the plates show that they arise from the connective-tissue-corpuscles. In Plate 89, fig. 45 B, some of the cells are epithelioid connective-tissue-corpuscles, others contain a granular mass in the centre of the cell. This transition of epithelioid connective-tissue-corpuscles to granular cells can also be seen in Plate 89, figs. 45 A and C (see also figs. 41 and 45 A, *g*), where there is a very small granular mass in an epithelioid cell. The same thing is shown by the series of figs. 44 A to F, in which various epithelioid cells are seen with larger or smaller granular masses in their interior. An argument might, however, be put forth, that the drawings would be equally explained by supposing that the granular cells are the first formed, and that they are afterwards transformed into epithelioid cells. This argument is answered very completely by the thymus of the Bird, where we find very many epithelioid, and very few granular cells (see Plate 95, figs. 100 and 101). The thymus of the Tortoise also contains many epithelioid, and very few granular cells.

Another origin of the granular cells which might be suggested would be either the giant cells or the concentric corpuscles. But there are two fatal objections to the view that either of these is the forerunner of the granular cells: first, that sometimes in the inner edge of the cortex solitary granular cells are seen (see Plate 89, fig. 39), and such solitary cells could not have arisen from either of these sources; secondly, that the granular cells are found in much greater number than would be the case if they took their origin from either the concentric corpuscles or the giant cells. Again, the granular

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* The papers of ZIEGLER (89) and TILLMANNS (90), who described the formation of fibrous tissue in pathological new formations, by an exactly similar process, will be referred to afterwards.

cells met with in the cortex are usually attached to the connective-tissue network (see Plate 89, fig. 39, *a* and *b*).

The giant cells.

These appear to be capable of division into two kinds, which differ somewhat in their appearance, the period of life at which they are formed, and their probable origin.

In the fœtal thymus, giant cells are found in considerable numbers (see Plate 90, fig. 50). They are large, very granular, and multinuclear; they are met with near granular cells, and appear to arise from them; for in some cases we find granular cells of very irregular size and shape (see Plate 89, figs. 40 E and F, and 39 C and D); in other cases, granular cells containing two or three nuclei (see Plate 89, fig. 48; and Plate 90, fig. 49, *gr*); finally, we meet with preparations like those in Plate 90, fig. 50, *gr*, where a granular cell with three nuclei seems to be coalescing with other granular cells to form a giant cell.

In animals, during involution, when fibrous-tissue is present in the organ, giant cells are met with close to the newly-formed tissue. They are much branched; the branches lie in several planes (see Plate 90, fig. 56; and Plate 91, fig. 57). In other specimens we find giant cells containing granular masses in their interior (see Plate 90, figs. 51, 52, and 53). Others contain cavities or gaps looking something like the lumen of a vessel (see Plate 90, figs. 51 and 55). This form of giant cell seems to arise from the network of connective-tissue-corpuscles, and we can trace the transition from masses of protoplasm containing many nuclei, to these giant cells. In Plate 88, fig. 30, at the left side of the figure, there are masses of protoplasm containing large granular cells. In Plate 90, fig. 54, is seen a newly-formed band of connective-tissue, *ct*, and at right angles to it, and above it, multinuclear masses of protoplasm containing oval gaps. These masses of protoplasm in Plate 90, fig. 54, appear to be intermediate forms between the branched network of connective-tissue-corpuscles and the giant cells, and to account for the manner in which the gaps are formed in the giant cells, and for the presence of granular cells in the giant cells.

Concentric corpuscles.

The appearance of the concentric corpuscles, when large and fully formed, is very peculiar (see Plate 91, figs. 59, *a*, and 63). They consist of a central part and of outer layers, which latter we will for convenience call the capsule.

The central mass is composed: sometimes (1) of small cells: at other times (2) of granular masses, which have many small nuclei imbedded in them; these nuclei are concentrically arranged, as at Plate 85, fig. 7, *b*: or again (3) of a mass of fine granules, as at Plate 85, fig. 7, *g*: but in general (4) of a homogeneous and strongly refractive substance, which presents no definite structure, and consists of two, three, or more parts (see Plate 86, fig. 15; Plate 90, fig. 50; and Plate 91, fig. 63). These central masses are often of a yellow colour, and contain hæmoglobin granules (see Plate 86, fig. 17). The central mass does not always completely fill the capsule (see Plate 91, figs. 63 and 59, *a*), the interval having been formed by the shrinking of the contained substance. The central part is sometimes found passing down the vessel-like prolongations, which are attached to the concentric corpuscles (see Plate 91, fig. 59, *a*; and Plate 85, fig. 7, *ce*).

The outer part of the concentric corpuscle, which we have called the capsule, is formed of epithelioid cells (see Plate 91, fig. 59, *b*; Plate 90, fig. 49; and Plate 85, fig. 7). These cells are flattened towards the inside of the capsule; on the outer side they are polygonal, and are anatomically continuous with the branching connective-tissue-corpuscles, which form the network of the medulla (see Plate 86, fig. 15; Plate 91, fig. 64; and Plate 92, fig. 69).

The larger concentric corpuscles, besides being connected to the network of the medulla, have often projections formed of epithelioid cells, very like vessels in appearance (see Plate 91, figs. 59, 64, and 65). Others show sharply-defined processes, formed of epithelioid cells, such as are shown in the lower part of fig. 17, Plate 86. The cells composing these processes are generally very large, and contain large nuclei; and the cells of the network to which they are attached are larger than in the other parts of the medulla. As a rule, the nearer the connective-tissue-corpuscles are to the concentric corpuscles, the larger they are, the coarser are their processes, and the larger their nuclei. Again, we often find the concentric corpuscles attached to one another, or to the surrounding network, by long coarse threads, which have nuclei embedded in them at intervals. Such are shown from teased preparations in Plate 86, fig. 15, *th*; and Plate 91, figs. 61 and 62; and from sections of the tissue in Plate 90, fig. 50, *th*; and Plate 92, fig. 69, *th*. These coarse processes often run for long distances across the medulla, and mark out the position of the future bands of connective-tissue. In other specimens the concentric corpuscles are attached to one another, or to fibrous trabeculæ, by bundles of these coarse threads (see Plate 91, fig. 60) or by bands of fibrillated-tissue (see Plate 91, figs. 58 and 63).

Having considered the structure of the concentric corpuscles, when fully formed, let us trace their origin. In Plate 90, figs. 50 and 49, where sections are made through large portions of tissue, we find the concentric corpuscles near those parts of the medulla where there are granular cells. We have also seen that the epithelioid cells, forming the capsule of the concentric corpuscles, are continuous with the epithelioid connective-tissue-corpuscles of the medulla (see Plate 90, fig. 49), and it is from these two elements, the granular cells and the epithelioid cells, that the concentric corpuscles are formed. As the granular cells take their origin primarily in the connective-tissue-corpuscles, we may say that the concentric corpuscles arise from connective-tissue-corpuscles; and we shall be able to support this latter proposition in considering the somewhat different concentric corpuscles of the Bird and the Tortoise.

In Plate 92, fig. 68, is shown a medium-sized concentric corpuscle; the central part and the capsule are not yet differentiated from one another; and therefore we are able to perceive the tissues from which the corpuscle is formed. Nearly in the centre there is a granular cell lying in a vacuole, which has a thickened border, deeply stained by hæmatoxylin; around the vacuole are epithelioid connective-tissue-corpuscles, with many nuclei. These cells enclose two other granular cells.

In the upper part of Plate 92, fig. 69, another and smaller concentric corpuscle is shown; the structure is essentially the same. In the centre there is a granular cell, lying in a vacuole with a thickened border. The vacuole is surrounded by epithelioid connective-tissue-corpuscles, and the neighbouring cells are very large.

Plate 91, fig. 64, shows nearly the same; two small concentric corpuscles are in process of formation; the centre of each contains a granular mass, lying in a vacuole with a thickened border. The simplest concentric corpuscle of all may be seen in Plate 89, figs. 46 E and G, and Plate 91, figs. 66 and 67, where the granular mass, or in fig. 66 the three granular masses, are surrounded by protoplasm with proliferating nuclei.

It is thus seen that the concentric corpuscles and the giant cells arise from the same sources, *i.e.*, from the granular cells, and the epithelioid connective-tissue-corpuscles.

In considering the function of the concentric corpuscles, and their behaviour during involution, one point must be apparent to all, *i.e.*, that many of them disappear; for if the thymus of a very old animal be examined, very few will be found, and these as a rule will be small. It is also not difficult to prove that the greater part of most of the concentric-corpuscles, connective-tissue threads, and fibrous processes, become fibrous-tissue; the threads and processes becoming thicker, in consequence of the deposition of new tissue, by the granular cells, attached to their sides and lying between them. In these bundles of connective-tissue we find vessels which are either the pre-existing vessels enclosed by the newly-formed connective-tissue, or newly-formed vessels. Finally, the concentric corpuscles and the fibrous bands are changed into connective-tissue containing large vessels, which generally have one or two perivascular membranes. These changes are most easily traced in the thymus of the Calf and of the Child.

We shall find in some animals that the concentric corpuscles are transformed into cysts; this is notably the case in the Dog, Bird, and Tortoise.

Great differences will be noticed in sections of the thymus of different animals, both in the number and in the size of the concentric corpuscles. In the Child they are very numerous, and are often large and compound. In the Kitten they are still larger. In the Calf, on the other hand, they are small, and comparatively few in number. As a rule, the very large concentric corpuscles are compound; the layers of epithelioid cells, which in the single concentric corpuscle form only a capsule, are here extended over a large area, and embrace two or three, or even five concentric corpuscles (see Plate 85, fig. 7). At times we meet with abortive concentric corpuscles, where the capsule has been formed; but the central parts are composed of unchanged lymphoid cells of the connective-tissue network, and of traces of the reticulum. These abortive concentric corpuscles are sometimes of considerable size; and here, also, the capsule is finally converted into fibrous tissue. The concentric corpuscles are not found when the thymus is first formed, but can be seen during the middle and later periods of fœtal life (see Plate 90, figs. 49 and 50). They are most numerous during the first period of involution, *e.g.*, in the Calf, when the animal is from six to eighteen months old; a few are found even in very old animals, as in the Ox, twenty years old.

It is important for us to understand the relations of the concentric corpuscles to the blood vessels, since CORNIL and RANVIER (50) and AFANASSIEW (51 and 33) have concluded that these bodies are formed from the endothelium of the vessels. It is true that they are attached to the vessels, as has been noticed by HIS (46) and other observers; the connexion of the concentric corpuscles to the network ensures their connexion to the vessels; but they have a more intimate connexion where the capsule of epithelioid cells has, in its growth, surrounded a vessel. This has partially taken place in Plate 91, fig. 63, *v*, and completely in Plate 92, fig. 69. It has already been mentioned that in that part of the medulla where the granular cells and concentric corpuscles are formed there are only a few vessels, and these are very fine. In injected specimens the vessels may be seen to pass through the large compound concentric corpuscles in every direction but one, never through the central opaque masses. This is well shown in Plate 85, fig. 7. The very large compound concentric corpuscle fills nearly one-third of the medulla; the central part of the mass has fallen out, but the remainder shows five concentric bodies. The blood vessels are found in the outer part of the compound concentric corpuscle, but do not enter the granular masses, although they may wind around them as at *d*. I have two other preparations which show very similar appearances.

Various views have been held as to the origin of the concentric corpuscles (see p. 1067). It appears to me that there can be no doubt but that they arise from the epithelioid and granular cells; the question therefore resolves itself into this—from what do the epithelioid and granular cells arise? I think I have shown in pages 1084 and 1087, that they take origin from connective-tissue-corpuscles. Another source which might be suggested, which has not been already discussed (see page 1087) would be the remains of epithelial cells, of which the thymus is supposed (by some authors) to be at first formed; this is the view of HIS (52) and STIEDA (53). They, however, do not attempt to support their view by evidence, although STIEDA says that the concentric corpuscles cannot be injected; AFANASSIEW (33) showed however that the injection penetrates the concentric corpuscles, and it has been shown in the preceding paragraph, and in Plate 85, fig. 7, that vessels undoubtedly do penetrate into these corpuscles.

The proposition that the concentric corpuscles arise from the remains of the epithelial cells presents some probability, because of the undoubted epithelioid character of the cells found in the medulla of the thymus, and of those taking part in the formation of the concentric corpuscles. This similarity to epithelial cells, however, is met with in other places, as in pathological new formations. Thus epithelioid cells take part in the formation of tubercle, as has been pointed out by SCHÜPPEL (90) and others. Again, in the observations which were made by ZIEGLER (88) on the formation of pathological connective tissue (his method consisted in introducing small glass plates, carefully cemented at the edges, into the abdominal cavity or, by preference, into the subcutaneous connective tissue); he found at first a number of small cells, which in his view gave origin to epithelioid cells and to granular cells (Bildungszellen); these two new forms of cell in turn become, in some cases, giant cells. Without expressing any opinion as to the actual origin of the epithelioid and granular cells which he found, it is at least quite evident that they could not have arisen from the remains of epithelial cells, either in the peritoneal cavity or in the subcutaneous connective tissue. There is therefore no reason to believe because there are epithelioid cells in the thymus that they must have arisen from remains of epithelial cells which formed the original gland.

Again, there is a remarkable similarity in the formation of fibrous tissue, as described by ZIEGLER (88) and TILLMANNS (89)—the latter author introduced portions of hardened tissues into the abdominal cavities of Rabbits—to that which we have described in the thymus at pages 1086 and 1087, and represented in Plate 88, figs. 36 and 38; the granular cells, in many cases, are seen attached by long processes, to newly-forming fibrous-tissue, and are evidently taking part in its formation. And there is an equally great similarity in the way in which the giant cells are formed in these pathological new formations to that in which they are formed in the thymus, as described at page 1088, and represented in Plate 90, fig. 50.

ZIEGLER (88) and TILLMANNS (89) consider that the granular and epithelioid cells are formed from colourless blood corpuscles. We have already shown at page 1087 that we consider that in the thymus they arise from connective-tissue-corpuscles; but the after-steps of the formation of giant cells and fibrous-tissue from the epithelioid and granular cells appear to take place, in the thymus, in a very similar manner to that which has been described by those authors in pathological new formations.

We may further add, that the study of the involution of the thymus, with its gradual transition into fibrous-tissue, will be, no doubt, of considerable value to pathologists.

CILIATED EPITHELIUM IN THE THYMUS OF THE DOG.

REMAK (57) noticed (see History, p. 6) that in the Cat ciliated cysts are attached to the upper border of the thymus.

In the thymus of the lower forms of Vertebrates, cysts are not uncommon. The ciliated cells in the Dog occur only in connexion with cysts, which are found in the interior of the follicles.* These cysts are never met with during the earlier periods of growth; they are always formed either from concentric corpuscles, or from masses of epithelioid cells, which become cysts without passing through the stage of concentric

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* I have examined the thymus of Cats of various ages, but have never found in them any ciliated cysts.

corpuscles. If sections are made of the thymus of a Dog from six to nine years old, some of the follicles will be found to contain large cysts (see Plate 94, fig. 84); these cysts are lined by ciliated epithelium. In the thymus of Dogs from eight to thirty months old we shall find (1) small concentric corpuscles, (2) small cysts lined by epithelioid cells, and (3) small cysts lined by ciliated epithelium. If the thymus of a young Dog from one to four months old be examined, the follicles will be found, both as regards the cortex and the medulla, to resemble those met with in the Calf or other animals, and to contain small concentric corpuscles; no cysts can be found.

We will first describe the cysts as met with in older animals (see Plate 94, fig. 84). The follicles are small; there is a considerable amount of fat and connective tissue between them, which has to some extent penetrated into them (see Plate 94, fig. 84, F). In the follicle, cysts are present of such size as to be visible to the naked eye. They contain, in most cases, a deep yellow mass, somewhat resembling extravasated blood (see Plate 94, fig. 84); although this mass never contains any coloured corpuscles, even in preparations where the corpuscles can be clearly seen in the neighbouring blood vessels. The borders of the cysts are lined, over the greater part, by ciliated epithelium; in other places, apparently by a thickening of the tissues of the thymus, although it is possible that the cysts are lined throughout by ciliated epithelium, but that the cells have been lost in making the specimen.

These epithelial cells are either columnar, as in Plate 94, fig. 91, or sub-columnar, as in Plate 94, fig. 90. The individual cells can be separated by teasing in saline solution, and are very similar to the epithelial cells of the trachea (see Plate 94, fig. 92). I have attempted, in five instances, to find out if any active movement of the cilia takes place in these cells during life, but without success, except in one case, in which the trachea had been accidentally opened, and where the result was of no great value.

The chief point is to trace the origin of the cysts and of the ciliated epithelium.

If we make sections of the thymus of a Dog from eight to sixteen months old, we shall find (*a*) quite small concentric corpuscles with small vacuoles (see Plate 94, fig. 85); or (*b*) small concentric corpuscles, containing cysts or vacuoles, partially filled with degenerated cells which often show traces of vacuolation (see Plate 94, fig. 86). Further, we find in Dogs between these ages (*c*) small cysts containing either (1) degenerated epithelioid cells (see Plate 94, figs. 88, *dg*, and 89)—the masses of degenerated cells show traces of vacuolation (see Plate 94, fig. 89, *va*), and they are at times attached to the sides of the cysts (see Plate 94, figs. 88 and 89);—or (2) masses of granules, showing only traces of cell origin (see Plate 94, fig. 87); or (3) masses of hæmoglobin. The cysts are lined wholly, or in part (1), by ciliated epithelial cells; or (2) by masses of protoplasm, not, apparently, completely differentiated into cells, although these masses bear cilia on their surface (see Plate 94, figs. 88 and 89); or (3) by epithelioid cells (see Plate 94, fig. 89, *ep*). (Compare the epithelioid cells, *ep*, in Plate 94, figs. 87 and 88.) The ciliated cells, when they are found in these smaller cysts, are sometimes sub-columnar, but frequently cubical or even flattened. It will further be noticed that there are all transitional forms between flattened epithelioid cells and ciliated epithelium (see the smaller left-hand cyst in Plate 94, fig. 89).

It is quite easy to show gradual transitions, from small concentric corpuscles to the large cysts containing columnar ciliated epithelium. The various steps may be artificially classed as follows: First, concentric corpuscles, the peripheral cells of which are connected to, and continuous with, the network of the medulla (see Plate 94, fig. 85); secondly, concentric corpuscles, containing in their centre a mass of degenerated cells, which have been separated from the surrounding cells by a process of vacuolation (see Plate 94, fig. 86); thirdly, small cysts which are bordered by epithelioid cells, and which contain either a degenerated cell mass or a mass of hæmoglobin (see Plate 94, fig. 87); fourthly, cysts partly lined by epithelioid cells, and partly by ciliated epithelium (see Plate 94, figs. 88 and 89); and fifthly, large cysts, in some cases of nearly half the width of the follicle, lined by columnar ciliated cells, and generally containing masses of hæmoglobin in their interior (see Plate 94, fig. 84).

The ciliated cysts can therefore be shown to arise from concentric corpuscles; and these, as we have seen above, always primarily from connective-tissue-corpuscles. We therefore come to the conclusion *that the connective-tissue-corpuscles can undergo certain changes, which finally transform them into ciliated epithelium.* There is no doubt a considerable *à priori* improbability in this statement, as we should expect ciliated epithelium to be found in association with a higher organization, and not as one of the methods of involution or degeneration of the tissues; and further, the view stated above is incompatible with those which have been so long held concerning the respective functions of mesoblast and hypoblast. There seems, however, no other conclusion left to us, unless we grant that in the development of the thymus, certain traces of epiblastic tissue have been left: a view which we have discussed somewhat at pages 1091 and 1092, and which, as we shall afterwards see in studying the development, is not established.*

CHANGES IN THE THYMUS DURING INVOLUTION.

Some of the changes occurring in the thymus have been mentioned at pages 1079 and 1080; others, of which we have to treat, are: the formation of connective and fibrous-tissue in the follicle; the increase of the interfollicular connective-tissue, the invasion of this tissue into the follicle, the deposition of fat in this invading tissue and between the

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* Ciliated cells have long been known to occur on the surface of the peritoneal cavity of Frogs. WALDEYER (91) considered them only a continuation of the genital mucous membrane. KLEIN (92) described ciliated endothelium in the peritoneal cavity of Frogs, and figured (see his fig. 18) a vacuolated cell, containing fine cilia projecting into the vacuole. NEUMANN (93) noticed a transition between the endothelium of the peritoneal cavity of the Frog and ciliated epithelium; and NICOLSKY (94) found that in all Frogs ciliated epithelium exists on the serous cavities, and in young male Frogs on the pericardium. This ciliated epithelium is not to be classed as belonging to the female genital organs, but as transformed endothelial cells.

follicles; the changes in the blood vessels; and the final disappearance of the gland and its replacement by fat and connective-tissue.

The involution of the gland, though to the naked eye it appears to take place at a definite period of the gland's existence, does not really do so, but is a very gradual process; for we must consider the formation of epithelioid cells and of granular cells, as the primary indication of involution, and this first takes place during fœtal life. Again, the giant cells and the concentric corpuscles also aid in involution, and they are present during the middle and latter end of fœtal life, and during the period of the growth of the organ; while the formation of fibrous-tissue takes place during quite the latter end of growth, and during all the period of decrease in size. Further, various animals of the same species differ greatly, in accordance with the activity of their life, and the amount of food they have taken. And again, various parts of the same thymus differ considerably; thus in the Calf, the upper part of the thymus in the neck undergoes involution some time before the lower part.

The formation of fibrous-tissue within the follicle.

We have already stated that the granular cells become transformed directly into fibrous-tissue (see p. 1087). This transformation also takes place in the thymus of old animals. If we study a section of the medullary portion of the thymus of an Ox from six to eight years old, we find, among the lymphoid cells, and to a great extent replacing them, bundles of interlacing fibrous-tissue, with a considerable number of granular cells attached to their edges. If the thymus of a very old animal be examined, the whole of the medulla is seen to be transformed into fibrous-tissue and blood vessels, few granular cells remaining.

The giant cells also appear, to some extent, to take part in the formation of fibrous-tissue, their long processes being in some instances transformed into it.

In the medulla, the fibrous-tissue is formed also by connective-tissue-corpuscles. They increase in size, and become spindle-shaped (see Plate 93, fig. 79), and these spindle-shaped cells are then fibrillated, and become directly transformed into fibrous-tissue (see Plate 93, fig. 79, *f*, and 78, *f*).

The vessels of the medulla have often a very thick adventitia of newly-formed connective-tissue. This seems to be formed in two ways: partly by the action of the granular cells, and partly by the proliferation of the pre-existing adventitia of the vessel; such a vessel is shown in Plate 93, fig. 81. The vessel is seen to be surrounded by newly-formed connective-tissue. In the lower part of the figure, fibrous-tissue is attached to the vessel. The large veins surrounding the medulla have often a very thick covering of fibrous-tissue, and in shaking the specimens roughly, it sometimes happens that the whole of the medulla shakes out, and leaves a ring formed of trabeculæ of fibrous-tissue and vessels, the ring surrounding an apparent central cavity.

In the cortex, very few granular cells or giant cells are present; in the majority of specimens not one can be found; it is therefore clear that any formation of connective-tissue cannot here arise from them, and in the cortex the network of connective-tissue-corpuscles is directly transformed into connective-tissue (see Plate 93, fig. 80). The network is thickened, until the breadth of the threads is as great as or greater than that of the meshes; at the same time the adventitia of the vessel enlarges. These changes are gradually continued, until we obtain, lastly, a few lymphoid corpuscles, shut in by bands of newly-formed tissue, in which later on fibrillæ often appear.

Growth of the interfollicular tissue, and invasion of this tissue into the follicle. Plasma cells and formation of fat.

An important factor in involution is the growth of the connective-tissue between the follicles (see Plate 83, fig. 2; and Plate 84, figs. 3 and 4). In this tissue there is a deposition of plasma cells (see Plate 86, fig. 13*). Each plasma cell has a considerable zone of protoplasm surrounding its nucleus; this protoplasm contains granules, somewhat resembling the granules of hæmoglobin; they, however, stain very differently, and are not composed of hæmoglobin or of fat. The cells stain in a very characteristic manner in indigo-carmine and carmine (see Plate 86, fig. 13), the nucleus staining deep pink and the granules of a blue colour; the hæmoglobin of the coloured blood corpuscles in the preparation is green. These cells exactly resemble, both in staining properties and general appearance, many of the cells met with in the medulla of bone. They are evidently here the forerunners of fat cells.† By this growth of interfollicular-connective-tissue and fat, involution is probably assisted, the follicle being compressed.

It has been stated (see p. 1079) that some of the blood vessels, surrounded to a slight extent by connective-tissue, pass from the outside of the follicle, between the portions of cortical tissue, to reach the medulla; and it is found that these vessels, with their surrounding connective-tissue, in their passage through the cortical-tissue, are separated from it by the capsule of the follicle, the vessels piercing the capsule to enter the medulla. These vessels, as they enter the medulla, have at this point a perivascular sheath, *i.e.*, where they are surrounded by the endothelium of the capsule. The connective-tissue seems to press the capsule further in on the vessel, so that we get finger-like projections of the interfollicular-tissue, with an extension of the perivascular space. It is the extension of this perivascular space along the vessels, with the invasion of the interfollicular-connective-tissue into the follicle, which is of such importance in involution.

These projections of connective-tissue gradually widen and invade the follicle,

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* For an account of the views of various authors concerning plasma cells, see p. 1074.

† In the thymus stained by dahlia, after the manner of EHRLICH (79), cells staining deeply with the dahlia are met with in the centre of the follicle (see Plate 86, fig. 12). The nucleus becomes purple, and the granules of the same colour, but of a much deeper tint. The cells are often near the granular cells, which are themselves unacted upon by the blue solutions. They are of the same size as those met with in the interfollicular tissue. It is difficult to state the significance or function of these deeply staining cells, the method of EHRLICH not being very satisfactory except as a method of detection.

pressing further into it (see Plate 86, fig. 10), and sending out lateral processes which join on to the other invading processes, they isolate portions of the cortex, or even large parts of the follicle (see Plate 86, fig. 10). The processes have generally a narrow neck, and widen out considerably at the end. In these projections we find at first some retiform tissue, stretching from the vessel to the perivascular sheath. This soon gives place to cells of two kinds: plasma and granular cells. The plasma cells are the forerunners of fat cells. The granular cells are here often found on the walls of large veins (see Plate 88, figs. 33 and 34), they form fibrous-tissue and blood vessels. Thus we have penetrating into the follicle bands of fibrous-tissue containing fat cells; and by this means fat invades the follicle; but there is never any fatty metamorphosis of the lymphoid corpuscles.

Changes in the vessels.

In the invading processes of connective-tissue there is a considerable formation of new vessels: they are very peculiar. They are enclosed first of all between the membranes formed by the capsule of the follicle; they often have, in addition, a perivascular membrane, in some cases a membrane inside this again, like a double perivascular sheath; but the most remarkable feature is their adventitia, which often looks like a second layer of endothelium outside the vessel (see Plate 94, fig. 95). In some cases, there are two or even three layers of cells, forming such an adventitia (see Plate 94, fig. 96). These vessels are often seen to be filled with blood, and the lumen does not appear to differ from that of other vessels; they do not become concentric corpuscles, as Afanassiew (51) has stated; for they are not found in numbers until the thymus is undergoing involution, nor found to any extent except in the invading processes of connective-tissue.* These vessels are much enlarged during involution, and it is possible that this adventitia is the means by which the vessel increases in size.

To sum up the changes in involution: we find a large formation of connective-tissue and fat between the follicles; this tissue invades and divides the follicles. At the same time there is a change taking place in both the cortex and medulla—produced in the cortex by the connective-tissue-corpuscles, in the medulla by the granular cells, giant cells, and connective-tissue-corpuscles, the cortex disappearing much the more rapidly of the two.

The gland, diminished in size, though with enlarged blood vessels, is transformed into connective-tissue, and in Mammals is finally buried in fat. (*"In adipe circumfuso sepelitur,"* Haller (35)).

The Thymus of Birds, Reptiles, Amphibia, and Fish.

During the growth of the organ the microscopical characters of the thymus of all animals are very similar. When the point of full development has arrived, then, and

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* Except in the Tortoise, where all the vessels in involution partake of this nature.

during involution, there are considerable differences in the thymus of different animals. Thus, in the Bird and the Tortoise the concentric corpuscles are often enlarged, and transformed into small cysts. In the Dog the cysts are lined in great part (as we have already seen) with ciliated epithelium. In Mammals, fat invades the follicle; while in the other classes of Vertebrates none is found.

Thymus of the Bird.

The thymus of the Fowl, the Turkey, and the Pelican have been examined.

In the Bird, the thymus is attached by connective-tissue to the vein and the nerve of the neck; so that in cutting longitudinal sections, the follicles are found to be attached to a central string, composed of fat, some small amount of connective-tissue, and sometimes including the large vein and the nerve. In the Chicken of five or six weeks old, the microscopical characters of the gland, as regards both cortex and medulla, are very similar to those of the fully-formed organ in Mammals. In involution, which takes place at an early period of life in the Bird, the distinction between cortex and medulla is lost, as the cortical portion soon disappears, and the great number of concentric corpuscles gives the appearance of numerous medullary portions in the follicle.

The concentric corpuscles differ somewhat in their origin, and in their arrangement, from those of Mammals. They are formed principally of large epithelioid cells (see Plate 95, fig. 99), only a few granular cells taking part in their formation. These epithelioid and granular cells form masses, which are not so much compressed, nor are they so definitely arranged in concentric layers as in the case of Mammals (see Plate 95, figs. 100 and 101). They contain vacuoles of various sizes, with thickened borders. The vacuoles are formed and increase by the coalescence of the vacuoles of the individual epithelioid and granular cells. (Compare Plate 95, figs. 99, 100, and 101.) Within them, we often meet with granular masses and masses of hæmoglobin. On teasing and examining the thymus, we find many vacuolated cells (see Plate 95, fig. 98). Sometimes the cells exist as only a small ring of nucleated protoplasm, surrounding the vacuole (see Plate 95, fig. 98, b'').

A few giant cells are present (see Plate 95, fig. 97), where one is shown enclosing a granular mass.

The fibrous-tissue is formed in two ways: either from the granular cells or from the concentric corpuscles, the edge of the vessels being thickened in involution. Fat does not invade the follicles, nor is the gland finally surrounded by fat. The essential process in involution is, however, similar to that in Mammals; the points of contrast are: the rapid diminution of the cortex, the large number of epithelioid cells, and the large number of concentric corpuscles, the rapid involution, and the absence of invading connective-tissue and of fat. The gland is finally replaced by a small amount of connective-tissue.

Thymus of the Reptile.

The thymus of the Tortoise will be described first, as specimens of that animal were procured of various ages. The thymus of one young American Alligator, three months old, and of a fully-grown Python, were also examined.

The thymus of the Tortoise, when the animal is quite small, resembles that of the Mammal; and then the follicle is composed of cortex and medulla; but as the animal grows older the follicle is composed of three parts: an outer cortical part, which is formed of flattened cells, with a delicate reticulum stretching between them (see Plate 95, fig. 105); a medullary portion, containing most of the concentric corpuscles; and between these two a zone of tissue, looking as if it were compressed, containing a ring or rings of vessels, and many lymphoid cells. This peculiarity of the thymus of the Tortoise depends on the fact that the cortex does not disappear more rapidly than the medulla, as in the Mammal or Bird; but the lymphoid cells disappear, and the network of connective-tissue-corpuscles increases, and takes their place. A change, something like this, is shown in the Mammal in Plate 93, fig. 80, but it does not there proceed to anything like the same extent, as in the Tortoise, where the cortex finally contains hardly any lymphoid cells. Many pigmented cells can be seen in the connective-tissue at the edge of the follicle, and surrounding the invading blood vessels; these appear to correspond to the plasma cells of the thymus of Mammals.

Further, there are follicles (as has been noticed by AFANASSIEW (33)) which are entirely formed of flattened, spindle-shaped cells. These follicles will be more completely noticed in the description of the thymus of the Alligator.

The concentric corpuscles of the thymus of the Tortoise present intermediate forms between the concentric corpuscles of the Bird and the ciliated cysts of the Dog. There are at times found in the thymus a few granular cells (see Plate 95, figs. 104 and 107). The protoplasm of the largest of these cells is marked by concentric rings (see Plate 95, fig. 104). The concentric corpuscles are formed in great measure by epithelioid cells; they contain vacuoles of various sizes, in which sometimes a granular cell is present (see Plate 95, fig. 106). In Plate 95, fig. 107, two granular cells are seen to be enclosed in a large vacuole.

The fully-formed concentric corpuscles contain either a mass of cells balled together in the interior of the vacuole, evidently undergoing degeneration, or masses and granules of hæmoglobin (see Plate 95, fig. 108).

The blood vessels of the medulla are very large; most of the vessels in the thymus of old animals have perivascular sheaths and thickened adventitia, and are very similar to those which we have described in the thymus of the Calf. A few giant cells are present. As involution proceeds the ring of vessels between the cortex and the medulla enlarges.

The thymus of the young Alligator is composed of cortex and medulla, the cortex much resembles that of the Tortoise (see Plate 95, fig. 105). In the medulla there are a few concentric corpuscles formed of epithelioid cells.

In the thymus there is a follicle composed entirely of flattened, spindle-shaped cells; this exactly resembles those which are met with in the Tortoise, and which have been supposed by AFANASSIEW (33) to be individual follicles, undergoing involution before the rest of the gland; there is no evidence that this is the case; on the contrary, all the facts point to its being some other gland which is situated among the thymic follicles. As in the first place, it is found in quite young Reptiles, as in the small Tortoise and the young Alligator; and in Mammals and the other classes of Vertebrates we do not find individual follicles undergoing involution before the rest of the gland. Secondly, when a follicle undergoes involution the process is a very gradual one, and is never characterised by a change of the whole follicle, equally and everywhere, into one tissue. Thirdly, these follicles are composed everywhere of flattened, spindle-shaped cells, and no such tissue is met with in the undoubted thymic follicles of either old or young Reptiles. Fourthly, no such tissue is met with in the thymus of Fish, Amphibia, or Birds; exactly such follicles are, however, found in Mammals, where hibernating glands exist, as in the Hedgehog. We conclude that these follicles are those of some other gland, probably of the nature of the hibernating gland of Mammals.

In the large Snake the thymus is small and to a great extent transformed into fibrous-tissue. The follicles contain many concentric corpuscles of large size, and a large number of granular cells, and the steps of transition between granular cells and fibrous-tissue are well seen.

This gland also contains a follicle, composed entirely of flattened, spindle-shaped cells, such as we have just spoken of, and there is no resemblance between this follicle and the thymic follicles which are undergoing involution by the invasion and new formation of fibrous tissue.

Thymus of Amphibia.

In the Frog the medulla contains many granular cells, as many as forty to sixty can often be seen in a section of the small organ. The granular cells are clup-shaped, and are attached by their smaller ends to fibrous tissue.

The thymus of the Axolotl is situated in nearly the same position as in the Ray-fish. It often contains cysts of considerable size. Its cells are very large.

Thymus of Fish.

In the Ray-fish the thymus begins as one follicle (see Plate 84, fig. 5), which is afterwards divided into numerous follicles. These are, in the later stages of involution, widely separated from one another. The thymus is very soft, and requires hardening with great care, lest in making sections the greater part of the lymphoid tissue should be lost.

Many colourless blood corpuscles are found in the connective-tissue and in the follicles. They are easily distinguished from the lymphoid cells by their granular appearance and greater size. The Ray fishes used in this research (with the exception of the smaller ones) had all been dead about ten or twelve hours before they were procured. The presence of colourless corpuscles in the tissues may possibly be only due to wandering of colourless corpuscles from the vessels after the death of the animal.

The cortical tissue does not differ from that of Mammals, and is represented in Plate 95, fig. 103. During involution there are great changes in the vessels; they become surrounded by one or more perivascular sheaths.

The tissue found in the Cod-fish, by cutting through the lining membrane of the upper and back part of the gill cavity, is much harder than the thymus of the Ray-fish. The whole tissue is encapsulated with connective-tissue, from which proceed trabeculæ, dividing the lymphoid masses into definite follicles, of which the outer are larger than the inner.

The lymphoid tissue does not show any differentiation between cortex and medulla, nor any central ring of vessels; and the trabeculæ are covered on each side by endothelial cells.

This tissue is probably a lymphatic gland.

DEVELOPMENT.

In the History (p. 1073) an account of the various views which have been held concerning the development of the thymus has been given. On the one hand are the observations of FRIEDLEBEN (3) and AFANASSIEW (33), who describe the thymus as arising independently in connective-tissue; and on the other hand, the observations of the authors who believe that the thymus arises from the respiratory tract, or the pharynx, or from the branchial clefts.

The development has been traced in the thymus of the human embryo, and in that of the Rabbit and the Chick.

In the human embryo about two and a-half inches long, it is found in the thorax and the neck, as a double gland; the upper part of each half consists of a single follicle, not differentiated into cortex and medulla. The wall of the gland has projections on its exterior (see Plate 87, fig. 23), and the connective-tissue has slightly invaded the gland.

In the human embryo twelve weeks old, the thymus consists of several follicles, partially differentiated into cortex and medulla; it also contains a few concentric corpuscles (see Plate 86, fig. 11).

In the fœtal Rabbit sixteen days old, we find the thymus in the neck, lying in front and to the inner side of the carotid artery. It is seen, at its upper end, as a single tube, and below as a number of tubes, some of which are solid and others hollow; the hollows are of very different sizes. The cells in cross-sections of the tube vary from two or three to many in number; they are epithelioid in character, and differ in appearance from the epithelial cells of the thyroid, which lie in approximation to the upper part of the tube. The lumen of the tube is in some cases closely packed with blood corpuscles; and appears to be continuous with vessels which pass out through the epithelioid cells.

In the fœtal Rabbit eighteen days old there are present in the neck three glands: first, the thyroid; secondly, a gland similar in appearance to the thyroid, and lying on its outer side, and in front and to the inner side of the carotid artery; this gland does not extend so high up in the neck as the upper part of the thyroid, but reaches down nearly to the upper end of the thymus; and in the embryo of twenty days, it extends still further, and lies behind and to the outer side of the upper end of the thymus. We also find in embryoes eighteen days old, thirdly, the thymus, which in the upper part is composed of a tube or tubes, and in the lower part is more branched. The upper part of the thymus retains its fœtal form much more than the lower part, as was noticed by KÖLLIKER (76). Vessels and some connective-tissue are seen to pass into the follicle, and the outer cells forming the border of the follicle are more epithelioid in character than the inner cells. The growth of the thymus can be easily traced in specimens of this age; and it will be seen that long processes of epithelioid cells are pushed out in the surrounding connective-tissue (see Plate 95, fig. 109, *a*).

In the fœtal Rabbit of twenty-one or twenty-two days old the cells are much smaller, and do not resemble epithelioid cells, except at quite the upper part of the gland. In the embryo of thirty days old the cells are still smaller, and the thymus nearly resembles that of newly-born animals.

In sections of the Chick, early in the seventh day of incubation, two small circular collections of cells are found, behind, and to the outer side of the nerve and vein of the neck (see Plate 87, fig. 29). The first growth is the enlargement of the gland, pushing out the wall in projections, similar to those in Plate 87, fig. 23. This can be well seen on the eleventh day. On the fourteenth day the connective-tissue has invaded the gland, and divided it into separate follicles; and on the eighteenth day these are differentiated into cortex and medulla. The large epithelioid cells are not seen until the cortex has been differentiated from the medulla.

It will be seen from the above that we do not accept the epithelial origin of the thymus.

If, however, we accepted the views of those authors who conclude that the thymus arises from epithelial cells, it appears to me that the view of KÖLLIKER (76) must be maintained, and that the opinion of STIEDA (53) is incorrect. The latter author considers that the epithelium forms the concentric corpuscles, and that the lymphoid tissue is formed from the middle layer which invades the gland; if, however, sections of the thymus are made, at various stages of growth, it will be seen that the epithelioid cells become smaller and smaller, and that they are pressed to the outside of the follicles, while the connective-tissue and blood vessels invade the interior of the follicle; and it is certain that the concentric corpuscles are found chiefly in the interior of the follicle, and that the cortical part of the follicle is composed of lymphoid cells and retiform tissue.

THE SO-CALLED HIBERNATING GLANDS.

The situation of these glands has been well-described by BARKOW (21). He divides them into four: cervical, thoracic, axillary, and dorsal. Their structure is somewhat foreign to our present purpose of describing the thymus; as these glands only occur, to any extent, in a few animals, most of which hibernate. It was, however, necessary to examine them, as there has been so much confusion between hibernating glands and thymus.

They are well seen in the Hedgehog, and we have also examined them in the Alpine Marmot and in the Rat, and have found microscopical traces of them in the Cat. The hibernating gland in the last animal appears to have been seen by REMAK (57), as he says that at the upper end of the thymus of newly-born Kittens there is a small yellow gland, which consists of granular yellow cells (like liver cells) lying close together.

As seen by the naked eye, or by low powers, the shape of the lobules, in the hibernating gland and in the thymus, is identical; but the situation and the colour of the glands are so different, that at once the eye can distinguish between them; the one being of a red-brown, and the other of a pale grey colour. Their microscopical appearances are very different. The glands are composed of granular polygonal cells, placed close to one another, often arranged in rows, and having narrow capillaries between the cells. In their minute anatomy these glands rather resemble the liver, than any lymphoid tissue.*

HÆMOGLOBIN CONTAINED IN CYSTS AND IN COLOURLESS CELLS.

In the thymus of all animals, hæmoglobin is found, either contained within cysts (the cysts, as we have noticed before, arise from the concentric corpuscles), or in cells; these cells appear to be always situated near to the concentric corpuscles, and often form part of them.

The cysts in the Dog, Bird, and Tortoise are, when small, filled with degenerated cells, and when large with hæmoglobin masses and granules (see pages 1093, 1098, 1099).

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* In the Rat, the thymus is contiguous to the hibernating gland, which in its turn runs up to the salivary glands.

In the Hedgehog, there are present in the thorax the thymus, a lymphatic gland, and the hibernating gland, in contiguity to one another. The hibernating gland has two kinds of alveoli, one composed of spindle-shaped cells, and the other of polygonal cells, which have been described above.

The thymus of the Rat and of the Hedgehog, therefore, present unusual difficulties; and conclusions drawn from them are not of great weight, unless shown to be in accordance with the structure of the gland in other animals.

In what way this hæmoglobin arises, seems uncertain; whether from a conversion of the protoplasm of the cells into hæmoglobin, or from an extravasation of the colouring matter of the blood into the cysts.

Hæmoglobin is also found in cells. These cells are of three kinds: (1) spherical or oval cells of various sizes, in which the hæmoglobin is met with (*a*) in the form of granules (see Plate 86, figs. 19 and 20, *c*, *d*); or (*b*) in the form of spherules or masses, some of which exactly resemble the coloured blood corpuscles (see Plate 86, fig. 21). These masses of hæmoglobin enclosed in cells are oval in the Bird, Reptile, and Fish; but circular in all Mammals, except in the Camel (see Plate 86, fig. 21; and Plate 92, fig. 77, *d*). These cells will be referred to afterwards. (2) The branched cells of the reticulum contain hæmoglobin, generally in fine granules, at times in large masses (see Plate 86, figs. 18, *h*, and 14). (3) Cells or masses of protoplasm attached to the concentric corpuscles also often contain hæmoglobin (see Plate 86, figs. 16, *h.h*, and 15, *h*), and the concentric corpuscles themselves often contain this substance (see Plate 86, fig. 17, *h*).

THE LYMPHOID CELLS OF THE THYMUS, WHEN EXAMINED IN THE FRESH STATE.

In teasing the thymus of any animal, either with or without the addition of saline solution, the first point which attracts attention is the great number of small cells, varying somewhat in size (see Plate 92, figs. 74 and 76, *Thc*). The smallest are nearly the same size as the coloured blood corpuscles if the animal be a Mammal, (compare Plate 92, fig. 76, *bc* and *Thc*). These thymic cells are nearly spherical, and have a very low refractive power. They are seen in many cases to be composed of a nucleus and of a delicate zone of protoplasm; in the very smallest, no zone of protoplasm can be found. If they are treated with saline solution, they appear to contain from three to six dark granules, which look like nucleoli; but on using a high power, and focussing carefully, and making the cells roll over under the microscope, the protoplasm is seen to be irregularly heaped up, and the corpuscles have somewhat of the horse-chesnut shape (see Plate 92, fig. 70).

CHARACTERS OF THE LYMPH AND BLOOD ISSUING FROM THE THYMUS.

The lymph and blood, as they issue from the thymus, were obtained from Calves and Oxen of various ages, from fourteen weeks to thirty months. The animals, seven in number, were killed in the ordinary manner by the pole-axe. The tissues of the neck were then divided, down to the lower part of the cervical portion of the thymus; the largest of the veins issuing from the thymus were tied, including in the ligature some slight amount of additional tissue; and the animal was left for ten or fifteen minutes. By that time, in every case but one, the lymphatic vessel issuing by the side of the veins was easily perceptible, and was seen to be filled with a very pale-pink fluid. The lymph vessels, before being opened, were slightly washed with saline solution, which was carefully collected, to be afterwards tested; then the lymphatic was cut, and the issuing lymph collected in a pipette. This method was employed, as the lymph-vessels are so small that it is impossible to introduce any capillary tube into them with certainty of success. The lymph of the large lymphatic of the neck, the blood of the thymic veins and of the jugular vein, were also collected.

It was found that the lymph collected from the lymphatics of the thymus contained many colourless corpuscles of three varieties, of which the first is by far the most common. There are (1) small lymphoid cells, exactly resembling in every respect the small cells obtained by teasing the thymus (see Plate 92, fig. 71, *Thc*); they show very faint amœboid movements, have a grey homogeneous aspect, and contain fine granules; (2) spherical or oval cells of various sizes, containing granules, spherules, or masses of hæmoglobin; some of these hæmoglobin masses exactly resembling coloured blood corpuscles (see Plate 92, fig. 71 C, D, E); (3) large cells with many radiating processes (see Plate 92, fig. 71 B). Some of these last cells contain hæmoglobin granules, and others do not; they are more numerous than the second variety, but much fewer in number than the first.

The lymph issuing from the thymus contained a very small proportion of coloured blood corpuscles, less than 1 per cent.; whereas that of the lymphatic of the neck contained a larger proportion.

We therefore conclude that *there are present in the lymph issuing from the thymus cells containing coloured blood corpuscles, and hæmoglobin granules; and that in the lymphatics of the thymus there are more colourless cells than in the lymphatics of the neck.* Whether there are more colourless cells in the blood of the thymic veins, than in the blood of the jugular vein, could not be determined; in two cases out of six there seemed to be some increase in number, but in the other four, no increase could be detected. However, the blood of the thymic vein contained many masses of granular matter; the so-called *Zimmermann's* corpuscles.

PHYSIOLOGICAL CONCLUSIONS.

In considering the uses of the colourless cells which issue from the thymus by the lymphatics, the first thought that presents itself is that these cells form colourless blood corpuscles. Two circumstances must, however, be borne in mind: the first, that though in Mammals there is no great difference in size and appearance between some of the colourless blood corpuscles and the thymic cells (see Plate 92, figs. 71 and 76, *cbc* and *Thc*), there are considerable differences in Amphibia and Reptiles (see Plate 92, figs. 73 and 74, *cbc* and *Thc*), still greater differences in the Bird (see Plate 92, fig. 75), and greatest of all in the Ray-fish (see Plate 92, fig. 72). The second circumstance is, that the fluid which passes along the lymphatics from the thymus does not, in Mammals, pass directly to the blood, but first into lymphatic glands or even into a succession of glands, where it is possible that these colourless cells may be changed. It is therefore difficult to form any very certain conclusions as to the functions of these colourless cells, although probably *they form one source of the colourless blood corpuscles.*

The difference in size between the thymic corpuscles and the coloured blood corpuscles is seen to be very great in some classes of Vertebrates, and we have no reason to believe that the one are ever immediately transformed into the other (see Plate 92, figs. 72, 73, 74, and 75, *bc* and *Thc*).

There is one fact which presents considerable difficulties—*i.e.*, that the blood of the fœtus and of the newly-born animal contains a considerably larger proportion of coloured to colourless blood corpuscles than is the case in the blood of adults. This was first noticed by DENIS (95), and then by PANUM (96), and yet the thymus is certainly more active during fœtal life and during the first years of growth than it can be during adult life, when it is composed in a great measure of connective-tissue.

In considering the uses of the cells containing hæmoglobin, it has been stated above that we find large colourless cells containing masses in shape exactly identical with, though in size generally smaller than, the coloured blood corpuscles; and this is true, not only in all Mammals with circular corpuscles (see Plate 86, fig. 21), but in the Camel (see Plate 92, fig. 77, *d*), and also in the Bird, Reptile, and Fish. It is considered by KÖLLIKER (97) and most authors that these cells, as found in the spleen where they have long been known, are large colourless cells which have absorbed coloured blood corpuscles; but there are very great difficulties in holding that view—first, because we have found these cells in the lymphatics, in the blood in two cases of leucocythemia, in the thymus, the lymphatic glands, and the medulla of young bone,* and in no one of these situations have we any knowledge of any destruction of coloured blood corpuscles taking place; and secondly, because if such a destruction were taking place, it would seem somewhat difficult to understand how these contained masses of hæmoglobin, in undergoing solution, retain their form. And it seems to me that the older view of GERLACH (106) and SCHAFFNER (107) as to the origin of coloured blood corpuscles, is quite as worthy of support as any other, in the present state of our knowledge; and more especially so when we consider that the coloured blood corpuscles in the embryo in early life are formed from large colourless cells, as seen by KLEIN (108), BALFOUR (109), and others, in the Chick, and from connective tissue cells or network, as seen by SCHÄFER (110), WISSOZKY (111), and RANVIER (112) in young Mammals.

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* Further, in two cases of pernicious anæmia, which were examined, the medulla of the long bones was found to contain large cells enclosing spherules and masses of hæmoglobin, some of them enclosing masses exactly resembling coloured blood corpuscles. PONFICK (98), FEDE (99), COHNHEIM (100), OSLER and GARDNER (101), LITTEN and ORTH (102), and RIESS (103) have described the occurrence of cells containing blood corpuscles in the medulla of patients who had died from pernicious anæmia, and NEUMANN (104), BIZZOZERO (105), PONFICK (98), LITTEN and ORTH (102) have seen them in the medulla after typhoid fever or other diseases.

We therefore conclude that *the coloured blood corpuscles take origin, in part, at least, from large colourless cells.*

I wish to thank Mr. DENT, of St. George's Hospital, and Mr. COMPTON for their kind help.

EXPLANATION OF PLATES.

The drawings, except where the contrary is stated, were made from sections of tissue hardened in chromic acid, and then in alcohol, and stained in hæmatoxylin.

The outlines of many of the drawings were made by means of an OBERHÄUSER'S camera lucida, with a large HARTNACK'S stand; in such cases the lens (HARTNACK'S) used with the camera lucida, and the approximate magnification, are given. The ocular and objective used in completing the drawings are also given.

In other cases where the camera lucida was not used, the ocular and lens used in making the drawing are given with the magnification. These figures, however, are not drawn to scale.

PLATES 83–95.

Figs. 1, 2, 3, 4. Camera lucida, with a simple lens, magnified about 12½ times.

The four drawings show the appearance of the follicles in various stages of growth and involution. In each case the tissue was hardened for the same length of time in bichromate of potash and then in alcohol.

The cortical portion of the follicles is distinguished in the drawings, as in the original, by its darker colour.

The follicles are united by cortical-tissue at (*a*), by medullary-tissue at (*b*). In some follicles there are two or more portions of medullary-tissue, as at (*c*).

ct. Interfollicular connective-tissue; in figs. 2, 3, and 4 this tissue contains fat.

c′t′. Processes of the interfollicular connective-tissue, penetrating the cortex.

Fig. 1. Section of the greater part of a lobule of the thymus of Calf three months old.

Fig. 2. Section showing about two-thirds of a lobule of the thymus of Heifer two years old.

Fig. 3. Section of thymus of Ox six years old.

Fig. 4. Section of thymus of Cow, probably twenty years old. The vessels in the connective-tissue are very large.

Fig. 5. Camera lucida, simple lens × 12½.

From Ray-fish, 4½ inches across. The section is vertical, parallel to the greatest breadth of the fish, and made a little behind the spout cavity.

ca. Central cartilage.

NG. Spinal ganglia.

M. Muscles cut transversely.

M′. Muscle cut obliquely.

N′G′. Nerves and ganglia.

Th. Thymus, showing slight difference between cortex and medulla.

G. The gills.

c′a′. Cartilage supporting gills.

Fig. 6. Camera lucida, obj. 2, × 75, oc. III., obj. 4.

From Calf four days old. Section of follicle. The thymus was injected with carmine and gelatine, and hardened in alcohol.

V. Veins which lie just within the medullary portion.

V′. Veins passing out of the follicle in the interfollicular connective-tissue.

ar. Arteries.

gr. Granular cells.

cc. Concentric corpuscle.

Fig. 7. Camera lucida, obj. 4, × 150, oc. III., obj. 5.

From Kitten four months old. Section of thymus injected with carmine and gelatine, hardened in alcohol.

M. Margin of medulla.

d.d′. Two vessels are seen to penetrate the very large compound concentric corpuscle, the central portion of which has fallen out.

b.g. and *ce*. The central parts of the individual concentric corpuscles; they are not penetrated by the injection.

Fig. 8. Camera lucida, obj. 7, × 450, oc. III., obj. 9.

Shaken section of thymus of Calf twelve days old.

The drawing represents the method of double staining by hæmatoxylin granular cells of a brown colour, attached to blood vessel: the nuclei of the granular cells and vessels, purple; lymphoid cells, deep purple.

Fig. 9. Camera lucida, obj. 8, × 600. oc. III., obj. 9.

From young Calf; section of cortical portion of thymus, hardened in chromic acid and then in alcohol. The specimen was stained with indigo-carmine and carmine (method of MORRIS and SHAKESPEARE, see reference No. 87) in the proportion of one of the former to ten of the latter. The reticulum is blue-green. The lymphoid corpuscles are of a carmine colour.

Fig. 10. Camera lucida, with upper part of No. 2 lens, × about 36.

Section of a follicle of the thymus of Calf one year old. The thymus was hardened in chloride of gold and chromic acid.

ct. Connective-tissue surrounding the follicle, and penetrating into the interior, cutting off portions of the cortical-tissue.

c′t′. One of the invading processes of connective-tissue.

F. Fat in the invading connective-tissue.

Fig. 11. Camera lucida and a simple lens × $12\frac{1}{2}$.

Transverse section of the thymus of human fœtus of about the twelfth week.

cc. Concentric corpuscles.

ct. Connective-tissue which surrounds the gland, and has invaded it and divided up the follicle. The cortex is distinguished by its darker colour.

Fig. 12. Camera lucida, obj. 8, × 600, oc. III., obj. 9.

Plasma cells, from medulla of thymus of Calf twenty-eight days old.

Tissue hardened in alcohol, stained in dahlia (method of EHRLICH, see reference No. 79).

Fig. 13. Camera lucida, obj. 7 × 450, oc. III., obj. 8.

Section of thymus of Calf. The tissue was hardened in bichromate of potash, and afterwards in alcohol, and stained in indigo-carmine and carmine, in the proportion of one of the former to seven of the latter (method of MORRIS and SHAKESPEARE, see reference No. 87).

v. Blood vessel showing perivascular sheath, and containing coloured blood corpuscles, which are stained green.

pl. Plasma cells.

Fig. 14. Oc. III., obj. 8, × 400.

Teased specimens.

(*a*). From thymus of human fœtus five months old.

(*b*). From Dog one year old. Connective-tissue network containing hæmoglobin.

Fig. 15. Camera lucida, obj. 7, × 450, oc. III., obj. 9.

Teased specimen of thymus of Calf seven months old.

In the lower part there is a concentric corpuscle ; above, a large mass of protoplasm containing (*h.*) hæmoglobin.

th. Coarse protoplasmic threads, with imbedded nuclei.

Fig. 16. Oc. III., obj. 8, × 400.

Teased specimen of thoracic portion of the thymus of Calf one year old.

Mass, probably part of a concentric corpuscle containing large masses of hæmoglobin (*h. h. h.*).

Fig. 17. Camera lucida, obj. 5, × 300, oc. III., obj. 7.

Teased specimen of thymus of Calf seven months old.

Concentric corpuscle containing (*h.h.*) hæmoglobin granules.

p. Vessel-like prolongation. The centre is filled with a mass of protoplasm, ending in a rounded extremity.

Fig. 18. Oc. III., obj. 8, × 400.

Teased specimen of thymus of Calf one year old.

Nucleated connective-tissue cells of medulla, a long process which contains hæmoglobin granules (*h.*).

Fig. 19. Camera lucida, obj. 7, × 450, oc. III., obj. 9.

Teased specimen of thymus of Calf one year old.

lc. Lymphoid corpuscles.

b. Coloured blood corpuscles.

c. and *d.* Cells containing hæmoglobin granules.

Fig. 20. Camera lucida, obj. 7, × 450, oc. III., obj. 8.

Teased specimen of thymus of Calf seven months old.

c.d. Cells containing hæmoglobin granules.

Fig. 21. Oc. III., obj. 8, × 400.

Teased specimens.

a.b.c. From thymus of Dog one year old.

d. From thymus of Calf twenty days old.

The cells contain spherules, and masses of hæmoglobin, some of them as large as the coloured blood corpuscles.

Fig. 22. To illustrate the colours of the hæmatoxylin staining solutions.

They were all made with potash alum and extract of logwood, in the proportion of three to one.

(*a*). With old powdered alum.

(*b*). With old crystals of alum.

(*c*). With old alum. (This solution had been made four years, all the others only a few hours.)

(*d*). With old dried alum.

(*e*). With fresh crystals of alum.

(*f*). With freshly-made dried alum.

Fig. 23. Camera lucida, obj. 2, × 75, oc. III., obj. 4.

Transverse section of one-half of the thymus of human fœtus six to ten weeks old.

Only about one-third of the drawing is filled in.

There is no difference traceable between the medullary and cortical tissue.

ct. Connective-tissue surrounding the gland.

v. Blood vessels.

The lymphoid cells are drawn a little too small.

Fig. 24. Natural size, represents three lobules of the thymus of Calf seven months old. Separated after the tissue had lain for some hours in dilute bichromate of potash.

Figs. 25 and 26. From the cortex of follicle of the thymus of Calf twelve days old.

In fig. 25, the specimen was shaken very carefully; in fig. 26, roughly for a long time.

Fig. 25. Camera lucida, obj. 7, × 450, oc. III., obj. 8.

c. Capsule with the blood vessels running obliquely through it.

th. Strong threads.

A network, formed of star-shaped cells and their processes, unites the blood vessels and the threads, and forms an adventitia to the blood vessels.

Fig. 26. Camera lucida, obj. 5, × 300, oc. III., obj. 8.

c. and *th.* As in fig. 25.

Figs. 27 and 28. Cortex of follicle, from the thymus of fœtus three months old.

Fig. 27. Camera lucida, obj. 9, × 750, oc. III., obj. 11, immersion.

The reticulum is seen as a fine network, with deeply-stained, irregularly-shaped nodes; the lymphoid cells are nearly spherical, and are not so deeply stained.

Fig. 28. Camera lucida, obj. 8, × 600, oc. III., obj. 9.

The section was somewhat shaken.

a.a. The nuclei of connective-tissue-corpuscles.

v. Blood vessel showing two nuclei.

r. The reticulum which is deeply stained.

l.c. Lymphoid corpuscles.

Fig. 29. Camera lucida, simple lens, × $12\frac{1}{2}$.

Section through the neck of Chick on the seventh day of incubation.

c. Medullary canal.

ao. Aortæ.

Th. Thymus.

Jv. Jugular vein, behind it is seen the vagus nerve.

œ. Œsophagus.

Tr. Trachea.

Fig. 30. Camera lucida, obj. 8, × 600, oc. III., obj. 11, immersion.

From thymus of Calf twelve days old.

ct. Star-shaped connective-tissue-corpuscles.

c′t′. Epithelioid cells.

To the left of the figure the protoplasmic network has left spaces, which are partly filled with large granular spheroidal masses.

Fig. 31. Camera lucida, obj. 7, × 450, oc. III., obj. 8.

Section from thymus of Calf twenty-eight days old. The tissue was hardened in osmic and chromic acids.

Spheroidal cells are seen, partly overlapping one another. The centres of the cells are composed of spheroidal granular masses. The nuclei lie at the borders of the cell. At (*g*) there are four nuclei.

v. Blood vessel.

r. Reticulum of which only traces are seen here and there.

lc. Lymphoid corpuscles.

Fig. 32. Oc. III., obj. 8, × 400.

Teased specimen from thymus of Calf one month old.

Spheroidal cells containing granular masses.

Figs. 33 and 34. Camera lucida, obj. 7, × 450, oc. III., obj. 9.

From the upper end of cervical portion of thymus of Calf fully one year old.

The figures show granular cells.

In fig. 33 the central part only is granular.

gr. Granular cell with very coarse granules.

In fig. 34 all parts of the cells are granular.

V. Vein.

Figs. 35, 36, 37, and 38. Camera lucida, obj. 7, × 450, oc. III., obj. 9.

Drawings to demonstrate the attachment of the granular cells to the blood vessels, and to the connective-tissue trabeculæ.

Shaken sections of the thymus of Calf twelve days old.

Fig. 35. The granular cell is attached to a capillary blood vessel; the vessel has a perivascular sheath.

Figs. 36 and 38. Granular cells attached by processes to newly-formed connective tissue.

In fig. 36 the processes of some of the granular cells are fibrillated.

Fig. 37. The granular cells are attached, in the upper part of the figure, to newly-formed connective-tissue (*ct.*) and below to (*v.*) a capillary blood vessel.

Figs. 39 and 40. Camera lucida, obj. 7, × 450, oc. III., obj. 8.

Fig. 39. From shaken section of thymus of Calf twelve days old.

a and *b.* Granular cells attached to network.

c and *d.* Granular cells of large size and various shapes.

Fig. 40. From thymus of Calf nearly one year old.

e and *f.* Granular cells with processes.

Fig. 41. Camera lucida, obj. 7, × 450, oc. III., obj. 9.

Shaken section of thymus of Calf twelve days old.

ctc. Connective-tissue-corpuscle, containing a nucleus and granular mass.

Fig. 42. Camera lucida, obj. 7, × 450, oc. III., obj. 9.

Shaken section of thymus of Calf twelve days old.

Granular mass, attached to part of network of connective-tissue-corpuscles.

Fig. 43. Camera lucida, obj. 7, × 450, oc. III., obj. 9.

Shaken section of thymus of Calf nearly one year old.

A granular cell attached to a blood vessel.

Figs. 44, 45, and 46. Camera lucida, obj. 7, × 450, oc. III., obj. 9.

Sections of thymus of fœtal Lamb six inches long. The tissue was hardened in osmic and chromic acids.

Fig. 44. Epithelioid connective-tissue-corpuscles, containing granular masses of various sizes. The granular masses are apparently nucleated in D. and E.

A. Epithelioid cell, containing a small granular mass, and a nucleus.

Fig. 45, A, B, C. Epithelioid cells, some showing nuclei, others containing smaller or larger granular masses.

va. Vacuolated cells, containing granular mass.

Fig. 45, A, *gr.* Granular cell containing two nuclei.

g. A cell containing a nucleus and a small granular mass.

Fig. 46, A, B, C. Granular cells, which are more or less vacuolated.

Fig. 46, D. Granular cell with two nuclei and containing a large vacuole.

Fig. 46, E. Multinucleated protoplasm, surrounding a vacuole which contains a vacuolated granular cell.

This is one of the earliest stages of formation of a concentric corpuscle.

Fig. 46, F. Epithelioid cell which is much enlarged, the protoplasm is very transparent; the outer edge of the cell is somewhat thickened.

Fig. 46, G. Multinucleated protoplasm surrounding an epithelioid cell. The whole forms a commencing concentric corpuscle.

Fig. 47. Camera lucida, obj. 7, × 450, oc. III., obj. 9.

Very large transparent epithelioid cell.

From section of thymus of fœtal Lamb eleven inches long. The tissue was hardened in osmic and chromic acids.

Fig. 48. Camera lucida, obj. 7, × 450, oc. III., obj. 9.

Granular cell with three nuclei.

From section of thymus of Calf nearly one year old, stained by solution of Brazil wood.

Fig. 49. Camera lucida, obj. 7, × 450, oc. III., obj. 9.

Section of thymus of fœtal Lamb six inches long. The tissue was hardened in osmic and chromic acids.

In the lower part of the figure two concentric corpuscles are seen; above them, lymphoid and epithelioid cells; and in the upper part of the figure, many granular cells.

cc. Commencing concentric corpuscle.

gr. Granular cell with three nuclei.

v. Blood vessel.

Fig. 50. Camera lucida, obj. 7, × 450, oc. III., obj. 9.

Section of thymus of fœtal Lamb eleven inches long. The tissue was hardened in osmic and chromic acids.

In the lower part of the figure a concentric corpuscle is seen. To the left of the figure, two giant cells; to the right, at (*cc.*) commencing concentric corpuscles.

gr. Granular cells, probably forming a giant cell.

th. Coarse threads attached to the concentric corpuscles.

Figs. 51, 52, and 53. Camera lucida, obj. 7, × 450, oc. III., obj. 9.

Multinuclear masses of protoplasm, enclosing granular masses.

Fig. 51. Section of thymus of Calf three days old.

The tissue was hardened in chloride of gold and chromic acid.

Figs. 52 and 53. From thymus of Calf one year old.

Fig. 54. Camera lucida, obj. 7, × 450, oc. III., obj. 8.

From thymus of Calf nearly one year old.

Multinuclear masses of protoplasm, at right angles to newly-formed band of fibrous-tissue (*ct.*).

The protoplasmic masses are arranged so as to leave oval spaces or gaps.

Fig. 55. Camera lucida, obj. 7, × 450, oc. III., obj. 9.

Giant cell with central cavity, from thymus of Calf twelve days old.

Fig. 56. Camera lucida, obj. 8, × 600, oc. III., obj. 9.

Giant cell with various processes, from teased specimen of thymus of Calf six months old.

Fig. 57. Camera lucida, obj. 8, × 600, oc. III., obj. 9.

Teased specimen from thymus of Calf one year old.

Giant cell with processes in various planes.

Figs 58, 59 *a* and *b*, 60, 61, 62. Oc. III., obj. 8, × 400.

Teased specimens from thymus of a Child.

Fig. 58. *cc.* Concentric corpuscle, whose outline only is drawn; it ends in a vessel-like prolongation; the prolongation branches, and at this point there is a granular cell.

Fig. 59 *a.* Concentric corpuscle, containing a dark mass, which extends down the vessel-like prolongation of the concentric corpuscle.

Fig. 59 *b.* One of the cells of the periphery of the concentric corpuscle.

Fig. 60. A compound concentric corpuscle.

In the lower part of the figure, coarse connective-tissue threads.

Figs. 61 and 62. From the periphery of a concentric corpuscle to show the coarse threads with imbedded nuclei.

Fig. 63. Camera lucida, obj. 5, × 300, oc. III., obj. 8.

From thymus of Child, stillborn.

Three concentric corpuscles, connected to one another by fibrillated processes. Attached to one of the concentric corpuscles is a long thread.

v. Blood vessel.

Fig. 64. Camera lucida, obj. 7, × 450, oc. III., obj. 9.

Section of thymus of Calf three days old, hardened in chloride of gold and chromic acid.

Two commencing concentric corpuscles. The upper one consists of vacuolated protoplasm containing a granular cell. The two commencing concentric corpuscles are united by connective-tissue cells; so that the whole has somewhat the appearance of a vessel containing granular cells.

Fig. 65. Camera lucida, obj. 7, × 450, oc. III., obj. 9.

From thymus of Calf one year old. In the upper part of the figure a network is seen, containing one granular cell.

Below, is a large granular mass in continuity with a vessel-like formation. The whole is attached to the cells forming the network.

Fig. 66. Camera lucida, obj. 7, × 450, oc., III., obj. 9.

Section of thymus of Calf one year old, hardened in chloride of gold and chromic acid.

Two granular cells, and a small cell surrounded by a thick border; the whole forming a commencing concentric corpuscle.

Fig. 67. Camera lucida, obj. 7, × 450, oc. III., obj. 9.

From thymus of Calf one year old.

Multinuclear protoplasm surrounding a vacuole, which contains a granular cell. The whole forms an early stage in the formation of a concentric corpuscle.

Fig. 68. Camera lucida, obj. 7, × 450, oc. III., obj. 9.

From thymus of Calf twelve days old. In the lower part of the figure a commencing concentric corpuscle is seen, composed of epithelioid and granular cells. The central granular cell (*gr*) is surrounded by a vacuole.

v. Blood vessel, to which is attached a network of connective-tissue-corpuscles and granular cells.

f. A bundle of newly formed fibrous-tissue.

Fig. 69. Camera lucida, obj. 7, × 450, oc. III., obj. 8.

From thymus of Calf twelve days old. Shaken specimen. At the upper part of the figure a small concentric corpuscle is seen; the centre is formed by a granular cell.

v. Blood-vessel.

The neighbouring connective-tissue-corpuscles are very large.

f. Newly formed fibrous-tissue, attached to the wall of the vessel, in the lower part of the figure.

th. Coarse connective-tissue threads, with imbedded nuclei.

gr. Granular cells.

Fig. 70. Oc. III., obj. 11, immersion × 850.

A thymic corpuscle treated by $\frac{3}{4}$ per cent. saline solution.

Fig. 71. Camera lucida, obj. 8, × 600, oc. III., obj. 9.

cbc. The outlines of the colourless blood corpuscles, from one of the thymic veins of an Ox twenty-two months old.

Thc. The outline of the thymic corpuscles, from one of the lymphatics of the thymus of the same animal.

B, C, D, E, not drawn to scale.

B. Large cell, with processes containing fine granules, but no hæmoglobin.

C, D, E. Various cells containing larger or smaller masses of hæmoglobin.

Figs. 72, 73, 74, 75, and 76. Camera lucida, obj. 8, × 600, oc. III., obj. 9. These figures show the exact comparative sizes of the outlines of the coloured and of the colourless blood corpuscles, and of the Thymic corpuscles in the various classes of Vertebrates.

cb. and *cbc.* Coloured and colourless blood corpuscles, examined without reagent.

Thc. Thymic cells, examined in $\frac{3}{4}$ per cent. saline solution.

Fig. 72. From Ray-fish two feet seven inches across.

Fig. 73. From Frog about one-fourth grown.

Fig. 74. From Tortoise half-grown.

Fig. 75. From Pigeon three months old.

Fig. 76. From Dog eighteen months old.

Fig. 77. Camera lucida, obj. 7, × 450, oc. III., obj. 8.

bc. Coloured blood corpuscles of adult Camel.

cbc. Colourless blood corpuscle of the same.

d. Large cell, not drawn to scale, containing small oval coloured blood corpuscles from the lymphatic gland of the same.

Fig. 78. Camera lucida, obj. 7, × 450, oc. III., obj. 9.

Shaken section of medulla of follicle, from thymus of Calf twelve days old.

f. Commencing fibrous-tissue; at this point, the connective-tissue-corpuscles are much closer together, and the nuclei are more abundant than in other parts of the figure. A little above the middle, three granular cells are seen; to the left, connective-tissue-corpuscles forming a network. They are drawn as if they branched only in one plane.

A few lymphoid cells are seen.

Fig. 79. Camera lucida, obj. 7, × 450, oc. III., obj. 9.

Section of medulla of follicle, from thymus of Calf one year old, hardened in chloride of gold and chromic acid.

To show the formation of fibrous-tissue from spindle-shaped cells.

Figs. 80 and 81. Camera lucida, obj. 7, × 450, oc. III., obj. 9.

Shaken section of thymus of Calf nearly one year old.

Fig. 80. From cortex of a follicle.

v. A blood vessel, with adventitia of newly-formed tissue.

The network is composed of broad, closely-meshed connective-tissue.

Fig. 81. From centre of medulla of a follicle.

A blood vessel, with adventitia of newly-formed connective-tissue. The vessel is unchanged.

f. Fibrous-tissue attached to the adventitia.

Fig. 82. Camera lucida, obj. 7, × 450, oc. III., obj. 9.

From thymus of Calf twelve days old.

Three granular cells, attached to newly-formed fibrous-tissue; the largest has a distinctly fibrillated process.

Fig. 83. Camera lucida, obj. 7, × 450, oc. III., obj. 9.

Shaken section of the thymus of Calf nearly one year old.

Granular cell, whose process is in connexion with a dark reticulum.

Epithelioid cells are also seen.

Fig. 84. Camera lucida and simple lens, × $12\frac{1}{2}$.

From thymus of Dog nine years old. One follicle is shown, divided into two parts by invading fat tissue (*F*).

ce. Five cysts lined by ciliated epithelium. The cysts contain hæmoglobin.

Figs. 85, 86, 87, 88, and 89, demonstrate the formation of ciliated epithelium in the thymus of the Dog.

Camera lucida, obj. 7, × 450, oc. III., obj. 8.

Fig. 85. From thymus of Dog thirty months old.

A small giant cell, whose edges are attached to connective-tissue-corpuscles.

Fig. 86. From thymus of Dog five months old.

Concentric corpuscle.

ce. Cyst containing degenerated cells.

Fig. 87. From thymus of Dog five months old.

Concentric corpuscle with large cavity. The interior is filled with degenerated cells. The border of the cavity is lined with epithelioid cells (*ep*).

Fig. 88. From thymus of Dog sixteen months old.

Two cysts, partially lined by sub-columnar ciliated cells.

dg. Mass of degenerated cells.

ep. Epithelioid cells, partially lining one cyst.

Fig. 89. From thymus of Dog sixteen months old.

Two small cysts. That on the right is lined in great measure by ciliated epithelium. The cyst on the left is lined by ciliated epithelium in its upper part, but below is bordered by large epithelioid cells (*ep*).

va. Vacuolated and degenerated cells.

Fig. 90. Camera lucida, obj. 5, × 300, oc. III., obj. 7.

From thymus of Dog nine years old, showing a row of ciliated cells, which line a large cyst. The cells are in more than one layer. Only part of the drawing is filled in.

v. Blood vessel.

Fig. 91. Camera lucida, obj. 8, × 600, oc. III., obj. 9.

From thymus of Dog sixteen months old, showing columnar ciliated epithelium.

v. Blood vessel.

Fig. 92. Camera lucida, obj. 7, × 450, oc. III., obj. 8.

Teased specimen, from thymus of Dog about two years old.

Two ciliated epithelial cells.

Figs. 93 and 94. Camera lucida, obj. 7, × 450, oc. III., obj. 8.

To illustrate the formation of vessels.

Fig. 93. From thymus of Calf one year old.

The vessel is inter-epithelial.

Fig. 94. From thymus of human fœtus, probably three months old.

The new vessel is intra-epithelial and granular, and is formed from a granular cell.

Figs. 95 and 96. Camera lucida, obj. 7, × 450, oc. III., obj. 8.

Fig. 95. Section of thymus of Calf one year old.

The tissue was hardened in chloride of gold and chromic acid.

A vessel with thickened adventitia.

Fig. 96. From thymus of Calf one year old.

Vessel with adventitia composed of more than one layer of cells.

pe. Perivascular sheath.

Figs. 97, 98, 99, 100, and 101. Camera lucida, obj. 7, × 450, oc. III., obj. 8.

From thymus of Turkey seven and a-half months old.

Fig. 97. Giant cell, whose central part is formed by a granular mass.

lc. Lymphoid cell.

Fig. 98. Teased specimen.

a. Granular cell.

b, *b′*, *b″*, *b‴*. Various forms of vacuolated cells.

c. Cell with hæmoglobin granules.

Fig. 99. Shows part of the medullary portion of follicle.

Epithelioid cells, showing traces of reticulum between the cells.

gr. Two granular cells, one of which has four nuclei.

lc. Lymphoid corpuscles.

Figs. 100 and 101. Concentric corpuscles with large central cavity formed by vacuolation.

In fig. 101 individual vacuolated cells are seen.

Fig. 102. Camera lucida, obj. 8, × 600, oc. III., obj. 9.

From thymus of Turkey seven months old.

Shows small epithelioid cells, the lymphoid corpuscles, and the reticulum.

Fig. 103. Camera lucida, obj. 8, × 600, oc. III., obj. 9.

From thymus of Ray-fish 4½ inches across.

The lymphoid corpuscles and traces of the reticulum are seen.

ea. Large endothelial cell of the capsule of the follicle.

Figs. 104, 105, 106, 107, and 108. Camera lucida, obj. 7, × 450, oc. III., obj. 8.

From thymus of Tortoise.

Fig. 104. From medulla of follicle of Tortoise, probably twenty-one months old.

Granular cells, the largest shows concentric rings.

Fig. 105. From Tortoise, probably eighteen months old.

From cortex of the follicle, showing epithelioid cells and the reticulum.

Figs. 106 and 107. From thymus of medium-sized Tortoise.

Two concentric corpuscles which contain granular cells.

r. Traces of reticulum.

lc. Lymphoid corpuscles.

Fig. 108. From thymus of medium-sized Tortoise.

A large cyst, lined by columnar and sub-columnar epithelium.

h. Masses of hæmoglobin.

Fig. 109. Camera lucida, obj. 2, × 75, oc. III., obj. 4.

Transverse section of one-half of the thymus of fœtal Rabbit eighteen days old.

a. Process formed of epithelioid cells, continuous with the thymus.

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XXVI. *On the Effects of Heat on certain Haloid Compounds of Silver, Mercury, Lead, and Copper.*

By G. F. RODWELL, *Science Master in Marlborough College.*
Communicated by Professor A. W. WILLIAMSON, *For. Sec. R.S.*

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[PLATE 96.]

CONTENTS.

1. INTRODUCTION.

THE researches described in the following pages, which were commenced in 1874, were suggested by observations made by Professor CLERK MAXWELL and M. FIZEAU. The former ('Theory of Heat,' p. 8), in discussing the effects of heat upon matter, writes: "The body generally expands (the only exception among solid bodies, as far as I am aware, is iodide of silver, which has been found to contract as the tempera-

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* The author has been engaged at intervals during the last five years on an investigation of the effect of heat in producing expansion and contraction in iodide of silver and some other allied iodides and their alloys. A series of papers on the subject, written as the results were successively got out, have been published in the Proceedings of the Royal Society.

The paper which forms the last of the series, and which contains the results obtained with an extended series of alloys, by the methods which were ultimately adopted as the best, was read before the Royal Society on December 8, 1881, and an abstract only of it has appeared in the Proceedings. The present memoir is made up of the paper in question, preceded by a general account of the whole investigation.

ture rises)." The latter ('Nouvelles observations relatives à l'Iodure d'Argent'), remarks: "Ce corps, en effet, paraît offrir l'exemple d'une inversion complète des phénomènes ordinaires de la dilatation par la chaleur, car son volume diminue très-certainement pendant l'echauffement, et augmente pendant le refroidissement."

It was thought that results of interest might be obtained as to the molecular constitution of this anomalous body, by examining the effects of heat upon it at higher temperatures than those employed by FIZEAU (which in no case exceeded 100° C.); also by forming compounds, or alloys, with it, and other bodies, and determining the effect of its presence upon the coefficients of expansion and the physical structure of such bodies.

In this connexion we must remember that the phenomena exhibited by the iodide of silver on heating, are approximated to the anomalous behaviour of ice, bismuth, and some other substances, which at the moment of fusion, and for a few degrees above that point, exhibit contraction.

Moreover, certain crystals contract in the direction of one of their axes on heating; while garnets and some other minerals undergo an increase of specific gravity on being strongly heated, and slowly recover their original density. ERMAN asserts that the alloy of 2 parts bismuth, 1 part lead, and 1 part tin, which fuses at 94° C., expands when heated from 0° to 44° C., but on further heating contracts so that at 56° C. the density is the same as at 0°, and at 69° greater than at 0°. At a higher temperature expansion again sets in, and at 87·5 the alloy has again the same density as at 0°, while the density just before fusion is the same as that at 44° C. A number of metallic alloys of somewhat similar constitution, also possess anomalous coefficients.*

2. METHOD OF RESEARCH, AND THE APPARATUS EMPLOYED.

The first series of experiments were made with a view of determining the volume occupied by iodide of silver at different temperatures between 0° and 300° C. Preliminary experiments showed the nature of the changes to be examined. A mass of iodide of silver fused in a glass tube contracted while cooling so that it could be withdrawn from the tube, but on returning it to the tube and permitting it to cool further, it was seen to suddenly change in colour and structure, and simultaneously to expand, and violently break the tube.

Again, a thin spherical flask, which contained 10 cubic centims., was fitted with a long capillary tube attached to an accurately ground stopper. Iodide of silver was placed in the flask, which was then filled with mercury, and the capillary tube attached, forming in fact a large thermometer, the bulb of which contained, in addition to the mercury, a quantity of iodide of silver. On heating such an arrangement the mercury was observed to expand, until the contraction of the iodide exceeded the expansion of the mercury, when on further heating, the column of mercury descended in the tube.

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* See a paper "On the Expansion and Specific Heat of Fusible Alloys," by W. SPRING, Ann. Chim. Phys., [5], vii., 178-228.

On cooling, the mercury rose in the tube, until the contraction of the mercury exceeded the expansion of the iodide.

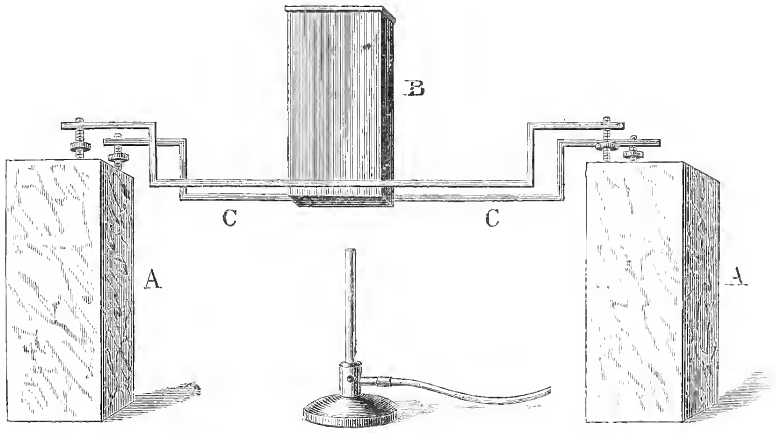
Thus it was obvious that some method of examination must be adopted, which should be equally applicable to the indication, and measurement, of either expansion or contraction.

Three experimental methods were tried. The last (γ) was finally adopted.

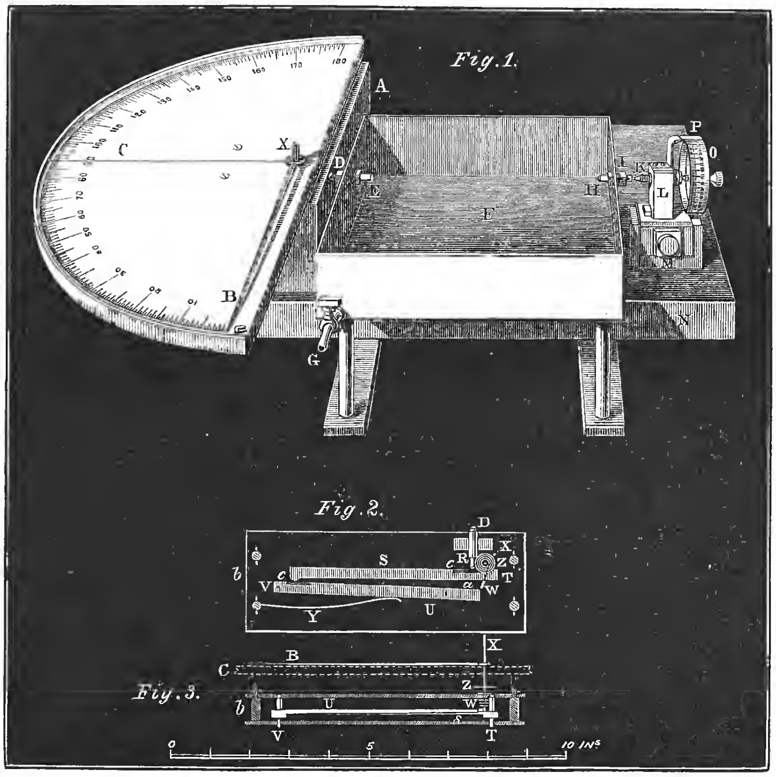
α. *Method of the specific gravity flask.*—Professor GUTHRIE suggested to me that a convenient mode of determining the contraction of the iodide might be to fill a specific gravity flask with mercury, and to determine the amounts of mercury driven through the capillary tube of the stopper for every ten or twenty degrees increase of temperature; then to place in the flask a known weight of fused iodide of silver, together with a known weight of mercury, and to repeat the determinations for similar ranges. 440 grms. of mercury were thus heated, and weighings were made at intervals of 10° C.; afterwards 38·3680 grms. of iodide of silver were introduced, and the heating maintained through similar ranges; but it was found that as the temperature approached 70° C. the iodide of silver was slightly decomposed by the hot mercury, green protoxide of mercury, HgI, being formed. Thus this method had to be abandoned.

β. *Method of the microscope.*—It was thought that rods of the iodide might be placed in a vertical bath of paraffine, and examined in the field of a microscope fitted with a micrometer eyepiece, during heating through various ranges of temperatures. The following apparatus was constructed for this purpose:—Two blocks of white marble (A, A, fig. A), 8 inches high by 4 inches by 3 inches, were placed 15 inches apart. Between them the paraffine bath B of copper was supported on massive iron bars C, C, terminated by levelling screws which rested on the marble blocks. The length of the bend in the iron bars, and the height of the brass levelling screws between the marble and the iron bars were so arranged that the upward expansion of the one compensated the downward expansion of the other. The bath contained ceresine capable of being heated to 340° C. A frame fitted into the bath containing

Fig. A.



supports for the rod of iodide of silver, and a system of vanes for agitating the liquid during heating. The rod of iodide was cast, sawn plane at the ends, smoothed by sand-paper, and furnished at one end with a cross wire arrangement for viewing in the field of the microscope. A cover fitted upon B through which a thermometer passed, also the head of the cross wire arrangement. The microscope was horizontally attached to a massive iron pillar, which was fixed in a block of white marble. Of course numerous precautions were taken to avoid the acquisition of heat by any portion of the microscope or stand, and every attempt was made to eliminate errors. But the method was unsatisfactory and had to be abandoned. Walking on the floor, resting an arm on the table, and other inevitable disturbing causes, produced tremors of such magnitude that all accurate observation came to an end.



γ. *Method of the expansion apparatus.*—I finally decided to attempt to enormously multiply a small motion by means of a system of levers; to cause bars of the iodide of silver, or other substance under examination, to act upon these levers while being heated through various known ranges of temperature; and to measure the movements by means of a micrometer-screw. This plan was adopted for all the determinations hereinafter given.

The following apparatus was devised in order to carry out the idea. The box A, fig. 1 (drawn to scale), contains the levers, shown in plan and section in figs. 2 and 3; B is an index moving through 180° of arc over the graduated half-circle C. B is attached to the axis X, which is in connexion with the second lever U. D is a sliding bar, one end of which bears upon the first lever, while the other is in contact with a short rod of glass E, which moves freely, but water-tight, in a metal stuffing-box. The rod of glass thus enters a brass trough F, which can be filled with water or melted paraffine, and heated by means of a BUNSEN'S burner. The water or paraffine is allowed to run off at the end of an experiment by the tap G. H is a rod of glass similar to E, which moves freely, but water-tight, in the metal stuffing-box I. Between E and H is placed the rod of substance, the expansion or contraction of which, we desire to determine. The end of H external to the trough F rests against the end of the micrometer-screw K, which is tipped with agate, and which moves steadily in the support L firmly clamped to the iron bar M, which is screwed to the base of the instrument N. The head of the micrometer-screw, O, is graduated into 250 divisions, and is figured to 500; the graduations are read off against the cross arm P. A plan of the levers is shown in fig. 2, where D (the end of which is seen in fig. 1) is the movable bar, sliding in a socket R and bearing against the first lever S, which is pivoted at T. The lever S bears against a second lever U, pivoted at V. From the opposite extremity of U a very fine steel chain W, such as is employed in watches, passes to a vertical axis X, which carries the needle B. The needle is brought back to zero by the pressure of the spring Y against the lever U, and also by a fine steel mainspring Z, which is fixed to the vertical axis X. *a* is a small pin to stop the lever U as soon as the needle has passed the zero-point by a few degrees. The levers are shown in section in fig. 3; the lettering is the same throughout. The framework *b* is of brass, as are also the levers; the pivots are of steel. Pieces of glass, *c c*, are let into the levers at the bearing points to diminish the friction.

The micrometer has threads $\frac{1}{100}$th of an inch apart: hence one complete revolution of the micrometer-head is equal to $\frac{1}{100}$th of an inch, and a movement of the head through one division is equal to $\frac{1}{25000}$th of an inch; but it is quite easy to read to half a division, and hence to $\frac{1}{50000}$th of an inch. The rods inserted between the micrometer and the lever are 6 inches long, and from $\frac{1}{4}$ inch to $\frac{1}{3}$ inch in diameter; they rested in the earlier experiments on light glass rollers placed in the trough F. A rod of the substance to be examined is inserted between E and H; perfect continuity of the parts is established, and the micrometer-head is turned until the needle stands at zero; the reading on the micrometer is registered, and the head is then turned until the index B has passed to 180° of arc; by again reading the micrometer, the relation between the micrometric divisions and the divisions on the scale of C is established.

By this means it was found that a movement of ·0035 inch in the micrometer moves the index through 180° of arc. Now since the index is 6 inches long, in moving through half a circle its extremity passes over 18·84 inches, and this motion is produced by a movement of ·0035 inch; hence the levers multiply any motion communicated to them 5382 times. The precise value of this index-scale having been ascertained, the determination of the expansion of a body by heat becomes an easy matter, provided that we are careful to secure perfect rigidity in all the fixed parts of the apparatus (by no means an easy task), and are further careful to prevent the conduction of heat from the short glass rods (E and H) to the interior of the apparatus. The coefficient of expansion of E and H must also be ascertained, and allowed for in all determinations. If, on the other hand, we wish to determine the contraction of a body, the index is pushed up to 180° of arc by means of the micrometer-screw, and is pushed back to zero as the bar contracts by the spring Y and the helical mainspring Z. The precise value of the return movement must be, of course, ascertained by means of the micrometer-head. It is obvious that in moving from 0° to 180° of arc the spring Y acts *against* the motion of the expanding body; while in moving from 180° to 0° it moves *with* the motion of the contracting body. All the earlier determinations were made with the apparatus in this form.

Subsequently, and for the later determinations, the following changes were made mainly with a view of reducing the resistance by diminishing friction :—

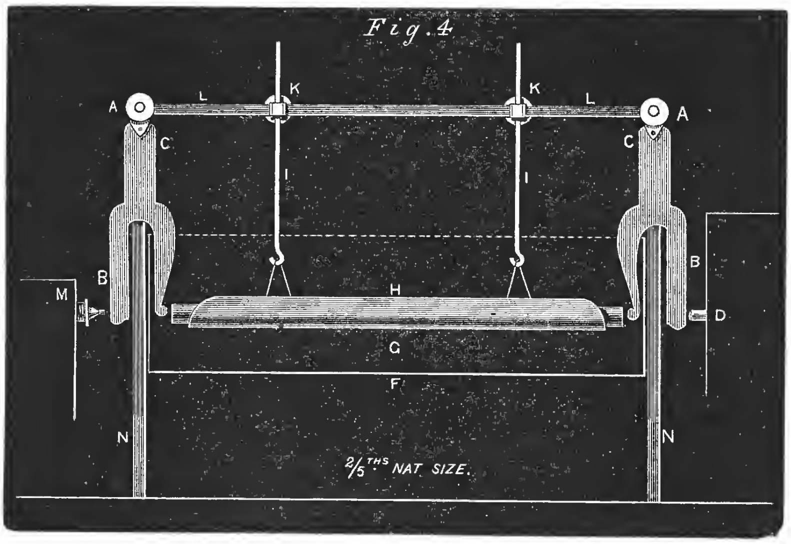
1. The wooden base N was replaced by a massive stone block, to which the box containing the levers, and the upright carrying the micrometer-head were firmly bolted.
2. The levers S, U (fig. 2) were reduced in weight.
3. The spring Y (fig. 2) was removed, as it was found that the recoil of the helical spring Z was quite sufficient to bring the index back to zero, and the presence of Y served only to increase the resistance and general strain.
4. The vertical axis X (figs. 1 and 3) was no longer allowed to work in the upper confining plate of the framework *b*, but it was caused to turn lightly in a bent arm above, while below it rested upon a slightly hollowed ruby.
5. The steel chain W communicating motion from the lever U to the vertical axis X, which carries the index, was shortened and caused to wind upon the barrel in such a manner that when unwound to the extent of half a single coil, it moved the index through its entire range, viz.: from 0° to 180° of arc. By this means any possibility of the chain doubling upon itself was obviated.
6. But perhaps the most important changes were the removal of the stuffing boxes I and the rods H from the trough F, and the substitution of levers working over the rim of the trough; and the suspension of the rod of substance under examination in a cradle between the levers. This was effected in the following manner :—

A horizontal bar L, L (fig. 4) was supported by rods N, N, strengthened by cross bars (not shown) let into the stone base of the instrument; it carried Y-shaped brass levers B, B, moving about axes at C, C (figs. 4 and 5), attached at the points A, A. F is the trough in which hot ceresine is used for heating the bar under examination, H, which is supported by the cradle G. Two rods I, I, which slide in holes K, K, and are capable of being held at any height by screws, support the cradle G. D is the rod (figs. 1 and 2) which bears upon the lever S, and M the point of the micrometer-screw.

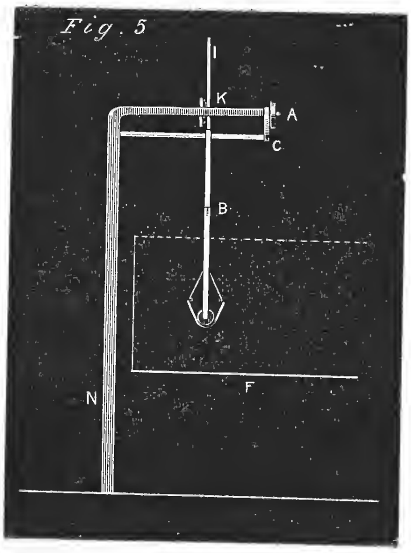
A bar of fine homogeneous silver was used in order to test the delicacy of the apparatus. It was placed between the levers B, B, and cold water was poured into the trough. The temperature was indicated by two thermometers reading well together, placed near the opposite extremities of the silver bar, and the needle was brought to zero. The water was then heated, being constantly stirred to produce uniformity of temperature throughout the whole mass, until the needle had been forced round to 180° of arc. The number of degrees of heat being read off, showed the heat necessary to produce a certain known expansion, and from this the coefficient of expansion was easily ascertained. Five results gave 10° C., 10°, 10°, 10°, 9°·5. The coefficient deduced from the mean of the determinations was for 1° C.

·0000193083.

Now LAVOISIER gives it as ·00001910; DANIELL ·00001951; FIZEAU (determined by the same method as that which he applied to the iodide, chloride, and bromide of



Section through the trough longitudinally, showing the mode of suspension of the bar, and the position of the levers.



End section of the trough, showing one of the levers, and the bar which carries it.

silver) gives ·00001921; and more recently MATTHIESSEN (Phil. Trans., 1866) found it to be ·000019436.

Rods of other metals gave the following results:—

	Expansion-apparatus.	Various observers.
Iron	·000011025	·00001182 ·00001194
Copper	·0000174433	·00001666 ·00001678 ·00001722
Lead	·0000302121	·00002924
Zinc	·0000288761	·00002918

It is thus evident that the apparatus is capable of considerable accuracy, especially when we remember that we are dealing with rods of substance only 6 inches in length. I venture to think that such an apparatus would prove a useful adjunct to a Physical Laboratory; for it would not only afford a means of determining coefficients of expansion with rapidity and accuracy, but it might be used for determining thicknesses in thousandths and ten-thousandths of an inch. For if the substance were introduced between the agate face of the micrometer-screw and the lever B, and if the index were then driven round to a certain point (say 20° of arc), and so with the other substances tested, an exact uniformity of pressure of the face of the screw on the substance would be secured, and minute thicknesses might thus be measured with accuracy.

In constructing such an apparatus, extreme rigidity of the fixed parts is a main necessity; the base should be of thick marble, and the micrometer-screw support should be deeply let into it and firmly secured. Massive supports, apparently fixed with the utmost firmness, sometimes yield with surprising readiness to the extent of $\frac{1}{500}$th of an inch. The elasticity of the metal often causes great inconveniences. The levers must be firm and strong, but not unnecessarily heavy; they should work very smoothly. Agate should be let into the brass at the points of contact. Care should be taken to avoid any transference of heat from the hot trough to other parts of the apparatus. By the use of a paraffine of high boiling-point, the temperature of a bar submitted to heat in the trough (F) can be raised to 340° C. Under such circumstances great care is requisite to prevent heat from being radiated or otherwise communicated to other parts of the apparatus.

Means employed to determine the expansion in passing from the solid to the liquid condition.

The determination of the amount of expansion which the various bodies examined undergo in passing from the solid to the liquid condition was effected in the following manner:—A conical tube of platinum weighing 44·844 grms. was carefully filled with mercury at a known temperature. It contained 105·43 grms. of mercury. It was then filled with the substance under examination at its melting-point; the substance was allowed to solidify, and the whole was weighed. The cavity due to the contraction of the mass in solidifying was then filled with mercury to the level of the mouth of the conical tube, and the mercury was weighed. Then, knowing the capacity of the tube, the specific gravity of mercury, and of the substance under examination, the temperature of the fusing-point, and the coefficient of expansion of the platinum cone, we have all the data requisite for the determination. A glass tube was sometimes employed. The real difficulty is the determination of temperatures above the boiling-point of mercury; and until a trustworthy method of general application has been devised, such determinations must be regarded as approximations. The expressions "below a red heat," "a dull red heat," &c., are still common in text-books and in memoirs; but the very definitions of what is meant by a "dull red heat" vary, as also do the temperatures assigned to it by different writers. The method of Mr. CARNELLEY (Jour. Chem. Soc., 1876) appears to be the most accurate yet devised.

3. IODIDE OF SILVER, AgI.

Experiments were made in order to determine certain general physical properties of iodide of silver before placing it in the expansion apparatus.

It was prepared:—

(α) By precipitation according to the equation

$$AgNO_3+KI=AgI+KNO_3.$$

A dilute solution of pure iodide of potassium was added to a dilute solution of nitrate of silver. The precipitated iodide was thoroughly washed with boiling distilled water, slowly dried, fused rapidly in a porcelain crucible, and cast into cylindrical rods in warm thin tubes of well-annealed glass.

(β) By dissolving pure silver (for which my thanks are due to Professor CHANDLER ROBERTS) in strong hydriodic acid, evaporating to dryness, and fusing.

(γ) By exposing silver leaf to the spontaneous evaporation of iodine.

The iodide prepared by precipitation was commonly used for all the purposes described in the following pages, with the exception of the experiments relating to the effect of light upon the iodide.

General physical properties.—The molten iodide which forms a mobile bromine-red liquid, solidifies at 527° C. to a perfectly transparent, very flexible, claret-coloured solid; as it cools it becomes lighter in colour, and the colour of amber, and as 163° C. is approached it becomes pale yellow. All this time it has been slowly contracting on cooling. At 163° C. the contraction ceases, and the mass neither expands nor contracts till the temperature has fallen to 156°·5 C., when expansion commences, slowly at first, but becoming very rapid between 151°·3 C. and 148° C. The body simultaneously changes from a plastic transparent, amorphous state, to that of a crystalline, opaque, pale green, solid. The more rapid expansion on cooling ceases at 142° C., and as it continues to cool an extremely slight expansion continues, until, according to FIZEAU'S calculation, it attains a minimum density and maximum volume at −60° C.

It is well known that plastic sulphur in becoming yellow, opaque, and crystalline, gives out a good deal of heat. Transparent plastic iodide of silver was also found to give out heat in changing from the amorphous to the crystalline condition.

The iodide of silver obviously possesses at least two allotropic modifications: the one plastic, amorphous, and transparent above 156°·5 C.; the other brittle, crystalline, and opaque below 142° C. If it be melted and poured into cold water a very brittle, yellow, opaque mass is the result.

Successive fusions appeared to make the iodide more brittle, but did not affect the specific gravity materially. Thus a specimen which had been fused many times had a specific gravity of 5·675, while another specimen after a single fusion gave 5·66. A specimen prepared by the solution of silver in hydriodic acid was found to have a specific gravity of 5·812. Several specimens of precipitated iodide, fused, gave 5·681. DEVILLE found 5·687, and in the case of the unfused precipitated iodide 5·807. DAMOUR found the native crystals to have a specific gravity of 5·667.

When a mass of iodide passes from the amorphous into the crystalline condition, the molecular commotion is so considerable that portions of the mass are sometimes jerked off from the ends of the bar, and large fissures a millimetre broad and several centimetres in length appear in the rod. Within these, crystals are sometimes seen projecting from the sides of the fissure. Under the microscope they present the appearance shown in fig. 6.

Fig. 6.

The expanding force is considerable. Attempts to burst an iron bottle did not succeed, but a thick porcelain tube was violently broken.

An attempt to determine with accuracy the relative conductivities, for heat, of iodide, bromide, and chloride of silver was not successful; but the result showed that the bromide conducts somewhat better than the chloride, and more than twice as well as the iodide.

We have seen above that iodide of silver passes at between 156°·5 C. and 142° C. from the transparent amorphous variety to the opaque crystalline state. WERNICKE (Pogg. Ann., cxliii., p. 560) mentions that prisms of fused iodide when cooled to 138° C. exhibit a sudden alteration of colour and transparency; and he considers that at a high temperature "part of the iodine is separated from its combination with the silver, and is absorbed by the remaining substance in the liquid state, for the spectrum, like that of solid and liquid iodine, contains no blue or violet light. In the normal state, below 138° C., silver iodide gives a spectrum less bright but twice as long, and particularly developed in the blue-violet portion." The supposed redistribution of the iodine at a high temperature was not borne out by any of our results.

Many experiments were made in order to decide between the very contradictory statements in regard to the action of light upon the iodide of silver, with the general result that the pure iodide is quite unaffected. Even when the iodide was precipitated in a cell at the apex of rays of the electric light concentrated by a large lens, it did not darken.

Determination of the coefficients of expansion and contraction.—FIZEAU "by a method depending on the accuracy with which extremely minute movements can be appreciated, by observing the changes they produce in a system of NEWTON's rings," determined the coefficient of cubic contraction of iodide of silver between −10° C. and +70° C., and found it to be ·00000417 for 1° C. This possession of what FIZEAU terms a "negative coefficient of expansion" is the more remarkable when we remember that the chlorides, bromides, and iodides of potassium, sodium, and ammonium, and the chloride and bromide of silver, expand considerably when heated, more so, indeed, than the most expansible metals, such as lead, tin, and zinc. The contraction of the iodide of silver is, according to FIZEAU, quite regular between −10° C. (14° F.) and +70° C. (158° F.); and he calculates that the contraction is equal to about $\frac{1}{7000}$ of its volume at 0° C. for 100° C., or, again, equal to one-sixth the expansion of platinum for 100° C. He also found that a large hexagonal crystal exhibited a very considerable contraction in the direction of the axis of symmetry, while a slight expansion was produced in a direction normal to the axis of the crystal.* The contraction was observed in the case both of the crystal, a confused crystalline mass, and an amorphous mass produced by strongly compressing the precipitated iodide until it became a hard mass capable of receiving a fine polish, and possessing a specific gravity of 5·569. FIZEAU considers

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* "Sur le propriété qui possède l'iodure d'argent de se contracter par la chaleur et de se dilater par le froid." Comptes Rendus, 1867. (Tom. lxiv. p. 314.)

that the iodide possesses its maximum of volume or minimum of density at a temperature of −60° C. (−76° F.).

M. Fizeau's determination was adopted for the range of temperature between −10° C. and +70° C.

For higher temperatures the iodide was examined in the expansion apparatus (p. 1128). Careful castings were made of the iodide in tubes of thin warm glass, 9 inches long by 0·3 inch diameter. The ends were sawn plane and carefully worked on fine sand-paper so as to produce a rod 6 inches in length. Copper caps were fitted on the ends of the bar, and it was supported in the cradle G (fig. 4) of the apparatus. By turning the micrometer-screw the index was forced up to 135° of arc. Water was used for heating the bars up to 90° C., and ceresine for higher temperatures. On heating the bar the index began to retreat very slowly, showing contraction, and this continued until a temperature of 142° C. was attained, when the contraction became more rapid; between 148° C. and 151°·3 C. it was very rapid; at 156°·5 C. it ceased altogether, and the needle was motionless, showing that although heat was being absorbed neither contraction nor expansion was taking place; and at 163° C. expansion commenced and continued to the melting point.

The contraction, as stated, commenced at 142° C. and terminated at 156°·5 C.; but it mainly took place between 148° C. and 151°·3 C. Thus in all for the 14°·5 C., during which the mass contracted, the index retreated through 18 revolutions of 135° to 0° of arc of the instrument, and these were related to the temperatures in the following manner:—

1	revolution of the index took place during heating from	142°	to	148° C.
14	,, ,, ,,	148°	,,	151°·3 C.
2	,, ,, ,,	151°·3	,,	153° C.
1	,, ,, ,,	153°	,,	156°·5 C.

Coefficients of cubical contraction and expansion for 1° C.

Between	0°	and	70° C.	=−·00000417
,,	70°	,,	142° C.	=−·00001749
,,	142°	,,	148° C.	=−·00016363
,,	148°	,,	151°·3 C.	=−·00420000
,,	151°·3	,,	153° C.	=−·00120000
,,	153°	,,	156°·5 C.	=−·00030000
,,	156°·5	,,	163° C.	= ·00000000
,,	163°	,,	527° C.	=+·00006921

Plus has been placed before the coefficient of expansion on heating, *minus* before the coefficients of contraction. The increase of volume in passing from the solid to the liquid state was determined by the method described on p. 1132.

If we take the volume at 0° C. as unity, the following volumes correspond to the temperatures given between 0° and the melting point:—

Volume at 0 °C.	=1·000000	Phase of contraction on heating. Expansion on cooling.
,, 124	= ·998765	
,, 133	= ·998608	
,, 142	= ·998450	
,, 148	= ·997469	
,, 151·3	= ·983609	
,, 153	= ·981560	
,, 156·5	= ·980510	
,, 163	= ·980510	
,, 200	= ·982377	Phase of expansion on heating. Contraction on cooling.
,, 300	= ·989298	
,, 400	= ·996219	
,, 500	=1·003140	
,, 527 (solid)	=1·005008	
,, 527 (liquid)	=1·040908	

Obviously the iodide possesses two points of similar density at different temperatures. Thus the density at 124° C. is the same as that at about 430° C.

If 5·675 be the specific gravity at 0° C., the specific gravity at the maximum density will be 5·771; at the minimum density 5·673, and in the molten condition at 527° C., the specific gravity will be 5·522.

4. Bromide of Silver.

Precipitated bromide of silver was fused, and cast in warm glass tubes. The mass when warm was found to be somewhat tenacious; the surface of the fused rod was smooth and brilliant, the fracture crystalline. Repeated fusion seemed to render the substance more crystalline. Although crystalline, the bromide was very compact, and altogether unlike the crystalline condition of the iodide; the rod contracted a good deal in cooling, and easily came out of the tube in which it was fused. Rods 6 inches long by ·3 inch diameter had their ends levelled by means of a fine steel saw, and were placed in the expansion-apparatus, and tested as described in the case of the iodide of silver bars. The coefficient of cubical expansion for 1° C. was found to be

·00010500.

Fizeau by his optical method found the coefficient to be ·000104061. The coefficient increases with the temperature, at least to the extent of ·000004 for each 100° C. increase of range. The expansion is very considerable in passing from the solid to the liquid condition.

The volumes corresponding to different temperatures and certain physical properties are tabulated with those of the chloride of silver (see p. 1138).

The bromide fuses to a reddish-brown liquid, not unlike bromine, and it solidifies to a bright yellow transparent solid when seen in thin layers. In thick layers it appears to be brownish-yellow. It is brittle even before it is quite cold. During the process of cooling loud cracking noises, like the cracking of a piece of porcelain or thick glass, are produced. The solidified solid produces a metallic ring when struck; but this is less marked than in the case of the chloride of silver.

5. Chloride of Silver.

Precipitated chloride of silver was fused and cast in warm glass tubes. It had been very carefully prepared in the dark, and the fused mass in thin layers was colourless and quite transparent. When warm, a rod of the chloride was very flexible, and to some extent malleable; when cold, the bar did not bend without fracture, and repeated fusings seemed to render it more liable to fracture. The rod contracted a good deal on cooling, and easily came out of the tube in which it was cast. Rods of 6 inches long by ·3 inch in diameter had their ends levelled by means of a fine steel saw, and were placed in the expansion-apparatus, and tested as described in the case of the iodide of silver bars. The coefficient of cubical expansion for 1° C. was found to be

·000095454.

Fizeau by his optical method found the coefficient to be ·000098814. The coefficient increases with the temperature, at least to the extent of ·000003 for each 100° C. increase of range. The expansion is considerable in passing from the solid to the liquid condition.

The volumes corresponding to the different temperatures and certain physical properties are tabulated below with those of the bromide of silver.

The chloride fuses to a dull light-red liquid, and becomes faintly yellow when solidified; in thin layers, colourless and quite transparent. When cold, it forms a pale greenish-yellow solid, also transparent in thin layers. No sounds were emitted during cooling. In thin layers the chloride is very flexible when cold; and thick rods of it are flexible while hot. It has a decided metallic ring if struck when cold.

	Chloride of silver.	Bromide of silver.
Fusing point	451° C.	427° C.
Specific gravity at 0° C.	5·505	6·245
,, ,, fusing point	4·919	5·595
Volume at −60° C.	1·000000	1·000000
,, −10° C.	..	..
,, 0° C.	1·005547	1·006060
,, 70° C.	..	..
,, 100° C.	1·015092	1·016560
,, 142° C.	..	..
,, 200° C.	1·024937	1·027460
,, 300° C.	1·035082	1·038760
,, 400° C.	1·045227	1·050460
,, fusing point (solid)	1·050319	1·053470
,, ,, ,, (liquid)	1·116427	1·122840
Physical structure, &c., of fused mass	Crystalline fracture; thin layers transparent and flexible; thick rods very flexible when hot.	Crystalline and brittle.

6. CHLOROBROMIODIDES OF SILVER, HAVING THE COMPOSITION :—AgI, Ag_2Br_2, Ag_2Cl_2; AgI, AgBr, AgCl; Ag_2I_2, AgBr, AgCl; Ag_3I_3, AgBr, AgCl; Ag_4I_4, AgBr, AgCl.

Compounds of silver with iodine, bromine, and chlorine are found in nature: *embolite* (ἐμβόλιον) is a chlorobromide of silver, and minerals having respectively the composition Ag_3BrCl_2, $Ag_5Br_2Cl_3$, Ag_4Br_3Cl, $Ag_9Br_4Cl_5$, and Ag_4BrCl_3, have been analysed by DOMEYKO, FIELD, MÜLLER, RICHTER and others. They occur chiefly in Chili, and constitute the principal ores of the silver mines of Chañarcillo. They possess specific gravities which vary from 5·75 to 6·2, and according to DANA the colour is "greyish-green, and asparagus-green to pistachio or yellowish-green, and yellow; often dark, becoming darker externally on exposure." Von LASAULX has described in the 'Jahresberichte für Mineralogie,' for 1878, a new silver haloid mineral having the composition Ag_2Br_2, Ag_2Cl_2, AgI which he cites as the first instance of the three haloid salts occurring crystallized together in nature. Two years previously I had prepared, for the purposes of this enquiry, a substance having the same composition, and its properties were described in Proc. Roy. Soc., No. 174, 1876. It was found by Von LASAULX, associated with BEAUDANTITE, in a mine in the district of Ems, Nassau, in the form of small yellow, or olive-green octohedra, never exceeding 3 millims. in size.

Five compounds of the halogens with silver were prepared, the first having the composition AgI, Ag_2Br_2, Ag_2Cl_2, and the others increasing by one molecule of AgI to Ag_4I_4, AgBr, AgCl. The substances were fused together in a porcelain crucible, and cast in glass tubes. They were examined in the expansion apparatus in the usual manner. The coefficients and more prominent physical characteristics are given in the following table. Generally speaking, of all the compounds it may be said that they fused to a bromine-red transparent liquid, becoming successively brick-red, dull orange, and yellow or yellowish-brown on cooling. They furnished bright yellow powders, turning green on exposure to light.

CHLOROBROMIODIDES of Silver.

	1. AgI, Ag_2Br_2, Ag_2Cl_2.	2. AgI, AgBr, AgCl.	3. Ag_2I_2. AgBr, AgCl.	4. Ag_3I_3, AgBr, AgCl.	5. Ag_4I_4, AgBr, AgCl.
Fusing point	383° C.	331° C.	326° C.	354° C.	380° C.
Specific gravity at 0° C.	6·152	6·1197	6·503	5·9717	5·907
,, ,, fusing point .	5·5118	5·5673	5·6971	5·6430	5·680
,, ,, 0° C. calculated on the assumption that no change of volume occurs.	5·836	5·8010	5·762	5·741	5·725
Volume at 0° C.	1·000000	1·000000	1·000000	1·000000	1·000000
,, 124° C.	1·015331*	1·012037	1·010301	1·007440	1·006696
,, 133° C.	1·015037	1·006637	·993201	·984041	·979696
,, of solid at fusing point .	1·054986	1·046646	1·032283	1·009612	1·006372
,, liquid at solidification point	1·112376	1·097486	1·059998	1·057645	1·040513
Certain physical properties . . .	Crystalline fracture; brittle both when hot and cold. Resembled bromide of silver.	Compact, hard, homogeneous; semi-crystalline fracture, very tenacious when cold; bends slightly at 250° C., but breaks easily.	Compact, hard, homogeneous, very tenacious when cold; at 250° C. flexible enough to be bent through an angle of 40° without breaking.	Brittle when cold; at 250° C. flexible enough to be bent through a right angle.	Crystalline fracture, more brittle when cold than any of the preceding, except No. 1. Very flexible at 250° C., and capable of being twisted without fracture.
Composition in 100 parts:—					
Iodide of silver	26·1692	41·484	58·6404	68·0178	73·9285
Bromide of silver	41·8708	33·186	23·4557	18·1379	14·7856
Chloride ,, ...	31·9600	25·330	17·9039	13·8450	11·2859

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* In the case of No. 1, the temperatures at which contraction began and finished respectively were 125·5° and 131·5°, not, as in the case of the others, 124° and 133° C.

Several facts require to be noted in regard to these compounds.

(1) No. 1, containing 26 per cent. of AgI is almost unaffected by it, and closely resembles bromide of silver, save that a very slight contraction occurs on heating between 125°·5 and 131°·5 C., and the orange coloured powder produced by pulverisation turns green on exposure to light. In No. 3, which contains a little more than half its weight of iodide of silver, we find the greatest divergence in every respect from the properties of its constituents, although the influence of the AgI is very marked both in the plasticity of the substance above 250° C., and in the considerable contraction which takes place between 124° C. and 133° C. It possesses the lowest melting point (viz. : 326° C., which is 101° C. lower than that of AgBr, which constitutes nearly one-fourth of its weight, and 201° C. lower than that of AgI, which constitutes more than half its weight), and the highest specific gravity. In No. 5, the influence of the AgI is most marked, but the melting point is still 147° C. lower than that of the AgI, which constitutes nearly three-fourths of its weight.

(2) We may note that the fusing points diminish from 1 to 3, and increase from 3 to 5 ; while the specific gravities diminish on both sides of No. 3.

(3) While iodide of silver commences its considerable contraction consequent upon change of state at 142° C., and finishes it at 156°·5 C. (mainly, however, between 148° and 151°·3 C.), the chlorobromiodides all commence their contraction at 124° C., viz. : 18° C. lower, and finish at 133° C., or 23°·5 C. lower. The lead-silver iodide (p. 1149) practically does the same thing. This matter is alluded to in greater detail in the general discussion of results (pp. 1162, 1163).

(4) It is a curious fact that until the percentage of iodide of silver in the compounds becomes considerable, its chief influence is exerted between a narrow range of temperature ; and more than this, so soon as the contraction is over the mass undergoes far more rapid expansion than any of its constituents when heated through the same range of temperature. The condition is obviously one of great complexity. Take the case, for example, of the chlorobromiodide No. 3. Between 0° C. and 124° C. any mass of 100 molecules consists of 58 which are undergoing slight contraction on heating, while 42 are undergoing rapid expansion. Then between 124° C. and 133° C. 58 molecules are rapidly contracting ; heat is disappearing in internal work and is changing the crystalline into the amorphous iodide, *i.e.*, converting an opaque, brittle, highly crystalline body into a transparent, plastic, denser body ; while the 42 molecules of chloride and bromide are still expanding. Finally, from 133° C. to the fusing point, all the molecules are expanding, 42 of them quickly, and 58 of them slowly.

7. DI-IODIDE OF MERCURY, HgI_2.

This substance is dimorphous. In the amorphous condition it presents the appearance of a brilliant scarlet powder, which, if heated, fuses at 200° C. and volatilises just above the fusing point to a vapour more than twice as dense as that of mercury.

The vapour condenses to rhombic prismatic crystals, which frequently become scarlet while cooling, but which, if they still remain yellow when cold, instantly become scarlet if rubbed or otherwise mechanically agitated.

On heating a mass of the crimson amorphous iodide it turns yellow at 126° C., and just before the melting point is attained, the yellow changes to a deep red-brown. The liquid resulting from the fusion has the appearance of liquid iodide of silver—that is to say, it has the exact colour of bromine. The liquid when cooled solidifies to a red-brown solid, which speedily becomes yellow, and at 126° C. it changes to the crimson octohedral variety. Distinct cracking sounds, due to inter-molecular movements, were heard during the continuance of the change. Heat is absorbed when the red iodide changes to yellow, and is given out when the yellow iodide changes to the red.

A bar of the iodide was placed in the expansion apparatus, melted paraffine was poured upon it, and when the index had become steady a gentle heat was applied to the paraffine. The index showed a regular and slow expansion until a temperature of 126° C. was reached, when the bar began to change from the octohedral to the prismatic condition, and without further rise of temperature rapid expansion took place. The temperature was kept constant until the change was complete, and was then slowly raised. A regular expansion now took place under a higher coefficient than before the molecular change, and this continued until the melting point was attained.

The coefficient of cubical expansion for 1° C. from 0° C. to the point of change—126° C.—was found to be—

·0000344706.

At 126° C., during the change from the red octohedral to the yellow prismatic condition, the body increased in bulk to the extent of—

·00720407.

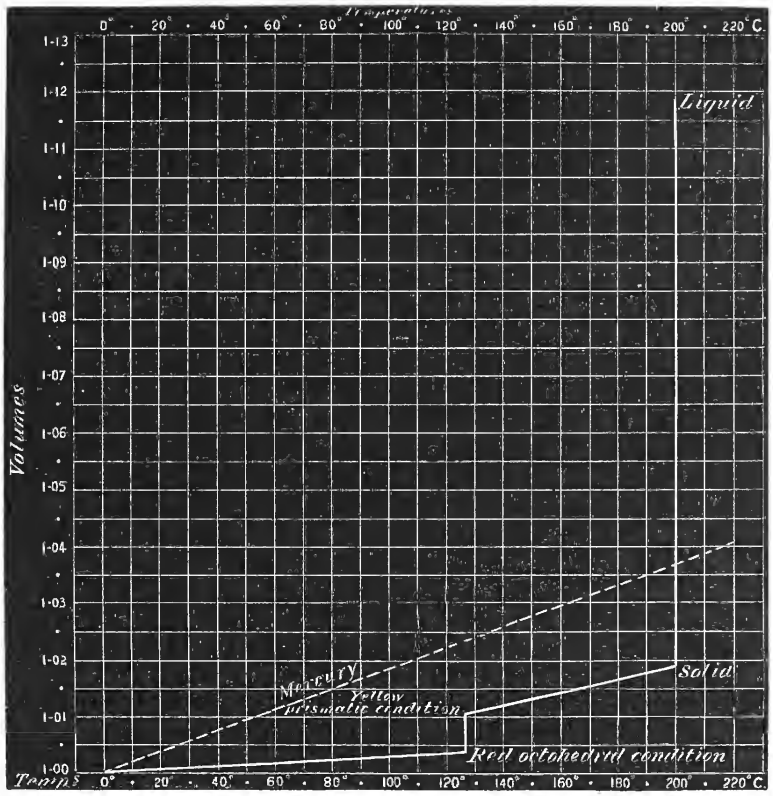
The coefficient of cubical expansion for 1° C. from 126° C., after the change to the melting point, 200° C., was—

·0001002953.

Thus, if we suppose a molten mass of the iodide of mercury to be cooling down from 200° C. to 0° C., the following would be the volumes under the conditions indicated:—

Volume at 200° C. of the liquid mass . . . . =1·1191147
,, ,, ,, solid mass . . . . =1·0190453
,, 126° C. (yellow prismatic condition). . . . . . =1·0115378
,, ,, (red octohedral condition) . . . . . =1·0043337
,, 0° C. . . . . =1·0000000

The changes are shown at one view in the accompanying curve table, in which the expansion of mercury is given for comparison.



According to Schiff the specific gravity of the octohedral iodide is 5·91, while Karsten makes it 6·2009, and Boullay 6·320.

Two distinct specimens with which we worked gave respectively—

(1) 6·3004
(2) 6·2941.

The specific gravity of the fused iodide was found by the method before described to be—

5·2865.

Thus the specific gravities corresponding to the five marked conditions shown in the curve table are as follows:—

Specific gravity at	0° C.	=6·297
,, ,,	126° C. (octohedral condition)	=6·276
,, ,,	,, (prismatic condition)	=6·225
,, ,,	200° C. solid	=6·179
,, ,,	,, liquid	=5·286

In the above experiments on the di-iodide of mercury I have to acknowledge the assistance of Mr. H. M. Elder, at that time a pupil in Marlborough College.

8. Di-iodide of Lead, PbI_2.

Pure iodide of lead was cast into rods one-third of an inch in diameter and 6 inches in length. The ends were made plane by a fine steel saw, and they were furnished with copper caps. The iodide underwent the same changes of colour as were observed in the iodide of silver; that is to say, it fused to a bromine-red liquid, which, when solidified, became red-brown, and, while cooling, brick-red, reddish-yellow, and, when completely cool, orange-yellow. Harsh noises, like those produced by bending tin, were heard during the cooling of the mass, and the fracture was highly crystalline.

Differences of opinion exist as to the effect of fusing iodide of lead in the air. Experiments showed that it may be fused out of contact with air, with scarcely appreciable loss. When, however, it was fused in contact with air, rapid decomposition ensued.

When the iodide was heated in a current of carbonic anhydride, it sublimed unchanged in crystals; while if it was heated in a current of dry oxygen it rapidly decomposed, fine crystals of iodine collecting in the fore part of the tube.

The specific gravity of iodide of lead, in common with the iodides of copper, silver, and potassium, is less than the mean specific gravity of its constituents. Karsten found it to be 6·0282, Boullay 6·11, and my own determinations gave 6·12. The calculated specific gravity is 6·629.

The fusing point as determined by Mr. Carnelley is 383° C.

The coefficient of cubical expansion for 1° C. was found to be

$$\cdot 00007614$$

for temperatures between 0° C. and 205° C. It increased to

$$\cdot 00008317$$

between 205° C. and 253° C.

Between 253° C. and 265° C. the mass expanded rapidly, with a coefficient nearly eight times greater than the previous, viz.:—

$$\cdot 0006378.$$

After the subsidence of this rapid expansion it no longer retained the original coefficient, but assumed one of more than double the amount, viz.:—

$$\cdot 000180.$$

At temperatures some distance from the melting point the rod began to bend, and it became necessary to assume that this last coefficient continues to the melting point. The expansion in passing from the solid to the liquid condition was determined by the method previously described.

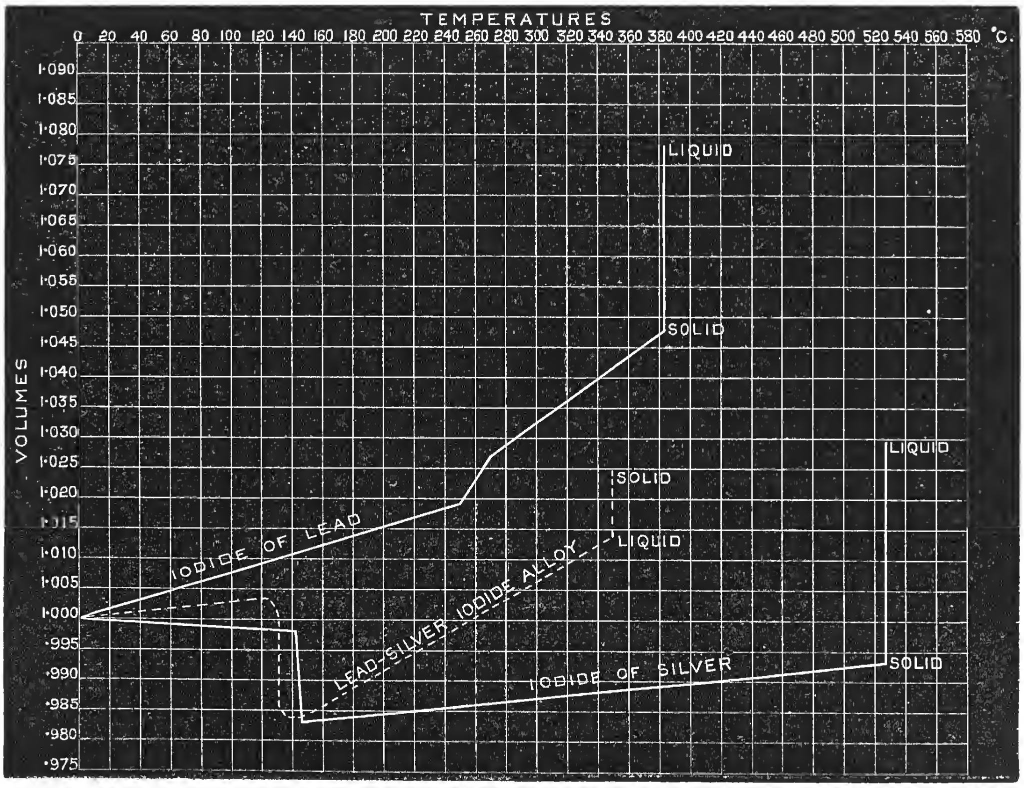
It will be observed that the iodide of lead, as in the case of the iodide of mercury (*vide* p. 1142), has three coefficients of expansion, viz.:—(*a*) a coefficient somewhat less than that of chloride of silver up to 253° C.; (*b*) a coefficient during 12° C., nearly eight times greater than the preceding; and (*c*) finally a coefficient somewhat more than twice as great as that between 0° and 253° C., at temperatures above 265° C. Undoubtedly the iodide of lead, as in the case of the iodide of mercury, undergoes a molecular change, while rapidly expanding between 253° C. and 265° C., and before assuming the higher coefficient. This is supported by the fact that the highly brittle and crystalline rod showed itself capable of bending after having undergone the rapid expansion. It will be remembered that the iodide of silver, which is very crystalline and brittle below 145° C., becomes amorphous and plastic above that temperature. The familiar example of sulphur will also recur to the mind.

If we suppose a mass of iodide of lead to be heated from 0° C. to the melting point (383° C.) the following will be the volumes at the respective temperatures:—

Volume at 0° C. . . . . =1·000000
,, 205 . . . . =1·015608
,, 253 . . . . =1·019595
,, 265 . . . . =1·027248
,, 383 (solid) . . . . =1·048488
,, 383 (liquid) . . . . =1·078080

The curve is shown in Table A.

Table A.—Table showing the relationship between the temperature and volume of iodide of lead, iodide of silver, and of the lead-silver iodide alloy, $PbI_2.AgI$.



The specific gravity of the iodide in the molten condition is 5·6247.

The fact that a substance may possess two or three different coefficients of expansion has apparently only been observed hitherto in the case of such substances as fusible alloy, because in determining the coefficients of solid bodies temperatures exceeding 100° C. have rarely been employed. Paraffine or ceresine if used as a heating medium will allow the determination of coefficients to a temperature of 340° C., and, undoubtedly, many bodies if thus examined would be found to present anomalies similar to those remarked in the case of the iodides of lead and mercury.

9. The Lead-Silver Iodide.

Bearing in mind the peculiar nature of the coefficients of the chlorobromiodides of silver (*vide* p. 1140), it was thought to be advisable to determine the coefficients of a lead-silver iodide.

These bodies were accordingly fused together in the proportion of one molecule of each, viz.: $PbI_2.AgI$. This contains in 100 parts

Iodide of lead	=66·206	Lead	=29·7449
Iodide of silver	=33·794	Silver	=15·5642
		Iodine	=54·6909
	100·000		100·0000

The substances were fused together in a porcelain crucible, and cast in thin glass tubes 9 inches long by one-third of an inch in diameter. The molten mass underwent the same changes of colour in cooling as either one of its constituents, and ultimately became a dull orange-coloured compact mass. Although composed of two substances which are highly crystalline and brittle, the alloy was found to be hard and tenacious. Although the constituents are coarsely crystalline in structure, the alloy is finely granular. During the cooling of the mass it expanded with sufficient force to break the glass tube. Harsh noises were emitted during cooling, and the whole mass was sometimes jerked from its position; while, if held in the hand, it was felt to be agitated by strong tremors.

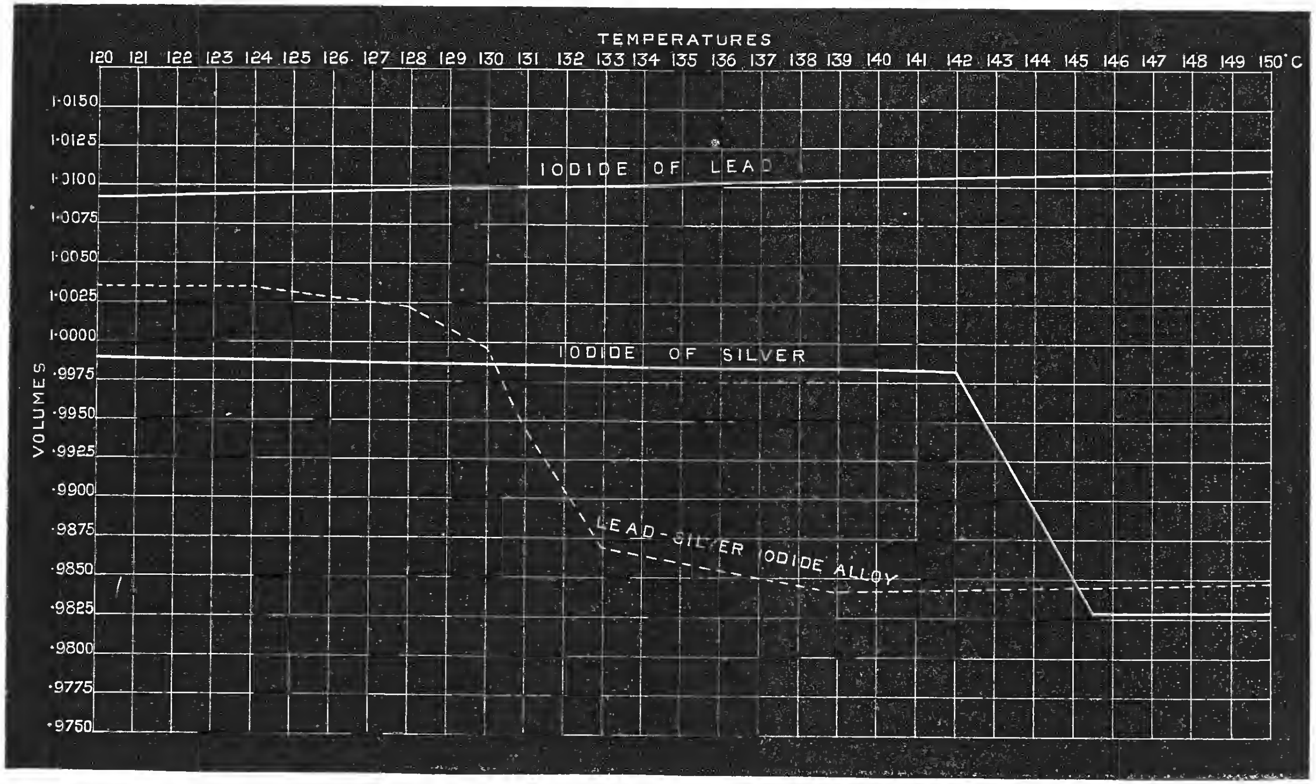
Mr. T. Carnelley has determined for me the melting point of the alloy, which he finds to be 350° C.

The specific gravity is 5·923.

By repeated digestion with large volumes of boiling water the alloy is decomposed, the iodide of lead being dissolved, while the iodide of silver remains as a dull green powder.

On examination in the expansion apparatus the alloy was found to undergo slow expansion to a temperature of 118° C., then, for 6° C., it simply absorbed heat without either contracting or expanding. At 124° C. contraction commenced, and continued

TABLE B.—Details of the contraction by heat of iodide of silver, and of the lead-silver iodide alloy.



at unequal rates till a temperature of 139° C. was attained. Again, the mass underwent neither contraction nor expansion during heating through 5° C., and then it commenced to expand somewhat rapidly. The most rapid contraction on heating took place between 130° and 133° C. Thus, in all, for the temperatures during which the mass contracted, the index moved through 15 revolutions of 180° to 0° of arc, and these were related to the temperatures in the following manner:—

1 revolution of index took place during heating from	124° to 128° C.
2	,, ,, ,,	128° ,, 130° C.
4	,, ,, ,,	130° ,, 131° C.
6	,, ,, ,,	131° ,, 133° C.
2	,, ,, ,,	133° ,, 139° C.

The details of these contractions are shown in Table B.

The heating, especially at these temperatures, was excessively slow, and so moderated that a complete observation of the behaviour of the substance in the expansion apparatus lasted from three to four hours. Above 144° C. the alloy expanded with a coefficient about three times greater than that which it possessed betweed 0° and 118° C.

COEFFICIENTS of cubical expansion and contraction of the alloy for 1° C.

Between	0°	and	118° C.	= +·0000306
,,	124°	,,	128° C.	= −·0003240
,,	128°	,,	130° C.	= −·0012990
,,	130°	,,	131° C.	= −·0017330
,,	131°	,,	133° C.	= −·0039000
,,	133°	,,	139° C.	= −·0004329
,,	144°	,,	350° C.	= +·0001150

Plus has been placed before the coefficients of expansion on heating; *minus* before the coefficients of contraction. The expansion in passing from the solid to the liquid condition was determined as before.

The curve is shown in Table A (p. 1145).

The coefficient between 144° C. and the fusing point increased rapidly with the temperature.

If we take the volume at 0° C. as unity, we have the following volumes corresponding to the temperatures given:—

Volume at 0° C. . . . . =1·000000
" 118 . . . . =1·003610
" 124 . . . . =1·003610
" 128 . . . . =1·002314
" 130 . . . . = ·999716
" 131 . . . . = ·994517
" 133 . . . . = ·986717
" 139 . . . . = ·984120
" 144 . . . . = ·984120
" 150 . . . . = ·984810
" 300 . . . . =1·006500
" 350 (solid) . . . . =1·013790
" 350 (liquid) . . . . =1·024370

In regard to this alloy the following points may be noted :—

1. It possesses a similar density at three different temperatures. Thus, it is obvious that the density is the same at 0° C., at just below 130° C., and at 282° C.

2. Although the alloy contains only 33·794 per cent. of iodide of silver, it contracts as considerably *during heating* as the iodide itself.

3. While the iodide of silver commences its contraction at 142° C., and finishes it at 145°·5, the alloy commences to contract 18° C. lower (viz.: at 124° C.), and finishes 6°·5 C. lower (viz., at 139° F.).

4. The chlorobromiodides of silver also began to contract on heating (an effect which, of course, we must attribute solely to the presence of iodide of silver) at 124° C., but they finished at 133° C.

5. The harsh sounds emitted by the alloy during cooling, and the tremors simultaneously propagated through the mass, prove that violent molecular agitation is going on, at such time as the iodide of silver is passing from the amorphous plastic condition, to the brittle crystalline condition, within the mass of the iodide of lead.

6. The fusing point of the alloy is 177° C. lower than that of the iodide of silver, which constitutes one-third of its weight, and 33° C. lower than that of the iodide of lead, which constitutes two-thirds of its weight.

7. If the lowering of the fusing point (also markedly apparent in the case of the chlorobromiodides of silver) is due to the fact that similar particles of matter attract each other more powerfully than dissimilar, and hence, when the particles of two bodies are mutually diffused, the attraction becomes less, and the molecular motion is consequently more readily assimilated, the same cause may serve to explain the commencement of the phase of contraction on heating the alloy, at a temperature 18° C. lower than the substance to which it owes this property.

8. It is interesting to compare one of the chlorobromiodides of silver with the lead-silver iodide. For this purpose we will take the chlorobromiodide which contains the nearest approach to the same quantity of iodide of silver as the alloy. The second of the chlorobromiodides before described (p. 1140) contains 41·484 per cent. of iodide of

silver, and 58·5160 per cent. of the chloride and bromide of silver, which latter, from the heat point of view, may be regarded as the same substance, because their coefficients of expansion are practically the same. It may be noted (*vide* below) that while the expansion of the bromide (which is slightly greater than that of the chloride) scarcely exceeds that of the iodide of lead, and while, moreover, the chlorobromiodide contains 8 per cent. more iodide of silver than the lead-silver iodide, the amount of *contraction* by heat of the latter is nearly four times greater than that of the former, although we must believe this effect to be solely due to the iodide of silver in each case.

Comparison of the coefficients of the iodide of lead and the bromide of silver, used in conjunction with iodide of silver in the formation of the two compounds given below.

	Iodide of lead.	Bromide of silver.
Melting point	383° C.	427° C.
Volume at 0° C.	1·000000	1·000000
,, 205	1·015608	1·021945
,, 253	1·019594	1·027369
,, 265	1·027248	1·028725
,, 383 (solid)	1·048488	1·042531
,, 383 (liquid)	1·078080	..
,, 427 (solid)	..	1·047855
,, 427 (liquid)	..	1·107225

Comparison of the coefficients of the lead-silver iodide alloy with those of a chlorobromiodide of silver, and of iodide of silver.

	Lead-silver iodide alloy, containing 33·794 per cent. of iodide of silver.	Chlorobromiodide of silver, containing 41·484 per cent. of iodide of silver.	Iodide of silver.
Composition	$PbI_2.AgI$.	AgI,AgBr,AgCl.	AgI.
Volume at 0° C.	1·000000	1·000000	1·000000
,, 118	1·0036108	..	..
,, 124	1·0036108	1·012037	·998765
,, 128	1·002314	..	..
,, 130	·999716	..	..
,, 131	·994517	..	..
,, 133	·986717	1·006637	·998608
,, 139	·984120	..	·998503
,, 142	..	..	·998450
,, 144	·984120	..	..
,, 144·5	..	..	..
,, 150	·984810	..	..
,, 156·5	..	..	·980510
,, 300	1·002060	1·039239	·989298
Volume of solid at melting point	1·013790	1·046646	1·005008
,, liquid at solidification point	1·024370	1·097486	1·040908
Melting point	350° C.	331° C.	527° C.

The Tables (A, p. 1145, B, p. 1147) show the curves of contraction and expansion of the lead-silver iodide alloy, and of its constituents. In Table B the scale has been enlarged in order to show the details of the contraction of the alloy on being heated from 124° C. to 139° C.

Other alloys of iodide of lead with iodide of silver were made, having the following composition:—

(1)	$2AgI.PbI_2$,	containing	50·517	per cent.	of iodide of silver.
(2)	$3AgI.2PbI_2$	,,	43·360	,,	,,
(3)	$4AgI.3PbI_2$	,,	40·497	,,	,,
(4)	$5AgI.4PbI_2$	,,	38·950	,,	,,
(5)	$10AgI.9PbI_2$	,,	36·190	,,	,,

They all possessed the same general appearance as the alloy $AgI.PbI_2$ described above, which contains 33·794 per cent. of iodide of silver. But with the exception of No. 5 they were so brittle that they could not be cast into rods suitable for use with the expansion apparatus. During cooling large rifts appeared in the rod at right angles to its length, at the time when the iodide of silver commenced to expand. In the case of No. 1 the rod was violently broken during its cooling by the expanding iodide of silver; even when slowly annealed in hot paraffine. It may be noted that no such effect was produced in the case of the chlorobromiodides of silver, having the composition respectively: $Ag_2I_2.AgBr.AgCl$; $Ag_3I_3.AgBr.AgCl$; $Ag_4I_4.AgBr.AgCl$; and containing in each case a larger percentage of iodide of silver (viz.: 58·6404; 68·0171; and 73·9285) than the silver-lead iodide alloy No. 1. The chlorobromiodides, although, of course, their brittleness increased with the percentage of iodide of silver, formed less brittle rods than the iodide of silver, and than the first of the silver-lead iodide alloys, although the latter contains 23 per cent. less iodide of silver than the chlorobromiodide $Ag_4I_4.AgBr.AgCl$.

10. Iodide of Copper.

The cuprous iodide was prepared by the method of Soubeiran, in which an aqueous solution of two molecules of cupric sulphate and two molecules of ferrous sulphate is precipitated by potassic iodide, according to the equation

$$2(CuSO_4)+2(FeSO_4)+2KI=K_2SO_4+Fe_23SO_4+Cu_2I_2.$$

The precipitated iodide was washed with dilute sulphuric acid and with water, and was completely dried at 200° C. It then presented the appearance of a nearly white powder, not of a dead white colour like oxide of zinc, but white with a tinge of grey. On heating to a temperature considerably exceeding 200° C. it becomes pale canary-yellow, which just before its fusion changes to orange. When fused it forms a bromide-red liquid, which solidifies to a steel-grey crystalline mass. If the fused mass be powdered, the powder presents the same grey-white colour as the original precipitate. SOUBEIRAN asserts that cuprous iodide fuses to a brown mass, which yields a green powder, and I imagine from this that he did not entirely wash out all traces of ferrous and ferric sulphate from the precipate. In thin laminæ it is translucent, transmitting a greenish light. It fuses at 601° C. according to the determinations of Mr. THOMAS CARNELLEY. When heated in the air it begins to give off iodine soon after the temperature has passed 230° C., but it gives it off most copiously when fused in the air. Oxide of copper is left.

When heated in a current of dry carbonic anhydride the iodide gives off a small amount of iodine, and afterwards remains in tranquil fusion without undergoing decomposition. At a bright red heat it slowly volatilises, the sublimate condensing immediately above the molten mass which is producing it. When heated in a current of dry oxygen, iodine is freely evolved, and oxide of copper ultimately remains in the form of a fine crystalline powder.

A quantity of iodide weighing 9·0080 grms. was heated for many hours in an air-bath to a temperature of 190° C. It lost ·4551 per cent. On raising the temperature and maintaining it between 200° and 300° C. for several hours the loss increased to 3·3303. It was then ignited over a one-jet BUNSEN burner for 16 minutes, the cover being retained on the crucible. The total loss amounted to 4·6514. The crucible was now strongly heated over a four-jet BUNSEN burner; the mass of course freely fused, and evolved iodine although the crucible was covered. After heating for 10 minutes the loss due to this heating was 1·3987, and the total loss was equal to 6·0501 per cent. A further ignition of five minutes only raised the total loss to 6·3277. The ignition was then continued with the cover of the crucible removed. Iodine was freely evolved. After more than an hour's ignition no more iodine was evolved, and the weight became constant. The loss amounted to 40·4639 per cent. of the whole, leaving in the crucible 3·6450 grms. of cupric oxide in minute crystals. The calculated weight of the residue, on the supposition that all the Cu_2I_2 is converted into CuO, is 3·7590 grms. Hence there can be no doubt that the iodine is completely expelled and replaced by oxygen. Subsequent testing of the residue failed to reveal the presence of any iodine.

16·0130 grms. of the iodide only lost 0·6106 per cent. by heating for about 15 hours to a temperature of 230° C. 2·8050 grms. heated for some time to 190° C. lost ·2495 per cent.; at 231° C. it lost 1·2830; on raising the temperature to 260° C. for some hours the loss increased to 17·7540. Ignition for 20 minutes over a one-jet BUNSEN burner (the cover being kept on the crucible) raised the loss to 25·7932. The cover was then removed, and after strong ignition for more than an hour no more iodine was evolved, and 1·1703 grm. of oxide of copper remained in the crucible. The calculated amount is 1·1755.

The loss of weight on ignition is equal to the difference between the atomic weight of iodine and of oxygen, for the change is expressed by $Cu_2I_2+O_2=2CuO+I_2$. Hence for every 126·53 of iodine lost 16 of oxygen is gained. No definite oxyiodide seems to be formed midway, but merely a mixture of CuO and Cu_2I_2. Cuprous iodide cannot be fused without some loss of iodine and oxidation of the surface. This mixture of iodide and oxide forms a black scum on the surface of the molten iodide, which is less fusible than the iodide, and which effectually protects it from the air. In casting, the more fluid iodide easily runs from beneath this partially oxidised surface. Some of the latter was fused after running off the pure iodide from beneath, and its specific gravity was taken, and was found, as we should expect, to be lower than that of the iodide; for while the specific gravity of the latter is 5·6936, that of the partially oxidised surface was found to be only 5·6030.

The iodide was found to contract considerably when it solidified. A deep hollow cone sinks into the mass in the act of solidifying. The change of volume which occurs in the passage from the solid to the liquid state was determined by the method of the platinum cone before described.

Good rods of the iodide were cast by quickly fusing the powdered precipitate and pouring the fused iodide from beneath the semi-oxidised crust into warm glass tubes. The resulting cast rod was brittle and possessed a dark steel-grey colour, no doubt due to surface oxidation. The fracture was highly crystalline and the rods somewhat brittle. The rods were reduced by sawing to a length of 6 inches; the ends were rendered plane by rubbing on a level surface of sand-paper, and were furnished with copper caps. The expansion was determined in the apparatus described above (p. 1128). Unlike the iodides of lead and silver, the iodide of copper does not possess more than one coefficient of expansion, at least for temperatures below 300° C.

The coefficient of cubical expansion for 1° C. was found to be

·00007317.

It increases but slightly with the temperature.

If the cast rod is plunged into hot paraffine and allowed to cool very slowly the coefficient is somewhat lower. The determinations of the volume in the molten state were not satisfactory, owing to the high temperature at which the iodide fuses and the readiness with which it then superficially undergoes oxidation.

The following volumes correspond to the temperatures given :—

Volume at	0° C.	=1·000000
,,	100	=1·007317
,,	200	=1·014634
,,	300	=1·021951
,,	400	=1·029268
,,	500	=1·036585
,,	600	=1·043902
,,	601 (solid)	=1·043975
,,	601 (liquid)	=1·118401

11. Copper-Silver Iodides.

1. *Copper-silver iodide,* $Cu_2I_2.AgI$.

Iodide of silver and iodide of copper were fused together in the proportion of one molecule of each. The fusing point was obviously lower than that of either of the constituents; the fusion was tranquil, and the mass did not oxidise at the surface. The alloy contains in 100 parts:—

Iodide of copper . . .	=61·7767	Copper	=20·5515
Iodide of silver	=38·2233	Silver	=17·5430
		Iodine	=61·9055
	100·0000		100·0000

Specific gravity of the cast-rod =(1)5·6290, (2)5·6526. When fused in a tube, instead of being cast into it, the specific gravity was a little higher—

(1) 5·7302 (2) 5·7500.

The melting point of this, and of each of the other alloys, was determined for me by Mr. Carnelley. $Cu_2I_2.AgI$ melts at 514° C. Although containing 38 per cent. of iodide of silver the alloy contracted on cooling, and came easily out of the glass tube. It was brown in colour in the mass, but when powdered it was a brilliant yellow, unaffected by light. In thin layers it was yellow and perfectly transparent. Fracture resinous.

Rods one-third of an inch in diameter and six inches long were cast in hot glass tubes. The ends were sawn plane by a fine steel saw, and were afterwards worked smooth on sand-paper, and furnished with copper caps. They were examined in the expansion apparatus in the usual way.

The alloy expanded on heating under a definite coefficient of ·00004998 until a temperature of 223° C. was attained, when the coefficient began to diminish rapidly, and between 223° C. and 256° C. it fell to

·00001999.

Between 256° C. and 284° C. the mass simply absorbed heat without undergoing either contraction or expansion.

At 284° C. it began to contract, and between that temperature and 309° C. it underwent slight contraction under a negative coefficient of

·00003999.

At 309° C. the contraction ceased and the mass simply absorbed heat without either contracting or expanding, till a temperature of 319° C. had been attained, at which point rapid expansion set in under a coefficient of

·00016665

which is more than three times greater than the coefficient of expansion between 0° C. and 223° C.

The following volumes correspond to the temperatures given :—

Volume at	0° C.	=1·000000
,,	100	=1·004998
,,	200	=1·009996
,,	223	=1·011145
,,	256	=1·011804
,,	284	=1·011804
,,	309	=1·010805
,,	319	=1·010805
,,	400	=1·024303
,,	500	=1·040968
,,	melting point (514° C.) solid	=1·043301
,,	,, ,, (514 C.) liquid	=1·103307

The fact that the alloy does not commence to expand again after its phase of contraction till a temperature of 324° C. has been attained, necessitated the raising of the temperature to an unusual height. The ceresine employed for heating, ignites when air has free access to it at 300° C. Moreover, ordinary thermometers which contain a vacuum above the mercury cannot be heated above 330° C. as the mercury then commences to boil. But these difficulties were overcome, and the ceresine was even carried to a temperature of 340° C. by closely covering the experimental trough with a brass plate so as to shut out direct access of air, leaving room only for the thermometer and for the working of the levers. Great care was taken to screen the rest of the apparatus by bright metal plates from the effects of radiation. The thermometer difficulty was got over by leaving a little air in the tube above the mercury, by which means it could be heated to 350° C. with readiness, but, of course, it must be borne in mind that the readings at these high temperatures are less reliable than those at lower ranges. It was not found to be practicable to work above 340° C., as at such a temperature the ceresine is liable at any moment to ignite; moreover it fumes furiously, and begins to decompose. The dense acrid vapours are particularly irritating to the lungs and eyes. Nevertheless for temperatures up to 340° C., ceresine or a paraffine of high boiling point, is, I believe, the most convenient medium that we possess.

2. *Copper-silver iodide alloy*, $Cu_2I_2.2AgI$.

The percentage of iodide of silver was now augmented to 55 by the addition of another molecule..

$Cu_2I_2.2AgI$ contains :—

Iodide of copper	= 44·6934	Copper	= 14·8624
Iodide of silver	= 55·3066	Silver	= 25·3939
		Iodine	= 59·7437
	100·0000		100·0000

The alloy fused at 496° C.

Specific gravity =5·7225 to 5·7377.

It did not expand on solidifying. The fracture was resinous, and it was more brittle than the preceding. Annealing in paraffine did not diminish the brittleness. In the mass it was brown; in thin layers yellow and transparent; in powder a brilliant yellow. Unaffected by light.

The alloy expanded on heating from 0° C. to 221° C. with a cubical coefficient for 1° C. of

·00003750.

Between 221° C. and 233° C. it simply absorbed heat without either expanding or contracting.

At 233° C. it commenced to contract, with a negative coefficient of

·00010587.

This continued until a temperature of 298° C. had been attained, viz.: through 65° C., when expansion commenced with a coefficient of

·00009474 for 1° C.

and continued until the melting point had been attained.

The following volumes correspond to the temperatures given :—

Volume at	0° C.	=1·000000
,,	221	=1·008287
,,	233	=1·008287
,,	298	=1·001406
,,	300	=1·001595
,,	400	=1·011069
,,	melting point (496° C.) (solid)	=1·020164
,,	,, ,, (496° C.) (liquid)	=1·062958

3. *Copper-silver iodide alloy*, $Cu_2I_2.3AgI$.

The percentage of iodide of silver was further increased to 65 by the addition of another molecule.

$Cu_2I_2.3AgI$ contains :—

Iodide of copper	= 35·0116
Iodide of silver	= 64·9884
	100·0000

Copper	= 11·6558
Silver	= 29·8335
Iodine	= 58·5107
	100·0000

The alloy was more brittle than the preceding. Fracture, as before, resinous. Colour: in the mass, brown; in thin laminæ, yellow and transparent; in powder, a brilliant yellow. Unaffected by light.

Melting point =494° C.

Specific gravity 5·716.

The alloy expanded on heating from 0° C. to 194° C., then it simply absorbed heat without either expanding or contracting till it attained a temperature of 214° C. Rapid contraction now took place, and continued through 66° C., viz.: to 280° C. At this temperature the mass began to expand with the same coefficient as that possessed by the previous alloy (No. 2) above 298° C.

The coefficient of cubical expansion for 1° C. from 0° C. to 177° C. was

·00002307.

Between 177° C. and 194° C. the coefficient diminished to

·000012855.

Then the alloy simply absorbed heat without change of volume.

At 214° C. it commenced to contract under a negative coefficient of

·00017424.

This continued until a temperature of 280° C. had been reached, when expansion commenced under the coefficient of

·00009474 for 1° C.

This continued to the melting point.

The following volumes correspond to the temperatures given :—

Volume at	0° C.	=1·000000
,,	177	=1·004083
,,	194	=1·004301
,,	214	=1·004301
,,	280	= ·992902
,,	300	= ·994796
,,	400	=1·004270
,,	melting point (494° C.) (solid)	=1·013225
,,	,, ,, (494° C.) (liquid)	=1·081637

4. *Copper-silver iodide alloy*, $Cu_2I_2.4AgI$.

The percentage of silver iodide was further increased to 71 by the addition of a fourth molecule.

$Cu_2I_2.4AgI$ contains :—

Cu_2I_2	= 28·7775	Copper	= 9·5820
AgI	= 71·2225	Silver	= 32·6994
		Iodine	= 57·7186
	100·0000		100·0000

The alloy resembled the preceding. It was, however, slightly more brittle, and the fracture was more crystalline. The colour in the mass was brownish-yellow; in thin laminæ bright yellow; and in powder a brilliant yellow. Unaffected by light.

Melting point =493° C.

Specific gravity =5·7064.

The alloy expanded on heating from 0° C. to 159° C. under a coefficient of

·00001999.

Between 159° C. and 180° C. the coefficient fell to

·00001056.

From 180° C. to 199° C. it simply absorbed heat without undergoing either expansion or contraction.

At 199° C. contraction commenced, and continued under a negative coefficient of

·00007200

till 213° C., when it increased to

·0003798,

and so continued till 234° C., when it fell again to

·00007200,

under which coefficient it contracted till a temperature of 282° C. was attained.

Somewhat rapid expansion then set in, with a coefficient of

·0002050.

The following volumes correspond to the temperatures given :—

Volume at	0° C.	=1·000000
„	159	=1·003180
„	180	=1·003296
„	199	=1·003296
„	213	=1·002288
„	234	= ·994313
„	282	= ·990857
„	300	= ·994547
„	400	=1·015047
„	melting point (493° C.) (solid)	=1·034112
„	„ „ (493° C.) (liquid)	=1·065601

5. *Copper-silver iodide alloy*, $Cu_2I_2.12AgI$.

The percentage of silver iodide was finally increased to 88 by alloying one molecule of iodide of copper with twelve of iodide of silver. This extreme was employed in order to test the extent to which a small percentage of iodide of copper may mask for a while the prominent characteristics of the iodide of silver. None of the alloys hitherto prepared contain so large a percentage of iodide of silver. $Ag_4I_4.AgCl.AgBr$ contains 74 per cent.

$Cu_2I_2.12AgI$ contains :—

Cu_2I_2	= 11·8696	Copper	= 3·9535
AgI	= 88·1304	Silver	= 40·4770
		Iodine	= 55·5695
	100·0000		100·0000

The alloy possessed a yellowish-green colour in the mass, but thin layers of it were yellow and transparent. The powder was yellow: much brighter than that of iodide of silver, but not quite so brilliant as that of the other alloys. It turned slightly green on exposure to light. The fracture was crystalline; the grain being much finer than that of the iodide of silver. The alloy violently broke the tube in which it was cast by expansion during cooling, but no rifts appeared in the rod; nor was it nearly so brittle as iodide of silver.

Melting point =513° C.

Specific gravity =5·6950.

The alloy expanded on heating from 0° C. to 124° C. under a coefficient of

·00000636.

Between 124° C. and 153° C. it simply absorbed heat without undergoing either expansion or contraction.

At 153° C. contraction commenced, and continued under a negative coefficient of

·0000831

till a temperature of 168° C. had been attained, when it increased to

·00028899,

and so continued till the temperature had risen to 225° C., when expansion commenced under the coefficient

·00006666.

The following volumes correspond to the temperatures given :—

Volume at	0° C.	=1·000000
,,	124	=1·000788
,,	153	=1·000788
,,	168	= ·998985
,,	225	= ·982512
,,	300	= ·987511
,,	400	= ·994177
,,	500	=1·000843
,,	502 (solid)	=1·000976
,,	502 (liquid)	=1·042612

DISCUSSION OF THE RESULTS.

1. The copper-silver iodide alloys described in the foregoing pages vary in regard to the quantity of iodide of silver which they possess to the extent of 50 per cent.; the lowest, $Cu_2I_2.AgI$, containing 38 per cent. of iodide of silver, and the highest, $Cu_2I_2.12AgI$, 88 per cent.

2. It will be noticed, however, that the percentage of iodine varies but little, for while $Cu_2I_2.AgI$ contains 61·9055 per cent., $Cu_2I_2.12AgI$ contains 55·5695.

3. The specific gravity of the alloys varies slightly, and is a little above the mean specific gravity of their constituents. This was also observed in the case of the five chlorobromiodides of silver previously described (p. 1140), and in that of the lead-silver iodide alloy (p. 1146), and while the specific gravity of Cu_2I_2 is 5·6936, and of AgI 5·6750, that of the alloys is as follows:—

Cu_2I_2. AgI	5·7302
Cu_2I_2. 2AgI	5·7225
Cu_2I_2. 3AgI	5·7160
Cu_2I_2. 4AgI	5·7064
Cu_2I_2.12AgI	5·6950

4. The melting points do not differ much from each other, in fact those of $Cu_2I_2.2AgI$, $Cu_2I_2.3AgI$, $Cu_2I_2.4AgI$, and $Cu_2I_2.12AgI$, are practically the same; but they are considerably lower than those of their constituents: *i.e.*, more than 100° C. lower than that of iodide of copper, and more than 30° C. lower than that of iodide of silver. This was also noticed in the case of the chlorobromiodides of silver, and in that of the lead-silver iodide alloy.

5. While both the iodide of copper and the iodide of silver are highly crystalline in structure, even in thin layers, the copper-silver iodides are resinous in fracture and transparent in thin layers, even when they contain as much as 65 per cent. of iodide of silver. With greater percentages the structure becomes slightly crystalline.

6. It is obvious that some of the alloys possess three points of similar density; and others two, at different temperatures. Thus, in the case of $Cu_2I_2.2AgI$, the density is the same at 133° C., 265° C., and 334° C.

7. When strongly heated in a current of carbonic anhydride the alloys remained in tranquil fusion, undergoing an extremely slow volatilization. Heated in a current of dry oxygen, iodine was freely evolved, and oxide of copper appeared on the surface of the mass. When heated in dry hydrogen, hydriodic acid was formed, and the iodides were reduced.

8. The general colour of the alloys was in the mass from brown-yellow to green-yellow, as the percentage of iodide of silver was increased. In thin laminæ bright yellow, and in a powder a brilliant dark yellow, becoming slightly lighter as the percentage of iodide of silver was increased.

9. The coefficient of expansion of the alloys below the point at which contraction on heating commences, was found to decrease as the percentage of iodide of silver was augmented :—

Percentage of AgI.	Coefficient of expansion.
38·2232	·00004998
55·3066	·00003750
64·9884	·00002307
71·2225	·00001998
88·1304	·00000636

It was in all cases considerably less than that of iodide of copper.

The same fact was observed in the case of the chlorobromiodides of silver :—

Percentage of AgI.	Coefficient of expansion.
26·1692	·00012216
41·4840	·00009529
58·6404	·00008307
68·0171	·00006000
73·9285	·00005400

10. On the other hand, the expansion above the point at which contraction on heating finishes, was greater than that of either the iodide of silver or the iodide of copper.

11. MATTHIESSEN asserts that the coefficient of expansion of a metallic alloy is the mean of the coefficients of expansion of the several volumes of the metals composing it, and in the case of ordinary alloys this law undoubtedly holds good. But in the iodide of silver alloys the conditions are so complex that we can scarcely hope for the same result. Nor do we find it. We are dealing, on the one hand, with a substance which possesses several allotropic modifications; several distinct coefficients of contraction on heating, and one coefficient of expansion; and, on the other hand, with bodies, which like the iodides of lead and mercury also possess allotropic modifications, each with its own coefficient of expansion, or with a single coefficient of expansion and no allotropic modification like the iodide of copper.

12. MATTHIESSEN considers that in nearly all cases two-metal alloys may be regarded as solidified solutions of the one metal in the other, as glass is a solidified solution of different silicates, or gold and silver in the gold-silver alloys.

13. The lowering of the fusing point of alloys undoubtedly means that the cohesion of the particles in the alloy is less than that of the particles in its constituents; and

admitting the supposition that alloys are solidified solutions of one body in another (12), this lessening of cohesion is not difficult to explain. For it is well known that particles of matter of similar composition attract each other (*cohesion*) more readily, and with greater force, than particles of different composition (*adhesion*), and hence the homogeneous diffusion of one substance in another must make the molecular attraction less than that of either body by itself. Hence, molecular motion is more readily assimilated, and sooner produces any given effect, such as the fusion of the mass.

14. It was thought that the same explanation would apply to the curious fact that the five chlorobromiodides of silver, and the lead-silver iodide alloy, all commenced their period of contraction on heating at a lower temperature than the iodide of silver, to which substance alone can they owe their contraction on heating, but the results obtained with the copper-silver iodide alloys appear to discredit this.

15. For it is most noteworthy in this connexion to observe that while the iodide of silver commences to contract considerably on heating, that is to pass from its crystalline into its amorphous plastic condition, at 142° C., the five chlorobromiodides of silver, the percentage of iodide of silver in which varies from 26·1692 to 73·9285, and the lead-silver iodide alloy, the percentage of iodide of silver in which amounts to 33·794, all commence their contraction at 124° C., that is 18° C. lower, although the coefficients of expansion of the associated bodies necessarily differ. It would thus appear that 124° C. is the temperature at which iodide of silver commences its passage from the crystalline into the amorphous condition when freed from the attraction of its own molecules, provided no other attraction or influence supervenes; while the attraction exerted when it exists unalloyed with any other substance, and when its molecules are hence much nearer to each other, raises the point of commencement of change 18° C. higher, viz.: to 142° C.

16. The same result was looked for in the case of the copper-silver iodide alloys, but it was not found. The temperature at which contraction commences was in all cases raised, instead of being lowered, so that the presence of the iodide of copper hinders the assimilation of molecular motion by the molecules, although the coefficient of expansion of the iodide of copper is lower than that of either the chloride or bromide of silver, or of the iodide of lead which enter into the composition of the other alloys.

Percentage of iodide of copper in the silver-copper iodide alloys.	Temperature at which expansion on heating finishes.
61·7767	256° C.
44·6934	221° C.
35·0116	194° C.
28·7775	180° C.
11·8696	124° C.

But the contraction did not begin when the expansion ceased; the mass simply absorbed heat without undergoing change of volume through a varying range of degrees, which removes the point of contraction still further from that of the iodide of silver.

Thus, while in the case of the chlorobromiodide of silver containing only 26·1692 per cent. of iodide of silver, and of the lead-silver iodide alloy containing 33·794 per cent., the contraction commenced at 124° C., the following temperatures were those at which the contraction commenced in the case of the copper-silver iodide alloys containing in all cases more iodide of silver than either of the alloys mentioned above:—

Percentage of iodide of silver.	Temperature at which contraction on heating commenced.
38·2232	284° C.
55·3066	233° C.
64·9884	214° C.
71·2225	199° C.
88·1304	153° C.

Thus the effect of 61·7767 per cent. of iodide of copper was to raise the point at which the iodide of silver commences to pass from the crystalline to the amorphous condition 142° C., viz.: from 142° C. to 284° C.; and the influence of 28·7775 of iodide of copper was sufficient to raise the temperature of the point of change to 199° C., while the presence of 66·206 per cent. of iodide of lead, and of 74 per cent. of chloride and bromide of silver, not only did not raise the temperature at which contraction commences, but lowered it 18° C. When the percentage of iodide of silver in the copper-silver iodide alloys was very considerable the temperature at which contraction commenced fell to 153° C.

17. This difference in the commencement of the period of contraction on heating would lead us to look for differences of structure in the case of those alloys which begin to contract at 124° C., and those in which the contraction is masked and annulled until a higher temperature is attained. Such differences we have no difficulty in finding. If we compare the lead-silver iodide alloy, $PbI_2.AgI$, previously described (p. 1146), which contains 33·794 per cent. of iodide of silver, with the copper-silver iodide alloy, $Cu_2I_2.AgI$, which contains 38·2232 per cent. of iodide of silver, and which, hence, most nearly approaches it in composition, the differences are very conspicuous. $PbI_2.AgI$ is crystalline in structure, and in cooling it expands considerably and breaks the tube in which it is cast. Harsh noises are emitted during cooling, and strong tremors are propagated through the mass. The expansion of the alloy ceases at 118° C., contraction commences at 124° C., and continues to 139° C. rapidly, almost equalling that of iodide of silver itself; the mass decreasing in volume to ·984120. $Cu_2I_2.AgI$ is resinous in structure, and in cooling it contracts considerably, and comes out of the tube in which it is cast. No harsh sounds are emitted during cooling, and no tremors are propagated through the mass. The expansion of the alloy ceases at

256° C., contraction commences feebly and under a very low coefficient at 284° C., and continues till a temperature of 309° C. has been attained, when rapid expansion sets in. The difference in the volume due to this contraction is very slight: thus, volume at 284° C. = 1·011804, and volume at 309° C. when the contraction is finished = 1·010805.

18. The microscope, when applied to the examination of thin layers, also shows differences. $PbI_2.AgI$ in a thin layer is opaque owing to the presence of multitudes of small crystals. $Cu_2I_2.AgI$ is transparent. It would therefore appear that the presence of the Cu_2I_2 has prevented the iodide of silver from changing from the amorphous plastic condition to the crystalline condition: a change which, as above stated, is usually accomplished at 142° C.

19. The researches of petrologists in connexion with the examination of thin slices of minerals under the microscope, have shown that if rocks be fused and suddenly cooled no structure is developed, while, if slowly cooled, they not only become crystalline, but may be built up entirely of crystals. Thus basalt perfectly fused and suddenly cooled in water forms a black glass, which exhibits no structure under the highest power. Less rapid cooling develops *microliths* or *crystallites*, and as the cooling is still further retarded the microliths group themselves into gem-like forms, into radial forms about a centre, into small definite crystals, and into large definite crystals. Thus the glassy basalt or tachylyte, passes into ordinary basalt; and the ordinary basalt, into the highly crystallised gabbro. Also it is noted that the specific gravity increases as the crystalline structure is more and more developed. Thus glassy basalt has a lower specific gravity than basalt, and the latter than gabbro. We are also very familiar with the difference in the size of the crystals of any substance which separates out from solution if the evaporation be rapid or slow.

Now AgI above 142° C. corresponds to the glassy basalt.
,, below ,, ,, gabbro.
Or octohedral sulphur corresponds to the gabbro.
,, plastic ,, ,, ,, tachylyte.

The crystals of felspar, quartz, &c., doubtless separate out from solution in the ground mass of the mineral, as a soluble salt separates from its solution in water.

20. We have obviously to deal with—

(*a*) The crystalline forces which tend to build up the substance into crystals.

(*b*) The force of cohesion exerted by the crystals whether microliths or large crystals for each other.

(*c*) The attraction subsisting between the surrounding mass or medium and the crystals.

On the other hand we have the molecular motion tending to disunite them.

21. It would therefore appear that in the case of the lead-silver iodide alloy, $PbI_2.AgI$, the ground mass of PbI_2, with which the iodide of silver is surrounded, offers no opposition to the passage of the iodide from the plastic to the crystalline condition, but rather, by separating the crystals from the influence of their own attraction, promotes it. While, on the other hand, that in the case of the copper-silver iodide alloy the ground mass of Cu_2I_2, by its attraction or by other influence, prevents the passage of the iodide of silver from the plastic to the crystalline condition, or at least considerably hinders it, and allows it only to take place at a temperature far above the usual point, or when the percentage of iodide of silver is so considerable that the adverse influence of the Cu_2I_2 is overcome, as in the last alloy containing 88 per cent. of AgI.

22. If a small thin layer of sulphur be melted on a piece of mica and suddenly cooled in cold water, and placed under the microscope, it is seen to be perfectly transparent; gradually, as we watch it from day to day, crystals are seen forming themselves out of the ground mass, and these increase rather in number than in size, until the whole is converted into a crystalline opaque brittle mass. In this case molecular motion being, as we know, slowly lost, the crystalline forces become more and more able to build up crystals out of the ground mass. On the other hand, if a thin layer of iodide of silver be melted on a slip of mica, and placed, while in the plastic condition, under the microscope, it is seen to be transparent, but at the moment when the temperature sinks to 142° C. the mass suddenly crystallises all over. Between these two extremes we, of course, have instances in which the passage from the amorphous to the crystalline state takes place with greater or lesser readiness. If iodide of lead be fused on a piece of mica, and placed under the microscope, it is seen to crystallise rapidly, and it presents the appearance of crystalline plates with numerous veined markings (fig. 7). When alloyed with iodide of silver the mass is seen to be made up of a multitude of small tabular crystals (fig. 8), altogether different from either the iodide of lead, or the iodide of silver when similarly and separately treated. The copper-silver iodide alloys (except when the percentage of iodide of silver is very large) present the network appearance under the microscope shown in fig. 9. They are transparent moreover in thin layers, and translucent in thicker layers, while the chlorobromiodides, and the iodide of lead, and the lead-silver iodide alloy are opaque from the presence of multitudes of crystals.

23. It is conceivable that the iodide of lead possesses so strong a tendency to crystallise, and so little attraction for the molecules of iodide of silver, that it in no way interferes with the crystallisation of the latter; while the iodide of copper may possess but little tendency to crystallise itself and considerable attraction for the iodide of silver homogeneously diffused throughout. Or, in the one case we may have an actual chemical attraction of the molecule of the one iodide for that of the other;

while in the other case they may exist as a literal alloy—a solidified solution of the one substance in the other, like glass.

Fig. 7.

Fig. 8.

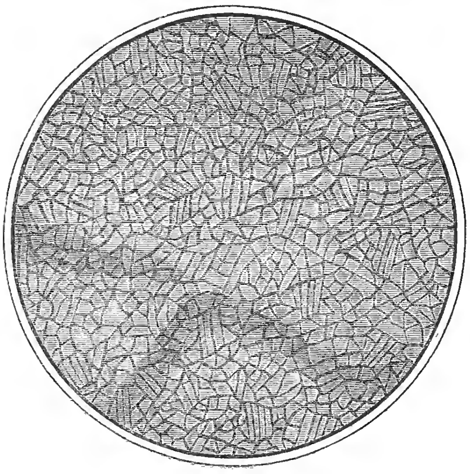
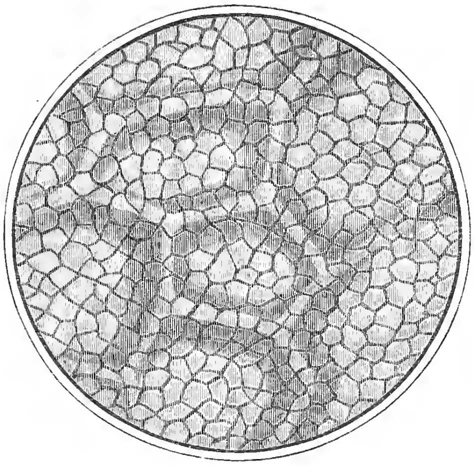


Fig. 9.



24. In the case of the alloy $Cu_2I_2.AgI$, which contains 61·7767 per cent. of iodide of copper, the effect of the 38·2233 per cent of iodide of silver has been—

(*a*) To diminish the coefficient of expansion below 256° C. from ·00007317 to ·00004998.

(*b*) To annul the expansion altogether between 256° C. and 284° C. and between 309° C. and 319° C.

(*c*) To produce contraction on heating between 284° C. and 309° C., viz.: through 25° C.

(*d*) To augment more than two-fold the coefficient of expansion above 319° C., viz.: from ·00007317 to ·00016665.

25. The effect of the addition of the second molecule of AgI, raising the percentage to 55·3066, has been—

(*a*) To diminish the coefficient of expansion below 221° C. to ·00003750.

(*b*) To annul the expansion between 221° C. and 233° C.

(*c*) To produce contraction on heating between 233° C. and 298° C., viz.: through 65° C.

(*d*) To augment slightly the coefficient of expansion above 298° C., viz.: from ·00007317 to ·00009474.

26. The addition of the third molecule of AgI in $Cu_2I_2.3AgI$ raises the percentage of iodide of silver to 64·9884, and the effect has been—

(*a*) To diminish the coefficient of expansion below 177° C. to ·00002307, and between 177° C. and 194° C. to ·00001285.

(*b*) To annul the expansion between 194° C. and 214° C.

(*c*) To produce contraction on heating between 214° C. and 280° C., viz.: through 66° C.

(*d*) To augment to the same extent as the preceding the coefficient of expansion above 280° C., viz.: from ·00007317 to ·00009474.

It will be noticed in the case of this alloy, which differs from the preceding one ($Cu_2I_2.2AgI$) by containing nearly 10 per cent. more of iodide of silver and nearly 10 per cent. less of iodide of copper—(1) that the contraction takes place through the same range of temperature (65° C.), beginning, however, 19° C. lower; (2) that the coefficient of expansion above the point at which contraction finishes is the same; (3) that the specific gravity and fusing point are practically the same. In fact, the almost sole effect of the additional 10 per cent. of iodide of silver has been to increase the coefficient of contraction between 214° C. and 280° C.

27. The addition of the fourth molecule of AgI in $Cu_2I_2.4AgI$ raises the percentage of AgI to 71·2225, and the effect has been—

(*a*) To diminish the coefficient of expansion below 159° C. to ·00001999, and between 159° C. and 180° C. to ·00001056.

(*b*) To annul the coefficient of expansion between 180° C. and 199° C.

(*c*) To produce contraction on heating between 199° C. and 282° C., viz.: through 83° C.

(*d*) To augment more than two-fold the coefficient of expansion above 282° C., viz.: from ·00007317 to ·0002050.

28. The addition of the twelfth molecule of AgI in $Cu_2I_2.12AgI$ raised the percentage of AgI to 88·1304, and, as might be expected, altered the character of the alloy from that of the preceding, approximating it more to the iodide of silver, and diminishing the influence of the iodide of copper. The effect has been—

(*a*) To diminish the coefficient of expansion below 124° C. to ·00000636.

(*b*) To annul the coefficient of expansion between 124° C. and 153° C.

(*c*) To produce contraction on heating between 153° C. and 225° C., viz.: through 72° C.

(*d*) To produce a coefficient of expansion above 225° C. almost identical with that of iodide of silver.

Plate 96 shows the effects of heat upon the copper-silver iodide alloys.

Professor M. BELLATI and Dr. R. ROMANESE, of the University of Padua, have lately determined the specific heat, and heat of transformation, of the iodide of silver and the copper-silver iodides above described, using for their determinations the same specimens as those with which I worked, and which were forwarded to Padua at their request. Their results are given in this volume (page 1169).

I beg to express my indebtedness to the Royal Society for several grants which it has allowed for the prosecution of these researches.

XXVII. *On the Specific Heat and Heat of Transformation of the Iodide of Silver,* AgI, *and of the Alloys, or Compounds,* $Cu_2I_2.AgI$; $Cu_2I_2.2AgI$; $Cu_2I_2.3AgI$; $Cu_2I_2.4AgI$; $Cu_2I_2.12AgI$; $PbI_2.AgI$.

By Professor MANFREDO BELLATI *and Dr.* R. ROMANESE, *of the University of Padua.*

Communicated by Professor A. W. WILLIAMSON, *Foreign Secretary R.S.*

Received June 7,—Read June 15, 1882.

IN a series of papers printed in the Proceedings of the Royal Society for the years 1877, 1879, 1881, Mr. G. F. RODWELL has investigated the expansion and contraction by heat of the silver iodide, AgI; of certain chlorobromiodides of silver; and of certain alloys of silver iodide, with lead iodide, and copper iodide. Analogous researches were made by us on the substances $HgI_2.2AgI$; $HgI_2.3AgI$; $HgI_2.Cu_2I_2$;* which, when heated to a certain temperature, change their colour. $HgI_2.2AgI$ and $HgI_2.3AgI$, at about 50° C., change from canary-yellow to red, and $HgI_2.Cu_2I_2$, at about 70° C., changes from red to a chocolate colour. For temperatures below and beyond the colour-change the co-efficient of expansion and the specific heat are regular, but during a range of a certain number of degrees of heat in which colour-change and corresponding modification of structure take place, the substances undergo a very notable expansion and absorb a great quantity of heat.

It seemed to Mr. RODWELL, and to us, that it would be interesting to make a calorimetric study of those substances whose expansion and contraction he had determined, and Mr. RODWELL having kindly furnished us with the specimens which he employed in his researches, we have briefly described in this paper the method and results of our calorimetric determinations.

During the operation of heating, the substance was contained in a vertical double cylinder of brass, 3 centims. in diameter and about 12 centims. long (fig. 1, p. 1176). The substance to be examined, made into small rods, was arranged round the reservoir of a thermometer, whose stem passed through a cork which closed the upper end of the brass cylinder. At the lower end this cylinder was closed by two small double doors which could be opened by a spring, and thus allowed the substance to fall out. The

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* M. BELLATI and R. ROMANESE. Atti del R. Istit. Veneto (1880), ser. v., vol. vi.; Nuovo Cimento (1880), ser. iii., vol. viii.

brass cylinder was introduced into another very long brass cylinder, heated externally and at the lower end by water or paraffine, and closed by a cork. The stem of the thermometer, passing through this cork, supported at a convenient height the inner cylinder.

This manner of heating the substance is, indeed, somewhat slow; but we adopted it because it is very easy to maintain the temperature constant for a long time, and because it allowed us to carry the substance, protected by the cylinder, from the heating apparatus to the calorimeter, in which it was allowed to fall, without any appreciable loss of heat.

We used a GEISSLER'S thermometer graduated to $\frac{1}{2}$° C.; but the temperatures were referred to the air-thermometer after having accurately compared our GEISSLER'S thermometer with a JOLLY'S air-thermometer.

The calorimeter (fig. 2) was a copper cylinder containing about 125 grms. of water, furnished with a movable cover, and was contained in a tin cylinder surrounded by water. For stirring the water a horizontal frame of copper wire covered by a brass net was moved up and down by a handle. The substance fell upon this net, and was moved in the water with the stirrer.

The thermometer in the calorimeter was graduated to $\frac{1}{10}$° C., and was accurately compared with a standard thermometer. The thermal capacity of the calorimeter, stirrer, and thermometer was 2·660 grms. Calorimetric experiments and calculations were made according to REGNAULT'S method, improved by WÜLLNER.* In order to diminish corrections the temperature of the water in the calorimeter was initially cooled below the outer temperature.

We have not described the physical properties of the bodies we have studied, because Mr. RODWELL has done so in his papers. We have only given the temperature range of the contraction and expansion by heat of each substance, derived from Mr. RODWELL'S private communications, and afterwards the results of our calorimetrical experiments.

Iodide of silver, AgI.

From 0° to 142° C., slight contraction.
,, 142° ,, 156°·5 C., great contraction.
,, 156°·5 ,, 527° C., expansion.

The calorimetric results are given in the following table, in which Q denotes the number of calories given out by unit weight of the substance in cooling from T to t° C. The rate of change of the specific heat of water for temperature is no doubt small, but still uncertain, and therefore we did not introduce any correction for it; but we give in the table the initial water temperature τ, so that the eventual correction is always possible.

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* A. WÜLLNER, Wiedemann, Ann. (1880), vol. x., p. 284.

AgI.

No.	*t*.	T.	Q.		Difference.	τ.
			Observed.	Calculated.		
	° C.	° C.				° C.
1	15·67	67·68	2·994	2·990	+0·004	14·1
2	14·72	75·28	3·489	3·497	−0·008	12·9
3	14·03	110·68	5·713	5·705	+0·008	11·6
4	14·97	138·66	7·430	7·434	−0·004	11·0
5	20·68	163·15	14·90	14·87	+0·03	13·0
6	19·13	162·7	14·90	14·93	−0·03	11·4
7	21·01	264·0	20·66	20·67	−0·01	11·4
8	22·11	259·5	20·38	20·35	+0·03	11·0
9	21·87	259·7	20·35	20·37	−0·02	11·6

From these data we have calculated the mean specific heat c of AgI between any two temperatures t and T below 142°, the specific heat c_1 after change of structure, and the latent heat of change λ, supposing that the change happens at 150° C.

$$c=0·054389+0·0000372\ (\mathrm{T}+t)$$
$$c_1=0·0577$$
$$\lambda=6·25.$$

The table shows also the calculated values for Q.

Cu_2I_2.12AgI.

From 0° to 95° C., very slight expansion.
" 95° " 124° C., neither expansion nor contraction.
" 124° " 228° C., notable contraction.
" 228° " 502° C., rapid expansion.

Our calorimetric results are given in the following table :—

Cu_2I_2.12AgI.

No.	*t*.	T.	Q.		Difference.	τ.
			Observed.	Calculated.		
	° C.	° C.				° C.
10	15·13	88·51	4·321	4·316	+0·005	12·7
11	16·69	88·81	4·238	4·242	−0·004	14·3
12	23·67	233·2	20·53	20·58	−0·05	12·2
13	24·00	234·3	20·67	20·62	+0·05	12·5
14	26·27	338·4	26·56	26·52	+0·04	11·5
15	24·81	327·9	25·97	26·00	−0·03	10·4

$$c=0·05882\ \text{(mean, from 16° to 89°)}$$
$$c_1=0·0580$$
$$\lambda=8·31\ \text{(at 160°)}.$$

$Cu_2I_2.4AgI$.

From 0° to 180° C., slight expansion.
,, 180° ,, 199° C., neither expansion nor contraction.
,, 199° ,, 282° C., contraction.
,, 282° ,, 493° C., expansion.

The following table gives a summary of our experiments :—

$Cu_2I_2.4AgI$.

No.	t.	T.	Q.		Difference.	τ.
			Observed.	Calculated.		
	° C.	° C.				° C.
16	15·47	98·94	5·129	5·110	+0·019	13·8
17	15 61	102·29	5·298	5·319	−0·021	13·9
18	16·83	173·72	10·093	10·094	−0·001	13·6
19	15·91	168·25	9·764	9·761	+0·003	12·8
20	17·67	293·5	26·62	26·57	+0·05	9·0
21	20·07	295·5	26·52	26·57	−0·05	11·6
22	20·91	339·9	29·58	29·64	−0·06	11·4
23	21·61	337·6	29·49	29·44	+0·05	12·2

$$c=0{\cdot}056526+0{\cdot}0000410\ (T+t)$$
$$c_1=0{\cdot}0702$$
$$\lambda=7{\cdot}95\ \text{(at 230° C.)}.$$

$Cu_2I_2.3AgI$.

From 0° to 194° C., expansion.
,, 194° ,, 214° C., neither expansion nor contraction.
,, 214° ,, 280° C., contraction.
,, 280° ,, 494° C., expansion.

Our results are as follows :—

$Cu_2I_2.3AgI$.

No.	t.	T.	Q.		Difference.	τ.
			Observed.	Calculated.		
	° C.	° C.				° C.
24	14·48	60·59	2·846	2·846	. .	13·7
25	13·85	179·12	10·781	10·747	+0·034	10·4
26	11·88	179·63	10·867	10·901	−0·034	8·4
27	17·00	290·0	26·27	26·27	. .	8·7
28	19·31	289·9	26·13	26·13	. .	11·2
29	20·28	338·3	29·59	29·58	+0·01	10·9
30	19·30	343·2	29·99	30·00	−0·01	9·8

$$c=0{\cdot}059624+0{\cdot}0000280\ (T+t)$$
$$c_1=0{\cdot}0726$$
$$\lambda=7{\cdot}74\ \text{(at 240° C.)}.$$

$Cu_2I_2.2AgI$.

From 0° to 221° C., expansion.
,, 221° ,, 233° C., neither expansion nor contraction.
,, 233° ,, 298° C., contraction.
,, 298° ,, 496° C., expansion.

We obtained :—

$Cu_2I_2.2AgI$.

No.	t.	T.	Q.		Difference.	τ.
			Observed.	Calculated.		
	° C.	° C.				° C.
31	11·70	63·86	3·299	3·300	−0·001	10·6
32	13·10	64·06	3·227	3·226	+0·001	12·0
33	13·36	209·4	13·23	13·26	−0·03	8·7
34	15·12	209·3	13·17	13·14	+0·03	10·5
35	20·85	305·7	27·99	28·01	−0·02	11·3
36	18·24	306·9	28·29	28·27	+0·02	8·5

$$c=0{\cdot}061035+0{\cdot}0000295\,(T+t)$$
$$\lambda=7{\cdot}88.$$

$Cu_2I_2.AgI$.

From 0° to 256° C., expansion.
,, 256° ,, 284° C., neither expansion nor contraction.
,, 284° ,, 309° C., contraction.
,, 309° ,, 324° C., neither expansion nor contraction
,, 324° ,, 514° C., expansion.

Our results are as follows :—

$Cu_2I_2.AgI$.

No.	t.	T.	Q.		Difference.	τ.
			Observed.	Calculated.		
	° C.	° C.				
37	11·66	63·59	3·378	3·378	. .	10·4
38	15·05	231·1	15·05	15·02	+0·03	9·2
39	15·46	229·2	14·81	14·85	−0·04	9·7
40	20·19	333·0	31·29	31·28	+0·01	8·1
41	22·26	332·7	31·11	31·12	−0·01	10·3

$$c=0{\cdot}063099+0{\cdot}0000260(T+t)$$
$$\lambda=8{\cdot}67.$$

Some experiments were also made with Cu_2I_2, but this substance, when heated, emits iodine vapour, and is therefore slowly decomposed. Its surface was covered by some oxide. The mean specific heat between 13° C. and 65° C. would be found 0·0684, and between 13° C. and 148° C. 0·0686. The increase for temperature is therefore very little.

$PbI_2.AgI$.

From 0° to 118° C., expansion.
,, 118° ,, 124° C., neither expansion nor contraction.
,, 124° ,, 139° C., contraction, more rapid between 130° C. and 133° C.
,, 139° ,, 144° C., neither expansion nor contraction.
,, 144° ,, fusion, expansion.

Our determinations gave the following results:—

$PbI_2.AgI$.

No.	t.	T.	Q.		Difference.	r.
			Observed.	Calculated.		
	° C.	° C.				
42	12·65	65·82	2·566	2·558	+0·008	11·7
43	10·75	62·56	2·484	2·491	−0·007	9·9
44	11·93	113·0	4·909	4·904	+0·005	10·2
45	13·45	112·9	4·815	4·823	−0·008	11·8
46	13·75	171·1	10·531	10·529	+0·002	10·2
47	15·66	171·4	10·455	10·456	−0·001	12·2
48	15·09	242·4	14·52	14·51	+0·01	10·5
49	13·90	242·0	14·52	14·54	−0·02	9·3

$$c=0·047458+0·00000839\ (T+t)$$
$$c_1=0·0567$$
$$\lambda=2·556.$$

The following table recapitulates our results. θ_1 and θ_2 are the temperatures at which the structure-change commences and ends according to Mr. RODWELL'S experiments.

Composition of the substance.	Percentage of AgI.	θ_1.	θ_2.	c.	c_1.	λ.
		° C.	° C.			
AgI	100	142	156·5	$0·054389+0·0000372\ (T+t)$	0·0577	6·25
$Cu_2I_2.12AgI$	88·1	95	228	0·05882 (from 16° to 89°)	0·0580	8·31
$Cu_2I_2.4AgI$	71·2	180	282	$0·056526+0·0000410\ (T+t)$	0·0702	7·95
$Cu_2I_2.3AgI$	65·0	194	280	$0·059624+0·0000280\ (T+t)$	0·0726	7·74
$Cu_2I_2.2AgI$	55·3	221	298	$0·061035+0·0000295\ (T+t)$	. .	7·88
$Cu_2I_2.AgI$	38·2	256	324	$0·063099+0·0000260\ (T+t)$	. .	8·67
$PbI_2.AgI$	33·8	118	144	$0·047458+0·00000839\ (T+t)$	0·0567	2·556

From these data we may deduce the following conclusions.

I. The specific heat of the bodies we experimented on, below the point at which a change of structure commences, increases with the temperature.

II. This specific heat for the alloys of silver- and copper-iodide diminishes when the percentage of silver-iodide increases, but if we calculate the specific heat of the alloys from the specific heats and the proportion of components, we approach only roughly to the true values. It seems that the coefficient of $T+t$ generally increases with the percentage of AgI. The coefficient 0·000028, for $Cu_2I_2.3AgI$, appears to be less than that for $Cu_2I_2.2AGI$; but, perhaps, that is due to some uncertainty in the experiments, notably Nos. 25 and 26. The coefficient for $Cu_2I_2.12AgI$ was not determined, owing to the low temperature of change of this substance.

III. The value of the heat of change, λ, for $PbI_2.AgI$ is *less,* and for the alloys of silver- and copper-iodide is *greater* than for pure iodide of silver. This singular result leads to the assumption that some or all of the alloys of copper- and silver-iodide *are real chemical or molecular compounds.* Starting from the values of λ for AgI and for $Cu_2I_2.AgI$, and calculating the values of λ for the other alloys, considered as mixtures of AgI and $Cu_2I_2.AgI$, we find the numbers 7·99, 7·62, 7·38, 6·72 respectively for $Cu_2I_2.2AgI$, $Cu_2I_2.3AgI$, $Cu_2I_2.4AgI$, $Cu_2I_2.12AgI$. These numbers differ from those given above, but making allowance for errors of observation, for want of the values of c_1 respecting $Cu_2I_2.AgI$ and $Cu_2I_2.2AgI$, and for the uncertainty of the temperature for which λ is computed, it is perhaps probable that at least $Cu_2I_2.2AgI$ and $Cu_2I_2.3AgI$ might be considered as mixtures of AgI and $Cu_2I_2.AgI$. The uncertainty in the temperature of change is chiefly influential in $Cu_2I_2.4AgI$ and $Cu_2I_2.12AgI$, for which bodies the interval of temperature between the commencement and end of the change of structure exceeds 100° C. Moreover, for these substances c_1 is small, and the coefficient of $T+t$ in the value of c has not been determined for $Cu_2I_2.12AgI$, and is somewhat uncertain for $Cu_2I_2.4AgI$, owing to discrepancy in the experiments Nos. 16 and 17. For these reasons the values of λ deduced from observation for those two substances do not pretend to great accuracy.

IV. The values of c_1 increase with the percentage of Cu_2I_2; and in the case of AgI and those alloys of AgI and Cu_2I_2 for which c_1 was determined, the values of c_1 are smaller than the values of c computed for corresponding temperatures. The difference between c_1 and c, extended beyond the temperature of change, diminishes as the percentage of Cu_2I_2 increases. It is therefore probable that c_1 does not much differ from c in $Cu_2I_2.2AgI$ and $Cu_2I_2.AgI$, for which the values of λ are computed on this assumption.

V. For $PbI_2.AgI$ the value of c_1 is greater than the value of c for corresponding temperatures.

Perhaps other conclusions may be derived by comparing our calorimetric results with those of thermic expansion and contraction obtained by Mr. RODWELL; but

Mr. RODWELL's last paper had not been published *in extenso*, when the present paper was written.

Before concluding, we beg to express our deepest gratitude to Mr. RODWELL, who kindly sent us the substances he had studied, and to Professor FR. ROSSETTI, Director of the Physical Institute of Padua, who afforded us every facility for the execution of our experiments.

Fig. 1.

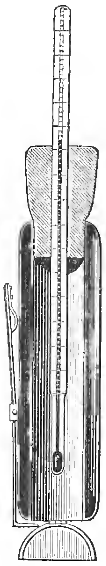
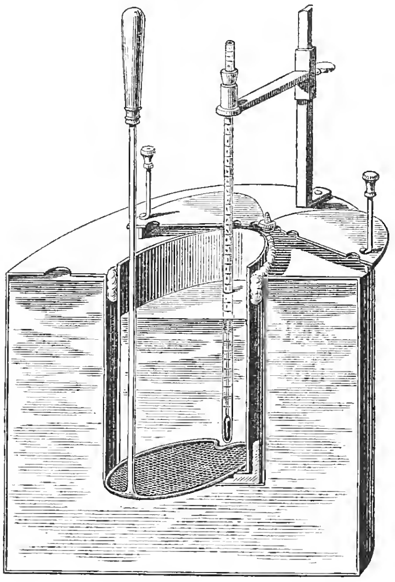


Fig. 2.



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By Sir J. B. Lawes, *Bart., LL.D., F.R.S., F.C.S.,*
J. H. Gilbert, *Ph.D., F.R.S., F.C.S., F.L.S.,*
and M. T. Masters, *M.D., F.R.S., F.L.S.*

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The Individual Plants.

THE BOTANY OF EACH SEPARATE PLOT, IN EACH SEASON OF COMPLETE SEPARATION.

PART II.—THE BOTANICAL RESULTS.

Introduction; Scope of the Experiments, and Mode of Experimenting.

IN Part I. (Phil. Trans., Part I., 1880), under the title of "*The Agricultural Results*," a general description of the experiments, with full particulars of the conditions of manuring of each of the more than 20 plots, was given. The effects of each condition of manuring on the character of growth of the herbage, as illustrated in the quantities of produce yielded, and in the amounts of nitrogen and of mineral matter taken up, on each plot, were also fully considered. But, so varied were the components of the mixed herbage, both as to the species grown, and as to the character of development of the plants, that, to render the *"Agricultural Results"* sufficiently intelligible, and to prevent misconception, if the element of quantity only were taken into account, it was found necessary to describe, in general terms, the differences—in the botanical composition, in the character of development, and in some points in the chemical composition of the produce also. The object of the present section is to describe and discuss, more in detail, what may be called the *botany* of the plots;—that is, to show both the normal botanical composition of the herbage, and the changes induced by the application of the different manuring agents, and by variation in the climatal conditions of the different seasons; and, as far as may be, to ascertain what are the special characters of growth, above-ground or under-ground, normal or induced, by virtue of which the various species have dominated, or have been dominated over, in the struggle which has ensued.

The first season of the experiments was 1856. In that year 13 plots, in 1858 four others, in 1865 one, and in 1872 two more, were brought under experiment; thus making 20 in all. But, of these, two have been subdivided, so that the number may now be reckoned as 22.

As already stated, even in the first years of the experiments it was observed that those manures which are the most effective with wheat, barley, or oats grown on arable land—that is with gramineous species grown separately—were also the most effective in bringing forward the grasses proper, in the mixed herbage. Again, those manures which were the most beneficial to beans or clover the most developed the leguminous species of the mixed herbage, and *vice versâ.* It was further observed that there was great variation in the predominance of individual species among the grasses, and also among the representatives of other orders.

Indeed, in the second year, 1857, the differences in the floras were so marked that a first attempt was then made to separate and determine the proportion of each separate species, in carefully averaged and weighed samples taken from several of the plots at the time the crops were cut; and, taking advantage of the experience thus gained, more careful separations were undertaken in the case of samples of the produce of seven of the most characteristically different plots in the third season, 1858. The results relating to the produce of 1858 were published in Vol. xx. of the Journal of the Royal Agricultural Society of England, in 1859. In these early trials, for the superintendence of which we were indebted to the late Dr. EVAN PUGH of Pennsylvania, the samples were separated into:—

1. Gramineous herbage; stems bearing flower or seed.
2. Gramineous herbage; detached leaves and indeterminate stems.
3. Leguminous herbage.
4. Miscellaneous herbage.

In all, only 11 grasses, three leguminous plants, and nine plants of other orders, were then identified in the samples, though undoubtedly many more were present;

and, under the head of "Gramineous herbage, detached leaves, and indeterminate stems," in one case as little as 15 per cent., and in another more than 53 per cent. of the total was recorded. This result at once illustrates both the difficulty of the work, and the great difference in the character of growth on the different plots.

From year to year the plots became more and more characteristic; and in the seventh season, 1862, it was decided to undertake much more complete botanical separations. To this end it was necessary, not only to expend much more time and labour in the work, but to obtain the services of a competent assistant to devote attention exclusively to it. Accordingly, we applied to Dr. (now Sir JOSEPH) HOOKER, who kindly recommended to us such an assistant; as he has also done on three subsequent occasions, at intervals of five years—namely, in 1867, 1872, and 1877. Thus, the botanical work was superintended in 1862 by Mr. W. SUTHERLAND, in 1867 by Mr. RICHARD L. KEENAN, in 1872 by Mr. W. B. HEMSLEY, and in 1877 by Mr. WALTER DAVIS. In each of these four seasons of more complete separations, Mr. J. J. WILLIS, of the Rothamsted Laboratory, assisted whenever able to do so, and from the experience thus gained was on each subsequent occasion enabled to afford substantial aid to the new comer. Mr. WILLIS has also, especially in recent years, annually made notes, at intervals, on the herbage of the growing crops, and he has conducted partial separations on several occasions. On each occasion, whether of more complete or of only partial separation, from three to six boys have also been occupied in the work.

The Samples: their botanical analysis.—The mode of taking and treating the samples may be briefly described as follows: Eight or ten mowers being put upon the experimental plot, the botanical assistant followed, taking small quantities of grass immediately after the scythe from each swath, until nearly the whole of the plot was down; boys also followed, with hampers or sacks, for the collection of the samples so taken. The quantities thus collected amounted to very many times more than was required for botanical analysis. They were at once carefully mixed on a cloth, so as to shake out seed, or otherwise damage the herbage, as little as possible; and from the bulk a sample of 10, 12½, 15, or even 20 lbs., according to circumstances, was then immediately weighed. The weighed samples were spread out on cloths, at the ordinary temperature, to dry, and were then carefully preserved for future examination.

In the conduct of the separations each worker had a small handful of the mixed herbage placed before him, which he separated into its various component species as far as he was able. From time to time the superintendent revised each of the so separated portions. But there always remained an undetermined residue, which varied in amount exceedingly, according as the herbage was simple or complex, stemmy or leafy, mature or immature, and so on. These at first undetermined residues, after some reduction in the hands of the superintendent, were next separated into portions of different character by means of sieves of various gauges, by which the examination and the identification of the various components were much facilitated.

Still, these further separations were always very tedious and laborious; and, as will be seen hereafter, the amount of finally undetermined residue varied very considerably according to the description, and the character of development, of the herbage. It will be readily understood that a very luxuriant and stemmy gramineous produce would be much more easily separated into its components than a mass consisting chiefly of fine leafy matter. To quote an extreme case in illustration of the difference in the character of the herbage, and of the difference in the degree of difficulty of separation accordingly, it may be mentioned that whilst a sample of 20 lbs. from one plot in 1872 only occupied from four to five days in botanical analysis, a sample of equal weight from another plot in the same year occupied 30 days.

In further explanation of the mode of procedure it may be stated that, in addition to the examination of the growing plants at intervals during the season, prior to the cutting of the crops the botanical assistant, after spending some days on the ground to become acquainted with the character of the plots, made systematic notes upon them. The instructions given to him were—to enumerate, in the order of their apparent predominance on each plot, 1, the Gramineous species; 2, the Leguminous species 3, the species of other Orders—classed together for convenience as "Miscellaneous" species; to estimate, as far as practicable, the probable proportion in which the most prominent species occurred, stating, at the same time, on what basis the estimate of relative predominance was formed—whether merely on the quantity of flower stems or of probable total plant, &c., noting the circumstances probably affecting the degree of accuracy of the estimate; also to state the general characteristics of the distribution, and of the growth, on each plot—what plants were flowering or showing greater or less flowering and seeding tendency, the order of forwardness, &c., and also any special point observable.

After thus describing the herbage on each plot, the next point was to consider the normal characteristics of growth of each important species, and to compare its predominance, and its distinctive characters of development, if any, on the different plots; especially noting differences in tendency to form more leaf, more stem, &c.; also differences in colour, degree of forwardness, fertility, barrenness, and the like. Careful observations were also to be made as to what particular "Natural Orders," if any, seemed to be characteristic of the herbage of the different plots.

Then, within a few days of the time of cutting, the general order of ripeness of the components of each plot was noted, stating particularly what plants—gramineous, leguminous, or miscellaneous—were flowering or seeding, and especially which would probably self-sow the ground.

Again, at the time of and after mowing, further observations were made with a view to the confirmation or correction of the notes previously recorded; and also to ascertain what plants, if any, occurred which had not been previously observed.

Lastly, after the removal of the first crops, and during the growth of the second crops, and especially just before these were cut or fed, as the case might be, notes were made on the progress and character of growth on the different plots; and sometimes, when the crops were fed by sheep, observations were made as to what species were preferred and what were discarded by the animals.

Instructions were also given carefully to look for, and to note, throughout the whole enquiry, any characters of growth, normal or induced, above-ground or under-ground, by virtue of which it was probable that any one species dominated over others, or in consequence of which the plants dominated over had succumbed.

In 1862, 20 samples of the mixed herbage, of 10 lbs. each, were submitted to botanical separation; and the work occupied Mr. SUTHERLAND, several boys, and occasionally Mr. WILLIS, for about four months; and Mr. WILLIS and the boys for about two months more. The results so obtained were published in Vol. xxiv., Part I. of the Journal of the Royal Agricultural Society of England (1863). But, even in the case of these separations, which were so much more elaborate than the earlier ones, the quantity of undetermined stem, leaf, and shedded flowers and seeds varied from under 5 to nearly 28 per cent. of the total sample. These at first undetermined residues have, however, since been much farther worked down; and the results given in the present paper relating to the produce of 1862 are, accordingly, considerably amended as compared with those originally published as above referred to.

In 1867, 20 samples of 10 lbs. each and one of 20 lbs. were operated upon. The herbage was generally more stemmy and riper than that of 1862. The separations were, therefore, less difficult; but they were carried further, about five months being devoted to the work; and the proportions of undetermined residue were less than in even the revised separations of the produce of 1862.

In 1872 most of the herbage was finer than usual, larger samples were taken than previously, the amount of matter left undetermined was very small, and the time devoted to the separations was much longer than formerly. Thus, in all, 23 samples were operated upon: 14 of 15 lbs., and 9 of 20 lbs. each; and the period occupied in the analysis of the samples of that year was between 10 and 11 months. Nearly three months were afterwards expended in the revision of the results relating to the produce of 1862, as above referred to; when the previously undetermined residues were on the average reduced to about one-third of the original amounts, and in many cases much lower.

In 1877, 24 samples of 12½ lbs. each were taken. In the work of separation, besides some new hands, two, and sometimes three, who had taken part in the detailed separations of 1872, and in partial ones in 1874, 1875, and 1876, were engaged. The work was more completely done than on any previous occasion, no undetermined residue whatever being left; and the time occupied was not quite five months.

Besides the complete separations into individual species, in samples from every plot in the four seasons of 1862, 1867, 1872, and 1877, partial separations, as above referred to, that is into three main divisions of—

Gramineous herbage,
Leguminous herbage,
Miscellaneous herbage,

have been executed in the case of selected plots, first and second crops, as under :—

	First crops.	Second crops.	Total.
1871	5	—	5
1874	5	—	5
1875	9	9	18
1876	8	—	8
1877	(Complete)	11	11
1878	14	11	25
1879	12	8	20
1880	12	6	18

These partial separations have been superintended by Mr. WILLIS, excepting those of the second crop of 1877, which were conducted by Mr. WALTER DAVIS.

Finally in regard to the samples, although there can be no doubt that they do represent the actual vegetation of the plots at the time of cutting with close approximation to correctness, it is to be remembered that some plants may by that time have grown up and already to a great extent disappeared, whilst others may escape the scythe by reason of their dwarf and lowly habit. It is obvious, however, that any inaccuracies in the indications of the botanical separations arising from such causes must, from the very nature of the case, be but small, and that they will practically be confined to the results relating to plants of only scanty occurrence or meagre development. The notes taken on the ground do, indeed, show that the range of error due to the causes referred to is not material.

Characters of the Seasons in which Complete Botanical Separations were conducted.

In the section on the "Influence of Season on the Produce of Hay," Part I., pp. 390 to 405, it has been shown how very great was the difference in the amount of the mixed produce dependent on the climatal conditions, and also what widely different seasons might yield large, and again, what widely different seasons might yield small crops. In fact, a given quantity of the mixed produce grown under the same conditions as to manuring, might be composed very differently in two different seasons. This difference was stated, in general terms, to consist not only in the different proportions in which the various Orders, genera, or species were represented, but also in the character of development of the plants in regard to leafiness or stemminess, luxuriance and succulence, or maturation, &c. It is obvious that, if this be so, it is essential to consider the characters of the seasons themselves, in which the botanical separations were made; and to come to some conclusion as to their probable influence on the botanical composition, and the character of development, of the mixed herbage, apart from that of progressive exhaustion of the soil, or the continued application of different manuring substances; so as in some degree to discriminate between results due to the

TABLE XXXIII.

	Plot 3. Unmanured continuously.	Plot 7. Mixed mineral manure, alone.	Plot 9. Mixed mineral manure, and 400 lbs. ammonia-salts.	Plot 11. Mixed mineral manure, and 800 lbs. ammonia-salts.	Plot 14. Mixed mineral manure, and 550 lbs. nitrate soda.	Means.
TOTAL NUMBER OF SPECIES FOUND.						
1862	50	44	28	25	28	35
1867	43	42	29	19	30	33
1872	49	41	30	16	30	33
1877	52	44	27	16	27	33
Average of the 4 years . .	49	43	29	19	29	34
GRAMINEOUS HERBAGE—PER CENT. (BY WEIGHT) IN TOTAL PRODUCE.						
1862	68·65	63·21	88·43	91·72	88·83	80·17
1867	62·46	58·69	76·68	94·89	93·60	77·26
1872	67·92	47·85	92·08	99·05	92·77	79·93
1877	71·15	74·38	94·65	98·00	87·81	85·20
Average of the 4 years . .	67·55	61·03	87·96	95·91	90·75	80·64
LEGUMINOUS HERBAGE—PER CENT. (BY WEIGHT) IN TOTAL PRODUCE.						
1862	8·59	25·42	0·13	0·01	0·13	6·85
1867	5·73	12·84	0·16	0·01	0·39	3·83
1872	9·17	40·26	0·02	0·01	1·36	10·17
1877	8·54	13·71	0·41	0·00	0·76	4·68
Average of the 4 years . .	8·01	23·06	0·18	0·01	0·66	6·38
MISCELLANEOUS HERBAGE—PER CENT. (BY WEIGHT) IN TOTAL PRODUCE.						
1862	22·76	11·37	11·44	8·27	11·04	12·98
1867	31·81	28·47	23·16	5·10	6·01	18·91
1872	22·91	11·89	7·90	0·94	5·87	9·90
1877	20·31	11·91	4·94	2·00	11·43	10·12
Average of the 4 years . .	24·44	15·91	11·86	4·08	8·59	12·98
GRAMINEOUS HERBAGE—LBS. PER ACRE.						
1862	2095	2796	5661	6525	5080	4431
1867	2081	2616	4127	5511	6728	4213
1872	1116	2027	5210	7075	5765	4238
1877	1682	3793	5730	7495	5495	4839
Average of the 4 years . .	1743	2808	5182	6651	5767	4430
LEGUMINOUS HERBAGE—LBS. PER ACRE.						
1862	262	1125	8	1	7	281
1867	191	573	9	1	28	160
1872	151	1705	1	1	85	390
1877	202	699	25	0	48	195
Average of the 4 years . .	202	1026	11	1	42	257
MISCELLANEOUS HERBAGE—LBS. PER ACRE.						
1862	695	503	733	588	631	630
1867	1060	1269	1246	296	432	860
1872	377	504	447	67	365	352
1877	480	608	299	153	715	451
Average of the 4 years . .	653	721	681	276	536	573
TOTAL MIXED HERBAGE—LBS. PER ACRE.						
1862	3052	4424	6402	7114	5718	5342
1867	3332	4458	5382	5808	7188	5233
1872	1644	4236	5658	7143	6215	4980
1877	2364	5100	6054	7648	6258	5485
Average of the 4 years . .	2598	4555	5874	6928	6345	5260
Average 20 years 1856-75 .	2383	3958	5711	6726	6407	5037

TABLE XXXIV.—Abstract of meteorological conditions of the four seasons in which 1866–8, 1871–2, and 1876–7; and comparison of each

	Monthly.							
	July.	August.	September.	October.	November.	December.	January.	February.
RAINFALL AT ROTHAMSTED.—INCHES.								
Average 22 years 1855–6 to 1876–7 . . .	2·56	2·45	2·64	3·13	2·20	2·18	2·85	1·66
1861–1862	3·19	0·89	1·63	1·46	3·99	1·58	1·77	0·60
1866–1867	3 01	3·44	4·10	1·82	2·16	2·70	2·56	1·94
1871–1872	4·00	0·77	4·07	1·79	0·66	1·42	4·68	1·47
1876–1877	1·46	2·98	5·02	1·52	4·20	6·00	4·99	2·10
More + or less − than average of 22 years 1855–6 to 1876–7 1862 . . .	+0·63	−1·56	−1·01	−1·67	+1·79	−0·60	−1·08	−1·06
1867 . . .	+0·45	+0·99	+1·46	−1·31	−0·04	+0·52	−0·29	+0·28
1872 . . .	+1·44	−1·68	+1·43	−1·34	−1·54	−0·76	+1·83	−0·19
1877 . . .	−1·10	+0·53	+2·38	−1·61	+2·00	+3·82	+2·14	+0·44
RAINFALL AT ROTHAMSTED.—NUMBER OF DAYS WHEN 0·01 INCH, OR MORE, FELL.								
Average 22 years 1855–6 to 1876–7 . . .	12	13	14	16	15	15	16	13
1861–1862	21	11	14	13	20	14	16	7
1866–1867	12	18	24	11	12	15	12	11
1871–1872	18	5	13	10	12	13	21	16
1876–1877	11	14	25	15	20	27	26	18
More + or less − than average of 22 years 1855–6 to 1876–7 1862 . . .	+9	−2	0	−3	+5	− 1	0	−6
1867 . . .	0	+5	+10	−5	−3	0	− 4	−2
1872 . . .	+6	−8	− 1	−6	−3	− 2	+ 5	+3
1877 . . .	−1	+1	+11	−1	+5	+12	+10	+5
MEAN MAXIMUM TEMPERATURE AT GREENWICH (FAHRENHEIT DEGREES).								
Average 22 years 1855–6 to 1876–7 . . .	76·8	73·6	67·9	59·0	48·4	44·5	44·0	46·4
1861–1862	72·3	75·6	68·3	64·1	47·3	45·9	43·9	46·5
1866–1867	72·6	69·4	65·1	58·2	50·5	47·6	39·5	50·7
1871–1872	72·6	78·1	67·5	58·6	43·2	42·2	46·3	51·7
1876–1877	77·7	74·5	65·3	60·2	49·9	48·6	49·5	50·4
More + or less − than average of 22 years 1855–6 to 1876–7 1862 . . .	−4·5	+2·0	+0·4	+5·1	−1·1	+1·4	−0·1	+0·1
1867 . . .	−4·2	−4·2	−2·8	−0·8	+2·1	+3·1	−4·5	+4·3
1872 . . .	−4·2	+4·5	−0·4	−0·4	−5·2	−2·3	+2·3	+5·3
1877 . . .	+0·9	+0·9	−2·6	+1·2	+1·5	+4·1	+5·5	+4·0
MEAN MINIMUM TEMPERATURE AT GREENWICH (FAHRENHEIT DEGREES).								
Average 22 years 1855–6 to 1876–7 . . .	53·3	53·2	49·5	43·9	36·6	35·0	33·7	34·6
1861–1862	53·4	53·8	48·2	47·7	34·1	36·0	34·3	36·7
1866–1867	52·5	52·3	50·6	45·7	38·0	37·4	28·5	39·5
1871–1872	54·0	53·8	50·3	41·9	32·7	34·2	37·0	39·2
1876–1877	56·1	54·2	48·9	46·9	38·3	39·5	36·8	38·0
More + or less − than average of 22 years 1855–6 to 1876–7 1862 . . .	+0·1	+0·6	−1·3	+3·8	−2·5	+1·0	+0·6	+2·1
1867 . . .	−0·8	−0·9	+1·1	+1·8	+1·4	+2·4	−5·2	+4·9
1872 . . .	+0·7	+0·6	+0·8	−2·0	−3·9	−0·8	+3·3	+4·6
1877 . . .	+2·8	+1·0	−0·6	+3·0	+1·7	+4·5	+3·1	+3·4
MEAN TEMPERATURE AT GREENWICH (FAHRENHEIT DEGREES).								
Average 22 years 1855–6 to 1876–7 . . .	62·5	61·8	57·4	50·6	42·3	40·0	38·9	40·1
1861–1862	60·9	63·2	57·1	54·9	40·8	41·0	39·0	41·1
1866–1867	61·0	59·4	56·4	51·3	44·3	42·9	34·2	44·7
1871–1872	61·7	64·8	57·4	49·4	37·6	38·3	41·3	44·8
1876–1877	65·0	62·9	55·8	52·7	44·0	44·3	43·1	44·0
More + or less − than average of 22 years 1855–6 to 1876–7 1862 . . .	−1·6	+1·4	−0·3	+4·3	−1·5	+1·0	+0·1	+1·0
1867 . . .	−1·5	−2·4	−1·0	+0·7	+2·0	+2·9	−4·7	+4·6
1872 . . .	−0·8	+3·0	0·0	−1·2	−4·7	−1·7	+2·4	+4·7
1877 . . .	+2·5	+1·1	−1·6	+2·1	+1·7	+4·3	+4·2	+3·9
MEAN RANGE OF TEMPERATURE AT GREENWICH (FAHRENHEIT DEGREES).								
Average 22 years 1855–6 to 1876–7 . . .	21·6	20·4	18·3	14·8	11·8	9·6	9·8	11·4
1861–1862	18·9	21·8	20·1	16·4	13·2	9·9	9·6	9·8
1866–1867	20·1	17·1	14·5	12·5	12·5	10·2	11·0	11·2
1871–1872	18·6	24·3	17·2	16·7	10·4	8·0	9·2	12·5
1876–1877	21·6	20·3	16·4	13·3	11·6	9·1	12·7	12·4
More + or less − than average of 22 years 1855–6 to 1876–7 1862 . . .	−2·7	+1·4	+1·8	+1·6	+1·4	+0·3	−0·2	−1·6
1867 . . .	−1·5	−3·3	−3·8	−2·3	+0·7	+0·6	+1·2	−0·2
1872 . . .	−3·0	+3·9	−1·1	+1·9	−1·4	−1·6	−0·6	+1·1
1877 . . .	0·0	−0·1	−1·9	−1·5	−0·2	−0·5	+2·9	+1·0

complete botanical separations of the mixed produce were conducted; viz.: 1861–2, with the average of 22 years, 1855–6 to 1876–7.

Monthly (continued).				12 months, July to June, inclusive.	4 months, July, August, September, October.	4 months, November, December, January, February,	4 months, March, April, May, June.	3 months, April, May, June.	2 months, April and May.
March.	April.	May.	June.						
RAINFALL AT ROTHAMSTED.—INCHES (continued).									
1·77	1·97	2·16	2·40	27·97	10·78	8·89	8·30	6·53	4·13
3·06	2·84	2·91	3·41	27·33	7·17	7·94	12·22	9·16	5·75
2·17	2·82	3·35	1·06	31·13	12·37	9·36	9·40	7·23	6·17
2·15	1·63	2·89	3·09	28·62	10·63	8·23	9·76	7·61	4·52
2·55	2·76	2·82	1·43	37·83	10·98	17·29	9·56	7·01	5·58
+1·29	+0·87	+0·75	+1·01	−0·64	−3·61	−0·95	+3·92	+2·63	+1·62
+0·40	+0·85	+1·19	−1·34	+3·16	+1·59	+0·47	+1·10	+0·70	+2·04
+0·38	−0·34	+0·73	+0·69	+0·65	−0·15	−0·66	+1·46	+1·08	+0·39
+0·78	+0·79	+0·66	−0·97	+9·86	+0·20	+8·40	+1·26	+0·48	+1·45
RAINFALL AT ROTHAMSTED.—NUMBER OF DAYS WHEN 0·01 INCH, OR MORE, FELL (continued).									
14	11	12	12	163	55	59	49	35	23
18	14	18	16	182	59	57	66	48	32
12	18	15	10	170	65	50	55	43	33
10	13	18	15	164	46	62	56	46	31
23	13	18	7	217	65	91	61	38	31
+4	+3	+6	+4	+19	+ 4	− 2	+17	+13	+ 9
−2	+7	+3	−2	+ 7	+10	− 9	+ 6	+ 8	+10
−4	+2	+6	+3	+ 1	− 9	+ 3	+ 7	+11	+ 8
+9	+2	+6	−5	+54	+10	+32	+12	+ 3	+ 8
MEAN MAXIMUM TEMPERATURE AT GREENWICH (FAHRENHEIT DEGREES) (continued).									
49·9	58·4	64·0	71·4	58·7	69·3	45·8	60·9	64·6	61·2
50·0	57·5	66·4	67·1	58·8	70·1	45·9	60·3	63·7	62·0
44·5	58·7	64·7	70·2	57·6	66·3	47·1	59·5	64·5	61·7
53·5	59·3	62·1	71·3	58·9	69·2	45·9	61·6	64·2	60·7
49·3	54·3	59·1	74·4	59·4	69·4	49·6	59·3	62·6	56·7
+0·1	−0·9	+2·4	−4·3	+0·1	+0·8	+0·1	−0·6	−0·9	+0·8
−5·4	+0·3	+0·7	−1·2	−1·1	−3·0	+1·3	−1·4	−0·1	+0·5
+3·6	+0·9	−1·9	−0·1	+0·2	−0·1	+0·1	+0·7	−0·4	−0·5
−0·6	−4·1	−4·9	+3·0	+0·7	+0·1	+3·8	−1·6	−2·0	−4·5
MEAN MINIMUM TEMPERATURE AT GREENWICH (FAHRENHEIT DEGREES) (continued).									
34·9	39·6	43·6	50·2	42·4	50·0	35·0	42·1	44·5	41·6
38·4	41·7	47·9	49·3	43·5	50·8	35·3	44·3	46·3	44·8
33·0	42·3	44·7	49·1	42·8	50·3	35·9	42·3	45·4	43·5
37·7	40·1	42·5	50·0	42·8	50·0	35·8	42·6	44·2	41·3
34·1	39·5	41·1	51·5	43·8	51·5	38·4	41·6	44·0	40·3
+3·5	+2·1	+4·3	−0·9	+1·1	+0·8	+0·3	+2·2	+1·8	+3·2
−1·9	+2·7	+1·1	−1·1	+0·4	+0·3	+0·9	+0·2	+0·9	+1·9
+2·8	+0·5	−1·1	−0·2	+0·4	0·0	+0·8	+0·5	−0·3	−0·3
−0·8	−0·1	−2·5	+1·3	+1·4	+1·5	+3·4	−0·5	−0·5	−1·3
MEAN TEMPERATURE AT GREENWICH (FAHRENHEIT DEGREES) (continued).									
41·6	47·6	52·4	58·9	49·5	58·1	40·3	50·1	53·0	50·0
43·1	48·4	55·4	56·3	50·1	59·0	40·5	50·8	53·4	51·9
37·7	49·0	53·4	58·1	49·4	57·0	41·5	49·6	53·5	51·2
46·6	48·3	50·9	59·2	49·9	58·3	40·5	50·8	52·8	49·6
40·8	45·6	48·7	61·4	50·7	59·1	43·9	49·1	51·9	47·2
+1·5	+0·8	+3·0	−2·6	+0·6	+0·9	+0·2	+0·7	+0·4	+1·9
−3·9	+1·4	+1·0	−0·8	−0·1	−1·1	+1·2	−0·5	+0·5	+1·2
+3·0	+0·7	−1·5	+0·3	+0·4	+0·2	+0·2	+0·7	−0·2	−0·4
−0·8	−2·0	−3·7	+2·5	+1·2	+1·0	+3·6	−1·0	−1·1	−2·8
MEAN RANGE OF TEMPERATURE AT GREENWICH (FAHRENHEIT DEGREES) (continued).									
14·6	18·9	20·4	21·1	16·1	18·8	10·7	18·7	20·1	19·7
11·6	15·8	18·5	17·8	15·3	19·3	10·6	15·9	17·4	17·1
11·5	16·4	20·0	21·1	14·9	16·0	11·2	17·3	19·2	18·2
15·7	19·2	19·7	21·3	16·1	19·2	10·0	19·0	20·1	19·4
15·2	14·8	18·0	22·9	15·7	17·9	11·5	17·7	18·6	16·4
−3·0	−3·1	−1·9	−3·3	−0·8	+0·5	−0·1	−2·8	−2·7	−2·6
−3·1	−2·5	−0·4	0·0	−1·2	−2·8	+0·5	−1·4	−0·9	−1·5
+1·1	+0·3	−0·7	+0·2	0·0	+0·4	−0·7	+0·3	0·0	−0·3
+0·6	−4·1	−2·4	+1·9	−0·4	−0·9	+0·8	−1·0	−1·5	−3·3

incidental characters of the seasons merely, and those properly attributable to the artificial conditions induced by exhaustion, or those supplied by manure. It is proposed, therefore, before entering upon the consideration of the botany of the different plots, as illustrated by the results of the detailed botanical separations, to point out what were the characters of the seasons themselves in which the separations were made, and what was the general character of their influence on the vegetation of the different plots.

The results given in Tables XXXIII. and XXXIV. supply the chief basis for the illustrations which follow in relation to these points.

Table XXXIII., p. 1189, gives the total number of species found, and both the per cent. by weight in the total produce, and the lbs. per acre, of the gramineous, the leguminous, and the miscellaneous herbage, respectively, in each of the four seasons of complete botanical separation, on each of the five very characteristically differently manured plots which were selected for illustration when considering the characters of the seasons of the highest and of the lowest productiveness (Part I., Phil. Trans., 1880, p. 399). There is also given the total amount of mixed herbage per acre, on each plot, in each of the four seasons, and the average for each plot, over the four years of separation and over the 20 years 1856–1875 inclusive. From these results some idea can be formed of the general and comparative characters of each of the four seasons, so far as activity of vegetation or productiveness merely is concerned. It should be explained that, for the purposes of this very summary statement of the botanical characteristics of the produce of the four different seasons, the "undetermined" herbage, if any, that is the amount which could not be referred to individual species, has, nevertheless, after careful examination been apportioned by judgment among the three main divisions given in the table—namely, the gramineous, the leguminous, and the "miscellaneous" herbage.

Table XXXIV., pp. 1190–1191, shows some of the meteorological conditions of the four seasons of the complete botanical separations. The same particulars as to rainfall and temperature are given, and in the same form, as in the tables illustrating the characters of the seasons of the highest and of the lowest productiveness (Part I., pp. 400–1). They are given for each of the months from July to June separately, and for series of months collectively, for the average of the 22 years, 1855–6 to 1876–7 inclusive, and for each of the four years (July to June inclusive), 1861–2, 1866–7, 1871–2, and 1876–7. The variations of each of the four seasons + or − the average of the 22 are also given.

Season 1861–2.

There was a considerable deficiency of rain throughout the autumn of 1861 and the winter of 1861–2, excepting in November, when there was a considerable excess. The autumn and winter were also warmer than the average, excepting again in November, when, with the excess of rain, there were also prevailing low temperatures, the month being generally inclement. The autumn dryness would appear not to have been favourable for the second growth of 1861; but there had been a considerable excess of rain in June and July, and the second crops were estimated to be rather above the average on all the plots, excepting on that with the highest quantity of ammonia-salts. Thus, both as to previous growth and measurable meteorological factors, the conditions antecedent to spring growth in 1862 would not seem to have been favourable to luxuriance. In each of the months of March, April, May, and June, however, there was more than the average amount of rain ; and the excess was the greatest, and considerable, in March, June coming next in this respect, so far favouring both an early start and protracted luxuriance. The mean temperatures of March, April, and May were also above the average ; due in March and April to high minimum readings, and in May to both high maximum and minimum readings. In June, however, with the excess of rain, there was considerably lower than the average maximum, and lower also than the average minimum temperatures, giving, upon the whole, a wet and cold month at the time when the mixed herbage should mature.

Thus, with antecedent conditions certainly not specially favourable, but, on the other hand, not specially unfavourable, the period of most active vegetation was, both as to moisture and temperature, propitious for luxuriance, whilst the concluding period was not suitable for maturation.

Under these circumstances, the crop of 1862 was considerably over average without manure, with mineral manure alone, and with mineral manure and ammonia-salts ; but it was below the average with mineral manure and nitrate of soda.

Without manure the excess was in the largest proportion in the gramineous herbage, but partly, also, in that furnished by the leguminous and miscellaneous plants.

With mixed mineral manure, including potass, the increase over the produce without manure was in the grasses and the Leguminosæ, whilst miscellaneous plants were in reduced amount, and the increase compared with the average of the four on the same plot was entirely in the Leguminosæ.

With the mixed mineral manure and the smaller quantity of ammonia-salts the increase was mainly in the grasses, but considerably, also, in miscellaneous herbage, especially in *Rumex Acetosa*. With the mixed mineral manure and the double quantity of ammonia-salts the yield of grasses was less than on the average of the four years, but, judging from the amount of total produce, probably more than the average of the 20 years ; and there was a considerable excess of miscellaneous plants, and again more especially of *Rumex*. On both these mineral and ammonia-plots *Conopodium denudatum* and *Achillea Millefolium* were also abundant.

With the mixed mineral manure and nitrate of soda the *deficiency*, compared with the average, was chiefly in the grasses, the Leguminosæ also being in very small amount ; but the miscellaneous plants were fairly abundant, and here again the chief weed was *Rumex Acetosa*, *Conopodium* coming next, and *Ranunculus* (*repens* and *bulbosus*) third.

Thus, especially on the plots with ammonia, the wet and warm-growing period was favourable to the luxuriance of gramineous herbage, the freer growing and comparatively surface-rooting and moisture-loving species there prevailing; whilst on the plot with nitrate of soda where, besides the free-growing and chiefly surface-rooting *Poa trivialis* and *Lolium perenne*, the deeper-rooting and comparatively drought-resisting *Bromus mollis* had already established predominance, the gramineous herbage, as a whole, was not specially luxuriant. The season was not unfavourable for the Leguminosæ on the plots where the grasses were not forced by nitrogenous manures; but where they were so the Leguminosæ were much discouraged. Miscellaneous plants were a good deal favoured. Where there was no nitrogenous manure *Rumex, Plantago,* and *Ranunculus* were very prominent; where there was nitrogenous manure and coincident free growth of grasses, *Rumex* was still the most prominent weed, *Conopodium denudatum* coming next, and with ammonia-salts *Achillea Millefolium*, but with nitrate of soda *Ranunculus* (*repens* and *bulbosus*) came third.

The notes made on the crop before and at the time of cutting show that the herbage generally was characterised by abundance of foliage, dense undergrowth and backwardness, with comparatively little tendency to form stems or to produce flower or seed.

The season of 1862 was, therefore, upon the whole, favourable for luxuriance of the freer growing species, yielding, however, for the most part, leafy and immature produce.

Season 1866–7.

In the preceding year, 1866, the first crops had been, upon the whole, over average, and the second crops were estimated to be so; but, owing to the wetness of the autumn, they had been cut and left to decay on the land. There had been a considerable excess of rain in June, July, August, and September, 1866, with on the average low temperatures. The concluding three months of 1866 were, however, upon the whole, deficient in rain, with higher than the average temperatures, though in November a good deal of rain fell within a short period, causing floods in some localities. In January, 1867, the fluctuations were very great. Extreme cold and heavy falls of snow alternated with rapid thaws, warm weather, heavy gales, and a good deal of rain, though in the aggregate there was not an excess of fall, but there was a considerable defect in temperature. The last week of January and almost the whole of February were unusually warm, with, at the beginning of February, a large amount, and during the month an excess, of rain. March, on the other hand, was almost to the conclusion very cold and wintry, with a good deal of snow. There was an excess of fall during the month, and on the average very low temperatures, and vegetation was much checked. April and the beginning of May were very unsettled—stormy, rainy, and changeable as to temperature, but on the whole warmer than the average. Later in May, besides some very warm, there was a longer period of extremely cold weather, with a dry atmosphere and frosty nights, vegetation being again much checked, though, during the month, both the fall of rain and the temperatures were above average.

Lastly, June showed considerable deficiency of rain, was very changeable as to temperature, but upon the whole colder than the average.

Thus the season of 1866–7 was marked by very great fluctuations. The early winter was generally warmer than usual with about the average fall of rain. Then came intervals of severe frost, snow, and heavy gales, which were followed by several weeks of warm weather, with a good deal of rain—conditions favourable for an early start of vegetation. But the early spring was very wintry and stormy, and growth was much checked. The remainder of the growing period was very changeable as to temperature, and frequently wet and stormy. June, the maturing period, though changeable, was upon the whole dry, but not so warm as usual.

With such extremely varying climatal conditions we could hardly expect other than irregularity of result with very different plants, and with very different manuring conditions. Without manure especially, and with purely mineral manure in a less degree, there was more than the average amount of produce. And, exactly reversing the results of 1862, there was with the mineral manure and ammonia-salts a considerable deficiency, and with the mineral manure and nitrate of soda a considerable excess, of produce, compared with the average of either the four years or the 20.

Without manure, and with mineral without nitrogenous manure, the excess of produce over the average was not proportionally great in the grasses, leguminous herbage was deficient, but miscellaneous plants contributed an unusually large proportion of the crops. Without manure by far the most prominent weed was *Plantago lanceolata*. *Luzula campestris*, *Pimpinella Saxifraga*, *Conopodium denudatum*, *Ranunculus* (*repens* and *bulbosus*), and *Rumex Acetosa* were also prominent. With the purely mineral manure *Conopodium denudatum* and *Rumex Acetosa* each contributed about one-third of the miscellaneous herbage ; *Achillea Millefolium* was also prominent ; and, as without manure, *Luzula campestris*, *Pimpinella Saxifraga*, and *Plantago lanceolata*, were somewhat so, but in a much less degree.

Owing to the wetness of February and the general inclemency of March, the sowing of the ammonia-salts was not commenced until March 11 ; the process was even then interrupted by snow, and the plots here referred to were not sown until March 21.

With the mineral manure and the smaller quantity of ammonia-salts there was a considerable deficiency of total produce compared with the average. This deficiency was due to a generally restricted growth of the grasses ; probably accounted for in part by the adverse weather of March, and in part by the less effect or even damage done by the late sown ammonia-salts, under such conditions. Leguminosæ were scarcely represented. But, again, miscellaneous herbage was unusually abundant. Among the grasses the poor and meagrely yielding, but hardy *Festuca ovina* contributed the most, *Agrostis vulgaris*, *Poa pratensis*, and *Holcus lanatus* a good deal ; whilst the more freely yielding *Dactylis* and others were in but small amount. Among the miscellaneous plants, *Rumex Acetosa* and *Conopodium denudatum* contributed nearly nine-tenths of the whole ; *Achillea Millefolium* being next in prominence.

With the mineral manure and the double amount of ammonia-salts, there was again great deficiency in the yield of the grasses; but here *Dactylis glomerata* was by far the most prominent, and gave a large amount of both leafy and stemmy growth. *Agrostis vulgaris* came second in prominence, and both *Alopecurus pratensis* and *Poa pratensis* were fairly represented. With this predominance of a few free-growing grasses, the amount of miscellaneous herbage was considerably less than half as much as on the same plot in 1862, and only about a quarter as much as on the plot with the smaller quantity of ammonia-salts in the same year, 1867. The weed which maintained the most prominent place in the struggle was *Rumex Acetosa; Conopodium denudatum* coming next. Only two other species were found in the sample, and these in only very insignificant amount.

The nitrate of soda was not sown until April 10, when growth would be fairly established, so that the loss by drainage which would otherwise result from the excess of rain would be checked, and active vegetation favoured. The excess instead of deficiency of produce which was thus obtained with the mineral manure and nitrate of soda consisted almost exclusively of gramineous herbage. *Poa trivialis* contributed about one-third, and *Bromus mollis* more than one-sixth of the whole produce; whilst *Lolium perenne, Dactylis glomerata, Avena flavescens,* and *Holcus lanatus* were also in fair proportion. The crop was characteristically very stemmy, the most so of any in the series, and *Bromus* especially was shedding ripe seeds. It will be remembered that whilst June, the maturing month, was in 1862 very wet and unusually cold, it was in 1867 very dry, and though not so warm as usual, by no means so cold as in 1862. Hence, the grasses which had been brought so rapidly forward under the influence of the nitrate of soda were unable to ripen. With this free and forward growth of grasses, leguminous plants occurred in quite insignificant amount, and miscellaneous species in less than average quantity. Of the latter, *Conopodium denudatum, Anthriscus sylvestris,* and *Rumex Acetosa* were in the greatest prominence.

Thus, with the very fluctuating climatal characters of the season of 1867, we have, with the varying floras, and the varying and irregularly and late-sown manures, widely varying characters of growth. There was in some cases an excess, and in others a deficiency of produce compared with the average. The different gramineous species were very differently affected; Leguminosæ were generally deficient, and miscellaneous plants were for the most part in excess. The growth was, upon the whole, much more characteristically leafy than stemmy; but with nitrate of soda it was very characteristically stemmy.

Season 1871–2.

Between the separation season of 1867 and that of 1872 a remarkable period intervened, so far as the growth on the experimental plots was concerned. One of those four intervening years (1869) was the one of highest productiveness in the whole series; another (1868) came only second in this respect; and a third (1870) gave by far the lowest produce in the entire series. It happens, therefore, that these three out of the four intermediate seasons came under detailed discussion when considering the seasons of highest, and of lowest, productiveness, in Part I. of this paper. As the conditions and results of growth referred to undoubtedly exercised considerable influence on the struggle from year to year, it seems desirable to make brief reference to the characters and results of those four seasons, before describing those of 1872, the season of separation itself.

The first of the four seasons, 1868, was on the whole the second in productiveness throughout the series. "Luxuriant early growth was followed by both unusual drought and unusual heat, yielding quantity by virtue of high development and maturation, as distinguished from succulence and immaturity." And the heavy and ripe first crops were succeeded by very meagre second growth.

The season of 1869 was the one of highest productiveness of all throughout the series. "The period prior to that of most active above-ground growth had brought the herbage into an unusual state of forwardness; when .... abundance of rain, with, upon the whole, low temperatures, gave great luxuriance, but comparatively leafy, succulent, and immature produce." And, as under the very different conditions in 1868, the second growth was considerably below average.

The season of 1870 was the one of lowest productiveness throughout the series. "The winter and early spring of 1870 had not upon the whole been deficient in rain, but the period had been extremely variable as to temperature, frequently very inclement; and on the average colder than usual. The herbage was, from these causes, very backward at the commencement of the active growing period. April, May, and June followed with a great deficiency of rain, very high day and low night temperatures, yielding very stunted, and prematurely ripened produce." The drought and heat still prevailing several weeks after the cutting of the first crops, the second crops (which were cut and spread on the plots) were also very meagre.

The season of 1871 gave more than the average produce of first crops under all conditions of manuring. The second crops were also uniformly above the average, and on the nitrate of soda plots very considerably so. The greater part of the winter, 1870–1, had been extremely severe, with a great deal of snow, and very cold winds. At the commencement of the growing time the plots were very bare and backward. With the exception of the latter half of April, that month, May and June were unusually cold, and there was an excess of rain in April and June, but a deficiency in May. June especially was very unseasonably cold and wet. The dryness of May checked undergrowth, and favoured the formation of flowering stems; but with the wetness of June a kind of second growth succeeded; and when the crop was cut there was, besides the early-formed flower stems and some seed, a large amount of leafy matter produced, much contributing to the weight of the crops.

Thus the first two years since the botanical separations of 1867 had given the two heaviest first crops in the series, the third the lightest, and the fourth more than the average. The heavy crop of the first year had been characterised by great stemminess and maturity, and the heavy ones of the second and fourth years by luxuriant leafy growth. The first (1868), and the third and smallest crop (1870), were the result of seasons of drought and heat; in the case of the heavy crop the drought and heat coming late, and succeeding upon previous luxuriance, and in the case of the light crop commencing early, and succeeding upon previous backward conditions.

Although the freer growth of the wetter seasons must obviously affect the relative predominance of the different components of the mixed herbage, seasons of heat and drought, like those of 1868 and 1870 (though they were so very different from one another), have a much greater influence in this respect. In fact, there is no doubt that the distribution and predominance of species in 1872 was influenced, not only by the continuous application of the different manures from year to year, but materially also by the characters and the growth of the several preceding seasons, especially by those of the years of drought and heat.

We come now to the characters and the growth of the season of 1872 itself. July 1871 was characterised by a considerable excess of rain and low temperatures; August and the first half of September were warm and dry; but the end of September was very wet, cold, and stormy. October, November, and December were considerably deficient in rain, with lower than average temperatures. Mr. Glaisher states, indeed, that November and the first half of December were characterised by the longest continuance of low temperatures in the century for that period of the year. After the long continued dry and very cold weather, the three months from the middle of December 1871 to the middle of March 1872 were almost continuously very unusually warm, with a considerable excess of rain in January, a deficiency in February, but again some excess in March. The latter half of March was exceedingly variable as to temperature; and this character prevailed until early in May; periods of unusually high and unusually low temperatures alternating, the higher, however, prevailing; whilst April was deficient in rain. Then followed about five weeks of mostly cold and cloudy weather with hoar frosts and frequent rain; the fall being in excess in both May and June, as it had been also in March. The last fortnight or so before the cutting of the grass was, however, warmer, and the period included some heavy thunderstorms.

Thus the early winter was dry and extremely cold; next followed nearly three months of prevailing high temperatures for the period, with a sufficiency of rain, so far favouring an early start of growth. Then came some weeks very variable as to temperature, with some deficiency of rain; the remainder of the growing period was unseasonably cold and cloudy, but with a sufficiency of rain; and, finally, the maturing period was warm, but interspersed with heavy showers.

These very changeable characters of season are obviously those of fair luxuriance, but of varying tendency to stem and seed formation, and to maturation, according to the already established botanical and other characteristics of the herbage, and the conditions of manuring supplied. The general result was less than average produce with defective conditions of manuring, but over average with high manuring where the nitrogen was applied as ammonia-salts, but less than average where it was applied as nitrate of soda.

Without manure the produce was very considerably deficient. The herbage was short, even, and scanty in development of both leaf and stem. Among the grasses, the hardy and surface- and fibrous-rooting *Festuca ovina* was the first, and the creeping-rooted *Agrostis vulgaris* the second, in order of predominance.

With the purely mineral manure there was, notwithstanding a considerable deficiency of grasses, more than average total produce, due to the growth of a very unusual proportion and amount of leguminous herbage, which consisted almost exclusively of *Lathyrus pratensis.* There was a fairly mixed, though deficient, undergrowth of grasses, *Festuca ovina* and *Agrostis vulgaris* being, however, by far the most prominent. Miscellaneous plants were in less amount than usual; the most prominent being *Achillea Millefolium,* which contributed nearly half; whilst *Luzula campestris, Conopodium denudatum,* and *Rumex Acetosa* were the next in prominence.

With the mixed mineral manure and the smaller quantity of ammonia-salts scarcely the average produce was yielded. The grasses were in about, or perhaps over, average amount. Leguminosæ were almost absent. Miscellaneous species were in less than average quantity. By far the most prominent grass was *Poa pratensis*—*Agrostis vulgaris, Dactylis glomerata, Avena elatior, Festuca ovina,* and *Holcus lanatus* being also fairly prominent. The growth was characterised by coarseness of both leaf and stem. The most prominent weed was *Rumex Acetosa,* which contributed nearly two-thirds of the miscellaneous herbage. The only other miscellaneous plants occurring in any noticeable quantity were *Conopodium denudatum* and *Achillea Millefolium.*

With the mixed mineral manure and the double quantity of ammonia-salts there was considerably more than average produce, and the excess was exclusively in grasses. Nearly two-fifths of the whole produce was made up of the free-growing and heavy-yielding *Dactylis glomerata;* and most of the remainder consisted of *Agrostis vulgaris, Alopecurus pratensis, Avena elatior, Poa pratensis,* and *Holcus lanatus,* in the order enumerated, but in fairly equal proportions. The herbage comprised therefore a fair quantity of a number of free-growing, and in some cases good meadow grasses. The growth was, however, very luxuriant and coarse. The only weed in any prominence was *Rumex Acetosa.*

With the mixed mineral manure and nitrate of soda there was rather less than average produce. The grasses were in about average amount. Leguminosæ were in more than average, but still in insignificant quantity. Miscellaneous plants were in considerably less amount than usual. Nearly half the gramineous herbage was made up of *Bromus mollis,* which was in full flower early in June; *Poa trivialis* was also very prominent. In much less quantity followed *Avena flavescens, Lolium perenne, Alopecurus pratensis, Holcus lanatus,* and *Dactylis glomerata.* The only leguminous plant in any quantity was *Lathyrus pratensis.* *Anthriscus sylvestris* was the only really prominent weed ; *Rumex Acetosa* and *Conopodium denudatum* coming next.

Thus, by virtue of the characters and results of the intermediate seasons since 1867, and especially by the effects of the two years of drought, 1868 and 1870, the distribution of plants on the different plots had become considerably modified. Among the grasses *Festuca ovina* had become very much more prevalent, especially on the less liberally manured plots; as also had *Agrostis vulgaris,* and even so with higher manuring. *Festuca ovina,* indeed, which had been first in predominance on only two plots in 1862, and on eight in 1867, was so on 13 in 1872; *Agrostis vulgaris* again, which had been second on seven plots in 1862, and on eight in 1867, was so on 14 in 1872. With mineral manure and ammonia-salts the free-growing *Dactylis* or *Poa pratensis,* had, however, maintained the first place ; *Agrostis vulgaris* and some others being also prominent, but *Festuca ovina* much less so. *Bromus mollis,* which had seeded very freely in 1870 and 1871 on the plot with mineral manure and nitrate of soda, was again first with that mixture in 1872, *Poa trivialis* coming second on that plot, but having considerably gone down on the deficiently manured and ammonia manured plots. A few other moderately good grasses also maintained a place. Leguminous herbage had only gained ground materially with mineral manure alone. Miscellaneous herbage had been generally reduced. The plants which had maintained their ground the most prominently were *Rumex Acetosa, Achillea Millefolium* (which had indeed considerably increased), and *Conopodium denudatum.*

To conclude,—with considerably modified floras and vegetation on the differently manured plots, due materially to recent seasons of luxuriance or drought, or both, and with very variable weather in the season of separation itself (1872), the greater part of the growing period being more favourable for luxuriance than maturation, and the ripening weather being late, broken by heavy showers and of short duration, we have very variable characteristics of produce, according to the conditions of manuring supplied.

Season 1876–7.

In the last section it has been shown that the flora and vegetation of the different plots had become considerably modified by the characteristics of the seasons prior to the separation year of 1872; and that the most potent of these influences had been the occurrence of seasons of drought and heat; whilst the character of the period of growth itself of the first crop of 1872 had been that of variableness, thus differently affecting the mixed herbage on the differently manured plots according to the conditions which had been induced under the influence of the several preceding seasons.

As to amount of produce, the first crops of 1872 had been below average with defective manuring, but average, or above average, with liberal manuring. After the cutting of the first crops there was a considerable amount of rain; the growth was somewhat dense, and the second crops were pretty uniformly over average.

The winter of 1872–3 was upon the whole very wet; in the earlier part of it warmer than the average, but in February, and also in March, there were deep snows and sharp frosts. Then followed a dry grass-growing season, with, for the most part, lower than average temperatures. The result was backward growth and deficient first crops of hay under every condition of manuring, and very considerable deficiency under most conditions. The grasses were dwarfish, yielding but little stem and rather thin undergrowth, thus favouring the luxuriance of a few strong growing weeds, such as *Rumex Acetosa*, *Centaurea nigra*, and *Achillea Millefolium;* whilst leguminous herbage was less prominent than usual. After the removal of the first crops there was more than the average amount of rain, there was fairly dense growth on most of the plots, yielding two cuttings, one in the middle of August and the other in the middle of October, the produce in both cases being spread on the respective plots. On many plots *Festuca ovina* contributed a large proportion of the dense grassy undergrowth.

The winter and early spring of 1873–4 were upon the whole considerably warmer than usual, but with a considerable deficiency of rain, so that vegetation remained very backward. Then followed continued drought, with unusual cold both day and night, and the already backward herbage was very materially damaged, yielding not only checked and stunted, but really injured crops. Indeed, the grass-season of 1874 was the second in order of unproductiveness among the first 20 of the experiments, and as such has already been considered in more detail in Part I. of this paper. July was hotter than the average, with, however, a sufficiency of rain; August was dry but cold; September and October were wet, with rather above average temperatures. There was, nevertheless, but little growth after the first cutting, excepting on a few of the highly-manured plots. A second crop was cut in the middle of August, and a third at the end of September, both being spread on the respective plots.

The winter of 1874–5 was variable, but included a good deal of severe weather, with more than the average fall of rain. There was a deficiency of rain, with lower than average temperatures, in February, March, and April; the period being upon the whole inclement, cold, and dry, and vegetation was accordingly backward. There was an excess of rain in May and June, with at the same time higher than average temperatures. May especially was warm, showery, and genial, stimulating the growth of most species; whilst June improved some, but retarded others, the middle of the month being stormy and sunless. The produce was, on most of the experimental plots, below average; but on a few of those most highly-manured, above average. The wet and warm May was specially favourable to such moisture-loving and surface-rooting plants as *Agrostis vulgaris* and *Poa trivialis,* which came much to the front; and the grasses showed such a dense leafy-growth that the leguminous and miscellaneous species were less prominent than usual. After the cutting of the first crop there was a great excess of rain in July, though with lower than average temperatures in that month, but higher afterwards. The result was that the short yield of the first crop was succeeded by an exceedingly luxuriant aftermath. Among the grasses on the highly-manured plots, the large free-growing species were the most favoured, and they yielded flower-stems in abundance. On the plots with mineral manure and ammonia-salts *Dactylis glomerata* and *Avena elatior* were specially prominent; and with mineral manure and nitrate of soda many seedlings of *Bromus mollis* were observed. Leguminous and miscellaneous plants were also generally very prolific in flower-stems, and in many cases seeded. At the time of cutting the second crops, before the middle of September, the unmanured and the mineral-manured plots showed great variety of colour owing to the number of species in flower. On the plots heavily dressed with mineral manures and ammonia-salts the general herbage ranged from 9 to 18 inches in height, whilst the flowering stems of the grasses were from 18 inches to 3 feet high, according to the quantity of nitrogenous manure employed. The mineral manure and nitrate of soda, on the other hand, which had yielded the heaviest first crop gave less excessive aftergrowth, and but few flowering stems. Under all conditions as to manure the second cuttings were, however, much heavier than the average; indeed, excepting with the nitrate, generally very heavy.

Owing to the amount of the crops, to the difficulty which had been found in dealing with the second crops without removing them, and to the fact that a period of 20 years had now been completed, during which only the first crops had been removed, it was decided henceforward to cut and remove the second crops whenever practicable; and, accordingly, the heavy second crops of 1875 were so treated.

It is obvious that the removal of the second crops materially affects the condition of the land, and in the case of the deficiently-manured plots tends to much more rapid exhaustion. It is also obvious that, with this change, a new element in the struggle among the components of the mixed herbage is brought into play, or at any rate the effects of an old one are considerably intensified.

The late autumn, winter, and early spring of 1875–6 were, excepting in December and January, very wet; and the two comparatively dry winter months were notably colder than the average. In the middle of April, 1876, there was a very unusually heavy snow fall; and, succeeding upon this unseasonable weather, May and June were both unusually dry and unusually cold. The result was a backward and generally scanty and thin crop, with a deficiency of bottom grass. The leguminous and miscellaneous species, as well as the gramineous, were deficient. It was only with the highest manuring that the weight of produce was average, or over average; the result in these cases being due to a few of the freer-growing species, with comparatively extended root-range, attaining considerable luxuriance under those conditions; whilst, on most of the plots, the amount of produce was much below the average. The first crops were not cut until the end of June. July was both warm and dry, August was warm and wet, and September cold and very wet. The second crops were for the most part patchy; but the wet weather of September brought forward a fair amount of leafy bottom growth. The quantity of produce was, however, very much less than in

1875, and in contrast to the second growth of that season, there was, in 1876, scarcely any tendency to formation of stem, and both leguminous and miscellaneous species were deficient and backward. The crops were cut towards the end of September; but owing to the wetness and coldness of the weather they could not be made into hay and removed, and were, therefore, spread upon the respective plots.

We have now to consider the characters of the season, and the general characters of the produce, of 1877: the fourth and last of the years of complete botanical separation.

In November, December, and January, 1876–7, there was a great excess of rain; the total excess in the three months amounting to about 8 inches. There was again an excess in each of the months of February, March, April, and May; the total excess in these four months amounting to about 2½ inches. With this great excess of rain over seven consecutive months, from the beginning of the winter to May inclusive, there was, at the same time, in the first four months of the period, November to February inclusive, a considerable excess of temperature, both maximum and minimum; but in the next three months, March, April, and May, the temperatures were for the most part unusually low.

Thus, during the winter and spring the soil was saturated with water, and with the warmth as well as wetness of the winter, grass was very green, forward, and promising at the commencement of the usual active growing period; but the cold weather of April and May greatly checked vegetation. About the middle of April piercingly cold and boisterous winds prevailed, and, although they tended to dry the saturated soil, they were otherwise adverse to growth. Early in May the weather was extremely cold, with north-east winds, storms of sleet, and severe night frosts. Almost all species seemed to suffer, and it was not until about the third week of the month that there was genial growing weather, brightening the prospects of the hay harvest, but coming too late entirely to overcome the effects of the previous adverse conditions. Hence, at the commencement of June the mixed herbage generally presented a very backward appearance, grasses were short and benty, with little undergrowth; but, with the abundance of moisture within the soil, and the warm forcing weather of the early part of June, rapid growth then set in, and the mixed herbage thickened surprisingly, and the grasses especially came quickly to maturity. Eventually the weight of produce was over average on most of the experimental plots, and considerably over average on a few of those the most liberally manured. But, as the following comments on the herbage of a few typical plots, and the detailed results given further on will show, there was considerable irregularity in the condition of development of the different components of the mixed herbage.

Without manure, there was about the average total produce, a full number of species, a rather higher than average percentage, and about an average amount of grasses; there was a fair proportion of leguminous species, but less than the average amount of miscellaneous herbage. *Festuca ovina* was the most prominent grass, *Agrostis vulgaris* coming second, and *Holcus lanatus* third; whilst *Briza media, Anthoxanthum odoratum,* and *Lolium perenne* were fairly represented. Of the leguminous herbage, the deep-rooting *Lotus corniculatus* contributed nearly half, and *Lathyrus pratensis* and *Trifolium pratense* most of the remainder. Of miscellaneous species, *Ranunculus* (*repens* and *bulbosus*) and *Plantago lanceolata* together contributed about a third, *Achillea Millefolium, Conopodium denudatum, Rumex Acetosa,* and *Luzula campestris* making up most of the remainder. The crop was dwarf, both in leaf and stem, and mostly backward; but the miscellaneous species were upon the whole rather more advanced than either the grasses or the Leguminosæ.

With the mixed mineral manure alone there was considerably more than the average quantity of total produce, the grasses being in larger, but the leguminous and miscellaneous species in less percentage as well as less actual amount than usual. As without manure, *Festuca ovina* was by far the most prominent grass, *Holcus lanatus* and *Agrostis vulgaris* together contributing about as much, and *Dactylis glomerata, Avena flavescens, Anthoxanthum odoratum,* and *Lolium perenne* each contributed several per cent. to the produce. Of the total leguminous growth, *Lathyrus pratensis* contributed more than seven-eighths; of the miscellaneous herbage, *Rumex Acetosa* contributed more than half. Next in order was *Conopodium denudatum,* yielding less than a quarter as much, and five or six other species made up most of the remainder. The crop was dense and moderately tall, with a fair amount of both stem and undergrowth. Among the grasses *Festuca ovina* gave the largest proportion of stem, and *Agrostis vulgaris* the largest proportion of leaf. The herbage was upon the whole more matured than without manure, though the leguminous plants were rather more backward than usual.

With the mixed mineral manure and the smaller amount of ammonia-salts, the weight of produce was more than the average. Nearly 95 per cent. by weight consisted of grasses, less than a half per cent. of leguminous, and less than 5 per cent. of miscellaneous species; the grasses contributing a larger, and the miscellaneous species a much smaller proportion than usual. *Festuca ovina* was again the most prominent grass; but *Poa pratensis, Dactylis glomerata, Avena elatior, Agrostis vulgaris,* and *Holcus lanatus* were also in considerable amount, and the six grasses enumerated contributed nearly 90 per cent. of the total produce. The only weed in any prominence was *Rumex Acetosa.* The crop was heavy and dense, with a large proportion of tall, luxuriant, and matured flowering stems. *Avena elatior* was the most, and *Holcus lanatus* and *Agrostis vulgaris* were the least, advanced of the prominent grasses.

With the mixed mineral manure and the double quantity of ammonia-salts there was much more than the average amount of total produce. There was a smaller number of species than usual. The grasses contributed about 98 per cent. of the whole; no Leguminosæ were found, only four miscellaneous species were observed, and *Rumex Acetosa* was the only one in any prominence. Among the grasses the freely-growing *Agrostis vulgaris, Holcus lanatus, Avena elatior, Alopecurus pratensis,* and *Dactylis glomerata* all greatly exceeded in amount the *Festuca ovina,* which was here in but small proportion, and was nearly equalled by *Poa pratensis.* Upon the whole the growth was rank and tufty, and the free-growing grasses yielded a considerable amount of flowering stems.

With the mixed mineral manure and the larger quantity of nitrate of soda (equal in nitrogen to the smaller quantity of ammonia-salts) there was scarcely the average weight of total produce. The number of species was lower than usual; the grasses contributed about 88 per cent. of the total weight; the Leguminosæ gave less than 1 per cent.; but the miscellaneous species between 11 and 12 per cent., which was considerably more than the average on that plot. *Poa trivialis* and *Alopecurus pratensis* were by far the most prominent grasses, *Holcus lanatus* and *Dactylis glomerata* coming next, whilst *Bromus mollis* and *Poa pratensis* were also fairly represented. *Lathyrus pratensis* was the only leguminous plant observed. *Anthriscus sylvestris* and *Rumex Acetosa* contributed more than three-fourths of the miscellaneous herbage. The crop on this plot was the most evenly luxuriant, and the most matured, of the series, yielding a large bulk of stem, with comparatively little leaf, and a large proportion of the plants were either in flower or seed, the base of the stems turning brown with ripeness.

Between the years of separation of 1867 and 1872, as far as the influences of season were concerned in modifying the vegetation of the different plots, there had been years of great luxuriance of growth, and years of unusual drought with heat, and it was the latter which had the most marked effect in modifying the struggle established under the different manuring conditions.

Between the years of separation of 1872 and 1877, the influences of season were of a different kind. The first crops of the intermediate years were only average or under average in amount, and those of the second year were not only much under average, but the herbage was really damaged by the dryness and cold of the growing period; whilst in each of the years excepting the third, that period was considerably deficient in rain, and marked by unseasonably cold intervals, much checking vegetation. Nevertheless the *hay-year*, reckoned from July of one year to the end of June of the year of growth, showed in each case excepting that of the second year, an excess of rain over the average, which, however, chiefly affected the aftermath. It is, indeed, obvious that an excess of rain prior to the period of active growth of the first crops may leave the soil and subsoil in such a moist condition as to render the herbage less dependent on the fall of the actual period of growth itself; but, in so far as the excess of rain increases the second crops, the condition of the herbage will be affected for the growth of the succeeding first crops. It happens that, whilst the third and fifth series of autumn crops grown since the separation of 1872 were deficient and variable in character, owing partly to the unseasonable weather of the period, and partly to the previously induced condition of the herbage, the first, second, and fourth seasons gave more or less luxuriant second growth; and the fourth season not only gave the heaviest second crops of the series of years, but the produce was for the first time removed from the land.

We thus had, prior to the season of separation of 1877, several seasons of more or less ungenial weather for the growth of the first crops, actually damaged herbage, in one case affecting even the second crop, with, in other cases, comparatively luxuriant second growth, and one heavy second crop removed; and such were the conditions preparatory to the growth of the crop to be submitted to botanical analysis. Lastly, the season of separation itself, following these conditions, was characterised by a wet and mild winter and spring, giving early promise; growth was then checked by cold, but eventually luxuriant and heavy crops were obtained, which, however, were very variable in character of development according to the character of the manures employed.

The general result was a tendency to an increased proportion of gramineous herbage, and a diminished proportion, especially of the miscellaneous species, on most of the plots. There was also a tendency to a diminution in the proportion of the leguminous herbage on those plots which had been under the same treatment from the beginning; but a tendency to increase on some where the manures favourable to such plants had not been employed until more recently. Further, with the prevailing wetness of the intervening period, the freer-growing grasses gained ground on those plots where the manures were the most favourable for the luxuriant growth of gramineous herbage.

The foregoing review of the characters of the seasons preparatory to, and during the actual growth of the four crops submitted to detailed botanical separation, will sufficiently show that, independently of the very much more marked influences of the different manures in determining the results of the struggle between the members of the different Orders, or between the different species referable to the same Order, or even to the same genus—the effects of season must be taken account of, as constituting one important element among the many influences involved in bringing about the final result.

THE FLORA OF THE PLOTS; ORDERS, GENERA, AND SPECIES REPRESENTED.

Under this heading we propose to consider the number and the general character of the plants which have been observed during the course of the experiments on all the plots collectively.

Number of Orders, Genera, and Species.—The total number of species observed upon the plots is 89, comprised in 63 genera, and 22 Orders. The Dicotyledons number 59 species; the Monocotyledons 26; and the Acotyledons (including three Mosses but excluding Fungi) 4. The following is a list of the Orders, and of the number of genera and species within each, represented on the plots. As will be seen, the sequence of the Orders is according to the number of species they respectively contributed to the herbage of the plots (see Table XXXV., p. 1207).

Table XXXV.

Orders.	Number of genera.	Number of species.
1. Gramineæ	14	20
2. Compositæ	12	13
3. Leguminosæ	5	10
4. Rosaceæ	5	6
5. Ranunculaceæ	1	5
6. Umbelliferæ	5	5
7. Labiatæ	3	3
8. Polygonaceæ	1	3
9. Liliaceæ	3	3
10. Caryophyllcæ	2	3
11. Scrophulariaceæ	1	3
12. Musci	1	3
13. Rubiaceæ	1	2
14. Plantaginaceæ	1	2
15. Cruciferæ	1	1
16. Hypericaceæ	1	1
17. Dipsaceæ	1	1
18. Primulaceæ	1	1
19. Orchidaceæ	1	1
20. Juncaceæ	1	1
21. Cyperaceæ	1	1
22. Filices	1	1
Total	63	89

The number of species detailed in the foregoing table does not, however, give any idea of the degree of predominance or of the absolute quantities of any particular species, or of the number of species, on any individual plot. It may happen that one species, *e.g.*, *Dactylis glomerata*, may, in one season, or under one particular manurial condition, all but monopolise the area of the plots. An estimate of the relative preponderance of the different species is only arrived at by observation during growth, and after the removal of the crop, and by the careful analysis of the samples from the results of which the percentage and the weight of each species is deduced, as explained in the introduction.*

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* In reference to this subject the following statements relating to the absolute number of individual plants in a given area may be cited.

Darwin, 'Origin of Species,' chapter 3, mentions 357 seedlings of various kinds as having come up upon an area 3 feet long and 2 feet wide, previously cleared. Of these no less than 295 were destroyed chiefly by slugs or insects.

Sinclair, in his 'Hortus Gramineus Woburnensis' (1824), p. 244, 8th edition, gives a table showing the number of distinct rooted plants of various species found in one square foot in nine separate localities. The number varied from 1,798 to 634; and where particular species only were grown by themselves, as in the case of rye grass, to 75 only. Of the 1,798 plants in one foot square of a well managed water meadow, 1,702 were grasses, and 96 clover and other plants. Of 910 plants in a foot square of old pas-

It is desirable in this place to give a complete list of the species observed, together with a statement of the number of plots on which their presence was ascertained, in whatever amount, large or small, in each of the four separation years. In some cases a particular species may be very largely represented on a given plot, while on another, although it may have been observed on the ground during the course of the season, it may have disappeared prior to the samples being taken, or it may have been present in such small proportion as not to come in appreciable quantity into the samples taken at one particular time. This remark applies especially to those Leguminosæ and miscellaneous plants whose presence on particular plots is all but infinitesimal. Although it has been thought desirable to enumerate all the species which have, at any time, been noticed on the plots, yet it is to be observed that the total number of species occurring in such quantity as to be represented in the samples, even from plots where the vegetation is the most varied, does not exceed an average of 50.

In the complete list which here follows, Table XXXVI., the species are arranged in the three groups of *Grasses, Leguminosæ,* and *Miscellaneous Plants.*

ture in Woburn Park, 880 were grasses, 30 clover and other plants, the number of separate species being only 12.

Drs. Oemler and Fuchs (Die Landwirth. Versuchs-Stat., Bd. xvii, 211) give the following table showing the number of plants growing in one square foot of meadow-land in Schleswig :—

	No. of Plants.
Festuca pratensis	100
Holcus lanatus	66
Poa pratensis	64
Phleum pratense	59
Avena elatior	41
Dactylis glomerata	32
Lolium perenne, fine	24
Lolium perenne	22
Alopecurus pratensis	14
Rumex Acetosa	4
Ranunculus repens	3
Poa annua	2
Per square foot	431

These figures suffice to show the great range of variation as to the number of individual plants found under different circumstances according to the intensity of the struggle and competition between them on a given area; and they bring out the fact that the number is less in proportion as the number of species is less. That is to say, complexity of herbage is an indication of feeble competition, and hence the greater number of individual plants; whilst a small number of species is most probably associated with more active struggle, greater luxuriance of individual plants, and therefore a smaller number in a given area.

Darwin, in his 'Origin of Species,' chapter 3, alludes to the same fact in these words: "The struggle almost invariably will be most severe between the individuals of the same species, for they frequent the same districts, require the same food and are exposed to the same dangers."

TABLE XXXVI.—Complete List of Species that have been either found in the samples, or observed on the plots.

	Number of Plots on which found in :—				Mean.
	1862. (1)	1867. (1)	1872. (1)	1877. (1)	
GRAMINEÆ.					
1. Anthoxanthum odoratum, *L.*	19	20	22	22	21
2. Alopecurus pratensis, *L.*	19	20	22	22	21
3. Phleum pratense, *L.*	6	5	6	3	5
4. Agrostis vulgaris, *With.*	19	20	22	22	21
5. Aira cæspitosa, *L.*	5	3	17	9	9
6. Holcus lanatus, *L.*	19	20	22	22	21
7. Avena elatior, *L.*	18	20 (2)	20	21 (3)	20
8. ,, pubescens, *L.*	20 (4)	20	20	20	20
9. ,, flavescens, *L.*	19	20	22	22	21
10. Poa pratensis, *L.*	19	20	22	22	21
11. ,, trivialis, *L.*	19	20	22	22	21
12. Briza media, *L.*	18 (5)	15 (6)	17	12	16
13. Dactylis glomerata, *L.*	19	20	22	22	21
14. Cynosurus cristatus, *L.*	19 (7)	14	16	16 (8)	16
15. Festuca ovina, *L.*	19	20	22	22	21
16. ,, pratensis, *Huds.*	17	16	18	16	17
17. ,, elatior, *L.*	0	0	0	1	1
18. ,, loliacea, *Huds.*, a var. of pratensis	0	0	0	2	1
19. Bromus mollis, *L.*	19	18	19	12	17
20. Lolium perenne, *L.*	19	20	21	21	21
LEGUMINOSÆ.					
1. Trifolium repens, *L.*	17	17	18	16 (9)	17
2. ,, pratense, *L.*	17 (10)	14	17	18 (11)	17
3. ,, minus, *Sm.*	0	0	0	1	1
4. ,, procumbens, *L.*	1	1	1	0	1
5. Lotus corniculatus, *L.*	16 (12)	14	15	20 (13)	16
6. ,, major, *Scop.*	0	0	0	1	1
7. Lathyrus pratensis	17	19 (14)	19	20 (15)	19
8. Ononis arvensis, *L.*	1 (16)	1 (17)	1	1	1
9. Vicia sepium, *L.*	0	0	1 (18)	0	1
10. ,, cracca, *L.*	0	1 (19)	1 (20)	1	1

(1) Total number of plots in 1862, 19; in 1867, 20; and in 1872 and 1877, 22.

(2) In quantity sufficient to be represented in the samples from 16 plots only, though observed on 4 others during growth.

(3) In 19 samples only.
(4) In 19 samples only.
(5) In 9 samples only.
(6) In 9 samples only.
(7) In 14 samples only.
(8) In 14 samples only.
(9) In 15 samples only.
(10) In 15 samples only.
(11) In 15 samples only.
(12) In 10 samples only.
(13) In 15 samples only.
(14) In 16 samples only.
(15) In 18 samples only.
(16) In no sample.
(17) In no sample.
(18) In no sample.
(19) In no sample.
(20) In no sample.

TABLE XXXVI. (continued).—Complete List of Species that have been either found in the samples, or observed on the plots.

	Number of Plots on which found in:—				Mean.
	1862. (1)	1867. (1)	1872. (1)	1877. (1)	
MISCELLANEOUS SPECIES.					
Ranunculaceæ—					
1. Ranunculus acris, *L.*	19 (2)	17 (3)	16	21 (4)	18
2. ,, bulbosus, *L.*	19	17	18	19 (5)	18
3. ,, repens, *L.*					
4. ,, auricomus, *L.*	0	1 (6)	1 (7)	1	1
5. ,, Ficaria (8)	0	0	0	0	0
Cruciferæ—					
6. Cardamine pratensis, *L.*	0	0	2 (9)	0	1
Caryophylleæ—					
7. Stellaria graminea, *L.*	16 (10)	13	14	15 (11)	15
8. ,, Holostea, *L.*	0	0	0	1	1
9. Cerastium triviale, *Linn.*	15	15	16	16 (12)	16
Hypericineæ—					
10. Hypericum perforatum, *L.*	0	1	0	1	1
Rosaceæ—					
11. Potentilla reptans, *L.*	6	6	9	16 (13)	9
12. ,, Fragariastrum, *Ehr.*	0	1	1	1	1
13. Alchemilla vulgaris, *L.*	0	0	0	1 (14)	1
14. Agrimonia Eupatoria, *L.*	2	4	5	6 (15)	4
15. Poterium Sanguisorba, *L.*	1	2	2	2	2
16. Spiræa Ulmaria, *L.*	1	2	3	6	3
Umbelliferæ—					
17. Conopodium denudatum, *Koch*	19	20	22	22	21
18. Pimpinella Saxifraga, *L.*	18	17	20	20 (16)	19
19. Heracleum Sphondylium, *L.*	15 (17)	3 (18)	10	14	11
20. Anthriscus sylvestris, *Hoffm.*	2 (19)	3	4	3 (20)	3
21. Daucus Carota, *L.*	0	0	1 (21)	0	1
Rubiaceæ—					
22. Galium verum, *L.*	14 (22)	14 (23)	15	19 (24)	16
23. ,, Aparine, *L.*	1	0	2	7	3
Dipsaceæ—					
24. Scabiosa arvensis, *L.*	7	6 (25)	10	11	9
Compositæ—					
25. Centaurea nigra, *L.*	15 (26)	17 (27)	18	17	17
26. Carduus arvensis, *Curtis*	2 (28)	2	1	2 (29)	2
27. Bellis perennis, *L.*	5 (30)	5	6	8 (31)	6
28. Achillea Millefolium, *L.*	19	20	20	21	20
29. Chrysanthemum Leucanthemum, *L.*	1 (32)	4 (33)	1	4 (34)	3
30. Senecio erucifolius, *L.*	0	0	1 (35)	0	1
31. Hypochæris radicata, *L.*	0	5 (36)	1 (37)	0	2

(1) Total number of plots in 1862, 19; in 1867, 20; and in 1872 and 1877, 22.
(2) In quantity sufficient to be represented in the samples from 10 plots only.
(3) In 14 samples only.
(4) In 15 samples only.
(5) In 16 samples only.
(6) In no sample.
(7) In no sample.
(8) In no sample, but observed on 11 plots in 1871.
(9) In no sample.
(10) In 8 samples only.
(11) In 13 samples only.
(12) In 15 samples only.
(13) In 10 samples only.
(14) In no sample.
(15) In 4 samples only.
(16) In 19 samples only.
(17) In 1 sample only.
(18) In 2 samples only.
(19) In no sample.
(20) In 2 samples only.
(21) In no sample.
(22) In 5 samples only.
(23) In 8 samples only.
(24) In 13 samples only.
(25) In 5 samples only.
(26) In 12 samples only.
(27) In 15 samples only.
(28) In 1 sample only.
(29) In 1 sample only.
(30) In 2 samples only.
(31) In 5 samples only.
(32) In 1 sample only.
(33) In no sample.
(34) In 2 samples only.
(35) In no sample.
(36) In no sample.
(37) In no sample.

TABLE XXXVI (continued).—Complete List of Species that have been either found in the samples, or observed on the plots.

	Number of Plots on which found in :—				Mean.
	1862. (1)	1867. (1)	1872. (1)	1877. (1)	
MISCELLANEOUS SPECIES (continued).					
Compositæ (continued)—					
32. Tragopogon pratensis, *L.*	4	5	6	7 (2)	5
33. Leontodon hispidus, *L.*	5	8	6	6	6
34. ,, autumnalis, *L.*	1	3	1	1	2
35. Taraxacum officinale, *Wigg.*	17	17 (3)	13	10	14
36. Sonchus oleraceus, *L.*	0	0	1 (4)	0	1
37. Hieracium Pilosella, *L.*	3	4	3	4 (5)	4
Plantagineæ—					
38. Plantago lanceolata, *L.*	16	14	14	16	15
39. ,, media, *L.*	0	1 (6)	0	0	1
Scrophularineæ—					
40. Veronica Chamædrys, *L.*	14	15	16	16 (7)	15
41. ,, serpyllifolia, *L.*	0	1 (8)	2 (8)	2	1
42. ,, officinalis, *L.*	0	0	0	1 (9)	1
Labiatæ—					
43. Thymus Serpyllum, *L.*	0	0	0	2	1
44. Prunella vulgaris, *L.*	3	6	6	12 (10)	7
45. Ajuga reptans, *L.*	3	5	7	11 (11)	7
Primulaceæ—					
46. Primula veris, *L.*	6 (12)	6 (13)	1	8 (14)	5
Polygonaceæ—					
47. Rumex Acetosa, *L.*	19	20	22	22	21
48. ,, crispus, *L.*	0	2 (15)	2 (16)	2 (17)	2
49. ,, obtusifolius, *L.*	0	1 (18)	1 (19)	1 (20)	1
Orchideæ—					
50. Orchis Morio, *L.*	3 (21)	0	1 (22)	1 (23)	1
Liliaceæ—					
51. Scilla nutans, *Sm.*	1	0	3	6	3
52. Fritillaria Meleagris, *L.*	0	2 (24)	1 (25)	0	1
53. Ornithogalum umbellatum, *L.*	0	0	1	0	1
Juncaceæ—					
54. Luzula campestris, *Willd.*	16	20	19	19 (26)	19
Cyperaceæ—					
55. Carex præcox, *Jacq.*	12	13	16	10	13
Filices—					
56. Ophioglossum vulgatum, *L.*	4	4 (27)	4 (27)	3 (28)	4
Musci—					
57. Hypnum squarrosum, *L.* 58. ,, rutabulum, *L.* 59. ,, hians, *Hedwig.?*	10	8	15	20	13

(1) Total number of plots in 1862, 19; in 1867, 20; and in 1872 and 1877, 22.
(2) In quantity sufficient to be represented in the samples from 6 plots only.
(3) In 16 samples only.
(4) In no sample.
(5) In 3 samples only.
(6) In no sample.
(7) In 13 samples only.
(8) In no sample.
(9) In no sample.
(10) In 7 samples only.
(11) In 10 samples only.
(12) In 4 samples only.
(13) In 2 samples only.
(14) In 6 samples only.
(15) In no sample.
(16) In no sample.
(17) In 1 sample only.
(18) In no sample.
(19) In no sample.
(20) In no sample.
(21) In no sample.
(22) In no sample.
(23) In no sample.
(24) In no sample.
(25) In no sample.
(26) In 18 samples only.
(27) In no sample.
(28) In 1 sample only.

In addition to the foregoing, it may be mentioned that seedlings of the common oak, *Quercus Robur L.*, occasionally come up in small numbers near the trees, but they are never able to maintain themselves, and may be passed over without further notice.

The nomenclature of the species which has been adopted is that of Sir J. HOOKER's 'Student's Flora.'

The classification into gramineous, leguminous, and miscellaneous plants has many practical advantages for our special purpose. Grasses constitute by far the largest proportion of the plants found on the plots; Leguminosæ are very distinct in many aspects, and, as is more fully shown in the sections treating of the effects of the various manures, they often manifest contrary tendencies to those of the grasses—a circumstance not to be wondered at when it is remembered how great is the difference between the leaves and the roots of most grasses and those of Leguminosæ, and coincidently how different are the requirements of the two.

The relation of the various miscellaneous orders to the grasses, and to the leguminous plants, and to one another, cannot be dealt with in a few generalising paragraphs. The mode of growth, and the root-development, of most of them are, speaking generally, much more like those of the leguminous plants than those of the grasses. That their greater or less relative prevalence is very much an affair of season, encouraging or discouraging, as the case may be, the growth of their competitors, the grasses, is shown in the subsequent sections relating to the several plants and plots. Again, though present conditions avail much in regulating the distribution and comparative prevalence of various groups of plants, it is now well recognised that causes anterior to the existing order of things have determined the existence of larger or smaller number of the species of each particular family.

The alleged antagonisms between plants.—DUREAU DE LAMALLE* was one of the first to call attention to the apparent antagonism of certain plants, and to their alternate predominance, the one over the other—a fact frequently observed in the case of forests. He pointed out that grasses were in his experience the most powerful enemies of Saintfoin and of Lucerne; that they overcome them when growing together, without however being able to destroy them utterly. Moreover, he remarked that in some isolated plateaux never manured or irrigated, he saw, five or six times in the course of 30 years, grasses and Leguminosæ lose and regain the prominence one over the other. Similar phenomena have been observed at Rothamsted, as will be illustrated in the sequel.

When, however, as at Rothamsted, investigation is pushed further, and when particular species of grasses, or of Leguminosæ, &c., are examined as to their behaviour

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* "Mémoire sur l'alternance, ou sur ce problème: la succession alternative dans la reproduction des espèces végétales, vivantes en société, est elle une loi générale de la nature?"—Ann. Sc. Nat., Ser. 1, 1825, vol. 5, p. 50. See also ALPHONSE DE CANDOLLE, 'Géographie Botanique,' tom. i., p. 472.

with particular manures, then it is found that just as certain orders, taken collectively, say grasses, and Leguminosæ, have opposite tendencies and different requirements, so certain species of the same Order and even of the same genus may exhibit contrary phenomena. Just as Leguminosæ as a whole may be practically banished, and Gramineæ made to prevail, so certain species of Gramineæ or of Leguminosæ can be driven out, or, at least, some particular species may be so greatly favoured that the others are banished, either by the superior vigour exhibited by the victorious species, or by the directly injurious agency of the manure applied. From this point of view the comparative details hereafter given as to various members of the same genus which exhibit opposite tendencies, especially *Poa pratensis* and *Poa trivialis*, and the different species of *Avena*, which nevertheless manifest very opposite tendencies, may profitably be studied.

This opposite tendency between nearly allied plants (further illustrated by the occasional impossibility of grafting, or of mutually fertilising, allied species), is a most interesting phenomenon, and of great importance from both a scientific and a practical point of view. Several illustrations of it will appear further on; meanwhile we may, in passing, allude to the necessity for caution in the interpretation of isolated experiments which this fact demonstrates. Because a particular grass, or other plant, is little benefited by ammonia-salts, for instance, it does not follow that it will not be favoured by nitrates; nor, because if while growing in association with other species it may not be specially benefited by a particular manure, does it follow that it would not derive advantage from the same substance when growing separately.

The alleged antagonism of certain plants has been supposed to be due to the injurious effects of root-excretions;* but this supposition is, to say the least, not a sufficient explanation of the observed phenomena. The true explanation of the facts recorded lies probably in the varying conformation and requirements of different plants, as will be illustrated in subsequent sections. The advantages derived from the practice of rotation also receive illustration from these same circumstances.

Habit and Conformation of Plants: Organisation by means of which they maintain or improve their position, or succumb in competition with others.

Before passing to the consideration of individual species, a few general remarks on the structure and mode of life of the plants with which we have to deal are requisite. All the plants are, indeed, well known, and many of the facts we have to call attention to are equally familiar. Nevertheless, it is important to note the points of greatest significance in the consideration of the endowments by virtue of which plants growing in association maintain their position, prevail over, or succumb to their competitors.

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* See A. P. De Candolle, 'Physiologie Végétale,' p. 1474; Duchartre's 'Eléments de Botanique,' p. 246; Braconnot, 'Ann. Phys. et Chimie,' 1839, t. 72, p. 27.

Almost all the plants on the plots are perennials—very few are annual. The duration of the plant in some cases is indeed increased by the rarity with which seeds are perfected. The hay-crop being cut when but few plants have had time to ripen seed, and some have not even been able to flower, it is clear that the annual plants have little chance of maintaining their ground. In other cases, however, seeds are produced abundantly.

Hardiness, &c.—The hardiness or power of resisting frost or drought, especially during the growing season, is a matter evidently of foremost consequence. Although most of the plants can bear a very low temperature with comparative impunity, if in a quiescent state, yet the young growing shoots or leaves are liable to suffer injury from frost. Some of the plants on the plots are protected from the action of frost by the remains of the dead leaves of the former year; but while this protection may be useful in winter, it may be disadvantageous in spring by promoting premature growth. The early or late development in spring then becomes an element of considerable importance among the causes tending to the supremacy of certain plants over others. Supposing two plants to have started into growth together, it is obvious that the more hardy of the two would have a better chance of enduring the onslaught of frost at this critical period than the more tender one, and would hence gain the advantage over it.

Apart from the varying effect of frost or other injurious circumstance on different plants, it has to be taken into consideration that different plants have different seasons of growth. All other conditions being equal, some species, or individuals by hereditary tendency, start into growth earlier or later than others. It is clear that the late-growing species would be at a disadvantage when growing in association with more early developed plants of equal hardihood; because the latter would have occupied the ground, to some extent, before the former had had time to develop themselves.

So far as ability to withstand drought is concerned, it is certain that it depends most materially on the underground development, either of the stock or of the roots proper. A striking illustration of this has been given at p. 334, *et seq.*, of Part I. In considering the competition that is going on between one description of plant and another, the form, size, and direction of the leaves, the manner in which they are protected from the effects of excessive radiation, and their tendency to transpire much or little, according to the surface they expose to the air, &c., have also to be taken into account, as well as the variation in their anatomical organisation according to the circumstances under which they grow. These points are referred to in more or less detail under the head of the individual species; but some general remarks may appropriately be given in this place.

Roots and root-hairs.—It is necessary to draw particular attention to the generally recognised difference between the true feeding roots and those organs also commonly called roots, but whose office is that of transmitting or storing nutrient matter or substances capable of conversion into such. The true feeding roots, those portions which absorb the watery solutions from the soil, are, as is well known, the minute fibrils, and even of these it is the portions near to the distal ends only which are active. From the sides of these fibrils often protrude the delicate root-hairs which have the same powers of absorbing watery solutions from the soil that the fibrils themselves have. These root-hairs vary greatly in number even in the same plant, and they differ in dimensions in different individuals, some plants being wholly destitute of them. Their number depends upon the nature of the medium in which they are placed.* Where the soil is porous and charged with moisture the root-hairs are abundant, so much so that they give a white cobwebby appearance to the roots in such situations. Their relation to the temperature of the soil has not been properly worked out, although it is clear that a relatively warm soil is favourable to their development. They are developed on all sides from above downwards.

The production of root-hairs appears to be influenced by much the same circumstances as that of the root-fibrils themselves. Moreover, the root-hairs are often specially abundant in the case of seedling plants, containing little or no reserve of water, and at the point where growth is going on actively, and a large demand for water and nutriment consequently exists. The duration of these minute hairs is often very short. Once the necessity for their presence is passed, or the conditions for absorption become unfavourable, they shrivel and die.

So far then as absorption of nutritive matter is concerned, it is the minutest fibrils with their hair like out-growths that we have to consider. We know, in a general way, that their number is much greater in the same species of plant under some circumstances than under others. If they are growing in very fertile, moist, well aërated and well drained soil, the production of feeding roots (and of root-hairs in the case of the plants producing them), is greater than under opposite circumstances. Under such favourable conditions they are short and densely matted, whereas in sterile soil they are elongated and produce but few fibrils. The dense leash of feeding roots made by trees whose roots have access to water is well known ; as also the network of root-hairs formed around any food-yielding substance, such as a piece of leather or bone.

NOBBE and SACHS have shown as a result of well devised experiments, that where the fertilizing material was thoroughly mixed with the soil, the root-fibrils were most abundant, and relatively few in the intervening less rich portions. The experiments were varied in method, but in all cases the results showed that the "principal development of the roots occurred in the immediate vicinity of the material which could furnish

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* "Notes on Root-hairs and Root-Growth," by M. T. MASTERS, Journal of Royal Horticultural Society, vol. v. (1879), p. 173. [A paper of M. MER, "De la constitution et des fonctions des poils radicaux," read before the French Association for the Advancement of Science at Reims in 1880, but of which we have only seen a very brief abstract, should be consulted.—Note added, October, 1882.]

them the nutriment."[*] HELLRIEGEL[†] found that a plant of barley, in rich porous soil had made roots of the aggregate length of 128 feet, while a similar specimen in coarser, heavier land had only 80 feet; and numerous measurements of a similar character have been made by NOBBE in the course of his water-culture experiments.

In a mixed pasture where a great variety of species is growing, there is a great diversity in the natural habit of the plants, irrespective of the direct effect of soil or manure. Some roots go much deeper than others; some branch more than others, and in this way different plants, as it were, tap different layers of soil, and so utilize portions that would be left unused by others. As a general rule, if the root-range of any given plant be restricted, the appropriate food may be presumed to be near at hand, and in sufficient quantities. If, on the contrary, the root-range is wide, then the food is distributed over a wider area.[‡]

There is evidence to show that roots by virtue of the acid fluid with which the membrane of the root-hairs or of the absorbent cells is saturated have, at least in some cases, the power of dissolving, and it may be presumed directly absorbing, substances with which they come in contact;[§] but this point has not been made a subject of investigation at Rothamsted.

It stands to reason that a large production of root-fibrils and root-hairs would exhaust the soil in their immediate neighbourhood sooner, and more thoroughly, than a more limited development would do. But it must be remembered that long, deeply-rooting plants, or plants with thick root-branches, penetrating downwards to a considerable depth, may, in the aggregate, produce quite as many or more feeding roots, though they may be more widely diffused and not so apparent to the eye. These differences are, in a general sense, denoted by the ordinary terms of surface-rooting or deep-rooting plants. The surface-rooting plants have usually finely branched root-fibres in great numbers, not penetrating very deeply, and liable to be injured by drought or frost. The more deeply-rooting species are usually of longer duration, with coarser, stronger, less branched, but much more deeply-penetrating roots. Such plants, therefore, are better able to withstand drought; first, because they are not so much affected by surface supply, and next, because they can obtain moisture from a greater depth and can accumulate a larger store of it in their tissues. These differences were well illustrated in 1870, a year very remarkable for the drought which prevailed. In that year, as has been more fully explained in Part I. (p. 334, *et seq.*), samples of the surface soil and turf, and of the subsoil of some of the experimental plots were taken, and the distribution of the roots was examined at various depths down to 54 inches. The plots selected

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[*] SACHS, 'Physiologie Végétale,' ed. MICHELI, p. 196.

[†] HELLRIEGEL, HOFFMANN's Jahresbericht, 1864, p. 106.

[‡] DUREAU DE LAMALLE, *op. cit.*

[§] DIETRICH, in HOFFMANN's Jahresbericht, vi., 3; SACHS, 'Physiologie Végétale,' ed. MICHELI, p. 208. See also VAN TIEGHEM, 'Traité de Botanique,' p. 256 [1882], where the action of the root on the soil is compared to a process of digestion, a process carried on only in the part of the root covered with root-hairs as a result of the contact of their membrane charged with "sucs" and the particles of soil.

were one of the two unmanured ones, Plot 9 (mixed mineral manure and ammonia-salts), and Plot 14 (mixed mineral manure and nitrate of soda).

In connexion with this subject it must be borne in mind that in many perennial plants, while the main body and principal branches of the root persist from year to year, the fibrils and root-hairs, as before mentioned, die off, and are renewed only when circumstances are favourable. The relatively early or late period at which root-growth of this character takes place would naturally affect the vigour of the plant and influence it for good or ill in its competition with others.*

Root-stock, rhizome, &c.—Under this head we may include, for our present purpose, those portions of the root not directly concerned with the absorption of moisture or of food from without; those which are more or less woody or succulent, and which serve as holdfasts, as conduits, or as depositories for water and nutritive matters. More often than not these organs, in a strict botanical sense, are underground stems, having the internal structure and mode of growth of stems or of branches, and not of roots proper. It is not necessary to go into detail on these points here; suffice it to say that, as a general rule, any portion of the plant, subterranean or not, which bears leaves or scales (which are the rudiments of leaves) or buds, is to be considered as a stem or branch, and not as a root.

Speaking broadly, the object of these structures is to fix the plant in the soil, to enable it to avail itself of the food in the deeper layers of the soil, by emitting feeding roots when the conditions are favourable, and at greater depths than is, as a rule, possible in the case of fibrous roots, to store up water and nutritive matters, to force their way by progressive growth at their tips into fresh and unoccupied territory, or to make their way into that occupied by their neighbours if these be too weak, either intrinsically or from force of circumstances, to resist the intrusion. Often the subterranean stem or branches serve the purpose of propagation or of reproduction. Often, too, they serve, to some extent, as organs of locomotion; for instance, the stock continues to grow at one end, year after year, the opposite end gradually dying away. In the course of a few years the plant, therefore, occupies quite a different position from that which it had at first.

The details of these differences in form and structure are given in the ordinary text-books of botany, though little is therein said upon them from our present point of view. For our purposes we may speak of these "stocks" and "root-stocks" as: 1. Elongated and more or less branched, either descending vertically or spreading, for

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* On the general question of root-growth and root absorption the most important paper known to us is that of SACHS in the "Land. Versuch. Stat.," 1859, iv., 1, wherein the researches of other experimenters are summarised. See also MICHELI's French translation of SACHS' 'Physiologie Végétale,' p. 187. The more recent papers of the same author, "Ueber das Wachsthum der Haupt und Nebenwurzel" (Arbeiten des Bot. Institut in Wurzburg, 1873, 1874) should also be consulted. [A. paper of M. MER "Des modifications de structure et de forme qu'éprouvent les racines suivant les milieux ou ils végètent," read before the French Association for the Advancement of Science at Rheims in 1880, is only known to us by the brief abstract in the Bulletin of the Botanical Society of France, 1881, Rev. Bibl., p. 213.—Note added, October, 1882.]

a relatively short distance, at a greater or less angle on, or slightly beneath, the surface of the soil; 2. As dilated and tuberous; 3. As contracted and tufted, as in many of the grasses, whose tufts or "hummocks" consist of the short contracted stocks emitting plentiful root-fibres below, and dense tufts of leaves above; 4. As creeping; as in the case of Twitch or Bent (*Triticum repens*), where the stock gives off slender creeping branches on or beneath the surface, such branches sometimes having the power of rooting freely at the nodes, as in Twitch; at others not rooting freely, as in the "stolons," or offshoots of *Achillea Millefolium*.* In any case they are usually terminated by a bud or cluster of leaves, surrounding a "growing point," and which is capable of rooting and forming a distinct plant.

Stem.—Under this head we may include all those organs whose office it is to bear well-developed leaves and flowers. It is not necessary to discuss the many forms the stem may assume. It will suffice to recall that some plants have but a single stem springing from the root-stock, while, in other cases, as in many grasses, the stems or culms are numerous and tufted. Of course, if the root-stock branches there will be stems corresponding to those branches. The number of the stems and of their branches, their habit, their direction, whether erect or decumbent, their texture—all have an important influence on the terms of competition between different plants. The production of numerous stems, as in those grasses which "tiller," seems connected with the necessity for securing sufficient nutriment from the air, solar light, &c., in a short space of time.

Leaves.—On ordinary meadow land a great variety in the shape, size, and direction of leaves, as well as in the mode in which they are packed in the bud, may at once be seen. There are the grasses with their long, thin, narrow, folded or convolute leaves, at first more or less erect, afterwards, according to their position on the stem or culm, spreading. There are the Plantains and Clovers, with their broad, flat leaves; there is the Daisy, with its spoon-shaped leaves pressed flat to the soil; and between these extremes every conceivable variation in size, shape, and direction occurs. These variations are, it is to be supposed, sometimes associated with corresponding differences in internal structure, and more especially with diverse functional endowments, as a result of which the life-history of the plants presents the differences which are so obvious, but the explanation of which is at present so incomplete.

The influence that the mere shape and direction of leaves may have upon the struggle between different plants is strikingly illustrated in early spring, before they have started into growth. At that time, many of the "weeds" of grass-land have the form of flat rosettes, resting immediately on the surface of which they occupy circular patches of considerable area. Such is the case in the Plantain, in the Dandelion, in the Hawkweed, in the Daisy, and many others. Now, if these plants start into

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* A note of Mr. DARWIN's may be quoted here as showing the beneficial effects of cross fertilisation on *Origanum vulgare*:—"The superiority of the crossed plants was truly wonderful, owing to their increase by stolons."—('Fertilisation of Flowers,' p. 96.)

growth earlier than the neighbouring grasses they will necessarily have an advantage over them, and will more or less crowd them out. If on the contrary the grasses begin to grow first, or the growth of the broad-appressed leaved plants be checked, then the grasses, or other plants, will get the start and overpower the rest.* In other cases, the flat leaves closely pressed against the soil prevent the growth of other plants; for instance, on an ill-kept lawn, the Daisies increase partly by virtue of their much divided stock, partly by their flat leaves closely pressed to the soil, and preventing the growth of other plants. In the Buttercups the leaves, which in early spring form a rosette appressed to the ground, become speedily lifted from it by the elongating leaf-stalks, and room is thereby left for the encroachments of the grasses or other plants which may be in the neighbourhood. In plants like *Conopodium denudatum*, the leaf in its initial stage is packed into very small compass, almost like a furled flag. In this state it is thrust up amid other leaves till it gets space to unfold. In other cases, as in the clovers, the two halves of the young leaves are folded like a sheet of note-paper and remain so till by the lengthening of the stalks they grow beyond the obstruction. In the *Festuca ovina* the thin, slender, cylindrical leaves easily make their way amongst the surrounding foliage, and this may be one reason for the predominance of the plant, while its comparatively small evaporating surface may also stand it in good stead. In fine, there are endless adaptations such as those cited, well known to the botanist, and familiar to the most superficial observer of plants. It is not needful to dwell upon them here at any greater length, our object being not to call attention to special arrangements, but to the general fact that all these variations, in the growth and "habit" of plants, are elements to be considered in studying the nature and the force of the struggle for existence among them, and the circumstances that determine the "survival of the fittest."

With reference to the minute anatomical structure of the leaves and herbaceous parts of plants, it is well known that external conditions have no inconsiderable influence upon them. The structure of aquatic plants is a case in point; the anatomical construction being profoundly modified as a consequence of the medium in which the plants grow. So, also, the increased succulence—due to augmented size or number of cells—seen in plants grown by the sea-side is matter of common observation. The densely woody contracted habit of shrubby plants growing on the higher mountains has also often been noted. Amongst the most recent investigations on this subject are those of M. DUVAL JOUVE, who has examined the structure of the leaves of grasses as modified

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* In the spring of 1875 one of us was a witness of a struggle of this kind between *Cerastium triviale* and *Plantago lanceolata*. The frosts of March had very severely injured the outermost leaves of the Plantain, a large number of which were growing on an exposed railway bank where little else grew. Each plant was surrounded by a growth of *Cerastium triviale* which was much less damaged, if at all, by the frost, and which consequently invaded and, to a considerable extent, overpowered the Plantain. As the spring went on, however, the new leaves of the Plantain, when starting into growth, assumed an upward direction instead of the usual horizontal one, and the plant quickly outgrew the *Cerastium*, and so regained its supremacy.

by external conditions. Referring the reader to his paper* for full details, it may here be stated on his authority, that the hard woody fibres beneath the epidermis of the leaves of grasses—"the hypodermic fibres"—which form a protecting sheath to the fibrovascular bundles of which they are indeed a dependence, and which must greatly contribute to the rigidity of the leaf, are fostered in their growth in hot dry situations. Shade and moisture, on the contrary, are not favourable to their development. It is easy to see the bearing of these facts on the question of the struggle for existence. Speaking of the cellular or parenchymatous structure of the leaves of grasses the same botanist remarks that there are, speaking generally, three categories of cells: 1, those with green chlorophyll; 2, colorless cells without chlorophyll; 3, stelliform cells. The green cells predominate in grasses growing in cool shady localities; the colorless cells are relatively abundant in species growing on sandy sea-coasts; while the stelliform cells are especially abundant in aquatic species.

Flowers, &c.—It forms no part of our purpose to describe in detail the inflorescence or the floral structure of such well known plants as those which constitute the mixed herbage of the plots. A few general observations may, however, appropriately be made in illustration of the competition that is going on between them. Some of the plants on the plots never get an opportunity of flowering; others flower, but do not perfect their seed before they are cut by the scythe. It might be expected that the duration of such plants would be prolonged, as the flowering, and still more the seeding, is so exhausting to the plant. Some plants which do not get an opportunity of flowering or seeding before the first crop is cut, do so before the aftermath is taken. The early or late flowering is, therefore, an element to be considered.

Other matters relating to fertilisation, such as the question whether particular flowers are self-fertilised or fecundated by wind-wafted pollen, or by pollen carried by insects, are amply deserving of study, with special reference to the advantage or injury resulting to some of the competing plants by such complex interactions.† For instance, if in any given season the particular insects adapted to fertilise any particular flower were from any cause absent, or present in diminished numbers, the number of seedlings of that plant would be proportionately diminished, and *vice versâ.*

Some plants, again, like *Rumex Acetosa,* seed more abundantly than others. This is a diœcious, wind-fertilised flower, whose seeds are dispersed in abundance, and as

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* 'Ann. Sc. Nat. (Botanique),' 1875, vol. i., p. 367. [See VESQUE in the same publication, vol. xiii., 1882, p. 24, and STAHL as quoted by VESQUE; also HACKEL'S 'Monograph of European Fescues,' of which an abstract is given in TRIMEN'S 'Journal of Botany,' September and October, 1882.]

† With reference to the "relation of organism to organism, the most important of all relations," see DARWIN, 'Origin of Species,' chapter 3, and as to the relative advantages to the species of self or of cross-fertilisation, see DARWIN 'On the Effects of Cross and Self-Fertilisation in the Vegetable Kingdom,' and on 'The Different Forms of Flowers on Plants on the same Species.' As to the fertilisation of grasses, see BIDARD, Comptes Rendus, 1869, p. 1486; SYME, in 'Journal of Botany,' 1871, p. 173, and in 'Journal of the Royal Horticultural Society,' 1872, vol. iii., p. 7; SPRUCE in same publication, 1872, vol. iii., p. 4; A. S. WILSON, 'Gardeners' Chronicle', March 14, 1874.

will be seen in the sections relating to the plots, germinate freely, especially availing themselves of bare patches, occasioned by frost or drought killing the herbage. Doubtless the seedlings of other plants would grow equally well on such places, and sometimes indeed they do so, but the greater number of seedling plants of the *Rumex* give it an advantage over others which are less prolific.

The early seeding tendency would necessarily give an advantage to the plants that possessed it, as others would be cut prior to the ripening of the seed. As the plants on the plots are almost all perennials, this is of the less consequence; nevertheless, it is an important element in estimating the results of the competition between species which must not be overlooked. Among those species which ripen their seed before, or about the time of cutting, and whose seeds are therefore liable to be dispersed, are—*Anthoxanthum odoratum, Bromus mollis, Alopecurus pratensis, Poa trivialis, Ranunculus bulbosus, Pimpinella Saxifraga* (in second crop), *Conopodium denudatum* (sometimes), *Trifolium pratense* (occasionally), *Rumex Acetosa, Luzula campestris,* &c. Of course, allowance must be made for season and for the influence of manures.

General Effects of Manures on Vegetation.—In the experiments under consideration, the effects of the various manures on the vegetation were ascertained by observation of the plants during growth, noting the characteristic tendencies of development, by the botanical separation or analysis of the mixed herbage, and by the weight and chemical composition of the produce. The various results have been or will be brought out in detail elsewhere. But it should be called to mind here that a general tendency of nitrogenous manures is to favour luxuriant and continuous growth as distinguished from arrest and consolidation of that already formed; whilst that of mineral manures is to favour consolidation rather than luxuriance. Or, to put it in another way, a characteristic effect of nitrogenous manures is to favour the extension of foliage and give it depth of colour, whereas that of the mineral manures is to tend to stem-formation and production of seed. In fact, a relative excess of nitrogenous supply favours the extended growth of the organs of vegetation, prolonging their development, it may be until the resources of the plant are exhausted or the season is past. On the other hand, a relative excess of mineral manures may bring on premature ripening. It is the proper adaptation of the two descriptions of supply to the current requirements of the plant and of the season that gives both full, properly proportioned, and well-matured growth. It is obvious, therefore, that when a number of plants of different habits, and different periods of flowering and seeding are growing in association as in the mixed herbage of grass land, the character of the manure must exert a very potent influence on the struggle.*

It may be stated, however, that neither in the experiments at Rothamsted whether on individual plants or on plants growing in association, nor in those made at Chiswick

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* For the effects of manures on individual species grown separately, see "Reports of Experiments on the Influence of Various Manures on Different Species of Plants," Journal of the Royal Horticultural Society, new series, vol. iii., p. 19, and "Second Report," vol. iii., p. 124, 1870; also 'Rothamsted Reports of Experiments on Different Crops.'

with individual species growing separately, has there been absolute change in distinctive form by the use of various manures. Changes of degree are indeed sometimes very marked, as, for instance, in the tufts of *Dactylis glomerata*. Again, such changes as there are are chiefly in the organs of vegetation—roots, leaves, and stems, which show greater or less relative amount, precocity, permanence, and so on; whilst the changes in the parts of the flower are much less. It is probable that more attentive observation would reveal differences in the processes of fertilization and in the germinating power as in the maturation of the seed.

General Occurrence of Certain Species:—Hostile Competition or Peaceful Association.

Certain plants occur with such frequency, and under such widely differing conditions, that it would seem as if they were in a great measure indifferent to the character of the soil or to that of the manure. There are indeed certain plants which if not crowded out by their more powerful neighbours seem to thrive on almost any soil and under almost any condition of manuring. The great extension of *Festuca ovina* under the most varied conditions of manuring is a case in point. We see illustrations of the constancy of occurrence, or of the same power of adaptation to varying conditions of soil, &c., in the case of wild plants generally. It is easy to pick out certain plants peculiar to limestone, or sandy or clay soils, as the case may be, but these are relatively few when compared with the large numbers that seem indifferent to the nature of the soil.* Again, observations of this character tend to show that it is the physical nature of the soil, its capacity for holding water and its permeability to roots, that are, in most cases, of greater importance than its more strictly chemical composition. It may also be stated, that plants growing in association with others of different species do not necessarily grow where the conditions of soil and climate seem to be, or are really most favourable to them individually, but in those localities where they can best maintain themselves in the competition with others. In a word, they do best where they can best adapt themselves to the combination of surrounding circumstances.† Any peculiarity of organisation which will help them in their struggle is of course a decided advantage. Such peculiarities, although they cannot be induced, may be, and are, enhanced by the action of manures.

In the continual internecine strife between plants, it is not always the resources of the plant itself which enables it to maintain or extend its ground. Its success

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* MASTERS, "On the Flowering Plants of Oxfordshire," Transactions of the Ashmolean Society, 1857. For a summary of our knowledge on this matter, see ALPH. DE CANDOLLE, 'Géographie Botanique,' vol. i., p. 423; and the more recent researches of CONTEJEAN, Ann. Sc. Nat. Bot., ser. 5, t. xx., p. 266, and ser. 6, t. ii., p. 223. While there is great difference of opinion as to the relative influence of various soils on plants, all are agreed that, with certain obvious limitations, the majority of plants are "indifferent" to the chemical or physical nature of the soil.

† For some general remarks on the conditions of the struggle, see the chapter on the extinction of species in the late Sir CHARLES LYELL'S 'Principles of Geology,' and that on the struggle for existence in DARWIN'S 'Origin of Species.'

depends, as pointed out by A. P. CANDOLLE,* in a great measure on the number of its foes or of its allies among the animals and plants inhabiting the same region. Thus, for example, a plant which loves the shade may multiply, if some tree with spreading boughs and dense foliage flourish in the neighbourhood. Another, which if unassisted would be overpowered by the rank growth of some hardy competitor, is secure because its leaves are unpalatable to cattle, whilst they annually crop down its antagonist, and rarely suffer it to ripen its seed.

The same author, in his 'Physiologie Végétale,' tom. iii., p. 1471, in alluding to the relative development of the root fibrils in different plants, points out that when in any given area there are several individual plants of different degrees of vigour, the roots of the stronger will deprive those of the weaker of their nourishment, and that this inequality of action will confer an advantage on the first comer.

Some experiments bearing directly on the struggle among plants have been made by Professor HOFFMANN, of Giessen,† who watched the changes among plants growing together, and endeavoured to ascertain what were the circumstances which contributed to the success of the victors. In a previous set of experiments it had been found that the particular plants under observation grew equally well in different kinds of soil, provided due care were taken to prevent the growth of intruding weeds. Having arrived at this result, the several plants were left to themselves, with a view of determining how they would comport themselves without assistance against the inroads of weeds. It resulted that the weeds completely gained the ascendancy. The species which held out longest was *Asperula cynanchica*. This plant, after having been grown in a bed for three years, and protected from weed invasion by the use of the hoe, was then left to take care of itself. It held out for some four years, but was ultimately elbowed out by the intruders.

Professor HOFFMANN then set himself to observe the results of the internecine struggle between the weeds themselves, thinking that the ultimate survivors would perhaps prove to have special affinities for the soil in which they grew. Thus left to themselves the beds became so densely covered that in a square foot 460 living plants, and the remnants of many others which had succumbed, were counted. Every year in July the beds were examined, and every year the number of species was found to have diminished, till at length only a few were left; and these not only persisted, but slowly gained ground from year to year, and ultimately remained in possession of the plot.

Of 107 species under observation, all, or nearly all, found the most essential requisites of their existence equally well in all the varieties of soil, so that, other conditions being equal, the nature of the soil was indifferent. The species which remained victors, all the others being ultimately dispossessed, were *Triticum repens*, *Poa pratensis*, *Potentilla reptans*, *Acer pseudo-platanus*, *Cornus sanguinea*, native plants;

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* Cited by LYELL, 'Principles,' vol. ii., ed. 10 (1868), p. 435.

† See 'Botanische Zeitung' (Beilage), 1865, and 'Landwirthschaftliches Wochenblatt des Ackerbauministeriums in Wien,' 1870; see also 'Gardeners' Chronicle,' 1870, p. 664.

and *Aster salignus, A. parviflorus, Euphorbia virgata,* and *Prunus Padus,* derived from the garden.

It may, therefore, be inferred that the district in which these experiments were made would in process of time, if no obstacle were afforded, become covered with meadows and woods—meadows in the low-ground and woods in elevated places. Again, the experiments show that the survival of certain plants has not been materially influenced by the nature of the soil. Thus, *Triticum repens* was ultimately spread over all the plots, whether of sand or of loam or of lime; whether drained or undrained. So also with *Poa pratensis* and *Potentilla reptans.*

As to the action of shade, it was found that low-growing plants, especially if annuals, disappeared rapidly; while taller-growing ones, such as the *Triticum, Prunus Padus,* &c., survived. The general results at Rothamsted and at Chiswick are consistent with those established by Professor HOFFMANN, and combine to show that the survival of certain plants is due much less directly to external conditions than to the "habit" of the plant itself; that is, as already stated, to the facility the plant has of adapting itself to varying external conditions, and thus of triumphing over others less favourably endowed in this wise.

The immediate source of victory lies very generally in the powerful root-growth of the survivors; including under the general term root, not only the root proper, but the offshoots and runners which are given off just below or on the surface of the ground. Indeed, this habit of growth is more advantageous to plants in such a struggle than the development of the true root downwards would be.

With reference to these questions of struggle, competition, and association, it is requisite to distinguish clearly between those cases in which the competition or the struggle is direct, and those in which it is indirect. In some cases where plants grow in association the strong overpower the weak by virtue of their superior endowments, in others one plant gains the advantage because it was the first to take possession of the ground; or because, for some reason or another, no resistance is offered to its spread. But, while among individual plants of the same species, having the same requirements and growing in association, there must necessarily be hostile competition which would disturb the balance of vegetation, and result in the survival of the fittest and the destruction of the weakest, there is also an opposite process which tends to maintain an equilibrium. Different plants have different requirements, and if these be supplied at the same time and place, the vegetation may be of a very mixed character, and no one plant or set of plants gain preponderance over the rest, till some circumstance arises to disturb the equilibrium. We have then to discriminate between the effects of hostile competition and those of peaceful concomitance or association. DE CANDOLLE, in the paper previously cited, asserts that *all* plants within a given area are at war with one another; but this assertion, in view of what has just been stated, requires to be taken in a qualified sense.

We have already alluded to the fact that, in the case of various plants growing in association, the number of individual plants within a given area is generally small in proportion as the number of species is also small. The diminution not only in the number of species, but in that of individual plants, is usually the result of conditions favouring the luxuriance of some species more than others; and hence the competition between plant and plant becomes more severe. The variety and the number are reduced, but the strength and size of those which remain are enhanced. The experimental details given in the sequel will afford some striking illustrations of these facts.

NAEGELI* considers the internecine war to be the most severe between those species and varieties which are the most nearly allied in character and organisation, because they require the same conditions of existence. He cites in illustration the case of three species of *Achillea*, two of which, *A. moschata* and *A. atrata*, make similar demands on their environment, and so come into competition one with another, with the result that the two are rarely found together, because one overpowers the other. The third species, *A. Millefolium*, having a different organisation from the other two is not brought into competition with them because its requirements are different; and it is, in consequence, found growing with either or both of the others. It must be remembered, however, that plants very closely allied morphologically, so far as their outward conformation is concerned, may, nevertheless, vary greatly in physiological endowments. We see this in the very varied constitutional and physiological peculiarities sometimes manifested in seedlings derived from the same parent plant. We shall hereafter have to dwell at length on the marked physiological diversities between two nearly allied species of *Poa*, *P. trivialis* and *P. pratensis*, as also between three species of *Avena* already mentioned, but in these cases there are structural differences between the species which indicate a more distant relationship between them than is apparent at first sight.

NAEGELI further alludes in some detail to the manner in which the preponderance of one species and the consequent ultimate extinction of another nearly allied form may be brought about. "It might be assumed," says NAEGELI, "that this result would always take place, and that one of two plants would always be crowded out, because the two could hardly be precisely equally hardy. But this conclusion would be unsound, because it would hold good only for plants whose conditions of existence were as nearly as possible alike. We can imagine another case in which the two species suffer injury from altogether dissimilar external influences (one, *e.g.*, from spring frost, the other from dry heat,) so that sometimes the number of individuals of one species, sometimes that of the other species diminishes, and where, moreover, the production and the germination of the seeds are affected by altogether dissimilar external influences, so that sometimes the one, sometimes the other species, increases most rapidly and occupies the vacant spots. The numerical proportion of the two species must in this case be variable, but neither is able to expel the other." The fluctuations from year to year of particular species on some of the plots at Rothamsted will afford similar illustrations.

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* Sitzungsb. der K. Bay. Acad. der Wissensch., Dec. 15, 1865. See also SACHS' 'Text Book,' English edition, 1875, p. 833.

Not only may the varying mechanical or chemical nature of the soil influence the issue of the struggle, but also the varied requirements of plants as to water, light, and heat. NAEGELI gives some examples of this. When *Primula officinalis* and *P. elatior* occur together in the same district they are sometimes sharply separated from one another; *P. officinalis* preferring the dry, *P. elatior* the damp spots. Each is most vigorous in its own special habitat, and may expel the other. But if only one species occurs in the district it is not so exacting; *P. officinalis* may then exist in damp, *P. elatior* in dry situations.

The influence, direct or indirect, that may be exerted by insects and other forms of animal life on the vegetation of the plots is too large a subject to be more than incidentally mentioned in this place. That influence may be detrimental or it may be favourable to plant growth or plant fertilisation; and its importance, from the latter point of view especially, has been more fully recognised since the publication of Mr. DARWIN's observations and inferences; but the general question was not overlooked by LINNÆUS and the older naturalists, who recognised the interdependence of insects and plants, and pointed out how the equilibrium between the several classes was maintained.*

The foregoing illustrations will convey some idea of the manifold conditions affecting the competition between neighbouring plants. Plants growing in association are subjected to the competition of their immediate allies having the same requirements as to food, moisture, space, light, &c., or to the more active antagonism of less nearly related plants, which, though making somewhat different demands on the environment, yet overcome their neighbours by their superior hardiness or force of growth. Or the equilibrium may be disturbed by causes external to the plants themselves. On the other hand, plants may grow together in comparatively peaceful association, because their requirements are different, or because the conditions are so unfavourable for the luxuriance of any in particular that many exist on somewhat equal terms of limited growth.

The competition may be chiefly among the roots which form a network more or less dense and to a greater or less depth, according to the particular plant. The roots extend or ramify at different angles, and in different directions and to varying depths, according to the mechanical, chemical, and hygroscopic qualities of the soil. Just beneath the surface are the rootstocks, firm and more or less woody, jostling their neighbours, and pushing them aside, or emitting slender creeping stolons to twine their devious course among other plants. Or the competition may be chiefly above-ground. The dense tufts of some grasses occupy the surface, and prevent the growth of neighbouring species. In other cases, the creeping stems, the offsets, the runners—a hundred different forms and adaptations, provided for the benefit of the individual plant, as weapons of offence or defence, or as store houses for nutriment—contribute to the result. So again, also, is it with the form, the arrangement, the development of leaves, and also of the flowers and seeds. Indeed, the whole plant, in all its parts, and in all

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* See Amœn. Acad., vi., *passim*, cited in LYELL's 'Principles of Geology,' vol. ii., ed. 10, p. 436.

its functions, is better or worse adapted to the circumstances in which it is placed—the better, the more probable is its victory; the worse, the greater the probability that it will give place to others.

Absence or Predominance of Certain Species.—When in the discussion of the particular plants we speak of their absence from, or of their being driven out from, particular plots, our language must not be interpreted too strictly. Such statements, though based on careful observation on the plots during growth, and on botanical analysis of samples, are of necessity approximately correct only, for it can hardly be imagined, that in all cases of disappearance the plants are absolutely banished from the plot, either by the direct action of manures or by internecine conflict. They may be present, but in such diminished numbers, or in such limited growth, that they are not obvious on the plots at the times when the observations are made, and still less are they found in the samples. That this may be the case is shown by the fact that when circumstances become favourable they again appear. As has been said, many of the plants on the plots seldom, if ever, ripen their seed. The reappearance of such plants after an interval is not likely, therefore, to be due to the transport of seed. It is more probable that the root-stocks of such plants have all along existed on the plots, but, owing to unfavourable circumstances, have not been able to develop themselves. Some plants, it is well known—*e.g.*, some of the Orchids—have the power of maintaining an underground existence for several years, producing at most only a few minute leaves, and no flowers at all; so that their presence is not suspected till at length, in a favourable season, flowers are produced freely, and hundreds may be seen where the year before not one was observed. Such cases are familiar to botanists.

It is now generally recognised that the "characters" possessed by plants are either congenital—*i.e.*, ancestral, or physiological and adaptive. The former which are more particularly represented in the organs of reproduction are not subject to be changed by external conditions in short periods of time. Indeed, the hereditary tendency seems to preserve certain characteristics relatively unchanged, while, on the other hand, physiological characters (though of course also inherited) are much more liable to change under the influence of changed external conditions. As has been said, manures affect the degree of vigour of the foliage, roots, stems, &c., in a very marked degree; but they do not bring about any absolute difference in the nature of the organ or part affected. Certain characteristics are enhanced or degraded, as the case may be, but no absolutely new characters are engendered. The "struggle," dependent as it is partly on inherited and relatively invariable characters, and partly on other more pliable characteristics, can never be thoroughly appreciated until the structure of the plant, and its adaptation to the work it has to do, and to the conditions under which it has to do it, are better known.

Then as to the predominance of certain species, whilst some are more or less generally distributed on most or all of the plots, others are found only on some, and in small proportion and amount. The "predominance," like the "absence," is determined by observation during growth each season, as well as by the less frequent botanical analyses of samples of the produce. By such observations and analyses a much more correct estimate of the actual degree of preponderance of individuals of particular species in a given area is gained than is indicated by the usual terms of "common," "rather common," "rare," "very rare," and the like, which give but a very imperfect idea of actual or relative predominance.* Moreover, the species predominant in one season are not necessarily so in another. We have, therefore, a means of studying the special influence of climate, of manures, or of the innate characters of the plant itself, which is of great value in the consideration of the struggle for existence among plants. If, for instance, in the same season a species is dominant on a great number of differently manured plots, it may be inferred that its dominance is due not directly to the agency of a particular manure, or to the climatal conditions alone, which were common to all the plots, but largely to the habit of the plant itself. The predominance of *Festuca ovina*, for instance, already referred to, and its tendency to spread, as shown by the circumstance that it was dominant in the first year of separation on two plots, in the second on eight, and in the third and fourth on thirteen, are obviously due rather to a superior power of endurance in the plant itself compared with its competitors, than to directly favourable conditions of manuring or of season. On the other hand, some species are dominant on some plots, but are absent from, or exist only in small proportions on others. In such cases it may be assumed that the particular manure has been either directly favourable or prejudicial to the plant, or indirectly so by favouring or discouraging the growth of others. Again, if a species be dominant on a particular plot, or on a group of plots more or less similarly manured, the inference is that the manure is directly beneficial to the plant.

THE GRASSES.

Passing to the consideration of the particular plants met with on the various experimental plots, first in order come the grasses, which constitute the most numerous and most important of their occupants.

Before describing the individual gramineous species occurring on the plots, it will be well to make a few general observations on them collectively.

The species which occur in the experimental field are, almost without exception, perennials, and they have generally a tufted habit of growth, especially under the influence of suitable manures. They have all more or less branched fibrous roots, consisting of a multitude of fine threads, which descend to various depths, and whose degree and manner of ramification vary according to the species, and to the conditions of manuring, association, and season. They never form a thick tap root, as some of the Leguminosæ and many of the miscellaneous plants do. Nor have they much

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* See ALPH. DE CANDOLLE, 'Géog. Bot.,' vol. ii., p. 1154.

tendency to form "store houses" for the accumulation of nutritive materials in the shape of tubers or other dilatations of the root or stem. Although some grasses, *e.g.*, *Alopecurus bulbosus*, have such organs, yet none of the species found on the plots have been observed to develop such formations, with the exception of *Alopecurus pratensis* and *Bromus mollis*, in which the formation of small tuberous swellings on the root fibres has occasionally been observed. Many grasses produce offsets, stolons, or runners, either below or just above the surface of the ground, by means of which they are enabled to establish possession of an extended area. The stems or culms of the grasses are usually erect and hollow, frequently tufted, and occasionally branched at the base, as in the case of the species which "tiller," such branches being of essentially the same character as the creeping shoots or stolons just referred to, but differing mainly by their erect direction and in their office.

"Adventitious" roots spring from the base of the culms in many cases, and are produced in abundance from the nodes or joints of the creeping shoots, especially under the influence of moisture.

The blade of the leaf differs considerably in form and dimensions in various grasses, and also according to season and conditions of manuring. The amount and character of the pubescence also varies according to locality and moisture, and, in a less degree, according to manure. For our purpose it may suffice to mention *Dactylis glomerata* as the type of the broad-leaved, and *Festuca ovina* of the narrow-leaved species.

The internal structure of the leaves, as elsewhere mentioned, though on the same general plan, varies in different species of the same genus, and even in the same species, according to locality and other circumstances.

It would be out of place to enter here into those details as to the conformation of particular plants which may be found in any text-book of descriptive botany. In the following sub-sections relating to the individual species, attention will be confined to those points of structure which appear to have more or less direct relation to the varying external conditions of season, soil, manuring, &c., to which the plants thus growing in association are subjected. Such structural characteristics are only treated of very generally in botanical works. They have, however, been made the subject of special observation at Rothamsted, and the more important points will be noted under the head of each plant. In like manner those physiological endowments and life-habits which appear to be of more or less importance to the plant in its struggle with its associates will be briefly alluded to, whilst others whose bearing on the subject is less obvious will be passed over.[*]

Anthoxanthum odoratum.

This fragrant perennial grass has fine, much branched, dense root fibres, forming a dense mat close to the surface, but not deeply penetrating. Its culms are tufted, erect, never creeping, 1 to 2 feet in height, and they bear flat, narrow, and ascending leaves. It begins to grow very early in spring and again soon after cutting, though it

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[*] As to the fertilisation of grasses, see footnote at p. 1220.

yields but a scanty herbage. It is protogynous, and flowers so early that it is enabled to ripen seed before most other grasses. It is ordinarily found on dry pastures.

The characters specially favourable to it are its hardy constitution and its early growth, which enable it to occupy the ground before many of its associates, and to secure an early crop of seedlings. Its comparatively superficial roots enable it to avail itself of the food materials in the surface soil.

The following Table (XXXVII.) shows the plots upon which this grass was second or third (it was never first) in order of predominance (see p. 1227) *among the grasses* in either of the years of complete botanical separation : it also shows the actual percentages on those plots on which in the same years it yielded 5 per cent., or more, of the *total mixed herbage.*

In this and all subsequent tables of the kind relating to individual grasses, it is the *order of predominance* in the *total gramineous produce,* not in the *total mixed herbage of all orders,* that is represented. But as the total grasses predominate so uniformly over the species referable to all other Orders put together, and as, with only one exception, the grass which is first, second, or third among the grasses is also first, second, or third in the total herbage, the result would be practically the same in the tables relating to the grasses, whichever standard of predominance were adopted. As will be seen further on, however, it would be quite otherwise in the case of the Leguminosæ and of plants belonging to other Orders. The percentages are, however, those in the total mixed herbage. When a dash thus — appears in the tables the plots in question had not then been brought under experiment.

Table XXXVII.—Relative predominance of *Anthoxanthum odoratum.*

	Anthoxanthum odoratum							
	Was First, Second, or Third, among the total Grasses, as under.				Yielded 5 per cent. and over, to the total Mixed Herbage, as under.			
	1862.	1867.	1872.	1877.	1862.	1867.	1872.	1877.
Plots 1 . . .	. .	. .	. .	3	. .	. .	5·78	13·88
2 . . .	. .	. .	. .	. .	. .	. .	6·67	7·20
3 . . .	. .	2	. .	. .	. .	8·66	5·20	5·12
4–1 . .	. .	3	. .	. .	. .	7·16	. .	5·11
4–2 . .	. .	. .	. .	. .	. .	5·52	. .	. .
5 . . .	. .	3	3	3	5·77	5·51	. .	. .
6 . . .	. .	. .	3	. .	. .	. .	6·22	. .
8 . . .	. .	3	3	. .	. .	6·98	7·94	7·55
10 . . .	. .	. .	. .	. .	. .	5·27	. .	5·84
12 . . .	. .	2	3	. .	. .	7·66	6·67	5·47
17 . . .	. .	. .	. .	. .	. .	. .	. .	5·32
18 . . .	—	. .	. .	. .	—	. .	7·11	8·64
19 . . .	—	—	. .	. .	—	—	6·39	. .
20 . . .	—	—	. .	. .	—	—	7·17	. .
Total First . .	0	0	0	0	1	7	9	9
Second .	0	2	0	0				
Third . .	0	3	4	2				

Thus in no single instance did this grass attain the first place. It was second in only one of the seasons (1867), and then only on the unmanured plots. It was third in the same year, and also in the subsequent separation years on the plots that were either unmanured or defectively manured; where, consequently, there was relatively little growth of the herbage generally, and little activity of struggle.

It yielded more than 5 per cent. on a larger number of plots, but did so in no case where the vegetation of its associates was generally luxuriant and healthy. It reached 7 to 8 per cent. in a few cases; but only once, and then only on a single plot, has it attained to nearly 14 per cent. This grass, therefore, only becomes prominent under conditions which do not induce special luxuriance in its competitors; and, on the Rothamsted soil, it seems to be more injured by association with more luxuriant grasses than by the direct action of manures.

In the Chiswick experiments, where *Anthoxanthum* was grown separately, it did not thrive so well as when grown in association, whether from want of shelter or from more rapid drying up is uncertain. Be this as it may, in one year the highest degree of vigour was observed where mineral manures and nitrate of soda were applied; in the second year where mineral manures and ammonia-salts were used. This result seems to confirm what has already been stated as to the effect of association on this plant; for when grown separately it would seem to thrive under conditions of manuring which when in association favour its competitors, and thereby limit its own luxuriance.

The observation made by GRISEBACH in his 'Vegetation der Erde' (1872), p. 150, is opposed to our ordinary experience with this grass. He states that he once saw *Anthoxanthum odoratum* dispossess almost all the other grasses in some thoroughly irrigated meadows. Irrigation doubtless tends to the simplification of the herbage, but it is opposed to general experience that a grass usually preferring dry situations, and one so unable to compete with others of more luxuriant habit, should thrive so remarkably under precisely opposite conditions.

Alopecurus pratensis.

This grass has a long, deeply-penetrating, more or less vertical subterranean rootstock, from which proceed creeping offsets or stolons, capable of giving off fibrils. In addition to the main vertical root fibres, there is usually, close beneath the surface, a dense leash of horizontal fibrils, which is however much less marked in the unmanured plots. On some of the plots a formation of small tubers on the fibrils has been noticed. The culms attain a height of 1 to 3 feet. The leaves are rather broad. It has not a tufted habit of growth, but grows in detached patches by the extension of its creeping, subterranean off-shoots. It is protogynous, flowers early, and often a second time. It withstands the severest frosts, and grows rapidly after mowing or grazing.

In this country it generally grows upon rather rich soils, and is injuriously affected by extremes of moisture or of drought.

The favouring conditions for its growth are: its hardiness, rapid, vigorous, and early growth, which enable it to get the start of its competitors. These qualities, combined with its deep roots and numerous subterranean offsets, cause it to spread rapidly, and to exhaust the soil for the smaller, less deeply-rooting grasses. Its early seeding tendency is also a favourable circumstance in its competition with others.

The conditions of its predominance are indicated in the following table.

TABLE XXXVIII.—Relative predominance of *Alopecurus pratensis*.

		Alopecurus pratensis							
		Was First, Second, or Third, among the total Grasses, as under.				Yielded 5 per cent. and over, to the total Mixed Herbage, as under.			
		1862.	1867.	1872.	1877.	1862.	1867.	1872.	1877.
Plots	2	. .	. .	. .	. .	. .	. .	5·61	. .
	3	. .	. .	. .	. .	. .	5·82	. .	. .
	4–2	. .	2	. .	. .	. .	14·75	. .	. .
	10	. .	. .	. .	2	. .	. .	10·35	16·51
	11–1	. .	3	3	. .	. .	13·11	12·35	9·91
	11–2	. .	. .	2	2	. .	6·33	22·65	20·11
	13	. .	. .	. .	. .	. .	. .	5·89	6·78
	14	. .	. .	. .	2	. .	. .	. .	20·18
	15	. .	. .	. .	. .	6·90	5·95	. .	7·17
	16	. .	. .	2	. .	. .	8·27	15·22	12·23
	17	1	1	2	2	23·94	21·71	16·25	12·72
	19	—	—	. .	. .	—	—	. .	5·40
	20	—	—	. .	. .	—	—	. .	6·80
Total	First	1	1	0	0	2	7	7	10
	Second	0	1	3	4				
	Third	0	1	1	0				

It is seen that this plant was only first on plot 17, with nitrate of soda alone, and that only in the first two separation years. It occupied a second or a third place on plots all of which were more or less liberally manured; it was, indeed, second on three of the nitrate of soda plots.

It has exceeded 5 per cent. of the produce on an increasing number of plots from one separation year to another; and in the last it did so only on highly-manured plots, and especially on those with nitrate of soda. As is seen, it did not, on the majority of the plots, reach 5 per cent.; in fact, on many it was quite insignificant in amount; but under the influence of liberal manuring favourable to it, it reached from 10 to 15, and even to nearly 24 per cent.

From the foregoing results it appears that the deep and freely-rooting *Alopecurus pratensis*, with its numerous underground stolons, thrives best with high manuring, supplying much nitrogen. In this respect it comes into competition with *Dactylis glomerata*; which, however, maintains its prominence on the ammonia but not on the nitrate plots; whereas the *Alopecurus* does so on both the nitrate and the ammonia plots.

Agrostis vulgaris.

This is a very variable, tufted grass, emitting numerous stems, which creep along the surface of the ground, and produce roots and shoots at the nodes. The roots are exceedingly fine, much branched, and descend to a depth of 4 to 5 feet, but they also form a mat just beneath the surface. The root hairs are very abundant in damp situations, forming a dense white cobweb-like mass. The culms are slender, ascending, rooting at the base. The leaves are flat, narrow, and ascending. It flowers late in June and in July, and generally a second time in the autumn, and yields an abundant aftermath.

The characters favourable to its development are its hardiness and comparative indifference to the nature of the soil, its tufted habit, creeping stems and stolons, which enable it to extend itself, also its minutely branched roots, which gain extensive possession of the upper and richer layers of the soil.

The following table indicates its predominance on the various plots.

TABLE XXXIX.—Relative predominance of *Agrostis vulgaris.*

		Agrostis vulgaris							
		Was First, Second, or Third, among the total Grasses, as under.				Yielded 5 per cent. and over, to the total Mixed Herbage, as under.			
		1862.	1867.	1872.	1877.	1862.	1867.	1872.	1877.
Plots	1 . . .	. .	. .	1	1	. .	6·45	20·77	23·53
	2 . . .	. .	. .	2	1	. .	. .	11·02	17·97
	3 . . .	2	3	2	2	11·36	8·63	16·14	13·28
	4–1 . .	. .	. .	2	3	7·21	6·08	13·88	9·87
	4–2 . .	1	3	2	2	19·38	14·00	20·59	24·39
	5 . . .	1	2	2	2	24·30	20·97	26·62	29·46
	6 . . .	1	2	2	3	21·43	14·41	23·37	8·58
	7 . . .	3	3	2	3	7·14	5·69	11·72	12·02
	8 . . .	2	. .	2	3	10·01	. .	9·32	12·40
	9 . . .	1	2	2	. .	12·81	13·43	15·46	12·23
	10 . . .	. .	. .	3	3	9·37	8·61	14·17	16·29
	11–1 . .	3	2	2	1	18·81	19·27	13·56	29·20
	11–2 . .	2	2	. .	. .	13·17	24·16	10·16	17·09
	12 . . .	3	. .	2	2	9·43	5·55	11·13	13·29
	13 . . .	2	1	2	2	12·02	20·35	13·44	13·40
	15 . . .	2	. .	3	3	7·65	6·86	7·66	12·90
	16 . . .	2	2	3	2	12·49	13·62	12·40	14·58
	17 . . .	2	. .	3	1	11·01	7·05	10·60	17·92
	18 . . .	—	2	2	3	—	7·29	21·95	16·40
	19 . . .	—	—	2	3	—	—	25·56	10·83
	20 . . .	—	—	1	3	—	—	31·35	10·36
Total	First . .	4	1	2	4	16	17	21	21
	Second .	7	7	14	6				
	Third . .	3	3	4	9				

Of the 22 plots there is only one, viz.: that with the larger quantity of nitrate of soda and the mixed mineral manure (14), on which this plant has not been first, second, or third in order of prominence. It has been second much more frequently than first; indeed, more frequently so than any other species. It gained greatly in prominence up to the third separation year, 1872; and it has remained very prominent up to the present time, especially in the wetter seasons.

It has yielded more than 5 per cent. of the total produce on every plot except plot 14 (nitrate and minerals); and with very few exceptions it has done so in each of the four separation years. Indeed, in many cases it has formed a large proportion of the total herbage, amounting in a few to nearly 30 per cent. It is only very prominent where there is a liberal supply of nitrogen, under which condition its tufted habit becomes more conspicuous; but it is beaten by some freer-growing grasses where the manurial conditions are favourable to high maturation as well as to luxuriance. It is late in coming to maturity; and in a dry season is so backward that it scarcely flowers in the first crop. Hence, other things being equal, it yields a favourable percentage in the produce of different seasons. In 1870 it suffered greatly from the drought, but speedily recovered, even in the autumn of that year when the rain fell.

Holcus lanatus.

The roots of this grass do not penetrate deeply, but form a dense mat just beneath the surface. The rootstock has a slight tendency to creep. The culms are closely tufted, 1 to 2 feet in height, rooting freely from the lower nodes. The whole plant is more or less downy, and therefore perhaps less injuriously affected by frost or radiation.

It is moderately early, flowering in June and July, and again in the autumn. It yields a plentiful aftermath which, however, is said not to be relished by cattle. Under natural conditions it grows most freely in rather damp soils.

The characters favourable to its growth are its hardiness, its tufted habit, its tendency to produce roots from the base of the stems, and its densely-matted roots beneath the surface.

Table XI. shows the conditions of its predominance.

TABLE XL.—Relative predominance of *Holcus lanatus*.

	Holcus lanatus							
	Was First, Second, or Third, among the total Grasses, as under.				Yielded 5 per cent. and over, to the total Mixed Herbage, as under.			
	1862.	1867.	1872.	1877.	1862.	1867.	1872.	1877.
Plots 1	..	2	2	2	..	10·70	14·06	17·55
2	..	3	..	2	..	10 94	6·85	11·36
3	..	..	..	3	5·04	7·97	..	12·55
4–1	1	2	..	1	11·82	9·16	..	19·35
4–2	2	..	..	3	16·21	10·53	..	6·03
5	3	..	..	..	10·08	5·15	..	..
6	..	..	..	2	8·17	..	5·31	14·89
7	..	1	..	2	5·06	11·81	..	13·16
8	..	2	..	2	..	10·25	..	18 22
9	2	..	..	..	12·14	9·84	7·61	10·37
10	..	..	..	..	9 50	8·24	..	..
11–1	..	..	..	2	9·92	..	10·33	20·29
11–2	..	..	..	3	7·37	..	10·59	19·48
12	..	..	..	3	..	5·57	..	10·12
13	..	..	..	..	5·35	..	..	..
14	..	..	..	3	6·60	6·63	..	12·75
15	3	3	..	2	7·61	11·81	5·32	14·95
16	..	3	..	3	10·45	11·69	5·08	12·64
17	..	..	..	..	8·23	8·13	5·87	10·91
18	—	1	3	2	—	12·78	7·32	17·45
19	—	—	3	2	—	—	14·03	21·19
20	—	—	3	1	—	—	12·39	29·47
Total First	1	2	0	2	15	16	12	19
Total Second	2	3	1	9				
Total Third	2	3	3	6				

This comparatively shallow-rooting plant has seldom attained the first place, and only on plots the manurial conditions of which do not tend to general luxuriance. It has been more frequently second; but only twice under conditions of high manuring, both nitrogenous and mineral, and of general luxuriance. It has also in some cases attained the third place, and in a few even where the growth was in a high degree luxuriant. The grasses which it has had chiefly to compete with under the circumstances have been *Agrostis vulgaris, Festuca ovina, Anthoxanthum odoratum, Avena pubescens*, and *A. flavescens.*

It has contributed more than 5 per cent. to the produce on the majority of the plots and in a greater number of cases in the fourth separation year than previously.

It has reached its highest percentage with liberal nitrogenous and mineral manures; but it has not, upon the whole, been largely developed on so many plots as either *Agrotis vulgaris* or *Festuca ovina.* It has only in a few cases contributed 20 per cent. of the total produce, but in one case nearly 30 per cent.

Avena elatior.

This tall-growing, tufted grass, has vertical, very deeply penetrating, fibrous, yellowish roots, densely covered with root-hairs, but not so much branched as in the case of the Poas. From the lower part of the culm, stout fleshy adventitious roots are often emitted, which form a close mat near the surface. The culms are 2 to 4 feet in height, and bear abundant foliage. It begins to grow early, and flowers in June.

The endowments favourable to the *Avena elatior* are its hardiness, its comparative indifference to the character of the soil, its particularly ample root growth, both deep and superficial, its strong tufted habit, and its early flowering tendency.

Its degree of predominance is shown in Table XLI.

TABLE XLI.—Relative predominance of *Avena elatior.*

		Avena elatior							
		Was First, Second, or Third, among the total Grasses, as under.				Yielded 5 per cent. and over, to the total Mixed Herbage, as under.			
		1862.	1867.	1872.	1877.	1862.	1867.	1872.	1877.
Plots	6 . . .	. .	3	. .	. .	. .	6·50	. .	. .
	9 . . .	. .	. .	. .	. .	. .	. .	11·40	13·23
	10 . . .	. .	3	. .	. .	. .	11·71	13·22	9·58
	11-1 . .	. .	. .	. .	. .	. .	. .	10·41	14·86
	11-2 . .	. .	. .	3	1	6·40	. .	12·73	21·14
	13 . . .	. .	. .	. .	3	. .	. .	9·17	11·08
Total	First . .	0	0	0	1	1	2	5	5
	Second .	0	0	0	0				
	Third . .	0	2	1	1				

Thus, *Avena elatior* only once attained to the first place, and only in the fourth year of separation, or 22nd year of the experiments; but then it did so on the most highly-manured plot of the series. In no case was it second; but on four occasions it took a third place on plots where there was liberal nitrogenous manuring, on two without, and on two with liberal mineral supply as well.

It was only in the two later separation years that this grass yielded more than 5 per cent. of the produce on as many as five plots, each of which was highly manured with ammonia-salts, and in four of the cases in conjunction with liberal mineral manuring. In only one case, but then with the highest manuring, both nitrogenous and mineral, did it contribute more than 20 per cent. of the total produce. It is strikingly deficient on the nitrate of soda plots. On the whole it has rather gained ground on the high ammonia and mineral plots, where also its tufted habit has been observed to be most developed.

Avena pubescens.

Avena pubescens has short, creeping, underground rootstocks, and fine thread-like roots, generally superficial, but having the power to descend to a great depth under favourable circumstances. Its tufted culms are few in number, 1 to 3 feet in height. The leaves are narrow, downy, especially on poor soils, but on richer ones the plant becomes glabrescent. It reproduces itself abundantly by means of its short underground stock. It begins to grow early, flowers in June, and ripens an abundance of seed.

The favouring conditions are its hardiness, tufted habit, free seeding tendency, and power of reproduction by its slightly creeping stock. Its degree of prominence is shown, in the usual form, in the subjoined table.

TABLE XLII.—Relative predominance of *Avena pubescens.*

		Avena pubescens							
		Was First, Second, or Third, among the total Grasses, as under.				Yielded 5 per cent. and over, to the total Mixed Herbage, as under.			
		1862.	1867.	1872.	1877.	1862.	1867.	1872.	1877.
Plots	2 . . .	. .	. .	3	. .	. .	. .	10·28	7·68
	3 . . .	3	. .	. .	. .	9·65	. .	. .	. .
	4–1 . .	3	. .	. .	. .	9·42	. .	. .	. .
	4–2 . .	. .	. .	. .	. .	7·38	. .	. .	. .
	5 . . .	. .	. .	. .	. .	7·31	. .	. .	. .
	6 . . .	2	. .	. .	. .	14·54	. .	. .	. .
	7 . . .	1	. .	. .	. .	13·81	. .	. .	. .
	8 . . .	1	. .	. .	. .	12·68	. .	. .	. .
	9 . . .	. .	. .	. .	. .	10·22	. .	. .	. .
	10 . . .	2	. .	. .	. .	10·64	. .	. .	. .
	12 . . .	2	3	. .	. .	9·87	5·64	. .	. .
	13 . . .	. .	. .	. .	. .	5·48	. .	. .	. .
Total	First . .	2	0	0	0	11	1	1	1
	Second .	3	0	0	0				
	Third . .	2	1	1	0				

Thus, this plant only twice assumed the first place, then only in the first separation-year, and on plots where no nitrogenous manure was applied, and where, therefore, there was no general luxuriance of other grasses. It was three times second, but again only in the first year of separation, and again only on plots characterised by limited general growth. It was third twice in the first, once in the second, and once in the third separation-years; in each case without, or with deficient manuring.

In the first year of separation it yielded more than 5 per cent. of the produce on 11 plots, but on each succeeding occasion on one plot only.

Avena pubescens has, in fact, gone down under almost every condition of manuring and has maintained moderate prominence only on a few plots where there was generally a very limited degree of luxuriance among its associates; its nearer competitors have been *Agrostis vulgaris, Festuca ovina, Anthoxanthum odoratum, Holcus lanatus,* and *A. flavescens.*

Avena flavescens.

Avena flavescens is a perennial, tufted grass, of rather weakly habit, with branched, fibrous roots, that do not descend deeply into the ground. The culms are few, slender, 12 to 24 inches in height. It starts into growth early, and flowers in June and July, and sometimes again in the autumn. It is chiefly found in cool dry pastures, and light soils.

The favouring conditions for the growth of the plant may be said to be its hardiness, and power of producing fertile seeds before the first cutting, which gives it an advantage over *A. pubescens;* but, on the other hand, its weak habit and absence of creeping shoots tell against it.

The following figures show its position as to degree of prominence.

TABLE XLIII.—Relative predominance of *Avena flavescens.*

		Avena flavescens							
		Was First, Second, or Third, among the total Grasses as under.				Yielded 5 per cent. and over, to the total Mixed Herbage, as under.			
		1862.	1867.	1872.	1877.	1862.	1867.	1872.	1877.
Plots	1	..	..	..	..	..	6·85	6·23	..
	2	3	..	1	..	6·02	5·88	11·62	..
	4-1	..	..	3	..	..	..	6·09	..
	7	..	..	3	..	..	..	..	..
	8	..	..	..	..	5·42	..	6·94	..
	9	..	..	..	..	9·08	..	5·30	..
	10	..	..	..	..	10·05	..	..	..
	11-1	..	..	..	..	5·28	..	..	..
	14	..	..	3	..	..	7·12	5·67	..
	16	1	1	1	..	18·37	14·86	18·80	6·67
	18	—	..	..	..	—	..	5·75	..
	20	—	—	..	..	—	—	..	5·30
Total	First	1	1	2	0	6	4	8	2
	Second	0	0	0	0				
	Third	1	0	3	0				

On only two plots did *Avena flavescens* attain the first place; once on the plot which had formerly received farm-yard manure, but which had then been nine years without manure; and three times on plot 16 with the smaller amount of nitrate of soda and the mixed mineral manure. In no case did it come second. It was only on four occasions third; once on the plot then receiving farm-yard manure, once with superphosphate of lime alone, and once with the complete mixed mineral manure alone, but once also with the larger amount of nitrate of soda and the mixed mineral manure.

It contributed more than 5 per cent. to the produce six times in the first separation year, only four times in the second, eight in the third, and only twice in the fourth. The conditions under which it has most conspicuously maintained its position are where both nitrate of soda and mineral manure was employed.

It attained to between 11 and 12 per cent. in 1872 on the plot which had formerly received farm-yard manure, and it reached nearly 19 per cent. in the same year, on the plot with the mineral manure and the smaller amount of nitrate of soda. On no other plot has it shown any marked prominence.

Of the three species of Avena, *A. elatior* is the only one that may be said to have gained ground on the experimental plots; and it has done so were there was liberal nitrogenous manuring, in the form of ammonia-salt, with mineral manure in addition. Both *A. pubescens* and *A. flavescens* have, on the other hand, gone down; and neither of them has maintained even moderate prominence where ammonia-salts were used; but both, and especially *A. flavescens,* have been more favoured under the influence of the nitrate.

The comparatively robust habit and superior physical endowments of *A. elatior* are sufficient to account for its superiority over its congeners when growing in association. It may here be remarked that many botanists place *A. elatior* in a distinct genus—*Arrhenatherum,* by reason of the lower flower of each spikelet being male only, and also from the presence of certain rudimentary organs, indicative of a different line of descent of this plant from the true Avenas.

Poa trivialis.

This plant has closely matted minute fibrous roots, not penetrating deeply, but forming a dense mat beneath the surface, producing no underground offshoots, but developing, especially in moist weather, creeping branches from the base of the culms. The culms are tufted, 1 to 2 feet in height, rather rough, and bear linear leaves, narrower than those of *P. pratensis.* It is early in growth, though rather tender, and liable to injury from frost or drought. It grows rapidly, flowers in June, and again in the autumn. It seeds abundantly in the first crop, especially on plot 14 (mineral and high nitrate), where numerous seedlings have been observed. Like *P. pratensis* it prefers a moist rich soil.

The endowments propitious to the growth of this plant are its dense root-development, its rapid growth under favourable conditions as to soil, manure, and moisture, its formation of stolons, and its free seeding tendency.

Table XLIV. registers its characters as to predominance.

TABLE XLIV.—Relative predominance of *Poa trivialis.*

		Poa trivialis							
		Was First, Second, or Third, among the total Grasses, as under.				Yielded 5 per cent. and over, to the total Mixed Herbage, as under.			
		1862.	1867.	1872.	1877.	1862.	1867.	1872.	1877.
Plots	1 . . .	1	1	..	..	31·90	22·32	..	..
	2 . . .	1	2	..	..	28·18	15·75	..	..
	4–1. . .	..	..	..	..	5·16	5·65	..	..
	4–2. . .	3	..	..	..	8·14	..	..	..
	8 . . .	..	..	..	..	5·48	..	..	..
	9 . . .	..	..	..	..	8·72	..	..	..
	10 . . .	3	..	..	..	10·18	..	..	..
	11–1 . .	2	..	..	..	13·25	..	..	..
	11–2 . .	3	..	..	..	17·04	..	..	..
	13 . . .	3	..	..	..	11·84	6·68	..	..
	14 . . .	1	1	2	1	22·48	32·93	24·76	21·59
	15 . . .	..	1	2	..	6·53	23·67	7·95	6·05
	16 . . .	..	..	..	..	6·87	8 96	6·53	..
	17 . . .	...	2	..	..	5·21	12·08	..	..
Total	First . .	3	3	0	1	14	8	3	2
	Second . .	1	2	2	0				
	Third . .	4	0	0	0				

This Poa has only been first on four plots: on the two dung plots in 1862, and on one of them in 1867, after the discontinuance of the manure, but on neither since. It has three times been first on the plot with mixed mineral manure and the larger amount of nitrate, and once on the plot with the same amount of nitrate without minerals. It was second on one of the high ammonia and mineral plots in 1862, and in the same year it was third on four other ammonia plots. In 1867 it was second only on the dung plot and on one of the nitrate plots. Since that date it has been neither first, second, nor third on any of the ammonia plots. In 1872 it was second on two of the nitrate plots, but in the last year of separation it was first on only one—a nitrate plot, and not second nor third on any.

The number of plots on which it contributed 5 per cent. and upwards of the produce was 14 in 1862, only eight in 1867, and was reduced to three in 1872, and to two in 1877. The plots on which it maintained this degree of prominence were all nitrate plots. In fact, although at first extremely prominent on many plots it strikingly declined wherever ammonia was applied, and almost as strikingly retained its position on the plot with the mineral manure and the larger amount of nitrate of soda (14); yielding on it 22·48 per cent. of the total produce in 1862, 32·93 in 1867, 24·76 in 1872, and 21·59, in 1877; whilst in 1874 it was estimated to form about one-half of the total herbage of the plot.

Poa pratensis.

This grass, in addition to its tufted stems, has numerous underground stolons, which form new plants at their extremities. The roots, which are given off from the short underground stems as well as from the stolons, are fibrous, intricately branched, not penetrating very deeply, but forming a dense mat beneath the surface. The culms are 1 to 4 feet in height, erect, and smooth. The leaves are broadish. It begins to grow early in spring, which is advantageous on economic grounds, and it grows also rapidly after cutting. It flowers in June. It is found usually on rich, somewhat damp or shaded soil, but it is by no means confined to localities of this description.

This grass possesses advantages in its hardihood, free growth, its development of stolons, its dense root growth, its power of growing early, and again after cutting, &c. It is stated, however, to be particularly liable to the attacks of a fungus ("rust" or *Puccinia*).

Table XLV. records the relative degree of predominance of this species.

TABLE XLV.—Relative predominance of *Poa pratensis*.

		Poa pratensis							
		Was First, Second, or Third, among the total Grasses, as under.				Yielded 5 per cent. and over, to the total Mixed Herbage, as under.			
		1862.	1867.	1872.	1877.	1862.	1867.	1872.	1877.
Plots	1 . . .	. .	. .	3	. .	. .	6·56	7·41	. .
	4–2 . .	. .	. .	3	. .	. .	. .	5·11	. .
	9 . . .	3	3	1	2	10·68	13·02	22·67	18·03
	10 . . .	. .	1	2	. .	. .	14·81	19·62	6·48
	11–1 . .	. .	. .	. .	. .	9·43	12·86	10·40	. .
	11–2 . .	. .	3	. .	. .	5·12	10·38	12·43	. .
	13 . . .	. .	3	3	. .	. .	10·26	11·45	10·09
Total	First . .	0	1	1	0	3	6	7	3
	Second .	0	0	1	1				
	Third . .	1	3	3	0				

On only two occasions did *Poa pratensis* take the first place among grasses, on one plot in 1867, and on one in 1872, both manured with ammonia-salts and mineral manure. On each of the same two plots it was also once second. It came third rather more frequently, but in each case where ammonia-salts were applied in conjunction with mineral manure.

It was again only on the ammonia plots that it exceeded 5 per cent. of the total produce, and maintained its position in that respect; and it did so the most markedly where the ammonia-salts were used in the smaller quantity; other free-growing grasses overcoming it where the larger quantity of ammonia was employed. This was especially the case in the later years.

In only one instance did it yield more than 20 per cent. of the total produce, and then it was with the mixed mineral manure and the smaller quantity of ammonia-salts. On each of the nitrate plots, on the other hand, it occurred in only insignificant amount.

Comparison between the two species of Poa.—From what has been said of the two species of this genus, it is evident that the stolon-bearing *Poa pratensis* is specially benefited by nitrogenous manure in the form of ammonia-salts (in combination with mineral manure), but not at all by nitrate of soda; whereas the more finely rooted and non-stoloniferous *Poa trivialis* has declined markedly on the ammonia plots, but has remained very prominent on the nitrate plots, especially where the larger amount of nitrate was used with the mixed mineral manure. This is strikingly illustrated in the following summary table, giving side by side the percentage (in the total produce) of each of the two Poas on two comparable plots; plot 9 with mineral manure and nitrogen as ammonia-salts, and plot 14 with the same mineral manure and the same amount of nitrogen, but as nitrate of soda. It may be added that there is the same contrasted tendency of the two species to increase or decrease respectively on the other ammonia and nitrate plots of the series.

TABLE XLVI.

Separation years.	Plot 9. Mineral Manure and Ammonia Salts.		Plot 14. Mineral Manure and Nitrate of Soda.	
	Poa pratensis.	Poa trivialis.	Poa pratensis.	Poa trivialis.
	Per cent.	Per cent.	Per cent.	Per cent.
1862	10·68	8·72	1·45	22·48
1867	13·02	2·14	1·05	32·93
1872	22·67	0·64	2·57	24·76
1877	18·03	0·11	4·01	21·59

The interesting question arises whether the relatively shallow-rooting *Poa trivialis* predominates on the nitrate plots by reason of its fine surface roots arresting and taking up the nitrate before it has had time to penetrate too deeply. This view is favoured by the fact of the rapid growth which this plant invariably makes upon the application of the nitrate of soda in the spring. Another point worthy of remark is the contrast observable on plot 14, between the shallow-rooted *Poa trivialis* and the deeper and wiry-rooted *Bromus mollis*. In moist seasons, when the latter is relatively deficient, *Poa trivialis* is luxuriant, and *vice versâ*.

Dactylis glomerata.

This is a robust, free-growing grass, with strong, stout, fibrous roots, descending more or less vertically to a considerable depth, and together with other more horizontally disposed fibres matting the soil into a dense ball. The root-hairs, under favourable circumstances, are numerous, long, and delicate. In some cases we have observed the root fibres protruding from the base of the tufts, and ascending for 2 to 3 inches before they could find an outlet through which to descend to the soil. The stock is more or less tufted, especially so when well developed, as on the ammonia plots. The culms are 2 to 3 feet in height. The broad leaves vary greatly in colour and luxuriance, according to the manure. The plant flowers in June and is dichogamous. It makes growth by means of offsets from the base of the last year's stems, early in spring, and again after cutting or browsing. Hence it is valued as a pasture grass, but if allowed to grow too old it becomes too woody, and is then of less value as food. It grows in almost any soil, but shows a preference for those which are rich, and is considerably different in habit and appearance on different soils; the "hummocky" habit being specially noticeable where the food is abundant.

The circumstances favouring the success of the plant are—its robust constitution, general luxuriance and power of adaption; its deeply-penetrating, abundant and greatly branched root fibres; its densely tufted habit, and its growth after cutting or browsing.

Table XLVII. shows its predominance.

TABLE XLVII.—Relative predominance of *Dactylis glomerata.*

		Dactylis glomerata							
		Was First, Second, or Third, among the total Grasses, as under.				Yielded 5 per cent. and over, to the total Mixed Herbage, as under.			
		1862.	1867.	1872.	1877.	1862.	1867.	1872.	1877.
Plots	1 . . .	3	. .	. .	. .	16·43	6·43	. .	. .
	2 . . .	. .	. .	. .	. .	. .	6·51	. .	. .
	9 . . .	. .	. .	3	3	5·58	. .	11·88	14·07
	10 . . .	1	. .	. .	. .	12·51	5·44	. .	. .
	11–1 . .	1	1	1	3	24·16	39·31	39·28	17·11
	11–2 . .	1	1	1	. .	23·34	38·30	27·23	13·38
	13 . . .	1	2	1	1	27·88	20·29	43·05	40·75
	14 . . .	. .	. .	. .	. .	10·00	7·28	. .	12·48
Total	First . .	4	2	3	1	7	7	4	5
	Second .	0	1	0	0				
	Third . .	1	0	1	2				

Thus, *Dactylis* was first on four plots in 1862, on two in 1867, on three in 1872, and on one in 1877. Each of the plots on which it was so received ammonia-salts and mineral manure together. It was second on one of the same plots in 1867. It was third on one ammonia plot in 1862, on one in 1872, and on two in 1877. It has thus asserted itself only under the influence of liberal supplies of ammonia, in conjunction with mineral manure. It yielded 5 per cent. and more to the produce on seven plots in 1862, on seven in 1867, on four in 1872, and on five in 1877. The only plots on which it has been so far prominent in the two latter separation years are those receiving nitrogenous with mineral manures. But the only plots on which it has contributed a really large proportion of the herbage are those where the nitrogenous manure was in the form of ammonia-salts, and where there was liberal application of mineral manure as well. Even on some of these plots the plant has not been so prominent in recent years as formerly. Thus, whilst on one of the plots having the double amount of ammonia-salts and mineral manure, the *Dactylis* yielded nearly 40 per cent. of the produce in 1867 and in 1872; on the same plot it only furnished 17 per cent. in 1877. Again, on the other double-ammonia plot it declined from 38·3 per cent. in 1867, to 27·23 in 1872, and to 13·38 in 1877. The only plot on which it has yielded excessively in the more recent years is the one (13) with ammonia salts, mineral manure, and cut wheat straw, where it gave 27·88 per cent. in 1862, 20·29 per cent. in 1867, 43·05 per cent. in 1872, and 40·75 per cent. in 1877. On the other hand, it always occurs in very much less amount, and generally in quite insignificant quantity, where the nitrate of soda is used. In a word, it is really prominent only with a liberal supply of ammonia, associated with a correspondingly liberal supply of mineral constituents, together favouring general luxuriance, much stem formation, and tendency to mature.

Owing to the dense hummocky mode of growth of the *Dactylis* on plots suitable to its development, and to its deep roots, it is a formidable opponent to other grasses, where it has once got possession. But it is at a disadvantage in competition with the shallow, densely-matted roots of *Poa trivialis,* and the quick-growing, close stemmed, *Bromus mollis.* On the unmanured plots it is but of weakly habit, and is relatively deficient in the mat of roots beneath the surface, and in the dense tufts, which it produces under conditions of high manuring.

Festuca ovina.

This is a perennial, densely-tufted grass, often provided with numerous fine, creeping, underground stolons, by means of which the plant is widely spread, especially where the flowering tendency is restricted. The root fibres are very numerous (relatively more so than in any other grass on the plots), fine, with but few hairs, branching in all directions, and forming a very dense network just beneath the surface, thereby obtaining full possession of the soil to a depth of from 6 to 10 inches, but not usually penetrating more deeply. The culms are slender, angular, 6 inches to 2 feet in height, and decumbent at the base, giving off roots from the under surface. The leaves are mostly radical, very slender, erect, conduplicate, varying in colour and size. The plant flowers in June and July, and again in the autumn.

The form usually met with in the experimental plots is called *F. duriuscula,* which is larger than the typical *F. ovina,* and its upper leaves are flat, not folded. Some botanists have considered that there are several varieties of *F. ovina,* whilst others have ranked them as separate species. That considerable differences exist is obvious, but it is equally obvious that these differences are by no means so great as to preclude the idea that they may all have originated from a common stock. SINCLAIR ('Hortus Gramineus Woburnensis,' p. 155, 8th ed.) says, and his opinion is worthy of great consideration, that, "All these grasses (*F. glauca, glabra, cambrica, duriuscula,* and *rubra*) vary much from change of soil and situation; the flowers are particularly apt to vary in number, as well as in the length of their awns; there is one character, however, which I have never found to change under any variety of culture, which is the creeping root; and this is also an agricultural character of distinction which is never to be lost sight of, as it always produces a specific effect upon the soil, very distinct indeed from that of the fibrous rooted kinds."

The records of observation at Rothamsted do not enable us to speak so decidedly as SINCLAIR does as to the permanence of the character afforded by the "creeping root." According to him it is the form called *F. rubra* alone which possesses this characteristic, while the fine *F. ovina,* and its allies, or varieties, *duriuscula, glabra, cambrica,* do not show it. We are inclined to look on the "creeping root" as to a large extent an induced physiological character, depending on conditions of soil and competition, &c., and not as a permanent hereditary characteristic; and this seems to be the view of Mr. BENTHAM and Sir JOSEPH HOOKER, neither of whom admit the forms above mentioned to higher rank than that of sub-species or varieties. On the unmanured plots the form met with corresponds most closely with the typical *F. ovina;* while on the highly manured plots greater development and luxuriance is met with, and the plant assumes more of the habit of *F. duriuscula.*

In this country the grass is very abundant at all elevations, especially on dry poor soils. It is subject, under varying external conditions, to great variations in size and colour of parts, presence or absence of awns, &c.

The characters favourable to it are its dense rooting faculty, its tufted habit, abundant and fine offsets, threading their way among other roots, fine slender leaves, capable of expanding or closing according to hygroscopic conditions, enabling the plant to insinuate itself among its competitors, &c. Its restricted evaporating surface, robust constitution, and power of adaptation to diverse circumstances, as shown by its world-wide distribution, also give it a great advantage. Its ability to withstand drought is likewise a faculty greatly in its favour in dry seasons.

TABLE XLVIII.—Relative predominance of *Festuca ovina*.

		Festuca ovina							
		Was First, Second, or Third, among the total Grasses, as under.				Yielded 5 per cent. and over, to the total Mixed Herbage, as under.			
		1862.	1867.	1872.	1877.	1862.	1867.	1872.	1877.
Plots	1	..	..	..	..	..	6·17	6·40	10·75
	2	..	..	..	3	..	..	9·89	11·23
	3	1	1	1	1	13·30	15·20	21·67	21·89
	4–1	2	1	1	2	10·20	16·75	20·44	16·02
	4–2	..	1	1	1	6·80	26·09	49·29	55·20
	5	2	1	1	1	21·99	30·57	46·56	53·31
	6	3	1	1	1	13·33	25·93	31·15	38·02
	7	2	2	1	1	13·73	11·38	14·86	26·59
	8	3	1	1	1	7·51	17·74	23·95	19·76
	9	..	1	..	1	5·21	18·42	8·68	21·80
	10	..	2	1	1	..	14·74	19·80	26·34
	12	..	1	1	1	7·17	11·59	16·05	20·88
	13	..	..	..	..	..	6·82	..	..
	15	1	2	1	1	13·69	12·08	34·71	20·77
	16	3	..	..	1	11·13	10·44	10·33	16·66
	17	3	3	1	3	9·43	11·18	18·05	12·04
	18	—	3	1	1	—	7·05	23·76	24·65
	19	—	—	1	1	—	—	29·65	21·40
	20	—	—	2	2	—	—	25·54	18·69
Total	First	2	8	13	13	12	16	18	18
	Second	3	3	1	2				
	Third	4	2	0	2				

Thus, *Festuca ovina* gave the highest percentage in the total gramineous herbage on two plots in 1862, on eight in 1867, on 13 in 1872, and on 13 in 1877. It was second on three plots in 1862, on three in 1867, on one in 1872, and on two in 1877. It was third on four plots in 1862, on two in 1867, on none in 1872, and on two in 1877. It is clear, therefore, that this very poor, "common-land," but hardy and drought-resisting grass, has gained in prominence in a very marked degree on some plots during the progress of the experiments. It has only lost in second and third place to gain the first in the later years. It was first, second, or third on nine plots in 1862, on 13 in 1867, on 14 in 1872, and on 17 in 1877. It yielded 5 per cent. and over of the total produce on 12 plots in 1862, on 16 in 1867, on 18 in 1872, and on 18 in 1877.

The only other grass which has shown anything like such general prominence is *Agrostis vulgaris;* but although the latter has been more frequently first, second, or third, especially more frequently, and increasingly, second or third, it has been much less frequently first than *Festuca ovina.* *Agrostis vulgaris* has also yielded more than 5 per cent. on a greater number of plots in each separation year than the *Festuca,* but it has by no means contributed so high a percentage on some plots. Thus, on plot

4-2 (ammonia and superphosphate) *Festuca ovina* contributed 49·29 per cent. in 1872 and 55·20 per cent. in 1877: and on plot 5, with ammonia-salts alone, it gave 46·56 per cent. in 1872, and 53·31 per cent. in 1877; whereas *Agrostis vulgaris* has in only one case exceeded 30 per cent. The table shows that *Festuca ovina* was neither first, second, nor third on either plot 11–1, 11–2, 13 or 14, the four plots of the highest luxuriance of gramineous herbage, and especially of the freer-growing species. *Agrostis vulgaris*, however, contributed a much higher proportion of the produce on these plots (excepting 14) of high gramineous luxuriance, and maturation also, than did *Festuca ovina*.

The general result is, that *Festuca ovina* has gained in prominence on all the deficiently or only moderately manured plots; and it has done so the most, other things being equal, where, with a sufficiency of nitrogenous manure favouring gramineous luxuriance, there has been a deficiency of mineral supply preventing maturation, and consequently limiting the power of competition, of the freer-growing grasses. In fact, where the *Festuca ovina* has yielded the highest percentage of the produce, it has been with low total yield, and with a very large proportion of dark green, leafy, immature growth. The *Festuca ovina* also gains where other plants suffer for want of moisture; whilst the *Agrostis vulgaris* gains (if the other conditions are favourable to it) in the wetter seasons.

Festuca pratensis.

The stock of this grass is subterranean, perennial, somewhat creeping, but rarely producing offsets. The culms are tufted, 2 to 4 feet high; the leaves flat and broadish. The root fibres penetrate more deeply than those of *F. ovina*, but are not so much branched. It begins to grow somewhat early and rapidly, and flowers in June.

In this country this grass is partial to rich, moist soils, and is said to be an excellent pasture grass, much relished by cattle. It varies in stature, form of inflorescence, and presence or absence of stolons. *F. elatior*, sometimes considered a variety of this, has more stolons, and a more spreading flower-panicle.

This species of *Festuca*, instead of being first, second, or third in prominence on a large number of plots, and yielding in some cases half of the entire produce as did *F. ovina*, in only one case was first, and that was on the unmanured plot (12), in the first separation-year, 1862; and it was only on that occasion that it yielded more than 5 per cent. of the produce. On some plots in the first separation year it was absent, but on most it occurred in only small quantity; and on almost every plot it has gone down to a quite insignificant amount. The only plot on which it persistently gained is 15, with nitrate of soda alone up to 1875 inclusive, and the mixed mineral manure, including potass, each year since; and even there it only contributed 1·47 per cent. of the produce in the fourth separation-year.

Festuca elatior has only been found in the sample from one plot, and then in very insignificant amount.

Festuca loliacea, which is hardly more than a variety of *F. pratensis*, has only been found in the samples from two plots, and, like *Festuca elatior*, in quite insignificant quantity.

As compared with *Festuca ovina*, which is one of the most prevalent grasses on the plots, *F. pratensis* is rapidly and greatly diminishing. This may possibly arise from the fact that *F. pratensis* is not so hardy as *F. ovina*, is less sturdy in habit, has a less branched root-growth, fewer stolons, a greater evaporating surface from its broader leaves, which, together with its less extensive root-development, may afford a reason why *F. pratensis* does not resist drought so well as *F. ovina*.

Bromus mollis.

This grass is annual or biennial; all the other grasses on the plots being perennial. Its roots are stiff, wiry, not much branched, but sometimes very deeply penetrating. On plot 14 (nitrate and mineral), the roots were discovered at a depth of 4 to 5 feet, having lost their wiriness and become fleshy. Small tubers were also observed on the root-fibres at the depth of 9 inches, and more; the tubers measuring sometimes as much as a quarter of an inch in length. The culms are 1 to 3 feet high, and generally die away after seeding or cutting, when new growth, but not so highly developed, is formed from the stock. If constantly mown or fed off, it generally disappears. It flowers early, so that its seeds ripen before cutting, and many seedlings are produced.

The endowments favourable to it are its deeply rooting powers, which enable it to withstand drought, the tubers probably aiding as a store; also its abundant and early production of seed, which enable the plant to diffuse itself freely when circumstances are propitious.

The following table shows its predominance.

TABLE XLIX.—Relative predominance of *Bromus mollis*.

		Bromus mollis							
		Was First, Second, or Third, among the total Grasses, as under.				Yielded 5 per cent., and over, to the total Mixed Herbage, as under.			
		1862.	1867.	1872.	1877.	1862.	1867.	1872.	1877.
Plots	1 . . .	2	3	. .	. .	21·92	10·55	. .	. .
	2 . . .	2	1	. .	. .	17·77	16·39	. .	. .
	14 . . .	2	2	1	. .	18·04	17·69	42·10	8·02
	15 . . .	. .	. .	. .	. .	. .	6·27	. .	. .
Total	First . .	0	1	1	0	3	4	1	1
	Second . .	3	1	0	0				
	Third . .	0	1	0	0				

The position of this plant on the experimental plots is very marked. It was not first on any of the plots in 1862; but in that year it was second on plots 1 and 2, the first with farmyard manure and ammonia, and the second with farmyard manure alone. In 1867, however, it had reached the first place on plot 2, with farmyard manure alone up to 1863 inclusive, but no manure since; and it was third on plot 1, where the farmyard manure and ammonia-salts had been applied up to 1863, and the ammonia-salts alone since. On neither of these plots was it either first, second, or third in either 1872 or 1877; nor did it reach 5 per cent. of the produce in either of these two later separation-years. On plot 1 (dung and ammonia-salts the first eight years) it yielded 21·92 per cent. in 1862, 10·55 per cent. in 1867, 4·55 per cent. in 1862, and only 0·83 per cent. in 1877. On plot 2, again (dung alone the first eight years and no manure afterwards), it went down from 17·77 per cent. in 1862, to 16·39 per cent. in 1867, 3·89 per cent. in 1872, and to only 0·18 per cent. in 1877.

As the table shows, on only one other plot (14), with the larger amount of nitrate of soda and the mixed mineral manure, did it attain to the first or second place; and it was only on that plot that it yielded more than 5 per cent. of the produce in each of the four separation-years. Indeed, here it gave 18·04 per cent. in 1862, 17·69 per cent. in 1867, 42·1 per cent. in 1872, but only 8·02 per cent. in 1877. In 1870, however, the year of extreme spring and summer drought, plot 14 yielded an enormous crop of *Bromus mollis;* estimated at half, or more, of the total produce. In 1871, again, this plant was estimated to yield the greater part of the crop on this same plot (14), owing probably in great part to the large number of seedlings produced in the previous year. As more fully discussed in Part I., p. 334, *et seq.*, the predominance of *Bromus* on this plot, in years of drought, is attributed to its wiry and deep roots gaining possession of the lower layers of the soil, and thus rendering it comparatively independent of surface moisture, and able to arrest the deeply percolating nitrate. Its chief rival on the plot is *Poa trivialis*, especially in wet seasons; whilst on the plots with the same mineral manure, and the same amount of nitrogen, but supplied as ammonia-salts, *Bromus mollis* occurs in very insignificant amount, *Poa trivialis* is in much less amount than *Poa pratensis*, and *Dactylis* and other free-growing grasses attain a high place.

It should be observed that plot 14, on which such large quantities of *Bromus* occur, is contiguous to plot 1 on one side, and to plot 15 on the other; so that its freer occurrence on those plots than anywhere else, excepting on plot 14, may in part be accounted for by this circumstance. Upon the whole, *Bromus mollis* has somewhat gone down on plot 14 in the later, frequently wet years; *Poa trivialis*, as already said, proving its most successful rival, and *Alopecurus* coming next. It remains to be seen whether *Bromus*, a plant of evil reputation with the farmer, will not re-assert itself in years of comparative drought.

Lolium perenne.

The culms of this grass are more or less tufted, 1 to 2 feet high, decumbent, and subterranean at the base, then ascending, rooting at the nodes, and giving off leafy stolons. The roots are much branched, fibrous, penetrate to a considerable depth, and give off a dense network of root-fibrils near the surface. The root-hairs are stoutish, sometimes short, numerous; at other times very fine and intricate.

This grass shoots early, flowers in May and June, and again in the autumn; and it is observed to grow well after trampling down or cutting, and to vary greatly in character according to external conditions. It is most favoured by a moist, warm season. Mr. WHITWORTH is recorded by SINCLAIR to have collected as many as 60 varieties. It does not produce much aftermath.

The endowments specially favourable to it are its hardiness, creeping offshoots, growth after cutting, &c.

TABLE L.—Relative predominance of *Lolium perenne.*

		Lolium perenne							
		Was First, Second, or Third, among the total Grasses, as under.				Yielded 5 per cent. and over, to the total Mixed Herbage, as under.			
		1862.	1867.	1872.	1877.	1862.	1867.	1872.	1877.
Plots	3 . . .	. .	. .	. .	. .	6·37	. .	. .	. .
	4–1 . .	. .	. .	. .	. .	9·28	5·24	. .	. .
	4–2 . .	. .	. .	. .	. .	6·47	. .	. .	. .
	8 . . .	. .	. .	. .	. .	5·92	. .	. .	7·63
	14 . . .	3	3	. .	. .	13·80	9·36	5·55	. .
	15 . . .	. .	. .	. .	. .	7·49	. .	. .	7·32
	16 . . .	. .	. .	. .	. .	5·85	6·23	. .	. .
	17 . . .	. .	. .	. .	. .	5·09	. .	. .	6·68
	18 . . .	—	. .	. .	. .	—	5·15	. .	6·45
Total	First . .	0	0	0	0	8	4	1	4
	Second .	0	0	0	0				
	Third . .	1	1	0	0				

The table shows that this grass was neither first nor second on any plot in either of the separation-years; but it was third in the first two separation-years, 1862 and 1867, on plot 14, with the larger amount of nitrate of soda and the mixed mineral manure. It yielded more than 5 per cent. of the produce on eight plots in 1862, on four in 1867, on one only in 1872, but again on four in 1877. It was, nevertheless, found in greater or less quantity in the samples from every plot in each separation-year, excepting plot 11–1 in 1872, and 11–2 in 1877; these being the two plots most highly manured with ammonia-salts, in conjunction with mixed mineral manure. It will be observed that it yielded 5 per cent. and over to the produce on plot 3 (unmanured), on plot 4–1 (superphosphate alone) and 4–2 (superphosphate and ammonia-salts) in 1862, but only on one of them in 1867, and not again on either. On plot 8 (mineral manure, then without potass) it gave more than 5 per cent. in 1862, and again in 1877. But the most favourable conditions for it in the struggle are obviously those supplied on the nitrate of soda plots, on four of which it yielded more than 5 per cent. in 1862, on two in 1867, on one in 1872, and on two in 1877. This is consistent with the results obtained at Chiswick, where the plant was grown separately; and it was where the nitrate was applied that the root development of this plant was greatest. It also yielded somewhat more than 5 per cent. on plot 18, manured with the ash-constituents and the nitrogen of one ton of hay, in the second and fourth separation-years.

It will be observed, however, that, even on those plots the most favourable to it, there was generally a decline in the percentage amount of it from the first to the third separation-year, but generally some recovery in 1877, though not on the plot which upon the whole yielded the highest percentage, namely, plot 14, with the nitrate of soda and mineral manure; to the produce of which it contributed 13·8 per cent. in 1862, 9·36 per cent. in 1867, 5·55 per cent. in 1872, and only 2·63 per cent. in 1877, whilst on several other plots it yielded a higher percentage than this in 1877, and on some between 6 and 8 per cent.

It is obvious that *Lolium perenne* is a plant of relatively weakly habit, and that it did not flourish where ammonia-salts were used, but that where the nitrogen was supplied as nitrate of soda, it was much more able to maintain some, though still a rather low position in the struggle. This may be due either to a want of favourable manurial conditions for the plant itself where ammonia-salts are applied, or to the competition of stronger species being thus stimulated, whilst with the nitrate of soda, the favourable conditions for the plant itself have probably something to do with the results. When growing separately the plant is reputed to attain considerable luxuriance with liberal nitrogenous manuring, and plenty of moisture and warmth; and its partial recovery of prominence in the later years may be due to the prevailing wetness of the seasons.

There are certain other grasses which occur only on some of the plots, and then only in very small proportions, and which therefore demand merely passing notice.

Phleum pratense.—The *Cat's-tail*, or *Timothy grass*, occurs in moist pastures, and has a slightly creeping root which occasionally becomes bulbous, a tufted stock whence arise erect stems bearing cylindrical flower-spikes. It produces in spring an abundance of leaves at the base of the stems. This grass was observed on, or found in the samples from, six plots in 1862, five in 1867, six in 1872, and three in 1877. The plots on which it was found were for the most part those treated either with ammonia-salts or nitrate of soda, but in no case did it reach 0·2 of the total produce, and generally much less.

Aira cæspitosa.—The *Tufted-hair grass* is a large, coarse-growing grass, forming thick tufts or hummocks, which throw up tall stems bearing coarse foliage, and light panicles of flowers. It is a common plant in moist shady woods, and is abundant on Harpenden Common, though very sparingly represented on the plots. Its presence was noted on five plots in 1862, on three in 1867, on 17 in 1872, and on nine in 1877. It occurred in slightly greater proportionate amount on the ammonia and the nitrate plots, but in every case its produce was quite insignificant.

Briza media.—The *Quaking grass* is tufted, slightly creeping, and usually grows in dry open situations. It occurred in the samples taken from nine plots in 1862, in nine in 1867, in 17 in 1872, and in 12 in 1877. It was in largest proportion on the unmanured plots, on which it has on the whole increased, and in one instance it contributed more than 7 per cent. of the produce.

Of all the manured plots the one with superphosphate of lime alone was the most favourable to it, but even there it furnished little more than 2 per cent. of the produce. It was generally absent from the samples taken from the ammonia plots, or, if represented at all, it was in quite insignificant amount. It was, however, much more general, though still in very small quantity, where nitrate of soda was used. It would thus seem that ammonia-salts are either directly prejudicial to this grass, or that they are so indirectly, by favouring the more luxuriantly growing species.

Cynosurus cristatus.—The *crested Dog's-tail* is a tufted grass, the finely branched roots of which are reputed to penetrate deeply. It was found in the samples from, or observed during growth on, 14 plots in 1862, 14 in 1867, 16 in 1872, and 14 in 1877. It was absent, or in very small proportion, on the ammonia plots, but was generally present on the nitrate plots, and on those without mineral manure, or with mineral manure alone.

In one case where nitrate of soda was used it reached 2·54 per cent., and in three others it yielded more than 1 per cent. It also exceeded 1 per cent. on one occasion without manure, and in three with mineral manure alone; but in the large majority of the samples it was in very much smaller proportion.

It is obvious that neither of these four last-mentioned grasses was able to maintain even a moderate degree of prominence where the conditions were favourable for the luxuriance of other gramineous species.

THE LEGUMINOSÆ.

Next in importance to the grasses in the mixed herbage of meadow-land are the leguminous plants. The species which occur on the plots all belong to the Papilonaceous sub-division of the Order, the only one indeed represented in temperate climates. They are all low-growing or trailing perennials, varying considerably in "habit" one from another.

The roots seldom or never produce such a dense mass of fine fibrils as do those of the grasses, but the underground stocks if less finely branched penetrate deeply or spread widely. Their fibres, which are usually thicker than those of grasses, are very generally provided with little tubercles or nodules. Microscopic examination of the tubercles in *Trifolium repens* has shown us that they consist of an epidermal covering, investing two or three layers of cortical cells, within which is a circle of very minute fibro-vascular bundles with delicate spiral vessels. The central portion within the vessels is occupied by a large mass of cellular tissue, the innermost cells of which are larger than the outer polygonal, and often containing "crystalloid" contents, perhaps of aleurone or of some albuminoid substance. The fibro-vascular bundles are directly continuous with the vascular cylinder of the root fibre. There seems, then, no doubt, that these nodules are modified root fibres.* When fully formed no root-cap is visible, nor is this surprising, as their function seems to be not to extend the growth of the root but to furnish a magazine of nutritive matter. For the same reasons, probably, all the specimens that we have examined have been entirely destitute of root-hairs.

[Owing to the peculiar conformation of the flowers, the intervention of insects becomes almost absolutely essential to the fertilisation of these plants, as illustrated by Mr. DARWIN'S remarks on the interdependence of the red-clover, humble-bees, field-mice, and cats.]

Although the occurrence and amount of leguminous species materially affect the quality of the mixed herbage of meadow land, and are very dependent on the manurial conditions supplied, the proportion they contribute to the total produce is comparatively small, even when they are considered to be well represented. Indeed, if we were to attempt to represent the predominance of the leguminous species in the *total mixed herbage*, it happens that the most prominent of them would only once be first in predominance among the total species on the plot. It is, therefore, necessary in the tables of predominance to show those which take the first, the second, or the third place among the total leguminous, and not among the total mixed herbage of the plot. It is essential to bear this in mind in looking at the tables, otherwise the frequency with which one or other of the four leguminous species occurring in any material quantity on the plots appear as first, second, or third, would be quite misleading. The columns showing when the respective plants contributed 5 per cent. or over to the total produce, clearly show, by the very few entries, and the many blanks, the relatively small proportion of such herbage.

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* See WORONIN, Ann. Sc. Nat. 5me. sér., vol. 7, p. 84, tab. 6, who shows that these tubercles are associated with the presence of Bacteria, but in our experience they may occur without them, at least it is only occasionally that we have been able to detect them. The subject of root tubers has attracted the more attention of late from the presence of somewhat similar nodules on the roots of vines affected with Phylloxera, but the resemblance to the leguminous tubercles is merely superficial. Other similar swellings have been attributed to the presence of a myxomycetous fungus.

Trifolium repens.

The common white clover has a long, deeply penetrating, tap-root, giving off numerous rather thick, wiry, flexuous branches, studded with little tubercles, and descending obliquely into the soil. The stock gives off numerous, slender branches, creeping on the ground over and between the leaves and stems of neighbouring plants, and producing abundance of fine fibrous adventitious roots. The leaves are small, consisting each of three rounded flat leaflets, which in the young state are folded in halves, and so occupy comparatively little space. It flowers from May to October, but is rarely seen in fruit on the plots, its propagation being mainly secured by means of the creeping branches. The plant, moreover, is peculiarly subject in some seasons to have its floral organs more or less completely replaced by leaves.

The common white clover is one of those plants which is particularly liable to make its appearance on reclaimed land from estuaries and the like; also on land over which fire has passed, and often on new-made ground, seeming thus to have special capacities for availing itself of the conditions so afforded.

The structural endowments specially favourable to its diffusion are its hardiness and power of adaptation, as shown by its wide geographical distribution, its two-fold root power—the long tap root enabling it to derive moisture and nourishment from the lower layers of the soil, and thus to be comparatively independent of drought, while the abundant adventitious roots emitted from the creeping branches enable it to avail itself largely also of the surface-soil supplies. The trailing branches and closely folded young leaves also give the plant the advantage of making its way in between its neighbours, and so enabling it to avail itself readily of any vacant space. Its hard resisting seed-coat enables the seed to lie in the ground for long periods until circumstances become favourable for germination.

The following table shows the relative degree of predominance of this plant.

TABLE LI.—Relative predominance of *Trifolium repens.*

		Trifolium repens							
		Was First, Second, or Third, among the total Leguminosæ, as under.				Yielded 5 per cent. and over, to the total Mixed Herbage, as under.			
		1862.	1867.	1872.	1877.	1862.	1867.	1872.	1877.
Plots	1	2	2	2	2	..	..	..	..
	2	2	..	2	..	..	..	..	..
	4–1	2	..	3	..	..	..	..	..
	4–2	2	2	..	2	..	..	..	..
	5	..	2	3	..	..	..	..	..
	6	3	2	..	..	..	..	..	..
	7	3	..	2	..	..	..	..	..
	8	3	..	..	..	..	..	..	..
	9	2	2	..	..	..	..	..	..
	10	2	..	1	..	..	..	..	..
	11–1	..	..	1	..	..	..	..	..
	13	2	2	2	..	..	..	..	..
	14	2	2	..	..	..	..	..	..
	15	2	2	1	3	..	..	..	..
	16	2	3	3	..	..	..	..	..
	17	2	1	3	3	..	..	..	..
	18	..	..	3	..	..	..	..	..
	19	..	..	..	2	..	..	..	..
	20	..	..	3	2	..	..	..	..
Total	First	0	1	3	0	0	0	0	0
	Second	11	8	4	4				
	Third	3	1	6	2				

On only two plots, those receiving mineral manure including potass, did this plant ever contribute more than 2 per cent. of the total produce, and the highest amount was 3·08 per cent. It is shown to be first in quantity among the Leguminosæ on one plot in 1867, and on three in 1872. It was second on 11 plots in 1862, on eight in 1867, on four in 1872, and on four in 1877; and it was third on three plots in 1862, on one in 1867, on six in 1872, and on two in 1877. Yet this plant on no plot in either year yielded 5 per cent. of the total produce. Indeed, the cases in which it showed prominence among its allies were only those in which the total Leguminosæ were in comparatively insignificant amount; and where it was in the largest actual quantity it was so overpowered by other leguminous species that it did not attain to any relative predominance. It occurred in the smallest actual amount where ammonia-salts were used; it was rather more favoured on the nitrate of soda plots; but it gave a higher percentage without manure, and the highest of all with purely mineral manures containing potass.

On the whole it would appear that this plant is diminishing on the greater number of the plots; but there are several circumstances to be taken into account. For instance, it is certain that neither the percentage tables, nor those showing the weight of the produce yielded by this plant, give a wholly correct idea of the real quantities present. Its procumbent habit renders it liable to be concealed by taller-growing plants and overlooked, and may also prevent it from being reached by the scythe. Hence it may readily exist to some extent beyond what the records show. In this manner, in a measure, as SINCLAIR long ago pointed out ('Hort. Gram. Woburn.,' p. 223), may be accounted for those statements of the sudden appearance of the white clover on land which has been cleared or broken up, where it had not previously been noticed. Some instances of this have been observed at Rothamsted. Its great variability and power of adaptation to different circumstances also conduce to this result; but probably the result is due largely to the nature of the seed, which is so constructed as to lie uninjured in the soil for a long period of time, until the conditions are favourable for germination, when it would spread rapidly, by means of its offshoots, and quickly occupy vacant spaces.

Trifolium pratense.

The ordinary red clover of grass land is a tufted, very deeply-rooting perennial, (sometimes annual) with numerous long, thick tap-shaped roots, more or less branched, and with rather thick wavy fibres, provided with nodules, but almost destitute of root-hairs.* The stock is divided into numerous short, stout, spreading branches, with no true runners, but the central portion of the stock or crown ultimately dies, and the branches then form independent plants. Its growth begins early in spring; it flowers and ripens seed both in the first and second crops, though seedlings are rarely observed on the plots.

The structural characteristics favourable to the growth of this plant are: its powerful deep-rooting habit, enabling it to get food from considerable depths, and thereby also preserving it from the effects of drought; its lateral roots availing themselves of the nutriment in the upper layers of the soil, while the fleshy substance of the main roots acts as a storehouse; its downy leaflets serving as a protection from frost, or excessive radiation, and the way in which they are folded in the young state enabling them to insinuate themselves between its competitors; lastly, its extensive distribution in the northern hemisphere of the Old World shows that it bears low temperatures, but in spite of these properties it is gradually declining on the experimental plots.

The following table shows the relative degree of predominance of this plant.

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* The characters of the roots of this plant under treatment with various manures, during two seasons at Chiswick, are noted in the Journal of the Royal Horticultural Society, vol. iii., 1870, pp. 49 and 140.

TABLE LII.—Relative predominance of *Trifolium pratense*.

		Trifolium pratense							
		Was First, Second, or Third, among the total Leguminosæ, as under.				Yielded 5 per cent. and over, to the total Mixed Herbage, as under.			
		1862.	1867.	1872.	1877.	1862.	1867.	1872.	1877.
Plots	1 . . .	2	3	2	. .	. .	. .	. .	. .
	2 . . .	3	2	3	2	. .	. .	. .	. .
	3 . . .	1	2	2	3	. .	. .	. .	. .
	4–1 . .	1	3	. .	. .	. .	. .	. .	. .
	4–2 . .	2	. .	2	. .	. .	. .	. .	. .
	5 . . .	2	2	. .	. .	. .	. .	. .	. .
	6 . . .	2	2	3	2	. .	. .	. .	. .
	7 . . .	2	2	3	2	6·84	. .	. .	. .
	8 . . .	2	2	3	3	7·71	. .	. .	. .
	9 . . .	. .	. .	. .	3	. .	. .	. .	. .
	10 . . .	. .	. .	. .	1	. .	. .	. .	. .
	12 . . .	1	2	3	3	. .	. .	. .	. .
	13 . . .	3	. .	2	. .	. .	. .	. .	. .
	14 . . .	2	2	2	. .	. .	. .	. .	. .
	15 . . .	1	3	2	2	. .	. .	. .	. .
	16 . . .	1	1	2	2	. .	. .	. .	. .
	17 . . .	1	2	2	2	. .	. .	. .	. .
	18 . . .	—	1	. .	3	—	. .	. .	. .
	19 . . .	—	—	1	3	—	—	. .	. .
	20 . . .	—	—	2	. .	—	—	. .	. .
Total	First . .	6	2	1	1	2	0	0	0
	Second. .	7	9	9	6				
	Third . .	2	3	5	6				

This plant was first among four leguminous species on six plots in 1862, on two in 1867, and on one only in 1872 and 1877. It was second on seven plots in 1862, on nine in 1867, on nine in 1872, and on six in 1877, and it was third on two in 182, on three in 1867, on five in 1872, and on six in 1877. But it contributed as much as 5 per cent. to the total produce on only two plots, with purely mineral manure, and on these only in the first of the four separation-years.

It is practically absent from the samples taken from the plots where ammonia-salts are applied, whether these be used alone or in conjunction with mineral manures. It has gone down, but in a less degree, where nitrate of soda has been used. It has also gone down on the plots most favourable to Leguminosæ in general, and where it increased considerably in the early years—namely, those where mixed mineral manure, including potass, was employed. Finally, it has maintained a more uniform position on the unmanured plots, where there is no luxuriance of any species, than anywhere else; but here, in the later years, it has only contributed about 2 per cent. of the total produce. It has, in fact, not maintained even moderate prominence where there was any luxuriance of the grasses; and where the conditions were more favourable for the

Leguminosæ collectively, *Trifolium pratense,* like *T. repens,* has given place to other plants of the same family. It has, however, sustained the competition much better than *T. repens.*

There are other species of this genus which occur on the plots, but only in insignificant proportions, viz. :—

Trifolium minus.—This plant has only once been found in the samples, and then only in very insignificant amount.

Trifolium procumbens has twice been found in the samples, but, like *T. minus,* in most insignificant quantity.

Both *T. minus* and *T. procumbens* are, however, frequent in the immediate neighbourhood; and it is probable, from the appearance of the plots during growth, as well as from the creeping habit of the plants, that the samples do not adequately represent the amount of these plants on the plots, though in any case it must be very small, such plants having little or no capabilities of maintaining themselves amid their more powerful rivals.

Lotus corniculatus.

The root or subterranean stock of this robust perennial is tap-shaped, stout, fleshy, whitish, descending deeply into the sub-soil, its main branches are slender, horizontal, with numerous nodules, the lower branches descending, destitute, or nearly so, of nodules. The above-ground stem forms tufts of wiry decumbent or erect branches, bearing leaves of the same general shape as those of the clovers, but smaller. It flowers, as a rule, too late to perfect its seed before mowing. It is very hardy, bears drought well, is not very particular as to soil, but is stated to be variable on different soils, &c. At Rothamsted the variety which occurs is that called *vulgaris* in HOOKER's 'Student's Flora,' and it is pretty constant in its characteristics.

The conditions propitious to it are: its powerful root-stock, with extensive storage of nutritive matter, deep range, and power of availing itself of the food and moisture in the lower layers of the soil; the fact that the seeds are rarely produced before the first cutting, thus obviating exhaustion, and tending to prolong the life of the individual; its hardy constitution, as shown by its wide geographical distribution, its relative indifference to the nature of the soil, climate, &c.

For its predominance see the subjoined table.

TABLE LIII.—Relative predominance of *Lotus corniculatus.*

		Lotus corniculatus							
		Was First, Second, or Third, among the total Leguminosæ, as under.				Yielded 5 per cent. and over, to the total Mixed Herbage, as under.			
		1862.	1867.	1872.	1877.	1862.	1867.	1872.	1877.
Plots	2	..	3	..	3	..	..	..	..
	3	2	1	1	1	..	..	5·94	..
	4-1	3	2	2	2	..	..	..	..
	5	1	1	1	1	..	..	..	..
	6	3	1	2	3	..	..	..	..
	7	..	3	..	3	..	..	..	..
	8	..	3	2	2	..	..	..	..
	10	..	..	..	1	..	..	..	..
	11–1	..	1	..	..	..	..	..	..
	12	3	1	1	1	..	..	5·16	..
	13	..	..	2	..	..	..	..	..
	15	3	1	2	3	..	..	..	..
	16	..	..	..	3	..	..	..	..
	17	2	3	1	1	..	..	..	..
	18	—	3	1	2	—	..	..	..
	19	—	—	2	..	—	—	..	..
	20	—	—	1	3	—	—	..	..
Total	First	1	6	6	5	0	0	2	0
	Second	2	1	6	3				
	Third	4	5	0	6				

Lotus corniculatus was first amongst its few allies on one plot in 1862, on six in 1867, on six in 1872, and on five in 1877. It was second on two plots in 1862, on one in 1867, on six in 1872, and on three in 1877. It was third on four plots in 1862, on five in 1867, on none in 1872, but on six in 1877. Notwithstanding this considerable predominance among Leguminosæ, it only yielded more than 5 per cent. of the total mixed herbage on the two unmanured plots, and there only once, viz.: in the third separation-year. Like the other leguminous species, it has gone down in a very marked manner on all the plots where ammonia-salts were used; but it has done so less where these were used alone, and where only poor surface-rooting grasses were stimulated against it, and again on plot 18, where only a small quantity of ammonia-salts, with a complete mineral manure, was employed. It has been less reduced on the plots only moderately manured with nitrate of soda. On the unmanured plots alone has it shown any marked increase; but with superphosphate of lime alone, and with the mixed mineral manure without potass, it has also shown a tendency to increase rather than to diminish. Where the mixed mineral manure, including potass, is applied, it is the more surface-rooting *Lathyrus pratensis* that prominently represents leguminous growth; and it is only where there is no luxuriance of grasses, or of more surface-feeding Leguminosæ, that the *Lotus corniculatus*, with its deeper roots, hardy habit, and comparative independence of surface-food and surface-moisture, appears to be able to maintain or to improve its position.

Lathyrus pratensis.

This is a perennial plant with a root-stock greatly differing in character from that of the *Lotus corniculatus,* being very long, slender, wiry, black in colour, creeping, or in some cases descending vertically to considerable depths, slightly branched, but never forming thick fleshy branches like the *Lotus.* On the other hand, it produces adventitious buds, and roots much more freely. By these comparatively superficial roots it is enabled to avail itself more fully than the *Lotus* does of the food in the surface soil. The stems are slender, weak, angled or flattish, trailing, or supporting themselves by means of the leaf-tendrils. It begins to grow late in the spring, and flowers abundantly, but it does not as a rule ripen its seeds on the plots before cutting.

The characteristics which favour its growth are its hardiness and robust constitution, its creeping root-stock, and its tendrils which enable it to avail itself, at little cost to itself, of the stems of its neighbours, and it may be to strangle them. Its duration is also probably increased by the circumstance that it so rarely perfects seeds on the plots. The peculiar manner in which the leaves are folded flat in the bud, so as to occupy little space and enable the plant to push its way through and between its competitors, is also noteworthy.

The following table shows the relative degree of prominence of this plant.

TABLE LIV.—Relative predominance of *Lathyrus pratensis.*

		Lathyrus pratensis							
		Was First, Second, or Third, among the total Leguminosæ, as under.				Yielded 5 per cent. and over, to the total Mixed Herbage, as under.			
		1862.	1867.	1872.	1877.	1862.	1867.	1872.	1877.
Plots	1 . . .	1	1	1	1	. .	. .	. .	. .
	2 . . .	1	1	1	1	. .	. .	. .	5·25
	3 . . .	3	3	3	2	. .	. .	. .	. .
	4-1. . .	. .	1	1	1	. .	. .	. .	. .
	4-2 . .	1	1	1	1	. .	. .	. .	. .
	5 . . .	3	2	2	2	. .	. .	. .	. .
	6 . . .	1	. .	1	1	. .	. .	. .	6·56
	7 . . .	1	1	1	1	13·51	6·78	36·68	12·11
	8 . . .	1	1	1	1	8·76	6·82	. .	. .
	9 . . .	1	1	1	2	. .	. .	. .	. .
	10 . . .	1	1	1	. .	. .	. .	. .	. .
	11-1 . .	1	. .	. .	. .	. .	. .	. .	. .
	12 . . .	2	3	2	2	. .	. .	. .	. .
	13 . . .	1	1	1	. .	. .	. .	. .	. .
	14 . . .	1	1	1	1	. .	. .	. .	. .
	15 . . .	. .	3	. .	1	. .	. .	. .	. .
	16 . . .	3	2	1	1	. .	. .	6 68	9·22
	17 . . .	. .	. .	. .	3	. .	. .	. .	. .
	18 . . .	—	2	2	1	—	. .	. .	. .
	19 . . .	—	—	3	1	—	—	. .	6·92
	20 . . .	—	—	. .	1	—	—	. .	. .
Total	First . .	11	10	12	13	2	2	2	5
	Second . .	1	3	3	4				
	Third . .	3	3	2	1				

Among the four leguminous species of any distinct prominence at all, this plant takes the first place in the greatest number of instances. It was the first of the four on 11 plots in 1862, on 10 in 1867, on 12 in 1872, and on 13 in 1877. It was second on only one plot in 1862, on three in 1867 and 1872, and on four in 1877. It was third on three plots in 1862 and 1867, on two in 1872, and on one only in 1877. It, however, yielded more than 5 per cent. of the total produce on only two plots in 1862, two in 1867, two in 1872, but on five in 1877. Like its three allies it has almost disappeared on most of the ammonia plots; but has even increased where the smaller quantity of nitrate of soda was employed, especially when in conjunction with mineral manure containing potass. Without manure, although the actual quantity is small, it has increased rather than diminished; but in a considerably less degree than the *Lotus.*

The most striking point in the development of *Lathyrus* on the experimental plots is its very great increase where mineral manure, including potass, is used, without nitrogenous manure. The table shows that on plot 6, where ammonia salts had been applied during the first 13 years, but a mixed mineral manure, including potass (without ammonia), in 1869, and each year since, *Lathyrus* did not contribute 5 per cent. to the produce in either 1862, 1867, or 1872, but that it did so in 1877, that is some years after the change. The plant yielded more than 5 per cent. to the produce in each separation-year on plot 7, where the mixed mineral manure, including potass, was applied alone every year. And it did so in 1862 and 1867, but not afterwards, on plot 8, to which the same mixed mineral manure, including potass (in considerable excess), was applied for six years, to 1861 inclusive, but without potass in 1862 and afterwards.

The actual percentage of *Lathyrus* in the total herbage was increased from 0·23 in the first, to 6·56 in the last separation-year on plot 6, where the ammonia-salts were applied in the earlier years, and the potass manure in the later. It diminished from 8·76 to 2·37 per cent. on plot 8, which received potass during the earlier, but not during the later years; and on plot 7, where the mineral manure including potass was applied every year, *Lathyrus* contributed 13·51 per cent. in the warm moist season of 1862, 6·78 per cent. in the cold and changeable season of 1867, 36·68 per cent. in the variable season of 1872, and 12·11 per cent. in the tardy spring of 1877. It also increased up to 9·22 per cent. in 1877 on plot 16, where the smaller quantity of nitrate with the mineral manure, including potass, was every year applied. It has likewise increased in a very remarkable degree, in recent years, on plot 15, where nitrate of soda alone was applied up to 1875 inclusive, but the mixed mineral manure, including potass (without nitrate), each year since. It is obvious that potass manure has remarkably increased the development of this species, whilst it has not done so in the case of either of its three allies.

Somewhat detailed reference has already been made to this result in the section relating to plot 7 (Part I., p. 307, *et seq.*); and the subject will come up again when discussing the botany, and again when discussing the chemistry, of that plot. But it may here be remarked that, of the four leguminous species which have been considered, the *Lathyrus* feeds by its adventitious roots very much more than the rest in the surface-soil; that the supplied potass descends comparatively little into the sub-soil; and that the nitrogen of the surface-soil where this leguminous plant has so remarkably developed, has diminished in a greater degree than on any other plot. It would appear that under the influence of the liberal potass supply, the plant has been enabled to obtain the large amount of nitrogen it requires in a greater degree from the accumulated stores within the surface-soil.

It seems probable, from the foregoing data, that some of the fluctuations in the amount of produce yielded in different seasons by this plant may be explained by its peculiar habit of growth. Its underground development, either of root or stock, is relatively small, as compared either with that of most of the grasses or with that of *Lotus* and the *Clovers*. On the other hand, its above-ground growth, its creeping stems and erect branches, insinuating themselves between their competitors, and clinging to them by means of their tendrils, &c., constitute differences which may go far to explain the fluctuations above recorded. The great increase on those plots where this plant predominated was in the year 1872, a year when the spring temperature was above average but the rainfall below it—conditions which may have acted partly by discouraging its competitors, and partly by directly favouring its own growth.

Three other species of the leguminous family, *Ononis arvensis, Vicia cracca,* and *Vicia sepium,* have been found on the plots; but only one of them, the *Ononis arvensis,* has been so far developed as to come into the samples, then only on one plot—that with superphosphate of lime alone, and in quite insignificant amount.

The Miscellaneous Plants.

The plants included under this heading comprise, in all, 59 species (exclusive of Fungi), distributed through 44 genera and 20 families. A detailed list of these Orders, genera, and species is given at pp. 1210–11. Representatives of some of these families form a conspicuous portion of the herbage on some of the plots, while on others they are not found at all. The relative proportions also in which they occur are exceedingly different.

The plants which have occurred at some time in the samples from all the plots, or which have been observed during growth on all, are *Ranunculus acris, Conopodium denudatum, Achillea Millefolium, Rumex Acetosa, Luzula campestris,* and *Hypnum squarrosum.* The following have occurred on a large number of the plots: *Ranunculus bulbosus, R. repens, R. Ficaria, Stellaria graminea, Cerastium triviale, Pimpinella Saxifraga, Heracleum Sphondylium, Galium verum, Scabiosa arvensis, Centaurea nigra, Taraxacum officinale, Plantago lanceolata, Veronica Chamædrys,* and *Carex præcox.* The plants which have only been observed on a small number of plots are: *Ranunculus auricomus, Cardamine pratensis, Stellaria Holostea, Hypericum perforatum, Potentilla reptans, P. Fragariastrum, Alchemilla vulgaris, Agrimonia Eupatoria, Poterium Sanguisorba, Spiræa Ulmaria, Anthriscus sylvestris, Daucus Carota, Galium Aparine, Carduus arvensis, Bellis perennis, Chrysanthemum Leucanthemum, Senecio erucifolius, Hypochæris radicata, Tragopogon pratensis, Leontodon hispidus, L. autumnalis, Sonchus oleraceus, Hieracium Pilosella, Plantago media, Veronica serpyllifolia, V. officinalis, Thymus Serpyllum, Prunella vulgaris, Ajuga reptans, Primula veris, Rumex crispus, R. obtusifolius, Orchis Morio, Scilla nutans, Fritillaria Meleagris, Ornithogalum umbellatum,* and *Ophioglossum vulgatum.*

The occurrence of certain plants on all or on a majority of the plots seems to indicate that the plants in question are in a considerable degree independent of manurial influence, or are able to adapt themselves to circumstances better than others—a power arising probably from diverse characteristics as to habit and internal structure (see *ante* p. 1222). The proportions, however, in which particular plants are found depend very greatly on manurial agency and association.

In the list given at pp. 1210–11 the plants are arranged in botanical order; but in discussing the constituents of the miscellaneous herbage in this place, the plan followed is, to mention first those which have been respectively first, second or third in order of predominance among their associates, or those which have contributed not less than 5 per cent. of the total produce. Other plants which are not so well represented, either in relative predominance or in percentage, and concerning which it is not necessary to speak at so much length, are treated of subsequently; while those Orders in which none of the members are represented in the degree above-mentioned are briefly alluded to at the end of the section.

RANUNCULACEÆ.

The species of this Order found on the plots are few in number. Three of them, *R. acris*, *R. bulbosus*, and *R. repens*, are met with in greater or less abundance on all the plots. *R. Ficaria* occurs on 11, but it does not figure in the samples taken in the separation-years; *R. auricomus*, also, has only been found on two plots, and in only one sample (that from plot 16 in 1877), although it is not uncommon in the adjacent woods and shrubberies.

Ranunculus bulbosus and *R. repens.*

It is unfortunate that, owing to the difficulty of determining to which of these two species fragments of leaves and stems such as occur in the samples belong, the two plants are taken together in the tables showing the percentage and the weight of the several constituents of the herbage. This is the more to be regretted as the habit of the two species is so different.

R. bulbosus is an erect perennial, with a somewhat globose root-stock, destitute of runners, but reproduced by the formation of a new bulb-like bud at the top of the old one. The root-fibres are thick, whitish, unbranched, and descend vertically. The hairs on the stem and foliage are usually longer and coarser than in *R. repens.* It flowers and produces seed abundantly.

R. repens has, as the name implies, creeping, prostrate stems, which give off runners or creeping shoots, which root at the nodes, where they form fresh plants, and thus enable the plant to reproduce itself freely without seeding, which rather rarely happens. The roots are similar to those of *R. bulbosus*, but more slender.

The following table illustrates the predominance of the two species collectively.

TABLE LV.—Relative predominance of *Ranunculus bulbosus* and *R. repens*.

		Ranunculus bulbosus and *R. repens*							
		Were First, Second, or Third, among the total Miscellaneæ, as under.				Yielded 5 per cent. and over, to the total Mixed Herbage, as under.			
		1862.	1867.	1872.	1877.	1862.	1867.	1872.	1877.
Plots	1 . . .	2	. .	. .	3	. .	. .	. .	. .
	2 . . .	2	. .	3	2	. .	. .	. .	. .
	3 . . .	2	. .	2	1	. .	. .	. .	. .
	4–1 . .	1	. .	2	1	5·87	. .	. .	5·84
	4–2 . .	2	. .	. .	. .	. .	. .	. .	. .
	8 . . .	3	. .	. .	. .	. .	. .	. .	. .
	12 . . .	3	. .	. .	1	. .	. .	. .	6·45
	14 . . .	3	. .	. .	. .	. .	. .	. .	. .
	16 . . .	2	. .	. .	. .	. .	. .	. .	. .
	18 . . .	—	2	2	3	—	5·73	. .	. .
	19 . . .	—	—	1	. .	—	—	. .	. .
	20 . . .	—	—	1	1	—	—	. .	. .
Total	First . .	1	0	2	4	1	1	0	2
	Second .	5	1	3	1				
	Third . .	3	0	1	2				

In 1862 *R. bulbosus* and *R. repens*, taken together, were once first, five times second, and three times third, among the total miscellaneous herbage; in 1867 they were not first at all, only once second, and not third at all; in 1872 they were twice first, but only on the two plots brought under experiment for the first time in that year, but they were three times second and once third; in 1877 they were four times first, once second, and twice third.

Notwithstanding this seeming prominence, they together only yielded a very little over 5 per cent. of the total produce on one plot in 1862, on one in 1867, on none in 1872, and on two plots in 1877. The only plots on which they supplied from 5 to 6 per cent. were three of small total produce, viz.: 4–1 (superphosphate alone), in 1862 and in 1867, 12 (unmanured), and 18 (minerals and nitrogen contained in 1 ton of hay).

The plots on which these plants came first, second, or third among the miscellaneous herbage, but on which they did not contribute 5 per cent. to the total produce, were mostly characterised by meagre luxuriance; the exceptions being plot 16 (mineral manure and the smaller quantity of nitrate of soda), plot 14 (mineral manure and the larger quantity of nitrate of soda), and the two newer nitrate and mineral plots, 19 and 20. They were absent from, or present in very small quantities, in the samples from the plots treated with ammonia-salts in conjunction with mineral manure where the gramineous herbage was luxuriant. They were, however, it is seen, more abundant on the nitrate plots.

It may be observed that of the two species *R. bulbosus* was by far the most frequent and prominent; but *R. repens* is in relatively larger proportion on plot 1 (ammonia-salts with, or after farmyard manure), and on plot 5 (ammonia-salts alone). It is obvious, therefore, that, on the whole, these plants have only retained a comparatively prominent place where the growth of those associated with them has been relatively small.

Ranunculus acris.—This is a tall-growing species, with a thick root-stock, from which proceed a number of white, rather fleshy, root-fibres, branching obliquely, and horizontally, destitute of root-hairs, and not descending very deeply. It is destitute alike of the thickened stem of *R. bulbosus* and of the creeping shoots of *R. repens,* and has, apart from its constitutional or physiological characteristics, no very obvious structural endowments to account for its frequency of occurrence, unless it be its tall stature and abundant production of seed, both in the first and second crops. It did not attain a first, second, or third position among the miscellaneous herbage on any plot; nor did it in any case yield 5 per cent. of the total produce. It was absent, or nearly so, wherever ammonia-salts with mineral manures were applied, and where, accordingly, there was considerable luxuriance, especially of certain grasses. Thus on plot 4–1 (superphosphate only) there is commonly in spring a large quantity of this plant, while on plot 4–2, to which ammonia-salts as well as superphosphate are applied, there is little or none. It was in small, but nevertheless in larger quantity on the unmanured, and on the purely mineral manured plots, on which the herbage generally was not luxuriant. It was considerably more prominent on the nitrate of soda plots, and was especially so where the smaller quantity of nitrate was used without mineral manure, and again on the plot where the larger quantity of nitrate of soda had been applied alone up to 1875, and mixed mineral manure afterwards. On the former it reached 2·95, and on the latter 3·74 per cent. of the total produce in the fourth separation year 1877.

Ranunculus Ficaria.—This, the lesser Celandine, is a low-growing species, producing a number of fleshy root-like tubers, by means of which the plant is propagated, as it is also by means of numerous sub-globose thickened buds, formed under certain circumstances in the axils of the leaves, whence they detach themselves and, falling to the ground, grow into distinct plants. It flowers so early (April) that it is rarely noticed in the summer months, and its leaves form no distinguishable part of the samples, as the foliage is withered before the hay is cut; or, from the leaves being so near the ground, it is passed over by the scythe. It has been noticed on 11 plots, and in largest quantity on plot 16 (nitrate of soda and minerals).

Ranunculus auricomus has the general habit of *R. acris,* but is not so tall and is much more glabrous. Its flowers are rarely, if ever, found in a perfect condition, the petals being more or less defective in size or number. It is conjectured that this plant does not stand in the same need for insects to insure its fertilization, as do the other more conspicuous flowered species. It has been seen on plot 16, and on no other, and, like the Celandine, it withers before the crop is cut, and thus forms no part of the samples.

The structural characters of the foregoing five species of *Ranunculus* (all more or less acrid) are thus seen to be very different, so far at least as their vegetative organs are concerned. Their floral structure is much more uniform. The peculiar structural endowments of *R. bulbosus*, *repens*, and *Ficaria* give them great advantages, while the others are able specially to assert themselves when the competition with associated plants is not severe.

CARYOPHYLLACEÆ.

This Order is only represented by three species, viz.: *Stellaria graminea*, *S. Holostea*, and *Cerastium triviale*. *S. graminea* and *C. triviale* occur on a large number of plots, but *S. Holostea* has only once been found in the samples, and neither *S. graminea* nor *S. Holostea* has ever been first, second, or third in order of predominance; nor has either of them, on any occasion, yielded as much as 5 per cent. of the produce. All are weak-stemmed plants, with relatively small true roots. *Cerastium triviale*, however, has a great tendency, when placed under suitable conditions, to form both roots and shoots at the nodes, so that, under such circumstances, it increases rapidly.

Cerastium triviale.

This perennial weed forms intricate masses of slender shallow roots, and numerous prostrate slender leafy branches, which produce roots and shoots at the nodes. Root-hairs are abundant. It flowers early, and ripens seeds on a great number of the plots.

The circumstances propitious to its growth are its hardiness, close habit, which enables it to insinuate itself between the branches and leaves of other plants, and to occupy any vacant ground, and its early and free seeding property, by reason of which the seeds may get dispersed by the mowers and haymakers. The following table shows that, although this plant occurs so generally on the plots, it very seldom attains a really prominent position as to amount of produce.

TABLE LVI.—Relative predominance of *Cerastium triviale*.

		Cerastium triviale							
		Was First, Second, or Third, among the total Miscellaneæ, as under.				Yielded 5 per cent. and over, to the total Mixed Herbage, as under.			
		1862.	1867.	1872.	1877.	1862.	1867.	1872.	1877.
Plots	12 . . .	. .	. .	3	. .	. .	. .	. .	. .
	15 . . .	. .	3	1	. .	. .	. .	9·26	. .
	17 . . .	. .	. .	2	. .	. .	. .	. .	. .
Total	First . .	0	0	1	0	0	0	1	0
	Second .	0	0	1	0				
	Third . .	0	1	1	0				

Thus, *Cerastium triviale* was only once first among the miscellaneous plants, viz.: in 1872 on plot 15 (with the larger quantity of nitrate of soda alone), on which it had previously been third in 1867. In 1872, the third separation-year, it was also second on plot 17 (with the smaller quantity of nitrate alone). In the same year it was third on one of the unmanured plots, 12. On only one occasion, however, has it contributed more than 5 per cent. of the total produce, and this was in 1872, on the plot with the larger amount of nitrate of soda alone, when it gave 9·26 per cent.

This plant was, therefore, most favoured under the influence of nitrate of soda alone. It was all but absent from the ammonia plots, and was very sparingly represented on the mineral manure plots. Next to the nitrate plots it succeeded best without any manure at all. It is obvious that it cannot sustain itself in competition where there is any general luxuriant growth among its associates; the only apparent exception being under the conditions of partial luxuriance induced by nitrate of soda alone. The explanation of this may be that it is specially fitted to take advantage of the direct supply in the upper layers of the soil.

UMBELLIFERÆ.

This is a large and well marked Order, morphologically speaking, but its chemical products vary greatly. The species are almost without exception herbaceous perennials, and they occur in very various localities in most regions of the globe, except the intertropical. The plants of this family represented on the plots are: *Conopodium denudatum*, found on all the plots in each separation-year; *Pimpinella Saxifraga*, found on most of the plots; *Heracleum Sphondylium* on a large number, especially in the earlier years; *Anthriscus sylvestris* on a few, and *Daucus Carota* on one.

Conopodium denudatum.

This plant is a perennial, with a blackish sub-globose tuberous root-stock (whence the name "earth nut") at a slight depth below the surface, and studded with protuberances or buds, from which proceed slender, erect stems, 1 to 3 feet high, and minute thread-like roots, which do not penetrate very deeply. The leaves are ternately divided, the segments again finely cut.

It flowers in May, ripens and sheds seeds previous to the first mowing, on which account its weight in the samples is less than it otherwise would be in proportion to the number of plants on the plots.

The structural features favourable to the plant are its tuberous root-stock, which secures it from injury from drought or frost, and ensures a supply of buds and fresh shoots. These tubers are, however, sought out by wood pigeons and other creatures. Its diffusion is also favoured by the abundance of seed which it produces. The manner in which its thin stems and folded leaves push themselves up between other plants, also confers an advantage on this plant.

The following table shows its relative degree of prominence.

TABLE LVII.—Relative predominance of *Conopodium denudatum.*

	Conopodium denudatum							
	Was First, Second, or Third, among the total Miscellaneæ, as under.				Yielded 5 per cent., and over, to the total mixed Herbage, as under.			
	1862.	1867.	1872.	1877.	1862.	1867.	1872.	1877.
Plots 1 . . .	3	3	. .	. .	. .	. .	. .	. .
2 . . .	3	3	. .	. .	. .	. .	. .	. .
3 . . .	. .	. .	3	. .	. .	. .	. .	. .
4–2 . .	. .	2	. .	. .	. .	. .	. .	. .
5 . . .	3	2	. .	3	. .	5·74	. .	. .
6 . . .	. .	2	3	. .	. .	7·87	. .	. .
7 . . .	2	1	3	2	. .	9 22	. .	. .
8 . . .	2	2	. .	. .	. .	6 84	. .	. .
9 . . .	2	2	2	2	. .	9·35	. .	. .
10 . . .	2	2	. .	. .	. .	. .	. .	. .
11–1 . .	2	2	2	3	. .	. .	. .	. .
11–2 . .	2	2	2	3	. .	. .	. .	. .
12 . . .	. .	2	. .	2	. .	5·42	. .	. .
13 . . .	3	1	2	2	. .	8·04	. .	. .
14 . . .	2	1	2	. .	. .	. .	. .	. .
16 . . .	3	2	1	. .	. .	. .	. .	. .
18 . . .	—	3	3	2	—	. .	. .	. .
19 . . .	—	—	2	1	—	—	. .	. .
20 . . .	—	—	2	2	—	—	. .	. .
Total First . .	0	3	1	1	0	7	0	0
Second .	7	10	7	6				
Third . .	5	3	4	3				

It is seen from this table that *Conopodium denudatum,* perhaps owing partly to the relative delicacy of its foliage, and partly to its early ripening and loss of seed before cutting, very seldom came first in predominance by weight among the total miscellaneous herbage. In 1862 it was second on seven, and third on five plots; in 1867 it was three times first, ten times second, and three times third; in 1872 it was once first, seven times second, and four times third; and in 1877 it was once first, six times second, and three times third. Thus, although it was very seldom first, it was very frequently second, and rather frequently third.

Yet only in the second separation-year, when the cold spring checked the grasses, did the amount ever reach 5 per cent. of the total produce, but then it did so on seven plots. The only plots upon which it was neither first, second, nor third, nor yielded 5 per cent. to the produce in either separation-year, were plot 4–1 (superphosphate of lime alone), plot 15 (with the larger quantity of nitrate of soda alone), and plot 17 (with the smaller quantity of nitrate of soda alone).

In the second separation-year, 1867, when it reached its highest amounts, it was most productive on the two plots 9 and 13 highly dressed with both mineral manure and ammonia-salts, and on plot 7, with the mixed mineral manure, including potash, but without ammonia. It is obvious that this plant can, probably by reason of its tuber, and its thin stems which are not overpowered by the grasses, sustain the competition of free-growing grasses, even when these latter are stimulated by ammonia-salts in conjunction with mineral manures; but a liberal supply of the latter would seem to be essential to its luxuriance. It is apparently much less favoured by nitrate of soda than by ammonia-salts. The table shows that it occurred in very much less amount in the third and fourth separation-years; although in some seasons it has been very conspicuous on many plots. During the three years (1878, 1879, 1880) it was estimated to be either first, second, or third in order on the unmanured plots, and on those treated with ammonia-salts. On plot 9 (minerals and ammonia) it has been conspicuous and abundant, but with the minerals and the double quantity of ammonia (plot 11) it has not thriven; and during the three years above-mentioned it has occurred very rarely on this plot—but abundantly on plot 20, with the nitrate of potass and superphosphate.

With regard to the great development of this plant in 1867, it may be observed that it corresponded nearly with that of *Rumex Acetosa* in the same season. Perhaps the relatively low spring rainfall and temperature of that season may have discouraged the growth of the grasses, and thereby allowed of the superior growth of *Conopodium*, *Rumex*, and other miscellaneous plants.

Pimpinella Saxifraga.

This has a short, stout stock, rather slender, erect stems, and pinnately divided broad leaves. It was found in the samples from almost all the plots, excepting those liberally manured with ammonia-salts, from which it was frequently absent. It was also in very small amount in the samples from the nitrate plots; and it was the most prominent on the unmanured plots, and on those receiving an only partial mineral manure. Excepting that it was more prominent in the second than in either of the other separation-years, it may be said to have gone down, even on the plots the most favourable to it; and it has done so more markedly on the unmanured plots, than on the one treated with superphosphate of lime alone. It is obvious that this Umbellifer is not able to hold its own under the influence of nitrogenous manures, whether alone or in conjunction with mineral manures, that is under conditions which favour the grasses, nor when mineral manures are used alone, in which case the luxuriance of Leguminosæ, and other plants, is promoted.

Heracleum Sphondylium.

This is a tall, coarse-growing plant, with very large, thick, fleshy, deeply-penetrating root-fibres, and broad rough foliage. It is a biennial, and occurs generally in isolated examples, not in groups. It seeds freely, and if the seedlings spring up upon any vacant patches, they speedily avail themselves of the space to the exclusion of other species.

Owing to the plant generally occurring as isolated specimens it was frequently noted as existing on the plots although it was not included in the samples. It is obvious that, from the character of the plant, there must be some uncertainty as to the amount coming into the sample being a true representation of its proportion in the total herbage of the plot. From the notes taken on the ground it would appear probable that there was a greater number of plants on some of the plots, where there was comparatively little general luxuriance, but it was only where there was liberal nitrogenous and mineral manuring, and specially where the nitrogen was applied as ammonia-salts, that isolated individuals acquired great vigour, and ripened seed more freely than elsewhere; though, as it would seem, from the effects of competition injurious to seedlings in general, the plant continues to show itself in isolated specimens. Under the manurial conditions in which the *Heracleum* flourishes there is a great luxuriance of the grasses, especially of a few very free-growing ones, and the question obviously arises—how far its success under these circumstances depends on the directly favourable influence of the manure on its own growth, enabling it to displace its neighbours, or how far the generally somewhat tufted and patchy character of the grasses in such cases affords space for its development?

Anthriscus sylvestris.

This is a common hedge-row biennial, with a thick, fleshy tap-root, descending some distance into the soil; it has erect stems, finely cut foliage, sometimes bright green, at other times claret-red, even in the case of two plants growing close together. It is a plant not common in pastures unless where overshadowed by trees.

It forms a constituent of but few of the samples, and indeed is observed on very few of the plots. It is absent, or nearly so, from the unmanured, the purely mineral manured, and the ammonia plots. The only conditions under which it has acquired any real prominence are those where the larger amount of nitrate of soda in conjunction with the mixed mineral manure is employed. Here, owing to the abundance of seed produced, it increased from 1·52 per cent. in the second, to 3·86 in the third, and to 4·64 per cent. in the fourth separation-years respectively. In the season of 1880 it was a very prominent plant on the same plot. Taking the whole of the plots, and the four separation-years, there is only one other instance in which the amount found in the sample reached 0·1 per cent., namely, in that from a plot adjoining the one above referred to.

Daucus Carota.—The Wild Carrot was found only in the sample from one plot, where its appearance was probably accidental.

COMPOSITÆ.

This, the largest group of flowering plants known, comprises a vast assemblage of species greatly alike as to their floral characters, but differing much in the appearance of their stem, leaves, &c. The chemical products also vary greatly. The species represented on the plots are: *Centaurea nigra,* found on a majority of the plots, *Carduus arvensis* on very few, *Bellis perennis* on more, but still only on a few, *Achillea Millefolium* on almost all, *Chrysanthemum Leucanthemum* on very few, *Senecio erucifolius* only once, *Hypochœris radicata* occasionally, *Tragopogon pratensis* on several, *Leontodon hispidus* also on several, *L. autumnalis* less frequently, *Taraxacum officinale* on a considerable number, *Sonchus oleraceus* only once, and *Hieracium Pilosella* in a few instances.

Centaurea nigra.

This common weed of pasture land forms a densely-tufted, much-branched, perennial, with a tough, woody stock, from which descend long, stout, deeply-penetrating roots. The stems are erect, rigid, 1 to 2 feet high, and branched, the branches ending in hard globose flower-heads. It flowers too late to produce seed in the first crop.

The qualities favourable to its persistence or spread are its hardiness and power of withstanding drought, and, where permitted to ripen, its free-seeding.

The following table shows the relative degree of prominence of this plant.

TABLE LVIII.—Relative predominance of *Centaurea nigra.*

		Centaurea nigra							
		Was First, Second, or Third, among the total Miscellaneæ, as under.				Yielded 5 per cent. or over, to the total Mixed Herbage, as under.			
		1862.	1867.	1872.	1877.	1862.	1867.	1872.	1877.
Plots	1	..	..	3	..	..	..	..	..
	4-2	..	..	3	2	..	..	..	..
	5	..	3	3	..	..	..	..	..
	6	..	3	..	..	..	..	..	..
	10	..	..	3	3	..	..	..	..
	15	..	..	3	3	..	..	..	..
	17	1	3	1	3	..	..	10·28	..
Total	First	1	0	1	0	0	0	1	0
	Second	0	0	0	1				
	Third	0	3	5	3				

Thus this plant was first in quantity among the miscellaneous flora on one plot in 1862, but was not second or third in that year at all. It was third three times in 1867, but not once first or second. It was once first and five times third in 1872; and it was once second and three times third in 1877. It was, therefore, seldom first or second among miscellaneous plants; but was more frequently third. Yet it only once contributed more than 5 per cent. to the total herbage, namely, in 1872, on plot 17 (with the smaller quantity of nitrate of soda alone), and it was on this plot that the plant came first in 1862 and in 1872, and here also it was third in 1867 and in 1877. The actual percentages which it yielded to the total herbage on the plot were 4·41 in 1862, 4·10 in 1867, 10·28 in 1872, but only 2·82 in 1877. It also yielded as much as 2·58 per cent. to the produce on plot 15, with the larger amount of nitrate of soda alone, in the third separation-year. The only other plots on which it came second or third were those to which ammonia-salts were applied, but without any, or with deficient mineral manuring. With ammonia-salts and full mineral manuring it was almost completely banished. Next to the plots with nitrate of soda alone, it gave the highest percentage on the unmanured ones, where it increased up to the third separation-year, but afterwards declined.

Achillea Millefolium.

This perennial herb is specially well endowed, having a woody, more or less branching stock, which emits slender, creeping offshoots or runners, and rather thick vertical roots giving off more or less horizontal branches destitute of root-hairs. The flowering stems are erect, terminal, dying down after flowering, thus necessitating the formation of lateral shoots (see Jour. Roy. Hort. Soc., vol. iii., 1870, p. 53). Each "runner" or stolon forms a tuft of leaves at the extremity, which ultimately becomes an independent plant by the decay of the runner which connects it to the parent stock. The leaves are crowded, spreading horizontally, and are very finely cut, whence the specific name. It does not, as a rule, flower before the first mowing, but does so before the second cutting, the flowers being probably fertilized by insects. As a rule, in this country it frequents dry pastures and banks, and bears drought well. Cattle and sheep eat it readily, and few plants sprout more freely after the browsing than this does, by reason of its mode of growth.

The characters favourable to the milfoil are its hardy constitution, and ability to withstand drought, its powerful under-ground growth, its divided stock, its habit of propagating itself by runners, and its free seeding character. These are all endowments likely to favour it in competition.

The following table shows the relative degree of prominence of this plant.

TABLE LIX.—Relative predominance of *Achillea Millefolium*.

		Achillea Millefolium							
		Was First, Second, or Third, among the total Miscellaneæ, as under.				Yielded 5 per cent. and over, to the total Mixed Herbage, as under.			
		1862.	1867.	1872.	1877.	1862.	1867.	1872.	1877.
Plots	1	..	2	2	..	..	..	5·01	..
	2	..	..	1	..	..	..	..	..
	3	..	..	..	3	..	..	..	..
	4–1.	..	..	1	..	..	..	5·38	..
	4–2.	3	3	2	..	..	..	..	..
	5	2	..	..	..	..	..	..	..
	6	2	..	2	2	..	..	..	..
	7	3	3	1	3	..	..	5·23	..
	8	..	3	1	2	..	..	9·75	..
	9	3	3	3	..	..	..	..	..
	10	3	3	2	..	..	..	..	..
	11–1	3	3	..	3	..	..	..	..
	11–2	3	..	..	..	..	..	..	..
	12	..	..	1	3	..	..	..	..
	13	3	3	3	..	..	..	..	..
	15	3	..	2	..	..	..	..	..
	16	..	3	2	2	..	..	..	..
	17	..	..	3	..	..	..	..	..
	18	—	..	1	..	—	..	..	..
Total	First	0	0	6	0	0	0	4	0
	Second	2	1	6	3				
	Third	8	8	3	4				

The milfoil was found in the samples from all the plots in 1862 and 1867; and in those from all but two in 1872, and all but one in 1877. In the first separation-year it was not first among the Miscellaneæ on any plot, but it was second on two, and third on eight. In the second separation-year (1867) it was again not first anywhere, only once second, but it was third on eight plots. In 1872 it came first on six plots, second on six, and third on three. In 1877 it was not first at all, second only on three plots, and third on four. It was only in the third separation-year (1872), when it had attained the first and second place more frequently, that it, on any plots, yielded 5 per cent. and more to the total produce. Then it did so only on four, three of which received only mineral manure; and the one of these which yielded by far the highest amount of it (9·75 per cent.) was that from which the potass had been excluded for some years, and on which the growth of the Leguminosæ, and the maturation of the herbage generally, had, in consequence, become much restricted. The plant was more favoured on the nitrate of soda than on the ammonia plots; but where the nitrate was employed in the larger quantity, and in conjunction with the mineral manures, it was in much reduced amount. Where, however, the ammonia-salts were used with the mineral manures, it was nearly banished. The conditions most favourable to it, when thus growing in association, are, therefore, those under which the luxuriance of its neighbours is restricted. In recent seasons, probably owing to the wetness, and to the increased growth of the grasses, it has decreased on all the plots; and on the whole it is the most abundant on plot 6 (mineral manures after ammonia).

Carduus arvensis.

This, the commonest of the Thistles, on arable or on waste land, has a perennial, creeping rootstock, sending up annual stems, with prickly leaves, and heads of flowers, male on one plant, female on another. It was not, in any one separation-year, observed on more than two plots; and the only samples in which it was found were those from plot 18 (with the mineral constituents and nitrogen of one ton of hay), in 1867, and from plot 7 (mixed mineral manure including potass), in each of the four separation-years; and it only once reached as much as 0·26 per cent. of the total herbage. It would appear that this plant requires liberal supplies of mineral constituents near the surface, but that it cannot withstand the competition of luxuriantly growing associates.

Bellis perennis.

The common Daisy is well fitted to hold its own, or to spread, where circumstances permit. It has a perennial tufted stock, emitting numerous, rather fleshy, root-fibres, and branching at the top into numerous subdivisions. Sometimes, moreover, it gives off long shoots, appressed to the ground, and with a tuft of leaves at the end, capable of becoming a distinct plant. The leaves being flatly pressed to the ground, prevent the growth of plants beneath their shade.

This persistent lawn-weed has only been observed on from five to eight plots out of the 22; and it only came into the samples in 1862 from two, in 1867 from five, in 1872 from six, and in 1877 from five plots. It was found in extremely insignificant amount on three of the nitrate of soda plots, but not on any ammonia plot, or even where there was full mineral manuring. It occurred but in extremely small amount in each separation-year in the samples from one of the unmanured plots; again, in each separation-year, on the plot with superphosphate of lime alone; and in the last three separation-years on plot 1, after the discontinuance of the dung. It thus did not hold its own wherever there was any luxuriant growth of other plants. The contrary result on lawns is coincident with precisely contrary conditions of growth of the general herbage; in addition to which, it may be pointed out that the scythe or the mowing machine mutilates the appressed daisy leaves much less than it does the herbage of taller-growing species.

Chrysanthemum Leucanthemum.

This frequent ornament to cornfields and railway banks is occasionally met with in pastures. It has a tap-root, an erect, slightly-branched stem, and rather broad, coarsely-toothed leaves.

This plant was observed on only one plot in 1862, on four in 1867, on one in 1872, and on four in 1877. It did not come into the samples from any plot either in 1862 or 1867; but it was found in one in 1872, and in two in 1877, in each case in a small fraction of 1 per cent. One of the plots on which it occurred was an unmanured one. It is, however, abundant in neighbouring meadows under ordinary treatment.

Senecio erucifolius has been observed on only one plot, and only in 1872; and it has in no case been met with in the samples.

Hypochœris radicata was observed on five plots in 1867, but on one only in 1872, and has never been found in the samples.

Tragopogon pratensis.—This is a biennial plant, with a thick tap-root and long, sharply-pointed grass-like leaves. The yellow flowers are succeeded by rather large seed vessels, which are readily dispersed by means of a large pappus.

This, the Goat's-beard, was found in four samples in 1862, in five in 1867, in six in 1872, and in six in 1877; and in the last-mentioned year it was observed on one other plot. It was by far the most frequent and prominent on the unmanured plots; but it has there never yielded more than a third of 1 per cent. of the produce. The only case in which it furnished more than this was on plot 9 (ammonia-salt and mixed mineral manure), in the first separation-year, since which time it has not been found in the samples from that plot. It occurred in very small amount in each of the four samples from plot 16, with the smaller quantity of nitrate of soda and mineral manure; but otherwise, beyond what has been stated, only on deficiently-manured plots, and then in insignificant amounts.

Leontodon hispidus is a perennial herb, with spreading hispid root leaves, leafless flower-stems and yellow flowers, succeeded by feathery pappus.

The rough Hawkbit was found in 1862 in the samples from five plots, in 1867 from eight, in 1872 from six, and in 1877 from six. It only exceeded 1 per cent. of the total produce on one of the unmanured plots, on which it gradually increased. Next to the unmanured, it was most prominent on plot 4–1 (superphosphate of lime alone), but it did not there reach to 1 per cent. of the total herbage. The other plots on which it occurred were those characterised by more or less exhaustion, and little or no luxuriant growth.

Leontodon autumnalis is a Composite with perennial root-stock, branching at the top and bearing tufts of spreading, nearly glabrous, narrow, lobed leaves. The flower-heads are small and relatively few, and the pappus feathery. *L. autumnalis* was much less frequent in the samples than *L. hispidus.* It was found in only one sample in

1862, in three in 1867, in one in 1872, and in one in 1877. In every case it was present in extremely insignificant amount.

Taraxacum officinale.—The root-stock of this plant is very stout, descending deeply into the ground, and dividing at the summit into numerous subdivisions, each with its tuft of rather broad appressed leaves. The flower-stalks have no leaves, and the brilliant yellow flowers are succeeded by light feathery pappus. The common Dandelion was found in 1862 in 17 samples, in 1867 in 16 (but observed on one more plot), in 1872 in 13, and in 1877 in 10—in a decreasing number, therefore, from period to period. In only one case did it contribute more than 1 per cent. to the produce, and that was with the larger amount of nitrate of soda alone (plot 15) in the first separation-year, after which it nearly disappeared from that plot. In one instance it reached to 0·63 per cent., viz.: on the plot with the larger quantity of nitrate of soda and mineral manure, and in the last separation-year. This was the only plot on which it increased. The other plots on which it was the most frequent were those without manure, or with deficient mineral manuring. It was all but absent from the plots liberally manured with ammonia-salts. Thus, although in no case was the amount large, the only conditions under which this plant increased were those of a liberal supply of nitrate of soda and mineral manure; whereas when with the same mineral manure the nitrogen was applied as ammonia-salt the plant was practically banished. It would seem that the maintenance of its position, though dependent on liberal manuring, was also much influenced both by the character of the seasons and by association.

Sonchus oleraceus.—This cornfield weed never occurred in sufficient amount to be found in the samples, and was only once observed, viz.: on plot 14, in the third separation-year (1872).

Hieracium Pilosella.—A low-growing perennial, with a tufted stock, and long creeping barren shoots, which enable the plant to insinuate itself between others, and take possession of vacant spaces. This plant was found in 1862 in three samples, in 1867 in four, in 1872 in three, and in 1877 in three (but was also observed in that year on one other plot, 1). It was the most frequent and prominent on the unmanured plots, but in no case did it yield 0·1 per cent. of the total produce.

The foregoing statements show that only three species of the Order *Compositæ* have maintained any special prominence on the plots, viz.: *Centaurea nigra, Achillea Millefolium,* and *Taraxacum officinale;* but the conditions under which these plants, respectively, have been the most developed, are by no means so uniform as to allow of any satisfactory generalisation in regard to the Order as a whole. This, indeed, could hardly be expected when the varying habit and organisation of the plants constituting this widely-diffused family are considered.

PLANTAGINACEÆ.

This is a small Order, the species of which are very similar in habit. *Plantago lanceolata* and *P. media* are the only species found on the plots. The species generally affect dry places, and are often found by waysides. They are natives almost exclusively of temperate regions.

Plantago lanceolata.

The common Rib-grass is a perennial, tufted plant, bearing close rosettes of lanceolate leaves, which spread flat on the ground at first, or where there is no impediment; but subsequently they assume a more or less erect direction. The stock is thick, strong, descending to a considerable depth, giving off numerous long, rather fleshy, root-fibres. The root-hairs are thin, long, delicate, sometimes consisting of more than one cell. It flowers in May, and continues to do so in succession throughout the summer. It is protogynous, and generally wind-fertilised; its seeds are matured before the first, and again before the second crops are removed.

It is a variable plant, and in the Chiswick experiments (Jour. Roy. Hort. Soc., vol. iii., 1870, p. 157) numerous self-sown seedlings appeared under the different conditions of manuring, which presented considerable variations in shape and colour of leaf, pubescence, &c.; variations which could not be attributed to the immediate operation of external conditions, seeing that they appeared in all the boxes indifferently, and in each box under the same conditions. At Rothamsted, in like manner, we have seen a hairy and a glabrous-leaved variety in juxtaposition on the same plot. In severe frosts the outermost leaves of the tuft are liable to be injured, the younger central ones being unhurt. One consequence of this is, that a hardier plant in proximity to it has an immediate advantage, and commences to occupy the space left by the dead or decaying foliage, before the growth of the young leaves in spring. These latter, from the pressure exerted on them, are made to assume an ascending direction.

The endowments favourable to the Rib-grass are its generally hardy constitution, as shown by its general distribution over more than a third of the surface of the globe. Its dense, somewhat thick foliage, and its powerful and persistent root-stock also tell in its favour. The freedom with which seeds are produced also tends to perpetuate the species, even if the individual plants succumbed in the struggle. The following table shows the relative degree of prominence of this plant.

TABLE LX.—Relative predominance of *Plantago lanceolata.*

		Plantago lanceolata							
		Was First, Second, or Third, among the total Miscellaneæ, as under.				Yielded 5 per cent. and over, to the total Mixed Herbage, as under.			
		1862.	1867.	1872.	1877.	1862.	1867.	1872.	1877.
Plots	2 . . .	. .	1	. .	1	. .	. .	. .	. .
	3 . . .	1	1	. .	2	7·34	10·73	. .	. .
	4-1 . .	2	1	. .	2	5·63	9·66	. .	. .
	12 . . .	1	1	. .	. .	7·74	8·25	. .	. .
	15 . . .	1	2	. .	. .	6·92	. .	. .	. .
	17 . . .	2	2	. .	1	. .	. .	. .	7·99
Total	First . .	3	4	0	2	4	3	0	1
	Second .	2	2	0	2				
	Third . .	0	0	0	0				

Plantago lanceolata was found in the samples from 16 plots in 1862, 14 in 1867, 14 in 1872, and 16 in 1877; in 1862 it came first on three plots, second on two, but third on none; in 1867 it came first on four plots, and second on two; in 1872 it did not come either first, second, or third on any plots, but in 1877 it was again first on two, and second on two.

It yielded more than 5 per cent. to the produce on four plots in 1862, on three in 1867, on none in 1872, and on only one in 1877.

The conditions under which it asserted prominence to the degree shown by the table are significant. It so occurred on only six plots: the two unmanured ones, the plot unmanured for some years after the application of farmyard manure, the plot with superphosphate of lime alone, and the two plots with nitrate of soda without mineral manure. Four of the six plots were therefore characterised by extremely restricted growth of the herbage in general, and the remaining two (nitrate of soda alone), by imperfect development and maturation.

Even with the nitrate of soda and mineral manure together, the more vigorous growth induced in its associates was adverse to this plant.

With ammonia-salts, both with and without mineral manures, it was very much reduced, and in many cases banished—at least to such an extent as not to appear in the samples.

Upon the whole, then, this plant has decreased, or even disappeared entirely, on all plots where there was general luxuriance; and whether from direct action of the manure, or from competition, it has suffered much more under the influence of ammonia-salts than under that of nitrate of soda. The results of the observations at Chiswick (see Jour. Roy. Hort. Soc., vol iii., 1873, p. 144) would seem to show that the action of ammonia was directly adverse to the plant, whereas that of nitrate of soda was not so.

The general result would then appear to be due, partly to the direct effect of manures, and partly to association. In any case the plant seems to be steadily declining, and it is, at any rate, satisfactory to know that it may be eliminated by encouraging the growth of more valuable species.

Plantago media, a weed of dry lime-stone districts, was not found in any of the samples, and was only observed on one occasion, viz.: on plot 4–1 in 1867.

POLYGONACEÆ.

The Docks and Sorrels are the only representatives on the plots of this Order whose members are so readily recognised by their "ochreate" stipules and floral characteristics. The herbaceous species are most abundant in temperate regions, extending to the arctic regions on the one hand, and into sub-tropical countries on the other. A few, and these of arborescent habit, occur in tropical and sub-tropical America. It would seem, then, on the whole, that these plants can adapt themselves to almost any climatal condition except the excessive heat and moisture of the tropics.

Rumex Acetosa.

The Common Sorrel is a perennial, with a thick, somewhat woody, much branched rhizome, giving off stout, yellowish, relatively little branched, but deeply penetrating roots, often forming a dense mass of fine fibres, but destitute of root-hairs. The flowers are diœcious, the males on one plant, the females on another. It would be interesting to observe whether any, and if so what, differences exist on the several plots, or in different seasons, in the relative abundance of male or female plants. It flowers in May, and continues to do so throughout the summer, producing seed abundantly. On the bare places, on almost all the plots, seedlings of this plant were observed in the spring of 1875, as also in other years, but less conspicuously.

The characteristics which are likely to be of advantage to this plant when growing in association are its hardiness, robust habit, deep and free-rooting, ample storage of water and nutritive matters in its root-stock, and its abundant production of seed.

The following table shows the relative degree of prominence of this plant.

Table LXI.—Relative predominance of *Rumex Acetosa.*

		Rumex Acetosa							
		Was First, Second, or Third, among the total Miscellaneæ, as under.				Yielded 5 per cent. and over, to the total Mixed Herbage, as under.			
		1862.	1867.	1872.	1877.	1862.	1867.	1872.	1877.
Plots	1 . . .	1	1	1	1	6·09	5·74	9·29	10·49
	2 . . .	1	2	2	3	12·44	. .	. .	. .
	4–1 . .	3	2	. .	3	. .	5·47	. .	. .
	4–2 . .	1	1	1	1	13·39	8·42	6·85	. .
	5 . . .	1	1	1	1	9·15	15·94	7·13	. .
	6 . . .	1	1	1	1	12·11	24·27	7·51	7·66
	7 . . .	1	2	. .	1	. .	8·88	. .	6·67
	8 . . .	1	1	3	1	. .	7·86	. .	5·84
	9 . . .	1	1	1	1	5·40	10·89	. .	. .
	10 . . .	1	1	1	1	10·38	12·61	. .	5·99
	11–1 . .	1	1	1	1	7·02	. .	. .	. .
	11–2 . .	1	1	1	1	. .	. .	. .	. .
	12 . . .	2	3	2	. .	. .	. .	. .	. .
	13 . . .	1	2	1	1	5·48	. .	. .	6·84
	14 . . .	1	3	2	2	6·88	. .	. .	. .
	15 . . .	2	1	. .	1	6·64	7·34	. .	5·79
	16 . . .	1	1	3	1	5·46	5·62	. .	. .
	17 . . .	3	1	. .	. .	. .	7·53	. .	. .
	18 . . .	—	1	. .	1	—	24·33	. .	. .
	19 . . .	—	—	. .	2	—	—	. .	. .
	20 . . .	—	—	. .	3	—	—	. .	. .
Total	First . .	14	13	9	14	12	13	4	7
	Second .	2	4	3	2				
	Third . .	2	2	2	3				

Evidences of the existence of this plant were found in the samples taken from all the plots (19) in 1862, from all (20) in 1867, from all (22) in 1872, and from all (22) in 1877, that is, on every plot in each separation-year. In 1862 it was first in percentage among the total miscellaneous herbage on fourteen plots, second on two plots, and third also on two; in other words, it was either first, second, or third on eighteen out of nineteen plots in that year. In 1867 it was first on thirteen plots, second on four, and third on two, or so far prominent on 19 out of 20 plots. In 1872 it was first on nine plots, second on three, and third on two, or first, second, or third on only 14 out of 20. Lastly, in 1877 it was first on fourteen plots, second on two, and third on three, or again very prominent on 19 out of 22 plots.

It yielded more than 5 per cent. of the total produce on 12 plots in 1862, on 13 in 1867, on four only in 1872, and on seven in 1877.

The only plots upon which it did not come first in one or more of the four years were the two unmanured plots, and plot 4–1 with superphosphate of lime alone. It was in 1867 that it acquired its greatest predominance, and both in that year and in the others it was on the plots manured with ammonia-salts that it contributed the largest proportion of the total miscellaneous herbage. On two ammonia plots in 1867 it contributed nearly a fourth of the whole produce, and on several others a very large amount. But it yielded less where the combination of mineral manures with the ammonia-salts was such as to favour the luxuriance and maturation of the grasses. With nitrate of soda the produce of this plant was much less than with ammonia-salts, and especially when mineral manures were used in conjunction with the nitrate.

It is obvious that the *Rumex Acetosa* is a gross feeder, and that it is favoured by ammonia-salts rather than by nitrate, but that it is restricted in development when the conditions of manuring are favourable to the luxuriant growth of the grasses. It has, disregarding some exceptions, yielded less to the produce in recent years than formerly. It continues to find its most favourable conditions on the ammonia plots, particularly when they are used in conjunction with minerals. It also exists to a large extent on the nitrate plots. Its most prominent associates among the grasses are —*Festuca ovina, Dactylis glomerata, Poa trivialis, Holcus lanatus, Avena pubescens, A. flavescens,* and *Bromus mollis.* Among Leguminosæ, its most conspicuous associate has been *Lathyrus pratensis;* and among the Miscellaneæ, *Achillea Millefolium.*

It would seem that this objectionable weed is by no means so readily displaced by high manuring, and the competition consequent on it, as the Plantago and some others; for it flourishes under the influence of ammonia-salts, and it is only when these are associated with a liberal admixture of mineral constituents that the grasses seem to have the power of overcoming it

Rumex crispus.—The curled Dock, like its congeners, has a thick, almost woody, root-stock, which descends into the ground, and, while thus securing a position and a hold, stores up nutriment and water in its tissues, which give it a great advantage in the battle of life. It is, however, not found in any quantity on the plots. It was observed on two plots in 1867, on two in 1872, and on two in 1877 ; but it was only once found in a sample, viz.: in that from plot 14, in 1877.

Rumex obtusifolius has almost the same habit as the preceding, from which it differs mainly in the leaves. It was observed on one plot in 1867, on one in 1872, and on one in 1877, but was never found in any of the samples.

Of the three species above mentioned, *Rumex Acetosa* is more especially a moist pasture plant, growing in association with other plants ; while the Docks are more particularly plants of waste places, waysides, and arable land, growing in masses where other plants less robust in character do not find the conditions suitable for their well-doing.

JUNCACEÆ.

The species of this Order are distributed throughout the world, the true Rushes (*Juncus*) being found chiefly in wet, marshy places, and having relatively small development of their leaves ; while the Woodrushes (*Luzula*) generally occur in drier localities, and have broader and more grass-like foliage. The latter genus alone is represented on the plots.

Luzula campestris.

The field Woodrush is a small tufted perennial, with a creeping root-stock, sometimes giving off offsets. The true roots are fine, and do not penetrate deeply. The leaves are linear and grass-like. It flowers in early spring, is protogynous, is wind-fertilised, and produces abundance of seed, which sheds before mowing.

Although nowhere forming a very prominent feature of the vegetation, it is one of the most widely distributed plants known, being found in the northern hemisphere from the Arctic regions to the Mediterranean ; in America, from Canada to South Carolina ; as well as in Australia, New Zealand, and the Cape of Good Hope. Any climate, therefore, except an extremely hot or dry one, will suit the plant. It seems to be equally indifferent to the nature of the soil ; but in this country it is usually found in association with grasses, &c., in pastures and meadows. The circumstances that would seem to favour it in its struggle with other denizens of meadow land are the same as those which enable it to exist over so wide a geographical area. Its hardiness, creeping root-stock, and abundance of seed, all give it an advantage ; though only once has it been found to take the most prominent place among miscellaneous plants, and then on the unmanured plot.

TABLE LXII.—Relative predominance of *Luzula campestris.*

		Luzula campestris							
		Was First, Second, or Third, among the total Miscellaneæ, as under.				Yielded 5 per cent. and over, to the total Mixed Herbage, as under.			
		1862.	1867.	1872.	1877.	1862.	1867.	1872.	1877.
Plots	3 . . .	3	2	1	. .	. .	. .	. .	. .
	4-1 . .	. .	. .	3	. .	. .	. .	. .	. .
	6 . . .	3	. .	. .	3	. .	. .	. .	. .
	7 . . .	. .	. .	2	. .	. .	. .	. .	. .
	8 . . .	. .	. .	2	. .	. .	. .	. .	. .
	19 . . .	—	—	3	. .	—	—	. .	. .
	20 . . .	—	—	2	. .	—	—	. .	. .
Total	First . .	0	0	1	0	0	0	0	0
	Second .	0	1	3	0				
	Third . .	2	0	2	1				

It was found in the samples from 16 out of 19 plots in 1862, from all (20) in 1867, from 19 out of 22 in 1872, and from 18 out of 22 in 1877, but it was observed on one more in that year.

In 1862 it came neither first nor second among miscellaneous plants on any plot, but was third on two. In 1867 it was second on one; in 1872 it was first on one, second on three, and third on two; in 1877 it was neither first nor second on any plot, and third only on one. But notwithstanding this apparent predominance on several occasions, it in no case contributed 5 per cent. to the total produce.

Its highest percentages were on the plots without manure, with superphosphate of lime alone, and with the mixed mineral manure, excluding potass. On these plots it, for the most part, increased in amount up to the third separation-year, but afterwards diminished. It was almost excluded from all the ammonia plots, especially where the manuring was highest. With nitrate of soda alone it fared better than with ammonia-salts alone, but with nitrate of soda and mineral manure together it speedily became reduced to an insignificant amount. Of recent years it has been most abundant on the unmanured plots, and on the plot with superphosphate of lime only.

Thus this plant fortunately only becomes at all prominent under poor conditions of manuring, and restricted growth of the herbage generally.

We now return to the consideration of those Orders no member of which has attained such a degree of prominence as to come first, second, or third among the miscellaneous plants, or to yield 5 per cent. of the total produce.

CRUCIFERÆ.

The only representative of this well marked and important family is *Cardamine pratensis*, a plant usually found in damp, rich meadows, and sometimes in great profusion, but which has only been observed on two of the experimental plots, and only in the third separation-year. That it does not occur more frequently, and in greater quantity, is probably due to the good natural drainage afforded by the chalk a few feet below the surface.

HYPERICACEÆ.

Hypericum perforatum is the only member of this group found on the plots. It has only been found in the samples from the plot receiving the smaller quantity of nitrate of soda alone, then only in two of the four separation-years, and in each case in very insignificant amount.

ROSACEÆ.

The species of this Order, though so numerous and so widely diffused, do not, as a rule, contribute in any very marked proportion to the herbage of pasture land in this country. The following species have been observed on the experimental plots:—

Potentilla reptans.—This plant was found in six samples in 1862, in six in 1867, in nine in 1872, and in 10 in 1877, but it was observed on 16 plots in that year. In only one case did it contribute 0·25 per cent. to the produce; in only two 0·1, and in all the rest extremely small amounts. It was most frequent on the unmanured plots; but it gave the largest amount with nitrate of soda alone. From its creeping habit it might have been expected to take a more prominent position.

Potentilla Fragariastrum was found in only one sample in 1867, one in 1872, and one in 1877, in each case from the unmanured plot. It is not a characteristic pasture plant.

Alchemilla vulgaris did not come into any sample, was only observed on one of the unmanured plots (3), and there only in the fourth separation-year. Nevertheless, it is not uncommon in the adjoining pastures.

Agrimonia Eupatoria occurred in the samples from two plots in 1862, four in 1867, five in 1872, and four in 1877, but it was observed on six in that year. Excepting twice in very small amount on one of the nitrate plots, it was only found on the unmanured plots, or on those supplied with a partial mineral manure; in every case in insignificant quantity.

Poterium Sanguisorba was found in one sample in 1862, and in two in 1867, 1872, and 1877 respectively. The only plots on which it was thus found were one of the unmanured, and the adjacent one with superphosphate of lime alone. On the latter it rather diminished in amount, while on the unmanured plot it increased, but not to such an extent as to contribute in the last separation-year as much as 1 per cent. of the produce.

Spiræa Ulmaria occurred in the samples from one plot in 1862, two in 1867, three in 1872, and six in 1877. It has not occurred on either of the unmanured plots, and has maintained its position the best on plot 7 (with the mixed mineral manure including potass). It has not been found on any of the nitrate plots, or on either of those with the highest quantity of ammonia-salts; and on the others where it occurred it has declined to an insignificant amount. In no case has it reached $\frac{1}{2}$ a per cent. of the produce, and it has been generally represented by less than 0·1 per cent.

Rubiaceæ.

With the exception of some species of *Galium*, the members of this Order are not characteristic of pastures.

Galium verum occurred in five samples in 1862, but was observed on 14 plots; in eight samples in 1867, and was again observed on 14 plots; in 10 samples in 1872, and observed on 15 plots; and in 14 in 1877, but it was observed on 19 plots. It was present in very small quantity, or not at all, on the unmanured plots, scarcely at all on the plots with the purely mineral manure, but it was in larger amount, and sometimes increased, where ammonia-salts were used, either alone or with deficient mineral manuring. But where the ammonia-salts were combined with full mineral manuring it was practically banished. Its highest yield was 2·4 per cent. with ammonia-salts alone in the third separation-year; and it twice yielded more than 1 per cent. on the plot with the larger amount of nitrate of soda alone. In all other cases it yielded under, and in most very much under, 1 per cent.

Galium Aparine was much less frequent than *G. verum*. It was found in only one sample in 1862, in none in 1867, in two in 1872, and in seven in 1877. The amount in which it occurred was in all cases insignificant. On none of the plots highly manured with both nitrogen and mineral constituents did it occur at all; but, nevertheless, it did so more frequently with than without nitrogenous manure. The plant, by reason of its rampant growth, hooked prickles, and great facilities for diffusion by means of its bur-like seed-vessels, would seem to be well fitted for the struggle with its associates. It is clear, however, that it has no chance against them except under conditions unfavourable to their growth.

DIPSACEÆ.

Few of the species of this small Order are pasture plants.

Scabiosa arvensis is the only one that has been met with on the plots. It is a coarse, strong-growing perennial, with a stout root-stock. It occurred in the samples taken from seven plots in 1862, five in 1867 (but was observed on one more), 10 in 1872, and 11 in 1877. It was generally found on the unmanured plots, but in very insignificant amount. It was absent, or very nearly so, wherever there was liberal nitrogenous and mineral manuring, but it reached as much as 0·3 per cent. with the smaller quantity of nitrate of soda alone; and it reached its highest percentage on the plot with ammonia-salts alone, where in 1862 it yielded 0·03 per cent., in 1867 0·28, in 1872 0·62, and in 1877 1·67 per cent.

SCROPHULARIACEÆ.

The only representatives of this large family found on the plots are two species of *Veronica*. None of the parasitic species of *Rhinanthus*, *Bartsia*, or *Melampyrum* occur, though they are not wanting in the neighbourhood.

Veronica Chamædrys occurred in 13 samples in 1862, and was observed on 14 plots; in 14 samples in 1867, and was observed on 15 plots; in 14 samples in 1872, and 13 in 1877; but in both these years it was observed on 16 plots. It was scarcely represented at all on the ammonia plots, but was slightly more so on the nitrate plots. It occurred in largest quantity on the plots without manure after farmyard manure, without manure from the commencement, and with purely mineral manure. It only exceeded 1 per cent. of the produce on the plot which had previously received farmyard manure, and was afterwards left unmanured. As the foregoing statements will show, this plant occurred pretty frequently on the plots, notwithstanding its weak and humble growth, and it was sometimes plentiful among the bottom herbage, its creeping habit enabling it to co-exist with the grasses.

Veronica serpyllifolia was observed on one plot only in 1867 and in 1872, and was found in two samples in 1877, and then only in small fractional proportion ; otherwise not at all. The plots on which it thus occurred were 3, 4–1, 15, and 17 ; that is, without manure, with superphosphate alone, with mineral manure after nitrate, and with nitrate of soda alone.

LABIATÆ.

This large and well-characterised Order contributes but few species to the Rothamsted pastures, and the proportion in which they occur is but trifling.

Prunella vulgaris has procumbent or trailing stems, and from its hardiness, exceedingly wide geographical distribution, and frequency of occurrence in the immediate neighbourhood, might have been expected to form a more prominent constituent of the herbage of the plots than it does. It was found in three samples in 1862, in six in 1867, in six in 1872, and in seven in 1877. In the last-named year it was also observed during growth on 12 of the plots. It was met with in each separation-year in the samples from the unmanured and the superphosphate plots ; though in no case did it contribute 0·1 per cent. to the herbage. Its amount in any other samples was quite insignificant.

Thymus Serpyllum, the common Wild Thyme, from its perennial duration, hard, wiry, procumbent, and much-branched stems, and dense habit, might have been expected to occur more frequently on the plots, particularly as it is not uncommon in the neighbourhood. It was found only in two samples, those from the unmanured plots, in the fourth separation-year (1877).

Ajuga reptans, the common Bugle, is a pasture plant, occurring throughout the whole of Europe and temperate Asia. Its stock produces somewhat fleshy, relatively unbranched root-fibres, and dense tufts of leaves, from amid which creeping offsets are given off, which root at the nodes, and form distinct plants. It was found in three samples in 1862, in five in 1867, in seven in 1872, in 10 in 1877, and was observed on one other plot in that year. It occurred most frequently in the samples from the unmanured plots, from the plot with superphosphate of lime, and from that with the smaller quantity of nitrate of soda. In the samples from the last-named plot it reached its highest percentage ; but, like the *Prunella*, it never contributed as much as 0·1 to the total herbage.

All three of the above-mentioned Labiates possess in their organisation and constitutional characters attributes which would lead us to expect to meet with them in larger proportions on the experimental plots ; but it is evident they cannot surmount the opposition offered by other plants, when these are stimulated by conditions of high manuring.

PRIMULACEÆ.

Primula veris, the common Cowslip, is the only member of this family found on the plots, and it is so only in small quantities. It was found in four samples in 1862 (but observed on six plots); in two samples in 1867 (and observed on six plots); in one sample only in 1872; and in six in 1877 (but observed on eight plots). It was most frequent on the unmanured and the mineral manured plots; but in all cases it yielded an extremely small quantity, and it was all but absent from all plots receiving nitrogenous manure.

ORCHIDACEÆ.

Orchis Morio is the only species of this family observed on the plots. It was present on three in 1862, on none in 1867, on one in 1872, and on one in 1877, these being the unmanured, and one of the mineral manured plots; but it was never in sufficient quantity to be represented in the samples.

LILIACEÆ.

Three species represent this Order, viz.: *Scilla nutans, Fritillaria Meleagris,* and *Ornithogalum umbellatum.*

Scilla nutans, the Blue Bell or Wild Hyacinth, was found in one sample in 1862, in none in 1867, in three in 1872, and in six in 1877. It occurred on plots very variously manured, but in no case exceeded 0·01 per cent. of the total herbage.

Fritillaria Meleagris has not been met with in any of the samples, but a few plants were observed on two plots in 1867, on one in 1872, and on one in 1877; in each year on the plot with the smaller quantity of nitrate of soda alone. The plant occurs with some frequency in the Park, under some elm-trees at a little distance from the plots, but appears to have quite disappeared from the plots of late years.

Ornithogalum umbellatum was found in extremely small amount in one sample (from plot 9) in 1872, but has otherwise not been observed on the plots.

All three of these *Liliaceæ* are bulbous plants, flowering early, and are not of a character to be represented in the samples, even in the small proportion in which they exist in the growing herbage. The organisation of the bulbs is such as to provide for reproduction independently of the seed, and thus it not unfrequently happens that the plants persist for some years, forming new bulbs and leaves each season, but not producing flower. The same holds good in the case of the Orchids. The profusion of Blue Bells in woods and shady places probably arises from the circumstance that they are enabled to grow and maintain themselves under conditions in which other plants cannot so well do so.

CYPERACEÆ.

The Sedges are well nigh as widely distributed as the Grasses; but they are more especially plants of wet places. The contrast between the plants of the two Orders is very interesting, but the experimental plots afford but little opportunity for illustrating it, inasmuch as there is only one of the Order met with on them, viz.:—

Carex præcox, which is a slow-growing, tufted, early-flowering Sedge, with creeping root-stock, giving off fibrous roots. It is very generally diffused in meadows, even on those which are relatively dry. It was found in 12 samples in 1862, in 13 in 1867, in 16 in 1872, and in 10 in 1877. It was most frequent on the unmanured plots and on those with purely mineral manures. It was all but absent from the ammonia plots; but it was more general on the nitrate plots. It yielded by far the largest quantity to the herbage on the unmanured plots. In only two cases, however, did it contribute more than 1 per cent. to the produce.

FILICES.

Ophioglossum vulgatum, the Adder's Tongue Fern, is the only representative of this order which has been met with on the plots. It is a low-growing species, with a small, perennial, creeping root-stock, throwing up a single ovate leaf, which is apt to escape observation amid the more luxuriant herbage, and even if it be not withered before the grass is cut, is of such lowly stature as not often to come into the samples. It was found in four samples in 1862, in none in 1867 or 1872, and in one only in 1877. Its presence was, however, noted on four plots in 1867, on four in 1872, and on three in 1877. Its amount in every instance was so small as to require no further comment.

MUSCI.

Three species of *Hypnum*—*H. squarrosum, H. rutabulum,* and *H. hians*—occur on the plots. One or other of these was found in 10 samples in 1862, in eight in 1867, in 15 in 1872, and in 20 in 1877. They were by far the most frequent on the unmanured plots, and on those receiving mineral manure alone. They were scarcely represented at all on the ammonia plots, but were more general on the nitrate plots. In only two cases did the amount reach or exceed 0·1 per cent., and these were on the plots without manure and with superphosphate of lime alone. In all others the quantity was extremely insignificant. The evidence is, that their presence in the mixed herbage is more directly dependent upon the amount of water at command, and on the absence of luxuriance of their associates, than on manurial conditions specially favourable to themselves.

THE BOTANY OF EACH SEPARATE PLOT, IN EACH SEASON OF COMPLETE SEPARATION.

Thus far we have described the arrangement of the experiments, the mode of experimenting, and the characters of the seasons preparatory to and during the periods of growth of the four crops selected for complete botanical separation; we have given a complete list of all the plants, either found in the samples or observed growing on any one of the whole series of 20 (or more) plots; we have called attention to the general prevalence of certain plants, and to the characters, structural or physiological, above-ground or under-ground, which may be supposed to influence the results of the struggle, when numerous species are growing in association, and when conditions are provided which unequally affect the activity of growth of the different components of the mixed herbage. Finally, the actual or relative characteristics of growth of each species found in the samples, or observed on the plots, have been briefly described; and it has been pointed out, under what conditions of manuring each has maintained or improved its position, or has succumbed in the combat with its associates under the more active competition which has been induced.

It remains to describe the botanical composition of the collective herbage of each of the differently manured plots, in each of the four seasons of botanical separation; to direct attention to the changes in the flora, from one separation-year to another, under the different conditions; and to consider the character of the changes—whether apparently progressive and regular, and probably due almost exclusively to the conditions induced by exhaustion or by manuring; or whether they are probably materially influenced by the characters of the seasons to which the associated species have been exposed.

It is obviously essential, as a means of discriminating between the results primarily due to the conditions of manuring, and those greatly dependent on the characters of the seasons of growth, to bear in mind what were those characters, and what was the nature of their influence in determining to luxuriance or to maturation, and so on. It will be desirable, therefore, as a preliminary to the discussion which is now to follow, very briefly to summarise in this place the climatal characters of the different seasons, referring for more detail to the fuller consideration of the subject at pp. 1188–1206.

First season of separation, 1862.—This, the first season of complete botanical separation, was the seventh in the course of the experiments. Dependent on previous conditions, there were full average second crops in 1861. The autumn and winter of 1861–2 were warmer and drier than usual, excepting November, which was cold and wet. In 1862 the grass-growing months were wetter than the average, and also warmer, excepting June, which was wet and cold, protracting growth, but not maturing. The experimental crops were generally considerably over the average. The climatal conditions were favourable to the prominence of the grasses, especially of the freer-growing ones, also of free-growing weeds, and especially *Rumex Acetosa*, *Conopodium denudatum*, and *Achillea Millefolium*. The herbage was characterised by abundance of foliage, dense undergrowth, backwardness, and little tendency to form stem, or to flower and seed.

Second season of separation, 1867.—In 1866 both the first and second crops were over average, and the second were cut and left on the ground. The autumn, winter, and spring of 1866–7 were very changeable, and growth was much checked. June, the maturing period, was again changeable; upon the whole drier, but not so warm as usual. With very varying climatal conditions, there was great irregularity in the character of growth of the different plants, and with the different manures. Without manure, and with deficient manuring, there was more than average produce; with mineral manure and ammonia-salts together there was a considerable deficiency; and with mineral manure and nitrate of soda a considerable excess. With the nitrate the excess was almost all gramineous, and the most forward grasses produced much stem, under the influence of the dry, though not warm, weather of June. With deficient manuring, and with ammonia-salts alone, there was, on the other hand, restricted growth of the grasses, but miscellaneous plants were unusually abundant. Upon the whole, the different grasses were very differently affected; Leguminosæ were generally deficient; miscellaneous plants were in excess; and, excepting with the nitrate, the growth was much more characteristically leafy than stemmy.

Third season of separation, 1872.—The characters of the seasons intermediate between the separation-year of 1867 and that of 1872 undoubtedly exercised considerable influence on the struggle from year to year. The first of the four intermediate years gave very heavy produce, characterised by great stemminess and maturity; the second, again, gave very heavy produce, but characterised by great luxuriance and leafy growth; the third yielded the smallest crop of the whole series, prematurely ripened; the fourth was one of variable characters, giving more than average amounts of produce, with great unevenness of development of the plants. It is obvious that these various conditions were calculated to influence the character and results of the struggle in very various ways; and, in the facts adduced, we have illustrated in a striking manner how greatly the question is complicated by the element of season.

The season of 1872 itself was extremely changeable; favourable for fair luxuriance, but of very varying tendency to stem and seed formation, and to maturation, according to the already established character of the herbage, and to the conditions of manuring supplied. There was generally less than average produce with deficient, and more than average with high, manuring. By virtue of the characters of the four preceding seasons, and especially of the two years of heat and drought, 1868 and 1870, the distribution of plants had become considerably modified. With deficient manuring *Festuca ovina* especially, but *Agrostis vulgaris* also, had much increased. With more liberal manuring, when the nitrogen was supplied as ammonia, *Dactylis glomerata* and *Poa pratensis*, and when as nitrate, *Bromus mollis* and *Poa trivialis* had become more prominent. The season of 1872 was favourable for superficially feeding Leguminosæ where the manures were appropriate. Miscellaneous plants had been generally reduced; but *Rumex Acetosa* had maintained its position, *Achillea Millefolium* had considerably increased, and *Conopodium denudatum* was also prominent.

To conclude,—with considerably modified floras, induced by recent seasons of luxuriance, or of drought, or of both, and with very variable climatal conditions in the season of separation itself, for the most part more favourable for luxuriance than maturation, and the ripening period both late and much broken, there were very variable characteristics of produce on the differently manured plots.

Fourth season of separation, 1877.—In the first crops of 1873 the grasses were dwarfish, with little stem and thin undergrowth; some strong weeds—such as *Rumex Acetosa, Centaurea nigra*, and *Achillea Millefolium*—were luxuriant, but Leguminosæ were less prominent than usual. There was fairly dense aftergrowth yielding two cuttings, which were spread on the respective plots. *Festuca ovina* contributed a large proportion of the dense undergrowth.

The winter and early spring of 1873–4 were warmer but drier than usual, and vegetation was very backward. With continued drought and low temperatures the herbage was materially damaged, yielding stunted and injured crops. There was comparatively little aftergrowth, excepting with high manuring; second and third crops were, however, cut, and spread on the respective plots.

In the winter and early spring of 1874–5 the weather was very variable; but there was an excess of rain, and there were higher than average temperatures in May and June. The produce was generally below, but with high manures it was above average. *Agrostis vulgaris* and *Poa trivialis* were very prominent; and the dense, leafy growth of the grasses kept back the leguminous and miscellaneous species. The comparatively meagre first growth was followed by great luxuriance of free-growing species, such as, according to the plot, *Dactylis glomerata, Avena elatior*, and *Bromus mollis.* Leguminous and miscellaneous plants were also prominent and prolific. The second cuttings were much heavier than usual, and they were, for the first time, removed from the land and weighed—a procedure obviously affecting the condition of the land, and at the same time introducing a new element into the struggle among the components of the mixed herbage.

The winter and early spring of 1875–6, and the usual growing period, were variable and unseasonable, yielding backward, scanty, and thin crops, excepting with the highest manuring, where a few free-growing grasses were luxuriant. Leguminous and miscellaneous species were deficient. The autumn was wet, but variable as to temperature, and the second growth of the grasses was accordingly patchy, but with a good deal of leafy bottom; whilst both leguminous and miscellaneous species were deficient. Owing to the wetness and coldness of the weather, the second crops could not be weighed and removed, but were cut and spread on the respective plots.

In 1877, the year of botanical separation itself, grass gave great early promise, but it was much checked by a cold spring, and it remained backward, short, and benty, with little undergrowth, until, with warm weather in June, the herbage thickened surprisingly, yielding, eventually, more than average produce, especially on the highly-manured plots, where free-growing grass prevailed. There was, however, considerable irregularity of development, excepting with the nitrate of soda.

Thus, between the third and fourth separation-years, 1872 and 1877, there were several seasons of more or less ungenial weather for the growth of first crops, which were only average or under average; and those of the second year were both under average and much damaged by the dryness and coldness of the growing period. The second crops of the period were, however, over average, and those of the third intermediate season (1875) were not only very heavy, but were removed from the land. Finally, the year of separation itself was one of early promise, then checked growth, but eventually luxuriant and heavy produce, with, however, very uneven development on the different plots. The general result was an increased proportion of gramineous herbage, especially of the freer-growing grasses, where there was liberal manuring.

1. *Without manure; Plots* 3 *and* 12.

Before describing in detail the botany of the manured plots, it is essential to study carefully that of the unmanured plots 3 and 12, both separately and comparatively. We shall thus gain an idea of the natural vegetation of the experimental area. By comparing the composition of the herbage on one plot with that on the other, we shall be able to form an approximate estimate of the range and the limits of variation on ostensibly duplicate areas; and by comparing the results of one year with those of another, we shall learn something of the character of the changes induced by variations of season, by the constant competition and struggle, and by progressive exhaustion, irrespectively of the influence of manure. Further, we shall acquire standards with which to compare the results obtained on the manured plots.

The following Tables LXIII. and LXIV., pp. 1294, 1295, give a complete numerical record of the botany of the two unmanured plots, in each of the four separation-years. In Table LXIII. is given the number of species found in the samples, grouped as "Gramineæ," "Leguminosæ," and "Other Orders" (all the miscellaneous species being arranged under this heading); in it the percentage of each species in the total mixed herbage is also recorded. In Table LXIV. the actual quantity of each species, in lbs. per acre, is given. The mean results for each plot over the four years, as well as the difference between the one plot and the other, are also shown.

TABLE LXIII.—Number of Species, and Proportion per Cent. (by weight) of each Species, Without Manure; Plots 3 and 12.

		Plot 3.				Plot 12.				Mean of 1862, 1867, 1872, 1877.		
		1862.	1867.	1872.	1877.	1862.	1867.	1872.	1877.	Plot 3.	Plot 12.	Plots 3 and 12.
NUMBER OF SPECIES.												
Gramineæ		18	15	17	17	16	16	17	16	17	16	17
Leguminosæ		4	4	4	4	4	4	4	4	4	4	4
Other Orders		28	24	28	31	24	30	29	31	28	29	28
Total		50	43	49	52	44	50	50	51	49	49	49
GRAMINEÆ; PER CENT.												
Gramineæ	Anthoxanthum odoratum	4·28	8·66	5·20	5·12	4·09	7·66	6·67	5·47	5·81	5·97	5·89
	Alopecurus pratensis	4·49	5·82	0·52	0·30	3·12	2·62	2·74	0·96	2·78	2·36	2·57
	Phleum pratense	0·01	...	...	...	...	...	...	...	0·01	...	0·01
	Agrostis vulgaris	11·36	8·63	16·14	13·28	9·43	5·55	11·13	13·29	12·35	9·60	10·98
	Aira cæspitosa	0·01	...	0·01	0·04	...	...	0·01	...	0·01	0·01	0·01
	Holcus lanatus	5·04	7·97	3·60	12·55	4·63	5·57	3·81	10·12	7·29	6·03	6·66
	Avena elatior	0·07	0·21	0·13	0·05	0·78	0·67	1·84	0·76	0·11	1·01	0·56
	Avena pubescens	9·65	3·07	3·55	2·69	9·87	5·64	4·85	2·87	4·74	5·81	5·28
	Avena flavescens	2·37	1·86	3·49	1·08	2·25	1·76	2·38	0·55	2·20	1·74	1·97
	Poa pratensis	0·29	0·17	0·09	0·07	0·72	0·68	0·72	0·27	0·15	0·60	0·37
	Poa trivialis	1·54	1·17	0·50	0·55	2·68	1·93	0·91	0·80	0·99	1·58	1·28
	Briza media	1·89	0·68	6·40	7·25	1·37	1·66	4·22	3·72	4·05	2·74	3·40
	Dactylis glomerata	1·76	1·74	0·90	0·70	2·82	3·19	1·90	2·64	1·27	2·64	1·95
	Cynosurus cristatus	0·17	0·13	1·11	0·98	0·49	0·43	0·96	0·44	0·59	0·81	0·70
	Festuca ovina	13·30	15·20	21·67	21·89	7·17	11·59	16·05	20·88	18·01	13·92	15·96
	Festuca pratensis	0·04	...	0·01	0·01	10·11	3·94	2·25	3·27	0·01	4·89	2·45
	Festuca loliacea	...	...	...	0·03	...	...	...	...	0·01	...	0·01
	Bromus mollis	0·13	0·05	0·01	...	0·08	0·04	0·01	0·01	0·05	0·04	0·05
	Lolium perenne	6·37	4·03	2·37	4·55	4·46	3·09	1·91	2·28	4·33	2·94	3·63
	Undetermined (chiefly Gramineæ)	7·84	6·14	2·96	...	8·37	3·00	1·34	...	4·23	3·18	3·70
	Total	70·61	65·53	68·66	71·15	72·44	59·02	63·70	68·33	68·99	65·87	67·43
LEGUMINOSÆ; PER CENT.												
Leguminosæ	Trifolium repens	0·53	0·21	0·38	0·13	0·53	1·65	1·10	0·03	0·31	0·83	0·57
	Trifolium pratense	4·48	2·11	1·68	2·09	2·08	2·87	1·58	2·07	2·59	2·15	2·37
	Lotus corniculatus	1·83	2·35	5·94	3·95	1·64	4·26	5·16	3·09	3·52	3·54	3·53
	Lathyrus pratensis	1·26	0·68	0·98	2·37	1·95	2·01	2·33	2·29	1·32	2·14	1·73
	Total	8·10	5·35	8·98	8·54	6·20	10·79	10·17	7·48	7·74	8·66	8·20
OTHER ORDERS; PER CENT.												
Ranunculaceæ	Ranunculus acris	0·01	0·01	0·07	...	...	0·35	0·04	0·09	0·02	0·12	0·07
	Ranunculus repens et bulbosus	4·88	2·01	2·94	3·45	2·62	1·72	3·22	6·45	3·32	3·50	3·41
Caryophyllaceæ	Stellaria graminea	0·01	0·02	0·02	0·02	0·07	0·42	0·90	0·94	0·02	0·58	0·30
	Stellaria Holostea	...	...	...	...	...	..	...	0·01	...	0·01	0·01
	Cerastium triviale	0·46	0·37	0·85	0·82	0·39	1·02	3·44	0·81	0·62	1·41	1·01
Rosaceæ	Potentilla reptans	...	...	0·01	0·01	0·01	0·01	0·01	0·02	0·01	0·01	0·01
	Potentilla Fragariastrum	...	...	...	...	...	0·01	0·01	0·01	...	0·01	0·01
	Agrimonia Eupatoria	0·01	...	0·02	0·01	...	0·01	0·01	...	0·01	0·01	0·01
	Poterium Sanguisorba	...	0·21	0·49	0·88	...	...	...	...	0·39	...	0·19
Umbelliferæ	Conopodium denudatum	0·97	2·95	2·85	1·90	1·58	5·42	2·17	2·84	2·16	3·00	2·58
	Pimpinella Saxifraga	1·53	3·44	0·86	0·79	0·99	1·20	0·44	0·49	1·65	0·78	1·21
	Heracleum Spondylium	...	...	0·02	...	...	...	...	...	0·01	...	0·01
Rubiaceæ	Galium verum	...	...	...	...	0·01	0·11	0·01	0·01	...	0·03	0·02
	Galium Aparine	...	...	0·01	...	...	...	...	...	0·01	...	0·01
Compositæ	Centaurea nigra	0·31	0·59	2·11	1·06	1·20	0·94	3·35	2·47	1·02	1·99	1·50
	Bellis perennis	0·01	0·04	0·01	0·02	...	...	...	0·01	0·02	0·01	0·02
	Achillea Millefolium	1·53	1·16	1·78	1·99	1·13	1·53	3·77	2·81	1·61	2·31	1·96
	Tragopogon pratensis	0·01	...	0·01	0·01	0·30	0·36	0·33	0·01	0·01	0·25	0·13
	Leontodon hispidus	0·06	0·64	1·27	1·32	0·11	0·10	0·13	0·07	0·82	0·10	0·46
	Leontodon autumnalis	...	...	...	...	...	...	...	0·01	...	0·01	0·01
	Taraxacum officinale	0·05	0·16	0·15	0·06	0·12	0·13	0·01	0·14	0·10	0·10	0·10
	Hieracium Pilosella	0·01	0·01	0·01	0·04	...	0·01	0·02	0·08	0·02	0·03	0·02
	Chrysanthemum Leucanthemum	...	...	...	0·12	...	...	...	...	0·03	...	0·01
Dipsaceæ	Scabiosa arvensis	0·01	...	0·02	0·12	0·05	0·01	0·09	0·06	0·04	0·05	0·04
Plantaginaceæ	Plantago lanceolata	7·34	10·73	2·66	3·16	7·74	8·25	0·41	1·43	5·97	4·46	5·21
Scrophulariaceæ	Veronica Chamædrys	0·48	0·32	0·14	0·24	0·39	0·46	0·23	0·57	0·29	0·41	0·35
Labiatæ	Prunella vulgaris	0·01	0·03	0·01	0·09	0·03	0·12	0·01	0·04	0·03	0·05	0·04
	Thymus Serpyllum	...	...	...	0·09	...	...	...	...	0·02	...	0·01
	Ajuga reptans	0·01	0·01	...	0·02	...	0·02	0·01	0·01	0·01	0·01	0·01
Primulaceæ	Primula veris	0·01	...	...	0·01	0·03	0·03	0·03	0·03	0·01	0·03	0·02
Polygonaceæ	Rumex Acetosa	1·40	1·76	1·77	1·87	3·02	3·61	3·52	2·06	1·70	3·05	2·37
Liliaceæ	Scilla nutans	...	...	...	0·01	...	...	...	...	0·01	...	0·01
Juncaceæ	Luzula campestris	1·91	3·61	3·42	1·77	1·14	2·96	2·98	1·50	2·68	2·14	2·41
Cyperaceæ	Carex præcox	0·18	1·02	0·84	0·37	0·31	1·35	0·90	0·68	0·60	0·81	0·71
Filices	Ophioglossum vulgatum	0·01	...	...	...	...	...	...	...	0·01	...	0·01
Musci	Hypnum squarrosum	0·06	0·03	0·01	0·06	0·10	0·04	0·07	0·54	0·05	0·20	0·12
	Hypnum rutabulum	0·01		0·01		0·01		0·01				
	Hypnum hians	0·01		...		0·01		0·01				
	Total	21·29	29·12	22·36	20·31	21·36	30·19	26·13	24·19	23·27	25·47	24·37
SUMMARY.—PER CENT.												
Gramineæ		70·61	65·53	68·66	71·15	72·44	59·02	63·70	68·33	68·99	65·87	67·43
Leguminosæ		8·10	5·35	8·98	8·54	6·20	10·79	10·17	7·48	7·74	8·66	8·20
Other Orders		21·29	29·12	22·36	20·31	21·36	30·19	26·13	24·19	23·27	25·47	24·37
Total		100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00

TABLE LXIV.—Quantity of each Species, per Acre, in Lbs., Without Manure; Plots 3 and 12.

		Plot 3.				Plot 12.				Means of 1862, 1867, 1872, and 1877.		
		1862.	1867.	1872.	1877.	1862.	1867.	1872.	1877.	Plot 3.	Plot 12.	Plots 3 and 12.
GRAMINEÆ; LBS. PER ACRE.												
	Number of Species	18	15	17	17	16	16	17	16	17	16	17
Gramineæ	Anthoxanthum odoratum	130·6	288·5	85·5	121·0	140·0	233·5	150·2	118·5	156·4	160·5	158·5
	Alopecurus pratensis	137·0	193·9	8·5	7·1	106·8	79·9	61·7	20·8	86·6	67·3	76·9
	Phleum pratense	0·3	...	...	...	...	...	...	...	0·1	...	0·1
	Agrostis vulgaris	346·7	287·5	265·3	313·9	322·8	169·2	250·7	287·8	303·4	257·6	280·5
	Aira cæspitosa	0·3	...	0·2	0·9	...	...	0·2	...	0·4	0·1	0·3
	Holcus lanatus	153·8	265·5	59·2	296·7	158·5	169·8	85·8	219·1	193·8	158·3	176·1
	Avena elatior	2·1	7·0	2·1	1·2	26·7	20·4	41·4	16·4	3·1	26·2	14·6
	Avena pubescens	294·5	102·3	58·4	63·6	338·8	171·9	109·2	62·1	129·7	170·5	150·1
	Avena flavescens	72·4	62·0	57·4	25·5	77·0	53·6	53·6	11·9	54·3	49·0	51·6
	Poa pratensis	8·9	5·7	1·5	1·7	24·6	20·7	16·2	5·8	4·4	16·8	10·6
	Poa trivialis	47·0	39·0	8·2	13·2	91·7	58·8	20·5	17·3	26·9	47·1	37·0
	Briza media	57·7	22·7	105·2	171·4	46·7	50·6	95·0	80·5	89·2	68·2	78·7
	Dactylis glomerata	53·7	58·0	14·8	16·6	96·5	97·2	43·8	57·2	35·8	73·7	54·7
	Cynosurus cristatus	5·2	4·3	18·2	23·2	16·7	13·1	21·6	9·5	12·7	15·2	14·0
	Festuca ovina	405·9	506·5	356·2	517·5	245·5	353·3	361·5	452·0	446·5	353·1	399·8
	Festuca pratensis	1·2	...	0·2	0·2	346·1	120·1	50·7	70·8	0·4	146·9	73·6
	Festuca loliacea	...	...	...	0·7	...	...	...	...	0·2	...	0·1
	Bromus mollis	4·0	1·7	0·2	...	2·7	1·2	0·2	0·2	1·5	1·1	1·3
	Lolium perenne	194·4	134·3	39·0	107·6	152·7	94·2	42·0	49·4	118·8	84·6	101·7
	Undetermined (chiefly Gramineæ)	239·3	204·5	48·7	...	286·5	91·4	30·2	...	123·1	102·0	112·5
	Total	2155·0	2183·4	1128·8	1682·0	2480·3	1798·9	1434·5	1479·3	1787·3	1798·2	1792·7
LEGUMINOSÆ; LBS. PER ACRE.												
	Number of Species	4	4	4	4	4	4	4	4	4	4	4
Leguminosæ	Trifolium repens	16·2	7·0	6·2	3·1	18·1	50·3	24·8	0·6	8·1	23·4	15·8
	Trifolium pratense	136·7	70·3	27·6	49·4	71·2	87·5	35·6	44·8	71·0	59·8	65·4
	Lotus corniculatus	55·9	78·3	97·7	93·4	56·2	129·8	116·2	66·9	81·3	92·3	86·8
	Lathyrus pratensis	38·4	22·7	16·1	56·0	66·8	61·3	52·4	49·6	33·3	57·5	45·4
	Total	247·2	178·3	147·6	201·9	212·3	328·9	229·0	161·9	193·7	233·0	213·4
OTHER ORDERS; LBS. PER ACRE.												
	Number of Species	28	24	28	31	24	30	29	31	28	29	28
Ranunculaceæ	Ranunculus acris	0·3	0·3	1·2	...	...	10·7	0·9	2·0	0·5	3·4	1·9
	Ranunculus repens et bulbosus	148·9	67·0	48·3	81·6	89·7	52·4	72·5	139·7	86·4	88·8	87·6
Caryophyllaceæ	Stellaria graminea	0·3	0·7	0·3	0·5	2·4	12·8	20·3	20·4	0·5	14·0	7·2
	Stellaria Holostea	...	...	...	...	...	...	...	0·2	...	0·1	0·1
	Cerastium triviale	14·1	12·2	14·0	19·4	13·4	30·8	77·5	17·5	14·9	34·8	24·8
Rosaceæ	Potentilla reptans	...	...	0·2	0·2	0·3	0·3	0·2	0·4	0·1	0·3	0·2
	Potentilla Fragariastrum	...	...	...	...	...	0·3	0·2	0·2	...	0·2	0·1
	Agrimonia Eupatoria	0·3	...	0·3	0·2	...	0·3	0·2	...	0·2	0·1	0·1
	Poterium Sanguisorba	...	7·0	8·0	20·8	...	...	...	...	8·9	...	4·4
Umbelliferæ	Conopodium denudatum	29·6	98·3	46·8	44·9	54·1	165·2	48·9	61·5	54·9	82·4	68·6
	Pimpinella Saxifraga	46·7	114·6	14·1	18·8	33·9	36·6	9·9	10·6	48·5	22·7	35·6
	Heracleum Sphondylium	...	...	0·3	...	...	...	...	...	0·1	...	0·1
Rubiaceæ	Galium verum	...	...	...	...	0·3	3·4	0·2	0·2	...	1·0	0·5
	Galium Aparine	...	...	0·2	...	...	...	...	...	0·1	...	0·1
Compositæ	Centaurea nigra	9·5	19·7	34·6	25·1	41·1	28·7	75·5	53·5	22·2	49·8	36·0
	Bellis perennis	0·3	1·3	0·2	0·5	...	...	...	0·2	0·6	0·1	0·3
	Achillea Millefolium	46·7	38·7	29·3	47·0	38·7	46·6	84·9	60·8	40·4	57·7	49·0
	Tragopogon pratensis	0·3	...	0·2	0·2	10·3	11·0	7·4	0·2	0·2	7·2	0·3
	Leontodon hispidus	1·8	21·3	20·9	31·2	3·8	3·0	2·9	1·5	18·8	2·8	3·7
	Leontodon autumnalis	...	...	...	...	...	...	...	0·2	...	0·1	0·1
	Taraxacum officinale	1·5	5·4	2·5	1·4	4·1	4·0	0·2	3·0	2·7	2·8	2·8
	Hieracium Pilosella	0·3	0·3	0·2	1·0	...	0·3	0·5	1·7	0·5	0·6	0·5
	Chrysanthemum Leucanthemum	...	...	...	2·8	...	...	...	...	0·7	...	0·3
Dipsaceæ	Scabiosa arvensis	0·3	...	0·3	2·8	1·7	0·3	2·0	1·3	0·8	1·3	1·1
Plantaginaceæ	Plantago lanceolata	224·0	357·5	43·7	74·7	265·0	251·5	9·3	31·0	175·0	139·2	157·1
Scrophulariaceæ	Veronica Chamædrys	14·7	10·7	2·3	5·7	13·4	14·0	5·2	12·4	8·4	11·3	9·8
Labiatæ	Prunella vulgaris	0·3	1·0	0·2	2·1	1·0	3·7	0·2	0·9	0·9	1·5	1·2
	Thymus Serpyllum	...	...	...	2·1	...	...	...	...	0·5	...	0·3
	Ajuga reptans	0·3	0·3	...	0·5	...	0·6	0·2	0·2	0·3	0·2	0·3
Primulaceæ	Primula veris	0·3	...	...	0·2	1·0	0·9	0·7	0·7	0·1	0·8	0·5
Polygonaceæ	Rumex Acetosa	42·7	58·7	29·1	44·2	103·4	110·1	79·3	44·6	43·7	84·3	64·0
Liliaceæ	Scilla nutans	...	...	...	0·2	...	...	...	...	0·1	...	0·1
Juncaceæ	Luzula campestris	58·3	120·3	56·2	41·8	39·0	90·3	67·1	32·5	69·1	57·2	63·1
Cyperaceæ	Carex præcox	5·5	34·0	13·8	8·8	10·6	41·2	20·3	14·7	15·5	21·7	18·6
Filices	Ophioglossum vulgatum	0·3	...	...	...	...	...	...	...	0·1	...	0·1
Musci	Hypnum squarrosum	1·9	1·0	0·2	1·4	3·4	1·2	1·6	11·7	1·3	1·2	0·6
	Hypnum rutabulum	0·3		0·2		0·4		0·2			3·3	2·3
	Hypnum hians	0·3		...		0·4		0·2			0·1	0·1
	Total	649·8	970·3	367·6	480·1	731·4	920·2	588·5	523·8	617·0	691·0	654·0
SUMMARY.—QUANTITIES PER ACRE, IN LBS.												
	Gramineæ	2155·0	2183·4	1128·8	1682·0	2480·3	1798·9	1434·5	1479·3	1787·3	1798·2	1792·7
	Leguminosæ	247·2	178·3	147·6	201·9	212·3	328·9	229·0	161·9	193·7	233·0	213·4
	Other orders	649·8	970·3	367·6	480·1	731·4	920·2	588·5	523·8	617·0	691·0	654·0
	Total	3052·0	3332·0	1644·0	2364·0	3424·0	3048·0	2252·0	2165·0	2598·0	2722·0	2660·0

Number of species.—According to the tables, the total number of species found in the samples from plot 3 was, in the four separation-years respectively—50, 43, 49, and 52; and in those from plot 12—44, 50, 50, and 51. These it is to be understood are the numbers of species found in the samples; but reference to Table XXXVI., pp. 1209–11, will show that generally some were observed on the plots that did not come into the samples, though such occurred in very insignificant amounts. The average number of species over the four years was, on plot 3, 49, on plot 12, 49, and of course on the two plots, also 49.

It is further seen that the number of species of grasses was, on plot 3—18, 15, 17, and 17; and on plot 12—16, 16, 17, and 16; the average number being therefore rather the higher on plot 3. Leguminosæ are uniformly represented by four species in the samples from both plots. The number of species contributed by other Orders were, in the four years, as follows: on plot 3—28, 24, 28, and 31; and on plot 12—24, 30, 29, and 30; the average number being practically the same on the two plots.

It may be said that there is, upon the whole, a slight tendency to an increase in the number of species with the continuance of the unmanured condition; and that such increase as there is, is among the miscellaneous plants. It might indeed have been expected that, as the exhaustion of the soil increased, the competition arising from the luxuriant growth of particular species would have diminished in intensity, and have induced the results observed.

Among the fluctuations from year to year the chief irregularity to be noted is that plot 3 had a large number of species in 1862, and a small number in 1867; whilst plot 12 had a small number in 1862, and a large one in 1867. The reduction in number of species on plot 3 in 1867 was about equally divided between the grasses and the miscellaneous plants; while the increase in number on plot 12 in the same year was entirely in the miscellaneous plants.

The table shows that there was a larger amount of undetermined matter in the earlier than in the later separations, and it is, of course, a question how such matter should be allotted. By careful examination in each case, it has been ascertained that by far the larger amount of the undetermined matter is referable to the grasses, and so, to bring the results of the different years into comparison, it has been thought fairer to include this matter among the gramineous herbage than either to exclude it from the record, or to allot it by judgment merely to the different divisions of Gramineæ, Leguminosæ, and Miscellaneæ. With this explanation we may proceed to consider the results as they stand.

Neither an examination of the tables now under discussion, nor of those including the results of the partial separations in other years, indicates any really marked increase or decrease in the percentage of the grasses in the total mixed herbage on the unmanured plots. There was, however, very considerable fluctuation in the percentage from year to year, according to season. Thus, in the first and the last of the complete separation-years 1862 and 1877, that is with an interval of 15 years, both the unmanured plots yielded high percentages of Gramineæ. In the second of the four years (1867) they both yielded the lowest, and in the third (1872) about medium percentages.

The percentages of the Leguminosæ also fluctuated from year to year somewhat differently on the two plots, and did so without manifesting any very distinct tendency either to increase or decrease, in relative proportion to the rest of the herbage; though the indications were perhaps the more towards increase with the continuance of the unmanured condition.

Of total Miscellaneæ there was also considerable fluctuation in percentage from year to year, and this was the case on both plots. In the second separation-year (1867), when the percentage of grasses was on both plots the lowest, that of the Miscellaneæ was on both plots the highest; and whilst in the two subsequent separation-years (1872 and 1877), the percentage of Miscellaneæ was on plot 3 fairly uniform, and much the same as in the first year, it was on plot 12 considerably higher than in the first year. On this plot, indeed, there was an obvious tendency to an increase in the proportion of such herbage.

Looking thus at the percentage merely which each of the three main groups has contributed to the herbage each year, it would seem that both the Leguminosæ and the Miscellaneæ, but especially the latter, have rather gained than lost in the competition with the grasses, as the exhaustion has proceeded. But, if we turn from the *percentage* to the *actual* amounts per acre of each description of herbage each year, a clearer idea of what has taken place will be gained. We now observe that each description of herbage—Gramineous, Leguminous or Miscellaneous—has considerably decreased in yield in the later years, and that this is so, though in different degrees, on both plots.

Referring first to plot 3, we find the total produce of the grasses pretty equal in 1862 and in 1867, little more than half as much in 1872, and only about three-fourths as much in the fourth separation-year, 1877, as in the first or second. There is thus, therefore, a manifest reduction in yield of such herbage independently of fluctuation of season. Of leguminous herbage there was much more in the first separation-year than afterwards; but more in the fourth than in either the second or the third; indicating, therefore, a less marked decline in actual yield than in the case of the Gramineæ. The partial separations of later years, indeed, indicate a tendency to increase rather than to decrease.

Lastly, of miscellaneous herbage, plot 3 yielded very fluctuating amounts in the four separation-years: in the second (1867) there was about one-and-a-half time as much as in 1862; in the third (1872) there was little more than half as much as in 1862, and very much less than half as much as in 1867; in the fourth year again (1877) there was very much less actual quantity of such herbage than in either the first or the second year. There was then, as with the grasses, a considerable reduction in the growth of these plants with the progress of the exhaustion, and this reduction of actual amount grown is much greater in both sets of plants than would be gathered from their percentage relations alone.

Turning to plot 12, the reduction in the actual yield of the grasses is seen to be very considerable. It is also more uniform from the first to the third separation-year than on plot 3, and the amount is much the same in the fourth as in the third, and only about three-fifths as much as in the first. Of leguminous herbage there was, on the average, more yielded on plot 12 than on plot 3. The fluctuations in amount from year to year were great, and not accordant in order with those on plot 3. Of miscellaneous herbage, again, there was, on the average, more on plot 12 than on plot 3, and there were also considerable fluctuations from year to year; but these were not so discordant in their order with those on plot 3, as in the case of the Leguminosæ. On both plots there was a considerable increase in the amount of miscellaneous herbage in the second year compared with the first, and on both there was much less in the third and fourth than in the first year. In the last two separation-years there was more on plot 12 than on plot 3.

The general result is, then, that on both plots there was a tendency to considerable reduction in the amount of both the Gramineæ and the Miscellaneæ in the later years, with less marked, if any, reduction in the much smaller actual quantity of leguminous herbage. The resultant is a considerable reduction in the yield of the total mixed herbage.

It is obvious that, considering the fluctuations from year to year according to season, the results obtained in four individual years at stated intervals might be misleading in considering whether there has or has not been a real increase or decrease in the growth with the continuance of the unmanured condition. But it may be here remarked that the decided indications of decrease in growth manifested in the tables under consideration are fully confirmed by the amounts of produce of hay, of nitrogen, and of mineral matter, obtained in each of the first 20 years of the experiments (see Part I., p. 300, and context).

It will be observed that the actual quantities yielded are, on the average, considerably greater on plot 12 than on plot 3, and, as has been pointed out at p. 301, Part I., there is reason to believe that the soil of, at any rate, a part of plot 12 was "made ground," which had a greater depth of mould than in the case of the plots generally. For this reason the results obtained on plot 3, instead of the mean results of the two unmanured plots, will be taken as the standard by which to compare those obtained on the manured land.

We now turn to the percentage, and the actual amounts, of the different species occurring on the two plots. It must be remembered that the first year of complete separation (1862) was the seventh year of the experiments, and hence the distribution of plants then found cannot be taken as indicating the exact state of the flora at the commencement of the experiments. At the same time it must be borne in mind that whatever changes have occurred on these two plots have been the results of natural, not artificially induced, conditions.

Unfortunately, we have but little evidence as to the changes that occurred during those first seven years. As already alluded to, the first attempts at really quantitative botanical separation were made in the third year of the experiments (1858), but neither were the samples taken with so much care, nor were the botanical separations conducted with as much knowledge and experience, then as subsequently; perhaps, therefore, it will be sufficient to remark in regard to the results of the preliminary separations, that they showed *Lolium perenne* to be then the most prominent grass on the unmanured plot 3; *Holcus lanatus* was second, yielding a very high proportion; whilst the only other grasses that were at all prominent were *Avena elatior*, *Anthoxanthum odoratum*, *Agrostis vulgaris*, *Briza media*, and *Festuca ovina*, which were so somewhat in the order in which they are here mentioned.

Reference to Table LXIII., p. 1294, will show that the order of prevalence was very different from this in each of the four complete separation-years. *Lolium perenne* is shown to be in very much smaller amount than the earlier separation indicated; but with, notwithstanding some irregularity, a tendency to still further reduction in amount, thus confirming the conclusion of its former greater prominence. *Holcus lanatus* is also in each of the subsequent separations very much less prominent than was indicated in 1858, though on both plots it was more prominent in the last of the four than in either of the preceding separation-years. *Avena elatior*, too, has gone down very considerably. On the other hand, compared with 1858, *Anthoxanthum odoratum* has, in the subsequent years, maintained its position, or has even increased in amount on the unmanured plots. *Agrostis vulgaris* has done so in a very marked degree, as also has *Briza media*.

Turning from the comparison of the more recent and more detailed results with the initiative ones of 1858, and confining attention to those of the four complete separation-years alone, the marked features of condition and change may be described as follows:—

The most prominent plant on both plots, and the one which has increased in greatest degree (both in percentage and in actual quantity) with the continuous unmanured condition, is the hardy, waste-land grass, *Festuca ovina*. This grass has acquired a greater degree of prominence on the poorer plot 3 than on the better-conditioned plot 12.

The plant second in order in prominence, and again on both plots, is the creeping, self-asserting, *Agrostis vulgaris*, and again it has been the more prominent on the poorer plot 3.

Holcus lanatus, though fluctuating considerably in proportion from year to year, is also doubtless again taking a prominent place.

Next in order comes *Anthoxanthum odoratum*, which, though variable in amount from year to year, is rather on the increase than on the decrease, in proportion to its associates, but, with the decreasing total herbage somewhat on the decrease in actual quantity per acre.

Briza media, though yielding a less average amount than *Anthoxanthum*, is nevertheless obviously on the increase, both in percentage and in actual quantity. *Avena pubescens* has contributed a considerable average percentage proportion and actual amount on both plots, but on both it has greatly decreased from the first to the fourth separation-year. And only now comes in order, and as already observed upon the whole decreasing both in percentage and in actual amount, *Lolium perenne*, which it will be remembered was the most prominent grass in the examination made in the third year of the experiments (1858). The other grasses, which showed moderate prominence in the earlier separations, but which have declined considerably in the later, under the unmanured condition, are the more valued and freer-growing grasses, *Alopecurus pratensis, Avena flavescens, Poa trivialis*, and *Dactylis glomerata*.

Looking at the mean results of the four years for each of the two plots, as given in the first two columns in the right-hand division of each of the two tables, it will be observed that the grasses which are in higher average proportion, and actual amount, on the more exhausted plot 3 are *Festuca ovina, Agrostis vulgaris, Holcus lanatus, Briza media*, and *Lolium perenne;* whereas those which are more prominent on the better-conditioned plot 12 are *Avena pubescens, Dactylis glomerata*, and *Poa trivialis;* the poorer grasses obviously the more prevailing on the poorer plot.

On reference to the tables, it cannot fail to be observed that, upon the whole, there is surprising general uniformity in the components of the herbage on the duplicate unmanured plots; and the differences which have been referred to, connected as they obviously are with some differences in the conditions of the land, are a confirmation of the trustworthiness of the results rather than otherwise.

One remarkable discrepancy between the herbage of the two plots remains, however, to be mentioned, viz.: that on the poorer plot 3, scarcely a trace of *Festuca pratensis* was observed at either separation; whilst on plot 12 there was 10 per cent. of it in the first, and more than 3 per cent. in the fourth, separation-year. Its very prominent occurrence on plot 12 in the earlier years must doubtless have been due to some accidental circumstance, probably connected with the somewhat abnormal condition of the land before referred to; but, with the continuous exhaustion, it would seem to be rapidly disappearing.

As already said, of leguminous species four only have been found in the samples from the unmanured plots, viz.: *Trifolium repens, T. pratense, Lotus corniculatus*, and *Lathyrus pratensis*. In the third year of the experiments, 1858, the most prominent of these was *Lathyrus pratensis, Lotus corniculatus* coming second, and *Trifolium pratense* third.

Trifolium repens has occurred in insignificant proportion and amount on both plots in the more complete separations; irregularly in the different years, becoming gradually reduced on plot 3 ; increasing in the second and third years on plot 12, but going down to less in the fourth year, however, than on plot 3.

Trifolium pratense has, in each year, and on each plot, occurred in considerably larger amount than *T. repens,* still the quantity of it has been small on both plots; decreasing almost regularly on plot 3, but less so on plot 12.

The only leguminous plant which has, in a marked degree, gained ground on the unmanured plots is the deep-rooting, drought-resisting *Lotus corniculatus;* and although not without exception in this respect, it has, perhaps, increased with more regular progression, though occurring in less average amount, on the poorer plot 3 than on the less exhausted plot 12.

Lastly, *Lathyrus pratensis* has yielded a fluctuating but considerably smaller percentage and actual amount than *Lotus corniculatus.* It will be seen further on that this plant is very dependent upon superficial supply of certain constituents; and, in accordance with this, we find the percentage, and the yield of it, the higher on the richer of the two plots, 12.

Thus, of the leguminous plants occurring on the unmanured plots, the only one which has improved its position in the struggle is *Lotus corniculatus;* which has a deeply-penetrating, thick, fleshy root, by virtue of which it must be supposed that it is less dependent on the superficial layers of the soil for either food or moisture, and is able to store up material so as the better to withstand drought or exhaustion of the surface-soil. By these means it maintains its position when many of its associates fail.

In the early separation, in 1858, *Plantago lanceolata* was by far the most prominent of the miscellaneous plants on the unmanured plots, where it contributed nearly 11 per cent. of the mixed herbage. No other species yielded 2 per cent., and those which were at all prominent at that time were so in the following order: *Conopodium denudatum, Achillea Millefolium, Rumex Acetosa* (*Lychnis Flos-cuculi**), and various species of *Ranunculus.* The more recent and more complete separations show that in most cases there is very great irregularity in the occurrence of the different miscellaneous species from year to year, dependent on the characters of the seasons, and doubtless on the degree of intensity of the competition engendered accordingly. Without manure, *Plantago lanceolata* may still be considered the most prominent weed, at any rate on the poorer of the two unmanured plots (3) ; but on both plots it was in considerably less amount in the two later, than in the two earlier separations. The plants which, in proportion to the total herbage, have shown the most marked tendency to increase under the conditions of exhaustion are the several species of *Ranunculus,* and next, perhaps, in this respect, comes the undesirable, and poverty-indicating, *Luzula campestris.* In less and very variable proportions and amounts, but still in some seasons very prominent, and some of them increasing, are *Conopodium denudatum, Pimpinella*

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* This, though not included in any sample, is said to have been found on this plot in 1858, but has never been seen since, although it occurs in a plantation within 100 yards of the plot.

Saxifraga, Achillea Millefolium, Centaurea nigra, and *Rumex Acetosa.* In insignificant amounts, but still in most seasons present and notable, are *Cerastium triviale, Poterium Sanguisorba* (on plot 3), *Leontodon hispidus, Veronica Chamædrys,* and *Carex præcox.* The remainder of the miscellaneous species occur less regularly, and in still smaller proportions, as will be seen by an examination of the tables.

Comparing the mean results for each plot, it may be noted that *Plantago lanceolata, Luzula camprestris, Pimpinella Saxifraga,* and *Leontodon hispidus* are, on the average, more prominent on the poorer plot 3, whilst *Conopodium denudatum, Rumex Acetosa, Achillea Millefolium, Centaurea nigra,* and *Cerastium triviale* are the more prevalent on the plot 12.

Referring now, not to the prevalence of individual species, but of the species collectively of different families, it is seen that *Plantaginaceæ* have, on the average of the two plots, yielded the largest both percentage and acreage amounts among the miscellaneous plants. The *Umbelliferæ* and *Compositæ* came second and third, the *Ranunculaceæ* fourth, and, after these, *Polygonaceæ* and *Juncaceæ.* Comparing the two plots, the *Plantaginaceæ* are decidedly the more prominent on the poorer plot 3; whilst the *Compositæ* and the *Polygonaceæ* are so on the less exhausted plot 12.

Upon the whole it may be said that there is a tendency to increase in the number of species on the unmanured plots; and the greater the exhaustion, the less will the grasses, and the more will the Leguminosæ, but especially the miscellaneous plants, predominate. Among the latter, those which become the more prominent are referable to families noteworthy for the large numerical proportion in which either their specific or their individual representatives occur in temperate climates, under natural conditions; that is in the absence of stimulated luxuriance and consequent intensified struggle. Thus, of the widely distributed families, we have nine species of *Compositæ,* three of *Umbelliferæ,* four of *Rosaceæ,* four of *Ranunculaceæ,* and three of *Labiatæ;* whilst some of the smaller families, which have fewer species, are remarkable for the large quantity yielded by their individual representatives, as *Polygonaceæ* and *Juncaceæ.*

Natural rotation.—Having now considered the climatal conditions of the four separation-years, the characteristics of the chief species found on the various plots, and in detail the number, percentages, and actual weights, of the species occurring on the unmanured plots, we are in a position to appreciate the facts in reference to what has been called natural alternation or rotation, as a result of the struggle uninfluenced by the artificial conditions induced by manuring.

The conditions of success are so variable, even for the same plant at different stages of growth, and in different seasons, that we can scarcely predicate with any certainty whether any individual species will gain or lose in the conflict; although we may, perhaps, form a fair conclusion as to the prevalence of certain groups of species: as, for instance, the poorer grasses—one or other of them according to the wetness or dryness of the season or series of seasons, and also according to the decline of the freer-growing competitors. It is not, however, safe to assume that the intrinsically strongest, the hardiest, or the most prolific, whether in seed or offshoot, will, in virtue of any one of these endowments, necessarily be the victor under all circumstances. The factors are so numerous, so complex, and so interdependent, that the "survival of the fittest" depends not on any one quality, but on a capacity for adaptation to a combination of conditions some favourable, others detrimental.

The idea of a natural alternation, or rotation, as broached by M. DUREAU DE LAMALLE (previously referred to at p. 1212), is not strictly applicable to the case before us. That there is some such alternation is evident. It is not, however, the regular rhythmic process assumed by M. DUREAU, but a phenomenon influenced by the many varying circumstances to which reference has been made, and it is, therefore, in itself very variable.* It is to be borne in mind that in the Rothamsted experiments, even in the case of the unmanured plots, there are some artificial conditions introduced. Thus, the cutting of the crops interferes with the normal course of the vegetation, whilst the removal of them from the land induces a gradual exhaustion. Further, the variations of season from year to year will have their varied effect, accordingly as the condition of the herbage is affected by such artificial treatment. It fact, it will be seen that variations of season have a most potent influence on the result, not only under the less artificial conditions of the unmanured, but also under the more artificial ones of the manured plots.

In most cases we are without data for ascertaining the duration of life of a perennial plant, especially when it is subjected to cutting; but we know that there is a limit, and that, when this is reached, one plant will die, and another will take its place; and so we may have a fluctuating relative predominance of Gramineæ, of Leguminosæ, or representatives of other families.

THE MANURED PLOTS.

We now come to discuss the results due to the increased luxuriance induced by characteristically different conditions of manuring. We must refer to Part I., pp. 293–300, for the previous history and general character of the area selected for experiment, for the arrangement of the plots, for the detailed description of the manures employed, the mode of their application, &c. At p. 300, *et seq.*, will be found a discussion of the results obtained on each individual plot, so far as the amounts of total produce, of nitrogen, and of mineral matter yielded, are concerned; and it will be convenient now to consider the *botanical* results obtained on the different plots, in the same order as to plots as in discussing the so-designated *agricultural* results.

It is, therefore, proposed to consider the effects produced by ammonia-salts alone; nitrate of soda alone; mixed mineral manure alone (including potass); superphosphate

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* The remarks of Mr. DARWIN, on the "struggle for existence," chapter 3 of the 'Origin of Species,' should be consulted with reference to the points raised in this section.

of lime alone; mixed mineral manure (without potass); ammonia-salts, in various amounts, with mixed mineral manure (including potass); nitrate of soda, in different quantities, with mixed mineral manure (including potass); ammonia-salts and superphosphate of lime; ammonia-salts and mixed mineral manure (with and without potass); ammonia-salts alone (13 years), succeeded by mixed mineral manure; equal nitrogen and equal potass, in nitrate of soda and sulphate of potass, and in nitrate of potass, in each case with superphosphate of lime; mixture supplying the ash constituents, and the nitrogen, of one ton of hay; and, lastly, farmyard manure alone, and with ammonia-salts in addition.

The Tables.—The plan adopted is—referring to Tables X. and XI. in the Appendix for full particulars of the occurrence of each species, however small in amount, in each separation-year, on each plot—to embody with the text relating to each plot a table, showing, in the case of the grasses and of the Miscellaneæ, the full details for those species which have exceeded 1 per cent. of the total herbage in any one of the separation-years; for those species which have occurred in less amount than 1 per cent. (bracketed together) the collective result only; and, lastly, a simple enumeration of all those which, though they occurred on the unmanured plot 3, were not found at all in any of the samples from the manured plot to which the table refers. This plan reduces the record in the tables with the text, and facilitates their study. In the case of the Leguminosæ, the species of which are few, but of which the occurrence or absence is always important, the results are given in full detail. For each plot the particulars given are—1, the percentage of each species, or group of species, in the total mixed herbage; 2, the produce per acre, in lbs., of each species or group of species; 3, the increase or the decrease in the actual amounts of each species, or group of species, on the manured plot under consideration, compared with the amounts on the unmanured plot 3, as given in Table LXIII., p. 1294.

With reference to this arrangement of the results in the tables, followed also in the discussion of them, it should be clearly recognised that, comparing plot with plot, the percentages of the different species, and the actual amounts of them per acre, may represent very different relations. Thus, supposing any particular species contributed 5 per cent. to the total mixed herbage of a plot yielding say only 2,000 lbs. of total produce per acre, this would represent the growth of only 100 lbs. per acre of that species on that plot. Supposing, however, the same species contributed only the same percentage (5) to the herbage of a plot yielding say 6,000 lbs. of total produce per acre, this would represent the growth of 300 lbs. instead of only 100 lbs. of that species on the same area. Or, to put the converse case, supposing a species contributed 300 lbs. to a total produce of 2,000 lbs., this would represent 15 per cent. of that species; but supposing the same species contributed the same amount, 300 lbs. per acre, to a total produce of 6,000 lbs., the identical amount of actual yield would only represent on that plot 5, instead of 15 per cent., of the total produce. Again, the increase or the decrease in the actual yield of the different species compared with their amount on the unmanured, or on any other plot, being the resultant of the actual amounts on two different plots, may show quite different relations between one species and another, from those indicated either in the columns of percentage, or in those of actual amount. These distinctions should be borne in mind; otherwise, illustrations drawn from the tables may be taken to be repetitions which are in reality not such.

2. *Ammonia-salts, alone; Plot* 5.

On this plot 400 lbs. of ammonia-salts have been applied per acre, per annum, each year, from 1856 up to the present time. Formerly (and in Part I.) we estimated that this amount of the ammonia salts supplied about 82 lbs. of nitrogen; but of late years they have occurred in commerce in a state of greater purity, and the quantity of nitrogen now supplied may be 88 lbs., or probably for the whole period 86 lbs.

It is obvious that, under the condition of this experiment, any increased growth would be dependent for its necessary mineral constituents (sulphuric acid and chlorine excepted) on the supplies derived from the soil itself. The result has been a rapidly-decreasing amount of produce, and of increase over that without manure, in the later as compared with the earlier years. The details show that there was also a reduced amount of mineral matter taken up in the later years. Indeed, in the later years there has been actually less of some important mineral constituents taken up than on the unmanured plot, 3.

Table LXV., p. 1306, shows the botanical composition of the herbage of plot 5:—the percentage, and the actual amount per acre, of the different species and groups, and the increase or deficiency of each compared with the unmanured plot, 3.

The change in the flora of this plot (5), under the influence of a relative plethora of nitrogen, and exhaustion of available mineral constituents, has been very great.

Unfortunately, the evidence available relating to the period of the experiments prior to the first complete separation-year (1862) is not such as to enable us to speak as definitely as might be desired as to the relative predominance of grasses, Leguminosæ, and Miscellaneæ, during that period. Starting from 1862, the table shows, in the main, a very great increase in the percentage of the grasses, a very small amount of, and an immaterial change in, the Leguminosæ, and on the whole a very great reduction in the percentage proportion, and especially in the actual amount, of miscellaneous species. The second separation-year was, however, a very marked exception in these respects. In that year, the grasses were in unusually small, and the Miscellaneæ in unusually large, proportion. The autumn, winter, and spring had been very changeable, whereby early vegetation was much checked; the usual maturing period was also very changeable. The consequent restricted growth of the grasses was, of course, an element in favour of such Miscellaneæ as could take advantage of the climatal and manurial conditions. The percentage of total grasses was, in 1862 rather more than 86, in 1867 not quite 72, in 1872 nearly 85, and in 1877 more than 94. In neither year did the Leguminosæ contribute ½ per cent. But the Miscellaneæ gave less than 14 per cent. in 1862; more than double, or nearly 28 per cent., in 1867; less than 15 per cent. in 1872; and less than 6 per cent. in 1877.

TABLE LXV.—Number of Species, Percentage, Quantity per Acre, &c.,

	Number of species, and proportion per cent.				
	1862.	1867.	1872.	1877.	Mean.
NUMBER OF SPECIES.					
Gramineæ	17	15	15	13	15
Leguminosæ	4	4	3	2	3
Other Orders	17	17	13	14	15
Total	38	36	31	29	33
GRAMINEÆ.					
	p. c.	p. c.	p. c.	p. c.	p. c.
Anthoxanthum odoratum	5·77	5·51	3·04	4·09	4·60
Agrostis vulgaris	24·30	20·97	26·62	29·46	25·34
Holcus lanatus	10·08	5·15	1·90	3·01	5·03
Avena elatior	3·93	2·78	1·49	0·23	2·11
Avena pubescens	7·31	0·63	0·24	0·12	2·07
Poa pratensis	1·07	0·65	0·61	0·23	0·64
Dactylis glomerata	2·39	1·39	0·70	3·25	1·93
Festuca ovina	21·99	30·57	46·56	53·31	38·11
Lolium perenne	3·33	1·21	0·97	0·09	1·40
None yielding 1 per cent. { Alopecurus pratensis, Aira cæspitosa, Avena flavescens, Poa trivialis, Briza media, Cynosurus cristatus, Festuca pratensis, Bromus mollis }	2·70	1·38	1·62	0·27	1·50
Absent—Phleum pratense, Festuca loliacea	...	...	...	...	...
Undetermined (chiefly Gramineæ)	3·45	1·61	0·95	...	1·50
Total	86·32	71·85	84·70	94·06	84·23
LEGUMINOSÆ.					
Trifolium repens	0·01	0·01	0·01	...	0·01
Trifolium pratense	0·04	0·01	...	...	0·01
Lotus corniculatus	0·05	0·31	0·41	0·14	0·23
Lathyrus pratensis	0·02	0·01	0·04	0·05	0·03
Total	0·12	0·34	0·46	0·19	0·28
OTHER ORDERS.					
Conopodium denudatum	1·15	5·74	1·02	0·65	2·14
Galium verum	..	0·61	2·40	0·22	0·81
Scabiosa arvensis	0·03	0·28	0·62	1·67	0·65
Centaurea nigra	0·01	2·43	2·18	0·53	1·29
Achillea Millefolium	1·33	1·09	1·05	0·16	0·91
Rumex Acetosa	9·15	15·94	7·13	2·13	8·58
Luzula campestris	1·11	0·62	0·15	0·06	0·48
None yielding 1 per cent. { Ranunculus acris, R. repens et bulbosus, Stellaria graminea, Cerastium triviale, Spiræa Ulmaria, Pimpinella Saxifraga, Heracleum Sphondylium, Galium Aparine, Taraxacum officinale, Plantago lanceolata, Veronica Chamædrys, Carex præcox, Hypnum squarrosum, H. rutabulum, H. hians }	0·78	1·10	0·29	0·33	0·63
Absent { Potentilla reptans, Poterium Sanguisorba, Agrimonia Eupatoria, Bellis perennis, Chrysanthemum Leucanthemum, Tragopogon pratensis, Leontodon hispidus, Hieracium sphondylium, Prunella vulgaris, Thymus Serpyllum, Ajuga reptans, Primula veris, Scilla nutans, Ophioglossum vulgatum }	...	...	...	...	...
Total	13·56	27·81	14·84	5·75	15·49
SUMMARY.					
Gramineæ	86·32	71·85	84·70	94·06	84·23
Leguminosæ	0·12	0·34	0·46	0·19	0·28
Other Orders	13·56	27·81	14·84	5·75	15·49
Total	100·00	100·00	100·00	100·00	100·00

of each Species, by 400 lbs. Ammonia-salts alone; Plot 5.

NUMBER OF SPECIES (continued).

Quantity per acre.					Increase + or decrease − compared with plot 3, without manure.				
1862.	1867.	1872.	1877.	Mean.	1862.	1867.	1872.	1877.	Mean.
...	...	..	...	...	− 1	0	− 2	− 4	− 2
...	...	...	...	...	0	0	− 1	− 2	− 1
...	...	...	...	...	−11	−7	−15	−17	−13
...	...	...	...	...	−12	−7	−18	−23	−16

GRAMINEÆ (continued).

Quantity per acre.					Increase + or decrease − compared with plot 3, without manure.				
1862.	1867.	1872.	1877.	Mean.	1862.	1867.	1872.	1877.	Mean.
lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
222·4	182·2	76·4	120·7	150·4	+ 91·8	−106·3	− 9·1	− 0·3	− 6·0
936·5	693·3	669·2	869·7	792·2	+ 589·8	+405·8	+ 403·9	+ 555·8	+488·8
388·5	170·3	47·7	88·9	173·8	+ 234·7	− 95·2	− 11·5	− 207·8	− 20·0
151·4	91·9	37·4	6·8	71·9	+ 149·3	+ 84·9	+ 35·3	+ 5·6	+ 68·8
281·7	20·8	6·0	3·5	78·0	− 12·8	− 81·5	− 52·4	− 60·1	− 51·7
41·2	21·5	15·3	6·8	21·2	+ 32·3	+ 15·8	+ 13·8	+ 5·1	+ 16·8
92·1	46·0	17·6	95·9	62·9	+ 38·4	− 12·0	+ 2·8	+ 79·3	+ 27·1
847·5	1010·6	1170·5	1573·7	1150·5	+ 441·6	+504·1	+ 814·3	+1056·2	+704·0
128·3	40·0	24·3	2·7	48·8	− 66·1	− 94·3	− 14·7	− 104·9	− 70·0
104·2	45·5	41·1	8·0	49·8	− 220·6	−278·1	− 157·0	− 233·5	−222·1
...	...	...	...	...	− 0·3	...	...	− 0·7	− 0·3
133·0	53·3	23·8	...	52·5	− 106·3	−151·2	− 24·9	...	− 70·6
3326·8	2375·4	2129·3	2776·7	2652·1	+1171·8	+192·0	+1000·5	+1094·7	+864·7

LEGUMINOSÆ (continued).

Quantity per acre.					Increase + or decrease − compared with plot 3, without manure.				
1862.	1867.	1872.	1877.	Mean.	1862.	1867.	1872.	1877.	Mean.
0·4	0·3	0·3	...	0·3	− 15·8	− 6·7	− 5·9	− 3·1	− 7·8
1·5	0·3	...	...	0·4	− 135·2	− 70·0	− 27·6	− 49·4	− 70·6
1·9	10·3	10·3	4·1	6·7	− 54·0	− 68·0	− 87·4	− 89·3	− 74·6
0·8	0·3	1·0	1·5	0·9	− 37·6	− 22·4	− 15·1	− 54·5	− 32·4
4·6	11·2	11·6	5·6	8·3	− 242·6	−167·1	− 136·0	− 196·3	−185·4

OTHER ORDERS (continued).

Quantity per acre.					Increase + or decrease − compared with plot 3, without manure.				
1862.	1867.	1872.	1877.	Mean.	1862.	1867.	1872.	1877.	Mean.
44·3	189·7	25·6	19·2	69·7	+ 14·7	+ 91·4	− 21·2	− 25·7	+ 14·8
...	20·2	60·3	6·5	21·6	...	+ 20·2	+ 60·3	+ 6·5	+ 21·6
1·2	9·3	15·6	49·3	18·8	+ 0·9	+ 9·3	+ 15·3	+ 46·5	+ 18·0
0·4	80·3	54·8	15·6	37·7	− 9·1	+ 60·6	+ 20·2	− 9·5	+ 15·5
51·2	36·0	26·4	4·7	29·5	+ 4·5	− 2·7	− 2·9	− 42·3	− 10·9
352·6	527·0	179·2	62·9	280·4	+ 309·9	+468·3	+ 150·1	+ 18·7	+236·7
42·8	20·5	3·8	1·8	17·2	− 15·5	− 99·8	− 52·4	− 40·0	− 51·9
30·1	36·4	7·4	9·7	21·3	− 428·6	−567·0	− 133·7	− 202·6	−332·9
...	...	...	...	...	− 4·0	− 31·2	− 30·2	− 62·0	− 32·0
522·6	919·4	373·1	169·7	496·2	− 127·2	− 50·9	+ 5·5	− 310·4	−121·2

SUMMARY (continued).

Quantity per acre.					Increase + or decrease − compared with plot 3, without manure.				
1862.	1867.	1872.	1877.	Mean.	1862.	1867.	1872.	1877.	Mean.
3326·8	2375·4	2129·3	2776·7	2652·1	+1171·8	+192·0	+1000·5	+1094·7	+864·7
4·6	11·2	11·6	5·6	8·3	− 242·6	−167·1	− 136·0	− 196·3	−185·4
522·6	919·4	373·1	169·7	496·2	− 127·2	− 50·9	+ 5·5	− 310·4	−121·2
3853	3306	2514	2952	3157	+ 802	− 26	+ 865	+ 588	−558

It will thus be seen that the botany of the plot has varied exceedingly at the different periods. In addition to the influence of the manure, this has been due in part to the character of the intermediate seasons, and of their vegetation, and in part to the characters of the seasons of growth themselves. In further illustration of this, it will be observed that, whilst there was a reduction in the total weight of gramineous herbage per acre from the first to the third separation-year, there was much more than the average weight of such herbage in the fourth. With regard to the Miscellaneæ again, whilst the first of the four years gave more than 500 lbs., the second contributed more than 900 lbs., the third less than 400 lbs., and the fourth less than 170 lbs.

Notwithstanding these great fluctuations in the general characters of the herbage according to season, an examination of the details relating to the proportion, and the amount, of the individual species, not only further illustrates the point, but shows clearly a progressive change referable to the condition of manuring.

It may here be premised that, according to the separations of 1858, *Lolium perenne* and *Holcus lanatus* were then very prominent grasses, *Festuca ovina* being about equally abundant; *Agrostis vulgaris, Dactylis glomerata,* and *Avena elatior* were also prominent, but in a less degree. Perhaps the most characteristic feature is the very great increase in the percentage of *Festuca ovina,* and notwithstanding the reduced amount of total produce, a gradually increasing quantity per acre of this individual poor grass from year to year. The percentage has increased from under 22 in the first, to more than 53 in the fourth separation-year; and the produce of it per acre from under 850 lbs. in 1862, to nearly 1,600 lbs. in 1877.

Next in order of prominence is *Agrostis vulgaris,* which, excepting in the second year (1867), and in a much less degree than *Festuca ovina,* has also gradually increased in percentage proportion; but which, with diminishing yield of total produce per acre on the plot, has not itself given an uniformly increased actual amount in the later years.

With the exception of these two inferior grasses, the *Festuca* maintaining itself in competition better under dry, and the *Agrostis* under wet conditions, every other grass, and it may also be said every other plant, has diminished, both in percentage, and amount per acre, under the peculiar conditions of manuring of this plot. *Holcus lanatus,* one of the most prominent of the grasses in 1858, has gone down from about 10 per cent. in the first, to only about 3 per cent. in the fourth separation-year; *Avena pubescens* from more than 7 to a fraction of 1 per cent.; *Avena elatior* from nearly 4, also to a fraction of 1 per cent.; and *Lolium perenne,* which was first in order of prominence in 1858, has gone down from more than 3 per cent. to practically nothing in the last separation-year. *Anthoxanthum odoratum* has also declined, but in a less degree; whilst *Dactylis glomerata,* which was in each of the four years in comparatively small amount, had somewhat gained ground in the fourth. The decrease in the actual yield per acre of these grasses is more marked than is that of their percentage. For example, *Holcus lanatus* has gone down from a yield of nearly 400 lbs. to less than 90 lbs. per acre.; *Avena pubescens* from nearly 300 lbs. to little more than 3 lbs.; *Avena elatior* from more than 150 lbs. to less than 7 lbs.; and *Lolium perenne* from nearly 130 lbs. to less than 3 lbs.; whilst even *Anthoxanthum odoratum* is reduced in amount from 222 lbs. per acre in 1862, to scarcely 121 lbs. in 1877; and *Dactylis glomerata*, though yielding rather more in the fourth than in the first separation-year, has in neither case contributed 100 lbs.

Of the four Leguminosæ, not one is in any degree prominent; but it is noteworthy that the only one which has shown any appreciable tendency to increase is the deep-rooting, self-dependent, *Lotus corniculatus.*

Of Miscellaneous species, the only one in any prominence is *Rumex Acetosa;* but, excepting that it was in much larger proportion and amount in the second separation-year (1867) than in the first, it also has in the main much declined, yielding more than 9 per cent. in the first, and little more than 2 per cent. in the fourth separation-year; and, in actual weight, more than 350 lbs. per acre in the first, and scarcely 63 lbs. in the fourth separation-year. *Achillea Millefolium* has declined from 1·33 in the first, to 0·16 in the fourth year, and *Luzula campestris* from more than 1 to a very small fraction. *Conopodium denudatum* was in small proportion, and amount, and also declining, though, as in the case of *Rumex Acetosa*, it increased remarkably in the second separation-year (1867), when, from the characters of the season, the growth of the Gramineæ was so restricted. The only miscellaneous species which uniformly increased, though still occurring in insignificant amount, was *Scabiosa arvensis.*

The table shows that a large number of miscellaneous species, which occurred in moderate amounts on the unmanured plot, did not collectively contribute 1 per cent. to the herbage of this plot, and about an equal number were not represented at all.

Compared with the produce without manure, the columns on the right-hand of the table show that, taking the average of the four years, there was an increase of about 865 lbs. per acre per annum of gramineous herbage; more than this amount in the first, third, and fourth years, but less than one-fourth as much in the second separation-year. In each of the four years there was actually less leguminous herbage, and on the average 185 lbs. less on the manured plot, 5, than on the unmanured plot, 3. That is to say, the leguminous species, which are characterised by containing a very high percentage of nitrogen, developed and yielded actually less under the influence of highly nitrogenous manure than they did without manure continuously.

Of the collective miscellaneous herbage again, the members of which are also generally richer in nitrogen than the Gramineæ, there was each year, with one slight exception, less on this manured plot than on the unmanured one; and, taking the average of the four years, there was 121 lbs. less. The actual quantity of *Rumex Acetosa* was, however, considerably more on the manured plot, and that of some other species slightly more; but the balance against the manured plot was due to a large number of species that were represented on the unmanured plot occurring either not at all, or in very small quantity, on the manured one. Among those which were comparatively prominent on the unmanured, but which were scarcely represented on the manured plot, 5, *Plantago lanceolata* especially, and the several species of *Ranunculus*, may be noted.

Upon the whole, the result of the annual application of a relative excess of ammonia-salts, without mineral or ash-constituents, the plants having thus to rely exclusively on the resources of the soil for these, is greatly to reduce the total number of species, especially of the Miscellaneæ, and to reduce the actual quantity grown, not only of the collective Miscellaneæ, but of every individual species occurring in any degree of prominence. Leguminosæ are almost banished; and although the number of species of grasses occurring in greater or less amount is nearly the same as without manure, by far the larger proportion of the whole produce consists of two grasses only—*Festuca ovina* and *Agrostis vulgaris.* These two have increased in both proportion and amount, whilst all others have diminished in a greater or less degree; and of these two, which have increased, the poorer *Festuca ovina*, has done so in by far the larger proportion, contributing in the fourth separation-year more than half the total produce, whilst *Agrostis vulgaris* supplied nearly 30 per cent. of it.

Thus, with a supply of nitrogen in the form of ammonia-salts, in which it distributes in an available condition much less rapidly and widely than when applied as nitrate of soda, the herbage has come to consist almost exclusively of a very few comparatively superficially rooting species, and the result was, as has been already referred to, and as will be illustrated in detail in Part III., that the collective herbage was able to take up even less of some of the mineral constituents in the later years than that without manure, which comprised a very much more uniform mixture of a large number of species, of varying habits of growth, and of varying range of food collection.

3. *Nitrate of soda, alone; Plots* 15 *and* 17.

The application of nitrate of soda did not commence until the third year of the experiments (1858). For 18 consecutive years, 1858 to 1875 inclusive, plot 15 received 550 lbs. of nitrate of soda per acre per annum, estimated to contain the same amount of nitrogen (about 86 lbs.) as the ammonia-salts applied to plot 5. Over the same period of 18 years, plot 17 received half the amount of nitrate, as it was then found that, with the larger amount of nitrate, there was scarcely any more increase than with the smaller, and that the proportion of the nitrogen supplied which was recovered in the increase of crop was much less where the larger amount was used; the application of nitrate to that plot was discontinued after 1875, the eighteenth year, and in, and since, 1876, a mixed mineral manure, including potass, has been applied instead. The object of this was, first, to determine whether or not, under these circumstances, any material proportion of the hitherto unrecovered supplied nitrogen would now be recovered; and, secondly, to ascertain how far the character of the vegetation in regard to the distribution of species would revert to its former condition. Thus, the fourth separation-year (1877) is the second year of the application of the mineral manure instead of the nitrate on plot 15. On plot 17, however, where the smaller quantity of nitrate was employed from the commencement, no change has been made in the manuring up to the present time.

As in the case of plot 5, the following table (Table LXVI., pp. 1312–13) gives for plot 15 the particulars of the per cent., and of the actual amount yielded, of each species among the grasses and the Miscellaneæ, which contributed in any one year more than 1 per cent. to the produce, the results for those species yielding less than this being given collectively; whilst those occurring on the unmanured plot, but not on the manured one at all, are again enumerated and bracketed together. As before, for the few species of Leguminosæ, the full details are given. But, as there was a fundamental change of manure between the third and fourth separation-years, the mean for the first three separation-years is given, and the results for the fourth (1877) separately, instead of the mean for the four years.

It should be observed that much more increase of produce was obtained on plot 15 by nitrate of soda than on plot 5 by ammonia-salts containing the same amount of nitrogen. The reduction in produce was much less in the later years by the nitrate alone than by the ammonia-salts alone; much more nitrogen was yielded in the produce by the nitrate, and the decline in the yield of it was much less; one and a-half time as much mineral matter was taken up by the herbage on the nitrate plot, including, besides soda which was supplied in the manure, more lime, more magnesia, and much more potass, phosphoric acid, and silica; all of which must have been derived from the stores of the soil itself. Lastly, the decline in the amount of mineral matter taken up in the later years was less than with the ammonia-salts.

For an explanation of the much greater result in almost every particular of production with the nitrate of soda than with ammonia-salts containing the same quantity of nitrogen, we must look to the difference in the condition and distribution of the nitrogen within the soil, and to the coincident great difference in the flora in the two cases. The nitrogen of the nitrate distributes much more rapidly than does that of the ammonia-salts; much of the ammonia being probably in the first instance retained as such in the upper layers of the soil, and only gradually oxidating and distributing as nitrites or nitrates. The result is that many more species are favoured by the nitrate; a greater variety of grasses contribute to the bulk of the produce; and leguminous, and especially miscellaneous species, are more favoured. Accordingly, there is a much greater variety of "habit" developed under the influence of the nitrate, and a much more extended range of soil is commanded by the roots of the more varied herbage. Hence much more mineral matter is derived from the stores of the soil, and the different plants, especially the grasses, yield much more stem and mature much better. Indeed, as will be seen, some of the most deeply-penetrating grasses flourish under these conditions.

TABLE LXVI.—Number of Species, Percentage, Quantity per Acre, &c.,

	Number of species, and proportion per cent.					Quantity per acre.				
	1862.	1867.	1872.	Mean.	1877.	1862.	1867.	1872.	Mean.	1877.
NUMBER OF SPECIES.										
Gramineæ	17	16	17	17	15	...	...	...	...	...
Leguminosæ	4	4	3	4	4	...	...	...	...	...
Other Orders	18	19	19	18	24	...	...	...	...	...
Total	39	39	39	39	43	...	...	...	...	...
GRAMINEÆ.										
	p. c.	p. c.	p. c.	p. c.	p. c.	lbs.	lbs.	lbs.	lbs.	lbs.
Anthoxanthum odoratum	1·82	1·83	4·49	2·71	4·16	74·4	96·7	164·3	111·8	157·4
Alopecurus pratensis	6·90	5·95	2·46	5·10	7·17	281·9	314·5	90·0	228·8	271·3
Agrostis vulgaris	7·65	6·86	7·66	7·39	12·90	312·6	362·6	280·3	318·5	488·1
Holcus lanatus	7·61	11·81	5·32	8·25	14·95	311·0	624·3	194·6	376·6	565·7
Avena pubescens	3·53	0·70	1·56	1·93	3·13	144·2	37·0	57·1	79·4	118·4
Avena flavescens	3·86	4·26	3·83	3·98	2·98	157·7	225·2	140·1	174·3	112·8
Poa trivialis	6·53	23·67	7·95	12·72	6·05	266·8	1251·2	290·9	603·0	228·9
Dactylis glomerata	2·09	0·21	0·11	0·80	0·36	85·4	11·1	4·0	33·5	13·6
Festuca ovina	13·69	12·08	34·71	20·16	20·77	559·3	638·5	1270·0	822·6	786·0
Festuca pratensis	0·03	0·27	0·63	0·31	1·47	1·2	14·3	23·1	12·9	55·6
Bromus mollis	2·12	6·27	4·00	4·13	1·65	86·6	331·4	146·4	188·2	62·4
Lolium perenne	7·49	3·24	4·42	5·05	7·32	306·1	171·3	161·7	213·1	277·0
None yielding 1 per cent. { Phleum pratense, Aira cæspitosa, Poa pratensis, Briza media, Cynosurus cristatus, Avena elatior, Festuca pratensis }	0·37	0·36	0·66	0·47	0·54	15·1	19·1	24·2	19·4	20·5
Absent—Festuca loliacea	...	...	...	...	...	...	...	...	...	...
Undetermined (chiefly Gramineæ)	14·57	2·51	0·96	6·01	...	595·4	132·7	35·1	254·4	...
Total	78·26	80·02	78·76	79·01	83·45	3197·7	4229·9	2881·8	3436·5	3157·7
LEGUMINOSÆ.										
Trifolium repens	0·04	0·08	0·06	0·06	0·01	1·6	4·2	2·2	2·7	0·4
Trifolium pratense	0·20	0·04	0·03	0·09	0·31	8·2	2·1	1·1	3·8	11·7
Trifolium procumbens	0·01	...	...	0·01	...	0·4	...	...	0·1	...
Lotus corniculatus	0·02	0·35	0·03	0·13	0·01	0·8	18·5	1·1	6·8	0·4
Lathyrus pratensis	...	0·04	...	0·01	1·47	...	2·1	...	0·7	55·6
Total	0·27	0·51	0·12	0·30	1·80	11·0	26·9	4·4	14·1	68·1
OTHER ORDERS.										
Ranunculus acris	0·46	0·25	1·24	0·65	3·74	18·8	13·2	45·4	25·8	141·5
Ranunculus repens et bulbosus	1·80	0·39	0·39	0·86	0·49	73·6	20·6	14·3	36·2	18·5
Cerastium triviale	1·07	3·68	9·26	4·67	0·59	43·7	194·5	338·8	192·4	22·3
Galium verum	...	1·06	1·21	0·76	0·23	...	56·1	44·3	33·5	8·7
Centaurea nigra	...	0·17	2·58	0·92	0·90	...	9·0	94·4	34·5	34·0
Achillea Millefolium	2·53	1·13	2·60	2·08	0·58	103·4	59·7	95·1	86·1	22·0
Taraxacum officinale	1·20	0·11	0·10	0·47	0·08	49·0	5·8	3·7	19·5	3·0
Plantago lanceolata	6·92	4·67	0·28	3·95	0·56	282·8	246·9	10·3	180·0	21·2
Rumex Acetosa	6·64	7·34	2·06	5·34	5·79	271·3	388·0	75·4	244·9	219·1
None yielding 1 per cent. { Stellaria graminea, Potentilla reptans, Conopodium denudatum, Pimpinella Saxifraga, Heracleum Sphondylium, Galium Aparine, Scabiosa arvensis, Leontodon hispidus, L. autumnalis, Veronica Chamædrys, V. serpyllifolia, Ajuga reptans, Scilla nutans, Luzula campestris, Carex præcox, Hypnum squarrosum, H. rutabulum, H. hians }	0·85	0·67	1·40	0·99	1·79	34·7	35·4	51·1	40·2	67·9
Absent { Agrimonia Eupatoria, Poterium Sanguisorba, Spiræa Ulmaria, Bellis perennis, Chrysanthemum Leucanthemum, Tragopogon pratensis, Hieracium Pilosella, Prunella vulgaris, Thymus Serpyllum, Primula veris, Ophioglossum vulgatum }	...	...	...	...	...	...	...	...	...	...
Total	21·47	19·47	21·12	20·69	14·75	877·3	1029·2	772·8	893·1	558·2
SUMMARY.										
Gramineæ	78·26	80·02	78·76	79·01	83·45	3197·7	4229·7	2881·8	3436·5	3157·7
Leguminosæ	0·27	0·51	0·12	0·30	1·80	11·0	26·9	4·4	14·1	68·1
Other Orders	21·47	19·47	21·12	20·69	14·75	877·3	1029·2	772·8	893·1	558·2
Total	100·00	100·00	100·00	100·00	100·00	4086	5826	3659	4344	3784

of each Species, by 550 lbs., Nitrate of Soda alone; Plot 15.

Increase + or decrease − compared with:—									
Plot 3, without manure.					Plot 5, ammonia-salts alone.				
1862.	1867.	1872.	Mean.	1877.	1862.	1867.	1872.	Mean.	1877.
Number of Species (continued).									
− 1	+1	0	0	−2	0	+1	+2	+1	+ 2
0	0	− 1	0	0	0	0	0	0	+ 2
−10	−5	− 9	−9	−7	+1	+2	+6	+3	+10
−11	−4	−10	−9	−9	+1	+3	+8	+4	+14
Gramineæ (continued).									
lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
− 56·2	− 191·8	+ 78·8	− 56·4	+ 36·4	−148·0	− 85·5	+ 87·9	− 48·5	+ 36·7
+ 144·9	+ 120·6	+ 81·5	+ 115·7	+ 264·2	+256·9	+ 299·0	+ 69·2	+ 208·4	+264·5
− 34·1	+ 75·1	+ 15·0	+ 18·7	+ 174·2	−623·9	− 330·7	− 388·9	− 447·9	−381·6
+ 157·2	+ 358·8	+ 135·4	+ 217·1	+ 269·0	− 77·5	+ 454·0	+ 146·9	+ 174·4	+476·8
− 150·3	− 65·3	− 1·3	− 72 3	+ 54·8	−137·5	+ 16·2	+ 51·1	− 23·5	+114·9
+ 85·3	+ 163·2	+ 82·7	+ 110·4	+ 87·3	+132·7	+ 210·0	+ 135·5	+ 159·4	+112·5
+ 219·8	+1212·2	+ 282·7	+ 571·6	+ 215·7	+232·3	+1241·3	+ 276·1	+ 583·3	+228·6
+ 31·7	− 46·9	− 10·8	− 8·7	− 3·0	− 6·7	− 34·9	− 13·6	− 18·4	− 82·3
+ 153·4	+ 132·0	+ 913·8	+ 399·8	+ 268·5	−288·2	− 372·1	+ 99·5	− 187·0	−787·7
...	+ 14·3	+ 22·9	+ 12·4	+ 55·4	− 12·3	+ 10·0	+ 22·6	+ 6·8	+ 55·6
+ 82·6	+ 329·7	+ 146·2	+ 186·2	+ 62·4	+ 83·5	+ 331·4	+ 146·4	+ 187·2	+ 62·4
+ 111·7	+ 37·0	+ 122·7	+ 90·5	+ 169·4	+177·8	+ 131·3	+ 137·4	+ 148·9	+274·3
− 59·4	− 20·6	− 103·0	− 61·1	− 177·9	−180·6	− 94·9	− 28·9	− 101·5	+ 6·3
...	...	...	...	− 0·7	...	...	...	...	...
+ 356·1	− 71·8	− 13·6	+ 90·2	...	+462·4	+ 79·4	+ 11·3	+ 184·3	...
+1042·7	+2046·5	+1753·0	+1614·1	+1475·7	−129·1	+1854·5	+ 752·5	+ 826·0	+381·0
Leguminosæ (continued).									
− 14·6	− 2·8	− 4·0	− 7·1	− 2·7	+ 1·2	+ 3·9	+ 1·9	+ 2·4	+ 0·4
− 128·5	− 68·2	− 26·5	− 74·4	− 37·7	+ 6·7	+ 1·8	+ 1·1	+ 3·2	+ 11·7
+ 0·4	...	...	+ 0·1	...	+ 0·4	...	...	+ 0·1	...
− 55·1	− 59·8	− 96·6	− 70·5	− 93·0	− 1·1	+ 8·2	− 9·2	− 0·7	− 3·7
− 38·4	− 20·6	− 16·1	− 25·0	− 0·4	− 0·8	+ 1·8	− 1·0	...	+ 54·1
− 236·2	− 151·4	− 143·2	− 176·9	− 133·8	+ 6·4	+ 15·7	− 7·2	+ 5·0	+ 62·5
Other Orders (continued).									
+ 18·5	+ 12·9	+ 44·2	+ 25·2	+ 141·5	+ 18·4	+ 12·5	+ 44·6	+ 25·2	+139·4
− 75·3	− 46·4	− 34·0	− 51·9	− 63·1	+ 61·3	+ 18·0	+ 12·3	+ 30·6	+ 15·9
+ 29·6	+ 182·3	+ 324·8	+ 179 0	+ 2·9	+ 43·3	+ 191·5	+ 338·8	+ 191·3	+ 22·3
...	+ 56·1	+ 44·3	+ 33·5	+ 8·7	...	+ 35·9	− 16·0	+ 6·7	+ 2·2
− 9·5	− 10·7	+ 59·8	+ 13·2	+ 8·9	− 0·4	− 71·3	+ 39·6	− 10·7	+ 18·4
+ 56·7	+ 21·0	+ 65·8	+ 47·9	− 25·0	+ 52·2	+ 23·7	+ 68·7	+ 48·2	+ 17·3
+ 47·5	+ 0·4	+ 1·2	+ 16·4	+ 1·6	+ 48·6	+ 5·5	+ 3·2	+ 19·1	+ 3·0
+ 58·8	− 110·6	− 33·4	− 28·4	− 53·5	+279·0	+ 246·2	+ 10·3	+ 178·5	+ 21·2
+ 228·6	+ 329·3	+ 46·3	+ 201·4	+ 174·9	− 81·3	− 139·0	− 103·8	− 108·1	+156·2
− 125·3	− 365·8	− 104·7	− 198·7	− 88·9	− 66·4	− 213·2	+ 2·0	− 92·7	− 4·8
− 2·1	− 9·6	− 9·1	− 7·1	− 29·9	...	...	...	...	− 2·6
+ 227·5	+ 58·9	+ 405·2	+ 230·5	+ 78·1	+354·7	+ 109·8	+ 399·7	+ 288·1	+388·5
Summary (continued).									
+1042·7	+2046·5	+1753·0	+1614 1	+1475·7	−129·1	+1854·5	+ 752·5	+ 826·0	+381·0
− 236·2	− 151·4	− 143·2	− 176·9	− 133·8	+ 6·4	+ 15·7	− 7·2	+ 5·0	+ 62·5
+ 227·5	+ 58·9	+ 405·2	+ 230·5	+ 78·1	+354·7	+ 109·8	+ 399·7	+ 288·1	+388·5
+1034	+1954	+2015	+1668	+1420	+232	+1980	+1145	+1119	+832

Whilst on plot 5, with ammonia-salts, the number of species found in the samples declined in the first three separation-years from 38 to 36 and 31, on plot 15, with the nitrate of soda, it was 39 in each of the three years; and whilst it declined to 29 in the fourth separation-year with ammonia-salts, it rose to 43 on plot 15 after the mixed mineral manure had been substituted for the nitrate.

The table shows that there was much less fluctuation, and a lower mean percentage of gramineous herbage, on plot 15 than on plot 5; the highest amount was little over 80 per cent., the fact being that a much larger number of miscellaneous species contributed their share to the yield.

A glance at the figures more in detail brings to view the fact that no one grass contributed such an overwhelming percentage as on plot 5, and that a much larger number yielded a fair proportion. Here again, however, *Festuca ovina* furnished in the third separation-year rather more than one-third of the whole; but in neither year did *Agrostis vulgaris* contribute an excessive proportion as it did on plot 5. The most prominent among the other grasses were—*Poa trivialis* and *Holcus lanatus*, both of which increased in the second year (1867), *Poa trivialis* very considerably, although it diminished in the third year (1872); *Alopecurus pratensis* and *Lolium perenne*, both of which however declined; *Avena flavescens* in smaller but comparatively uniform percentage; and *Anthoxanthum* and *Bromus mollis* which increased. Lastly, those which were in fair proportion in the first year, but which were reduced very considerably subsequently, were *Avena pubescens* and *Dactylis glomerata*. The mere length of this enumeration is sufficient to show how many more grasses were enabled to maintain or improve their position in the struggle under the difference of condition, and more extended distribution, of the manurial agent within the soil, the correspondingly increased root-range, and the greater command of the mineral food of the soil and subsoil. It is due to the same causes that the grasses showed a much greater tendency to the formation of stem; and an examination of the right hand columns of the table, in which the results are compared with those relating to plot 5 with the ammonia-salts, will show that several species of freer growth, and higher agricultural repute, were brought into greater prominence.

As with the ammonia-salts so with the nitrate of soda, the highly nitrogenous Leguminosæ were much discouraged, but in a somewhat less degree.

Of miscellaneous plants one only, *Rumex Acetosa*, was really prominent on plot 5 with the ammonia-salts, *Conopodium denudatum* coming second, and *Centaurea nigra* developed more occasionally. On plot 15, with the nitrate of soda, both *Rumex Acetosa* and *Plantago lanceolata* were prominent in the first and second separation-years; but each went down considerably in the third; *Conopodium denudatum*, which, as already said, was the second in prominence with the ammonia-salts, is in quite insignificant amount with the nitrate of soda; *Centaurea nigra* is also less prominent with the nitrate. The plant which has developed in the most striking degree under the influence of the nitrate is *Cerastium triviale*, which increased from little over

1 per cent. in the first, to nearly 4 in the second, and to more than 9 in the third separation-year; contributing in that season not far from half of the total miscellaneous herbage. Lastly, whilst *Ranunculus repens* and *R. bulbosus* have diminished, the more deeply-rooting *Ranunculus acris* has increased.

Thus, as is the case with the gramineous herbage so with the miscellaneous, it is very much more mixed under the influence of the nitrate than under that of the ammonia-salts.

Looking to the columns of produce, and of increase, per acre over the unmanured of each group, or of each species of plants, it is observed that there was a very great increase in the total gramineous herbage—nearly twice as great in the second year as in the first, but somewhat less in the third than in the second. In the first year there was even less of such increase than with the ammonia-salts (plot 5), but in the second year there was very much more, and in the third considerably more by the nitrate than by the ammonia.

Of leguminous herbage there was actually much less each year with the nitrate alone than in the same years without manure, and on the average there was scarcely more of such produce than with the ammonia-salts.

Of total miscellaneous herbage there was actually more in the second than in the first or third year, but owing to the very large amount without manure also, in the second year, there was much less increase by the nitrate in that year than in either the first or the third; indeed, very little. There was also a larger actual quantity of total miscellaneous herbage with the nitrate than with the ammonia-salts, the excess being, however (notwithstanding the large actual quantity), considerably less in the second year (1867).

Referring to the actual yield, that is to the quantity per acre, of the individual species among the grasses, *Festuca ovina* contributes by far the largest amount—more than without manure, and in the third year very much more; less, however, in the first and second years than by ammonia-salts. *Agrostis vulgaris,* which contributed so much on the ammonia plot, yielded very much less with the nitrate. Next in order of actual amount to *Festuca ovina* comes *Poa trivialis,* nearly the whole of the produce of which was, each year, in excess of the amounts of it on either the unmanured plot 3, or the ammonia plot 5, on both of which it occurred in very insignificant quantity; whilst it is to be observed that the nearly allied *Poa pratensis* was scarcely represented on the nitrate plot at all. *Holcus lanatus* gave on the average a fair yield, but much more in the second than in the first or third year; considerably more in each year than on the unmanured plot, and in the second year much more, and in the third more, than on the ammonia plot 5. *Alopecurus pratensis, Avena flavescens,* and *Lolium perenne* also contributed notably to the produce, and each of them materially more than either without manure or with ammonia-salts. In a word, the gramineous herbage was considerably more in actual amount, and very much more mixed than under the influence of the ammonia-salts. It may, however, be noted that

Phleum pratense, *Avena elatior*, *Cynosurus cristatus*, *Briza media*, and also *Poa pratensis*, were in very insignificant quantities.

The yield per acre of leguminous herbage was in every case insignificant, but it was greater in the second than in either the first or the third year, and the diminution in the total yield of such herbage compared with that of the unmanured plot was due to a decrease in each of the four species—*Trifolium repens*, *T. pratense*, *Lotus corniculatus*, and *Lathyrus pratensis*. On the nitrate plot, however, *Trifolium procumbens* occurred, but in very small amount, and it was not observed on either the unmanured or the ammonia plot, although the plant is not uncommon in the neighbourhood.

Of miscellaneous herbage *Rumex Acetosa* contributed the largest actual amount, much more than without manure, but in each year considerably less than with ammonia-salts. *Plantago lanceolata* also contributed largely in the first and second separation-years, but very little in the third; indeed, less in both the second and third years than without manure, but in the first and second years considerably more, and in the third somewhat more than with ammonia-salts. As already observed, the only miscellaneous plant which increased in a marked manner on this nitrate plot was *Cerastium triviale*, which contributed in the third year nearly half the total miscellaneous herbage, and in each year almost the whole was in excess of that yielded either without manure or with ammonia-salts. The great abundance of *Cerastium* may perhaps be partially accounted for, amongst other characteristics, by its early and free seeding, as already referred to. *Centaurea nigra* yielded practically nothing in the first and second separation-years, and less in those years on this plot than either without manure or with ammonia-salts, but it gave a fair amount in the third year, and then more than either without manure or with ammonia-salts. It should be borne in mind, however, that from the size and character of the plant there is some difficulty in securing a fair proportion of it in the samples. *Achillea Millefolium* perhaps came next in order of yield to *Cerastium*, not indeed contributing much, but still considerably more than without manure or with ammonia-salts. Then come the various species of *Ranunculus* collectively, only the more deeply-rooting *R. acris* increasing, and giving more each year than either without manure or with ammonia-salts, while the generally shallower rooting *R. repens* and *R. bulbosus* decline in yield, and give much less than without manure, though still more than with ammonia-salts. The only other plant in any noticeable quantity was *Galium verum*, which was practically unrepresented on the unmanured plot, but which in the third separation-year was even in larger amount on the ammonia than on the nitrate plot. Like *Cerastium triviale*, *Galium* is a plant of relatively meagre development, rooting chiefly at, or at a short distance beneath, the surface.

We have then, under the influence of excessive application of nitrate of soda alone, a very much more uniformly mixed herbage than with an amount of ammonia-salts containing the same quantity of nitrogen. There is not only much more produce, both of Gramineæ and of Miscellaneæ, but there is a less excessive proportion of the grasses, and a greater number, both of them and of the miscellaneous species, contribute a fair proportion to the mixed herbage. Of the grasses, several of freer habit of growth, and of better character, are fairly represented, whilst among the Miscellaneæ there is not the almost exclusive predominance of *Rumex Acetosa* as was the case with the ammonia-salts, other plants, both of superficial and deeper foraging tendencies, finding a due place.

During the 18 years' application of the nitrate of soda the plot exhibited a very different general aspect from that of the ammonia plot. The herbage of the ammonia plot was almost exclusively leafy, and extremely dark green, indicating, as is known, a very high percentage of nitrogen in the dry substance, and, it is to be supposed, a liberal formation of chlorophyll; yet, as has been seen, there was extremely restricted vegetation (carbon assimilation), and scarcely any tendency to maturation. The nitrate plot (15) showed somewhat similar characters, but in a very much less degree; the herbage was much more characteristically leafy than where mineral manures are employed in conjunction with the nitrate, but there was much more luxuriance, much more tendency to form stem, and a much lighter and healthier colour than with the ammonia. Though more nitrogen was taken up, there was a less percentage of it in the dry substance of the produce. In other words, for a given amount of nitrogen taken up there had been much more carbon assimilation, that is to say, much more growth, due doubtless in great part to the greater supply of mineral constituents derived from the soil by the more varied and more deeply-rooting herbage.

We have now to call attention to the changes induced in the botanical composition of plot 15 by the cessation of the application of the nitrate, and the substitution of a mixed mineral manure, including potass. Although 1877, the fourth year of separation, was only the second after the change of manuring, the table shows that a considerable change in the flora was then already indicated, and it has been much more marked in subsequent years, as has been shown by the partial separations which have since been undertaken.

Referring first to the results for 1877, as given in the table, perhaps the most marked feature is the tendency already developed to increase in the Leguminosæ, and especially in *Lathyrus pratensis*, which in subsequent years continued to increase in a rapid ratio.

There was, after the change, so much fluctuation in the proportion to one another of the grasses and the Miscellaneæ, according to season, that it is difficult to say with regard to either group as a whole, whether the tendency was to increase or decrease as the direct effect of the altered condition of manuring.

It will be remembered that, with the ammonia-salts alone, *Festuca ovina* and *Agrostis vulgaris* almost entirely displaced the other grasses; and that, with the nitrate of soda alone, *Festuca ovina* also gained ground, and was by far the most prominent of the gramineous species, whilst *Agrostis vulgaris* was in very much less amount, a number of other species contributing a due proportion. The table shows that in the second year only (1877) of the substitution of the mixed mineral manure (including potass) for the nitrate, *Festuca ovina* had already considerably declined, though still remaining the most prominent species; *Agrostis vulgaris*, on the other hand, had considerably gained ground, both in percentage and in actual yield per acre. Of other grasses, *Holcus lanatus* increased very much, both in percentage and in actual quantity, under the influence of the change to mineral manure, and *Alopecurus pratensis*, *Avena pubescens*, and *Lolium perenne* did so in a less degree; *Poa trivialis*, *Avena flavescens*, and *Bromus mollis*, however, declined.

It should be observed that the difference in proportion and amount of the different grasses indicated by the figures by no means represents the whole of the change in the herbage, resulting from the change of manure; for, under the influence of the mineral manure, there was a much lighter colour, and a much greater tendency to form stem and to mature, than under the influence of the nitrate, and the question arises whether it may not be owing to this different tendency of growth induced by the manure that certain species have yielded a larger proportion and amount.

Referring to the Leguminosæ, *Lathyrus pratensis*, as already intimated, showed a tendency to considerable increase, and it has done so in a very marked degree since. It will be seen further on, that this result is quite consistent with that obtained where mixed mineral manure, including potass, has been applied every year from the commencement.

Among the Miscellaneous species, perhaps the most marked change resulting from the application of the mineral manure instead of the nitrate is the great increase in *Ranunculus acris*, a recurrence of a large proportion and amount of *Rumex Acetosa*, and a very great diminution in *Cerastium triviale*, which had, it will be remembered, become so remarkably prominent under the influence of the nitrate. Of other species, all were more or less discouraged, *Achillea Millefolium* perhaps the most conspicuously so.

We now turn to the results obtained on plot 17, receiving only half as much nitrate of soda as plot 15; and, therefore, only half as much nitrogen, and only half as much as in the ammonia-salts supplied to plot 5.

Before going into the detail of the botany of the plot, it is necessary to call to mind some of the particulars of production compared with those on plots 5 and 15 respectively. There was, with the smaller quantity of nitrogen as nitrate, much more produce than with double the quantity as ammonia-salts, and very little less than with double the quantity as nitrate of soda. There was much more nitrogen taken up in the produce than with the double quantity as ammonia, and though a less actual amount than with the double quantity as nitrate, a larger proportion of that supplied was recovered in the increase. Much more mineral matter was taken up than with the ammonia, nearly as much as with the double nitrate, and even more of lime, potass, and silica, which were, of course, derived from the soil itself.

This brief enumeration of general characters of produce is sufficient to indicate that there was a difference in the flora also, as Table LXVII., pp. 1320–21, clearly illustrates.

An examination of the figures in the table shows that, as compared with double the quantity of nitrogen as ammonia-salts, the increase obtained by the smaller quantity of nitrate of soda was chiefly in the Gramineæ, but considerable also in the Leguminosæ and in the miscellaneous herbage. As compared with the results obtained with the double quantity of nitrate, the small total deficiency was chiefly in the grasses, whilst there was, on the average, an increase in the amount of both the leguminous and the miscellaneous species.

The columns of percentage of the individual species show at a glance much less excess of any single plant—in fact, a much more generally mixed herbage than with the double amount of nitrogen, whether as ammonia-salts or as nitrate of soda. Now, neither *Festuca ovina* nor *Agrostis vulgaris* takes the first place over the average of the four separation-years; *Alopecurus pratensis* is here, on the average, the most prominent plant, though, it is true, declining from the first to the fourth separation-year. *Agrostis vulgaris* and *Festuca ovina* come next in average proportion, each varying considerably in prominence, but the two somewhat in opposite directions, from year to year. An examination of the climatal characters of the seasons of separation, and of the intervening ones, will show that the fluctuations in the relative prevalence of these two plants were such as might be expected, the freer-growing, creeping, and moisture-loving *Agrostis* being the more prominent in the years of comparatively luxuriant growth, and *Festuca ovina* in those of generally more restricted gramineous luxuriance. Next in order to these, the three most prominent of the grasses on plot 17, come *Holcus lanatus, Lolium perenne, Anthoxanthum odoratum,* and *Avena pubescens,* each of which contributed a fair proportion. *Avena flavescens* was, upon the whole, less prominent than *A. pubescens;* and *Poa trivialis,* which was prominent in the earlier, greatly declined in the later years. Upon the whole, however, the gramineous herbage was, as has been said, of a mixed character.

Although Leguminosæ were much less discouraged with the smaller than with the larger quantity of nitrate, and especially less than with the ammonia-salts, no species of this Order gained any degree of prominence, all being in considerably less proportion, and amount, than even without manure. But the leguminous plant which best maintained its position was the deeply-rooting *Lotus corniculatus,* which is also the most prominent of the Leguminosæ on the unmanured plots.

The miscellaneous herbage was not only more in quantity than either without manure, with the ammonia-salts, or even than with the double quantity of nitrate, but different plants became prominent. For example, as will be remembered, with the double nitrate *Cerastium triviale* increased in an extraordinary degree, whereas, with the smaller quantity, in neither corresponding year did it yield one-third as much. With

TABLE LXVII.—Number of Species, Percentage, Quantity per Acre,

	Number of species, and proportion per cent.					Quantity per acre.				
	1862.	1867.	1872.	1877.	Mean.	1862.	1867.	1872.	1877.	Mean.
NUMBER OF SPECIES.										
Gramineæ	16	16	17	16	16	...	...	...	...	...
Leguminosæ	4	3	4	4	4	...	...	...	...	...
Other Orders	13	23	22	29	22	...	...	...	...	...
Total	33	42	43	49	42	...	...	...	...	...
GRAMINEÆ.										
	p. c.	p. c.	p. c.	p. c.	p. c.	lbs.	lbs.	lbs.	lbs.	lbs.
Anthoxanthum odoratum	2·06	2·31	4·50	5·32	3·55	91·6	117·4	149·5	198·9	139·4
Alopecurus pratensis	23·94	21·71	16·25	12·72	18·66	1064·4	1103·4	540·0	475·5	795·8
Agrostis vulgaris	11·01	7·05	10·60	17·92	11·64	489·5	358·3	352·2	669·8	467·4
Holcus lanatus	8·23	8·13	5·87	10·91	8·28	365·9	413·2	195·1	407·8	345·5
Avena pubescens	4·24	1·15	4·09	4·27	3·44	188·5	58·5	135·8	159·6	135·6
Avena flavescens	1·45	3·18	4·96	1·98	2·89	64·5	161·6	164·8	74·0	116·2
Poa trivialis	5·21	12·08	2·74	1·59	5·40	231·6	613·4	91·1	59·4	248·9
Dactylis glomerata	1·80	0·57	0·64	0·58	0·90	80·0	29·0	21·3	21·7	38·0
Festuca ovina	9·43	11·18	18·05	12·04	12·67	419·3	568·2	599·8	450·0	509·3
Bromus mollis	0·18	2·26	0·81	0·15	0·85	8·0	114·9	26·9	5·6	38·9
Lolium perenne	5·09	3·23	2·94	6·68	4·49	226·3	164·2	97·7	249·7	184·5
None yielding 1 per cent. { Aira cæspitosa, Avena elatior, Poa pratensis, Briza media, Cynosurus cristatus, Festuca pratensis }	1·32	1·01	1·24	1·71	1·32	58·7	51·5	41·3	64·0	53·9
Absent—Phleum pratense, Festuca loliacea	...	...	...	...	...	...	...	...	...	...
Undetermined (chiefly Gramineæ)	7·40	1·86	0·58	...	2·47	328·9	94·5	19·2	...	110·6
Total	81·36	75·72	73·27	75·87	76·56	3617·2	3848·1	2434·7	2836·0	3184·0
LEGUMINOSÆ.										
Trifolium repens	0·05	0·32	0·09	0·01	0·11	2·2	16·3	3·0	0·4	5·5
Trifolium pratense	0·31	0·26	0·12	0·11	0·17	13·8	13·2	4·0	4·1	8·8
Lotus corniculatus	0·05	0·12	1·16	0·78	0·34	2·2	6·1	38·6	29·1	19·0
Lathyrus pratensis	0·01	...	0·01	0·01	0·23	0·5	...	0·3	0·4	0·3
Trifolium procumbens	...	...	...	...	...	...	...	...	...	...
Total	0·42	0·70	1·38	0·91	0·85	18·7	35·6	45·9	34·0	33·6
OTHER ORDERS.										
Ranunculus acris	0·14	0·29	1·15	2·95	1·13	6·2	14·7	38·2	110·3	42·4
Ranunculus repens et bulbosus	1·95	1·04	1·33	1·96	1·57	86·7	52·8	44·2	73·3	64·2
Cerastium triviale	0·29	1·00	3·05	0·39	1·18	12·9	50·8	101·4	14·6	44·9
Conopodium denudatum	1·48	2·44	1·51	0·72	1·53	65·8	124·4	50·2	26·9	66·8
Centaurea nigra	4·41	4·10	10·28	2·82	5·40	196·1	208·3	341·6	105·4	212·8
Achillea Millefolium	2·14	1·39	2·91	1·39	1·95	95·1	70·6	96·7	51·9	78·6
Plantago lanceolata	3·85	4·83	2·41	7·99	4·77	171·2	245·4	80·1	298·7	198·8
Rumex Acetosa	3·57	7·53	1·58	2·56	3·81	158·7	382·7	52·5	95·7	172·4
None yielding 1 per cent. { Stellaria graminea, Hypericum perforatum, Potentilla reptans, Pimpinella Saxifraga, Heracleum Sphondylium, Scabiosa arvensis, Bellis perennis, Chrysanthemum Leucanthemum, Tragopogon pratensis, Leontodon hispidus, L. autumnalis, Taraxacum officinale, Hieracium Pilosella, Veronica Chamædrys, V. serpyllifolia, Prunella vulgaris, Ajuga reptans, Primula veris, Luzula campestris, Carex præcox, Hypnum squarrosum, H. hians, H. rutabulum }	0·39	0·96	1·13	2·44	1·25	17·4	48·6	37·5	91·2	48·8
Absent { Agrimonia Eupatoria, Poterium Sanguisorba, Galium Aparine, Thymus Serpyllum, Scilla nutans, Ophioglossum vulgatum }	...	...	...	...	...	...	...	...	...	...
Total	18·22	23·58	25·35	23·22	22·59	810·1	1198·3	842·4	868·0	929·7
SUMMARY.										
Gramineæ	81·36	75·72	73·27	75·87	76·56	3617·2	3848·1	2434·7	2836·0	3184·0
Leguminosæ	0·42	0·70	1·38	0·91	0·85	18·7	35·6	45·9	34·0	33·6
Other Orders	18·22	23·58	25·35	23·22	22·59	810·1	1198·3	842·4	868·0	929·7
Total	100·00	100·00	100·00	100·00	100·00	4446	5082	3323	3738	4147

&c., of each Species, by 275 lbs. Nitrate of Soda alone; Plot 17.

Increase + or decrease − compared with:—														
Plot 3, without manure.					Plot 15, 550 lbs. nitrate, to 1875.					Plot 5, ammonia-salts alone.				
1862.	1867.	1872.	1877.	Mean.	1862.	1867.	1872.	Mean.*	1877.	1862.	1867.	1872.	1877.	Mean.
NUMBER OF SPECIES (continued).														
−2	+1	0	−1	−1	−1	0	0	0	+1	−1	+1	+2	+3	+1
0	−1	0	0	0	0	−1	+1	0	0	0	−1	+1	+2	+1
−15	−1	−6	−2	−6	−5	+4	+3	+1	−2	−4	+6	+9	+15	+7
−17	−1	−6	−3	−7	−6	+3	+4	+1	−1	−5	+6	+12	+20	+9
GRAMINEÆ (continued).														
lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
−39·0	−171·1	+64·0	+77·9	−17·0	+17·2	+20·7	−14·8	+7·7	+41·5	−130·8	−64·8	+73·1	+78·2	−11·0
+927·4	+909·5	+531·5	+468·4	+709·2	+782·5	+788·9	+450·0	+673·8	+204·2	+1039·4	+1087·9	+519·2	+468·7	+778·8
+142·8	+70·8	+86·9	+355·9	+164·0	+176·9	−4·3	+71·9	+81·5	+181·7	−447·0	−335·0	−317·0	−199·9	−324·8
+212·1	+147·7	+135·9	+111·1	+151·7	+54·9	−211·1	+0·5	−51·9	−157·9	−22·6	+242·9	+147·4	+318·9	+171·7
−106·0	−43·8	+77·4	+96·0	+5·9	+44·3	+21·5	+78·7	+48·2	+41·2	−93·2	+37·7	+129·8	+156·1	+57·6
−7·9	+99·6	+107·4	+48·5	+61·9	−93·2	−63·6	+24·7	−44·0	−38·8	+39·5	+146·4	+160·2	+73·7	+104·9
+184·6	+574·4	+82·9	+46·2	+222·0	−35·2	−637·8	−199·8	−291·0	−169·5	+197·1	+603·5	+76·3	+59·1	+234·0
+26·3	−29·0	+6·5	+5·1	+2·2	−5·4	+17·9	+17·3	+9·9	+8·1	−12·1	−17·0	+3·7	−74·2	−24·9
+13·4	+61·7	+243·6	−67·5	+62·8	−140·0	−70·3	−670·2	−293·5	−336·0	−428·2	−442·4	−570·7	−1123·7	−641·2
+4·0	+113·2	+26·7	+5·6	+37·4	−78·6	−216·5	−119·5	−138·2	−56·8	+4·9	+114·9	+26·9	+5·6	+38·1
+31·9	+29·9	+58·7	+142·1	+65·7	−79·8	−7·1	−64·0	−50·3	−27·3	+98·0	+124·2	+73·4	+247·0	+135·7
−16·7	+11·8	−86·1	−134·6	−56·3	+45·3	+27·6	−3·4	+23·2	−12·1	−150·5	−66·8	−12·3	+49·8	−45·0
−0·3	...	...	−0·7	−0·3	−2·9	−9·5	−2·6	−5·0	...	...	...	...	...	...
+89·6	−110·0	−29·5	...	−12·5	−266·5	−38·2	−15·9	−106·9	...	+195·9	+41·2	−4·6	...	+58·1
+1462·2	+1664·7	+1305·9	+1154·0	+1396·4	+419·5	−381·8	−447·1	−136·5	−321·7	+290·4	+1472·7	+305·4	+59·3	+531·9
LEGUMINOSÆ (continued).														
−14·0	+9·3	−3·2	−2·7	−2·6	+0·6	+12·1	+0·8	+4·5	...	+1·8	+16·0	+2·7	+0·4	+5·2
−122·9	−57·1	−23·6	−45·3	−62·2	+5·6	+11·1	+2·9	+6·5	−7·6	+12·3	+12·9	+4·0	+4·1	+8·4
−53·7	−72·2	−59·1	−64·3	−62·3	+1·4	−12·4	+37·5	+8·8	+28·7	+0·3	−4·2	+28·3	+25·0	+12·3
−37·9	−22·7	−15·8	−55·6	−33·0	+0·5	−2·1	+0·3	−0·4	−55·2	−0·3	−0·3	−0·7	−1·1	−0·6
...	...	...	...	...	−0·4	...	...	−0·1	...	...	...	...	...	...
−228·5	−142·7	−101·7	−167·9	−160·1	+7·7	+8·7	+41·5	+19·3	−34·1	+14·1	+24·4	+34·3	+28·4	+25·3
OTHER ORDERS (continued).														
+5·9	+14·4	+37·0	+110·3	+41·9	−12·6	+1·5	−7·2	−6·1	−31·2	+5·8	+14·0	+37·4	+108·2	+41·4
−62·2	−14·2	−4·1	−8·3	−22·2	+13·1	+32·2	+29·9	+25·1	+54·8	+74·4	+50·2	+42·2	+70·7	+59·4
−1·2	+38·6	+87·4	−4·8	+30·0	−30·8	−143·7	−237·4	−137·3	−7·7	+12·5	+47·8	+101·4	+14·6	+44·1
+36·2	+26·1	+3·4	−18·0	+11·9	+42·1	+113·8	+34·5	+63·5	−2·2	+21·5	−65·3	+24·6	+7·7	−2·9
+186·6	+188·6	+307·0	+80·3	+190·6	+196·1	+199·3	+247·2	+214·2	+71·4	+195·7	+128·0	+286·8	+89·8	+175·1
+48·4	+31·9	+67·4	+4·9	+38·2	−8·3	+10·9	+1·6	+1·4	+29·9	+43·9	+34·6	+70·3	+47·2	+49·1
−52·8	−112·1	+36·4	+224·0	+23·8	−111·6	−1·5	+69·8	−14·4	+277·5	+167·4	+244·7	+80·1	+298·7	+197·7
+116·0	+324·0	+23·4	+51·5	+128·7	−112·6	−5·3	−22·9	−46·9	−123·4	−193·9	−144·3	−126·7	+32·8	−108·0
−116·0	−262·3	−74·6	−28·7	−120·3	−42·6	+18·0	−1·6	−8·7	+50·2	−39·4	−10·6	+13·5	+38·0	+0·1
−0·6	−7·0	−8·5	−23·3	−9·9	...	−56·1	−44·3	−33·5	−9·5	−0·4	−20·2	−60·3	−9·4	−22·5
+160·3	+228·0	+474·8	+387·9	+312·7	−67·2	+169·1	+69·6	+57·2	+309·8	+287·5	+278·9	+469·3	+698·3	+433·5
SUMMARY (continued).														
+1462·2	+1664·7	+1305·9	+1154·0	+1396·4	+419·5	−381·8	+447·1	−136·5	−321·7	+290·4	+1472·7	+305·4	+59·3	+531·9
−228·5	−142·7	−101·7	−167·9	−160·1	+7·7	+8·7	+41·5	+19·3	−34·1	+14·1	+24·4	+34·3	+28·4	+25·3
+160·3	+228·0	+474·8	+387·9	+312·7	−67·2	+169·1	+69·6	+57·2	+309·8	+287·5	+278·9	+469·3	+698·3	+433·5
+1394	+1750	+1679	+1374	+1549	+360	−204	−336	−60·0	−46	+592	+1776	+809	+786	+991

* Mean of three separations only, the conditions of manuring on plot 15 having been changed after 1875.

the double nitrate *Centaurea nigra,* though it increased, occurred in only small average proportion, but with the smaller quantity it became, in the third separation-year, not only the most prominent of the miscellaneous plants, but in that year it contributed about 10 per cent. of the total produce of the plot. With the double nitrate both *Rumex Acetosa* and *Plantago lanceolata* yielded considerable average proportions, but each declined very much in the third separation-year. With the single nitrate they also declined in the third, but again rose in the fourth separation-year; the *Plantago* yielding nearly 8 per cent. of the total produce in that year. Of the species of *Ranunculus, R. acris* gained ground, and *R. repens* and *R. bulbosus* declined, with the double nitrate, whilst with the smaller quantity *R. repens* and *R. bulbosus* yielded fairly average amounts throughout. Of other miscellaneous species, *Achillea Millefolium* was about equally favoured on the two nitrate plots; but *Conopodium denudatum* was much less prominent with the double than with the single nitrate.

The columns showing the increase or decrease in the actual yield of each species under the influence of the smaller amount of nitrate (plot 17), as compared with the larger amount (plot 15), bring to view the difference in the herbage in the two cases still more strikingly. It is there seen how much larger an actual amount of *Alopecurus pratensis,* and how much smaller an amount, especially of *Festuca ovina,* but also of *Poa trivialis, Bromus mollis,* and *Lolium perenne,* was grown with the smaller than with the larger amount of nitrate. Again, among the miscellaneous plants, *Centaurea nigra* in each year contributed much more, *Conopodium denudatum, Ranunculus repens* and *R. bulbosus* somewhat more, but *Cerastium triviale* much less, with the smaller quantity of nitrate.

The general result is that there was a more mixed and better description of herbage with the smaller than with the larger amount of nitrate of soda, as there was a better with the larger amount than with the same amount of nitrogen applied as ammonia-salts.

From the particulars of the amounts of produce, and of their chemical and botanical characters, which have been enumerated and compared, it is to be concluded that the much more favourable conditions of growth which were induced under the influence of the larger amount of nitrogen as nitrate than with the same amount as ammonia-salts, also obtained when the smaller amount was used. Indeed, they were obviously developed in a greater degree proportionally to the amount of nitrate used with the smaller than with the larger quantity, and in some particulars, especially in so far as the botany of the plots is concerned, not only proportionally, but actually more with the smaller than with the larger amount. This result is doubtless due to a less forced luxuriance with the less excessive application of nitrate, and coincidently a more favourable balance of available mineral constituents.

4. *Mixed mineral manure, alone (including potass); Plot* 7.

Having considered the effects on the mixed herbage of applying purely nitrogenous manures, in various forms and quantities, for many years in succession, under which conditions the plants have to rely upon the soil itself for the mineral constituents required, we now come to discuss the effects of applying annually, from the commencement, a full mineral manure alone, including potass, the same as was applied to plot 15 after the application of nitrate for 18 years. Of course, now the vegetation is left to obtain its nitrogen from other sources than manure; that is, from the soil itself, or from the atmosphere, as the case may be.

Referring to Part I., p. 307, *et seq.*, for more detailed statements of the results as to the amounts of produce, of nitrogen, and of mineral matter, yielded, it may be here repeated that such a mixed mineral manure gave much more produce than ammonia-salts alone, and nearly as much as with either the smaller or the larger amount of nitrate of soda. There was also an increase in gross produce, in nitrogen, and in mineral matter, taken up over the later as compared with the earlier years. Even the gramineous herbage, so markedly influenced by nitrogenous manures, increased considerably, especially in the later years, under the influence of the purely mineral manure. Leguminous herbage, though in less actual amount than the gramineous, increased in much greater proportion, indeed, in a very striking degree. Miscellaneous plants, however, rather declined than increased.

Although no nitrogen was supplied, there was a great increase in the amount taken up as compared with the produce without manure, especially by the increased leguminous growth. Upon the whole, without nitrogenous manure, the amount of nitrogen taken up under the influence of the purely mineral manure was one and two-thirds as much as without manure, and nearly three times as much as by applying the same manure to either wheat or barley on arable land. Further, with this greatly increased yield of nitrogen in the produce, without the supply of any by manure, there was found a great reduction in the percentage of nitrogen in the upper layers of the soil, indicating the source whence the nitrogen of the vegetation had been derived.

Bearing in mind these striking facts, we now turn to the botanical characteristics of the mineral manured plot, as shown in Table LXVIII., pp. 1324–5.

As already referred to, it has been found that the number of species compared with that without manure is diminished, whatever the description of manure applied. Taking the average of the four years of complete separation, there were 49 species found in the samples from the unmanured plot, 33 only by ammonia-salts alone, 39 (over three separation-years only) with the larger quantity of nitrate, and 42 with the smaller quantity. We have now with the mixed mineral manure alone an average of 43, which is higher than on any plot with nitrogenous manure, whether used alone or in conjunction with minerals. The fact is that, with nitrogenous manure, there is more of forced luxuriance of individual species, varying, it is true,

TABLE LXVIII.—Number of Species, Percentage, Quantity per Acre, &c.,

	Number of species, and proportion per cent.				
	1862.	1867.	1872.	1877.	Mean.
NUMBER OF SPECIES.					
Gramineæ	18	16	17	17	17
Leguminosæ	4	4	4	4	4
Other Orders	22	22	20	23	22
Total	44	42	41	44	43
GRAMINEÆ.					
	p. c.	p. c.	p. c.	p. c.	p. c.
Anthoxanthum odoratum	3·06	3·93	2·73	3·18	3·22
Alopecurus pratensis	0·34	0·88	1·17	0·48	0·72
Agrostis vulgaris	7·14	5·69	11·72	12·02	9·14
Holcus lanatus	5·06	11·81	3·16	13·16	8·29
Avena elatior	2·41	0·06	0·46	1·29	1·05
Avena pubescens	13·81	3·90	2·36	2·25	5·58
Avena flavescens	4·02	4·84	3·72	3·65	4·07
Poa pratensis	1·13	1·05	2·27	1·75	1·55
Poa trivialis	3·81	4·38	2·30	2·11	3·15
Dactylis glomerata	2·57	4·67	1·68	3·67	3·15
Festuca ovina	13·73	11·38	14·86	26·59	16·64
Bromus mollis	1·26	0·98	0·04	0·01	0·57
Lolium perenne	3·12	2·40	0·59	3·02	2·28
None yielding 1 per cent. { Phleum pratense, Aira cæspitosa, Briza media, Cynosurus cristatus, Festuca pratensis }	0·30	0·92	0·30	1·20	0·69
Absent—Festuca loliacea	...	...	...	...	...
Undetermined (chiefly Gramineæ)	2·89	2·40	1·46	...	1·68
Total	64·65	59·29	48·82	74·38	61·78
LEGUMINOSÆ.					
Trifolium repens	3·08	0·47	1·77	0·01	1·33
Trifolium pratense	6·84	4·75	1·13	1·55	3·56
Lotus corniculatus	1·27	0·69	0·19	0·04	0·55
Lathyrus pratensis	13·51	6·78	36·68	12·11	17·27
Total	24·70	12·69	39·77	13·71	22·71
OTHER ORDERS.					
Ranunculus repens et bulbosus	1·31	0·38	0·21	0·32	0·55
Conopodium denudatum	2·03	9·22	1·31	1·58	3·53
Pimpinella Saxifraga	0·86	1·20	0·22	0·28	0·64
Achillea Millefolium	1·69	3·10	5·23	0·64	2·66
Plantago lanceolata	0·23	1·10	0·07	0·09	0·37
Rumex Acetosa	2·10	8·88	1·16	6·67	4·69
Luzula campestris	1·41	1·17	1·64	0·50	1·18
None yielding 1 per cent. { Ranunculus acris, Stellaria graminea, Cerastium triviale, Potentilla reptans, Agrimonia Eupatoria, Spiræa Ulmaria, Heracleum Sphondylium, Galium verum, Scabiosa arvensis, Centaurea nigra, Carduus arvensis, Tragopogon pratensis, Leontodon hispidus, Taraxacum officinale, Hieracium Pilosella, Veronica Chamædrys, Prunella vulgaris, Ajuga reptans, Primula veris, Carex præcox, Ophioglossum vulgatum, Hypnum squarrosum, H. rutabulum, H. hians }	1·02	2·97	1·57	1·83	1·89
Absent { Poterium Sanguisorba, Galium Aparine, Bellis perennis, Chrysanthemum Leucanthemum, Thymus Serpyllum, Scilla nutans }	...	...	...	...	...
Total	10·65	28·02	11·41	11·91	15·51
SUMMARY.					
Gramineæ	64·65	59·29	48·82	74·38	61·78
Leguminosæ	24·70	12·69	39·77	13·71	22·71
Other Orders	10·65	28·02	11·41	11·91	15·51
Total	100·00	100·00	100·00	100·00	100·00

of each Species, by Mixed Mineral Manure alone (including Potass); Plot 7.

Quantity per acre.					Increase + or decrease − compared with plot 3, unmanured.				
1862.	1867.	1872.	1877.	Mean.	1862.	1867.	1872.	1877.	Mean.
NUMBER OF SPECIES (continued).									
...	...	...	...	...	0	+1	0	0	0
...	...	...	...	...	0	0	0	0	0
...	...	...	...	...	−6	−2	−8	−8	−6
...	...	...	...	...	−6	−1	−8	−8	−6
GRAMINEÆ (continued).									
lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
135·4	175·2	115·2	162·2	147·0	+ 4·8	− 113·3	+ 29·7	+ 41·2	− 9·4
15·0	39·2	49·6	24·5	32·1	− 122·0	− 154·7	+ 41·1	+ 17·4	− 54·5
315·9	253·7	496·4	613·0	419·8	− 30·8	− 33·8	+ 231·1	+ 299·1	+ 116·4
223·9	526·5	133·8	671·2	388·8	+ 70·1	+ 261·0	+ 74·6	+ 374·5	+ 195·0
106·6	2·7	19·5	65·8	48·6	+ 104·5	− 4·3	+ 17·4	+ 64·6	+ 45·5
611·0	173·9	100·0	114·7	249·9	+ 316·5	+ 71·6	+ 41·6	+ 51·1	+ 120·2
177·8	215·8	157·6	186·1	184·3	+ 105·4	+ 153·8	+ 100·2	+ 160·6	+ 130·0
50·0	46·8	96·2	89·3	70·6	+ 41·1	+ 41·1	+ 94·7	+ 87·6	+ 66·2
168·6	195·3	97·4	107·6	142·2	+ 121·6	+ 156·3	+ 89·2	+ 94·4	+ 115·3
113·7	208·2	71·2	187·2	145·1	+ 60·0	+ 150·2	+ 56·4	+ 170·6	+ 109·3
607·4	507·4	629·5	1356·1	775·1	+ 201·5	+ 0·9	+ 273·3	+ 838·6	+ 328·6
55·7	43·5	1·7	0·5	25·3	+ 51·7	+ 41·8	+ 1·5	+ 0·5	+ 23·8
138·0	107·0	25·0	154·0	106·0	− 56·4	− 27·3	− 14·0	+ 46·4	− 12·8
13·3	41·0	13·1	61·1	32·2	− 51·4	+ 14·0	− 110·7	− 134·6	− 70·6
...	...	...	...	...	...	...	...	− 0·7	− 0·2
127·9	107·0	61·8	...	74·1	− 111·4	− 97·5	+ 13·1	...	− 49·0
2860·2	2643·2	2068·0	3793·3	2841·1	+ 705·2	+ 459·8	+ 939·2	+2111·3	+1053·8
LEGUMINOSÆ (continued).									
136·2	21·0	75·0	0·5	58·2	+ 120·0	+ 14·0	+ 68·8	− 2·6	+ 50·1
302·6	211·9	47·9	79·1	160·4	+ 165·9	+ 141·6	+ 20·3	+ 29·7	+ 89·4
56·2	30·8	8·0	2·0	24·3	+ 0·3	− 47·5	− 89·7	− 91·4	− 57·0
597·7	302·0	1553·8	617·6	767·7	+ 559·3	+ 279·3	+1537·7	+ 561·6	+ 734·4
1092·7	565·7	1684·7	699·2	1010·6	+ 845·5	+ 387·4	+1537·1	+ 497·3	+ 816·9
OTHER ORDERS (continued).									
58·0	16·9	8·9	16·3	25·0	− 90·9	− 50·1	− 39·4	− 65·3	− 61·4
89·8	411·0	55·5	80·6	159·2	+ 60·2	+ 312·7	+ 8·7	+ 35·7	+ 104·3
38·0	53·5	9·3	14·3	28·8	− 8·7	− 61·1	− 4·8	− 4·5	− 19·7
74·9	138·2	221·5	32·6	116·8	+ 28·2	+ 99·5	+ 192·2	− 14·4	+ 76·4
10·2	49·0	3·0	4·6	16·7	− 213·8	− 308·5	− 40·7	− 70·1	− 158·3
92·9	395·9	49·1	340·2	219·5	+ 50·2	+ 337·2	+ 20·0	+ 296·0	+ 175·8
62·4	52·2	69·5	25·5	52·4	+ 4·1	− 68·1	+ 13·3	− 16·3	− 16·7
44·9	132·4	66·5	93·4	84·4	− 7·7	+ 25·5	− 25·2	− 7·3	− 3·7
...	...	...	...	...	− 0·3	− 8·3	− 9·4	− 26·4	− 10·9
471·1	1249·1	483·3	607·5	702·8	− 178·7	+ 278·8	+ 115·7	+ 127·4	+ 85·8
SUMMARY (continued).									
2860·2	2643·2	2068·0	3793·3	2841·1	+ 705·2	+ 459·8	+ 939·2	+2111·3	+1053·8
1092·7	565·7	1684·7	699·2	1010·6	+ 845·5	+ 387·4	+1537·1	+ 497·3	+ 816·9
471·1	1249·1	483·3	607·5	702·8	− 178·7	+ 278·8	+ 115·7	+ 127·4	+ 85·8
4424	4458	4236	5100	4554	+1372	+1126	+2592	+2736	+1956

according to the amount, the description, and the combination in which the nitrogen is employed. The characteristic tendency of the various mineral manures, on the other hand, is to favour consolidation or maturation rather than luxuriance, and, hence, when used alone, the competition becomes less severe, and a larger number of species is enabled to maintain a place.

The lines in the table showing the percentages of the total Gramineæ, total Leguminosæ, and total Miscellaneæ, exhibit such extremely wide ranges in their proportion to one another in the four complete separation-years, that it is difficult from the figures there given to form a definite conclusion as to the tendency of either group to increase or diminish in relative proportion. Calling to our aid the results of the partial separations made in several intermediate and succeeding years, it must be concluded that the grasses have increased in their proportion during the later years, whilst the Leguminosæ have not sustained the tendency to increase so prominently in the later as in the earlier years. It may also be said of the Miscellaneæ that the tendency with them is to decline in proportion. In actual amount per acre, too, the grasses have of late years increased, and the Leguminosæ and the Miscellaneæ diminished. Without manure, on the other hand, though the actual quantities per acre are much less than with the mineral manure, the grasses have diminished very considerably, the Leguminosæ have even increased, but the Miscellaneæ have diminished even in greater degree than with the mineral manure.

Whilst referring to the relation to one another of our three main groups, some reference should be made to the characters of the second crops in this respect. The indication is that in the second crops the percentage of Gramineæ is generally higher, and that of the Leguminosæ and Miscellaneæ lower ; and this is especially the case on the mixed mineral manure plot now under consideration.

Among the grasses, *Festuca ovina* is by far the most prominent species, and is increasing ; but it is, nevertheless, in by no means such excessive proportion, or amount, as on the plots with an excess of nitrogenous manure. *Agrostis vulgaris* and *Holcus lanatus* are also prominent, and, upon the whole, increasing ; though the latter shows considerable fluctuation, according to season. Taking the average of the four complete separation-years, these three most prominent grasses yield collectively only about one-third of the total herbage ; and, consistently with the character of comparatively little forced luxuriance of individual species, a considerable number of other grasses contributes a fair proportion to the produce. The most prominent of these, on the average, though declining, is *Avena pubescens,* then *A. flavescens ;* next come *Dactylis glomerata, Poa trivialis,* and *Anthoxanthum odoratum,* in about equal amounts ; then *Lolium perenne,* and, in smaller quantities, but still apparently maintaining some position, come *Poa pratensis, Avena elatior,* and *Alopecurus pratensis.*

The final column in the table further indicates the general character of the change induced by the mixed mineral manure as compared with the produce without manure. It is seen that, of *Alopecurus pratensis, Lolium perenne, Anthoxanthum odoratum,* and a few grasses of very limited habit of growth, there was actually less with the manure than without manure; but of no fewer than 10 species there was greater or less increase with the manure. That is to say, there was comparatively little relative predominance of any particular species, but a generally increased development of the numerous plants constituting the flora of the unmanured plot, a result which is strikingly different from that obtained with the increased luxuriance induced by the application of nitrogenous manures.

Of the four Leguminosæ enumerated as found in the samples, it will be seen that, whilst *Trifolium repens*, *T. pratense*, and *Lotus corniculatus*, have contributed a decreasing, and, in the later years, a very insignificant amount, *Lathyrus pratensis* has, on the average, furnished three-fourths of the whole of the leguminous herbage; with, however, very fluctuating quantities according to season, and, as has been already alluded to, probably a decreasing amount in the more recent years. It is of interest to observe in the columns showing the increase over the unmanured produce, that there is actually less of the deep-rooting and self-reliant *Lotus corniculatus* with than without the manure, and that almost the whole of the increase is due to the comparatively shallow-rooting *Lathyrus*. It is, however, worthy of note that the partial separations of the second crops indicate a lower proportion, and scarcely any more actual produce, of the mixed leguminous herbage with the manure than without manure; and it is also significant that, in these second crops, the *Trifoliums* bear a much larger proportion to the *Lathyrus* than in the first. An examination of the samples of the partial separations leads to the conclusion that this is explained by the fact that, owing to the habit and conformation of the plants, new and stronger shoots are more freely produced from the thick-branched stock of the *Trefoils* which remain after cutting, than from the comparatively thinner and less branching stock of the *Lathyrus*, in which the relatively few shoots that are produced after cutting are of very slender and feeble character.

Among the Miscellaneæ, *Rumex Acetosa* is the most prominent on the mineral manured plot, as it was also on the ammonia plot, and on that with the larger amount of nitrate of soda. But the mineral manured plot was more favourable to *Conopodium denudatum*, and to *Achillea Millefolium*, both of which, however, fluctuated very greatly in amount, according to season. No other miscellaneous plant, except *Luzula campestris*, was in any noticeable degree of prominence. The last division of the table shows that *Plantago lanceolata*, and the various species of *Ranunculus* taken collectively, were in each year in actually less amount with the mineral manure than without manure; so also, but in a less degree, was *Pimpinella Saxifraga*, and, in two out of the four years, *Luzula campestris*.

The only plants which uniformly yielded more with the mineral manure than without manure, were *Rumex Acetosa* and *Conopodium denudatum*, *Achillea Millefolium* doing so in three out of the four years. Thus, under the conditions of maturation rather than of luxuriance, that is of tendency to consolidation of tissue, and to reproduction, rather than to vegetation merely, the miscellaneous plants which most prominently gained ground are the thick-stocked and free-seeding *Rumex,* the tuberous-rooted *Conopodium,* and the fleshy-stocked, much branched, and also free-seeding *Achillea.* It may here be stated that in the years subsequent to the last complete separation, *Centaurea nigra* appears to be gaining ground, this being especially manifest in the second crops.

It will be observed from the foregoing enumeration that, independently of the striking increase in the growth of *Lathyrus,* which, however, would seem to be less marked in recent years, the herbage grown under the influence of mixed mineral manure, including potass, is very complex, and the distribution is very similar in general character to that without manure; the difference between the two being, with the exception of the altered relation of the different Leguminosæ, due more to a different condition of development, than to any marked distinction in the relative predominance of different orders, genera, or species.

The general appearance of the mineral manured plot is also quite characteristic. Instead of the dark colour, irregularity, and prominent leafiness, exhibited by the plants of the ammonia and the nitrate plots, there is here a lighter, healthier hue, and general evenness, with a very large proportion of fine but comparatively matured stems, intermixed nevertheless with much leafy bottom-growth. This character of stemminess, and tendency to consolidation of tissue, applies not only to the grasses, but also to the Leguminosæ and to the miscellaneous plants; among the latter, especially to the *Rumex* and the *Conopodium,* whilst the *Achillea,* being later in development, is not so markedly stemmy in the first crop, its main development of above-ground, or at least of erect-growing, stem taking place subsequently to the first cutting. These remarks apply to the first crops. The second crops show, in general, a more grassy growth than the first, with a greater predominance of fine leaf; and although usually fairly mixed, still *Festuca ovina* and *Agrostis vulgaris* are, upon the whole, the most prominent.

Notwithstanding that a characteristic of the herbage of this plot is to include a larger amount of Leguminosæ, and especially of *Lathyrus,* than any other (unless, indeed, it has been overtaken in this respect in quite recent years on plot 15, where the same mineral manure was first applied in 1876), there is no doubt that, according to the figures, there has been some decline as compared with the earlier years. Unfortunately, however, we have much less frequent determinations in the earlier than in the later years, the first being in the third season (1858), when about 23 per cent. of the total produce consisted of Leguminosæ, of which about 18 consisted of *Trifolium pratense,* and only 4½ of *Lathyrus pratensis.* The first complete separation-year was the seventh of the experiments, when the Leguminosæ contributed nearly 25 per cent. to the produce, *Trifolium pratense* now yielding less than 7, but *Lathyrus* about 13½ per cent. Five years later (1867) the total Leguminosæ were under 13 per cent. *Trifolium pratense* yielding under 5, and *Lathyrus* nearly 7 of this. In 1871 the total Leguminosæ amounted to a little over 16 per cent. In 1872, the seventeenth year of the experiments, the Leguminosæ contributed nearly 40 per cent., *Trifolium pratense* giving little more than 1 of this, and *Lathyrus* nearly 37. Since that time the percentage of total leguminous herbage has been—under 13 in 1874, under 9 in 1875, about 9½ in 1876, 13¾ in 1877 (of which more than 12 was *Lathyrus*), 13½ in 1878, little more than 8 in 1879, and only 7½ in 1880.

It is thus obvious, not only that the Leguminosæ were very much more prominent in the earlier than in the later years, but that the deeper-rooting *Trifolium pratense* rapidly gave place to the more superficially-rooting *Lathyrus;* whilst this, in its turn, would appear to be now declining.

It has been assumed that the source of the large amount of nitrogen essential to, and taken up by, the *Lathyrus*, was the accumulation within the surface soil, which has been shown to have diminished considerably in recent years. The question arises, how far the decline in yield is to be attributed to the amount of nitrogen available from this source from year to year being, under the influence of the mineral manure (including potass) and the exhausting growth of the Leguminosæ, gradually reduced ; or whether, in addition to or apart from this, there is anything in the treatment of the vegetation which would tend to lessen the recuperative power of these leguminous species. On this point it should be observed, as has been explained in detail elsewhere, that owing to the injury done to the animals when the second crops were consumed on the land by sheep without any other food being given, it became necessary to abandon the practice and to cut the aftermath instead. Thus, in 1866, 1870, 1873 (twice), 1874 (twice), and in 1876, the after-growth was cut and left on the land to decay; whilst in 1875, 1877, 1878, 1879, and 1880 the second crops were not only cut, but were removed from the land, thus considerably enhancing the tendency to exhaustion of the soil.

Taking it for granted that the reduction in leguminous growth in recent years under the influence of the continuous application of the mineral manure is not merely a temporary effect due to the characters of the seasons, but a permanent tendency set in, we may, perhaps, safely conclude that reduction in the amount of nitrogen annually available from the accumulated resources of the soil is one cause of the decline, but we may, perhaps, also assume, that the altered treatment of the second crops has not been without influence. Reference has already been made to the comparatively injurious effect of the first cutting upon the *Lathyrus*, as exhibited in the character of the second growth of that plant, and it would seem only reasonable to suppose that such injury, clearly manifest after the first cuttings, would be enhanced when the second growth is annually treated in the same way. The question suggests itself whether, as the *Trifoliums* and the *Lotus* suffer less than the *Lathyrus* by cutting, they will, in consequence, show any tendency to recover their position. There is already, perhaps a slight indication of this, so far as the *Trifolium* is concerned. But in so far as the relative prominence of the *Lathyrus* has been due to the supplies of the surface-soil becoming available under the influence of the mineral manure, it would hardly seem probable that the deeper-rooting plants would thoroughly regain their former position.

With regard to the undoubted tendency to gradual increase, not only in the relative but in the actual amount of the grasses in recent years, the explanation may perhaps be open to question; and the evidence bearing upon it will be considered more in detail in the chemical section of our report. As has been referred to, the growth of a leguminous crop on arable land, frequently leaves the surface soil richer in nitrogen in a degree determinable by analysis; and it is known that after the removal of a highly nitrogenous leguminous crop a larger gramineous one, taking up more nitrogen than if it had succeeded another of the same description, will be obtained. It has been already suggested, whether the increased growth of the grasses here, in the mixed herbage, is due to similar conditions being induced by the increased growth of the Leguminosæ, now in association, instead of separately in alternation, with them; that is, whether it be due to a residue of combined nitrogen left after the growth of the leguminous plants in such condition of combination and distribution as to be now available for the associated gramineous ones, the subsequent growth of which is thereby enhanced. Another explanation obviously may be, that under the influence of the mixed mineral manure, including potass, the richer nitrogenous accumulations of the surface soil of the grassland itself, yield up their stores in a condition in which the Gramineæ as well as the Leguminosæ of the mixed herbage can take them up. Against this supposition is the fact, that such a potass manure will increase the growth of Leguminosæ in the poorer in nitrogen ordinary arable land, whilst it will not, in any marked degree, increase the growth of the Gramineæ on such arable land. If, therefore, the result be due to the direct liberation, in an available form, of nitrogen from the resources of the soil itself, under the influence of the mineral manure, it would seem that it must be provided in different conditions of combination for these two very distinct descriptions of herbage, and that a supply in the condition required by the Gramineæ is in a very much greater degree available from the richer in nitrogen pasture, than from the poorer in nitrogen arable, surface soil.

5. *Superphosphate of lime, alone; Plot* 4–1.

The experiment with superphosphate of lime alone did not commence until the fourth year; sawdust alone, but without effect, having been applied in each of the first three years. It has been seen that in the case of the mixed mineral manure of plot 7, very marked results were produced, but from the complex character of the manure, it is only by comparison with those on other plots, and by other collateral evidence, that it can be decided to which of the constituents of the mixed manure the specially characteristic results were mainly due. Hence it is of considerable interest to compare the results obtained with superphosphate of lime alone with those from the mixed mineral manure, which included, besides the same amount of superphosphate, salts of potass, soda, and magnesia. In the next sub-section will be compared the results yielded with superphosphate of lime in conjunction with salts of soda and magnesia, but without potass.

The superphosphate of lime alone yielded more gross produce, but only a little more, than was obtained without manure, and the excess was the less in the later years. Indeed, not only the actual amount of produce, but the yield per acre of both nitrogen and mineral matter declined more rapidly with the superphosphate alone than without manure. In fact, the superphosphate of lime alone yielded, over the first 17 years of its application, less than two-thirds as much produce as the mixed mineral manure, and not three-fifths as much nitrogen. It is obvious, therefore, that the results which were obtained on plot 7 were in only a very small degree due to the superphosphate which the manure supplied. Let us see how the botany of the superphosphate plot bears out this conclusion. (See Table LXIX., pp. 1332–3.)

It will be seen that the effect of the manure was, as already said, upon the whole, extremely meagre. In fact, there was scarcely any increased luxuriance induced; and such increase in produce as there was, was mainly due to an increased tendency to stem-formation and consolidation of tissue in the case of some species. There was thus but little artificial struggle induced, and we have an average of 47 species as compared with 49 without manure. Both the percentage in the total herbage, and the actual quantity per acre, of the total Gramineæ rather declined, whilst those of the Leguminosæ rather increased; and the percentage, but not the actual quantity, of the miscellaneous herbage also showed some tendency to increase.

Compared with the produce without manure, there was, however, on the average, a slight increase of gramineous, a decrease of leguminous, and an increase of miscellaneous herbage, the result being but insignificant increase of total produce.

The column showing the average increase or decrease in the yield per acre of each individual species by superphosphate of lime alone, compared with the unmanured plot 3, shows a slight increase in quantity, in the order named, of *Holcus lanatus*, *Poa trivialis*, *Avena flavescens*, *Lolium perenne*, and *Avena pubescens*, a very slight increase of *Cynosurus cristatus* and *Dactylis glomerata*, and an actual deficiency of *Briza media*, *Agrostis vulgaris*, *Alopecurus pratensis*, *Anthoxanthum odoratum*, and, though very trifling, even of *Festuca ovina*. Upon the whole, therefore, the increase is partly, though not entirely, among some of the better agricultural grasses, and as has been said, it was due rather to consolidation of tissue than to increased luxuriance or succulence; in fact, to stemmy development but with dwarf growth.

Of the Leguminosæ, *Trifolium pratense* and *Lotus corniculatus* yielded less in each separation-year than without manure. On the other hand, in the three later of the four years, *Lathyrus pratensis* gave some increase, and on the average *Trifolium repens* gave a very slight increase. Lastly, on this plot of languid competition, and coincident great variety of species, we have in the last two separation-years a small quantity of the hard-wooded, somewhat shrubby *Ononis arvensis*, which is scarcely observable on any other plot. It is also on this plot of languid competition, and on one other plot only, that fairy rings develop in any marked degree.

TABLE LXIX.—Number of Species, Percentage, Quantity per Acre,

	Number of Species, and proportion per cent.					Quantity per acre.				
	1862.	1867.	1872.	1877.	Mean.	1862.	1867.	1872.	1877.	Mean.
NUMBER OF SPECIES.										
Gramineæ	16	15	16	16	16	...	...	...	...	...
Leguminosæ	4	4	5	5	5	...	...	...	...	...
Other Orders	26	27	28	24	26	...	...	...	...	...
Total	46	46	49	45	47	...	...	...	...	...
GRAMINEÆ.										
	p. c.	p. c.	p. c.	p. c.	p. c.	lbs.	lbs.	lbs.	lbs.	lbs.
Anthoxanthum odoratum	3·66	7·16	4·74	5·11	5·17	119·0	253·5	84·0	158·0	153·6
Alopecurus pratensis	1·32	1·84	0·86	1·40	1·36	42·9	65·1	15·2	43·2	41·6
Agrostis vulgaris	7·21	6·08	13·88	9·87	9·26	234·5	215·2	245·9	305·2	250·2
Holcus lanatus	11·82	9·16	4·71	19·35	11·26	384·4	324·3	83·5	598·3	347·6
Avena pubescens	9·42	4·97	4·09	4·02	5·62	306·4	175·9	72·5	124·3	169·8
Avena flavescens	4·12	4·28	6·09	2·47	4·24	134·0	151·5	107·9	76·4	117·5
Poa trivialis	5·16	5·65	3·79	4·72	4·83	167·8	200·0	67·1	146·0	145·2
Briza media	0·58	0·32	2·12	2·16	1·29	18·9	11·3	37·6	66·8	33·7
Dactylis glomerata	2·25	0·99	0·57	1·41	1·31	73·2	35·0	10·1	43·6	40·5
Cynosurus cristatus	0·34	0·84	1·03	0·80	0·75	11·0	29·7	18·2	24·7	20·9
Festuca ovina	10·20	16·75	20·44	16·02	15·85	331·7	592·9	362·2	495·4	445·5
Lolium perenne	9·28	5·24	3·12	4·35	5·50	301·8	185·7	55·3	134·5	169·3
None yielding 1 per cent. { Avena elatior Poa pratensis Festuca pratensis Bromus mollis }	1·34	0·87	0·77	0·10	0·77	43·5	30·8	13·7	3·1	22·8
Absent—Phleum pratense, Aira cæspitosa, Festuca loliacea	...	...	...	...	...	...	...	...	...	...
Undetermined (chiefly Gramineæ)	8·26	2·73	0·82	...	2·95	268·6	96·6	14·5	...	94·9
Total	74·96	66·88	67·03	71·78	70·16	2437·7	2367·5	1187·7	2219·5	2053·1
LEGUMINOSÆ.										
Trifolium repens	0·61	0·09	0·48	0·35	0·38	19·8	3·2	8·5	10·8	10·6
Trifolium pratense	1·45	0·17	0·11	0·30	0·51	47·2	6·0	2·0	9·2	16·1
Lotus corniculatus	0·41	1·23	3·71	0·86	1·55	13·3	43·6	65·7	26·6	37·3
Lathyrus pratensis	0·32	1·34	4·19	3·38	2·31	10·4	47·4	74·3	104·5	59·2
Ononis arvensis	...	...	0·12	0·64	0·19	...	...	2·1	19·8	5·4
Total	2·79	2·83	8·61	5·53	4·94	90·7	100·2	152·6	170·9	128·6
OTHER ORDERS.										
Ranunculus repens et bulbosus	5·87	1·29	4·16	5·84	4·29	190·9	45·7	73·7	180·6	122·7
Conopodium denudatum	0·97	2·32	1·17	0·64	1·22	31·5	82·1	20·7	19·8	38·5
Pimpinella Saxifraga	0·87	3·21	1·28	1·48	1·71	28·3	113·6	22·7	45·8	52·6
Centaurea nigra	0·43	0·36	1·01	0·66	0·62	14·0	12·7	17·9	20·4	16·2
Achillea Millefolium	1·42	1·88	5·38	3·19	2·96	46·2	66·5	95·3	98·7	76·7
Plantago lanceolata	5·63	9·66	3·13	3·78	5·55	183·1	342·0	55·4	116·9	174·4
Rumex Acetosa	3·94	5·47	2·81	3·37	3·89	128·1	193·6	49·8	104·2	118·9
Luzula campestris	1·22	2·41	3·57	0·84	2·01	39·7	85·3	63·2	26·0	53·4
None yielding 1 per cent. { Ranunculus acris, Stellaria graminea, Cerastium triviale, Potentilla reptans, Agrimonia Eupatoria, Poterium Sanguisorba, Heracleum spondylium, Galium Aparine, G. verum, Scabiosa arvensis, Bellis perennis, Tragopogon pratensis, Leontodon hispidus, L. autumnalis, Taraxacum officinale, Hieracium Pilosella, Veronica Chamædrys, Prunella vulgaris, Ajuga reptans, Carex præcox, Ophioglossum vulgatum, Hypnum squarrosum, H. rutabulum, H. hians }	1·90	3·69	1·85	2·89	2·65	61·8	130·8	33·0	89·2	78·9
Absent { Chrysanthemum Leucanthemum, Thymus Serpyllum, Scilla nutans, Spiræa Ulmaria, Carduus arvensis }	...	...	...	...	...	...	...	...	...	...
Total	22·25	30·29	24·36	22·69	24·90	723·6	1072·3	431·7	701·6	732·3
SUMMARY.										
Gramineæ	74·96	66·88	67·03	71·78	70·16	2437·7	2367·5	1187·7	2219·5	2053·1
Leguminosæ	2·79	2·83	8·61	5·53	4 94	90·7	100·2	152·6	170·9	128·6
Other Orders	22·25	30·29	24·36	22·69	24·90	723·6	1072·3	431·7	701·6	732·3
Total	100·00	100·00	100·00	100·00	100·00	3252	3540	1772	3092	2914

&c., of each Species, by Superphosphate of Lime alone; Plot 4–1.

Increase + or decrease − compared with:—									
Plot 3, without manure.					Plot 7, mixed mineral manure (with potass).				
1862.	1867.	1872.	1877.	Mean.	1862.	1867.	1872.	1877.	Mean.
Number of Species (continued).									
−2	0	−1	−1	−1	−2	−1	−1	−1	−1
0	0	+1	+1	+1	0	0	+1	+1	+1
−2	−3	0	−7	−2	+4	+5	+8	+1	+4
−4	−3	0	−7	−2	+2	+4	+8	+1	+4
Gramineæ (continued).									
lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
−11·6	−35·0	−1·5	+37·0	−2·8	−16·4	+78·3	−31·2	−4·2	+6·6
−94·1	−128·8	+6·7	+36·1	−45·0	+27·9	+25·9	−34·4	+18·7	+9·5
−112·2	−72·3	−19·4	−8·7	−53·2	−81·4	−38·5	−250·5	−307·8	−169·6
+230·6	+58·8	+24·3	+301·6	+153·8	+160·5	−202·2	−50·3	−72·9	−41·2
+11·9	+73·6	+14·1	+60·7	+40·1	−304·6	+2·0	−27·5	+9·6	−80·1
+61·6	+89·5	+50·5	+50·9	+63·2	−43·8	−64·3	−49·7	−109·7	−66·8
+120·8	+161·0	+58·9	+132·8	+118·3	−0·8	+4·7	−30·3	+38·4	+3·0
−38·8	−11·4	−67·6	−104·6	−55·5	+17·6	+8·6	+33·4	+59·7	+29·9
+19·5	−23·0	−4·7	+27·0	+4·7	−40·5	−173·2	−61·1	−143·6	−104·6
+5·8	+25·4	...	+1·5	+8·2	+7·5	+19·9	+14·4	+19·6	+15·3
−74·2	+86·4	+6·0	−22·1	−1·0	−275·7	+85·5	−267·3	−860·7	−329·6
+107·4	+51·4	+16·3	+26·9	+50·5	+163·8	+78·7	+30·3	−19·5	+63·3
+27·3	+16·4	+9·7	...	+13·4	−176·3	−90·7	−108·4	−200·9	−144·0
−0·6	...	−0·2	−1·6	−0·7	−1·0	...	−0·4	−0·5	−0·5
+29·3	−107·9	−34·2	...	−28·2	+140·7	−10·4	−47·3	...	+20·8
+282·7	+184·1	+58·9	+537·5	+265·8	−422·5	−275·7	−880·3	−1573·8	−788·0
Leguminosæ (continued).									
+3·6	−3·8	+2·3	+7·7	+2·5	−116·4	−17·8	−66·5	+10·3	−47·6
−89·5	−64·3	−25·6	−40·2	−54·9	−255·4	−205·9	−45·9	−69·9	−144·3
−42·6	−34·7	−32·0	−66·8	−44·0	−42·9	+12·8	+57·7	+24·6	+13·0
−28·0	+24·7	+58·2	+48·5	+25·9	−587·3	−254·6	−1479·5	−513·1	−708·5
...	...	+2·1	+19·8	+5·4	...	...	+2·1	+19·8	+5·4
−156·5	−78·1	+5·0	−31·0	−65·1	−1002·0	−465·5	−1532·1	−528·3	−882·0
Other Orders (continued).									
+42·0	−21·3	+25·4	+99·0	+36·3	+132·9	+28·8	+64·8	+164·3	+97·7
+1·9	−16·2	−26·1	−25·1	−16·4	−58·3	−328·9	−34·8	−60·8	−120·7
−18·4	−1·0	+8·6	+27·0	+4·1	−9·7	+60·1	+13·4	+31·5	+23·8
+4·5	−7·0	−16·7	−4·7	−6·0	+12·7	−22·5	+6·5	+15·3	+2·9
−0·5	+27·8	+66·0	+51·7	+36·3	−28·7	−71·7	−126·2	+66·1	−40·1
−40·9	−15·5	+11·7	+42·2	−0·6	+172·9	+293·0	+52·4	+112·3	+157·7
+85·4	+134·9	+20·7	+60·0	+75·2	+35·2	−202·3	+0·7	−236·0	−100·6
−18·6	−35·0	+7·0	−15·8	−15·7	−22·7	+33·1	−6·3	+0·5	+1·0
+18·4	+35·3	−32·5	−7·7	+3·4	+23·1	+79·9	−8·5	+14·7	+27·6
...	...	...	−5·1	−1·3	−4·9	−46·3	−13·6	−13·8	−19·7
+73·8	+102·0	+64·1	+221·5	+115·3	+252·5	−176·8	−51·6	−94·1	+29·5
Summary (continued).									
+282·7	+184·1	+58·9	+537·5	+265·8	−422·5	−275·7	−880·3	−1573·8	−788·0
−156·5	−78·1	+5·0	−31·0	−65·1	−1002·0	−465·5	−1532·1	−528·3	−882·0
+73·8	+102·0	+64·1	+221·5	+115·3	+252·5	−176·8	−51·6	−94·1	+29·5
+200	+208	+128	+728	+316	−1172	−918	−2464	−2008	−1640

The Miscellaneous plants which on the average have contributed rather more to the produce than without manure are—*Rumex Acetosa, Achillea Millefolium, Ranunculus repens,* and *R. bulbosus,* and, in a very insignificant degree, *Pimpinella Saxifraga;* whilst *Conopodium denudatum, Luzula campestris,* and *Centaurea nigra,* actually yield less than without manure.

The general result by superphosphate of lime shows, then, comparatively immaterial change from the condition of the unmanured herbage, whilst that which is observed, is mainly due to increased maturation, and not to increased luxuriance. The general aspect of the plot is that of comparatively puny and stemmy development of freer-growing species, or the predominance of those of small habit, according to the season. In the case of the grasses, the second growth is much more characterised by a large proportion of fine-leaf than is the first; the smaller grasses predominating. Among the Leguminosæ, *Lathyrus* shows less of prominence in the second than in the first crops. Among the Miscellaneæ the chief weeds attain a greater degree of maturity.

Thus, an examination of the botany of the superphosphate plot, and a comparison of it with that without manure, fully confirms the indications of the yield of hay, of nitrogen, and of mineral matter, and shows conclusively that the striking effects, both in increased yield and in development of individual species, which were produced on plot 7, with the *mixed* mineral manure, including potass, were but little, if at all, attributable to the superphosphate of lime which that complex manure contained.

6. *Mixed mineral manure, alone, with and without potass; Plot* 8.

As the superphosphate of lime of the mixed mineral manure was obviously not the potent agent, at least when used alone, the results must be traceable to the potass, the soda, or the magnesia salt, separately or conjointly, which were employed with it. For the first six years of the experiments, plots 7 and 8 each received the same mixed mineral manure, including potass; but from that time (1862), and since, the same manure has been still annually applied to plot 7, and the same, *excluding the potass,* to plot 8; an increased amount of soda salt being used instead. From the time of this change, not only the produce of hay, but the yield of nitrogen, and of mineral matter, diminished very greatly on plot 8, now without potass. Taking the results of the partial as well as of the complete separations, it appears that the *percentage* of total gramineous herbage has been almost uniformly greater in the smaller amount of produce yielded on plot 8 without, than in the larger amount on plot 7 with the potass. Without the potass the percentage of the Leguminosæ diminished exceedingly; whilst that of the miscellaneous species collectively rather increased. The yield per acre, however, of each of the three descriptions of herbage considerably diminished after the exclusion of the potass. As referred to more fully at pp. 316, *et seq.*, of Part I., and as will be shown in detail in Part III., from the time of the cessation of the application of the potass-salt, the amount of potass annually taken up by the crop declined in an extraordinary degree; and not only so, but the amount taken up of most of the other mineral constituents, soda and silica excepted, also declined. The percentage in the dry substance was, however, much the same in the produce of the two plots; whilst that of the potass was much less where it was not supplied, though still higher than without manure.

There is thus direct evidence of the influence of the potass of the manure on the yield, on the botany, and on the chemical composition, of the produce. But there is evidence of various kinds that there was a residue of the potass applied in the first six years, which had, at first, a considerable, and which has still, some effect on the results obtained on plot 8. Table LXX., pp. 1336–7, shows in detail the botanical composition of the herbage; and as the special point under consideration is the result as compared with that on plot 7, where the application of potass has been continued, there is given, not only the percentage botanical composition of the mixed herbage, the lbs. per acre of each species, and the increase or reduction compared with the unmanured plot 3, but also the increase or reduction in the yield of each, in each separation-year, as compared with plot 7.

It happens that 1862, the first year of complete botanical separation, was also the first year of the exclusion of the potass from the manure of plot 8, so that it is not to be expected that the results of that year will show so markedly the characteristic effects of the change as those of the subsequent periods.

Referring first to the number of species, there was in each year a less number than without manure; on the average of the four separation-years, six less. Compared with plot 7 there was a falling off in number in the first separation-year, that is in the first year of the exclusion of the potass, but a gradual increase subsequently; presumably due to a less active competition, with the less free growth, in the absence of the potass. There were, however, generally fewer species found in the samples than in those from plot 4–1, with superphosphate of lime alone, where there was still less of growth, and consequently still less of active competition.

An examination of the lines of total Gramineæ, total Leguminosæ, and total Miscellaneæ, will show that, in the second separation-year, that is after the effects of the change had become thoroughly established, there was a diminution, both in the percentage and in the yield per acre, of the grasses, a very great diminution, both in percentage and yield, of the leguminous herbage, and a greater increase in those of the miscellaneous herbage. But it will be remembered that the second separation-year was one pre-eminently favourable to a relative excess of the miscellaneous species; still, in the third and fourth separation-years, there was also a higher percentage proportion, and amount, of such herbage than in the first. There was at the same time a continued diminution in both the percentage and actual amount of the leguminous herbage; in fact, only about one-sixth as much was grown in the fourth as in the first separation-year. The total yield of the Gramineæ also diminished very much from the first to the third, but increased again considerably in the fourth separation-year. The fluctuations according to season were, indeed, very considerable. Still, especially as

TABLE LXX.—Number of Species, Percentage, Quantity per Acre, &c.,

	Number of species, and proportion per cent.					Quantity per acre.				
	1862.	1867.	1872.	1877.	Mean.	1862.	1867.	1872.	1877.	Mean.
NUMBER OF SPECIES.										
Gramineæ	17	17	17	16	17	...	...	...	...	...
Leguminosæ	4	4	4	4	4	...	...	...	...	...
Other Orders	17	21	21	28	22	...	...	...	...	...
Total	38	42	42	48	43	...	...	...	...	...
GRAMINEÆ.										
	p. c.	p. c.	p. c.	p. c.	p. c.	lbs.	lbs.	lbs.	lbs.	lbs.
Anthoxanthum odoratum	3·72	6·98	7·94	7·55	6·54	168·3	266·9	203·9	273·6	228·2
Agrostis vulgaris	10·01	4·32	9·32	12·40	9·01	452·9	165·2	239·3	449·4	326·7
Holcus lanatus	4·51	10·25	4·61	18·22	9·40	204·0	391·9	118·4	660·3	343·7
Avena elatior	4·52	3·16	4·40	3·17	3·81	204·5	120·9	113·0	114·9	138·3
Avena pubescens	12·68	3·44	3·66	1·67	5·36	573·6	131·5	94·0	60·5	214·9
Avena flavescens	5·42	3·52	6·94	2·45	4·58	245·2	134·6	178·2	88·8	161·7
Poa pratensis	1·72	1·50	2·11	1·03	1·59	77·8	57·4	54·2	37·3	56·7
Poa trivialis	5·48	3·48	1·62	3·20	3·45	247·9	133·1	41·6	116·0	134·7
Briza media	0·07	0·08	1·16	0·57	0·47	3·2	3·0	29·8	20·6	14·1
Dactylis glomerata	3·50	1·48	0·66	0·98	1·66	158·3	56·6	16·9	35·5	66·8
Cynosurus cristatus	0·25	0·24	1·02	1·14	0·66	11·3	9·2	26·2	41·3	22·0
Festuca ovina	7·51	17·74	23·95	19·76	17·24	339·8	678·4	615·0	716·1	587·3
Festuca pratensis	2·21	0·42	0·33	0·54	0·88	100·0	16·1	8·5	19·6	36·1
Bromus mollis	1·38	0·43	0·09	0·01	0·48	62·4	16·4	2·3	0·4	20·4
Lolium perenne	5·92	2·61	1·92	7·63	4·52	267·8	99·8	49·3	276·5	173·3
Not yielding 1 per cent. } Alopecurus pratensis, Aira cæspitosa	0·40	0·89	0·53	0·87	0·68	18·1	34·0	13·7	31·5	24·3
Absent. Phleum pratense, Festuca loliacea	...	...	...	...	...	...	...	...	...	...
Undetermined (chiefly Gramineæ)	2·39	2·49	1·30	...	1·54	108·2	95·3	33·3	...	59·1
Total	71·69	63·03	71·56	81·19	71·87	3243·3	2410·3	1837·6	2942·3	2608·3
LEGUMINOSÆ.										
Trifolium repens	2·70	0·10	0·25	0·10	0·78	122·1	3·8	6·4	3·6	34·0
Trifolium pratense	7·71	1·13	0·27	0·36	2·37	348·8	43·2	6·9	13·0	103·0
Lotus corniculatus	0·15	0·83	3·51	1·18	1·42	6·8	31·7	90·2	42·8	42·9
Lathyrus pratensis	8·76	6·82	3·94	2·37	5·47	396·3	260·8	101·2	85·9	211·0
Ononis arvensis	...	...	...	...	...	...	...	...	...	...
Total	19·32	8·88	7·97	4·01	10·04	874·0	339·5	204·7	145·3	390·9
OTHER ORDERS.										
Ranunculus acris	...	0·66	0·47	1·09	0·55	...	25·3	12·0	39·5	19·2
Ranunculus repens et bulbosus	1·16	0·55	1·00	0·65	0·84	52·5	21·0	25·7	23·5	30·7
Conopodium denudatum	1·59	6·84	1·73	1·07	2·81	71·9	261·3	44·4	38·8	104·1
Pimpinella Saxifraga	0·81	1·91	1·05	0·67	1·11	36·6	73·1	27·0	24·3	40·2
Achillea Millefolium	0·93	4·89	9·75	2·76	4·58	42·1	187·0	250·4	100·0	144·9
Plantago lanceolata	0·71	1·53	0·34	0·26	0·71	32·1	58·5	8·7	9·4	27·2
Rumex Acetosa	1·93	7·86	1·96	5·84	4·39	87·3	300·6	50·3	211·6	162·5
Luzula campestris	0·75	1·76	2·66	0·48	1·41	33·9	67·3	68·3	17·4	46·7
None yielding 1 per cent. { Stellaria graminea, Cerastium triviale, Potentilla reptans, Agrimonia Eupatoria, Spiræa Ulmaria, Heracleum Sphondylium, Galium verum, G. Aparine, Scabiosa arvensis, Centaurea nigra, Tragopogon pratensis, Taraxacum officinale, Veronica Chamædrys, Prunella vulgaris, Ajuga reptans, Primula veris, Carex præcox, Hypnum squarrosum, H. rutabulum, H. hians	1·11	2·09	1·51	1·98	1·69	50·3	80·1	38·9	71·9	60·3
Absent { Poterium Sanguisorba, Bellis perennis, Chrysanthemum Leucanthemum, Leontodon hispidus, Hieracium Pilosella, Thymus Serpyllum, Scilla nutans, Ophioglossum vulgatum	...	...	...	...	...	...	...	...	...	...
Total	8·99	28·09	20·47	14·80	18·09	406·7	1074·2	525·7	536·4	635·8
SUMMARY.										
Gramineæ	71·69	63·03	71·56	81·19	71·87	3243·3	2410·3	1837·6	2942·3	2608·3
Leguminosæ	19·32	8·88	7·97	4·01	10·04	874·0	339·5	204·7	145·3	390·9
Other Orders	8·99	28·09	20·47	14·80	18·09	406·7	1074·2	525·7	536·4	635·8
Total	100·00	100·00	100·00	100·00	100·00	4524	3824	2568	3624	3635

of each Species, by Mixed Mineral Manure, without Potass; Plot 8.

Increase + or decrease − compared with:—														
Plot 3, without manure.					Plot 7, mixed mineral manure (with potass).					Plot 4-1, superphosphate, alone.				
1862.	1867.	1872.	1877.	Mean.	1862.	1867.	1872.	1877.	Mean.	1862.	1867.	1872.	1877.	Mean.
NUMBER OF SPECIES (continued).														
−1	+2	0	−1	0	−1	+1	0	−1	0	+1	+2	+1	0	+1
0	0	0	0	0	0	0	0	0	0	0	0	−1	−1	−1
−11	−3	−7	−3	−6	−5	−1	+1	+5	0	−9	−6	−7	+4	−4
−12	−1	−7	−4	−6	−6	0	+1	+4	0	−8	−4	−7	+3	−4
GRAMINEÆ (continued).														
lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
+ 37·7	− 21·6	+118·4	+ 152·6	+ 71·8	+ 32·9	+ 91·7	+ 88·7	+ 111·4	+ 81·2	+ 49·3	+ 13·4	+119·9	+115·6	+ 74·6
+ 106·2	−122·3	− 26·0	+ 135·5	+ 23·3	+137·0	− 88·5	− 257·1	− 163·6	− 93·1	+ 218·4	− 50·0	− 6·6	+144·2	+ 76·5
+ 50·2	+126·4	+ 59·2	+ 363·6	+ 149·9	− 19·9	−134·6	− 15·4	− 10·9	− 45·1	− 180·4	+ 67·6	+ 34·9	+ 62·0	− 3·9
+ 202·4	+113·9	+110·9	+ 113·7	+ 135·2	+ 97·9	+118·2	+ 93·5	+ 49·1	− 89·7	+ 200·3	+114·5	+110·3	+113·4	+134·6
+ 279·1	+ 29·2	+ 35·6	− 3·1	+ 85·2	− 37·4	− 42·4	− 6·0	− 54·2	− 35·0	+ 267·2	− 44·4	+ 21·5	− 63·8	+ 45·1
+ 172·8	+ 72·6	+120·8	+ 63·3	+ 107·4	+ 67·4	− 81·2	+ 20·6	− 97·3	− 22·6	+ 111·2	− 16·9	+ 70·3	+ 12·4	+ 44·2
+ 68·9	+ 51·7	+ 52·7	+ 35·6	+ 52·3	+ 27·8	+ 10·6	− 42·0	− 52·0	− 13·9	+ 59·6	+ 48·2	+ 46·2	+ 36·7	+ 47·7
+ 200·9	+ 94·1	+ 33·4	+ 102·8	+ 107·8	+ 79·3	− 62·2	− 55·8	+ 8·4	− 7·5	+ 80·1	− 66·9	− 25·5	− 30·0	− 10·5
− 54·5	− 19·7	− 75·4	− 150·8	− 75·1	+ 1·9	+ 0·3	+ 25·6	+ 13·5	+ 10·3	− 15·7	− 8·3	− 7·8	− 46·2	− 19·6
+ 104·6	− 1·4	+ 2·1	+ 18·9	+ 31·0	+ 44·6	−151·6	− 54·3	− 151·7	− 78·3	+ 85·1	+ 21·6	+ 6·8	− 8·1	+ 26·3
+ 6·1	+ 4·9	+ 8·0	+ 18·1	+ 9·3	+ 7·8	− 0·6	+ 22·4	+ 36·2	+ 16·4	+ 0·3	− 20·5	+ 8·0	+ 16·6	+ 1·1
− 66·1	+171·9	+258·8	+ 198·6	+ 140·8	−267·6	+171·0	− 14·5	− 640·0	−187·8	+ 8·1	+ 85·5	+252·8	+220·7	+141·8
+ 98·8	+ 16·1	+ 8·3	+ 19·4	+ 35·7	+ 92·5	− 12·4	+ 3·8	− 28·8	+ 13·8	+ 95·8	+ 16·1	+ 7·1	+ 19·0	+ 34·5
+ 58·4	+ 14·7	+ 2·1	+ 0·4	+ 18·9	+ 6·7	− 27·1	+ 0·6	− 0·1	− 4·9	+ 45·5	+ 1·2	+ 0·7	0·0	+ 11·9
+ 73·4	− 34·5	+ 10·3	+ 168·9	+ 54·5	+129·8	− 7·2	+ 24·3	+ 122·5	+ 67·3	− 34·0	− 85·9	− 6·0	+142·0	+ 4·0
− 119·2	−159·9	+ 5·0	+ 23·5	− 62·7	+ 2·6	− 5·2	− 36·3	+ 6·5	− 8·2	− 24·8	− 31·1	− 1·5	− 11·7	− 17·3
− 0·3	...	...	− 0·7	− 0·3	− 0·5	...	...	...	− 0·1	...	...	...	...	...
− 131·1	−109·2	− 15·4	...	− 64·0	− 19·7	− 11·7	− 28·5	...	− 15·0	− 160·4	− 1·3	+ 18·8	...	− 35·8
+1088·3	+226·9	+708·8	+1260·3	+ 821·0	+383·1	−232·9	− 230·4	− 851·0	−232·8	+ 805·6	+ 42·8	+649·9	+722·8	+555·2
LEGUMINOSÆ (continued).														
+ 105·9	− 3·2	+ 0·2	+ 0·5	+ 25·9	− 14·1	− 17·2	− 68·6	+ 3·1	− 24·2	+ 102·3	+ 0·6	− 2·1	− 7·2	+ 23·4
+ 212·1	− 27·1	− 20·7	− 36·4	+ 32·0	+ 46·2	−168·7	− 41·0	− 66·1	− 57·4	+ 301·6	+ 37·2	+ 4·9	+ 3·8	+ 86·9
− 49·1	− 46·6	− 7·5	− 50·6	− 38·4	− 49·4	+ 0·9	+ 82·2	+ 40·8	+ 18·6	− 6·5	− 11·9	+ 24·5	+ 16·2	+ 5·6
+ 357·9	+238·1	+ 85·1	+ 29·9	+ 177·7	−201·4	− 41·2	−1452·6	− 531·7	−556·7	+ 385·9	+213·4	+ 26·9	− 18·6	+151·8
...	...	...	...	...	...	...	...	...	...	...	...	− 2·1	− 19·8	− 5·4
+ 626·8	+161·2	+ 57·1	− 56·6	+ 197·2	−218·7	−226·2	−1480·0	− 553·9	−619·7	+ 783·3	+239·3	+ 52·1	− 25·6	+262·3
OTHER ORDERS (continued).														
− 0·3	+ 25·0	+ 10·8	+ 39·5	+ 18·7	− 3·5	+ 20·0	+ 3·9	+ 20·6	+ 10·3	− 1·0	+ 21·8	+ 9·9	+ 31·5	+ 15·5
− 96·4	− 46·0	− 22·6	− 58·1	− 55·7	− 5·5	+ 4·1	+ 16·8	+ 7·2	+ 5·7	− 138·4	− 24·7	− 48·0	−157·1	− 92·0
+ 42·3	+163·0	− 2·4	− 6·1	+ 49·2	− 17·9	−149·7	− 11·1	− 41·8	− 55·1	+ 40·4	+179·2	+ 23·7	+ 19·0	+ 65·6
− 10·1	− 41·5	+ 12·9	+ 5·5	− 8·3	− 1·4	+ 19·6	+ 17·7	+ 10·0	+ 11·4	+ 8·3	− 40·5	+ 4·3	− 21·5	− 12·4
− 4·6	+148·3	+221·1	+ 53·0	+ 104·5	− 32·8	+ 48·8	+ 28·9	+ 67·4	+ 28·1	− 4·1	+120·5	+155·1	+ 1·3	+ 68·2
− 191·9	−299·0	− 35·0	− 65·3	− 147·8	+ 21·9	+ 9·5	+ 5·7	+ 4·8	+ 10·5	− 151·0	−283·5	− 46·7	−107·5	−147·2
+ 44·6	+241·9	+ 21·2	+ 167·4	+ 118·8	− 5·6	− 95·3	+ 1·2	− 128·6	− 57·0	− 40·8	+107·0	+ 0·5	+107·4	+ 43·6
− 24·4	− 53·0	+ 12·1	− 24·4	− 22·4	− 28·5	+ 15·1	− 1·2	− 8·1	− 5·7	− 5·8	− 18·0	+ 5·1	− 8·6	− 6·7
+ 0·4	− 2·9	− 30·7	+ 3·4	− 8·0	+ 14·6	− 11·3	− 8·5	+ 2·5	− 0·7	− 4·1	− 8·8	− 5·4	+ 6·1	− 3·2
− 2·7	− 29·9	− 29·3	− 58·6	− 30·2	− 5·7	− 35·7	− 11·0	− 5·1	− 14·4	− 20·4	− 51·1	− 4·5	− 35·8	− 27·9
− 243·1	+103·9	+158·1	+ 56·3	+ 18·8	− 64·4	−174·9	+ 42·4	− 71·1	− 67·0	− 316·9	+ 1·9	+ 94·0	−165·2	− 96·5
SUMMARY (continued).														
+1088·3	+226·9	+708·8	+1260·3	+ 821·0	+383·1	−232·9	− 230·4	− 851·0	−232·8	+ 805·6	+ 42·8	+649·9	+722·8	+555·2
+ 626·8	+161·2	+ 57·1	− 56·6	+ 197·2	−218·7	−226·2	−1480·0	− 553·9	−619·7	+ 783·3	+239·3	+ 52·1	− 25·6	+262·3
− 243·1	+103·9	+158·1	+ 56·3	+ 18·8	− 64·4	−174·9	+ 42·4	− 71·1	− 67·0	− 316·9	+ 1·9	+ 94·0	−165·2	− 96·5
+1472	+492	+924	+1260	+1037	+100	−634	−1668	−1476	−919	+1272	+284	+796	+532	+721

shown by the intermediate and recent partial separations, the tendency to decrease in total yield of Gramineæ, to decrease in a greater degree (in proportion to their amount) in the yield of the Leguminosæ, and to decrease somewhat in that of the miscellaneous species also, cannot be doubted. The most prominent and unquestionable of all these results was, however, the great and continuous decline in leguminous growth after the exclusion of the potass.

Looking to the particulars relating to the individual species, the first point to note is the large number of species, especially of the grasses, contributing fairly to the yield, and the comparatively small number asserting very marked predominance, these being the indications of languid struggle. Among the grasses, *Festuca ovina* and *Holcus lanatus* are the only species which have very obviously gained ground; the latter, however, showing considerable fluctuation from one separation-year to another. *Agrostis vulgaris* has also yielded a comparatively large proportion, and has, upon the whole, increased in relative amount. The meagre-growing *Anthoxanthum odoratum,* though giving a less average percentage than any of the foregoing, has, nevertheless, continuously increased.

Among the grasses which have the most obviously diminished in percentage are, *Avena pubescens,* which has done so the most strikingly of all, *Poa trivialis, Dactylis glomerata, Festuca pratensis,* and *Bromus mollis; Lolium perenne* doing so up to the third separation-year, but afterwards regaining ground. Those which have shown comparative indifference, fluctuating from season to season without any very obvious tendency, independently of season, either to increase or diminish, are *Avena elatior, Avena flavescens,* and, though occurring in much smaller amount, *Poa pratensis.*

Comparing the actual yield per acre with that without manure, it is seen that, among the grasses yielding more than one per cent. to the herbage, there was, on the average, and almost uniformly in detail, an increased amount of every species, excepting *Briza media,* which alone was in each separation-year in less quantity than without manure. The grasses which give the largest increase compared with the unmanured plot are *Holcus lanatus, Festuca ovina, Avena elatior, Avena flavescens,* and *Poa trivialis.* It is thus obvious that the conditions, either owing to the constituents still supplied, or to the residue of the previous applications of potass, were still favourable to an increased growth of Gramineæ.

Compared with the produce of plot 7, with the continuous application of potass, there are, on the other hand, nine grasses that give, on the average, less on plot 8, and only six which give, on the average, more; the balance as to total quantity being considerably and increasingly against plot 8. Indeed, if the results of the first separation-year—that is, of the first year of change—were not included in the general mean, the result would be very much more strikingly against plot 8, as compared with plot 7, in the yield of grasses. As the table stands, the grasses which yield actually more without than with the potass are *Avena elatior, Anthoxanthum odoratum, Lolium perenne,* and, in much less degree, *Cynosurus cristatus, Festuca pratensis,* and

Briza media; whilst those which are the most prominently reduced on plot 8 are *Festuca ovina, Agrostis vulgaris, Dactylis glomerata, Holcus lanatus,* and, in a less degree, *Avena pubescens, Avena flavescens, Poa pratensis, Poa trivialis,* and *Bromus mollis.*

On the other hand, as compared with plot 4–1, with superphosphate of lime alone, plot 8, with its continuous supply of soda and magnesia, and its residue from previous applications of potass, in addition to the superphosphate, yielded, on the average, more, and sometimes considerably more, of almost every gramineous species, the excess being the most prominent in the case of *Festuca ovina* and *Avena elatior*.

The results of the complete separations show (and the fact is fully borne out by observation in other years) that the great reduction in leguminous herbage, consequent on the exclusion of the potass, is mainly due to a greatly reduced growth of *Lathyrus pratensis,* but also to a decreased growth of *Trifolium pratense* and *Trifolium repens;* whilst, though in less amount than without manure, there is on plot 8, compared with plot 7, even a slight increase in the yield of the deep-rooting, self-reliant *Lotus corniculatus.* In fact, it will be found throughout the experiments that the Leguminosæ are only specially favoured by purely mineral manures containing potass, and that, under the influence of such comparatively superficial supply of potass, it is the *Lathyrus pratensis* which becomes by far the most prominent.

As has been said, the proportion in the mixed herbage of the collective miscellaneous species has been, upon the whole, increased since the exclusion of the potass; but, as with the grasses and the Leguminosæ, the yield per acre of the Miscellaneæ has declined compared with plot 7, though there is still an excess compared with plot 3 without manure. The miscellaneous species which have maintained the greatest prominence throughout are *Rumex Acetosa* and *Achillea Millefolium, Conopodium* coming next in this respect; but there has been very great fluctuation in the proportion and amount of collective and individual miscellaneous species from year to year, according to season. In the first separation-year (1862), there was a small proportion and amount of the collective Miscellaneæ, and no one individual species was in marked excess. In the second separation-year there was more than three times as high a percentage, and more than two and a-half times as much actual yield, of such produce, *Rumex Acetosa, Conopodium denudatum,* and *Achillea Millefolium* each being in excessive amount. In the third separation-year there was less percentage, and only about half as much actual yield, as in the second; and now *Achillea Millefolium* yielded nearly half the total miscellaneous herbage, and no other species was very specially prominent. Lastly, in the fourth separation-year, with a fair total amount, *Rumex Acetosa* contributed about two-fifths of the whole, *Achillea Millefolium* about one-fifth, and no other species any material amount. Taking the average of the four separation-years, the order of prominence in actual yield among the Miscellaneæ on plot 8 was as follows: *Rumex Acetosa, Achillea Millefolium, Conopodium denudatum,* and in much less quantity, *Luzula campestris, Pimpinella Saxifraga, Ranunculus repens* and *bulbosus, Plantago lanceolata,* and *Ranunculus acris.*

Compared with the produce without manure, plot 8 gave in each separation-year a greater or less excess of *Rumex Acetosa,* and in three out of the four years of *Achillea Millefolium;* whilst in each year it gave considerably less of *Plantago lanceolata,* and, in a less degree, of *Ranunculus repens* and *bulbosus.*

Compared with plot 7 (the result, therefore, being due to the exclusion of potass), *Achillea Millefolium, Plantago lanceolata, Pimpinella Saxifraga,* and the various species of *Ranunculus* were generally in excess, whilst *Rumex Acetosa* and *Conopodium denudatum* were mostly in reduced amount.

Lastly, compared with the results by superphosphate of lime alone, plot 8 gave, on the average, some excess of *Achillea Millefolium, Conopodium denudatum, Rumex Acetosa,* and in less degree of *Ranunculus acris;* but a considerable deficiency of *Plantago lanceolata* and of *Ranunculus repens* and *bulbosus,* and, to some extent, of *Pimpinella Saxifraga* and *Luzula campestris.*

Upon the whole, the produce by the mixed mineral manure, including potass during the earlier years, but excluding it during the greater part of the total period, has presented fairly even growth, with considerably mixed herbage; and, since the exclusion of the potass, both restricted luxuriance and maturation as compared with plot 7, and a gradual but very marked diminution in the proportion and amount of the Leguminosæ, especially of the more superficially-feeding *Lathyrus pratensis; Lotus,* on the other hand, having increased.

Among the Grasses a large number contributed to the herbage, and although *Agrostis vulgaris, Festuca ovina,* and *Holcus lanatus* are in the highest relative prominence, each has considerably reduced in actual yield since the exclusion of the potass, as also have *Dactylis glomerata* and a number of others in less degree; whilst plants of such opposite characters as *Avena elatior, Anthoxanthum odoratum,* and even *Lolium perenne,* have increased in actual yield as compared with their produce on plot 7.

Among the Miscellaneæ the most prominent, though each very variable in amount from year to year according to season, are *Achillea Millefolium, Rumex Acetosa,* and *Conopodium denudatum,* the first being in excess of the amount on plot 7, the other two considerably in defect.

Lastly, comparing the results without manure, with superphosphate of lime alone, with the mixed mineral manure including potass, and with the same excluding potass, it is manifest that the effects, both as to quantity and to botanical and chemical composition, are more dependent upon the supply of the potass than of any of the other constituents.

One more striking point in regard to the botany of this plot is that it is here, where there is no nitrogenous manure to induce luxuriance, and where there is relative deficiency of potass, that, as on plot 4–1, where there was equally no nitrogenous supply and less potass still, fairy-rings are of very frequent occurrence, whilst they are scarcely observed on any other plot; and this is so notwithstanding the fact that fungi are among the most highly nitrogenous of plants, and are also very rich in potass. The principal fungus met with on the rings is the common Champignon, *Marasmius oreades;* and the grass of the rings is extremely luxuriant, the species most favoured being *Poa trivialis* and *Holcus lanatus;* whilst among the Leguminosæ *Lathyrus pratensis* is the most prominent, and among the Miscellaneæ *Rumex Acetosa* and *Conopodium denudatum* are particularly luxuriant. On the comparatively exhausted space within the rings every description of herbage is much less luxuriant, and the finer-leaved grasses again become more predominant. Further particulars on this subject will be found in a note by one of us, "On the Occurrence of Fairy-rings," Jour. Linn. Soc. (1875), Botany, vol. 15, p. 17, *et seq.;* and the chemical aspects of the question will be further treated of in the chemical section (Part III.) of our report.

7. *Ammonia-salts* (400 *lbs. per acre*), *with mixed mineral manure, including potass; Plot* 9.

Thus far we have considered the results obtained without manure, by various nitrogenous manures alone, and by various mineral manures alone. We have now to call attention to those yielded by mixtures of the nitrogenous and mineral manures. Among these the first to consider are the effects produced by a mixture of the same description and amount of ammonia-salts as were used alone on plot 5, and the same mixed mineral manure (including potass) as was used alone on plot 7. Accordingly, in the table (LXXI., pp. 1342–3), besides the percentage of each species, its produce per acre, and its greater or less amount than without manure, in each separation-year, there is given the increase or decrease of each species, in each separation-year, compared with the corresponding results obtained, on the one hand on plot 5 with the ammonia-salts alone, and on the other on plot 7 with the mixed mineral manure alone. It should also be stated that there has been no change in the manuring of either of these three plots from the commencement.

With the mixture of both nitrogenous and mineral manure there was more than twice as much produce as without manure, nearly twice as much as with ammonia-salts alone, and nearly one and a half time as much as with the mineral manure alone. There was some falling-off in the later compared with the earlier years of the first 20, though in a less degree than with the ammonia-salts alone; whilst with the mineral manure alone there was increase rather than diminution. With the mixture, however, there was, during the next five years, a higher rate of produce, even of first crops, than during the first 20, notwithstanding that in the later years the second crops have been removed from the land. With the nitrogen supplied in conjunction with minerals there was a considerably greater yield of nitrogen in the produce than with either the ammonia-salts alone, or the mixed mineral manure alone; and of mineral matter taken up per acre, there was more than twice as much as with the ammonia-salts alone, and considerably more than with the mineral manure alone. In a word, with the combination of both nitrogenous and mineral manure, there was more hay,

TABLE LXXI.—Number of Species, Percentage, Quantity per Acre, &c., of each Species,

	Number of species, and proportion per cent.					Quantity per acre.				
	1862.	1867.	1872.	1877.	Mean.	1862.	1867.	1872.	1877.	Mean.
NUMBER OF SPECIES.										
Gramineæ	13	14	16	13	14	...	...	...	...	...
Leguminosæ	2	2	1	4	2	...	...	...	...	...
Other Orders	13	13	13	10	12	...	...	...	...	...
Total	28	29	30	27	28	...	...	...	...	...
GRAMINEÆ.										
	p. c.	p. c.	p. c.	p. c.	p. c.	lbs.	lbs.	lbs.	lbs.	lbs.
Anthoxanthum odoratum	1·24	3·59	2·25	2·94	2·51	79·4	193·2	127·3	178·0	144·5
Alopecurus pratensis	0·27	0·07	2·76	0·96	1·01	17·3	3·8	156·2	58·2	58·9
Agrostis vulgaris	12·81	13·43	15·46	12·23	13·48	820·1	722·8	874·7	740·4	789·5
Holcus lanatus	12·14	9·84	7·61	10·37	9·99	777·2	529·6	430·5	627·8	591·3
Avena elatior	...	2·50	11·40	13·23	6·78	...	134·6	645·0	801·0	395·1
Avena pubescens	10·22	1·41	0·49	0·07	3·04	654·3	75·9	27·7	4·2	190·5
Avena flavescens	9·08	3·78	5·30	0·67	4·71	581·3	203·4	299·8	40·5	281·2
Poa pratensis	10·68	13·02	22·67	18·03	16·10	683·7	700·7	1282·7	1091·5	939·7
Poa trivialis	8·72	2·14	0·64	0·11	2·90	558·3	115·2	36·2	6·7	178·1
Dactylis glomerata	5·58	4·64	11·88	14·07	9·04	357·2	249·7	672·2	851·8	532·8
Festuca ovina	5·21	18·42	8·68	21·80	13·52	333·5	991·4	491·1	1319·8	784·0
Festuca pratensis	1·42	0·07	0·03	...	0·38	90·9	3·8	1·7	...	24·1
Bromus mollis	4·46	0·11	0·10	...	1·17	285·5	5·9	5·7	...	74·3
Lolium perenne	4·20	1·01	1·11	0·16	1·62	268·9	54·3	62·8	9·7	98·9
None yielding 1 per cent. { Aira cæspitosa, Briza media, Cynosurus cristatus }	...	...	0·02	0·01	0·03	...	...	1·2	0·6	0·3
Absent—Phleum pratense, Festuca loliacea.	...	...	...	...	...	...	...	...	...	...
Undetermined (chiefly Gramineæ).	2·56	3·03	1·79	...	1·84	164·0	163·1	101·3	...	107·1
Total	88·59	77·06	92·19	94·65	88·12	5671·6	4147·4	5216·1	5730·2	5191·3
LEGUMINOSÆ.										
Trifolium repens	0·01	0·01	...	...	0·01	0·6	0·5	...	...	0·3
Trifolium pratense	...	...	...	0·01	0·01	...	...	...	0·6	0·1
Trifolium minus	...	...	...	0·01	0·01	...	...	...	0·6	0·1
Lathyrus pratensis	0·12	0·15	0·02	0·06	0·08	7·7	8·1	1·1	3·6	5·2
Ononis arvensis	...	...	...	0·33	0·07	...	...	...	20·0	5·0
Lotus corniculatus	...	...	...	...	...	...	...	...	...	...
Total	0·13	0·16	0·02	0·41	0·18	8·3	8·6	1·1	24·8	10·7
OTHER ORDERS.										
Conopodium denudatum	2·92	9·35	1·51	0·75	3·62	186·9	503·2	85·4	45·4	205·2
Achillea Millefolium	1·95	2·03	1·50	0·04	1·37	124·8	109·3	84·8	2·4	80·3
Rumex Acetosa.	5·40	10·89	4·60	3·60	6·12	345·7	586·2	260·2	218·0	352·5
None yielding 1 per cent. { Ranunculus repens et bulbosus, Stellaria graminea, Potentilla reptans, Spiræa Ulmaria, Pimpinella Saxifraga, Heracleum Sphondylium, Galium verum, Scabiosa arvensis, Centaurea nigra, Tragopogon pratensis, Taraxacum officinale, Plantago lanceolata, Veronica Chamædrys, Scilla nutans, Ornithogalum umbellatum, Luzula campestris, Carex præcox, Hypnum squarrosum, H. rutabulum, H. hians. }	1·01	0·51	0·18	0·55	0·59	64·7	27·3	10·4	33·2	34·0
Absent { Ranunculus acris, Cerastium triviale, Agrimonia Eupatoria, Poterium Sanguisorba, Galium Aparine, Bellis perennis, Chrysanthemum Leucanthemum, Leontodon hispidus, Hieracium Pilosella, Prunella vulgaris, Thymus Serpyllum, Ajuga reptans, Primula veris, Ophioglossum vulgatum. }	...	...	...	...	...	...	...	...	...	...
Total	11·28	22·78	7·79	4·94	11·70	722·1	1226·0	440·8	299·0	672·0
SUMMARY.										
Gramineæ	88·59	77·06	92·19	94·65	88·12	5671·6	4147·4	5216·1	5730·2	5191·3
Leguminosæ	0·13	0·16	0·02	0·41	0·18	8·3	8·6	1·1	24·8	10·7
Other Orders	11·28	22·78	7·79	4·94	11·70	722·1	1226·0	440·8	299·0	672·0
Total	100·00	100·00	100·00	100·00	100·00	6402·0	5382·0	5658·0	6054·0	5874·0

by 400 lbs. Ammonia-salts, with Mixed Mineral Manure, including Potass ; Plot 9.

Increase + or decrease − compared with :—														
Plot 3, without manure.					Plot 5, ammonia-salts alone.					Plot 7, mixed mineral manure (with potass) alone.				
1862.	1867.	1872.	1877.	Mean.	1862.	1867.	1872.	1877.	Mean.	1862.	1867.	1872.	1877.	Mean.
Number of Species (continued).														
− 5	− 1	− 1	− 4	− 3	− 4	−1	+1	0	−1	− 5	− 2	− 1	− 4	− 3
− 2	− 2	− 3	0	− 2	− 2	−2	−2	+2	−1	− 2	− 2	− 3	− 0	− 2
−15	−11	−15	−21	−16	− 4	−4	0	−4	−3	− 9	− 9	− 7	−13	−10
−22	−14	−19	−25	−21	−10	−7	−1	−2	−5	−16	−13	−11	−17	−15
Gramineæ (continued).														
lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
− 51·2	− 95·3	+ 41·8	+ 57·0	− 11·9	− 143·0	+ 11·0	+ 50·9	+ 57·3	− 5·9	− 56·0	+ 18·0	+ 12·1	+ 15·8	− 2·5
− 119·7	− 190·1	+ 147·7	+ 51·1	− 27·7	− 7·7	− 11·7	+ 135·4	+ 51·4	+ 41·9	+ 2·3	− 35·4	+ 106·6	+ 33·7	+ 26·8
+ 473·4	+ 435·3	+ 609·4	+ 426·5	+ 486·1	− 116·4	+ 29·5	+ 205·5	− 129·3	− 2·7	+ 504·2	+ 469·1	+ 378·3	+ 127·4	+ 369·7
+ 623·4	+ 264·1	+ 371·3	+ 331·1	+ 397·5	+ 388·7	+ 359·3	+ 382·8	+ 538·9	+ 417·5	+ 553·3	+ 3·1	+ 296·7	− 43·4	+ 202·5
− 2·1	+ 127·6	+ 642·9	+ 799·8	+ 392·0	− 151·4	+ 42·7	+ 607·6	+ 794·2	+ 323·2	− 106·6	+ 131·9	+ 625·5	+ 735·2	+ 346·5
+ 359·8	− 26·4	− 30·7	− 59·4	+ 60·8	+ 372·6	+ 55·1	+ 21·7	+ 0·7	+ 112·5	+ 43·3	− 98·0	− 72·3	− 110·5	− 59·4
+ 508·9	+ 141·4	+ 242·4	+ 15·0	+ 226·9	+ 556·3	+ 188·2	+ 295·2	+ 40·2	+ 269·9	+ 403·5	− 12·4	+ 142·2	− 145·6	+ 96·9
+ 674·8	+ 695·0	+1281·2	+1089·8	+ 935·3	+ 642·5	+ 679·2	+1267·4	+1084·7	+ 918·5	+ 633·7	+ 653·9	+1186·5	+1002·2	+ 869·1
+ 511·3	+ 76·2	+ 28·0	− 6·5	+ 152·2	+ 523·8	+ 105·3	+ 21·4	+ 6·4	+ 164·2	+ 389·7	− 80·1	− 61·2	− 100·9	+ 36·9
+ 303·5	+ 191·7	+ 657·4	+ 835·2	+ 497·0	+ 265·1	+ 203·7	+ 654·6	+ 755·9	+ 469·9	+ 243·5	+ 41·5	+ 601·0	+ 694·6	+ 387·7
− 72·4	+ 484·9	+ 134·9	+ 802·3	+ 337·5	− 514·0	− 19·2	− 679·4	− 253·9	− 366·5	− 273·9	+ 484·0	− 138·4	− 36·3	+ 8·9
+ 89·7	+ 3·8	+ 1·5	− 0·2	+ 23·7	+ 77·4	− 0·5	+ 1·2	...	+ 19·5	+ 83·4	− 24·7	− 3·0	− 48·4	+ 1·8
+ 281·5	+ 4·2	+ 5·5	...	+ 72·8	+ 282·4	+ 5·9	+ 5·7	...	+ 73·5	+ 229·8	− 37·6	+ 4·0	− 0·5	+ 49·0
+ 74·5	− 80·0	+ 23·8	− 97·9	− 19·9	+ 140·6	+ 14·3	+ 38·5	+ 7·0	+ 50·1	+ 130·9	− 52·7	+ 37·8	− 144·3	− 7·1
− 63·2	− 27·0	− 122·4	− 194·9	− 102·0	− 3·1	− 0·6	+ 0·8	...	− 0·9	− 5·3	− 12·5	− 7·2	− 12·1	− 9·5
− 0·3	...	...	− 0·7	− 0·3	...	...	...	...	...	− 0·5	...	...	...	− 0·1
− 75·3	− 41·4	+ 52·6	...	− 16·0	+ 31·0	+ 109·8	+ 77·5	...	+ 54·6	+ 36·1	+ 56·1	+ 39·5	...	+ 33·0
+3516·6	+1964·0	+4087·3	+4048·2	+3404·0	+2344·8	+1772·0	+3086·8	+2953·5	+2539·2	+2811·4	+1504·2	+3148·1	+1936·9	+2350·2
Leguminosæ (continued).														
− 15·6	− 6·5	− 6·2	− 3·1	− 7·8	+ 0·2	+ 0·2	− 0·3	...	...	− 135·6	− 20·5	− 75·0	− 0·5	− 57·9
− 136·7	− 70·3	− 27·6	− 48·8	− 70·9	− 1·5	− 0·3	...	+ 0·6	− 0·3	− 302·6	− 211·9	− 47·9	− 78·5	− 160·3
...	...	...	+ 0·6	+ 0·1	...	...	...	+ 0·6	+ 0·1	...	...	...	+ 0·6	+ 0·1
− 48·2	− 70·2	− 96·6	− 89·8	− 76·1	+ 6·9	+ 7·8	+ 0·1	+ 2·1	+ 4·3	− 590·0	− 293·9	−1552·7	− 614·0	− 762·5
...	...	...	+ 20·0	+ 5·0	...	...	...	+ 20·0	+ 5·0	...	...	...	+ 20·0	+ 5·0
− 38·4	− 22·7	− 16·1	− 56·0	− 33·3	− 1·9	− 10·3	− 10·3	− 4·1	− 6·7	− 56·2	− 30·8	− 8·0	− 2·0	− 24·3
− 238·9	− 169·7	− 146·5	− 177·1	− 183·0	+ 3·7	− 2·6	− 10·5	+ 19·2	+ 2·4	−1084·4	− 557·1	−1683·6	− 674·4	− 999·9
Other Orders (continued).														
+ 157·3	+ 404·9	+ 38·6	+ 0·5	+ 150·3	+ 142·6	+ 313·5	+ 59·8	+ 26·2	+ 135·5	+ 97·1	+ 92·2	+ 29·9	− 35·2	+ 46·0
+ 78·1	+ 70·6	+ 55·5	− 44·6	+ 39·9	+ 73·6	+ 73·3	+ 58·4	− 2·3	+ 50·8	+ 49·9	− 28·9	− 136·7	− 30·2	− 36·5
+ 303·0	+ 527·5	+ 231·1	+ 173·8	+ 308·8	− 6·9	+ 59·2	+ 81·0	+ 155·1	+ 72·1	+ 252·8	+ 190·3	+ 211·1	− 122·2	+ 133·0
− 447·8	− 703·6	− 206·8	− 230·0	− 396·9	− 8·6	− 135·7	− 130·7	− 47·3	− 80·6	− 137·0	− 233·9	− 127·3	− 95·4	− 148·4
− 18·3	− 43·7	− 45·2	− 80·8	− 47·1	− 1·2	− 3·7	− 0·8	− 2·4	− 2·0	− 11·8	− 42·8	− 19·5	− 25·5	− 24·8
+ 72·3	+ 255·7	+ 73·2	− 181·1	+ 55·0	+ 199·5	+ 306·6	+ 67·7	+ 129·3	+ 175·8	+ 251·0	− 23·1	− 42·5	− 308·5	− 30·8
Summary (continued).														
+3516·6	+1964·0	+4087·3	+4048·2	+3404·0	+2344·8	+1772·0	+3086·8	+2953·5	+2539·2	+2811·4	+1504·2	+3148·1	+1936·9	+2350·2
− 238·9	− 169·7	− 146·5	− 177·1	− 183·0	+ 3·7	− 2·6	− 10·5	+ 19·2	+ 2·4	−1084·4	− 557·1	−1683·6	− 674·4	− 999·9
+ 72·3	+ 255·7	+ 73·2	− 181·1	+ 55·0	+ 199·5	+ 306·6	+ 67·7	+ 129·3	+ 175·8	+ 251·0	− 23·1	− 42·5	− 308·5	− 30·8
+3350	+2050	+4014	+3690	+3276	+2548	+2076	+3144	+3102	+2717	+1978	+ 924	+1422	+ 954	+1319

more nitrogen, and more mineral matter removed, than with either description of manure used separately; but whilst with the mineral manure alone there was, in the later years, an increase rather than a diminution in all three items of yield, there was with the combination, at any rate within the first 20 years, a diminution in all, but in a much less degree than with the nitrogenous manure alone. An adequate conception of these differences can, however, only be attained on a detailed comparison of the botany of the respective plots as affected by the very characteristically different descriptions of manures.

The first point to remark is the very great reduction in the number of species under the influence of this mixed nitrogenous and mineral manure (Table LXXI., pp. 1342–3). Compared with plot 3 without manure, and taking the four separation-years, there is an average of three fewer species of grasses, two fewer of Leguminosæ, and as many as 16 fewer of miscellaneous plants; or of 21 less total number of species found in the samples. The number only varied from 27 to 30 in the four separation-years, and averaged only 28; whilst, without manure, there was an average of 49. Compared with plot 5 with ammonia-salts alone, there was on the average, and almost in every individual instance, a slight reduction in the number of species of each description of herbage; and there were, on the average of the four separation-years, five fewer species with the mixture than with the ammonia-salts alone. Compared with the results on plot 7 with the mixed mineral manure alone, there was, with one exception, a reduced number of species of each description of herbage in each separation-year. The average reduction was of grasses three, of Leguminosæ two, and of miscellaneous species 10; or, in all, an average of 15 fewer species by the addition of the ammonia-salts to the mineral manure.

A glance at the lines of totals in the table will show that the herbage became almost exclusively gramineous under the influence of the luxuriance induced by the nitrogenous manure, and the tendency to stem-formation and maturation favoured by the associated mineral constituents. Thus, in the last separation-year nearly 95 per cent. of the mixed herbage was referable to gramineous species, not half a per cent. to Leguminosæ, and not 5 per cent. to Miscellaneæ.

Taking the average of the four separation-years there was, in fact, nearly three times as much gramineous herbage per acre as without manure, nearly twice as much as by the ammonia-salts alone, and not far from twice as much as with the mineral manure alone. Reckoned in the same way, there was, on the average, not much more than one-twentieth as much leguminous herbage as without manure, rather more than with ammonia-salts alone, and only about one-hundredth as much as with the mineral manure alone. Turning to the total miscellaneous species a little more is yielded on the average by the mixed nitrogenous and mineral manure than without manure, about one-third more than by the ammonia-salts alone, but even less than by the mineral manure alone.

Looking to the record of the percentage of the individual gramineous species in the total mixed herbage, *Poa pratensis* is perhaps the most uniformly predominating, *Festuca ovina* and *Agrostis vulgaris* coming next in this respect; the one or the other being relatively the more prevalent in the different separation-years according to the characters of the season. A number of other grasses have, however, generally contributed a fair proportion to the herbage; thus, *Holcus lanatus* has yielded a considerable average percentage and actual amount, *Dactylis glomerata* about the same average proportion, but with more tendency to increase, *Avena elatior* also has shown considerable tendency to increase, whilst *Anthoxanthum odoratum* has yielded a small but not greatly varying quantity, and *Alopecurus pratensis* less still. On the other hand, whilst *Poa pratensis* has much increased in both percentage and actual amount, *Poa trivialis* has, in a considerably greater degree, diminished, in fact almost become excluded; and, whilst *Avena elatior* has greatly increased, *A. pubescens* and *A. flavescens* have each very greatly diminished, indeed almost disappeared. Of the other grasses which have gone down in this struggle among free, luxuriantly-growing, stem-forming, and maturing species, are *Bromus mollis*, *Lolium perenne*, and in much smaller actual quantity *Festuca pratensis*.

Under the influence of such gramineous luxuriance, as already said, the Leguminosæ occurred in most insignificant amount, so much so that it is of little interest to go into any detail as to which of such species was the most persistent and prominent. *Lathyrus pratensis* was so, indeed, but in most immaterial quantity each year. *Ononis arvensis*, which may be designated as a weed, gave, however, the largest, though a very small percentage and actual quantity, in the last separation-year. There was, on the average of the four separation-years, only between 10 and 11 lbs. of total leguminous herbage per acre—a quantity which gives to its occurrence on the plot an almost accidental character.

The division of the table relating to the Miscellaneæ shows that under the conditions of luxuriant growth of the grasses on this plot, only three species of the miscellaneous group come under the category of those yielding more than one per cent. to the total herbage in any one of the separation-years. These are, in the order of their average prevalence, *Rumex Acetosa*, *Conopodium denudatum*, and *Achillea Millefolium*. Each of these varied considerably in amount from year to year according to the character of the season; but each of the three gave by far the most in the second separation-year, which, it will be remembered, was extremely favourable for Miscellaneæ generally; indeed, in that year the quantity both of *Rumex* and of *Conopodium* was extremely large. In each of the other years the *Rumex* gave more than the other two together. In fact, it is obvious that the conditions were, notwithstanding the severe competition of the free-growing grasses, very specially favourable for this vigorous, free-seeding, and objectionable weed.

In actual yield, each of these three most prominent of the miscellaneous species gave much more, on the average, than without manure, especially the *Rumex;* but of the numerous species, taken collectively, none of which have yielded one per cent. to the total herbage, there was much less growing on this plot of great luxuriance of grasses than without manure. Compared with the yield of the three prominent species by ammonia-salts alone, the mixture gave considerably more, especially of *Conopodium denudatum;* and compared with the produce from the mineral manure alone, the mixture gave an excess of *Rumex Acetosa*, a less excess of *Conopodium denudatum*, an actual deficiency of *Achillea Millefolium*, and also a considerable deficiency of those species taken collectively, each of which only occurred in fractional quantity.

Upon the whole, then, the mixture of nitrogenous and mineral manure has given a produce characterised by great luxuriance of gramineous species, by an almost total exclusion of Leguminosæ, and by the considerable prevalence of only very few miscellaneous species, representing three very distinct natural orders: the *Polygonaceæ*, the *Umbelliferæ*, and the *Compositæ*, the species themselves being of very marked, as well as of very different, characters of growth.

The general aspect of the plot was also very distinct from that of either of the plots with which it is compared. Not only was the herbage almost exclusively gramineous, but the grasses were very luxuriant, generally developing broad leaves and strong stems, with considerable tendency to consolidation of tissue and to flowering, seeding, and maturation. The total absence of Leguminosæ, and of most of the usually prevalent Miscellaneæ, only two or three of which show any degree of prominence, also give to the appearance of the plot a marked character. Indeed, the herbage is too exclusively gramineous, and too coarse, to constitute even moderately good hay.

8. *Ammonia-salts* (400 *lbs. per acre*), *with mixed mineral manure including potass, and* 2000 *lbs. cut wheat straw; Plot* 13.

The mixture annually supplied to this plot contained precisely the same description and amount of ammonia-salts, and precisely the same complex mineral manure, including potass, as that applied to plot 9, but with 2000 lbs. of cut wheat straw in addition. The object was to try the effect of silica, and of carbonaceous organic matter, supplied somewhat in the same condition as in dung. The straw would, of course, contain a certain amount of other mineral constituents besides silica, and the amount annually supplied contained also about 9 lbs. of nitrogen. Undoubtedly, however, the manurial constituents of the straw would be for the most part very slowly available.

Compared with plot 9 without the straw, plot 13 with it gave an average of several hundred pounds more produce per acre annually; and, like plot 9, it has yielded rather more during the last five years than during the first 20. Over the 20 years, and probably since, plot 13 has yielded more mineral matter, and more nitrogen, per acre per annum; in fact, more of each individual mineral constituent than plot 9, especially of potass and phosphoric acid. Of silica, plot 13 furnished rather less in the later than in the earlier years; but whilst in the earlier years the two plots yielded practically the same amount, the amount annually taken up fell off in the later years very considerably on plot 9, but much less on plot 13; so that the increased yield of silica over plot 9 was all in the later years. Over the 20 years, however, the excess of silica, taken up where the straw was used, was only about one-eighth as much as it would contain; and the fact that the excess was all in the later years is consistent with the supposition that the silica of the straw would be only very gradually available. A similar argument applies to the increased assimilation of nitrogen, which was much more in the later than in the earlier years.

An examination of the comparative botany of the two plots will afford useful evidence in aid of the explanation of these results.

The Table LXXII., pp. 1348–9, shows the percentage of each species at each separation, the produce per acre, the increase over that without manure; and, finally, the increase or decrease over that on plot 9 with the same manure, excepting the straw.

As with the ammonia-salts and mineral manure without the straw, so with it, we have, but even in a greater degree, a very marked reduction in the number of species found in the samples. We have, on the average, three fewer Gramineæ, two fewer Leguminosæ, and 18 fewer Miscellaneæ, in all 23 fewer species, than without manure; and there are two fewer Miscellaneæ than on plot 9 without the straw. The average number of species found on plot 13 has been 26, instead of 49 without manure; and the range in the several separation-years has been from 30 in the first (1862), to 22 only in the last (1877).

With the straw, the average percentage of Gramineæ in the total herbage is higher, that of Leguminosæ equally low, and that of Miscellaneæ lower, than without it. With the straw the highest average percentage of Gramineæ was more than 95 in the third separation-year, and it was 91 taking the average of the four years. The highest average percentage of Leguminosæ was also in the third separation-year, but it did not then reach to one-third, and averaged less than one-fifth, of 1 per cent. The highest percentage of Miscellaneæ was 13·56, in the second separation-year, the lowest 4·29, in the third year, and the average 8·77.

Compared with the unmanured plot 3, there was an average of more than 4000 lbs. per acre per annum excess of Gramineæ, of nearly 200 lbs. deficiency of Leguminosæ, and a small deficiency of total Miscellaneæ also. Compared with plot 9, there were between 600 and 700 lbs. average increase of gramineous herbage, practically the same average amount of leguminous herbage, but a deficiency of total miscellaneous.

Looking to the composition of the herbage in more detail, it is to be observed that this almost exclusively gramineous produce was characterised by containing a very large and increasing percentage of *Dactylis glomerata*. Next in order, but with less than half the average quantity, came *Agrostis vulgaris*, which was in fairly uniform amount throughout, and next *Poa pratensis* in less, but in increasing proportion. Though yielding less average quantities than the foregoing, *Avena elatior* and *Alopecurus pratensis* have gained ground in a marked degree; whilst *Poa trivialis*, instead of increasing, as did *Poa pratensis*, has in a striking degree diminished, indeed, almost disappeared. *Avena pubescens*, *Avena flavescens*, and *Festuca pratensis*, have

TABLE LXXII.—Number of Species, Percentage, Quantity per Acre, &c., of Potass, and 2000 lbs.

		Number of species, and proportion per cent.					Quantity per acre.				
		1862.	1867.	1872.	1877.	Mean.	1862.	1867.	1872.	1877.	Mean.
	NUMBER OF SPECIES.										
	Gramineæ	14	15	15	14	14	...	...	...	...	...
	Leguminosæ	3	2	3	0	2	...	...	...	...	...
	Other Orders	13	8	10	8	10	...	...	...	...	...
	Total	30	25	28	22	26	...	...	...	...	...
	GRAMINEÆ.										
		p. c.	p. c.	p. c.	p. c.	p. c.	lbs.	lbs.	lbs.	lbs.	lbs.
	Anthoxanthum odoratum	0·69	1·89	0·42	0·33	0·83	43·7	112·8	29·1	20·5	51·5
	Alopecurus pratensis	3·36	4·53	5·89	6·78	5·14	210·2	274·8	388·7	419·8	323·4
	Agrostis vulgaris	12·02	20·34	13·44	13·40	14·80	753·3	1243·3	927·7	841·2	941·4
	Holcus lanatus	5·35	4·70	2·71	4·57	4·33	339·1	280·7	195·1	286·6	275·4
	Avena elatior	0·42	2·49	9·17	11·08	5·79	26·1	147·6	647·7	696·9	379·6
	Avena pubescens	5·48	0·59	0·14	0·02	1·56	340·1	35·5	10·0	1·2	96·7
	Avena flavescens	4·84	4·27	1·72	0·56	2·85	306·3	258·8	123·0	35·9	181·0
	Poa pratensis	3·91	10·25	11·44	10·09	8·92	243·6	612·4	772·3	629·5	564·4
	Poa trivialis	11·83	6·68	2·11	0·64	5·31	749·2	412·0	154·7	39·9	338·9
	Dactylis glomerata	27·88	20·28	43·05	40·75	32·99	1765·4	1240·6	3090·7	2565·1	2165·4
	Festuca ovina	2·27	6·82	3·60	3·57	4·06	142·6	406·2	237·4	222·2	252·1
	Festuca pratensis	3·88	0·57	0·05	0·03	1·13	245·8	34·7	3·3	1·6	71·4
	Lolium perenne	2·75	1·21	0·16	0·11	1·06	175·4	73·3	11·3	6·8	66·7
None yielding 1 per cent.	Phleum pratense, Aira cæspitosa, Cynosurus cristatus, Bromus mollis	0·63	0·09	0·11	0·11	0·24	39·8	4·7	7·5	6·5	14·7
Absent	Briza media, Festuca loliacea	...	...	...	...	...	...	...	...	...	...
	Undetermined (chiefly Gramineæ)	5·12	1·61	1·38	...	2·03	325·5	98·0	96·2	...	129·9
	Total	90·43	86·32	95·39	92·04	91·04	5706·1	5235·4	6694·7	5773·7	5852·5
	LEGUMINOSÆ.										
	Trifolium repens	0·04	0·03	0·01	...	0·02	2·1	1·8	0·3	...	1·0
	Trifolium pratense	0·03	...	0·01	...	0·01	1·6	...	0·8	...	0·6
	Trifolium minus	...	...	...	...	...	...	...	...	...	...
	Lathyrus pratensis	0·23	0·09	0·29	...	0·15	14·5	5·4	19·2	...	9·8
	Ononis arvensis	...	...	...	...	...	...	...	...	...	...
	Lotus corniculatus	...	...	0·01	...	0·01	...	...	0·4	...	0·1
	Total	0·30	0·12	0·32	...	0·19	18·2	7·2	20·7	...	11·5
	OTHER ORDERS.										
	Conopodium denudatum	1·99	8·04	0·66	1·02	2·91	125·6	487·1	46·6	63·6	180·7
	Achillea Millefolium	1·31	1·46	0·63	0·01	0·85	83·6	89·2	41·9	0·6	53·8
	Rumex Acetosa	5·48	3·86	2·73	6·84	4·71	346·7	233·3	188·7	433·1	300·4
None yielding 1 per cent.	Ranunculus acris, R. repens et bulbosus, Stellaria graminea, Cerastium triviale, Potentilla reptans, Pimpinella Saxifraga, Heracleum Sphondylium, Anthriscus sylvestris, Galium verum, Centaurea nigra, Taraxacum officinale, Plantago lanceolata, Veronica Chamædrys, Primula veris, Scilla nutans, Luzula campestris, Carex præcox, Hypnum squarrosum, H. rutabulum, H. hians	0·46	0·20	0·27	0·09	0·30	27·8	11·8	17·4	5·0	15·6
Absent	Agrimonia Eupatoria, Poterium Sanguisorba, Galium Aparine, Scabiosa arvensis, Bellis perennis, Chrysanthemum Leucanthemum, Tragopogon pratensis, Leontodon hispidus, Hieracium Pilosella, Prunella vulgaris, Thymus Serpyllum, Ajuga reptans, Ophioglossum vulgatum	...	...	...	...	...	...	...	...	...	...
	Total	9·27	13·56	4·29	7·96	8·77	583·7	821·4	294·6	502·3	550·5
	SUMMARY.										
	Gramineæ	90·43	86·32	95·39	92·04	91·04	5706·1	5235·4	6694·7	5773·7	5852·5
	Leguminosæ	0·30	0·12	0·32	...	0·19	18·2	7·2	20·7	...	11·5
	Other Orders	9·27	13·56	4·29	7·96	8·77	583·7	821·4	294·6	502·3	550·5
	Total	100·00	100·00	100·00	100·00	100·00	6308	6064	7010	6276	6414

each Species, by 400 lbs. Ammonia-salts, with Mixed Mineral Manure, including cut Wheat-straw; Plot 13.

Increase + or decrease − compared with:—									
Plot 3, without manure.					Plot 9; as plot 13, but without straw.				
1862.	1867.	1872.	1877.	Mean.	1862.	1867.	1872.	1877.	Mean.
NUMBER OF SPECIES (continued).									
− 4	0	− 2	− 3	− 3	+1	+1	−1	+1	0
− 1	− 2	− 1	− 4	− 2	+1	0	+2	−4	0
−15	−16	−18	−23	−18	0	−5	−3	−2	−2
−20	−18	−21	−30	−23	+2	−4	−2	−5	−2
GRAMINEÆ (continued).									
lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
− 86·9	− 175·7	− 56·4	− 100·5	− 104·9	− 35·7	− 80·4	− 98·2	− 157·5	− 93·0
+ 73·2	+ 80·9	+ 380·2	+ 412·7	+ 236·8	+ 192·9	+ 271·0	+ 232·5	+ 361·6	+ 264·5
+ 406·6	+ 955·8	+ 662·4	+ 527·3	+ 638·0	− 66·8	+ 520·5	+ 53·0	+ 100·8	+ 151·9
+ 185·3	+ 15·2	+ 135·9	− 10·1	+ 81·6	− 438·1	− 248·9	− 235·4	− 341·2	− 315·9
+ 24·0	+ 140·6	+ 645·6	+ 695·7	+ 376·5	+ 26·1	+ 13·0	+ 2·7	− 104·1	− 15·5
+ 45·6	− 66·8	− 48·4	− 62·4	− 33·0	− 314·2	− 40·4	− 17·7	− 3·0	− 93·8
+ 233·9	+ 196·8	+ 65·6	+ 10·4	+ 126·7	− 275·0	+ 55·4	− 176·8	− 4·6	− 100·2
+ 234·7	+ 606·7	+ 770·8	+ 627·8	+ 560·0	− 440·1	− 88·3	− 510·4	− 462·0	− 375·3
+ 702·2	+ 373·0	+ 146·5	+ 26·7	+ 312·0	+ 190·9	+ 296·8	+ 118·5	+ 33·2	+ 159·8
+1711·7	+1182·6	+3075·9	+2548·5	+2129·6	+1408·2	+ 990·9	+2418·5	+1713·3	+1632·6
− 263·3	− 100·3	− 118·8	− 295·3	− 194·4	− 190·9	− 585·2	− 253·7	−1097·6	− 531·9
+ 244·6	+ 34·7	+ 3·1	+ 1·4	+ 71·0	+ 154·9	+ 30·9	+ 1·6	+ 1·6	+ 47·3
− 19·0	− 61·0	− 27·7	− 100·8	− 52·1	− 93·5	+ 19·0	− 51·5	− 2·9	− 32·2
+ 30·0	− 1·3	− 11·1	− 17·6	...	− 245·7	− 1·2	+ 1·2	+ 5·9	− 59·8
− 57·7	− 22·7	− 105·2	− 172·1	− 89·4	...	...	− 0·6	...	− 0·1
+ 86·2	− 106·5	+ 47·5	...	+ 6·8	+ 161·5	− 65·1	− 5·1	...	+ 22·8
+3551·1	+3052·0	+5565·9	+4091·7	+4065·2	+ 34·5	+1088·0	+1478·6	+ 43·5	+ 661·2
LEGUMINOSÆ (continued).									
− 14·1	− 5·2	− 5·9	− 3·1	− 7·1	+ 1·5	+ 1·3	+ 0·3	...	+ 0·7
− 135·1	− 70·3	− 26·8	− 49·4	− 70·4	+ 1·6	...	+ 0·8	− 0·6	+ 0·5
...	...	...	...	...	...	...	...	− 0·6	− 0·1
− 23·9	− 17·3	+ 3·1	− 56·0	− 23·5	+ 6·8	− 2·7	+ 18·1	− 3·6	+ 4·6
...	...	...	...	...	...	...	...	− 20·0	− 5·0
− 55·9	− 78·3	− 97·3	− 93·4	− 81·2	...	...	+ 0·4	...	+ 0·1
− 229·0	− 171·1	− 126·9	− 201·9	− 182·2	+ 9·9	− 1·4	+ 19·6	− 24·8	+ 0·8
OTHER ORDERS (continued).									
+ 96·0	+ 388·8	− 0·2	+ 18·7	+ 125·8	− 61·3	− 16·1	− 38·8	+ 18·2	− 24·5
+ 36·9	+ 50·5	+ 12·6	− 46·4	+ 13·4	− 41·2	− 20·1	− 42·9	− 1·8	− 26·5
+ 304·0	+ 174·6	+ 159·6	+ 388·9	+ 256·7	+ 1·0	− 352·9	− 71·5	+ 215·1	− 52·1
− 498·8	− 731·6	− 214·5	− 274·8	− 429·8	+ 9·2	+ 2·3	+ 9·9	− 16·7	+ 1·3
− 4·2	− 31·2	− 30·5	− 64·2	− 32·6	− 46·1	− 17·8	− 2·9	− 11·5	− 19·7
− 66·1	− 148·9	− 73·0	+ 22·2	− 66·5	− 138·4	− 404·6	− 146·2	+ 203·3	− 121·5
SUMMARY (continued).									
+3551·1	+3052·0	+5565·9	+4091·7	+4065·2	+ 34·5	+1088·0	+1478·6	+ 43·5	+ 661·2
− 229·0	− 171·1	− 126·9	− 201·9	− 182·2	+ 9·9	− 1·4	+ 19·6	− 24·8	+ 0·8
− 66·1	− 148·9	− 73·0	+ 22·2	− 66·5	− 138·4	− 404·6	− 146·2	+ 203·3	− 121·5
+3256	+2732	+5366	+3912	+3816	− 94	+ 682	+1352	+ 222	+ 540

also gone down very much. *Lolium perenne,* occurring in smaller amount, has also declined. The grasses which have shown but little decided tendency either to increase or decrease are *Festuca ovina* and *Anthoxanthum odoratum,* the latter being, however, in very small amount.

But it is a consideration of the actual yield per acre rather than of the percentage, and the comparison of the amounts with those without manure, and with the nitrogenous and mineral manure without the straw, that bring out the change in the herbage the most strikingly.

There was, on the average, 2165 lbs. of *Dactylis glomerata* annually contributed to the produce, nearly the whole of which was in excess of the yield of the same grass without manure. There was an average of 941 lbs. of *Agrostis vulgaris,* about two-thirds of which was in excess of that yielded without manure. There was an average of 564 lbs. of *Poa pratensis,* all but a fraction of which was in excess of that without manure. There was also a notable excess of *Avena elatior, Poa trivialis, Alopecurus pratensis,* and *Avena flavescens;* but there was an actual deficiency of *Festuca ovina, Anthoxanthum odoratum,* and a few other grasses.

The comparison with plot 9, however, brings out the most prominently the difference in the botanical composition of the herbage without and with the straw.

In the last column of the table it is seen that there was, on the average of the four separation-years, 1633 lbs. more *Dactylis* with than without the straw. There was also an average increase, but much less marked, of *Alopecurus pratensis, Poa trivialis, Agrostis vulgaris,* and *Festuca pratensis;* whilst there was a deficiency, with the straw, of more than 500 lbs. of *Festuca ovina,* a large deficiency of *Poa pratensis* (instead of excess, as of *P. trivialis*), also of *Holcus lanatus,* and a less one of *Avena flavescens, A. pubescens, Anthoxanthum odoratum,* and *Lolium perenne;* a small deficiency of *Avena elatior,* and collectively of *Phleum pratense, Aira cæspitosa, Cynosurus cristatus,* and *Bromus mollis.* Thus, the grass which is in very marked excess with the straw is *Dactylis glomerata,* whilst *Alopecurus pratensis, Poa trivialis,* and *Agrostis vulgaris* are also notably in excess. The grasses so brought into prominence are all free-growing, and three out of the four of good agricultural repute. On the other hand, the grasses most notably in relative defect are, *Festuca ovina, Poa pratensis,* and *Holcus lanatus,* whilst the several species of *Avena,* and *Anthoxanthum odoratum,* are also in decreased amount. Compared with the grasses which have become more prominent, several of those which have diminished are plants of more limited growth. The *Festuca ovina* especially, which is in the greatest degree in defect, is comparatively very superficial-rooted and fine-leaved. In fact, the most striking character is the enormous increase of *Dactylis,* and the very large diminution of *Festuca ovina.*

With regard to the Leguminosæ, the yield on both plots, 13 as well as 9, is so small that the differences are of little moment; but, so far as the figures go, the conditions seem to be rather more favourable for *Lathyrus pratensis* on the plot with the straw.

As on plot 9, so on plot 13, the only three miscellaneous plants which have contributed as much as 1 per cent. to the mixed herbage in any one year, and which were in relative prominence in the order named, are *Rumex Acetosa*, *Conopodium denudatum*, and *Achillea Millefolium*. There was, however, on the average, and in almost every case excepting in the last separation-year, a deficiency of each on the straw plot compared with plot 9 without straw; *Rumex*, however, gave a marked excess with the straw in the last, though the excess was much less than was the deficiency in the second separation-year.

Upon the whole, then, the plot with the straw has given somewhat more produce, a larger percentage and actual amount of gramineous herbage, a great excess of *Dactylis*, and a fair excess of a few other grasses, with a considerable deficiency of *Festuca ovina* and several other Gramineæ. The Leguminosæ have been scarcely affected, but the Miscellaneæ have, upon the whole, diminished in amount, though the same species are prominent as on the plot without the straw.

The general characters and aspect of this luxuriant grassy herbage were those of tufted habit, broad leaves, large stems, and considerable tendency to flowering and seeding, these characters being developed in a greater degree than on the plot without the straw. The herbage, in fact, like that of plot 9, although composed of a fair proportion of grasses of good repute, was far too stemmy and coarse to constitute good hay.

The question obviously arises—what was the explanation of the marked change in quantity, but especially in the relative predominance, of different species, and also in the character of development of the herbage on plot 13, with the straw?

As already intimated, the straw contained some nitrogen, some of most of the mineral constituents required, a great excess of silica, and a large amount of carbonaceous organic matter; but all of these would, presumably, become only slowly available, and consistently with this the relatively greater amount of produce was much more manifest in the later years. It can hardly be to the gradually available increased supplies of food that the whole of the effects are to be attributed.

As referred to in Part I., plot 13 was somewhat earlier shaded from the afternoon sun than was plot 9, and it adjoins plot 12, part of which is supposed to have been "made ground." But neither does the earlier shade seem sufficient to account for the effects produced, nor did the observations of the sections of the soil when sampling plot 13 indicate anything abnormal in its condition as they did on plot 12. Further, a consideration of the exact circumstances of the experiment, and its results, would seem to lead to more natural explanations.

Observation showed that, probably owing to the mulching effect of the straw chaff, and the consequent protection from the effects of spring frosts, there was a more active growth of young foliage quite early in the spring, and the question arises whether it is due to this that the grasses which have asserted the greater prominence owe their relatively favourable position in the struggle. With reference to this point, observations made on the plots have shown that occasionally when *Dactylis* has suffered from early frost, then the other grasses have come to the front. The usually great prevalence of *Dactylis* on plot 13 seems, therefore, to be reasonably attributable, at any rate in some degree, to protection in the early stages of its growth. Assuming this to be so, and that the result was the establishment of a few freer-growing, and the retardation of more meagrely-developing species, we have obviously, in the fact of the predominance of plants of greater feeding and growing capacities, conditions suited to the gathering up of more food, even with the same supplies; whilst, this being the case, the comparatively small increased supply would not be without its influence. In other words, the increased produce on the plot, and the increased accumulation of nitrogen, silica, and mineral constituents generally, is not to be referred exclusively to the increased supply of constituents from without, but largely to the special conditions favouring the development of species having a greater power of food collection, and therefore a greater power of growth.

9. *Ammonia-salts* (800 *lbs. per acre*), *and mixed mineral manure, including potass; Plots* 11–1 *and* 11–2.

It has been shown that on plots 9 and 13, with the mixed mineral manure (including potass), and 400 lbs. of ammonia-salts per acre per annum, the herbage became extremely prominently gramineous, and was, upon the whole, coarse and stemmy. Still, it seemed desirable to determine whether or not the limit of growth attainable with the soil in question, and with average seasons, was reached, and to ensure that this limit should be secured. Accordingly, on plot 11, with the same mineral manure as on plot 9, double the quantity of ammonia-salts, that is 800 lbs. per acre per annum, supplying on the average about 172 lbs. of nitrogen (see p. 1305), was applied. During the first three years, however, this appearing from the results to be excessive, the quantity of ammonia-salts was reduced to one-half, that is, to the same as on plot 9; but the results obtained indicating that the limit of possible growth was not reached, the quantity was again doubled, and the double application has been continued up to the present time.* After the first six years, three with the double, and three with the single amount of ammonia-salts, and when it was decided again to use the double quantity, the plot was divided, one half, 11–1, having besides the ammonia-salts the same mineral manure as before; and the other, 11–2, the same, with the addition of artificial silicates of soda and lime for nine years, and afterwards of silicate of soda alone.

Compared with plot 9, there was a considerable increase of hay produced, and of nitrogen taken up, on the plots 11, with increased amount of ammonia salts; and in each item there was more increase on 11–2 with the silicates than on 11–1 without. There was, too, with the silicates, a less falling off in hay, and in mineral matter taken up, and a greater increase in the nitrogen gathered, over the later than over the earlier years. And, although there was less increase of herbage produced for the second increment of ammonia-salts applied than for the first, yet there was a greater proportion of the nitrogen supplied recovered in the increase. That is to say, the

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* Note, January, 1883.—The above was written in 1881; for the crop of 1882 the quantity of ammonia-salts was reduced from 800 to 600 lbs.

percentage of nitrogen was relatively, indeed abnormally, high; or, in other words, less carbon was assimilated in proportion to the nitrogen accumulated. This might be due to a deficiency of available mineral matter, or to a limitation of the climatal characters for more active carbon assimilation, or partly to the one and partly to the other.

These several points are considered in Part I. (pp. 327–332), and will be discussed in their chemical aspects in Part III.; but an examination of the actual and comparative botanical characteristics of the plots will contribute useful data to the discussion.

In Table LXXIII., pp. 1354–5, will be found for plot 11–1, with the mixed mineral manure and the double quantity of ammonia-salts each year (excepting in the fourth, fifth, and sixth), the percentage of each species at the different separations, the produce of each per acre, the increase of each over the unmanured plot 3, and also the increase over plot 9, with the mineral manure and the smaller quantity of ammonia-salts.

With still more forced luxuriance than on plot 9, we have at the same time a still greater reduction in the number of species. Taking the average of the four separation-years, we have, compared with the produce without manure, five fewer Gramineæ, three fewer Leguminosæ, and as many as 22 fewer Miscellaneæ; or, in all, 30 fewer species. There was, indeed, on 11–1 an average of only 19 species against 49 without manure; and the number diminished in the four separation-years from 28 to 18, 16, and to 15 only in 1877. Compared with plot 9 even, there was an average reduction of two Gramineæ, one Leguminosæ, and six Miscellaneæ: in all, of nine species.

With the excessive luxuriance of individual species, and consequently greatly reduced numbers, the herbage is still more exclusively gramineous, the percentage of grasses reaching nearly 99 in 1872, and, on the average, nearly 95. Leguminosæ were scarcely represented at all. Again, only three Miscellaneæ yielded more than 1 per cent. to the produce in any one year, and these were the same as were persistent on plots 9 and 13, viz.: *Rumex Acetosa*, *Conopodium denudatum* and *Achillea Millefolium;* and they are in relative prominence in the same order as on those plots, but each in smaller and decreasing quantity. Thus, in the first separation-year there was more than 10 per cent. of total Miscellaneæ; in the second, not 6; in the third, little over 1; in the fourth, not 2½; and, on the average, only 5 per cent.

Among the grasses, though considerably fluctuating in amount, and in much smaller quantity in the last separation-year than previously, *Dactylis glomerata* gave by far the highest average percentage in the produce. Indeed, in the second and third of the four years it contributed nearly 40 per cent., less than half as much in the fourth, and the general average was about 30 per cent. This plant was, in each separation-year, in very much higher proportion than on plot 9, with half the quantity of ammonia-salts. Next in order as to average percentage came *Agrostis vulgaris*, *Holcus lanatus*, *Alopecurus pratensis*, and *Avena elatior;* each of which showed considerable tendency to increase, especially *Agrostis*, which in the fourth separation-year yielded nearly twice as much as the *Dactylis*, and has since maintained, perhaps, the first place among its competitors. *Poa pratensis* also yielded a fair average, but in the last

TABLE LXXIII.—Number of Species, Percentage, Quantity per Acre, &c., of each Species,

	Number of species, and proportion per cent.					Quantity per acre.				
	1862.	1867.	1872.	1877.	Mean.	1862.	1867.	1872.	1877.	Mean.
NUMBER OF SPECIES.										
Gramineæ	15	13	11	11	12	...	...	...	...	...
Leguminosæ	1	1	1	...	1	...	...	...	...	...
Other Orders	12	4	4	4	6	...	...	...	...	...
Total	28	18	16	15	19	...	...	...	...	...
GRAMINEÆ.										
	p. c.	p. c.	p. c.	p. c.	p. c.	lbs.	lbs.	lbs.	lbs.	lbs.
Alopecurus pratensis	2·80	13·11	12·35	9·91	9·54	199·0	702·7	880·7	673·5	614·0
Agrostis vulgaris	13·17	19·27	13·56	29·20	18·80	936·1	1032·9	967·0	1984·4	1230·1
Holcus lanatus	9·92	2·86	10·33	20·29	10·85	705·1	153·3	736·6	1378·8	743·4
Avena elatior	0·77	4·55	10·41	14·86	7·65	54·7	243·9	742·4	1009·8	512·7
Avena pubescens	1·66	0·01	...	...	0·42	118·0	0·5	...	...	29·6
Avena flavescens	5·28	0·46	0·09	0·01	1·46	375·3	24·7	6·4	0·7	101·8
Poa pratensis	9·43	12·86	10·40	1·47	8·54	670·3	689·3	741·6	99·9	550·3
Poa trivialis	13·25	0·14	0·09	0·33	3·45	941·8	7·5	6·4	22·4	244·5
Dactylis glomerata	24·16	39·31	39·28	17·11	29·96	1717·3	2107·0	2801·1	1163·0	1947·1
Festuca ovina	1·46	0·50	0·38	4·15	1·62	103·8	26·8	27·1	282·0	109·9
Festuca pratensis	1·90	...	...	...	0·47	135·1	...	...	...	33·8
Bromus mollis	1·39	0·04	0·01	...	0·36	98·8	2·1	0·7	...	25·4
Lolium perenne	1·37	0·08	...	0·01	0·37	97·4	4·3	...	0·7	25·6
None yielding 1 per cent. { Anthoxanthum odoratum, Aira cæspitosa }	0·10	0·06	0·78	0·19	0·29	7·1	3·2	55·6	12·9	19·7
Absent { Phleum pratense, Briza media, Cynosurus cristatus, Festuca loliacea. }	...	...	...	...	...	...	...	...	...	...
Undetermined (chiefly Gramineæ)	2·72	0·87	1·16	...	1·18	193·3	46·6	82·7	...	80·7
Total	89·38	94·12	98·84	97·53	94·96	6353·1	5044·8	7048·3	6628·1	6268·6
LEGUMINOSÆ.										
Trifolium repens	...	...	0·01	...	{ 0·01 }	...	...	0·7	...	0·2
Lotus corniculatus	...	0·01	...	...		...	0·6	...	...	0·1
Lathyrus pratensis	0·01	...	...	...		0·7	...	...	...	0·2
Trifolium pratense, T. minus, Ononis arvensis	...	...	...	...	...	...	...	...	...	...
Total	0·01	0·01	0·01	...	0·01	0·7	0·6	0·7	...	0·5
OTHER ORDERS.										
Conopodium denudatum	1·79	1·84	0·04	0·01	0·91	127·2	98·6	2·9	0·7	57·3
Achillea Millefolium	1·45	0·06	...	0·01	0·38	103·1	3·2	...	0·7	26·7
Rumex Acetosa	7·02	3·96	1·09	2·25	3·57	499·0	212·3	77·7	152·9	235·5
None yielding 1 per cent. { Ranunculus repens et bulbosus, Pimpinella Saxifraga, Heracleum Sphondylium, Centaurea nigra, Taraxacum officinale, Prunella vulgaris, Luzula campestris, Carex præcox, Ophioglossum vulgatum, Hypnum squarrosum, H. rutabulum, H. hians }	0·35	0·01	0·02	0·20	0·17	24·9	0·5	1·4	13·6	10·2
Absent { Ranunculus acris, Stellaria graminea, Cerastium triviale, Potentilla reptans, Agrimonia Eupatoria, Poterium Sanguisorba, Galium Aparine, Scabiosa arvensis, Bellis perennis, Chrysanthemum Leucanthemum, Tragopogon pratensis, Leontodon hispidus, Hieracium Pilosella, Plantago lanceolata, Veronica Chamædrys, Thymus Serpyllum, Ajuga reptans, Primula veris, Scilla nutans }	...	...	...	...	...	...	...	...	...	...
Total	10·61	5·87	1·15	2·47	5·03	754·2	314·6	82·0	167·9	329·7
SUMMARY.										
Gramineæ	89·38	94·12	98·84	97·53	94·96	6353·1	5044·8	7048·3	6628·1	6268·6
Leguminosæ	0·01	0·01	0·01	...	0·01	0·7	0·6	0·7	...	0·5
Other Orders	10·61	5·87	1·15	2·47	5·03	754·2	314·6	82·0	167·9	329·7
Total	100·00	100·00	100·00	100·00	100·00	7108	5360	7131	6796	6598

by 800* lbs. Ammonia-salts, and Mixed Mineral Manure, including Potass; Plot 11–1.

Increase + or decrease − compared with:—									
Plot 3, without manure.					Plot 9, 400 lbs. ammonia-salts, and mixed mineral manure.				
1862.	1867.	1872.	1877.	Mean.	1862.	1867.	1872.	1877.	Mean.
NUMBER OF SPECIES (continued).									
− 3	− 2	− 6	− 6	− 5	+2	− 1	− 5	− 2	−2
− 3	− 3	− 3	− 4	− 3	−1	− 1	0	− 4	−1
−16	−20	−24	−27	−22	−1	− 9	− 9	− 6	−6
−22	−25	−33	−37	−30	0	−11	−14	−12	−9
GRAMINEÆ (continued).									
lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
+ 62·0	+ 508·8	+ 872·2	+ 666·4	+ 527·4	+ 181·7	+ 698·9	+ 724·5	+ 615·3	+ 555·1
+ 589·4	+ 745·4	+ 701·7	+1670·5	+ 926·7	+ 116·0	+ 310·1	+ 92·3	+1244·0	+ 440·6
+ 551·3	− 112·2	+ 677·4	+1082·1	+ 549·6	− 72·1	− 376·3	+ 306·1	+ 751·0	+ 152·1
+ 52·6	+ 236·9	+ 740·3	+1008·6	+ 509·6	+ 54·7	+ 109·3	+ 97·4	+ 208·8	+ 117·6
− 176·5	− 101·8	− 58·4	− 63·6	− 100·1	− 536·3	− 75·4	− 27·7	− 4·2	− 160·9
+ 302·9	− 37·3	− 51·0	− 24·8	+ 47·5	− 206·0	− 178·7	− 293·4	− 39·8	− 179·4
+ 661·4	+ 683·6	+ 740·1	+ 98·2	+ 545·9	− 13·4	− 11·4	− 541·1	− 991·6	− 389·4
+ 894·8	− 31·5	− 1·8	+ 9·2	+ 217·6	+ 383·5	− 107·7	− 29·8	+ 15·7	+ 65·4
+1663·6	+2049·0	+2786·3	+1146·4	+1911·3	+1360·1	+1857·3	+2128·9	+ 311·2	+1414·3
− 302·1	− 479·7	− 329·1	− 235·5	− 336·6	− 229·7	− 964·6	− 464·0	−1037·8	− 674·1
+ 133·9	...	− 0·2	− 0·2	+ 33·4	+ 44·2	− 3·8	− 1·7	...	+ 9·7
+ 94·8	+ 0·4	+ 0·5	...	+ 23·9	− 186·7	− 3·8	− 5·0	...	− 48·9
− 97·0	− 130·0	− 39·0	− 106·9	− 93·2	− 171·5	− 50·0	− 62·8	− 9·0	− 73·3
− 123·8	− 285·3	− 30·1	− 109·0	− 137·1	− 72·3	− 190·0	− 72·3	− 165·1	− 124·9
− 63·2	− 27·0	− 123·4	− 195·3	− 102·2	...	...	− 0·6	− 0·6	− 0·2
− 46·0	− 157·9	+ 34·0	...	− 42·4	+ 29·3	− 116·5	− 18·6	...	− 26·4
+4198·1	+2861·4	+5919·5	+4946·1	+4481·3	+ 681·5	+ 897·4	+1832·2	+ 897·9	+1077·3
LEGUMINOSÆ (continued).									
− 16·2	− 7·0	− 5·5	− 3·1	− 7·9	− 0·6	− 0·5	+ 0·7	...	− 0·1
− 55·9	− 77·7	− 97·7	− 93·4	− 81·2	...	+ 0·6	...	...	+ 0·1
− 37·7	− 22·7	− 16·1	− 56·0	− 33·1	− 7·0	− 8·1	− 1·1	− 3·6	− 5·0
− 136·7	− 70·3	− 27·6	− 49·4	− 71·0	...	...	...	− 21·2	− 5·2
− 246·5	− 177·7	− 146·9	− 201·9	− 193·2	− 7·6	− 8·0	− 0·4	− 24·8	− 10·2
OTHER ORDERS (continued).									
+ 97·6	+ 0·3	− 43·9	− 44·2	+ 2·4	− 59·7	− 404·6	− 82·5	− 44·7	− 147·9
+ 56·4	− 35·5	− 29·3	− 46·3	− 13·7	− 21·7	− 106·1	− 84·8	− 1·7	− 53·6
+ 456·3	+ 153·6	+ 48·6	+ 108·7	+ 191·8	+ 153·3	− 373·9	− 182·5	− 65·1	− 117·0
− 248·6	− 362·5	− 169·0	− 167·4	− 236·6	+ 11·4	− 6·4	− 5·5	+ 1·5	+ 0·5
− 257·3	− 411·6	− 92·0	− 163·0	− 231·2	− 51·2	− 20·4	− 3·5	− 21·1	− 24·3
+ 104·4	− 655·7	− 285·6	− 312·2	− 287·3	+ 32·1	− 911·4	− 358·8	− 131·1	− 342·3
SUMMARY (continued).									
+4198·1	+2861·4	+5919·5	+4946·1	+4481·3	+ 681·5	+ 897·4	+1832·2	+ 897·9	+1077·3
− 246·5	− 177·7	− 146·9	− 201·9	− 193·2	− 7·6	− 8·0	− 0·4	− 24·8	− 10·2
+ 104·4	− 655·7	− 285·6	− 312·2	− 287·3	+ 32·1	− 911·4	− 358·8	− 131·1	− 342·3
+4056	+2028	+5487	+4432	+4000	+ 706	− 22	+1473	+ 742	+ 724

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* 400 lbs. only in 1859, 1860, and 1861.

separation a much diminished, proportion; whilst *P. trivialis,* yielding largely in the earlier, contributed a mere fraction in the later years. *Avena flavescens* has also gone down very much; whilst *Festuca ovina,* the prevailing grass without manure, with ammonia-salts alone, or with mineral manures alone, occurred here in very small proportion; and several other grasses were only very meagrely represented.

On 11–1, with the mineral manure and the excess of ammonia-salts, *Dactylis glomerata* has on the average contributed nearly 2000 lbs. per acre per annum, nearly the whole of which is in excess of the amount grown without manure. The plot has also yielded an average of more than 900 lbs. per acre per annum of *Agrostis vulgaris* in excess of plot 3, and in the fourth separation-year much more than this. *Holcus lanatus, Alopecurus pratensis, Avena elatior,* and *Poa pratensis,* have also each yielded, on the average, an excess of more than 500 lbs.; and, again, the *Holcus, Alopecurus,* and *Avena elatior,* much more in the fourth separation-year; whilst *Poa pratensis* yielded much less in that year. *Poa trivialis* also shows an average excess, due, however, entirely to a large excess in the first separation-year. Owing to the same cause (excess in the first year), *Festuca pratensis,* and *Bromus mollis,* also show some slight average excess. On the other hand, with this excessively high manuring, and considerably increased yield of a number of free-growing grasses, there was an actual deficiency of more than 300 lbs. of *Festuca ovina,* of about 100 lbs. of *Avena pubescens,* and of 93 lbs. of *Lolium perenne,* besides a deficiency of between 200 and 300 lbs. of a number of comparatively meagre-growing species taken collectively.

Compared with plot 9 with half the quantity of ammonia-salts, plot 11–1 has given an average of more than 1400 lbs. excess of *Dactylis glomerata,* but much less in the fourth year; an excess of several hundred pounds each, of *Agrostis vulgaris,* and *Alopecurus pratensis,* and of more than a hundred pounds each, of *Holcus lanatus,* and *Avena elatior;* but the excess of *Agrostis, Holcus,* and *Alopecurus,* was much greater in the last separation-year than on the average. There is an actual average deficiency, compared with plot 9, of nearly 700 lbs. of *Festuca ovina,* of nearly 400 lbs. of *Poa pratensis,* and of each the deficiency was very much greater in the last separation-year. There was also an average deficiency of between 100 and 200 lbs. of *Avena pubescens* and *A. flavescens,* besides some deficiency in the quantities furnished by *Bromus mollis, Lolium perenne,* and a few other species taken collectively; but of the two species of *Avena,* the *Bromus,* and the *Lolium,* the deficiency was much the less in the later years.

As the table shows, the occurrence of the Leguminosæ may be said to be quite immaterial—indeed, all but accidental.

Of the three most prominent Miscellaneæ, *Rumex Acetosa, Conopodium denudatum,* and *Achillea Millefolium,* each has gone down in a very marked degree from the earlier to the later years; *Rumex Acetosa* being, in fact, the only one which was at all fairly represented in the fourth separation-year.

Thus, with the mixed mineral manure, including potass, and an undoubtedly excessive amount of ammonia-salts, the herbage has become almost exclusively gramineous, Leguminosæ are practically banished, and Miscellaneæ much reduced, both in number and in quantity. The grasses which have become the most prominent are of large habit and free growth, whilst those which are reduced are for the most part characteristically of an opposite description.

For a number of years *Dactylis* was by far the most prominent, *Agrostis, Alopecurus,* and *Poa pratensis,* coming next; but, latterly, *Dactylis* has lost ground, both *Poas* have very much reduced, and *Agrostis vulgaris, Holcus lanatus,* and *Avena elatior,* seem to be coming much more to the fore in the struggle.

The character of growth of the plot is essentially tufty and patchy; coarse strong seed-stems developing, with large, dark green, broad, "flag"-like leaves, and the overgrown plants are often "laid," and the crowns become more or less rotten before cutting, whilst the maturing is irregular and imperfect.

It is not surprising that, with these conditions, there is not the same amount of growth, represented by carbon-assimilation, in proportion to the nitrogen and the mineral matter taken up, as where the smaller quantity of nitrogen is used.

According to notes taken on the ground, the young plants, and perhaps those of *Dactylis, Festuca ovina,* and *Holcus,* more than others, are really injured by the direct action of the large application of the ammonia-salts. Indeed, it is probable that, whilst the deficient assimilation of carbon is partly due to deficient atmospheric conditions of light, heat, and moisture, and their favourable mutual adaptation to stage of growth, it is also, in part, attributable to some deficiency in the amount and capacity of the leaf surface over the total area of the plot, there being occasional blank, uncovered spaces, instead of a uniformly distributed close leafy "bottom herbage," and such plants as are prominent are forced to the extreme limit possible within the season-period of their growth, and do not attain maturity before the favourable climatic conditions for so doing have passed.

A fact of interest which should not be overlooked is that, with a high percentage of nitrogen in the produce, and a high percentage of mineral matter also, there is, at the same time, a great depth of green colour, indicating, it may be presumed, an abundant formation of chlorophyll. We have, therefore, with abundance of nitrogen, of mineral matter, and of chlorophyll, what may be called conditions of fuller potential growth; yet, in defect of the necessary climatal conditions, and, perhaps with a consequent limitation in the duration of the cycle of growth of the plants themselves, we have a deficient carbon-assimilation, in other words deficient growth over a given area.

In the next Table (LXXIV., pp. 1358–9) are given the botanical results relating to plot 11–2, which was manured precisely as 11–1 with the exception that, in the seventh year, and subsequently, artificial silicates of lime and soda, or of soda alone, were also applied. In addition to the usual particulars, the increase or decrease in actual yield per acre of each species compared with the produce on 11–1 is given.

TABLE LXXIV.—Number of Species, Percentage, Quantity per Acre, &c., of including Potass, and

	Number of species, and proportion per cent.					Quantity per acre.				
	1862.	1867.	1872.	1877.	Mean.	1862.	1867.	1872.	1877.	Mean.
NUMBER OF SPECIES.										
Gramineæ	14	14	13	11	13	...	...	...	...	...
Leguminosæ	0	0	0	0	0	...	...	...	...	...
Other Orders	7	5	3	5	5	...	...	...	...	...
Total	21	19	16	16	18	...	...	...	...	...
GRAMINEÆ.										
	p. c.	p. c.	p. c.	p. c.	p. c.	lbs.	lbs.	lbs.	lbs.	lbs.
Anthoxanthum odoratum	1·00	0·10	0·16	0·17	0·36	71·2	6·3	11·5	14·4	25·8
Alopecurus pratensis	1·50	6·33	22·65	20·11	12·65	106·8	396·0	1620·6	1709·3	958·2
Agrostis vulgaris	18·81	24·16	10·16	17·09	17·55	1339·3	1511·4	726·9	1452·7	1257·6
Holcus lanatus	7·37	4·79	10·59	19·48	10·56	524·7	299·6	757·7	1655·8	809·4
Avena elatior	6·40	4·83	12·73	21·14	11·27	455·7	302·2	910·8	1796·9	866·4
Avena flavescens	3·42	1·75	0·41	0·02	1·40	243·5	109·5	29·3	1·7	96·0
Poa pratensis	5·12	10·38	12·43	4·50	8·11	364·5	649·4	889·4	382·5	571·5
Poa trivialis	17·04	1·52	0·81	0·01	4·85	1213·3	95·1	58·0	0·9	341·8
Dactylis glomerata	23·34	38·30	27·23	13·38	25·56	1661·8	2396·0	1948·3	1137·3	1785·8
Festuca ovina	0·66	2·05	0·30	2·55	1·39	47·0	128·2	21·5	216·7	103·4
Festuca pratensis	2·28	0·06	...	...	0·57	162·3	3·8	...	...	41·5
Lolium perenne	1·46	0·38	0·16	...	0·50	104·0	23·8	11·5	...	34·8
None yielding 1 per cent. { Phleum pratense, Aira cæspitosa, Avena pubescens, Bromus mollis }	1·45	0·08	0·03	0·01	0·40	103·2	5·0	2·1	0·9	27·9
Absent. { Briza media, Cynosurus cristatus, Festuca loliacea }	...	...	...	...	...	...	...	...	...	...
Undetermined (chiefly Gramineæ)	4·39	0·94	1·60	...	1·73	312·6	58·8	114·5	...	121·5
Total	94·24	95·67	99·26	98·46	96·90	6709·9	5985·1	7102·1	8369·1	7041·6
LEGUMINOSÆ.										
Trifolium repens, T. pratense, T. minus Lotus corniculatus, Lathyrus pratensis, Ononis arvensis.	...	...	...	...	...	...	...	...	...	...
Total	...	...	...	...	...	...	...	...	...	...
OTHER ORDERS.										
Conopodium denudatum	1·19	0·70	0·03	0·01	0·48	84·7	43·8	2·1	0·8	32·8
Rumex Acetosa	4·08	3·57	0·70	1·34	2·41	290·5	223·4	50·1	113·9	169·5
None yielding 1 per cent. { Ranunculus repens et bulbosus, Pimpinella Saxifraga, Heracleum Sphondylium, Galium verum, Achillea Millefolium, Taraxacum officinale, Luzula campestris, Carex præcox, Hypnum squarrosum, H. rutabulum, H. hians }	0·49	0·06	0·01	0·19	0·21	34·9	3·7	0·7	16·2	13·9
Absent. { Ranunculus acris, Stellaria graminea, Cerastium triviale, Potentilla reptans, Agrimonia Eupatoria, Poterium Sanguisorba, Galium Aparine, Scabiosa arvensis, Centaurea nigra, Bellis perennis, Chrysanthemum Leucanthemum, Tragopogon pratensis, Leontodon hispidus, Hieracium Pilosella, Plantago lanceolata, Veronica Chamædrys, Prunella vulgaris, Thymus Serpyllum, Ajuga reptans, Primula veris, Scilla nutans, Ophioglossum vulgatum }	...	...	...	...	...	...	...	...	...	...
Total	5·76	4·33	0·74	1·54	3·10	410·1	270·9	52·9	130·9	216·2
SUMMARY.										
Gramineæ	94·24	95·67	99·26	98·46	96·90	6709·9	5985·1	7102·1	8369·1	7041·6
Leguminosæ	...	...	...	...	...	...	...	...	...	...
Other Orders	5·76	4·33	0·74	1·54	3·10	410·1	270·9	52·9	130·9	216·2
Total	100·00	100·00	100·00	100·00	100·00	7120	6256	7155	8500	7257

each Species, by 800 lbs. of Ammonia-salts, and Mixed Mineral Manure, Silicates; Plot 11–2.

Increase + or decrease − compared with:—									
Plot 3, without manure.					Plot 11–1; as 11–2, but without silicates.				
1862.	1867.	1872.	1877.	Mean.	1862.	1867.	1872.	1877.	Mean.
NUMBER OF SPECIES (continued).									
− 4	− 1	− 4	− 6	− 4	−1	+1	+2	0	+1
− 4	− 4	− 4	− 4	− 4	−1	−1	−1	0	−1
−21	−19	−25	−26	−23	−5	+1	−1	+1	−1
−29	−24	−33	−36	−31	−7	+1	0	+1	−1
GRAMINEÆ (continued).									
lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
− 59·4	− 282·2	− 74·0	− 106·6	− 130·6	+ 64·8	+ 3·1	− 44·1	+ 1·5	+ 6·3
− 30·2	+ 202·1	+1612·1	+1702·2	+ 871·6	− 92·2	−306·7	+739·9	+1035·8	+344·2
+ 992·6	+1223·9	+ 461·6	+1138·8	+ 954·2	+403·2	+478·5	−240·1	− 531·7	+ 27·5
+ 370·9	+ 34·1	+ 698·5	+1359·1	+ 615·6	−180·4	+146·3	+ 21·1	+ 277·0	+ 66·0
+ 453·6	+ 295·2	+ 908·7	+1795·7	+ 863·3	+401·0	+ 58·3	+168·4	+ 787·1	+353·7
+ 171·1	+ 47·5	− 28·1	− 23·8	+ 41·7	−131·8	+ 84·8	+ 22·9	+ 1·0	− 5·8
+ 355·6	+ 643·7	+ 887·9	+ 380·8	+ 567·1	−305·8	− 39·9	+147·8	+ 282·6	+ 21·2
+1166·3	+ 56·1	+ 49·8	− 12·3	+ 314·9	+271·5	+ 87·6	+ 51·6	− 21·5	+ 97·3
+1608·1	+2338·0	+1933·5	+1120·7	+1750·0	− 55·5	+289·0	−852·8	− 25·7	−161·3
− 358·9	− 378·3	− 334·7	− 300·8	− 343·1	− 56·8	+101·4	− 5·6	− 65·3	− 6·5
+ 161·1	+ 3·8	− 0·2	− 0·2	+ 41·1	+ 27·2	+ 3·8	...	...	+ 7·7
− 90·4	− 110·5	− 27·5	− 107·6	− 84·0	+ 6·6	+ 19·5	+ 11·5	− 0·7	+ 9·2
− 195·9	− 99·0	− 56·7	− 63·6	− 103·8	−114·3	+ 2·4	+ 1·4	+ 0·9	− 27·3
− 62·9	+ 27·0	− 123·4	− 195·3	− 102·1	...	...	...	...	...
+ 73·3	− 145·7	+ 65·8	...	− 1·6	+119·3	+ 12·2	+ 31·8	...	+ 40·8
+4554·9	+3801·7	+5973·3	+6687·1	+5254·3	+356·8	+940·3	+ 53·8	+1741·0	+773·0
LEGUMINOSÆ (continued).									
− 247·2	− 178·3	− 147·6	− 201·9	− 193·7	− 0·7	− 0·6	− 0·7	...	− 0·5
− 247·2	− 178·3	− 147·6	− 201·9	− 193·7	− 0·7	− 0·6	− 0·7	...	− 0·5
OTHER ORDERS (continued).									
+ 55·1	− 54·5	− 44·7	− 44·1	− 22·1	− 42·5	− 54·8	− 0·8	+ 0·1	− 24·5
+ 247·8	+ 164·7	+ 21·0	+ 69·7	+ 125·8	−208·5	+ 11·1	− 27·6	− 39·0	− 66·0
− 275·2	− 377·3	− 164·2	− 184·6	− 250·1	− 87·4	0·0	0·0	+ 1·9	− 21·4
− 267·4	− 432·3	− 126·8	− 190·2	− 254·4	− 5·7	...	− 0·7	...	− 1·6
− 239·7	− 699·4	− 314·7	− 349·2	− 400·8	−344·1	− 43·7	− 29·1	− 37·0	−113·5
SUMMARY (continued).									
+4554·9	+3801·7	+5973·3	+6687·1	+5254·3	+356·8	+940·3	+ 53·8	+1741·0	+773·0
− 247·2	− 178·3	− 147·6	− 201·9	− 193·7	− 0·7	− 0·6	− 0·7	...	− 0·5
− 239·7	− 699·4	− 314·7	− 349·2	− 400·8	−344·1	− 43·7	− 29·1	− 37·0	−113·5
+4068	+2924	+5511	+6136	+4659	+ 12	+896	+ 24	+1704	+ 659

There was even still more total produce, and still more nitrogen and mineral matter taken up, with the addition of the silicates; and, over the later as compared with the earlier period, the falling off in the yield of hay and its contents of mineral matter was less, and the increase in the assimilation of nitrogen was greater, than in the case of 11–1. There was, nevertheless, with an actual diminution in the amount of mineral matter, and an increase of that of nitrogen, taken up in the later years, evidence of a relative deficiency of available mineral matter compared with the available nitrogen. Under the influence of the silicates, there was an increase in the amount of the mineral constituents in the produce far beyond that of the increased quantity of silica taken up, which was extremely small in proportion to that supplied. Indeed, besides the increased amount of silica, lime, and soda, taken up, these being supplied, there was also more magnesia, potass, phosphoric acid, sulphuric acid, and chlorine, gathered; though none of these were supplied in greater amount than on 11–1. We have, therefore, with the same supplies of nitrogen, and of all the mineral constituents, except silica, lime, and soda, which were in increased amount, nevertheless a greater activity of accumulation, and greater than could be due to the direct supply of constituents by the silicates used. It is probable that the alkaline silicates reacted within the soil, serving to neutralise sulphuric acid and chlorine, and to liberate nitrogen and mineral constituents in an available condition; and also that, with the increased growth, the plants acquired increased root-range, and increased capacity of food-collection.

Referring to the table of botanical results, it will be seen that there were on the average rather fewer species on 11–2 with, than on 11–1 without the silicates, the average number found in the samples being only 18. There was also a somewhat higher average percentage of grasses in the produce; the proportion being more than 99 in the third, and nearly 97 on the average of the four separation-years. There was absolutely no leguminous plant found in any of the samples. This was so, notwithstanding that there was supplied as much potass as, and more of some other mineral constituents than, on plot 7, where the Leguminosæ were so prominent; there being there no nitrogenous manure in addition to give the Gramineæ such pre-eminent advantage in the struggle. Among Miscellaneæ only two, *Rumex Acetosa* and *Conopodium denudatum*, contributed as much as 1 per cent. to the produce in any one separation-year, and of these the *Rumex* has gone down to only about 1 per cent., and the *Conopodium* to a mere fraction in the later years; all other miscellaneous species collectively not yielding, on the average, a quarter of 1 per cent.

The chief point of interest in the detail of the botanical results is the difference of effect as compared with plot 11–1 without the silicates. There is seen to be in the later separation-years a very great increase in the relative amount of *Alopecurus pratensis*, a great increase also in that of *Avena elatior*, and some in *Poa pratensis* and *Holcus lanatus*; whilst *Agrostis vulgaris*, *Dactylis glomerata*, and most other grasses yielded, in the later years, less than on 11–1.

Thus, with the addition of silicates of lime and soda, we have more nitrogen, and more of other mineral constituents besides those supplied, taken up, and coincidently more total growth—that is, there was more carbon assimilated, and more dry substance produced, the result of a fuller development of some of the more vigorous, freer-growing, and early maturing grasses, as *Alopecurus pratensis, Avena elatior, Poa pratensis,* and *Holcus lanatus.* All of these are, especially compared with *Agrostis,* early ripeners; and with this character it is to be supposed that they would gain in consolidation, and with this in weight, at the same time that they would tend, both by their obtrusion and more rapid accumulation of material, to limit both the space and the resources at the command of their later maturing associates. With the increased growth there was great coarseness of herbage, but a less merely tufty and patchy condition of the plot, there being many fewer bare spaces of damaged herbage. The dark green and immature condition was also somewhat less marked; whilst some species, specially *Avena elatior* in some seasons, flowered and seeded more freely; in other words, matured better. This plant, it may be noted, on both plots and in most seasons formed stem and seed more freely than any other species in the second crops; indeed, in some cases it apparently contributed more stem to the second crop on plot 11–2 than all the other species over the whole series of plots. These circumstances must not be overlooked in accounting for the considerably increased predominance of *Avena elatior,* especially on plot 11–2, where the conditions for maturation were, as has been shown, somewhat more favourable than on 11–1. On the other hand, *Dactylis* was, as a rule, by no means the most prominent in the second crops, especially in the later years, during which they have for the most part been cut and removed from the land; increasing, therefore, the exhaustion of the soil, and obviously introducing an element which should be borne in mind in judging of the causes of the changes in the flora from the third to the fourth separation-years.

It has been pointed out in reference to the results on plot 11–1 that, so far as the nitrogen and the mineral constituents supplied and taken up were concerned, there was, so to speak, potential, beyond the degree of actual, growth attained. Now, on 11–2, with somewhat greater mineral supplies, more of nitrogen and of mineral matter taken up, and some more growth, we have still, as shown both by the insufficiently matured character of the herbage, and by its chemical composition, deficient carbon-accumulation in relation to the soil-supplies—nitrogen and mineral matter—taken up. In fact, it would seem that the limits of possible growth under the influence of the soil in question, and of the average climatic conditions of the seasons, had been reached.

10. *Nitrate of soda* (550 *lbs. per acre*), *with mixed mineral manure, including potass; Plot* 14.

The same mineral manure, including potass, is annually applied to this plot as to plot 9, and the same amount of nitrogen also, but in the form of nitrate of soda instead of ammonia-salts. It has been shown that, when each of these was used without mineral manure, a given amount of nitrogen as nitrate of soda yielded more hay, more nitrogen, and more mineral matter, and that there was a less reduction in the yield of each in the later years, than when the same amount was applied as ammonia-salts. Now, when each is applied in conjunction with mineral manure, there is again more hay produced, and more mineral matter taken up, but rather less nitrogen, with the nitrate. But, whilst there is a decrease in each item in the later years on plot 9, with the mineral manure and ammonia-salts, there is an increase of each on plot 14, with the mineral manure and nitrate of soda.

Referring to Part I. for a fuller consideration of these points, and to Part III. (to follow) for still more detailed evidence, it must suffice here to state very briefly some of the conditions of these differences of result on the two plots. The nitrogen of the nitrate distributes more rapidly both in the upper and in the lower layers of the soil than does that of the ammonia-salts. Accordingly, both free-surface-feeders and more deeply-rooting species are encouraged on the nitrate plot, and hence the collective herbage on that plot is less susceptible to the adverse influence of drought than that on plot 9. A striking instance of this occurred in 1870, and the conditions are somewhat fully described in Part I. already referred to; but it may be stated briefly that whilst in that year of very unusual drought there was a deficiency compared with the average of about 23 cwts. of hay on the mineral and ammonia plot 9, there was a deficiency of less than 1½ cwt. on the mineral and nitrate plot 14. On this plot, the deep and wiry-rooted *Bromus mollis* contributed nearly half the herbage in that year; and examinations and analyses of the soils and subsoils of the plots showed much the more extensive root-development in the lower layers on plot 14, and very much less moisture there remaining. It is obvious that the deeper-rooting species had drawn up much of their needed water from the subsoil; partly, perhaps, directly, and partly by virtue of increased capillary action induced by the pumping out of the upper layers. It was also found that, probably in part due to the action of the roots, and in part to chemical reactions, the subsoil of the nitrate plot was more disintegrated and ameliorated than that of either plot 9 or the unmanured plot 3.

With these few general remarks, indicating some of the characteristic differences of result on the two plots, we now turn to a detailed consideration of their comparative botany.

The following Table (LXXV., pp. 1364–5) gives, in the usual form, a record of the facts relating to the plot under consideration; it also shows the amounts yielded by the different orders and species compared with those on the unmanured plot 3, and on plot 9 with the mineral manure and the nitrogen supplied as ammonia-salts.

In the first place there were, on the average, two fewer Gramineæ, two fewer Leguminosæ, and 16 fewer Miscellaneæ, than without manure, the average number found in the four separations in the samples from plot 14 being 29. Compared with plot 9 there was but little difference in number; in fact, only one more species.

There is not very much difference in the average percentages of total Gramineæ, Leguminosæ, and Miscellaneæ, on plot 9 and on plot 14. On each, there is considerable fluctuation as between the Gramineæ and the Miscellaneæ in the different separation-years; and the relations are not the same in the same season under the different conditions of the two plots. On the average, the nitrate plot gives a higher percentage of Gramineæ, and a lower percentage of Miscellaneæ; but, on the other hand, though still in insignificant proportion, it yields notably more Leguminosæ, especially of *Lathyrus pratensis*.

Bearing in mind the main features of the botany of the plots hitherto considered which received a mixture of mineral and nitrogenous manure, but the latter as ammonia-salts, it will be observed that the species predominating are very widely different. Here, with the nitrate, the most prominent plant, on the average, and with one exception in each separation-year, is *Poa trivialis*, which in no case hitherto recorded has been the first in the list. The one exception was that in 1872 *Bromus mollis* contributed more than 42 per cent., whereas in some earlier years it was estimated to have even yielded more than this, whilst *Poa trivialis* gave in that year less than 25 per cent. In the fourth separation-year, however, whilst *Poa trivialis* was only reduced to 21½ per cent., *Bromus mollis* was reduced to 8 per cent. From the first to the fourth separation-year the percentages of *Alopecurus pratensis* increased in a very striking degree, and those of *Holcus lanatus* and *Dactylis glomerata* also increased, whilst *Lolium perenne* markedly decreased. Looking to the results in the fourth separation-year, *Poa trivialis* is the first on the list, *Alopecurus pratensis* coming second, each contributing more than 20 per cent., whilst *Holcus lanatus* and *Dactylis glomerata* each contribute more than 12 per cent. Of other grasses of any prominence, *Bromus mollis* gives about 8, and *Poa pratensis* about 4 per cent.; *Avena flavescens, Lolium perenne, Agrostis vulgaris*, considerably less, and *Festuca ovina, Avena elatior*, and others, quite insignificant proportions.

As already intimated, the only leguminous plant which was found in more than a trace, and which increased, was *Lathyrus pratensis;* the total Leguminosæ, however, yielding on the average considerably less than 1 per cent.

Of miscellaneous species yielding in any one year as much as 1 per cent. to the produce, there are with the mineral manure and nitrate four, whilst with the mineral and ammonia there were only three. However, the fourth on the nitrate plot is the *Ranunculus* (*repens* and *bulbosus* together), which owes its place in the average column to its yield in the first separation-year only, afterwards all but disappearing. Of the remaining three, as on plot 9, *Rumex Acetosa* and *Conopodium denudatum* have each a place, but on plot 9 the third was *Achillea Millefolium*, whilst here on plot 14 it is *Anthriscus sylvestris*, which has much increased in the later years, even more than *Rumex Acetosa*.

The main features of the miscellaneous herbage may be said to be, that *Rumex Acetosa* has given the highest average proportion, but has fluctuated very much, yielding very little in the second and third, and less in the fourth, than in the first

TABLE LXXV.—Number of Species, Percentage, Quantity per Acre, &c., of each Species,

	Number of species, and proportion per cent.					Quantity per acre.				
	1862.	1867.	1872.	1877.	Mean.	1862.	1867.	1872.	1877.	Mean.
NUMBER OF SPECIES.										
Gramineæ	15	14	14	15	15	...	...	...	...	...
Leguminosæ	3	3	2	1	2	...	...	...	...	...
Other Orders	10	13	14	11	12	...	...	...	...	...
Total	28	30	30	27	29	...	...	...	...	...
GRAMINEÆ.										
	p. c.	p. c.	p. c.	p. c.	p. c.	lbs.	lbs.	lbs.	lbs.	lbs.
Alopecurus pratensis	0·22	3·54	3·72	20·18	6·91	12·6	254·5	231·2	1262·9	440·3
Agrostis vulgaris	0·42	0·61	0·24	1·55	0·71	24·0	43·8	14·9	97·0	44·9
Holcus lanatus	6·60	6·63	3·67	12·75	7·41	377·4	476·5	228·1	797·9	470·0
Avena elatior	3·14	...	...	0·32	0·86	179·5	...	...	20·0	49·9
Avena flavescens	4·88	7·12	5·67	2·93	5·15	279·0	511·8	352·4	183·3	331·6
Poa pratensis	1·45	1·05	2·57	4·01	2·27	82·9	75·5	159·7	250·9	142·3
Poa trivialis	22·48	32·93	24·76	21·59	25·44	1285·4	2367·0	1538·8	1351·1	1635·5
Dactylis glomerata	10·00	7·28	3·33	12·48	8·27	571·8	523·3	207·0	781·0	520·8
Festuca ovina	0·88	1·58	0·16	0·48	0·77	50·3	113·6	10·0	30·0	51·0
Bromus mollis	18·04	17·69	42·10	8·02	21·46	1031·5	1271·6	2616·5	501·9	1355·1
Lolium perenne	13·80	9·36	5·55	2·63	7·84	789·1	672·8	344·9	164·6	492·9
None yielding 1 per cent. { Anthoxanthum odoratum, Phleum pratense, Aira cæspitosa, Avena pubescens, Cynosurus cristatus, Festuca pratensis }	2·13	1·25	0·31	0·87	1·15	121·9	89·8	19·3	54·5	71·6
Absent—Briza media, Festuca loliacea	...	...	...	...	...	...	...	...	...	...
Undetermined (chiefly Gramineæ)	5·48	5·21	0·79	...	2·87	313·4	374·5	49·1	...	184·2
Total	89·52	94·25	92·87	87·81	91·11	5118·8	6774·7	5771·9	5495·1	5790·1
LEGUMINOSÆ.										
Trifolium repens	0·01	0·01	...	...	0·01	0·6	0·7	...	...	0·3
Trifolium pratense	0·01	0·01	0·01	...	0·01	0·6	0·7	0·6	...	0·5
Lathyrus pratensis	0·11	0·37	1·35	0·76	0·64	6·2	26·6	83·9	47·6	41·1
Trifolium minus, Lotus corniculatus, Ononis arvensis	...	...	...	...	...	...	...	...	...	...
Total	0·13	0·39	1·36	0·76	0·66	7·4	28·0	84·5	47·6	41·9
OTHER ORDERS.										
Ranunculus repens et bulbosus	1·23	0·10	0·03	...	0·34	70·3	7·2	1·9	...	19·9
Conopodium denudatum	1·55	1·57	0·61	0·18	0·97	88·6	112·9	37·9	11·3	62·7
Anthriscus sylvestris	...	1·52	3·86	4·64	2·50	...	109·3	239·9	290·4	159·9
Rumex Acetosa	6·88	1·11	0·61	4·40	3·25	393·4	79·8	37·9	275·3	196·6
None yielding 1 per cent. { Ranunculus acris, Cerastium triviale, Pimpinella Saxifraga, Heracleum Sphondylium, Centaurea nigra, Achillea Millefolium, Leontodon hispidus, Taraxacum officinale, Plantago lanceolata, Rumex crispus, Scilla nutans, Luzula campestris, Carex præcox, Hypnum squarrosum, H. rutabulum, H. hians }	0·69	1·06	0·66	2·21	1·17	39·5	76·1	41·0	138·3	73·7
Absent { Ranunculus auricomus, Stellaria graminea, Potentilla reptans, Agrimonia Eupatoria, Poterium Sanguisorba, Spiræa Ulmaria, Galium verum, G. Aparine, Scabiosa arvensis, Bellis perennis, Chrysanthemum Leucanthemum, Tragopogon pratensis, Hieracium Pilosella, Veronica Chamædrys, Prunella vulgaris, Thymus Serpyllum, Ajuga reptans, Primula veris, Rumex crispus, Ornithogalum umbellatum, Ophioglossum vulgatum }	...	...	...	...	...	...	...	...	...	...
Total	10·35	5·36	5·77	11·43	8·23	591·8	385·3	358·6	715·3	512·8
SUMMARY.										
Gramineæ	89·52	94·25	92·87	87·81	91·11	5118·8	6774·7	5771·9	5495·1	5790·1
Leguminosæ	0·13	0·39	1·36	0·76	0·66	7·4	28·0	84·5	47·6	41·9
Other Orders	10·35	5·36	5·77	11·43	8·23	591·8	385·3	358·6	715·3	512·8
Total	100·00	100·00	100·00	100·00	100·00	5718	7188	6215	6258	6344

by 550 lbs. Nitrate of Soda and Mixed Mineral Manures, including Potass; Plot 14.

Increase + or decrease − compared with :—									
Plot 3, without manure.					Plot 9, ammonia-salts and mixed mineral manure.				
1862.	1867.	1872.	1877.	Mean.	1862.	1867.	1872.	1877.	Mean.
NUMBER OF SPECIES (continued).									
− 3	− 1	− 3	− 3	− 2	+2	0	−2	+2	+1
− 1	− 1	− 2	− 3	− 2	+1	+1	+1	−3	0
−18	−11	−14	−20	−16	−3	0	+1	+1	0
−22	−13	−19	−25	−20	0	+1	0	0	+1
GRAMINEÆ (continued).									
lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
− 124·4	+ 60·6	+ 222·7	+1255·8	+ 353·7	− 4·7	+ 250·7	+ 75·0	+1204·7	+ 381·4
− 322·7	− 243·7	− 250·4	− 216·9	− 258·5	−796·1	− 679·0	− 859·8	− 643·4	− 744·6
+ 223·6	+ 211·0	+ 168·9	+ 501·2	+ 276·2	−399·8	− 53·1	− 202·4	+ 170·1	− 121·3
+ 177·4	− 7·0	− 2·1	+ 18·8	+ 46·8	+179·5	− 134·6	− 645·0	− 781·0	− 345·2
+ 206·6	+ 449·8	+ 295·0	+ 157·8	+ 277·3	−302·3	+ 308·4	+ 52·6	+ 142·8	+ 50·4
+ 74·0	+ 69·8	+ 158·2	+ 249·2	+ 137·9	−600·8	− 625·2	−1123·0	− 840·6	− 797·4
+1238·4	+2328·0	+1530·6	+1337·9	+1608·6	+727·1	+2251·8	+1502·6	+1344·4	+1456·4
+ 518·1	+ 465·3	+ 192·2	+ 764·4	+ 485·0	+214·6	+ 273·6	− 465·2	− 70·8	− 12·0
− 355·6	− 392·9	− 346·2	− 487·5	− 395·5	−283·2	− 877·8	− 481·1	−1289·8	− 733·0
+1027·5	+1269·9	+2616·3	+ 501·9	+1353·6	+746·0	+1265·7	+2610·8	+ 501·9	+1280·8
+ 594·7	+ 538·5	+ 305·9	+ 57·0	+ 374·1	+520·2	+ 618·5	+ 282·1	+ 154·9	+ 894·0
− 310·2	− 305·3	− 143·2	− 154·4	− 228·1	−702·7	− 183·1	− 138·0	− 128·3	− 287·7
− 57·7	− 22·7	− 105·2	− 172·1	− 89·4	...	...	− 0·6	...	− 0·1
+ 74·1	+ 170·0	+ 0·4	...	+ 61·1	+149·4	+ 211·4	− 52·2	...	+ 77·1
+2963·8	+4591·3	+4643·1	+3813·1	+4002·8	+552·8	+2627·3	+ 555·8	− 235·1	+ 598·8
LEGUMINOSÆ (continued).									
− 15·6	− 6·3	− 6·2	− 3·1	− 7·8	...	+ 0·2	...	...	...
− 136·1	− 69·6	− 27·0	− 49·4	− 70·5	+ 0·6	+ 0·7	+ 0·6	− 0·6	+ 0·4
− 32·2	+ 3·9	+ 67·8	− 8·4	+ 7·8	− 1·5	+ 18·5	+ 82·8	+ 44·0	+ 35·9
− 55·9	− 78·3	− 97·7	− 93·4	− 81·3	...	...	...	− 20·6	− 5·1
− 239·8	− 150·3	− 63·1	− 154·3	− 151·8	− 0·9	+ 19·4	+ 83·4	+ 22·8	+ 31·2
OTHER ORDERS (continued).									
− 78·6	− 59·8	− 46·4	− 81·6	− 66·5	+ 61·3	+ 4·5	+ 1·9	− 2·4	+ 16·4
+ 59·0	+ 14·6	− 8·9	− 33·6	+ 7·8	− 98·3	− 390·3	− 47·5	− 34·1	− 142·5
...	+ 109·3	+ 239·9	+ 290·4	+ 159·9	...	+ 109·3	+ 239·9	+ 290·4	+ 159·9
+ 350·7	+ 21·1	+ 8·8	+ 231·1	+ 152·9	+ 47·7	− 506·4	− 222·3	+ 57·3	− 155·9
− 371·4	− 648·9	− 190·0	− 131·5	− 335·4	− 91·1	− 37·9	− 50·7	+ 126·2	− 13·3
− 17·7	− 21·3	− 12·4	− 39·6	− 22·9	− 49·9	− 19·9	− 3·5	− 21·1	− 23·8
− 58·0	− 585·0	− 9·0	+ 235·2	− 104·2	−130·3	− 840·7	− 82·2	+ 416·3	− 159·2
SUMMARY (continued).									
+2963·8	+4591·3	+4643·1	+3813·1	+4002·8	−552·8	+2627·3	+ 555·8	− 235·1	+ 598·8
− 239·8	− 150·3	− 63·1	− 154·3	− 151·8	− 0·9	+ 19·4	+ 83·4	+ 22·8	+ 31·2
− 58·0	− 585·0	− 9·0	+ 235·2	− 104·2	−130·3	− 840·7	− 82·2	+ 416·3	− 159·2
+2666	+3856	+4571	+3894	+3746	−684	+1806	+ 557	+ 204	+ 470

separation-year; *Anthriscus sylvestris* has gradually and considerably increased; *Conopodium denudatum* has very much diminished; and the several species of *Ranunculus* have become practically absent from the samples.

Comparing the actual yield per acre on plot 14 with that without manure, the table shows that on the average, and almost in every individual instance, there was a considerable excess of every grass occurring in more than 1 per cent. excepting *Festuca ovina* and *Agrostis vulgaris*, which are the two most prominent grasses of the locality under poor conditions of the soil. Those which have yielded the greatest average increase compared with the unmanured produce are most prominently and uniformly *Poa trivialis* and *Bromus mollis*, next in order coming *Dactylis glomerata, Lolium perenne, Alopecurus pratensis*, and, less prominently, *Holcus lanatus, Avena flavescens, Poa pratensis*, and *Avena elatior*.

Of Leguminosæ there was, with the one exception of *Lathyrus* in two of the years, a large deficiency of each species on plot 14, compared with plot 3 without manure.

Of Miscellaneæ there was, on plot 14 compared with plot 3, an excess of *Rumex Acetosa, Anthriscus sylvestris*, and, generally, of *Conopodium denudatum;* but there was a deficiency of the species of *Ranunculus*, and a very marked deficiency of a large number which occurred in too small quantity on plot 14 to be included in the list of those ever yielding 1 per cent. to the mixed herbage.

Compared with plot 9 (with the ammonia-salts), *Poa trivialis* and *Bromus mollis* are again the chief grasses in excess, and both are very largely so. In a much less degree, *Lolium perenne* and *Alopecurus pratensis* are in excess on plot 14; whilst *Festuca ovina, Poa pratensis*, and *Agrostis vulgaris*, are all largely, *Avena elatior* considerably, and *Holcus lanatus, Dactylis glomerata*, and some others in some degree, in defect.

Small as is the actual amount of leguminous herbage on plot 14, still about half of it is in excess of that on plot 9, and the excess is due, as already intimated, to *Lathyrus pratensis.*

Of miscellaneous species, the yield per acre of *Rumex Acetosa*, and of *Conopodium denudatum*, averaged considerably less on plot 14 than on plot 9; whilst the whole of the yield of *Anthriscus sylvestris* with the nitrate was in excess of that found in the samples from plot 9.

The foregoing details show—and the table should be further examined on the point—that there was a very striking difference in the growth, and in the botanical composition of the herbage, accordingly as the nitrogen (in conjunction with minerals) was applied as ammonia-salts or as nitrate of soda. With the nitrate there was on the average a notably greater quantity of gramineous herbage produced, somewhat more leguminous, but less miscellaneous herbage. With the ammonia-salts a greater number contributed a fair proportion to the herbage, and the most prominent grasses were *Poa pratensis, Festuca ovina, Agrostis vulgaris, Avena elatior*, and *Holcus lanatus.* With the nitrate of soda only one of these was in any special prominence; the most prominent were, instead of *Poa pratensis, P. trivialis* and *Bromus mollis;* and the others in any important amount were, *Dactylis glomerata, Lolium perenne, Holcus lanatus, Alopecurus pratensis*, and *Avena flavescens;* the Holcus, the only one of these which was in the list of prominence on plot 9, was, however, in less quantity than it was with the ammonia-salts.

But, apart from the evidence afforded by the record of the relative and the actual quantities in which different species occurred on the two plots, there was very great difference in the general aspect and tendency of development of the herbage in the two cases. On plot 14, with the nitrate, the growth was almost invariably very much more forward, with early and very prominent development of stem, but with comparatively little growth of leaf from the base. Another character incident to that of the coarse and luxuriant stemmy growth of a few individual species is that the turf of this plot (14) is less compact and looser than that of plot 9, and of most of the other plots. According to the season of growth, or of those preceding it, one or other of the most prominent plants attracted attention, and gave character to the plot. The produce was always ready to be cut much earlier than that on any other plot; the stems being generally ripe, and even dead at the bottom, before the herbage on most of the other plots was ready for cutting. Owing to this, and to the most prominent plants flowering and seeding early, and frequently shedding seed before cutting, their permanence was favoured. This was especially the case with *Bromus mollis*, which frequently produced seed in the first growth, and gave an abundance of seedling plants in the second. On the other hand, the fact of this plant being usually annual, or biennial only, may serve to account for the great fluctuation in its degree of prominence in the different seasons, dependent presumably on the climatal circumstances having been favourable or otherwise for its reproduction and persistence. Again, the fact of the occurrence of tubers on the roots, which has been observed in this plant on this plot, may perhaps be taken to indicate a tendency to assume a perennial duration.

In conclusion, the contrast brought to view between the occurrence and predominance of different species on two plots manured in many respects so very similarly as plots 9 and 14, the only difference being that of the condition of combination in which the nitrogen is supplied, is exceedingly remarkable. The result affords a striking illustration of the influence upon a flora, of variations of conditions of growth which would at first sight appear comparatively unimportant. There was also a difference in the character of growth, dependent in part on the diversity in the natural "habit" of the plants favoured, by virtue of which, again, it is that they have become susceptible to the influence of the conditions provided; but the result is also in part due, directly to the known characteristic action of the manure itself, in inducing great luxuriance and, in conjunction with sufficient mineral supply, great forwardness and tendency to consolidation and maturation.

11. *Nitrate of soda* (275 *lbs. per acre*) *with mixed mineral manure, including potass; Plot* 16.

This experiment, like the rest of the nitrate series (plots 14, 15, 16, and 17), did not commence until the third year (1858). Plot 16 received annually the same description and amount of mixed mineral manure, including potass, as plot 14, but in conjunction with only 275 lbs. instead of 550 lbs. of nitrate of soda, supplying, therefore, only half the quantity of nitrogen. The botanical details will show that this reduction in the supply of the element contributing characteristically to luxuriance, with at the same time the maintenance of the conditions tending rather to maturation, has very materially affected the results of the struggle among the component plants of the mixed herbage.

The first point to notice (Table LXXVI., pp. 1370–1) is that there was on the average a greater number of species found in the samples from plot 16, with the smaller than in those from plot 14, with the larger amount of nitrate. This was the case within each of the three groups; there being one more grass, two more Leguminosæ, and five more Miscellaneæ; but in the fourth separation-year there were as many as three more Leguminosæ, and 11 more Miscellaneæ, in the samples from plot 16 than in those from plot 14. Taking the average of the four separation-years, there were 37 species found, against 29 with the mineral manure and double quantity of nitrate, and 49 without manure. There was also, with a tendency to the further reduction in the number on plot 14 from one year to another, within the separation period, a greater tendency to regain in number over the same period on plot 16; the numbers found in the four separation-years being 34, 34, 36, and 41, respectively.

There was a considerably smaller average percentage of total Gramineæ with the smaller than with the larger amount of nitrate (and mineral manures). There was a much larger, and a greatly increasing, percentage of leguminous herbage, and a considerably higher average, but a gradually diminishing, proportion of total Miscellaneæ. The herbage was, in fact, less exclusively gramineous, and, upon the whole, considerably more mixed, with the smaller amount of nitrate, and the consequent less luxuriance of the freer-growing species.

Among the grasses, neither *Poa trivialis* nor *Bromus mollis*, which were so prominent on plot 14, was so in any degree on plot 16. *Poa trivialis*, however, maintained its position very much better than *Bromus mollis*, which yielded only a fraction of 1 per cent. in the fourth separation-year. Nor was any other grass so prominent on the average on plot 16 as both of these were on plot 14. On the other hand, on plot 16, the prevalent but poor grasses of the locality, *Agrostis vulgaris* and *Festuca ovina* are, upon the whole, the most prominent grasses; whilst each of these was in most insignificant amount on plot 14. In about equal prominence with *Agrostis vulgaris* and *Festuca ovina*, are *Avena flavescens*, with a tendency to diminish, *Holcus lanatus*, with fairly uniform occurrence, and *Alopecurus pratensis*, with marked tendency to increase; the last two of these, it will be remembered, also asserted considerable prominence in the later years on plot 14; whilst *Avena flavescens* was found in comparatively small and diminishing percentage on that plot. Of other grasses that were fairly prominent on plot 14, *Dactylis glomerata* is in very much less amount, though somewhat increasing, on plot 16. *Lolium perenne* declined on both plots, but gave the highest average amount on plot 14. *Anthoxanthum odoratum* and *Avena pubescens*, neither of which reached 1 per cent. in either separation-year on plot 14, each increased and averaged about 2 per cent. on plot 16.

The difference in the prevalence of Leguminosæ on the two plots is most striking. With the forced luxuriant growth of individual grasses with the larger amount of nitrate, there was not, on the average, 1 per cent. of leguminous herbage; but, with the smaller amount of nitrate there was, on the average, more than 5 per cent., rising from little over 2 in the first, to nearly 9½ in the fourth separation-year. The table will show that practically the whole of the increase was due to enhanced development of *Lathyrus pratensis.*

Whilst on plot 14, with the larger amount of nitrate, there were only four species of the miscellaneous group yielding in any one of the four separation-years 1 per cent. to the herbage, there were, on plot 16, with the smaller amount of nitrate, seven species which attained this degree of prominence. *Rumex Acetosa* was, on the whole, the most prominent on both plots; *Conopodium denudatum* was decidedly the more generally prominent with the smaller amount of nitrate, as also was *Achillea Millefolium*, which was not in the list of the first four on plot 14. The several species of *Ranunculus* were, again, more prominent on plot 16; and *Plantago lanceolata* and *Centaurea nigra* each came into the foremost list on that plot, but not on plot 14. On the other hand, *Anthriscus sylvestris*, which became so prominent in the later years on plot 14, in neither year reached anything like 1 per cent. on plot 16.

Upon the whole, the Miscellaneæ, like the Leguminosæ, yielded a higher average proportion to the mixed herbage on plot 16; but, unlike the Leguminosæ, the quantity of the Miscellaneæ diminished from one separation-year to another. This result was chiefly due to the diminishing proportions of *Rumex Acetosa*, *Conopodium denudatum*, and *Ranunculus repens* and *R. bulbosus.* Lastly, on this point, it is evident that, among the Miscellaneæ, as among the grasses, the herbage was considerably the more mixed with the smaller amount of nitrate, and the coincident less forced luxuriance, and less active struggle.

Looking to the columns of actual yield per acre of each species, and of the increase or decrease of each, compared with plot 3 without manure, and with plot 14 with the double quantity of nitrate, the more evenly mixed character of the herbage is again clearly brought to view. Compared with the produce without manure, it is seen that there is, on the average of the four separation-years, a more or less considerable increase of nine out of the eleven specially enumerated grasses; and there was a diminution, but in only a small degree, of the remaining two, namely, *Anthoxanthum*

TABLE LXXVI.—Number of Species, Percentage, Quantity per
Mixed Mineral Manure,

	Number of species, and proportion per cent.					Quantity per acre.				
	1862.	1867.	1872.	1877.	Mean.	1862.	1867.	1872.	1877.	Mean.
NUMBER OF SPECIES.										
Gramineæ	17	14	17	15	16	...	...	...	...	...
Leguminosæ	3	4	4	4	4	...	...	...	...	...
Other Orders	14	16	15	22	17	...	...	...	...	...
Total	34	34	36	41	37	...	...	...	...	...
GRAMINEÆ.										
	p. c.	p. c.	p. c.	p. c.	p. c.	lbs.	lbs.	lbs.	lbs.	lbs.
Anthoxanthum odoratum	1·42	1·78	1·40	2·22	1·70	73·5	105·7	62·7	135·3	94·3
Alopecurus pratensis	0·67	8·27	15·22	12·23	9·12	34·7	491·2	681·1	745·6	488·2
Agrostis vulgaris	12·49	13·62	12·40	14·58	13·27	646·7	809·0	554·9	888·8	724·9
Holcus lanatus	10·45	11·69	5·08	12·64	9·95	541·1	694·4	227·3	770·5	558·3
Avena pubescens	1·49	1·72	1·54	3·34	2·02	77·2	102·2	68·9	203·6	113·0
Avena flavescens	18·37	14·86	18·80	6·67	14·67	951·2	882·7	841·3	406·6	770·4
Poa trivialis	6·87	8·96	6·53	4·82	6·79	355·7	532·2	292·2	293·8	368·5
Dactylis glomerata	1·59	2·59	3·75	4·63	3·14	82·4	153·9	167·8	282·2	171·4
Festuca ovina	11·13	10·44	10·33	16·66	12·14	576·3	620·1	462·3	1015·6	668·6
Bromus mollis	2·04	2·64	2·03	0·57	1·82	105·7	156·8	90·9	34·7	97·0
Lolium perenne	5·85	6·23	3·10	3·75	4·73	302·9	370·1	138·7	228·6	260·1
None yielding 1 per cent. { Phleum pratense, Aira cæspitosa, Avena elatior, Poa pratensis, Briza media, Cynosurus cristatus, Festuca pratensis }	0·65	0·59	0·90	0·83	0·76	33·6	35·1	40·2	50·7	40·0
Absent—Festuca loliacea	...	...	...	...	...	...	...	...	...	...
Undetermined (chiefly Gramineæ)	4·99	1·04	0·50	...	1·63	253·3	61·8	22·4	...	85·6
Total	78·01	84·43	81·58	82·94	81·74	4039·3	5015·2	3650·7	5056·0	4440·3
LEGUMINOSÆ.										
Trifolium repens	0·35	0·50	0·32	0·01	0·29	18·1	29·7	14·3	0·6	15·7
Trifolium pratense	1·85	0·70	0·43	0·11	0·77	95·8	41·6	19·3	6·7	40·9
Lotus corniculatus	...	0·02	0·02	0·06	0·03	...	1·2	0·9	3·7	1·4
Lathyrus pratensis	0·04	0·57	6·68	9·22	4·13	2·1	33·8	298·9	562·0	224·2
Total	2·24	1·79	7·45	9·40	5·22	116·0	106·3	333·4	573·0	282·2
OTHER ORDERS.										
Ranunculus acris	0·95	0·05	0·79	1·24	0·76	49·2	3·0	35·4	75·6	40·8
Ranunculus repens et bulbosus	4·66	0·33	0·33	0·26	1·39	241·6	19·6	14·8	15·8	72·9
Conopodium denudatum	4·50	4·80	3·90	0·51	3·42	233·0	285·1	174·5	31·1	180·9
Centaurea nigra	...	0·08	1·14	0·31	0·38	...	4·8	51·0	18·9	18·7
Achillea Millefolium	2·47	1·76	3·29	1·55	2·26	127·9	104·5	147·2	94·5	118·5
Plantago lanceolata	1·25	0·84	0·07	0·22	0·58	64·7	49·9	3·1	13·4	32·7
Rumex Acetosa	5·46	5·62	1·19	2·19	3·61	282·7	333·8	53·3	133·5	200·8
None yielding 1 per cent. { Ranunculus auricomus, Stellaria graminea, Cerastium triviale, Potentilla reptans, Pimpinella Saxifraga, Heracleum Sphondylium, Anthriscus sylvestris, Galium verum, Tragopogon pratensis, Leontodon hispidus, Taraxacum officinale, Veronica Chamædrys, Ajuga reptans, Primula veris, Luzula campestris, Hypnum squarrosum, H. rutabulum, H. hians }	1·46	0·30	0·26	1·38	0·64	23·6	17·8	11·6	84·2	34·5
Absent { Agrimonia Eupatoria, Poterium Sanguisorba, Galium Aparine, Scabiosa arvensis, Bellis perennis, Chrysanthemum Leucanthemum, Hieracium Pilosella, Prunella vulgaris, Thymus Serpyllum, Rumex crispus, Scilla nutans, Carex præcox, Ophioglossum vulgatum }	...	...	...	...	...	...	...	...	...	...
Total	19·75	13·78	10·97	7·66	13·04	1022·7	818·5	490·9	467·0	699·8
SUMMARY.										
Gramineæ	78·01	84·43	81·58	82·94	81·74	4039·3	5015·2	3650·7	5056·0	4440·3
Leguminosæ	2·24	1·79	7·45	9·40	5·22	116·0	106·3	333·4	573·0	282·2
Other orders	19·75	13·78	10·97	7·66	13·04	1022·7	818·5	490·9	467·0	699·8
Total	100·00	100·00	100·00	100·00	100·00	5178	5940	4475	6096	5422

Acre, &c., of each Species, by 275 lbs. Nitrate of Soda, with including Potass; Plot 16.

Increase + or decrease − compared with :—									
Plot 3, without manure.					Plot 14; as 16, but with double quantity of nitrate.				
1862.	1867.	1872.	1877.	Mean.	1862.	1867.	1872.	1877.	Mean.
NUMBER OF SPECIES (continued).									
− 1	−1	0	− 2	− 1	+2	0	+3	0	+1
− 1	0	0	0	0	0	+1	+2	+ 3	+2
−14	−8	−13	− 9	−11	+4	+3	+1	+11	+5
−16	−9	−13	−11	−12	+6	+4	+6	+14	+8
GRAMINEÆ (continued).									
lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
− 57·1	− 182·8	− 22·8	+ 14·3	− 62·1	+ 53·5	+ 96·4	+ 61·4	+ 131·5	+ 85·7
− 102·3	+ 297·3	+ 672·6	+ 738·5	+ 401·6	+ 22·1	+ 236·7	+ 449·9	− 517·3	+ 47·9
+ 300·0	+ 521·5	+ 289·6	+ 574·9	+ 421·5	+ 622·7	+ 765·2	+ 540·0	+ 791·8	+ 680·0
+ 387·3	+ 428·9	+ 168·1	+ 473·8	+ 364·5	+ 163·7	+ 217·9	− 0·8	− 27·4	+ 88·3
− 217·3	− 0·1	+ 10·5	+ 140·0	− 16·7	+ 25·7	+ 36·1	+ 57·1	+ 174·2	+ 73·3
+ 878·8	+ 820·7	+ 783·9	+ 381·1	+ 716·1	+ 672·2	+ 370·9	+ 488·9	+ 223·3	+ 438·8
+ 308·7	+ 493·2	+ 284·0	+ 280·6	+ 341·6	− 929·7	−1834·8	−1246·6	−1057·3	−1267·0
+ 28·7	+ 95·9	+ 153·0	+ 265·6	+ 135·6	− 489·4	− 369·4	− 39·2	− 498·8	− 349·4
+ 170·4	+ 113·6	+ 106·1	+ 498·1	+ 222·1	+ 526·0	+ 506·5	+ 452·3	+ 985·6	+ 617·6
+ 101·7	+ 155·1	+ 90·7	+ 34·7	+ 95·5	− 925·8	−1114·8	−2525·6	− 467·2	−1258·1
+ 108·5	+ 235·8	+ 99·7	+ 121·0	+ 141·3	− 486·2	− 302·7	− 206·2	+ 64·0	− 232·8
− 42·1	− 4·6	− 87·2	− 147·8	− 70·3	− 279·2	− 54·8	− 125·7	− 241·5	− 175·5
...	...	...	− 0·7	− 0·2	...	...	...	...	...
+ 19·0	− 142·7	− 26·3	...	− 37·5	+ 55·1	− 312·7	+ 26·7	...	− 98·6
+1884·3	+2831·8	+2521·9	+3374·0	+2653·0	−1079·5	−1759·5	−2121·2	− 439·1	−1349·8
LEGUMINOSÆ (continued).									
+ 1·9	+ 22·7	+ 8·1	− 2·5	+ 7·6	+ 17·5	+ 29·0	+ 14·3	+ 0·6	+ 15·4
− 40·9	− 28·7	− 8·3	− 42·7	− 30·1	+ 95·2	+ 40·9	+ 18·7	+ 6·7	+ 40·4
− 55·9	− 77·1	− 96·8	− 89·7	− 79·9	...	+ 1·2	+ 0·9	+ 3·7	+ 1·4
− 36·3	+ 11·1	+ 282·8	+ 506·0	+ 190·9	− 4·1	+ 7·2	+ 215·0	+ 514·4	+ 183·1
− 131·2	− 72·0	+ 185·8	+ 371·1	+ 88·5	+ 108·6	+ 78·3	+ 248·9	+ 525·4	+ 240·3
OTHER ORDERS (continued).									
+ 48·9	+ 2·7	+ 34·2	+ 75·6	+ 40·3	+ 49·2	− 18·6	+ 23·0	+ 31·2	+ 21·2
+ 92·7	− 47·4	− 33·5	− 65·8	− 13·5	+ 171·3	+ 12·4	+ 12·9	+ 15·8	+ 5 ·0
+ 203·4	+ 186·8	+ 127·7	− 13·8	+ 126·0	+ 144·4	+ 172·2	+ 136·6	+ 19·8	+ 118·2
− 9·5	− 14·9	+ 16·4	− 6·2	− 3·5	...	+ 4·1	+ 51·0	+ 18·9	+ 18·5
+ 81·2	+ 65·8	+ 117·9	+ 47·5	+ 78·1	+ 114·2	+ 70·7	+ 134·1	+ 53·8	+ 93·2
− 159·3	− 307·6	− 40·6	− 61·3	− 142·3	+ 54·4	+ 49·2	+ 2·5	+ 13·4	+ 29·8
+ 240·0	+ 275·1	+ 24·2	+ 89·3	+ 157·1	− 110·7	+ 254·0	+ 15·4	− 141·8	+ 4·2
− 117·2	− 268·7	− 99·8	− 37·1	− 130·7	+ 8·1	− 110·8	− 242·6	− 250·6	− 148·8
− 7·3	− 43·6	− 23·2	− 41·3	− 28·9	...	...	− 0·6	− 8·8	− 2·3
+ 372·9	− 151·8	+ 123·3	− 13·1	+ 82·8	+ 430·9	+ 433·2	+ 132·3	− 248·3	+ 187·0
SUMMARY (continued).									
+1884·3	+2831·8	+2521·9	+3374·0	+2653·0	−1079·5	−1759·5	−2121·2	− 439·1	−1349·8
− 131·2	− 72·0	+ 185·8	+ 371·1	+ 88·5	+ 108·6	+ 78·3	+ 248·9	+ 525·4	+ 240·3
+ 372·9	− 151·8	+ 123·3	− 13·1	+ 82·8	+ 430·9	+ 433·2	+ 132·3	− 248·3	+ 187·0
+2126	+2608	+2831	+3732	+2824	− 540	−1248	−1740	− 162	− 922

odoratum and *Avena pubescens*. *Avena flavescens*, on the other hand, gave the highest average increase, and considerably higher than any other species on plot 16. Those next in order, and yielding considerably increased amounts, are *Alopecurus pratensis*, *Agrostis vulgaris*, *Holcus lanatus*, and *Poa trivialis;* whilst the meagrely-growing and poor grass *Festuca ovina*, the luxuriant *Dactylis glomerata*, as well as *Lolium perenne*, and *Bromus mollis*, yield increase in a much less degree.

Among the Leguminosæ, *Lathyrus* alone gives considerably, but *Trifolium repens* slightly, more on plot 16 than without manure; but *Trifolium pratense*, and *Lotus corniculatus*, give notably less with the manure than without any.

The increase of miscellaneous produce over the yield without manure is chiefly in *Rumex Acetosa* and *Conopodium denudatum*, in a less degree in *Achillea Millefolium* and *Ranunculus acris;* whilst there is, with the manure, considerably less of *Plantago lanceolata*, and somewhat less of *Ranunculus repens*, *R. bulbosus*, and *Centaurea nigra*. There is also, under the influence of the manure, an average deficiency of other miscellaneous species taken collectively.

The comparison which presents the greatest interest is, however, that between the actual yield of the different species and groups on plot 16 with the smaller, and on plot 14 with the larger amount of nitrate. With the smaller amount, there is a very considerable deficiency in the actual yield per acre of the grasses collectively. The greatest deficiency, and it is here very large, is in *Poa trivialis* and *Bromus mollis*. There is also a considerable deficiency of *Dactylis glomerata*, and of *Lolium perenne*. There is, on the other hand, with the smaller quantity of nitrate, a marked excess in the actual yield of *Agrostis vulgaris*, *Festuca ovina*, and *Avena flavescens*, and a less average excess of *Anthoxanthum odoratum*, *Avena pubescens*, *Alopecurus pratensis*, and *Holcus lanatus*, the last two giving a relative deficiency in the last separation-year.

Of Leguminous species there is, in proportion to the small total amount, a much greater yield with the smaller amount of nitrate. This increase is contributed by each of the four species enumerated, but in by far the greatest actual quantity by *Lathyrus pratensis;* whilst practically the whole of the much less actual yield of *Trifolium pratense*, *Trifolium repens*, and *Lotus corniculatus*, is in excess of the amount of them on plot 14.

Among the Miscellaneæ, again, the average actual increase over the quantity on plot 14 is contributed in a greater or less degree by each of the seven specially enumerated species; *Conopodium denudatum* yielding the largest amount of it, *Achillea Millefolium* and the various species of *Ranunculus* less, and *Plantago lanceolata*, *Centaurea nigra*, and *Rumex Acetosa*, less still. Of the large number collectively which do not come into the special list, the aggregate quantity yielded is, on the other hand, smaller on plot 16 than on plot 14.

So much for the detailed results obtained in the four years of complete separation. It should, however, be observed that the partial separations of subsequent years show a tendency to an increase in the proportion of total Gramineæ, to a decline in that of

Leguminosæ, and to an increase in that of the Miscellaneæ, on plot 16. How far this is merely a matter of season, or has to be taken as indicating permanent change, remains to be proved as time goes on.

The general result brought out by the botanical details is that, with the mineral manure and the smaller amount of nitrate of soda, that is with the less forcing conditions, no one species of grass is in any such prominence as are some on plot 14; a considerably greater number contributing somewhat largely, and a greater number fairly, to the produce. Moreover, those which were the most, and very characteristically, prominent on plot 14—*Poa trivialis* and *Bromus mollis*—are in insignificant amount on plot 16. Again, the two prevailing grasses of the locality, under usual conditions, *Agrostis vulgaris* and *Festuca ovina*, were all but banished with the larger, but were among the most prominent with the smaller, amount of nitrate; and a number of others which were in comparatively insignificant quantity on plot 14 characterised the herbage of plot 16. Thus, those which were the more prominent on plot 14 were of more rapid and freer growth than those which were so on plot 16; whilst, on the latter, a number of species of relatively meagre luxuriance maintained a fair position, yielding, upon the whole, with the less active struggle, a much more mixed gramineous herbage. Similar remarks, *mutatis mutandis*, apply also in some degree to the Leguminosæ, but to the Miscellaneæ in a greater degree than to them.

We have throughout spoken of the manurial conditions provided on plot 14 as being, compared with those on plot 16, such as to greatly increase luxuriance relatively to maturing tendency of growth. Yet, as a matter of fact, the herbage on plot 14 was always much more forward and riper than that on 16. This result, apparently so anomalous, is in reality by no means so. The excessive amount of nitrate forced into great prominence a few very free-growing grasses, and these were characterised by early maturity; whilst the smaller quantity of nitrate favoured a number of grasses of much more sluggish development, and later growth. It thus necessarily happened that, with the conditions of greater luxuriance, we had at the same time, owing to the very characteristically different flora encouraged, greater maturation also than with the conditions of less luxuriant growth, and, relatively to these, a greater supply of the constituents known to promote maturation.

As referred to in Part I., and as will be fully illustrated in Part III., the difference in the chemical composition of the produce of the two plots was quite consistent with the difference in the relative predominance of families and species, and with the relative condition of maturity described. Thus, *cæteris paribus*, the more mature a plant, the less will be the percentage of both mineral constituents and nitrogen in its dry substance. Again, speaking generally, the percentage of the mineral constituents is higher in the Leguminosæ than in the Gramineæ, and higher still in the Miscellaneæ, of the mixed herbage; and the percentage of nitrogen is somewhat higher in the Miscellaneæ, and considerably higher in the Leguminosæ, than in the Gramineæ. Quite consistently with these facts, the percentage, both of mineral constituents and of nitrogen, was lower in the dry substance of the more gramineous and the riper produce of plot 14, than in the more mixed and less matured herbage of plot 16.

Another point of interest brought out by a consideration of the mutual relations of the botanical and the chemical results is, that whilst with the heavier, the more simple, and the more gramineous herbage of plot 14, there was a greater actual quantity of both nitrogen and of mineral constituents taken up over a given area, the excess is comparatively small. In fact, with about 41 lbs. more nitrogen applied per acre per annum to plot 14 than to plot 16, the latter took up within about 7 lbs. per acre per annum as much as the former, and very nearly as much of several of the more important mineral constituents. The probable explanation of the obviously more complete utilization, either of the nitrogen supplied, or of the stores of it within the soil itself, is that, with the much greater variety of herbage, there was, at the same time, a more varied range of root-distribution, and a more varied food-collecting capacity. And, with these, there was, with actually less of the mineral constituents taken up with equal supplies provided, still very much more in proportion to the nitrogen supplied, and to the increased luxuriance induced.

12. *Ammonia-salts* (400 *lbs. per acre*), *and superphosphate of lime*; *Plot* 4–2.

In the experiments hitherto considered in which nitrogenous and mineral manures were used together, the mineral manure has been very complex, supplying more of all the mineral constituents, excepting silica, than were taken up. The effects of a given amount of nitrogenous manure with more or less partial mineral manures will now be described; and from the results a judgment can be formed as to which of the mineral constituents the characteristic effects, botanical or otherwise, are to be attributed. Among the series the experiment first to be noticed is that in which the mineral manure consisted of superphosphate of lime alone; and the results obtained with this in conjunction with ammonia-salts (plot 4–2) will be compared, not only with those without manure, but with those with the same amount of ammonia-salts used alone, that is to say, without any mineral manure whatever (plot 5). The experiment with the superphosphate of lime and ammonia-salts (plot 4–2), like that with the superphosphate alone (plot 4–1), did not commence until the fourth year (1859).

The addition of the superphosphate to the ammonia-salts increased the average amount of produce by more than one-third, the annual yield of nitrogen by about one-fifth, and that of the mineral matter taken up about in the proportion of from two to three. Not only was there very considerable increase in the amount taken up of those constituents which were supplied by the "superphosphate" (lime, some magnesia, phosphoric acid, and sulphuric acid), but there was also a considerably larger amount of both potass and silica, to say nothing of soda and chlorine, taken up, though neither of these was supplied. There is in these facts clear evidence of the defective supply of minerals on plot 5 compared with plot 4–2; but, inasmuch as the amounts of several of the mineral constituents taken up declined in the later years in about the same proportion on plot 4–2 as on plot 5, and the quantity of potass (which was not supplied) did so even in a much greater degree, it is obvious that there was still very considerable deficiency of some of the mineral constituents required.

The following Table (LXXVII., pp. 1376–7) gives the botanical results in the same form as usual; and it further shows the amount of yield of each species on plot 4–2 compared with that on plot 5 without the "superphosphate."

There is an average of 19 fewer total species on 4–2 with the ammonia-salts and superphosphate of lime, than without manure. The average number of grasses was reduced by three, that of Leguminosæ by two, and that of Miscellaneæ by 14. There were even, on the average, one grass, one Leguminosæ, and one Miscellaneous species, less than on plot 5 with the ammonia-salts without superphosphate. The average number of species found on the plot was 30, reducing from one separation-year to another, as follows: 35, 30, 28, 26.

The produce showed a higher proportion of gramineous herbage from one separation-year to another, and higher than with the ammonia-salts alone.

Leguminosæ were scarcely represented; *Lathyrus pratensis* being the only leguminous plant which has come into the samples in each separation-year, and then in only insignificant proportion.

Of Miscellaneous species only five came into the list of those yielding more than 1 per cent. in any one year, whilst there were seven such with the ammonia-salts alone.

The results relating to the Gramineæ are very striking. As with the ammonia-salts alone, so now with ammonia-salts and superphosphate of lime, *Festuca ovina* is not only, on the average, the most prominent grass, but it has enormously increased in predominance, yielding more than 50 per cent. to the produce on both plots in the last separation-year. On both plots, again, *Agrostis vulgaris* is the second in predominance; and it also increased, but in a much less degree, than *Festuca ovina*. It increased more in percentage, but not in actual weight, on plot 5 with the ammonia-salts alone, than where they were used in conjunction with superphosphate of lime; whilst the *Festuca ovina* increased the most under the latter condition. In the later, wetter seasons, the *Agrostis* has apparently gained in relative predominance. The only other grasses which yielded even moderate average percentages to the herbage on plot 4–2 were *Holcus lanatus*, *Alopecurus pratensis*, and *Poa trivialis*, each of which however, declined very considerably in the later years; whilst *Avena pubescens*, *Lolium perenne*, and *Avena flavescens*, which were fairly represented in the first separation-year, contributed scarcely anything in the last. Besides these, *Anthoxanthum odoratum*, *Avena elatior*, *Poa pratensis*, and *Dactylis glomerata* yielded, upon the whole, small but fluctuating percentages.

In the last separation-year more than 82 per cent. of the produce on plot 5, with the ammonia-salts alone, consisted of *Festuca ovina* and *Agrostis vulgaris* together, and nearly 80 per cent. did so on plot 4–2; but, as already said, the produce on the latter

TABLE LXXVII.—Number of Species, Percentage, Quantity per Acre, &c., of

	Number of species, and proportion per cent.					Quantity per acre.				
	1862.	1867.	1872.	1877.	Mean.	1862.	1867.	1872.	1877.	Mean.
NUMBER OF SPECIES.										
Gramineæ	14	14	15	13	14	...	...	...	...	...
Leguminosæ	3	3	2	2	2	...	...	...	...	...
Other Orders	18	13	11	11	14	...	...	...	...	...
Total	35	30	28	26	30	...	...	...	...	...
GRAMINEÆ.										
	p. c.	p. c.	p. c.	p. c.	p. c.	lbs.	lbs.	lbs.	lbs.	lbs.
Anthoxanthum odoratum	2·24	5·52	1·47	2·36	2·90	106·5	240·2	47·0	110·8	126·1
Alopecurus pratensis	0·66	14·75	3·94	1·58	5·23	31·4	641·9	125·9	74·2	218·4
Agrostis vulgaris	19·38	14·00	20·59	24·39	19·59	921·7	609·3	657·9	1145·3	833·5
Holcus lanatus	16·21	10·53	2·03	6·03	8·70	771·0	458·3	64·9	283·1	394·3
Avena elatior	2·46	0·41	2·48	1·02	1·59	117·0	17·8	79·2	47·9	65·5
Avena pubescens	7·38	3·94	0·28	0·03	2·90	351·0	171·5	9·0	1·4	133·2
Avena flavescens	2·09	0·41	0·09	0·03	0·65	99·4	17·8	2·9	1·4	30·4
Poa pratensis	0·67	3·87	5·11	1·56	2·80	31·9	168·4	163·3	73·3	109·2
Poa trivialis	8·14	2·15	2·10	0·31	3·17	387·1	93·6	67·1	14·6	140·6
Dactylis glomerata	2·28	0·38	0·16	1·83	1·16	108·4	16·6	5·1	85·9	54·0
Festuca ovina	6·80	26·09	49·29	55·20	34·35	323·4	1135·4	1574·6	2592·2	1406·5
Lolium perenne	6·47	1·36	0·70	0·21	2·19	307·7	59·2	22·4	9·9	99·8
None yielding 1 per cent. { Aira cæspitosa, Briza media, Festuca pratensis, F. elatior, Bromus mollis }	0·79	0·21	0·07	0·08	0·30	37·6	9·1	2·2	3·8	13·1
Absent { Phleum pratense, Cynosurus cristatus, Festuca loliacea }	...	...	...	...	...	...	...	...	...	...
Undetermined (chiefly Gramineæ)	4·74	2·51	0·34	...	1·90	225·4	109·3	10·8	...	86·4
Total	80·31	86·13	88·65	94·63	87·43	3819·5	3748·4	2832·3	4443·8	3711·0
LEGUMINOSÆ.										
Trifolium repens	0·01	0·01	...	0·01	0·01	0·5	0·4	...	0·5	0·4
Trifolium pratense	0·01	...	0·01	...	0·01	0·5	...	0·3	...	0·2
Lathyrus pratensis	0·07	0·03	0·02	0·03	0·03	3·3	1·3	0·7	1·4	1·6
Lotus corniculatus	...	...	...	...	...	...	...	...	...	...
Total	0·09	0·04	0·03	0·04	0·05	4·3	1·7	1·0	1·9	2·2
OTHER ORDERS.										
Ranunculus repens et bulbosus	2·14	0·04	0·01	...	0·54	101·8	1·8	0·3	...	25·9
Conopodium denudatum	1·29	2·65	0·39	0·11	1·11	61·3	115·3	12·5	5·2	48·6
Centaurea nigra	0·01	0·21	1·25	0·85	0·58	0·5	9·1	39·9	39·9	22·4
Achillea Millefolium	1·77	1·49	1·75	0·27	1·32	84·1	64·9	55·9	12·7	54·4
Rumex Acetosa	13·39	8·42	6·85	3·09	7·93	636·8	366·4	218·9	145·0	341·7
None yielding 1 per cent. { Ranunculus acris, Stellaria graminea, Cerastium triviale, Pimpinella Saxifraga, Heracleum Sphondylium, Galium verum, G. Aparine, Scabiosa arvensis, Taraxacum officinale, Plantago lanceolata, Veronica Chamædrys, Luzula campestris, Carex præcox, Hypnum squarrosum, H. rutabulum, H. hians }	1·00	1·02	1·07	1·01	1·04	47·7	44·4	34·2	47·5	43·5
Absent { Potentilla reptans, Agrimonia Eupatoria, Poterium Sanguisorba, Bellis perennis, Chrysanthemum Leucanthemum, Spiræa Ulmaria, Tragopogon pratensis, Leontodon hispidus, Hieracium Pilosella, Prunella vulgaris, Thymus Serpyllum, Ajuga reptans, Primula veris, Scilla nutans, Ophioglossum vulgatum }	...	...	...	...	...	...	...	...	...	...
Total	19·60	13·83	11·32	5·33	12·52	932·2	601·9	361·7	250·3	536·5
SUMMARY.										
Gramineæ	80·31	86·13	88·65	94·63	87·43	3819·5	3748·4	2832·3	4443·8	3711·0
Leguminosæ	0·09	0·04	0·03	0·04	0·05	4·3	1·7	1·0	1·9	2·2
Other Orders	19·60	13·83	11·32	5·33	12·52	932·2	601·9	361·7	250·3	536·5
Total	100·00	100·00	100·00	100·00	100·00	4756	4352	3195	4696	4250

each Species, by 400 lbs. Ammonia-salts, and Superphosphate of Lime; Plot 4–2.

Increase + or decrease − compared with :—									
Plot 3, without manure.					Plot 5, ammonia-salts alone.				
1862.	1867.	1872.	1877.	Mean.	1862.	1867.	1872.	1877.	Mean.
NUMBER OF SPECIES (continued).									
− 4	− 1	− 2	− 4	− 3	−3	−1	0	0	−1
− 1	− 1	− 2	− 2	− 2	−1	−1	−1	0	−1
−10	−11	−17	−20	−14	+1	−4	−2	−3	−1
−15	−13	−21	−26	−19	−3	−6	−3	−3	−3
GRAMINEÆ (continued).									
lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
− 24·1	− 48·3	− 38·5	− 10·2	− 30·3	−115·9	+ 58·0	− 29·4	− 9·9	− 24·3
− 105·6	+ 448·0	+ 117·4	+ 67·1	+ 131·8	+ 6·4	+ 626·4	+105·1	+ 67·4	+ 201·4
+ 575·0	+ 321·8	+ 392·6	+ 831·4	+ 530·1	− 14·8	− 84·0	− 11·3	+ 275·6	+ 41·3
+ 617·2	+ 192·8	+ 5·7	− 13·6	+ 200·5	+382·5	+ 288·0	+ 17·2	+ 194·2	+ 220·5
+ 114·9	+ 10·8	+ 77·1	+ 46·7	+ 62·4	− 34·4	− 74·1	+ 41·8	+ 41·1	− 6·4
+ 56·5	+ 69·2	− 49·4	− 62·2	+ 3·5	+ 69·3	+ 150·7	+ 3·0	− 2·1	+ 55·2
+ 27·0	− 44·2	− 54·5	− 24·1	− 23·9	+ 74·4	+ 2·6	− 1·7	+ 1·1	+ 19·1
+ 23·0	+ 162·7	+ 161·8	+ 71·6	+ 104·8	− 9·3	+ 146·9	+148·0	+ 66·5	+ 88·0
+ 340·1	+ 54·6	+ 58·9	+ 1·4	+ 113·7	+352·6	+ 83·7	+ 52·3	+ 14·3	+ 125·7
+ 54·7	− 41·4	− 9·7	+ 69·3	+ 18·2	+ 16·3	− 29·4	− 12·5	− 10·0	− 8·9
− 82·5	+ 628·9	+1218·4	+2074·7	+ 960·0	−524·1	+ 124·8	+404·1	+1018·5	+ 256·0
+ 113·3	− 75·1	− 16·6	− 97·7	− 19·0	+179·4	+ 19·2	− 1·9	+ 7·2	+ 51·0
− 25·6	− 15·3	− 103·6	− 168·7	− 78·4	+ 18·3	+ 4·5	+ 1·5	+ 3·2	+ 6·7
− 5·5	− 4·3	− 18·2	− 23·9	− 13·0	− 0·4	− 0·3	− 0·2	...	− 0·2
− 13·9	− 95·2	− 37·9	...	− 36·7	+ 92·4	+ 56·0	− 13·0	...	+ 33·9
+1664·5	+1565·0	+1703·5	+2761·8	+1923·7	+492·7	+1373·0	+703·0	+1667·1	+1058·9
LEGUMINOSÆ (continued).									
− 15·7	− 6·6	− 6·2	− 2·6	− 7·7	+ 0·1	+ 0·1	− 0·3	+ 0·5	+ 0·1
− 136·2	− 70·3	− 27·3	− 49·4	− 70·8	− 1·0	− 0·3	+ 0·3	...	− 0·2
− 35·1	− 21·4	− 15·4	− 54·6	− 31·7	+ 2·5	+ 1·0	− 0·3	− 0·1	+ 0·7
− 55·9	− 78·3	− 97·7	− 93·4	− 81·3	− 1·9	− 10·3	− 10·3	− 4·1	− 6·7
− 242·9	− 176·6	− 146·6	− 200·0	− 191·5	− 0·3	− 9·5	− 10·6	− 3·7	− 6·1
OTHER ORDERS (continued).									
− 47·1	− 65·2	− 48·0	− 81·6	− 60·5	+ 89·5	− 0·8	− 1·7	− 2·6	+ 21·1
+ 31·7	+ 17·0	− 34·3	− 39·7	− 6·3	+ 17·0	− 74·4	− 13·1	− 14·0	− 21·1
− 9·0	− 10·6	+ 5·3	+ 14·8	+ 0·2	+ 0·1	− 71·2	− 14·9	+ 24·3	− 15·3
+ 37·4	+ 26·2	+ 26·6	− 34·3	+ 14·0	+ 32·9	+ 28·9	+ 29·5	+ 8·0	+ 24·9
+ 594·1	+ 307·7	+ 189·8	+ 100·8	+ 298·0	+284·2	− 160·6	+ 39·7	+ 82·1	+ 61·3
− 320·5	− 612·3	− 115·1	− 127·8	− 293·9	− 14·1	− 39·4	− 50·9	− 14·6	− 29·9
− 4·2	− 31·2	− 30·2	− 62·0	− 32·0	...	...	...	− 2·6	− 0·7
+ 282·4	− 368·4	− 5·9	− 229·8	− 80·5	+409·6	− 317·5	− 11·4	+ 80·6	+ 40·3
SUMMARY (continued).									
+1664·5	+1565·0	+1703·5	+2761·8	+1923·7	+492·7	+1373·0	+703·0	+1667·1	+1058·9
− 242·9	− 176·6	− 146·6	− 200·0	− 191·5	− 0·3	− 9·5	− 10·6	− 3·7	− 6·1
+ 282·4	− 368·4	− 5·9	− 229·8	− 80·5	+409·6	− 317·5	− 11·4	+ 80·6	+ 40·3
+1704	+1020	+1551	+2332	+1652	+902	+1046	+681	+1744	+1093

plot became more gramineous than that on the former, and it included a larger number of other grasses in moderate proportion than did the produce with the ammonia-salts alone. In fact, with the ammonia-salts alone, with a somewhat larger total number of gramineous species represented in the samples, only nine came into the list as yielding more than 1 per cent. in any one year, whilst on plot 4–2, with the superphosphate in addition, there were 12 that came into this category. The result was a somewhat more mixed herbage so far as the gramineous components were concerned, due, however, it is to be supposed, rather to more of maturing tendency, and therefore of consolidation, giving weight, than to increased luxuriance of the species which came the more into prominence.

Among the five Miscellaneæ coming into the foremost list, *Rumex Acetosa* was, as in other cases, by far the most prominent; next in order coming *Achillea Millefolium, Conopodium denudatum, Centaurea nigra,* and various species of *Ranunculus.* But of these, *Rumex* declined from more than 13 to only 3 per cent.; the species of *Ranunculus* almost disappeared; *Conopodium denudatum* and *Achillea Millefolium* also declined, although in the second cuttings the *Achillea* has, in the later years, shown even more prominence than the *Rumex,* owing probably to the fact that it is less injured by cutting, and more particularly to the circumstance that the *Rumex* is more advanced in development at the time of the first cutting, and is therefore more exhausted by the profusion of seed that is formed, and is less ready to produce shoots in the aftermath. The only miscellaneous species that showed any tendency to increase, though occurring in comparatively small actual quantity, was *Centaurea nigra.* Upon the whole, however, the percentage of the Miscellaneæ diminished very greatly in the later years, as that of the Gramineæ increased.

Compared with the produce without manure, the ammonia-salts and superphosphate of lime gave a very large, and greatly increasing, actual amount of *Festuca ovina* and *Agrostis vulgaris.* This mixture also gave more of each of the *Poas,* more also, but in a rapidly decreasing ratio, of *Alopecurus pratensis* and *Holcus lanatus,* and slightly more of *Avena elatior* and *Dactylis glomerata.* There was, however, an actual deficiency of *Anthoxanthum odoratum, Avena flavescens, Lolium perenne,* and of a number of others taken collectively.

Of leguminous herbage there was so small a quantity that, practically, the whole of that without manure was in excess of that with the ammonia-salts and superphosphate of lime.

Of miscellaneous herbage there was, compared with the produce without manure, a great average, though diminishing, excess of *Rumex Acetosa,* a much smaller and iminishing excess of *Achillea Millefolium,* but a considerable deficiency of the various species of *Ranunculus,* and of *Conopodium denudatum; Centaurea nigra* showed an excess, but only in the later years.

Compared with the actual amount of each species grown by the ammonia-salts alone, the addition of the superphosphate gives, among the grasses, a greatly increasing excess of *Festuca ovina*, and a diminishing one of *Holcus lanatus* and *Alopecurus pratensis*. In the last separation-year only was there a considerable excess of *Agrostis*. There was also, in smaller, but in fluctuating amount, an excess of *Poa pratensis*, *P. trivialis*, *Lolium perenne*, *Avena pubescens*, and *Avena flavescens*, the excess in each case more or less declining. There was, on the other hand, generally an actual deficiency of *Anthoxanthum odoratum* and *Dactylis glomerata*.

The Leguminosæ were even in somewhat less actual amount with the "superphosphate" in addition, than with the ammonia-salts alone.

Of miscellaneous herbage, the mixture yielded more than the ammonia-salts alone, especially of *Rumex Acetosa* and *Achillea Millefolium;* whilst there was, excepting in the first year, a reduction in the amount of the species of *Ranunculus*, and also of *Conopodium; Centaurea nigra* was sometimes in deficiency, sometimes in excess.

The main effect of the addition of superphosphate to the ammonia-salts was, then, notably to increase the total gramineous, and slightly that of the total miscellaneous herbage. The chief increase was contributed by the poorer grasses—*Festuca ovina*, *Holcus lanatus*, and *Agrostis vulgaris;* a few of rather better character developing somewhat better with the mixture than with the ammonia-salts alone.

The general aspect and character of the herbage were, however, much the same on the two plots. It consisted mainly of very dark green, fine-leaved, tufted, and stunted grasses; but these characters were less marked under the influence of the "superphosphate," which aided the formation of stem, and tended somewhat more to maturity of a few of the superficially rooting species. However, in dry seasons, the herbage has seemed rather to die at the bottom than to ripen.

As referred to elsewhere, the excessively dark green colour is indicative of a high percentage of nitrogen, an abundant formation of chlorophyll, but deficient carbon-assimilation, due to a deficiency of certain mineral constituents, relatively to the amount of nitrogen taken up. In fact, there is, so far as the nitrogen available, and the chlorophyll formed, are concerned, so to speak potential, but not actual growth.

It is obvious, therefore, that the superphosphate used did not suffice for the full efficacy of the nitrogen supplied.

13. *Ammonia-salts* (400 *lbs. per acre*), *and mixed mineral manure, with and without potass; Plots* 9 *and* 10.

The results of the experiment last considered have shown that the mere addition to ammonia-salts of superphosphate of lime (supplying phosphoric acid, sulphuric acid, lime, and some magnesia) did not suffice very materially to affect either the quantity of the produce or the character of the herbage. We have now to consider the effects of the further supply of salts of potass, soda, and magnesia, and of salts of soda and magnesia, without potass. The point will be illustrated by a comparison of the results obtained on plot 10 with those on plot 9, those on the latter having been already separately considered.

During the first six years of the experiments, plots 9 and 10 each received, annually, the same amount of ammonia-salts (400 lbs. per acre), the same amount of superphosphate of lime (as on 4–2), and the same amount of the sulphates of potass, soda, and magnesia. The only difference between the manuring of the two plots was that, during those first six years (and the seventh), plot 10 received in addition to the manures mentioned a quantity of sawdust, which was, however, without effect. After the six years, the two plots continued to receive the same manure with the one important exception that the potass-salt was now excluded from the mixture applied to plot 10, and the quantity of the soda-salt was increased.

Referring to Part I., p. 345, for a fuller account of the facts, it may be briefly stated that, during the first six years, plot 10 had received about 900 lbs. of potass per acre, which calculation showed was considerably in excess of that removed in the crops. There was, therefore (if there were no loss by drainage, and there would certainly be but little, if any), an annually accumulating residue of potass. At any rate the result was that, compared with plot 9, there was but little falling off in the amount of total produce grown during the first five or six years after the cessation of the application of potass; and practically the same amount of nitrogen was taken up as on plot 9 with the continued supply of potass. Yet the amount of potass taken up declined even in the first year of the cessation; though it continued much in excess of that taken up on plot 4–2, where none had been supplied. The evidence was that the residue of the potass previously applied was not without effect; but the effects both as to amount of produce, and the botanical and chemical characters of the herbage, diminished considerably in the later years. The following Table (LXXVIII., pp. 1382–3) gives the botanical details in the same form as usual, and, in addition, the amount of actual yield of each species on plot 10, where the application of potass was discontinued, compared with that on plot 9, where the potass was continuously applied.

There were, on the average of the four separation-years, two fewer grasses, two fewer Leguminosæ, and 18 fewer Miscellaneæ, or 22 fewer total species in the samples from plot 10 with the discontinued supply of potass, than in those from the unmanured plot 3. The average number of species was 27, with a tendency to reduction; the numbers in the four years being 31, 27, 23, 28. It may here be mentioned that the first year of separation (1862) was the first year of the exclusion of the potass from the manure. There was even, on the average, a tendency to fewer species than on plot 9.

The percentage of total Gramineæ in the mixed herbage was not materially different on the two plots; but it was, upon the whole, higher on plot 10 with the smaller potass supply. Leguminosæ were in quite insignificant proportion on both plots; but in less without, than with, the continued supply of potass. On both, with general increase in the percentage of Gramineæ, there was general decrease in that of the Miscellaneæ.

Referring now more to detail, it is first to be noticed that as many as 14 species of grasses yielded more than 1 per cent. to the produce in one or other separation-year, and this is the same number as came into the list on plot 9. The next point to observe is, that no one grass is in anything like the same degree of prominence as was the case on plot 5, with ammonia-salts alone, or on plot 4–2, with the ammonia-salts and superphosphate of lime. The grass which is the most prominent, and increasing, and more so on plot 10 without than on plot 9 with continued supply of potass, is, however, *Festuca ovina*, which was so excessively prominent both with the ammonia-salts alone and the ammonia-salts and superphosphate. The next plant in order, and increasing on plot 10, is *Agrostis vulgaris*, which, it will be remembered, was also the second on the plot with ammonia alone, and on that with ammonia and "superphosphate." Succeeding to these, the grass which has gained most in prominence, and which has done so very strikingly on plot 10, is *Alopecurus pratensis*, whilst it has been but very meagrely represented on plot 9. Of the three *Avenas*, *A. elatior* has increased very much, whilst *A. pubescens* and *A. flavescens*, which were in large amount at first, have almost disappeared; and the result was very similar on plot 9. Of the two *Poas*, *P. pratensis* has fluctuated very much; upon the whole it has increased, but it has in the main been less prominent than on plot 9, whilst *Poa trivialis* has become almost extinct on both plots. *Dactylis*, which increased on plot 9 with the continuous supply of potass, diminished much on plot 10. The other grasses which maintained any degree of prominence are *Holcus lanatus*, which, however, has diminished, and *Anthoxanthum odoratum*, which has considerably increased; whilst *Bromus mollis*, *Lolium perenne*, and *Festuca pratensis* (occurring in less actual amount), have markedly declined.

Thus, the herbage was very prominently gramineous; the most prominent grasses were the inferior species of the locality, *Festuca ovina* and *Agrostis vulgaris*, but a considerable number of others contributed a fair proportion. The results show, however, curiously varying aptitude to the conditions supplied, among the different species of the same genus. This is illustrated in the increase of one, and the great decline of two of the *Avenas*; in the increase of one and the great decline of the other species of *Poa*, and in the very great increase of the one and the decline of the other species of *Festuca*.

The behaviour of *Avena elatior* being so strikingly contrasted with that of *A. flavescens* and of *A. pubescens* may perhaps suggest the question whether it should really be included in this genus, or whether its separation as a distinct genus under the name *Arrhenatherum*, as still maintained by some botanists of authority, is not more consistent with what is here elicited as to the physiological endowments of the plant, as it certainly is with its structural peculiarities. It is true that the equally different behaviour of the two *Poas* cannot be explained on any similar supposition.

TABLE LXXVIII.—Number of Species, Percentage, Quantity per Acre, &c., of each Potass;

	Number of species and proportion per cent.					Quantity per acre.				
	1862.	1867.	1872.	1877.	Mean.	1862.	1867.	1872.	1877.	Mean.
NUMBER OF SPECIES.										
Gramineæ	16	15	15	15	15	...	...	...	...	...
Leguminosæ	2	1	2	2	2	...	...	...	...	...
Other Orders	13	11	6	11	10	...	...	...	...	...
Total	31	27	23	28	27	...	...	...	...	...
GRAMINEÆ.										
	p. c.	p. c.	p. c.	p. c.	p. c.	lbs.	lbs.	lbs.	lbs.	lbs.
Anthoxanthum odoratum	1·46	5·27	3·26	5·84	3·96	90·2	262·7	141·2	285·0	194·8
Alopecurus pratensis	2·05	2·99	10·35	16·51	7·97	126·7	149·0	447·9	805·7	382·3
Agrostis vulgaris	9·37	8·61	14·17	16·29	12·11	579·1	429·1	613·3	794·9	604·1
Holcus lanatus	9·50	8·24	4·37	4·74	6·71	587·1	410·7	189·2	231·3	354·6
Avena elatior	0·08	11·71	13·22	9·58	8·64	4·9	583·6	572·1	467·5	407·0
Avena pubescens	10·64	1·64	0·44	0·24	3·24	657·6	81·7	19·0	11·7	192·5
Avena flavescens	10·05	2·00	0·77	0·18	3·25	621·1	99·7	33·3	8·9	190·8
Poa pratensis	4·06	14·81	19·62	6·48	11·24	250·9	738·1	849·1	316·2	538·6
Poa trivialis	10·18	2·78	1·21	0·47	3·66	629·1	138·5	52·4	22·9	210·7
Dactylis glomerata	12·51	5·44	3·12	4·88	6·49	773·1	271·1	135·0	238·1	354·3
Festuca ovina	4·08	14·74	19·80	26·34	16·24	252·1	734·7	856·9	1285·4	782·3
Festuca pratense	1·49	0·25	0·09	0·14	0·49	92·1	12·5	3·9	6·9	28·8
Bromus mollis	2·53	0·69	1·74	1·55	1·63	156·4	34·4	75·3	75·6	85·4
Lolium perenne	3·02	1·84	0·57	0·17	1·40	186·6	91·7	24·7	8·3	77·8
None yielding 1 per cent. { Phleum pratense, Aira cæspitosa, Cynosurus cristatus }	0·07	0·01	0·01	0·01	0·04	4·3	0·5	0·5	0·5	1·5
Absent—Briza media, Festuca loliacea	...	...	...	...	...	...	...	...	...	...
Undetermined (chiefly Gramineæ)	4·36	1·61	1·94	...	1·98	269·5	80·2	84·1	...	108·5
Total	85·45	82·63	94·68	93·42	89·05	5280·8	4118·2	4097·9	4558·9	4514·0
LEGUMINOSÆ.										
Trifolium repens	0·01	...	0·01	...	0·01	0·6	...	0·4	...	0·3
Trifolium pratense	...	...	...	0·01	0·01	...	...	...	0·5	0·1
Lotus corniculatus	...	...	...	0·01	0·01	...	...	...	0·5	0·1
Lathyrus pratensis	0·11	0·08	0·01	...	0·03	6·8	4·0	0·4	...	2·8
Trifolium minus, Ononis arvensis	...	...	...	...	...	...	...	...	...	...
Total	0·12	0·08	0·02	0·02	0·06	7·4	4·0	0·8	1·0	3·3
OTHER ORDERS.										
Conopodium denudatum	1·77	2·49	0·06	0·06	1·09	109·4	124·1	2·6	2·9	59·7
Achillea Millefolium	0·87	1·91	0·80	0·03	0·90	53·8	95·2	34·6	1·4	46·2
Rumex Acetosa	10·38	12·61	4·22	5·99	8·30	641·5	628·5	182·6	292·3	436·2
None yielding 1 per cent. { Ranunculus acris, R. repens et bulbosus, Cerastium triviale, Potentilla reptans, Spiræa Ulmaria, Pimpinella Saxifraga, Scabiosa arvensis, Centaurea nigra, Taraxacum officinale, Plantago lanceolata, Luzula campestris, Carex præcox, Hypnum squarrosum, H. rutabulum, H. hians }	1·41	0·28	0·22	0·48	0·60	87·1	14·0	9·5	23·5	33·6
Absent. { Stellaria graminea, Agrimonia Eupatoria, Poterium Sanguisorba, Heracleum Sphondylium, Galium Aparine, Bellis perennis, Chrysanthemum Leucanthemum, Tragopogon pratensis, Leontodon hispidus, Hieracium Pilosella, Veronica Chamædrys, Prunella vulgaris, Thymus Serpyllum, Ajuga reptans, Primula veris, Scilla nutans, Ornithogalum umbellatum, Ophioglossum vulgatum }	...	...	...	...	...	...	...	...	...	...
Total	14·43	17·29	5·30	6·56	10·89	891·8	861·8	229·3	320·1	575·7
SUMMARY.										
Gramineæ	85·45	82·62	94·68	93·42	89·05	5280·8	4118·2	4097·9	4558·9	4514·0
Leguminosæ	0·12	0·08	0·02	0·02	0·06	7·4	4·0	0·8	1·0	3·3
Other Orders	14·43	17·29	5·30	6·56	10·89	891·8	861·8	229·3	320·1	575·7
Total	100·00	100·00	100·00	100·00	100·00	6180	4984	4328	4880	5093

Species, by 400 lbs. Ammonia-salts and Mixed Mineral Manure, with and without Plot 10.

Increase + or decrease − compared with:—									
Plot 3, without manure.					Plot 9; as plot 10, but with potass every year.				
1862.	1867.	1872.	1877.	Mean.	1862.	1867.	1872.	1877.	Mean.
NUMBER OF SPECIES (continued).									
− 2	0	− 2	− 2	− 2	+3	+1	−1	+2	+1
− 2	− 3	− 2	− 2	− 2	0	−1	+1	−2	0
−15	−13	−22	−20	−18	0	−2	−7	+1	−2
−19	−16	−26	−24	−22	+3	−2	−7	+1	−1
GRAMINEÆ (continued).									
lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
− 40·4	− 25·8	+ 55·7	+ 164·0	+ 38·4	+ 10·8	+ 69·5	+ 13·9	+ 107·0	+ 50·3
− 10·3	− 44·9	+ 439·4	+ 798·6	+ 295·7	+109·4	+145·2	+ 291·7	+ 747·5	+323·4
+ 232·4	+ 141·6	+ 348·0	+ 481·0	+ 300·7	−241·0	−293·7	− 261·4	+ 54·5	−185·4
+ 433·3	+ 145·2	+ 130·0	− 65·4	+ 160·8	−190·1	−118·9	− 241·3	− 396·5	−236·7
+ 2·8	+ 576·6	+ 570·0	+ 466·3	+ 403·9	+ 4·9	+449·0	− 72·9	− 333·5	+ 11·9
+ 363·1	− 20·6	− 39·4	− 51·9	+ 62·8	+ 3·3	+ 5·8	− 8·7	+ 7·5	+ 2·0
+ 548·7	+ 37·7	− 24·1	− 16·6	+ 136·5	+ 39·8	−103·7	− 266·5	− 31·6	− 90·4
+ 242·0	+ 732·4	+ 847·6	+ 314·5	+ 534·2	−432·8	+ 37·4	− 433·6	− 775·3	−401·1
+ 582·1	+ 99·5	+ 44·2	+ 9·7	+ 183·8	+ 70·8	+ 23·3	+ 16·2	+ 16·2	+ 31·6
+ 719·4	+ 213·1	+ 120·2	+ 221·5	+ 313·5	+415·9	+ 21·4	− 537·2	− 613·7	−178·5
− 153·8	+ 228·2	+ 500·7	+ 767·9	+ 335·8	− 81·4	−256·7	+ 365·8	− 34·4	− 1·7
+ 90·9	+ 12·5	+ 3·7	+ 6·7	+ 28·4	+ 1·2	+ 8·7	+ 2·2	+ 6·9	+ 4·7
+ 152·4	+ 32·7	+ 75·1	+ 75·6	+ 83·9	−129·1	+ 28·5	+ 69·6	+ 75·6	+ 11·1
− 7·8	− 42·6	− 14·3	− 99·3	− 41·0	− 82·3	+ 37·4	− 38·1	− 1·4	− 21·1
− 1·5	− 3·8	− 17·9	− 23·6	− 11·7	+ 4·3	+ 0·5	− 0·1	− 0·1	+ 1·3
− 57·7	− 22·7	− 105·2	− 172·1	− 89·4	...	...	− 0·6	...	− 0·1
+ 30·2	− 124·3	+ 35·4	...	− 14·6	+105·5	− 82·9	− 17·2	...	+ 1·4
+3125·8	+1934·8	+2969·1	+2876·9	+2726·7	−390·8	− 29·2	−1118·2	−1171·3	−677·3
LEGUMINOSÆ (continued).									
− 15·6	− 7·0	− 5·8	− 3·1	− 7·8	...	− 0·5	+ 0·4	...	...
− 136·7	− 70·3	− 27·6	− 48·9	− 70·9	...	...	...	− 0·1	...
− 55·9	− 78·3	− 97·7	− 92·9	− 81·2	...	...	...	+ 0·5	+ 0·1
− 31·6	− 18·7	− 15·7	− 56·0	− 30·5	− 0·9	− 4·1	− 0·7	− 3·6	− 2·4
...	...	...	...	...	...	...	...	− 20·6	− 5·1
− 239·8	− 174·3	− 146·8	− 200·9	− 190·4	− 0·9	− 4·6	− 0·3	− 23·8	− 7·4
OTHER ORDERS (continued).									
+ 79·8	+ 25·8	− 44·2	− 42·0	+ 4·8	− 77·5	−379·1	− 82·8	− 42·5	−145·5
+ 7·1	+ 56·5	+ 5·3	− 45·6	+ 5·8	− 71·0	− 14·1	− 50·2	− 1·0	− 34·1
+ 598·8	+ 569·8	+ 153·5	+ 248·1	+ 392·5	+295·8	+ 42·3	− 77·6	+ 74·3	+ 83·7
− 424·5	− 718·0	− 219·8	− 252·5	− 403·4	+ 69·8	− 11·2	+ 3·1	+ 3·0	+ 16·2
− 19·2	− 42·6	− 33·1	− 68·0	− 41·0	− 47·4	− 2·1	− 4·0	− 12·7	− 16·6
+ 242·0	− 108·5	− 138·3	− 160·0	− 41·3	+169·7	−364·2	− 211·5	+ 21·1	− 96·3
SUMMARY (continued).									
+3125·8	+1934·8	+2969·1	+2876·9	+2726·7	−390·8	− 29·2	−1118·2	−1171·3	−677·3
− 239·8	− 174·3	− 146·8	− 200·9	− 190·4	− 0·9	− 4·6	− 0·3	− 23·8	− 7·4
+ 242·0	− 108·5	− 138·3	− 160·0	− 41·3	+169·7	−364·2	− 211·5	+ 21·1	− 96·3
+3128	+1652	+2684	+2516	+2495	−222	−398	−1330	−1174	−781

Before leaving this subject it may be mentioned that, both with the ammonia-salts alone, and with the ammonia-salts and superphosphate, all three of the above-enumerated *Avenas* almost disappear. Under the same conditions, again, both the *Poas* are discouraged. It is only under the influence of the further supply of more or less potass, a condition greatly favouring stem-formation and maturation, that we get the one so-called *Avena* (*elatior*), and the one *Poa* (*pratensis*), brought into prominence; the former especially being characterised by its great tendency to form stem.

The occurrence of Leguminosæ in the samples from plot 10 is quite insignificant, even more so than on plot 9 with the more potass. In both cases, owing to the associated ammonia so forcing the Gramineæ, and it being itself not directly favourable to Leguminosæ, such herbage was practically excluded, notwithstanding the greater or less potass supply.

Whilst with the ammonia-salts alone there were seven, and with the ammonia-salts and superphosphate five, Miscellaneæ which contributed 1 per cent. to the produce in one or other of the separation-years, there were only three on each of the plots 9 and 10; the one with salts of potass, soda, and magnesia, in addition every year, and the other with soda and magnesia every year, and potass for only a few of the earlier years. The three species coming into the separate list are the same, and occur in the same order of prominence, on both plots, viz.: *Rumex Acetosa*, *Conopodium denudatum*, and *Achillea Millefolium*. The *Rumex* has, upon the whole, been the more prominent, whilst the *Conopodium* and the *Achillea* have been the less so, with the smaller supply of potass. Each of the three species has, however, declined on both plots.

Compared with the produce without manure, the table shows on plot 10 a greatly increased actual yield of gramineous herbage, and a proportionally great reduction of leguminous, and a reduced amount also of miscellaneous, herbage.

Of the grasses, there was on the average of the four separations, indeed, with few exceptions in each separation-year, an increased actual yield of every species specially enumerated, excepting *Lolium perenne*. The average increase was the greatest of *Poa pratensis*, *Avena elatior*, *Festuca ovina*, *Agrostis vulgaris*, *Alopecurus pratensis*, and *Dactylis glomerata*; and of these, those which are the most gaining in ascendancy are *Festuca ovina*, *Alopecurus pratensis*, *Agrostis vulgaris*, and *Avena elatior*; whilst *Poa pratensis* has done so in a less marked degree, and *Dactylis glomerata* has declined. Among the grasses yielding less increase over plot 3, *Holcus lanatus*, *Avena pubescens*, *A. flavescens*, *Poa trivialis*, *Bromus mollis*, and *Festuca pratensis*, have considerably declined in relative predominance; whilst, though the quantity of it is small, *Anthoxanthum odoratum* shows a gradually augmenting increase.

Practically, the whole of the leguminous produce without manure was in excess of that on plot 10.

Of the Miscellaneæ, the three species specially enumerated as the most prominent, each gives, on the average, more than without manure, especially *Rumex Acetosa*, which is very greatly in excess, as it was also on plot 9. But of the large number occurring in only small amount, the total yield was very insignificant, and very largely in defect of that without manure; thus showing, notwithstanding the excess of *Rumex Acetosa*, a diminution in the total yield of Miscellaneæ as compared with that without manure.

It is, however, the comparison of the yield on plot 10 with the partial, with that on plot 9 with the continuous, supply of potass, which illustrates the most clearly the effects of the special conditions provided; and it brings strikingly to view the effects of potass on the mixed herbage.

The right hand division of the table shows that, with the reduced supply of potass, there was a great and increasing deficiency in total gramineous herbage, and a slight deficiency of both leguminous and miscellaneous herbage. Turning to the individual species it is seen that there was considerable reduction in the yield of *Poa pratensis*, *Dactylis glomerata*, *Holcus lanatus*, and *Agrostis vulgaris*, all more or less free-growing plants, and mostly free stem-producers; and all, excepting the *Agrostis*, show upon the whole a considerably increasing deficiency from one separation-year to another. The only grass which has shown a considerable excess compared with plot 9, and the excess of which has greatly increased in the later years, is *Alopecurus pratensis*. *Anthoxanthum odoratum* has shown a small but gradually increasing excess, whilst the stemmy *Avena elatior* especially, but *A. flavescens* in some degree, and *Festuca ovina*, have shown a deficiency. A number of other grasses have, fluctuating with the seasons, sometimes given excess and sometimes deficiency.

Lastly, it is worthy of note that whilst *Avena elatior* and *Avena flavescens* each show a deficiency, *Avena pubescens* does not; and whilst *Poa pratensis* has become the most largely deficient of the grasses on plot 10, *Poa trivialis* has, on the contrary, each year given a slight excess.

On both plots the yield of Leguminosæ was so small as to render detailed comparison superfluous.

Of Miscellaneæ, *Rumex Acetosa* yielded, on the average, more on the plot with the smaller supply of potass and the larger supply of soda; whilst, in each separation-year, both *Conopodium denudatum* and *Achillea Millefolium* were in less amount with the smaller supply of potass.

Upon the whole, then, the exclusion of the potass from the manure of plot 10 has much reduced the total growth. The reduction is more or less in each group, but by far the greatest among the grasses; and among these the tendency to decline in yield is chiefly among some of the freer-growing, and more especially stem-producing species. The most marked exception to this is that, with the decline of almost all competing species, *Alopecurus pratensis* largely increased, both in actual and in relative yield.

Apart from the differences which are brought to light in the records of percentage and actual yield of the different species, there were even more striking distinctions observable in the colour of the herbage, and in the characters of development of the plants. These characters were found to be quite consistent with some main differences in the chemical composition of the produce.

Thus, on plot 10, where the supply of potass was discontinued, the tendency to form stem has gradually, and at length very greatly, diminished; the herbage has come to consist in a much larger proportion of leaf; it maintains a darker green colour, does not eventually ripen as on plot 9, and is altogether more patchy and uneven. The chemical examination of the produce shows that, with these characters, substantially the same amount of nitrogen continues to be taken up, on plot 10 with the discontinued supply of potass and the diminished and deficiently matured produce, as on plot 9. On the other hand, the amount of potass taken up has, in the later years, been only about one-third as much on plot 10 as on plot 9. We have here again, then, so far as the nitrogen taken up is concerned, and also the abundance of chlorophyll formed, as indicated by the colour of the herbage, in a sense, potential growth. But, with the deficient supply of potass, there is deficient stem-formation and immaturity; in fact, deficient produce; in other words, deficient carbon-assimilation. Nor did the increased supply of soda compensate for the deficiency of potass, so far as these characters of growth are concerned.

From the various facts adduced, it is evident that it was the want of a liberal available supply of potass that restricted the growth on plot 4–2, with the ammonia-salts and superphosphate of lime, and on plot 10 with the ammonia-salts, superphosphate of lime, and a liberal supply of soda and magnesia, but only a partial one of potass; indeed, the agricultural, the botanical, and so far the chemical results, concur in showing how essential is a liberal provision of potass for the full effect of the nitrogen which was at the same time supplied. The comparison of the results obtained on plots 7 and 8, the one with the mixed mineral manure each year, including potass, and the other with the potass discontinued after the first few years (as on plot 10), but in both these cases without ammonia or other artificial nitrogenous supply, showed that, under those conditions also, the potass was the most essential to the growth among the mineral constituents supplied. But, whilst when the potass was used in conjunction with ammonia, its effect was greatly to develop the Gramineæ, and practically to exclude the Leguminosæ, it was, when used without ammonia, very characteristically to increase the Leguminosæ, and very much less the Gramineæ than when the ammonia was also applied.

14. *Ammonia-salts alone* (400 *lbs. per acre*), 13 *years* (1856–1868); *succeeded by mixed mineral manure, including potass, each year since*, *Plot* 6.

From the commencement of the experiments, plots 5 and 6 each received the same amounts of ammonia-salts without any mineral manure, and the plots were, therefore, so far, duplicates. The only difference was, that during each of the first seven years plot 6 received sawdust in addition, which, however, had little or no effect.

On both plots, the produce much diminished, the herbage greatly deteriorated, and calculation showed that not so much nitrogen was annually taken up in the total produce as was supplied in the manure, and the increased amount of nitrogen yielded over that in the unmanured produce was not more than one-fourth as much as was supplied. It is obvious, therefore, that a large proportion of the supplied nitrogen had been without effect.

After 13 years, it was decided to stop the application of ammonia-salts on plot 6, and to apply the mixed mineral manure, including potass, instead. The application of ammonia-salts alone was, however, continued on plot 5. It seemed of interest to determine what would be the effects of the change—on the amount of produce, on the distribution of species, on the character of development of the herbage, and on its chemical composition; also to ascertain whether, or in what degree, the hitherto unrecovered supplied nitrogen would be reclaimed in subsequent years, under the influence of the mineral manures.

Let us see by reference to the table which follows (LXXIX., pp. 1388–9), what have been the effects of the change in the manuring on the botany of the plot. Besides the usual particulars, the produce on plot 6, with the ammonia-salts succeeded by mixed mineral manure, is compared with that on plot 5, with the exclusive and continuous supply of ammonia-salts, and also with that on plot 7, with the exclusive and continued supply of the mixed mineral manure.

Inasmuch as plot 6 received the same amount of ammonia-salts as plot 5 for 13 years, from 1856–1868 inclusive, the first two botanical separations (of 1862 and 1867) show the character of the herbage under the influence of that manure; and it is only the separations of 1872 and 1877 that show the character and the degree of the change in the herbage induced by the substitution of mineral manure for ammonia-salts.

Compared with the produce without manure, that on plot 6 showed an average of 13 fewer total species; but the deficiency was less in the years after than before the change, the numbers being in the four separation-years, 34, 32, 39 and 38. Compared with plot 5, with the continuous supply of ammonia-salts, plot 6 gave four fewer total species in the first two years; but in the third it gave eight more, and in the fourth nine more—that is under the influence of the mineral manure. Lastly, compared with plot 7, where the mineral manure was applied continuously from the commencement, there were fewer species on plot 6, but a much less deficiency in number after than before the change of manure.

According to the results of the four complete separations, the percentage of the total Gramineæ in the total produce does not seem to have varied much as a direct result of the manures, though it has done so considerably as an effect of season. The partial separations of intermediate and succeeding years do, nevertheless, show a tendency to increase in the proportion of the grasses.

Since the change, however, the Leguminosæ have increased in percentage in a striking degree, whilst the Miscellaneæ have diminished.

TABLE LXXIX.—Number of Species, Percentage, Quantity per Acre, &c., of Mineral Manure alone,

	Number of species, and proportion per cent.					Quantity per acre.				
	1862.	1867.	1872.	1877.	Mean.	1862.	1867.	1872.	1877.	Mean.
NUMBER OF SPECIES.										
Gramineæ	15	14	17	14	15	...	...	...	...	...
Leguminosæ	4	3	4	4	4	...	...	...	...	...
Other Orders	15	15	18	20	17	...	...	...	...	...
Total	34	32	39	38	36	...	...	...	...	...
GRAMINEÆ.										
	p. c.	p. c.	p. c.	p. c.	p. c.	lbs.	lbs.	lbs.	lbs.	lbs.
Anthoxanthum odoratum	3·92	4·31	6·22	4·89	4·83	174·5	137·6	175·5	206·5	173·5
Alopecurus pratensis	1·70	0·02	0·03	0·09	0·46	75·7	0·6	0·8	3·8	20·2
Agrostis vulgaris	21·43	14·41	23·37	8·58	16·95	954·1	460·0	659·5	362·2	608·9
Holcus lanatus	8·17	3·07	5·31	14·89	7·86	363·8	98·0	149·8	628·7	310·1
Avena elatior	3·44	6·50	3·60	2·77	4·08	153·1	207·5	101·6	116·9	144·8
Avena pubescens	14·54	0·90	1·83	1·67	4·73	647·3	28·7	51·6	70·5	199·5
Avena flavescens	1·18	0·24	1·49	0·48	0·85	52·5	7·6	42·0	20·3	30·6
Poa pratensis	2·28	1·65	2·42	1·73	2·02	101·5	52·7	68·3	73·0	73·9
Poa trivialis	1·53	0·40	0·98	0·63	0·88	68·1	12·8	27·7	26·6	33·8
Dactylis glomerata	2·05	1·71	1·28	4·09	2·28	91·3	54·6	36·1	172·7	88·7
Festuca ovina	13·33	25·93	31·15	38·02	27·10	593·4	827·7	879·0	1605·2	976·3
Lolium perenne	4·58	1·39	0·69	1·97	2·16	203·9	44·4	19·5	83·2	87·7
None yielding 1 per cent. { Aira cæspitosa, Briza media, Cynosurus cristatus, Festuca pratensis, Bromus mollis }	0·60	0·04	0·05	0·15	0·22	26·7	1·2	1·5	6·3	9·0
Absent—Phleum pratense, Festuca loliacea	...	...	...	...	...	...	...	...	...	...
Undetermined (chiefly Gramineæ)	1·77	2·09	0·81	...	1·17	78·8	66·7	22·9	...	42·1
Total	80·52	62·66	79·23	79·96	75·59	3584·7	2000·1	2235·8	3375·9	2799·1
LEGUMINOSÆ.										
Trifolium repens	0·01	0·01	0·01	0·01	0·01	0·5	0·3	0·3	0·4	0·4
Trifolium pratense	0·03	0·01	0·04	0·08	0·04	1·3	0·3	1·1	3·4	1·5
Lotus corniculatus	0·01	0·08	0·06	0·03	0·05	0·5	2·6	1·7	1·3	1·5
Lathyrus pratensis	0·23	...	1·47	6·56	2·06	10·2	...	41·5	277·0	82·2
Total	0·28	0·10	1·58	6·68	2·16	12·5	3·2	44·6	282·1	85·6
OTHER ORDERS.										
Conopodium denudatum	0·55	7·87	2·47	0·88	2·94	24·5	251·2	69·7	37·2	95·6
Centaurea nigra	...	1·40	1·41	0·44	0·81	...	44·7	39·8	18·6	25·8
Achillea Millefolium	3·34	1·08	4·09	1·72	2·56	148·7	34·5	115·4	72·6	92·8
Rumex Acetosa	12·11	24·27	7·51	7·66	12·88	539·1	774·7	211·9	323·4	462·3
Luzula campestris	0·97	0·72	1·81	1·54	1·26	43·2	23·0	51·0	65·0	45·5
None yielding 1 per cent. { Ranunculus acris, R. repens et bulbosus, Stellaria graminea, Cerastium triviale, Potentilla reptans, Pimpinella Saxifraga, Heracleum Sphondylium, Galium verum, G. Aparine, Scabiosa arvensis, Taraxacum officinale, Plantago lanceolata, Veronica Chamædrys, Carex præcox, Hypnum squarrosum, H. rutabulum, H. hians }	2·23	1·90	1·90	1·12	1·80	99·3	60·6	53·8	47·2	65·3
Absent { Agrimonia Eupatoria, Poterium Sanguisorba, Spiræa Ulmaria, Bellis perennis, Carduus arvensis, Chrysanthemum Leucanthemum, Tragopogon pratensis, Leontodon hispidus, Hieracium Pilosella, Prunella vulgaris, Thymus Serpyllum, Ajuga reptans, Primula veris, Scilla nutans, Ophioglossum vulgatum }	...	...	...	...	...	...	...	...	...	...
Total	19·20	37·24	19·19	13·36	22·25	854·8	1188·7	541·6	564·0	787·3
SUMMARY.										
Gramineæ	80·52	62·66	79·23	79·96	75·59	3584·7	2000·1	2235·8	3375·9	2799·1
Leguminosæ	0·28	0·10	1·58	6·68	2·16	12·5	3·2	44·6	282·1	85·6
Other Orders	19·20	37·24	19·19	13·36	22·25	854·8	1188·7	541·6	564·0	787·3
Total	100·00	100·00	100·00	100·00	100·00	4452	3192	2822	4222	3672

each Species, by Ammonia-salts alone, 13 years (1856–1868), succeeded by Mixed each year since; Plot 6.

Increase + or decrease − compared with:—														
Plot 3, without manure.					Plot 5, ammonia-salts alone, every year.					Plot 7, mixed mineral manure, every year.				
1862.	1867.	1872.	1877.	Mean.	1862.	1867.	1872.	1877.	Mean.	1862.	1867.	1872.	1877.	Mean.
NUMBER OF SPECIES (continued).														
− 3	− 1	0	− 3	− 2	−2	−1	+2	+1	0	− 3	− 2	0	−3	−2
0	− 1	0	0	0	0	−1	+1	+2	+1	0	− 1	0	0	0
−13	− 9	−10	−11	−11	−2	−2	+5	+6	+2	− 7	− 7	−2	−3	−5
−16	−11	−10	−14	−13	−4	−4	+8	+9	+3	−10	−10	−2	−6	−7
GRAMINEÆ (continued).														
lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
+ 43·9	−150·9	+ 90·0	+ 85·5	+ 17·1	− 47·9	− 44·6	+ 99·1	+ 85·8	+ 23·1	+ 39·1	− 37·6	+ 60·3	+ 44·3	+ 26·5
− 61·3	−193·3	− 7·7	− 3·3	− 66·4	+ 50·7	− 14·9	− 20·0	− 3·0	+ 3·2	+ 60·7	− 38·6	− 48·8	− 20·7	− 11·9
+ 607·4	+172·5	+ 394·2	+ 48·3	+ 305·5	+ 17·6	−233·3	− 9·7	− 507·5	−183·3	+ 638·2	+ 206·3	+ 163·1	−250·8	+189·1
+ 210·0	−167·5	+ 90·6	+ 332·0	+ 116·3	− 24·7	− 72·3	+102·1	+ 539·8	+136·3	+ 139·9	− 428·5	+ 16·0	− 42·5	− 78·7
+ 151·0	+200·5	+ 99·5	+ 115·7	+ 141·7	+ 1·7	+115·6	+ 64·2	+ 110·1	+ 72·9	+ 46·5	+ 204·8	+ 82·1	+ 51·1	+ 96·2
+ 352·8	− 73·6	− 6·8	+ 6·9	+ 69·8	+365·6	+ 7·9	+ 45·6	+ 67·0	+121·5	+ 36·3	− 145·2	− 48·4	− 44·2	− 50·4
− 19·9	− 54·4	− 15·4	− 5·2	− 23·7	+ 27·5	− 7·6	+ 37·4	+ 20·0	+ 19·3	− 125·3	− 208·2	− 115·6	−165·8	−153·7
+ 92·6	+ 47·0	+ 66·8	+ 71·3	+ 69·5	+ 60·3	+ 31·2	+ 53·0	+ 66·2	+ 52·7	+ 51·5	+ 5·9	− 27·9	− 16·3	+ 3·3
+ 21·1	− 26·2	+ 19·5	+ 13·4	+ 6·9	+ 33·6	+ 2·9	+ 12·9	+ 26·3	+ 18·9	− 100·5	− 182·5	− 69·7	− 81·0	−108·4
+ 37·6	− 3·4	+ 21·3	+ 156·1	+ 52·9	− 0·8	+ 8·6	+ 18·5	+ 76·8	+ 25·8	− 22·4	− 153·6	− 35·1	− 14·5	− 56·4
+ 187·5	+321·2	+ 522·8	+1087·7	+ 529·8	−254·1	−182·9	−291·5	+ 31·5	−174·2	− 14·0	+ 320·3	+ 249·5	+249·1	+201·2
+ 9·5	− 89·9	− 19·5	− 24·4	− 31·1	+ 75·6	+ 4·4	− 4·8	+ 80·5	+ 38·9	+ 65·9	− 62·6	− 5·5	− 70·8	− 18·3
− 41·7	− 27·5	− 122·5	− 189·4	− 95·2	+ 7·0	− 3·7	+ 0·6	+ 5·7	+ 2·4	− 41·8	− 83·3	− 13·3	− 55·3	− 48·4
− 0·3	...	...	− 0·7	− 0·3	...	...	...	...	...	− 0·5	...	...	...	− 0·1
− 160·5	−137·8	− 25·8	...	− 81·0	− 54·2	+ 13·4	− 0·9	...	− 10·4	− 49·1	− 40·3	− 38·9	...	− 32·0
+1429·7	−183·3	+1107·0	+1693·9	+1011·8	+257·9	−375·3	+106·5	+ 599·2	+147·0	+ 724·5	− 643·1	+ 167·8	−417·4	− 42·0
LEGUMINOSÆ (continued).														
− 15·7	− 6·7	− 5·9	− 2·7	− 7·7	+ 0·1	...	...	+ 0·4	+ 0·1	− 135·7	− 20·7	− 74·7	− 0·1	− 57·8
− 135·4	− 70·0	− 26·5	− 46·0	− 69·5	− 0·2	...	+ 1·1	+ 3·4	+ 1·1	− 301·3	− 211·6	− 46·8	− 75·7	−158·9
− 55·4	− 75·7	− 96·0	− 92·1	− 79·8	− 1·4	− 7·7	− 8·6	− 2·8	− 5·2	− 55·7	− 28·2	− 6·3	− 0·7	− 22·8
− 28·2	− 22·7	+ 25·4	+ 221·0	+ 48·9	+ 9·4	− 0·3	+ 40·5	+ 275·5	+ 81·3	− 587·5	− 302·0	−1512·3	−340·6	−685·5
− 234·7	−175·1	− 103·0	+ 80·2	− 108·1	+ 7·9	− 8·0	+ 33·0	+ 276·5	+ 77·3	−1080·2	− 562·5	−1640·1	−417·1	−925·0
OTHER ORDERS (continued).														
− 5·1	+152·9	+ 22·9	− 7·7	+ 40·7	− 19·8	+ 61·5	+ 44·1	+ 18·0	+ 25·9	− 65·3	− 159·8	+ 14·2	− 43·4	− 63·6
− 9·5	+ 25·0	+ 5·2	− 6·5	+ 3·6	− 0·4	− 35·6	− 15·0	+ 3·0	− 11·9	− 1·3	+ 9·5	+ 28·4	+ 13·5	+ 12·5
+ 102·0	− 4·2	+ 86·1	+ 25·6	+ 52·4	+ 97·5	− 1·5	+ 89·0	+ 67·9	+ 63·3	+ 73·8	− 103·7	− 106·1	+ 40·0	− 24·0
+ 496·4	+716·0	+ 182·8	+ 279·2	+ 418·6	+186·5	+247·7	+ 32·7	+ 260·5	+181·9	+ 446·2	+ 378·8	+ 162·8	− 16·8	+242·8
− 15·1	− 97·3	− 5·2	+ 23·2	− 23·6	+ 0·4	+ 2·5	+ 47·2	+ 63·2	+ 28·3	− 19·2	− 29·2	− 18·5	+ 39·5	− 6·9
− 359·5	−542·8	− 87·8	− 168·1	− 289·5	+ 68·0	− 5·3	− 29·5	− 15·7	+ 4·3	− 44·4	− 99·4	− 3·4	− 61·5	− 52·3
− 4·2	− 31·2	− 30·0	− 61·8	− 31·9	...	...	...	− 2·6	− 0·7	− 6·1	− 56·6	− 19·1	− 14·8	− 24·0
+ 205·0	+218·4	+ 174·0	+ 83·9	+ 170·3	+332·2	+269·3	+168·5	+ 394·3	+291·1	+ 383·7	− 60·4	+ 58·3	− 43·5	+ 84·5
SUMMARY (continued).														
+1429·7	−183·3	+1107·0	+1693·9	+1011·8	+257·9	−375·3	+106·5	+ 599·2	+147·0	+ 724·5	− 643·1	+ 167·8	−417·4	− 42·0
− 234·7	−175·1	− 103·0	+ 80·2	− 108·1	+ 7·9	− 8·0	+ 33·0	+ 276·5	+ 77·3	−1080·2	− 562·5	−1640·1	−417·1	−925·0
+ 205·0	+218·4	+ 174·0	+ 83·9	+ 170·3	+332·2	+269·3	+168·5	+ 394·3	+291·1	+ 383·7	− 60·4	+ 58·3	− 43·5	+ 84·5
+1400	−140	+1178	+1858	+1074	+598	−114	+308	+1270	+515	+ 28	−1266	−1414	−878	−882

It will be observed that, as was the case on plot 5, *Festuca ovina* remained by far the most prominent grass, and that its proportion increased in the last two separation-years, that is after the cessation of the application of the ammonia-salts, and the substitution of mineral manure; but the increase was not by any means so marked as it was on plot 5, with the continued supply of ammonia-salts. Again, as on plot 5, *Agrostis vulgaris* was very prominent; and, although it showed an increase in the third separation-year, it had decreased very much in the fourth, *Holcus lanatus* having gradually increased, and much exceeded it in that year; whilst on plot 5 *Holcus* declined very much in the later years. The only other grasses which seemed to become distinctly more prominent after the change were *Anthoxanthum odoratum* and *Dactylis glomerata;* whilst *Avena elatior* and *A. flavescens, Poa pratensis* and *P. trivialis,* and *Lolium perenne,* yielded fluctuating and small quantities; and *Avena pubescens,* which had yielded largely in the first separation-year, gave very little in the second, and but little more in the third and fourth. Thus, the most prominent grasses were of low agricultural repute, nor were they characteristically free-growers; they were, on the other hand, such as have generally gained prominence under conditions of defective luxuriance and maturation combined.

Among Leguminosæ, *Lathyrus pratensis* very soon showed considerable increase after the application of the mineral manure; but, neither in the complete separation-years, nor in the intermediate and subsequent partial separations, has it acquired anything like the same degree of prominence as on plot 7, with the continuous supply of the same mineral manure, including potass, from the commencement.

Among the Miscellaneæ there were five species which yielded more than 1 per cent. to the total produce in one or other of the separation-years; whilst there were seven in this category on both plots 5 and 7. Of these, as usual, *Rumex Acetosa* was by far the most prominent; but it contributed very much less in the later years, though still more than on plot 5. Next in order is *Achillea Millefolium,* which has increased since the change of manure; whilst *Conopodium denudatum* has diminished, but again not so much as on plot 5. *Luzula campestris* has, on the other hand, become more prominent. Lastly, *Galium verum,* and *Scabiosa arvensis,* which by their increase in the later years came into the list of prominence on plot 5, with the ammonia-salts alone continuously, did not do so on plot 6. Although on both plots the proportion of total Miscellaneæ has gone down considerably in the later years, it has done so in a less degree on plot 6 than on plot 5.

Comparing the actual yield per acre of the different groups and species with the produce without manure, the fluctuations were, as the result of the fluctuations in the seasons themselves, so great in the first two separation-years, that it is difficult to determine how much of the subsequent change is to be set down to change in the manure.

The best indications of the effects of the change will be found in the comparison of the yield on plot 6 with that on plot 5. Referring to the first two separation-years, when the two plots were practically manured alike, it is seen that in the first, the Gramineæ, the Leguminosæ, and the Miscellaneæ, were each in excess on plot 6; whilst in the second the Gramineæ and Leguminosæ were in somewhat greater deficiency, but the Miscellaneæ were again in excess. After the change of manure on plot 6, there was in the first subsequent separation-year a slight, and in the second a great, increase of gramineous herbage. There was also in the first a slight, and in the second a greater, increase in the leguminous herbage; and, again, of miscellaneous herbage, there was a less amount of increase in the first, and a greater in the second.

Looking more to detail, the results show that in the fourth separation-year *Holcus lanatus* had then increased in yield, compared with plot 5, about as much as all the other grasses put together; and those which also increased in a greater or less degree in both the third and fourth separation-years are *Anthoxanthum odoratum*, *Avena elatior*, *A. pubescens*, *A. flavescens*, *Poa pratensis*, *P. trivialis*, and *Dactylis glomerata*, whilst *Lolium perenne* and *Festuca ovina* only gained in the fourth separation-year. On the other hand, *Agrostis vulgaris* declined in the fourth separation-year nearly as much as *Holcus lanatus* increased.

Of the leguminous produce on plot 6, which was much more in the fourth than in the third separation-year, and which in both cases consisted almost exclusively of *Lathyrus pratensis*, nearly the whole of the amount was in excess of that on plot 5.

Of Miscellaneæ there was, in each separation-year, more on plot 6 than on plot 5, so that the excess after the change of manure cannot be attributed exclusively to that change. The greatest excess on plot 6 is of *Rumex Acetosa*, next of *Achillea Millefolium*, then of *Luzula campestris*, and then of *Conopodium denudatum*.

Thus, the effect of the change from ammonia-salts to a complete mineral manure was to increase the yield of by far the majority of the grasses, and of *Holcus lanatus* especially, and to diminish that of only very few, but of these, *Agrostis vulgaris* considerably. It also increased the yield of the Leguminosæ, and somewhat perhaps that of the Miscellaneæ.

We have next to compare the yield of each group and species on plot 6, with the mixed mineral manure succeeding ammonia-salts, with that on plot 7, with the same mixed mineral manure every year from the commencement. The results contrast strikingly with those last referred to. Whilst, compared with plot 5, plot 6 gave an excess of almost every grass after the change, compared with plot 7 (with the continuous supply of the mineral manure including potass), there was a deficiency of almost every grass. Thus, in the third separation-year, plot 6 showed a deficiency in seven, but collectively a greater excess in five; and in the fourth separation-year a deficiency in nine, and an excess in three, the collective deficiency being very much greater than the excess in the third separation-year. The only grass which yielded considerably more with the supply of the mineral manure for the shorter period was the poorer *Festuca ovina;* whilst *Agrostis vulgaris* gave considerably less, as also did

Avena flavescens, and, in a less degree, *Poa trivialis, Lolium perenne, Avena pubescens, Holcus lanatus, Alopecurus pratensis, Poa pratensis,* and *Dactylis glomerata.* Thus, the grasses which are in defect with the less-continued supply of mineral manure are, for the most part, either of better quality, or of freer growth, than those which are in excess.

Of leguminous produce as a whole, there was very much less, as well as a greater or less deficiency of each species represented, with the less-continued supply of mineral manure, but the deficiency was by far the greatest in the case of *Lathyrus pratensis.* The yield of Leguminosæ on the plot previously receiving ammonia-salts—under the influence of which they were almost excluded—has, indeed, been much less during the subsequent 12 years of the application of the mixed mineral manure including potass, than it was during the first 12 years of the application of the same manure to plot 7, in this case succeeding upon the unmanured condition of soil, and the coincident more complex condition of the mixed herbage. It might be supposed that the reserve of nitrogen to be rendered available under the influence of the mineral manure would be greater during the 12 years succeeding the application of an excess of ammonia-salts than during the first 12 years of the application of the mineral manure on plot 7. The great relative deficiency in yield of Leguminosæ under the influence of the same manure applied upon a residue of ammonia-salts would seem to be explicable, therefore, on the supposition that the Leguminosæ had been to a great extent really banished during the 13 years' application of the ammonia-salts; whilst at the commencement of the experiment on plot 7 they were generally distributed, but of restricted growth, as is common without manure, or when the grasses are not forced into prominence by nitrogenous manures. That the deficient yield of Leguminosæ on plot 6 compared with plot 7 is due to a want of plant, and not (as it cannot be) to a corresponding deficiency of food material, is further rendered probable by the fact that, although the actual amount in each complete or partial separation-year since the change of manure on plot 6 is much less there than on plot 7, yet the amounts fluctuate somewhat correspondingly on the two plots from year to year according to season.

So much for the distinctions between the botany of plot 5 with the continuous supply of ammonia-salts, of plot 6 with ammonia-salts succeeded by mineral manure, and of plot 7 with the continuous supply of the same mineral manure from the commencement. The figures in the table, and the foregoing comments, however, bring to view very inadequately the very great differences between the herbage on the respective plots.

In the first season of the change of manure the herbage on plot 6 acquired a totally different aspect. Instead of the dark, almost blue, green colour, and restricted growth and development, which, in common with plot 5, it had, and which the latter still maintained, the Gramineæ showed a much less patchy and more uniform, though still not luxuriant, growth, grasses of small habit, as has been shown, still remaining prevalent; the colour was of a paler and more lively hue; there was more tendency to form stem and seed, and there was fuller and more even bottom growth. Leguminosæ showed some signs of recovery and increasing tendency to ripen seed, as also did some of the Miscellaneæ.

From year to year these characters developed, the plot losing all semblance to plot 5 and gradually assimilating in character to plot 7; the chief difference between plots 6 and 7 being that the former did not show so conspicuous a development of Leguminosæ.

It will be necessary to make brief reference to the chemical history of the plots, and of their produce, in order to attain a clearer conception of the changes of result effected by the change of manuring. In the first place, in the first and second crops of the last 12 years, there has been about one and a-half time as much dry substance grown on plot 6, where mineral manure was substituted for ammonia-salts, as on plot 5, where the application of ammonia-salts was continued. Then again, it is found that as much nitrogen has been removed in the first crops of the 12 years since the change, from plot 6 to which no nitrogen has, during that period, been applied, as from plot 5, to which the application has been continued. But in five of the last six seasons second crops have also been removed from the land, and in these rather less nitrogen has been removed from plot 6 without, than from plot 5 with the continuous supply. The reason of this doubtless is that, whilst on plot 5 the first crops were leafy and unmatured, those of plot 6 were much more stemmy and ripe, and, therefore, the herbage was the more exhausted; to add to which disadvantage, there was also doubtless a less available store of nitrogen remaining. The result is, however, that there is only an average of between 3 and 4 lbs. less nitrogen yielded per acre per annum on plot 6, in the first and second crops taken together, than on plot 5. It is, however, remarkable, that even more nitrogen has been taken off in the produce of plot 7, to which none has been applied from the commencement (but only a mixed mineral manure including potass), than in that of either plot 5, to which more nitrogen was annually applied as ammonia-salts than was removed in the crops, or in that of plot 6, which had the same amount of ammonia-salts during the first 13 of the 25 years.

Further, the percentage of nitrogen in the dry substance of the produce is, since the change, very much lower in that of plot 6 than in that of plot 5. This indicates very much greater maturation, that is, very much more consolidation, or, in other words, under the influence of the mineral manure very much more carbon-assimilation in proportion to a given amount of nitrogen taken up. The percentage of nitrogen in the dry substance of the produce of plot 6 is indeed somewhat lower than in that of plot 7, owing to its less proportion of the more highly nitrogenous leguminous herbage.

In regard to the source of the large amount of nitrogen taken up on plot 7, where none has been applied, it was stated (Part I., p. 312) that, after the experiments had proceeded 20 years, the soil of plot 7 showed a considerably lower percentage of nitrogen than that either of the unmanured plot, or of the ammonia plot 5. That of plot 6 also showed a considerable reduction. The obvious inference is that, on plot 7 at any rate, nitrogen must, under the influence of the mineral manure, have been obtained from the previously accumulated stores within the soil. It would seem, too, more than probable that the source of the continued large yield of nitrogen on plot 6, after the discontinuance of the application of the ammonia-salts, is also to be attributed, mainly at any rate, to a supply rendered available from earlier accumulations, rather than to the unrecovered residue of the previously applied ammonia-salts.

The mineral composition of the herbage as remarkably illustrates the difference in the character of growth as do the facts in regard to the nitrogen. Leaving fuller details on both points for the third or chemical part of our report, it may be stated that, even in the first year after the change, very much more total mineral matter, and especially very much more potass, and phosphoric acid also, was taken up on plot 6 than on plot 5. Indeed, over the 12 years since the change of manuring, nearly twice as much total mineral matter has been taken up in the first and second crops of plot 6 as in those of plot 5; but even on plot 6 less has been taken up than on plot 7 during the same period. Analytical details which are at command relating to the produce of the first seven of the last 12 years, show that it is of potass and phosphoric acid especially, that there has been a greatly increased accumulation by the growing plants; and it is these which, other things being equal, favour the tendency to stem- and seed-formation.

Thus, evidence of very various kinds concurs in showing that the effect of the change of manuring was not very strikingly to affect the distribution of species on the plot, and especially not to bring into prominence plants of free and luxuriant habit; but it was, on the other hand, very greatly to alter the character of development of the plants of meagre habit which had already possession of the soil. This remark applies especially to the grasses and to the Miscellaneæ, the effect on the Leguminosæ being, in a marked degree, to increase their predominance as well as to favour their maturing tendency.

15. *Equal nitrogen and equal potass, in nitrate of soda and sulphate of potass, and in nitrate of potass; in each case with superphosphate of lime; Plots* 19 *and* 20.

Having regard to the very marked effects of nitrate of soda, and of salts of potass, whether used separately or in conjunction, on the botanical composition, the luxuriance, and the character of development, of the mixed herbage, it seemed desirable to determine the comparative effects of a given amount of nitrate of potass, and of a mixture of nitrate of soda and sulphate of potass, containing the same amount of nitrogen and the same amount of potass. Accordingly, in 1872, plots 19 and 20 were set apart for experimenting on this point. The area appropriated had from the commencement been enclosed with the other experimental ground, and so protected from grazing by animals. It had been entirely unmanured for a number of years, and every year one crop of hay had been removed from it. To plot 19, 275 lbs. of nitrate of soda (the same amount as applied to plot 16) was applied; and to plot 20,

TABLE LXXX.—Number of Species, Percentage, Quantity per Acre, &c., of each Species, by equal Nitrogen and equal Potass, in Nitrate of Soda and Sulphate of Potass (Plot 19), and in Nitrate of Potass (Plot 20), in each case with Superphosphate of Lime; Plots 19 and 20.

	Number of Species, and proportion per cent.									Quantity per Acre.								
	Plot 16.		Plot 19.		Plot 20.		Mean of 1872 and 1877.			Plot 16.		Plot 19.		Plot 20.		Mean of 1872 and 1877.		
	1872.	1877.	1872.	1877.	1872.	1877.	Plot 16.	Plot 19.	Plot 20.	1872.	1877.	1872.	1877.	1872.	1877.	Plot 16.	Plot 19.	Plot 20.
NUMBER OF SPECIES.																		
Gramineæ	17	15	16	16	15	16	16	16	16	...	...	...	...	...	...	...	...	...
Leguminosæ	4	4	4	5	5	4	4	5	4	...	...	...	...	...	...	...	...	...
Other Orders	15	22	20	18	22	18	19	19	20	...	...	...	...	...	...	...	...	..
Total	36	41	40	39	42	38	39	40	40	...	...	...	...	...	...	...	...	...
GRAMINEÆ.																		
	p. c.	p. c.	p. c.	p. c.	p. c.	p. c.	p. c.	p. c.	p. c.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
Anthoxanthum odoratum	1·40	2·22	6·39	2·47	7·17	3·51	1·81	4·43	5·34	62·7	135·3	286·8	116·8	309·2	180·6	99·0	201·8	244·9
Alopecurus pratensis	15·22	12·23	0·23	5·40	1·22	6·80	13·73	2·81	4·01	681·1	745·6	10·3	255·3	52·6	349·8	713·3	132·8	201·2
Agrostis vulgaris	12·40	14·58	25·56	10·83	31·35	10·36	13·49	18·19	20·85	554·9	888·8	1147·1	512·0	1351·8	532·9	721·8	829·5	942·3
Holcus lanatus	5·08	12·64	14·03	21·19	12·39	29·47	8·86	17·61	20·93	227·3	770·5	629·7	1001·9	534·3	1515·9	498·9	815·8	1025·1
Avena pubescens	1·54	3·34	1·50	3·00	1·02	2·40	2·44	2·25	1·71	68·9	203·6	67·3	141·8	44·0	123·4	136·3	104·5	83·7
Avena flavescens	18·80	6·67	3·29	4·37	3·07	5·30	12·73	3·83	4·18	841·3	406·6	147·7	206·6	132·4	272·6	624·0	177·1	202·5
Poa trivialis	6·53	4·82	0·75	4·73	0·39	3·06	5·67	2·74	1·72	292·2	293·8	33·7	223·6	16·8	157·4	293·0	128·6	87·1
Dactylis glomerata	3·75	4·63	0·67	2·43	0·30	1·15	4·19	1·55	0·72	167·8	282·2	30·1	114·9	13·0	59·2	225·0	72·5	36·1
Cynosurus cristatus	0·05	0·18	1·17	2·54	1·15	1·51	0·12	1·86	1·33	2·2	11·0	52·5	120·1	49·6	77·7	6·6	86·3	63·6
Festuca ovina	10·33	16·66	29·65	21·40	25·54	18·69	13·49	25·52	22·12	462·3	1015·6	1330·7	1011·8	1101·3	961·5	738·9	1171·2	1031·4
Lolium perenne	3·10	3·75	4·14	1·23	2·03	3·61	3·42	2·68	2·82	138·7	228·6	185·8	58·2	87·5	185·7	183·7	122·0	136·6
Bromus mollis	2·03	0·57	...	...	...	...	1·30	...	...	90·9	34·7	...	...	...	...	62·8	...	...
None yielding 1 per cent. { Poa pratensis, Briza media, Festuca pratensis, Festuca loliacea, Phleum pratense, Aira cæspitosa, Avena elatior }	0·85	0·65	1·24	1·36	0·75	0·25	0·76	1·31	0·51	38·0	39·7	55·5	64·3	32·2	12·8	38·9	60·1	22·6
Absent { Phleum pratense (plots 16 and 19), Festuca loliacea (plots 16 and 20), Bromus mollis (plots 19 and 20) }	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Undetermined (chiefly Gramineæ)	0·50	...	0·73	...	0·74	...	0·25	0·37	0·37	22·4	...	32·8	...	31·9	...	11·2	16·4	15·9
Total	81·58	82·94	89·35	80·95	87·12	86·11	82·26	85·15	86·61	3650·7	5056·0	4010·0	3827·3	3756·6	4429·5	4353·4	3918·6	4093·0
LEGUMINOSÆ.																		
Trifolium repens	0·32	0·01	0·19	0·84	0·21	0·64	0·16	0·52	0·42	14·3	0·6	8·5	39·7	9·1	32·9	7·5	24·1	21·0
Trifolium pratense	0·43	0·11	1·30	0·80	0·63	0·05	0·27	1·05	0·34	19·3	6·7	58·4	37·8	27·2	2·6	13·0	48·1	14·9
Trifolium procumbens	...	...	...	...	0·01	...	...	...	0·01	...	...	...	...	0·4	...	...	...	0·2
Lotus corniculatus	0·02	0·06	0·79	0·12	1·09	0·24	0·04	0·45	0·66	0·9	3·7	35·5	5·7	47·0	12·3	2·3	20·6	29·7
Lotus major	...	...	...	0·04	...	...	...	0·02	...	...	...	...	1·9	...	...	...	1·0	...
Lathyrus pratensis	6·68	9·22	0·23	6·92	0·10	2·73	7·95	3·57	1·42	298·9	562·0	10·3	327·2	4·3	140·5	430·4	168·7	72·4
Total	7·45	9·40	2·51	8·72	2·04	3·66	8·42	5·61	2·85	333·4	573·0	112·7	412·3	88·0	188·3	453·2	262·5	138·2
OTHER ORDERS.																		
Ranunculus acris	0·79	1·24	0·30	0·61	0·01	0·19	1·01	0·45	0·10	35·4	75·6	13·5	28·8	0·4	9·8	55·5	21·2	5·1
Ranunculus repens et bulbosus	0·33	0·26	2·02	0·19	3·93	3·83	0·29	1·10	3·88	14·8	15·8	90·7	9·0	169·5	197·0	15·3	49·8	183·2
Conopodium denudatum	3·90	0·51	1·96	4·47	2·02	2·66	2·20	3·21	2·34	174·5	31·1	88·0	211·3	87·1	136·8	102·8	149·6	111·9
Centaurea nigra	1·14	0·31	0·08	0·07	0·01	0·16	0·72	0·08	0·08	51·0	18·9	3·6	3·3	0·4	8·2	34·9	3·5	4·3
Achillea Millefolium	3·29	1·55	0·68	0·66	0·78	0·80	2·42	0·67	0·79	147·2	94·5	30·5	31·2	33·7	41·1	120·8	30·8	37·4
Rumex Acetosa	1·19	2·19	0·72	2·65	1·26	1·51	1·69	1·68	1·38	53·3	133·5	32·3	125·3	54·4	77·7	93·4	78·8	66·0
Luzula campestris	0·06	0·16	1·42	0·33	2·02	0·36	0·11	0·87	1·19	2·7	9·8	63·8	15·6	87·1	18·5	6·3	39·7	52·8
None yielding 1 per cent. { Ranunculus auricomus, Stellaria graminea, Cerastium triviale, Potentilla reptans, Agrimonia Eupatoria, Pimpinella Saxifraga, Heracleum Sphondylium, Anthriscus sylvestris, Galium verum, G. Aparine, Bellis perennis, Tragopogon pratensis, Taraxacum officinale, Plantago lanceolata, Veronica Chamædrys, Prunella vulgaris, Ajuga reptans, Primula veris, Scilla nutans, Carex præcox, Ophioglossum vulgatum, Hypnum squarrosum, H. hians, H. rutabulum }	0·27	1·44	0·96	1·35	0·81	0·72	0·88	1·18	0·78	12·0	87·8	42·9	63·9	34·8	37·1	49·9	53·5	36·1
Absent { Poterium Sanguisorba, Scabiosa arvensis, Chrysanthemum Leucanthemum, Leontodon hispidus, Hieracium Pilosella, Thymus Serpyllum }	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Total	10·97	7·66	8·14	10·33	10·84	10·23	9·32	9·24	10·54	490·9	467·0	365·3	488·4	467·4	526·2	478·9	426·9	496·8
SUMMARY.																		
Gramineæ	81·58	82·94	89·35	80·95	87·12	86·11	82·26	85·15	86·61	3650·7	5056·0	4010·0	3827·3	3756·6	4429·5	4353·4	3918·6	4093·0
Leguminosæ	7·45	9·40	2·51	8·72	2·04	3·66	8·42	5·61	2·85	333·4	573·0	112·7	412·3	88·0	188·3	453·2	262·5	138·2
Other Orders	10·97	7·66	8·14	10·33	10·84	10·23	9·32	9·24	10·54	490·9	467·0	365·3	488·4	467·4	526·2	478·9	426·9	496·8
Total	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	4475	6096	4488	4728	4312	5144	5285	4608	4728

Increase + or Decrease − compared:—																		
With Plot 3, without manure.									With Plot 16.						With Plot 19.			
Plot 16.		Plot 19.		Plot 20.		Mean of 1872 and 1877.			Plot 19.		Plot 20.		Mean of 1872 and 1877.		Plot 20.			
1872.	1877.	1872.	1877.	1872.	1877.	Plot 16.	Plot 19.	Plot 20.	1872.	1877.	1872.	1877.	Plot 19.	Plot 20.	1872.	1877.	Mean.	
NUMBER OF SPECIES.																		
0	− 2	−1	− 1	−2	− 1	− 1	− 1	− 1	−1	+1	−2	+1	0	0	−1	0	0	Gramineæ.
0	0	0	+ 1	+1	0	0	+ 1	0	0	+1	+1	0	+1	0	+1	−1	−1	Leguminosæ.
−13	− 9	−8	−13	−6	−13	−11	−11	−10	+5	−4	+7	−4	0	+1	+2	0	+1	Other Orders.
−13	−11	−9	−13	−7	−14	−12	−11	−11	+4	−2	+6	−3	+1	+1	+2	−1	0	Total.
GRAMINEÆ.																		
lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	
− 22·8	+ 14·3	+ 201·3	− 4·2	+ 223·7	+ 59·6	− 4·2	+ 98·6	+ 141·7	+224·1	− 18·5	+246·5	+ 45·3	+102·8	+145·9	+ 22·4	+ 63·8	+ 43·1	Anthoxanthum odoratum.
+ 672·6	+ 738·5	+ 1·8	+ 248·2	+ 44·1	+ 342·7	+ 705·5	+ 125·0	+ 193·4	−670·8	− 490·3	−628·5	−395·8	−580·5	−512·1	+ 42·3	+ 94·5	+ 68·4	Alopecurus pratensis.
+ 289·6	+ 574·9	+ 881·8	+ 198·1	+1086·5	+ 219·0	+ 432·2	+ 539·9	+ 652·7	+592·2	− 376·8	+796·9	−355·9	+107·7	+220·5	+204·7	+ 20·9	+112·8	Agrostis vulgaris.
+ 168·1	+ 473·8	+ 570·5	+ 705·2	+ 475·1	+1219·2	+ 321·0	+ 637·9	+ 847·2	+402·4	+ 231·4	+307·0	+745·4	+316·9	+526·2	− 95·4	+514·0	+209·3	Holcus lanatus.
+ 10·5	+ 140·0	+ 8·9	+ 78·2	− 14·4	+ 59·8	+ 75·3	+ 43·5	+ 22·7	− 1·6	− 61·8	− 24·9	− 80·2	− 31·8	− 52·6	− 23·3	− 18·4	− 20·8	Avena pubescens.
+ 783·9	+ 381·1	+ 90·3	+ 181·1	+ 75·0	+ 247·1	+ 582·6	+ 135·7	+ 161·1	−693·6	− 200·0	−708·9	−134·0	−446·9	−421·5	− 15·3	+ 66·0	+ 25·4	Avena flavescens.
+ 284·0	+ 280·6	+ 25·5	+ 210·4	+ 8·6	+ 144·2	+ 282·3	+ 117·9	+ 76·4	−258·5	− 70·2	−275·4	−136·4	−164·4	−205·9	− 16·9	− 66·2	− 41·5	Poa trivialis.
+ 153·0	+ 265·6	+ 15·3	+ 98·3	− 1·8	+ 42·6	+ 209·3	+ 56·8	+ 20·5	−137·7	− 167·3	−154·8	−223·0	−152·5	−188·9	− 17·1	− 55·7	− 36·4	Dactylis glomerata.
− 16·0	− 12·2	+ 34·3	+ 96·9	+ 31·4	+ 54·5	− 14·1	+ 65·6	+ 42·9	+ 50·3	+ 109·1	+ 47·4	+ 66·7	+ 79·7	+ 57·0	− 2·9	− 42·4	− 22·7	Cynosurus cristatus.
+ 106·1	+ 498·1	+ 974·5	+ 494·3	+ 745·1	+ 444·0	+ 302·1	+ 734·4	+ 594·6	+868·4	− 3·8	+639·0	− 54·1	+432·3	+292·5	−239·4	− 50·3	−139·8	Festuca ovina.
+ 99·7	+ 121·0	+ 146·8	− 49·4	+ 48·5	+ 78·1	+ 110·4	+ 48·7	+ 63·3	+ 47·1	− 170·4	− 51·2	− 42·9	− 61·7	− 47·1	− 98·3	+127·5	+ 14·6	Lolium perenne.
+ 90·7	+ 34·7	...	...	...	...	+ 62·7	...	...	− 90·9	− 34·7	− 90·9	− 34·7	− 62·8	− 62·8	...	...	...	Bromus mollis.
− 71·2	− 135·7	− 53·7	− 111·8	− 77·0	− 162·6	− 103·5	− 82·7	− 119·9	+ 17·5	+ 24·6	− 5·8	− 26·9	+ 21·2	− 16·3	− 23·3	− 51·5	− 37·5	{ Poa pratensis, Briza media, Festuca pratensis, Festuca loliacea, Phleum pratense, Aira cæspitosa, Avena elatior } None yielding 1 per cent.
...	− 0·7	− 0·2	...	− 0·2	− 0·7	− 0·4	− 0·1	− 0·5	...	...	...	...	...	...	...	...	...	{ Phleum pratense (plots 16 and 19), Festuca loliacea (plots 16 and 20), Bromus mollis (plots 19 and 20). } Absent.
− 26·3	...	− 15·9	...	− 16·8	...	− 13·1	− 7·9	− 8·4	+ 10·4	...	+ 9·5	...	+ 5·2	+ 4·7	− 0·9	...	− 0·5	Undetermined (chiefly Gramineæ).
+2521·9	+3374·0	+2881·2	+2145·3	+2627·8	+2747·5	+2948·1	+2513·3	+2687·7	+359·3	−1228·7	+105·9	−626·5	−434·8	−260·4	−253·4	+602·2	+174·4	Total.
LEGUMINOSÆ.																		
+ 8·1	− 2·5	+ 2·3	+ 36·6	+ 2·9	+ 29·8	+ 2·8	+ 19·4	+ 16·3	− 5·8	+ 39·1	− 5·2	+ 32·3	+ 16·6	+ 13·5	+ 0·6	− 6·8	− 3·1	Trifolium repens.
− 8·3	− 42·7	+ 30·8	− 11·6	− 0·4	− 46·8	− 25·5	+ 9·6	− 23·6	+ 39·1	+ 31·1	+ 7·9	− 4·1	+ 35·1	+ 1·9	− 31·2	− 35·2	− 33·2	Trifolium pratense.
...	...	...	...	+ 0·4	...	...	...	+ 0·2	...	...	+ 0·4	...	...	+ 0·2	+ 0·4	...	+ 0·2	Trifolium procumbens.
− 96·8	− 89·7	− 62·2	− 87·7	− 50·7	− 81·1	− 93·3	− 75·0	− 65·9	+ 34·6	+ 2·0	+ 46·1	+ 8·6	+ 18·3	+ 27·4	+ 11·5	+ 6·6	+ 9·1	Lotus corniculatus.
...	...	...	+ 1·9	...	...	...	+ 1·0	...	...	+ 1·9	...	...	+ 1·0	...	...	− 1·9	− 1·0	Lotus major.
+ 282·8	+ 506·0	− 5·8	+ 271·2	− 11·8	+ 84·5	+ 394·4	+ 132·7	+ 36·4	−288·6	− 234·8	−294·6	−421·5	−261·7	−358·0	− 6·0	−186·7	− 96·3	Lathyrus pratensis.
+ 185·8	+ 371·1	− 34·9	+ 210·4	− 59·6	− 13·6	+ 278·4	+ 87·7	− 36·6	−220·7	− 160·7	−245·4	−384·7	−190·7	−315·0	− 24·7	−224·0	−124·3	Total.
OTHER ORDERS.																		
+ 34·2	+ 75·6	+ 12·3	+ 28·8	− 0·8	+ 9·8	+ 54·9	+ 20·6	+ 4·5	− 21·9	− 46·8	− 35·0	− 65·8	− 34·3	− 50·4	− 13·1	− 19·0	− 16·1	Ranunculus acris.
− 33·5	− 65·8	+ 42·4	− 72·6	+ 121·2	+ 115·4	− 49·6	− 15·1	+ 118·3	+ 75·9	− 6·8	+154·7	+181·2	+ 34·5	+167·9	+ 78·8	+188·0	+133·4	Ranunculus repens et bulbosus.
+ 127·7	− 13·8	+ 41·2	+ 166·4	+ 40·3	+ 91·9	+ 57·0	+ 103·8	+ 66·1	− 86·5	+ 180·2	− 87·4	+105·7	+ 46·8	+ 9·1	− 0·9	− 74·5	− 37·7	Conopodium denudatum.
+ 16·4	− 6·2	− 31·0	− 21·8	− 34·2	− 16·9	+ 5·1	− 26·3	− 25·5	− 47·4	− 15·6	− 50·6	− 10·7	− 31·4	− 30·6	− 3·2	+ 4·9	+ 0·8	Centaurea nigra.
+ 117·9	+ 47·5	+ 1·2	− 15·8	+ 4·4	− 5·9	+ 82·7	− 7·3	− 0·7	−116·7	− 63·3	−113·5	− 53·4	− 90·0	− 83·4	+ 3·2	+ 9·9	+ 6·6	Achillea Millefolium.
+ 24·2	+ 89·3	+ 3·2	+ 81·1	+ 25·3	+ 33·5	+ 56·7	+ 42·1	+ 29·3	− 21·0	− 8·2	+ 1·1	− 55·8	− 14·6	− 27·4	+ 22·1	− 47·6	− 12·8	Rumex Acetosa.
− 53·5	− 32·0	+ 7·6	− 26·2	+ 30·9	− 23·3	− 42·7	− 9·3	+ 3·8	+ 61·1	+ 5·8	+ 84·4	+ 8·7	+ 33·4	+ 46·5	+ 23·3	+ 2·9	+ 13·1	Luzula campestris.
− 63·5	− 33·8	− 46·4	− 68·7	− 56·9	− 96·9	− 48·7	− 57·7	− 76·9	+ 31·0	− 23·9	+ 22·8	− 50·7	+ 2·6	− 14·0	− 8·1	− 26·8	− 17·4	{ Ranunculus auricomus, Stellaria graminea, Cerastium triviale, Potentilla reptans, Agrimonia Eupatoria, Pimpinella Saxifraga, Heracleum Sphondylium, Anthriscus sylvestris, Galium verum, G. Aparine, Bellis perennis, Tragopogon pratensis, Taraxacum officinale, Plantago lanceolata, Veronica Chamædrys, Prunella vulgaris, Ajuga reptans, Primula veris, Scilla nutans, Carex præcox, Ophioglossum vulgatum, Hypnum squarrosum, H. hians, H. rutabulum } None yielding 1 per cent.
− 46·6	− 73·9	− 32·8	− 62·9	− 30·4	− 61·5	− 60·4	− 47·7	− 46·0	...	...	...	...	...	...	...	...	...	{ Poterium Sanguisorba, Scabiosa arvensis, Chrysanthemum Leucanthemum, Leontodon hispidus, Hieracium Pilosella, Thymus Serpyllum. } Absent.
+ 123·3	− 13·1	− 2·3	+ 8·3	+ 99·8	+ 46·1	+ 55·0	+ 3·0	+ 72·9	−125·6	+ 21·4	− 23·5	+ 59·2	− 52·0	+ 17·9	+102·1	+ 37·8	+ 69·9	Total.
SUMMARY.																		
+2521·9	+3374·0	+2881·2	+2145·3	+2627·8	+2747·5	+2948·1	+2513·3	+2687·7	+359·3	−1228·7	+105·9	−626·5	−434·8	−260·4	−253·4	+602·2	+174·4	Gramineæ.
+ 185·8	+ 371·1	− 34·9	+ 210·4	− 59·6	− 13·6	+ 278·4	+ 87·7	− 36·6	−220·7	− 160·7	−245·4	−384·7	−190·7	−315·0	− 24·7	−224·0	−124·3	Leguminosæ.
+ 123·3	− 13·1	− 2·3	+ 8·3	+ 99·8	+ 46·1	+ 55·0	+ 3·0	+ 72·9	−125·6	+ 21·4	− 23·5	+ 59·2	− 52·0	+ 17·9	+102·1	+ 37·8	+ 69·9	Other Orders.
+2831	+3732	+2844	+2364	+2668	+2780	+3281	+2604	+2724	+ 13	−1368	−163	−952	−677	−557	−176	+416	+120	Total.

TABLE LXXX.—Number of Species, Percentage, Quantity per Ac (Plot 20),

	Number of Species, and proportion per eent.											
	Plot 16.		Plot 19.		Plot 20.		Mean of 1872 and 1877.			Plot 16.		
	1872.	1877.	1872.	1877.	1872.	1877.	Plot 16.	Plot 19.	Plot 20.	1872.	1877	
Gramineæ	17	15	16	16	15	16	16	16	16	...	...	Gramineæ.
Leguminosæ	4	4	4	5	5	4	4	5	4	...	...	Leguminosæ.
Other Orders	15	22	20	18	22	18	19	19	20	...	...	Other Orders.
Total	36	41	40	39	42	38	39	40	40	...	...	Total.
	p. c.	p. c.	p. c.	p. c.	p. c.	p. c.	p. c.	p. c.	p. c.	lbs.	lbs.	
Anthoxanthum odoratum	1·40	2·22	6·39	2·47	7·17	3·51	1·81	4·43	5·34	62·7	135·	Anthoxanthum odoratum.
Alopecurus pratensis	15·22	12·23	0·23	5·40	1·22	6·80	13·73	2·81	4·01	681·1	745·	Alopecurus pratensis.
Agrostis vulgaris	12·40	14·58	25·56	10·83	31·35	10·36	13·49	18·19	20·85	554·9	888·	Agrostis vulgaris.
Holcus lanatus	5·08	12·64	14·03	21·19	12·39	29·47	8·86	17·61	20·93	227·3	770·	Holcus lanatus.
Avena pubescens	1·54	3·34	1·50	3·00	1·02	2·40	2·44	2·25	1·71	68·9	203·	Avena pubescens.
Avena flavescens	18·80	6·67	3·29	4·37	3·07	5·30	12·73	3·83	4·18	841·3	406·	Avena flavescens.
Poa trivialis	6·53	4·82	0·75	4·73	0·39	3·06	5·67	2·74	1·72	292·2	293·	Poa trivialis.
Dactylis glomerata	3·75	4·63	0·67	2·43	0·30	1·15	4·19	1·55	0·72	167·8	282·	Dactylis glomerata.
Cynosurus cristatus	0·05	0·18	1·17	2·54	1·15	1·51	0·12	1·86	1·33	2·2	11·	Cynosurus cristatus.
Festuca ovina	10·33	16·66	29·65	21·40	25·54	18·69	13·49	25·52	22·12	462·3	1015·	Festuca ovina.
Lolium perenne	3·10	3·75	4·14	1·23	2·03	3·61	3·42	2·68	2·82	138·7	228·	Lolium perenne.
Bromus mollis	2·03	0·57	...	...	...	...	1·30	...	...	90·9	34·	Bromus mollis.
None yielding 1 per cent. Poa pratensis, Briza media, Festuca pratensis, Festuca loliacea, Phleum pratense, Aira cæspitosa, Avena elatior	0·85	0·65	1·24	1·36	0·75	0·25	0·76	1·31	0·51	38·0	39·	Poa pratensis, Briza media, Festuca pratensis, Festuca loliacea, Phleum pratense, Aira cæspitosa, Avena elatior None yielding 1 per eent.
Absent Phleum pratense (plots 16 and 19), Festuca loliacea (plots 16 and 20), Bromus mollis (plots 19 and 20)	...	...	...	...	...	...	...	...	...	...	...	Phleum prateuse (plots 16 and 19), Festuca loliacea (plots 16 and 20), Bromus mollis (plots 19 and 20). Absent.
Undetermiued (chiefly Gramineæ)	0·50	...	0·73	...	0·74	...	0·25	0·37	0·37	22·4	...	Undetermined (chiefly Gramineæ).
Total	81·58	82·94	89·35	80·95	87·12	86·11	82·26	85·15	86·61	3650·7	5056	Total.
Trifolium repens	0·32	0·01	0·19	0·84	0·21	0·64	0·16	0·52	0·42	14·3	0	Trifolium repens.
Trifolium pratense	0·43	0·11	1·30	0·80	0·63	0·05	0·27	1·05	0·34	19·3	6	Trifolium pratense.
Trifolium procumbens	...	...	...	...	0·01	...	...	...	0·01	...	...	Trifolium procumbens.
Lotus cornieulatus	0·02	0·06	0·79	0·12	1·09	0·24	0·04	0·45	0·66	0·9	3	Lotus corniculatus.
Lotus major	...	...	...	0·04	...	...	...	0·02	...	...	...	Lotus major.
Lathyrus pratensis	6 68	9·22	0·23	6·92	0·10	2·73	7·95	3·57	1·42	298·9	562	Lathyrus pratensis.
Total	7·45	9·40	2·51	8·72	2·04	3·66	8·42	5·61	2·85	333·4	573	Total.
Ranunculus acris	0·79	1·24	0·30	0·61	0·01	0·19	1·01	0·45	0·10	35·4	75	Ranunculus acris.
Ranunculus repens et bulbosus	0·33	0·26	2·02	0·19	3·93	3·83	0·29	1·10	3·88	14·8	15	Banunculus repens et bulbosus.
Conopodium denudatum	3·90	0·51	1·96	4·47	2·02	2·66	2·20	3·21	2·34	174·5	31	Conopodium denudatum.
Centaurea nigra	1·14	0·31	0·08	0·07	0·01	0·16	0·72	0·08	0·08	51·0	18	Ceutaurea nigra.
Achillea Millefolium	3·29	1·55	0·68	0·66	0·78	0·80	2·42	0·67	0·79	147·2	94	Achillea Millefolium.
Rumex Acetosa	1·19	2·19	0·72	2·65	1·26	1·51	1·69	1·68	1·38	53·3	133	Rumex Acetosa.
Luzula campestris	0·06	0·16	1·42	0·33	2·02	0·36	0·11	0·87	1·19	2·7	9	Luzula campestris.
None yielding 1 per cent. Ranunculus auricomus, Stellaria graminea, Cerastium triviale, Potentilla reptans, Agrimonia Eupatoria, Pimpinella Saxifraga, Heracleum Sphondylium, Anthriscus sylvestris, Galium verum, G. Aparine, Bellis perennis, Tragopogon pratensis, Taraxacum officinale, Plantago lanceolata, Veronica Chamædrys, Prunella vulgaris, Ajuga reptans, Primula veris, Scilla nutans, Carex præcox, Ophioglossum vulgatum, Hypnum squarrosum, H. hians, H. rutabulum	0·27	1·44	0·96	1·35	0·81	0·72	0·88	1·18	0·78	12·0	8	Ranunculus auricomus, Stellaria graminea, Cerastium triviale, Potentilla reptans, Agrimonia Eupatoria, Pimpinella Saxifraga, Heracleum Sphondylium, Anthriscus sylvestris, Galium verum, G. Aparine, Bellis perennis, Tragopogon pratensis, Taraxacum officinale, Plantago lanceolata, Veronica Chamædrys, Prunella vulgaris, Ajuga reptans, Primula veris, Scilla nutans, Carex præcox, Ophioglossum vulgatum, Hypnum squarrosum, H. hians, H. rutabulum None yielding 1 per cent.
Absent Poterium Sanguisorba, Scabiosa arvensis, Chrysanthemum Leucanthemum, Leontodon hispidus, Hieracium Pilosella, Thymus Serpyllum	...	...	...	...	...	...	...	...	...	...	.	Poterium Sanguisorba, Scabiosa arvensis, Chrysanthemum Leucanthemum, Leontodon hispidus, Hieracium Pilosella, Thymus Serpyllum. Absent.
Total	10·97	7·66	8·14	10·33	10·84	10·23	9·32	9·24	10·54	490·9	46	Total.
Gramineæ	81·58	82·94	89·35	80·95	87·12	86·11	82·26	85·15	86·61	3650·7	50	Gramineæ.
Leguminosæ	7·45	9·40	2·51	8·72	2·04	3·66	8·42	5·61	2·85	333·4	5	Leguminosæ.
Other Orders	10·97	7·66	8·14	10·33	10·84	10·23	9·32	9·24	10·54	490·9	4	Other Orders.
Total	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	4475	60	Total.

327 lbs. of nitrate of potass, containing the same amount of nitrogen as the 275 lbs. of nitrate of soda. To plot 19 was also applied 290 lbs. of sulphate of potass, containing the same amount of potass as in the 327 lbs. of nitrate of potass supplied to plot 20. To both plots superphosphate, in the same amount as used on other plots of the series, was also applied.

Thus, so far as nitrogen, as nitrate, and superphosphate, are concerned, the two plots, 19 and 20, were manured precisely as plot 16 had been for 14 preceding years, and as it continued to be subsequently. Plot 16 received, however, about 5 lbs. per acre per annum more potass, and some sulphate of soda and sulphate of magnesia in addition. The only difference between the manuring of plots 19 and 20 was, that plot 19 received the soda of the nitrate, and the sulphuric acid of the sulphate of potass, in addition to the constituents supplied to plot 20. On the other hand, plot 20 received its nitric acid and its potass in combination.

The folding Table (LXXX.) shows the percentage and the acreage particulars of each group and species for plots 16, 19, and 20, in the last two complete separation-years (1872 and 1877), the experiments of 19 and 20 only commencing in 1872. There is also given, the difference between the produce on each, compared with that on plot 3 without manure; and plots 19 and 20 are each compared with plot 16, and with one another.

In each of the two separation-years each of the three plots 16, 19, and 20 shows considerably fewer species than the unmanured plot 3; the deficiency being almost exclusively in the Miscellaneæ. Plot 16, which has been the longer under treatment, shows a slightly greater average deficiency in number, and both 19 and 20 show a greater reduction in number in the second than in the first of the two separation-years. The actual number of species found in the samples in the two separation-years was, in those from plot 16, 36 and 41; in those from plot 19, 40 and 39, and in those from plot 20, 42 and 38.

According to the determinations in the complete separation-years, both plots 19 and 20 show a higher average percentage of Gramineæ than plot 16, but, according to the subsequent partial separations, plot 16 indicates the higher proportion in the first, but not in the second crops. Of Leguminosæ, the older plot, 16, which received rather more of potass, and some other mineral constituents, gave a higher percentage in the earlier seasons (especially in the complete separation-years), but according to the subsequent partial separations a less proportion in the later seasons. In the period prior to that of our comparison, plot 16 gave a much higher percentage of Miscellaneæ than subsequently, but in the later years it has, on the average, given less than on either plot 19 or 20; and plot 19, with the nitrate of soda and sulphate of potass, has each year given a less proportion than plot 20 with the nitrate of potass.

Referring to individual species, the table shows that on both plots 19 and 20 the two prevalent grasses of the locality under poor conditions, *Festuca ovina* and *Agrostis vulgaris,* were by far the most prominent in the first of the two separation-years. In the second, however, they had each declined considerably, whilst on both, but especially with the nitrate of potass, *Holcus lanatus* had very considerably gained ground. These three grasses were also among the most prominent on plot 16. On that plot, however, *Alopecurus* had become very characteristic; whilst on plots 19 and 20 it did, it is true, increase very considerably from the first to the second separation-year, but still yielded a much less proportion than on plot 16. The other grasses of chief prominence on plot 16 were *Avena flavescens*, *Poa trivialis*, and, in a less degree, *Dactylis glomerata* and *Lolium perenne*. Of these, *Avena flavescens*, *Poa trivialis*, and *Dactylis glomerata*, had each increased on plots 19 and 20, but neither had acquired the same prominence as on plot 16. In the later years, however, the notes indicated that, as on plot 16, *Avena flavescens* became more prominent on both plots; whilst *Lolium perenne* decreased on 19 with the nitrate of soda, but increased on 20 with the nitrate of potass. *Anthoxanthum odoratum* was much the higher on both the newer experimental plots, but on both it declined very much from the one separation-year to the other. *Avena pubescens* was meagrely, but about equally and increasingly represented on the three plots; whilst the poor *Cynosurus cristatus*, a poor-land grass, was more prominent on the two plots the most recently reclaimed from the unmanured condition.

According to the figures in the table, Leguminosæ considerably increased on both plots 19 and 20 from the first to the second separation-year; but much more on plot 19 with nitrate of soda than on plot 20 with nitrate of potass, and this relative excess on plot 19 continued, as is shown by the partial separations of later years. As in other cases of increase of Leguminosæ under the influence of a potass manure, *Lathyrus pratensis* is by far the most prominent constituent. As already intimated, however, the later partial separations further show that the Leguminosæ have considerably declined on plot 16, and are now more prominent on both 19 and 20, and more so again on 19 with the nitrate of soda, than on 20 with the nitrate of potass. According to the complete separations, *Lathyrus pratensis* is seen to be the most prominent leguminous species on all three plots, so far as final weight is concerned; but according to the notes taken on the ground during growth, *Trifolium pratense* or *T. repens* showed considerable prominence on the two newer plots; *T. pratense* being the more prominent with the more rapidly distributing nitrate of soda; and *T. repens* the more so with the probably more superficially retained, and less deeply distributing, nitrate of potass. The greater prominence of the *Lathyrus* in the samples is no doubt partly accounted for by the fact that a larger proportion of its produce would be included in the crop as mown, whereas a considerable proportion of the *Trifoliums* would be left uncut. At any rate, it is clear that Leguminosæ are favoured on all three plots; and it would seem more so with the nitrate of soda and sulphate of potass than with the nitrate of potass. Whether the decline on all three in the later years (subsequent to the last complete separation), but the more on the plot which has been the longer under treatment, be due to an exhaustion of the plants, and to the increased competition with the grasses, under the influence of the manure, and especially of the nitrate, or whether it is merely a temporary effect of seasons more favourable to their competitors, remains to be proved.

As the table shows, there are seven Miscellaneæ which have yielded 1 per cent. or more to the total produce in one or other of the separation-years on one or other of the three plots 16, 19, 20. But the species reaching this degree of prominence are not the same on the three plots. There were, in fact, in the two years in question, only five which did so on plot 16, and only four which did so on either 19 or 20. These were on the respective plots as shown below; and as the results of a single season might be misleading, they are in each case given in the order, not as in the latest separation-year, but of their average prominence over the two years:—

Plot 16.	*Plot* 19.	*Plot* 20.
Achillea Millefolium.	Conopodium denudatum.	Ranunculus repens and bulbosus.
Conopodium denudatum.	Rumex Acetosa.	Conopodium denudatum.
Rumex Acetosa.	Ranunculus repens and bulbosus.	Rumex Acetosa.
Ranunculus acris.	Luzula campestris.	Luzula campestris.
Centaurea nigra.		

It will be observed that, on the older and somewhat more fully mineral-manured plot 16, *Achillea Millefolium* is the first, and *Centaurea nigra* the last on the above list; whilst neither of them comes into the category on either 19 or 20. On the other hand, *Luzula campestris* is the fourth in average prominence (but much declining) on both 19 and 20, but does not come into the list of 16. Of the remaining plants enumerated, two are the same on the three plots, and the others belong to the same genus, though not to the same species. Among these, *Conopodium* is first on the nitrate of soda plots 16 and 19, and second on the nitrate of potass plot 20; *Rumex Acetosa* is second on 16 and 19, but third on 20; *Ranunculus acris* is third on 16; *R. repens* and *R. bulbosus* third on 19, and first on 20. Too much stress should not, however, be laid on the exact degree of prominence indicated by the figures, as these clearly show that there is great fluctuation according to season as well as according to manure.

As between plot 19, with the nitrate of soda and sulphate of potass, and plot 20 with the nitrate of potass, it is worthy of remark that, on the former, on which it is to be presumed the nitrate would distribute the more rapidly and the more deeply, the deeper-rooting *Ranunculus acris* maintains (at the second separation) a greater prominence than *R. repens* and *R. bulbosus;* whilst with the nitrate of potass, which would be less diffusible, the more superficially-rooting *R. repens* and *R. bulbosus* maintain much the higher degree of prominence. Consistently also, the deep-rooting *Conopodium denudatum* and *Rumex Acetosa* attain a greater prominence on the nitrate of soda plot. On neither of the newer plots is *Achillea Millefolium* so prominent as on the older one (16); whilst *Luzula*, as already referred to, has, so far, greatly declined on both.

So far as the Miscellaneæ are concerned, the general result on both plots would seem to be, as was the case with the Gramineæ, to approximate more nearly to the condition of plot 16. With the longer continuance of the treatment on plot 16, and with an increasing competition on the part of the grasses and Leguminosæ, the Miscellaneæ collectively declined to a lower point on it than on either of the two plots more recently brought under experiment. The previously most prominent species on plot 16, *Conopodium*, *Rumex*, and *Achillea*, on it declined, but, so far, they have either increased or remained stationary on the newer plots.

Referring to the actual yield per acre on the three plots, though there is considerable fluctuation in the amount of produce contributed by the three main groups of plants, plot 16 has, on the average, not only of the two complete separation-years, but also including those of partial separation, yielded considerably more total gramineous herbage, more leguminous herbage, and about the same amount of miscellaneous.

It would be of little use to compare the actual produce of individual species on the three plots with that without manure, though the results are given for reference in the table. The comparison of the produce of the different groups and species on each of the two new plots with that on 16, is of much more interest. The most striking points indicated are that, although there was a much greater amount of both *Agrostis vulgaris* and *Festuca ovina* on both plots 19 and 20 than on plot 16 in the first separation-year (1872, which was also the first year of the experiments on 19 and 20), there was, in the second separation-year (1877), on both a less amount, especially of *Agrostis vulgaris*. The fact is, that whilst both these grasses had increased considerably from the first to the second of these two separation-years on the older plot 16, each greatly diminished from its higher original amount on plots 19 and 20. *Alopecurus pratensis* again showed on both the new plots, and in both seasons, a great deficiency compared with plot 16, although on both it showed an actual and considerable increase. *Holcus lanatus*, on the other hand, was in much greater, and in more rapidly increasing, amount on the two plots 19 and 20 than on 16; and it increased very much the more largely on the nitrate of potass plot 20, where in the second separation-year it contributed more than a third of the total gramineous herbage. *Avena flavescens* also, especially in the first year, yielded considerably less on plots 19 and 20 than on the older plot 16, though on both it rather increased in actual amount. *Dactylis glomerata*, *Lolium perenne*, and *Avena pubescens*, were also each more or less in deficiency on plots 19 and 20. *Cynosurus cristatus* and *Anthoxanthum odoratum* were, on the other hand, in some excess. It was, in fact, in the poorer grasses—*Festuca ovina*, *Agrostis vulgaris*, and *Holcus lanatus*—that the two newer plots showed the greatest excess over plot 16 in the first of the two separation-years. In the second separation-year, however, there was either a deficiency, or a less excess of these, as compared with their yield on plot 16; whilst those which were in the greatest deficiency in the first separation-year on the newer plots compared with the older one, were the better grasses *Alopecurus pratensis* and *Avena flavescens*, each of which had gained, both in actual and relative amount, compared with 16, by the second separation-year. Indeed, so far as the actual yield of the different grasses is concerned, the two newer plots in the main approximated to the character of the older one.

So far as the actual yield of the leguminous species is concerned, the most marked distinction between the older plot and the two newer ones is, that on the latter, *Lathyrus pratensis* is in considerable deficiency, whereas the different species of *Trifolium* have been hitherto mostly in excess.

Looking to the actual yield of the miscellaneous species, *Conopodium denudatum* alone comes to yield markedly more on the newer than on the older nitrate of soda plot, most of the other species yielding less. With the nitrate of potass, on the other hand, *Ranunculus repens* and *bulbosus*, as well as *Conopodium denudatum*, yield an excess.

The last three columns of the table bring to view the excess or deficiency in actual yield of each group or species of plant on plot 20 as compared with plot 19. They show that on plot 20, with the nitrate of potass, there was among the grasses a strikingly greater tendency to increase in the comparatively shallow-rooting *Holcus lanatus*, and some tendency to increase in *Lolium perenne*, *Alopecurus pratensis*, *Avena flavescens*, *Agrostis vulgaris*, and *Anthoxanthum odoratum*. There was, however, with the nitrate of potass a tendency to less produce of *Poa trivialis*, *Dactylis glomerata*, *Festuca ovina*, *Cynosurus cristatus*, and *Avena pubescens*. But the differences are, excepting in the very marked instance of the *Holcus*, too small to be thoroughly relied on in forming a judgment as to the different tendencies on the two plots.

Of total Leguminosæ there was notably less on plot 20 than on plot 19. There was, somewhat characteristically, a deficiency of the deeply-rooting *Trifolium pratense*; but there was at the same time a greater deficiency in the more surface-feeding *Lathyrus pratensis*.

Comparing the actual yield of miscellaneous species on plot 20 with that on plot 19, the most striking differences are, that with the less rapidly diffusing nitrate of potass, the comparatively superficially-rooting *Ranunculus repens* and *bulbosus* give a considerable excess, whilst the more deeply-rooting *Conopodium denudatum* and *Rumex Acetosa* are relatively in defect.

Taking the whole period of nine years, the general result has been that, with equal nitrogen on the three plots, but with more mineral supply on plot 16, which has also been under the same conditions of experiment for 14 preceding years, it showed annually more total growth, that is, yielded more total produce, containing more nitrogen, and more total mineral constituents, than either of the newer plots 19 or 20. Again, plot 19, with rather fuller mineral supply than plot 20, has yielded slightly more produce, more nitrogen, and more mineral matter. The deficiency of total growth on the newer plots was chiefly in the grasses, but somewhat in the Leguminosæ also.

Upon the whole, the tendency of change on the new plots was such as to bring them more nearly to the condition of the older one. All three plots, with only moderate supply of nitrogen, in the form of nitrate (which is more favourable for plants of various habits and root-ranges than are ammonia-salts), and with, at the same time, an abundance of the most important mineral constituents, show fairly mixed herbage, with no excessive predominance of the poorer grasses of the locality, and a fair proportion, and amount, of a number of others of freer growth and better repute. There is at the same time a tendency to increase in the proportion and yield of the Leguminosæ. On the older plot there is a reduction in the proportion and amount of the Miscellaneæ ; and on the newer ones there are, it is true, fluctuations from one season to another, but as yet without very marked tendency either to increase or to decrease. The herbage is, upon the whole, more mixed on the older plot than upon either of the newer plots, and perhaps it is the more so on plot 19 with the nitrate of soda, than on plot 20 with the nitrate of potass.

Comparing the flora of the two newer plots, the differences, such as they are, are seen to be consistent with the slight difference in the character of the manures employed. The most marked points which have been brought out are that the less diffusible nitrate of potass has brought the superficially-rooting *Holcus lanatus* into very great relative prominence ; it has, however, much less favoured the comparatively surface-feeding *Lathyrus pratensis* than has the mixture of nitrate of soda and sulphate of potass. More consistently it has been less favourable to the deeper-rooting *Trifolium pratense.* Again, the more diffusible nitrate of soda has favoured the development of the deeply-rooting *Ranunculus acris ;* whilst the less diffusible nitrate of potass has, in a much greater degree, enhanced the growth of the more superficially-rooting *R. repens* and *R. bulbosus.* Another point of distinction between the action of the two closely allied manures, as will be brought out more fully when treating of the chemical results, is that with the nitrate of potass, more of potass, though less of most other mineral constituents, is taken up ; and, with this increase of potass, there is a somewhat greater tendency to stem-formation and maturation.

16. *Mixture supplying the ash-constituents, and the nitrogen, of one ton of hay; Plot* 18.

This experiment was not commenced until 1865, that is nine years later than most of the series. As explained more fully in Part I., p. 362, it had for its chief object to test the validity of the principle of manuring enunciated by LIEBIG, according to which all the constituents removed in crops, or contained in those it is wished to grow, and neither more nor less than are so removed or contained, should be returned to the soil, if the produce is to be maintained, or more should be supplied if it is to be increased. In his earlier application of the principle he limited this necessary return or supply to the mineral or ash-constituents of the crops removed or to be grown. Another object was to determine how much of the several constituents annually supplied would be recovered in the increase of crop.

The plan of experiment adopted was, to supply not only as much of each of the mineral constituents, but also as much nitrogen, as would be contained in one ton of hay, and to estimate the amount of increase of produce, and of the constituents contained in that increase. In carrying out the experiments, the constituents were supplied in chloride of potassium, sulphate of magnesia, bone-ash and sulphuric acid mixed as superphosphate, silicate of soda, silicate of lime, and ammonia-salts. The quantities of these supplied contained, as nearly as could be calculated, the full amount of potass, magnesia, lime, phosphoric acid, silica, and nitrogen, contained in one ton of hay of average composition; and the quantities of the various substances requisite for this supplied at the same time a considerable excess of soda, sulphuric acid, and chlorine.

Stated broadly, the result is that there was, up to 1880 inclusive, that is, over 16 years, so far as can be calculated, only about three-fourths of a ton instead of one ton of annual increase of produce due to the manure. This was so, notwithstanding that the nitrogen as well as the mineral constituents of a ton of hay had been annually supplied. Of the nitrogen supplied, only about half, but of most of the mineral constituents more than half of those supplied, are estimated to have been recovered in the increase. As pointed out in Part I., however, as the flora changed considerably compared with that without manure, there is some uncertainty in the estimation of the produce, and of its constituents, which are to be referred to increase due to the manure. Thus, under the influence of the manure, there is a much larger proportion and amount of gramineous herbage, which, though it contains a comparatively low percentage of nitrogen, is nevertheless greatly influenced by nitrogen artificially supplied; and hence the proportion of that supplied which was actually taken up, and contributed to the increase, may be greater than is estimated. But it may be observed that a less proportion of some of the more important mineral constituents—potass, for example—was recovered, with this carefully balanced but not excessive supply, than when larger quantities of mineral matter and nitrogen were applied, and when greater general luxuriance was, in consequence, induced.

The first year of the experiment being 1865, which was the tenth of the main series, we have the results of three complete botanical separations—in 1867, 1872, and 1877. The percentage and acreage results of these are given in the following Table, LXXXI., p. 1402; as also is the increase in actual yield of the various component species, over that on the unmanured plot 3, in the same seasons.

TABLE LXXXI.—Number of Species, Percentage, Quantity per Acre, &c., of each Species, by a Mixture supplying the Ash-constituents, and the Nitrogen, of 1 ton of Hay; Plot 18.

	Number of species, and proportion per cent.				Quantity per acre.				Increase + or decrease − compared with Plot 3.			
	1867.	1872.	1877.	Mean.	1867.	1872.	1877.	Mean.	1867.	1872.	1877.	Mean.
NUMBER OF SPECIES.												
Gramineæ	15	18	14	16	...	...	...	...	0	+1	− 3	0
Leguminosæ	4	4	4	4	...	...	...	...	0	0	0	0
Other Orders	21	22	21	21	...	...	...	...	−3	−6	−10	−7
Total	40	44	39	41	...	...	...	...	−3	−5	−13	−7
GRAMINEÆ.												
	p. c.	p. c.	p. c.	p. c.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
Anthoxanthum odoratum	4·94	7·11	8·64	6·89	214·0	265·9	388·8	289·6	− 74·5	+ 180·4	+ 267·8	+ 124·6
Agrostis vulgaris	7·29	21·95	16·40	15·21	315·8	820·9	738·0	624·9	+ 28·3	+ 555·6	+ 424·1	+ 336·0
Holcus lanatus	12·78	7·32	17·45	12·52	553·6	273·8	785·2	537·5	+ 288·1	+ 214·6	+ 488·5	+ 330·3
Avena pubescens	2·87	2·58	1·94	2·46	124·3	96·5	87·3	102·7	+ 22·0	+ 38·1	+ 23·7	+ 27·9
Avena flavescens	3·42	5·75	3·12	4·09	148·1	215·1	140·4	167·9	+ 86·1	+ 157·7	+ 114·9	+ 119·6
Poa trivialis	4·83	2·92	2·80	3·52	209·2	109·2	126·0	148·1	+ 170·2	+ 101·0	+ 112·8	+ 128·0
Dactylis glomerata	1·79	1·20	1·28	1·42	77·5	44·9	57·6	60·0	+ 19·5	+ 30·1	+ 41·0	+ 30·2
Festuca ovina	7·05	23·76	24·65	18·49	305·4	888·5	1109·2	767·7	− 201·1	+ 532·3	+ 591·7	+ 307·6
Lolium perenne	5·15	3·37	6·45	4·99	223·1	126·0	290·2	213·1	+ 88·8	+ 87·0	+ 182·6	+ 119·5
None yielding 1 per cent. { Alopecurus pratensis, Phleum pratense, Aira cæspitosa, Avena elatior, Poa pratensis, Briza media, Cynosurus cristatus, Festuca pratensis, Bromus mollis }	2·23	4·04	1·45	2·59	96·8	151·2	65·4	104·5	− 138·5	+ 15·1	− 140·3	− 87·9
Absent.—Festuca loliacea	...	...	...	...	...	...	...	...	...	...	− 0·7	− 0·2
Undetermined (chiefly Gramineæ)	3·11	0·80	...	1·30	134·8	29·9	...	54·9	− 69·7	− 18·8	...	− 29·5
Total	55·46	80·80	84·18	73·48	2402·6	3021·9	3788·1	3070·9	+ 219·2	+1893·1	+2106·1	+1406·1
LEGUMINOSÆ.												
Trifolium repens	0·42	0·58	0·03	0·34	18·2	21·7	1·4	13·8	+ 11·2	+ 15·5	− 1·7	+ 8·4
Trifolium pratense	3·05	0·54	0·20	1·27	132·1	20·2	9·0	53·8	+ 61·8	− 7·4	− 40·4	+ 4·7
Lotus corniculatus	0·71	1·38	0·26	0·78	30·7	51·6	11·7	31·3	− 47·6	− 46·1	− 81·7	− 58·5
Lathyrus pratensis	0·83	1·07	1·50	1·13	36·0	40·0	67·5	47·8	+ 13·3	+ 23·9	+ 11·5	+ 16·2
Total	5·01	3·57	1·99	3·52	217·0	133·5	89·6	146·7	+ 38·7	− 14·1	− 112·3	− 29·2
OTHER ORDERS.												
Ranunculus repens et bulbosus	5·73	3·11	2·56	3·80	248·2	116·3	115·2	159·9	+ 181·2	+ 68·0	+ 33·6	+ 94·3
Conopodium denudatum	3·69	2·42	3·18	3·09	159·9	90·5	143·0	131·2	+ 61·6	+ 43·7	+ 98·1	+ 67·9
Achillea Millefolium	0·76	4·06	1·32	2·04	32·9	151·8	59·4	81·4	− 5·8	+ 122·5	+ 12·4	+ 43·1
Plantago lanceolata	2·79	0·04	0·05	0·96	120·9	1·5	2·2	41·5	− 236·6	− 42·2	− 72·5	− 117·1
Rumex Acetosa	24·33	1·97	4·66	10·32	1054·0	73·7	209·7	445·8	+ 995·3	+ 44·6	+ 165·5	+ 401·8
Luzula campestris	0·44	1·72	0·04	0·73	19·1	64·3	1·8	28·4	− 101·2	+ 8·1	− 40·1	− 44·4
None yielding 1 per cent. { Ranunculus acris, Stellaria graminea, Cerastium triviale, Potentilla reptans, Pimpinella Saxifraga, Heracleum Sphondylium, Galium verum, G. Aparine, Centaurea nigra, Carduus arvensis, Bellis perennis, Chrysanthemum Leucanthemum, Tragopogon pratensis, Taraxacum officinale, Hieracium Pilosella, Veronica Chamædrys, Prunella vulgaris, Scilla nutans, Carex præcox, Hypnum sqarrosum, H. hians, H. rutabulum }	1·79	2·31	2·02	2·06	77·4	86·5	91·0	84·9	− 123·8	+ 1·8	+ 2·9	− 39·8
Absent { Agrimonia Eupatoria, Poterium Sanguisorba, Scabiosa arvensis, Leontodon hispidus, Thymus Serpyllum, Ajuga reptans }	...	...	...	...	...	...	...	...	− 28·6	− 29·5	− 57·8	− 38·7
Total	39·53	15·63	13·83	23·00	1712·4	584·6	622·3	973·1	+ 742·1	+ 217·0	+ 142·2	+ 367·1
SUMMARY.												
Gramineæ	55·46	80·80	84·18	73·48	2402·6	3021·9	3788·1	3070·9	+ 219·2	+1893·1	+2106·1	+1406·1
Leguminosæ	5·01	3·57	1·99	3·52	217·0	133·5	89·6	146·7	+ 38·7	− 14·1	− 112·3	− 29·2
Other Orders	39·53	15·63	13·83	23·00	1712·4	584·6	622·3	973·1	+ 742·1	+ 217·0	+ 142·2	+ 367·1
Total	100·00	100·00	100·00	100·00	4332	3740	4500	4190	+1000	+2096	+2136	+1744

The first separation was in the third year of the experiment, when 40 species were found in the sample, there being in that year in the sample from the unmanured plot (3) six fewer than the average, namely, only 43. In the second separation-year (1872) there were 44 species in the sample from plot 18, and 49 in that from plot 3. In the third year, or the thirteenth of the experiment, there were only 39 in the sample from plot 18, but 52 in that from plot 3. There was thus a deficiency in the number of species on the manured plot compared with the unmanured, in the first separation-year of three, in the second of five, and in the third of 13. This, as the actual number of species in each separation-year shows, does not represent anything like a corresponding decline in the actual number of species on the manured plot; the difference between the number on it and that on the unmanured plot being, in fact, due to a considerable increase in the later years in the number on the unmanured, doubtless owing to the decreasing intensity of the struggle without any manure. The increase thus resulting was especially among the Miscellaneæ; of which there were in the samples from the unmanured produce 24 in the first, 28 in the second, and 31 in the third, of the three separation-years; whilst in those from plot 18, the numbers ranged 21, 22, and 21; that is practically showing no change. Upon the whole, then, with somewhat intensified struggle on the manured plot, there are fewer species than without manure; though, with the competition comparatively limited, there is fluctuation according to season, rather than any marked diminution, from one separation-year to another, in the actual number of species coming into the samples.

It will be observed that, with comparatively limited manuring, the nitrogen supplied as ammonia-salts, and comparatively limited luxuriance and struggle, there were only nine grasses which yielded more than 1 per cent. to the produce in one or other of the separation-years. With the less intensity of struggle without manure, there were, however, 12; and on plot 16, with more nitrogen and more mineral matter supplied, but the nitrogen applied as nitrate of soda, and with much more active luxuriance than on plot 18 with the ammonia-salts, there were, nevertheless, 11. This is, no doubt, explained by the fact frequently before referred to, that the nitrogen of ammonia-salts distributes much less rapidly and freely than that of nitrate of soda; and, therefore, even with less luxuriance, and consequently less active competition, the growth of a more limited number is favoured.

The percentage of grasses in the total produce, nevertheless, increased considerably from one separation-year to another. The increase was, however, almost exclusively due to the increased proportion of the poor and comparatively surface-rooting *Festuca ovina, Agrostis vulgaris, Holcus lanatus,* and *Anthoxanthum odoratum;* which collectively contributed in 1867, 32; in 1872, 60; and in 1877, 67 per cent. to the total herbage. Besides these, *Lolium perenne, Avena flavescens* and *pubescens, Poa trivialis,* and *Dactylis glomerata,* contributed comparatively small but fluctuating amounts, without very obvious tendency either to increase or to decrease; though *Poa trivialis, Avena pubescens,* and *Dactylis glomerata,* showed the greater tendency to decrease.

On the other hand, both the Leguminosæ and the Miscellaneæ diminished in percentage from one separation-year to another.

Among the Leguminosæ, there was a slight tendency to increase in the proportion of *Lathyrus pratensis;* but there was a considerable decrease in that of *Trifolium pratense*, and some in that of *T. repens*, and *Lotus corniculatus.*

Among the six Miscellaneæ which contributed 1 per cent. to the produce in one or other separation-year, *Rumex Acetosa*, *Ranunculus repens* and *bulbosus*, *Plantago lanceolata*, and *Luzula campestris*, markedly declined from the first to the third of the three separation-years; whilst *Achillea Millefolium* showed some tendency to increase, and *Conopodium denudatum* but little either to increase or decrease. Among these, *Rumex Acetosa* was in very abnormally high amount in the first of the three separation-years (1867), in which it will be remembered there was a very large quantity of miscellaneous herbage on most plots. The *Rumex* continued, however, to maintain the first place, *Conopodium denudatum* and *Ranunculus repens* and *bulbosus* also remaining prominent.

The columns of actual yield per acre also show an increase in that of the Gramineæ, and a diminution in that of both Leguminosæ and Miscellaneæ.

Comparing the actual yield per acre of the different species and groups on plot 18 with that on plot 3, without manure, there is (with two exceptions in the first year) an increased yield of every grass in each of the three separation-years under the influence of the manure. The increase in the actual yield of the grasses collectively was very great from one separation-year to another, and it was much the greatest in the case of the poorer and superficially-rooting grasses already referred to, viz.: *Festuca ovina*, *Agrostis vulgaris*, *Holcus lanatus*, and *Anthoxanthum odoratum.*

Of Leguminosæ, there was on the manured plot less actual yield of the two *Trifoliums;* the most marked decline was in the deeply-rooting *Lotus corniculatus;* and there was a uniform but slight increase in the more superficially-feeding *Lathyrus* only. Confining attention to the two later separation-years, when the characteristics of the plot had become the more established, it is seen that there was, with the manure, a greater actual yield of *Rumex Acetosa*, *Conopodium denudatum*, *Ranunculus repens* and *bulbosus*, and *Achillea Millefolium;* but there was a decrease compared with the produce without manure, of *Plantago lanceolata* and *Luzula campestris*, two frequently very persistent elements on permanent grass land under poor conditions.

Upon the whole, it is evident that, with this mixed manure containing the different constituents approximately in the proportion in which they occur in the crop to be grown, with the supply of nitrogen limited, and in the form of ammonia-salts, the effect was by no means equally to favour the increased growth of the groups or species as they were found in the mixed herbage. The effect was, on the other hand, especially to favour the grasses, which came to be very prominent, and particularly the poorer and more superficially-rooting species, which are the most predominant under poor conditions, and which are reputed to be of inferior quality. Further, the amount of leguminous herbage grown on a given area was actually reduced; whilst that of some of the weeds was notably increased, and of these, that of *Rumex Acetosa* was the most prominently so.

Referring to what has been stated to be one of the main objects of the experiment—to determine whether the mineral constituents, and the nitrogen, contained in one ton of hay would yield one ton of increase, it has been stated generally that only about three-quarters of a ton instead of one ton of increase has been obtained. This is almost precisely the amount reached over the average of the 16 years of the experiment. Over the first 10 years, however, the increase amounted to an average of only half a ton; but during the succeeding six years, in five of which second crops were removed, it was much higher, giving the average of three-quarters of a ton increase for the whole period, as above stated. It is obvious that, with an excess of manure on the manured plot, the exhaustion by the removal of the second crops will be much less on it than on the unmanured plot; and hence the amount of the produce calculated as increase over that without manure will be the greater. There was also a considerably increased yield of both mineral matter and nitrogen over the later as compared with the earlier years, due to the removal of second crops. Still, there was, over the whole period, scarcely more than half of the supplied nitrogen estimated to be recovered; and even over the last six years, in five of which second crops were removed, there was, so far as can be estimated, only about two-thirds of the annually supplied nitrogen annually recovered. In fact, when, as is now done, the results of five more years, in four of which second crops were removed, are brought into the calculation, the general conclusions as stated in Part I., where the points under discussion, and the explanation of the facts, are more fully considered than would be appropriate here, are not invalidated.

Finally, as will be judged from the results of the separations which have been considered, the general aspect of this plot was not widely different from that of the unmanured plots; but it exhibited a greater preponderance of grasses, and some increased depth of colour, and luxuriance. The appearance of the herbage was, nevertheless, upon the whole, that of restricted growth.

17. *Farmyard manure, alone, and with ammonia-salts in addition; Plots* 2 *and* 1.

We now turn from the consideration of the effects, on the botany of the plots, of more or less complex artificial mixtures of the various constituents of manure, to those of the very complex and heterogeneous natural manure, farmyard dung. To plot 2, farmyard manure, at the rate of 14 tons per acre per annum, was applied for eight consecutive years, from 1856 to 1863 inclusive. To plot 1, the same amount of farmyard manure was annually applied over the same eight years, but with 200 lbs. of ammonia-salts (supplying about 43 lbs. of nitrogen) per acre per annum, in addition. After the eight years, the application of dung was discontinued on both plots; plot 2 remained from that time entirely unmanured; plot 1, however, still received the same annual application of ammonia-salts as previously.

Calculation showed that a comparatively insignificant proportion of both the nitrogen and the mineral constituents estimated to be annually supplied in the dung was annually recovered in the increase of crop yielded; nor did the addition of the ammonia-salts increase the produce so much as might have been expected. It seemed desirable, therefore, to attempt to determine the degree, and the permanence, of the effect of the large amounts of the nitrogen, and of the mineral constituents of the manure, which were calculated not to be recovered in the increase of crop during the years of the application, the greater part of which, at any rate, must be supposed to exist as residue within the soil. Further, so far as there would be a relatively greater available residue of mineral constituents than of nitrogen, it was to be supposed that the application of the ammonia-salts would greatly accelerate the recovery of them.

The main facts relating to the produce and the increase of the total mixed herbage over the first 20 years were considered in detail in Part I.; some reference was also there made to the amounts of the nitrogen, and of the mineral constituents, taken up, and these points will be more fully treated in Part III. It is, however, necessary in order to render intelligible the conditions under which the botanical changes have taken place, that the general results as to the produce, and some of its constituents, should be briefly stated here; and we are now able to give them for 25 years, in five out of the last six of which second as well as first crops were removed, the tendency to exhaustion being thereby considerably accelerated.

Whether we look to the period of the application of the farmyard manure, or to the succeeding periods of six, six, and five subsequent years of the action of the residue, there is a considerable increase compared with the unmanured plot; and there is further increase over each period by the action of the ammonia-salts. There was, however, on all three plots, a considerable reduction in produce during the second six as compared with the first six years after the cessation of the application of the dung on plots 2 and 1. Over the last five years, however, there was a slight excess of produce compared with that over the previous six, even reckoning the first crops only, and, including the second crops, there was a very considerably greater total produce, both on the unmanured and on the manured plots, than over the preceding six years; and over the whole period of 25 years, there was nearly as much annual excess on the manured plots compared with the unmanured, as over the first 20 years.

Much the same may be said of both the total mineral matter and the nitrogen taken up. Indeed, much more of the mineral matter has been annually removed in the first crops only, over the last five years, than over the preceding six, and including the second crops, very much more; in fact, annually more than over the first 20 years. Of nitrogen there was, including the first and second crops, more than one-third more annually taken off without manure, more on plot 2 with dung or its residue alone, and one-third more on plot 1 with the ammonia-salts in addition, over the last five, than over the preceding 20 years. It is obvious, therefore, that the removal of the second crops in the later years is, as before said, rapidly accelerating exhaustion, a fact which

TABLE LXXXII.—Number of Species, Percentage, Quantity per Acre, &c., of each Species, by Farmyard Manure, alone, Plot 2; and by Farmyard Manure and Ammonia-salts, Plot 1.

(The Farmyard Manure on both Plots the first 8 years only, 1856–63; Plot 2 afterwards unmanured, Plot 1 Ammonia-salts every year.)

	Number of Species, and proportion per cent.										Quantity per Acre.									
	Plot 2.					Plot 1.					Plot 2.					Plot 1.				
	1862.	1867.	1872.	1877.	Mean.	1862.	1867.	1872.	1877.	Mean.	1862.	1867.	1872.	1877.	Mean.	1862.	1867.	1872.	1877.	Mean.
NUMBER OF SPECIES.																				
Gramineæ	14	17	18	18	17	15	15	18	15	16	...	...	...	...	...	...	...	...	...	...
Leguminosæ	3	4	4	4	4	4	4	3	2	3	...	...	...	...	..	...	...	...	...	...
Other Orders	13	20	25	30	22	9	15	15	17	14	...	...	...	...	...	...	...	...	...	..
Total	30	41	47	52	43	28	34	36	34	33	...	...	...	...	...	...	...	...	...	...
GRAMINEÆ.																				
	p. c.	p. c.	p. c.	p. c.	p. c.	p. c.	p. c.	p. c.	p. c.	p. c.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
Anthoxanthum odoratum	0·20	2·86	6·67	7·20	4·23	0·07	1·35	5·78	13·88	5·27	10·1	163·5	188·8	260·1	155·6	3·9	79·4	204·1	656·8	236·1
Alopecurus pratensis	3·07	2·48	5·61	3·46	3·65	0·26	1·55	2·93	2·52	1·81	155·3	141·8	158·8	125·0	145·2	14·4	91·1	103·4	119·2	82·0
Agrostis vulgaris	2·64	4·94	11·02	17·97	9·14	0·59	6·45	20·77	23·53	12·83	133·6	282·4	312·0	649·1	344·3	32·8	379·3	733·4	1113·4	564·8
Holcus lanatus	1·94	10·94	6·85	11·36	7·77	3·88	10·70	14·06	17·35	11·50	98·2	625·3	193·9	410·3	331·9	215·6	629·2	496·4	821·0	540·6
Avena elatior	2·04	0·40	0·52	0·15	0·78	0·62	0·64	1·62	1·65	1·13	103·2	22·9	14·7	5·4	36·6	34·4	35·6	57·2	78·1	51·3
Avena pubescens	2·56	3·93	10·28	7·68	6·11	0·50	1·58	2·76	2·08	1·73	[129·5	224·6	291·0	277·4	230·6	27·8	92·9	97·4	98·4	79·1
Avena flavescens	6·02	5·88	11·62	2·92	6·61	3·99	6·85	6·23	1·37	4·61	304·6	336·1	329·0	105·5	268·8	221·7	402·8	220·0	64·8	227·3
Poa pratensis	1·66	3·56	2·25	1·20	2·17	1·45	6·56	7·41	1·38	4·20	84·0	203·5	63·7	43·3	98·6	80·6	387·7	261·6	65·3	198·8
Poa trivialis	28·18	15·75	3·07	2·37	12·34	31·90	22·32	4·35	2·71	15·32	1425·9	900·3	86·9	85·6	624·7	1772·4	1312·4	153·6	128·2	841·7
Dactylis glomerata	4·51	6·51	3·27	2·79	4·27	16·43	6·43	3·31	4·23	7·60	228·2	372·1	92·6	100·7	198·4	912·9	378·1	116·9	200·2	402·0
Festuca ovina	0·37	4·59	9·89	11·23	6·52	0·76	6·17	6·40	10·75	6·02	18·7	262·4	280·0	405·6	241·7	42·2	362·8	226·0	508·7	284·9
Bromus mollis	17·77	16·39	3·89	0·18	9·56	21·92	10·55	4·55	0·83	9·46	899·2	936·8	110·1	6·5	488·1	1217·8	620·3	160·7	39·3	509·5
Lolium perenne	1·43	3·63	3·17	4·92	3·29	1·42	3·19	1·71	1·66	1·99	72·4	207·5	89·8	177·7	136·8	78·9	187·6	60·4	78·6	101·4
None yielding 1 per cent. { Phleum pratense, Aira cæspitosa, Briza media, Cynosurus cristatus, Festuca pratensis	0·01	0·37	1·04	1·99	0 86	0·11	0·10	0·11	0·02	0·10	0·5	21·1	29·4	72·0	30·8	6·1	5·9	4·0	1·0	4·2
Absent—Festuca loliacea	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Undetermined (chiefly Gramineæ)	2·65	2·29	0·87	...	1·45	5·07	1·99	0·22	...	1·82	134·1	130·8	24·6	...	72·4	281·6	117·0	7·7	...	101·6
Total	75·05	84·52	80·02	75·42	78·75	88·97	86·43	82·21	83·96	85·39	3797·5	4831·1	2265·3	2724·2	3404·5	4943·1	5082·1	2902·8	3973·0	4225·3
LEGUMINOSÆ.																				
Trifolium repens	0·60	0·06	0·52	0·16	0·34	0·03	0·07	0·01	0·01	0·03	30·4	3·4	14·7	5·8	13·6	1·7	4·1	0·4	0·5	1·7
Trifolium pratense	0·34	0·30	0·28	0·96	0·47	0·03	0·05	0·01	...	0·02	17·2	17·2	7·9	34·7	19·2	1·7	2·9	0·4	...	1·3
Lotus corniculatus	...	0·10	0·17	0·17	0·11	0·01	0·01	...	...	0·01	...	5·7	4·8	6·1	4·2	0·6	0·6	...	...	0·3
Lathyrus pratensis	0·98	1·17	3·96	5·25	2·84	0·12	0·85	0·32	0·50	0·45	49·6	66·9	112·2	189·6	104·6	6·6	50·0	11·2	23·6	22·8
Total	1·92	1·63	4·93	6·54	3·76	0·19	0·98	0·34	0·51	0·51	97·2	93·2	139·6	236·2	141·6	10·6	57·6	12·0	24·1	26·1
OTHER ORDERS.																				
Ranunculus acris	...	0·02	1·02	1·49	0·63	...	0·26	0·14	1·30	0·42	...	1·1	28·9	53·8	20·9	...	15·3	4·9	61·5	20·4
Ranunculus repens et bulbosus	2·89	0·99	1·53	3·58	2·24	1·53	0·37	0·58	1·28	0·94	146·2	56·6	43·3	129·3	93·8	85·0	21·8	20·4	60·6	46·9
Cerastium triviale	0·01	0·05	1·22	0·40	0·42	...	0·11	0·24	0·47	0·20	0·5	2·9	34·5	14·4	13·1	...	6·5	8·4	22·2	9·3
Conopodium denudatum	2·83	2·85	1·15	0·96	1·94	1·46	1·59	0·55	0·56	1·04	143·2	162·9	32·6	34·7	93·3	81·1	93·5	19·4	26·5	55·1
Centaurea nigra	0·04	0·14	1·28	0·90	0·59	...	0·18	1·24	0·25	0·41	2·0	8·0	36·2	32·5	19·7	...	10·6	43·8	11·8	16·6
Achillea Millefolium	2·41	1·09	2·93	2·17	2·15	1·35	3·07	5·01	0·72	2·54	122·0	62·3	82·9	78·4	86·4	75·0	180·4	177·0	34·1	116·6
Plantago lanceolata	1·67	3·08	1·45	3·70	2·47	0·30	0·51	0·14	0·27	0·30	84·5	176·1	41·0	133·6	108·8	16·7	30·0	4·9	12·8	16·1
Veronica Chamædrys	0·28	1·66	1·25	0·72	0·98	...	0·28	0·03	0·01	0·08	14·2	94·9	35·4	26·0	42·6	...	16·5	1·1	0·5	4·5
Rumex Acetosa	12·44	3·07	1·93	2·46	4·97	6·09	5·74	9·29	10·49	7·90	629·5	175·5	54·6	88·9	237·1	338·4	337·4	328·0	496·4	375·1
None yielding 1 per cent. { Stellaria graminea, Potentilla reptans, Agrimonia Eupatoria, Spiræa Ulmaria, Pimpinella Saxifraga, Heracleum Sphondylium, Anthriscus sylvestris, Galium verum, Scabiosa arvensis, Bellis perennis, Tragopogon pratensis, Leontodon hispidus, Taraxacum officinale, Prunella vulgaris, Ajuga reptans, Primula veris, Scilla nutans, Luzula campestris, Carex præcox, Hypnum squarrosum, H. hians, H. rutabulum	0·46	0·90	1·29	1·66	1·10	0·11	0·48	0·23	0·18	0·27	23·2	51·4	36·7	60·0	43·0	6·1	28·3	8·3	8·5	12·8
Absent { Poterium Sanguisorba, Galium Aparine, Chrysanthemum Leucanthemum, Hieracium Pilosella, Thymus Serpyllum, Ophioglossum vulgatum	...	...	...	..	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Total	23·03	13·85	15·05	18·04	17·49	10·84	12·59	17·45	15·53	14·10	1165·3	791·7	426·1	651·6	758·7	602·3	740·3	616·2	734·9	673·4
SUMMARY.																				
Gramineæ	75·05	84·52	80·02	75·42	78·75	88·97	86·43	82·21	83·96	85·39	3797·5	4831·1	2265·3	2724·2	3404·5	4943·1	5082·1	2902·8	3973·0	4225·3
Leguminosæ	1·92	1·63	4·93	6·54	3·76	0·19	0·98	0·34	0·51	0·51	97·2	93·2	139·6	236·2	141·6	10·6	57·6	12·0	24·1	26·1
Other Order	23·03	13·85	15·05	18·04	17·49	10·84	12·59	17·45	15·53	14·10	1165·3	791·7	426·1	651·6	758·7	602·3	740·3	616·2	734·9	673·4
Total	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	5060	5716	2831	3612	4304	5556	5880	3531	4732	4924

	Increase + or Decrease − compared with Plot 3.										Increase + or Decrease − compared with Plot 2.					
	Plot 2.					Plot 1.					Plot 1.					
	1862.	1867.	1872.	1877.	Mean.	1862.	1867.	1872.	1877.	Mean.	1862.	1867.	1872.	1877.	Mean.	
NUMBER OF SPECIES.																
Gramineæ	−4	+2	+1	+1	0	−3	0	+1	−2	−1	+1	−2	0	−3	−1	Gramineæ.
Leguminosæ	−1	0	0	0	0	0	0	−1	−2	−1	+1	0	−1	−2	−1	Leguminosæ.
Other Orders	−15	−4	−3	−1	−6	−19	−9	−13	−14	−14	−4	−5	−10	−13	−8	Other Orders.
Total	−20	−2	−2	0	−6	−22	−9	−13	−18	−16	−2	−7	−11	−18	−10	Total.
GRAMINEÆ.																
	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	
Anthoxanthum odoratum	−120·5	−125·0	+103·3	+139·1	−0·8	−126·7	−209·1	+118·6	+535·8	+79·7	−6·2	−84·1	+15·3	+306·7	+80·5	Anthoxanthum odoratum.
Alopecurus pratensis	+18·3	−52·1	+150·3	+117·9	+58·6	−122·6	−102·8	+94·9	+112·1	−4·6	−140·9	−50·7	−55·4	−5·8	−63·2	Alopecurus pratensis.
Agrostis vulgaris	−213·1	−5·1	+46·7	+335·2	+40·9	−313·9	+91·8	+468·1	+799·5	+261·4	−100·8	+96·9	+421·4	+464·3	+220·5	Agrostis vulgaris.
Holcus lanatus	−55·6	+359·8	+134·7	+113·6	+138·1	+61·8	+363·7	+437·2	+524·3	+346·8	+117·4	+3·9	+302·5	+410·7	+208·7	Holcus lanatus.
Avena elatior	+101·1	+15·9	+12·6	+4·2	+33·5	+32·3	+28·6	+55·1	+76·9	+48·2	−68·8	+12·7	+42·5	+72·7	+14·7	Avena elatior.
Avena pubescens	−165·0	+122·3	+232·6	+213·8	+100·9	−266·7	−9·4	+39·0	+34·8	−50·6	−101·7	−131·7	−193·6	−179·0	−151·5	Avena pubescens.
Avena flavescens	+232·2	+274·1	+271·6	+80·0	+214·5	+149·3	+340·8	+162·6	+39·3	+173·0	−82·9	+66·7	−109·0	−40·7	−41·5	Avena flavescens.
Poa pratensis	+75·1	+197·8	+62·2	+41·6	+94·2	+71·7	+382·0	+260·1	+63·6	+194·4	−3·4	+184·2	+197·9	+22·0	+100·2	Poa pratensis.
Poa trivialis	+1378·9	+861·3	+78·7	+72·4	+597·8	+1725·4	+1273·4	+145·4	+115·0	+814·8	+346·5	+412·1	+66·7	+42·6	+217·0	Poa trivialis.
Dactylis glomerata	+174·5	+314·1	+77·8	+84·1	+162·6	+859·2	+320·1	+102·1	+183·6	+366·2	+684·7	+6·0	+24·3	+99·5	+203·6	Dactylis glomerata.
Festuca ovina	−387·2	−244·1	−76·2	−111·9	−204·8	−363·7	−143·7	−130·2	−8·8	−161·6	+23·5	+100·4	−54·0	+103·1	+43·2	Festuca ovina.
Bromus mollis	+895·2	+935·1	+109·9	+6·5	+486·6	+1213·8	+618·6	+160·5	+39·3	+508·0	+318·6	−316·5	+50·6	+32·8	+21·4	Bromus mollis.
Lolium perenne	−122·0	+73·2	+50·8	+70·1	+18·0	−115·5	+53·3	+21·4	−29·0	−17·4	+6·5	−19·9	−29·4	−99·1	−35·4	Lolium perenne.
None yielding 1 per cent. { Phleum pratense, Aira cæspitosa, Briza media, Cynosurus cristatus, Festuca pratensis	−64·2	−5·9	−94·4	−123·7	−72·0	−58·6	−21·1	−119·8	−194·7	−98·6	+5·6	−15·2	−25·4	−70·2	−26·6	Phleum pratense, Aira cæspitosa, Briza media, Cynosurus cristatus, Festuca pratensis. } None yielding 1 per cent.
Absent—Festuca loliacea	...	...	...	−0·7	−0·2	...	...	...	−0·7	−0·2	...	...	...	...	...	Absent—Festuca loliacea.
Undetermined (chiefly Gramineæ)	−105·2	−73·7	−24·1	...	−50·7	+42·3	−87·5	−41·0	...	−21·5	+147·5	−13·8	−16·9	...	+29·2	Undetermined (chiefly Gramineæ).
Total	+1642·5	+2647·7	+1136·5	+1042·2	+1617·2	+2788·1	+2898·7	+1774·0	+2291·0	+2438·0	+1145·6	+251·0	+637·5	+1248·8	+820·8	Total.
LEGUMINOSÆ.																
Trifolium repens	+14·2	−3·6	+8·5	+2·7	+5·5	−14·5	−2·9	−5·8	−2·6	−6·4	−28·7	+0·7	−14·3	−5·3	−11·9	Trifolium repens.
Trifolium pratense	−119·5	−53·1	−19·7	−14·7	−51·8	−135·0	−67·4	−27·2	−49·4	−69·7	−15·5	−14·3	−7·5	−34·7	−17·9	Trifolium pratense.
Lotus corniculatus	−55·9	−72·6	−92·9	−87·3	−77·1	−55·3	−77·7	−97·7	−93·4	−81·0	+0·6	−5·1	−4·8	−6·1	−3·9	Lotus corniculatus.
Lathyrus pratensis	+11·2	+44·2	+96·1	+133·6	+71·3	−31·8	+27·3	−4·9	−32·4	−10·5	−43·0	−16·9	−101·0	−166·0	−81·8	Lathyrus pratensis.
Total	−150·0	−85·1	−8·0	+34·3	−52·1	−236·6	−120·7	−135·6	−177·8	−167·6	−86·6	−35·6	−127·6	−212·1	−115·5	Total.
OTHER ORDERS.																
Ranunculus acris	−0·3	+0·8	+27·7	+53·8	+20·4	−0·3	+15·0	+3·7	+61·5	+19·9	...	+14·2	−24·0	+7·7	−0·5	Ranunculus acris.
Ranunculus repens et bulbosus	−2·7	−10·4	−5·0	+47·7	+7·4	−63·9	−45·2	−27·9	−21·0	−39·5	−61·2	−34·8	−22·9	−68·7	−46·9	Ranunculus repens et bulbosus.
Cerastium triviale	−13·6	−9·3	+20·5	−5·0	−1·8	−14·1	−5·7	−5·6	+2·8	−5·6	−0·5	+3·6	−26·1	+7·8	−3·8	Cerastium triviale.
Conopodium denudatum	+113·6	+64·6	−14·2	−10·2	+38·4	+51·5	−4·8	−27·4	−18·4	+0·2	−62·1	−69·4	−13·2	−8·2	−38·2	Conopodium denudatum.
Centaurea nigra	−7·5	−11·7	+1·6	+7·4	−2·5	−9·5	−9·1	+9·2	−13·3	−5·6	−2·0	+2·6	+7·6	−20·7	−3·1	Centaurea nigra.
Achillea Millefolium	+75·3	+23·6	+53·6	+31·4	+46·0	+28·3	+141·7	+147·7	−12·9	+76·2	−47·0	+118·1	+94·1	−44·3	+30·2	Achillea Millefolium.
Plantago lanceolata	−139·5	−181·4	−2·7	+58·9	−66·2	−207·3	−327·5	−38·8	−61·9	−158·9	−67·8	−146·1	−36·1	−120·8	−92·7	Plantago lanceolata.
Veronica Chamædrys	−0·5	+84·2	+33·1	+20·3	+34·2	−14·7	+5·8	−1·2	−5·2	−3·9	−14·2	−78·4	−34·3	−25·5	−38·1	Verouica Chamædrys.
Rumex Acetosa	+586·8	+116·8	+25·5	+44·7	+193·4	+295·7	+278·7	+298·9	+452·2	+331·4	−291·1	+161·9	+273·4	+407·5	+138·0	Rumex Acetosa.
None yielding 1 per cent. { Stellaria graminea, Potentilla reptans, Agrimonia Eupatoria, Spiræa Ulmaria, Pimpinella Saxifraga, Heracleum Sphondylium, Anthriscus sylvestris, Galium verum, Scabiosa arvensis, Bellis perennis, Tragopogon pratensis, Leontodon hispidus, Taraxacum officinale, Prunella vulgaris, Ajuga reptans, Primula veris, Scilla nutans, Luzula campestris, Carex præcox, Hypnum squarrosum, H. hians, H. rutabulum	−95·5	−248·5	−73·2	−50·8	−116·8	−106·5	−247·0	−78·8	−63·7	−124·0	−17·1	−23·1	−28·4	−51·5	−30·2	Stellaria graminea, Potentilla reptans, Agrimonia Eupatoria, Spiræa Ulmaria, Pimpinella Saxifraga, Heracleum Sphondylium, Anthriscus sylvestris, Galium verum, Scabiosa arvensis, Bellis perennis, Tragopogon pratensis, Leontodon hispidus, Taraxacum officinale, Prunella vulgaris, Ajuga reptans, Primula veris, Scilla nutans, Luzula campestris, Carex præcox, Hypnum squarrosum, H. hians, H. rutabulum. } None yielding 1 per cent.
Absent { Poterium Sanguisorba, Galium Aparine, Chrysanthemum Leucanthemum, Hieracium Pilosella, Thymus Serpyllum, Ophioglossum vulgatum	−0·6	−7·3	−8·4	−26·7	−10·8	−6·7	−31·9	−31·2	−65·3	−33·8	...	...	...	...	...	Poterium Sanguisorba, Galium Aparine, Chrysanthemum Leucanthemum, Hieracium Pilosella, Thymus Serpyllum, Ophioglossum vulgatum. } Absent.
Total	+515·5	−178·6	+58·5	+171·5	+141·7	−47·5	−230·0	+248·6	+254·8	+56·4	−563·0	−51·4	+190·1	+83·3	−85·3	Total.
SUMMARY.																
Gramineæ	+1642·5	+2647·7	+1136·5	+1042·2	+1617·2	+2788·1	+2898·7	+1774·0	+2291·0	+2438·0	+1145·6	+251·0	+637·5	+1248·8	+820·8	Gramineæ.
Leguminosæ	−150·0	−85·1	−8·0	+34·3	−52·1	−236·6	−120·7	−135·6	−177·8	−167·6	−86·6	−35·6	−127·6	−212·1	−115·5	Leguminosæ.
Other Order	+515·5	−178·6	+58·5	+171·5	+141·7	−47·5	−230·0	+248·6	+254·8	+56·4	−563·0	−51·4	+190·1	+83·3	−85·3	Other Orders.
Total	+2008	+2384	+1187	+1248	+1706	+2504	+2548	+1887	+2368	+2326	+496	+164	+700	+1120	+620	Total.

Table LXXXII.—Number of Species, Percentage, Qu
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	Number of Species, and proportion per cent.										
	Plot 2.					Plot 1.					
	1862.	1867.	1872.	1877.	Mean.	1862.	1867.	1872.	1877.	Mean.	1862.
Gramineæ	14	17	18	18	17	15	15	18	15	16	...
Leguminosæ	3	4	4	4	4	4	4	3	2	3	...
Other Orders	13	20	25	30	22	9	15	15	17	14	...
Total	30	41	47	52	43	28	34	36	34	33	...
	p. c.	p. c.	p. c.	p. c.	p. c.	p. c.	p. c.	p. c.	p. c.	p. c.	lbs.
Anthoxanthum odoratum	0·20	2·86	6·67	7·20	4·23	0·07	1·35	5·78	13·88	5·27	10·1
Alopecurus pratensis	3·07	2·48	5·61	3·46	3·65	0·26	1·55	2·93	2·52	1·81	155·3
Agrostis vulgaris	2·64	4·94	11·02	17·97	9·14	0·59	6·45	20·77	23·53	12·83	133·6
Holcus lanatus	1·94	10·94	6·85	11·36	7·77	3·88	10·70	14·06	17·35	11·50	98·2
Avena elatior	2·04	0·40	0·52	0·15	0·78	0·62	0·64	1·62	1·65	1·13	103·2
Avena pubescens	2·56	3·93	10·28	7·68	6·11	0 50	1·58	2·76	2·08	1·73	[129·5
Avena flavescens	6·02	5·88	11·62	2·92	6·61	3·99	6·85	6·23	1·37	4·61	304·6
Poa pratensis	1·66	3·56	2·25	1·20	2·17	1·45	6·56	7·41	1·38	4·20	84·0
Poa trivialis	28·18	15·75	3·07	2·37	12·34	31·90	22·32	4·35	2·71	15·32	1425·9
Dactylis glomerata	4·51	6·51	3·27	2·79	4·27	16·43	6·43	3·31	4·23	7·60	228·2
Festuca ovina	0·37	4·59	9·89	11·23	6·52	0·76	6·17	6·40	10·75	6·02	18·7
Bromus mollis	17·77	16·39	3·89	0·18	9·56	21·92	10·55	4·55	0·83	9·46	899·2
Lolium perenne	1·43	3·63	3·17	4·92	3·29	1·42	3·19	1·71	1·66	1·99	72·4
None yielding 1 per cent. { Phleum pratense, Aira cæspitosa, Briza media, Cynosurus cristatus, Festuca pratensis }	0·01	0·37	1·04	1·99	0 86	0·11	0·10	0·11	0·02	0·10	0·5
Absent—Festuca loliacea	...	...	...	...	...	...	...	...	...	...	...
Undetermined (chiefly Gramineæ)	2·65	2·29	0·87	...	1·45	5·07	1·99	0·22	...	1·82	134·1
Total	75·05	84·52	80·02	75·42	78·75	88·97	86·43	82·21	83·96	85·39	3797·5
Trifolium repens	0·60	0·06	0·52	0·16	0·34	0·03	0·07	0·01	0·01	0·03	30·4
Trifolium pratense	0·34	0·30	0·28	0·96	0·47	0·03	0·05	0·01	...	0·02	17·2
Lotus corniculatus	...	0·10	0·17	0·17	0·11	0·01	0·01	...	...	0·01	...
Lathyrus pratensis	0·98	1·17	3·96	5·25	2·84	0·12	0·85	0·32	0·50	0·45	49·6
Total	1·92	1·63	4·93	6·54	3·76	0·19	0·98	0·34	0·51	0·51	97·2
Ranunculus acris	...	0·02	1·02	1·49	0·63	...	0·26	0·14	1·30	0·42	...
Ranunculus repens et bulbosus	2·89	0·99	1·53	3·58	2·24	1·53	0·37	0·58	1·28	0·94	146·2
Cerastium triviale	0·01	0·05	1·22	0·40	0·42	...	0·11	0·24	0·47	0·20	0·5
Conopodium denudatum	2·83	2·85	1·15	0·96	1·94	1·46	1·59	0·55	0·56	1·04	143·2
Centaurea nigra	0·04	0·14	1·28	0·90	0·59	...	0·18	1·24	0·25	0·41	2·0
Achillea Millefolium	2·41	1·09	2·93	2·17	2·15	1·35	3·07	5·01	0·72	2·54	122·0
Plantago lanceolata	1·67	3·08	1·45	3·70	2·47	0·30	0·51	0·14	0·27	0·30	84·5
Veronica Chamædrys	0·28	1·66	1·25	0·72	0·98	...	0·28	0·03	0·01	0·08	14·2
Rumex Acetosa	12·44	3·07	1·93	2·46	4·97	6·09	5·74	9·29	10·49	7·90	629·5
None yielding 1 per cent. { Stellaria graminea, Potentilla reptans, Agrimonia Eupatoria, Spiræa Ulmaria, Pimpinella Saxifraga, Heracleum Sphondylium, Anthriscus sylvestris, Galium verum, Scabiosa arvensis, Bellis perennis, Tragopogon pratensis, Leontodon hispidus, Taraxacum officinale, Prunella vulgaris, Ajuga reptans, Primula veris, Scilla nutans, Luzula campestris, Carex præcox, Hypnum squarrosum, H. hians, H. rutabulum }	0·46	0·90	1·29	1·66	1·10	0·11	0·48	0·23	0·18	0·27	23·2
Absent { Poterium Sanguisorba, Galium Aparine, Chrysanthemum Leucanthemum, Hieracium Pilosella, Thymus Serpyllum, Ophioglossum vulgatum }	...	...	...	..	...	...	...	...	...	...	...
Total	23·03	13·85	15·05	18·04	17·49	10·84	12·59	17·45	15·53	14·10	1165·3
Gramineæ	75·05	84·52	80·02	75·42	78·75	88·97	86·43	82·21	83·96	85·39	3797·5
Leguminosæ	1·92	1·63	4·93	6·54	3·76	0·19	0·98	0·34	0·51	0·51	97·2
Other Order	23·03	13·85	15·05	18·04	17·49	10·84	12·59	17·45	15·53	14·10	1165·3
Total	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	5060

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ıthum odoratum.
us pratensis.
vulgaris.
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ıtior.
ıbescens.
ıvescens.
ensis.
alis.
glomerata.
ɔvina.
mollis.
ıerenne.
pratense. Aira cæspi- } None yielding 1 per cent.
riza media, Cynosurus
ıs, Festuca pratensis.
·Festuca loliacea.
nined (chiefly Gramineæ).

ı repens.
ı pratense.
rniculatus.
s pratensis.

llus acris.
llus repens et bulbosus.
m triviale.
lium denudatum.
a nigra.
Millefolium.
lanceolata.
Chamædrys.
Acetosa.
graminea, Potentilla } None yielding 1 per cent.
s, Agrimonia Eupatoria,
Ulmaria, Pimpinella
aga, Heracleum Sphon-
ı, Anthriscus sylvestris,
ı verum, Scabiosa
is, Bellis perennis, Tra-
on pratensis, Leontodon
ıs, Taraxacum offici-
Prunella vulgaris, Ajuga
s, Primula veris, Scilla
, Luzula campestris,
præcox, Hypnum squar-
, H. hians, H. ruta-
ı Sanguisorba, Galium } Absent.
ıe, Chrysanthemum
ıthemum, Hieracium
la, Thymus Serpyllum,
glossum vulgatum.

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should have a marked influence on the relative proportion, and the degree of luxuriance, of the various components of the mixed herbage.

The folding Table (LXXXII.) gives the percentage and per acre results of the four complete botanical separations, of the produce of plot 2 with dung alone, or its residue, and of that of plot 1 with ammonia-salts in addition. It also gives the increase in actual yield of each species or group over that on plot 3 without manure, and the increase on plot 1, with the ammonia-salts, over plot 2 without them.

The first separation-year (1862) was the seventh, and the last but one, of the application of the dung, so that the botanical condition of the plots at that period best represents the direct effects of the manure. The subsequent results on plot 2 show the effects of the discontinuance of the manure, and the gradual approximation to the unmanured condition. The results for plot 1 show the modifications induced during, or after the cessation of the application of dung, when ammonia-salts were also applied.

It will be observed that, in the first separation-year, the number of species was very much lower on the manured plots than on the unmanured one; and it was lower with the greater luxuriance, and the consequent more active struggle, where the ammonia-salts were applied.

After the cessation of the application of the dung, the number of species increased on both plots; but very much more without than with the ammonia-salts. Thus, the numbers were, in the four separation-years, on plot 2—30, 41, 47, and 52, and on plot 1, with the ammonia-salts, 28, 34, 36, and 34. In fact, under the influence of the residue of the dung alone the number of species rapidly increased, and finally reached that without manure; whilst, with the ammonia-salts in addition, the increase was comparatively very slight, and in the last separation-year there were 18 fewer species than without manure, or with the residue of the dung on plot 2. The reduction in the number, under the influence both of the direct application of the dung, and of that of the ammonia-salts, was almost exclusively in miscellaneous species; and the great relative deficiency of these in the later years, where the ammonia was supplied, is a striking illustration of the effects of increasing the luxuriance of the grasses in displacing their competitors.

We have thus far given an outline of the differences between the amounts of produce, the yield of some constituents, and the general botanical character of the herbage, on the two farmyard-manured plots as compared with the unmanured plot. In considering the botanical results in more detail, it will conduce to clearness to confine attention in the first place to the actual and comparative results on plot 2, with the dung or its residue, and afterwards to trace separately the effects of the addition of the ammonia-salts.

A glance at the table shows that, in 1862, the dung being still annually applied, there was a smaller percentage both of total Gramineæ, and of total Leguminosæ, and, notwithstanding the reduced number of miscellaneous species, a higher percentage of total miscellaneous herbage, than after the discontinuance of the dung.

In that, the first separation-year, by far the most prominent grasses were *Poa trivialis* and *Bromus mollis,* both of which had declined to a comparatively insignificant amount by the fourth separation-year. *Avena flavescens* and *Dactylis glomerata* were the next in order of prominence in the first year, and, though each afterwards fluctuated in amount, each yielded less in the fourth than in the first separation-year. The other grasses that yielded appreciable but comparatively small quantities were *Alopecurus pratensis, Agrostis vulgaris, Avena pubescens, Avena elatior, Holcus lanatus, Poa pratensis,* and *Lolium perenne.* Of these, after the cessation of the application of the dung, that is, as the plot gradually approached the unmanured condition, *Agrostis vulgaris, Holcus lanatus,* and *Avena pubescens* increased very largely. *Festuca ovina* also gradually increased from a fraction of 1 per cent. in the first to more than 11 per cent. in the fourth separation year, *Anthoxanthum odoratum* from a small fraction of 1 per cent. to more than 7 per cent., and *Lolium perenne* from less than 1½ per cent. to nearly 5 per cent. *Alopecurus* also, upon the whole, somewhat increased. But, besides *Poa trivialis* and *Bromus mollis,* which were so prominent at first but which decreased so strikingly, *Avena elatior* and *Poa pratensis,* at all times in small amount, also showed a tendency to diminish.

Thus, whilst *Poa trivialis, Bromus mollis, Avena flavescens,* and *Dactylis glomerata,* had been brought into greater or less prominence under the influence of the dung, the prevailing grasses of the locality under poor condition—*Festuca ovina, Agrostis vulgaris, Holcus lanatus,* and *Anthoxanthum odoratum*—were discouraged. But, on the cessation of the application of the dung, and the gradual assimilation to the unmanured condition, the certainly better grasses which had been previously encouraged were now discouraged, and the inferior ones which had previously been discouraged were now encouraged.

The table shows a much less percentage, and also actual amount, of leguminous herbage in the first year than after the cessation of the dung. The percentage increased from under 2 in the first to more than 6½ in the fourth separation-year; and the actual amount from under 100 lbs. per acre in the first to 236 lbs. in the fourth separation-year. The increase was almost exclusively of *Lathyrus pratensis;* but there was a tendency to increase rather than to decrease in *Trifolium pratense* also.

Of miscellaneous species, a much larger number than is usual on the manured plots comes into the list of those which have yielded more than 1 per cent. to the produce in one or other separation-year. There were in the first year only five, but taking together the three subsequent separation-years there have been nine in this category. In the first year, that is, under the immediate influence of the application of the dung, *Rumex Acetosa* was not only in very large amount, but it even contributed more than half the total miscellaneous herbage. *Ranunculus repens* and *bulbosus, Conopodium denudatum, Achillea Millefolium,* and, in a less degree, *Plantago lanceolata,* were also fairly prominent. *Veronica Chamædrys*, *Centaurea nigra*, and *Cerastium triviale*, occurred in only insignificant amounts. In the second and subsequent separation-years, notwithstanding that the second (1867) was very favourable to the growth of miscellaneous herbage over the plots generally, the Miscellaneæ collectively, and the *Rumex* especially, were in very much smaller proportion, and amount, than in the first separation-year. Of the other species, those which gained ground, or maintained a fair relative position, after the discontinuance of the dung, were *Plantago lanceolata*, *Ranunculus repens* and *bulbosus*, and *Achillea Millefolium*. *Ranunculus acris*, *Centaurea nigra*, *Veronica Chamædrys*, and *Cerastium triviale*, though in smaller amounts, also showed tendency to increase. *Conopodium denudatum*, on the other hand, gradually diminished. The tendency of change under the influence of the decreasing available residue of the dung was, in fact, to decrease the unusual prominence of *Rumex*, and to bring a larger number of species into position in the struggle.

Comparing the actual yield of the different groups and species of plants on plot 2 with their amounts without manure, it is seen that in the first separation-year there was a very considerable excess of total Gramineæ, a deficiency of total Leguminosæ, and a large excess of total Miscellaneæ, under the influence of the dung. Among the grasses, *Poa trivialis* and *Bromus mollis* were in very large excess. *Avena flavescens*, *Dactylis glomerata*, and *Avena elatior*, were in moderate, and *Poa pratensis* in small excess. On the other hand, those which were in marked deficiency, compared with the produce without manure, were *Festuca ovina*, *Agrostis vulgaris*, *Avena pubescens*, *Lolium perenne*, and *Anthoxanthum odoratum*. It is thus seen that the grasses which yielded an excess on the manured plot were, for the most part, of freer growth and of better repute, whilst those which were in deficiency are characteristic of conditions of limited competition. After the discontinuance of the dung *Poa trivialis* and *Bromus mollis*, which were previously in such large excess, declined to quite insignificant amounts in the fourth year. *Avena flavescens*, *A. elatior*, *Dactylis glomerata*, and *Poa pratensis*, also declined; whilst *Agrostis vulgaris* and *Festuca ovina* increased largely, both in actual amount and in relation to their yield without manure; and *Avena pubescens*, *Anthoxanthum odoratum*, *Holcus lanatus*, *Alopecurus pratensis*, and *Lolium perenne*, did so in a less degree.

Of total Leguminosæ, there was an actual deficiency on the dunged plot compared with the amount without manure; but the deficiency diminished from year to year after the cessation until, in the fourth separation-year, there was even a rather greater actual amount on the plot which had been for the shorter period without manure. In the first year, that is, during the application of the dung, the deficiency was chiefly in *Trifolium pratense*, and, in a less degree, in *Lotus corniculatus*. Subsequently, *Trifolium pratense* gained in relative position, *Lotus corniculatus* lost, *Trifolium repens* changed but little, but *Lathyrus pratensis* considerably increased. Thus it was the comparatively surface-rooting *Lathyrus* that chiefly gained under the influence of the abundant, but only slowly available residue of manurial constituents; and, under the same conditions, the also surface-rooting *Trifolium repens* fluctuated little. It was under the same conditions that the deeper-rooting *Trifolium pratense* continued, though decreasingly, in relative defect; whilst the also deep-rooting *Lotus corniculatus* showed somewhat greater relative deficiency under the influence of the residue than in the years of the application of the dung.

Comparing the produce of miscellaneous species with the manure with that without it, there was in the first separation-year, as already indicated, a very great excess of *Rumex Acetosa* on the manured plot, and some excess of *Conopodium denudatum* and *Achillea Millefolium* also. The species most markedly in defect under the influence of the manure was *Plantago lanceolata.* The difference in the amounts yielded on the two plots of *Cerastium triviale, Centaurea nigra, Veronica Chamædrys,* and the several species of *Ranunculus,* was too small to be of any significance. The plants, which remained, or became, in excess over the unmanured yield of them, after the application of the manure was discontinued, were *Rumex Acetosa, Achillea Millefolium,* the various species of *Ranunculus, Plantago lanceolata,* and *Veronica Chamædrys,* and in an insignificant degree *Centaurea nigra;* whilst *Conopodium denudatum* went down considerably, and *Cerastium triviale* fluctuated above and below the continuously unmanured produce of it. The general result was then, that a larger number maintained a moderate position, and none were so specially prominent, after the discontinuance of the manure—that is, as the activity of the struggle became less and less.

The point last referred to—that is, the decrease in the predominance of a few individual species, and the increase in the number showing moderate prominence, as the unmanured condition was approached—was equally observable in the case of the Gramineæ as in that of the Miscellaneæ. In the case of the Leguminosæ, the stoppage of the manure most prominently favoured the increased growth of the surface-rooting *Lathyrus,* but otherwise only slightly affected the growth of the leguminous plants, both *Trifolium pratense* and *Lotus corniculatus* remaining in obvious deficiency as compared with their produce without manure continuously.

Upon the whole, whether we look to the number of species which came to maintain a moderate position, or to the character of the species which did so, it is obvious that the general result was that of diminishing intensity of competition, and of gradual approach to the conditions on the continuously unmanured plot.

Thus, the tendencies of botanical change, as well as the particulars of produce which have been briefly referred to, concur in showing that, notwithstanding the enormous unrecovered amount of some of the most important constituents supplied in the dung during the first eight years of the experiments, the residue remaining in the soil was, after a few years, in a very slowly available condition, and, so far as it was available, chiefly so to the more superficially-rooting species. Of the nitrogen of the dung estimated to be unrecovered, part, at any rate, would probably remain in a condition very slowly liberated from its existing state of combination, whilst a portion would be subject to loss by drainage. On the other hand, the more important mineral constituents, though they might be but slowly available, would be little if at all subject to loss by drainage, and would probably remain within a comparatively limited depth from the surface.

We have now to consider whether an annual additional supply of nitrogen in the form of ammonia-salts increased luxuriance, and rendered the comparatively dormant residue of the mineral constituents of the dung more rapidly available, a necessary result of which would be increased intensity of competition. We have also to consider, what has been the effect of the addition on the relative predominance, and the actual yield, of the various species in the mixed herbage, as shown by the botanical separations. To this end we turn to the results in the folding Table LXXXII., facing p. 1407, relating to plot 1, with the ammonia-salts, and to the comparison of them with those of plot 2, without them.

It has been already shown, that one effect of the ammonia-salts was greatly to reduce the number of species, especially of miscellaneous plants. In the first separation-year (1862), the seventh of the application of the dung to plot 2, and the seventh of the application of dung and ammonia-salts together to plot 1, the main distinctions between the two plots were as follows :—

With the ammonia-salts there was a much larger percentage and actual amount of gramineous herbage, very much less leguminous, and also very much less miscellaneous herbage. Referring to individual species, *Poa trivialis* and *Bromus mollis*, which were in very great prominence on plot 2, were in greater prominence still with the ammonia. *Dactylis glomerata* was also in very much larger percentage and amount with than without the ammonia-salts. The only other grasses in any fair amount with the ammonia-salts were *Avena flavescens* and *Holcus lanatus;* whereas, without the ammonia, besides these two, *Alopecurus pratensis*, *Avena pubescens*, *A. elatior*, and *Agrostis vulgaris*, were also in fair quantity. In other words, with the ammonia-salts, a few individual grasses attained greater prominence, and the rest were more meagrely represented than without them. We have, in fact, with the ammonia, the constantly observed result of increased vegetative luxuriance.

Leguminous species were, under the influence of the dung and ammonia-salts together, almost banished ; *Lathyrus pratensis*, however, retaining the first place.

Of miscellaneous species, there were in 1862 only four on plot 1 with the ammonia-salts which contributed as much as 1 per cent., whilst there were five on plot 2 without them. The percentage of Miscellaneæ was, indeed, less than half, and the actual amount per acre little more than half, as much with as without the ammonia-salts. *Rumex Acetosa* was, on plot 1 as on plot 2, the most prominent, but in only about half the proportion and amount with the ammonia-salts. The other miscellaneous plants of moderate prominence on plot 1 were the same as were so on plot 2, viz. : *Ranunculus repens* and *bulbosus*, *Conopodium denudatum*, *Achillea Millefolium*, and *Plantago lanceolata ;* but each was in considerably less proportion and amount than on plot 2. The general result on this point is, then, that with the increased luxuriance of a few grasses, there is diminished predominance of other grasses, and of Miscellaneous plants, and a still greater diminution of Leguminosæ.

In the subsequent separation-years—that is, after the discontinuance of the dung on both plots, but with the application of the ammonia-salts continued on plot 1—a marked change in the botany of the plot became apparent. As on plot 2, the two previously most prominent grasses, *Poa trivialis* and *Bromus mollis*, declined rapidly, as also did *Dactylis glomerata*. Again, as on plot 2, but in a much more marked degree, *Agrostis vulgaris* and *Holcus lanatus* rapidly increased; as also, though attaining a somewhat less degree of prominence than the foregoing, did *Anthoxanthum odoratum* and *Festuca ovina*, the *Anthoxanthum* attaining a very exceptional degree of prominence for that plant. Of other species, *Alopecurus pratensis*, *Avena pubescens*, and *A. elatior*, increased in a small degree, whilst *Poa pratensis* and *Lolium perenne* fluctuated. The result was, that a larger number maintained a fair position on the ammonia plot after the cessation of the application of the dung.

Leguminosæ, especially *Lathyrus pratensis*, somewhat increased after the first separation-year, but in too small proportion and amount to be attributed with certainty to the changed condition of manuring.

Of Miscellaneæ, upon the whole, a greater number came into the list of those yielding 1 per cent. to the produce after the change. But, under the influence of the continued supply of ammonia-salts, and the declining effect of the dung-residue, *Rumex Acetosa* increased very considerably, and in each year yielded a large proportion of the total miscellaneous produce; whilst on plot 2, without the ammonia-salts, it greatly diminished. *Achillea Millefolium* also upon the whole increased in prominence, and the various species of *Ranunculus* were somewhat favoured. *Cerastium triviale* and *Centaurea nigra*, although in small proportions, also increased. *Conopodium denudatum*, on the other hand, diminished.

The comparison of the botany of the herbage on plot 1 with the ammonia-salts, with that on plot 2 without them, is shown in the last five columns of the table. It is seen that in the first separation-year, that is before the discontinuance of the farmyard manure, the plot with the ammonia-salts showed a large excess of produce of the grasses collectively, a considerable deficiency in that of the Miscellaneæ, and also a deficiency in that of the Leguminosæ.

The grasses which were the most prominently in excess under the influence of the ammonia-salts were *Dactylis glomerata*, *Poa trivialis*, and *Bromus mollis*. *Holcus lanatus* was so in a less degree, and *Festuca ovina* and *Lolium perenne* less still. The grasses in actual deficiency under the influence of the ammonia-salts, and of the increased luxuriance of the foregoing species, were *Alopecurus pratensis*, *Agrostis vulgaris*, and the three *Avenas—pubescens*, *flavescens*, and *elatior*. Thus, the grasses which were the most prominent were for the most part comparatively free-growing and free-feeding species.

The figures relating to the three subsequent separations show a marked change in the relation of the two plots after the cessation of the application of the farmyard manure. On plot 1, with the ammonia, there comes to be—but little excess of

Dactylis glomerata, *Poa trivialis*, or *Bromus mollis;* a very large excess of *Agrostis vulgaris*, *Holcus lanatus*, and *Anthoxanthum odoratum*, and some of *Festuca ovina*, and *Avena elatior;* and an actual deficiency, compared with plot 2, of two of the *Avenas*, *pubescens* and *flavescens*, and of *Lolium perenne*. That is to say, after the discontinuance of the dung, the greater growth of grasses with, than without, the ammonia-salts is chiefly due to a greater prominence, and a greater luxuriance, of the poorer species, *Agrostis vulgaris*, *Holcus lanatus*, *Anthoxanthum odoratum*, and *Festuca ovina*.

Whilst, without the ammonia-salts, Leguminosæ greatly increased in prominence, with them the quantity remained so small that the deficiency compared with plot 2 increased from year to year, the result being mainly due to the relatively deficient growth of *Lathyrus pratensis*.

Of total Miscellaneæ, with a great relative deficiency on plot 1 in the first separation-year, there was subsequently even a slight excess. This was in fact exclusively due to the very much greater growth of *Rumex Acetosa*. Indeed, almost all the other miscellaneous species show either an average or an increasing relative deficiency with the ammonia-salts; the most marked decline, compared with plot 2, being in *Plantago lanceolata*, *Achillea Millefolium*, and *Ranunculus repens* and *R. bulbosus*. We have then, where the ammonia-salts were applied, a greater prominence of individual miscellaneous species, and especially of *Rumex Acetosa*, as the excess of available nitrogen relatively to the available supply of other constituents became the more marked.

Upon the whole, notwithstanding the large unrecovered residue of the manurial constituents remaining in the soil, the approximation in the botanical composition of the herbage to that without manure is very obvious on plot 2 after the cessation of the application of the dung. It is, however, much less so on plot 1 with the ammonia-salts. It is true that, on both plots, some of the better and freer-growing grasses decline, and poorer and more meagrely-growing ones gain in prominence; but these do so in the main in a much greater degree with the partially forced, but at the same time restricted, growth under the influence of the ammonia-salts. That there was, on plot 1, a supply of available nitrogen in excess of that of the available mineral constituents, was quite obvious from the increased predominance of gramineous herbage, with at the same time dark green leafy growth, and relatively less tendency to mature. Under these circumstances, too—and the nitrogen being supplied in a condition in which it is much more rapidly available than the greater part of that in dung, but less rapidly distributing than in nitrate of soda—it is to be expected that the more superficially rooting species should be the more favoured; especially if they are also such as generally prevail under conditions of little competition with more free-growing species.

DETAILED results of Experiments at Rothamsted, illustrating the Domination of one Plant over another in the Mixed Herbage of Permanent Meadow Land.

APPENDIX–TABLE X.—Number of Species, and Percentage of each Species, and of each Group (Gramineæ, Leguminosæ, and Miscellaneæ), in the total Mixed Herbage of each Plot, in each year of complete botanical separation, 1862, 1867, 1872, and 1877.

	Description of Manure	Without Manure, 1856, and since.								Farmyard Manure, 8 years, 1856–63; discontinued 1864, and since.							
										With Ammonia-salts = 43 lbs. Nitrogen.				Alone.			
	Plot Numbers	3				12				1				2			
	Seasons	1862	1867	1872	1877	1862	1867	1872	1877	1862	1867	1872	1877	1862	1867	1872	1877
	GRAMINEOUS HERBAGE.																
	Number of Species	18	15	17	17	16	16	17	16	15	15	18	15	14	17	18	18
	Botanical Names:—																
	1. Anthoxanthum odoratum	4·28	8·66	5·20	5·12	4·09	7·66	6·67	5·47	0·07	1·35	5·78	13·88	0·20	2·86	6·67	7·20
	2. Alopecurus pratensis	4·49	5·82	0·52	0·30	3·12	2·62	2·74	0·96	0·26	1·55	2·93	2·52	3·07	2·48	5·61	3·46
	3. Phleum pratense	0·01	...	...	...	...	...	...	...	0·09	0·03	0·06	...	...	0·05	0·04	0·01
	4. Agrostis vulgaris	11·36	8·63	16·14	13·28	9·43	5·55	11·13	13·29	0·59	6·45	20·77	23·53	2·64	4·94	11·02	17·97
	5. Aira cæspitosa	0·01	...	0·01	0·04	...	...	0·01	...	...	...	0·01	0·01	...	...	0·01	0·01
	6. Holcus lanatus	5·04	7·97	3·60	12·55	4·63	5·57	3·81	10·12	3·88	10·70	14·06	17·55	1·94	10·94	6·85	11·36
	7. Avena elatior	0·07	0·21	0·13	0·05	0·78	0·67	1·84	0·76	0·62	0·64	1·62	1·65	2·04	0·40	0·52	0·15
	8. Avena pubescens	9·65	3·07	3·55	2·69	9·87	5·64	4·85	2·87	0·50	1·58	2·76	2·08	2·56	3·93	10·28	7·68
	9. Avena flavescens	2·37	1·86	3·49	1·08	2·25	1·76	2·38	0·55	3·99	6·85	6·23	1·37	6·02	5·88	11·62	2·92
	10. Poa pratensis	0·29	0·17	0·09	0·07	0·72	0·68	0·72	0·27	1·45	6·56	7·41	1·38	1·66	3·56	2·25	1·20
	11. Poa trivialis	1·54	1·17	0·50	0·56	2·68	1·93	0·91	0·80	31·90	22·32	4·35	2·71	28·18	15·75	3·07	2·37
	12. Briza media	1·89	0·68	6·40	7·25	1·37	1·66	4·22	3·72	...	...	0·01	...	...	0·02	0·17	0·65
	13. Dactylis glomerata	1·76	1·74	0·90	0·70	2·82	3·19	1·90	2·64	16·43	6·43	3·31	4·23	4·51	6·51	3·27	2·79
	14. Cynosurus cristatus	0·17	0·13	1·11	0·98	0·49	0·43	0·96	0·44	0·02	0·07	0·02	0·01	0·01	0·28	0·78	0·98
	15. Festuca ovina	13·30	15·20	21·67	21·89	7·17	11·59	16·05	20·88	0·76	6·17	6·40	10·75	0·37	4·59	9·89	11·23
	16. Festuca pratensis	0·04	...	0·01	0·01	10·11	3·94	2·25	3·27	...	...	0·01	...	...	0·02	0·04	0·34
	17. Festuca elatior	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	18. Festuca loliacea	...	...	...	0·03	...	...	...	...	...	...	...	...	...	...	...	...
	19. Bromus mollis	0·13	0·05	0·01	...	0·08	0·04	0·01	0·01	21·92	10·55	4·55	0·83	17·77	16·39	3·89	0·18
	20. Lolium perenne	6·37	4·03	2·37	4·55	4·46	3·09	1·91	2·28	1·42	3·19	1·71	1·66	1·43	3·63	3·17	4·92
	Gramineæ Determined	62·77	59·39	65·70	71·15	64·07	56·02	62·36	68·33	83·90	84·44	81·99	83·96	72·40	82·23	79·15	75·42
	Undetermined, chiefly Gramineæ	7·84	6·14	2·96	...	8·37	3·00	1·34	...	5·07	1·99	0·22	...	2·65	2·29	0·87	...
	Total	70·61	65·53	68·66	71·15	72·44	59·02	63·70	68·33	88·97	86·43	82·21	83·96	75·05	84·52	80·02	75·42
	LEGUMINOUS HERBAGE.																
	Number of Species	4	4	4	4	4	4	4	4	4	4	3	2	3	4	4	4
	1. Trifolium repens	0·53	0·21	0·38	0·13	0·53	1·65	1·10	0·03	0·03	0·07	0·01	0·01	0·60	0·06	0·52	0·16
	2. Trifolium pratense	4·48	2·11	1·68	2·09	2·08	2·87	1·58	2·07	0·03	0·05	0·01	...	0·34	0·30	0·28	0·96
	3. Trifolium minus	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	4. Trifolium procumbens	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	5. Lotus corniculatus	1·83	2·35	5·94	3·95	1·64	4·26	5·16	3·09	0·01	0·01	...	...	...	0·10	0·17	0·17
	6. Lotus major	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	7. Lathyrus pratensis	1·26	0·68	0·98	2·37	1·95	2·01	2·33	2·29	0·12	0·85	0·32	0·50	0·98	1·17	3·96	5·25
	8. Ononis arvensis	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	9. Vicia Cracca	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	Total	8·10	5·35	8·98	8·54	6·20	10·79	10·17	7·48	0·19	0·98	0·34	0·51	1·92	1·63	4·93	6·54
	MISCELLANEOUS HERBAGE.																
	Number of Species	28	24	28	31	24	30	29	31	9	15	15	17	13	20	25	30
Ranunculaceæ	1. Ranunculus acris	0·01	0·01	0·07	...	...	0·35	0·04	0·09	...	0·26	0·14	1·30	...	0·02	1·02	1·49
	2 and 3. Ranunculus repens et bulbosus	4·88	2·01	2·94	3·45	2·62	1·72	3·22	6·45	1·53	0·37	0·58	1·28	2·89	0·99	1·53	3·58
	4. Ranunculus auricomus	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Caryophylleæ	5. Stellaria graminea	0·01	0·02	0·02	0·02	0·07	0·42	0·90	0·94	...	...	0·01	...	0·04	0·01	0·08	0·05
	6. Stellaria Holostea	...	...	...	...	...	...	...	0·01	...	...	...	...	...	...	...	...
	7. Cerastium triviale	0·46	0·37	0·85	0·82	0·39	1·02	3·44	0·81	...	0·11	0·24	0·47	0·01	0·05	1·22	0·40
Hypericineæ	8. Hypericum perforatum	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Rosaceæ	9. Potentilla reptans	...	...	0·01	0·01	0·01	0·01	0·01	0·02	0·02	...	...	0·01	...	...	0·01	0·01
	10. Potentilla Fragariastrum	...	...	...	...	...	0·01	0·01	0·01	...	...	...	...	...	...	...	...
	11. Agrimonia Eupatoria	0·01	...	0·02	0·01	...	0·01	0·01	...	...	...	...	...	...	...	...	0·09
	12. Poterium Sanguisorba	...	0·21	0·49	0·88	...	...	...	...	...	...	...	...	...	...	...	...
	13. Spiræa Ulmaria	...	...	...	...	...	...	...	...	...	...	...	0·02	...	...	0·09	...
Umbelliferæ	14. Conopodium denudatum	0·97	2·95	2·85	1·90	1·58	5·42	2·17	2·84	1·46	1·59	0·55	0·56	2·83	2·85	1·15	0·96
	15. Pimpinella Saxifraga	1·53	3·44	0·86	0·79	0·99	1·20	0·44	0·49	0·07	0·03	0·03	0·02	0·36	0·48	0·17	0·27
	16. Heracleum Sphondylium	...	...	0·02	...	...	...	...	...	...	...	...	...	...	...	...	0·04
	17. Anthriscus sylvestris	...	...	...	...	...	...	...	...	...	0·26	...	...	...	0·02	0·02	...
Rubiaceæ	18. Galium verum	...	...	...	...	0·01	0·11	0·01	0·01	...	...	...	...	0·01	0·02	0·06	0·04
	19. Galium Aparine	...	...	0·01	...	...	...	...	...	...	...	...	...	...	...	...	...
Dipsaceæ	20. Scabiosa arvensis	0·01	...	0·02	0·12	0·05	0·01	0·09	0·06	...	...	...	...	...	...	0·01	0·10
Compositæ	21. Centaurea nigra	0·31	0·59	2·11	1·06	1·20	0·94	3·35	2·47	...	0·18	1·24	0·25	0·04	0·14	1·28	0·90
	22. Carduus arvensis	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	23. Bellis perennis	0·01	0·04	0·01	0·02	...	...	...	0·01	...	...	...	...	...	0·01	0·01	0·01
	24. Achillea Millefolium	1·53	1·16	1·78	1·99	1·13	1·53	3·77	2·81	1·35	3·07	5·01	0·72	2·41	1·09	2·93	2·17
	25. Chrysanthemum Leucanthemum	...	...	...	0·12	...	...	...	...	...	...	...	...	...	...	...	...
	26. Tragopogon pratensis	0·01	...	0·01	0·01	0·30	0·36	0·33	0·01	...	...	...	...	...	...	0·02	0·02
	27. Leontodon hispidus	0·06	0·64	1·27	1·32	0·11	0·10	0·13	0·07	...	...	...	...	...	0·02	0·02	0·04
	28. Leontodon autumnalis	...	...	...	...	...	...	...	0·01	...	...	...	...	...	...	...	...
	29. Taraxacum officinale	0·05	0·16	0·15	0·06	0·12	0·13	0·01	0·14	0·02	0·17	0·13	0·03	...	0·14	0·24	0·25
	30. Hieracium Pilosella	0·01	0·01	0·01	0·04	...	0·01	0·02	0·08	...	...	...	...	...	...	...	...
Plantagineæ	31. Plantago lanceolata	7·34	10·73	2·66	3·16	7·74	8·25	0·41	1·43	0·30	0·51	0·14	0·27	1·67	3·08	1·45	3·70
Scrophularineæ	32. Veronica Chamædrys	0·48	0·32	0·14	0·24	0·39	0·46	0·23	0·57	...	0·28	0·03	0·01	0·28	1·66	1·25	0·72
	33. Veronica serpyllifolia	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Labiatæ	34. Prunella vulgaris	0·01	0·03	0·01	0·09	0·03	0·12	0·01	0·04	...	...	...	...	...	0·01	...	0·02
	35. Thymus Serpyllum	...	...	...	0·09	...	...	...	...	...	...	...	...	...	...	...	...
	36. Ajuga reptans	0·01	0·01	...	0·02	...	0·02	0·01	0·01	...	0·01	...	0·02	...	...	0·01	0·03
Primulaceæ	37. Primula veris	0·01	...	...	0·01	0·03	0·03	0·03	0·03	...	...	...	...	...	...	...	0·04
Polygonaceæ	38. Rumex Acetosa	1·40	1·76	1·77	1·87	3·02	3·61	3·52	2·06	6·09	5·74	9·29	10·49	12·44	3·07	1·93	2·46
	39. Rumex crispus	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Liliaceæ	40. Scilla nutans	...	...	...	0·01	...	...	...	...	...	...	...	0·01	...	...	...	0·01
	41. Ornithogalum umbellatum	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Juncaceæ	42. Luzula campestris	1·91	3·61	3·42	1·77	1·14	2·96	2·98	1·50	...	0·01	0·05	0·07	0·05	0·18	0·53	0·62
Cyperaceæ	43. Carex præcox	0·18	1·02	0·84	0·37	0·31	1·35	0·90	0·68	...	...	0·01	...	...	0·01	0·01	0·01
Filices	44. Ophioglossum vulgatum	0·01	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Musci	45. Hypnum squarrosum	0·06		0·01		0·10		0·07		...	...	...	...	...	...	...	
	46. Hypnum rutabulum	0·01	0·03	0·01	0·06	0·01	0·04	0·01	0·54	...	...	...	...	...	...	0·01	0·01
	47. Hypnum hians	0·01		...		0·01		0·01		...	...	...	...	...	...	...	
	Total	21·29	29·12	22·36	20·31	21·36	30·19	26·13	24·19	10·84	12·59	17·45	15·53	23·03	13·85	15·05	18·04

	Description of Manure	Superphosphate of Lime each year, 1859, and since.								Ammonia-salts alone, = 86 lbs. Nitrogen.				"Mixed Mineral Manure"—Sulphates of Potass, Soda, and Magnesia, and Superphosphate of Lime.			
		Alone.				With Ammonia-salts = 86 lbs. Nitrogen.								1869 and since; previously (13 years, 1856–68) Ammonia-salts.			
	Plot Numbers	4–1				4–2				5				6			
	Seasons	1862	1867	1872	1877	1862	1867	1872	1877	1862	1867	1872	1877	1862	1867	1872	1877
	GRAMINEOUS HERBAGE.																
	Number of Species	16	15	16	16	14	14	15	13	17	15	15	13	15	14	17	14
	1. Anthoxanthum odoratum	3·66	7·16	4·74	5·11	2·24	5·52	1·47	2·36	5·77	5·51	3·04	4·09	3·92	4·31	6·22	4·89
	2. Alopecurus pratensis	1·32	1·84	0·86	1·40	0·66	14·75	3·94	1·58	0·65	0·47	0·83	0·23	1·70	0·02	0·03	0·09
	3. Phleum pratense	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	4. Agrostis vulgaris	7·21	6·08	13·88	9·87	19·38	14·00	20·59	24·39	24·30	20·97	26·62	29·46	21·43	14·41	23·37	8·58
	5. Aira cæspitosa	...	...	...	...	...	...	0·04	...	0·01	0·01	...	...	...	...	0·01	0·01
	6. Holcus lanatus	11·82	9·16	4·71	19·35	16·21	10·53	2·03	6·03	10·08	5·15	1·90	3·01	8·17	3·07	5·31	14·89
	7. Avena elatior	0·13	0·18	0·15	0·05	2·46	0·41	2·48	1·02	3·93	2·78	1·49	0·23	3·44	6·50	3·60	2·77
	8. Avena pubescens	9·42	4·97	4·09	4·02	7·38	3·94	0·28	0·03	7·31	0·63	0·24	0·12	14·54	0·90	1·83	1·67
	9. Avena flavescens	4·12	4·28	6·09	2·47	2·09	0·41	0·09	0·03	0·65	0·46	0·16	0·01	1·18	0·24	1·49	0·48
	10. Poa pratensis	0·56	0·26	0·45	0·02	0·67	3·87	5·11	1·56	1·07	0·65	0·61	0·23	2·28	1·65	2·42	1·73
	11. Poa trivialis	5·16	5·65	3·79	4·72	8·14	2·15	2·10	0·31	0·89	0·30	0·59	0·01	1·53	0·40	0·98	0·63
	12. Briza media	0·58	0·32	2·12	2·16	...	...	0·01	...	0·06	...	0·01	0·02	...	...	0·01	...
	13. Dactylis glomerata	2·25	0·99	0·57	1·41	2·28	0·38	0·16	1·83	2·39	1·39	0·70	3·25	2·05	1·71	1·28	4·09
	14. Cynosurus cristatus	0·34	0·84	1·03	0·80	...	...	...	...	0·01	0·01	0·01	...	0·01	0·03	0·01	...
	15. Festuca ovina	10·20	16·75	20·44	16·02	6·80	26·09	49·29	55·20	21·99	30·57	46·56	53·31	13·33	25·93	31·15	38·02
	16. Festuca pratensis	0·13	...	0·08	0·02	0·46	0·01	...	...	0·35	0·13	0·02	...	0·44	0·01	0·01	0·14
	17. Festuca elatior	...	...	...	...	...	...	...	0·08	...	...	...	...	...	...	...	...
	18. Festuca loliacea	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	19. Bromus mollis	0·52	0·43	0·09	0·01	0·33	0·20	0·02	...	0·08	...	...	...	0·15	...	0·01	...
	20. Lolium perenne	9·28	5·24	3·12	4·35	6·47	1·36	0·70	0·21	3·33	1·21	0·97	0·09	4·58	1·39	0·69	1·97
	Gramineæ Determined	66·70	64·15	66·21	71·78	75·57	83·62	88·31	94·63	82·87	70·24	83·75	94·06	78·75	60·57	78·42	79·96
	Undetermined, chiefly Gramineæ	8·26	2·73	0·82	...	4·74	2·51	0·34	...	3·45	1·61	0·95	...	1·77	2·09	0·81	...
	Total	74·96	66·88	67·03	71·78	80·31	86·13	88·65	94·63	86·32	71·85	84·70	94·06	80·52	62·66	79·23	79·96
	LEGUMINOUS HERBAGE.																
	Number of Species	4	4	5	5	3	3	2	2	4	4	3	2	4	3	4	4
	1. Trifolium repens	0·61	0·09	0·48	0·35	0·01	0·01	...	0·01	0·01	0·01	0·01	...	0·01	0·01	0·01	0·01
	2. Trifolium pratense	1·45	0·17	0·11	0·30	0·01	...	0·01	...	0·04	0·01	...	...	0·03	0·01	0·04	0·08
	3. Trifolium minus	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	4. Trifolium procumbens	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	5. Lotus corniculatus	0·41	1·23	3·71	0·86	...	...	...	...	0·05	0·31	0·41	0·14	0·01	0·08	0·06	0·03
	6. Lotus major	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	7. Lathyrus pratensis	0·32	1·34	4·19	3·38	0·07	0·03	0·02	0·03	0·02	0·01	0·04	0·05	0·23	...	1·47	6·56
	8. Ononis arvensis	...	...	0·12	0·64	...	...	...	...	...	...	...	...	...	...	...	...
	9. Vicia Cracca	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	Total	2·79	2·83	8·61	5·53	0·09	0·04	0·03	0·04	0·12	0·34	0·46	0·19	0·28	0·10	1·58	6·68
	MISCELLANEOUS HERBAGE.																
	Number of Species	26	27	28	24	18	13	11	11	17	17	13	14	15	15	18	20
Ranunculaceæ	1. Ranunculus acris	0·03	0·10	0·12	0·26	0·01	0·01	...	...	0·01	0·02	0·03	0·07	...	...	0·35	0·34
	2 and 3. Ranunculus repens et bulbosus	5·87	1·29	4·16	5·84	2·14	0·04	0·01	...	0·32	0·08	0·08	0·09	0·77	0·16	0·12	0·23
	4. Ranunculus auricomus	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Caryophylleæ	5. Stellaria graminea	0·03	0·17	0·07	0·15	0·01	0·02	...	...	0·20	0·09	...	...	...	0·03	0·01	0·01
	6. Stellaria Holostea	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	7. Cerastium triviale	0·43	0·97	0·85	0·74	0·01	0·01	0·01	...	0·01	0·09	...	...	0·07	0·04	0·02	0·01
Hypericineæ	8. Hypericum perforatum	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Rosaceæ	9. Potentilla reptans	0·01	...	...	...	...	...	...	...	...	...	...	...	...	0·01	0·01	0·01
	10. Potentilla Fragariastrum	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	11. Agrimonia Eupatoria	0·02	0·01	0·01	...	...	...	...	...	...	...	...	...	...	...	...	...
	12. Poterium Sanguisorba	0·01	0·75	0·09	0·21	...	...	...	...	...	...	...	...	...	...	...	...
	13. Spiræa Ulmaria	...	...	...	...	...	...	...	...	...	...	...	0·09	...	...	...	...
Umbelliferæ	14. Conopodium denudatum	0·97	2·32	1·17	0·64	1·29	2·65	0·39	0·11	1·15	5·74	1·02	0·65	0·55	7·87	2·47	0·88
	15. Pimpinella Saxifraga	0·87	3·21	1·28	1·48	0·60	0·39	0·36	0·01	0·10	0·77	0·15	0·01	0·06	0·61	0·26	0·13
	16. Heracleum Sphondylium	...	...	0·02	...	0·03	...	...	0·24	...	...	0·01	0·05	...	...	0·15	0·16
	17. Anthriscus sylvestris	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Rubiaceæ	18. Galium verum	...	...	...	0·01	0·09	0·50	0·66	0·68	...	0·61	2·40	0·22	0·97	0·33	0·90	0·16
	19. Galium Aparine	...	...	0·01	...	...	...	...	0·01	0·01	...	...	0·01	...	...	...	0·01
Dipsaceæ	20. Scabiosa arvensis	...	...	0·01	...	...	...	0·01	0·05	0·03	0·28	0·62	1·67	0·19	0·67	0·04	0·02
Compositæ	21. Centaurea nigra	0·43	0·36	1·01	0·66	0·01	0·21	1·25	0·85	0·01	2·43	2·18	0·53	...	1·40	1·41	0·44
	22. Carduus arvensis	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	23. Bellis perennis	0·01	0·05	0·04	0·02	...	...	...	...	...	...	...	...	...	...	...	...
	24. Achillea Millefolium	1·42	1·88	5·38	3·19	1·77	1·49	1·75	0·27	1·33	1·09	1·05	0·16	3·34	1·08	4·09	1·72
	25. Chrysanthemum Leucanthemum	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	26. Tragopogon pratensis	...	0·01	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	27. Leontodon hispidus	0·59	0·62	0·11	0·92	...	...	...	...	...	...	...	...	...	...	...	...
	28. Leontodon autumnalis	...	0·01	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	29. Taraxacum officinale	0·34	0·13	0·07	...	0·05	0·01	...	...	0·01	0·01	0·02	...	0·02	...	0·02	...
	30. Hieracium Pilosella	0·01	0·01	0·01	0·01	...	...	...	...	...	...	...	...	...	...	...	...
Plantagineæ	31. Plantago lanceolata	5·63	9·66	3·13	3·78	0·04	...	...	0·01	0·10	0·02	...	...	0·06	...	...	0·01
Scrophularineæ	32. Veronica Chamædrys	0·29	0·59	0·23	0·32	0·01	...	...	...	0·01	0·01	...	...	0·07	0·04	0·01	0·01
	33. Veronica serpyllifolia	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Labiatæ	34. Prunella vulgaris	0·03	0·04	0·01	0·02	...	...	...	...	...	...	...	...	...	...	...	...
	35. Thymus Serpyllum	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	36. Ajuga reptans	...	0·01	0·01	0·01	...	...	...	...	...	...	...	...	...	...	...	...
Primulaceæ	37. Primula veris	0·01	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Polygonaceæ	38. Rumex Acetosa	3·94	5·47	2·81	3·37	13·39	8·42	6·85	3·09	9·15	15·94	7·13	2·13	12·11	24·27	7·51	7·66
	39. Rumex crispus	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Liliaceæ	40. Scilla nutans	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	41. Ornithogalum umbellatum	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Juncaceæ	42. Luzula campestris	1·22	2·41	3·57	0·84	0·13	0·08	0·03	0·01	1·11	0·62	0·15	0·06	0·97	0·72	1·81	1·54
Cyperaceæ	43. Carex præcox	0·06	0·20	0·14	0·07	0·01	...	...	...	0·01	0·01	...	...	0·01	0·01	...	...
Filices	44. Ophioglossum vulgatum	0·01	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Musci	45. Hypnum squarrosum	0·01		0·03		0·01	...	...	...	...	...	...	0·01	0·01	...	0·01	0·01
	46. Hypnum rutabulum	0·01	0·02	0·01	0·15	...	...	...	...	...	...	...	...	...	...	...	0·01
	47. Hypnum hians	...		0·01		...	...	...	...	...	...	...	...	...	...	...	...
	Total	22·25	30·29	24·36	22·69	19·60	13·83	11·32	5·33	13·56	27·81	14·84	5·75	19·20	37·24	19·19	13·36

	Description of Manure	"Mixed Mineral Manure"—Sulphates of Potass, Soda, and Magnesia, and Superphosphate of Lime.															
		Alone, each year, 1856 and since.				With Potass, 6 years, 1856–61; without Potass, 1862 and since.				With Ammonia-salts = 86 lbs. Nitrogen.				Mineral Manures as Plot 8, and Ammonia-salts = 86 lbs. Nitrogen.			
	Plot Numbers	7				8				9				10			
	Seasons	1862	1867	1872	1877	1862	1867	1872	1877	1862	1867	1872	1877	1862	1867	1872	1877
	GRAMINEOUS HERBAGE.																
	Number of Species	18	16	17	17	17	17	17	16	13	14	16	13	16	15	15	15
	1. Anthoxanthum odoratum	3·06	3·93	2·72	3·18	3·72	6·98	7·94	7·55	1·24	3·59	2·25	2·94	1·46	5·27	3·26	5·84
	2. Alopecurus pratensis	0·34	0·88	1·17	0·48	0·39	0·88	0·52	0·87	0·27	0·07	2·76	0·96	2·05	2·99	10·35	16·51
	3. Phleum pratense	0·01	...	...	...	...	...	...	...	...	...	...	...	0·06	...	...	...
	4. Agrostis vulgaris	7·14	5·69	11·72	12·02	10·01	4·32	9·32	12·40	12·81	13·43	15·46	12·23	9·37	8·61	14·17	16·29
	5. Aira cæspitosa	0·01	...	0·01	0·01	0·01	0·01	0·01	...	...	...	0·01	...	...	...	0·01	0·01
	6. Holcus lanatus	5·06	11·81	3·16	13·16	4·51	10·25	4·61	18·22	12·14	9·84	7·61	10·37	9·50	8·24	4·37	4·74
	7. Avena elatior	2·41	0·06	0·46	1·29	4·52	3·16	4·40	3·17	...	2·50	11·40	13·23	0·08	11·71	13·22	9·58
	8. Avena pubescens	13·81	3·90	2·36	2·25	12·68	3·44	3·66	1·67	10·22	1·41	0·49	0·07	10·64	1·64	0·44	0·24
	9. Avena flavescens	4·02	4·84	3·72	3·65	5·42	3·52	6·94	2·45	9·08	3·78	5·30	0·67	10·05	2·00	0·77	0·18
	10. Poa pratensis	1·13	1·05	2·27	1·75	1·72	1·50	2·11	1·03	10·68	13·02	22·67	18·03	4·06	14·81	19·62	6·48
	11. Poa trivialis	3·81	4·38	2·30	2·11	5·48	3·48	1·62	3·20	8·72	2·14	0·64	0·11	10·18	2·78	1·21	0·47
	12. Briza media	0·03	0·06	0·10	0·14	0·07	0·08	1·16	0·57	...	...	0·01	...	...	...	...	...
	13. Dactylis glomerata	2·57	4·67	1·68	3·67	3·50	1·48	0·66	0·98	5·58	4·64	11·88	14·07	12·51	5·44	3·12	4·88
	14. Cynosurus cristatus	0·08	0·22	0·09	0·10	0·25	0·24	1·02	1·14	...	...	...	0·01	0·01	0·01	...	...
	15. Festuca ovina	13·73	11·38	14·86	26·59	7·51	17·74	23·95	19·76	5·21	18·42	8·68	21·80	4·08	14·74	19·80	26·34
	16. Festuca pratensis	0·17	0·64	0·11	0·95	2·21	0·42	0·33	0·54	1·42	0·07	0·03	...	1·49	0·25	0·09	0·14
	17. Festuca elatior	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	18. Festuca loliacea	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	19. Bromus mollis	1·26	0·98	0·04	0·01	1·38	0·43	0·09	0·01	4·46	0·11	0·10	...	2·53	0·69	1·74	1·55
	20. Lolium perenne	3·12	2·40	0·59	3·02	5·92	2·61	1·92	7·63	4·20	1·01	1·11	0·16	3·02	1·84	0·57	0·17
	Gramineæ Determined	61·76	56·89	47·36	74·38	69·30	60·54	70·26	81·19	86·03	74·03	90·40	94·65	81·09	81·02	92·74	93·42
	Undetermined, chiefly Gramineæ	2·89	2·40	1·46	...	2·39	2·49	1·30	...	2·56	3·03	1·79	...	4·36	1·61	1·94	...
	Total	64·65	59·29	48·82	74·38	71·69	63·03	71·56	81·19	88·59	77·06	92·19	94·65	85·45	82·63	94·68	93·42
	LEGUMINOUS HERBAGE.																
	Number of Species	4	4	4	4	4	4	4	4	2	2	1	4	2	1	2	2
	1. Trifolium repens	3·08	0·47	1·77	0·01	2·70	0·10	0·25	0·10	0·01	0·01	...	...	0·01	...	0·01	...
	2. Trifolium pratense	6·84	4·75	1·13	1·55	7·71	1·13	0·27	0·36	...	...	...	0·01	...	...	...	0·01
	3. Trifolium minus	...	...	...	...	...	...	...	...	...	...	...	0·01	...	...	...	...
	4. Trifolium procumbens	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	5. Lotus corniculatus	1·27	0·69	0·19	0·04	0·15	0·83	3·51	1·18	...	...	...	...	...	...	...	0·01
	6. Lotus major	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	7. Lathyrus pratensis	13·51	6·78	36·68	12·11	8·76	6·82	3·94	2·37	0·12	0·15	0·02	0·06	0·11	0·08	0·01	...
	8. Ononis arvensis	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	9. Vicia Cracca	...	...	...	...	...	...	...	...	...	...	...	0·33	...	...	...	...
	Total	24·70	12·69	39·77	13·71	19·32	8·88	7·97	4·01	0·13	0·16	0·02	0·41	0·12	0·08	0·02	0·02
	MISCELLANEOUS HERBAGE.																
	Number of Species	22	22	20	23	17	21	21	28	13	13	13	10	13	11	6	11
Ranunculaceæ	1. Ranunculus acris	0·08	0·12	0·19	0·37	...	0·66	0·47	1·09	...	...	...	...	0·01	...	...	...
	2 and 3. Ranunculus repens et bulbosus	1·31	0·38	0·21	0·32	1·16	0·55	1·00	0·65	0·14	0·05	...	0·04	0·36	0·03	...	...
	4. Ranunculus auricomus	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Caryophylleæ	5. Stellaria graminea	...	0·01	0·01	0·01	0·02	0·09	0·04	0·02	...	0·03	...	...	...	...	...	...
	6. Stellaria Holostea	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	7. Cerastium triviale	0·05	0·02	0·01	0·01	0·08	0·11	0·33	0·17	...	...	...	...	0·01	...	...	...
Hypericineæ	8. Hypericum perforatum	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Rosaceæ	9. Potentilla reptans	...	...	0·01	...	...	0·01	...	...	0·05	...	0·01	0·10	0·01	0·01	...	0·11
	10. Potentilla Fragariastrum	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	11. Agrimonia Eupatoria	...	0·02	...	...	...	0·01	0·01	0·01	...	...	...	...	...	...	...	...
	12. Poterium Sanguisorba	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	13. Spiræa Ulmaria	...	0·25	0·06	0·17	...	...	...	0·02	...	...	0·01	0·02	0·49	0·07	...	0·04
Umbelliferæ	14. Conopodium denudatum	2·03	9·22	1·31	1·58	1·59	6·84	1·73	1·07	2·92	9·35	1·51	0·75	1·77	2·49	0·06	0·06
	15. Pimpinella Saxifraga	0·86	1·20	0·22	0·28	0·81	1·91	1·05	0·67	0·01	0·01	0·01	...	0·17	0·12	0·01	0·01
	16. Heracleum Sphondylium	...	0·21	0·02	0·57	...	...	...	0·22	...	...	0·06	0·15	...	...	...	...
	17. Anthriscus sylvestris	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Rubiaceæ	18. Galium verum	...	...	...	0·01	0·03	0·40	0·36	0·03	...	...	...	0·06	...	...	...	...
	19. Galium Aparine	...	...	...	...	...	...	...	0·01	...	...	...	...	...	...	...	...
Dipsaceæ	20. Scabiosa arvensis	0·02	...	...	0·06	0·06	0·01	0·03	0·47	...	0·33	0·03	0·17	...	...	...	0·15
Compositæ	21. Centaurea nigra	0·03	0·79	0·27	0·10	0·22	0·45	0·24	0·76	0·04	...	0·02	...	...	...	0·20	0·14
	22. Carduus arvensis	0·11	0·79	0·26	0·10	...	...	...	...	...	...	...	...	...	...	...	...
	23. Bellis perennis	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	24. Achillea Millefolium	1·69	3·10	5·23	0·64	0·93	4·89	9·75	2·76	1·95	2·03	1·50	0·04	0·87	1·91	0·80	0·03
	25. Chrysanthemum Leucanthemum	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	26. Tragopogon pratensis	...	0·20	0·13	...	...	0·01	...	0·01	0·72	...	...	...	...	...	...	...
	27. Leontodon hispidus	...	0·01	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	28. Leontodon autumnalis	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	29. Taraxacum officinale	0·16	0·10	...	...	0·03	0·10	0·05	0·01	...	0·01	...	...	0·31	0·01	...	...
	30. Hieracium Pilosella	0·01	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Plantagineæ	31. Plantago lanceolata	0·23	1·10	0·07	0·09	0·71	1·53	0·34	0·26	0·01	0·01	...	...	...	...	...	0·01
Scrophularineæ	32. Veronica Chamædrys	0·49	0·42	0·59	0·38	0·60	0·86	0·36	0·14	0·01	0·01	...	...	...	...	...	...
	33. Veronica serpyllifolia	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Labiatæ	34. Prunella vulgaris	...	...	...	0·01	...	...	...	0·05	...	...	...	...	...	...	...	...
	35. Thymus Serpyllum	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	36. Ajuga reptans	0·01	...	...	...	...	...	...	0·02	...	...	...	...	...	...	...	...
Primulaceæ	37. Primula veris	...	...	...	0·01	...	...	...	0·01	...	...	...	...	...	...	...	...
Polygonaceæ	38. Rumex Acetosa	2·10	8·88	1·16	6·67	1·93	7·86	1·96	5·84	5·40	10·89	4·60	3·60	10·38	12·61	4·22	5·99
	39. Rumex crispus	...	...	...	...	...	...	...	...	0·01	...	...	...	...	...	...	...
Liliaceæ	40. Scilla nutans	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	41. Ornithogalum umbellatum	...	...	...	...	...	...	...	...	...	...	0·01	...	...	...	...	...
Juncaceæ	42. Luzula campestris	1·41	1·17	1·64	0·50	0·75	1·76	2·66	0·48	0·02	0·05	0·01	...	0·04	0·03	0·01	0·01
Cyperaceæ	43. Carex præcox	0·02	0·02	0·01	0·01	0·06	0·03	0·06	0·02	...	0·01	0·01	...	0·01	0·01	...	...
Filices	44. Ophioglossum vulgatum	0·01	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Musci	45. Hypnum squarrosum	0·01	...	0·01	0·01	0·01	...	0·01		...	...	0·01	0·01	...	...	...	0·01
	46. Hypnum rutabulum	0·01	0·01	...	0·01	...	0·01	0·01	0·01	...	...	...	...	...	...	...	...
	47. Hypnum hians	0·01	...	...	...	...	...	0·01		...	...	...	...	...	...	...	...
	Total	10·65	28·02	11·41	11·91	8·99	28·09	20·47	14·80	11·28	22·78	7·79	4·94	14·43	17·29	5·30	6·56

	Description of Manure	"Mixed Mineral Manure"—Sulphates of Potass, Soda, and Magnesia, and Superphosphate of Lime.															
		With Ammonia-salts = 172 lbs. Nitrogen; only 86 lbs. in 1859-60-61.				As Plot 11—1 and Silicates in addition, 1862, and since.				With Ammonia-salts = 86 lbs. Nitrogen and Cut Wheat Straw.							
	Plot Numbers	11–1				11–2				13–1				13–2			
	Seasons	1862	1867	1872	1877	1862	1867	1872	1877	1862	1867	1872	1877	1862	1867	1872	1877
	GRAMINEOUS HERBAGE.																
	Number of Species	15	13	11	11	14	14	13	11	14	16	15	14	14	14	15	14
	1. Anthoxanthum odoratum	0·09	0·06	0·78	0·19	1·00	0·10	0·16	0·17	0·61	2·44	0·48	0·43	0·77	1·33	0·36	0·23
	2. Alopecurus pratensis	2·80	13·11	12·35	9·91	1·50	6·33	22·65	20·11	4·73	4·49	9·81	9·57	2·00	4·57	1·97	3·99
	3. Phleum pratense	...	...	...	...	...	...	0·02	...	...	0·05	...	...	...	...	0·17	0·19
	4. Agrostis vulgaris	13·17	19·27	13·56	29·20	18·81	24·16	10·16	17·09	15·21	16·72	15·78	13·27	8·83	23·97	11·10	13·53
	5. Aira cæspitosa	0·01	...	...	...	...	...	...	0·01	...	0·01	...	...	...	...	...	...
	6. Holcus lanatus	9·92	2·86	10·33	20·29	7·37	4·79	10·59	19·48	4·26	6·35	1·94	4·81	6·44	3·05	3·49	4·34
	7. Avena elatior	0·77	4·55	10·41	14·56	6·40	4·83	12·73	21·14	0·68	3·65	8·44	10·47	0·16	1·32	9·91	11·70
	8. Avena pubescens	1·66	0·01	...	...	0·76	0·01	...	...	9·14	0·81	0·16	0·03	1·82	0·38	0·13	0·01
	9. Avena flavescens	5·28	0·46	0·09	0·01	3·42	1·75	0·11	0·02	4·20	4·34	1·38	0·37	5·48	4·20	2·07	0·76
	10. Poa pratensis	9·43	12·86	10·40	1·47	5·12	10·38	12·43	4·50	5·73	13·69	16·30	11·92	2·08	6·81	6·59	8·26
	11. Poa trivialis	13·25	0·14	0·09	0·33	17·04	1·52	0·81	0·01	10·09	4·16	1·02	0·77	13·58	9·21	3·20	0·51
	12. Briza media	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	13. Dactylis glomerata	24·16	39·31	39·28	17·11	23·34	38·30	27·23	13·38	23·40	16·29	31·26	37·04	32·36	24·28	54·84	44·46
	14. Cynosurus cristatus	...	...	...	...	...	...	...	...	...	...	0·01	...	...	...	...	...
	15. Festuca ovina	1·46	0·50	0·38	4·15	0·66	2·05	0·30	2·55	2·68	9·59	5·96	4·68	1·86	4·05	1·23	2·47
	16. Festuca pratensis	1·90	...	...	...	2·28	0·06	...	...	3·20	0·53	0·02	0·01	4·56	0·61	0·07	0·04
	17. Festuca elatior	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	18. Festuca loliacea	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	19. Bromus mollis	1·39	0·04	0·01	...	0·69	0·07	0·01	...	0·56	0·09	0·01	0·01	0·70	0·01	0·01	...
	20. Lolium perenne	1·37	0·08	...	0·01	1·46	0·38	0·16	...	1·73	1·24	0·08	0·15	3·78	1·18	0·23	0·07
	Gramineæ Determined	86·66	93·25	97·68	97·53	89·85	94·73	97·66	98·46	86·22	84·45	92·65	93·53	84·42	84·97	95·37	90·56
	Undetermined, chiefly Gramineæ	2·72	0·87	1·16	...	4·39	0·94	1·60	...	3·47	1·48	1·46	...	6·77	1·74	1·30	...
	Total	89·38	94·12	98·84	97·53	94·24	95·67	99·26	98·46	89·69	85·93	94·11	93·53	91·19	86·71	96·67	90·56
	LEGUMINOUS HERBAGE.																
	Number of Species	1	1	1	0	0	0	0	0	3	2	3	0	2	2	3	0
	1. Trifolium repens	...	...	0·01	...	...	...	...	...	0·04	0·04	0·01	...	0·03	0·02	...	...
	2. Trifolium pratense	...	...	...	...	...	...	...	...	0·05	...	0·01	...	...	...	0·01	...
	3. Trifolium minus	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	4. Trifolium procumbens	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	5. Lotus corniculatus	...	0·01	...	...	...	...	...	...	...	...	...	...	...	...	0·01	...
	6. Lotus major	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	7. Lathyrus pratensis	0·01	...	...	...	...	...	...	...	0·44	0·13	0·59	...	0·03	0·05	0·01	...
	8. Ononis arvensis	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	9. Vicia Cracca	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	Total	0·01	0·01	0·01	...	...	...	...	...	0·53	0·17	0·61	...	0·06	0·07	0·03	...
	MISCELLANEOUS HERBAGE.																
	Number of Species	12	4	4	4	7	5	3	5	14	9	12	8	12	6	8	8
Ranunculaceæ	1. Ranunculus acris	...	...	...	...	...	...	...	...	0·05	...	...	...	...	...	...	...
	2 and 3. Ranunculus repens et bulbosus	0·03	...	...	...	0·02	...	...	...	0·08	0·01	...	...	0·01	...	0·01	...
	4. Ranunculus auricomus	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Caryophylleæ	5. Stellaria graminea	...	...	...	...	...	...	...	...	0·04	0·09	0·01	0·01	0·02	0·05	...	...
	6. Stellaria Holostea	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	7. Cerastium triviale	...	...	...	...	...	...	...	...	0·07	...	...	...	0·01	...	...	...
Hypericineæ	8. Hypericum perforatum	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Rosaceæ	9. Potentilla reptans	...	...	...	...	...	...	...	...	0·01	0·01	...	...	...	...	...	...
	10. Potentilla Fragariastrum	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	11. Agrimonia Eupatoria	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	12. Poterium Sanguisorba	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	13. Spiræa Ulmaria	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Umbelliferæ	14. Conopodium denudatum	1·79	1·84	0·04	0·01	1·19	0·70	0·03	0·01	2·12	8·17	0·66	1·25	1·87	7·91	0·67	0·79
	15. Pimpinella Saxifraga	0·06	...	...	...	0·01	...	...	...	0·07	0·02	0·01	...	0·01	0·01	...	0·01
	16. Heracleum Sphondylium	...	...	...	0·20	...	...	0·01	0·18	...	0·07	0·08	0·04	...	...	0·27	0·01
	17. Anthriscus sylvestris	...	...	...	...	...	...	...	...	...	...	...	...	...	...	0·03	...
Rubiaceæ	18. Galium verum	...	...	...	...	...	0·04	...	...	...	...	0·01	...	...	...	...	...
	19. Galium Aparine	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Dipsaceæ	20. Scabiosa arvensis	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Compositæ	21. Centaurea nigra	0·07	...	...	...	...	...	...	...	...	...	...	...	0·01	...	...	...
	22. Carduus arvensis	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	23. Bellis perennis	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	24. Achillea Millefolium	1·45	0·06	...	0·01	0·41	0·01	...	...	1·93	1·20	1·12	0·01	0·75	1·72	0·16	0·01
	25. Chrysanthemum Leucanthemum	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	26. Tragopogon pratensis	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	27. Leontodon hispidus	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	28. Leontodon autumnalis	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	29. Taraxacum officinale	0·15	...	...	...	0·04	...	...	...	0·06	...	...	...	0·08	...	...	0·03
	30. Hieracium Pilosella	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Plantagineæ	31. Plantago lanceolata	...	...	...	...	...	...	...	...	0·08	...	...	...	0·10	...	...	...
Scrophularineæ	32. Veronica Chamædrys	...	...	...	...	...	...	...	...	...	...	0·01	...	...	...	...	...
	33. Veronica serpyllifolia	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Labiatæ	34. Prunella vulgaris	...	...	0·01	...	...	...	...	...	...	...	...	...	...	...	...	...
	35. Thymus Serpyllum	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	36. Ajuga reptans	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Primulaceæ	37. Primula veris	...	...	...	...	...	...	...	...	0·01	...	...	...	...	...	...	...
Polygonaceæ	38. Rumex Acetosa	7·02	3·96	1·09	2·25	4·08	3·57	0·70	1·34	5·09	4·30	3·34	5·12	5·88	3·43	2·15	8·57
	39. Rumex crispus	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Liliaceæ	40. Scilla nutans	...	...	...	...	...	...	...	...	...	...	0·01	0·01	...	...	...	...
	41. Ornithogalum umbellatum	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Juncaceæ	42. Luzula campestris	0·01	0·01	...	...	...	0·01	...	...	0·17	0·03	0·01	0·02	0·01	0·10	0·01	0·01
Cyperaceæ	43. Carex præcox	0·01	...	0·01	...	0·01	...	...	...	...	...	0·01	...	...	...	...	...
Filices	44. Ophioglossum vulgatum	0·01	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Musci	45. Hypnum squarrosum	0·01	...	...	...	...	...	...	0·01	...	...	0·01	0·01	...	...	...	...
	46. Hypnum rutabulum	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	0·01
	47. Hypnum hians	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	Total	10·61	5·87	1·15	2·47	5·76	4·33	0·74	1·54	9·78	13·90	5·28	6·47	8·75	13·22	3·30	9·44

	Description of Manure	"Mixed Mineral Manure"—Sulphates of Potass, Soda, and Magnesia, and Superphosphate of Lime.								Nitrate of Soda alone.							
		With Nitrate of Soda = 86 lbs. Nitrogen.				With Nitrate of Soda = 43 lbs. Nitrogen.				= 86 lbs. Nitrogen.				= 43 lbs. Nitrogen.			
	Plot Numbers	14				16				15				17			
	Seasons	1862	1867	1872	1877	1862	1867	1872	1877	1862	1867	1872	1877	1862	1867	1872	1877
	GRAMINEOUS HERBAGE.																
	Number of Species	15	14	14	15	17	14	17	15	17	16	17	15	16	16	17	16
	1. Anthoxanthum odoratum	0·35	0·13	0·02	0·06	1·42	1·78	1·40	2·22	1·82	1·83	4·49	4·16	2·06	2·31	4·50	5·32
	2. Alopecurus pratensis	0·22	3·54	3·72	20·18	0·67	8·27	15·22	12·23	6·90	5·95	2·46	7·17	23·94	21·71	16·25	12·72
	3. Phleum pratense	...	0·17	...	...	0·02	...	...	...	0·07	0·18	0·07	...	...	...	...	...
	4. Agrostis vulgaris	0·42	0·61	0·24	1·55	12·49	13·62	12·40	14·58	7·65	6·86	7·66	12·90	11·01	7·05	10·60	17·92
	5. Aira cæspitosa	...	...	0·01	0·01	...	...	0·01	...	...	...	0·01	...	...	...	0·02	...
	6. Holcus lanatus	6·60	6·63	3·67	12·75	10·45	11·69	5·08	12·64	7·61	11·81	5·32	14·95	8·23	8·13	5·87	10·91
	7. Avena elatior	3·14	...	...	0·32	0·12	...	0·17	0·08	0·04	...	...	...	0·68	0·23	0·48	0·01
	8. Avena pubescens	0·90	0·92	0·19	0·47	1·49	1·72	1·54	3·34	3·53	0·70	1·56	3·13	4·24	1·15	4·09	4·27
	9. Avena flavescens	4·88	7·12	5·67	2·93	18·37	14·86	18·80	6·67	3·86	4·26	3·83	2·98	1·45	3·18	4·96	1·98
	10. Poa pratensis	1·45	1·05	2·57	4·01	0·02	0·10	0·39	0·48	0·14	0·13	0·37	0·11	0·02	0·21	0·09	0·09
	11. Poa trivialis	22·48	32·93	21·76	21·59	6·87	8·96	6·53	4·82	6·53	23·67	7·95	6·05	5·21	12·08	2·74	1·59
	12. Briza media	...	...	...	...	0·01	...	0·01	...	0·05	0·02	0·20	0·30	0·01	0·07	0·31	0·72
	13. Dactylis glomerata	10·00	7·28	3·33	12·48	1·59	2·59	3·75	4·63	2·09	0·21	0·11	0·36	1·80	0·57	0·64	0·58
	14. Cynosurus cristatus	0·01	...	...	...	0·16	0·30	0·05	0·18	0·07	0·03	0·01	0·13	0·12	0·41	0·30	0·81
	15. Festuca ovina	0·88	1·58	0·16	0·48	11·13	10·44	10·33	16·66	13·69	12·08	34·71	20·77	9·43	11·18	18·05	12·04
	16. Festuca pratensis	0·87	0·03	0·09	0·33	0·32	0·19	0·27	0·09	0·03	0·27	0·63	1·47	0·49	0·09	0·04	0·08
	17. Festuca elatior	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	18. Festuca loliacea	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	19. Bromus mollis	18·04	17·69	42·10	8·02	2·04	2·64	2·03	0·57	2·12	6·27	4·00	1·65	0·18	2·26	0·81	0·15
	20. Lolium perenne	13·80	9·36	5·55	2·63	5·55	6·23	3·10	3·75	7·49	3·24	4·42	7·32	5·09	3·23	2·94	6·68
	Gramineæ Determined	84·04	89·04	92·08	87·81	73·02	83·39	81·08	82·94	63·69	77·51	77·80	83·45	73·96	73·86	72·69	75·87
	Undetermined, chiefly Gramineæ	5·48	5·21	0·79	...	4·99	1·04	0·50	...	14·57	2·51	0·96	...	7·40	1·86	0·58	...
	Total	89·52	94·25	92·87	87·81	78·01	84·43	81·58	82·94	78·26	80·02	78·76	83·45	81·36	75·72	73·27	75·87
	LEGUMINOUS HERBAGE.																
	Number of Species	3	3	2	1	3	4	4	4	4	4	3	4	4	3	4	4
	1. Trifolium repens	0·01	0·01	...	...	0·35	0·50	0·32	0·01	0·04	0·08	0·06	0·01	0·05	0·32	0·09	0·01
	2. Trifolium pratense	0·01	0·01	0·01	...	1·85	0·70	0·43	0·11	0·20	0·04	0·03	0·31	0·31	0·26	0·12	0·11
	3. Trifolium minus	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	4. Trifolium procumbens	...	...	...	...	...	...	...	...	0·01	...	...	...	...	...	...	...
	5. Lotus corniculatus	...	...	...	...	...	0·02	0·02	0·06	0·02	0·35	0·03	0·01	0·05	0·12	1·16	0·78
	6. Lotus major	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	7. Lathyrus pratensis	0·11	0·37	1·35	0·76	0·04	0·57	6·68	9·22	...	0·04	...	1·47	0·01	...	0·01	0·01
	8. Ononis arvensis	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	9. Vicia Cracca	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	Total	0·13	0·39	1·36	0·76	2·24	1·79	7·45	9·40	0·27	0·51	0·12	1·80	0·42	0·70	1·38	0·91
	MISCELLANEOUS HERBAGE.																
	Number of Species	10	13	14	11	14	16	15	22	18	19	19	24	13	28	22	29
Ranunculaceæ	1. Ranunculus acris	...	0·30	0·20	0·71	0·95	0·05	0·79	1·24	0·46	0·25	1·24	3·74	0·14	0·29	1·15	2·95
	2 and 3. Ranunculus repens et bulbosus	1·23	0·10	0·03	...	4·66	0·33	0·33	0·26	1·80	0·39	0·39	0·49	1·95	1·04	1·33	1·96
	4. Ranunculus auricomus	...	...	...	...	...	...	...	0·13	...	...	...	...	...	...	...	...
Caryophylleæ	5. Stellaria graminea	...	...	...	...	...	...	...	0·01	...	...	0·24	0·03	...	0·08	0·04	0·01
	6. Stellaria Holostea	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	7. Cerastium triviale	0·01	0·01	0·01	0·06	0·03	0·06	0·02	0·07	1·07	3·68	9·26	0·59	0·29	1·00	3·05	0·39
Hypericineæ	8. Hypericum perforatum	...	...	...	...	...	...	...	...	...	...	...	...	...	0·01	...	0·08
Rosaceæ	9. Potentilla reptans	...	...	...	...	...	...	...	0·01	...	...	0·02	0·25	...	0·02	0·01	0·01
	10. Potentilla Fragariastrum	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	11. Agrimonia Eupatoria	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	12. Poterium Sanguisorba	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	13. Spiræa Ulmaria	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Umbelliferæ	14. Conopodium denudatum	1·55	1·57	0·61	0·18	4·50	4·80	3·90	0·51	0·58	0·20	0·43	0·77	1·48	2·44	1·51	0·72
	15. Pimpinella Saxifraga	0·03	0·02	0·02	0·01	0·01	...	0·01	0·02	0·01	0·09	0·08	0·04	...	0·01	0·01	0·02
	16. Heracleum Sphondylium	0·04	...	...	...	...	...	...	0·49	...	...	0·11	0·16	...	...	...	0·48
	17. Anthriscus sylvestris	...	1·52	3·86	4·64	...	...	0·09	0·05	...	...	...	...	...	...	...	...
Rubiaceæ	18. Galium verum	...	...	...	...	...	...	...	0·01	...	1·06	1·21	0·23	...	...	...	...
	19. Galium Aparine	...	...	...	...	...	...	...	...	...	...	...	0·01	...	...	...	...
Dipsaceæ	20. Scabiosa arvensis	...	...	...	...	...	...	...	...	0·01	...	...	...	...	...	0·19	0·30
Compositæ	21. Centaurea nigra	...	0·01	...	...	...	0·08	1·14	0·31	...	0·17	2·58	0·90	4·41	4·10	10·28	2·82
	22. Carduus arvensis	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	23. Bellis perennis	...	...	...	...	...	...	...	...	...	...	...	...	...	0·02	0·01	0·01
	24. Achillea Millefolium	0·24	0·47	0·21	0·65	2·47	1·76	3·29	1·55	2·53	1·13	2·60	0·58	2·14	1·39	2·91	1·39
	25. Chrysanthemum Leucanthemum	...	...	...	...	...	...	...	...	...	...	...	...	...	...	0·04	...
	26. Tragopogon pratensis	...	...	...	...	0·01	0·02	0·01	0·11	...	...	...	...	...	...	...	0·08
	27. Leontodon hispidus	...	...	0·01	...	...	0·02	...	...	0·01	0·01	...	0·01	0·02	0·05	0·07	0·25
	28. Leontodon autumnalis	...	...	...	...	...	...	...	...	0·03	0·03	0·02	...	...	0·01	...	...
	29. Taraxacum officinale	0·19	0·23	0·18	0·63	0·16	0·01	...	...	1·20	0·11	0·10	0·08	0·15	0·05	0·09	0·12
	30. Hieracium Pilosella	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	0·01
Plantagineæ	31. Plantago lanceolata	0·18	0·01	0·01	...	1·25	0·84	0·07	0·22	6·92	4·67	0·28	0·56	3·85	4·83	2·41	7·99
Scrophularineæ	32. Veronica Chamædrys	...	...	...	...	0·12	0·05	0·06	0·29	0·08	0·04	...	...	0·10	0·15	0·10	0·21
	33. Veronica serpyllifolia	...	...	...	...	...	...	...	...	...	...	...	0·01	...	...	...	0·01
Labiatæ	34. Prunella vulgaris	...	...	...	...	...	...	...	...	...	...	...	...	...	0·01	...	0·01
	35. Thymus Serpyllum	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	36. Ajuga reptans	...	...	...	...	...	...	0·01	...	0·01	...	...	0·02	...	0·03	0·02	0·08
Primulaceæ	37. Primula veris	...	...	...	...	...	...	...	0·01	...	...	...	...	...	0·01	...	...
Polygonaceæ	38. Rumex Acetosa	6·88	1·11	0·61	4·40	5·46	5·62	1·19	2·19	6·64	7·34	2·06	5·79	3·57	7·53	1·58	2·56
	39. Rumex crispus	...	...	...	0·13	...	...	...	...	...	...	...	...	...	...	...	...
Liliaceæ	40. Scilla nutans	...	...	...	0·01	...	...	...	...	...	...	...	0·01	...	...	...	...
	41. Ornithogalum umbellatum	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Juncaceæ	42. Luzula campestris	...	0·01	...	...	0·12	0·13	0·06	0·16	0·10	0·28	0·47	0·43	0·12	0·49	0·49	0·71
Cyperaceæ	43. Carex præcox	...	...	0·01	...	...	...	...	...	0·01	0·01	0·02	0·03	...	0·02	0·05	0·02
Filices	44. Ophioglossum vulgatum	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Musci	45. Hypnum squarrosum	...	...	0·01	...	0·01		...	0·01	...		...	0·01	...	...	...	0·01
	46. Hypnum rutabulum	...	...	...	...	...	0·01	...	...	0·01	0·01	0·01	...	...	...	0·01	0·01
	47. Hypnum hians	...	...	...	0·01	...		...	0·01	...		...	0·01	...	...	...	0·01
	Total	10·35	5·36	5·77	11·43	19·75	13·78	10·97	7·66	21·47	19·47	21·12	14·75	18·22	23·58	25·35	23·22

	Description of Manure	Mineral Constituents and Nitrogen of 1 ton Hay.			Superphosphate, Nitrate Soda, and Sulphate Potass.		Superphosphate, and Nitrate Potass.		Description of Manure.	
	Plot Numbers	18			19		20		Plot Numbers.	
	Seasons	1867	1872	1877	1872	1877	1872	1877	Seasons.	
	GRAMINEOUS HERBAGE.									
	Number of Species	15	18	14	16	16	15	16	Number of Species.	
	Botanical Names:—								Ordinary English names:—	
	1. Anthoxanthum odoratum	4·94	7·11	8·64	6·39	2·47	7·17	3·51	Sweet-scented Vernal Grass	1
	2. Alopecurus pratensis	0·95	0·92	0·75	0·23	5·40	1·22	6·80	Meadow Fox Tail	2
	3. Phleum pratense	...	0·01	...	...	...	...	0·01	Meadow Cat's Tail	3
	4. Agrostis vulgaris	7·29	21·95	16·40	25·56	10·83	31·35	10·36	Common Bent	4
	5. Aira cæspitosa	...	0·01	...	0·01	0·01	0·01	...	Tufted Hair Grass	5
	6. Holcus lanatus	12·78	7·32	17·45	14·03	21·19	12·39	29·47	Woolly Soft Grass	6
	7. Avena elatior	...	0·42	...	0·01	...	0·01	0·01	False Oats	7
	8. Avena pubescens	2·87	2·58	1·94	1·50	3·00	1·02	2·40	Downy Oat	8
	9. Avena flavescens	3·42	5·75	3·12	3·20	4·37	3·07	5·30	Yellow Oat	9
	10. Poa pratensis	0·10	0·63	0·23	0·05	0·12	0·01	0·01	Smooth Stalked Meadow Grass	10
	11. Poa trivialis	4·83	2·92	2·80	0·75	4·73	0·39	3·06	Rough Stalked Meadow	11
	12. Briza media	0·02	0·45	0·12	0·75	0·04	0·72	0·19	Quaking	12
	13. Dactylis glomerata	1·79	1·20	1·28	0·67	2·43	0·30	1·15	Cock's Foot	13
	14. Cynosurus cristatus	0·18	0·97	0·32	1·17	2·54	1·15	1·51	Crested Dog's Tail	14
	15. Festuca ovina	7·05	23·76	24·65	29·65	21·40	25·54	18·69	Sheep Fescue	15
	16. Festuca pratensis	0·97	0·62	0·03	0·42	0·87	...	0·03	Meadow Fescue	16
	17. Festuca elatior	...	...	...	...	...	...	...	Tall Meadow Fescue	17
	18. Festuca loliacea	...	...	...	...	0·32	...	...	Spiked Meadow Fescue	18
	19. Bromus mollis	0·01	0·01	...	...	...	...	...	Soft Brome Grass	19
	20. Lolium perenne	5·15	3·37	6·45	4·14	1·23	2·03	3·61	Perennial Rye	20
	Gramineæ Determined	52·35	80·00	84·18	88·62	80·95	86·38	86·11	Gramineæ Determined.	
	Undetermined, chiefly Gramineæ	3·11	0·80	...	0·73	...	0·74	...	Chiefly Gramineæ Undetermined.	
	Total	55·46	80·80	84·18	89·35	80·95	87·12	86·11	Total.	
	LEGUMINOUS HERBAGE.									
	Number of Species	4	4	4	4	5	5	4	Number of Species.	
	1. Trifolium repens	0·42	0·58	0·03	0·19	0·84	0·21	0·64	White or Dutch Clover	1
	2. Trifolium pratense	3·05	0·54	0·20	1·30	0·80	0·63	0·05	Common Red Clover	2
	3. Trifolium minus	...	...	...	...	...	...	...	Lesser Clover	3
	4. Trifolium procumbens	...	...	...	...	...	0·01	...	Hop Trefoil	4
	5. Lotus corniculatus	0·71	1·38	0·26	0·79	0·12	1·09	0·24	Bird's Foot Trefoil	5
	6. Lotus major	...	...	...	...	0·04	...	...	Greater Bird's Foot Trefoil	6
	7. Lathyrus pratensis	0·83	1·07	1·50	0·23	6·92	0·10	2·73	Meadow Vetchling	7
	8. Ononis arvensis	...	...	...	...	...	...	...	Field Rest Harrow	8
	9. Vicia Cracca	...	...	...	...	...	...	...	Tufted Vetch	9
	Total	5·01	3·57	1·99	2·51	8·72	2·04	3·66	Total.	
	MISCELLANEOUS HERBAGE.									
	Number of Species	21	22	21	21	18	22	18	Number of Species.	
Ranunculaceæ	1. Ranunculus acris	0·31	0·73	0·46	0·30	0·61	0·01	0·19	Upright Crowfoot or Buttercup	1
	2 and 3. Ranunculus repens et bulbosus	5·73	3·11	2·56	2·02	0·19	3·93	3·83	Creeping and bulbous Buttercup	2 and 3
	4. Ranunculus auricomus	...	...	...	...	...	...	...	Goldilocks	4
Caryophylleæ	5. Stellaria graminea	0·08	0·26	0·19	0·02	0·01	0·01	...	Lesser Stitchwort	5
	6. Stellaria Holostea	...	...	...	...	...	...	...	Greater Stitchwort	6
	7. Cerastium triviale	0·12	0·15	0·08	0·41	0·20	0·45	0·20	Mouse-ear Chickweed	7
Hypericineæ	8. Hypericum perforatum	...	...	...	...	...	...	...	Perforated St. John's Wort	8
Rosaceæ	9. Potentilla reptans	...	0·01	...	...	...	...	...	Creeping Cinque Foil	9
	10. Potentilla Fragariastrum	...	...	...	...	...	...	...	Barren Strawberry	10
	11. Agrimonia Eupatoria	...	...	...	0·01	0·01	...	...	Common Agrimony	11
	12. Poterium Sanguisorba	...	...	...	...	...	...	...	Common Burnet	12
	13. Spiræa Ulmaria	...	...	...	...	...	...	...	Meadow Sweet	13
Umbelliferæ	14. Conopodium denudatum	3·69	2·42	3·18	1·96	4·47	2·02	2·66	Earth or Pig Nut	14
	15. Pimpinella Saxifraga	0·07	0·04	0·02	0·07	0·03	0·05	0·06	Burnet Saxifrage	15
	16. Heracleum Sphondylium	...	...	0·01	...	...	...	...	Cow Parsnip or Hogweed	16
	17. Anthriscus sylvestris	...	...	...	...	...	...	...	Beaked Parsley	17
Rubiaceæ	18. Galium verum	...	0·02	0·77	0·01	...	0·11	0·04	Yellow Bed Straw	18
	19. Galium Aparine	...	...	0·01	...	...	...	0·01	Cleavers or Goose Grass	19
Dipsaceæ	20. Scabiosa arvensis	...	...	...	...	...	...	...	Field Scabious	20
Compositæ	21. Centaurea nigra	0·79	0·84	0·87	0·08	0·07	0·01	0·16	Black Knapwood	21
	22. Carduus arvensis	0·01	...	...	...	...	...	...	Creeping Plume Thistle	22
	23. Bellis perennis	0·01	...	...	0·01	...	0·01	...	Daisy	23
	24. Achillea Millefolium	0·76	4·06	1·32	0·68	0·66	0·78	0·80	Yarrow or Milfoil	24
	25. Chrysanthemum Leucanthemum	...	...	0·12	...	...	...	...	Oxeye Daisy	25
	26. Tragopogon pratensis	...	0·01	...	...	...	...	...	Goat's Beard	26
	27. Leontodon hispidus	...	...	...	...	...	...	...	Rough Hawkbit	27
	28. Leontodon autumnalis	...	...	...	...	...	...	...	Autumnal Hawkbit	28
	29. Taraxacum officinale	0·08	0·05	0·10	...	...	0·02	...	Dandelion	29
	30. Hieracium Pilosella	0·01	...	...	...	...	...	...	Mouse-ear Hawkbit	30
Plantagineæ	31. Plantago lanceolata	2·79	0·04	0·05	0·17	0·29	0·04	0·02	Ribwort Plantain	31
Scrophularineæ	32. Veronica Chamædrys	0·22	0·12	0·06	0·12	0·69	0·04	0·32	Germander Speedwell	32
	33. Veronica serpyllifolia	...	...	...	...	...	...	...	Thyme-leaved Speedwell	33
Labiatæ	34. Prunella vulgaris	0·02	...	...	0·01	...	0·01	0·01	Self Heal	34
	35. Thymus Serpyllum	...	...	...	...	...	...	...	Wild Thyme	35
	36. Ajuga reptans	...	...	...	0·01	0·08	0·01	0·04	Bugle	36
Primulaceæ	37. Primula veris	...	...	...	...	...	...	...	Cowslip	37
Polygonaceæ	38. Rumex Acetosa	24·33	1·97	4·66	0·72	2·65	1·26	1·51	Sorrel	38
	39. Rumex crispus	...	...	...	...	...	...	...	Curl-leaved Dock	39
Liliaceæ	40. Scilla nutans	...	0·01	...	...	...	0·01	...	Bluebell	40
	41. Ornithogalum umbellatum	...	...	...	...	...	...	...	Star of Bethlehem	41
Juncaceæ	42. Luzula campestris	0·44	1·72	0·04	1·42	0·33	2·02	0·36	Wood Rush	42
Cyperaceæ	43. Carex præcox	0·06	0·04	0·01	0·04	0·01	0·03	...	Vernal Carex or Sedge	43
Filices	44. Ophioglossum vulgatum	...	...	...	...	...	...	0·01	Adder's Tongue	44
Musci	45. Hypnum squarrosum		0·01	0·01	0·07	0·02	0·01	0·01	Mosses	45
	46. Hypnum rutabulum	0·01	0·01	0·01	0·01	0·01	0·01	...		46
	47. Hypnum hians		0·01	...	...	...	...	...		47
	Total	39·53	15·63	13·83	8·14	10·33	10·84	10·23	Total.	

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s, 6 years, hout Potass, l since.		With Ammo = 86 lbs. Ni			. Nitrogen.		Mineral Constituents and Nitrogen of 1 ton Hay.			Superphosphate, Nitrate Soda, and Sulphate Potass.		Superphosphate, and Nitrate Potass.		. . . . Description of Manure.
		9			17		18			19		20		. . . . Plot Number.
1872	1877	1862	1867	1	1872	1877	1867	1872	1877	1872	1877	1872	1877	. . . . Seasons.
		GRAMINEO												
17	16	13	14		17	16	15	18	14	16	16	15	16	. . . . Number of Species.
														Ordinary English names:—
203·9	273·6	79·4	193·2	4	149·5	198·9	214·0	265·9	388·8	286·8	116·8	309·2	180·6	Sweet-scented Vernal Grass 1
13·4	31·5	17·3	3·8	4	540·0	475·5	41·2	34·4	33·8	10·3	255·3	52·6	349·8	Meadow Fox Tail 2
...	...	...	...		...	...	...	0·4	...	...	...	...	0·5	Meadow Cat's Tail 3
239·3	449·4	820·1	722·8	3	352·2	669·8	315·8	820·9	738·0	1147·1	512·0	1351·8	532·9	Common Bent 4
0·3	...	...	..		0·7	...	...	0·4	...	0·4	0·5	0·4	...	Tufted Hair-grass 5
118·4	660·3	777·2	529·6	2	195·1	407·8	553·6	273·8	785·2	629·7	1001·9	534·3	1515·9	Woolly Soft Grass 6
113·0	114·9	...	134·6	7	16·0	0·4	...	15·7	...	0·4	...	0·4	0·5	False Oat 7
94·0	60·5	654·3	75·9	5	135·8	159·6	124·3	96·5	87·3	67·3	141·8	44·0	123·4	Downy Oat 8
178·2	88·8	581·3	203·4	6	164·8	74·0	148·1	215·1	140·4	147·7	206·6	132·4	272·6	Yellow Oat 9
54·2	37·3	683·7	700·7	7	3·0	3·4	4·3	23·6	10·4	2·2	5·7	0·4	0·5	Smooth Stalked Meadow Grass . . 10
41·6	116·0	558·3	115·2	4	91·1	59·4	209·2	109·2	126·0	33·7	223·6	16·8	157·4	Rough Stalked Meadow 11
29·8	20·6	...	...	6	10·3	26·9	0·9	16·8	5·4	33·7	1·9	31·0	9·8	Quaking 12
16·9	35·5	357·2	249·7	0	21·3	21·7	77·5	44·9	57·6	30·1	114·9	13·0	59·2	Cock's Foot 13
26·2	41·3	...	...	9	10·0	30·3	8·0	36·3	14·4	52·5	120·1	49·6	77·7	Crested Dog's Tail 14
615·0	716·1	333·5	991·4	2	599·8	450·0	305·4	888·5	1109·2	1330·7	1011·8	1101·3	961·5	Sheep Fescue 15
8·5	19·6	90·9	3·8	6	1·3	3·0	42·0	23·2	1·4	18·8	41·1	...	1·5	Meadow Fescue 16
...	...	...	...		...	...	...	...	...	...	...	...	...	Tall Meadow Fescue 17
...	...	...	...		...	...	...	...	...	...	15·1	...	...	Spiked Meadow Fescue 18
2·3	0·4	285·5	5·9	9	26·9	5·6	0·4	0·4	...	...	...	...	...	Soft Brome Grass 19
49·3	276·5	268·9	54·3	2	97·7	249·7	223·1	126·0	290·2	185·8	58·2	87·5	185·7	Perennial Rye 20
1804·3	2942·3	5507·6	3984·3	6	2415·5	2836·0	2267·8	2992·0	3788·1	3977·2	3827·3	3724·7	4429·5	Gramineæ Determined.
33·3	...	164·0	163·1	5	19·2	...	134·8	29·9	...	32·8	...	31·9	...	Chiefly Gramineæ Undetermined.
1837·6	2942·3	5671·6	4147·4	1	2434·7	2836·0	2402·6	3021·9	3788·1	4010·0	3827·3	3756·6	4429·5	Total.
		LEGUMINO												
4	4	2	2		4	4	4	4	4	4	5	5	4	. . . . Number of Species.
6·4	3·6	0·6	0·5	3	3·0	0·4	18·2	21·7	1·4	8·5	39·7	9·1	32·9	White or Dutch Clover 1
6·9	13·0	...	...	2	4·0	4·1	132·1	20·2	9·0	58·4	37·8	27·2	2·6	Common Red Clover 2
...	...	...	...		...	...	...	...	...	...	...	...	...	Lesser Clover 3
...	...	...	...		...	...	...	...	...	...	...	0·4	...	Hop Trefoil 4
90·2	42·8	...	...	1	38·6	29·1	30·7	51·6	11·7	35·5	5·7	47·0	12·3	Bird's Foot Trefoil 5
...	...	...	...		...	...	...	...	...	...	1·9	...	...	Greater Bird's Foot Trefoil 6
101·2	85·9	7·7	8·1		0·3	0·4	36·0	40·0	67·5	10·3	327·2	4·3	140·5	Meadow Vetchling 7
...	..	...	...		...	...	...	...	...	...	...	...	...	Field Rest Harrow 8
...	...	...	...		...	...	...	...	...	...	...	...	...	Tufted Vetch 9
204·7	145·3	8·3	8·6	6	45·9	34·0	217·0	133·5	89·6	112·7	412·3	88·0	188·3	Total.
		MISCELLANE												
21	28	13	13		22	29	21	22	21	21	18	22	18	. . . . Number of Species.
12·0	39·5	...	...	7	38·2	110·3	13·4	27·3	20·7	13·5	28·8	0·4	9·8	Upright Crowfoot or Buttercup . . 1
25·7	23·5	9·0	2·7	3	44·2	73·3	248·2	116·3	115·2	90·7	9·0	169·5	197·0	Creeping and Bulbous Buttercup . 2 and 3
...	...	...	...		...	...	...	...	...	...	...	...	...	Goldilocks 4
1·0	0·7	...	1·6	1	1·3	0·4	3·5	9·7	8·5	0·9	0·5	0·4	...	Lesser Stichwort 5
...	...	...	...		...	...	...	...	...	...	...	...	...	Greater Stichwort 6
8·5	6·2	...	...	8	101·4	14·6	5·2	5·6	3·6	18·4	9·5	19·4	10·3	Mouse-ear Chickweed 7
...	...	...	...	5	...	3·0	...	...	...	...	...	...	...	Perforated St. John's Wort . . . 8
...	...	3·2	...	0	0·3	0·4	...	0·4	...	...	...	...	...	Creeping Cinque Foil 9
...	...	...	...		...	...	...	...	...	...	...	...	...	Barren Strawberry. 10
0·3	0·4	...	...		...	...	...	...	...	0·4	0·5	...	...	Common Agrimony 11
...	...	...	...		...	...	...	...	...	...	...	...	...	Common Burnet 12
...	0·7	...	...		...	...	...	...	...	...	...	...	...	Meadow Sweet 13
44·4	38·8	186·9	503·2	4	50·2	26·9	159·9	90·5	143·0	88·0	211·3	87·1	136·8	Earth or Pig Nut 14
27·0	24·3	0·6	0·5	5	0·3	0·7	3·0	1·5	0·9	3·2	1·4	2·2	3·1	Burnet Saxifrage 15
...	8·0	...	...		...	17·9	...	...	0·5	...	...	...	...	Cow Parsnip or Hogweed 16
...	...	...	...		...	...	...	...	...	...	...	...	...	Beaked Parsley 17
9·2	1·1	...	...		...	...	...	0·7	3·2	0·4	...	4·8	2·1	Yellow Bed Straw 18
...	0·4	...	...		...	...	...	...	0·5	...	...	...	0·5	Cleavers or Goose Grass 19
0·8	17·0	...	17·8		6·3	11·2	...	...	...	...	...	...	...	Field Scabious 20
6·2	27·5	2·6	...	3	341·6	105·4	34·2	31·4	39·0	3·6	3·3	0·4	8·2	Black Knapweed 21
...	...	...	...		...	...	0·4	...	...	...	...	...	...	Creeping Plume Thistle 22
...	...	...	...	0	0·3	0·4	0·4	...	...	0·4	...	0·4	...	Daisy 23
250·4	100·0	124·8	109·3	6	96·7	51·9	32·9	151·8	59·4	30·5	31·2	33·7	41·1	Yarrow or Milfoil 24
...	...	..	...		1·4	...	...	...	5·4	...	...	...	...	Oxeye Daisy 25
...	0·4	46·1	...		...	3·0	...	0·4	...	...	...	...	...	Goat's Beard 26
...	...	...	...	5	2·3	9·3	...	...	...	...	...	...	...	Rough Hawkbit 27
...	...	...	...	5	...	...	...	...	...	...	...	...	...	Autumnal Hawkbit 28
1·3	0·4	...	0·5	5	3·0	4·5	3·5	1·9	4·5	...	...	0·9	...	Dandelion 29
...	...	...	...		...	0·4	0·4	...	...	...	...	...	...	Mouse-ear Hawkbit 30
8·7	9·4	0·6	0·5	4	80·1	298·7	120·9	1·5	2·2	7·6	13·7	1·7	1·0	Ribwort Plantain 31
9·2	5·1	0·6	0·5	6	3·3	7·8	9·5	4·5	2·7	5·4	32·6	1·7	16·5	Germander Speedwell 32
...	...	...	...		...	0·4	...	...	...	...	...	...	...	Thyme-leaved Speedwell 33
...	1·8	...	...	5	...	0·4	0·9	...	...	0·4	...	0·4	0·5	Self Heal 34
...	...	...	...		...	...	...	...	...	...	...	...	...	Wild Thyme 35
...	0·7	...	...	5	0·7	3·0	...	...	...	0·4	3·8	0·4	2·1	Bugle 36
...	0·4	...	...	5	...	...	...	...	...	...	...	...	...	Cowslip 37
50·3	211·6	345·7	586·2	7	52·5	95·7	1054·0	73·7	209·7	32·3	125·3	54·4	77·7	Sorrel 38
...	...	...	...		...	...	...	...	...	...	...	...	...	Curled-leaved Dock 39
...	...	0·7	...		...	...	...	0·4	...	...	...	0·4	...	Bluebell 40
...	...	...	...		...	...	...	...	...	...	...	...	...	Star of Bethlehem 41
68·3	17·4	1·3	2·7	9	16·3	26·5	19·1	64·3	1·8	63·8	15·6	87·1	18·5	Wood Rush 42
1·5	0·7	...	0·5	0	1·7	0·7	2·6	1·5	0·5	1·8	0·5	1·3	...	Vernal Carex or Sedge 43
...	...	...	...		...	...	...	...	...	...	...	...	0·5	Adder's Tongue 44
0·3 0·3 0·3	0·4	... ...	... ...		... 0·3 ...	0·4 0·4 0·4	0·4	0·4 0·4 0·4	0·5 0·5 ...	3·2 0·4 ...	0·9 0·5 ...	0·4 0·4 ...	0·5	Mosses 45 46 47
525·7	536·4	722·1	1226·0	3	842·4	868·0	1712·4	584·6	622·3	365·3	488·4	467·4	526·2	Total.

Detailed results of Experiments at Rothamsted, illustrating the Domination of one Plant over another in the Mixed Herbage of Permanent Meadow Land.

Appendix–Table XI.—Number of Species, and Quantities in lbs. per acre, of each Species, and each Group (Gramineæ, Leguminosæ, and Miscellaneæ), in the total Mixed Herbage of each Plot, in each year of complete botanical separation, 1862, 1867, 1872, and 1877.

Gramineous Herbage.

Description of Manure	Without Manure, 1856, and since.								Farmyard Manure, 8 years, 1856-63; discontinued 1864, and since.							
									With Ammonia-salts = 43 lbs. Nitrogen.				Alone.			
Plot Number	3				12				1				2			
Seasons	1862	1867	1872	1877	1862	1867	1872	1877	1862	1867	1872	1877	1862	1867	1872	1877
Number of Species	18	15	17	17	16	16	17	16	15	15	18	15	14	17	18	18
Botanical Names:—																
1. Anthoxanthum odoratum	130·6	288·5	85·5	121·0	140·0	233·5	150·2	118·5	3·9	79·4	204·1	656·8	10·1	163·5	188·8	260·1
2. Alopecurus pratensis	137·0	193·9	8·5	7·1	106·8	79·9	61·7	20·8	14·4	91·1	103·4	119·2	155·3	141·8	158·8	125·0
3. Phleum pratense	0·3	...	...	...	...	...	...	...	5·0	1·8	2·1	...	...	2·9	1·1	0·4
4. Agrostis vulgaris	346·7	287·5	265·3	313·9	322·8	169·2	250·7	287·8	32·8	379·3	733·4	1113·4	133·6	282·4	312·0	649·1
5. Aira cæspitosa	0·3	...	0·2	0·9	...	...	0·2	...	...	...	0·4	0·5	...	...	0·3	0·4
6. Holcus lanatus	153·8	265·5	59·2	296·7	158·5	169·8	85·8	219·1	215·6	629·2	496·4	821·0	98·2	625·3	193·9	410·3
7. Avena elatior	2·1	7·0	2·1	1·2	26·7	20·4	41·4	16·4	34·4	35·6	57·2	78·1	103·2	22·9	14·7	5·4
8. Avena pubescens	294·5	102·3	58·4	63·6	338·8	171·9	109·2	62·1	27·8	92·9	97·4	98·4	129·5	224·6	291·0	277·4
9. Avena flavescens	72·4	62·0	57·4	25·5	77·0	53·6	53·6	11·9	221·7	402·8	220·0	64·8	304·6	336·1	329·0	105·5
10. Poa pratensis	8·9	5·7	1·5	1·7	24·6	20·7	16·2	5·8	80·6	387·7	261·6	65·3	84·0	203·5	63·7	43·3
11. Poa trivialis	47·0	39·0	8·2	13·2	91·7	58·8	20·5	17·3	1772·4	1312·4	153·6	128·2	1425·9	900·3	86·9	85·6
12. Briza media	57·7	22·7	105·2	171·4	46·7	50·6	95·0	80·5	...	...	0·4	...	...	1·1	4·8	23·5
13. Dactylis glomerata	53·7	58·0	14·8	16·6	96·5	97·2	43·8	57·2	912·9	378·1	116·9	200·2	228·2	372·1	92·6	100·7
14. Cynosurus cristatus	5·2	4·3	18·2	23·2	16·7	13·1	21·6	9·5	1·1	4·1	0·7	0·5	0·5	16·0	22·1	35·4
15. Festuca ovina	405·9	506·5	356·2	517·5	245·5	353·3	361·5	452·0	42·2	362·8	226·0	508·7	18·7	262·4	280·0	405·6
16. Festuca pratensis	1·2	...	0·2	0·2	346·1	120·1	50·7	70·8	...	...	0·4	...	...	1·1	1·1	12·3
17. Festuca elatior	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
18. Festuca loliacea	...	...	...	0·7	...	...	...	...	...	...	...	...	...	...	...	...
19. Bromus mollis	4·0	1·7	0·2	...	2·7	1·2	0·2	0·2	1217·8	620·3	160·7	39·3	899·2	936·8	110·1	6·5
20. Lolium perenne	194·4	134·3	39·0	107·6	152·7	94·2	42·0	49·4	78·9	187·6	60·4	78·6	72·4	207·5	89·8	177·7
Gramineæ Determined	1915·7	1978·9	1080·1	1682·0	2193·8	1707·5	1404·3	1479·3	4661·5	4965·1	2895·1	3973·0	3663·4	4700·3	2240·7	2724·2
Undetermined, chiefly Gramineæ	239·3	204·5	48·7	...	286·5	91·4	30·2	...	281·6	117·0	7·7	...	134·1	130·8	24·6	...
Total	2155·0	2183·4	1128·8	1682·0	2480·3	1798·9	1434·5	1479·3	4943·1	5082·1	2902·8	3973·0	3797·5	4831·1	2265·3	2724·2

Description of Manure	Superphosphate of Lime each year, 1859, and since.								Ammonia-salts alone, = 86 lbs. Nitrogen.				"Mixed Mineral Manure"—Sulphates of Potass, Soda, and Magnesia, and Superphosphate of Lime.			
	Alone.				With Ammonia-salts = 86 lbs. Nitrogen.								1869 and since; previously (13 years, 1856-68), Ammonia-salts.			
Plot Number	4-1				4-2				5				6			
Seasons	1862	1867	1872	1877	1862	1867	1872	1877	1862	1867	1872	1877	1862	1867	1872	1877
Number of Species	16	15	16	16	14	14	15	13	17	15	15	13	15	14	17	14
Botanical Names:—																
1. Anthoxanthum odoratum	119·0	253·5	84·0	158·0	106·5	240·2	47·0	110·8	222·4	182·2	76·4	120·7	174·5	137·6	175·5	206·5
2. Alopecurus pratensis	42·9	65·1	15·2	43·2	31·4	641·9	125·9	74·2	25·0	15·5	20·8	6·8	75·7	0·6	0·8	3·8
3. Phleum pratense	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
4. Agrostis vulgaris	234·5	215·2	245·9	305·2	921·7	609·3	657·9	1145·3	936·5	693·3	669·2	869·7	954·1	460·0	659·5	362·2
5. Aira cæspitosa	...	...	...	...	...	...	1·3	...	0·4	0·3	...	...	...	...	0·3	0·4
6. Holcus lanatus	384·4	324·3	83·5	598·3	771·0	458·3	64·9	283·1	388·5	170·3	47·7	88·9	363·8	98·0	149·8	628·7
7. Avena elatior	4·2	6·4	2·7	1·5	117·0	17·8	79·2	47·9	151·4	91·9	37·4	6·8	153·1	207·5	101·6	116·9
8. Avena pubescens	306·4	175·9	72·5	124·3	351·0	171·5	9·0	1·4	281·7	20·8	6·0	3·5	647·3	28·7	51·6	70·5
9. Avena flavescens	134·0	151·5	107·9	76·4	99·4	17·8	2·9	1·4	25·0	15·2	4·6	0·3	52·5	7·6	42·0	20·3
10. Poa pratensis	18·2	9·2	8·0	0·6	31·9	168·4	163·3	73·3	41·2	21·5	15·3	6·8	101·5	52·7	68·3	73·0
11. Poa trivialis	167·8	200·0	67·1	146·0	387·1	93·6	67·1	14·6	34·5	9·9	14·8	0·3	68·1	12·8	27·7	26·6
12. Briza media	18·9	11·3	37·6	66·8	...	...	0·3	...	2·3	...	0·2	0·6	...	...	0·3	...
13. Dactylis glomerata	73·2	35·0	10·1	43·6	108·4	16·6	5·1	85·9	92·1	46·0	17·6	95·9	91·3	54·6	36·1	172·7
14. Cynosurus cristatus	11·0	29·7	18·2	24·7	...	...	...	...	0·4	0·3	0·2	...	0·4	0·9	0·3	...
15. Festuca ovina	331·7	592·9	362·2	495·4	323·4	1135·4	1574·6	2592·2	847·5	1010·6	1170·5	1573·7	593·4	827·7	879·0	1605·2
16. Festuca pratensis	4·2	...	1·4	0·6	21·9	0·4	...	...	13·5	4·3	0·5	...	19·6	0·3	0·3	5·9
17. Festuca elatior	...	...	...	...	...	...	...	3·8	...	...	...	...	...	...	...	...
18. Festuca loliacea	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
19. Bromus mollis	16·9	15·2	1·6	0·4	15·7	8·7	0·6	...	3·1	...	...	...	6·7	...	0·3	...
20. Lolium perenne	301·8	185·7	55·3	134·5	307·7	59·2	22·4	9·9	128·3	40·0	24·3	2·7	203·9	44·4	19·5	83·2
Gramineæ Determined	2169·1	2270·9	1173·2	2219·5	3594·1	3639·1	2821·5	4443·8	3193·8	2322·1	2105·5	2776·7	3505·9	1933·4	2212·9	3375·9
Undetermined, chiefly Gramineæ	268·6	96·6	14·5	...	225·4	109·3	10·8	...	133·0	53·3	23·8	...	78·8	66·7	22·9	...
Total	2437·7	2367·5	1187·7	2219·5	3819·5	3748·4	2832·3	4443·8	3326·8	2375·4	2129·3	2776·7	3584·7	2000·1	2235·8	3375·9

Description of Manure	"Mixed Mineral Manure"—Sulphates of Potass, Soda, and Magnesia, and Superphosphate of Lime.															
	Alone, each year, 1856 and since.				With Potass, 6 years, 1856-61; without Potass, 1862 and since.				With Ammonia-salts = 86 lbs. Nitrogen.				Mineral Manures as Plot 8, and Ammonia-salts = 86 lbs. Nitrogen.			
Plot Number	7				8				9				10			
Seasons	1862	1867	1872	1877	1862	1867	1872	1877	1862	1867	1872	1877	1862	1867	1872	1877
Number of Species	18	16	17	17	17	17	17	16	13	14	16	13	16	15	15	15
Botanical Names:—																
1. Anthoxanthum odoratum	135·4	175·2	115·2	162·2	168·3	266·9	203·9	273·6	79·4	193·2	127·3	178·0	90·2	262·7	141·2	285·0
2. Alopecurus pratensis	15·0	39·2	49·6	24·5	17·6	33·6	13·4	31·5	17·3	3·8	156·2	58·2	126·7	149·0	447·9	805·7
3. Phleum pratense	0·5	...	...	...	...	...	...	...	...	...	...	...	3·7	...	...	...
4. Agrostis vulgaris	315·9	253·7	496·4	613·0	452·9	165·2	239·3	449·4	820·1	722·8	874·7	740·4	579·1	429·1	613·3	794·9
5. Aira cæspitosa	0·5	...	0·4	0·5	0·5	0·4	0·3	...	...	..	0·6	...	...	...	0·5	0·5
6. Holcus lanatus	223·9	526·5	133·8	671·2	204·0	391·9	118·4	660·3	777·2	529·6	430·5	627·8	587·1	410·7	189·2	231·3
7. Avena elatior	106·6	2·7	19·5	65·8	204·5	120·9	113·0	114·9	...	134·6	645·0	801·0	4·9	583·6	572·1	467·5
8. Avena pubescens	611·0	173·9	100·0	114·7	573·6	131·5	94·0	60·5	654·3	75·9	27·7	4·2	657·6	81·7	19·0	11·7
9. Avena flavescens	177·8	215·8	157·6	186·1	245·2	134·6	178·2	88·8	581·3	203·4	299·8	40·5	621·1	99·7	33·3	8·9
10. Poa pratensis	50·0	46·8	96·2	89·3	77·8	57·4	54·2	37·3	683·7	700·7	1282·7	1091·5	250·9	738·1	849·1	316·2
11. Poa trivialis	168·6	195·3	97·4	107·6	247·9	133·1	41·6	116·0	558·3	115·2	36·2	6·7	629·1	138·5	52·4	22·9
12. Briza media	1·3	2·7	4·2	7·1	3·2	3·0	29·8	20·6	...	...	0·6	...	...	...	...	...
13. Dactylis glomerata	113·7	208·2	71·2	187·2	158·3	56·6	16·9	35·5	357·2	249·7	672·2	851·8	773·1	271·1	135·0	238·1
14. Cynosurus cristatus	3·5	9·8	3·3	5·1	11·3	9·2	26·2	41·3	...	...	...	0·6	0·6	0·5	...	...
15. Festuca ovina	607·4	507·4	629·5	1356·1	339·8	678·4	615·0	716·1	333·5	991·4	491·1	1319·8	252·1	734·7	856·9	1285·4
16. Festuca pratensis	7·5	28·5	4·7	48·4	100·0	16·1	8·5	19·6	90·9	3·8	1·7	...	92·1	12·5	3·9	6·9
17. Festuca elatior	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
18. Festuca loliacea	...	...	...	...	...	...	...	...	...	...	...	...	..	...	...	...
19. Bromus mollis	55·7	43·5	1·7	0·5	62·4	16·4	2·3	0·4	285·5	5·9	5·7	...	156·4	34·4	75·3	75·6
20. Lolium perenne	138·0	107·0	25·0	154·0	267·8	99·8	49·3	276·5	268·9	54·3	62·8	9·7	156·6	91·7	24·7	8·3
Gramineæ Determined	2732·3	2536·2	2006·2	3793·3	3135·1	2315·0	1804·3	2942·3	5507·6	3984·3	5114·8	5730·2	5011·3	4038·0	4013·8	4558·9
Undetermined, chiefly Gramineæ	127·9	107·0	61·8	...	108·2	95·2	33·3	...	164·0	163·1	101·3	...	269·5	80·2	84·1	...
Total	2860·2	2643·2	2068·0	3793·3	3243·3	2410·3	1837·6	2942·3	5671·6	4147·4	5216·1	5730·2	5280·8	4118·2	4097·9	4558·9

Description of Manure	"Mixed Mineral Manure"—Sulphates of Potass, Soda, and Magnesia, and Superphosphate of Lime.															
	With Ammonia-salts = 172 lbs. Nitrogen; only 86 lbs. in 1859-60-61.				As Plot 11—1 and Silicates in addition, 1862, and since.				With Ammonia-salts = 86 lbs. Nitrogen and Cut Wheat Straw.							
Plot Number	11-1				11-2				13-1				13-2			
Seasons	1862	1867	1872	1877	1862	1867	1872	1877	1862	1867	1872	1877	1862	1867	1872	1877
Number of Species	15	13	11	11	14	14	13	11	14	16	15	14	14	14	15	14
Botanical Names:—																
1. Anthoxanthum odoratum	6·4	3·2	55·6	12·9	71·2	6·3	11·5	14·4	37·6	141·5	30·7	26·1	49·7	84·2	27·5	14·9
2. Alopecurus pratensis	199·0	702·7	880·7	673·5	106·8	396·0	1620·6	1709·3	291·2	260·4	627·1	581·1	129·2	289·2	150·3	258·6
3. Phleum pratense	...	...	...	...	...	...	1·4	...	...	2·9	...	...	...	...	13·0	12·3
4. Agrostis vulgaris	936·1	1032·9	967·0	1984·4	1339·3	1511·4	726·9	1452·7	936·3	969·8	1008·7	805·8	570·4	1516·8	846·7	876·7
5. Aira cæspitosa	0·7	...	...	...	...	...	...	0·9	...	0·6	...	...	...	...	...	...
6. Holcus lanatus	705·1	153·3	736·6	1378·8	524·7	299·6	757·7	1655·8	262·2	368·3	124·0	292·1	416·0	193·0	266·2	281·2
7. Avena elatior	54·7	243·9	742·4	1009·8	455·7	302·2	910·8	1796·9	41·8	211·7	539·5	635·7	10·3	83·5	755·9	758·2
8. Avena pubescens	118·0	0·5	...	...	54·1	0·6	...	...	562·7	47·0	10·2	1·8	117·6	24·0	9·9	0·6
9. Avena flavescens	375·3	24·7	6·4	0·7	243·5	109·5	29·3	1·7	258·6	251·7	88·2	22·5	354·0	265·8	157·9	49·2
10. Poa pratensis	670·3	689·3	741·6	99·9	364·5	649·4	889·4	382·5	352·7	794·0	1041·9	723·8	134·4	430·9	502·7	535·3
11. Poa trivialis	941·8	7·5	6·4	22·4	1213·3	95·1	58·0	0·9	621·1	241·3	65·2	46·7	877·3	582·8	244·1	33·1
12. Briza media	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
13. Dactylis glomerata	1717·3	2107·0	2801·1	1163·0	1661·8	2396·0	1948·3	1137·3	1440·5	944·8	1998·1	2249·1	2090·4	1536·4	4183·2	2881·0
14. Cynosurus cristatus	...	...	...	...	...	...	...	...	...	...	0·6	...	...	...	...	...
15. Festuca ovina	103·8	26·8	27·1	282·0	47·0	128·2	21·5	216·7	165·0	556·2	381·0	284·2	120·2	256·3	93·8	160·1
16. Festuca pratensis	135·1	...	...	...	162·3	3·8	...	...	197·0	30·8	1·3	0·6	294·6	38·6	5·3	2·6
17. Festuca elatior	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
18. Festuca loliacea	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
19. Bromus mollis	98·8	2·1	0·7	...	49·1	4·4	0·7	...	34·5	5·2	0·6	0·6	45·2	0·6	0·8	...
20. Lolium perenne	97·4	4·3	...	0·7	104·0	23·8	11·5	...	106·5	71·9	5·1	9·1	214·2	74·7	17·5	4·5
Gramineæ Determined	6159·8	4998·2	6965·6	6628·1	6397·3	5926·3	6987·6	8369·1	5307·7	4898·1	5922·2	5679·2	5453·5	5376·8	7274·8	5868·3
Undetermined, chiefly Gramineæ	193·3	46·6	82·7	...	312·6	58·8	114·5	...	213·6	85·8	93·3	...	437·4	110·2	99·1	...
Total	6353·1	5044·8	7048·3	6628·1	6709·9	5985·1	7102·1	8369·1	5521·3	4983·9	6015·5	5679·2	5890·9	5487·0	7373·9	5868·3

Description of Manure	"Mixed Mineral Manure"—Sulphates of Potass, Soda, and Magnesia, and Superphosphate of Lime.								Nitrate of Soda alone.							
	With Nitrate of Soda = 86 lbs. Nitrogen.				With Nitrate of Soda = 43 lbs. Nitrogen.				= 86 lbs. Nitrogen.				= 43 lbs. Nitrogen.			
Plot Number	14				16				15				17			
Seasons	1862	1867	1872	1877	1862	1867	1872	1877	1862	1867	1872	1877	1862	1867	1872	1877
Number of Species	15	14	14	15	17	14	17	15	17	16	17	15	16	16	17	16
Botanical Names:—																
1. Anthoxanthum odoratum	20·0	9·3	1·3	3·8	73·5	105·7	62·7	135·3	74·4	96·7	164·3	157·4	91·6	117·4	149·5	198·9
2. Alopecurus pratensis	12·6	254·5	231·2	1262·9	34·7	491·2	681·1	745·6	281·9	314·5	90·0	271·3	1064·4	1103·4	540·0	475·5
3. Phleum pratense	...	12·2	...	...	1·0	...	...	...	2·9	9·5	2·6	...	...	...	...	...
4. Agrostis vulgaris	24·0	43·8	14·9	97·0	646·7	809·0	554·9	888·8	312·6	362·6	280·3	488·1	489·5	358·3	352·2	669·8
5. Aira cæspitosa	...	...	0·6	0·6	...	...	0·4	...	...	...	0·4	...	...	...	0·7	...
6. Holcus lanatus	377·4	476·5	228·1	797·9	541·1	694·4	227·3	770·5	311·0	624·3	194·6	565·7	365·9	413·2	195·1	407·8
7. Avena elatior	179·5	...	...	20·0	6·2	...	7·6	4·9	1·6	...	...	...	30·2	11·7	16·0	0·4
8. Avena pubescens	51·5	66·1	11·8	29·4	77·2	102·2	68·9	203·6	144·2	37·0	57·1	118·4	188·5	58·5	135·8	159·6
9. Avena flavescens	279·0	511·8	352·4	183·3	951·2	882·7	841·3	406·6	157·7	225·2	140·1	112·8	64·5	161·6	164·8	74·0
10. Poa pratensis	82·9	75·5	159·7	250·9	1·0	6·0	17·5	29·3	5·7	6·9	13·5	4·2	0·9	10·7	3·0	3·4
11. Poa trivialis	1285·4	2367·0	1538·8	1351·1	355·7	532·2	292·2	293·8	266·8	1251·2	290·9	228·9	231·6	613·4	91·1	59·4
12. Briza media	...	...	...	...	0·5	...	0·4	...	2·0	1·1	7·3	11·4	0·5	3·6	10·3	26·9
13. Dactylis glomerata	571·8	523·3	207·0	781·0	82·4	153·9	167·8	282·2	85·4	11·1	4·0	13·6	80·0	29·0	21·3	21·7
14. Cynosurus cristatus	0·6	...	...	...	8·3	17·8	2·2	11·0	2·9	1·6	0·4	4·9	5·3	20·9	10·0	30·3
15. Festuca ovina	50·3	113·6	10·0	30·0	576·3	620·1	462·3	1015·6	559·3	633·5	1270·0	786·0	419·3	568·2	599·8	450·0
16. Festuca pratensis	49·8	2·2	5·6	20·7	16·6	11·3	12·1	5·5	1·2	14·3	23·1	55·6	21·8	4·6	1·3	3·0
17. Festuca elatior	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
18. Festuca loliacea	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
19. Bromus mollis	1031·5	1271·6	2615·5	501·9	105·7	156·8	90·9	34·7	86·6	331·4	146·4	62·4	8·0	114·9	26·9	5·6
20. Lolium perenne	789·1	672·8	344·9	164·6	302·9	370·1	138·7	228·6	306·1	171·3	161·7	277·0	226·3	164·2	97·7	249·7
Gramineæ Determined	4805·4	6400·2	5722·8	5495·1	3781·0	4953·4	3628·3	5056·0	2602·3	4097·2	2846·7	3157·7	3288·3	3753·6	2415·5	2836·0
Undetermined, chiefly Gramineæ	313·4	374·5	49·1	...	258·3	61·8	22·4	...	595·4	132·7	35·1	...	328·9	94·5	19·2	...
Total	5118·8	6774·7	5771·9	5495·1	4039·3	5015·2	3650·7	5056·0	3197·7	4229·9	2881·8	3157·7	3617·2	3848·1	2434·7	2836·0

Description of Manure	Mineral Constituents and Nitrogen of 1 ton Hay.			Superphosphate, Nitrate Soda, and Sulphate Potass.		Superphosphate, and Nitrate Potass.		Description of Manure.	
Plot Number	18			19		20		Plot Number.	
Seasons	1867	1872	1877	1872	1877	1872	1877	Seasons.	
Number of Species	15	18	14	16	16	15	16	Number of Species.	
Botanical Names:—								Ordinary English names:—	
1. Anthoxanthum odoratum	214·0	265·9	388·8	286·8	116·8	309·2	180·6	Sweet-scented Vernal Grass	1
2. Alopecurus pratensis	41·2	34·4	33·8	10·3	255·3	52·6	349·8	Meadow Fox Tail	2
3. Phleum pratense	...	0·4	...	...	...	...	0·5	Meadow Cat's Tail	3
4. Agrostis vulgaris	315·8	820·9	738·0	1147·1	512·0	1351·8	532·9	Common Bent	4
5. Aira cæspitosa	...	0·4	...	0·4	0·5	0·4	...	Tufted Hair-grass	5
6. Holcus lanatus	553·6	273·8	785·2	629·7	1001·9	534·3	1515·9	Woolly Soft Grass	6
7. Avena elatior	...	15·7	...	0·4	...	0·4	0·5	False Oat	7
8. Avena pubescens	124·3	96·5	87·3	67·3	141·8	44·0	123·4	Downy Oat	8
9. Avena flavescens	148·1	215·1	140·4	147·7	206·6	132·4	272·6	Yellow Oat	9
10. Poa pratensis	4·3	23·6	10·4	2·2	5·7	0·4	0·5	Smooth Stalked Meadow Grass	10
11. Poa trivialis	209·2	109·2	126·0	33·7	223·6	16·8	157·4	Rough Stalked Meadow	11
12. Briza media	0·9	16·8	5·4	33·7	1·9	31·0	9·8	Quaking	12
13. Dactylis glomerata	77·5	44·9	57·6	30·1	114·9	13·0	59·2	Cock's Foot	13
14. Cynosurus cristatus	8·0	36·3	14·4	52·5	120·1	49·6	77·7	Crested Dog's Tail	14
15. Festuca ovina	305·4	888·5	1109·2	1330·7	1011·8	1101·3	961·5	Sheep Fescue	15
16. Festuca pratensis	42·0	23·2	1·4	18·8	41·1	...	1·5	Meadow Fescue	16
17. Festuca elatior	...	...	...	...	...	...	...	Tall Meadow Fescue	17
18. Festuca loliacea	...	...	...	...	15·1	...	...	Spiked Meadow Fescue	18
19. Bromus mollis	0·4	0·4	...	...	...	...	...	Soft Brome Grass	19
20. Lolium perenne	223·1	126·0	290·2	185·8	58·2	87·5	185·7	Perennial Rye	20
Gramineæ Determined	2267·8	2992·0	3788·1	3977·2	3827·3	3724·7	4429·5	Gramineæ Determined.	
Undetermined, chiefly Gramineæ	134·8	29·9	...	32·8	...	31·9	...	Chiefly Gramineæ Undetermined.	
Total	2402·6	3021·9	3788·1	4010·0	3827·3	3756·6	4429·5	Total.	

Leguminous Herbage.

Plot Number	3				12				1				2			
Seasons	1862	1867	1872	1877	1862	1867	1872	1877	1862	1867	1872	1877	1862	1867	1872	1877
Number of Species	4	4	4	4	4	4	4	4	4	4	3	2	3	4	4	4
1. Trifolium repens	16·2	7·0	6·2	3·1	18·1	50·3	24·8	0·6	1·7	4·1	0·4	0·5	30·4	3·4	14·7	5·8
2. Trifolium pratense	136·7	70·3	27·6	49·4	71·2	87·5	35·6	44·8	1·7	2·9	0·4	...	17·2	17·2	7·9	34·7
3. Trifolium minus	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
4. Trifolium procumbens	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
5. Lotus corniculatus	55·9	78·3	97·7	93·4	56·2	129·8	116·2	66·9	0·6	0·6	...	...	...	5·7	4·8	6·1
6. Lotus major	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
7. Lathyrus pratensis	38·4	22·7	16·1	56·0	66·8	61·3	52·4	49·6	6·6	50·0	11·2	23·6	49·6	66·9	112·2	189·6
8. Ononis arvensis	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
9. Vicia Cracca	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Total	247·2	178·3	147·6	201·9	212·3	328·9	229·0	161·9	10·6	57·6	12·0	24·1	97·2	93·2	139·6	236·2

Plot Number	4-1				4-2				5				6			
Seasons	1862	1867	1872	1877	1862	1867	1872	1877	1862	1867	1872	1877	1862	1867	1872	1877
Number of Species	4	4	5	5	3	3	2	2	4	4	3	2	4	3	4	4
1. Trifolium repens	19·8	3·2	8·5	10·8	0·5	0·4	...	0·5	0·4	0·3	0·3	...	0·5	0·3	0·3	0·4
2. Trifolium pratense	47·2	6·0	2·0	9·2	0·5	...	0·3	...	1·5	0·3	...	...	1·3	0·3	1·1	3·4
3. Trifolium minus	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
4. Trifolium procumbens	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
5. Lotus corniculatus	13·3	43·6	65·7	26·6	...	...	...	...	1·9	10·3	10·3	4·1	0·5	2·6	1·7	1·3
6. Lotus major	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
7. Lathyrus pratensis	10·4	47·4	74·3	104·5	3·3	1·3	0·7	1·4	0·8	0·3	1·0	1·5	10·2	...	41·5	277·0
8. Ononis arvensis	...	...	2·1	19·8	...	...	...	...	...	...	...	...	...	...	...	...
9. Vicia Cracca	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Total	90·7	100·2	152·6	170·9	4·3	1·7	1·0	1·9	4·6	11·2	11·6	5·6	12·5	3·2	44·6	282·1

Plot Number	7				8				9				10			
Seasons	1862	1867	1872	1877	1862	1867	1872	1877	1862	1867	1872	1877	1862	1867	1872	1877
Number of Species	4	4	4	4	4	4	4	4	2	2	1	4	2	1	2	2
1. Trifolium repens	136·2	21·0	75·0	0·5	122·1	3·8	6·4	3·6	0·6	0·5	...	...	0·6	...	0·4	...
2. Trifolium pratense	302·6	211·9	47·9	79·1	348·8	43·2	6·9	13·0	..	...	...	0·6	...	...	...	0·5
3. Trifolium minus	...	...	...	...	...	...	...	...	...	...	...	0·6	...	...	...	...
4. Trifolium procumbens	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
5. Lotus corniculatus	56·2	30·8	8·0	2·0	6·8	31·7	90·2	42·8	...	...	...	...	...	...	...	0·5
6. Lotus major	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
7. Lathyrus pratensis	597·7	302·0	1553·8	617·6	396·3	260·8	101·2	85·9	7·7	8·1	1·1	3·6	6·8	4·0	0·4	...
8. Ononis arvensis	...	...	...	...	...	...	...	..	...	...	...	20·0	...	...	...	...
9. Vicia Cracca	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Total	1092·7	565·7	1684·7	699·2	874·0	339·5	204·7	145·3	8·3	8·6	1·1	24·8	7·4	4·0	0·8	1·0

Plot Number	11-1				11-2				13-1				13-2			
Seasons	1862	1867	1872	1877	1862	1867	1872	1877	1862	1867	1872	1877	1862	1867	1872	1877
Number of Species	1	1	1	0	0	0	0	0	3	2	3	0	2	2	3	0
1. Trifolium repens	...	...	0·7	...	...	...	...	...	2·4	2·3	0·6	...	1·9	1·3	...	...
2. Trifolium pratense	...	...	...	...	...	...	...	...	3·1	...	0·7	...	...	...	0·8	...
3. Trifolium minus	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
4. Trifolium procumbens	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
5. Lotus corniculatus	...	0·6	...	...	...	...	...	...	...	...	...	...	...	...	0·8	...
6. Lotus major	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
7. Lathyrus pratensis	0·7	...	...	...	...	...	...	...	27·1	7·6	37·7	...	1·9	3·1	0·8	...
8. Ononis arvensis	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
9. Vicia Cracca	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Total	0·7	0·6	0·7	...	...	...	...	...	32·6	9·9	39·0	...	3·8	4·4	2·4	...

Plot Number	14				16				15				17			
Seasons	1862	1867	1872	1877	1862	1867	1872	1877	1862	1867	1872	1877	1862	1867	1872	1877
Number of Species	3	3	2	1	3	4	4	4	4	4	3	4	4	3	4	4
1. Trifolium repens	0·6	0·7	...	...	18·1	29·7	14·3	0·6	1·6	4·2	2·2	0·4	2·2	16·3	3·0	0·4
2. Trifolium pratense	0·6	0·7	0·6	...	95·8	41·6	19·3	6·7	8·2	2·1	1·1	11·7	13·8	13·2	4·0	4·1
3. Trifolium minus	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
4. Trifolium procumbens	...	...	...	...	...	...	...	...	0·4	...	...	...	...	...	...	...
5. Lotus corniculatus	...	...	...	...	...	1·2	0·9	3·7	0·8	18·5	1·1	0·4	2·2	6·1	38·6	29·1
6. Lotus major	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
7. Lathyrus pratensis	6·2	26·6	83·9	47·6	2·1	33·8	298·9	562·0	...	2·1	...	55·6	0·5	...	0·3	0·4
8. Ononis arvensis	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
9. Vicia Cracca	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Total	7·4	28·0	84·5	47·6	116·0	106·3	333·4	573·0	11·0	26·9	4·4	68·1	18·7	35·6	45·9	34·0

Plot Number	18			19		20		Plot Number.	
Seasons	1867	1872	1877	1872	1877	1872	1877	Seasons.	
Number of Species	4	4	4	4	5	5	4	Number of Species.	
1. Trifolium repens	18·2	21·7	1·4	8·5	39·7	9·1	32·9	White or Dutch Clover	1
2. Trifolium pratense	132·1	20·2	9·0	58·4	37·8	27·2	2·6	Common Red Clover	2
3. Trifolium minus	...	...	...	...	...	...	...	Lesser Clover	3
4. Trifolium procumbens	...	...	...	...	...	0·4	...	Hop Trefoil	4
5. Lotus corniculatus	30·7	51·6	11·7	35·5	5·7	47·0	12·3	Bird's Foot Trefoil	5
6. Lotus major	...	...	...	...	1·9	...	...	Greater Bird's Foot Trefoil	6
7. Lathyrus pratensis	36·0	40·0	67·5	10·3	327·2	4·3	140·5	Meadow Vetchling	7
8. Ononis arvensis	...	...	...	...	...	...	...	Field Rest Harrow	8
9. Vicia Cracca	...	...	...	...	...	...	...	Tufted Vetch	9
Total	217·0	133·5	89·6	112·7	412·3	88·0	188·3	Total.	

Miscellaneous Herbage.

	Plot Number	3				12				1				2			
	Seasons	1862	1867	1872	1877	1862	1867	1872	1877	1862	1867	1872	1877	1862	1867	1872	1877
	Number of Species	28	24	28	31	24	30	29	31	9	15	15	17	13	20	25	30
Ranunculaceæ	1. Ranunculus acris	0·3	0·3	1·2	...	...	10·7	0·9	2·0	...	15·3	4·9	61·5	...	1·1	28·9	53·8
	2 and 3. Ranunculus repens et bulbosus	148·9	67·0	48·3	81·6	89·7	52·4	72·5	139·7	85·0	21·8	20·4	60·6	146·2	56·6	43·3	129·3
	4. Ranunculus auricomus	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Caryophylleæ	5. Stellaria graminea	0·3	0·7	0·3	0·5	2·4	12·8	20·3	20·4	...	...	0·4	...	2·0	0·6	2·3	1·8
	6. Stellaria Holostea	...	...	...	...	...	...	...	0·2	...	...	...	...	...	...	...	...
	7. Cerastium triviale	14·1	12·2	14·0	19·4	13·4	30·8	77·5	17·5	...	6·5	8·4	22·2	0·5	2·9	34·5	14·4
Hypericineæ	8. Hypericum perforatum	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Rosaceæ	9. Potentilla reptans	...	...	0·2	0·2	0·3	0·3	0·2	0·4	1·1	...	...	0·5	...	...	0·3	0·4
	10. Potentilla Fragariastrum	...	...	...	...	...	0·3	0·2	0·2	...	...	...	...	...	...	...	...
	11. Agrimonia Eupatoria	0·3	...	0·3	0·2	...	0·3	0·2	...	...	...	...	...	...	...	...	3·3
	12. Poterium Sanguisorba	...	7·0	8·0	20·8	...	...	...	...	...	...	...	...	...	...	...	...
	13. Spiræa Ulmaria	...	...	...	...	...	...	...	...	...	...	...	0·9	...	...	2·5	...
Umbelliferæ	14. Conopodium denudatum	29·6	98·3	46·8	44·9	54·1	165·2	48·9	61·5	81·1	93·5	19·4	26·5	143·2	162·9	32·6	34·7
	15. Pimpinella Saxifraga	46·7	114·6	14·1	18·8	33·9	36·6	9·9	10·6	3·9	1·8	1·1	0·9	18·2	27·4	4·8	9·8
	16. Heracleum Sphondylium	...	...	0·3	...	...	...	...	...	...	...	...	...	...	...	...	1·4
	17. Anthriscus sylvestris	...	...	...	...	...	...	...	...	...	15·3	...	...	...	1·1	0·6	...
Rubiaceæ	18. Galium verum	...	...	...	...	0·3	3·4	0·2	0·2	...	...	...	...	0·5	1·1	1·7	1·4
	19. Galium Aparine	...	...	0·2	...	...	...	...	...	...	...	...	...	...	...	...	...
Dipsaceæ	20. Scabiosa arvensis	0·3	...	0·3	2·8	1·7	0·3	2·0	1·3	...	...	...	...	...	...	0·3	3·6
Compositæ	21. Centaurea nigra	9·5	19·7	34·6	25·1	41·1	28·7	75·5	53·5	...	10·6	43·8	11·8	2·0	8·0	36·2	32·5
	22. Carduus arvensis	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	23. Bellis perennis	0·3	1·3	0·2	0·5	...	...	...	0·2	...	...	...	...	...	0·6	0·3	0·4
	24. Achillea Millefolium	46·7	38·7	29·3	47·0	38·7	46·6	84·9	60·8	75·0	180·4	177·0	34·1	122·0	62·3	82·9	78·4
	25. Chrysanthemum Leucanthemum	...	...	...	2·8	...	...	...	...	...	...	...	...	...	...	...	...
	26. Tragopogon pratensis	0·3	...	0·2	0·2	10·3	11·0	7·4	0·2	...	...	...	...	...	...	0·6	0·7
	27. Leontodon hispidus	1·8	21·3	20·9	31·2	3·8	3·0	2·9	1·5	...	...	...	...	...	1·1	0·6	1·4
	28. Leontodon autumnalis	...	...	...	...	...	...	...	0·2	...	...	...	...	...	...	...	...
	29. Taraxacum officinale	1·5	5·4	2·5	1·4	4·1	4·0	0·2	3·0	1·1	10·0	4·6	1·4	...	8·0	6·8	9·0
	30. Hieracium Pilosella	0·3	0·3	0·2	1·0	...	0·3	0·5	1·7	...	...	...	...	...	...	...	...
Plantagineæ	31. Plantago lanceolata	224·0	357·5	43·7	74·7	265·0	251·5	9·3	31·0	16·7	30·0	4·9	12·8	84·5	176·1	41·0	133·6
Scrophularineæ	32. Veronica Chamædrys	14·7	10·7	2·3	5·7	13·4	14·0	5·2	12·4	...	16·5	1·1	0·5	14·2	94·9	35·4	26·0
	33. Veronica serpyllifolia	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Labiatæ	34. Prunella vulgaris	0·3	1·0	0·2	2·1	1·0	3·7	0·2	0·9	...	...	...	...	...	0·6	...	0·7
	35. Thymus Serpyllum	...	...	...	2·1	...	...	...	...	...	...	...	...	...	...	...	...
	36. Ajuga reptans	0·3	0·3	...	0·5	...	0·6	0·2	0·2	...	0·6	...	0·9	...	...	0·3	1·1
Primulaceæ	37. Primula veris	0·3	...	...	0·2	1·0	0·9	0·7	0·7	...	...	...	...	...	...	...	1·4
Polygonaceæ	38. Rumex Acetosa	42·7	58·7	29·1	44·2	103·4	110·1	79·3	44·6	338·4	337·4	328·0	496·4	629·5	175·5	54·6	68·9
	39. Rumex crispus	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Liliaceæ	40. Scilla nutans	...	...	...	0·2	...	...	...	...	...	...	...	0·5	...	...	...	0·4
	41. Ornithogalum umbellatum	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Juncaceæ	42. Luzula campestris	58·3	120·3	56·2	41·8	39·0	90·3	67·1	32·5	...	0·6	1·8	3·4	2·5	10·3	15·0	22·4
Cyperaceæ	43. Carex præcox	5·5	34·0	13·8	8·8	10·6	41·2	20·3	14·7	...	...	0·4	...	...	0·6	0·3	0·4
Filices	44. Ophioglossum vulgatum	0·3	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Musci	45. Hypnum squarrosum	1·9	1·0	0·2	1·4	3·4	1·2	1·6	11·7	...	...	...	...	...	...	...	0·4
	46. Hypnum rutabulum	0·3		0·2		0·4		0·2		...	...	...	...	...	...	0·3	
	47. Hypnum hians	0·3		...		0·4		0·2		...	...	...	...	...	...	...	
	Total	649·8	970·3	367·6	480·1	731·4	920·2	588·5	523·8	602·3	740·3	616·2	734·9	1165·3	791·7	426·1	651·6

Plot Number	4-1				4-2				5				6			
Seasons	1862	1867	1872	1877	1862	1867	1872	1877	1862	1867	1872	1877	1862	1867	1872	1877
Number of Species	26	27	28	24	18	13	11	11	17	17	13	14	15	15	18	20
1. Ranunculus acris	1·0	3·5	2·1	8·0	0·5	0·4	...	...	0·4	0·7	0·8	2·1	...	...	9·8	14·4
2 and 3. Ranunculus repens et bulbosus	190·9	45·7	73·7	180·6	101·8	1·8	0·3	...	12·3	2·6	2·0	2·6	34·3	5·1	3·4	9·7
4. Ranunculus auricomus	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
5. Stellaria graminea	1·0	6·0	1·2	4·6	0·5	0·9	...	...	7·7	3·0	...	...	...	0·9	0·3	0·4
6. Stellaria Holostea	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
7. Cerastium triviale	14·0	34·3	15·1	22·9	0·5	0·4	0·3	...	0·4	3·0	...	...	3·1	1·3	0·5	0·4
8. Hypericum perforatum	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
9. Potentilla reptans	0·3	...	...	...	...	...	...	...	...	...	...	...	...	0·3	0·3	0·4
10. Potentilla Fragariastrum	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
11. Agrimonia Eupatoria	0·7	0·4	0·2	...	...	...	...	...	...	...	...	...	...	...	...	...
12. Poterium Sanguisorba	0·3	26·6	1·6	6·5	...	...	...	...	...	...	...	...	...	...	...	...
13. Spiræa Ulmaria	...	...	...	...	...	...	...	...	...	...	...	2·6	...	...	...	...
14. Conopodium denudatum	31·5	82·1	20·7	19·8	61·3	115·3	12·5	5·2	44·3	189·7	25·6	19·2	24·5	251·2	69·7	37·2
15. Pimpinella Saxifraga	28·3	113·6	22·7	45·8	28·5	17·0	11·5	0·5	3·9	25·5	3·8	0·3	2·7	19·5	7·7	5·5
16. Heracleum Sphondylium	...	...	0·4	...	1·4	...	...	11·3	...	...	0·3	1·5	...	...	4·2	6·8
17. Anthriscus sylvestris	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
18. Galium verum	...	...	...	0·3	4·3	21·8	21·1	31·9	...	20·2	60·3	6·5	43·2	10·5	25·4	6·8
19. Galium Aparine	...	...	0·2	...	...	...	...	0·5	0·4	...	...	0·3	...	...	...	0·4
20. Scabiosa arvensis	...	...	0·2	...	...	...	0·3	2·3	1·2	9·3	15·6	49·3	8·5	21·4	1·1	0·8
21. Centaurea nigra	14·0	12·7	17·9	20·4	0·5	9·1	39·9	39·9	0·4	80·3	54·8	15·6	...	44·7	39·8	18·6
22. Carduus arvensis	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
23. Bellis perennis	0·3	1·8	0·7	0·6	...	...	...	...	...	...	...	...	...	...	...	...
24. Achillea Millefolium	46·2	66·5	95·3	98·7	84·1	64·9	55·9	12·7	51·2	36·0	26·4	4·7	148·7	34·5	115·4	72·6
25. Chrysanthemum Leucanthemum	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
26. Tragopogon pratensis	...	0·4	...	...	...	...	...	...	...	...	...	...	...	...	...	...
27. Leontodon hispidus	19·2	21·9	2·0	28·4	...	...	...	...	...	...	...	...	...	...	...	...
28. Leontodon autumnalis	...	0·4	...	...	...	...	...	...	...	...	...	...	...	...	...	...
29. Taraxacum officinale	11·1	4·6	1·2	...	2·4	0·4	...	...	0·4	0·3	0·5	...	0·9	...	0·5	...
30. Hieracium Pilosella	0·3	0·4	0·2	0·3	...	...	...	...	...	...	...	...	...	...	...	...
31. Plantago lanceolata	183·1	342·0	55·4	116·9	1·9	...	...	0·5	3·8	0·7	...	...	2·7	...	...	0·4
32. Veronica Chamædrys	9·4	20·9	4·1	9·9	0·5	...	...	...	0·4	0·3	...	...	3·1	1·3	0·3	0·4
33. Veronica serpyllifolia	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
34. Prunella vulgaris	1·0	1·4	0·2	0·6	...	...	...	...	...	...	...	...	...	...	...	...
35. Thymus Serpyllum	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
36. Ajuga reptans	...	0·4	0·2	0·3	...	...	...	...	...	...	...	...	...	...	...	...
37. Primula veris	0·3	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
38. Rumex Acetosa	128·1	193·6	49·8	104·2	636·8	366·4	218·9	145·0	352·6	527·0	179·2	62·9	539·1	774·7	211·9	323·4
39. Rumex crispus	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
40. Scilla nutans	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
41. Ornithogalum umbellatum	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
42. Luzula campestris	39·7	85·3	63·2	26·0	6·2	3·5	1·0	0·5	42·8	20·5	3·8	1·8	43·2	23·0	51·0	65·0
43. Carex præcox	2·0	7·1	2·5	2·2	0·5	...	...	...	0·4	0·3	...	...	0·4	0·2	...	...
44. Ophioglossum vulgatum	0·3	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
45. Hypnum squarrosum	0·3	0·7	0·5	4·6	0·5	...	...	...	...	...	...	0·3	0·4	...	0·3	0·4
46. Hypnum rutabulum	0·3		0·2		...	...	...	...	...	...	...	...	...	...	...	0·4
47. Hypnum hians	...		0·2		...	...	...	...	...	...	...	...	...	...	...	...
Total	723·6	1072·3	431·7	701·6	932·2	601·9	361·7	250·3	522·6	919·4	373·1	169·7	854·8	1188·7	541·6	564·0

Plot Number	7				8				9				10			
Seasons	1862	1867	1872	1877	1862	1867	1872	1877	1862	1867	1872	1877	1862	1867	1872	1877
Number of Species	22	22	20	23	17	21	21	28	13	13	13	10	13	11	6	11
1. Ranunculus acris	3·5	5·3	8·1	18·9	...	25·3	12·0	39·5	...	...	...	...	0·6	...	...	...
2 and 3. Ranunculus repens et bulbosus	58·0	16·9	8·9	16·3	52·5	21·0	25·7	23·5	9·0	2·7	...	2·4	22·2	1·5	...	...
4. Ranunculus auricomus	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
5. Stellaria graminea	...	0·4	0·4	0·5	0·9	3·5	1·0	0·7	...	1·6	...	...	...	...	...	...
6. Stellaria Holostea	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
7. Cerastium triviale	2·2	0·9	0·4	0·5	3·6	4·2	8·5	6·2	...	...	...	...	0·6	...	...	...
8. Hypericum perforatum	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
9. Potentilla reptans	...	...	0·4	...	...	0·4	...	...	3·2	...	0·6	6·0	0·6	0·5	...	5·4
10. Potentilla Fragariastrum	...	...	...	...	..	...	...	...	...	...	...	...	...	...	...	...
11. Agrimonia Eupatoria	...	0·9	...	...	...	0·4	0·3	0·4	...	...	...	...	...	...	...	...
12. Poterium Sanguisorba	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
13. Spiræa Ulmaria	...	11·1	2·6	8·7	...	...	...	0·7	...	...	0·6	1·2	30·3	3·5	...	2·0
14. Conopodium denudatum	89·8	411·0	55·5	80·6	71·9	261·3	44·4	38·8	186·9	503·2	85·4	45·4	109·4	124·1	2·6	2·9
15. Pimpinella Saxifraga	38·0	53·5	9·3	14·3	36·6	73·1	27·0	24·3	0·6	0·5	0·6	...	10·5	6·0	0·4	0·5
16. Heracleum Sphondylium	...	9·4	0·9	29·1	...	...	...	8·0	...	...	3·4	9·1	...	...	...	...
17. Anthriscus sylvestris	...	...	...	...	...	..	...	...	...	...	...	...	...	...	...	...
18. Galium verum	...	...	...	0·5	1·4	15·3	9·2	1·1	...	...	...	3·6	...	...	...	...
19. Galium Aparine	...	...	...	...	...	...	...	0·4	...	...	...	...	...	...	...	...
20. Scabiosa arvensis	0·9	...	...	3·1	27·1	0·4	0·8	17·0	...	17·8	1·7	10·3	...	...	...	7·3
21. Centaurea nigra	1·3	35·2	11·4	5·1	10·0	17·2	6·2	27·5	2·6	...	1·1	...	...	...	8·7	6·8
22. Carduus arvensis	4·9	35·2	11·0	5·1	...	...	...	...	...	...	...	...	...	...	...	...
23. Bellis perennis	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
24. Achillea Millefolium	74·9	138·2	221·5	32·6	42·1	187·0	250·4	100·0	124·8	109·3	84·8	2·4	53·8	95·2	34·6	1·4
25. Chrysanthemum Leucanthemum	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
26. Tragopogon pratensis	...	8·9	5·5	...	...	0·4	...	0·4	46·1	...	...	...	...	...	...	...
27. Leontodon hispidus	...	0·5	...	...	...	...	...	...	...	...	...	...	...	...	...	...
28. Leontodon autumnalis	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
29. Taraxacum officinale	7·1	4·5	...	...	1·4	3·8	1·3	0·4	...	0·5	...	...	19·2	0·5	...	...
30. Hieracium Pilosella	0·4	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
31. Plantago lanceolata	10·2	49·0	3·0	4·6	32·1	58·5	8·7	9·4	0·6	0·5	...	...	...	...	...	0·5
32. Veronica Chamædrys	21·7	18·7	25·0	19·4	2·7	32·9	9·2	5·1	0·6	0·5	...	...	...	...	...	...
33. Veronica serpyllifolia	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
34. Prunella vulgaris	...	...	...	0·5	...	...	...	1·8	...	...	...	...	...	...	...	...
35. Thymus Serpyllum	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
36. Ajuga reptans	0·4	...	...	...	...	...	...	0·7	...	...	...	...	...	...	...	...
37. Primula veris	...	...	...	0·5	...	...	...	0·4	...	...	...	...	...	...	...	...
38. Rumex Acetosa	92·9	395·9	49·1	340·2	87·3	300·6	50·3	211·6	345·7	586·2	260·2	218·0	641·5	628·5	182·6	292·3
39. Rumex crispus	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
40. Scilla nutans	...	...	...	...	...	...	...	...	0·7	...	...	...	...	...	...	...
41. Ornithogalum umbellatum	...	...	...	...	...	...	...	...	...	...	0·6	...	...	...	...	...
42. Luzula campestris	62·4	52·2	69·5	25·5	33·9	67·3	68·3	17·4	1·3	2·7	0·6	...	2·5	1·5	0·4	0·5
43. Carex præcox	0·9	0·9	0·4	0·5	2·7	1·2	1·5	0·7	...	0·5	0·6	...	0·6	0·5	...	...
44. Ophioglossum vulgatum	0·4	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
45. Hypnum squarrosum	0·4	0·5	0·4	0·5	0·5	0·4	0·3	0·4	...	...	0·6	0·6	...	...	...	0·5
46. Hypnum rutabulum	0·4		...	0·5	...		0·3		...	...	...	...	...	...	...	...
47. Hypnum hians	0·4		...	...	...		0·3		...	...	...	...	...	...	...	...
Total	471·1	1249·1	483·3	607·5	406·7	1074·2	525·7	536·4	722·1	1226·0	440·8	299·0	891·8	861·8	229·3	320·1

Plot Number	11-1				11-2				13-1				13-2			
Seasons	1862	1867	1872	1877	1862	1867	1872	1877	1862	1867	1872	1877	1862	1867	1872	1877
Number of Species	12	4	4	4	7	5	3	5	14	9	12	8	12	6	8	8
1. Ranunculus acris	...	...	...	...	...	...	...	...	3·1	...	...	...	...	...	...	...
2 and 3. Ranunculus repens et bulbosus	2·1	...	...	...	1·4	...	...	...	4·9	0·6	...	...	0·6	...	0·8	...
4. Ranunculus auricomus	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
5. Stellaria graminea	...	...	...	...	...	...	...	...	2·5	5·2	0·7	0·6	1·3	3·2	...	...
6. Stellaria Holostea	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
7. Cerastium triviale	...	...	...	...	...	...	...	...	4·3	...	...	...	0·6	...	...	...
8. Hypericum perforatum	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
9. Potentilla reptans	...	...	...	...	...	...	...	...	0·6	0·6	...	...	...	...	...	...
10. Potentilla Fragariastrum	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
11. Agrimonia Eupatoria	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
12. Poterium Sanguisorba	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
13. Spiræa Ulmaria	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
14. Conopodium denudatum	127·2	98·6	2·9	0·7	84·7	43·8	2·1	0·8	130·5	473·8	42·2	75·9	120·8	500·5	51·1	51·2
15. Pimpinella Saxifraga	4·3	...	...	...	0·7	...	...	...	4·3	1·2	0·7	...	0·6	0·6	...	0·7
16. Heracleum Sphondylium	...	...	...	13·6	...	...	0·7	15·3	...	4·1	5·1	2·4	...	...	20·5	0·7
17. Anthriscus sylvestris	...	...	...	...	...	...	...	...	...	...	...	...	...	...	2·3	...
18. Galium verum	...	...	...	...	...	2·5	...	...	...	...	0·7	...	...	...	...	...
19. Galium Aparine	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
20. Scabiosa arvensis	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
21. Centaurea nigra	5·0	...	...	...	...	...	...	...	...	...	...	...	0·6	...	...	...
22. Carduus arvensis	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
23. Bellis perennis	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
24. Achillea Millefolium	103·1	3·2	...	0·7	29·2	0·6	...	...	118·8	69·6	71·6	0·6	48·5	108·9	12·2	0·6
25. Chrysanthemum Leucanthemum	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
26. Tragopogon pratensis	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
27. Leontodon hispidus	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
28. Leontodon autumnalis	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
29. Taraxacum officinale	10·7	...	...	...	2·9	...	...	...	3·7	...	...	...	5·2	...	...	1·9
30. Hieracium Pilosella	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
31. Plantago lanceolata	...	...	...	...	...	...	...	...	4·9	...	...	...	6·5	...	...	...
32. Veronica Chamædrys	...	...	...	...	...	...	...	...	...	...	0·6	...	...	...	...	...
33. Veronica serpyllifolia	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
34. Prunella vulgaris	...	...	0·7	...	...	...	...	...	...	...	...	...	...	...	...	...
35. Thymus Serpyllum	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
36. Ajuga reptans	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
37. Primula veris	...	...	...	...	...	...	...	...	0·6	...	...	...	...	...	...	...
38. Rumex Acetosa	499·0	212·3	77·7	152·9	290·5	223·4	50·1	113·9	313·4	249·4	213·5	310·9	380·0	217·1	164·0	555·4
39. Rumex crispus	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
40. Scilla nutans	...	...	...	...	...	...	...	...	...	...	0·6	0·6	...	...	...	...
41. Ornithogalum umbellatum	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
42. Luzula campestris	0·7	0·5	...	...	...	0·6	...	...	10·5	1·7	0·6	1·2	0·6	6·3	0·8	0·6
43. Carex præcox	0·7	...	0·7	...	0·7	...	...	...	...	...	0·6	...	...	...	...	...
44. Ophioglossum vulgatum	0·7	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
45. Hypnum squarrosum	0·7	...	...	...	...	...	...	0·9	...	...	0·6	0·6	...	...	...	...
46. Hypnum rutabulum	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	0·6
47. Hypnum hians	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Total	754·2	314·6	82·0	167·9	410·1	270·9	52·9	130·9	602·1	806·2	337·5	392·8	565·3	836·6	251·7	611·7

Plot Number	14				16				15				17			
Seasons	1862	1867	1872	1877	1862	1867	1872	1877	1862	1867	1872	1877	1862	1867	1872	1877
Number of Species	10	13	14	11	14	16	15	22	18	19	19	24	13	23	22	29
1. Ranunculus acris	...	21·6	12·4	44·4	49·2	3·0	35·4	75·6	18·8	13·2	45·4	141·5	6·2	14·7	38·2	110·3
2 and 3. Ranunculus repens et bulbosus	70·3	7·2	1·9	...	241·6	19·6	14·8	15·8	73·6	20·6	14·3	18·5	86·7	52·8	44·2	73·3
4. Ranunculus auricomus	...	...	...	...	...	...	...	7·9	...	...	...	...	...	...	...	...
5. Stellaria graminea	...	...	...	...	...	...	...	0·6	...	...	8·8	1·1	...	4·1	1·3	0·4
6. Stellaria Holostea	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
7. Cerastium triviale	0·6	0·7	0·6	3·8	1·5	3·6	0·9	4·3	43·7	194·5	338·8	22·3	12·9	50·8	101·4	14·6
8. Hypericum perforatum	...	...	...	...	...	...	...	...	...	...	...	...	...	0·5	...	3·0
9. Potentilla reptans	...	...	...	...	...	...	...	0·6	...	...	0·7	9·5	...	1·0	0·3	0·4
10. Potentilla Fragariastrum	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
11. Agrimonia Eupatoria	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
12. Poterium Sanguisorba	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
13. Spiræa Ulmaria	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
14. Conopodium denudatum	88·6	112·9	37·9	11·3	233·0	285·1	174·5	31·1	23·7	10·6	15·7	29·1	65·8	124·4	50·2	26·9
15. Pimpinella Saxifraga	1·7	1·4	1·3	0·6	0·5	...	0·5	1·2	0·4	4·8	2·9	1·5	...	0·5	0·3	0·7
16. Heracleum Sphondylium	2·3	...	...	...	...	...	...	29·9	...	...	4·0	6·1	...	...	...	17·9
17. Anthriscus sylvestris	...	109·3	239·9	290·4	...	...	4·0	3·1	...	...	...	...	...	...	...	...
18. Galium verum	...	...	...	...	...	...	...	0·6	...	56·1	44·3	8·7	...	...	...	...
19. Galium Aparine	...	...	...	...	...	...	...	...	...	...	...	0·4	...	...	...	...
20. Scabiosa arvensis	...	...	...	...	...	...	...	...	0·4	...	...	...	...	...	6·3	11·2
21. Centaurea nigra	...	0·7	...	...	...	4·8	51·0	18·9	...	9·0	94·4	34·0	196·1	208·3	341·6	105·4
22. Carduus arvensis	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
23. Bellis perennis	...	...	...	...	...	...	...	...	...	...	...	...	...	1·0	0·3	0·4
24. Achillea Millefolium	13·7	33·8	13·1	40·7	127·9	104·5	147·2	94·5	103·4	59·7	95·1	22·0	95·1	70·6	96·7	51·9
25. Chrysanthemum Leucanthemum	...	...	...	...	...	...	...	...	...	...	...	...	...	...	1·4	...
26. Tragopogon pratensis	...	...	...	...	0·5	1·2	0·4	6·7	...	...	...	...	...	...	...	3·0
27. Leontodon hispidus	...	...	0·6	...	...	1·2	...	...	0·4	0·5	...	0·4	0·9	2·5	2·3	9·3
28. Leontodon autumnalis	...	...	...	...	...	...	...	...	1·2	1·6	0·7	...	...	0·5	...	...
29. Taraxacum officinale	10·9	16·5	11·2	39·4	8·2	0·6	...	...	49·0	5·8	3·7	3·0	6·7	2·5	3·0	4·5
30. Hieracium Pilosella	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	0·4
31. Plantago lanceolata	10·3	0·7	0·6	...	64·7	49·9	3·1	13·4	282·8	246·9	10·3	21·2	171·2	245·4	80·1	298·7
32. Veronica Chamædrys	...	...	...	...	6·2	2·9	2·7	17·7	3·3	2·1	...	...	4·5	7·6	3·3	7·8
33. Veronica serpyllifolia	...	...	...	...	...	...	...	...	...	...	...	0·4	...	...	...	0·4
34. Prunella vulgaris	...	...	...	...	...	...	...	...	...	...	...	...	...	0·5	...	0·4
35. Thymus Serpyllum	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
36. Ajuga reptans	...	...	...	...	...	...	0·4	...	0·4	...	...	0·8	...	1·5	0·7	3·0
37. Primula veris	...	...	...	...	...	...	...	0·6	...	...	...	...	...	0·5	...	...
38. Rumex Acetosa	393·4	79·8	37·9	275·3	282·7	333·8	53·3	133·5	271·3	388·0	75·4	219·1	158·7	382·7	52·5	95·7
39. Rumex crispus	...	...	...	8·2	...	...	...	...	...	...	...	...	...	...	...	...
40. Scilla nutans	...	...	...	0·6	...	...	...	...	...	...	...	0·4	...	...	...	...
41. Ornithogalum umbellatum	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
42. Luzula campestris	...	0·7	...	...	6·2	7·7	2·7	9·8	4·1	14·8	17·2	16·3	5·3	24·9	16·3	26·5
43. Carex præcox	...	...	0·6	...	...	...	...	...	0·4	0·5	0·7	1·1	...	1·0	1·7	0·7
44. Ophioglossum vulgatum	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
45. Hypnum squarrosum	...	...	0·6	...	0·5	0·6	...	0·6	...	...	0·4	0·4	...	...	...	0·4
46. Hypnum rutabulum	...	...	...	...	...		...	...	0·4	0·5	...	...	...	...	0·3	0·4
47. Hypnum hians	...	...	...	0·6	...		...	0·6	...	...	...	0·4	...	...	...	0·4
Total	591·8	385·3	358·6	715·3	1022·7	818·5	490·9	467·0	877·3	1029·2	772·8	558·2	810·1	1198·3	842·4	868·0

Plot Number	18			19		20		Plot Number.	
Seasons	1867	1872	1877	1872	1877	1872	1877	Seasons.	
Number of Species	21	22	21	21	18	22	18	Number of Species.	
1. Ranunculus acris	13·4	27·3	20·7	13·5	28·8	0·4	9·8	Upright Crowfoot or Buttercup	1
2 and 3. Ranunculus repens et bulbosus	248·2	116·3	115·2	90·7	9·0	169·5	197·0	Creeping and Bulbous Buttercup	2 and 3
4. Ranunculus auricomus	...	...	...	...	...	...	...	Goldilocks	4
5. Stellaria graminea	3·5	9·7	8·5	0·9	0·5	0·4	...	Lesser Stichwort	5
6. Stellaria Holostea	...	...	...	...	...	...	...	Greater Stichwort	6
7. Cerastium triviale	5·2	5·6	3·6	18·4	9·5	19·4	10·3	Mouse-ear Chickweed	7
8. Hypericum perforatum	...	...	...	...	...	...	...	Perforated St. John's Wort	8
9. Potentilla reptans	...	0·4	...	...	...	...	...	Creeping Cinque Foil	9
10. Potentilla Fragariastrum	...	...	...	...	...	...	...	Barren Strawberry	10
11. Agrimonia Eupatoria	...	...	...	0·4	0·5	...	...	Common Agrimony	11
12. Poterium Sanguisorba	...	...	...	...	...	...	...	Common Burnet	12
13. Spiræa Ulmaria	...	...	...	...	...	...	...	Meadow Sweet	13
14. Conopodium denudatum	159·9	90·5	143·0	88·0	211·3	87·1	136·8	Earth or Pig Nut	14
15. Pimpinella Saxifraga	3·0	1·5	0·9	3·2	1·4	2·2	3·1	Burnet Saxifrage	15
16. Heracleum Sphondylium	...	...	0·5	...	...	...	...	Cow Parsnip or Hogweed	16
17. Anthriscus sylvestris	...	...	...	...	...	...	...	Beaked Parsley	17
18. Galium verum	...	0·7	3·2	0·4	...	4·8	2·1	Yellow Bed Straw	18
19. Galium Aparine	...	...	0·5	...	...	...	0·5	Cleavers or Goose Grass	19
20. Scabiosa arvensis	...	...	...	...	...	...	...	Field Scabious	20
21. Centaurea nigra	34·2	31·4	39·0	3·6	3·3	0·4	8·2	Black Knapweed	21
22. Carduus arvensis	0·4	...	...	...	...	...	...	Creeping Plume Thistle	22
23. Bellis perennis	0·4	...	...	0·4	...	0·4	...	Daisy	23
24. Achillea Millefolium	32·9	151·8	59·4	30·6	31·2	33·7	41·1	Yarrow or Milfoil	24
25. Chrysanthemum Leucanthemum	...	...	5·4	...	...	...	...	Oxeye Daisy	25
26. Tragopogon pratensis	...	0·4	...	...	...	...	...	Goat's Beard	26
27. Leontodon hispidus	...	...	...	...	...	...	...	Rough Hawkbit	27
28. Leontodon autumnalis	...	...	...	...	...	...	...	Autumnal Hawkbit	28
29. Taraxacum officinale	3·5	1·9	4·5	...	...	0·9	...	Dandelion	29
30. Hieracium Pilosella	0·4	...	...	...	...	...	...	Mouse-ear Hawkbit	30
31. Plantago lanceolata	120·9	1·5	2·2	7·6	13·7	1·7	1·0	Ribwort Plantain	31
32. Veronica Chamædrys	9·5	4·5	2·7	5·4	32·6	1·7	16·5	Germander Speedwell	32
33. Veronica serpyllifolia	...	...	...	...	...	...	...	Thyme-leaved Speedwell	33
34. Prunella vulgaris	0·9	...	...	0·4	...	0·4	0·5	Self Heal	34
35. Thymus Serpyllum	...	...	...	...	...	...	...	Wild Thyme	35
36. Ajuga reptans	...	...	...	0·4	3·8	0·4	2·1	Bugle	36
37. Primula veris	...	...	...	...	...	...	...	Cowslip	37
38. Rumex Acetosa	1054·0	73·7	209·7	32·3	125·3	54·4	77·7	Sorrel	38
39. Rumex crispus	...	...	...	...	...	...	...	Curled-leaved Dock	39
40. Scilla nutans	...	0·4	...	...	...	0·4	...	Bluebell	40
41. Ornithogalum umbellatum	...	...	...	...	...	...	...	Star of Bethlehem	41
42. Luzula campestris	19·1	64·3	1·8	63·8	15·6	87·1	18·5	Wood Rush	42
43. Carex præcox	2·6	1·5	0·5	1·8	0·5	1·3	...	Vernal Carex or Sedge	43
44. Ophioglossum vulgatum	...	...	...	...	...	...	0·5	Adder's Tongue	44
45. Hypnum squarrosum	0·4	0·4	0·5	3·2	0·9	0·4	0·5	Mosses	45
46. Hypnum rutabulum		0·4	0·5	0·4	0·5	0·4	...		46
47. Hypnum hians		0·4	...	...	...	...	...		47
Total	1712·4	584·6	622·3	365·3	488·4	467·4	526·2	Total.	

INDEX

TO THE

PHILOSOPHICAL TRANSACTIONS

FOR THE YEAR 1882.

Fig 6.

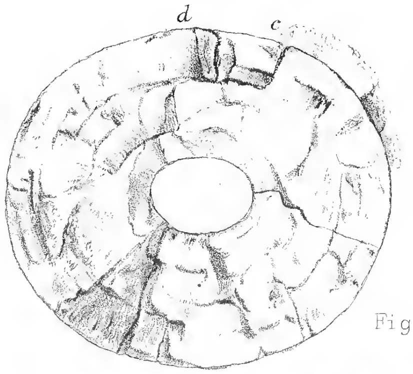


Fig. 4.

Fig. 5.

Fig. 7.

Fig. 8

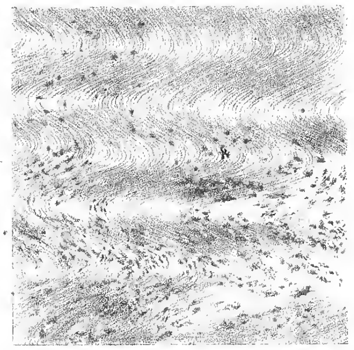


Fig. 9.

Fig. 10.

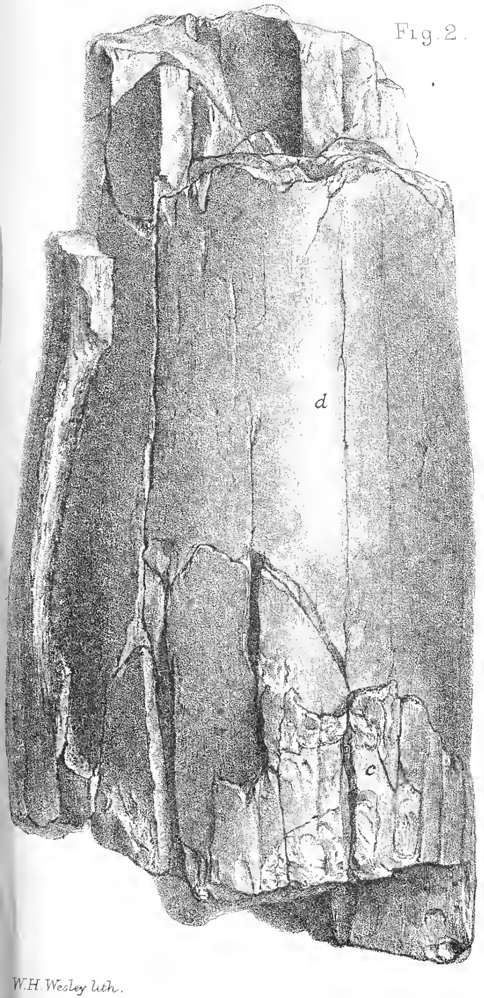


Fig. 2.

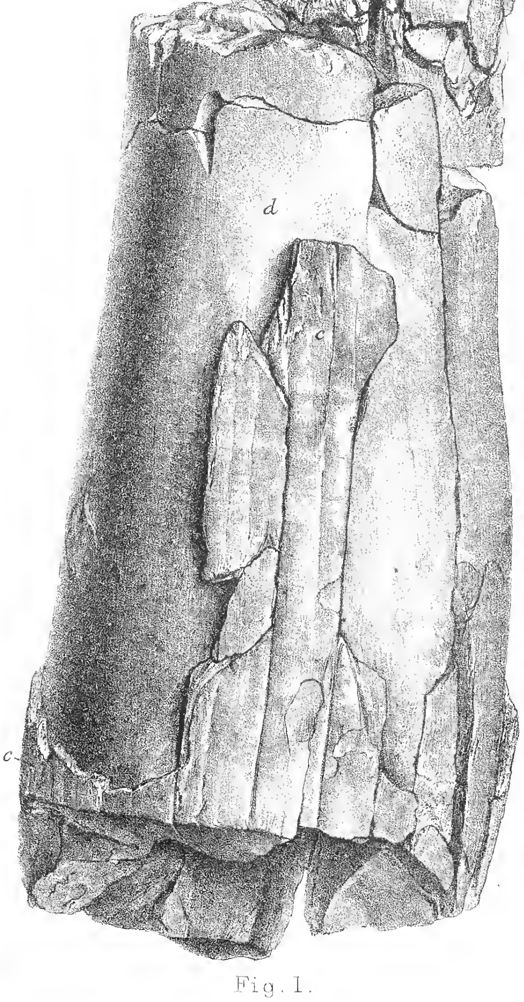


Fig. 1.

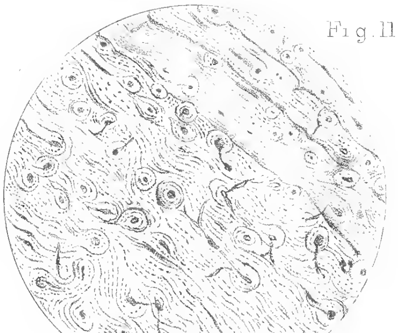


Fig. 11.

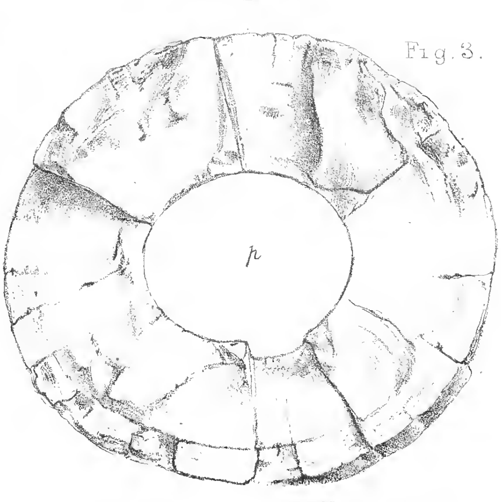


Fig. 3.

W.H. Wesley lith.

West, Newman & Co. imp.

Fig. 4.

Fall 25. (See Fifth set of Experiments.)

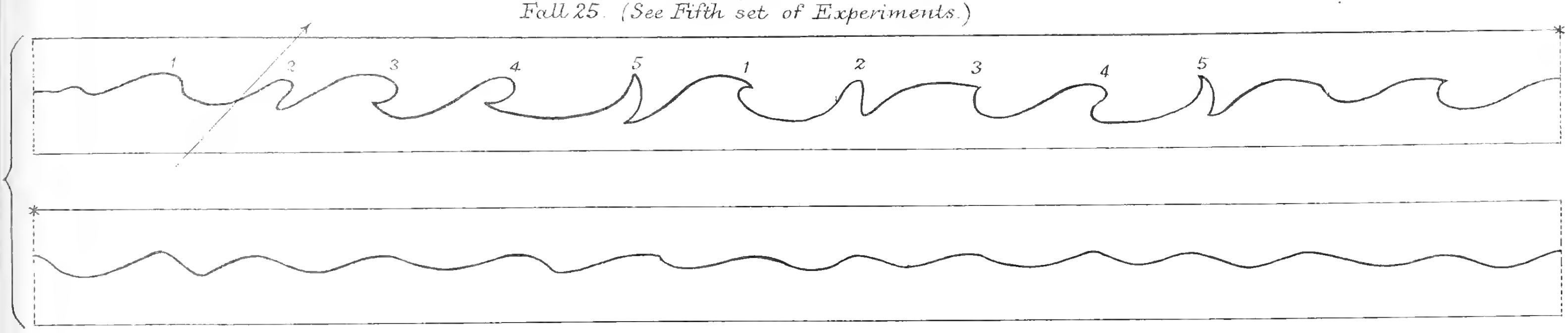


Fig. 5.

See Fifth set of Experiments. Fall 17.

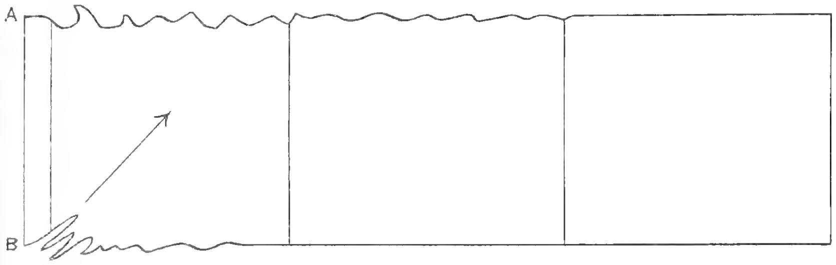


Fig 6.

See Seventh set of Experiments. Fall 36.

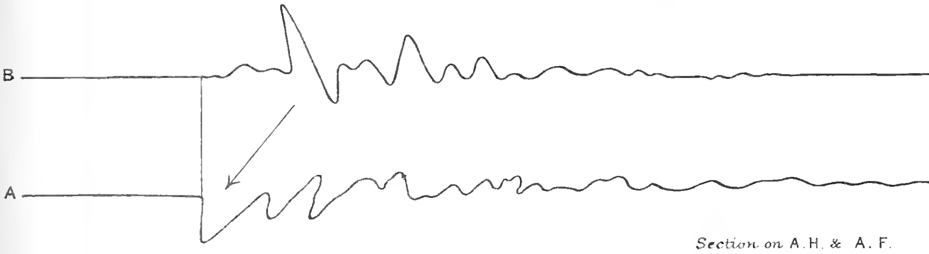
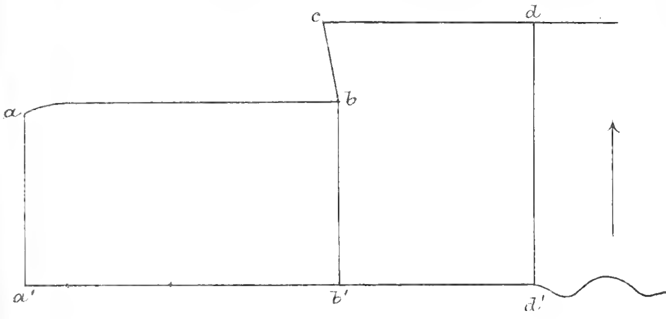
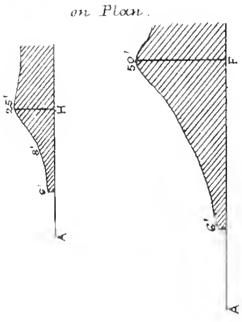


Fig. 7.

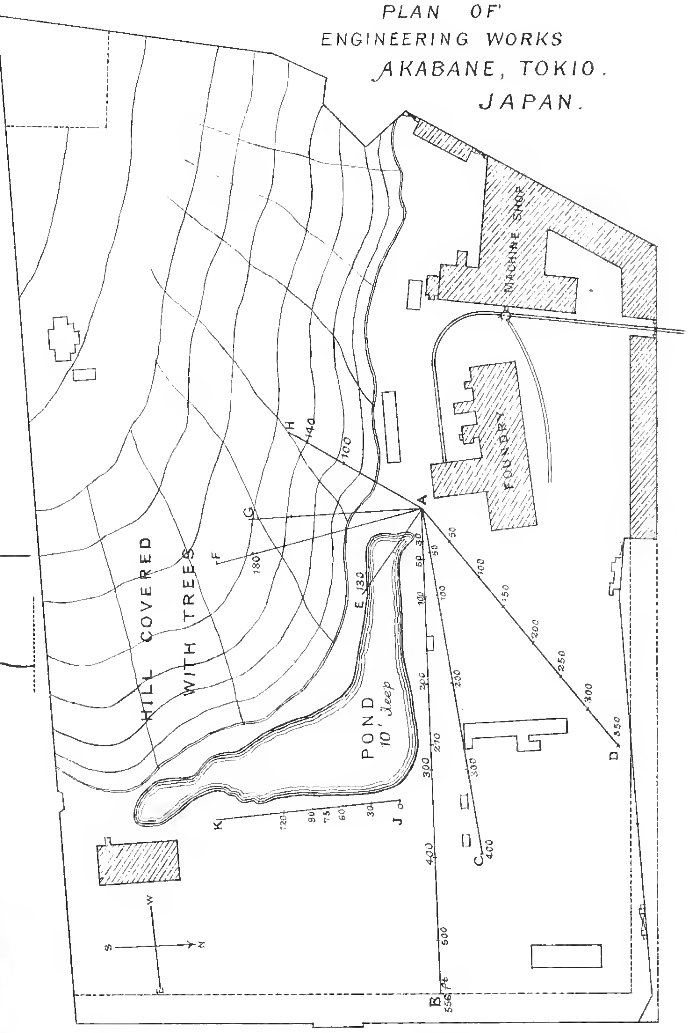
See Sixth set of Experiments. Fall 33.



Section on A.H. & A.F. on Plan.



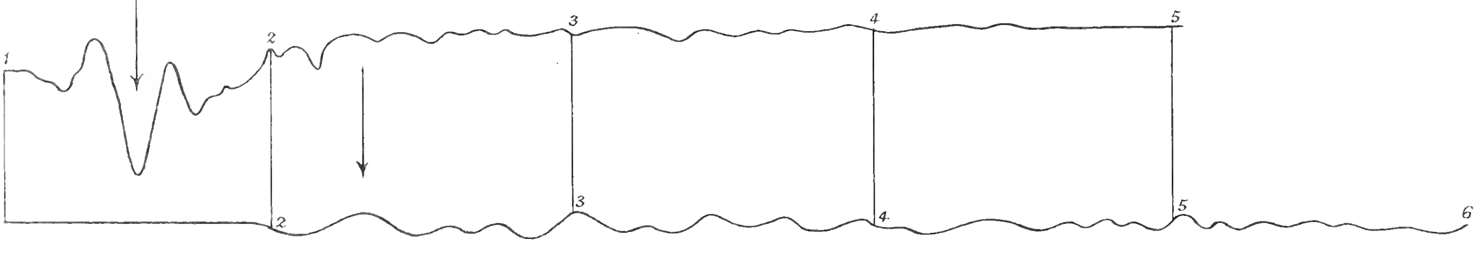
PLAN OF ENGINEERING WORKS AKABANE, TOKIO. JAPAN.



SCALE: $\frac{3}{32}$ inches = 20 feet for Plan.
" = 40 " for vertical Scale in Sections.

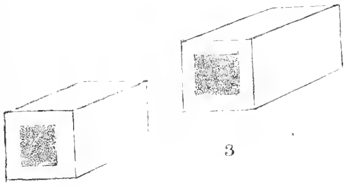
Fig. 8.

See Seventh set of Experiments. Fall 37 or 38



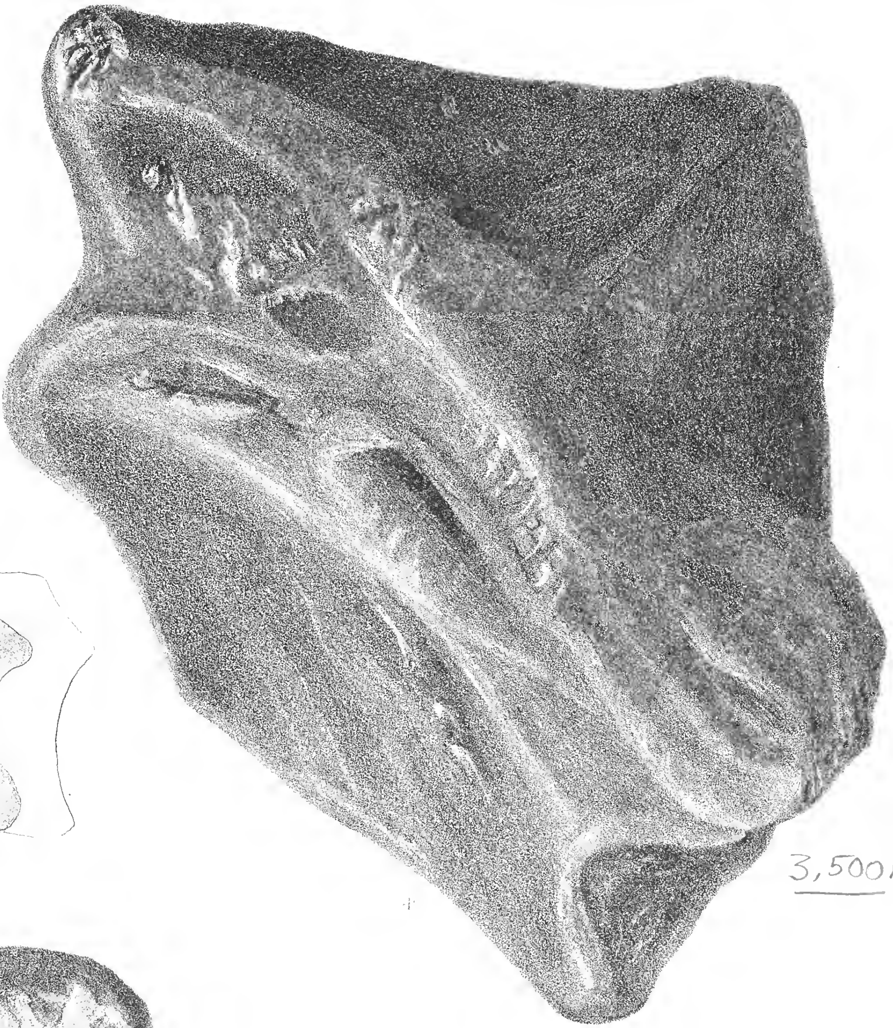
West Newman & Co. lith

6



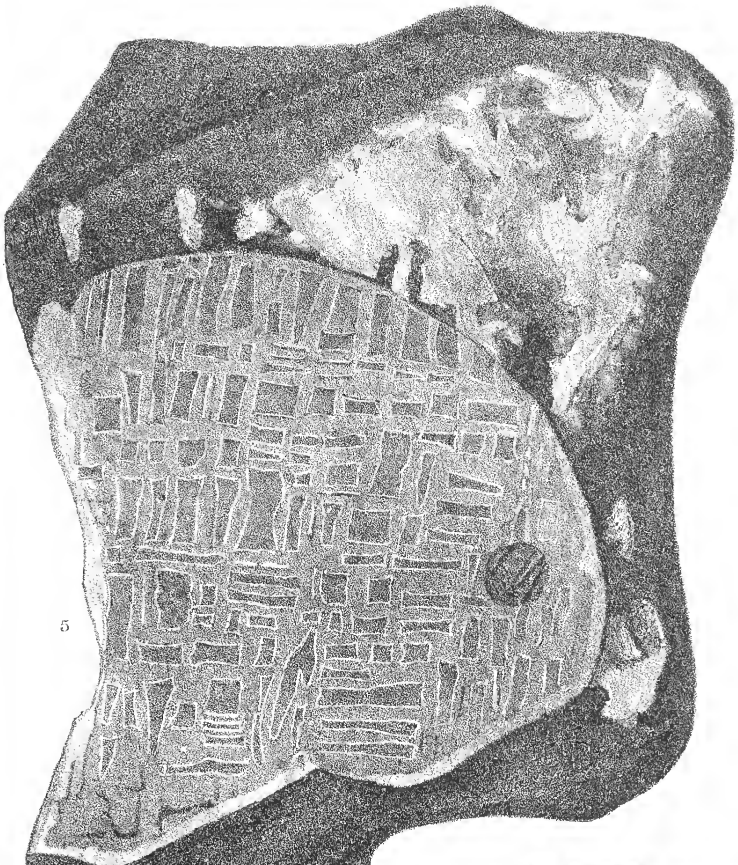
3

2



4

3,500 kg



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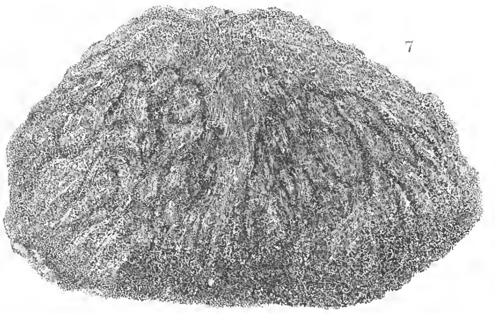

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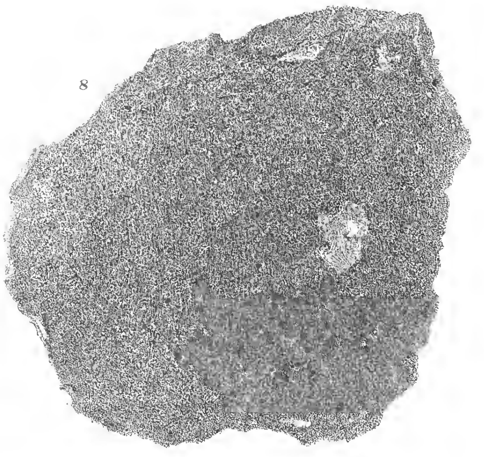

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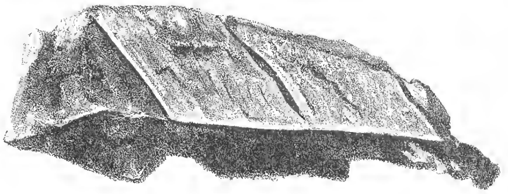


1

1.2.3 & 5. K.F. } del. ad nat.
4. C.M. }
6.7 & 8 photograph.

West, Newman & Co. lith.

Fig. 1.

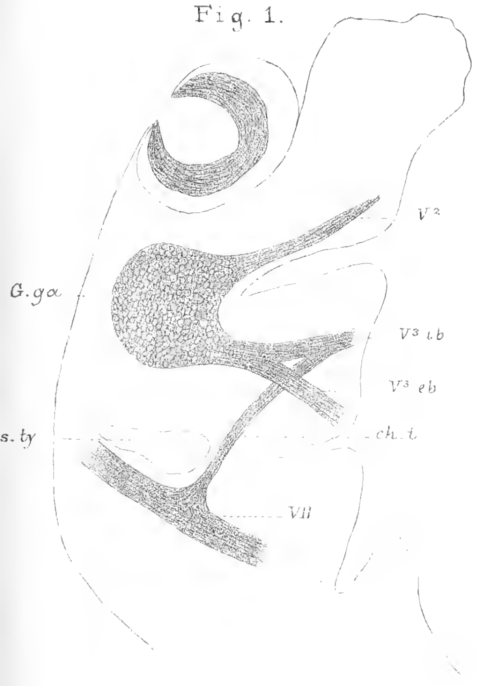


Fig. 4.

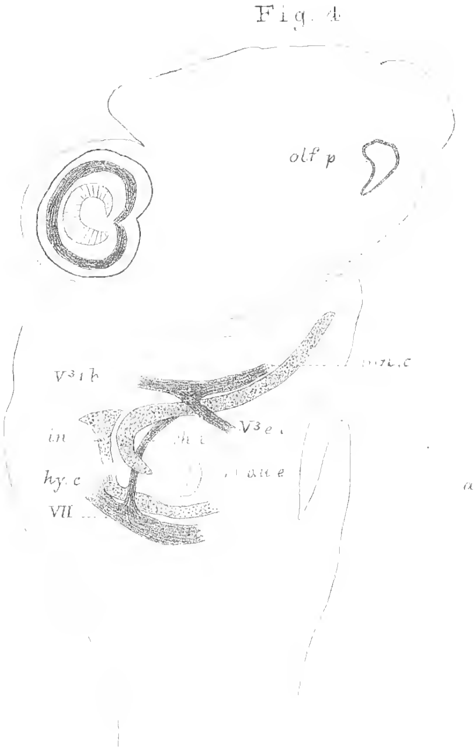


Fig. 2.

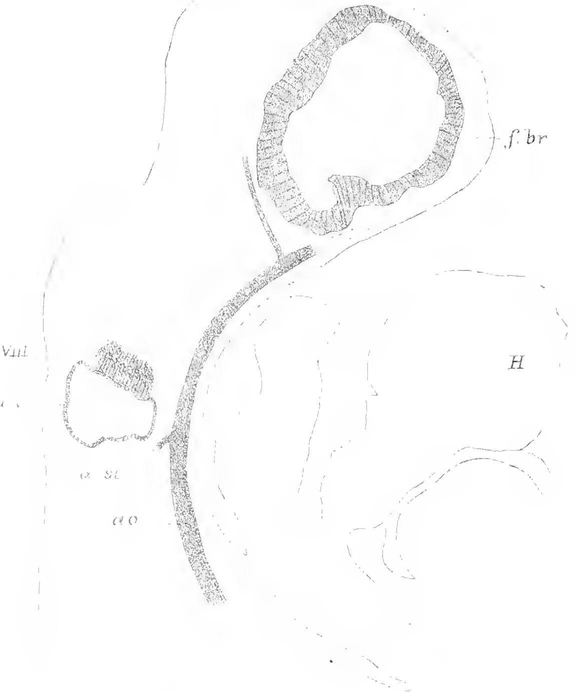


Fig. 5.

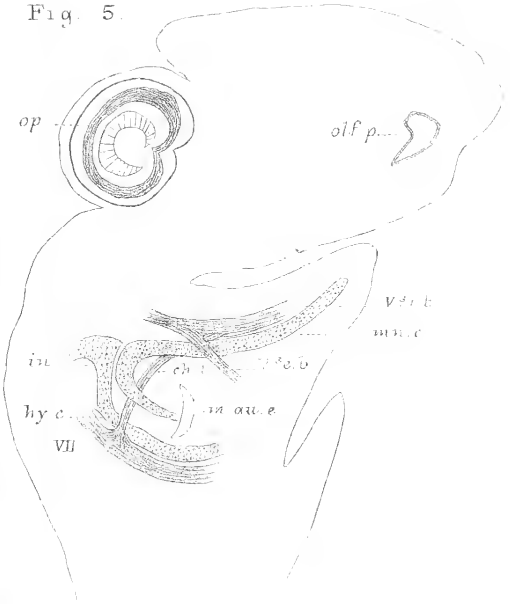


Fig. 3.

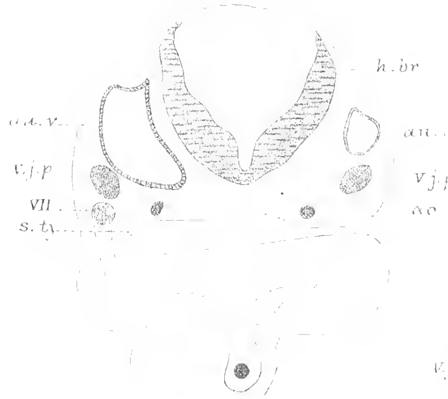
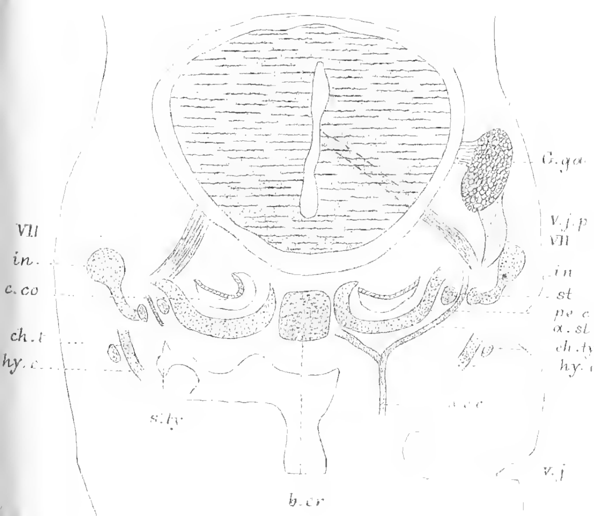


Fig. 6.

Fig. 7.



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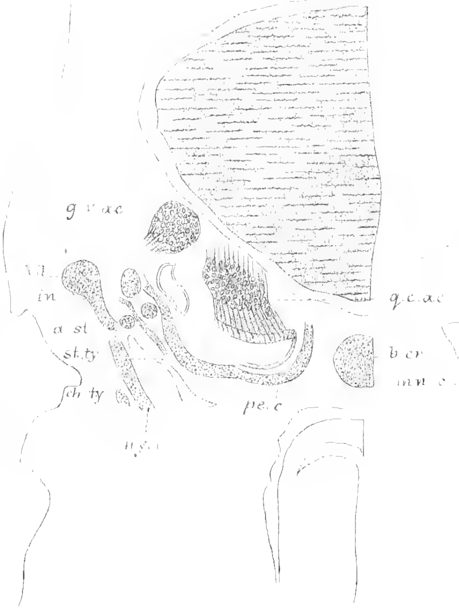

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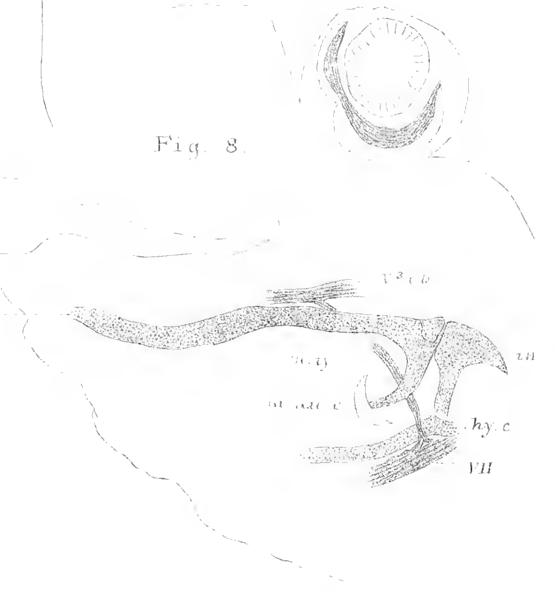
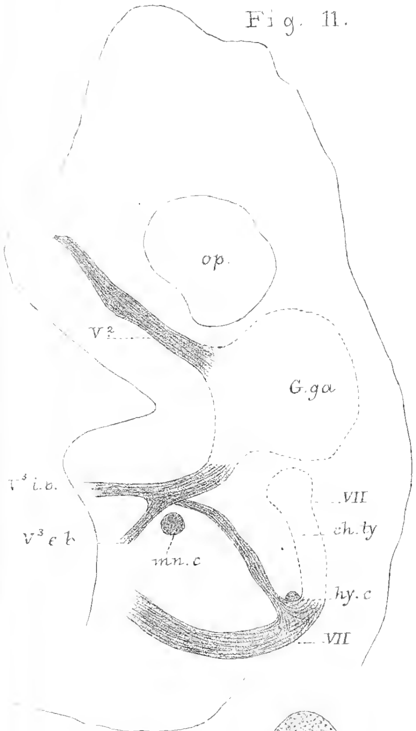


A. Fraser del!

All Figs × 25

West Newman & Co lith.

Fig. 11.



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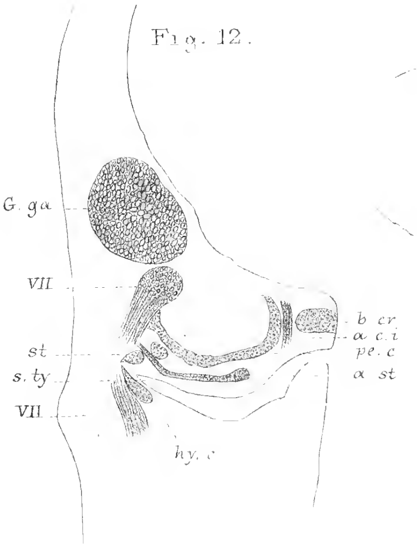

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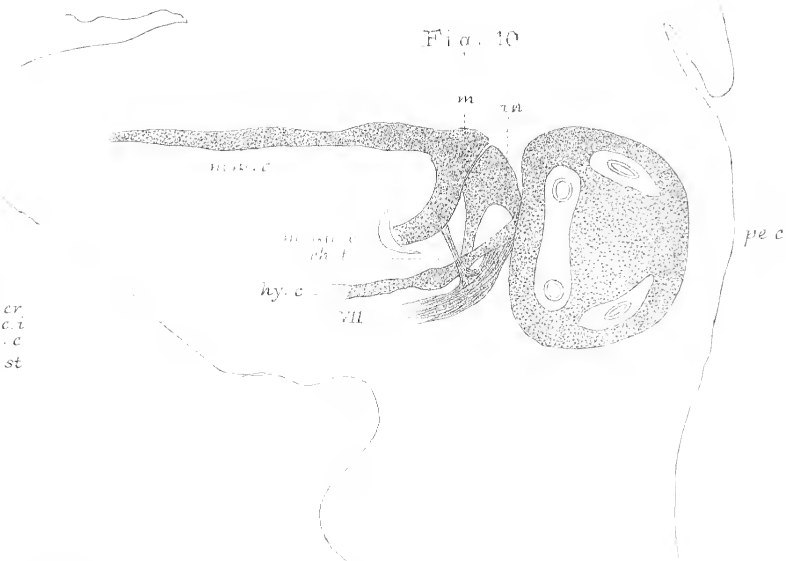


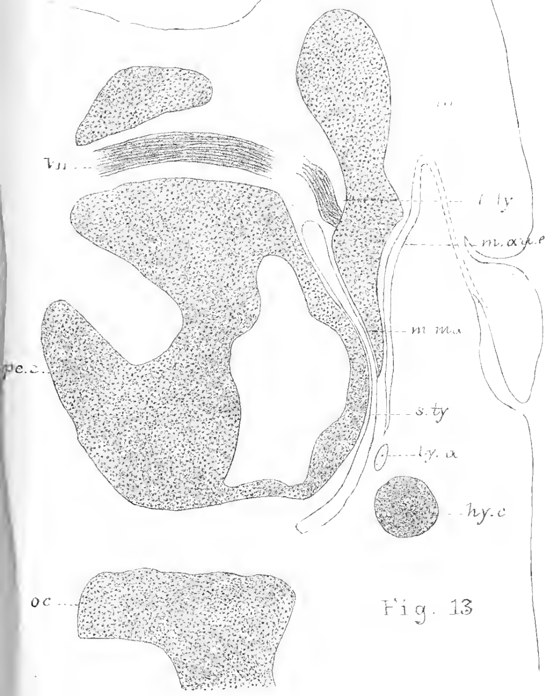
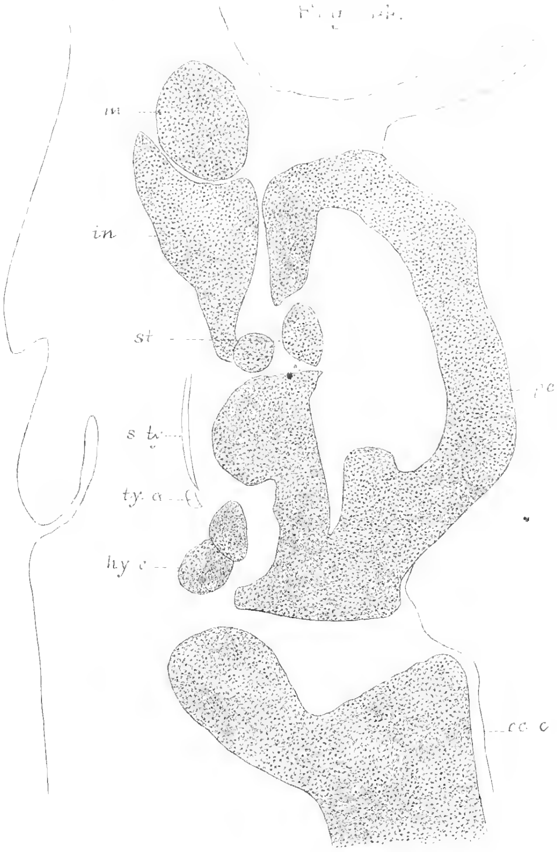


Fig. 13

Fig. 14.



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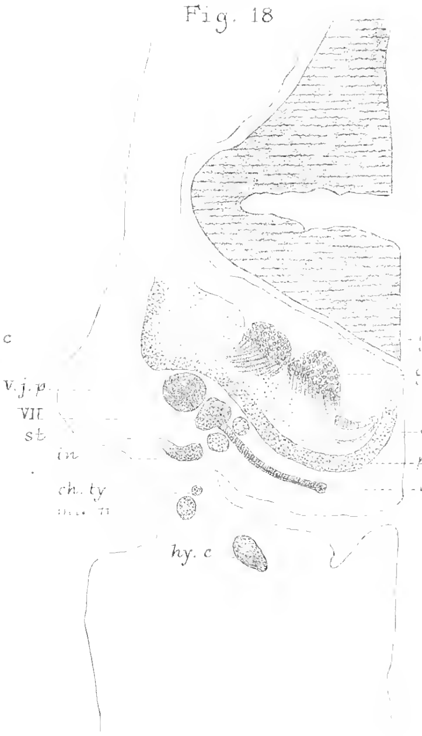

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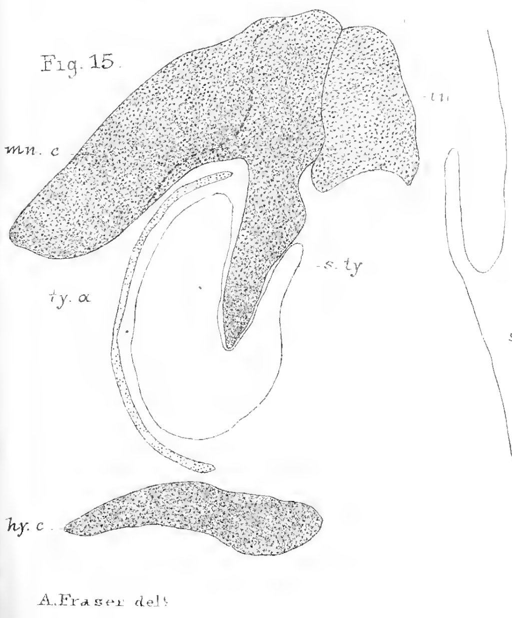

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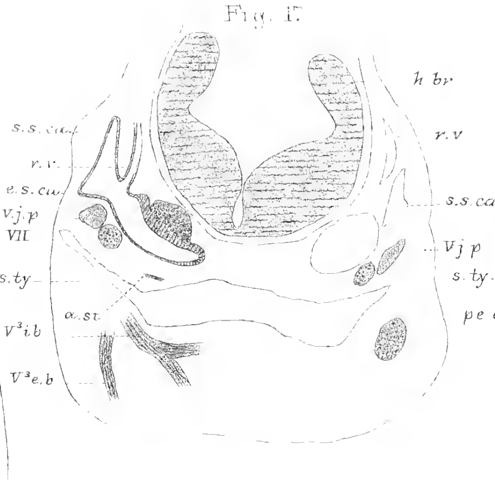

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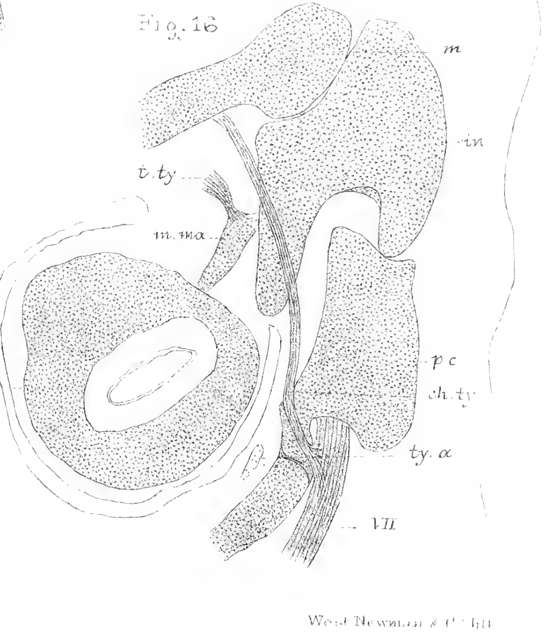
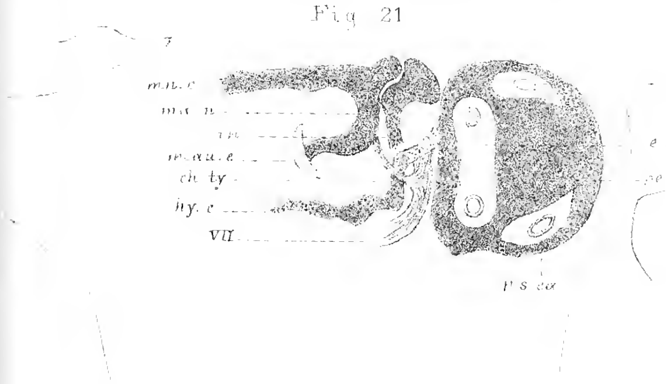


A. Fraser del.t

All Fig.s × 25.

West Newman & C.o lith

Fig 21



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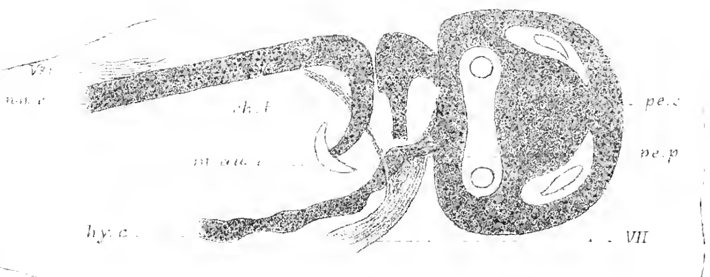

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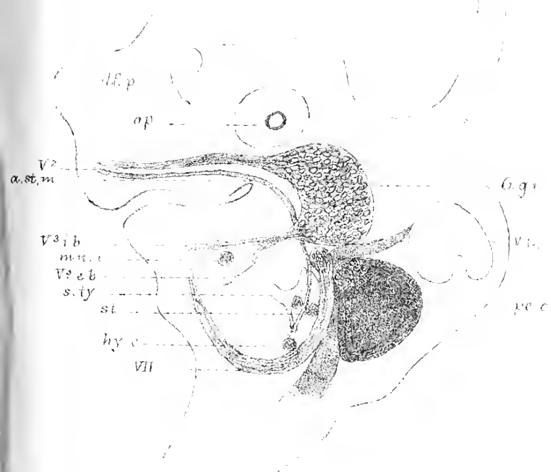

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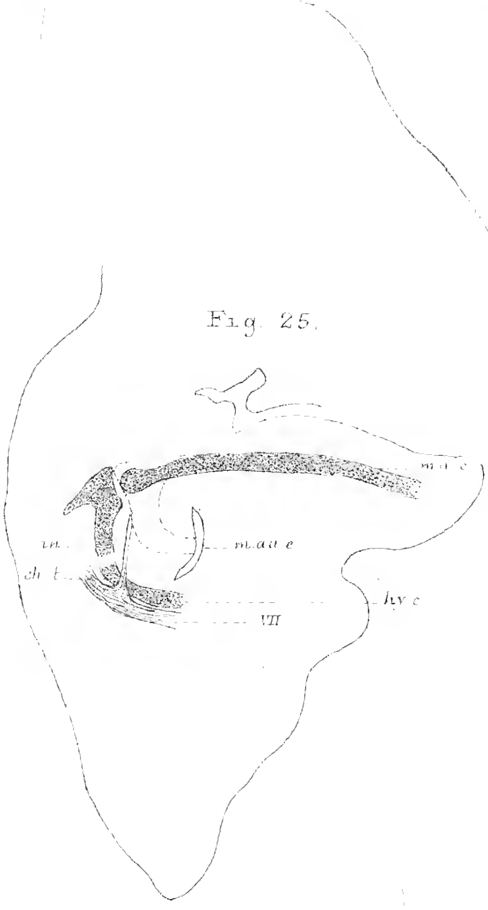

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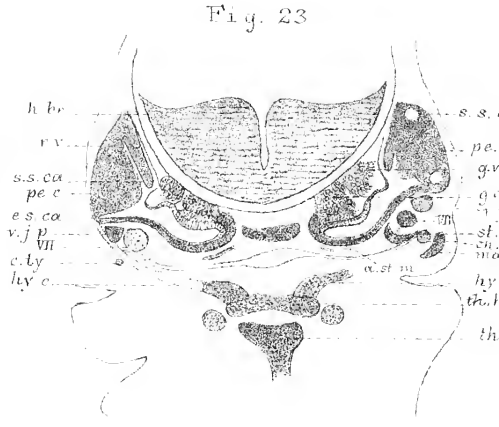

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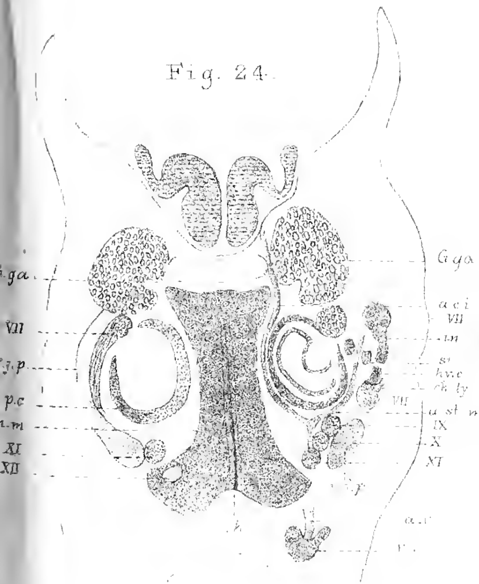
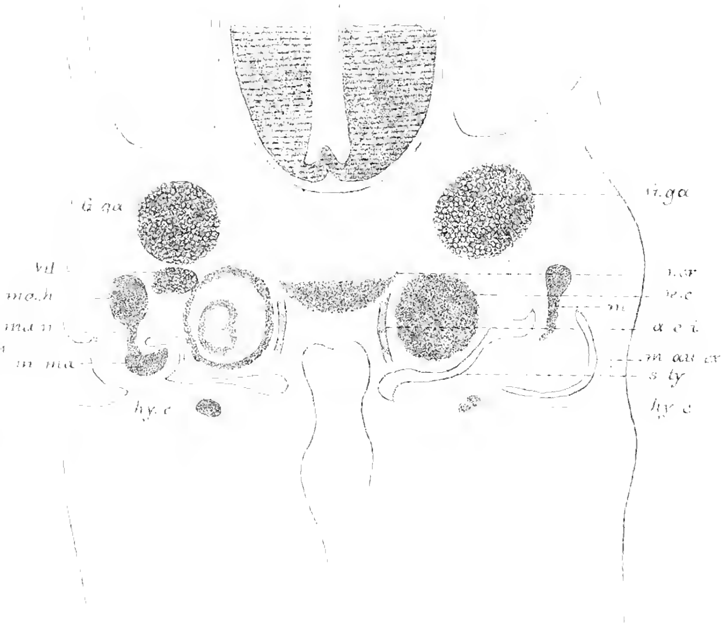


Fig 20

Fig. 26



All Figs × 25

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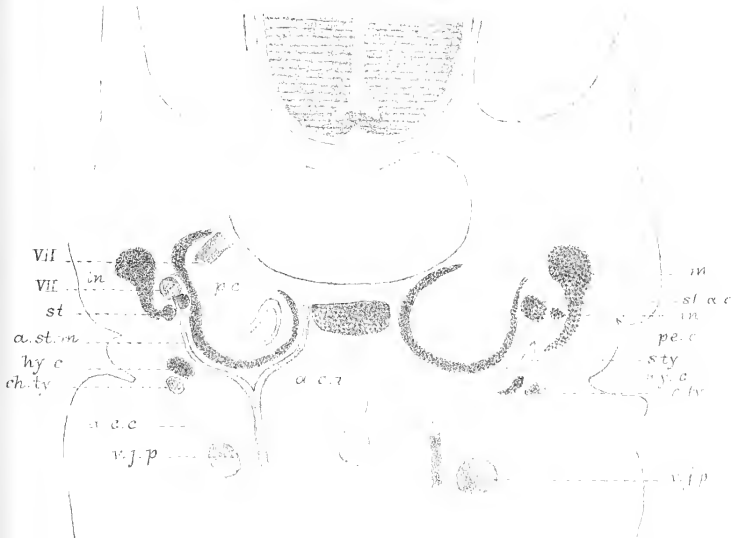

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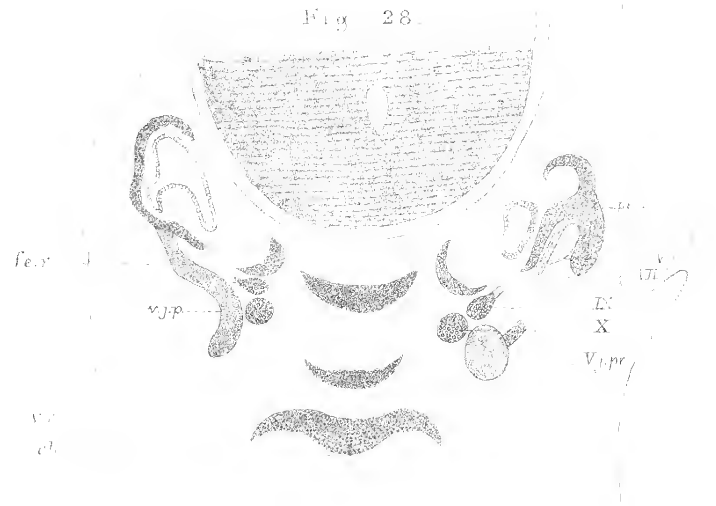
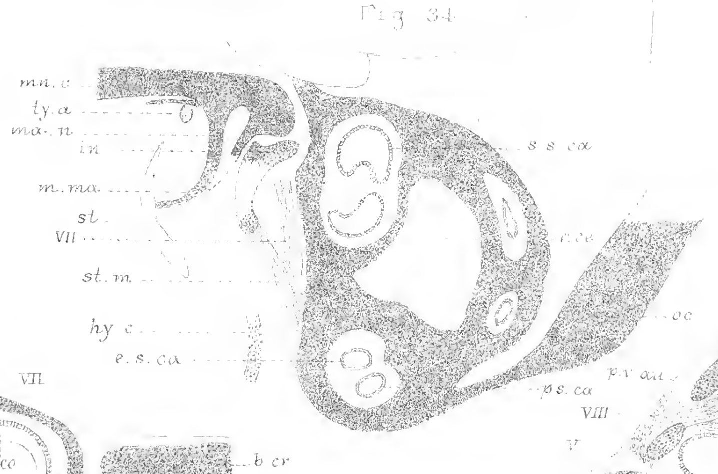


Fig. 29.

Fig. 33.

Fig. 30.

Fig. 34.



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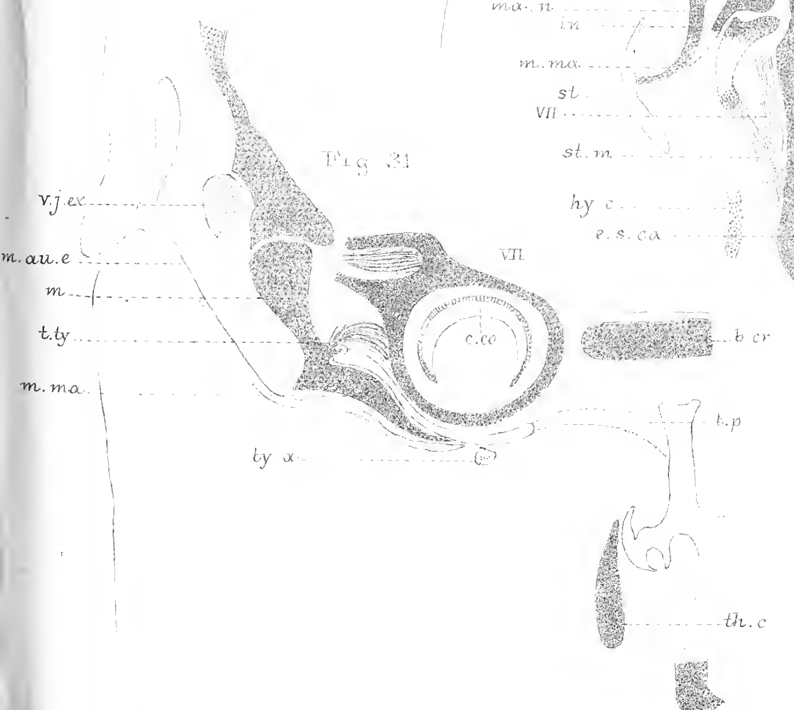

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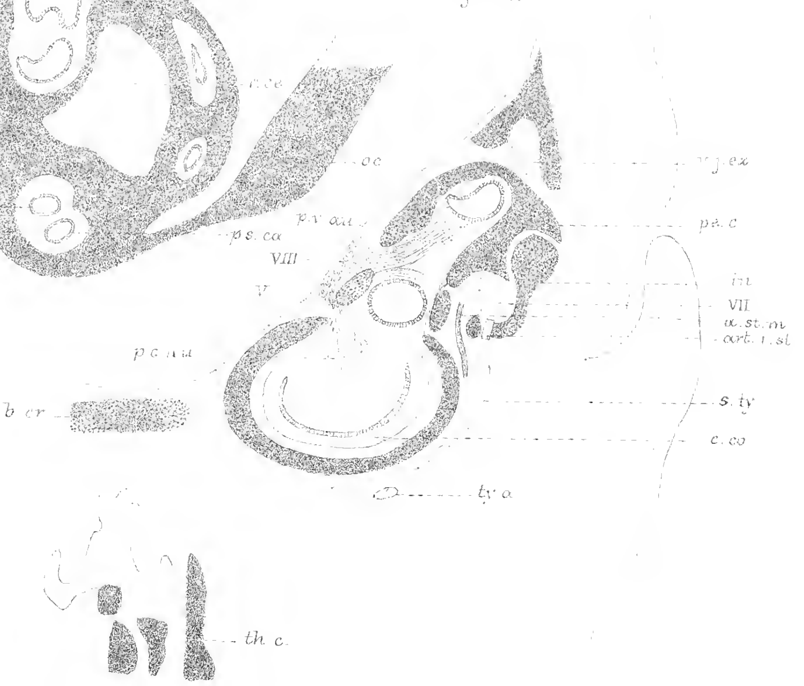


A. Fraser del.t

All Fig.s × 25.

West Newman & C.o Lith

Fig. 36 × 25.

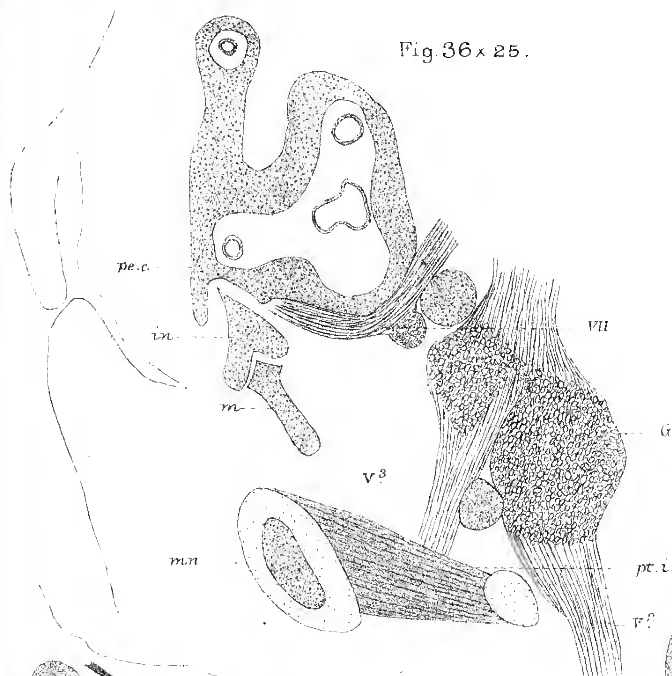
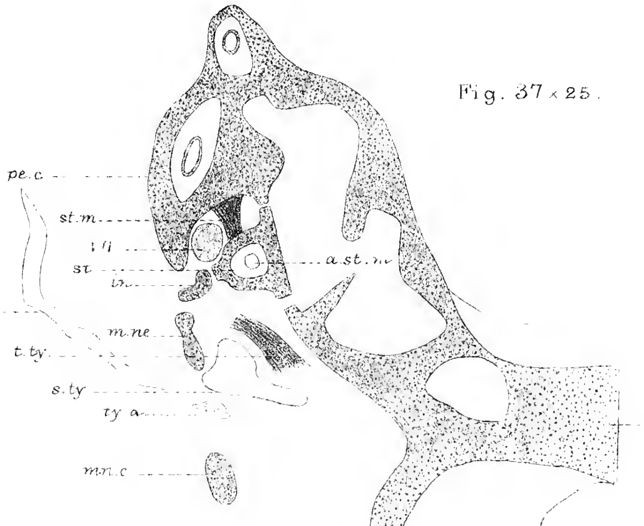


Fig. 37 × 25.



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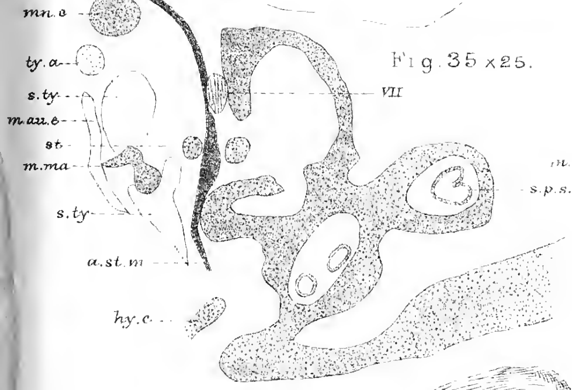


Fig. 38 × 25.

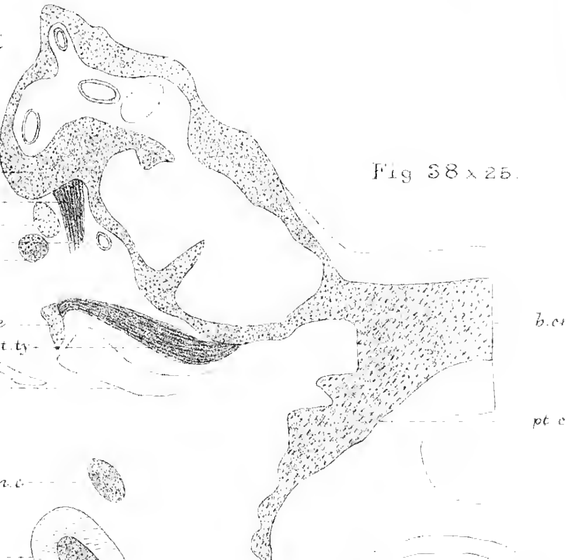


Fig. 40 × 10.

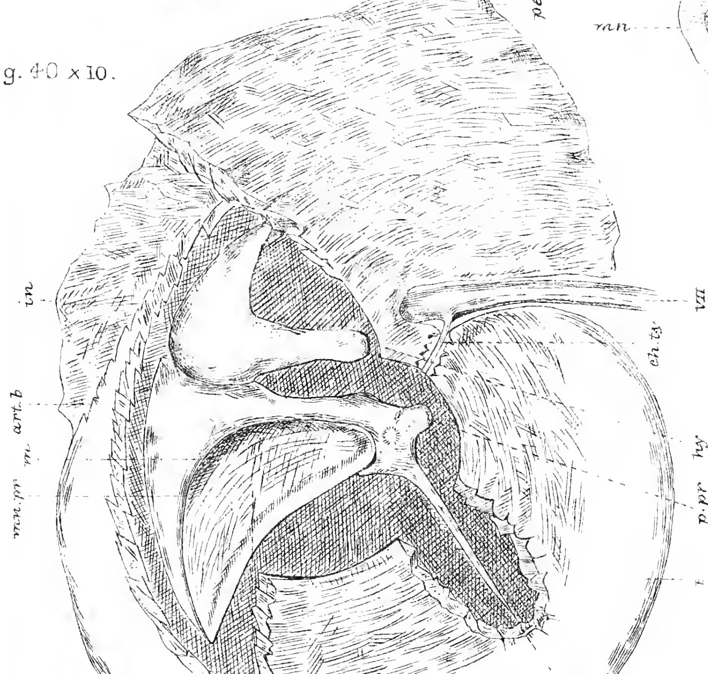


Fig. 39 × 14.

Fig. 40.A × 10.

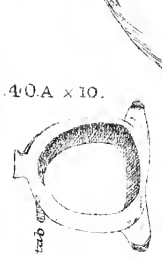


Fig. 40.B × 10.

A. Fraser del.t

West, Newman & Co. lith.

Fig. 1

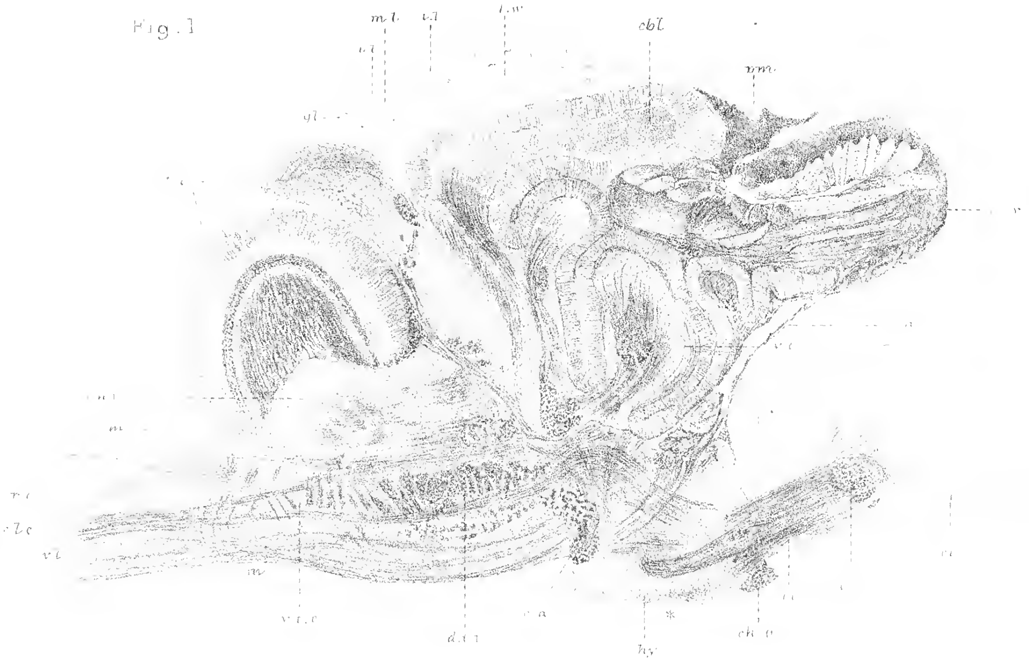


Fig. 2

Fig. 3.

A. Sanders del.

West, Newman & Co. lith.

Sanders.

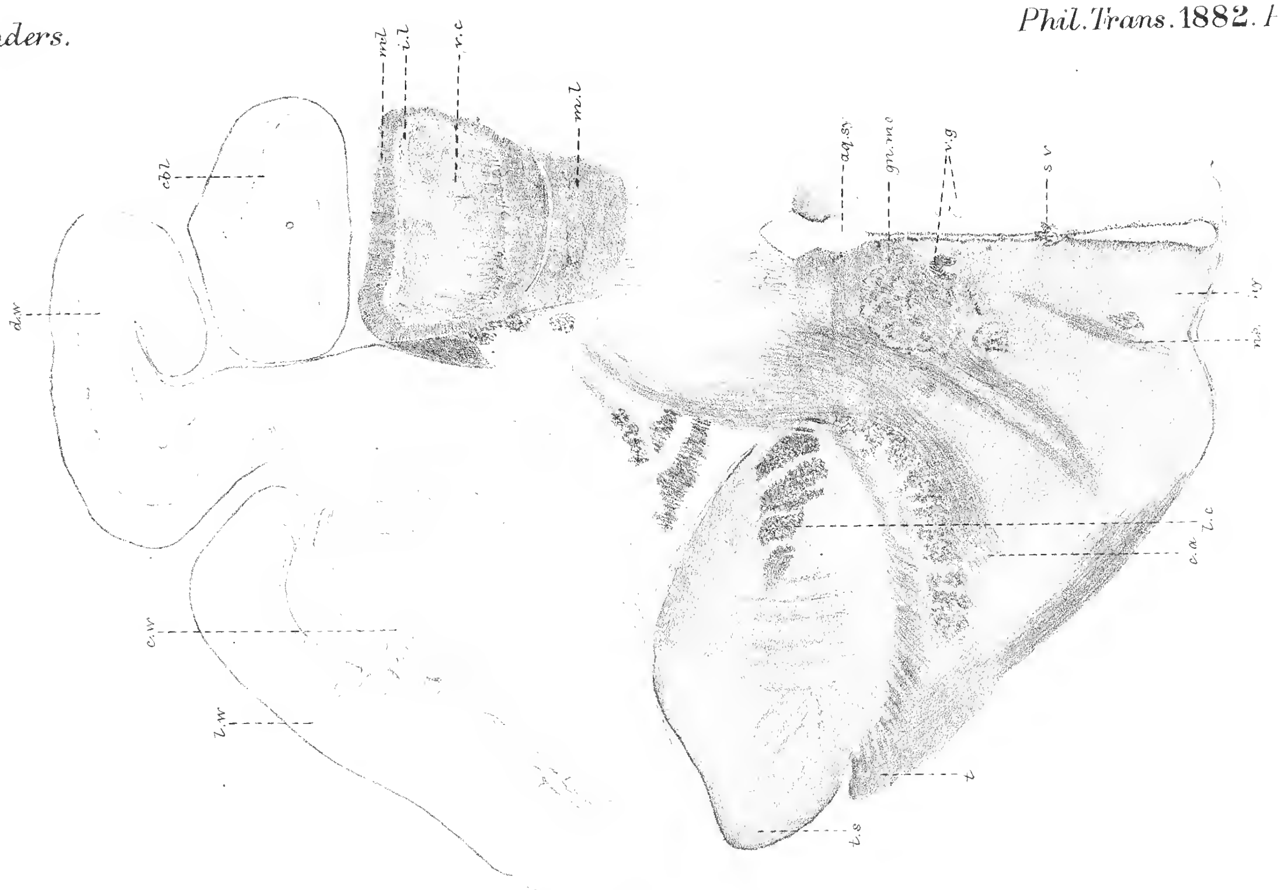
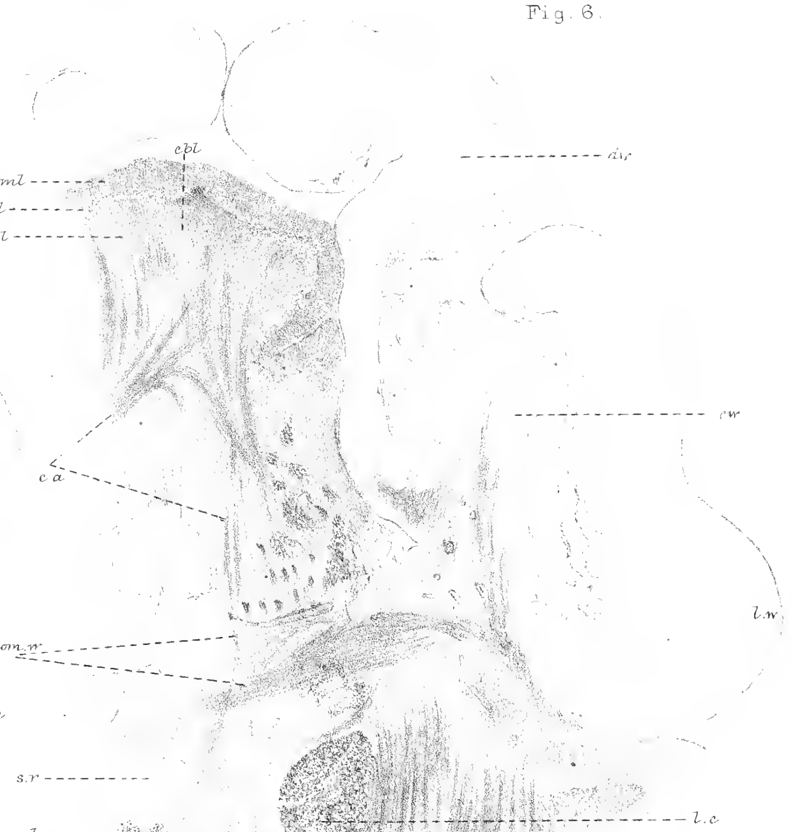


Fig. 4.

Fig. 5.

Fig. 6.



A. Sanders del^t

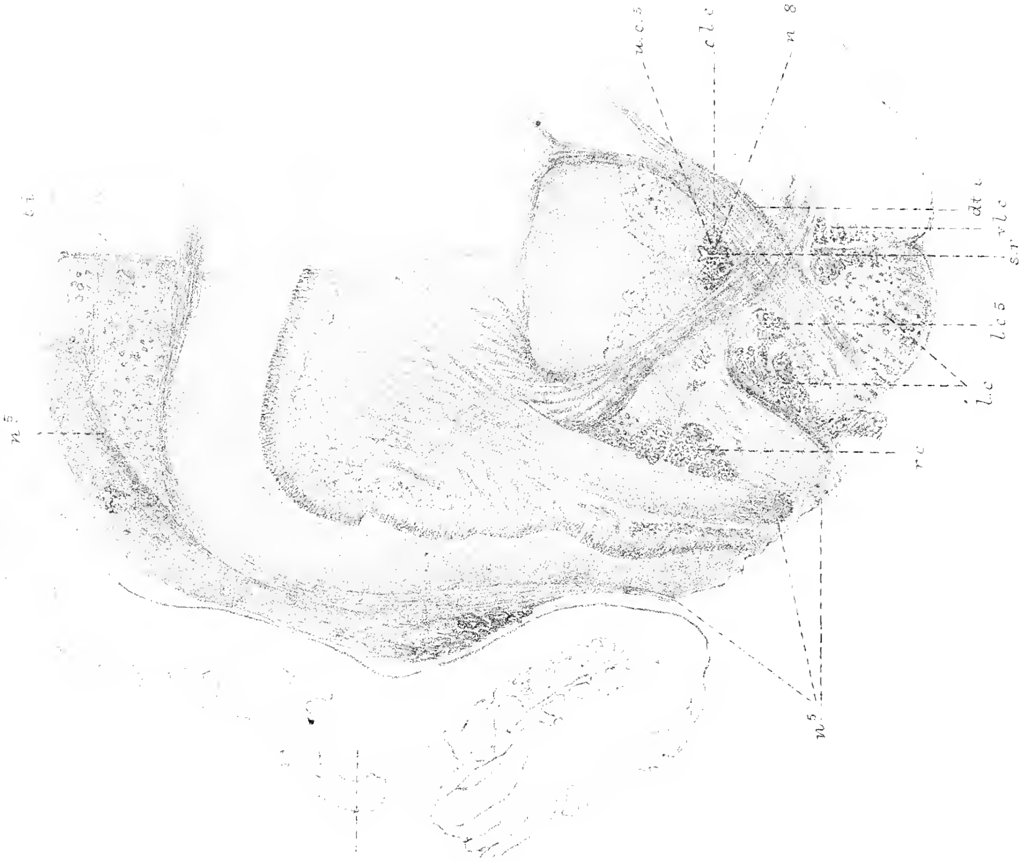
West Newman & Co. lith.

Sanders.

Phil. Trans. 1882. Plate 61.

Fig 8.

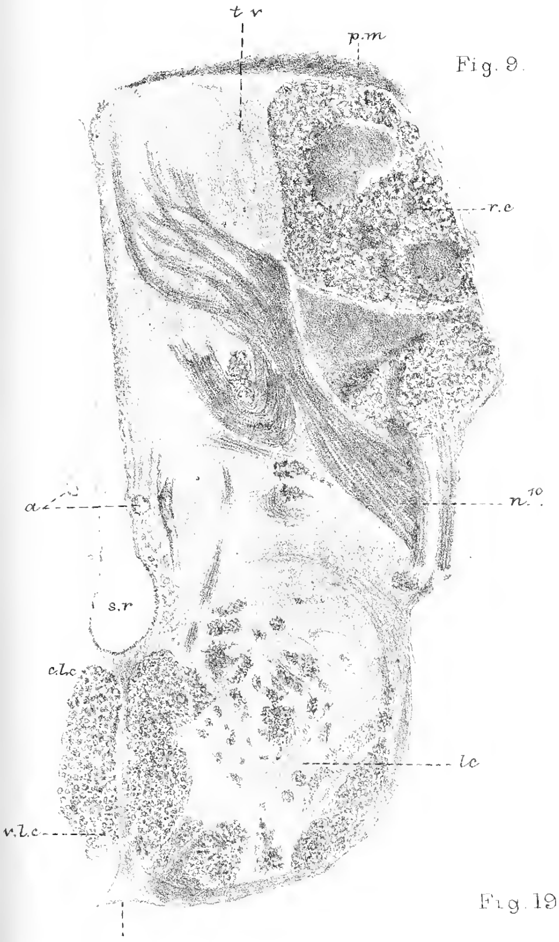
Fig 7.



A. Sanders del^t

West Newman & C^o lith.

Fig. 9.



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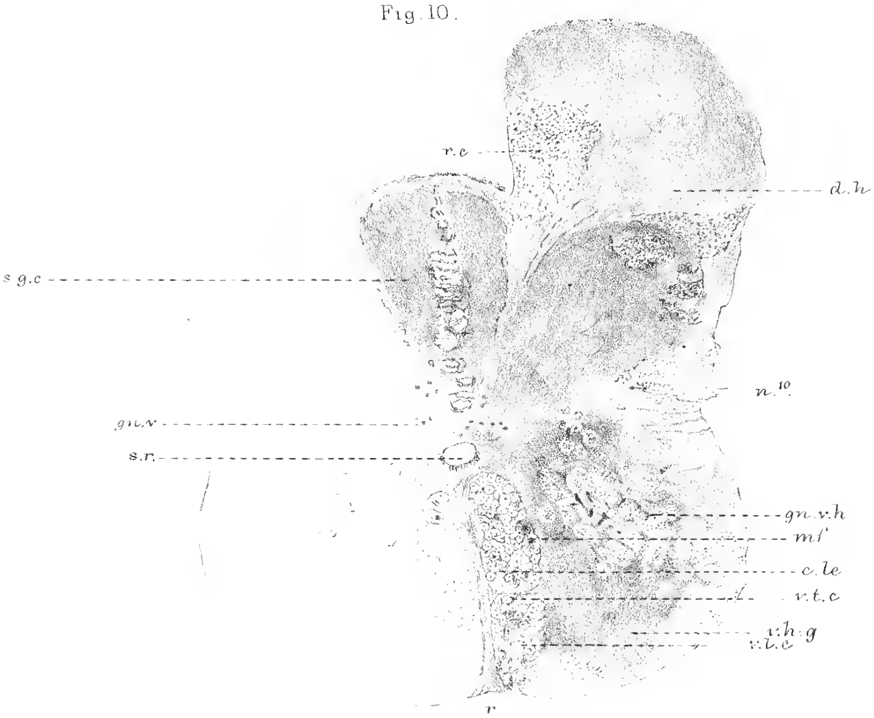

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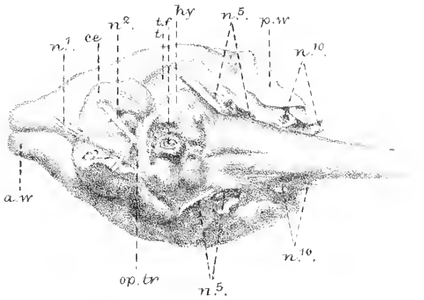

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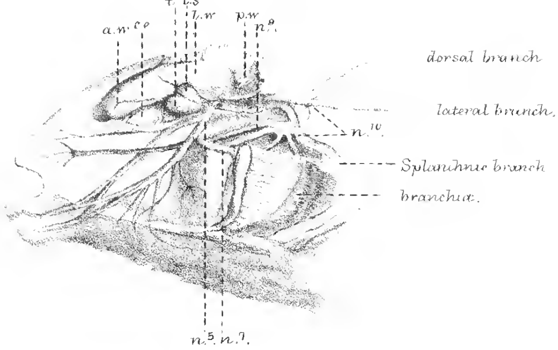

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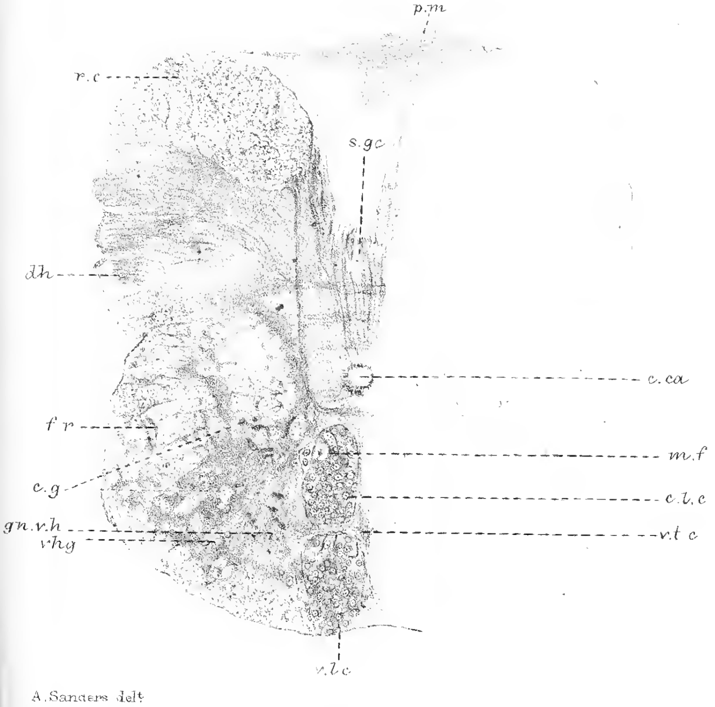


Fig. 12.

A. Sanders delt.

West Newman & Co. lith.


Fig 15

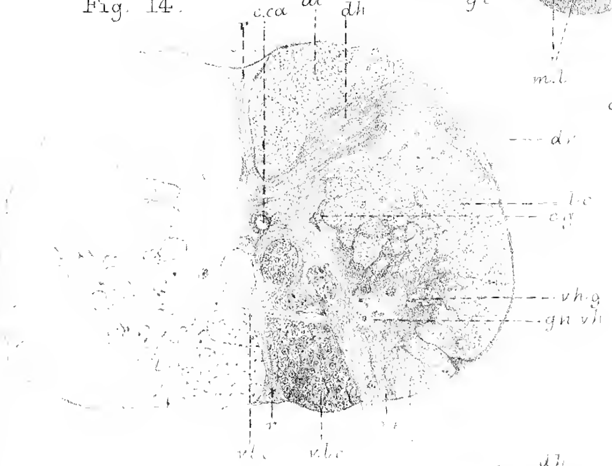
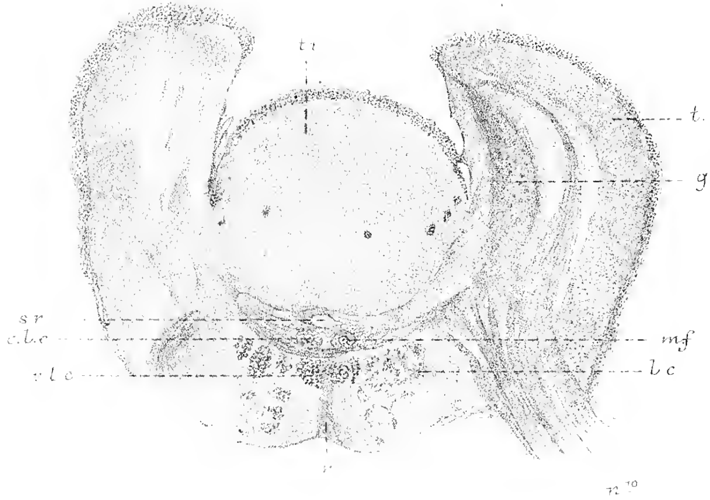
Fig. 14.


Fig 16


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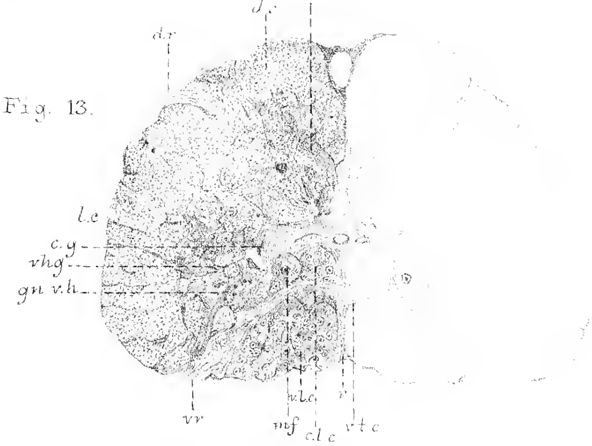

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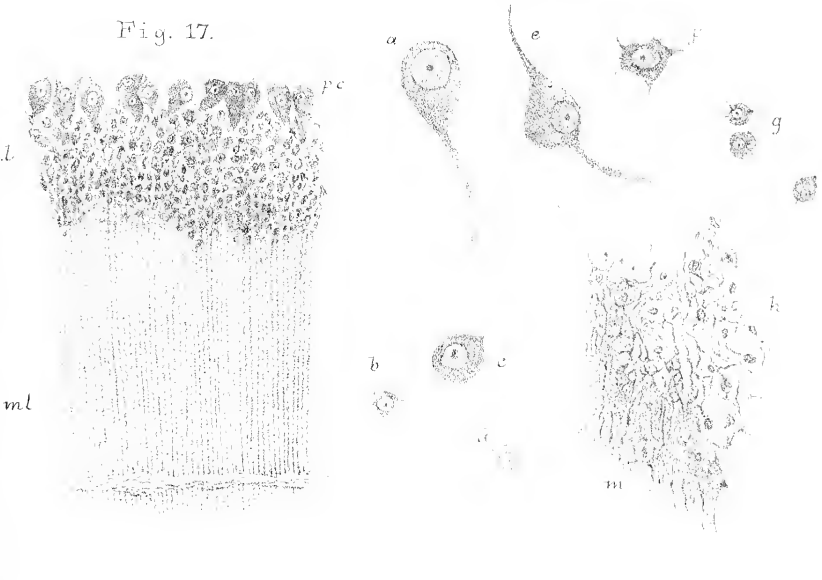
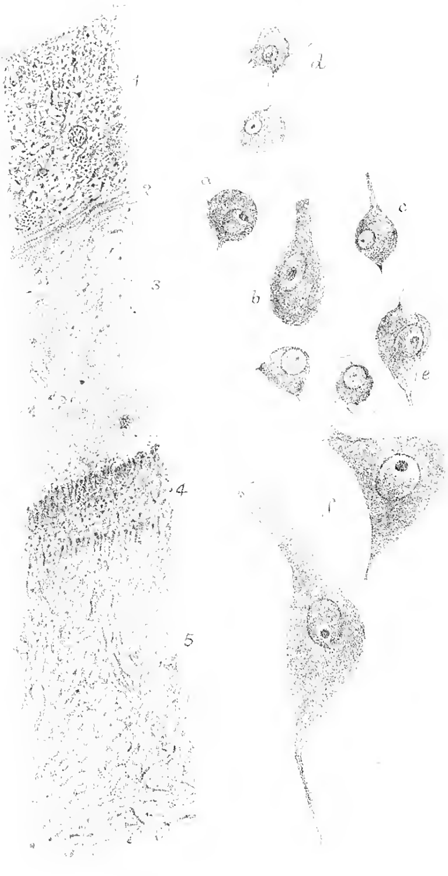
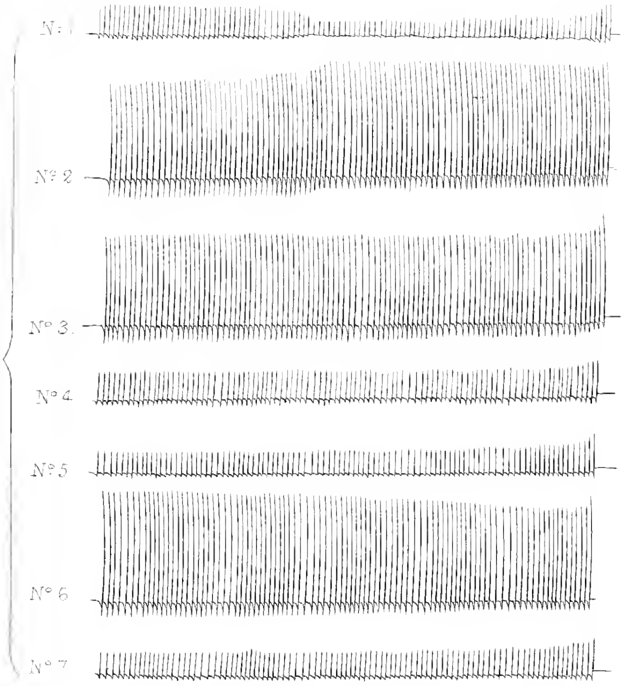
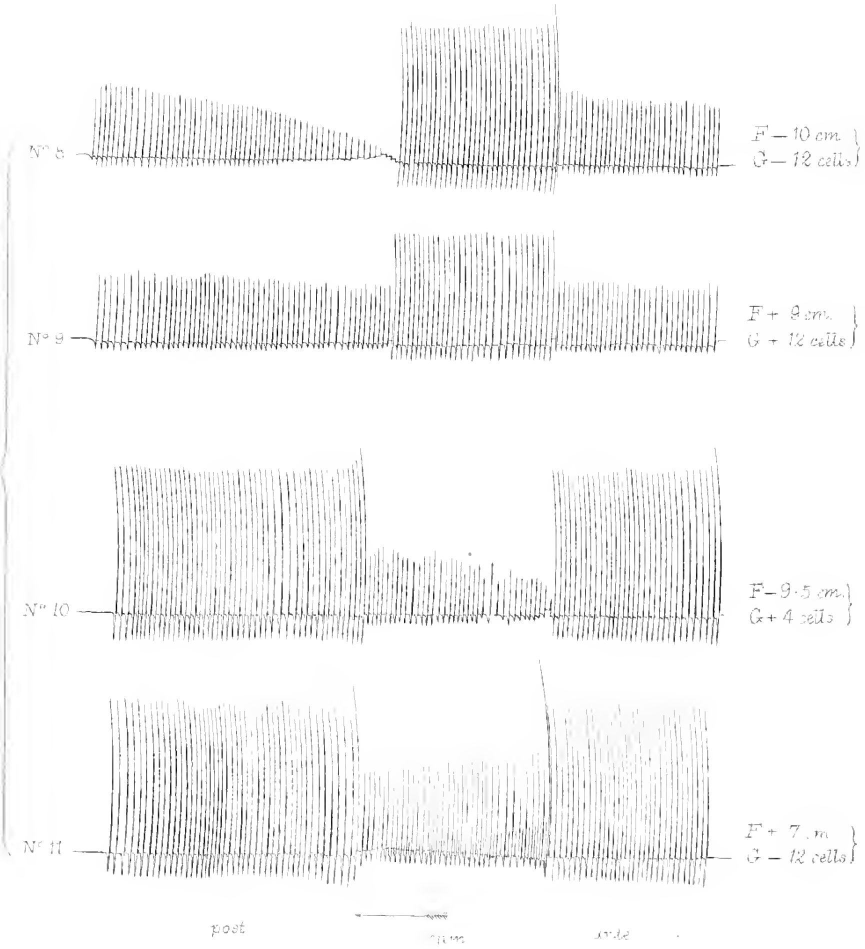
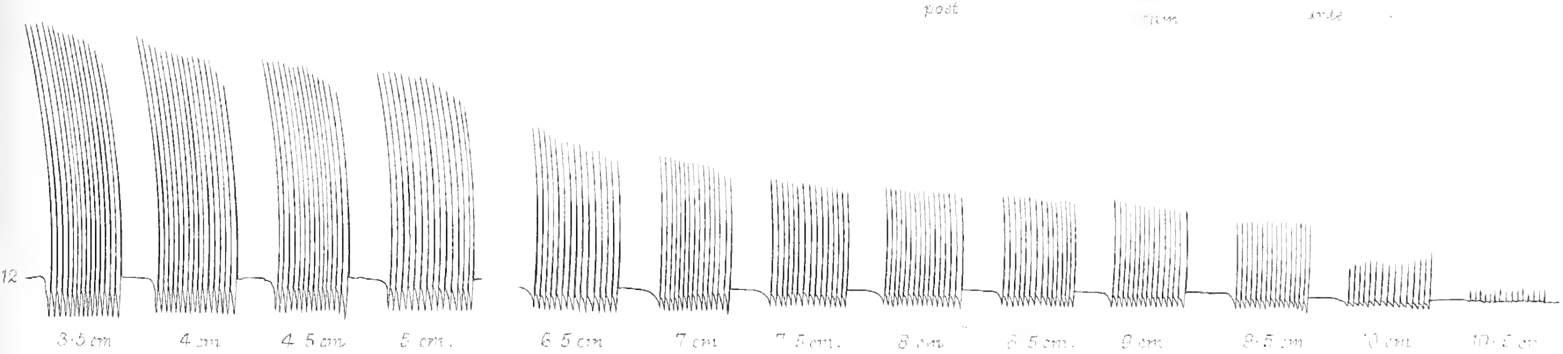
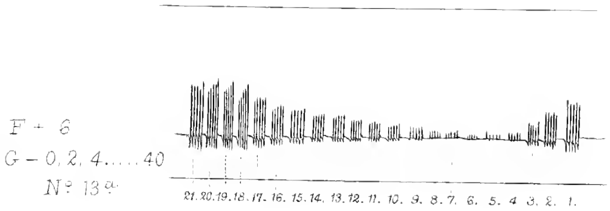


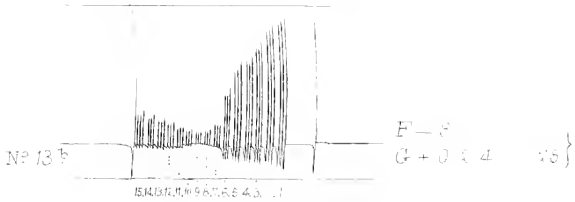
Fig 18


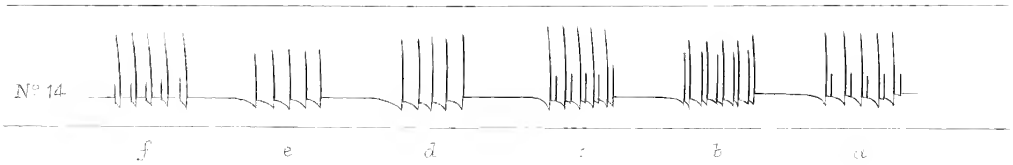












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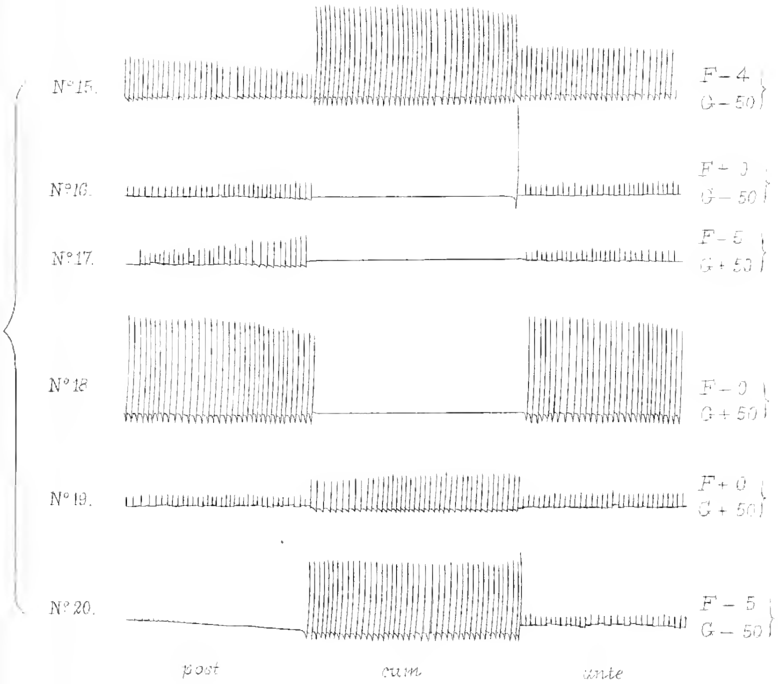

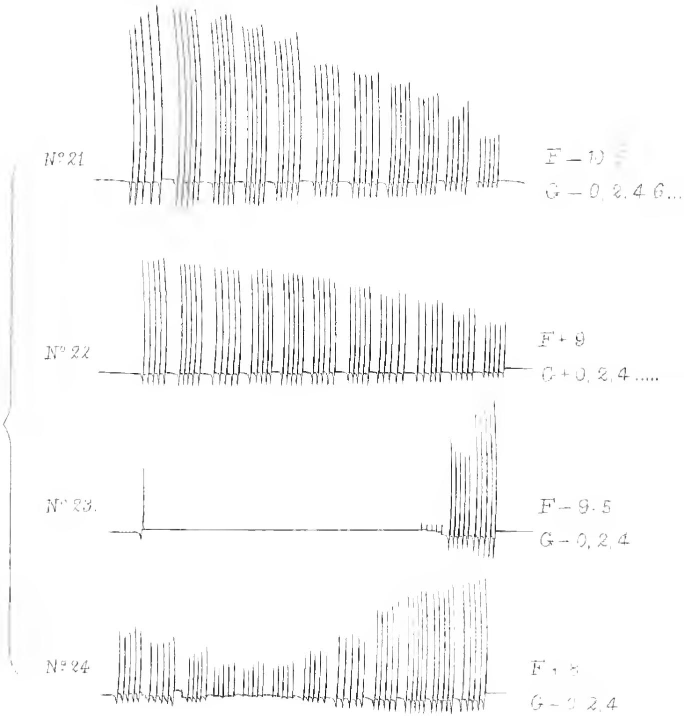

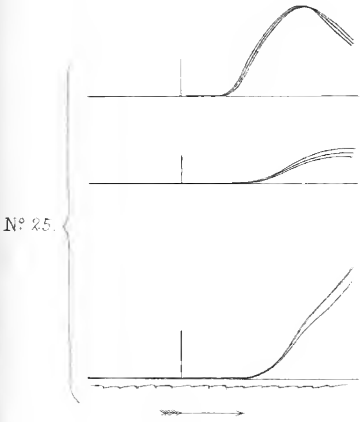

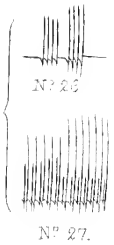

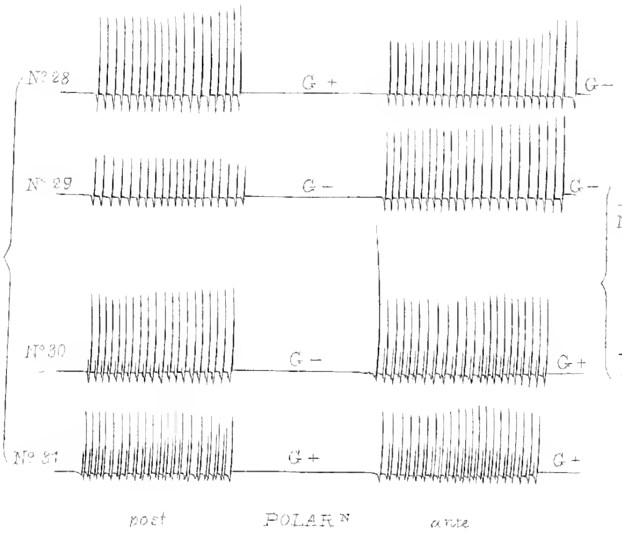

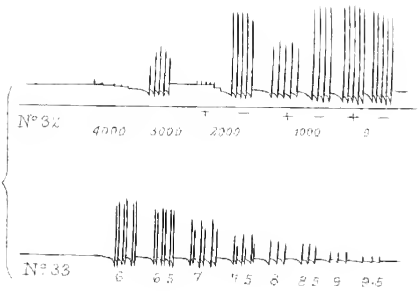

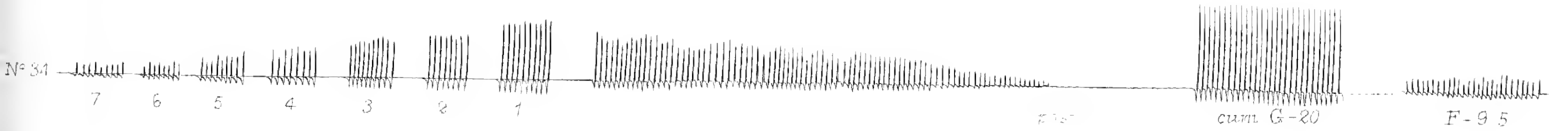

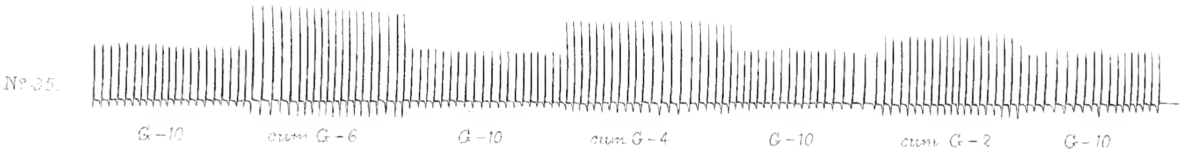

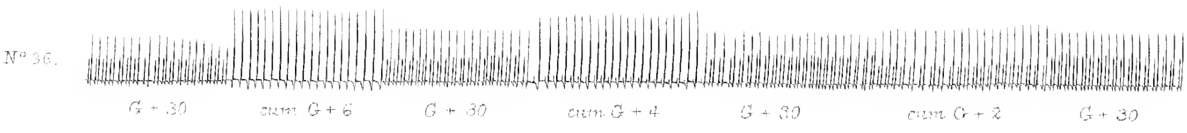

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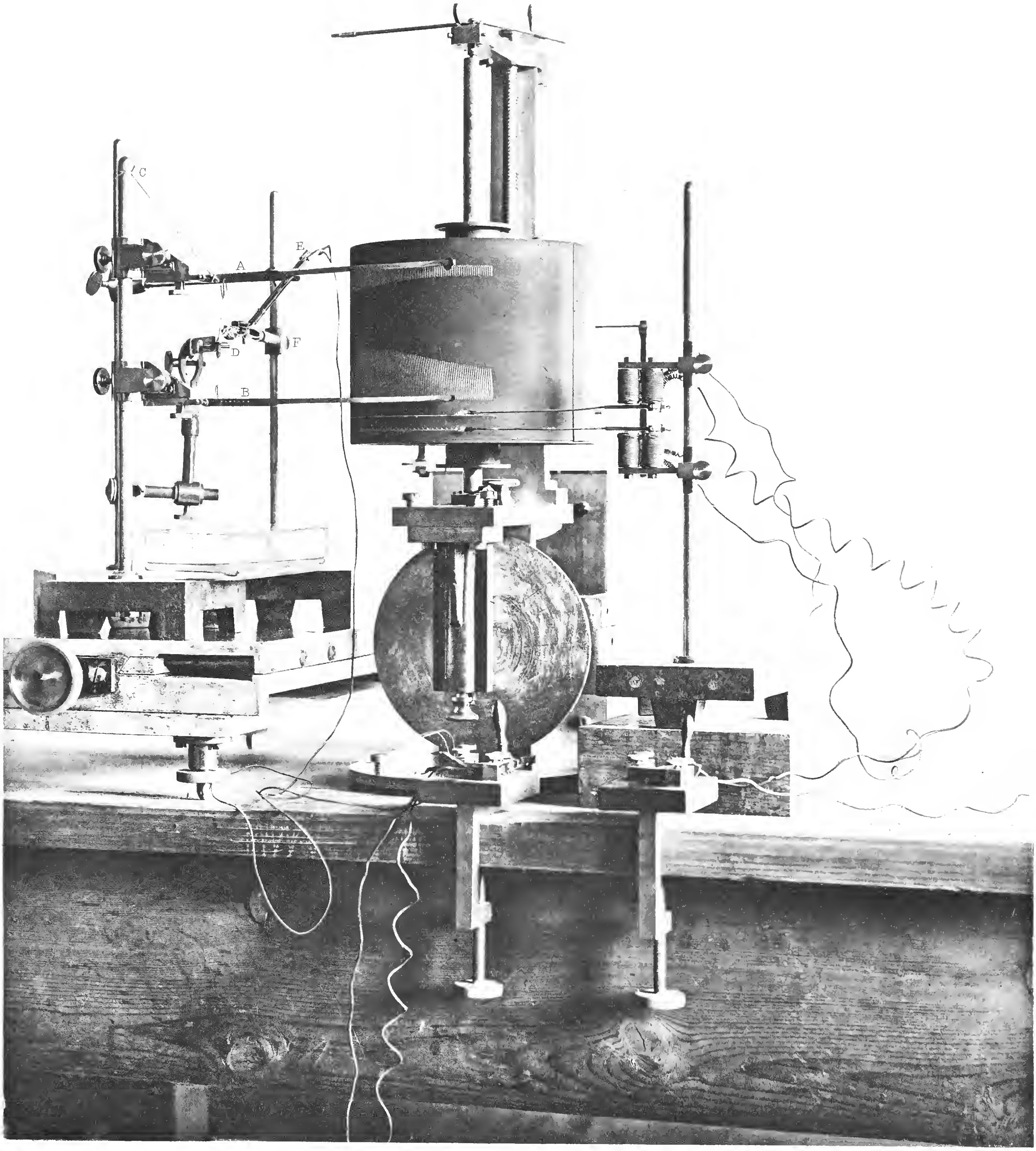

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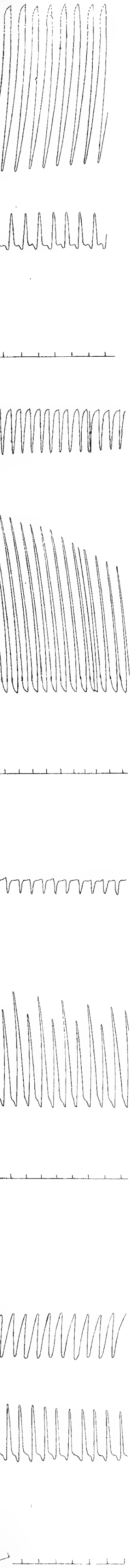

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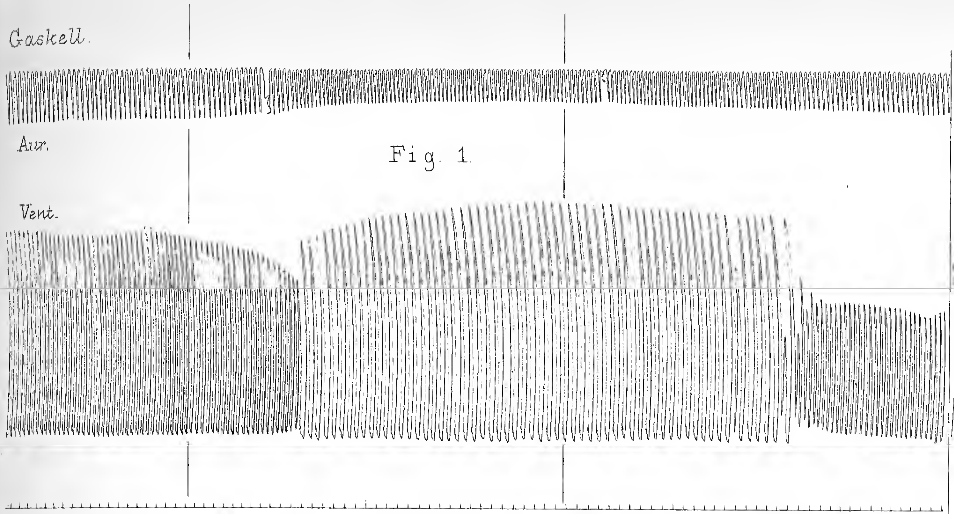
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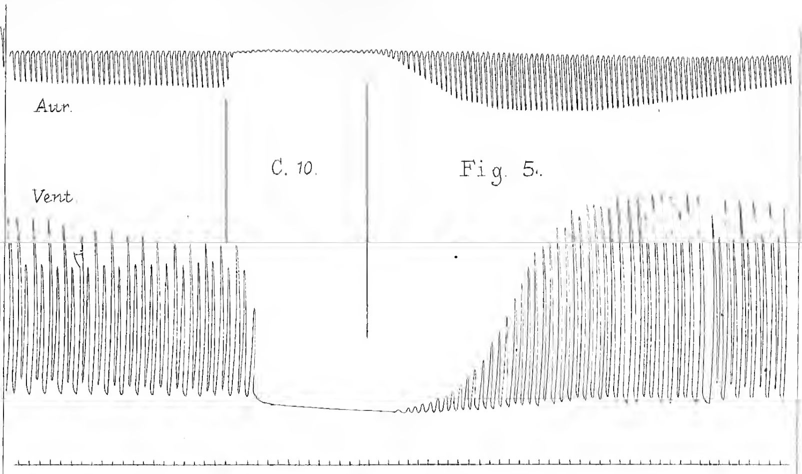
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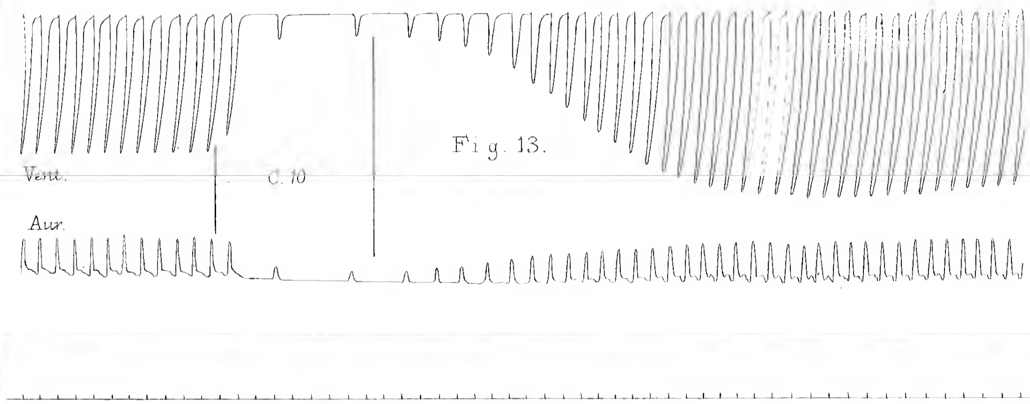
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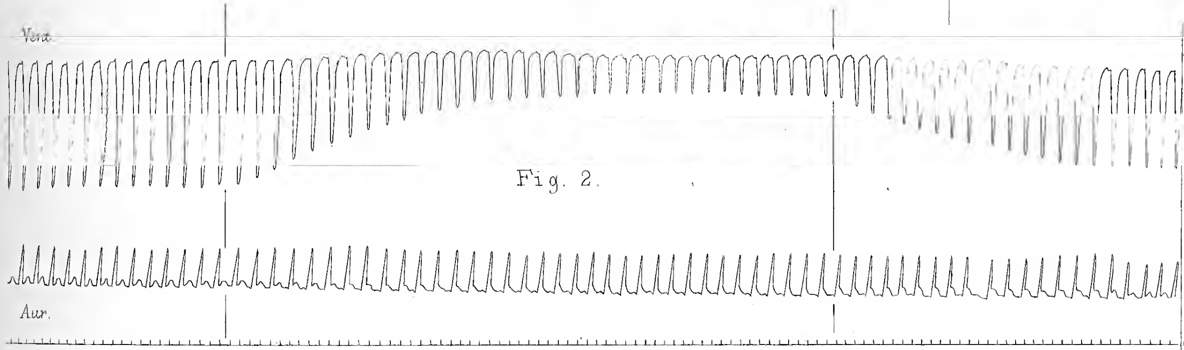
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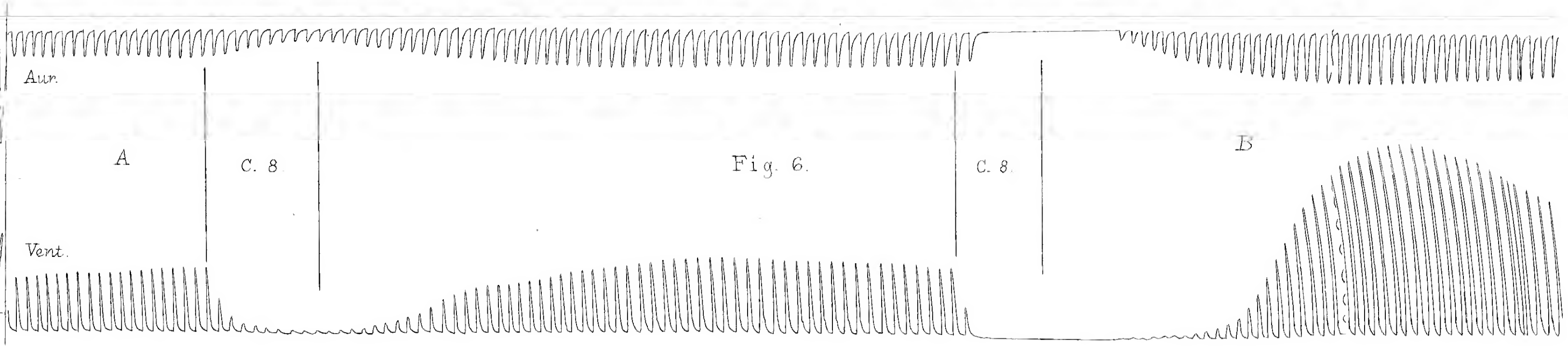
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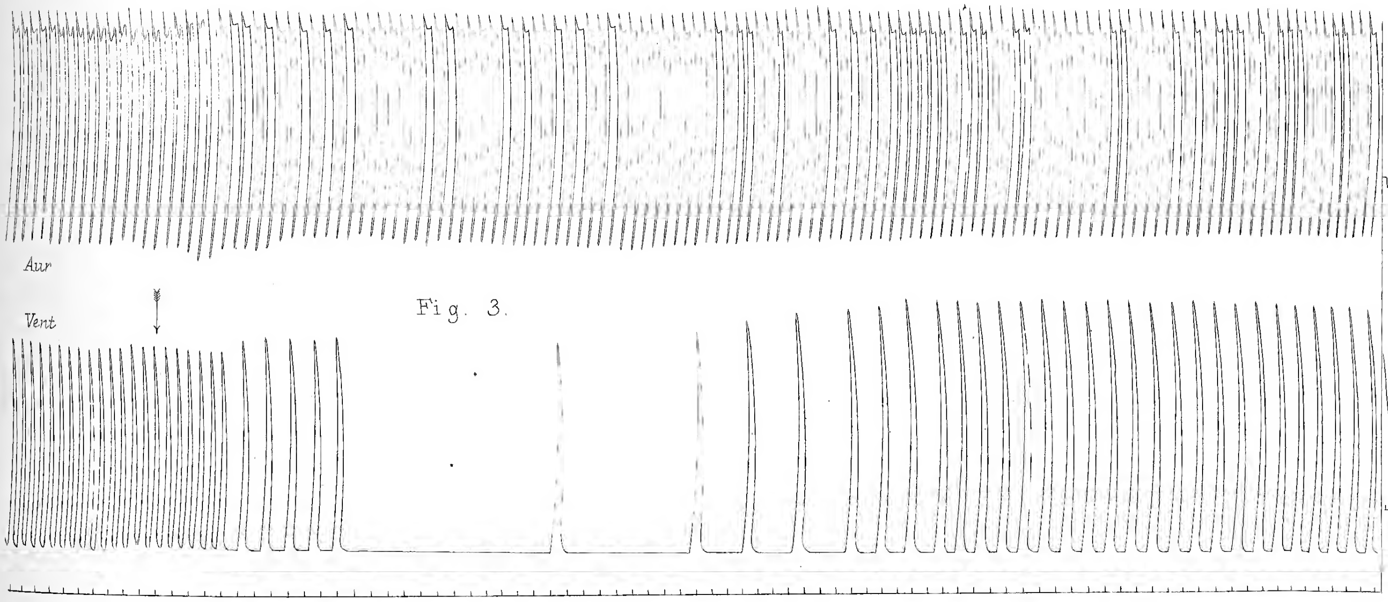
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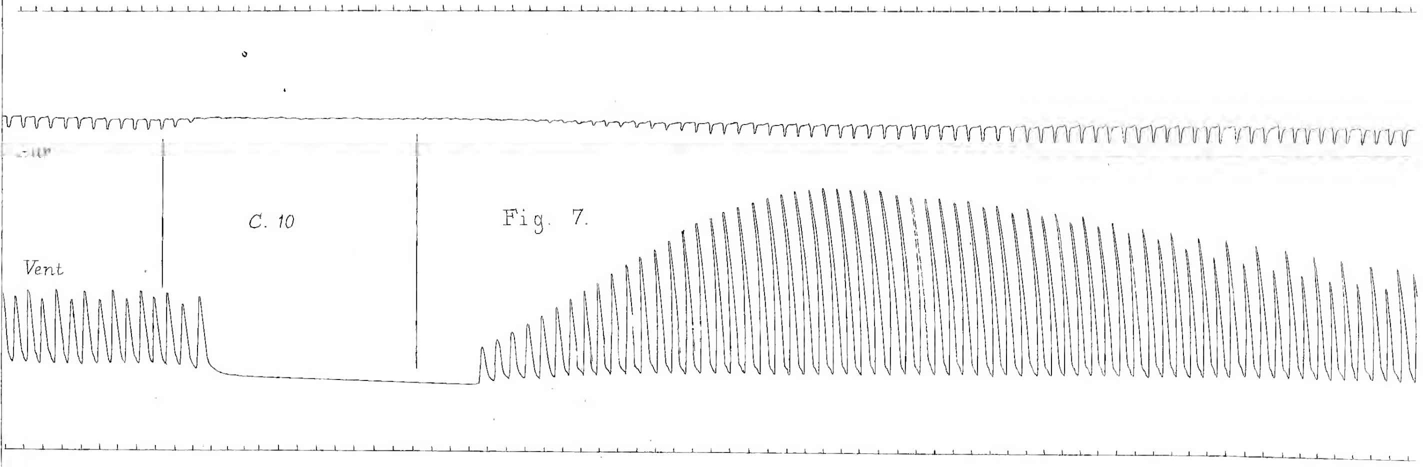
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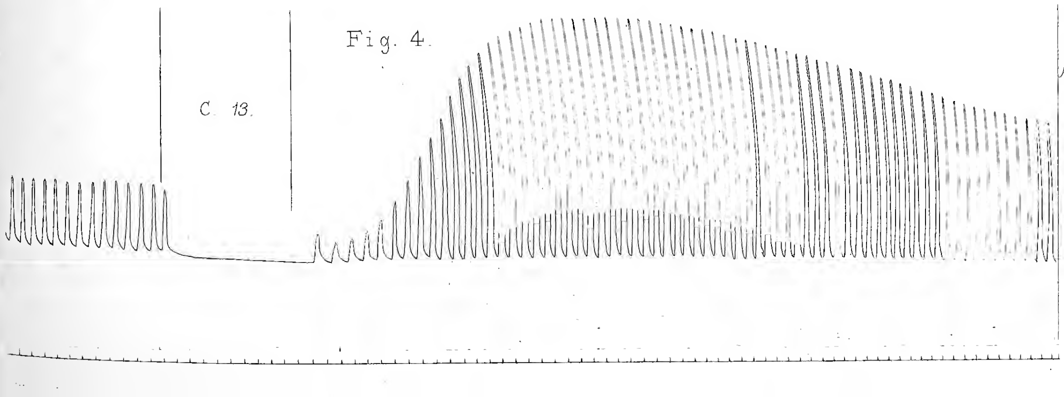
Fig. 11.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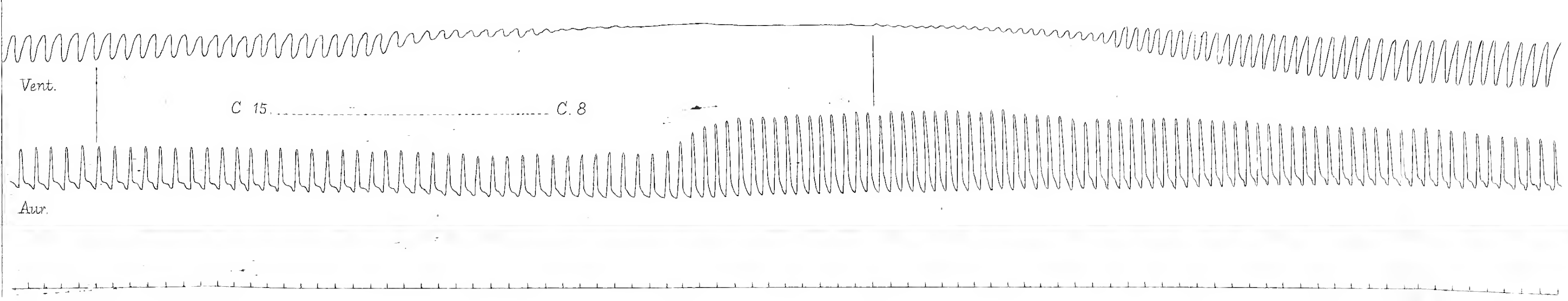
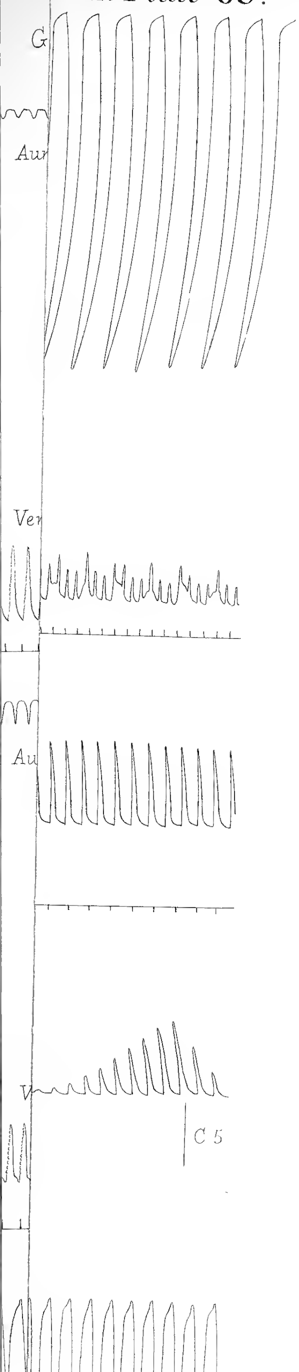


382. *Plate* 68.



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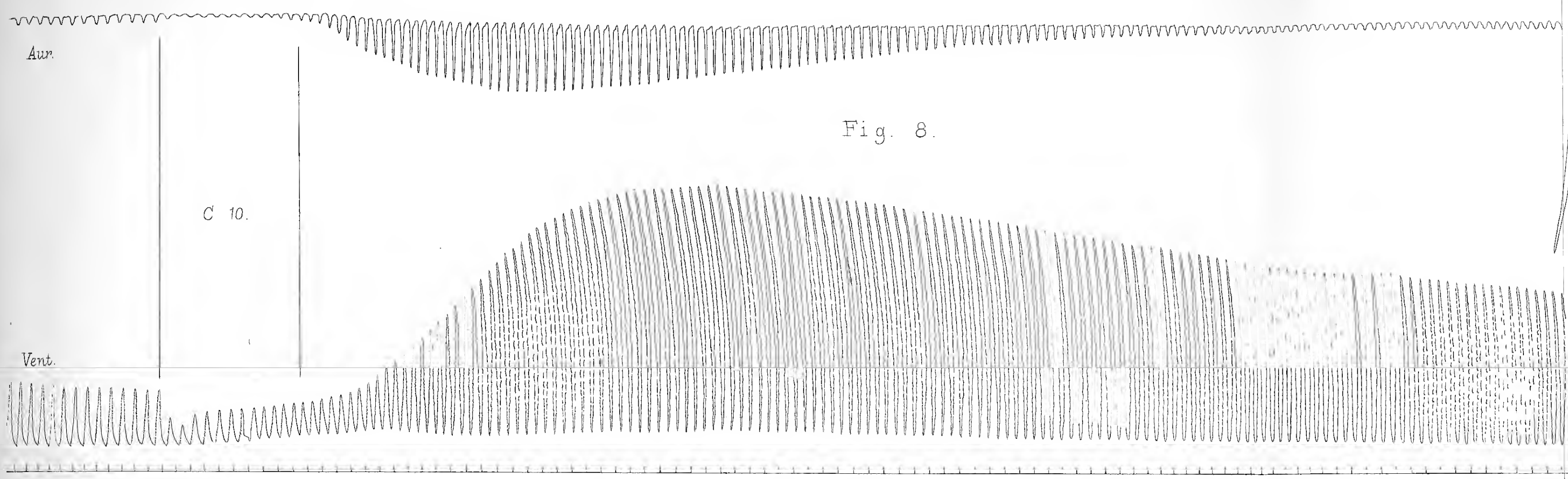

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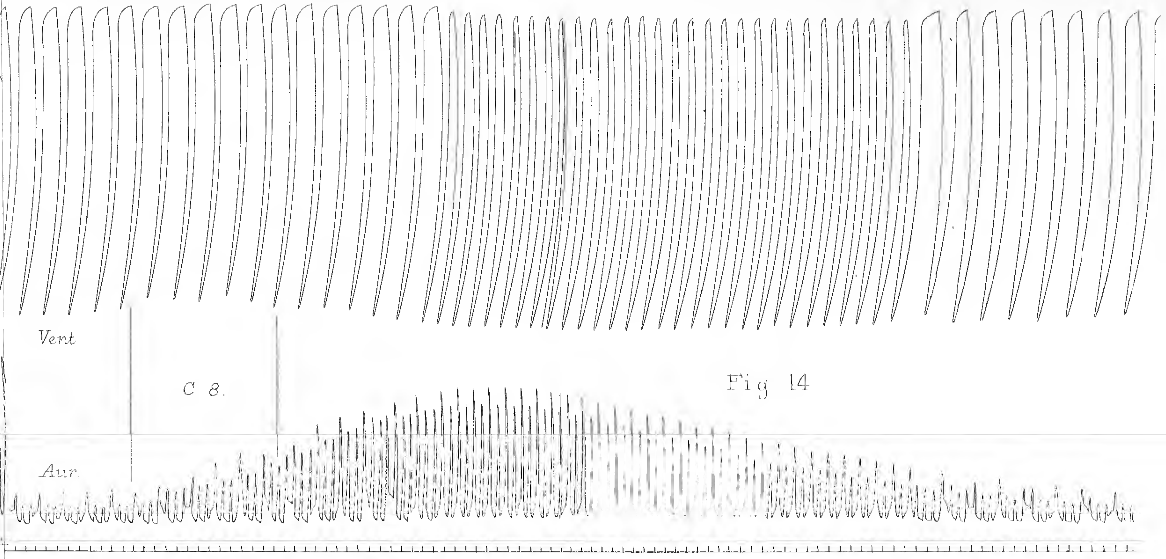

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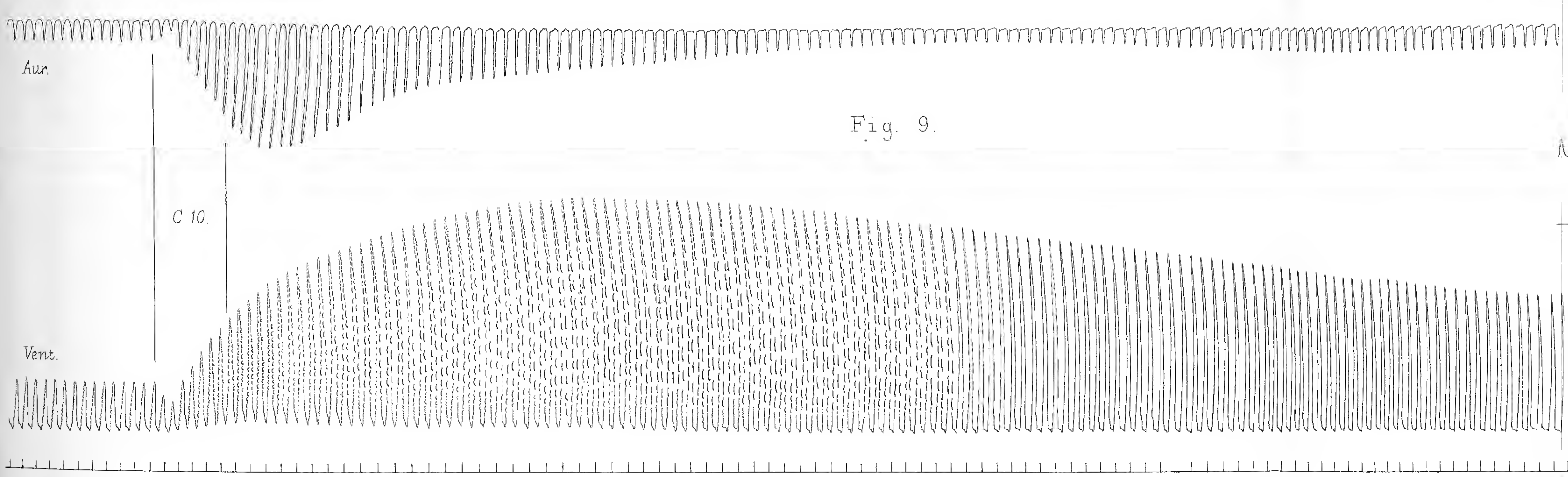

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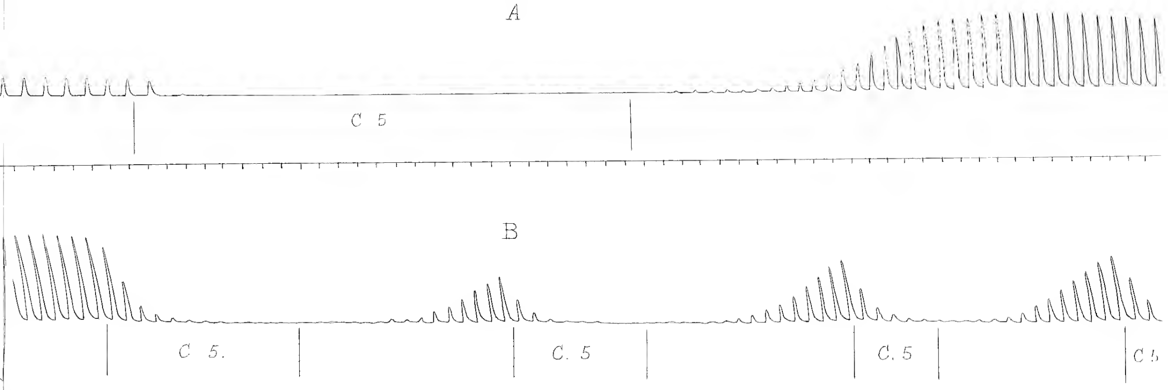
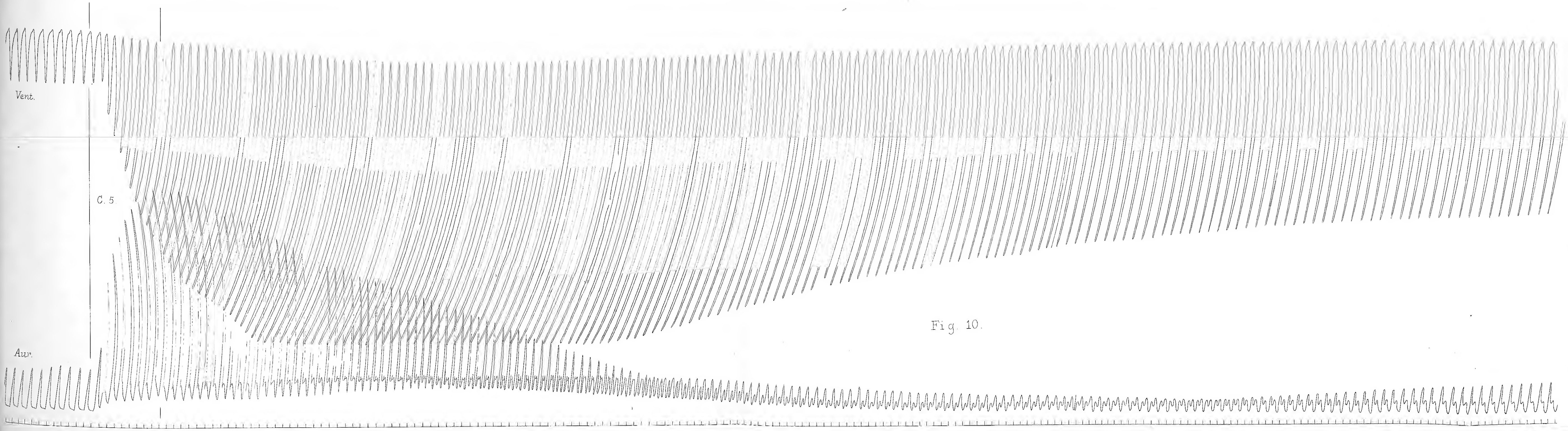
Fig. 12.





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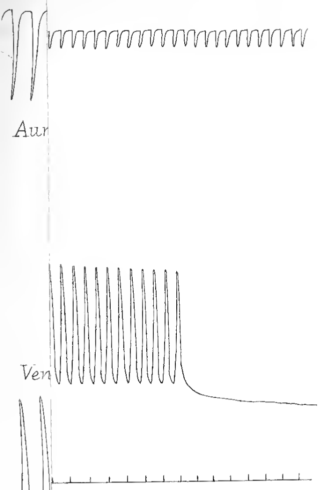

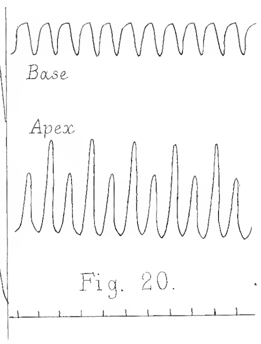

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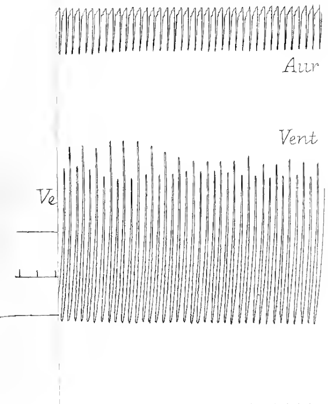

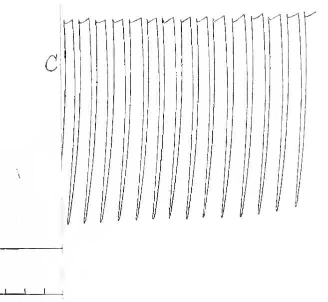

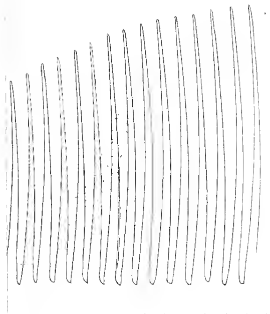

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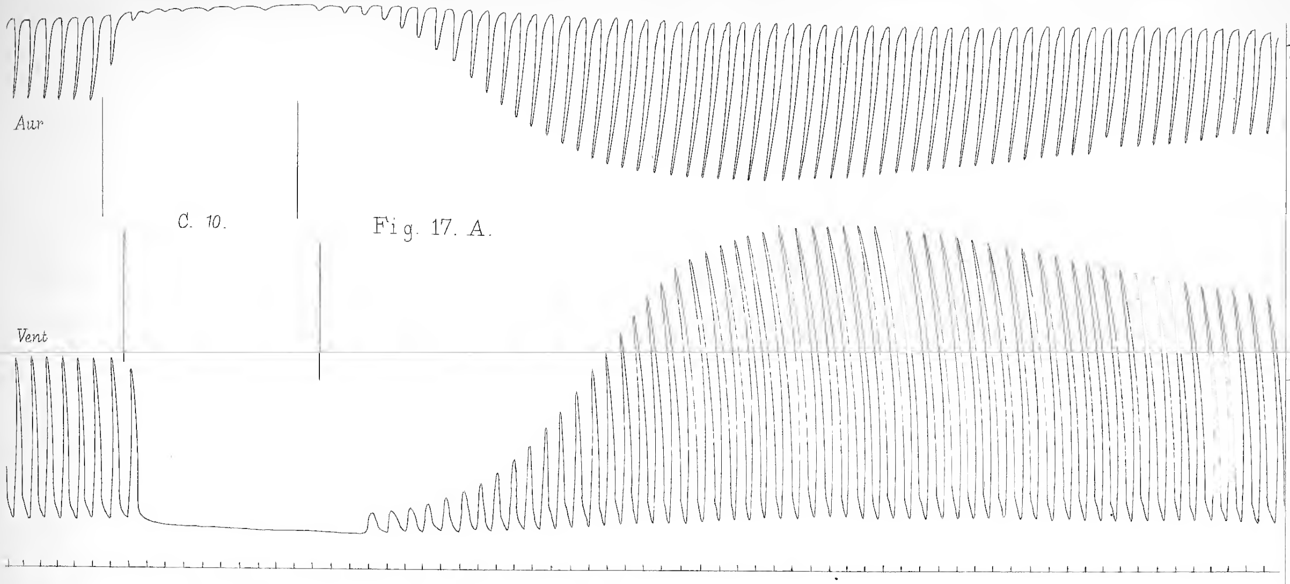

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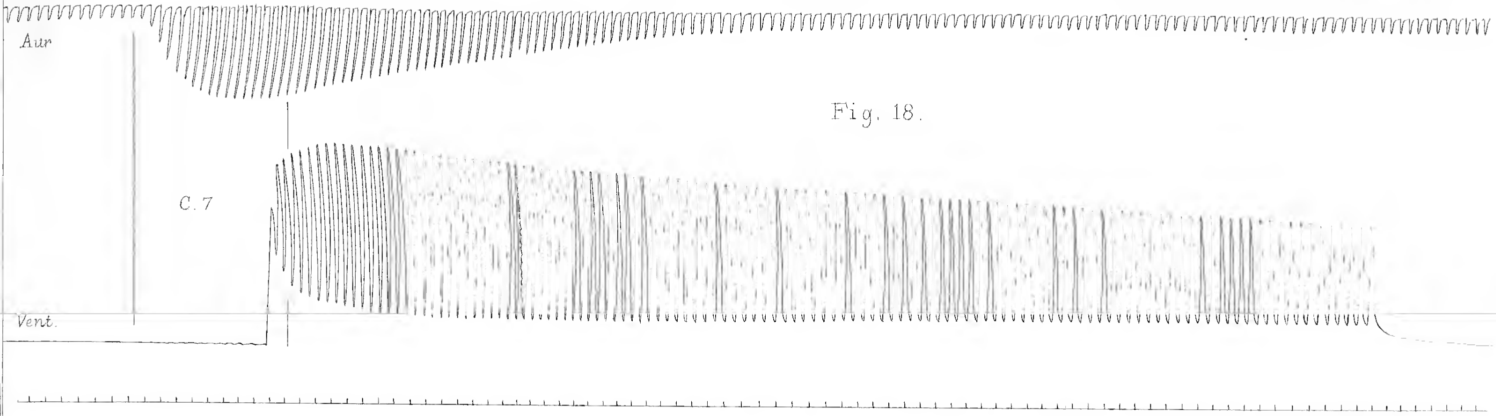

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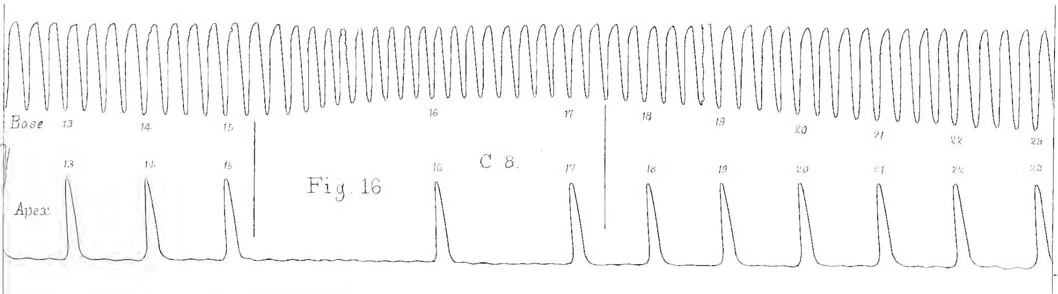

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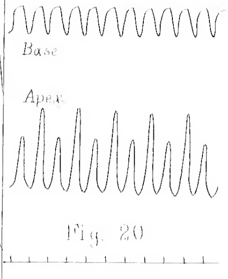

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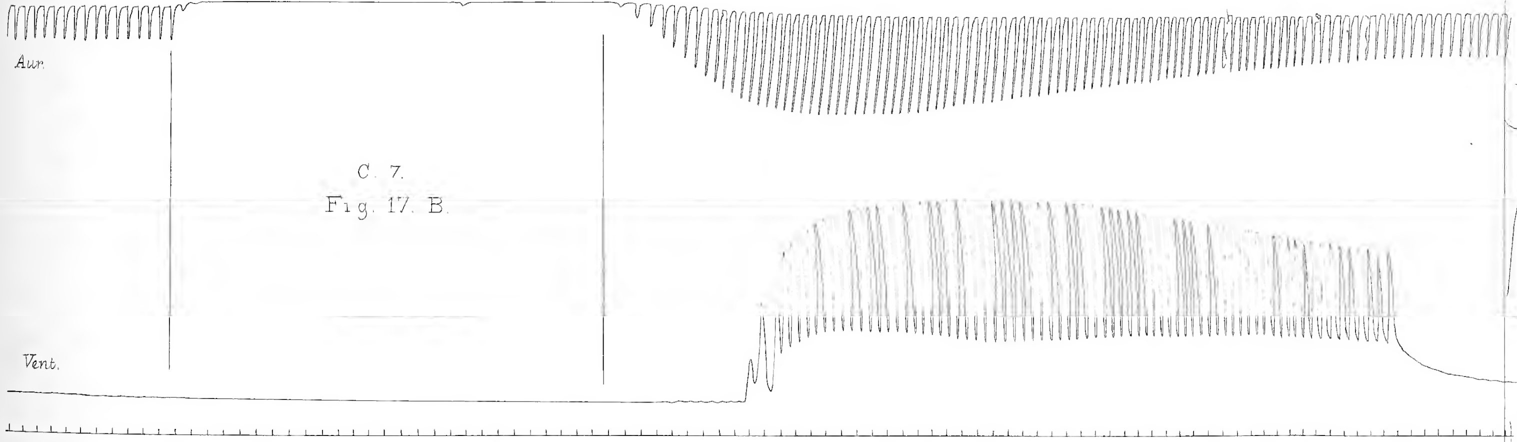

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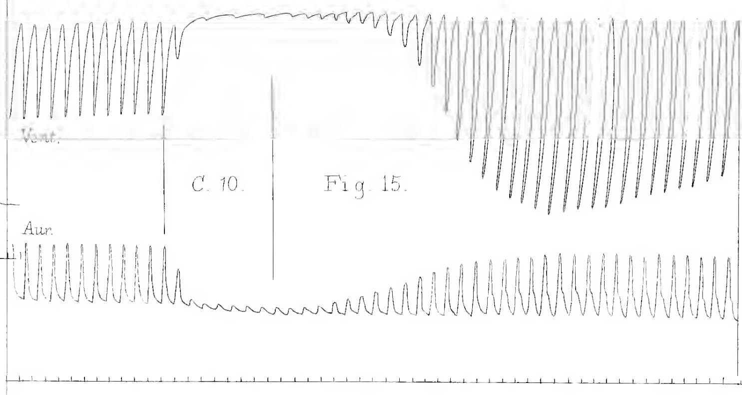

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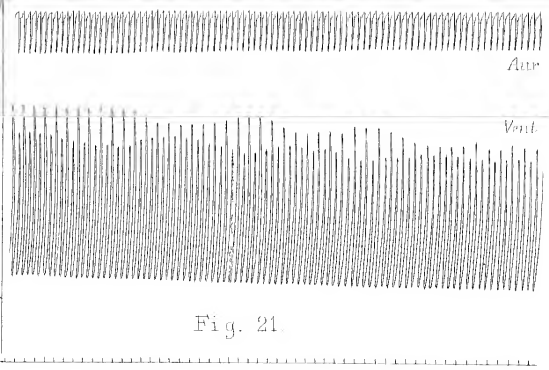

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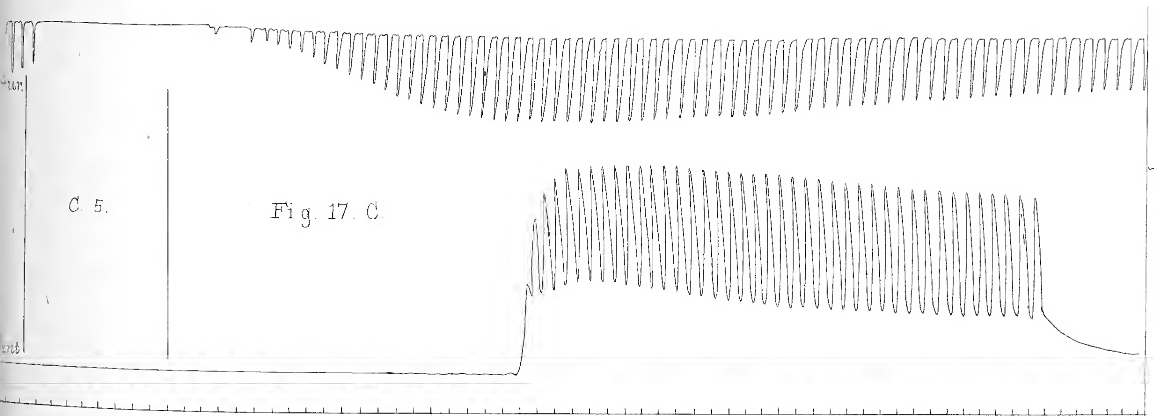

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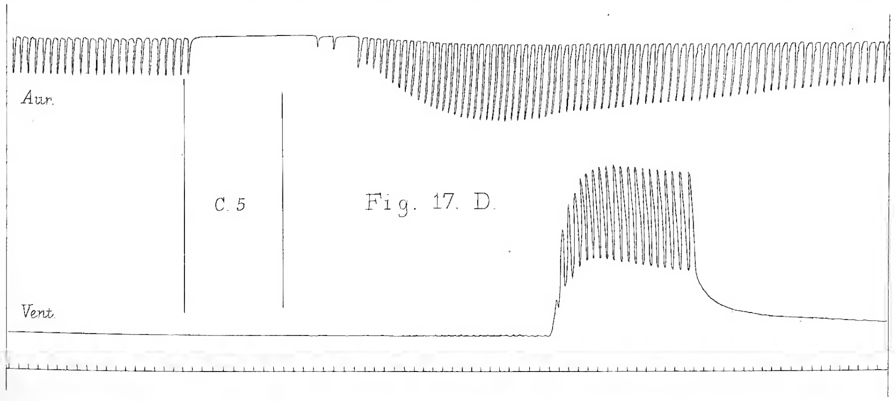

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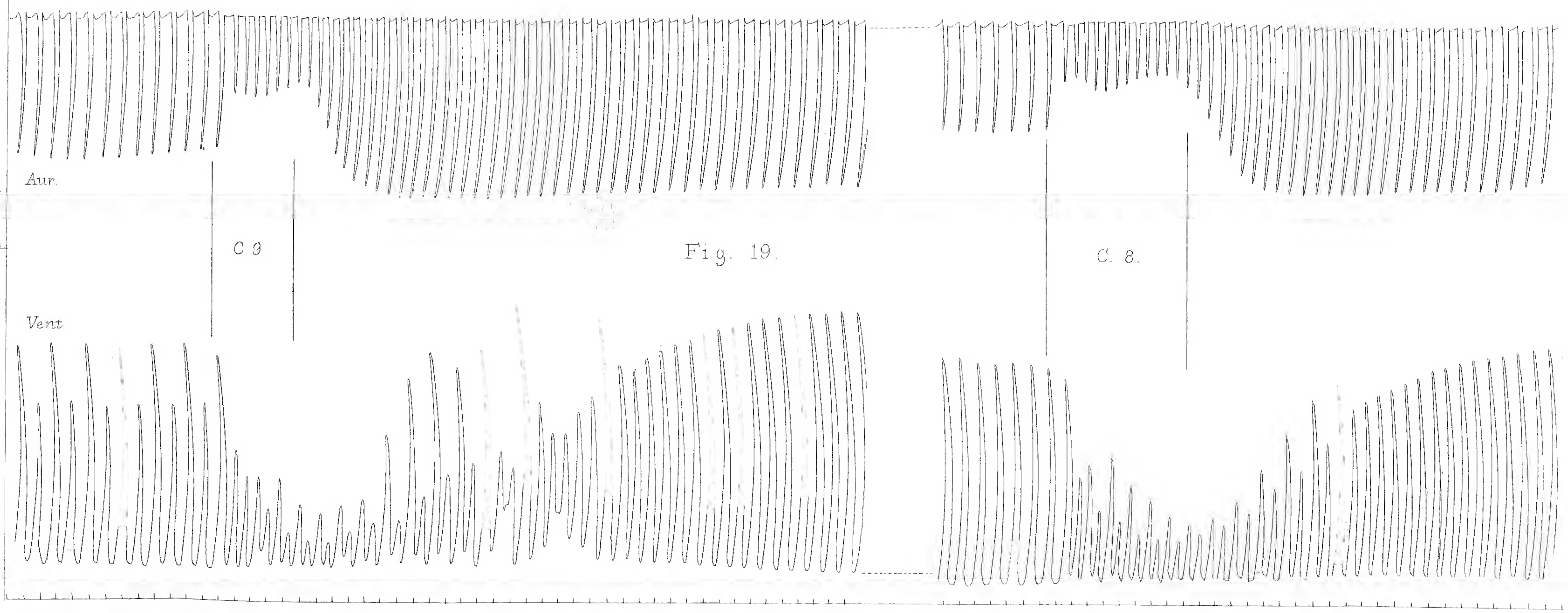
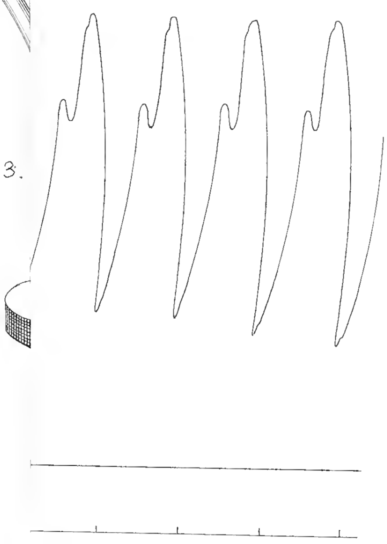


Fig. 19.

Fig. 23.



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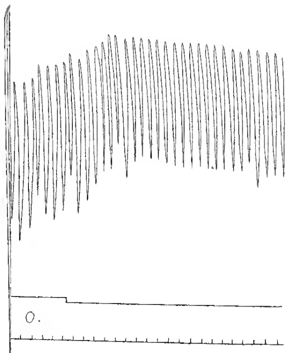

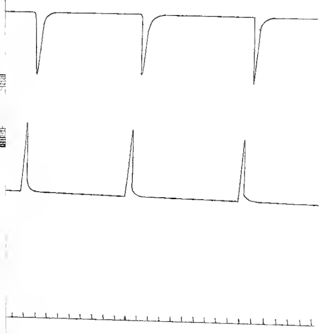

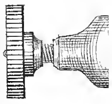

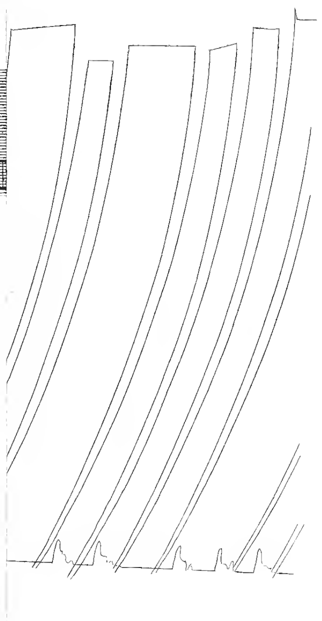

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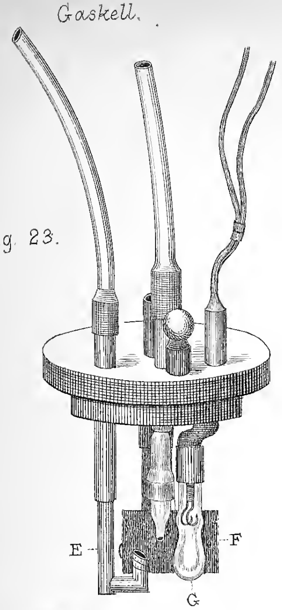

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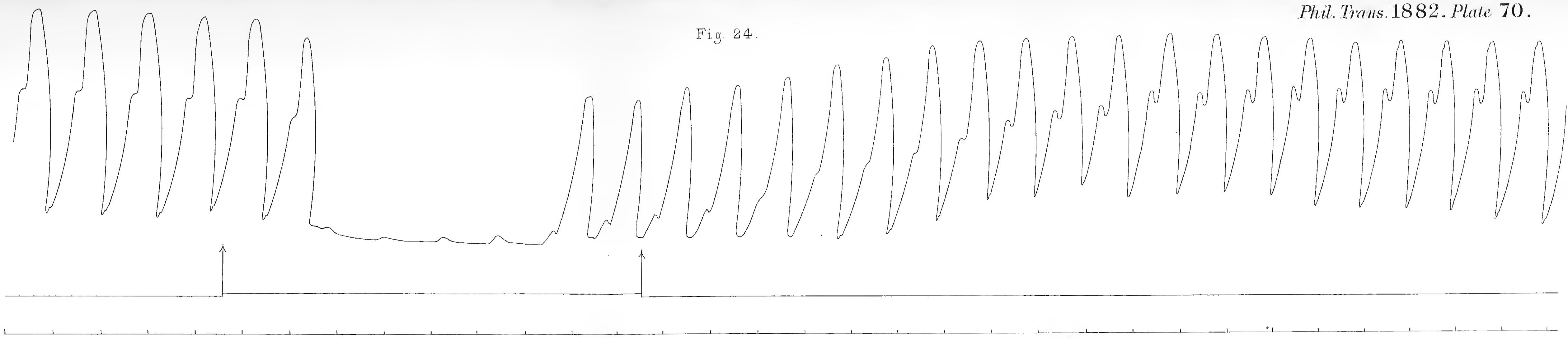

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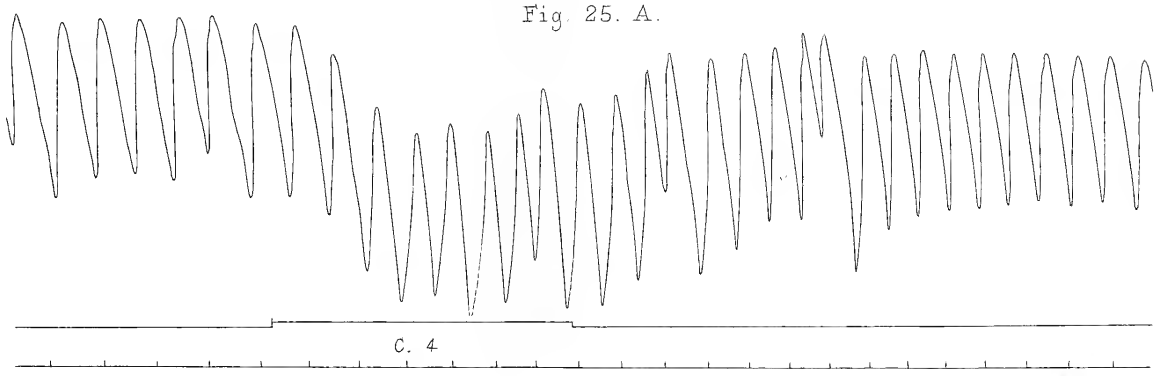

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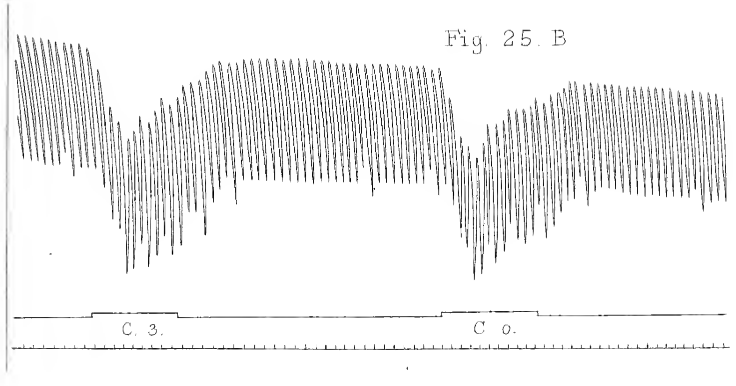
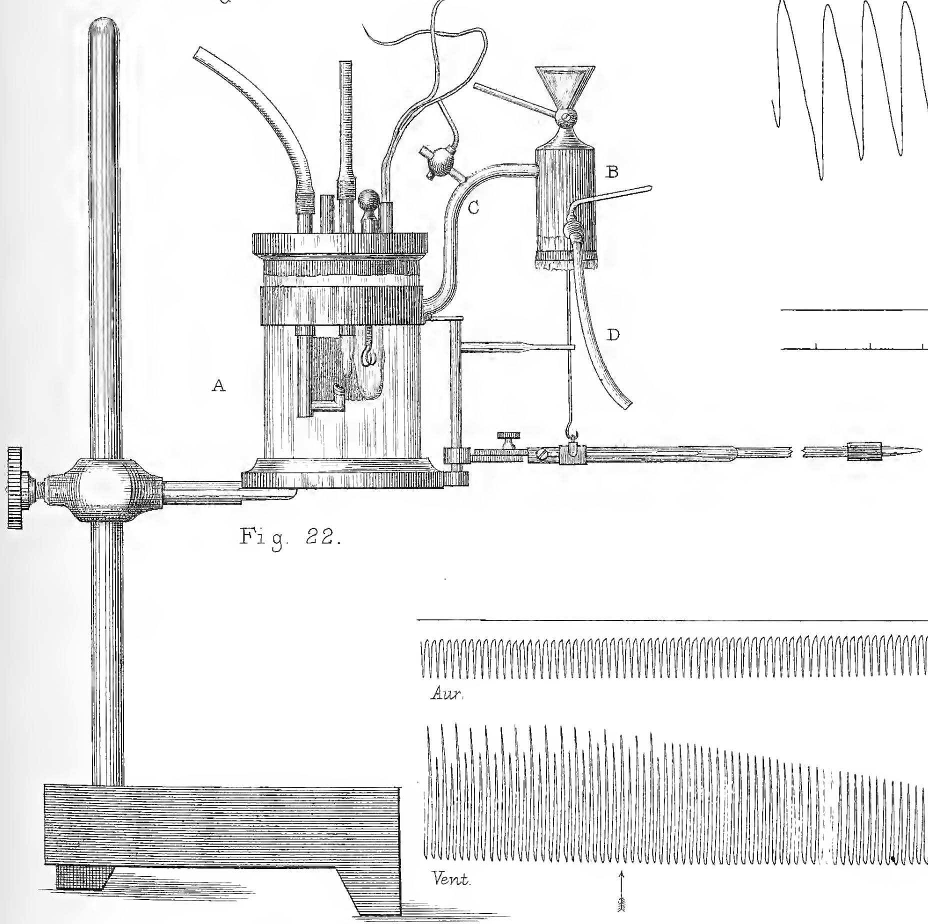


Fig. 25. B



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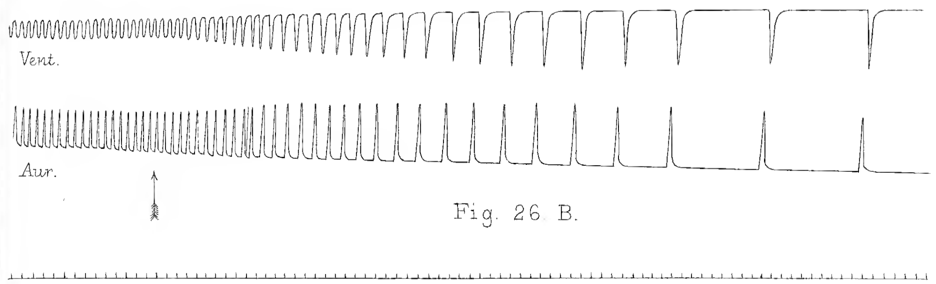

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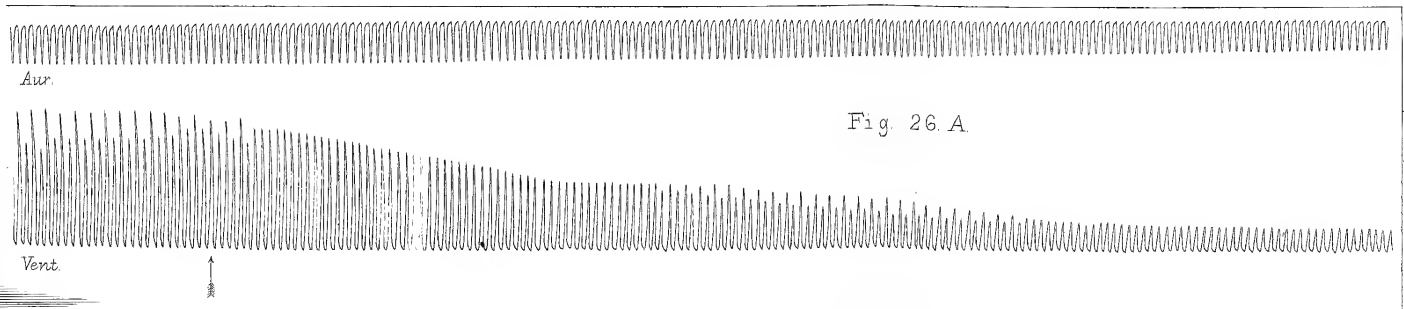

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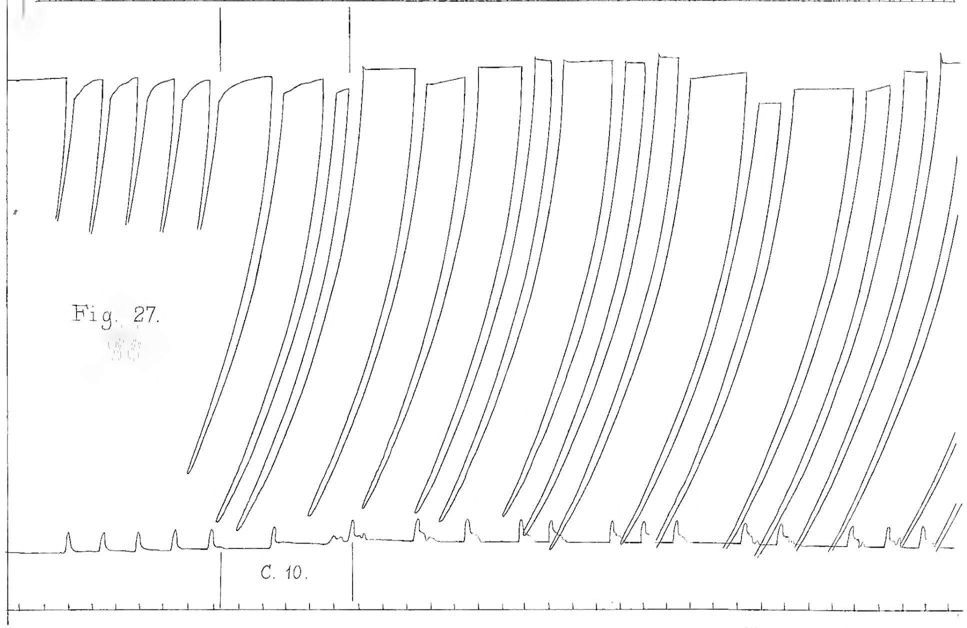

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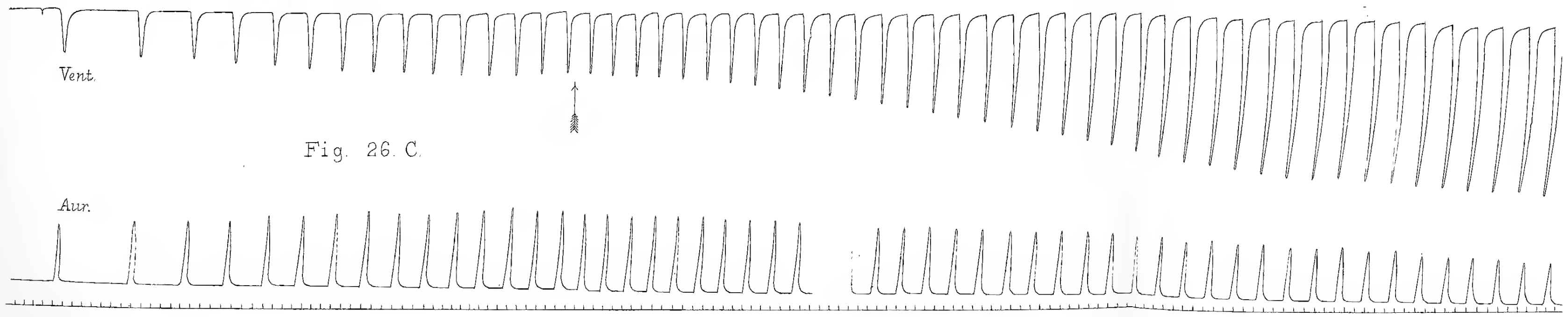

Fig. 26. C.

West Newman & Cº sc.

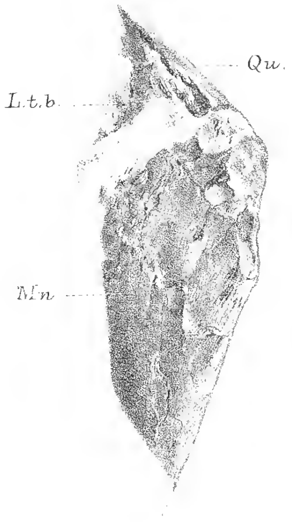

Fig. 4.

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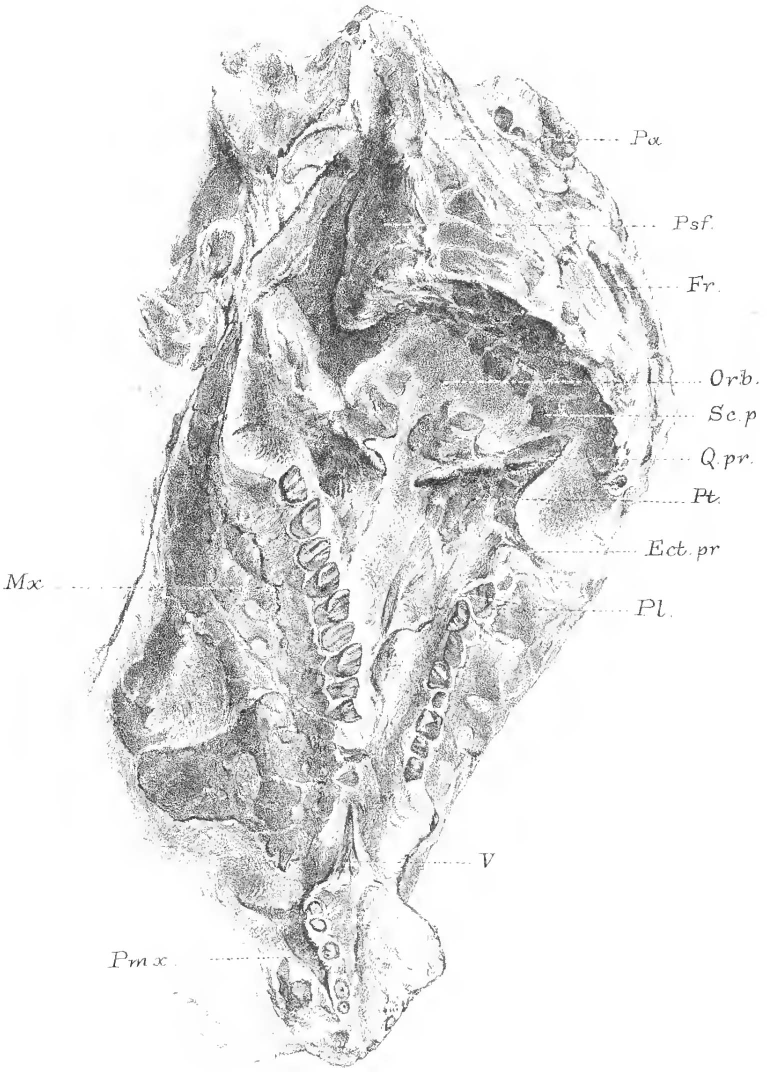

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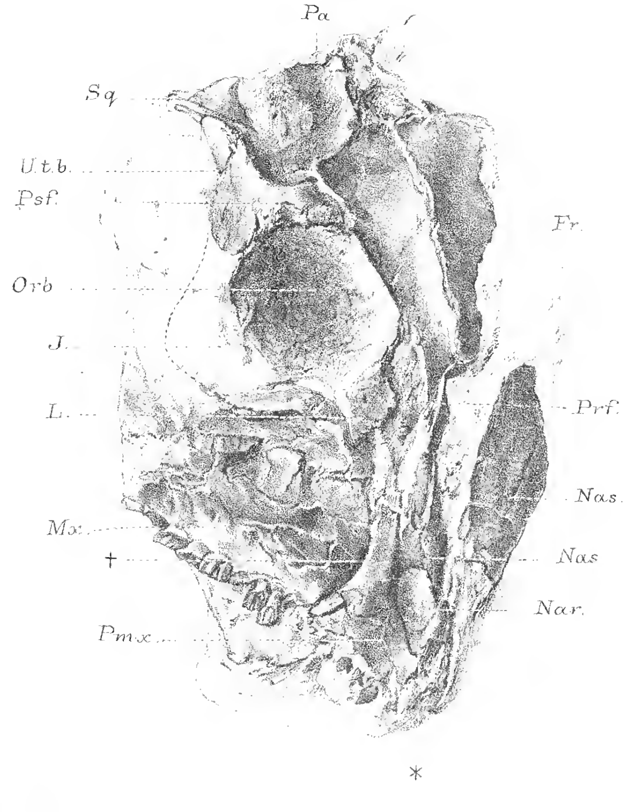

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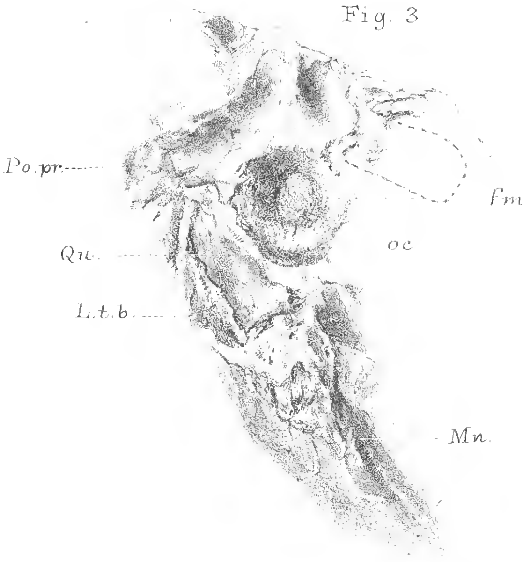

W. H. Wesley ad nat. del.

West Newman & Cº imp.

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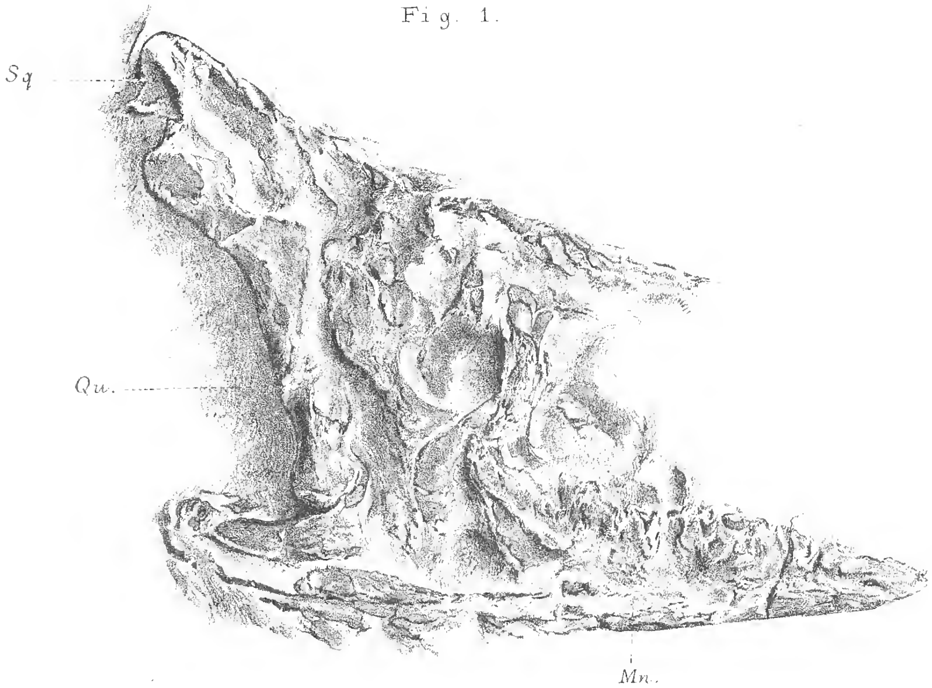
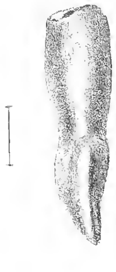


Fig. 2.

Fig. 3.

4.



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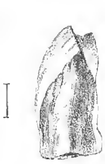

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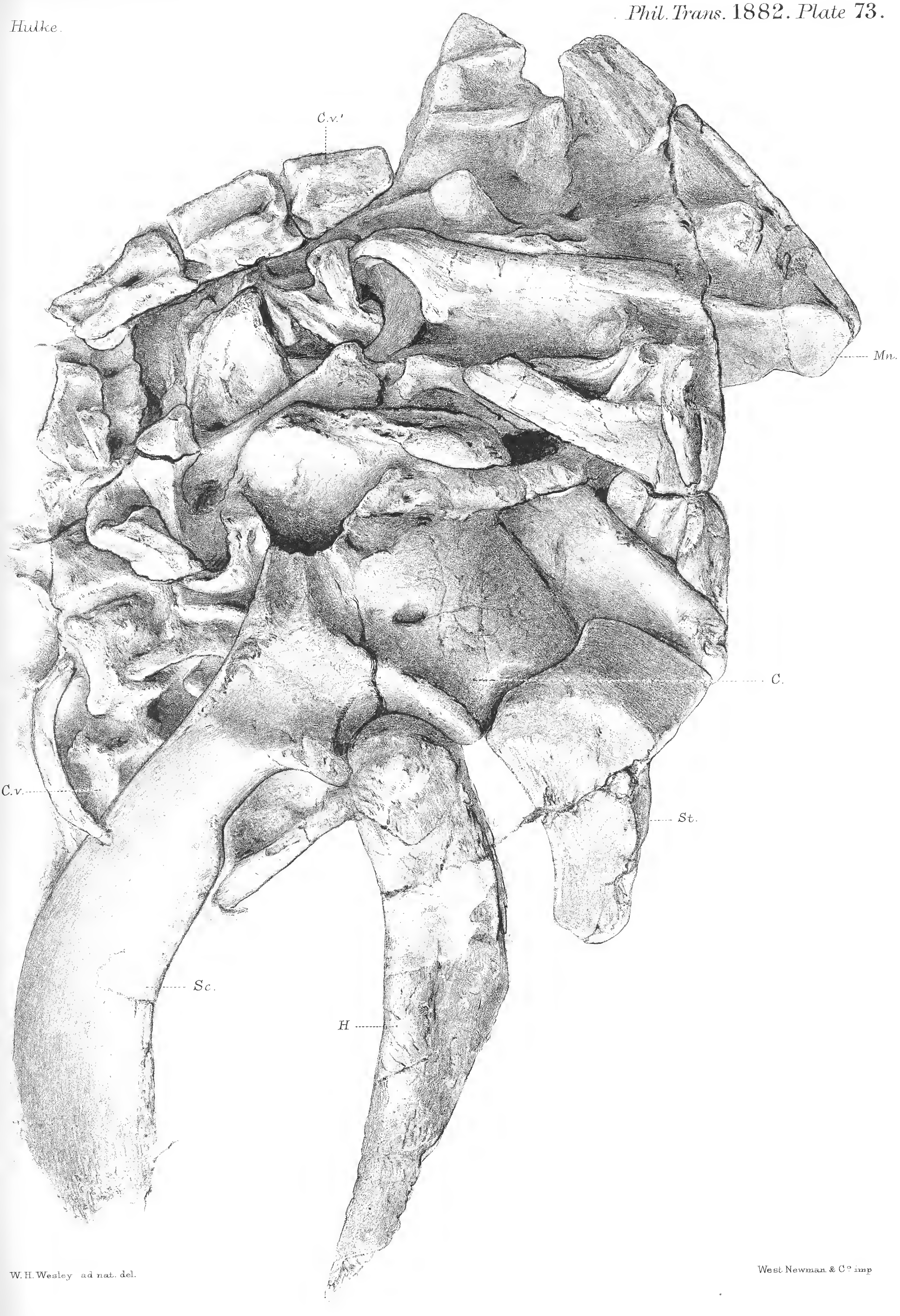
5.

W.H. Wesley ad nat. del.

West Newman & [illegible]



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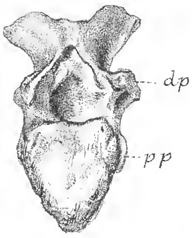
West Newman & C° imp

Fig. 4.



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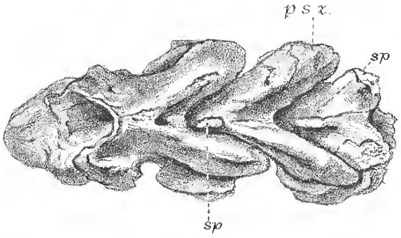

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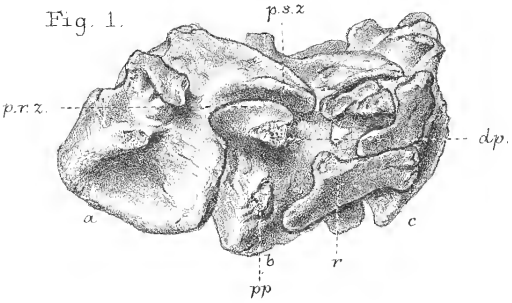

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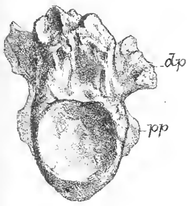

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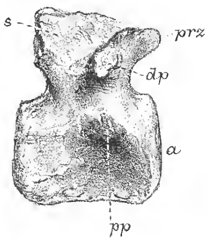

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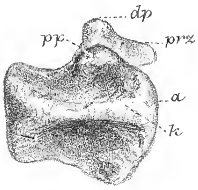
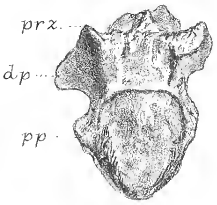


Fig. 7.



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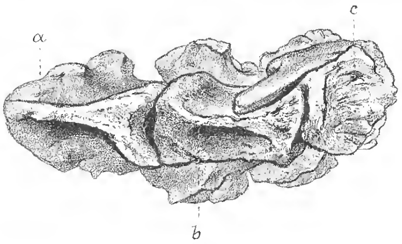

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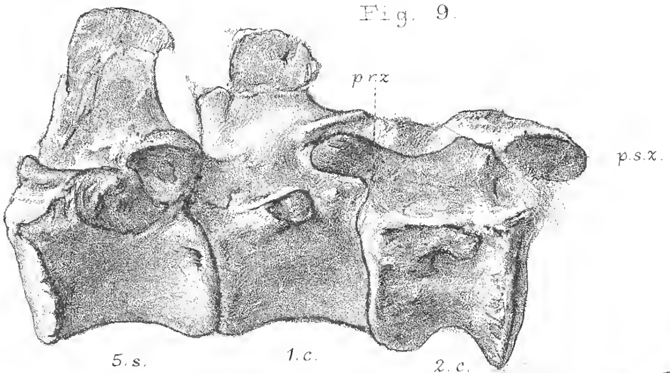

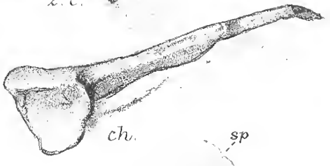

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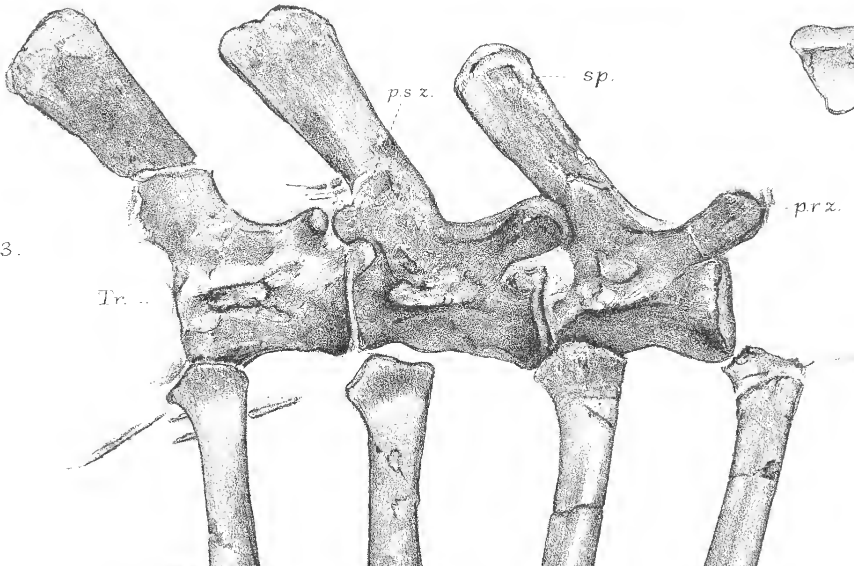

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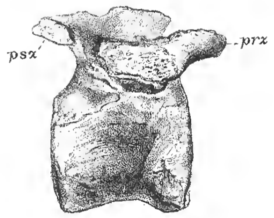
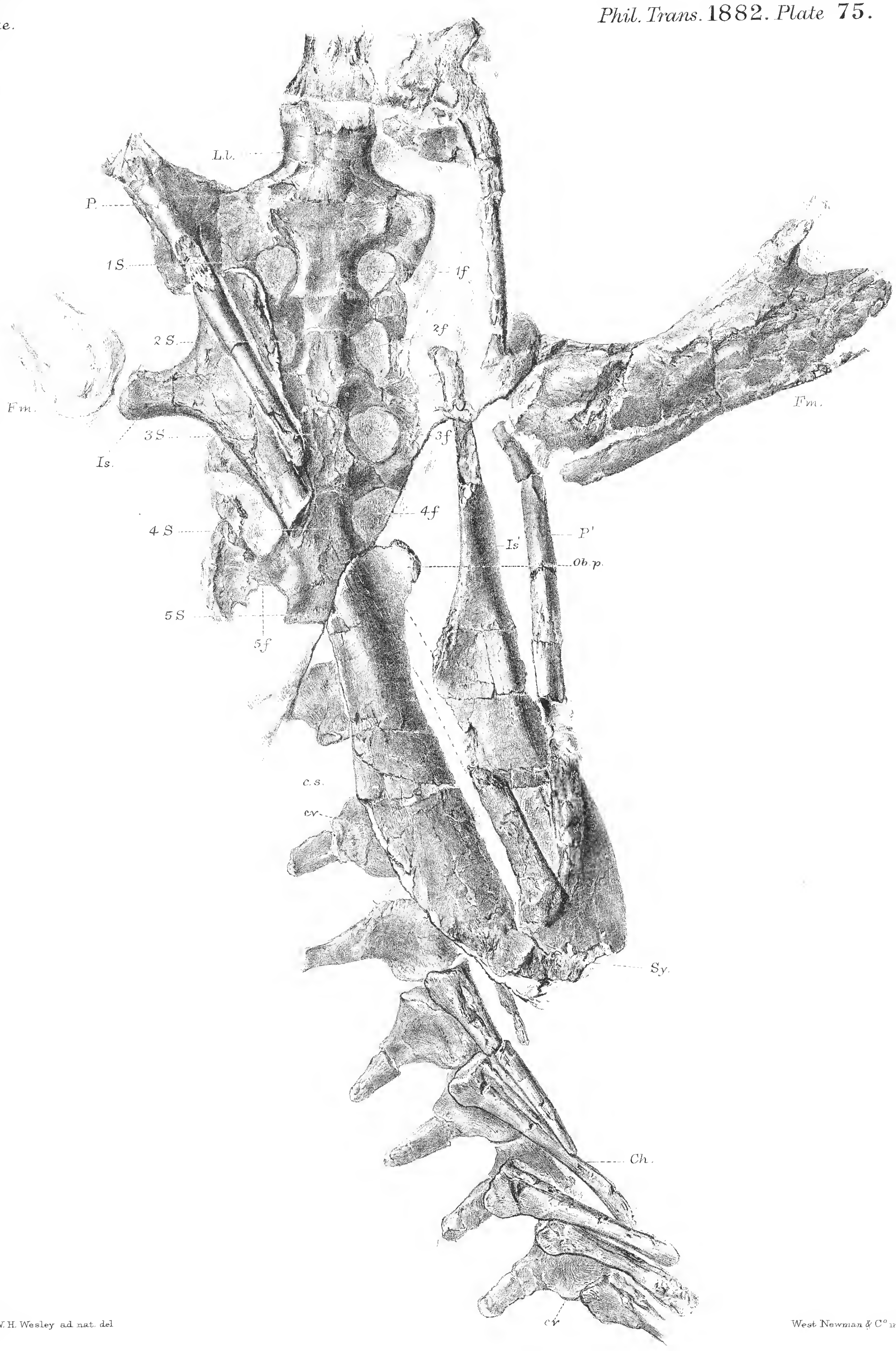


W.H. Wesley ad nat. del.
West Newman & Co. imp.



W.H. Wesley ad nat. del

West Newman & Co imp

Hulke.

Phil. Trans. 1882. *Plate* 76.

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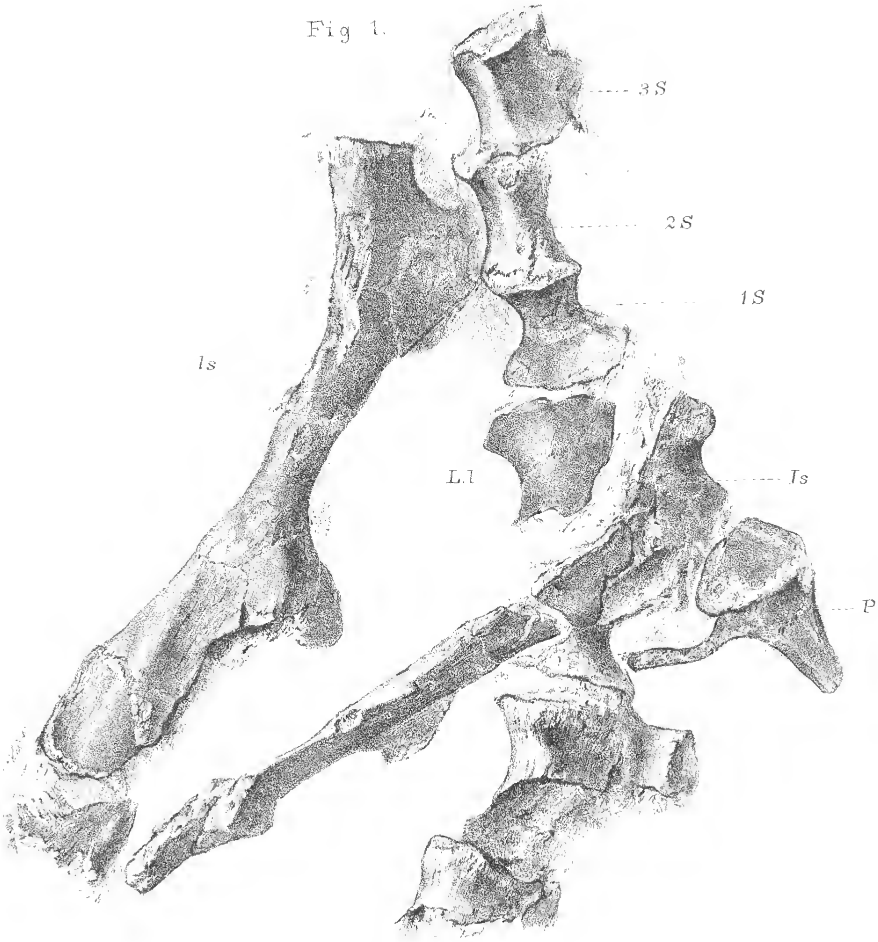

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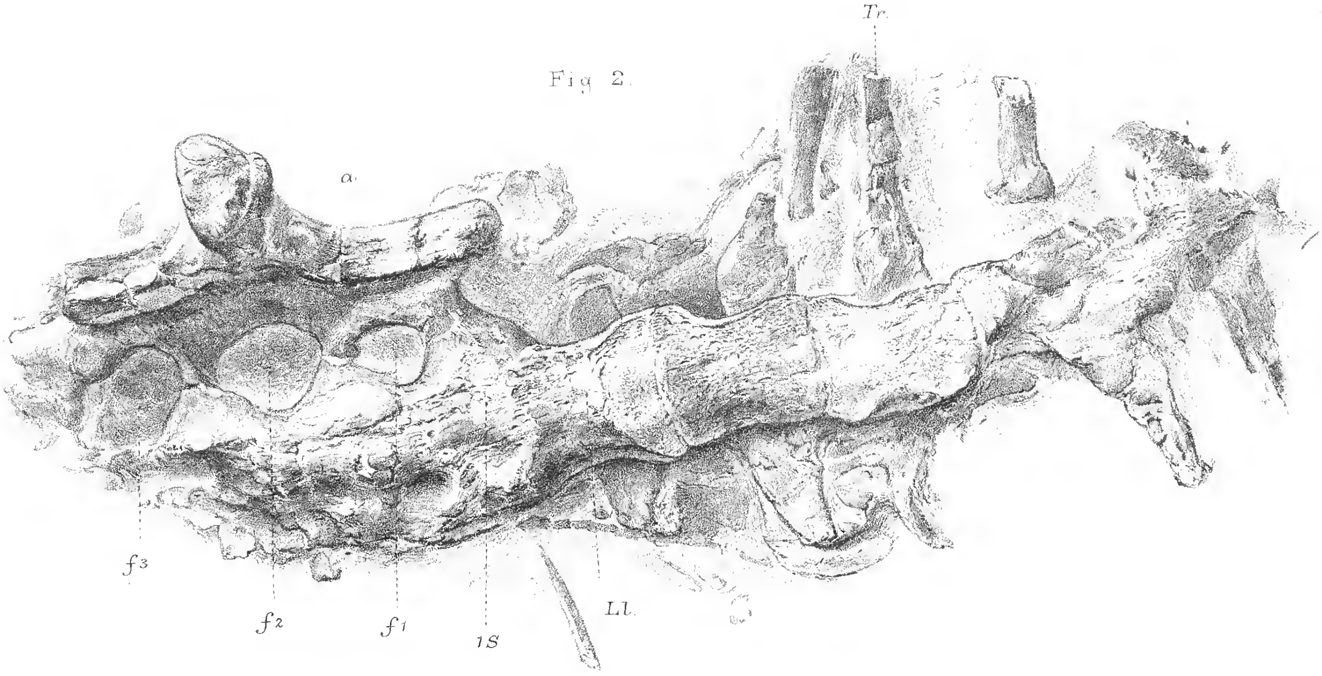
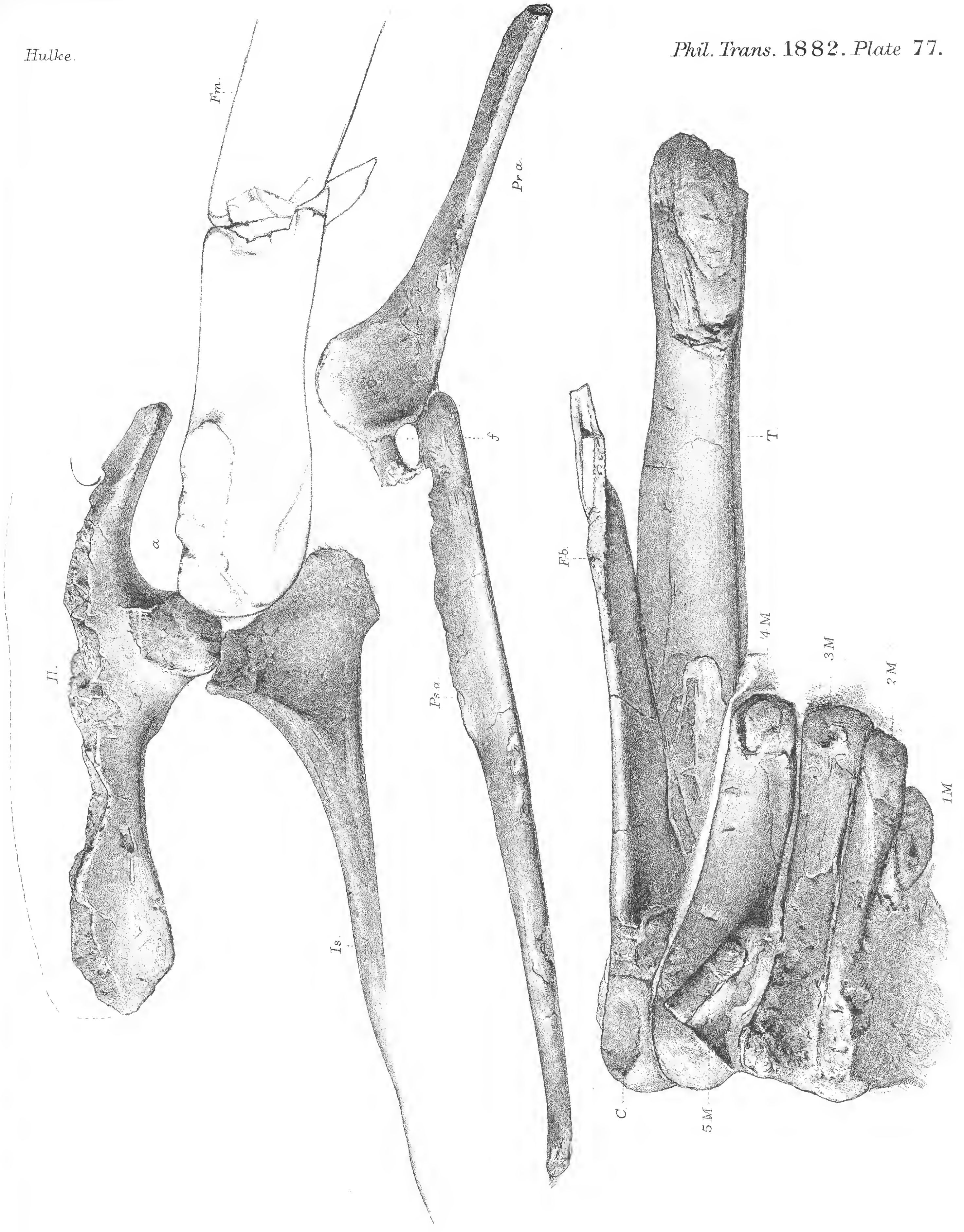


W.H. Wesley ad. nat. del

West Newman & C° imp



W.H.Wesley, ad nat. del.

West Newman & Co. imp.

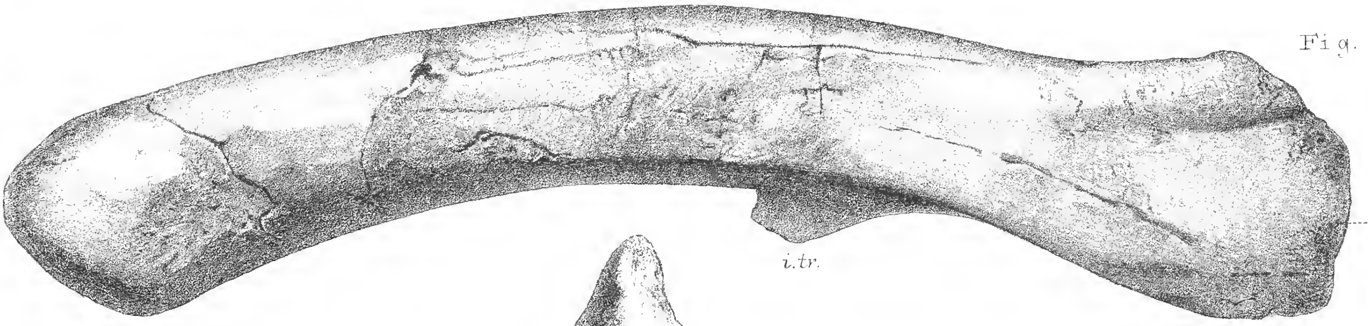


Fig. 1.

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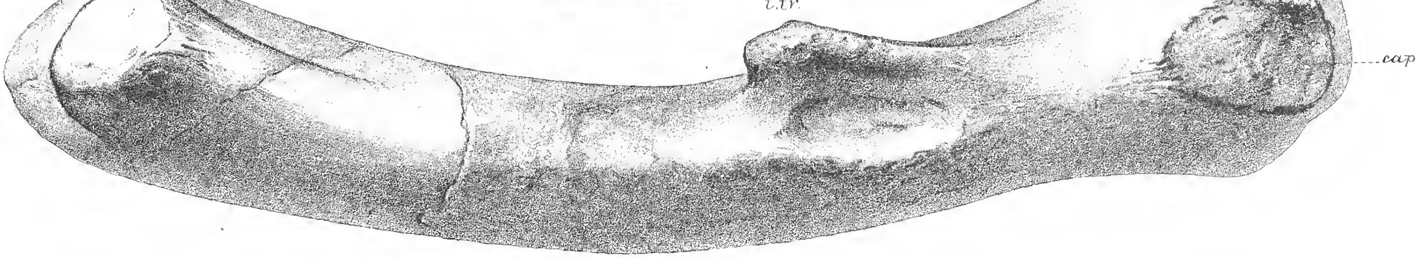

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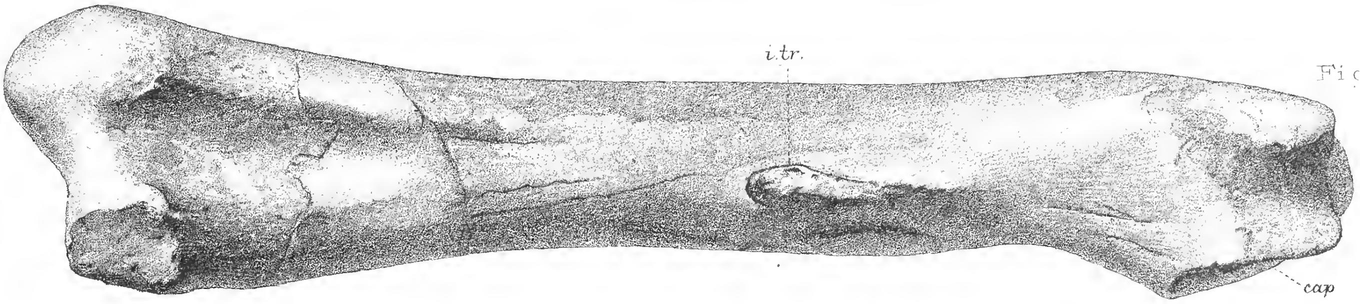
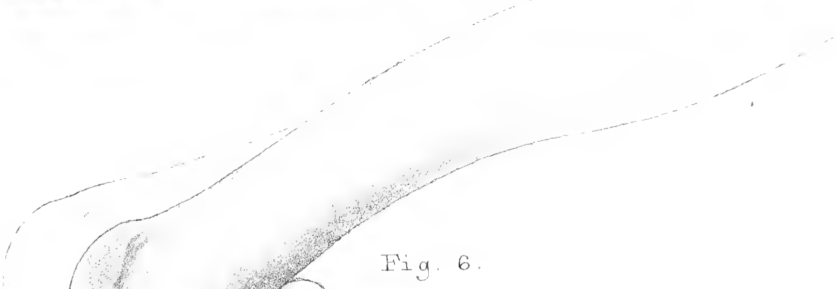


Fig. 3

Fig. 4



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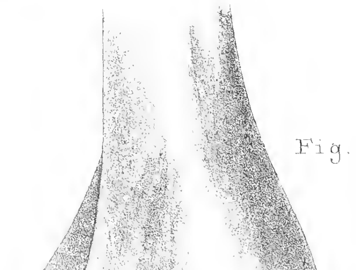

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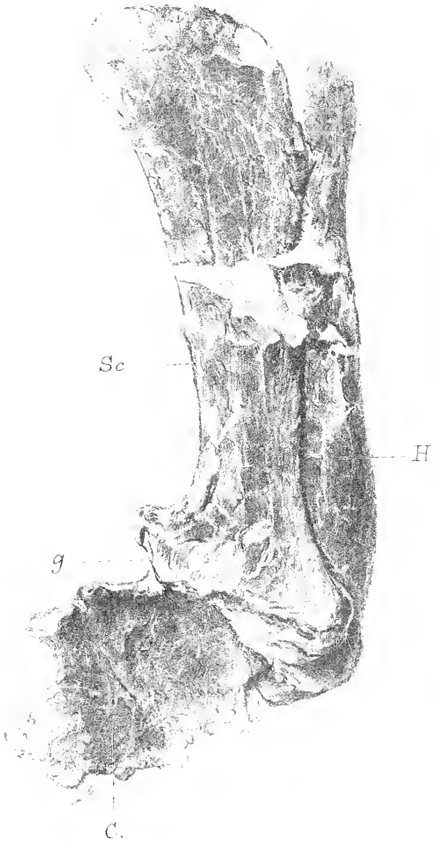

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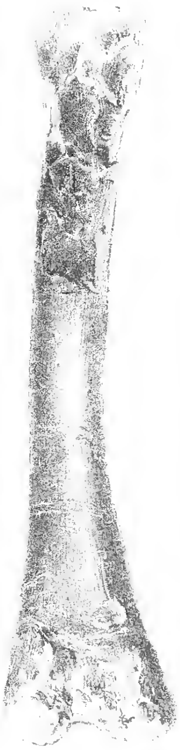

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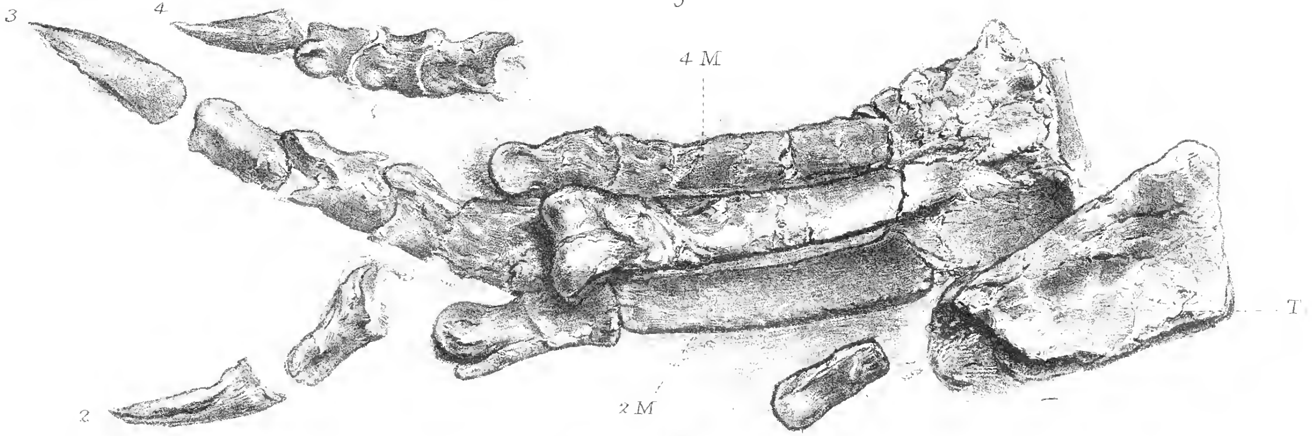

W.H Wesley ad nat. del

West Newman & Co imp.

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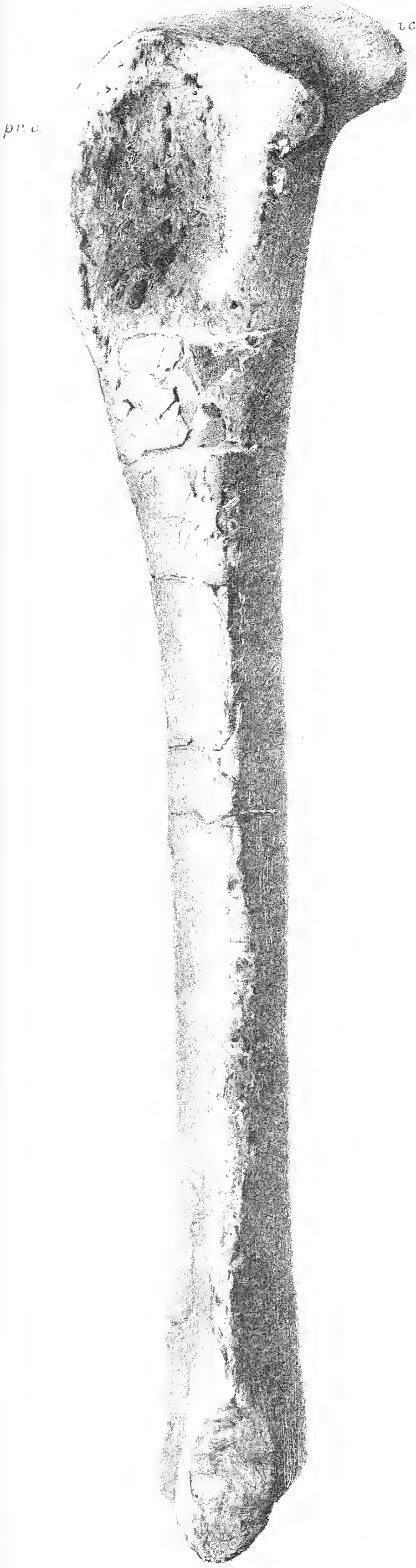

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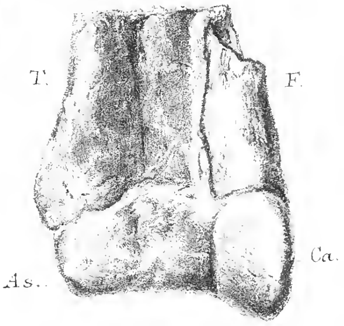

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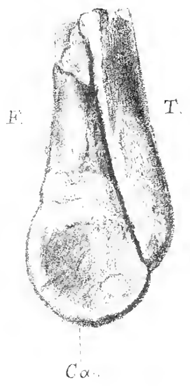

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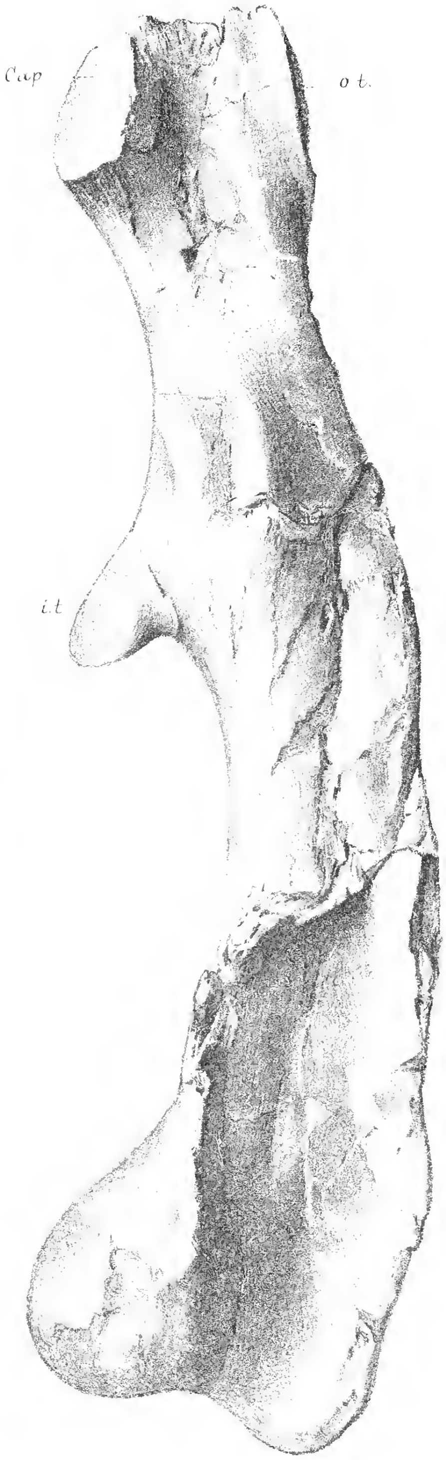

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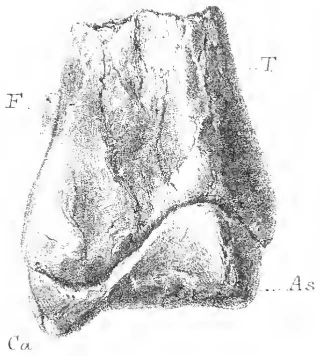

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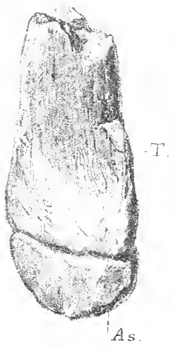

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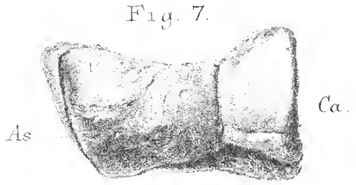

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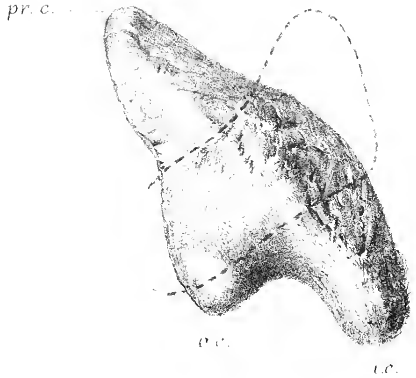

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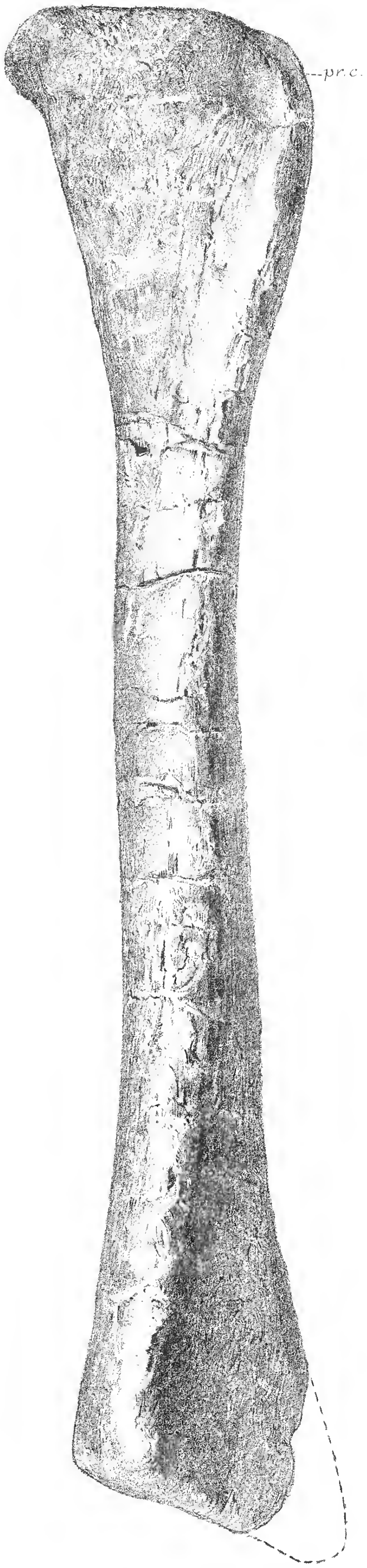

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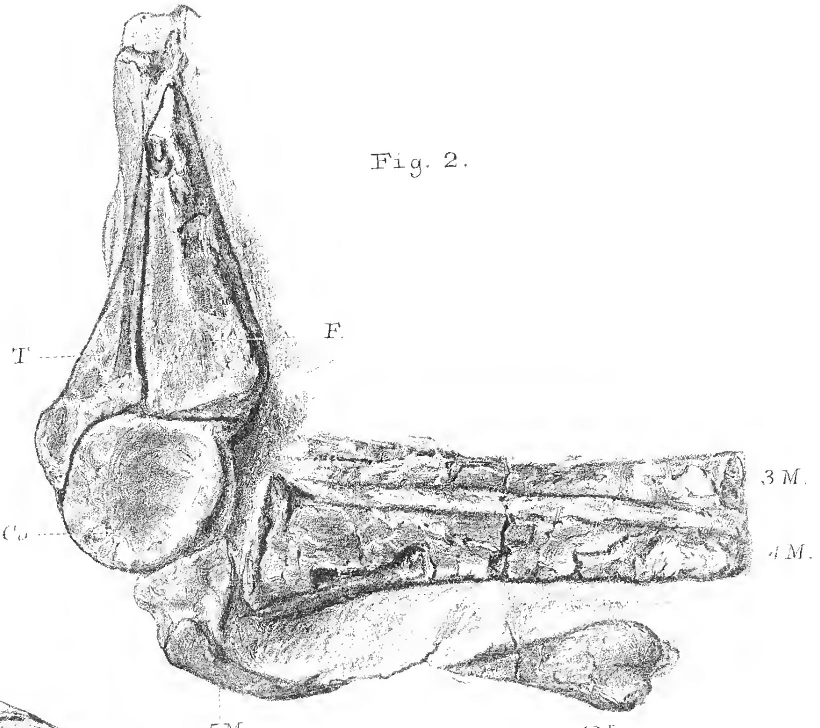

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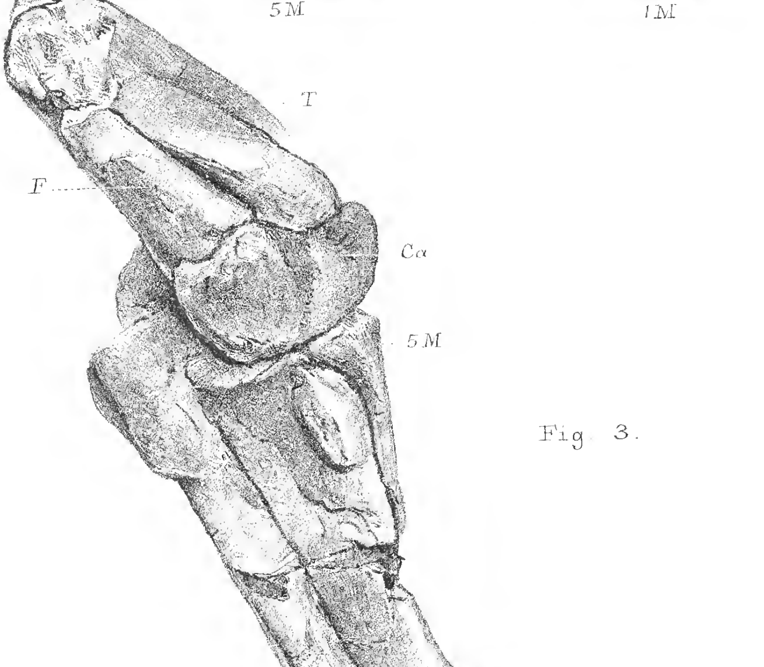
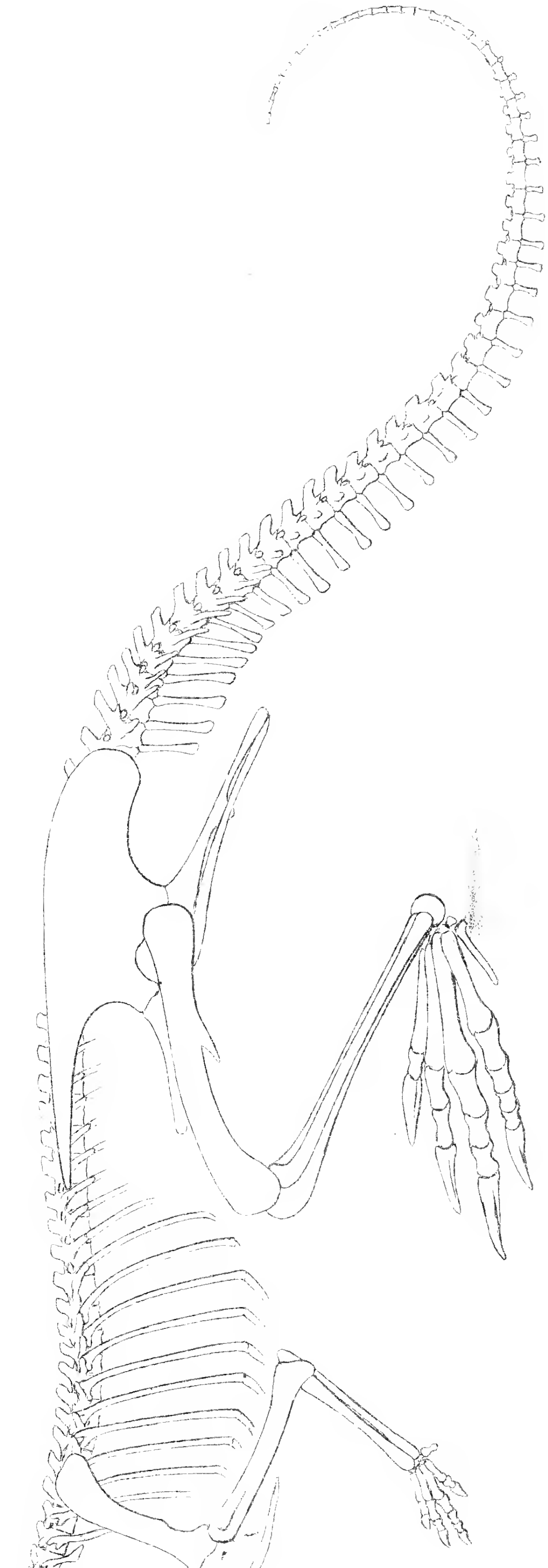


W.H Wesley ad nat. del.

West Newman & C° imp.

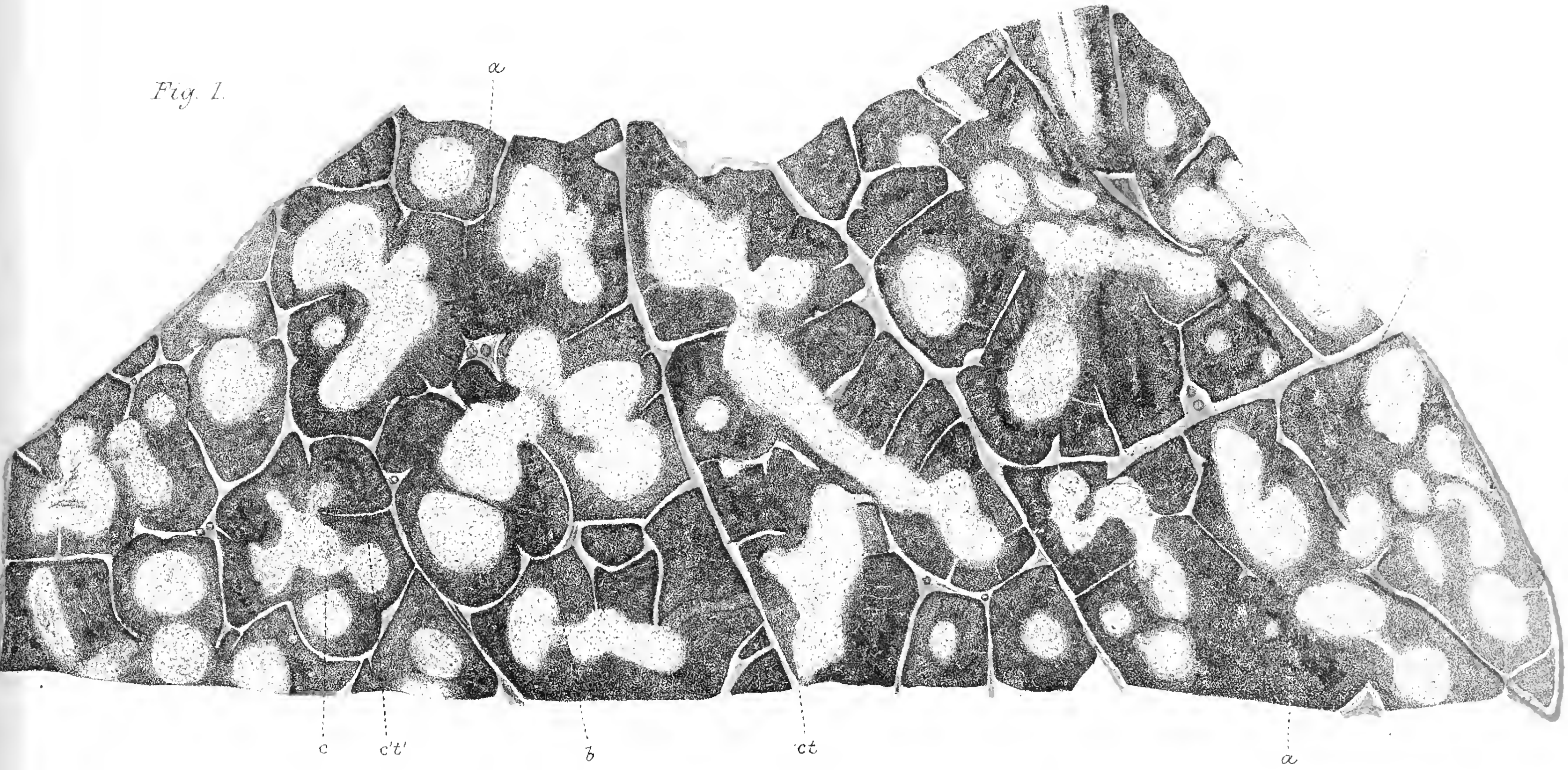


RESTORATION OF SKELETON OF HYPSILOPHODON.

W. H. Wesley lith.

West Newman & C° imp.

Fig. 1.



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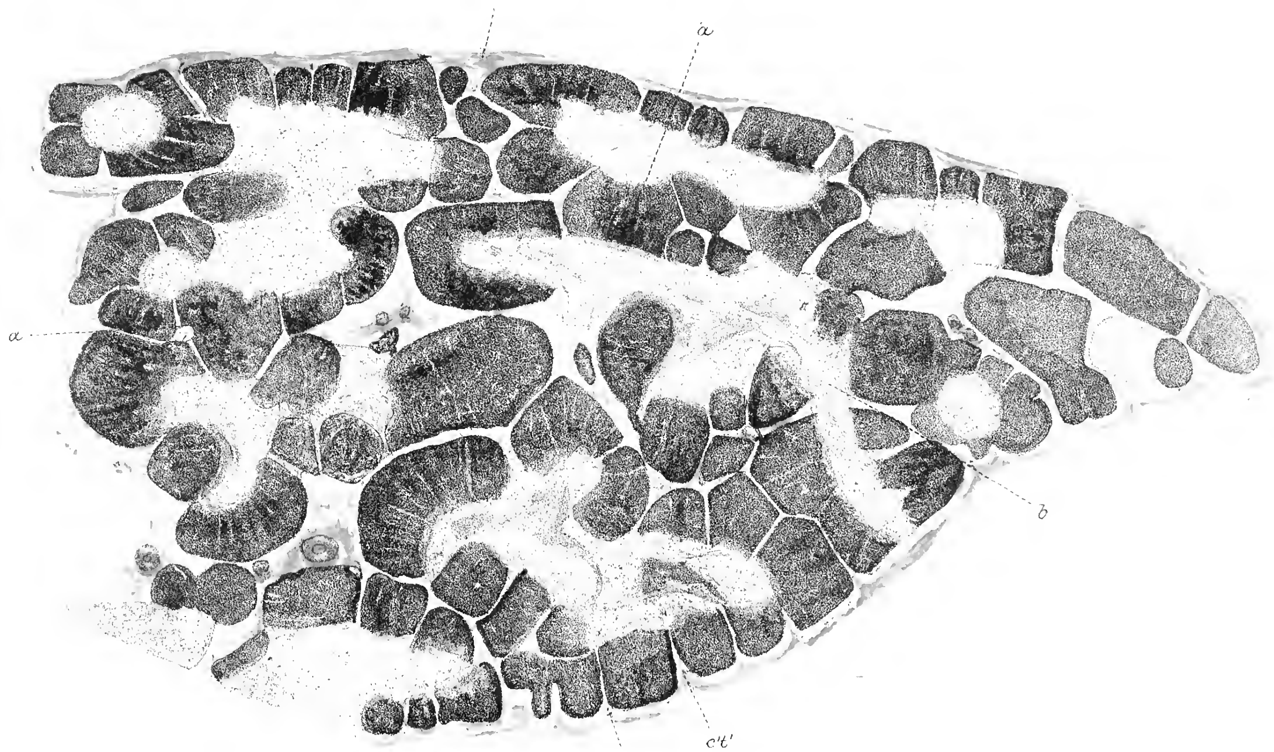
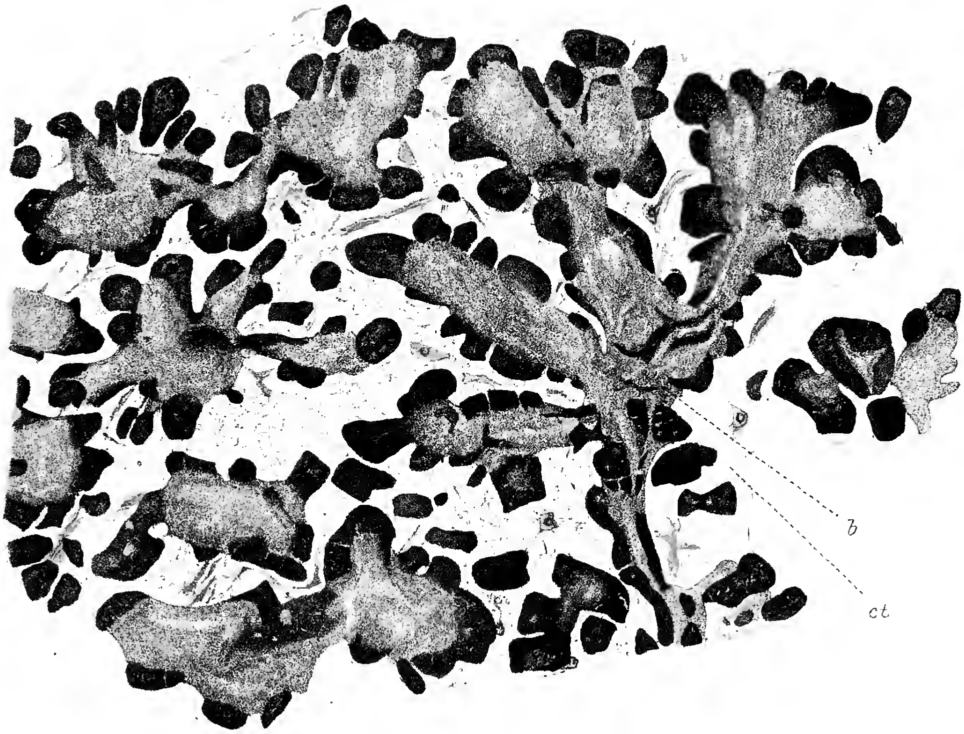


F Huth, Lith! Edin'

Fig. 3.



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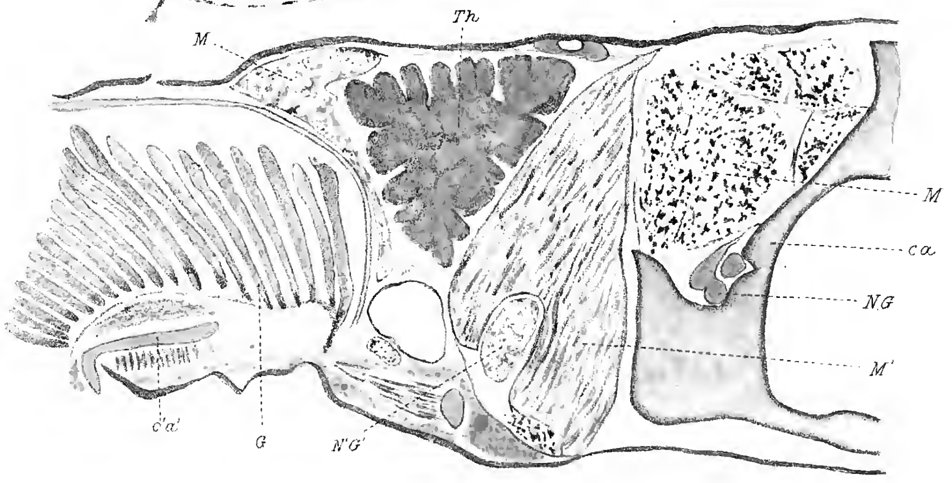

F Huth, Lith^r Edin^r

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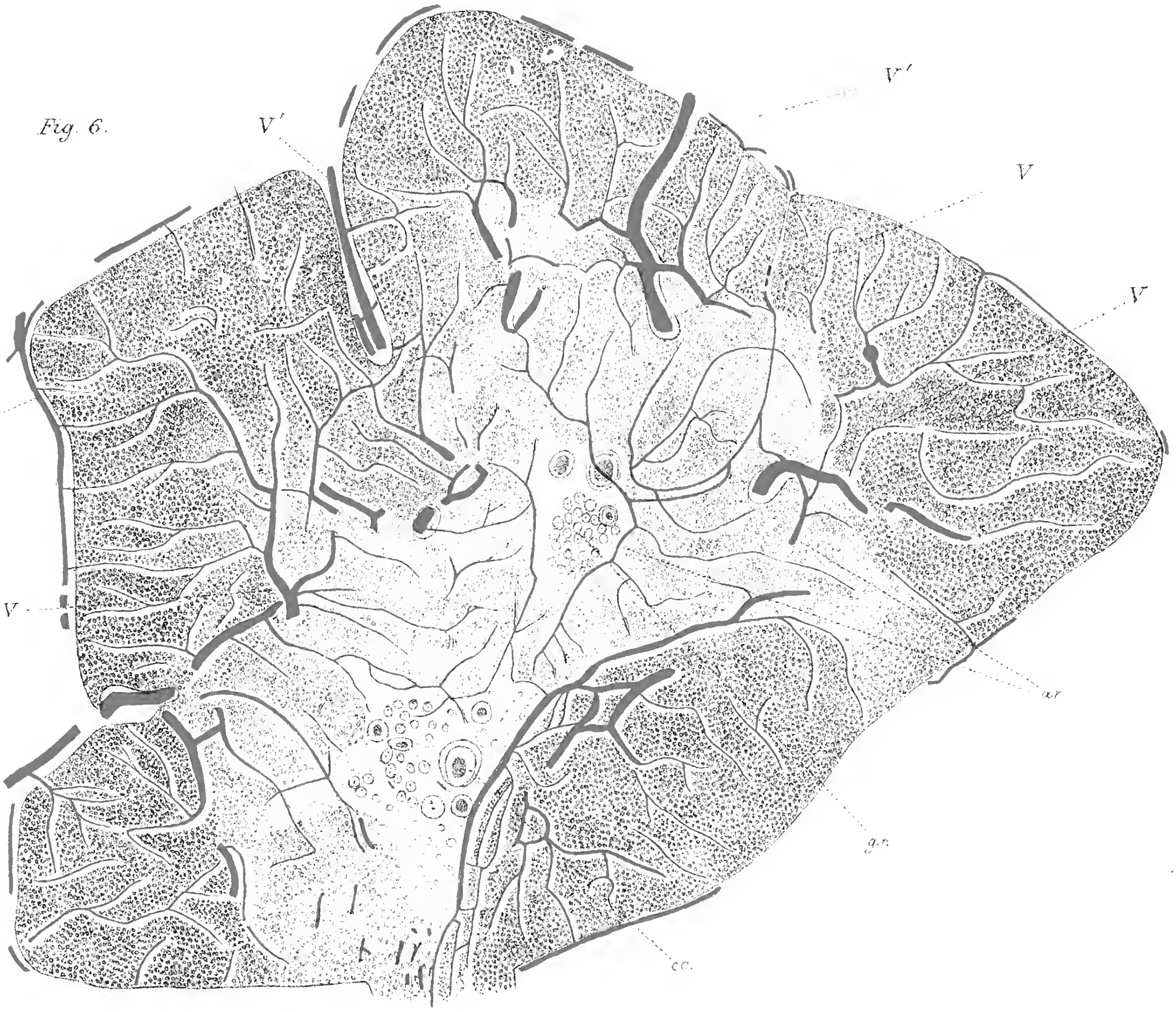

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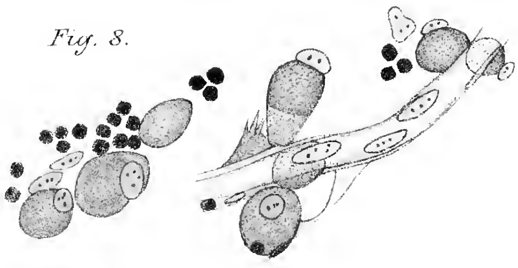

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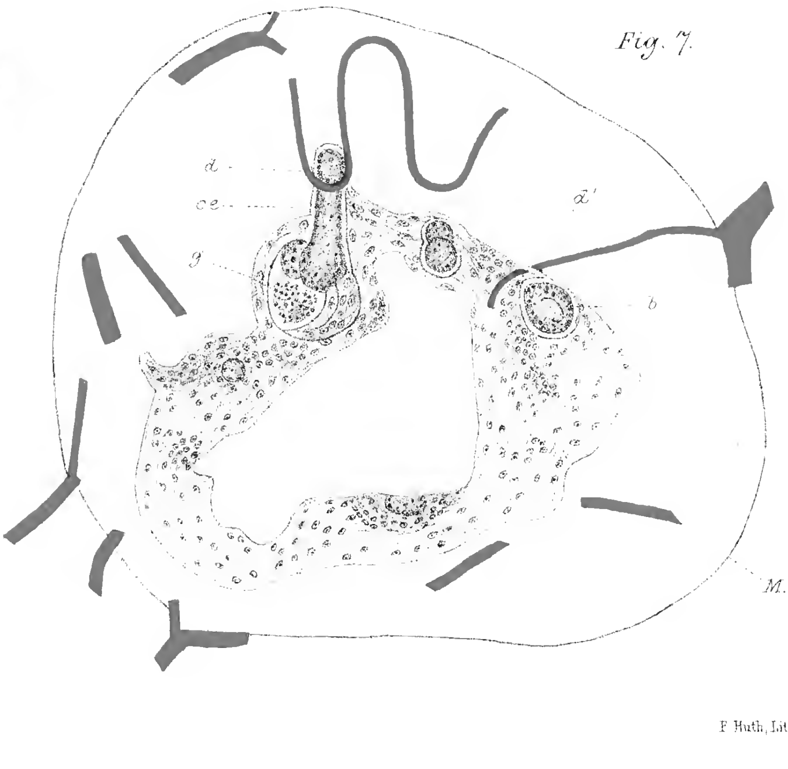
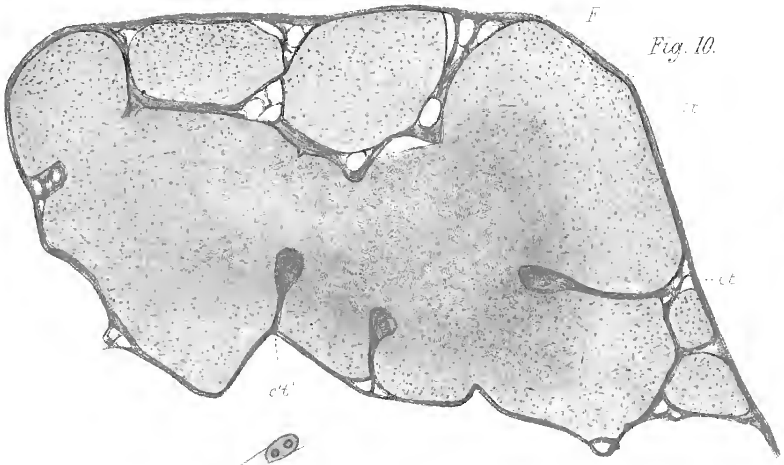


F. Huth, Lith^r Edin^r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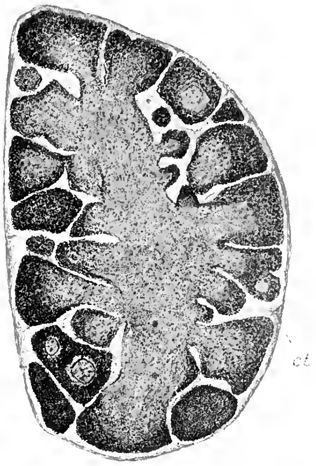
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Fig. 12.


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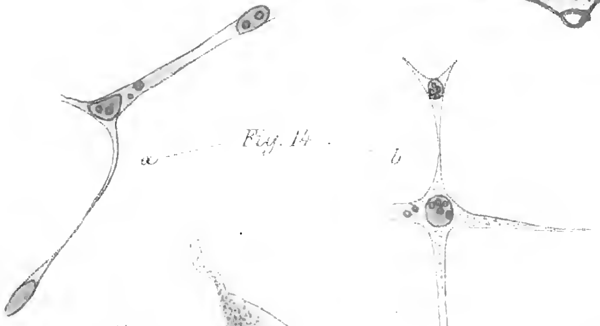
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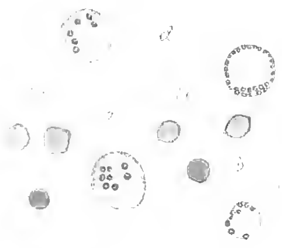
Fig. 15.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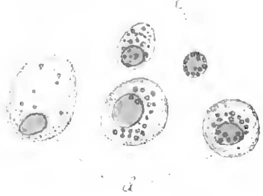
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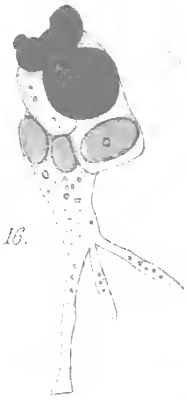
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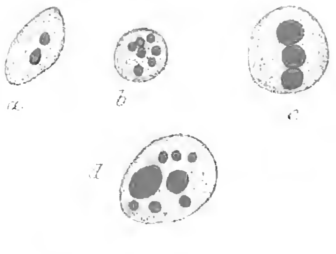
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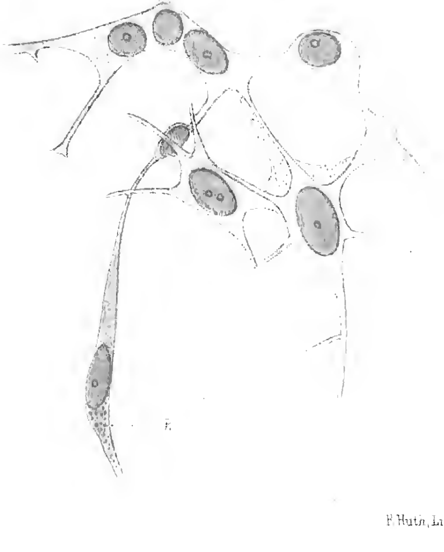
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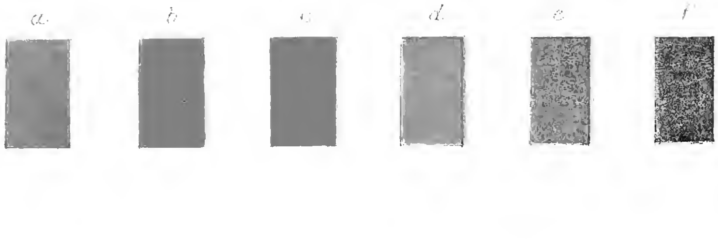
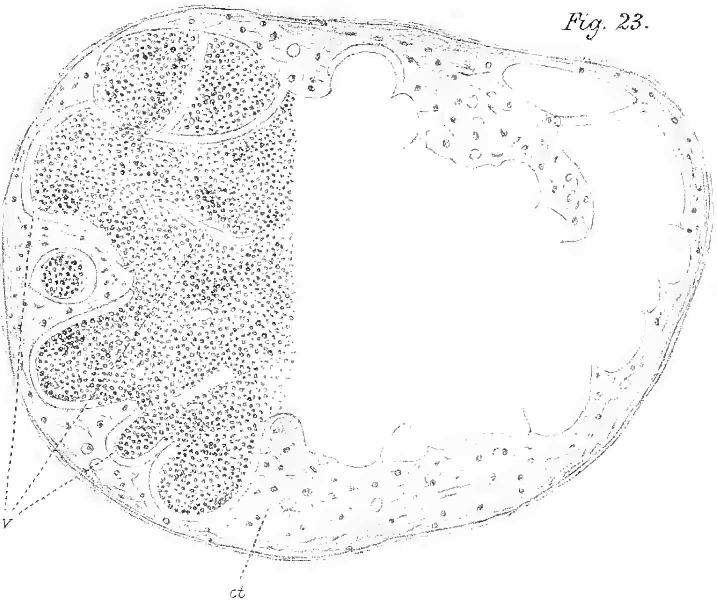


Fig. 22.

F. Huth, Lith^r Edin^r

Fig. 23.



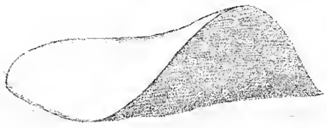


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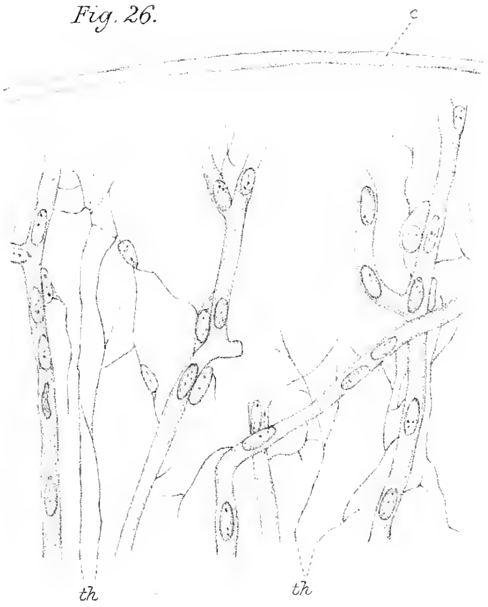
Fig. 26.



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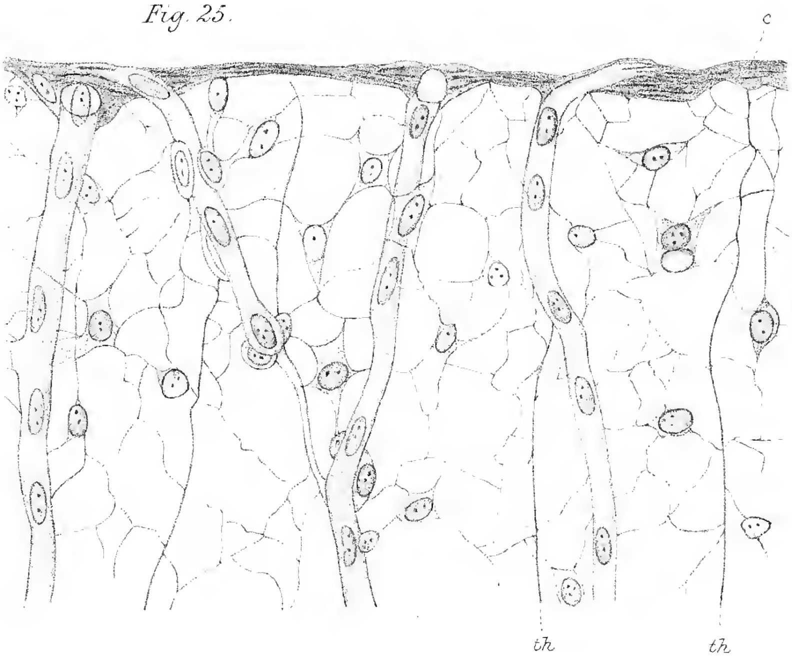

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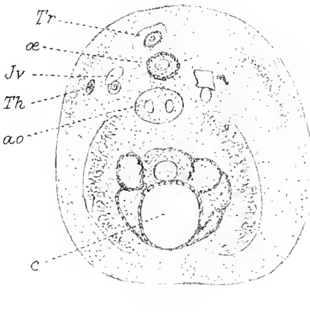

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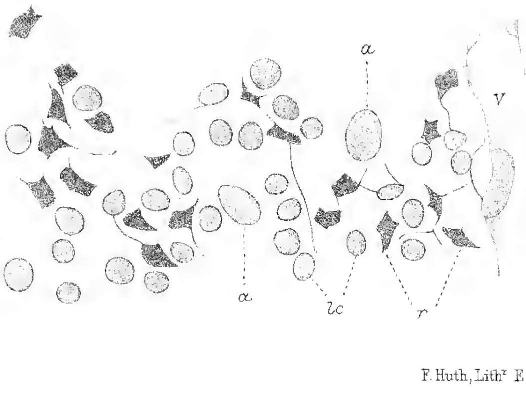
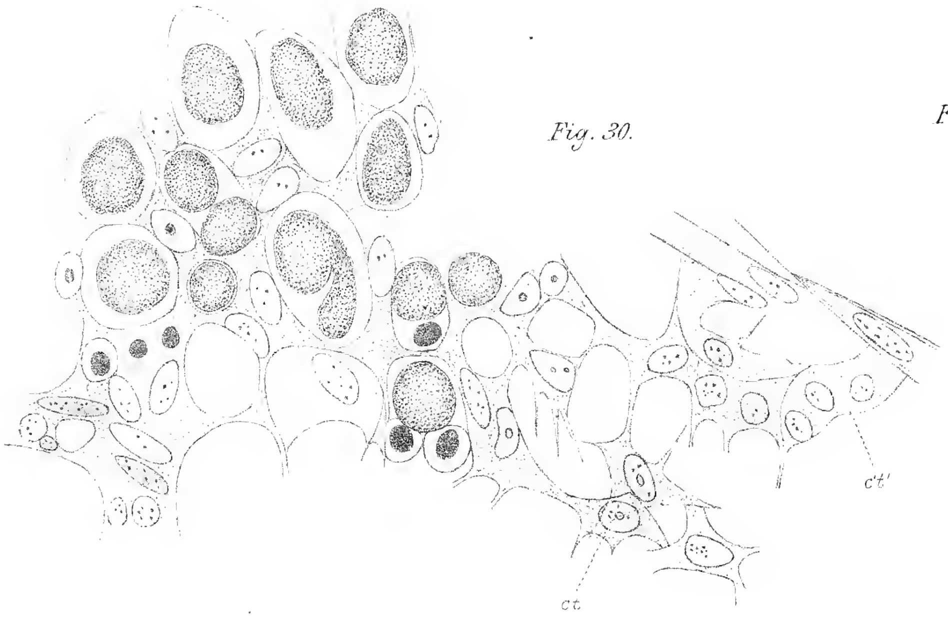


F. Huth, Lith^r Edin^r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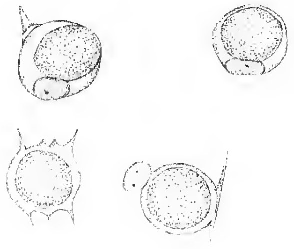

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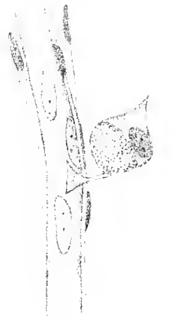

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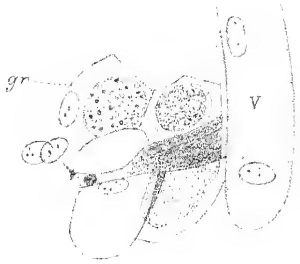

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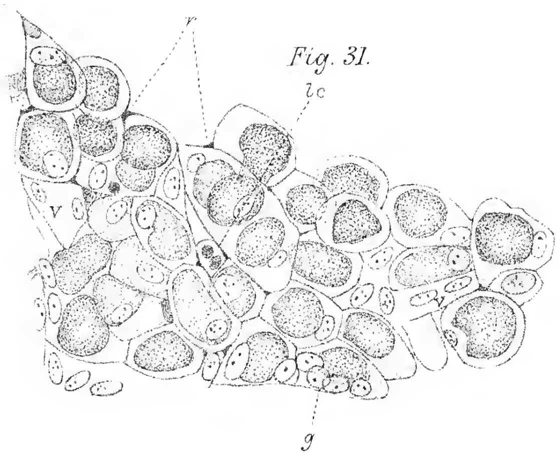

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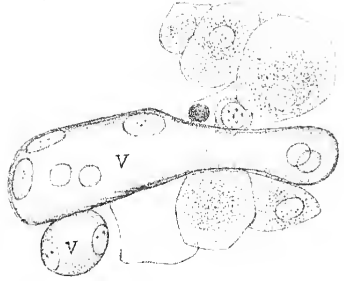

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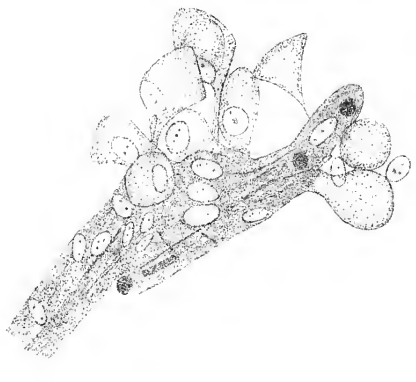

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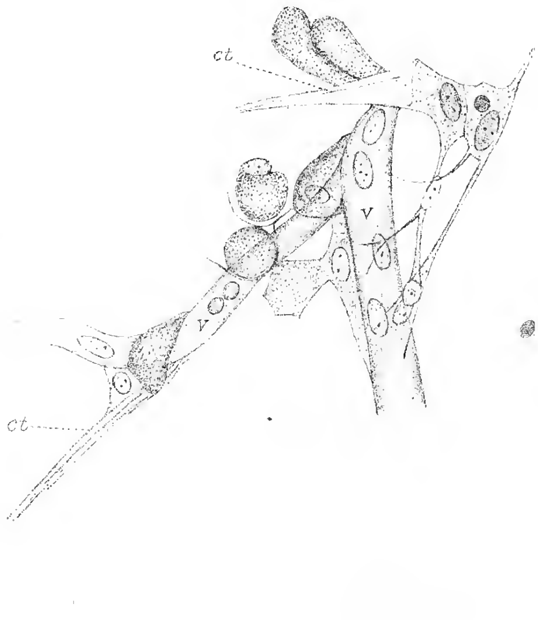

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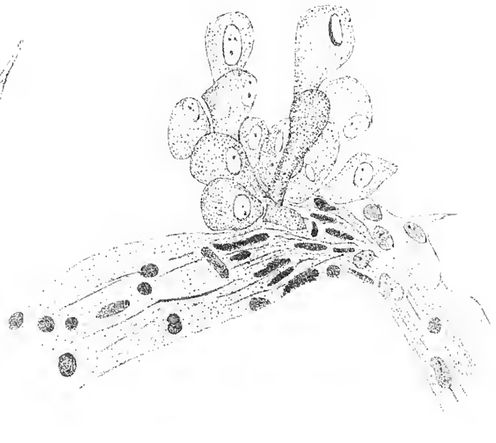
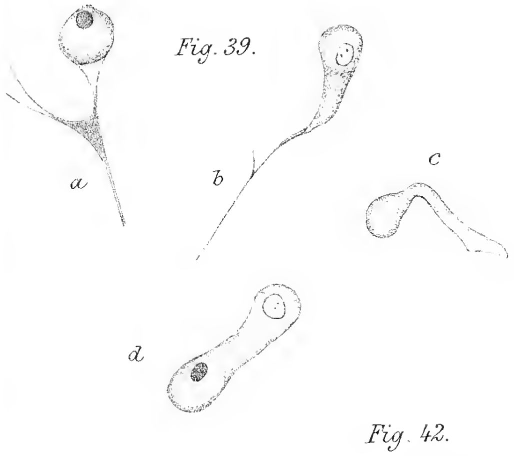


F. Huth, Lith^r Edin^r



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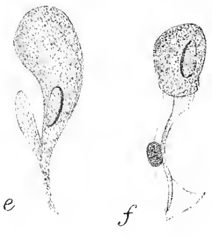

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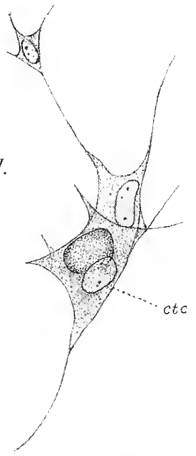

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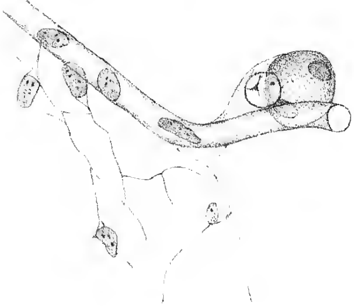

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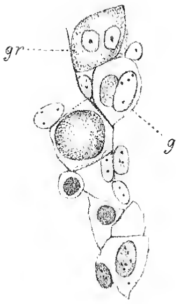
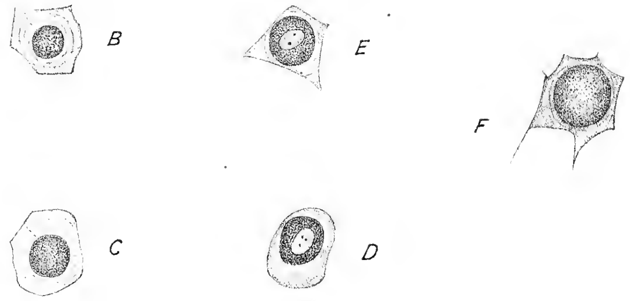
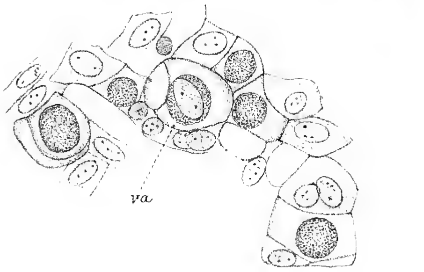


Fig. 45 A.

Fig. 44.





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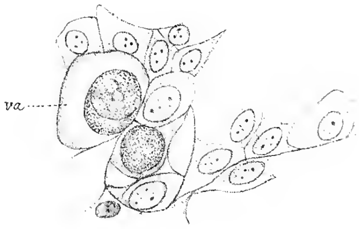
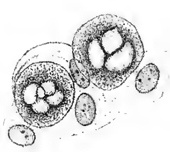


Fig. 45 C.



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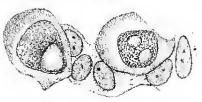

Fig. 46 B.

Fig. 46 C.

Fig. 46 D.

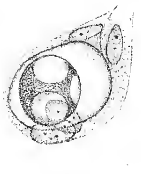

Fig. 46 E.

Fig. 46 F.

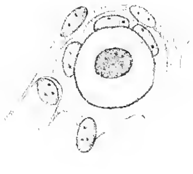

Fig. 46 G.

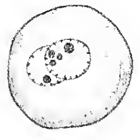

Fig. 47.

Fig. 48.

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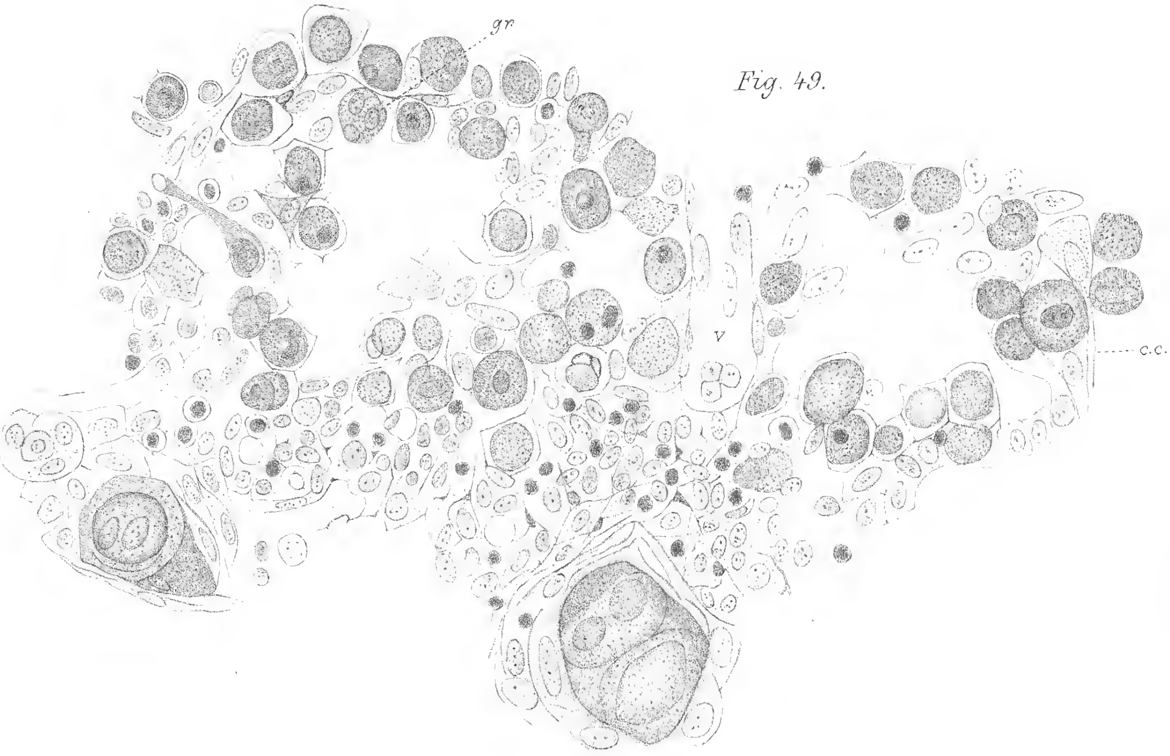


Fig. 49.

Fig. 52.

Fig. 50.

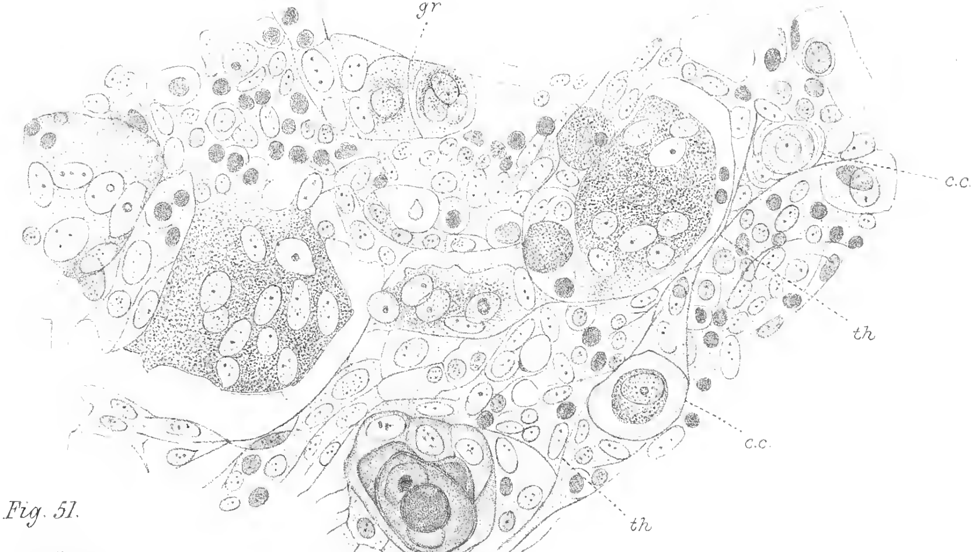
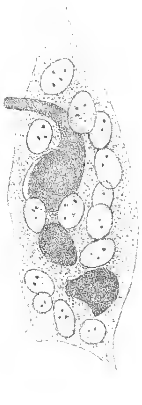


Fig. 56.

Fig. 51.

Fig. 53.



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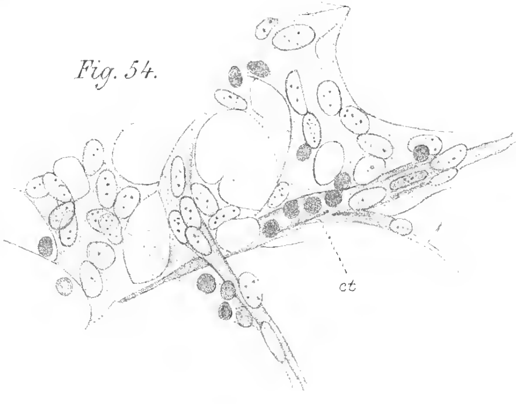

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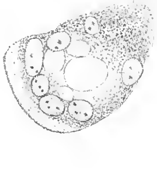

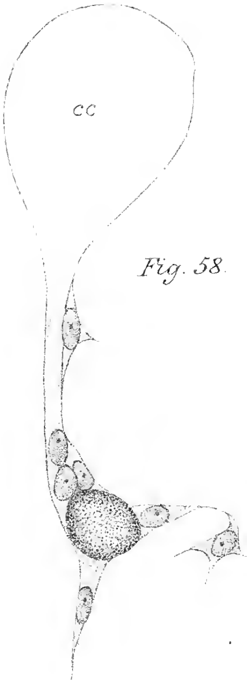

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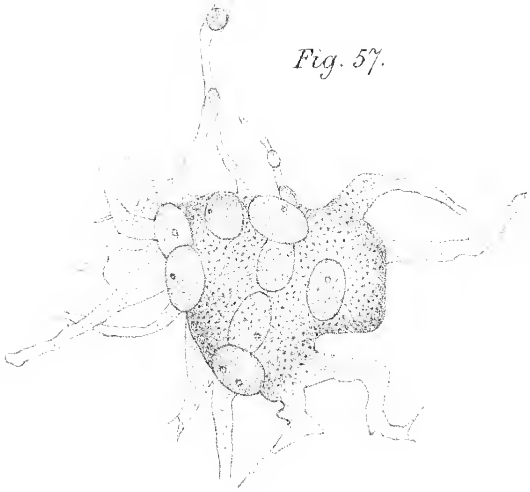

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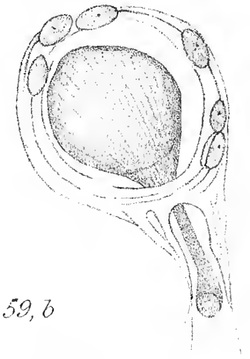

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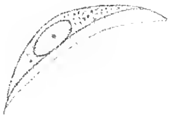
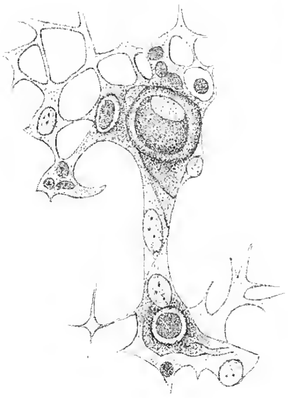


Fig. 59, b

Fig. 65.



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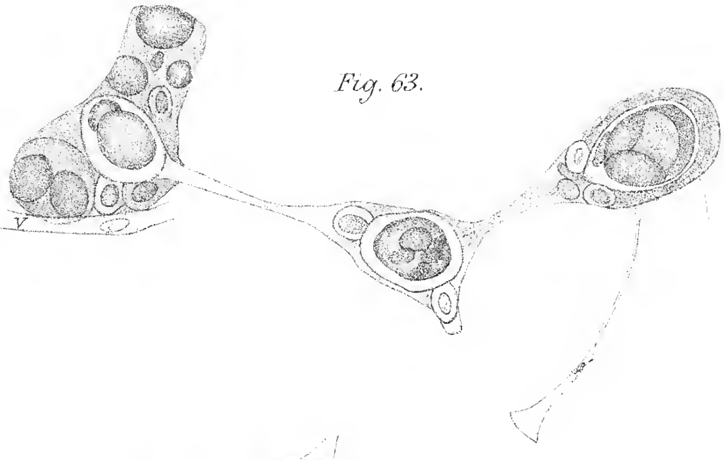

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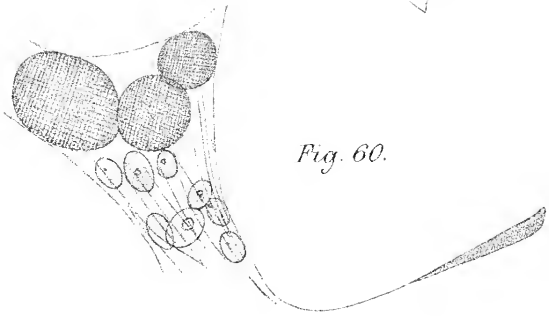

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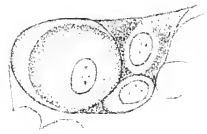

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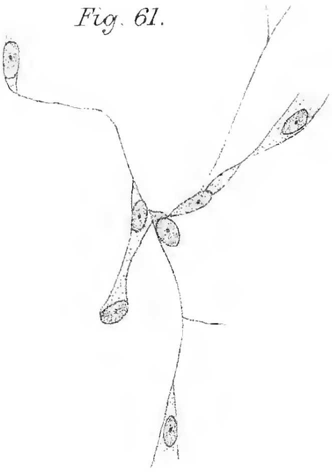

Fig. 61.

Fig. 67.

Fig. 62.

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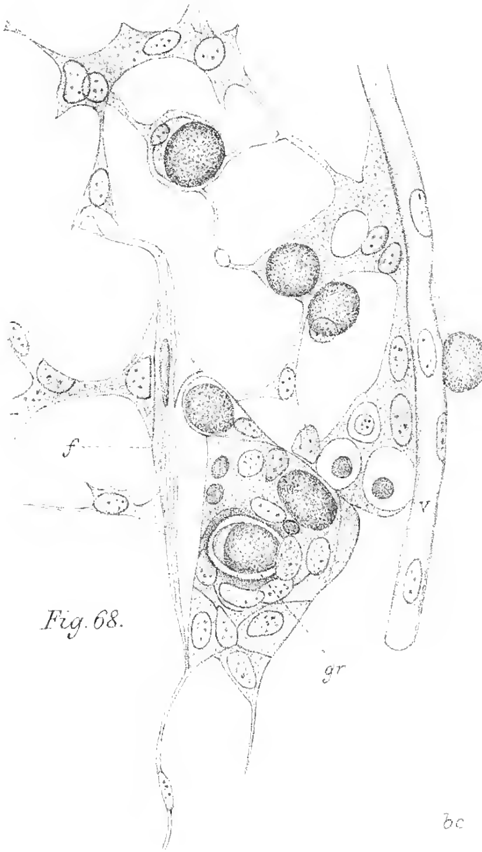
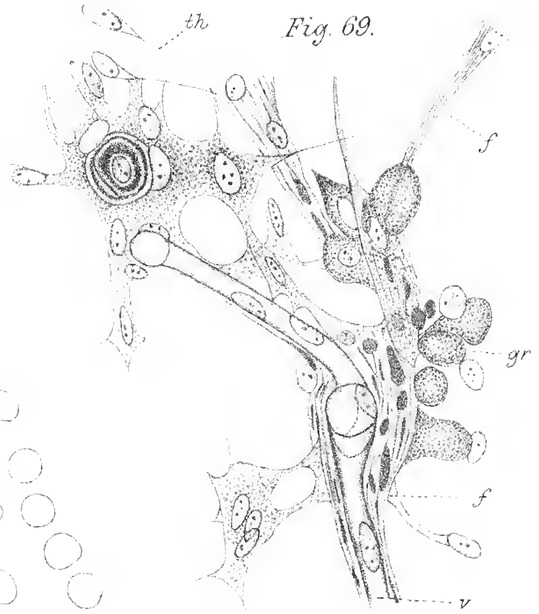


Fig. 68.

Fig. 70.



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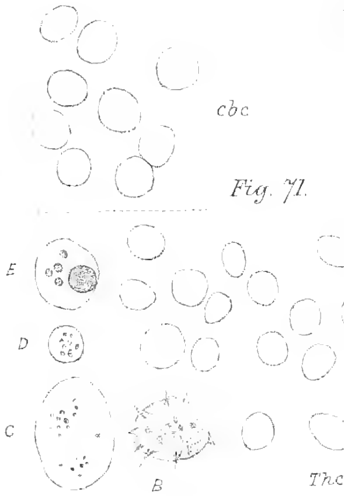

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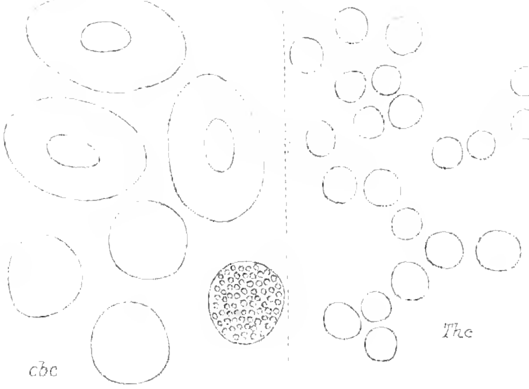
Fig. 72.



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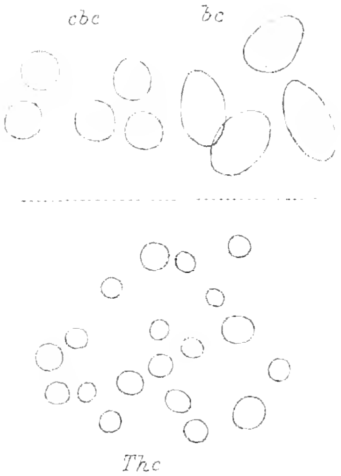

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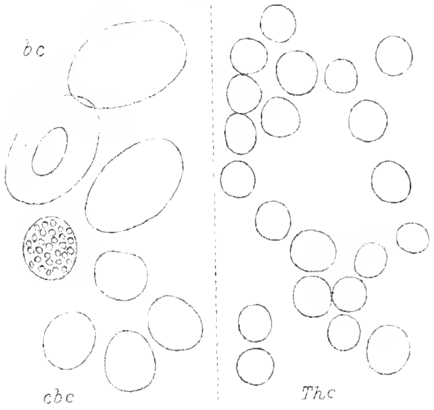

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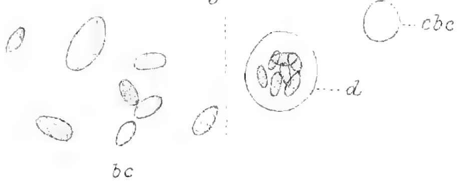

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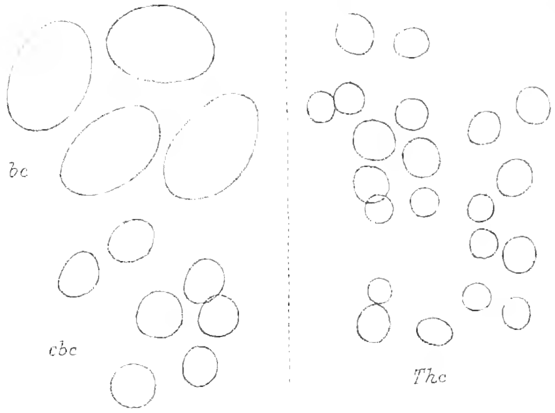

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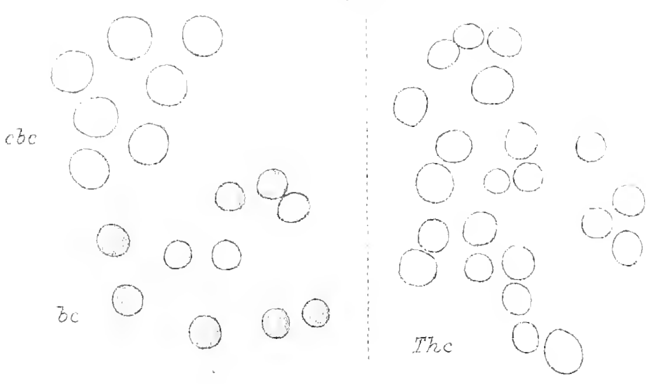

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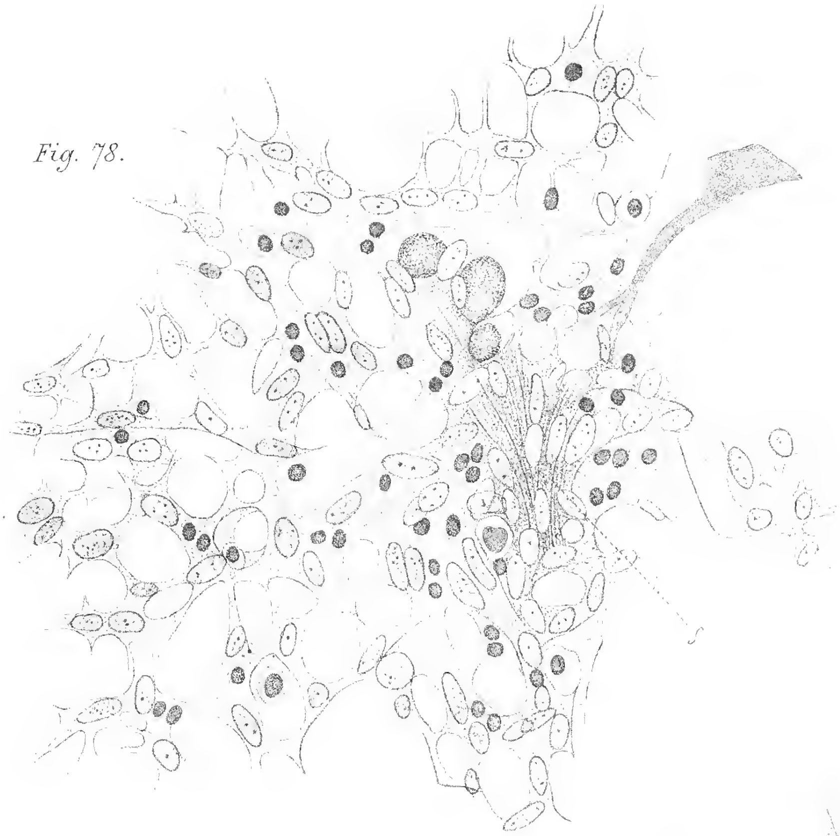
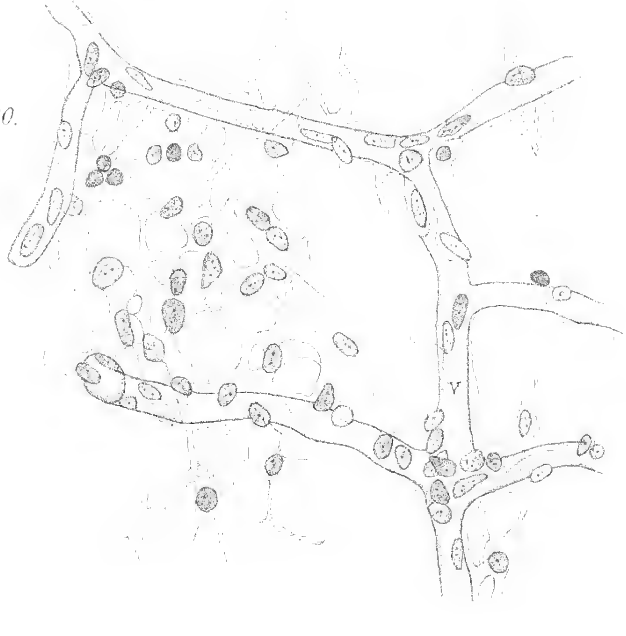


Fig. 82.

Fig. 83.

Fig. 79.

Fig. 80.



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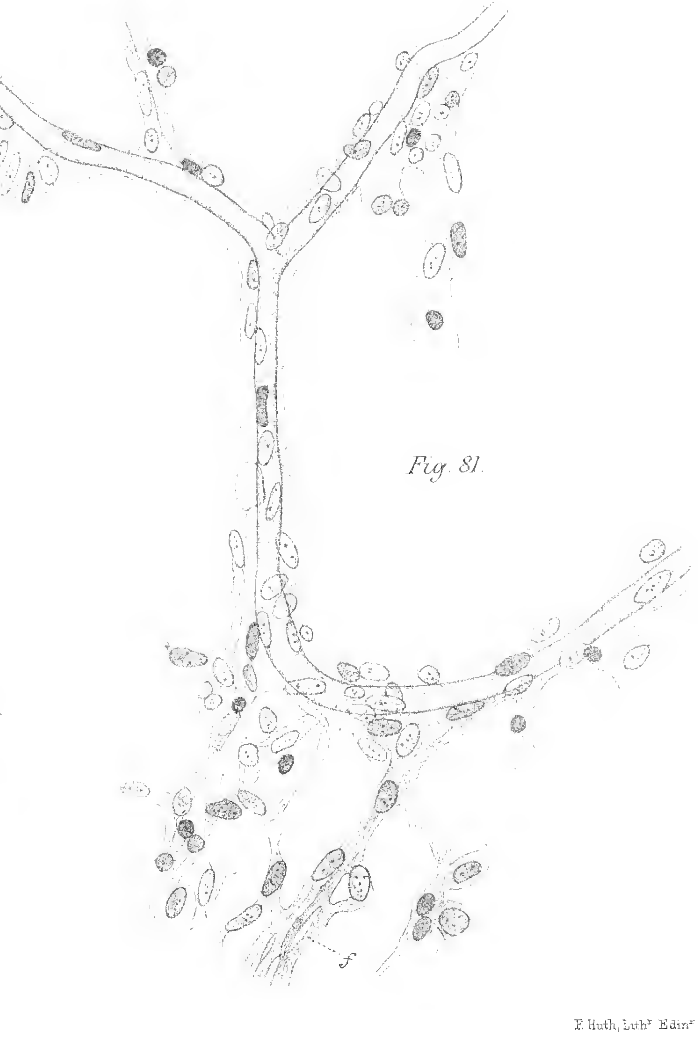

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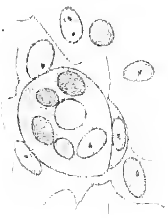
Fig. 84.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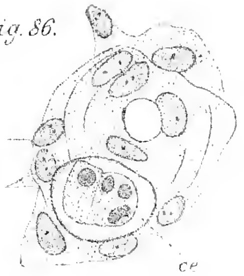
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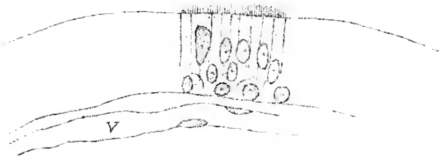
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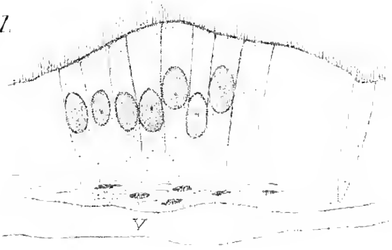
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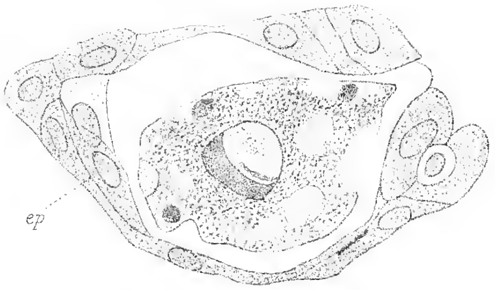
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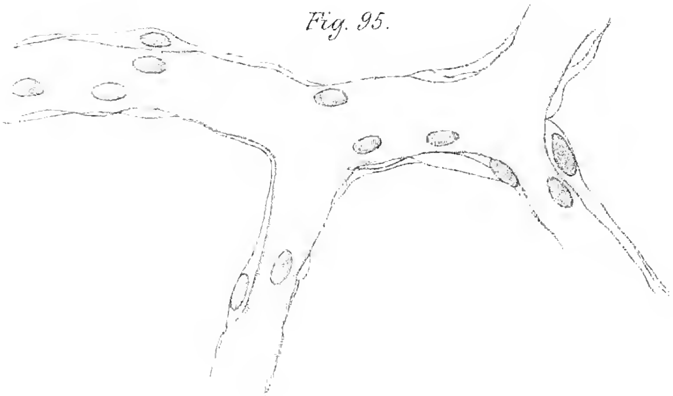
Fig. 89.


Fig. 88.


Fig. 92.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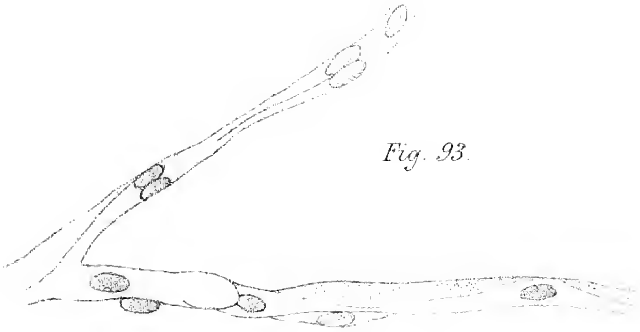
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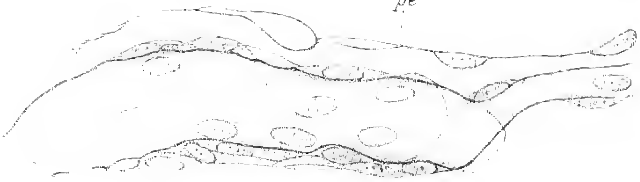
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Fig. 99.

Fig. 109.

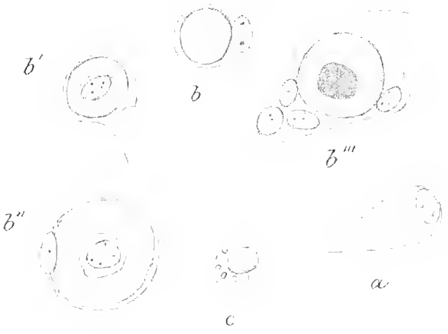
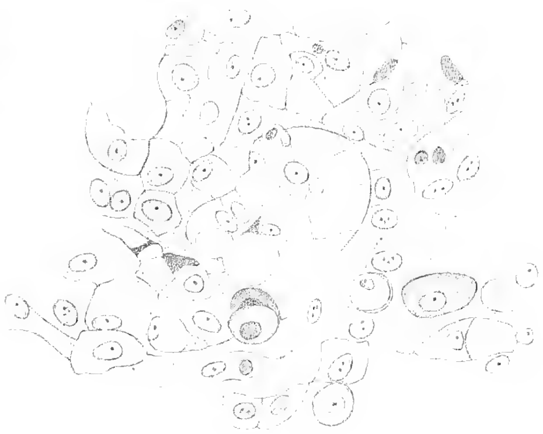


Fig. 98.

Fig. 103.

Fig. 102.

Fig. 101.



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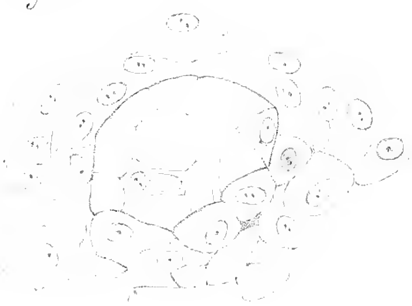

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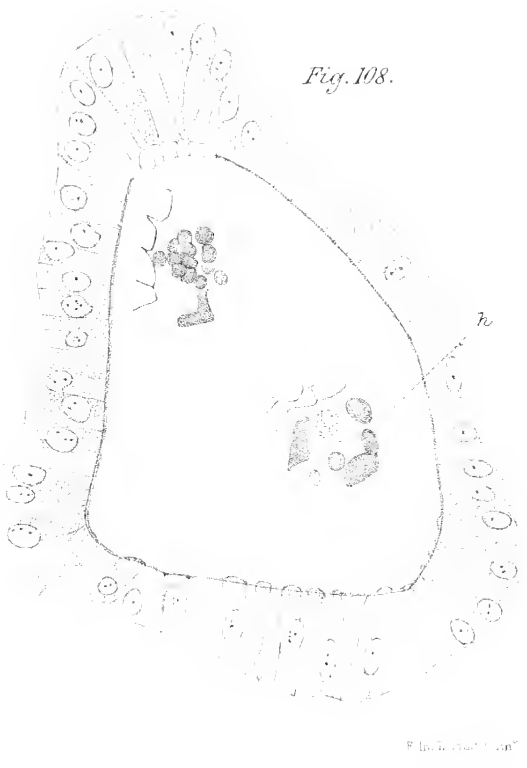

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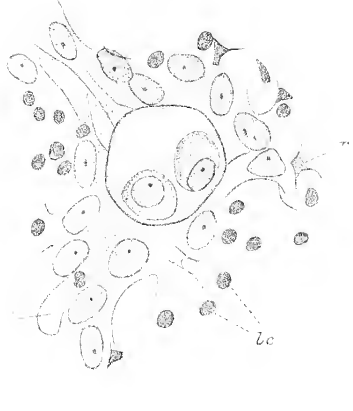
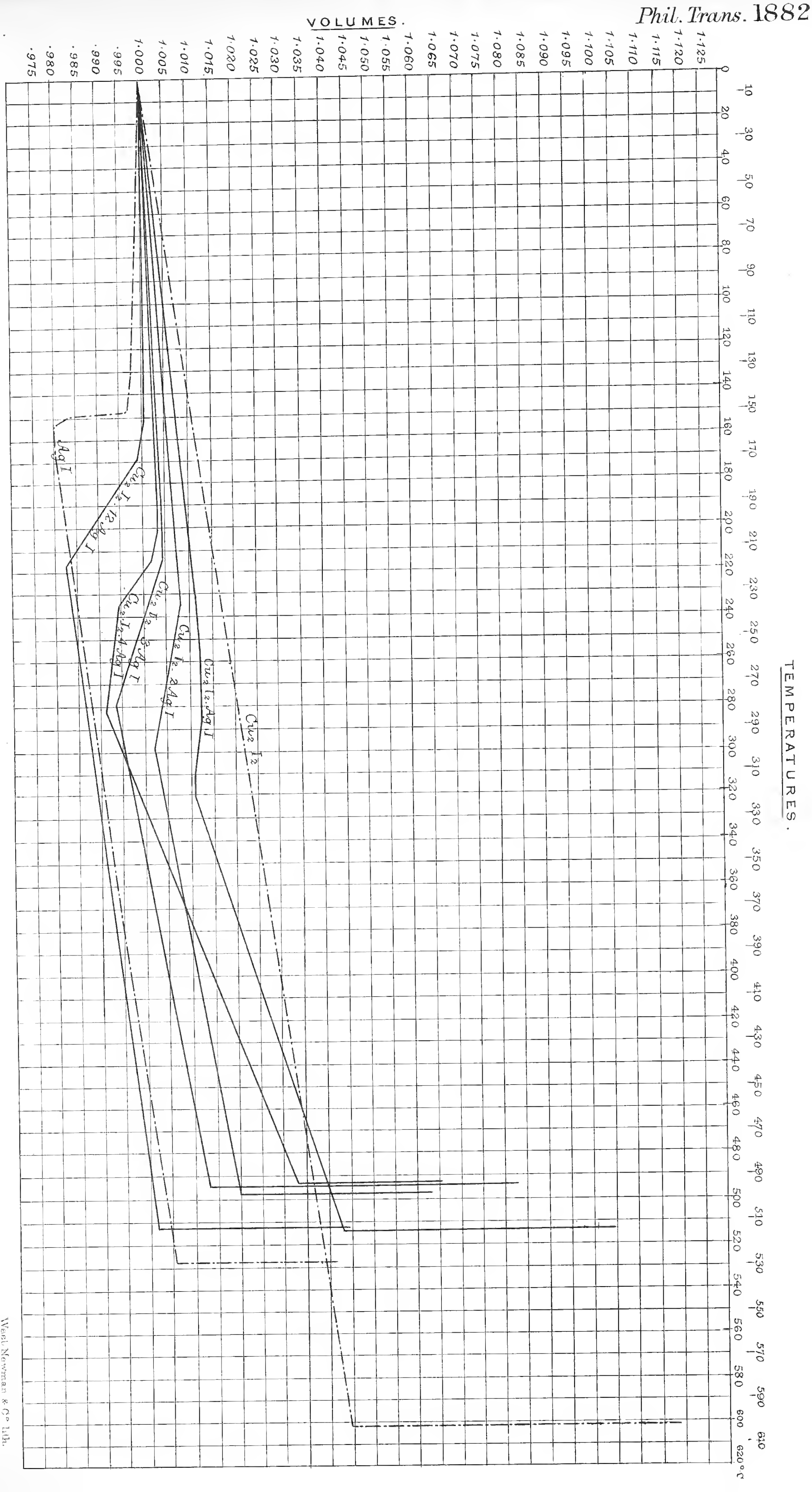


Fig. 106.

F. Huth, Lith.r Edin.r



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